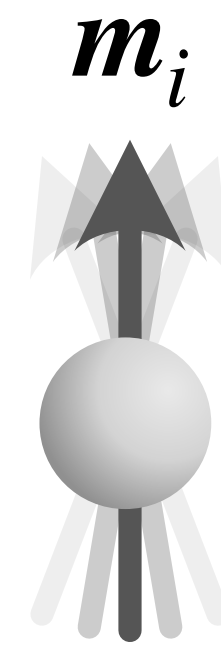


Domains & domain walls

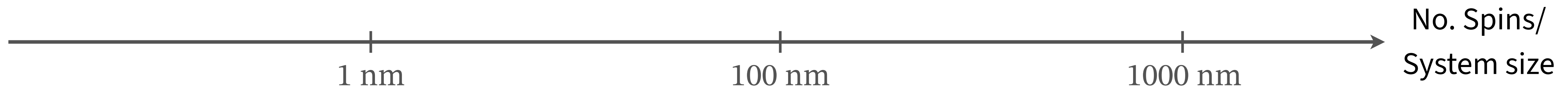
Lukas Körber

19.07.2026 — European School on Magnetism 2026, Uppsala

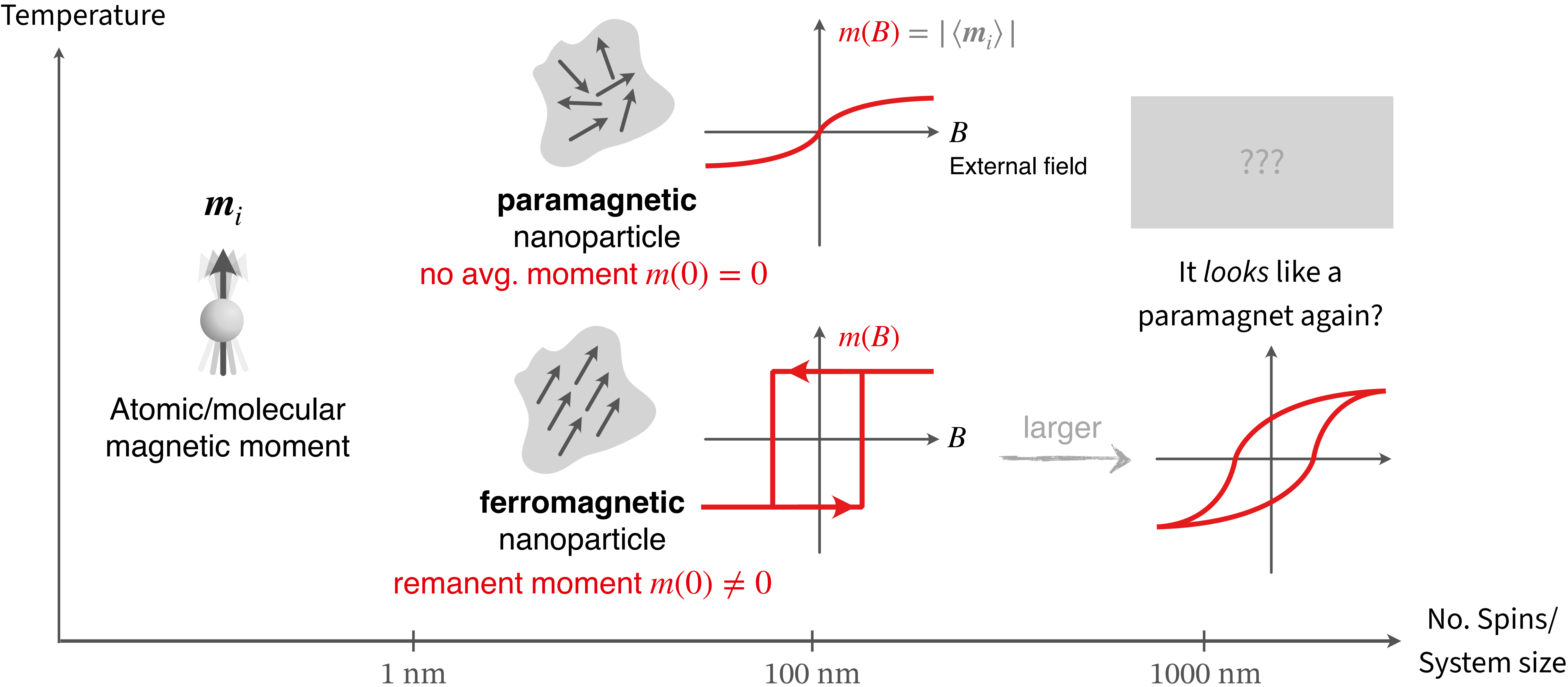
Domains and domain walls are archetypical structures in ordered materials



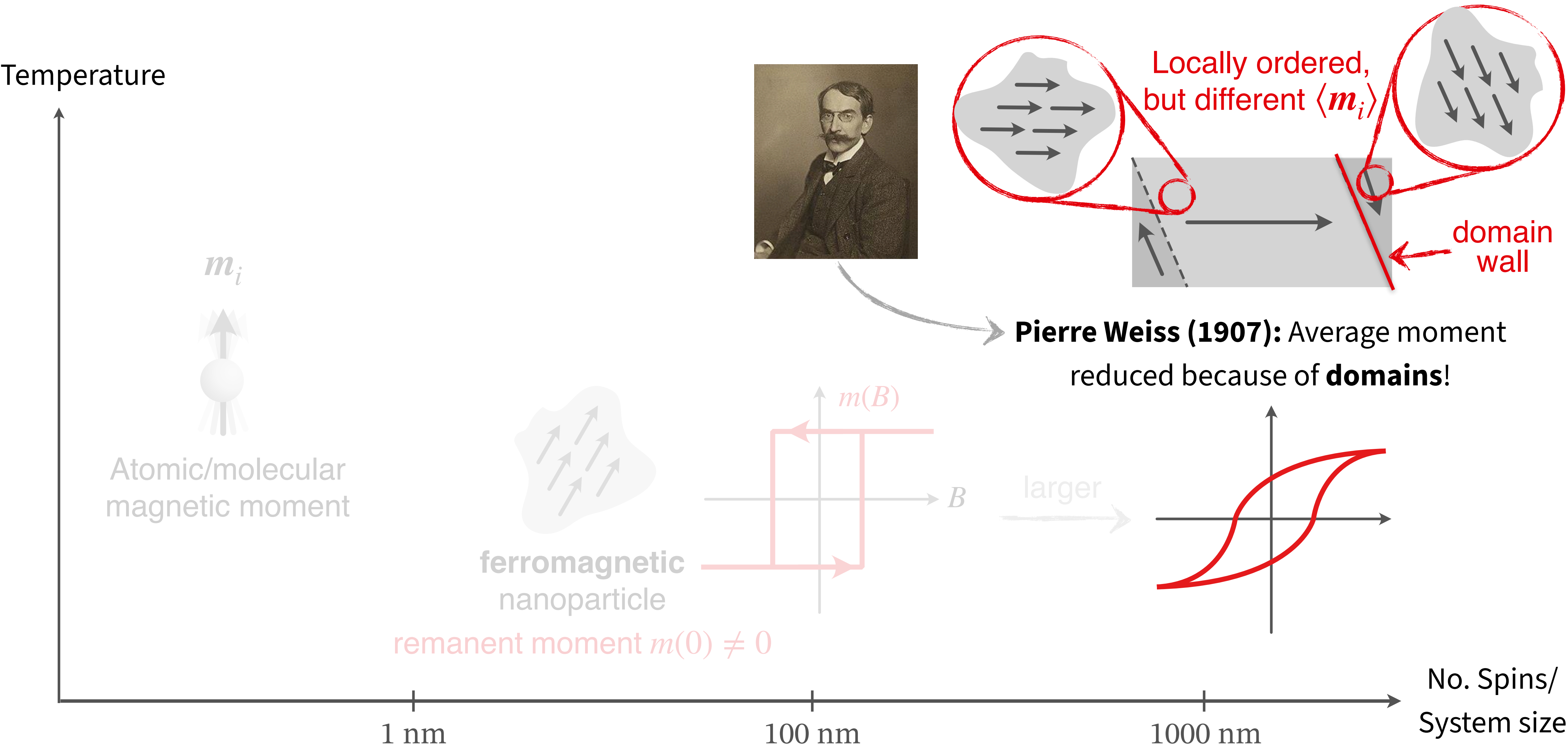
Atomic/molecular
magnetic moment



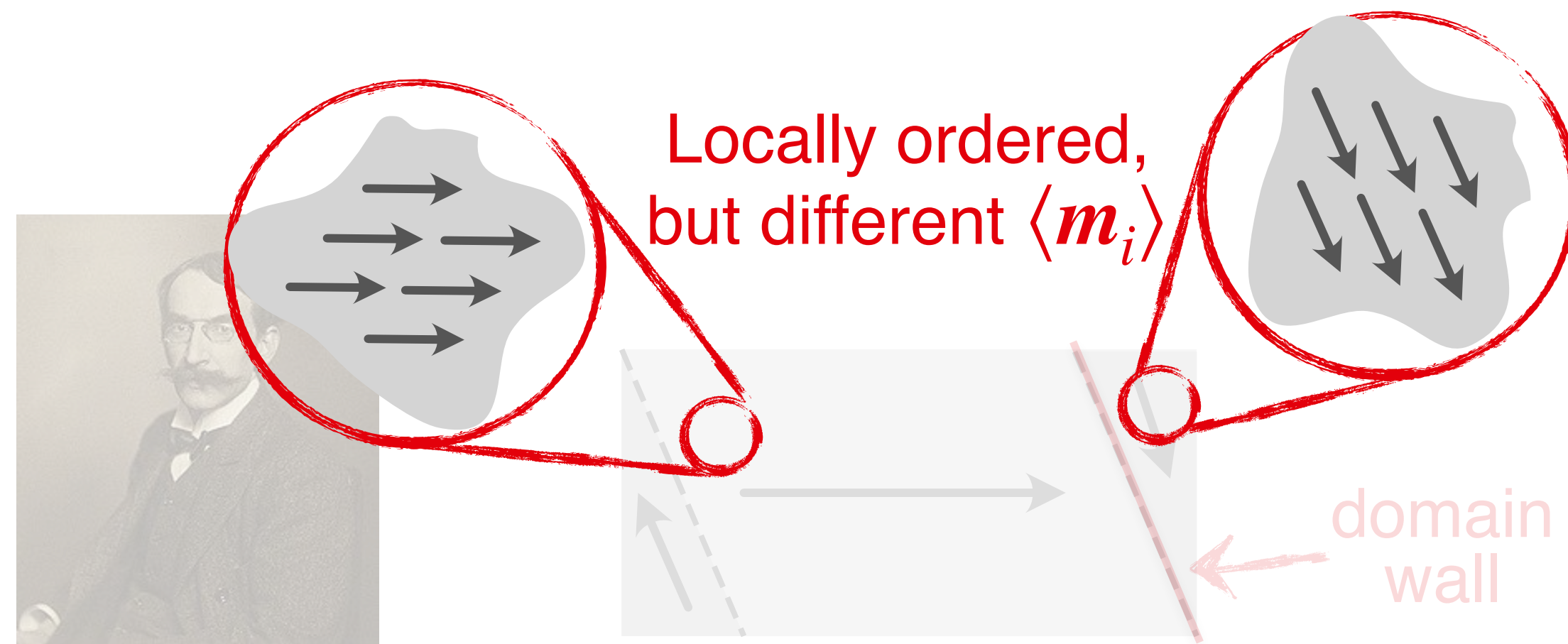
Domains and domain walls are archetypical structures in ordered materials



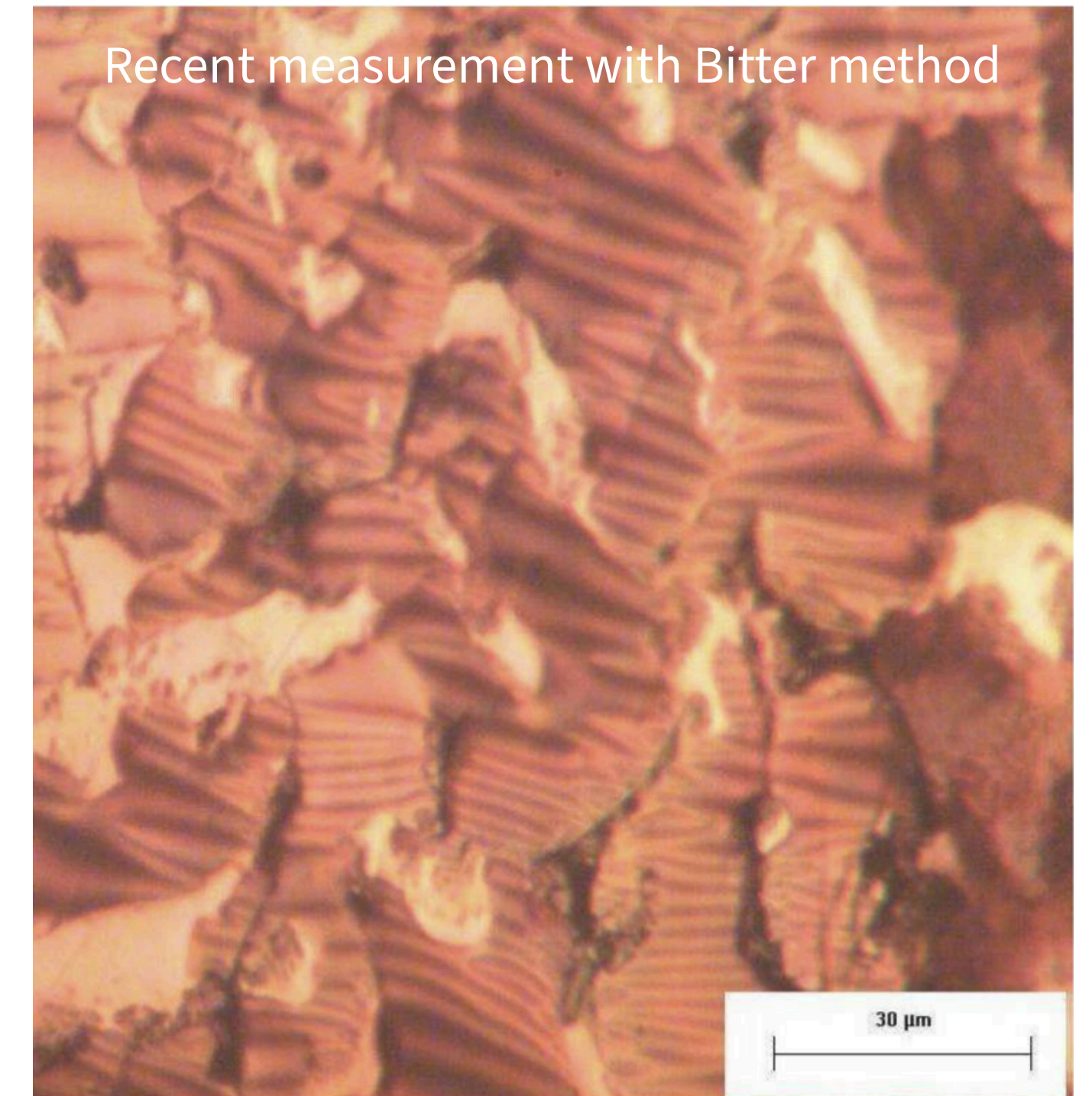
Domains and domain walls are archetypical structures in ordered materials



Domains and domain walls are archetypical structures in ordered materials



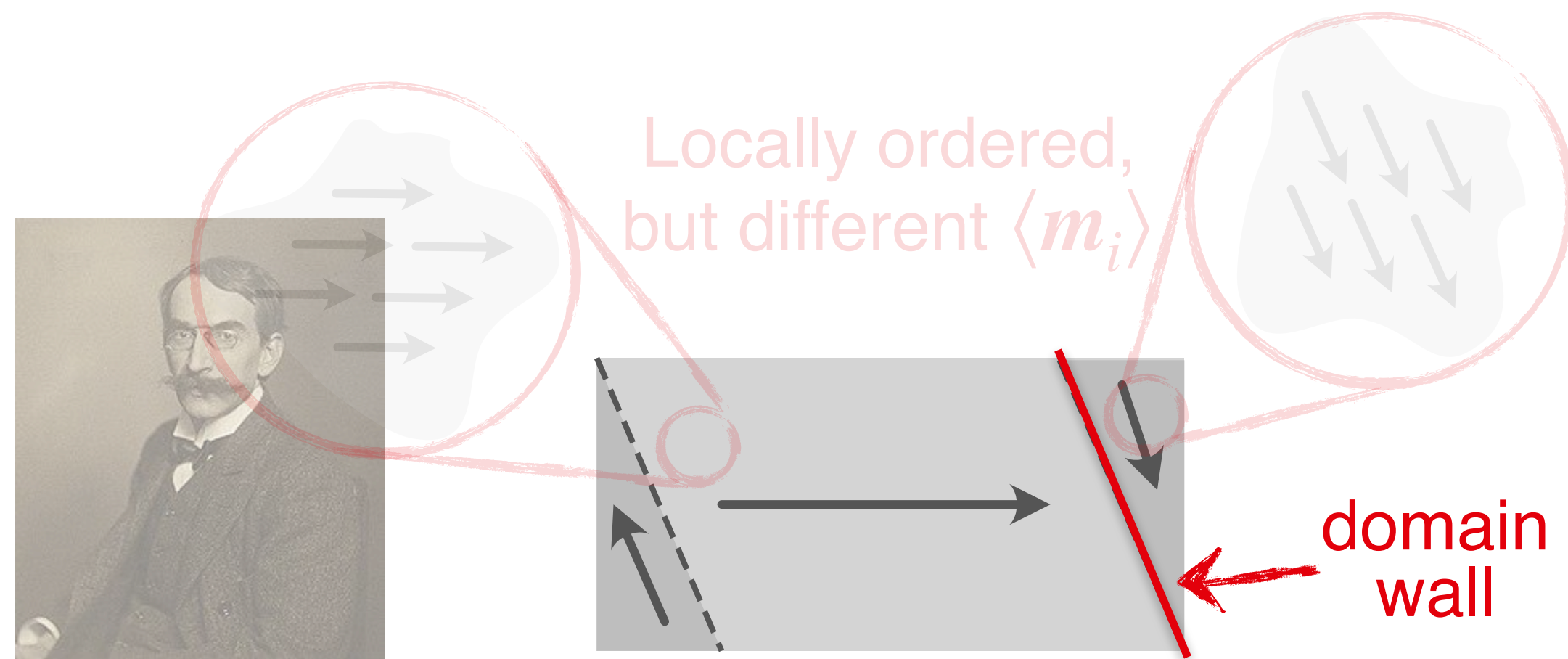
Pierre Weiss (1907): Average moment reduced because of **domains**!



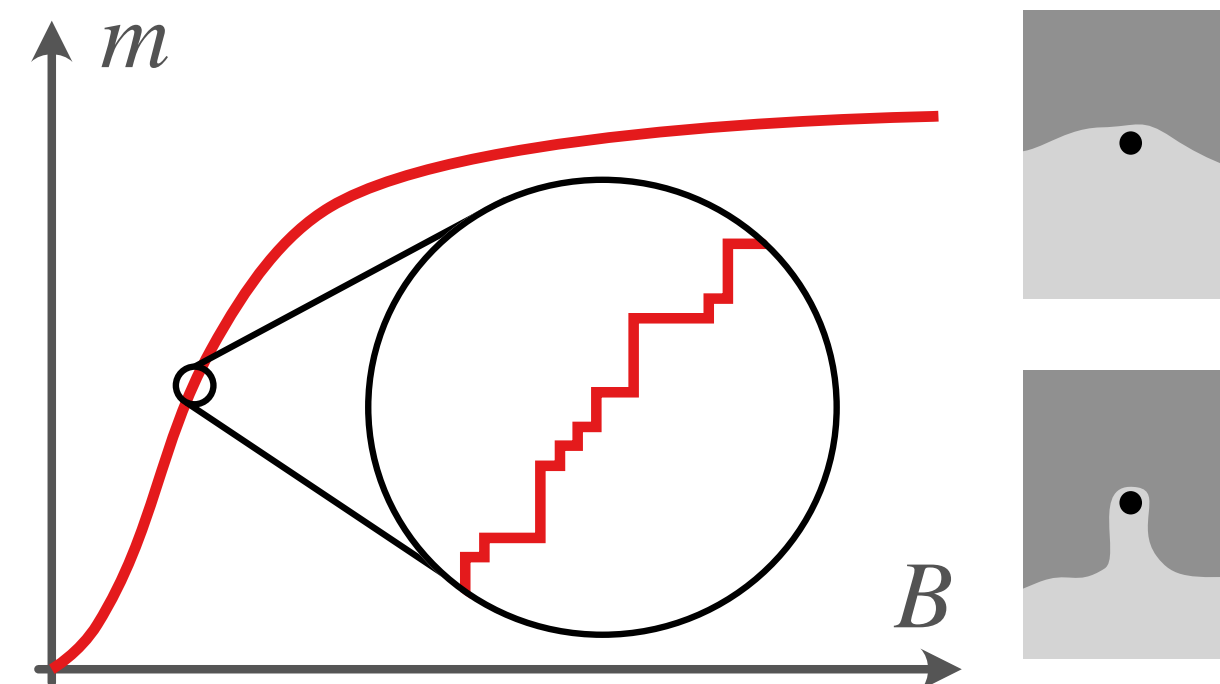
Lemos et al., Mat. Sci. Forum Vol. 802 (2014)

Francis Bitter (1930/1930) observed domains with ferrofluid on top of a magnet (Bitter method).

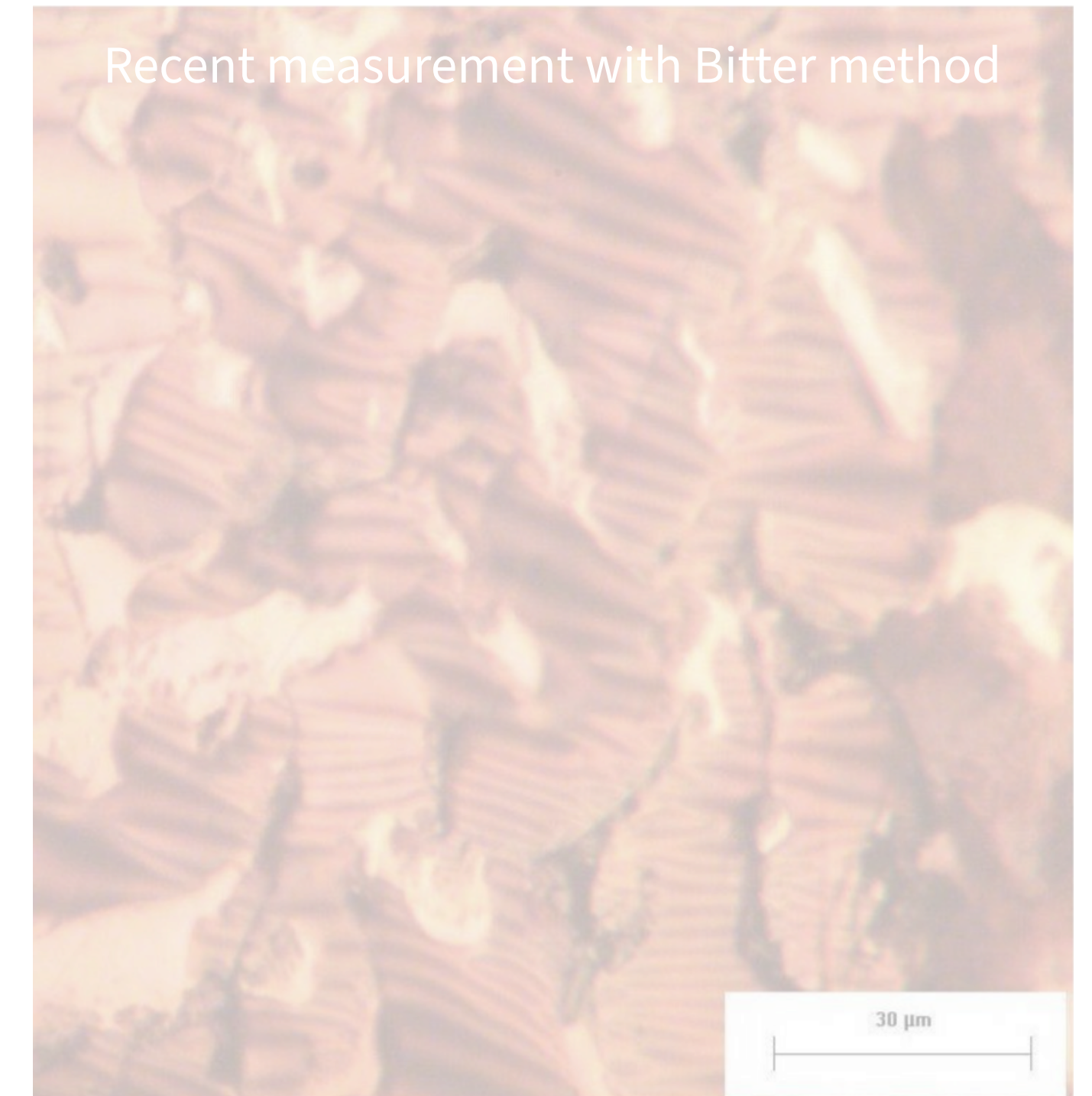
Domains and domain walls are archetypical structures in ordered materials



Pierre Weiss (1907): Average moment reduced because of **domains**!



Heinrich Barkhausen (1917) measured domain walls indirectly through their Barkhausen jumps.

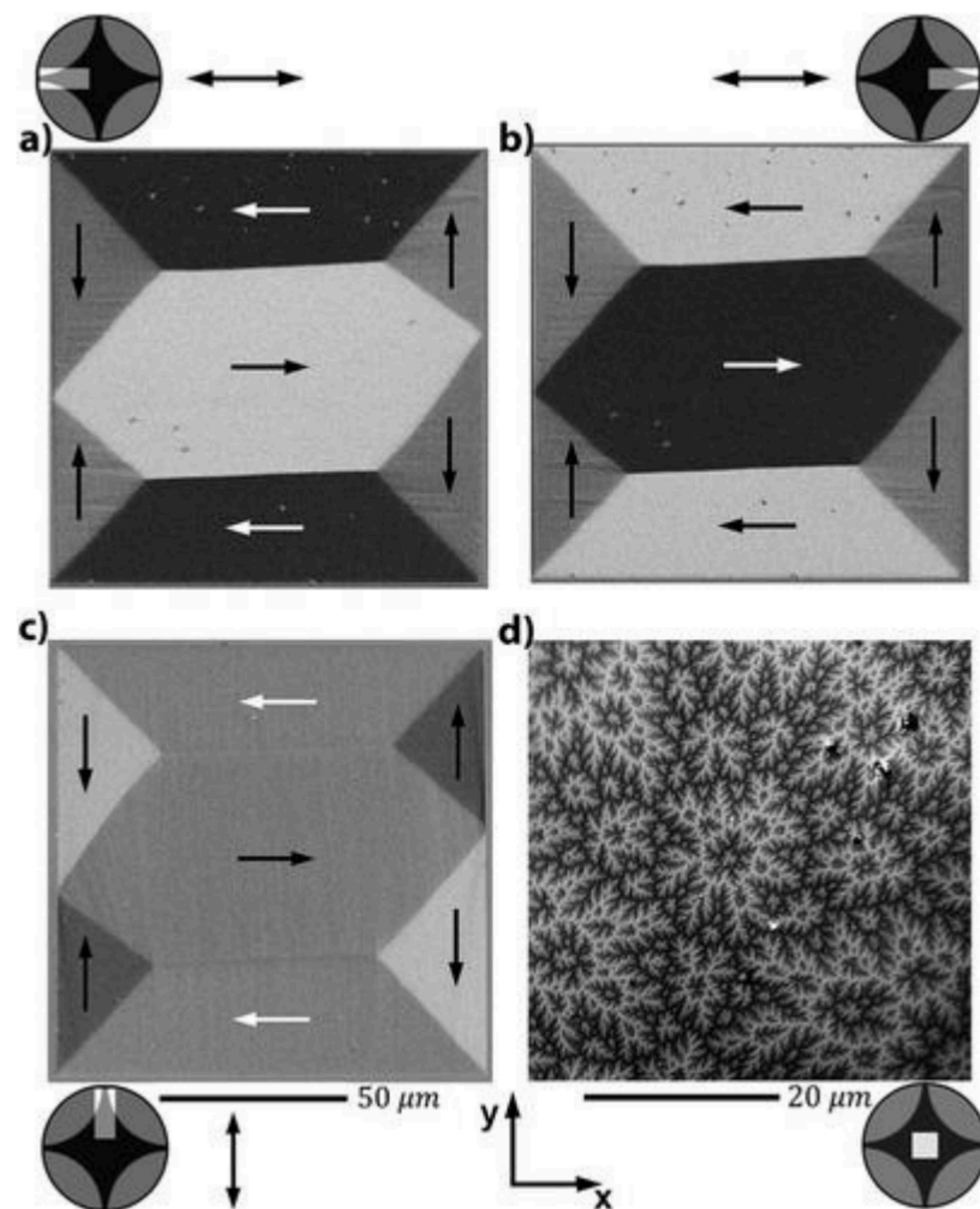


Lemos et al., Mat. Sci. Forum Vol. 802 (2014)

Francis Bitter (1930/1930) observed domains with ferrofluid on top of a magnet (Bitter method).

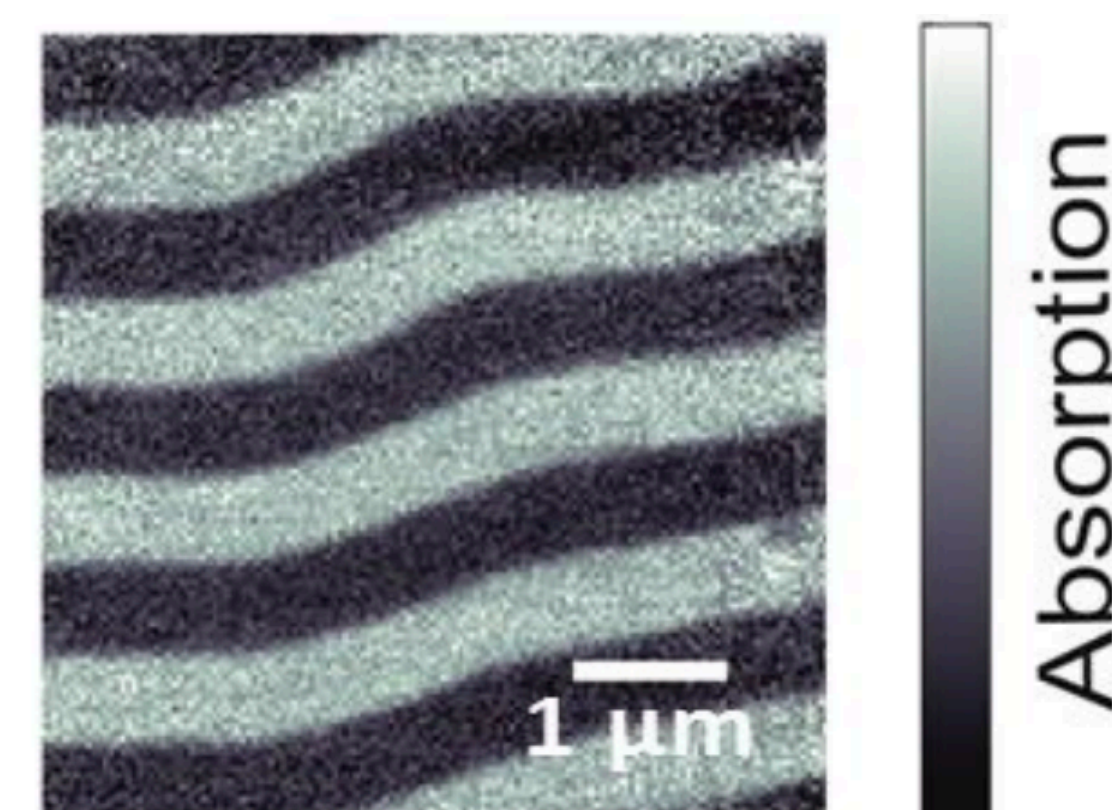
Today we have more advanced techniques to observe domains

Magneto-Optical-Kerr-Effect (MOKE) microscopy

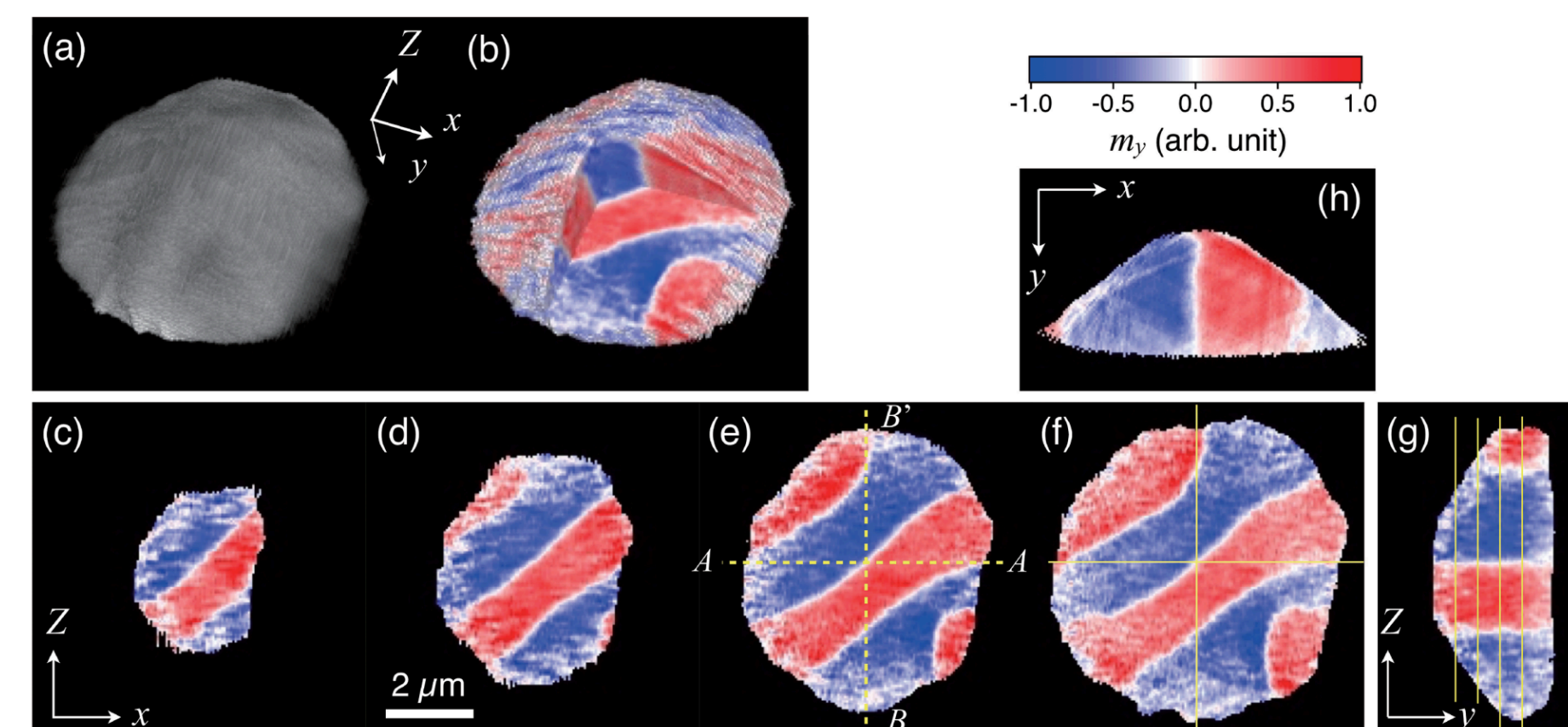


Soldatov & Schäfer,
Review of Scientific Instruments 88, 073701 (2017)

X-ray Magnetic Circular Dichroism (XMCD) microscopy & X-ray tomography



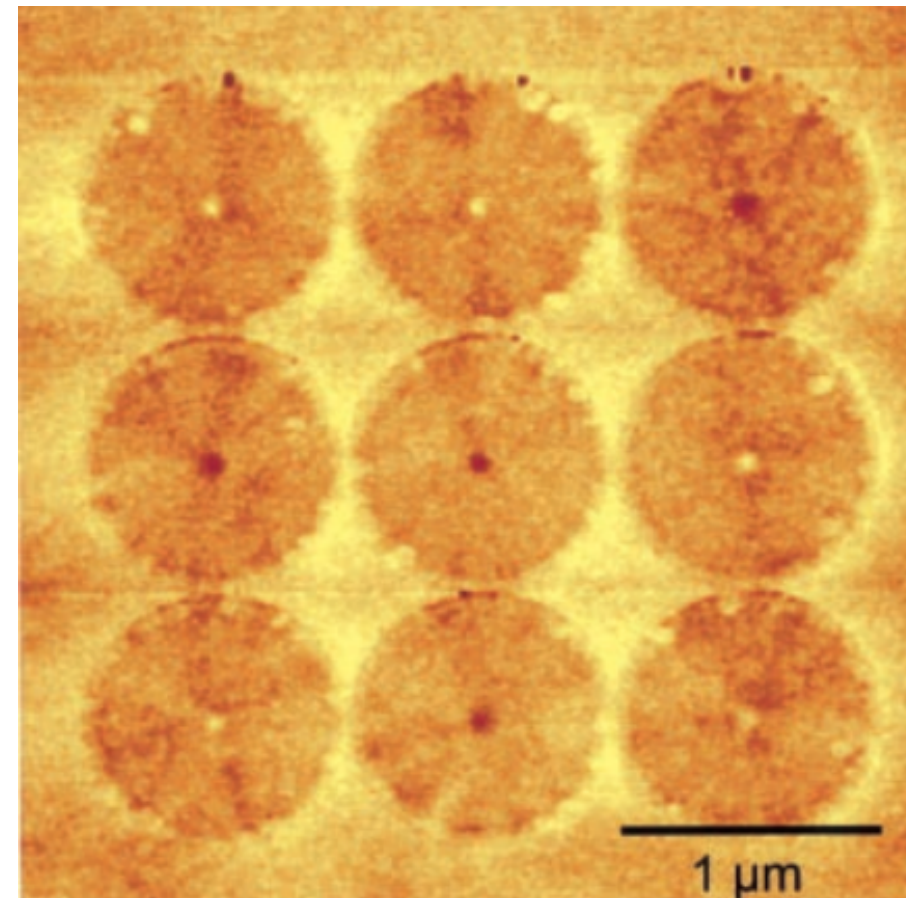
Donnelly et al., PRB 94, 064421 (2016)



Suzuki et al., Appl. Phys. Expr. 11, 036601 (2018)

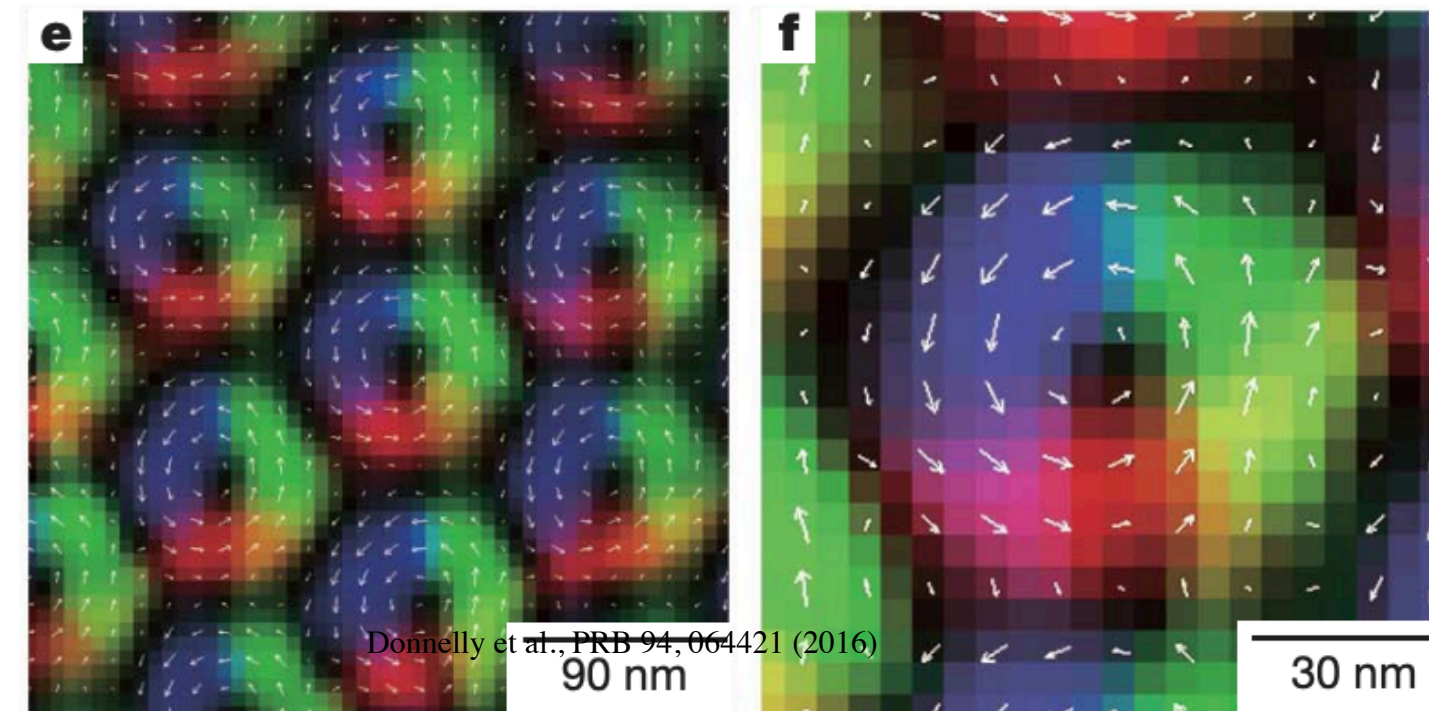
And even techniques to measure more complex spin textures

Vortex cores observed with
Magnetic-force microscopy (MFM)



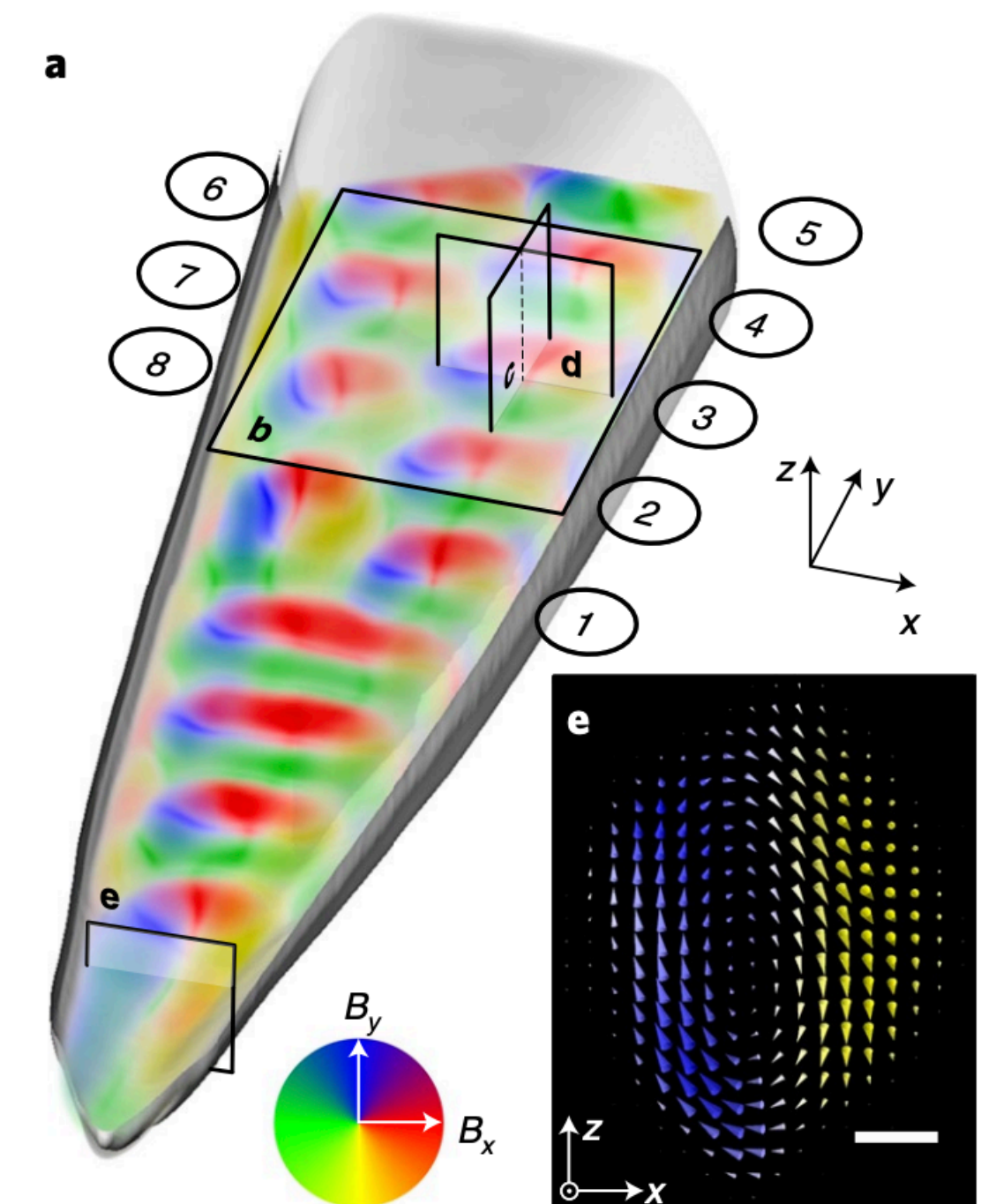
Shinjo, T., et al., *Science* 289.5481 (2000): 930-932.

Skyrmions observed with
Lorentz-transmission electron microscopy (LTEM)



Yu et al., *Nature* 465, 901–904 (2010)

Skyrmion tubes observed with
Holographic vector-field electron tomography



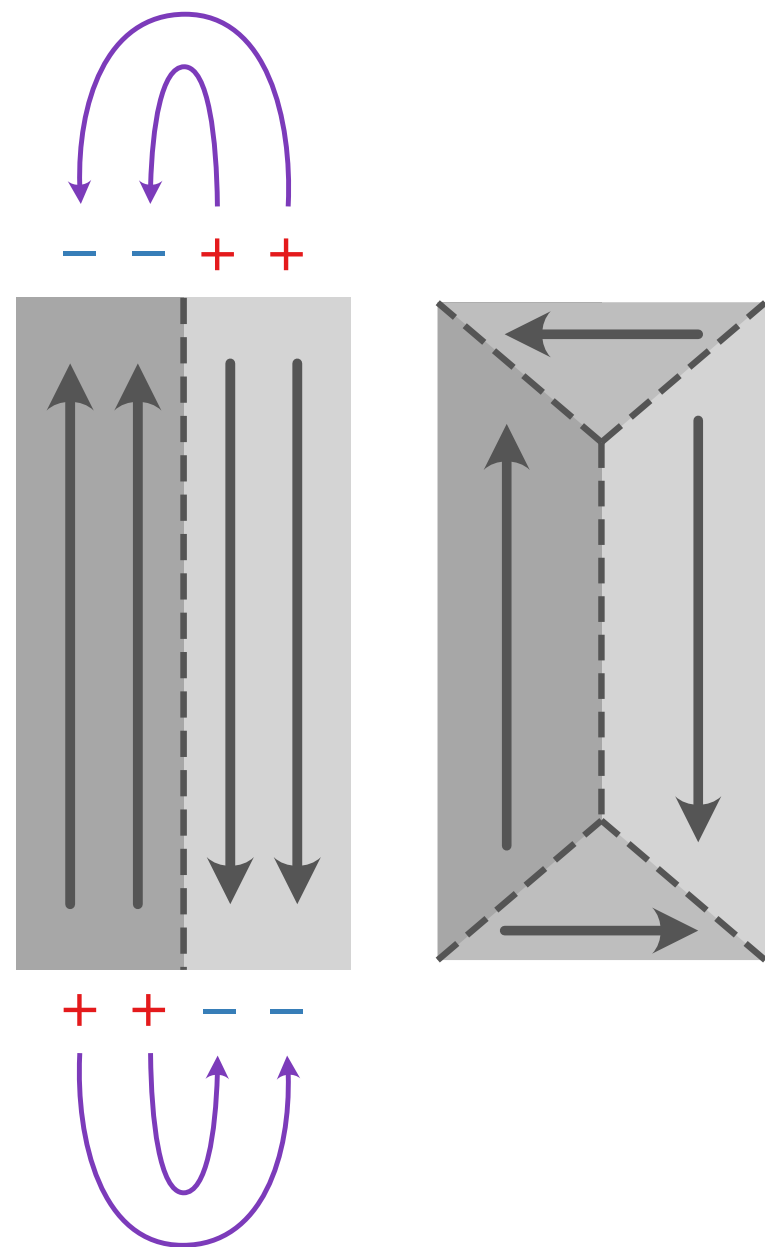
Wolf et al., *Nature Nano.* 17, 250–255 (2022)

Different techniques with different trade-offs:
Spatial/temporal resolution, number of vector components, ...

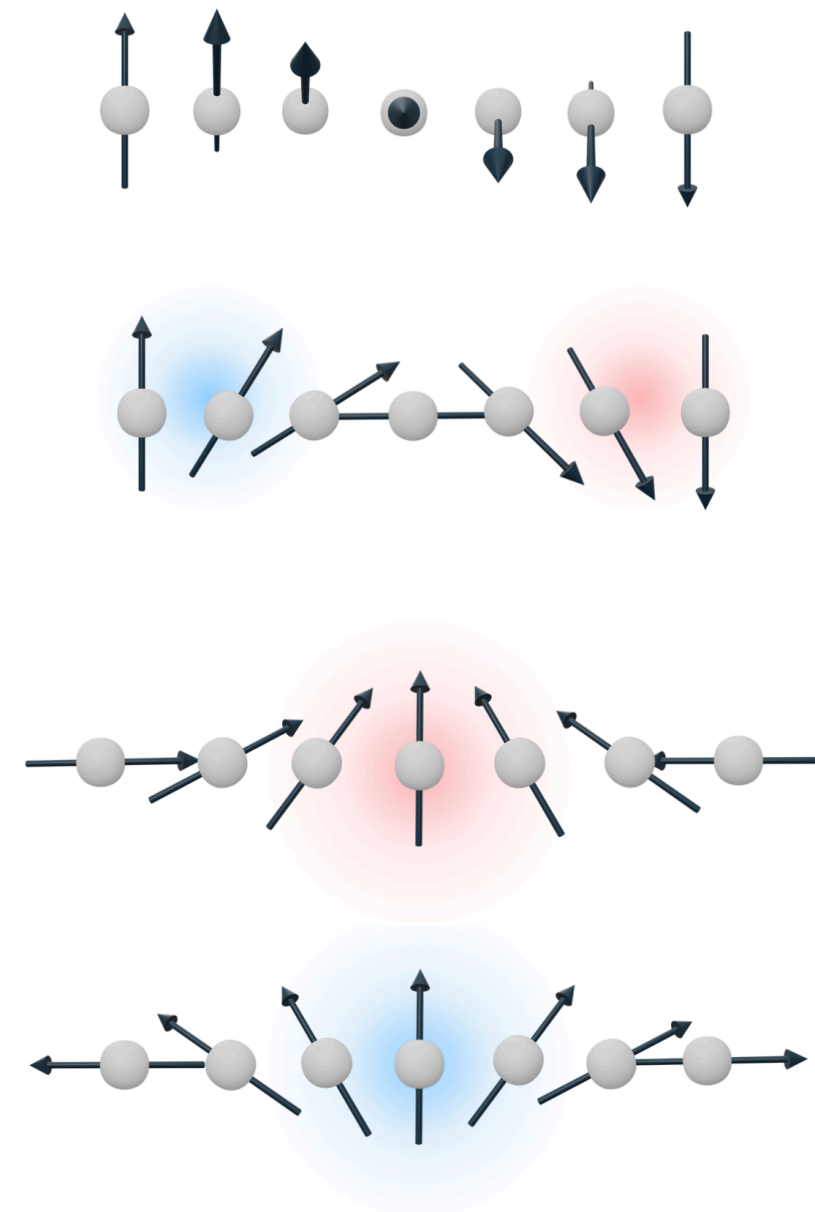
Here: Theoretical groundwork for these observations

What we are going to talk about in this lecture:

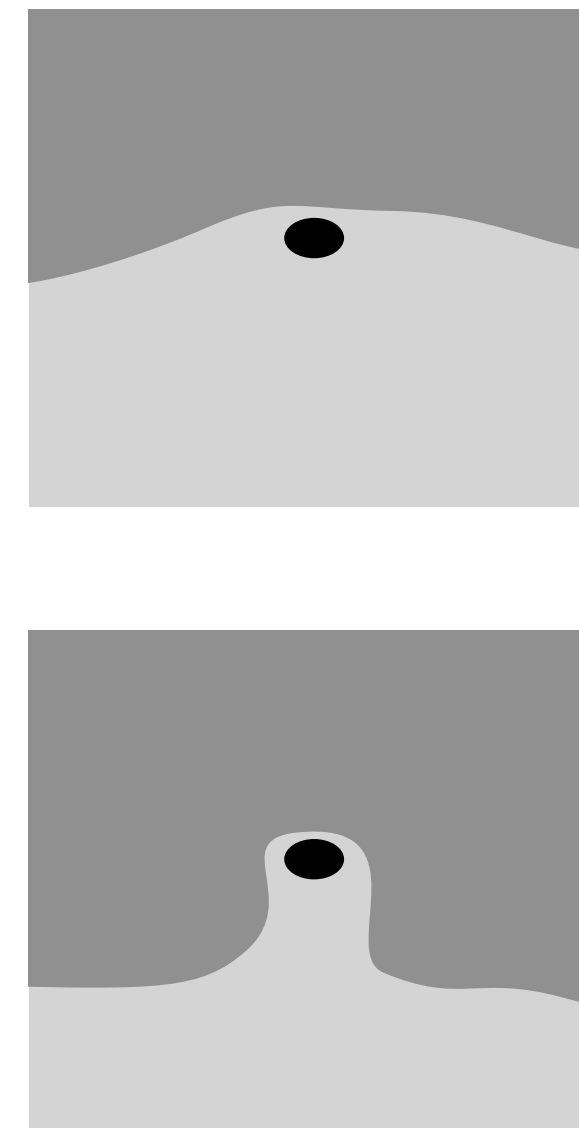
When and why do domains form?



Structure and types of domain walls



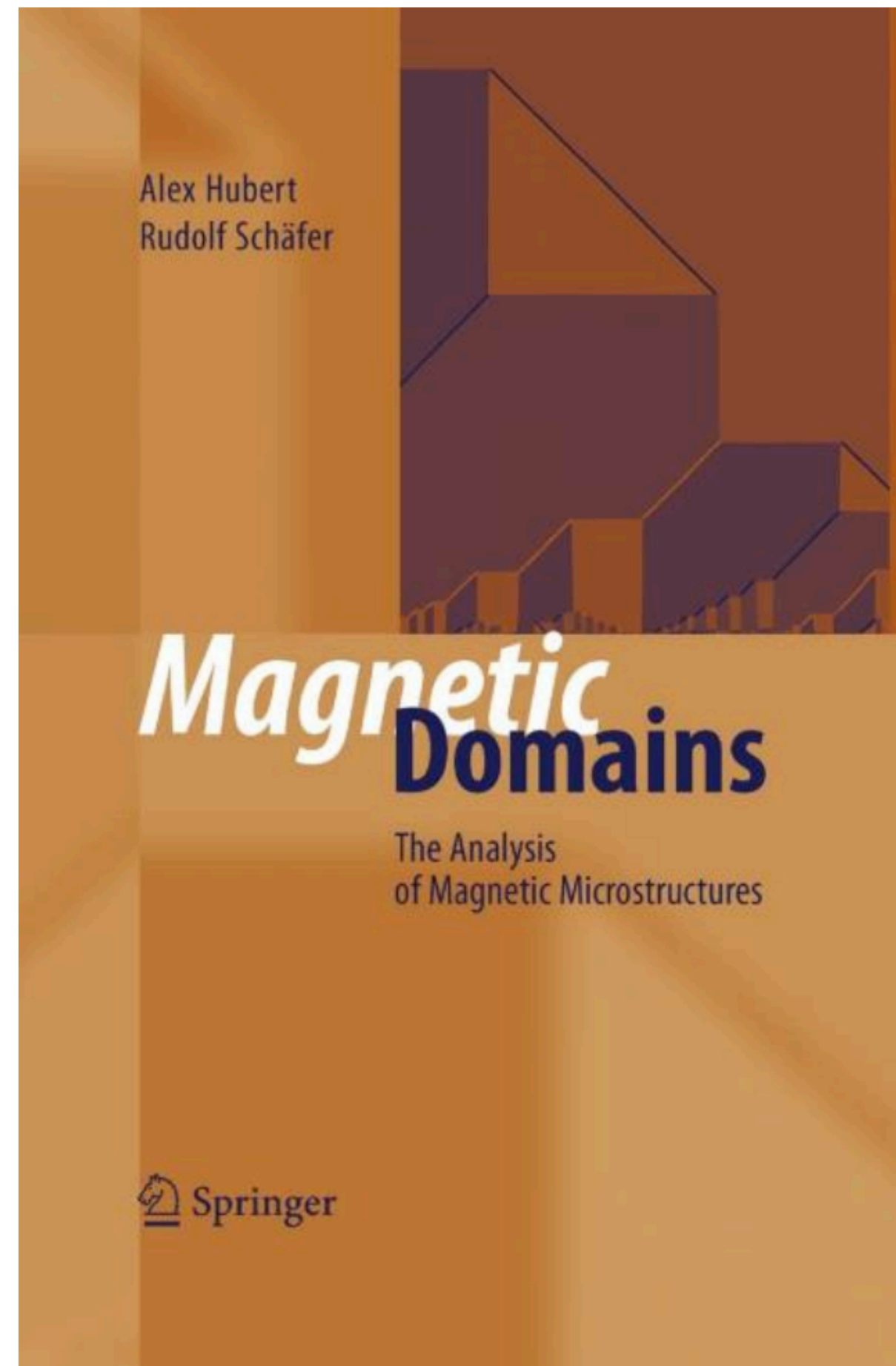
Domain-wall motion



Role of topology



Literature



All you need to know is in here...

Magnetic Domains – The analytics of magnetic microstructures

Alex Hubert and Rudolf Schäfer

Springer, 1998, ISBN: 9783540641087, pp 724

For theoretical problems and solutions

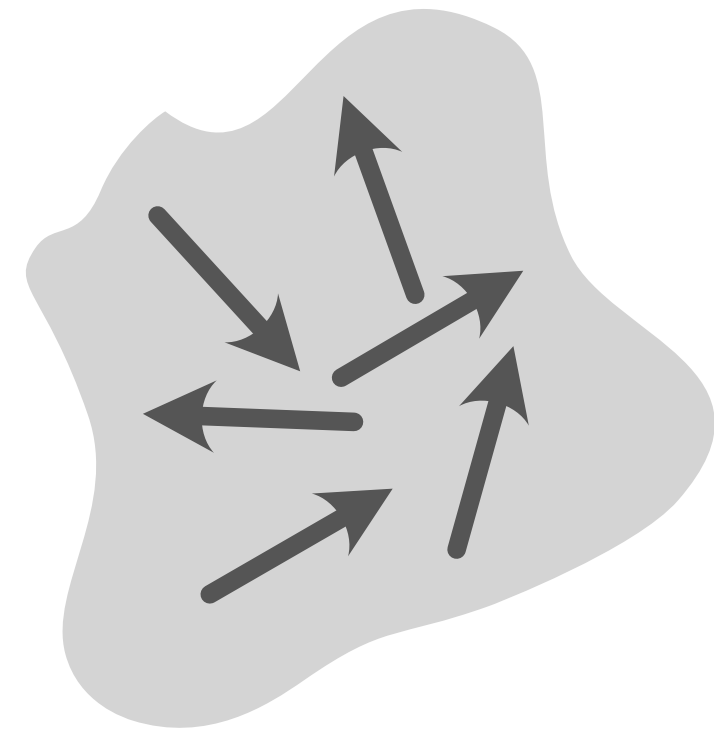
Problems in micromagnetism

Volodymyr Kravchuk

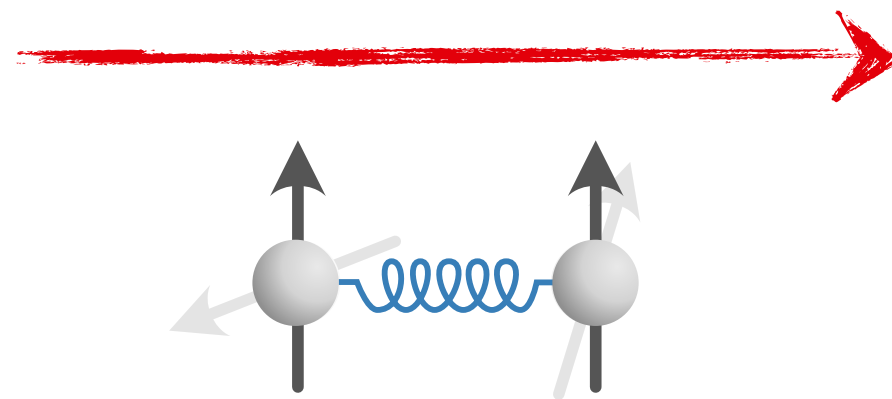
DOI: 10.13140/RG.2.2.22653.74728

Domains are extended regions of uniform order parameter

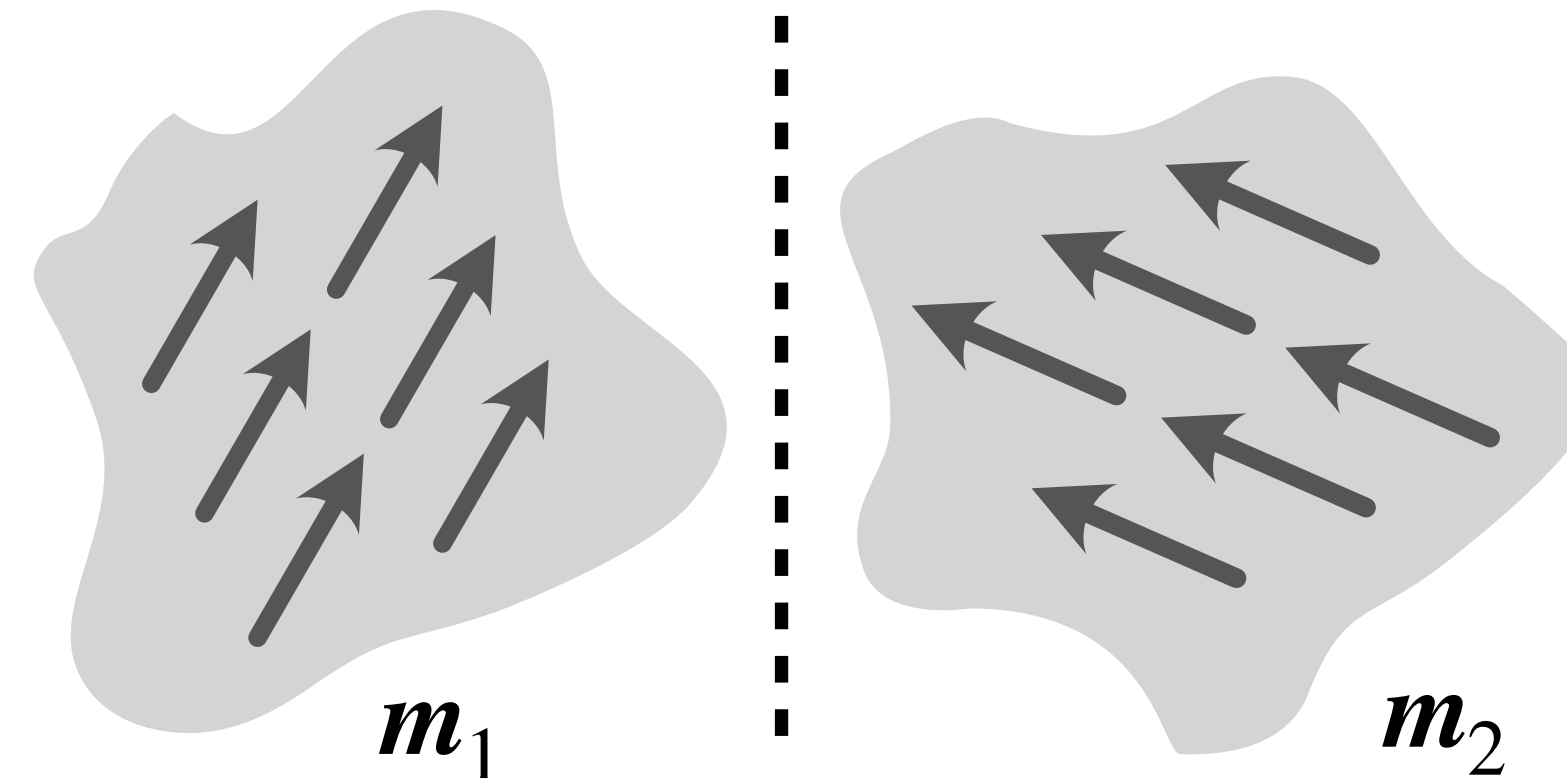
disordered, $m \approx 0$



„Turn on“ magnetism
(e.g. by cooling)



magnetically ordered, $m \neq 0$

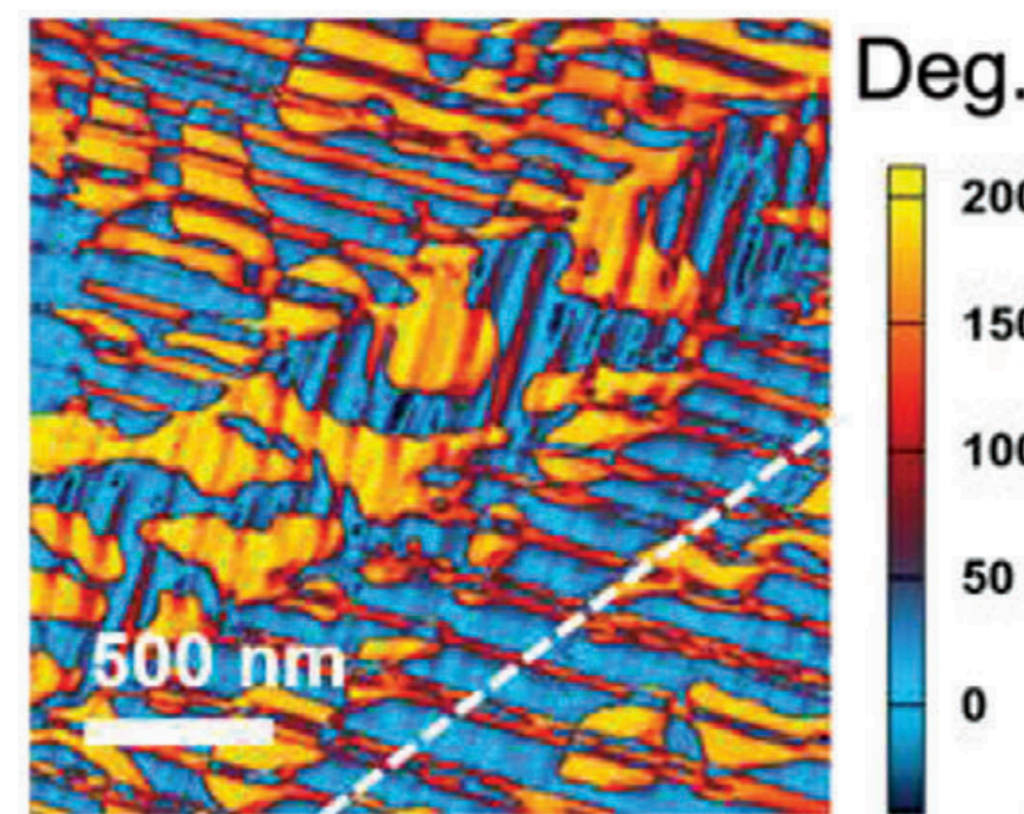


domain wall

$E(m_1) = E(m_2)$,
from symmetries
(system specific)

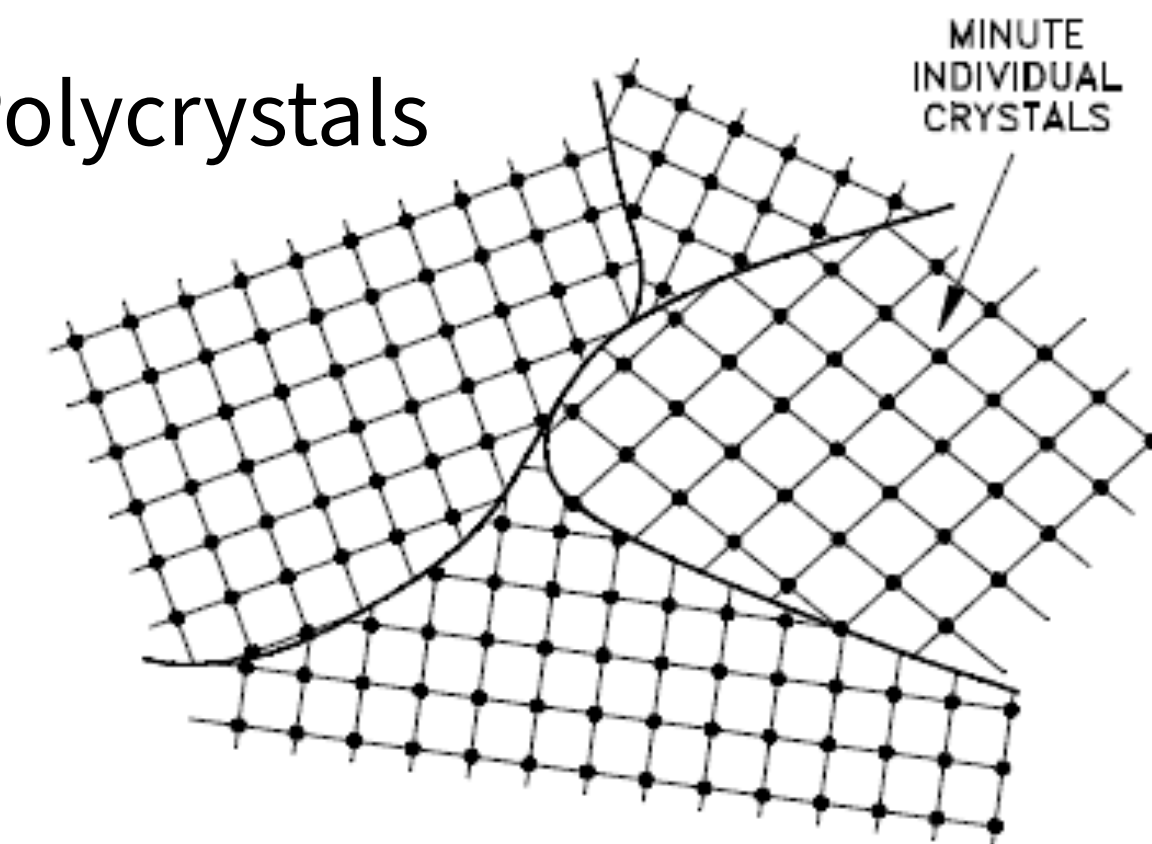
Not restricted to magnetism...

Ferroelectric domains
of polarization, $\mathbf{P}$



Chen et al., Adv. Sci. 9, 2105980 (2022)

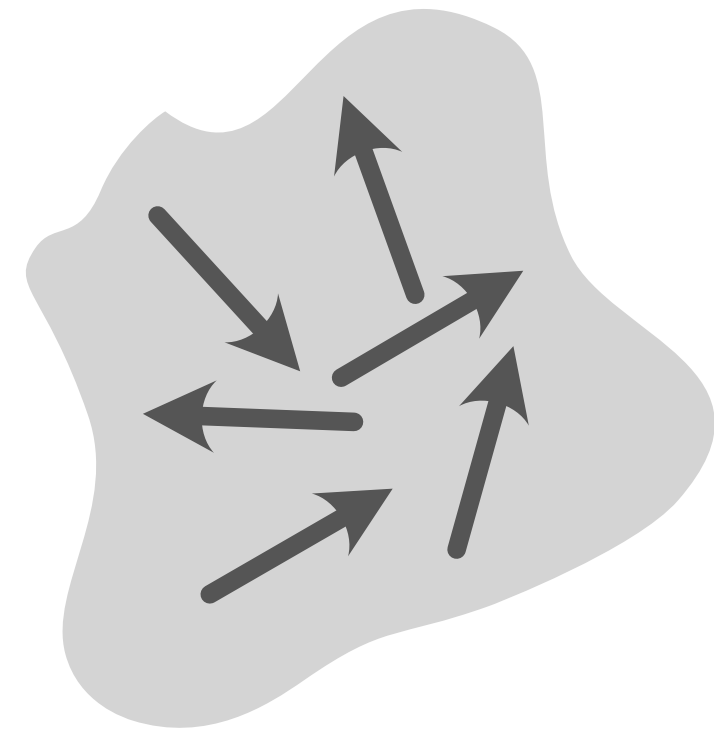
Polycrystals



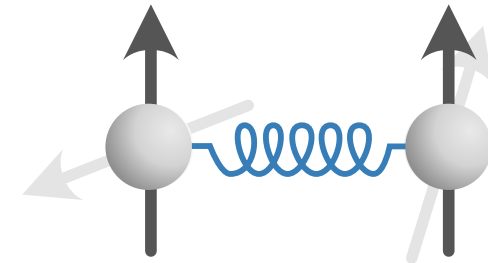
DOE Fundamentals Handbook (1993)

Domains are extended regions of uniform order parameter

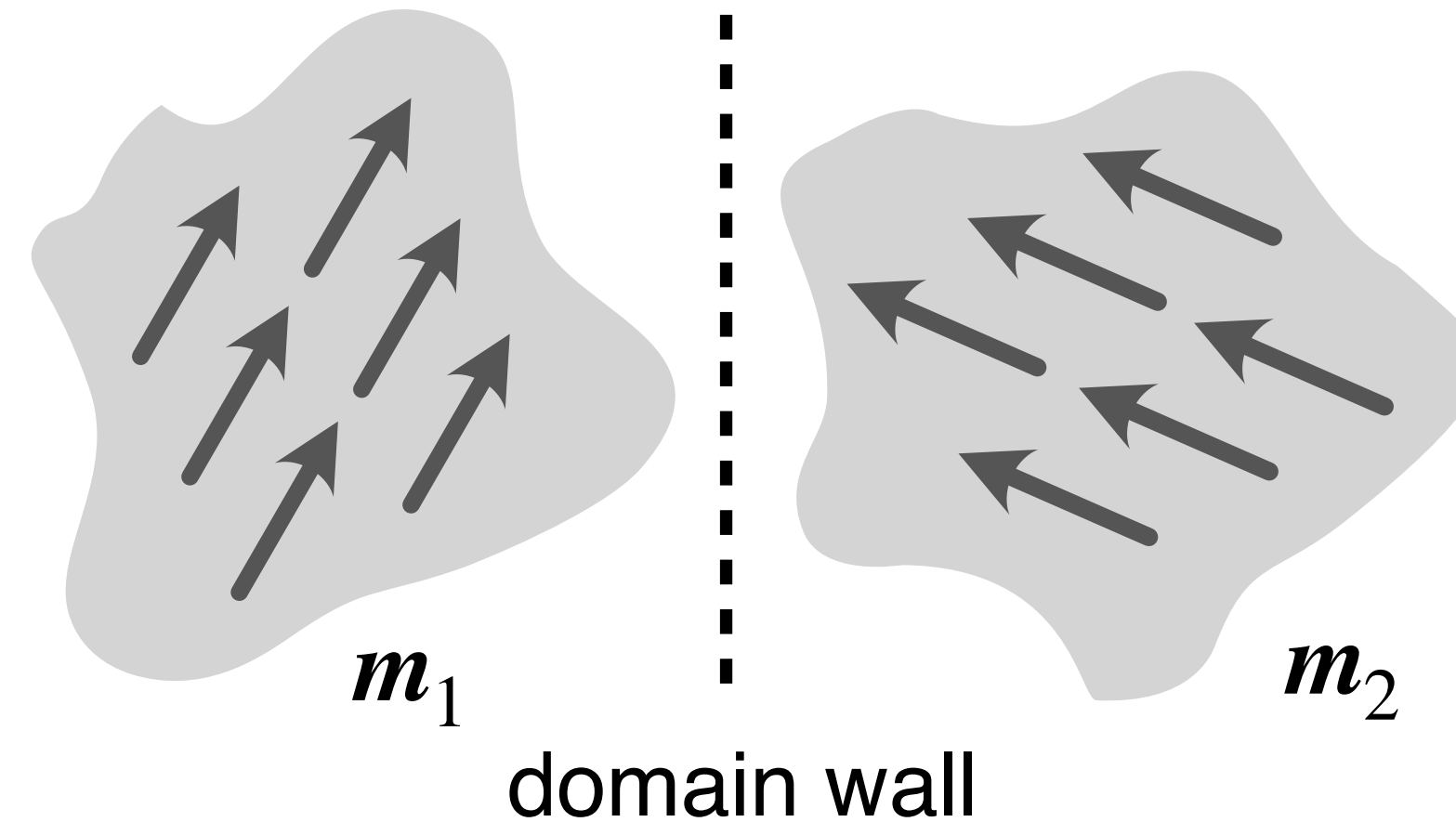
disordered, $m \approx 0$



„Turn on“ magnetism
(e.g. by cooling)



magnetically ordered, $m \neq 0$

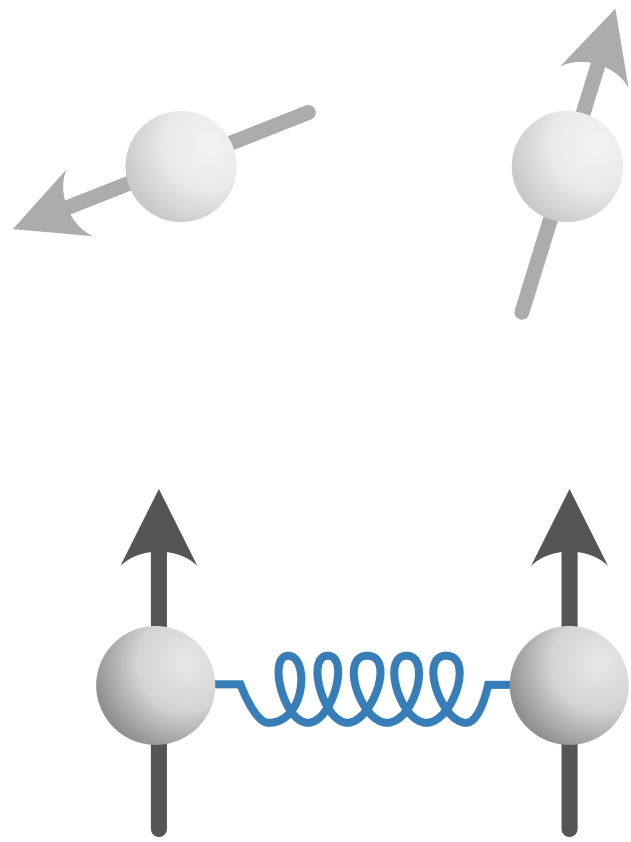


$E(m_1) = E(m_2)$,
from symmetries
(system specific)

Will magnets also form domains as ground states?

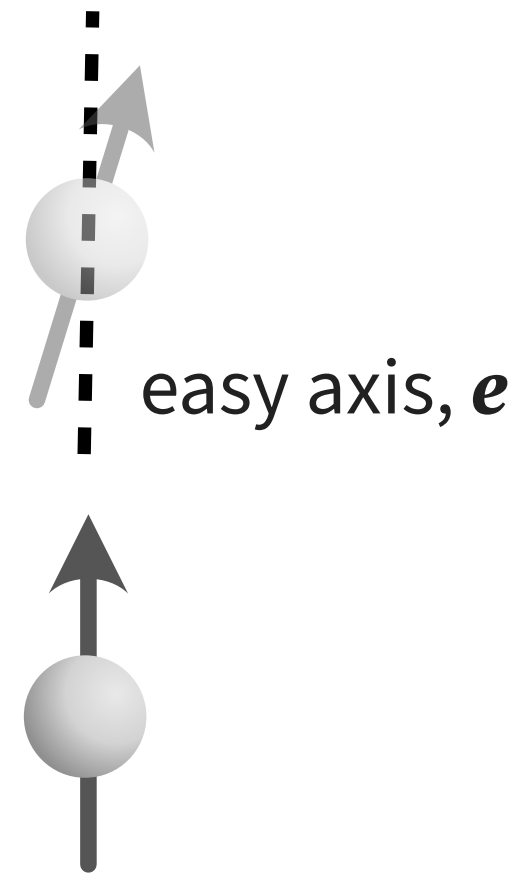
Domain patterns from balance of magnetic interactions

Exchange



Energy $\sum_{ij} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j$

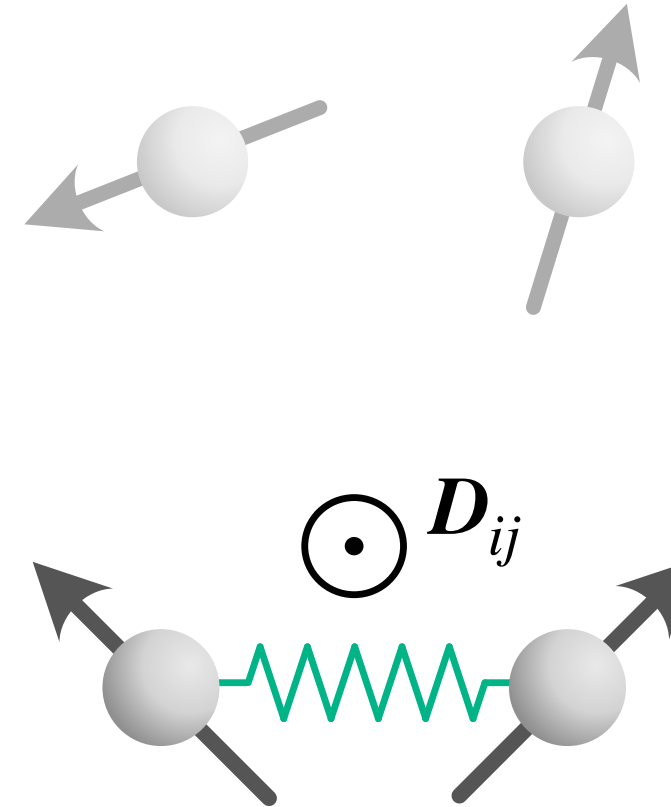
Crystalline anisotropy



*Also: multiple axes,
easy plane, ...*

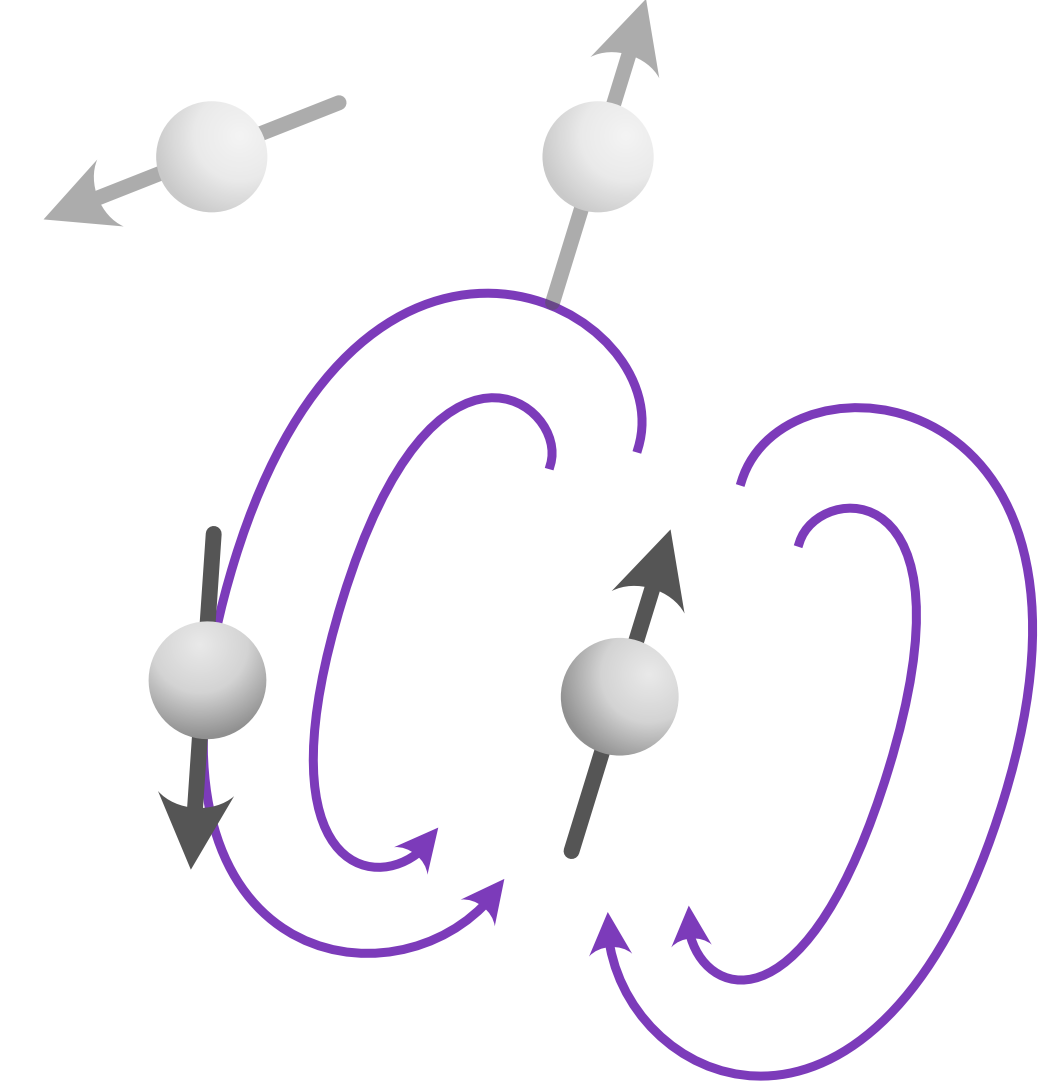
$\sum_{ij} \mathbf{m}_i \cdot \hat{\mathbf{K}}_{ij} \cdot \mathbf{m}_i$

Dzyaloshinskii-Moriya Interaction (DMI)



$\sum_{ij} \mathbf{D}_{ij} \cdot (\mathbf{m}_i \times \mathbf{m}_j)$

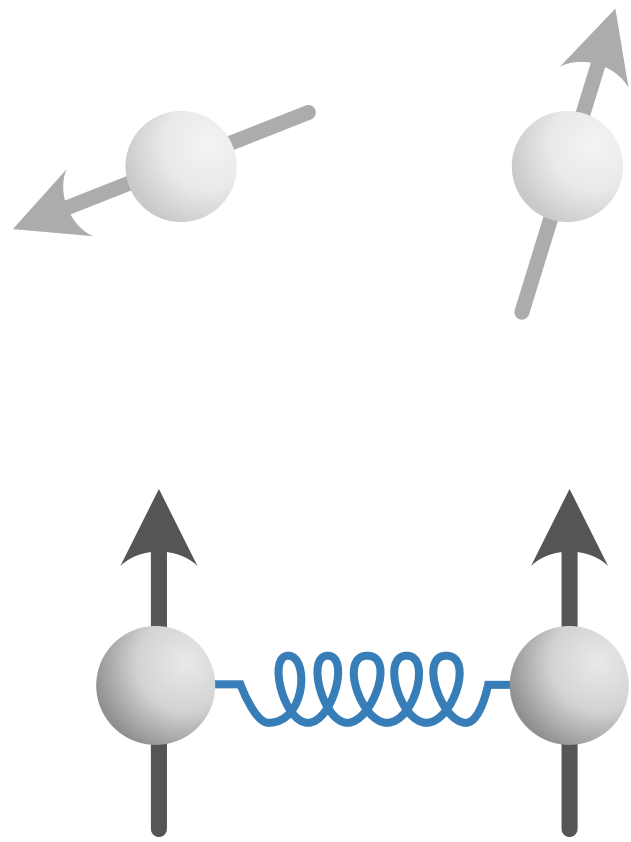
Dipole-dipole



$\propto \sum_{ij} \frac{(\mathbf{m}_i \cdot \mathbf{r}_{ij})(\mathbf{m}_j \cdot \mathbf{r}_{ij}) - \mathbf{m}_i \cdot \mathbf{m}_j}{(r_{ij})^2}$

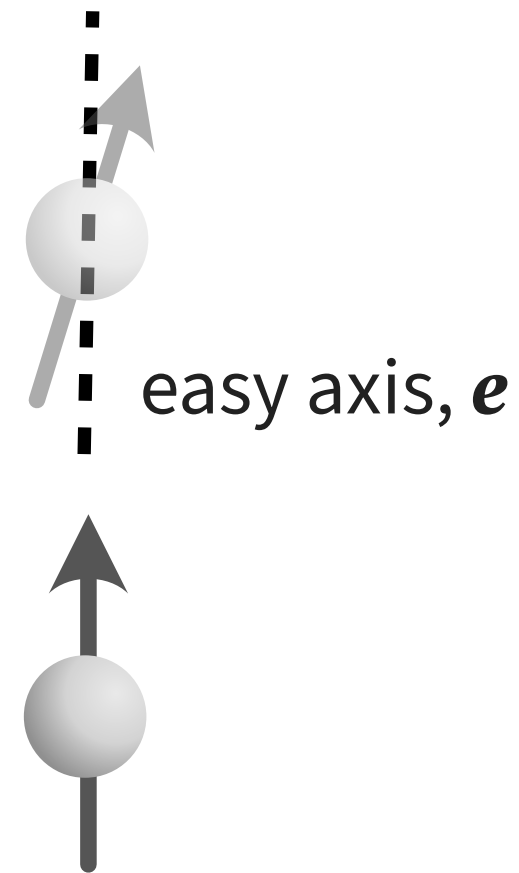
Domain patterns from balance of magnetic interactions

Exchange



Energy $\sum_{ij} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j$

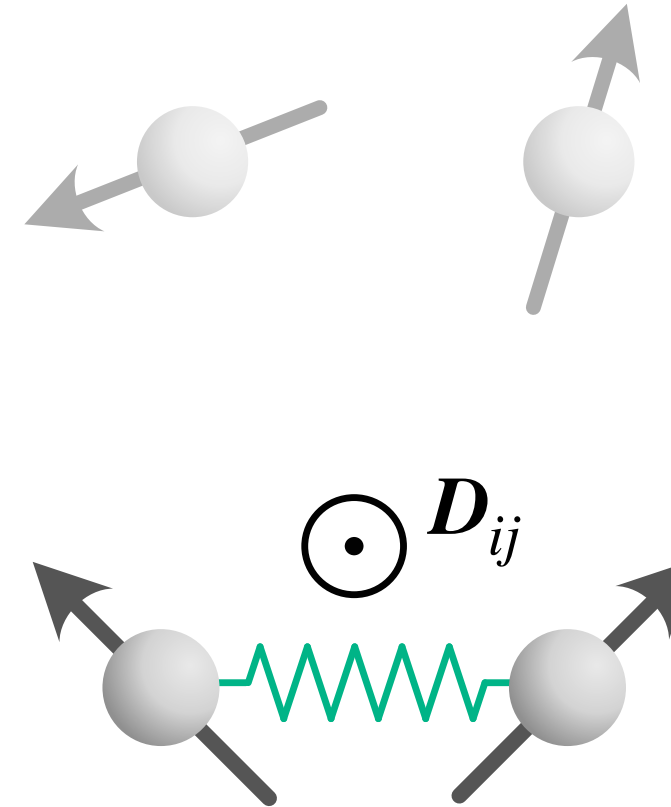
Crystalline anisotropy



*Also: multiple axes,
easy plane, ...*

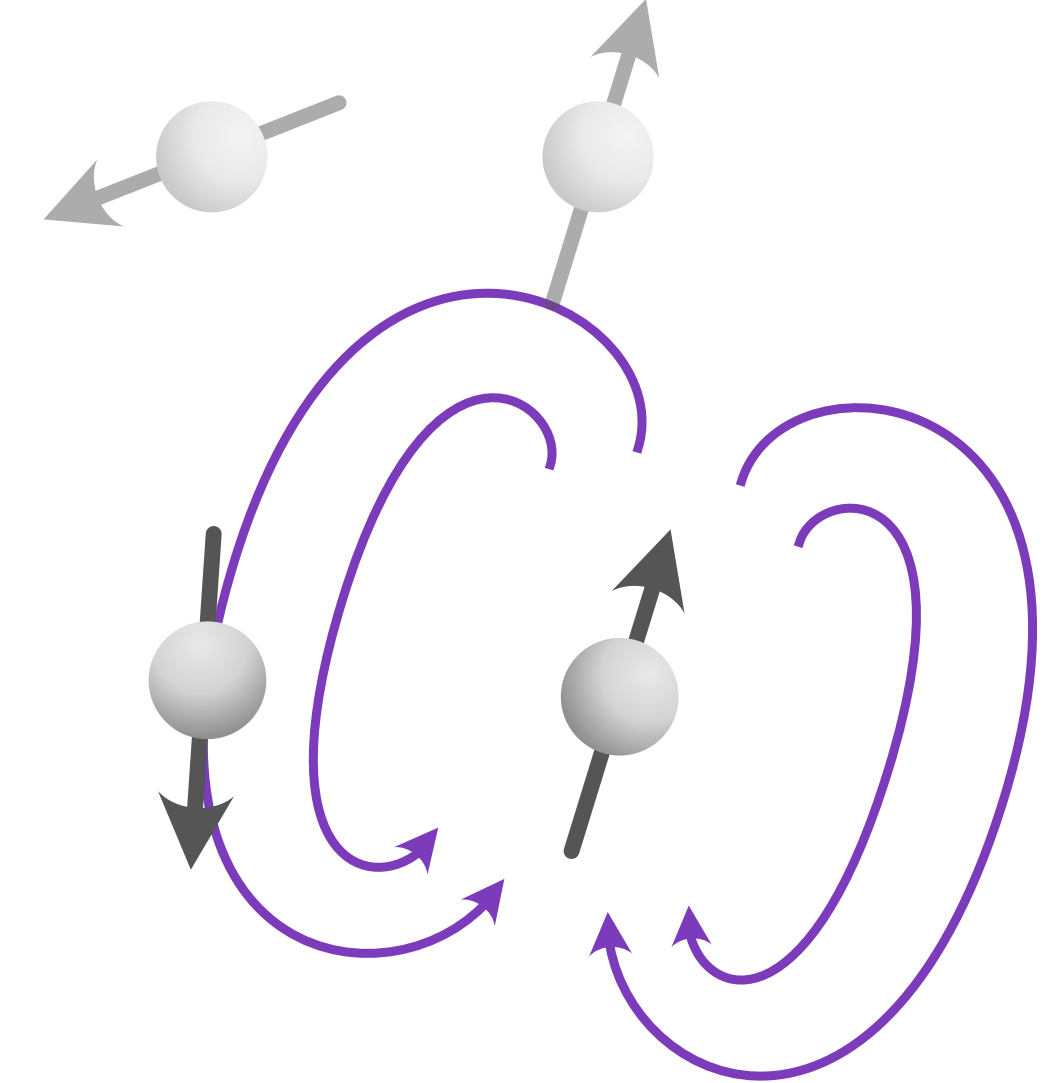
$\sum_{ij} \mathbf{m}_i \cdot \hat{\mathbf{K}}_{ij} \cdot \mathbf{m}_i$

Dzyaloshinskii-Moriya Interaction (DMI)



$\sum_{ij} \mathbf{D}_{ij} \cdot (\mathbf{m}_i \times \mathbf{m}_j)$

Dipole-dipole

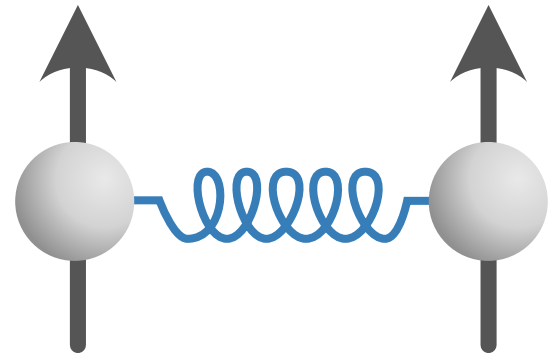


$\propto \sum_{ij} \frac{(\mathbf{m}_i \cdot \mathbf{r}_{ij})(\mathbf{m}_j \cdot \mathbf{r}_{ij}) - \mathbf{m}_i \cdot \mathbf{m}_j}{(r_{ij})^2}$

Very complicated, but very important.

Domain patterns are easier to understand in continuum limit

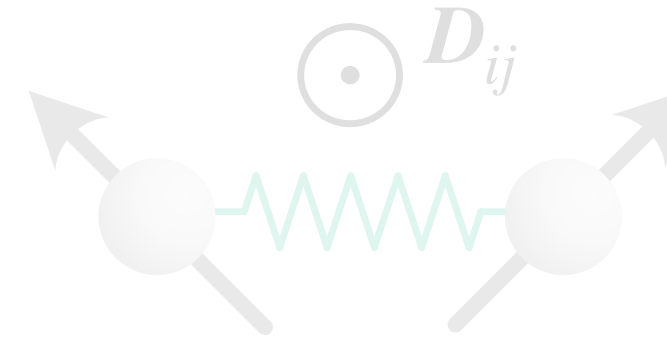
Exchange



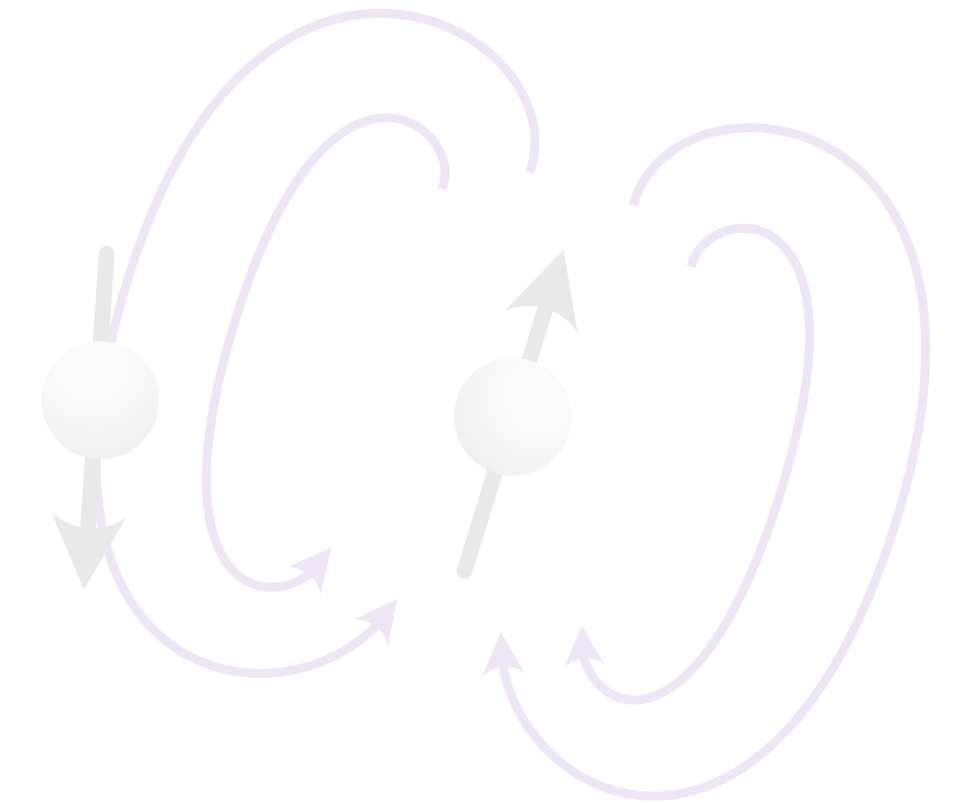
Crystalline anisotropy



Dzyaloshinskii-Moriya Interaction (DMI)



Dipole-dipole

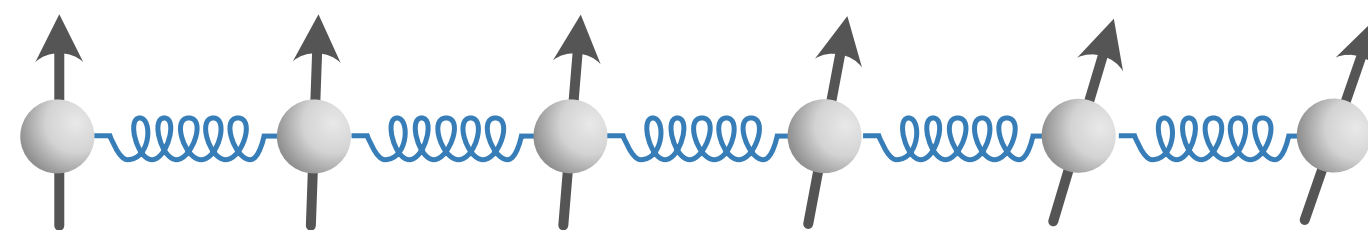


Energy $\sum_{ij} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j \approx A \int dV [\nabla \mathbf{m}(\mathbf{r})]^2 + \text{const.}$

$$\propto \frac{(\mathbf{m}_i \cdot \mathbf{r}_{ij})(\mathbf{m}_j \cdot \mathbf{r}_{ij}) - \mathbf{m}_i \cdot \mathbf{m}_j}{(r_{ij})^2}$$

Often strongest

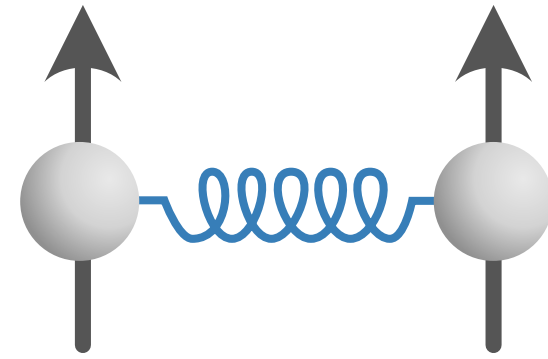
 Moments $\mathbf{m}_i$ rotate smoothly



$\approx \mathbf{m}(\mathbf{r})$ smooth magnetization (micromagnetism)

Domain patterns are easier to understand in micromagnetism

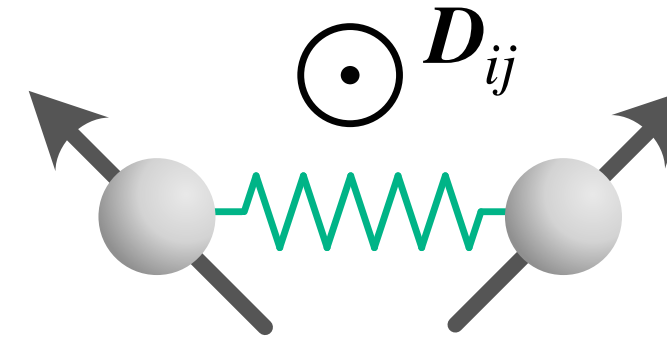
Exchange



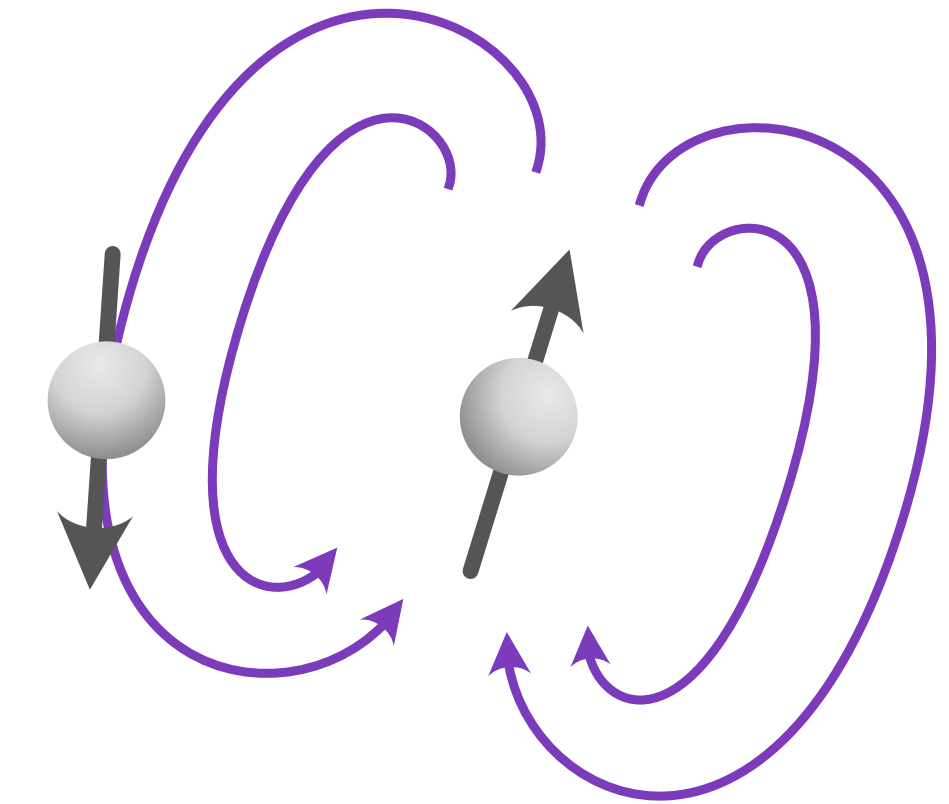
Crystalline anisotropy



Dzyaloshinskii-Moriya Interaction (DMI)

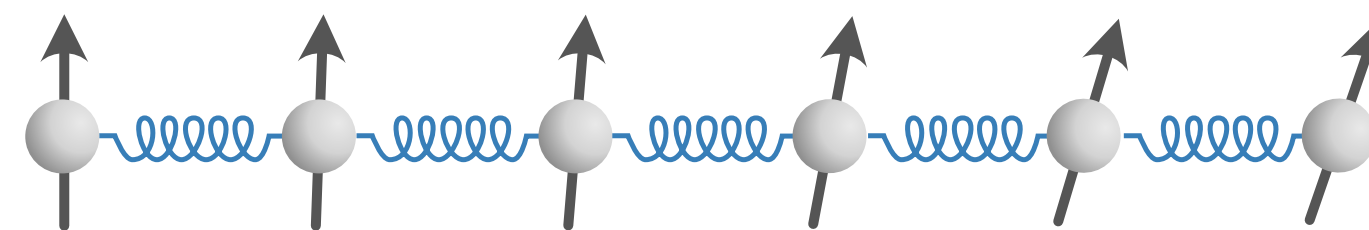


Dipole-dipole



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Micromagnetic energy functional



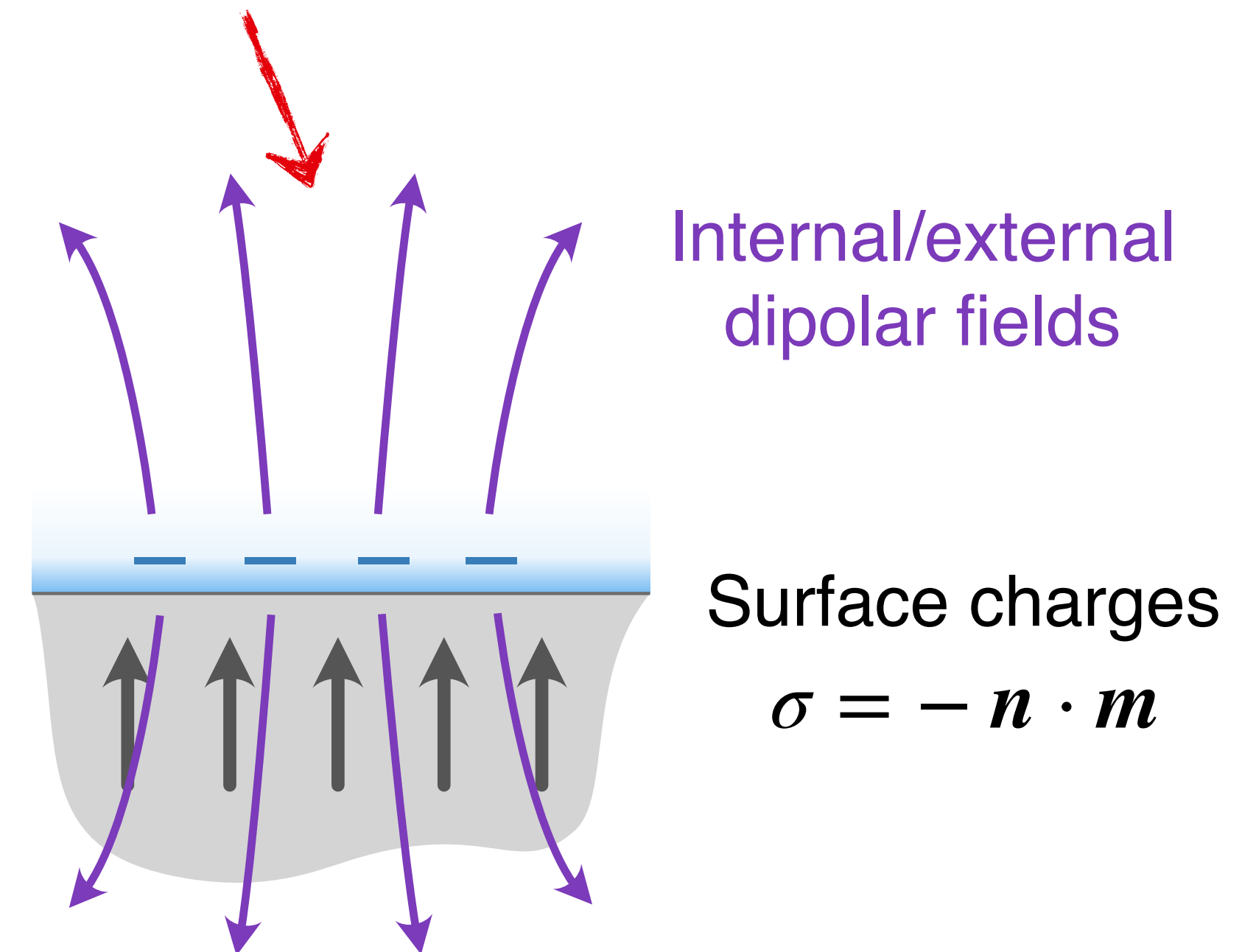
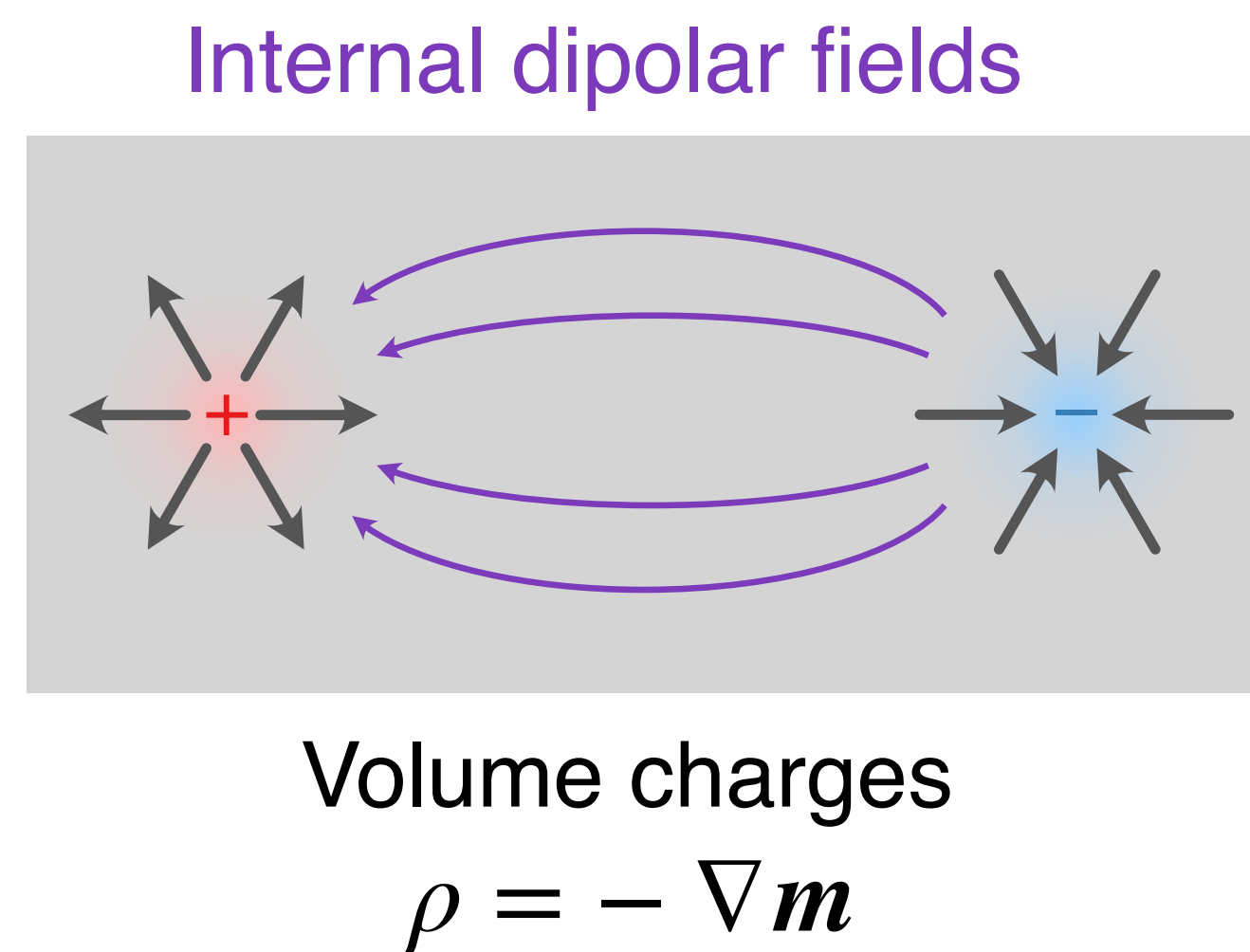
$\approx \mathbf{m}(\mathbf{r})$ smooth magnetization (micromagnetism)

Dipolar fields can be understood in the language of pseudocharges

$$\mathbf{B}^{\text{dip}} = M_s \nabla \phi$$

magnetostatic / dipolar potential

$$\phi(\mathbf{r}) = \frac{1}{4\pi} \int_V dV' \frac{\nabla' \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|} + \frac{1}{4\pi} \int_{\partial V} dA' \frac{\mathbf{n} \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|}$$



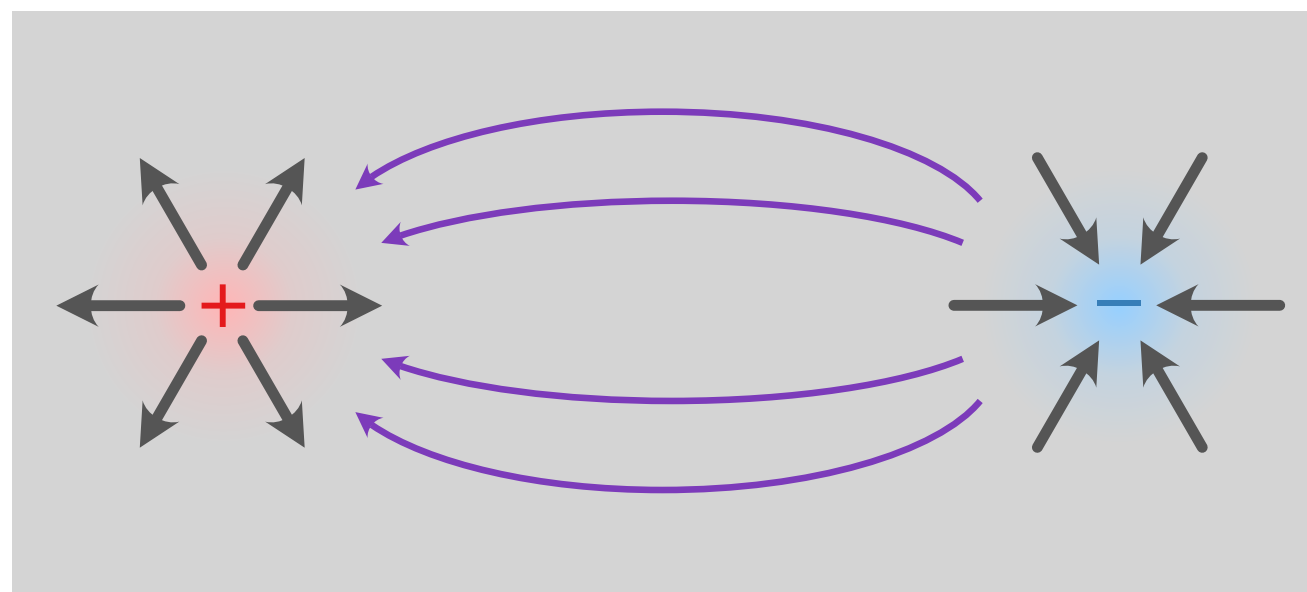
Dipolar fields can be understood in the language of pseudocharges

$$\mathbf{B}^{\text{dip}} = M_s \nabla \phi$$

magnetostatic / dipolar potential

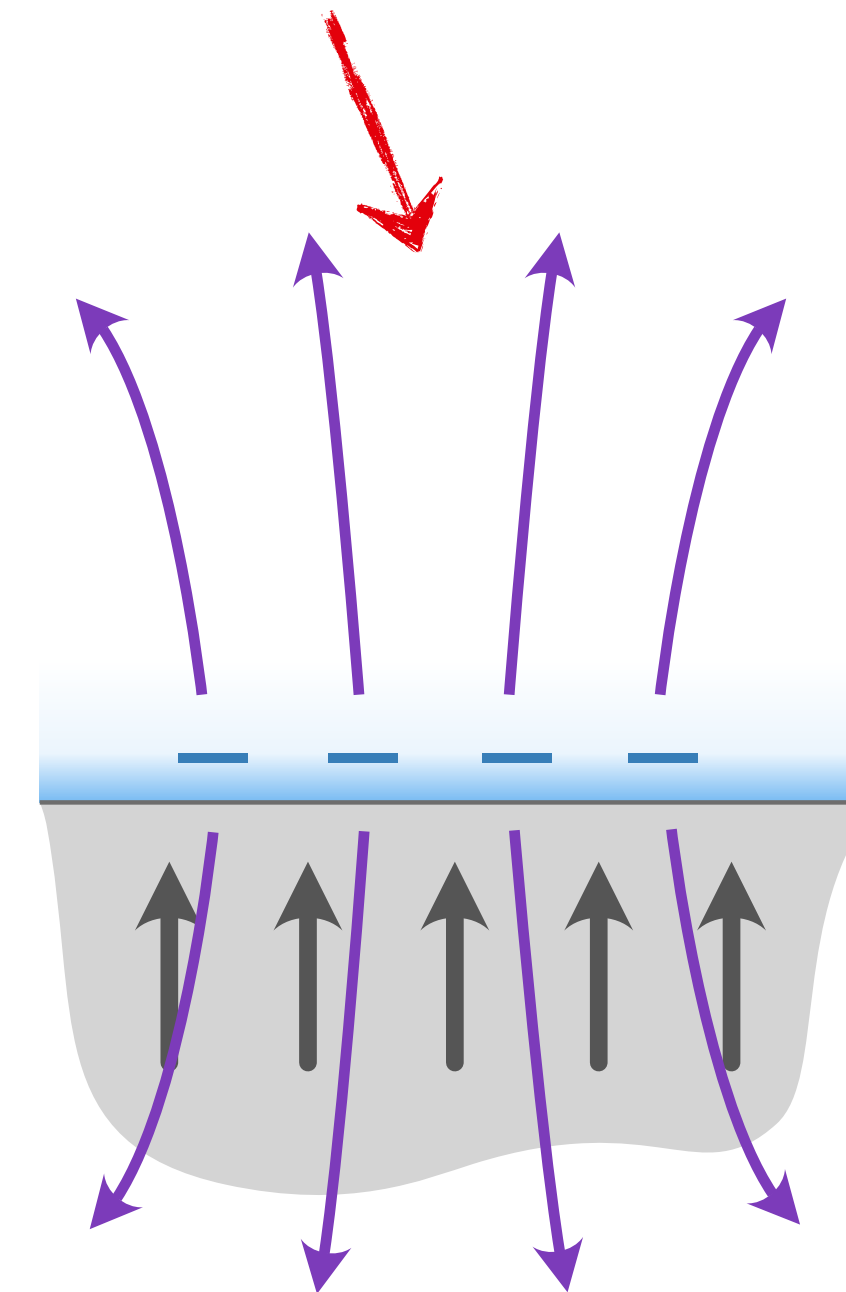
$$\phi(\mathbf{r}) = \frac{1}{4\pi} \int_V dV' \frac{\nabla' \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|} + \frac{1}{4\pi} \int_{\partial V} dA' \frac{\mathbf{n} \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|}$$

Internal dipolar fields



Volume charges
 $\rho = -\nabla \cdot \mathbf{m}$

Internal/external dipolar fields

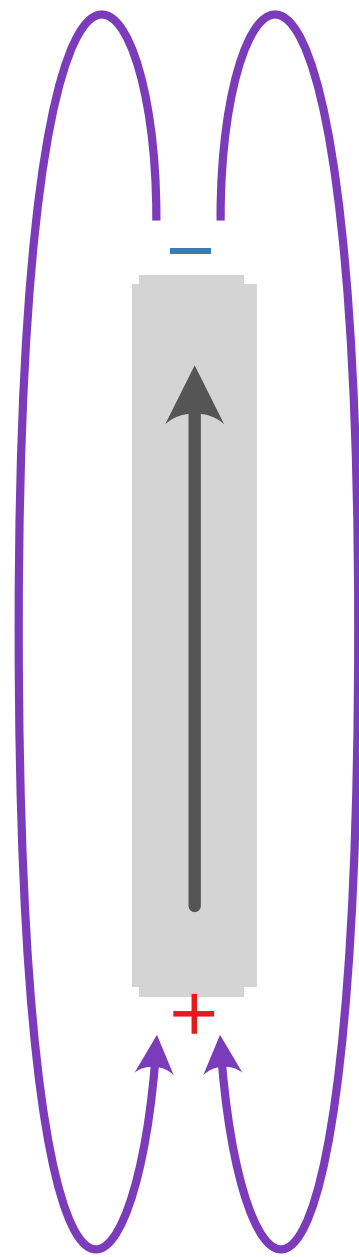


Surface charges
 $\sigma = \mathbf{n} \cdot \mathbf{m}$

Avoid costly strayfields / net charges.

Stray fields give rise to shape anisotropy

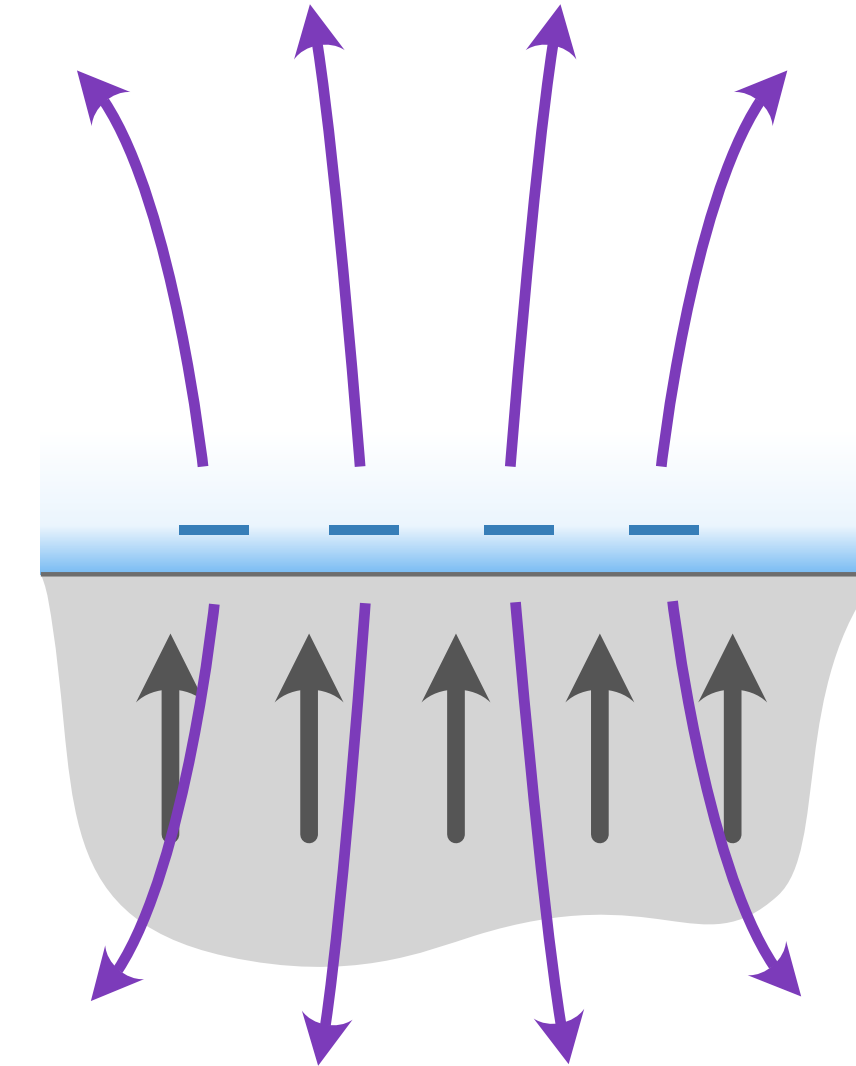
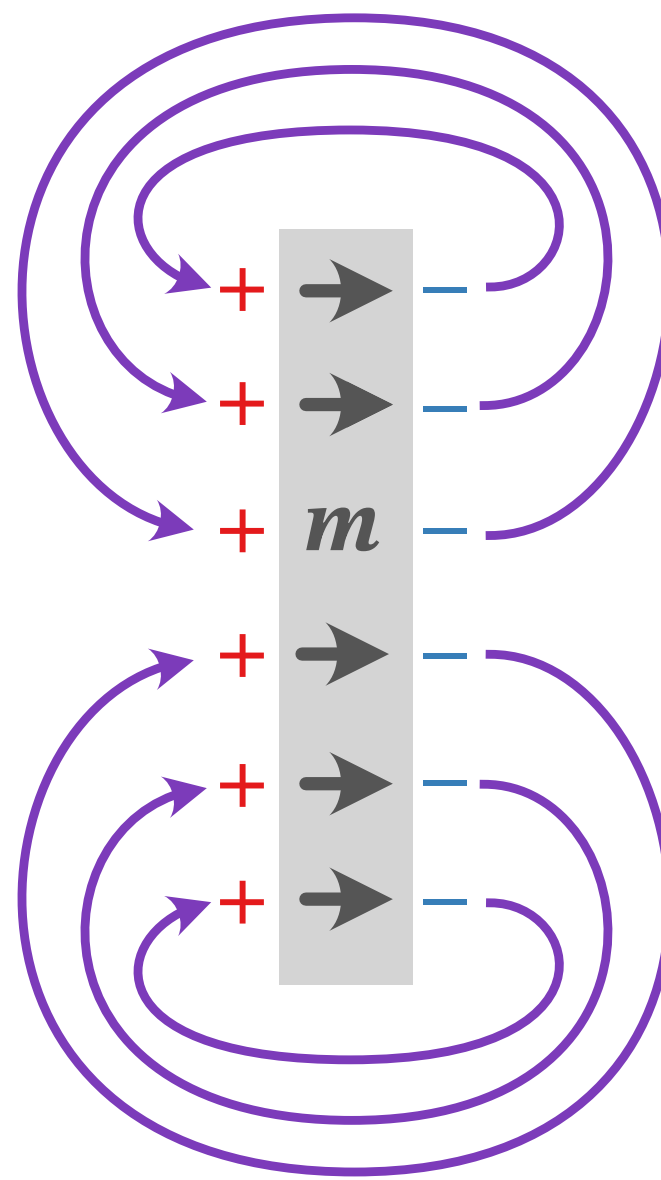
Less stray field



Shape anisotropy

stray field energy

$$E = \frac{1}{2\mu_0} \int dV B^2$$



Surface charges

$$\sigma = -\mathbf{n} \cdot \mathbf{m}$$

Stray fields give rise to shape anisotropy

Less stray field

Shape anisotropy

Demagnetizing tensor

$$\mathbf{B}^{\text{dip}} = -\mu_0 M_s \hat{\mathbf{N}} \cdot \mathbf{m} \quad \hat{\mathbf{N}} = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$$

Anisotropy energy

$$E(\mathbf{m}) = \int dV \left[\mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} \right] \quad \text{with} \quad \hat{\mathbf{K}} = \mu_0 M_s^2 \hat{\mathbf{N}}$$

$$\hat{\mathbf{N}} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

Sphere

isotrop

$$\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Infinite cylinder

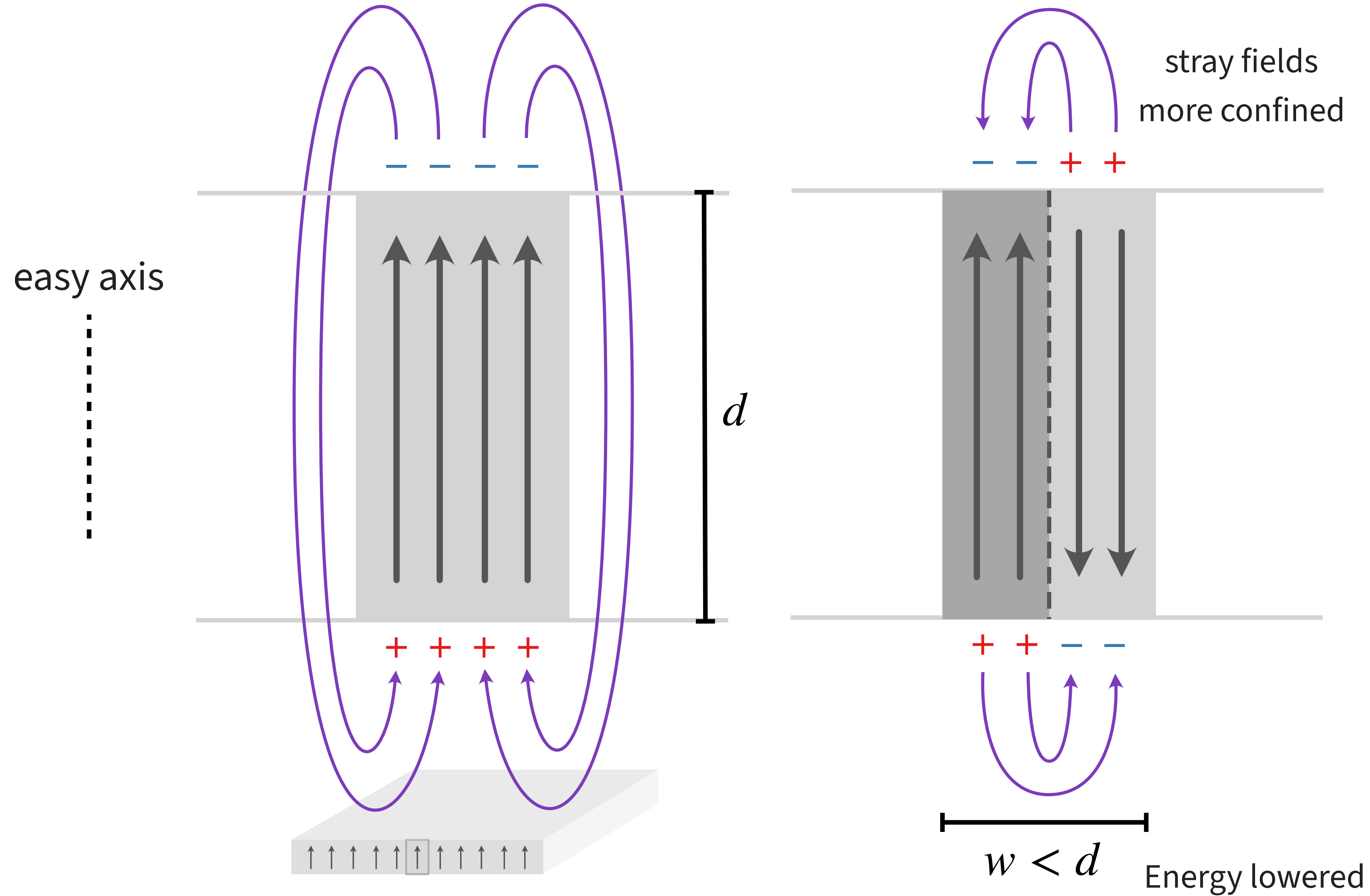
easy axis

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Infinite layer

easy plane

Domains are formed in larger magnets to avoid dipolar stray fields



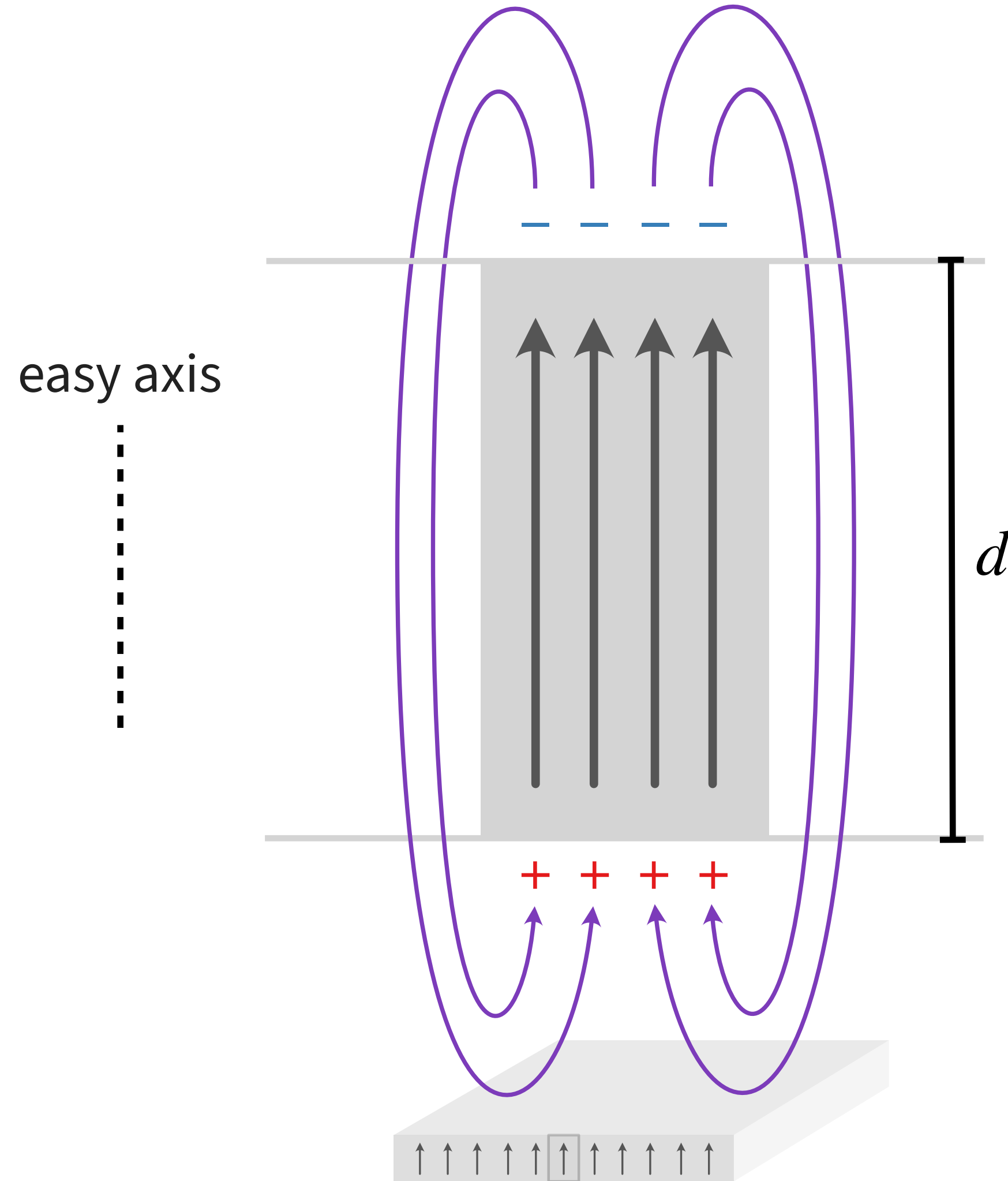
Stray-field energy
per unit area

$$\sigma_{\text{dip}} = \frac{1}{2} \mu_0 M_s^2 d$$

$$\sigma_{\text{dip}} = 0.136 \mu_0 M_s^2 w$$

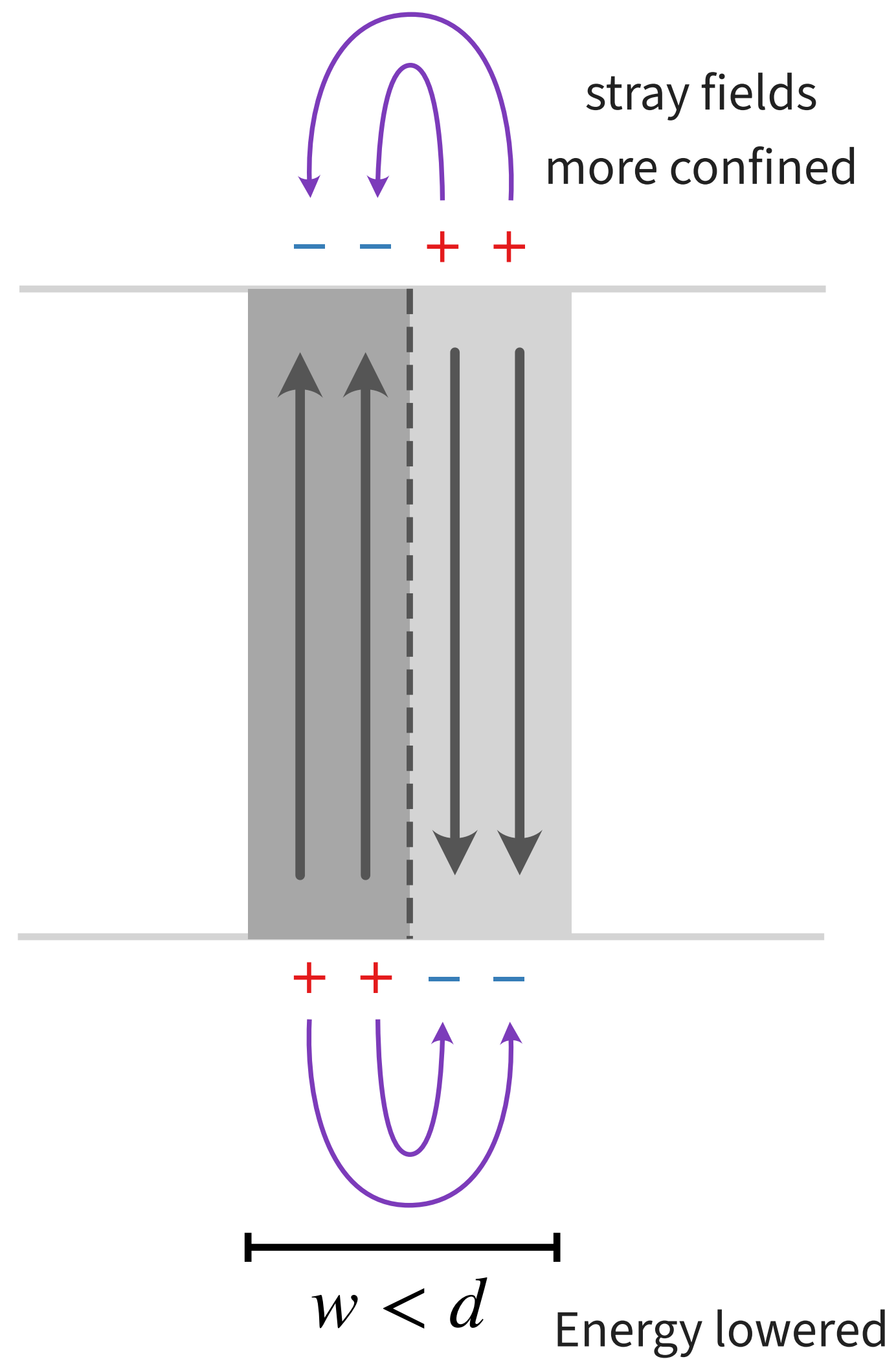
Kittel, Phys. Rev. 70, 11-12, 965 (1946)

Domains are formed in larger magnets to avoid dipolar stray fields

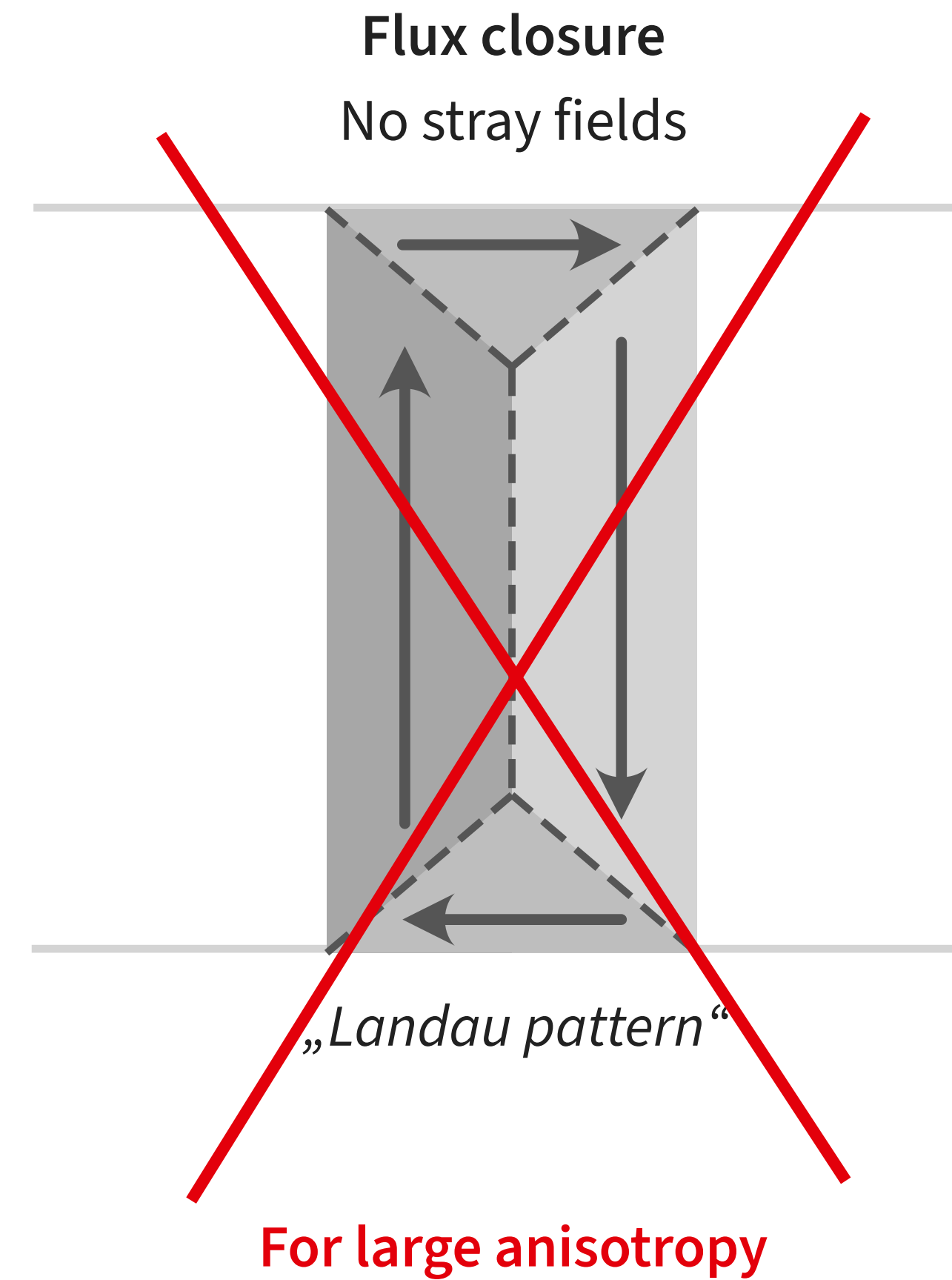


Stray-field energy
per unit area

$$\sigma_{\text{dip}} = \frac{1}{2} \mu_0 M_s^2 d$$

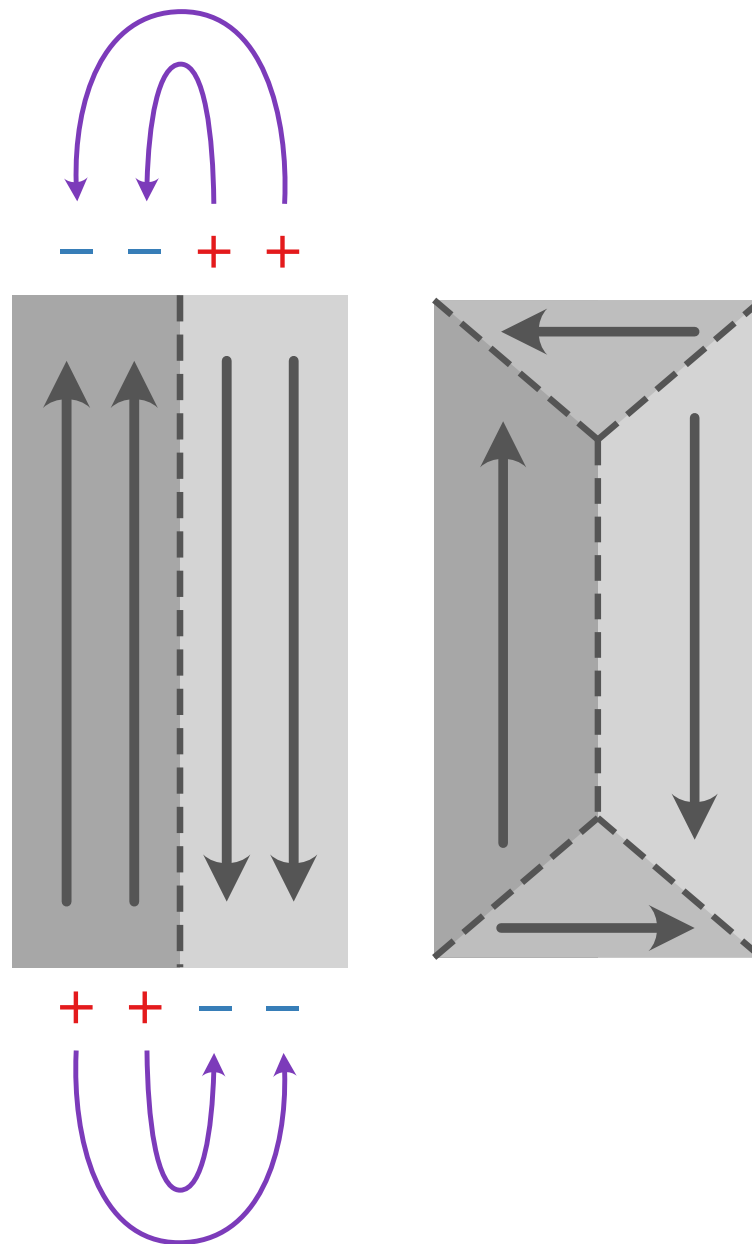


$$\sigma_{\text{dip}} = 0.136 \mu_0 M_s^2 w$$

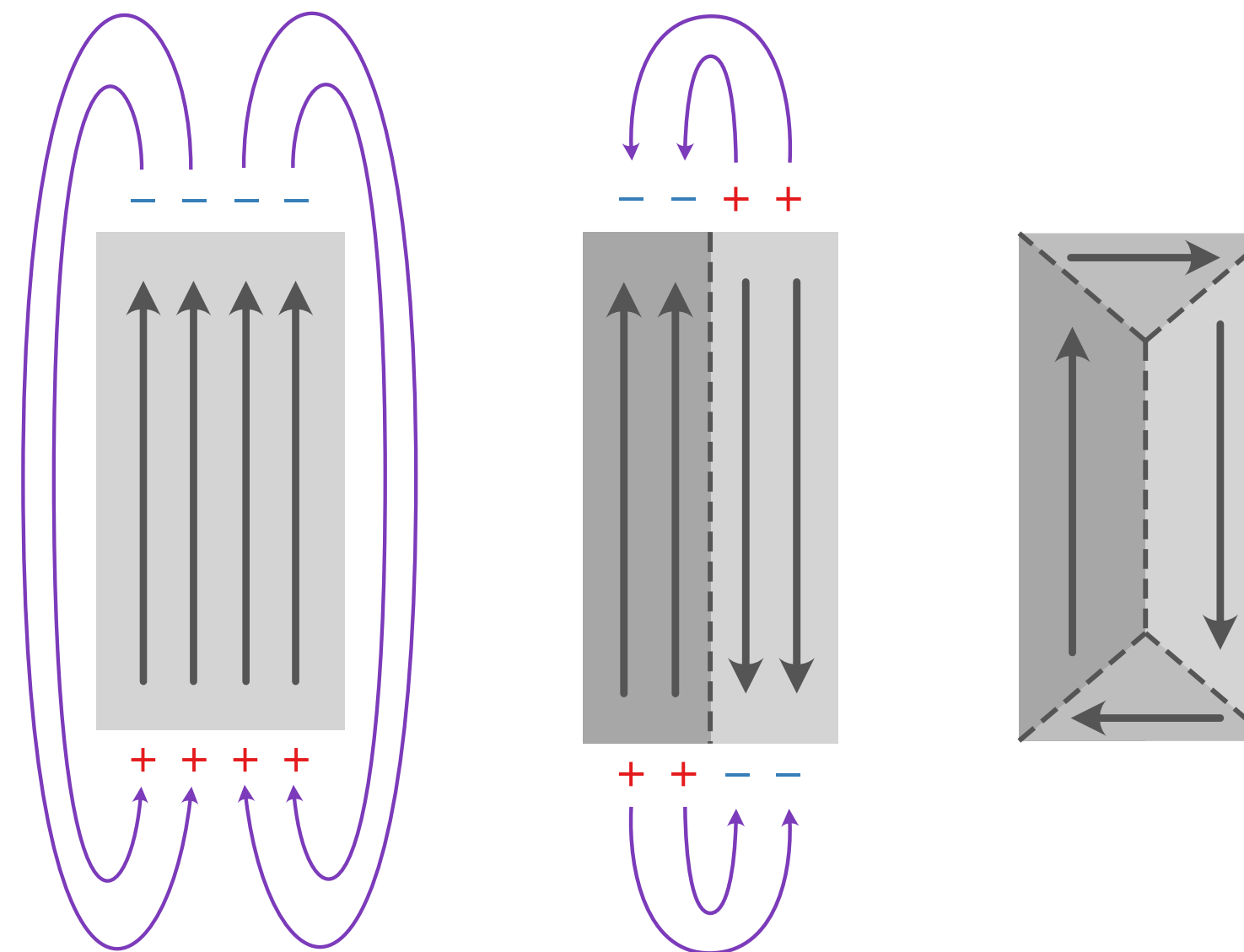


$$\sigma_{\text{dip}} = 0$$

When and why do domains form?



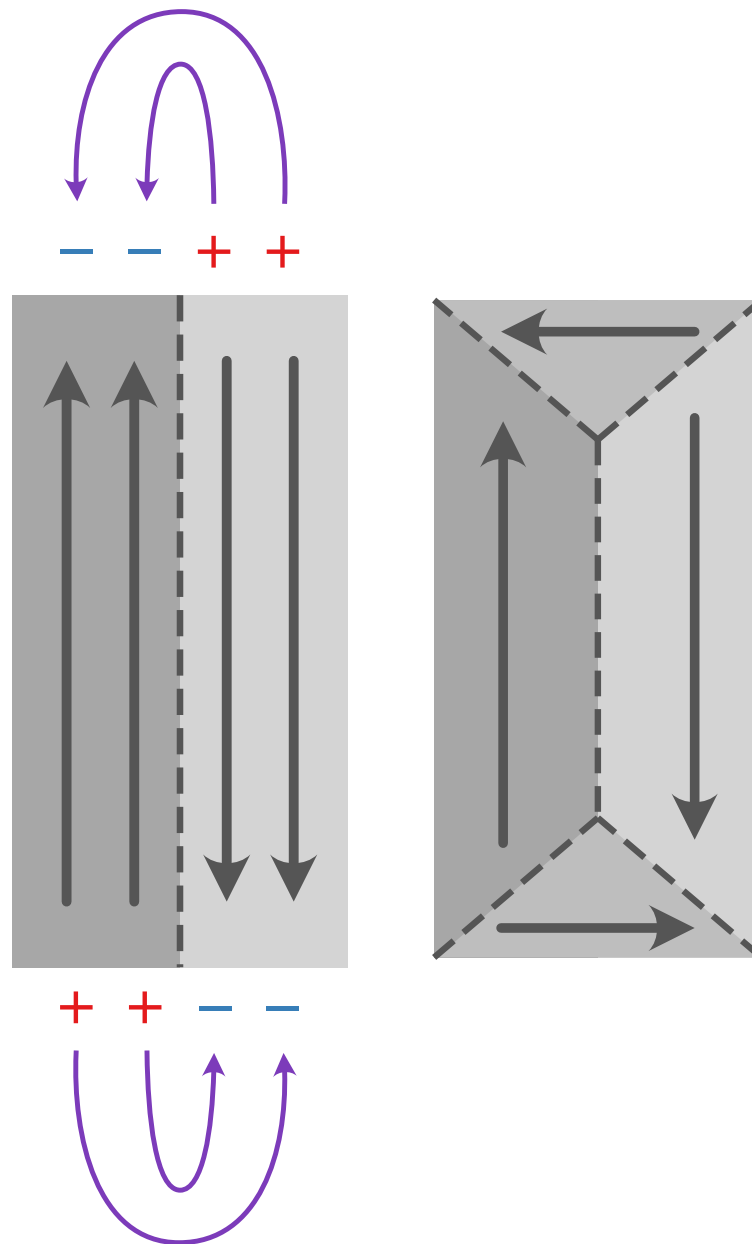
Favored in large enough magnets to avoid stray fields.



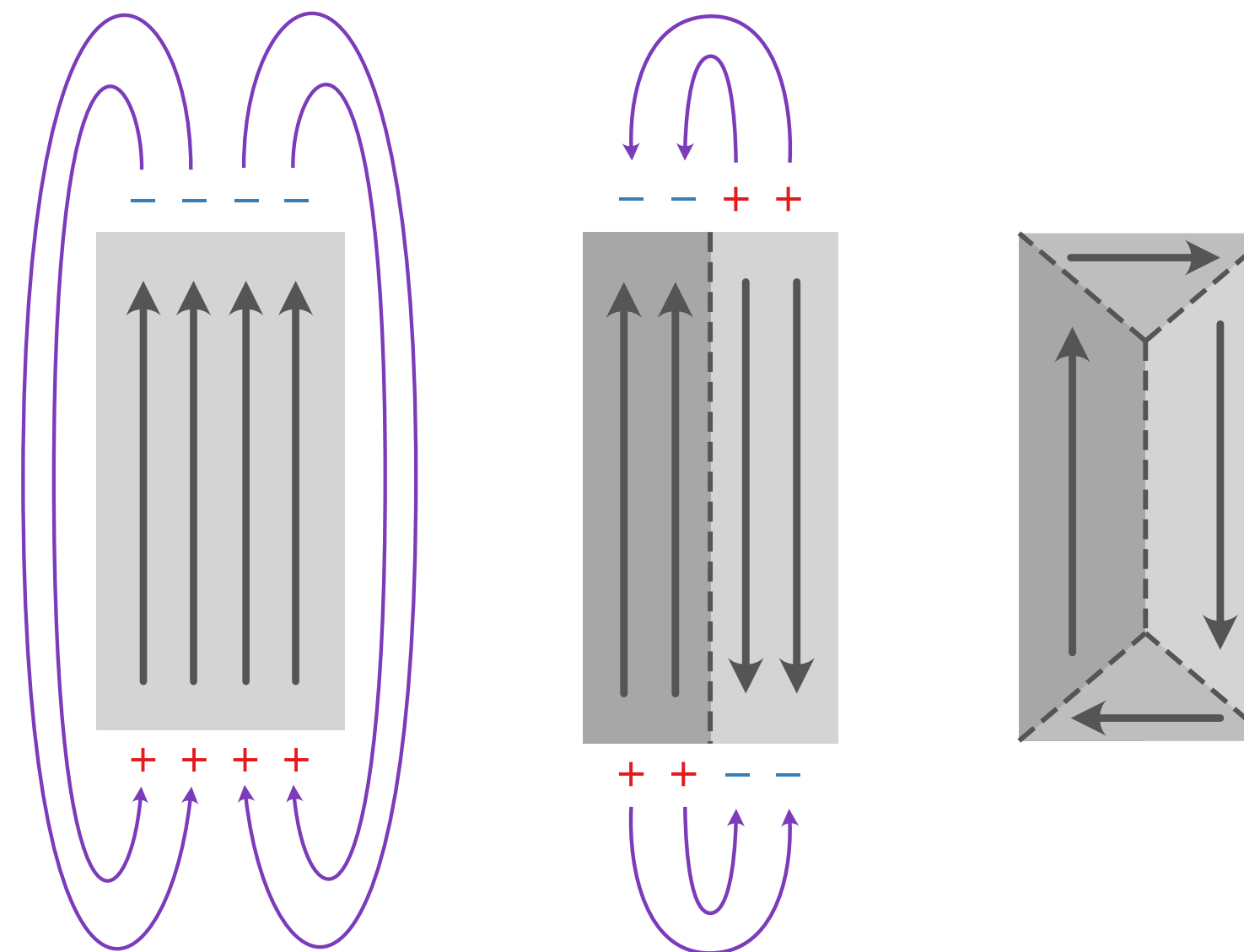
Need to be allowed by symmetry!

$$E(\mathbf{m}_1) = E(\mathbf{m}_2)$$

When and why do domains form?



Favored in large enough magnets to avoid stray fields.

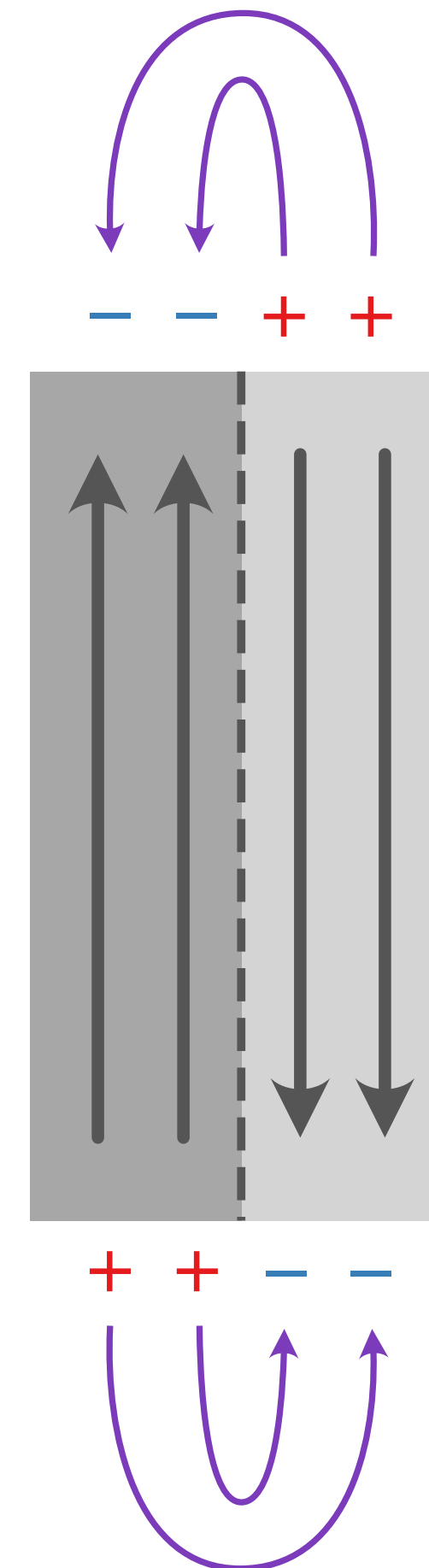
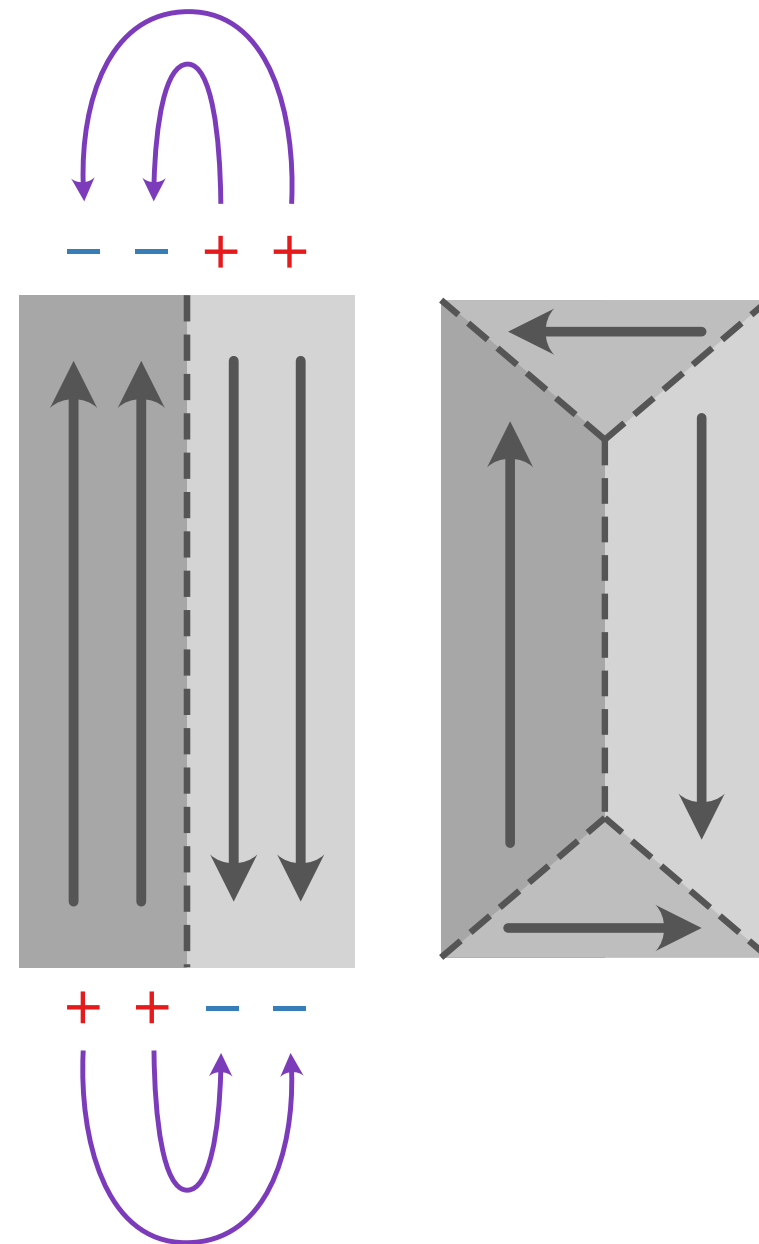


Need to be allowed by symmetry!

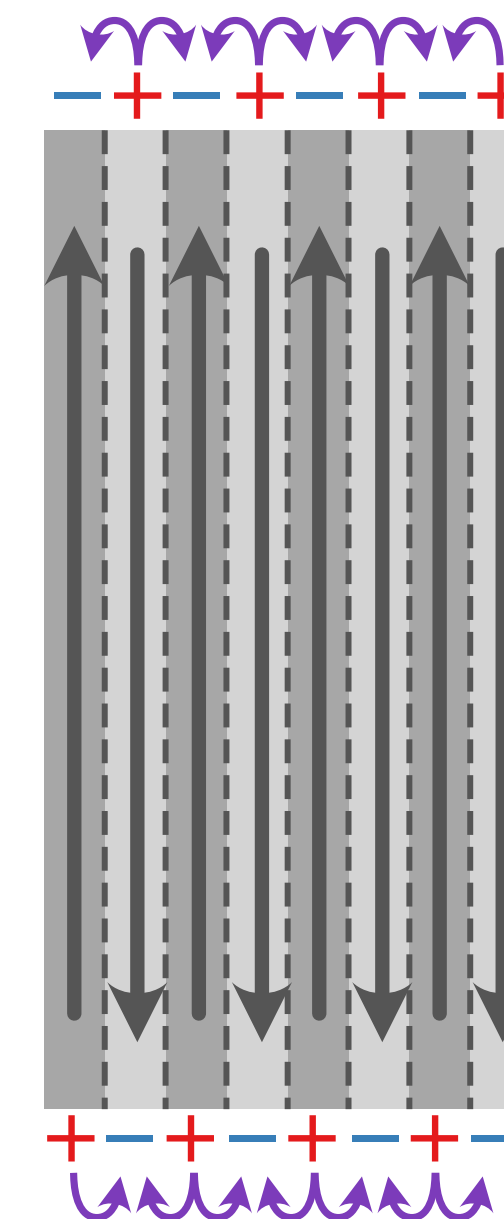
$$E(\mathbf{m}_1) = E(\mathbf{m}_2)$$

But what is large enough?

When and why do domains form?

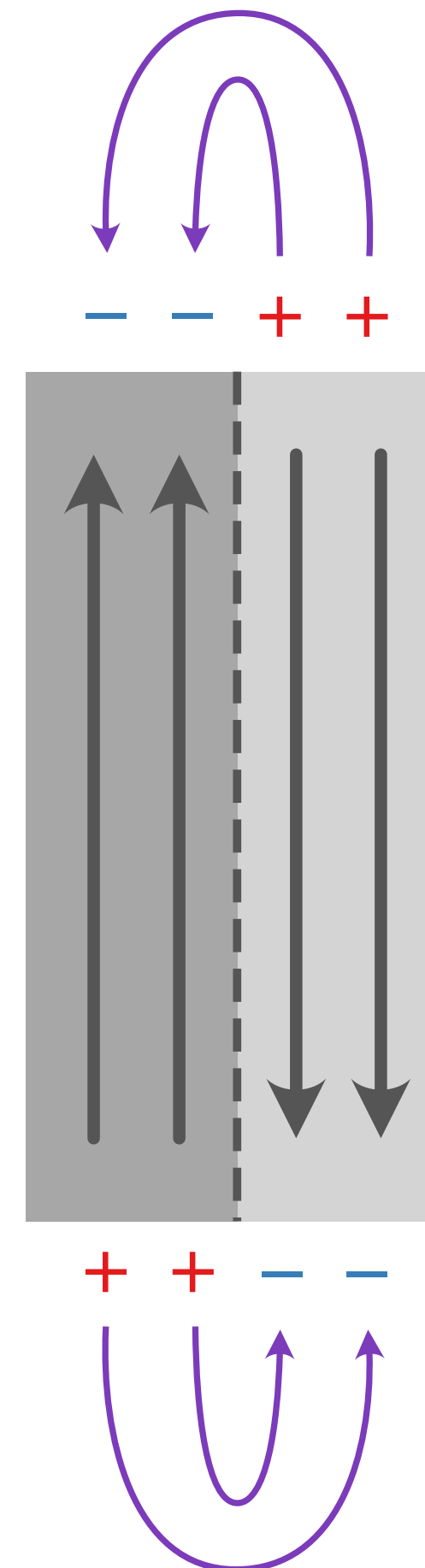
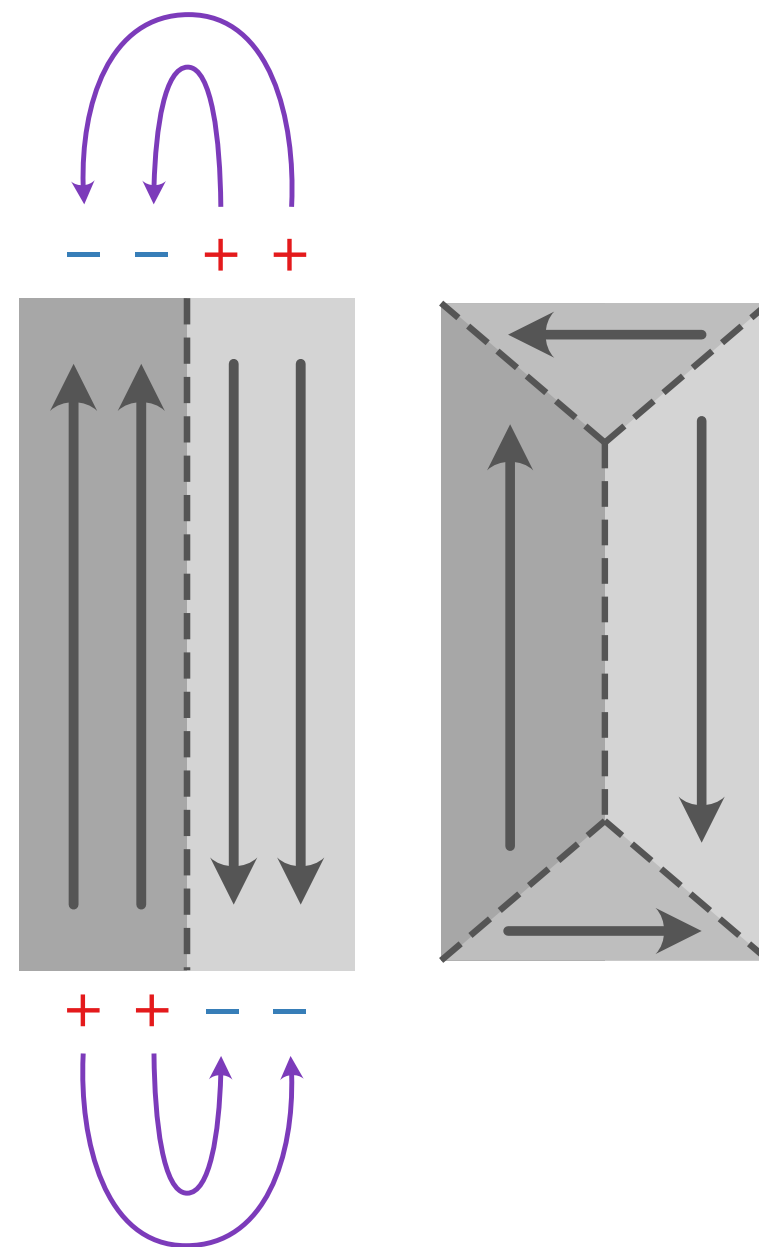


**Just make smaller
and smaller domains?**

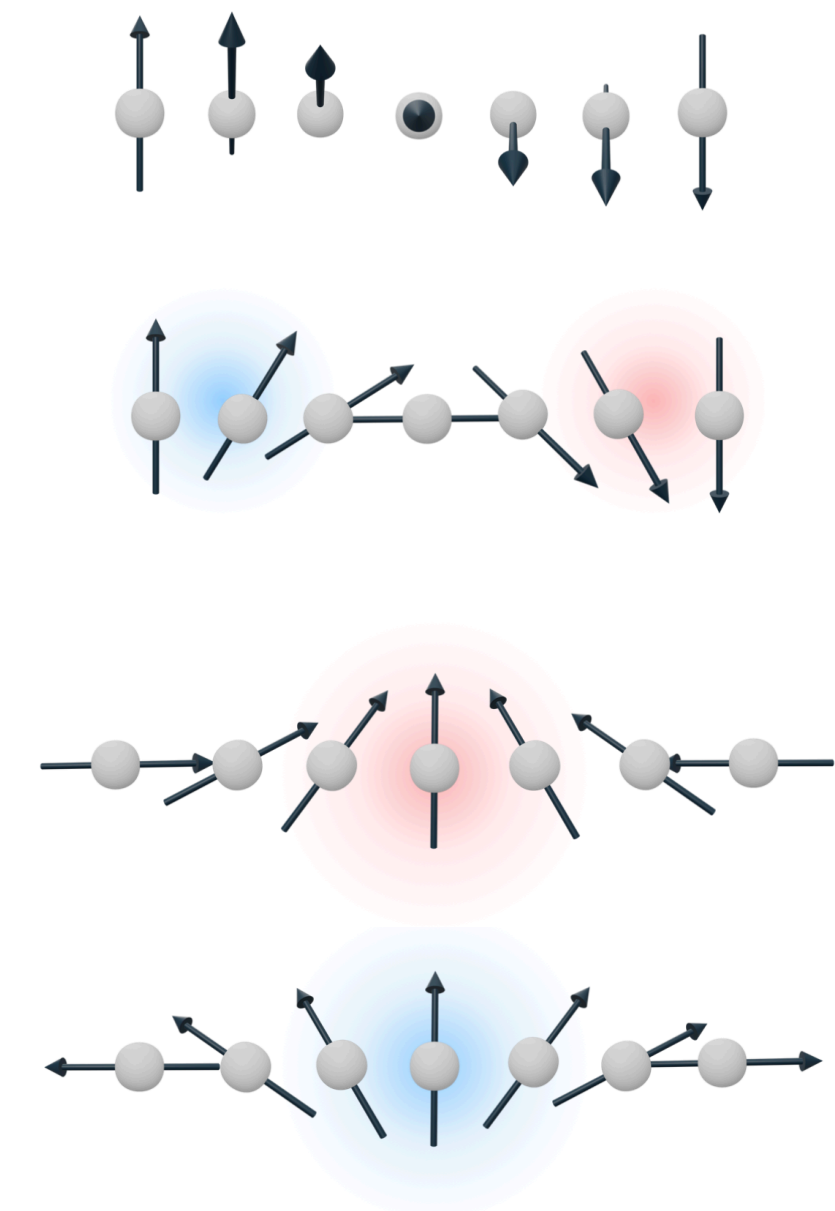


**Domain walls do
not come for free!**

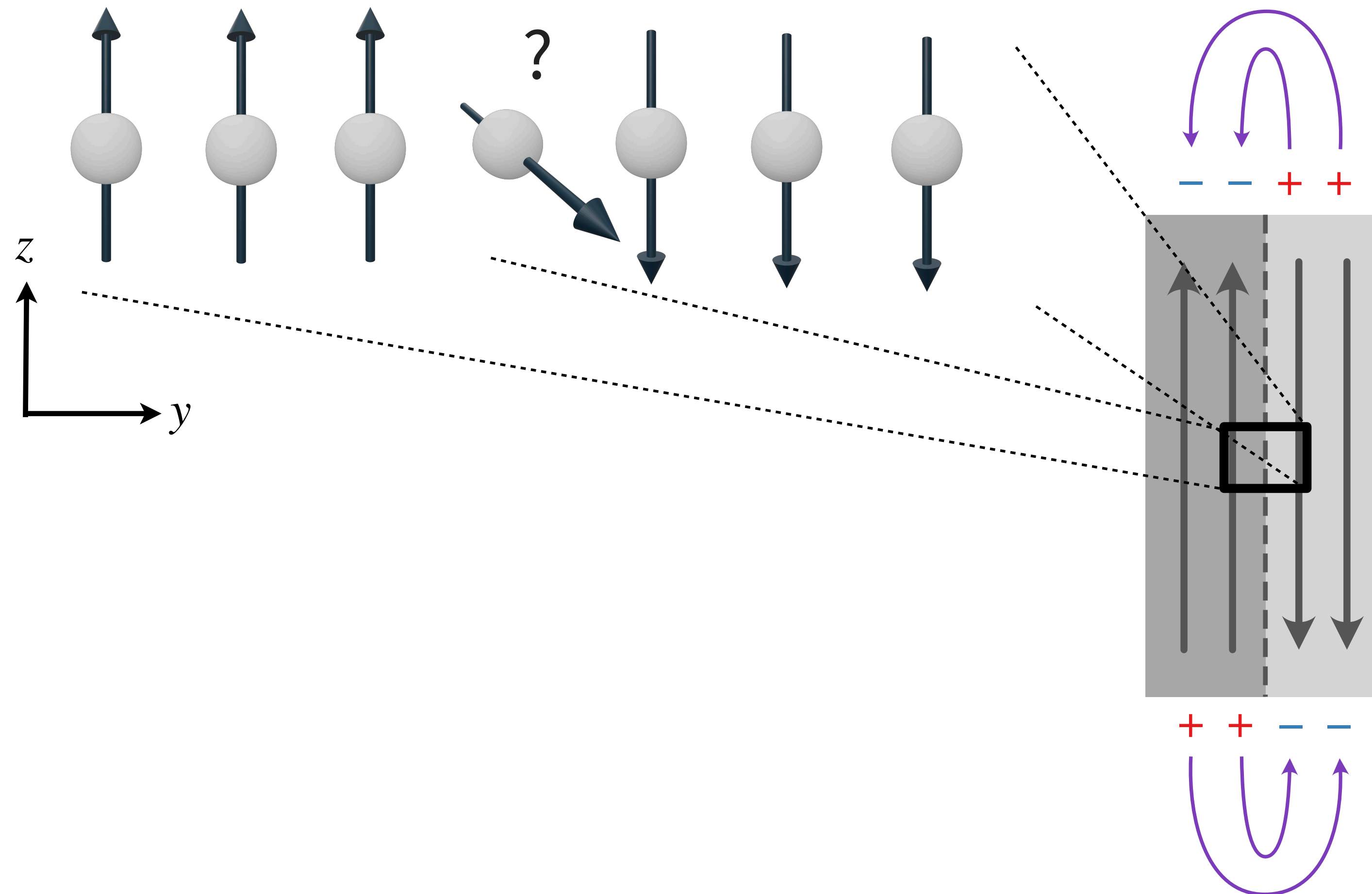
When and why do domains form?



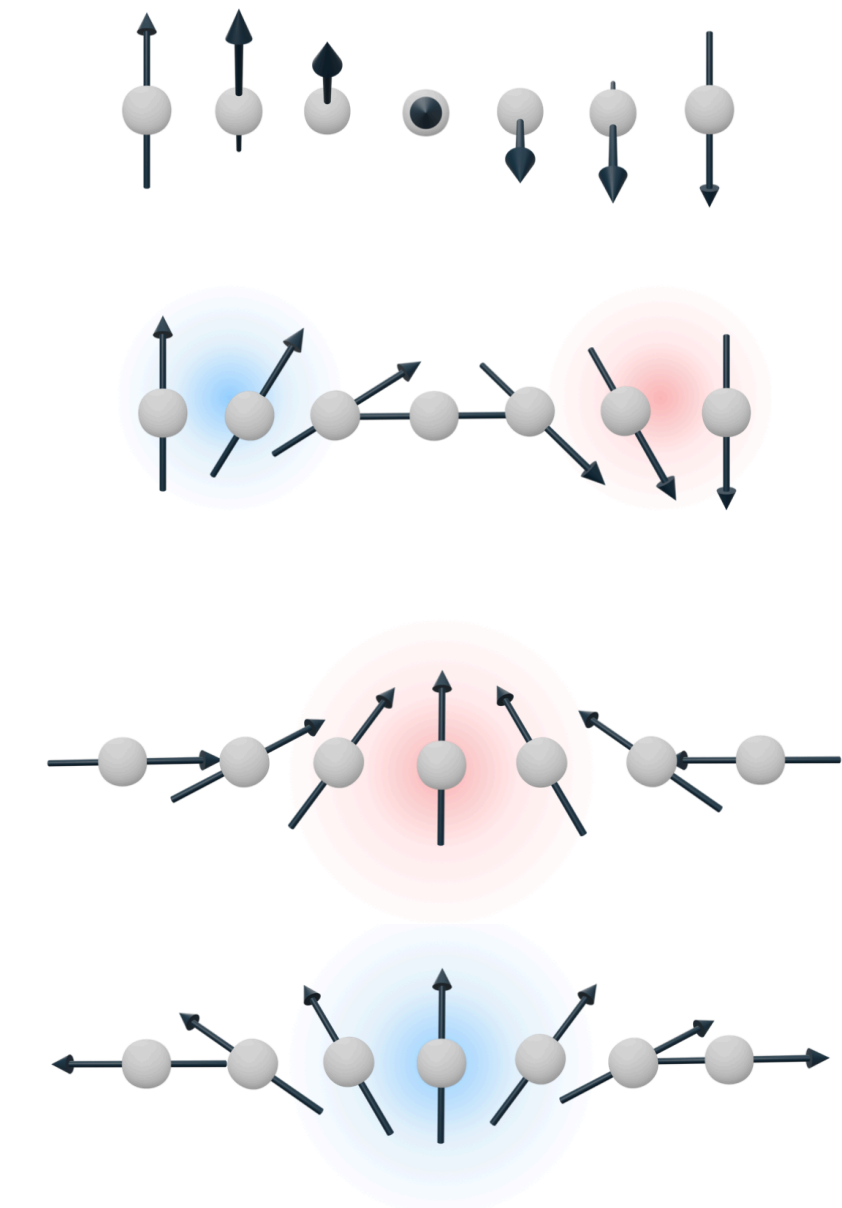
Structure and types of domain walls



What is the nature of the domain walls?

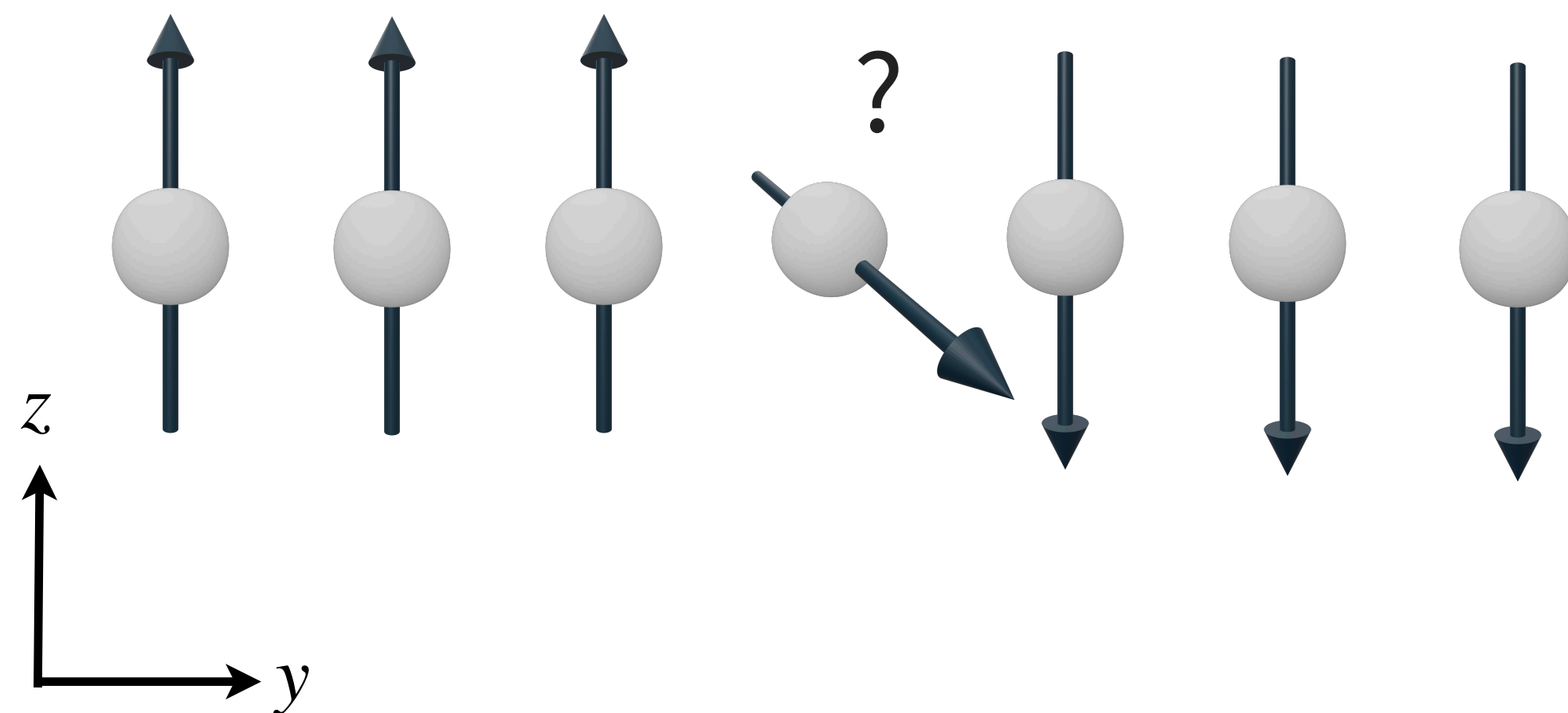


Structure and types of domain walls



Structure and types of domain walls

What is the structure of the domain walls?



Anisotropy

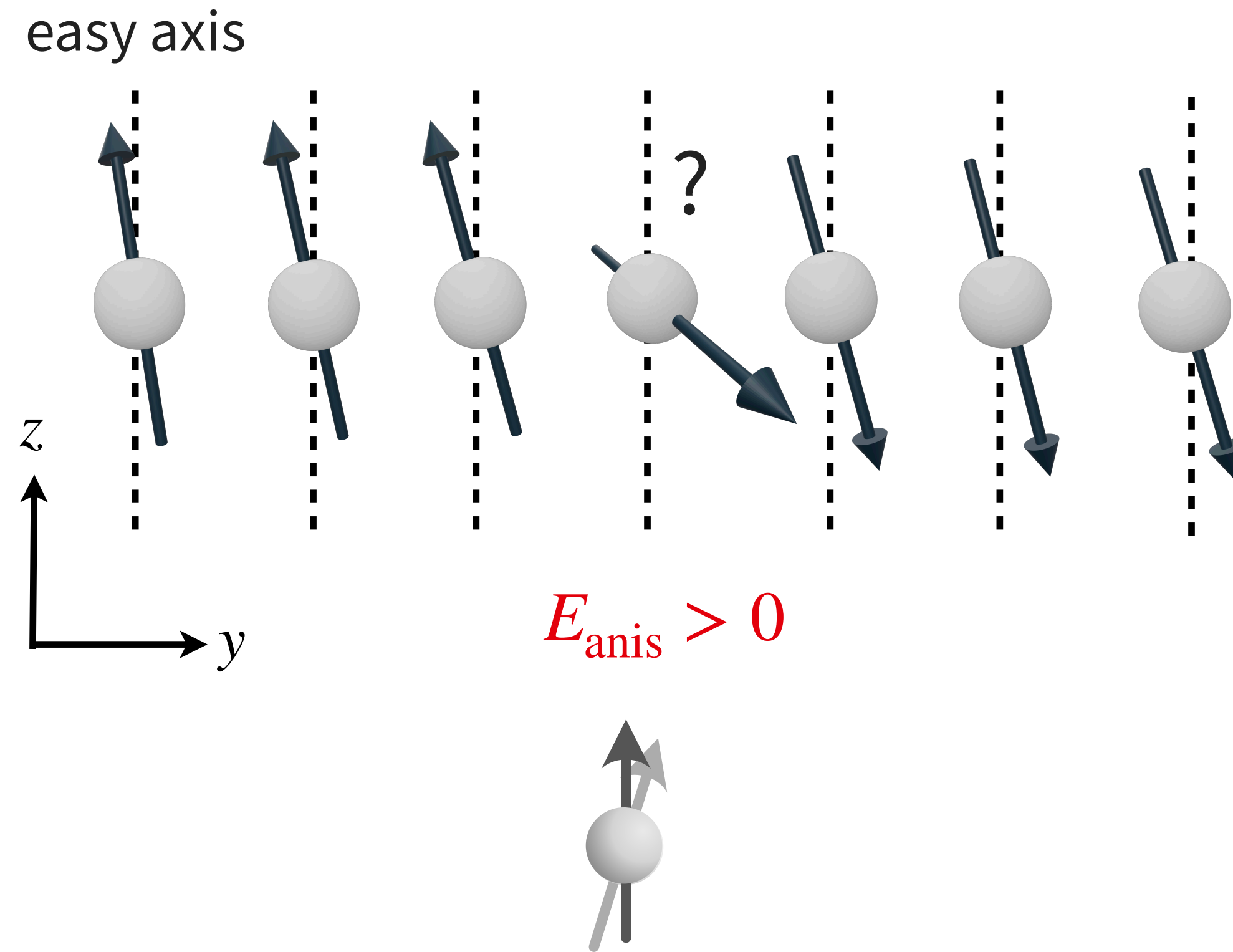
Exchange

Dipole-Dipole

DMI

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A(\nabla \mathbf{m})^2 - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?



Anisotropy

Sets domain alignment

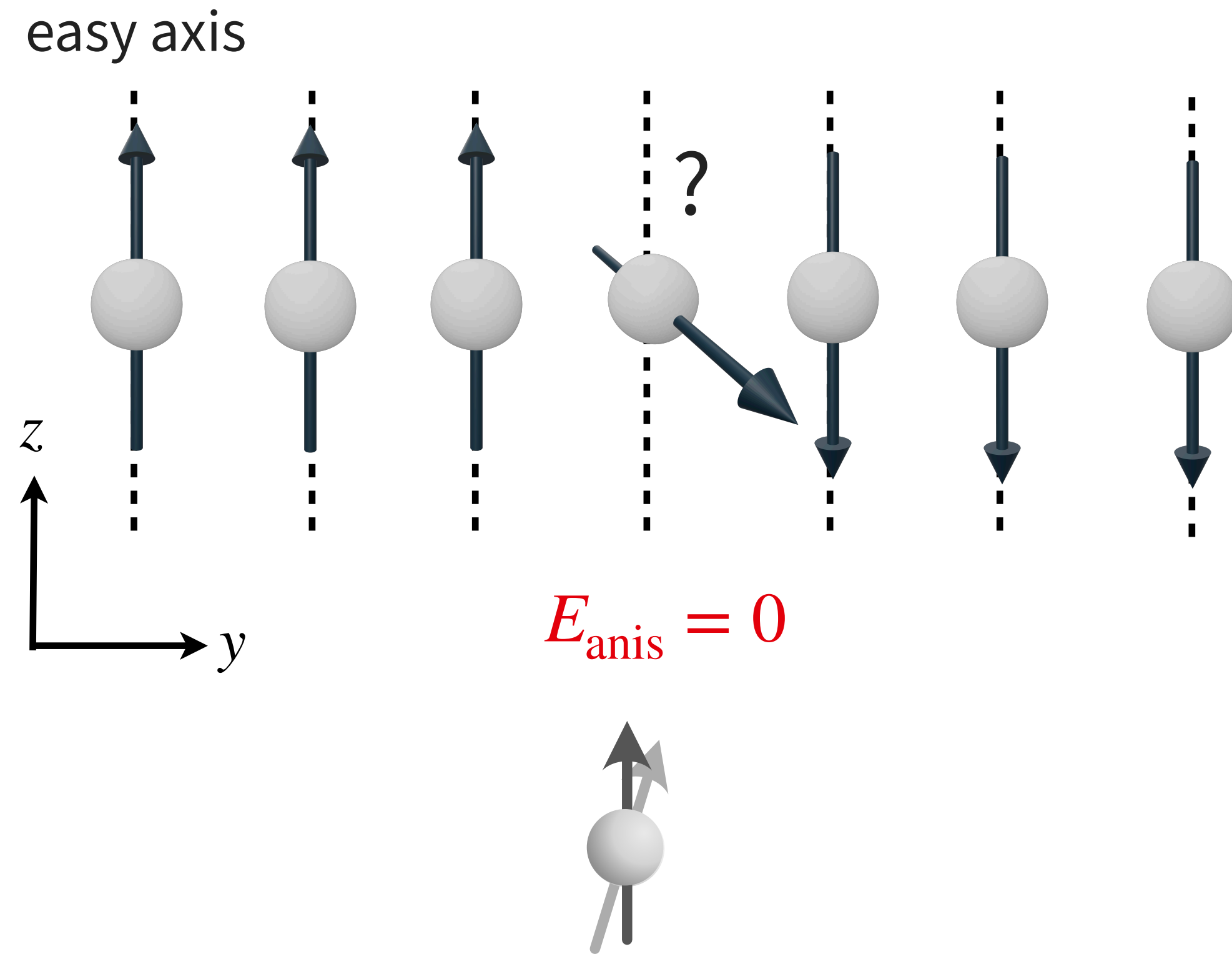
Exchange

Dipole-Dipole

DMI

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A(\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?



Anisotropy

Sets domain alignment

Exchange

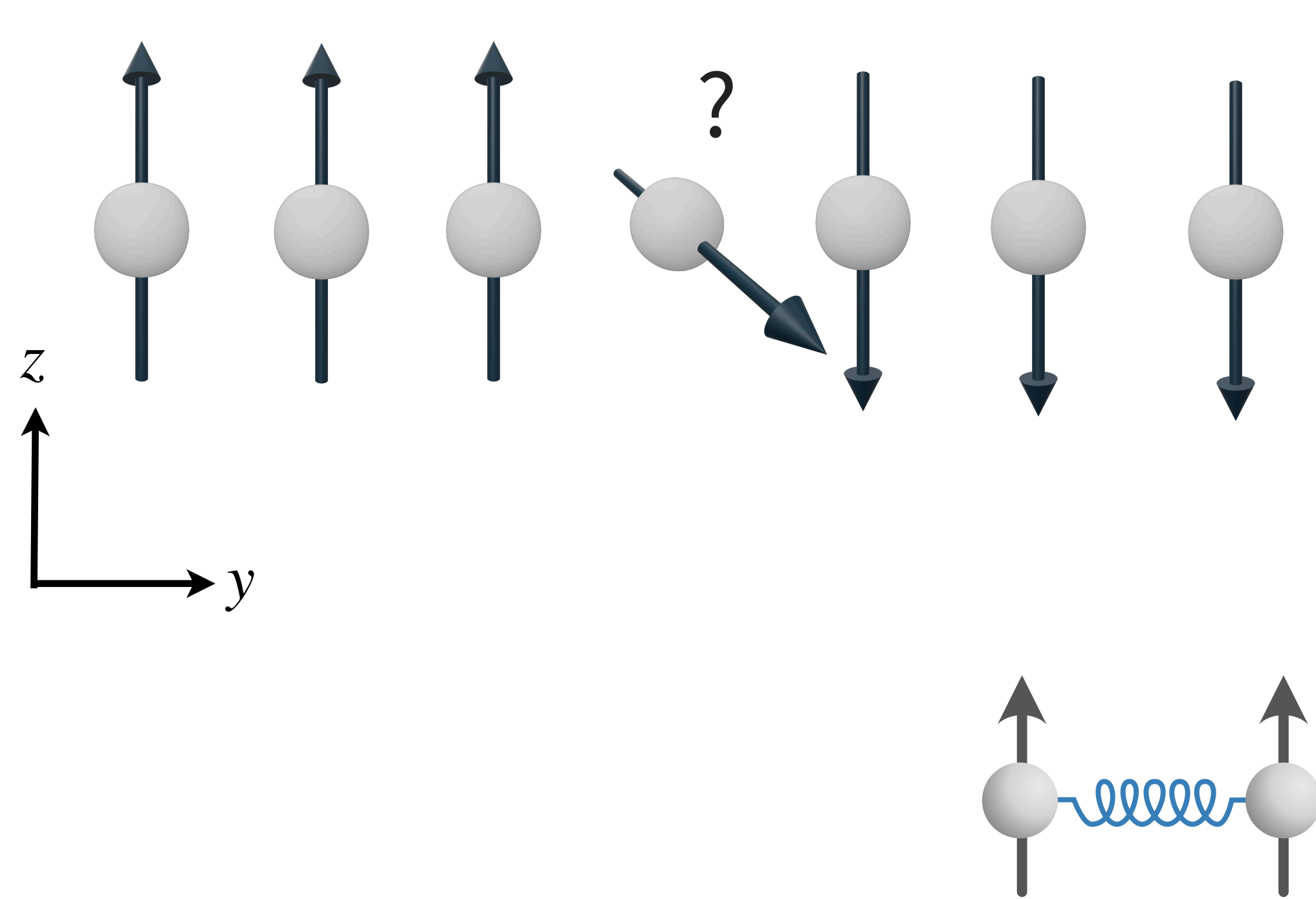
Dipole-Dipole

DMI

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A(\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis



Anisotropy

Exchange

Dipole-Dipole

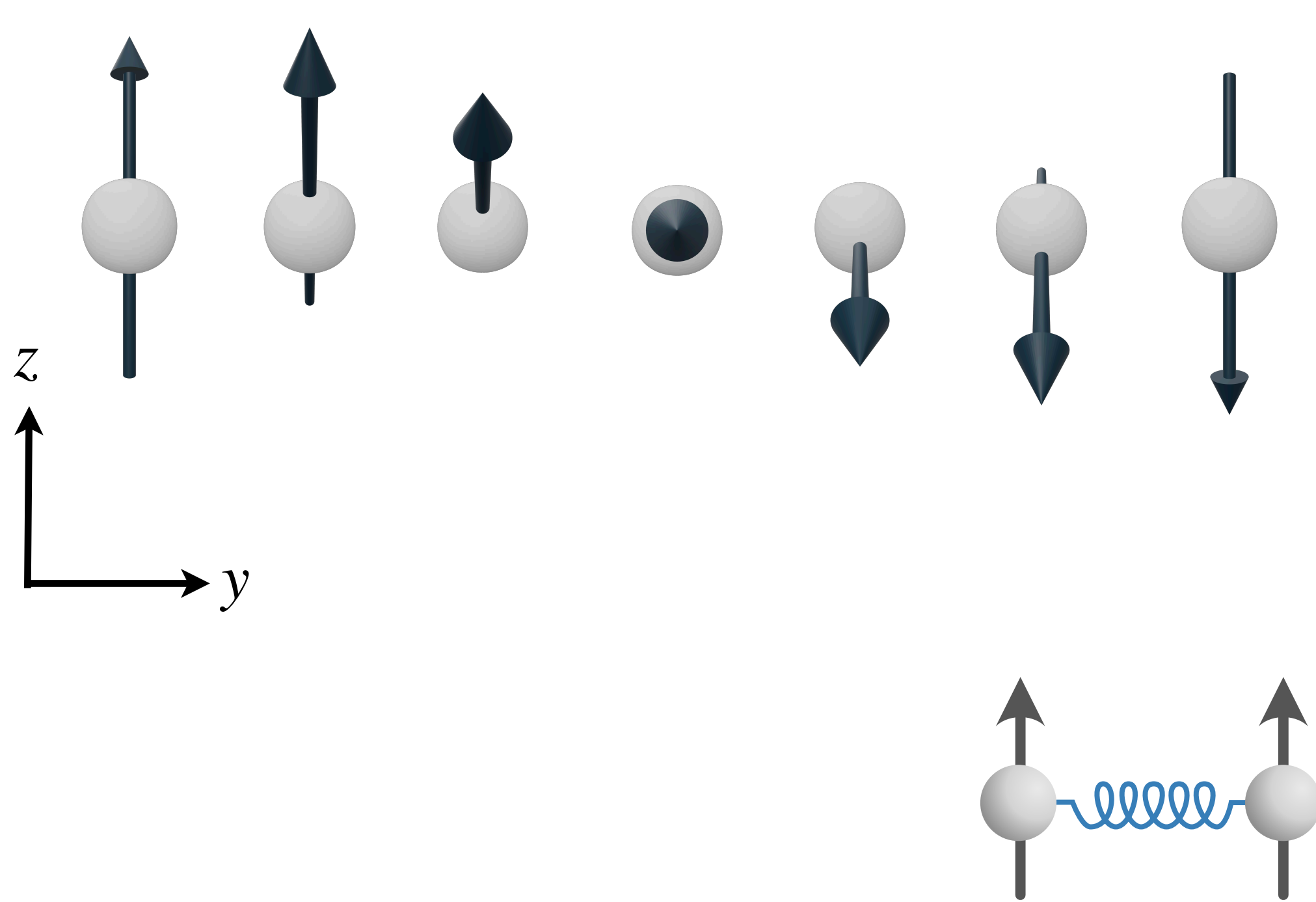
DMI

Prefers smooth transitions

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis



Anisotropy

Exchange

Dipole-Dipole

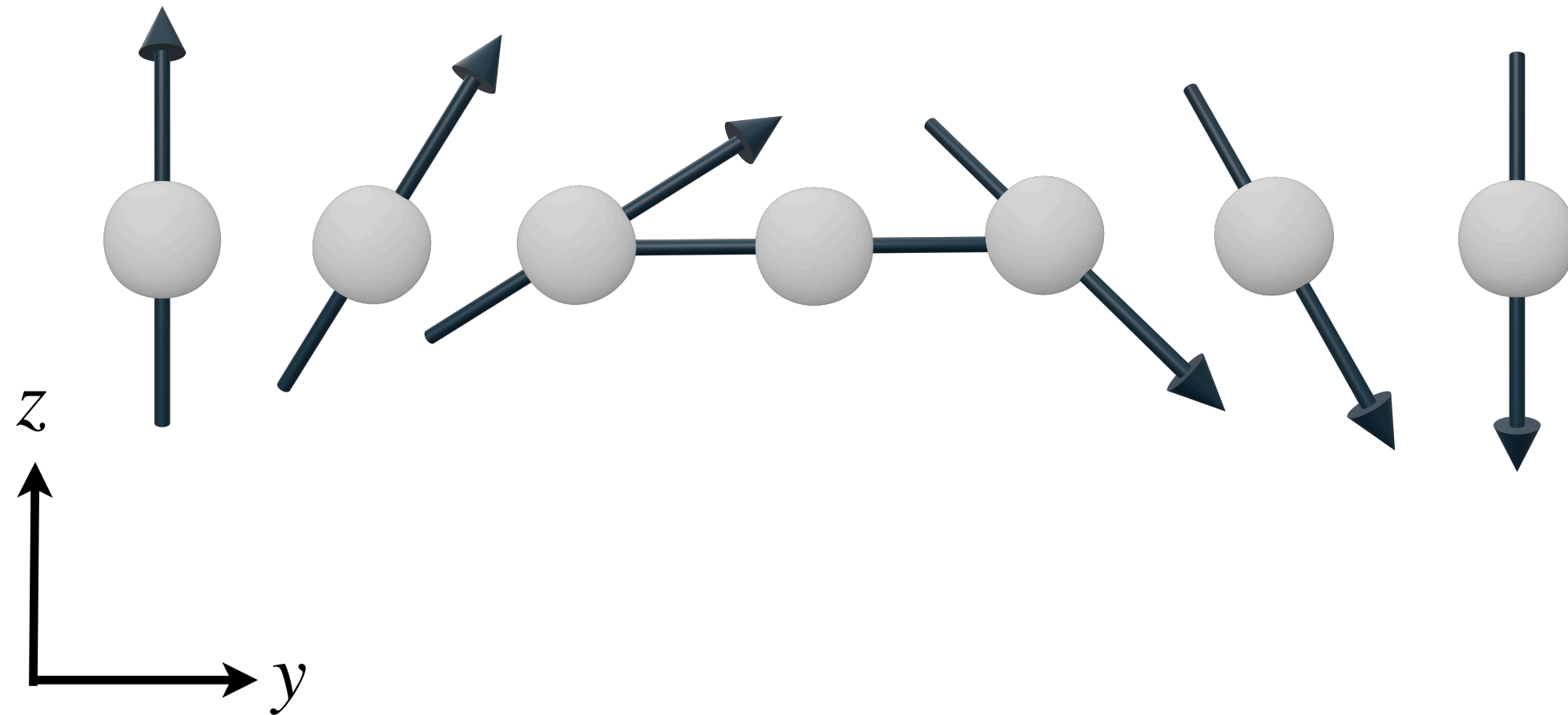
DMI

Prefers smooth transitions

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis

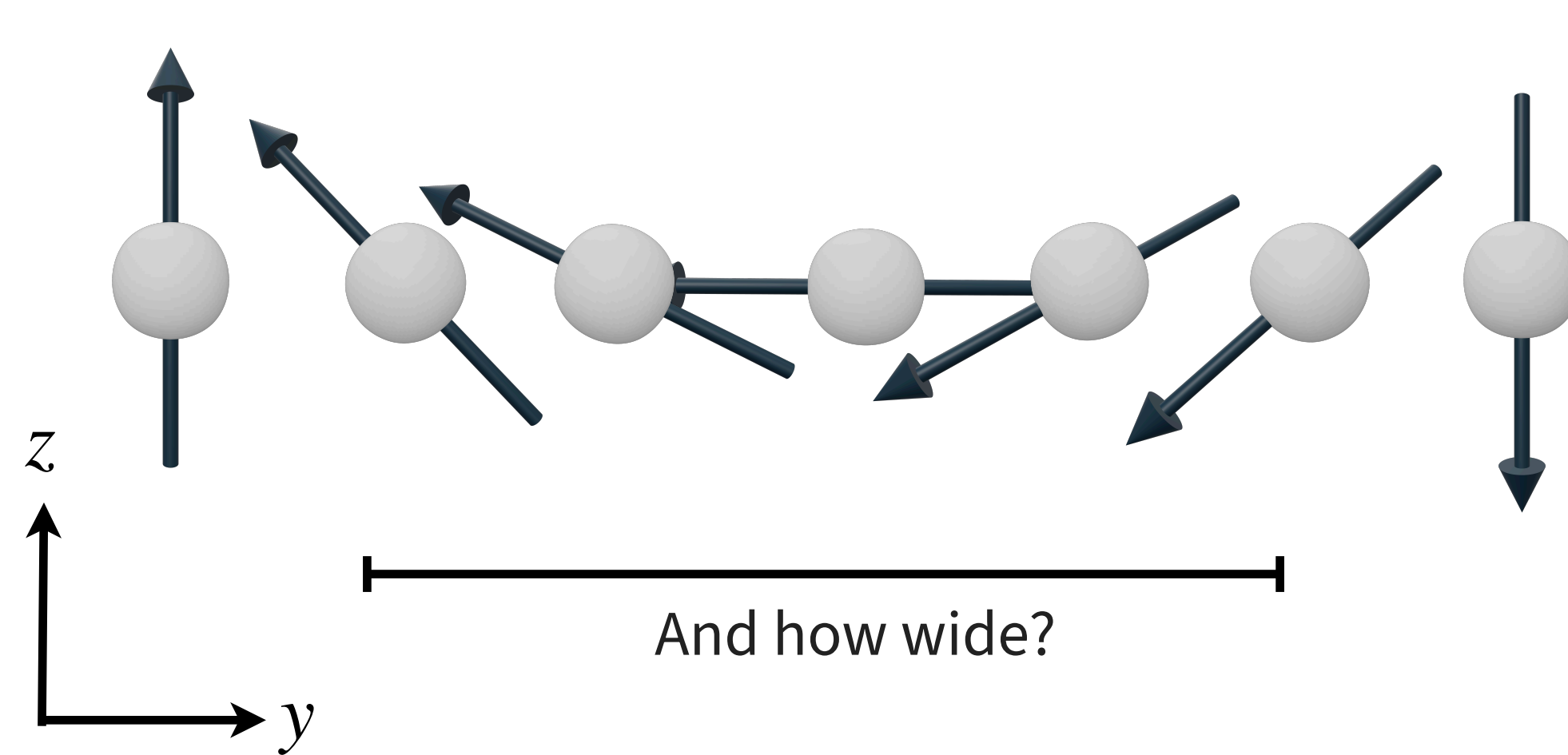


But why not like this?

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis

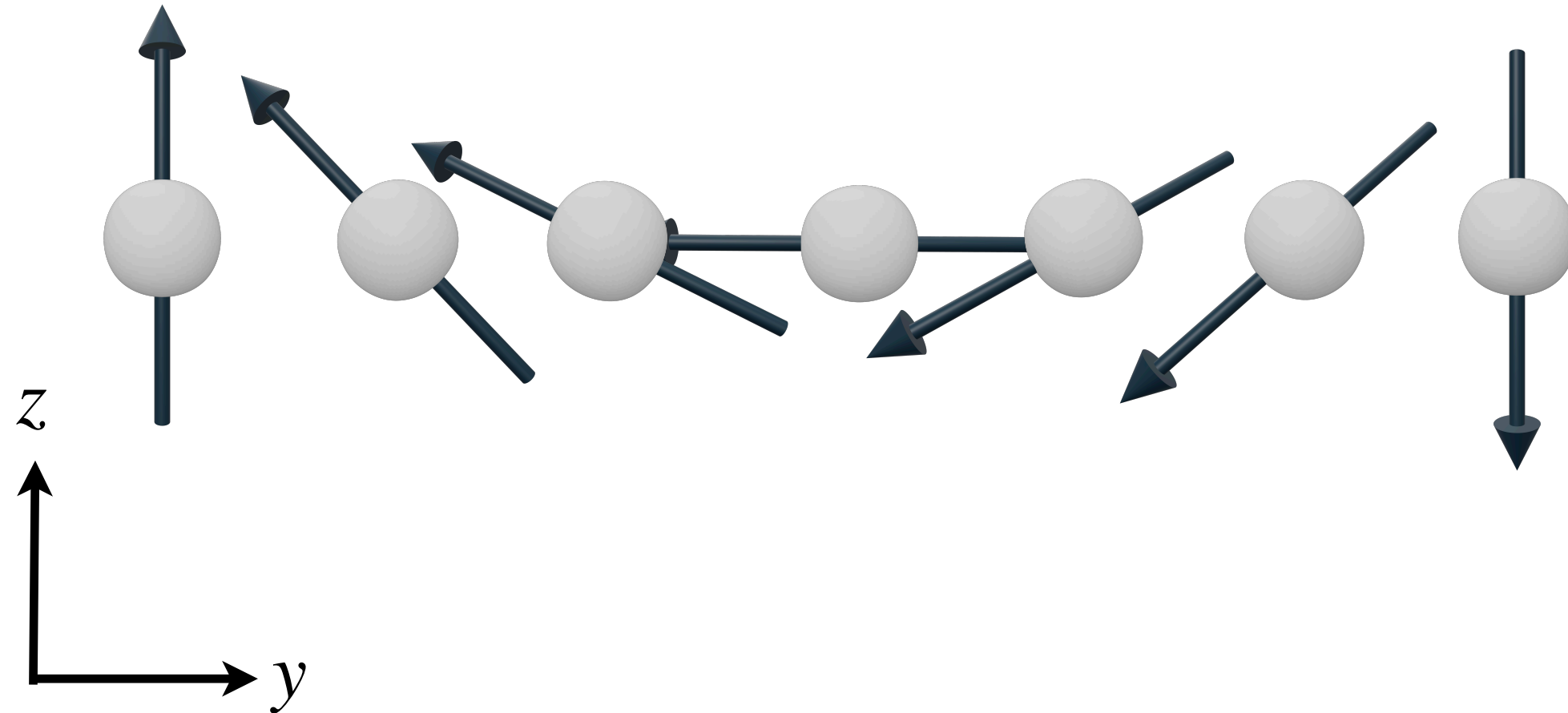


Or this?

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A(\nabla \mathbf{m})^2 - M_s B^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis



Néel-type domain wall
 $\mathbf{m}$ rotates perpendicular to wall.

Competition of all interactions possible!

Anisotropy

Exchange

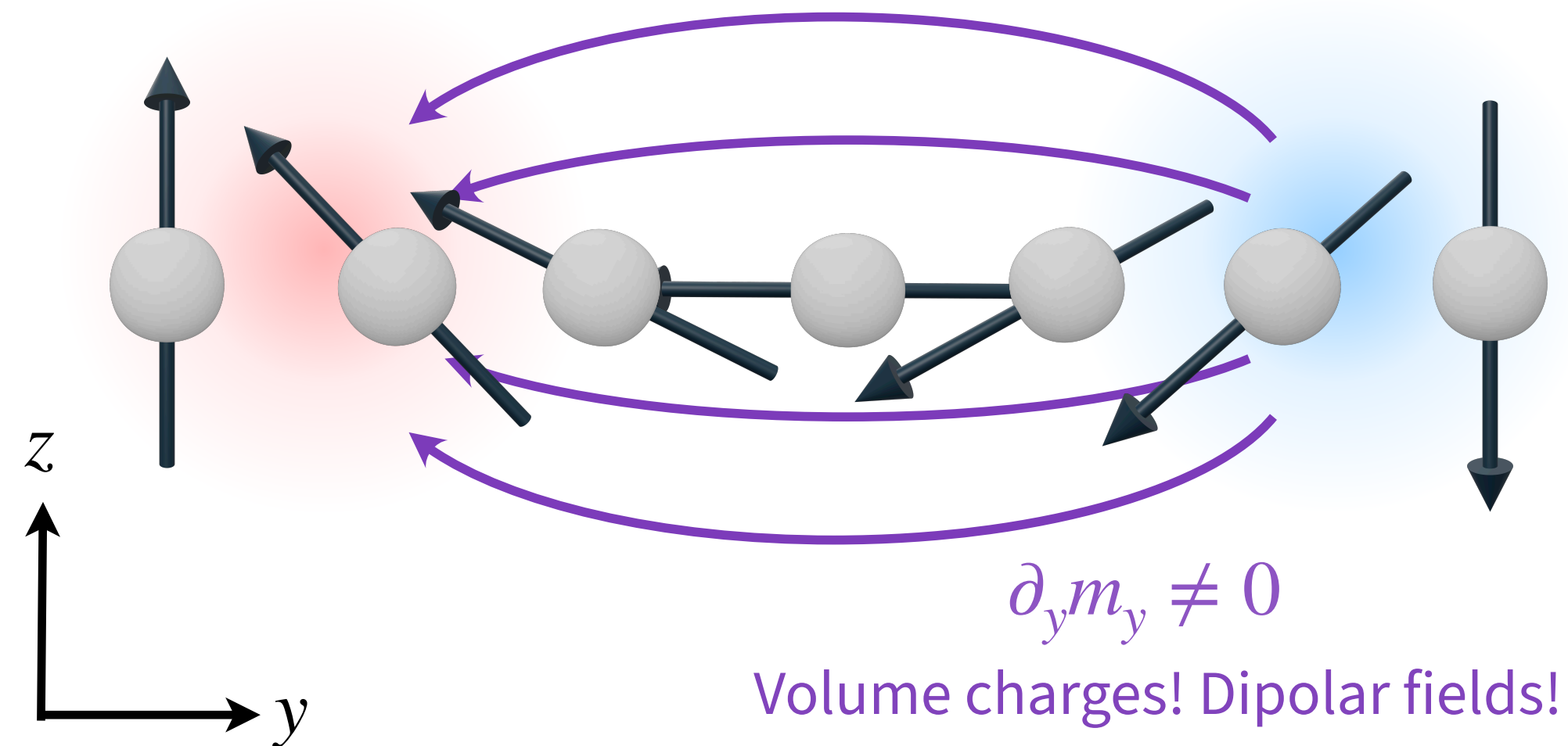
Dipole-Dipole

DMI

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A(\nabla \mathbf{m})^2 - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis



Néel-type domain wall
 $\mathbf{m}$ rotates perpendicular to wall.

Anisotropy

Exchange

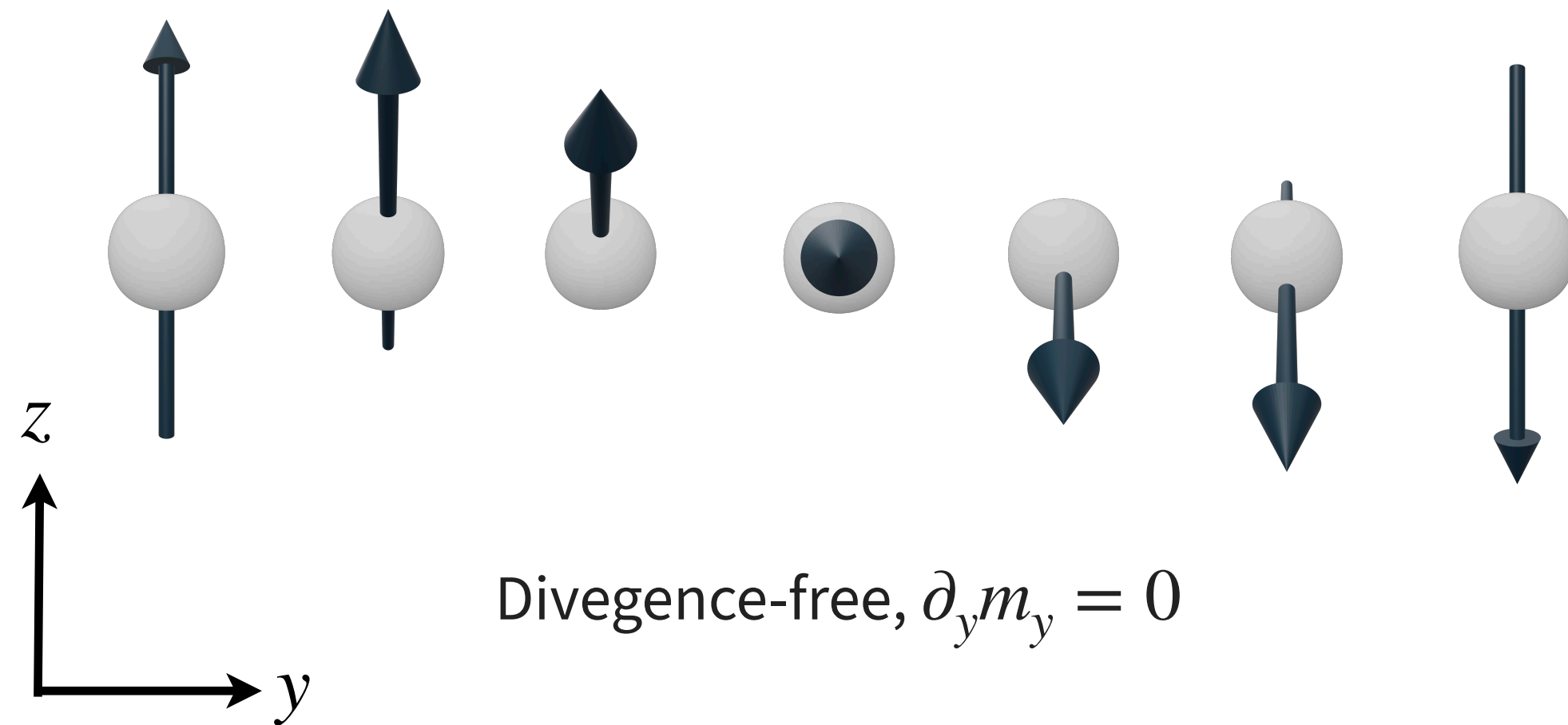
Dipole-Dipole

DMI

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) \right]$$

What is the structure of the domain walls?

easy axis



Bloch-type domain wall
m rotates *parallel* to wall.

Anisotropy

Exchange

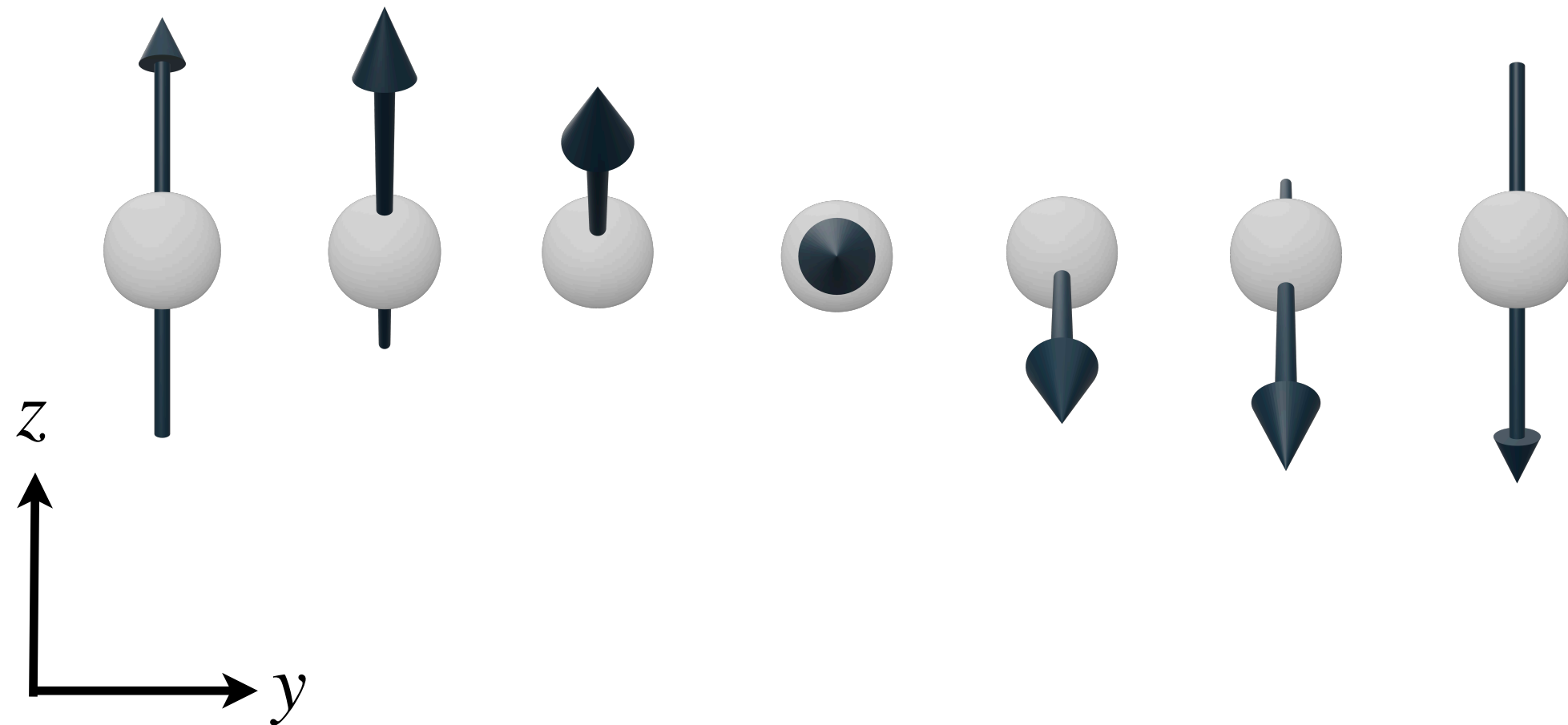
is zero
~~Dipole-Dipole~~

consider later
~~DMI~~

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 - \cancel{M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m}} + \cancel{\mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m})} \right]$$

Bloch wall profile can be obtained analytically

easy axis



Bloch-type domain wall
m rotates *parallel* to wall.

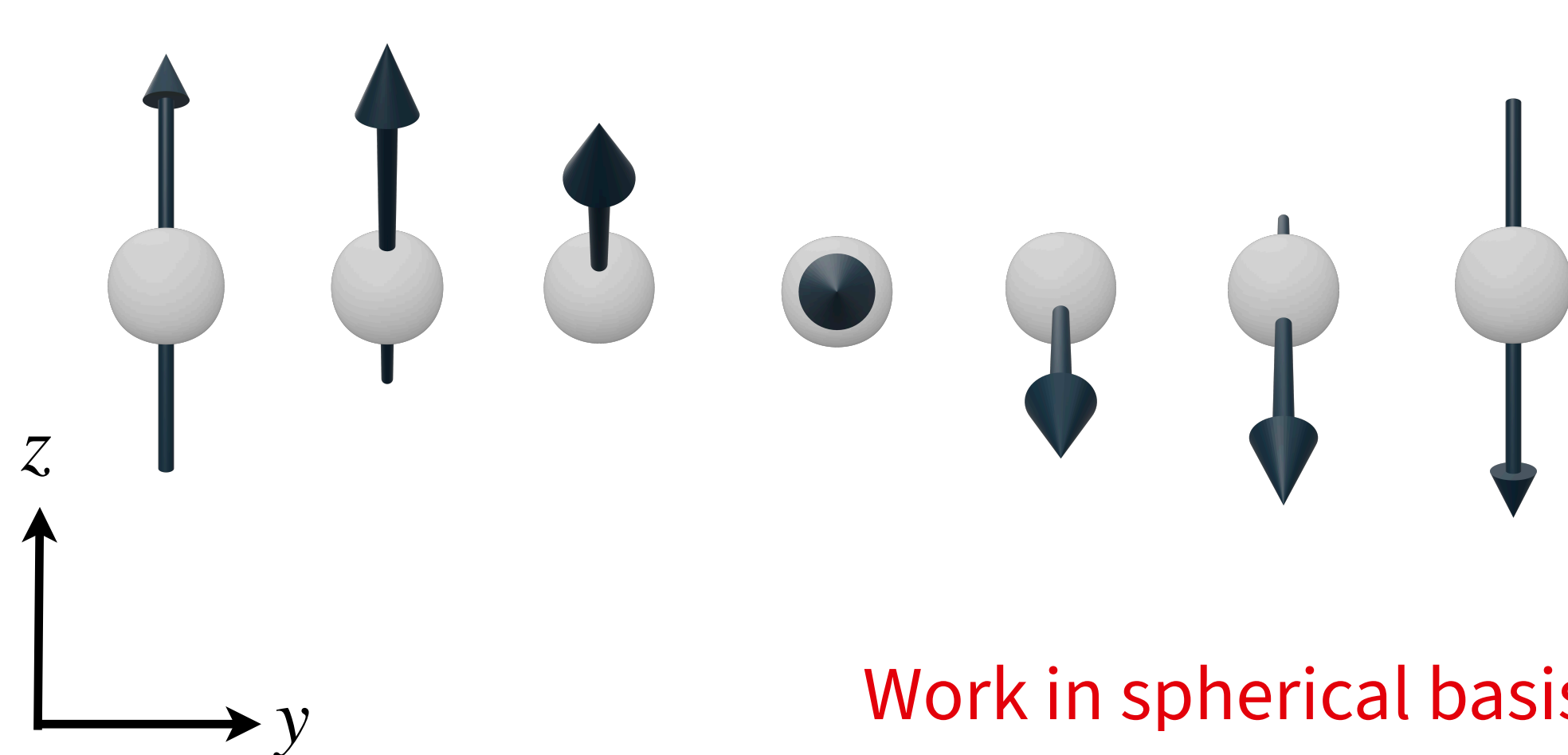
Anisotropy

Exchange

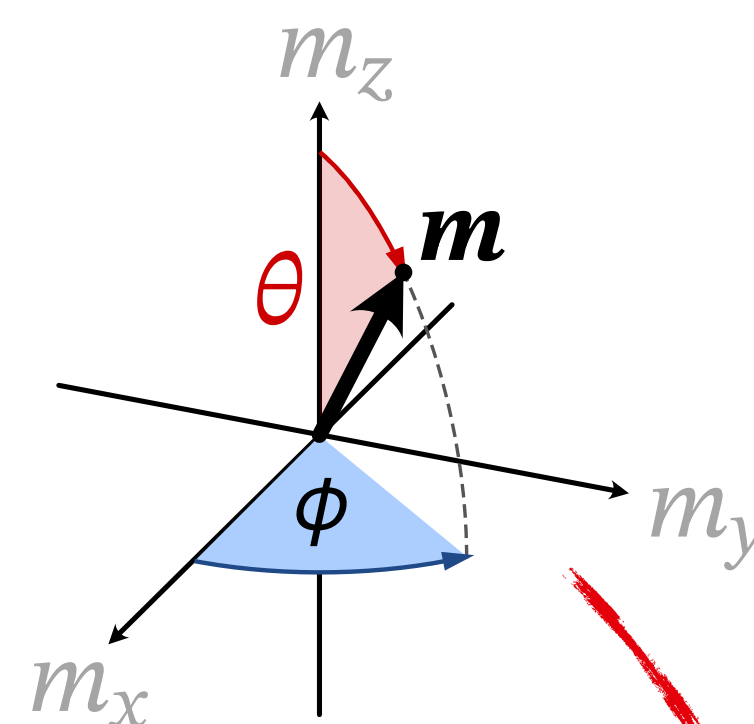
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right]$$

Bloch wall profile can be obtained analytically

easy axis



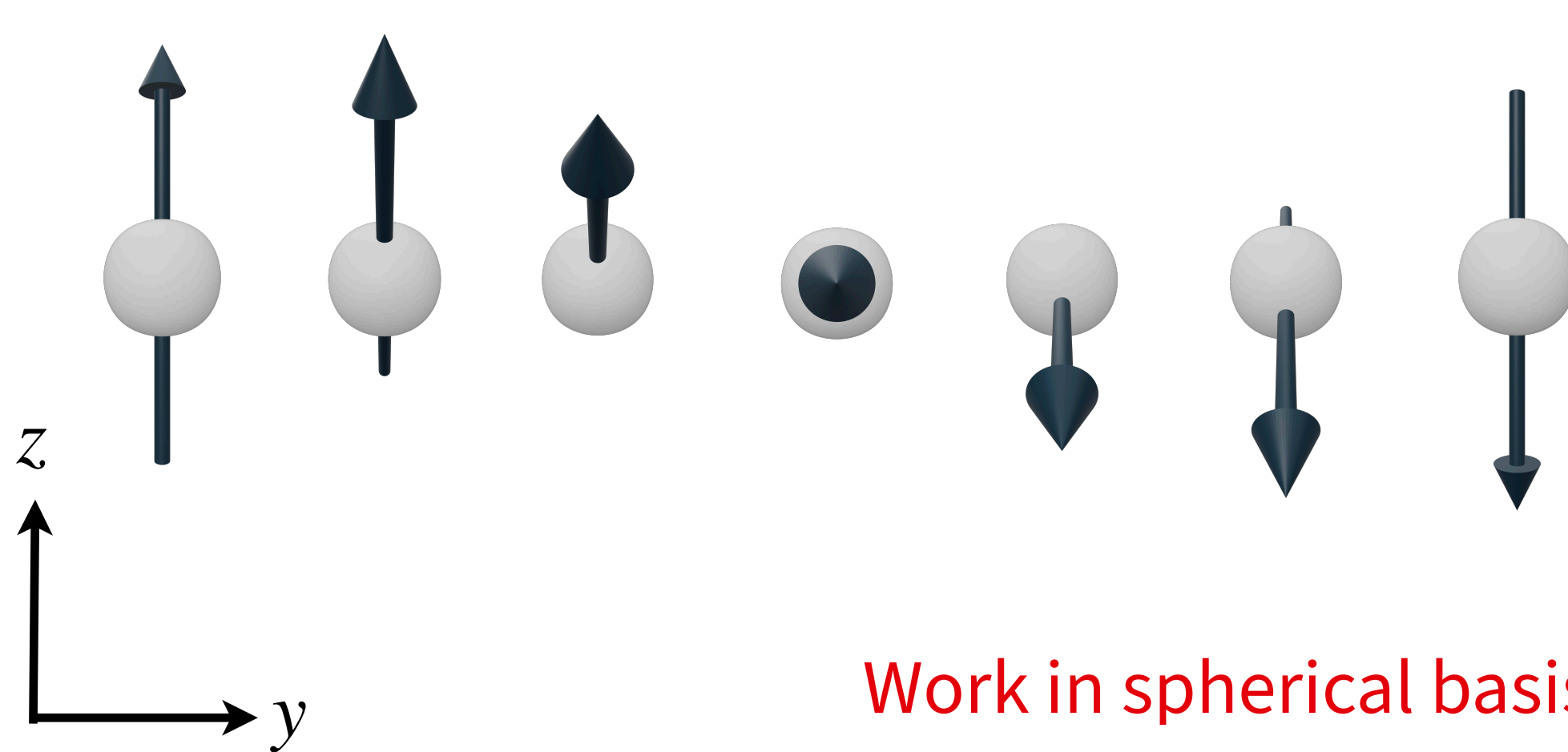
Work in spherical basis



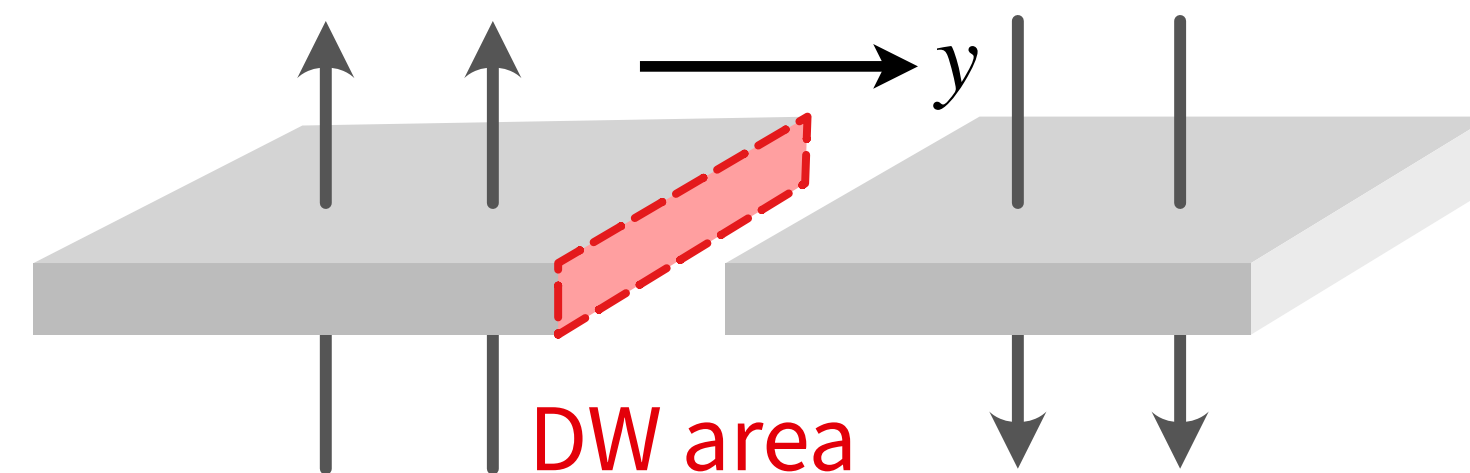
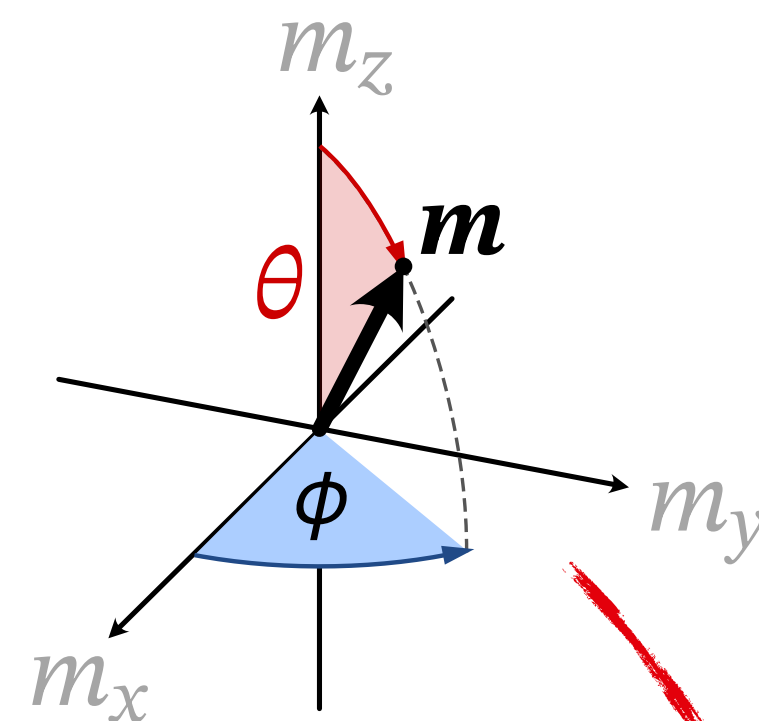
$$E(\mathbf{m}) = \int dV \left[\begin{array}{cc} \text{Anisotropy} & \text{Exchange} \\ K(1 - m_z^2) & + A(\nabla \mathbf{m})^2 \end{array} \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A(\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

easy axis



Work in spherical basis

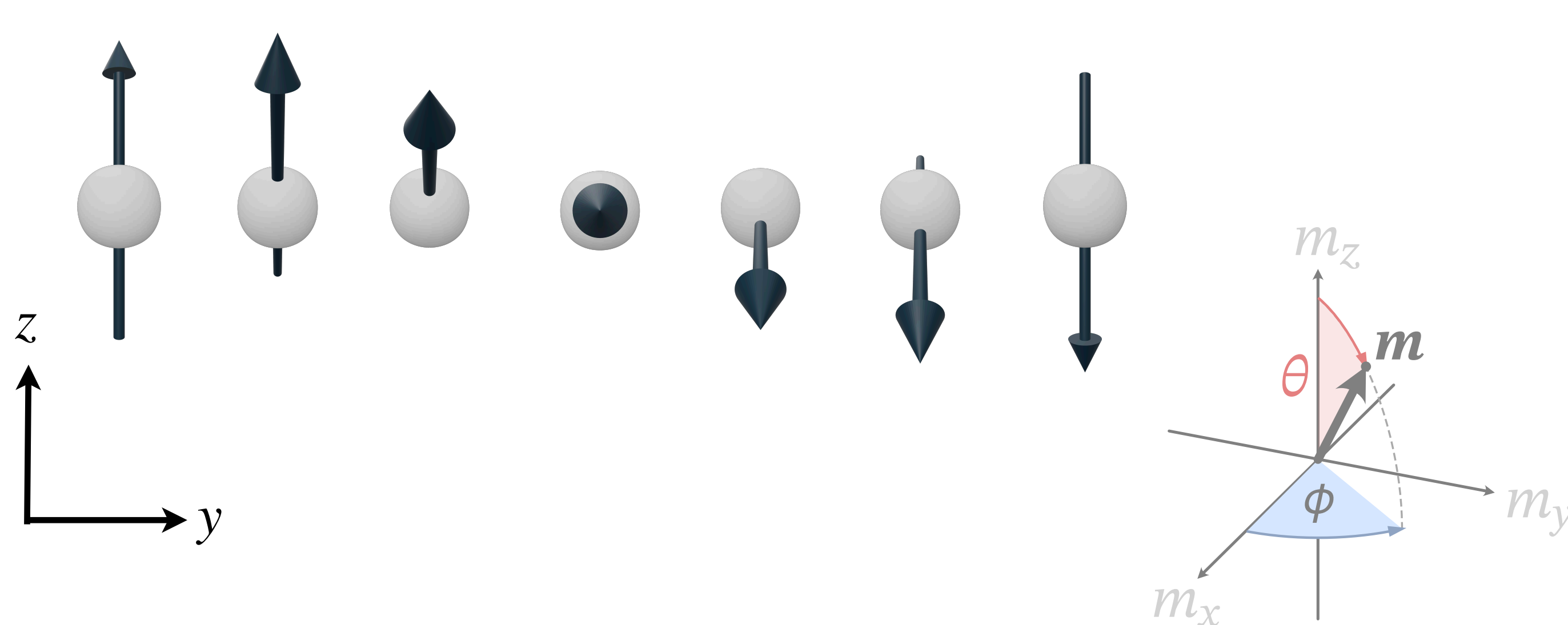


energy per area

$$E(\mathbf{m}) = \int dV \left[\text{Anisotropy } K(1 - m_z^2) + \text{Exchange } A(\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A(\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

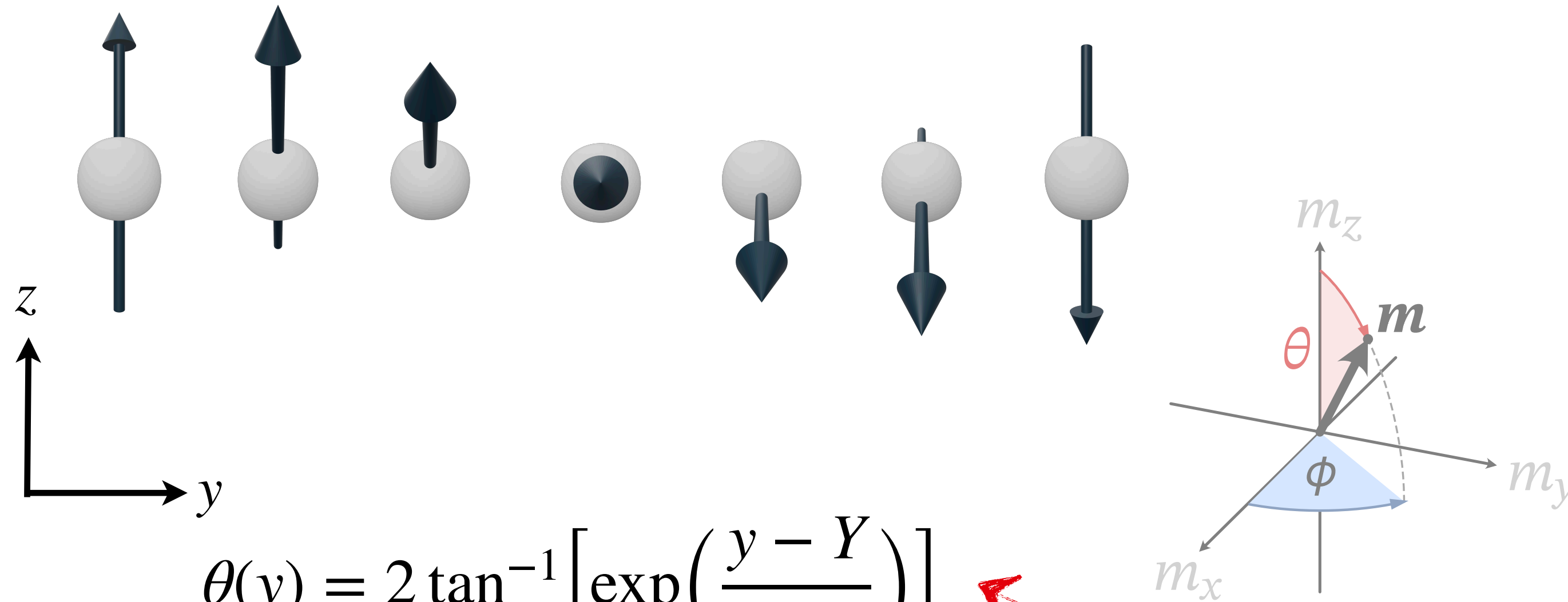
easy axis



$$E(\mathbf{m}) = \int dV \left[\overset{\text{Anisotropy}}{K (1 - m_z^2)} + \overset{\text{Exchange}}{A (\nabla \mathbf{m})^2} \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

easy axis



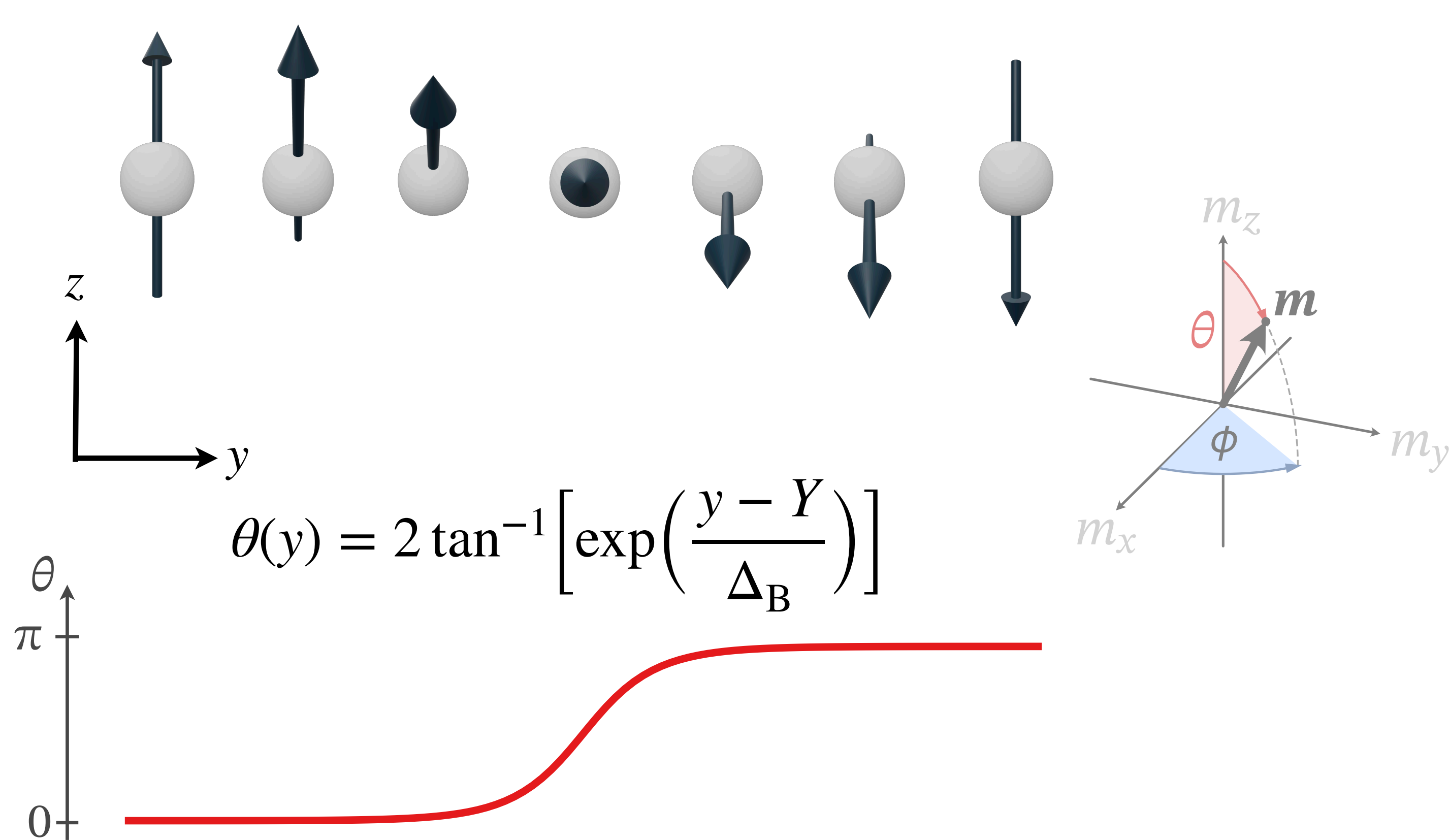
$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$

Minimize with boundary conditions
(here $\theta(\mp \infty) = 0, \pi$)

$$E(\mathbf{m}) = \int dV \left[\overset{\text{Anisotropy}}{K (1 - m_z^2)} + \overset{\text{Exchange}}{A (\nabla \mathbf{m})^2} \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

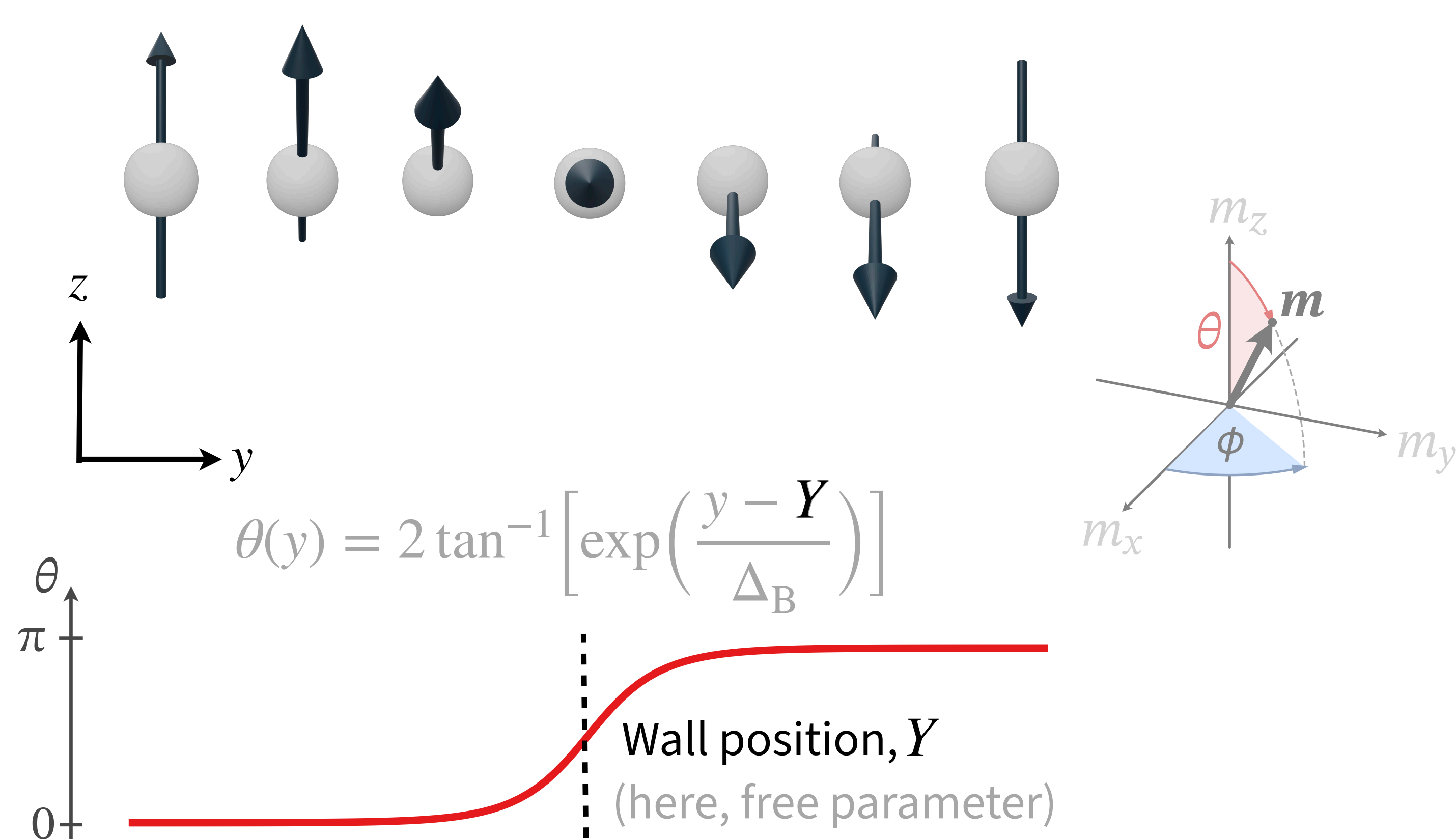
easy axis



$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

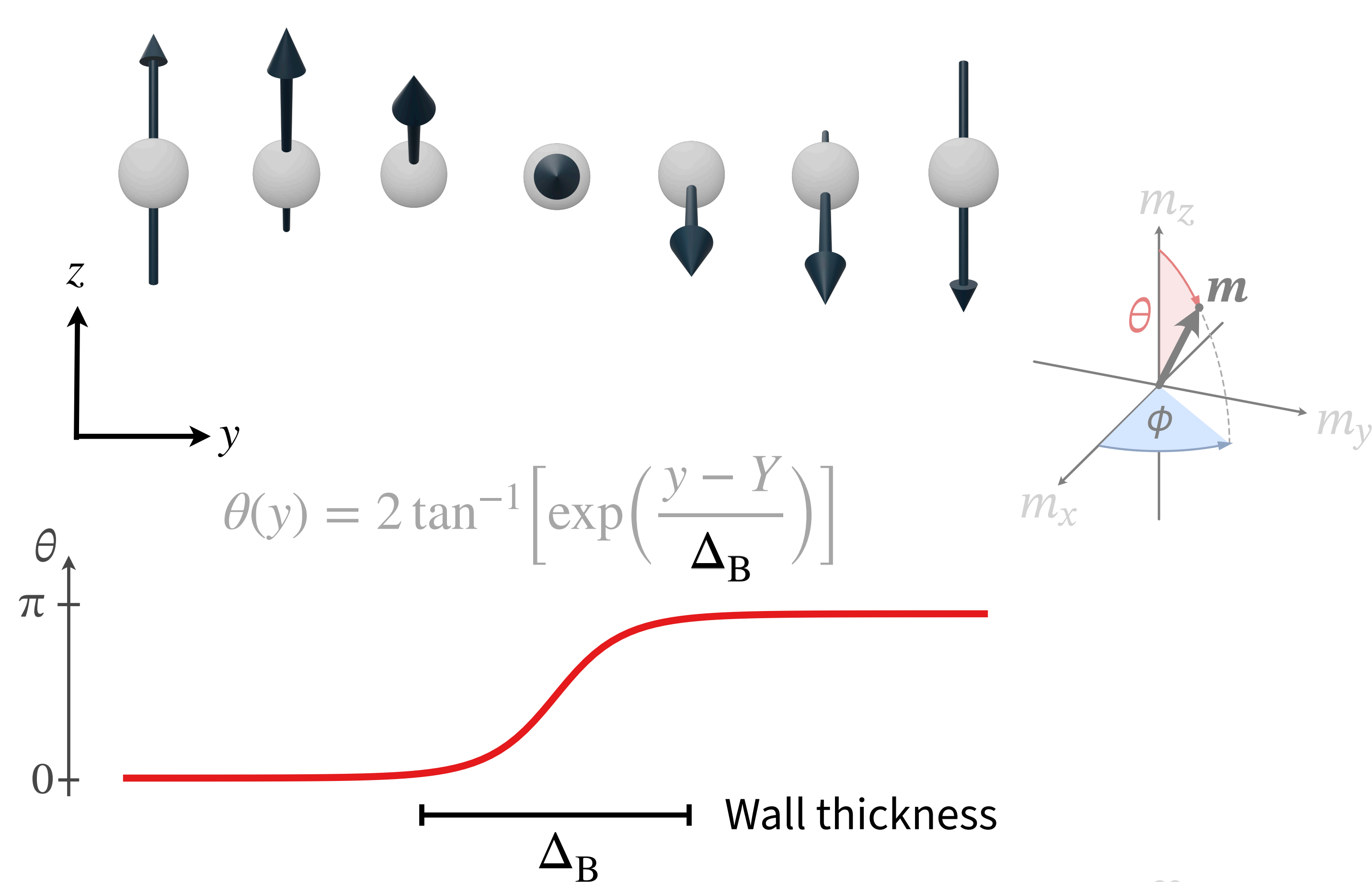
easy axis



$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Bloch wall profile can be obtained analytically

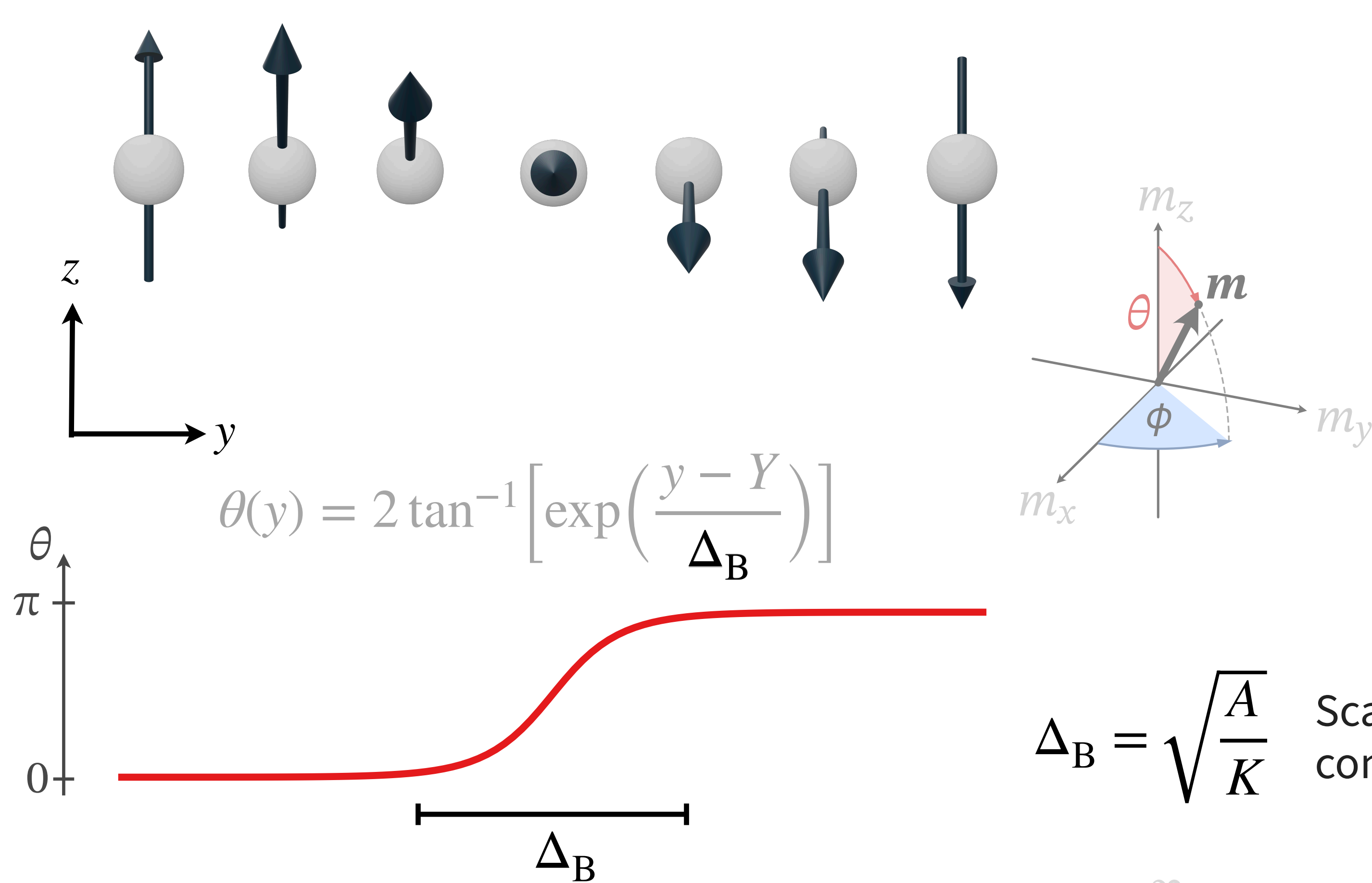
easy axis



$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain wall thickness is determined by competing interactions

easy axis

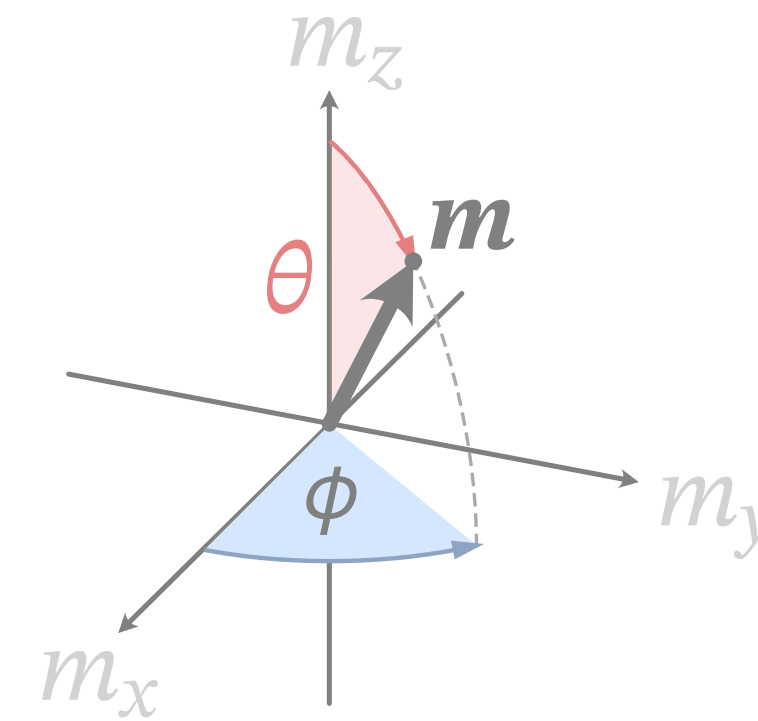
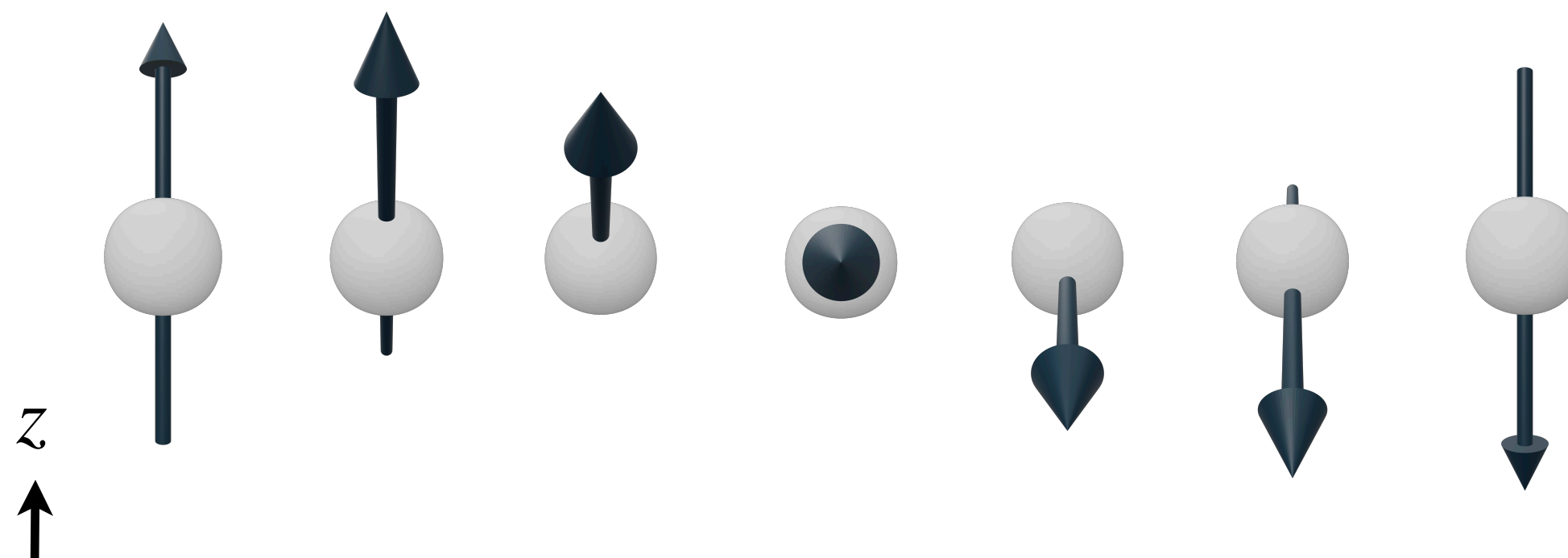


$$\Delta_B = \sqrt{\frac{A}{K}} \quad \text{Scale determined by competing interactions!}$$

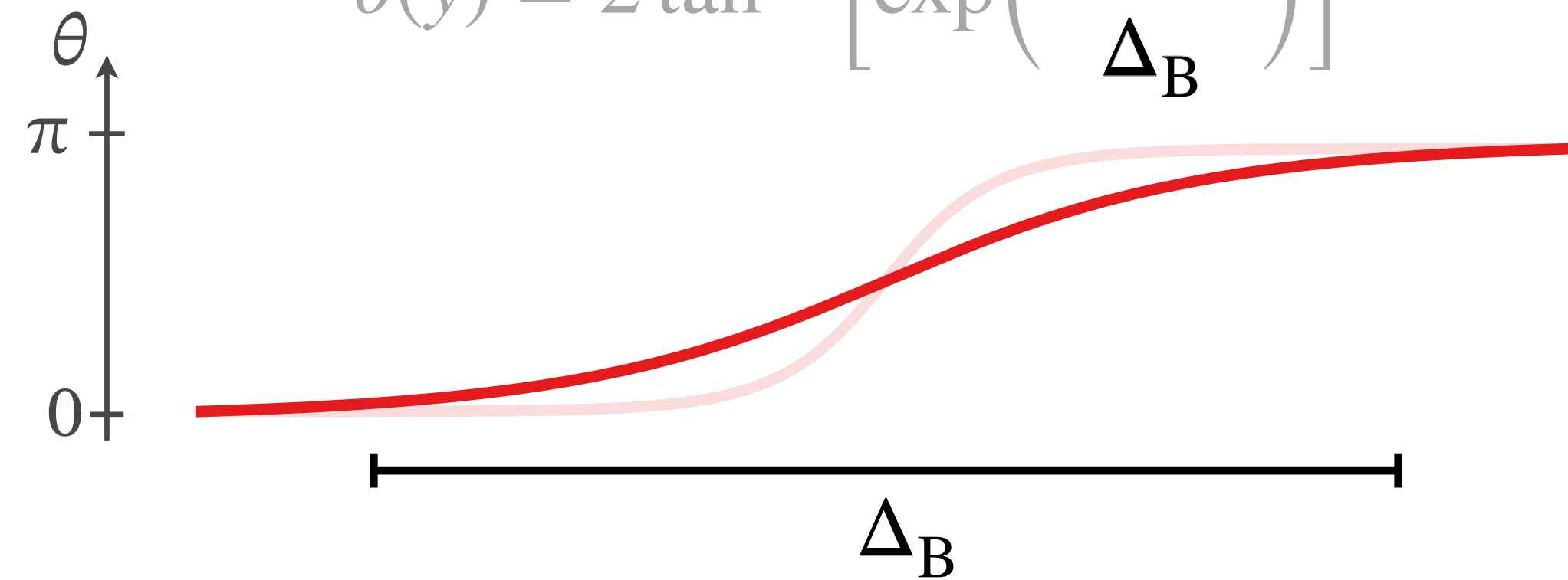
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain wall thickness is determined by competing interactions

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



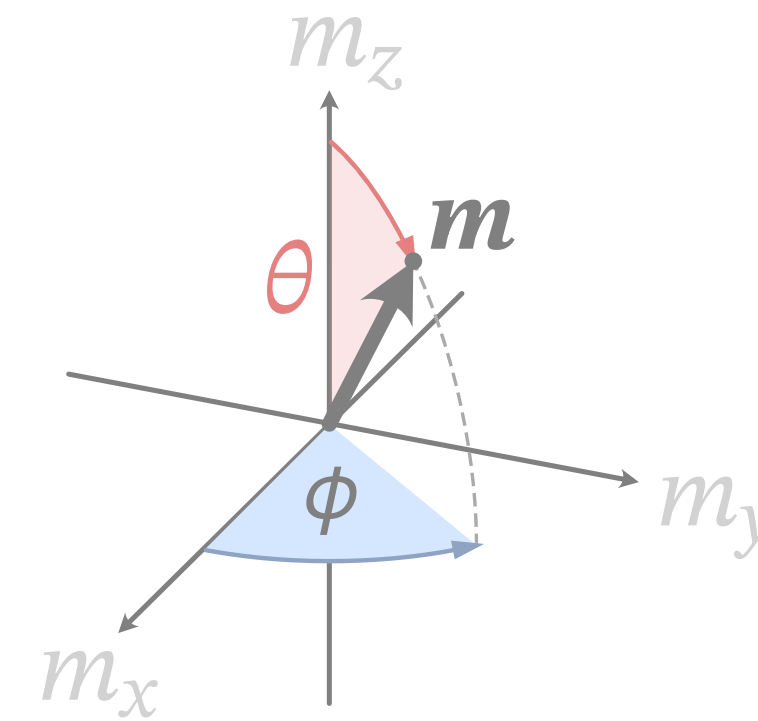
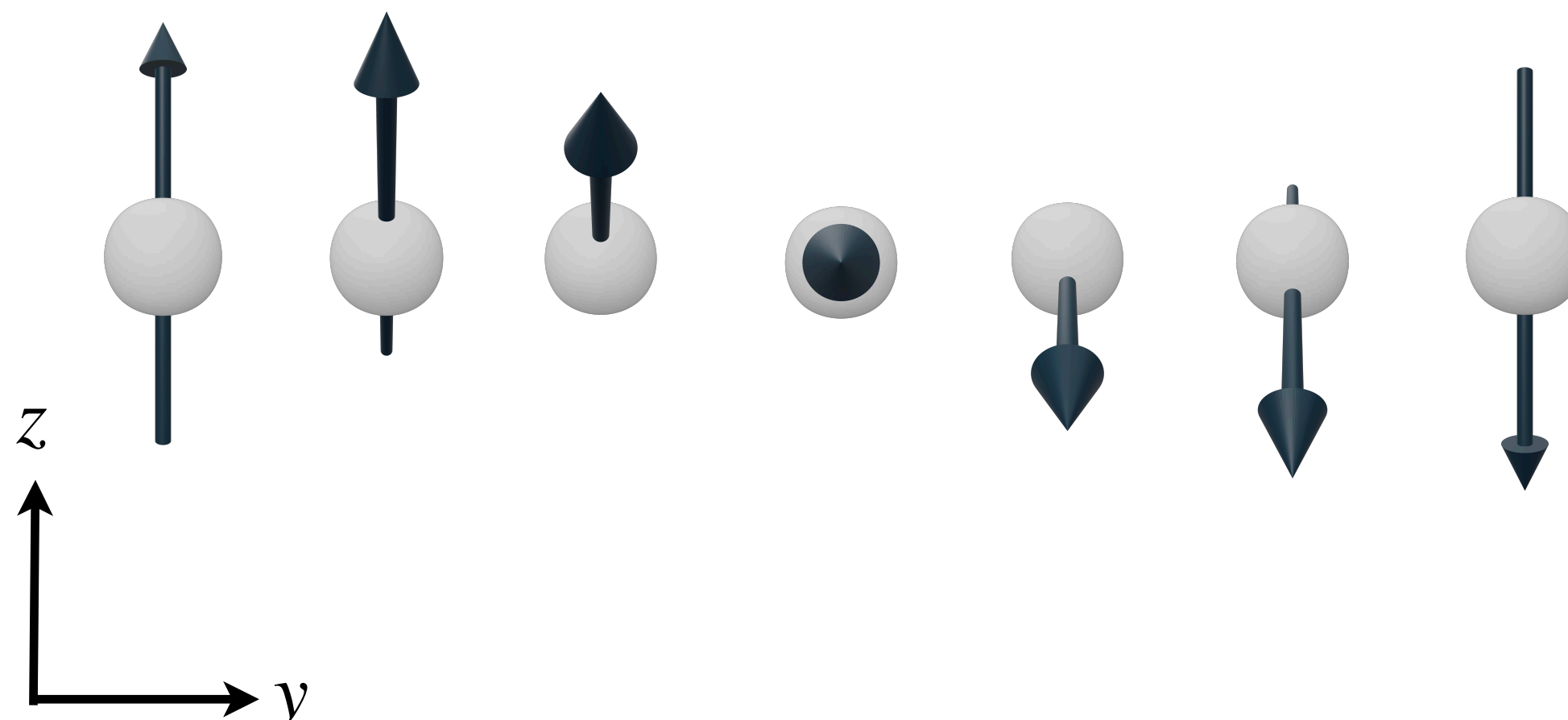
Large exchange $A \rightarrow$ rotate slower

$$\Delta_B = \sqrt{\frac{A}{K}}$$

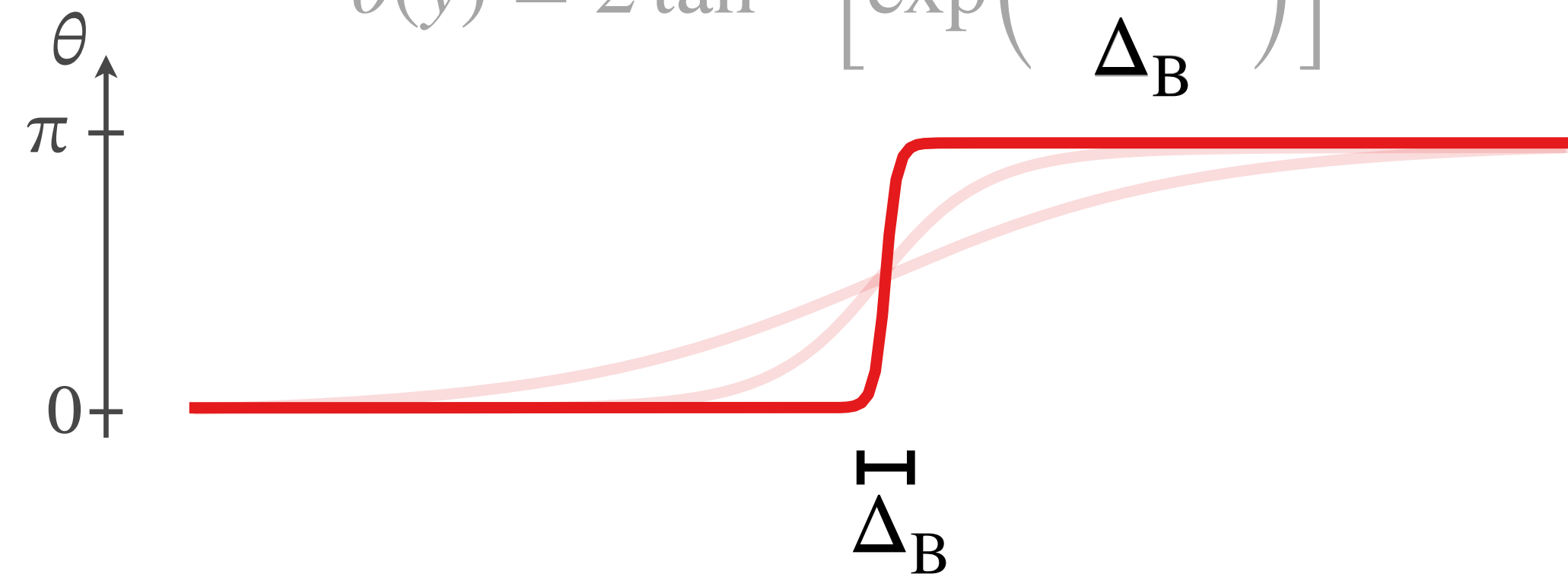
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain wall thickness is determined by competing interactions

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



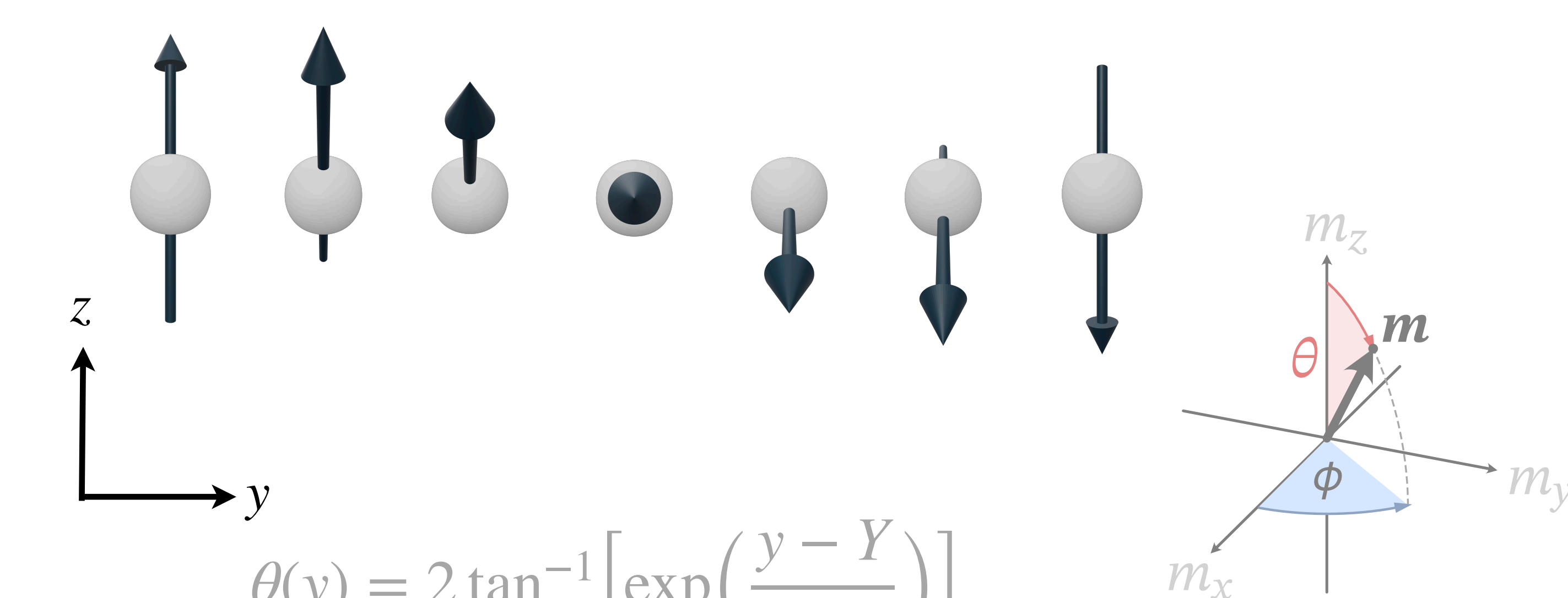
Large anisotropy $K \rightarrow$ remain closer to easy axis

$$\Delta_B = \sqrt{\frac{A}{K}}$$

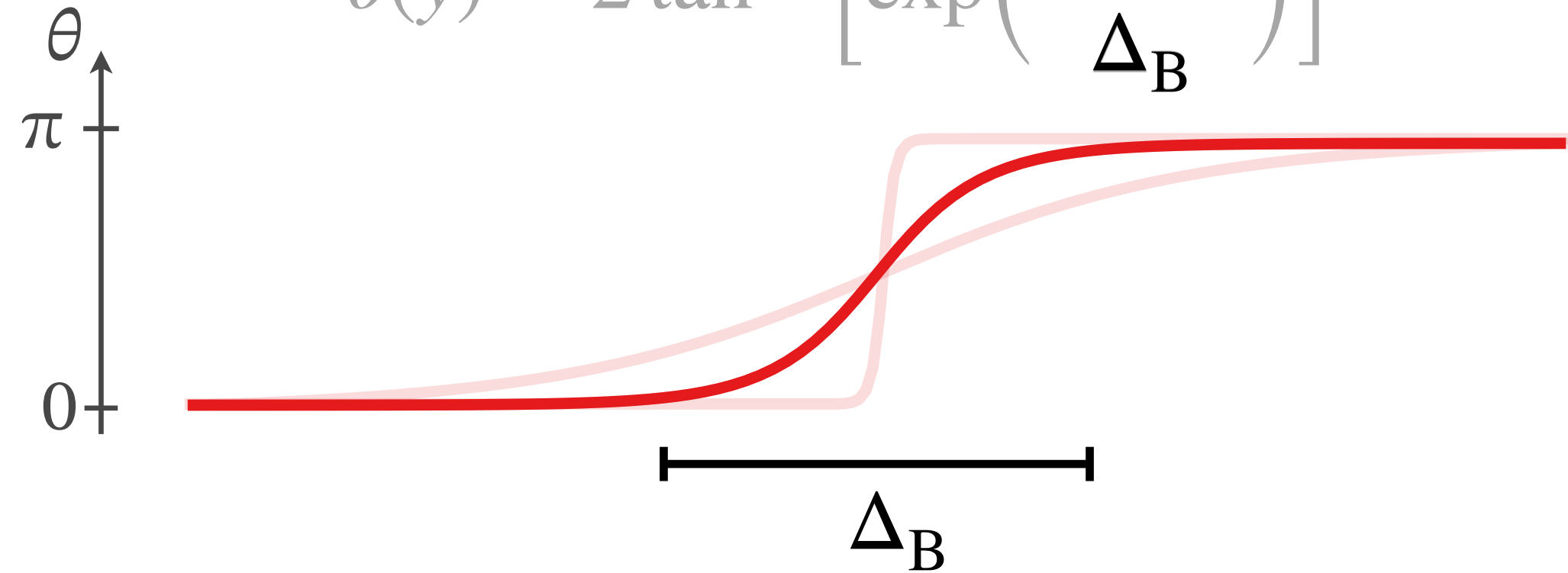
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain wall thickness is determined by competing interactions

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



Depends on material...

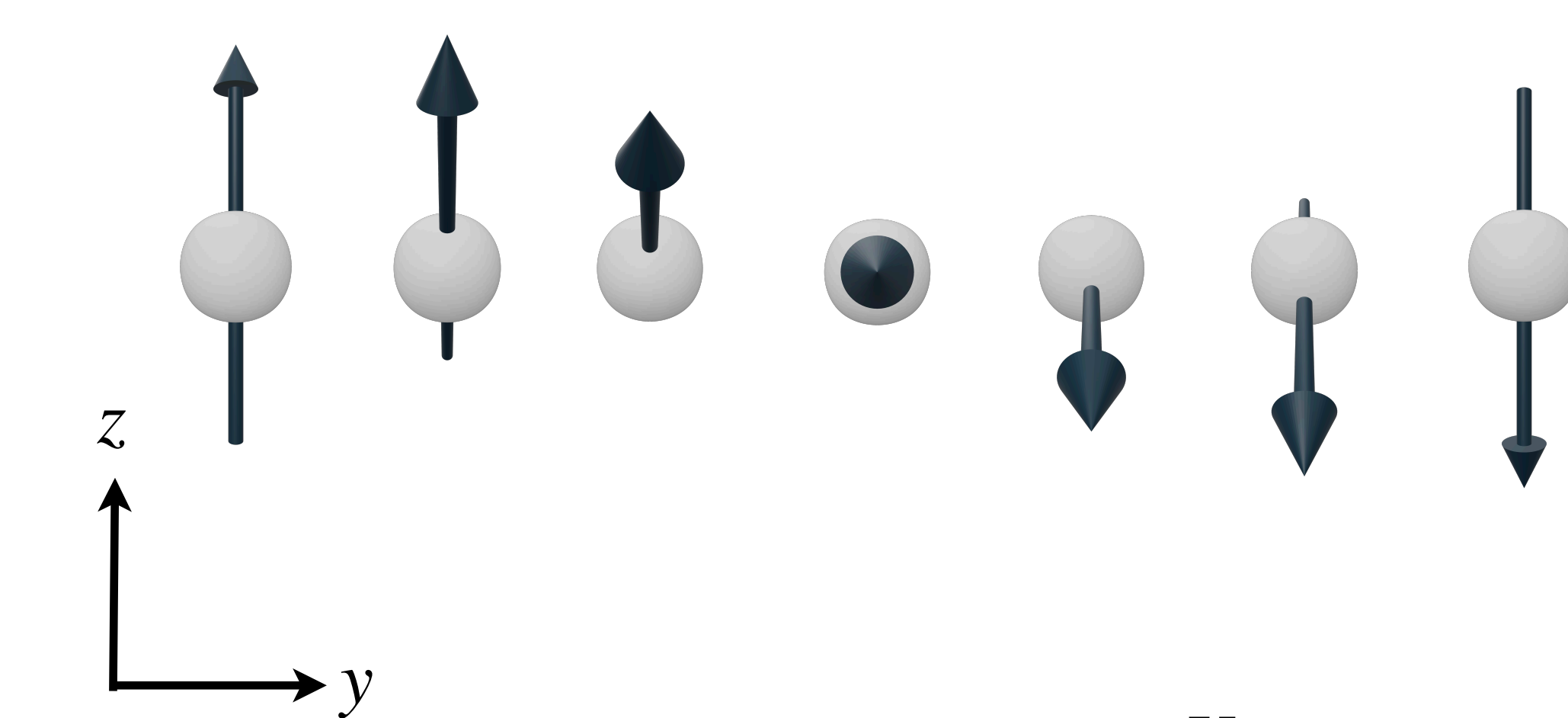
$$\Delta_B = \sqrt{\frac{A}{K}}$$

2 - 10 nm hard-magnetic materials
10 - 100 nm soft-magnetic materials

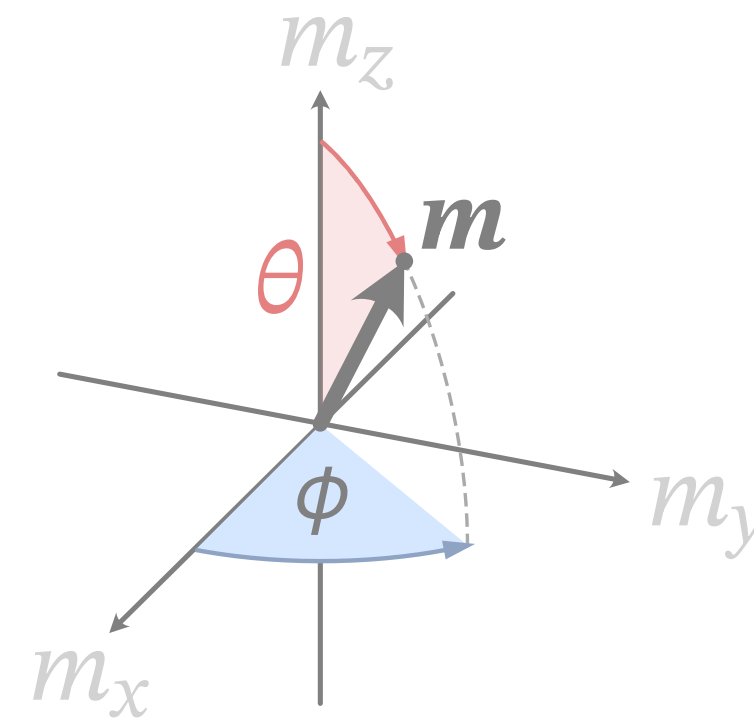
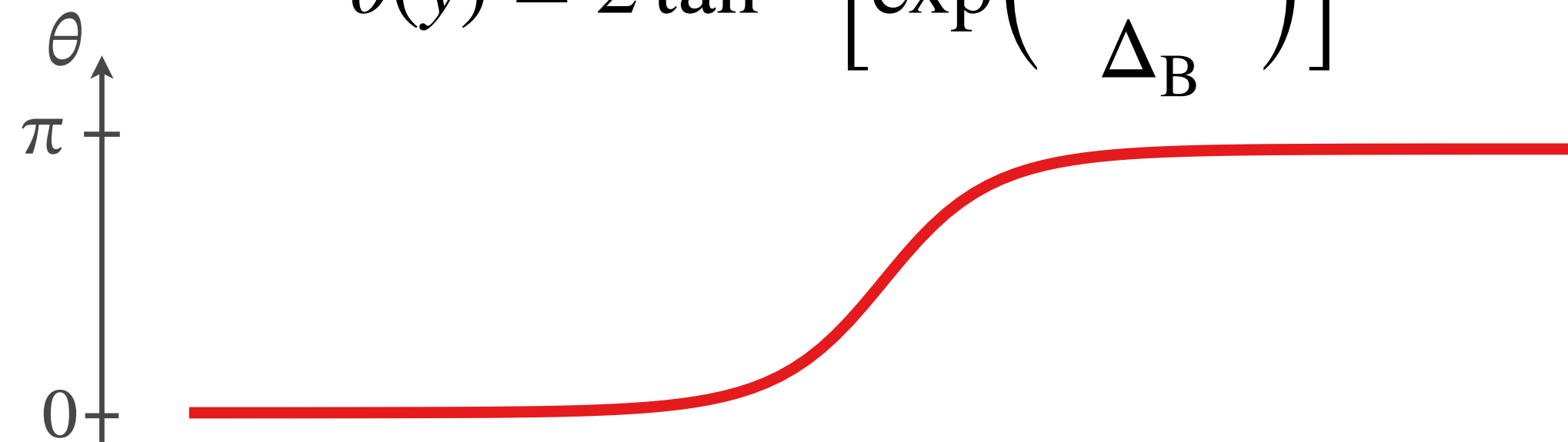
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain walls cost energy

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$

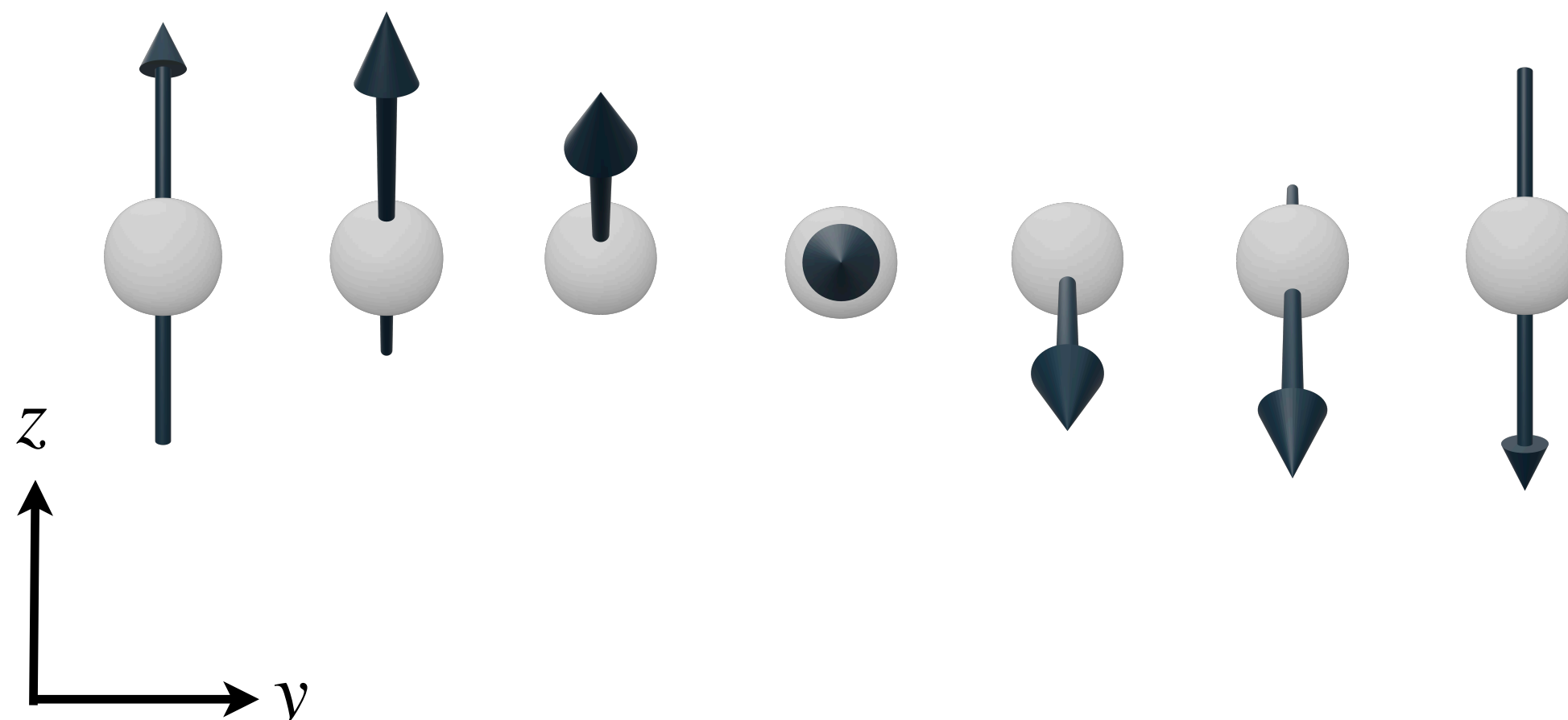


Energy cost of this wall?

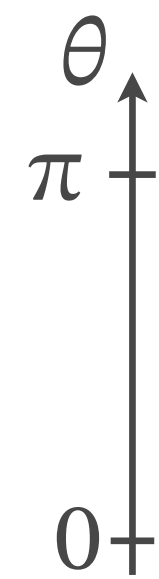
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

Domain walls cost energy

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$

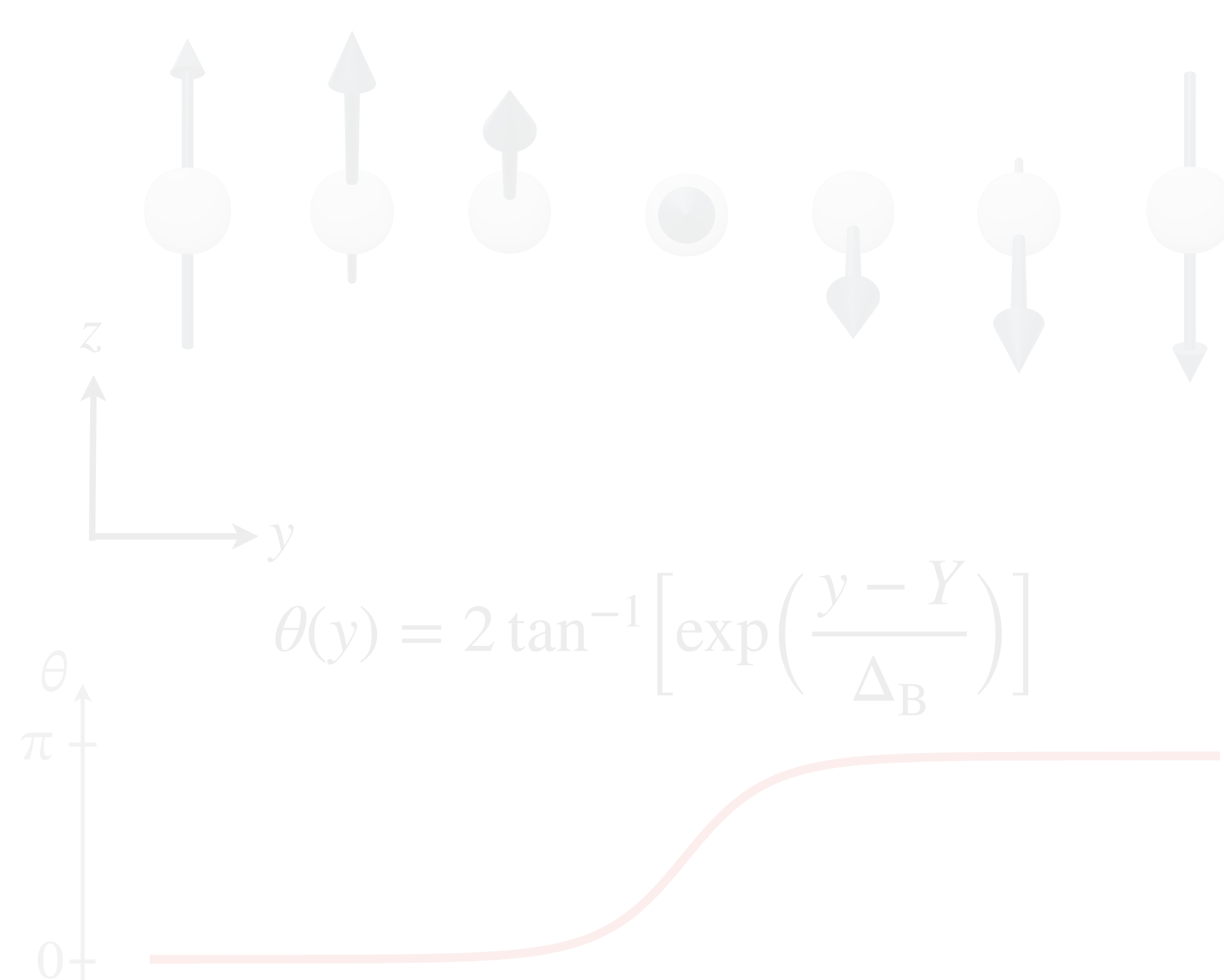


Plug in

$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S \int_{-\infty}^{\infty} dy \left[K \sin^2(\theta) + A (\partial_y \theta)^2 \right]$$

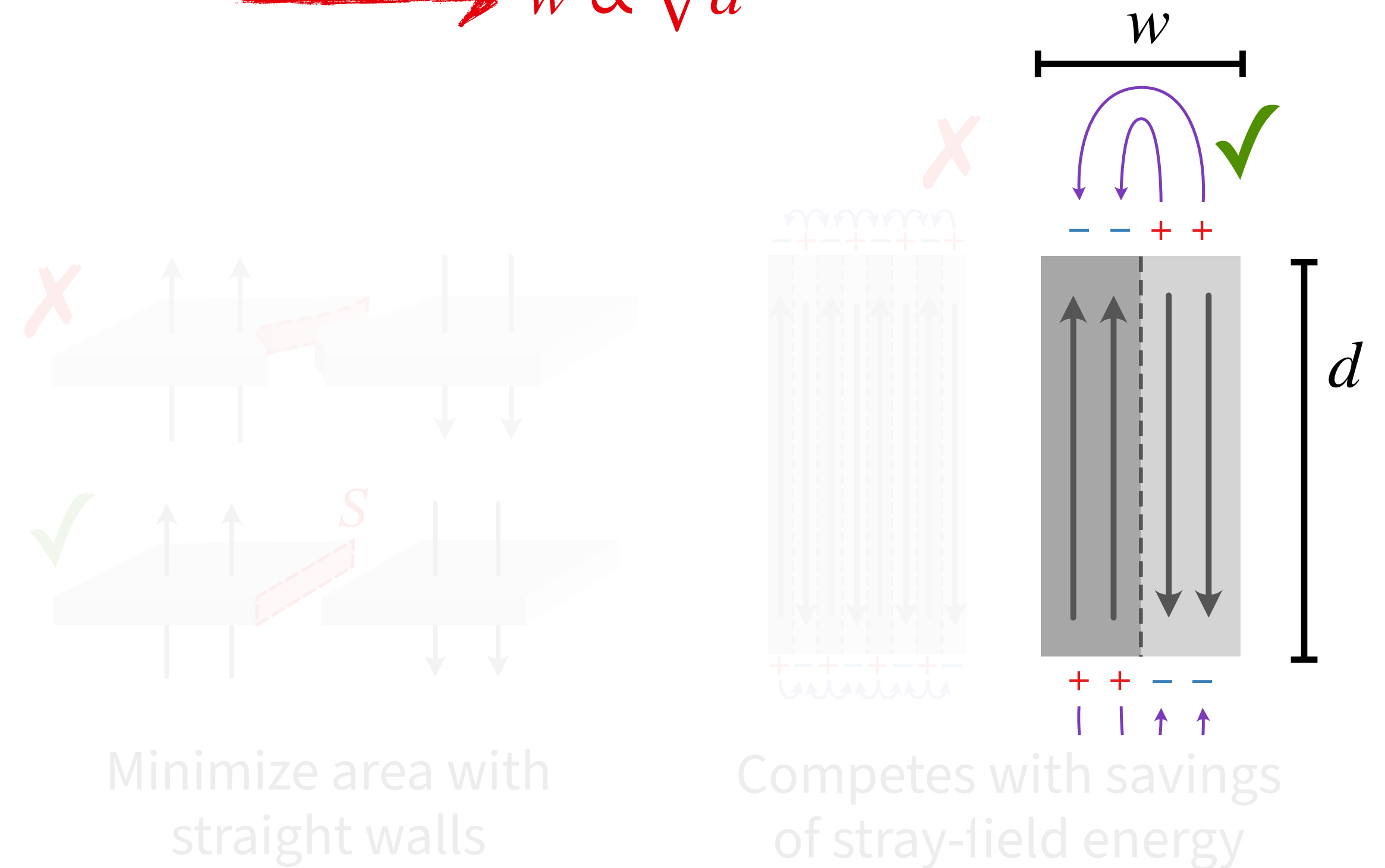
Domain walls cost energy

easy axis



$$\sigma_{\text{wall}} = E_{\text{wall}} \frac{d}{w} \quad \text{vs} \quad \sigma_{\text{stray}} = 0.136 \mu_0 M_s^2 w$$

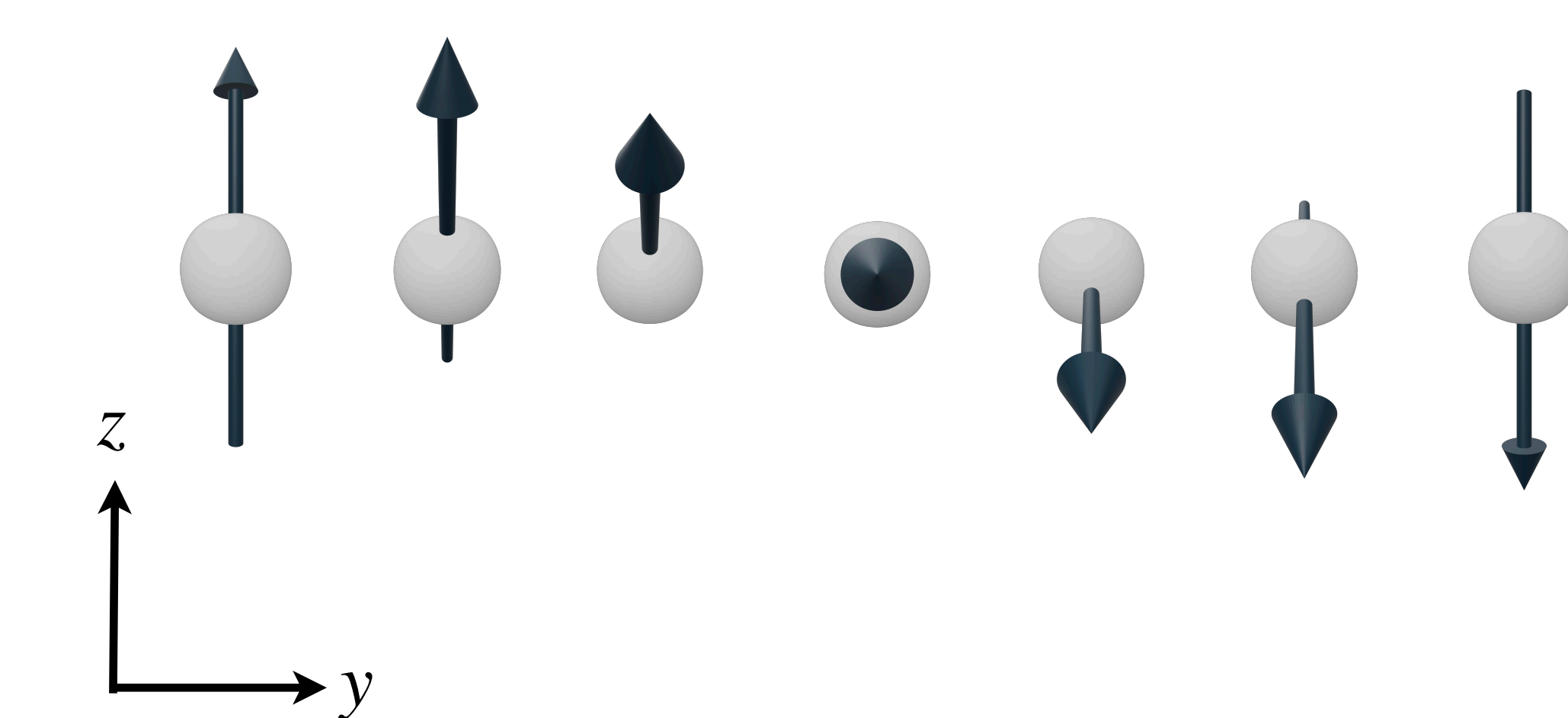
$$\longrightarrow w \propto \sqrt{d}$$



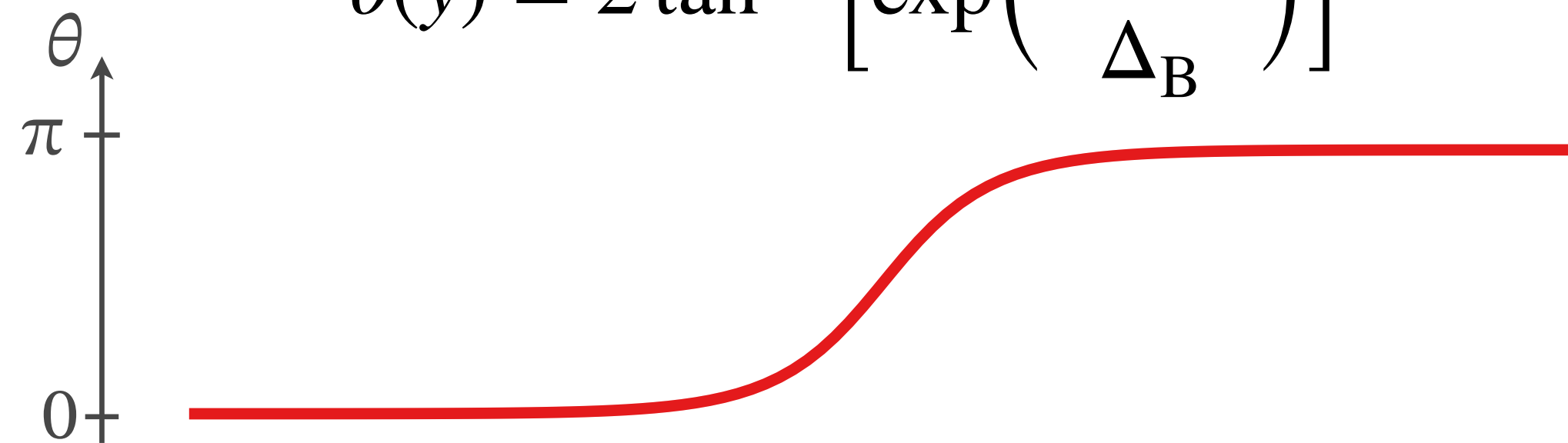
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S 4\sqrt{AK} > 0 \text{ always!}$$

Domain walls cost energy

easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



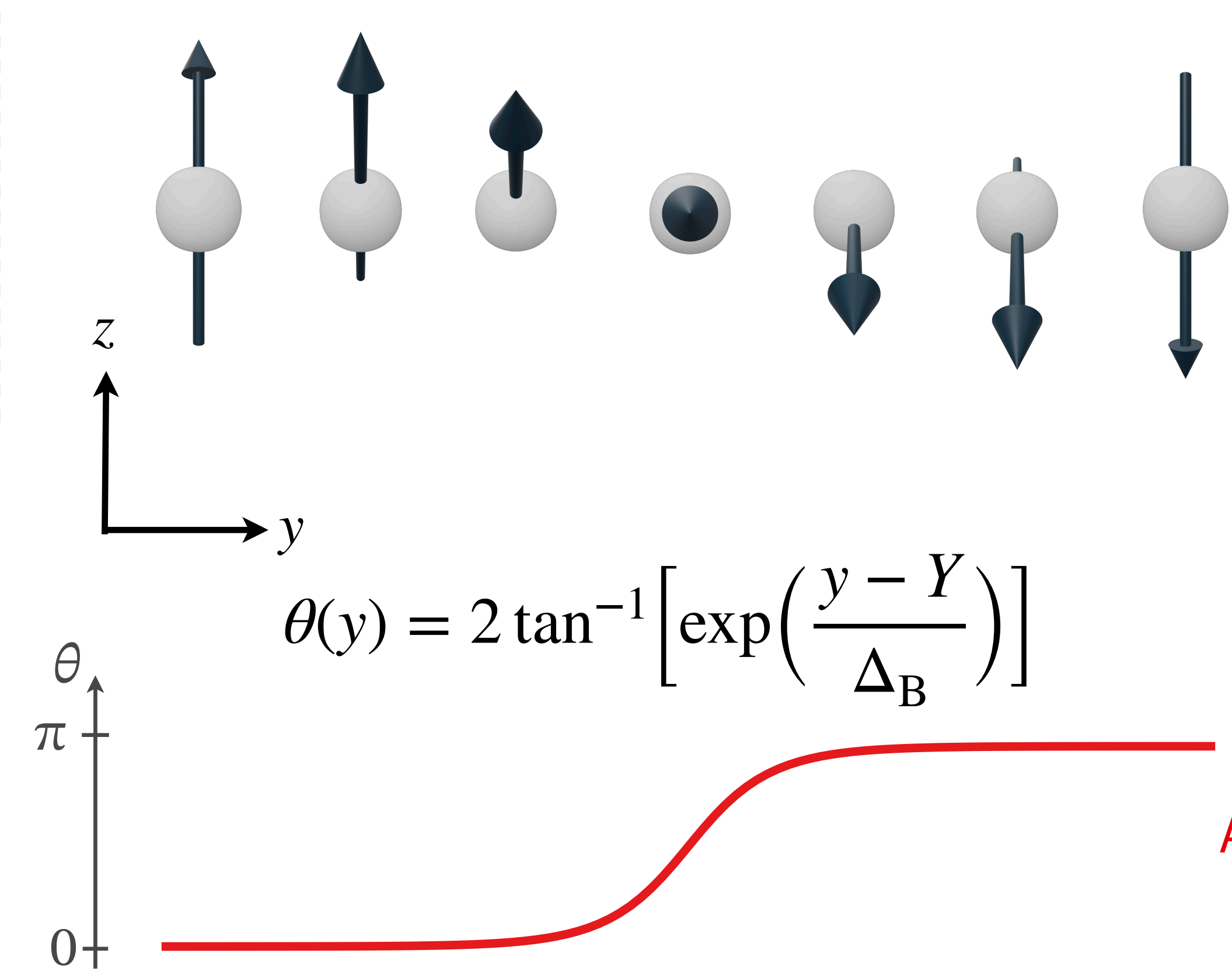
So does not depend
on position Y



$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 \right] = S 4\sqrt{AK}$$

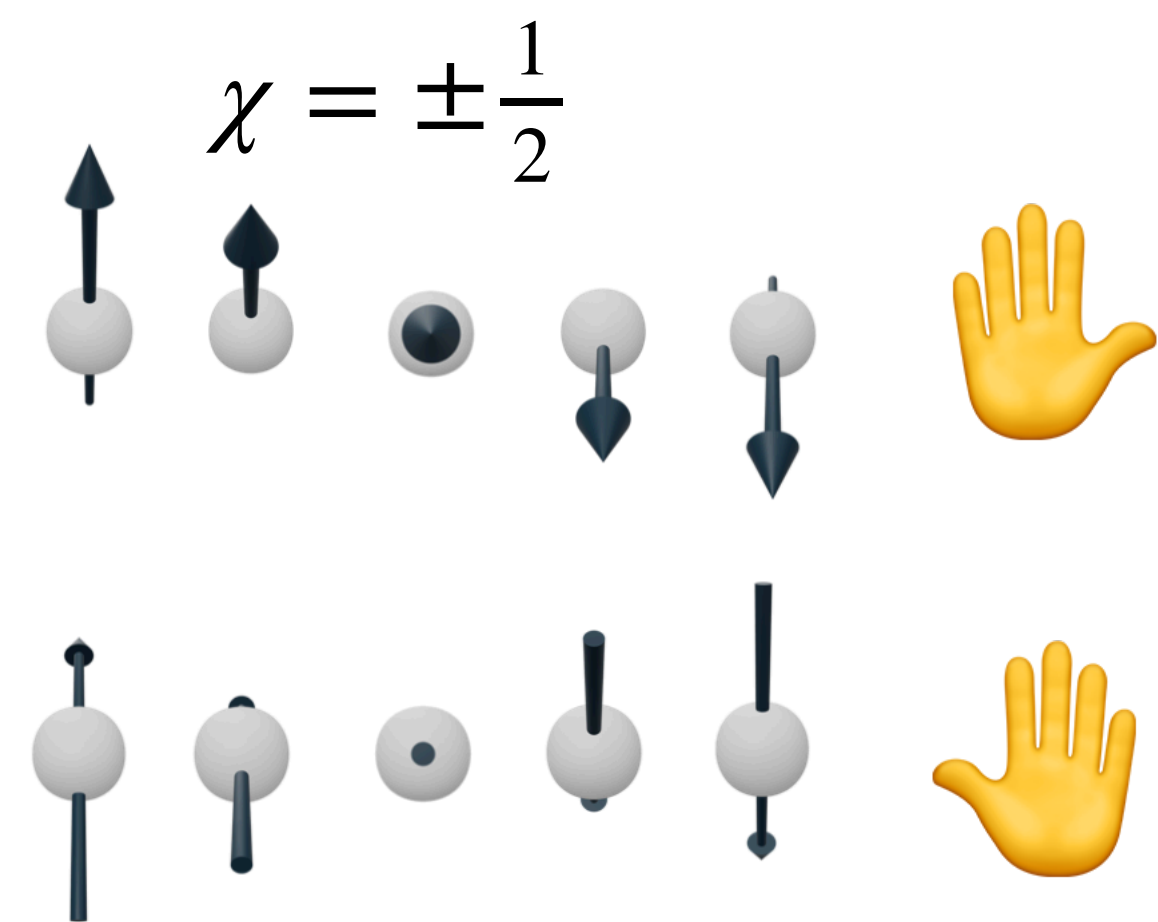
Domain walls come with different winding numbers

easy axis



And there is another free parameter here!

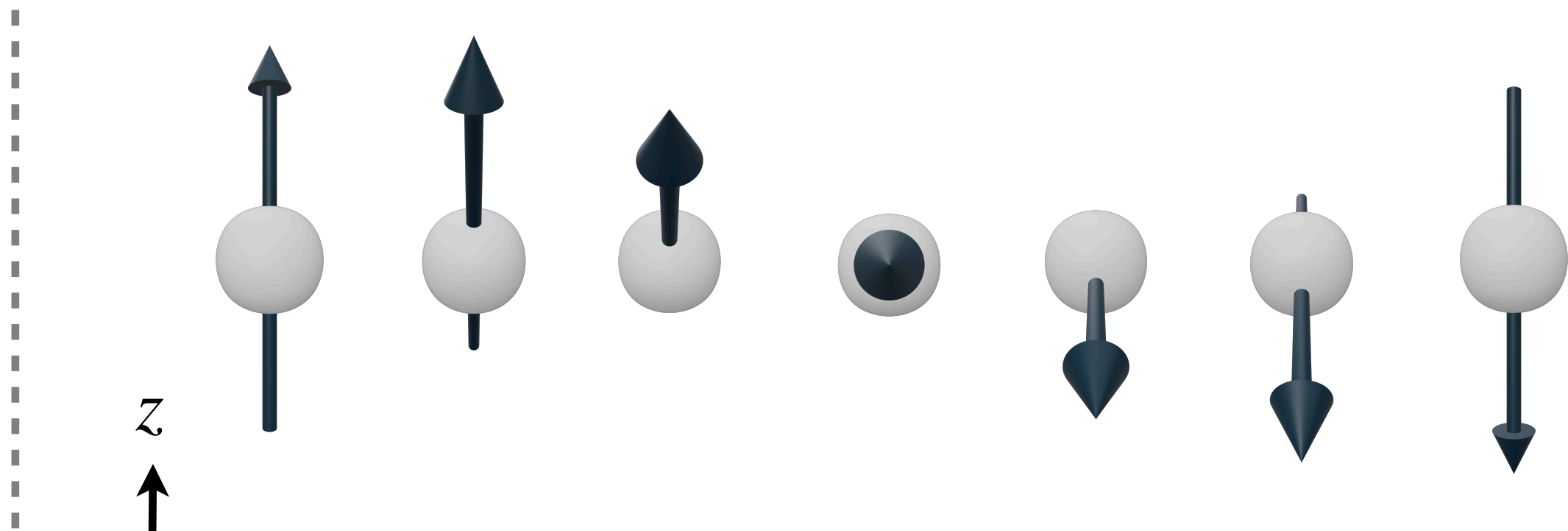
Chirality / winding number



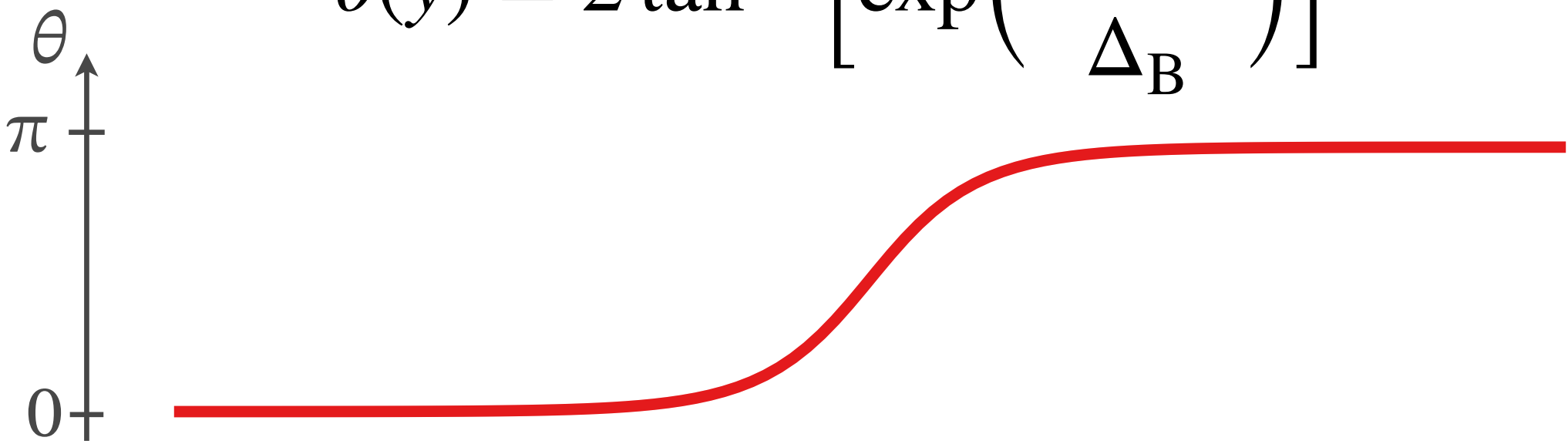
$$E(\boldsymbol{m}) = \int \mathrm{d}V \left[K \left(1 - m_z^2 \right) + A (\nabla \boldsymbol{m})^2 \right] = S 4 \sqrt{AK}$$

DMI prefers domain walls with particular winding number

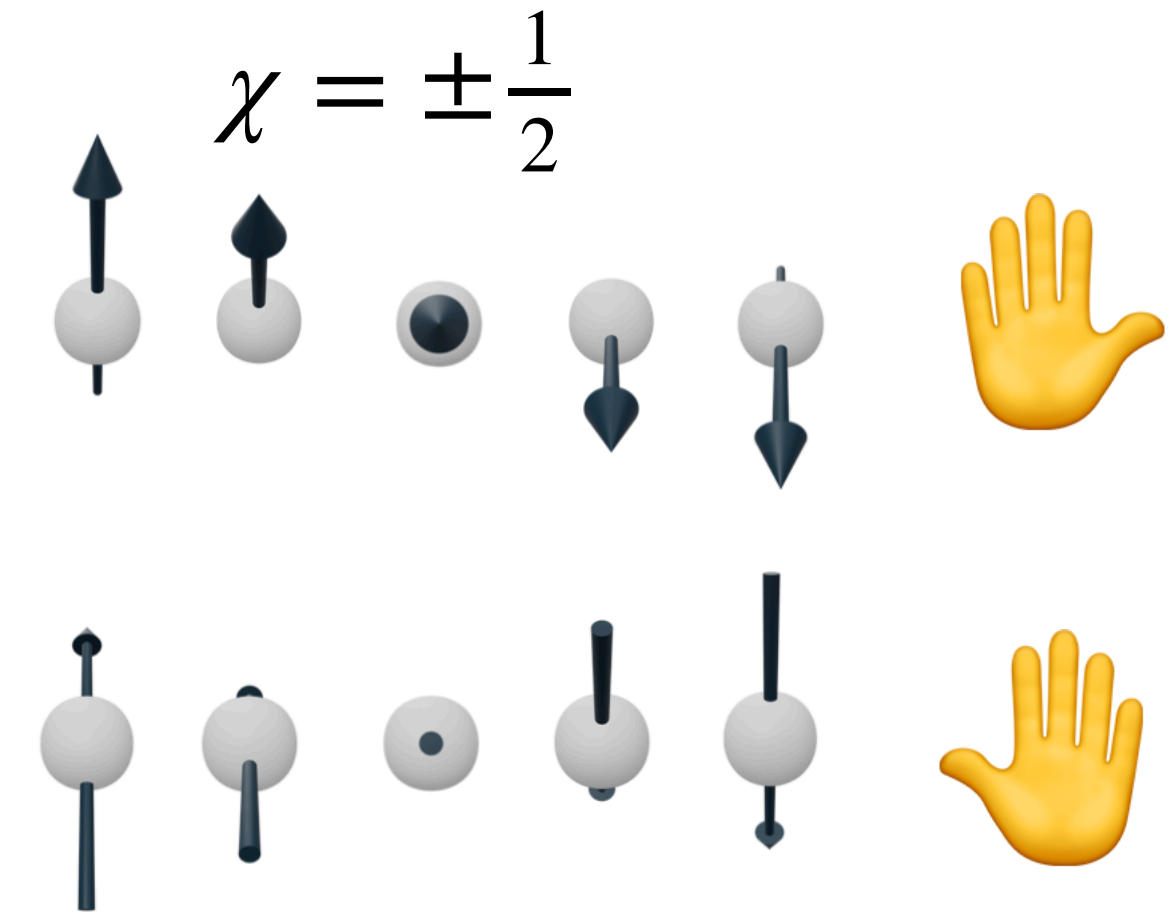
easy axis



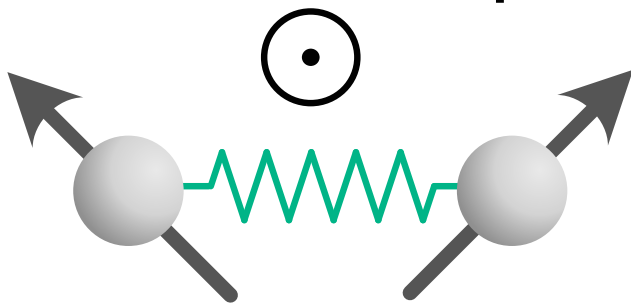
$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



Chirality / winding number



prefers particular chirality 🖐️ ≠ 🖐️



Dzyaloshinskii-Moriya Interaction (DMI)

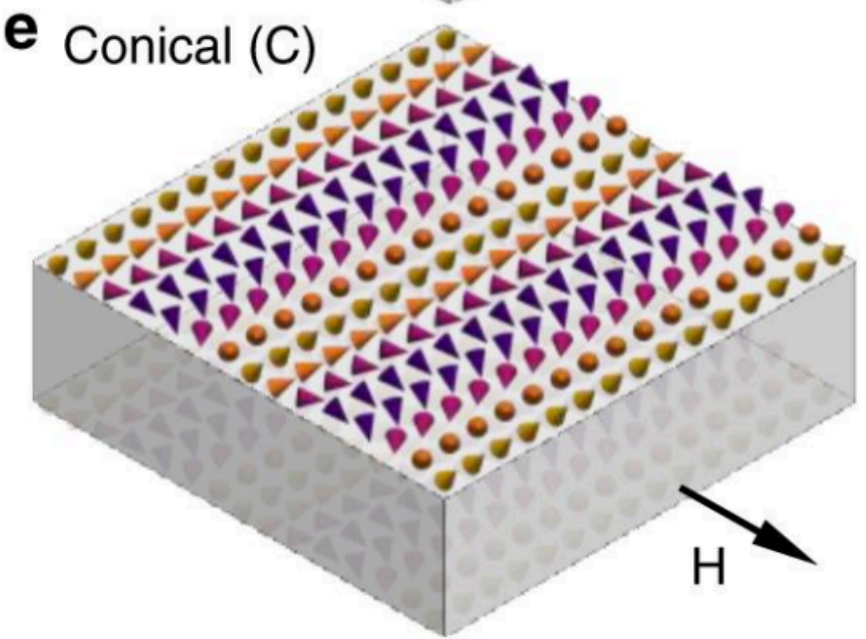
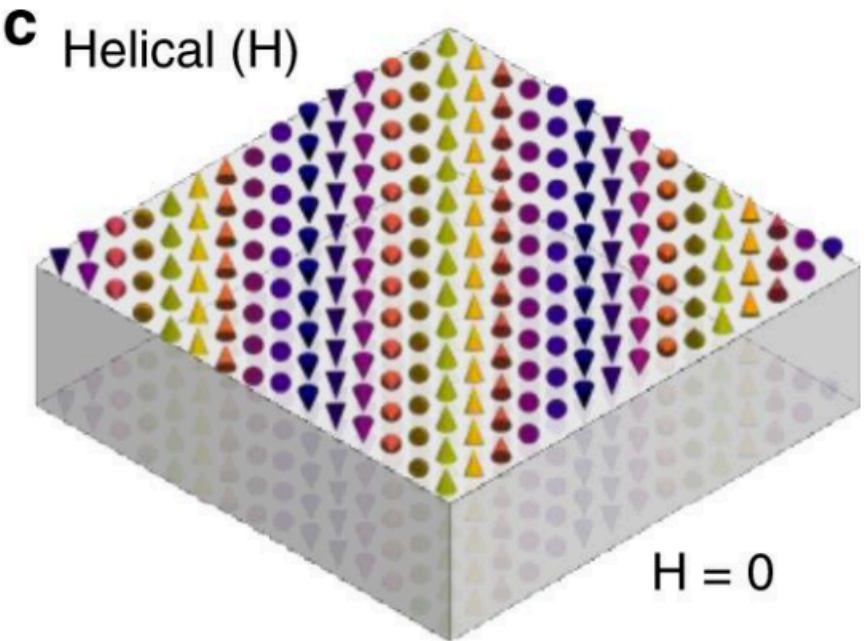
$$E(\boldsymbol{m}) = \int \mathrm{d}V \left[K \left(1 - m_z^2 \right) \quad + \quad A (\nabla \boldsymbol{m})^2 \quad + \quad \mathbf{D} : (\nabla \boldsymbol{m} \times \boldsymbol{m}) \right]$$

DMI can lead to „domains made of domain walls“

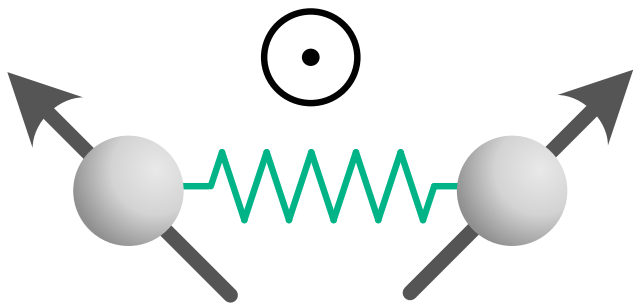
easy axis



$$\theta(y) = 2 \tan^{-1} \left[\exp \left(\frac{y - Y}{\Delta_B} \right) \right]$$



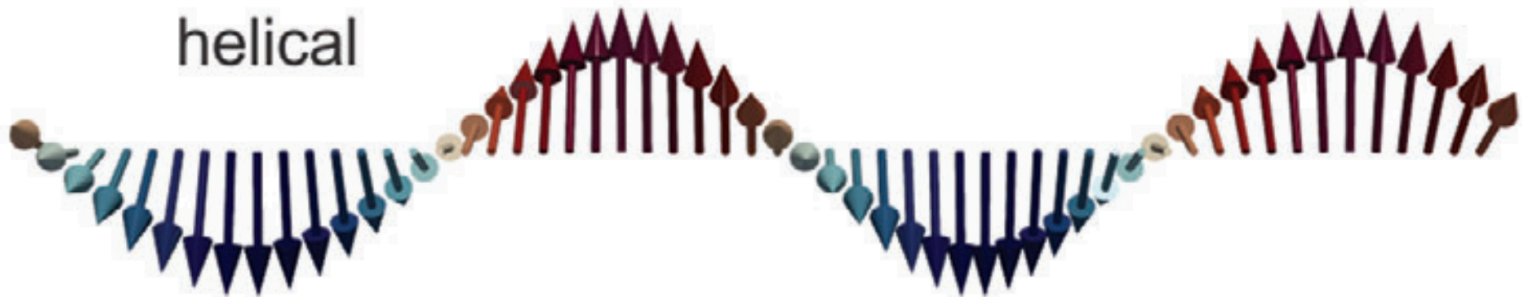
Birch et al.,
Nat. Comm. 11, 1726 (2020)



Dzyaloshinskii-Moriya Interaction (DMI)

$$E(\boldsymbol{m}) = \int \mathrm{d}V \left[K \left(1 - m_z^2 \right) + A (\nabla \boldsymbol{m})^2 + \boldsymbol{D} : (\nabla \boldsymbol{m} \times \boldsymbol{m}) \right]$$

Can lead to conical/helical order!



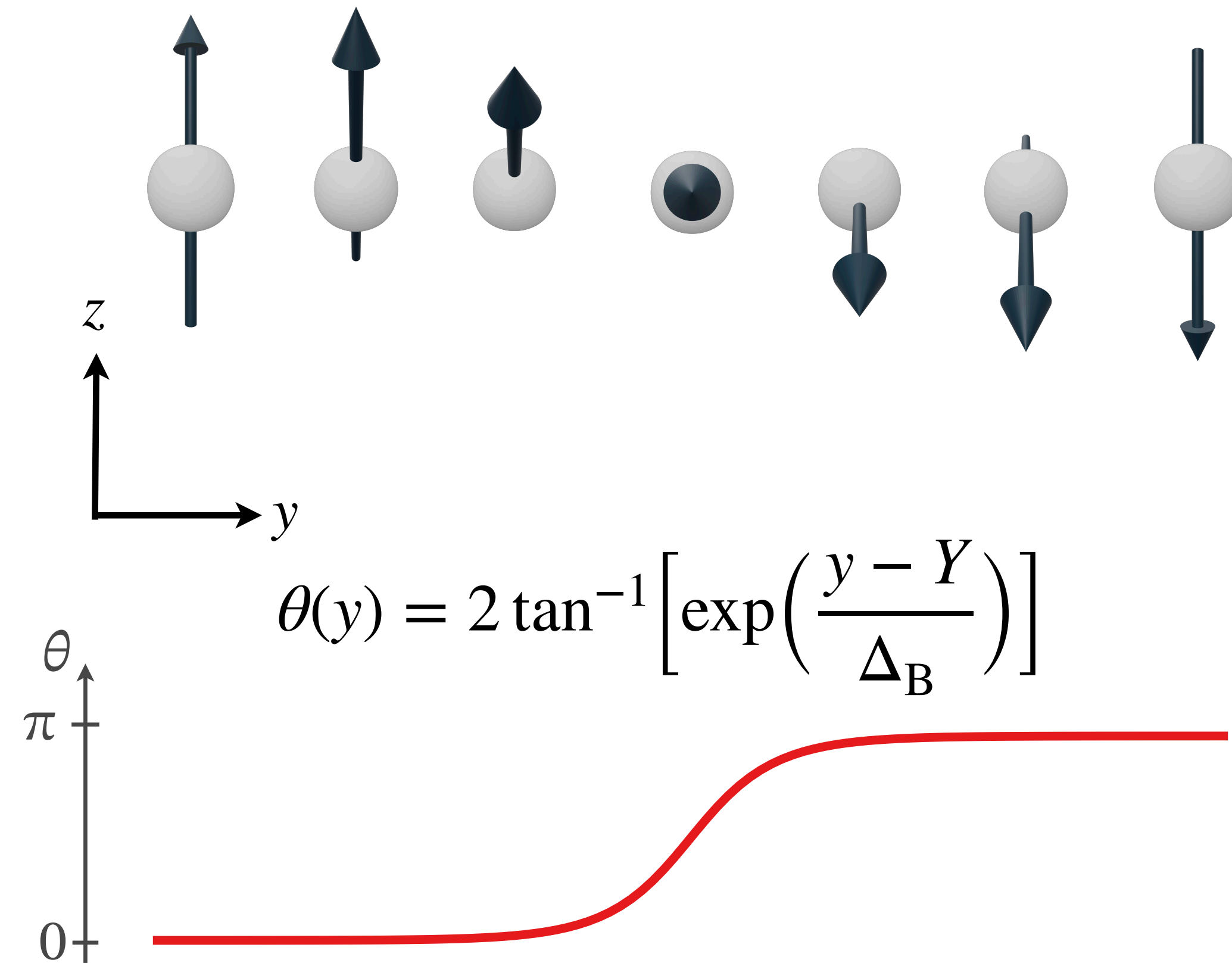
Order parameter:
Fourier component $m(Q)$



$\longrightarrow Q$

Brearton et al.,
Adv. Physics Res. 2, 2200116 (2023)

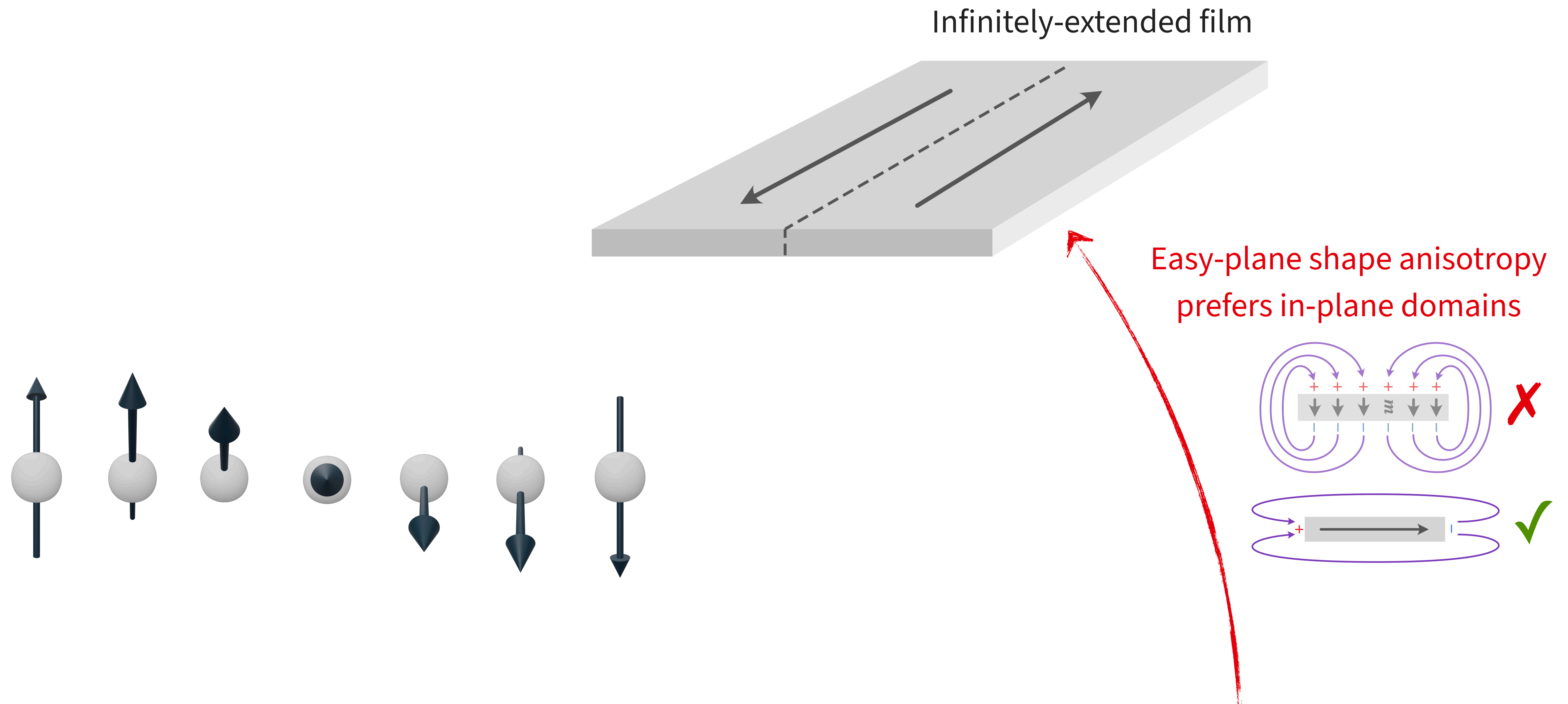
Does dipolar interaction influence domain walls?



What about dipolar interaction?

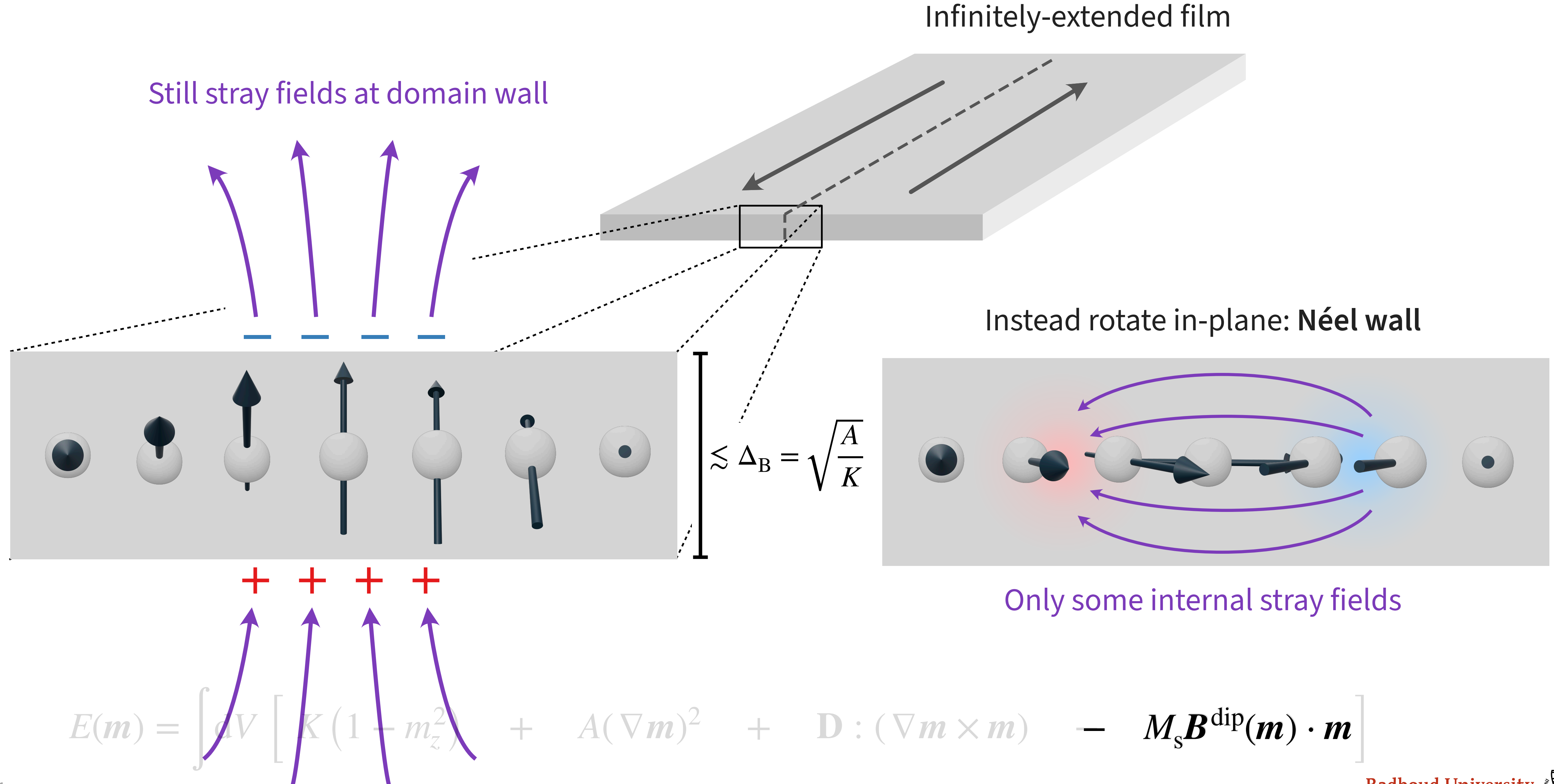
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Dipolar interactions become important in films



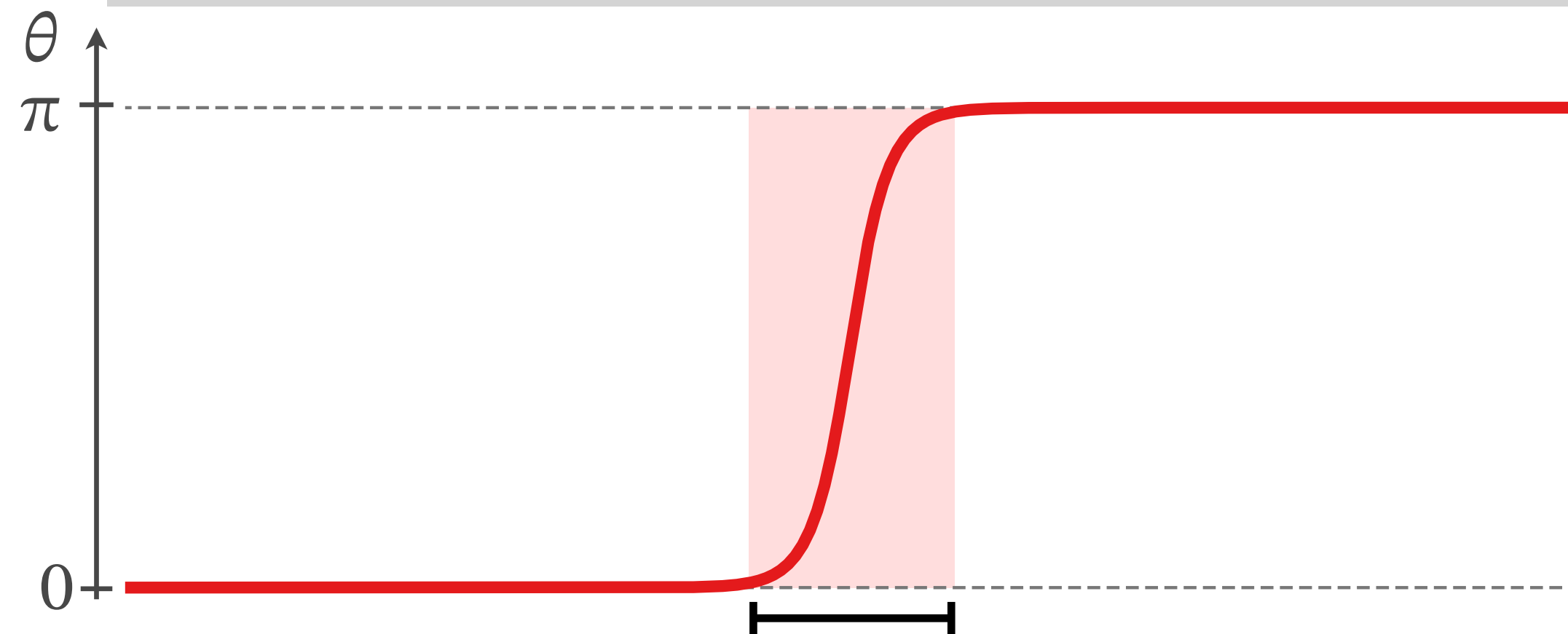
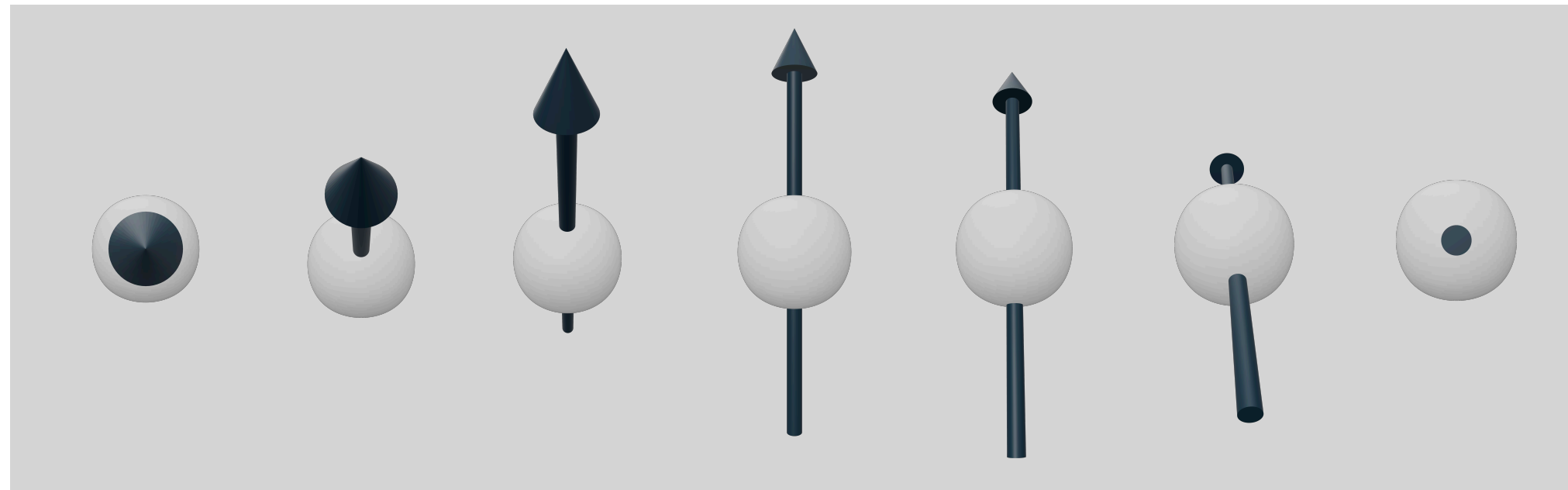
$$E(\mathbf{m}) = \int dV \left[K (1 - m_z^2) + A (\nabla \mathbf{m})^2 + \mathbf{D} : (\nabla \mathbf{m} \times \mathbf{m}) - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Dipolar interactions become important in films



Néel walls are preferred in thin magnetic fields

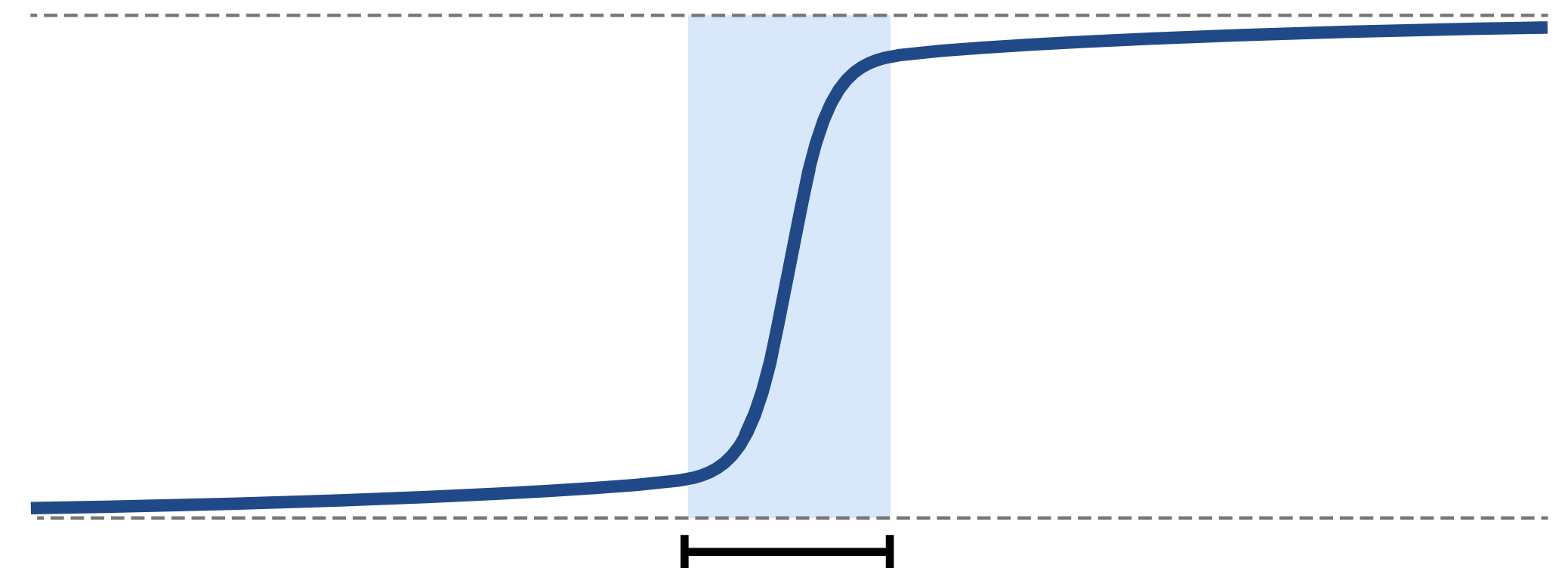
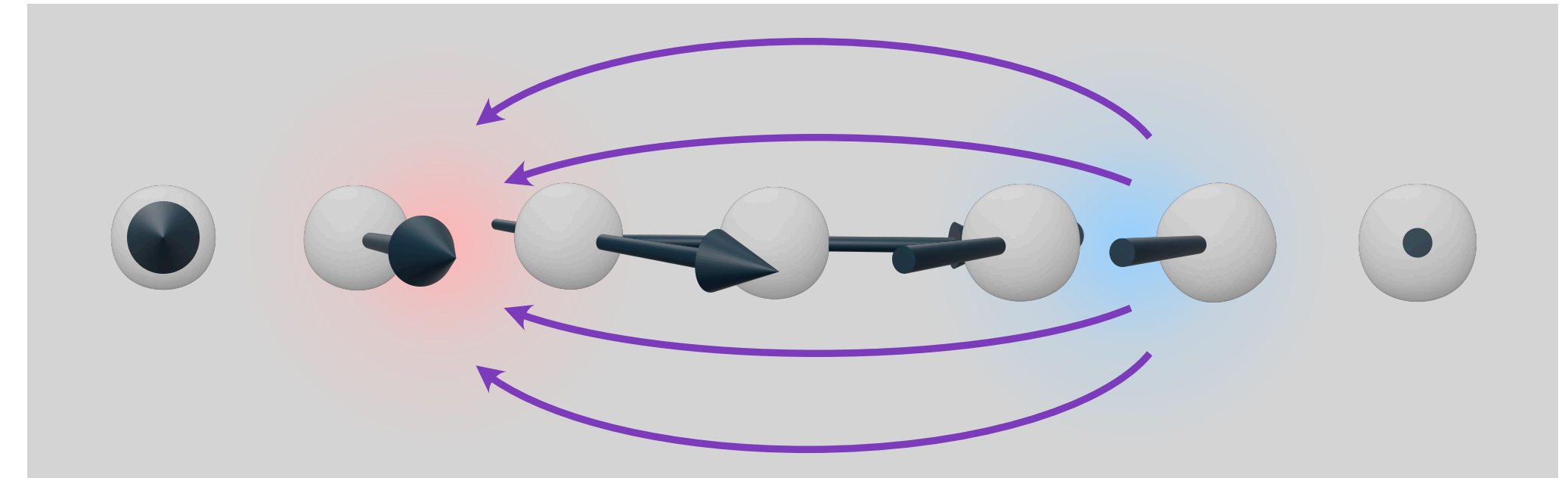
Bloch wall



$$\Delta_B = \sqrt{\frac{A}{K}}$$

Exchange-anisotropy length
exchange vs. anisotropy

Néel wall

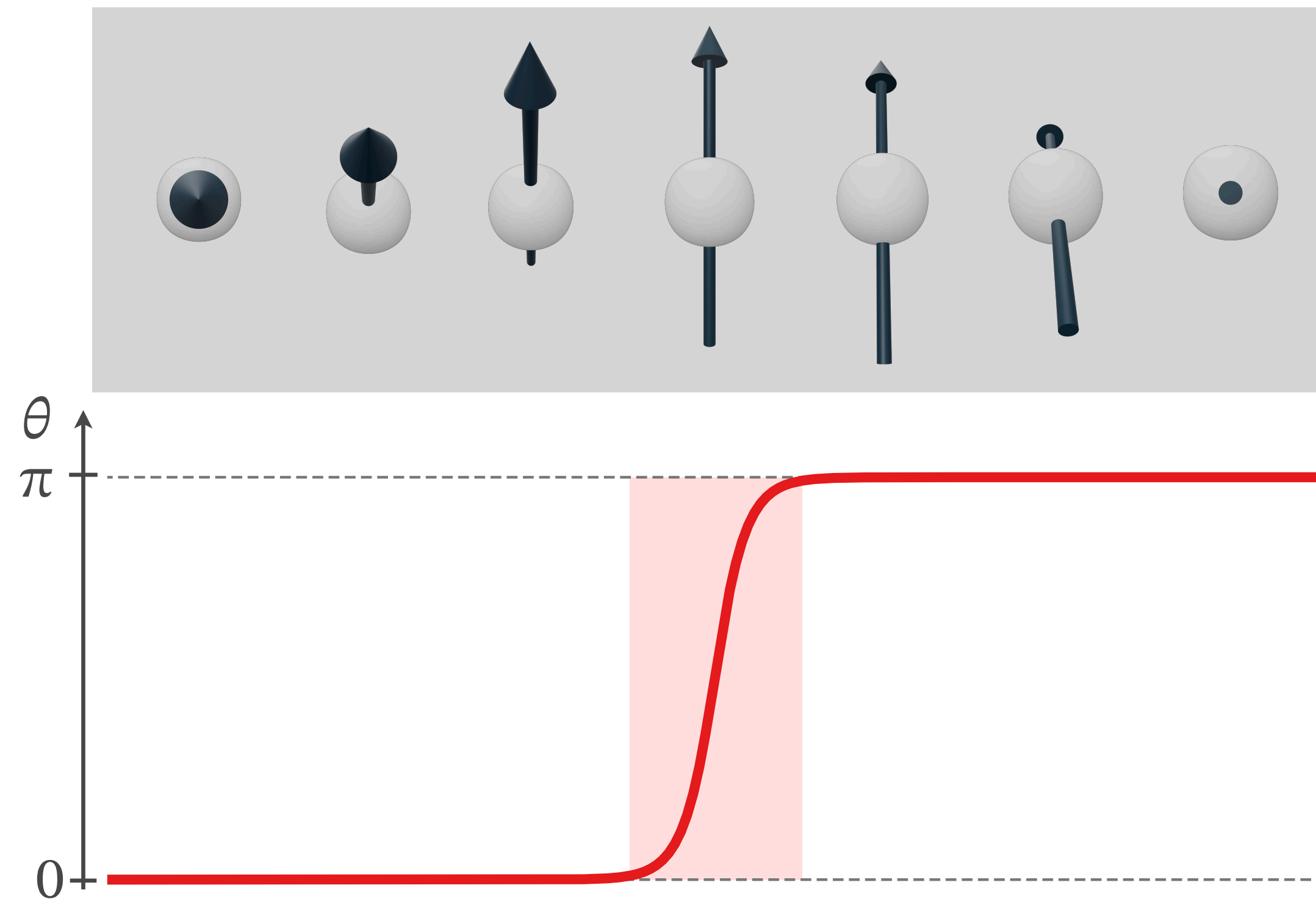


$$\Delta_N = \sqrt{\frac{2A}{\mu_0 M_s^2}}$$

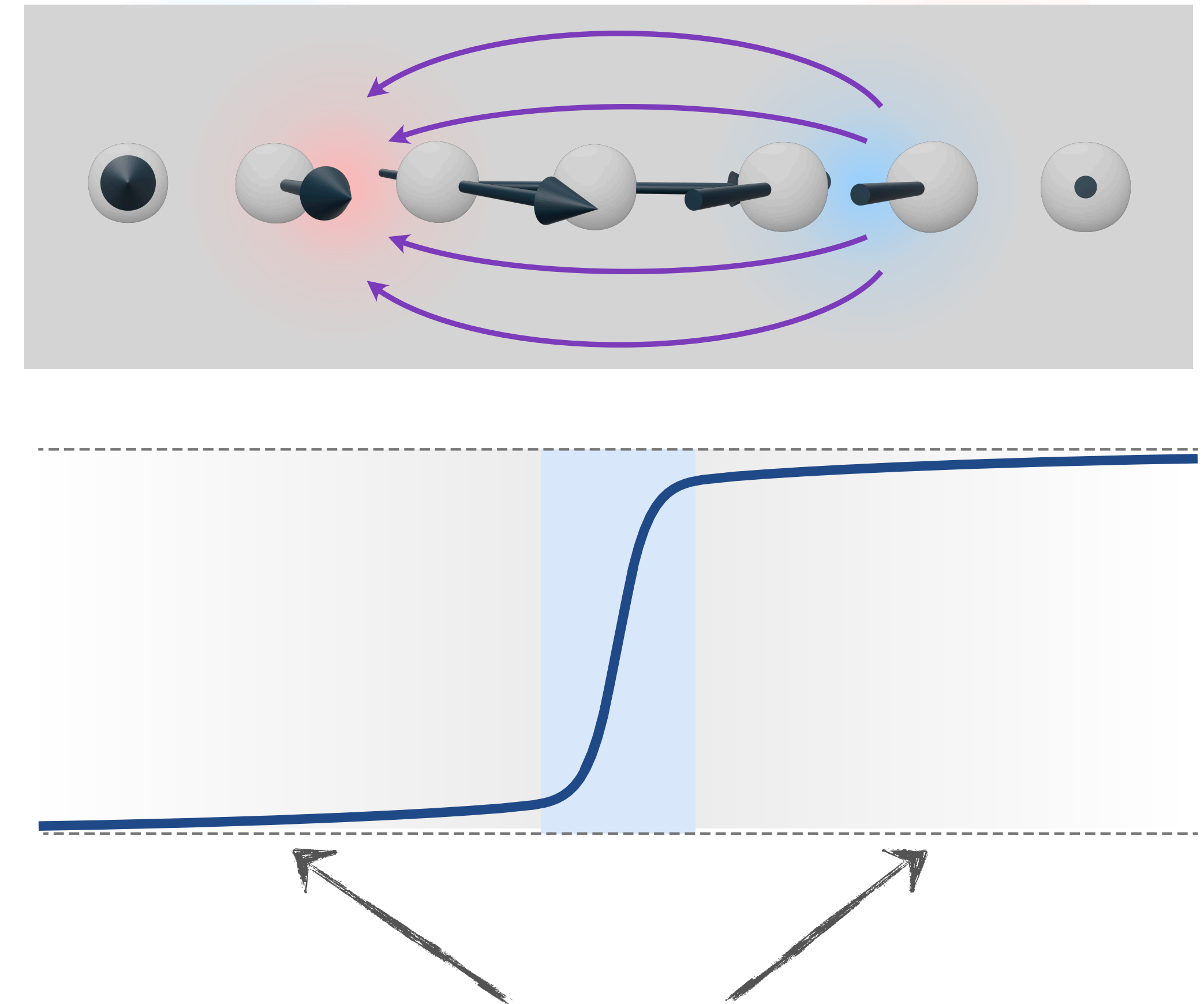
Dipole-exchange length
exchange vs. dipolar

Néel walls are preferred in thin magnetic fields

Bloch wall

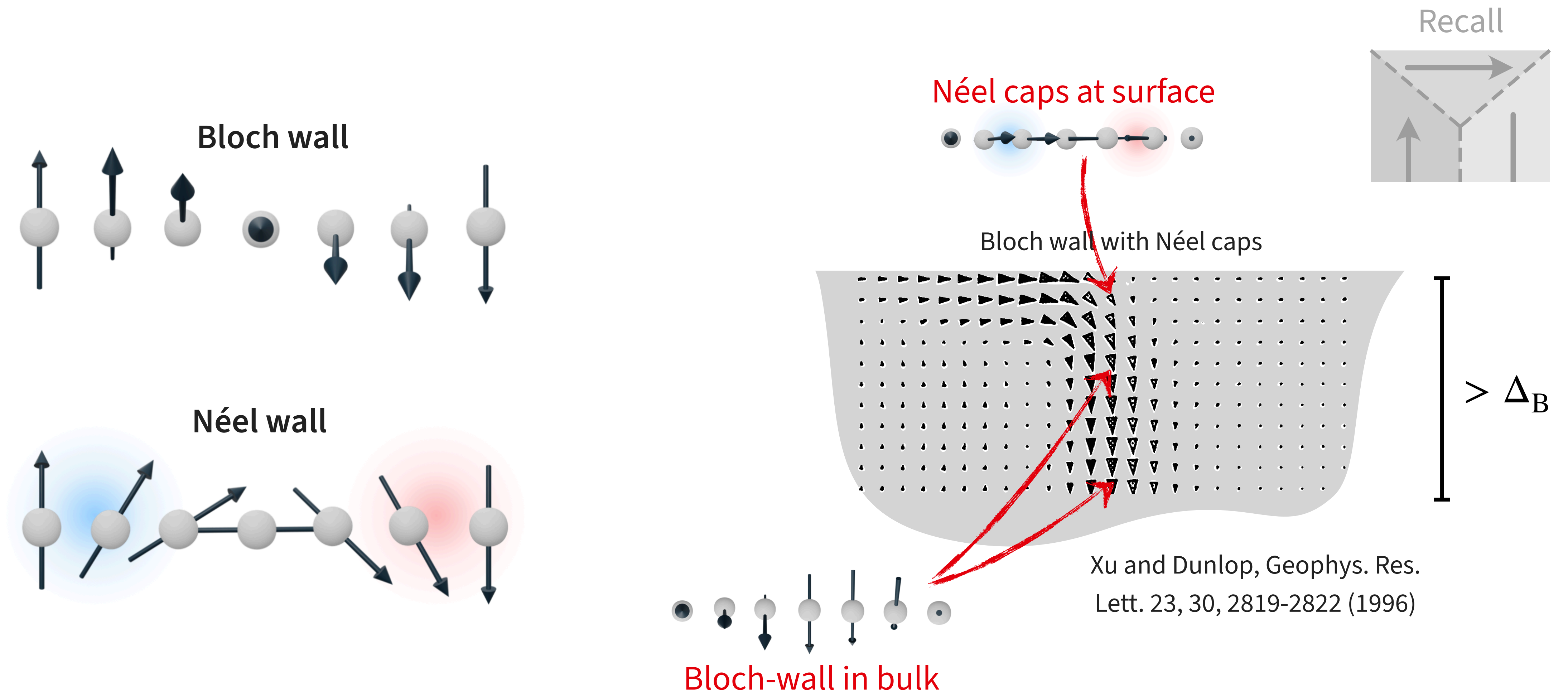


Néel wall

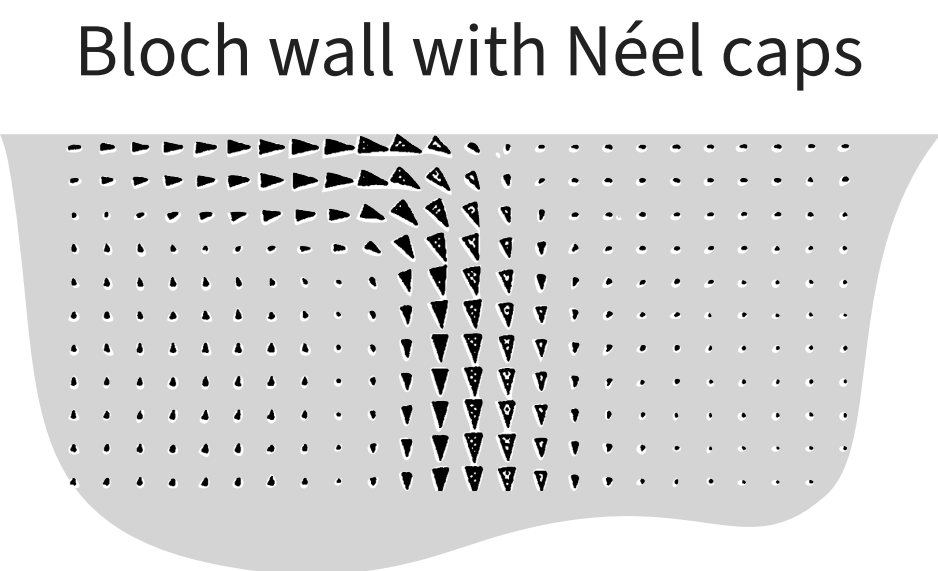
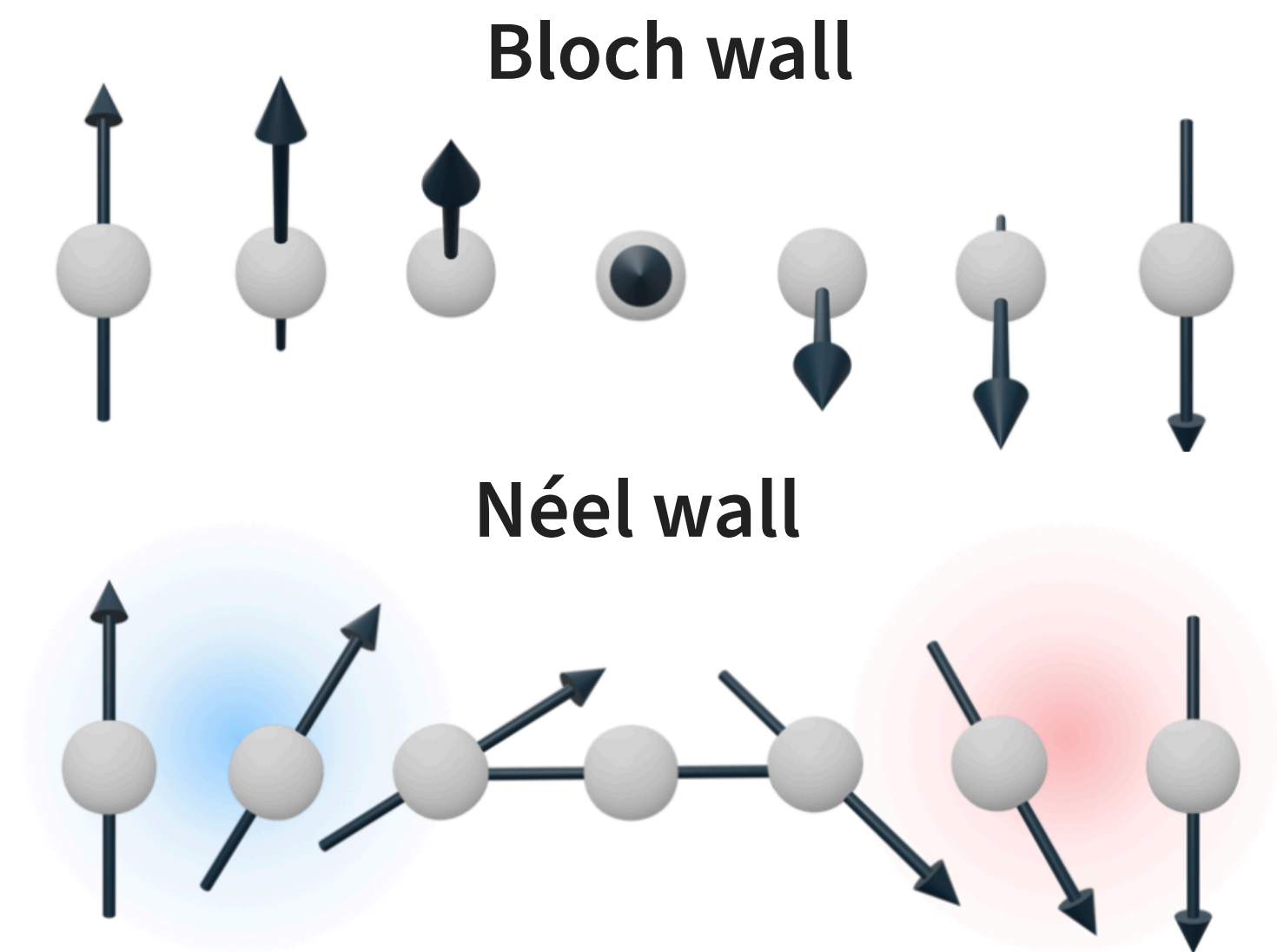


Careful: Néel is more complicated than Bloch!
Slow rotation outside of core (from dipolar interactions)

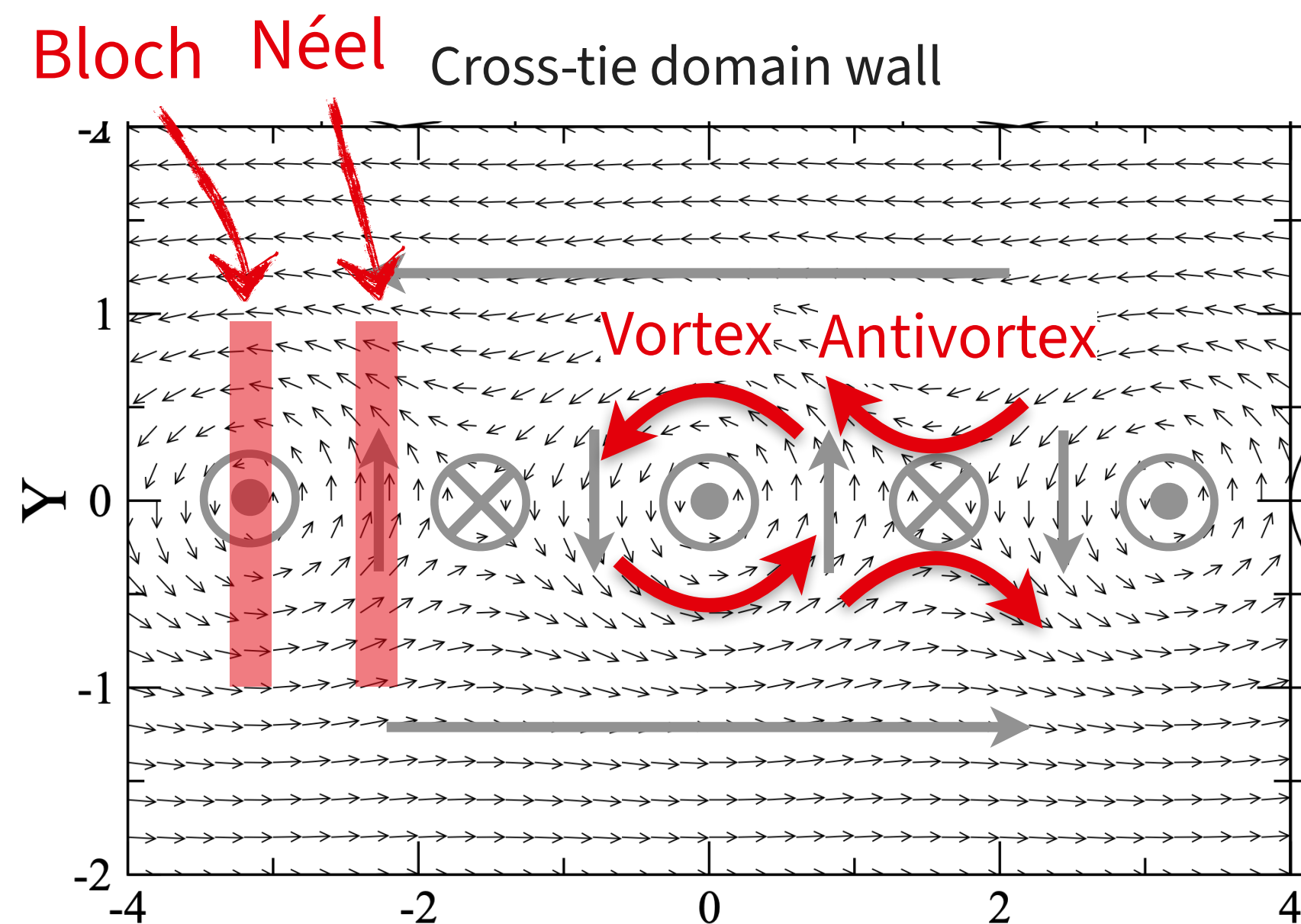
There is no sharp boundary between Néel walls and Bloch walls



There is no sharp boundary between Néel walls and Bloch walls

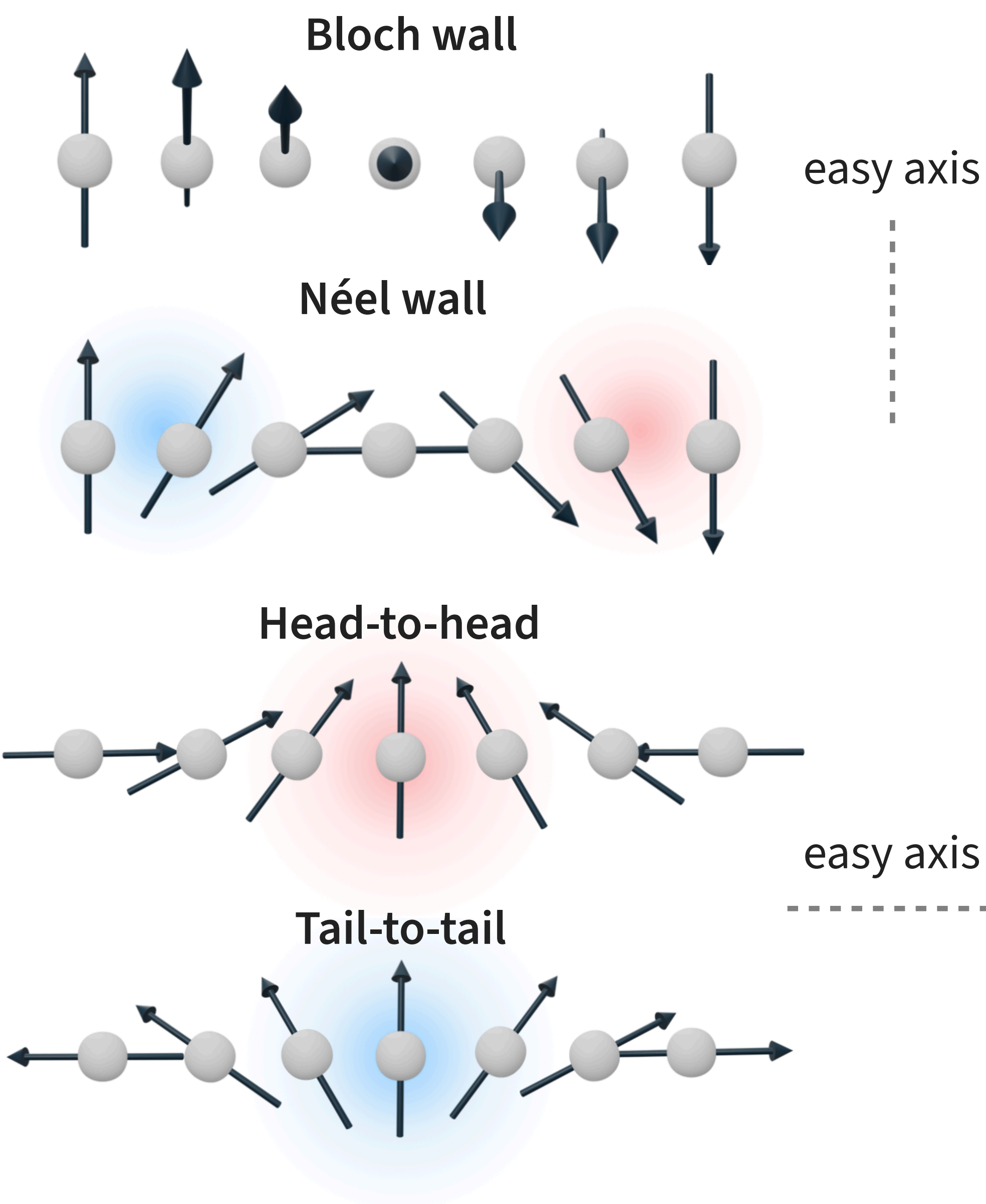


Xu and Dunlop, Geophys. Res. Lett. 23, 30, 2819-2822 (1996)

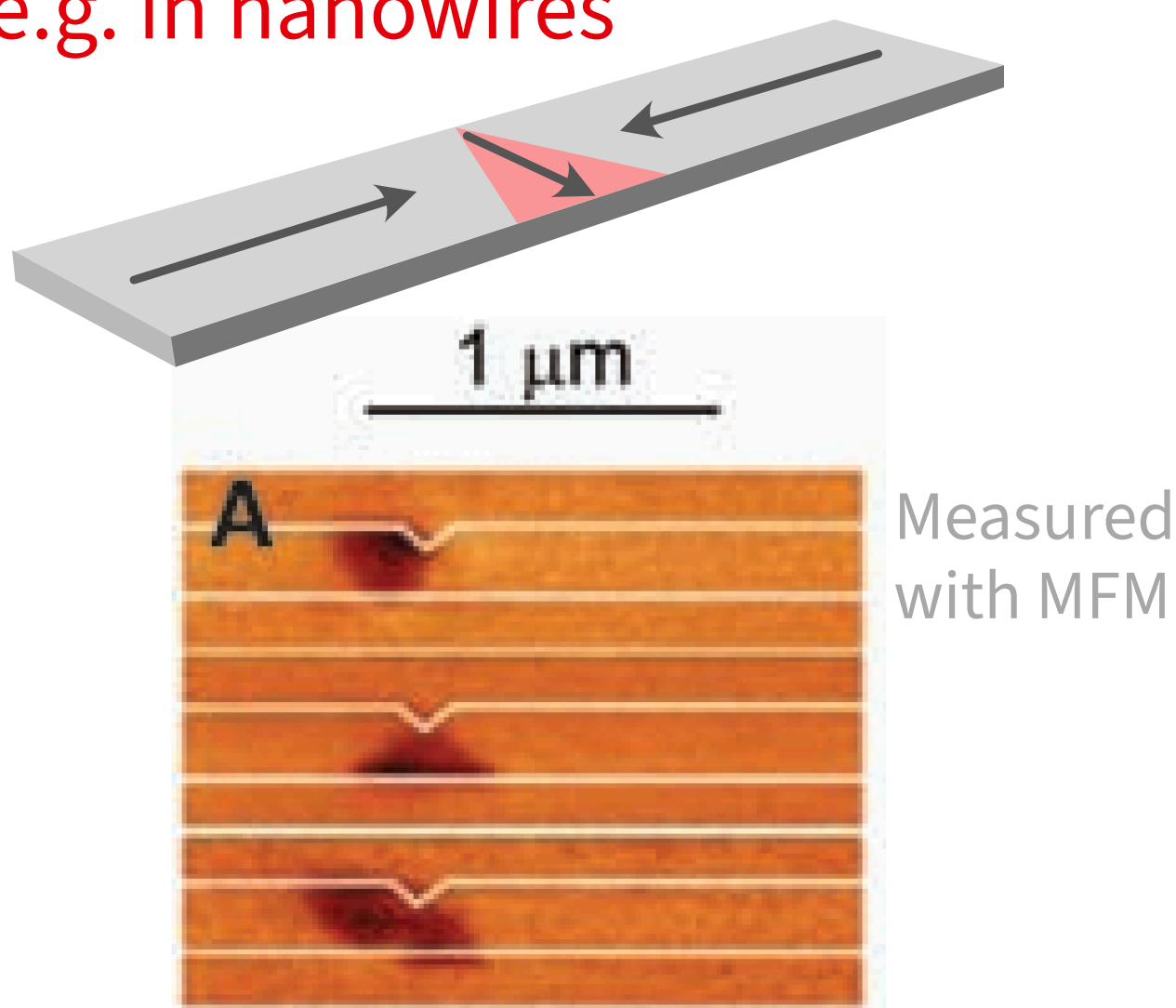


edited from Metlov,
APL 79, 2609-2611 (2001)

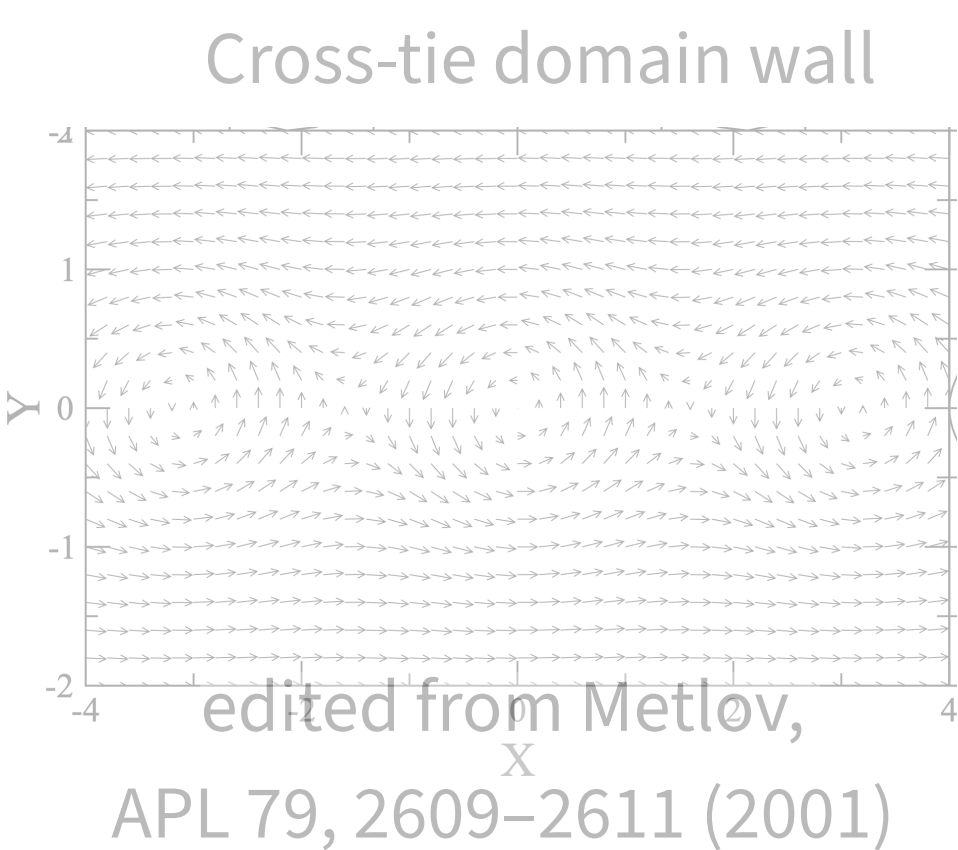
Other types of domain walls are also possible



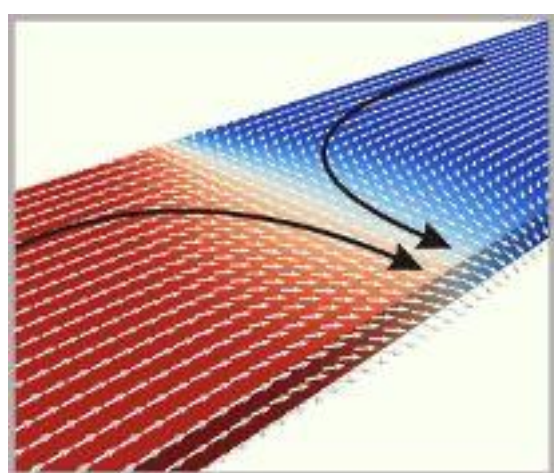
e.g. in nanowires



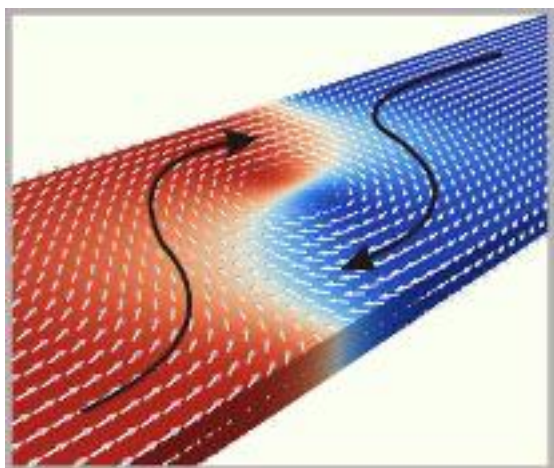
Parkin et al., Science 320, 5873 (2008)



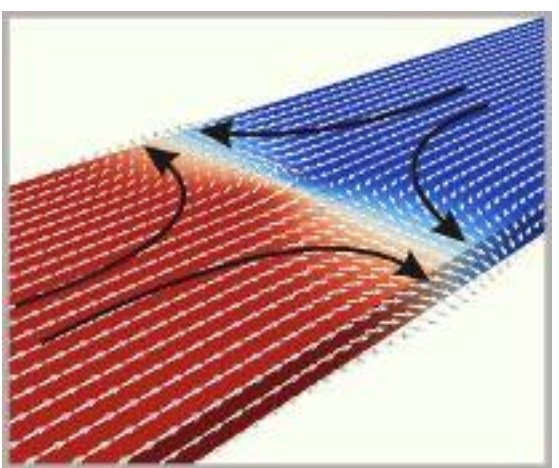
Transverse head-to-head



Vortex wall

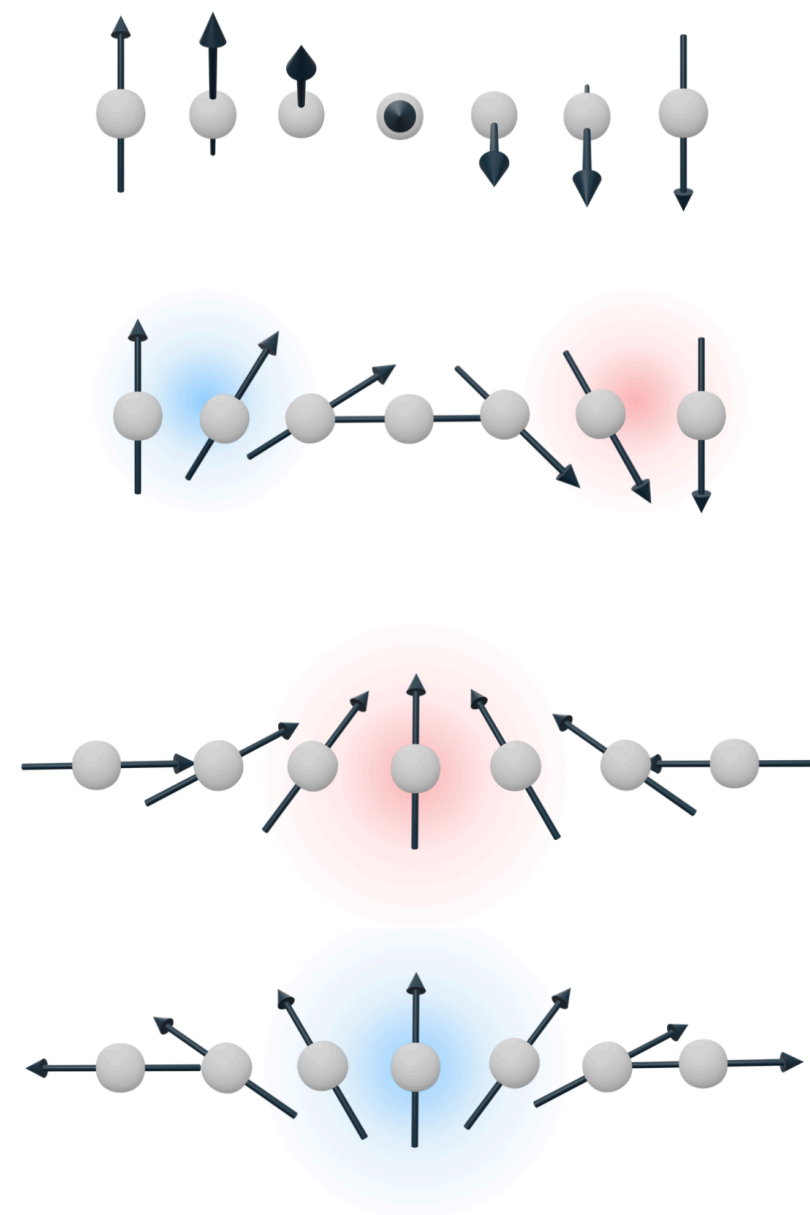


Antivortex wall



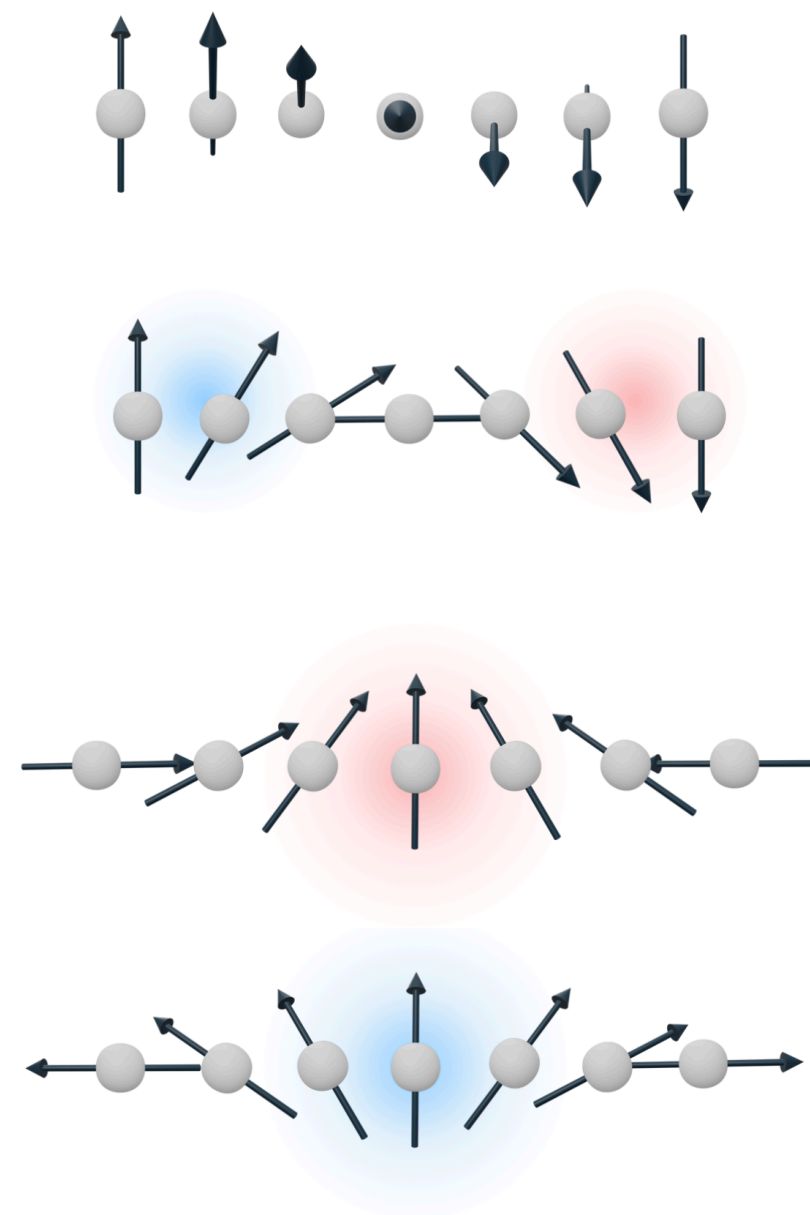
© Attila Kákay

Structure and types of domain walls

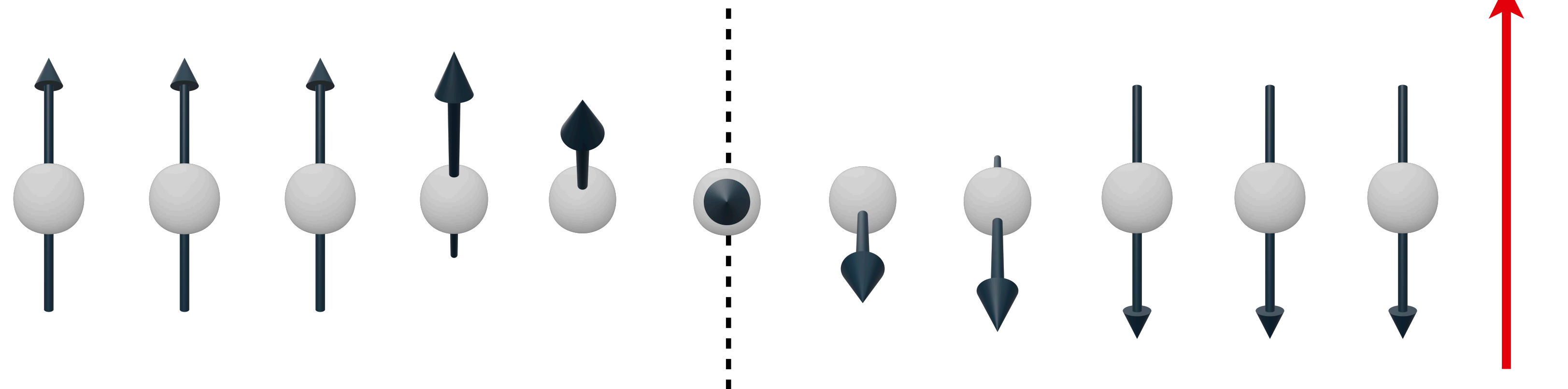


- Often smooth transitions between neighboring domains
- Scale by competition exchange vs. anisotropy/dipolar
- Domain vs. domain wall energy $\rightarrow$ number of domains
- Different types, depending on interactions
 - Bulk and/or high-anisotropy magnets $\rightarrow$ Bloch
 - Thin films and/or dominating dipolar interactions $\rightarrow$ Néel
 - Other types and mixtures possible

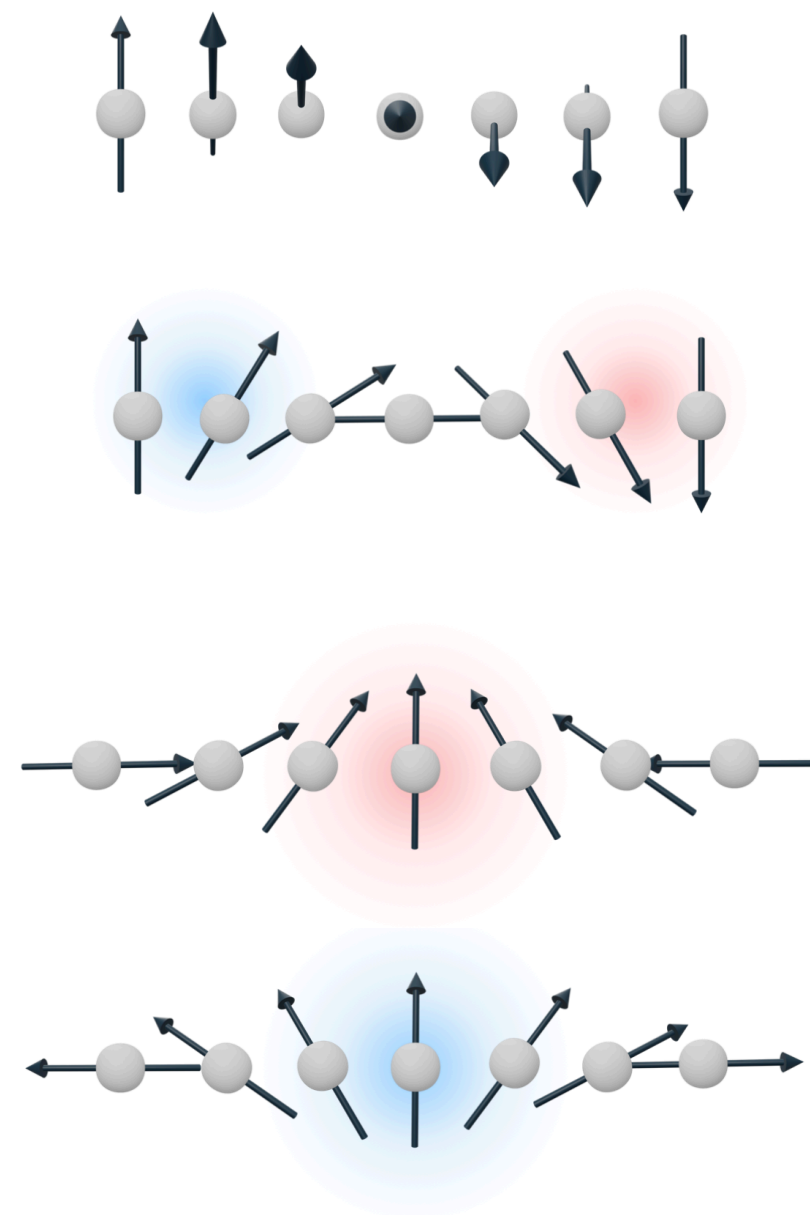
Structure and types of domain walls



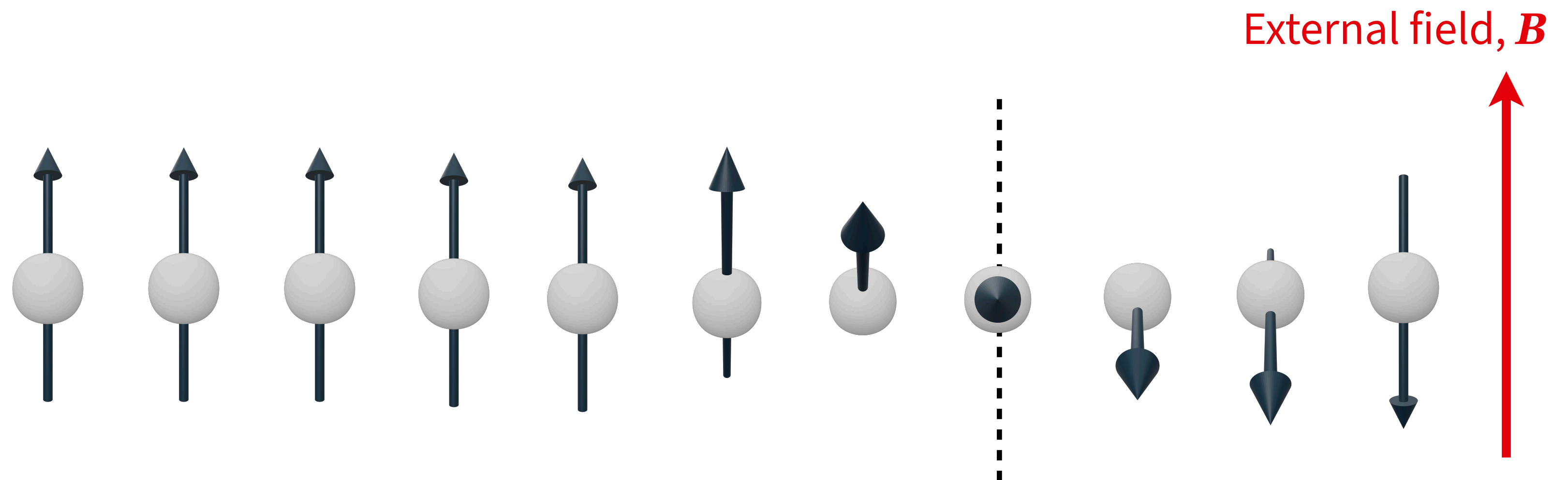
So far, we left one interaction out...



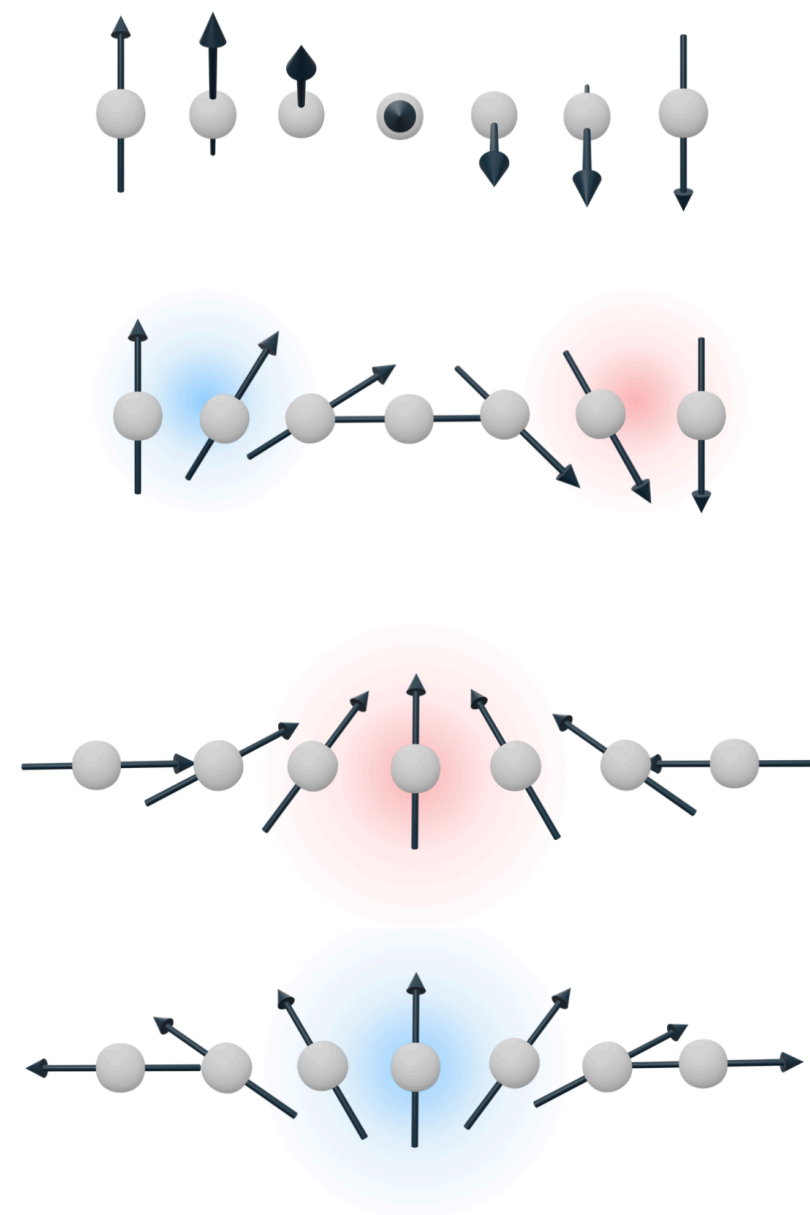
Structure and types of domain walls



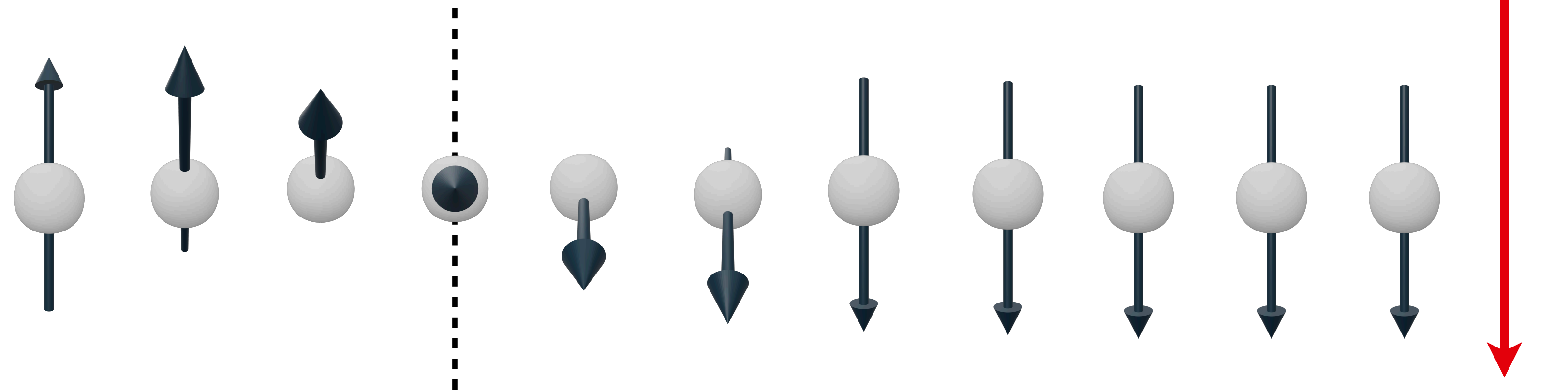
External fields move domain walls



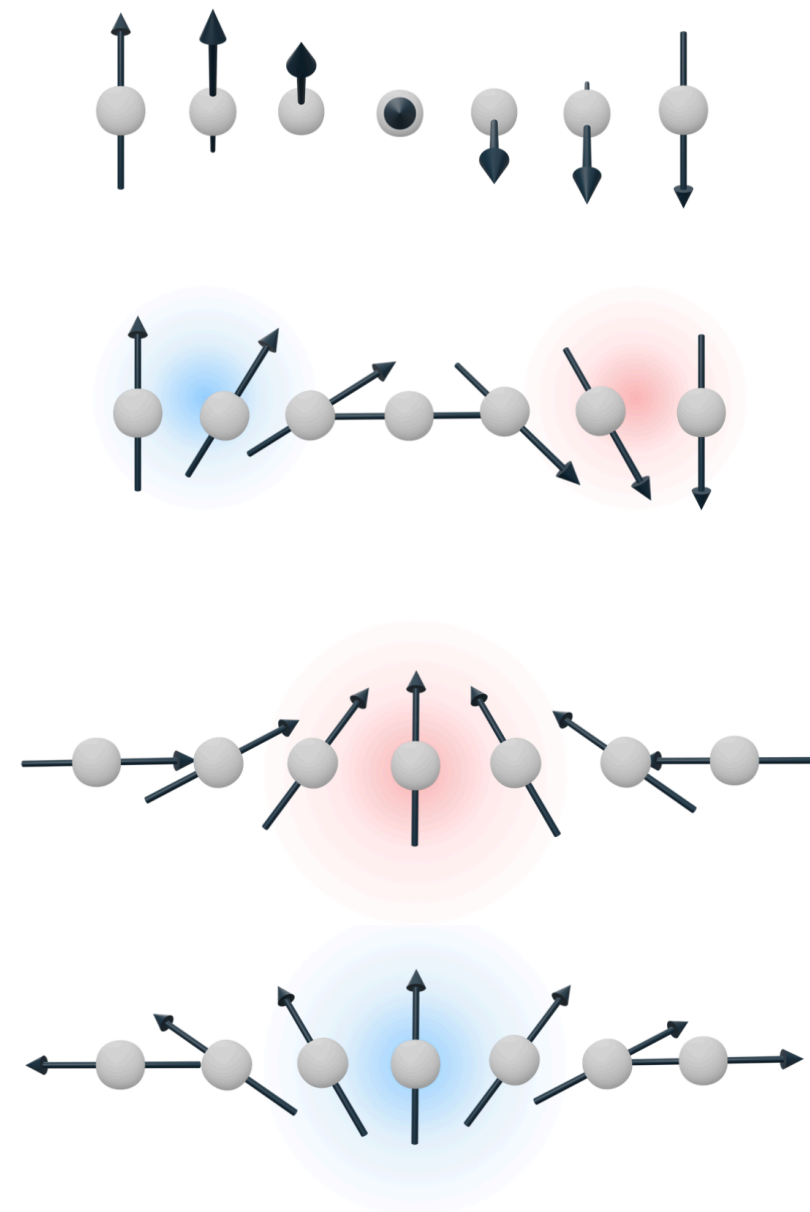
Structure and types of domain walls



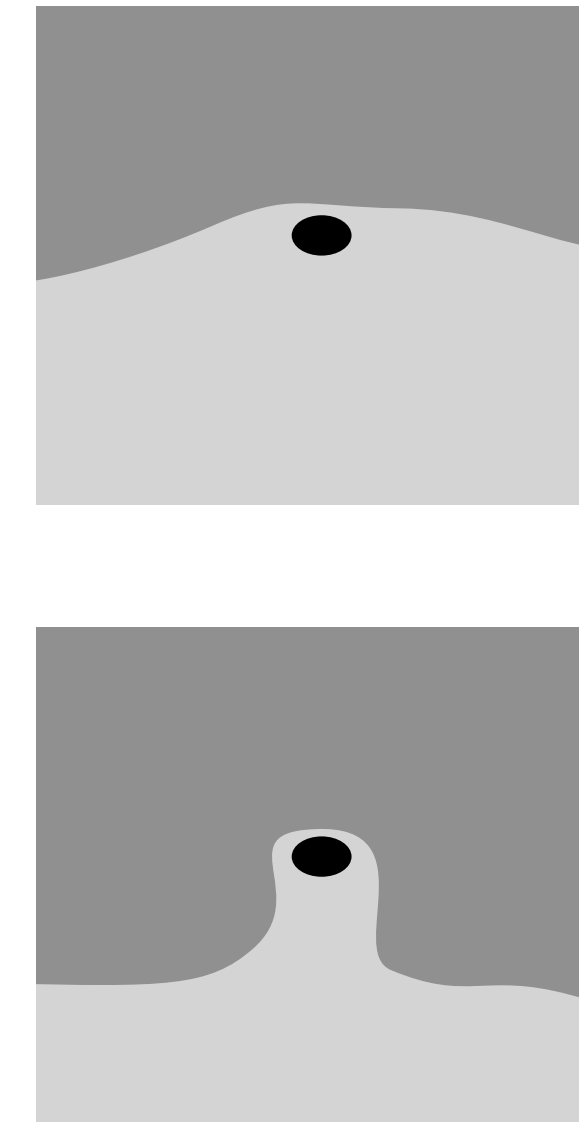
External fields move domain walls



Structure and types of domain walls

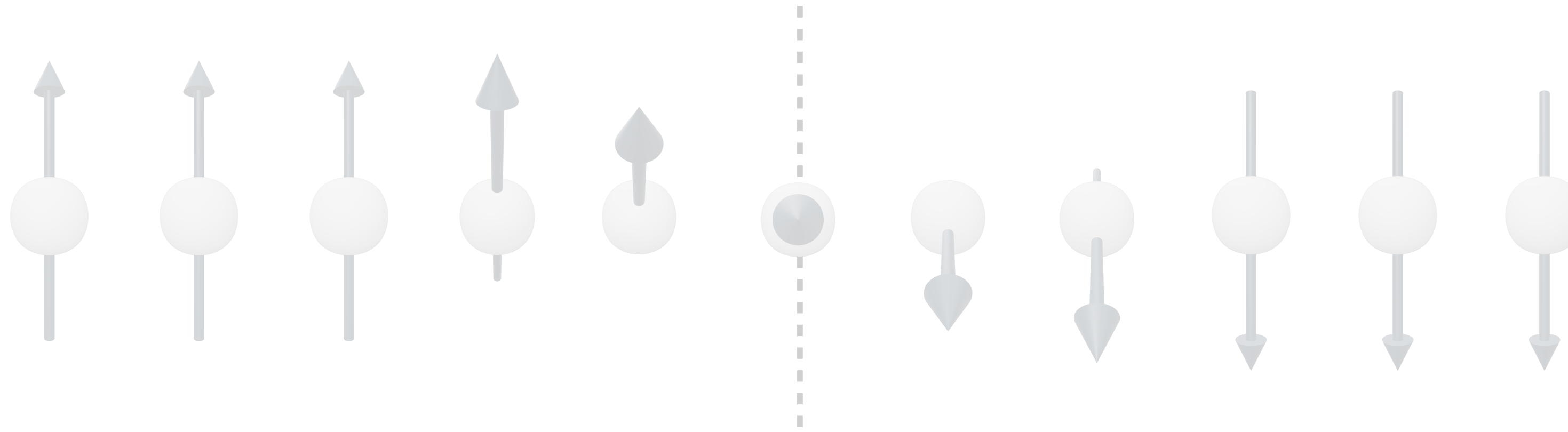


Domain-wall motion

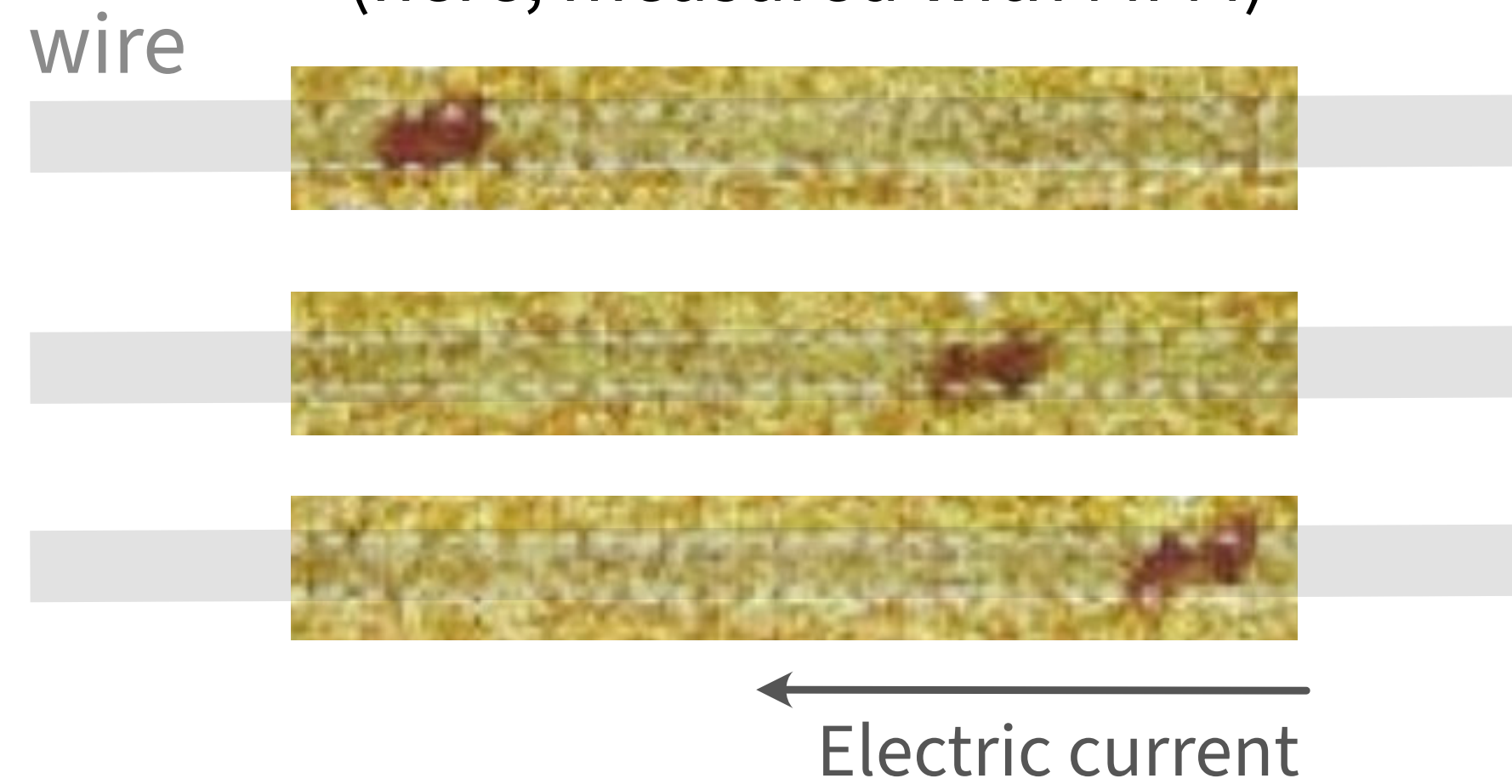


Domain-wall motion

Domain walls can also be moved by currents



Also: current-induced DW motion
(here, measured with MFM)

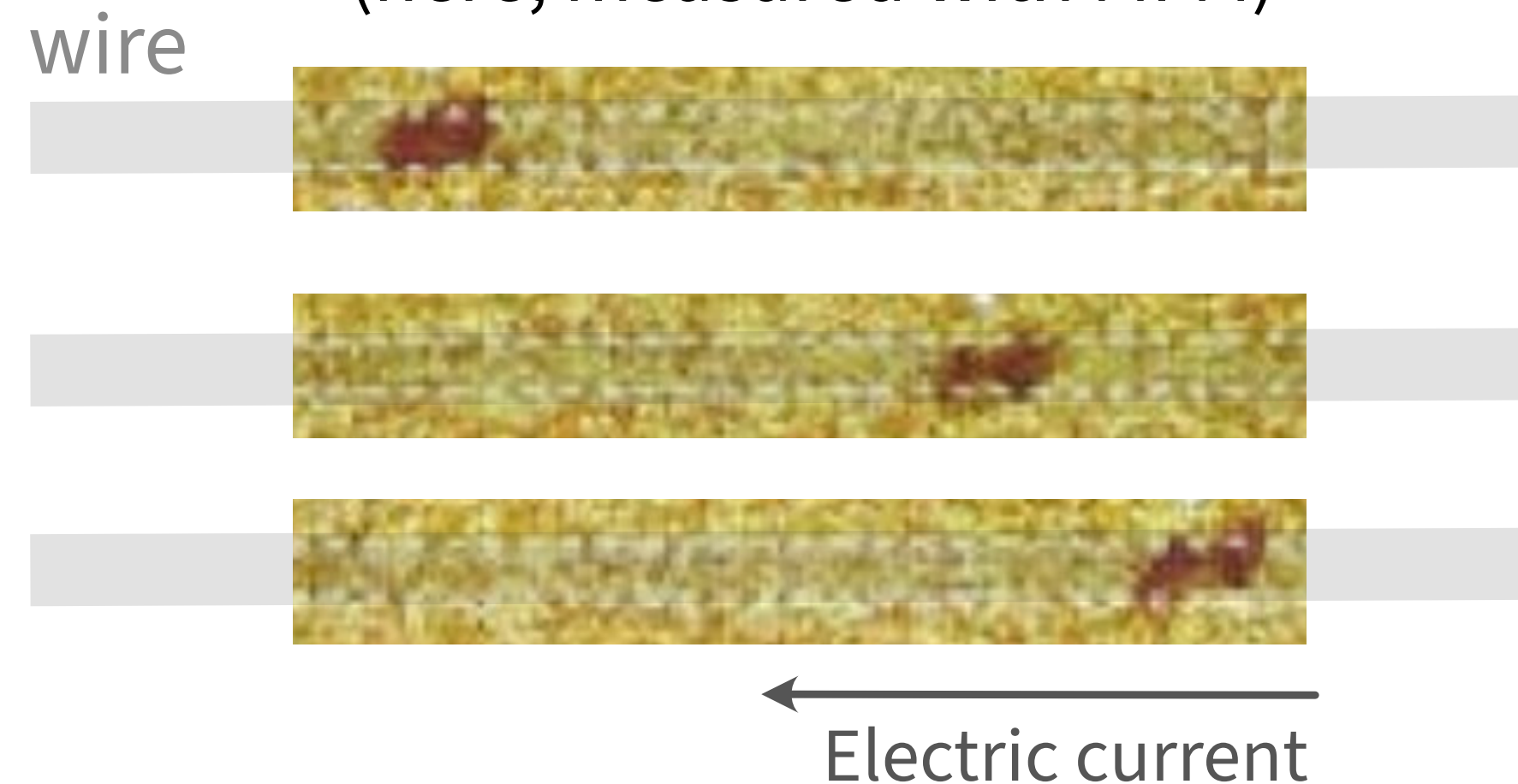


Yamaguchi, et al. PRL 92.7, 077205 (2004)
(edited)

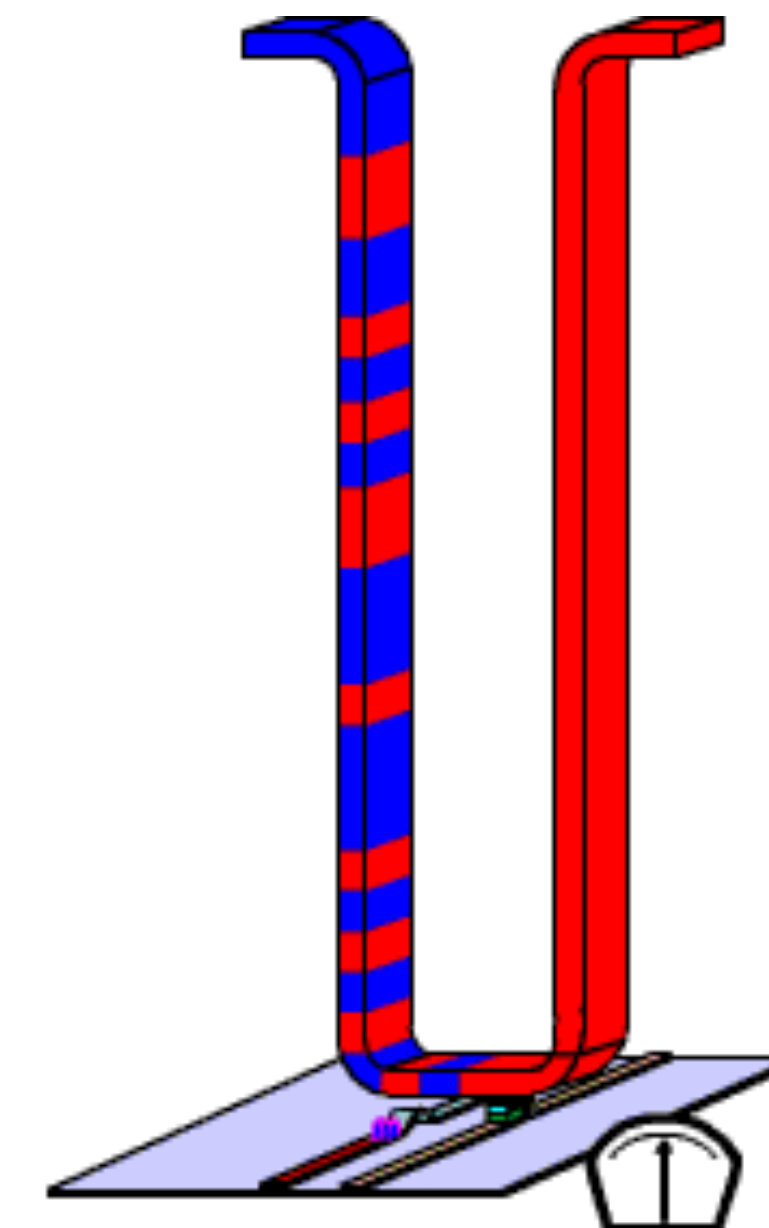
Domain walls can also be moved by currents (interesting for memory)



Also: current-induced DW motion
(here, measured with MFM)



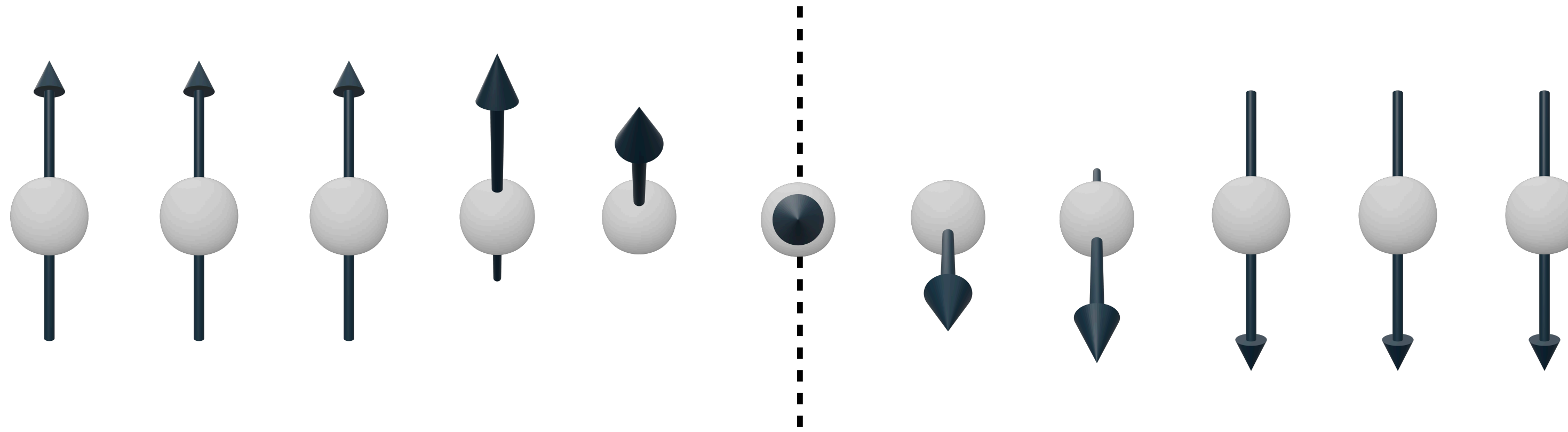
Yamaguchi, et al. PRL 92.7, 077205 (2004)
(edited)



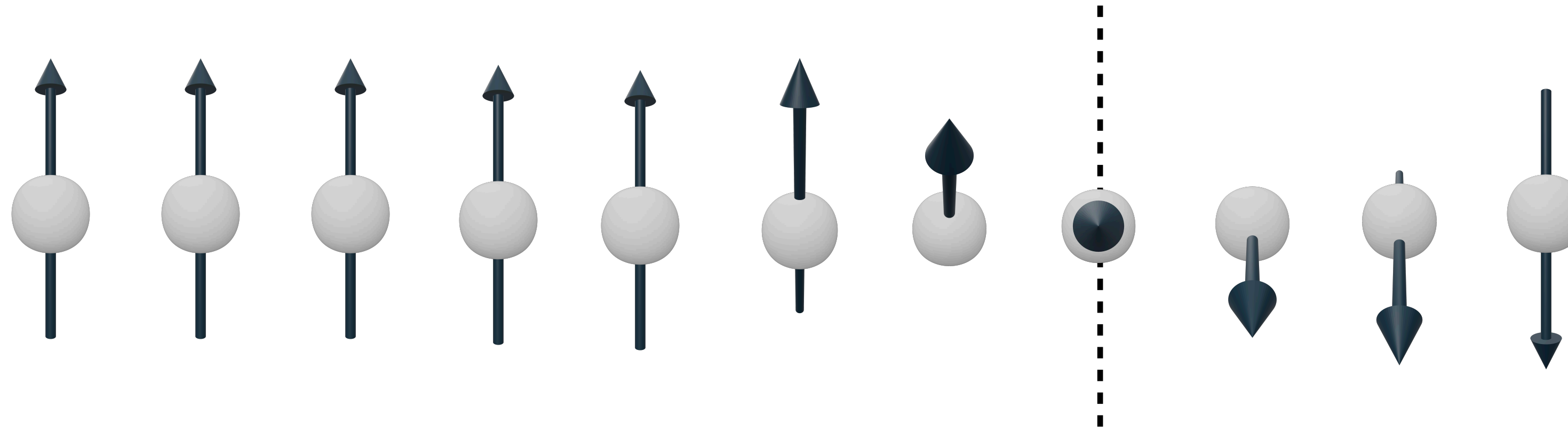
**Domain-wall based
racetrack memory**

Parkin et al., Science 320, 5873 (2008)
Zhang et al., Sci. Rep. 5, 7643 (2015)

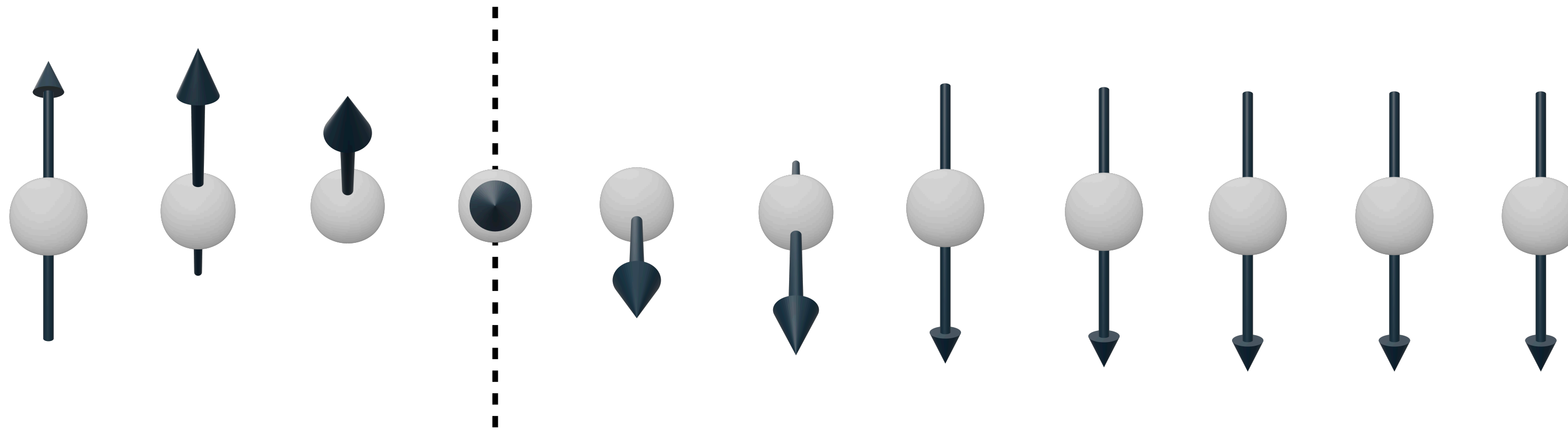
This is not actually how domain walls move...



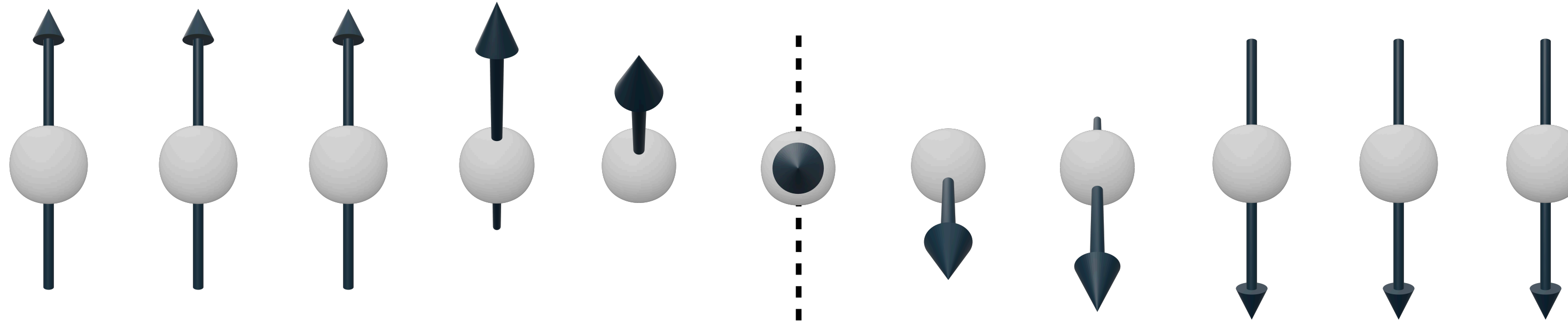
This is not actually how domain walls move...



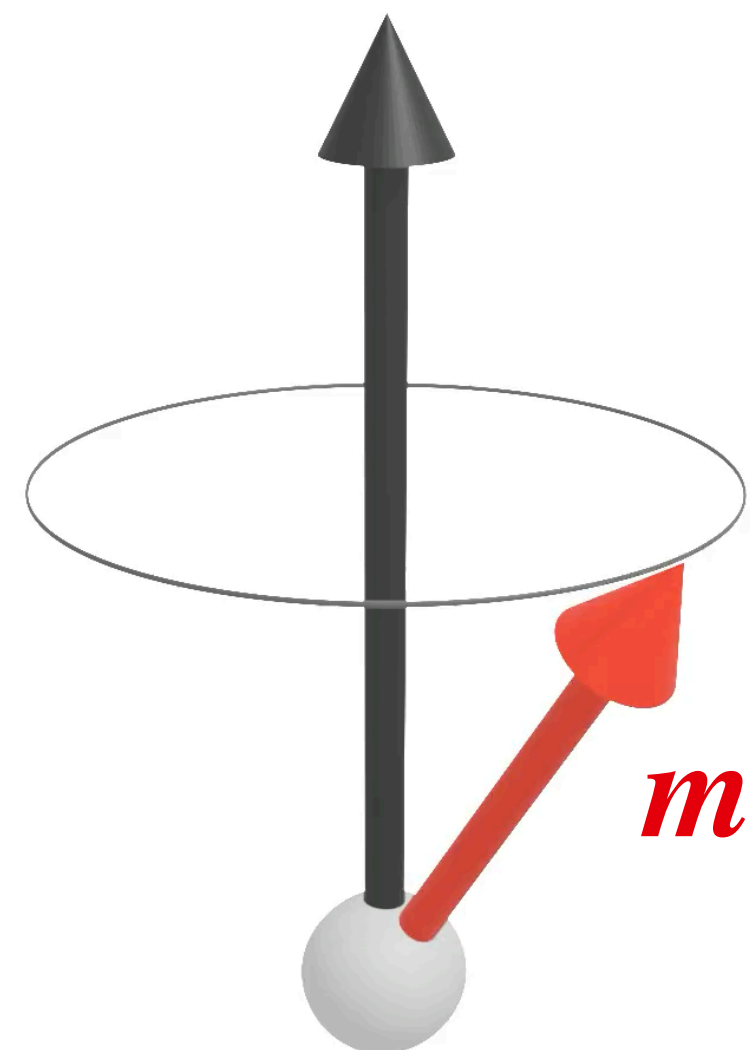
This is not actually how domain walls move...



Magnetic moments / magnetization need to satisfy Landau-Lishitz equation



$$B_{\text{eff}} = -\frac{1}{M_s} \frac{\delta E}{\delta m}$$



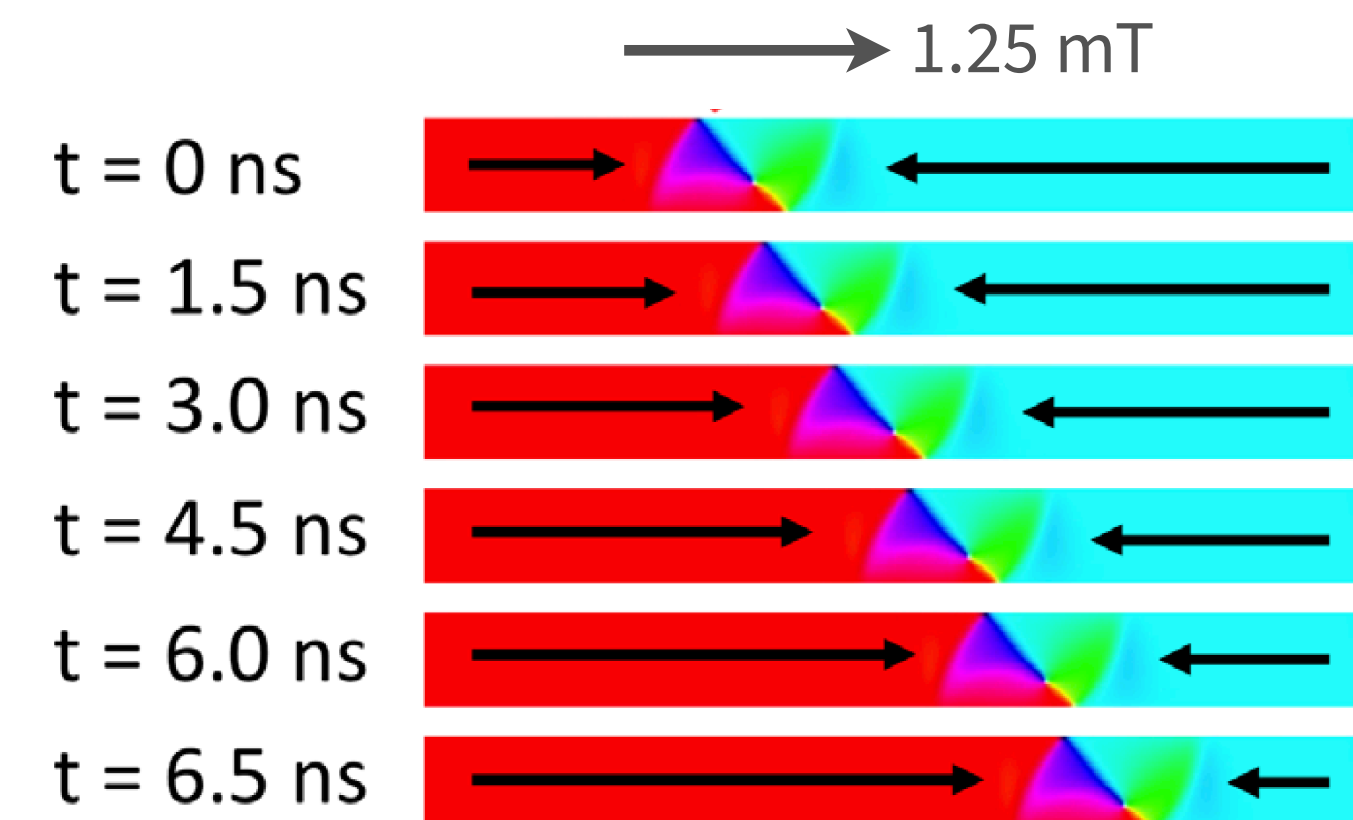
Dynamics need to satisfy
Landau-Lifshitz-Gilbert equation

$$\frac{dm}{dt} = \frac{\gamma}{M_s} \left[m \times \frac{\delta E}{\delta m} \right] + \alpha m \times \frac{dm}{dt}$$

See Lecture „Spin waves and Magnonics“

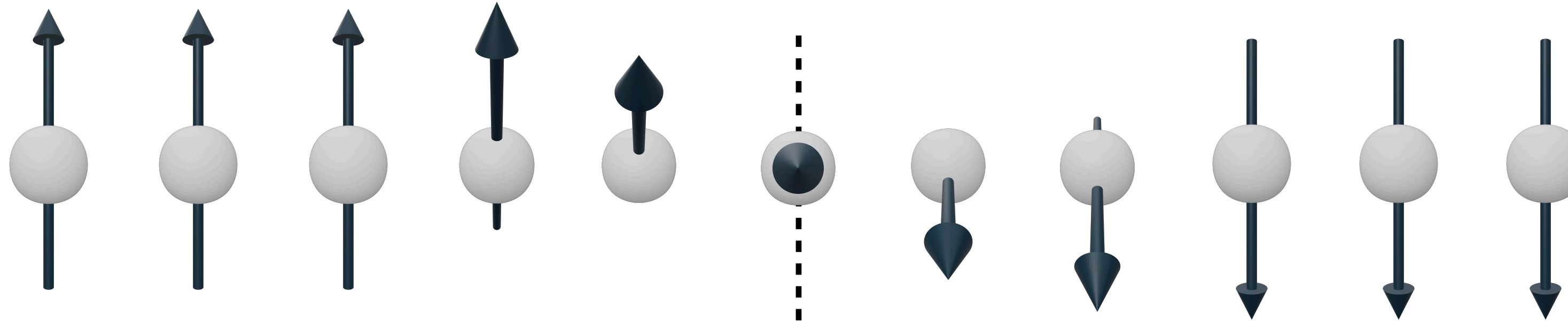
Solve numerically

$m(r, t)$ Micromagnetic simulation

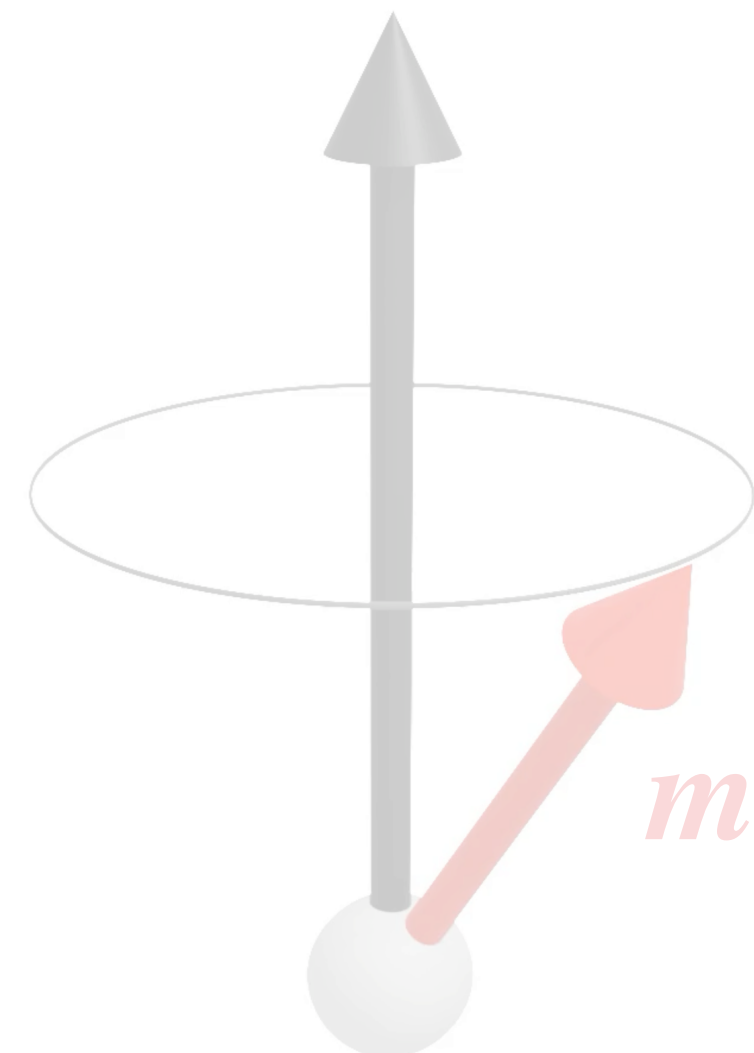


Hayward and Omari,
J. Phys. D: Appl. Phys. 50, 084006 (2017)

Magnetic moments / magnetization need to satisfy Landau-Lifshitz equation



$$B_{\text{eff}} = -\frac{1}{M_s} \frac{\delta E}{\delta m}$$



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$$\frac{dm}{dt} = \frac{\gamma}{M_s} \left[m \times \frac{\delta E}{\delta m} \right] + \alpha m \times \frac{dm}{dt}$$

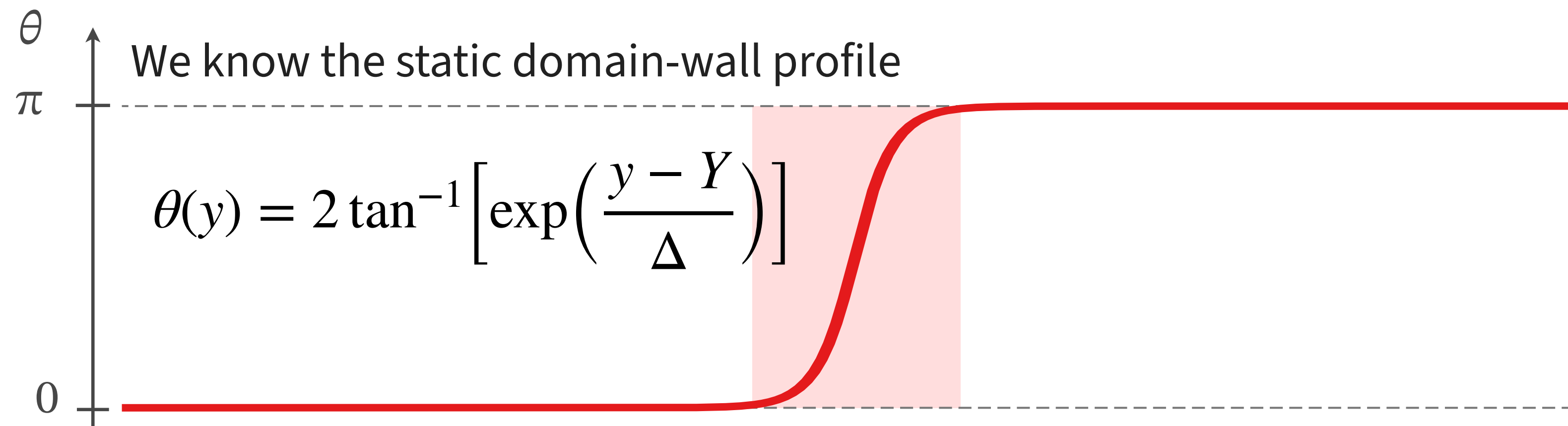
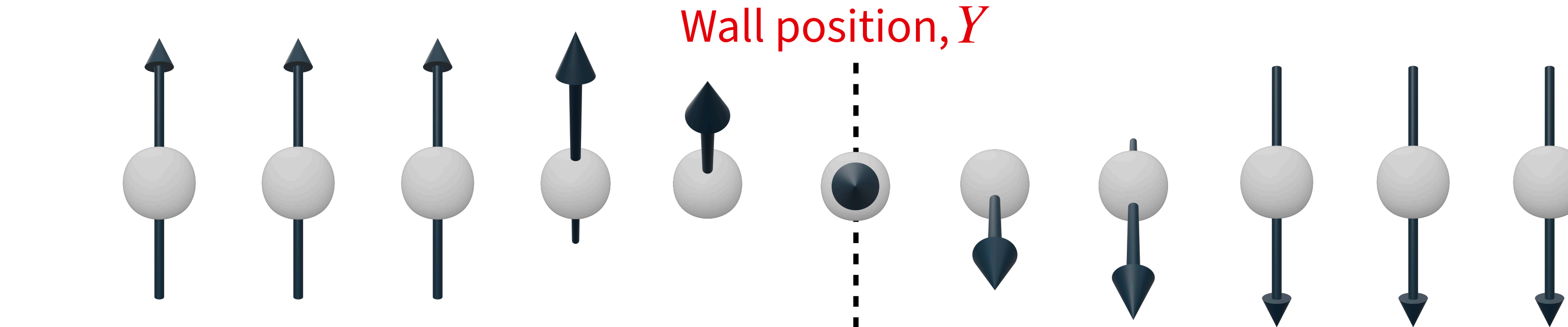
See Lecture „Spin waves and Magnonics“

Solve numerically

$m(r, t)$ Micromagnetic simulation

Effective theory?

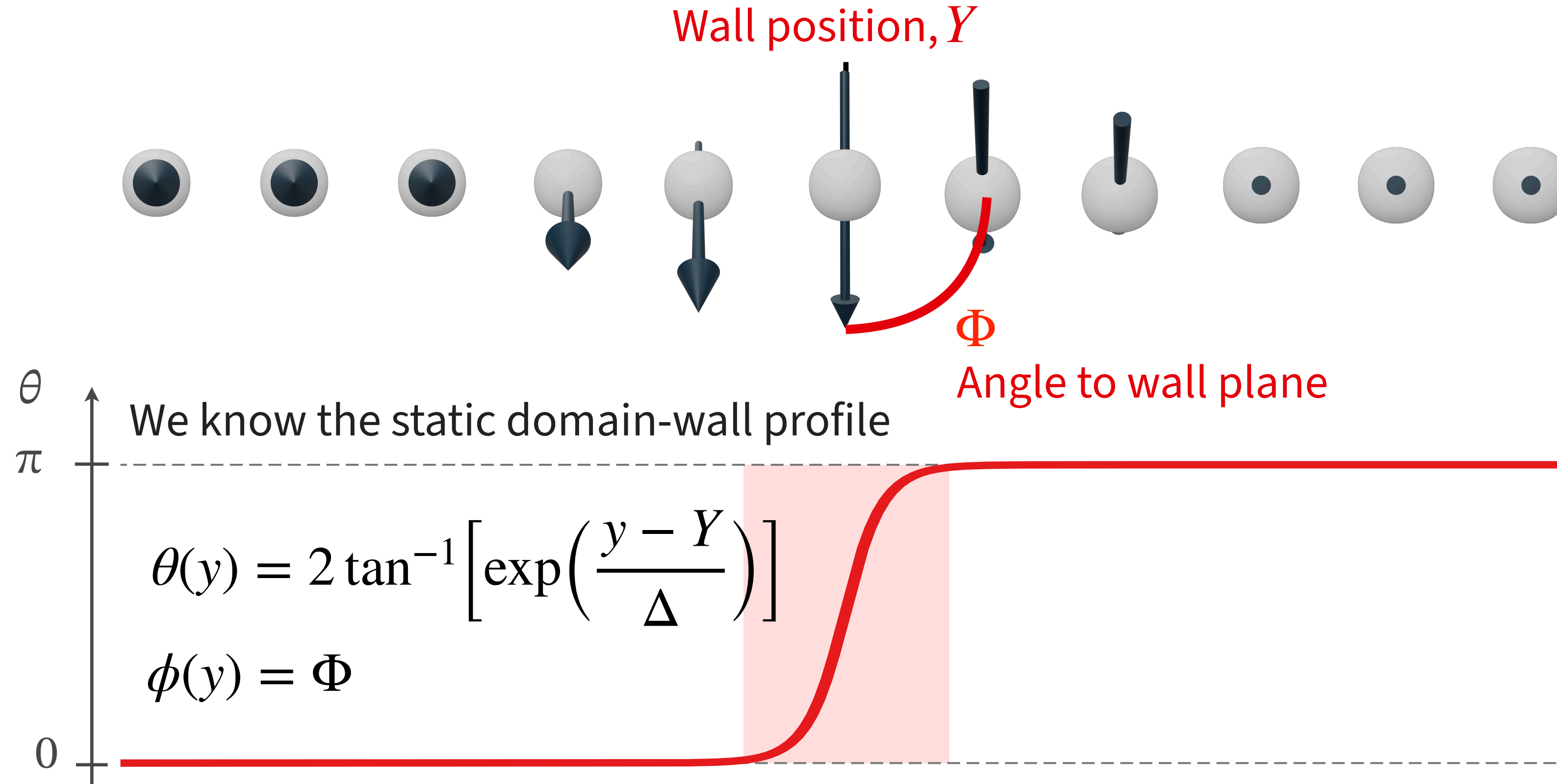
Effective domain wall motion understood using collective variables



Dynamics need to satisfy
Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{m}}{dt} = \frac{\gamma}{M_s} \left[\mathbf{m} \times \frac{\delta E}{\delta \mathbf{m}} \right] + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

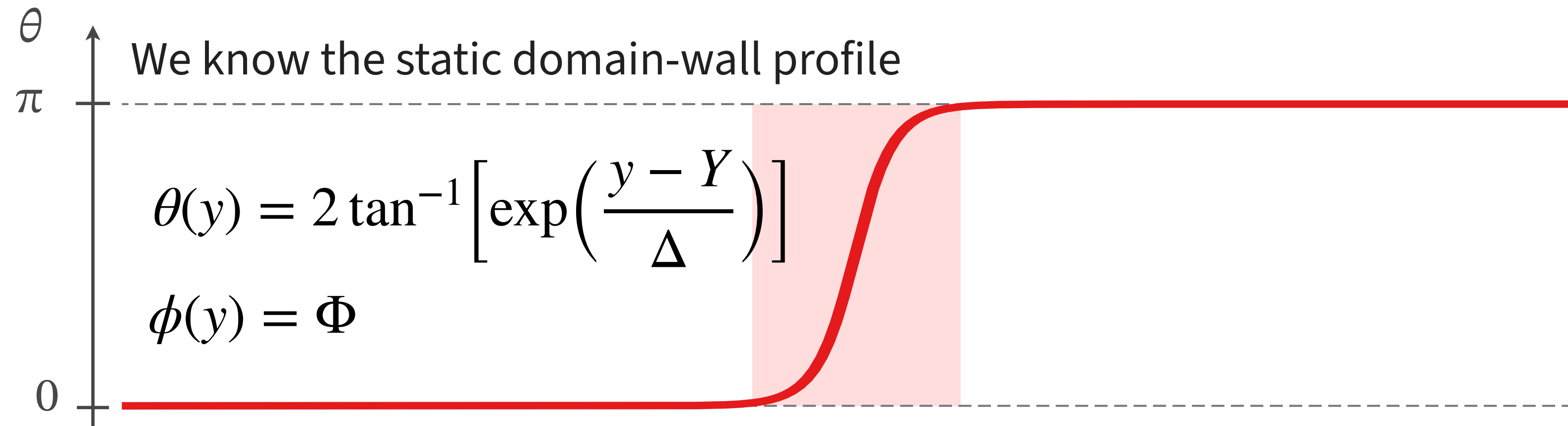
Effective domain wall motion understood using collective variables



Dynamics need to satisfy
Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{m}}{dt} = \frac{\gamma}{M_s} \left[\mathbf{m} \times \frac{\delta E}{\delta \mathbf{m}} \right] + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Effective domain wall motion understood using collective variables



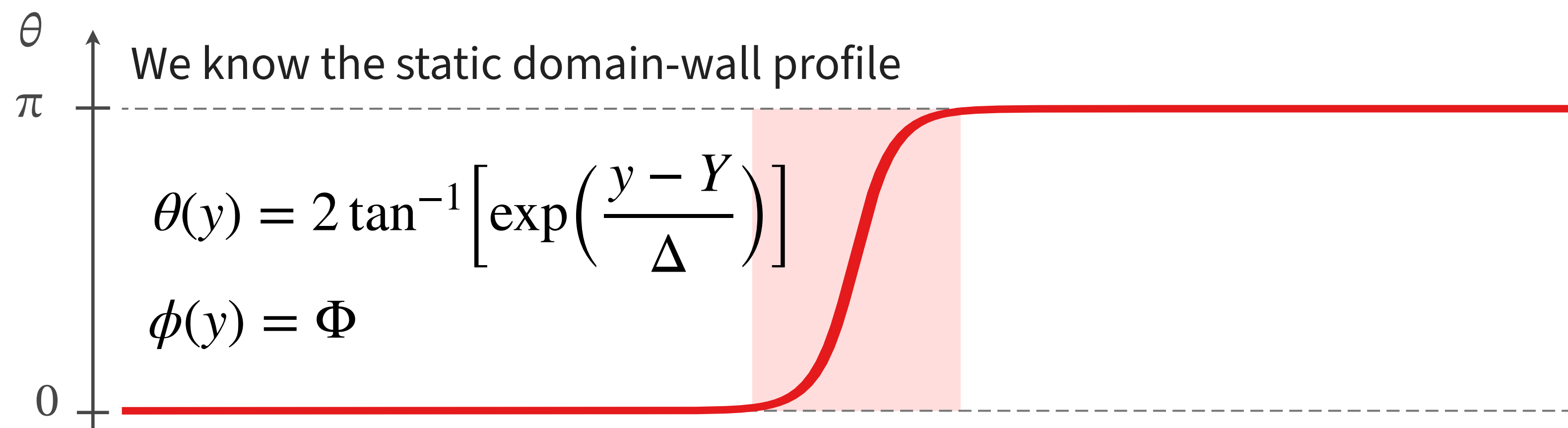
Assume rigid motion with collective variables $Y \rightarrow Y(t)$
 $\Phi \rightarrow \Phi(t)$

Dynamics need to satisfy

Landau-Lifshitz-Gilbert equation

$$\frac{d\mathbf{m}}{dt} = \frac{\gamma}{M_s} \left[\mathbf{m} \times \frac{\delta E}{\delta \mathbf{m}} \right] + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Effective domain wall motion understood using collective variables



Assume rigid motion with collective variables $Y \rightarrow Y(t)$
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Dynamics need to satisfy

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Integrate out $\mathbf{r}$ dependence

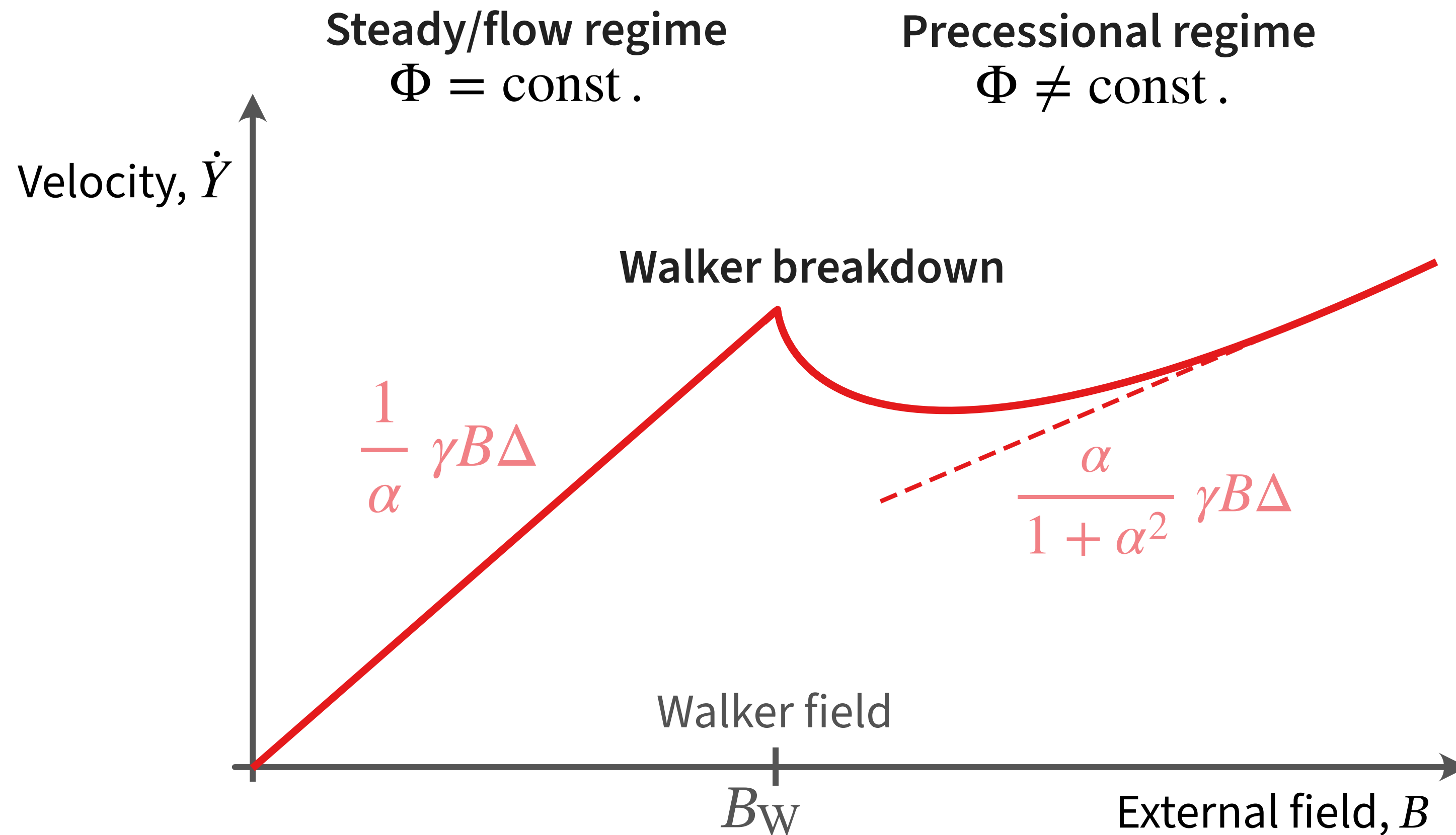
$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(Y(t), \Phi(t))$$

Thiele equations

$$\dot{Y} = \frac{\gamma}{2M_s} \frac{\partial E}{\partial \Phi} + \alpha \dot{\Phi} \Delta$$

$$-\dot{\Phi} = \frac{\gamma}{2M_s} \frac{\partial E}{\partial Y} + \alpha \frac{\dot{Y}}{\Delta}$$

Domain wall motion has different regimes

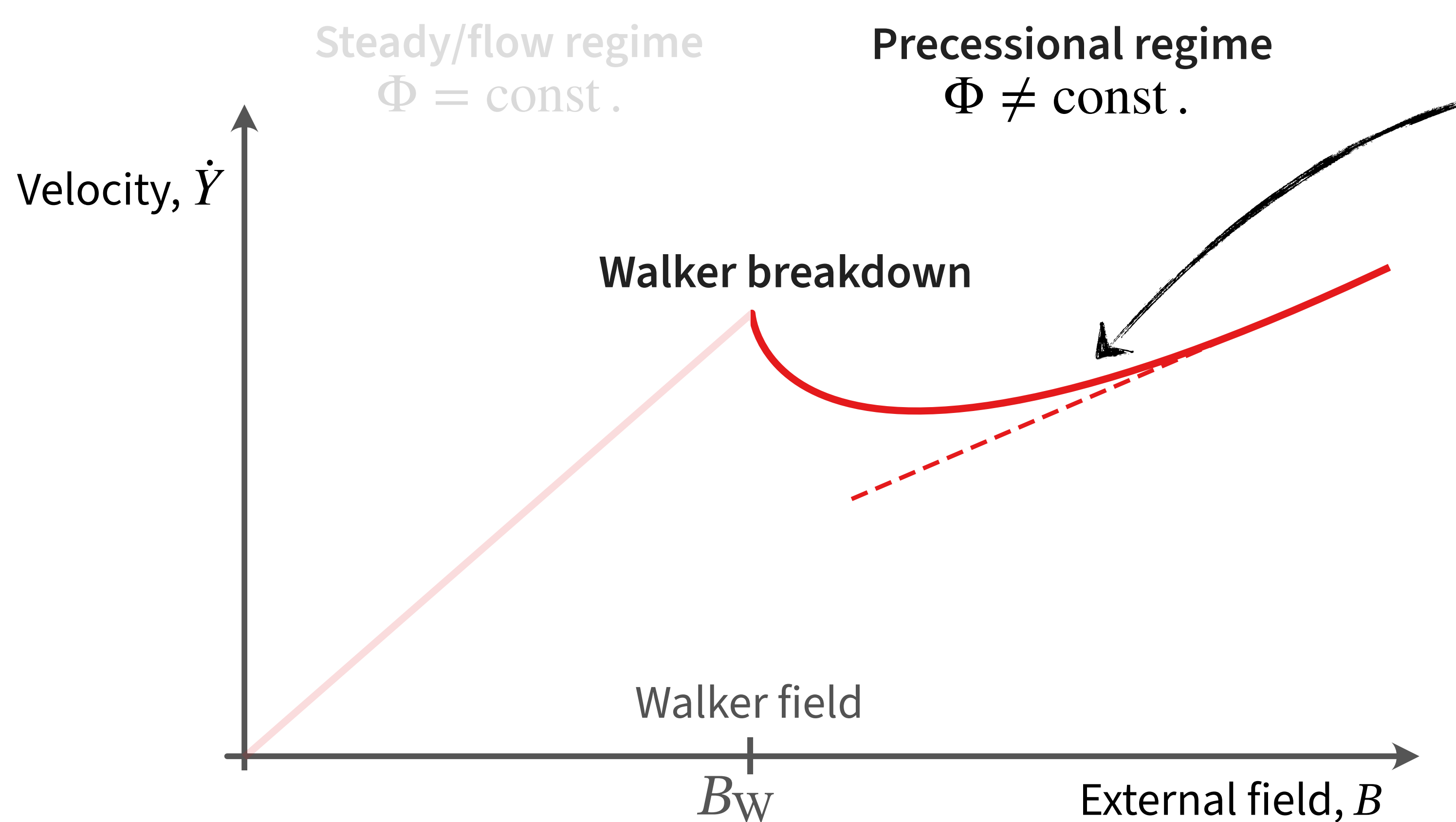


Thiele equations

$$\begin{aligned}\dot{Y} &= \frac{\gamma}{2M_s} \frac{\partial E}{\partial \Phi} + \alpha \dot{\Phi} \Delta \\ -\dot{\Phi} &= \frac{\gamma}{2M_s} \frac{\partial E}{\partial Y} + \alpha \frac{\dot{Y}}{\Delta}\end{aligned}$$

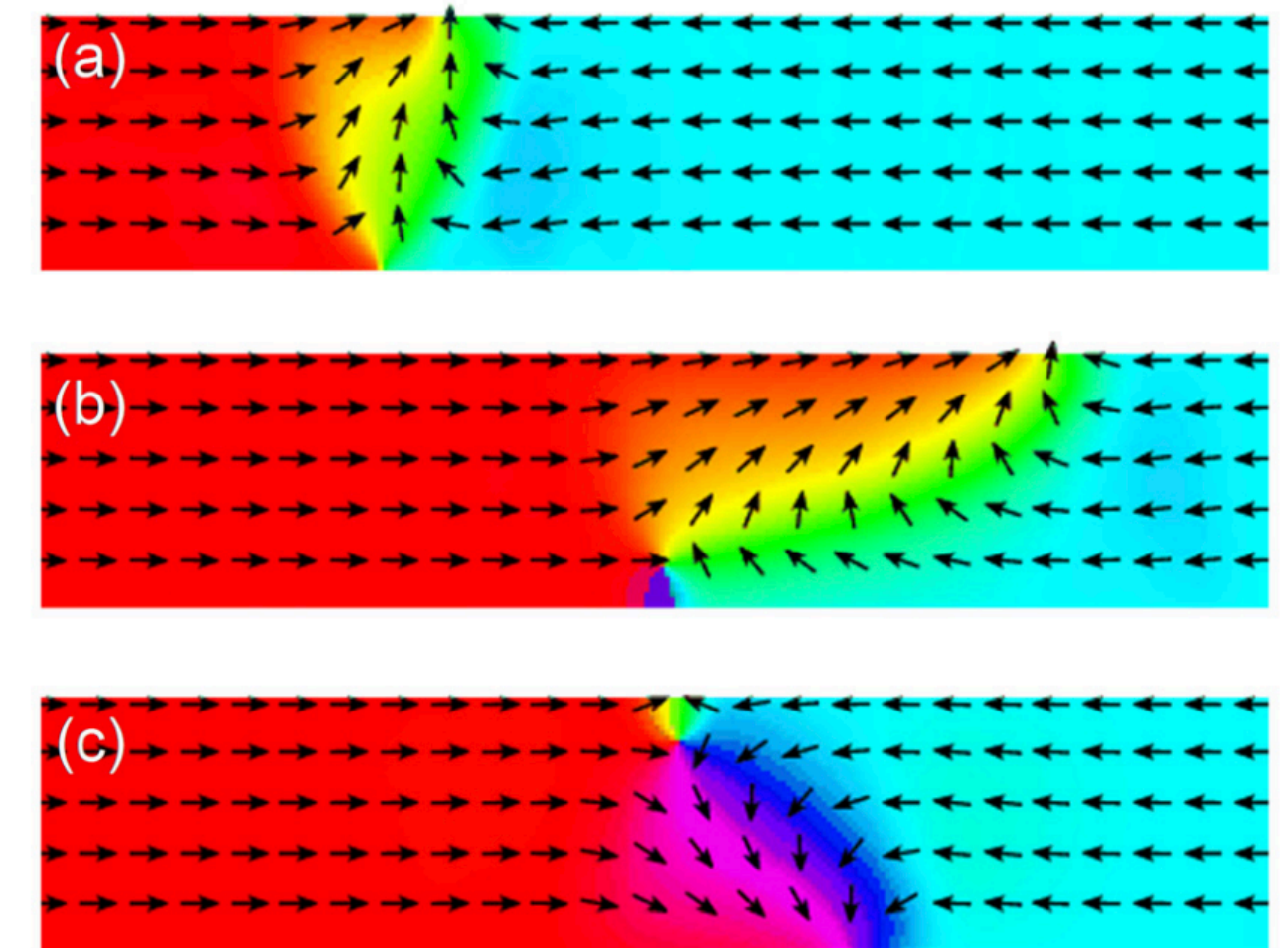
Solve

Domain wall motion has different regimes



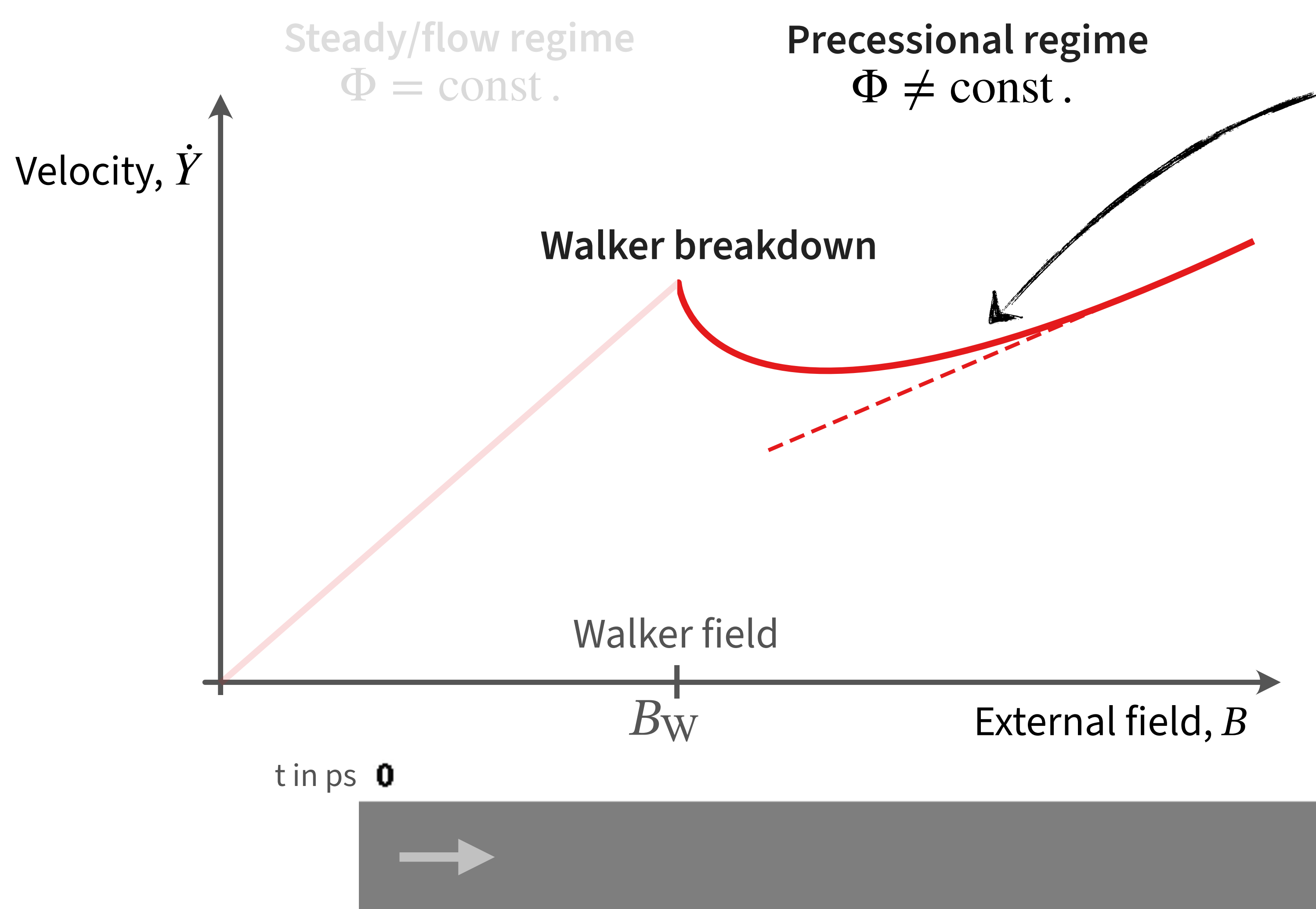
Can be more complicated in more realistic systems!

e.g. wall transforms from edges



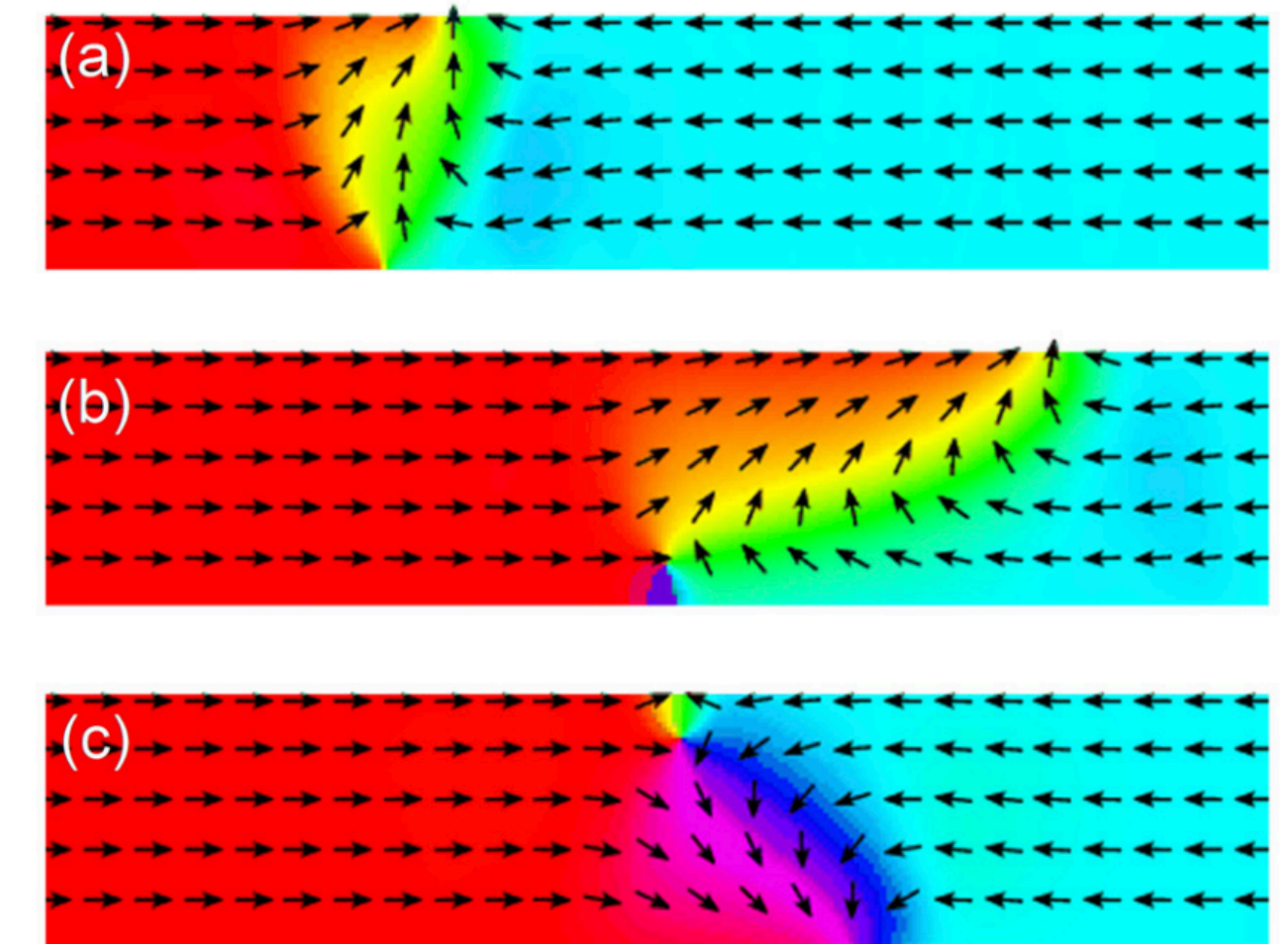
Burn et al., APL 102, 242414 (2013)

Domain wall motion has different regimes



Can be more complicated in more realistic systems!

e.g. wall transforms from edges

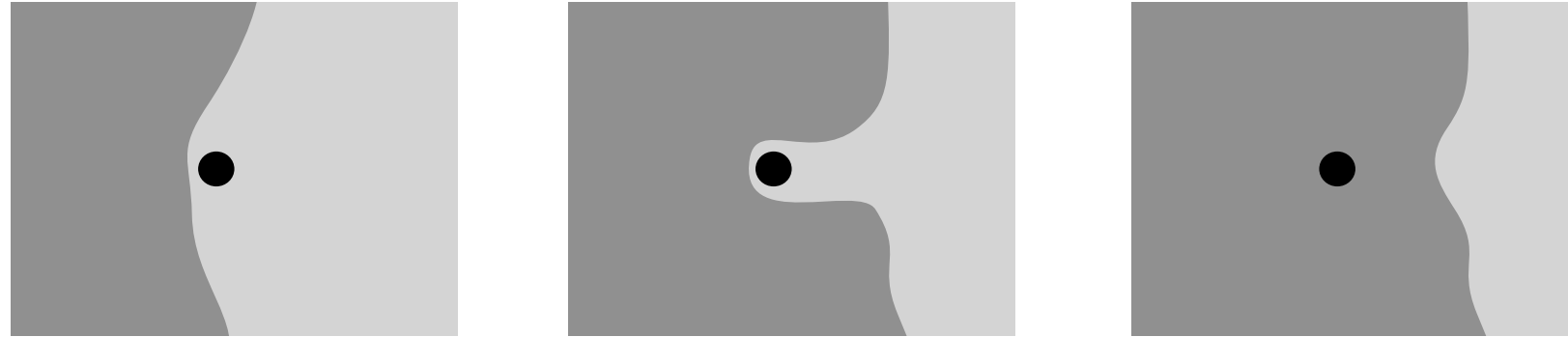


Burn et al., APL 102, 242414 (2013)

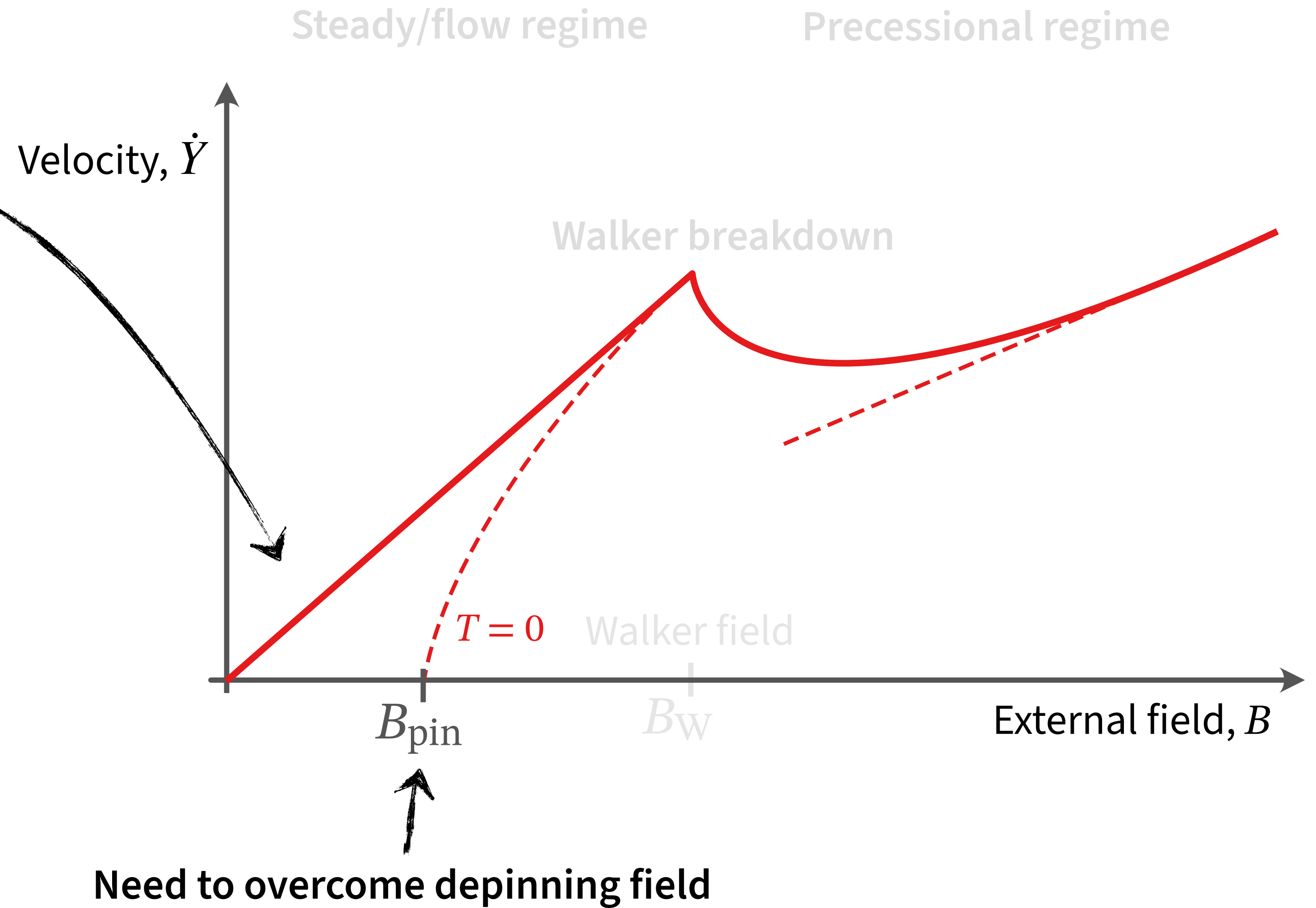
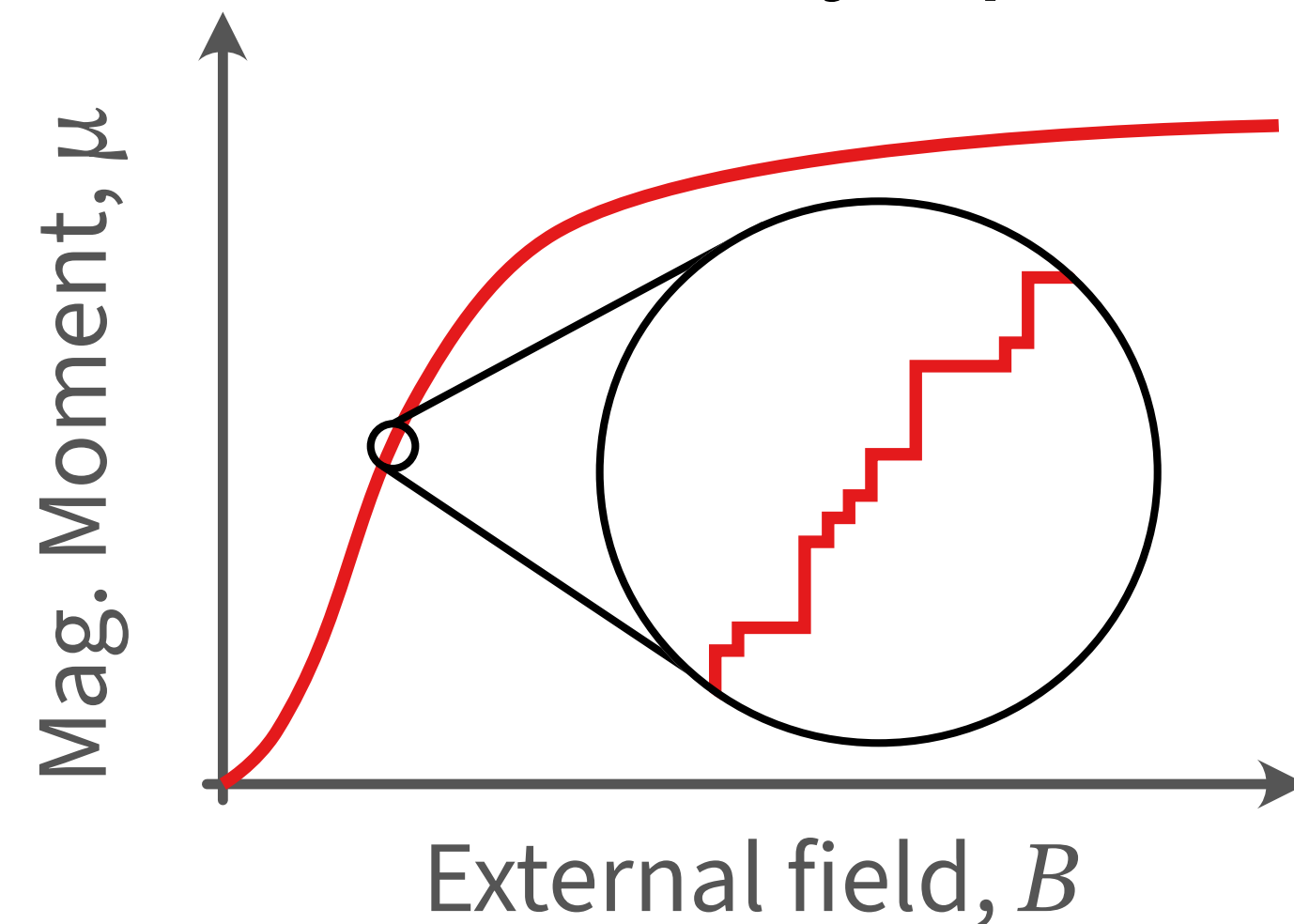
Breadown of head-to-head wall during current-induced motion © Attila Kákay

Defects are important at small fields

Obstructed by pinning of domain walls at defects



Barkhausen jumps!

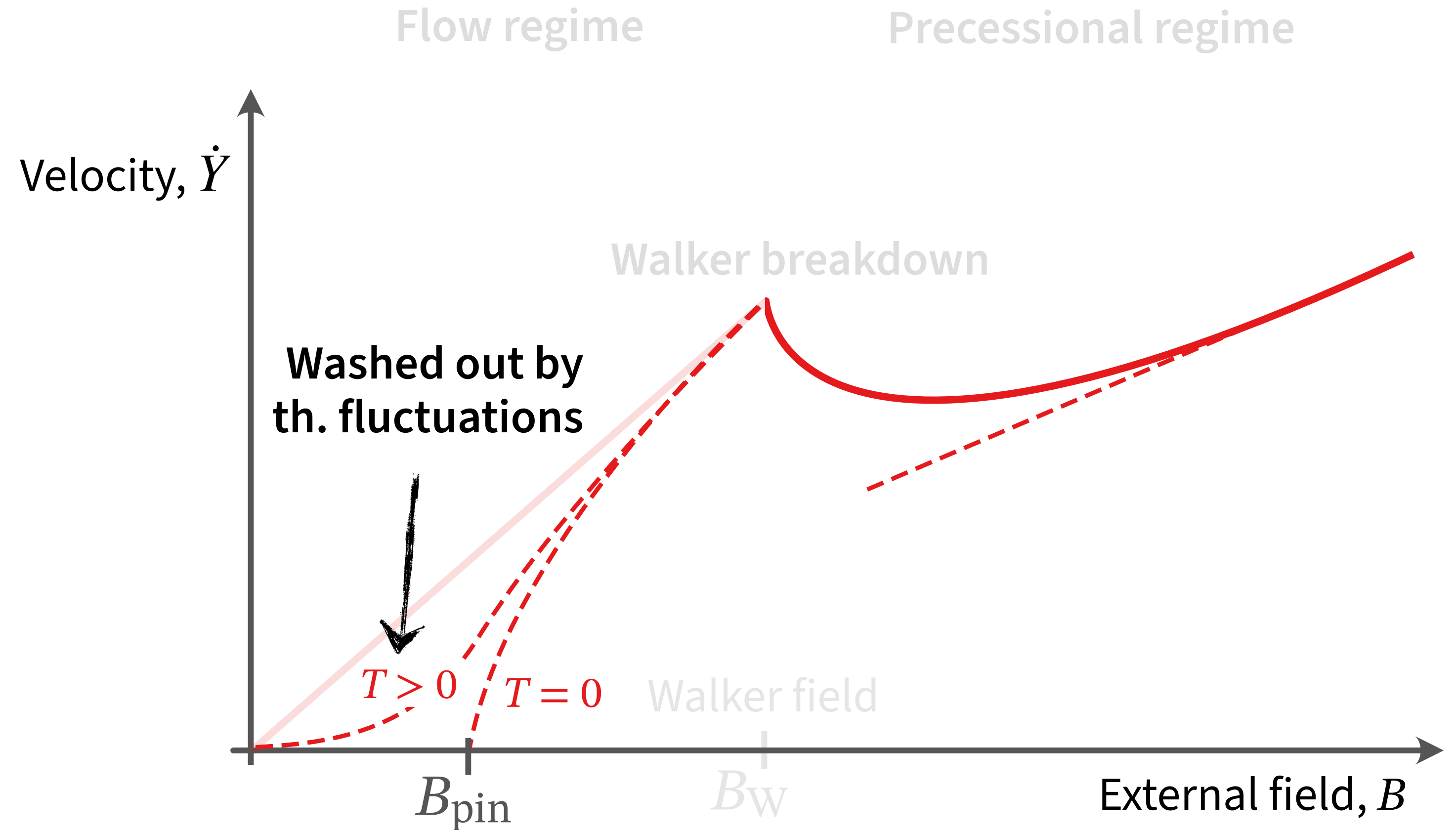
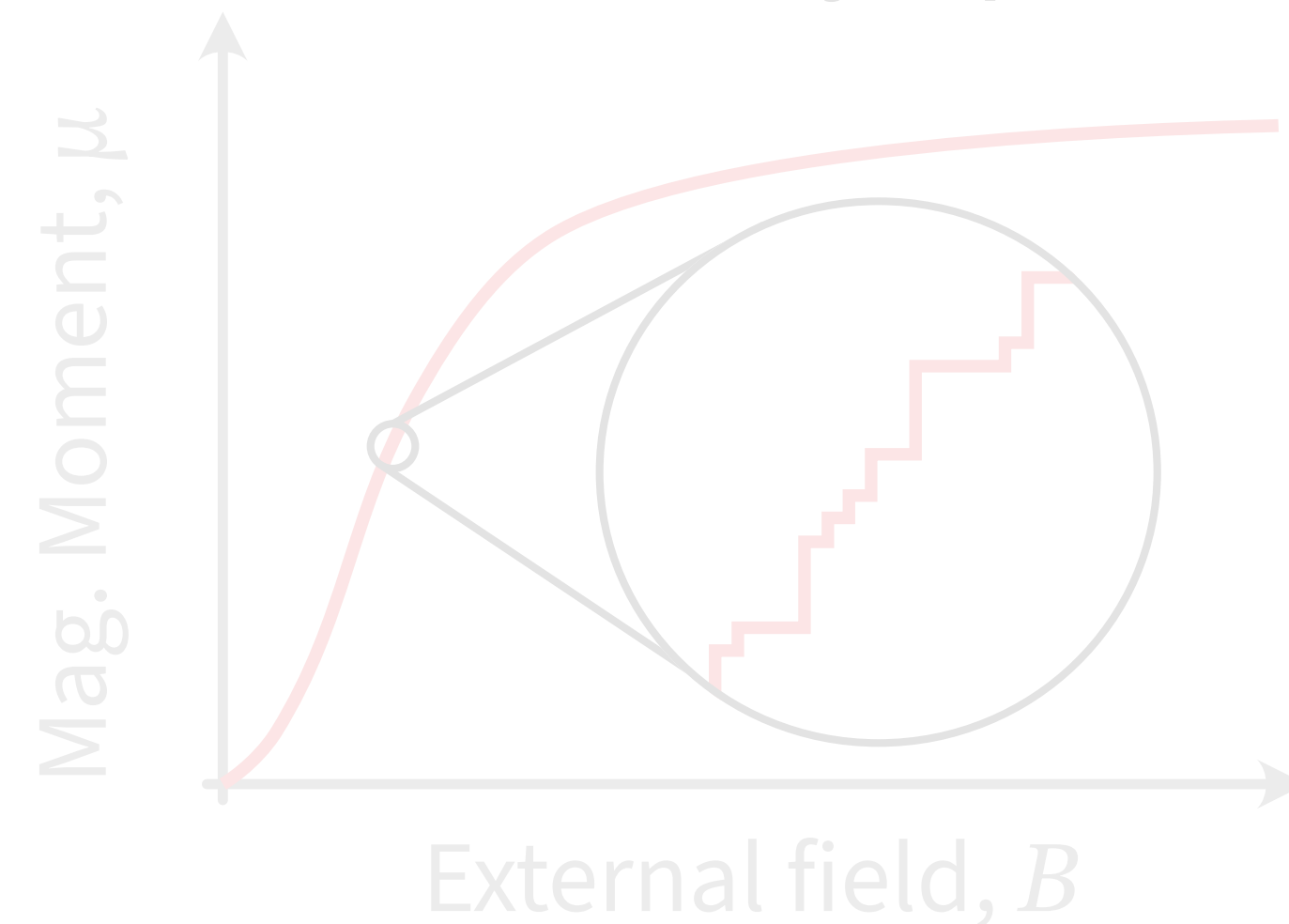


Defects are important at small fields

Obstructed by pinning of domain walls at defects



Barkhausen jumps!

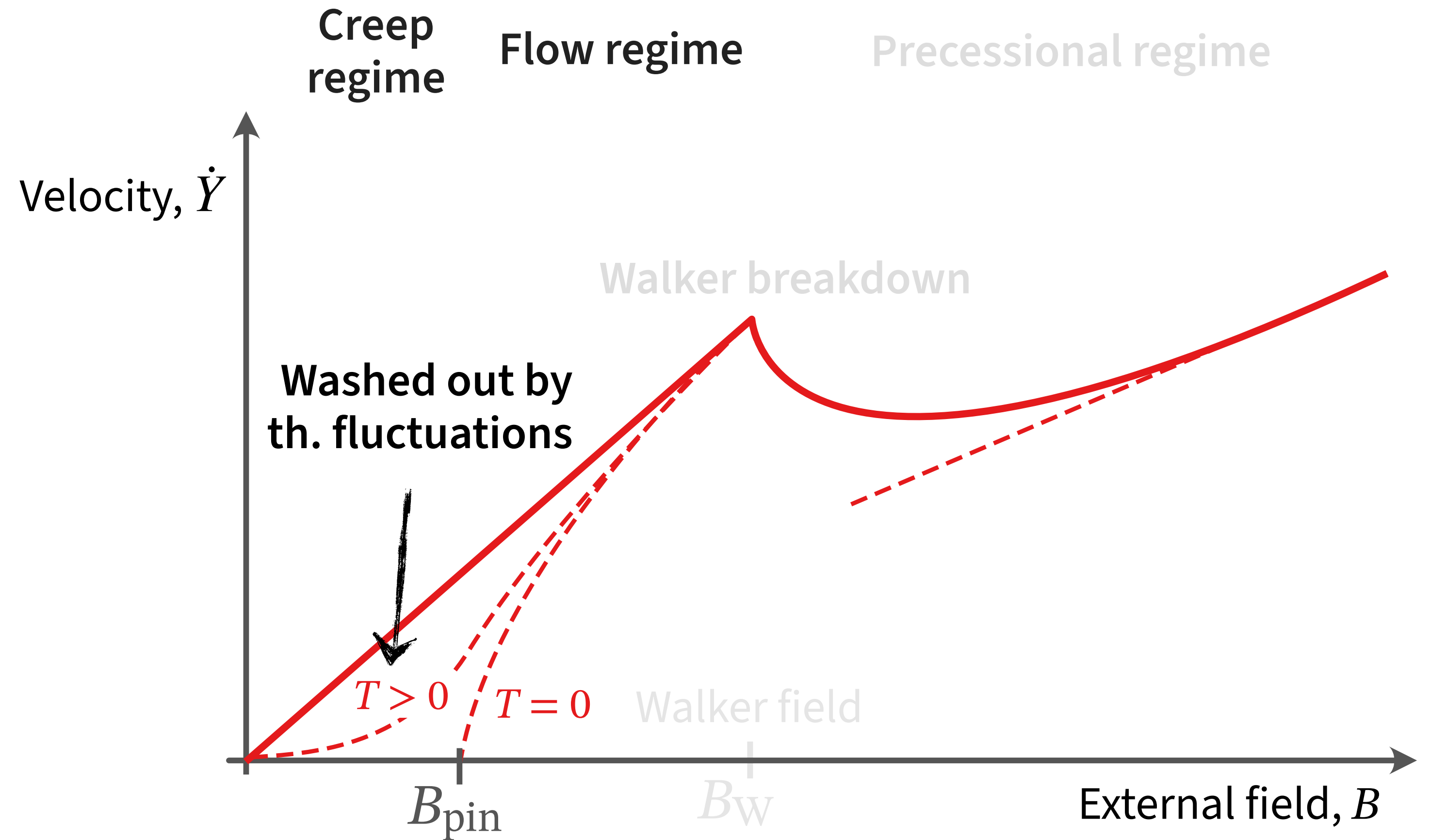
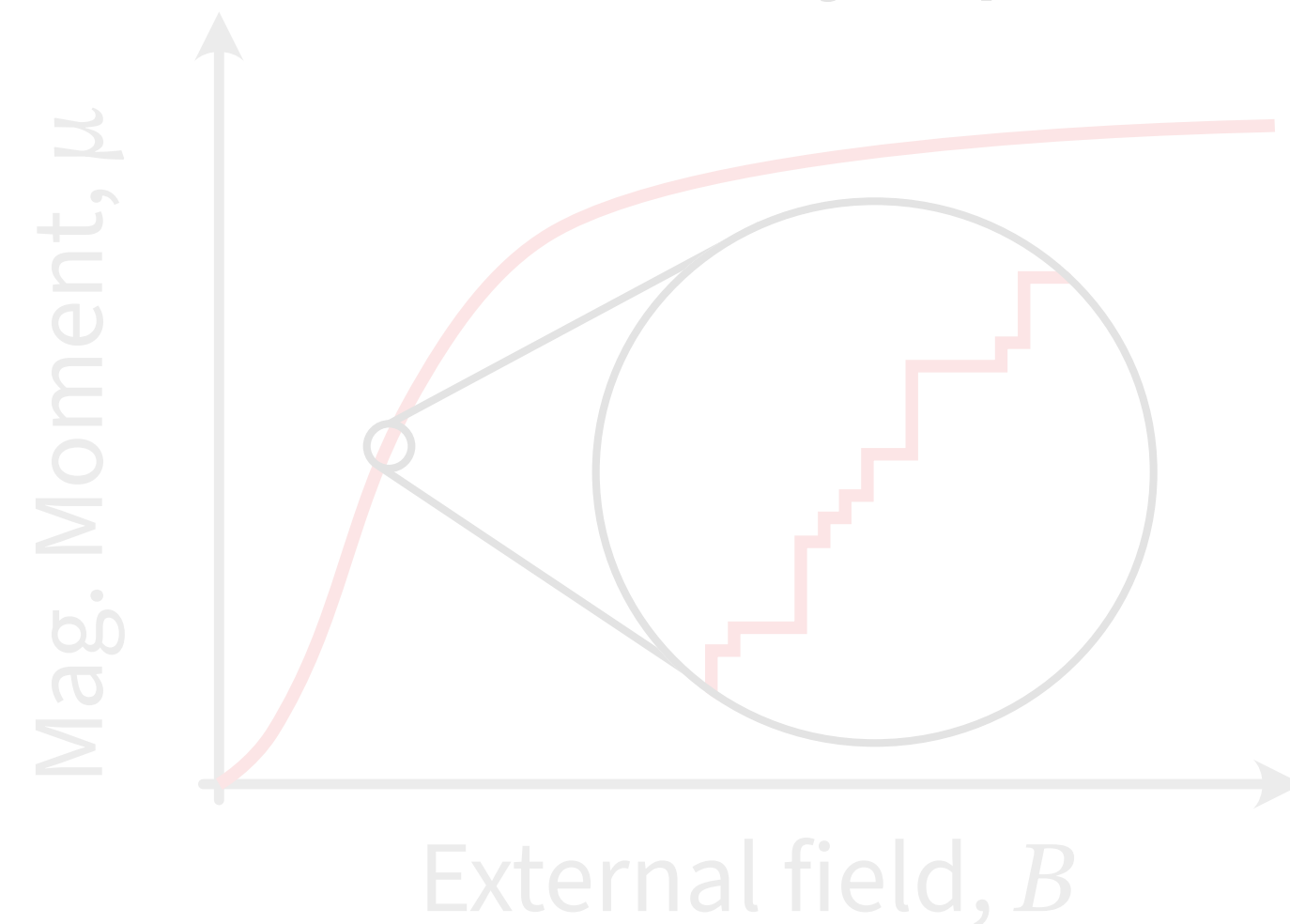


Defects are important at small fields

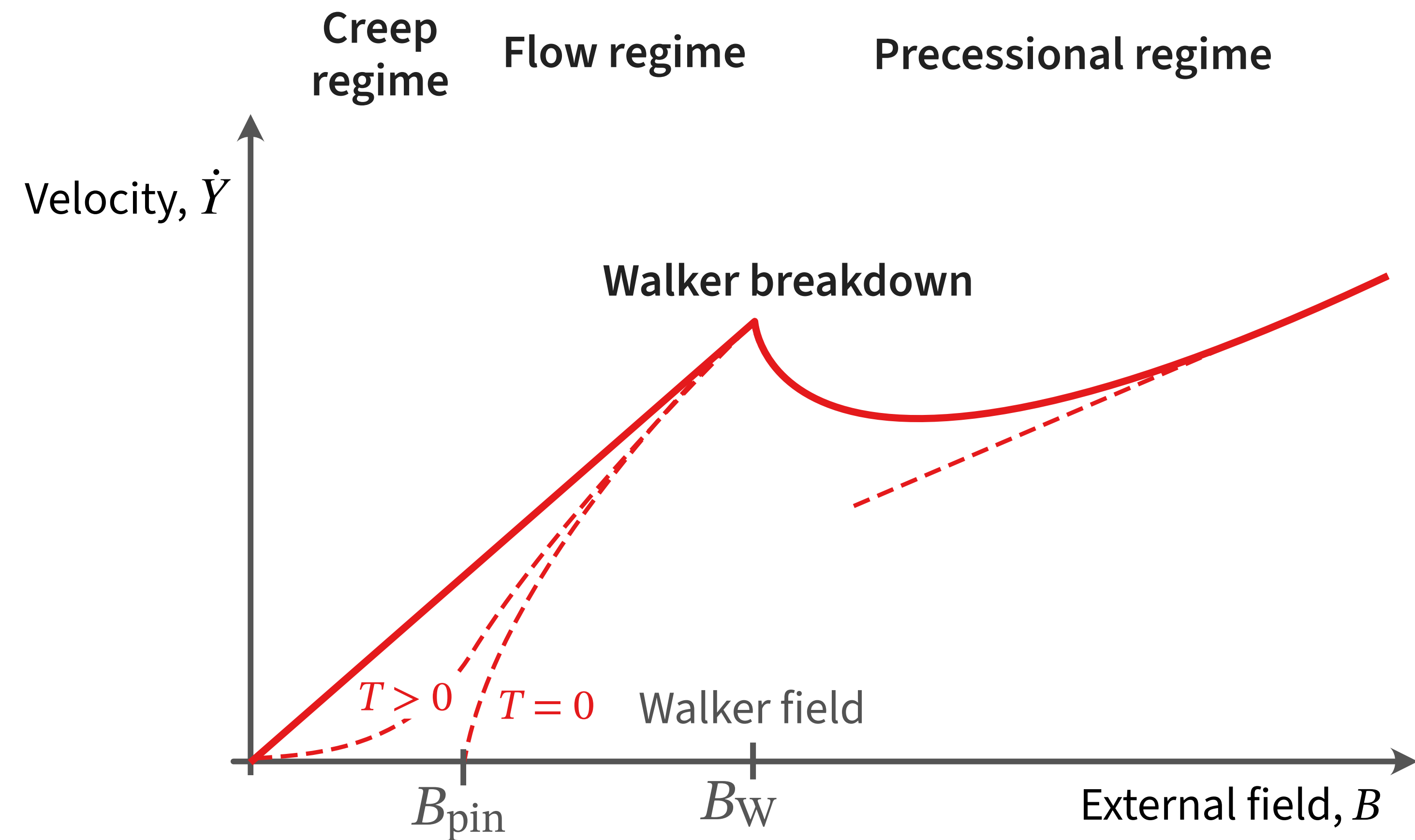
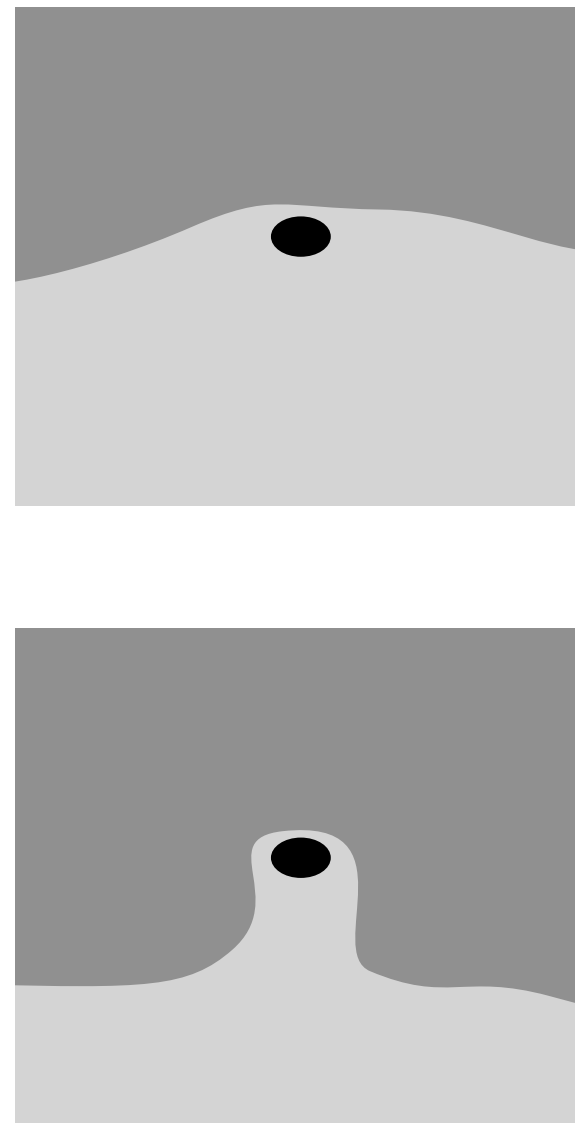
Obstructed by pinning of domain walls at defects



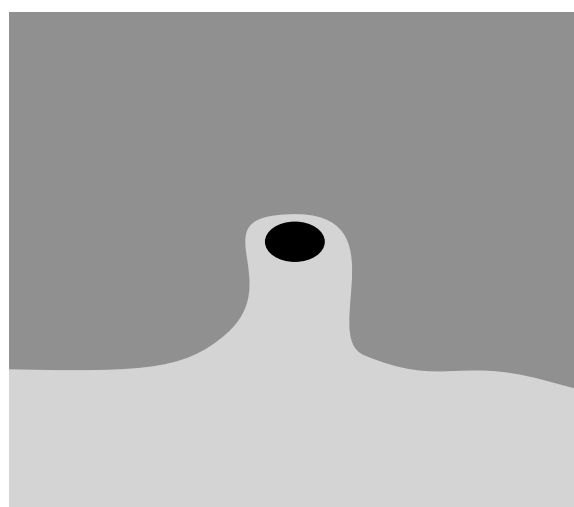
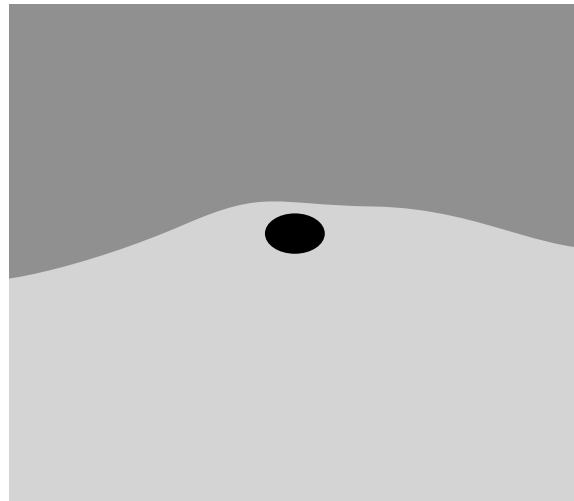
Barkhausen jumps!



Domain-wall motion



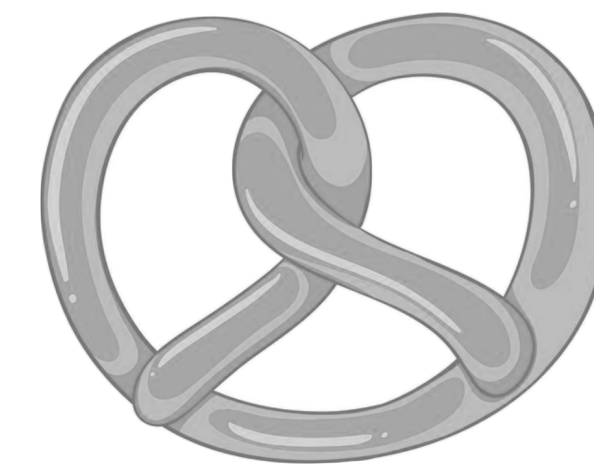
Domain-wall motion



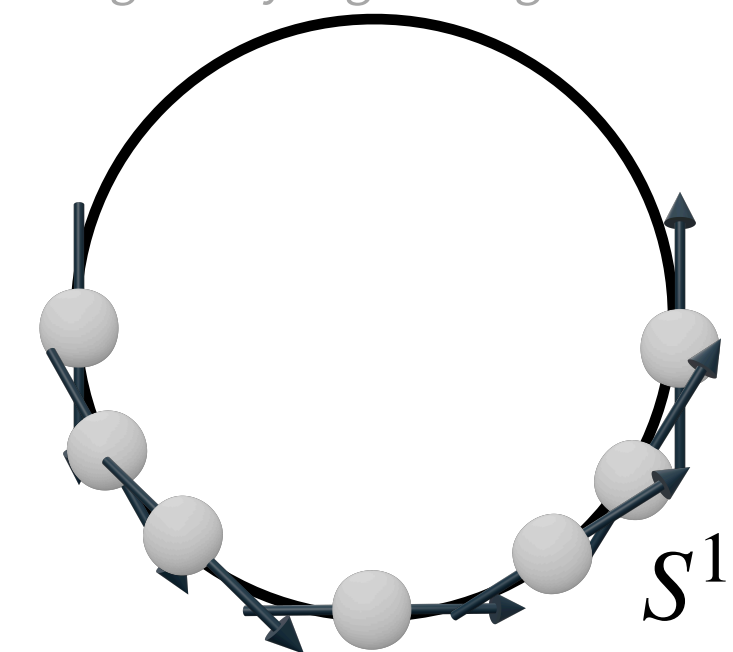
How else can they transform?



Role of topology

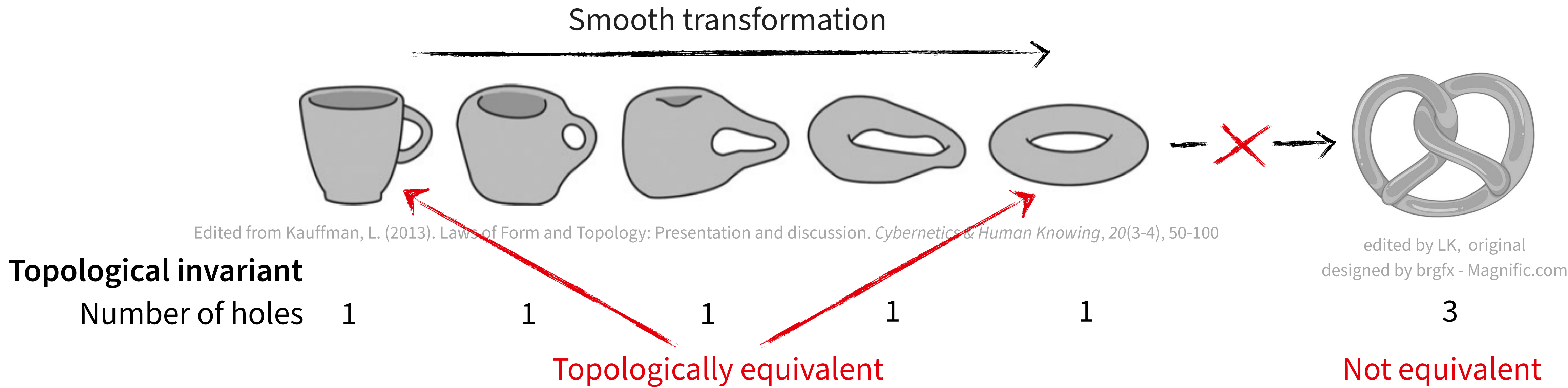


edited by LK, original
designed by brgfx - Magnific.com



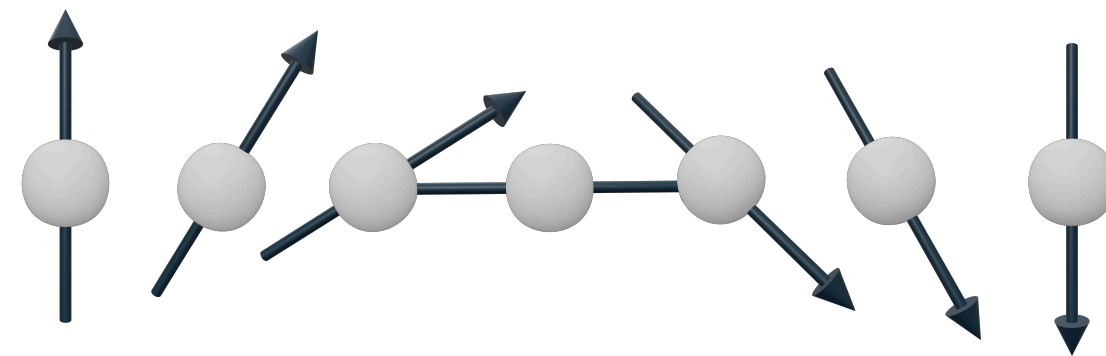
Domain walls as topological spin textures

In topology, objects differ only up to smooth transformations



In topology, objects differ only up to smooth transformations

Spin textures $\approx$ smooth fields $\mathbf{m}$



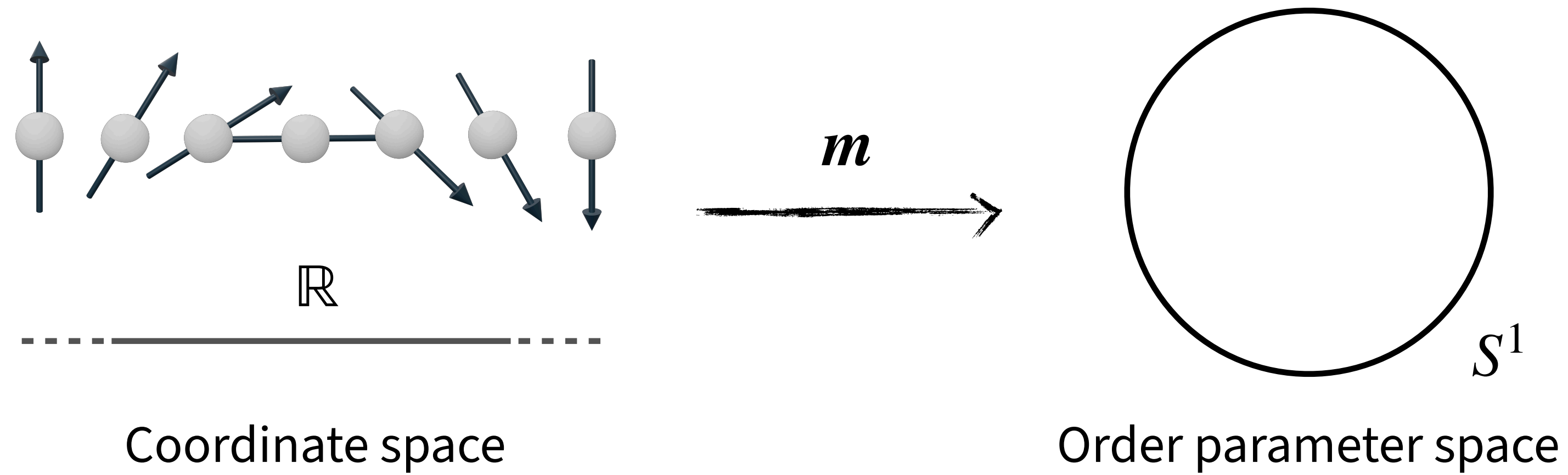
(remember: exchange)

Evolve smoothly in time

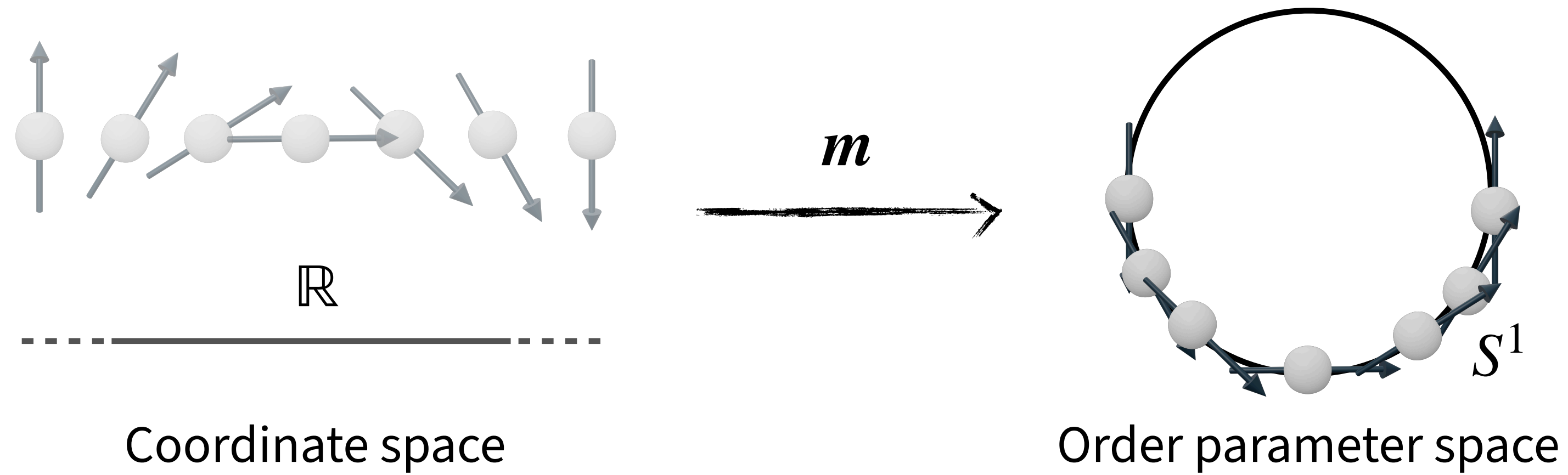
$$\frac{d\mathbf{m}}{dt} = \frac{\gamma}{M_s} \left[\mathbf{m} \times \frac{\delta E}{\delta \mathbf{m}} \right] + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Allowed deformations?
Invariants?

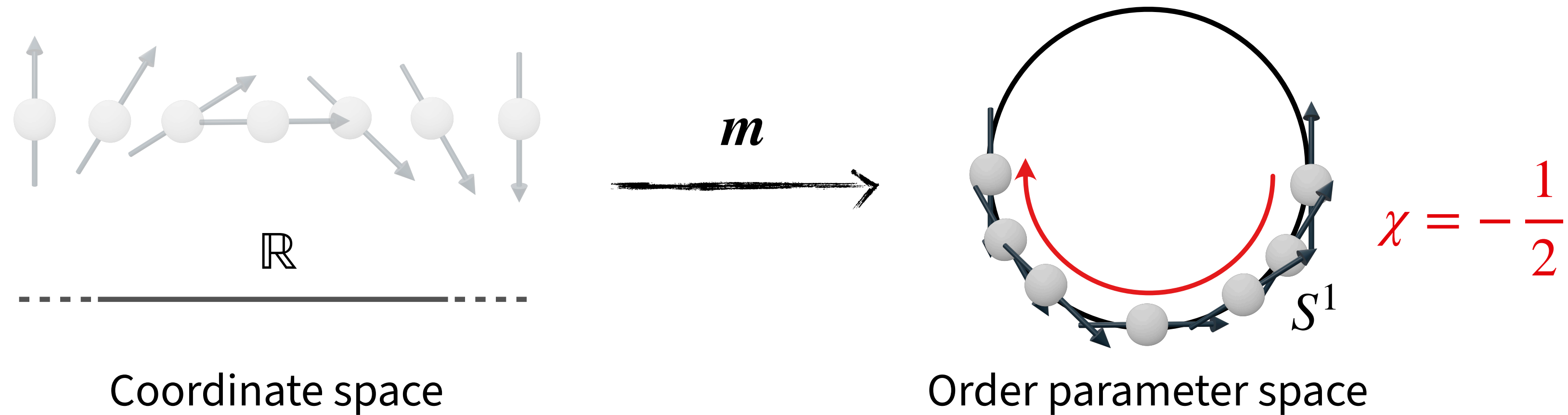
Domain walls are topological in terms of their winding number



Domain walls are topological in terms of their winding number



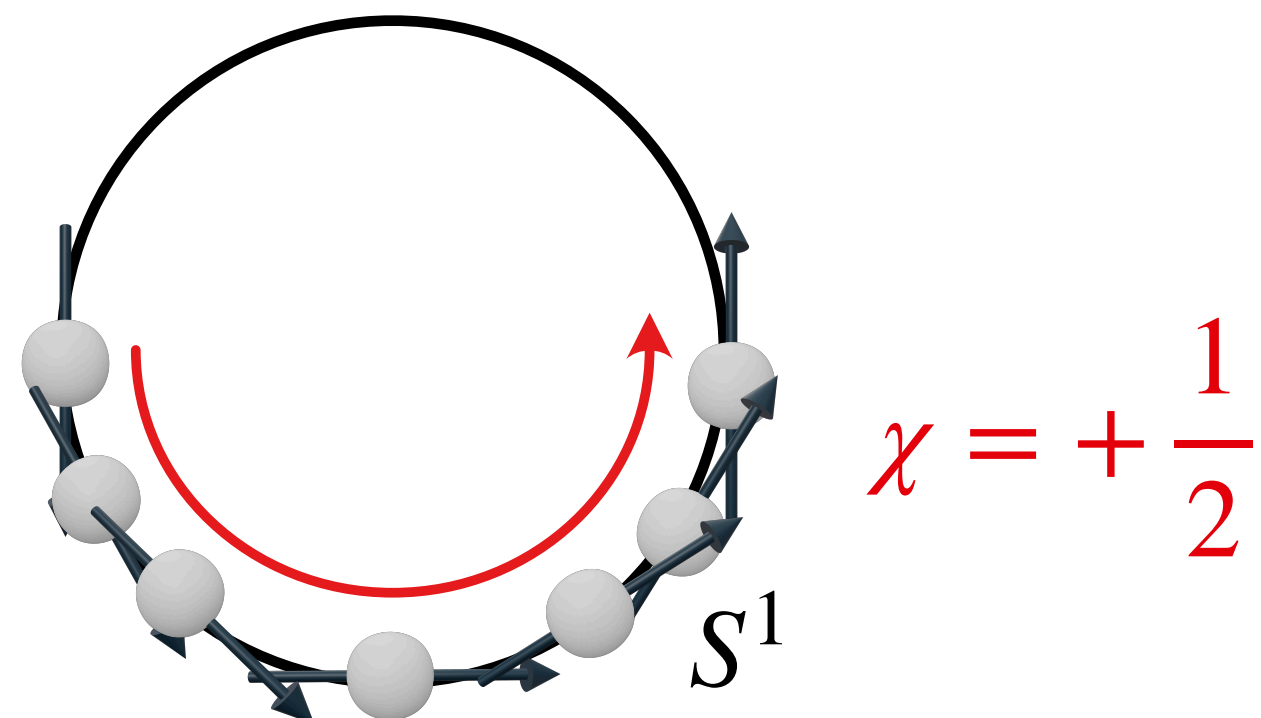
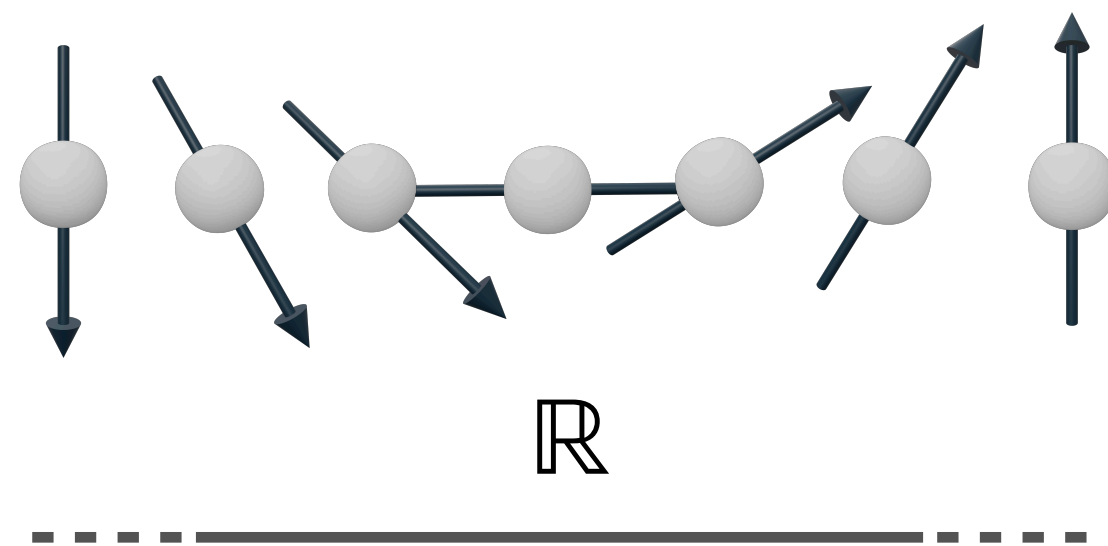
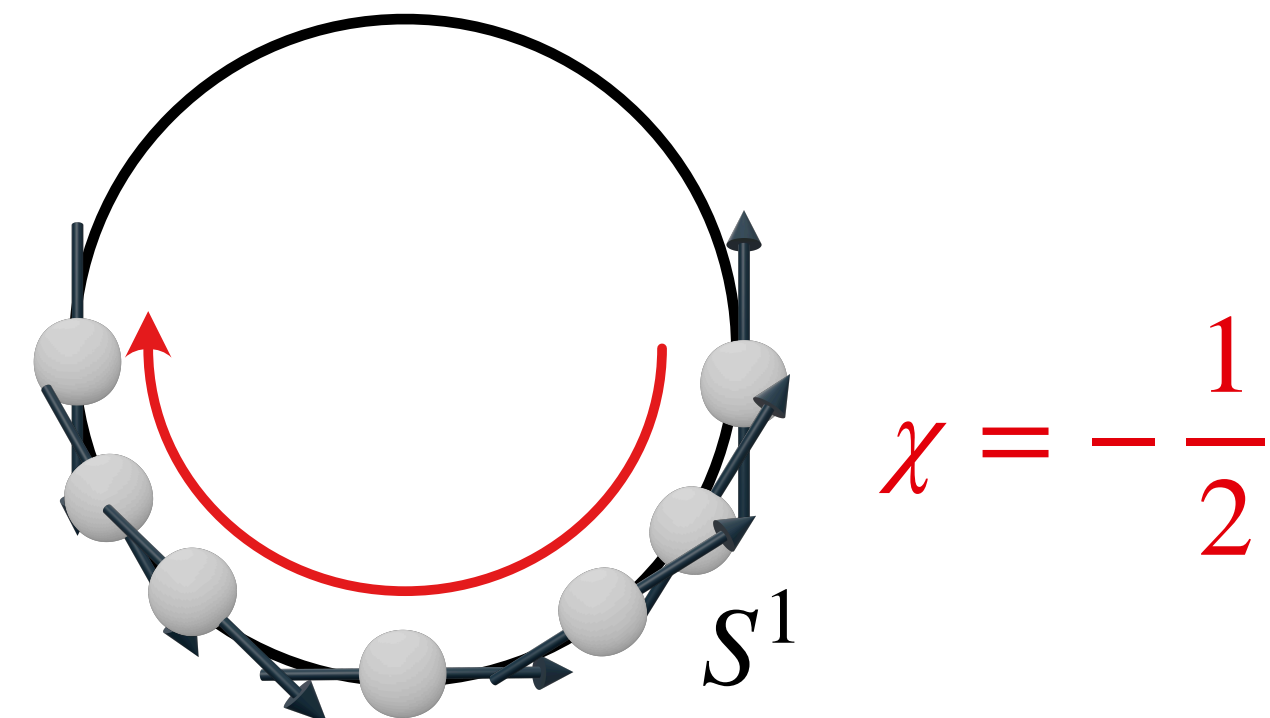
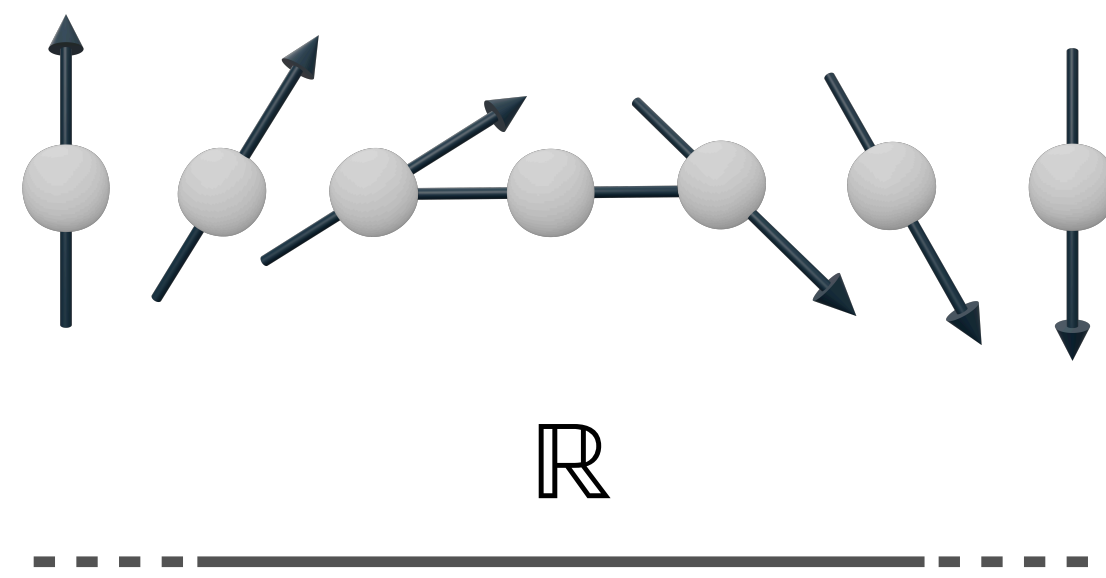
Domain walls are topological in terms of their winding number



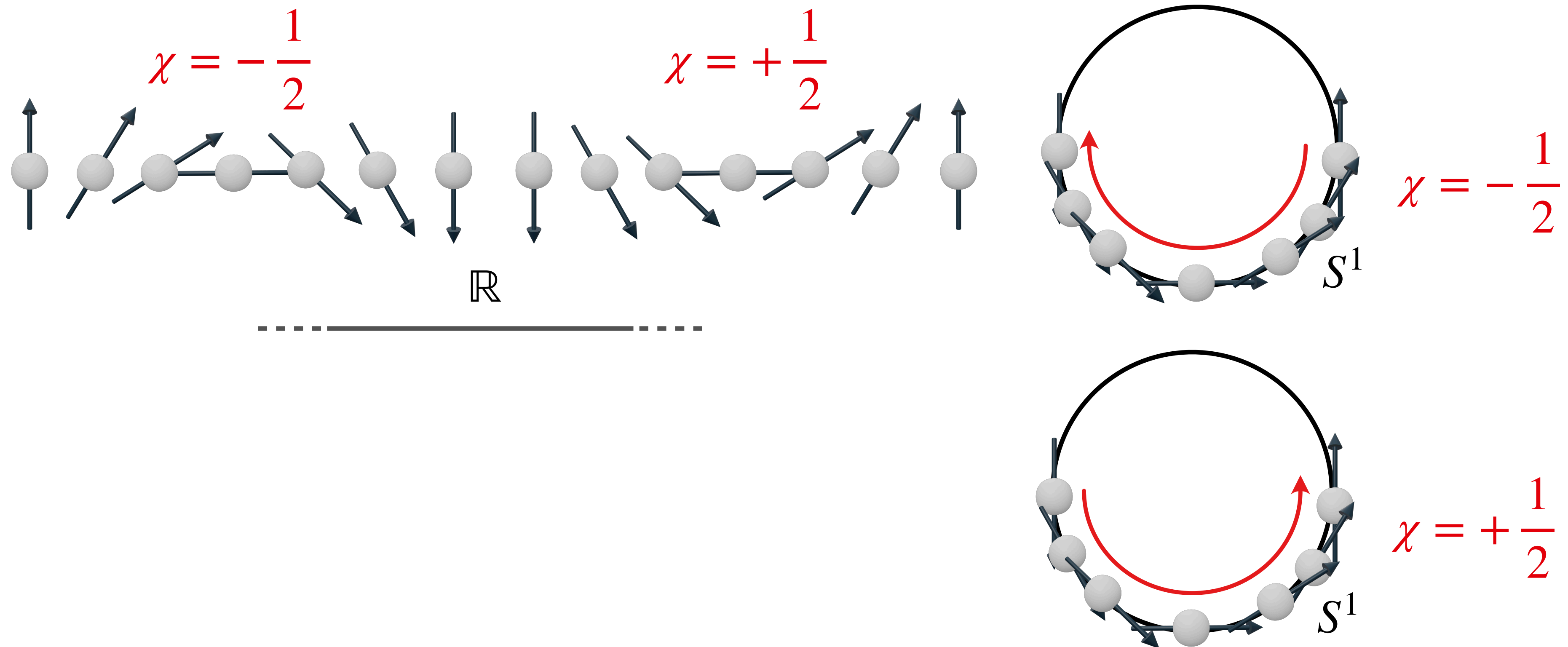
Topological invariant
Winding number

$$\chi = \frac{1}{2\pi} \int_{-\infty}^{\infty} dx \partial_x \theta(x)$$

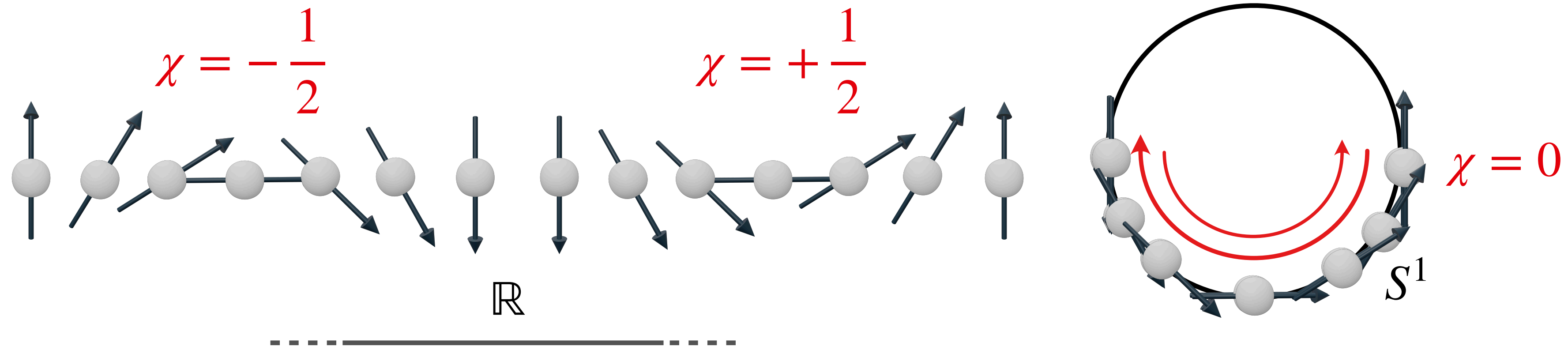
Domain walls are topological in terms of their winding number



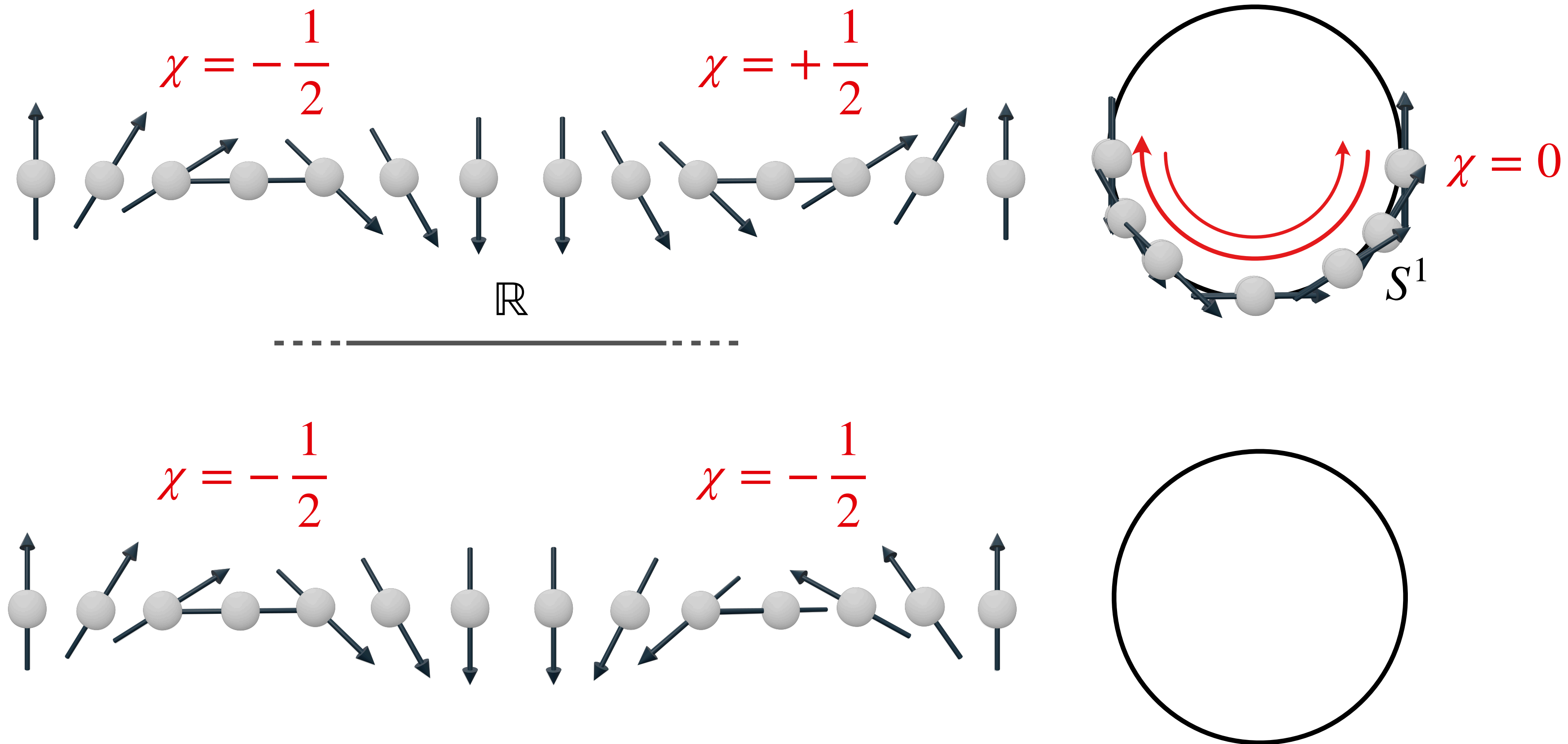
Adding domain walls means adding their winding numbers



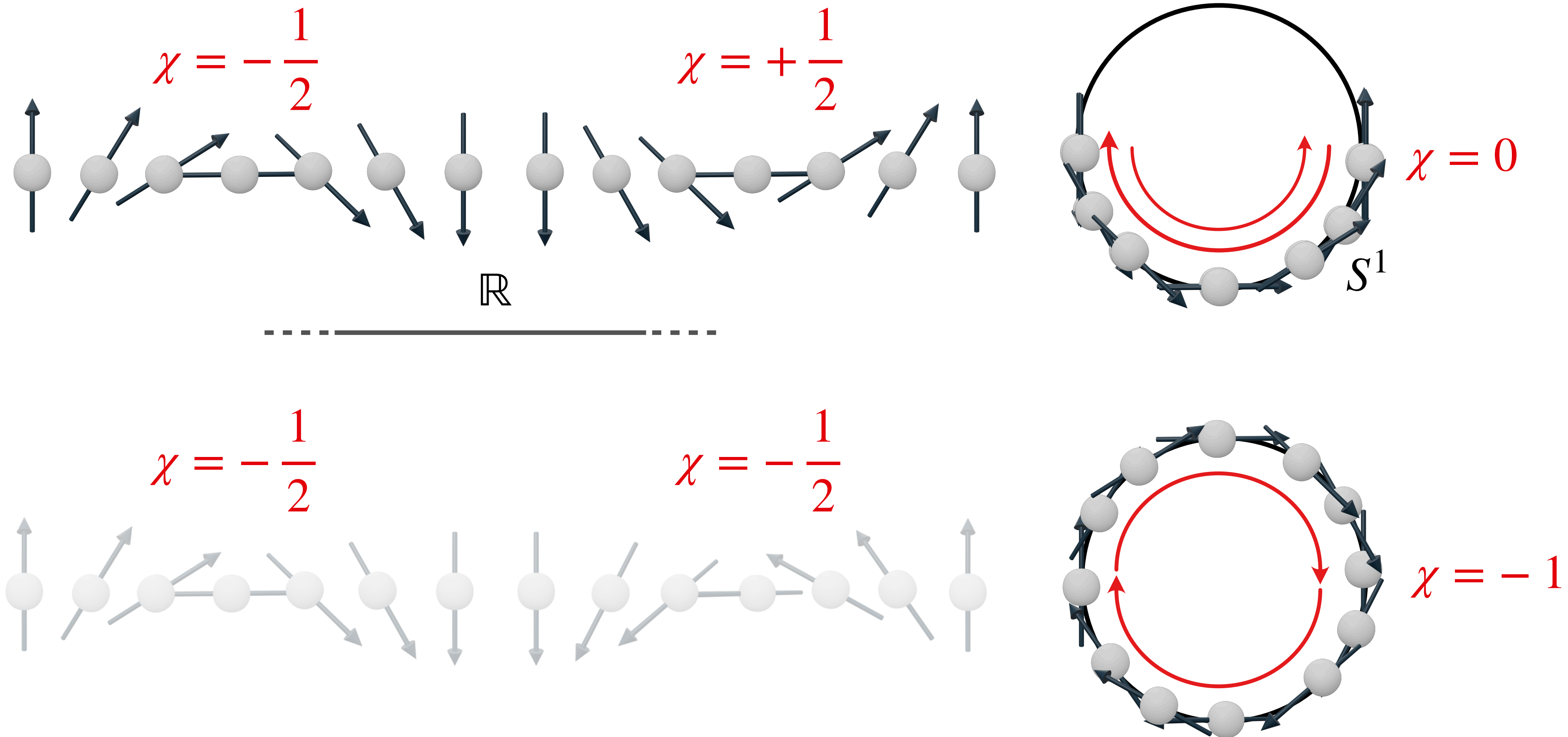
Adding domain walls means adding their winding numbers



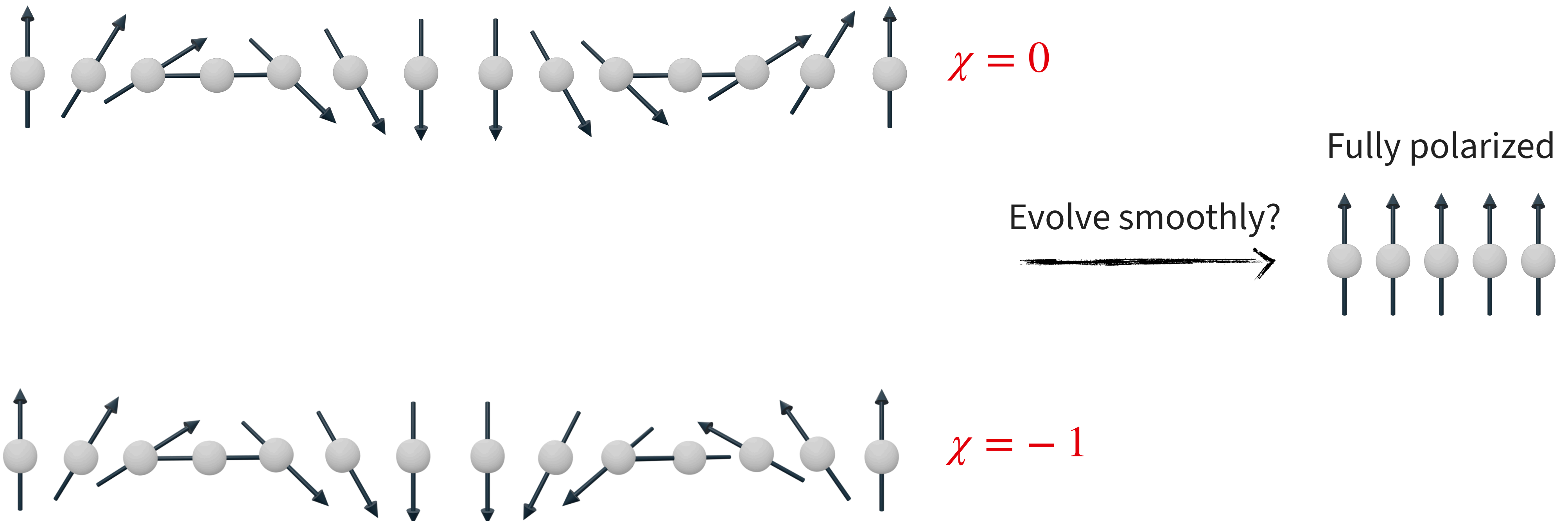
Adding domain walls means adding their winding numbers



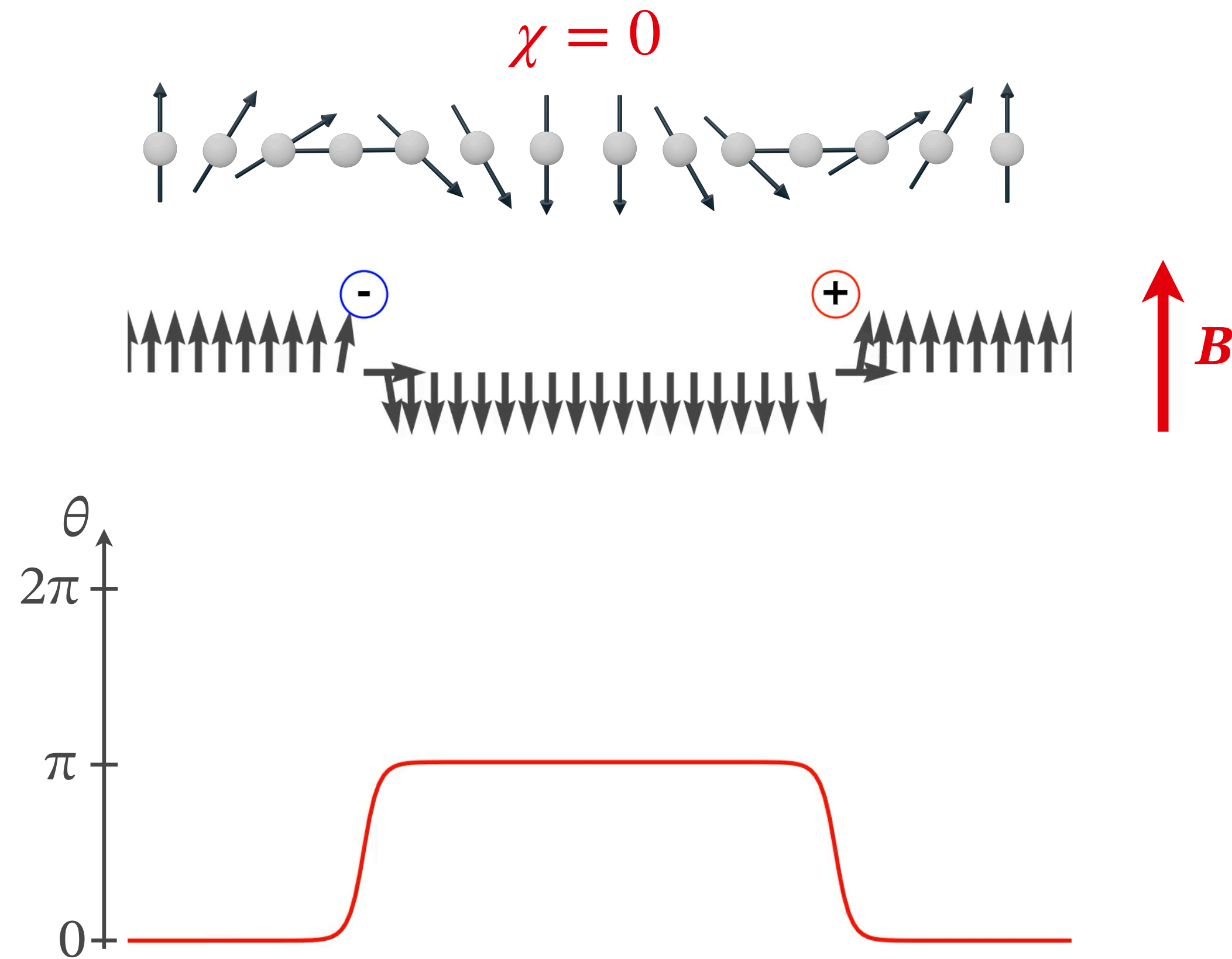
Adding domain walls means adding their winding numbers



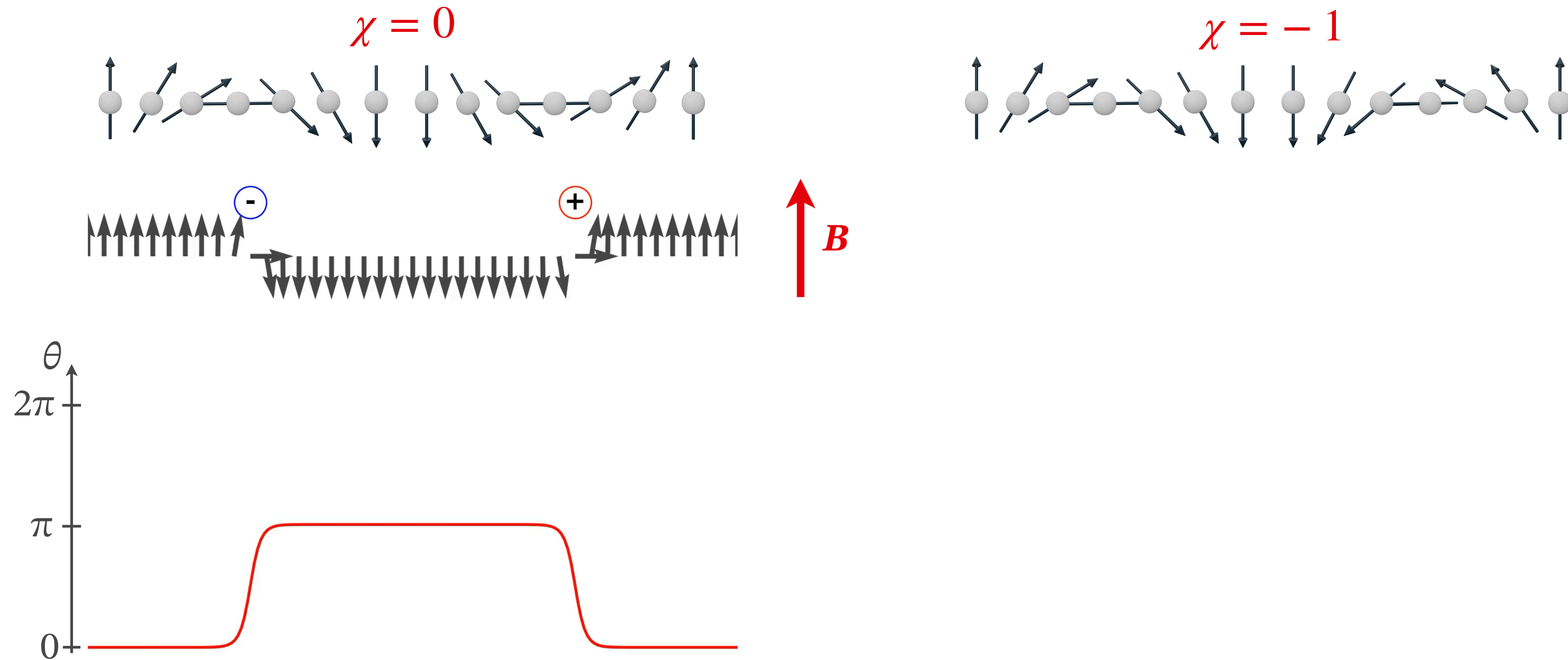
Total winding number determines the allowed transformations



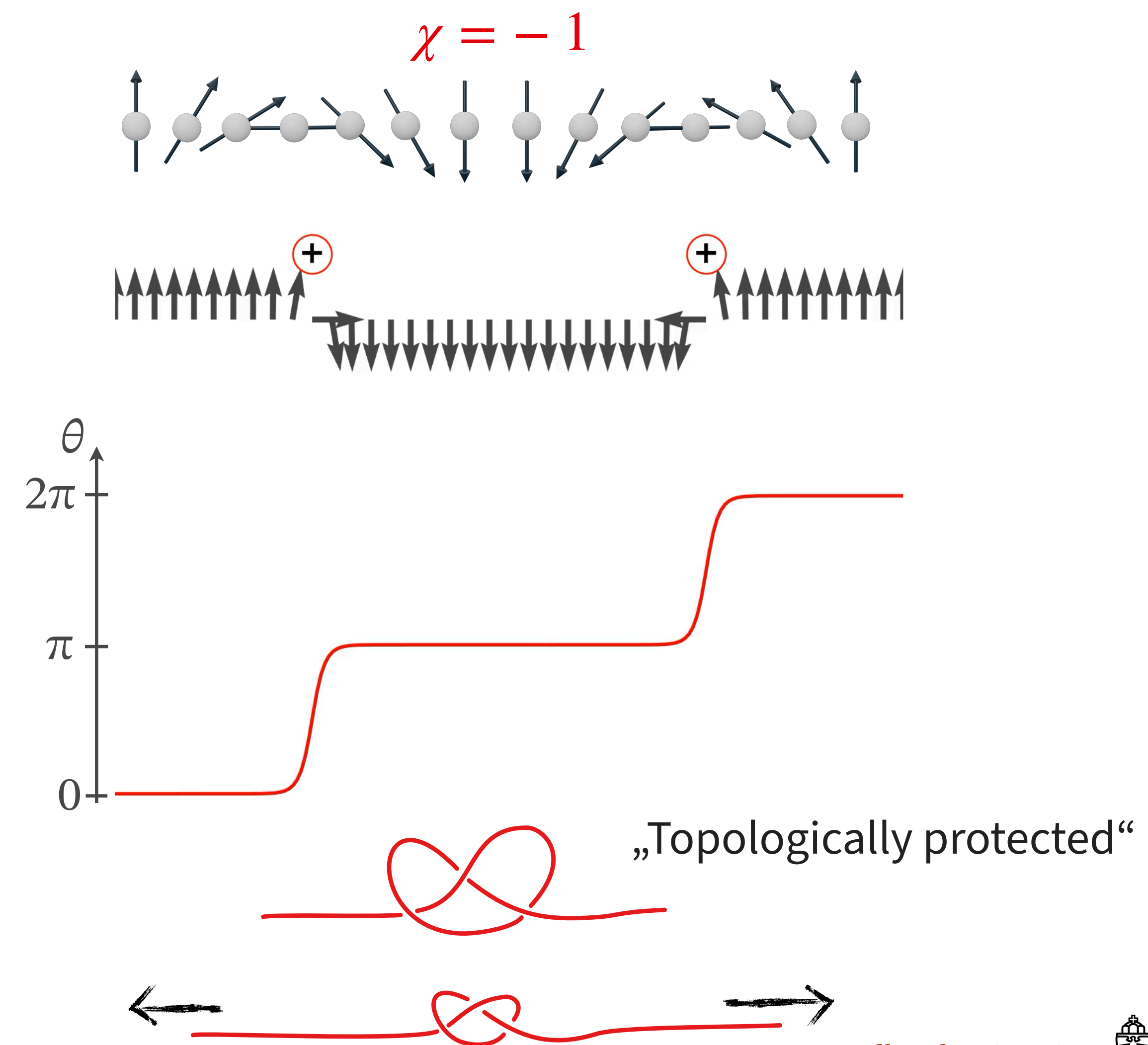
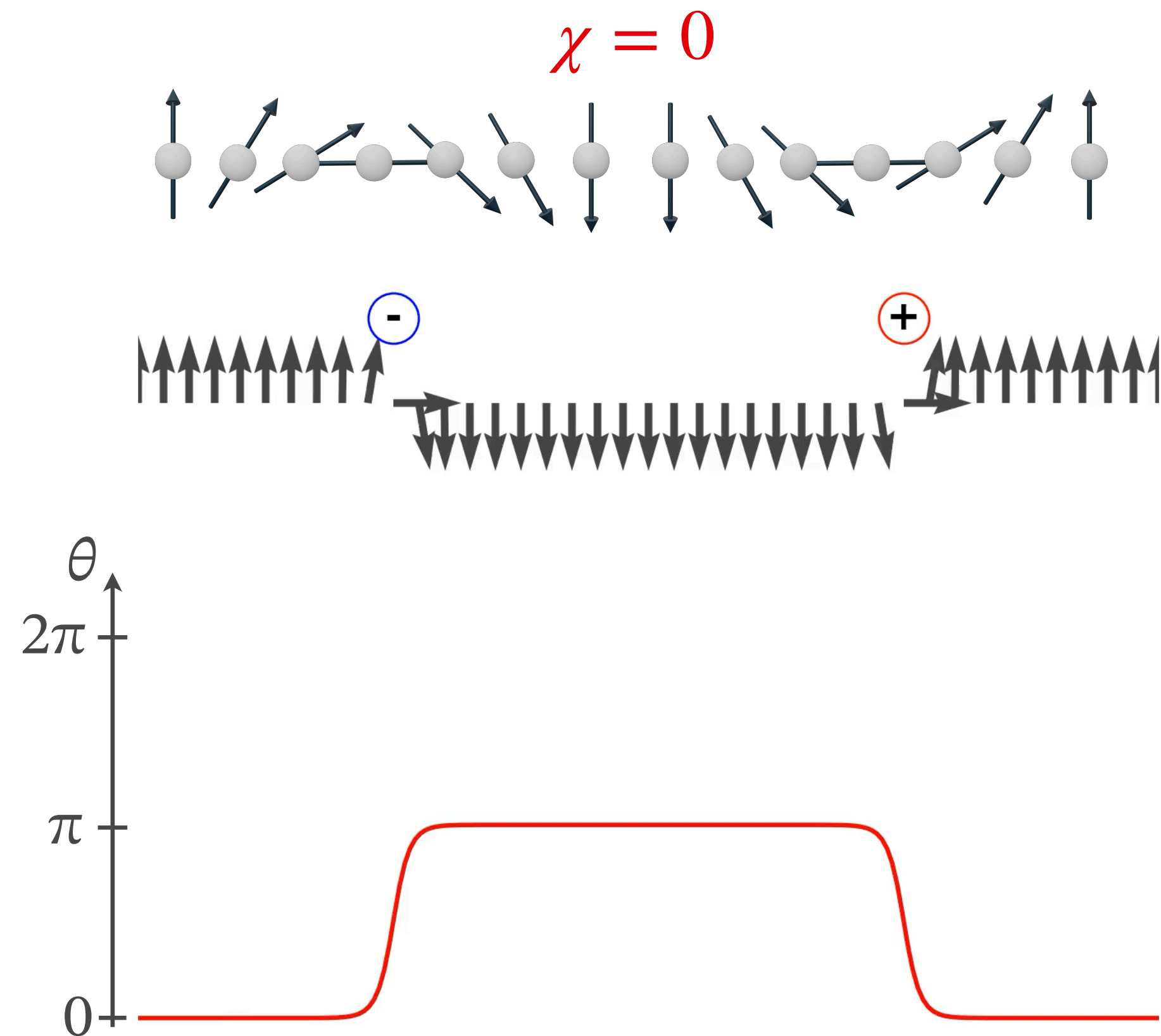
Which domain-wall pair can be removed by a field?



Which domain-wall pair can be removed by a field?



Configurations with integer winding number are topologically protected

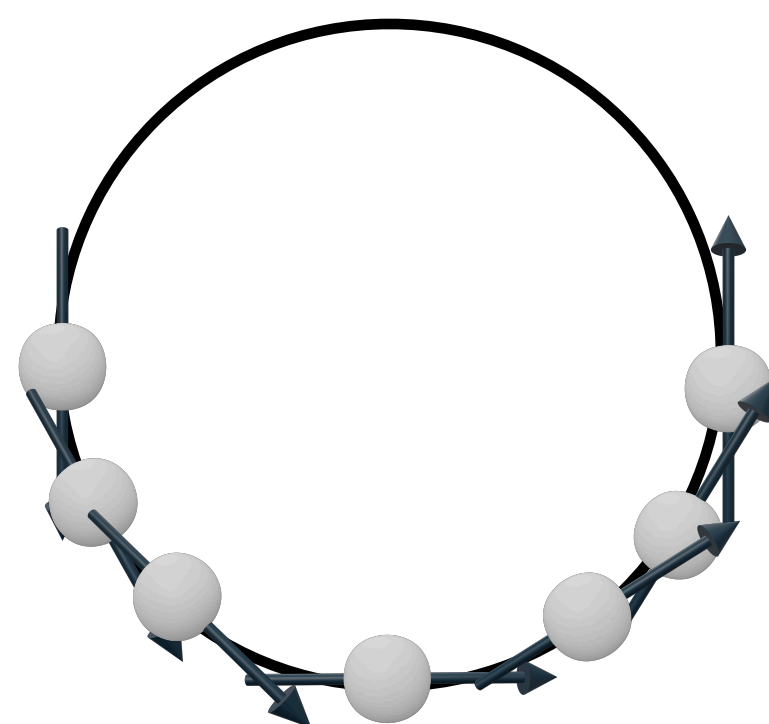
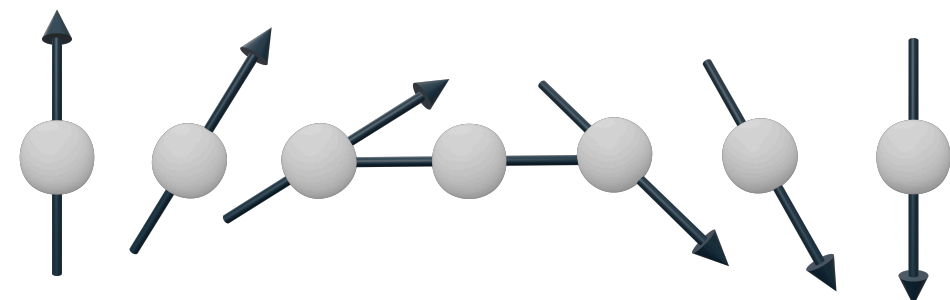


Topological textures also exist in higher dimensions

1D

Domain walls

$$\mathbf{m} : \mathbb{R} \rightarrow S^1$$

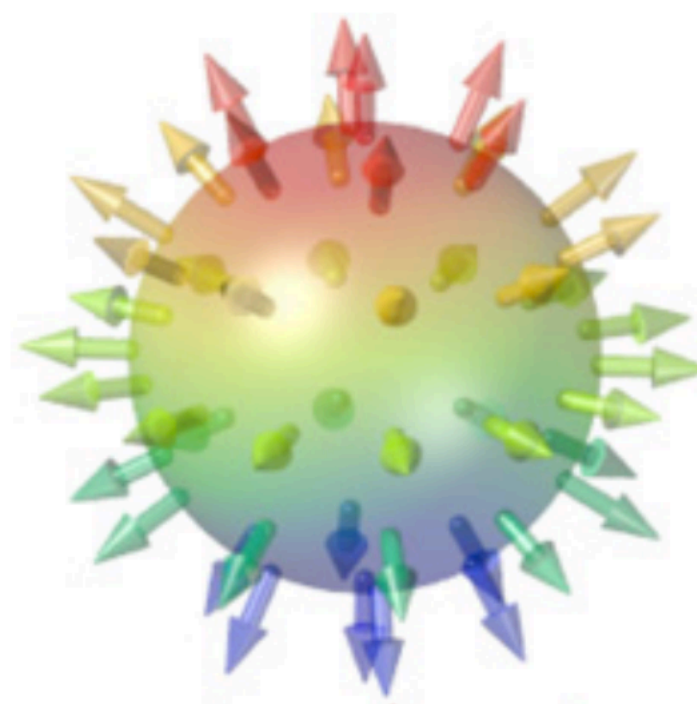
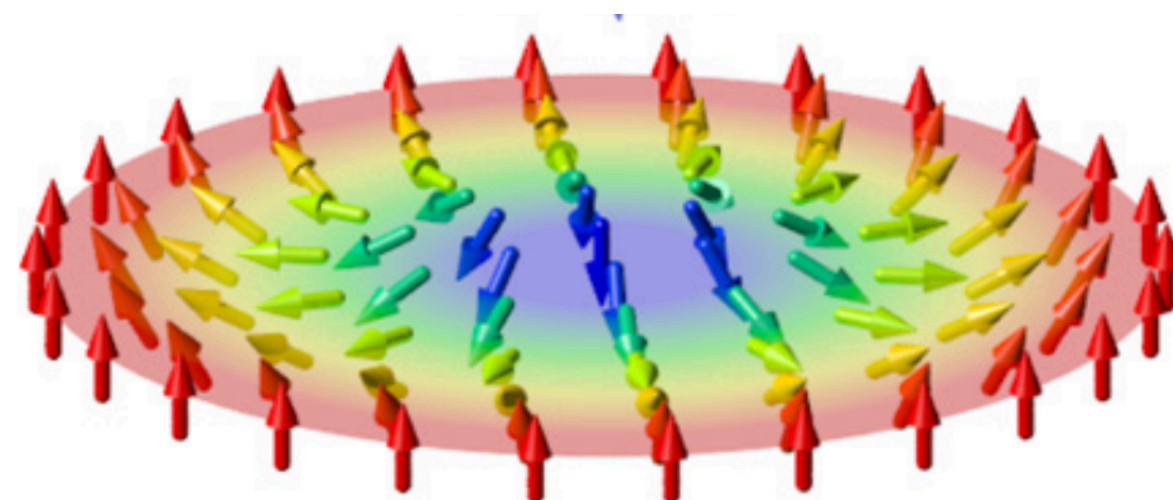


$$\chi = \frac{1}{2\pi} \int_{\mathbb{R}} dx \partial_x \theta(x)$$

2D

Vortices/merons, Skyrmions, ...

$$\mathbf{m} : \mathbb{R}^2 \rightarrow S^2$$



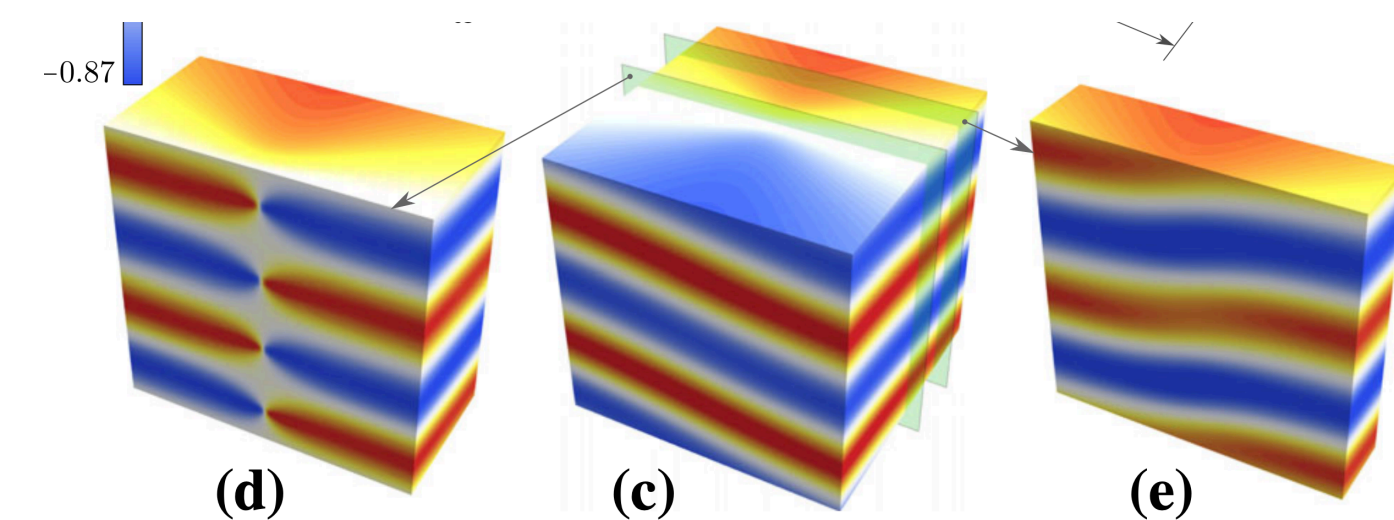
Everschor-Sitte et al.,
JAP 115, 172602 (2014)

$$Q = \frac{1}{4\pi} \int_{\mathbb{R}^2} dx dy \left(\frac{\partial \mathbf{m}}{\partial x} \times \frac{\partial \mathbf{m}}{\partial y} \right) \cdot \mathbf{m}$$

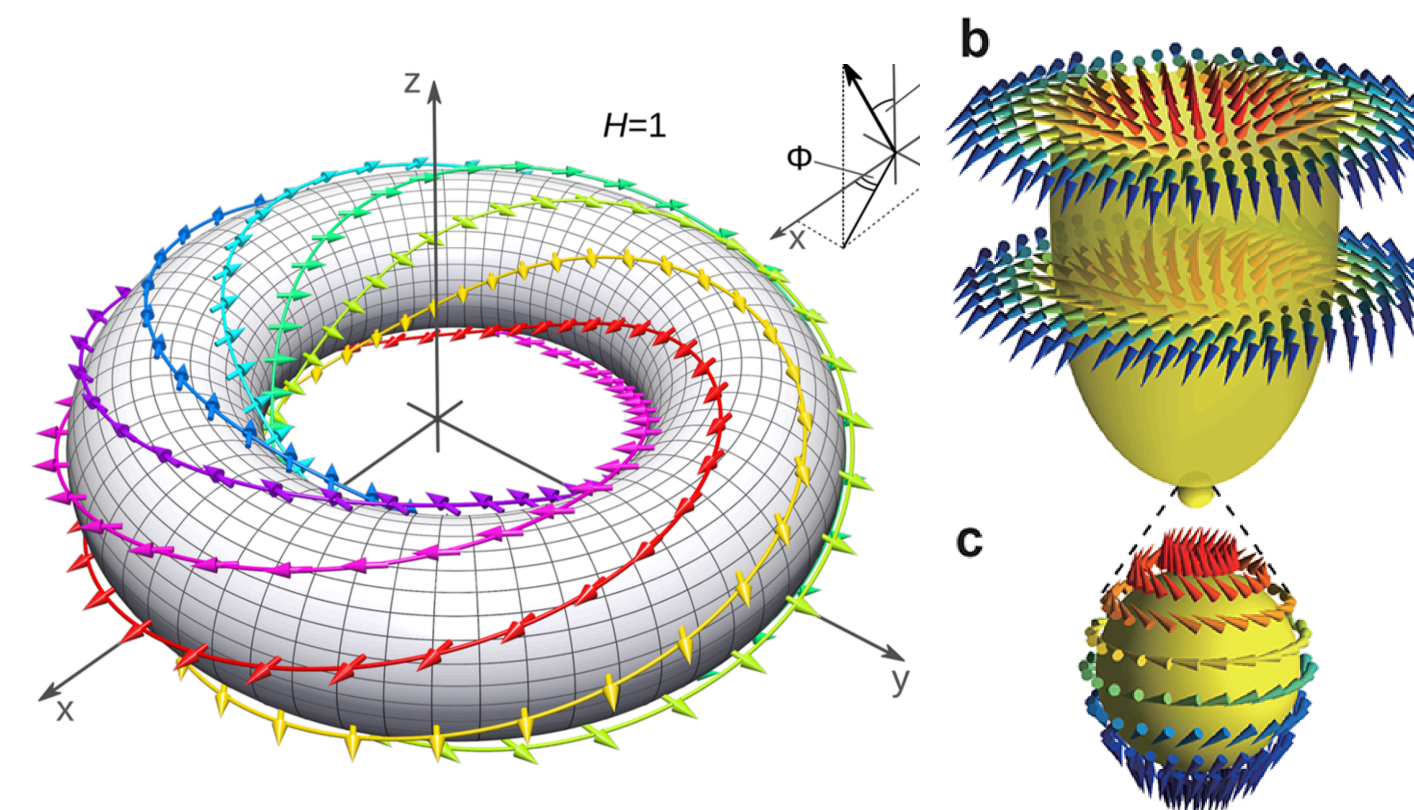
3D

Screw disclinations, Hopfions, Bobbers, ...

$$\mathbf{m} : \mathbb{R}^3 \rightarrow S^2$$



Azhar et al., PRL 128, 157204 (2022)

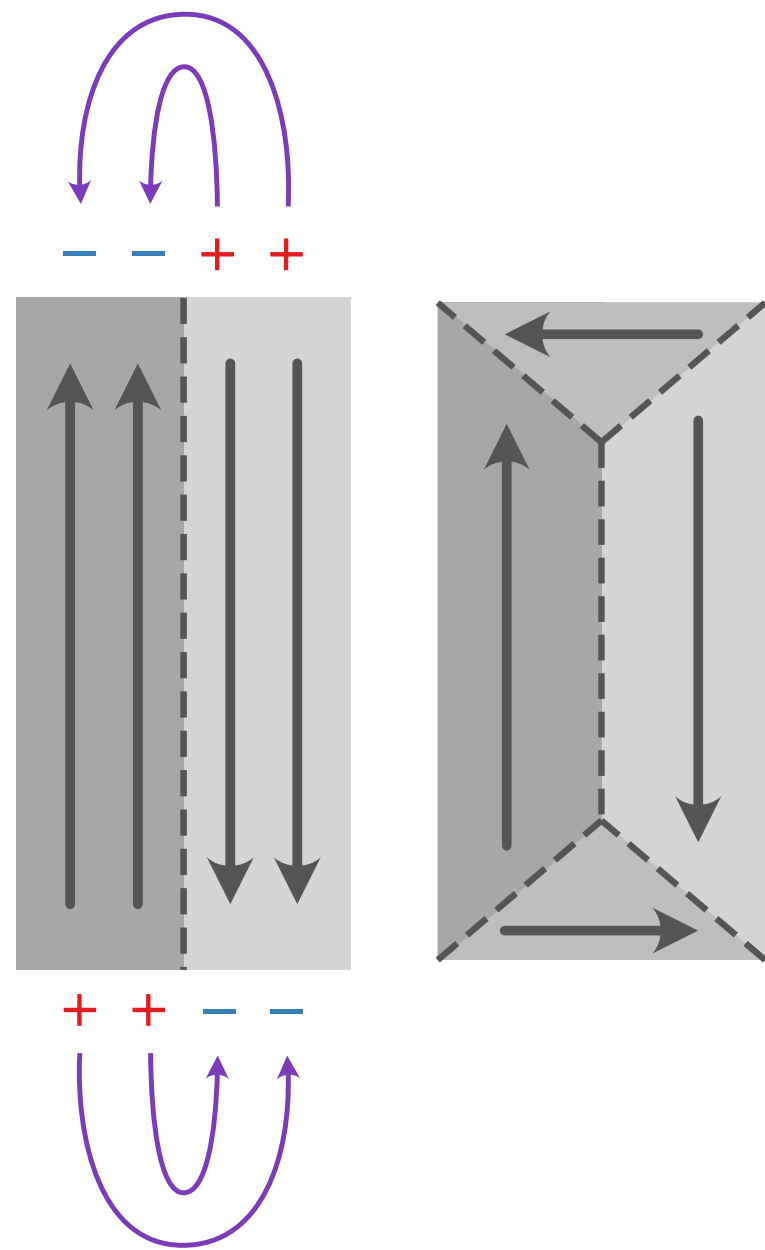


APL Mater. 10,
111113 (2022)

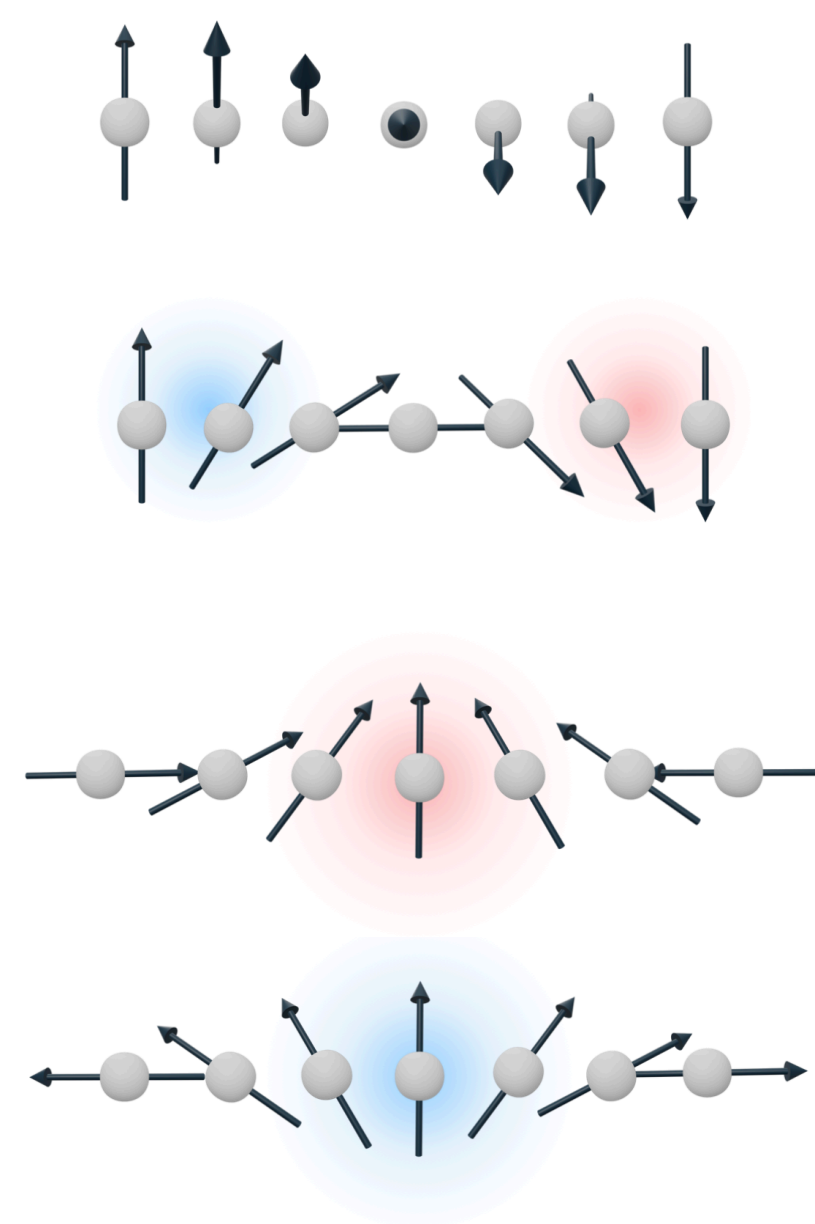
Nano Lett. 2023,
23, 5164–5170

Summary of whole lecture

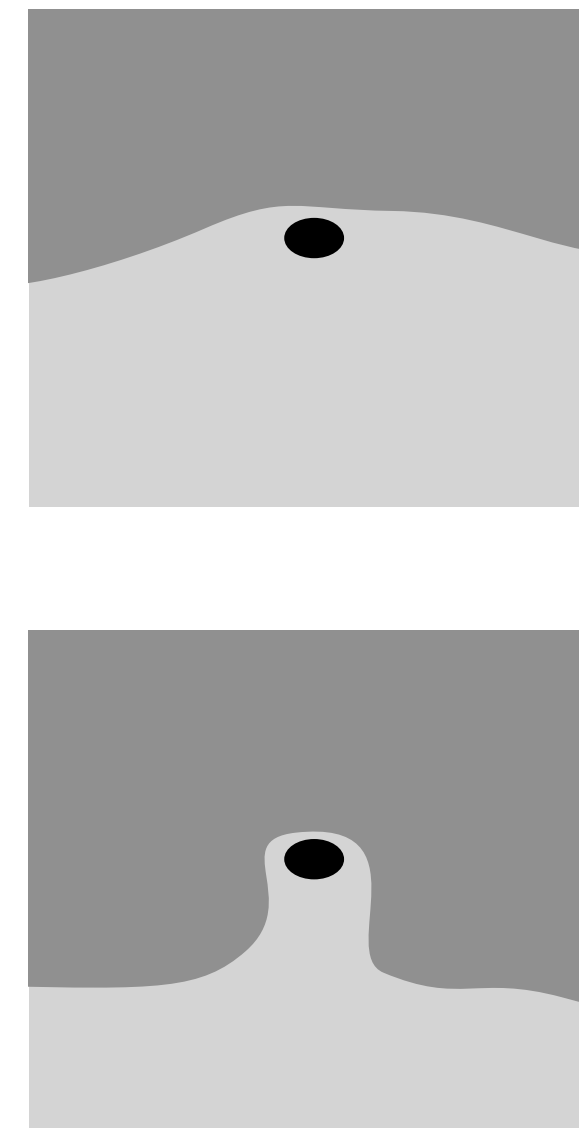
When and why do domains form?



Structure and types of domain walls



Domain-wall motion



Role of topology



Thank you for your attention!