Domains and Domain Walls

Stefan Blügel

Peter Grünberg Institut and Institute for Advanced Simulation,
Forschungszentrum Jülich and JARA

JÜLICH
Forschungszentrum

THE BOOK

Magnetic Domains – The analytics of magnetic microstructures
Alex Hubert and Rudolf Schäfer, Springer, 1998, ISBN: 9783540641087, pp 724

MULTISCALE MODELING

❖ Micromagnetic-model:

$$E(\mathbf{m}) = \int_{\mathbb{R}^2} \left[A\,|\nabla\mathbf{m}|^2 + \underline{\mathbf{D}} : (\nabla\mathbf{m} \times \mathbf{m}) + \mathbf{m}\cdot\underline{\mathbf{K}}\cdot\mathbf{m} - B\,\mathbf{m}\cdot\hat{\mathbf{e}}_z\right] d\mathbf{r} \quad + \ldots$$

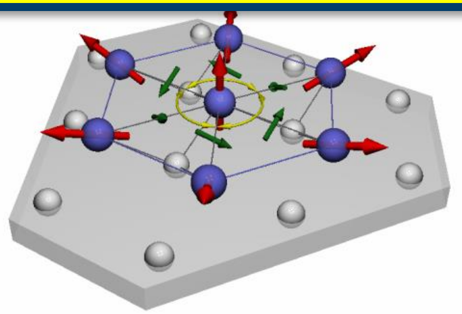
❖ Atomistic Spin-Lattice Model:

$$H = \frac{1}{2}\sum_{ij} J_{ij}\,\mathbf{m}_i\mathbf{m}_j + \sum_{ij} \mathbf{D}_{ij}\,\overbrace{\mathbf{m}_i \times \mathbf{m}_j}^{c} + \sum_i \mathbf{m}_i\,\underline{\mathbf{K}}\,\mathbf{m}_i + \sum_{ij}\frac{1}{r_{ij}^3}\left[\mathbf{m}_i\mathbf{m}_j - (\mathbf{m}_i\hat{\mathbf{e}}_i)(\mathbf{m}_j\hat{\mathbf{e}}_i)\right]$$

Long wave length limit

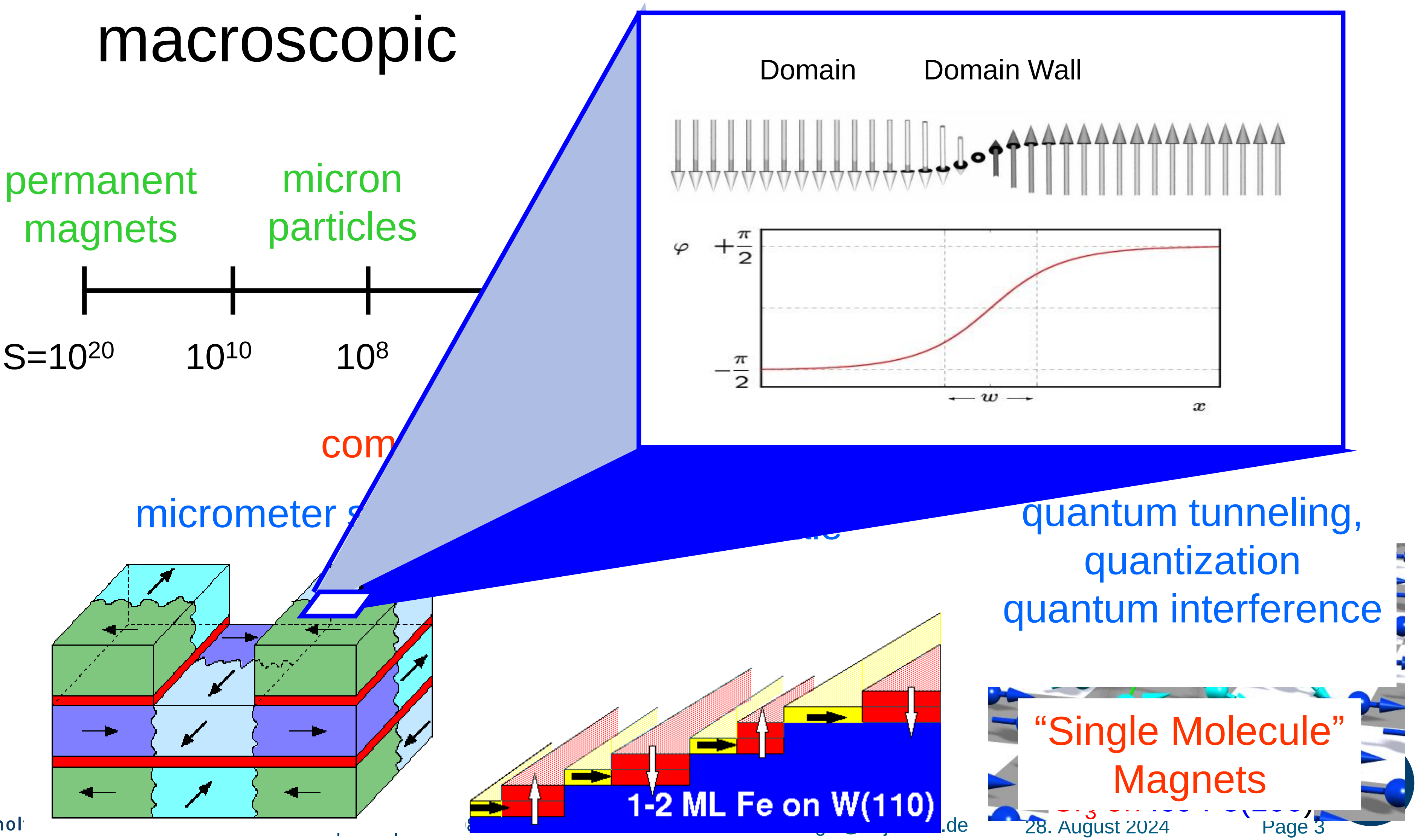
$$\mathbf{m}_j = \mathbf{m}_i + \sum_\alpha \mathcal{R}_{ij,\alpha}\,\partial_\alpha \mathbf{m}_i + \ldots$$

- Spin Stiffness: $A \propto \sum_{j>0} J_{0j} R_{0j}^2$
- Spiralization: $\underline{\mathbf{D}} \propto \sum_{j>0} \mathbf{D}_{0j} \otimes \mathbf{R}_{0j}$



MAGNETISM ON ALL SCALES:

WHAT HAPPENS TO MAGNETISM ON THE WAY FROM LARGE TO SMALL

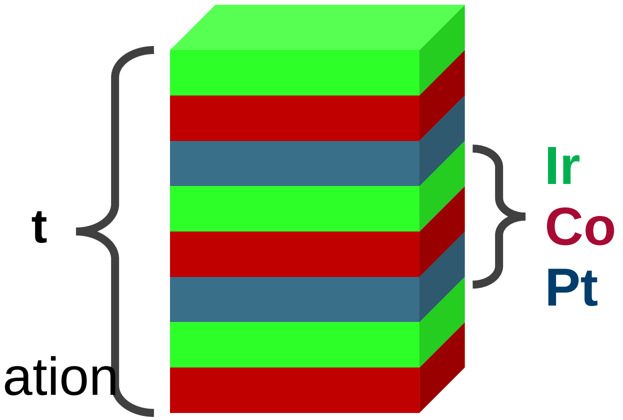


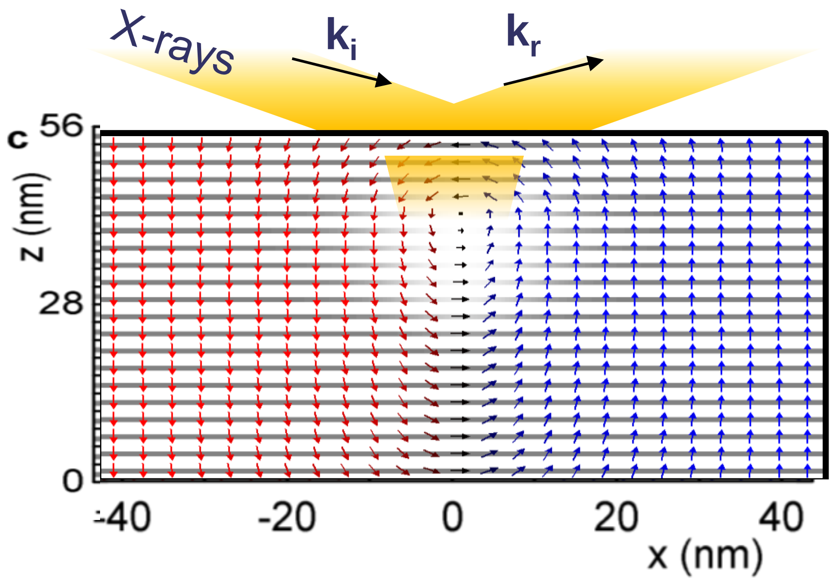
MAGNETOSTATICS

- Limit: thickness large to exchange length

$$t > \ell_{ex} = \sqrt{2A/\mu_0 M_s^2}$$

Diploar-field induced inhomogeneous magnetization
Dipolar-field induced twisting of DW chirality



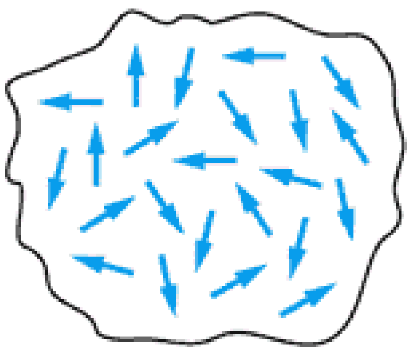


||**Ta** 10 | **Pt** 7 | (**Co** 0.8 | **X** 1| **Y** 1)$_{x20}$ | **Pt** 7

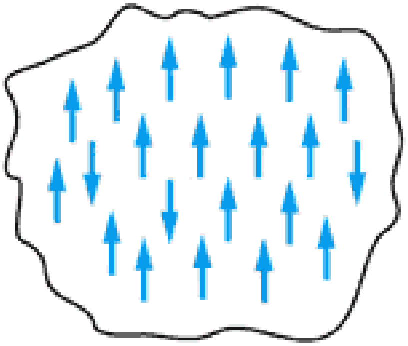
W. Legrand, *et al.*, Science Adv. (2018), *arXiv:***1712**.05978

MAGNETIC DOMAINS

Magnetic field absent

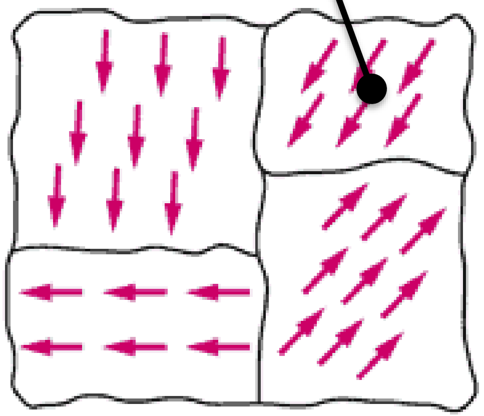


In presence of magnetic field

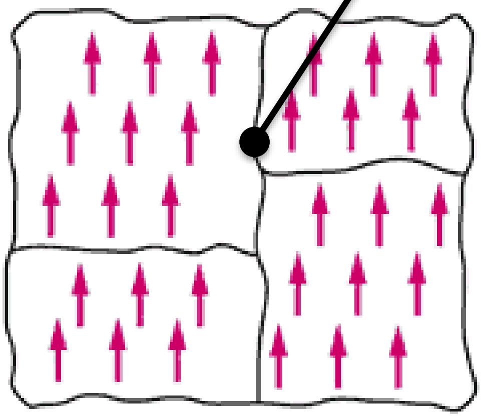


Paramagnetism

Magnetic domain



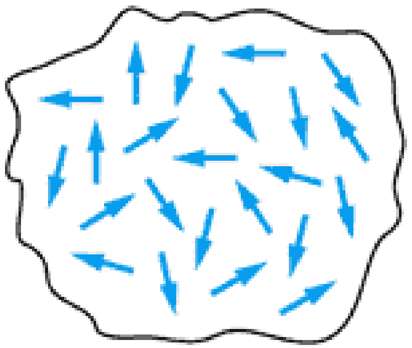
Domain Wall



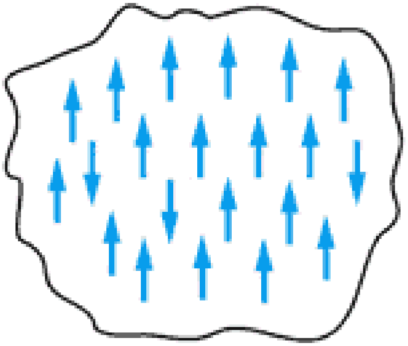
Ferromagnetism

2) MAGNETIC DOMAINS

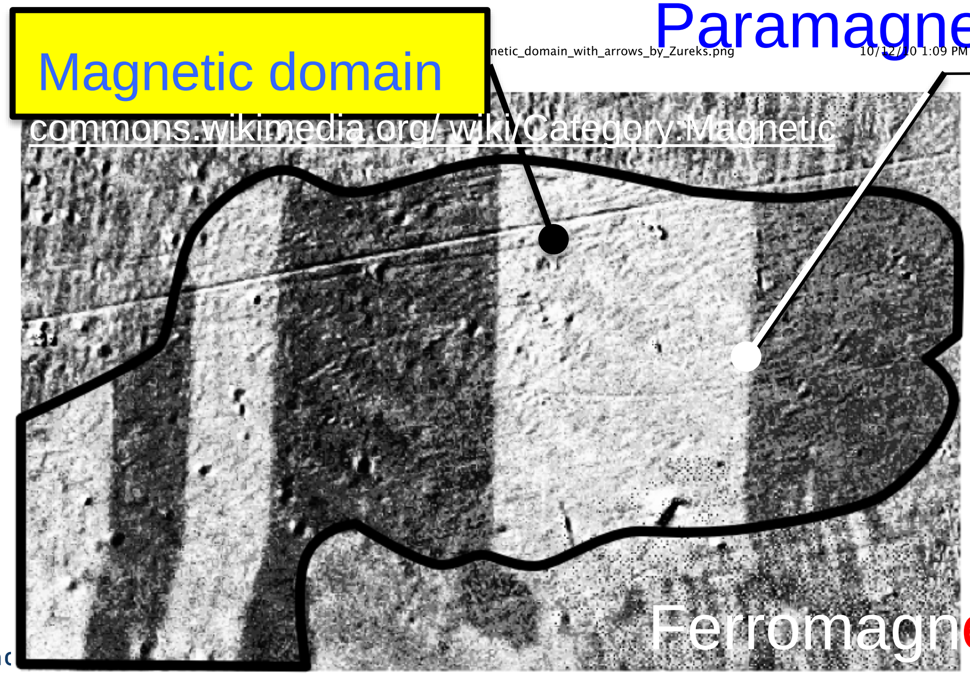
Magnetic field absent

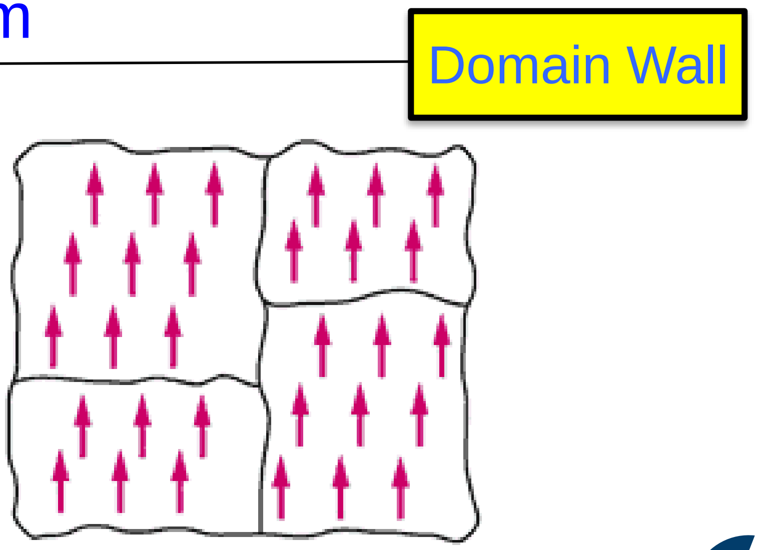


In presence of magnetic field



Paramagnetism

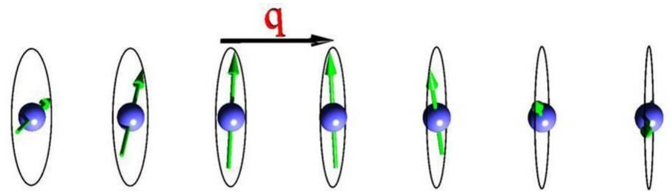




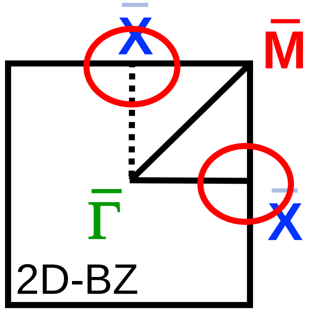
Ferromagnetism

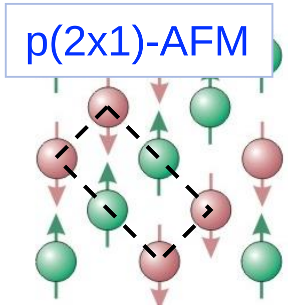
EXAMPLE: SQUARE LATTICE

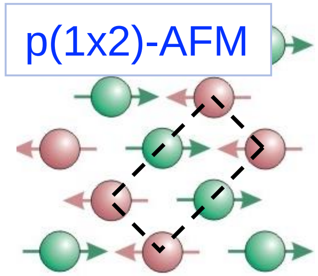
Domains of single-q spirals



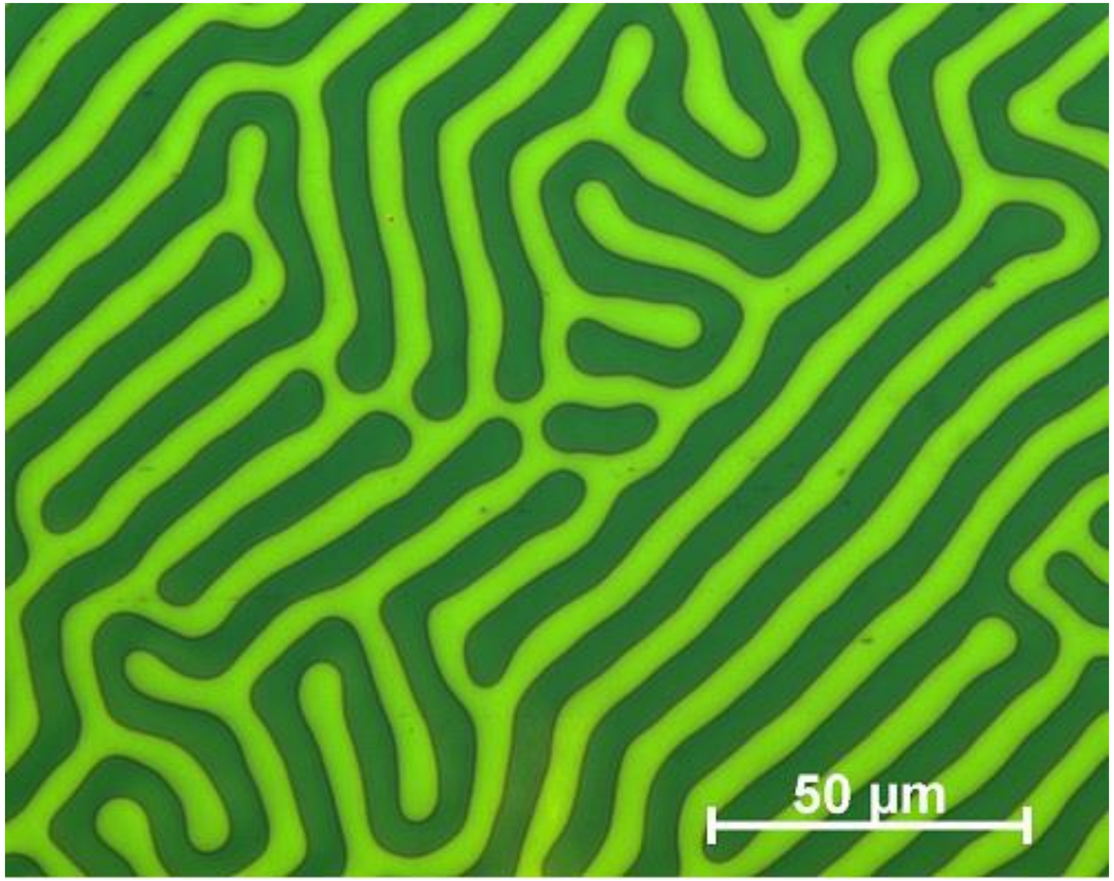
Spin spirals







Magnetic Domains



ONE-DIMENSIONAL DOMAIN WALL

Micromagnetic Theory

Domain Wall energy (1D): described in terms of a classical field theory

$$E_{tot} = \int \left\{ A \left(\frac{d f}{dx} \right)^2 + K_{eff} \sin^2 f \right\} dx$$

$$m_z(x) = m \cdot \varphi(x)$$

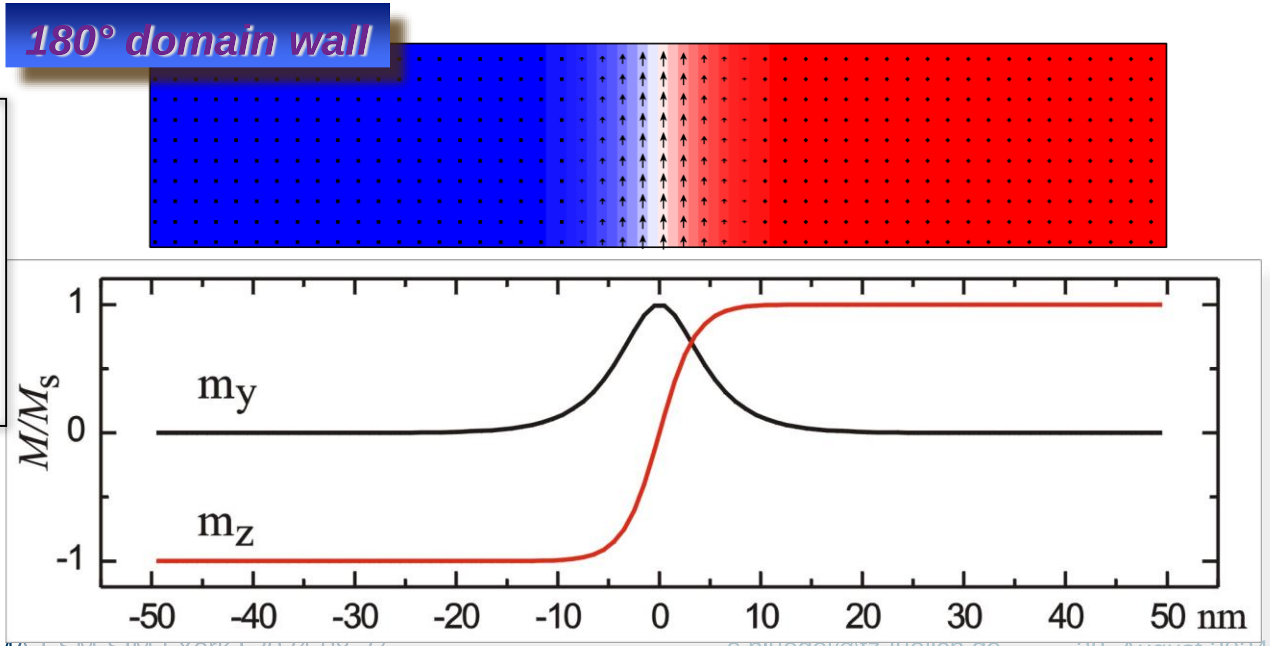
Planar approximation
\+ DW boundary condition:

$$\sin\varphi \xrightarrow{x \to -\infty} -1$$

$$\sin\varphi \xrightarrow{x \to +\infty} +1$$

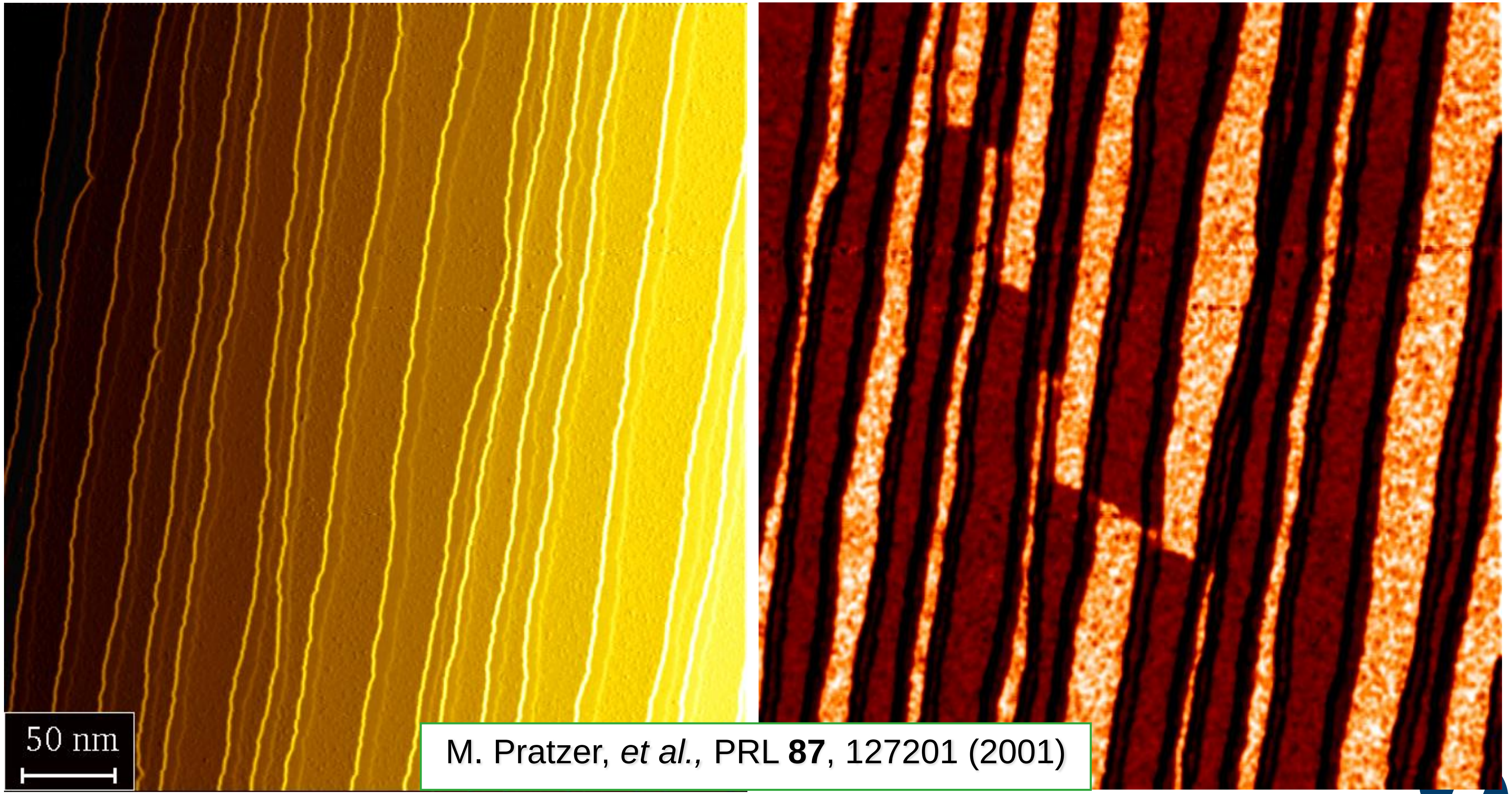
$$\sin\varphi(x) = \tanh\left(\frac{x - x_0}{w/2} \right)$$

with $\quad w = 2L = 2\sqrt{\frac{A}{K}}$



DOMAIN CONTRAST FOR FE NANOSTRIPES ON W(110)

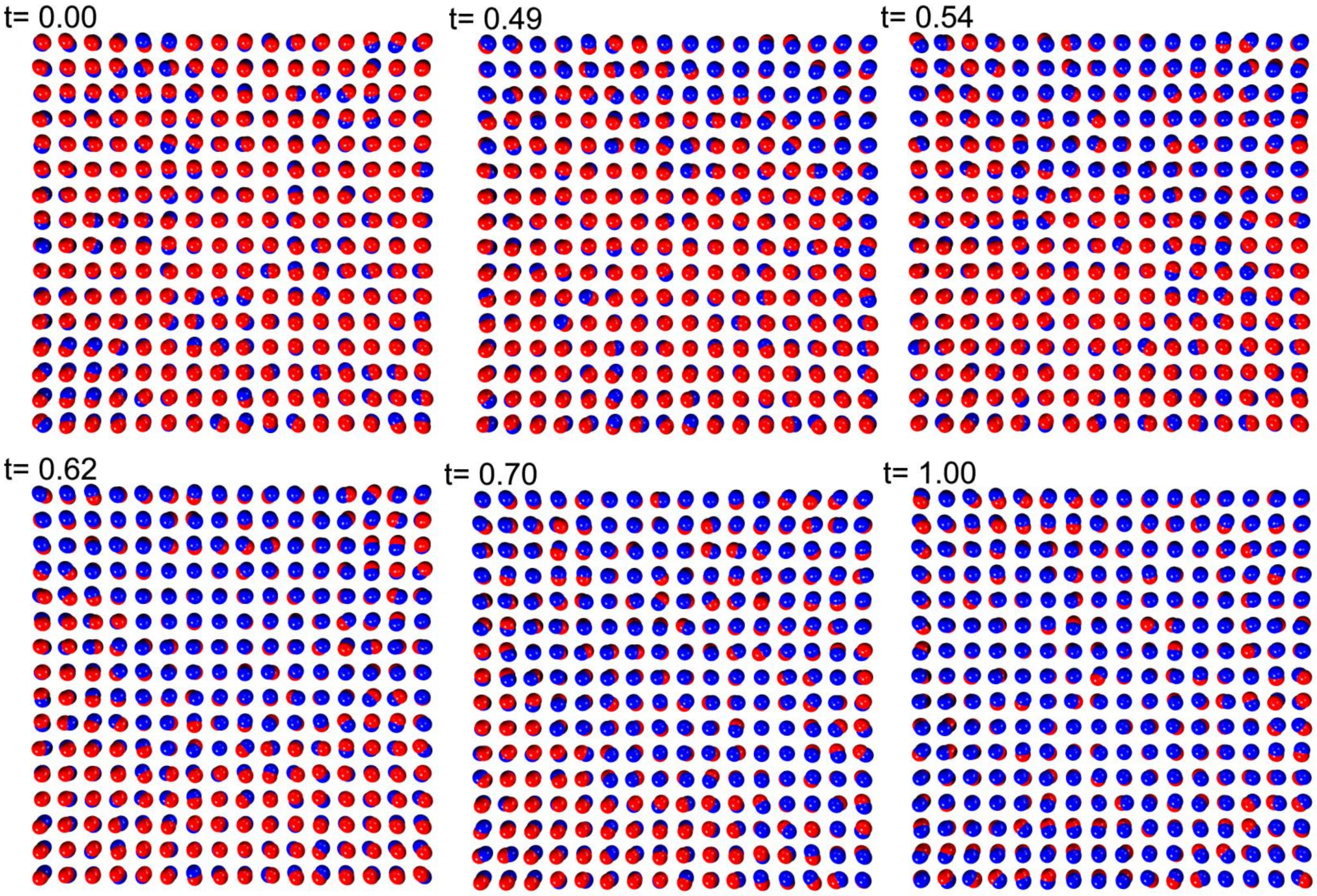
coverage Θ = 1.25 ML   Fe-tip   grown at T = 500 K U = +130 mV



M. Pratzer, *et al.*, PRL **87**, 127201 (2001)

MAGNETIZATION SWITCHING BY DOMAIN WALL MOVE

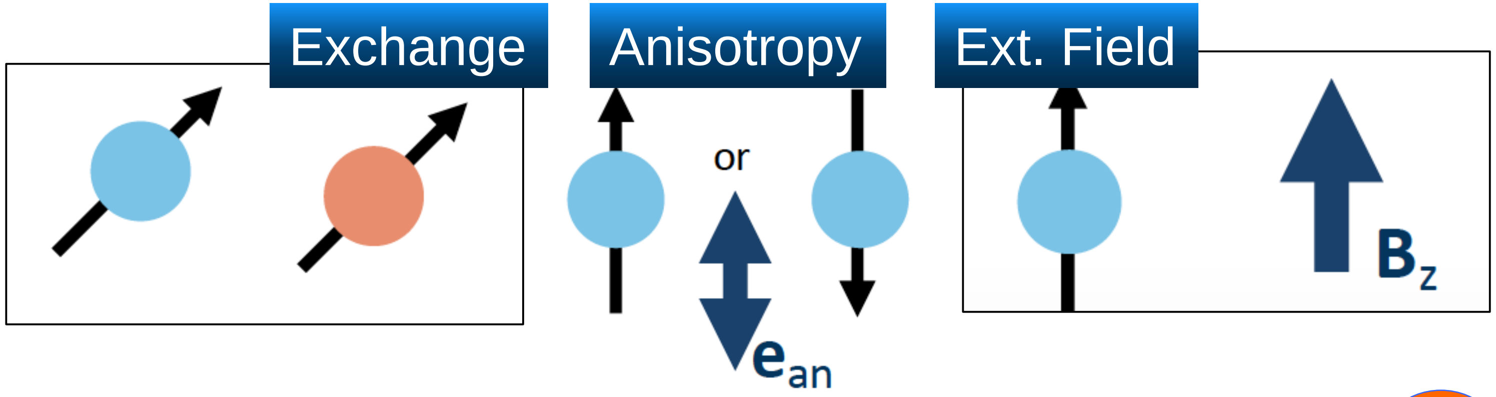
Simulation 15x15 Atoms K/J=0.9

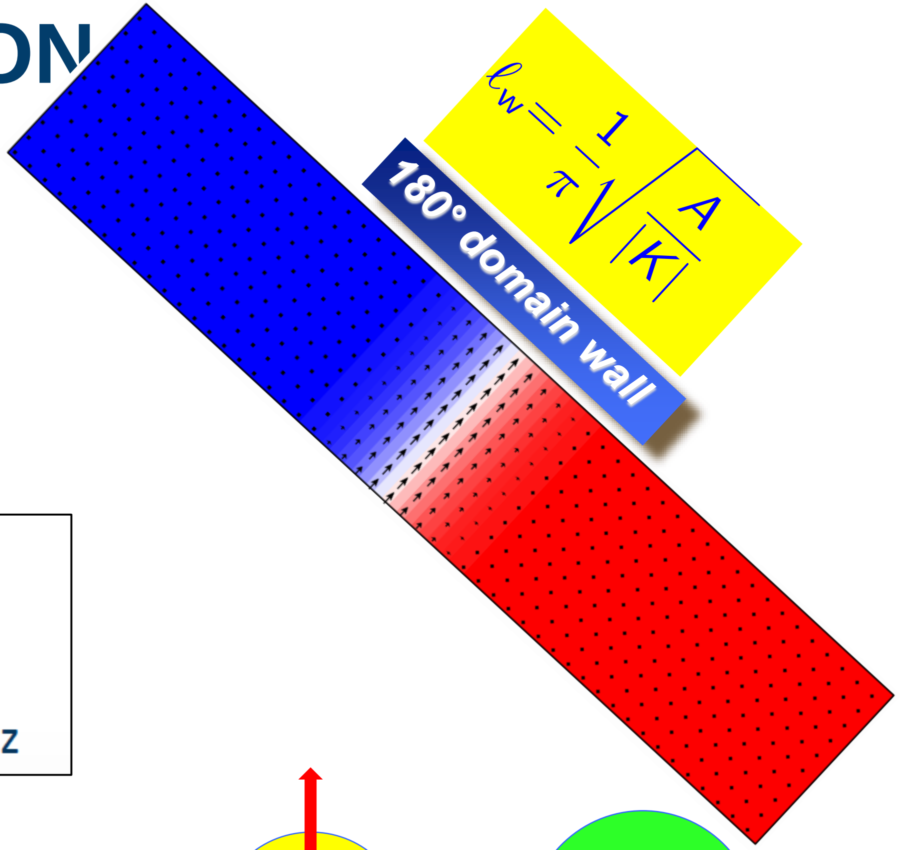


STABILITY OF DW – ENERGY FUNCTION

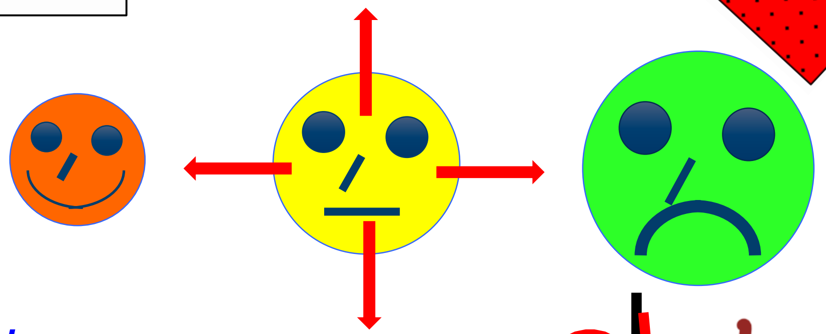
❖ Micromagnetic energy functional **1D**: :

$$E(\mathbf{m}) = \int_{\mathbb{R}^1} \left[A\,|\nabla\mathbf{m}|^2 + \mathbf{m}\cdot\underline{\mathbf{K}}\cdot\mathbf{m} - B\,\mathbf{m}\cdot\hat{\mathbf{e}}_z\right] dr$$

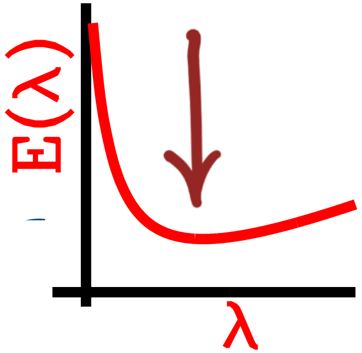




❖ Stretching transformation: $\mathbf{m}(\mathbf{r}) \rightarrow \mathbf{m}_\lambda(\mathbf{r}) = \mathbf{m}(\lambda\mathbf{r})$



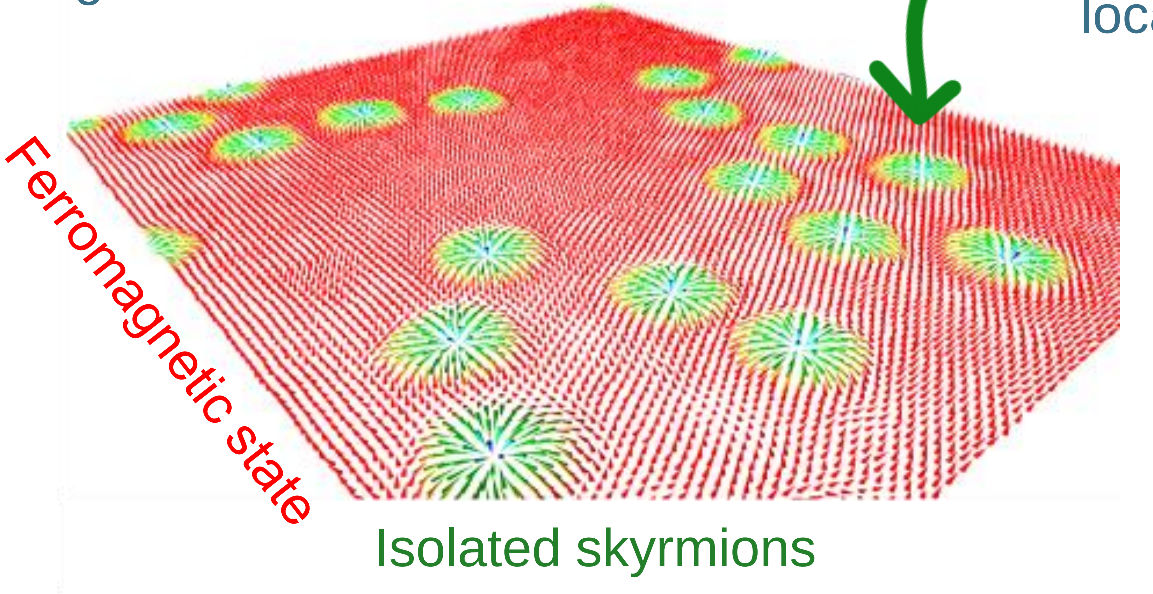
$$E(\lambda) = \int_{\mathbb{R}^1} \left[\frac{A}{\lambda^2}\,|\nabla\mathbf{m}|^2 + \mathbf{m}\cdot\underline{\mathbf{K}}\cdot\mathbf{m} - B\,\mathbf{m}\cdot\hat{\mathbf{e}}_z\right] \lambda dr$$



MAGNETIC DOMAIN VS TEXTURE

Skyrmion
two-dimensional
topological solition

three-dimensional
spin-texture
two-dimensionally
localized in space



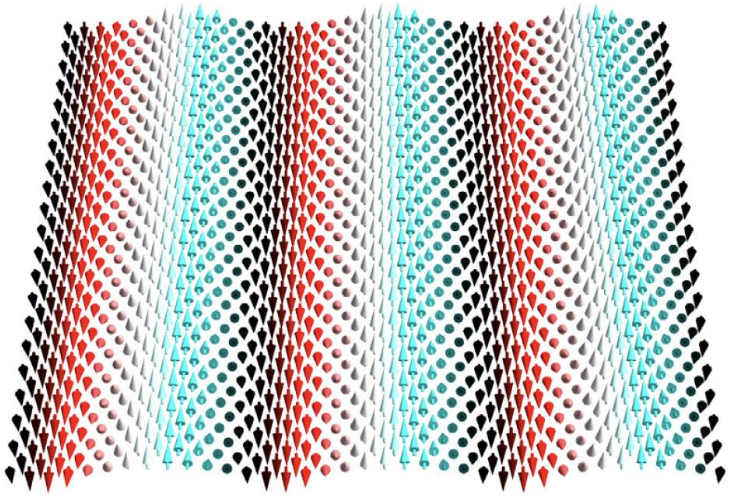
Isolated skyrmions

from Bertrand Dupé

MANY EXCITING SPIN TEXTURES

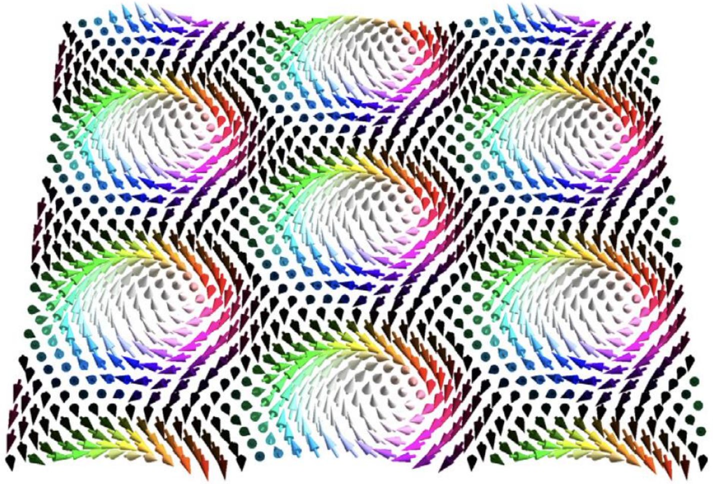
Lattice type textures

Trivial textures



Helical phase

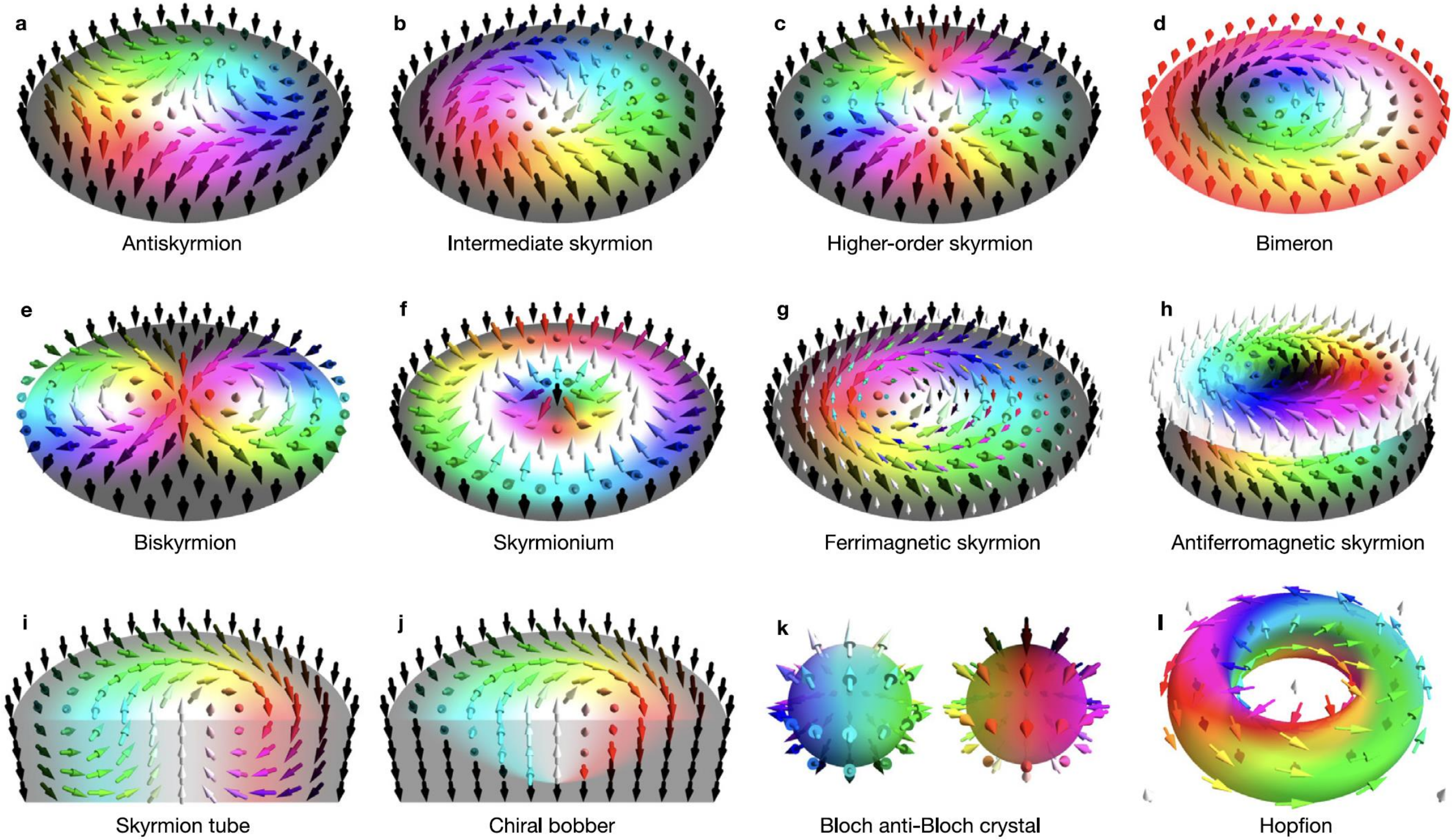
Topological textures



Skyrmion lattice

MANY EXCITING SPIN TEXTURES

Particle type textures



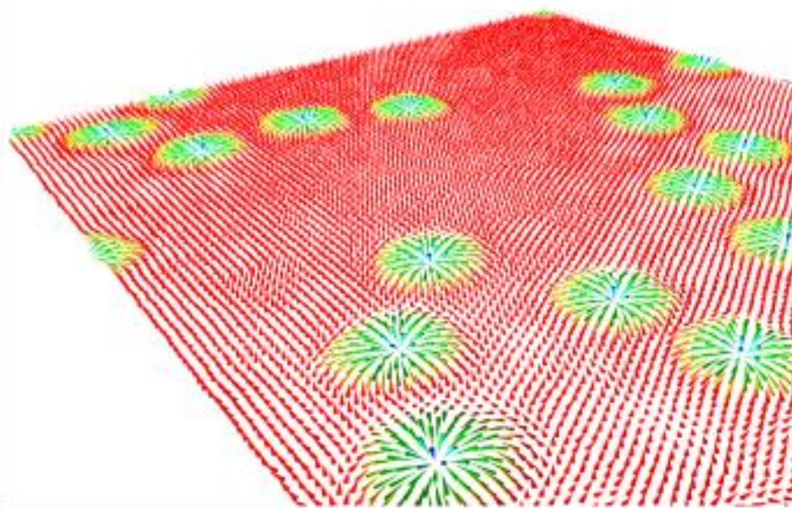
B. Göbel *et al.* Physics Reports **895,** 1-28 (2021)

Questions

- What makes them stable ?
- In which materials?
- Creation and annihilation
- Detection by microscopy and electrical transport
- Manipulation by current
- Transport and Dynamics
- Realspace ←→ Momentum Space
- Size optimization
- Fit for spintronics

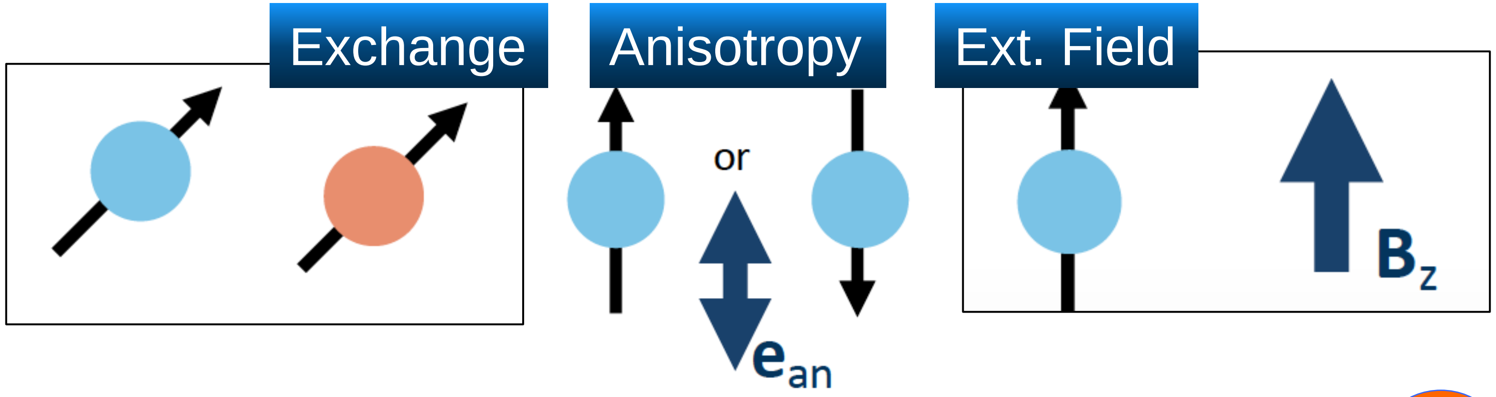
JÜLICH Forschungszentrum

ASPECT 2 – ENERGY FUNCTIONAL

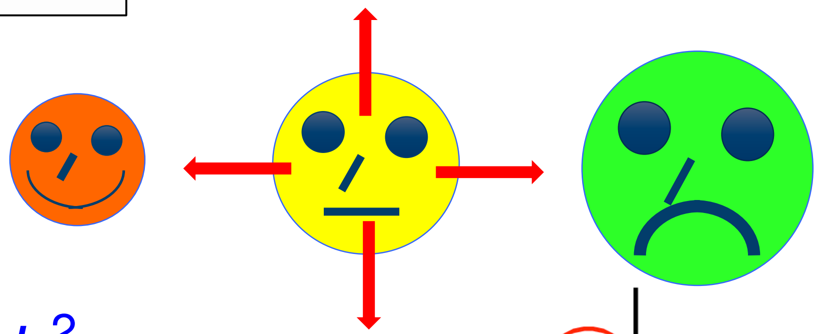


- Micromagnetic energy functional **2D**: :

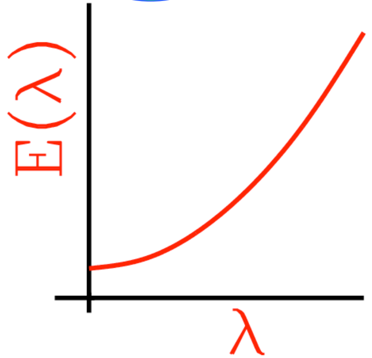
$$E(\mathbf{m}) = \int_{\mathbb{R}^2} \left[A\,|\nabla \mathbf{m}|^2 + \mathbf{m}\cdot\underline{\mathbf{K}}\cdot\mathbf{m} - B\,\mathbf{m}\cdot\hat{\mathbf{e}}_z \right] dr^2$$



- Stretching transformation: $\mathbf{m}(\mathbf{r}) \to \mathbf{m}_\lambda(\mathbf{r}) = \mathbf{m}(\lambda\mathbf{r})$



$$E(\lambda) = \int_{\mathbb{R}^2} \left[\frac{A}{\lambda^2}\,|\nabla \mathbf{m}|^2 + \mathbf{m}\cdot\underline{\mathbf{K}}\cdot\mathbf{m} - B\,\mathbf{m}\cdot\hat{\mathbf{e}}_z \right] \lambda^2 dr^2$$



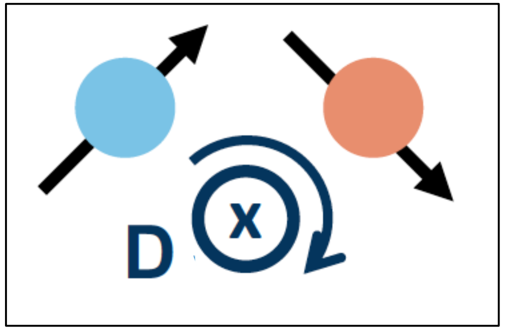
No nontrival / localized static solution (Derrick/Hobart theorem)

CHIRAL MAGNETIC SKYRMIONS – ENERGY FUNCTIONAL

- Micromagnetic energy functional **2D**:

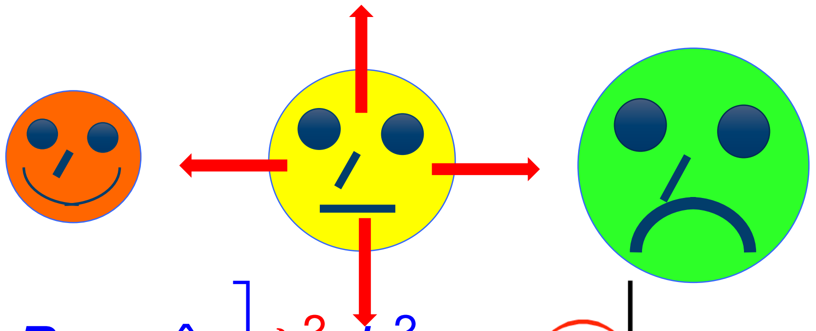
$$E(\mathrm{m}) = \int_{\mathbb{R}^2} \left[A\,|\nabla \mathrm{m}|^2 + \underline{\mathrm{D}} : (\nabla \mathrm{m} \times \mathrm{m}) + \mathrm{m} \cdot \underline{\mathrm{K}} \cdot \mathrm{m} - B\,\mathrm{m} \cdot \hat{\mathrm{e}}_z\right] dr^2$$

Dzyaloshinskii-Moriya

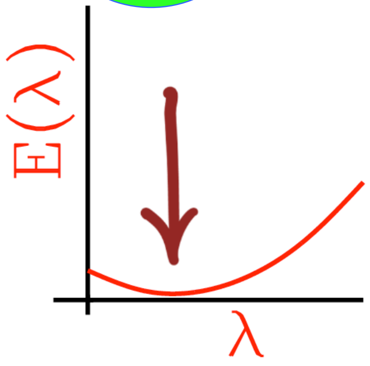


- Necessary conditions for DMI ($\underline{D} \neq 0$):
  (1) Spin-Orbit Interaction
  (2) Broken Inversion symmetry

- Stretching transformation: $\mathbf{m}(\mathbf{r}) \rightarrow \mathbf{m}_\lambda(\mathbf{r}) = \mathbf{m}(\lambda \mathbf{r})$



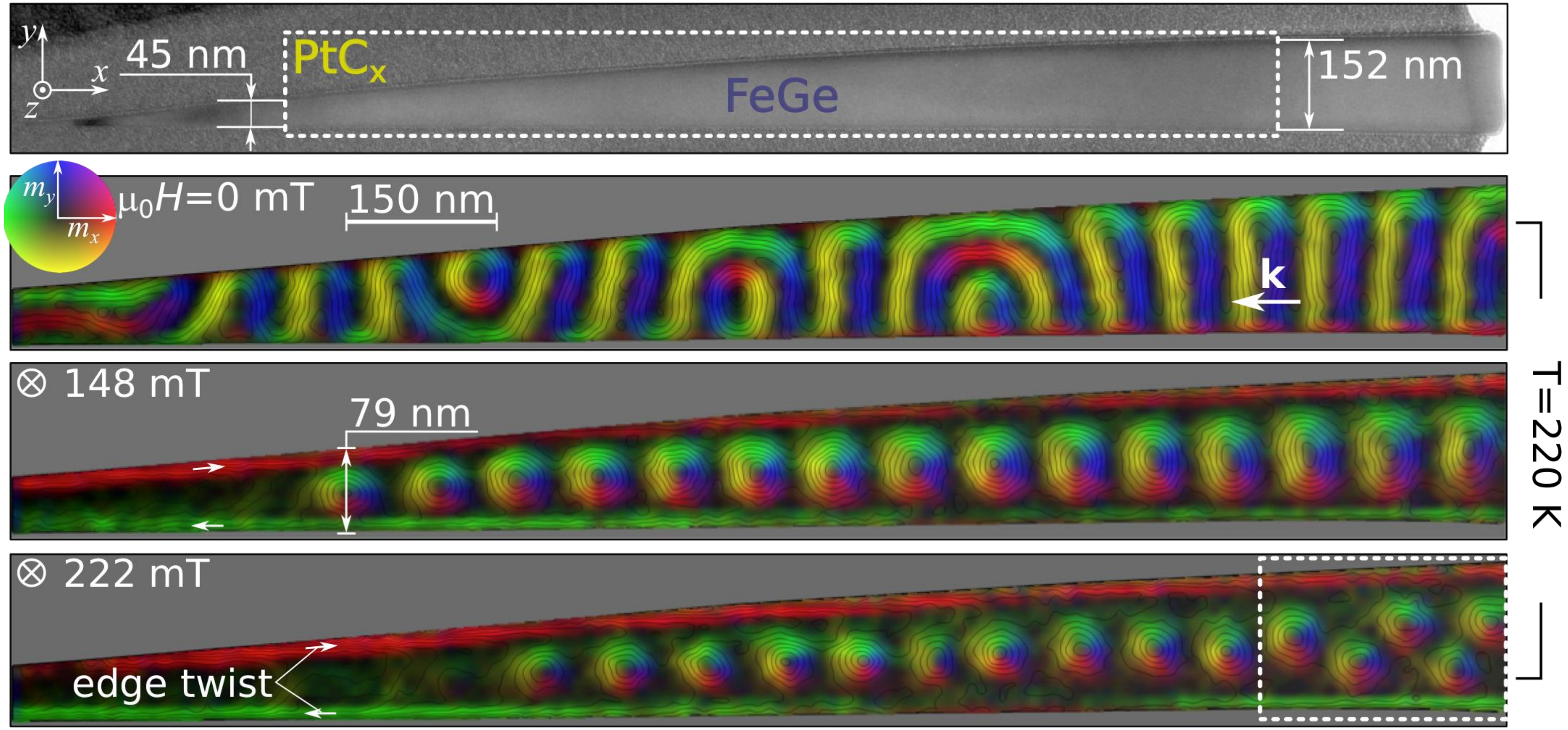
$$E(\lambda) = \int_{\mathbb{R}^2} \left[\frac{A}{\lambda^2}\,|\nabla \mathbf{m}|^2 + \frac{\mathbf{D}}{\lambda} \cdot (\nabla \mathbf{m} \times \mathbf{m}) + \mathbf{m} \cdot \underline{\mathbf{K}} \cdot \mathbf{m} - B\,\mathbf{m} \cdot \hat{\mathbf{e}}_z\right] \lambda^2 dr^2$$



Linear chiral symmetry breaking stabilizes skyrmions

Skyrmions in FeGe

Magnetic field dependence at 220 K after zero field cooling



Chiming Jin et al., Nature Communications **8**, 15569 (2017)

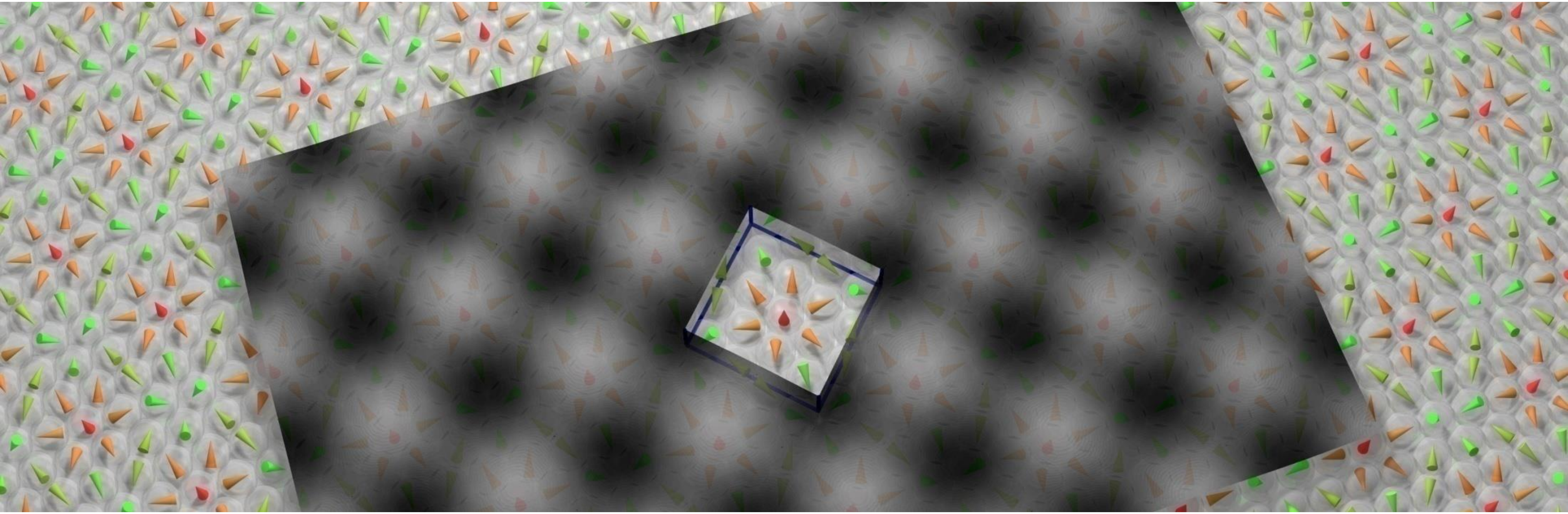
Three-dimensional spin/magnetization texture

spin texture: $\mathbf{R}_i \longmapsto \mathbf{S}_i = \mathbf{S}(\mathbf{R}_i)$

Spin-lattice representation

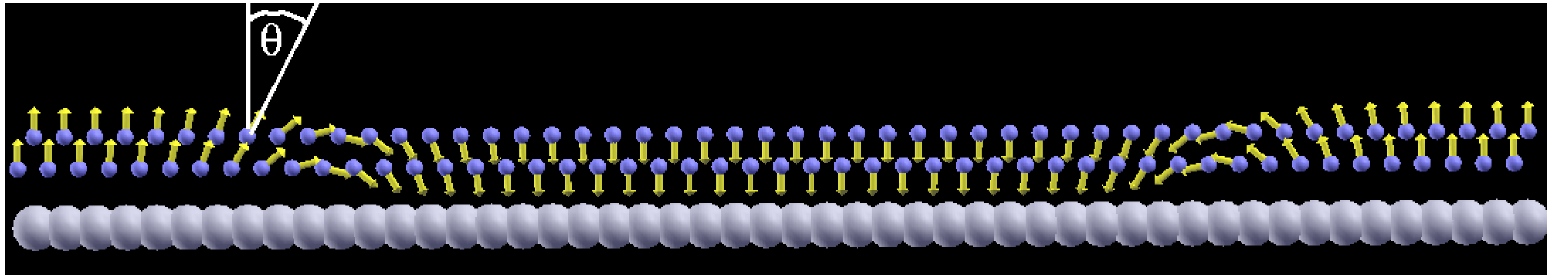
magnetization texture: $\mathbf{r} \longmapsto \mathbf{m}(\mathbf{r})$

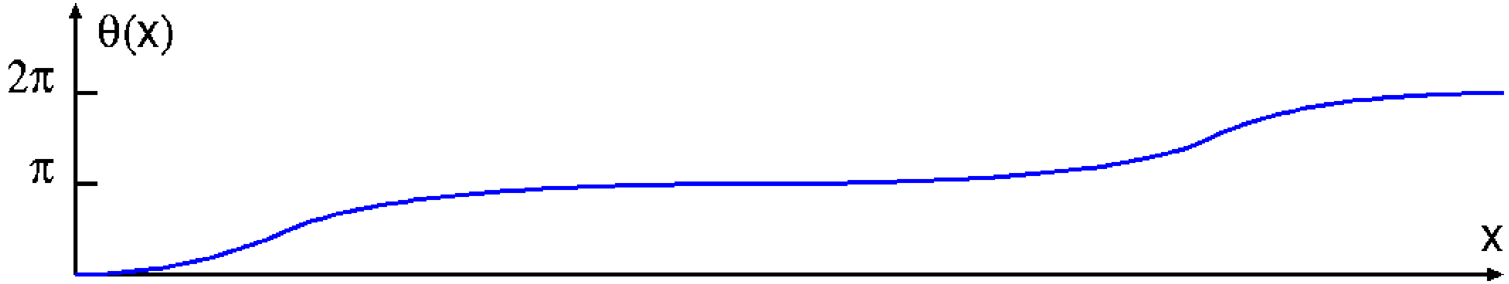
Continuum representation $|\mathbf{m}(\mathbf{r})| = 1$



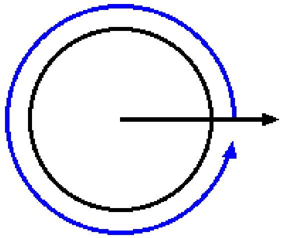
S. Heinze, K. v. Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, S. Blügel, Nat. Phys. **7**, 713 (2011)

TOPOLOGY IN DOMAIN WALLS: 2 DOMAIN WALLS





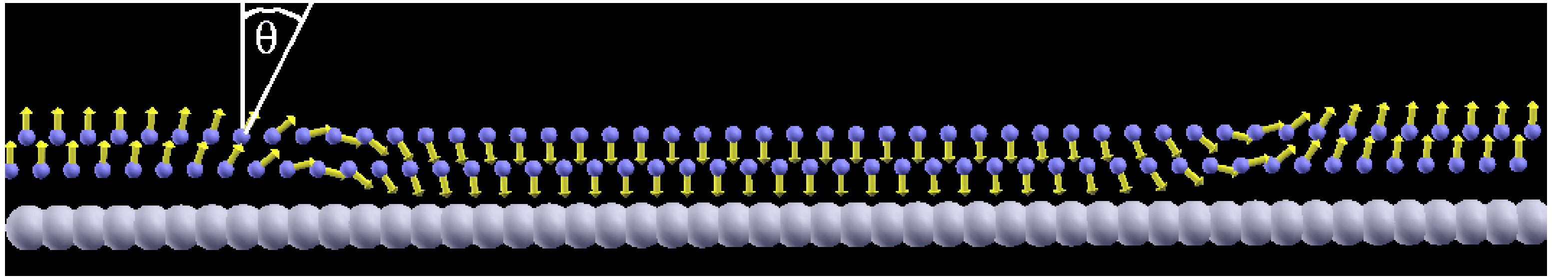
$$S = \frac{1}{2\pi}\oint \frac{\partial \theta(x)}{\partial x}\, dx = 1$$

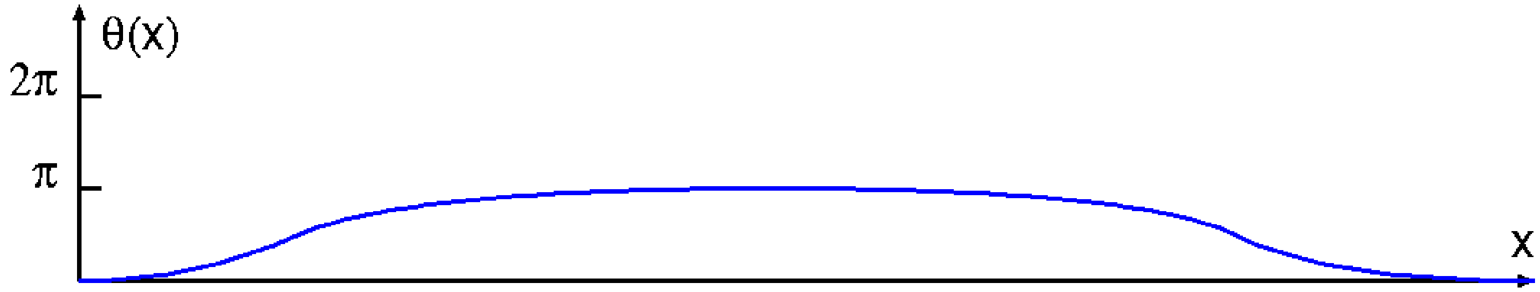


topological index, winding number

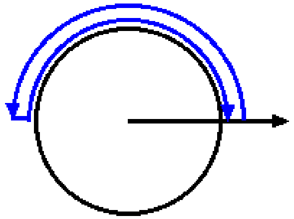
JÜLICH Forschungszentrum

TOPOLOGY IN DOMAIN WALLS: 2 DOMAIN WALLS





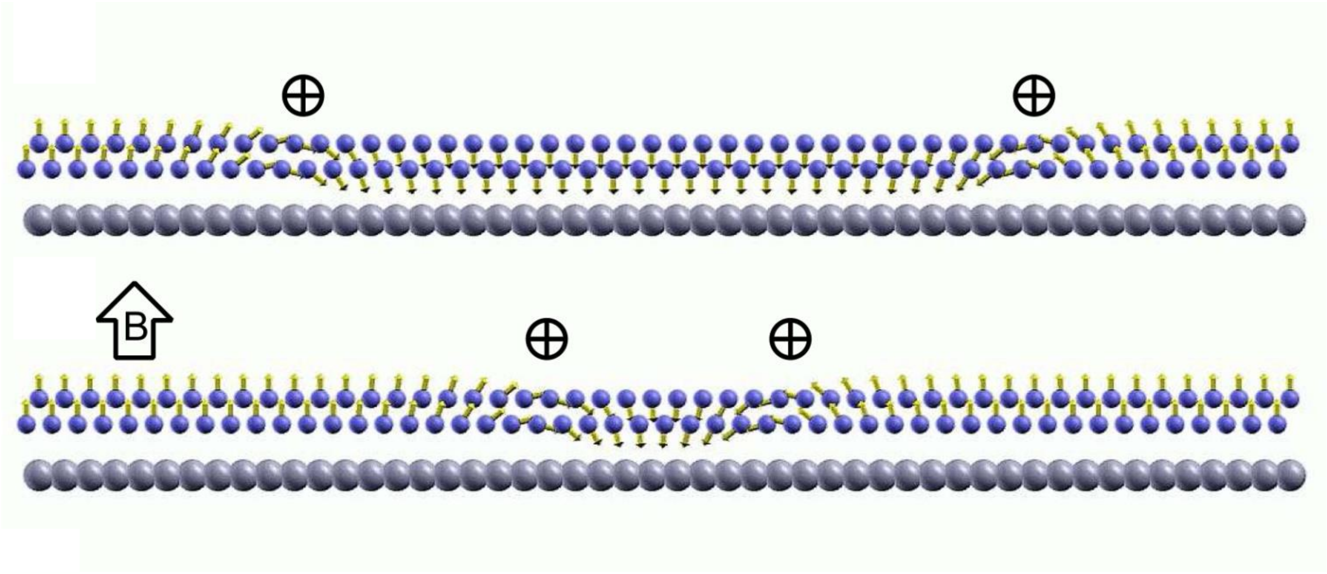
$$S = \frac{1}{2\pi}\int \frac{\partial\theta(x)}{\partial x}\,dx = 0$$



topologically trivial structure

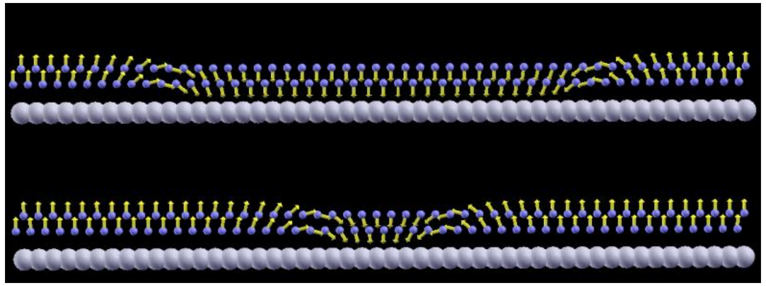
PARTICLE LIKE PROPERTIES

domain walls with
same
rotational sense



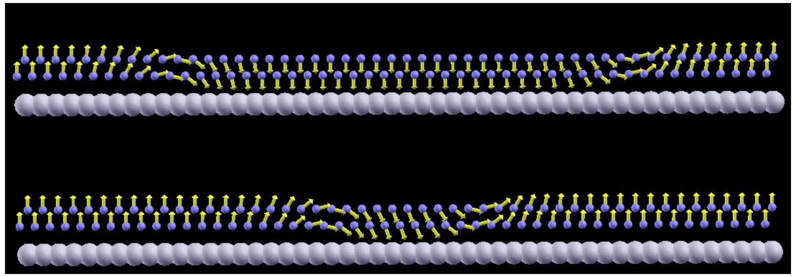
Topologically protected states: 1D

2 DOMAIN WALLS IN MAGNETIC FIELD:



B=0

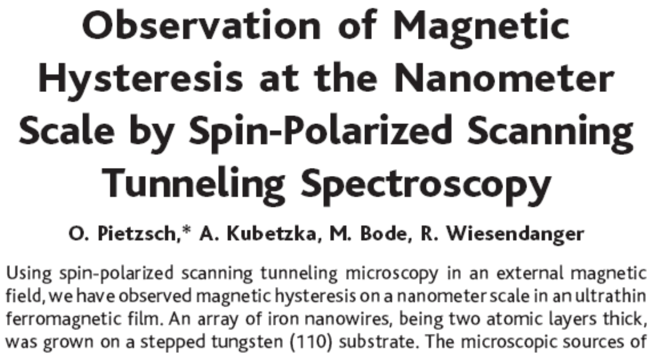
B



topologically protected:
B-field cannot destroy the inner domain (in 1D case)

topologically trivial:
B-field destroys the inner domain easily

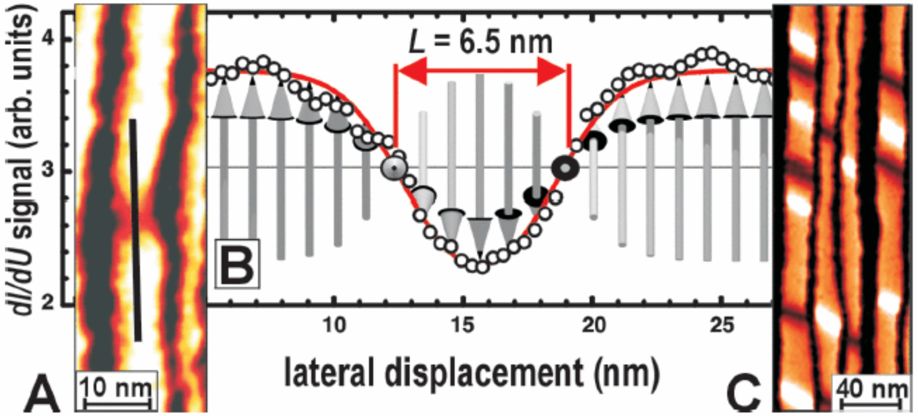
Example: Science **292,** 2053 (2001)



Observation of Magnetic Hysteresis at the Nanometer Scale by Spin-Polarized Scanning Tunneling Spectroscopy

O. Pietzsch,* A. Kubetzka, M. Bode, R. Wiesendanger

Using spin-polarized scanning tunneling microscopy in an external magnetic field, we have observed magnetic hysteresis on a nanometer scale in an ultrathin ferromagnetic film. An array of iron nanowires, being two atomic layers thick, was grown on a stepped tungsten (110) substrate. The microscopic sources of



B = 0.4T

B = 0T

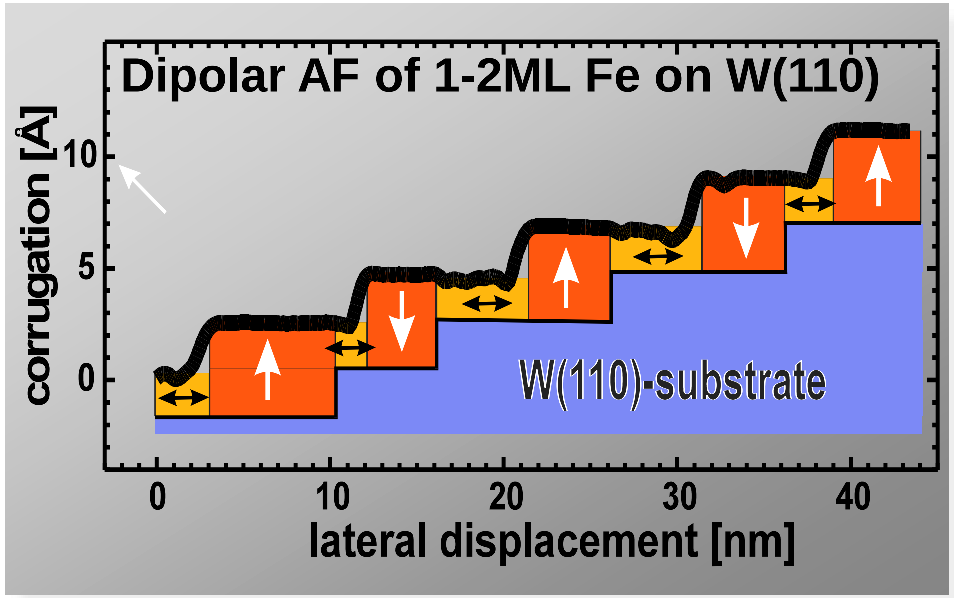
Domain Walls: 2 ML Fe on W(110)

Dzyaloshinskii-Moriya Interaction:

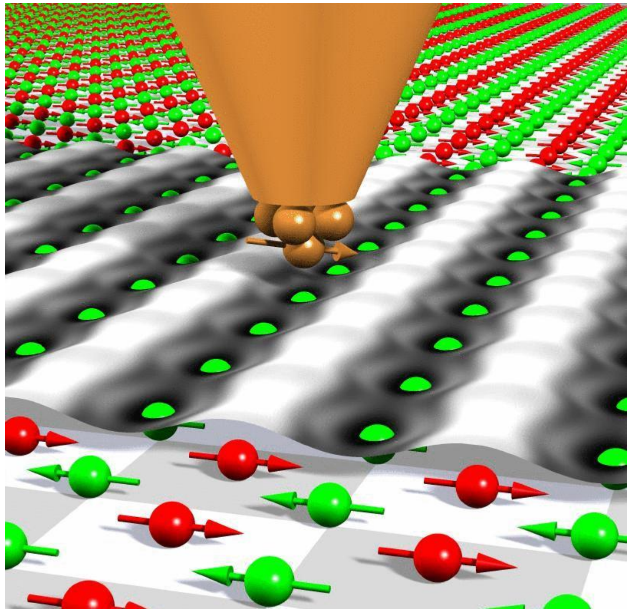
- Orientation of Domain Wall
- Uni-rotationality of Domain Wall
- Type of Domain Wall

MAGNETIC NANOSTRIPES OF FE/W(110)

Magnetic Structure on the nanometer scale:



Spin-Polarized STM at the atomic scale



$$I(\vec{r}_\parallel, z, V, \Theta) = I_0(\vec{r}_\parallel, z, V) + I_P(\vec{r}_\parallel, z, V, \Theta)$$

Tersoff-Hamann model

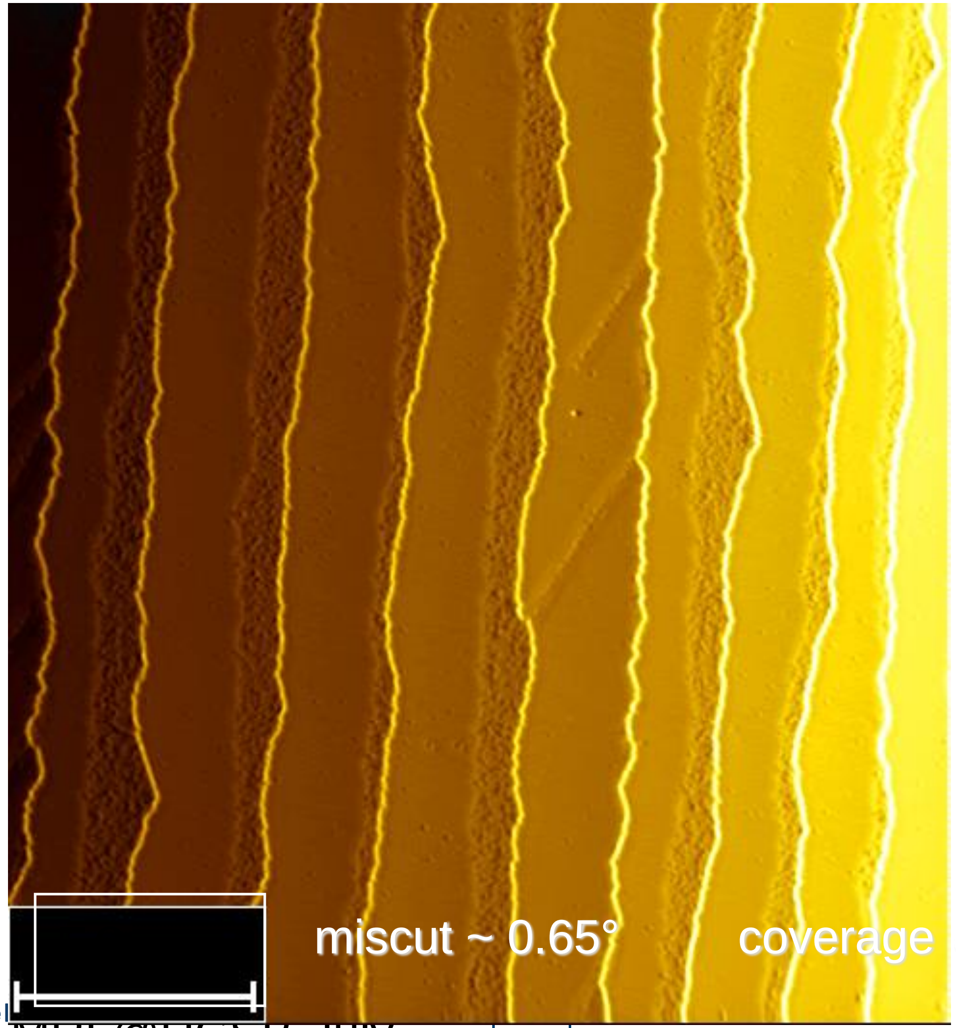
$$\propto n_T \int_{E_F}^{E_F+eV} n_S(\vec{r}_\parallel, z, \epsilon)\, d\epsilon + \vec{m}_T \int_{E_F}^{E_F+eV} \vec{m}_S(\vec{r}_\parallel, z, \epsilon)\, d\epsilon$$

D. Wortmann, S. Heinze, Ph. Kurz, G. Bihlmayer and S. Blügel, PRL **86** (2001)

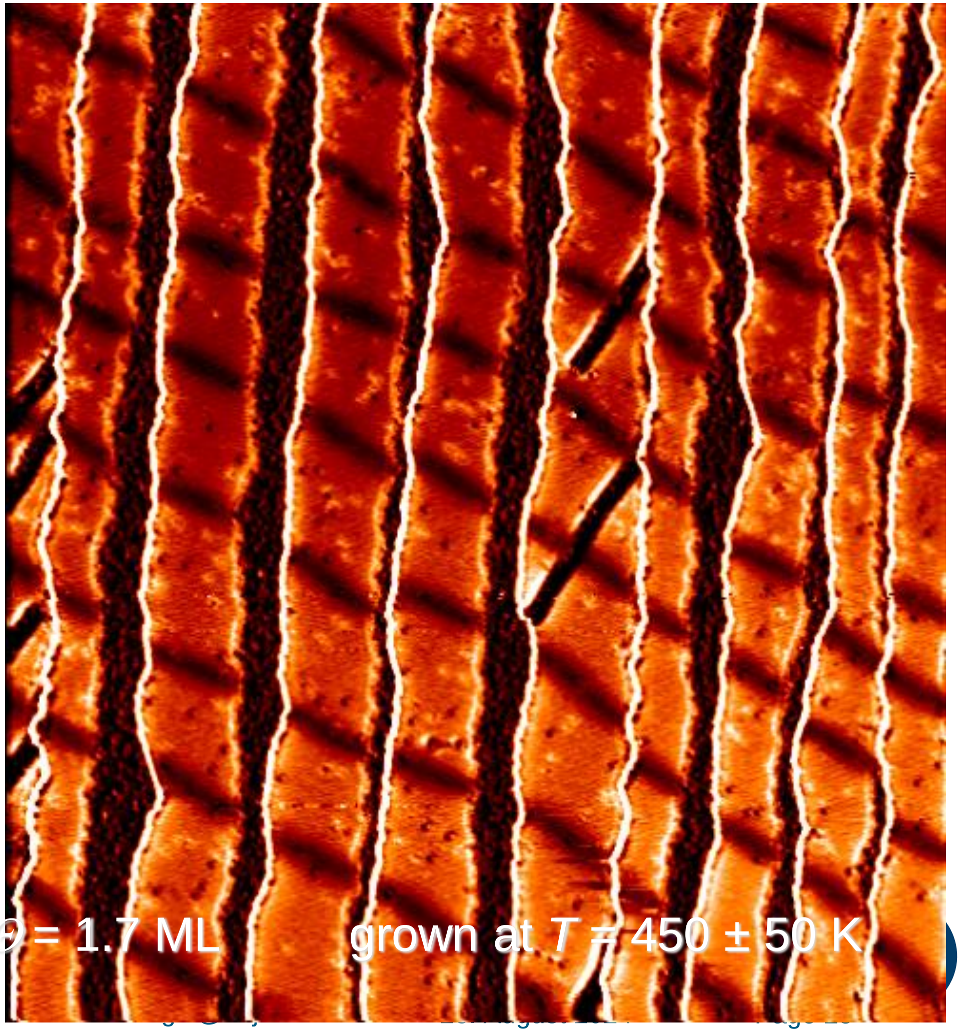
ELECTRONIC SPIN-DIRECTION CONTRAST AT FE NANOWIRES

W-substrate

topography



dI/dU map at U = 50 mV



miscut ~ 0.65°   coverage Θ = 1.7 ML   grown at T = 450 ± 50 K

FIRST HINT ON DMI AT SURFACES

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **67**, 020401(R) (2003)

Spin-polarized scanning tunneling microscopy study of 360° walls in an external magnetic field

A. Kubetzka,* O. Pietzsch, M. Bode, and R. Wiesendanger

Institute of Applied Physics and Microstructure Research Center, University of Hamburg, Jungiusstrasse 11, 20355 Hamburg, Germany

(Received 4 September 2002; published 9 January 2003)

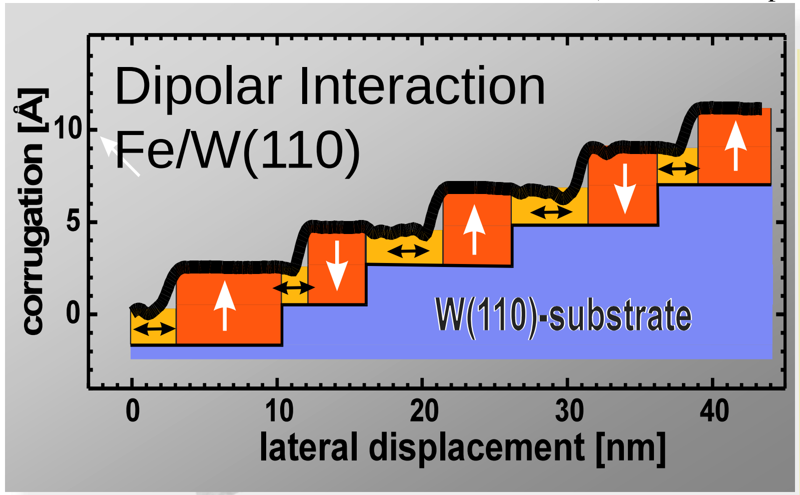


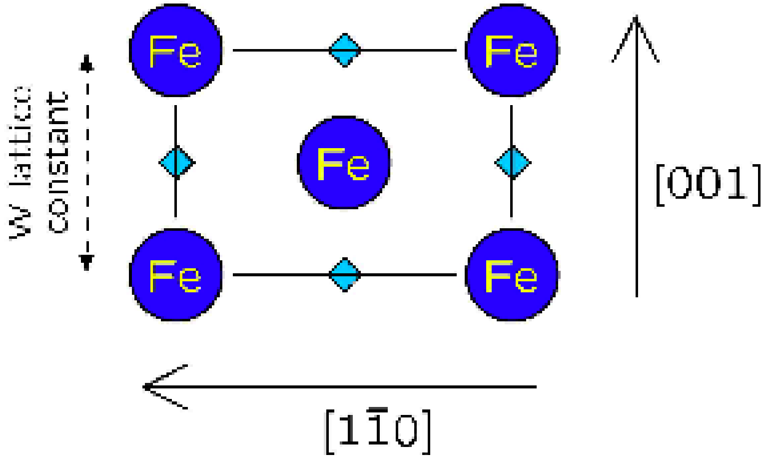
FIG. 1. 200×200 nm^2 constant-current (topography) image of 1.8 ML Fe on W(110), colorized with dI/dU map, recorded with a ferromagnetically coated W tip at $U = -0.3$ V, $I = 0.3$ nA, and $T = 14$ K. Two types of 180° domain walls can be distinguished by their in-plane magnetization component (see arrows).

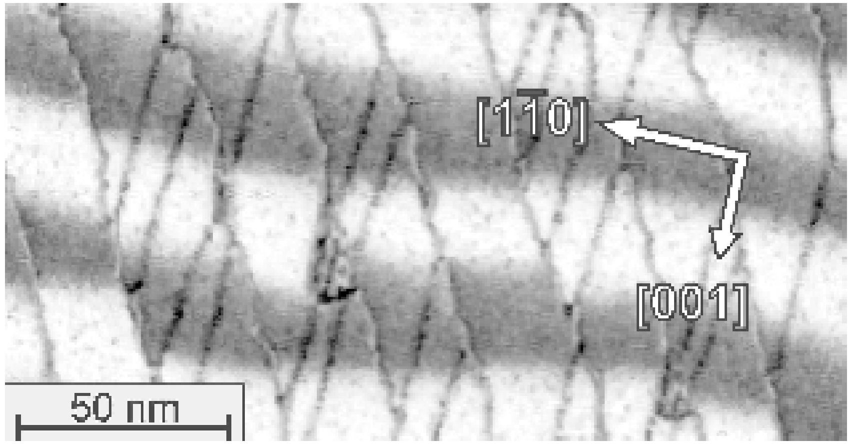
ments reveals that (i) the magnetization rotates along every single nanowire with a defined chirality, and that (ii) the rotational sense is the same in each of the 12 wires within the imaged area. These findings are consistent with data from a

which is effectively frozen in a metastable state. Observation (ii) is not yet fully understood. It might be connected to the miscut of the sample and/or the deviation of the axis of the wires from the [001] direction.

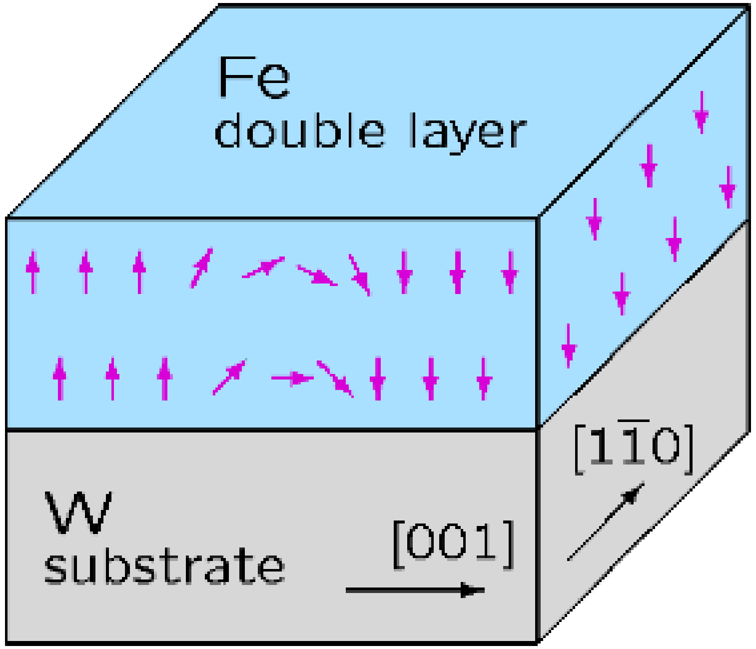
JÜLICH Forschungszentrum

DOMAIN-WALLS: 2 ML FE ON W(110)





Kubetzka, Bode, Pietzsch, Wiesendanger
PRL **88**, 057201 (2002)



Domain walls always oriented normal to [001] !

(PRL **92**, 077207 (2004))

Dzyaloshinskii-Moriya Interaction (DMI)

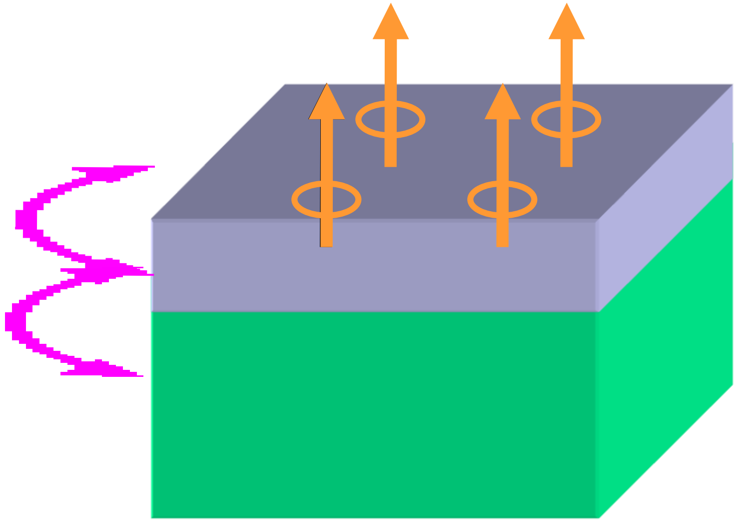
DZYALOSHINSKII-MORIYA INTERACTION

E. Dzyaloshinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 960 (1964) ; I. E. Dzyaloshinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 665 (1965)\
T. Moriya, PRL **4**, 228 (1960) ; T. Moriya, PR **120**, 91 (1960)

Break of inversion symmetry

$P(z) \neq P(-z)$

Chiral magnetic interaction
(Dzyaloshinskii-Moriya)

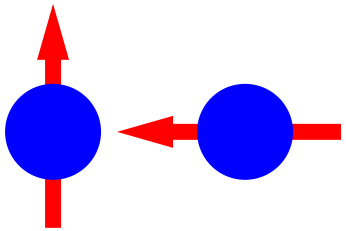


Magnetic Orientation

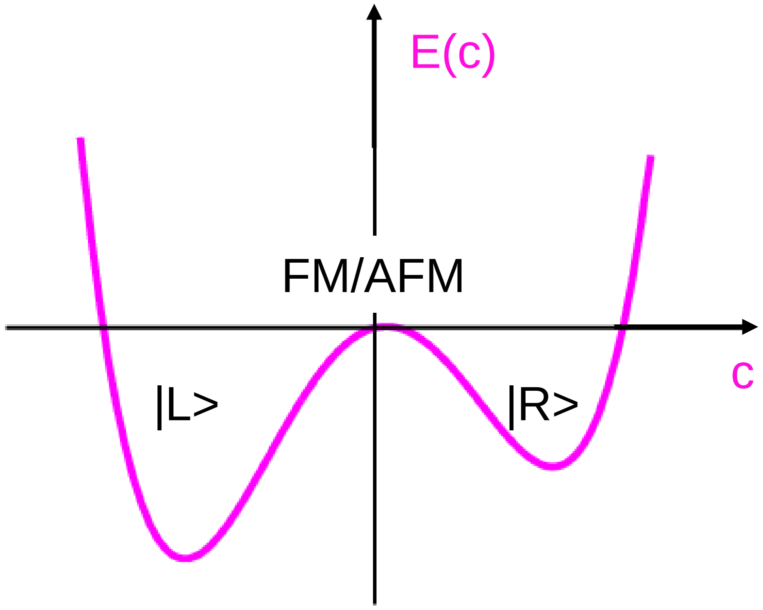
In-plane ⟺ Out-of-plane

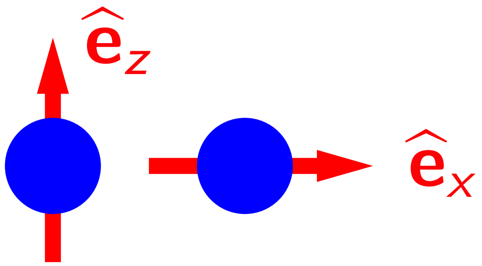
CHIRALITY OF DZYALOSHINSKII-MORIYA INTERACTION

I. E. Dzialoshinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **5,** 1259 (1957); J. Phys. and Chem. Sol. **4**, 241 (1958)



$\mathbf{c} = -\hat{\mathbf{e}}_y$





$\mathbf{c} = \mathbf{m}_i \times \mathbf{m}_j = \hat{\mathbf{e}}_y$

$$\mathcal{H}_{\mathrm{DM}} = -\sum_{ij} \mathbf{D}_{ij} \underbrace{(\mathbf{S}_i \times \mathbf{S}_j)}_{\mathbf{c}}$$

DZYALOSHINSKII-MORIYA INTERACTION

$$\mathcal{H}_{\mathrm{DM}} = -\mathbf{D}_{12} \underbrace{(\mathbf{S}_1 \times \mathbf{S}_2)}_{\mathbf{c}} \quad / \quad e_{\mathrm{DM}}(\underline{\mathbf{D}};\mathbf{m}) = \underline{\mathbf{D}} : (\nabla \mathbf{m} \times \mathbf{m})$$

- DMI in centro-symmetric systems: $\sum_{ij} \mathbf{D}_{ij} = \mathbf{0}$

  I. E. Dzialoshinskii, J. Exptl. Theoret. Phys. (U.S.S.R.) **5**, 1259 (1957), J. Phys. and Chem. Sol. **4**, 241 (1958)

  (nowadays popular in 2D systems , sometimes also termed hidden DMI)

- DMI in **non-**centro-symmetric systems $\sum_{ij} \mathbf{D}_{ij} \neq \mathbf{0}$

  J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 960 (1964); J. Exptl. Theoret. Phys. (U.S.S.R.) **20**, 665 (1965)

  ➔ leads to ordered structure with spatial modulation

MAGNETIC INTERACTIONS

$$H = \sum_{i,j} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j + \sum_{i} \mathbf{S}_i^T \cdot \underline{K}_i \cdot \mathbf{S}_i \; \mathbf{S}_j)$$

Isotropic symmetric exchange

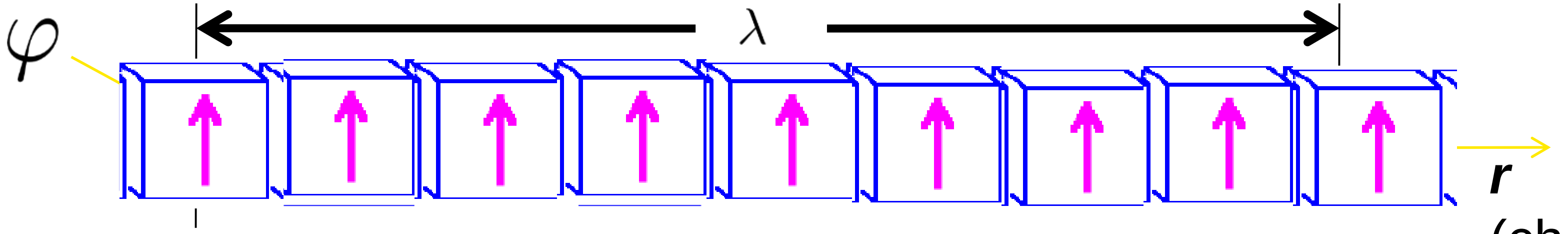
Magnetic Anisotropy Energy

ge

oriya

SOC effects

- Heisenberg-type interaction
- relativistic correction (SOC)
- surfaces and chains (breaking of inv. symmetry)



(chain direction)

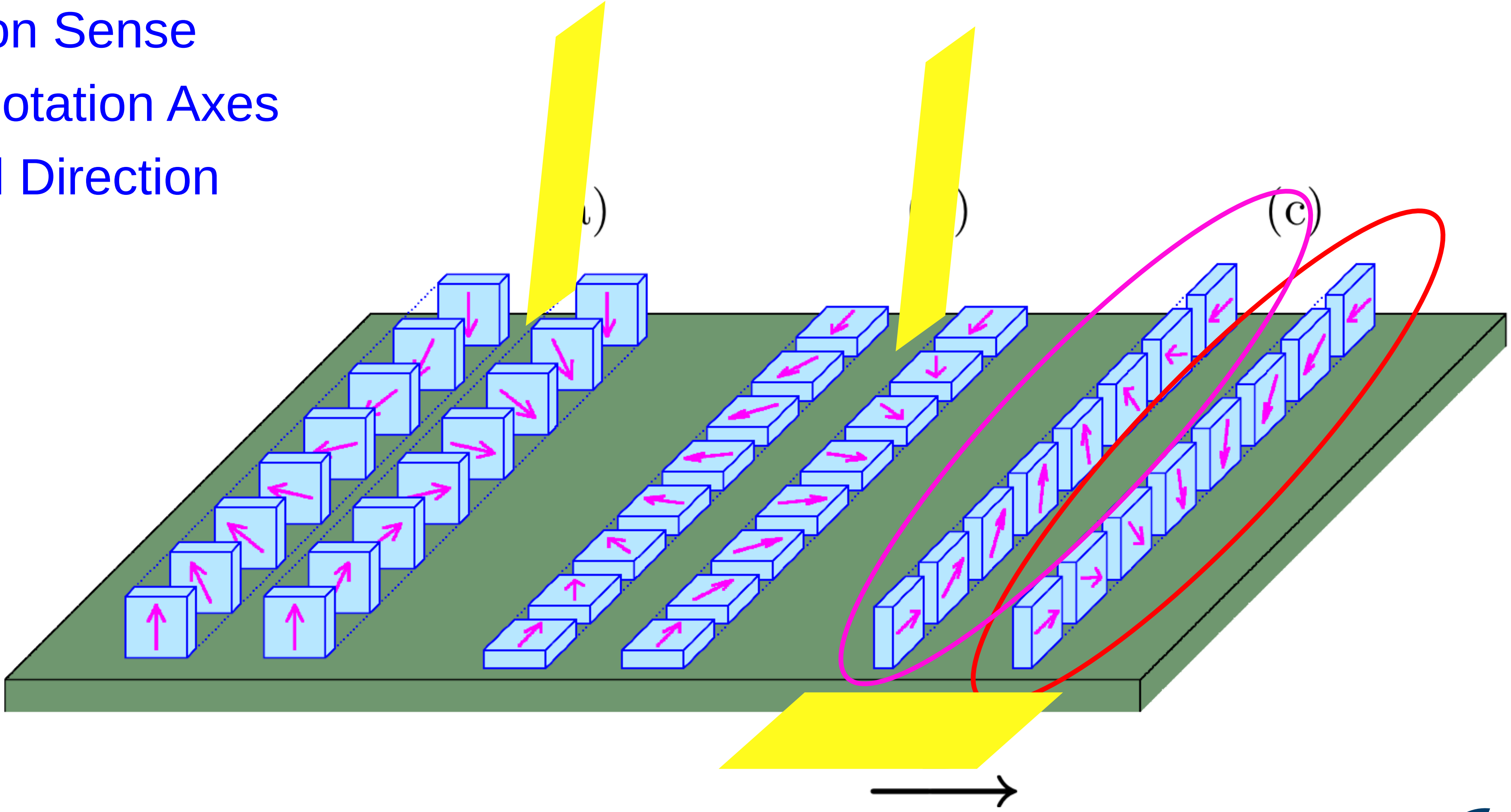
- now: homogeneous flat spiral: $\frac{d\varphi}{dr} = 2\pi/\lambda = \text{const.}$

$$\Rightarrow E(\lambda) = A\lambda^{-2} + D\lambda^{-1} + \overline{K}$$

JÜLICH Forschungszentrum

SPIN-SPIRALS IN MAGNETIC WIRES

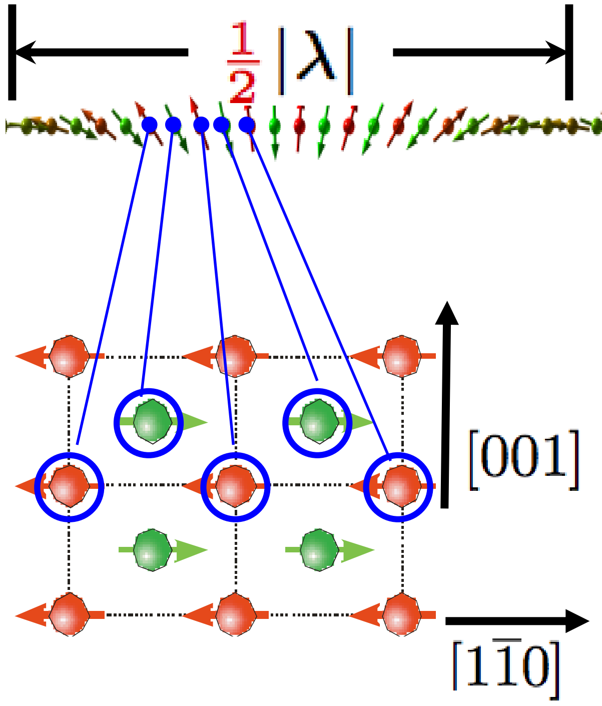
- Rotation Sense
- Spin Rotation Axes
- Spatial Direction



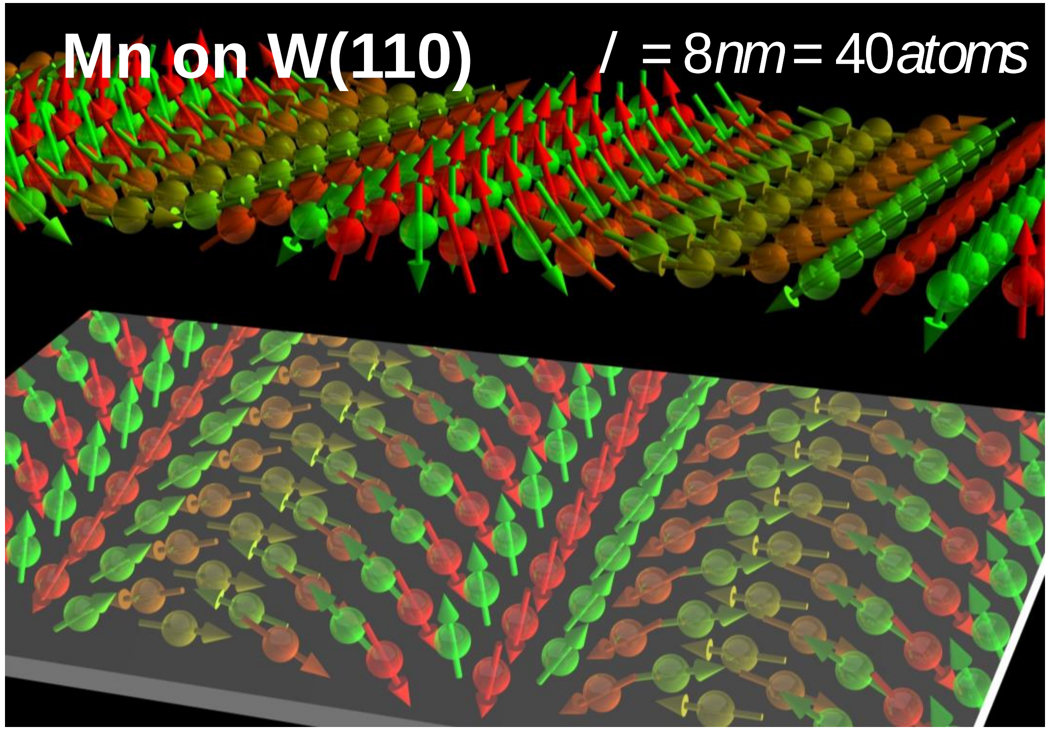
HOMOCHIRAL MAGNETIC SPIRAL: 1ML Mn on W(110)

Bode, Heide, von Bergmann, Ferriani, Heinze, Bihlmayer, Kubetzka, Pietzsch, Blügel, Wiesendanger, Nature **447,** 190 (2007)

Magnetic Configuration:

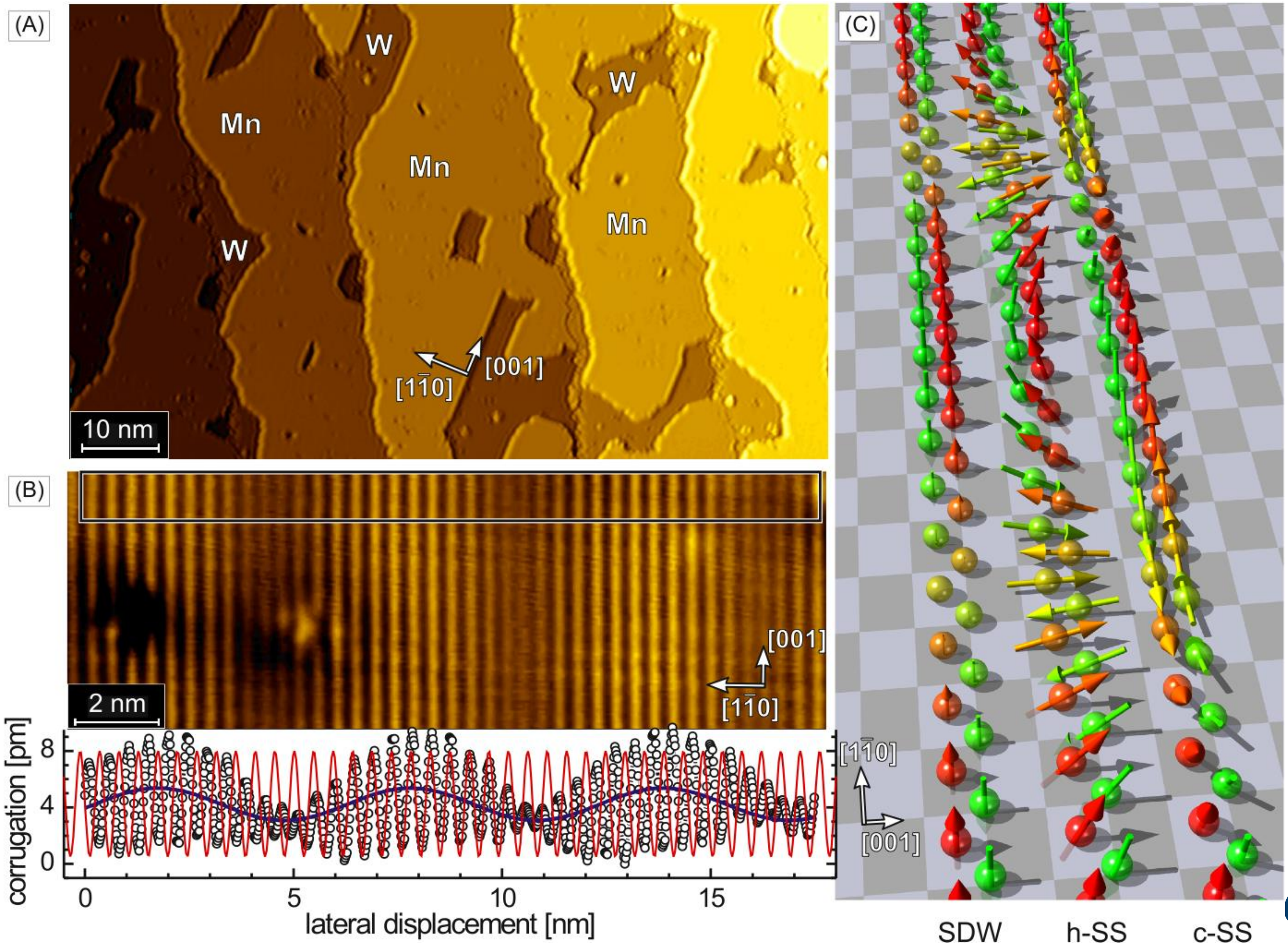


homochiral magnetism



HOMOCHIRAL MAGNETIC SPIRAL: 1ML Mn on W(110)

Bode, Heide, von Bergmann, Ferriani, Heinze, Bihlmayer, Kubetzka, Pietzsch, Blügel, Wiesendanger, Nature **447,** 190 (2007)



CHIRAL DOMAIN WALL

Micromagnetic Model:

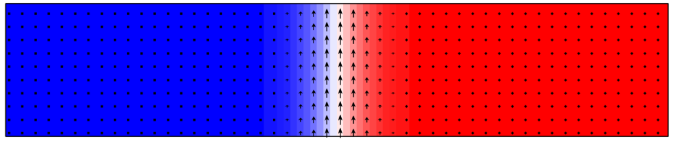
$$E[\vec{m}] = \int \mathrm{d}\vec{r} \left[J\, \dot{\vec{m}}(\vec{r})^2 + \vec{D}\cdot(\, \vec{m}(\vec{r}) \times \dot{\vec{m}}(\vec{r})\,) + \vec{m}(\vec{r})^{\dagger}\cdot \mathsf{K} \cdot \vec{m}(\vec{r}) \right]$$

+ planar approximation:

$$\vec{m}(\vec{r}) = m \cdot \varphi(x)$$

+ DW boundary condition:

$$\sin\varphi \xrightarrow{x\to -\infty} -1$$
$$\sin\varphi \xrightarrow{x\to +\infty} +1$$



$$E[\varphi] = \int_{-\infty}^{+\infty} \mathrm{d}x \left(A(J)\, \dot{\varphi}(x)^2 + D\, \dot{\varphi}(x) + K \, \cos^2 \varphi(x) \right)$$

2 ML FE/W(110) SPIN ROTATION PATHS

Micromagnetic Model:

$$E[\varphi] = \int_{-\infty}^{+\infty} \mathrm{d}x \left(K \cos^2\varphi + A\,\dot{\varphi}^2 + D\,\dot{\varphi} \right) \quad \text{with} \quad \begin{cases} \sin\varphi \xrightarrow{x \to -\infty} -1 \\ \sin\varphi \xrightarrow{x \to +\infty} +1 \end{cases}$$

$$E = 4\sqrt{A\,K} \pm \pi\,D \quad \text{if } E[\varphi] \text{ stationary}$$

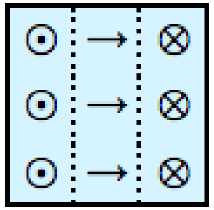
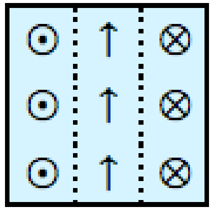
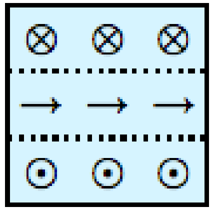
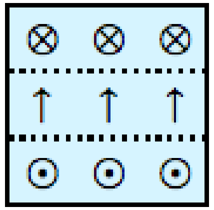
2 ML FE/W(110) SPIN ROTATION PATHS

$$E[\varphi] = \int_{-\infty}^{+\infty} \mathrm{d}x \left(K \cos^2\varphi + A\,\dot{\varphi}^2 + D\,\dot{\varphi} \right) \quad \text{with} \quad \begin{cases} \sin\varphi \xrightarrow{x\to-\infty} -1 \\ \sin\varphi \xrightarrow{x\to+\infty} +1 \end{cases}$$

$$E = 4\sqrt{A\,K} \pm \pi D \quad \text{if } E[\varphi] \text{ stationary}$$

(the term $\pm \pi D$ is crossed out)

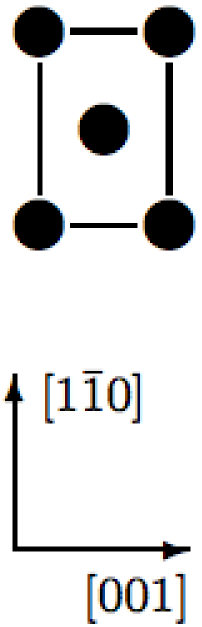
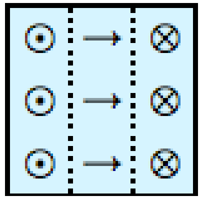
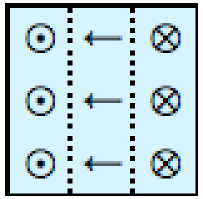
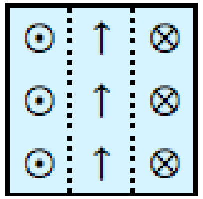
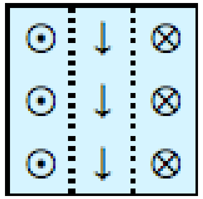
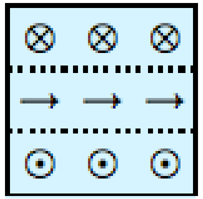
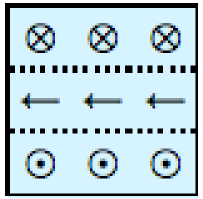
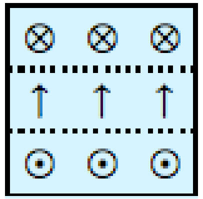
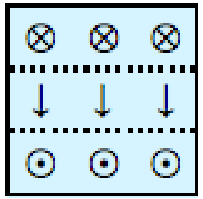
without D ⟶

	K_{001}	$K_{1\bar{1}0}$	
walls normal to [001] A_{001}			
walls normal to [1$\bar{1}$0] $A_{1\bar{1}0}$			

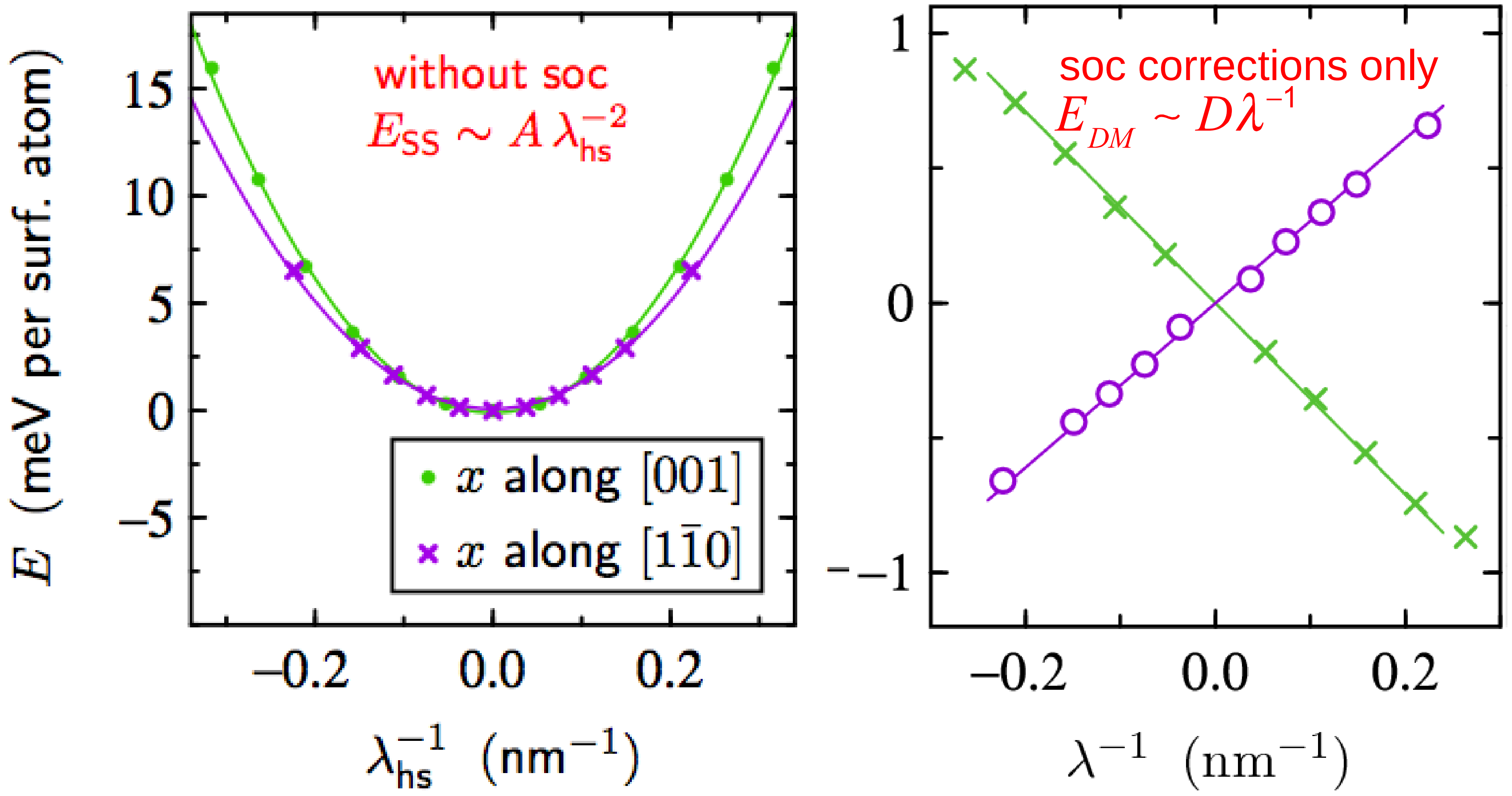
2 ML FE/W(110) SPIN ROTATION PATHS

$$E[\varphi] = \int_{-\infty}^{+\infty} \mathrm{d}x \left(K \cos^2\varphi + A\,\dot{\varphi}^2 + D\,\dot{\varphi} \right) \quad \text{with} \quad \begin{cases} \sin\varphi \xrightarrow{x \to -\infty} -1 \\ \sin\varphi \xrightarrow{x \to +\infty} +1 \end{cases}$$

$$E = 4\sqrt{AK} \pm \pi D \quad \text{if } E[\varphi] \text{ stationary}$$

	K_{001}		$K_{1\bar{1}0}$		
walls normal to [001] A_{001}	 $+D_{001}$	 $-D_{001}$	 $D=0$	 $D=0$	
walls normal to [1$\bar{1}$0] $A_{1\bar{1}0}$	 $D=0$	 $D=0$	 $+D_{1\bar{1}0}$	 $-D_{1\bar{1}0}$	

AB INITIO DETERMINATION OF EXCHANGE PARAMETERS A, D



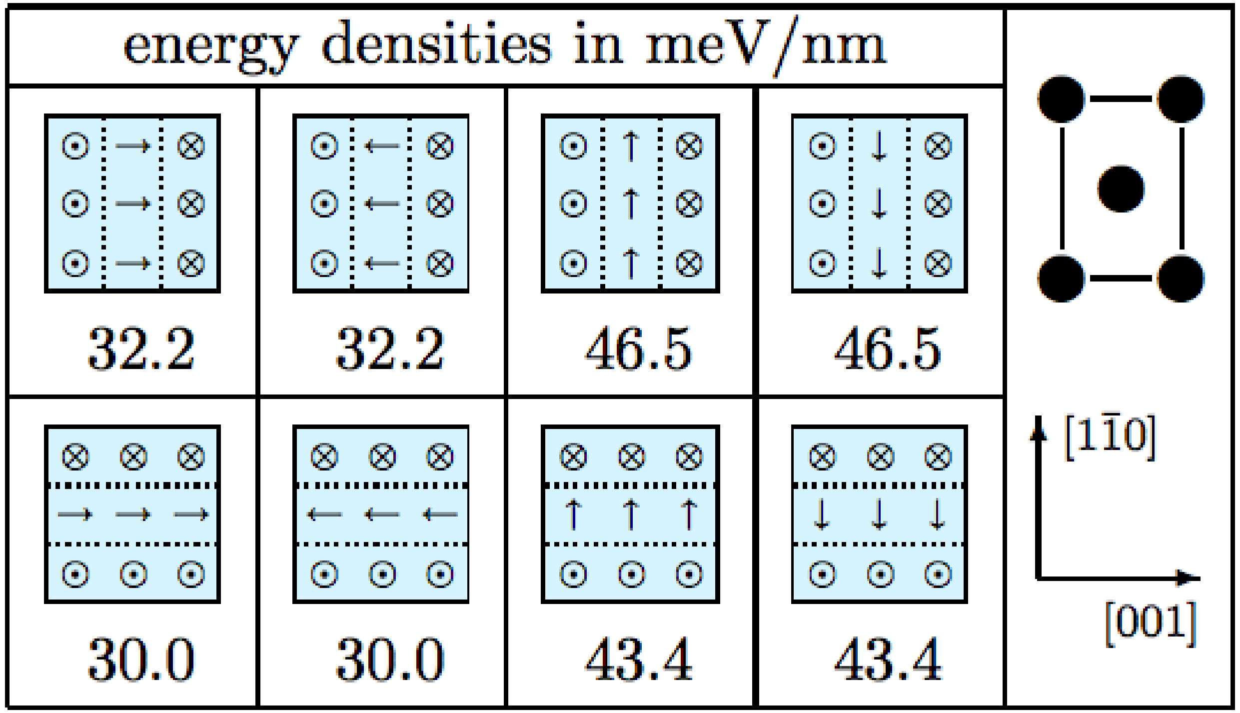
A_{001}=+ 58.8 meV
A_{110}= +51.1 meV

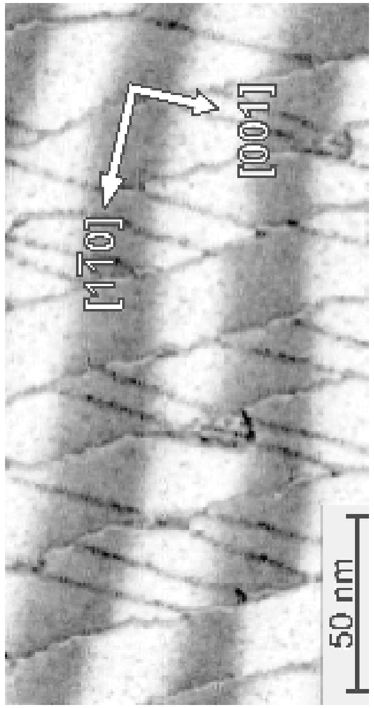
D_{001}=- 8.0 meVnm^{-1}
D_{110}= +6.7 meVnm^{-1}

2 ML FE/W(110) SPIN ROTATION PATHS

		001	1$\bar{1}$0
anisotropy energy	K / (meV nm^{-2})	1.1	2.3
spin stiffness	A / (meV)	58.8	51.1

$$E = 4\sqrt{AK}$$



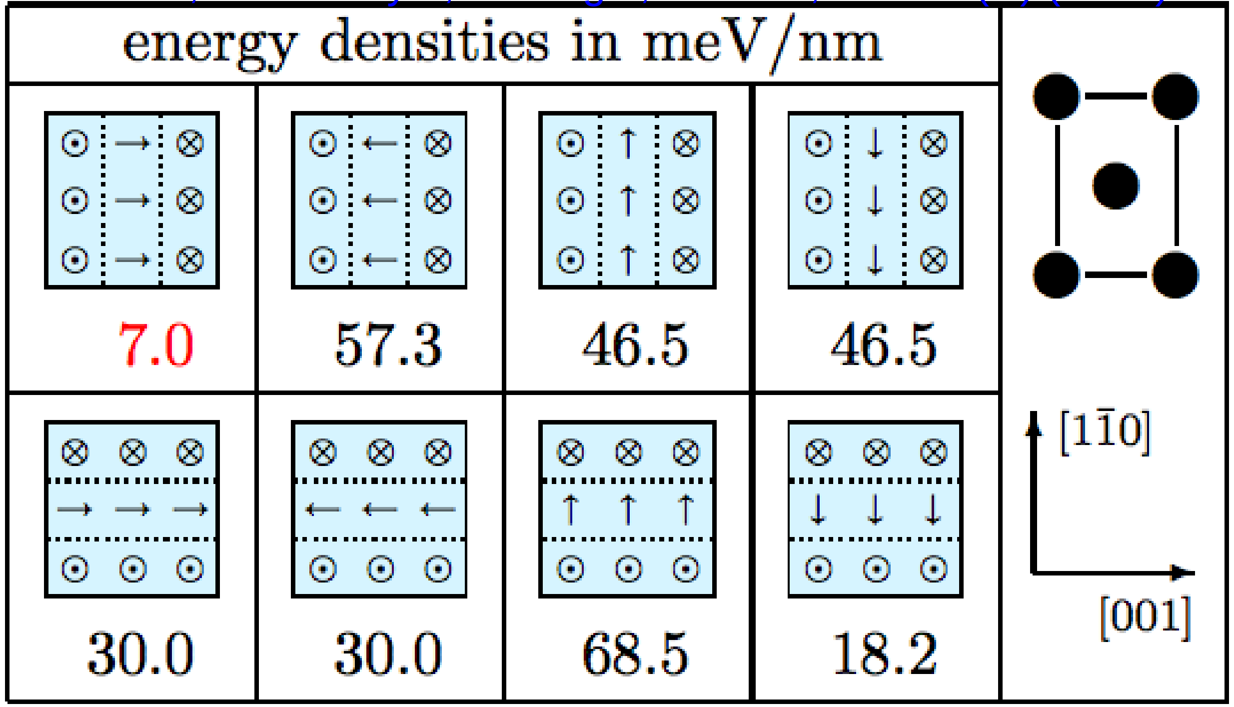


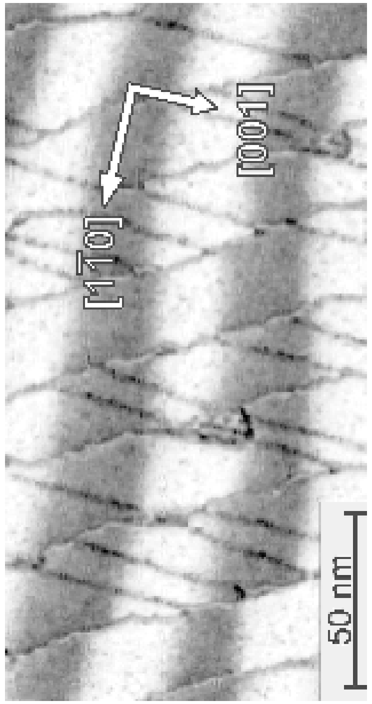
2ML FE/W(110) SPIN ROTATION PATHS

		001	1$\bar{1}$0
anisotropy energy	K / (meV nm^{-2})	1.1	2.3
spin stiffness	A / (meV)	58.8	51.1
DM interaction	D / (meV nm^{-1})	–8.0	6.7

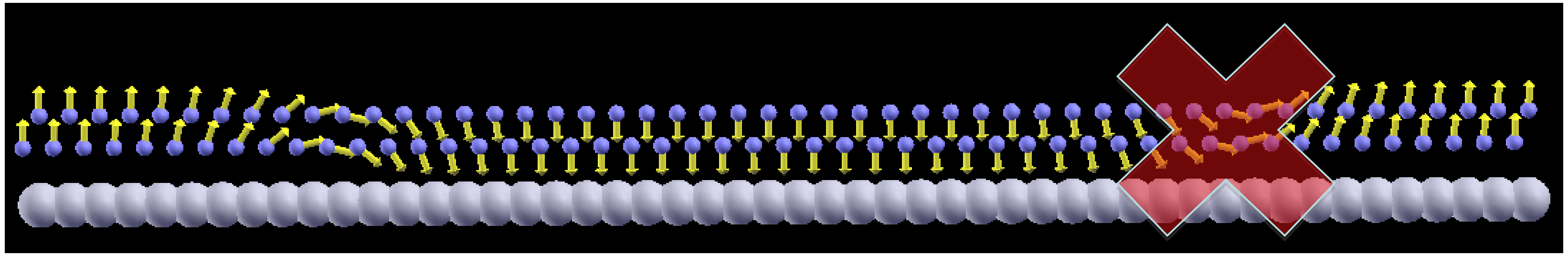
$$E = 4\sqrt{A\,K} \pm \pi\,D$$

M. Heide, G.Bihlmayer, S. Blügel, PRB **78**, 140403(R) (2008)





HOMOCHIRAL DOMAIN WALLS IN 2ML FE ON W(110)



Due to the Dzyaloshinskii-Moriya interaction this domain-wall is stabilized

This domain wall does not exist!

$$H = -\sum_{i,j} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j + \sum_i \mathbf{S}_i^T \mathcal{K} \mathbf{S}_i + \sum_{i,j} \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

domain wall width (first two terms)

rotational sense (last term)

Theory: M. Heide, G. Bihlmayer and S. Blügel, PRB **78**, 140403(R) (2008)

1D: DOMAIN WALL + DZYALOSHINSKII-M

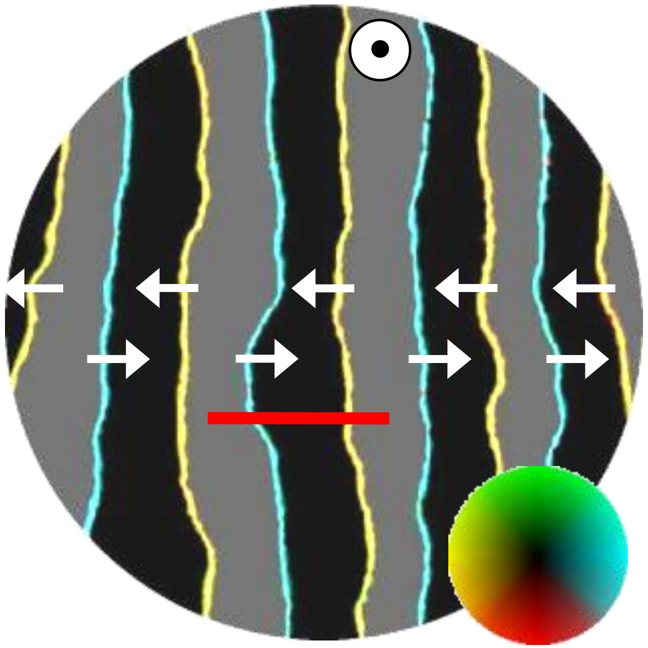
Micromagnetic Theory

Dzyaloshinskii-Moriya (DMI) can lead to chiral domain walls (1D)

$$E_{tot} = \int \left\{ A \left(\frac{df}{dx} \right)^2 + D \left(\frac{df}{dx} \right) + K_{eff} \sin^2 f \right\} dx$$

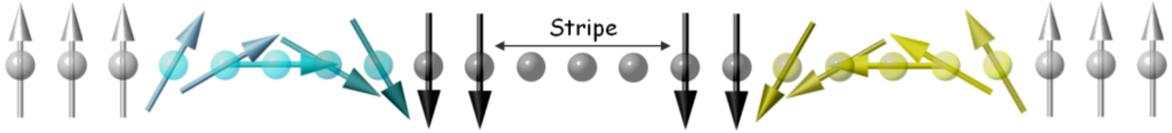
$$m_z(x) = m \cdot \varphi(x)$$

Heide, Bihlmayer, Blügel, PRB **78**, 140403(R) (‘08)



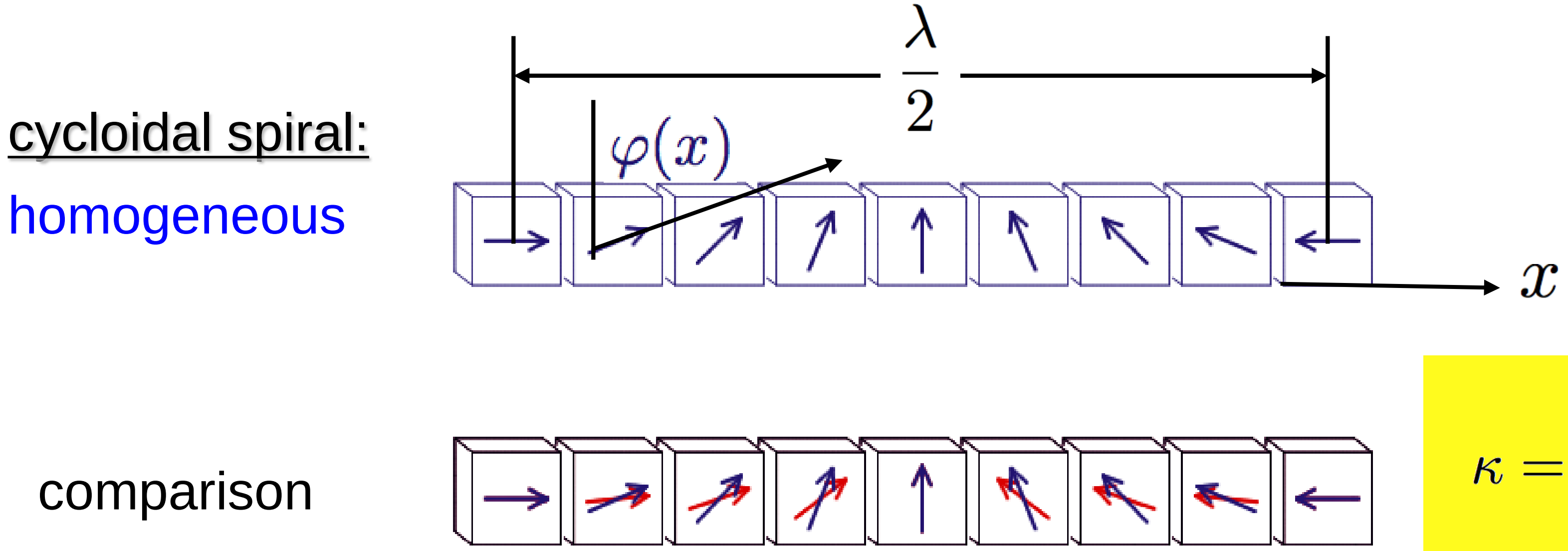
Spin Polarized Low Energy Electron Microscopy (SPLEEM)

Right-handed Cycloidal Chirality

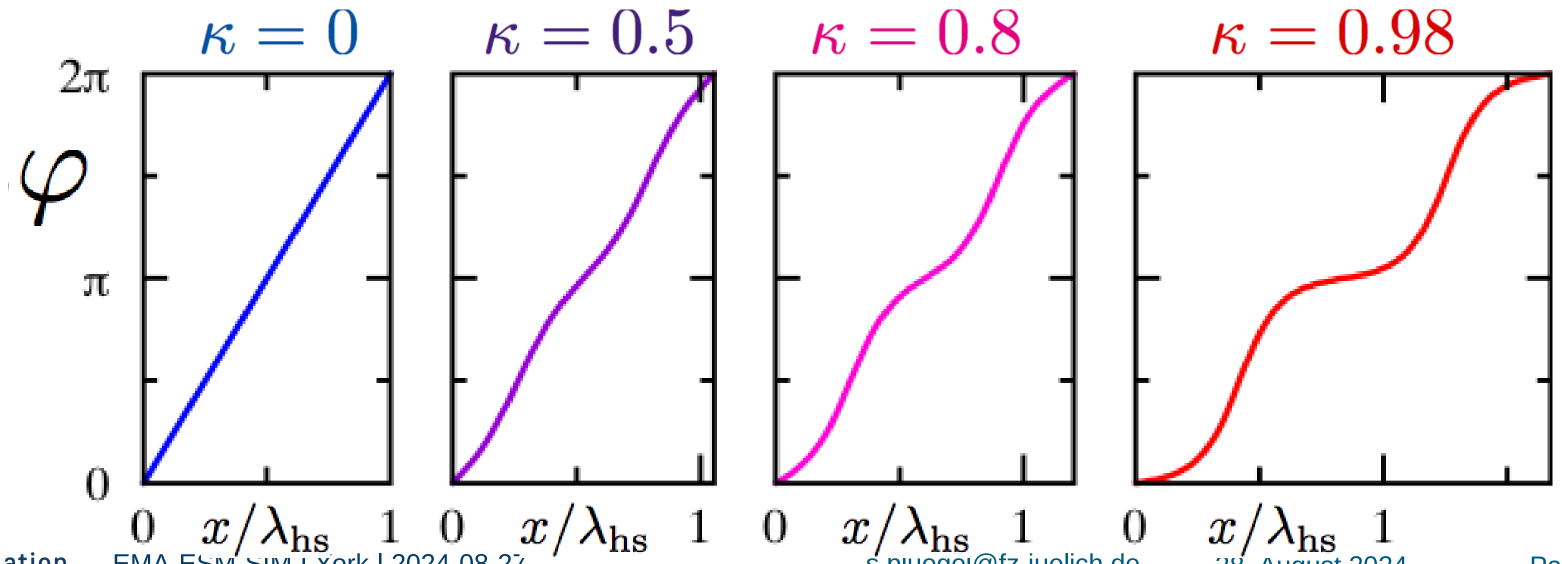


G. Chen, J. Zhu, A. Quesada, J. Li, A. T. N’Diaye, Y. Huo, T. P. Ma, Y. Chen, H.Y. Kwon, C. Won, Z. Q. Qiu, A. K. Schmid, and Y. Z. Wu, PRL **110**, 177204 (2013)

RELATION TO INHOMOGENEOUS CYCLOIDAL SPIRAL



$$\kappa = \frac{4}{\pi^3}\lambda_{\mathrm{hs}}\frac{K}{D}$$



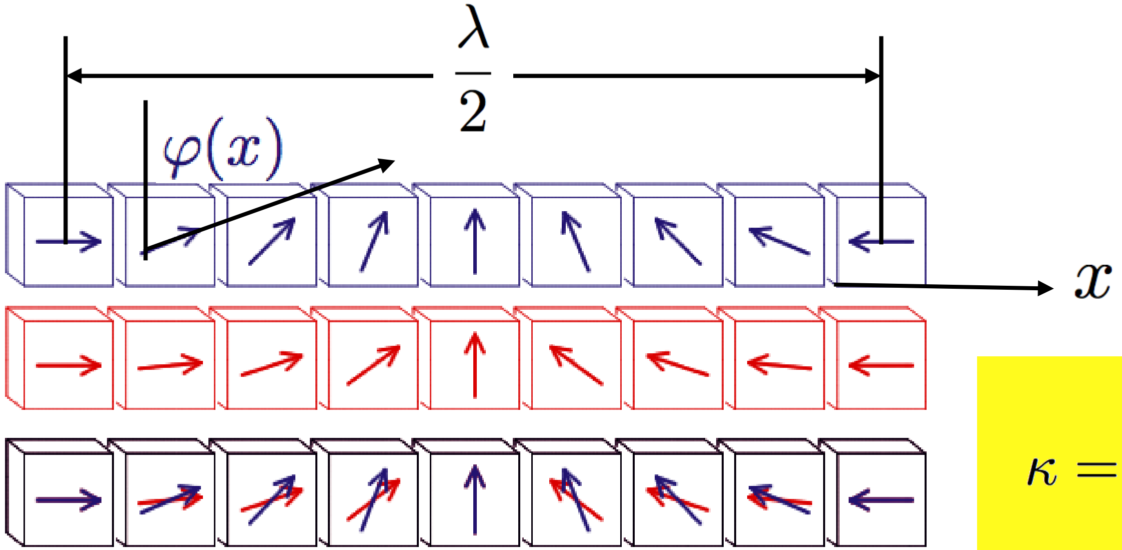
RELATION TO INHOMOGENEOUS CYCLOIDAL SPIRAL

<u>cycloidal spiral:</u>

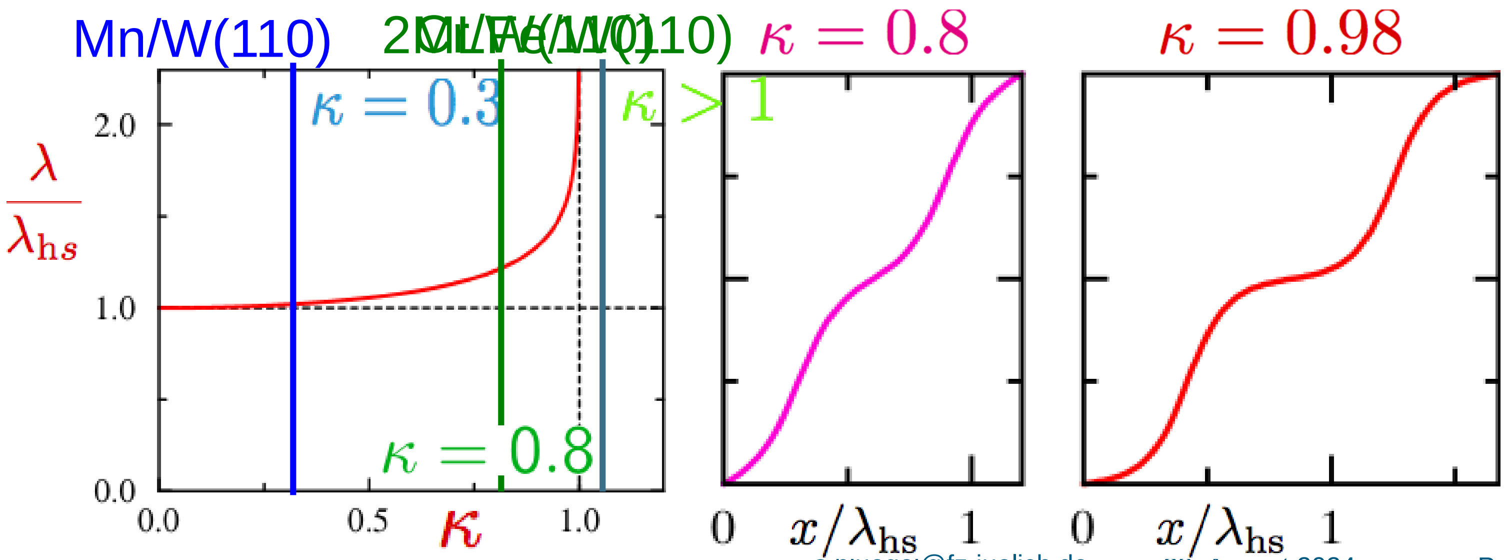
homogeneous

inhomogeneous

comparison



$$\kappa = \frac{4}{\pi^3}\lambda_{\mathrm{hs}}\frac{K}{D}$$



Skyrmion Radius

Micromagnetic analysis of skyrmion radius

Energy functional

$$E[\mathbf{m}] - E_{\mathrm{FM}} = t \int_{\mathbb{R}^2} \mathrm{d}r^2 \Big\{ \underbrace{A\,(\nabla \mathbf{m})^2}_{\text{exchange}} + \underbrace{D\ f(\nabla, \mathbf{m})}_{\text{DMI}} - \underbrace{K\,(m_z^2 - 1)}_{\text{aniso}} - \underbrace{M_0\,B\,(m_z - 1)}_{\text{Zeeman}} \Big\} + \Delta E_{\mathrm{demag}}[\mathbf{m}]$$

discuss later

Ansatz for magnetization field: Axial symmetric skyrmion tube $\mathbf{m}(\mathbf{r}) = \sin\Theta(\rho)\hat{\mathbf{e}}_\rho + \cos\Theta(\rho)\hat{\mathbf{e}}_z$

cylindrical coordinate system $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$

$\varrho = 2\rho/\ell_{\mathrm{w}}$

$b_k = B/K$

$\ell_{\mathrm{ss}} = 2A/|D|$

$\ell_{\mathrm{ih}} = |D|/2|K|$

$\kappa = 4/\pi^2(\ell_{\mathrm{hs}}/\ell_{\mathrm{ih}})$

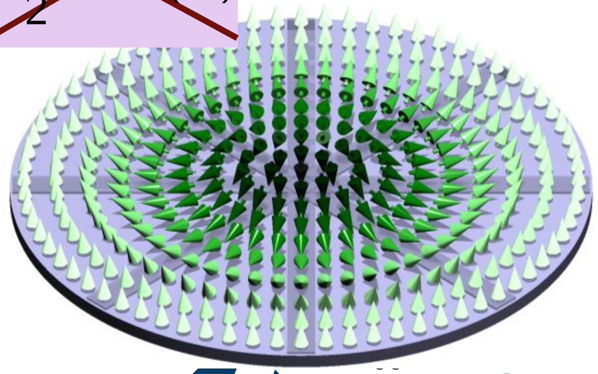
Optimal skyrmion profile $\Theta(\rho)$ $\delta E = E(\ldots) = 0$ → Euler-Lagrange eq.

dimensionless parameter $\kappa = \left(\frac{4}{\pi}\right)^2 \frac{AK}{D^2}$

zero field

Skyrmion profile equation: $\Theta'' + \frac{1}{\varrho}\Theta' - \frac{1}{2\varrho^2}\sin(2\Theta) + \frac{2}{\varrho}\left[\frac{2}{\pi}\frac{1}{\sqrt{\kappa}}\right]\sin^2\Theta - \frac{1}{2}\sin(2\Theta) - \frac{1}{2}b_k\sin(\Theta) = 0$

Solution by shooting method with boundary conditions: $\Theta(0) = \pi$ and $\lim_{\xi\to\infty}\Theta(\xi) = 0$



Skyrmion Radius R_{SK} [ℓ_w]: $m_z(R_{\mathrm{Sk}}) = 0$ or $\Theta(R_{\mathrm{Sk}}) = \frac{\pi}{2}$

domain-wall width $\ell_{\mathrm{w}} = \frac{1}{\pi}\sqrt{\frac{A}{|K|}}$

H. Jia, B. Zimmermann, M. Sallermann, G. Bihlmayer, S. Blügel, PRM **4**, 094407 (2020)

Micromagnetic analysis of skyrmion radius

Energy functional

$$E[\mathbf{m}] - E_{\mathrm{FM}} = t \int_{\mathbb{R}^2} \mathrm{d}r^2 \Big\{ \underbrace{A\,(\nabla\mathbf{m})^2}_{\text{exchange}} + \underbrace{D\; f(\nabla,\mathbf{m})}_{\text{DMI}} - \underbrace{K\,(m_z^2 - 1)}_{\text{aniso}} - \underbrace{M_0\,B\,(m_z - 1)}_{\text{Zeeman}} \Big\} + \Delta E_{\mathrm{demag}}[\mathbf{m}]$$

discuss later

Ansatz for magnetization field: Axial symmetry skyrmion tube $\mathbf{m}(\mathbf{r}) = \cos\Theta(\rho)\widehat{\mathbf{e}}_\rho + \sin\Theta(\rho)\widehat{\mathbf{e}}_z$

cylindrical coordinate system $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$

$\varrho = 2\rho/\ell_{\mathrm{w}}$

$b_k = B/K$

$\ell_{\mathrm{ss}} = 2A/|D|$

$\ell_{\mathrm{ih}} = |D|/2|K|$

$\kappa = 4/\pi^2(\ell_{\mathrm{hs}}/\ell_{\mathrm{ih}})$

Minimization $\dfrac{\delta E(\mathbf{m})}{\delta\mathbf{m}} = 0$ or $\dfrac{\delta E(\mathbf{m})}{\delta\Theta} = 0$

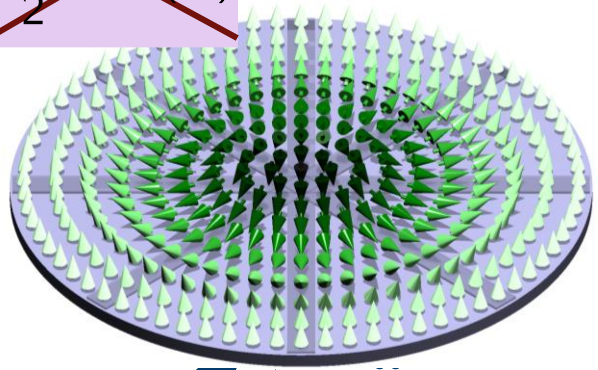
dimensionless parameter $\kappa = \left(\dfrac{4}{\pi}\right)^2 \dfrac{AK}{D^2}$

zero field

Skyrmion profile equation: $\Theta'' + \dfrac{1}{\varrho}\Theta' - \dfrac{1}{2\varrho^2}\sin(2\Theta) + \dfrac{2}{\varrho}\left[\dfrac{2}{\pi}\dfrac{1}{\sqrt{\kappa}}\right]\sin^2\Theta - \dfrac{1}{2}\sin(2\Theta) - \dfrac{1}{2}b_k\sin(\Theta) = 0$

Solution in limit of large ϱ : $\Theta'' = \dfrac{1}{2}\sin(2\Theta)$

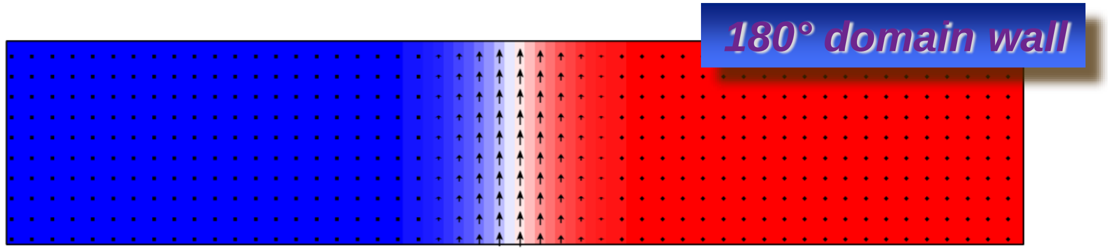
Domain wall solution [ℓ_{w}]: $\Theta(\rho) = 2\arctan\left(\exp\left(-2\dfrac{\rho \pm c}{\ell_{\mathrm{w}}}\right)\right)$

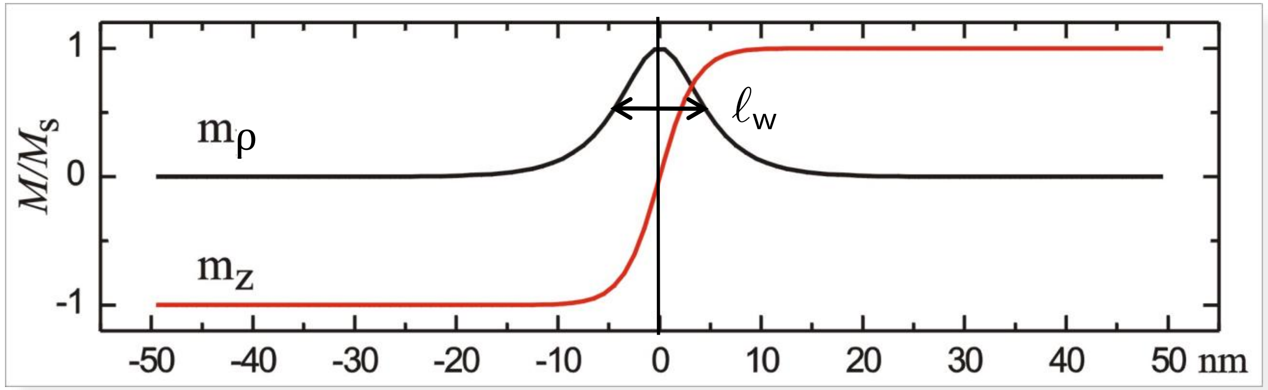


Micromagnetic analysis of skyrmion radius

Ansatz for magnetization field: Axial symmetry skyrmion tube $\mathbf{m}(\mathbf{r}) = \cos\Theta(\rho)\hat{\mathbf{e}}_\rho + \sin\Theta(\rho)\hat{\mathbf{e}}_z$

cylindrical coordinate system $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$

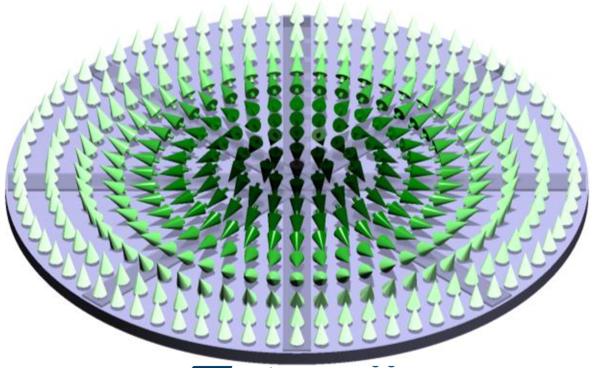




ρ_0 ⟶ ρ

Solution in limit of large ϱ : $\Theta'' = \frac{1}{2}\sin(2\Theta)$

Domain wall solution [ℓ_w]: $\Theta(\rho) = 2\arctan\left(\exp\left(-2\frac{\rho \pm c}{\ell_w}\right)\right)$



Skyrmion radius calculator

https://juspin.de/skyrmion-radius/

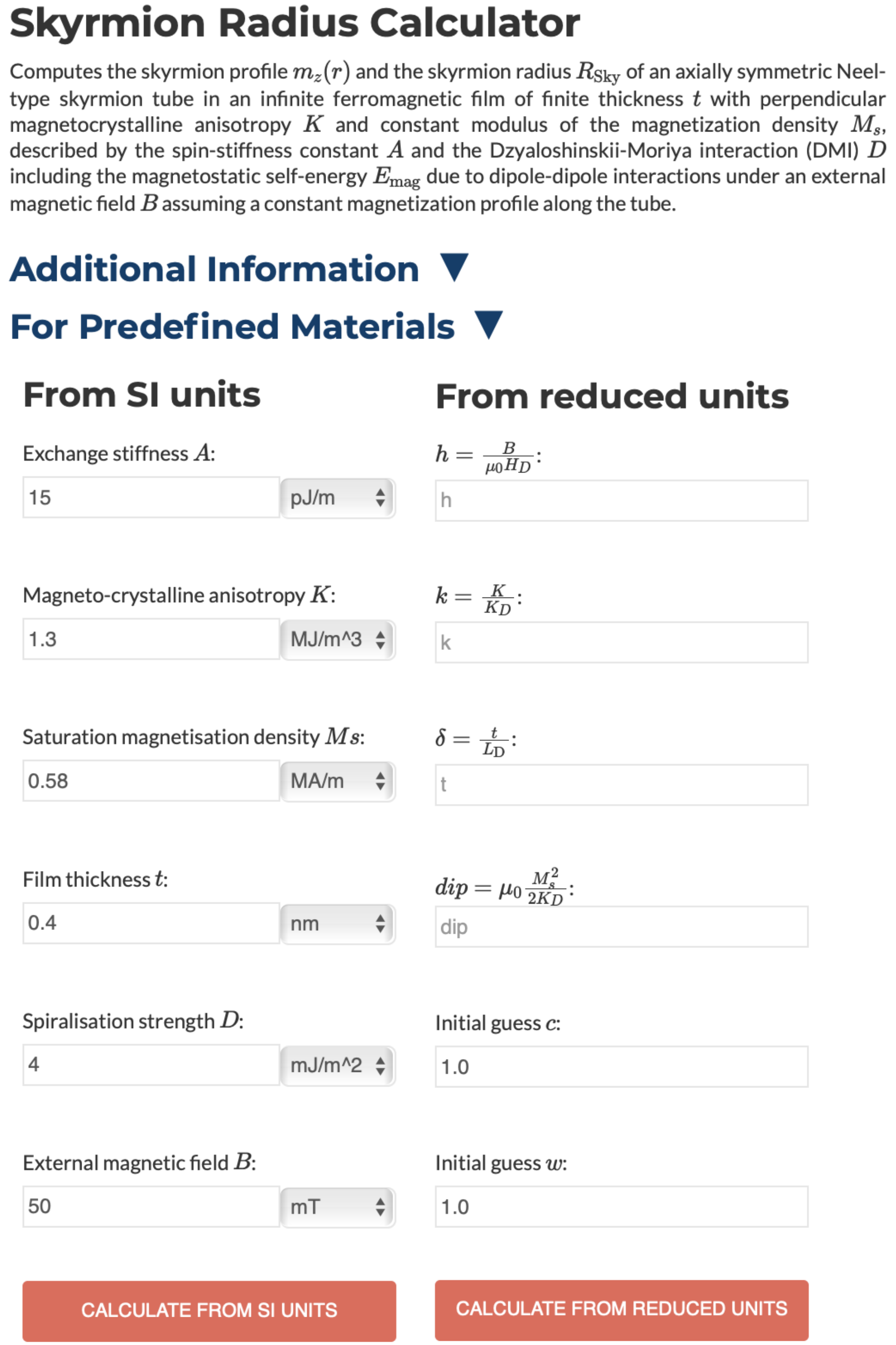
M. Sallermann, B. Zimmermann, F. Lux, S. Blügel, *in preparation*

Ansatz for magnetization field:

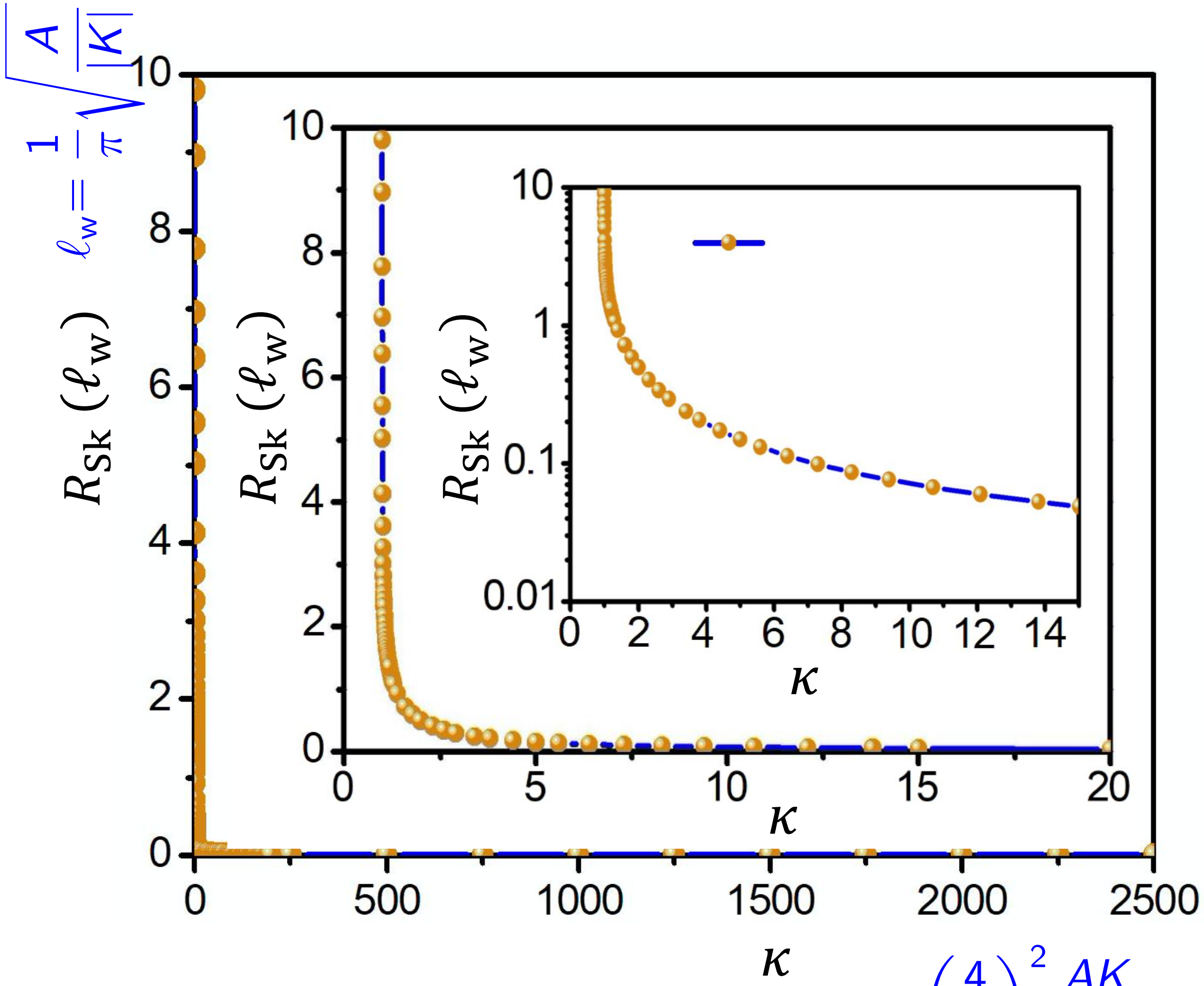
Axial symmetry skyrmion tube $\mathbf{m}(\mathbf{r}) = \cos\Theta(\rho)\hat{\mathbf{e}}_\rho + \sin\Theta(\rho)\hat{\mathbf{e}}_z$

cylindrical coordinate system $\mathbf{r} = (\rho\cos\varphi, \rho\sin\varphi, z)$

Domain wall solution [ℓ_w]: $\Theta(\rho) = 2\arctan\left(\exp\left(-2\frac{\rho \pm c}{\ell_w}\right)\right)$



Micromagnetic solution of skyrmion radius

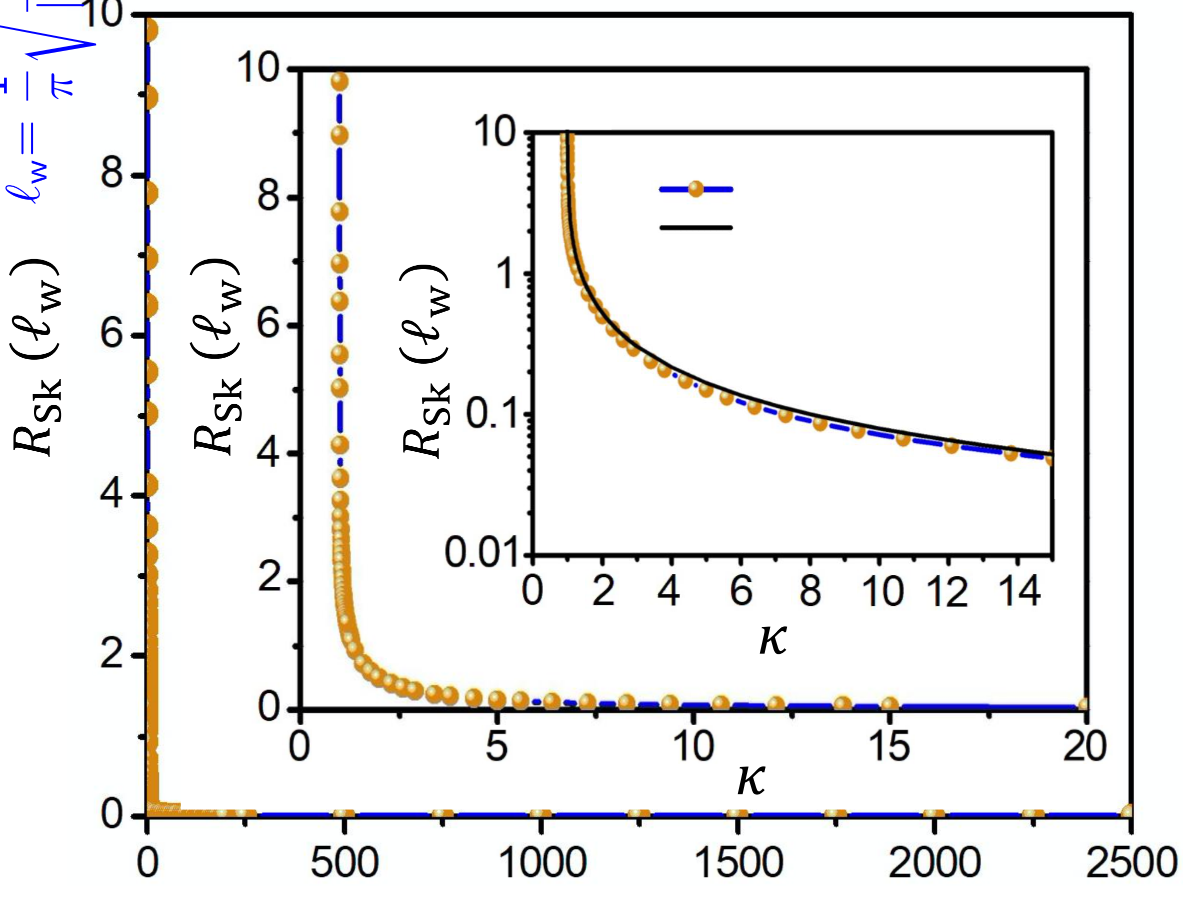


$\kappa = \left(\frac{4}{\pi}\right)^2 \frac{AK}{D^2}$

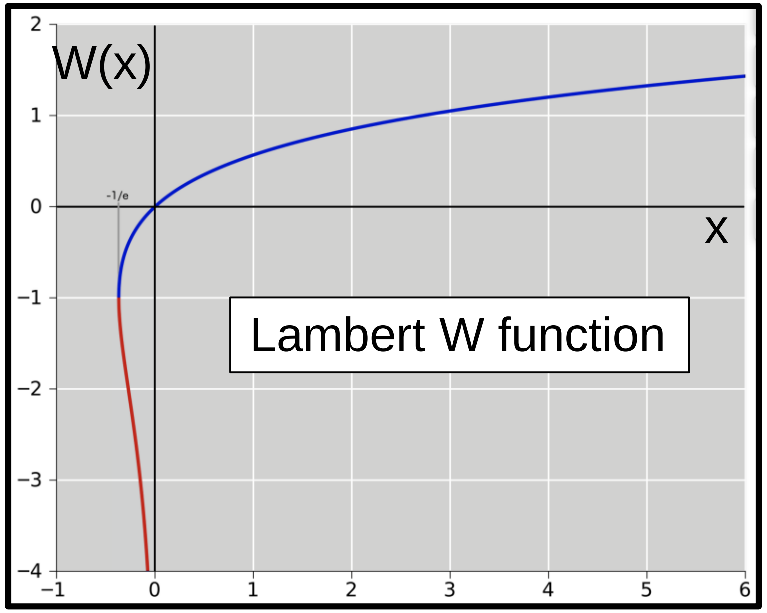
H. Jia, B. Zimmermann, M. Sallermann, G. Bihlmayer, S. Blügel, PRM **4**, 094407 (2020)

Micromagnetic solution of skyrmion radius

$\ell_w = \frac{1}{\pi}\sqrt{\frac{A}{|K|}}$



$\kappa = \left(\frac{4}{\pi}\right)^2 \frac{AK}{D^2}$



$W(y) = x$

$y(x) := xe^x$

Analytic approximation of R_{sk}:

$$R_{Sk}(\kappa)[\ell_w] \simeq \frac{1}{\pi\sqrt{\kappa}} \frac{1}{\ln(\alpha\pi\sqrt{\kappa})} \quad \text{with} \quad \alpha = 0.35$$

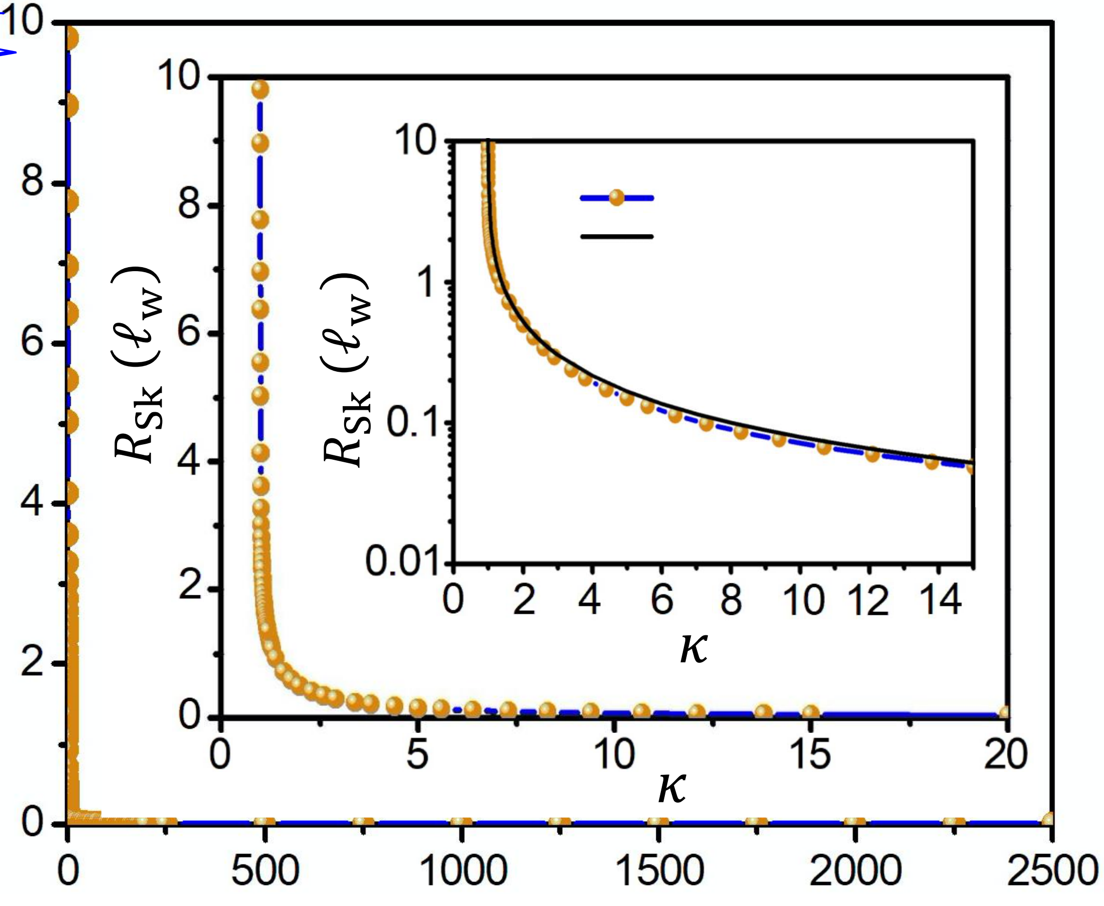
$$\text{for} \quad R_{Sk} \lesssim 2 \quad \text{or} \quad \kappa > 1.1$$

$$R_{Sk}(\kappa)[\ell_w] \simeq \frac{1}{2\sqrt{\kappa}} \frac{1}{W(\alpha\pi\sqrt{\kappa - 1})} \quad \text{with} \quad \alpha = 0.35$$

$$\text{for} \quad R_{Sk} \gtrsim 1 \quad \text{or} \quad \kappa \lesssim 1.6\,,$$

B. Zimmermann, F. Lux, M. Sallermann, S. Blügel,
to be pubished

Micromagnetic solution of skyrmion radius



R_{SK}: quickly diverging for **κ** < 2

radius variation for **κ** < 5

bad for transport

Analytic approximation of R_{sk}:

$$R_{Sk}(\kappa)[\ell_w] \simeq \frac{1}{\pi\sqrt{\kappa}}\frac{1}{\ln(\alpha\pi\sqrt{\kappa})} \quad \text{with} \quad \alpha = 0.35$$

for $R_{Sk} \lesssim 2$ or $\kappa > 1.1$

$$R_{Sk}(\kappa)[\ell_w] \simeq \frac{1}{2\sqrt{\kappa}}\frac{1}{W(\alpha\pi\sqrt{\kappa - 1})} \quad \text{with} \quad \alpha = 0.35$$

for $R_{Sk} \gtrsim 1$ or $\kappa \lesssim 1.6$,

B. Zimmermann, F. Lux, M. Sallermann, S. Blügel, to be pubished

Thanks !