

# Atomistic spin dynamics Micromagnetics

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# Few questions awaiting answers



- **What is a physical system?**
- **What is a model (in physics)?**
- **What is expected from a model?**
- **What are the means to validate a model?**
- **What is the strategy for building a model?**

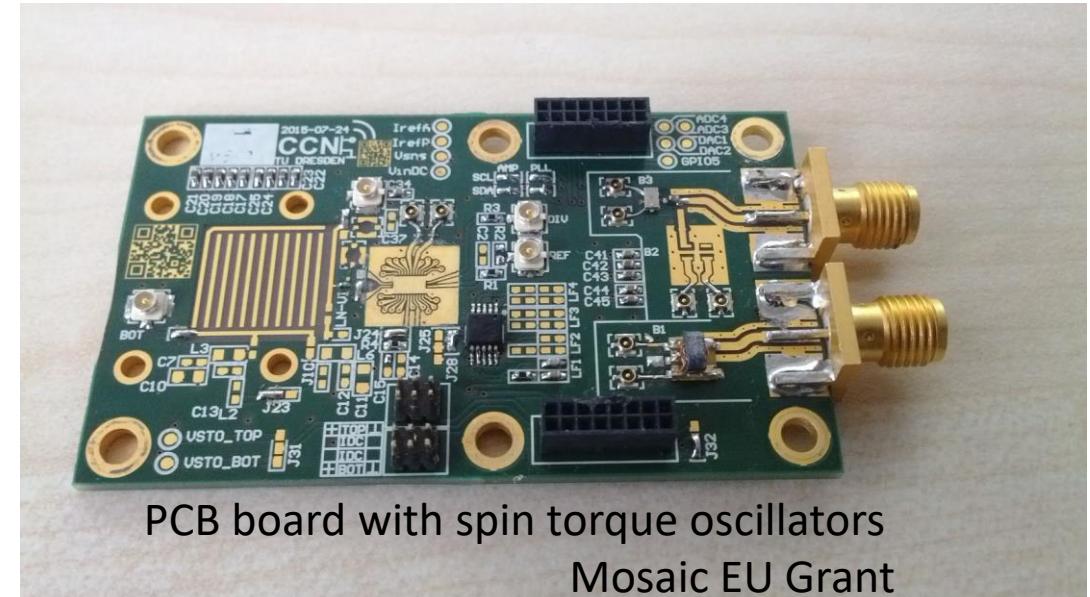
# Modelling - research and debate

## ❑ What is a physical system?

The systems under investigation are complex:

- more than one constituent in the system
- the constituents cannot be considered as independent

Experimental characterization is challenging



PCB board with spin torque oscillators  
Mosaic EU Grant

# Modelling - research and debate

## ■ What is a model (in physics)?

Describe as faithfully as possible the system under investigation  
Define the physical quantities of interest  
An idealized version of a system, simple enough so that  
it can be solved but no too simple to preserve essential physics



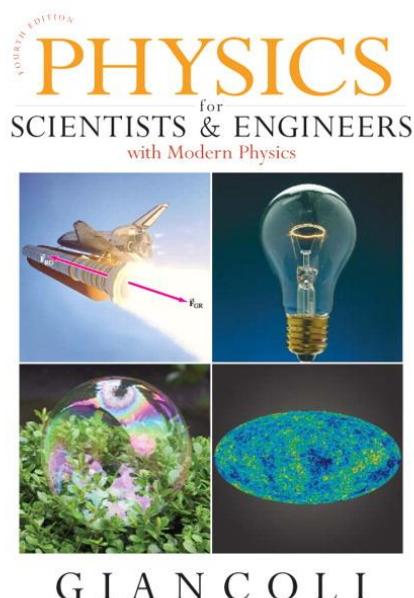
Model 1



Model 2

....

# Modelling - research and debate



## ■ What is expected from a model?

Analysis of the solution leads to increased understanding of a phenomenon or process, which can lead to radical improvements and knowledge progress.

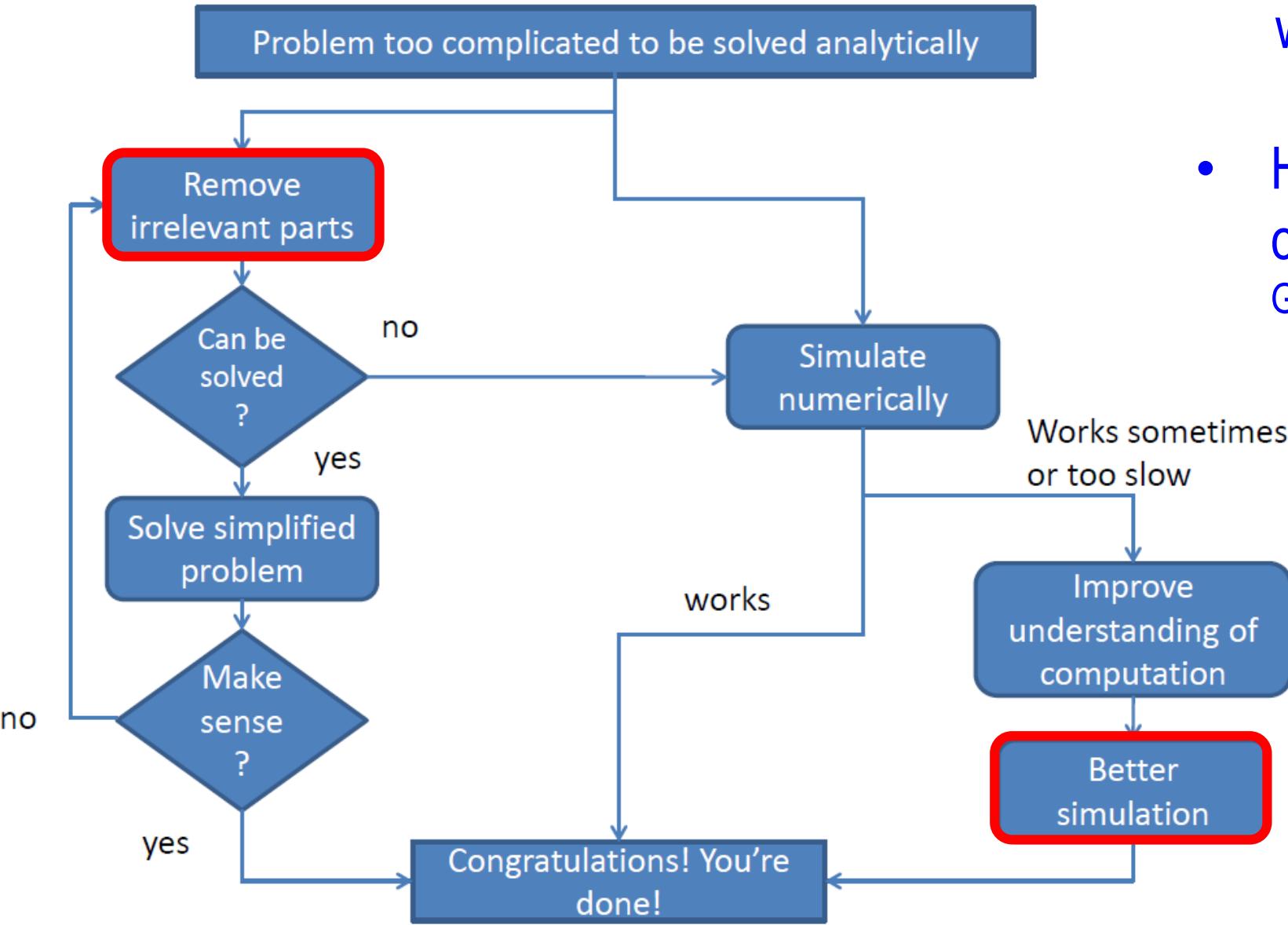
## ■ What are the means to validate a model?

Comparison with experiments and other models (it makes sense 😊)

## ■ What is the strategy for building a model?

Solve the appropriate equations, either analytically or numerically.  
→ this allows to calculate the physical quantities of interest.

## What is the strategy for building a model?



- Step by step, iteratively with rigor and critical thinking
- Hard but most of the time collective work - literature, GNU General Public License, github, [openscience](#)



# Outline



## ■ Atomistic spin dynamics

- Formalism
- Illustrations : alloy GdFe, Fe nanodot, AOS [Tb/Co] multilayers

## ■ Micromagnetism

- Formalism
- Macrospin limit
- Illustrations

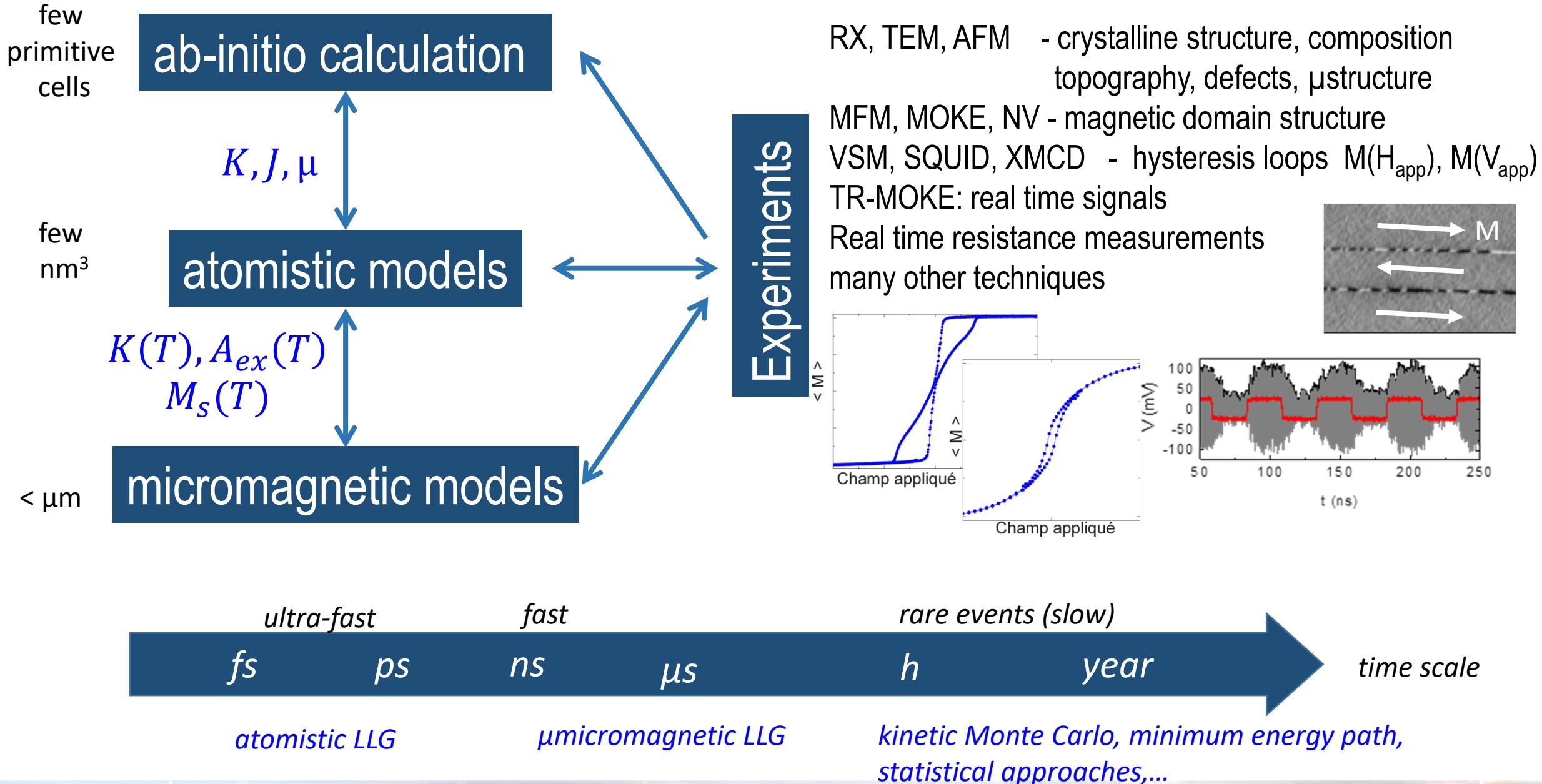
## ■ Micromagnetism and spin related phenomena

- Formalism
- Illustrations

## ■ Conclusions

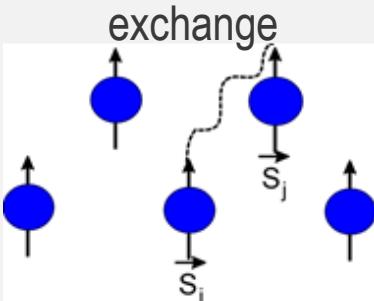


# Hierarchy of models in magnetism: space & time

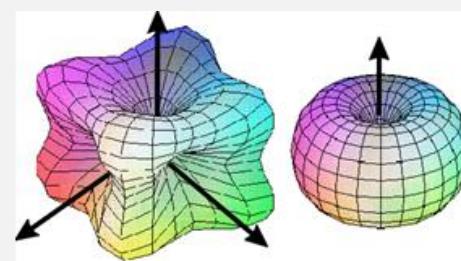


# Main “ingredients” in magnetism & spintronics

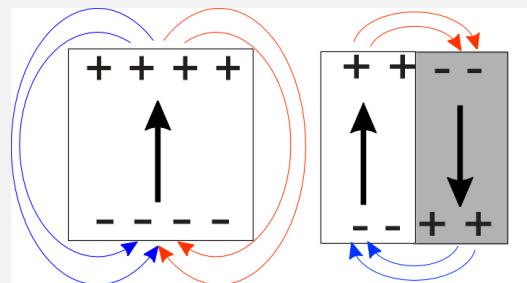
## Typical interactions



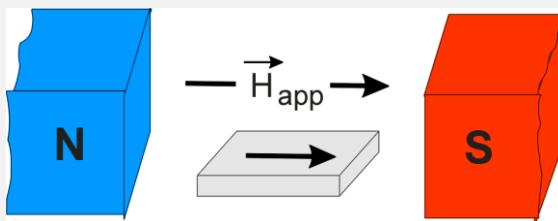
magnetocrystalline anisotropy



magnetostatic

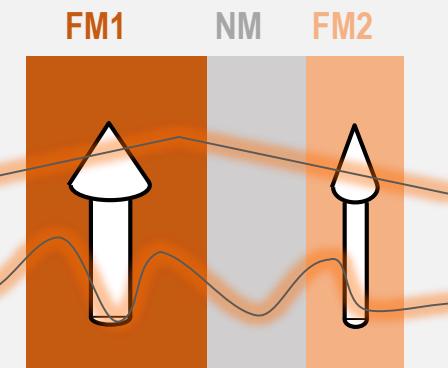
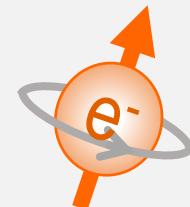


Zeeman



## Special role of the spin

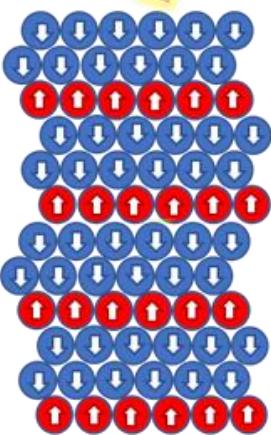
charge + spin



Spin dependent transport

# Atomistic spin dynamics

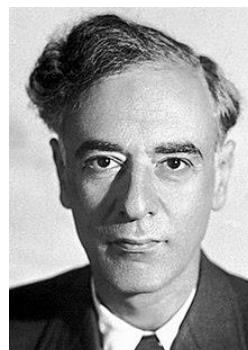
Formalism : atomistic spin model is a discrete description of magnetism (each atom possess a localized magnetic moment or spin); the time evolution of a single atomic spin given by the phenomenological Landau-Lifshitz-Gilbert equation



$$\frac{d\mathbf{S}_i}{dt} = - \frac{\gamma_i}{(1 + \lambda_i^2)} [\mathbf{S}_i \times \mathbf{B}_i + \lambda_i \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{B}_i)]$$

📖 Landau LD, Lifshitz EM (1935) *Theory of the dispersion of magnetic permeability in ferromagnetic bodies*. Phys Z Sowjetunion 8:153

📖 Gilbert TL (1955) *A Lagrangian formulation of the gyromagnetic equation of the magnetic field*. Phys Rev 100:1243.  
<https://doi.org/10.1103/PhysRevB.100.1235>

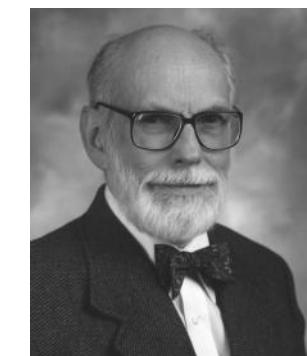


Lev D. Landau

Department of Theoretical Physics  
National Scientific Center - Kharkiv



Evgeny M. Lifshitz



Thomas L. Gilbert  
Argonne National Laboratory

# Atomistic spin dynamics

$$\frac{d\mathbf{S}_i}{dt} = -\frac{\gamma_i}{(1 + \lambda_i^2)} [\mathbf{S}_i \times \mathbf{B}_i + \lambda_i \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{B}_i)]$$

*precession term*

*relaxation term*

$\gamma_i$  - gyromagnetic ratio

$\lambda_i$  - Gilbert damping is a coupling of the spin to the electronic system and lattice (loss of energy)

Hamiltonian:  $\hat{\mathcal{H}} = -\sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i k_u (\mathbf{S}_i \cdot \mathbf{u}_K)^2 - \sum_i \mu_i \mathbf{S}_i \cdot \mathbf{B}_{app} + \dots$

*exchange term*      *anisotropy term*      *Zeeman term*      *dipolar magnetostriiction*

Thermal field:  $\langle \zeta_i \rangle = 0, \langle \zeta_{i\eta}(0) \zeta_{j\theta}(t) \rangle = 2\delta_{ij}\delta_{\eta\theta}\delta(t) \frac{\lambda_i k_B T_e \mu_i}{\gamma_i}$

- correctly simulate the static and dynamic properties of ferrimagnetic and antiferromagnetic materials as alloys of transition metals (TM) and rare – earth elements (RE)

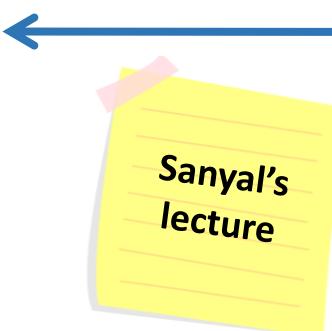
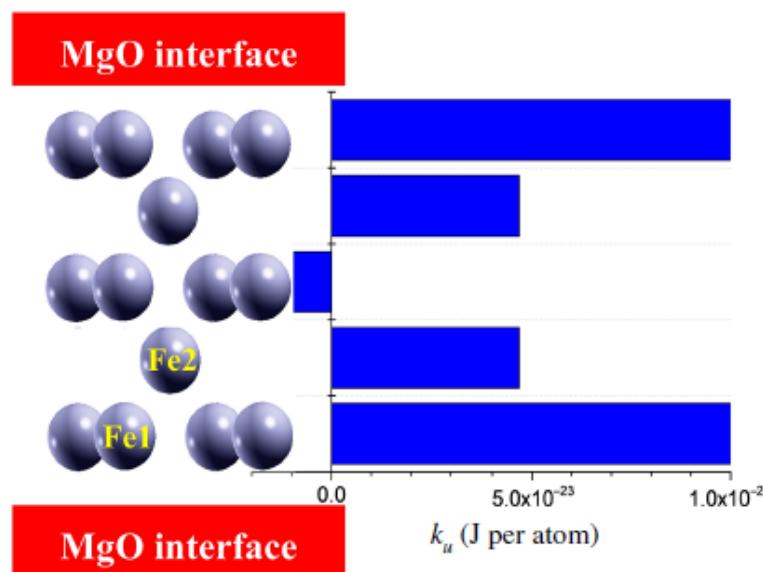
$$\mathbf{B}_i = -\frac{1}{\mu_i} \frac{\partial \hat{\mathcal{H}}}{\partial \mathbf{S}_i} + \zeta_i$$

# Atomistic spin dynamics : link with first principle computation

ab-initio calculation

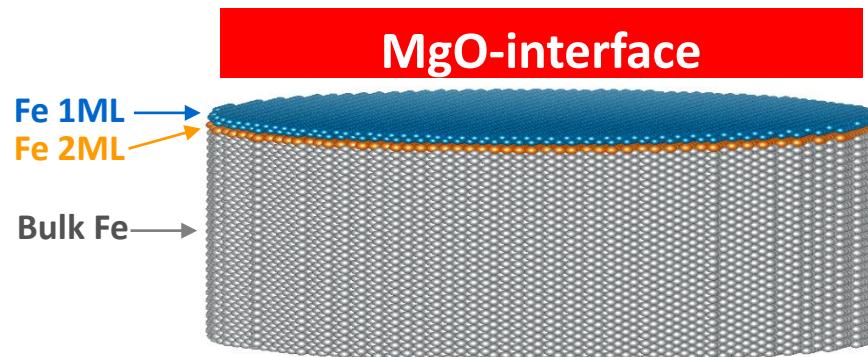
atomistic models

- Spin moments per layer
  - $\mu_s(\text{Fe1}) = 2.76 \mu_B$
  - $\mu_s(\text{Fe2}) = 2.49 \mu_B$
  - $\mu_s(\text{Fe\_bulk}) = 2.2 \mu_B$
- Layer-resolved  $K_s$  and  $\mu_s$  in Fe/MgO structures



- Temperature dependence of  $K_s$  and  $M_s$
- $K_s$  scaling with  $M_s$
- Curie temperature  $T_c$
- Temperature dependence of dead layer thickness

$$\hat{\mathcal{H}} = - \sum_{i < j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j - \sum_i k_u (\mathbf{S}_i \cdot \mathbf{u}_K)^2$$



<https://www.vasp.at>

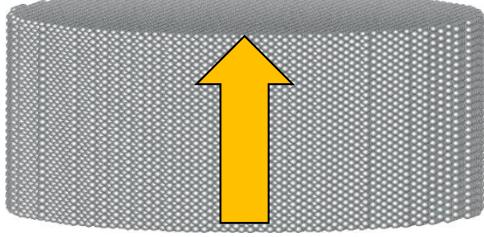


V A M P I R E  
[vampire.york.ac.uk](http://vampire.york.ac.uk)

Ibrahim et al . Phys. Rev. B (2022)

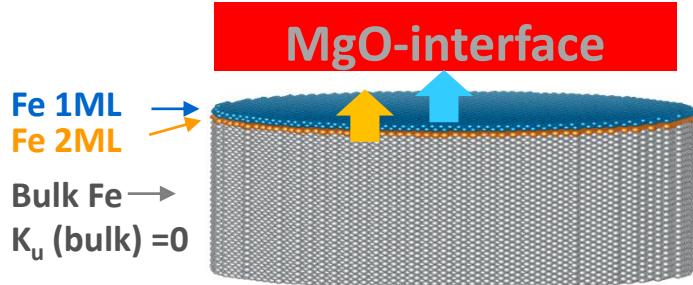
# Atomistic spin dynamics

## ➤ Bulk Fe bcc structure

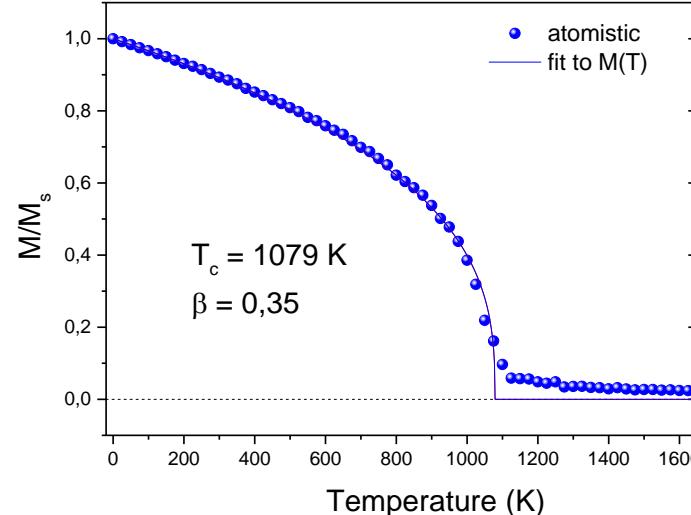


- ❑ Uniform uniaxial anis:  $K_u = 1.0e-24 \text{ J/atom}$
- ❑ Exchange constant:  $J = 7.05e-21 \text{ J/link}$
- ❑  $\mu_s = 2.2 \mu_B$

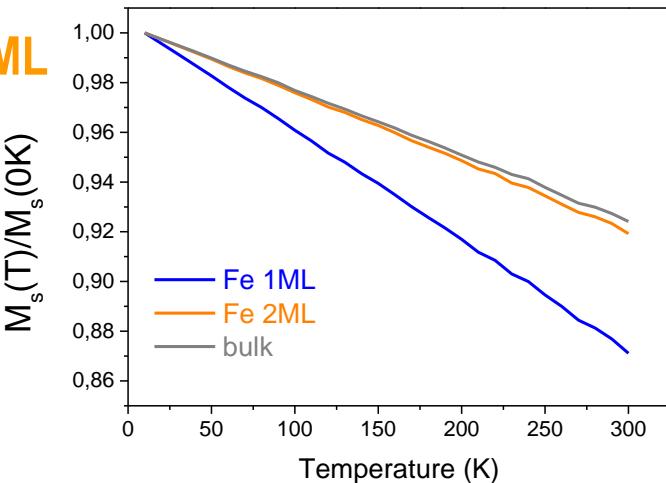
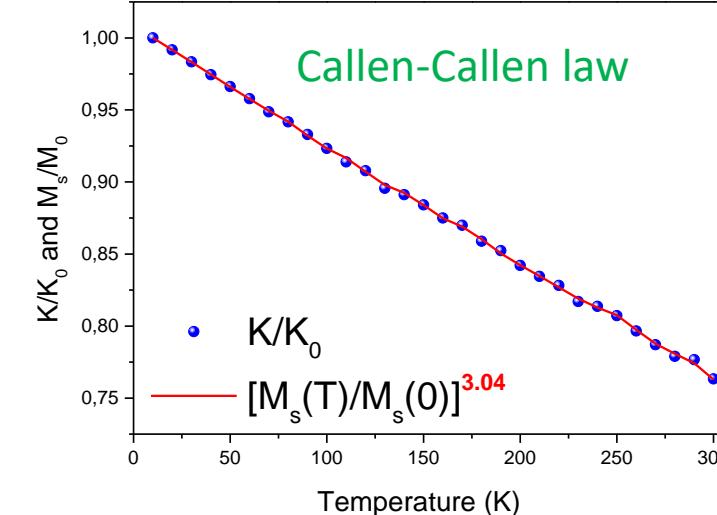
## ➤ Model: introduce $K_s$ for Fe 1ML & Fe 2ML



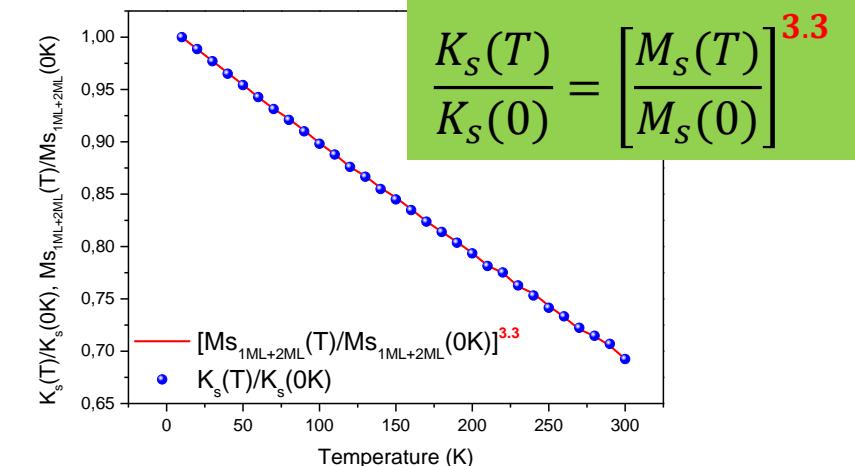
$$\frac{M_s(T)}{M_s(0)} = \left[ 1 - \frac{T}{T_c} \right]^{0.35}$$



$$\frac{K_s(T)}{K_s(0)} = \left[ \frac{M_s(T)}{M_s(0)} \right]^{3.04}$$



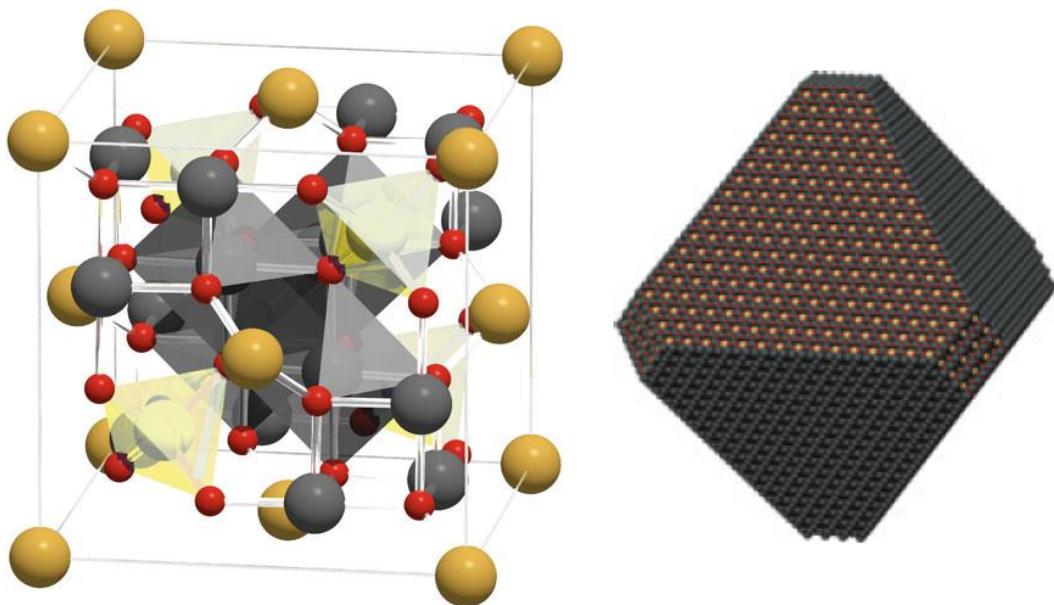
- $M_s$  of Fe 1ML decreases more rapidly
- $M_s$  of Fe 2ML behaves as Fe bulk



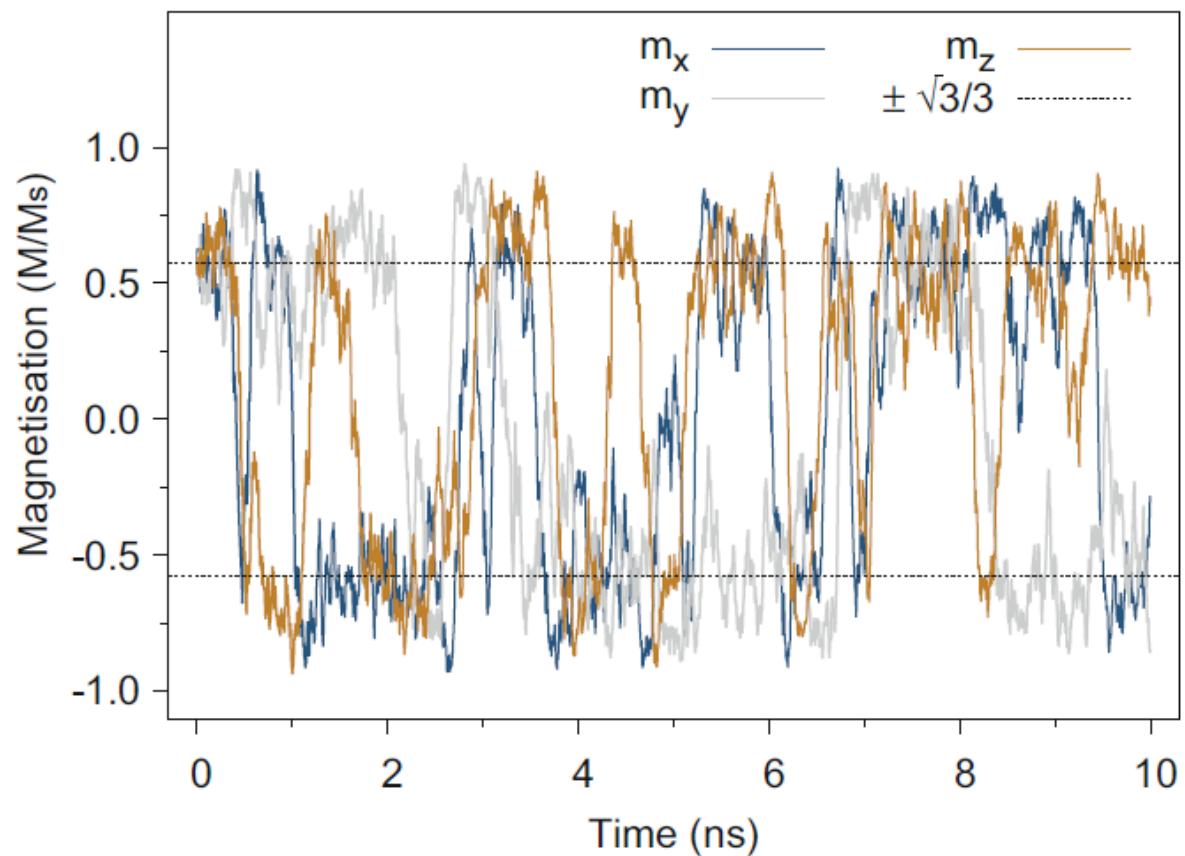
Ibrahim et al . Phys. Rev. B (2022)

# Atomistic spin dynamics : finite size $\text{Fe}_3\text{O}_4$ nanoparticle

12 nm octahedral single crystal magnetite  $\text{Fe}_3\text{O}_4$  nanoparticle



magnetization dynamics at 300K



- ✓ Fluctuations of different magnetization components between 8 minima along [111] crystal directions due to the cubic magnetocrystalline anisotropy

# Atomistic spin dynamics : bulk ferrimagnets

## Static properties of amorphous GdFe Alloys

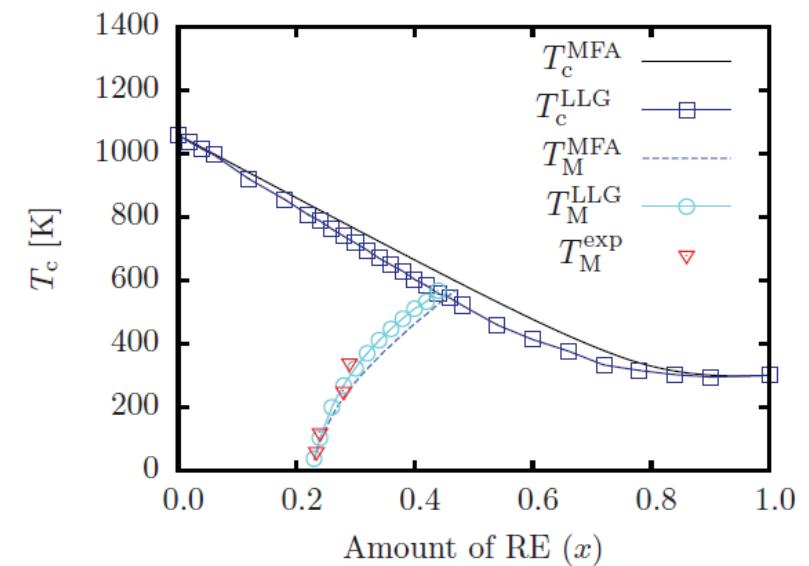
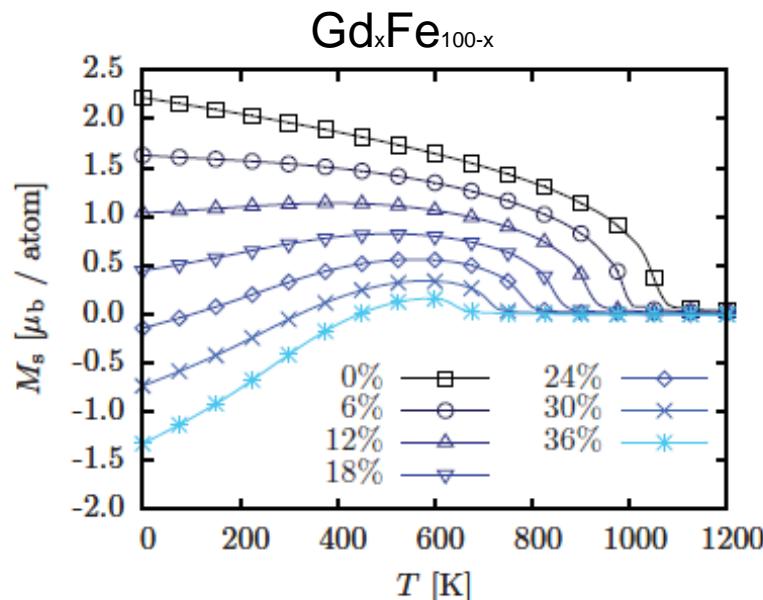
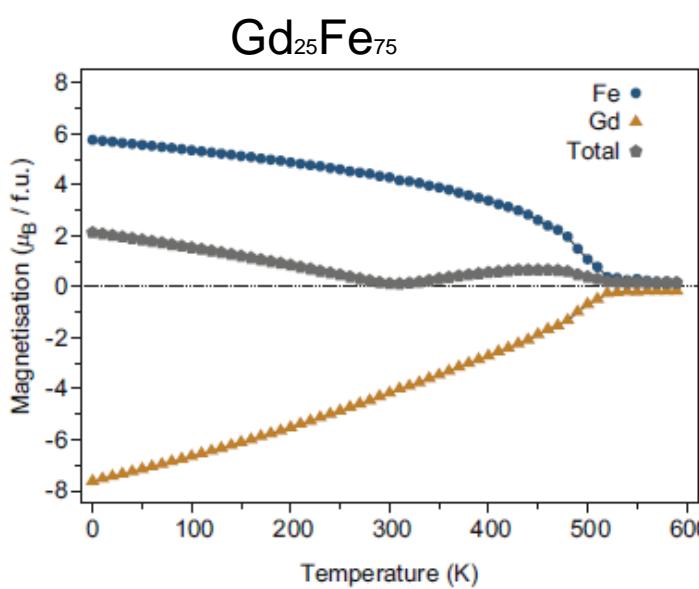
→ media of magnetic hard disks

62 500 spins & periodic boundary conditions (fcc)



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vampire.york.ac.uk

<https://vampire.york.ac.uk/>

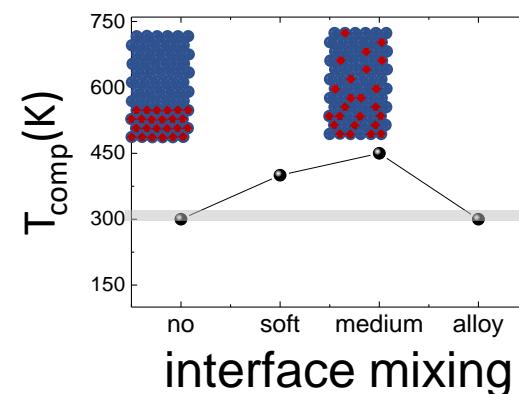
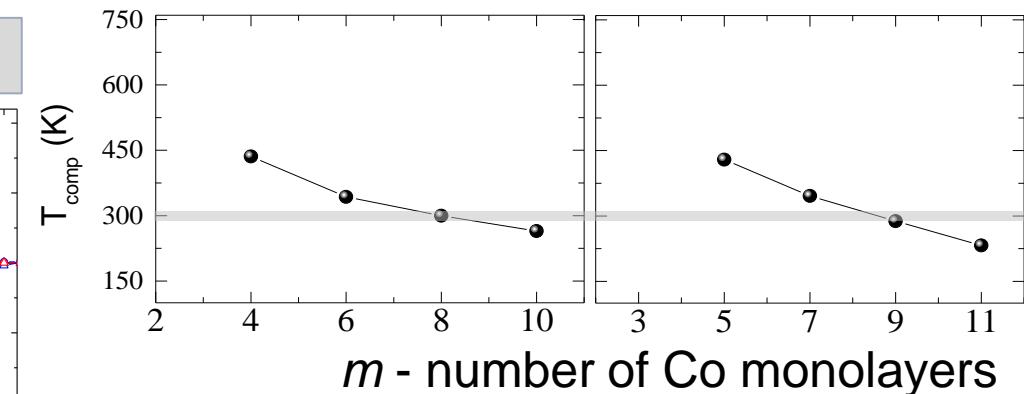
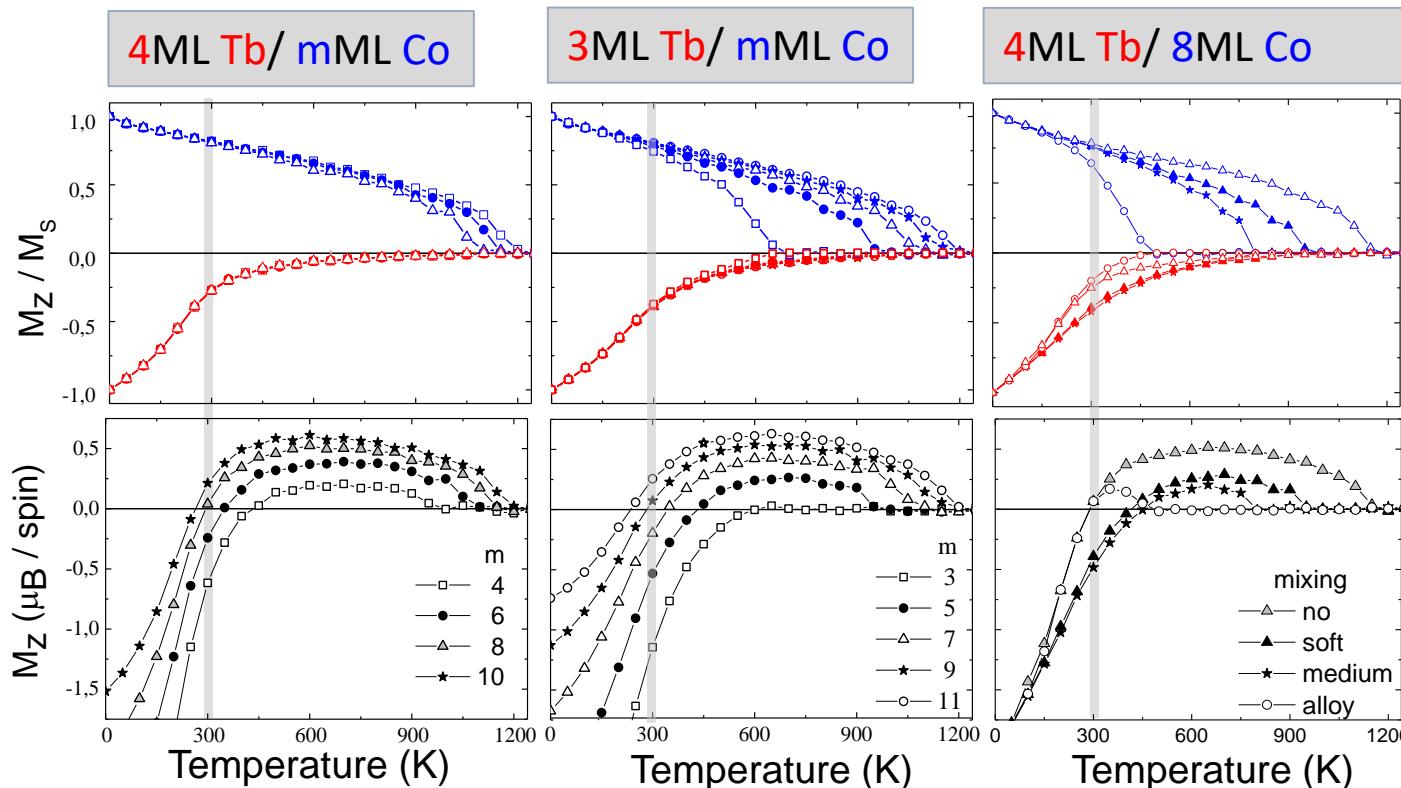


- ✓ The model allows to reproduce experimental observations  
→ validation of the model calibration

Ostler et al . Phys. Rev. B (2011)

# Atomistic spin dynamics- static properties [Tb/Co] layers

Ultrafast reversal in Tb/Co multilayers → storage layer in magnetic RAM



- ✓ The model allows to finely tune composition, the structure and to explore many possible combination → prediction and prospection

# Atomistic spin dynamics & 2 Temperature model

## LLG & 2 temperature model

$$\frac{dS_i}{dt} = -\frac{\gamma_i}{(1 + \lambda_i^2)} [S_i \times B_i + \lambda_i S_i \times (S_i \times B_i)]$$

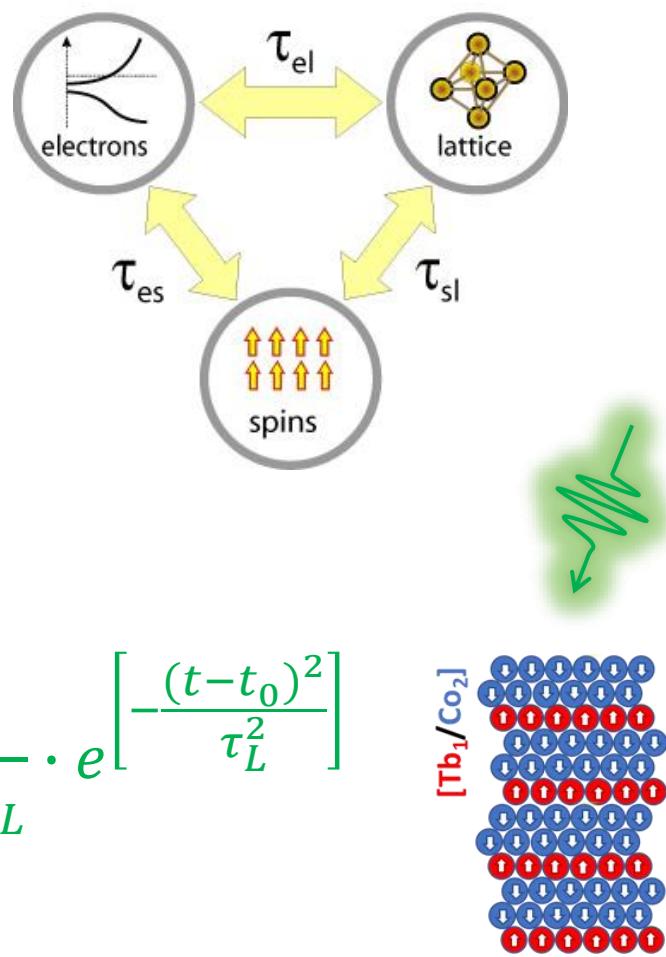
$$B_i = -\frac{1}{\mu_i} \frac{\partial \hat{\mathcal{H}}}{\partial S_i} + \zeta_i$$

$$\langle \zeta_i \rangle = 0, \langle \zeta_{i\eta}(0) \zeta_{j\theta}(t) \rangle = 2\delta_{ij}\delta_{\eta\theta}\delta(t) \frac{\lambda_i k_B T_e \mu_i}{\gamma_i}$$

$$\left\{ \begin{array}{l} C_e \frac{dT_e(t)}{dt} = -G[T_e(t) - T_{ph}(t)] + P(t) \\ C_{ph} \frac{dT_{ph}(t)}{dt} = G[T_e(t) - T_{ph}(t)] \end{array} \right.$$

$$P(t) = \frac{F}{t_{FM} \tau_L} \cdot e^{\left[ -\frac{(t-t_0)^2}{\tau_L^2} \right]}$$

*laser heating*

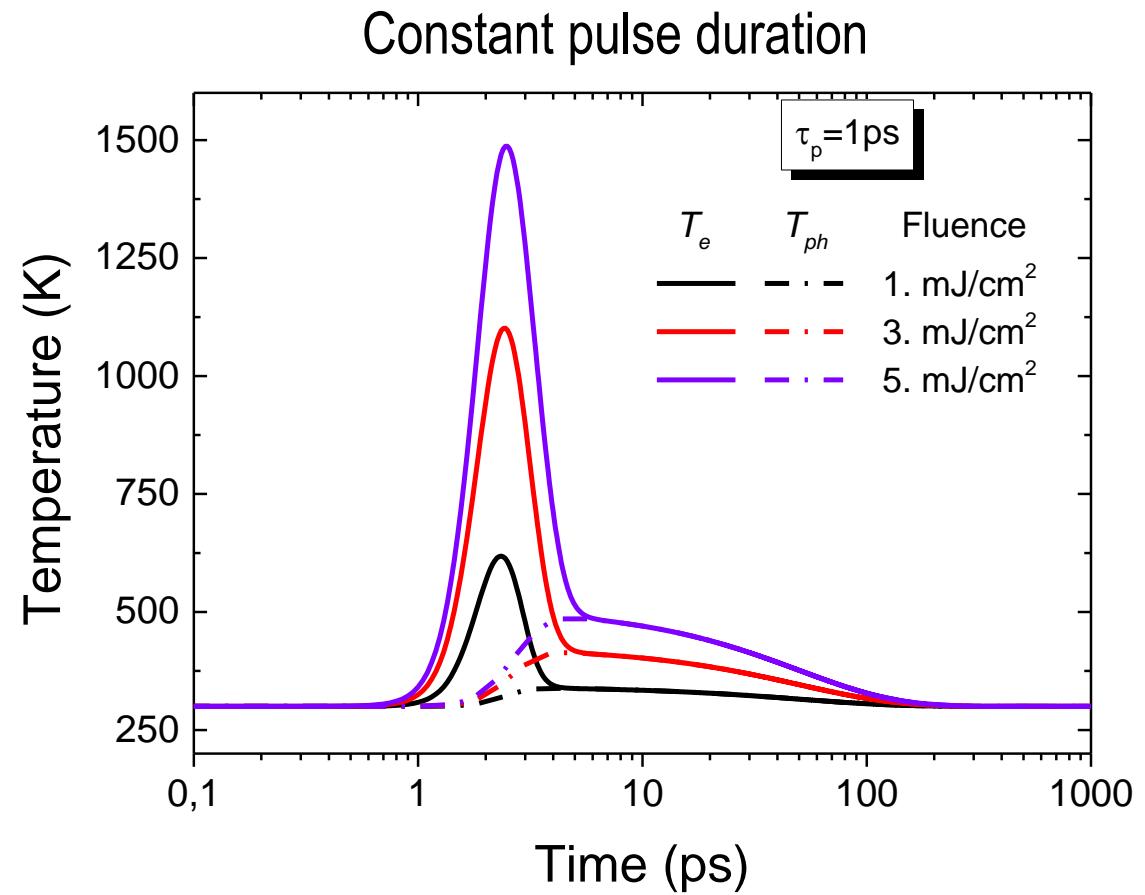
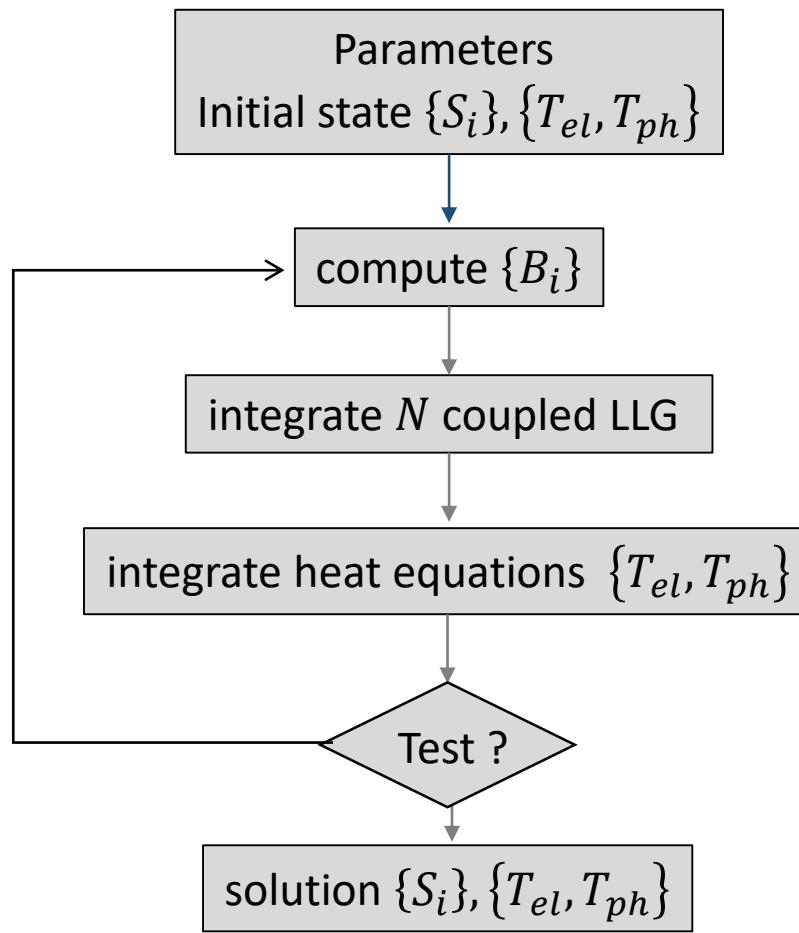


- ✓ Multiphysics coupling : spin dynamics and heat equations
- ASD Spintec solver

- 📖 L. Aviles-Felix et al., Sci. Rep. (2021)
- 📖 R. Moreno et al. Phys Rev B (2017)

# Atomistic spin dynamics : AOS of MTJ based on [Tb/Co] layers

- ✓ More than  $50 \times 50 \times 50$  exchange coupled spins
- ✓ Heun's integration scheme (predictor corrector scheme)
- ✓ Parallelization of the solver (CPU, GPU releases)



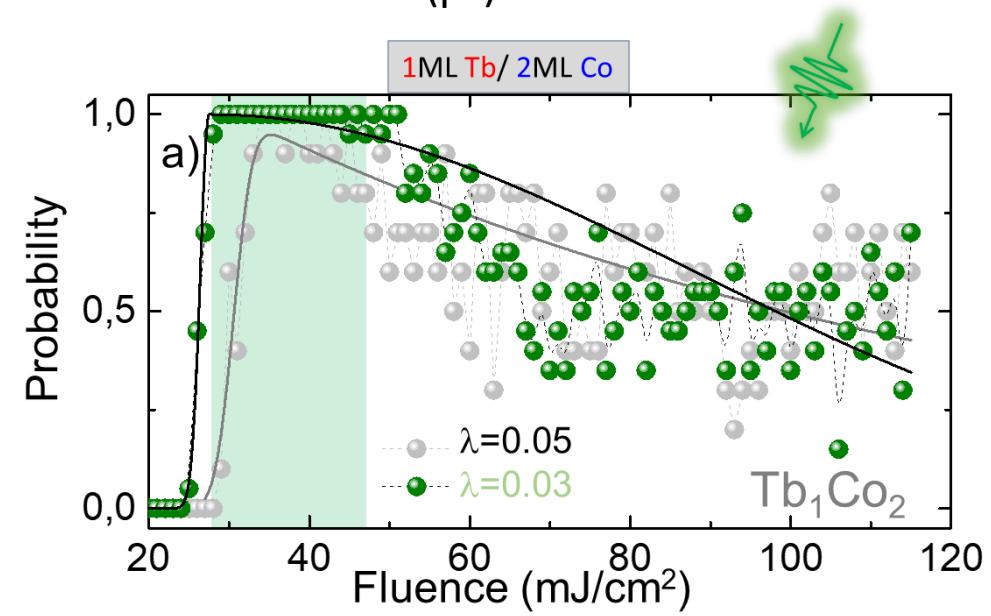
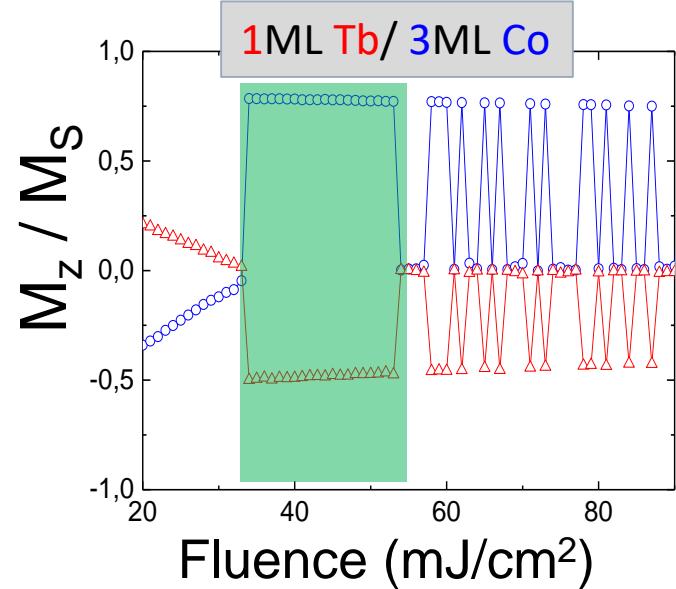
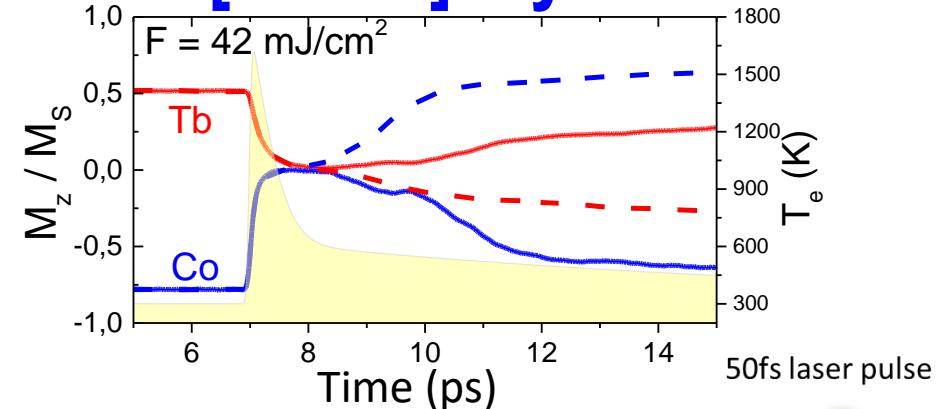
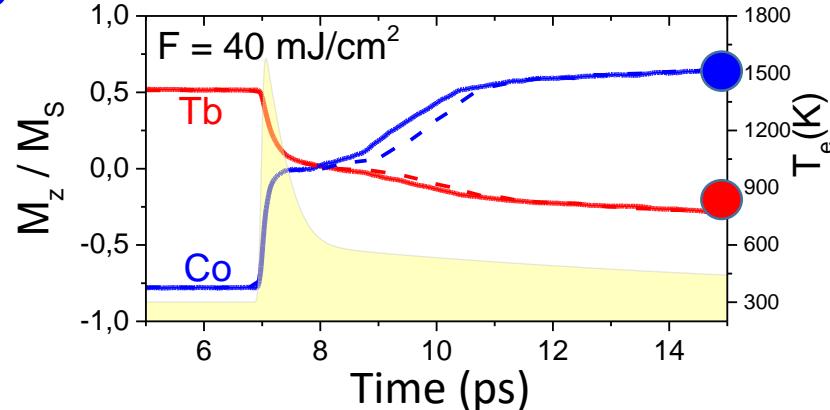
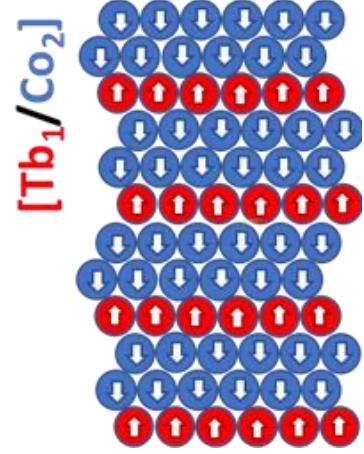
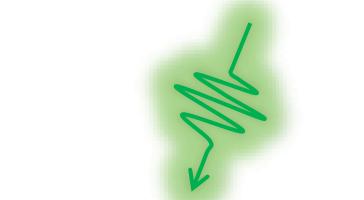
Spindynamics (LLG)



SpinSwift (LLB)

# Atomistic spin dynamics : AOS of MTJ based on [Tb/Co] layers

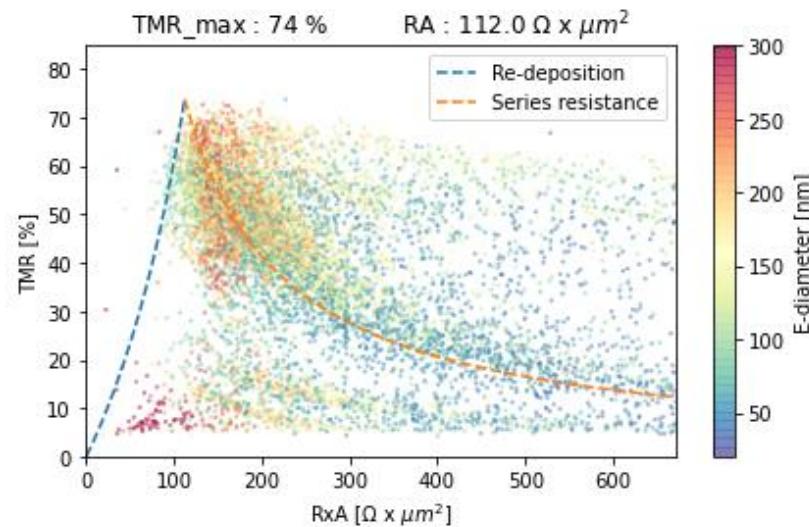
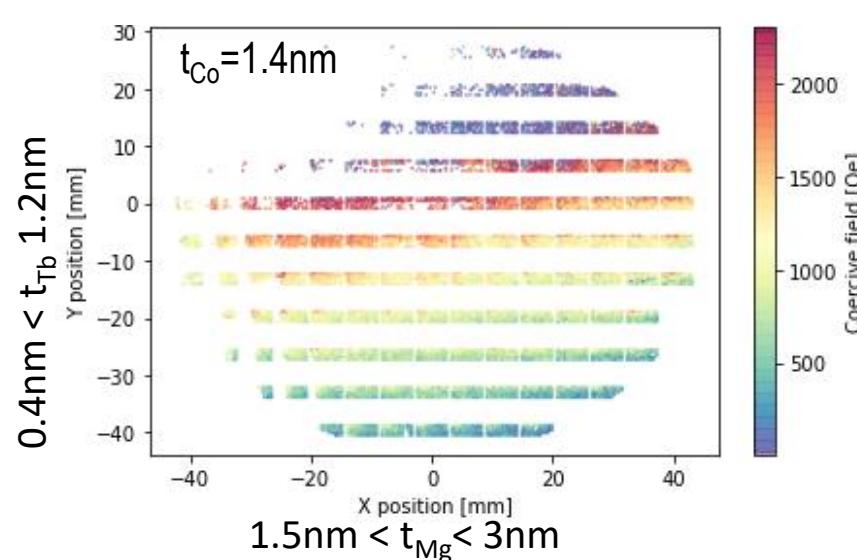
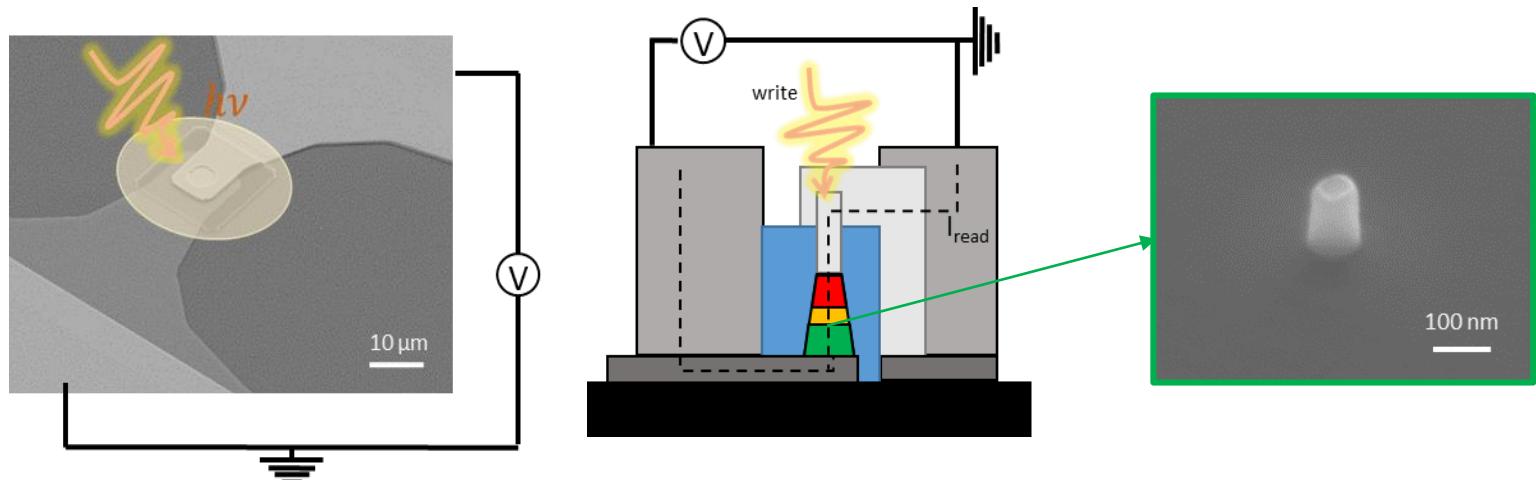
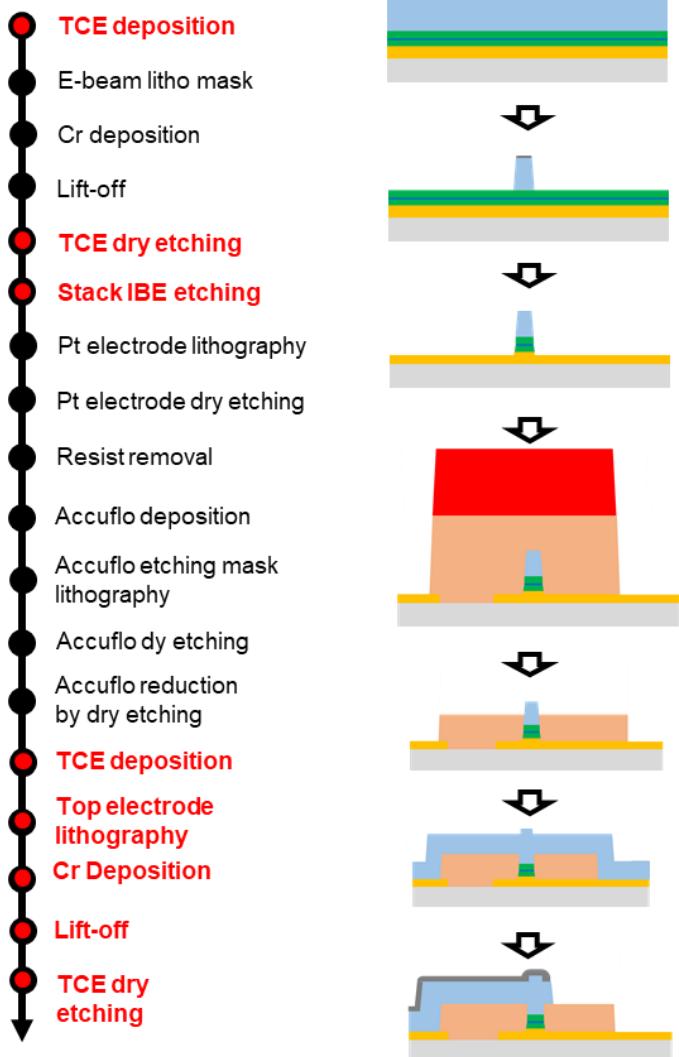
50fs laser pulse



- ✓ Mechanism of reversal : transient ferromagnetic state linked to the difference of demagnetizing time of Tb and Co.

- ✓ Key role of the material parameters (e.g. damping)
- ✓ Extracted diagrams to be compared with experiments

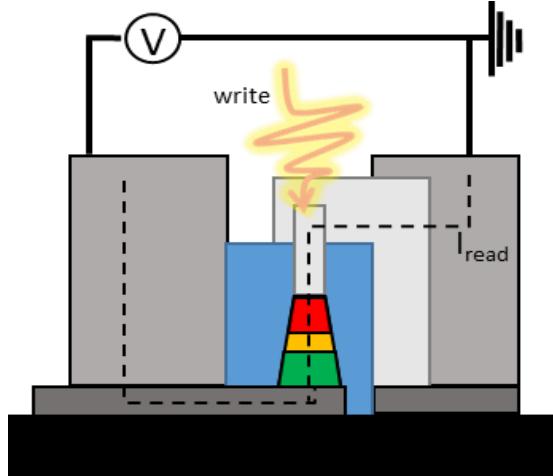
# Atomistic spin dynamics : AOS of MTJ based on [Tb/Co] layers



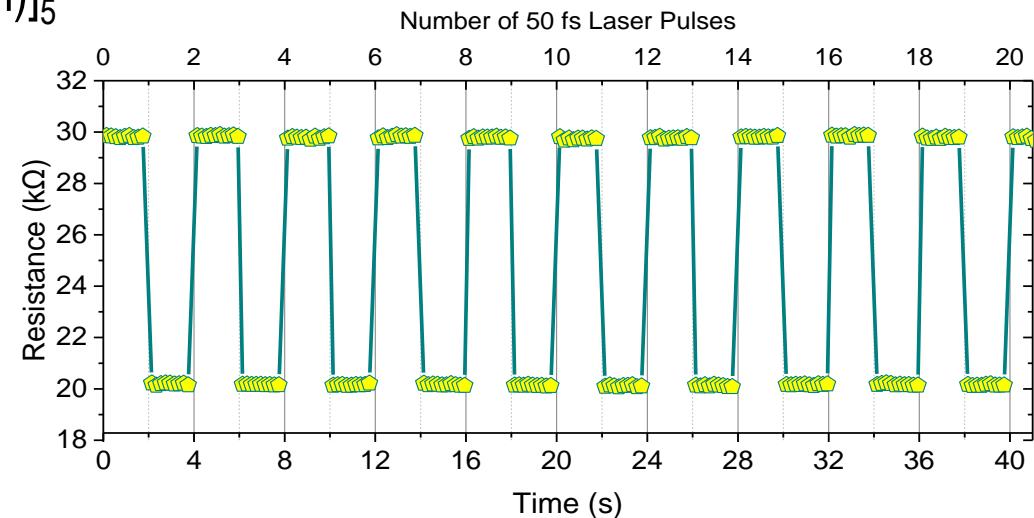
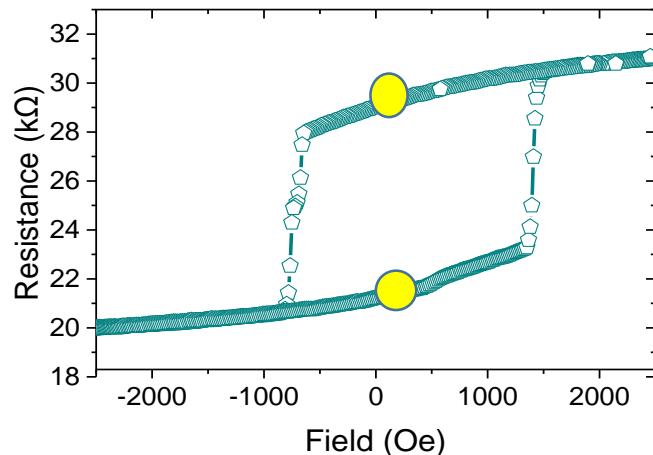
A. Olivier et al., Nanotechnology (2020)

David Salomoni's PhD

# Atomistic spin dynamics : AOS of MTJ based on [Tb/Co] layers



FeCoB1.3nm/ [Tb(0.6nm)/Co(1.4nm)]<sub>5</sub>

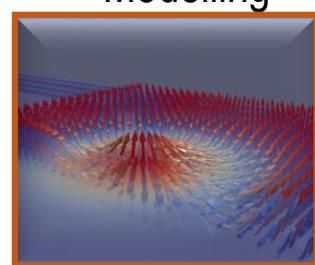


- ✓ **Reproducible helicity independent AOS with single shot:**
  - MTJ 100nm large with a TMR of 50%
  - field free
  - 50fs laser pulses, 5.5 mJ/cm<sup>2</sup>

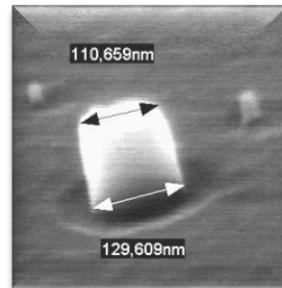
$$Ae^{ik_{\uparrow}x}|\uparrow\rangle$$

$$+Be^{ik_{\downarrow}x}|\downarrow\rangle$$

Theory



Materials

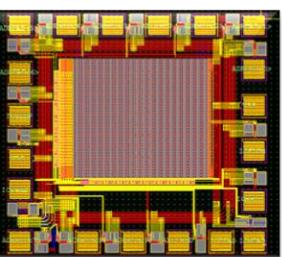


Nanofabrication

concepts

devices

Test



Design



**Importance of continuous exchanges between experiments and theory/modelling:**

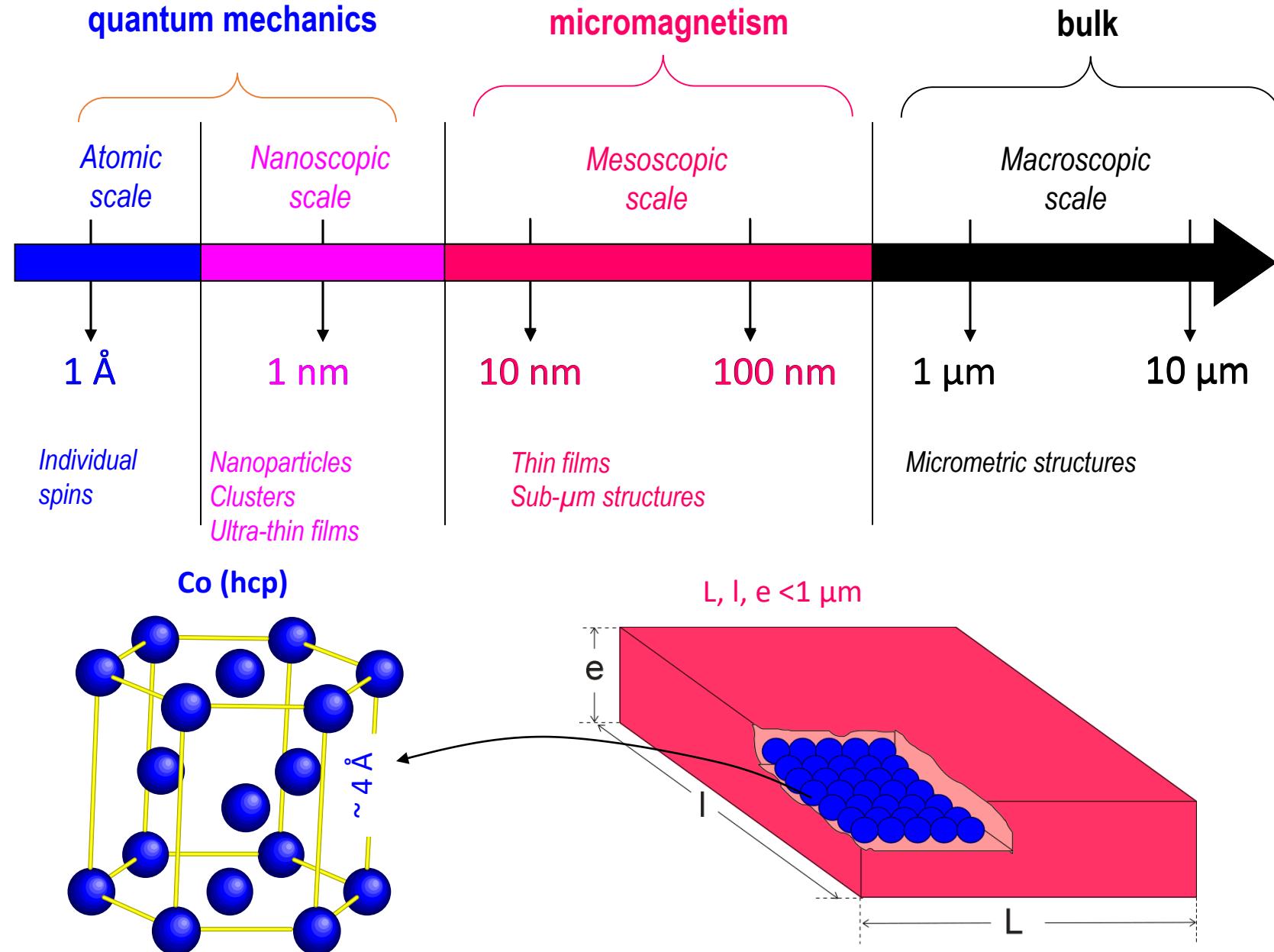
- ✓ Validation
- ✓ Calibration
- ✓ Accurate /reliable prediction
- ✓ Feed / challenge the models

# Outline



- Atomistic spin dynamics
  - Formalism
  - Illustrations : alloy GdFe, Fe nanodot, AOS [Tb/Co] multilayers
- Micromagnetism
  - Formalism
  - Macrospin limit
  - Illustrations
- Micromagnetism and spin related phenomena
  - Formalism
  - Illustrations
- Conclusions

# Micromagnetism



# Micromagnetism : the beginning & hypothesis

Classical theory of continuous ferromagnetic material

1963 - W. F. Brown Jr. { 1907 P. Weiss / magnetic domains  
1937 Landau-Lifshitz / domain walls

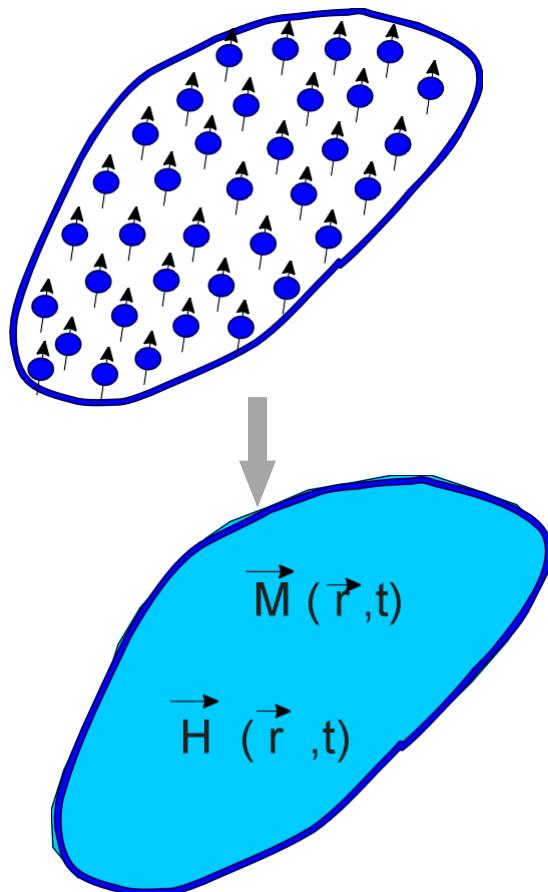
Continuous functions (space & time) { magnetization  $\mathbf{M}(\mathbf{r}, t)$   
field  $\mathbf{H}(\mathbf{r}, t)$   
energy  $E(\mathbf{M}(\mathbf{r}, t))$

Magnetization – constant amplitude vector  $|\mathbf{M}(\mathbf{r}, t)| = M_S$

Thermal effects- temperature dependent parameters

$$M_S(T), A_{ex}(T), K_u(T)$$

Very large number of individual spins



Continuous media

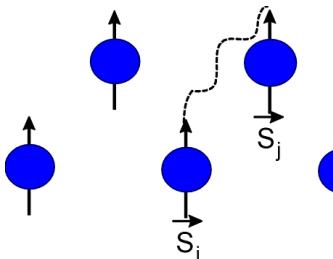
# Micromagnetism: Gibb's free energy

## Exchange interaction

magnetic order ( $T < T_c$ )

parallel alignment of spins

short range interaction but very strong



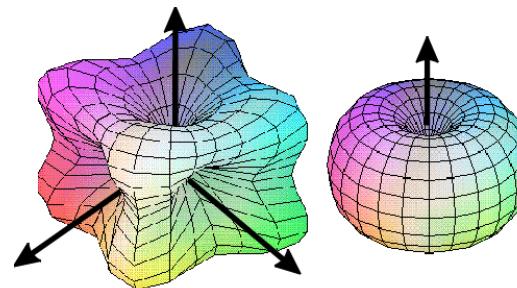
$$\int_{\Omega} A_{ex} (\nabla \mathbf{m}(\mathbf{r}, t))^2 dV$$

## Magnetocrystalline anisotropy

easy axis and hard axis

Impact of the crystal symmetry

Local interaction

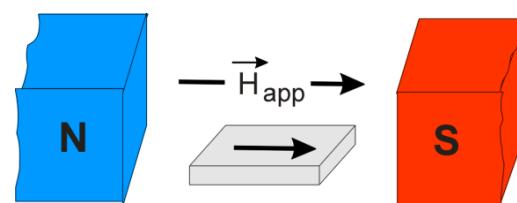


$$\int_{\Omega} K_u [1 - (\mathbf{u}_k \cdot \mathbf{m}(\mathbf{r}, t))^2] dV$$

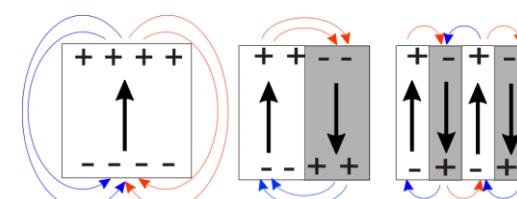
## Zeeman coupling

Externally applied field

Local interaction



$$-\mu_0 M_S \int_{\Omega} \mathbf{m}(\mathbf{r}, t) \cdot \mathbf{H}_{app}(\mathbf{r}, t) dV$$



$$-\frac{1}{2} \mu_0 M_S \int_{\Omega} \mathbf{m}(\mathbf{r}, t) \cdot \mathbf{H}_D(\mathbf{r}, t) dV$$

# Micromagnetism : magnetic stable state

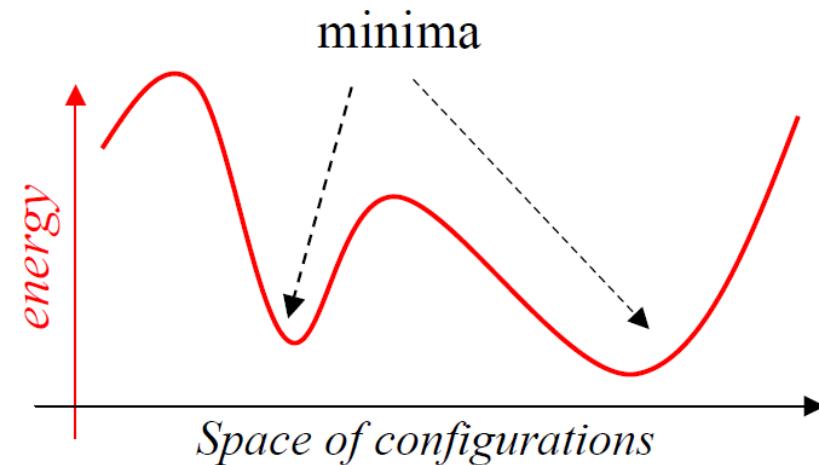
Magnetization distribution:

$$\mathbf{M}(\mathbf{r}, t) = M_S \mathbf{m}(\mathbf{r}, t) \quad \mathbf{r} \in \Omega$$

$$|\mathbf{m}(\mathbf{r}, t)|=1$$

Gibb's free energy:

$$E_{total} = \int_{\Omega} [\varepsilon_{ex} + \varepsilon_K + \varepsilon_{app} + \varepsilon_D] dV$$



Magnetic stable state = minimum of the Gibb's free energy functional

$$\mathbf{m} \rightarrow \mathbf{m} + \delta \mathbf{m}$$

$$\delta E_{total}(\mathbf{m}) = 0$$

$$\delta^2 E_{total}(\mathbf{m}) > 0$$

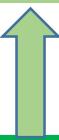
← variational principle

# Micromagnetism : static equilibrium equations

✓ Possible thus to find the magnetic stable states

Brown's equations

$$(\mathbf{m} \times \mathbf{H}_{eff}) = \mathbf{0}, \forall \mathbf{r} \in \Omega$$



$$\delta E_{total} = -\mu_0 M_S \int_{\Omega} (\mathbf{m} \times \mathbf{H}_{eff}) \cdot \delta \boldsymbol{\theta} dV + 2A_{ex} \oint_S \left( \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial \mathbf{n}} \right) \cdot \delta \boldsymbol{\theta} dS$$

$$\frac{\partial \mathbf{m}}{\partial \mathbf{n}} = \mathbf{0}, \forall \mathbf{r} \in S$$



Effective field:  $\mathbf{H}_{eff} = -\frac{1}{\mu_0 M_S} \frac{\delta E_{total}}{\delta \mathbf{m}} = \mathbf{H}_{ex} + \mathbf{H}_K + \mathbf{H}_D + \mathbf{H}_{app} + \mathbf{Cm}$

A. Hubert, R. Schafer Magnetic Domains

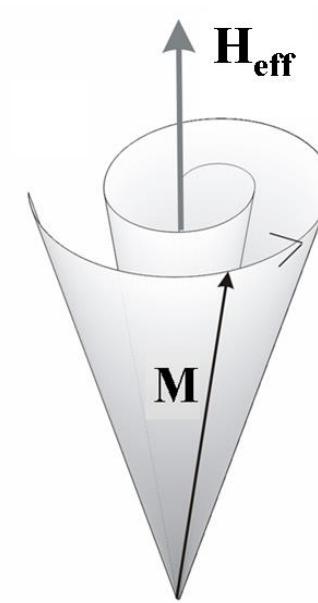
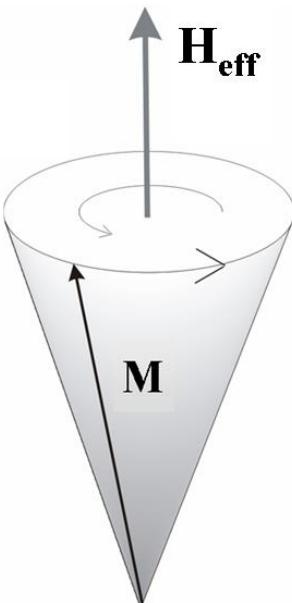
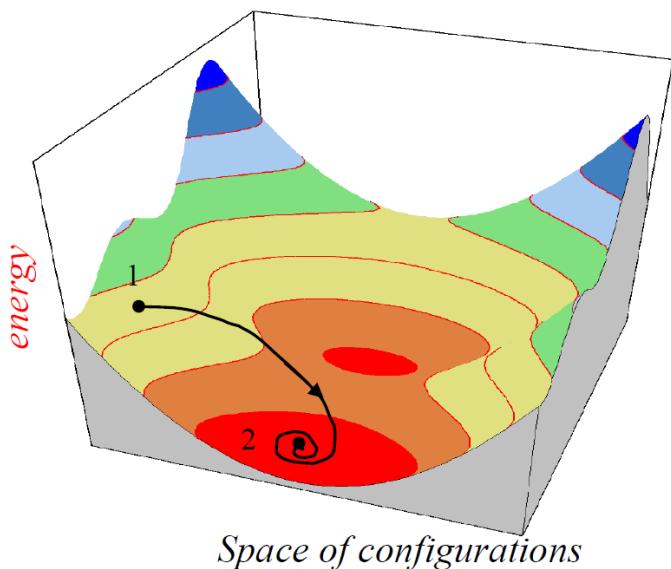
# Micromagnetism : dynamic equation

Equation de Landau-Lifshitz-Gilbert + more interactions

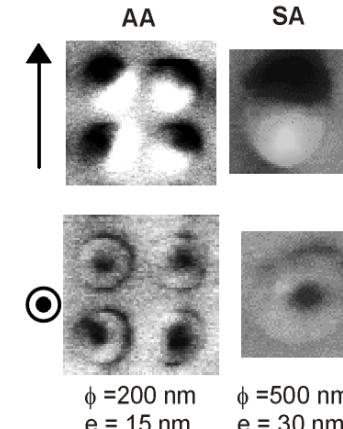
$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma_0(\mathbf{m} \times \mathbf{H}_{\text{eff}}) + \alpha \left( \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \right) + \left( \frac{\partial \mathbf{m}}{\partial t} \right)_{\text{add}}$$

macrospin

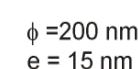
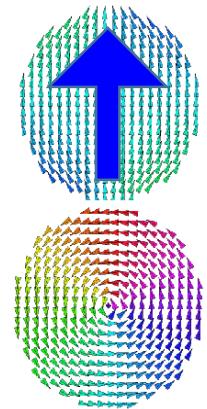
Space & time dependence  
 $\mathbf{m}(\mathbf{r}, t), \mathbf{r} \in \Omega$



Experiments



Simulation

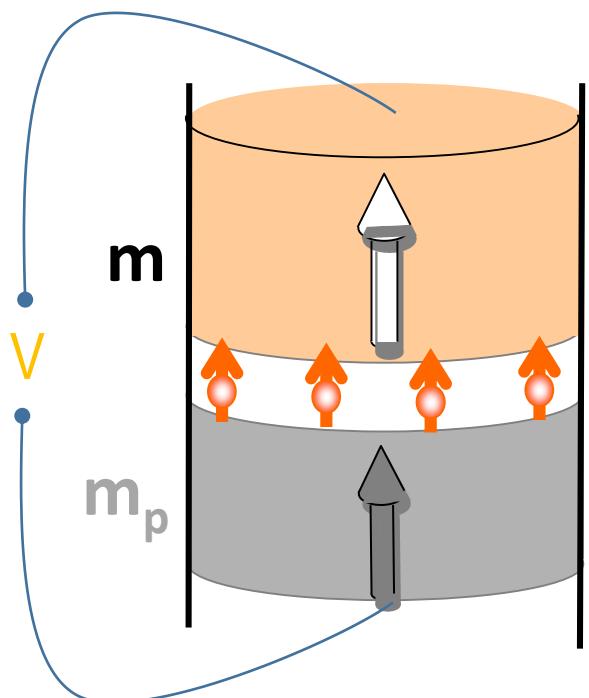


✓ Magnetization trajectory between two magnetic states

# Macrospin: micromagnetic limit if the exchange interaction is infinite

Circular magnetic tunnel junction pillar: polarizer  $m_p$  / insulator (MgO) / CoFeB ( $\mathbf{m}$ )

$\mathbf{m}$  is supposed to be uniform in each point of the CoFeB and have coherent dynamic.



$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma_0 (\mathbf{m} \times \mathbf{H}_{eff}) + \alpha \left( \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t} \right)$$

damping-like

LLG equation

$$-\gamma_0 a_{DL} V (\mathbf{m} \times (\mathbf{m} \times \mathbf{m}_P)) + \gamma_0 a_{FL} V^2 (\mathbf{m} \times \mathbf{m}_P)$$

field-like

Spin Transfer Torques

Slonczewski terms

$$\mathbf{H}_{eff} = \begin{pmatrix} 0 \\ 0 \\ H_K m_z + M_S m_z + H_{app} \end{pmatrix}$$

the effective field includes contributions due to the anisotropy, demagnetizing and applied field

$$\mathbf{m}_p = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

the spin current polarization is parallel with the out-of-plane direction

# Macrospin: micromagnetic limit if the exchange interaction is infinite

$$(1 + \alpha^2) \frac{\partial \mathbf{m}}{\partial t} = -\gamma_0 (\mathbf{m} \times \mathbf{H}_{eff}) - \gamma_0 (a_{FL} V^2 - \alpha a_{DL} V) (\mathbf{m} \times \mathbf{m}_P) \\ - \alpha \gamma_0 (\mathbf{m} \times (\mathbf{m} \times \mathbf{H}_{eff})) - \gamma_0 (-\alpha a_{FL} V^2 + a_{DL} V) (\mathbf{m} \times (\mathbf{m} \times \mathbf{m}_P))$$

$\mathbf{m}^2 = 1$

Holstein –Primakoff transformation :

$$c = \frac{m_x - im_y}{\sqrt{2(1 + m_z)}}$$

$$c^* = \frac{m_x + im_y}{\sqrt{2(1 + m_z)}}$$

$$\left\{ \begin{array}{l} \left( \frac{\partial \mathbf{c}}{\partial t} \right)_{prec} = -ic[\omega_H + (\omega_D - \omega_K)(2|c|^2 - 1)] \\ \left( \frac{\partial \mathbf{c}}{\partial t} \right)_{damp} = -\alpha c(1 - |c|^2)[\omega_H + (\omega_D - \omega_K)(2|c|^2 - 1)] \\ \left( \frac{\partial \mathbf{c}}{\partial t} \right)_{FL} = i[\alpha \omega_{DL} + \omega_{FL}]c \\ \left( \frac{\partial \mathbf{c}}{\partial t} \right)_{DL} = -[\omega_{DL} - \alpha \omega_{FL}](1 - |c|^2)c \end{array} \right.$$

$\omega_H = \frac{\gamma_0 H_{app}}{1 + \alpha^2}$   
 $\omega_K = \frac{\gamma_0 H_K}{1 + \alpha^2}$   
 $\omega_D = \frac{\gamma_0 M_s}{1 + \alpha^2}$   
 $\omega_{DL} = \frac{\gamma_0}{1 + \alpha^2} \frac{a_{DL} V}{\mu_0}$   
 $\omega_{FL} = \frac{\gamma_0}{1 + \alpha^2} \frac{a_{FL} V^2}{\mu_0}$

 Timopheev et al. Phys Rev B (2017)

# Macrospin: micromagnetic limit if the exchange interaction is infinite

$$\frac{\partial c}{\partial t} = -ic[\omega_H - (\alpha\omega_{DL} + \omega_{FL}) + (\omega_D - \omega_K)(2|c|^2 - 1)] \\ -\alpha c(1 - |c|^2) \left[ \omega_H + \left( \frac{\omega_{DL}}{\alpha} - \omega_{FL} \right) (\omega_D - \omega_K)(2|c|^2 - 1) \right]$$

$$c = \sqrt{p}e^{-i\phi}$$

Power equation

$$\frac{\partial p}{\partial t} = -2\alpha \left[ \omega_H + \left( \frac{\omega_{DL}}{\alpha} - \omega_{FL} \right) + (\omega_D - \omega_K)(2p - 1) \right] (1 - p)p$$

Phase equation

$$\frac{\partial \phi}{\partial t} = -[\omega_H - (\alpha\omega_{DL} + \omega_{FL}) + (\omega_D - \omega_K)(2p - 1)]$$

$$\omega_H = \frac{\gamma_0 H_{app}}{1 + \alpha^2}$$

$$\omega_K = \frac{\gamma_0 H_K}{1 + \alpha^2}$$

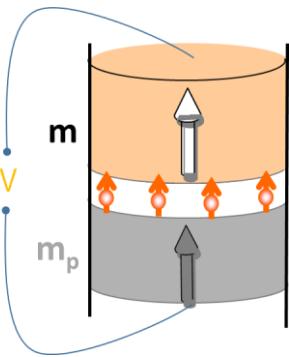
$$\omega_D = \frac{\gamma_0 M_s}{1 + \alpha^2}$$

$$\omega_{DL} = \frac{\gamma_0}{1 + \alpha^2} \frac{a_{DL} V}{\mu_0}$$

$$\omega_{FL} = \frac{\gamma_0}{1 + \alpha^2} \frac{a_{FL} V^2}{\mu_0}$$

- ✓ **Stability analysis around the equilibrium state**  $p_{eq} \rightarrow p_{eq} + \delta p$
- ✓ **Extraction of the critical lines, draw state diagrams**

# Macrospin: micromagnetic limit if the exchange interaction is infinite



State "up"  $\mathbf{m} = (0 \quad 0 \quad 1)$  stable if :

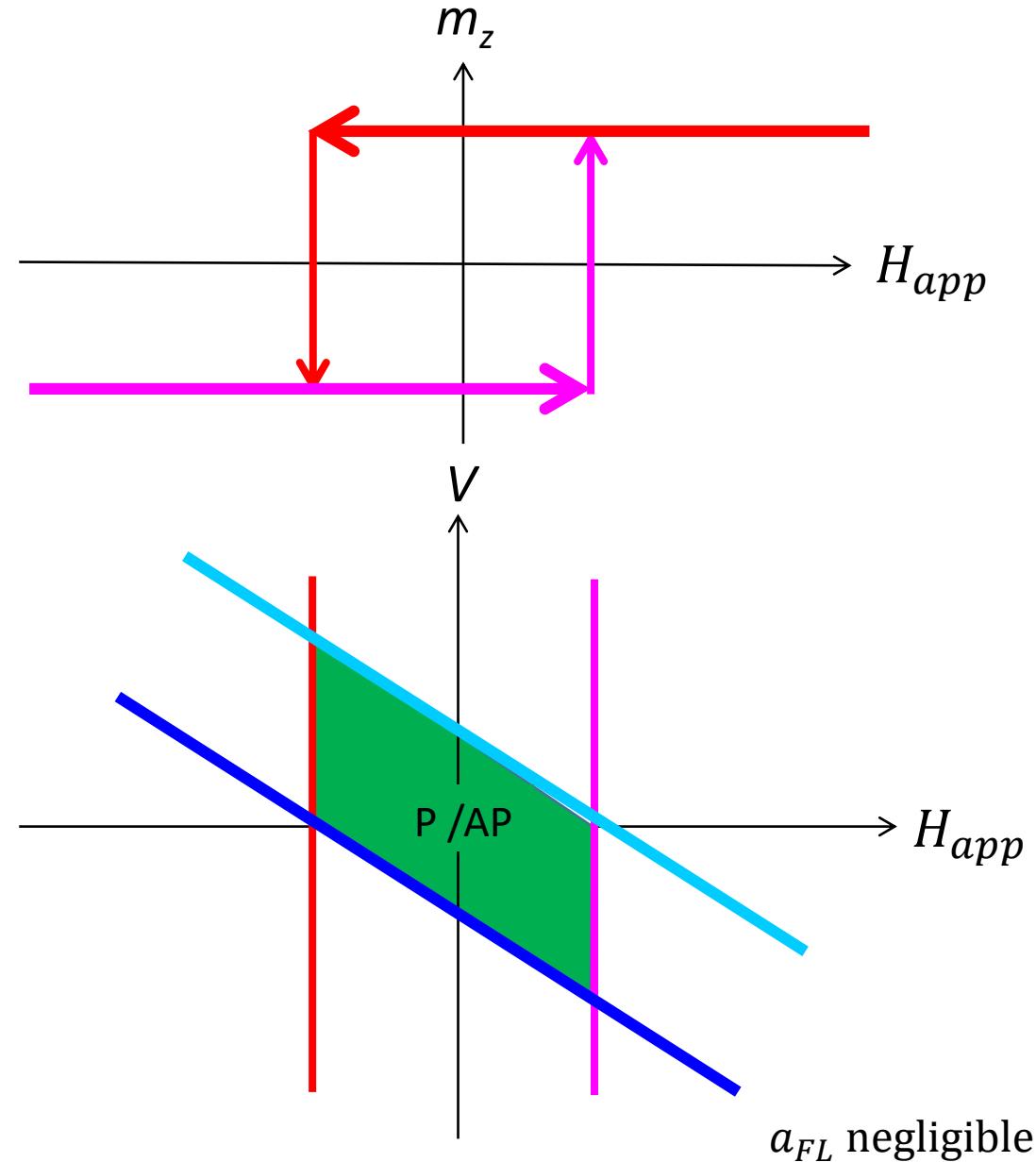
$$V = 0 \quad H_{app} > - (H_K - M_s)$$

$$V \neq 0 \quad H_{app} + \left( \frac{a_{DL}V}{\alpha\mu_0} - \frac{a_{FL}V^2}{\mu_0} \right) - (M_s - H_K) > 0$$

State "down"  $\mathbf{m} = (0 \quad 0 \quad -1)$  stable if :

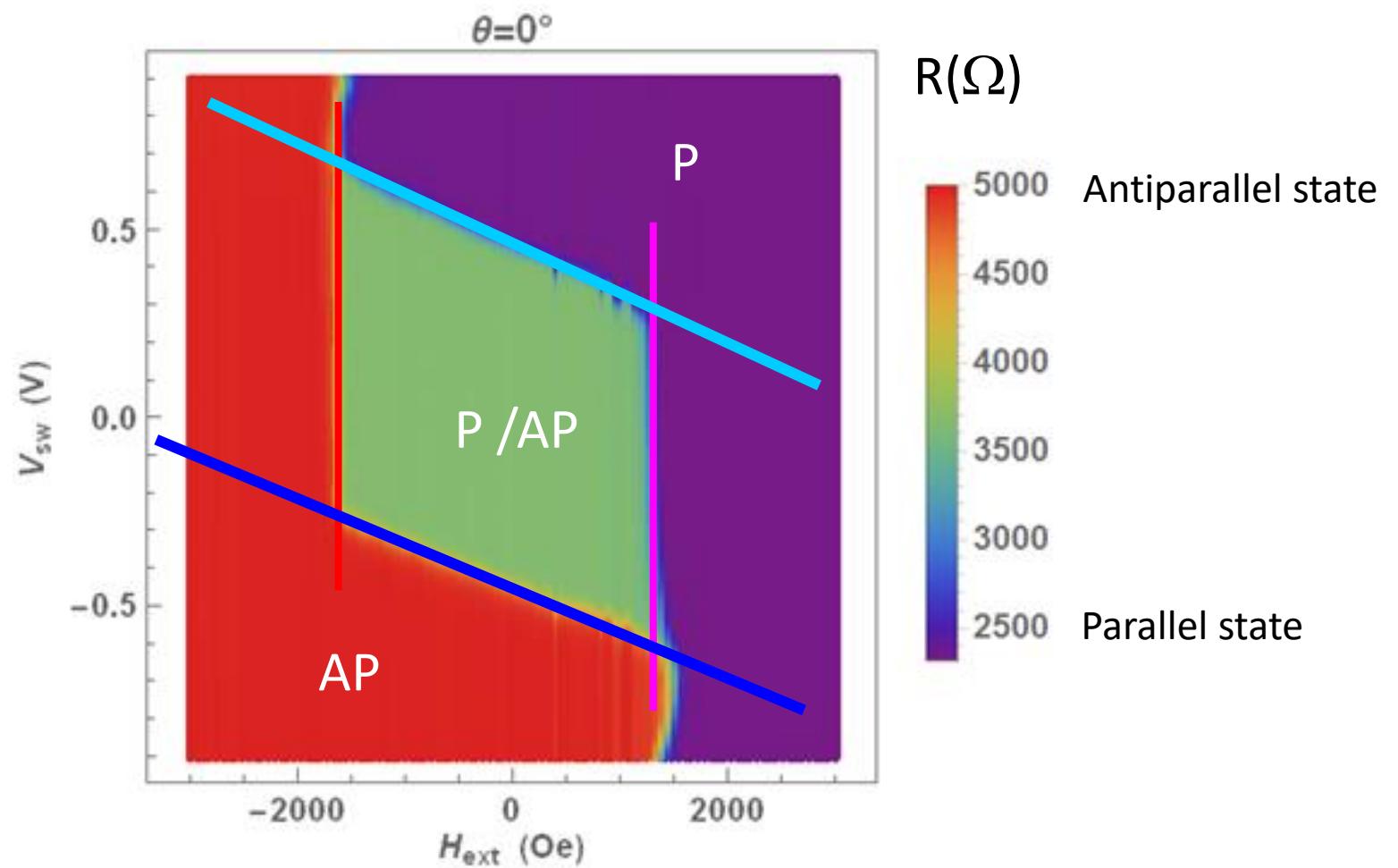
$$V = 0 \quad H_{app} < + (H_K - M_s)$$

$$V \neq 0 \quad H_{app} + \left( \frac{a_{DL}V}{\alpha\mu_0} - \frac{a_{FL}V^2}{\mu_0} \right) + (M_s - H_K) < 0$$



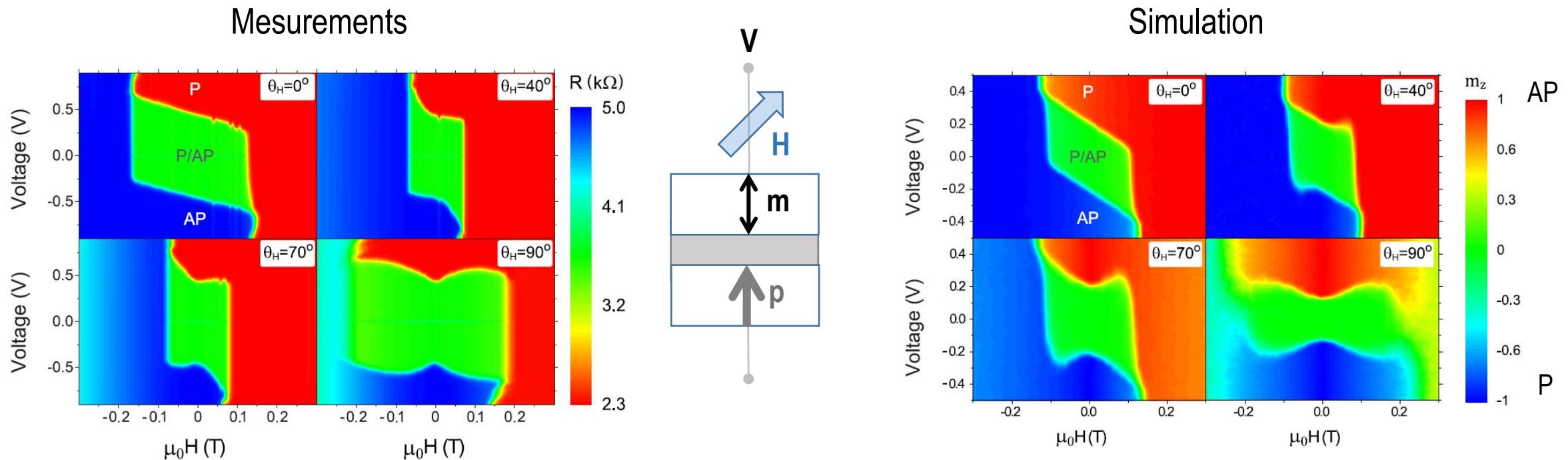
# Macrospin: micromagnetic limit if the exchange interaction is infinite

Experimental stability  $V$ - $H$  diagrams of 80 nm diameter MTJ at room temperature



Strelkov et al. Phys Rev B (2017)

# Macrospin: micromagnetic limit if the exchange interaction is infinite



- ✓ The model pointed out the necessity to add a new term to the crystalline anisotropy:

$$E_K = -K_1(\mathbf{u}_k \cdot \mathbf{m})^2 - K_2(\mathbf{u}_k \cdot \mathbf{m})^4$$

- ✓ The macrospin model is simple but enables a straightforward analysis

Strelkov et al. Phys Rev B (2017)

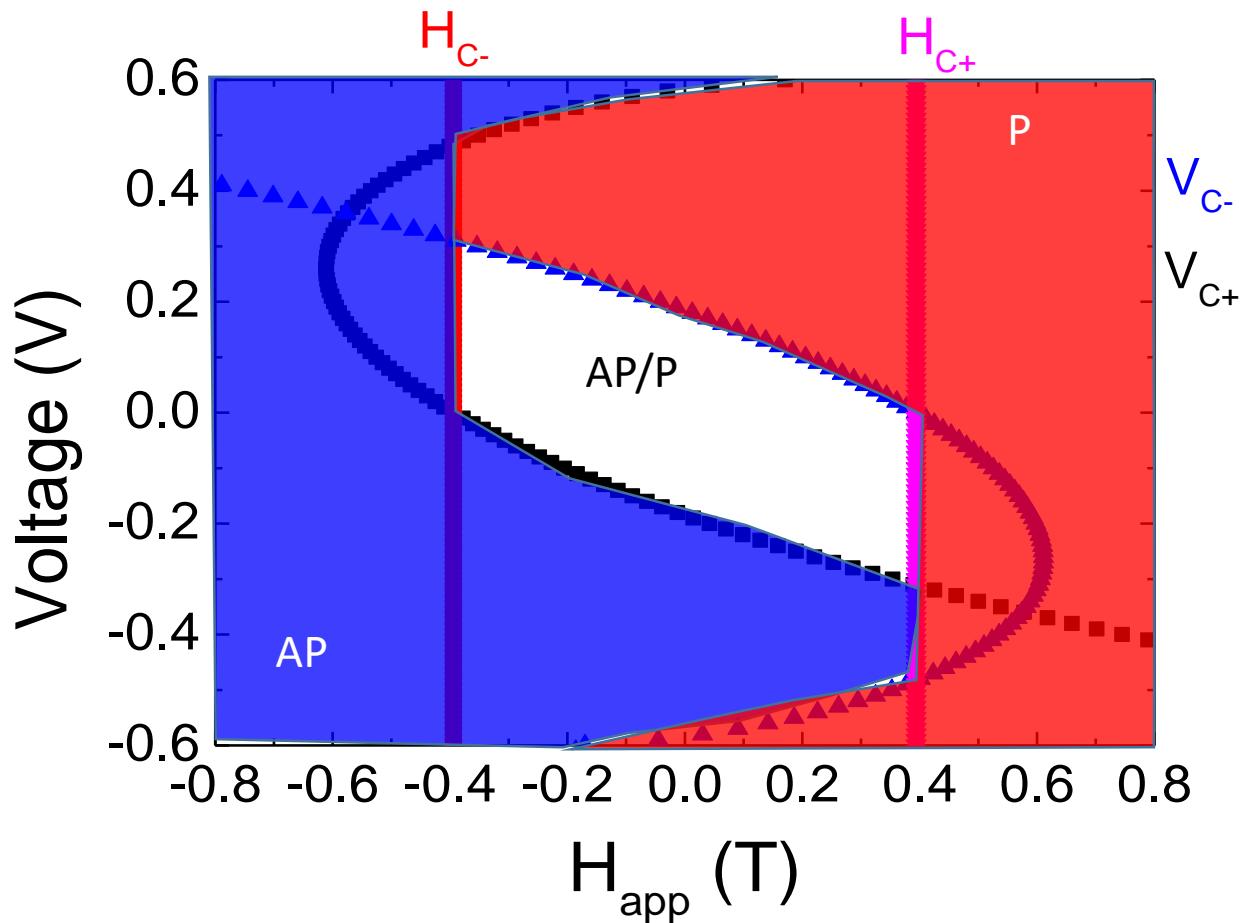
# Macrospin: micromagnetic limit if the exchange interaction is infinite

Joule heating changes the temperature during the operation

$$C \frac{dT}{dt} = -Q(T - T_0) + \frac{V^2}{R}$$

$$\frac{M_s(T)}{M_{s0}} = \frac{a_{DL}(T)}{a_{DL,0}} = \left[ 1 - \left( \frac{T}{T_c} \right)^{1.73} \right]$$

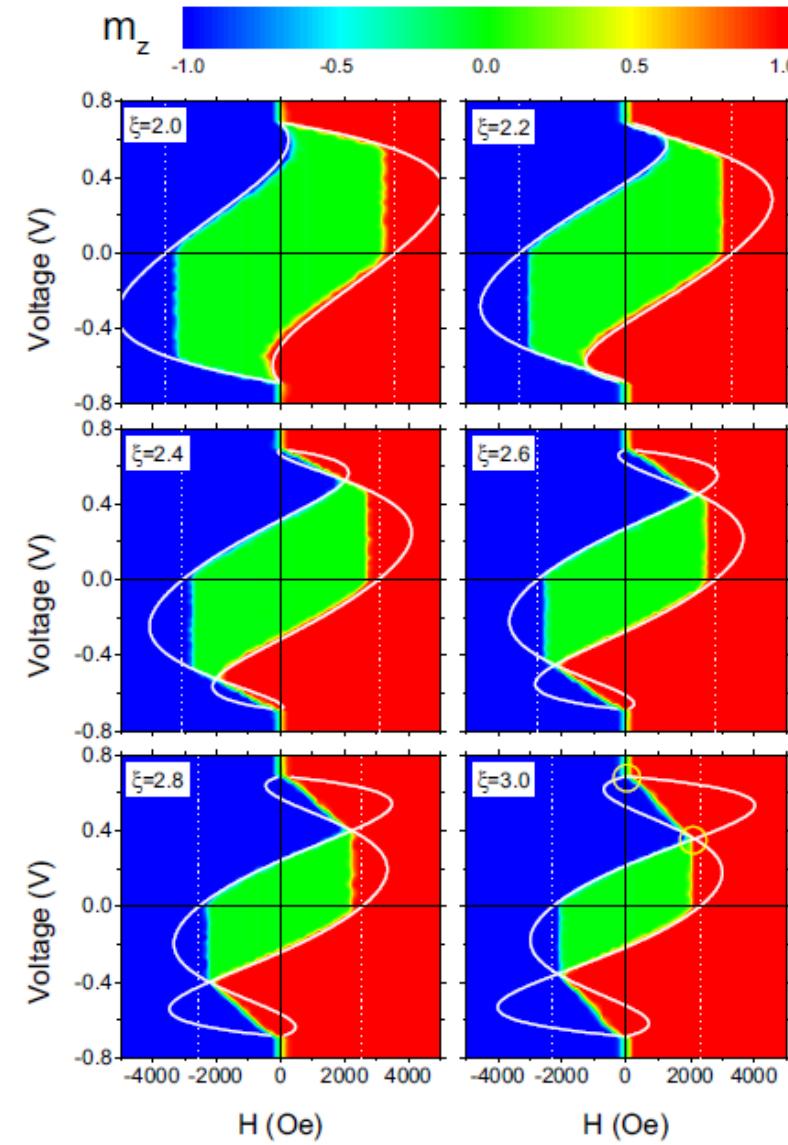
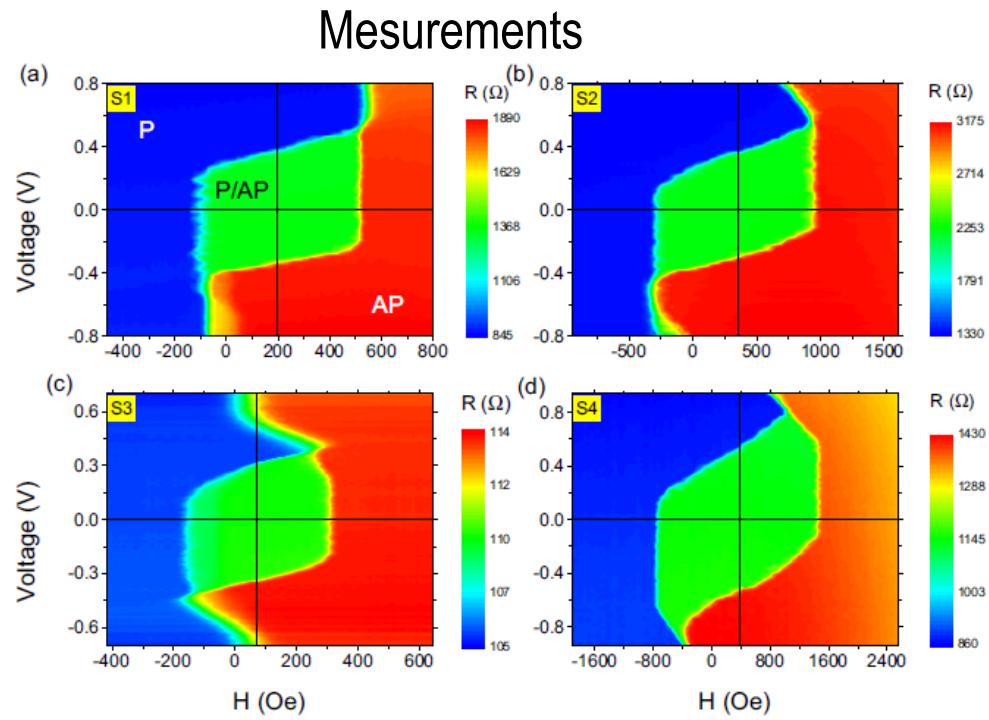
$$K_u(T) = K_0 \left[ \frac{M_s(T)}{M_{s0}} \right]^{\xi}$$



Strelkov et al. Phys Rev B (2017)

# Macrospin: micromagnetic limit if the exchange interaction is infinite

Simulation



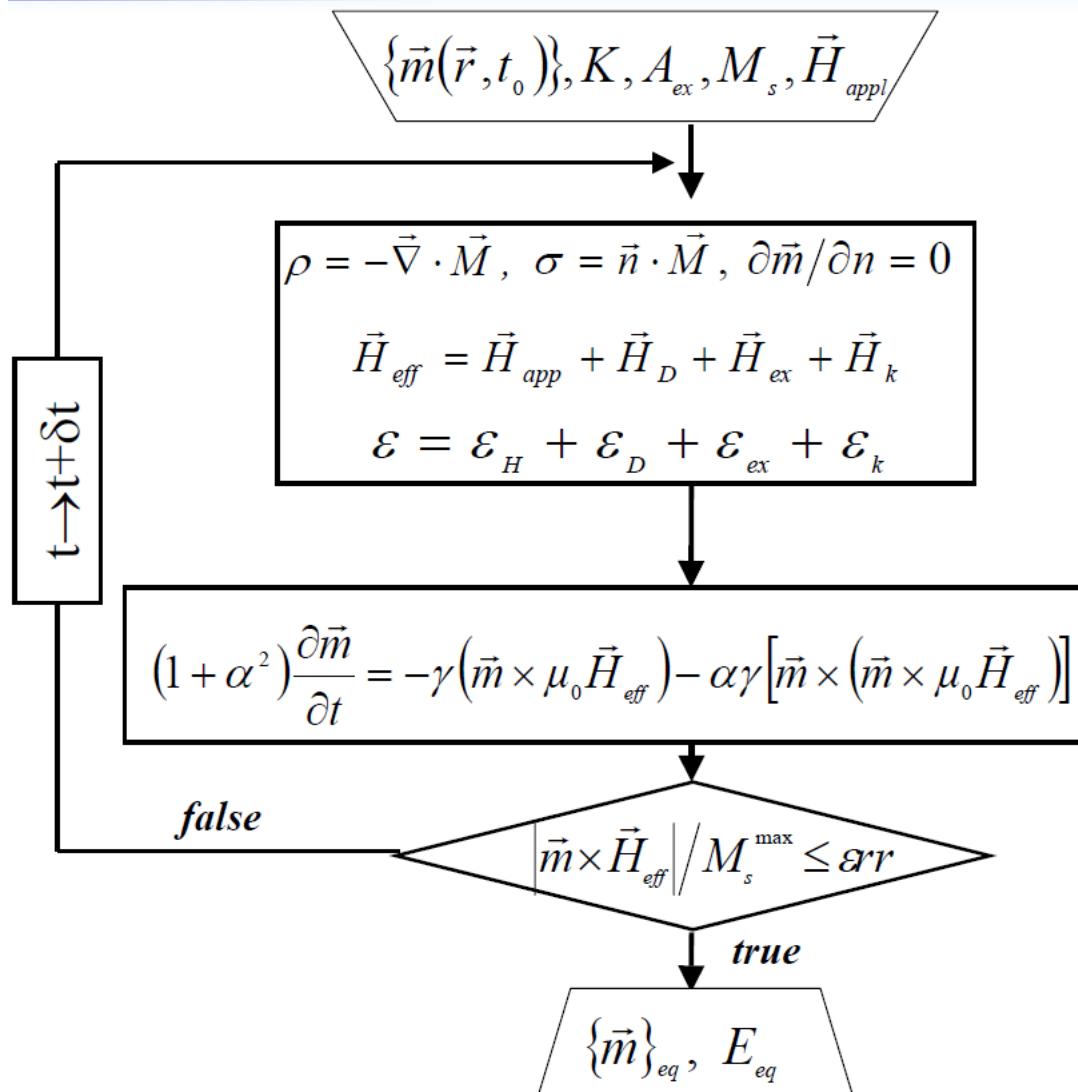
✓ Sample dependent features

# Outline



- Atomistic spin dynamics
  - Formalism
  - Illustrations : alloy GdFe, Fe nanodot, AOS [Tb/Co] multilayers
- Micromagnetism
  - Formalism
  - Macrospin limit
  - Illustrations
- Micromagnetism and spin related phenomena
  - Formalism
  - Illustrations
- Conclusions

# Micromagnetisme



■ Defining the problem  
-geometry description  
-material parameters  
-initial conditions  
(time & space)

■ State characterization  
-magnetic charges  
-magnetostatic field  
-fields & energy terms

■ LLG time integration  
- amplitude conservation

■ Check equilibrium criteria

■ Stable state- numerical solution

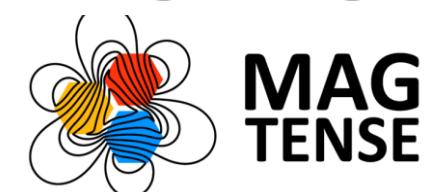
# Micromagnetisme



micro3D



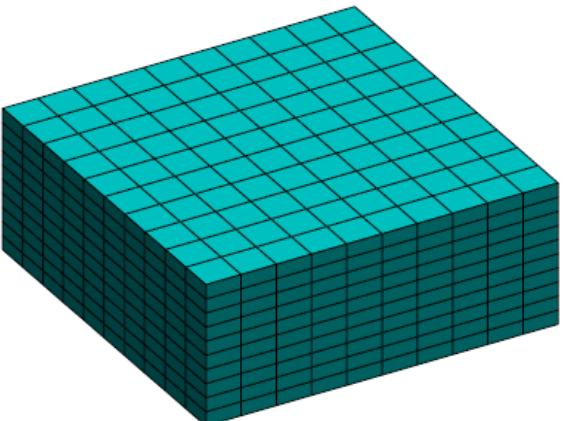
mumax<sup>3</sup>



MAGNUM<sup>fd</sup>



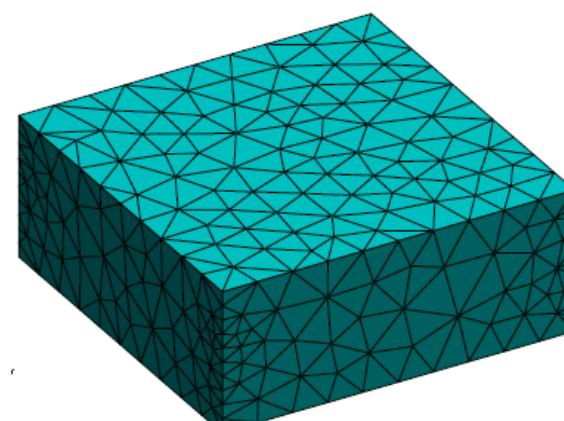
Finite difference  
approximation (FDA)



- regular mesh
- restrictive geometry
- W.F. Brown Jr (1965)
- Schabes et al., (1988)
- Berkov et al.
- Bertram et al.
- Donahue et al.
- Miltat et al.
- Nakatani et al.
- Toussaint et al.
- Scheinfein et al.
- J. -G. Zhu et al.....



Finite element method  
(FEM / BEM)



- irregular meshes
- adaptive mesh refinement
- Fredkin & Koehler
- Fidler & Schrefl
- Hertel & Kronmuller
- Ramstöck et al.
- .....

magpar

nmag

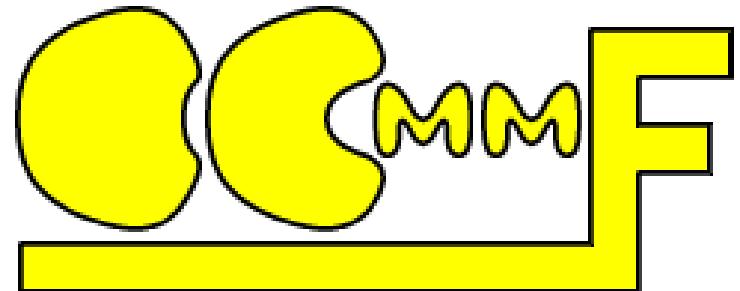


MAGNUM<sup>fe</sup>

feeLLGood  
finite element LLG object oriented development

FastMag

# Micromagnetisme



## Tomorrow's micromagnetic simulations

Cite as: J. Appl. Phys. 125, 180901 (2019); doi: 10.1063/1.5093730

Submitted: 24 February 2019 · Accepted: 25 April 2019 ·

Published Online: 10 May 2019



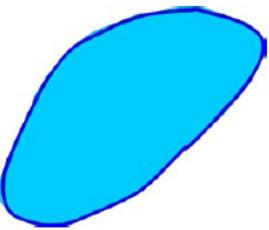
J. Leliaert<sup>a,b)</sup>  and J. Mulkers<sup>a,c)</sup> 

### AFFILIATIONS

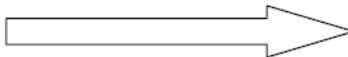
Department of Solid State Sciences, Ghent University, 9000 Ghent, Belgium

Name	Release	FE/FD	GPU capable?	Free?	References
LLG micromagnetics simulator	1997	FD	No	Commercial	<a href="#">29</a>
OOMMF	1998	FD	No	Free	<a href="#">27</a>
micromagus	2003 <sup>a</sup>	FD	No	Commercial	<a href="#">30</a>
magpar	2003	FE	No	Free	<a href="#">31</a>
Nmag	2007	FE	No	Free	<a href="#">32</a>
GPMagnet	2010	FD	Yes	Commercial	<a href="#">26</a>
FEMME	2010	FE	No	Commercial	<a href="#">33</a>
tetramag <sup>b</sup>	2010	FE	Yes	Commercial	<a href="#">34</a>
finmag <sup>c</sup>	2011	FE	No	Free	<a href="#">35</a>
Fastmag	2011	FE	Yes	Commercial	<a href="#">36</a>
Mumax	2011	FD	Yes	Free	<a href="#">37</a>
micromagnum	2012	FD	Yes	Free	<a href="#">38</a>
magnum.fd <sup>d</sup>	2014	FD	Yes	Free	<a href="#">39</a>
magnum.fe	2013	FE	No	Commercial	<a href="#">40</a>
mumax <sup>3</sup>	2014	FD	Yes	Free	<a href="#">41</a>
LLG micromagnetics simulator v4.	2015	FD	Yes	Commercial	<a href="#">29</a>
Grace	2015	FD	Yes	Free	<a href="#">42,43</a>
OOMMF (GPU version)	2016	FD	Yes	Free	<a href="#">44</a>
fidimag	2018	FD	No	Free	<a href="#">45</a>
commics	2018	FE	No	Free	<a href="#">46</a>

# Micromagnetisme

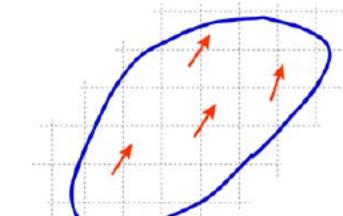


Numerical discretisation



$$\vec{m} = \left\{ \vec{m}(\vec{r}) \mid \vec{r} \in V, |\vec{m}|=1 \right\}$$

$$\vec{H}_{eff} = \left\{ \vec{H}_{eff}(\vec{r}) \mid \vec{r} \in V \right\}$$

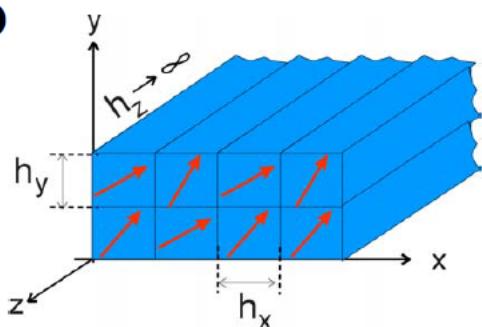


$$\vec{m} = \left\{ \vec{m}_i \mid i = 1..N, |\vec{m}_i| = 1 \right\}$$

$$\vec{H}_{eff} = \left\{ \vec{H}_{eff,i} \mid i = 1..N \right\}$$

*N – total number of mesh nodes*

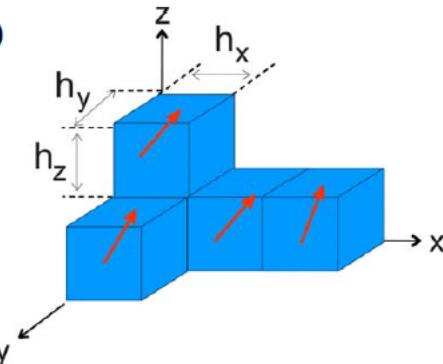
**2D**



-infinite prisms

: e.g. thin films

**3D**



-orthorhombic cells

: dots, wires, platelets, ...

# Micromagnetisme

## Finite Difference Aproximation (FDA)

$$E[\vec{m}] = \int_V \left\{ A_{ex} [\vec{\nabla} \cdot \vec{m}(\vec{r})]^2 + K_1 [1 - (\vec{u}_K \cdot \vec{m}(\vec{r}))^2] - \right. \\ \left. - \mu_0 M_s [\vec{m}(\vec{r}) \cdot \vec{H}_{app}(\vec{r})] - \frac{1}{2} \mu_0 M_s [\vec{m}(\vec{r}) \cdot \vec{H}_{dem}(\vec{m}(\vec{r}))] \right\} dV$$

$$\vec{H}_{eff} = \frac{2A_{ex}}{\mu_0 M_s} \Delta \vec{m} + \frac{2K_1}{\mu_0 M_s} (\vec{u}_K \cdot \vec{m}) \vec{u}_K + \vec{H}_H + \vec{H}_D$$

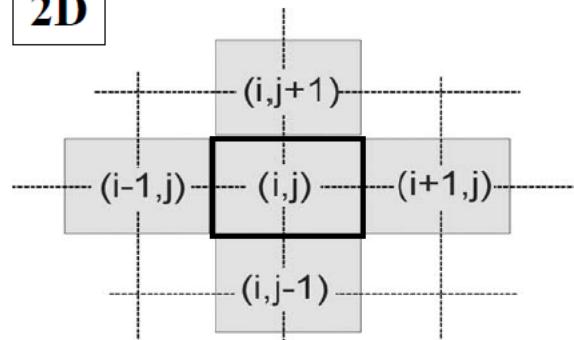
magnetic charges

$$\rho_m = -M_s (\vec{\nabla} \cdot \vec{m})$$

$$\sigma_m = M_s (\vec{m} \cdot \vec{n})$$

### Taylor expansion

2D



$$m(i+1, j) = m(i, j) + \frac{\partial m}{\partial x}(i, j) h_x + \frac{1}{2} \frac{\partial^2 m}{\partial x^2}(i, j) h_x^2 + O(h_x^3)$$

$$m(i-1, j) = m(i, j) - \frac{\partial m}{\partial x}(i, j) h_x + \frac{1}{2} \frac{\partial^2 m}{\partial x^2}(i, j) h_x^2 + O(h_x^3)$$

$$\frac{\partial m}{\partial x}(i, j) \cong \frac{m(i+1, j) - m(i-1, j)}{2h_x}$$

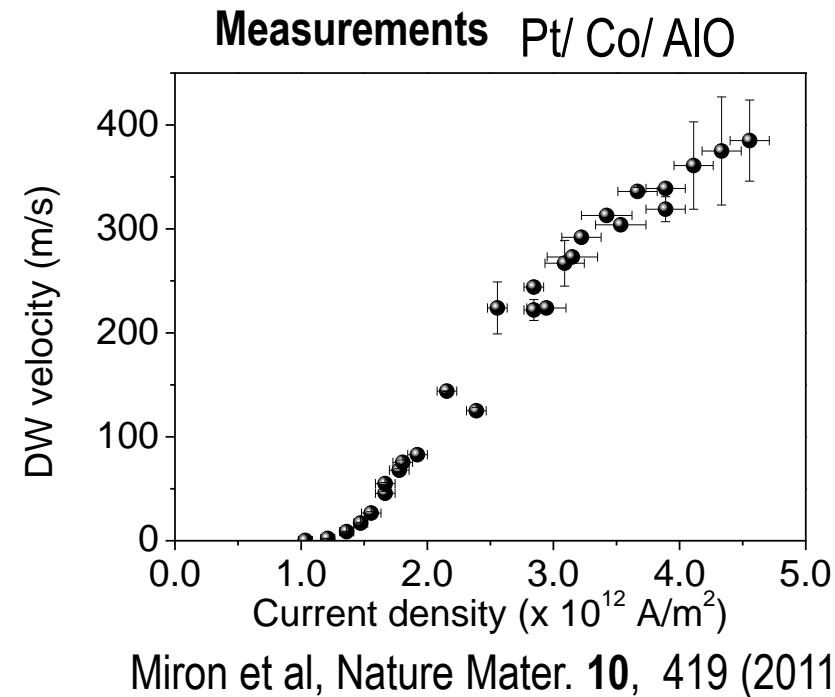
$$\frac{\partial^2 m}{\partial x^2}(i, j) \cong \frac{m(i+1, j) - 2m(i, j) + m(i-1, j)}{h_x^2}$$

The accuracy is dependent on the Taylor expansion order !



M. Labrune, J. Miltat, JMMM 151, 231 (1995).

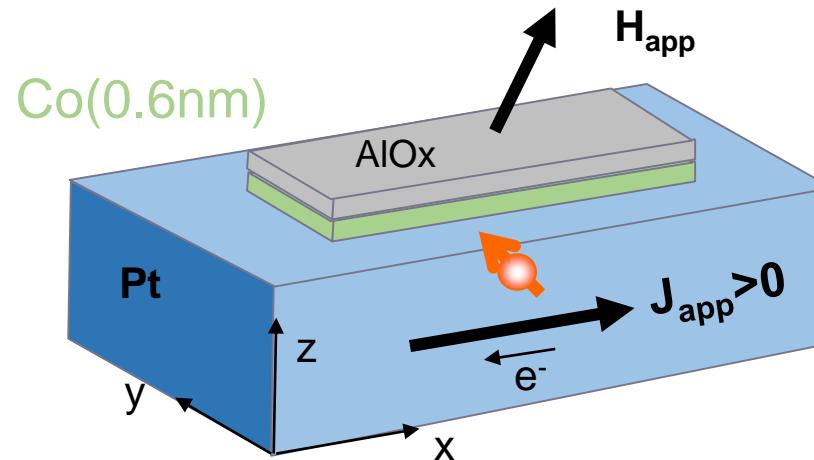
# Micromagnetism and spin related phenomena : DMI and domain walls



Eq. Landau-Lifshitz-Gilbert + DMI

$$\mathbf{H}_{\text{DM}}(\mathbf{m}) = \frac{2D}{\mu_0 M_s} \left( \frac{\partial m_z}{\partial x}, \quad \frac{\partial m_z}{\partial y}, \quad -\frac{\partial m_x}{\partial x} - \frac{\partial m_y}{\partial y} \right)$$

A. Thiaville et al *Europhys. Lett.* **100**, 57002 (2012)

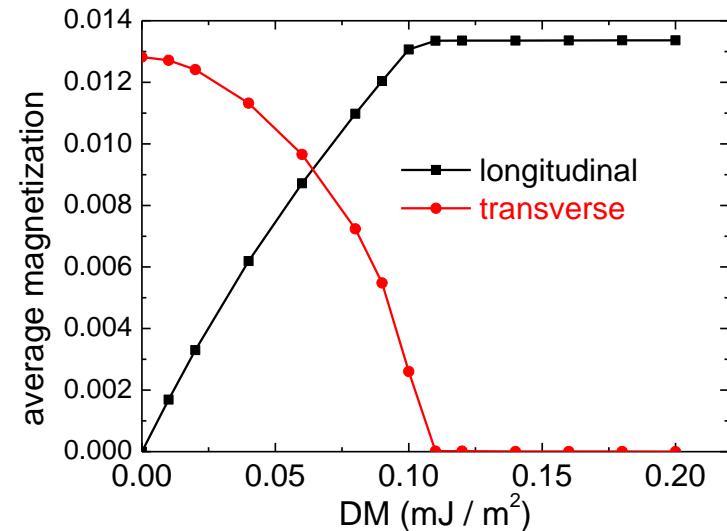
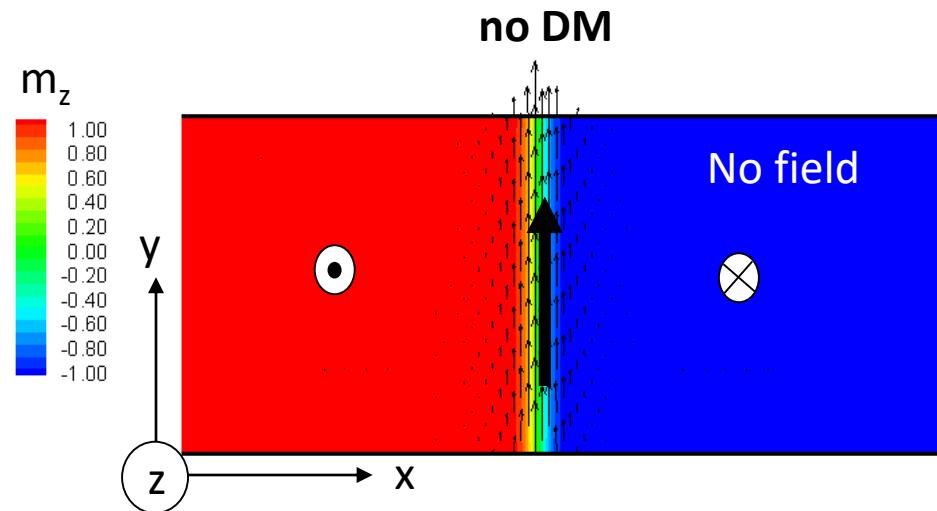


- 1) Chiral exchange Dzyaloshinskii-Moriya
- 2) Spin polarized current //Oy (Spin Hall effect & Rashba effect)

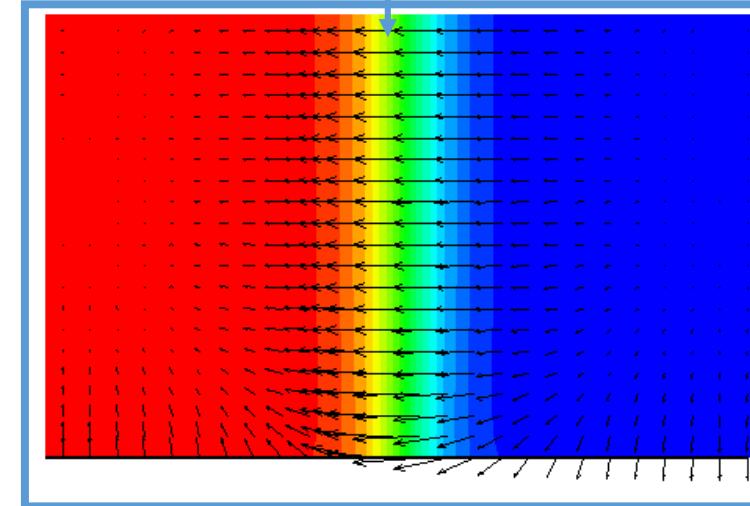
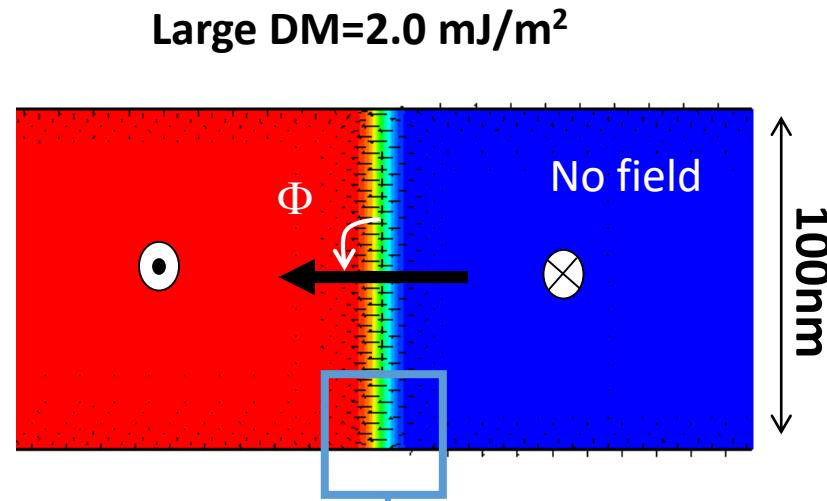
+ spin orbit torques

$$\begin{aligned} \left( \frac{\partial \mathbf{m}}{\partial t} \right)_{\text{SOT}} &= \gamma_0 C_{FL} J_{app} [\mathbf{m} \times (\mathbf{u}_J \times \hat{\mathbf{z}})] \\ &+ \gamma_0 C_{DL} J_{app} \mathbf{m} \times [\mathbf{m} \times (\mathbf{u}_J \times \hat{\mathbf{z}})] \end{aligned}$$

# Micromagnetism and spin related phenomena : DMI and domain walls

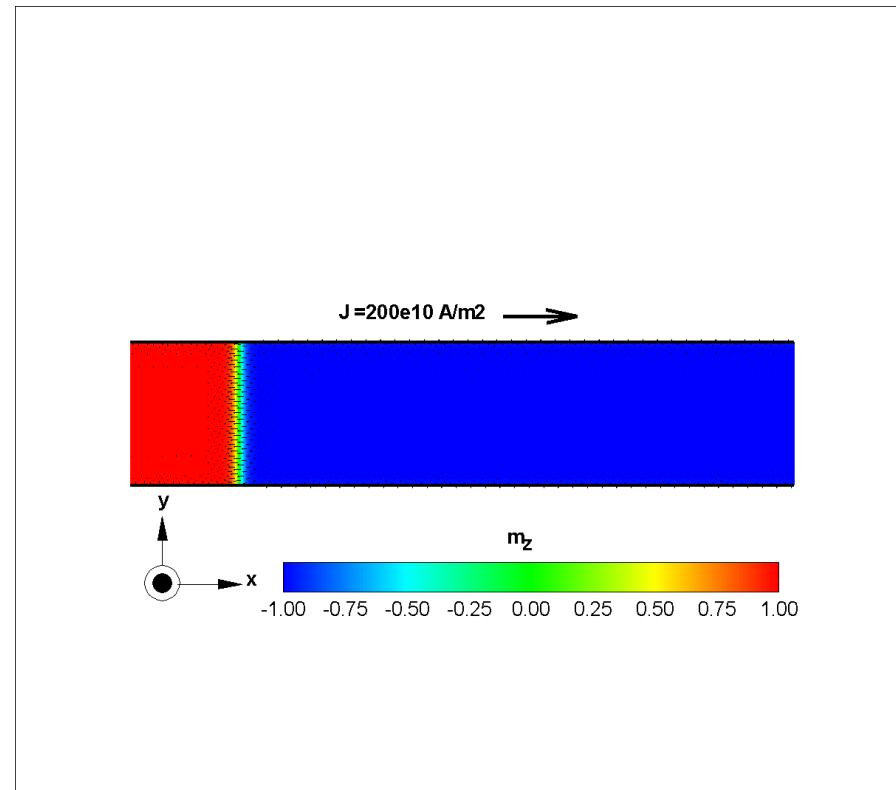
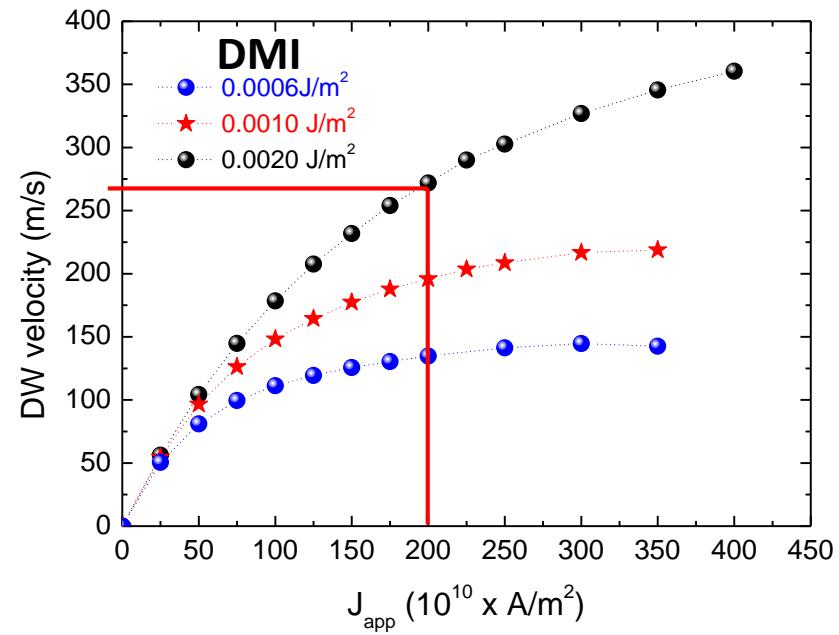


A. Thiaville et al., EPL, 100 (2013)



O. Boule et al, Phys. Rev. Lett (2013)

# Micromagnetism and spin related phenomena : DMI and domain walls



- ✓ DMI+SOT : the key to explain the fast current induced DW motion in these materials
- ✓ DW velocity  $\sim J_{app}$

# Micromagnetism and spin related phenomena : skyrmion Hall effect

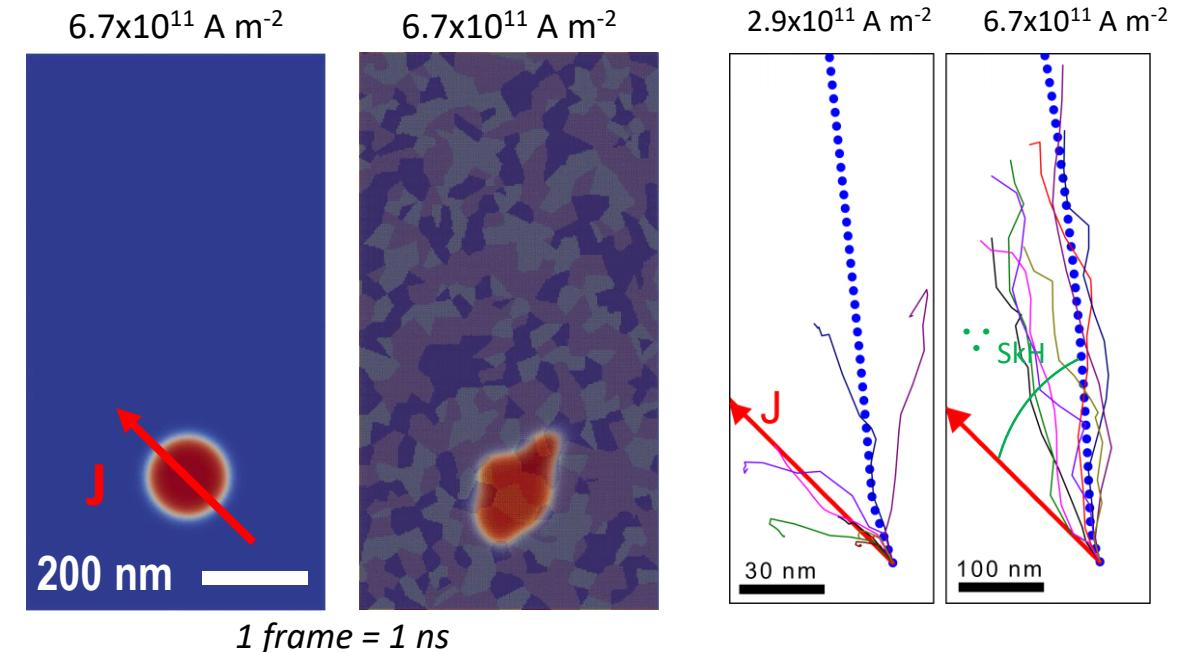
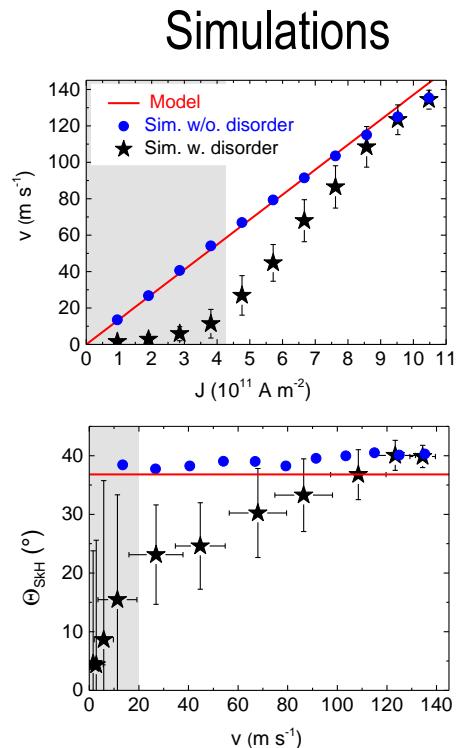
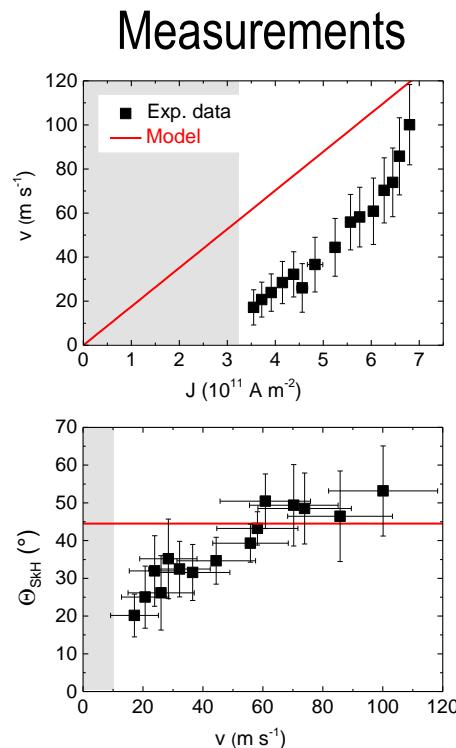


DARPA - TEE

Juge et al. PR Appl. (2019)

10.1103/PhysRevApplied.12.044007

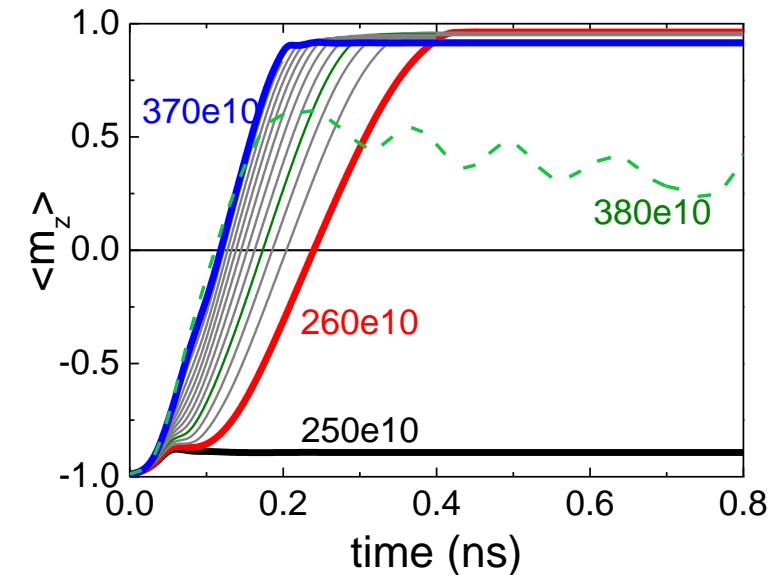
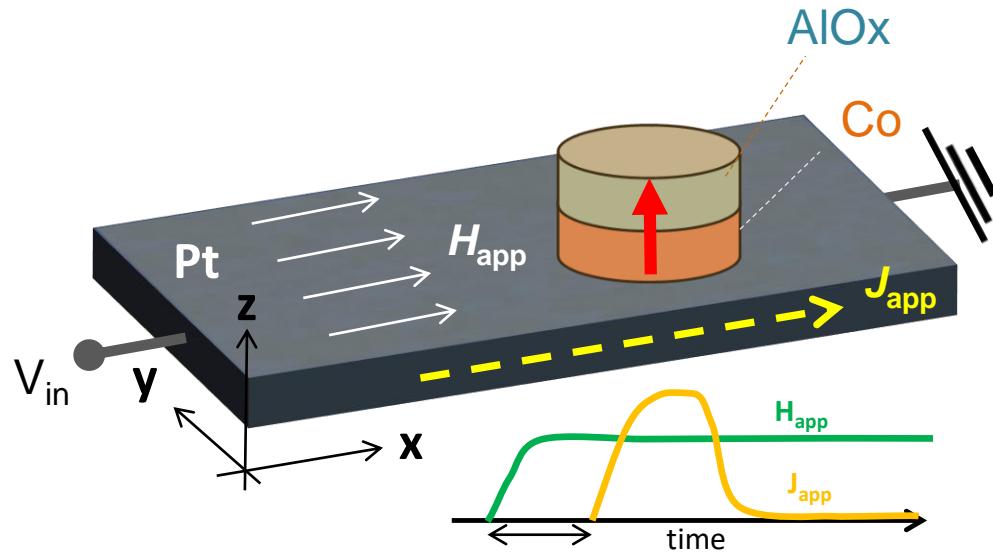
## Thin film of Pt / Co / MgO



✓ SOT current driven motion of the skyrmions depends strongly on the pinning

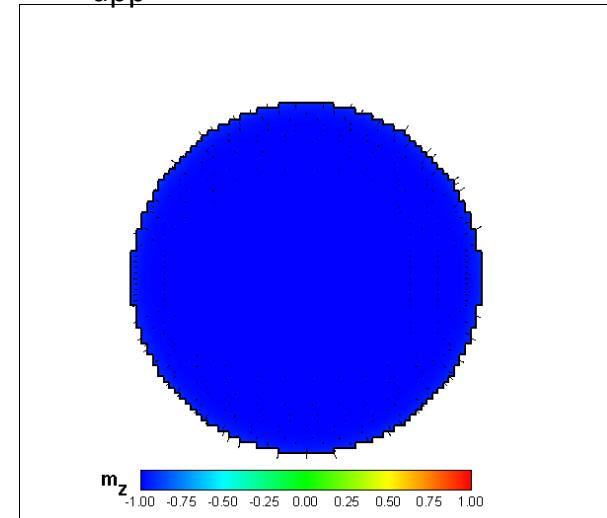
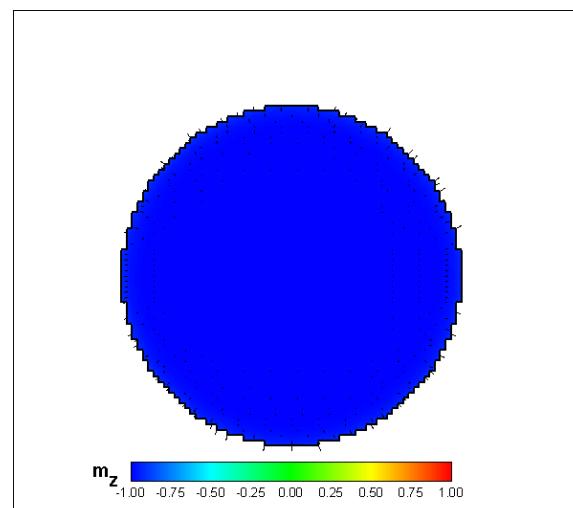
# Micromagnetism and spin related phenomena : SOT MRAM

SPOT



$$J_{app} = 260 \times 10^{10} \text{ A/m}^2$$

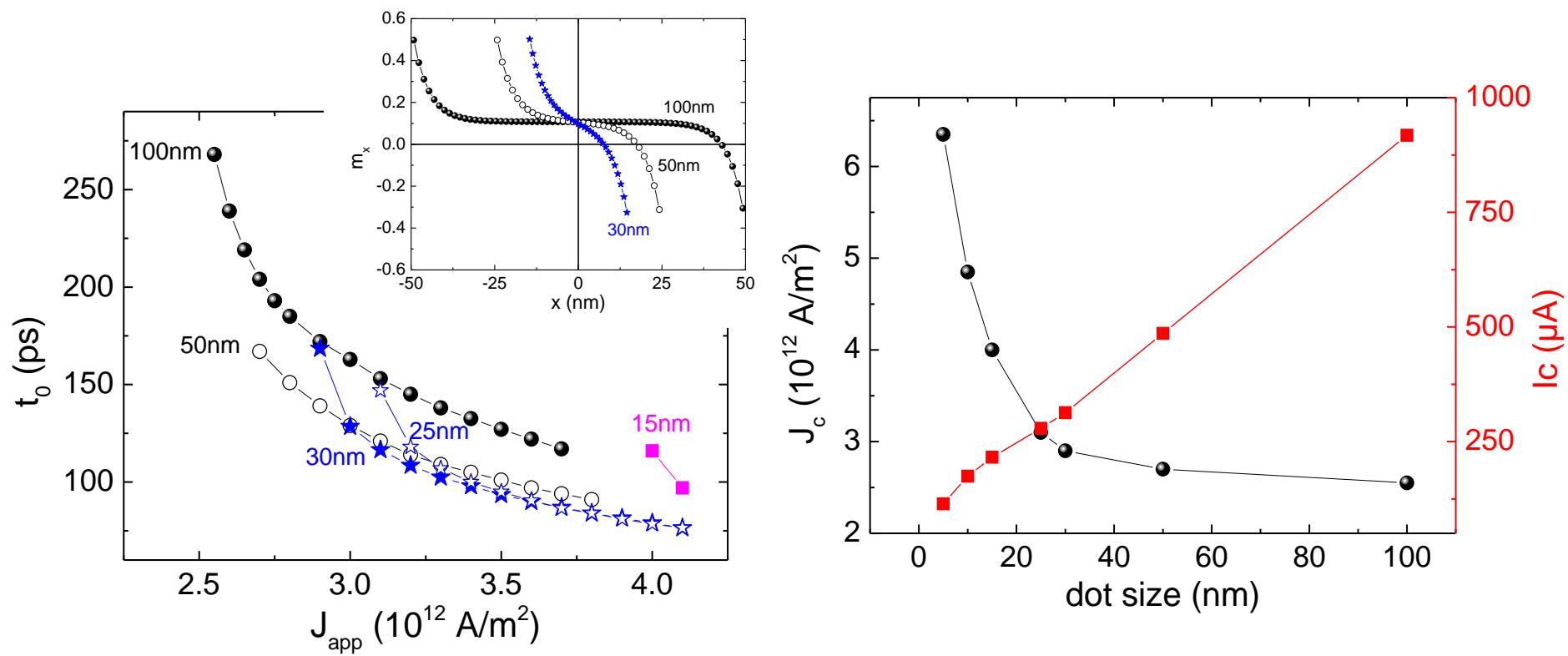
$$J_{app} = 350 \times 10^{10} \text{ A/m}^2 \text{ No field}$$



- ✓ Bipolar switching
- ✓ Very short pulse < 250ps
- ✓ Reversal by nucleation/propagation of DW

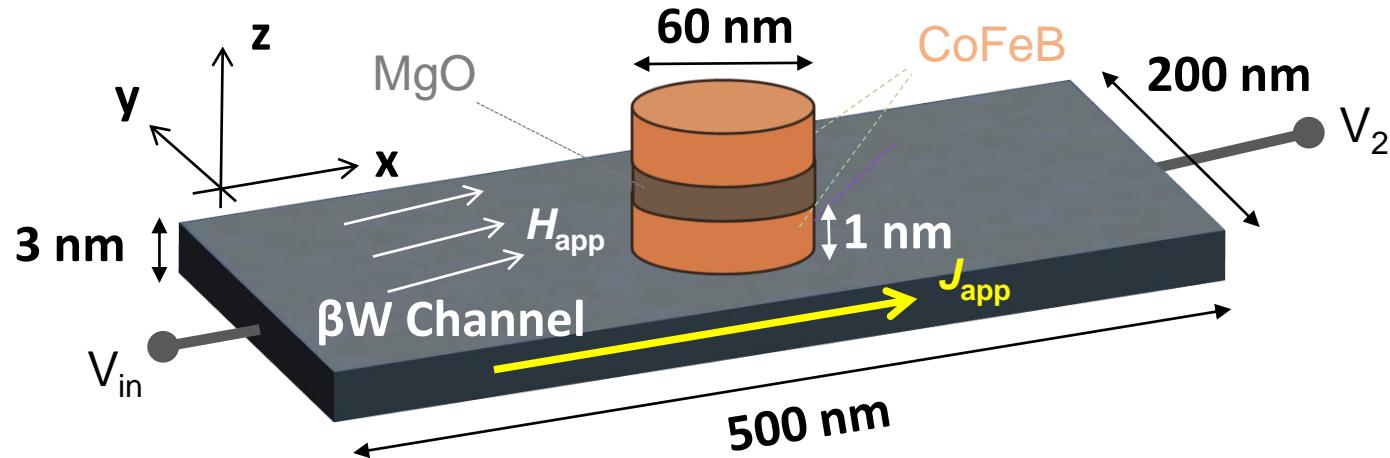
K. Garello et al Appl. Phys. Lett. (2014)  
N. Mikuszeit et al. Phys. Rev. B (2015)

# Micromagnetism and spin related phenomena : SOT MRAM



- ✓ DW reversal mechanism if nanopillar size > 30nm
- ✓ Critical current varies linearly with nanopillar size
- ✓ Writing energy ~20fJ  $E_w = 1k\Omega \cdot (250\mu A)^2 \cdot 300ps = 20fJ$

# Micromagnetism and spin related phenomena : SOT MRAM + heating



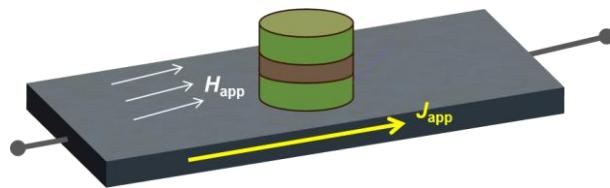
## Key elements:

### Interfaces HM/FM and FM/Ox

- perpendicular anisotropy – **binary states & memory stability**
- Dzyaloshinskii-Moriya interaction

**Spin orbit torques** → to **switch** between the two stable states

# Micromagnetism and spin related phenomena : SOT MRAM + heating



## Intrinsic issues

- ✓ Parameters sensitive to temperature  $\rightarrow K_i(T), M_s(T), A_{ex}(T), \dots$
- ✓ Impact on the magnetization dynamics
- ✓ Impact on the operation of the storage layer

Develop appropriate modelling approach

coupling : current  $\rightarrow$  temperature  $\rightarrow$  magnetization dynamics

Electric solver

$$J_{app}(\mathbf{r}, t)$$

COMSOL

Joule Heating

$$\mathbf{T}(\mathbf{r}, t)$$

3D

LLG + SOT

$$\mathbf{M}(J_{app}(\mathbf{r}, t), \mathbf{T}(\mathbf{r}, t))$$

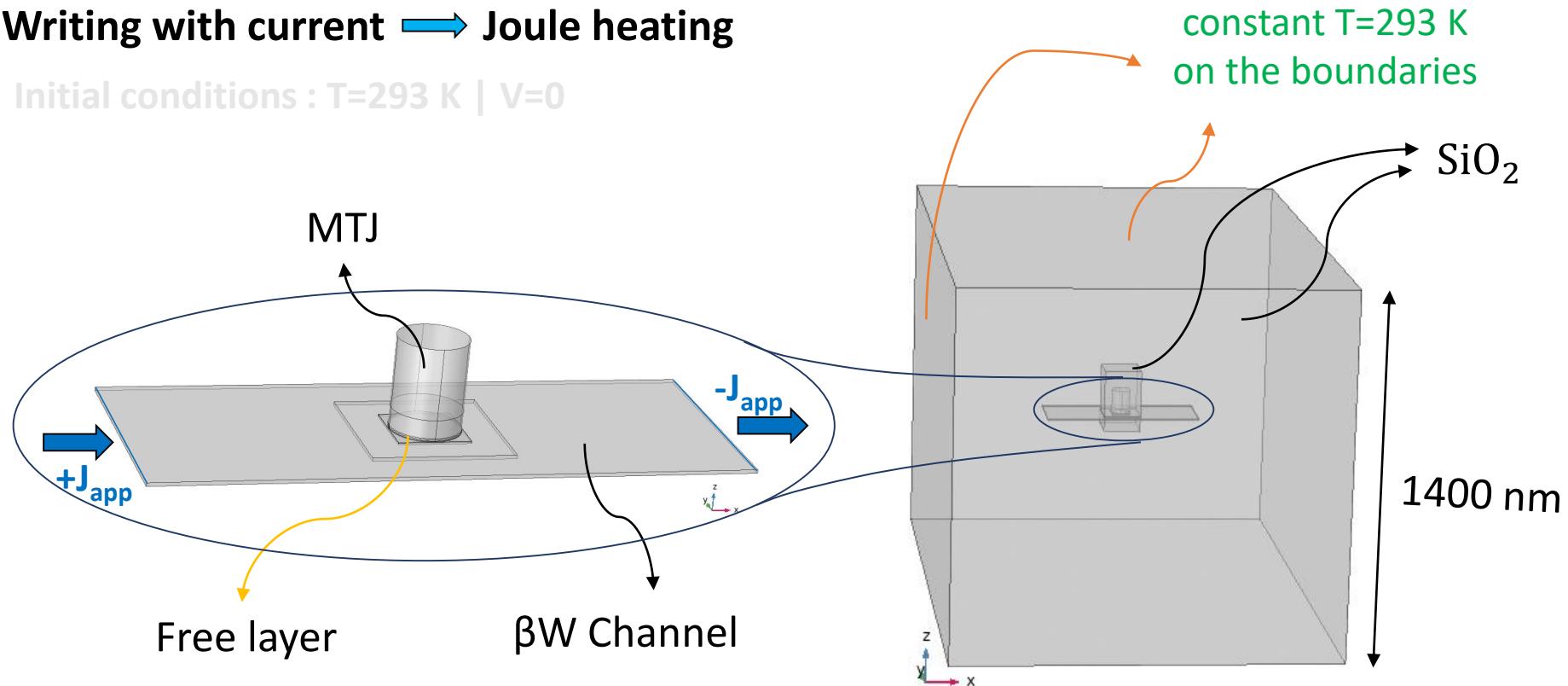
micromagnetic model

E. Grimaldi et al, Nature Nanotechnology (2020)

# Micromagnetism and spin related phenomena : SOT MRAM + heating

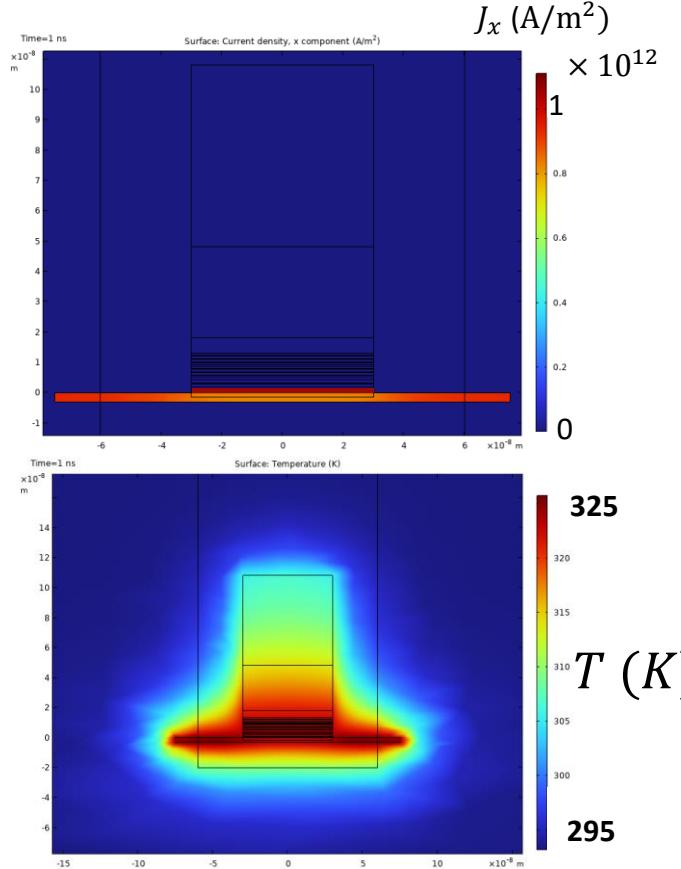
Writing with current  $\rightarrow$  Joule heating

Initial conditions :  $T=293\text{ K}$  |  $V=0$

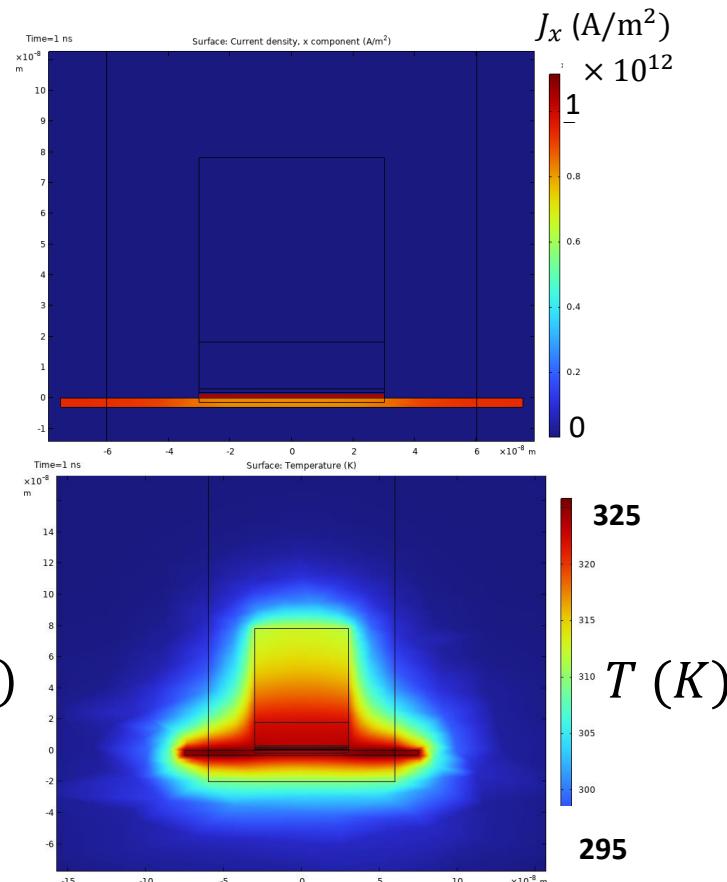


# Micromagnetism and spin related phenomena : SOT MRAM + heating

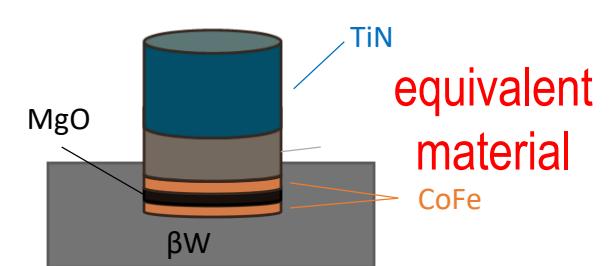
Full structure MTJ



Equivalent MTJ



$t = 1 \text{ ns}$



# Micromagnetism and spin related phenomena : SOT MRAM + heating

Material parameters follow Callen-Callen laws

Spontaneous magnetization

$M_{s0}$  the value at 0K

$T_c$  – critical temperature

$$M_s(T) = M_{s0} \left[ 1 - \left( \frac{T}{T_c} \right)^a \right]^b$$

$\Delta T = 30 K$

7 % variation

Magnetocrystalline anisotropy

interface & bulk

$$K(T) = K_0 \left[ \frac{M_s(T)}{M_{s0}} \right]^p$$

16 % variation

Exchange stiffness

$$A_{ex}(T) = A_{ex_0} \left[ \frac{M_s(T)}{M_{s0}} \right]^q$$

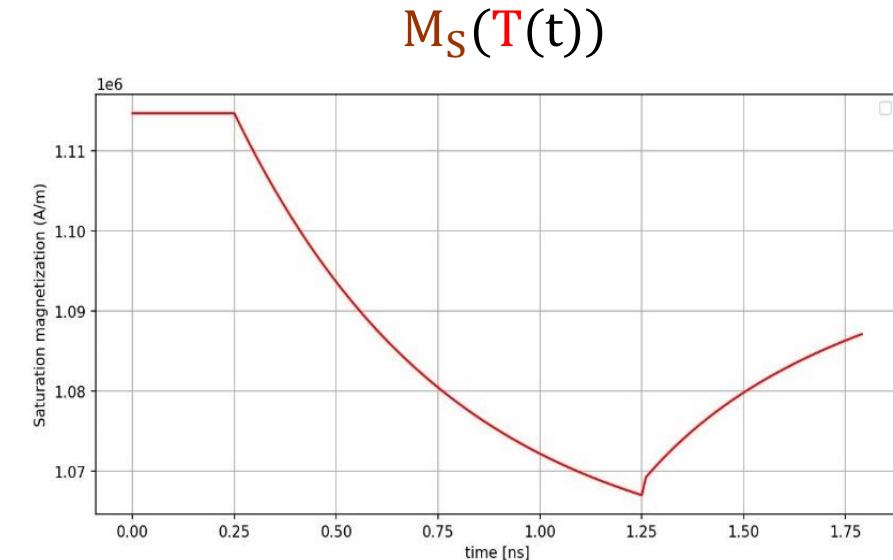
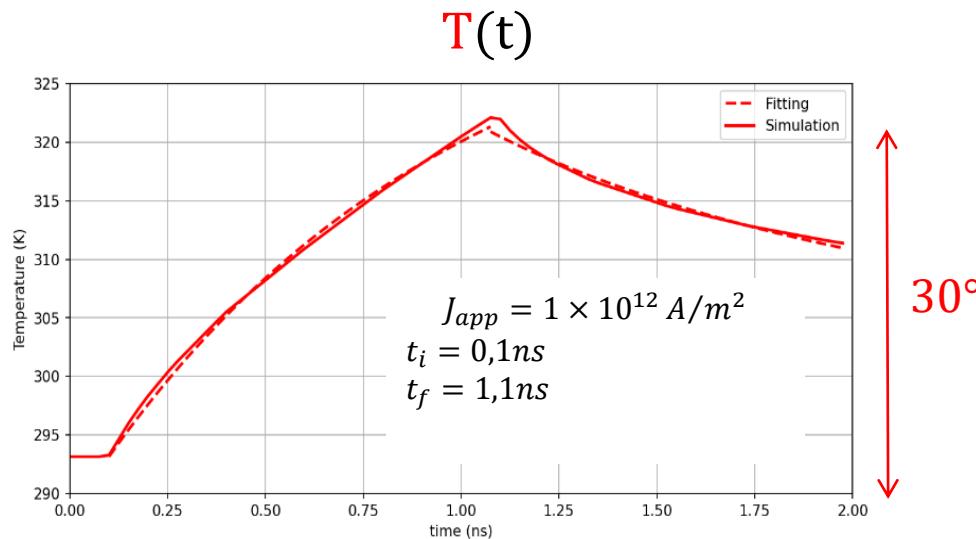
11 % variation

FeCoB

$M_{s0}$ (A/m)	1.09e6
$K_0$ (J/m <sup>3</sup> )	1.25e6
$A_{ex_0}$ (J/m)	1.5e-11
a	1
b	1
p	2.5
q	1.7
$T_c$ (K)	750
DMI (J/m <sup>2</sup> )	1.5e-4
$H_{app}$ (T)	0.06

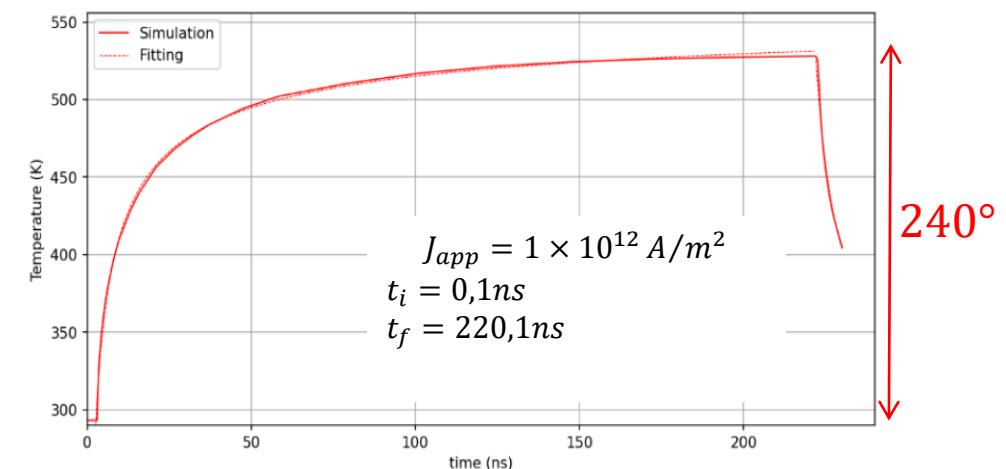
Lee et al., AIP Adv., (2017)

# Micromagnetism and spin related phenomena : SOT MRAM + heating

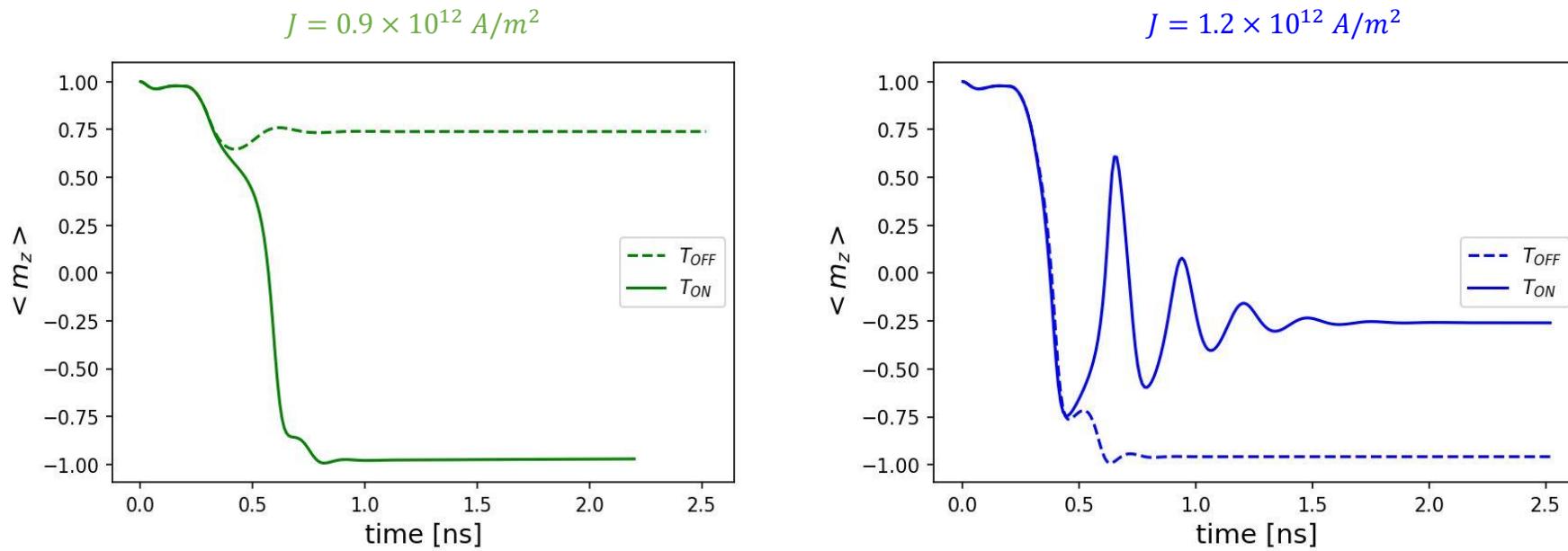


Fitting parameters :  $c$ ,  $t'$ ,  $T_r$ ,  $u$  and  $t^*$  depend on the simulation conditions

$$T(t) = \begin{cases} -\frac{c}{\sqrt{t - t'}} + T_r & t_i < t < t_f \\ -\frac{u}{t - t^*} + 293 & t > t_f \end{cases}$$



Given current pulse  $\rightarrow T(t) \rightarrow$  fitting  $\rightarrow$  micromagnetics



# Modelling – take home messages

- ❑ Model versus Reality

- ❑ Validity of the model

- ❑ Utility of the model

- ❑ Critical thinking on numerical solutions

A solver gives always a solution!



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