Diplolar and exchange spin waves

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APPLIED SPIN PHENOMENA
OPTIMAS light-spin-matter
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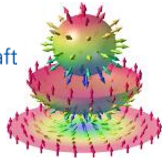
SPIN+X SFB/TRR 173 Kaiserslautern • Mainz
DFG Deutsche Forschungsgemeinschaft
priority programme
2137 Skyrmionics



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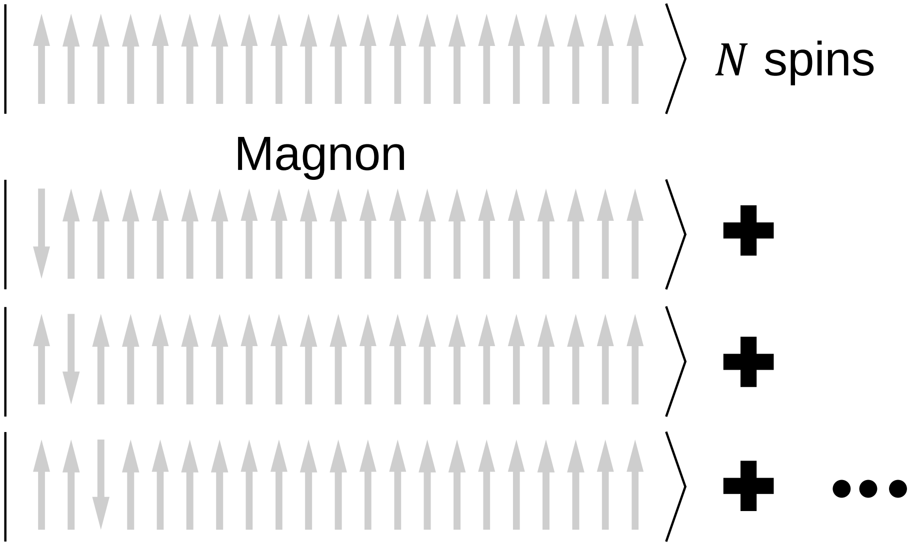
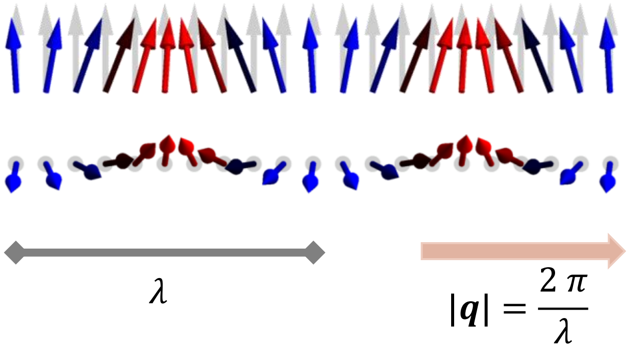
What are magnons and spin waves?

Ferromagnetic ground state



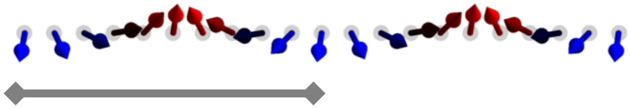
Spin wave



What is magnonics?

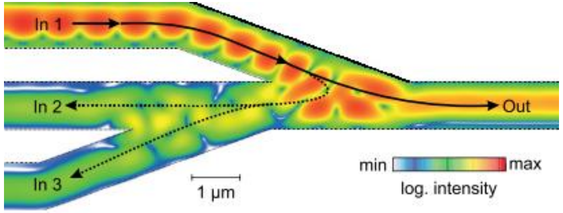
RPTU

Spin wave (quanta of excitation: magnons)



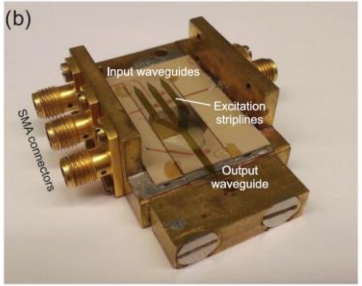
$\lambda \approx 10$ nm to 10 µm

Wave-based logic

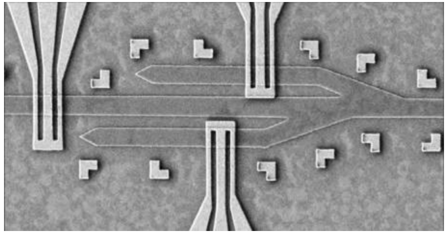


Klingler *et al.*, APL **106,** 212406 (2015)

Prototype magnon logic device



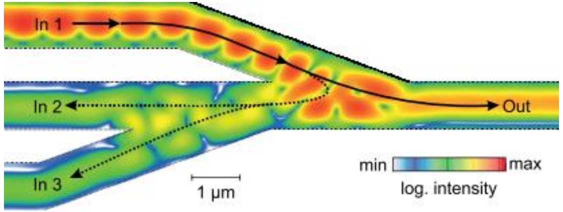
T. Fischer *et al.*, APL **110**, 152401 (2017)



Heinz et al., Nano Lett. **20**, 4220 (2020)
Sci. Adv. **6,** eabb4042 (2020)

Why is magnonics (potentially) useful?

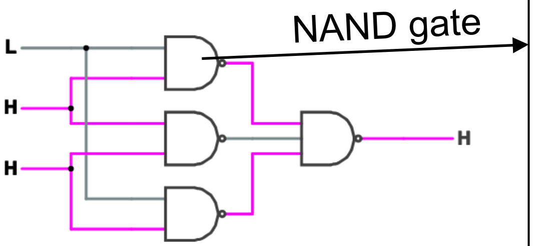
Magnonic majority gate

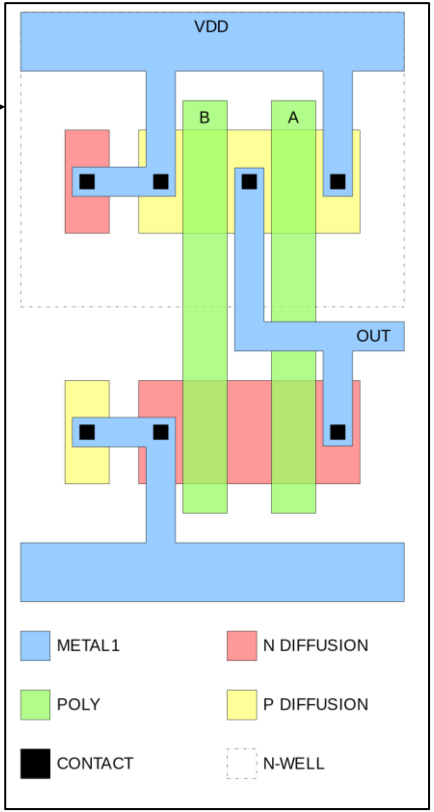


Klingler *et al.*, APL **106,** 212406 (2015)

Majority Logic Gate			
Input 1	Input 2	Input 3	Output
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

CMOS majority gate

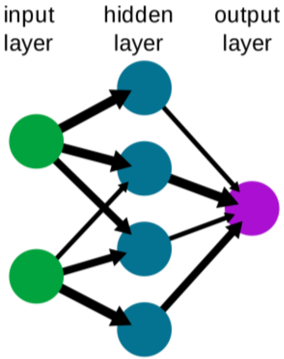




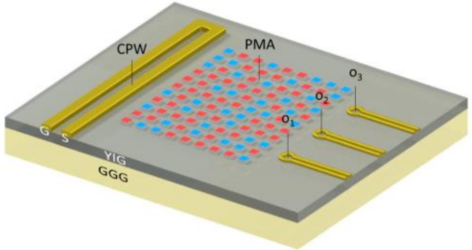
- Downscaling
- Direct input of GHz signals
- CMOS integration possible
- Advanced functionalities

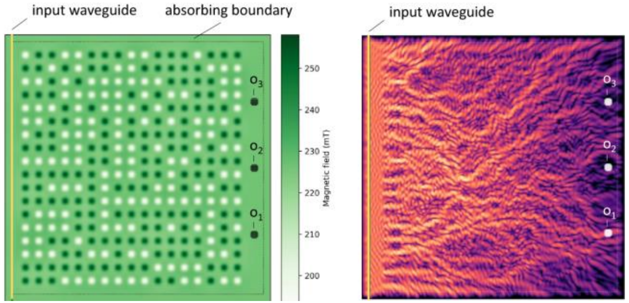
See review article:
J. Appl. Phys. **128**, 161101 (2020)

Magnonics beyond binary logic

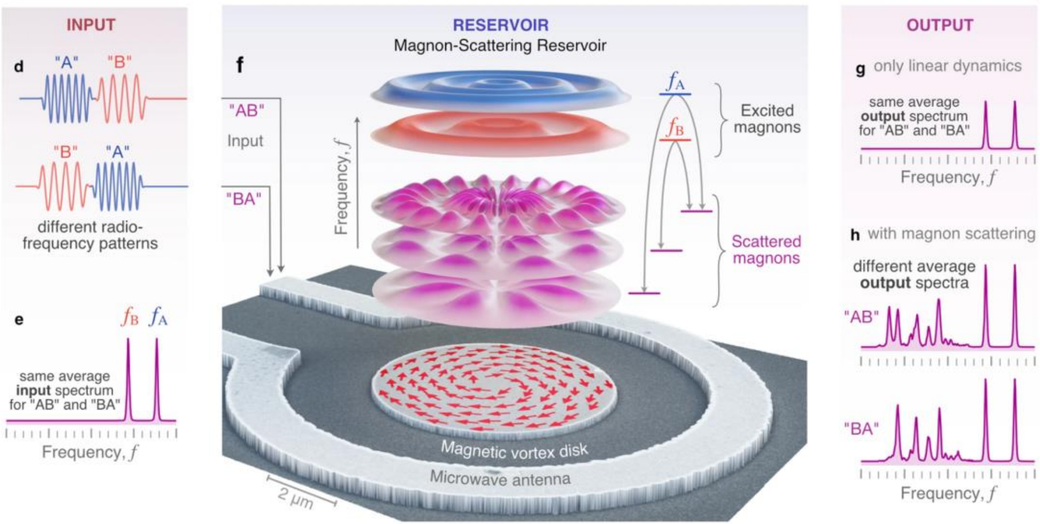


Magnonic
neural networks





Nature Communications **12**, 6422 (2021)

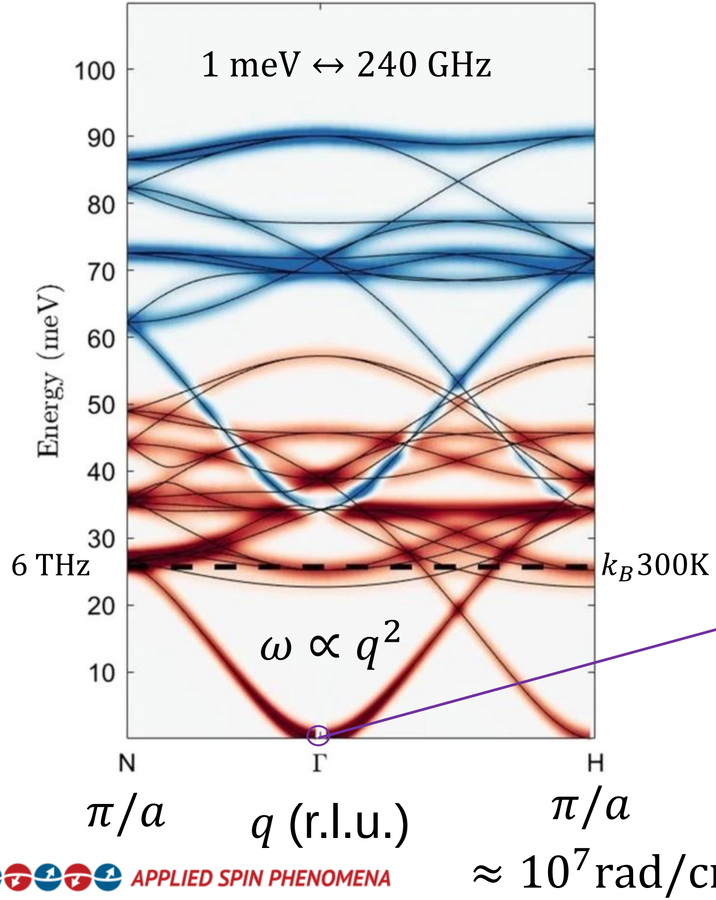


Magnonic reservoir computing

Nature Communications **14**, 3954 (2023)

Magnon dispersion

npj Quantum Materials **2**, 63 (2017)



Dipolar-Exchange Spin-Wave dispersion

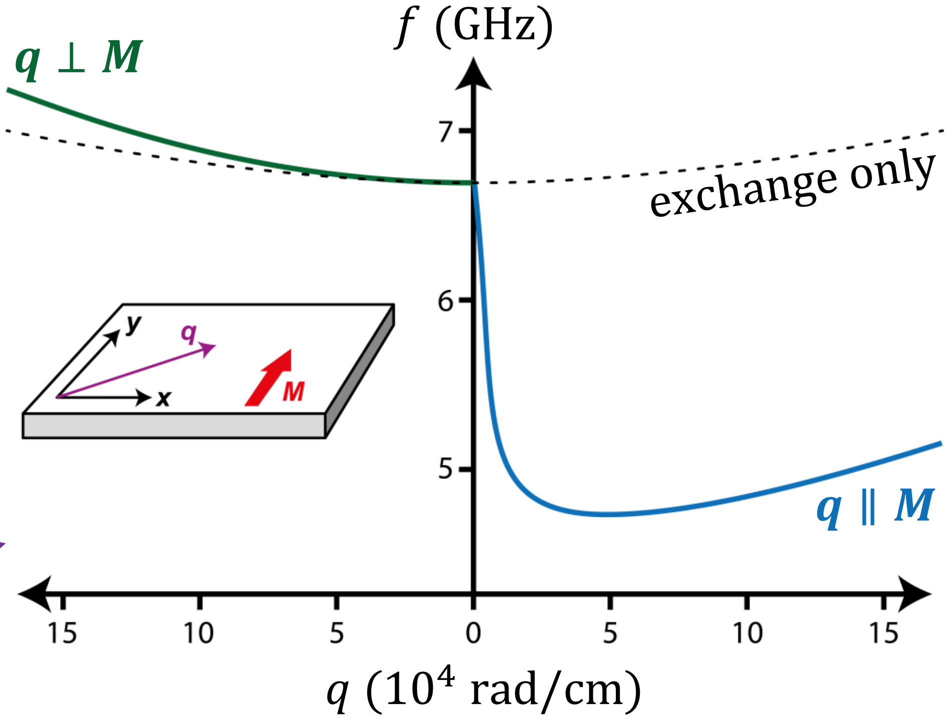
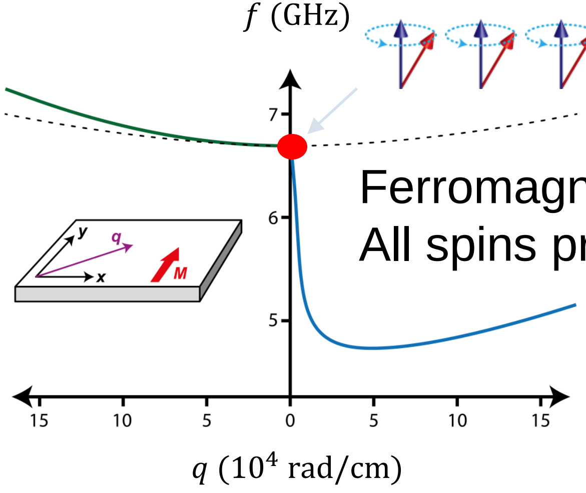
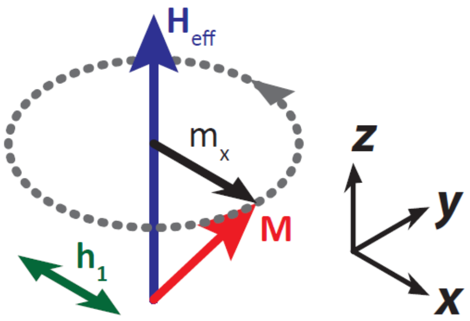


Figure adapted from: Nature Physics **12**, 1057 (2016)

Ferromagnetic resonance



Ferromagnetic resonance (q=0):
All spins precess in phase



$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\mu_0\,\boldsymbol{m}\times\boldsymbol{H}_{\text{eff}} + \alpha\,\boldsymbol{m}\times\frac{\partial \boldsymbol{m}}{\partial t}$$

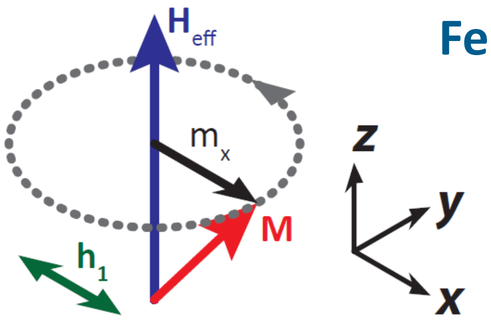
Landau-Lifshitz-Gilbert (LLG) equation

Consider only shape anisotropy:

$$\mu_0\boldsymbol{H}_{\text{eff}} = \mu_0\boldsymbol{H}_0 - \mu_0 M_s \tilde{\boldsymbol{N}}\,\boldsymbol{m}$$

$$\tilde{\boldsymbol{N}} \approx \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$$

Ferromagnetic resonance



$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\mu_0\, \boldsymbol{m} \times \boldsymbol{H}_{\mathrm{eff}} + \alpha\, \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t}$$

$$\mu_0 \boldsymbol{H}_{\mathrm{eff}} = \mu_0 \boldsymbol{H}_0 - \mu_0 M_s \tilde{\mathbf{N}}\, \boldsymbol{m}$$

$m_x, m_y \ll 1$ & $m_z \approx 1$

We describe only the transversal components of **m** and **h**!

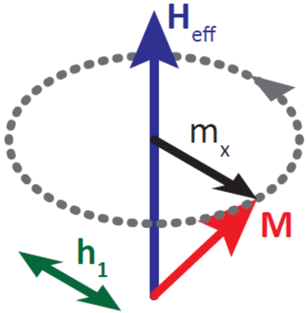
$$m_x(t) = m_x\, e^{i\omega t}$$
$$m_y(t) = m_y\, e^{i\omega t}$$
$$h_x(t) = h_x\, e^{i\omega t}$$
$$h_y(t) = h_y\, e^{i\omega t}$$

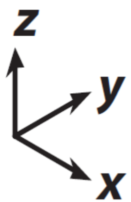
$$h_x = m_x\left[H_0 + M_s(N_x - N_z)\right] - i\frac{(m_y - \alpha m_x)\omega}{\mu_0\gamma}$$

$$h_y = m_y\left[H_0 + M_s(N_y - N_z)\right] + i\frac{(m_x + \alpha m_y)\omega}{\mu_0\gamma}$$

small angle dynamics are described by a system of two linear coefficient equations

Ferromagnetic resonance





$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\mu_0\, \boldsymbol{m} \times \boldsymbol{H}_{\text{eff}} + \alpha\, \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t}$$

$$\mu_0 \boldsymbol{H}_{\text{eff}} = \mu_0 \boldsymbol{H}_0 - \mu_0 M_s \tilde{\mathbf{N}}\, \boldsymbol{m}$$

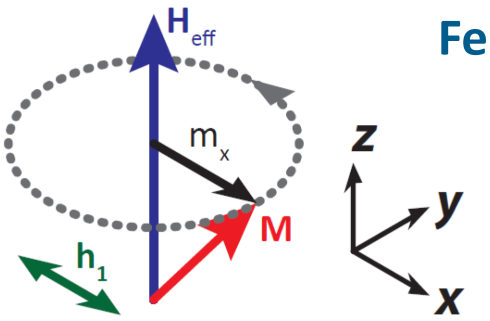
$$h_x = m_x \left[H_0 + M_{\text{s}}(N_x - N_z)\right] - i\frac{(m_y - \alpha m_x)\omega}{\mu_0\gamma}$$

$$h_y = m_y \left[H_0 + M_{\text{s}}(N_y - N_z)\right] + i\frac{(m_x + \alpha m_y)\omega}{\mu_0\gamma}$$

$$\begin{pmatrix} h_x \\ h_y \end{pmatrix} = \chi^{-1} \begin{pmatrix} m_x \\ m_y \end{pmatrix}$$

$$\chi^{-1} = \begin{pmatrix} H_0 + M_{\text{s}}(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & -\frac{i\omega}{\mu_0\gamma} \\ \frac{i\omega}{\mu_0\gamma} & H_0 + M_{\text{s}}(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix}$$

Coefficient matrix



Ferromagnetic resonance

$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\mu_0\, \boldsymbol{m} \times \boldsymbol{H}_{\mathrm{eff}} + \alpha\, \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t}$$

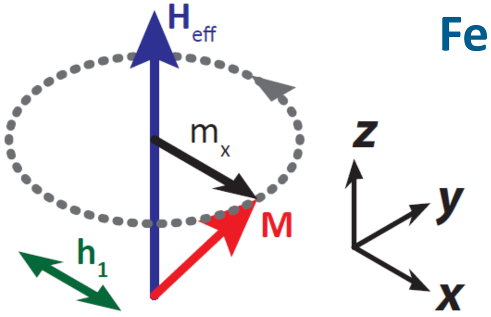
$$\mu_0 \boldsymbol{H}_{\mathrm{eff}} = \mu_0 \boldsymbol{H}_0 - \mu_0 M_{\mathrm{s}} \tilde{\mathbf{N}}\, \boldsymbol{m}$$

$$\chi^{-1} = \begin{pmatrix} H_0 + M_{\mathrm{s}}(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & -\frac{i\omega}{\mu_0\gamma} \\ \frac{i\omega}{\mu_0\gamma} & H_0 + M_{\mathrm{s}}(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix}$$

$$\boldsymbol{m}\, M_s = \chi\, \boldsymbol{h}_1 \qquad \chi = \frac{M_{\mathrm{s}}}{\det(\chi^{-1})} \begin{pmatrix} H_0 + M_{\mathrm{s}}(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & \frac{i\omega}{\mu_0\gamma} \\ -\frac{i\omega}{\mu_0\gamma} & H_0 + M_{\mathrm{s}}(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix}$$

(unitless) Polder susceptibilty χ relates transversal components of ***m*** and ***h***

Ferromagnetic resonance

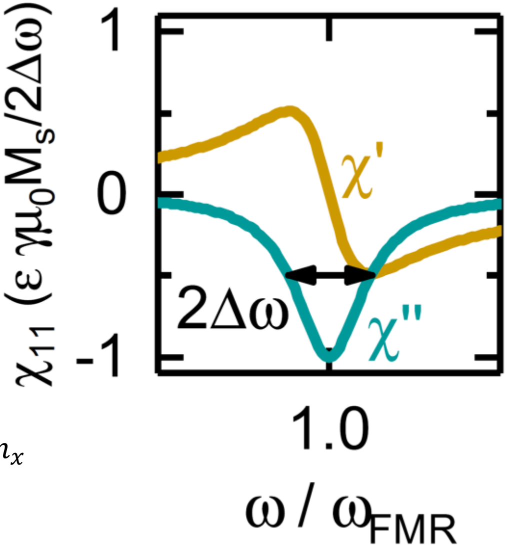


$$\frac{\partial \boldsymbol{m}}{\partial t} = -\gamma\mu_0\, \boldsymbol{m} \times \boldsymbol{H}_{\text{eff}} + \alpha\, \boldsymbol{m} \times \frac{\partial \boldsymbol{m}}{\partial t}$$

$$\mu_0 \boldsymbol{H}_{\text{eff}} = \mu_0 \boldsymbol{H}_0 - \mu_0 M_s \tilde{\mathbf{N}}\, \boldsymbol{m}$$

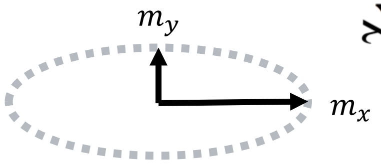
$$\boldsymbol{m}\, M_s = \chi\, \boldsymbol{h}_1$$

$$\chi = \frac{M_s}{\det(\chi^{-1})} \begin{pmatrix} H_0 + M_s(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & \frac{i\omega}{\mu_0\gamma} \\ -\frac{i\omega}{\mu_0\gamma} & H_0 + M_s(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix}$$



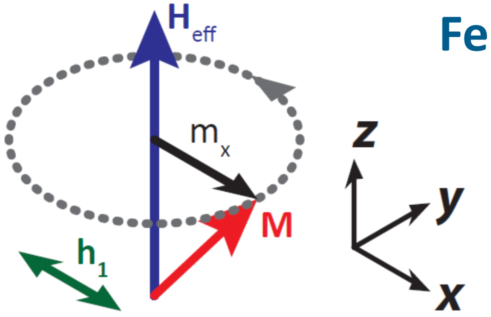
$$\chi = \chi' + i\,\chi''$$

$$\varepsilon = \left|\frac{m_x}{m_y}\right| \approx \sqrt{\frac{H_0 + M_s(N_y - N_z)}{H_0 + M_s(N_x - N_z)}}$$



ellipticity

Ferromagnetic resonance



$$\chi = \frac{M_s}{\det(\chi^{-1})} \begin{pmatrix} H_0 + M_s(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & \frac{i\omega}{\mu_0\gamma} \\ -\frac{i\omega}{\mu_0\gamma} & H_0 + M_s(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix}$$

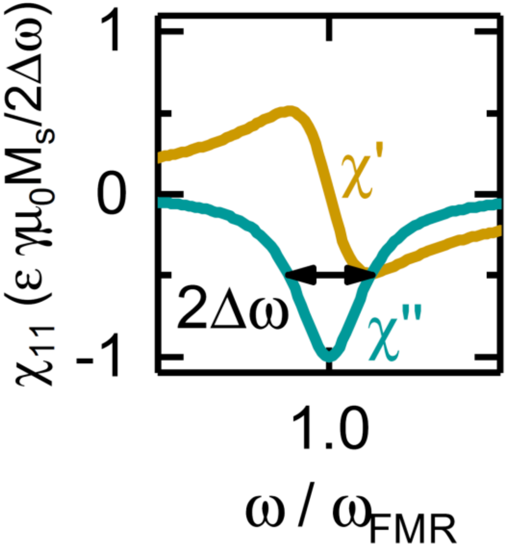
$$\boldsymbol{m}\, M_s = \chi\, \boldsymbol{h}_1$$

Resonance condition: $\det(\chi^{-1}) = 0$

For $\alpha \ll 1$:

$$\omega_{\mathrm{FMR}} = \mu_0\gamma\sqrt{[H_0 + M_s(N_x - N_z)]\,[H_0 + M_s(N_y - N_z)]}$$ Resonance

$$\Delta\omega = \alpha\sqrt{\left(\frac{1}{2}M_s\gamma\mu_0(N_x - N_y)\right)^2 + \omega^2} \qquad \Delta H = \alpha\frac{\omega}{\mu_0\gamma}$$ Linewidth

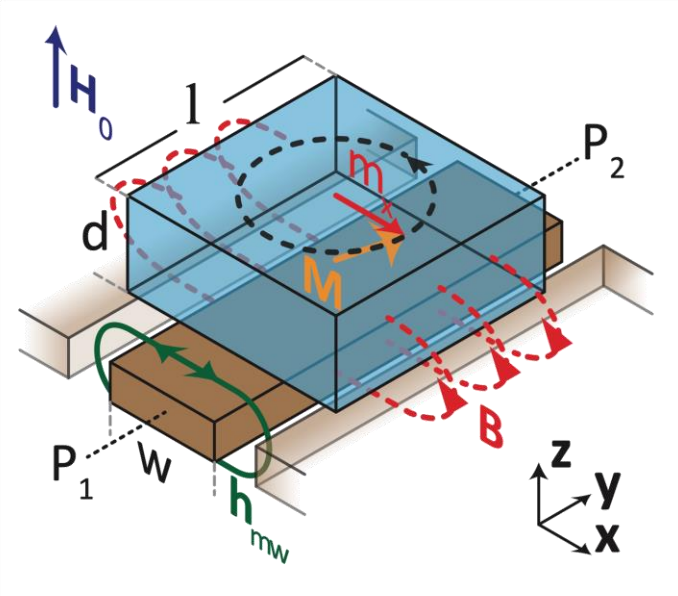


Broadband magnetic resonance spectroscopy

RPTU

$$\omega_{\mathrm{FMR}} = \mu_0 \gamma \sqrt{[H_0 + M_{\mathrm{s}}(N_x - N_z)]\,[H_0 + M_{\mathrm{s}}(N_y - N_z)]}$$

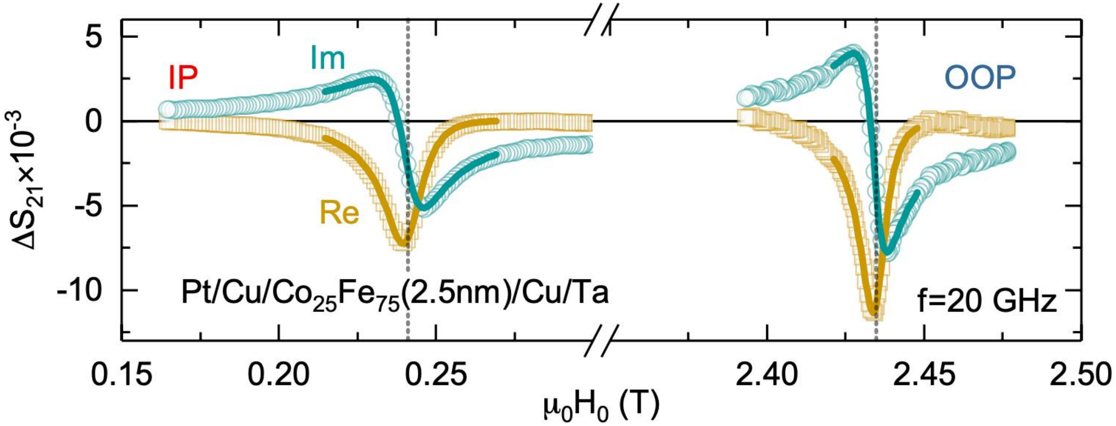
$$\Delta H = \alpha \frac{\omega}{\mu_0 \gamma}$$



oop geometry

$$S_{21} = \frac{V_2}{V_1}$$

$$V_{ind} \propto \frac{\partial m_x}{\partial t}$$

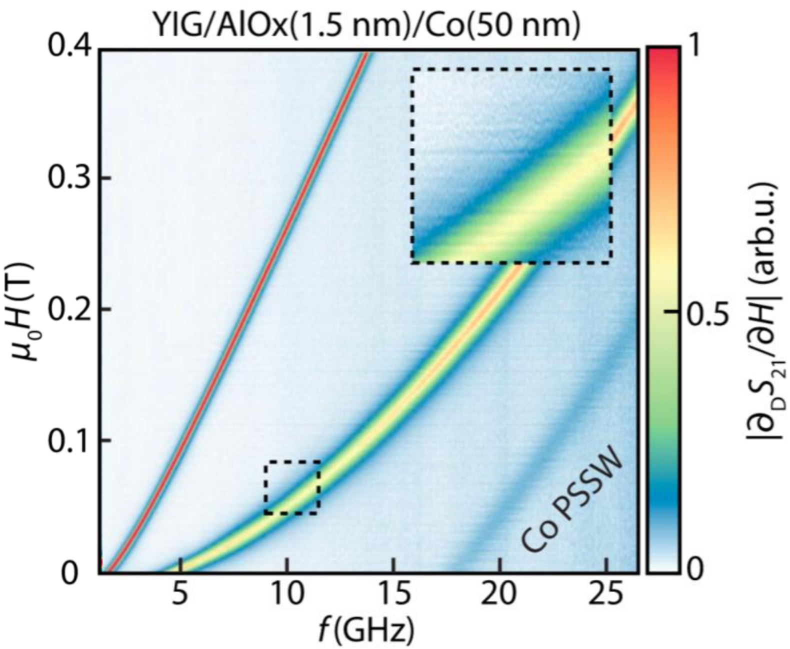


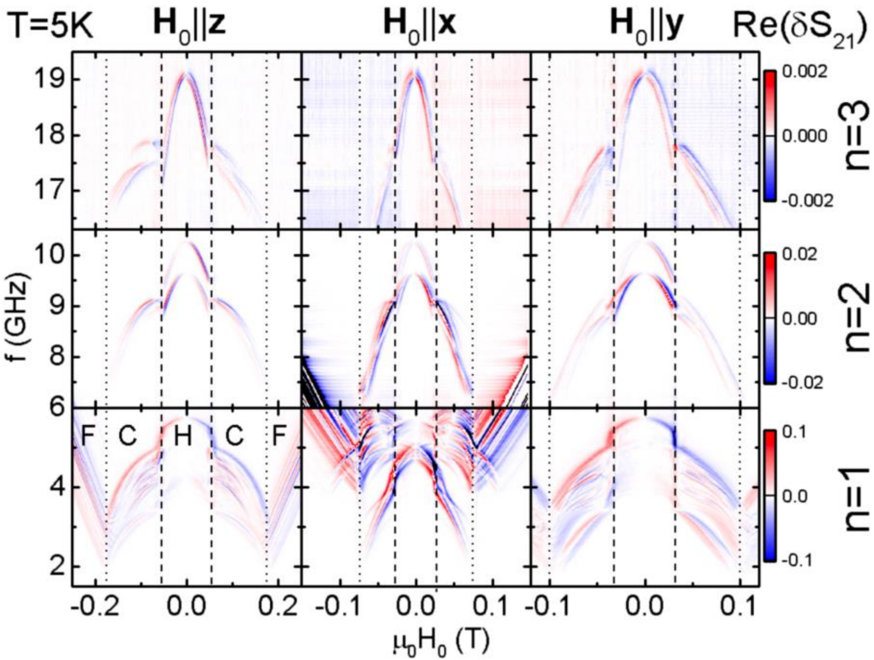
Broadband magnetic resonance spectroscopy

RPTU

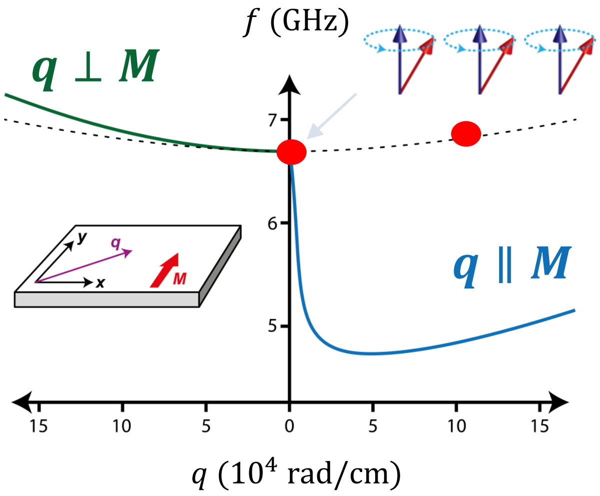
$$\omega_{\mathrm{FMR}} = \mu_0\gamma\sqrt{[H_0 + M_\mathrm{s}(N_x - N_z)]\,[H_0 + M_\mathrm{s}(N_y - N_z)]}$$

$$\Delta H = \alpha\frac{\omega}{\mu_0\gamma}$$



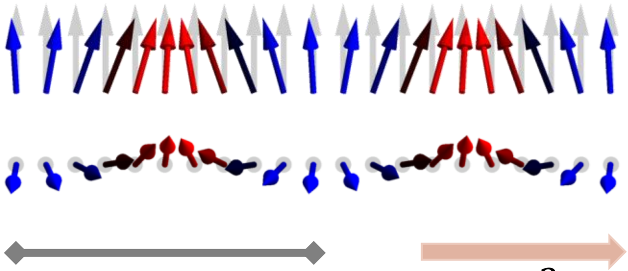


Exchange spin waves

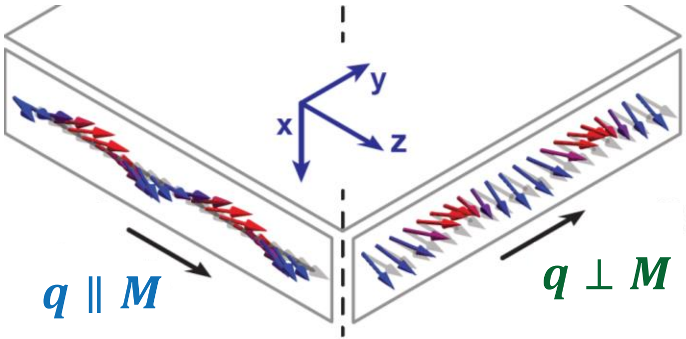


Ferromagnetic resonance (q=0):
All spins precess in phase

$q \neq 0$: Spin wave

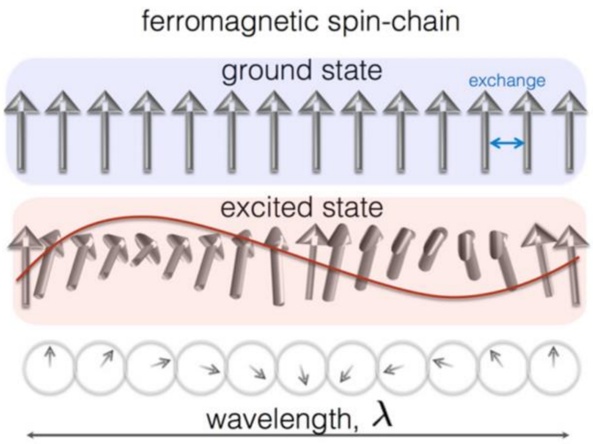


$$|\boldsymbol{q}| = \frac{2\,\pi}{\lambda}$$

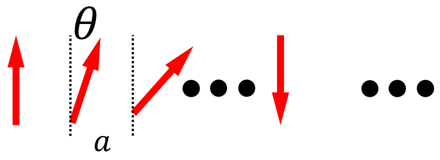


Exchange spin waves are isotropic!
f is independent of $\angle(\boldsymbol{q}, \boldsymbol{M})$

Exchange spin waves



$$\theta = a\,q \qquad \lambda = \frac{2\pi}{q}$$



J. Phys.: Condens. Matter **27** 243202 (2015)

$$H_{\mathrm{ex}} = -2\,J_{\mathrm{ex}} \sum_i \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1}$$

$$E_{\mathrm{ex}} = \hbar\omega_q = 4\,J_{\mathrm{ex}}S\,(1-\cos(qa)) \approx 2J_{\mathrm{ex}}S\,(qa)^2$$

$$\hbar\omega_q = D_s\,q^2$$

Parabolic dispersion

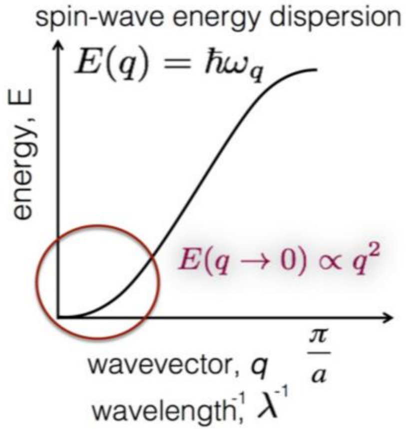
Spin wave stiffness:

$$D_s = 2\,J_{\mathrm{ex}}S\,a^2 = 2\,\frac{A\,g\mu_{\mathrm{B}}}{M_{\mathrm{s}}}$$

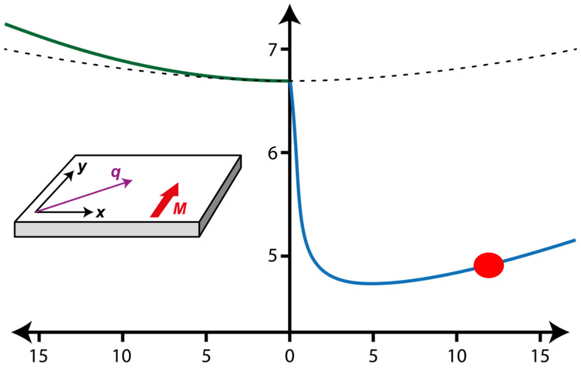
exchange constant A

exchange field:

$$\mu_0 H_{\mathrm{ex}} = \frac{D_s}{g\,\mu_B} q^2 = \frac{2A}{M_s}\,q^2$$



Dipolar spin waves



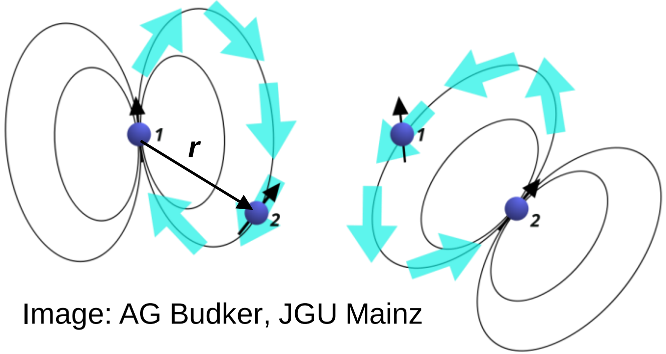


Image: AG Budker, JGU Mainz

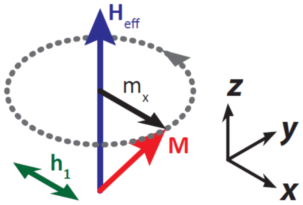
$$\mu_1 = \mu_2 = \mu_B$$

$$E_{\text{dipole}} = \frac{\mu_0}{4\,\pi\,r^3}\left[\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 - \frac{3}{r^2}(\boldsymbol{\mu}_1 \cdot \boldsymbol{r})(\boldsymbol{\mu}_2 \cdot \boldsymbol{r})\right]$$

- Dipolar interactions are anisotropic and long range
- Generally not easy to treat
- Propagating dipolar waves are propagating electromagnetic waves and thus need to satisfy Maxwell‘s equations

We assume that dipolar spin waves propagate much slower than light

Magnetostatic spin waves in infinite media



$\nabla \times \boldsymbol{h} = 0$ magnetostatic limit for dynamic fields in insulator

$\nabla \boldsymbol{b} = 0$ Gauss law

$$\bar{\chi} = \frac{M_s}{\det(\chi^{-1})} \begin{pmatrix} H_0 + M_s(N_y - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} & \frac{i\omega}{\mu_0\gamma} \\ -\frac{i\omega}{\mu_0\gamma} & H_0 + M_s(N_x - N_z) + \frac{i\alpha\omega}{\mu_0\gamma} \end{pmatrix} \Rightarrow \bar{\chi} = \begin{bmatrix} \chi & -i\kappa \\ i\kappa & \chi \end{bmatrix}$$

Infinite medium: $N_x = N_y = N_z = 0$

Neglect damping: $\alpha = 0$

$$\chi = \frac{\omega_0 \omega_M}{\omega_0^2 - \omega^2}$$

$$\kappa = \frac{\omega \omega_M}{\omega_0^2 - \omega^2}$$

$$\boldsymbol{b} = \mu_0(\boldsymbol{m} + \boldsymbol{h}) = \bar{\boldsymbol{\mu}}\, \boldsymbol{h} \qquad \bar{\boldsymbol{\mu}} = \mu_0 \begin{bmatrix} 1+\chi & -i\kappa & 0 \\ i\kappa & 1+\chi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\omega_M = \gamma \mu_0 M_s$$

$$\boldsymbol{h} = -\nabla \psi$$

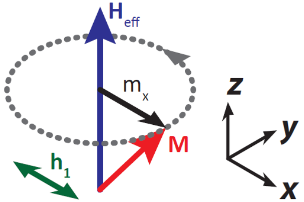
$$\omega_0 = \gamma \mu_0 H_0$$

Magnetostatic spin waves in infinite media

$\nabla \times \boldsymbol{h} = 0$ magnetostatic limit for dynamic fields

$\nabla \boldsymbol{b} = -\nabla(\overline{\boldsymbol{\mu}}\,\nabla\psi) = 0$ Gauss law

$$\chi = \frac{\omega_0 \omega_\mathrm{M}}{\omega_0^2 - \omega^2}$$



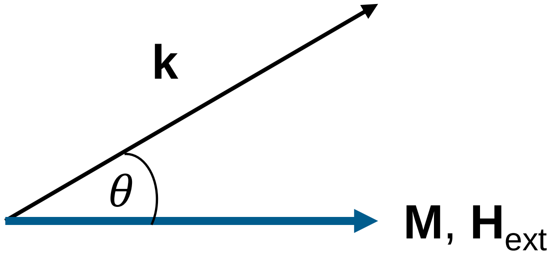
$$\omega_M = \gamma\mu_0 M_s$$

$$\omega_0 = \gamma\mu_0 H_0$$

$$\psi = \psi_0 e^{ikr}$$

$$\Rightarrow (1+\chi)\left[\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right] + \frac{\partial^2\psi}{\partial z^2} = 0$$

Walker equation

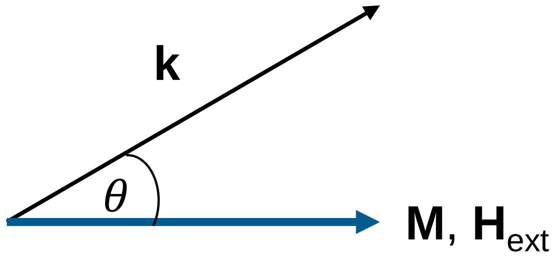


$$\psi = \psi_0 e^{ikr} \Rightarrow (1+\chi)\left(k_x^2 + k_y^2\right) + k_z^2 = 0$$

$$\omega = \sqrt{\omega_0[\omega_0 + \omega_M \sin^2\theta]}$$

Magnetostatic spin waves in infinite media

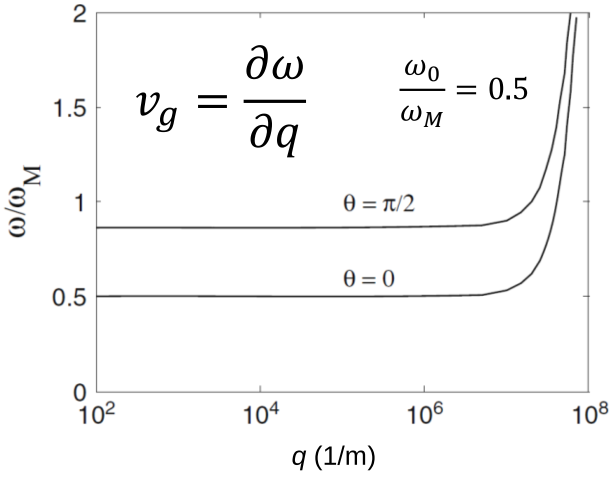
$$\omega = \sqrt{\omega_0[\omega_0 + \omega_M \sin^2\theta]} \qquad \omega_M = \gamma\mu_0 M_s \qquad \omega_0 = \gamma\mu_0 H_0$$



Adding exchange contribution:

$$\hbar\omega_{\text{ex}} = D_s\, q^2 \quad \Longrightarrow \quad \omega_{\text{ex}} = \gamma \frac{2A}{M_s} q^2$$

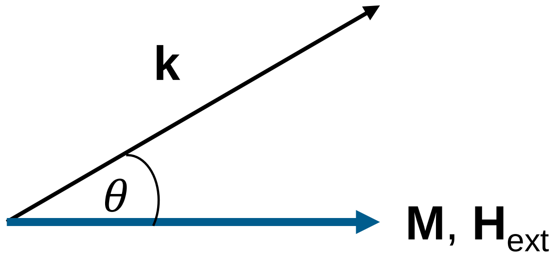
$$\omega = \sqrt{[\omega_0 + \omega_{\text{ex}}][\omega_0 + \omega_{\text{ex}} + \omega_M \sin^2\theta]}$$



Volume dipolar spin waves do not propagate!

Magnetostatic spin waves in infinite media

$$\omega = \sqrt{\omega_0[\omega_0 + \omega_M \sin^2\theta]} \qquad \omega_M = \gamma\mu_0 M_s \qquad \omega_0 = \gamma\mu_0 H_0$$



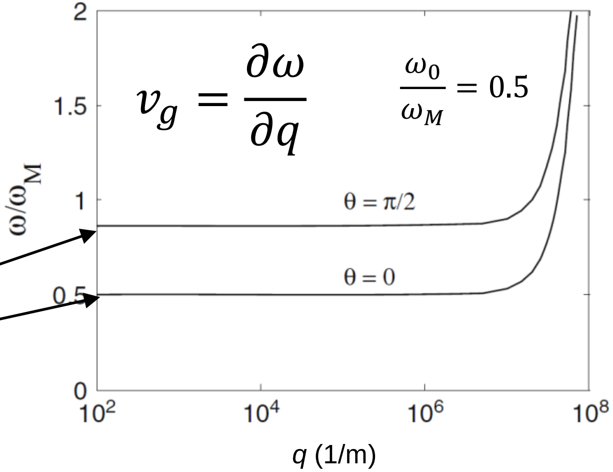
Adding exchange contribution:

$$\hbar\omega_{\mathrm{ex}} = D_s\, q^2 \quad \Rightarrow \quad \omega_{\mathrm{ex}} = \gamma \frac{2A}{M_s} q^2$$

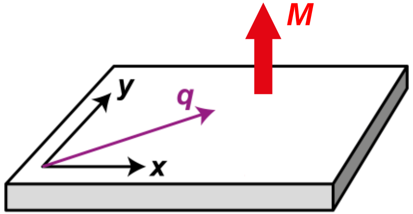
$$\omega = \sqrt{[\omega_0 + \omega_{ex}][\omega_0 + \omega_{\mathrm{ex}} + \omega_M \sin^2\theta]}$$

Dipolar wave bandwidth:

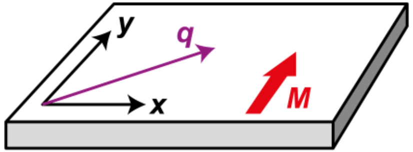
$$\omega_0 \leq \omega \leq \sqrt{\omega_0(\omega_0 + \omega_{\mathrm{M}})}$$



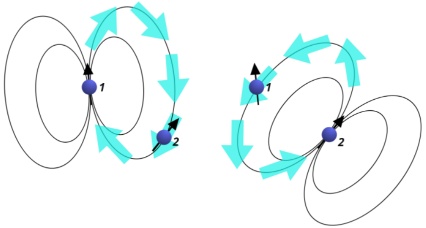
Spin waves in thin films



Thin film: $N_x = N_y = 0,\ N_z = 1$
Spin-waves can only propagate within the film plane



Out-of-plane magnetized thin film:
No symmetry breaking in the plane ($M \perp q$ always)
→ Isotropic spin-wave dispersion

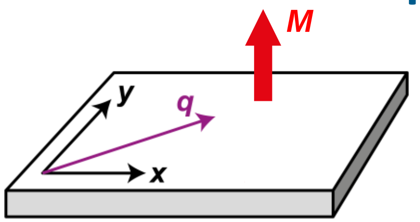


in-plane magnetized thin film:
($\angle(M, q)$ can be arbitrary)

→ Anisotropic spin wave dispersion

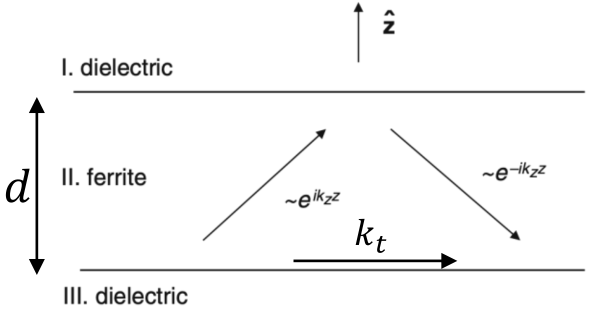
$$E_{\mathrm{dipole}} = \frac{\mu_0}{4\,\pi\, r^3}\left[\boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2 - \frac{3}{r^2}(\boldsymbol{\mu}_1 \cdot \boldsymbol{r})(\boldsymbol{\mu}_2 \cdot \boldsymbol{r})\right]$$

Forward volume spin waves



$$(1+\chi)\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right]+\frac{\partial^2\psi}{\partial z^2}=0$$

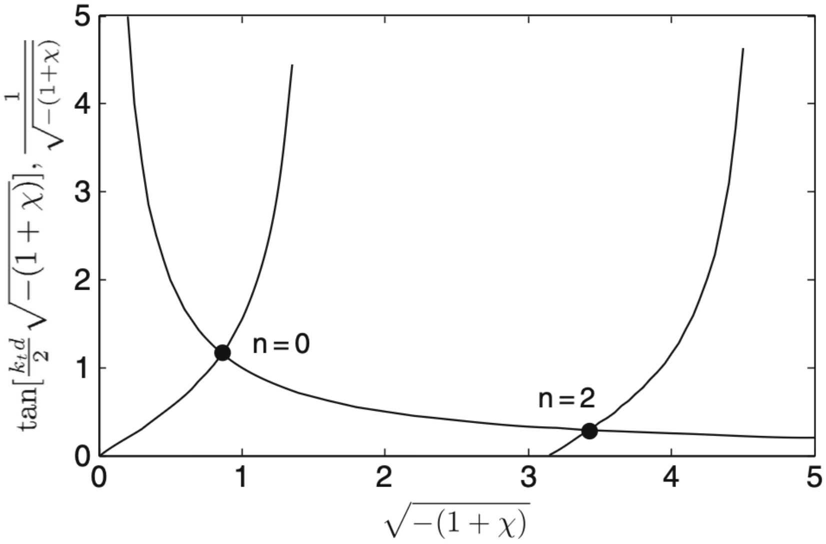
$$\psi_{\mathrm{II}} = \psi_0\, e^{i\mathbf{k}_t\cdot\mathbf{r}}\left[\frac{e^{ik_z z}+e^{-ik_z z}}{2}\right] = \psi_0 \cos(k_z z)\, e^{i\mathbf{k}_t\cdot\mathbf{r}},$$



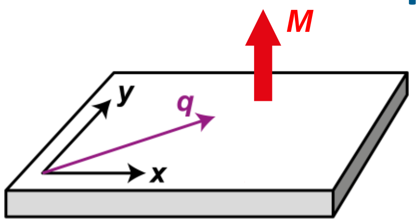
$$\omega_0 \le \omega \le \sqrt{\omega_0(\omega_0+\omega_M)} \qquad \chi = \frac{\omega_0\omega_M}{\omega_0^2-\omega^2} \rightarrow \chi < 0$$

Boundary cond: $\tan(k_z d/2) = \frac{k_t}{k_z}$

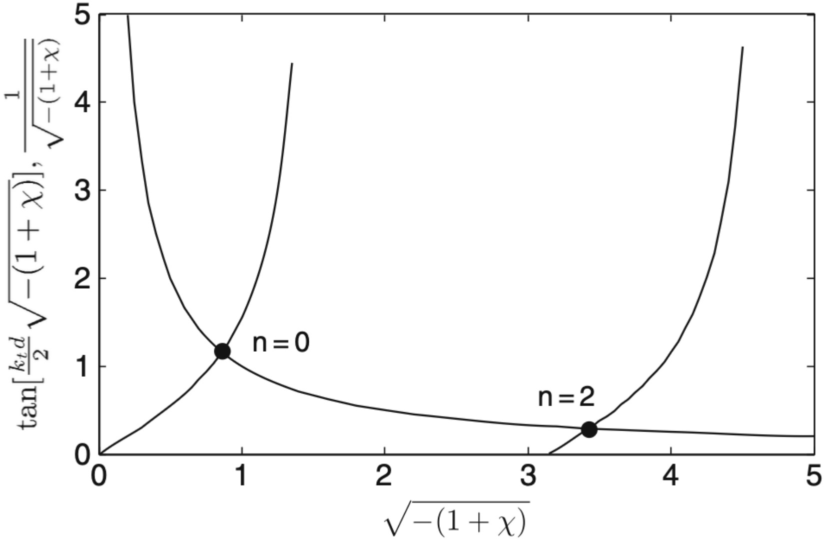
$$\tan\left[\frac{k_t d}{2}\sqrt{-(1+\chi)}\right] = \frac{1}{\sqrt{-(1+\chi)}}$$

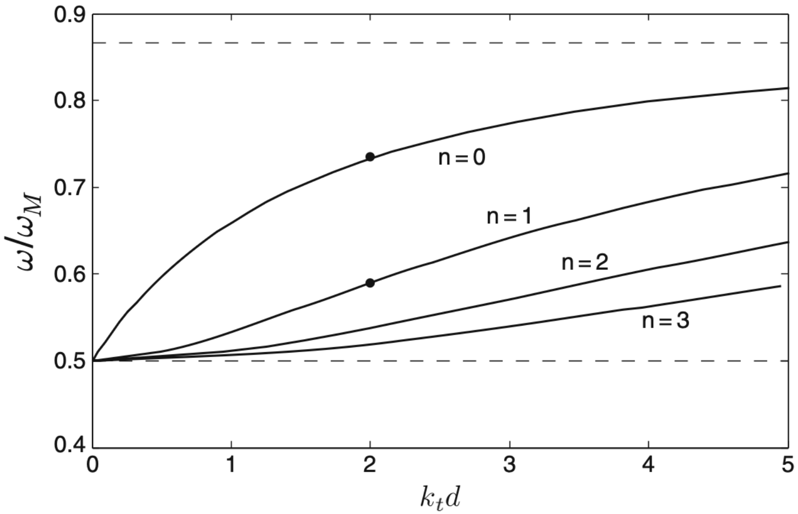


Forward volume spin waves

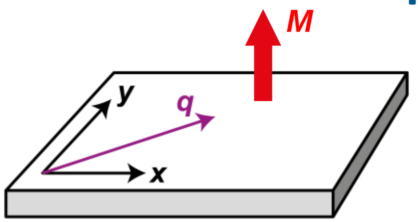


$$n = 0: \quad \omega^2 = \omega_0 \left[\omega_0 + \omega_M \left(1 - \frac{1 - e^{-k_t d}}{k_t d}\right)\right]$$

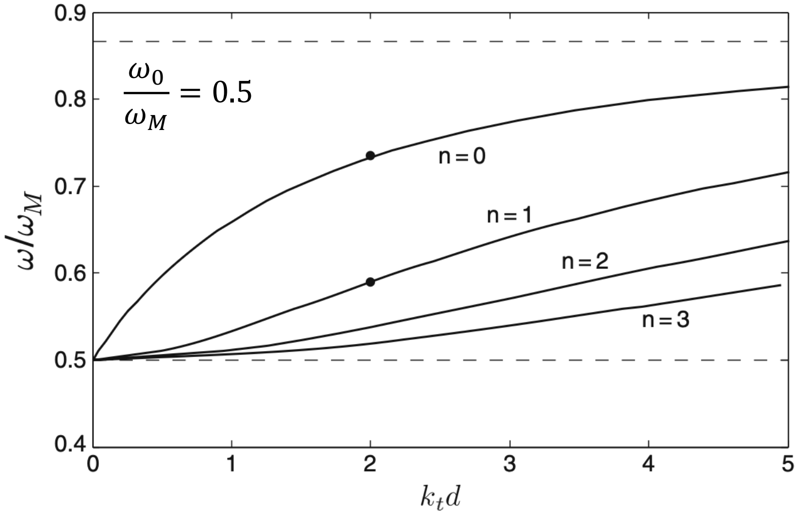


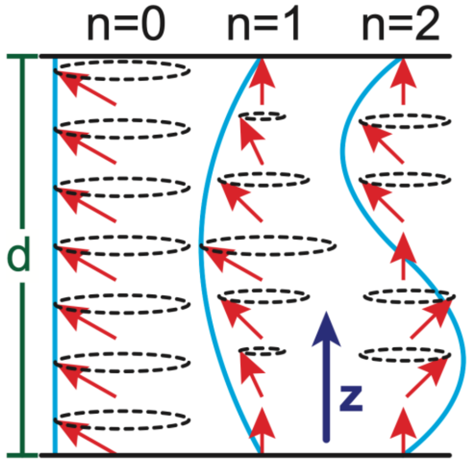


Forward volume spin waves

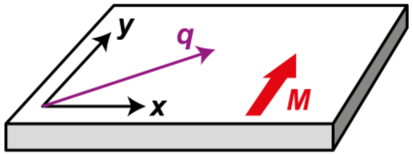


$$n = 0: \quad \omega^2 = \omega_0 \left[\omega_0 + \omega_M \left(1 - \frac{1 - e^{-k_t d}}{k_t d} \right) \right]$$

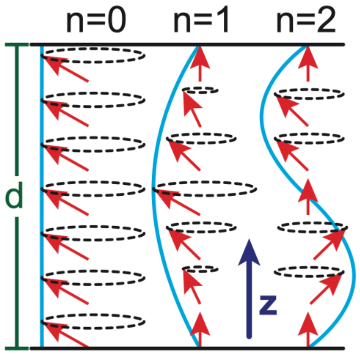




Backward volume spin waves



$\boldsymbol{q} \parallel \boldsymbol{M}$



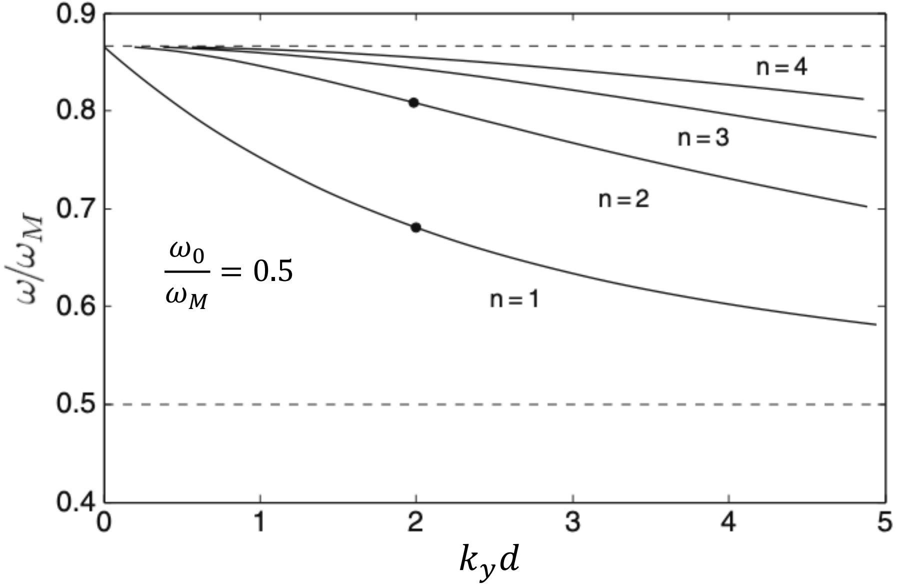
$$(1+\chi)\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}\right]+\frac{\partial^2\psi}{\partial y^2}=0$$

$$\psi_{II} = \psi_0 \sin(k_z z)\, e^{\pm i k_y y}$$

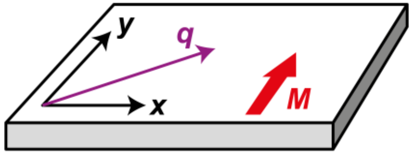
$n = 1$:

$$\omega^2 = \omega_0\left[\omega_0 + \omega_{\mathrm{M}}\left(\frac{1-e^{-k_z d}}{k_z d}\right)\right]$$

$$v_g = \frac{\partial \omega}{\partial q} < 0 \qquad v_p = \frac{\omega}{q} > 0$$



Damon-Eshbach spin waves



$\boldsymbol{q} \perp \boldsymbol{M}$

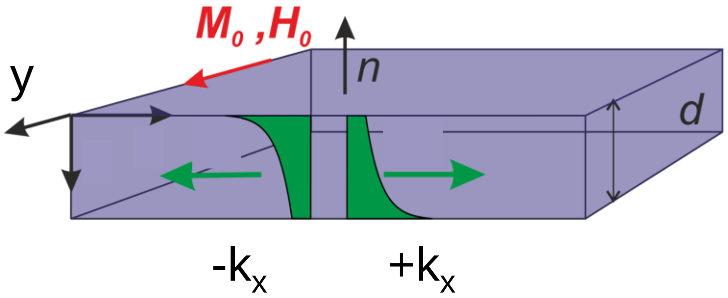
$$(1+\chi)\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}\right]+\frac{\partial^2\psi}{\partial y^2}=0$$

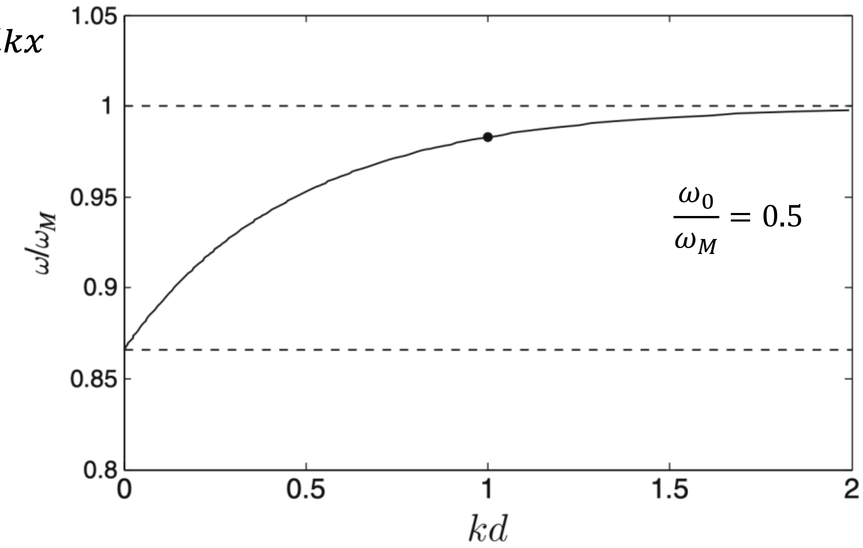
$$(1+\chi)(k_x^2+k_z^2)+k_y^2=0$$

Uniform orthogonal to propagation → $k_y = 0$

$$\rightarrow (k_x^2+k_z^2)=0$$
$$\rightarrow k_x=-k_y=k$$

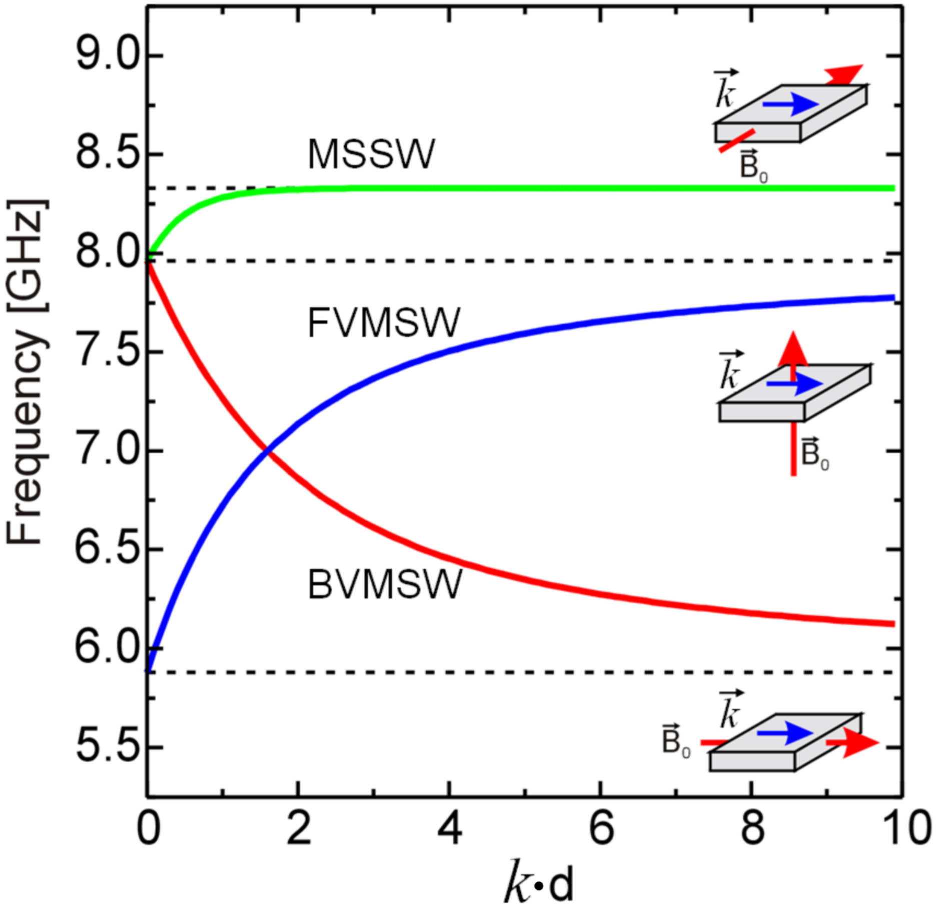
$$\psi_{II}=[\psi_{0+}e^{kz}+\psi_{0-}e^{-kz}]e^{\pm ikx}$$





$$\omega^2=\omega_0(\omega_0+\omega_\mathrm{M})+\frac{\omega_\mathrm{M}^2}{4}\left[1-e^{-2kd}\right]$$

Summary: Dipolar Spin Waves in Thin Films



Magnetostatic Surface Spin Waves

Forward Volume Magnetostatic Spin Waves

Backward Volume Magnetostatic Spin Waves

Image: Burkard Hillebrands

Backward Volume Magnetostatic Spin Waves

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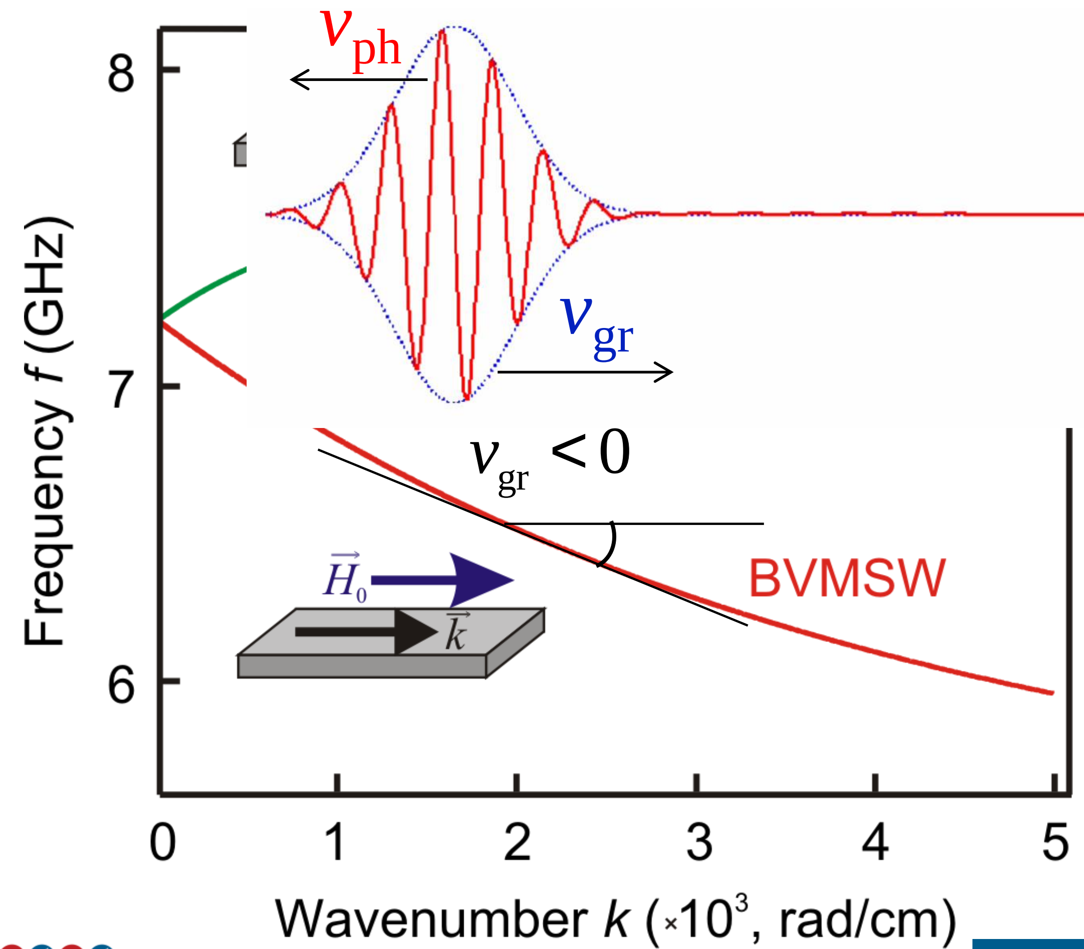


For BVMSWs:

$$v_{gr} < 0, \quad v_{ph} > 0$$

Backward Volume Spin Magnetostatic Waves

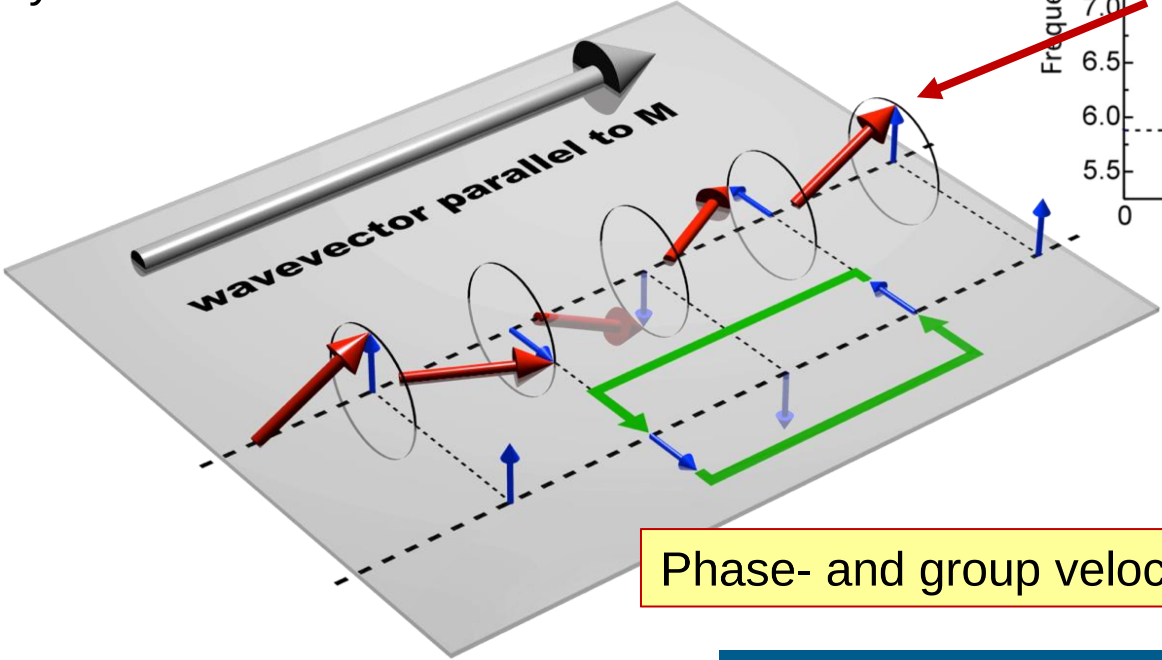
group and phase velocities have different signs

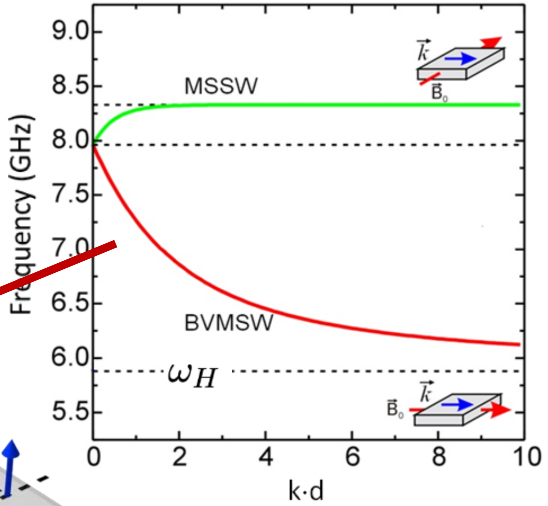
Backward Volume Magnetostatic Spin Waves

Magnetic surface charges due to finite film thickness!

Limit for $d \rightarrow \infty : \omega(k) = \omega_H$ $\qquad \omega_H = \gamma \mu_0 H_0$

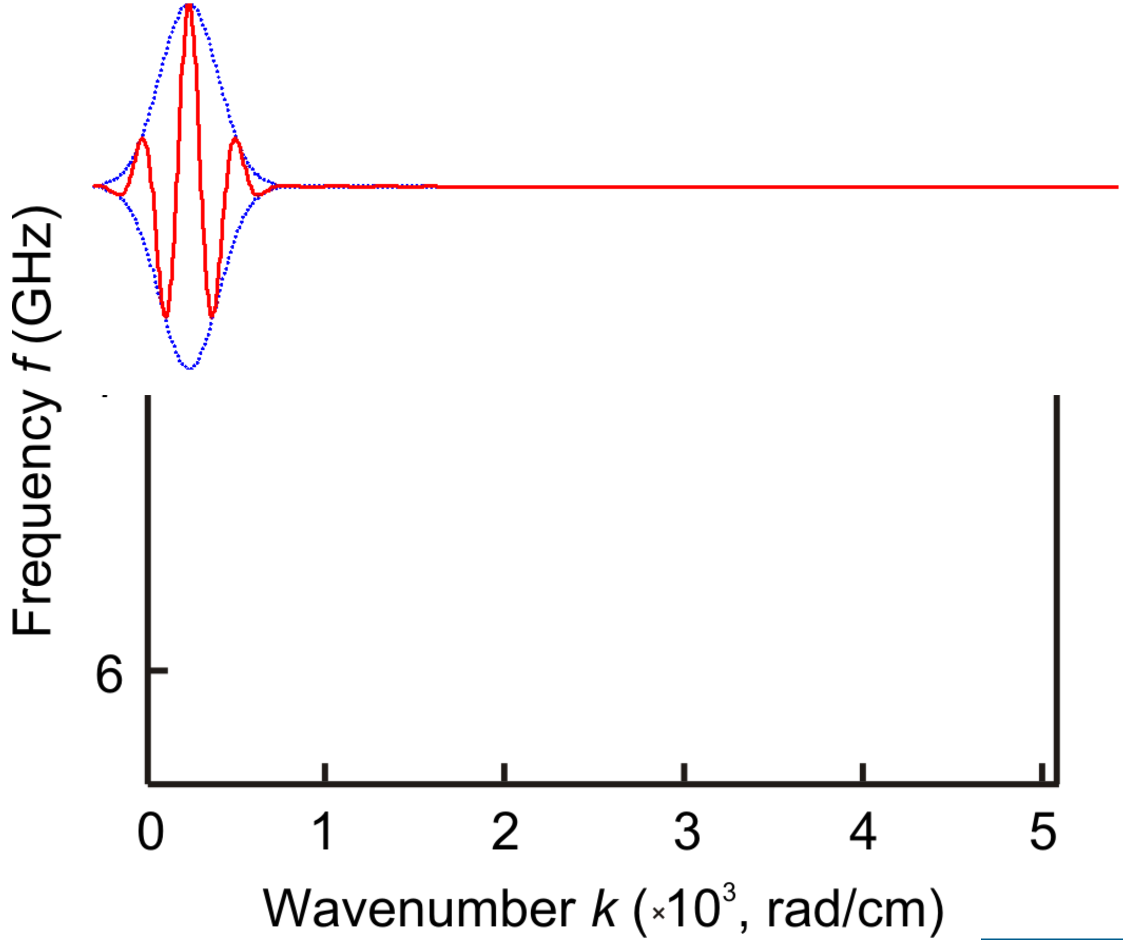
= frequency in the infinite Volume





Phase- and group velocity are antiparallel

Magnetostatic Surface Spin Waves



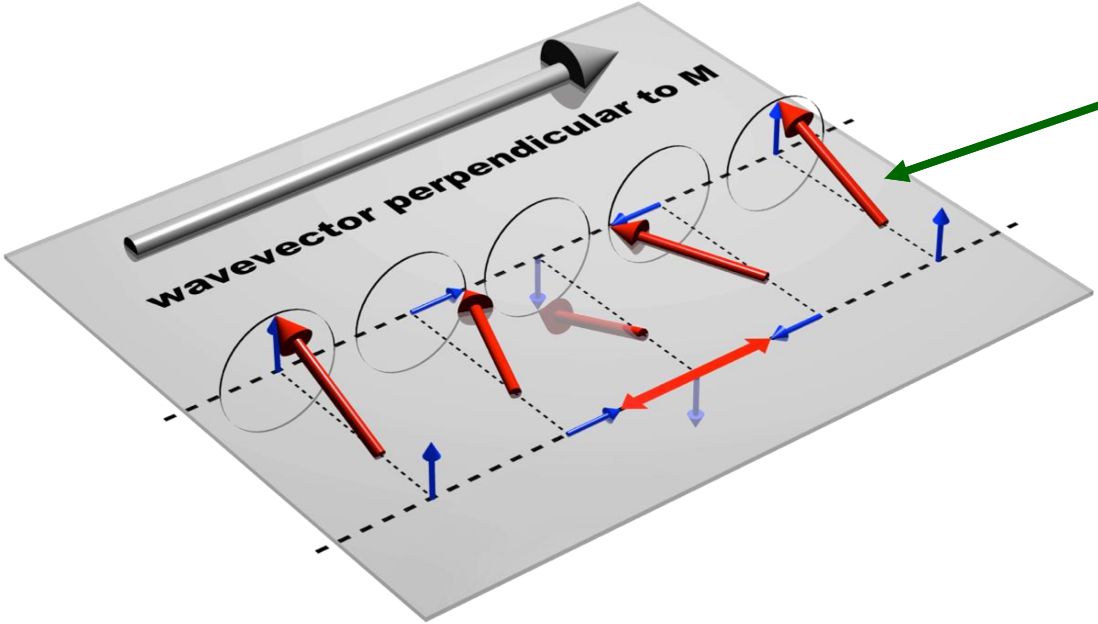
agnetostatic Surface
Spin Waves

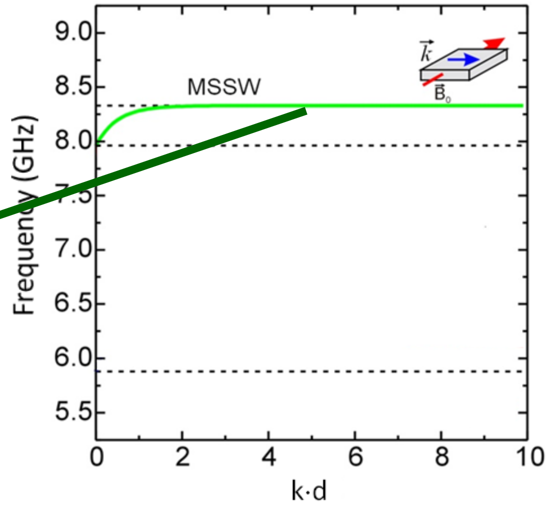
For spin waves:

$$v_{\text{ph}} \; {}^{1} \; v_{\text{gr}}$$

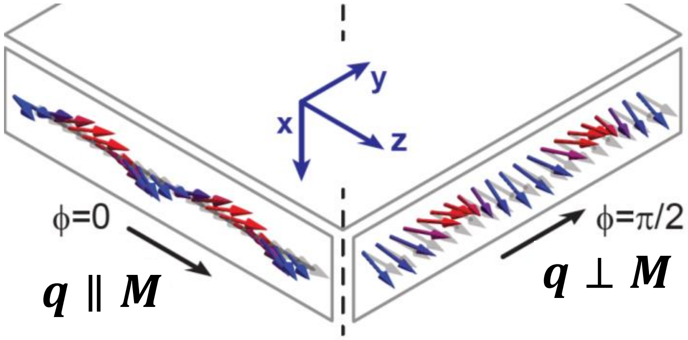
Magnetostatic Surface Spin Waves

Both volume and surface magnetic charges!
Volume charges dominate (due to ellipticity of precession)





Kalinikos-Slavin approximation



$\boldsymbol{q} \parallel \boldsymbol{M}$ $\boldsymbol{q} \perp \boldsymbol{M}$

- We have only discussed $\boldsymbol{q} \perp \boldsymbol{M}$ and $\boldsymbol{q} \parallel \boldsymbol{M}$ so far
- For arbitrary $\angle(\boldsymbol{q}, \boldsymbol{M})$ no analytical solution possible

$$H_x^{\mathrm{dip}} = M_s \frac{1 - e^{-q\,d}}{q\,d}$$

$$H_y^{\mathrm{dip}} = M_s \left(1 - \frac{1 - e^{-q\,d}}{q\,d}\right) \sin^2 \phi$$

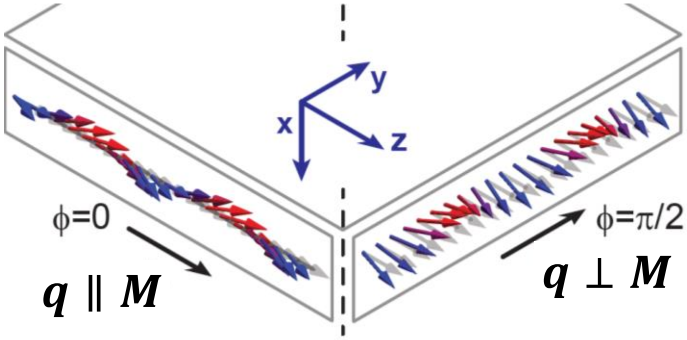
$$H_{\mathrm{ex}} = \frac{2A}{\mu_0 M_s}\, q^2$$

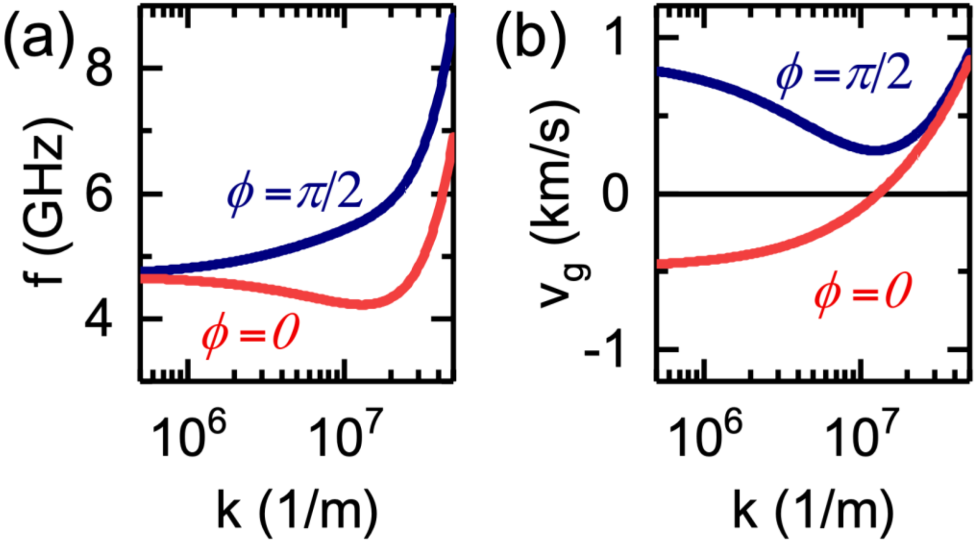
Kalinikos-Slavin approximation for dipolar-exchange spin waves in a tangentially magnetized thin film:

$$\omega = \mu_0 \gamma \sqrt{\left(H_0 + H_{\mathrm{ex}} + H_x^{\mathrm{dip}}\right)\left(H_0 + H_{\mathrm{ex}} + H_y^{\mathrm{dip}}\right)}$$

J. Phys. C: Solid State Phys. **19**, 7013 (1986).

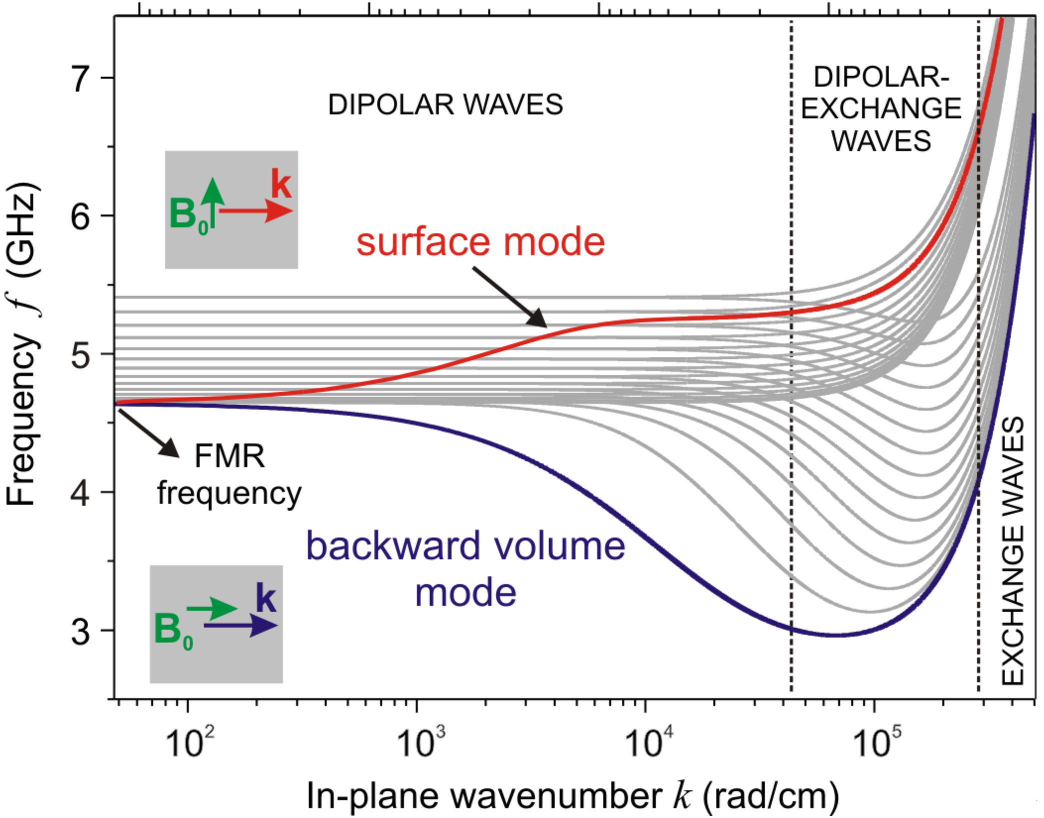
Kalinikos Slavin Solution

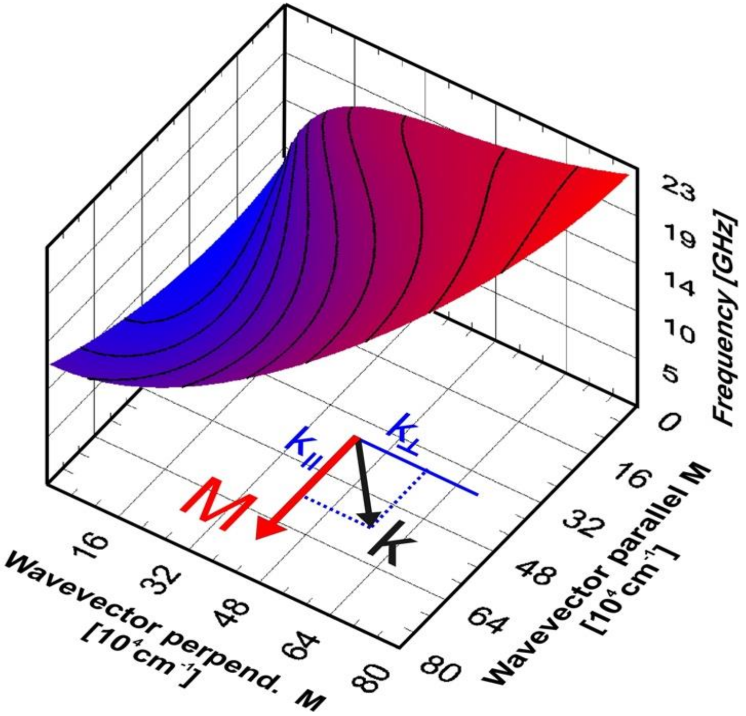




$$\omega = \mu_0 \gamma \sqrt{\left(H_0 + H_{\mathrm{ex}} + H_x^{\mathrm{dip}}\right)\left(H_0 + H_{\mathrm{ex}} + H_y^{\mathrm{dip}}\right)}$$

Full Spin Wave Dispersion of Tangentially Magnetized Thin Films





Two degenerate global minima at $q \neq 0$

Spin Waves and Chirality

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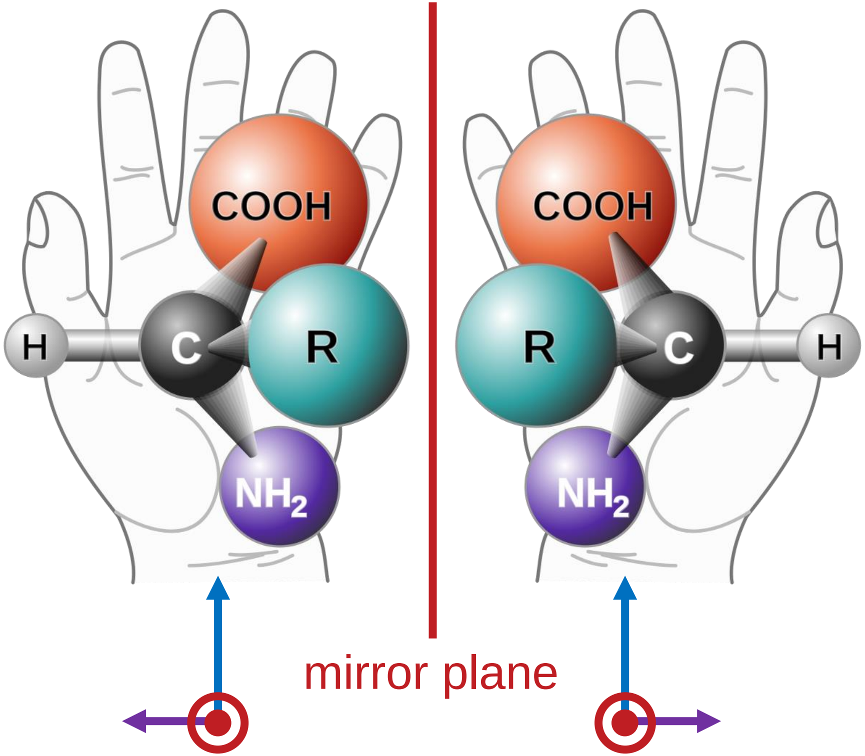
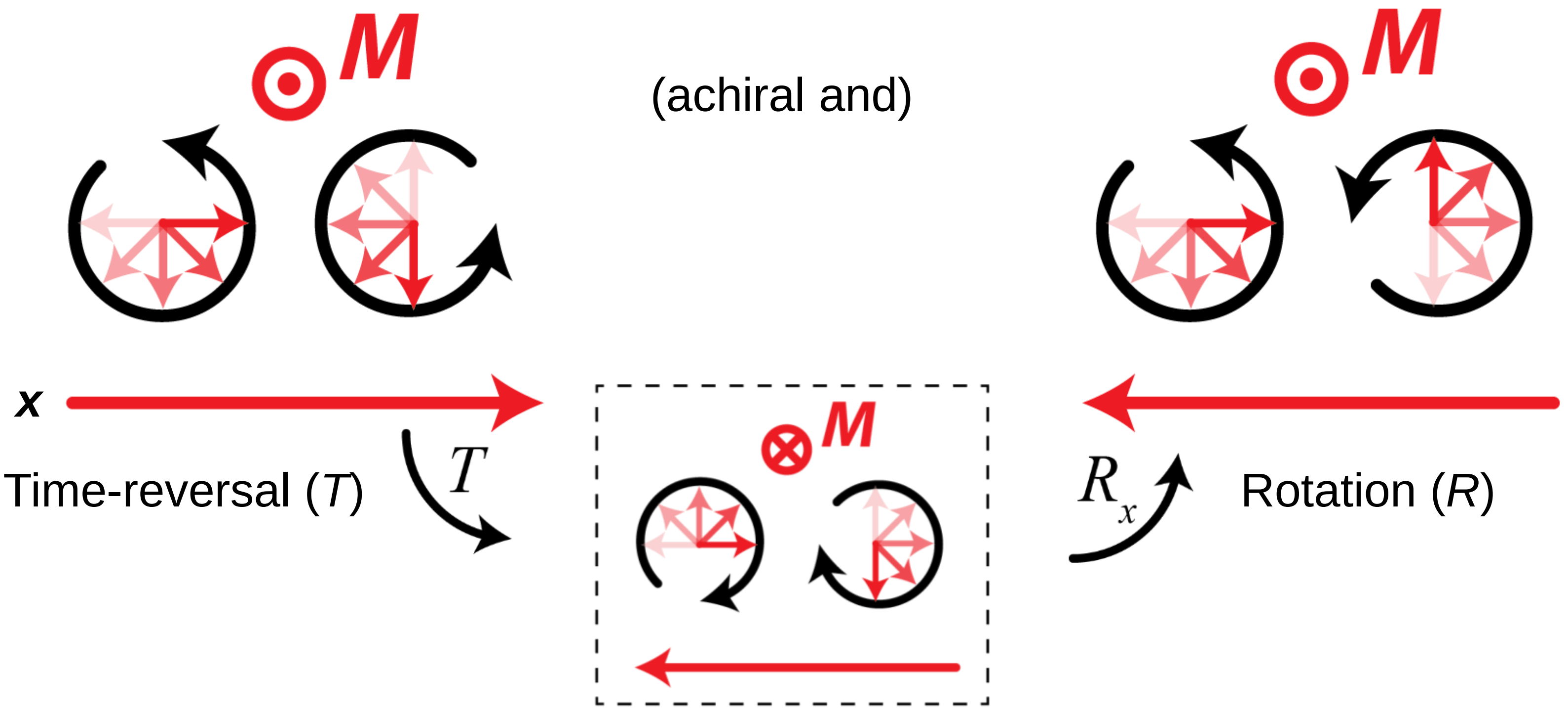
Chiral objects



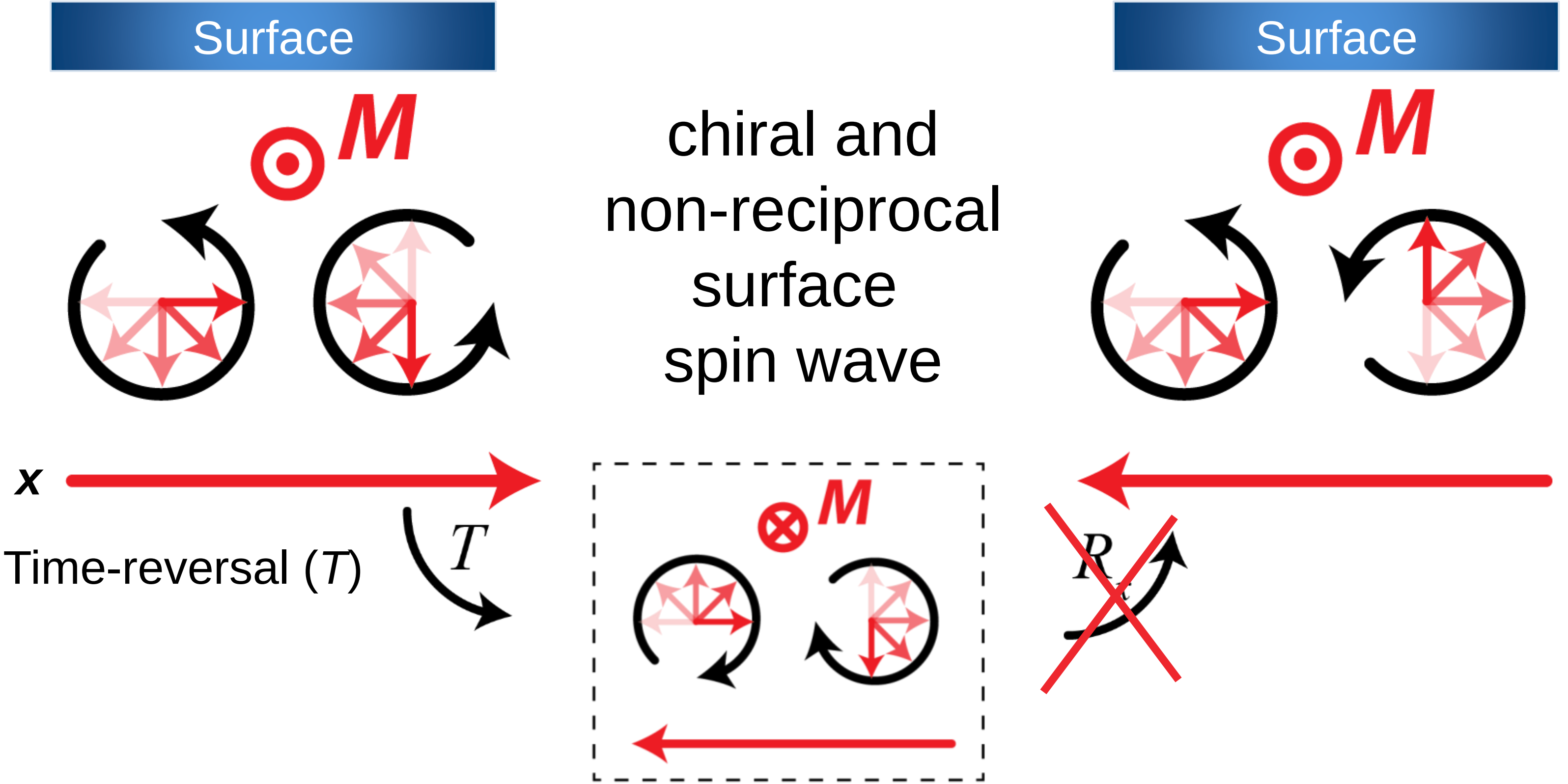
Achiral object

See also: T. Yu et al., Physics Reports **1009**, 1 (2023)

Spinwave (Non)-reciprocity



Spinwave (Non)-reciprocity



Chirality protects from backscattering

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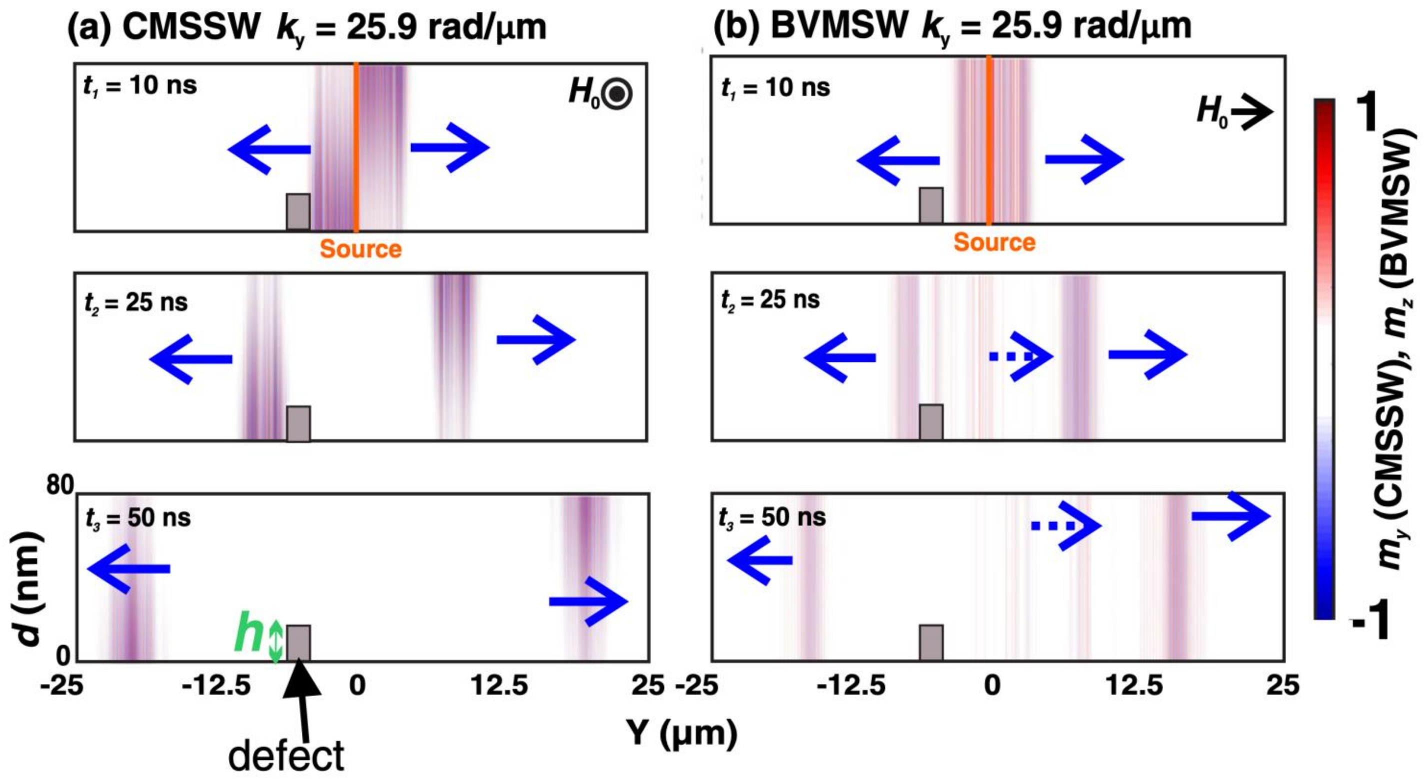
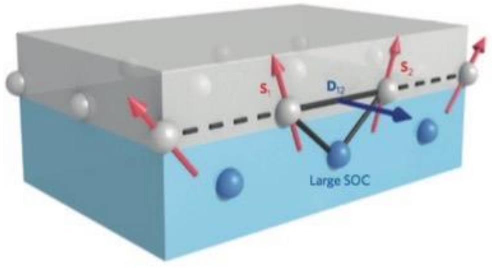
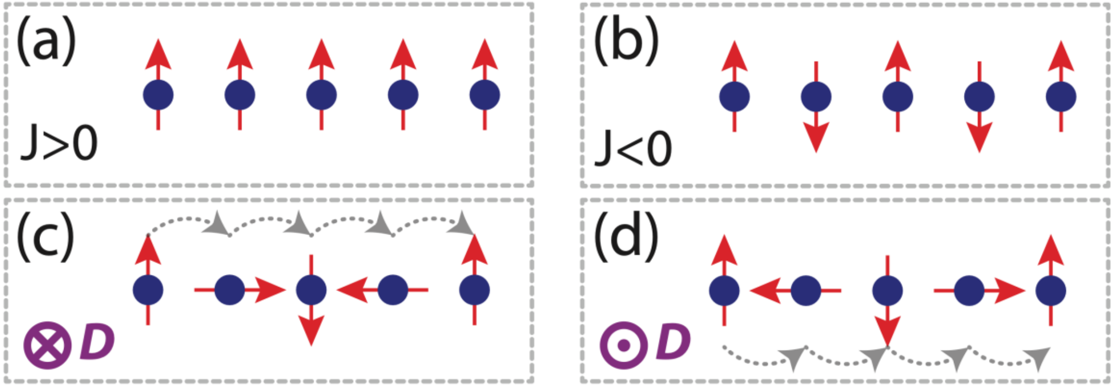


PRL **122**, 197201 (2019)

Chirality leads to non-reciprocity



Nat. Nanotech. **8**, 152 (2013).



$$H_{\mathrm{H}} = -J\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

$$H_{\mathrm{DMI}} = -\boldsymbol{D}\left(\boldsymbol{S}_1 \times \boldsymbol{S}_2\right)$$

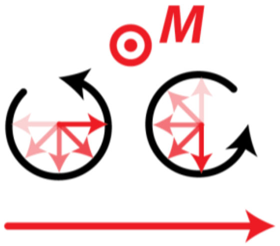
Dzyaloshinskii-Moriya Interaction (DMI):

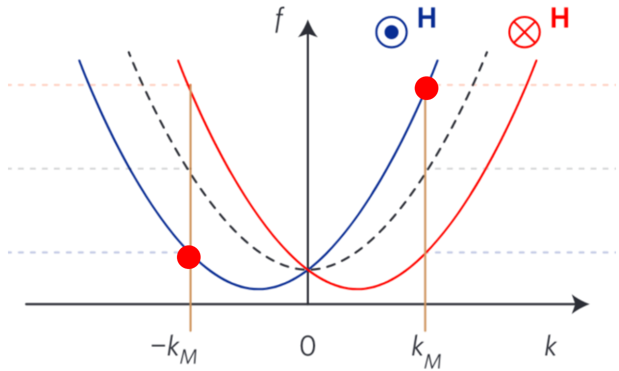
- Breaks degeneracy of orthogonal spin orientations
- Requires broken inversion symmetry and spin-orbit interaction

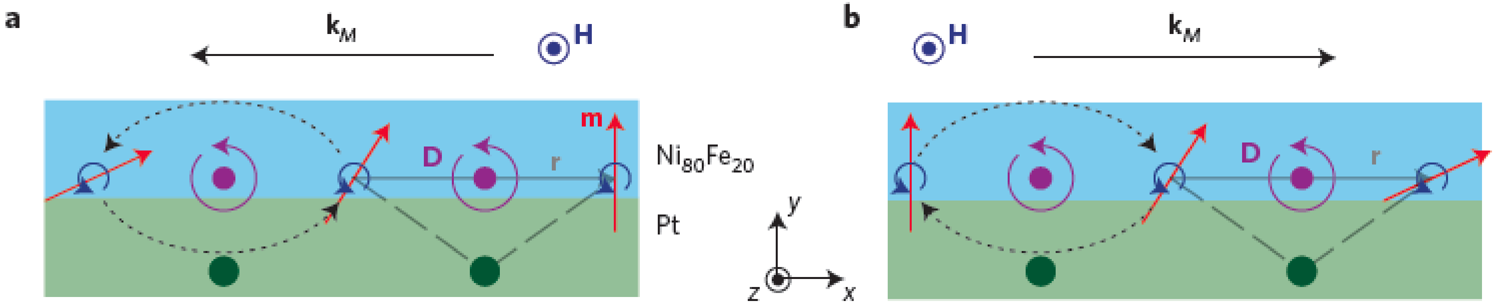
Chirality leads to non-reciprocity

RPTU

Surface







$$f = \frac{\omega}{2\pi} = f_0 + \boldsymbol{D}_{\mathrm{DMI}} \cdot \boldsymbol{q}$$

Nat. Phys. **11**, 825 (2015)

Spin wave damping

Kalinikos-Slavin approximation for dipolar-exchange spin waves in a tangentially magnetized thin film:

$$\omega_k = \mu_0\gamma\sqrt{\left(H_0 + H_{ex} + H_x^{dip}\right)\left(H_0 + H_{ex} + H_y^{dip}\right)}$$

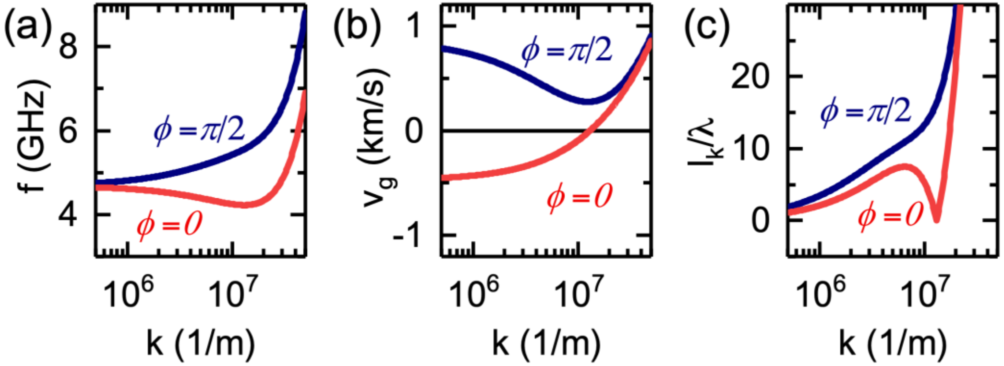
$$\Delta\omega_k = \alpha\mu_0\gamma\left(H_0 + H_{ex} + \frac{1}{2}\left(H_x^{dip} + H_y^{dip}\right)\right)$$

Spin-wave lifetime $\tau_k = \frac{1}{\Delta\omega_k}$

Spin-wave group velocity $v_g = \frac{\partial\omega_k}{\partial k}$

Spin-wave propagation length:

$$l_k = \tau_k |v_g|$$



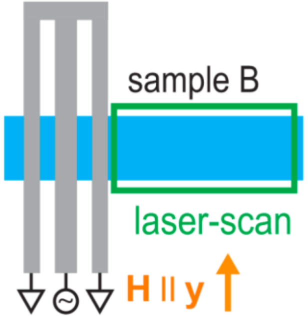
Spin wave damping

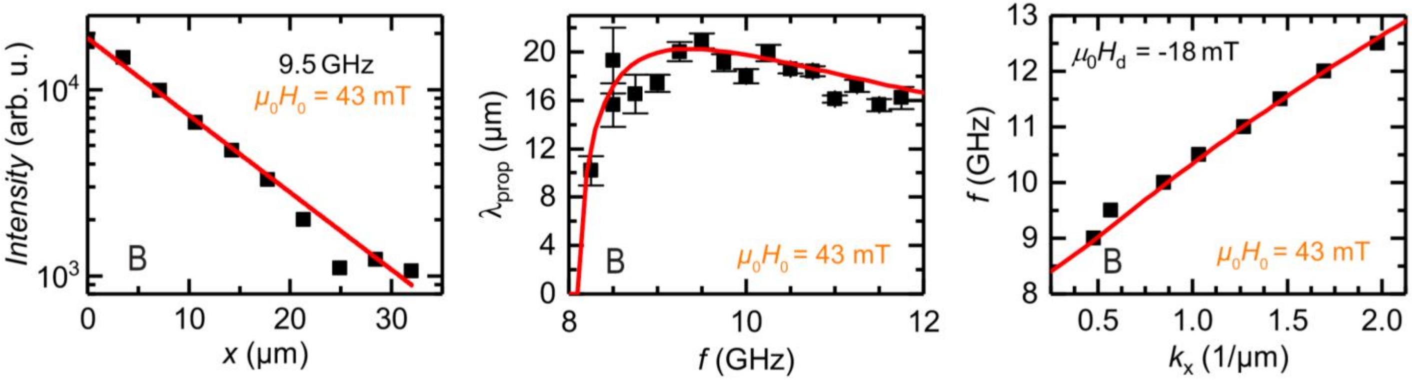
$$\omega_k = \mu_0\gamma\sqrt{\left(H_0 + H_{ex} + H_x^{dip}\right)\left(H_0 + H_{ex} + H_y^{dip}\right)}$$

$$\Delta\omega_k = \alpha\mu_0\gamma\left(H_0 + H_{ex} + \frac{1}{2}\left(H_x^{dip} + H_y^{dip}\right)\right)$$

$$\tau_k = \frac{1}{\Delta\omega_k}$$

$$l_k = \tau_k|v_g|$$

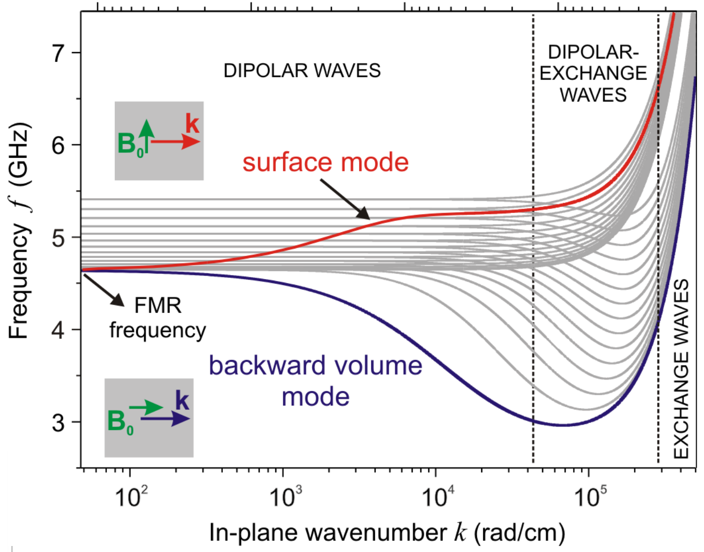




In metallic thin films, typically µm propagation length

Summary

- Spin waves can be used as information carriers
- Exchange spin waves have isotropic dispersion
- Dipolar spin waves have anisotropic dispersion
- Surface spin waves are chiral & non-reciprocal



Wave-based logic