



Magnetisation dynamics

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and

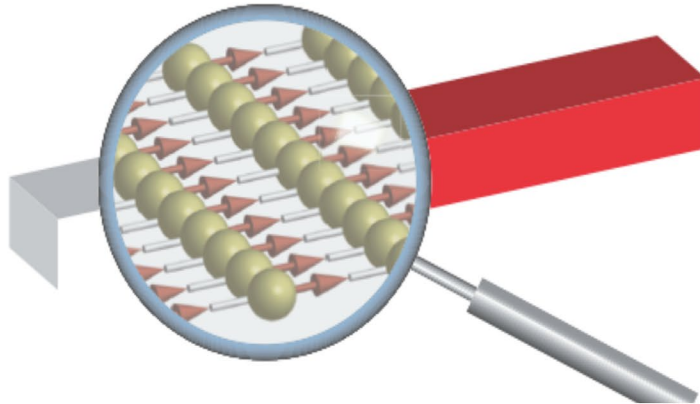
WISE-Wallenberg Initiative Materials Science



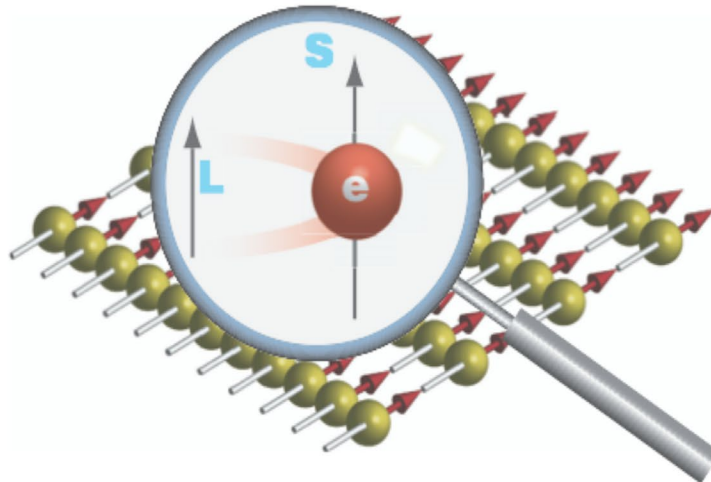
Spin dynamics



Micromagnetic simulations
Magnetization as $\mathbf{m}(\mathbf{r})$



Atomistic spin dynamics
Discrete moments $\mathbf{m}_i$

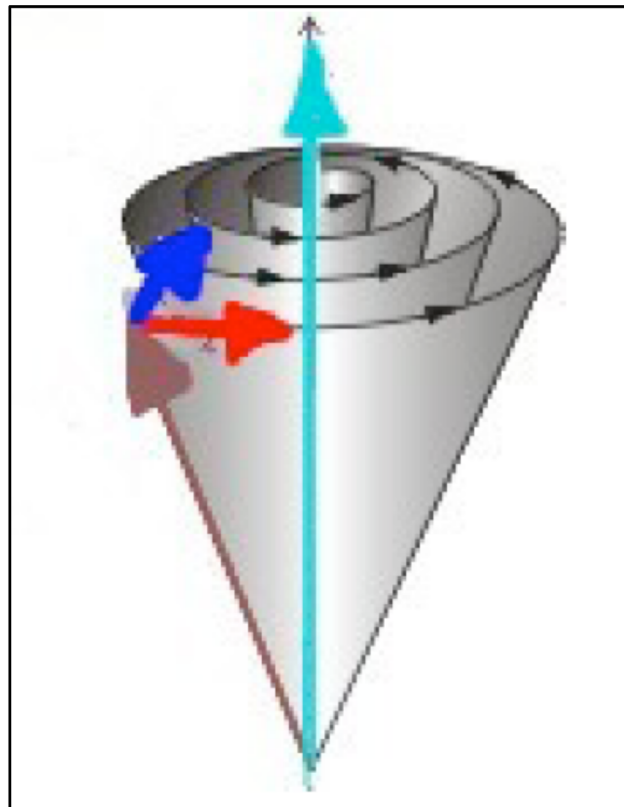


Time dependent DFT
Magnetization as $\mathbf{m}(\mathbf{r})$



Spin dynamics

Separate fast variables (electrons) and slow (atomic spins) and the EOM together with Landau-Lifshitz damping term gives:



$$\frac{\partial \mathbf{m}_i}{\partial t} = -\gamma \mathbf{m}_i \times \mathbf{B}_i - \gamma \frac{\alpha}{m} [\mathbf{m}_i \times [\mathbf{m}_i \times \mathbf{B}_i]]$$

Precession

Damping

The effective field $\mathbf{B}$ is given by

$$\mathbf{B}_i = -\frac{\partial \mathcal{H}}{\partial \mathbf{m}_i}$$



Spin dynamics

$$\mathbf{B}_i = -\frac{\partial \mathcal{H}}{\partial \mathbf{m}_i}$$

Heisenberg exchange

$$\mathcal{H}_{\text{ie}} = -\frac{1}{2} \sum_{i \neq j} J_{ij} \mathbf{m}_i \cdot \mathbf{m}_j$$

dipole-dipole

$$\mathcal{H}_{\text{dd}} = -\frac{1}{2} \sum_{i \neq j} Q_{ij}^{\mu\nu} m_i^\mu m_j^\nu$$

Magnetocrystalline anisotropy

$$\mathcal{H}_{\text{ma}} = K \sum_i (\mathbf{m}_i \cdot \mathbf{e}_K)^2$$

Dzyaloshinskii-Moriya

$$\mathcal{H}_{\text{DM}} = \sum_{i,j} \mathbf{D}_{ij} (\mathbf{m}_i \times \mathbf{m}_j)$$

Zeeman term

$$\mathcal{H}_{\text{ext}} = -\mathbf{B}_{\text{ext}} \cdot \sum_i \mathbf{m}_i$$



Langevin dynamics:

Stochastic contribution to
the effective field

$$\langle b_{i,\mu}(t) \rangle = 0, \quad D_{SLLG} = \frac{\alpha}{1 + \alpha^2} \frac{k_B T}{\gamma m}$$

$$\langle b_{i,\mu}(t) b_{j,\nu}(s) \rangle = 2D \delta_{\mu\nu} \delta_{ij} \delta(t - s)$$

Gaussian distribution

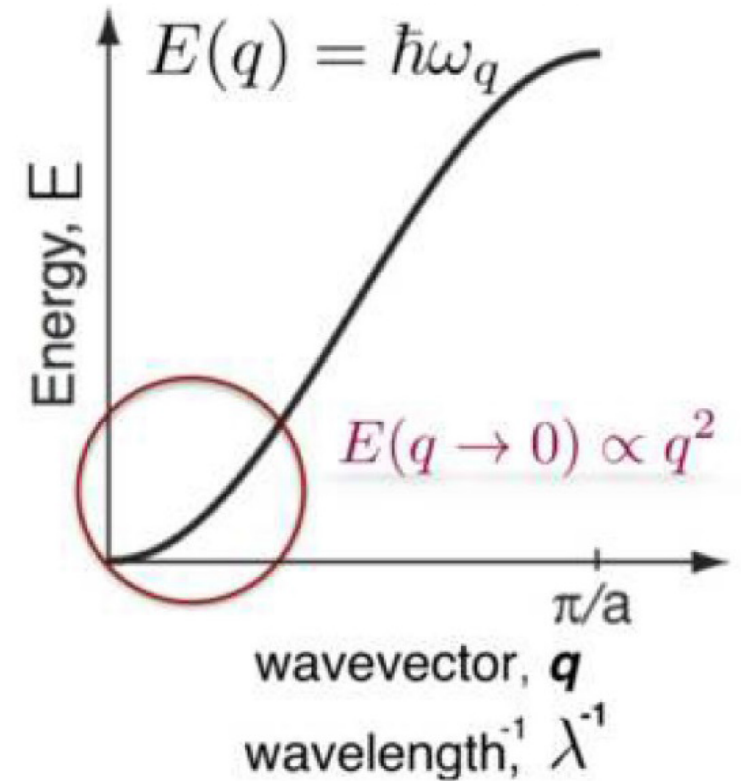
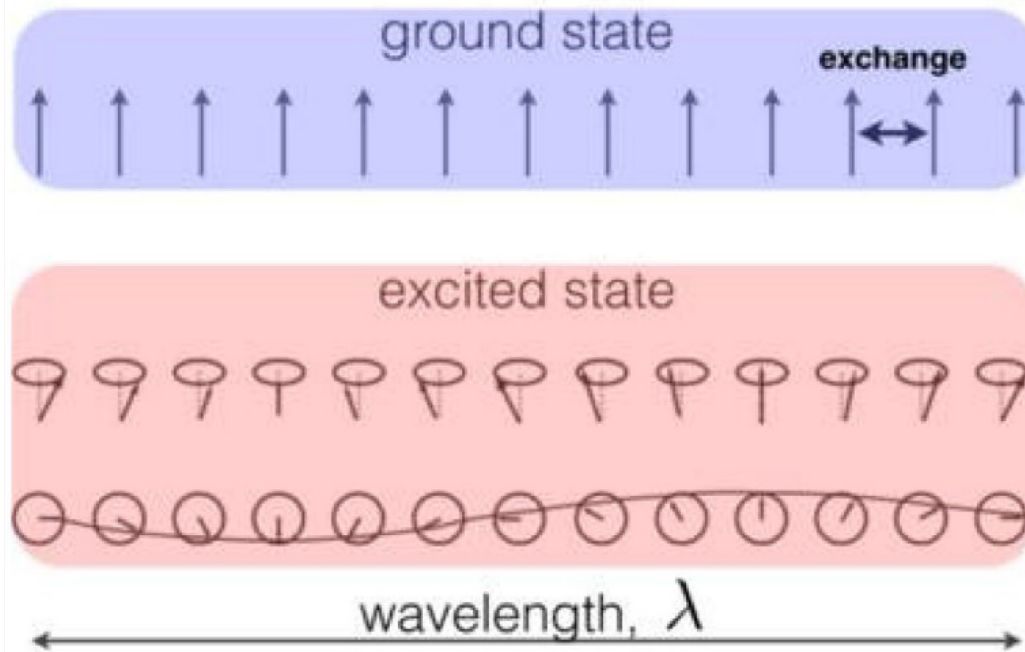
One thermal reservoir

$$\frac{\partial \mathbf{m}_i}{\partial t} = -\gamma [\mathbf{m}_i \times [\mathbf{B}_i + \mathbf{b}_i(t)]] - \gamma \frac{\alpha}{m} [\mathbf{m}_i \times [\mathbf{m}_i \times [\mathbf{B}_i + \mathbf{b}_i(t)]]]$$



Magnons

ferromagnetic spin-chain





Workflow, similar to MD simulations

- For given temperature equilibrate
- Sample a property, e.g. spin-spin correlations

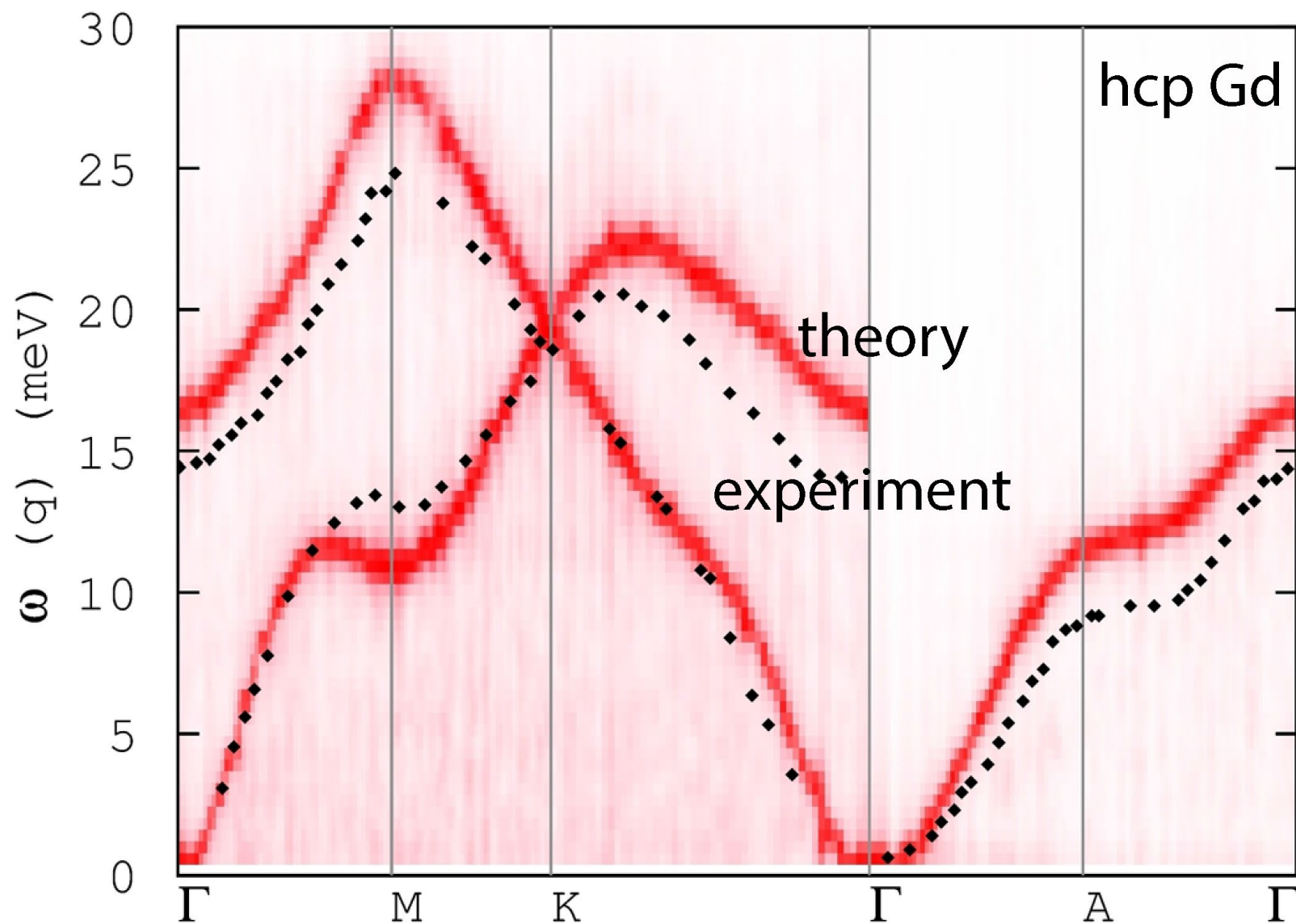
Spin-correlation function and dynamic structure factor

$$C^k(\mathbf{r} - \mathbf{r}', t) = \langle m_{\mathbf{r}}^k(t) m_{\mathbf{r}'}^k(0) \rangle - \langle m_{\mathbf{r}}^k(t) \rangle \langle m_{\mathbf{r}'}^k(0) \rangle,$$

$$S^k(\mathbf{q}, \omega) = \frac{1}{N\sqrt{2\pi}} \sum_{\mathbf{r}, \mathbf{r}'} e^{i\mathbf{q} \cdot (\mathbf{r} - \mathbf{r}')} \int_{-\infty}^{+\infty} e^{i\omega t} C^k(\mathbf{r} - \mathbf{r}', t) dt,$$

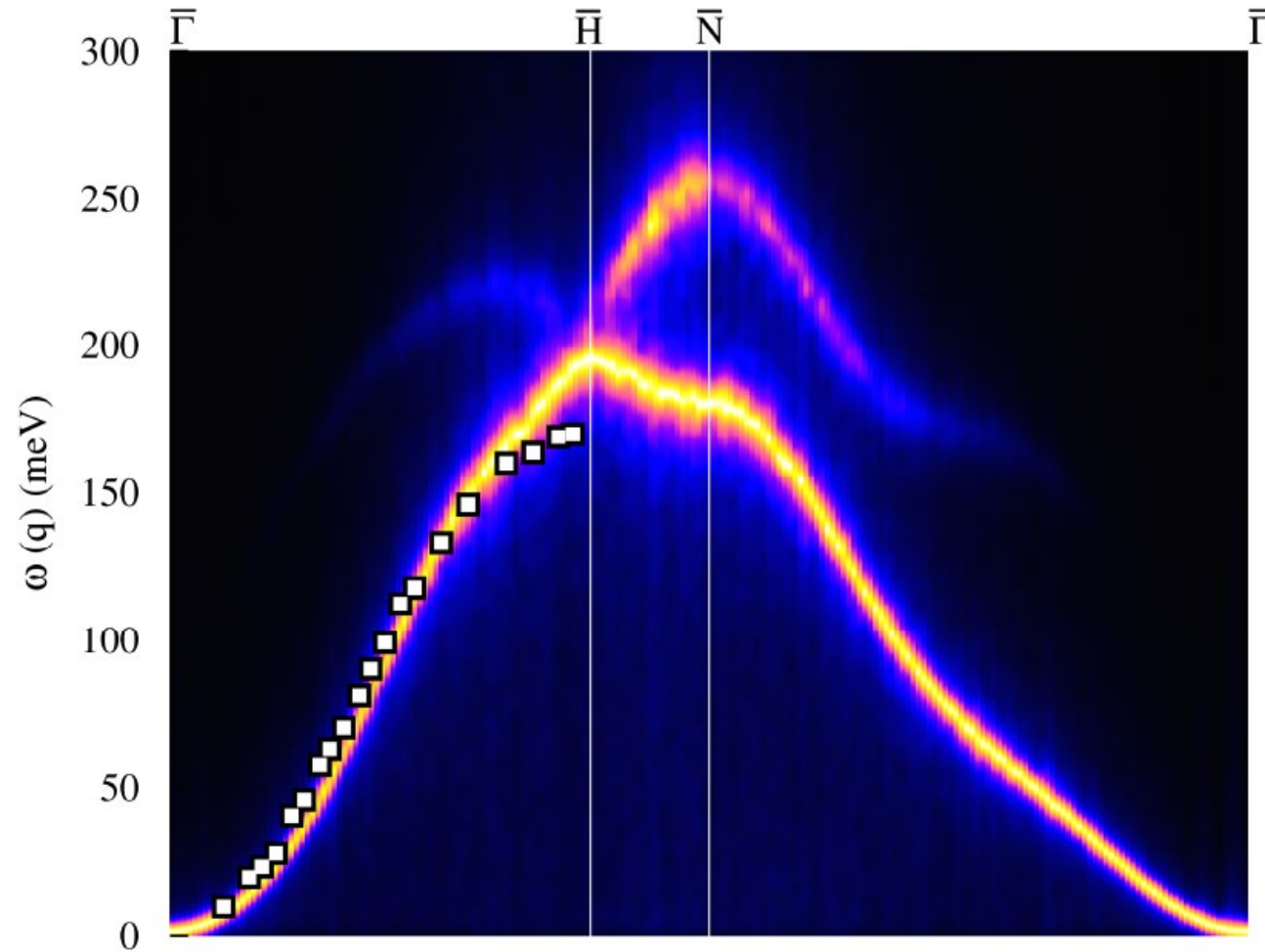


Magnons of Gd





Magnons of 2 ML Fe on W (110)



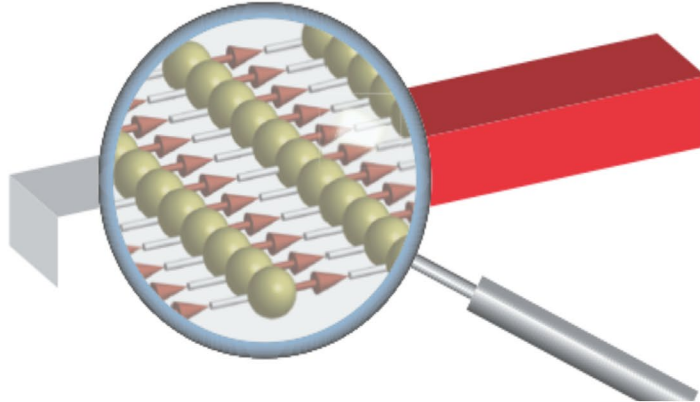


2

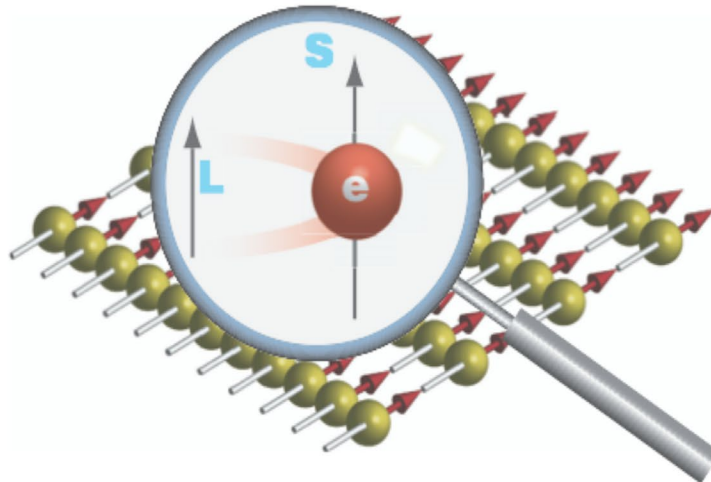
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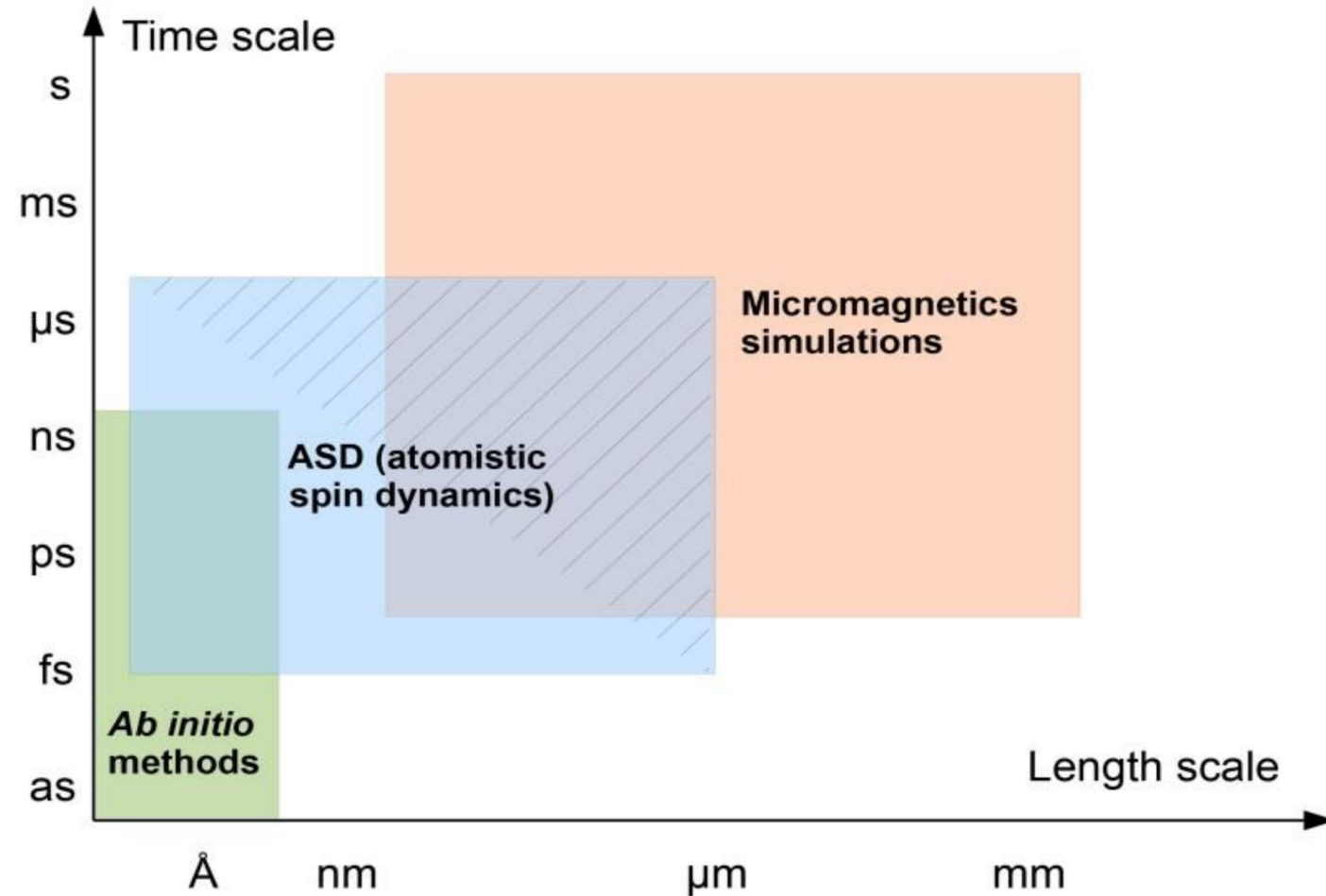
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Multiscale simulations





LLG equation for spin dynamics

Atomistic

$$\frac{d\vec{s}_i}{dt} = -\gamma \vec{s}_i \times \vec{H}_i - \alpha \vec{s}_i \times (\vec{s}_i \times \vec{H}_i)$$

$$\vec{H}_i^{\text{exc}} = \sum_{j \neq i} J_{ij} \vec{s}_j$$

$$\vec{H}_i^D = \sum_{j \neq i} \vec{D}_{ij} \times \vec{s}_j$$

$$\vec{H}_i^{\text{ani}} = K \vec{a} \cdot \vec{s}_i$$

$$\vec{H}^e = \mu \vec{H}^{\text{app}}$$

$$\vec{H}^i = \vec{H}_i^{\text{exc}} + \vec{H}_i^D + \vec{H}_i^{\text{ani}} + \vec{H}^e$$

Micromagnetics

$$\frac{\partial \vec{m}}{\partial t} = -\gamma \vec{m} \times \vec{H} - \alpha \vec{m} \times (\vec{m} \times \vec{H})$$

$$\vec{H}^{\text{exc}} = A \vec{e} \cdot \nabla \nabla \vec{m}$$

$$\vec{H}^D = \nabla \cdot (D \vec{e} \times \vec{m})$$

$$\vec{H}^{\text{ani}} = K \vec{a} \cdot \vec{m}$$

$$\vec{H}^e = \mu \vec{H}^{\text{app}}$$

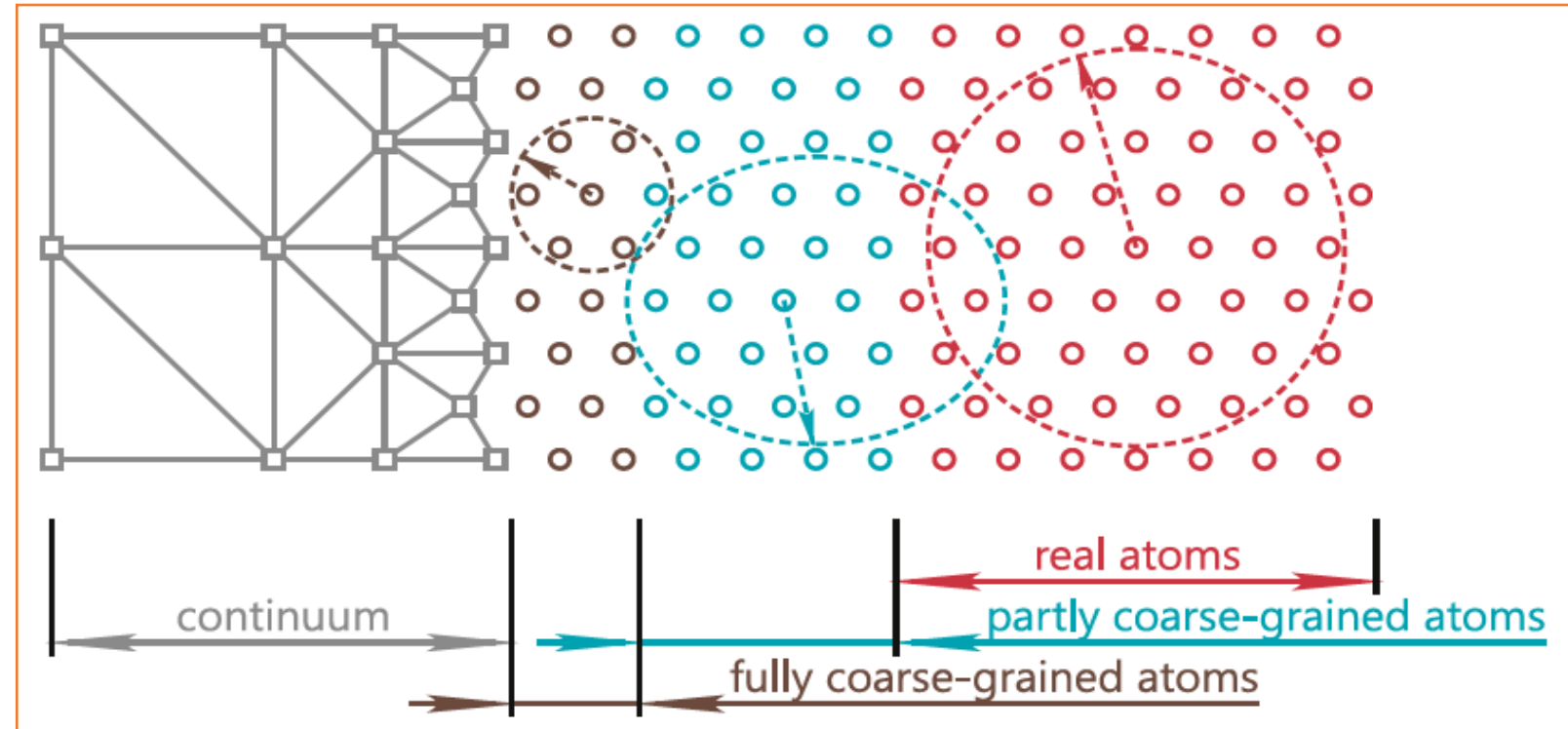
$$\vec{H} = \vec{H}^{\text{exc}} + \vec{H}^D + \vec{H}^{\text{ani}} + \vec{H}^e.$$

$$A \vec{e} = \frac{1}{2} \sum_{j \neq i} J_{ij} \vec{r}_{ij} \vec{r}_{ij}$$

$$D \vec{e} = \sum_{j \neq i} \vec{r}_{ij} \vec{D}_{ij}$$



Multiscale Mechanism



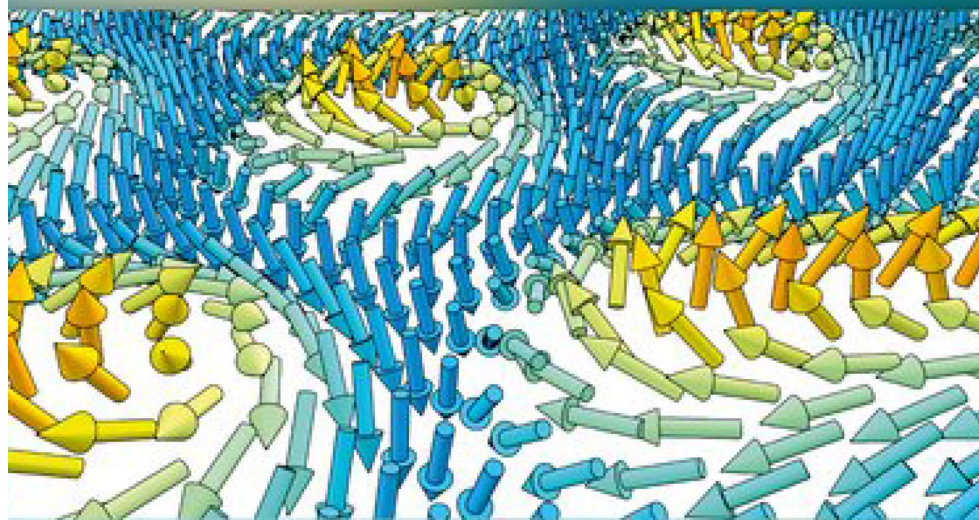
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E.Mendez, M.Pouletkov, G.Kreiss, O.Eriksson and M.Pereiro, Physical Review Research **2**, 013092 (2020)

N.Salehi, O.Eriksson and M.Pereiro, <https://arxiv.org/pdf/2504.14019>



OXFORD



Atomistic Spin Dynamics

Foundations and Applications

Olle Eriksson, Anders Bergman,
Lars Bergqvist, Johan Hellsvik