Quantities and units in Magnetism

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Slides: https://fruchart.eu/slides

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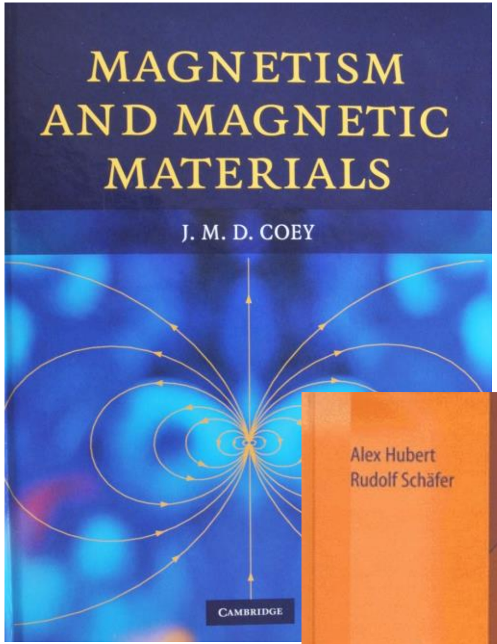
THE EUROPEAN SCHOOL ON MAGNETISM

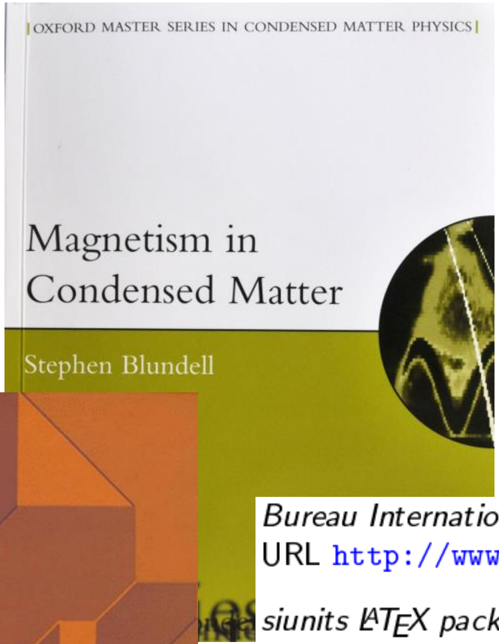
On va trop
rire (une fois)

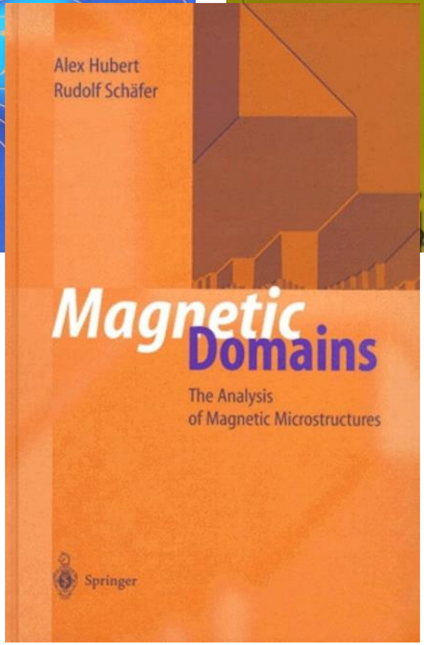
Il est givré ce
mec (ou quoi?)

Institut René Goscinny, https://www.institut-goscinny.org/

Some references







Bureau International des Poids et Mesures.
URL http://www.bipm.org/

siunits LaTeX package.
URL https://www.ctan.org/pkg/siunits

F. Cardarelli, Encyclopedia of Scientific units, weights and measures, Springer, London, 2003.

R. B. Goldfarb, *The Permeability of Vacuum and the Revised International System of Units*, IEEE Trans. Magn. 8, 1–3 (2017).

R. B. Goldfarb, *Electromagnetic Units, the Giorgi System, and the Revised InternationalSystem of Units*, IEEE Magn. Lett. 9, 1205905 (2018).

S. Schlamminger, Redefining the kilogram and other SI units, IOP, 2018.

What is a quantity?

What is a unit ?

Quantity

- Example: speed $\mathbf{v} = \delta\boldsymbol{\ell}/\delta t$
- Dimension: $\dim(\mathbf{v}) = \mathrm{L}\cdot\mathrm{T}^{-1}$

Units

- Why?
  - Provide a measure
  - Universality: share with others
- Possible formalism:

$$X = X_\alpha \langle X \rangle_\alpha$$

Quantity

Measure

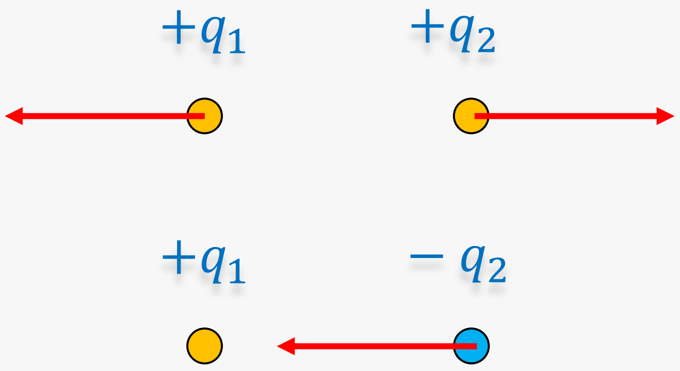
Reference quantity

$$\langle L \rangle_{\mathrm{SI}} = \text{meter} = 100 \langle L \rangle_{\mathrm{cgs}}$$

$$L = 50 \langle L \rangle_{\mathrm{SI}} = 5000 \langle L \rangle_{\mathrm{cgs}}$$

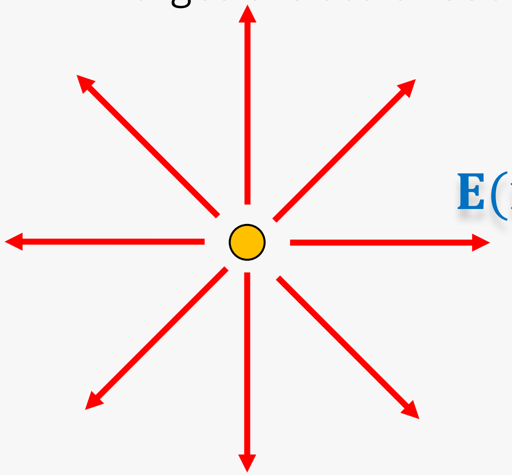
Facts: force between charges

$$\mathbf{F}_{1\to2} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\mathbf{u}_{12}$$



Modeling by the Physicist

- Electric field $\mathbf{E}_{1\to2}$ $\qquad \mathbf{F}_{1\to2} = \mathrm{q}_2\mathbf{E}_{1\to2}$
- Charges are scalar sources of electric field

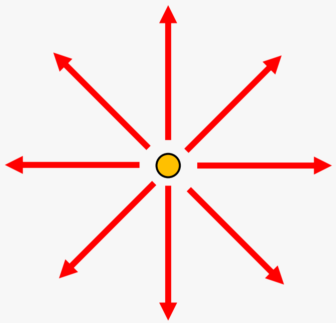


$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r^2}\mathbf{u}$$

The electric charge and the electric field

Macroscopic level: Gauss theorem

- Ostogradski theorem



$$\iiint_{\mathcal{V}} \boldsymbol{\nabla}\cdot\mathbf{E}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$

$$\Rightarrow \quad \frac{Q}{\epsilon_0} = \iiint_{\mathcal{V}} \frac{\rho}{\epsilon_0}\,\mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E}\cdot\mathbf{n}\,\mathrm{d}\mathcal{S}$$

Link

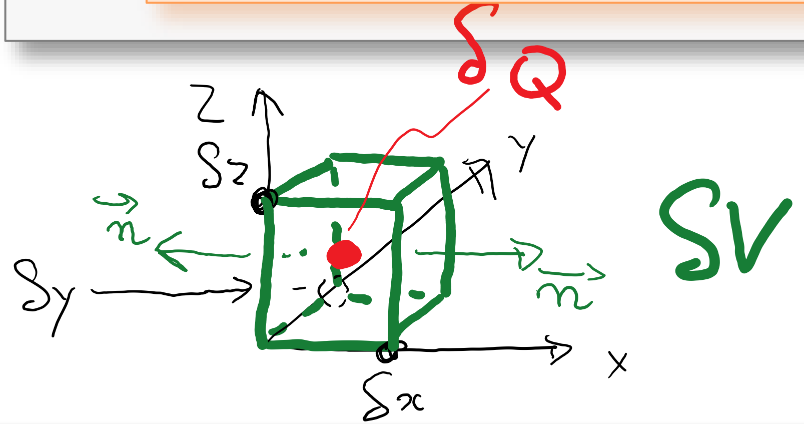
$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\partial E_x}{\partial x} + \ldots = \frac{E_x(x+\delta x) - E_x(x)}{\delta x} + \cdots$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla}\cdot\mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\delta Q}{\delta \mathcal{V}}$$ Volume density of electric charge

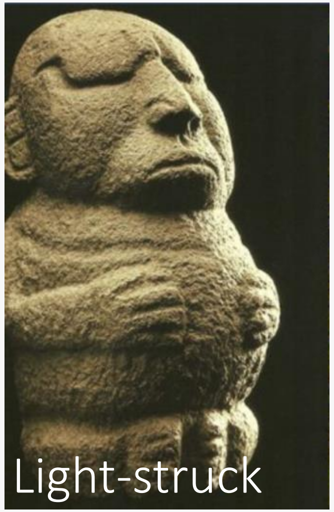
- ρ is the scalar source of **E**



Origin of magnetism

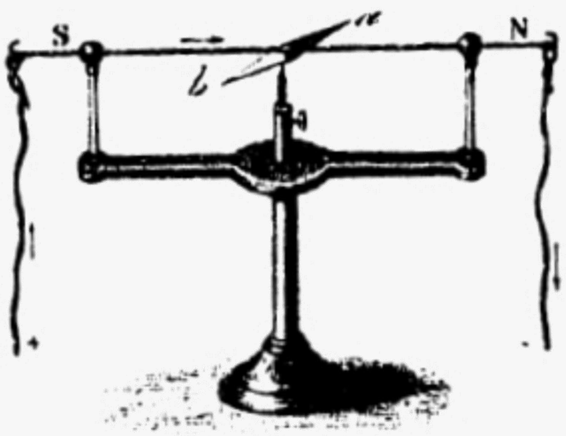
Century-old facts

- Magnetic materials (rocks)



- Magnetic field of the earth

Œrsted experiment in 1820



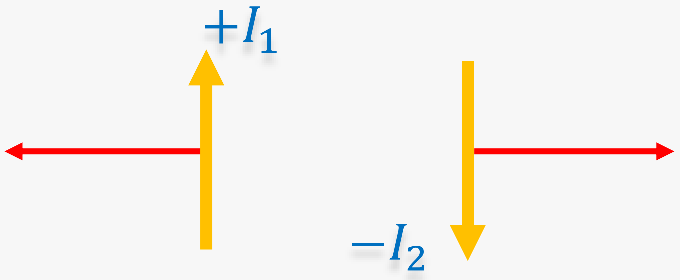
Hans-Christian Oersted, 1777–1851.

Birth of electromagnetism

The electric current and the magnetic induction field

Facts: force between charge currents

$$\delta\mathbf{F}_{1\to 2} = \mu_0 \frac{I_1 I_2[\delta\boldsymbol{\ell}_2 \times (\delta\boldsymbol{\ell}_1 \times \mathbf{u}_{12})]}{4\pi r_{12}^2}$$

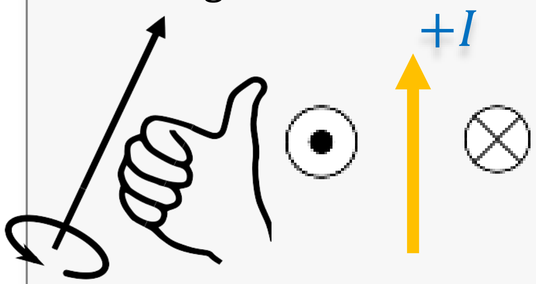


Note: former definition of the Ampère:

The force between two infinite wires 1m apart with current 1A ~~is~~ was 2×10^{-7} N/m

Modeling by the Physicist

- Magnetic induction field: Biot & Savart law



$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Retrieve the force (Laplace)

$$\delta\mathbf{F}_2 = I_2 \delta\boldsymbol{\ell} \times \mathbf{B}(\mathbf{r}_2)$$

$$\Rightarrow \mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

- Magnetic induction field defined through Lorentz Force

The electric current and the magnetic induction field

Macroscopic level: Ampere theorem

- Stokes theorem

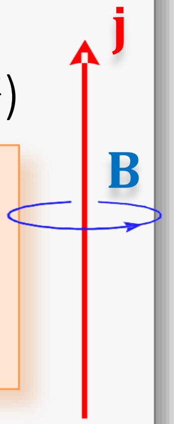
$$\iint_{\mathcal{S}} (\boldsymbol{\nabla} \times \mathbf{B}) \cdot \mathbf{n}\, \mathrm{d}\mathcal{S} = \oint_{\partial \mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

$$\Rightarrow \quad I = \mu_0 \iint_{\mathcal{S}} (\mathbf{j} \cdot \mathbf{n})\, \mathrm{d}\mathcal{S} = \oint_{\partial \mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j}$$

j: Volume density of current (A/m^2)

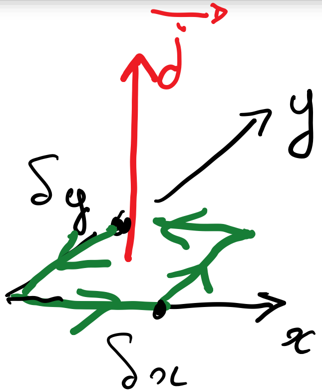


- j is the vectorial source of curl of B

Unit for B: tesla (T)

Link

$$\boldsymbol{\nabla} \times \mathbf{B} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y} \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{B_y(x+\delta x) - B_y(x)}{\delta x} - \dfrac{B_x(y+\delta y) - B_x(y)}{\partial y} \end{pmatrix}$$



Maxwell equations (in vacuum)

$$\mathbf{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$	⇒	Gauss theorem
$$\mathbf{\nabla} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$	⇒	Faraday law of induction
$$\mathbf{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$	⇒	Ampère theorem
$$\mathbf{\nabla} \cdot \mathbf{B} = 0$$	⇒	B is divergence free (no magnetic poles)

The magnetic point dipole (a magnetic moment)

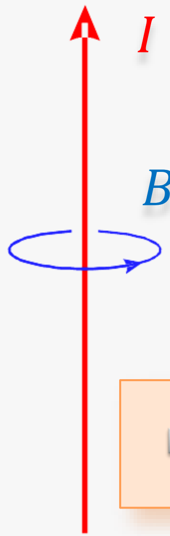
Biot and Savart

$$\delta \mathbf{B} = \frac{\mu_0 I \delta \boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Note: $1/r^2$ decay

Integrate

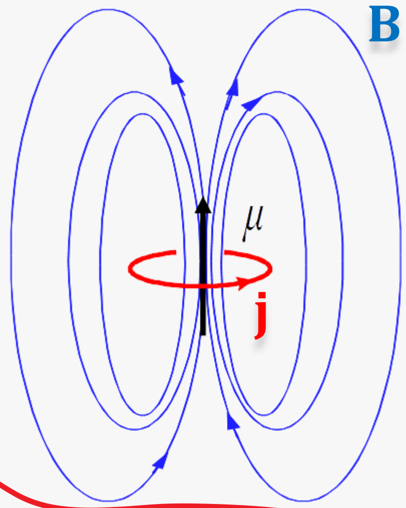
Ampere theorem and Œrsted field



$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

- Note: $1/r$ decay

The magnetic point dipole



- Simple loop

$$\boldsymbol{\mu} = I\mathcal{S}\,\mathbf{n} \qquad \textbf{Unit}: \mathrm{A}\cdot\mathrm{m}^2$$

- General definition

$$\boldsymbol{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbf{r} \times \mathbf{j}(\mathbf{r})\,\mathrm{d}\mathcal{V}$$

Derive

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - \boldsymbol{\mu}\right]$$

- Note: $1/r^3$ decay

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left(2\mu\cos\theta\,\mathbf{u}_r + \mu\sin\theta\,\mathbf{u}_\theta\right)$$

The magnetic point dipole in a magnetic induction field

Energy

$\mathcal{E} = -\boldsymbol{\mu} \cdot \mathbf{B}$ Zeeman energy J

Demonstration

- Work to compensate Lenz law during rise of **B**
- Integrate torque from Laplace force while flipping dipole in **B**

Force

$\mathbf{F} = \boldsymbol{\mu} \cdot (\overline{\overline{\boldsymbol{\nabla}\mathbf{B}}})$ N

- Valid only for fixed dipole
- No force in uniform magnetic induction field

Torque

$\boldsymbol{\Gamma} = \oint \mathbf{r} \times I(\mathrm{d}\boldsymbol{\ell} \times \mathbf{B}) = \boldsymbol{\mu} \times \mathbf{B}$ $\mathrm{N} \cdot \mathrm{m} = \mathrm{J}$

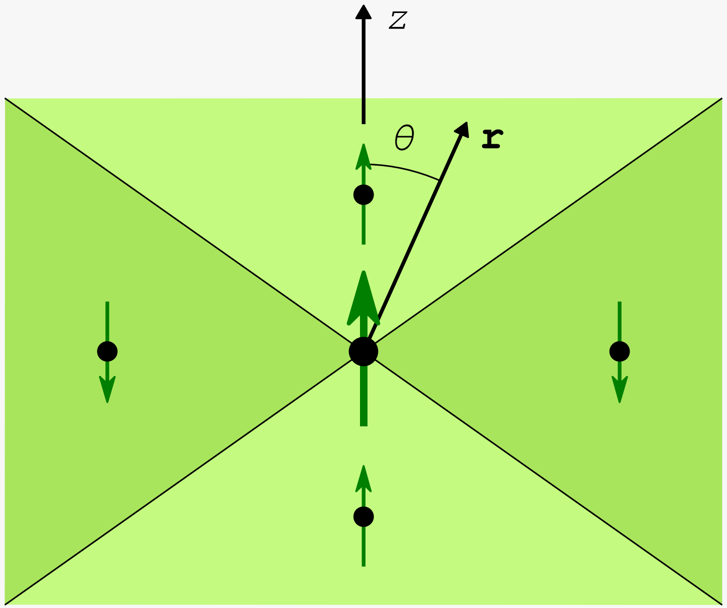
- Inducing precession of dipole around the field
- It is energy-conservative, as expected from Laplace (Lorentz) force

Two interacting magnetic point dipoles

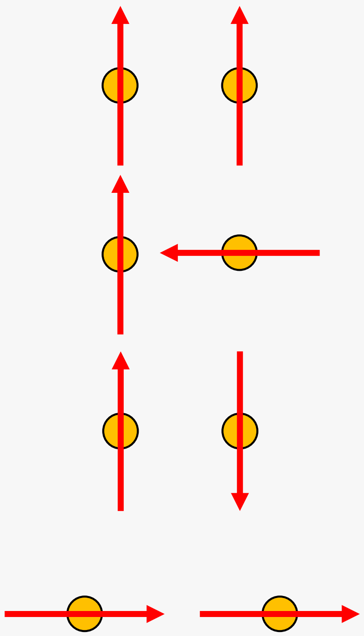
Energy

$$\mathcal{E} = -\frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}_1 \cdot \mathbf{r})(\boldsymbol{\mu}_2 \cdot \mathbf{r}) - \boldsymbol{\mu}_1 \cdot \boldsymbol{\mu}_2\right]$$

- The dipole-dipole interaction is anisotropic



Examples



$$\mathcal{E} = +2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = +\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = 0$$

$$\mathcal{E} = -\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = -2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

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Definition

- Volume density of magnetic point dipoles

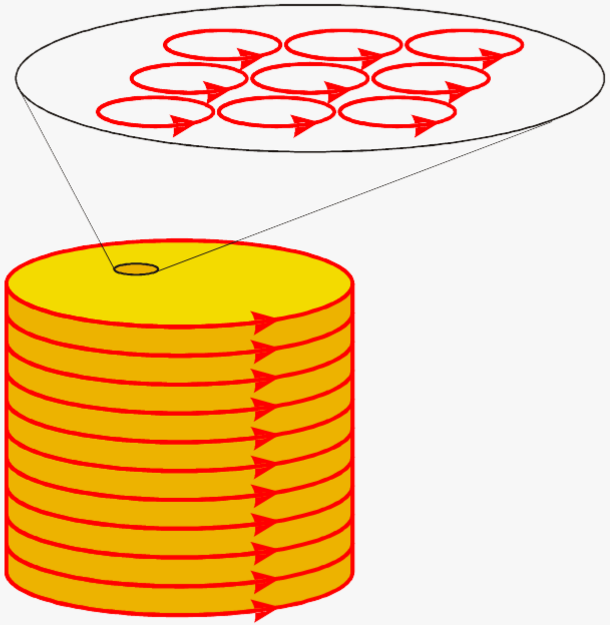
$$\mathbf{M} = \frac{\delta\boldsymbol{\mu}}{\delta\mathcal{V}} \qquad \mathrm{A/m}$$

- Total magnetic moment of a body

$$\boldsymbol{\mathcal{M}} = \int_{\mathcal{V}} \mathbf{M}\,\mathrm{d}\mathcal{V} \qquad \mathrm{A\cdot m^2}$$

- Applies to: ferromagnets, paramagnets, diamagnets etc.
- Must be defined at a length scale much larger than atoms
- Is the basis for the micromagnetic theory

Equivalence with surface currents



- Name: Amperian description of magnetism
- Surface current equals magnetization $\mathrm{A/m}$

Free currents and bound currents

Back to Maxwell equations

- Disregard fast time dependence: magnetostatics

$$\boldsymbol{\nabla}\times\mathbf{B}=\mu_0\left(\mathbf{j}+\cancel{\epsilon_0\frac{\partial\mathbf{E}}{\partial t}}\right)$$

- Consider separately real charge current, $\mathbf{j}_\mathrm{c}$ from fictitious currents of magnetic dipoles $\mathbf{j}_\mathrm{m}$

$$\boldsymbol{\nabla}\times\mathbf{B}=\mu_0(\mathbf{j}_\mathrm{c}+\mathbf{j}_\mathrm{m})$$

- One can show: $\boldsymbol{\nabla}\times\mathbf{M}=\mathbf{j}_\mathrm{m}$ $\quad\mathrm{A/m^2}$

$\mathbf{M}\times\mathbf{n}=\mathbf{j}_\mathrm{m,s}$ $\quad\mathrm{A/m}$

- Outside matter, $\mathbf{B}$ and $\mu_0\mathbf{H}$ coincide and have exactly the same meaning.

The magnetic field H

- One has: $\boldsymbol{\nabla}\times\left(\dfrac{\mathbf{B}}{\mu_0}-\mathbf{M}\right)=\mathbf{j}_\mathrm{c}$

- By definition: $\mathbf{H}=\dfrac{\mathbf{B}}{\mu_0}-\mathbf{M}$ $\quad\mathrm{A/m}$

$$\boldsymbol{\nabla}\times\mathbf{H}=\mathbf{j}_\mathrm{c}$$

B versus H : definition of the system

- **M**: local (infinitesimal) part in $\delta\mathcal{V}$ of the system defined when considering a magnetic material

- **H**: The remaining of B coming from outside $\delta\mathcal{V}$, liable to interact with the system

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The dipolar field $\mathbf{H}_\mathrm{d}$

- By definition: the contribution to $\mathbf{H}$ not related to free currents (possible to split as Maxwell equations are linear)

$$\nabla \times \mathbf{H}_\mathrm{d} = 0 \Rightarrow \mathbf{H}_\mathrm{d} = -\nabla \phi_\mathrm{d}$$

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{app}$$

External to magnetic body

Analogy with electrostatics

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla \phi$$

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

Volume density of charges

Derive the dipolar field

Maxwell equation $\nabla \cdot \mathbf{B} = \mathbf{0} \rightarrow \nabla \cdot \mathbf{H}_\mathrm{d} = -\nabla \cdot \mathbf{M}$

$$\Rightarrow \mathbf{H}_\mathrm{d}(\mathbf{r}) = -M_\mathrm{s} \iiint_{\mathcal{V}'} \frac{[\nabla \cdot \mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{V}' + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

$\rho(\mathbf{r}) = -M_\mathrm{s}\,\nabla \cdot \mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r}) = M_\mathrm{s}\,\mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

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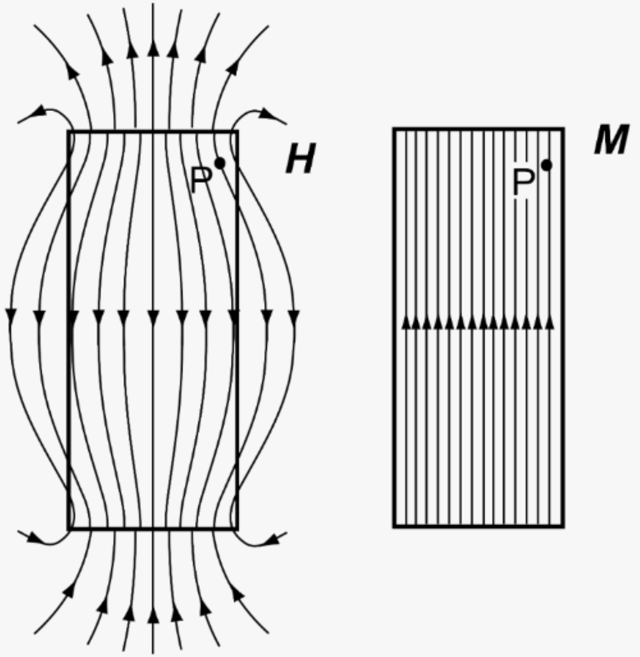
Example Permanent magnet (uniformly-magnetized)



- Surface charges

$$\sigma(\mathbf{r}) = M_s \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{S}'$$

Illustration from: M. Coey's book

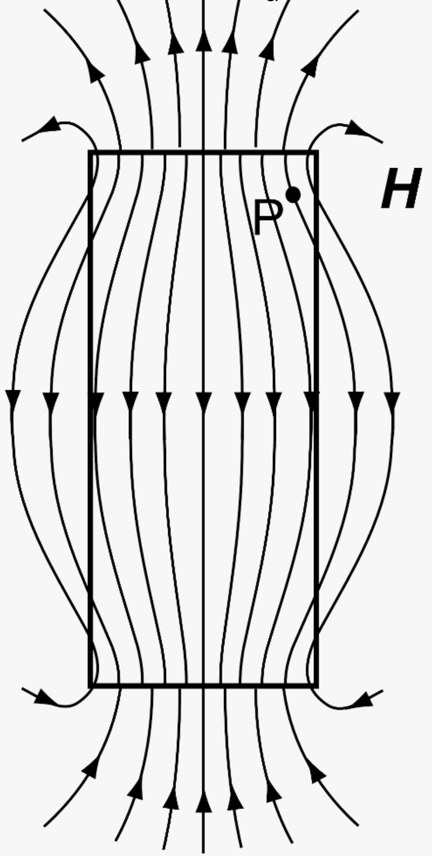
Vocabulary

- Generic names
  - Magnetostatic field
  - Dipolar field
- Inside material
  - Demagnetizing field
- Outside material
  - Stray field

B versus H – Amperian versus Coulombian – Continuity conditions

Coulombian

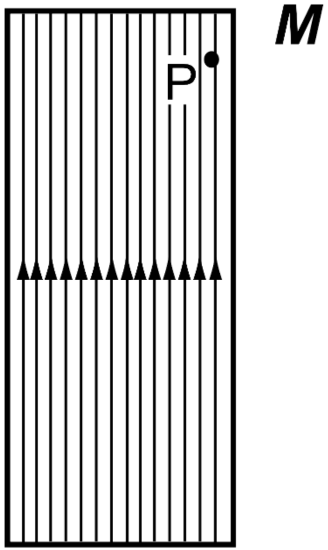
- Pseudo-charges source of H_d



$$\nabla \times \mathbf{H} = \mathbf{0}$$

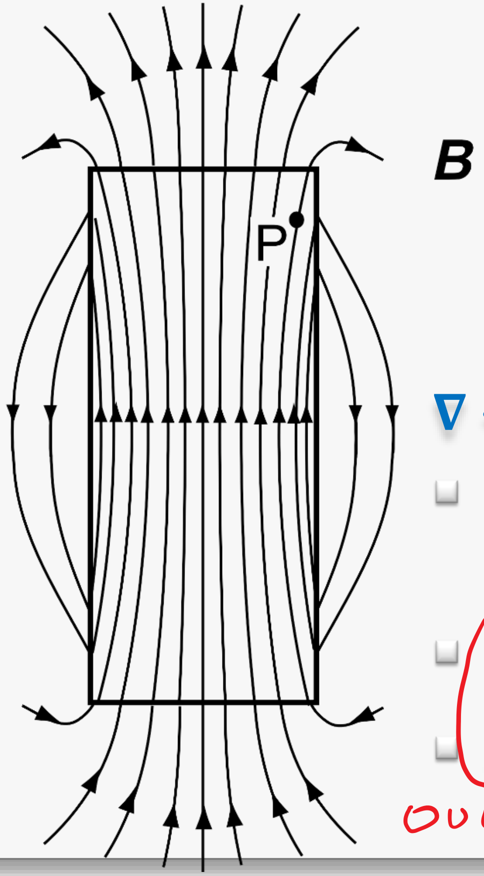
- No closed lines
- $\Delta H_{\parallel} = 0$
- $\Delta \mathbf{H} \cdot \mathbf{n} = \sigma$

out - in



Amperian

- Fictitious currents source of B



$$\nabla \cdot \mathbf{B} = \mathbf{0}$$

- No magnetic monopole
- $\Delta B_{\perp} = 0$
- $\Delta \mathbf{B} = \mu_0 \mathbf{j} \times \mathbf{n}$

out - in

From: M. Coey's book

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Dipolar energy

- Zeeman energy of microscopic volume

$$\delta\mathcal{E}_\mathrm{Z} = -\mu_0 \mathbf{M}\delta\mathcal{V} \cdot \mathbf{H}_\mathrm{ext}$$

- Elementary volume of a macroscopic system creating its own dipolar field

$$E_\mathrm{d} = \delta\mathcal{E}_\mathrm{d}/\delta\mathcal{V} = -\frac{1}{2}\mu_0 \mathbf{M} \cdot \mathbf{H}_\mathrm{d}$$

mutual energy /

- Total dipolar energy of macroscopic body

$$\mathcal{E}_\mathrm{d} = -\frac{1}{2}\mu_0 \iiint_\mathcal{V} \mathbf{M} \cdot \mathbf{H}_\mathrm{d} \, \mathrm{d}\mathcal{V}$$

$$\mathcal{E}_\mathrm{d} = \frac{1}{2}\mu_0 \iiint_\mathcal{V} \mathbf{H}_\mathrm{d}^2 \, \mathrm{d}\mathcal{V}$$

- Always positive. Zero means minimum

Size considerations

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \mathrm{Volume} + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\,\mathrm{d}\mathcal{S}'$$

- Unchanged if all lengths are scaled: homothetic. NB: the following is a solid angle:

$$\mathrm{d}\boldsymbol{\Omega} = \frac{(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathcal{S}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

- H_d does not depend on the size of the body
- Said to be a long-range interaction

Range of dipolar interactions in low dimensionality

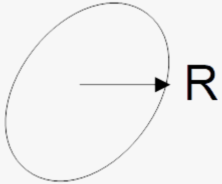
Range

- Upper bound of dipolar field in thin films

$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3} \mathrm{dr}$$

Integration

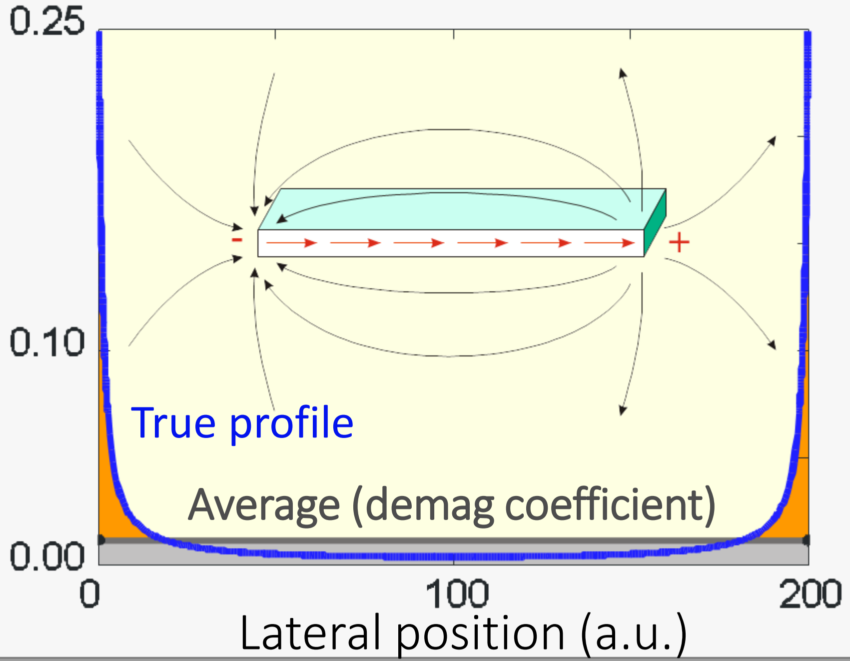
Hd for dipole



$$\|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

- Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged in low dimensions
- Dipolar fields are highly non-homogeneous in such large aspect ratio systems
- Consequences: non-uniform magnetization switching, edge modes etc.

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Dipolar energy for uniform magnetization

$$\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_s(m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + m_z\hat{\mathbf{z}})$$

Implicit $\sum_{i=x,y,z}$

- No volume charges: $\rho(\mathbf{r}) = -M_s\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = 0$
- Dipolar field: $\mathbf{H}_d(\mathbf{r}) = \oiint_{\partial\mathcal{V}} \dfrac{[\mathbf{M}(\mathbf{r}')\cdot\mathbf{n}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}' = M_s m_i \oiint_{\partial\mathcal{V}} \dfrac{\mathrm{n}_i(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$
- Dipolar energy:

$$\mathcal{E}_d = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M}(\mathbf{r})\cdot\mathbf{H}_d(\mathbf{r})\mathrm{d}\mathcal{V} = -\frac{1}{2}\mu_0 M_s^2 m_i \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\,\mathbf{m}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

$$\mathcal{E}_d = -K_d m_i m_j \iiint_{\mathcal{V}} \mathrm{d}\mathcal{V} \oiint_{\partial\mathcal{V}} \frac{n_i(\mathbf{r}')\,(r_j - r_j')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

Implicit $\sum_i \sum_{j=x,y,z}$

$$\langle\mathbf{H}_d(\mathbf{r})\rangle = -M_s\,\overline{\overline{\mathbf{N}}}\cdot\mathbf{m}$$

$$\mathcal{E}_d = K_d V\,\mathbf{m}\cdot\overline{\overline{\mathbf{N}}}\cdot\mathbf{m}$$

See more detailed approach: M. Beleggia et al., JMMM 263, L1-9 (2003)

Demagnetizing coefficients – Take-away messages

For any shape of body

$$\langle \mathbf{H}_{\mathrm{d}}(\mathbf{r}) \rangle = -M_{\mathrm{s}}\, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_{\mathrm{d}} = K_{\mathrm{d}} V\, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

Dipolar anisotropy is always of second order

- $\bar{\bar{\mathbf{N}}}$ demagnetizing tensor. Always positive, and can be diagonalized. $N_x + N_y + N_z = 1$

$$\mathcal{E}_{\mathrm{d}} = K_{\mathrm{d}} V \left(N_x m_x^2 + N_y m_y^2 + N_z m_z^2 \right)$$

- Along main directions

$$\langle H_{\mathrm{d},i}(\mathbf{r}) \rangle = -N_i M_{\mathrm{s}}$$

Hypothesis uniform **M** may be too strong

Remember: dipolar field is NOT uniform

For ellipsoids etc.

- Condition: boundary is a polynomial of the coordinates, with degree at most two

Slabs (thin films), cylinders, ellipsoids

$$z^2 = \left(\frac{t}{2}\right)^2 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \qquad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$$\mathbf{H}_{\mathbf{d}} = -M_{\mathrm{s}}\, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

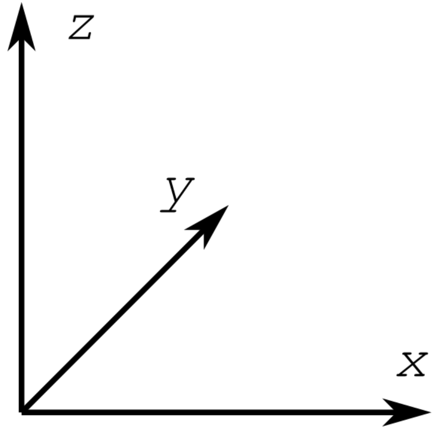
$$\mathcal{E}_{\mathrm{d}} = K_{\mathrm{d}} V\, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

- Along main directions

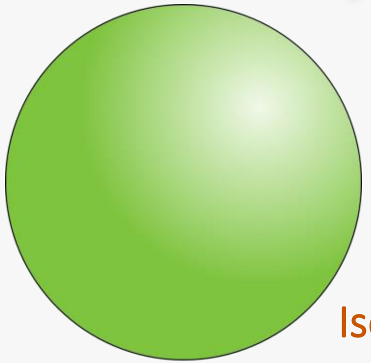
$$H_{\mathrm{d},i} = -N_i M_{\mathrm{s}}$$

M and **H** may not be colinear along non-main directions

Demagnetizing coefficient (examples)



Sphere $L_x = L_y = L_z = D$

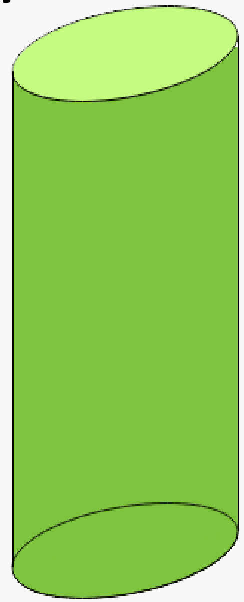


$$N_x = N_y = N_z = \frac{1}{3}$$

Isotropic

Cylinder $L_x = L_y = D$

$L_z = \infty$



$$N_x = N_y = \frac{1}{2}$$

$$N_z = 0$$

Favors **axial magnetization**

Slab (thin film) $L_x = L_y = \infty$

$$N_x = N_y = 0$$

$$N_z = 1$$

Favors in-plane magnetization

Take-away message

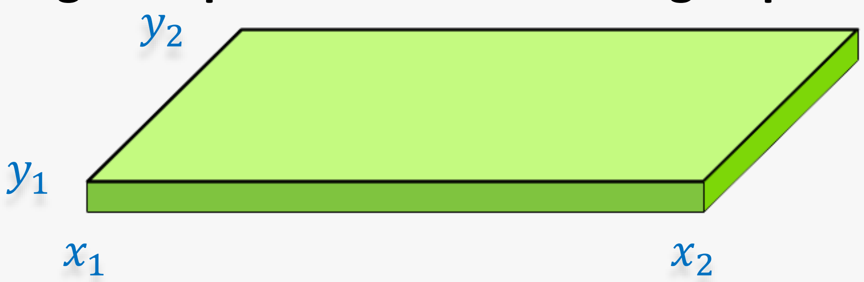
Dipolar energy favors alignment of magnetization with longest direction of sample

Core function for the magnetic scalar potential

$$F_{000}(x,y,z) = \frac{1}{r} = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$\phi(x,y,z) = \frac{Q}{4\pi} F_{000}(x,y,z)$$

Magnetic potential from a charged plate



$$\phi(x,y,z) = \frac{\sigma}{4\pi}\int_{x_1}^{x_2}\mathrm{d}x' \int_{y_1}^{y_2}\mathrm{d}y'\, F_{000}(x-x', y-y', z)$$

Sum of four terms involving here F_{100}

Definition for F_{ijk} function

F_{000} Can be integrated and/or derived analytically to any order, against x, y, z

$F_{ijk}(x,y,z)$

- Integrated *i* times versus *x*
- Integrated *j* times versus *y*
- Integrated *k* times versus *z*

Example

$$F_{100}(x,y,z) = \int F_{000}(x,y,z)\mathrm{d}x + Cste$$

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Magnetic scalar potential

F_{110} Magnetic scalar potential

Components of magnetic field

F_{11-1}	$H_{\mathrm{d},z}$
F_{010}	$H_{\mathrm{d},x}$
F_{100}	$H_{\mathrm{d},y}$

Gradients of magnetic field

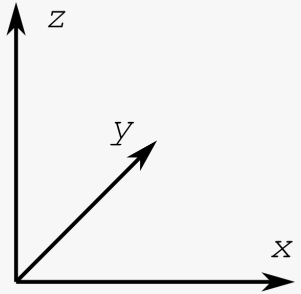
F_{11-2}	$\dfrac{\mathrm{d}H_{\mathrm{d},z}}{\mathrm{d}z}$
F_{01-1}	$\dfrac{\mathrm{d}H_{\mathrm{d},x}}{\mathrm{d}z}$
F_{10-1}	$\dfrac{\mathrm{d}H_{\mathrm{d},y}}{\mathrm{d}z}$

MFM contrast

Demag coefficients

F_{220}	N_z
F_{022}	N_x
F_{202}	N_y

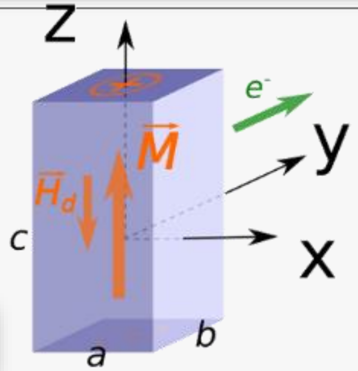
Examples of other specific cases



F_{12-1} *z* component of H, integrated along *y*

F_{120} *z*-averaged *z* component of H, integrated along *y*

Electron holography of a Pillar



The dipolar exchange length

When: anisotropy and exchange compete

$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K_\mathrm{d}\sin^2\theta$$

Exchange: $\mathrm{J/m}$ — Dipolar: $\mathrm{J/m^3}$

$$K_\mathrm{d} = \frac{1}{2}\mu_0 M_s^2$$

$$\boxed{\Delta_\mathrm{d} = \sqrt{A/K_\mathrm{d}} = \sqrt{2A/\mu_0 M_s^2}}$$

$$\Delta_\mathrm{d} \simeq 3 - 10\ \mathrm{nm}$$

Critical single-domain size, relevant for small particles made of soft magnetic materials

Often called: exchange length

The anisotropy exchange length

When: anisotropy and exchange compete

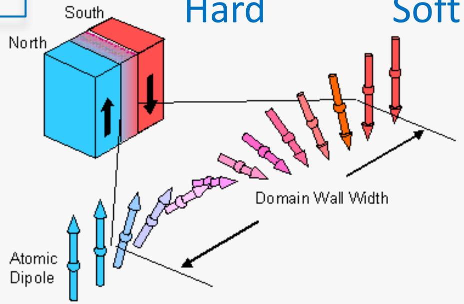
$$E = A\left(\frac{\partial m_i}{\partial x_j}\right)^2 + K\sin^2\theta$$

Exchange: $\mathrm{J/m}$ — Anisotropy: $\mathrm{J/m^3}$

$$\boxed{\Delta_\mathrm{u} = \sqrt{A/K}}$$

$\Delta_\mathrm{u} \simeq 1\ \mathrm{nm} \rightarrow 100\ \mathrm{nm}$

Hard — Soft



Sometimes called: Bloch parameter, or wall width

Note: Other length scales can be defined, e.g. with magnetic field

Units in SI versus cgs

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Definitions

	S.I.		cgs-Gauss	
	Meter	m	Centimeter	cm
	Kilogram	kg	Gram	g
	Second	s	Second	s
	Ampere	A	Ab-Ampere	ab-A = 10 A

$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ $\qquad$ $\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{S.I.}$ $\qquad$ $"\mu_0" = 4\pi.$

Problems with cgs versus SI

- The quantity for charge current is missing
  No check for homogeneity;
  paradox for spintronics
- Inconsistent definition of H
  Dimensionless quantities are affected:
  demag coefficients, susceptibility etc.

Conversion of measures for the same quantity

Field	$\mathbf{H}$	$1\ \mathrm{A/m}$	⟷	$4\pi \times 10^{-3}\ \mathrm{Oe}$	Œrsted
Moment	$\boldsymbol{\mu}$	$1\ \mathrm{A\cdot m^2}$	⟷	$10^3\ \mathrm{emu}$	
Magnetization	$\mathbf{M}$	$1\ \mathrm{A/m}$	⟷	$10^{-3}\ \mathrm{emu/cm^3}$	Electromagnetic Unit
Induction	$\mathbf{B}$	$1\ \mathrm{T}$	⟷	$10^4\ \mathrm{G}$	Gauss
Susceptibility	$\chi = M/H$	1	⟷	$1/4\pi$	

Tutorial on units
Questions: http://magnetism.eu/esm/2018/abs/fruchart-practical-abs1.pdf
Answers: http://magnetism.eu/esm/2018/abs/fruchart-practical-answers1.pdf

How to convert units from one system to another?

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Example: length $X = X_\alpha \langle X \rangle_\alpha$

Quantity

Reference quantity

Measure

The SI standard is 100 times **LARGER** than the cgs one

$$L = 50\langle L\rangle_{\mathrm{SI}} = 5000\langle L\rangle_{\mathrm{cgs}}$$

$50\ \mathrm{m}$ is equivalent to $5000\ \mathrm{cm}$

The SI measure is 100 times **SMALLER** than the cgs one

The ratio is opposite if one considers the standard for a quantity (a quantity) or the measure (a number) of a given quantity

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Process for converting units

1. Convert all basic units (MKSA)

$$\langle \mathrm{L}\rangle_{\mathrm{SI}} = \mathrm{meter} = 10^2 \langle \mathrm{L}\rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{M}\rangle_{\mathrm{SI}} = \mathrm{kilogram} = 10^3 \langle \mathrm{M}\rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{T}\rangle_{\mathrm{SI}} = \mathrm{second} = \langle \mathrm{T}\rangle_{\mathrm{cgs}}$$

$$\langle \mathrm{I}\rangle_{\mathrm{SI}} = \mathrm{Ampère} = 10^{-1} \langle \mathrm{I}\rangle_{\mathrm{cgs}}$$

2. Decompose any given quantity in fundamental quantities. In practice, identify a formula linking it to quantities already decomposed
3. Apply the formalism defining units and measures

$$X = X_\alpha \langle X\rangle_\alpha$$

Example Mechanics, force F

$$\mathbf{F} = m\,\mathbf{a}$$

$$\dim(\mathbf{F}) = \mathrm{M}\cdot\mathrm{L}\cdot\mathrm{T}^{-2}$$

$$\begin{aligned} F &= F_{\mathrm{SI}} \langle F\rangle_{\mathrm{SI}} \\ &= F_{\mathrm{SI}}\, \langle \mathrm{L}\rangle_{\mathrm{SI}}\, \langle \mathrm{M}\rangle_{\mathrm{SI}}\, \langle \mathrm{T}\rangle_{\mathrm{SI}}^{-2} \\ &= F_{\mathrm{SI}}\, 10^2 \langle \mathrm{L}\rangle_{\mathrm{cgs}}\, 10^3 \langle \mathrm{M}\rangle_{\mathrm{cgs}}\, (1)^{-2} \langle \mathrm{T}\rangle_{\mathrm{cgs}}^{-2} \\ &= F_{\mathrm{SI}}\, 10^5 \langle \mathrm{F}\rangle_{\mathrm{cgs}} = F_{\mathrm{cgs}}\, \langle \mathrm{F}\rangle_{\mathrm{cgs}} \end{aligned}$$

$$\Rightarrow \left| \begin{aligned} \langle \mathrm{F}\rangle_{\mathrm{SI}} &= 10^5\, \langle \mathrm{F}\rangle_{\mathrm{cgs}} \\ F_{\mathrm{SI}} &= 10^{-5}\, F_{\mathrm{cgs}} \end{aligned} \right.$$

$1\ \mathrm{N}$ is equivalent to $10^5\ \mathrm{erg}$

Proposed logarithmic formalism for dimensionality

$$\dim(\mathbf{X}) = \mathrm{L}^{\alpha} \cdot \mathrm{M}^{\beta} \cdot \mathrm{T}^{\gamma} \cdot \mathrm{I}^{\delta}$$

		$\langle X \rangle_{\mathrm{SI}} / \langle X \rangle_{\mathrm{cgs}}$	Log
M (meter)	$[\mathrm{L}] = [1\ 0\ 0\ 0]$	10^2	2
K (kg)	$[\mathrm{M}] = [0\ 1\ 0\ 0]$	10^3	3
S (second)	$[\mathrm{T}] = [0\ 0\ 1\ 0]$	1	0
A (Ampère)	$[\mathrm{I}] = [0\ 0\ 0\ 1]$	10^{-1}	-1

$$[\mathbf{X}] = \alpha[\mathrm{L}] + \beta[\mathrm{M}] + \gamma[\mathrm{T}] + \delta[\mathrm{I}]$$

$$[\mathbf{X}] = [\alpha\ \beta\ \delta\ \gamma]$$

Example Mechanics, force F

$$\mathbf{F} = m\,\mathbf{a}$$

$$[\mathbf{F}] = [m] + [\mathbf{a}] = [0\ 1\ 0\ 0] + [1\ 0\ -2\ 0]$$

$$[\mathbf{F}] = [1\ 1\ -2\ 0]$$

$$2\ \ 3\ \ \ \ 0$$

$$2\ \ 3\ \ \ \ 0\ \ \ \ \ \ \to 5$$

$1\ \mathrm{N}$ is equivalent to $10^5\ \mathrm{erg}$

Dimensionality

- A magnetic moment has the dimension of a pinpoint magnetic dipole $\boldsymbol{\mu} = I\mathbf{S}$. thus, $[\boldsymbol{\mu}] = [2\ 0\ 0\ 1]$.
- Magnetization is a volume density of magnetic moments: $\mathbf{M} = \boldsymbol{\mu}/V$, so: $[\mathbf{M}] = [-1\ 0\ 0\ 1]$. $\mathbf{M}$ and $\mathbf{H}$ have the same dimension as we can see from: $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. Thus: $[\mathbf{H}] = [-1\ 0\ 0\ 1]$.
- Magnetic induction B is what matters in Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, so that: $[\mathbf{B}] = [0\ 1\ \ -2\ \ -1]$.
- Magnetic flux is $\phi = BS$ so that: $[\phi] = [2\ 1\ \ -2\ \ -1]$.
- Finally, as in electricity, μ_0 makes the link between the source (current) and fields on one side, and energy and mechanics on the other side, as for the Lorentz force above: $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, or in vacuum: $\mathbf{curl}\,\mathbf{B} = \mu_0\mathbf{j}$, from which one derives: $[\mu_0] = [1\ 1\ \ -2\ \ -2]$.

Units (easy situations)

- Induction $\mathbf{B}$. 1 T is equivalent to 10^4 G, G standing for *Gauss*.
- Magnetization $\mathbf{M}$. 1 A/m is equivalent to 10^{-3} uem/cm^3, emu standing for *ElectroMagnetic Unit*.
- Flux ϕ. 1 Wb (Weber) is equivalent to 10^8 Mx, Mx standing for *Maxwell*.
- Moment $\boldsymbol{\mu}$. $1\,\mathrm{A}\cdot\mathrm{m}^2$ is equivalent to 10^3 emu.

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Tricky case 1: magnetic permeability

$$\mu_0 = \mu_{0_{\text{SI}}} \langle \mu_0 \rangle_{\text{SI}}$$

$$[\mu_0] = [1\ 1\ -2\ -2]$$

$$\rightarrow\ \langle \mu_0 \rangle_{\text{SI}} = 10^2 \cdot 10^3 \cdot (10^{-2})^{-1} \langle \mu_0 \rangle_{\text{cgs}}$$

$$\rightarrow\ \langle \mu_0 \rangle_{\text{SI}} = 10^7 \langle \mu_0 \rangle_{\text{cgs}}$$

$$\mu_0 = \mu_{0_{\text{SI}}} \langle \mu_0 \rangle_{\text{SI}} \rightarrow\ "\mu_{0_{\text{cgs}}}" = 4\pi$$

S.I.

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

cgs-Gauss

$$\mathbf{B} = \mathbf{H} + 4\pi \mathbf{M}$$

→ Unit for permeability dropped; H 4pi larger in cgs

Tricky case 2: magnetic field H

SI: $\mu_0 H = \mu_{0_{\text{SI}}} \langle \mu_0 \rangle_{\text{SI}}\, H_{\text{SI}} \langle H \rangle_{\text{SI}}$

$$\mu_0 H = 4\pi 10^{-7}\ 10^7 \langle \mu_0 \rangle_{\text{cgs}}\ 10^{-3} H_{\text{SI}} \langle H \rangle_{\text{cgs}}$$

Remember: $[\text{H}] = [1\ 0\ -2\ 0]$

cgs: $H = H_{\text{cgs}} \langle H \rangle_{\text{cgs}}$

$$\langle \mu_0 \rangle_{\text{cgs}} = 1$$

$$\rightarrow\ 4\pi\ 10^{-3}\ H_{\text{SI}} = H_{\text{cgs}}$$

$1\ \text{A/m}$ is equivalent to $4\pi\ 10^{-3}\ \text{Oe}$

Demagnetizing coefficients link H with M

$$\langle \mathbf{H}_\mathrm{d}(\mathbf{r}) \rangle = -M_\mathrm{s}\, \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\, \mathbf{m} \cdot \bar{\bar{\mathbf{N}}} \cdot \mathbf{m}$$

Unit:

dimensionless

$$N_x + N_y + N_z = 1$$

- Definition $H = -N\, M$

$$\rightarrow \left(N_x + N_y + N_z\right)_\mathrm{cgs} = 4\pi$$

- Definition $H = -4\pi N\, M$

$$\rightarrow \left(N_x + N_y + N_z\right)_\mathrm{cgs} = 1$$

Magnetic susceptibility links M with H

- Definition $\chi = \delta M / \delta H$

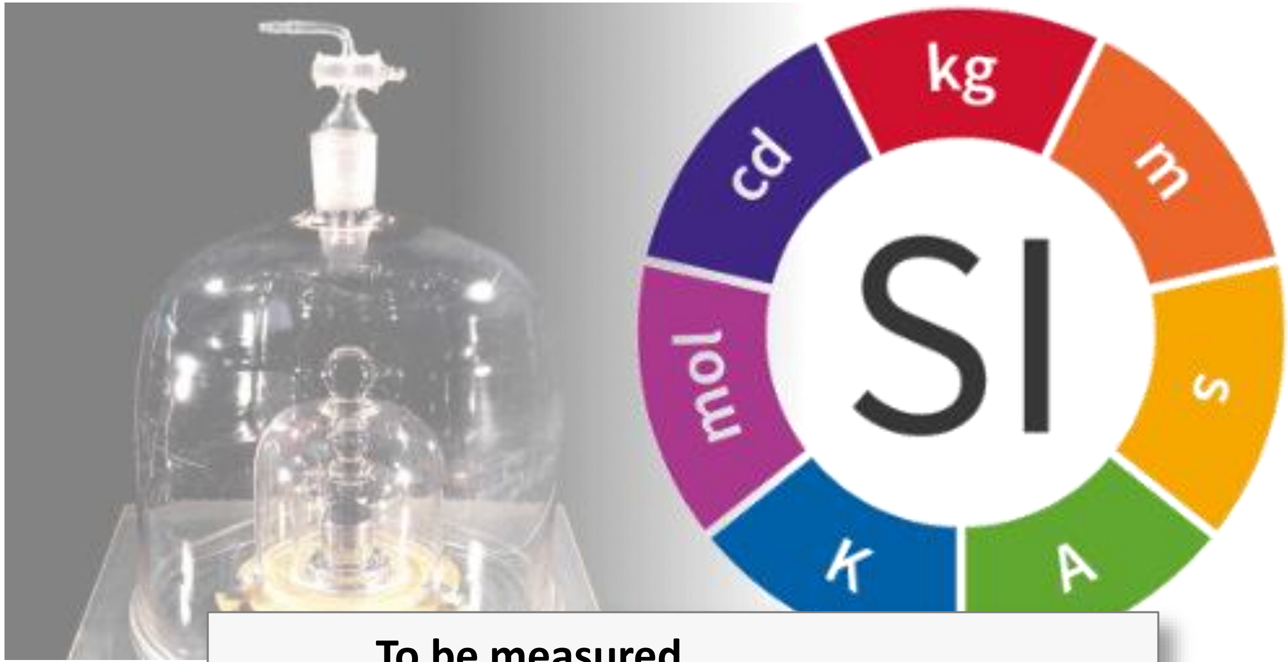
$$\rightarrow \chi_\mathrm{cgs} = \chi_\mathrm{SI} / 4\pi$$

- Definition $\chi = 4\pi\, \delta M / \delta H$

$$\rightarrow \chi_\mathrm{cgs} = \chi_\mathrm{SI}$$

Both definitions are used…

Quantum revolution in SI units in 2019



Define quantities

- Times
- Length
- Mass
- Electric charge

Fixed values

- Speed of light -> Defines m
- Hyperfine Cs transition -> Defines s
- Planck constant -> Defines kg
- Charge of the electron -> Defines A

To be measured

- Magnetic permeability of vacuum

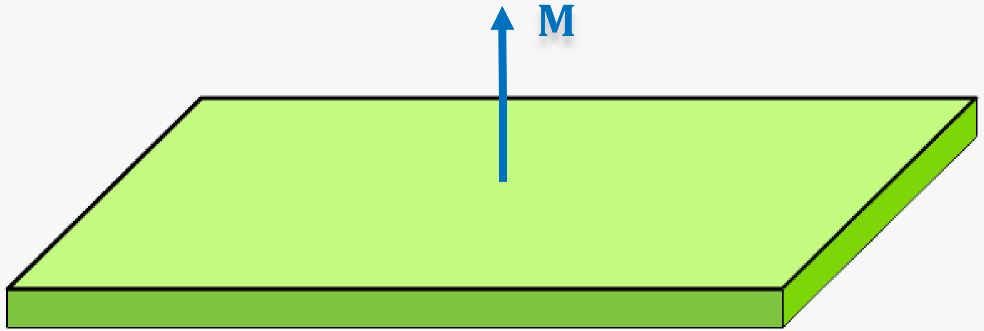
$$\mu_0 \neq 4\pi \times 10^{-7}\ \mathrm{S.I.}$$

$$\mu_0 = 4\pi[1 + 2.0(2.3) \cdot 10^{-10}] \times 10^{-7}\ \mathrm{S.I.}$$

R. B. Goldfarb, IEEE Trans. Magn. MAG. 8, 1-3 (2017); R. B. Goldfarb, IEEE Mag. Lett. 9, 1205905 (2018)
S. Schlamminger, Redefining the kilogram and other SI units, IOP Physics World Discovery (2018)

Quizz 1 – Stray field above thin film

What is the value of stray field above an extended perpendicularly-magnetized thin film?

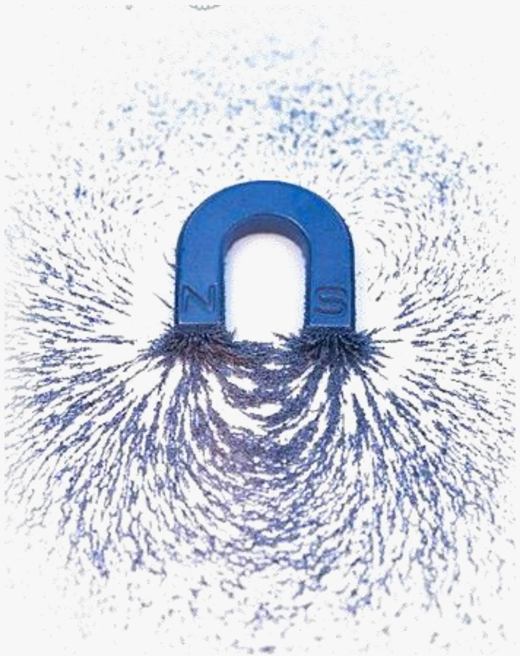


1. Zero
2. + M / 2
3. + M
4. Depends on the value of anisotropy

Quizz 2 – Maximum stray field

What is the maximum stray field (in free space) that can be obtained from a uniformly-magnetized system of arbitrary shape?

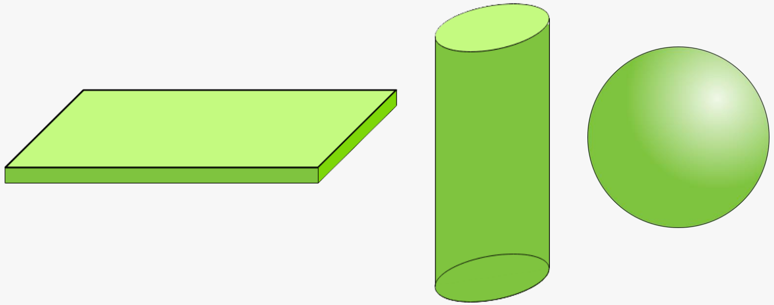
1. M /3
2. M /2
3. M
4. As high as we like, no limit



Quizz 3 – Demagnetizing coefficients

One can define and compute (analytics or numerics) demagnetizing coefficients for…

1. Ellipsoids, slabs, cylinders



2. Any finite-size body made up of one single piece

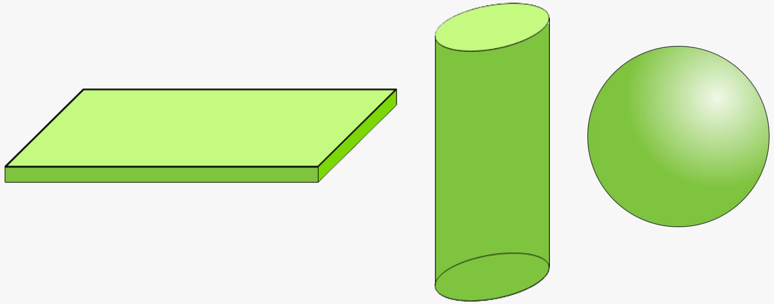
3. Any finite-size body of arbitrary shape

4. None, all is an approximation

Quizz 3 – Demagnetizing coefficients

One can define and compute (analytics or numerics) demagnetizing coefficients for…

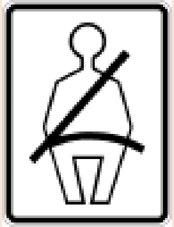
1. Ellipsoids, slabs, cylinders



2. Any finite-size body made up of one single piece

3. Any finite-size body of arbitrary shape

4. None, all is an approximation



I can "demonstrate" that the demagnetizing field inside a uniformly-magnetized sphere is zero

Quizz 4 – Sign of magnetostatic energy

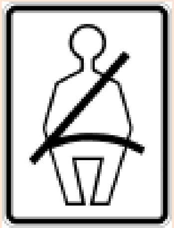
The magnetostatic (dipolar) energy of a system is…

1. Always negative
2. Always positive
3. Depends on the shape of the body
4. Depends on other energies: exchange, anisotropy etc.

Quizz 4 – Sign of magnetostatic energy

The magnetostatic (dipolar) energy of a system is…

1. Always negative
2. Always positive
3. Depends on the shape of the body
4. Depends on other energies: exchange, anisotropy etc.



I can “demonstrate” that magnetostatic energy can be positive and negative at the same time

Thank you for your attention !

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