

Spin waves & Magnonics Open Sans

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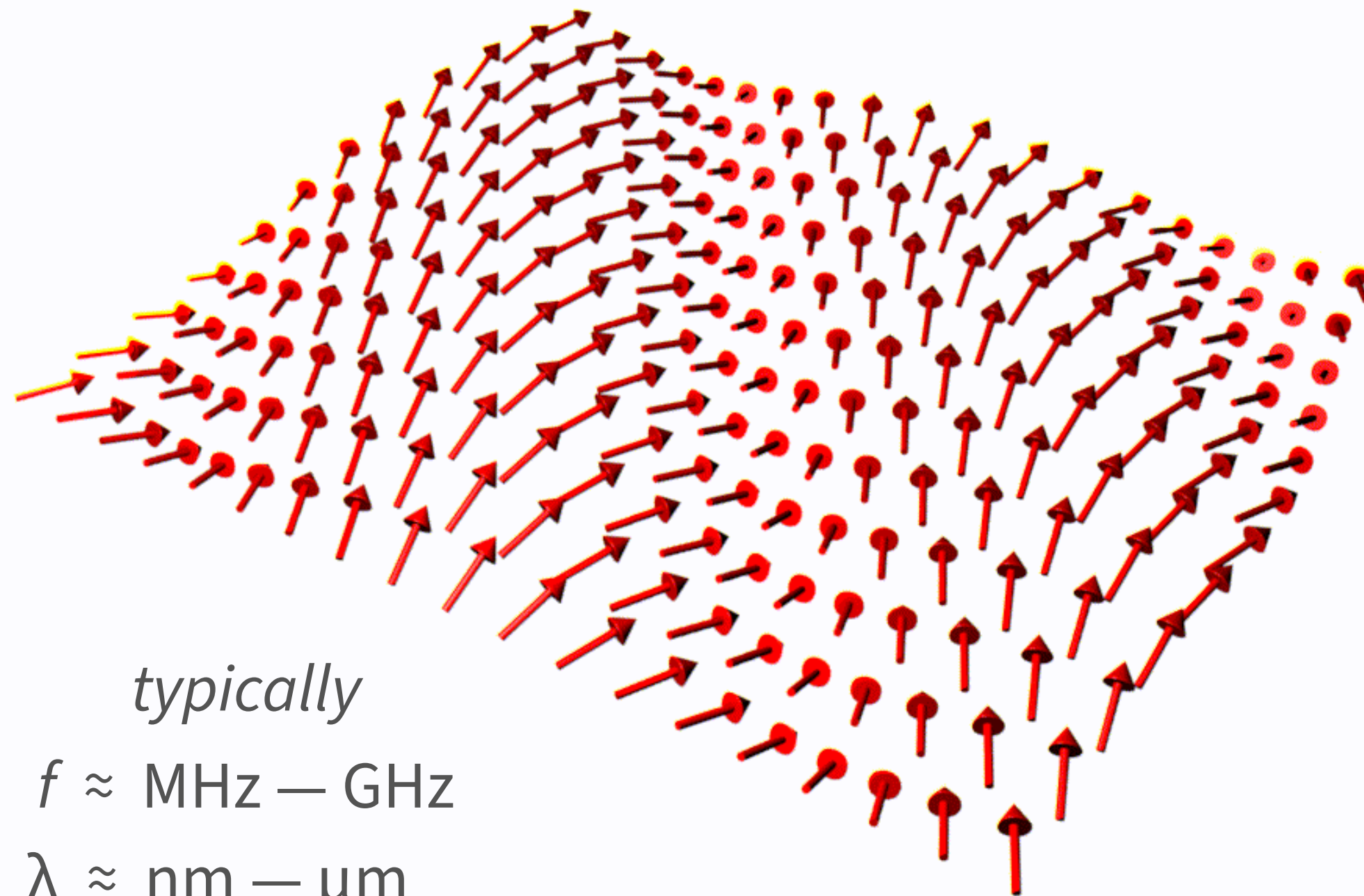
Spin waves & Magnonics Noto Serif

Lukas Körber

19.07.2026 — European School on Magnetism 2026, Uppsala

Magnons are the elementary excitations in magnetically ordered materials

magnon in
a ferromagnet



typically

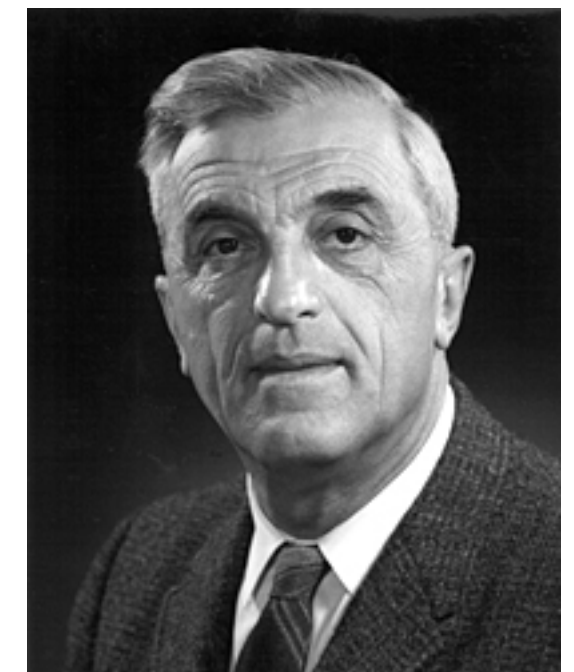
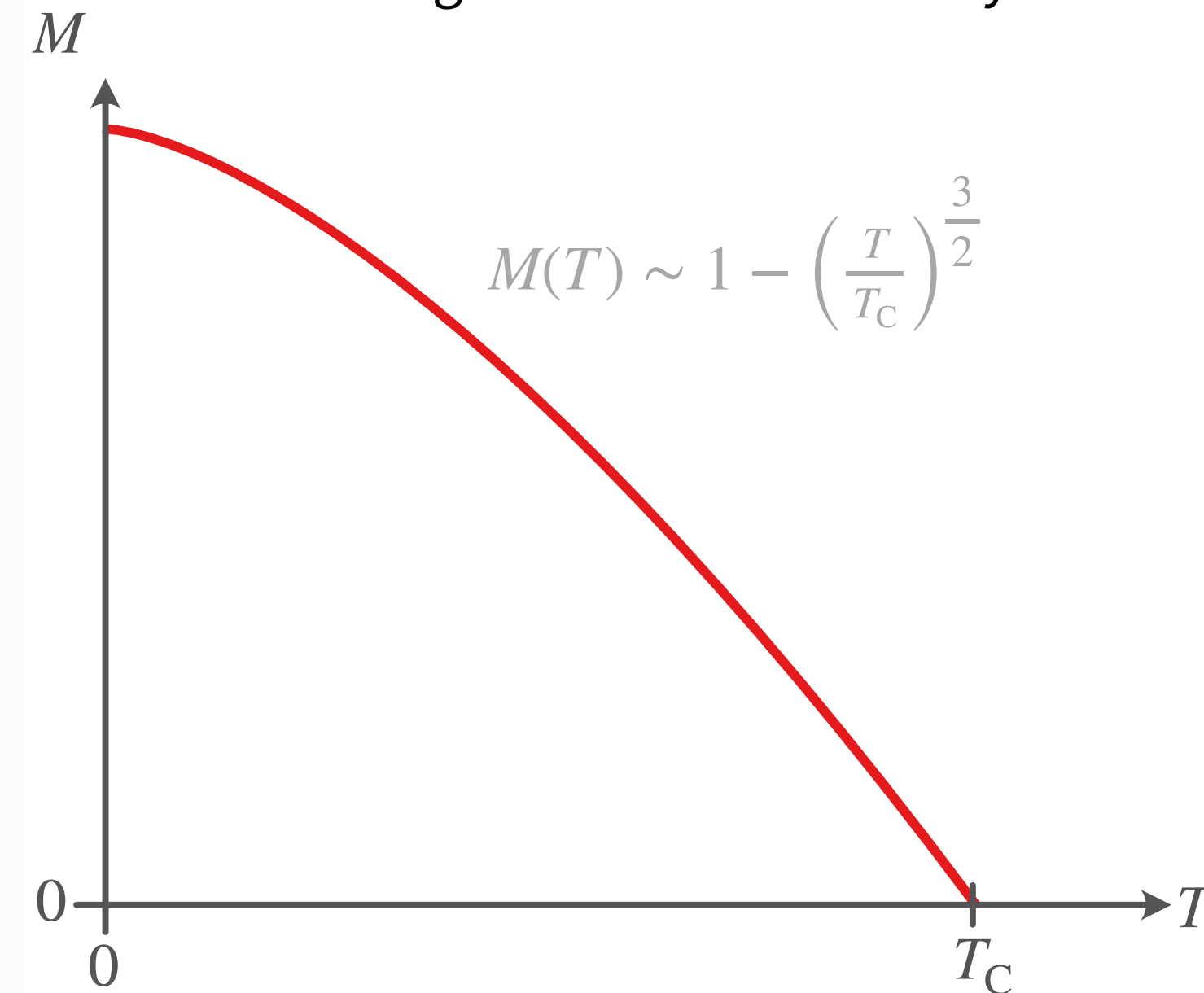
$f \approx \text{MHz} - \text{GHz}$

$\lambda \approx \text{nm} - \mu\text{m}$

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Bloch 3/2 law

Idea: Magnetization reduced by thermal magnons.



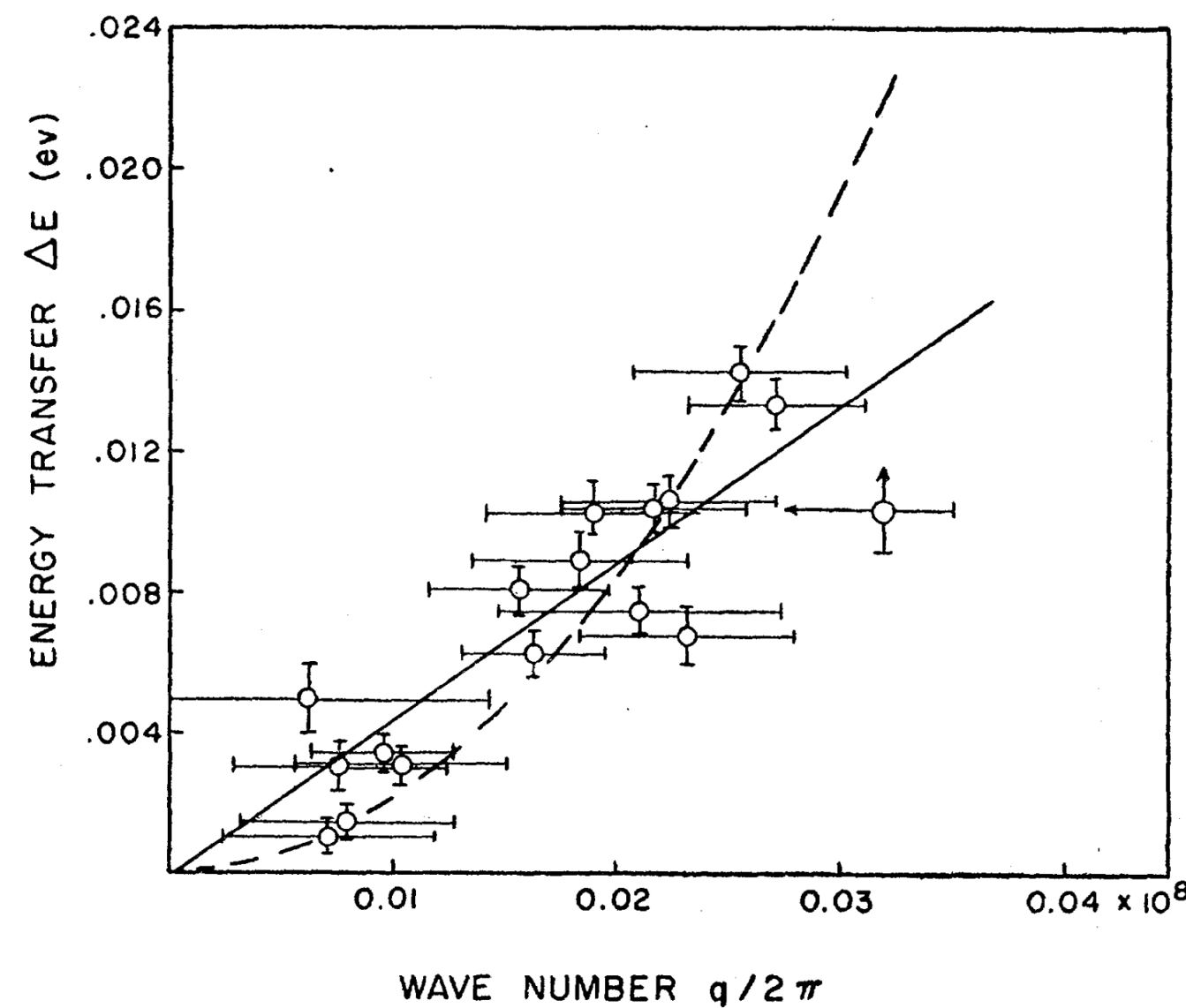
Felix Bloch (1930)*

Bloch, *Zeitschrift für Physik* **61** (3–4): 206–219 (1930)

*Image Source: Stanford University / Courtesy Stanford News Service (CC BY 3.0)

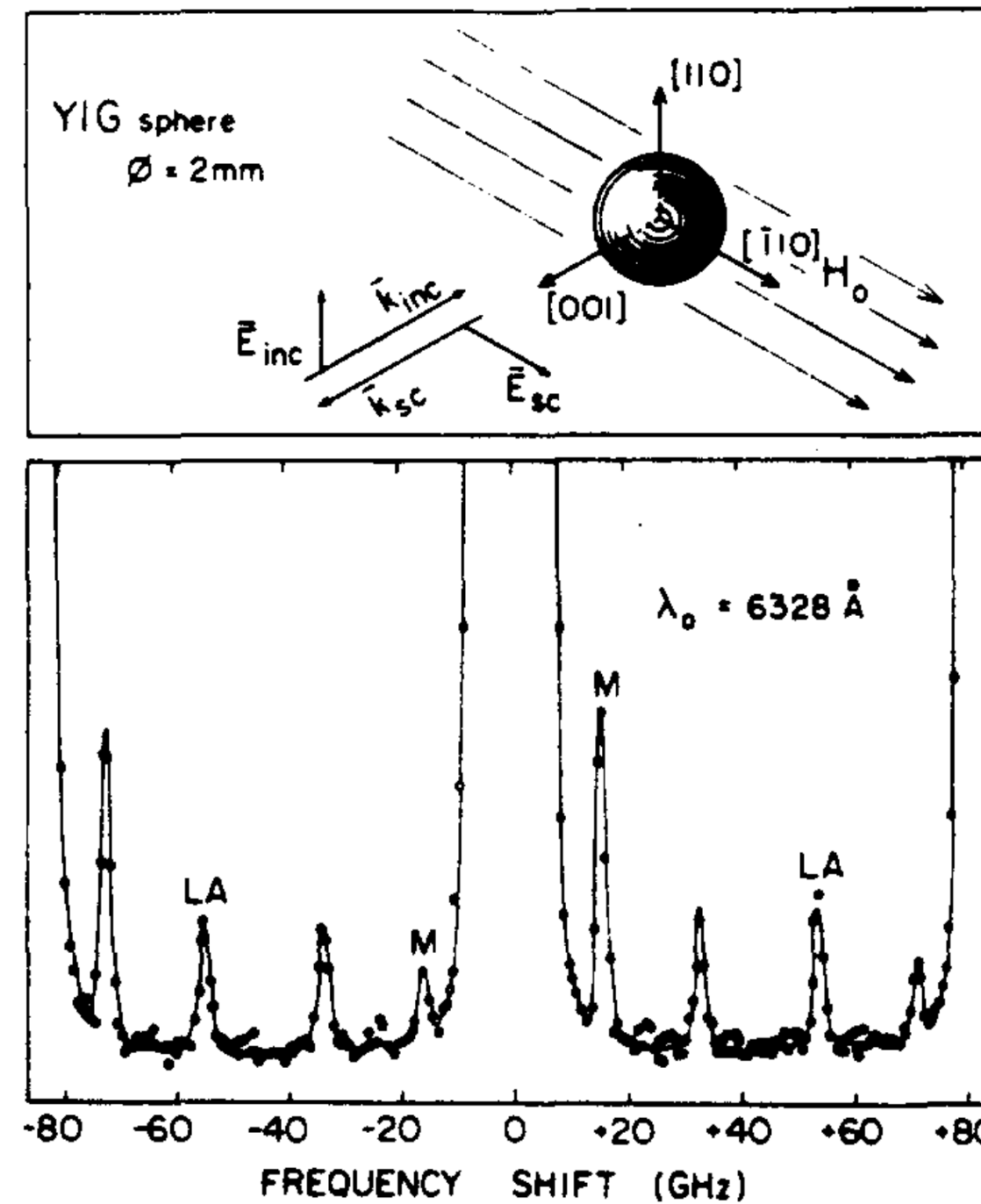
Magnons are the elementary excitations in magnetically ordered materials

First measurement by Neutron scattering



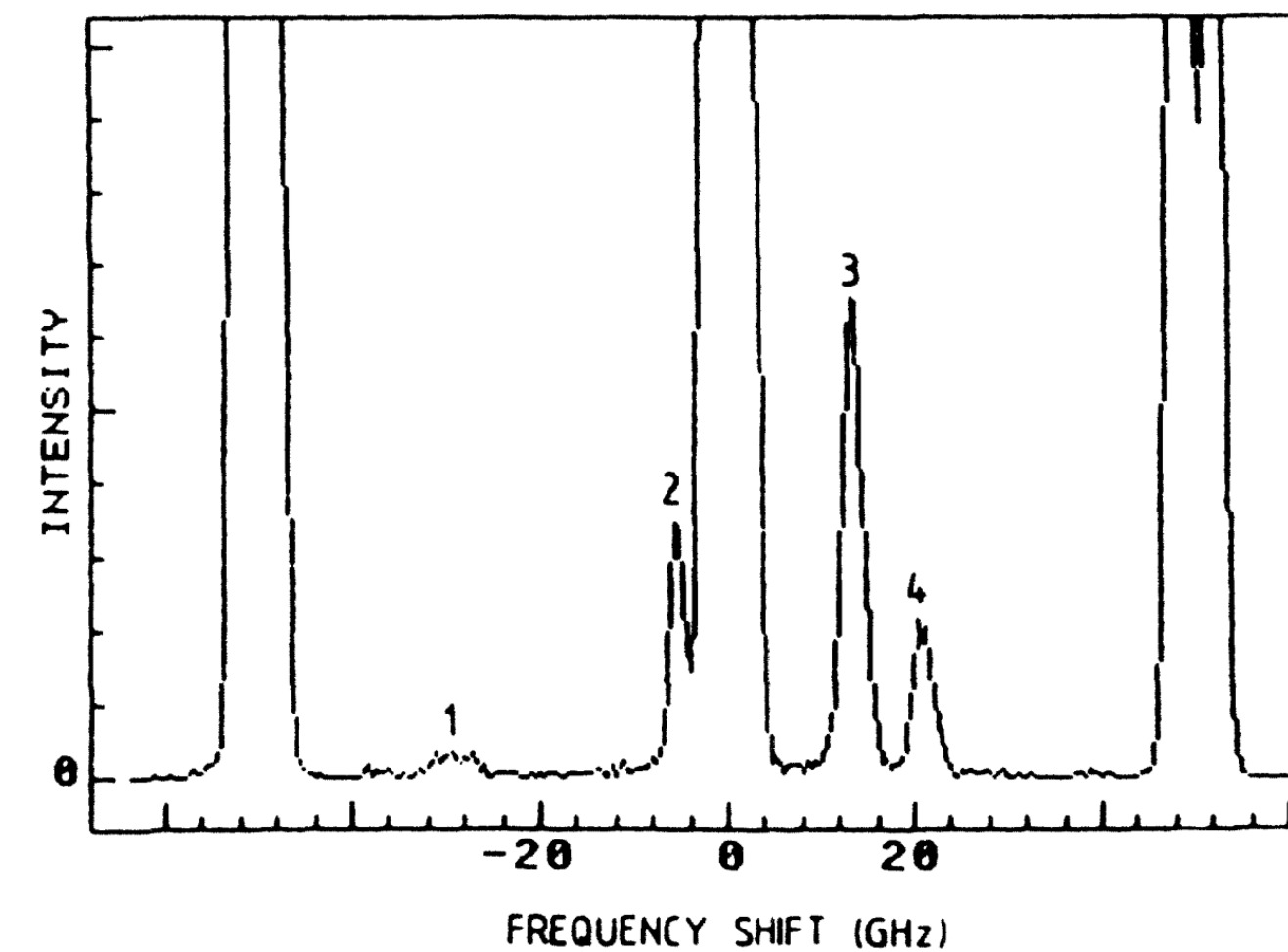
Brockhouse, Phys. Rev. 106, 859 (1957)

Historical measurements by Brillouin-light scattering



Sandercock & Wettling,
Solid State Comm., 13(10), 1729-1732. (1973).

Evidence of interlayer exchange

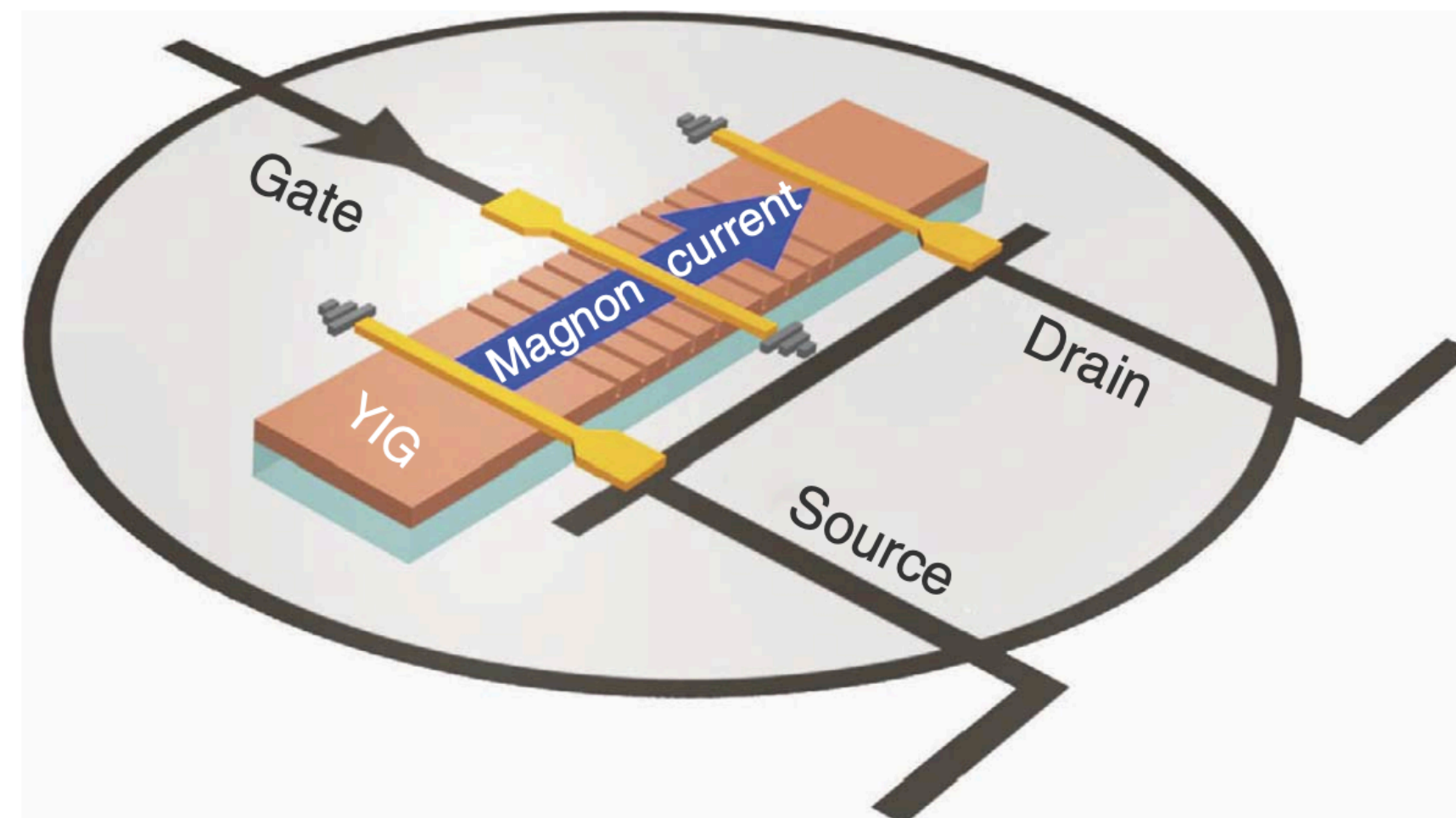


Grünberg et al.,
Phys. Rev. Lett. 57, 2442 (1986)

**Later lead to Nobel Prize
for GMR effect (2007)**

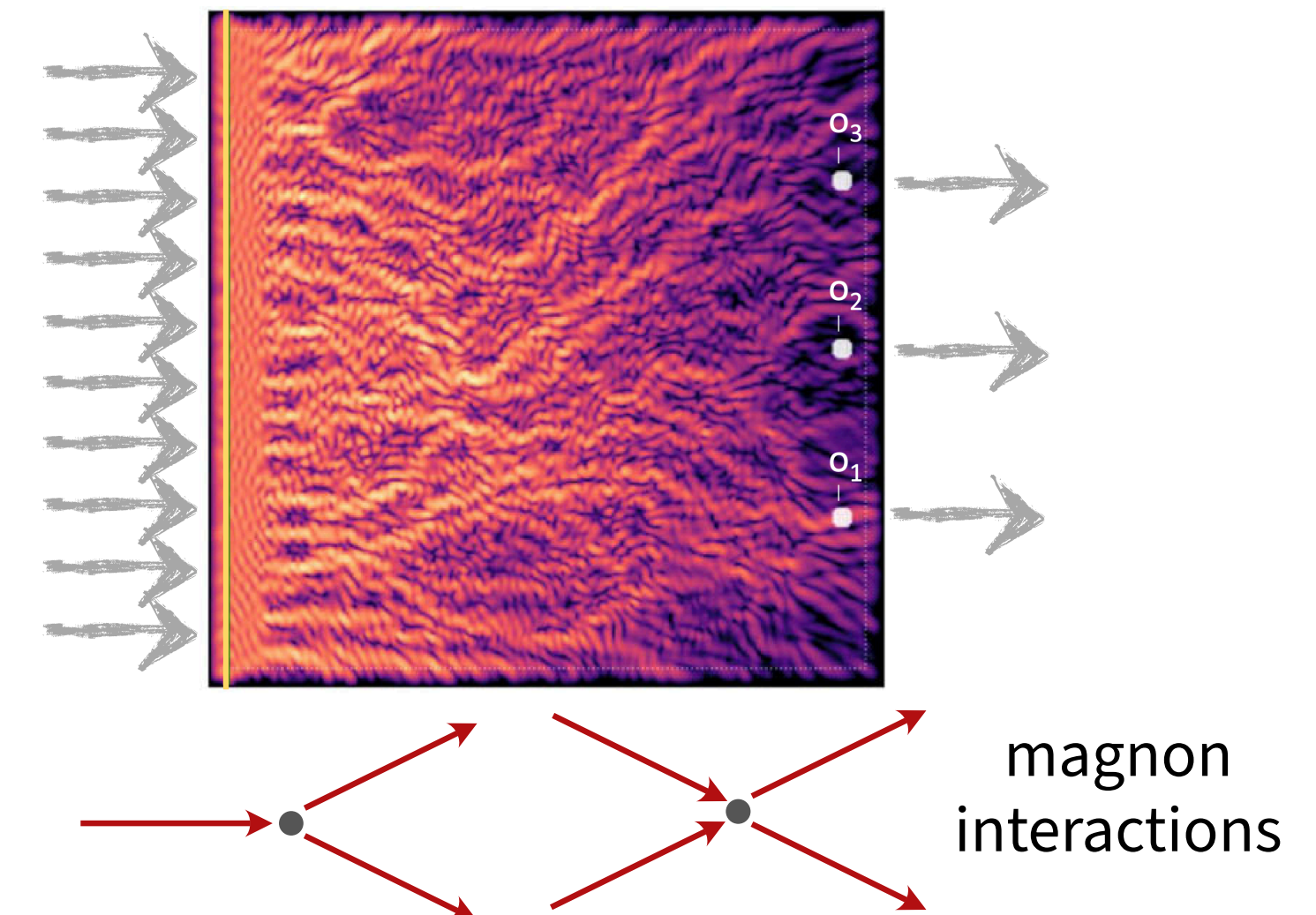
Today the field of magnonics is a vibrant research field

Wave-based logic (magnon transistor)



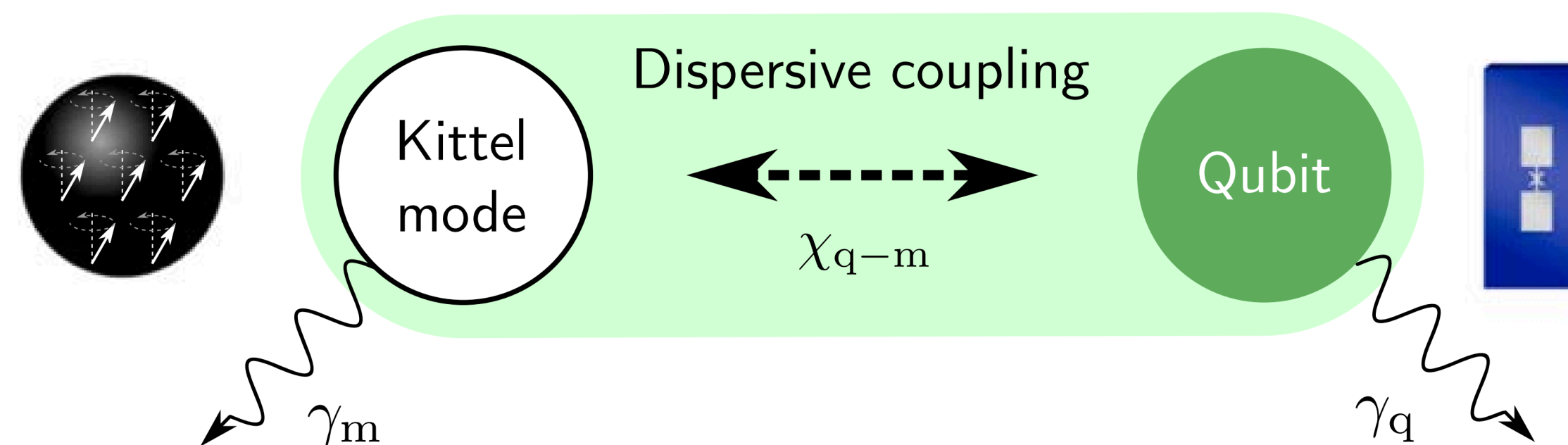
Chumak et al., Nat. Comm. 5:4700 (2024)

Neuromorphic computing using magnons



Papp et al., Nature Communications 12, 6422 (2021)

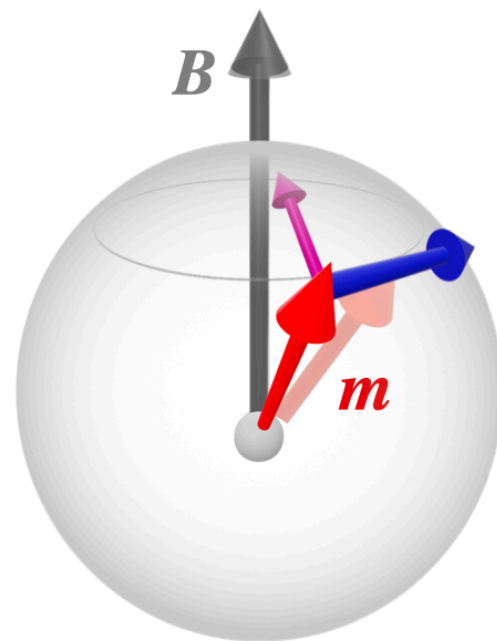
Interaction with spin qubits (quantum magnonics)



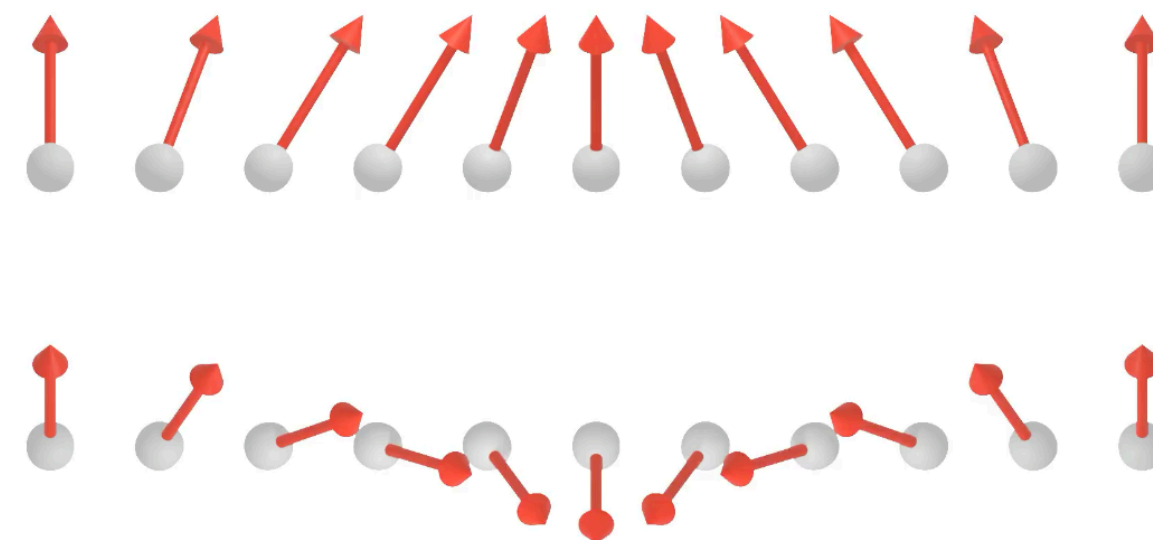
Lachance-Quirion et al., Appl. Phys. Express 12, 070101 (2019)

What we are going to talk about in this lecture:

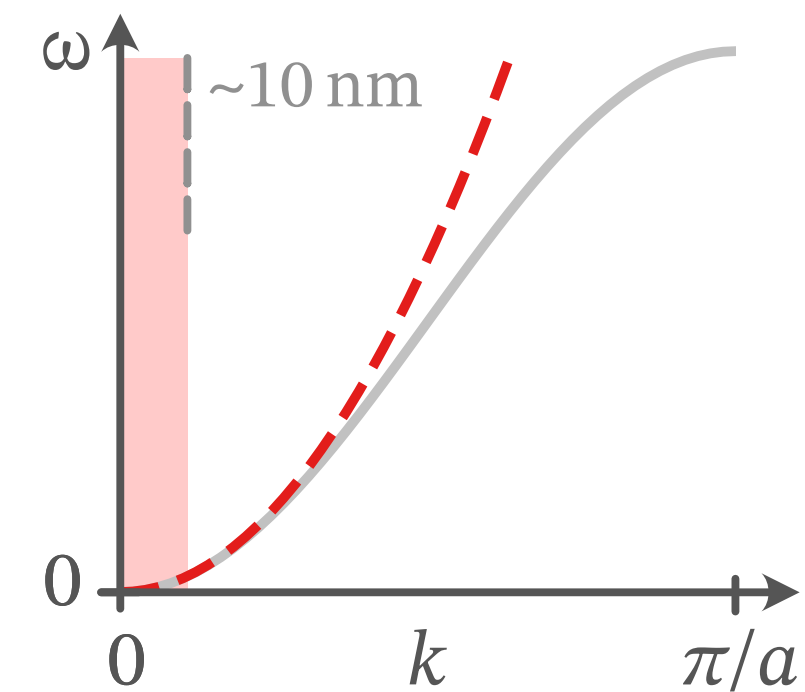
Precessional dynamics



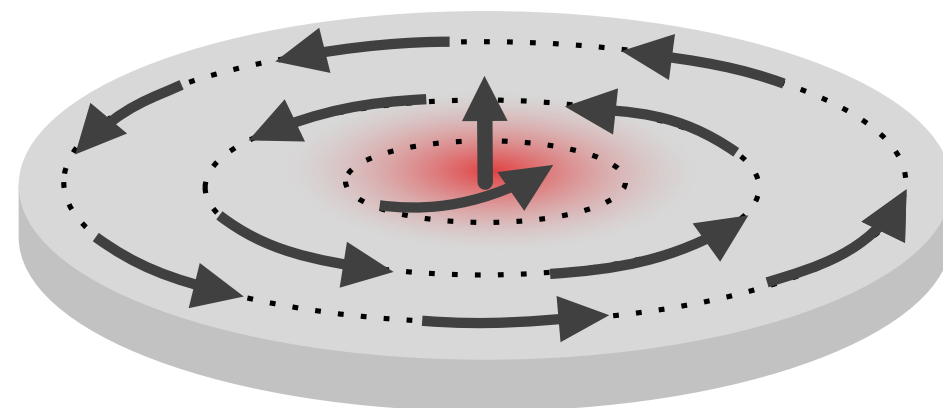
Magnons



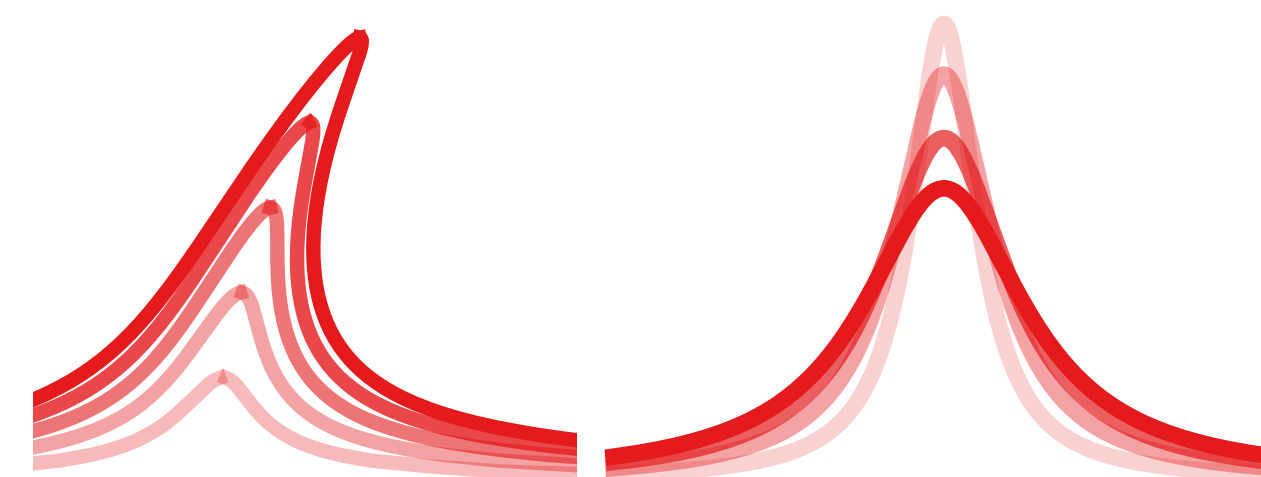
Long-wavelength limit



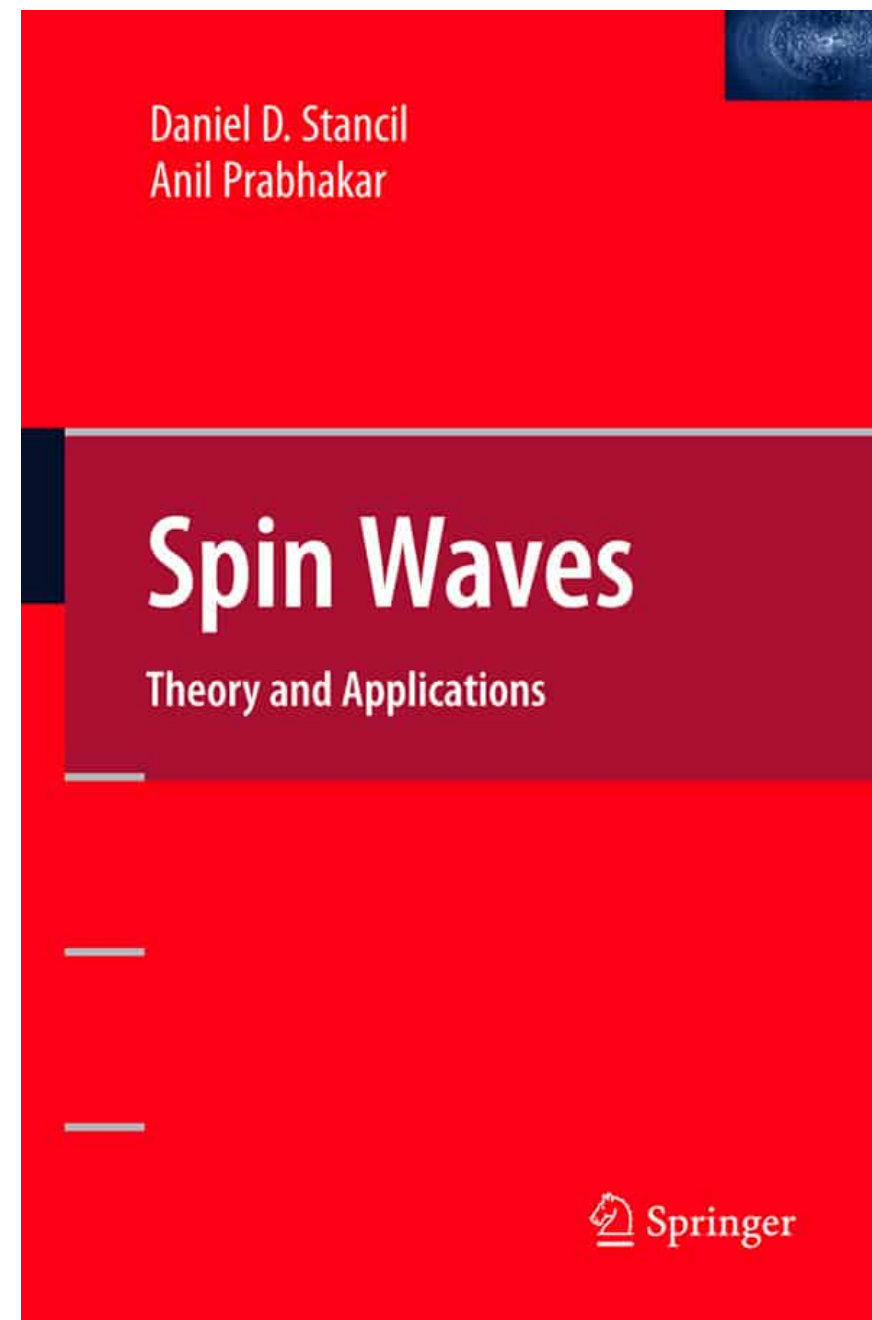
Confined magnons



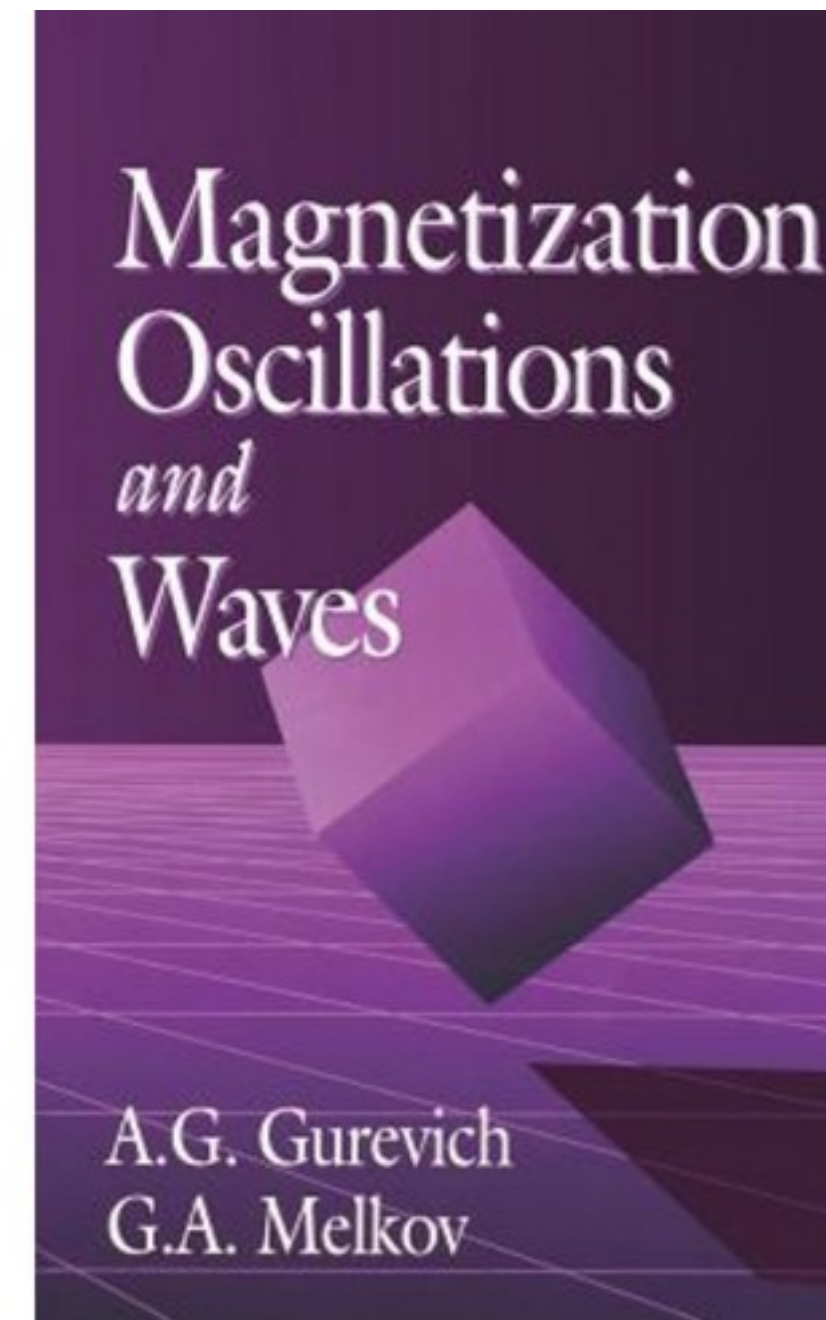
Damped & driven



Literature



Spin waves – Theory and applications
Daniel D. Stancil & Anil Prabhakar
Springer, 2009
ISBN: 978-0-387-77865-5



**Magnetization Oscillations
& Waves**
A.G. Gurevich & G.A. Melkov
CRC Press, 1996
ISBN: 978-0-8493-9460-7

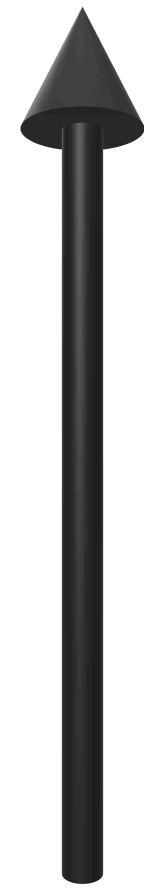
Precessional dynamics of magnetic moments

Magnetic moments originate from angular momenta

Zeemann energy

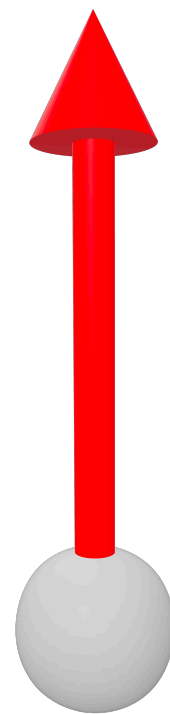
$$E = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

This is the lowest-energy
Configuration.



External field

$\boldsymbol{B}$



Magnetic moment

$\boldsymbol{\mu}$

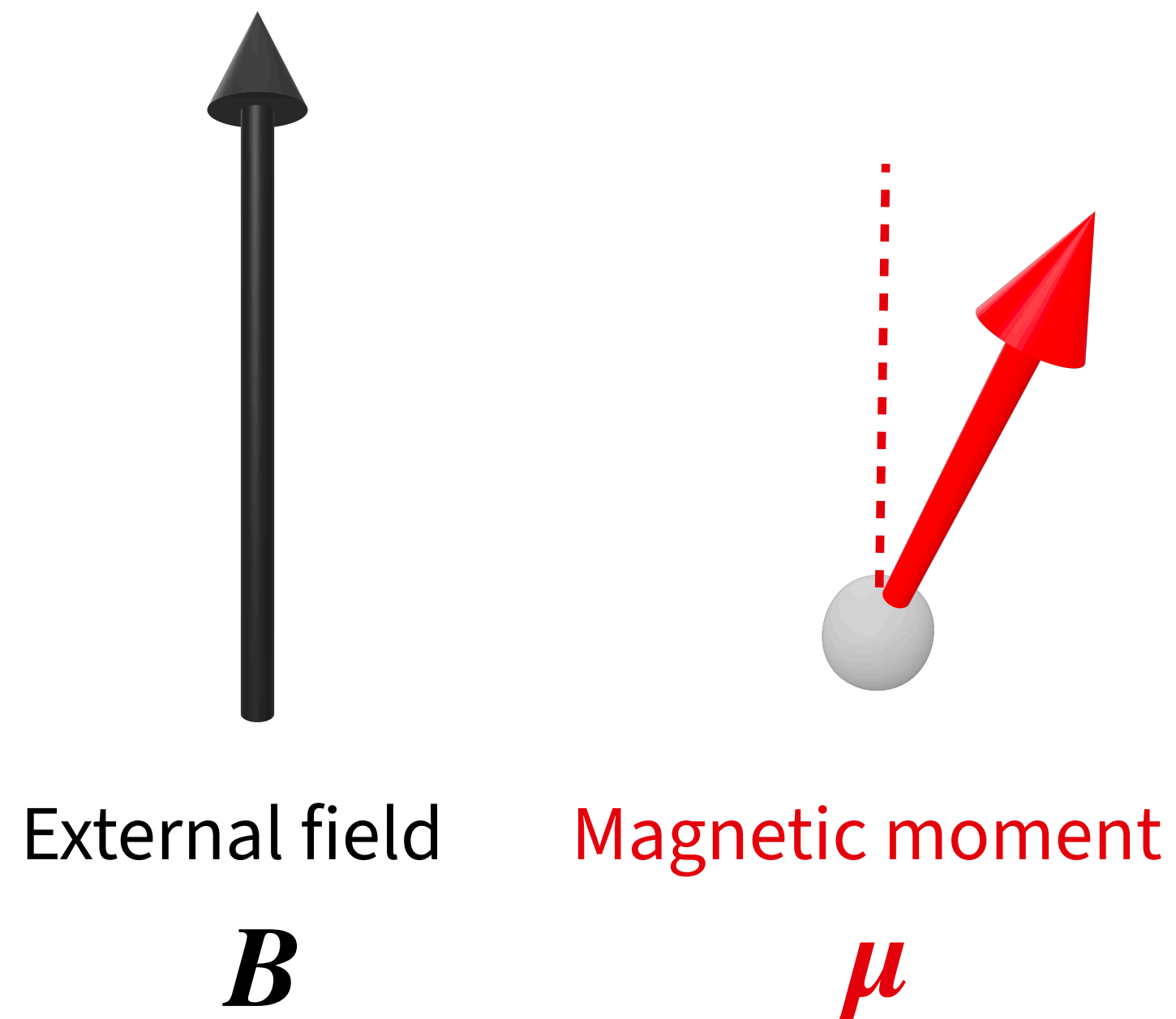
Magnetic moments originate from angular momenta

Zeemann energy

$$E = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

What happens in excited states?
Dynamics?

$$\frac{d\boldsymbol{\mu}}{dt} = ?$$

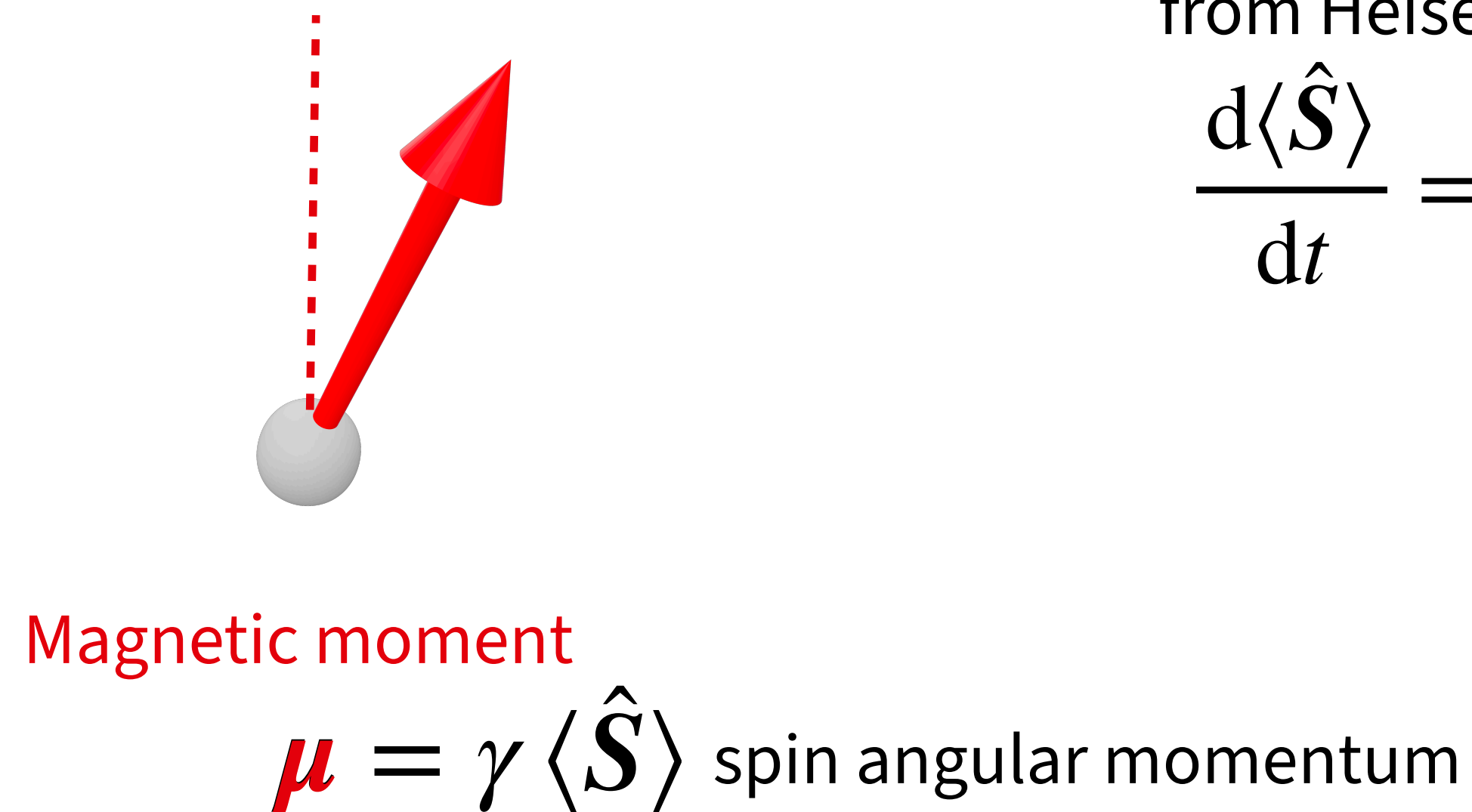


Magnetic moments originate from angular momenta

Hamiltonian

→ $\hat{H} = -\gamma \hat{\mathbf{S}} \cdot \mathbf{B}$

What happens in excited states?
Dynamics?



gyromagnetic ratio

$$\gamma = \frac{g q_e}{2m} < 0$$

👉 Orbital moment often *quenched* (= 0) by crystal symmetries.

from Heisenberg equation

$$\frac{d\langle \hat{\mathbf{S}} \rangle}{dt} = \frac{1}{i\hbar} \langle [\hat{\mathbf{S}}, \hat{H}] \rangle$$

$$[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

Commutator

And so do their dynamics

Hamiltonian

$$\hat{H} = -\gamma \hat{\mathbf{S}} \cdot \mathbf{B}$$

from Heisenberg equation

$$\frac{d\langle \hat{\mathbf{S}} \rangle}{dt} = \frac{1}{i\hbar} \langle [\hat{\mathbf{S}}, \hat{H}] \rangle$$

Work out for one component

$$[\hat{S}^x, \hat{H}] = -\gamma [\hat{S}^x, \hat{S}^x B^x + \hat{S}^y B^y + \hat{S}^z B^z]$$

$$= -\gamma \left([\hat{S}^x, \hat{S}^y] B^y + [\hat{S}^x, \hat{S}^z] B^z \right)$$

$$= -i\hbar\gamma (\hat{S}^z B^y - \hat{S}^y B^z)$$

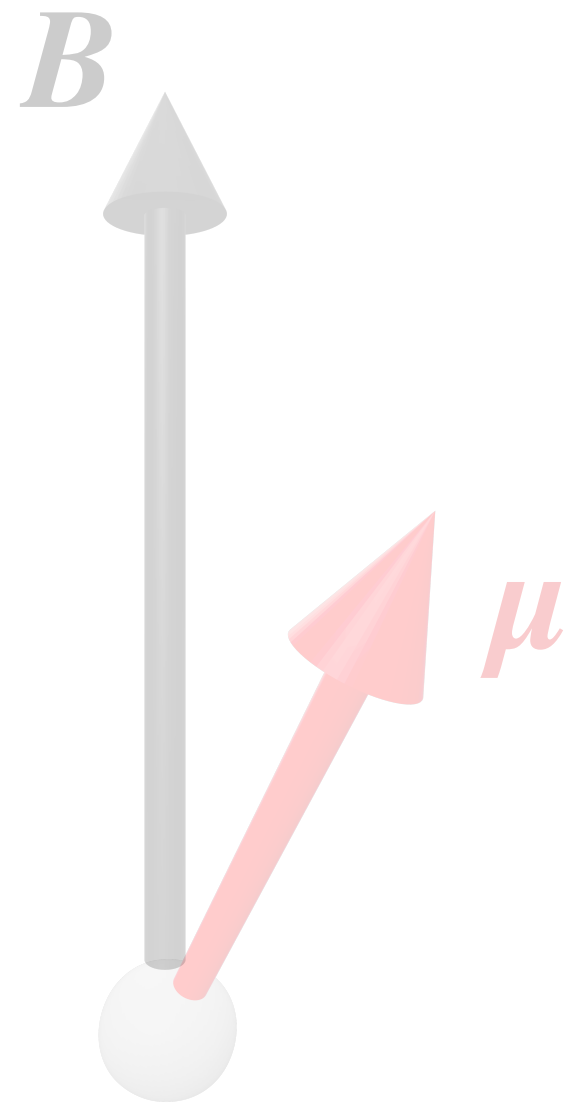
$$= +i\hbar\gamma (\mathbf{S} \times \mathbf{B})_x$$

Angular momenta satisfy

$$[\hat{S}^x, \hat{S}^y] = i\hbar \hat{S}^z$$

$$[\hat{S}^y, \hat{S}^z] = i\hbar \hat{S}^x$$

$$[\hat{S}^z, \hat{S}^x] = i\hbar \hat{S}^y$$



Magnetic moment $\boldsymbol{\mu} = \gamma \langle \hat{\mathbf{S}} \rangle$ spin angular momentum

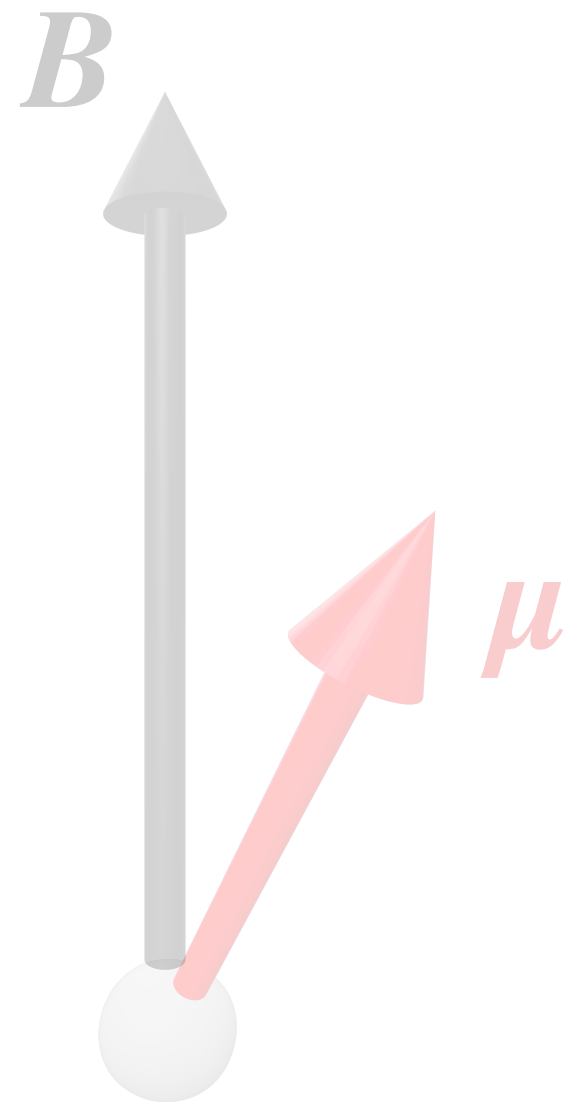
And so do their dynamics

Hamiltonian

$$\hat{H} = -\gamma \hat{\mathbf{S}} \cdot \mathbf{B}$$

from Heisenberg equation

$$\frac{d\langle \hat{\mathbf{S}} \rangle}{dt} = \gamma \langle \hat{\mathbf{S}} \rangle \times \mathbf{B}$$



Work out for one component

Angular momenta satisfy

$$[\hat{S}^x, \hat{S}^y] = i\hbar \hat{S}^z$$

$$[\hat{S}^y, \hat{S}^z] = i\hbar \hat{S}^x$$

$$[\hat{S}^z, \hat{S}^x] = i\hbar \hat{S}^y$$

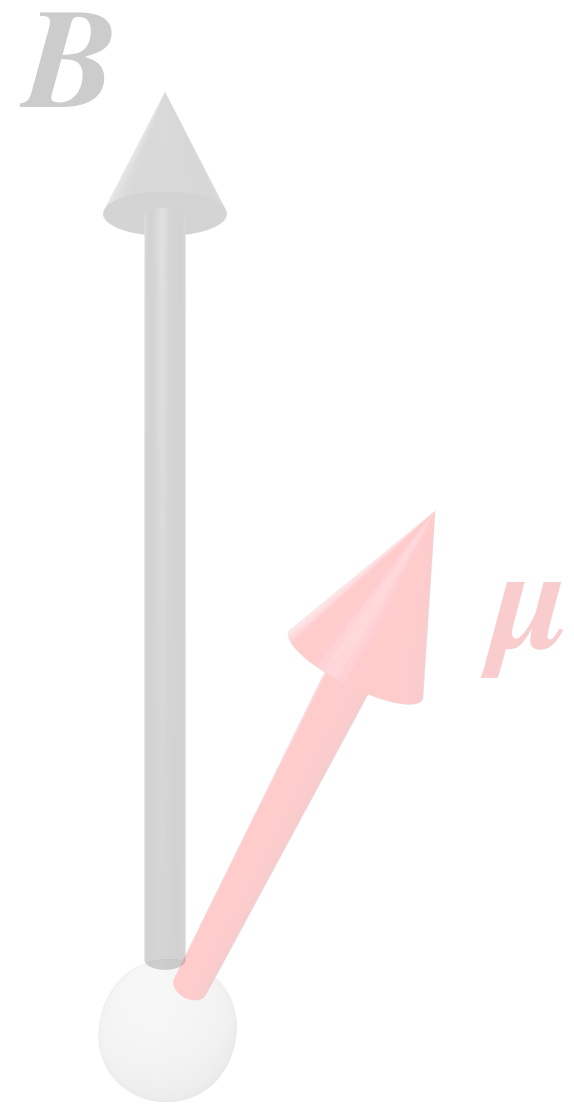
$$\begin{aligned} [\hat{S}^x, \hat{H}] &= -\gamma [\hat{S}^x, \hat{S}^x B^x + \hat{S}^y B^y + \hat{S}^z B^z] \\ &= -\gamma ([\hat{S}^x, \hat{S}^y] B^y + [\hat{S}^x, \hat{S}^z] B^z) \\ &= -\gamma (i\hbar \hat{S}^z B^y - i\hbar \hat{S}^y B^z) \\ &= +i\hbar \gamma (\mathbf{S} \times \mathbf{B})_x \end{aligned}$$

Magnetic moment $\boldsymbol{\mu} = \gamma \langle \hat{\mathbf{S}} \rangle$ spin angular momentum

And so do their dynamics

Hamiltonian

$$\hat{H} = -\gamma \hat{\mathbf{S}} \cdot \mathbf{B}$$



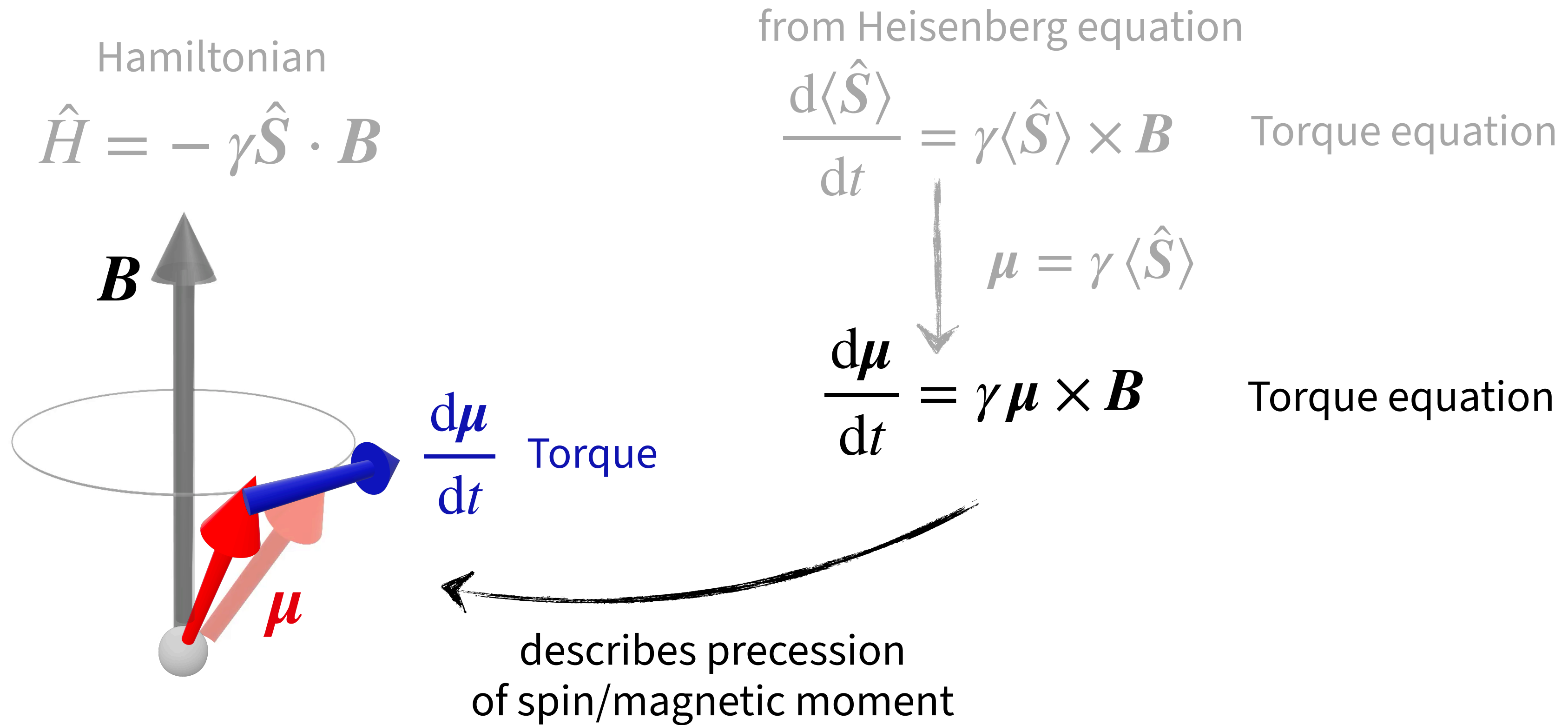
from Heisenberg equation

$$\frac{d\langle \hat{\mathbf{S}} \rangle}{dt} = \gamma \langle \hat{\mathbf{S}} \rangle \times \mathbf{B}$$

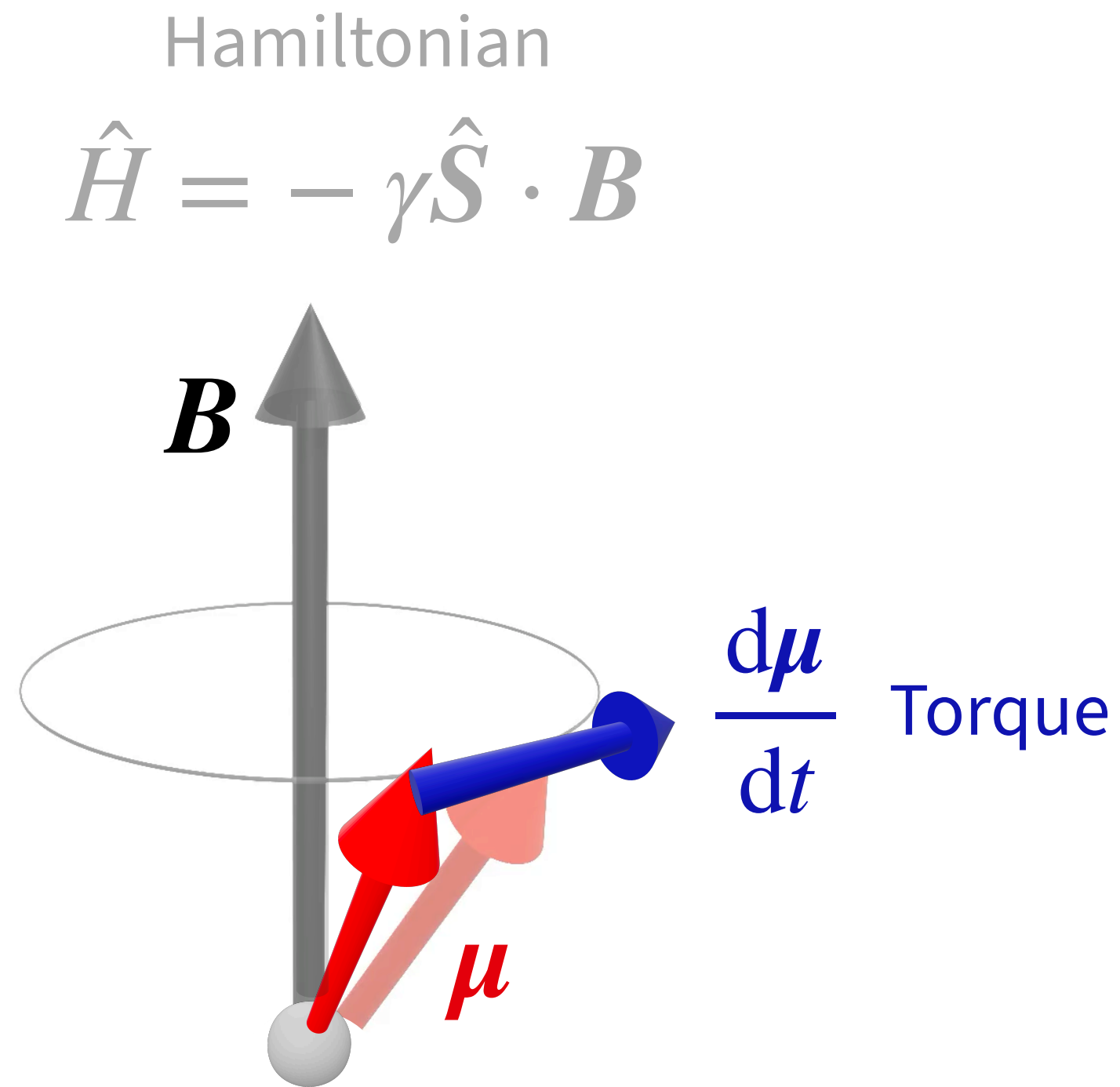
$$\downarrow \quad \mu = \gamma \langle \hat{\mathbf{S}} \rangle$$

$$\frac{d\mu}{dt} = \gamma \mu \times \mathbf{B} \quad \text{Torque equation}$$

Magnetic moments precess around magnetic fields



Magnetic moments precess around magnetic fields



from Heisenberg equation

$$\frac{d\langle \hat{S} \rangle}{dt} = \gamma \langle \hat{S} \rangle \times B \quad \text{Torque equation}$$

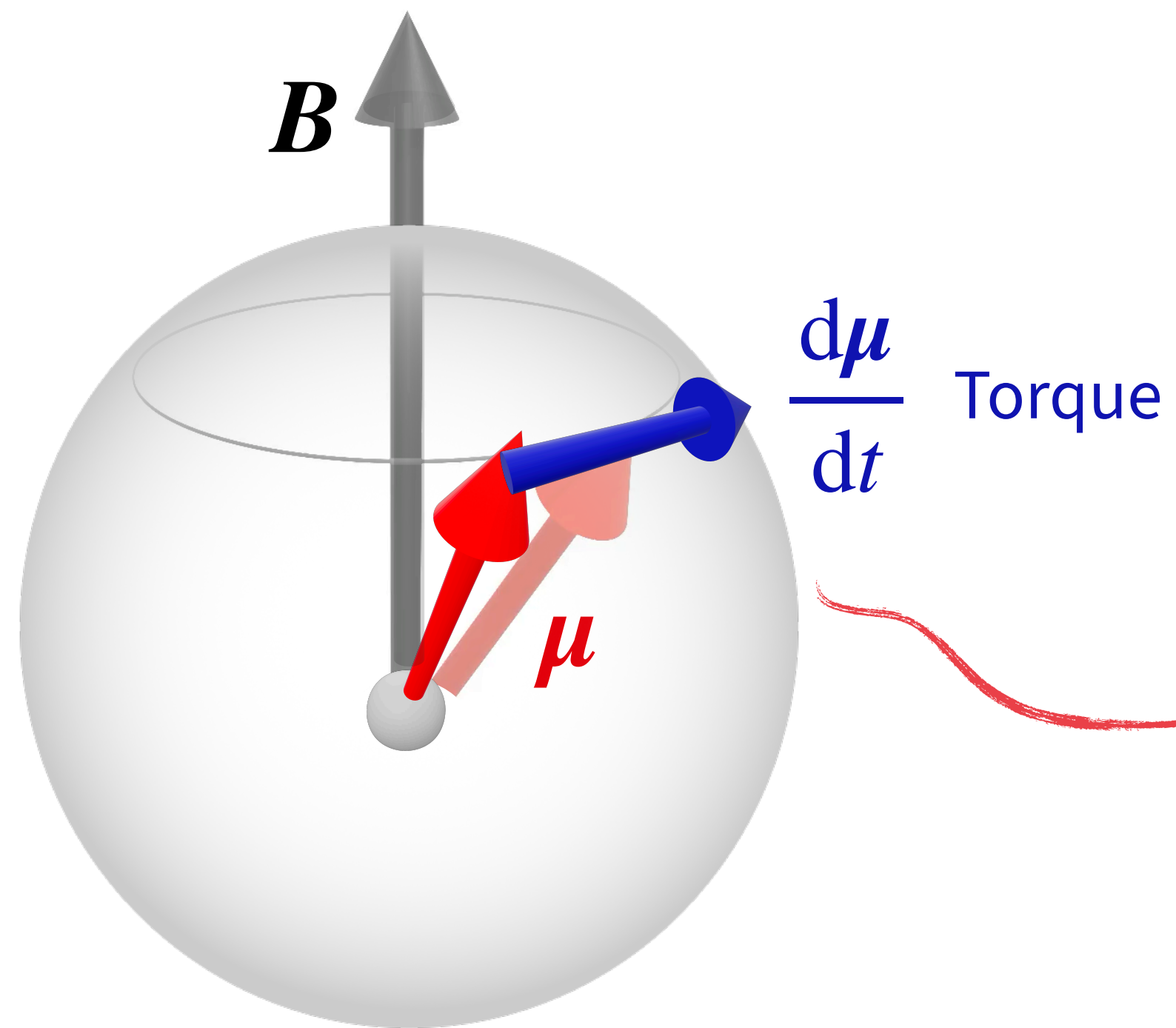
$$\mu = \gamma \langle \hat{S} \rangle$$

$$\frac{d\mu}{dt} = \gamma \mu \times B \quad \text{Torque equation}$$

Larmor precession with
angular frequency $\omega_B = \gamma B$

$$\frac{\gamma}{2\pi} \approx 28 \text{ GHz/T}$$

Magnetic moments precess around magnetic fields



$$\frac{d\mu}{dt} = \gamma \mu \times B$$

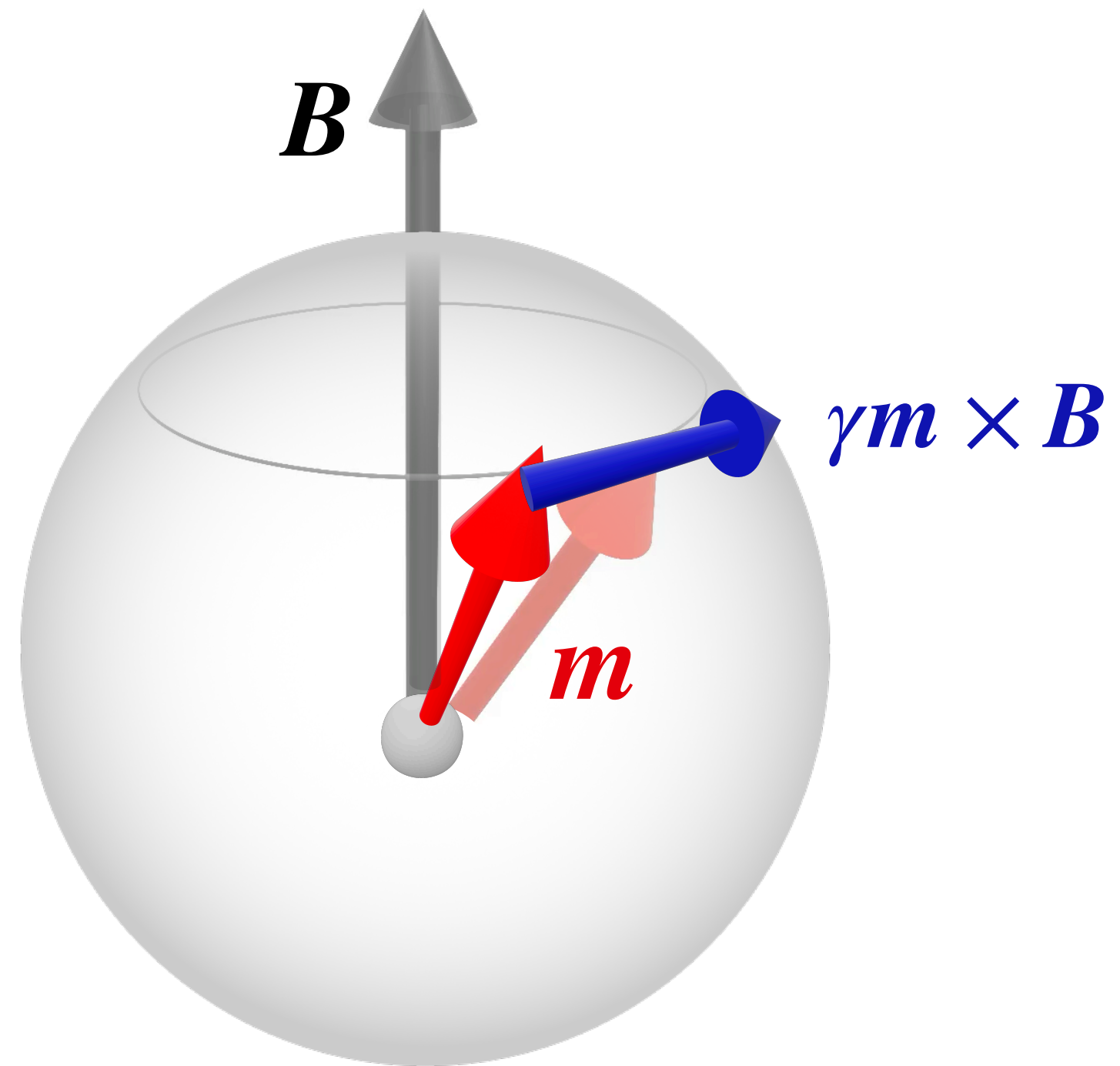
Dynamics restricted to a spherical surface!

$$\frac{d\mu}{dt} \perp \mu \quad |\mu| = \text{const.}$$

→ normalized moment $m = \frac{\mu}{|\mu|}$

Larmor precession with
angular frequency $\omega_B = \gamma B$

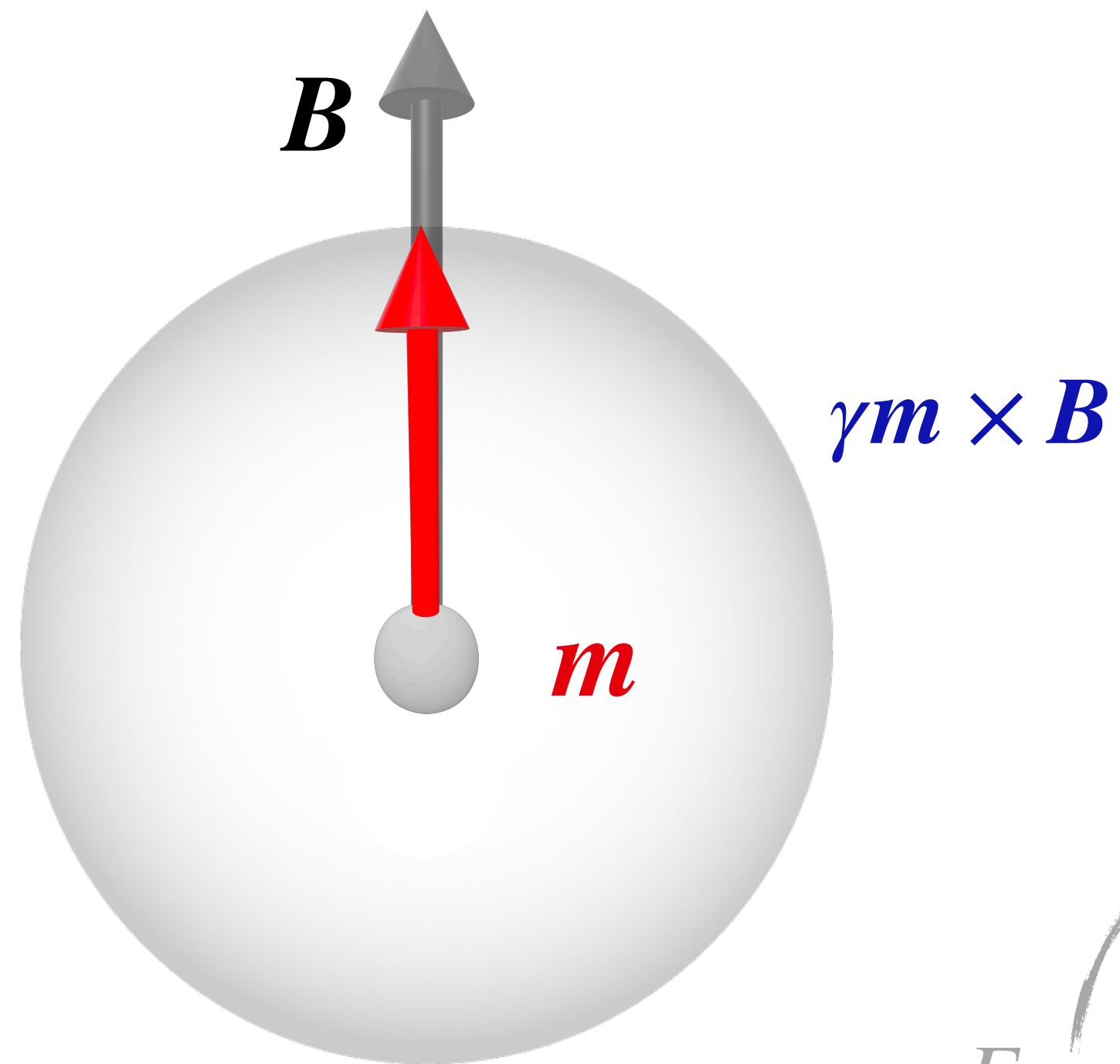
Magnetic moments precess around magnetic fields



$$\frac{dm}{dt} = \gamma m \times B$$

Larmor precession with
angular frequency $\omega_B = \gamma B$

The torque equation cannot bring us to equilibrium



Clearly no torque in equilibrium

$$\frac{d\mathbf{m}}{dt} = \gamma \mathbf{m} \times \mathbf{B} = 0 \quad \longleftrightarrow \quad \mathbf{m} \parallel \mathbf{B}$$

Relaxation? Equation conserves energy!

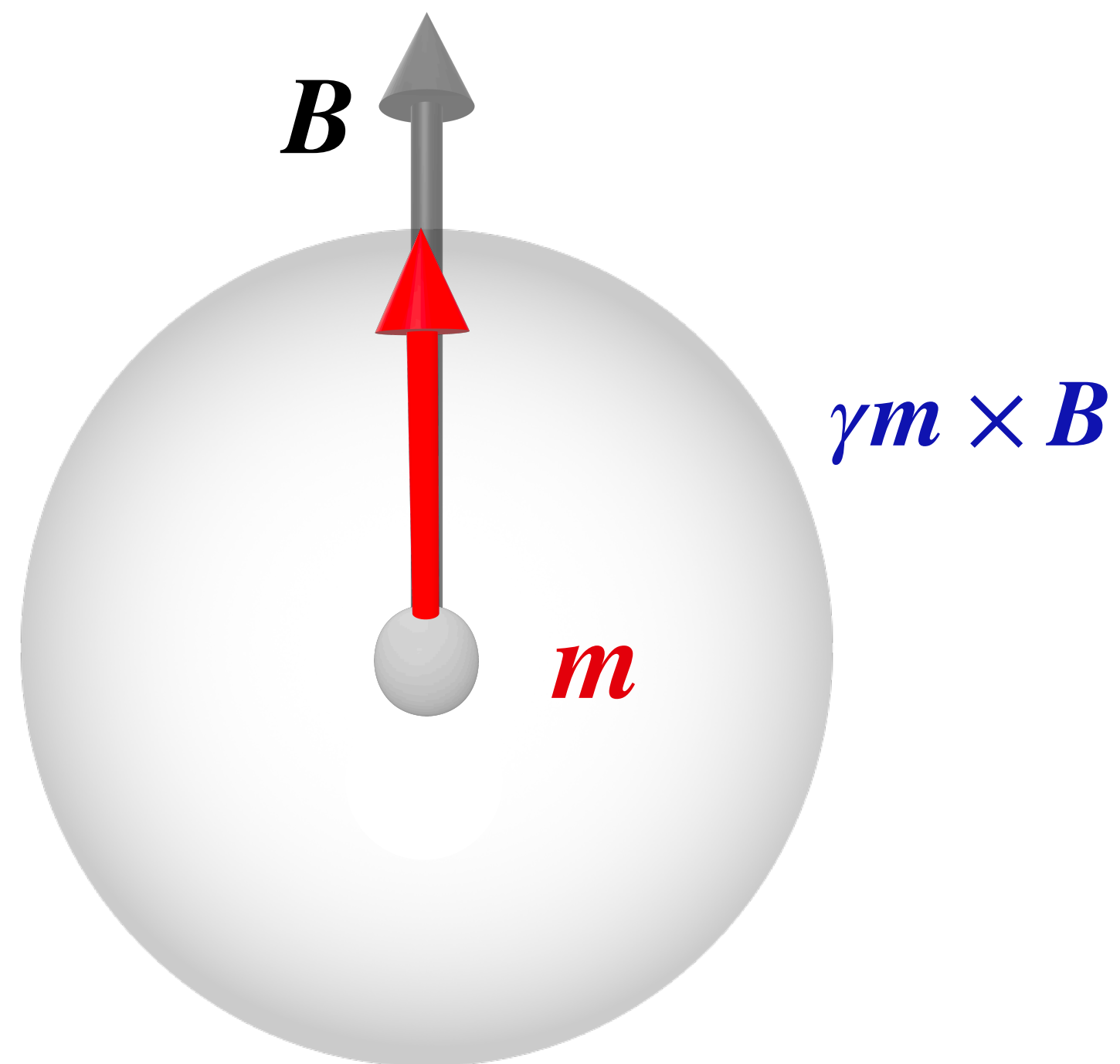
$$\frac{dE}{dt} = \frac{\partial E}{\partial \mathbf{m}} \cdot \frac{d\mathbf{m}}{dt} = 0$$

$E = -\mu \mathbf{m} \cdot \mathbf{B}$

$-\mu \mathbf{B} \quad \quad [\gamma \mathbf{m} \times \mathbf{B}]$

👉 No surprise, since we derived it from the Heisenberg equations.

Damping relaxes the magnetic moment to its equilibrium position



Additional torque required

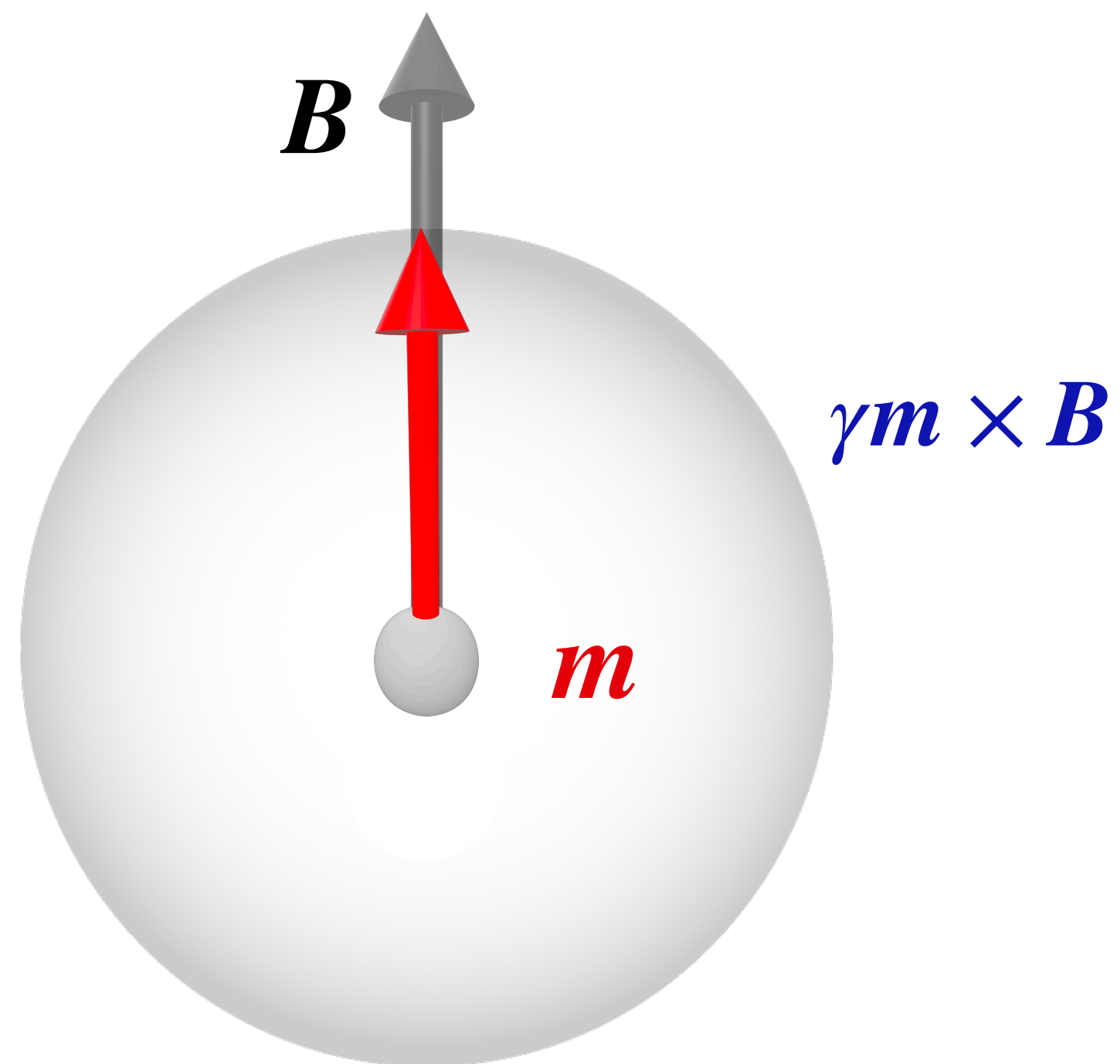
$$\frac{dm}{dt} = \gamma m \times B + T_{\text{damping}}$$

Relaxation? Equation conserves energy!

$$\frac{dE}{dt} = \frac{\partial E}{\partial m} \cdot \frac{dm}{dt} = 0$$

$$-\mu B \quad [\gamma m \times B]$$

Damping relaxes the magnetic moment to its equilibrium position

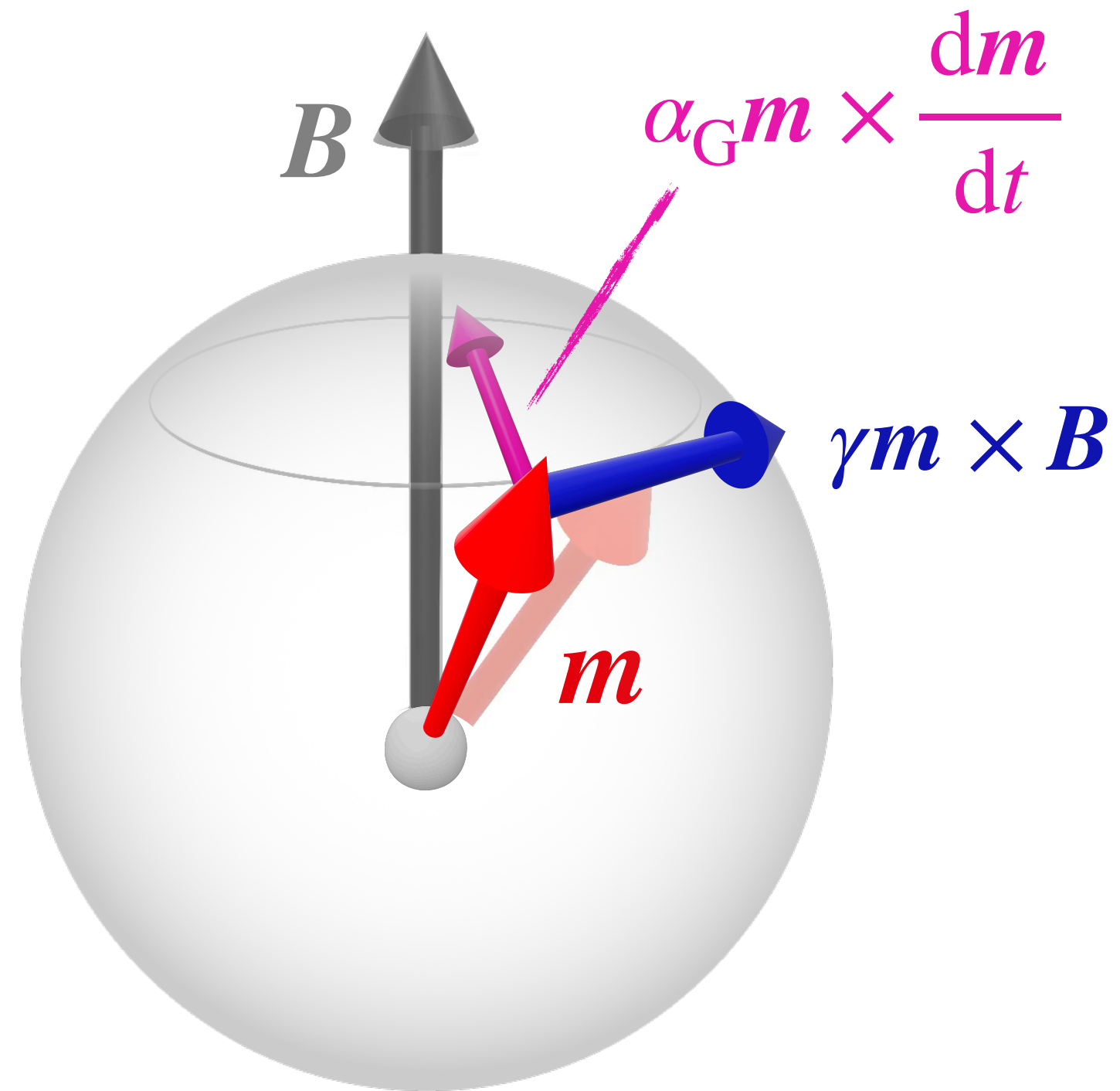


Landau-Lifshitz-Gilbert (LLG) equation

Most important for us:
Viscous **Gilbert damping**

$$\frac{dm}{dt} = \gamma m \times B + \underbrace{\alpha_G m \times \frac{dm}{dt}}_{\text{phenomenological damping parameter}}$$

Damping relaxes the magnetic moment to its equilibrium position



Damped spin precession

Landau-Lifshitz-Gilbert (LLG) equation

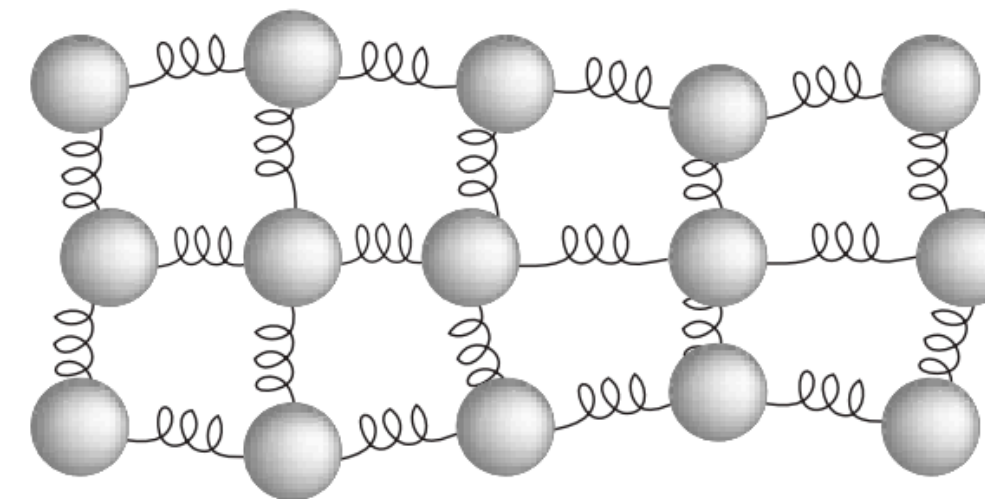
Most important for us:
Viscous **Gilbert damping**

$$\frac{dm}{dt} = \gamma m \times B + \alpha_G m \times \frac{dm}{dt}$$

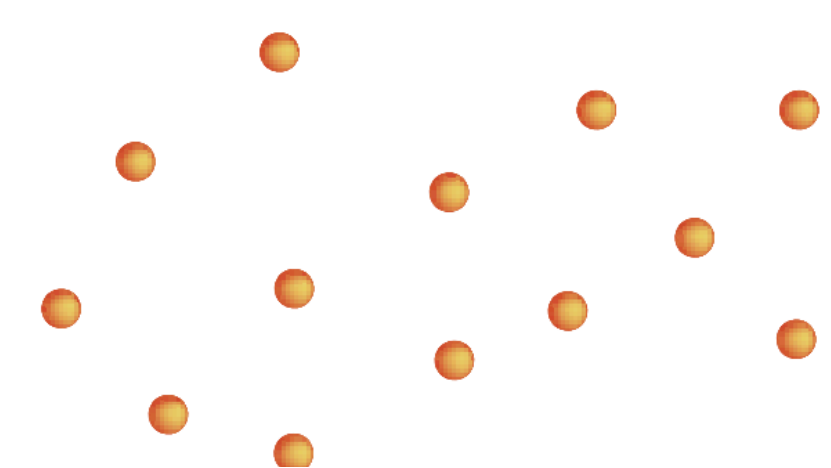
phenomenological damping parameter

Various origins of damping

Coupling to phonons



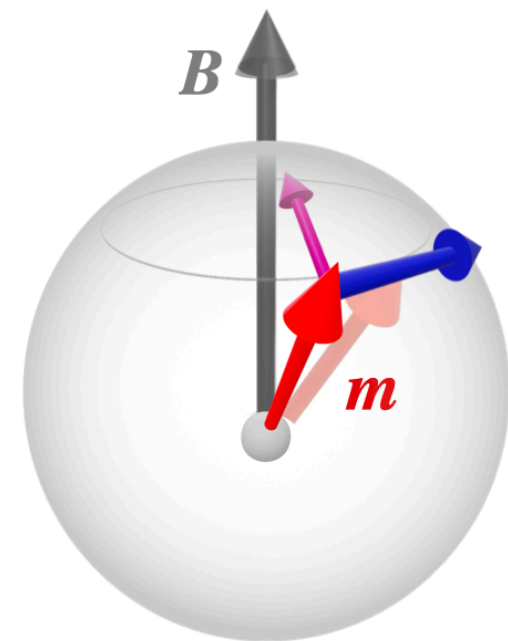
Coupling to electrons



© Joo-Von Kim

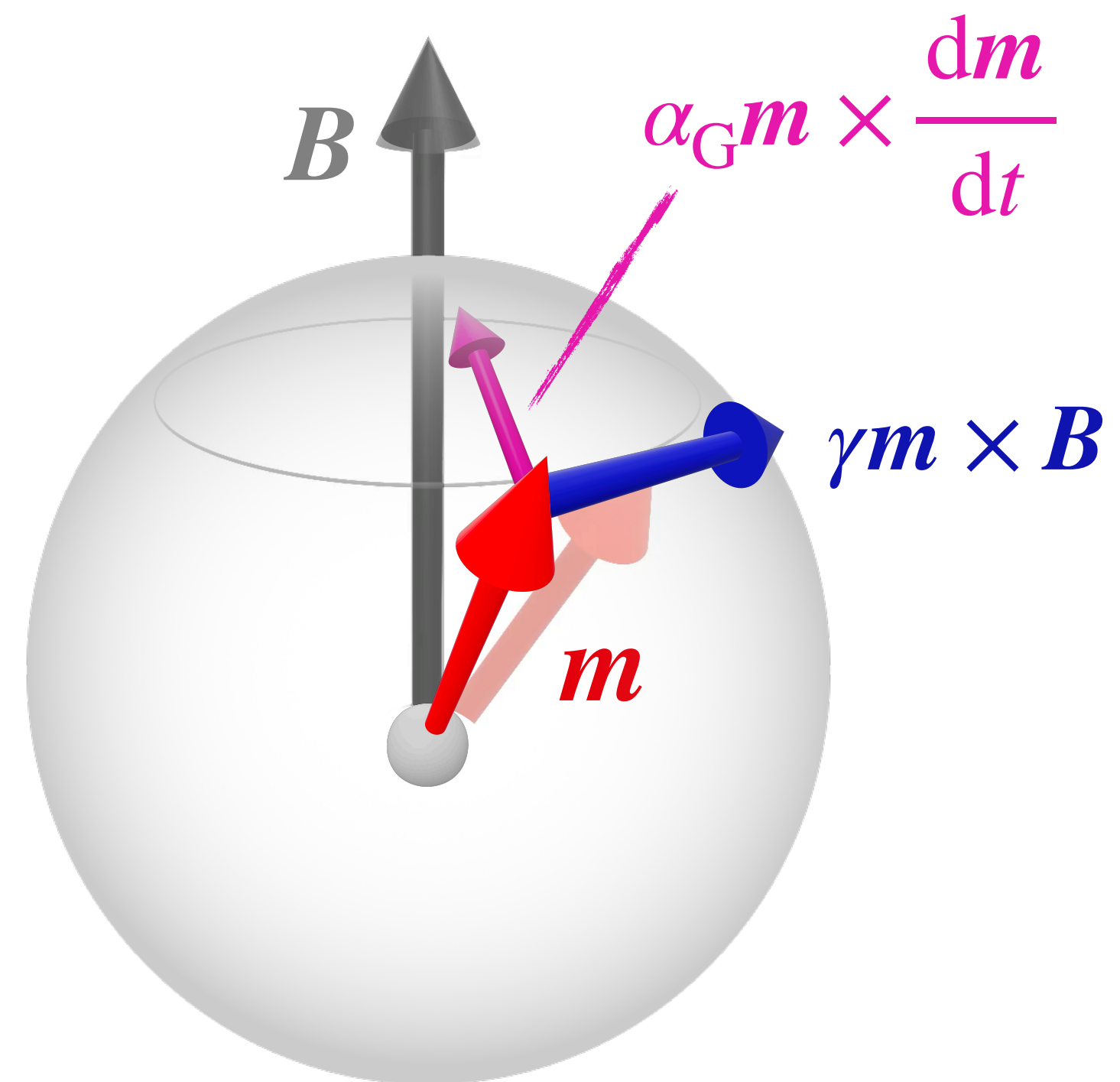
Depends on the type of material (e.g., metal vs. insulator, ...)

Precessional dynamics

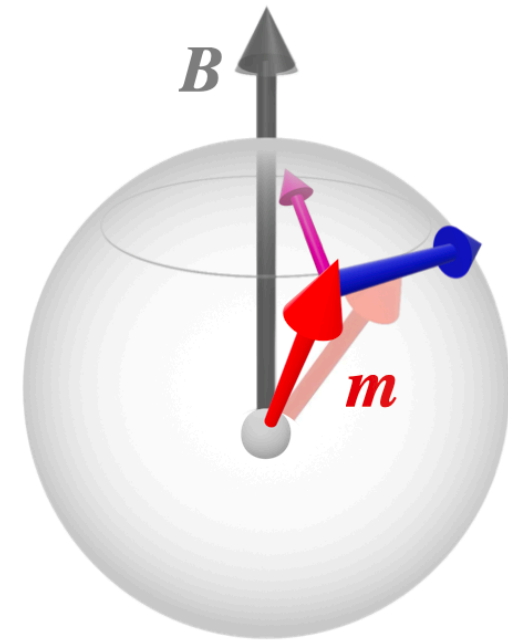


Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{dm}{dt} = \gamma m \times B + \alpha_G m \times \frac{dm}{dt}$$



Precessional dynamics



Single spin

$$E(m) = -\mu m \cdot B$$

$$B = -\frac{1}{\mu} \frac{\partial E}{\partial m}$$

+ interactions



Many spins

$$E(m_1, m_2, \dots)$$

$$B_i^{\text{eff}} = -\frac{1}{\mu} \frac{\partial E}{\partial m_i}$$

Effective field,
generated by other spins

coupled nonlinear EOM

$$\frac{dm_i}{dt} = \gamma m_i \times B_i^{\text{eff}}(m_j)$$

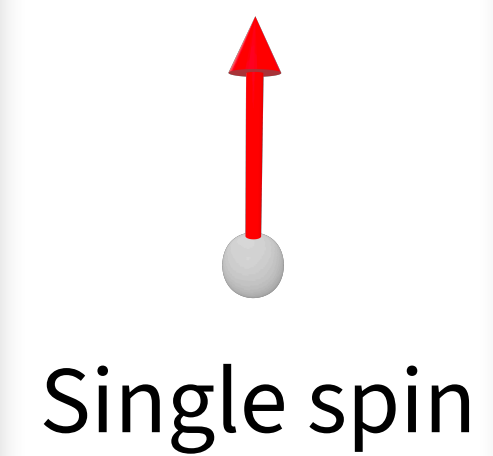
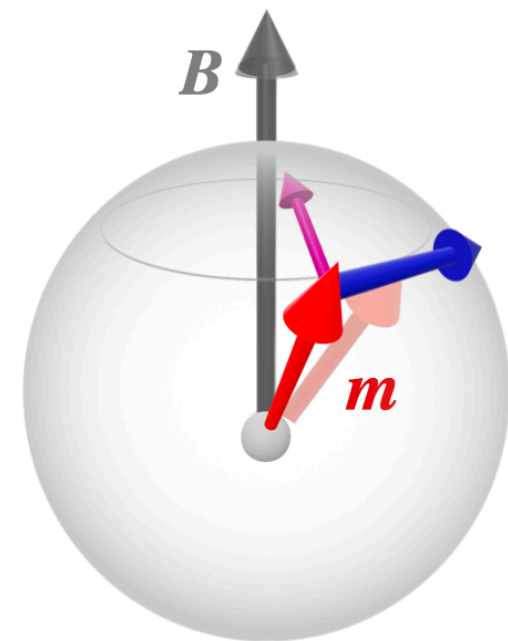
We want to keep

$$\frac{dE}{dt} = \frac{\partial E}{\partial m} \cdot \frac{dm}{dt} = 0$$

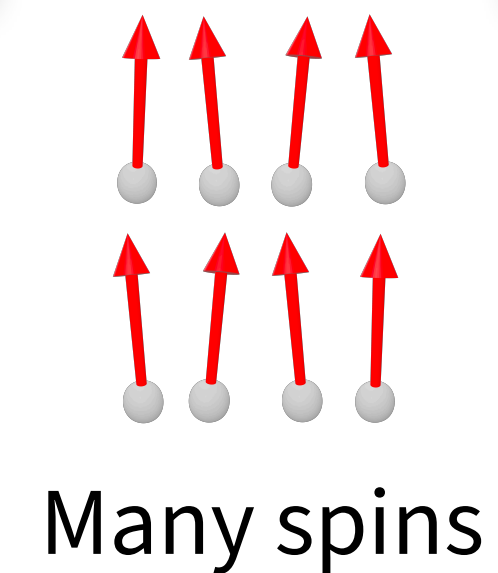
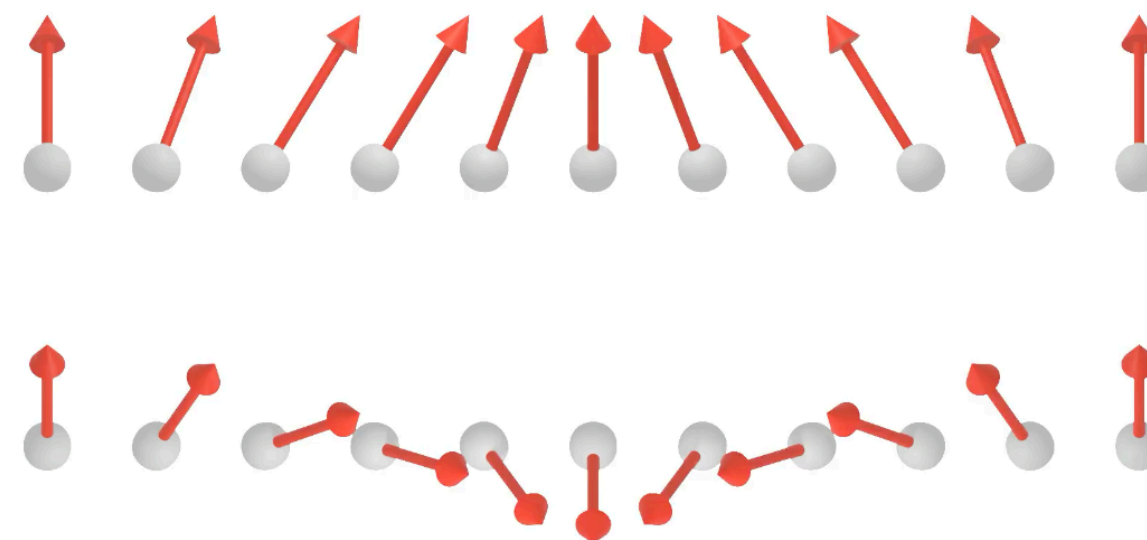
$\uparrow$
 $-\mu B_{\text{eff}}$

How to understand collective dynamics?

Precessional dynamics



Magnons

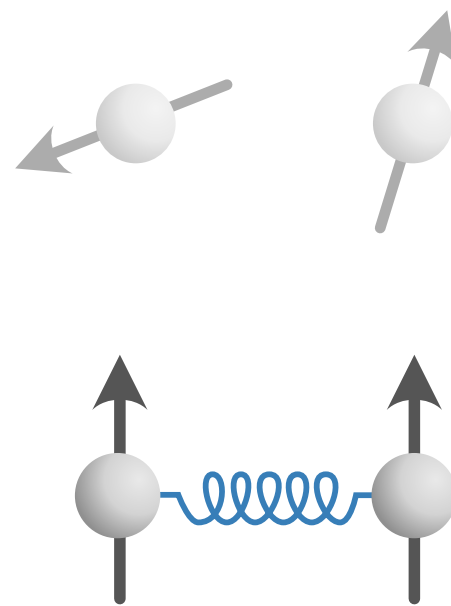


Look at simple dynamics (close to equ.) first!

Linear spin dynamics & Magnons

For now, focus on strongest interaction!

Exchange



Heisenberg spin Hamiltonian

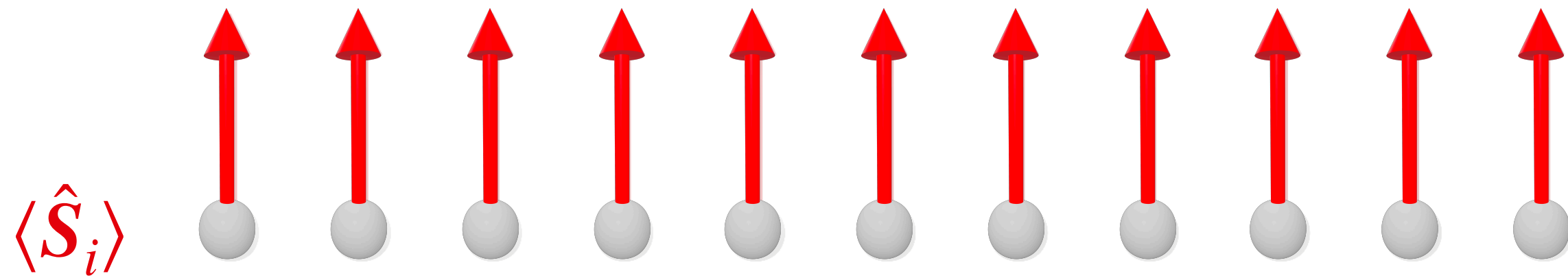
$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

What are the fundamental excitations of a ferromagnet?

ferromagnetic ($J > 0$) spin-1/2 chain

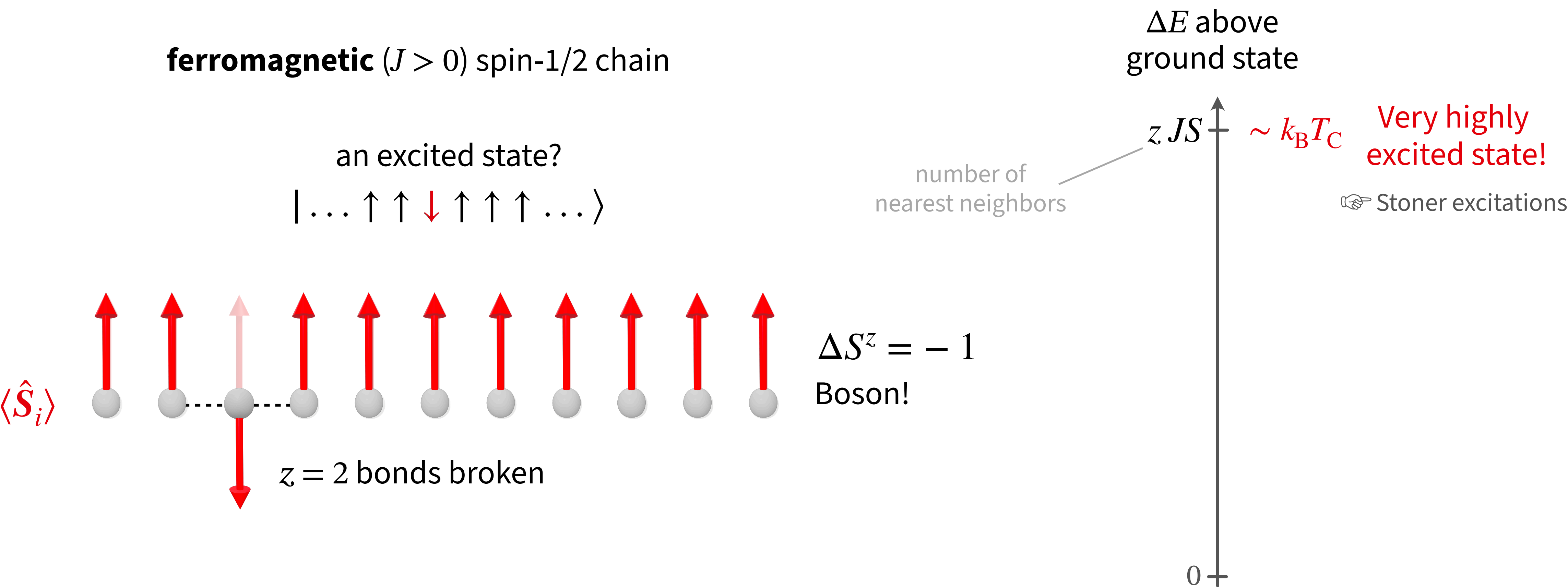
groundstate

$$|\phi_1\rangle = |\dots \uparrow \uparrow \uparrow \uparrow \uparrow \uparrow \dots\rangle$$



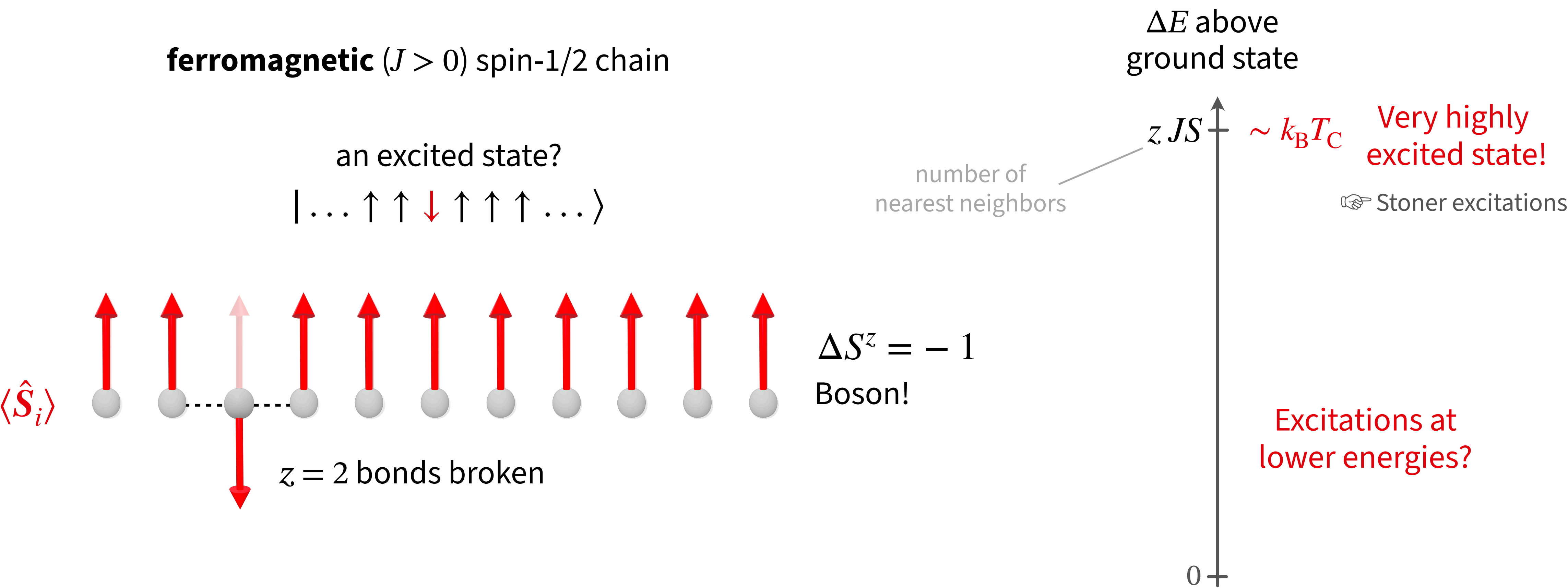
$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

Create a simple excited state by flipping a single spin



$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

Create a simple excited state by flipping a single spin

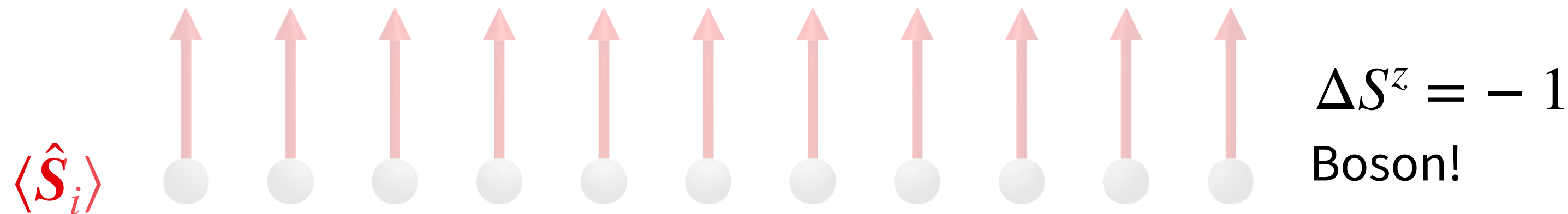


$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

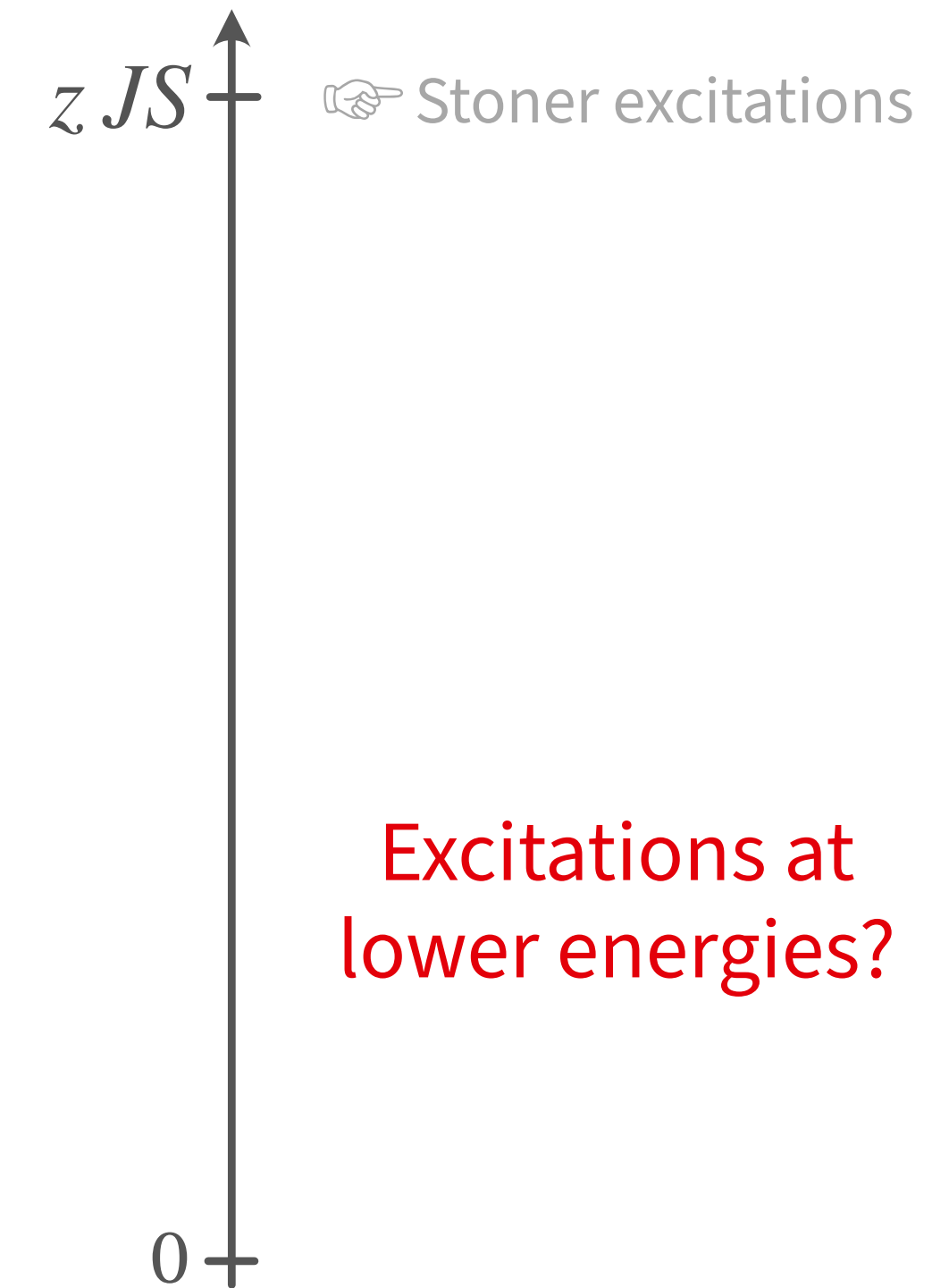
Distributed spin flips are lower in energy

Better: superposition of flips accross all sites

$$\begin{aligned} |\phi_1\rangle &= e^{ikr_1} |\dots \downarrow \uparrow \uparrow \uparrow \uparrow \uparrow \dots\rangle \\ &+ e^{ikr_2} |\dots \uparrow \downarrow \uparrow \uparrow \uparrow \uparrow \dots\rangle \\ &+ e^{ikr_3} |\dots \uparrow \uparrow \downarrow \uparrow \uparrow \uparrow \dots\rangle + \dots \end{aligned}$$



ΔE above
ground state

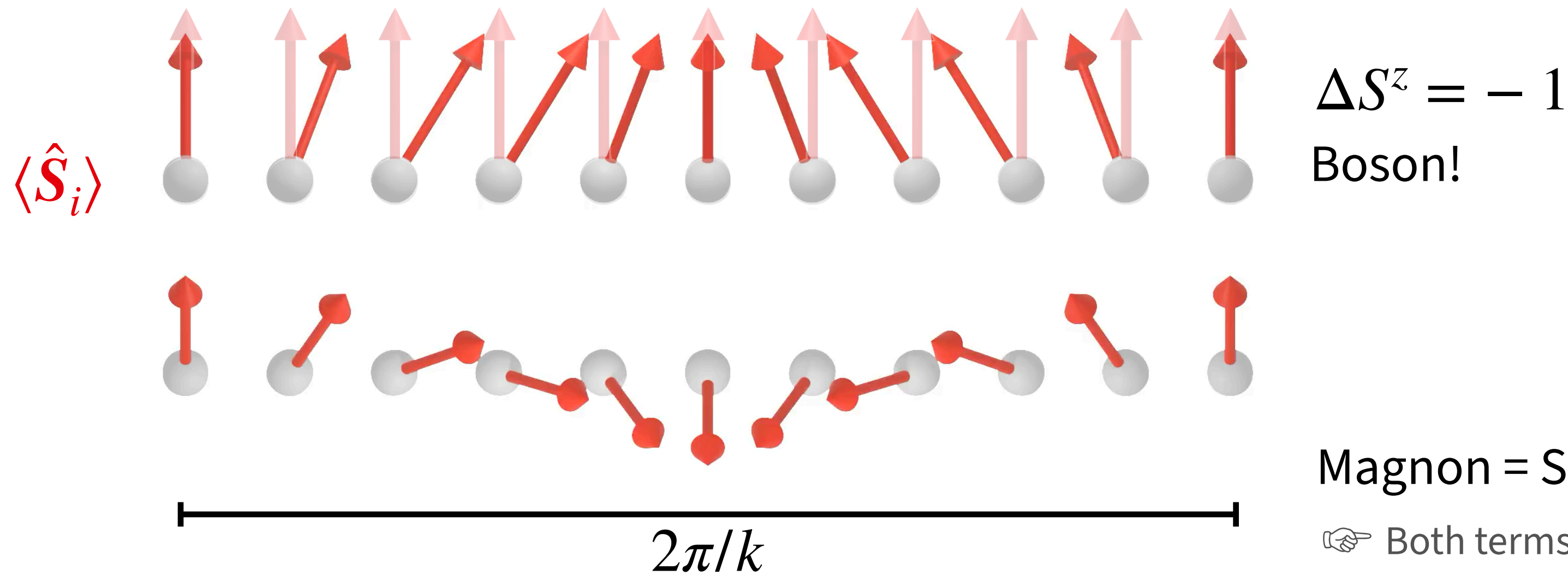


$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

Distributed spin flips correspond to a spin wave

Better: superposition of flips accross all sites

$$\begin{aligned} |\phi_1\rangle &= e^{ikr_1} |\dots \downarrow \uparrow \uparrow \uparrow \uparrow \uparrow \dots\rangle \\ &+ e^{ikr_2} |\dots \uparrow \downarrow \uparrow \uparrow \uparrow \uparrow \dots\rangle \\ &+ e^{ikr_3} |\dots \uparrow \uparrow \downarrow \uparrow \uparrow \uparrow \dots\rangle + \dots \end{aligned}$$



$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{S}_i \cdot \hat{S}_j$$

ΔE above
ground state

zJS Stoner excitations

Excitations at
lower energies?

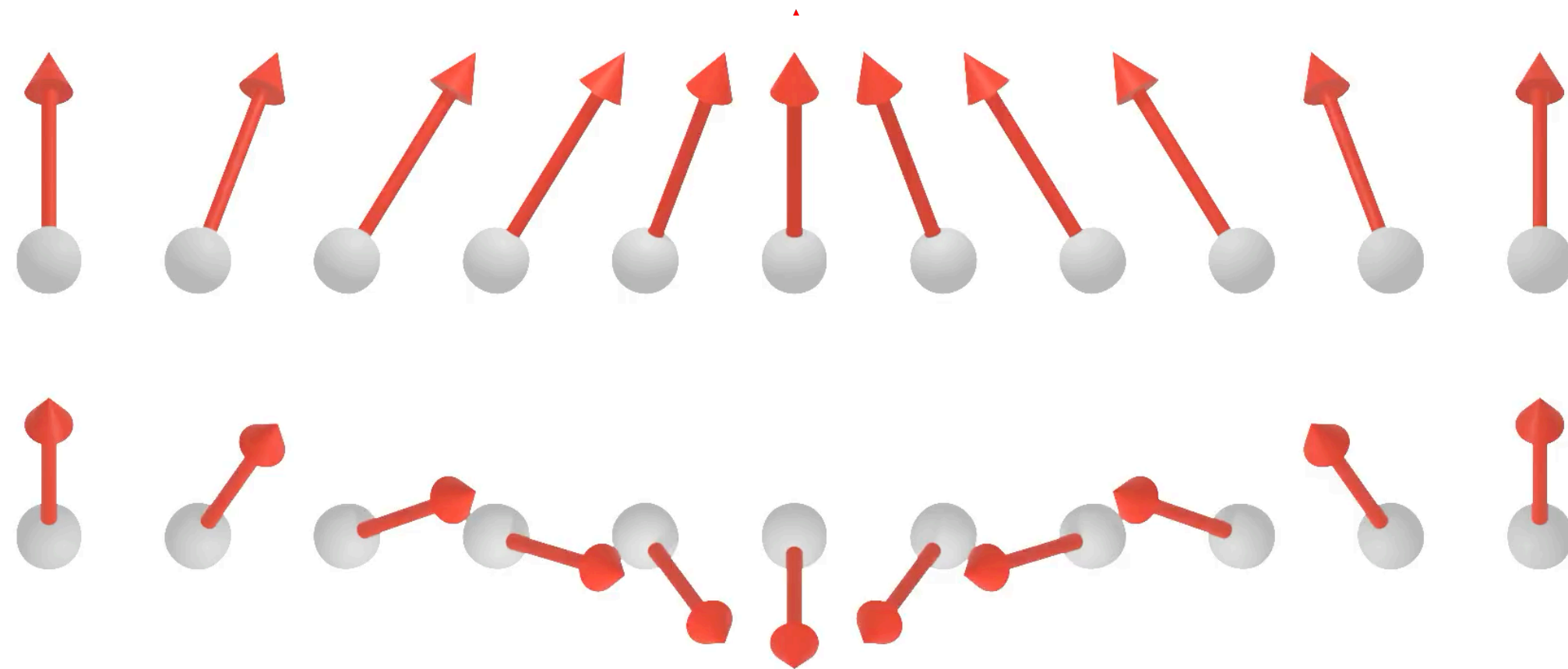
0

Both terms are often used interchangeably.

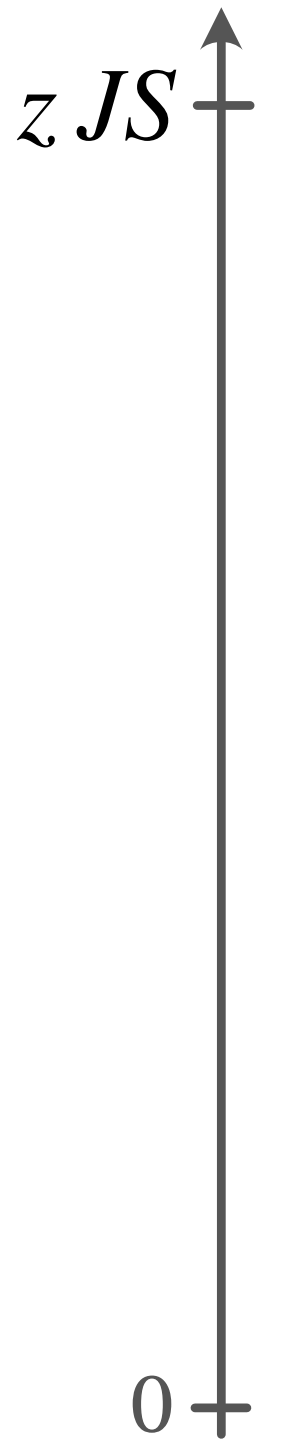
What are the frequencies of magnons?

Spin precession at each point?

$$\frac{d\mathbf{m}_i}{dt} = \gamma \mathbf{m}_i \times \mathbf{B}_i^{\text{eff}}$$



ΔE above
ground state



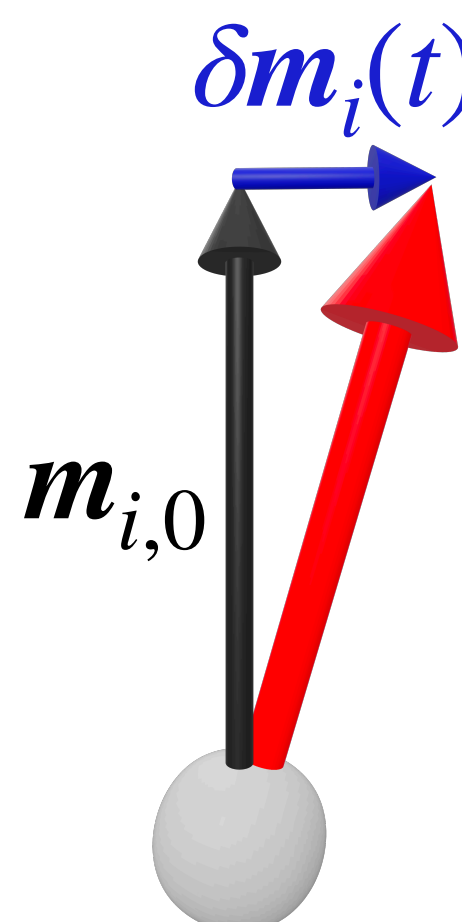
Consider as classical

$$\hat{H} = -J \sum_{\langle ij \rangle} \hat{\mathbf{S}}_i \cdot \hat{\mathbf{S}}_j \approx -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

We find magnon frequencies by linearizing the Landau-Lifshitz equation

$$\frac{d\mathbf{m}_i}{dt} = \gamma \mathbf{m}_i \times \mathbf{B}_i^{\text{eff}} = \gamma \left(\mathbf{m}_{i,0} \times \delta \mathbf{B}_i^{\text{eff}} + \delta \mathbf{m}_{i,0} \times \mathbf{B}_{i,0}^{\text{eff}} + \mathbf{m}_{i,0} \times \mathbf{B}_{i,0}^{\text{eff}} + \delta \mathbf{m}_{i,0} \times \delta \mathbf{B}_{i,0}^{\text{eff}} \right)$$

in equilibrium small



$\delta \mathbf{m}_i(t)$ Consider small variations from equilibrium direction

$\mathbf{m}_i(t) = \mathbf{m}_{i,0} + \delta \mathbf{m}_i(t)$
 $\mathbf{B}_i^{\text{eff}}(t) = \mathbf{B}_{i,0}^{\text{eff}} + \delta \mathbf{B}_i^{\text{eff}}(t)$

Time dependence only here

ΔE above ground state

$z JS$

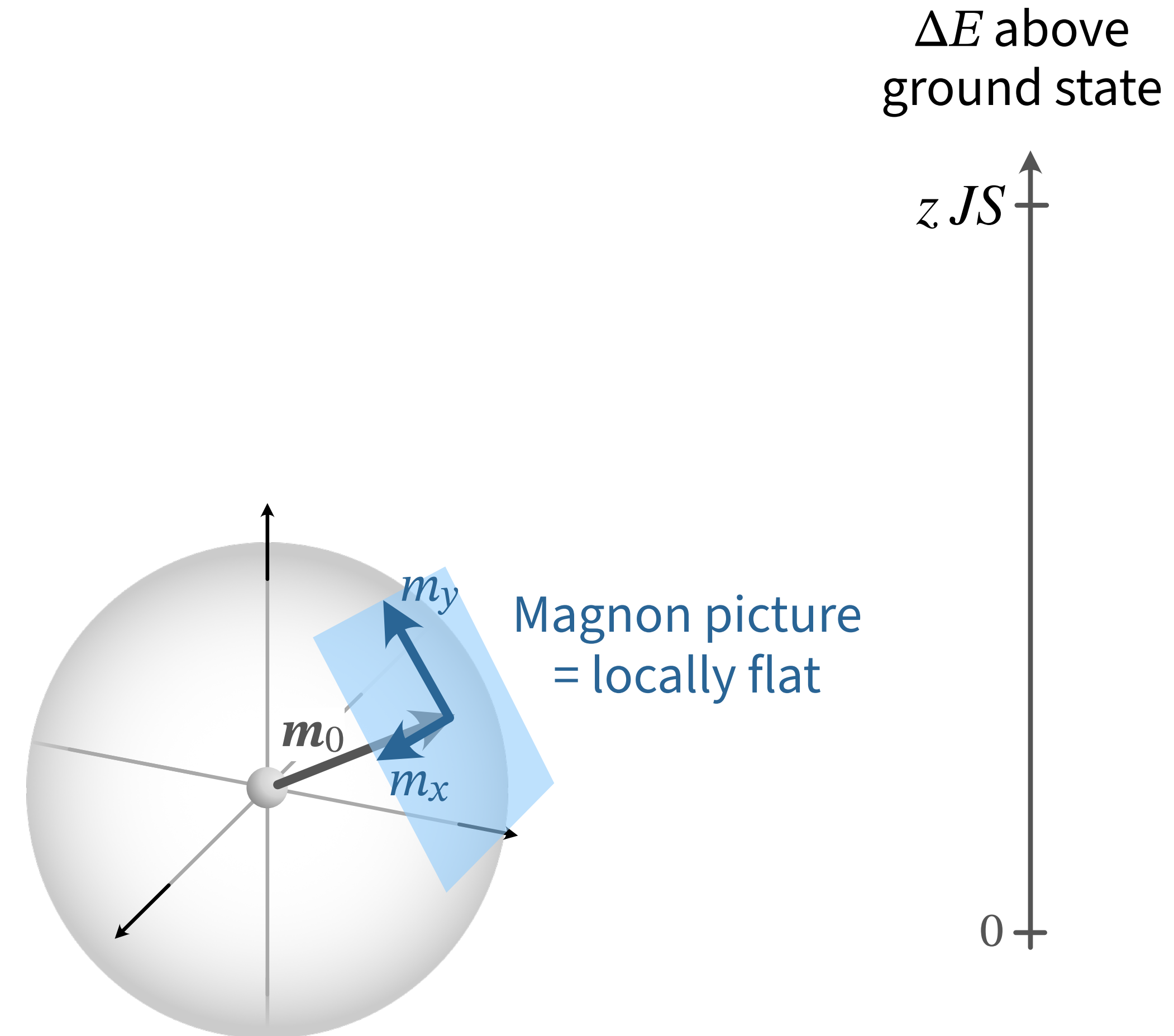
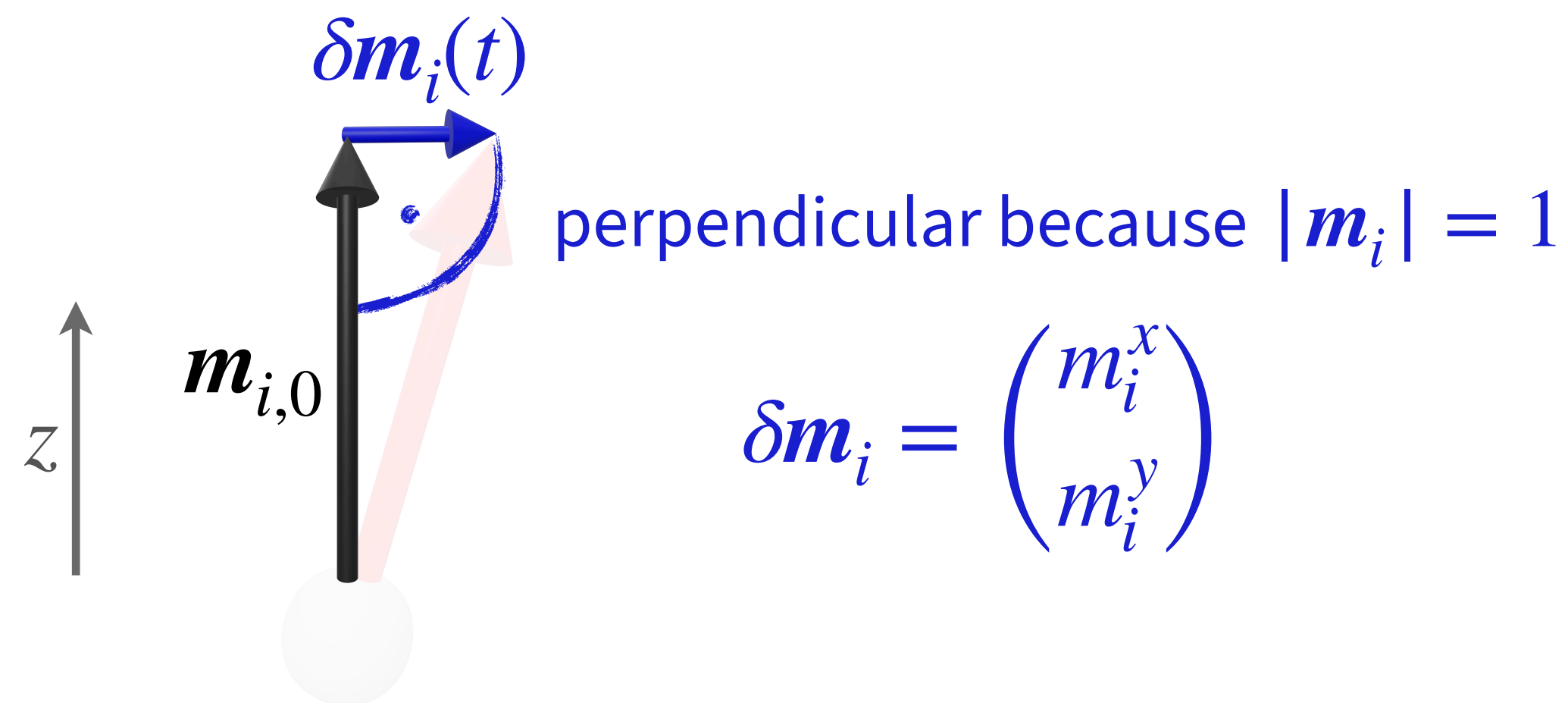
0

$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

We find magnon frequencies by linearizing the Landau-Lifshitz equation

Linearized equation of motion

$$\frac{d(\delta \mathbf{m}_i)}{dt} = \gamma \left(\mathbf{m}_{i,0} \times \delta \mathbf{B}_i^{\text{eff}} + \delta \mathbf{m}_{i,0} \times \mathbf{B}_{i,0}^{\text{eff}} \right)$$



$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

We find magnon frequencies by diagonalizing the dynamic matrix

Linearized equation of motion

$$\frac{d(\delta \mathbf{m}_i)}{dt} = \gamma \left(\mathbf{m}_{i,0} \times \delta \mathbf{B}_i^{\text{eff}} + \delta \mathbf{m}_{i,0} \times \mathbf{B}_{i,0}^{\text{eff}} \right)$$

$$i\omega \begin{pmatrix} m_1^x \\ m_2^x \\ \vdots \\ m_1^y \\ m_2^y \\ \vdots \end{pmatrix} = \begin{pmatrix} \text{Generally, numerics.} \\ \text{Atomistic} \\ \text{Spin} \\ \text{Micromagnetic} \\ \text{TetraX} \\ \text{MAGNUM} \\ \text{np} \\ \dots \end{pmatrix} \begin{pmatrix} m_1^x \\ m_2^x \\ \vdots \\ m_1^y \\ m_2^y \\ \vdots \end{pmatrix}$$

Spatial mode profile

all x components

all y components

ΔE above ground state

$z JS$

0

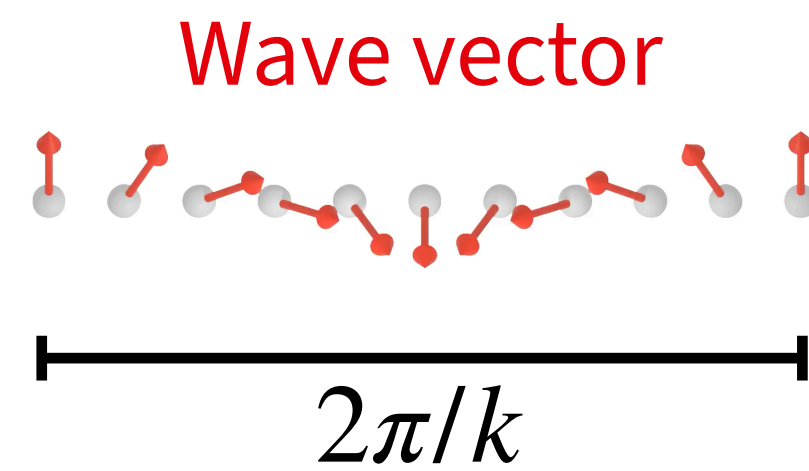
$$m_{x,y}(t) \sim e^{i\omega t}$$

Magnon frequency

But here, analytical solution.

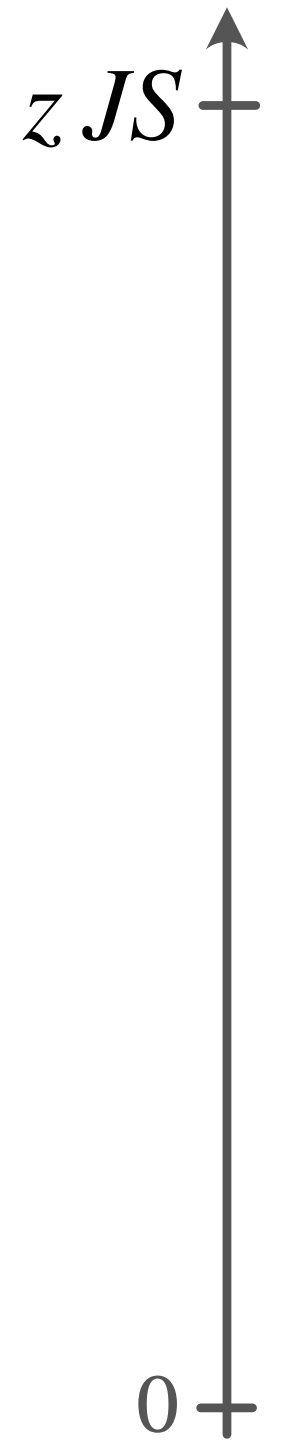
$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

For exchange magnons we can solve it analytically



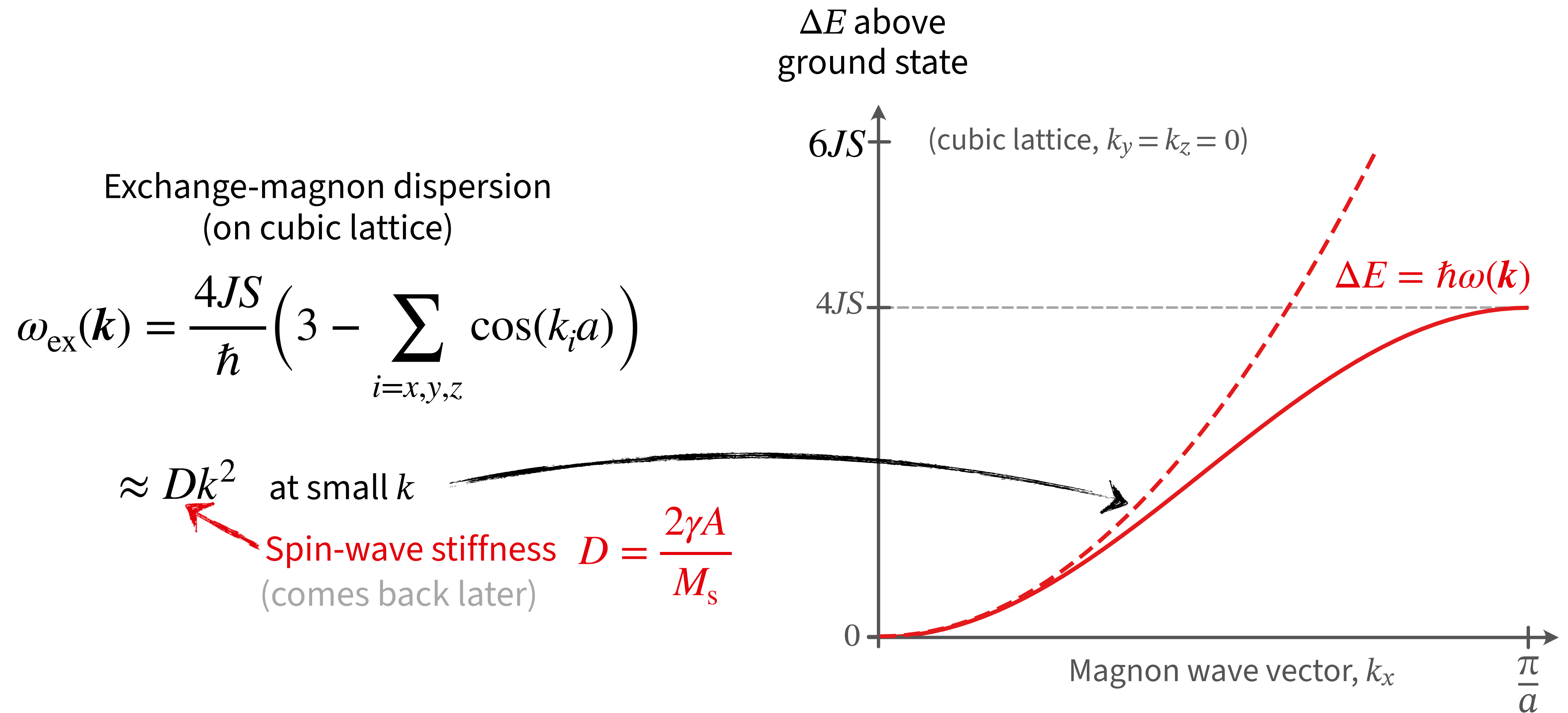
$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

ΔE above
ground state



$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Exchange magnon dispersion is quadratic at small wave vectors



$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Magnons are dispersive waves

Exchange-magnon dispersion
(on cubic lattice)

$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

$\approx Dk^2$ at small k

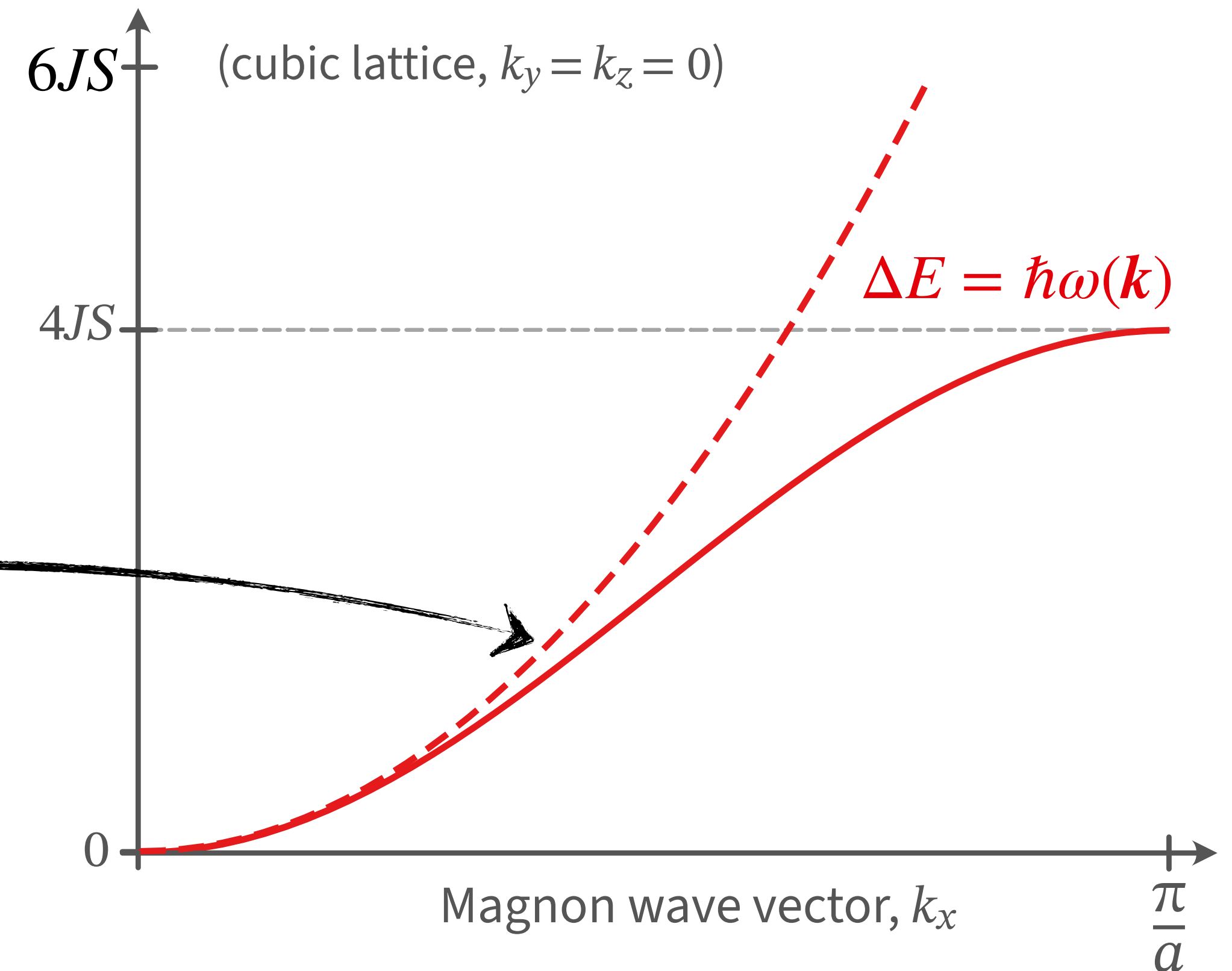
Spin-wave stiffness $D = \frac{2\gamma A}{M_s}$
(comes back later)

$\neq ck$

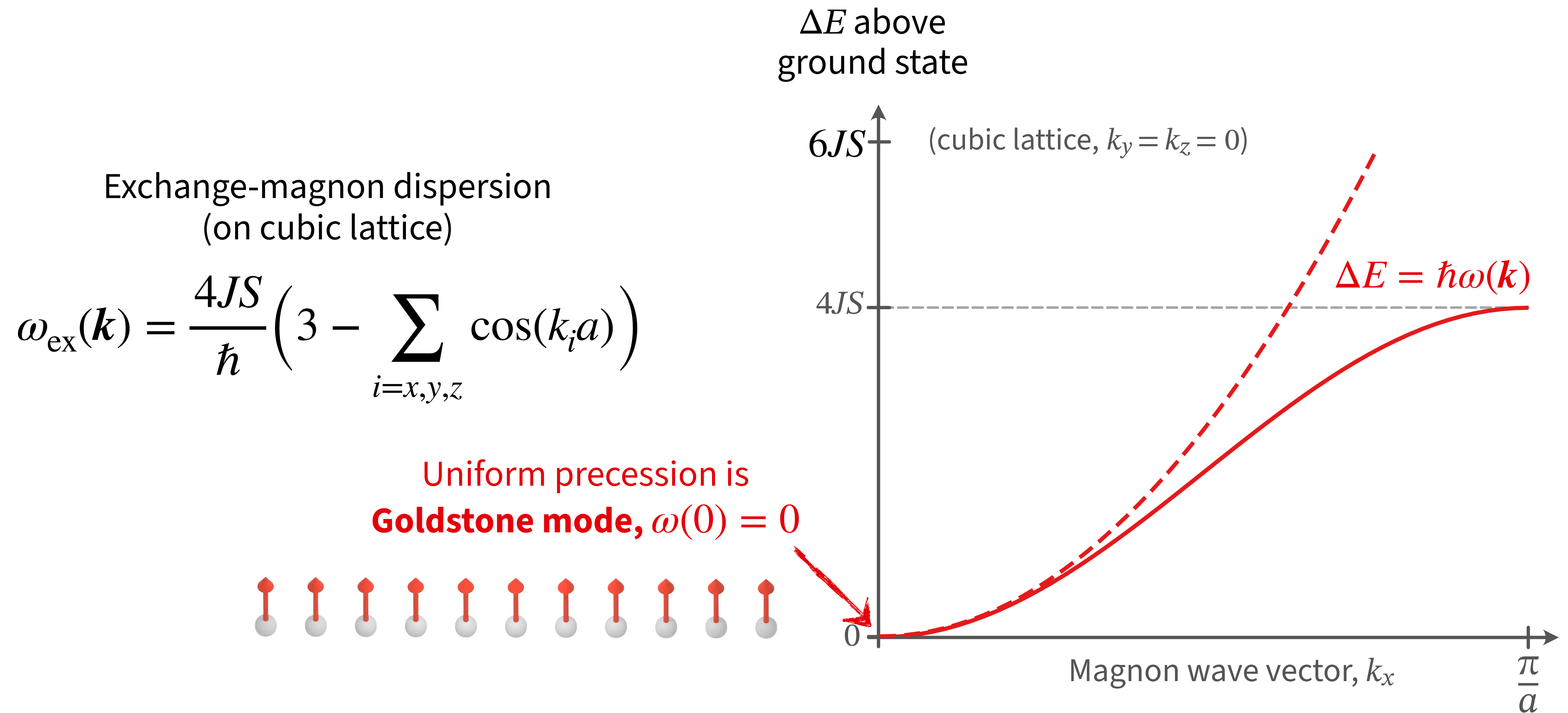
Magnons are dispersive!
 $\frac{\partial \omega}{\partial k} \neq \frac{\omega}{k}$



ΔE above
ground state



Uniform precession corresponds to a Goldstone mode

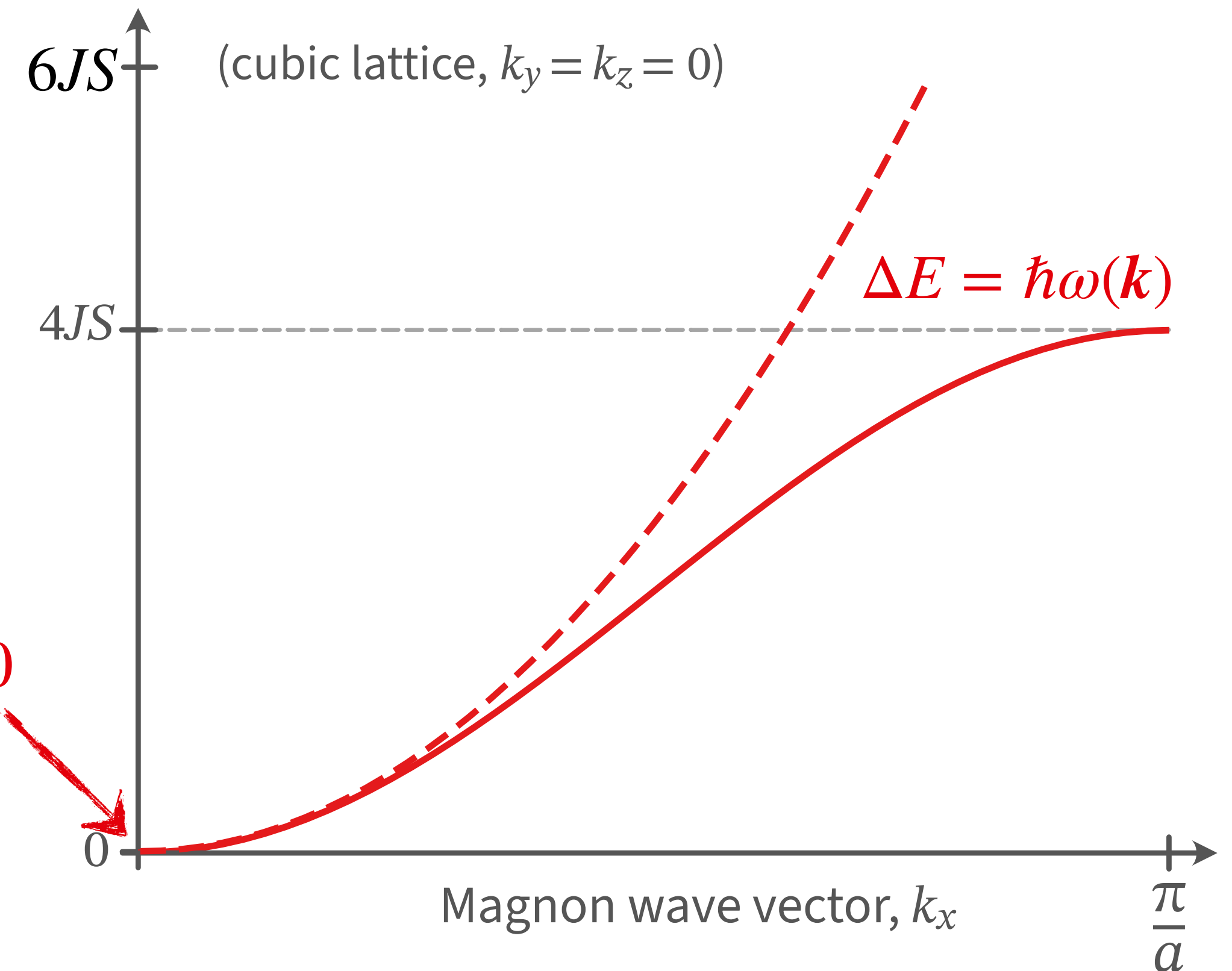


$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Uniform precession corresponds to a Goldstone mode

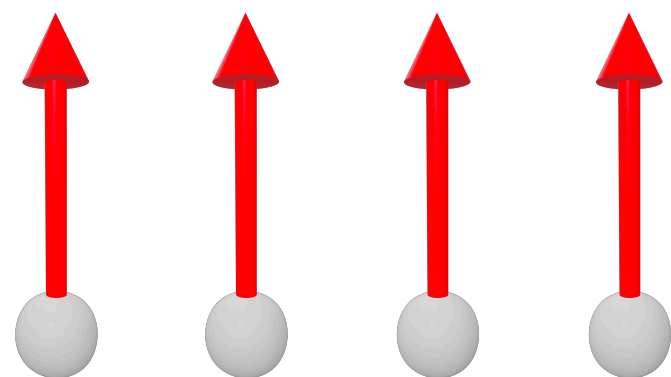
$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

ΔE above
ground state



Uniform precession is
Goldstone mode, $\omega(0) = 0$

Ferromagnetic state breaks
rotational symmetry
spontaneously

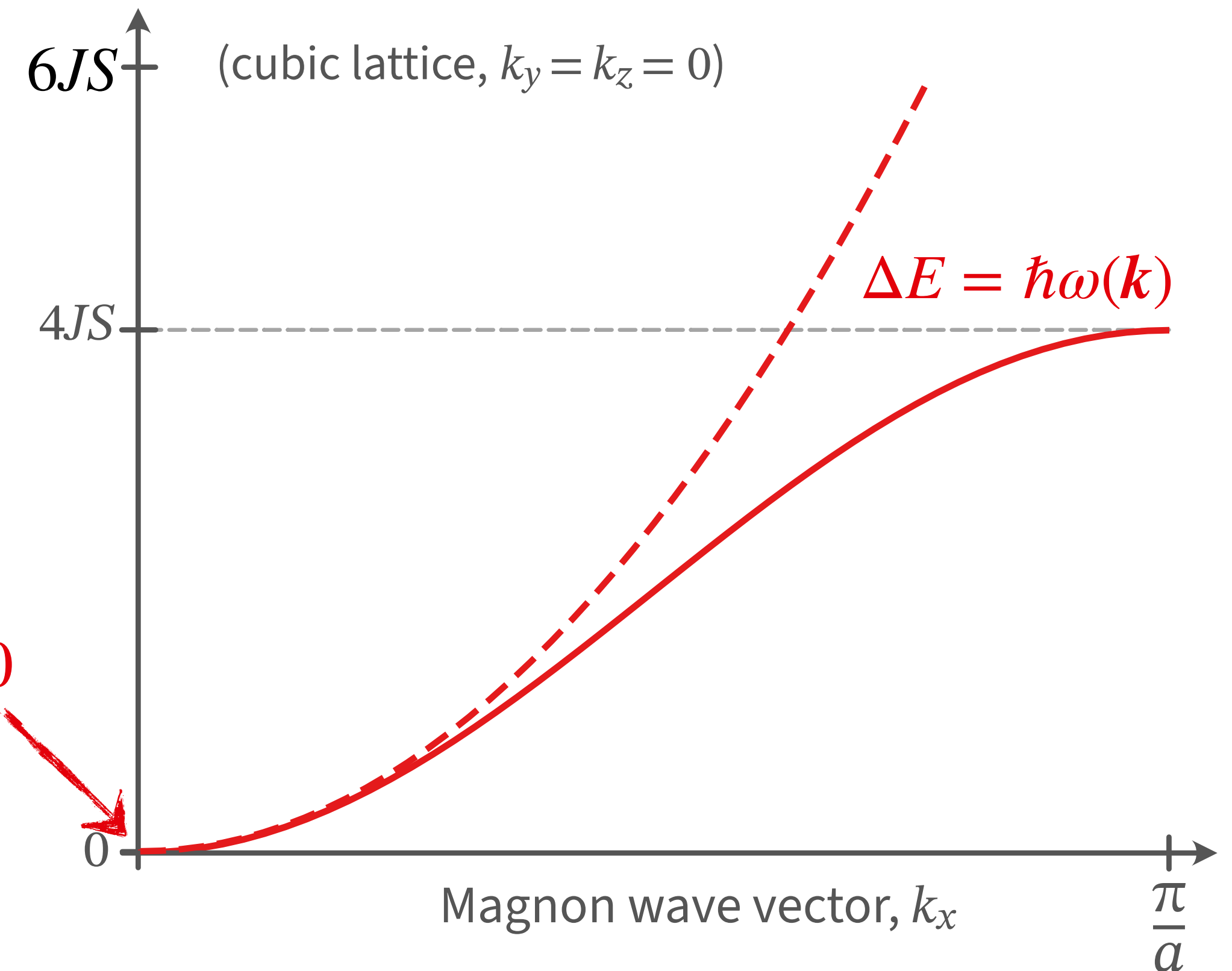


$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Uniform precession corresponds to a Goldstone mode

$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

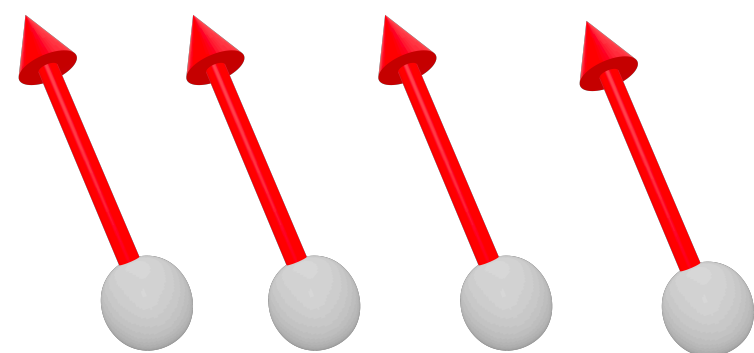
ΔE above
ground state



Uniform precession is
Goldstone mode, $\omega(0) = 0$

Ferromagnetic state breaks
rotational symmetry
spontaneously

Same energy

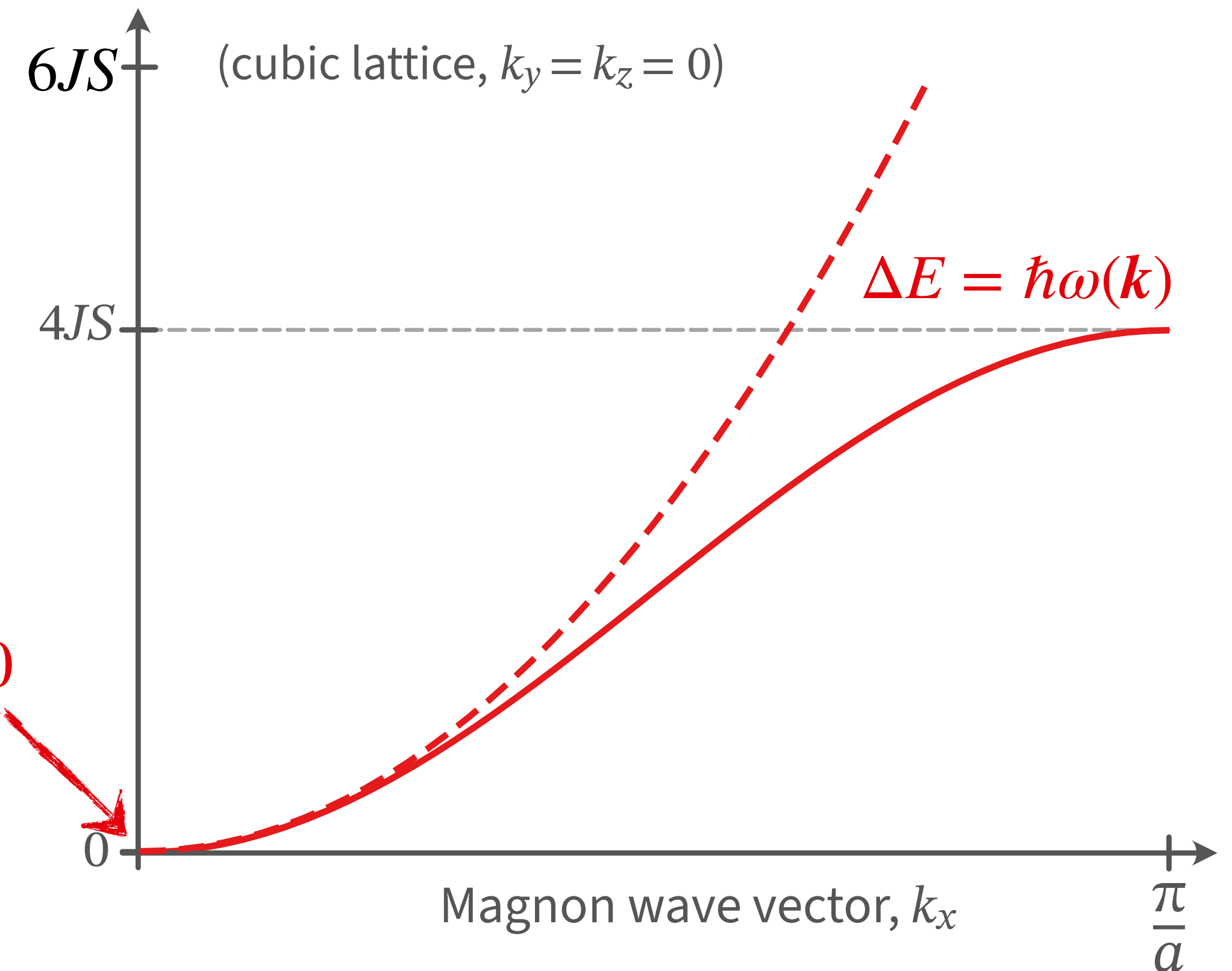


$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Uniform precession corresponds to a Goldstone mode

$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

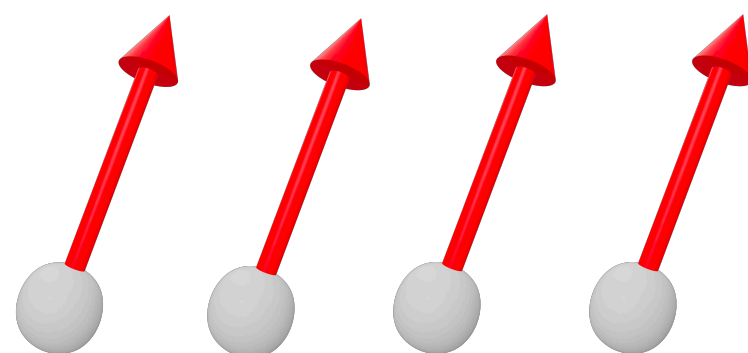
ΔE above
ground state



Uniform precession is
Goldstone mode, $\omega(0) = 0$

Ferromagnetic state breaks
rotational symmetry
spontaneously

Same energy



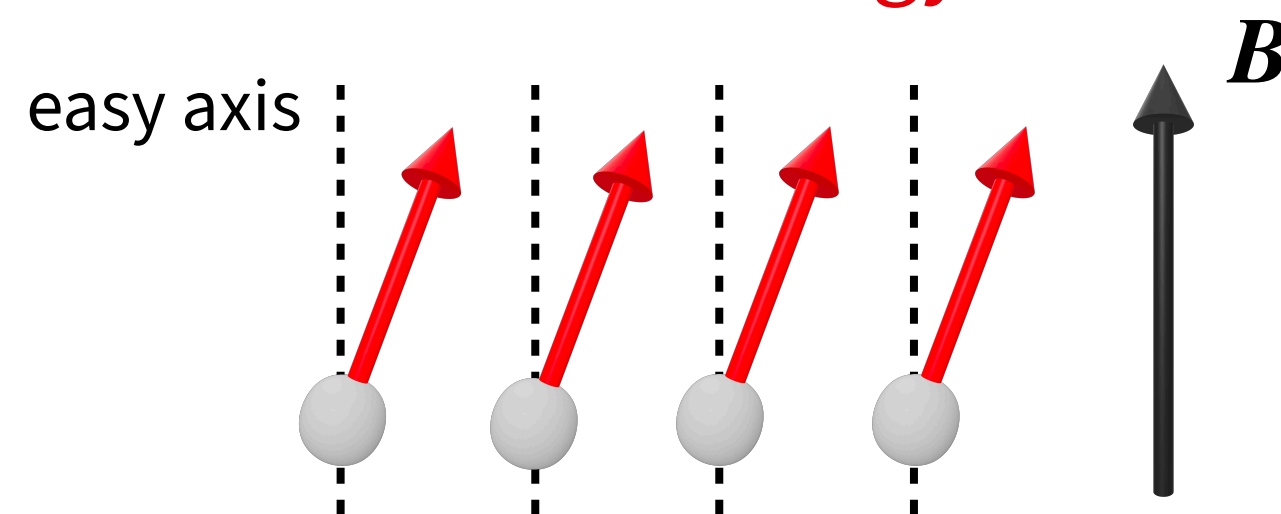
$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j$$

Explicitly breaking rotational symmetry (field or anisotropy) leads to gap

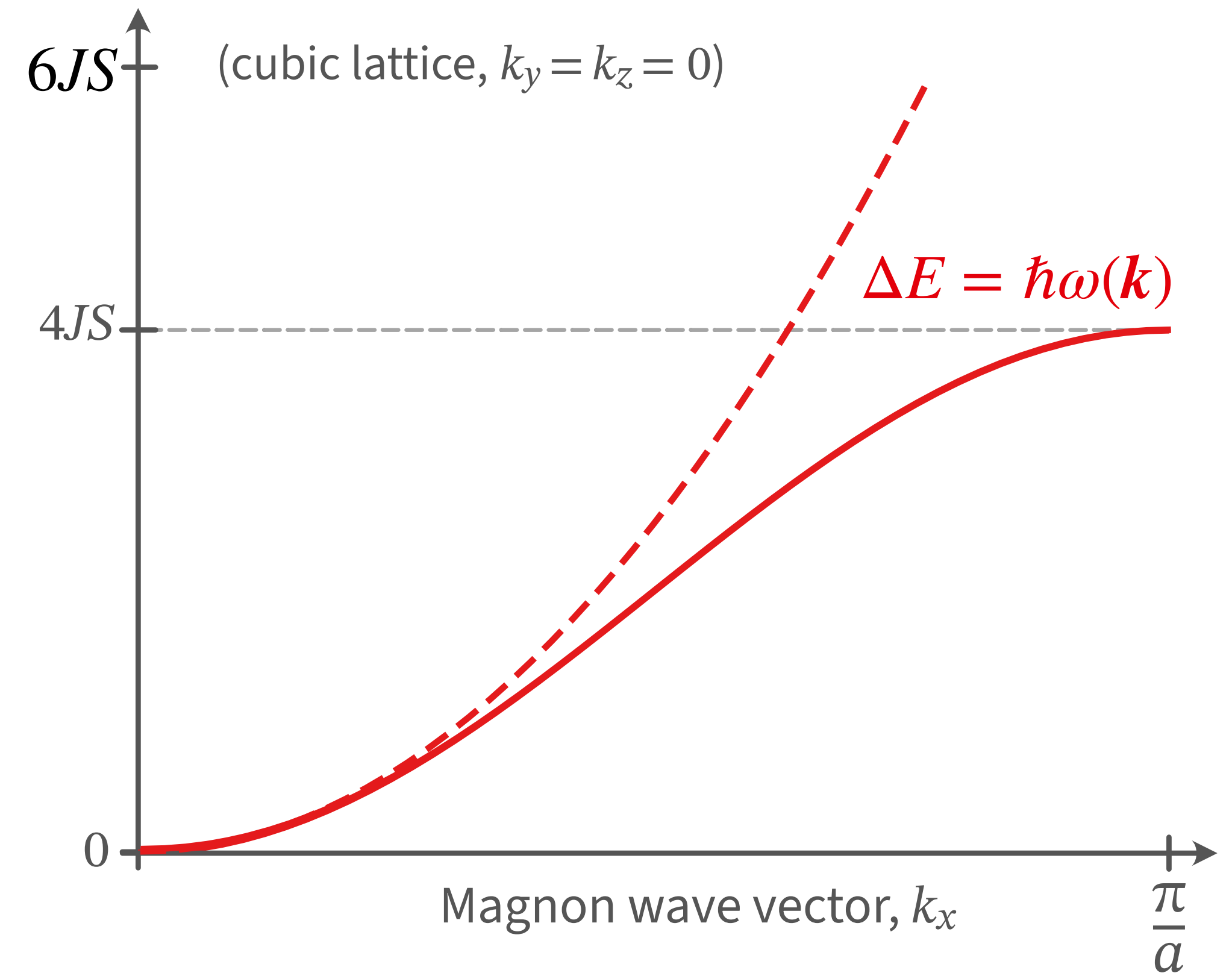
$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

Field or anisotropy
break it **explicitly!**

Costs energy!



ΔE above
ground state



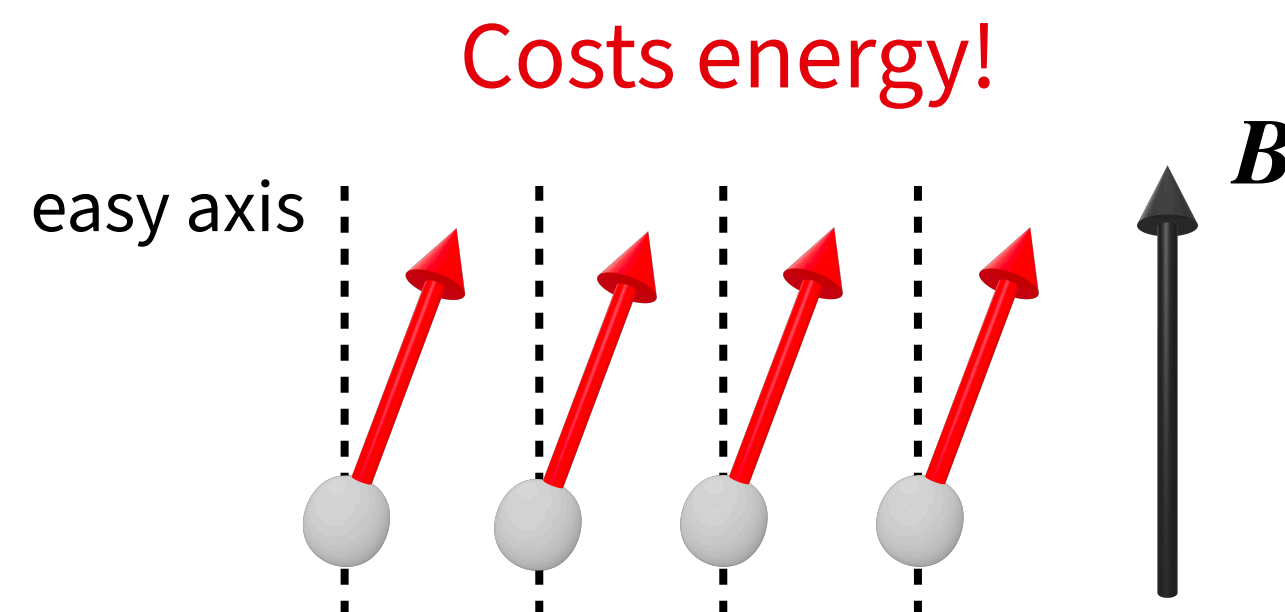
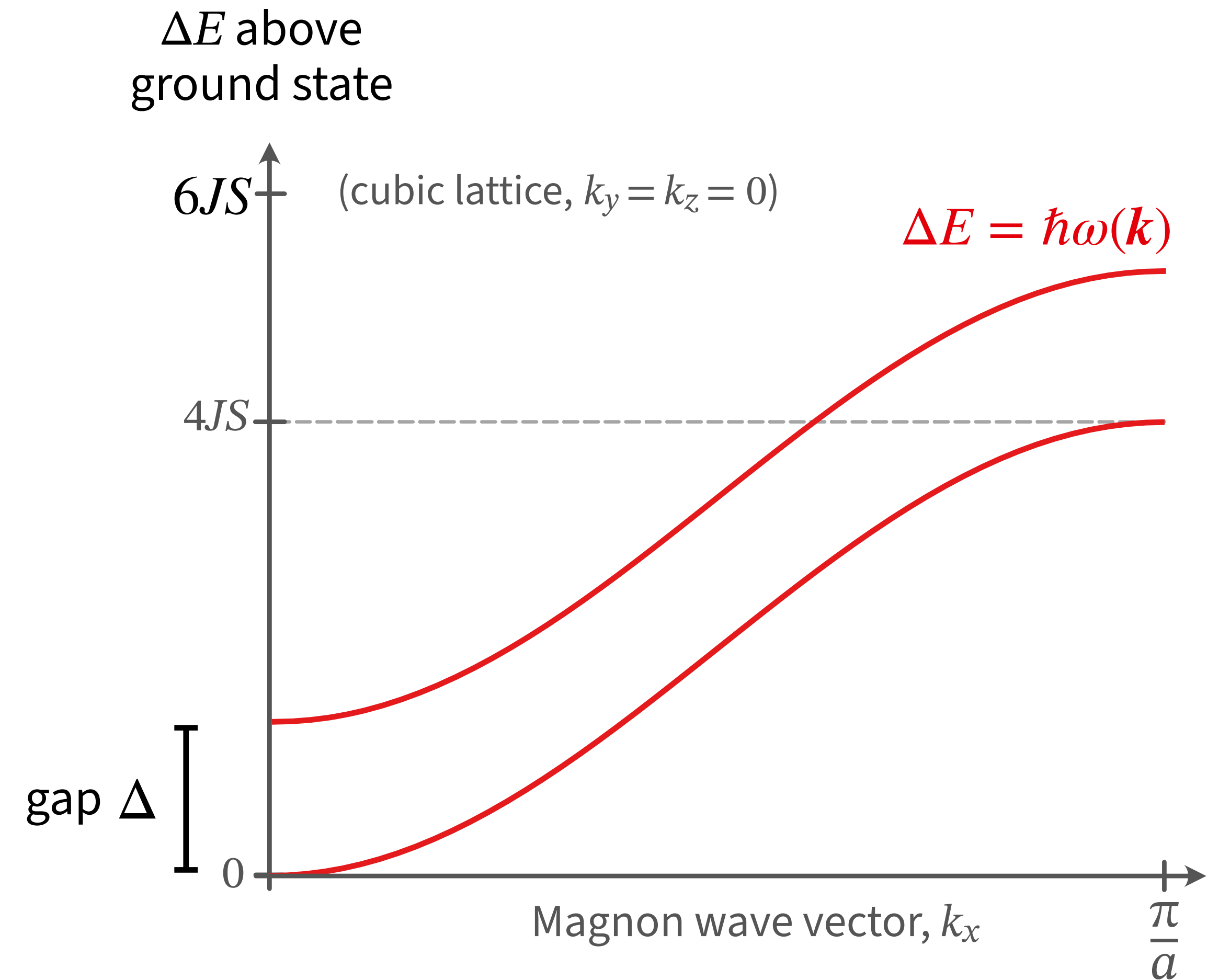
$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j \quad \text{External field} \quad - \mu \sum_i \mathbf{m}_i \cdot \mathbf{B} \quad \text{Anisotropy} \quad - K \sum_i (\mathbf{m}_i \cdot \mathbf{e})^2$$

Explicitly breaking rotational symmetry (field or anisotropy) leads to gap

(Our Larmor frequency from before)

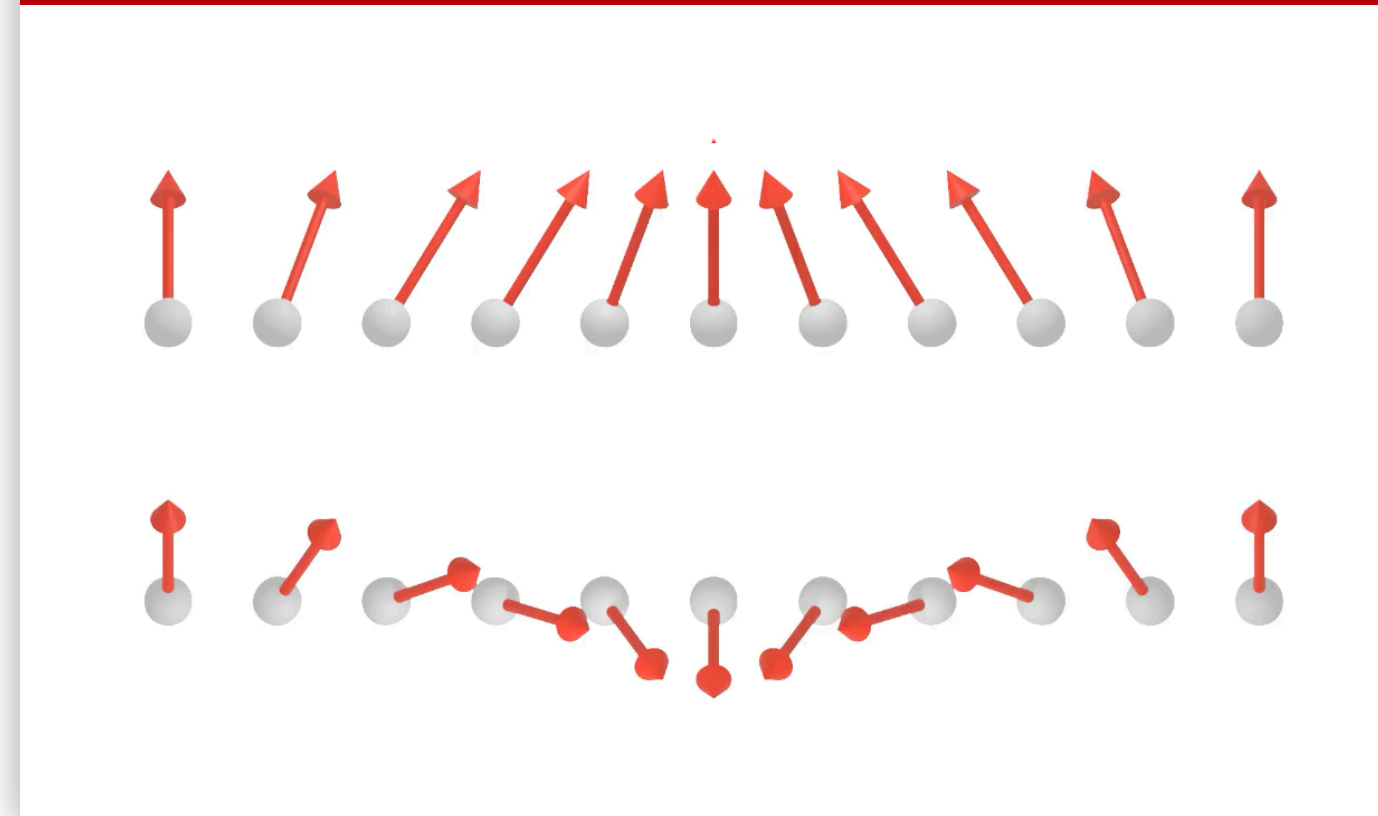
$$\omega_{\text{ex}}(\mathbf{k}) = \frac{4JS}{\hbar} \left(3 - \sum_{i=x,y,z} \cos(k_i a) \right)$$

$\omega_B = \gamma B$ $\omega_K = \gamma \frac{2K}{\mu}$



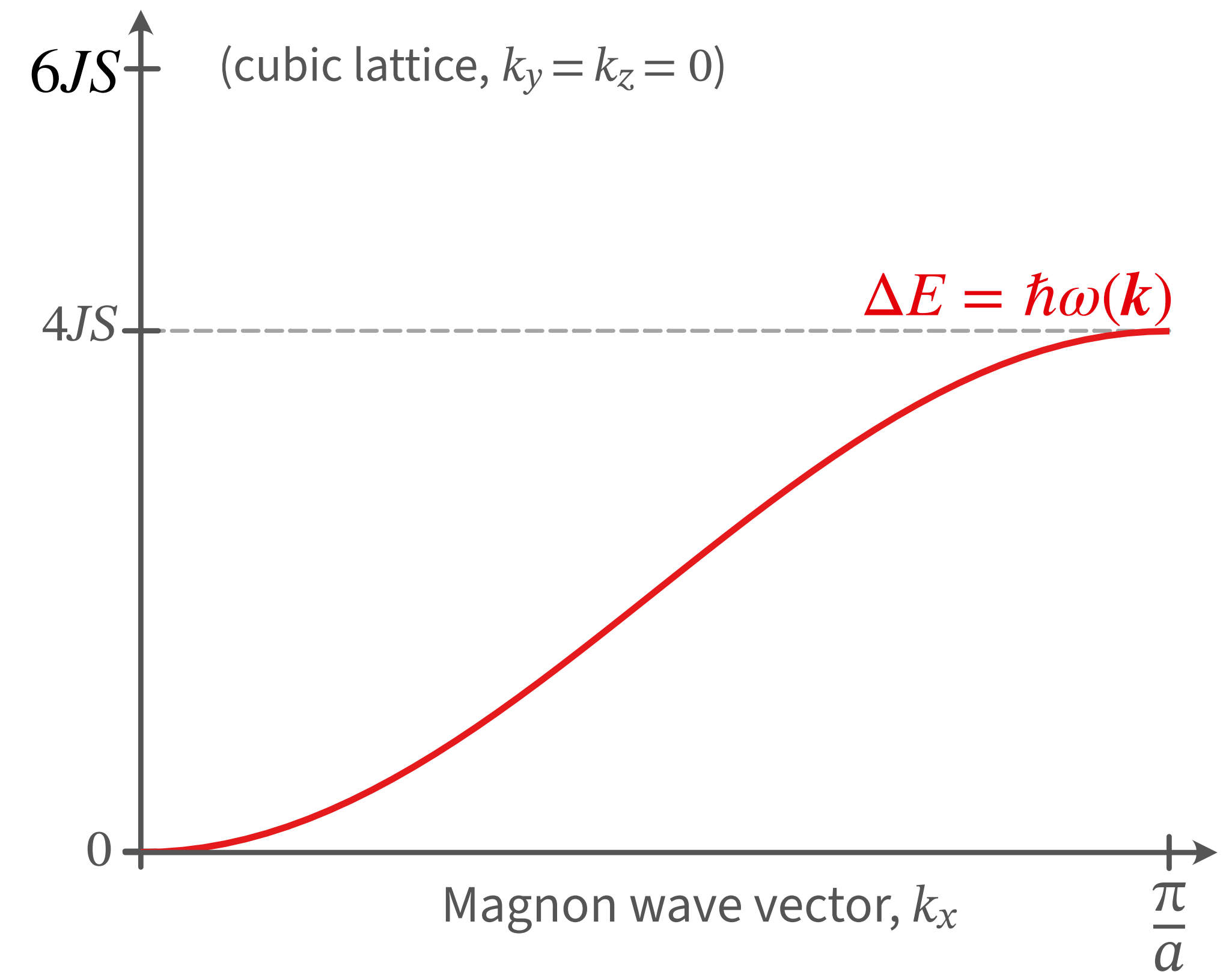
$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j \quad \text{External field} \quad - \mu \sum_i \mathbf{m}_i \cdot \mathbf{B} \quad \text{Anisotropy} \quad - K \sum_i (\mathbf{m}_i \cdot \mathbf{e})^2$$

Magnons

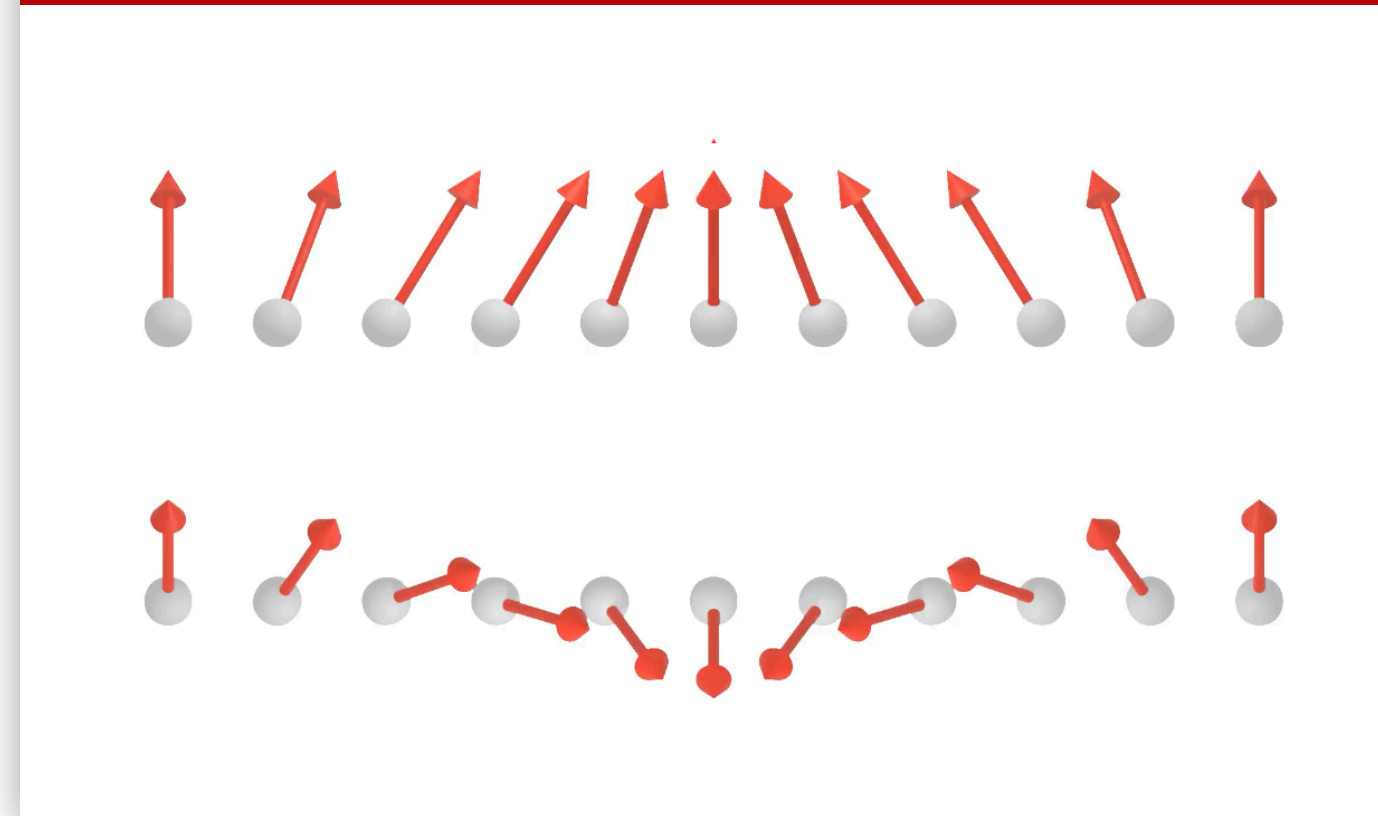


Wave-like bosonic excitations
in (most) magnetic systems

ΔE above
ground state



Magnons



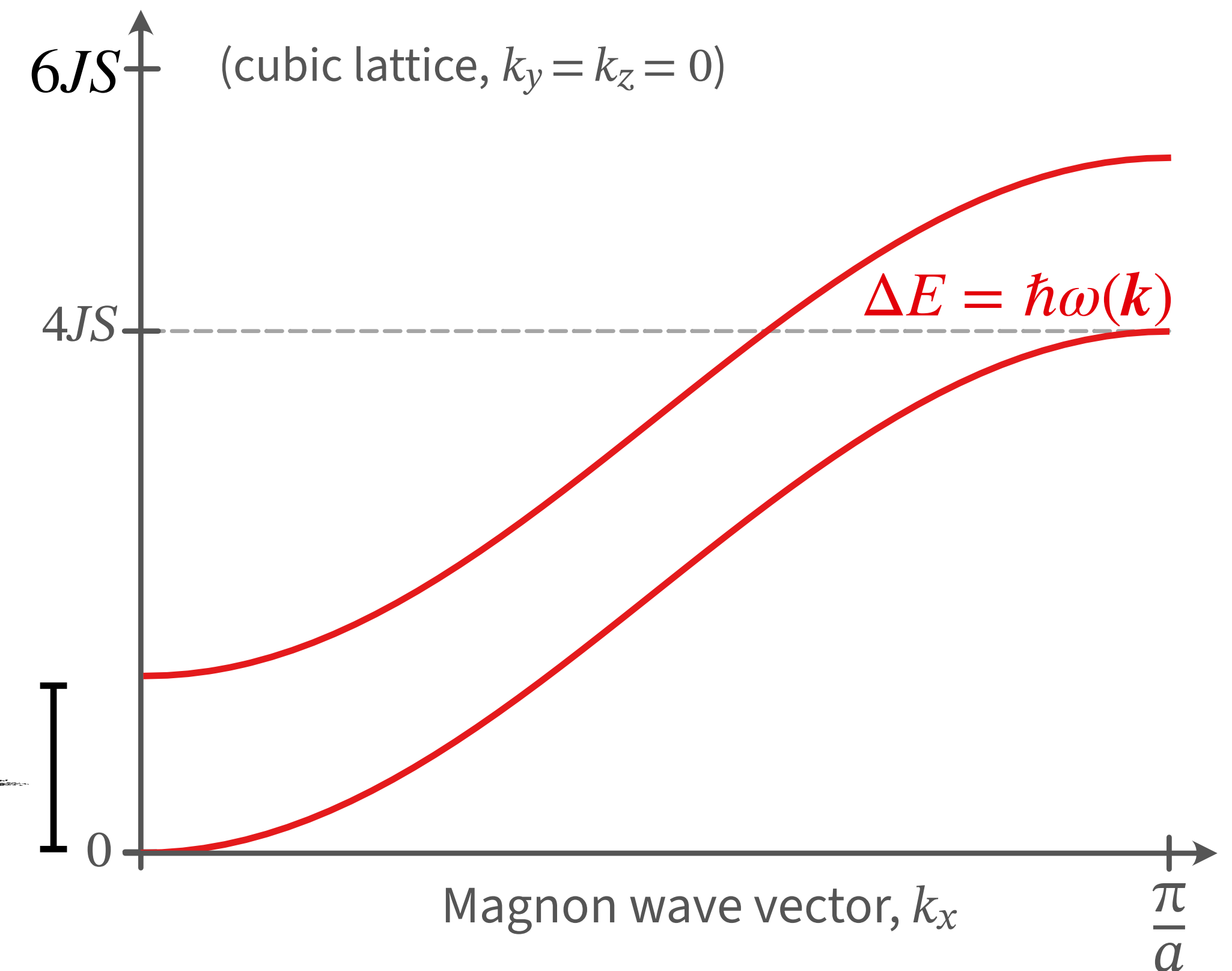
Wave-like bosonic excitations
in (most) magnetic systems



Dispersive in FM

Gapped by anisotropy and field

ΔE above
ground state



How to measure magnons (important examples)

Microwave absorption

- Ferromagnetic resonance (FMR) spectroscopy
- Propagating spin-wave spectroscopy (PSWS)

Optical & X-ray scattering

- Brillouin-light-scattering (BLS)/Raman scattering/MOKE
- (soft) X-rays

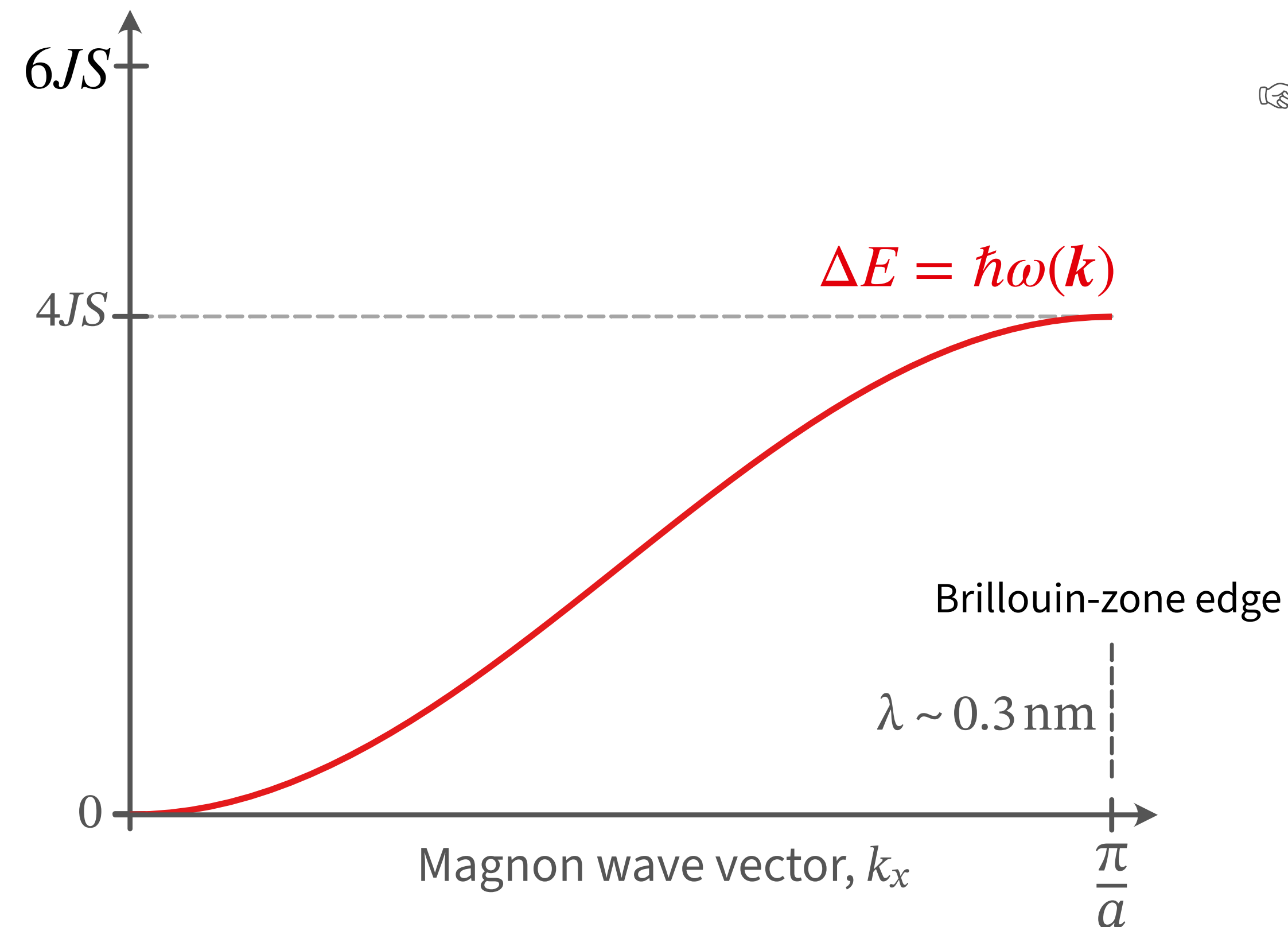
Neutron scattering



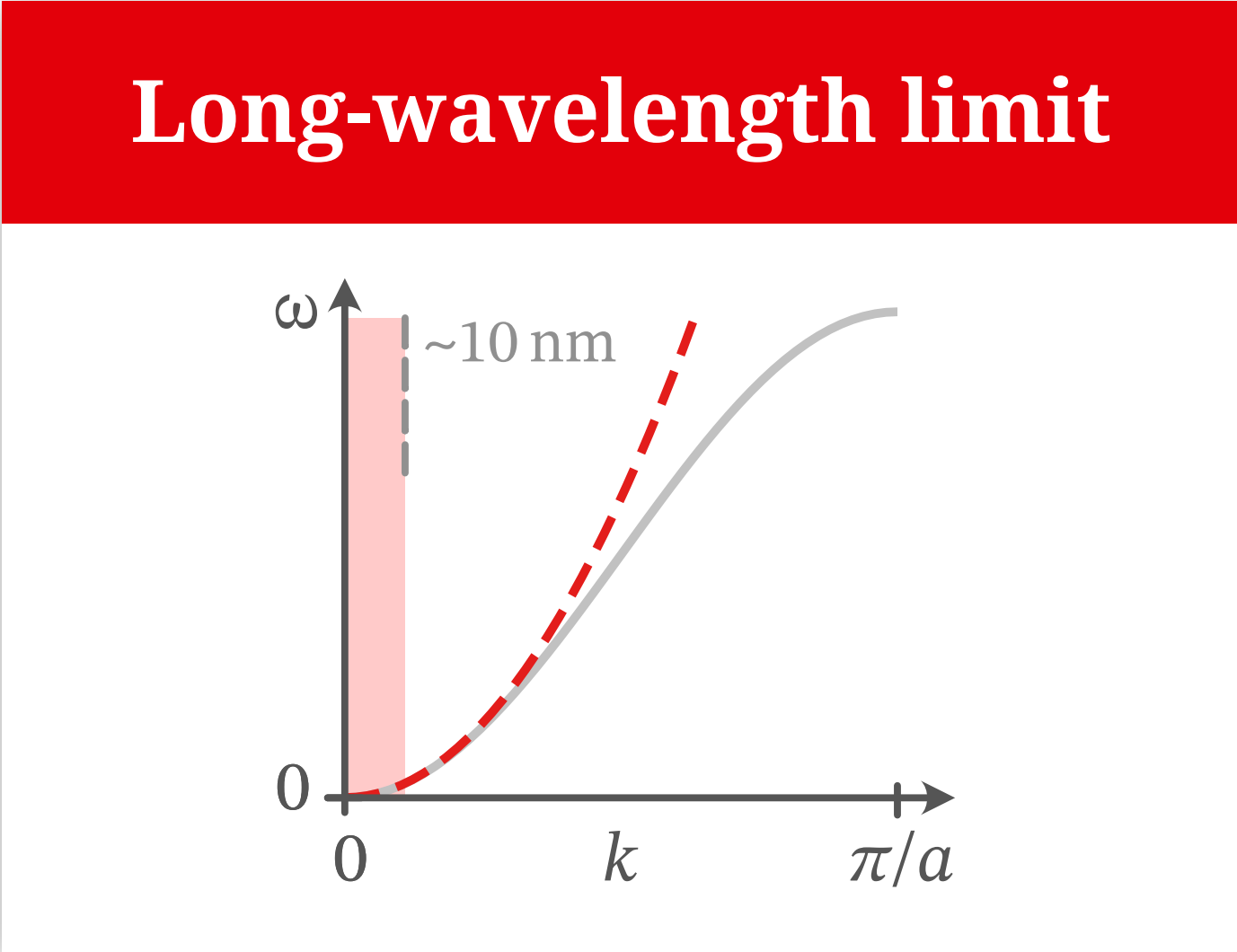
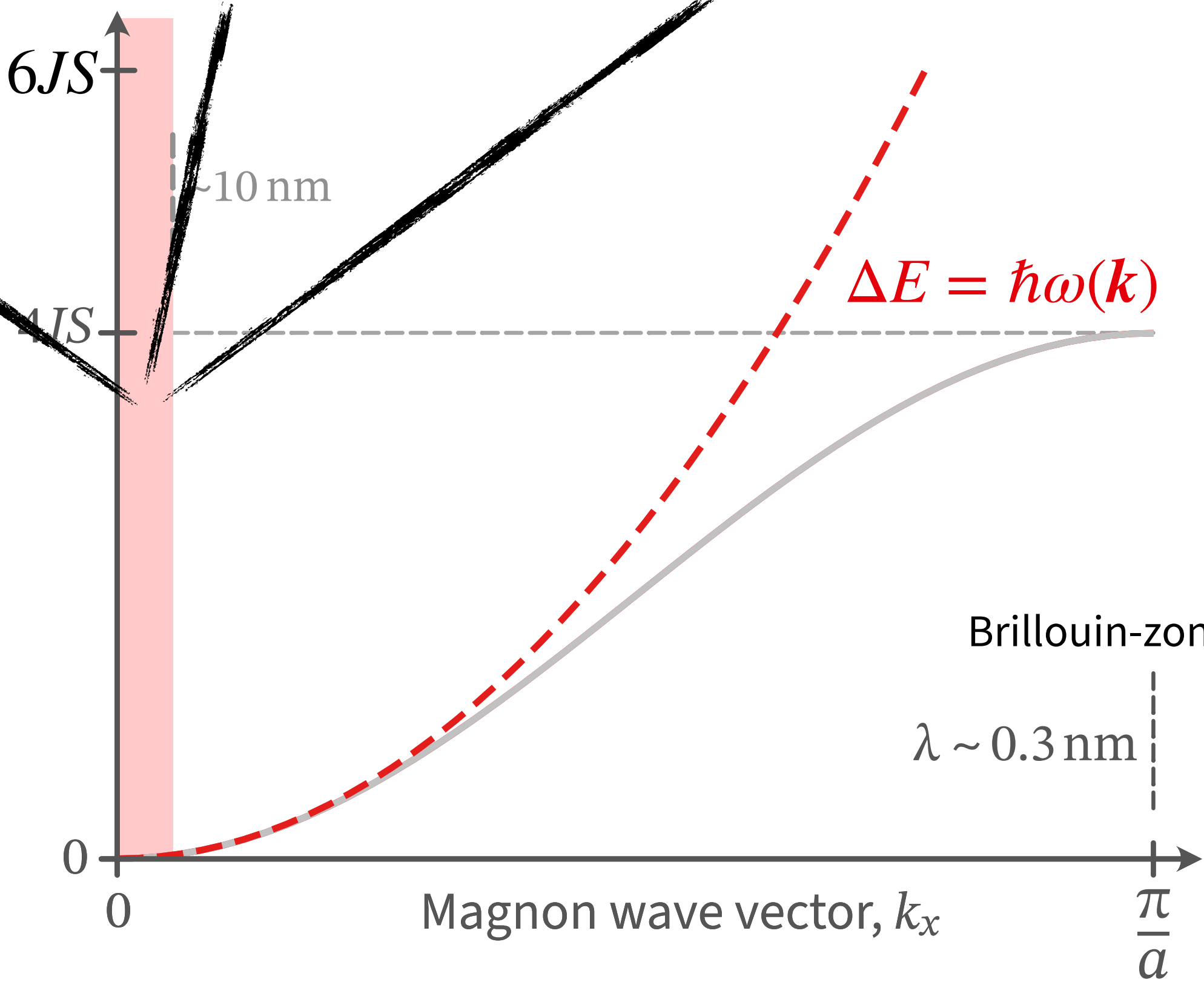
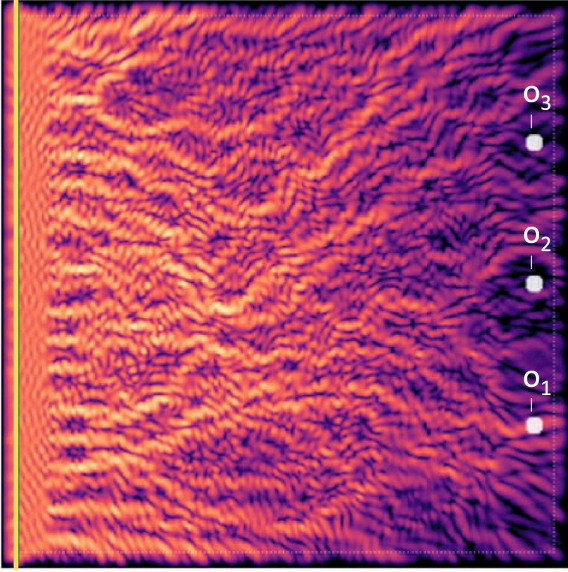
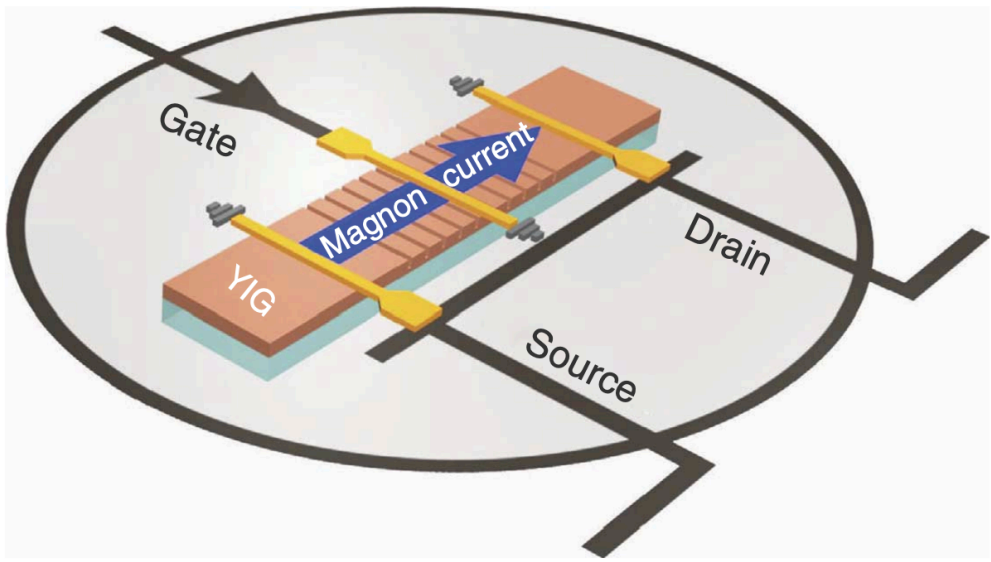
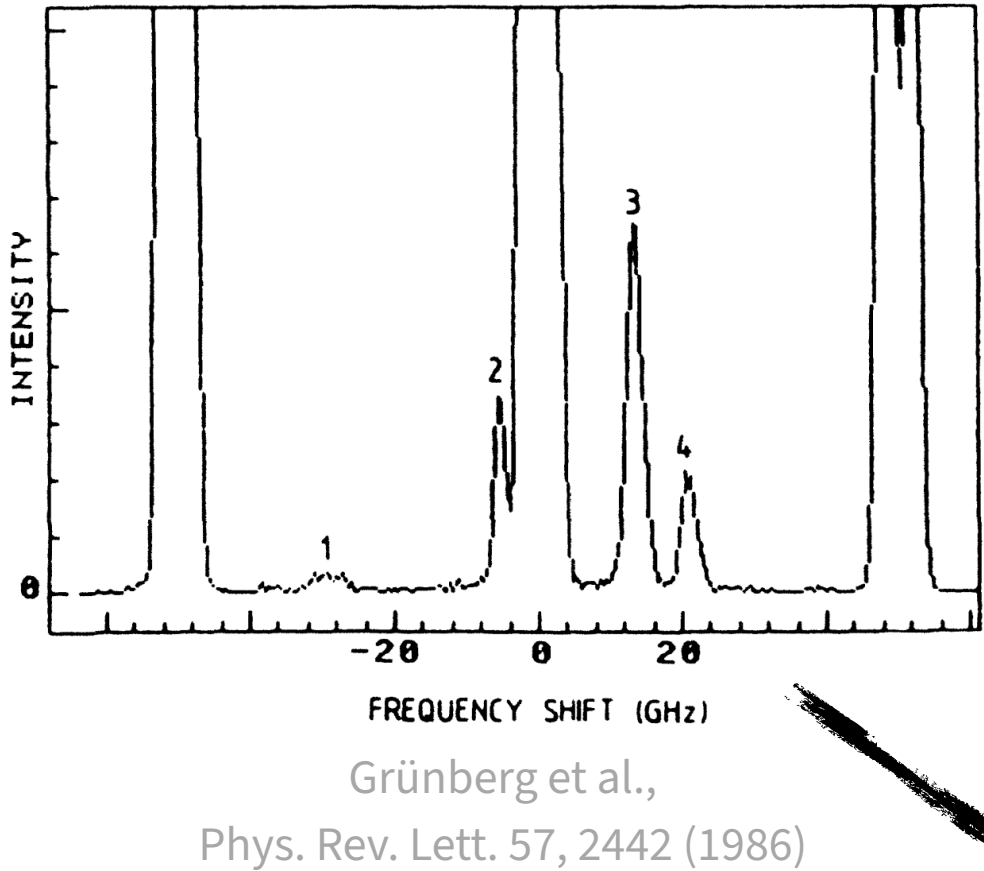
Laboratory (table-top)

Large facilities

👉 Other (e.g., indirect) methods, via spintronic or phononic effects available



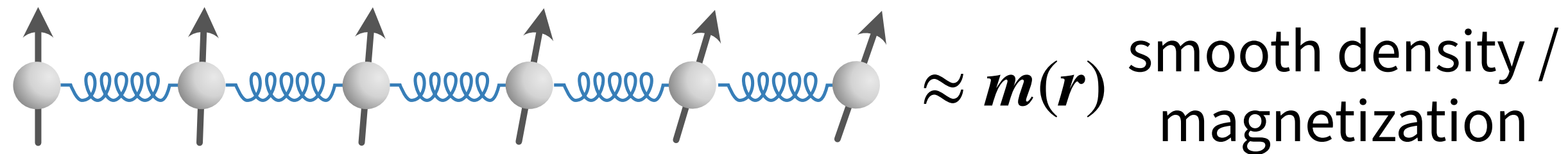
Long-wavelength magnons most relevant on nm to μm scale



Long-wavelength (dipole-exchange) magnons

Long-wavelength magnons are described in micromagnetism

Local moments $\mathbf{m}_i$



$$\mu [\text{Am}^2] \mapsto M_s [\text{A/m}]$$

Saturation
magnetization

Exchange in continuum approximation = micromagnetism

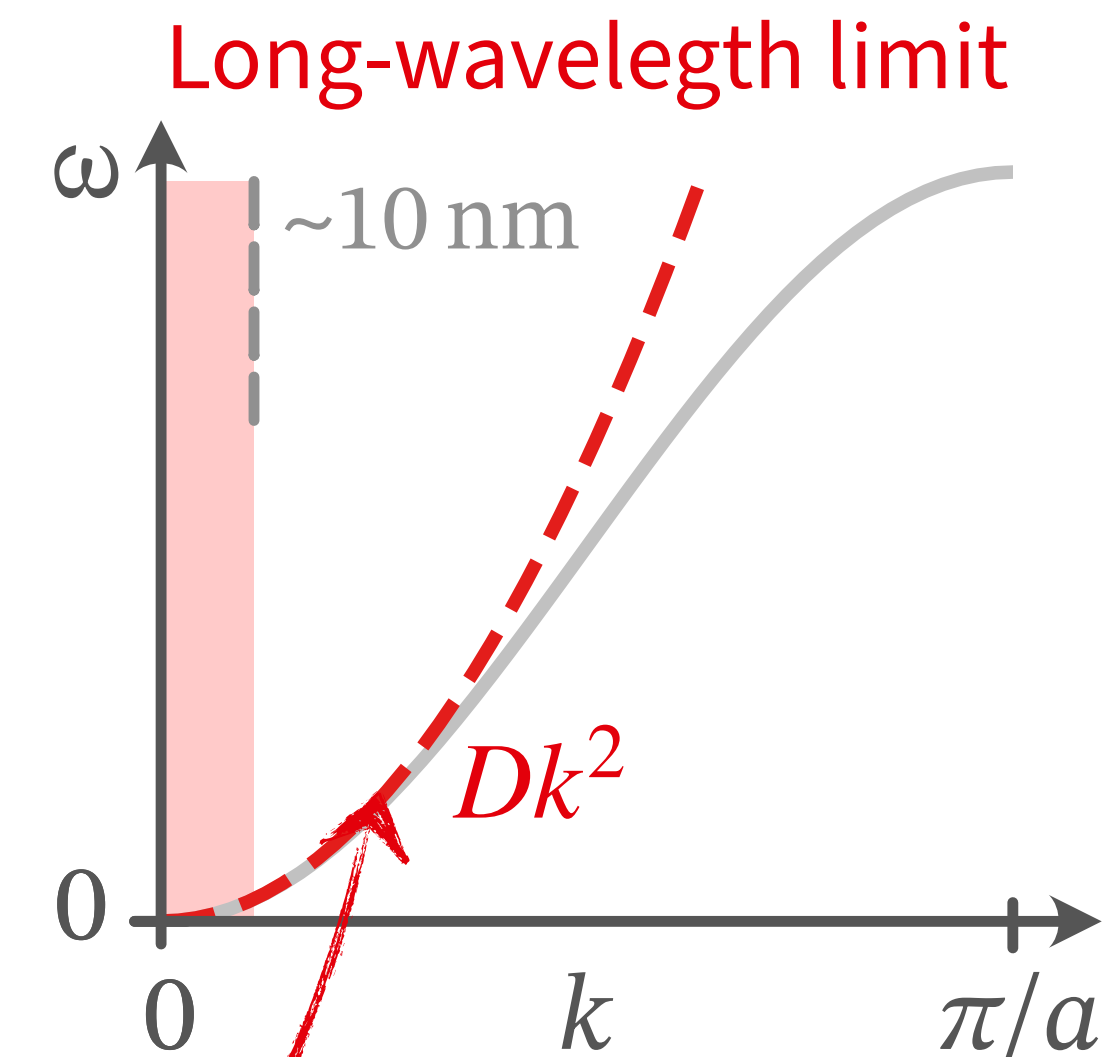
$$E = -JS^2\hbar^2 \sum_{\langle ij \rangle} \mathbf{m}_i \cdot \mathbf{m}_j \approx A \int dV [\nabla \mathbf{m}(\mathbf{r})]^2 + \text{const.}$$

$$\mathbf{B}^{\text{exc}} = -\frac{1}{\mu} \frac{\partial E}{\partial \mathbf{m}_i} \approx -\frac{1}{M_s} \frac{\delta E}{\delta \mathbf{m}} = \frac{2A}{M_s} \nabla^2 \mathbf{m}$$

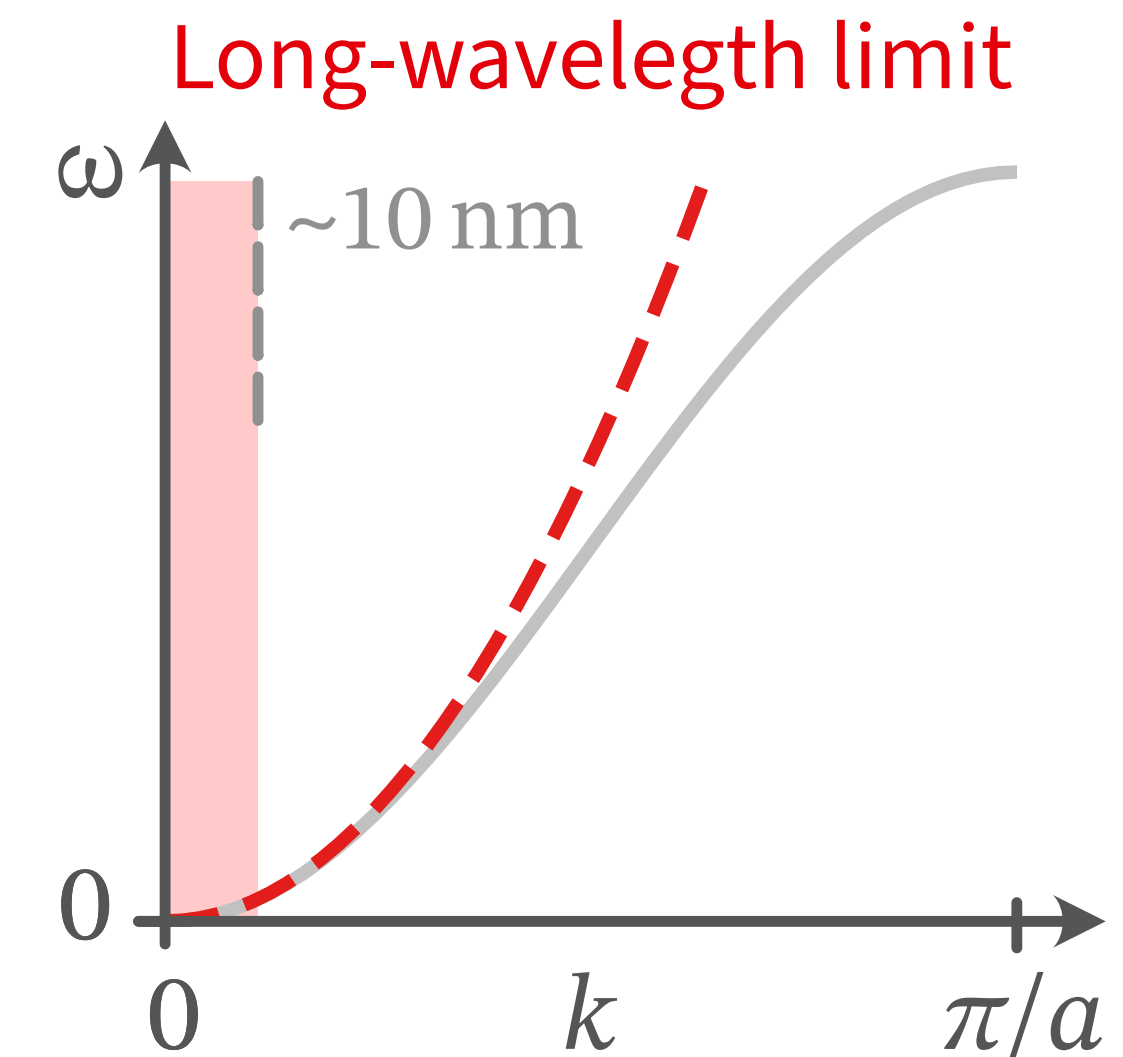
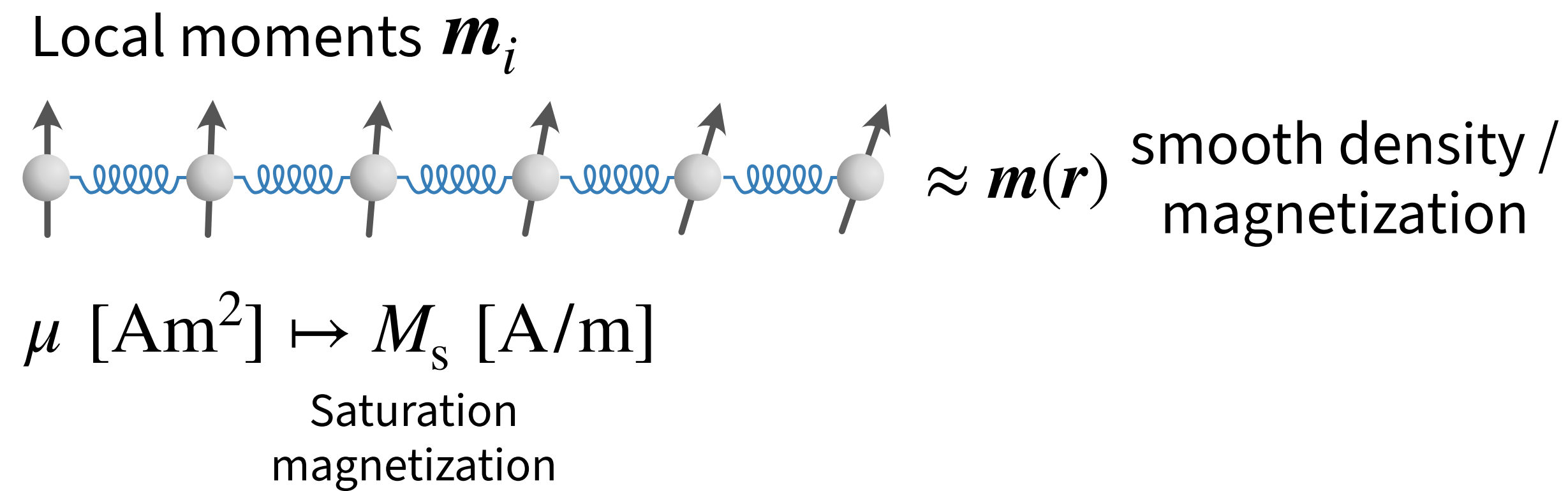
Partial
derivative

Functional
derivative

$\frac{D}{\gamma}$ We recover the
spin-wave stiffness



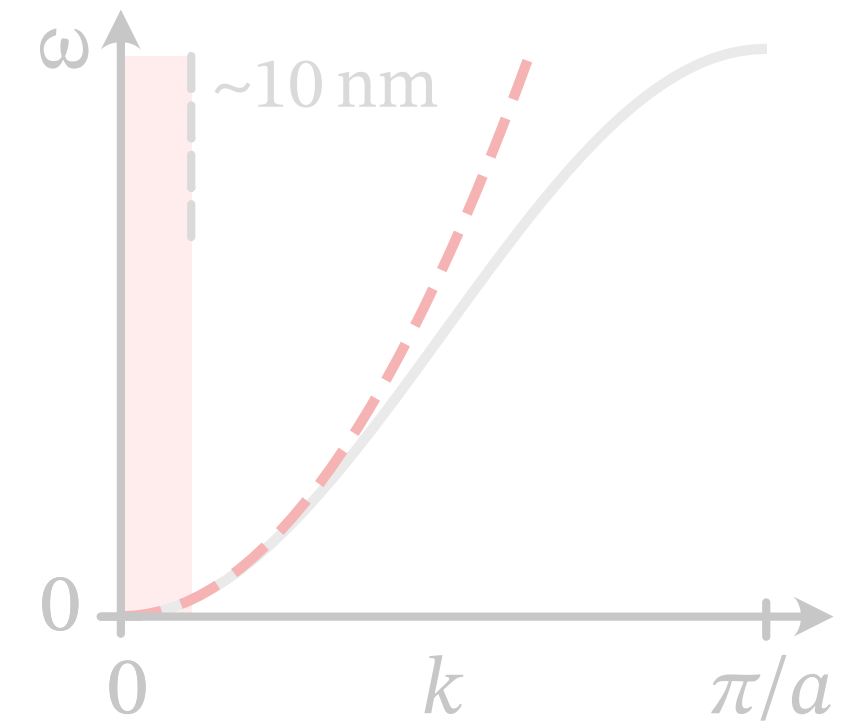
Long-wavelength magnons are described in micromagnetism



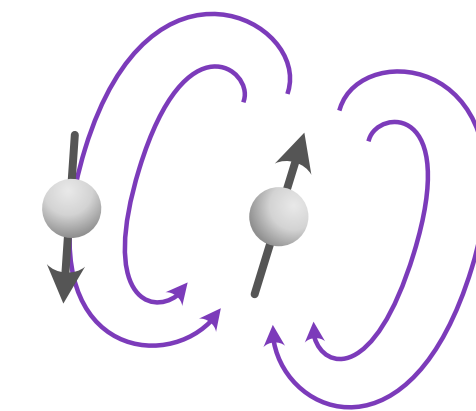
Micromagnetic energy functional

$$E(\mathbf{m}) = \int dV \left[\overset{\text{Exchange}}{A(\nabla \mathbf{m})^2} + \overset{\text{Anisotropy}}{\mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m}} - \overset{\text{Zeeman}}{M_s \mathbf{B} \cdot \mathbf{m}} - \overset{\text{Dipole-Dipole}}{M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m}} \right]$$

Dipolar fields become essential at large wavelengths



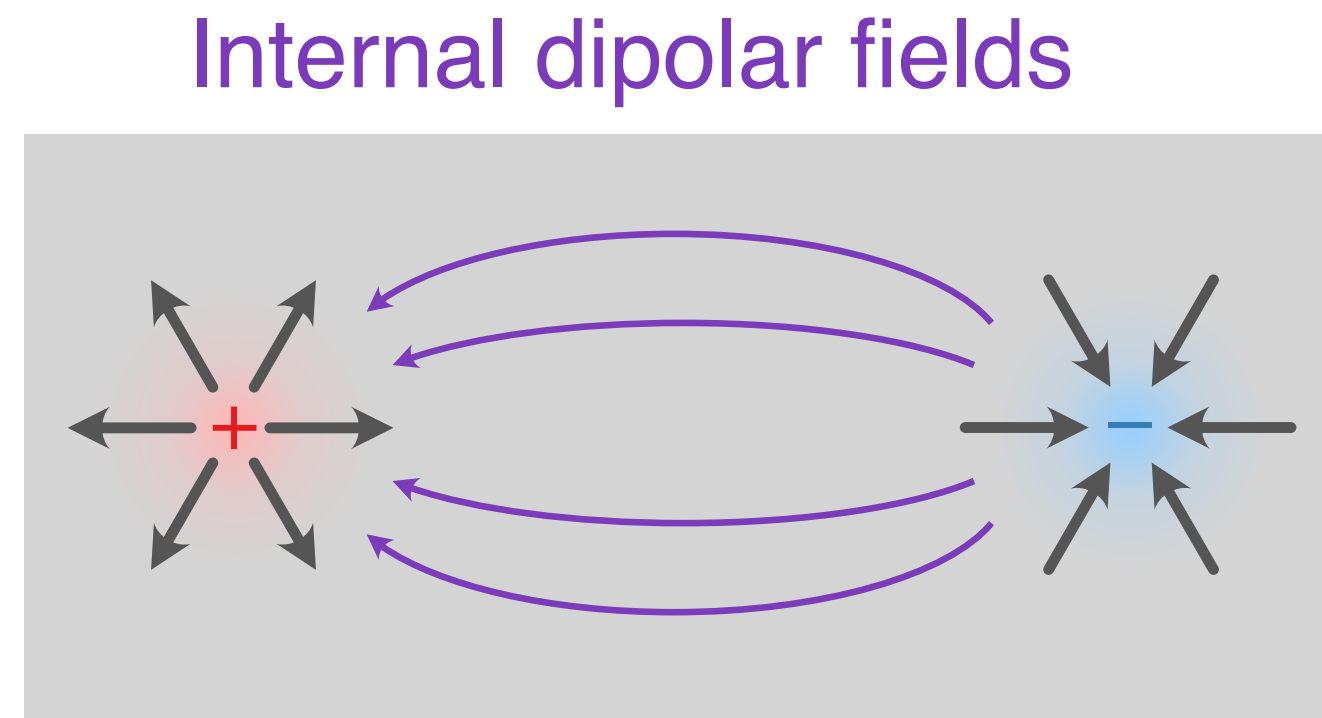
**Weaker, long-range
interaction
becomes relevant**



Dipole-Dipole

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

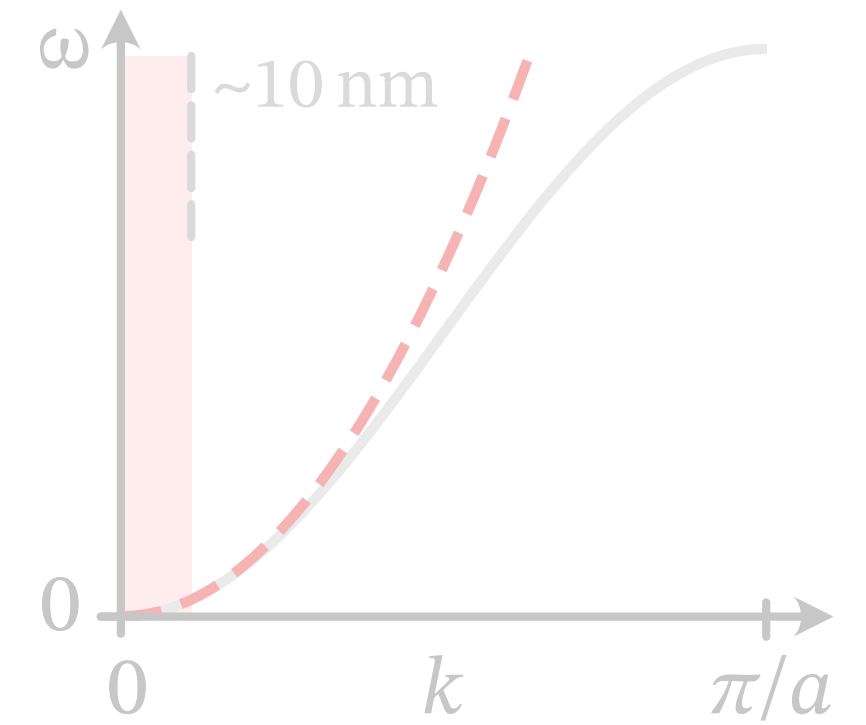
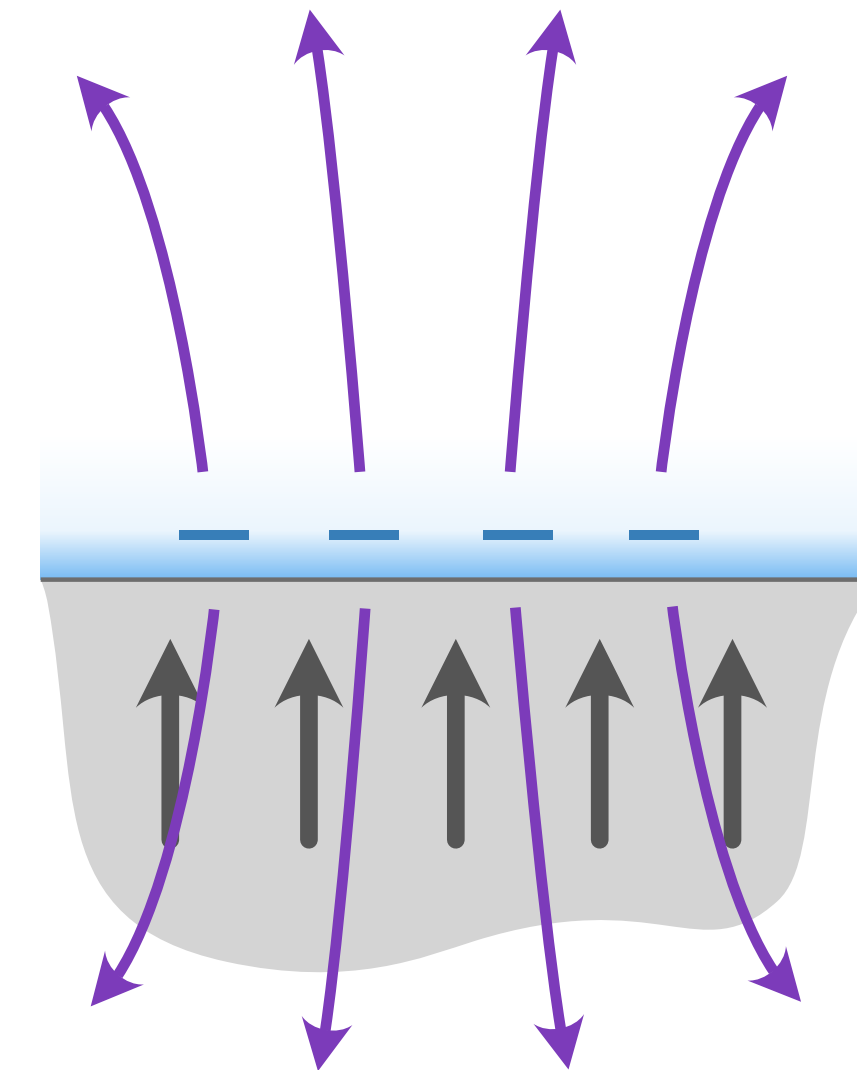
Dipolar fields become essential at large wavelengths



Volume charges
 $\rho = -\nabla \cdot \mathbf{m}$

Internal/external
 dipolar fields

Surface charges
 $\sigma = -\mathbf{n} \cdot \mathbf{m}$



$$\phi(\mathbf{r}) = \frac{1}{4\pi} \int_V dV' \frac{\nabla' \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|} + \frac{1}{4\pi} \int_{\partial V} dA' \frac{\mathbf{n} \cdot \mathbf{m}}{|\mathbf{r} - \mathbf{r}'|}$$

magnetostatic / dipolar potential

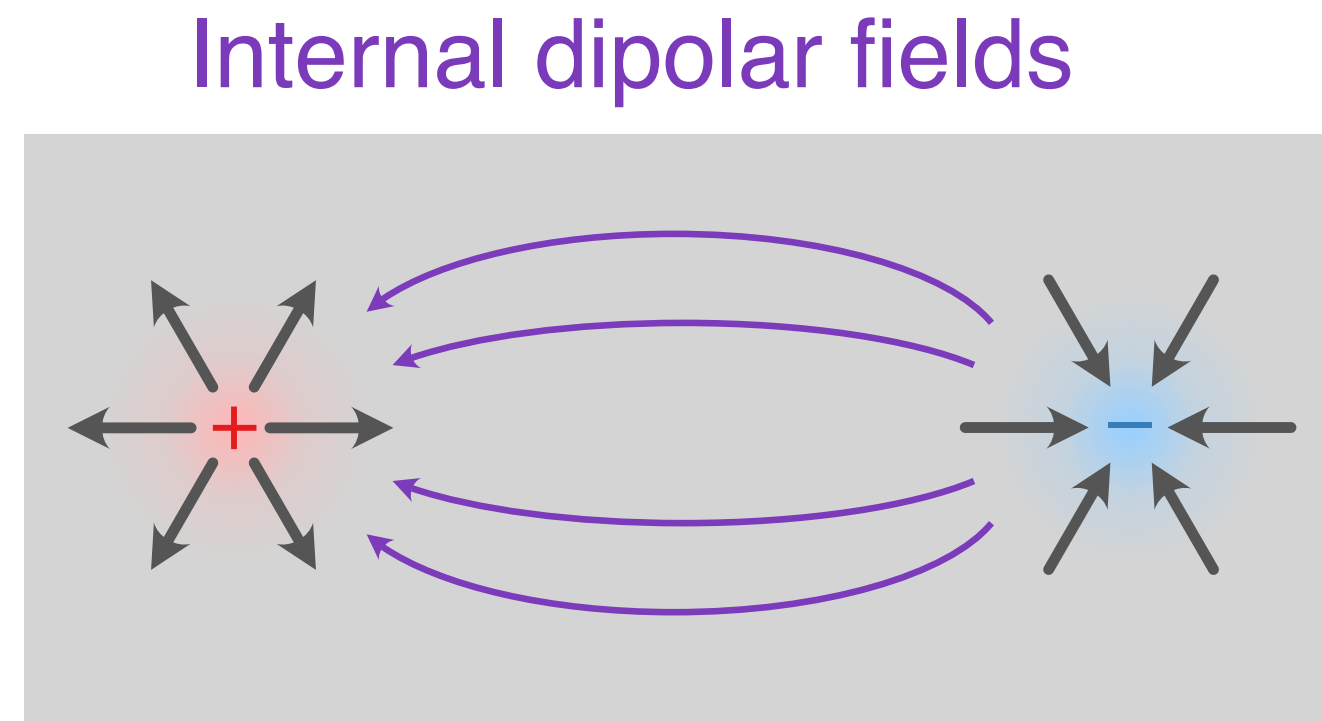
In magnetic static of
 Maxwell equations

$$\mathbf{B}^{\text{dip}} = \mu_0 \nabla \phi$$

Dipole-Dipole

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

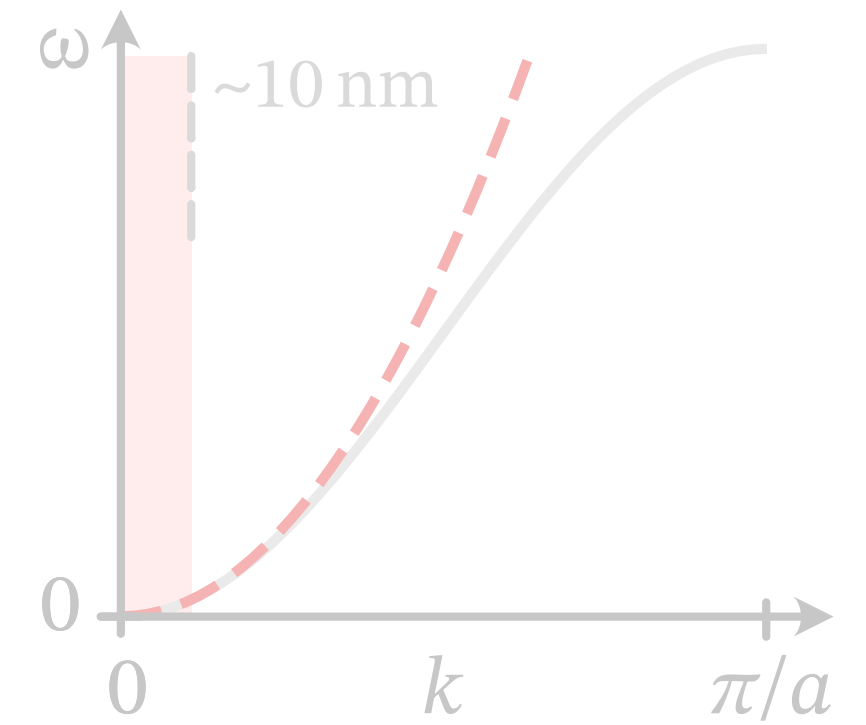
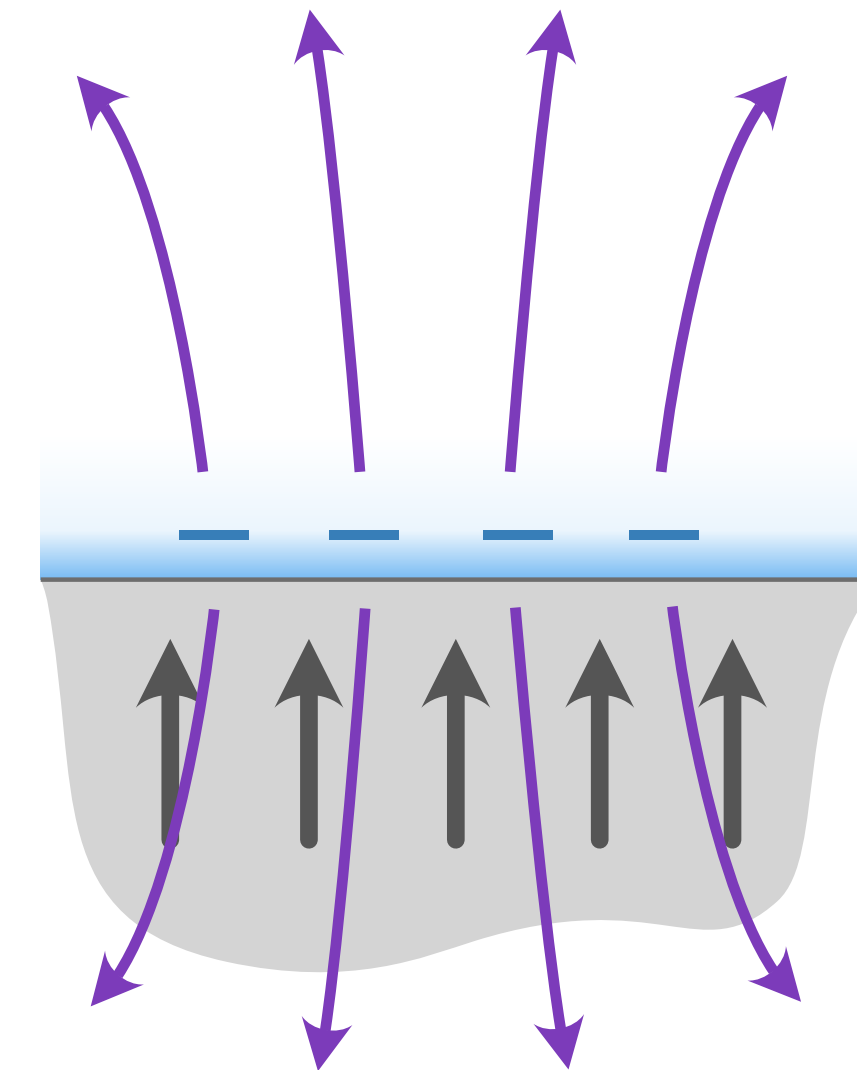
Dipolar fields become essential at large wavelengths



Volume charges
 $\rho = -\nabla \cdot \mathbf{m}$

Internal/external
 dipolar fields

Surface charges
 $\sigma = -\mathbf{n} \cdot \mathbf{m}$



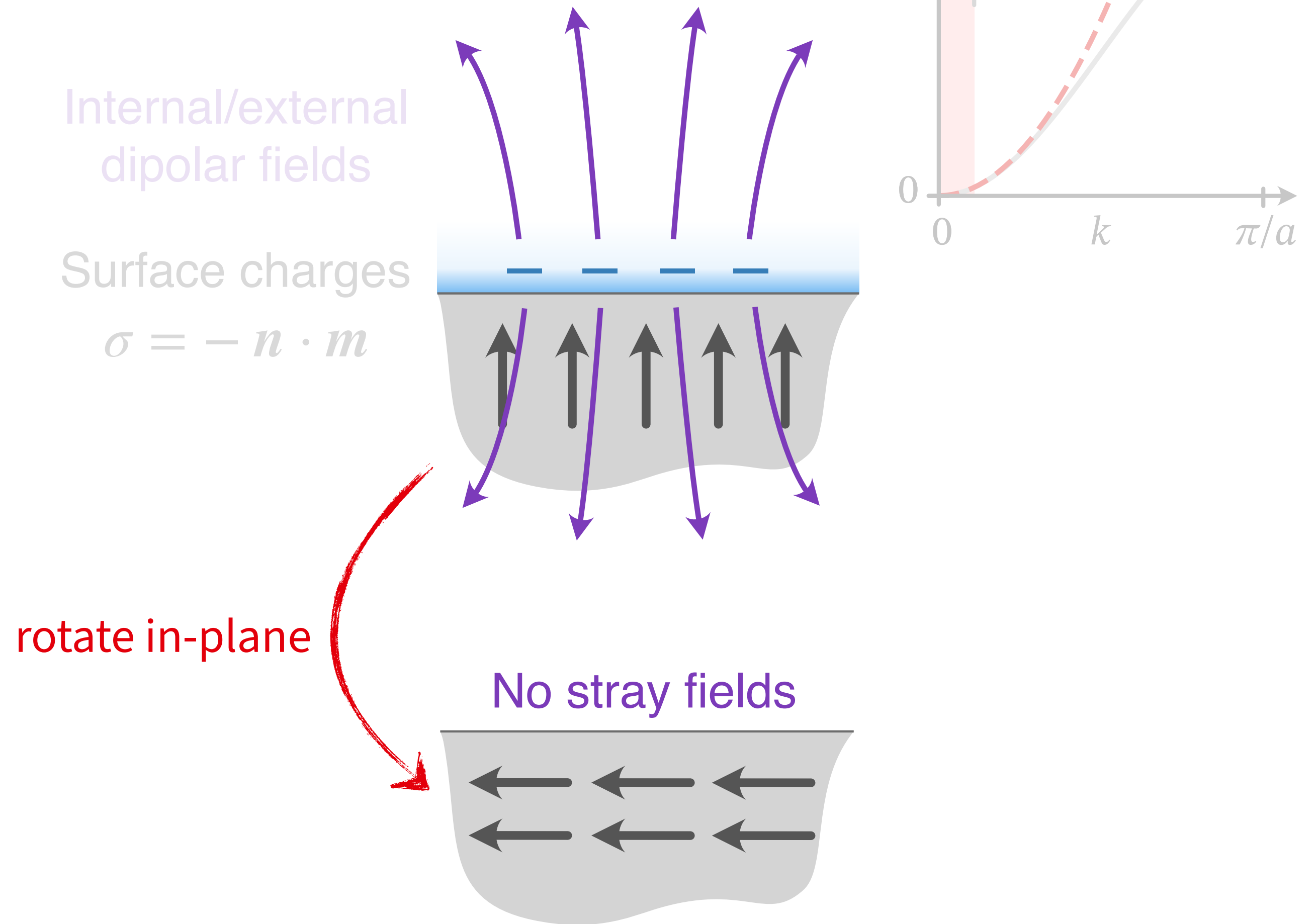
Avoid costly strayfields / net charges.

$$\mathbf{B}^{\text{dip}} = M_s \nabla \phi$$

Dipole-Dipole

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

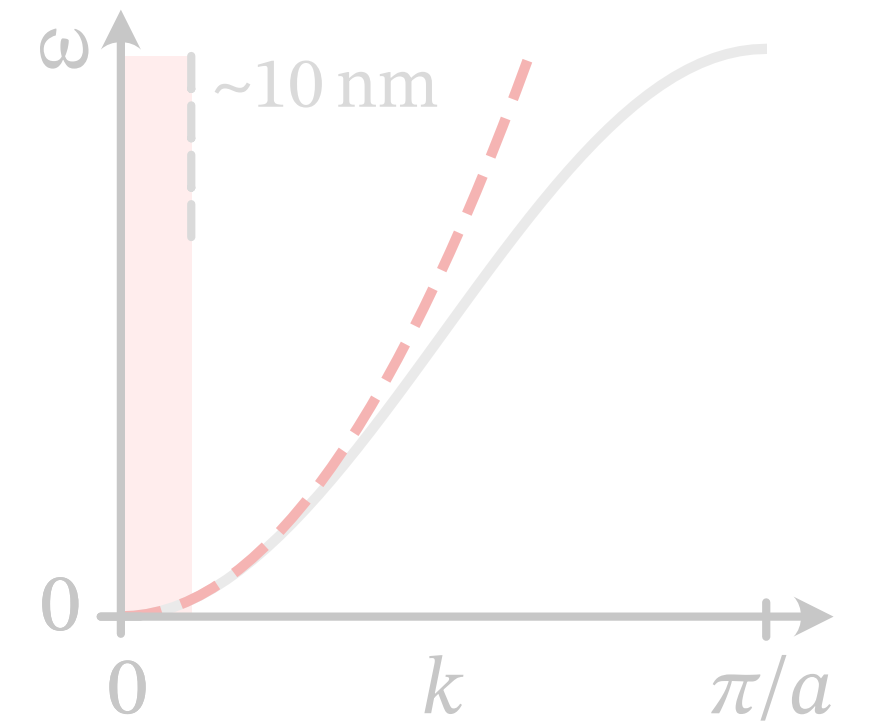
Avoiding surface charges leads to shape anisotropy



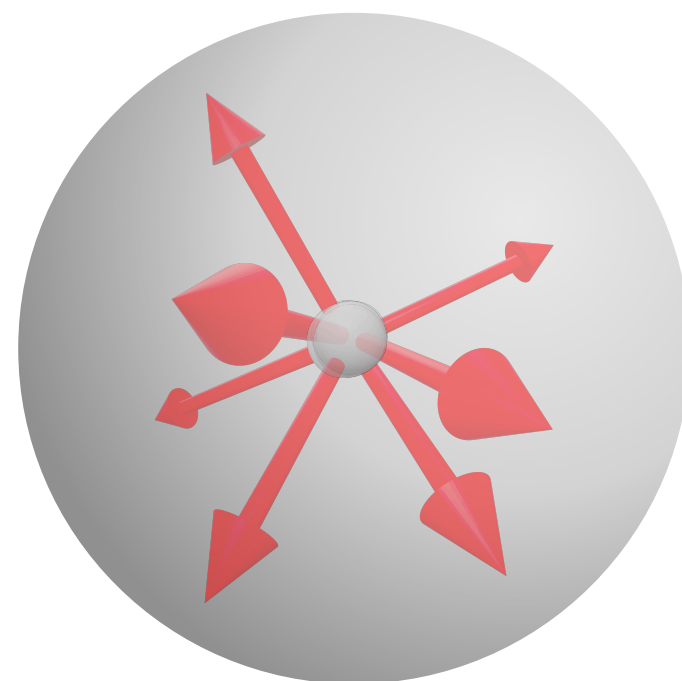
$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - \overset{\text{Dipole-Dipole}}{M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m}} \right]$$

Avoiding surface charges leads to shape anisotropy

Homogeneously magnetized
= macrospin

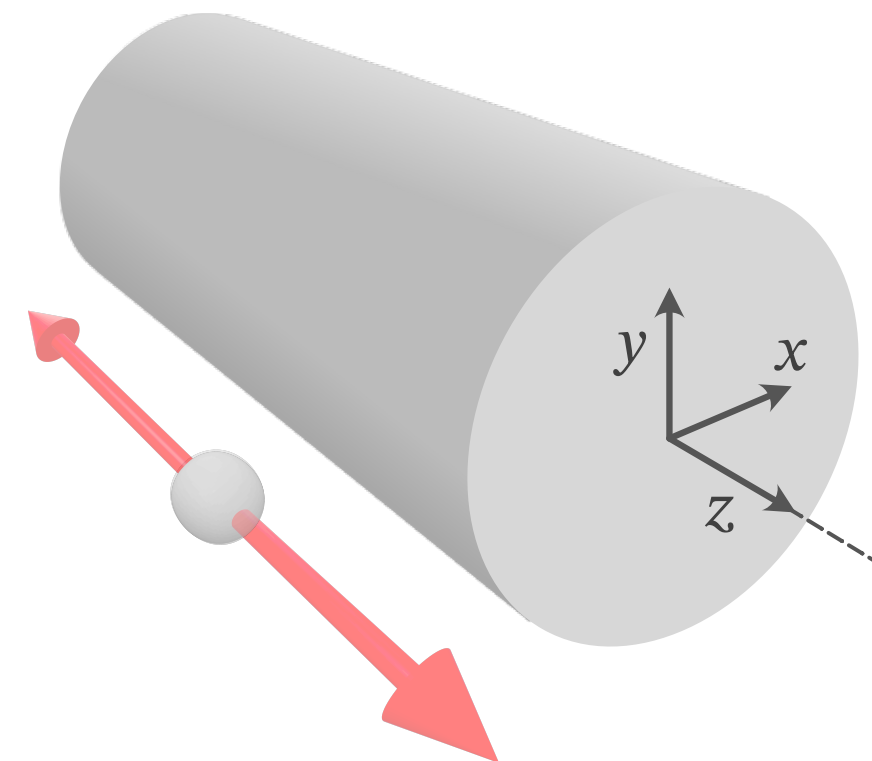


Sphere



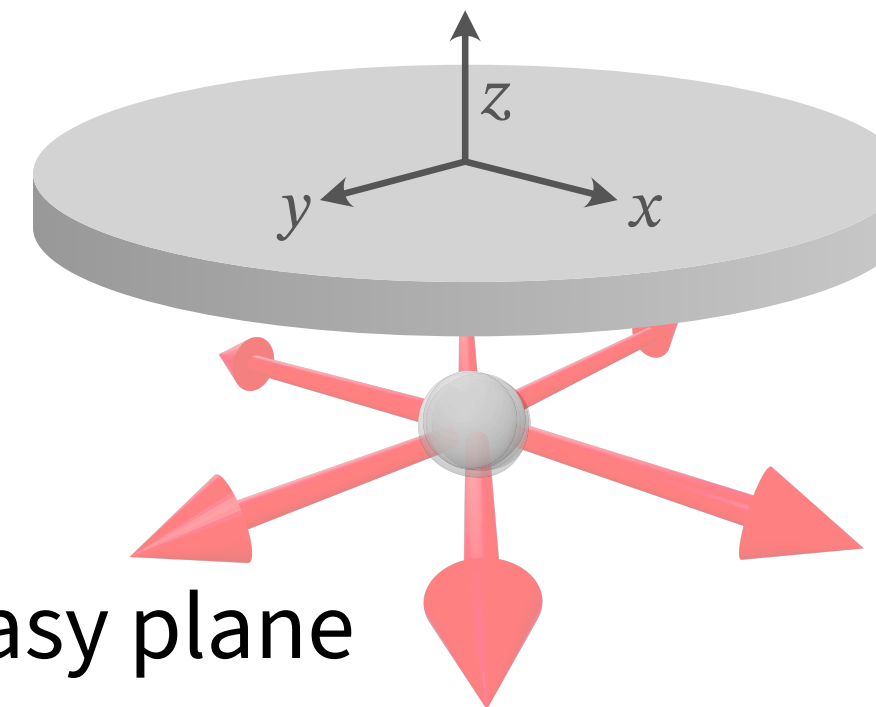
isotrop

Infinite cylinder

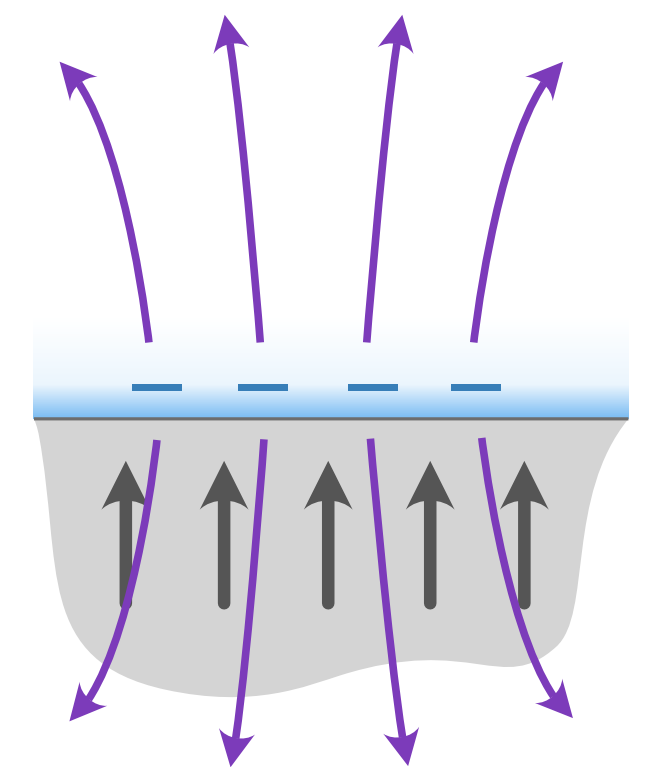


easy axis

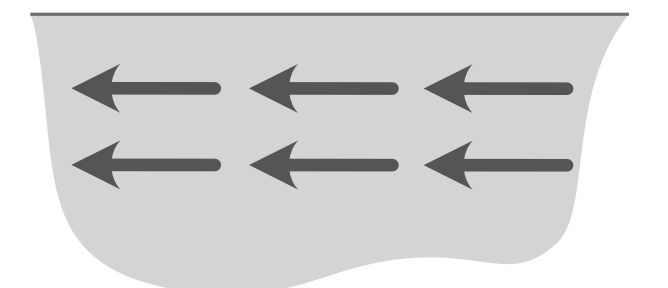
Infinite layer



easy plane



No stray fields



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Avoiding surface charges leads to shape anisotropy

Homogeneously magnetized
= macrospin

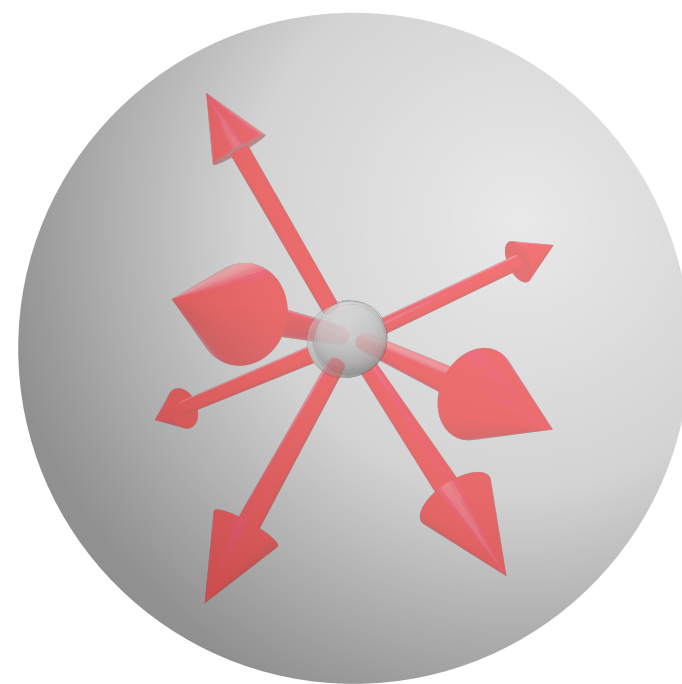
$$\longrightarrow \mathbf{B}^{\text{dip}} = -\mu_0 M_s \hat{\mathbf{N}} \cdot \mathbf{m}$$

dipolar tensor

$$\hat{\mathbf{N}} = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$$

$$\hat{\mathbf{N}} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

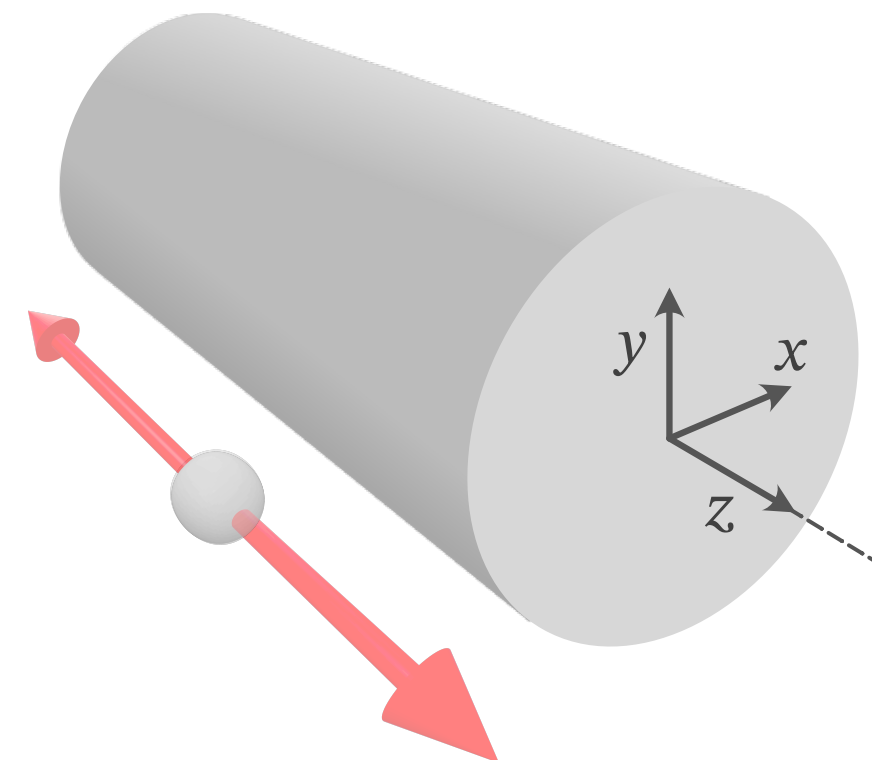
Sphere



isotrop

$$\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

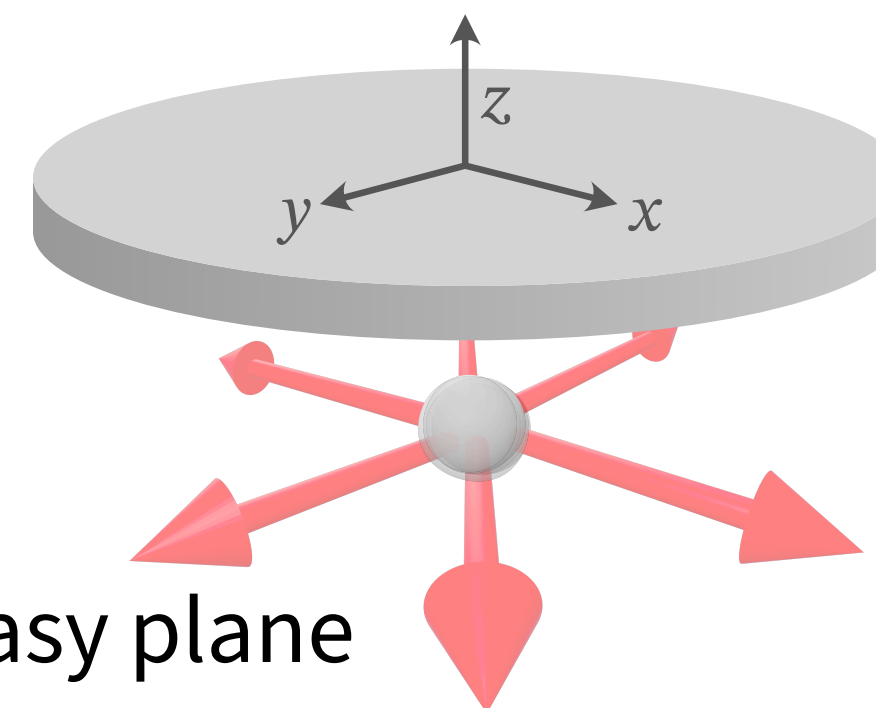
Infinite cylinder



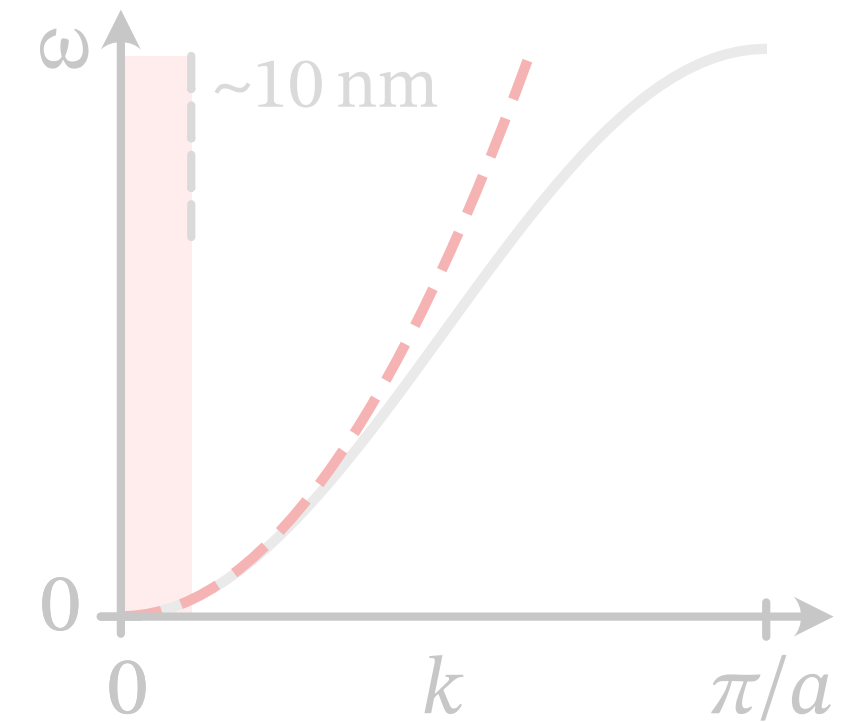
easy axis

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Infinite layer



easy plane



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Avoiding surface charges leads to shape anisotropy

Homogeneously magnetized
= macrospin

$$\longrightarrow \mathbf{B}^{\text{dip}} = -\mu_0 M_s \hat{\mathbf{N}} \cdot \mathbf{m}$$

dipolar tensor

$$\hat{\mathbf{N}} = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$$

$$\hat{\mathbf{N}} = \begin{pmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{pmatrix}$$

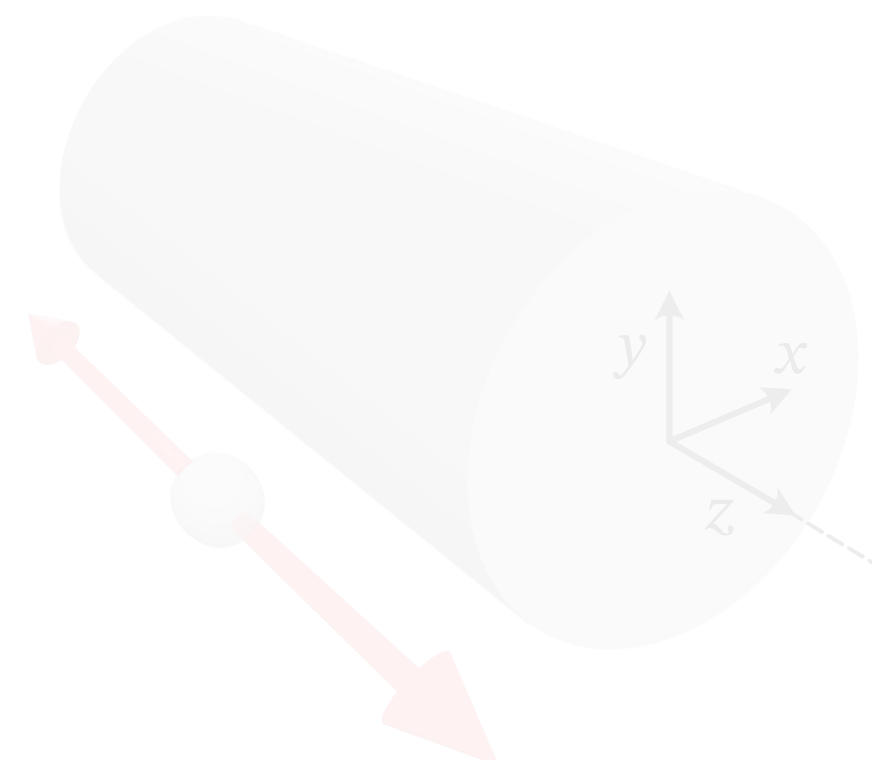
Sphere



isotrop

$$\begin{pmatrix} 1/2 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

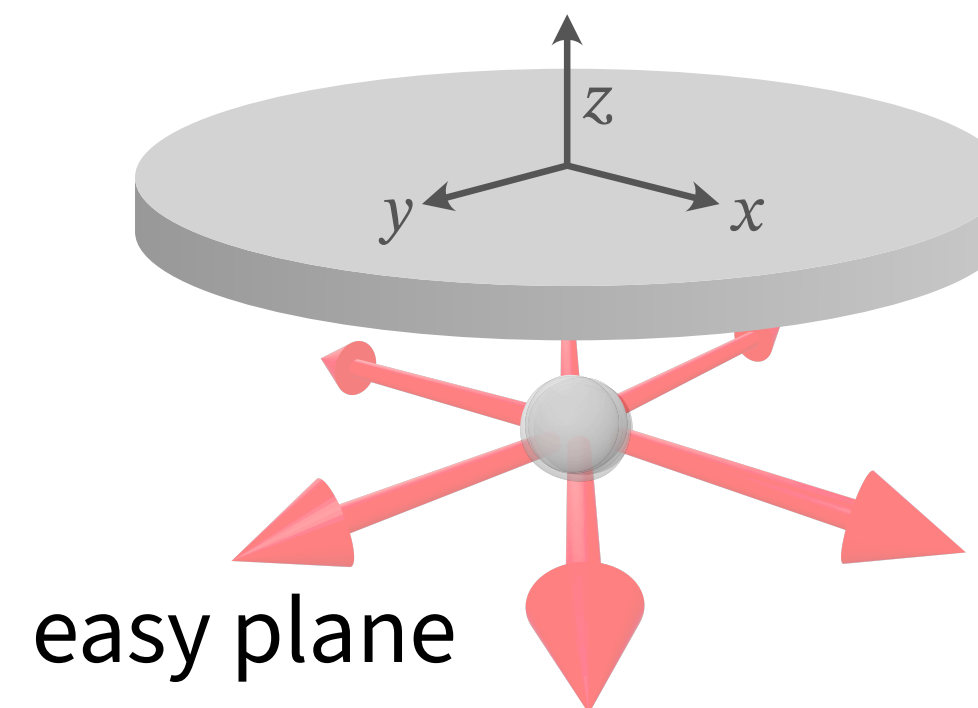
Infinite cylinder



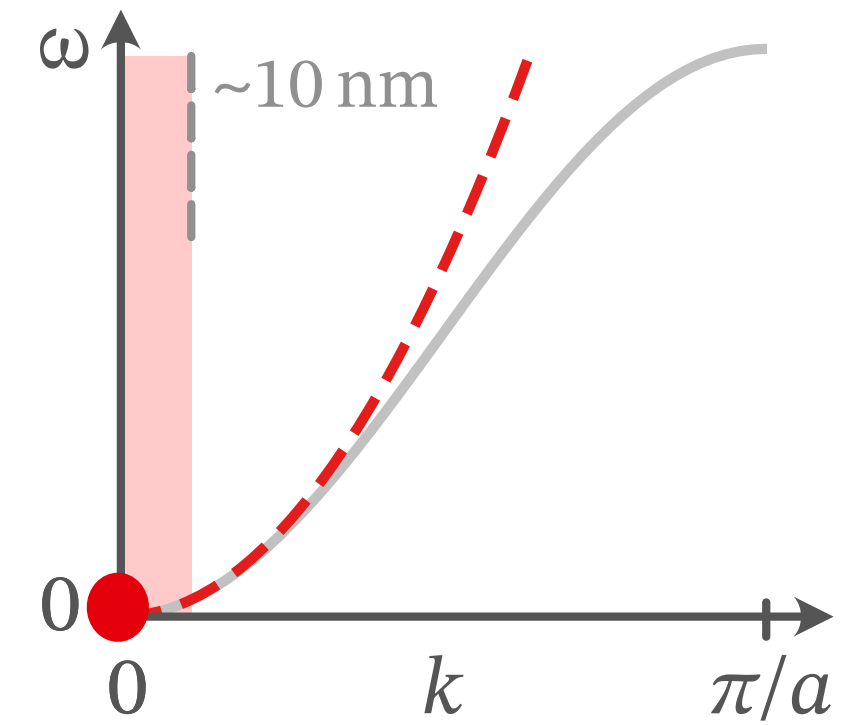
easy axis

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Infinite layer



easy plane



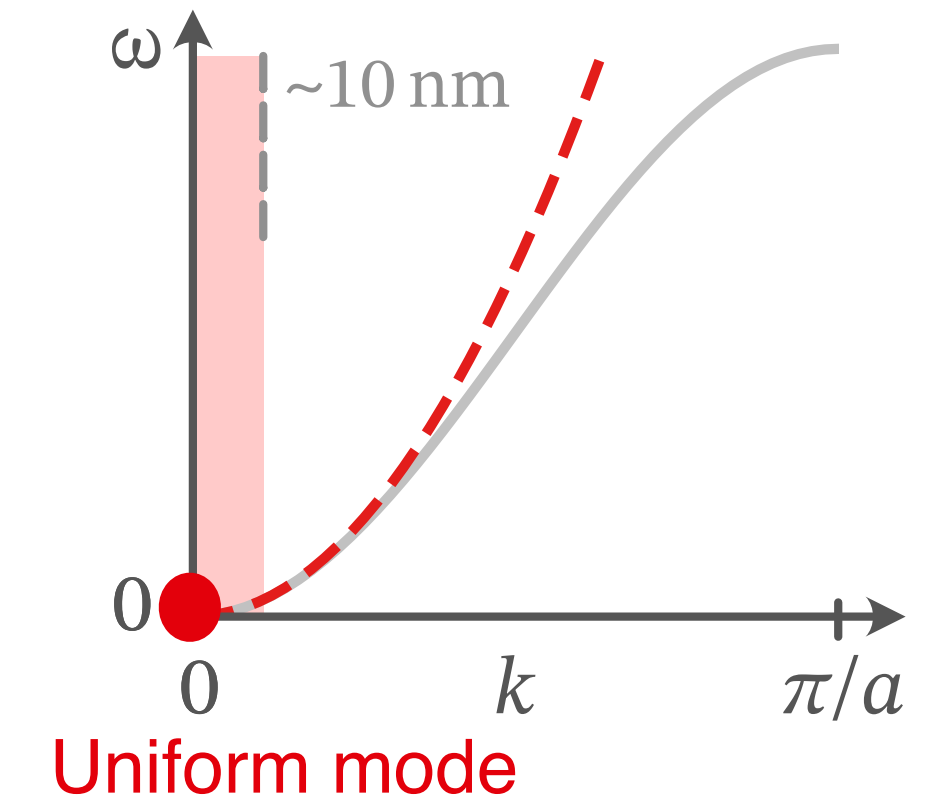
Macrospin

Understand uniform
mode, $k = 0$.

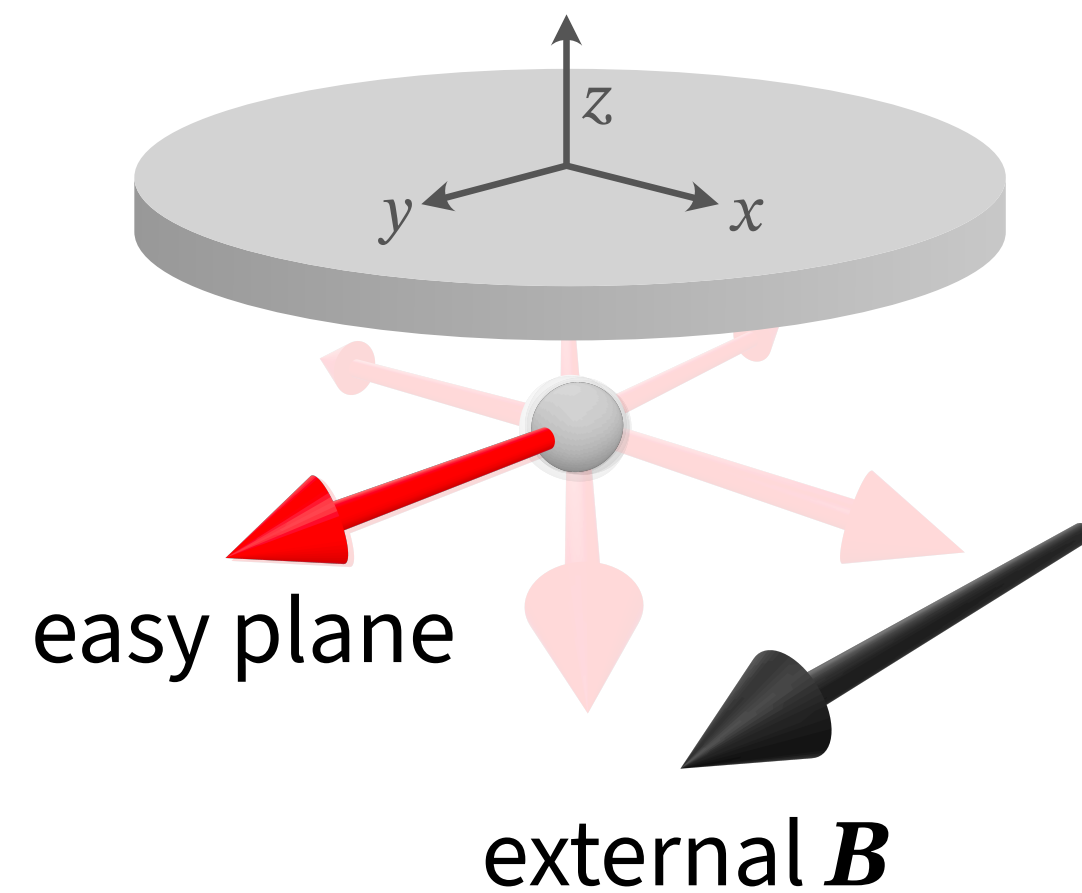
Gap?

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

Easy-plane shape anisotropy leads to elliptical precession



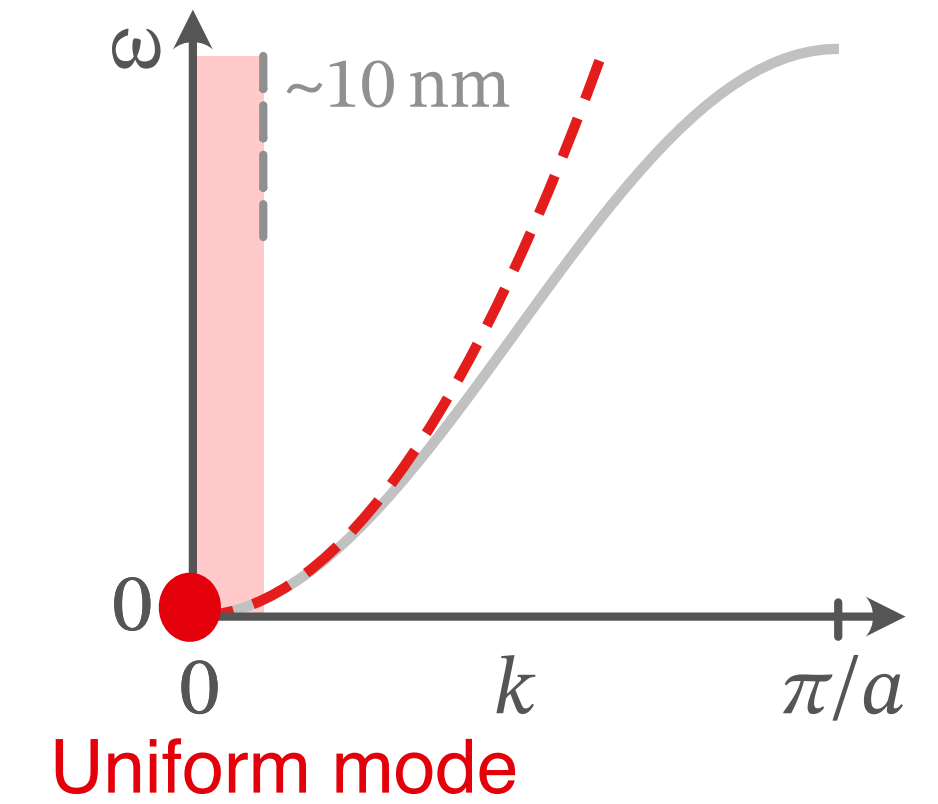
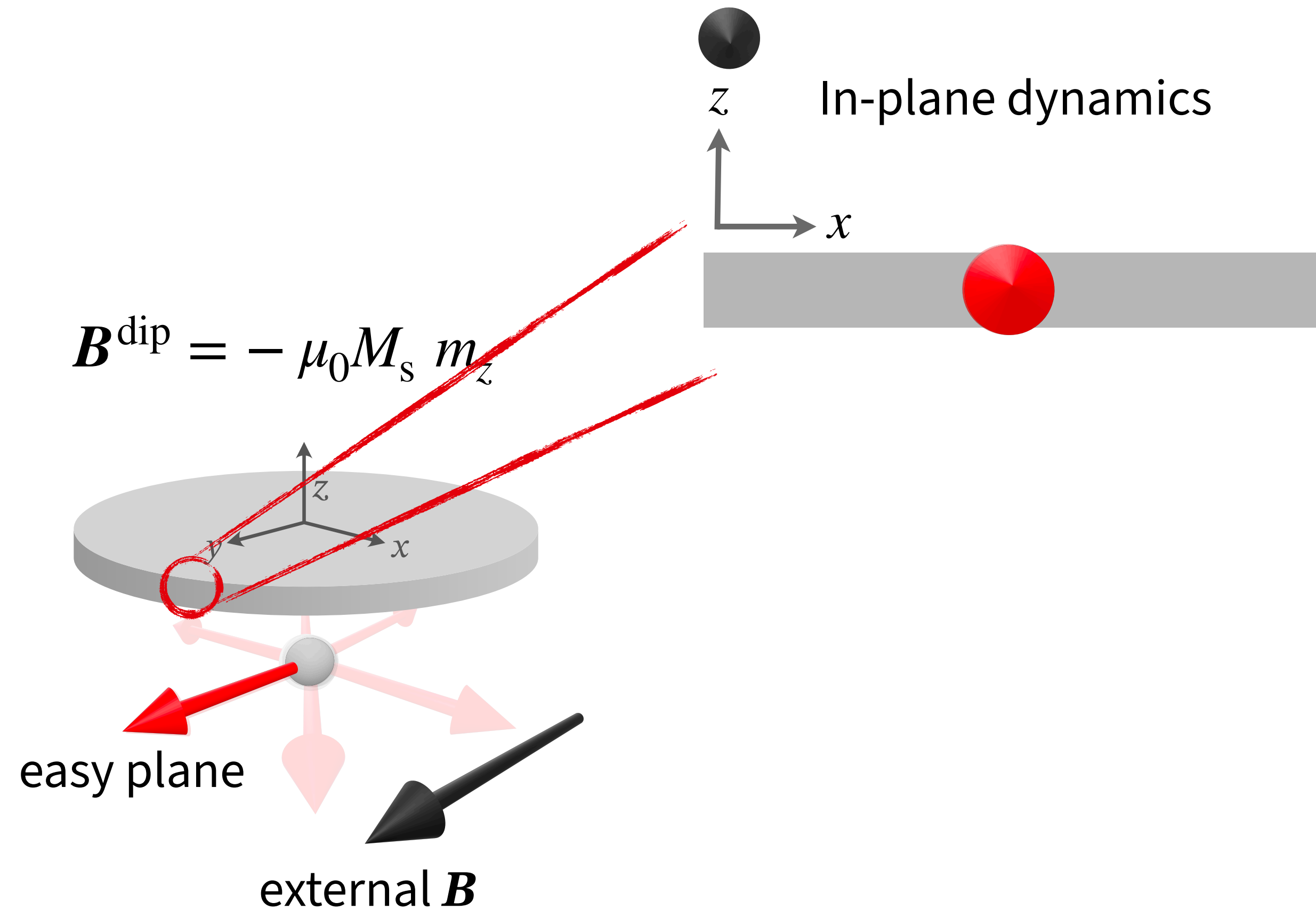
$$\mathbf{B}^{\text{dip}} = -\mu_0 M_s m_z$$



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_B = \gamma B$$

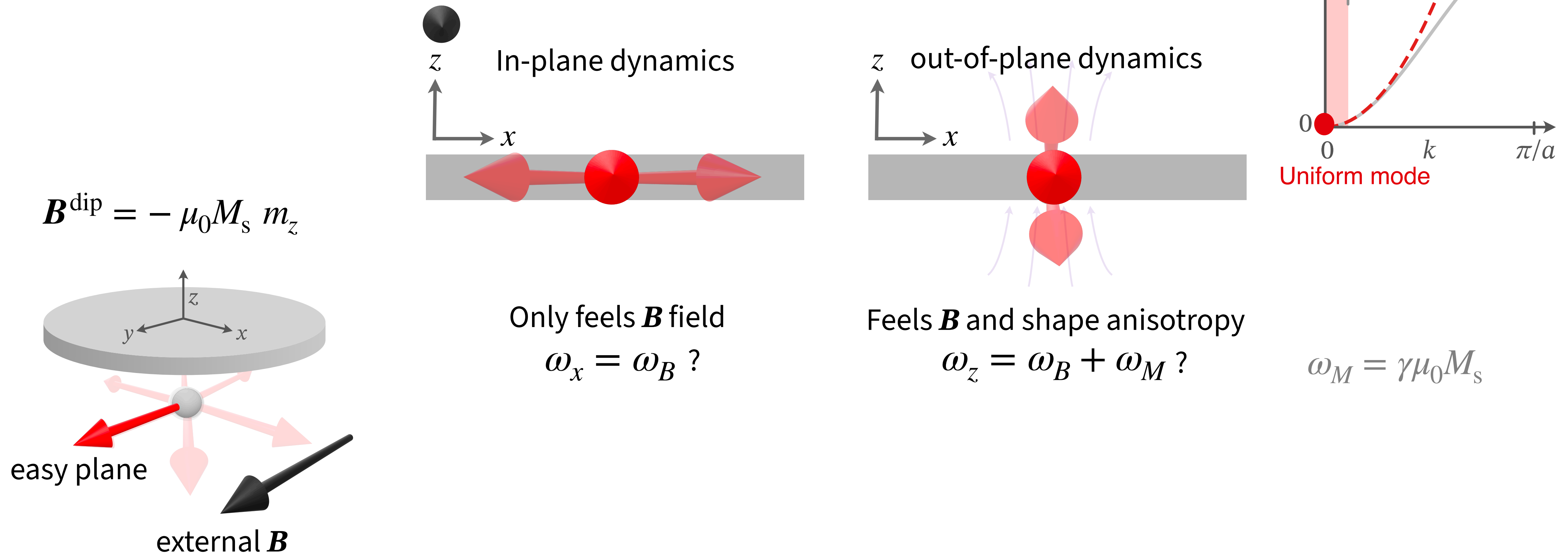
Easy-plane shape anisotropy leads to elliptical precession



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_B = \gamma B$$

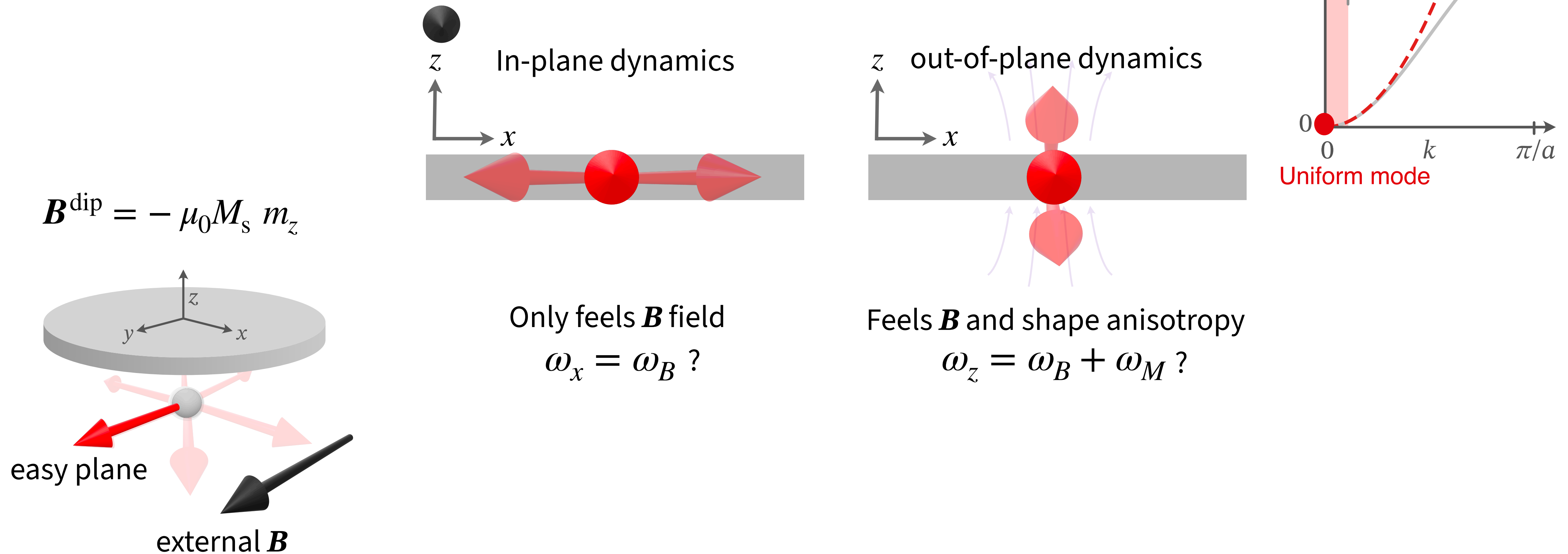
Easy-plane shape anisotropy leads to elliptical precession



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$\omega_B = \gamma B$

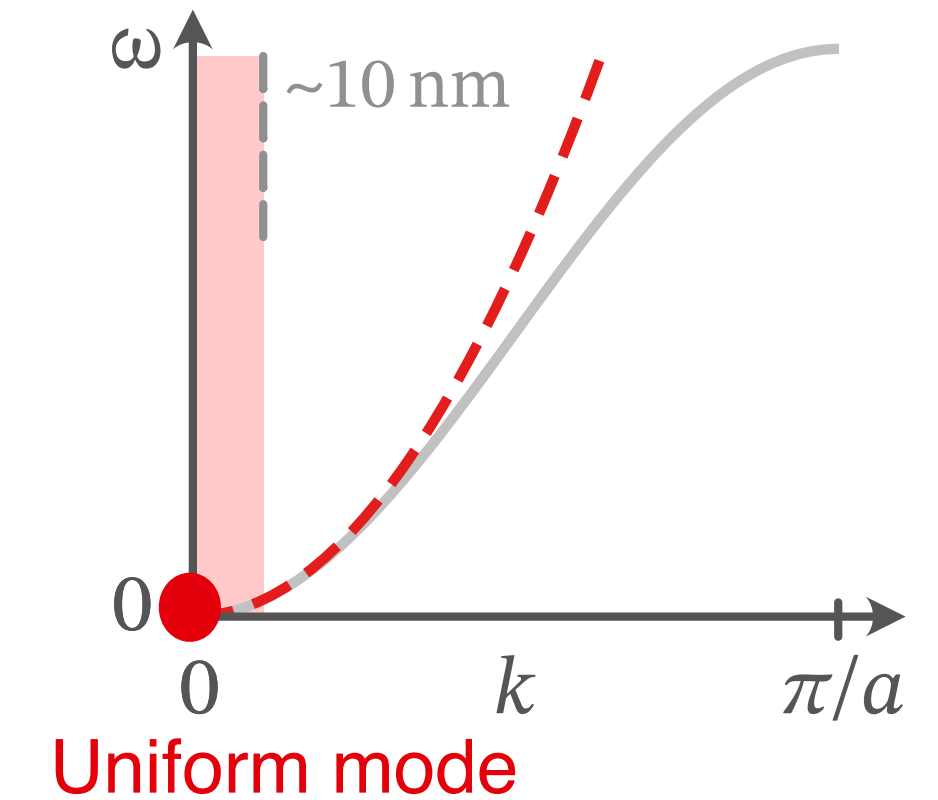
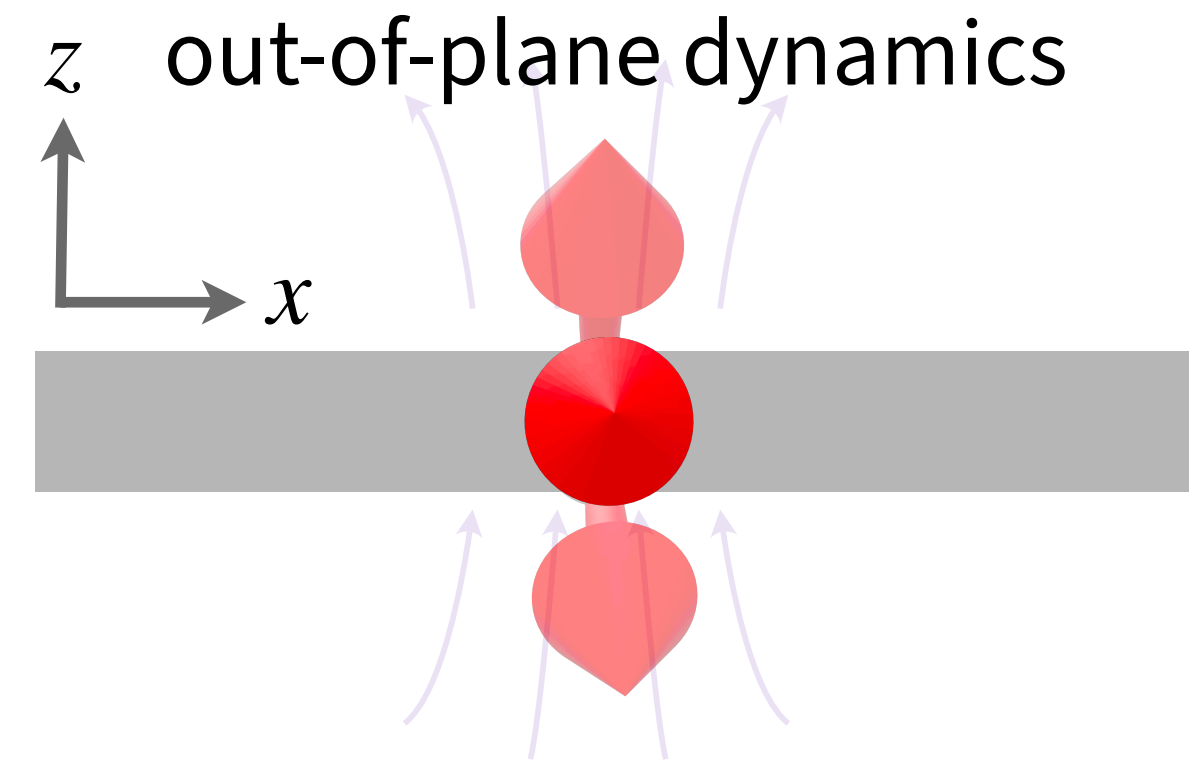
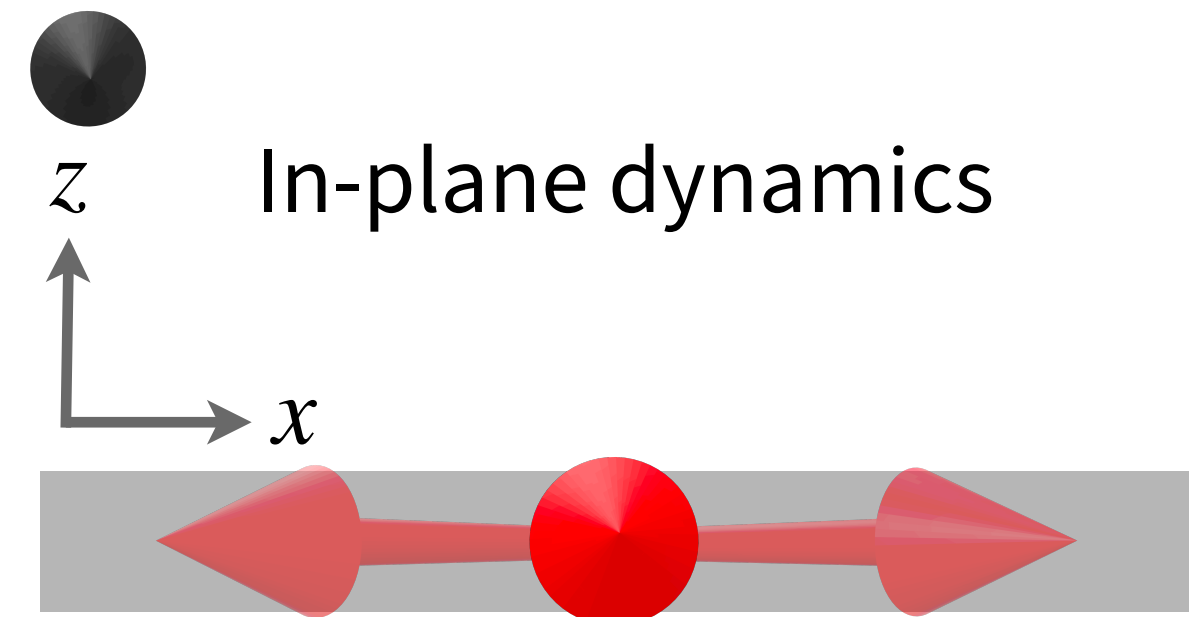
Easy-plane shape anisotropy leads to elliptical precession



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_B = \gamma B \qquad \omega_M = \gamma \mu_0 M_s$$

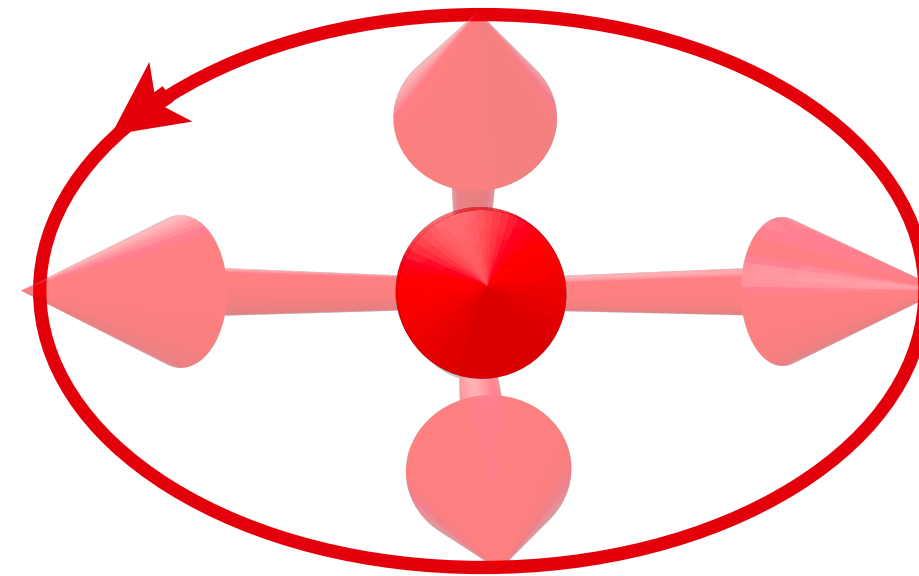
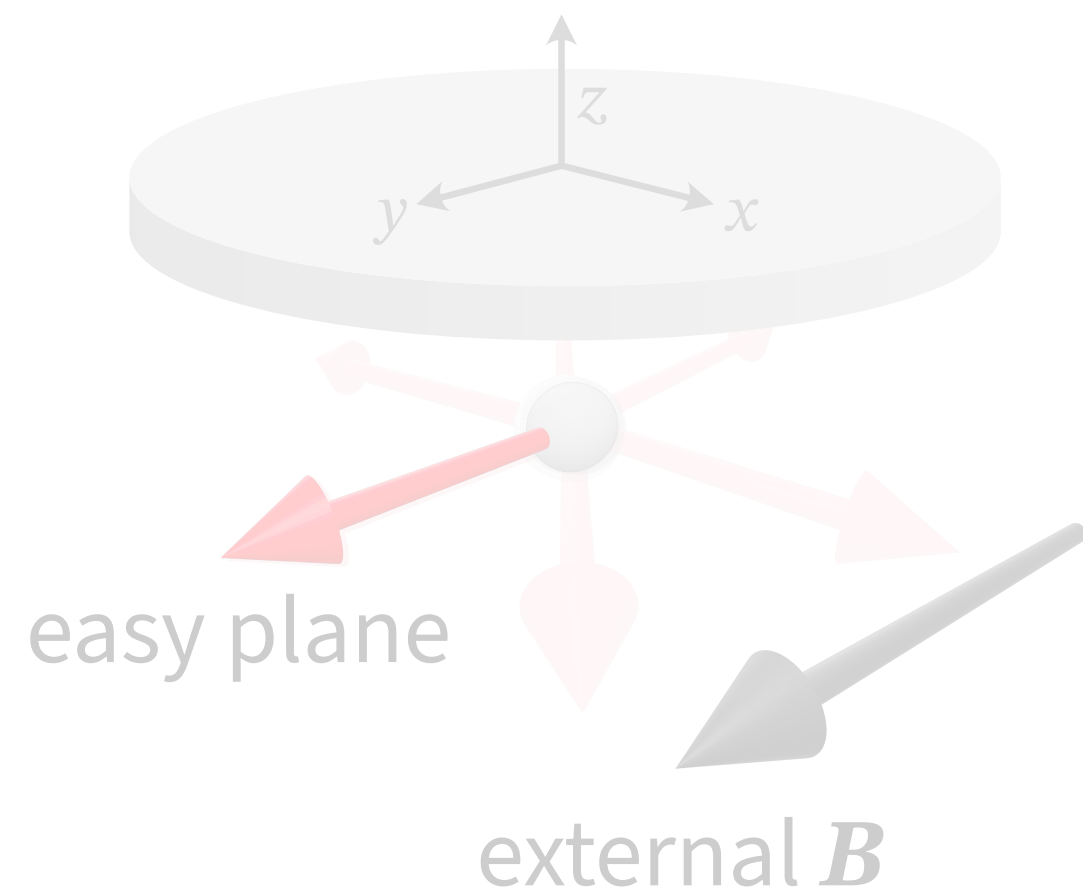
Easy-plane shape anisotropy leads to elliptical precession



$$\mathbf{B}^{\text{dip}} = -\mu_0 M_s \mathbf{m}_z$$

Only feels $\mathbf{B}$ field
 $\omega_x = \omega_B$?

Feels $\mathbf{B}$ and shape anisotropy
 $\omega_z = \omega_B + \omega_M$?



Elliptical precession orbit

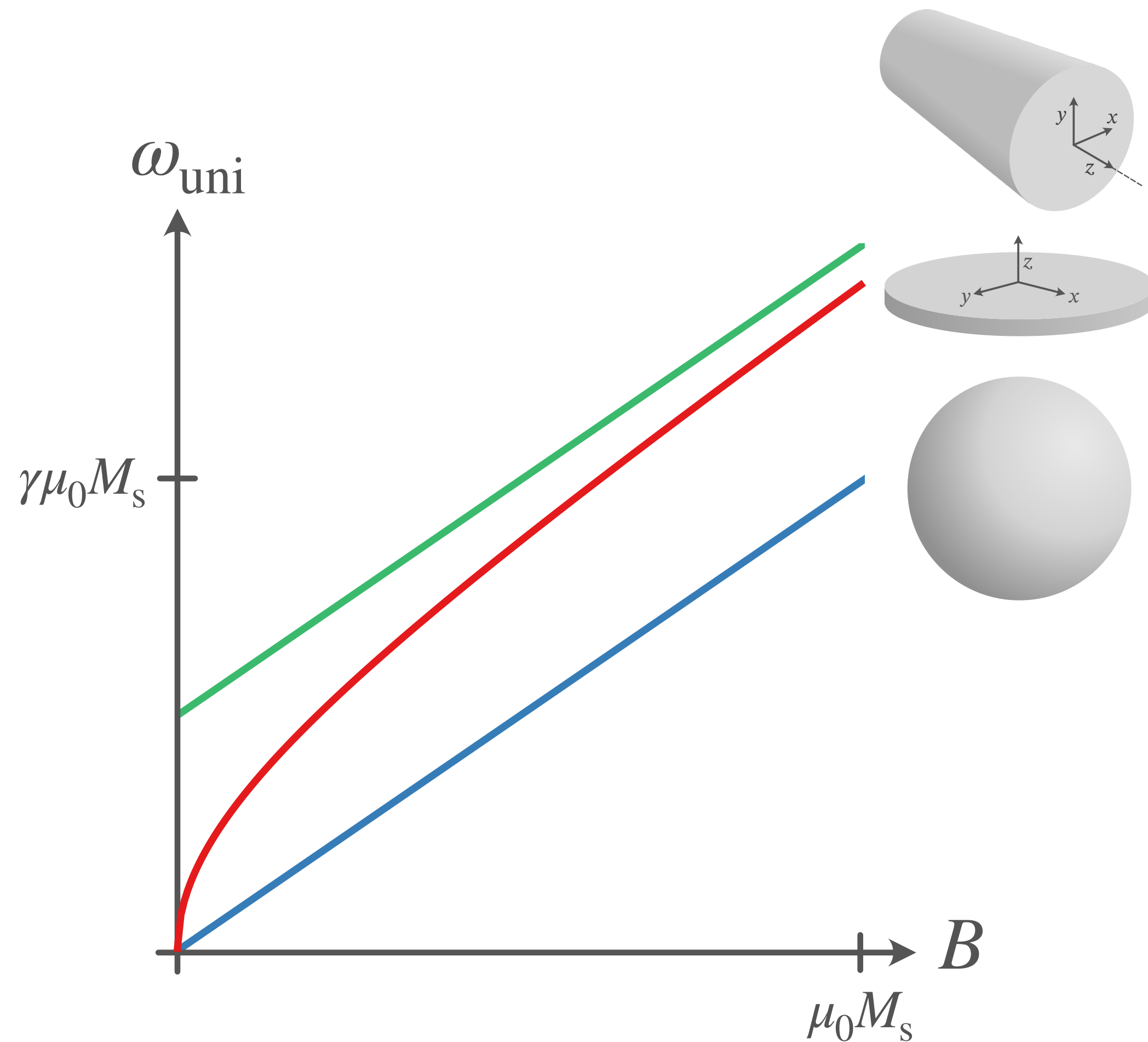
$$\omega = \sqrt{\omega_x \omega_z} \quad (\text{geometric mean})$$

$$= \sqrt{\omega_B [\omega_B + \omega_M]}$$

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

Avoiding surface charges leads to shape anisotropy



Charles Kittel

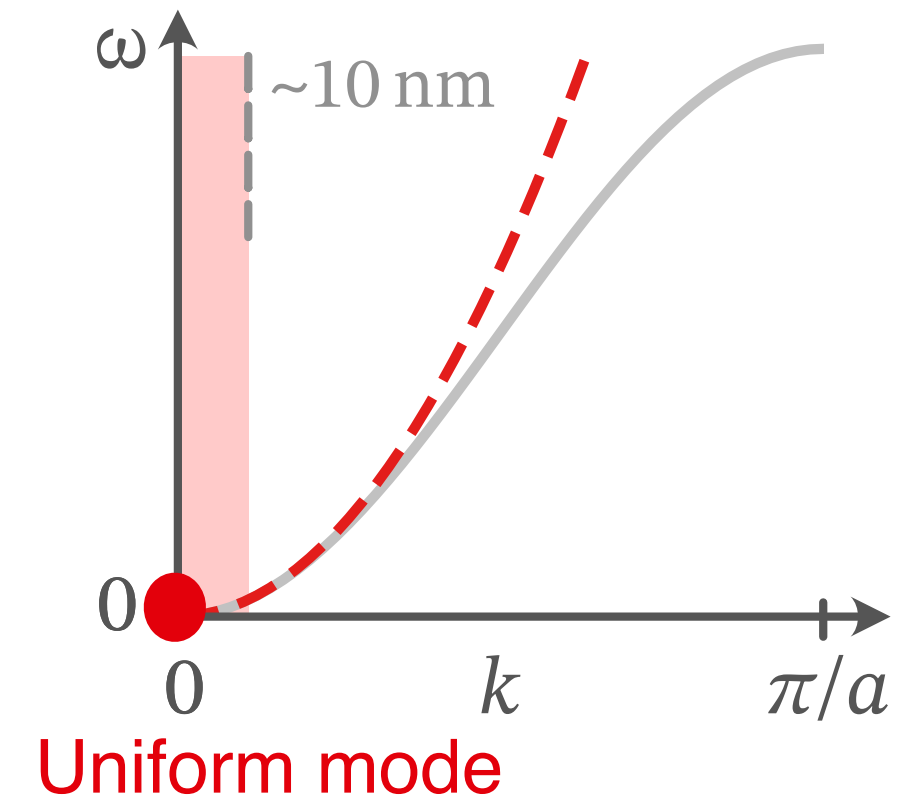
Kittel formula (for inf. layer)

$$\omega_{\text{uni}} = \sqrt{\omega_B[\omega_B + \omega_M]}$$

for general ellipsoid

$$= \sqrt{\left[\omega_B + \omega_M(N_x - N_z) \right] \left[\omega_B + \omega_M(N_y - N_z) \right]}$$

elements of $\hat{N} = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$

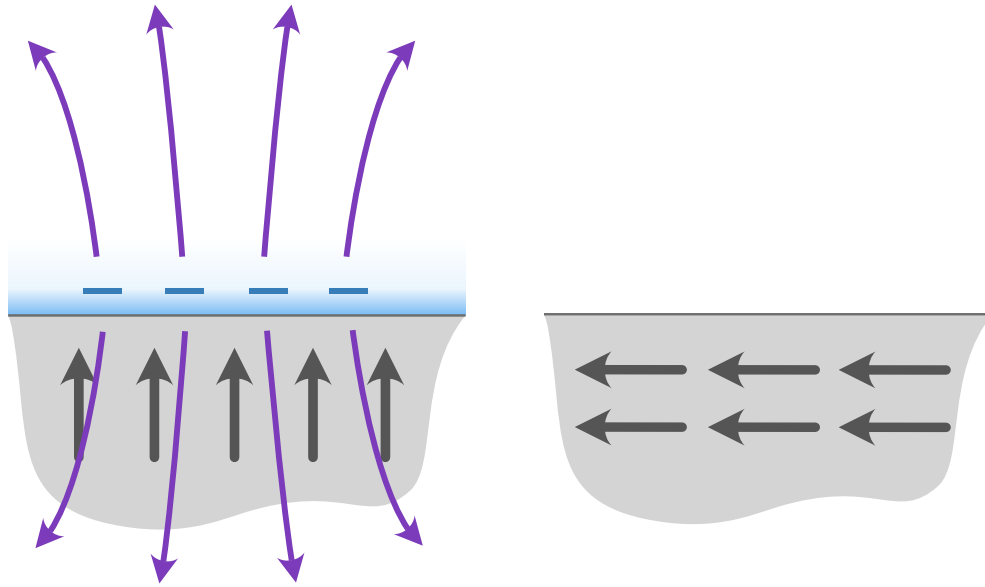
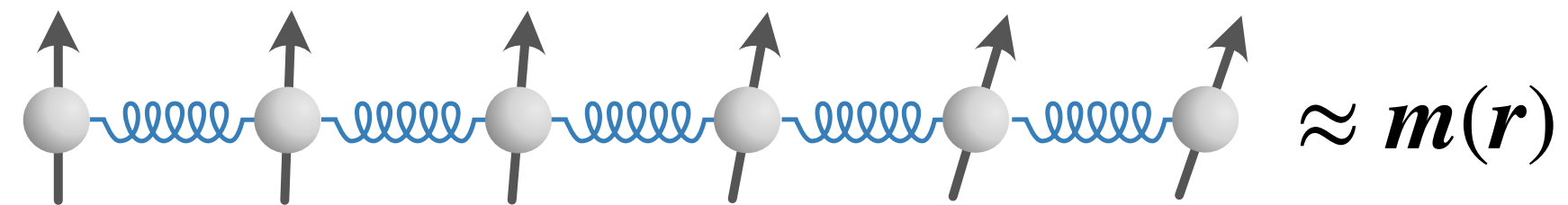


$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

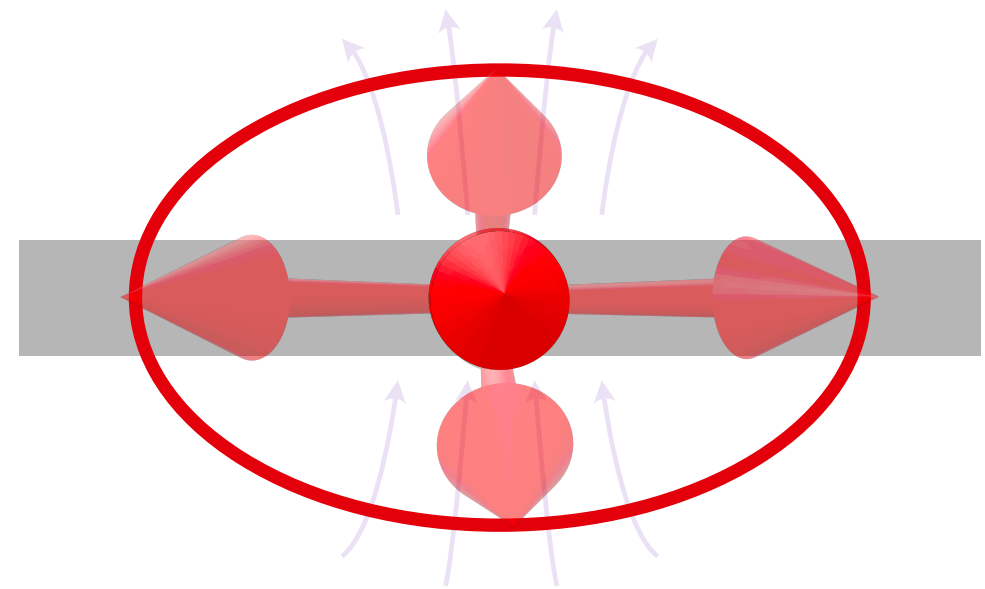
$\omega_B = \gamma B$
 $\omega_M = \gamma \mu_0 M_s$

Let's repeat what we already learned about dipole spin waves

Long-wavelength = continuum theory

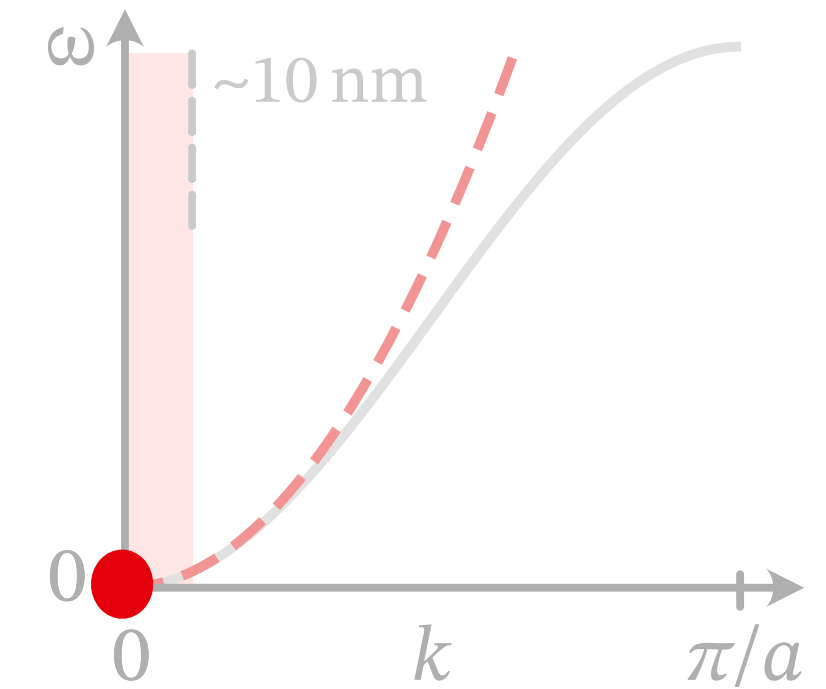


Dipolar interactions important,
can lead to shape anisotropy

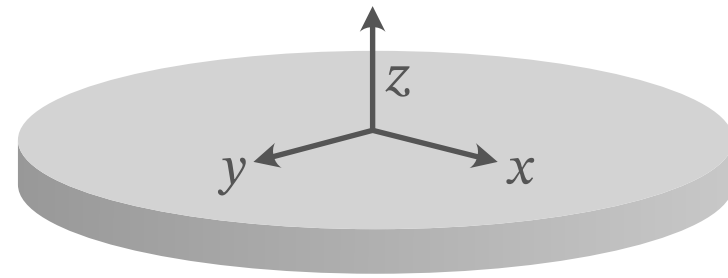


Can contribute to gap and
lead to elliptical precession

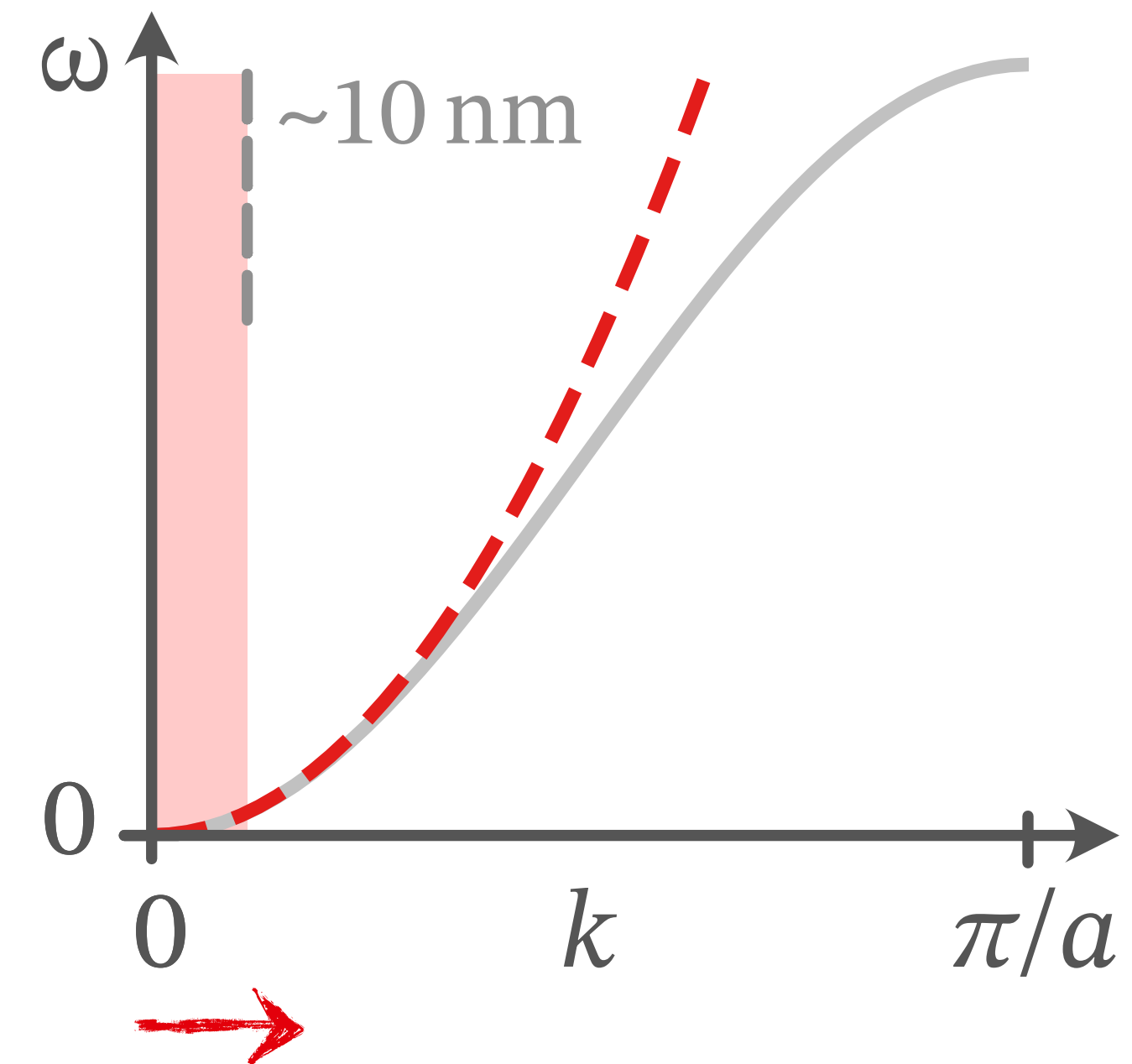
Long-wavelength limit



Let's come back to propagating modes



$$\omega_{\text{uni}} = \sqrt{\omega_B[\omega_B + \omega_M]}$$

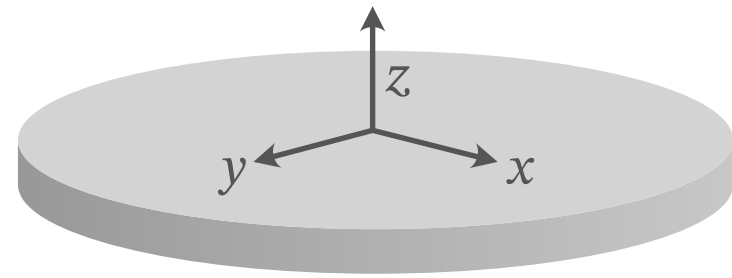


Back to propagating modes

$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

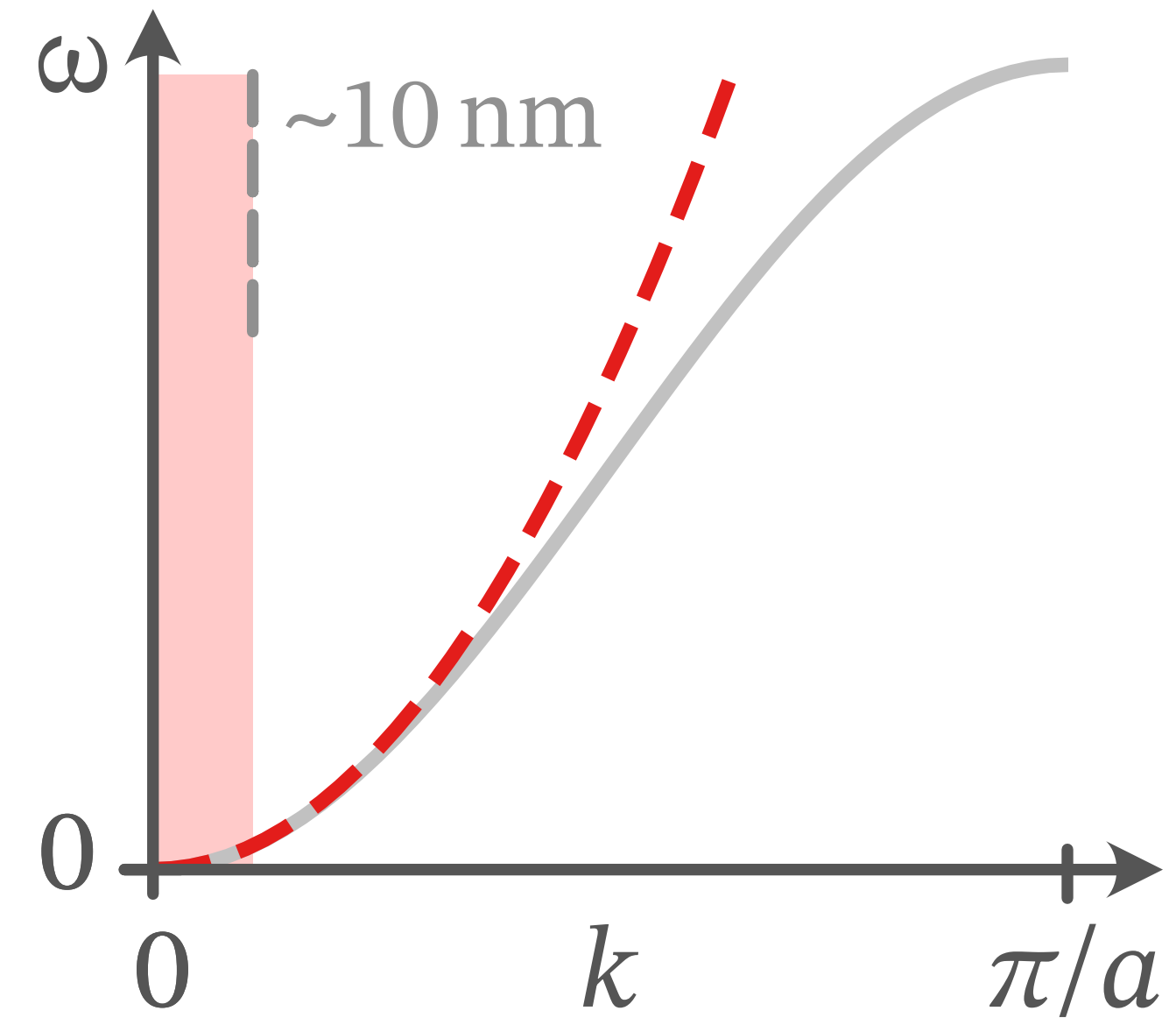
$$\omega_B = \gamma B \qquad \omega_M = \gamma \mu_0 M_s$$

Propagating modes require to consider exchange interaction again



$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M]}$$

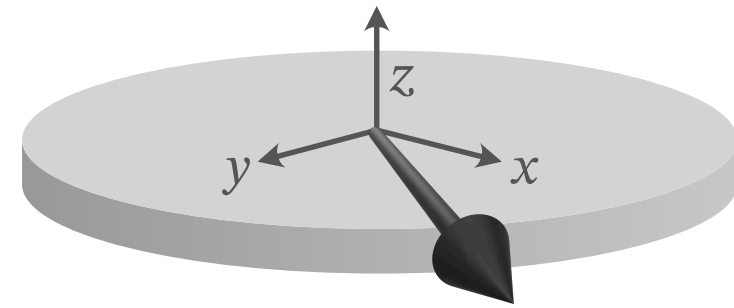
Need to include now



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_{\text{ex}} = (2\gamma A / M_s) k^2 \qquad \omega_B = \gamma B \qquad \omega_M = \gamma \mu_0 M_s$$

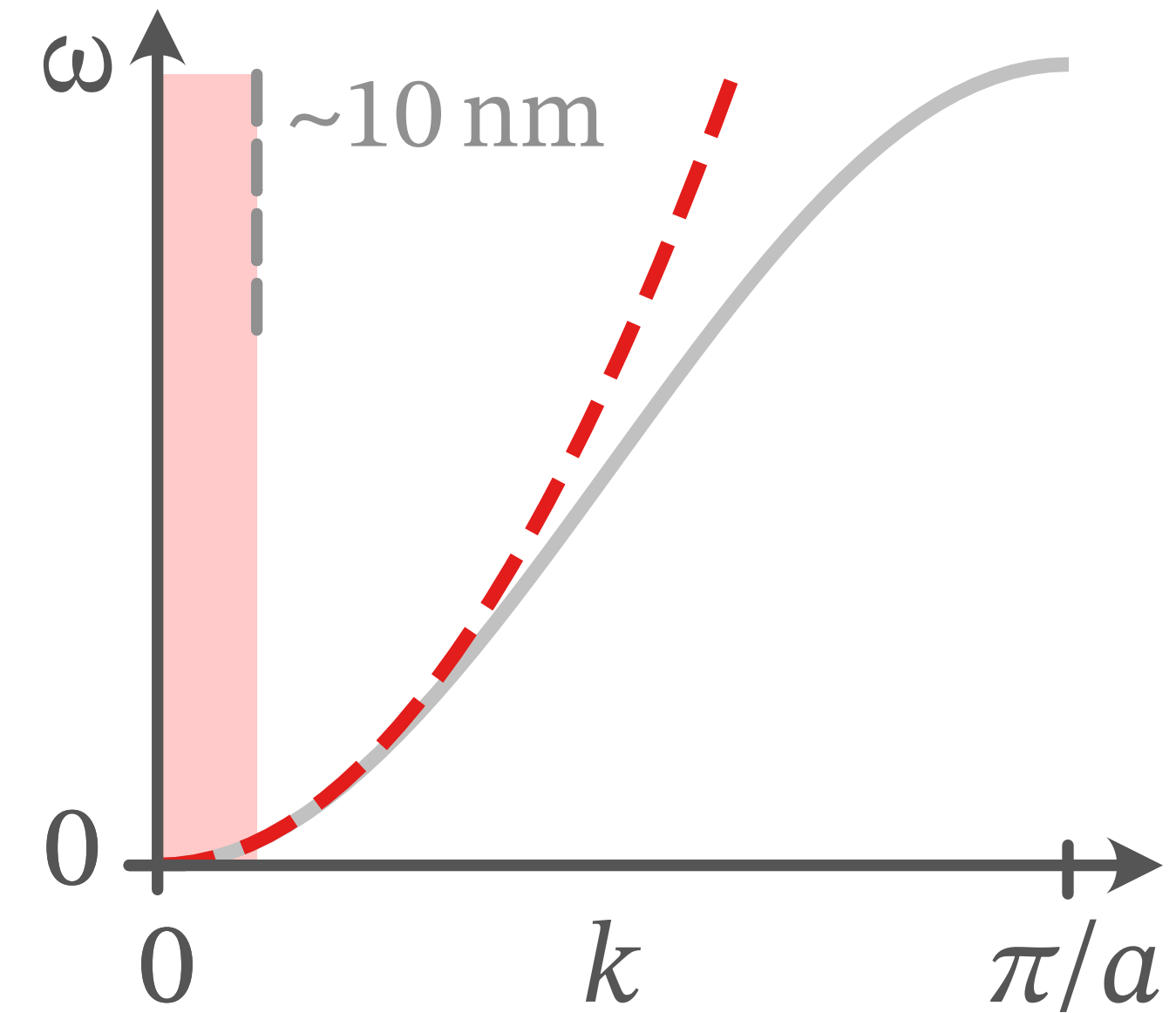
We consider only in-plane wave vectors for now



Only in-plane propagation

$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

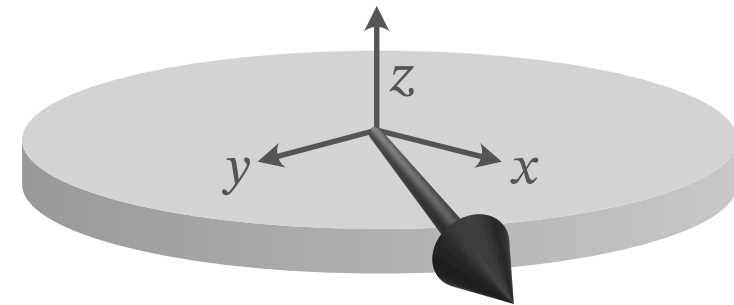
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M]}$$



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

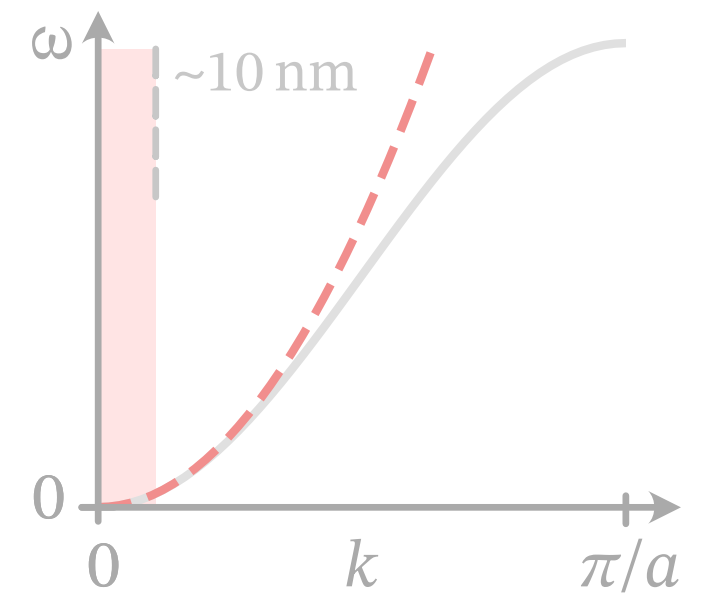
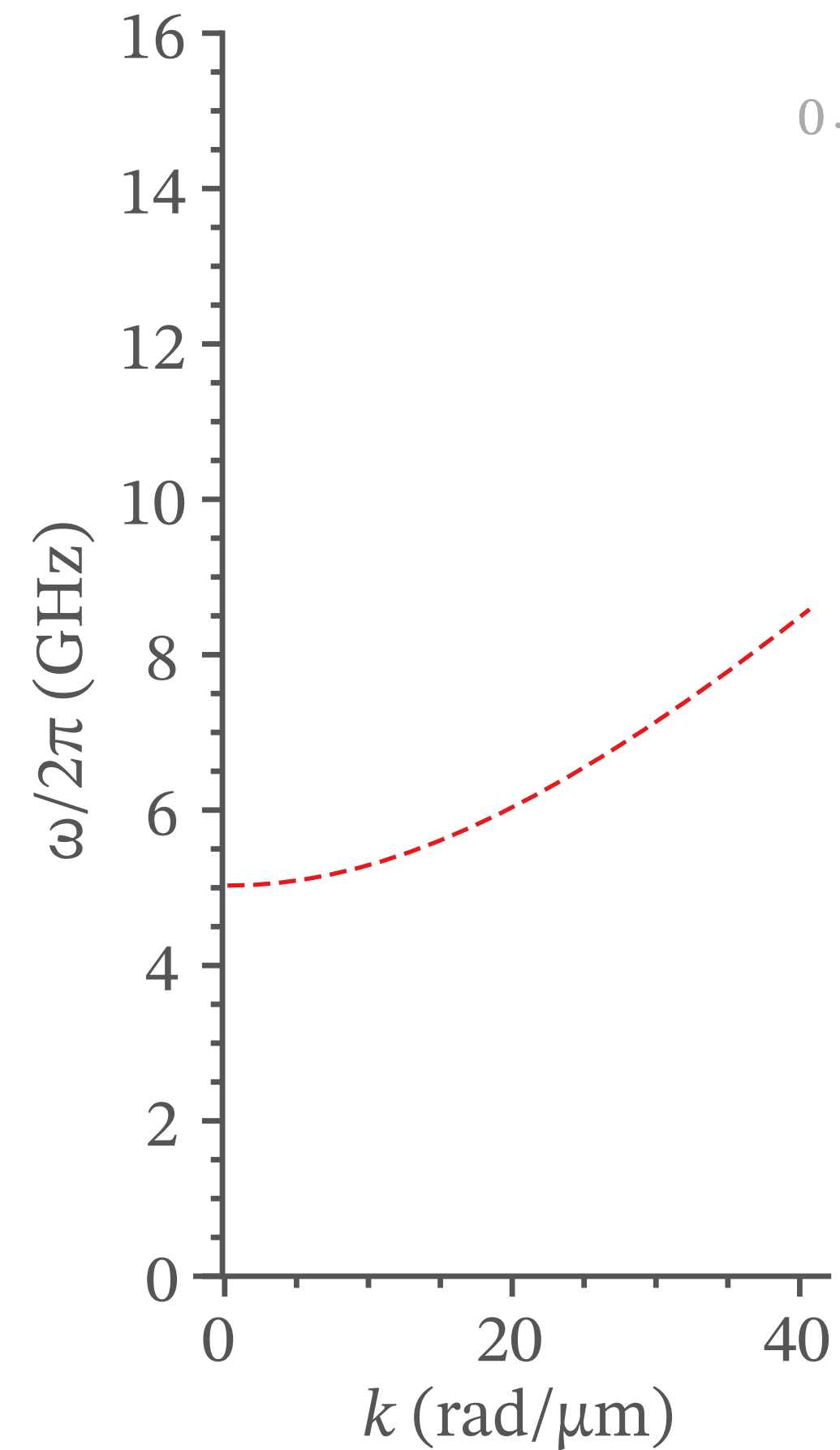
$$\omega_{\text{ex}} = (2\gamma A / M_s) k^2 \quad \omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

For thin enough films, consider dipolar fields only as anisotropy



$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

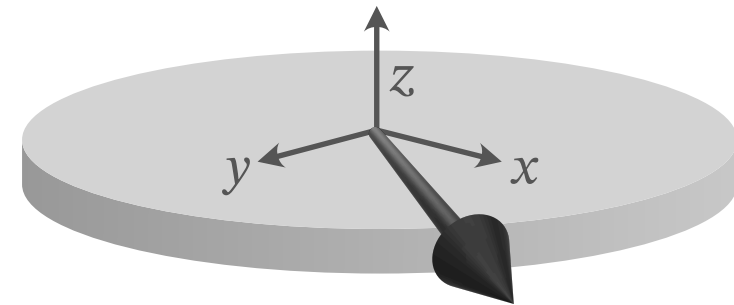
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M]}$$



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

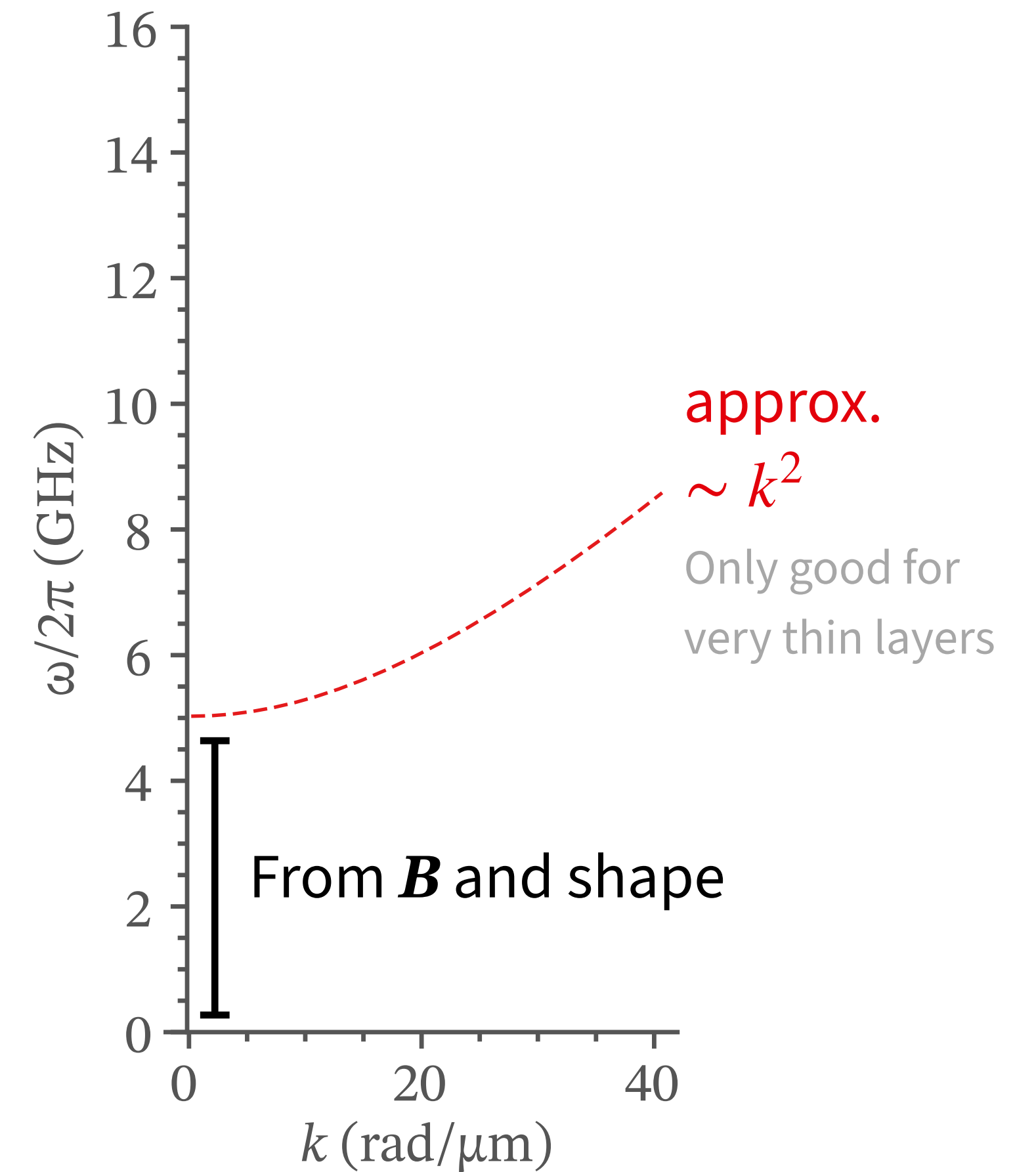
$$\omega_{\text{ex}} = (2\gamma A/M_s) k^2 \quad \omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

For thin enough films, consider dipolar fields only as anisotropy



$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

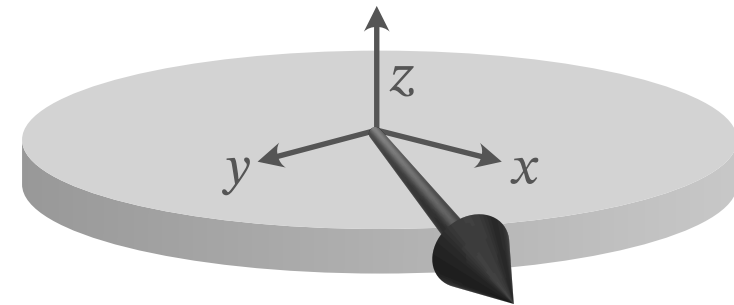
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M]}$$



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_{\text{ex}} = (2\gamma A/M_s) k^2 \quad \omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

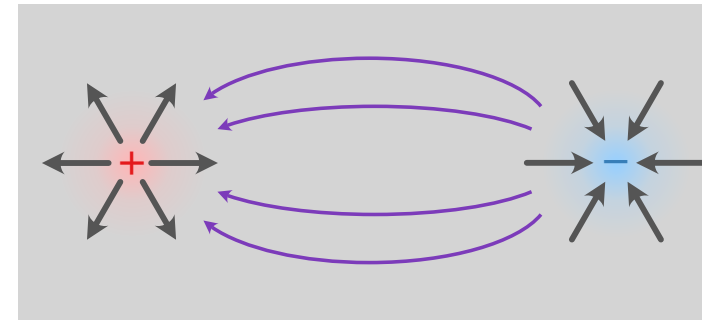
For thicker films we also need to consider the volume charges



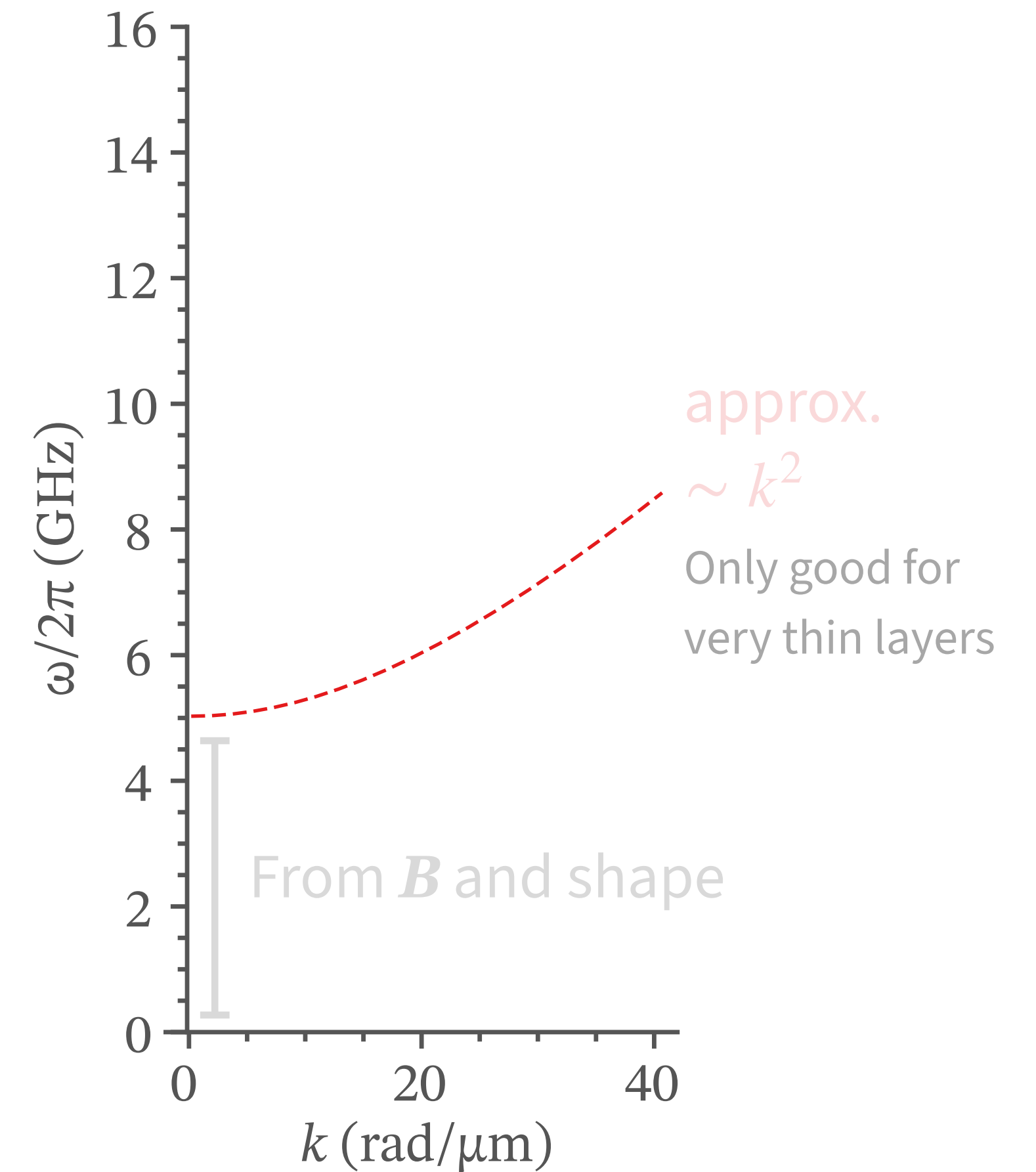
$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$

Also k -dependent!



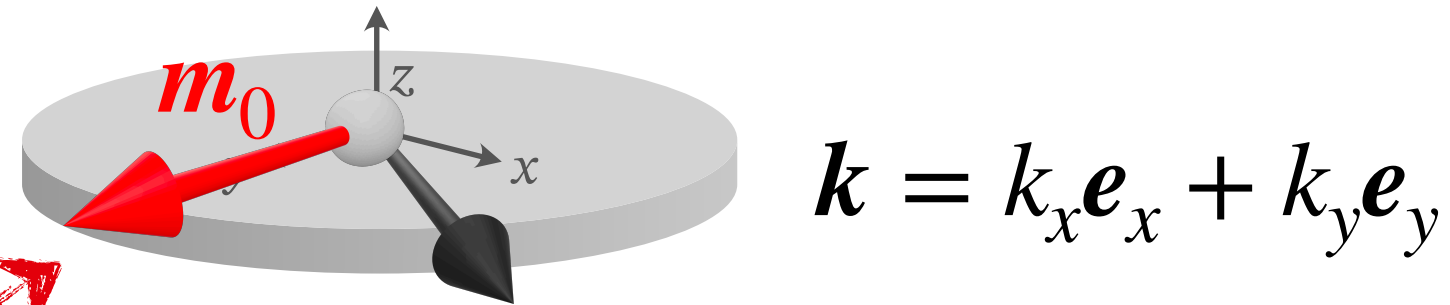
Waves produce charges!



$$E(m) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

$$\omega_{\text{ex}} = (2\gamma A/M_s) k^2 \quad \omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

Dipolar charges lead to dispersion anisotropy

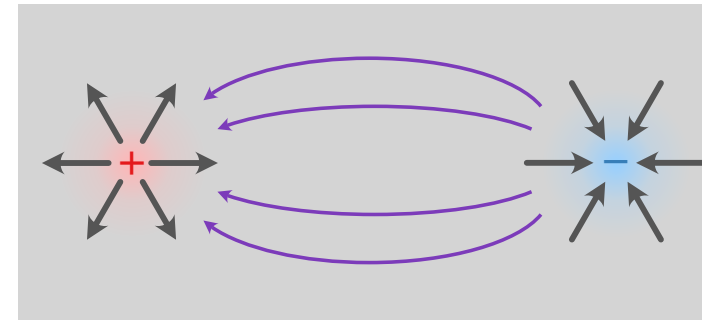


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

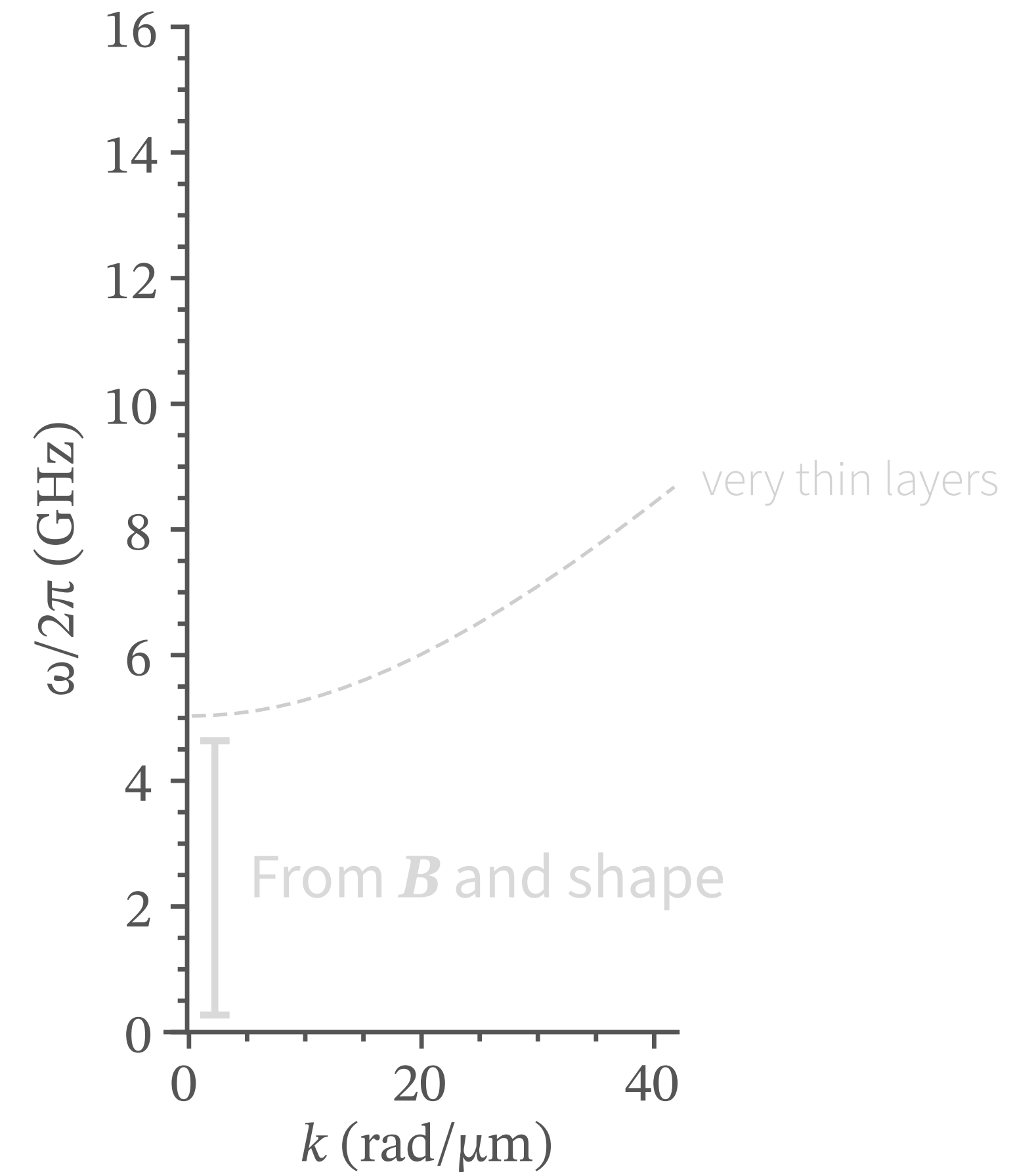
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$

Will also depend on $\angle(\mathbf{m}_0, \mathbf{k})$

Also k -dependent!



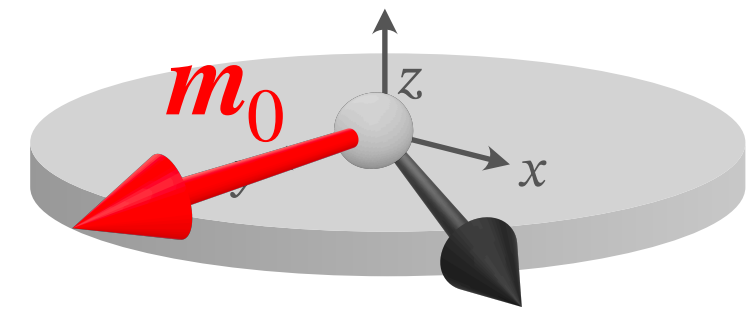
Waves produce charges!



$$E(\mathbf{m}) = \int dV \left[A(\nabla \mathbf{m})^2 + \mathbf{m} \cdot \mathbf{K} \cdot \mathbf{m} - M_s \mathbf{B} \cdot \mathbf{m} - M_s \mathbf{B}^{\text{dip}}(\mathbf{m}) \cdot \mathbf{m} \right]$$

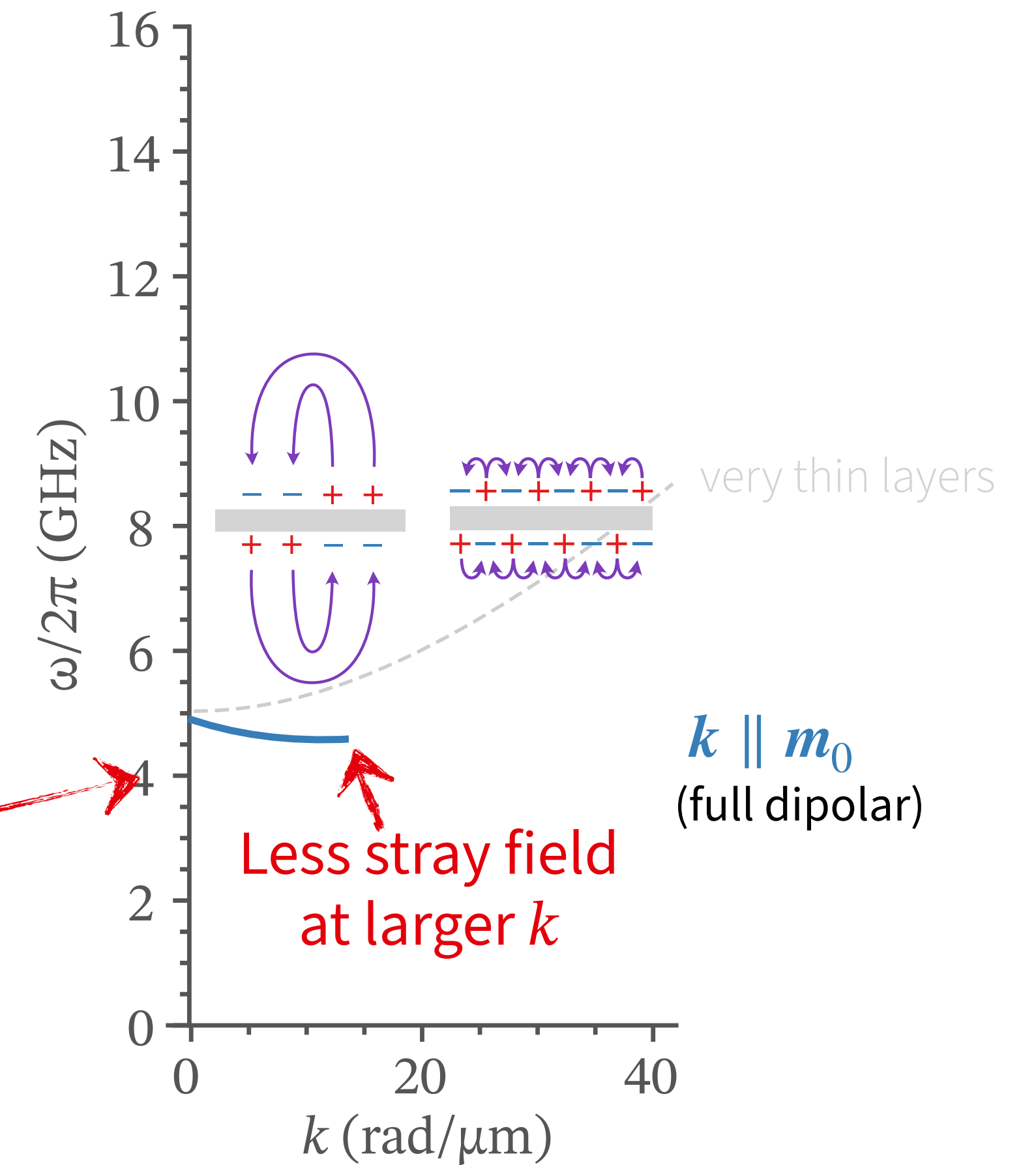
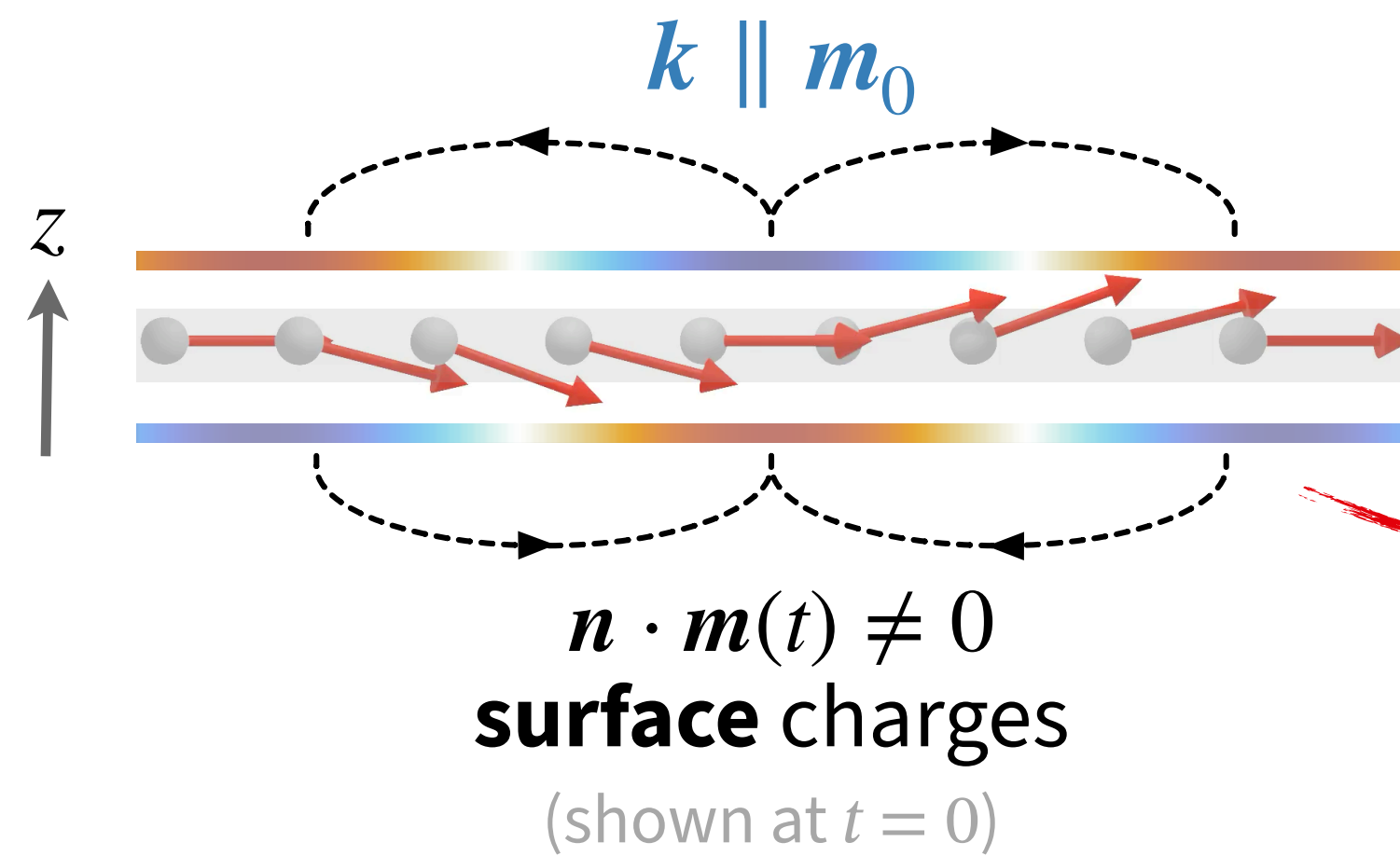
$$\omega_{\text{ex}} = (2\gamma A/M_s) k^2 \quad \omega_B = \gamma B \quad \omega_M = \gamma \mu_0 M_s$$

Dipolar charges lead to dispersion anisotropy

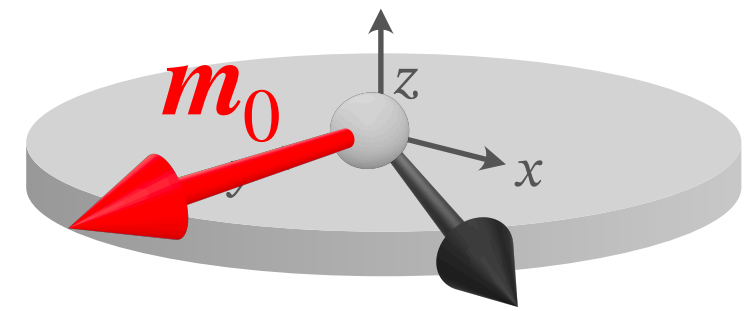


$$k = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$

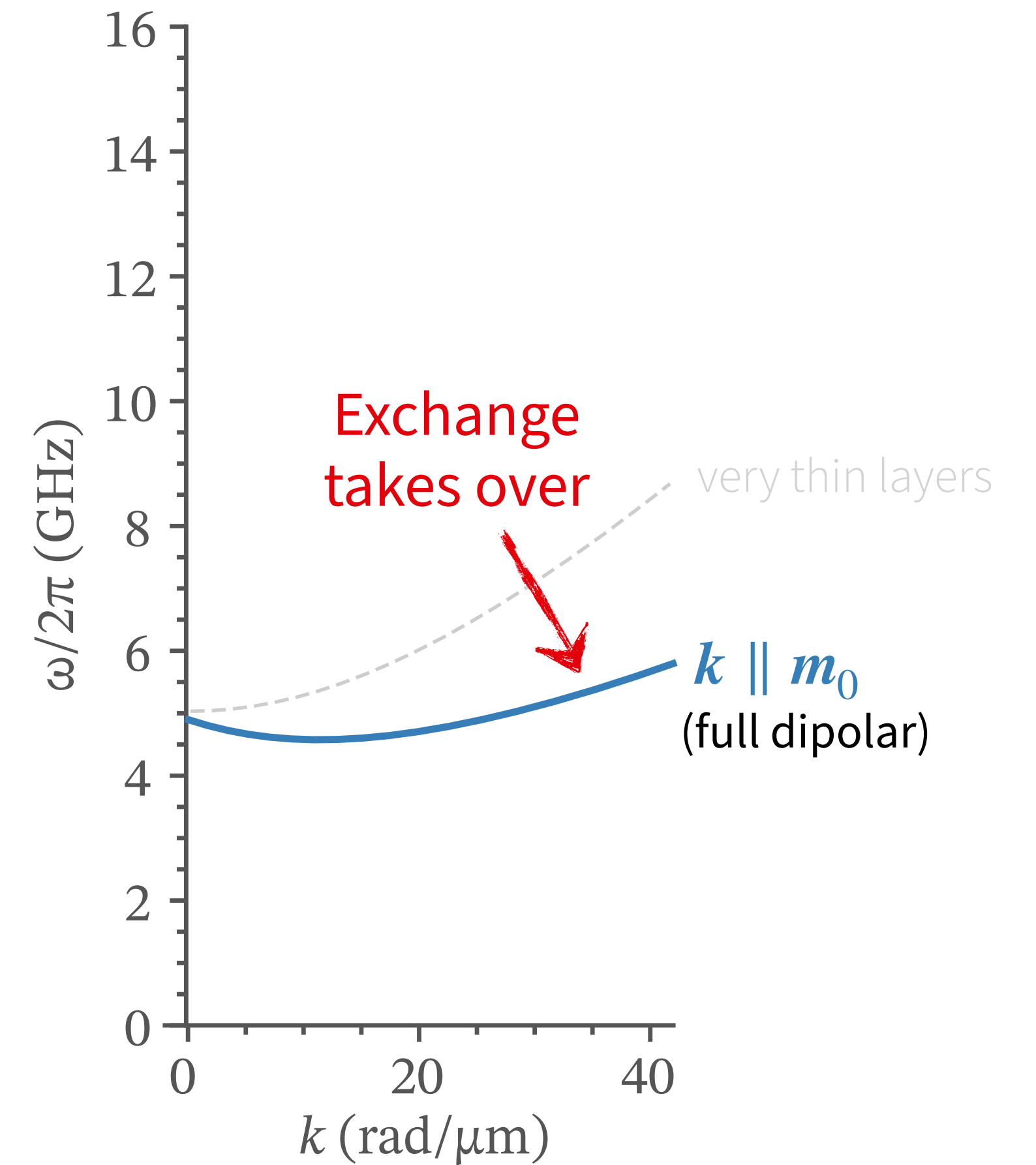
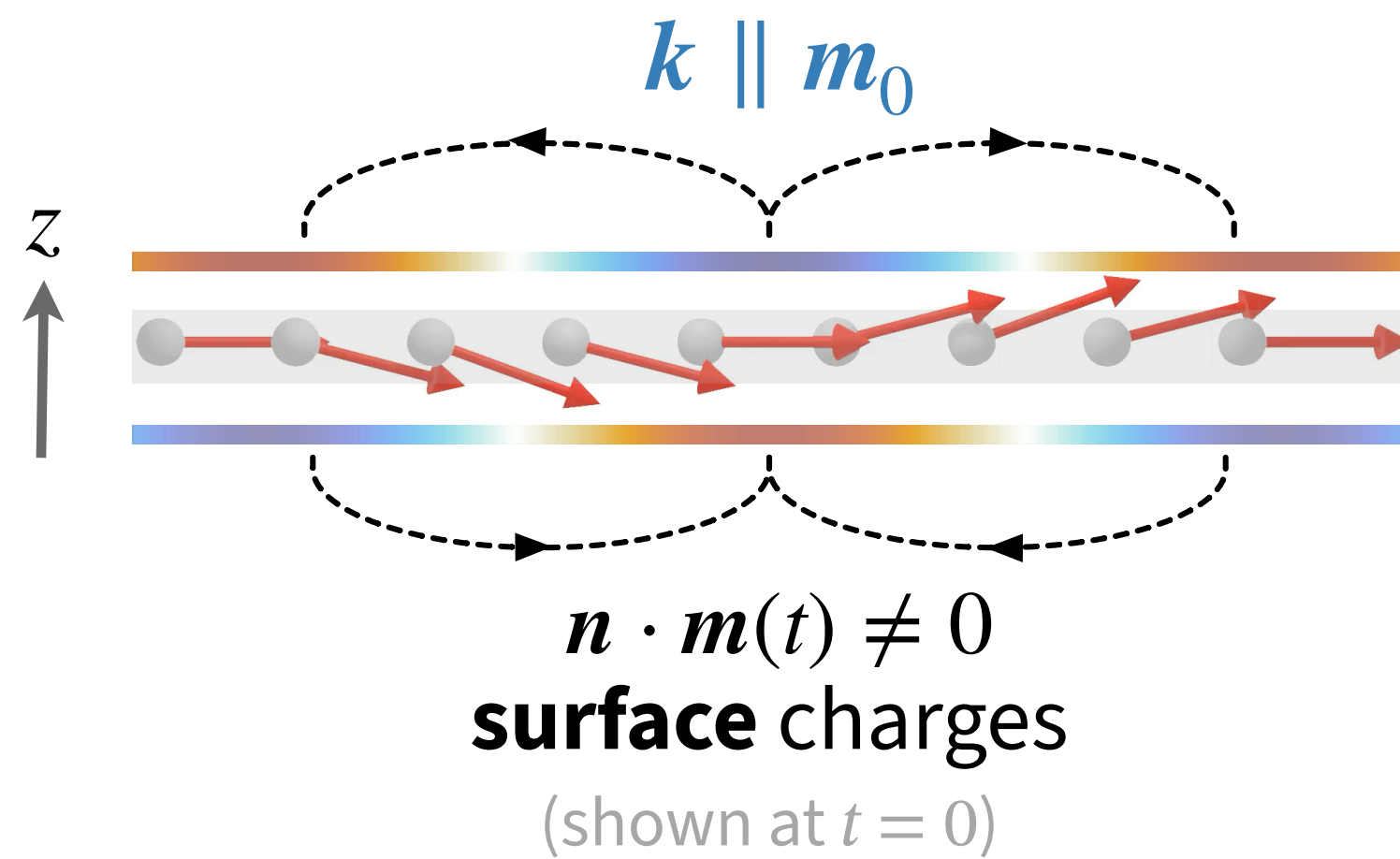


Dipolar charges lead to dispersion anisotropy

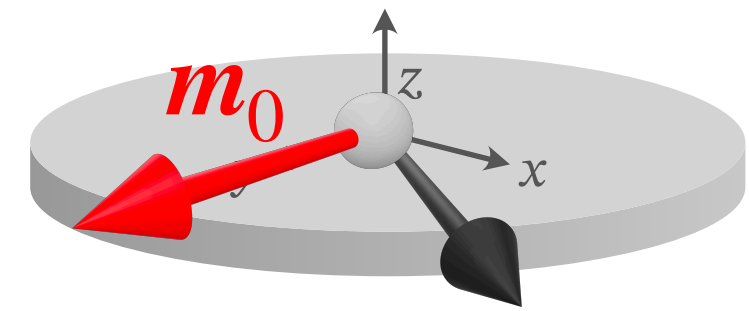


$$k = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$

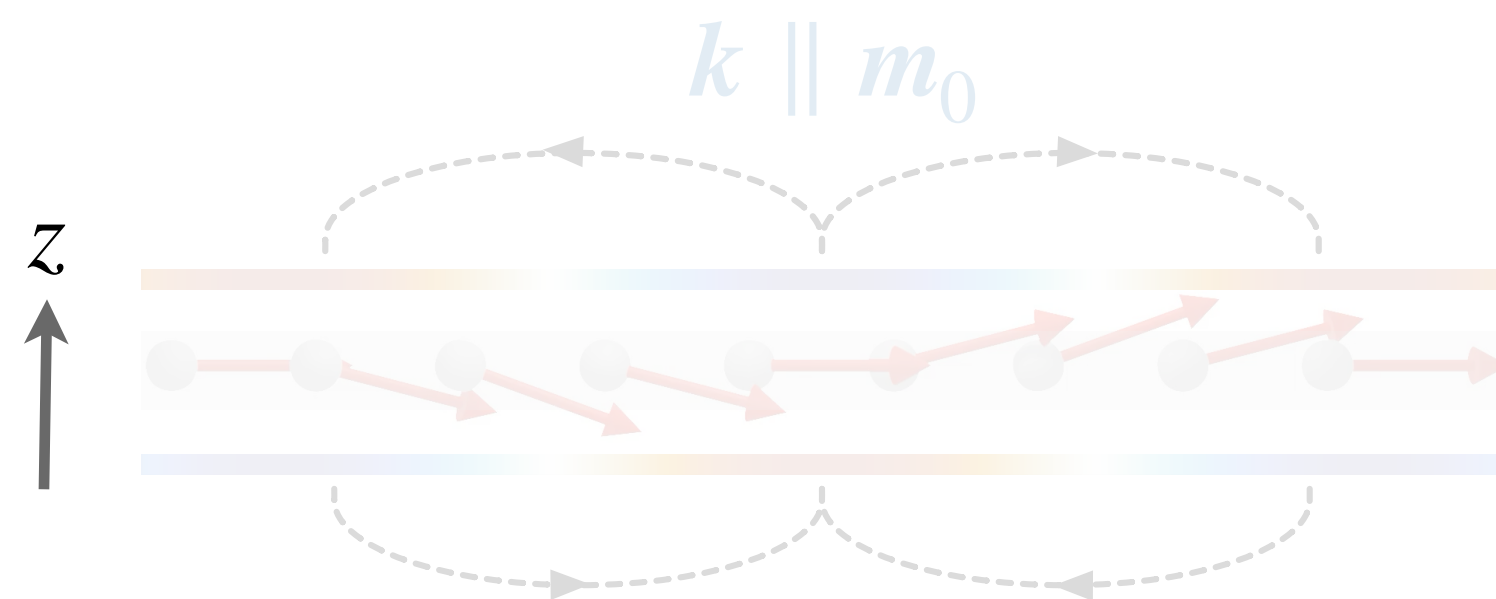


Dipolar charges lead to dispersion anisotropy

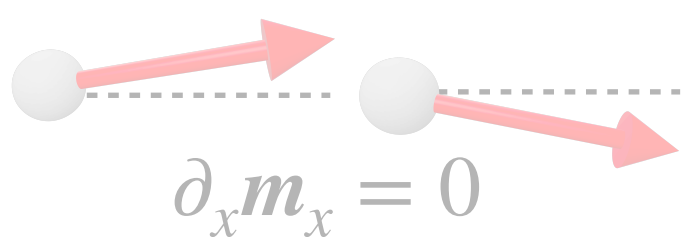


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

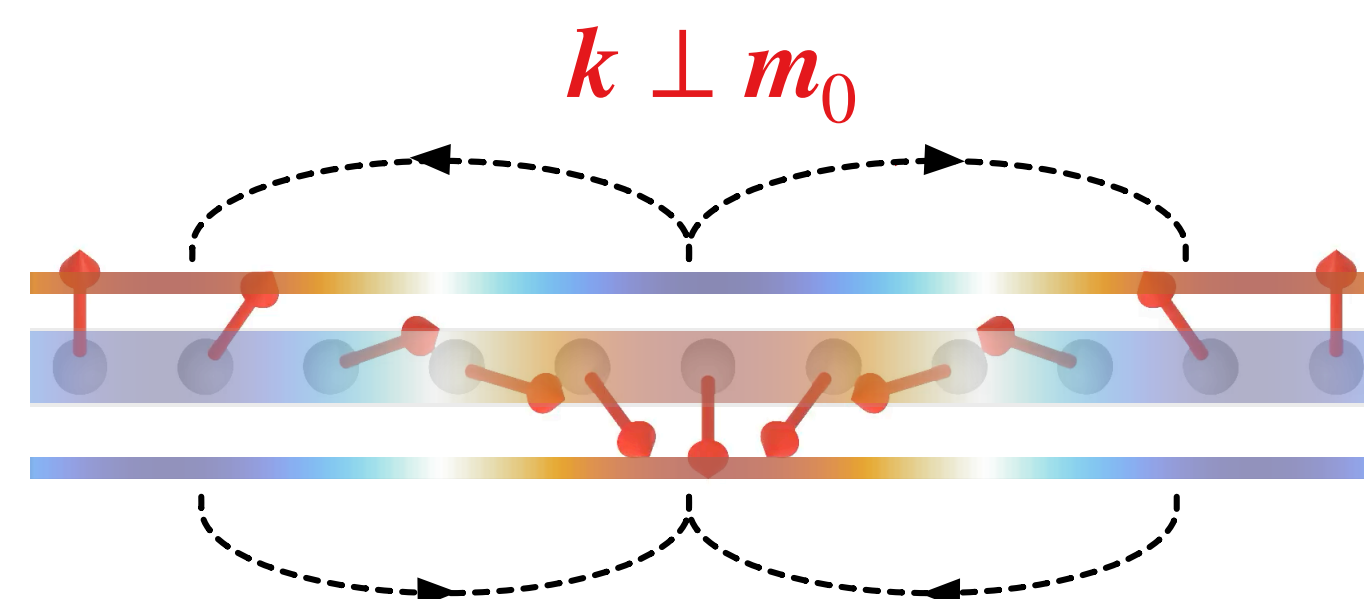
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$



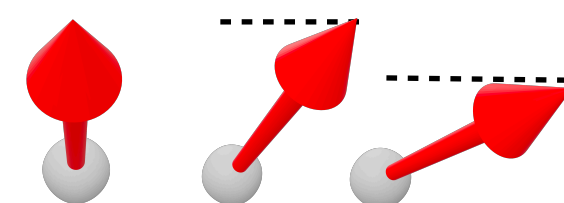
$\mathbf{n} \cdot \mathbf{m}(t) \neq 0$
surface charges
 (shown at $t = 0$)



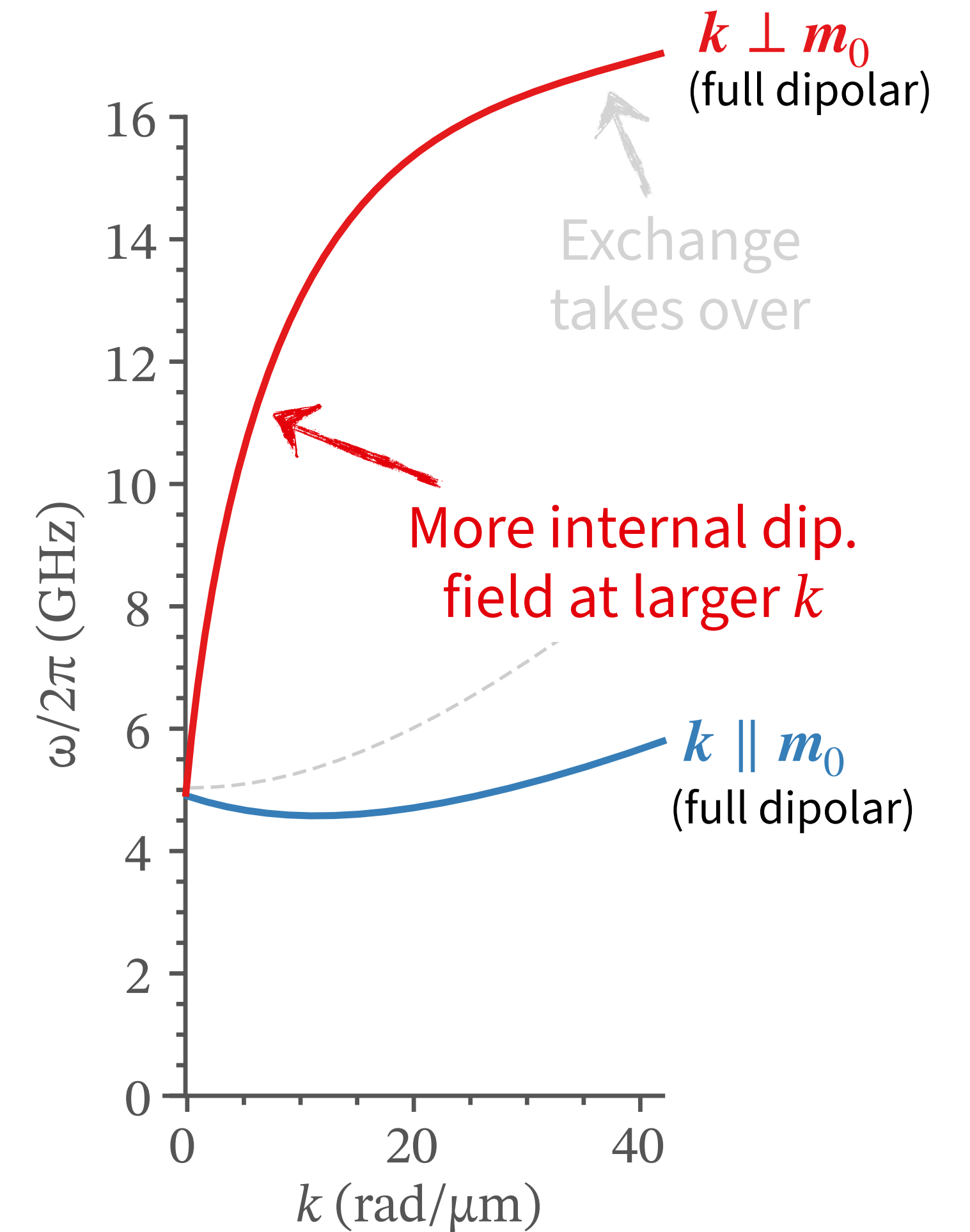
no volume charges



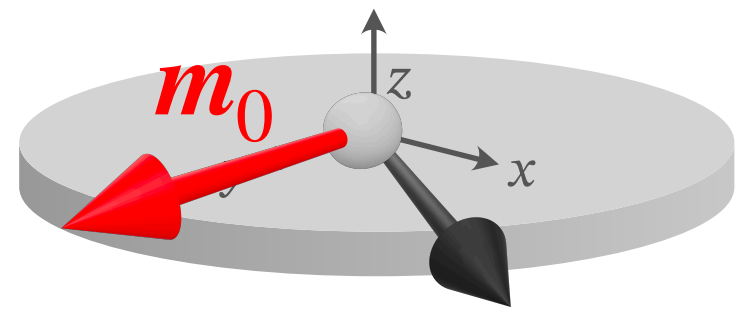
surface charges
 (shown at $t = 0$)



+ volume charges!

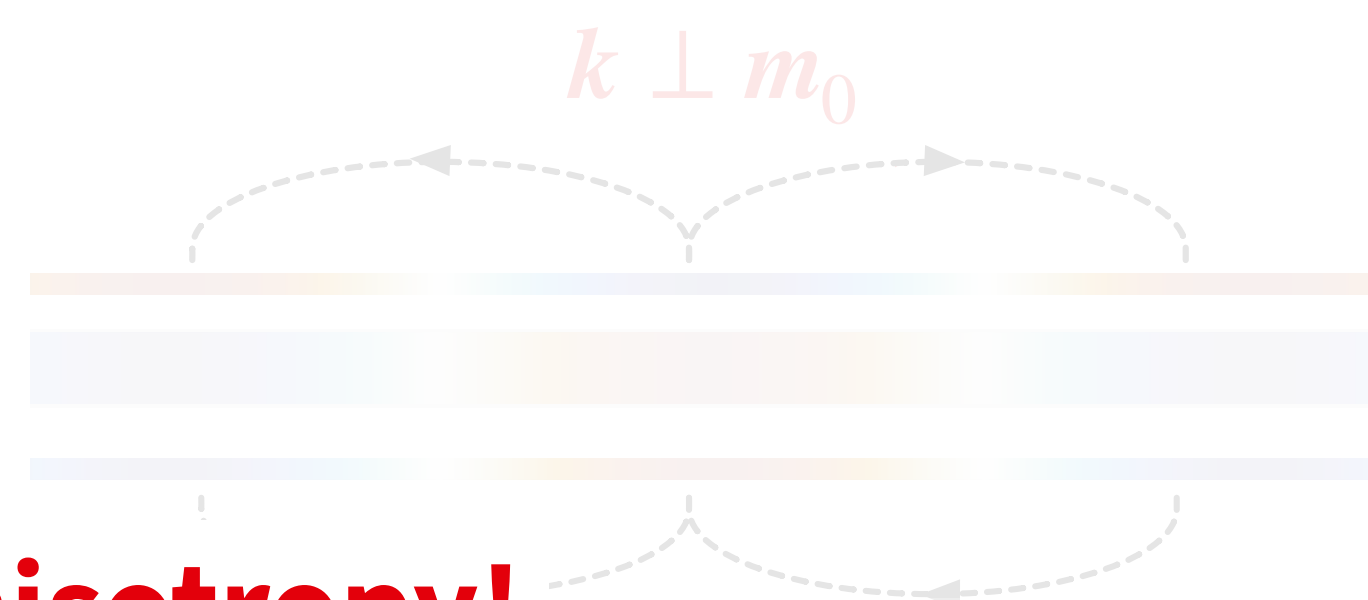


Dipolar charges lead to dispersion anisotropy

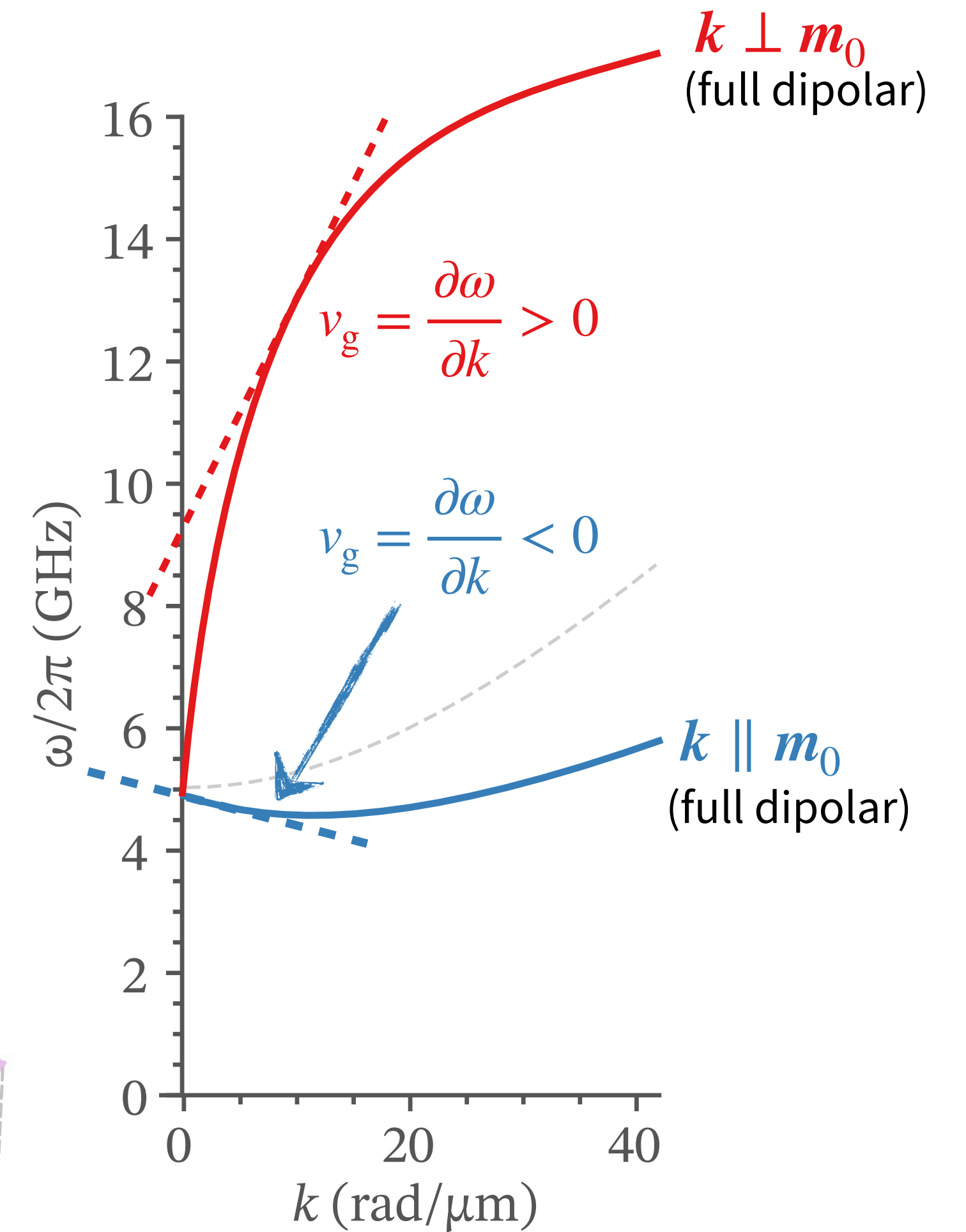
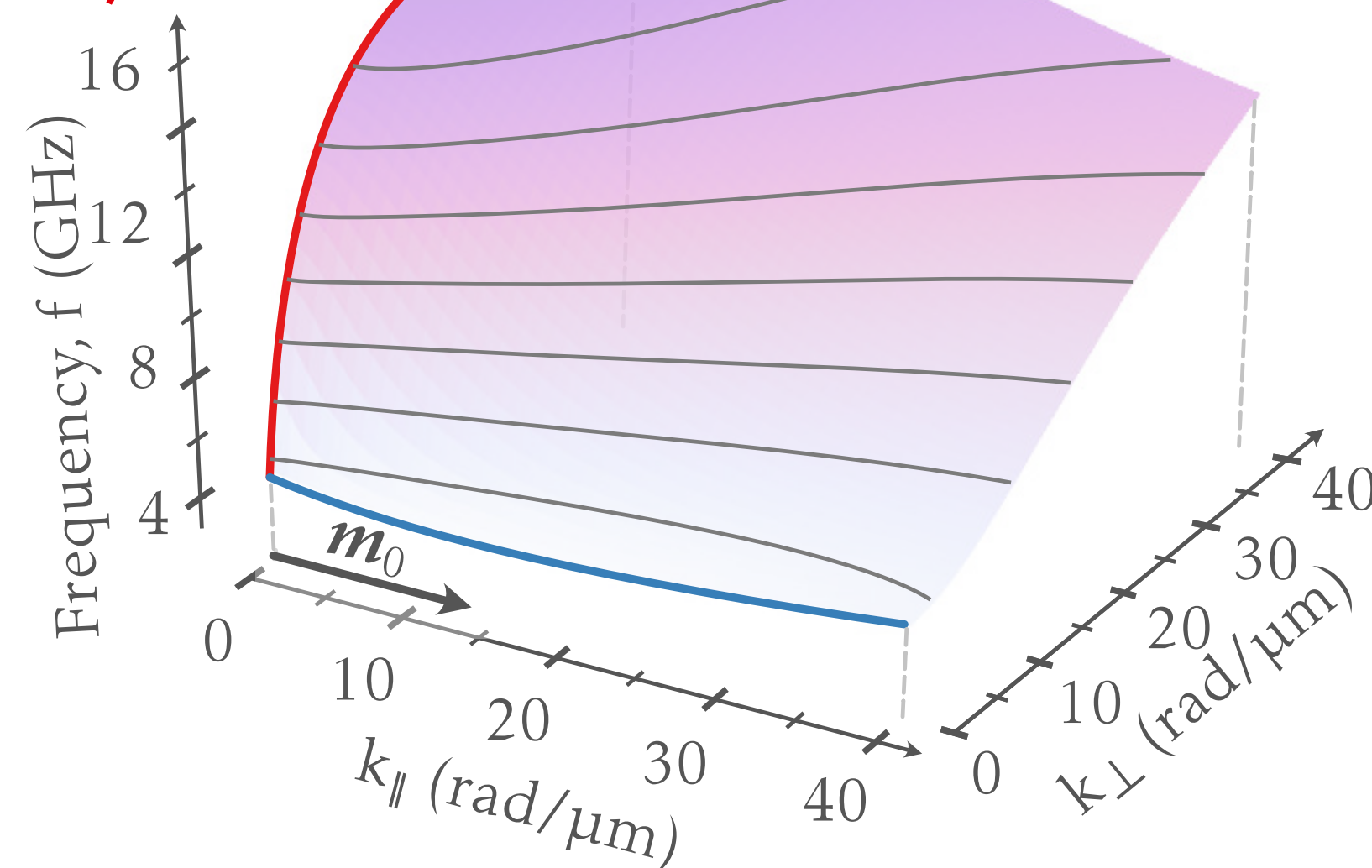


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

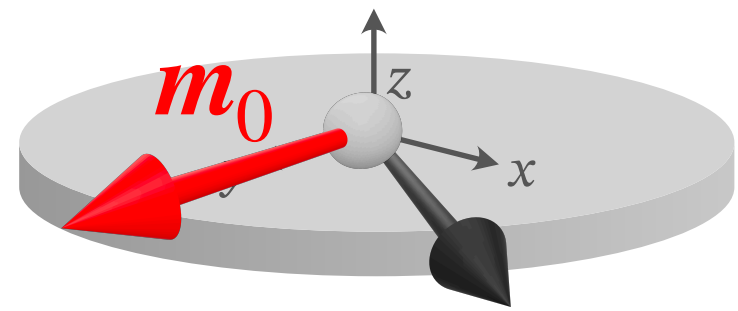
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$



Dispersion anisotropy!
(in k -space)

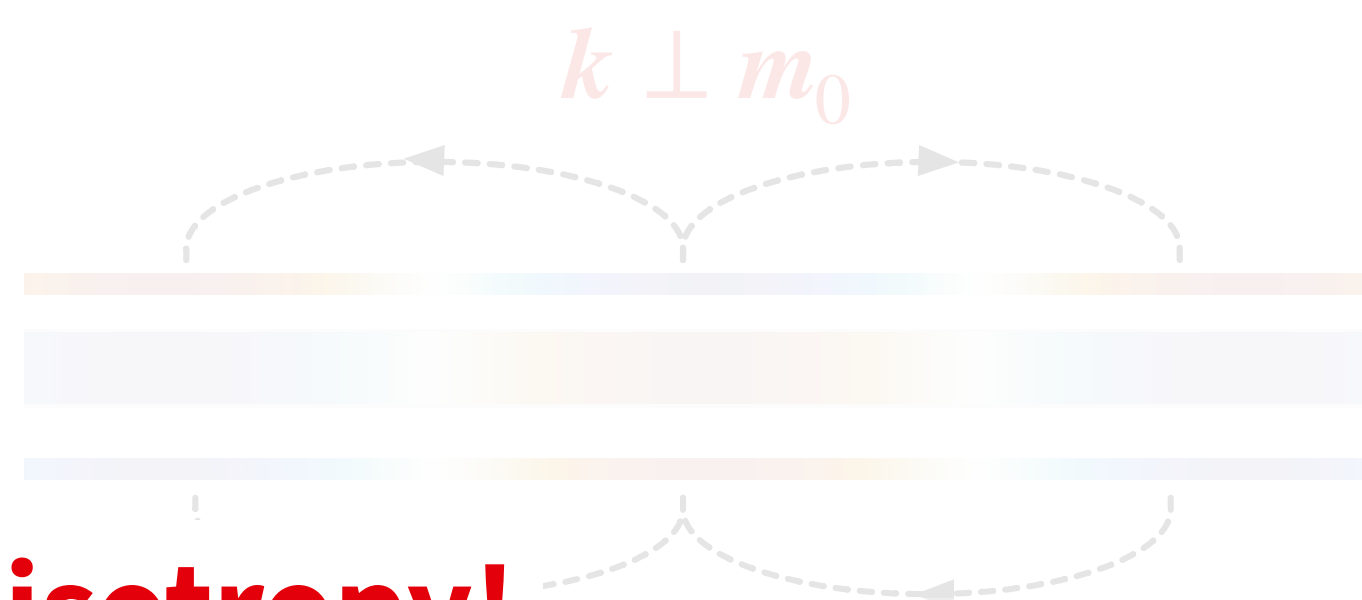


Dipolar charges lead to dispersion anisotropy

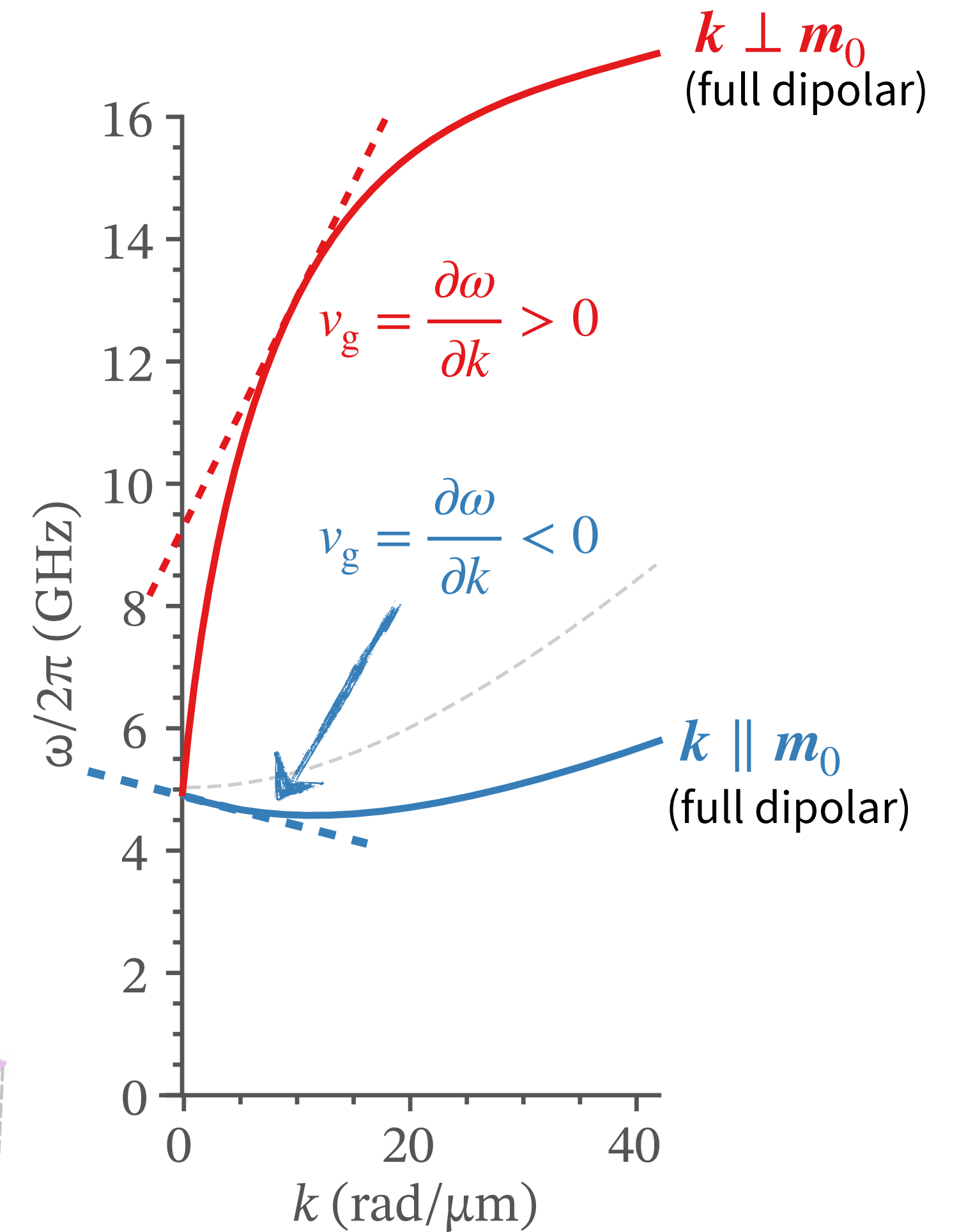
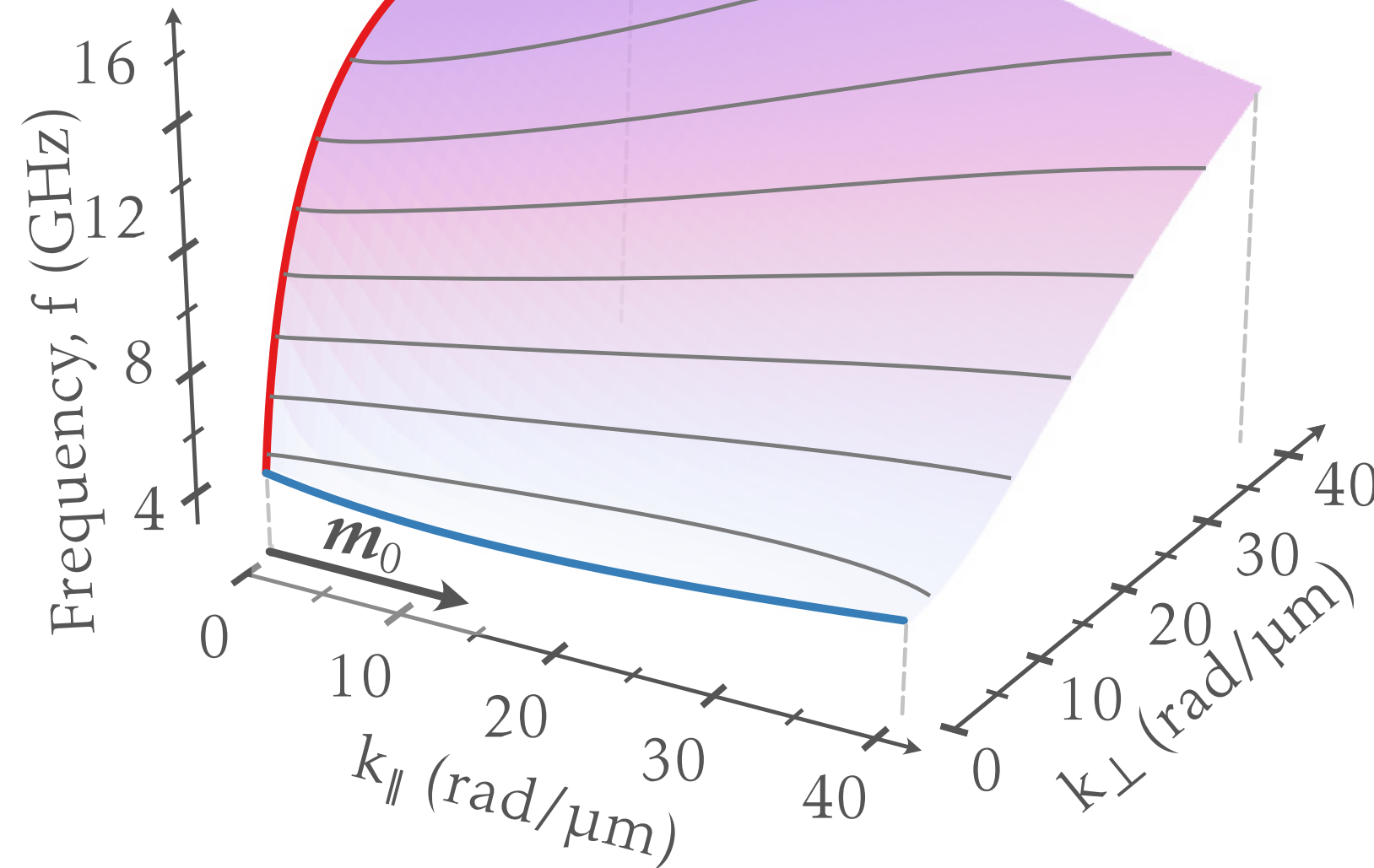
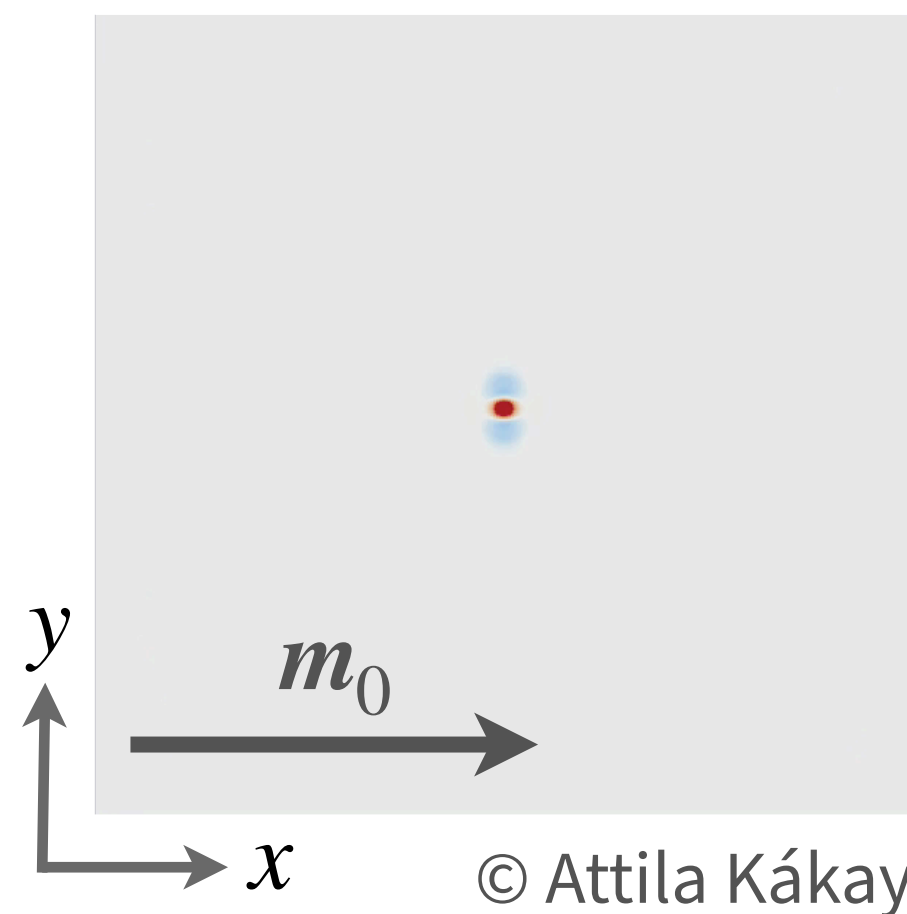


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

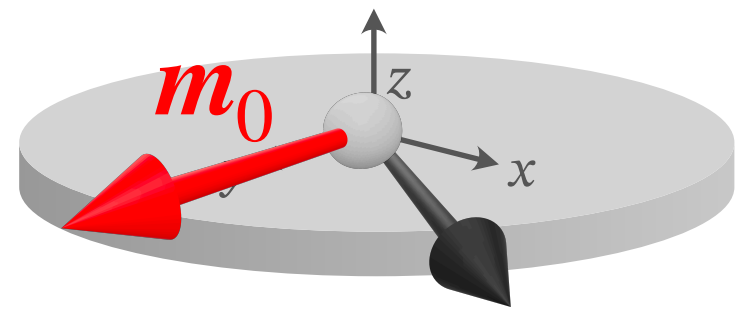
$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$



Dispersion anisotropy!
(in k -space)

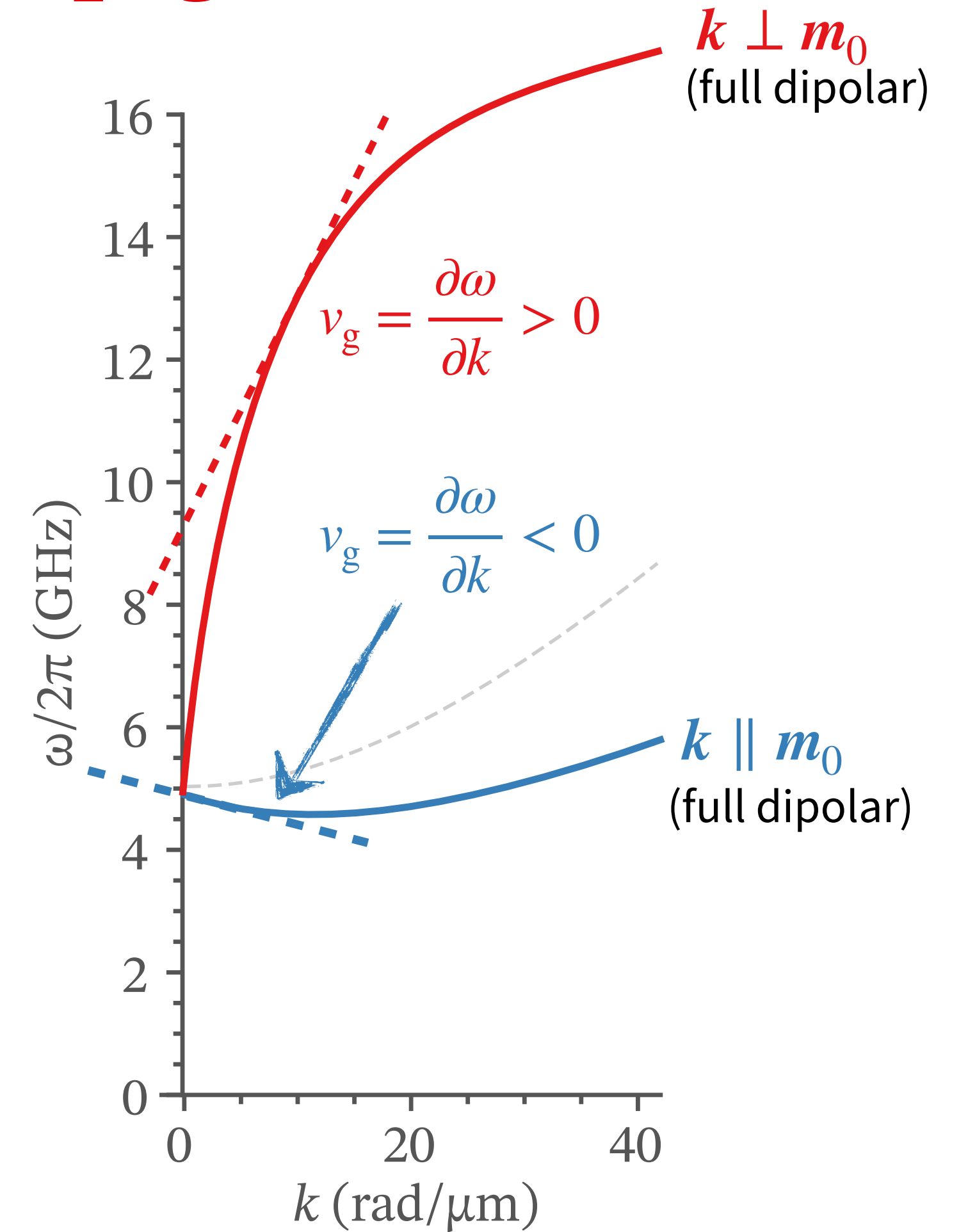
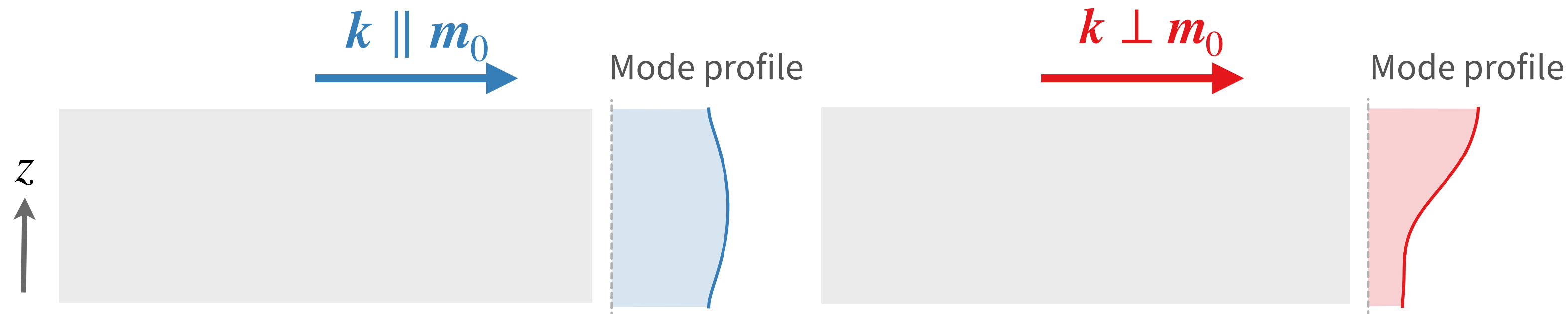


Modes have different localization depending on propagation direction

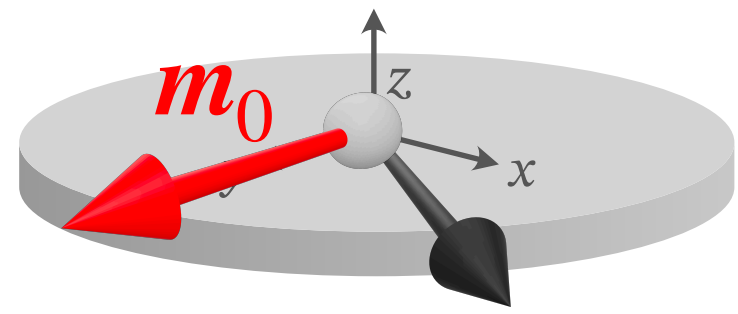


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$

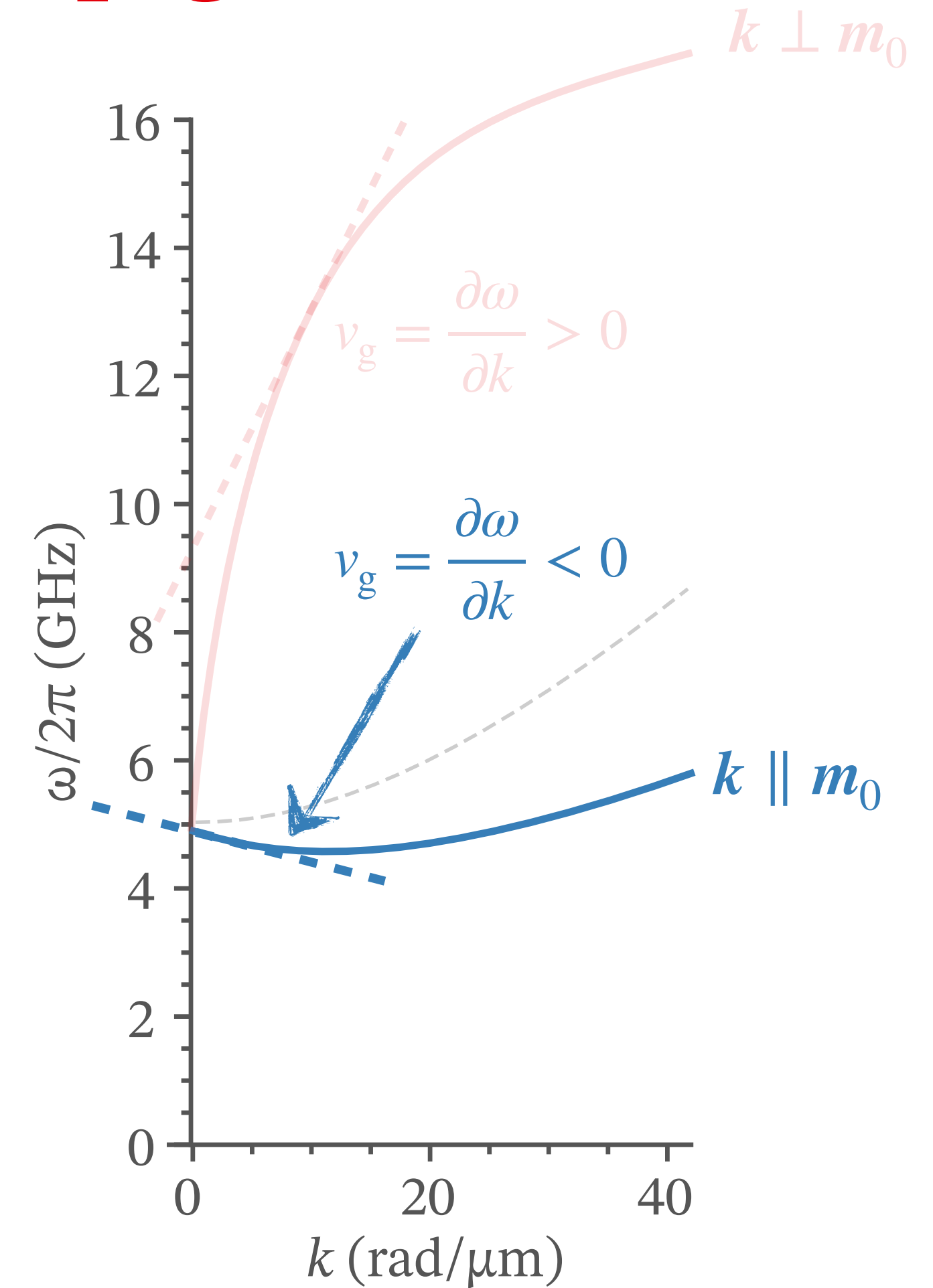
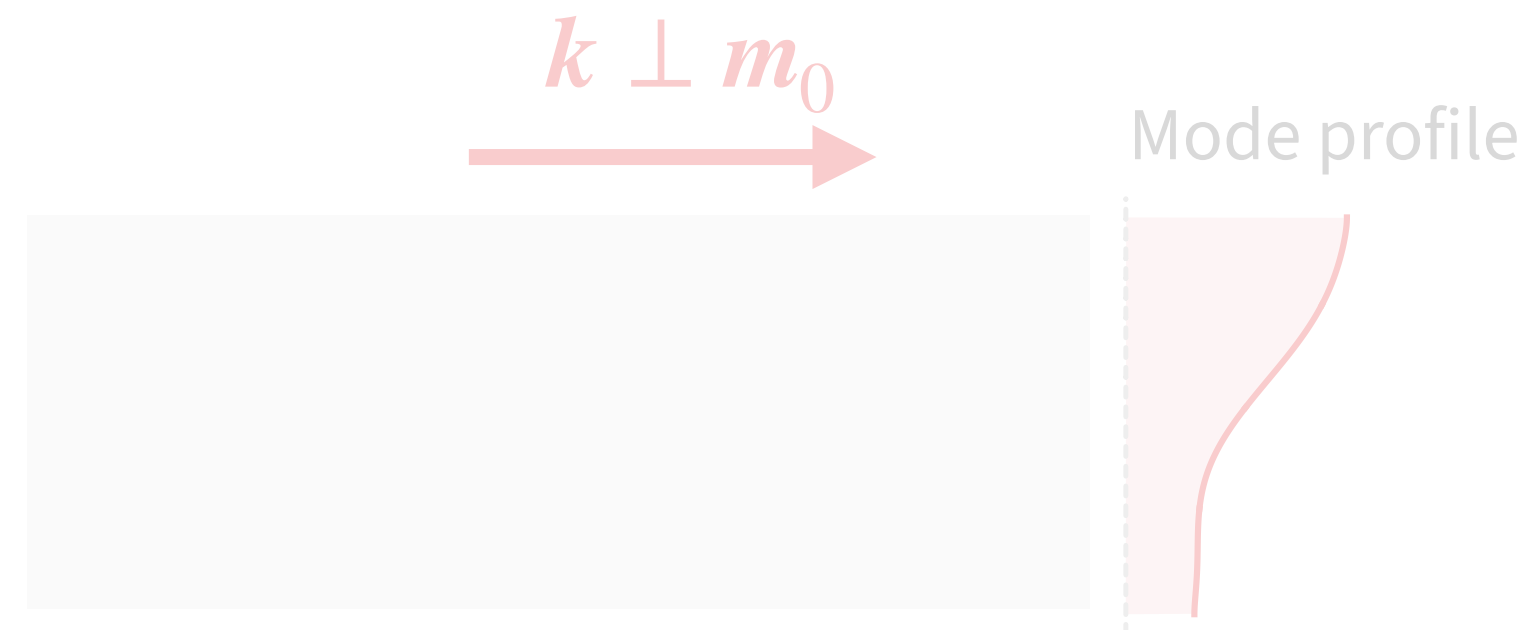
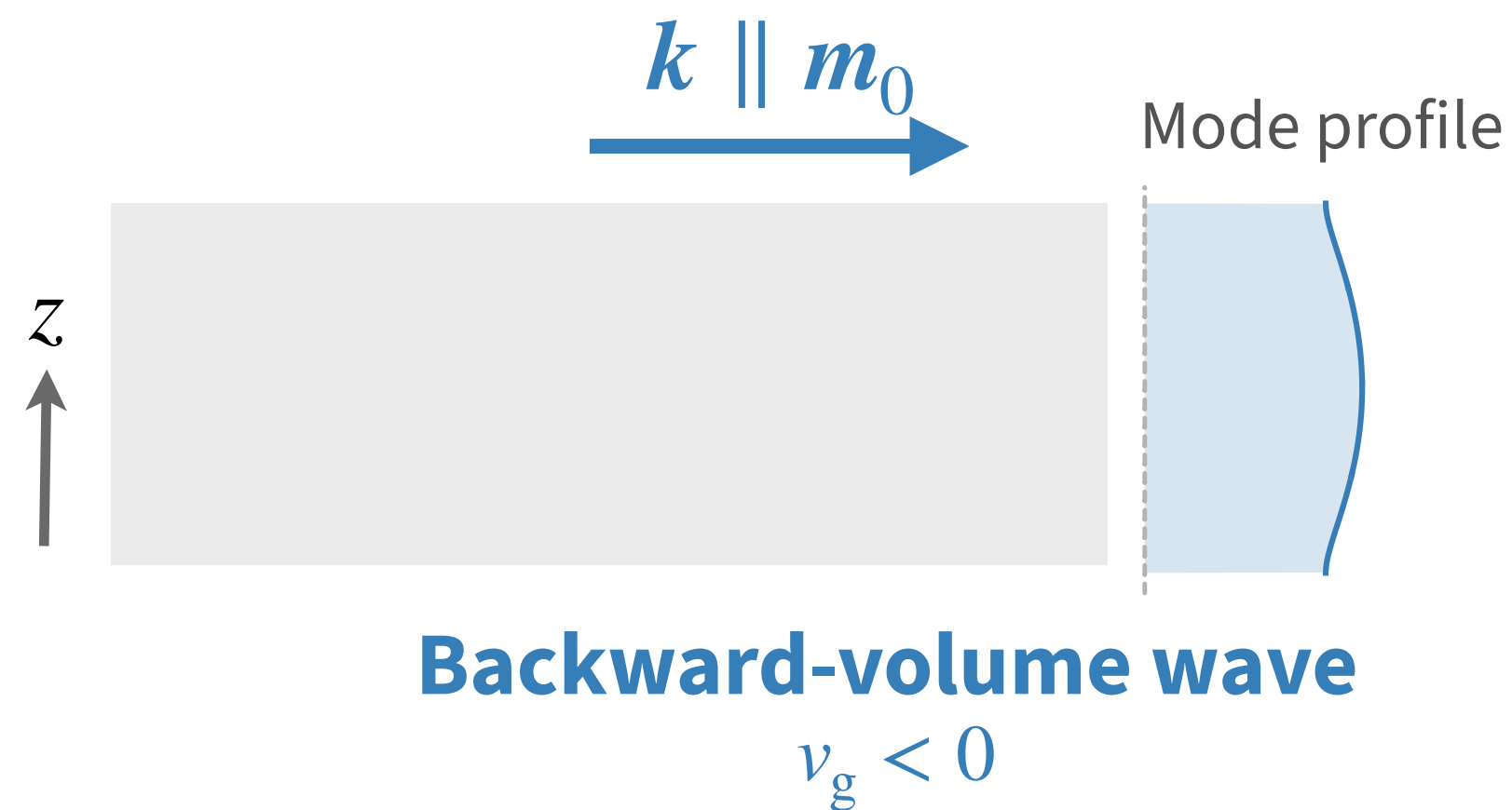


Modes have different localization depending on propagation direction

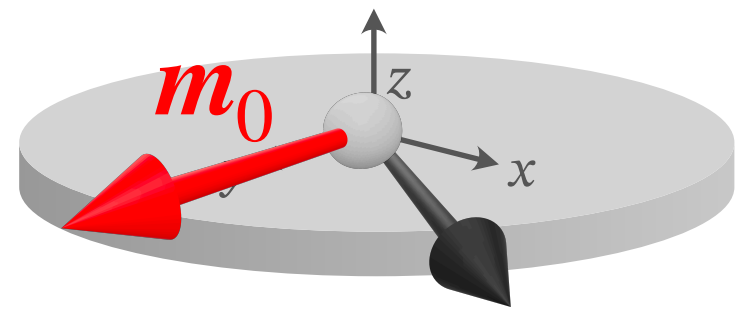


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$

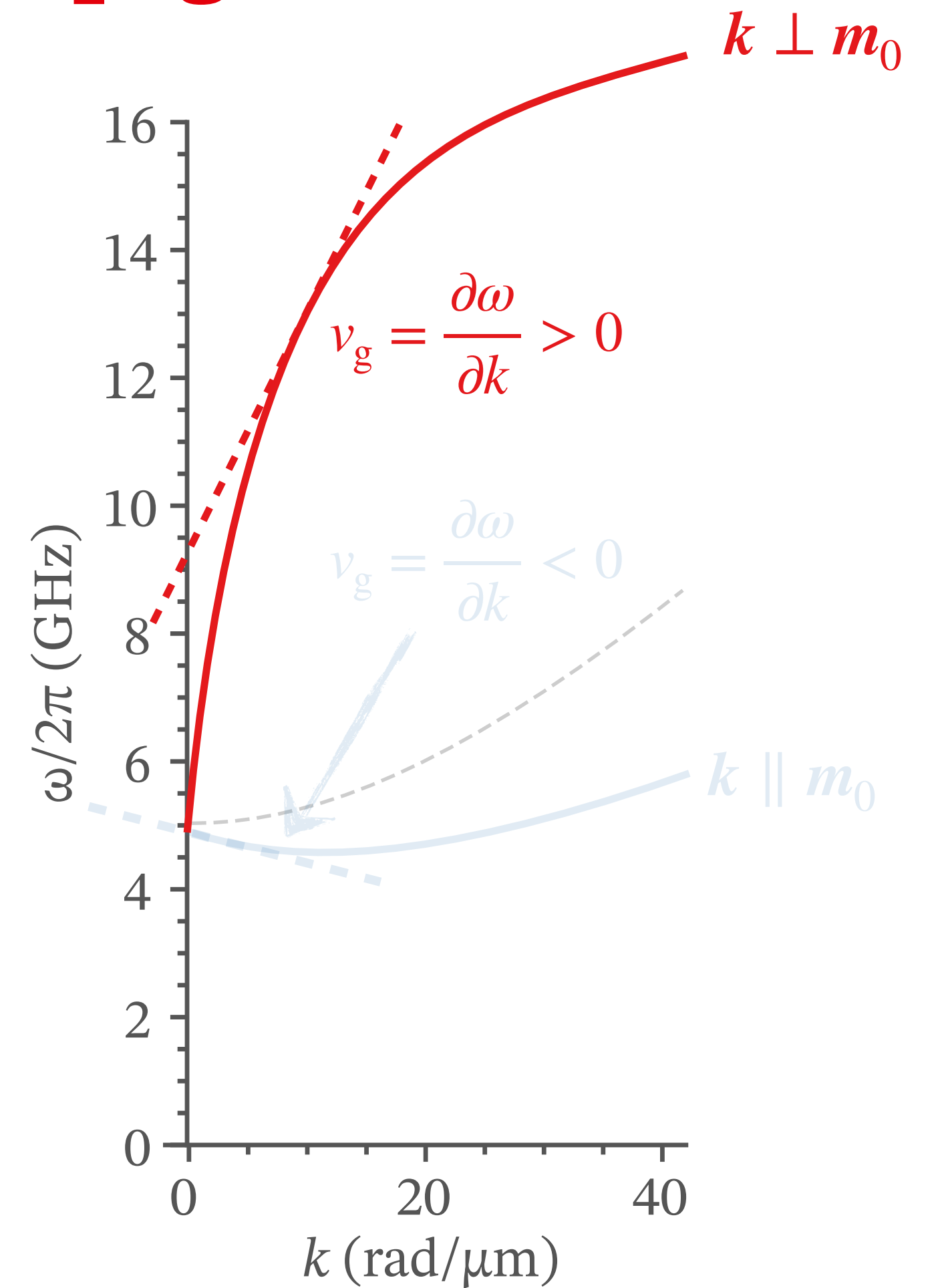
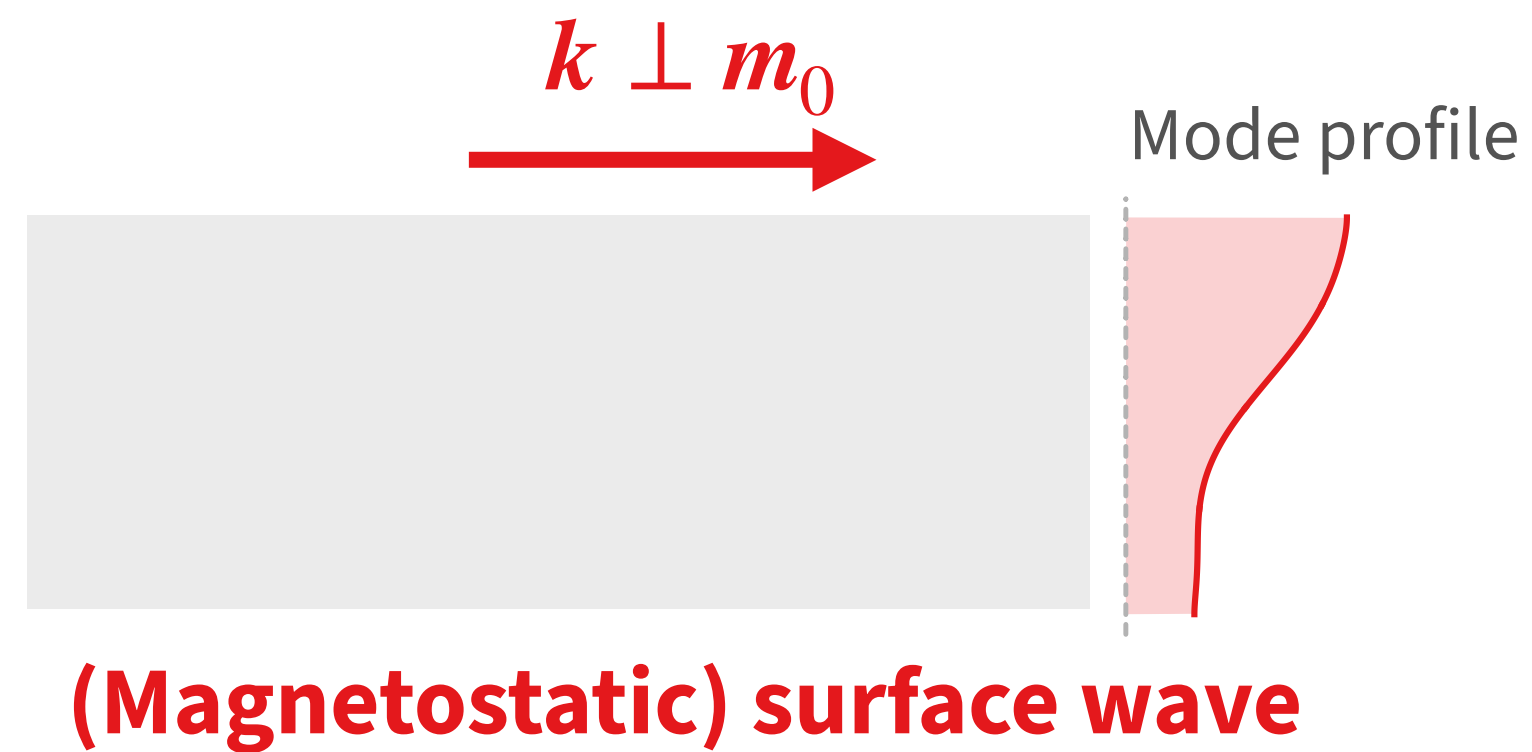
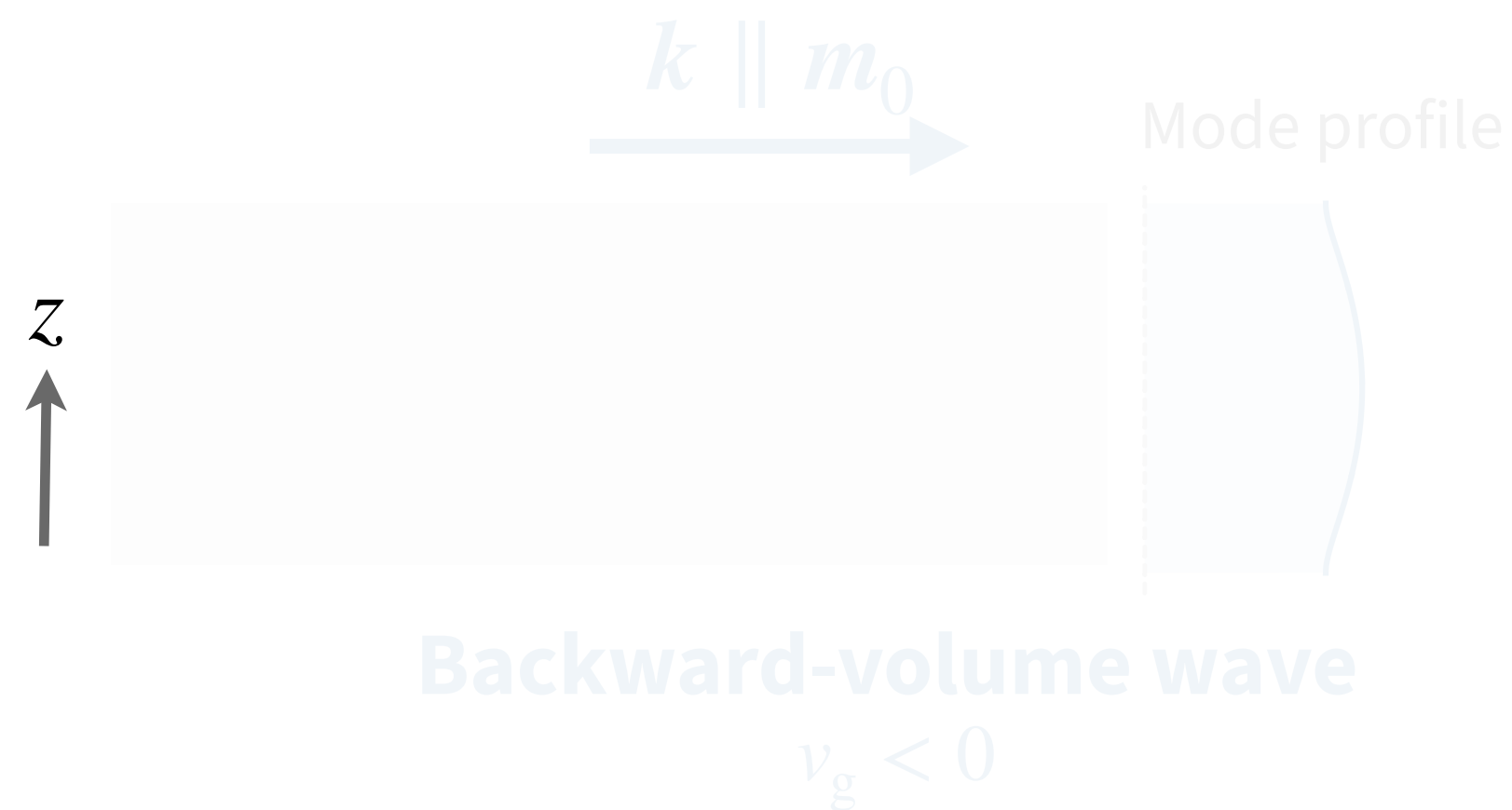


Modes have different localization depending on propagation direction

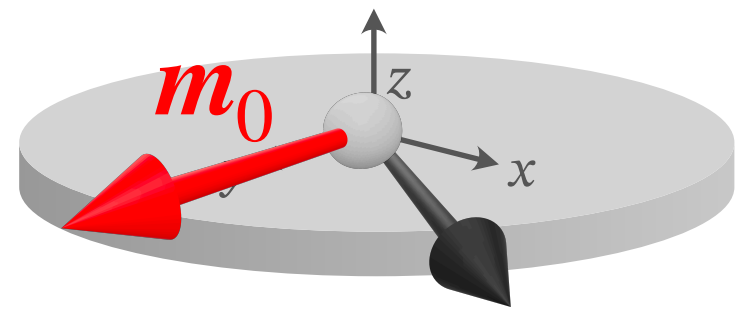


$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$



Modes have different localization depending on propagation direction

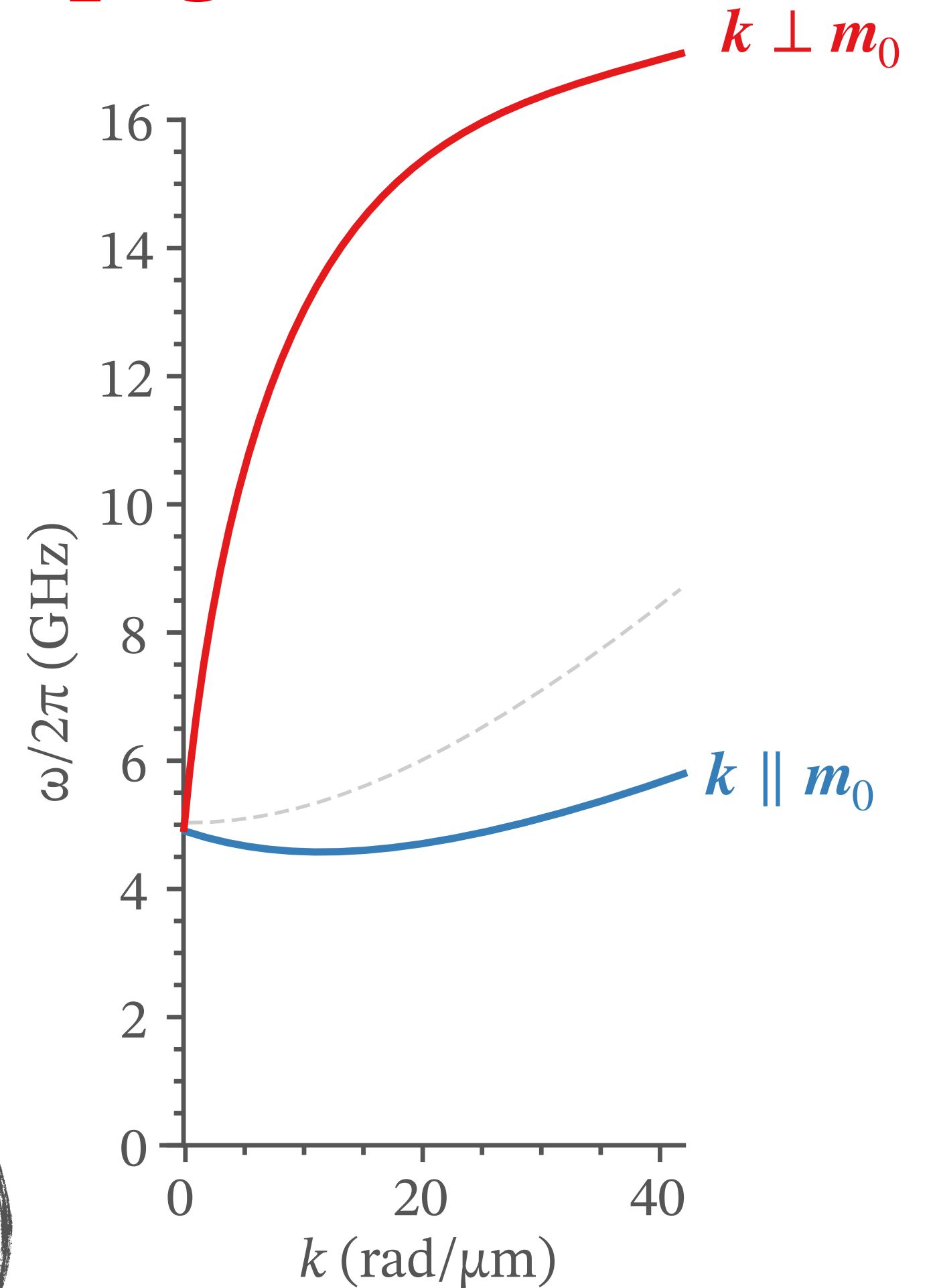
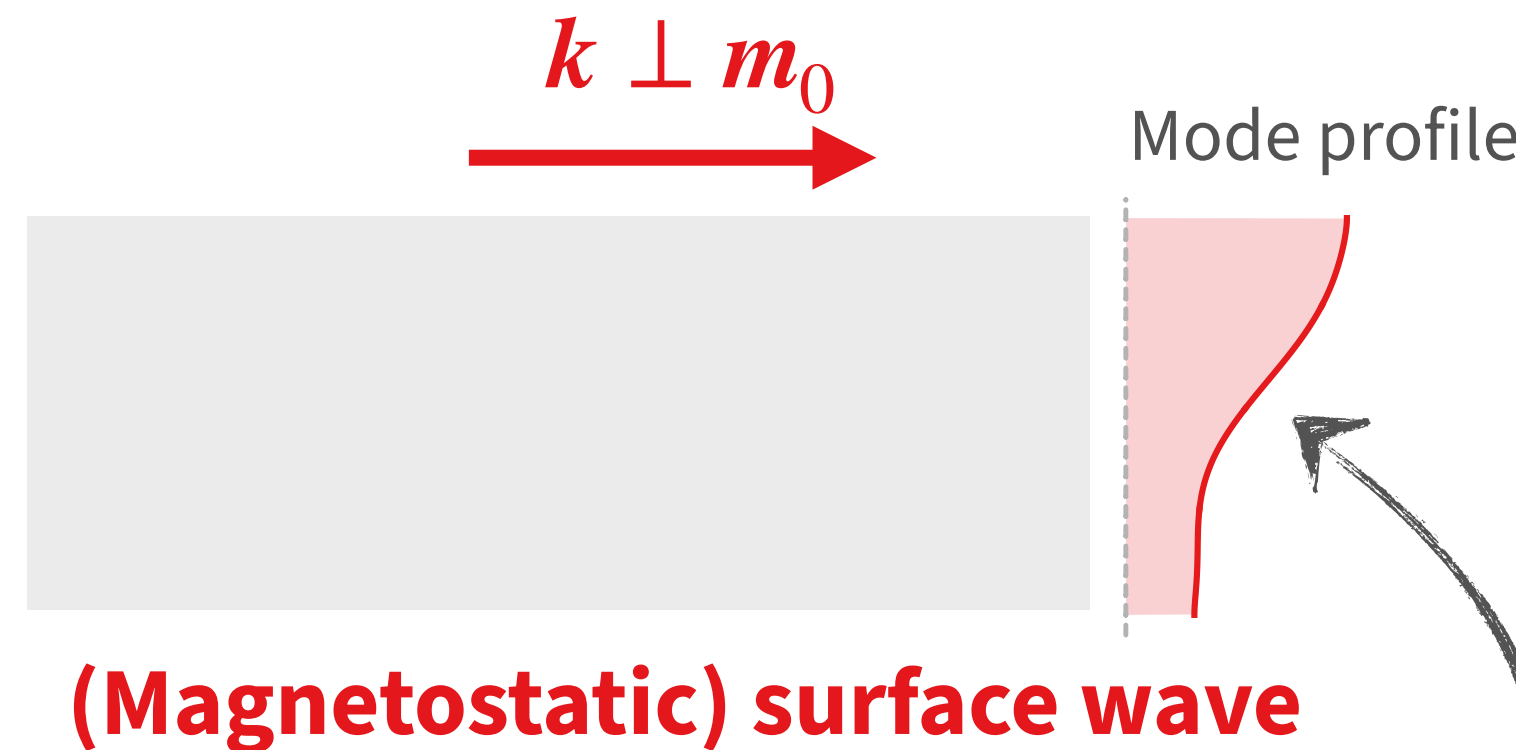
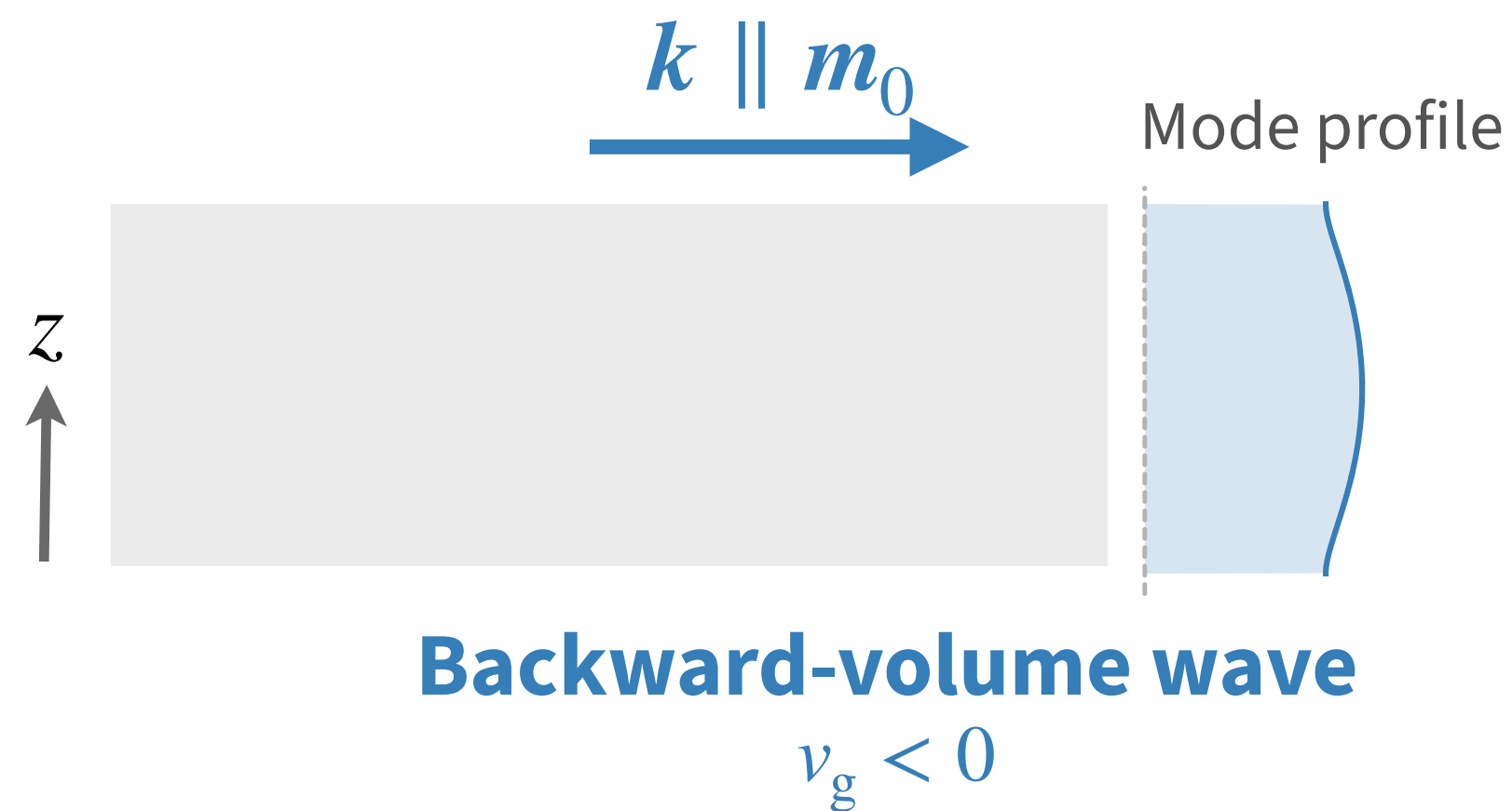


$$k = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(k) = \sqrt{[\omega_B + \omega_{\text{ex}}(k)] [\omega_B + \omega_{\text{ex}}(k) + \omega_M N_z(k)]}$$

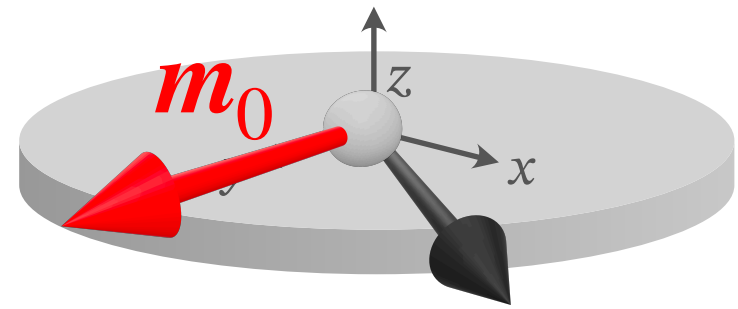
Cannot be given analytically anymore.

Approximate analytical formulas, e.g., by Kalinikos & Slavin



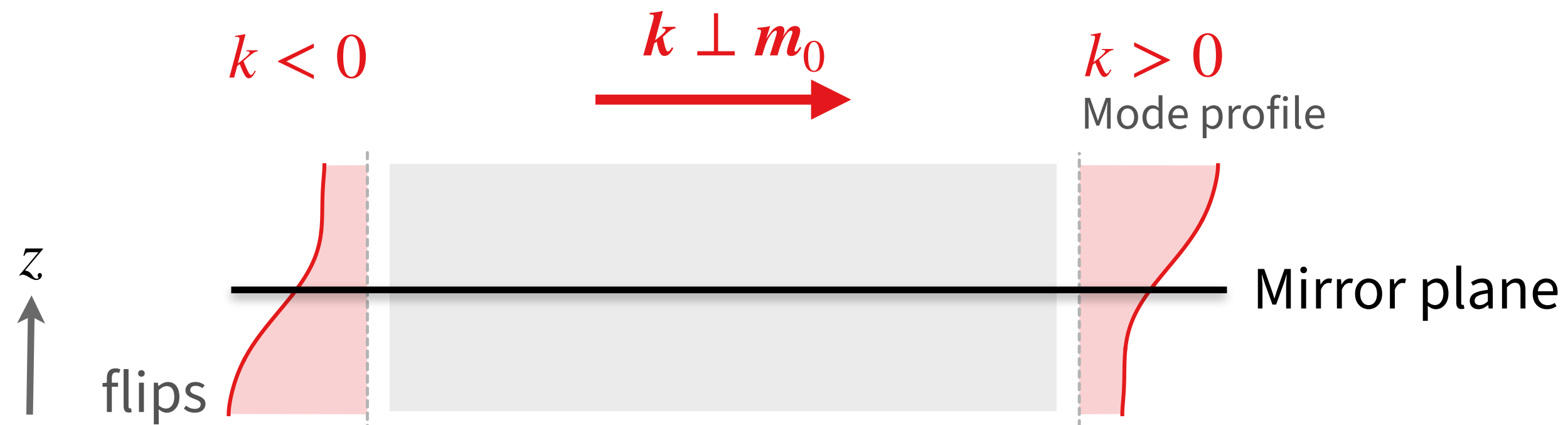
Computed with
 TetraX

Surfwave waves are chiral



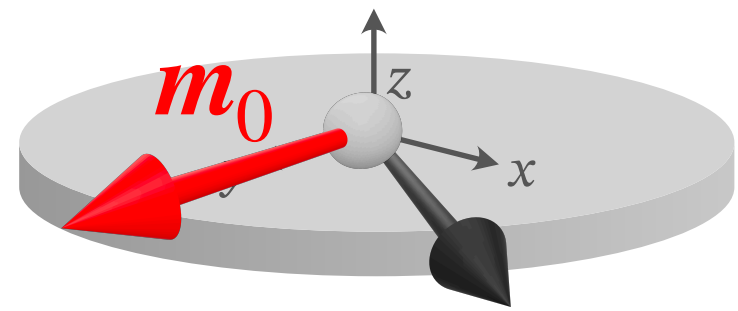
$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$



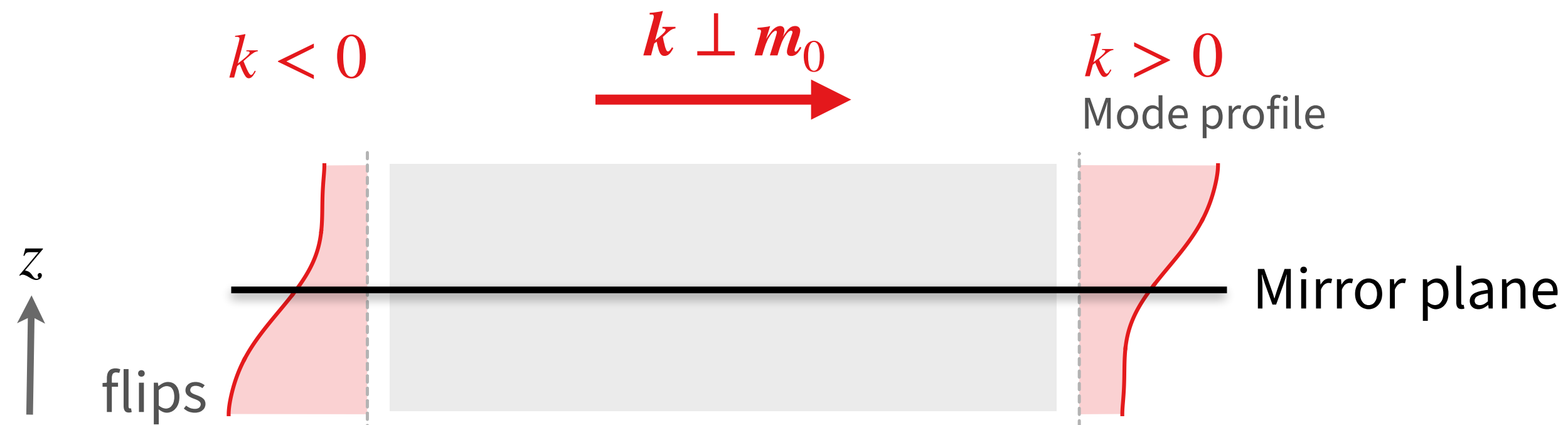
(Magnetostatic) surface wave

Surface waves are chiral



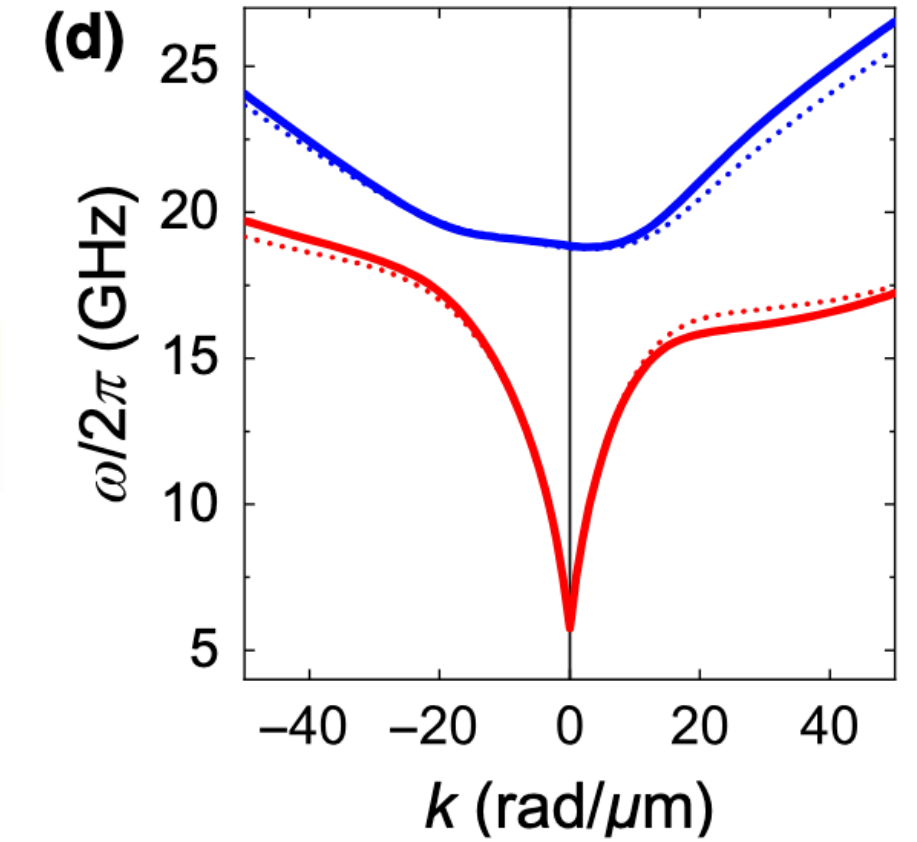
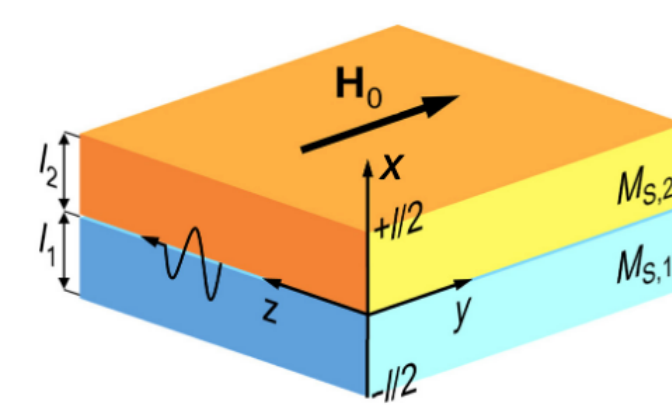
$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(\mathbf{k}) = \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$



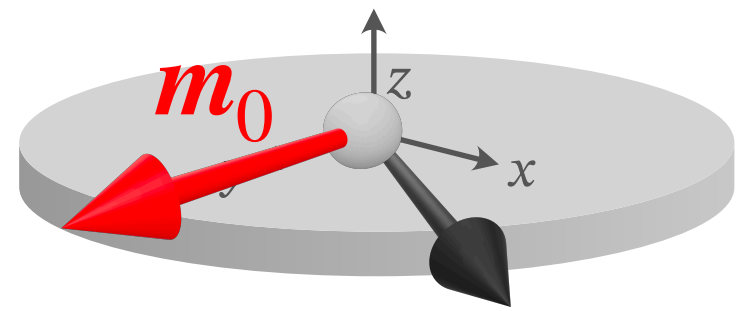
(Magnetostatic) surface wave

Magnetic bilayer



Grassi et al., PR Appl. 14, 024047 (2020)

Dipolar interaction can induce nonreciprocity



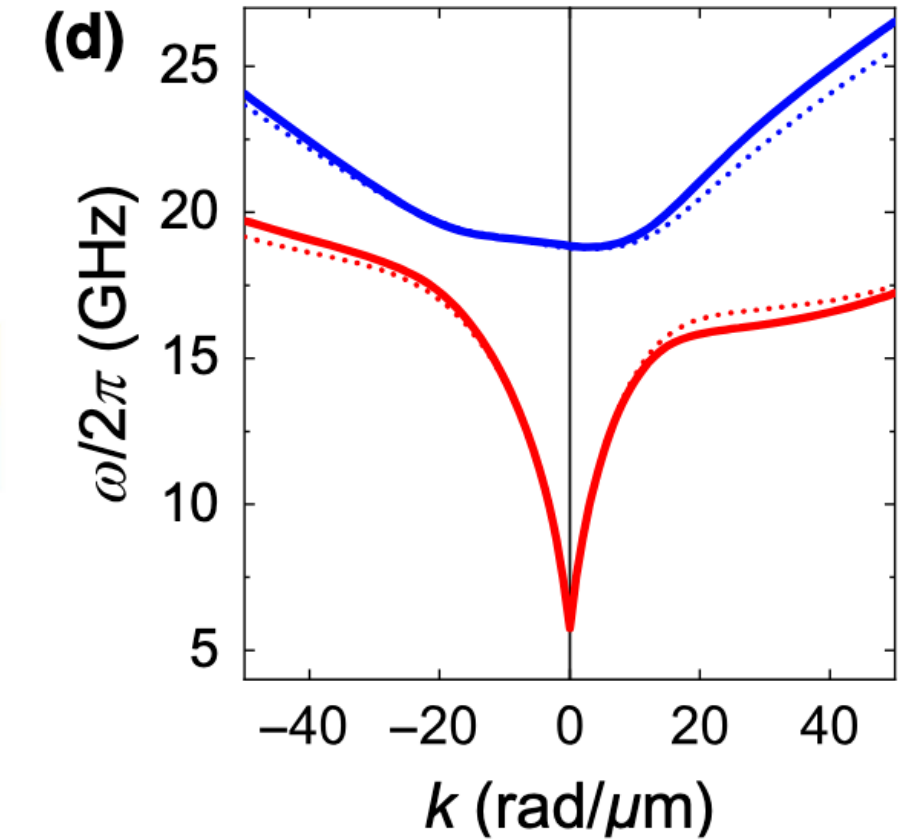
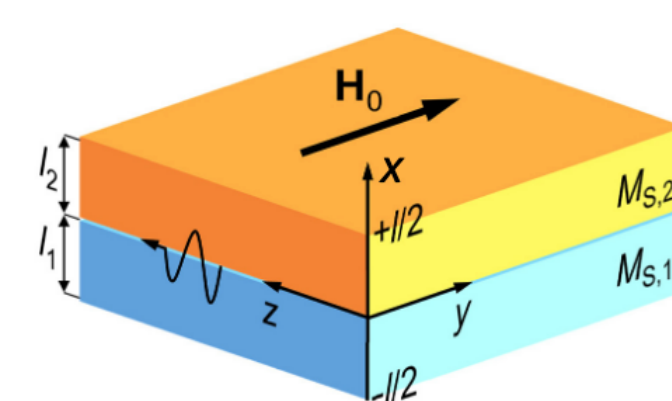
$$\mathbf{k} = k_x \mathbf{e}_x + k_y \mathbf{e}_y$$

$$\omega(\mathbf{k}) = \frac{\Delta\omega(\mathbf{k})}{2} + \sqrt{[\omega_B + \omega_{\text{ex}}(\mathbf{k})][\omega_B + \omega_{\text{ex}}(\mathbf{k}) + \omega_M N_z(\mathbf{k})]}$$

Nonreciprocity

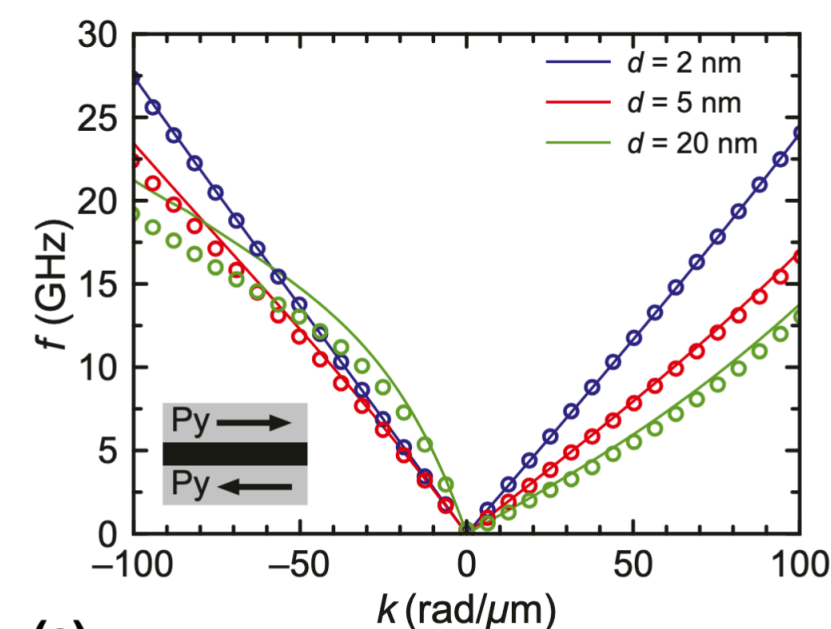
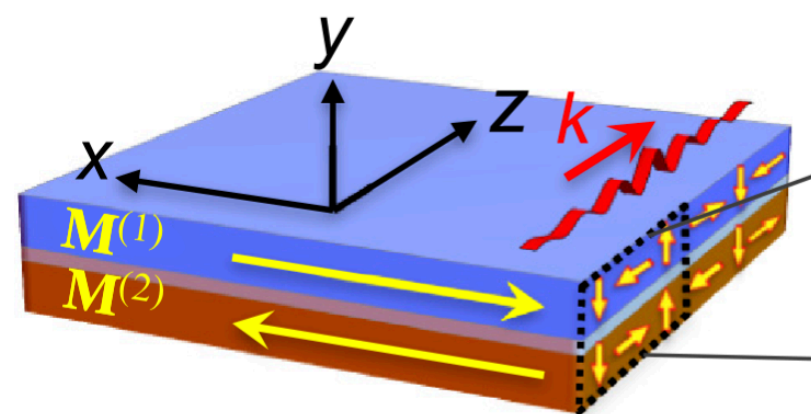
$$\omega(\mathbf{k}) \neq \omega(-\mathbf{k})$$

Magnetic bilayer



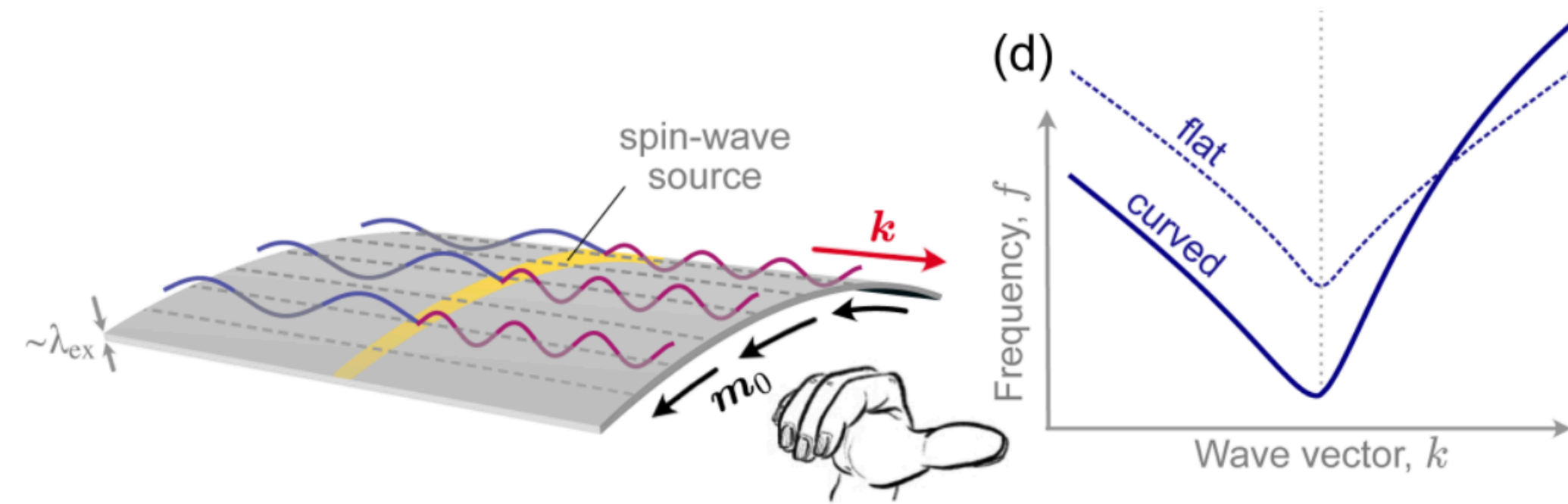
Grassi et al., PR Appl. 14, 024047 (2020)

Synthetic antiferromagnets



Gallardo et al., PR Appl. 12, 034012 (2019)

Curved magnetic shells

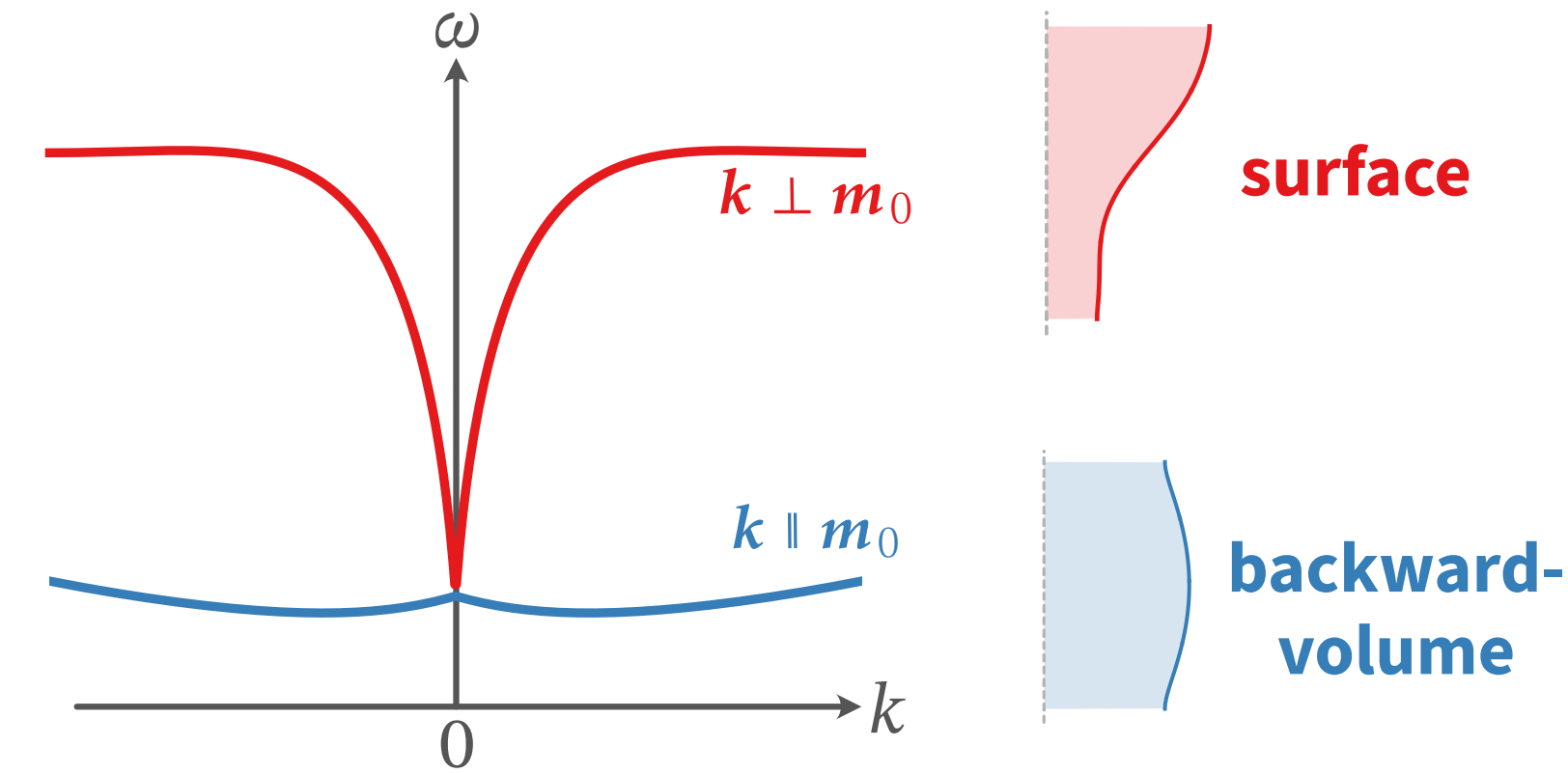
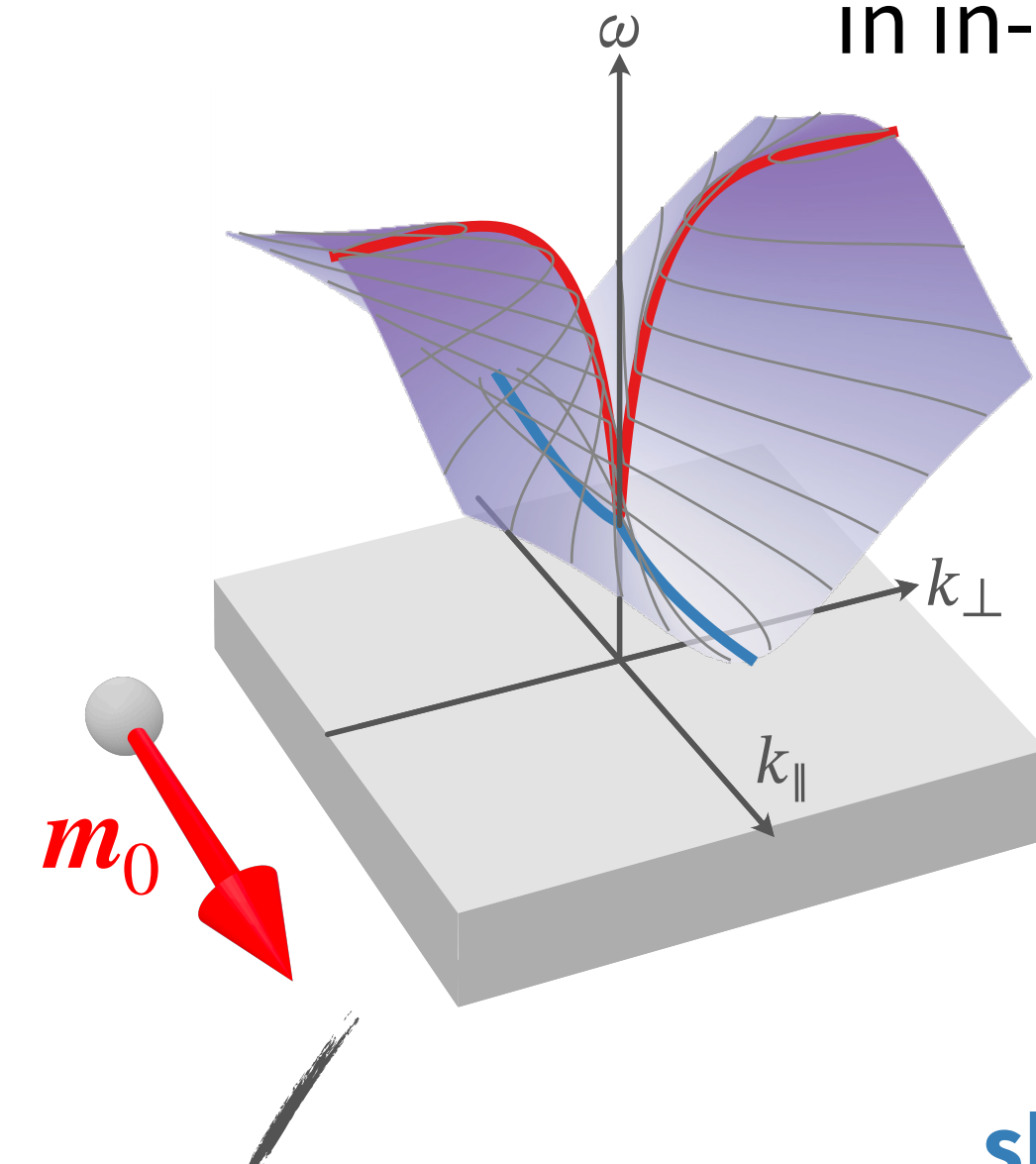


Körber et al., PRB 106, 014405 (2022)

Symmetry-breaking indicated by toroidal moment $\boldsymbol{\tau} = \int dA \, \mathbf{r} \times M_s \mathbf{m}_0$

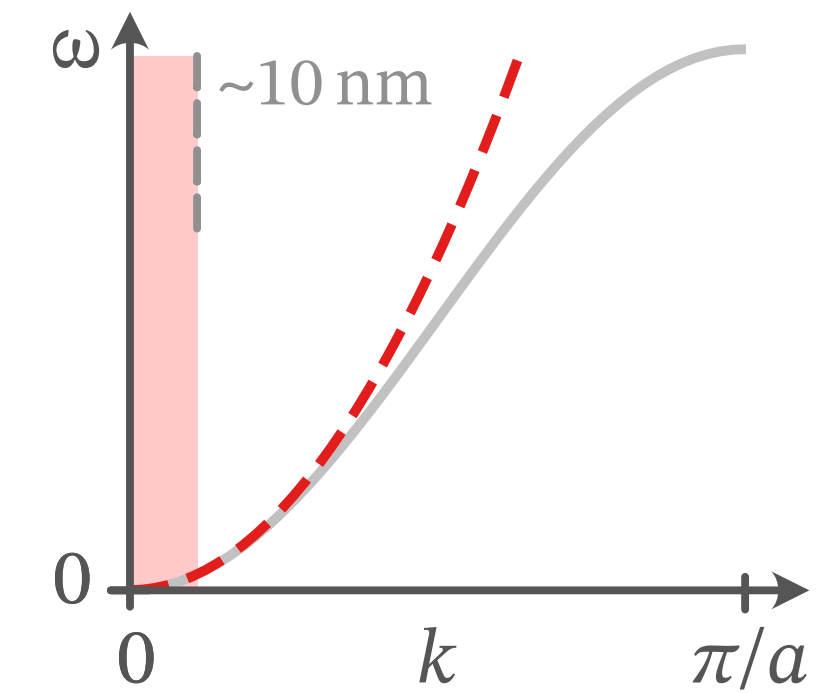
Brevis, LK et al., PR Appl. **24**, 024058 (2025)

Dispersion anisotropy from dipolar volume charges in in-plane magnetized films

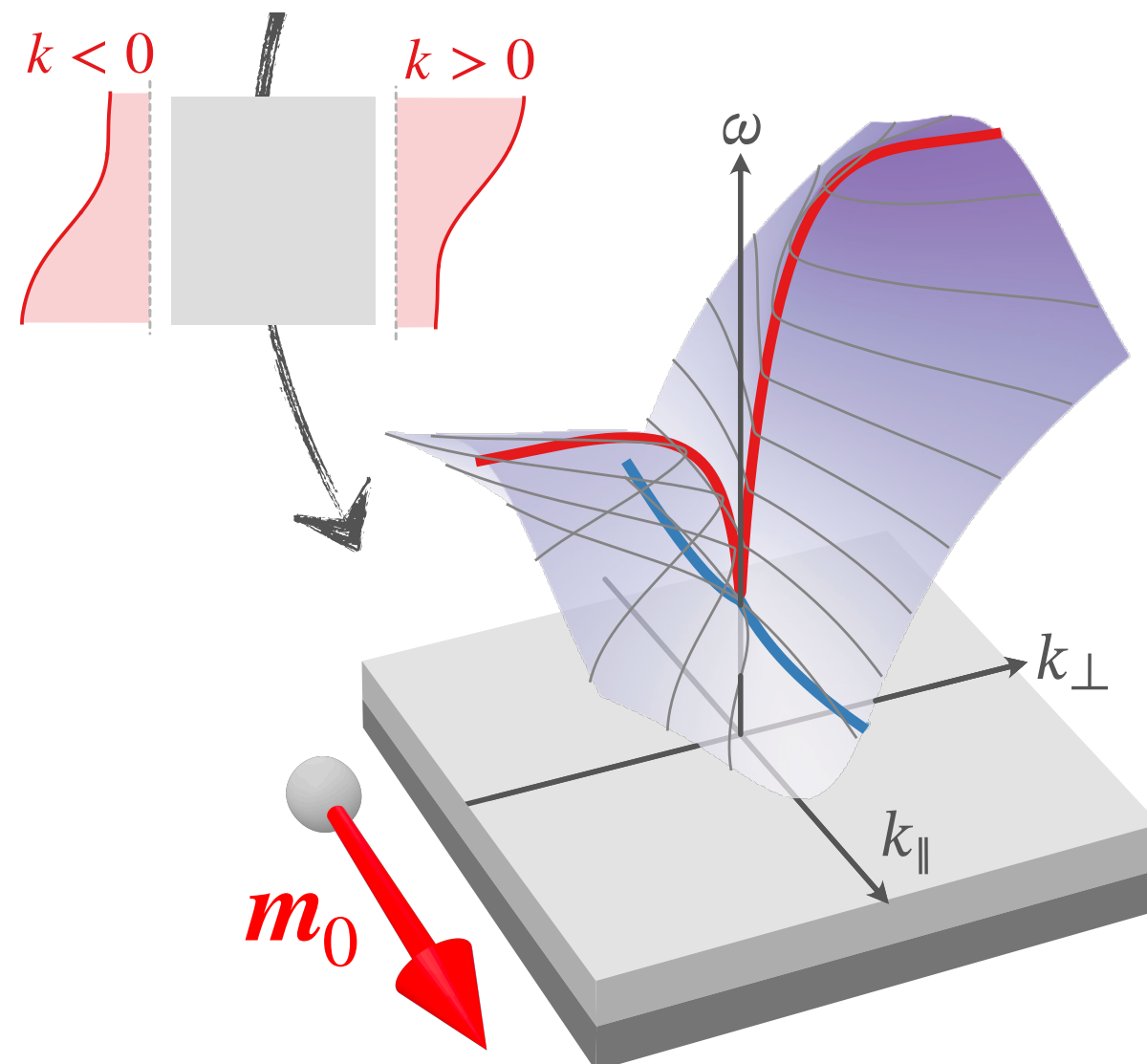


Fast surface waves and
slow backward-volume waves

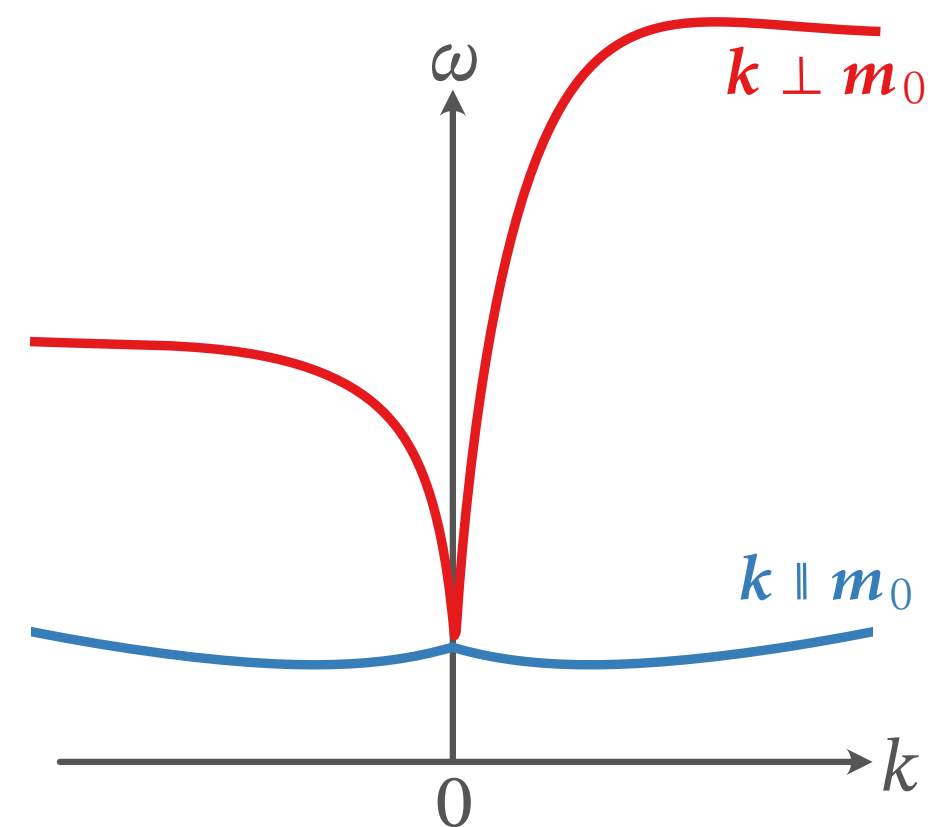
Long-wavelength limit



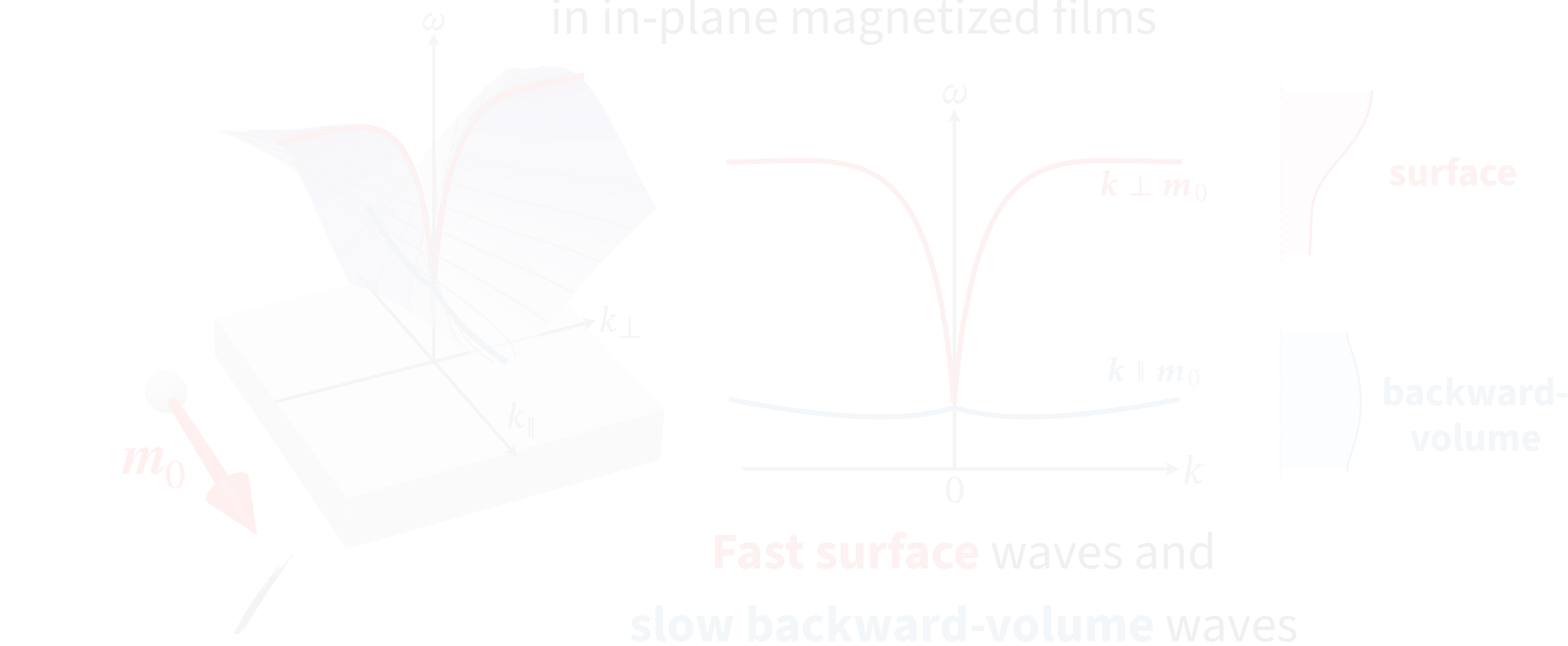
Directional localization
of surface waves



Dipole-induced
nonreciprocity/chirality



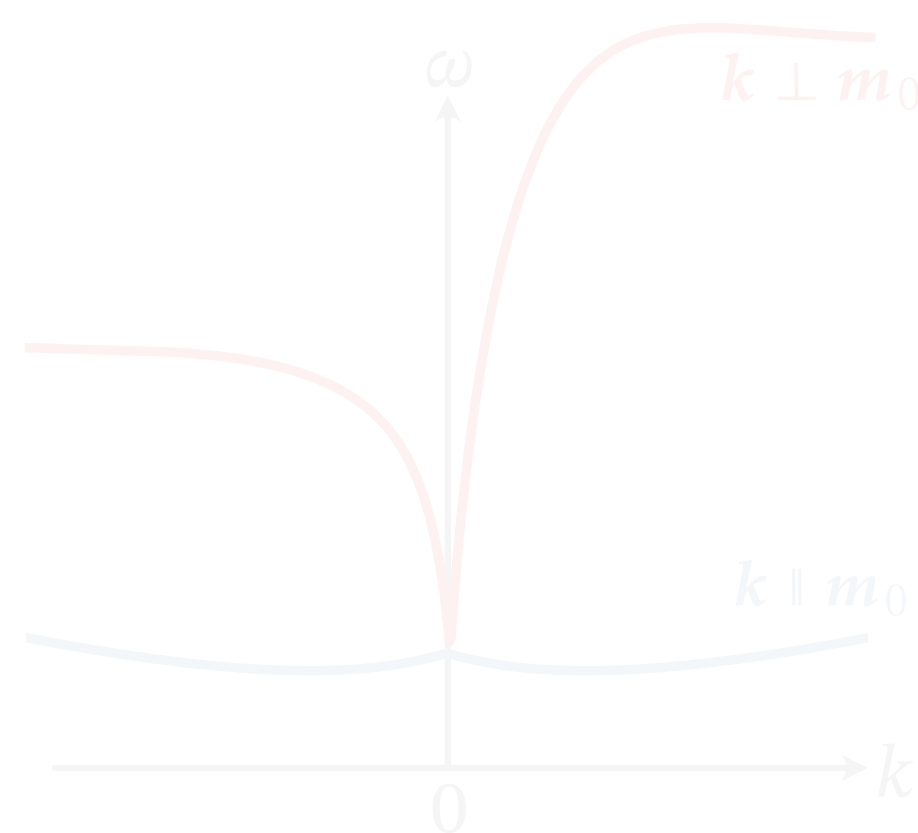
Dispersion anisotropy from dipolar charges in in-plane magnetized films



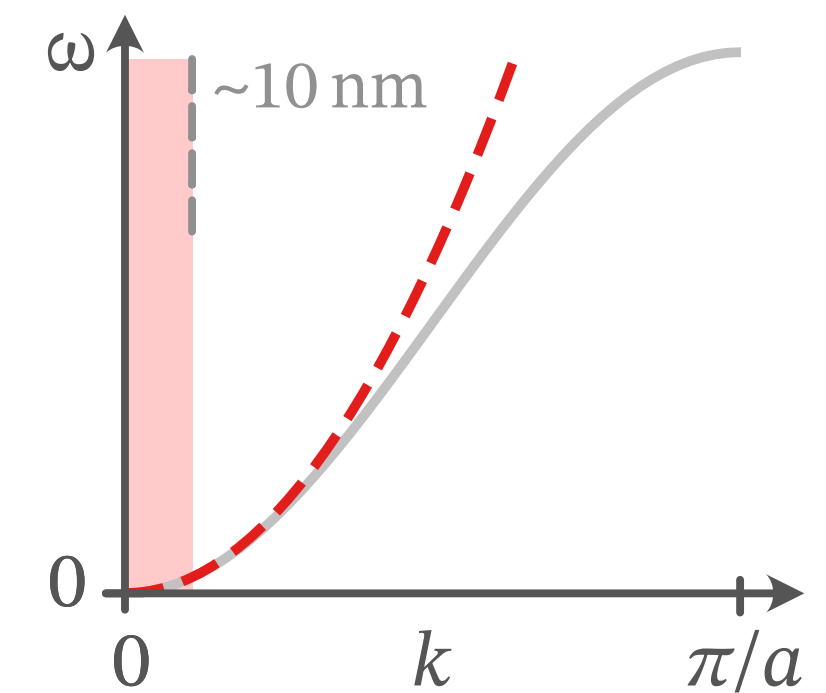
Directional localization of surface waves



Dipole-induced nonreciprocity/chirality

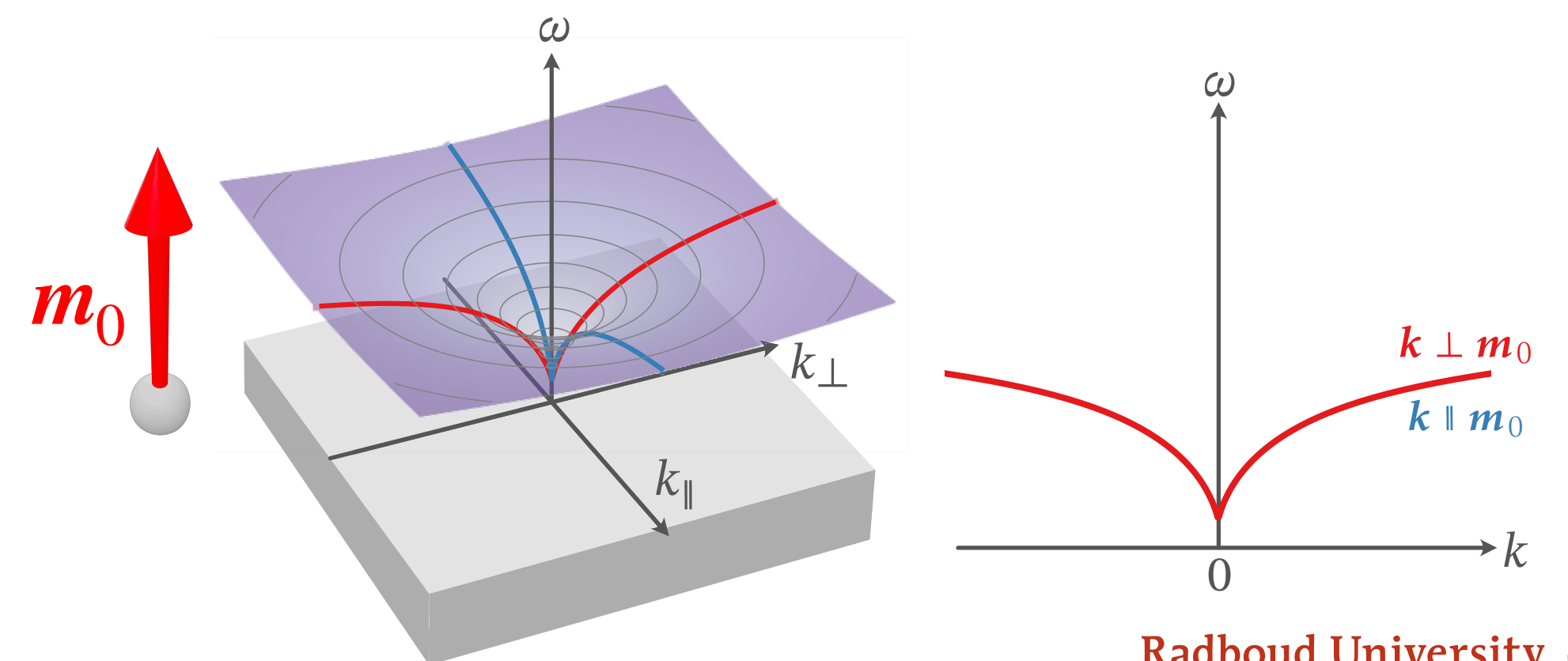


Long-wavelength limit



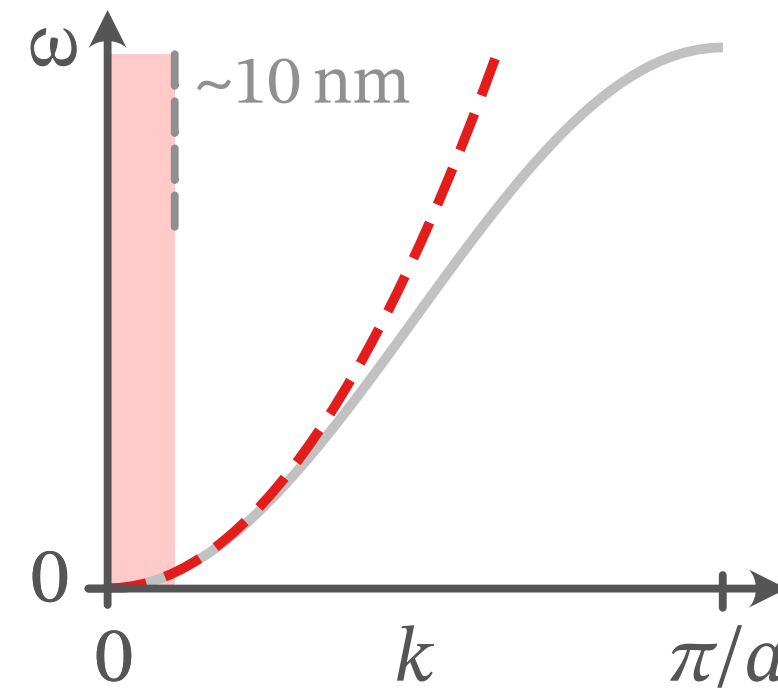
Note: Isotropic dispersion when out-of-plane magnetized

👉 Forward-volume waves

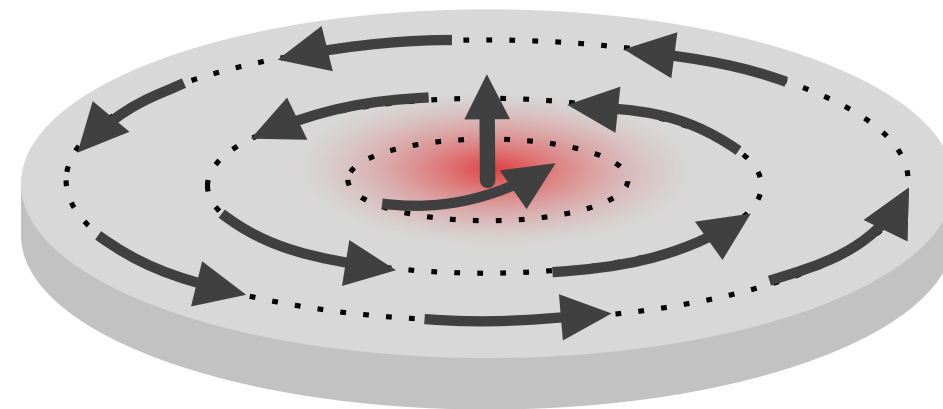


Long-wavelength magnons feel geometry, what about confinement?

Long-wavelength limit

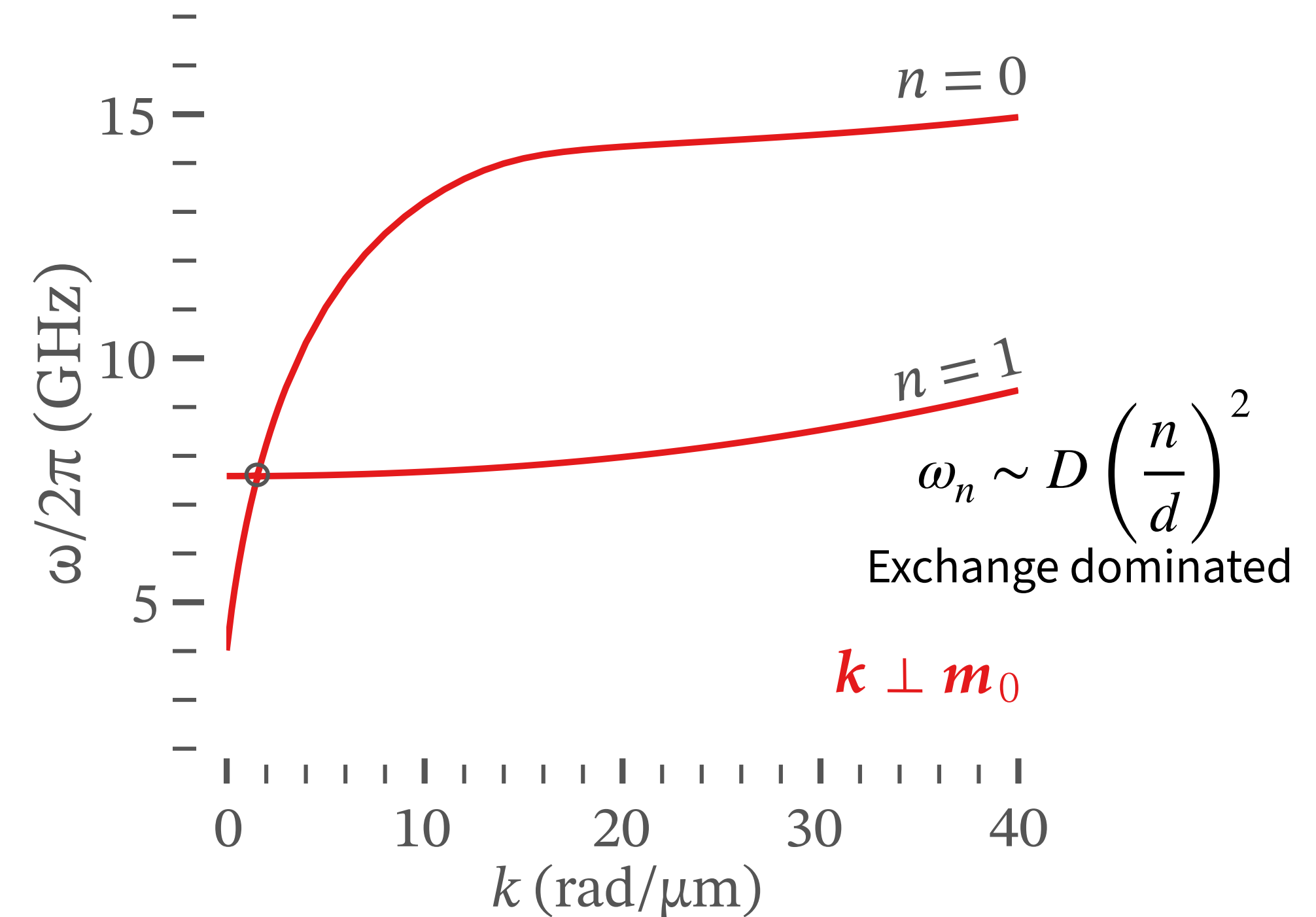
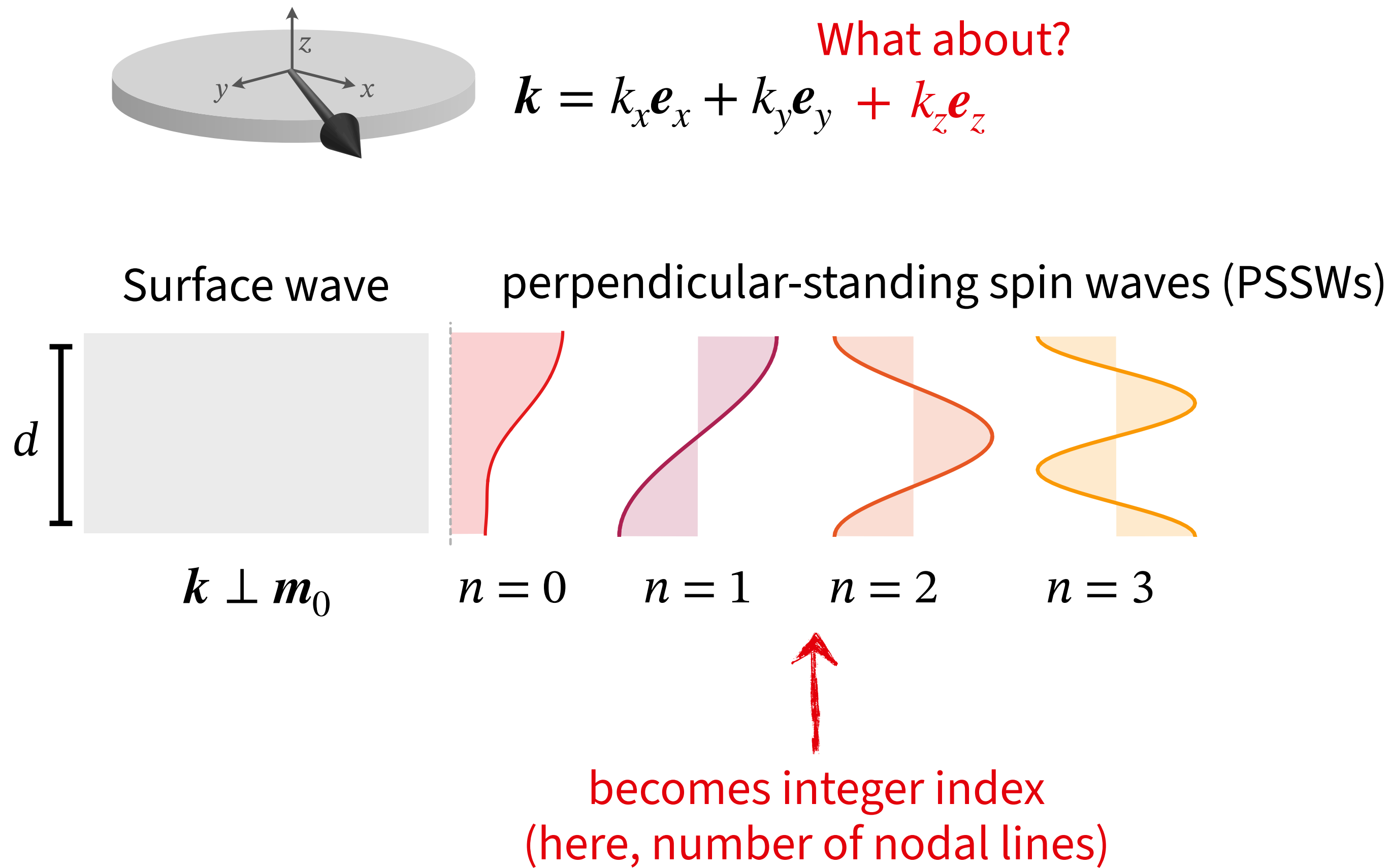


Confined magnons



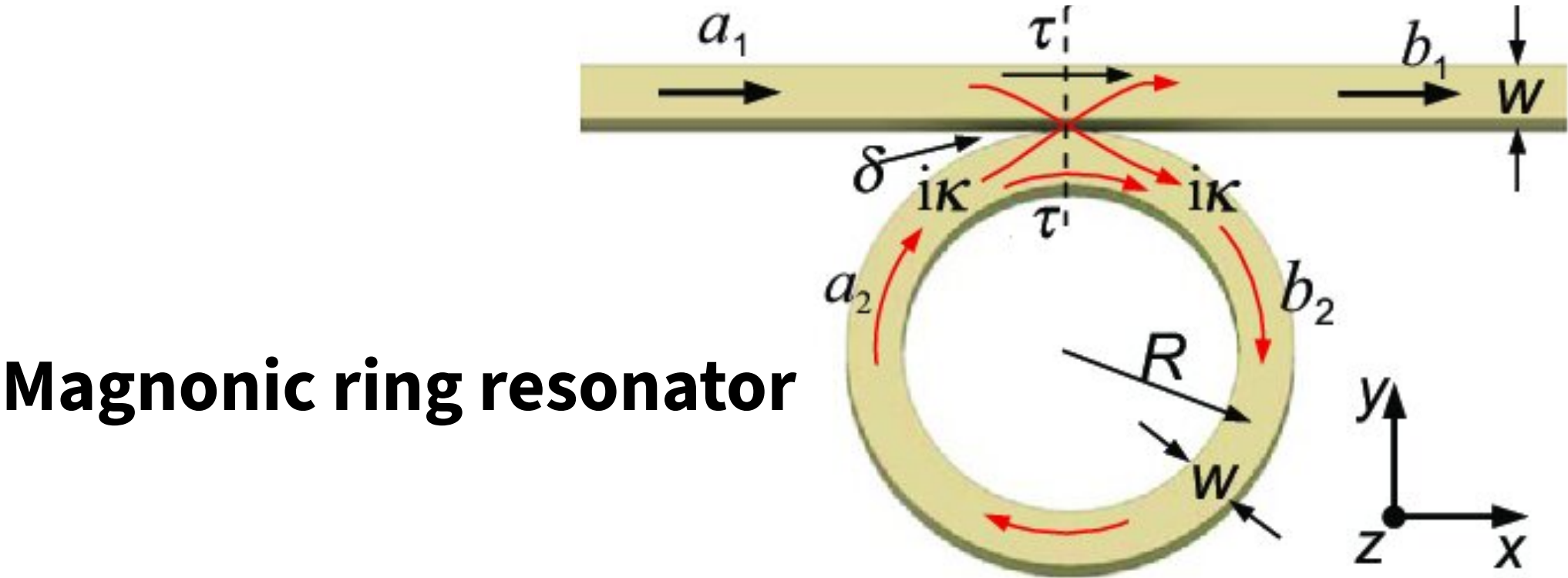
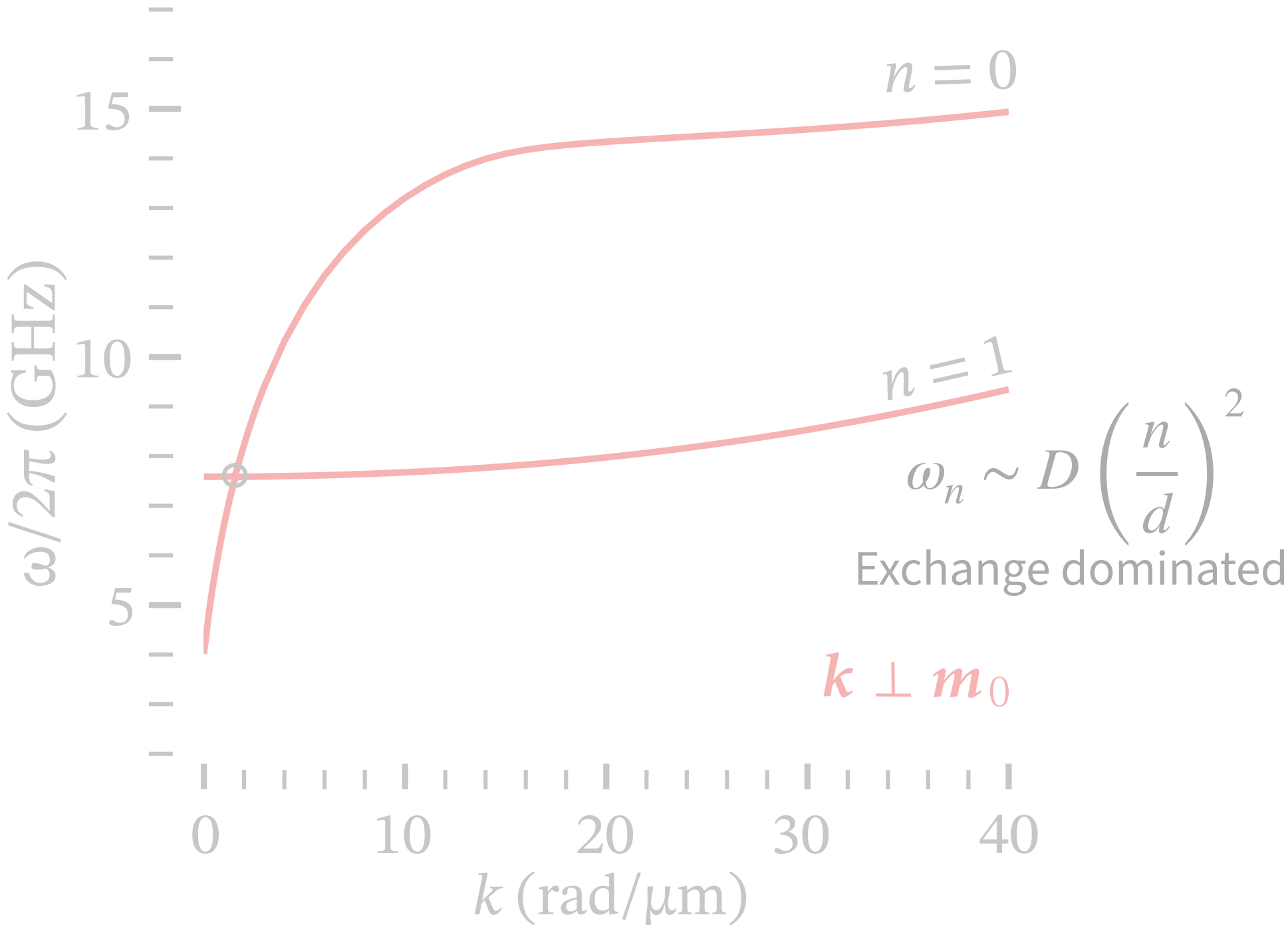
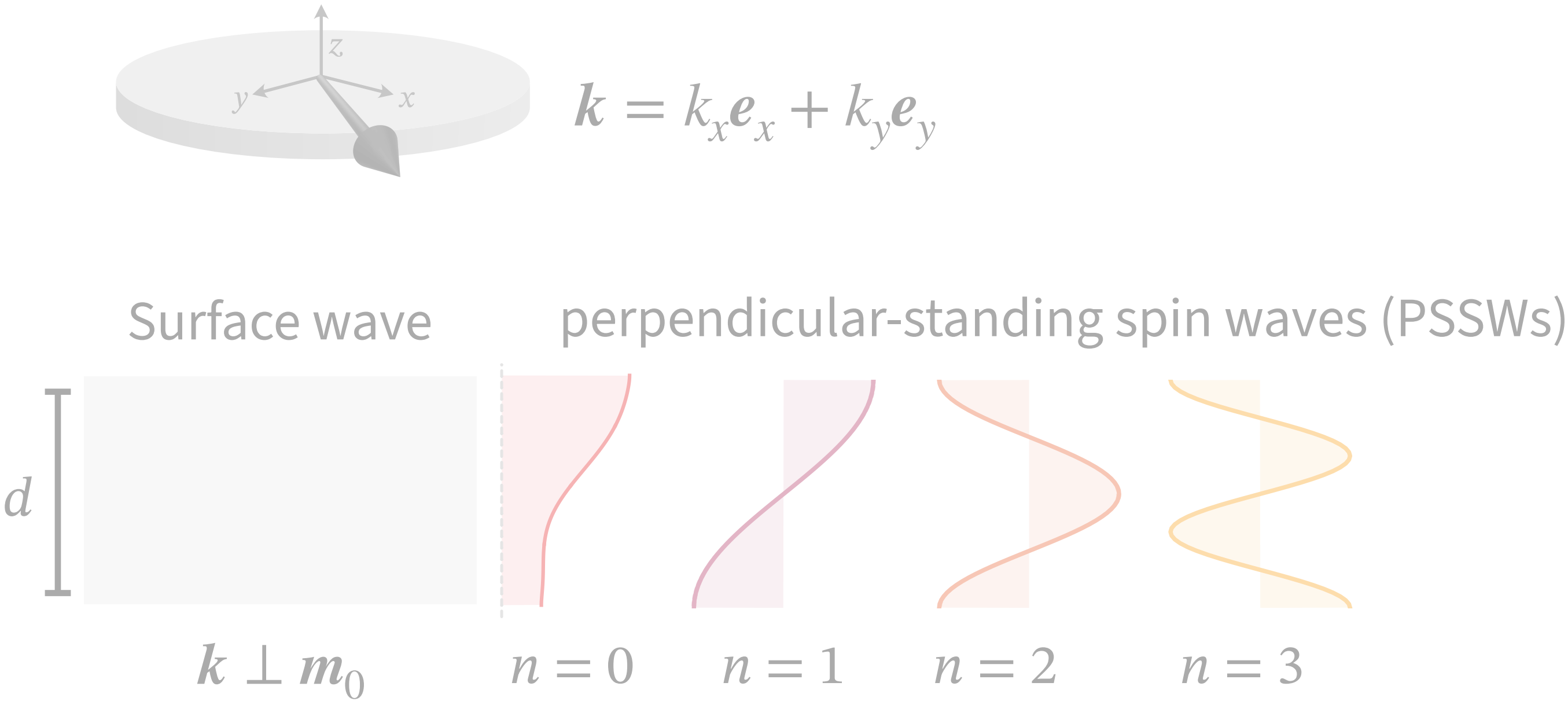
Magnons in confined and other structures

We have just talked about confined modes, without mentioning it

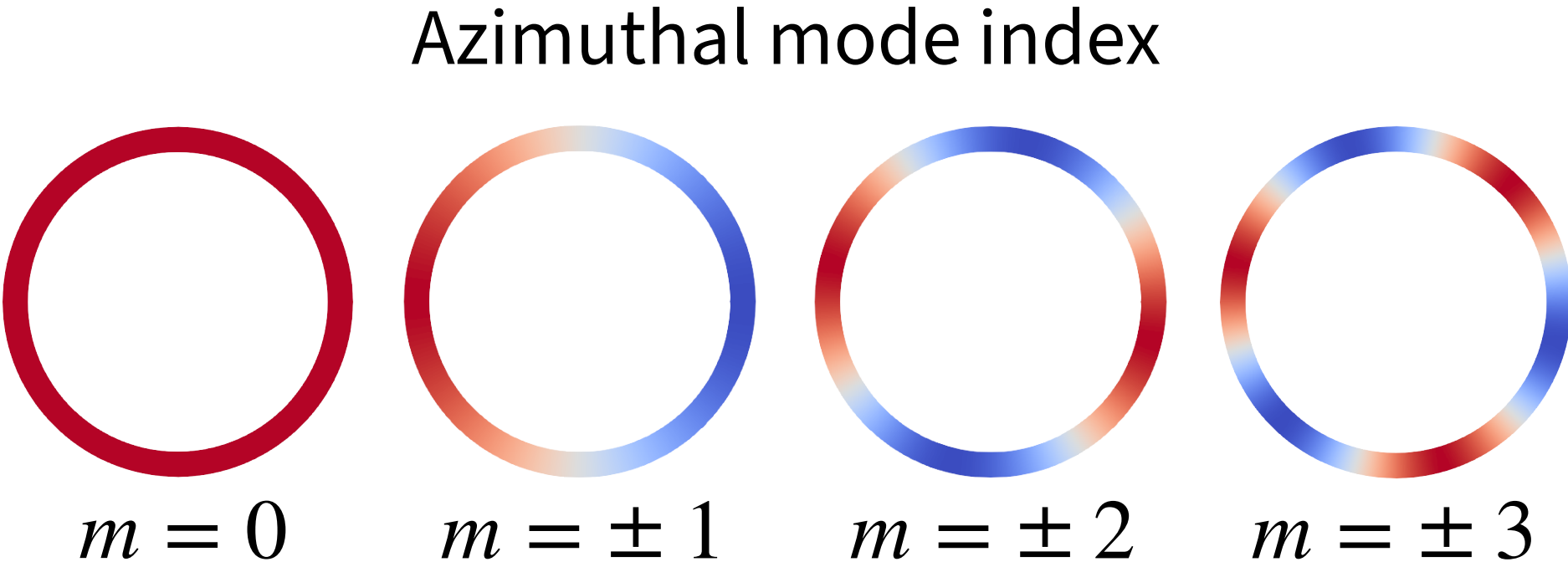


Edited from Körber et al.,
AIP Advances 12 (2022), p. 115206

Other geometries may imply other mode indices



Magnonic ring resonator

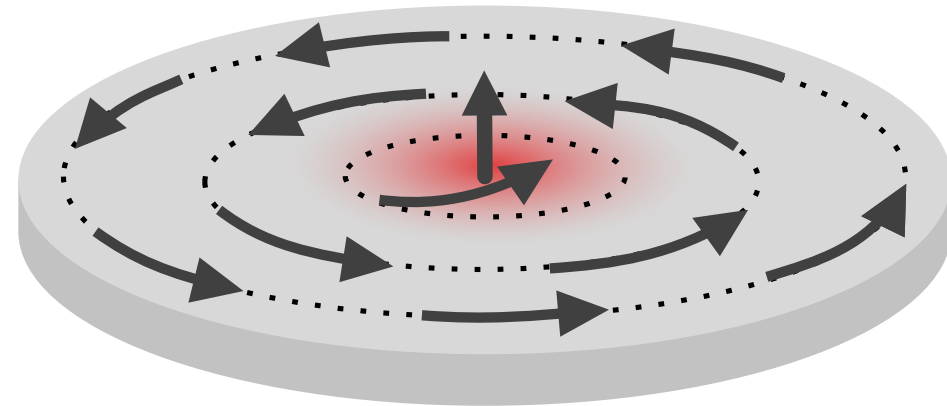


edited from Körber et al., PRB 106, 014405 (2022)

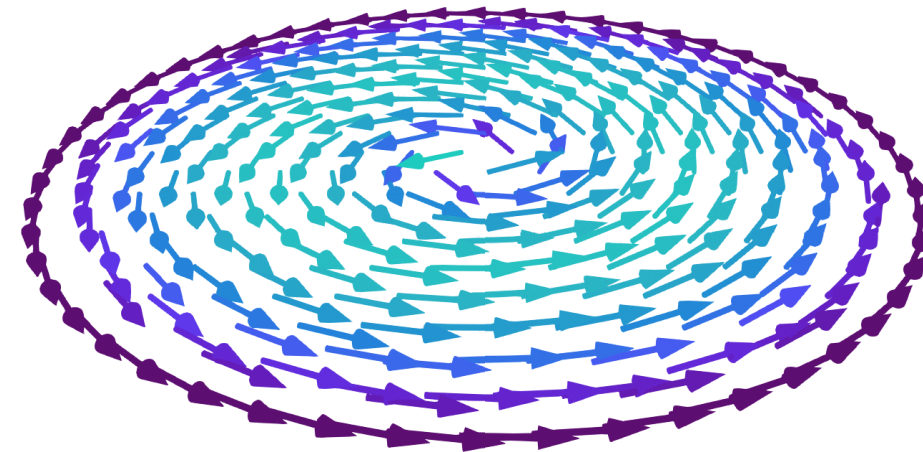
Magnons need not only live on a ferromagnetic background

e.g. azimuthal magnons in vortex

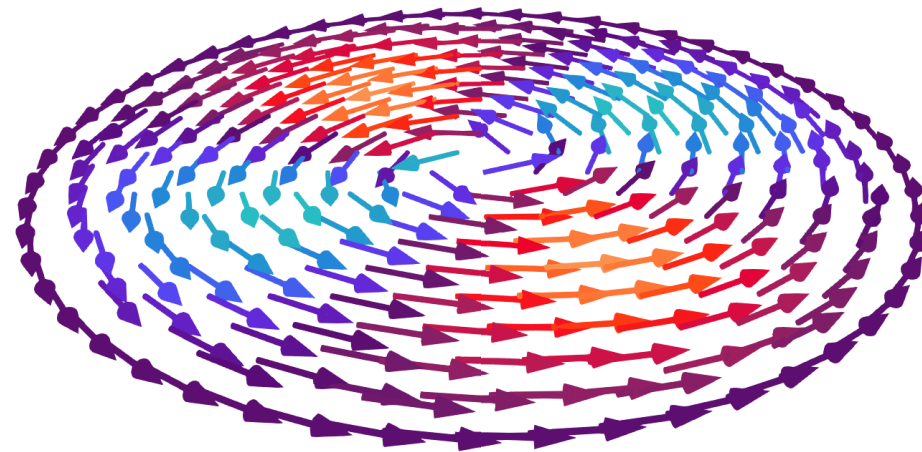
$$\mathbf{m}_0 \approx \mathbf{e}_\varphi$$



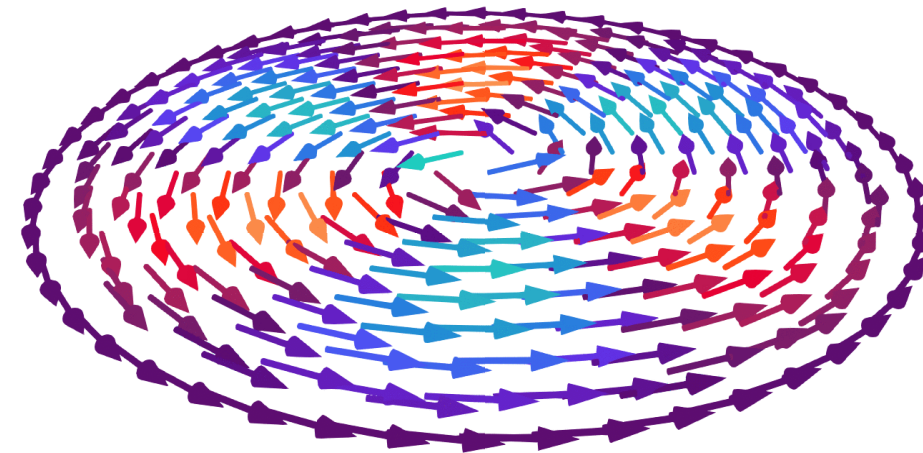
vortex



$$m = 0$$



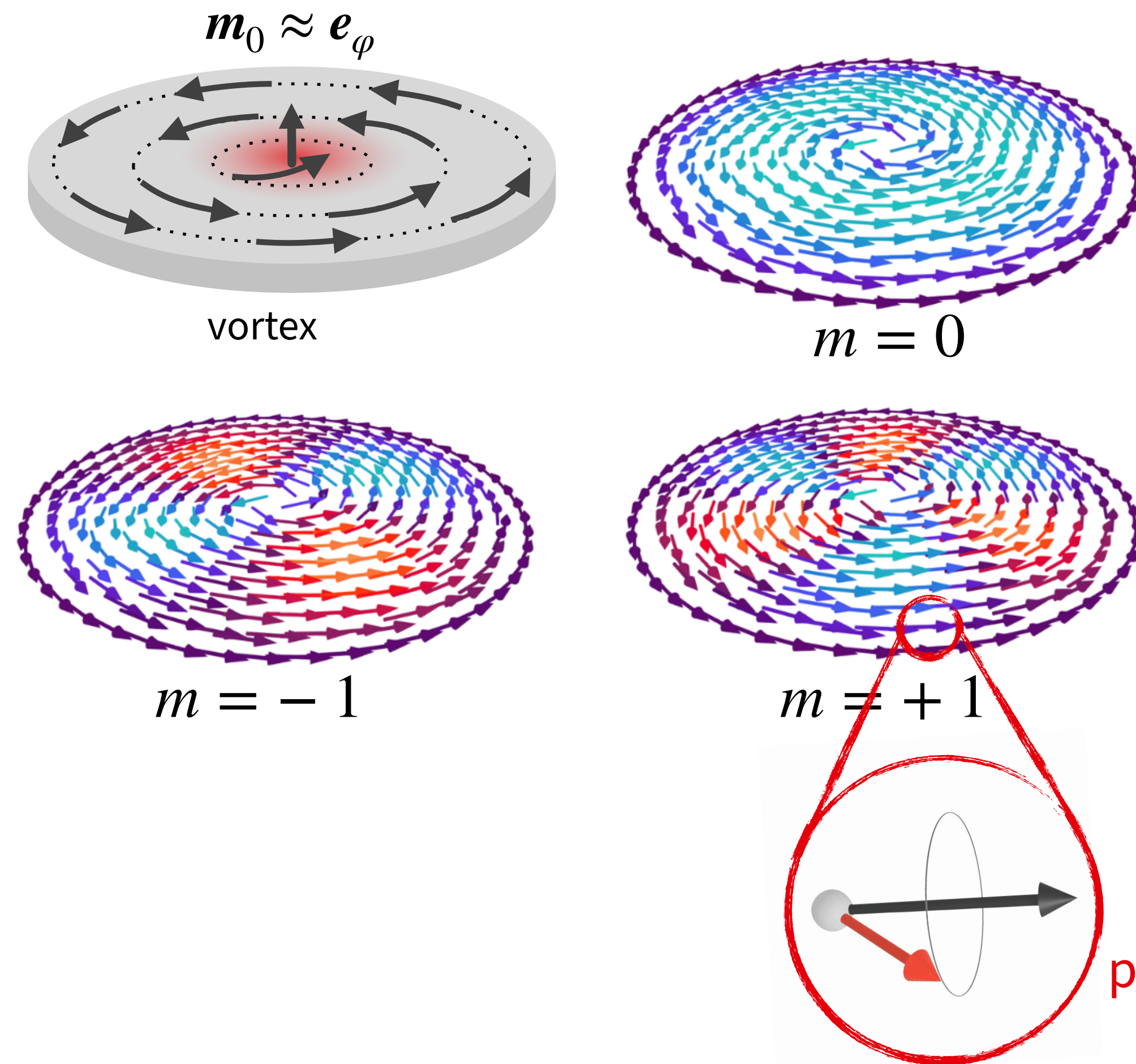
$$m = -1$$



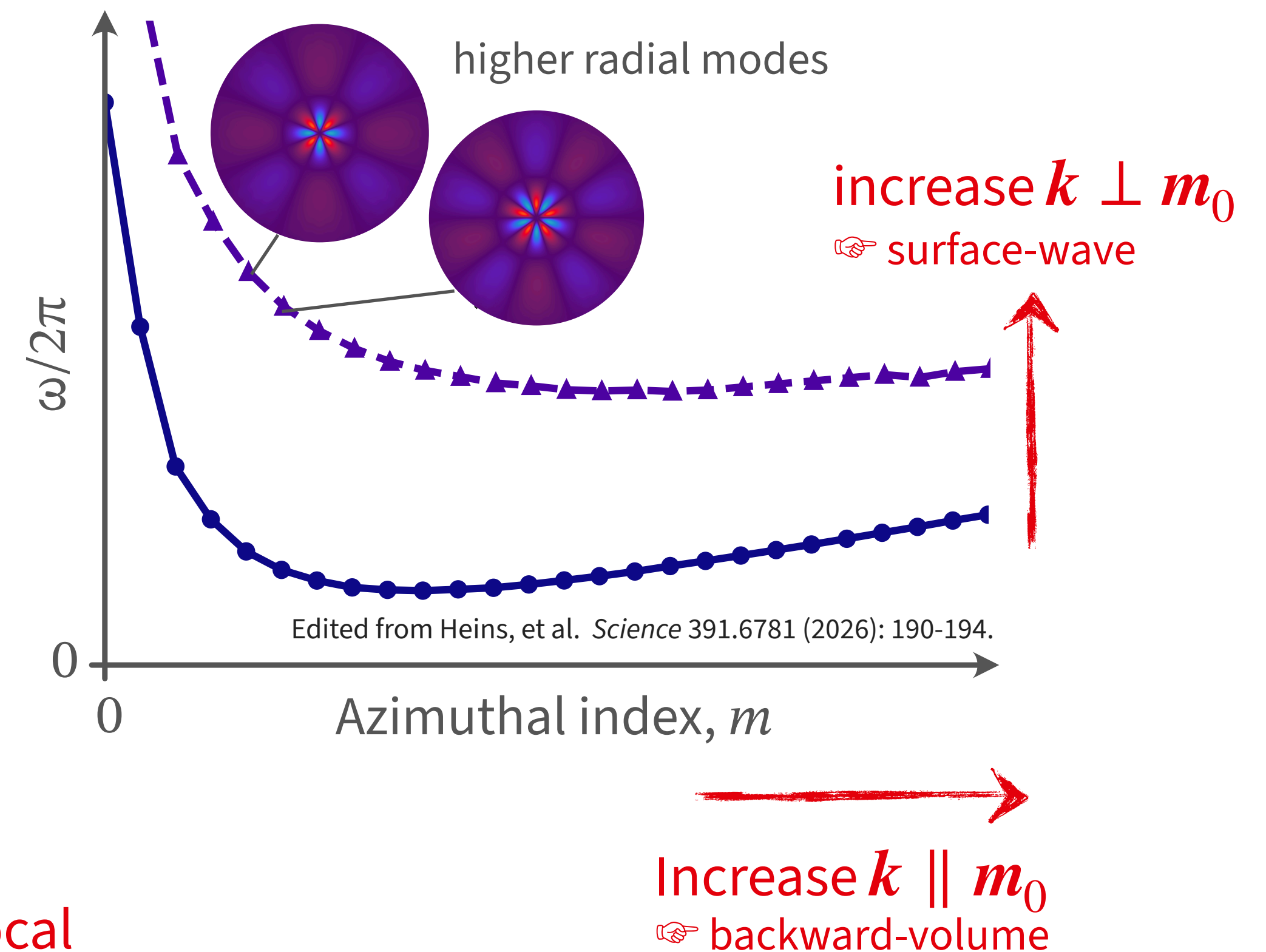
$$m = +1$$

Magnons need not only live on a ferromagnetic background

e.g. azimuthal magnons in vortex



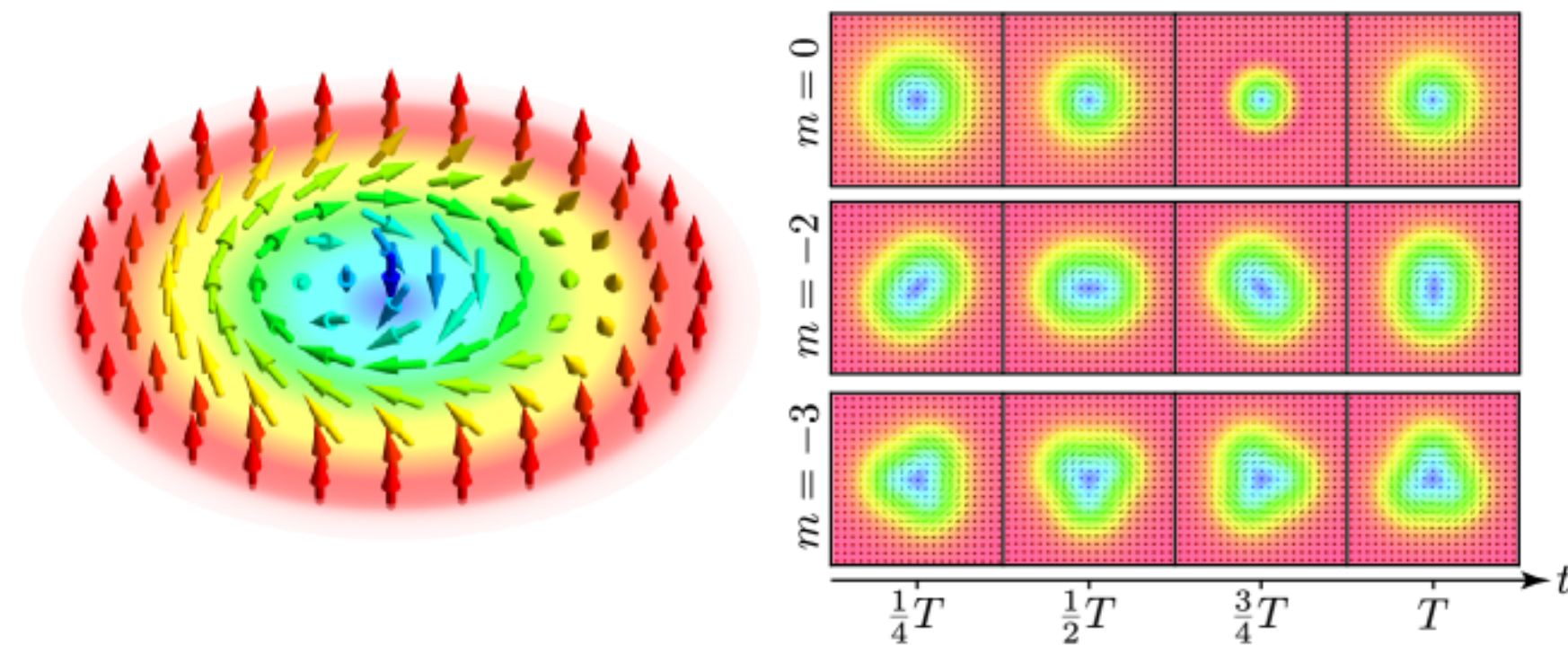
familiar dipole-exchange features recovered



Edited from Heins, et al. *Science* 391.6781 (2026): 190-194.

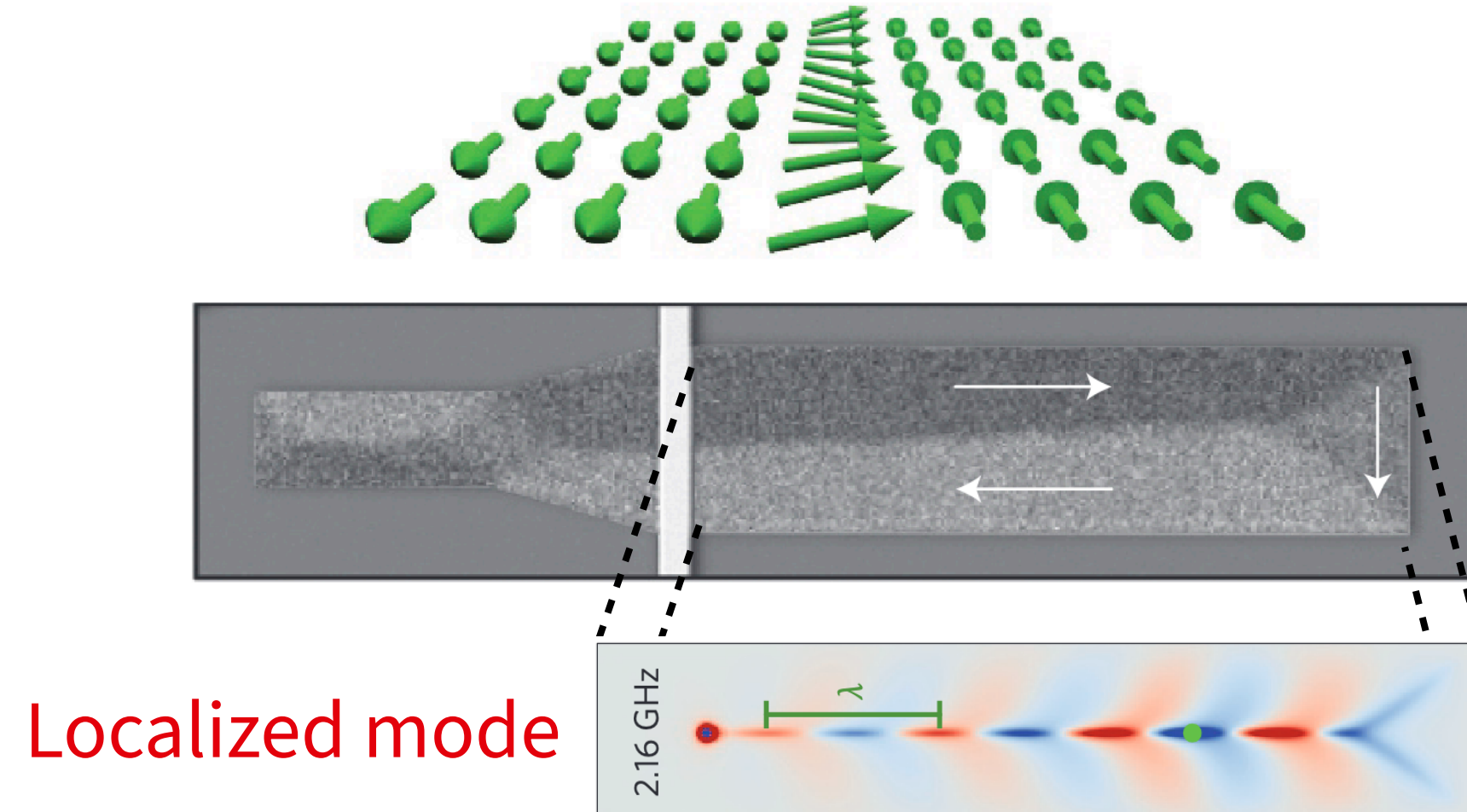
Magnons need not only live on a ferromagnetic background

Localized azimuthal modes in skyrmion



Schütte et al., Phys. Rev. B 90, 094423 (2014)

Localized propagating along domain walls

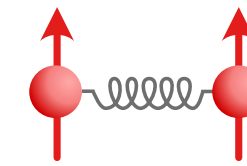


Localized mode

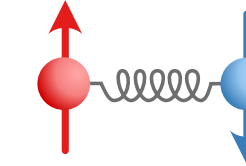
Wagner et al., Nat. Nano. 11, 432–436 (2016)

Multisublattice magnets

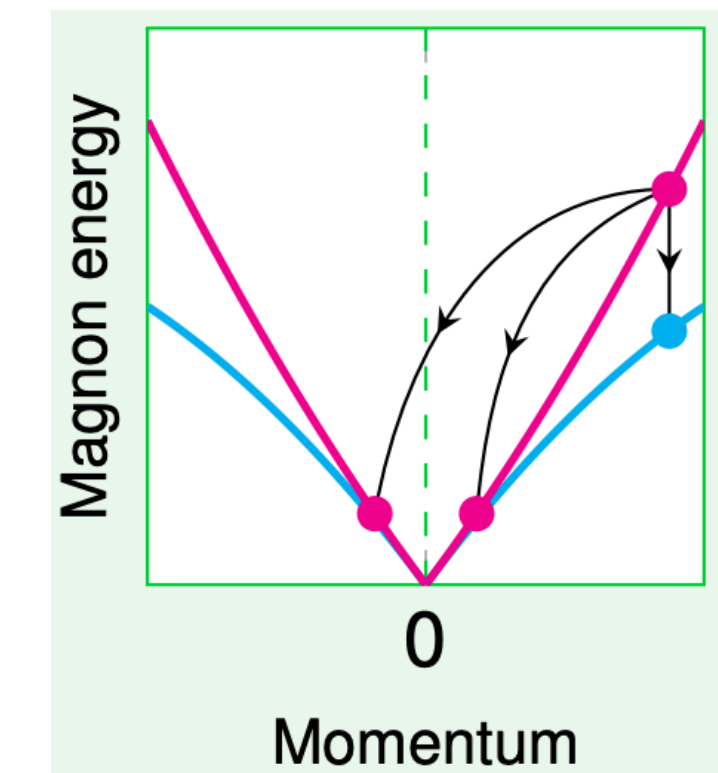
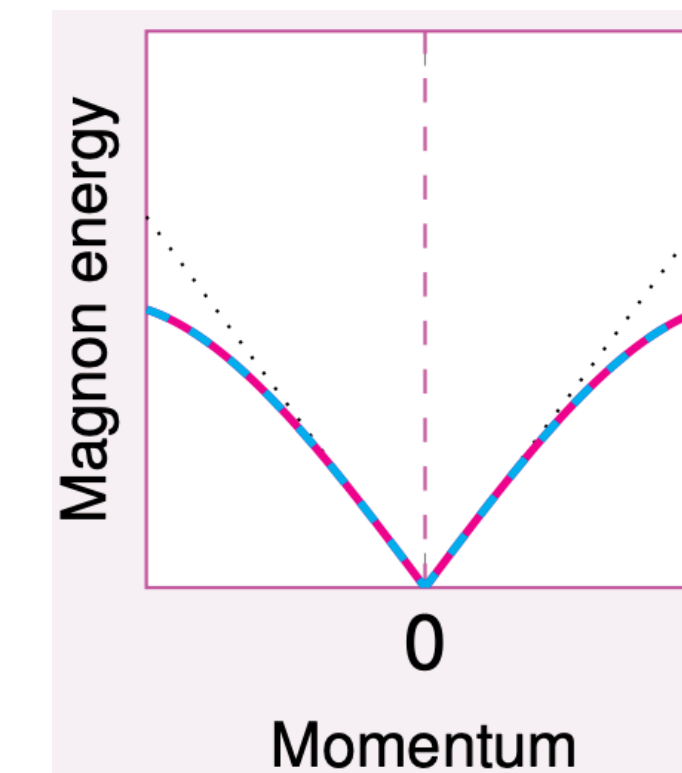
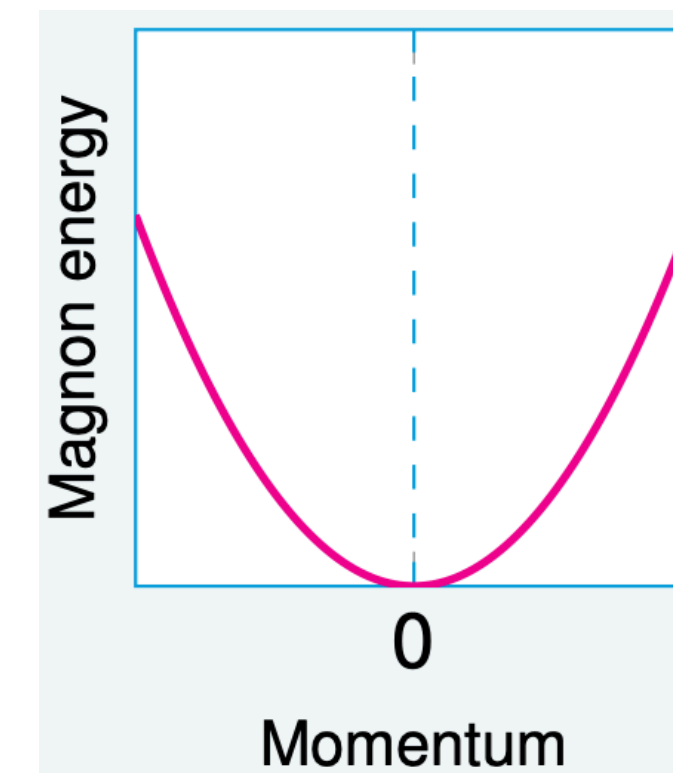
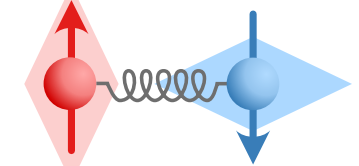
ferromagnet



antiferromagnet



altermagnet

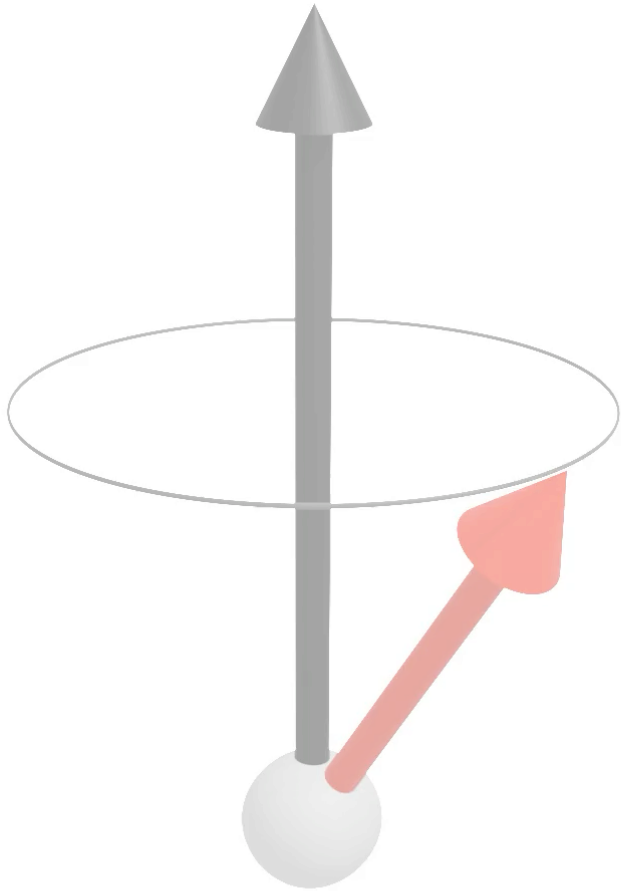


Eto et al., Phys. Rev. B **112**, 094442 (2025)

Severe quantum corrections or even quasi-particle instability possible!

Damped and driven magnon dynamics

Magnons generally do not have infinite lifetime



Effect of damping?

$$\frac{d\mathbf{m}}{dt} = \gamma \mathbf{m} \times \left(\mathbf{B}_{\text{eff}} + \frac{\alpha_G}{\gamma} \frac{d\mathbf{m}}{dt} \right)$$

$$\gamma \mathbf{B}_{\text{eff}} \rightarrow \gamma \mathbf{B}_{\text{eff}} + \alpha_G \frac{d\mathbf{m}}{dt}$$

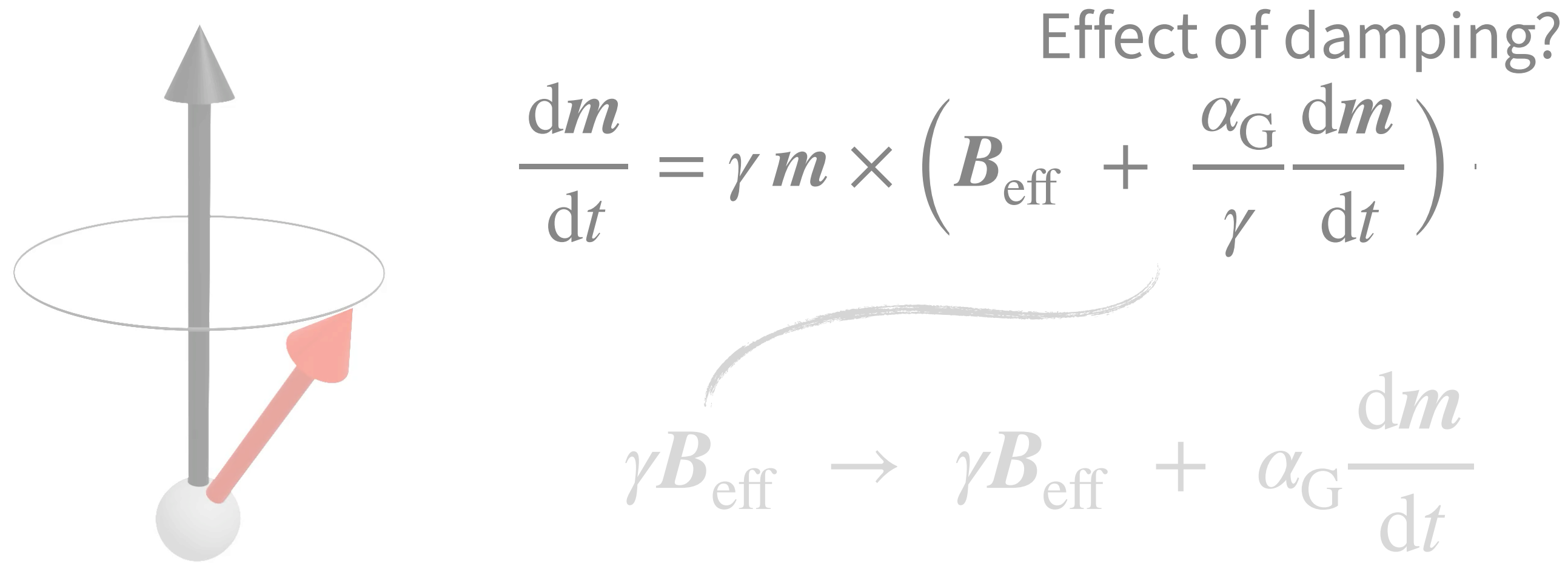
$$\frac{d(\delta\mathbf{m})}{dt} = \gamma \left(\mathbf{m}_0 \times \delta\mathbf{B}^{\text{eff}} + \delta\mathbf{m} \times \mathbf{B}_0^{\text{eff}} \right) + \alpha_G \mathbf{m}_0 \times \frac{d(\delta\mathbf{m})}{dt}$$

Linearized equation including damping

Mode frequencies acquire imaginary component (damping rate)

$$\omega \rightarrow \frac{1}{1 + \alpha_G^2} (\omega - i\Gamma) \approx \omega - i\Gamma$$

Magnons generally do not have infinite lifetime

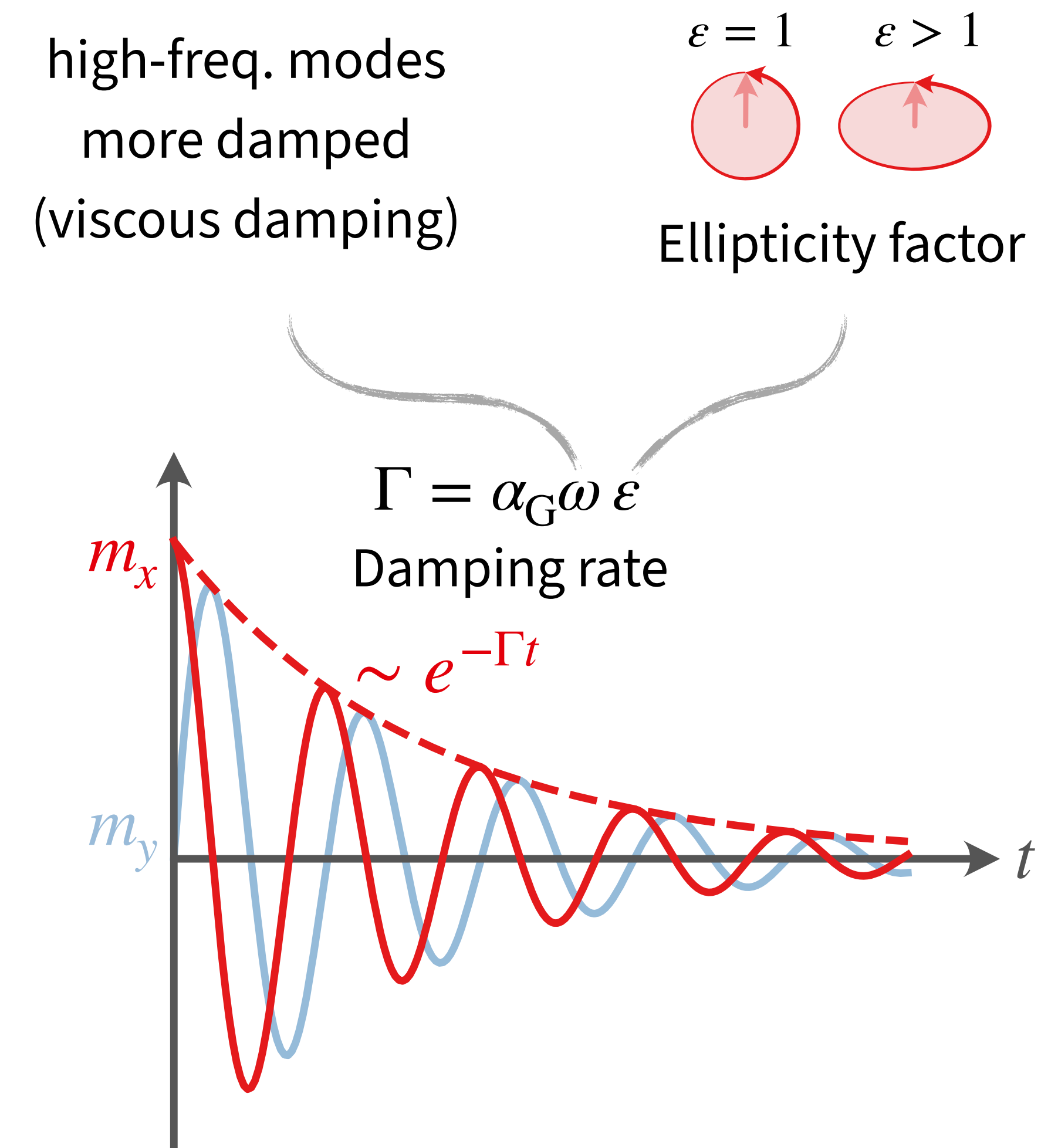


$$\frac{d(\delta \mathbf{m})}{dt} = \gamma \left(\mathbf{m}_0 \times \delta \mathbf{B}^{\text{eff}} + \delta \mathbf{m} \times \mathbf{B}_0^{\text{eff}} \right) + \alpha_G \mathbf{m}_0 \times \frac{d(\delta \mathbf{m})}{dt}$$

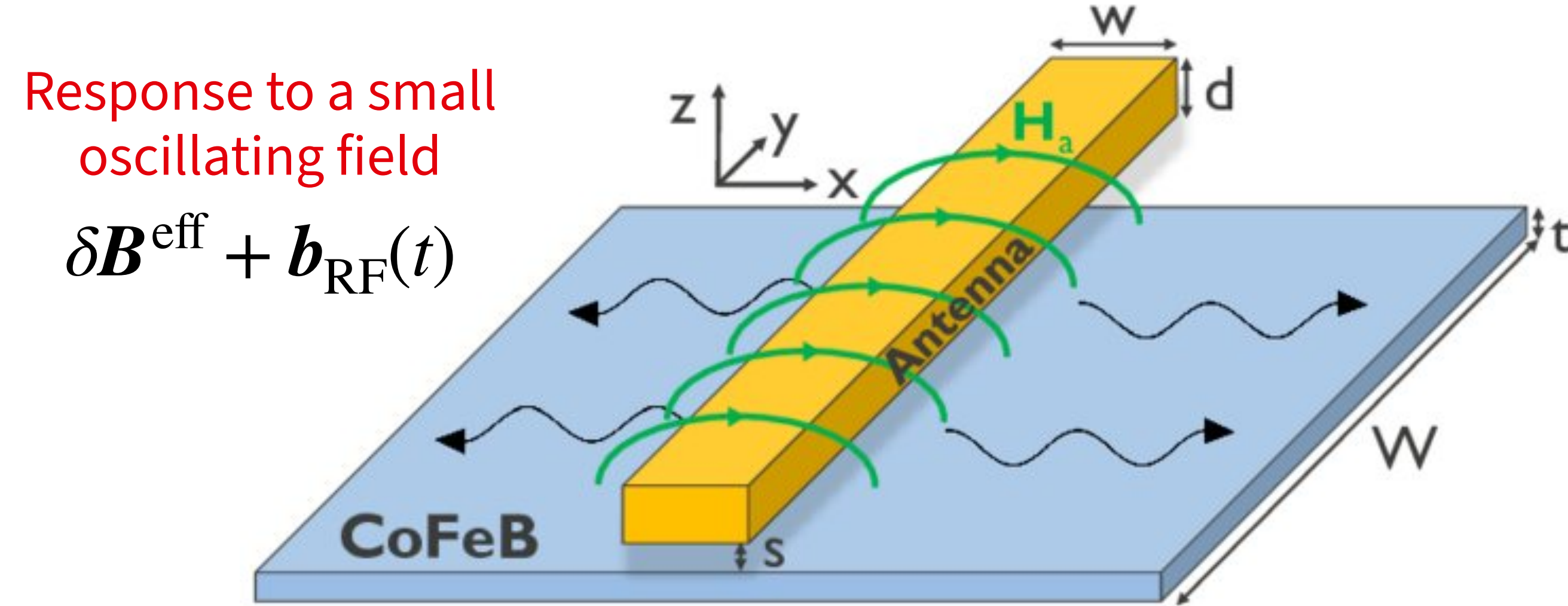
Linearized equation including damping

Mode frequencies acquire imaginary component (damping rate)

$$\omega \rightarrow \frac{1}{1 + \alpha_G^2} (\omega - i\Gamma) \approx \omega - i\Gamma$$



We can measure magnon lifetime, e.g., in absorption experiments



Vanderveken et al., Sci Rep 12, 3796 (2022)

We can measure magnon lifetime, e.g., in absorption experiments

Response to a small
oscillating field

$$\frac{d(\delta \mathbf{m})}{dt} = \gamma \left[\mathbf{m}_0 \times (\delta \mathbf{B}^{\text{eff}} + \mathbf{b}_{\text{RF}}(t)) + \delta \mathbf{m} \times \mathbf{B}_0^{\text{eff}} \right] + \alpha_G \mathbf{m}_0 \times \frac{d(\delta \mathbf{m})}{dt}$$

Linearized equation including damping

rewrite $\rightarrow$ Linear response

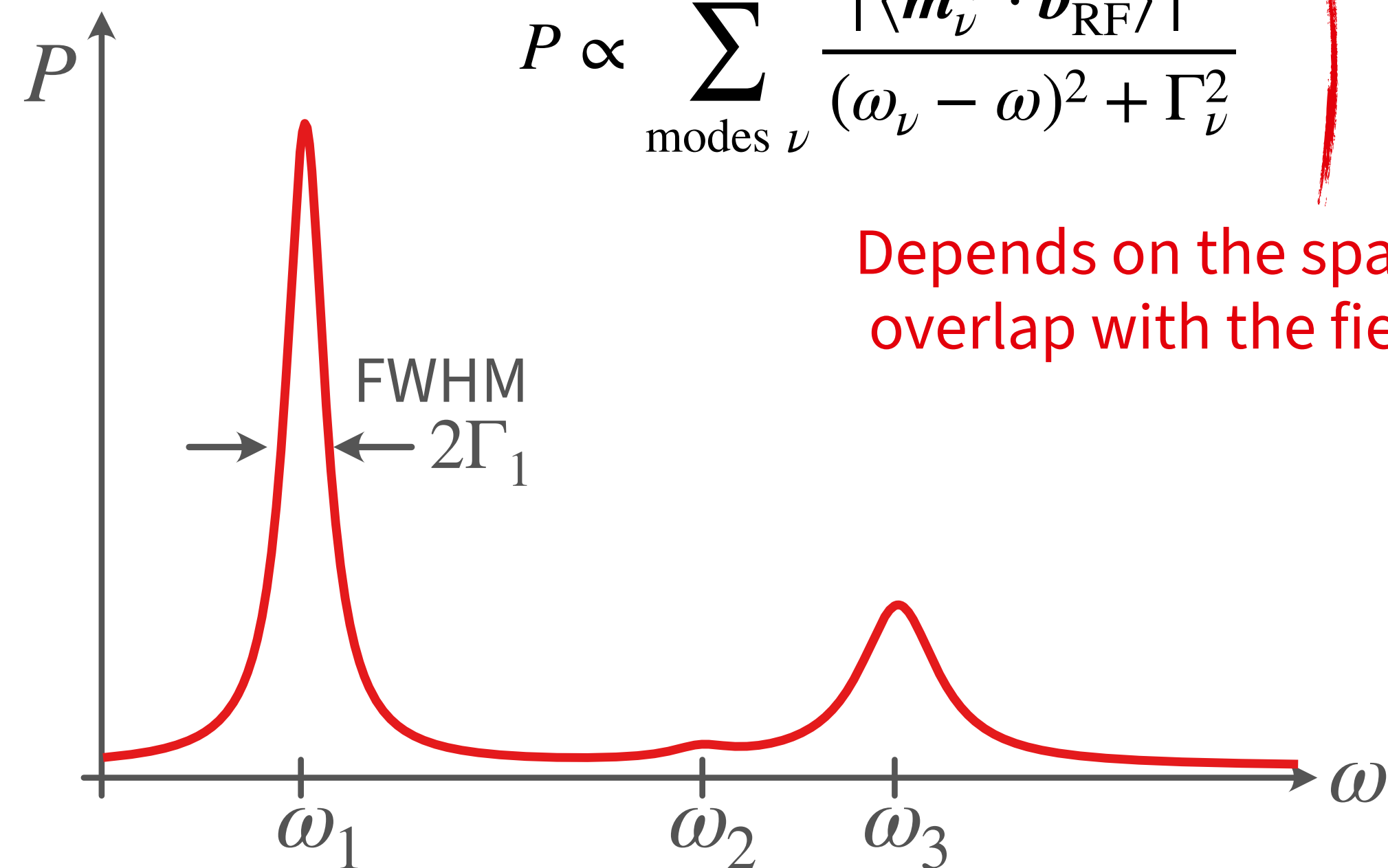
$$\delta \mathbf{m} = \hat{\chi}(\omega) \mathbf{b}_{\text{RF}}$$

**High-frequency (Polder)
susceptibility**

$$\hat{\chi} = \hat{\chi}' + i\hat{\chi}''$$

„Dispersion“ Absorption

Absorbed power



Depends on the spatial
overlap with the field!

We can measure magnon lifetime, e.g., in absorption experiments

Response to a small
oscillating field

$$\frac{d(\delta \mathbf{m})}{dt} = \gamma \left[\mathbf{m}_0 \times (\delta \mathbf{B}^{\text{eff}} + \mathbf{b}_{\text{RF}}(t)) + \delta \mathbf{m} \times \mathbf{B}_0^{\text{eff}} \right] + \alpha_G \mathbf{m}_0 \times \frac{d(\delta \mathbf{m})}{dt}$$

Linearized equation including damping

rewrite $\rightarrow$ Linear response

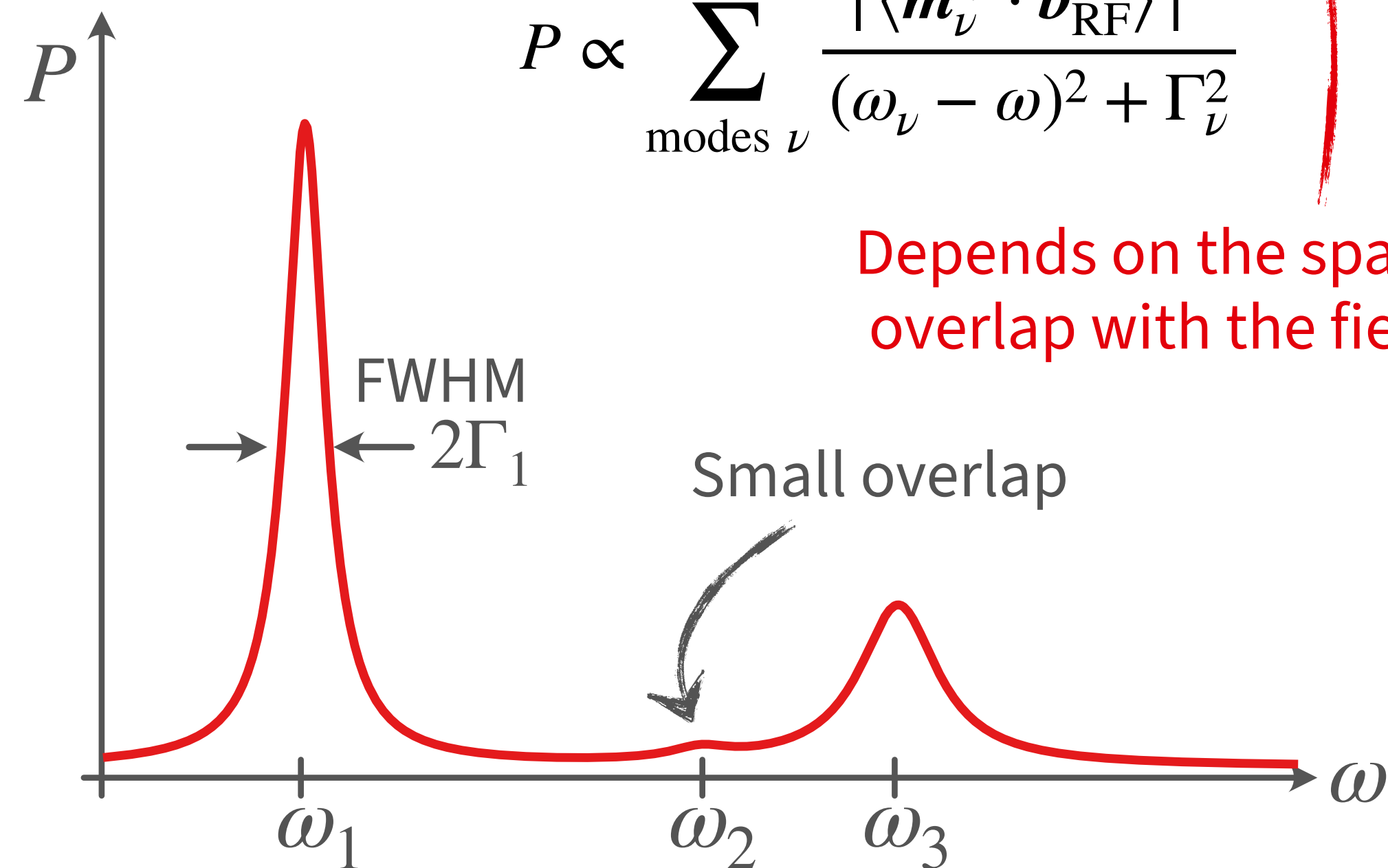
$$\delta \mathbf{m} = \hat{\chi}(\omega) \mathbf{b}_{\text{RF}}$$

**High-frequency (Polder)
susceptibility**

$$\hat{\chi} = \hat{\chi}' + i\hat{\chi}''$$

„Dispersion“ Absorption

Absorbed power



Depends on the spatial
overlap with the field!

Linear spin-wave theory fails at larger driving powers

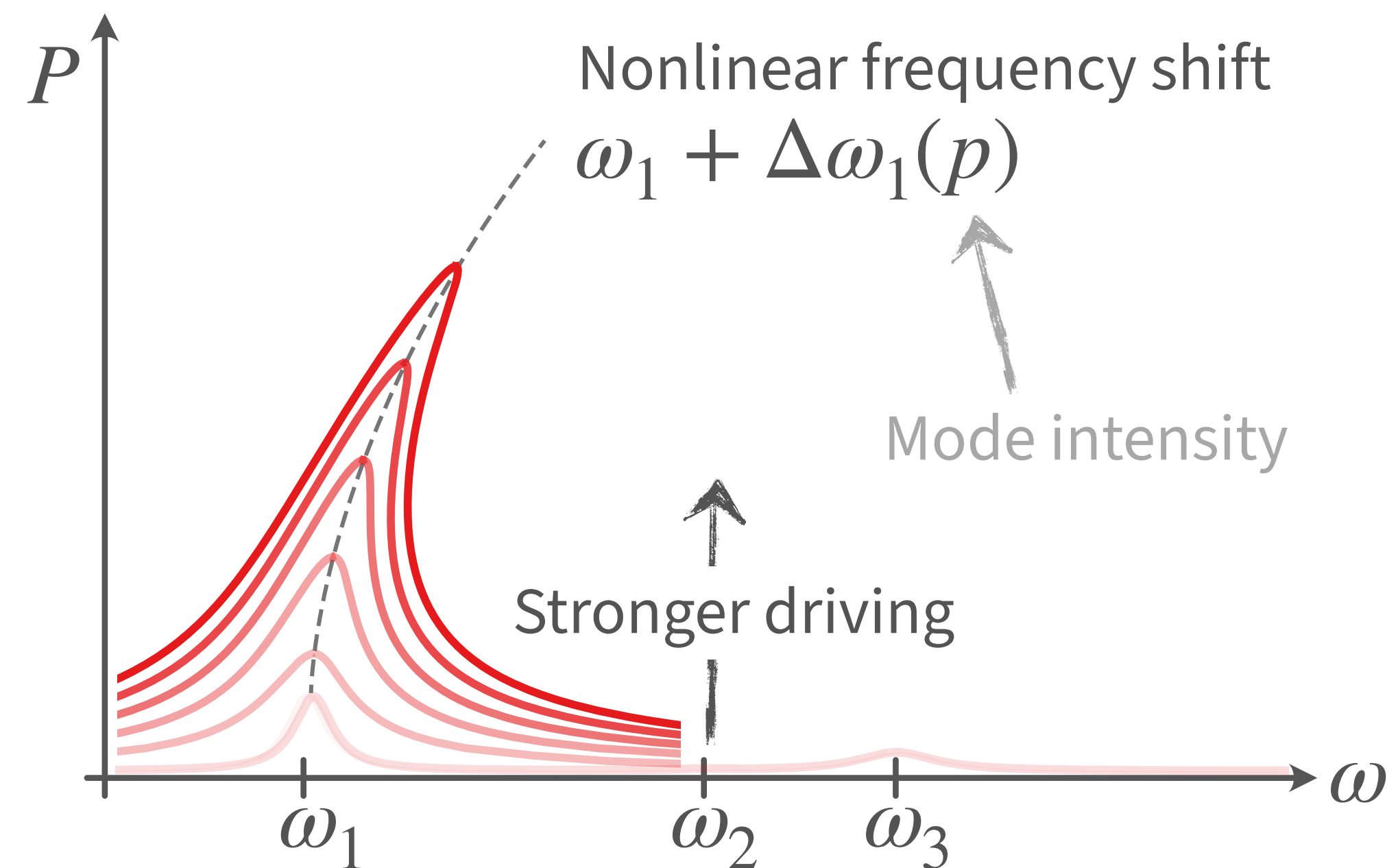
Now longer valid

$$\frac{d(\delta \mathbf{m})}{dt} = \gamma \left[\mathbf{m}_0 \times (\delta \mathbf{B}^{\text{eff}} + \mathbf{b}_{\text{RF}}(t)) + \delta \mathbf{m} \times \mathbf{B}_0^{\text{eff}} \right] + \alpha_G \mathbf{m}_0 \times \frac{d(\delta \mathbf{m})}{dt}$$

Linearized equation including damping

rewrite $\rightarrow$ Linear response
 $\delta \mathbf{m} = \hat{\chi}(\omega) \mathbf{b}_{\text{RF}}$

What happens at larger driving powers?



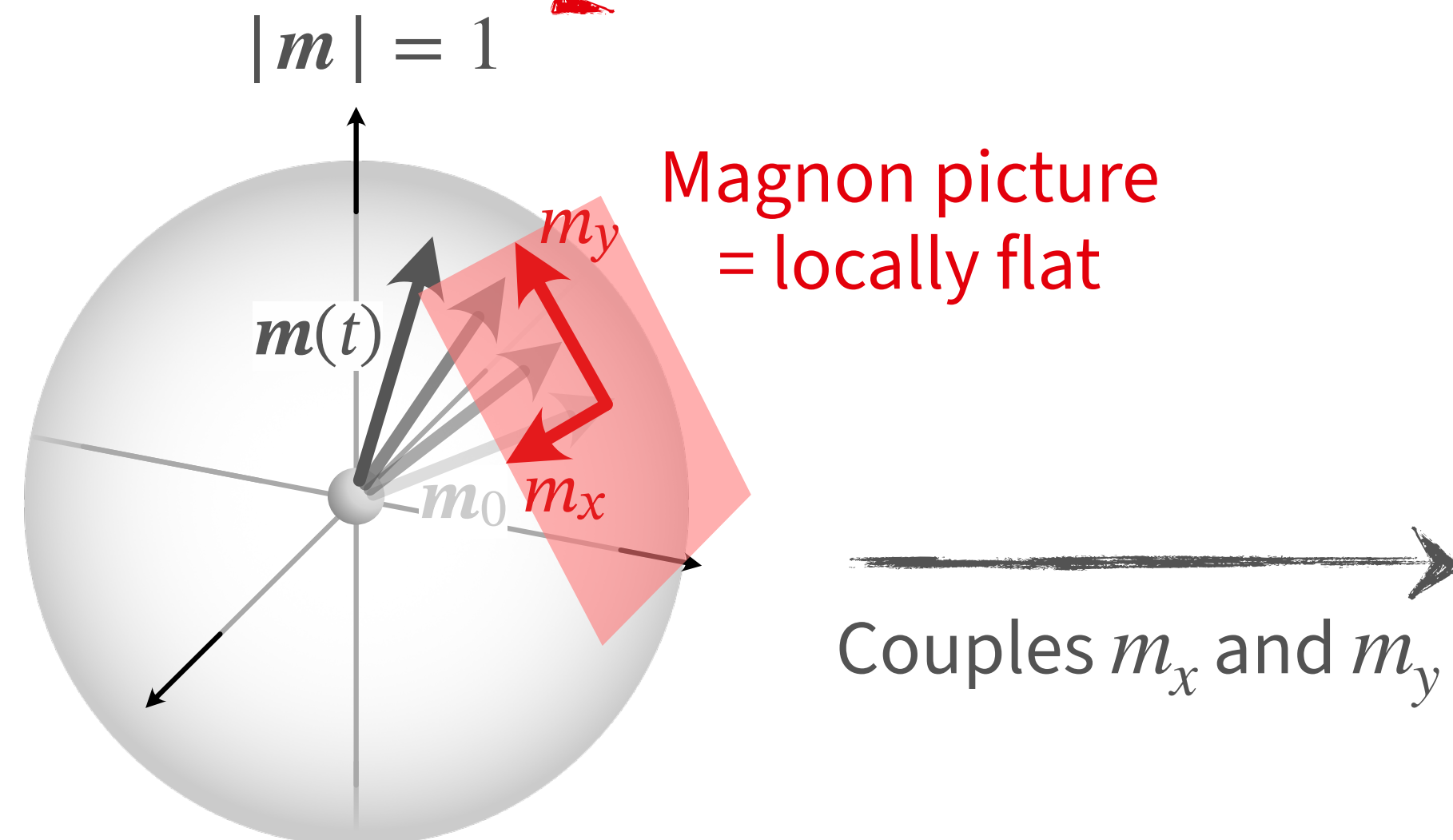
Spin/magnetization dynamics is intrinsically nonlinear

Full Landau-Lifshitz Gilbert equation

$$\rightarrow \frac{d\mathbf{m}}{dt} = \gamma \mathbf{m} \times \mathbf{B}_{\text{eff}}(\mathbf{m}) + \alpha_G \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Nonlinear!

because



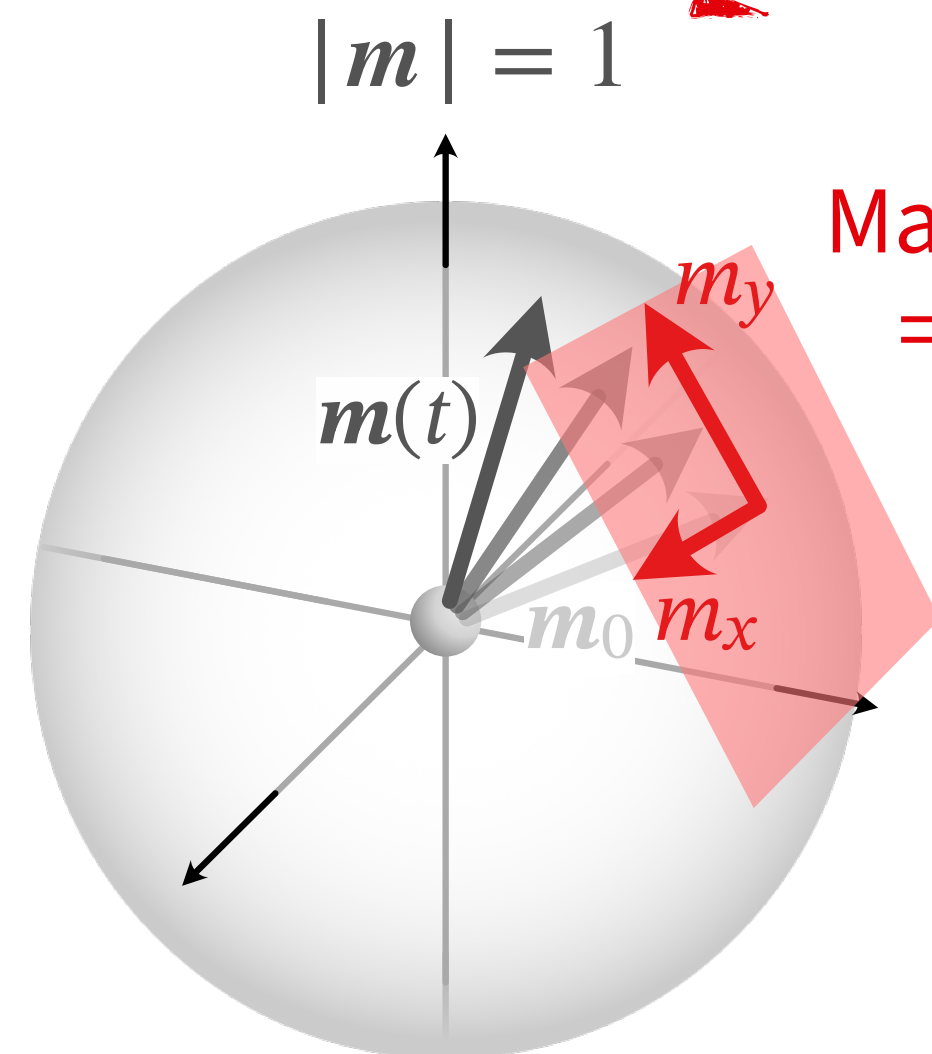
Spin/magnetization dynamics is intrinsically nonlinear

Full Landau-Lifshitz Gilbert equation

$$\frac{d\mathbf{m}}{dt} = \gamma \mathbf{m} \times \mathbf{B}_{\text{eff}}(\mathbf{m}) + \alpha_G \mathbf{m} \times \frac{d\mathbf{m}}{dt}$$

Nonlinear!

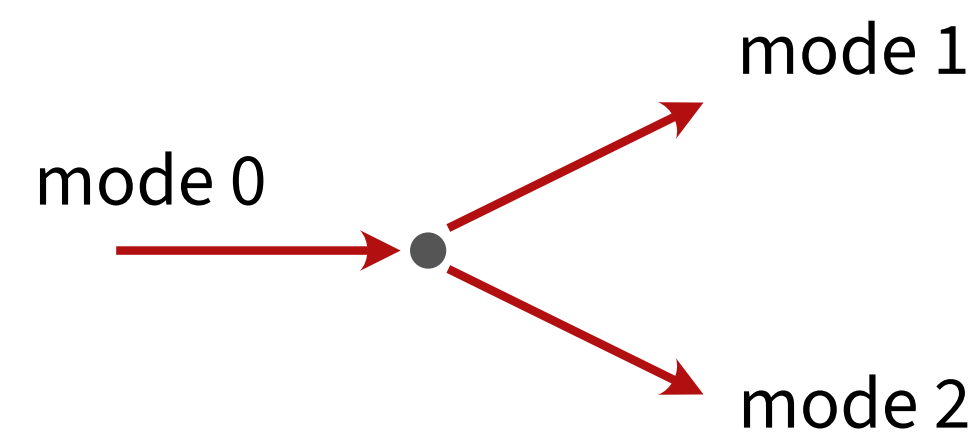
because



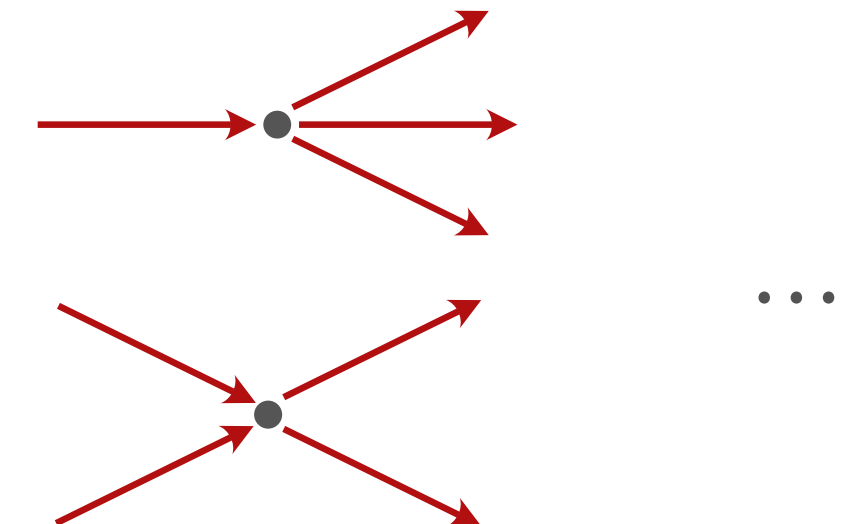
Magnon picture
= locally flat

Couples m_x and m_y

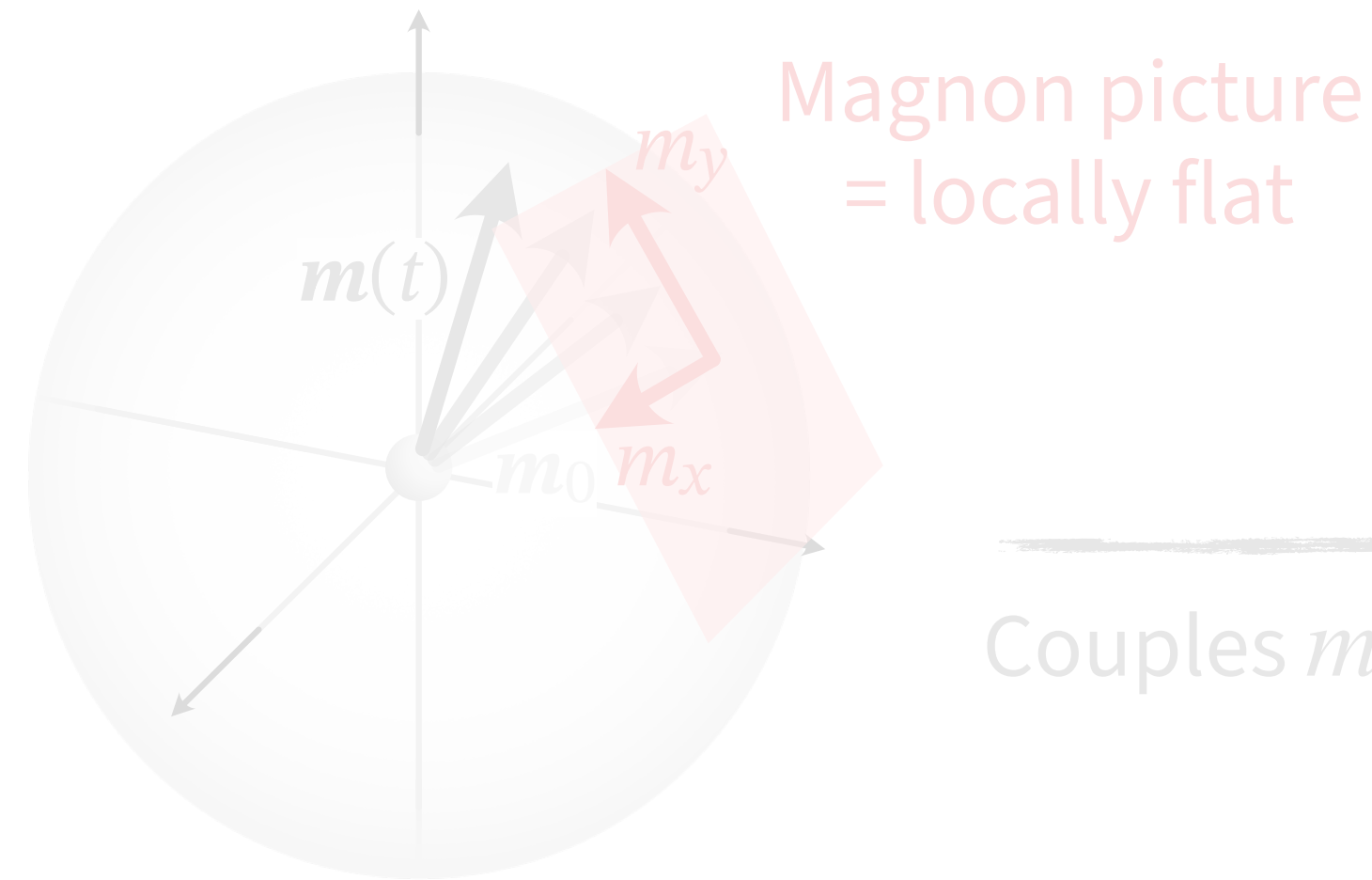
three-magnon
interactions



four-magnon
interactions

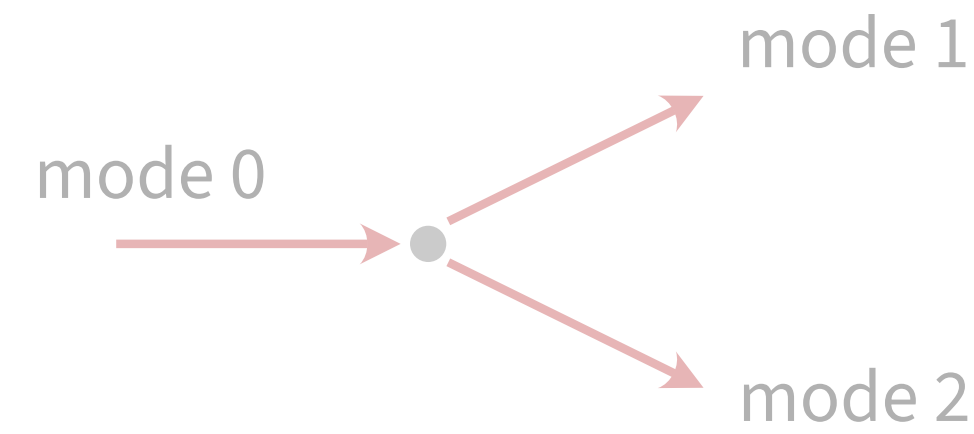


Nonlinear magnon interactions give rise to a wide zoo of phenomena

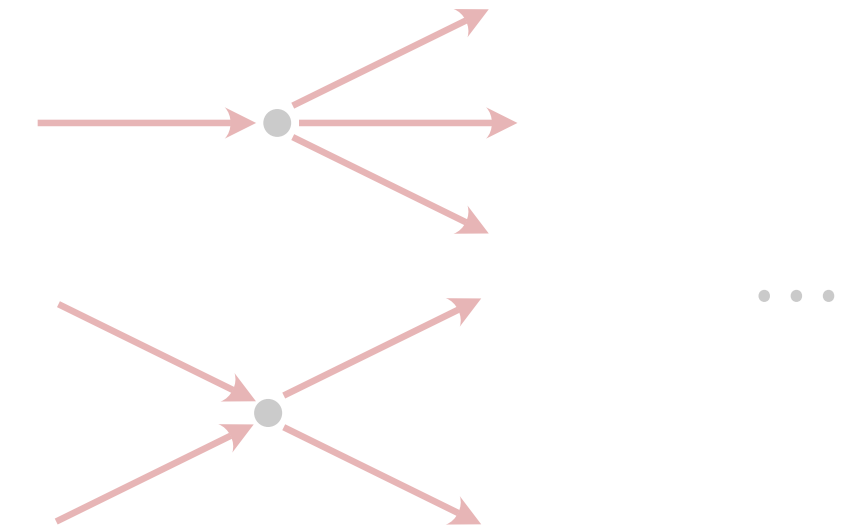


Couples m_x and m_y

three-magnon
interactions

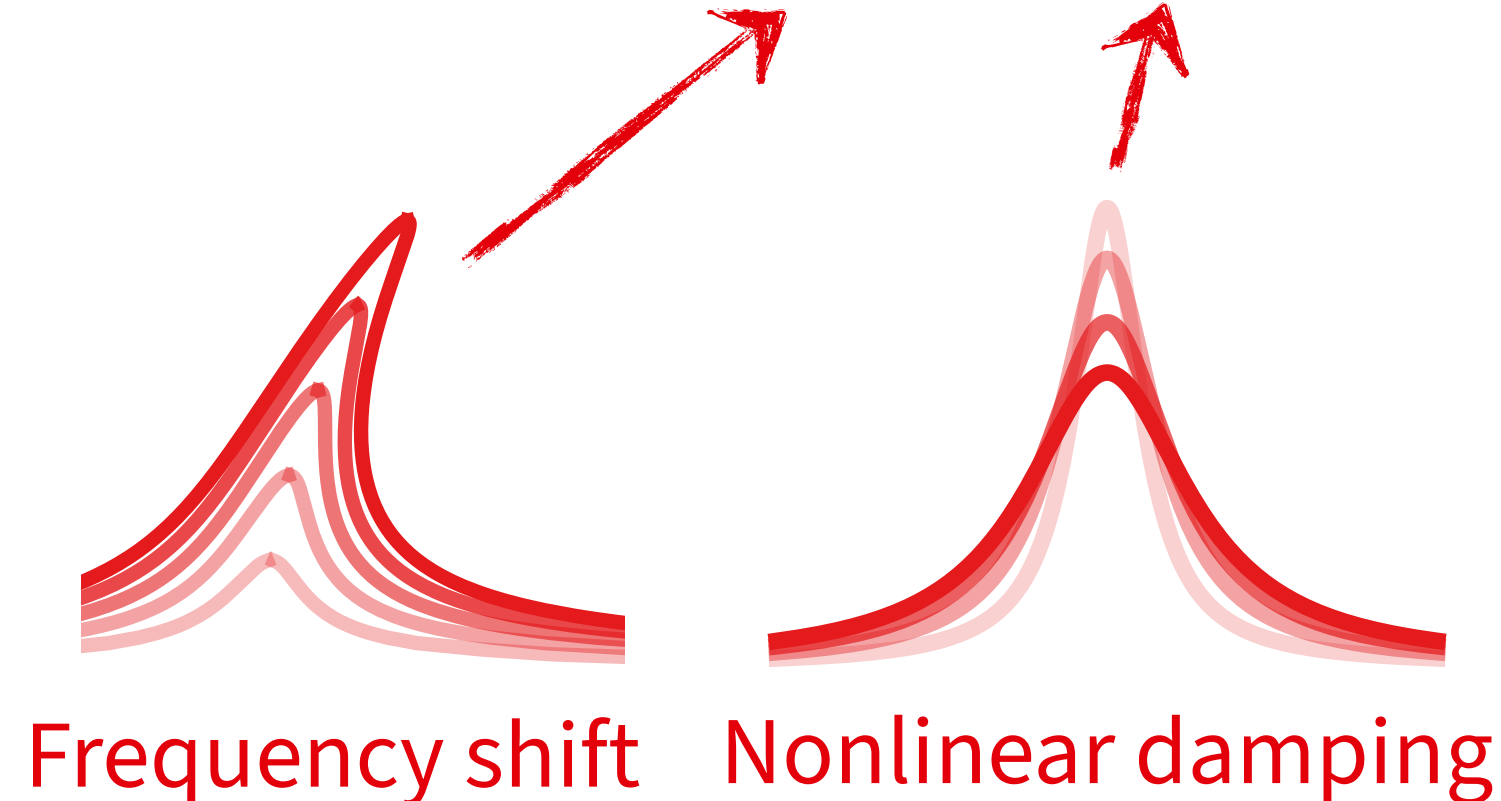


four-magnon
interactions

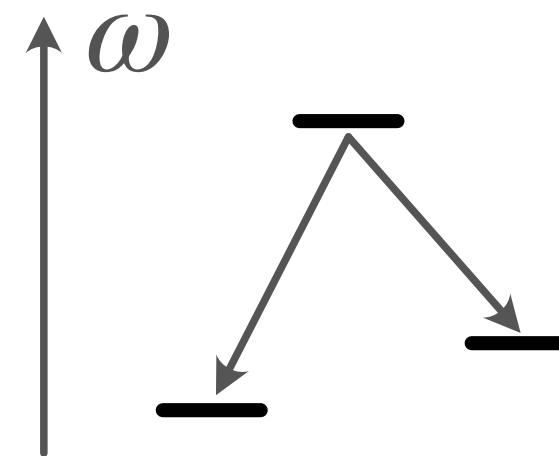


**Real + Imaginary
nonlinear corrections possible**

$$\omega_{\text{NL}} = \omega + (\Delta\omega - i\Delta\Gamma)$$



**Frequency/wave-
vector conversion**

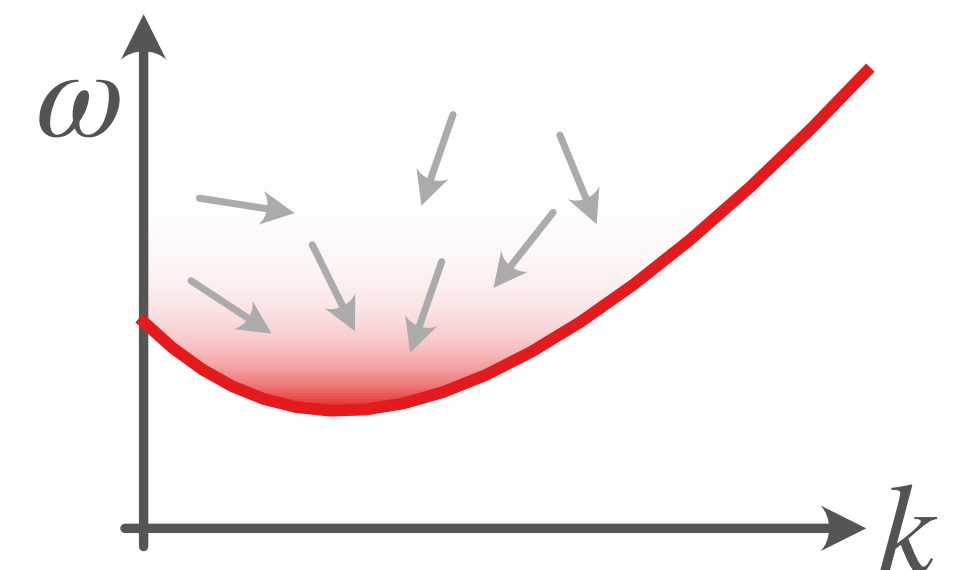


Conservation laws!
e.g., $\sum \omega_{\text{in}} = \sum \omega_{\text{out}}$

**Supercriticality,
Auto-oscillations**



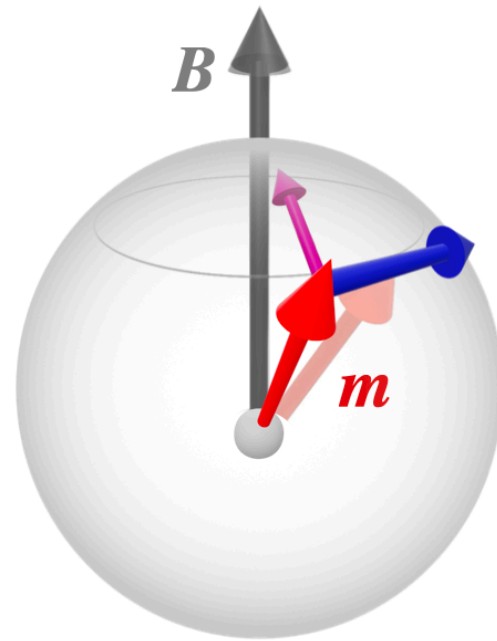
**Rayleigh-Jeans
wave condensation**



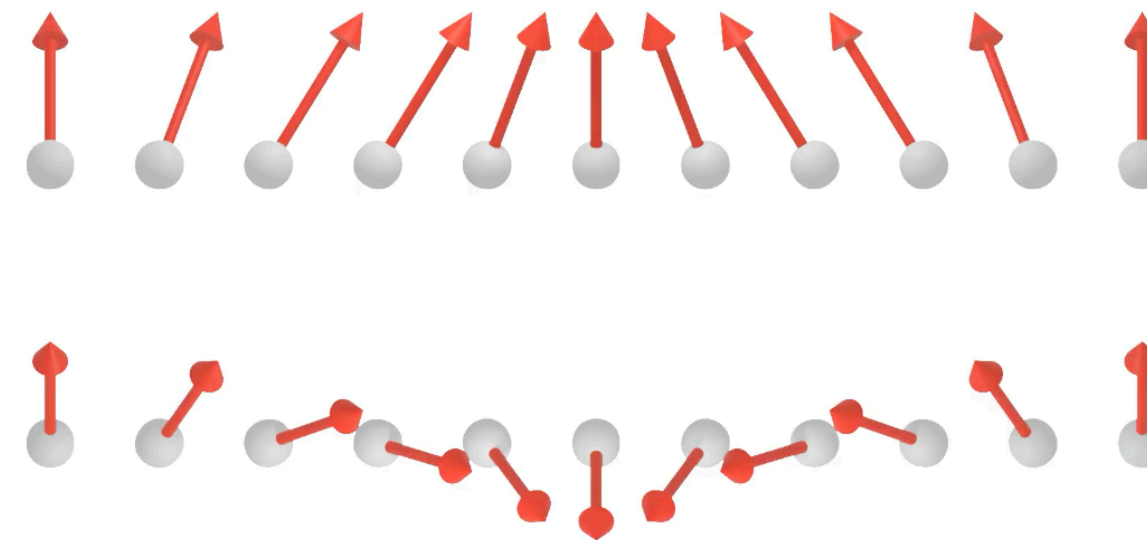
And much, much more...

Summary

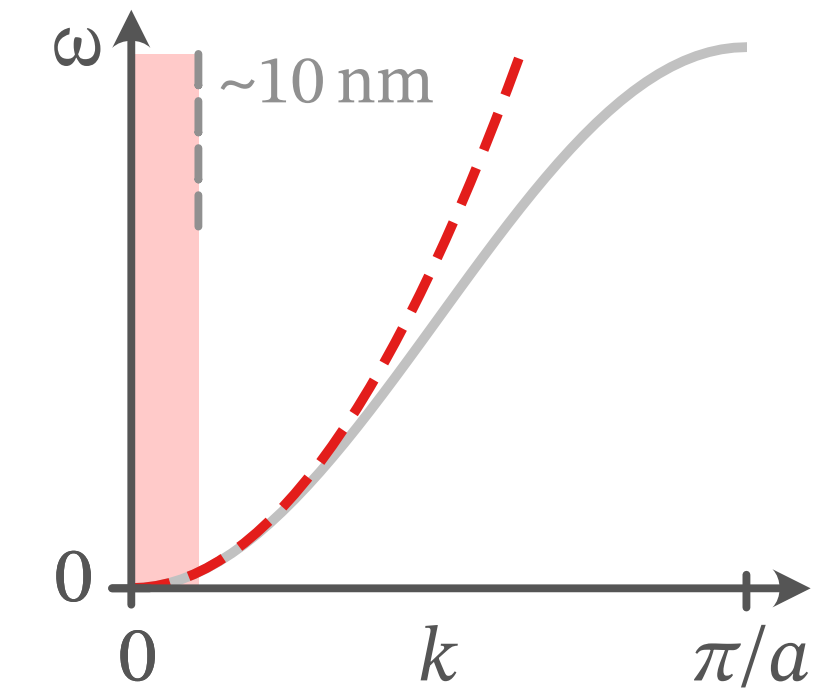
Precessional dynamics



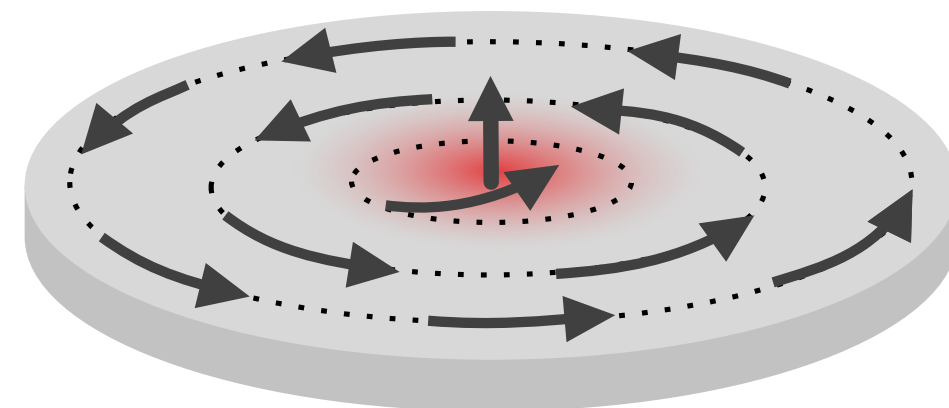
Magnons



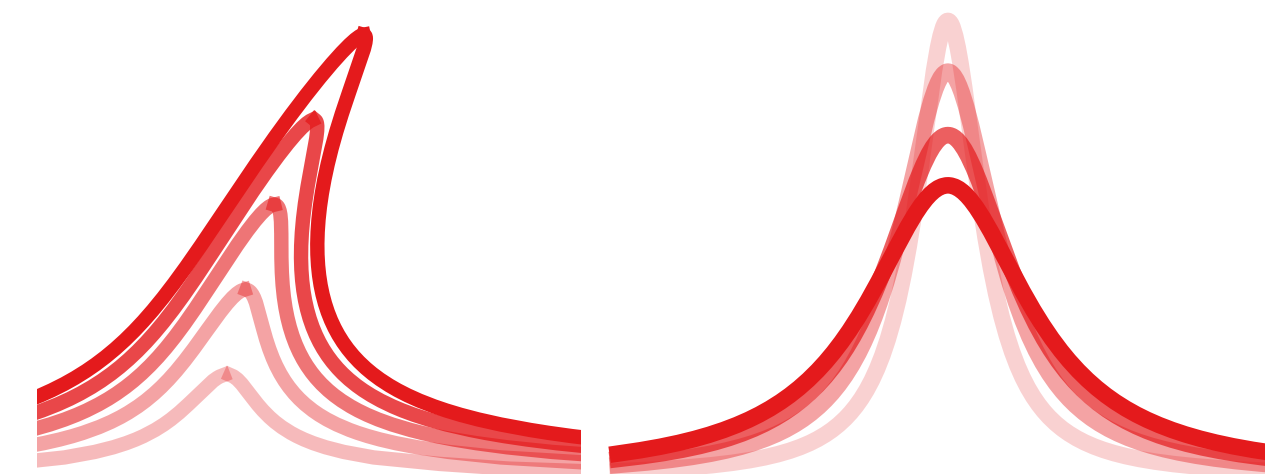
Long-wavelength limit



Confined magnons



Damped & driven



Thank you for your attention!