Dipolar and exchange spin waves

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OPTIMAS light-spin-matter

Definitions and relations

Matter in external magnetizing field $\vec{H}$ shows magnetic moment.

Magnetization $\vec{M}$ is the total magnetic moment per unit volume.

Dia- and paramagnetic materials: relation between $\vec{M}$ and $\vec{H}$ is linear: $\vec{M} = \chi\vec{H}$

Magnetic field: $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu\vec{H}$

with $\mu = \mu_0(1+\chi) = \mu_0\mu_r$, μ_r is the relative permeability of the material.

Magnetic polarization: $\vec{J}_{pol} = \mu_0\vec{M}$

Energy E_m, torque $\vec{T}$, and force $\vec{F}$ of a single magnetic dipole $\vec{\mu}_m$ in magnetic field $\vec{B}$:

$$E_m = -\vec{\mu}_m \cdot \vec{B}$$ (Zeeman energy of a magnetic dipole $\vec{\mu}_m$ in magnetic field $\vec{B}$)

$$\vec{T} = \vec{\mu}_m \times \vec{B}$$

$$\vec{F} = \vec{\nabla}(\vec{\mu}_m \cdot \vec{B})$$

Field $\vec{B}$ generated by magnetic dipole $\vec{\mu}_m$ in distance $\vec{r}$:

$$\vec{B} = \mu_0\left(\frac{3(\vec{\mu}_m \cdot \vec{r})\vec{r}}{r^5} - \frac{\vec{\mu}_m}{r^3}\right)$$

Definitions and relations

Interaction energy $E_{\text{dip-dip}}$ of two dipoles $\vec{\mu}_1$ and $\vec{\mu}_2$ at positions $\vec{r}_1$ and $\vec{r}_2$:

$$E_{\text{dip-dip}} = \mu_0 \left(\frac{\vec{\mu}_1 \cdot \vec{\mu}_2}{r_{12}^3} - 3\frac{(\vec{\mu}_1 - \vec{r}_{12})(\vec{\mu}_2 - \vec{r}_{12})}{r_{12}^5} \right) \text{ with } \vec{r}_{12} = \vec{r}_1 - \vec{r}_2$$

Magnetic moment $\vec{\mu}$:

can be viewed at a circular current I of an electron about an area $\vec{A} = A\hat{n}$,

$\hat{n}$: normal vector of area:

$$\vec{\mu} = I\vec{A}$$

Circular orbit with radius r, $A = \pi r^2$, $I = -|e| \cdot \frac{\omega}{2\pi}$:

$$\vec{\mu} = -\frac{1}{2}|e|\omega r^2 \hat{n}$$

Comparison to angular momentum $\vec{J}$ of an electron with mass m_e :

$$\vec{J} = \vec{r} \times m_e \vec{v} = m_e \omega r^2 \hat{n}$$

$$\Rightarrow \quad \vec{\mu} = -\frac{|e|}{2m_e}\vec{J}$$

Definitions and relations

Angular momentum $\vec{J}$ is quantized in units of $\hbar$

$\Rightarrow$ Elementary quantum of magnetic moment is Bohr magnetron

$$\mu_B = -\frac{|e|\hbar}{2m_e} = 9.2740 \cdot 10^{-24}\ \text{J/T}$$

Electrons also have spin. They have orbital momentum $\vec{\mu}_l$ and spin momentum $\vec{\mu}_s$:

$$\vec{\mu}_l = -\frac{|e|}{2m_e}\vec{l} \quad , \quad \vec{l}^{\,2} = l(l+1)\hbar^2$$

$$\vec{\mu}_s = -g_e \frac{|e|}{2m_e}\vec{s} \quad , \quad \vec{s}^{\,2} = s(s+1)\hbar^2$$

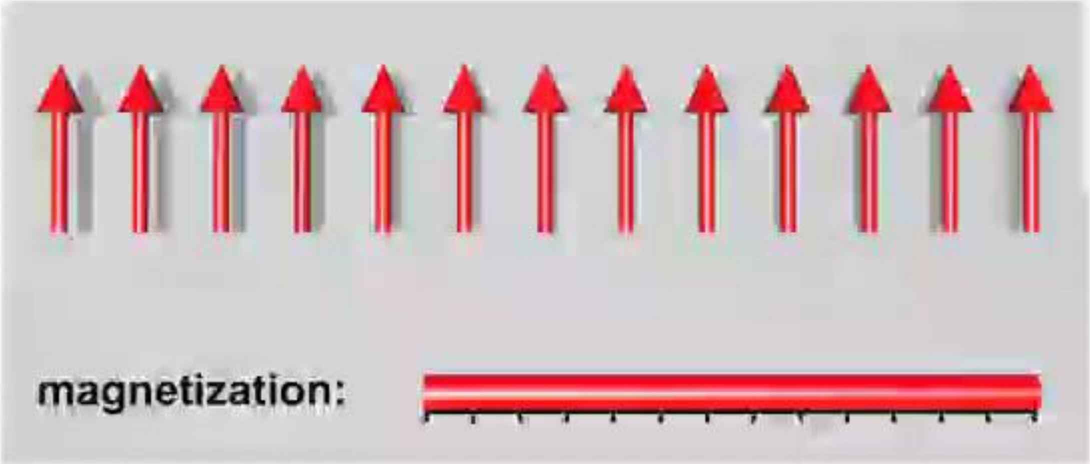
with $g_e = 2.0023$ the gyromagnetic ratio of the electron.

The spin moment is oriented (anti-) parallel to the external field with value

$$\mu_s = g_e \frac{|e|}{2m_e} \cdot |s_z|\hbar = g_e \mu_B |s_z| \approx \mu_B \quad , \quad s_z = \pm\frac{1}{2}$$

with s_z the magnetic spin quantum number

Ferromagnetic spin chain: magnon



Spin waves

RP TU Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

Spin dynamics

Torque $\vec{T}$ acting on moment $\vec{\mu}_{\mathrm{m}}$ in field $\vec{B}$:

$$\vec{T} = \vec{\mu}_{\mathrm{m}} \times \vec{B}$$

$\Rightarrow$ moment moves perpendicular to field direction $\Rightarrow$ precession

Angular momentum $\vec{L} = -\frac{\hbar}{g\mu_{\mathrm{B}}}\vec{\mu}_{\mathrm{m}} = -\frac{1}{\gamma}\vec{\mu}_{\mathrm{m}}$

$$\Rightarrow \vec{T} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t} = -\frac{1}{\gamma}\frac{\partial \vec{\mu}_{\mathrm{m}}}{\partial t} \stackrel{!}{=} \vec{\mu}_{\mathrm{m}} \times \vec{B}$$

with $\gamma = g\mu_{\mathrm{B}} / \hbar$ the gyromagnetic ratio, $g \approx 2$ (electron): $\gamma / 2\pi = 28$ GHz/T

Transition to magnetization:

$$\frac{1}{\gamma}\frac{\partial \vec{M}}{\partial t} = -\vec{M} \times \vec{B}_{\mathrm{eff}}$$

Spin dynamics

$$\frac{1}{\gamma}\frac{\partial \vec{M}}{\partial t} = -\vec{M}\times\vec{B}_{\text{eff}}$$ – Landau-Lifshitz torque equation

with $\vec{B}_{\text{eff}}$ the effective magnetic field, acting on the magnetization:

$$\vec{B}_{\text{eff}} = \vec{B}_0 + \vec{B}(t) + \vec{B}_{\text{ani}} + \vec{B}_{\text{exchange}} + \ldots$$

with

$\vec{B}_0 = \mu_0\vec{H}$: external magnetic field

$\vec{B}(t)$: time dependent field due to precession

$\vec{B}_{\text{ani}} = \frac{1}{M}\vec{\nabla}_{\hat{M}}\, g_{\text{ani,vol}}$: magnetic anisotropy field, $g_{\text{ani,vol}}$: enthalpy density

gradient with respect to the directions of $\hat{M}$

$\vec{B}_{\text{exchange}} = \frac{2J_{\text{ex}}}{M_s^2}\vec{\nabla}^2\vec{M}$: exchange field

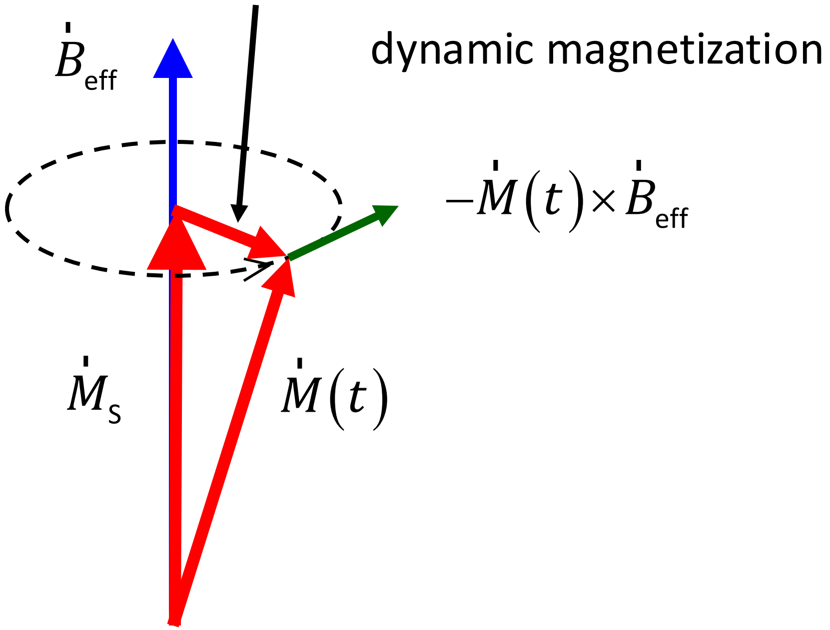
Coherent dynamics: spin waves

Spin wave: collective motion of magnetic moments

Landau-Lifshitz torque equation

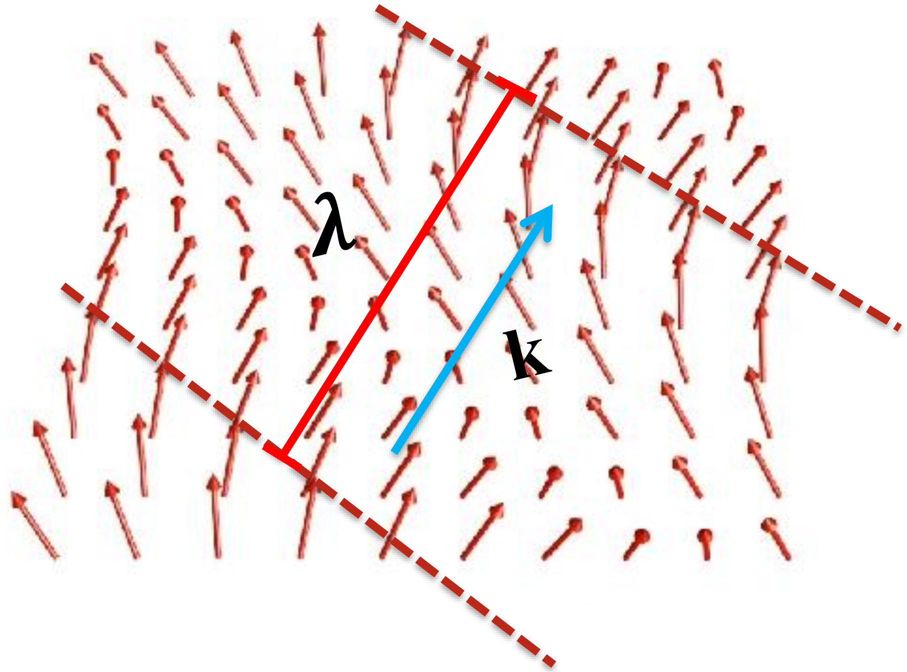
$$\frac{1}{\gamma}\frac{\partial \vec{M}(t)}{\partial t} = -\vec{M}(t)\times\vec{B}_{\mathrm{eff}}(t)$$

$$\vec{m}(\vec{r},t) = \vec{m}_0(\vec{r})\cdot e^{i(\vec{k}\vec{r}-\omega t)}$$



Coherent dynamics: spin waves

Spin wave: collective motion of magnetic moments



Landau-Lifshitz torque equation

$$\frac{1}{\gamma}\frac{\partial \vec{M}(t)}{\partial t} = -\vec{M}(t)\times\vec{B}_{\text{eff}}(t)$$

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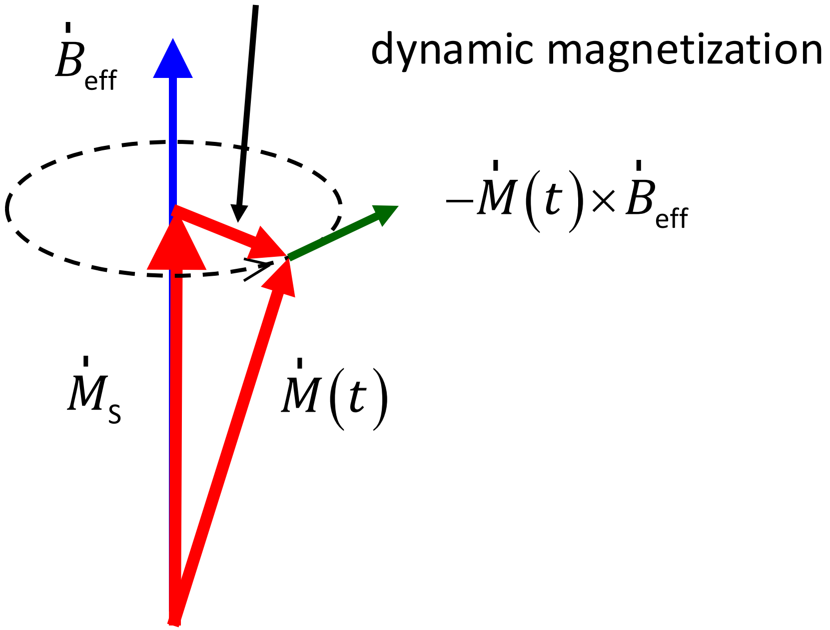
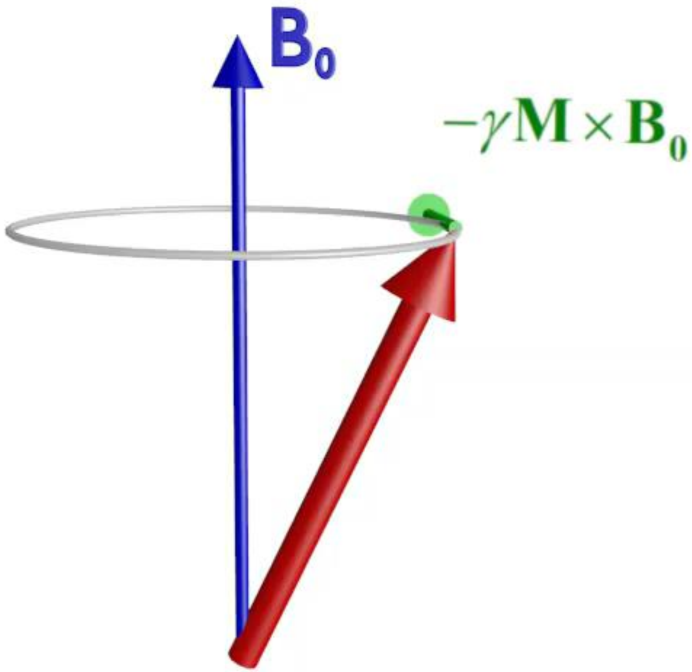


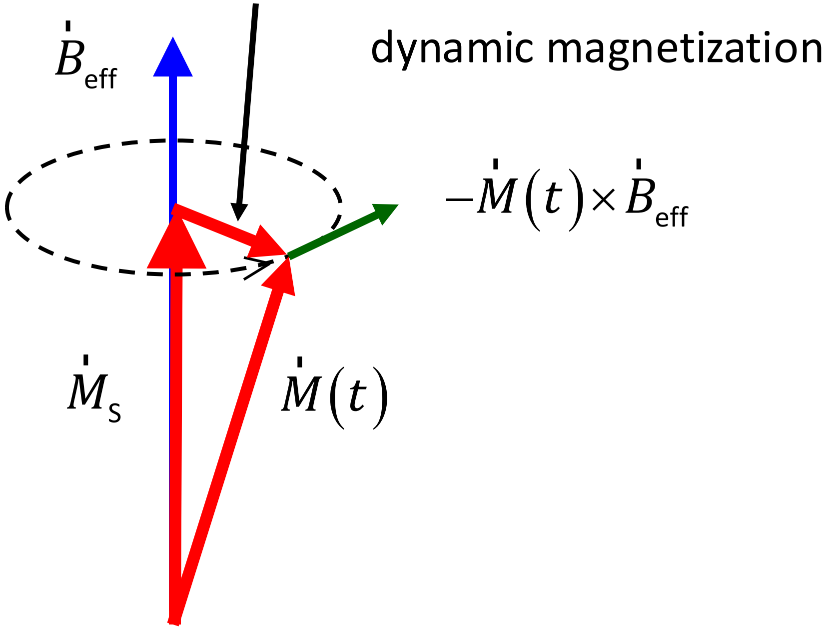
Coherent dynamics: spin-wave decay



Landau-Lifshitz-Gilbert torque equation with damping

$$\frac{1}{\gamma}\frac{\partial \vec{M}(t)}{\partial t} = -\vec{M}(t)\times\vec{B}_{\text{eff}}(t) + \frac{\alpha}{M_S}\vec{M}(t)\times\frac{\partial \vec{M}(t)}{\partial t}$$

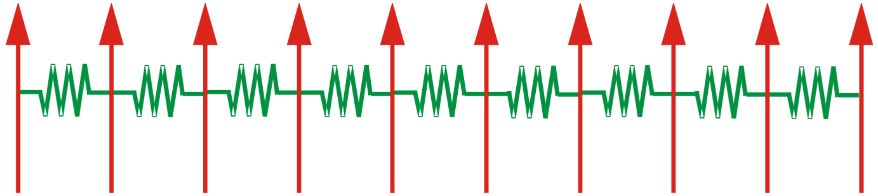
$$\vec{m}(\vec{r},t) = \vec{m}_0(\vec{r})\cdot e^{i\left(\vec{k}\vec{r}-\omega t\right)}$$



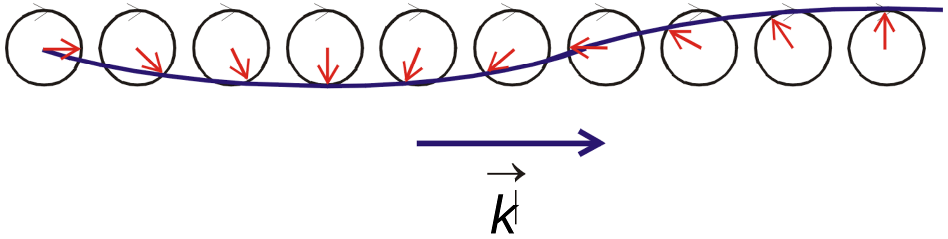
Spin waves

Two types of energy contributions

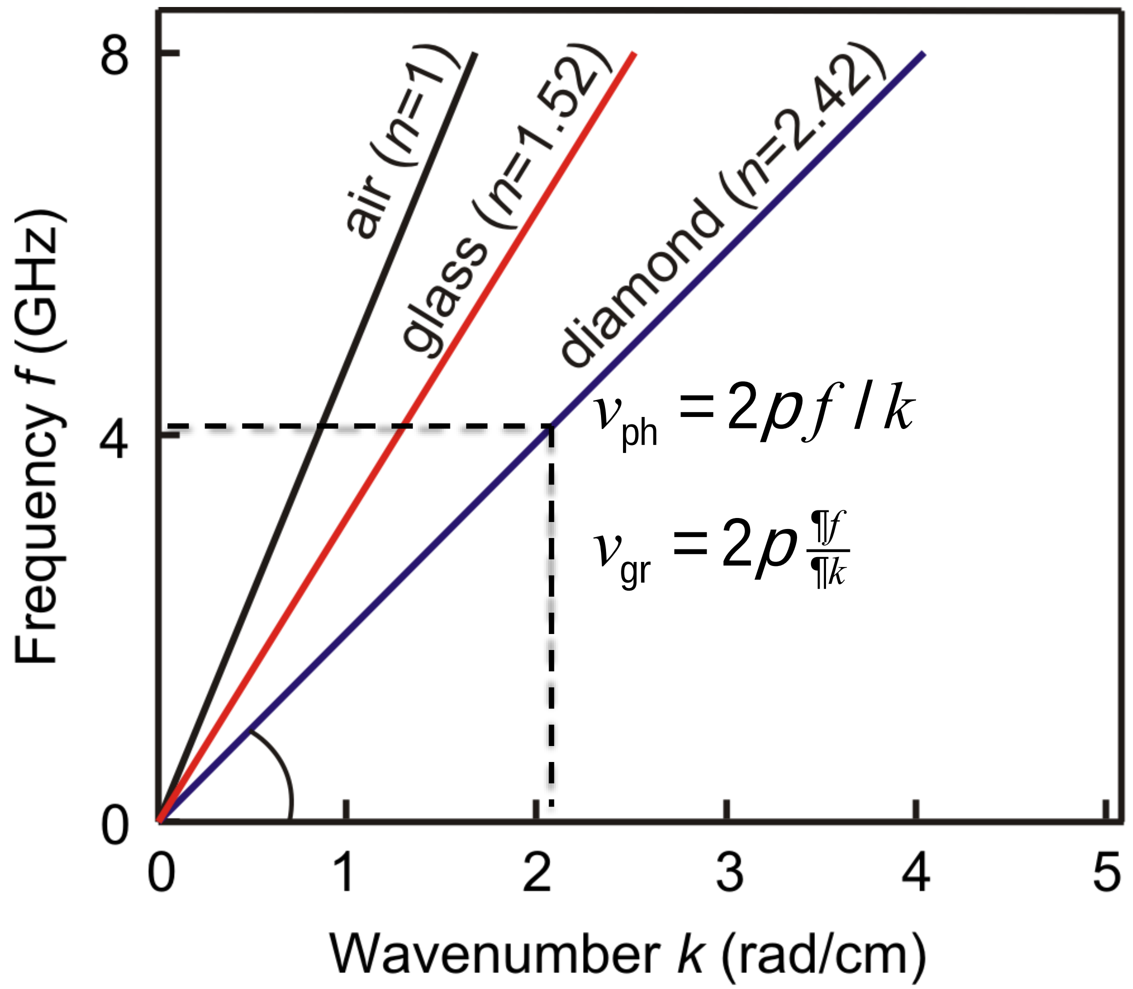
- exchange energy:
  generated by twist of neighbored spins



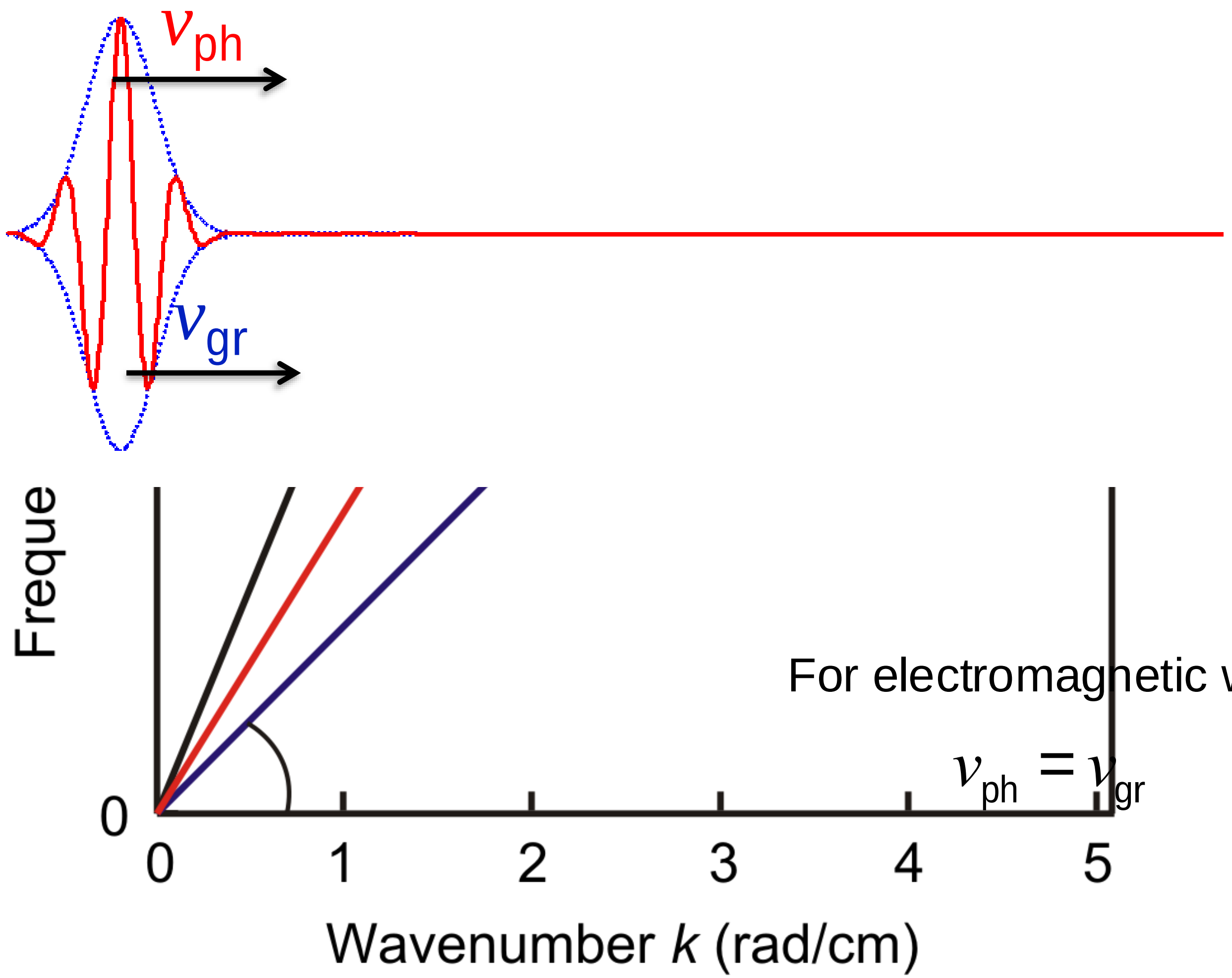
- dipolar energy:
  generated by magnetic poles in long-wavelength spin waves



Dispersion of electromagnetic wave



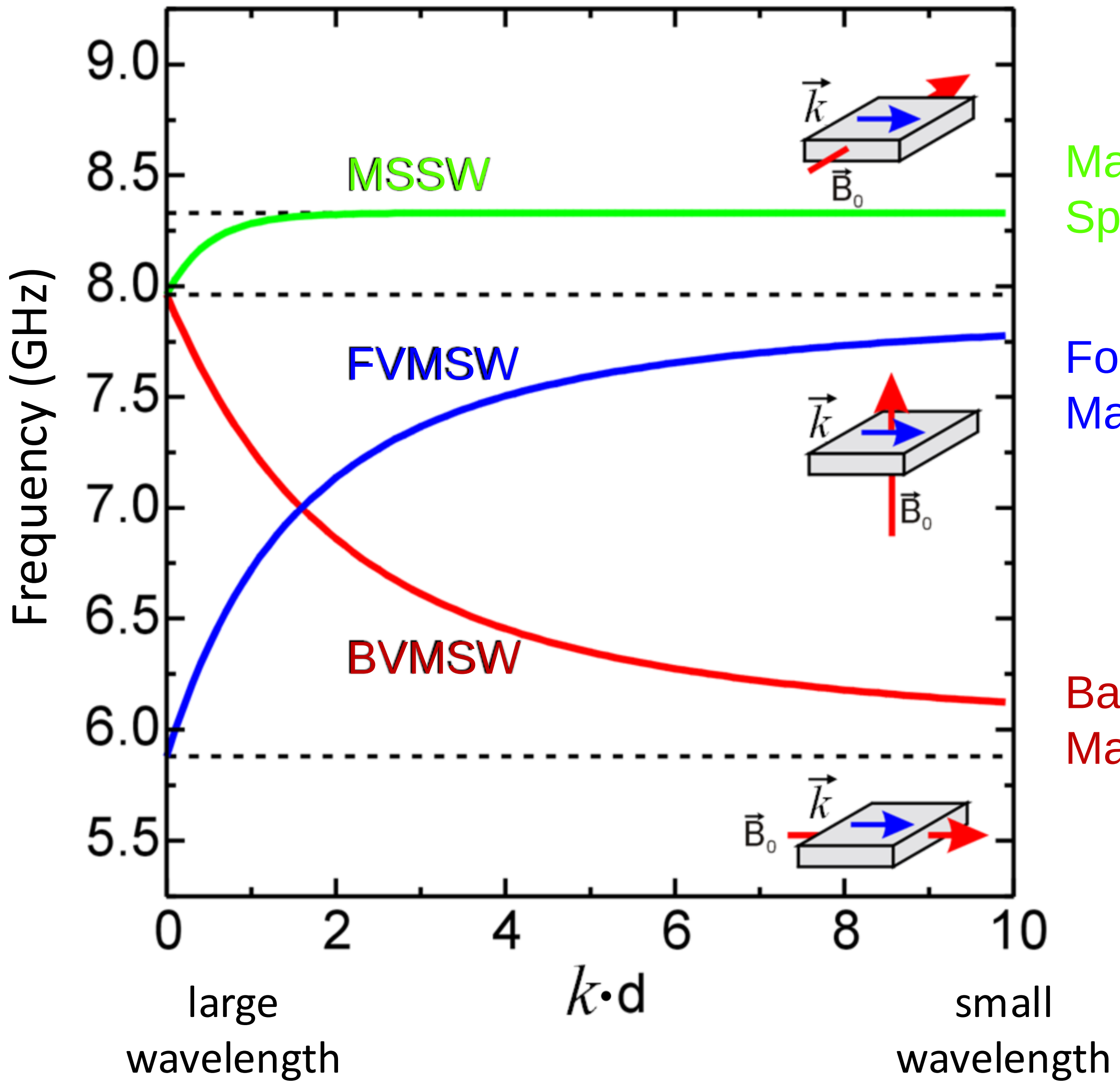
Dispersion of electromagnetic wave



For electromagnetic wave:

$$v_{ph} = v_{gr}$$

Dipolar spin waves

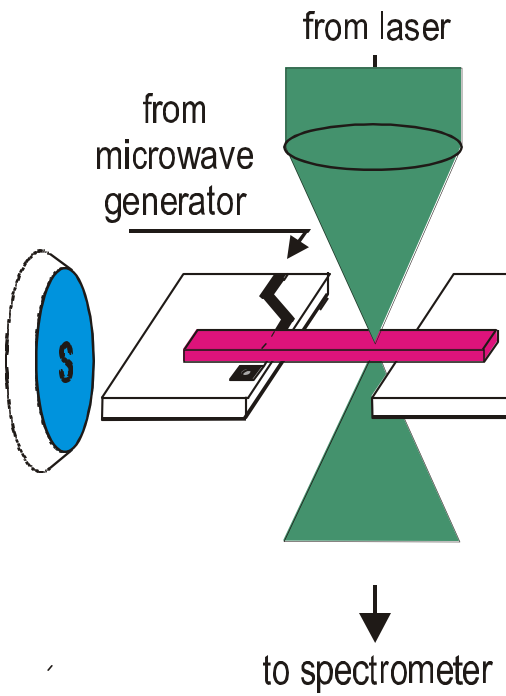


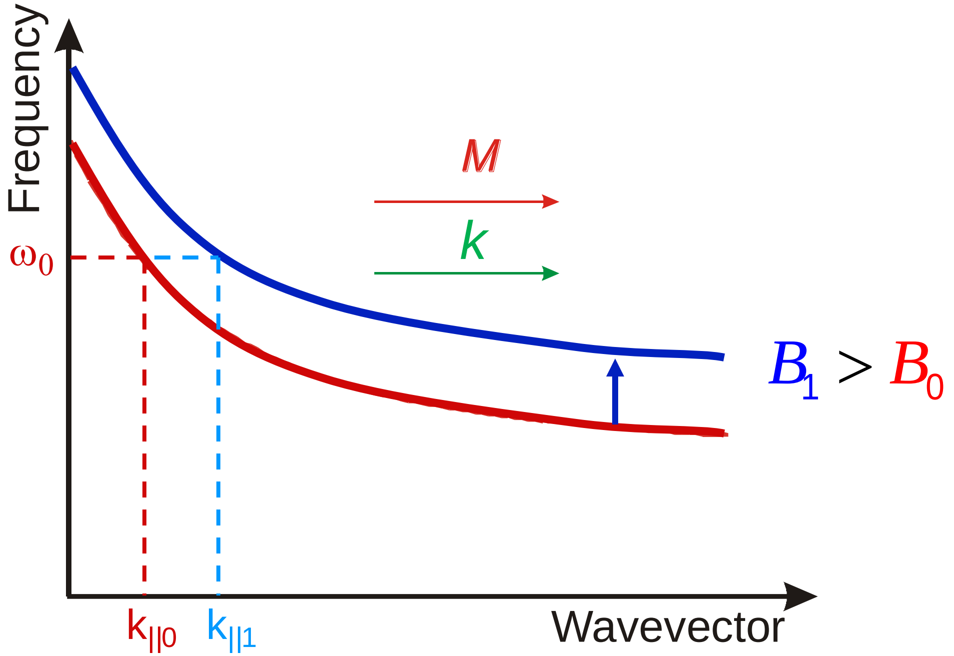
Magnetostatic Surface Spin Waves

Forward Volume Magnetostatic Spin Waves

Backward Volume Magnetostatic Spin Waves

Control of spin wave propagation



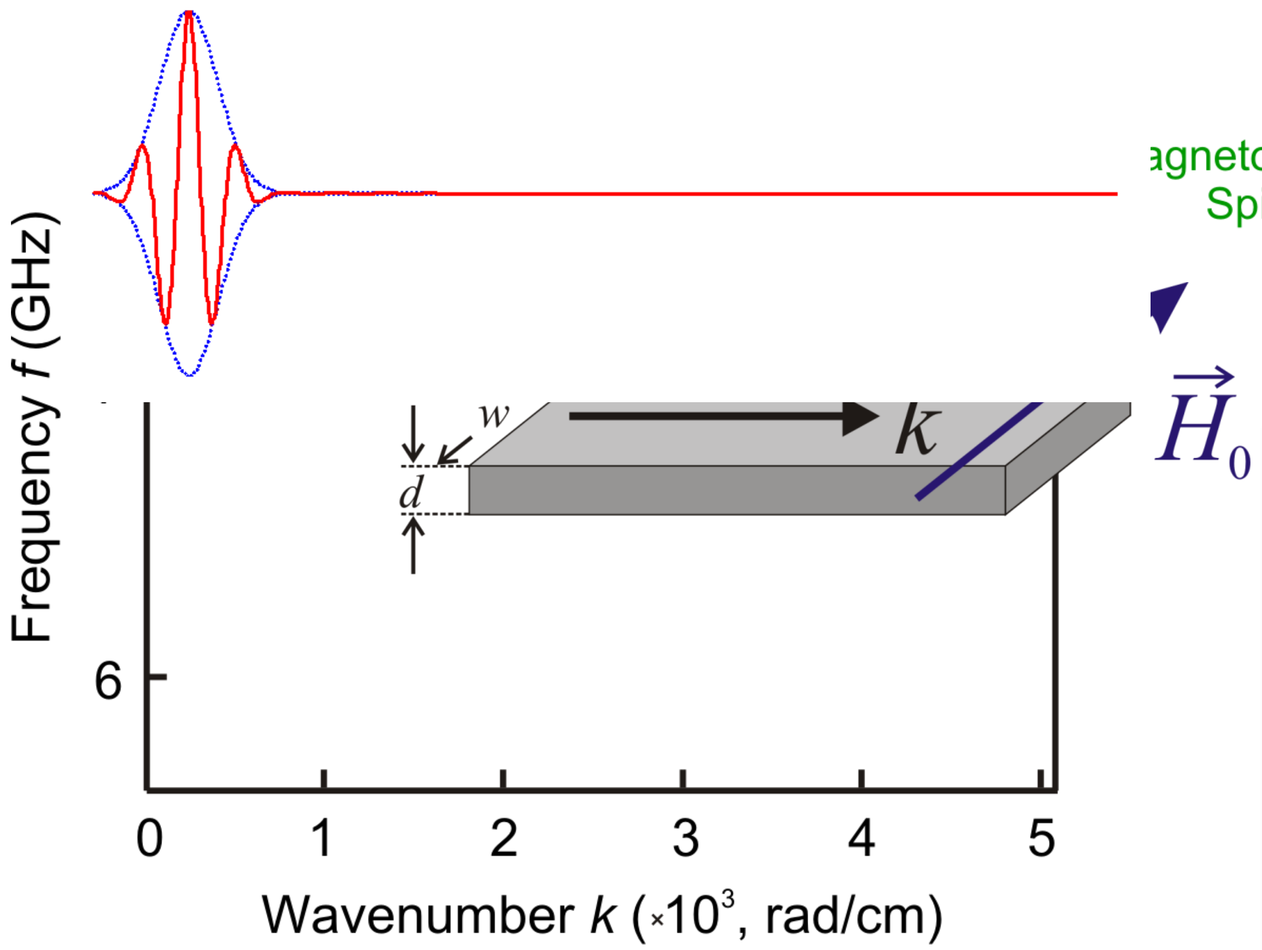


Wavevector **k**:

k_{parallel} defined by input frequency and dispersion

Dispersion shifted vertically by change in magnetic field

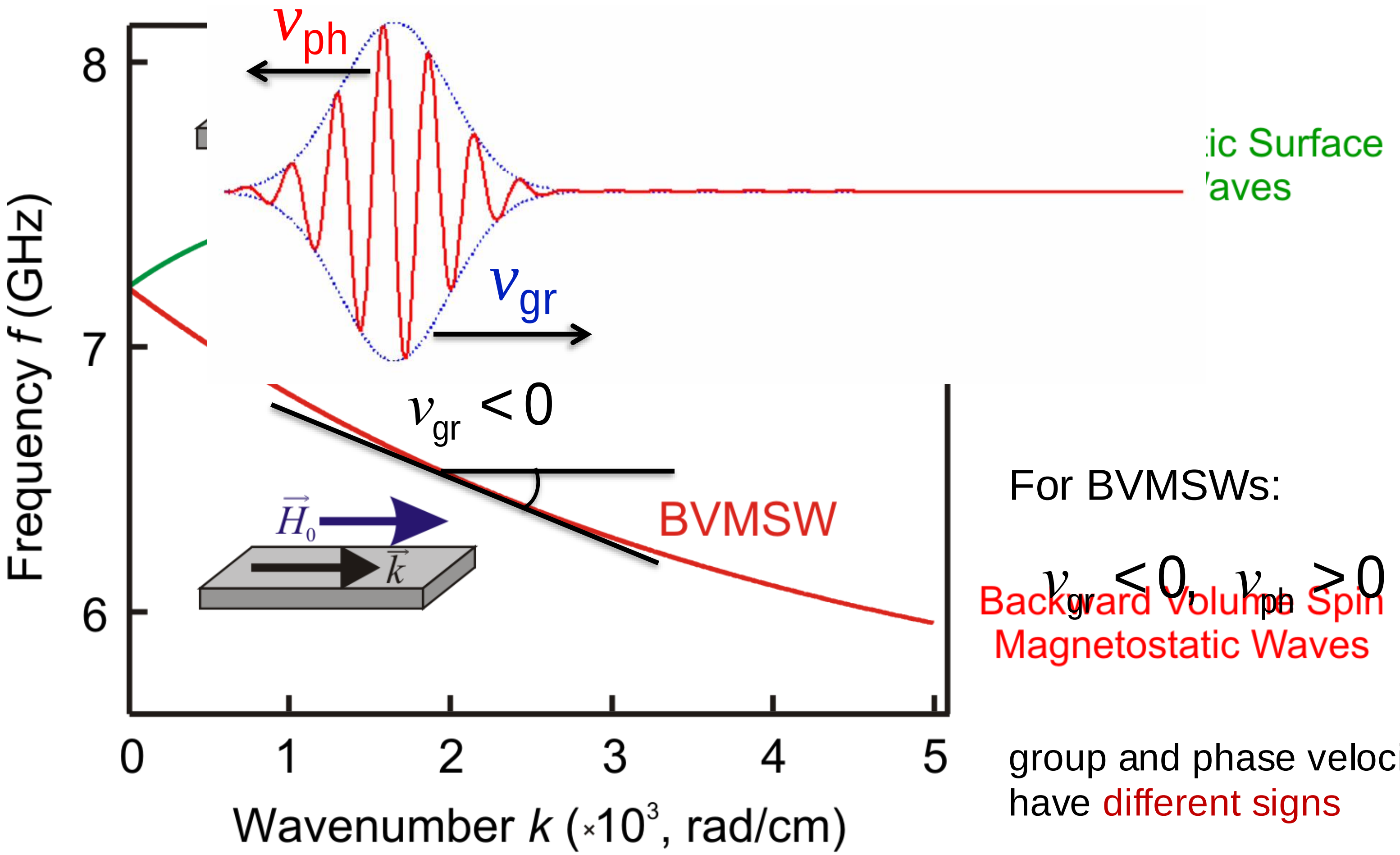
Dispersion curves for spin waves



Spin-wave dispersion characteristics:

- wavelengths are smaller (10^3 times)
- is not linear
- does not go to zero

Dispersion curves for spin waves

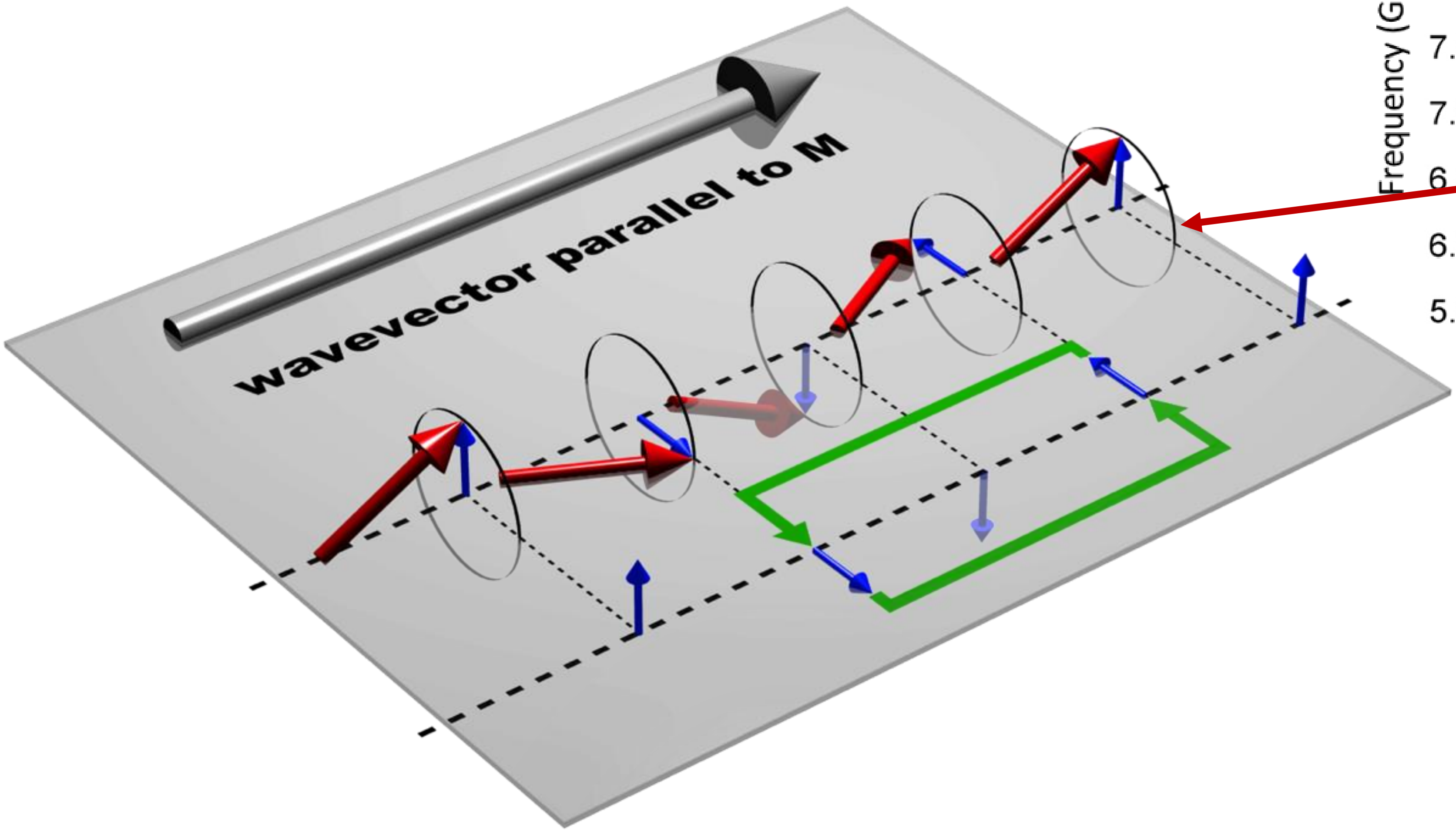


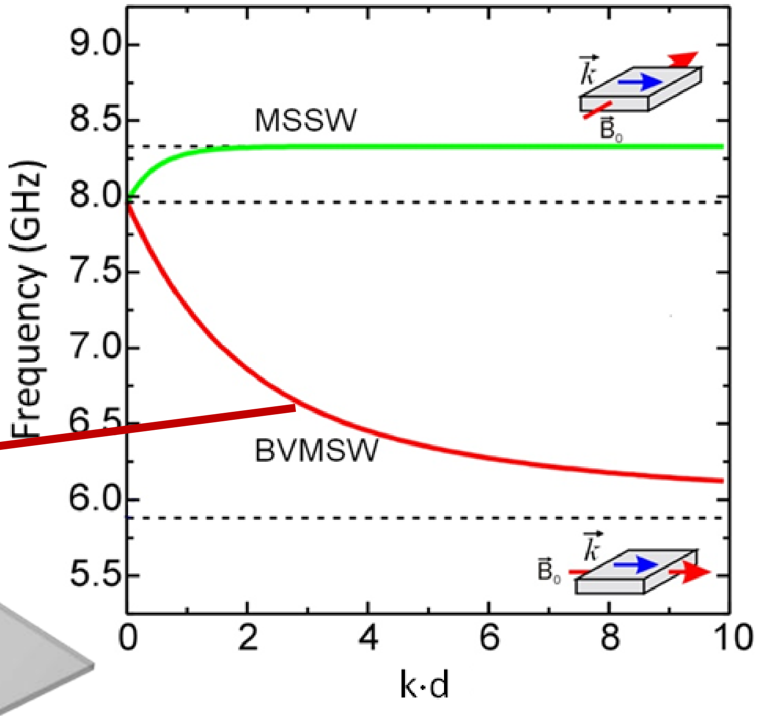
For BVMSWs:

$v_{gr} < 0, \; v_{ph} > 0$

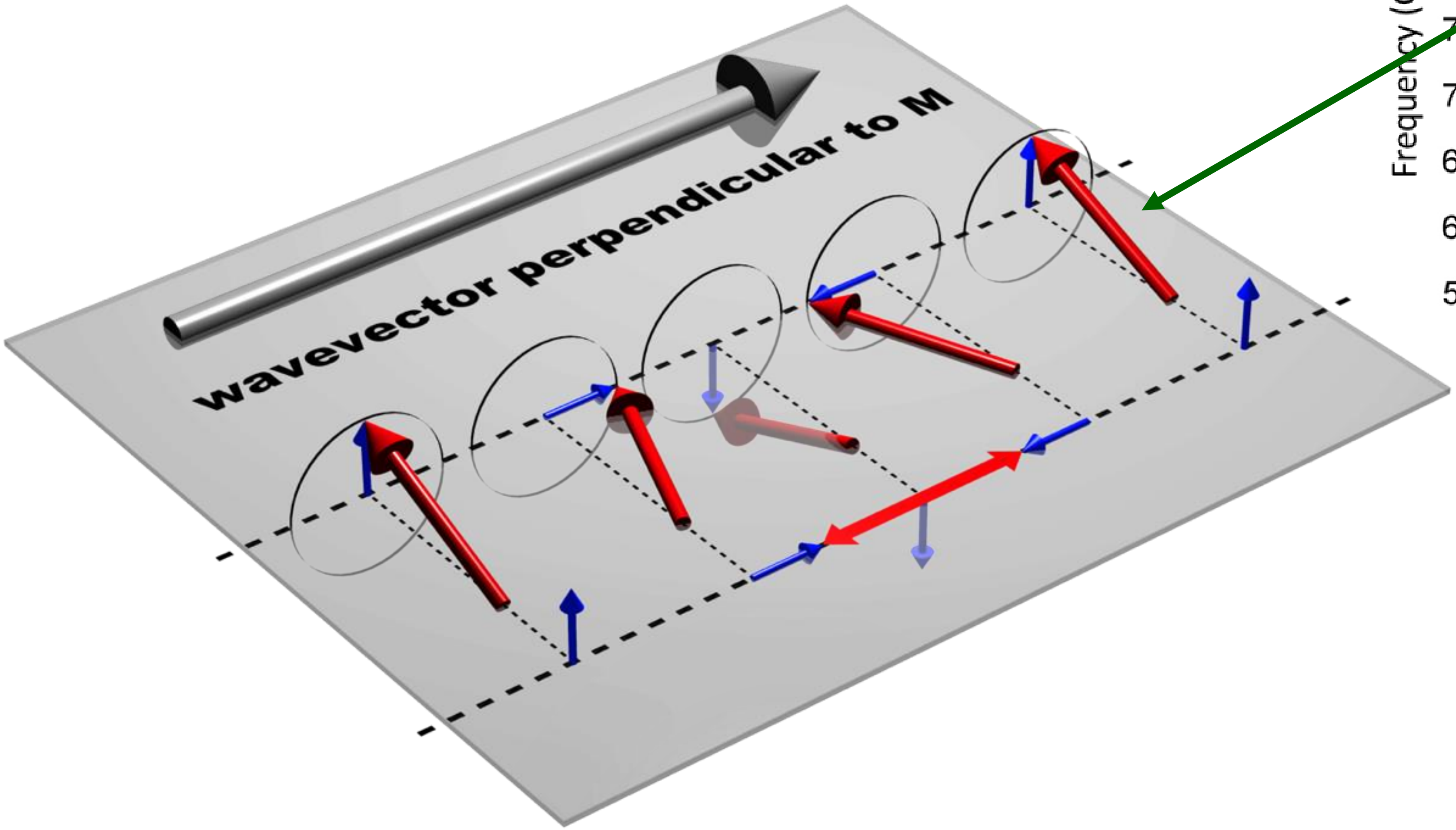
group and phase velocities have different signs

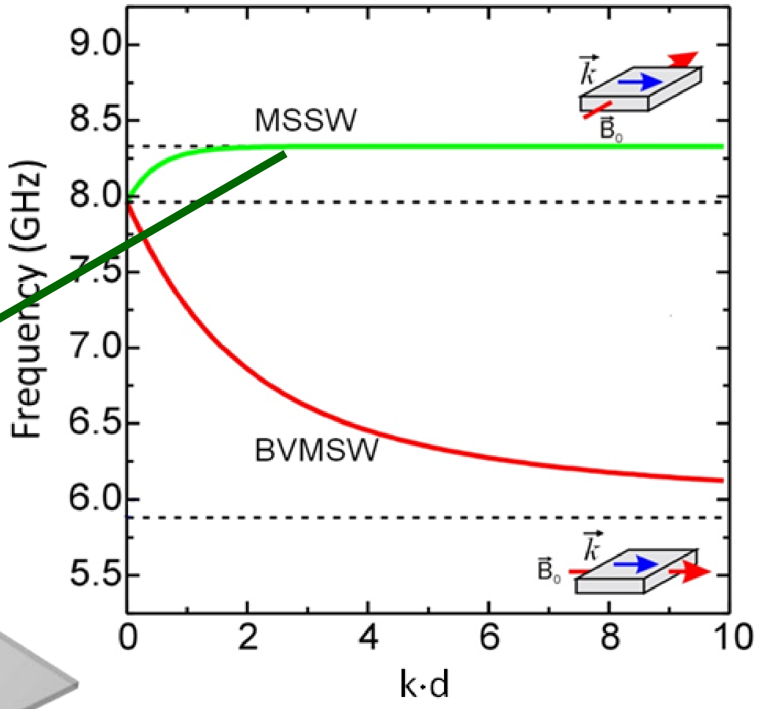
Backward volume magnetostatic spin wave





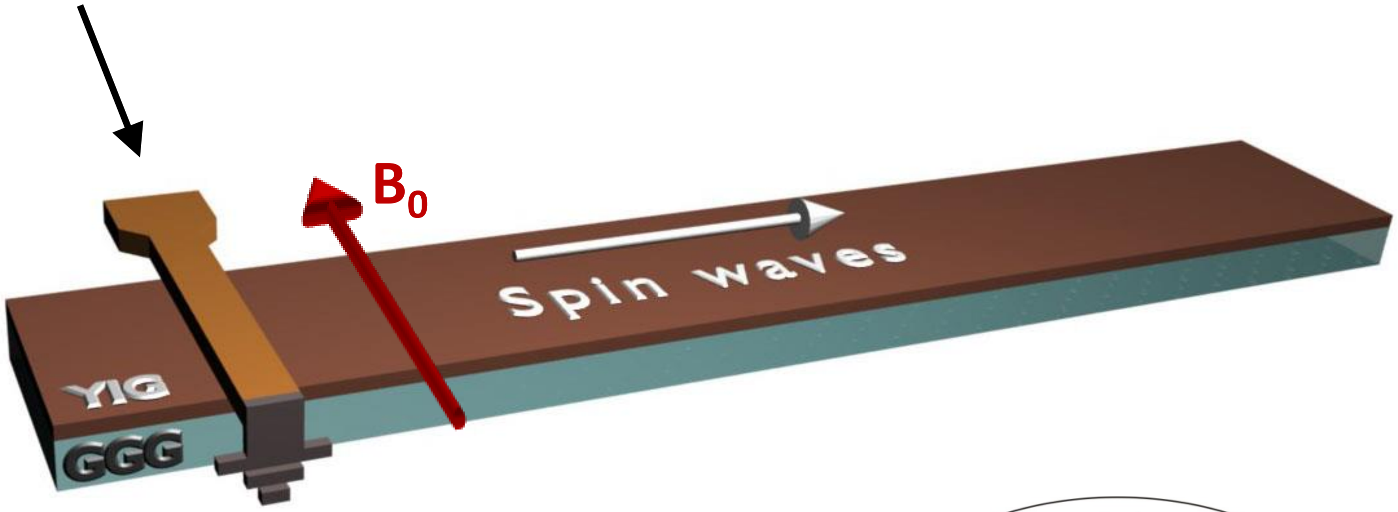
Magnetostatic surface spin wave

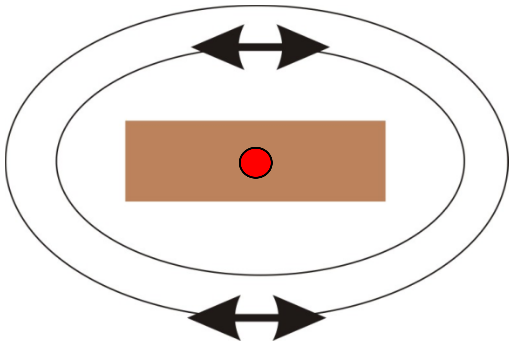




Excitation of dipolar spin waves

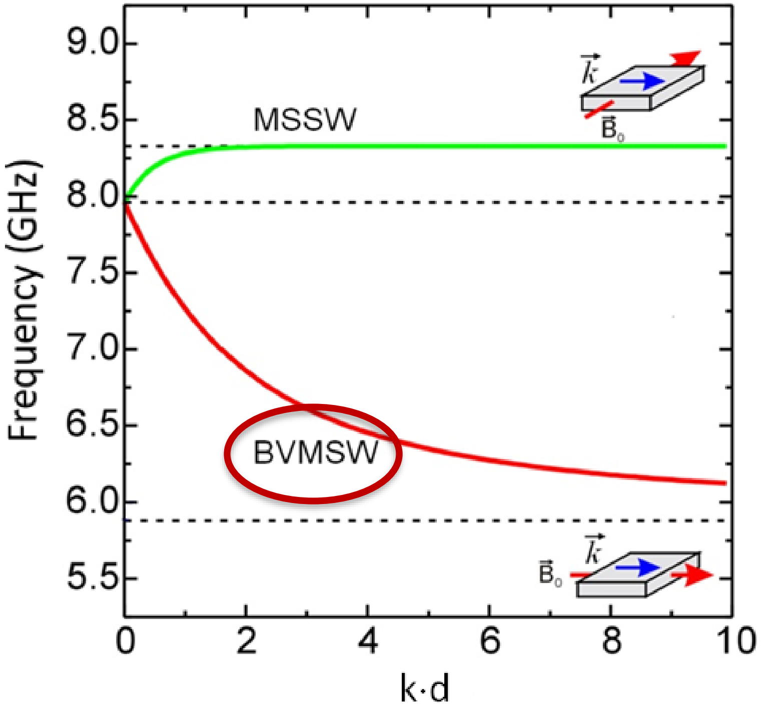
Input microwave signal





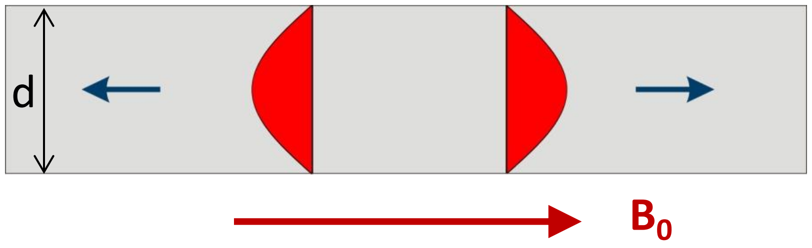
Alternating magnetic field

Backward volume magnetostatic spin waves (BVMSW)

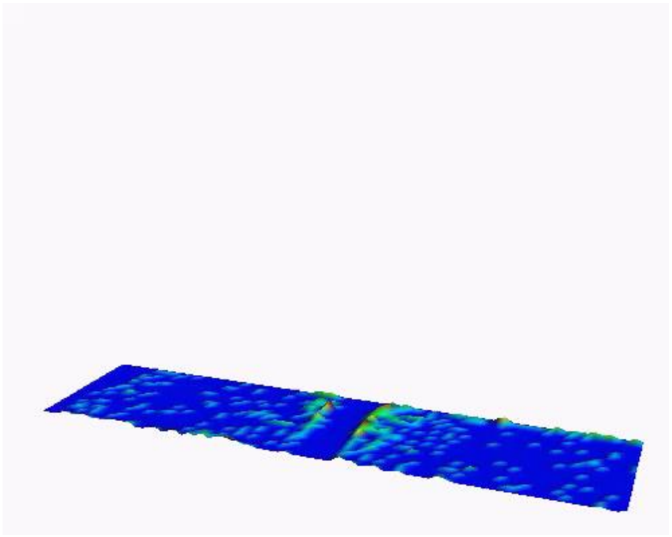


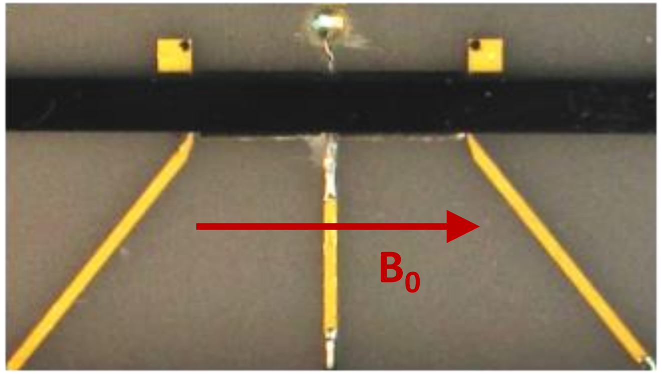
Dynamic magnetization profile

$$m_x \sim \cos(kx)$$

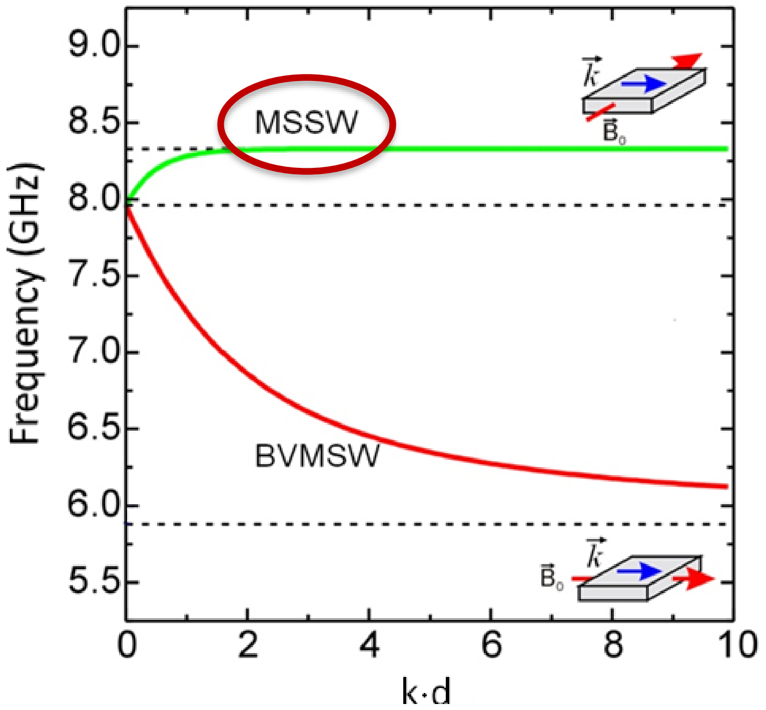


Excitation of BVMSW
measured with
Brillouin light scattering microscopy



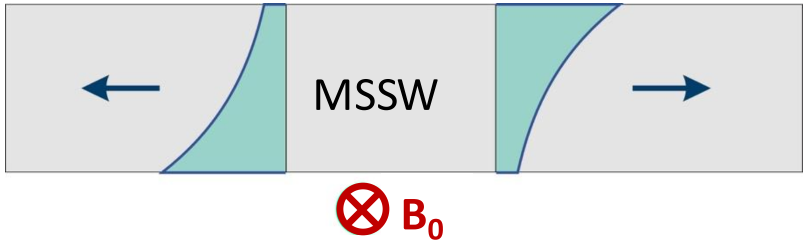


Magnetostatic surface spin waves (MSSW)



Dynamic magnetization profile

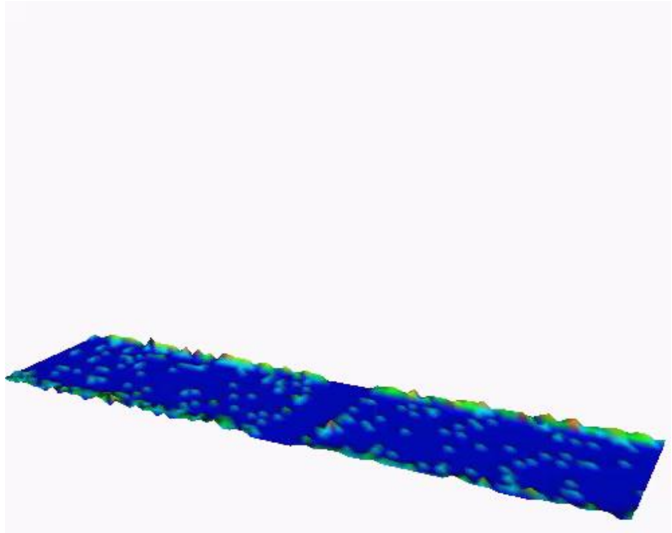
$$m_x \sim \exp(-kx)$$

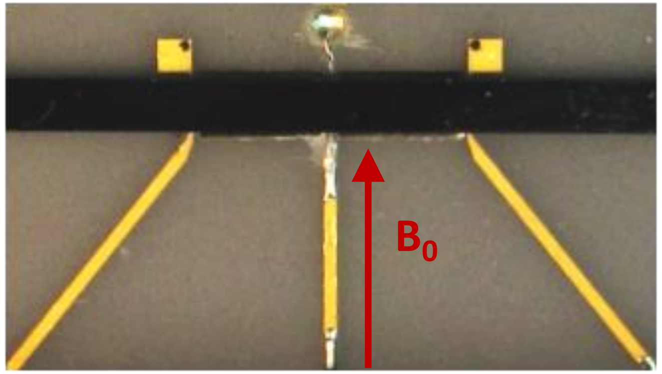


Excitation of MSSW

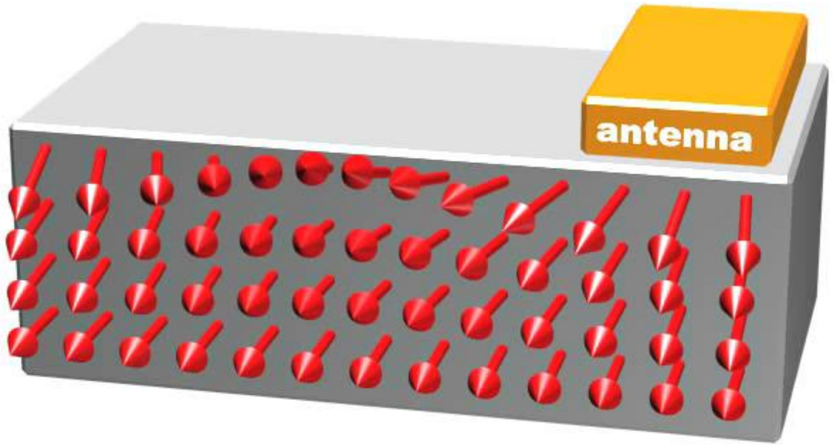
measured with

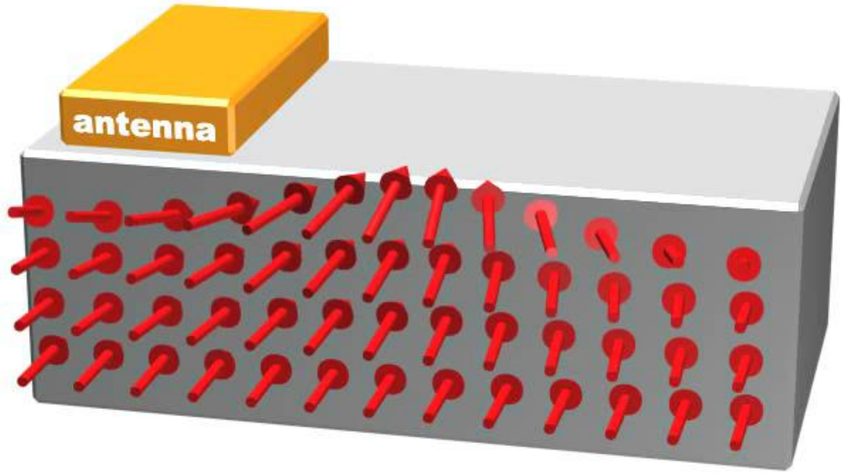
Brillouin light scattering microscopy





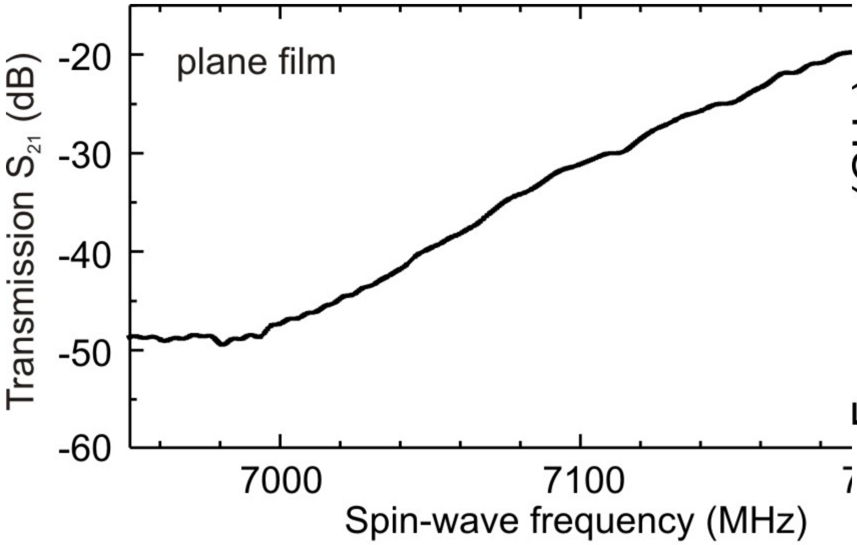
Magnetostatic surface spin wave

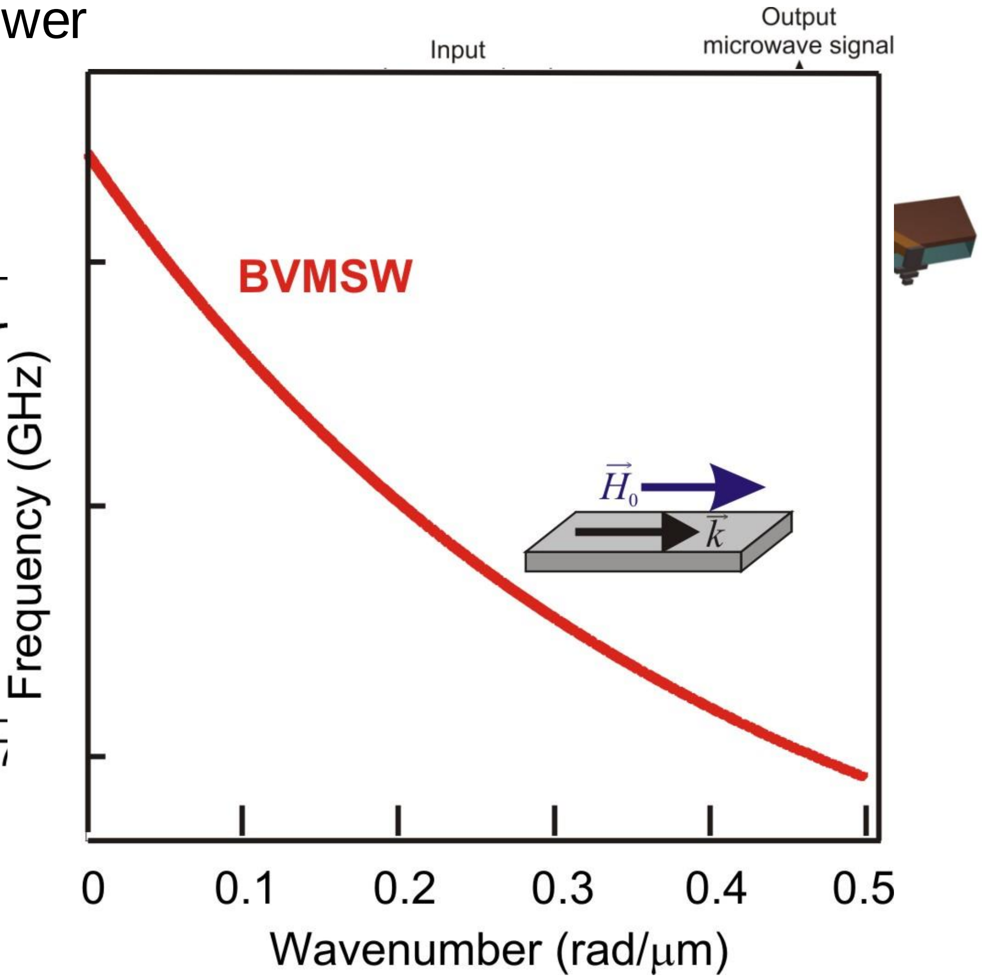




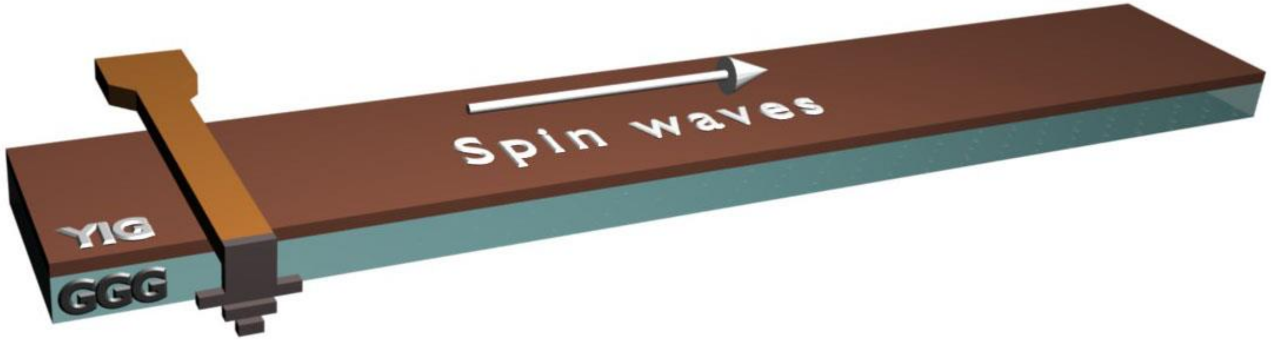
BVMSW transmission characteristics

Dependence of the transmitted power on frequency





Spin-wave waveguide



Frequency: $$f(k) = g\sqrt{H_0 + 4pM_0\frac{1 - \exp\left\{-\sqrt{(np/w)^2 + k^2}\,d\right\}}{\sqrt{(np/w)^2 + k^2}\,d}}$$

H_0 – magnetic field

M_0 – saturation magnetization

d – film thickness

w – waveguide width

n – transverse mode order

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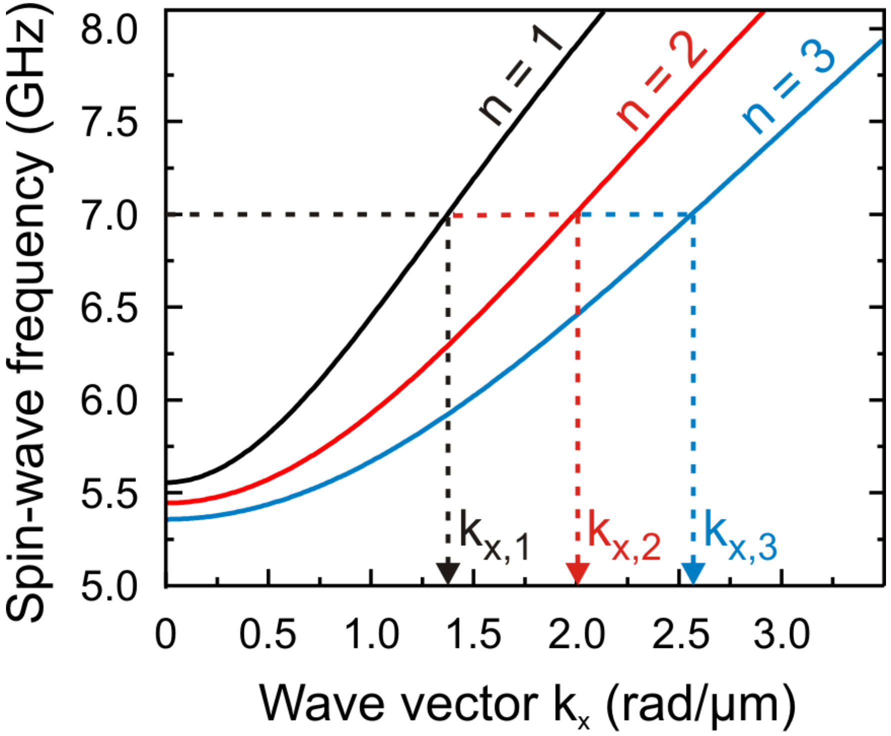
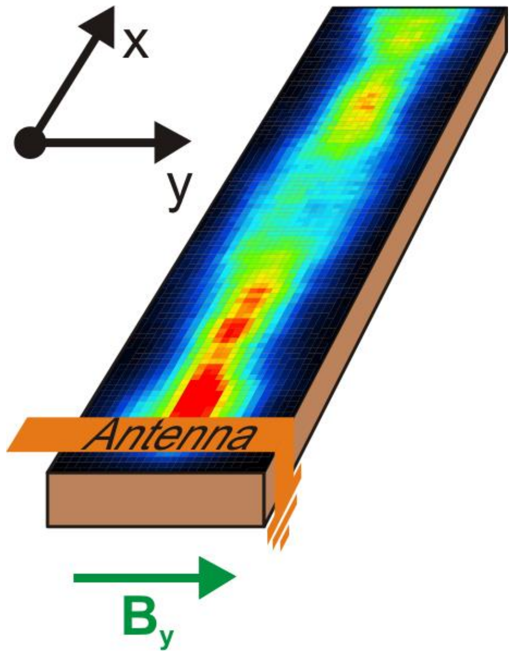
$Ni_{81}Fe_{19}$ waveguide



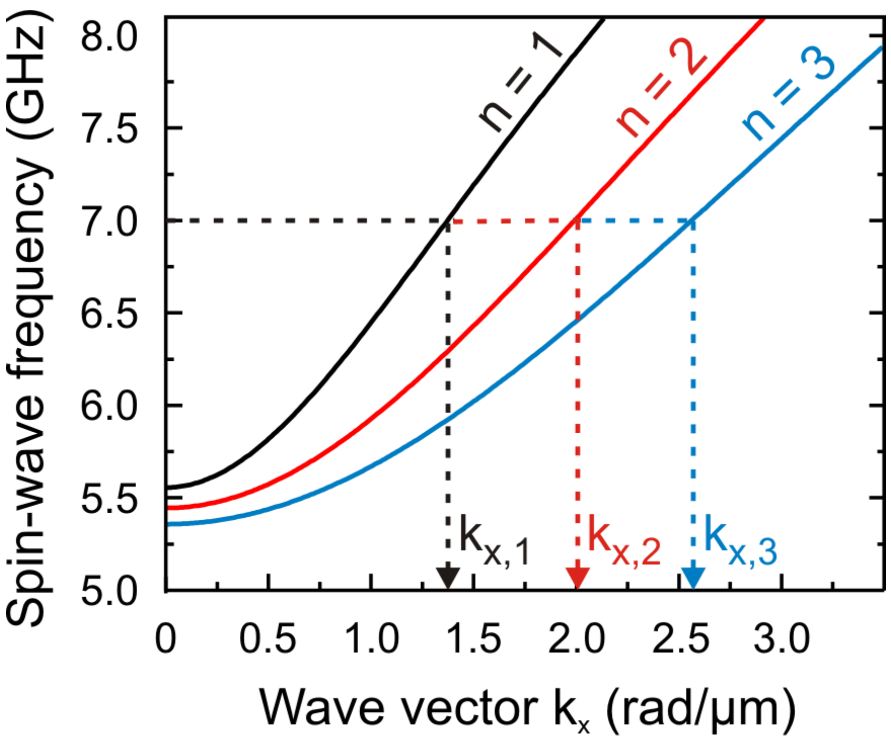


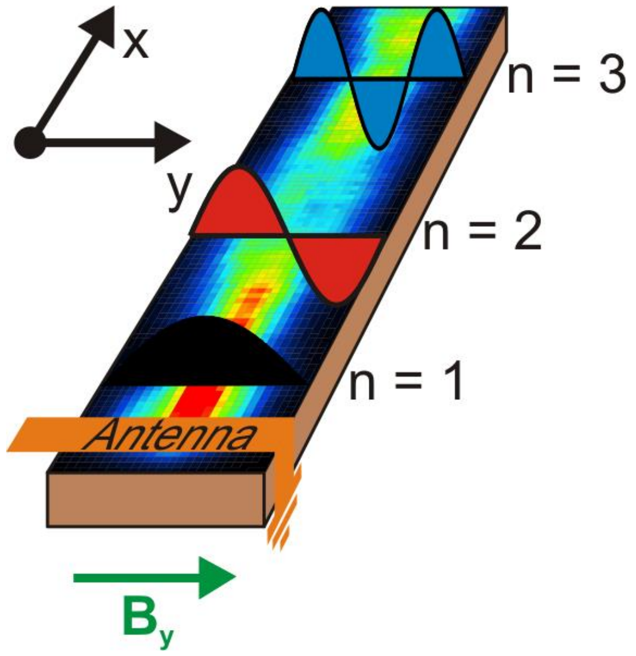
Material: $Ni_{81}Fe_{19}$

k_x: propagating spin wave

k_y: lateral standing spin wave with mode order n

$Ni_{81}Fe_{19}$ waveguide





Material: $Ni_{81}Fe_{19}$

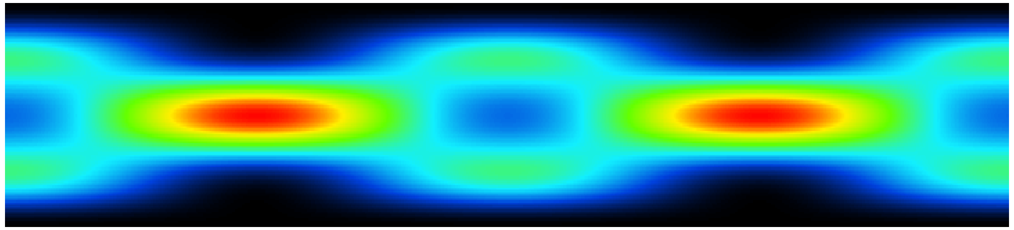
k_x: propagating spin wave

k_y: lateral standing spin wave with mode order n

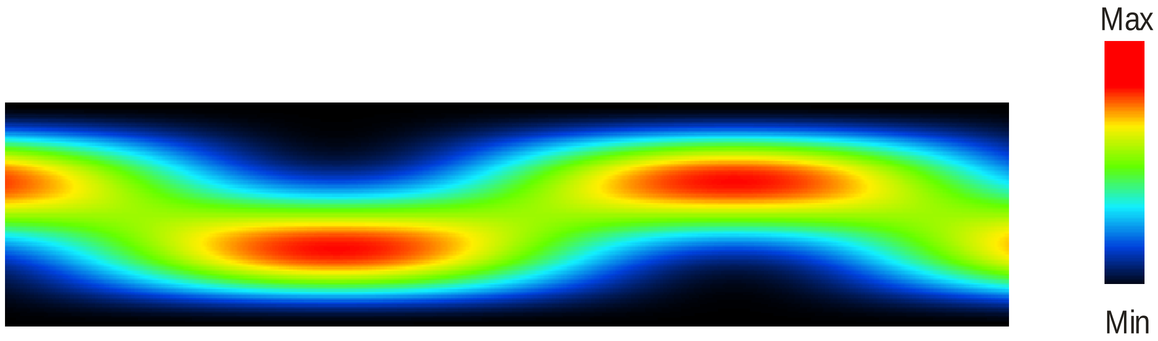
Wave superposition

$$I(x,y)=\left|A_1 e^{-i(k_1 x)}\cos(n_1\frac{p}{w}y)+A_2 e^{-i(k_2 x)}\cos(n_2\frac{p}{w}y)+\ldots\right|^2$$

Modes
n=1 & 3

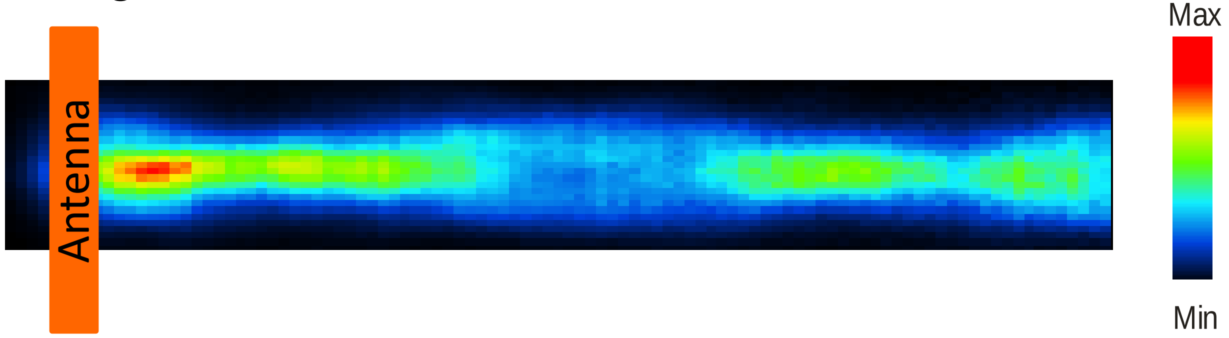


n=1 & 2



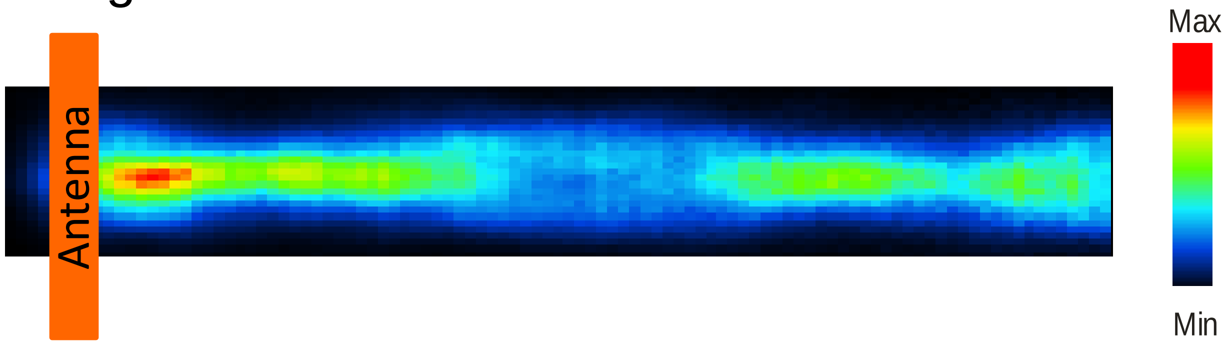
Influence of a skew

Reference waveguide

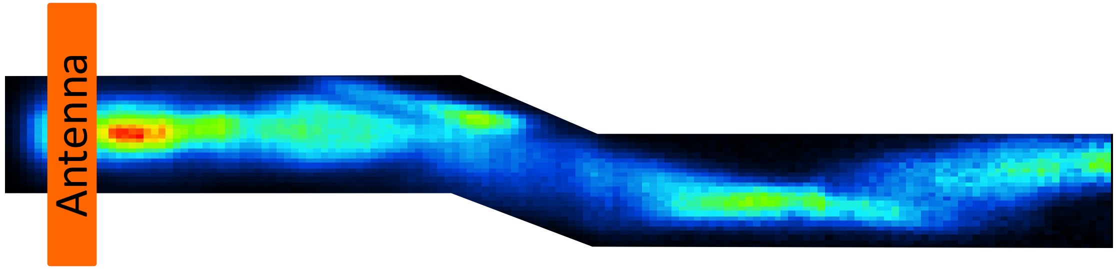


Influence of a skew

Reference waveguide

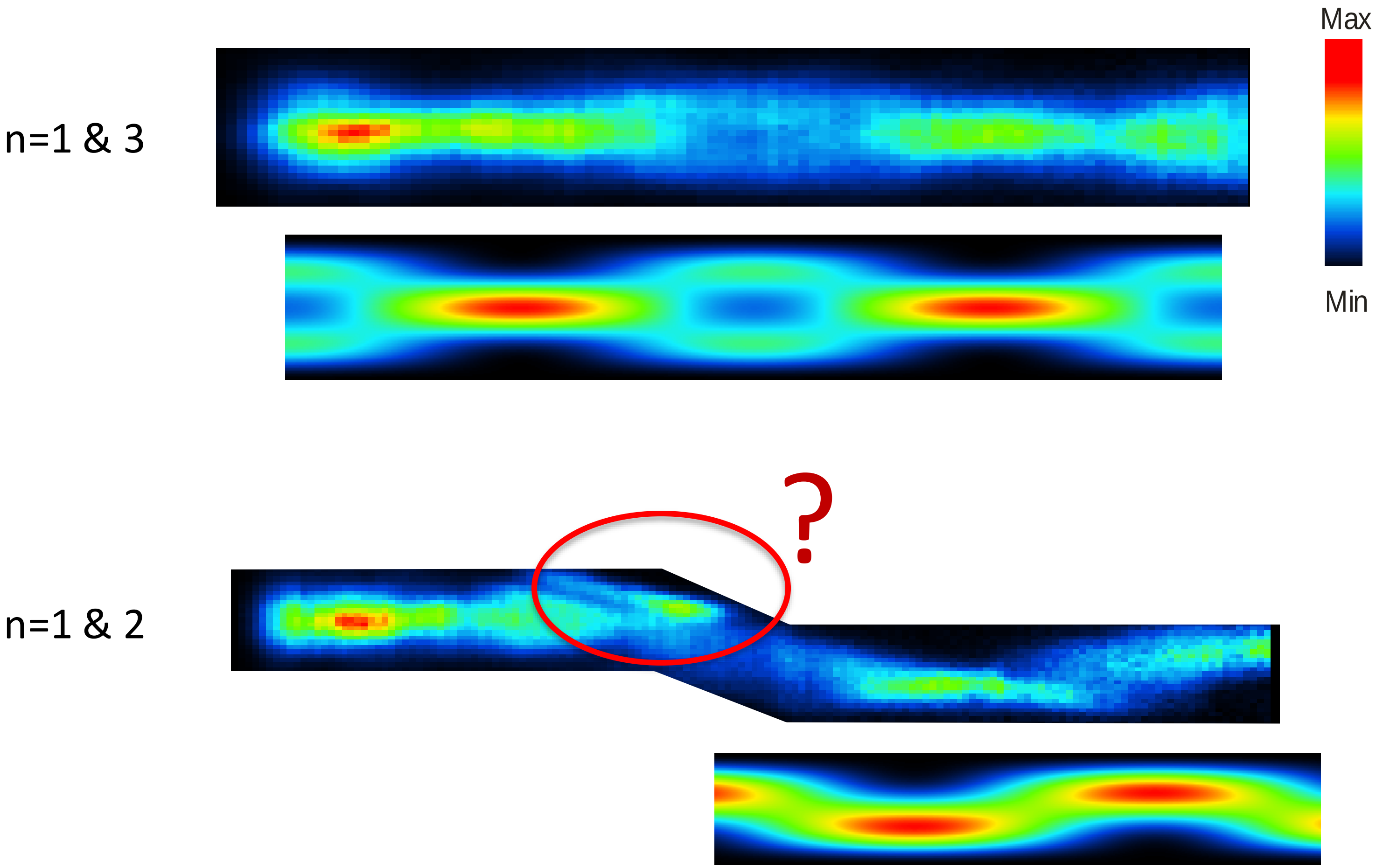


Waveguide with skew (1 µm)



- Changing interference patterns (n=1&3 to n=1&2)
- Edge mode: asymmetric source

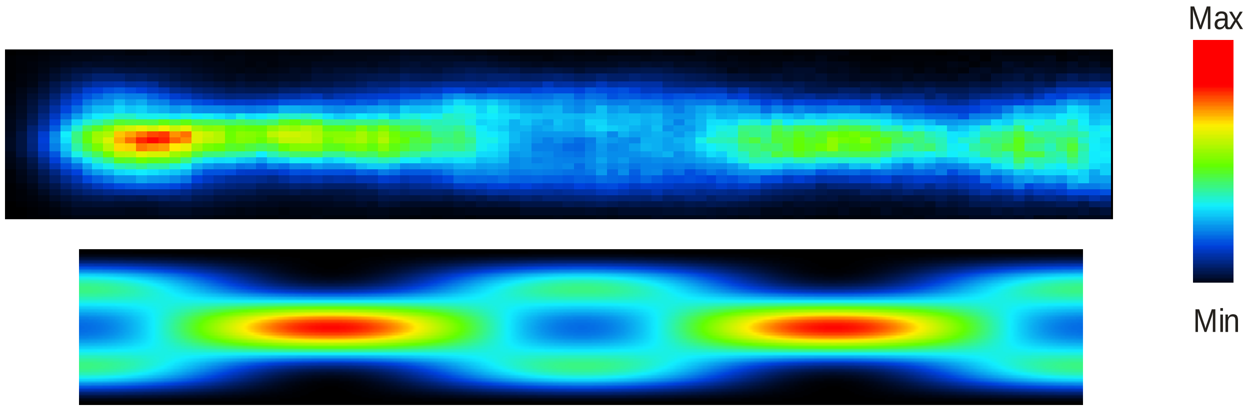
Supporting numerical simulations



Excitation of the second width mode

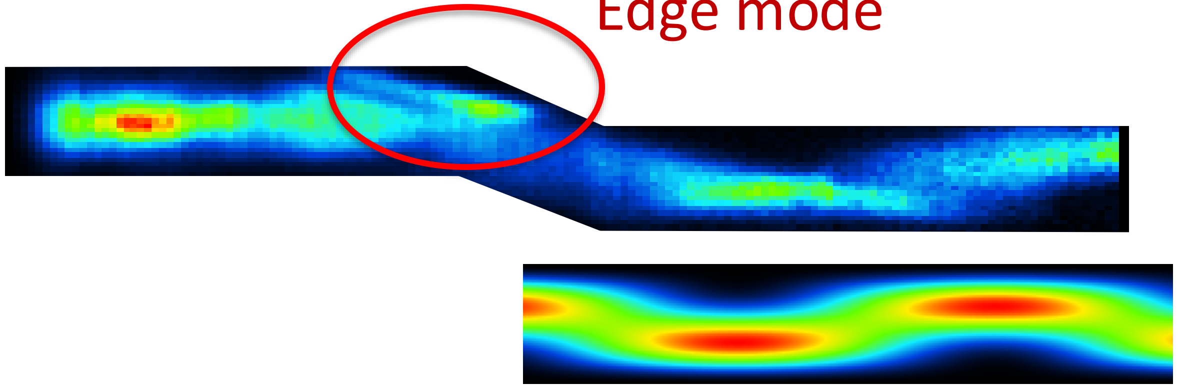
Supporting numerical simulations

n=1 & 3



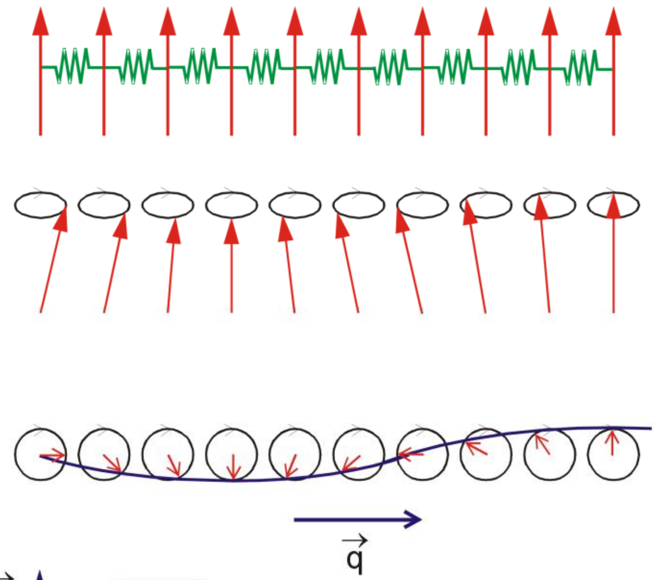
Edge mode

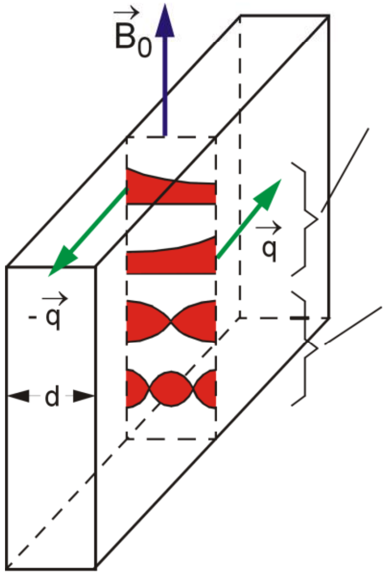
n=1 & 2



Excitation of the second width mode

Spin waves in a thin magnetic film





Dipolar Damon-Eshbach modes:

$$\frac{\omega}{\gamma} = B_0(B_0 + \mu_0 M_S) + \left(\frac{\mu_0 M_S}{2}\right)^2 \left(1 - e^{-2q_{\|}d}\right)$$

Standing spin waves:

$$\frac{\omega}{\gamma} = \frac{2J_{ex}}{M_S} \cdot q^2 = \frac{2J_{ex}}{M_S} \cdot \left(\frac{n\pi}{d}\right)^2 \quad , n = \pm 1, \pm 2, \pm 3, \ldots$$

J_{ex} : exchange constant

Exchange modes

Small wavelength: exchange interaction

$$\vec{B}_{\text{exch}} = \frac{2J_{\text{ex}}}{M_s^2}\vec{\nabla}^2\vec{M} = \frac{D}{M_s}\vec{\nabla}^2\vec{M}$$

must be considered.

Resonance condition for wavevector component perpendicular to film:

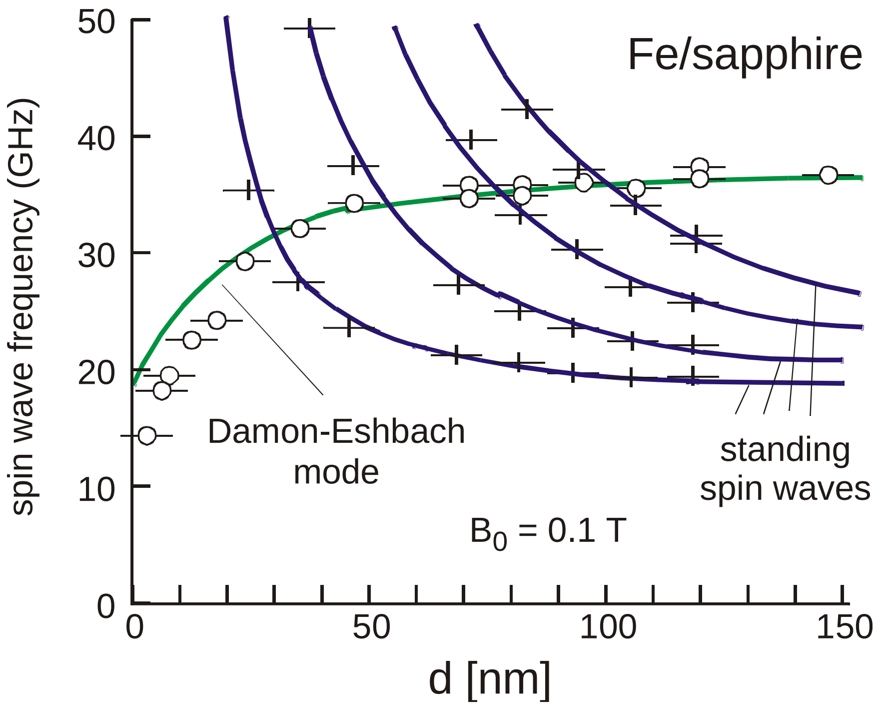
$$q_\perp = n\frac{\pi}{d}; \quad n = \pm 1, \pm 2, \pm 3\,\ldots$$

approximative calculation of exchange modes:

(outside crossing regimes with Damon-Eshbach modes)

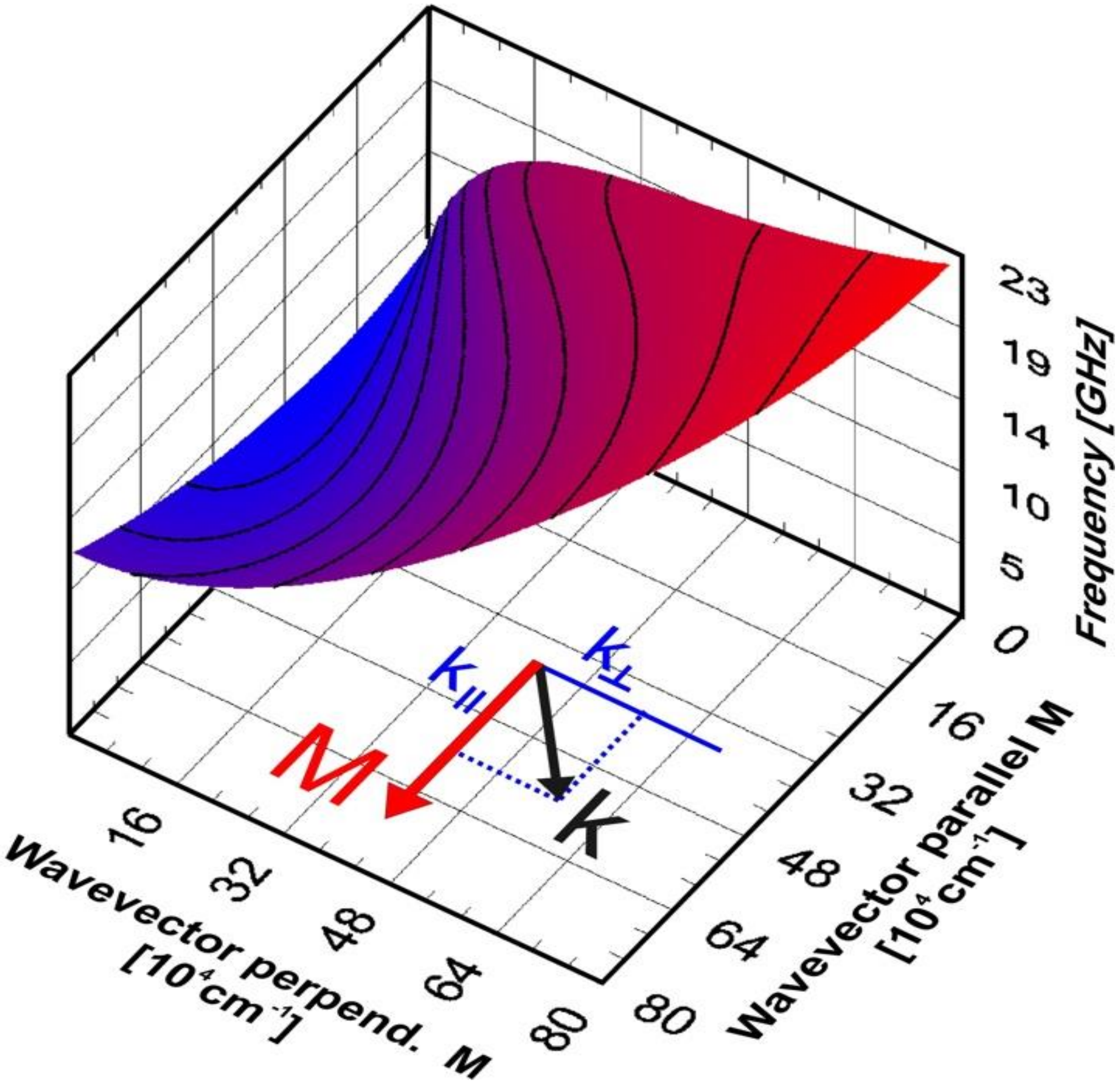
$$\left(\frac{\omega}{\gamma}\right)^2 = \left(B_0 + Dq^2\right)\left(B_0 + \mu_0 M_s + Dq^2\right)$$

with: $D = 2J_{\text{ex}}/M_s$



P. Grünberg et al., JMMM **28**, 319 (1982)

Propagation at oblique in-plane angle



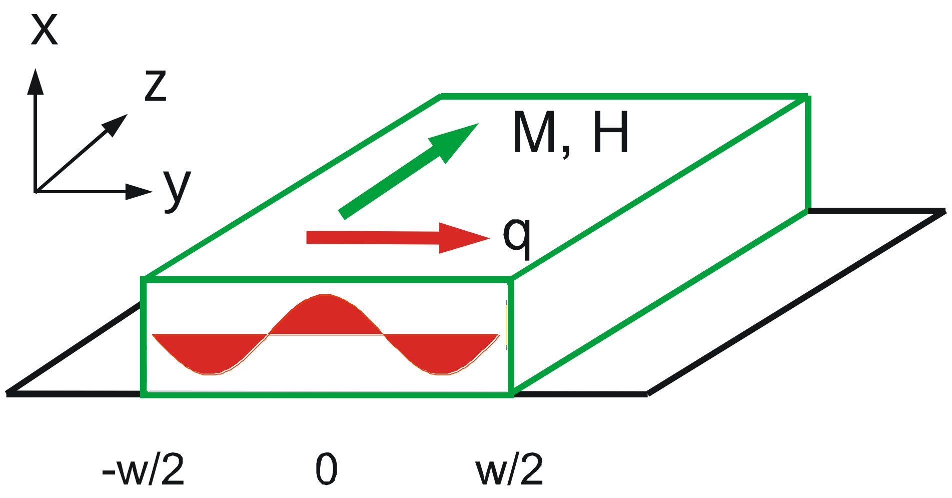
dipole-dipole interaction
and
exchange interaction

Permalloy film (15nm)
H_{ext} = 500 Oe

Magnetization dynamics

Confinement to magnetic objects:

quantized eigen modes („standing spin waves“)



➔ Find dynamic ground state, i.e., eigenmode spectrum

Problems:

- correct boundary conditions
- modes in inhomogeneously magnetized structures

Patterned magnetic films

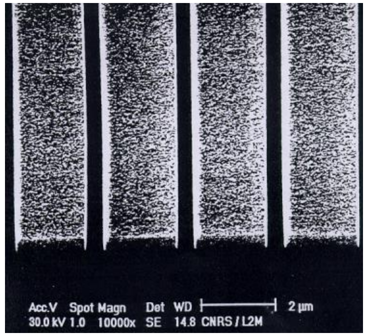
Au / $Ni_{81}Fe_{19}$ (220nm) / SiO_2 / Si

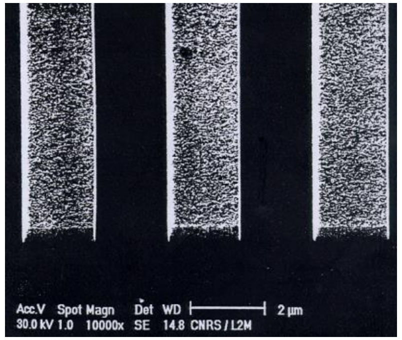
preparation: e-beam evaporation in UHV

coercivity: H_c = 1-2 Oe

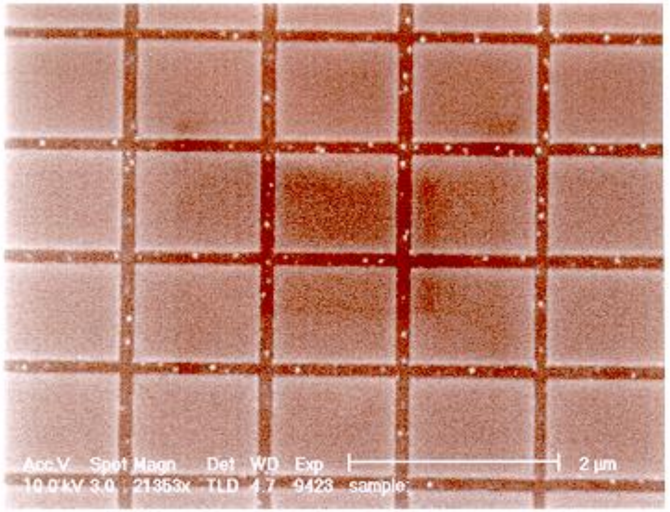
patterning: x-ray lithography (LURE, France)

Wires:



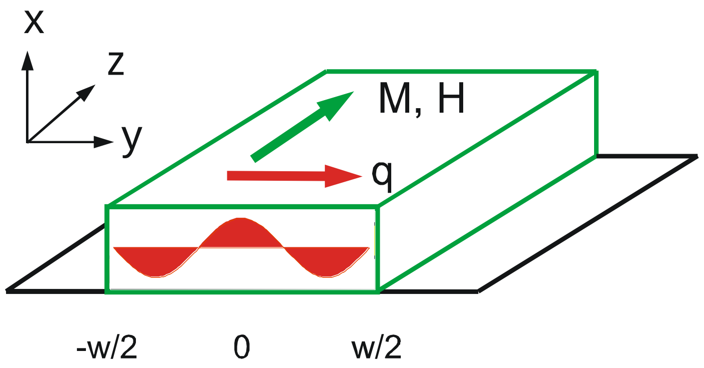


Dots:



Lateral quantum size effect

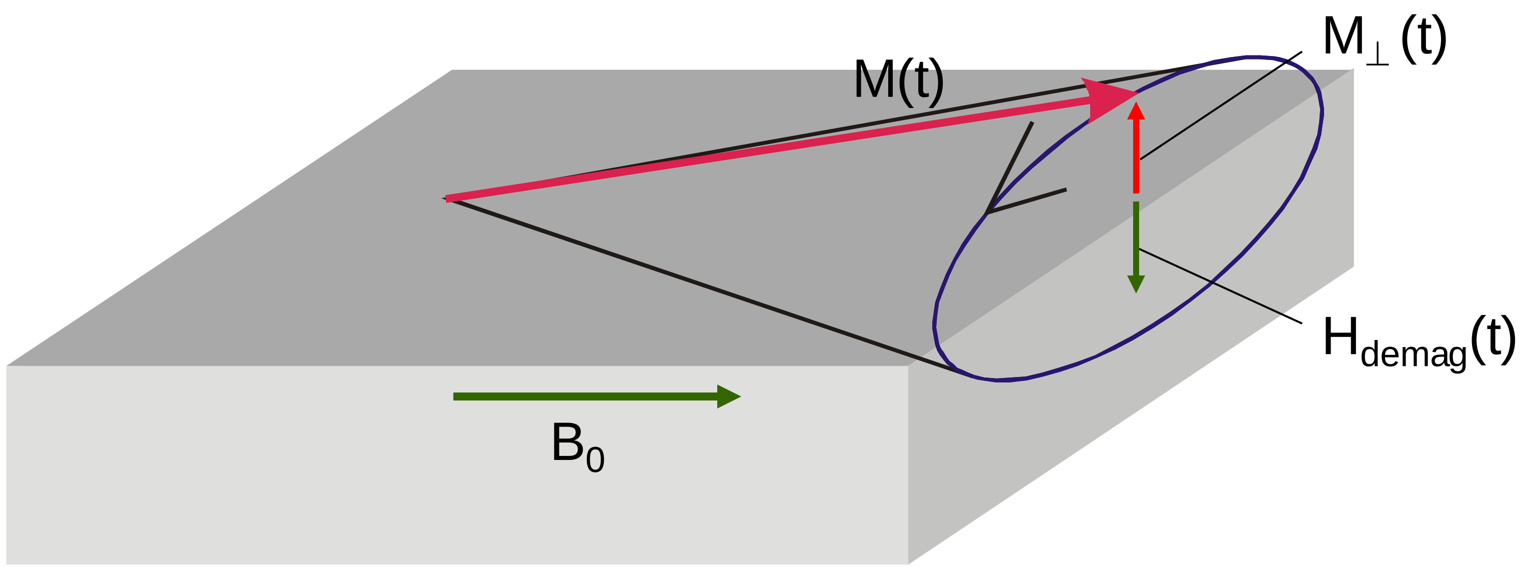
Standing lateral modes:



Standing lateral modes

- propagating dipolar modes (Damon-Eshbach modes) perpendicular to wires: "standing lateral modes”
- quantization condition due to the lateral edges:
  $w = n\,\lambda_{\text{spin wave}}/2$;
  $q_n = 2\pi/\lambda_{\text{spin wave}} = n\pi/w$; $n = 1,2,3,...$
- boundary conditions (open – pinned)
  - take dynamic stray fields into account
- calculation of frequencies by inserting q_n into Damon-Eshbach equation of motion

Boundary conditions for dynamic magnetization



Precessing magnetization has dynamic out-of-plane component
$\Rightarrow$ dynamic stray fields and thus dynamic surface torque on magnetization

Mode profiles

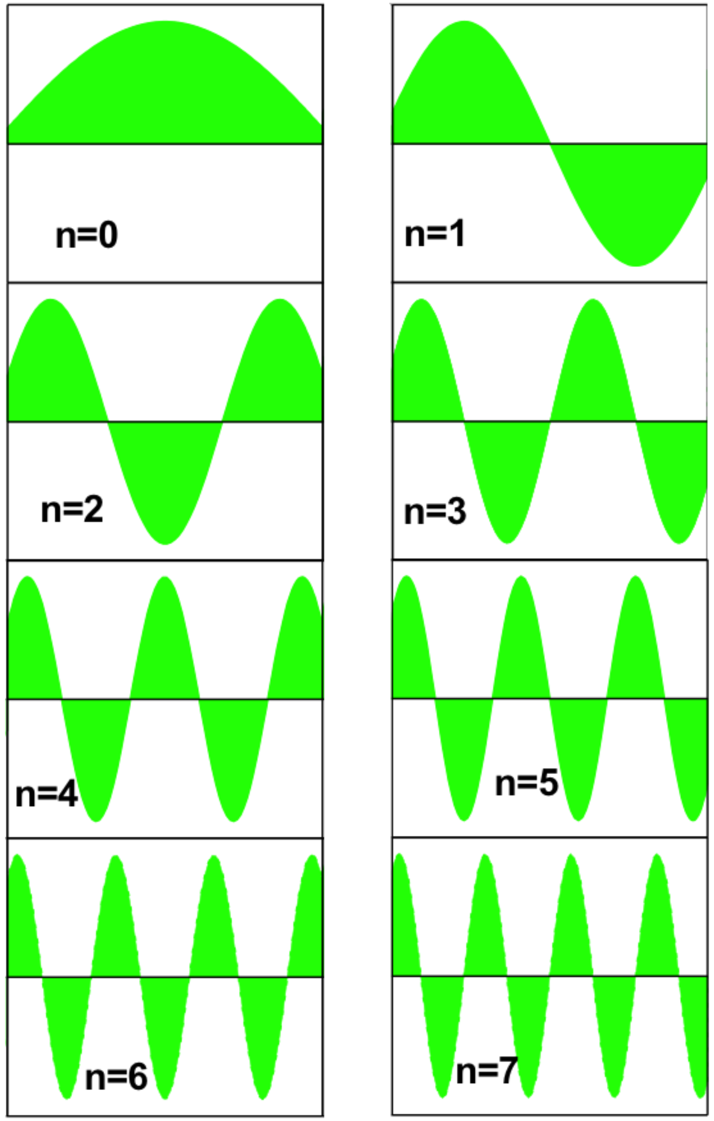
New dynamic dipole boundary condition for non-elliptical elements:

$$\frac{\partial \vec{m}}{\partial \hat{n}} + \frac{1}{\xi_D}\vec{m} = 0$$

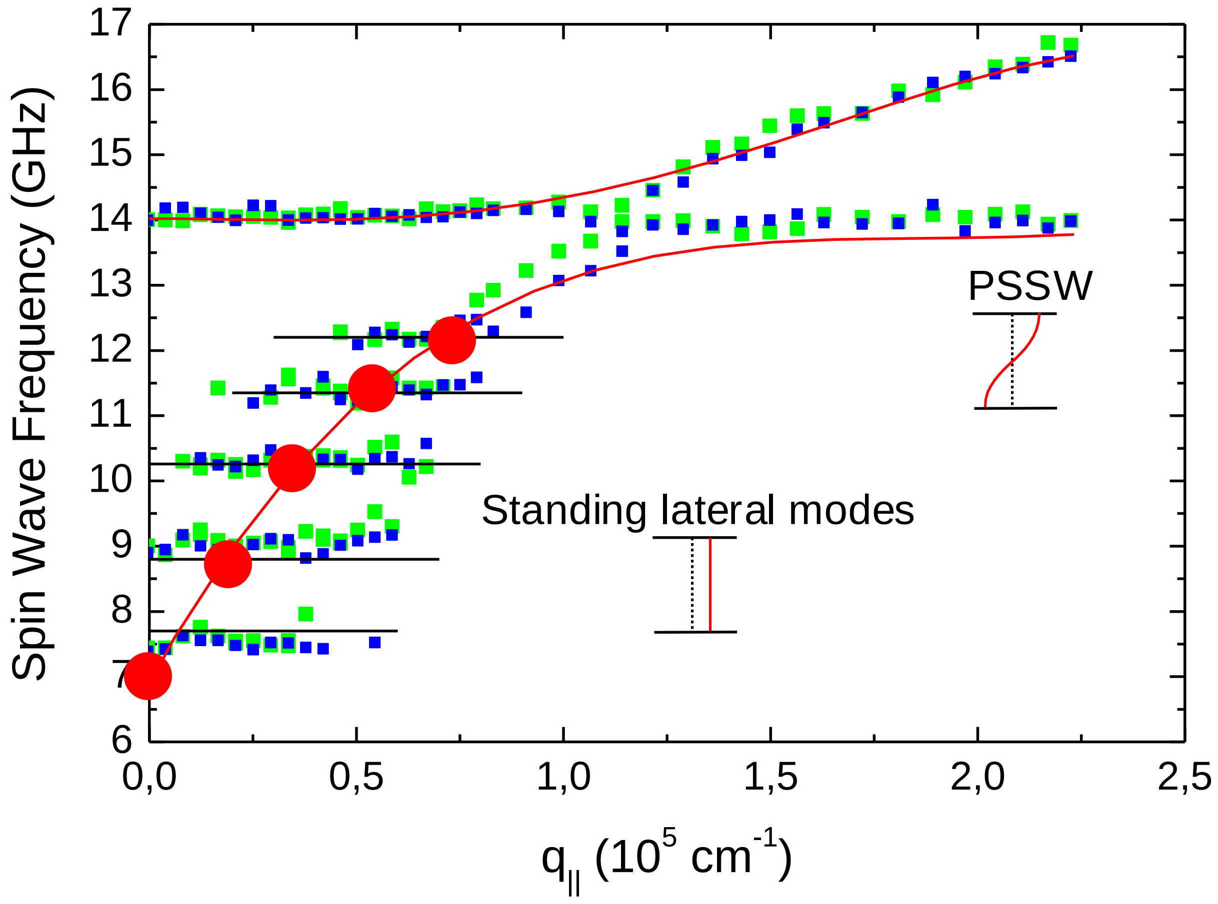
$$\xi_D = \frac{t}{2\pi}\left(1 + 2\ln\frac{w}{t}\right)$$

- Takes dynamic stray fields into account
- „Stray field induced pinning“

low-index modes ($\lambda >> \xi_D$)	„pinned“
high-index modes ($\lambda \approx \xi_D$)	„unpinned“



Frequencies of the quantized modes



width:
1.8 µm
separation:
0.7 µm (green)
2.2 µm (blue)
thickness: 50 nm

good quantitative agreement between the theory and the experiment is obtained

C. Mathieu et al., PRL **81,** 3968 (1998)

Magnon spectrum of in-plane magnetized YIG film

Landau-Lifshitz equation

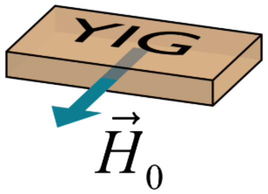
$$\frac{\partial \vec{M}}{\partial t} = -|\gamma| \mu_0 \vec{M} \times \vec{H}_{eff}$$

$$\vec{H}_{eff}(\vec{r}) = \vec{H}_0 + \int_V \tilde{G}(\vec{r},\vec{r}\,') \cdot \vec{M}(\vec{r}\,') dr'^3 + \frac{\eta}{\gamma M_S} \nabla^2 \vec{M} + \ldots$$

dipolar interaction

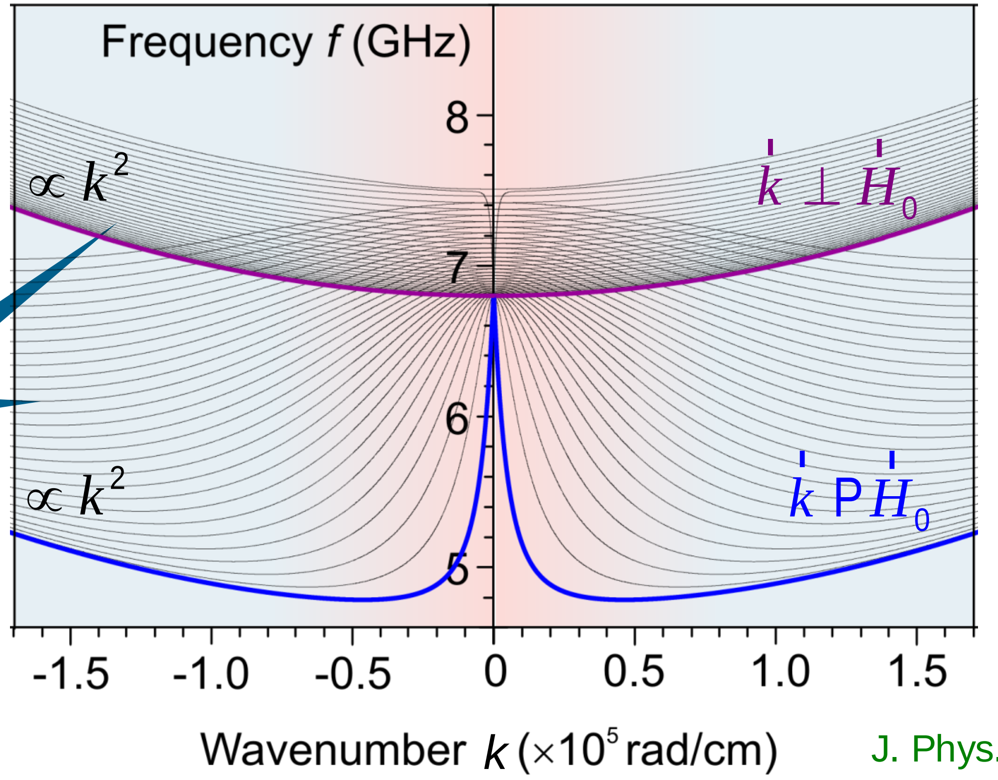
exchange interaction

$\mu_0 H_0 = 171$ mT



Thickness modes having a non-uniform harmonic distribution of dynamic magnetization along the film thickness

6 µm thick YIG film

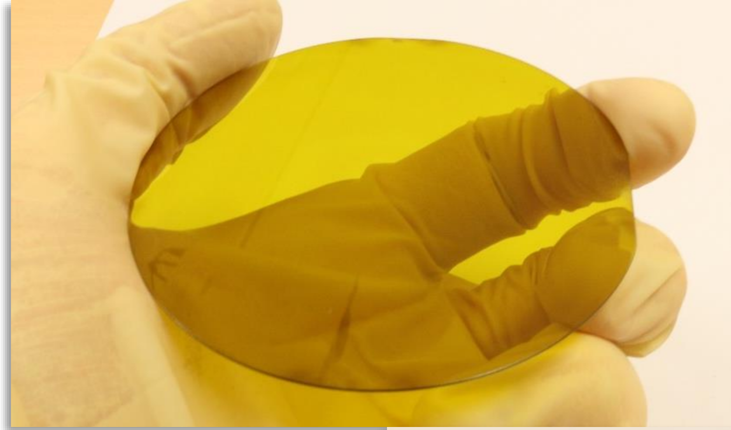


Calculations based on:
Kalinikos and Slavin,
J. Phys. C: Solid State Phys **19**, 7013 (1986)

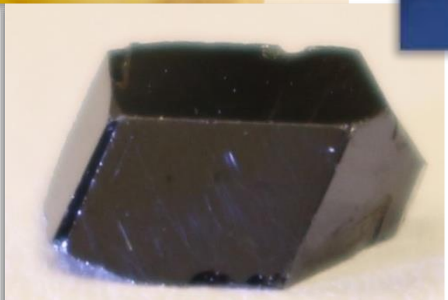
Yttrium Iron Garnet (YIG, $Y_3Fe_5O_{12}$)

- Room temperature ferrimagnet (T_C = 560 K)
- Cubic crystal
- **Low phonon damping**

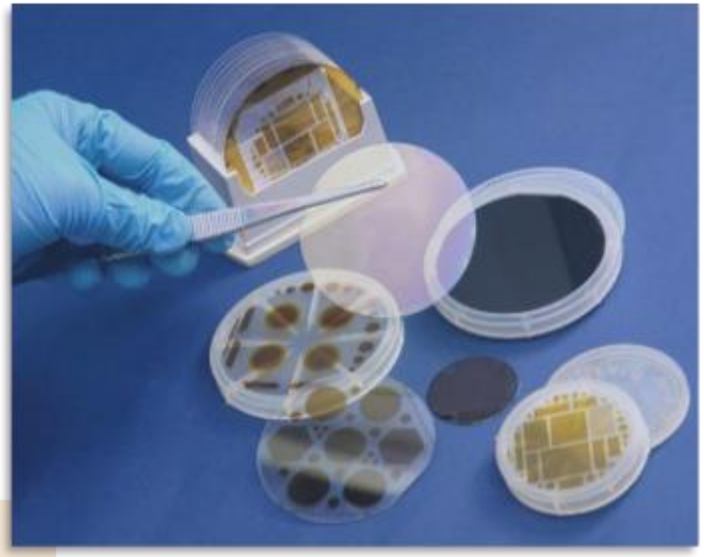
Cherepanov, Kolokolov, L'vov,
The saga of YIG, Phys. Rep. **229**, 81 (1993)

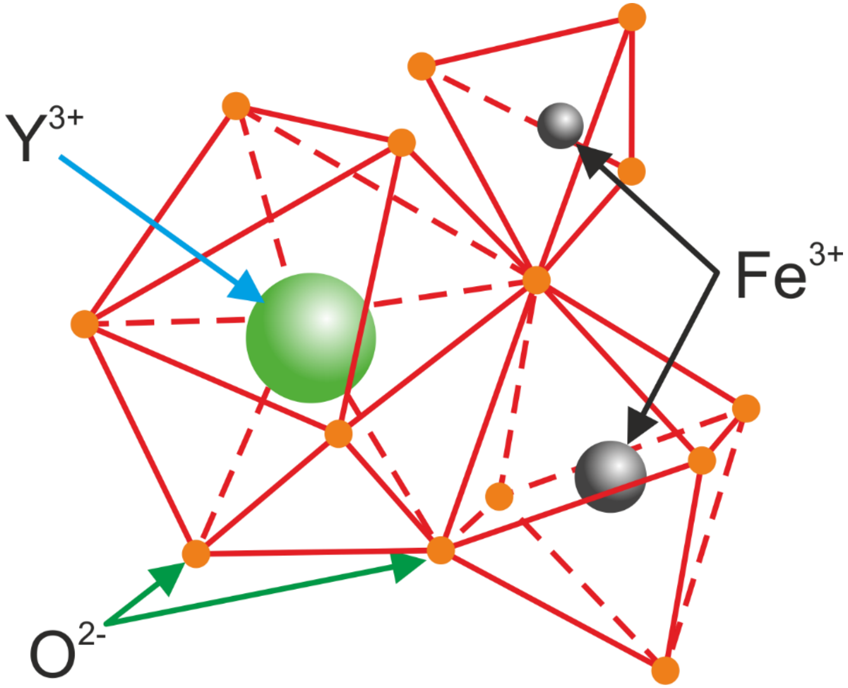


3" YIG wafer
SRC "Carat"
Lviv, Ukraine



YIG bulk crystal



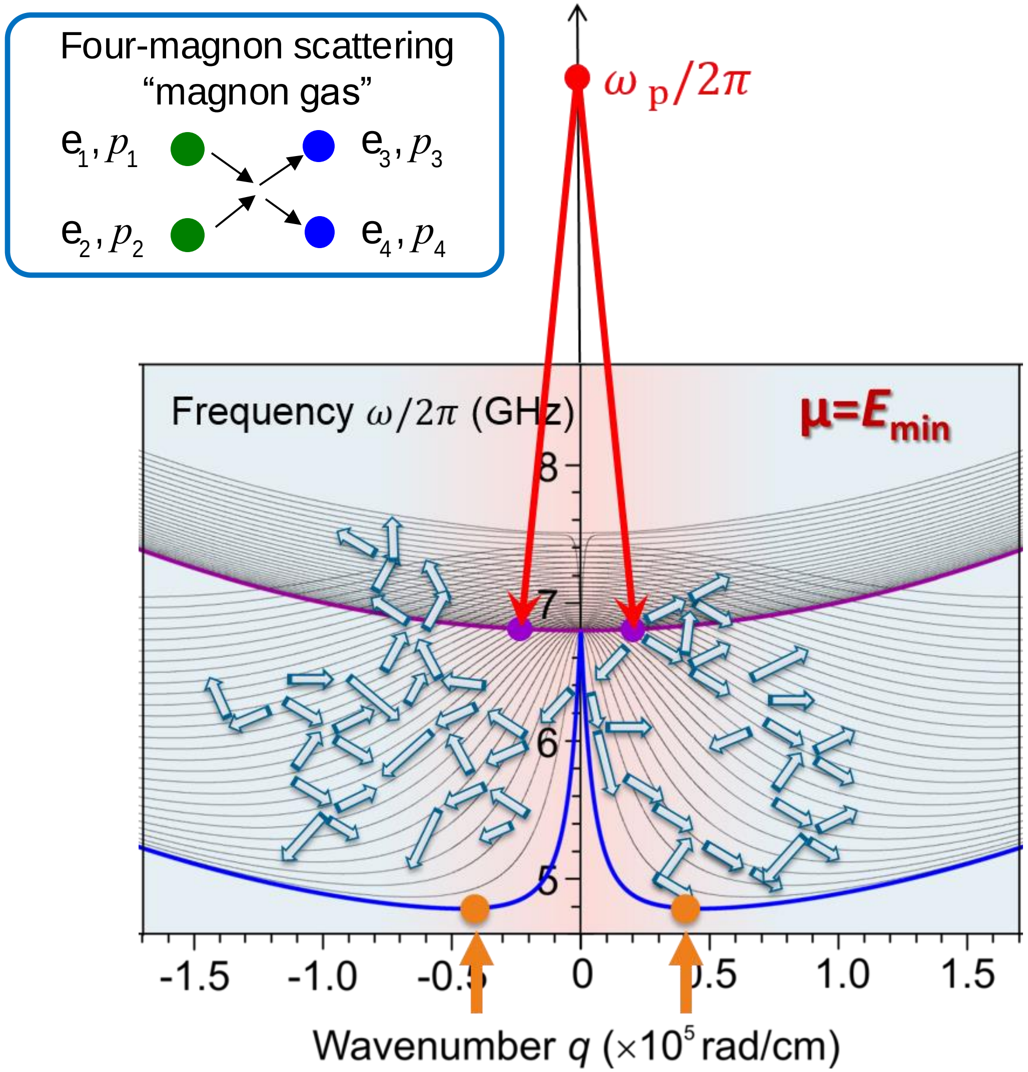


- Lattice constant 12.376 Å
- Unit cell – 80 atoms

Longest known spin-wave lifetime (up to 700 ns)

Knowledge box: Magnon Bose-Einstein condensate (BEC)

- BEC is macroscopic quantum state
  - Exists at bottom of the spin-wave spectrum with zero group velocity
- Fundamental scattering processes: four-magnon scattering
  - Excess magnons cannot relax within system relaxation time
  - Finite chemical potential μ
- Order parameter: coherency
  - Repulsive intermodal interaction leads to spatial stability of magnon condensates
- Methods to generate BEC: parametric pumping, spin-transfer torque, rapid cooling



Outlook: Magnon Bose–Einstein condensation

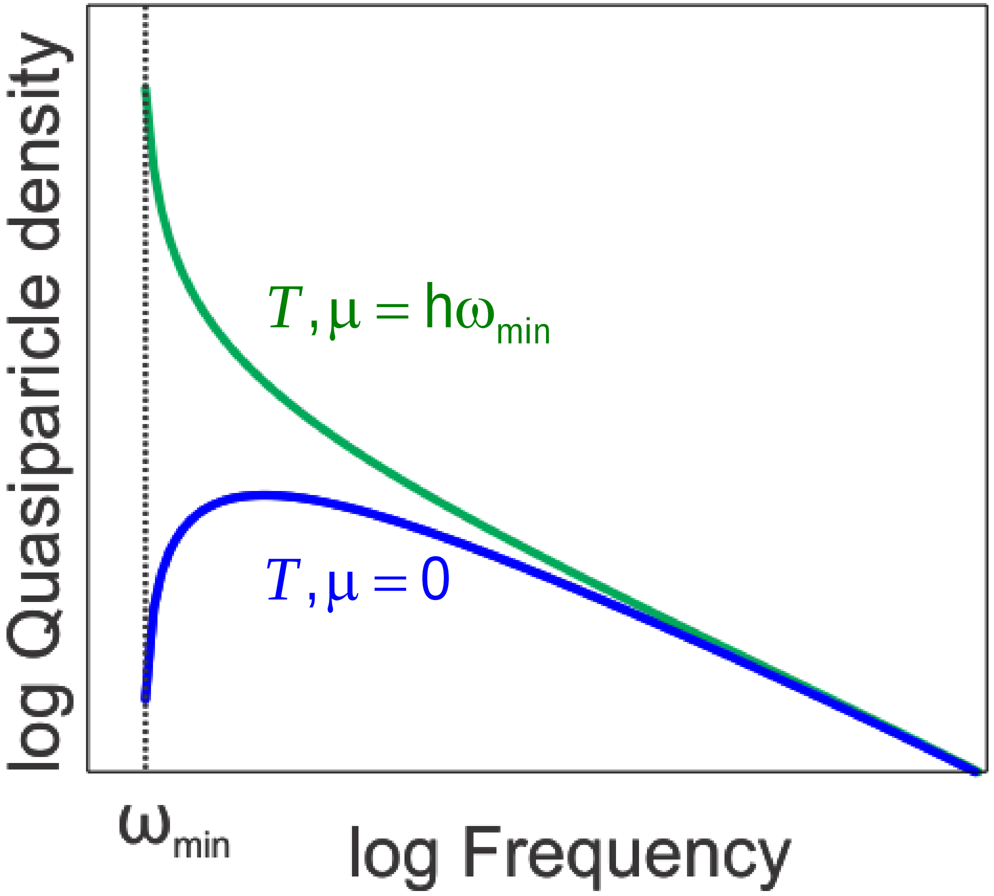
Bose-Einstein distribution

$$\rho(\omega) = \frac{D(\omega)}{\exp\left(\frac{\hbar\omega - \mu}{k_B T}\right) - 1}$$

μ: chemical potential

External injection of magnons beyond the thermal equilibrium level (about 3%) increases the chemical potential to the bottom of magnon spectrum and leads to Bose-Einstein condensation scenario even at room temperature

BEC of magnons – **macroscopic quantum phenomena** – spontaneously formed coherent wave in a chaotic magnon system



Demokritov *et al.*, *Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping*, Nature **443**, 430 (2006)

Outlook: Magnon Bose–Einstein condensation

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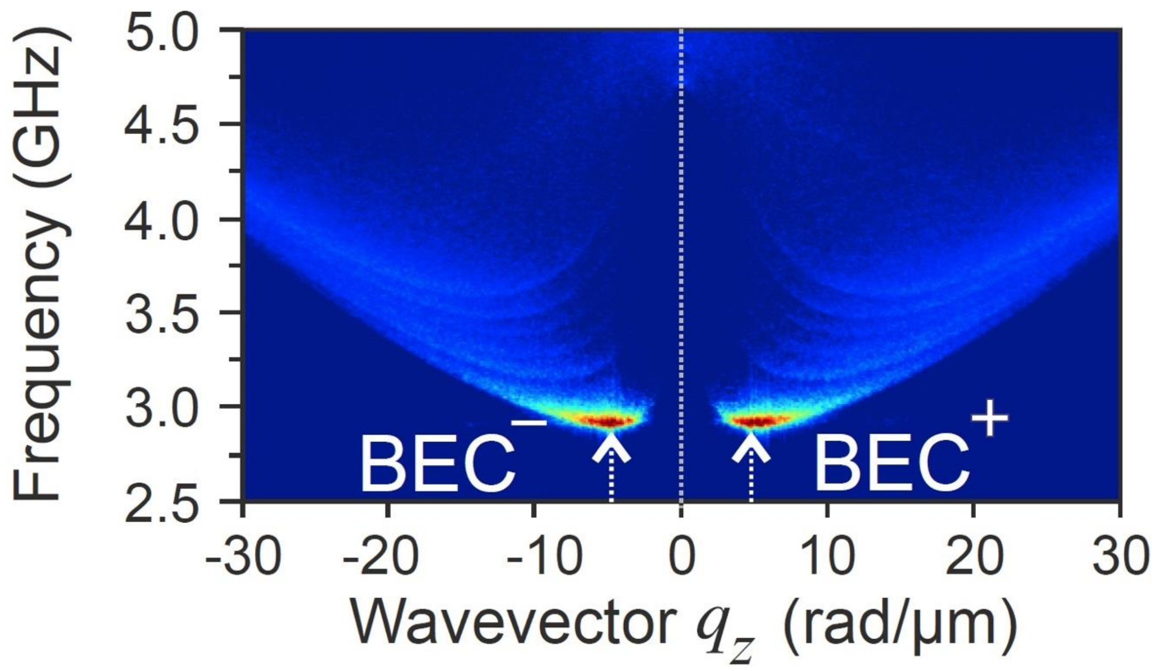
BEC of magnons – **macroscopic quantum phenomena** – spontaneously formed coherent wave in a chaotic magnon system

Numerical simulation of the condensation process of parametrically populated magnon gas in a YIG film



MuMax 3.0 numerical calculations

M. Mohseni et al., *Commun. Phys.* **5**, 196 (2022)

Summary: what we leaned in this lecture:

- Magnetization dynamics: torque equations & torque boundary conditions
- Energy contributions to spin-wave frequency and dispersion properties
- Backward volume magnetostatic spin waves
- Magnetostatic surface spin waves
- Quantized spin waves in confined structures
- Exchange spin waves
- Magnon Bose-Einstein condensation