MAGNETISATION PRECESSION AND SPIN WAVES

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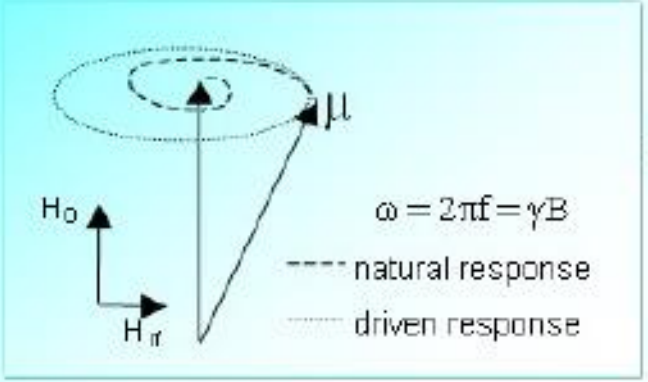
CSIC, SPAIN

Ferromagnetic resonance(FMR): (Arkadiev, 1911; Kittel, 1947)

A ferromagnetic body under applied field has a maximum absorption in frequencies:

$$\omega = \gamma\sqrt{[H + (N_x - N_z)M][H + (N_y - N_z)M]}$$

The absorption peak contains information about anisotropy field.

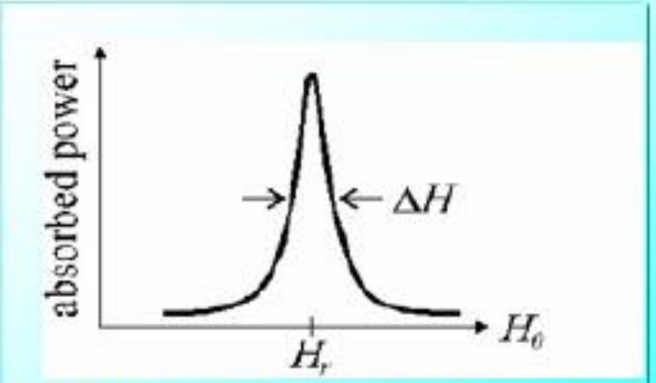


Precession and relaxation of **M** in response to an applied field **H**.

Torque on magnetisation

$$\frac{1}{\gamma}\frac{\partial M}{\partial t} = -[M \times H_0]$$

Lorentzian absorption line typical of FMR showing microwave power absorption as a function of swept bias field.



The absorption line width contains Information on damping processes

The Landau-Lifshitz (LL) and the Landau-Lifshitz-Gilbert (LLG) equations of motion

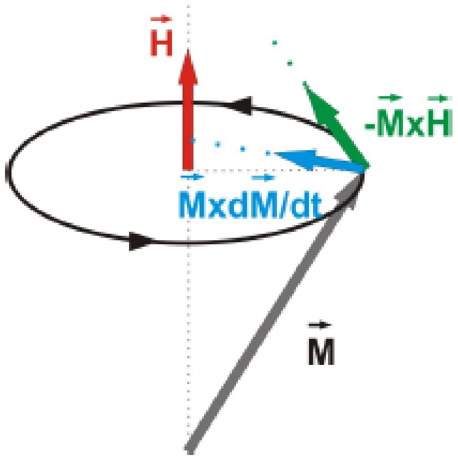
(for magnetization vector):

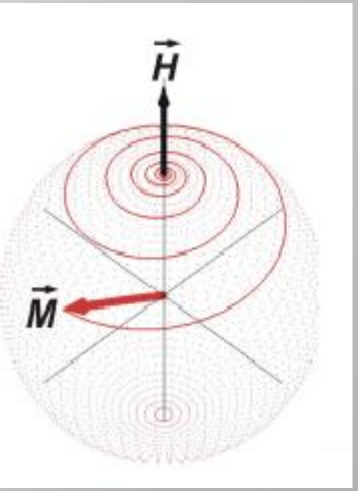
LL equation Landau-Lifshitz damping, 1935

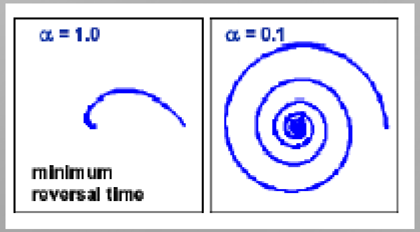
$$\frac{d\vec{M}}{dt} = -\gamma'_0\left[\vec{M}\times\vec{H}\right] - \frac{\alpha_{LL}\gamma_0}{M_s}\left[\vec{M}\times\left[\vec{M}\times\vec{H}\right]\right]$$

Gilbert equation
(physically more reasonable for large damping) Gilbert damping, 1955

$$\frac{d\vec{M}}{dt} = -\gamma_0\left[\vec{M}\times\vec{H}\right] + \frac{\alpha_G}{M_s}\left[\vec{M}\times\frac{d\vec{M}}{dt}\right]$$







Multiplying Gilber equation by x M and re-arranging
Landau-Lifshitz-Gilbert (LLG) equation

$$\frac{d\vec{M}}{dt} = -\frac{\gamma_0}{1+\alpha^2}\left[\vec{M}\times\vec{H}\right] - \frac{\gamma_0\alpha}{(1+\alpha^2)M_s}\left[\vec{M}\times\left[\vec{M}\times\vec{H}\right]\right]$$

Damping form is phenomenological
The value includes many intrinsic and extrinsic contributions

The Landau-Lifshitz equation: 80 years of history, advances, and prospects

V. G. Bar'yakhtar; B. A. Ivanov

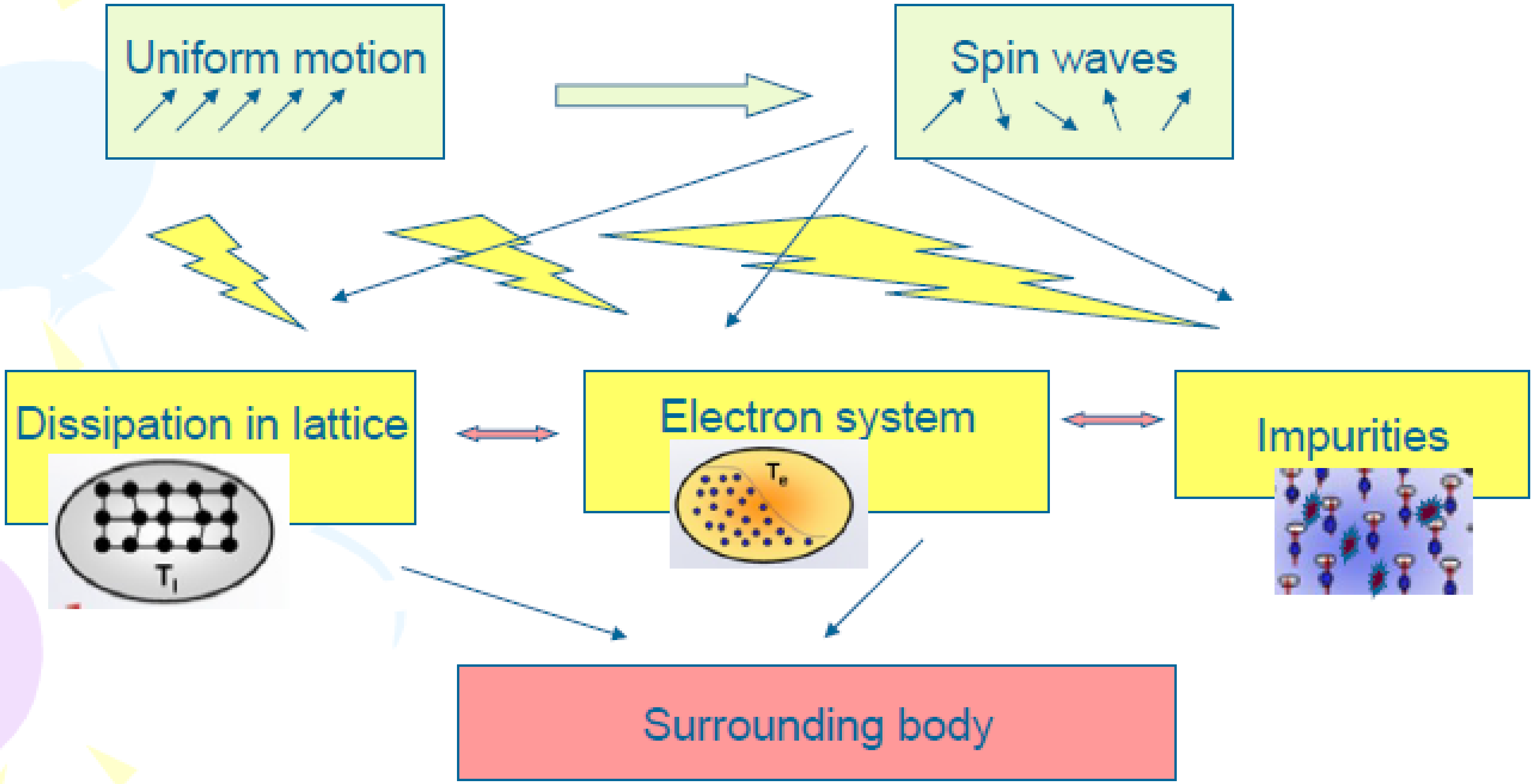
Low Temperature Physics 41, 663–669 (2015)

https://doi.org/10.1063/1.4931649

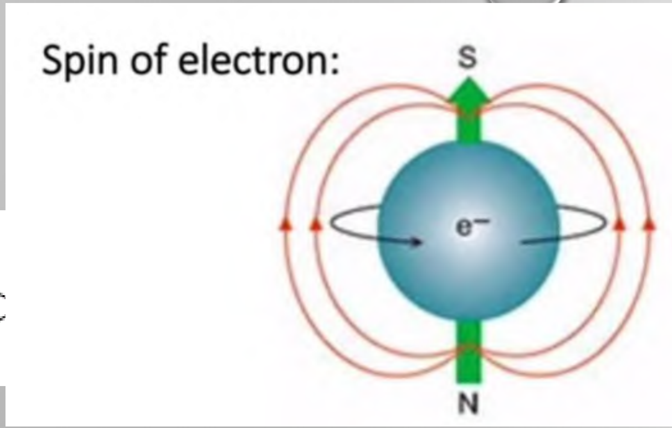
Translated by D. H. McNeill

Eighty years ago, the paper "On the theory of the dispersion of magnetic permeability in ferromagnetic bodies," by L. D. Landau and E. M. Lifshitz was published (Phys. Zs. Sowjetunion 8, 153 (1935)). For the modern reader it is easy to find the Russian translation of the paper in Landau's collected papers.[1] The page numbers from that translation are cited here. There is also a convenient English source.[2] The evolution of the physics of magnetic phenomena has shown that the results reported in that article turned out to be much broader than implied by its title. Some problems in the physics of magnetism that

Theory of magnetic damping constant (α):



QUANTUM MECHANICS OF ELECTRON SPIN



The time evolution of observable operator

$$i\hbar\frac{d}{dt} < \vec{S} >=< [\vec{S},\mathcal{H}] >$$

Hamiltonian (Zeeman term only)

Landé factor Borh magneton

$$\mathcal{H} = -\frac{g\mu_B}{\hbar}\vec{S}\cdot\vec{H}$$

$$[S_z,\mathcal{H}] = \frac{g\mu_B}{\hbar}i\hbar(S_yH_x - S_xH_y)$$

$$\frac{d}{dt} < \vec{S} >= \frac{g\mu_B}{\hbar}(\vec{S}\times\vec{H})$$

$$\vec{M} = -\mu_B g < \vec{S} >$$

Orbital magnetic moment is ignored

$$\frac{d}{dt}\vec{M} = -\gamma\vec{M}\times\vec{H} \quad \text{with} \quad \gamma = \frac{g\mu_B}{\hbar}$$

Larmor precession

$$\omega = \gamma H$$

Change of angular momentum == Torque

$$\frac{d\vec{L}}{dt} = [\vec{r}\times\vec{F}]$$

THE QUANTUM DERIVATION OF THE **LANDAU-LIFSHITZ-BLOCH (LLB) EQUATION /LANDAU-LIFSHITZ (LL) EQUATION**

D. A. Garanin. **Generalized Equation of Motion for a Ferromagnet** *Physica A*, **172**:470, 1991.

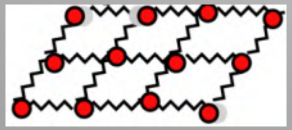
Hamiltonian:

Scattering probabilities

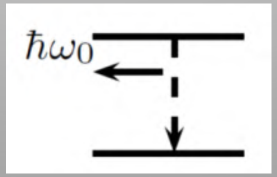
$$\hat{H} = -\mathbf{H}(t) \cdot \hat{\mathbf{S}} + \sum_q w_q \hat{a}_q^+ \hat{a}_q - \sum_q V_q (\boldsymbol{\eta} \cdot \hat{\mathbf{S}})(\hat{a}_q^+ + \hat{a}_{-q}) - \sum_{p,q} V_{p,q} (\boldsymbol{\eta} \cdot \hat{\mathbf{S}}) \hat{a}_p^+ \hat{a}_q$$

Zeeman

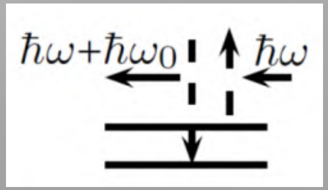
Phonon bath



Direct transformation



Raman processes



The derivation is based on the density matrix formalism

$$\rho = \sum_j p_j |\psi_j\rangle\langle\psi_j|,$$

Weak coupling with the bath:

$$\hat{\rho}(t) \cong \hat{\rho}_s(t)\hat{\rho}_b^{\mathrm{eq}}$$

The quantum derivation of the **Landau-Lifshitz-Bloch (LLB) equation /Landau-Lifshitz (LL) equation**

$$\frac{d}{dt}\hat{\rho}_s(t) = -\frac{i}{\hbar}\left[\hat{H}_s, \hat{\rho}_s(t)\right] - \frac{1}{\hbar^2}\int_0^t dt' \mathrm{Tr}_b \left[\hat{V}, \left[\hat{V}(t'-t)_I, \underbrace{\hat{\rho}_s(t)}_{\hat{\rho}_s(t')}\hat{\rho}_b^{eq}\right]\right]$$

Approximations for the LLB (LLG) derivation:

a) Markov approximation (short memory): $\hat{\rho}_s(t') \rightarrow \hat{\rho}_s(t)$

b) Secular approximation: neglect fast oscillating terms

c) Mean field for interactions (ferromagnet):

$$H \rightarrow H^{MFA} = \underbrace{H_{ex}}_{\text{Exchange}} + \underbrace{H}_{\text{Applied field}} + \underbrace{H_k}_{\text{Anisotropy}}$$

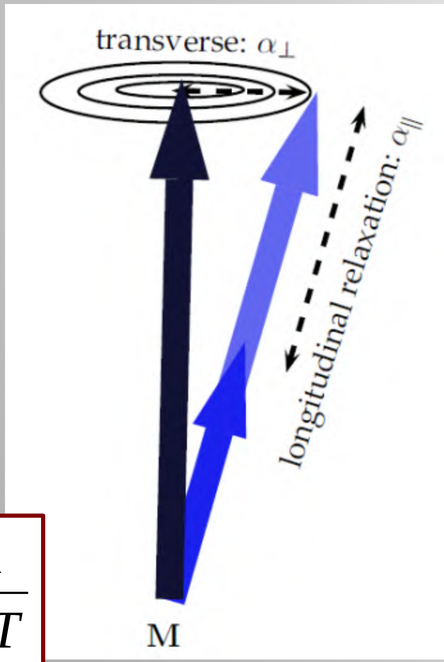
d) Strong exchange field:

$$|H_{ex}| >> |H + H_k|$$

P.Nieves et al PRB, 90 (2014) 104428

QUANTUM LLB (LL) EQUATION

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m}\times\mathbf{H}_{eff} + \gamma\alpha_{\parallel}\frac{\mathbf{m}\cdot\mathbf{H}_{eff}}{m^2}\mathbf{m} - \gamma\alpha_{\perp}\frac{\mathbf{m}\times(\mathbf{m}\times\mathbf{H}_{eff})}{m^2}$$



Effective field: $\mathbf{H}_{eff} = \frac{1}{2\tilde{\chi}_{\parallel}}\left(1-\frac{m^2}{m_e^2}\right)\mathbf{m}+\mathbf{h}$

Dampings:

$$\alpha_{\parallel} = \lambda\frac{2T}{3T_C}\frac{2q_s}{\sinh(2q_s)} \qquad q_s = \frac{3T_C m_e}{2(S+1)T}$$

$$\alpha_{\perp} = \lambda\left[\frac{\tanh(q_s)}{q_s}-\frac{T}{3T_C}+\frac{T(K_1-K_2)}{3T_C K_2}\right]$$

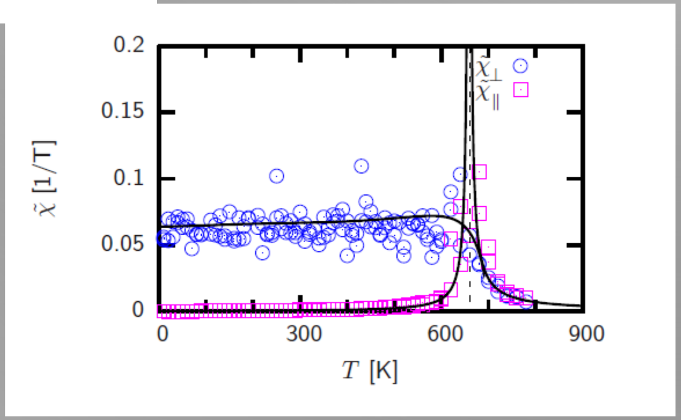
$$\lambda = K_2\left(\frac{S+1}{S}\right)\frac{\mu_{at}}{\gamma k_B T}$$

For S→∞ atomistic coupling to the bath parameter

Bose-Einstein distribution

$$K_1 = \sum_{p,q}\left|V_{p,q}\right|^2 n_p(n_q+1)\pi\delta(w_q-w_p)$$

Landau-Lifshitz limit:



Longitudinal field is very large (far from Tc)
It keeps $m = m_e$

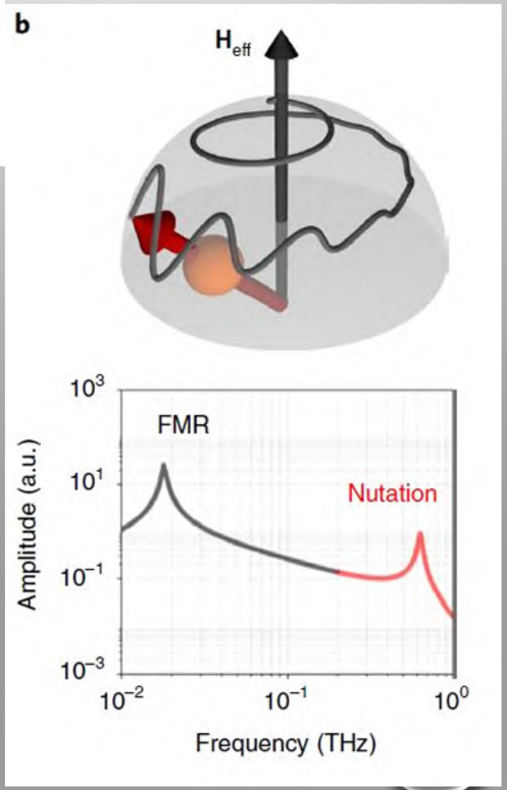
NUTATION

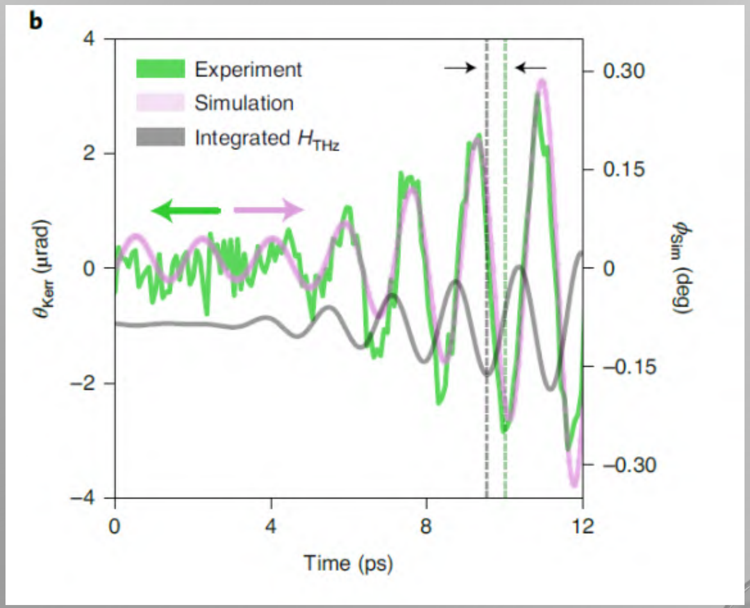
nature physics
ARTICLES
https://doi.org/10.1038/s41567-020-01040-y
Check for updates

Inertial spin dynamics in ferromagnets

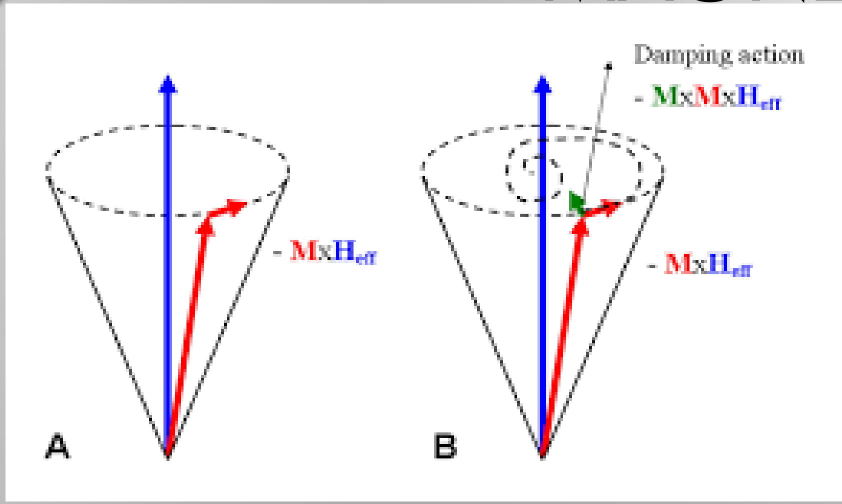
Kumar Neeraj[1,12], Nilesh Awari[2,12], Sergey Kovalev[2], Debanjan Polley[1], Nanna Zhou Hagström[1], Sri Sai Phani Kanth Arekapudi[3], Anna Semisalova[4,5], Kilian Lenz[5], Bertram Green[2], Jan-Christoph Deinert[2], Igor Ilyakov[2], Min Chen[2], Mohammed Bawatna[2], Valentino Scalera[6], Massimiliano d'Aquino[7], Claudio Serpico[6], Olav Hellwig[3,5], Jean-Eric Wegrowe[8], Michael Gensch[9,10] and Stefano Bonetti[1,11] ✉

$$\frac{d\mathbf{M}}{dt} = -|\gamma|\mathbf{M} \times \left[\mathbf{H}_{\text{eff}} - \frac{\alpha}{|\gamma|M_s}\left(\frac{d\mathbf{M}}{dt} + \tau\frac{d^2\mathbf{M}}{dt^2}\right)\right].$$





MAGNETIZATION PRECESSION



$$\frac{d\vec{M}}{dt} = -\frac{\gamma_0}{1+\alpha^2}\left[\vec{M}\times\vec{H}_{eff}\right] - \frac{\gamma_0\alpha}{(1+\alpha^2)M_s}[\vec{M}\times\left[\vec{M}\times\vec{H}_{eff}\right]]$$

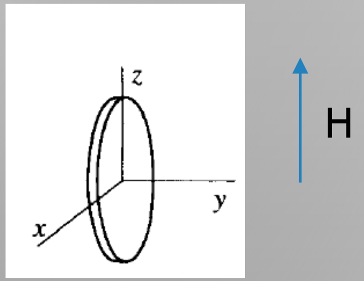
$$\mathcal{H} = \mathcal{H}_{\mathrm{ex}} + \mathcal{H}_{\mathrm{dmi}} + \mathcal{H}_{\mathrm{mc}} + \mathcal{H}_{\mathrm{me}} + \mathcal{H}_Z + \mathcal{H}_d$$

$$\vec{H}_{\mathrm{eff}} = -\frac{\delta\mathcal{H}}{\delta\vec{M}}$$

KITTEL FORMULA
(DIPOLAR INTERACTIONS IN THE SHAPE FACTOR APPROXIMATION FOR UNIFORM ELLIPSOID)

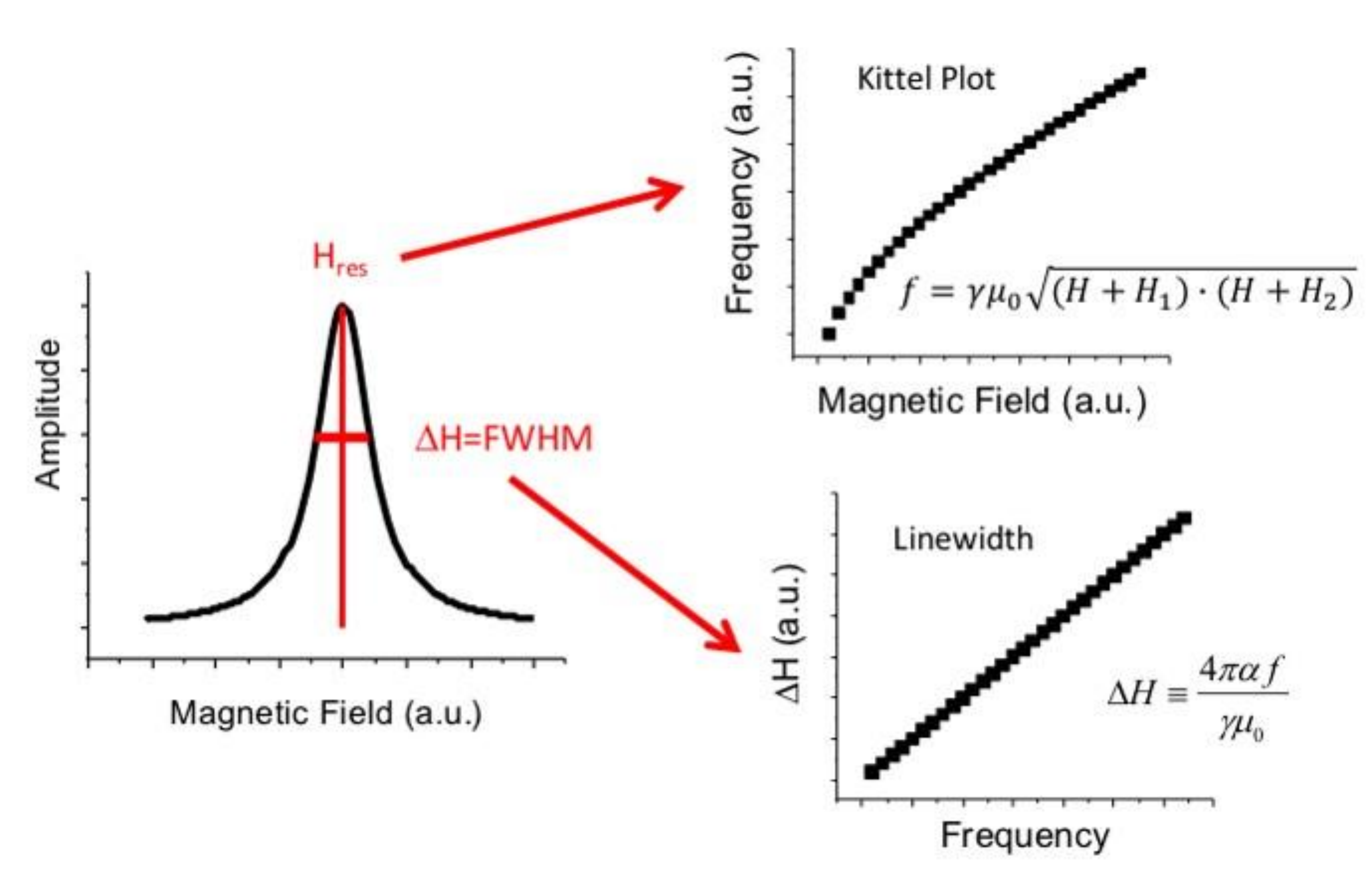
Linearizing LLG around the equilibrium position

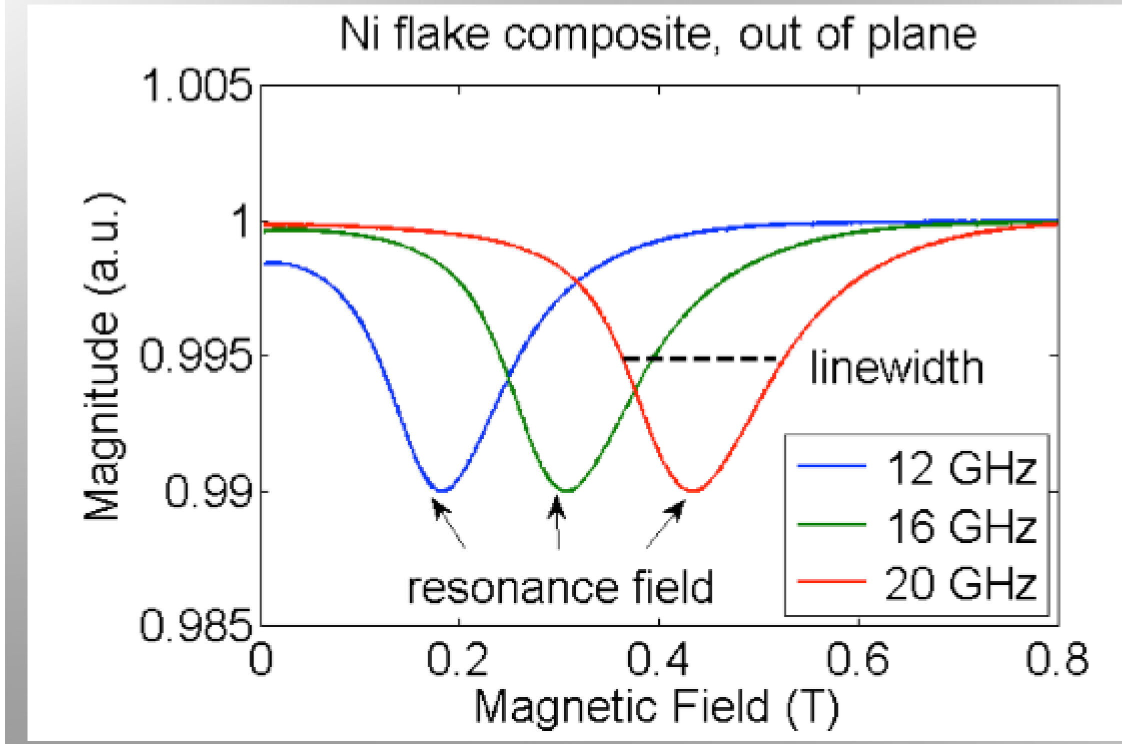
$$\mathrm{i}\omega \boldsymbol{m} + \gamma \boldsymbol{m}\left(\boldsymbol{H}_{e0} - \overleftrightarrow{N}\boldsymbol{M}_0\right) + \gamma\left(\overleftrightarrow{N}\boldsymbol{m}\right) \times \boldsymbol{M}_0 - \frac{\mathrm{i}\alpha\omega}{M_0}\boldsymbol{m} \times \boldsymbol{M}_0 = -\gamma \boldsymbol{M}_0 \times \boldsymbol{h}_e.$$



$$\omega_0 = \gamma\left\{\left[H_{e0} + (N_x - N_z)M_0\right]\left[H_{e0} + (N_y - N_z)M_0\right]\right\}^{1/2}$$

FERROMAGNETIC RESONANCE





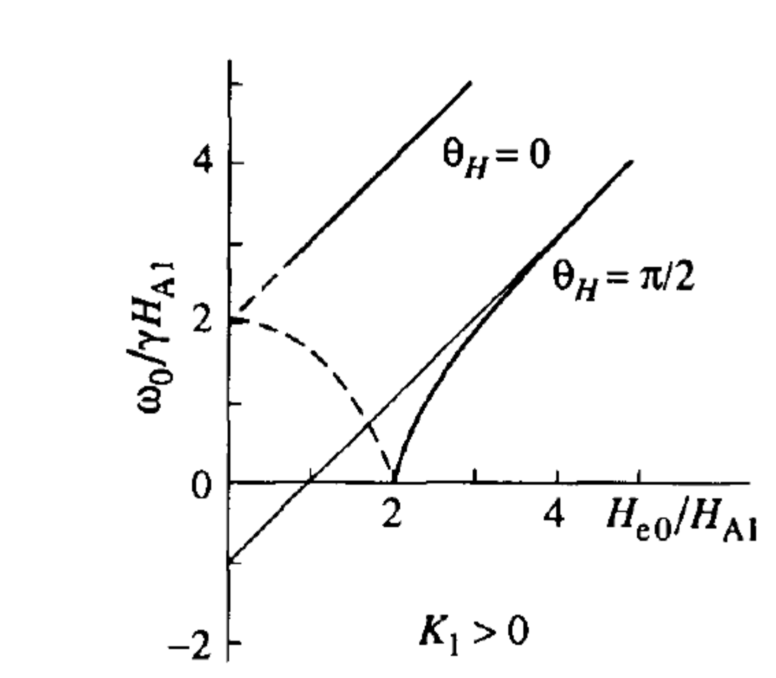
P.Dhagat Oregon State University webpage

FERROMAGNETIC RESONANCE FOR UNIAXIAL MAGNET

External field — Anisotropy field — Applied field angle

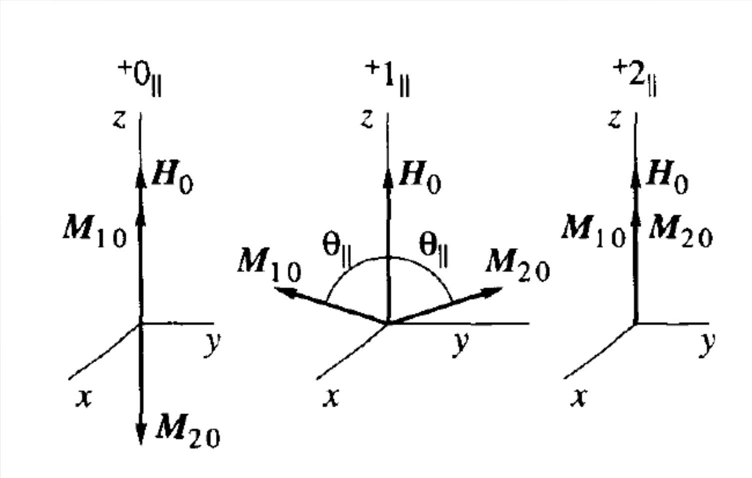
$$\frac{\omega_0^2}{\gamma^2} = \left[H_{e0}\cos(\theta_0 - \theta_H) + 2H_{A1}\cos 2\theta_0\right] H_{e0}\frac{\sin\theta_H}{\sin\theta_0}.$$

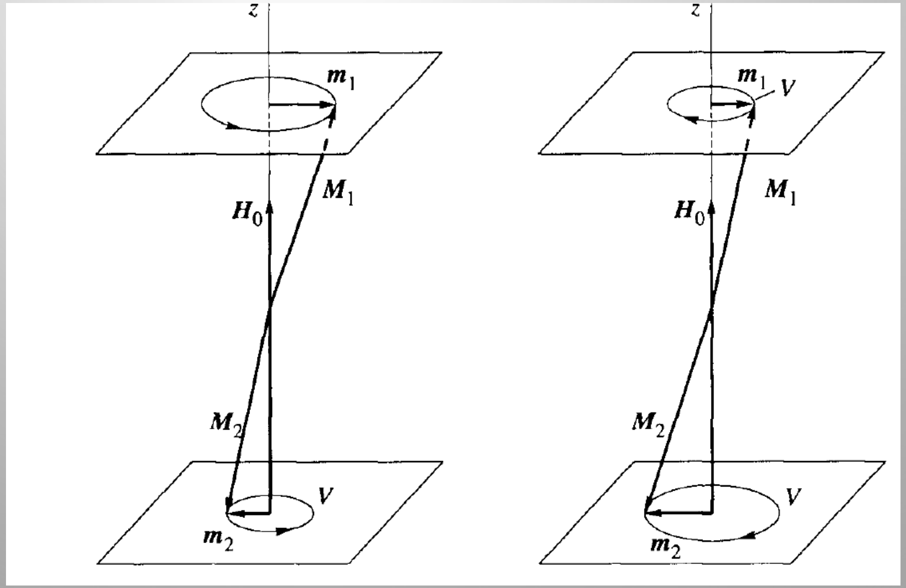
Equilibrium angle



A.G.Gurevich and G.A.Melkov
"Magnetisation oscillations and waves"

ANTIFERROMAGNETIC RESONANCE



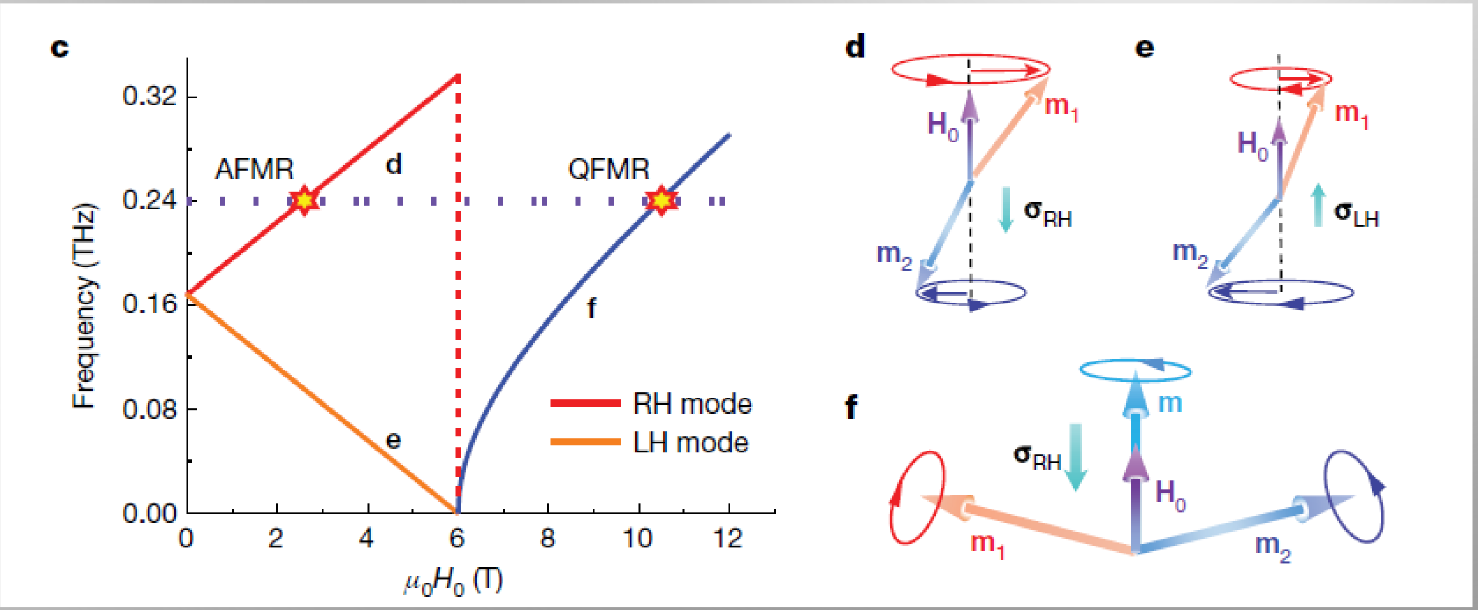


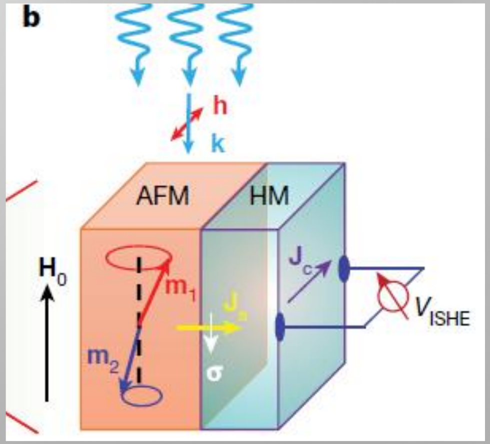
Modes in the antiparallel state

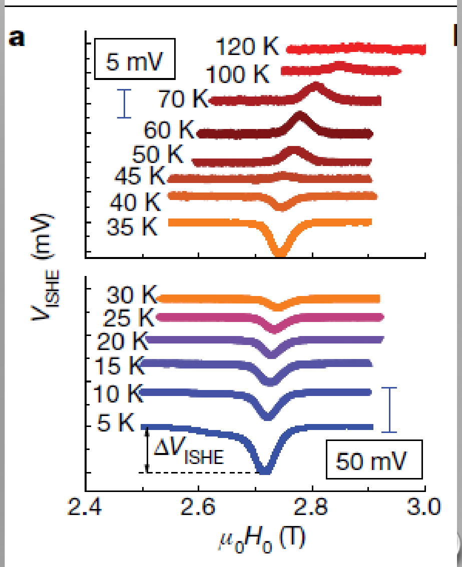
Spin-flop and spin-flip transitions

A.G.Gurevich and G.A.Melkov
“Magnetisation oscillations and waves”

ANTIFERROMAGNETIC RESONANCE

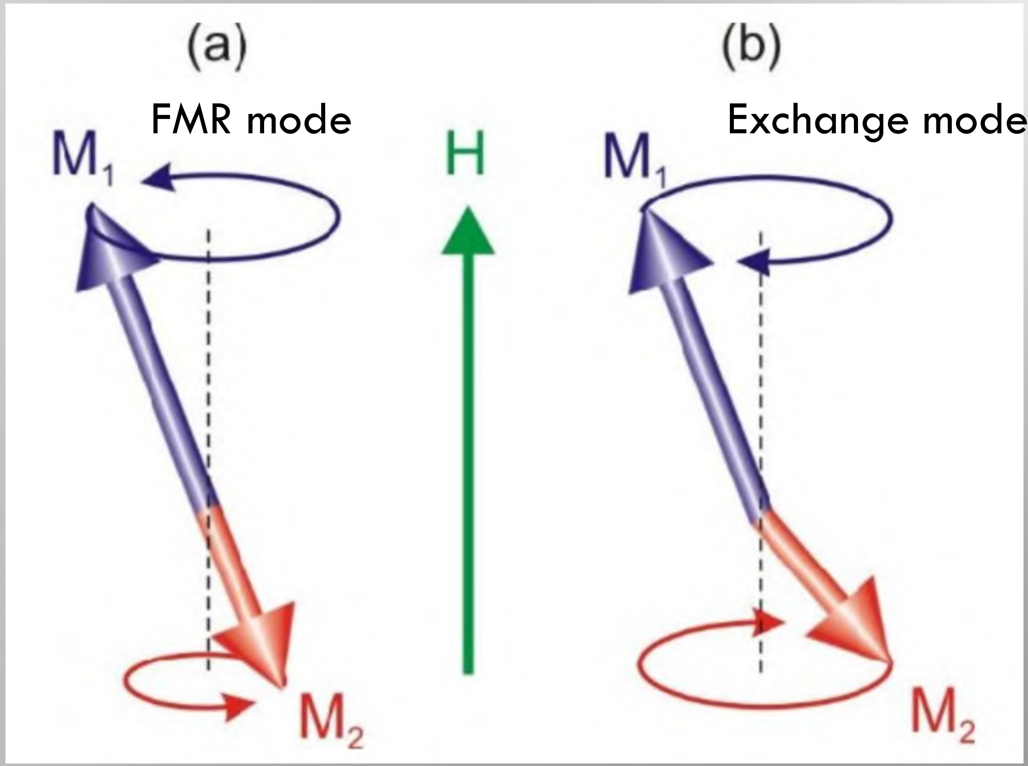


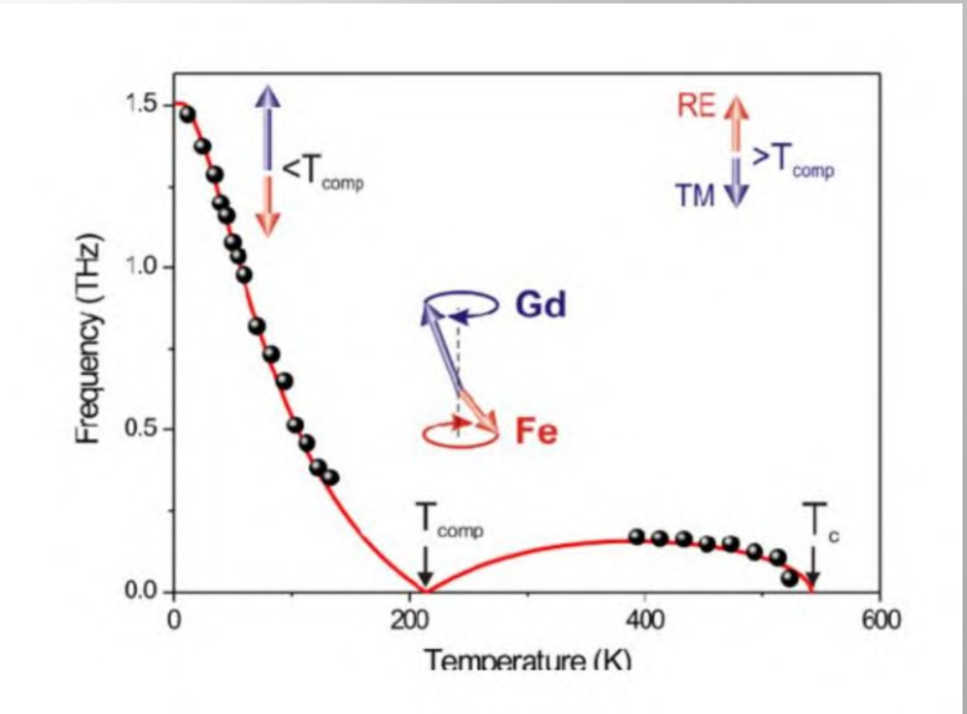




J.Li et al Nature 578 (2020)

FERRIMAGNETIC MATERIALS





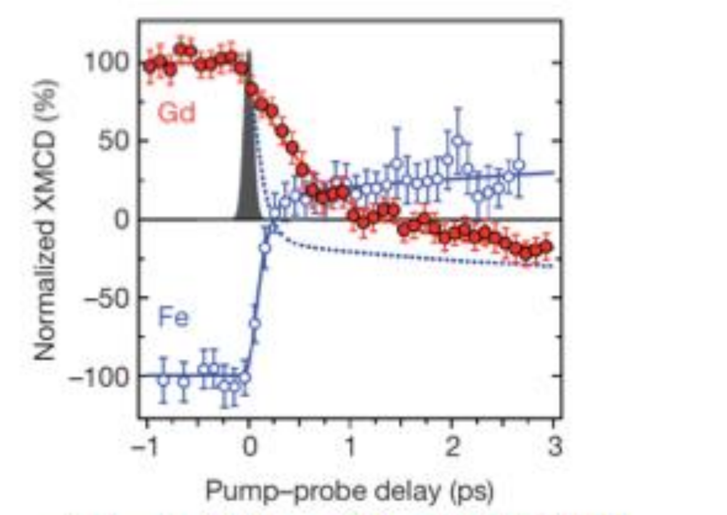
$$\omega_{\mathrm{FMM}} = \frac{\gamma_{\mathrm{T}}\gamma_{\mathrm{R}}\left(M_{\mathrm{T}}^{\mathrm{e}} - M_{\mathrm{R}}^{\mathrm{e}}\right)}{\left(\gamma_{\mathrm{R}} M_{\mathrm{T}}^{\mathrm{e}} - \gamma_{\mathrm{T}} M_{\mathrm{R}}^{\mathrm{e}}\right)} H^{0}$$

$$\omega_{\mathrm{EXM}} = A\left(\gamma_{\mathrm{T}} M_{\mathrm{R}}^{\mathrm{e}} - \gamma_{\mathrm{R}} M_{\mathrm{T}}^{\mathrm{e}}\right),$$

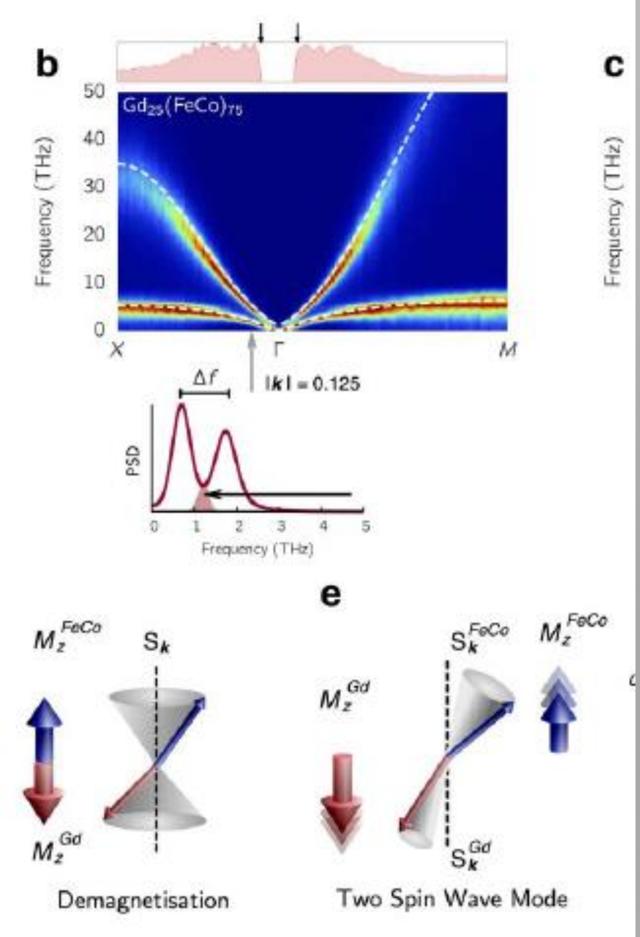
$$\alpha_{\mathrm{eff}} = \frac{M_{\mathrm{R}}^{\mathrm{e}}\gamma_{\mathrm{T}}\alpha_{\mathrm{R}} + \gamma_{\mathrm{R}}\alpha_{\mathrm{T}} M_{\mathrm{T}}^{\mathrm{e}}}{M_{\mathrm{R}}^{\mathrm{e}}\gamma_{\mathrm{T}} - M_{\mathrm{T}}^{\mathrm{e}}\gamma_{\mathrm{R}}},$$

Angular momentum compensation point

ULTRAFAST SWITCHING IN FEGD FROM SPINWAVE ANALYSIS

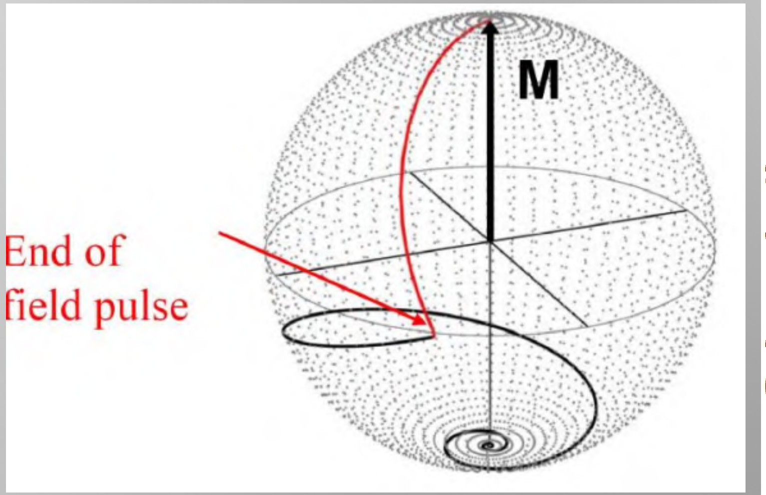


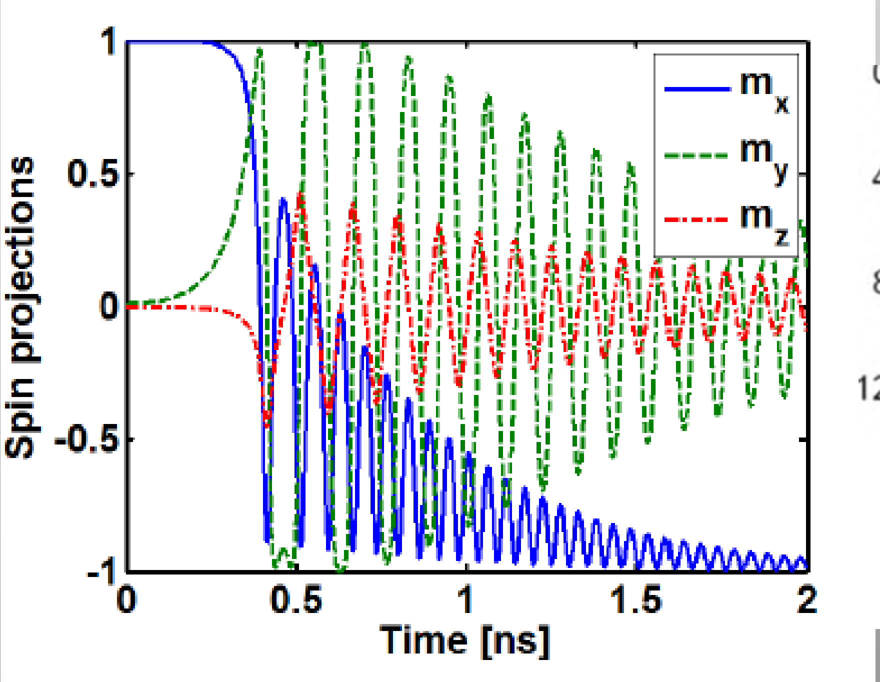
Radu *et al.*, Nature, **472**, 205-208 (2011).

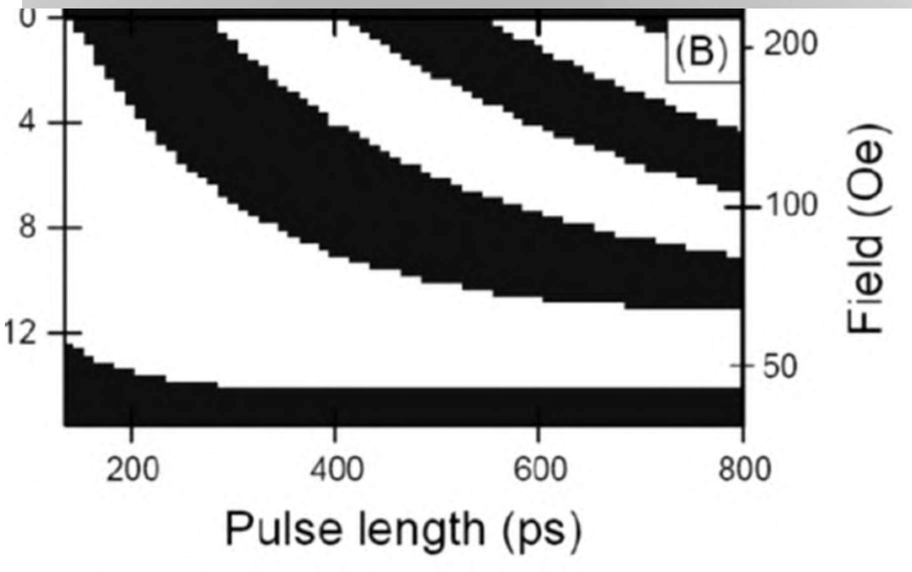


PRECESSIONAL SWITCHING

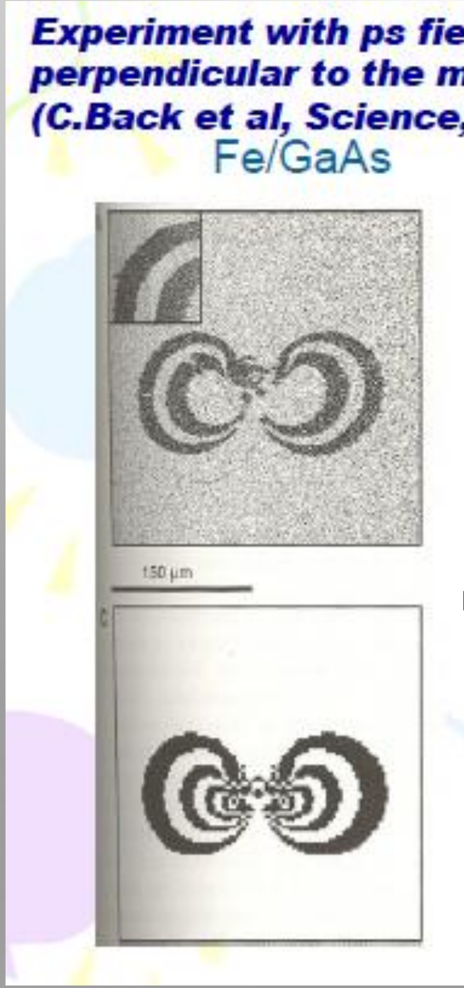
Perpendicular to M field







SIGNATURES OF PRECESSIONAL SWITCHING

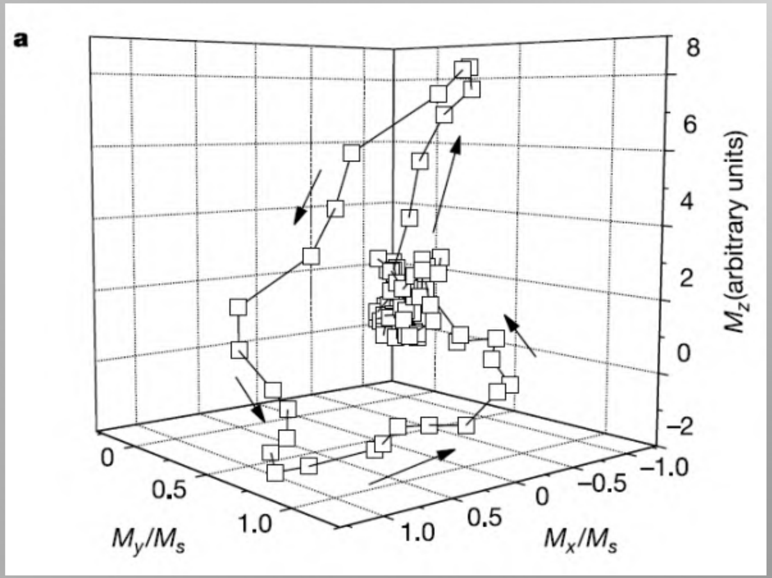


In-plane
magnetisation

Ultrafast precessional magnetization reversal by picosecond magnetic field pulse shaping

Th. Gerrits*, H. A. M. van den Berg*, J. Hohlfeld*, L. Bär† & Th. Rasing*

NATURE |VOL 418 | 1 AUGUST 2002



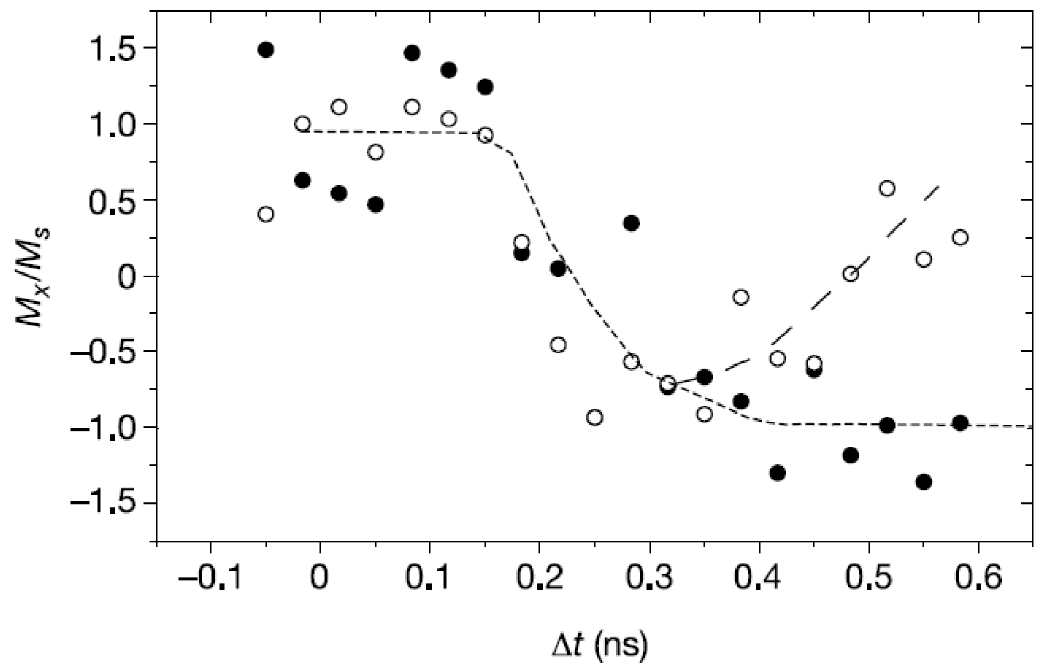
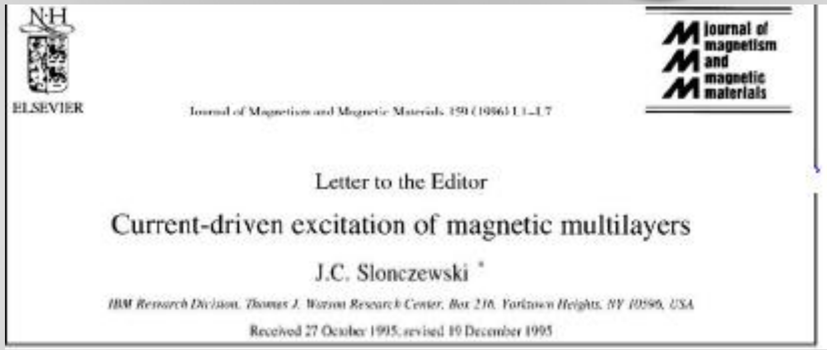


Figure 4 Switching by large-field excitation and suppression of ringing. Without a stop pulse, the system switches back to its initial state (open circles). After sending the stop pulse, the suppression of the ringing of the magnetization can clearly be observed (solid circles). The lines are guides to the eye. The low signal-to-noise ratio in the M_x

SPIN-TRANSFER TORQUE


ELSEVIER
Journal of Magnetism and Magnetic Materials 159 (1996) L1–L7
Journal of magnetism and magnetic materials

Letter to the Editor

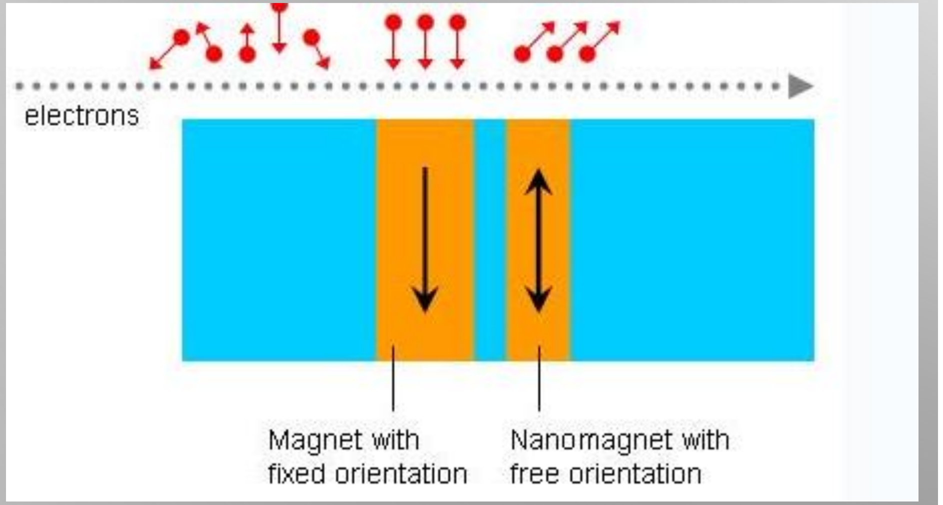
Current-driven excitation of magnetic multilayers

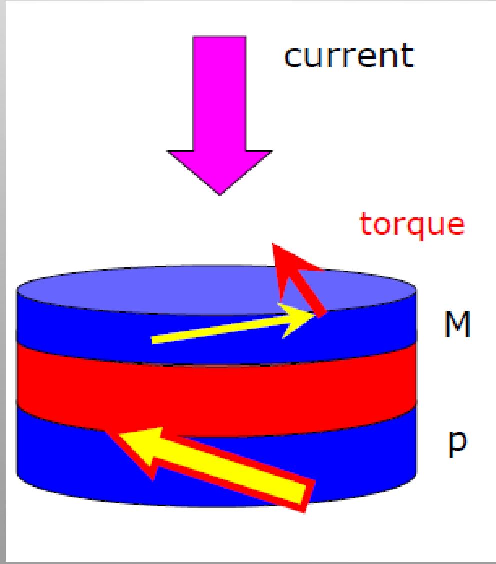
J.C. Slonczewski *

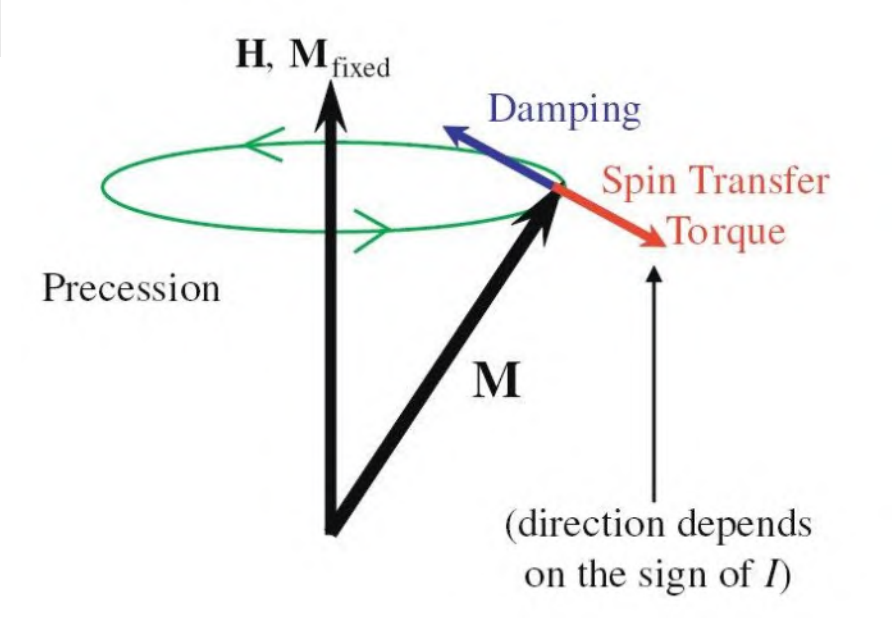
IBM Research Division, Thomas J. Watson Research Center, Box 218, Yorktown Heights, NY 10598, USA

Received 27 October 1995; revised 19 December 1995

- Electrons in magnetic material are spin polarised
- When electrons move through another thin magnetic layer with different M, they transfer their angular momentum exerting torque on magnetisation
- This can lead to magnetisation precession or switching





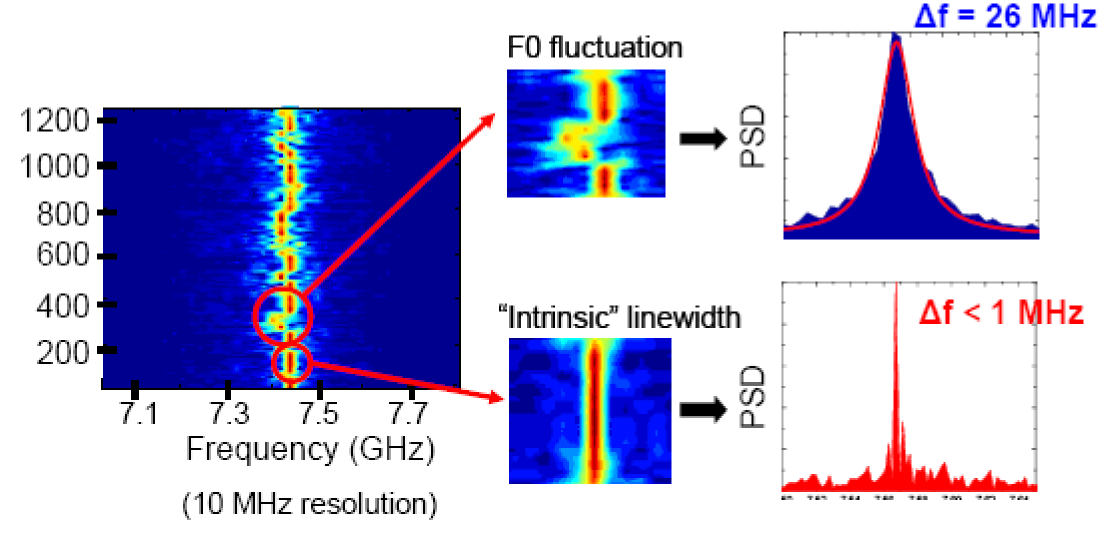


Magnetic random Access memories
Spin-torque nano-oscialltors

effective field — damping — spin transfer

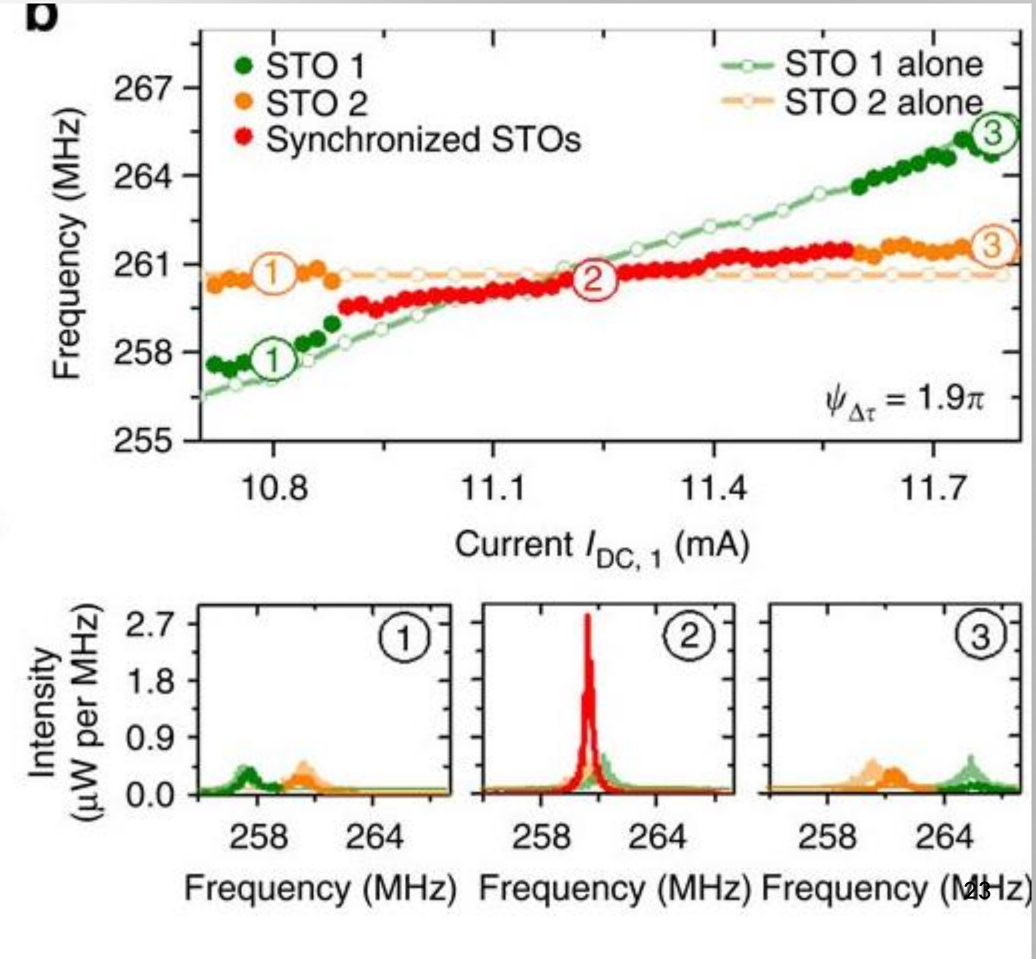
$$\frac{d\mathbf{M}}{dt} = -\gamma\left[\mathbf{M}\times\mathbf{B}_{eff}\right] + \frac{\alpha}{M_s}\left[\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right] - \frac{\Gamma}{g(\theta)}\left[\mathbf{M}\times\left[\mathbf{M}\times\mathbf{p}\right]\right]$$

SPIN-TORQUE NANO-OSCILLATORS



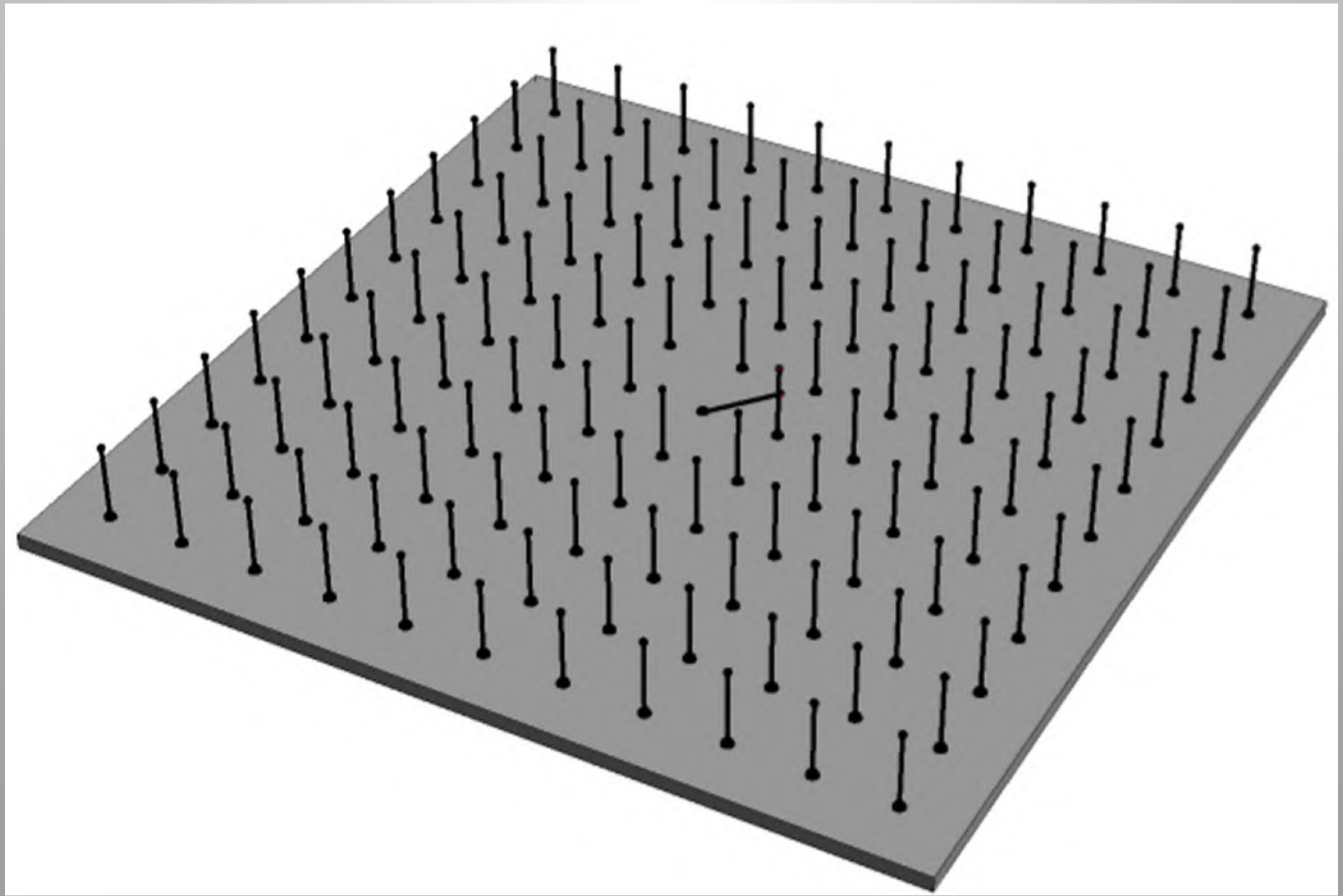
P.Villard et al., IEEE Journal Solid-State Circuits, VOL. 45, NO.1, Jan 2010

Disadvantages : intrinsic phase noise, frequency drift,



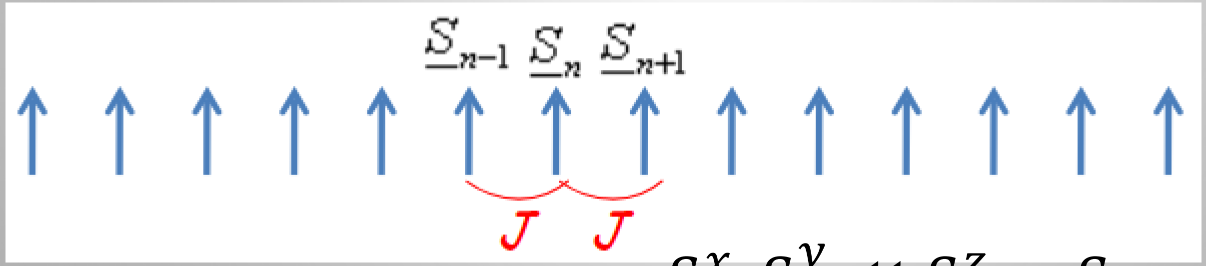
L.Lebrun Nature Comm Nature 8, 15825 (2017)

Spin waves == collective motion of spins



SPIN WAVES

Collective exchange motion of spins



$$S_n^x, S_n^y \ll S_n^z \approx S,$$

Spinwaves == Low-energy excitations (deviations fro the ground state) for temperatures T<<Tc

Quantized spinwaves ===magnons

$$\frac{d\underline{S}_n}{dt} = J\left(\underline{S}_n \times \underline{S}_{n-1} + \underline{S}_n \times \underline{S}_{n+1}\right)$$

$$\frac{dS_n^x}{dt} = J\left[S_n^y S_{n-1}^z - S_n^z S_{n-1}^y + S_n^y S_{n+1}^z - S_n^z S_{n+1}^y\right]$$

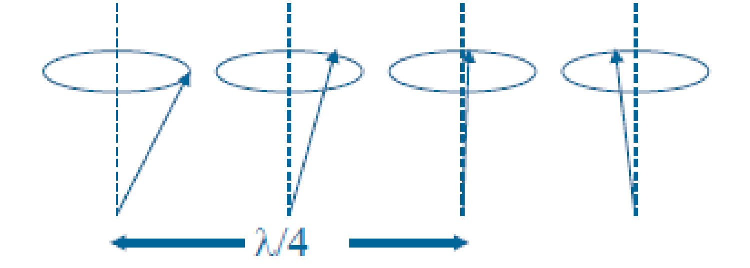
$$\frac{dS_n^x}{dt} = JS\left[2S_n^y - S_{n-1}^y - S_{n+1}^y\right]$$

$$\frac{dS_n^y}{dt} = -JS\left[2S_n^x - S_{n-1}^x - S_{n+1}^x\right]$$

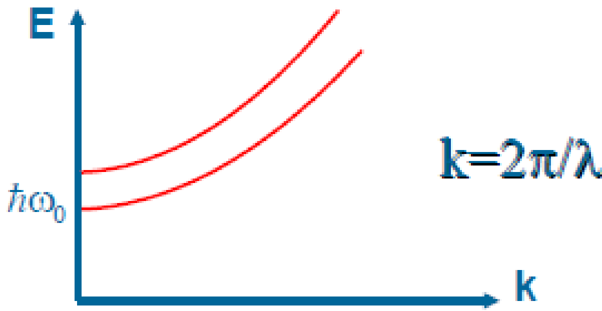
EXCHANGE SPIN WAVES

$$S_n^x = uSe^{i(nka-\omega t)}$$
$$S_n^y = vSe^{i(nka-\omega t)}$$

$$\omega = 2JS(1-\cos ka)$$



$$\omega(\mathbf{k}) = \omega(0) + Ak^2$$



SPIN WAVES

Kittel formula (dipolar interactions in the shape factor approximation)

Anisotropic single crystal ferromagnet:

$$\left(\frac{\omega}{\gamma}\right)^2 = \left(H_0 + H_A + Ak^2\right)\left(H_0 + H_A + Ak^2 + 2\pi M_s \sin^2\theta_k\right)$$

Angle between M and k

Applied field

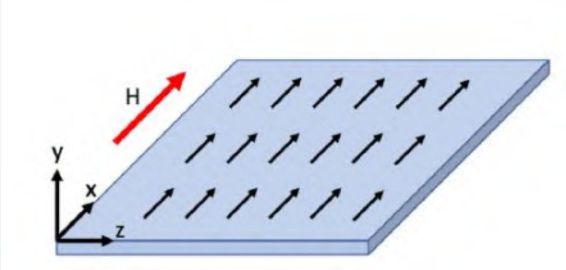
Anisotropy field

Exchange interaction

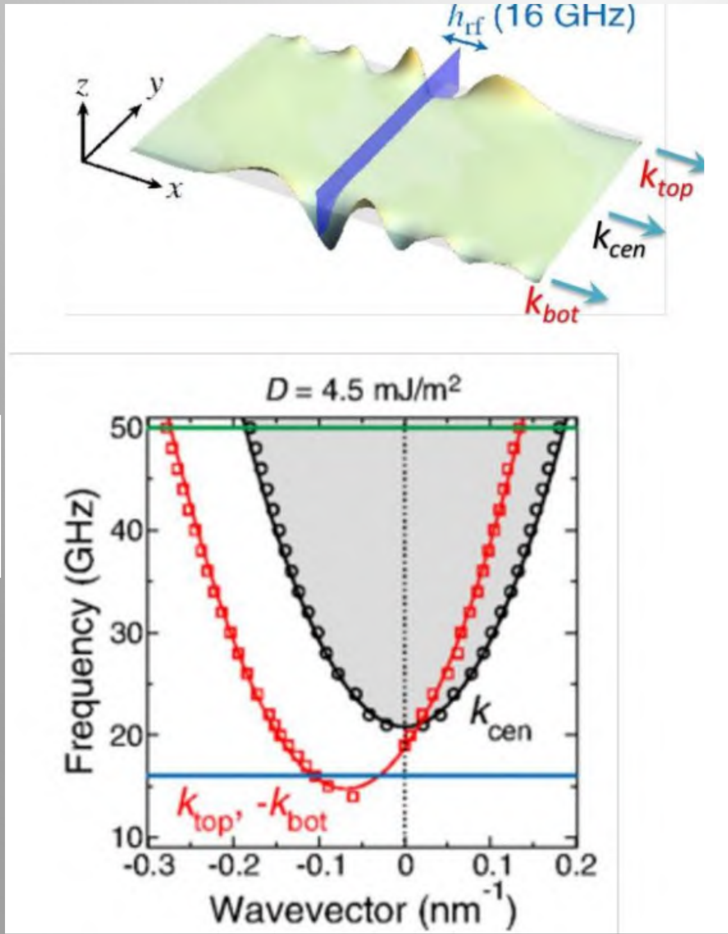
Magnetostatic interaction

SPINWAVES IN SYSTEMS WITH DMI

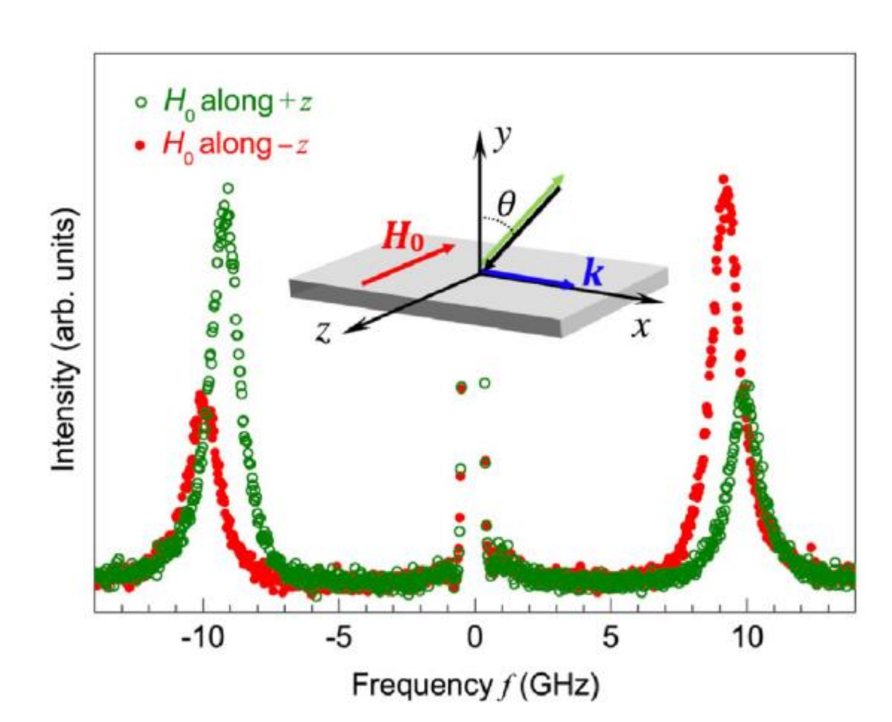
Non-reciprocal propagation



$$\omega(k) = |\gamma|\sqrt{H(H+M_s)} + \frac{|\gamma|D}{\mu_0 M_s}k.$$



P.Borys PhD 2015

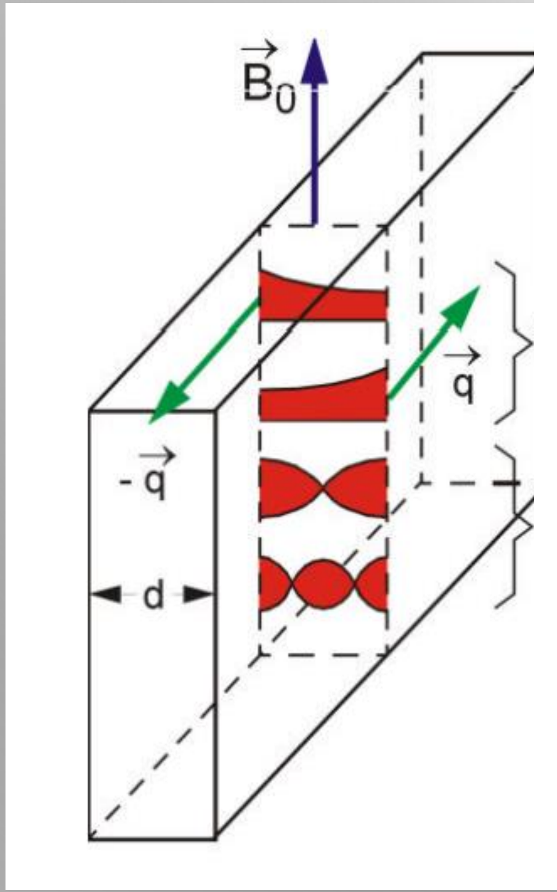


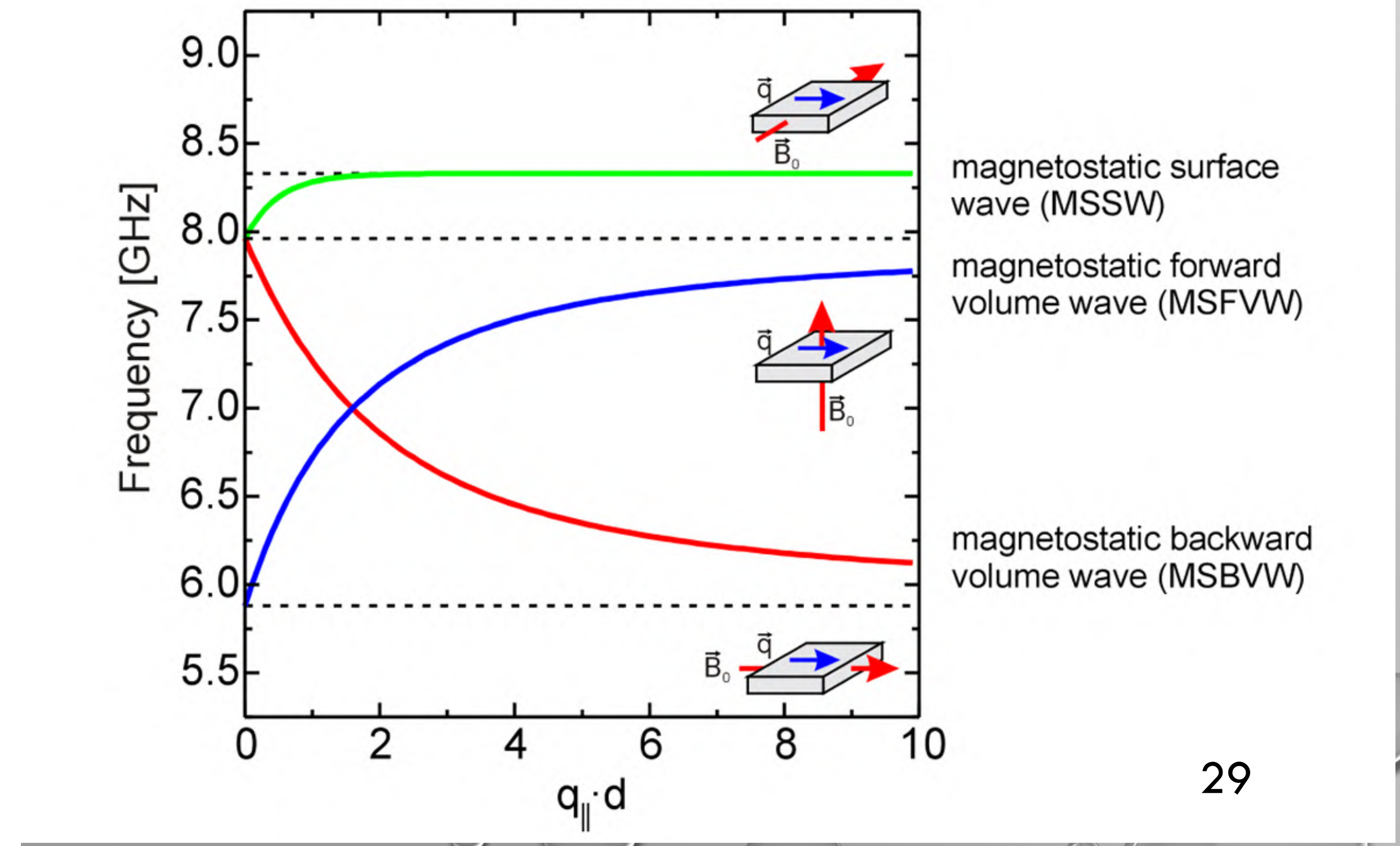
K.Di et al Phys. Rev. Lett. 114 (2015), 047201

MAGNETOSTATIC SPINWAVES

Magnetostatic energy generates magnetic poles (long wavelengths)

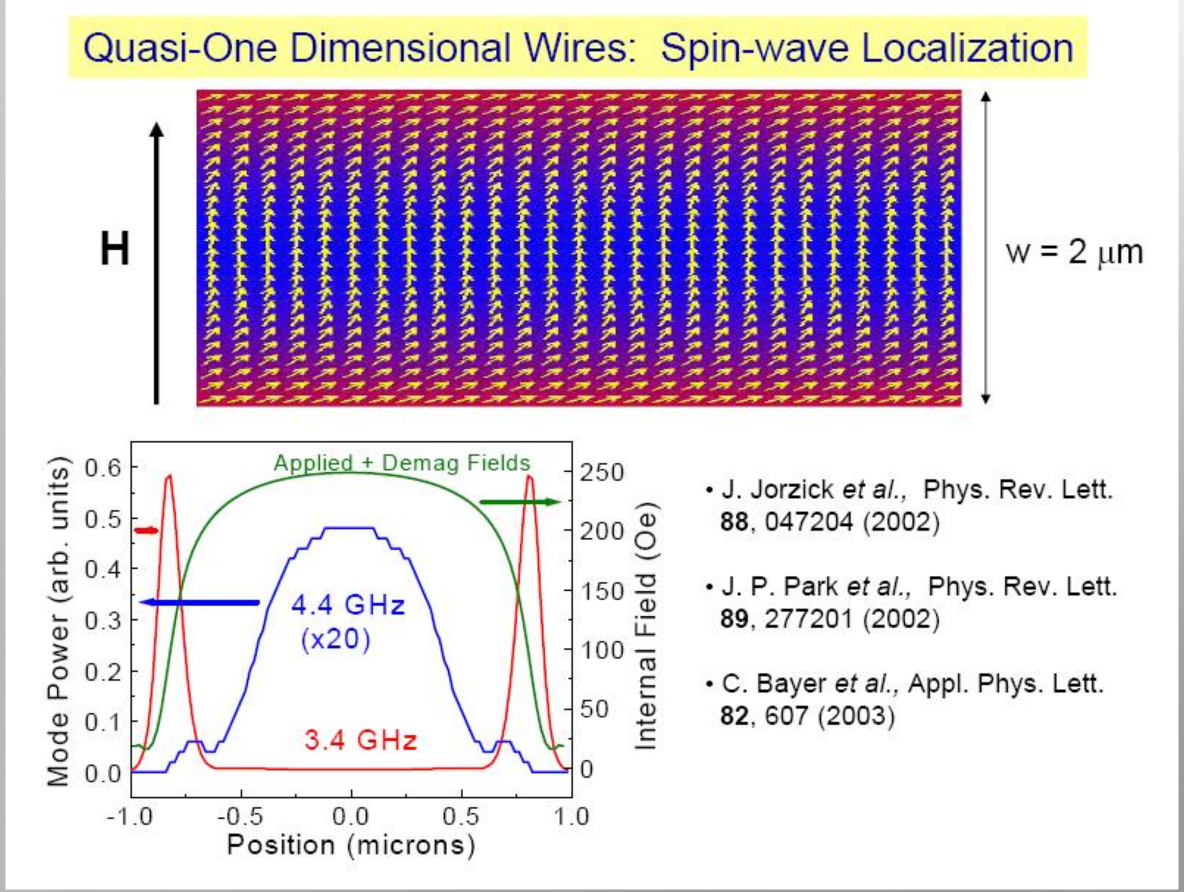
Dipolar
Damon-Eshbach mode

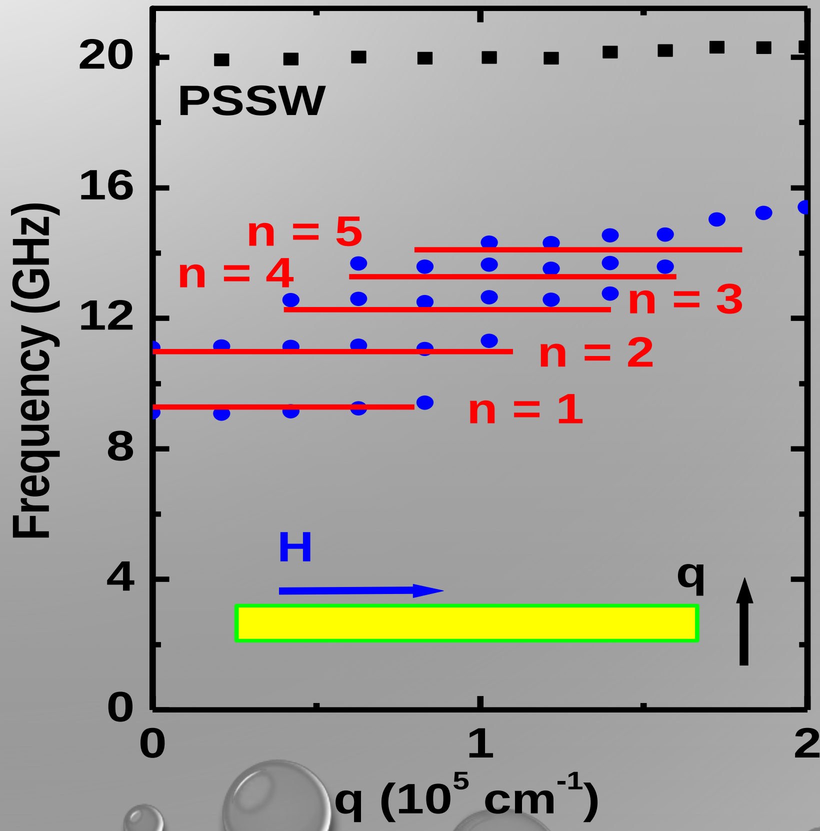




Group and phase velocities are not equal

QUANTIZED MODES IN NANOMAGNETS

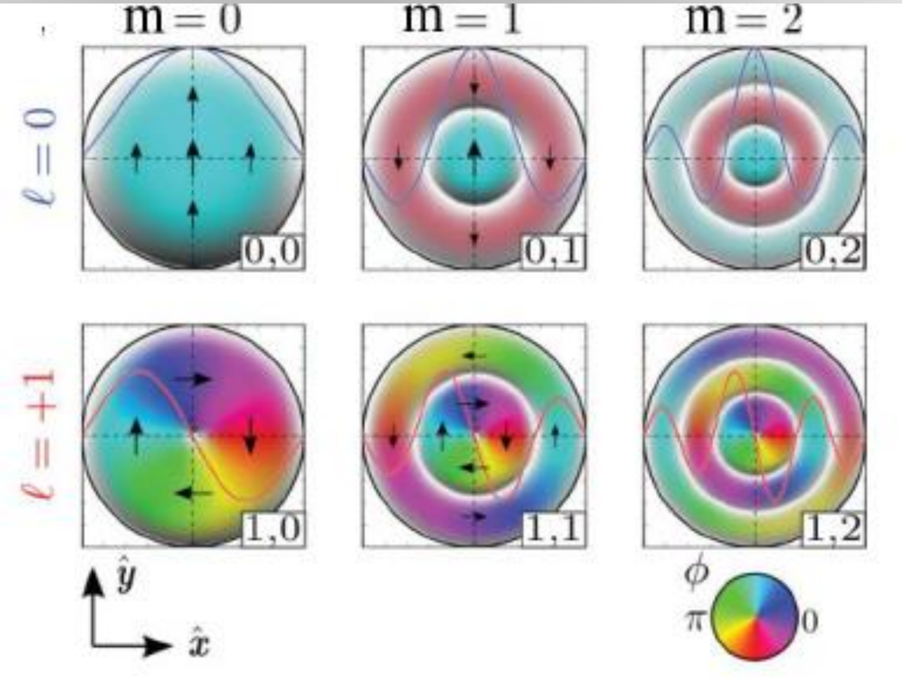


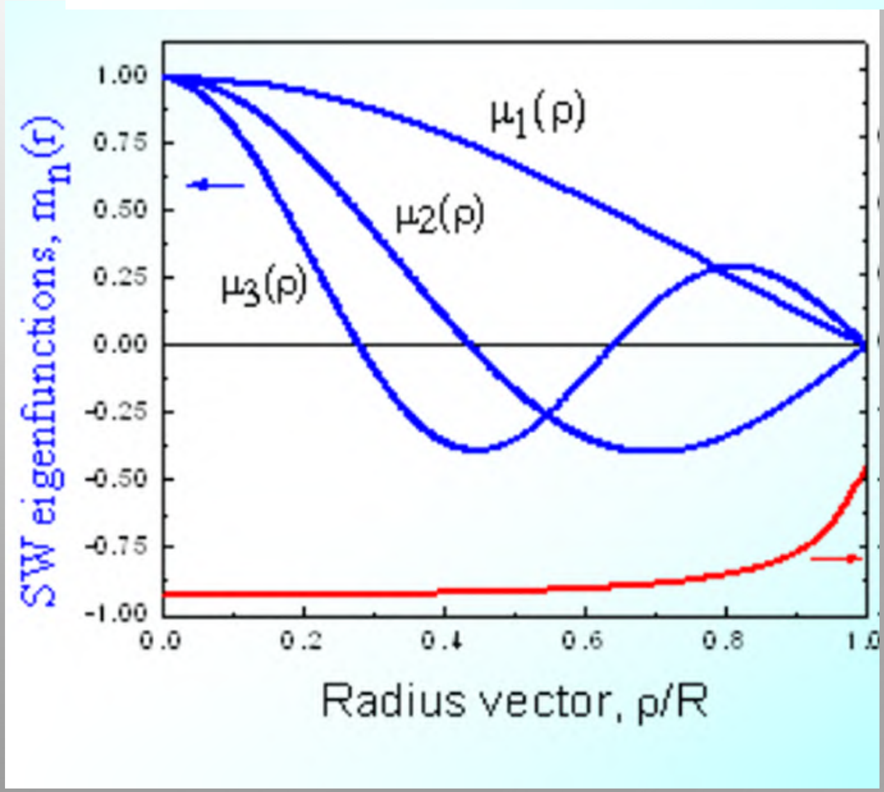


PY DOTS IN IN-PLANE STATE

B.Pegeau PhD

*K.Yu. Guslienko et.al.*Phys. Rev. **B 65**, 024414 (2002).

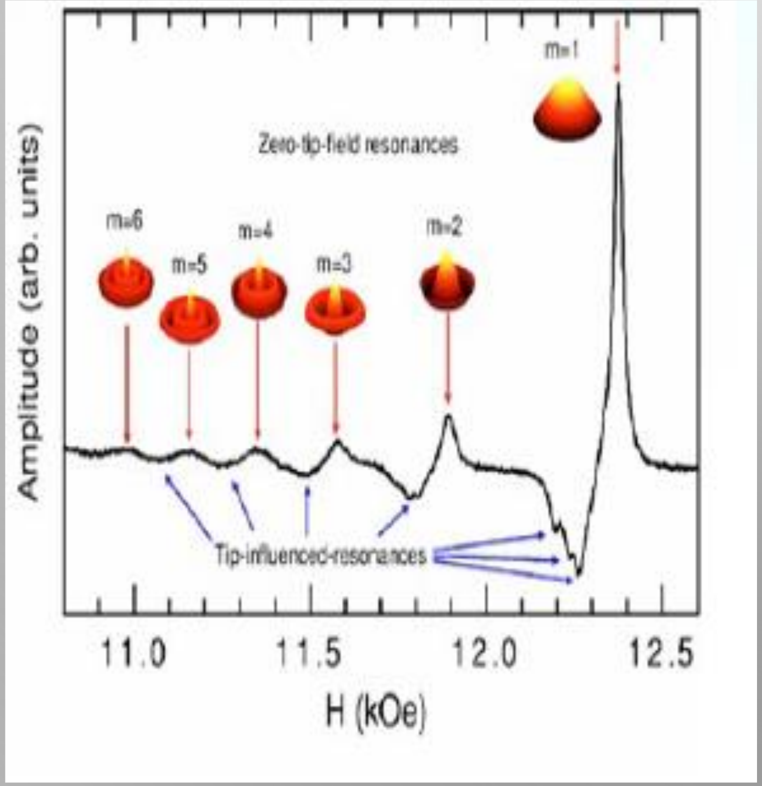




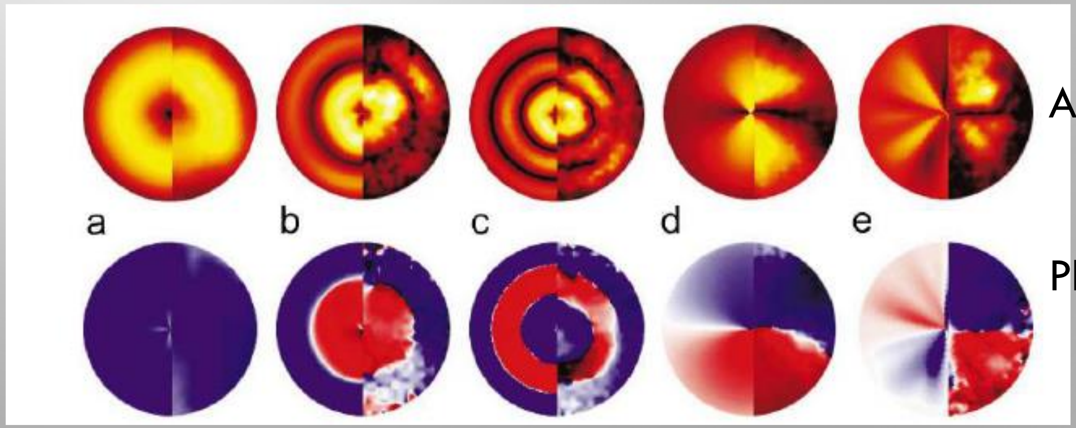
$$N_m = \frac{1}{A_m}\int_0^R N(\rho) J_0^2(k_m\rho)\rho d\rho, \quad A_m = \frac{1}{2}R^2 J_1^2(k_m).$$

EXCITATION MODES IN NANOMAGNETS

G.Kakazei et al



Vortex state in Py disc



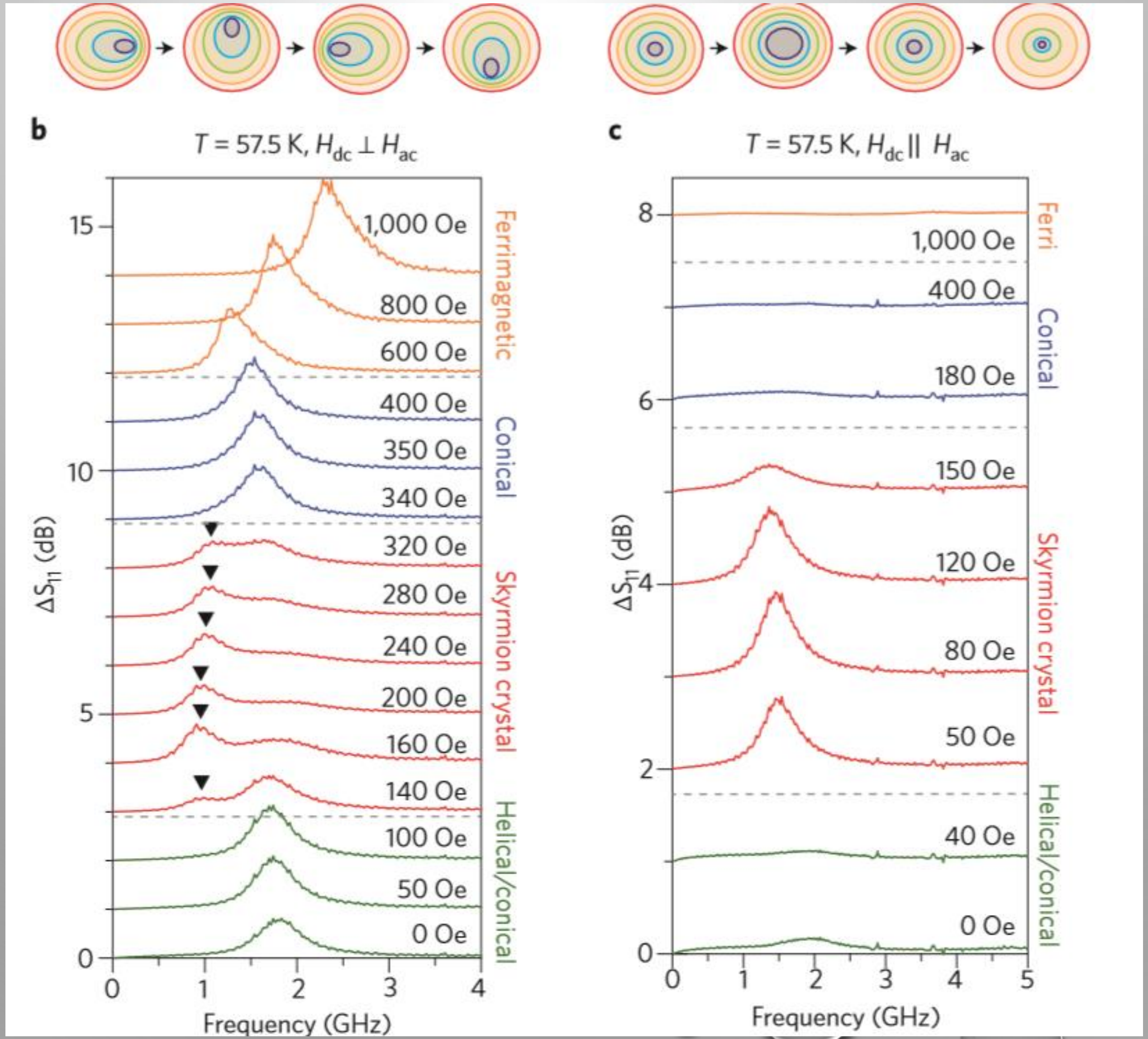
Amplitude

Phase

M.Bues PRL (2004)

Dynamics: two modes

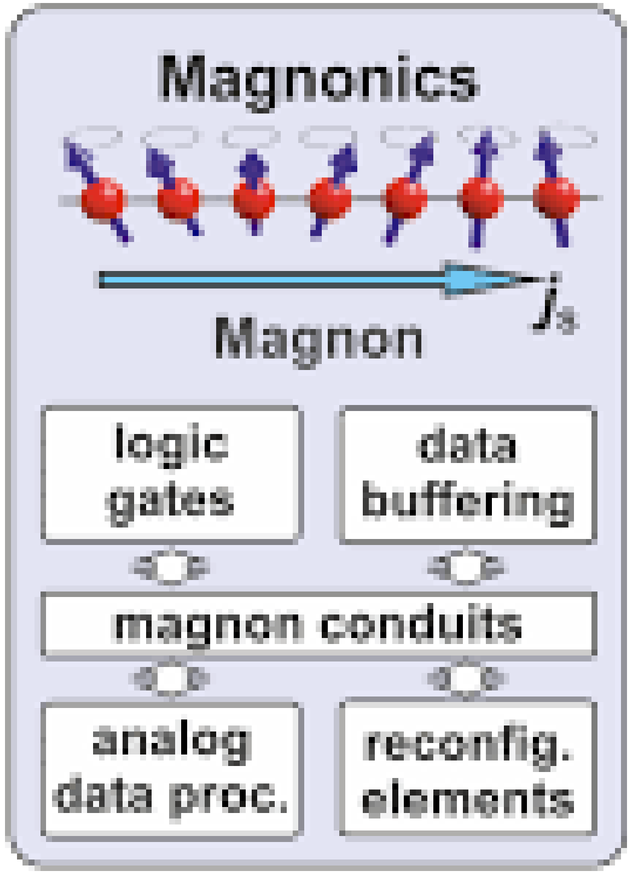
Gyrotropic & Breathing



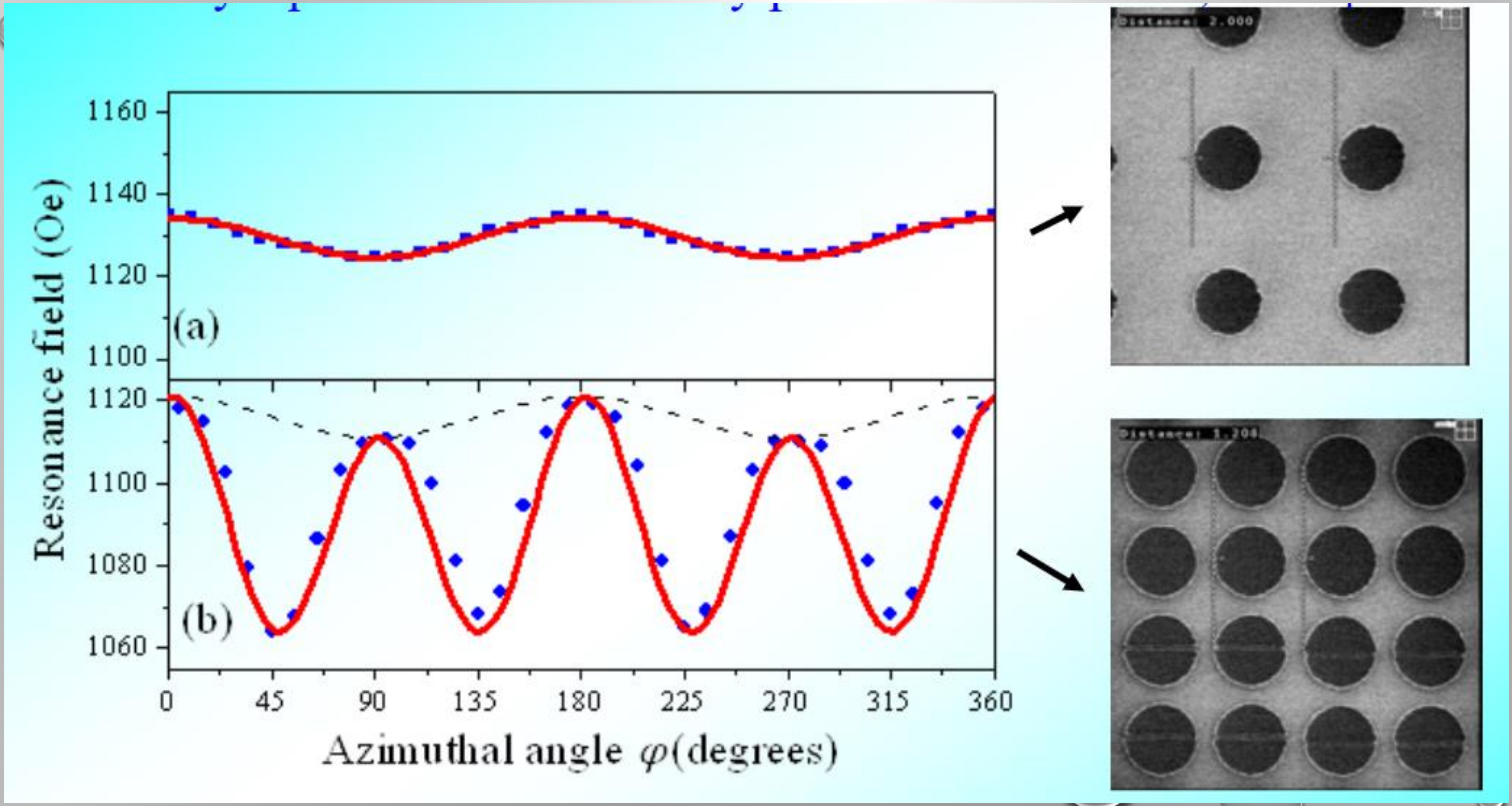
MAGNONICS

Spinwaves are used to transport and process information

Advantages: Easily reconfigurable spectrum
with external fields and spin-polarized currents

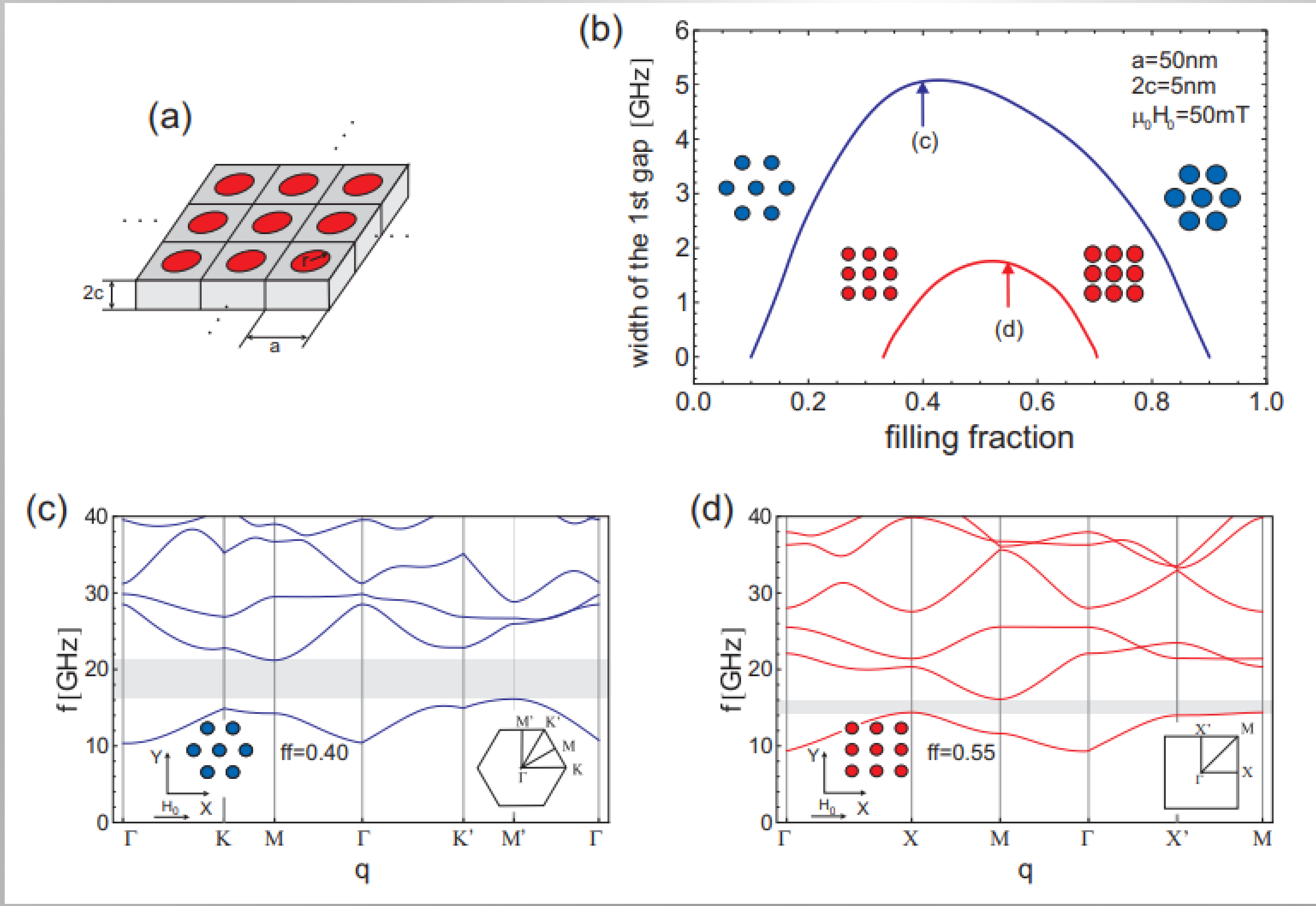


PY NANODOTS



G.Kakazei
Et al

MAGNONIC CRYSTALS



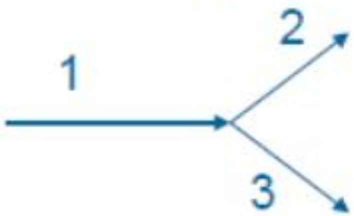
V.V. Kruglyak et al

Magnons and their interactions:

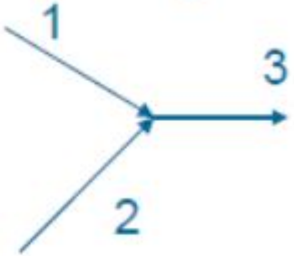
- Classical spinwaves correspond to quasiparticles called magnons.
- Linear normal modes (magnons) do not interact. Nonlinear processes correspond to magnon-magnon interactions.

magnon scattering

Magnon decay



Two magnon merging



These interactions define kinetic effects (e.x. heat conductivity) and width and shape of the FMR line and magnon lifetime

Thermalization processes occur via magnon-magnon interactions

THERMAL MAGNONS

Magnons are bosons (S=1) and fulfill Bose-Enstein statistics

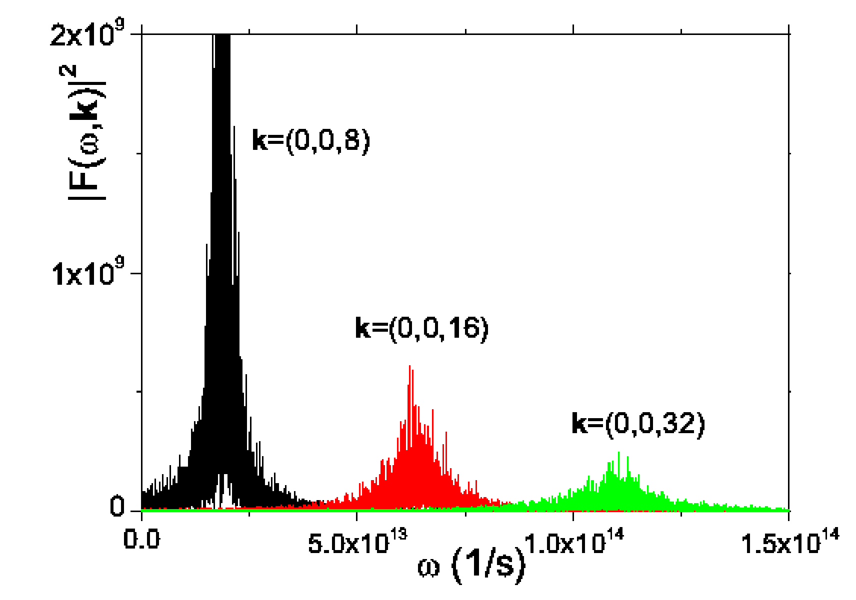
$$\rho(\nu) = D(\nu)n(\nu) = \frac{D(\nu)}{\exp\left(\frac{h\nu-\mu}{k_B T_0}\right) - 1}$$

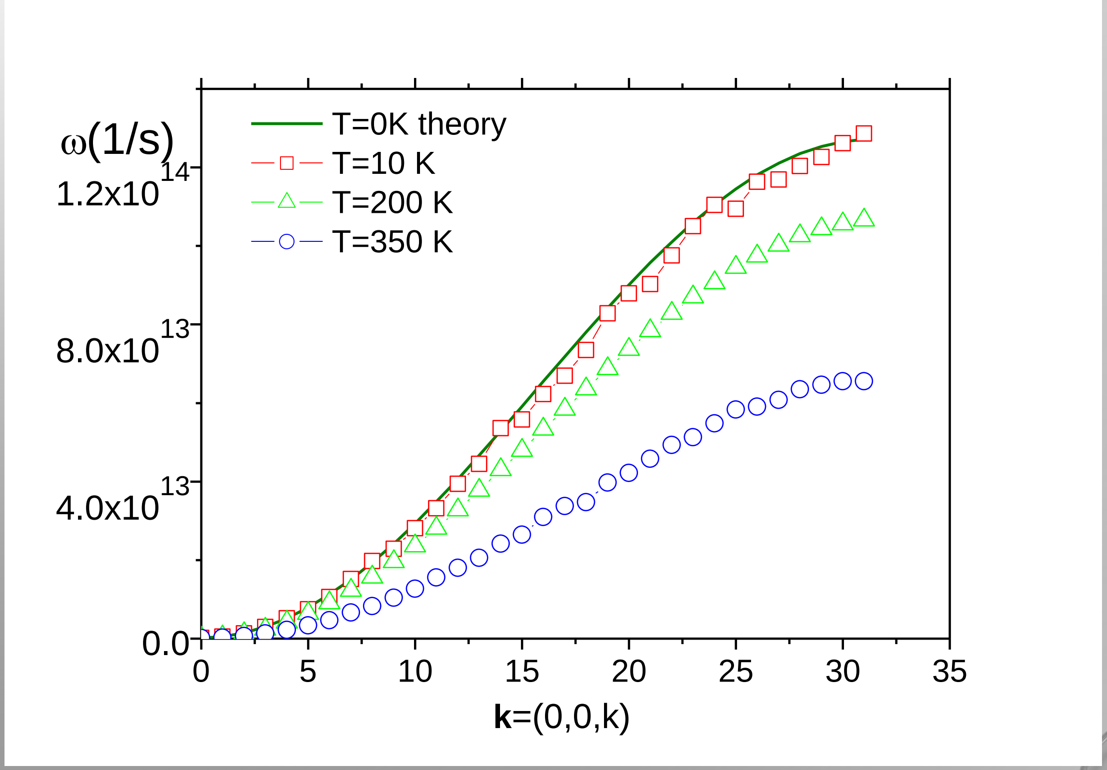
$$M(T) = \underbrace{M(T=0)}_{\frac{g\mu_B NS}{V}}\left(1 - \frac{V}{2\pi^2 NS}\left(\frac{k_B T}{D}\right)^{3/2} \frac{\sqrt{\pi}\,\varsigma(3/2)}{4}\right)$$

Felix Bloch
(1905 - 1983)
Nobel Prize in 1952 for NMR

TEMPERATURE-DEPENDENT SPECTRUM

Exchange stiffness calculation



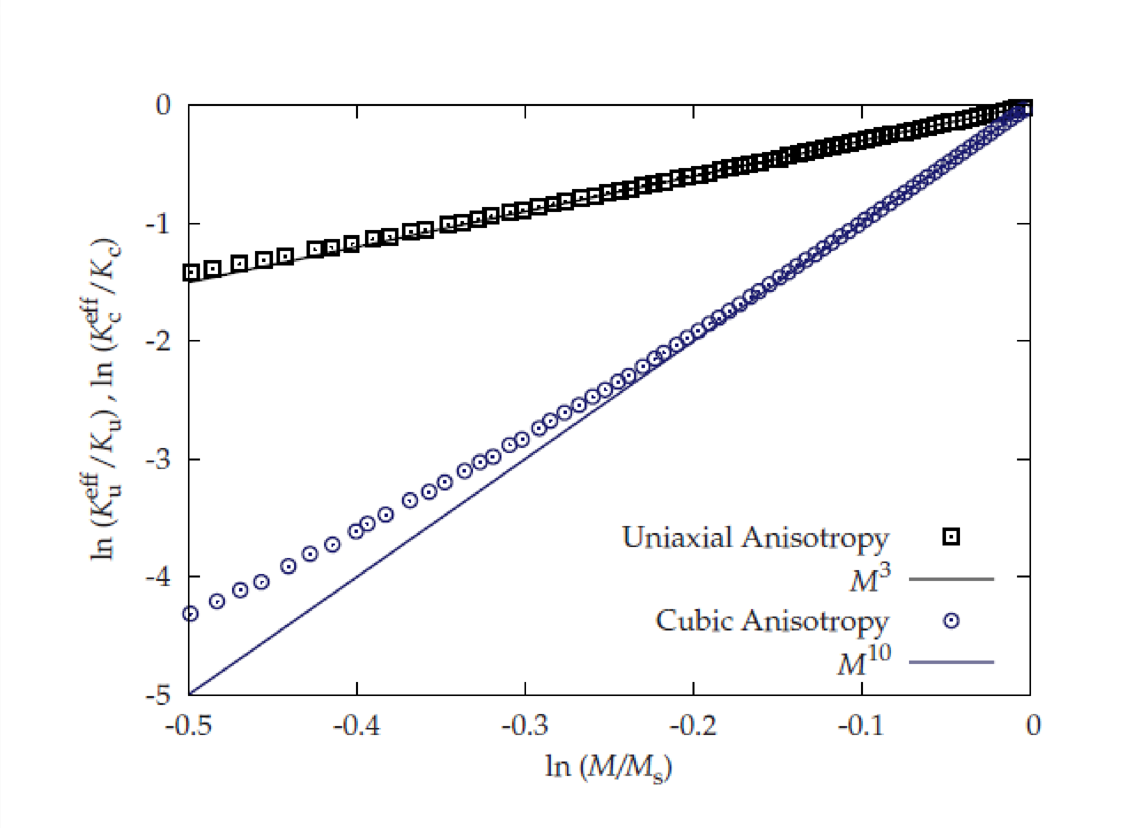


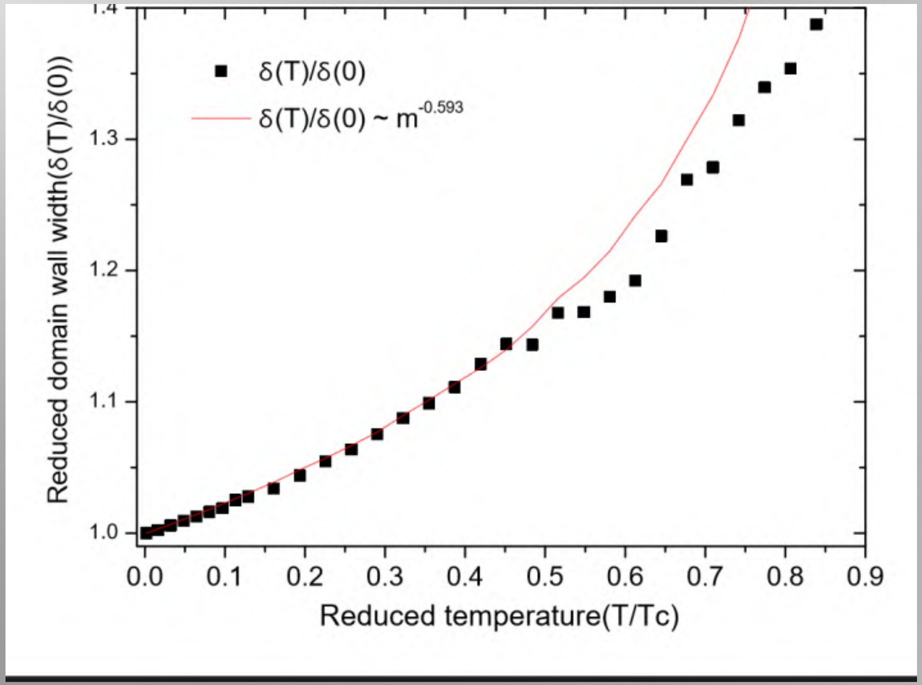
Different damping of different k-modes

$$\omega \propto A(T)\,k^2$$

U.Atxitia. … O.C.-F… PRB 82 (2010) 054415

CALLEN-CALLEN LAW FOR TEMPERATURE-DEPENDENCE OF MAGNETIC PARAMETERS

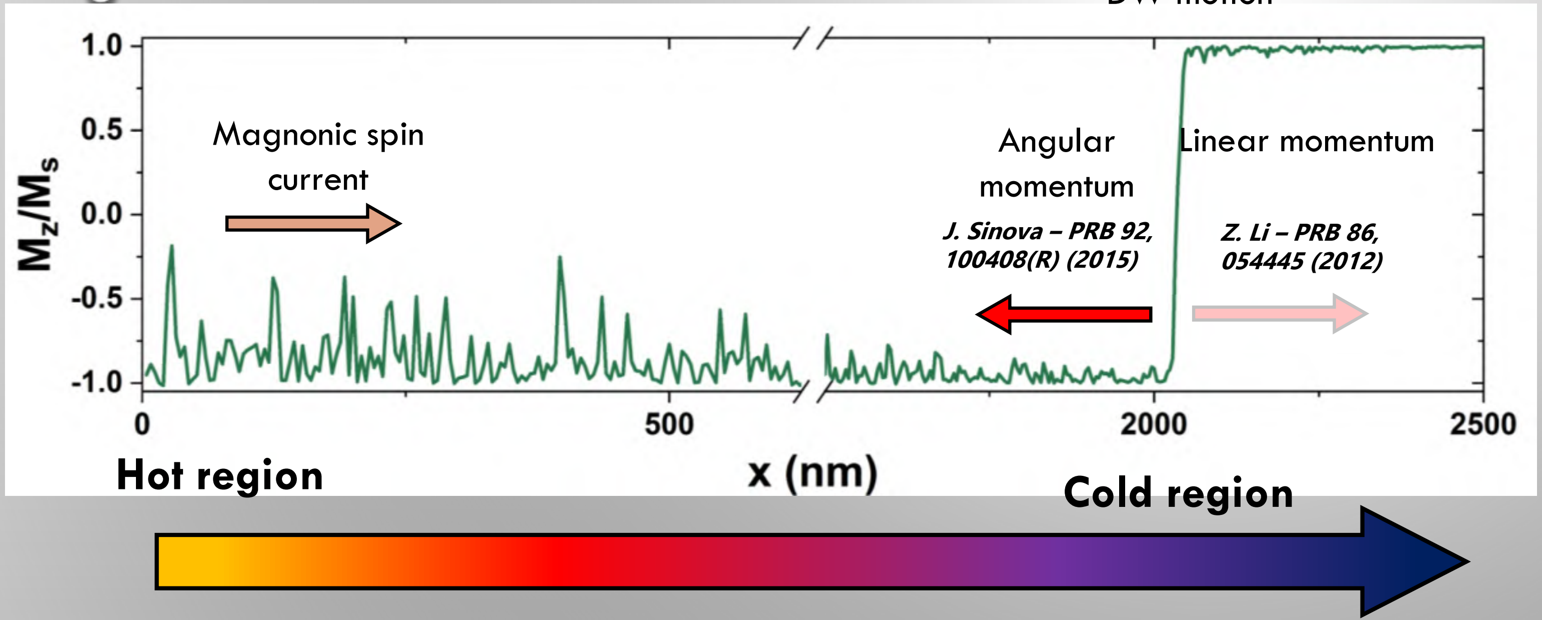




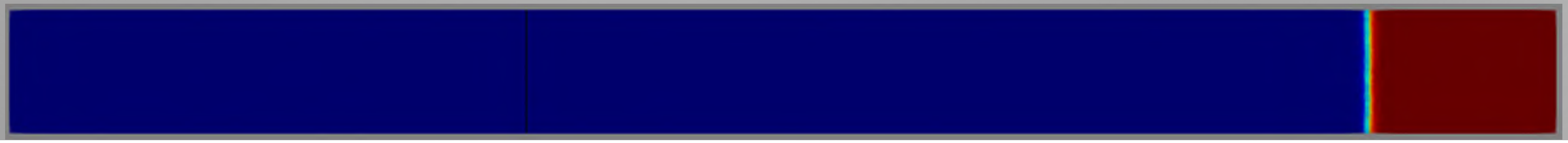
P.Asselin et al PRB B 82, 054415 (2010)

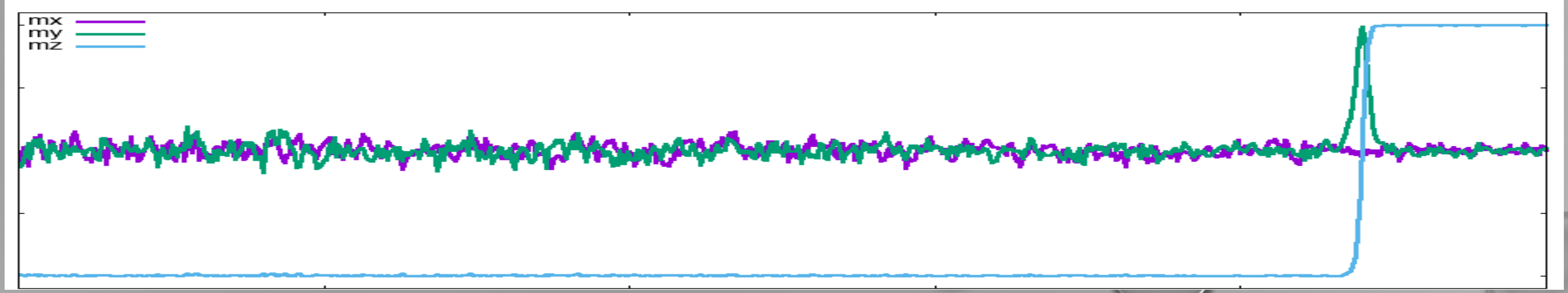
MOTION OD DOMAIN WALLS UNDER MAGNON CURRENTS

DW motion



$\nabla_x T = 0.008$ K/nm



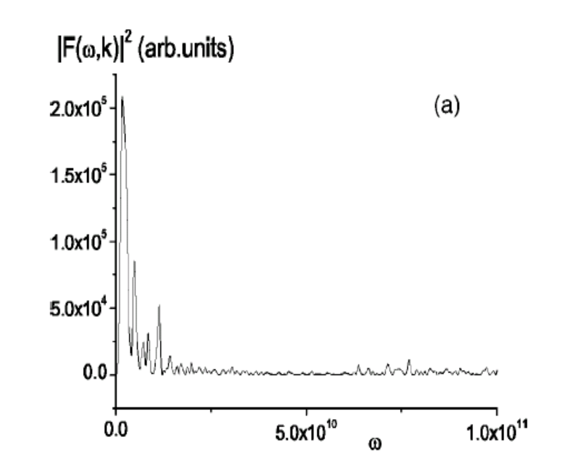


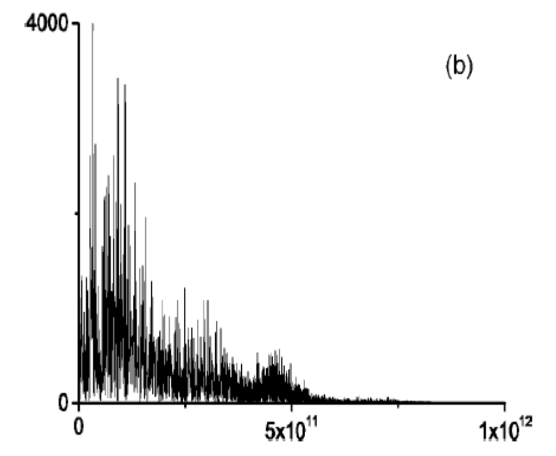
SPINWAVE ROLE DURING THE MAGNETISATION REVERSAL

- In the vicinity of the nucleation the spinwave instabilities occur.
- Spinwaves generation becomes chaotic and the system thermalizes through the distribution of energy over all degrees of freedom

- At the nucleation reversal the chaos is suppressed and the energy is transferred to the main eigenmode

k=0 k=6



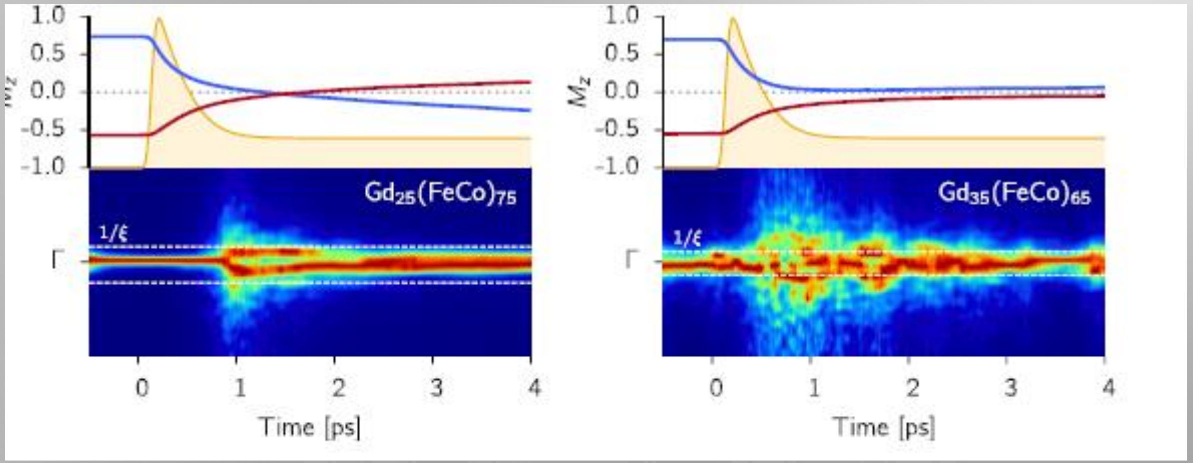


$H=-0.99\ H_K$

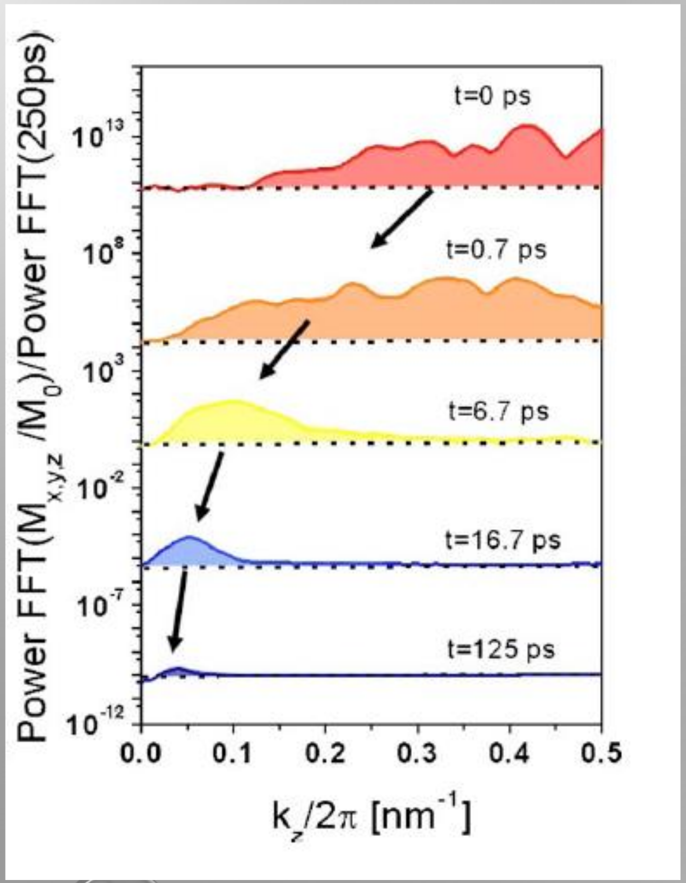
O.Chubykalo et al
Phys Rev B 65 (2002) 184428.

EQUILIBRATION OF MAGNONS AFTER STRONG (LASER) EXCITATION

GdFe switching



J.Barker, Sci Rep (2013)



Ni demagnetisation

M. Djordjevic PRB (2007)

LETTERS

Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping

S. O. Demokritov[1], V. E. Demidov[1], O. Dzyapko[1], G. A. Melkov[2], A. A. Serga[3], B. Hillebrands[3] & A. N. Slavin[4]

Creation of
Boze-Enstein condensate
By pumping of resonant phonons

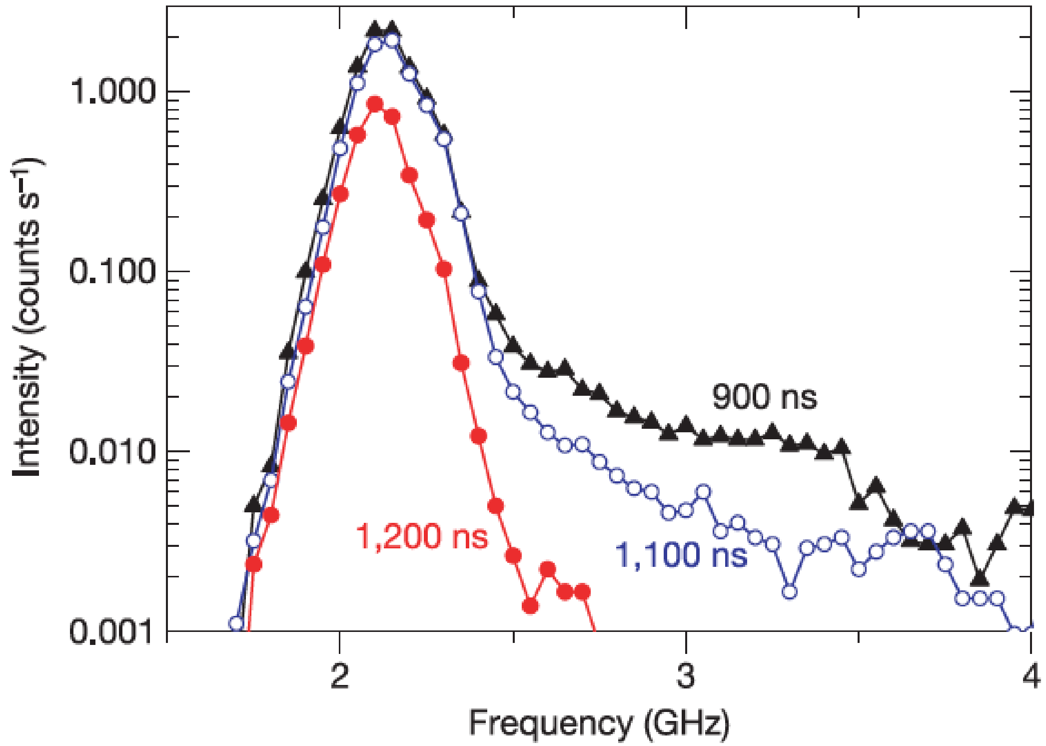


Figure 4 | Evolution of the magnon population after the pumping is switched off at τ = 1,000 ns. Open and filled circles, spectrum of light

MESSAGES

- MAGNETISAITON DYNAMICS IS CHARACTERIZED BY MAGNETISATION PRECESSION (FMR)
- SPINWAVES ARE MAGNETIC EXCITATIONS, THEIR SPECTRUM DEPEND ON GEOMETRYAND MAGNETIC PARAMETERS
- THEY CAN BE CONTROLLED BY MAGNETIC FIELDS AND GEOMETRY AND ARE PROMISING FOR MANY ICT APPLILCATIONS
- SPECIAL SPINWAVES IN NANOELEMENTS DUE TO MAGNETOSTATIC ENERGY AND DIFFERENT GROUND STATE (SD, VORTEX, SKYRMION), INFLUENCE OF GEOMETRY AND INTERACTIOSN
- MAGNONS ARE BOSONS AND OBEY THERMAL STATISTICS, RESPONSIBLE FOR DECREASE OF MAGNETISATION AND TEMPERATURE-DEPENDENT MAGNETIC PARAMETERS
- IMPORTANCE OF SPINWAVE NONLINEARITIES FOR THERMALIZATION AND MAGNETISATION REVERSAL