Fields, magnetostatics, units

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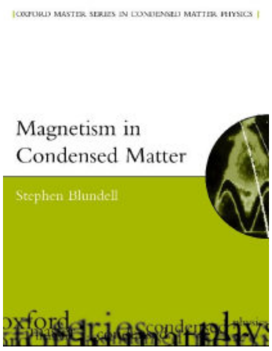
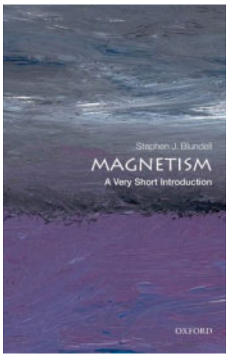
Books

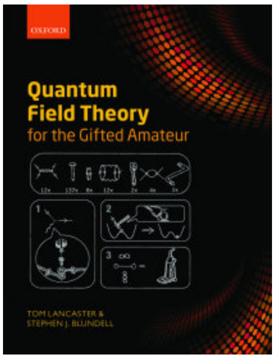
Magnetism in Condensed Matter, S.J. Blundell, OUP (2001)
Magnetism and Magnetic Materials, J.M.D. Coey, CUP (2009)
Basic aspects of the quantum theory of magnetism, D.I. Khomskii, CUP(2010)
Magnetism: A Very Short Introduction, S. J. Blundell, OUP (2012)



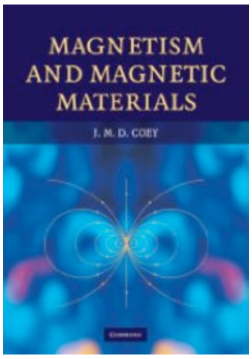
Magnetism in Condensed Matter,
S.J. Blundell,
32 Euros from amazon.de



Magnetism: VSI
S.J. Blundell,
11 Euros



Quantum Field Theory for the Gifted Amateur
T. Lancaster and S.J. Blundell,
35 Euros



Magnetism and Magnetic Materials
J. M. D. Coey
52 Euros from amazon.de

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Fields, magnetostatics, units

Lecture 1
1. Maxwell's equations
2. Poisson's equation and the Biot-Savart Law
3. Multipoles
4. Current loop and dipoles
5. Spin precession

Lecture 2
6. Field from a box of dipoles: B and H
7. Linear materials
8. Susceptibility and field calculations
9. Demagnetizing fields
10. Units

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Maxwell's equations:

(1) $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \int_S \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0}$ $\leftarrow \int_V \rho \, d\tau$

(2) $\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \int_S \vec{B}\cdot d\vec{S} = 0$

(3) $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{S}$

(4) $\vec{\nabla}\times\vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

$$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 \underbrace{\int_S \vec{J}\cdot d\vec{S}}_{I_{enclosed}} + \mu_0\varepsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{S}$$

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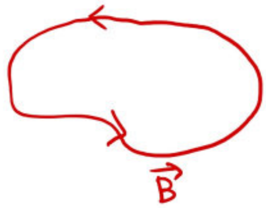
First two laws ⇒

Electric field starts and stops on charges



Magnetic field only goes round in loops. It doesn't start or stop on anything.

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B is the magnetic flux density or magnetic induction (officially), but it is often just the "magnetic field" or the "B-field".

Measured in tesla (T)   A BIG UNIT

e.g. interstellar space ~ nT
Earth's surface ~ 50 μT
strong electromagnet ~ 1 T
superconducting magnet ~ 1–20 T
pulsed field ~ 50–100 T
explosive flux compression ~ 2000 T
neutron star/magnetar ~ $10^9 - 10^{12}$ T

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EXPIRED

Old-fashioned units: B measured in Gauss.

[$10^4\,G \equiv 1\,T$.]

In free space, we can also use the H field (magnetic field strength) given by $\vec{B} = \mu_0 \vec{H}$ where $\mu_0 \overset{*}{=} 4\pi \times 10^{-7}\ H\,m^{-1}$ so that H is measured in $A\,m^{-1}$.

EXPIRED

Old-fashioned units: H measured in Oersted, and $B = \mu_0 H$ becomes $B = H$, so $1\,G = 1\,Oe$.

* Since 2018, this has no longer been definitional. But very close!

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What's the difference between B and H?

Answer #1: In free space, nothing!

There is just a scale factor between them. $\vec{B} = \mu_0 \vec{H}$

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Electrostatics:

$$\vec\nabla\cdot\vec E = \frac{\rho}{\varepsilon_0} \longrightarrow \nabla\cdot(-\nabla V) = \frac{\rho}{\varepsilon_0}$$

$$\vec\nabla\times\vec E = 0 \longrightarrow \vec E = -\nabla V \quad\Rightarrow\quad \boxed{\nabla^2 V = -\frac{\rho}{\varepsilon_0}}$$

$\frac{\partial\vec B}{\partial t} = 0$ because statics

POISSON'S EQUATION

Magnetostatics:

MAGNETIC VECTOR POTENTIAL

$$\vec\nabla\cdot\vec B = 0 \Rightarrow \vec B = \nabla\times\vec A$$

$$\vec\nabla\times\vec B = \mu_0\vec J \Rightarrow \underbrace{\vec\nabla\times(\vec\nabla\times\vec A)}_{-\nabla^2\vec A + \nabla(\nabla\cdot\vec A)} = \mu_0\vec J$$

choose $\vec\nabla\cdot\vec A = 0$ (can do this because $\vec A \to \vec A + \vec\nabla\chi$ gives same $\vec B$, so $\vec\nabla\cdot\vec A \to \vec\nabla\cdot\vec A + \nabla^2\chi$)

$$\Rightarrow \boxed{\nabla^2\vec A = -\mu_0\vec J}$$

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Simeon Poisson (1781-1840)

Life is good for only two things, discovering mathematics and teaching mathematics.

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Electrostatic Poisson equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon_0} \Rightarrow V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_V \frac{\rho(\vec{r}\,')\, d\tau'}{|\vec{r}\,' - \vec{r}|}$$

Magnetostatic Poisson equation

$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

think of as 3 independent Poisson equations

$$\nabla^2 A_x = -\mu_0 J_x \quad \text{etc}$$

$$\Rightarrow \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\vec{J}(\vec{r}\,')\, d\tau'}{|\vec{r}\,' - \vec{r}|}$$

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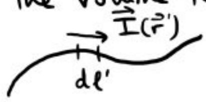
$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{r}')\,d\tau'}{|\vec{r}'-\vec{r}|}$$

confine the volume to a line



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}(\vec{r}')\,dl'}{|\vec{r}'-\vec{r}|}$$

$$\vec{B}(\vec{r}) = \nabla\times\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times(\vec{r}'-\vec{r})\,dl'}{|\vec{r}'-\vec{r}|^3}$$

$$= \frac{\mu_0}{4\pi} I \int \frac{\vec{dl}'\times(\vec{r}'-\vec{r})}{|\vec{r}'-\vec{r}|^3}$$

The Biot-Savart law.

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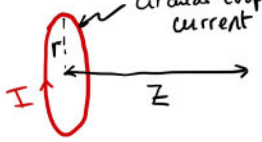
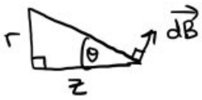
Examples

(1)



$$B(z) = \frac{\mu_0 I r^2}{2(r^2+z^2)^{3/2}}$$

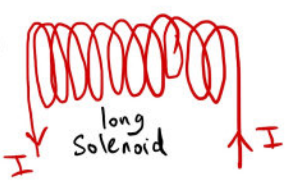
Proof:



$$\sin\theta = \frac{r}{(r^2+z^2)^{1/2}}$$

Biot-Savart: $B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi a I}{(r^2+z^2)}\sin\theta \rightarrow$ result

(2)



Field inside $B = \mu_0 n I$

NUMBER OF TURNS PER UNIT LENGTH

by direct integration of previous result or (quicker) by Ampère's theorem

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$

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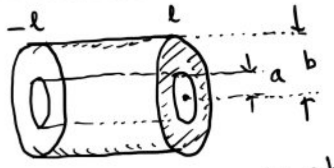
Hard examples (fill in the gaps, if keen!)

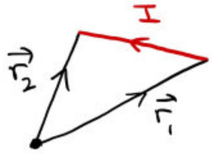
(3)



$$B \text{ at centre } = \frac{\mu_0 J}{2}\int_a^b r^2\,dr \int_{-\ell}^{\ell} \frac{dz}{(r^2+z^2)^{3/2}}$$

$$= \mu_0 J \ell \ln\left(\frac{\alpha+\sqrt{\alpha^2+\beta^2}}{1+\sqrt{1+\beta^2}}\right) \qquad \alpha=\frac{b}{a},\ \beta=\frac{\ell}{a}$$

(4)



straight-line section $\vec{r} = \vec{r}_1 + \lambda(\vec{r}_2 - \vec{r}_1)$

λ from 0 to 1

$$\vec{B} = \frac{\mu_0 I}{4\pi}\,\vec{r}_1\times\vec{r}_2 \int_0^1 \frac{d\lambda}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\,\frac{\vec{r}_1\times\vec{r}_2}{r_1 r_2}\,\frac{r_1+r_2}{(r_1 r_2 + \vec{r}_1\cdot\vec{r}_2)}$$

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Legendre polynomials

$$P_0(x) = 1 \qquad P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1) \qquad P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3) \qquad P_5(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

Adrien Marie-Legendre (1752-1833)

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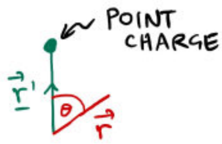
Point charge at $\vec{r}\,'$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{1}{|\vec{r}\,' - \vec{r}|} \qquad \text{Put } \underline{\vec{r}}\,' \parallel \underline{\hat{z}}$$



Solution to Laplace's equation

$$\Rightarrow V(\vec{r}) = \sum_{\ell=0}^{\infty} \left(A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}} \right) P_\ell(\cos\theta).$$

Match them, and put $\vec{r}$ up the z-axis too so $\theta = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \frac{1}{|r' - r|} = \sum_{\ell=0}^{\infty} A_\ell r^\ell + \frac{B_\ell}{r^{\ell+1}}$$

but $\displaystyle \frac{1}{|r' - r|} = \frac{1}{r} \sum_{\ell=0}^{\infty} \left(\frac{r'}{r}\right)^\ell$ for $r' < r$

$$\Rightarrow \text{in general } V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{\ell=0}^{\infty} \frac{1}{r} \left(\frac{r'}{r}\right)^\ell P_\ell(\cos\theta) \qquad r' < r$$

MULTIPOLE EXPANSION

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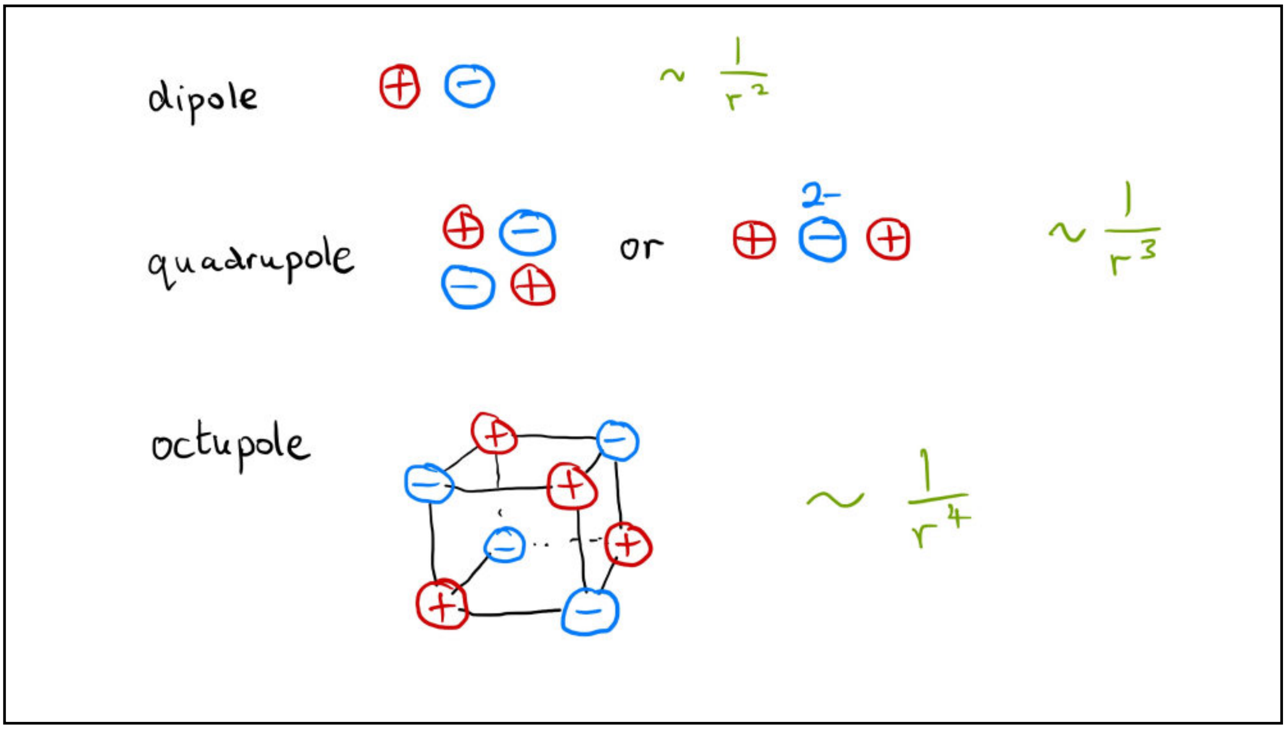

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Here is the proof of a useful vector identity. First recall Stokes' theorem which states that

$$\int \boldsymbol{\nabla} \times \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{S} = \oint \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{l}. \tag{1}$$

Now put $\boldsymbol{A}(\boldsymbol{r}) = f(\boldsymbol{r})\boldsymbol{c}$ where $\boldsymbol{c} \neq \boldsymbol{c}(\boldsymbol{r})$ is a constant vector. Hence we have

$$\boldsymbol{\nabla} \times \boldsymbol{A} = f \boldsymbol{\nabla} \times \boldsymbol{c} - \boldsymbol{c} \times \boldsymbol{\nabla} f = -\boldsymbol{c} \times \boldsymbol{\nabla} f \tag{2}$$

since $\boldsymbol{\nabla} \times \boldsymbol{c} = 0$ (because $\boldsymbol{c}$ is a constant vector, its derivative vanishes). Stokes' theorem then gives us

$$\oint \boldsymbol{A} \cdot \mathbf{d}\boldsymbol{l} = \boldsymbol{c} \cdot \oint f \, \mathbf{d}\boldsymbol{l} = -\int \boldsymbol{c} \times \boldsymbol{\nabla} f \cdot \mathbf{d}\boldsymbol{S} = -\boldsymbol{c} \cdot \int \boldsymbol{\nabla} f \times \mathbf{d}\boldsymbol{S}, \tag{3}$$

but this is true for any $\boldsymbol{c}$ and so

$$\boxed{\oint f \, \mathbf{d}\boldsymbol{l} = -\int \boldsymbol{\nabla} f \times \mathbf{d}\boldsymbol{S}.} \tag{4}$$

Now let us choose $f = \hat{\boldsymbol{r}} \cdot \boldsymbol{r}'$ and take the gradient and integral around a closed loop in the primed coordinates:

$$\oint (\hat{\boldsymbol{r}} \cdot \boldsymbol{r}') \, \mathbf{d}\boldsymbol{l}' = -\int \boldsymbol{\nabla}'(\hat{\boldsymbol{r}} \cdot \boldsymbol{r}') \times \mathbf{d}\boldsymbol{S} = -\hat{\boldsymbol{r}} \times \int \mathbf{d}\boldsymbol{S}' \tag{5}$$

which works because $\nabla'(\hat{\boldsymbol{r}} \cdot \boldsymbol{r}') = \hat{\boldsymbol{r}}$. In summary:

$$\boxed{\oint \hat{\boldsymbol{r}} \cdot \boldsymbol{r}' \, \mathbf{d}\boldsymbol{l}' = \left(\int \mathbf{d}\boldsymbol{S}' \right) \times \hat{\boldsymbol{r}}.} \tag{6}$$

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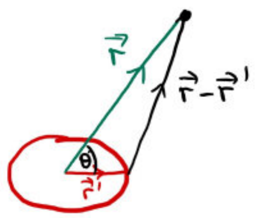
Current loop

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl}'}{|\vec{r} - \vec{r}'|}$$

$$= \frac{\mu_0 I}{4\pi} \sum_{n=0}^{\infty} \frac{1}{r^{n+1}} \oint \left(\frac{r'}{r}\right)^n P_n(\cos\theta') \vec{dl}'$$



$n=0$ term is zero

$$A_0(\vec{r}) = \frac{\mu_0 I}{4\pi r} \underbrace{\oint \vec{dl}'}_{0} = 0$$

$n=1$ term next most significant:

$$A_1(\vec{r}) = \frac{\mu_0 I}{4\pi r^2} \oint \underbrace{r' \cos\theta' \, \vec{dl}'}_{\vec{r}' \cdot \hat{\vec{r}} \, \vec{dl}'} = \frac{\mu_0}{4\pi r^2} \vec{m} \times \hat{\vec{r}}$$

where the magnetic dipole moment $\vec{m} = I \int dS' = I\vec{S}$

vector area of loop

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<u>Magnetic moment</u> = current × area

Units: $A\,m^2$

<u>Example</u>: Intrinsic magnetic moment of an electron is 1 Bohr magneton

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24}\ A\,m^2 .$$

NB $1\ A\,m^2 \equiv 1\ J\,T^{-1}$!

EXPIRED In cgs, use erg G^{-1} and $1\ \text{erg}\ G^{-1} = 10^{-3}\ J\,T^{-1}$!

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Put $\vec{m} /\!/ z$ in spherical polars:

$$\vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi r^2}\, m \sin\theta\ \hat{\vec{\phi}}$$

$$\vec{B}_1(\vec{r}) = \nabla \times \vec{A}_1(\vec{r}) = \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\, A_\phi)\,\hat{\vec{r}} - \frac{1}{r\sin\theta}\frac{\partial}{\partial r}(r\sin\theta\, A_\phi)\,\hat{\vec{\theta}}$$

$(A_r = A_\theta = 0)$

$$= \frac{\mu_0 m}{4\pi}\left(\frac{2\cos\theta}{r^3}\,\hat{\vec{r}} + \frac{\sin\theta}{r^3}\,\hat{\vec{\theta}}\right)$$

Exactly the same as for an electric dipole at sufficiently large r.

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We will use this to show that the magnetic vector potential from a magnetic dipole by

$$\boldsymbol{A} = \frac{\mu_0}{4\pi r^2}\boldsymbol{m} \times \hat{\boldsymbol{r}},$$

where $\boldsymbol{m}$ is the magnetic dipole moment.

For example, if we put $\boldsymbol{m}$ parallel to z and use spherical polars, we have that

$$\boldsymbol{A} = \frac{\mu_0}{4\pi r^2} m \sin\theta\, \hat{\boldsymbol{\phi}},$$

and so

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}(\boldsymbol{r}) = \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}(r\sin\theta\, A_\phi)\hat{\boldsymbol{r}} - \frac{1}{r\sin\theta}\frac{\partial}{\partial r}(r\sin\theta\, A_\phi)\hat{\boldsymbol{\theta}},$$

because $A_r = A_\theta = 0$ and hence

$$\boldsymbol{B} = \frac{\mu_0 m}{4\pi}\left(\frac{2\cos\theta}{r^3}\hat{\boldsymbol{r}} + \frac{\sin\theta}{r^3}\hat{\boldsymbol{\theta}}\right),$$

which is the same form as an electric dipole.

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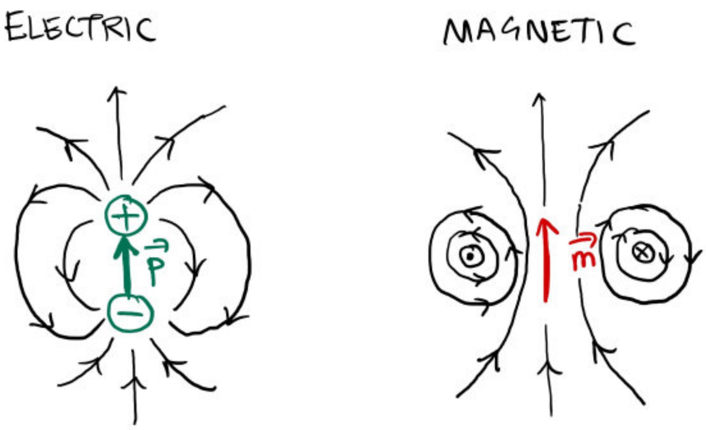

Field patterns look the same at large r
but not at small r.

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Magnetizable materials: in a field $\vec{B}$ acquire a magnetization $\vec{M}$ = magnetic dipole moment per unit volume

e.g. $\vec{M} = n\,\vec{m}$

magnetic moment of one atom/molecule

number of magnetic atoms/molecules per unit volume

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State of the system described by $|\psi\rangle$

$$|\psi\rangle = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}$$

ket

$$\langle\psi| = \begin{pmatrix} \psi_1^* & \psi_2^* & \dots & \psi_N^* \end{pmatrix}$$

bra

$$\langle\phi|\psi\rangle = \begin{pmatrix} \phi_1^* & \phi_2^* & \dots & \phi_N^* \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix} = \phi_1^*\psi_1 + \dots \phi_N^*\psi_N = \text{complex number}$$

bra-c-ket

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Magnetic moment and angular momentum

$$\vec{\mu} = \gamma \vec{L}$$

magnetic moment

angular momentum

gyromagnetic ratio $= \dfrac{q}{2m}$

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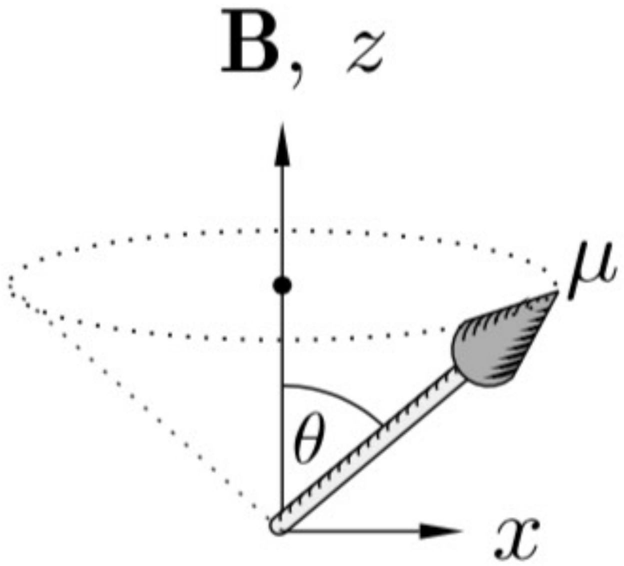

spin precession

E-fields don't do this to electric dipole moments

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Classical treatment of spin precession

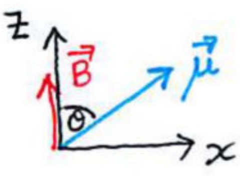
energy

$$E = -\vec{\mu}\cdot\vec{B}$$

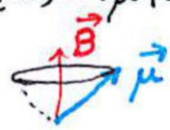
$$\vec{G} = \vec{\mu}\times\vec{B}$$

torque

$$\frac{d\vec{\mu}}{dt} = \gamma\,\vec{\mu}\times\vec{B}$$



$$\begin{pmatrix}\dot{\mu}_x\\ \dot{\mu}_y\\ \dot{\mu}_z\end{pmatrix} = \begin{pmatrix}\gamma B\mu_y\\ -\gamma B\mu_x\\ 0\end{pmatrix} \Rightarrow \begin{aligned}\mu_x(t) &= |\vec{\mu}|\sin\theta\cos\omega t\\ \mu_y(t) &= -|\vec{\mu}|\sin\theta\sin\omega t\\ \mu_z(t) &= |\vec{\mu}|\cos\theta\end{aligned}$$



$\Rightarrow$ SPIN PRECESSION $\qquad \omega = \gamma_\mu B$

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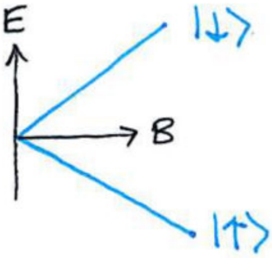
Quantum mechanics of spin-$\frac{1}{2}$

$$|\psi\rangle = a|\uparrow\rangle + b|\downarrow\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$$ SPINOR

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{H} = -\vec{\mu}\cdot\vec{B} = -\frac{\hbar}{2}\gamma B \sigma_z = -\frac{\hbar\omega}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$



$E = \frac{\hbar\omega}{2}$

$E = -\frac{\hbar\omega}{2}$

Zeeman splitting

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Pauli spin matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For a general direction $\hat{\vec{n}}$

$$\hat{n}\cdot\vec{\sigma} = \begin{pmatrix} n_z & n_x - in_y \\ n_x + in_y & -n_z \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta\, e^{-i\phi} \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}$$

Eigenstates are $|+\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2}\, e^{i\phi} \end{pmatrix}$ $\qquad |-\rangle = \begin{pmatrix} \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2}\, e^{i\phi} \end{pmatrix}$

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Pauli spin matrices

σ_x	σ_y	σ_z
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Eigenstates

$\lvert\uparrow_x\rangle$	$\lvert\downarrow_x\rangle$	$\lvert\uparrow_y\rangle$	$\lvert\downarrow_y\rangle$	$\lvert\uparrow_z\rangle$	$\lvert\downarrow_z\rangle$
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix}$	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -\mathrm{i} \end{pmatrix}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

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Quantum mechanical treatment of spin precession (1)

$B \parallel z$ Initial muon polarization

$$|\psi(0)\rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} = \cos\frac{\theta}{2}\,|\uparrow\rangle + \sin\frac{\theta}{2}\,|\downarrow\rangle$$

Time-dependence : $\hat{H}|\psi\rangle = i\hbar\frac{d}{dt}|\psi\rangle$

$$|\psi(t)\rangle = e^{i\omega t/2}\cos\frac{\theta}{2}\,|\uparrow\rangle + e^{-i\omega t/2}\sin\frac{\theta}{2}\,|\downarrow\rangle$$

$$\left.\begin{aligned} \langle\psi(t)|\sigma_x|\psi(t)\rangle &= \sin\theta\cos\omega t \\ \langle\psi(t)|\sigma_y|\psi(t)\rangle &= -\sin\theta\sin\omega t \\ \langle\psi(t)|\sigma_z|\psi(t)\rangle &= \cos\theta \end{aligned}\right\} \text{spin precession}$$

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Quantum mechanical treatment of spin precession (2)

Time-evolution operator

$$\hat{H} = -\frac{\hbar\omega}{2}\,\vec{n}\cdot\vec{\sigma}$$

$$\hat{U}(t) = e^{-i\hat{H}t/\hbar} = \cos\frac{\omega t}{2}\, I + i\sin\frac{\omega t}{2}\, \underbrace{\hat{n}\cdot\vec{\sigma}}_{\sigma_z}$$

$$|\psi(t)\rangle = \hat{U}(t)|\psi(0)\rangle = \begin{pmatrix} \cos\frac{\omega t}{2} + i\sin\frac{\omega t}{2} & 0 \\ 0 & \cos\frac{\omega t}{2} - i\sin\frac{\omega t}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} e^{i\omega t/2} \cos\frac{\theta}{2} \\ e^{-i\omega t/2} \sin\frac{\theta}{2} \end{pmatrix}$$

→ same result

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Fields, magnetostatics, units

Lecture 1
1. Maxwell's equations
2. Poisson's equation and the Biot-Savart Law
3. Multipoles
4. Current loop and dipoles
5. Spin precession

Lecture 2
6. Field from a box of dipoles: B and H
7. Linear materials
8. Susceptibility and field calculations
9. Demagnetizing fields
10. Units

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