e-ESM, an online higher-education Magnetism event

Transport and spintronics

Joo-Von Kim
Centre for Nanoscience and Nanotechnology
CNRS • Université Paris-Saclay
91120 Palaiseau, France

joo-von.kim@c2n.upsaclay.fr

Transport and spintronics

- Brief review of (some) concepts in spintronics

- Spin-dependent transport phenomena in ferromagnetic metals
  "How magnetism affects spin transport"

  - *Fermi surfaces*
    Two current model
    Giant magnetoresistance
    Tunnel magnetoresistance

- Spin transport torques
  "*How spin transport affects magnetism*"

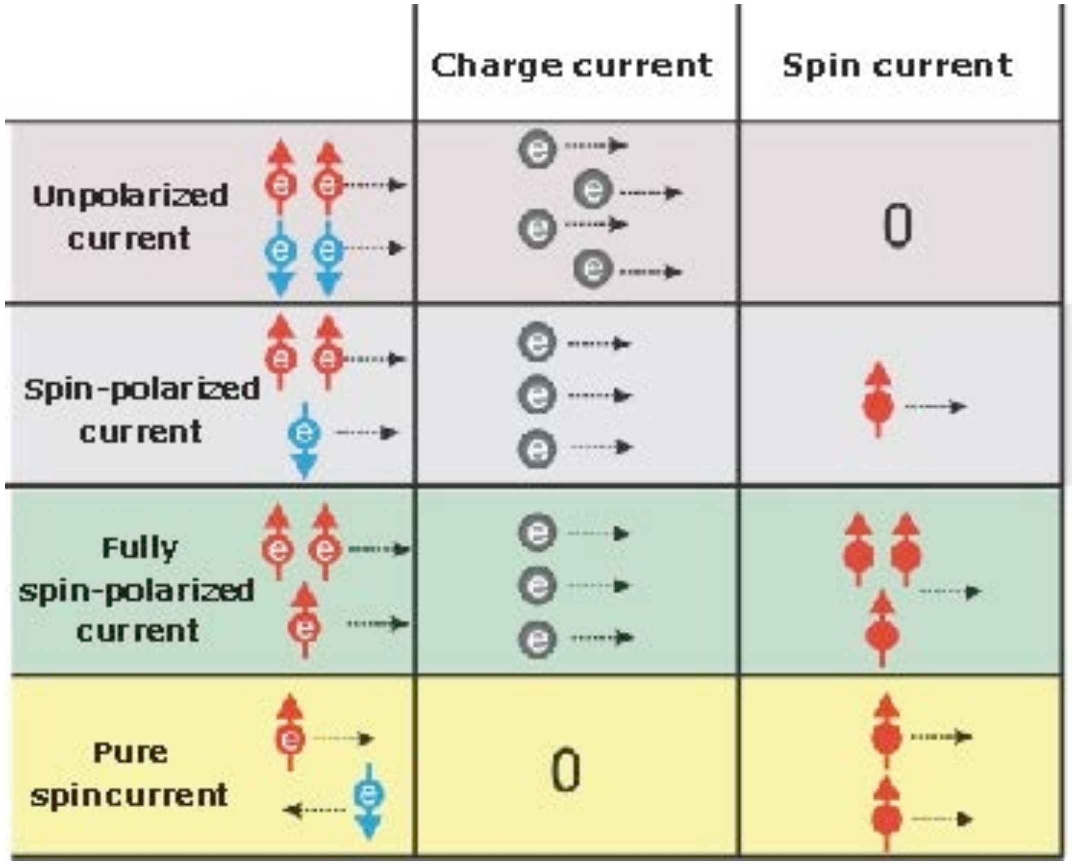
  - *Spin diffusion*
    Slonczewski model (CPP)
    Zhang-Li model (CIP)
    Spin-orbit torques

Lecture 1

Lecture 2

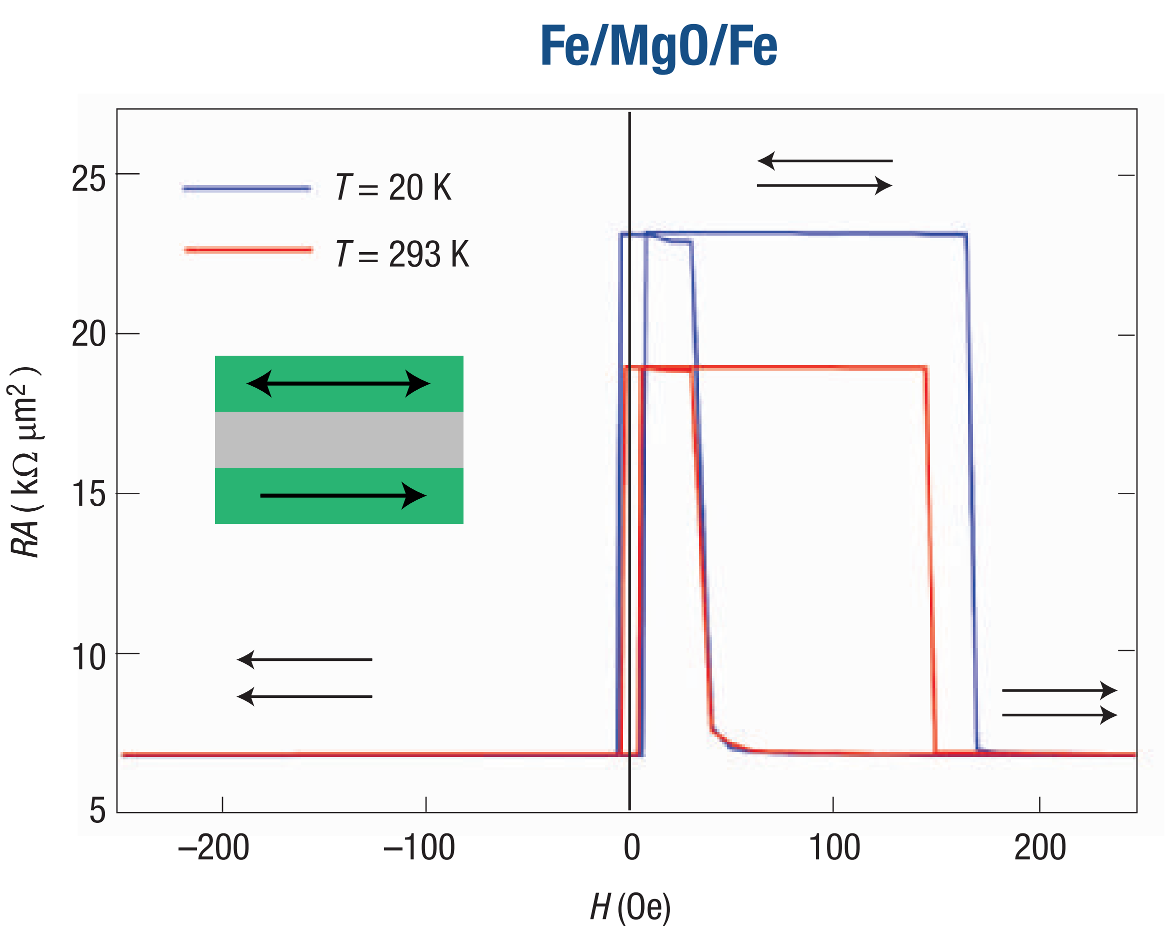
What is spintronics?

- Electronics: move electron charges (or holes) around in circuits
- Spin electronics: exploit spin degree of freedom
- In ferromagnetic transition metals, currents are naturally spin-polarized
  (Half-metals: potential for 100% spin polarization)
- **Magnetism** plays an important role in the manifestation of spin-dependent transport
- Pure spin currents do not cause Joule heating → Low power electronics

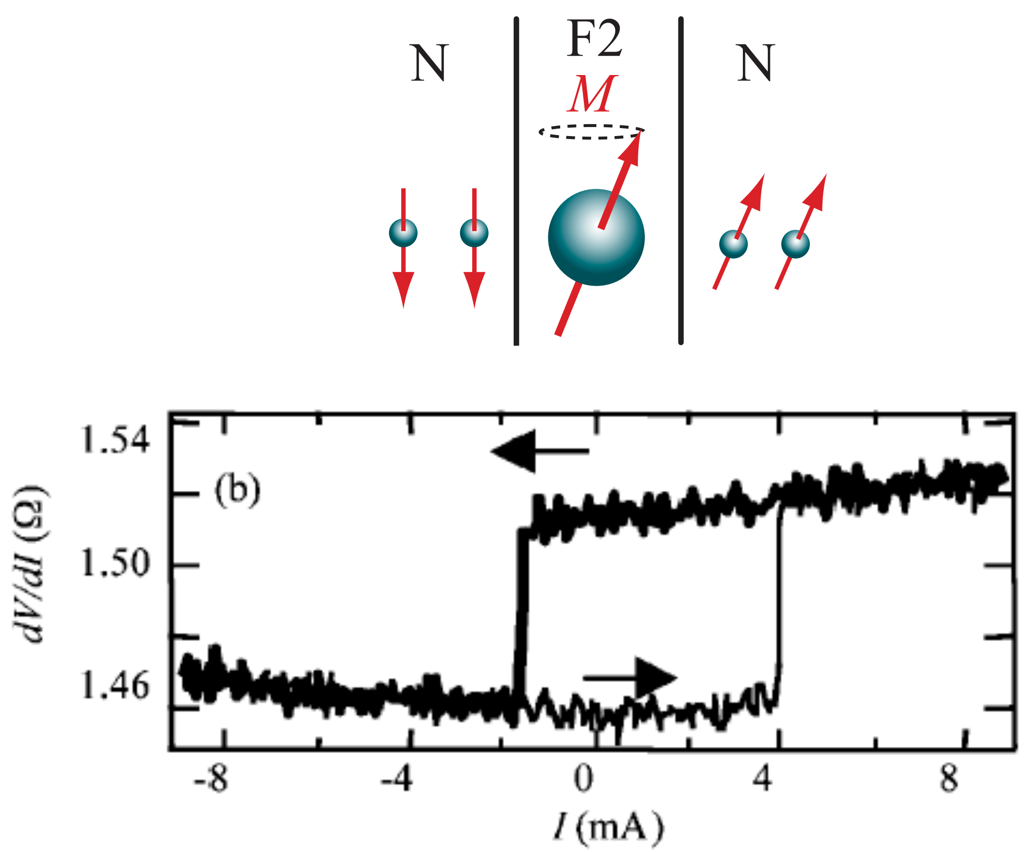


Interplay between magnetic order and transport

GMR, TMR



Spin transport torques



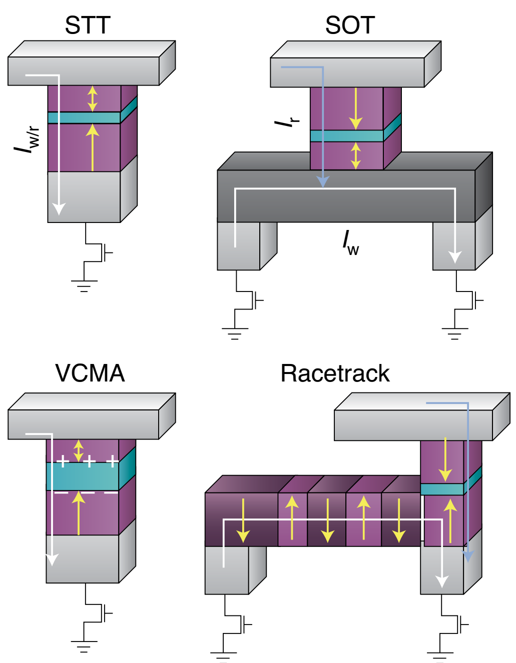
Hall effects

Many other spin-orbit related phenomena … no time to cover here

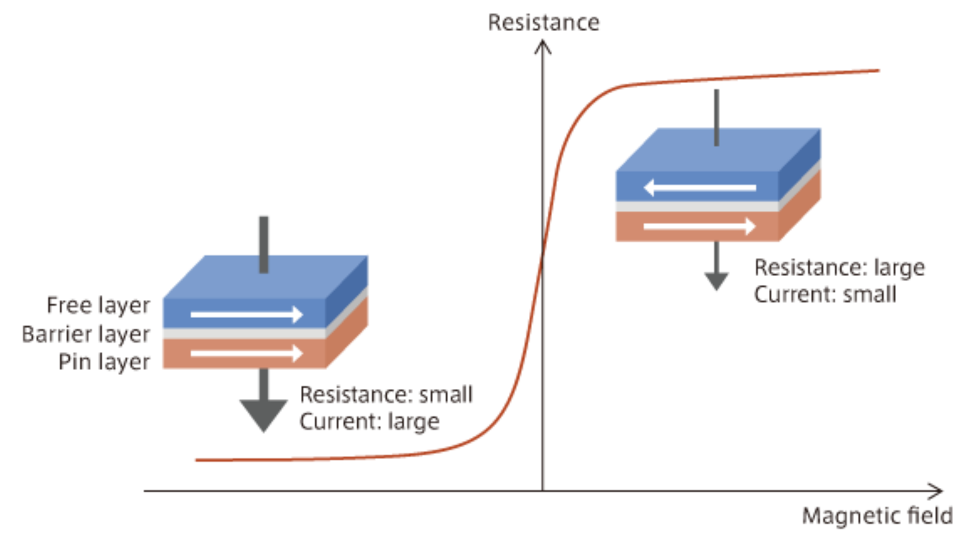
Some applications

see B Dieny *et al*, Nat Electron **3**, 446 (2020)

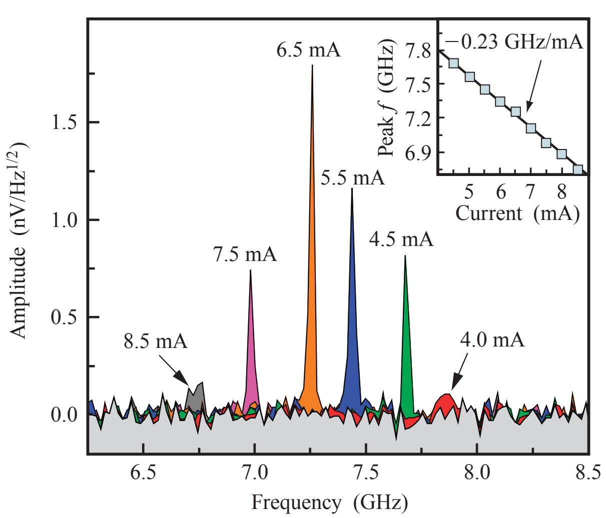
Novolatile memories

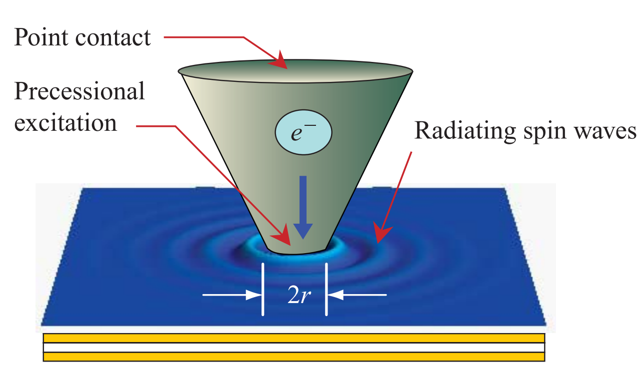


Field sensors

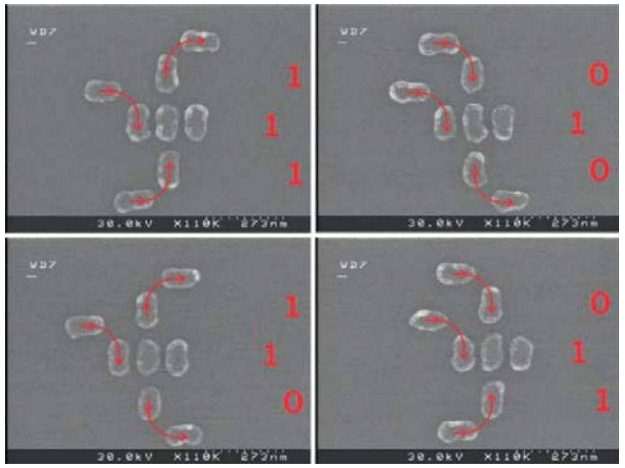


Microwave oscillators





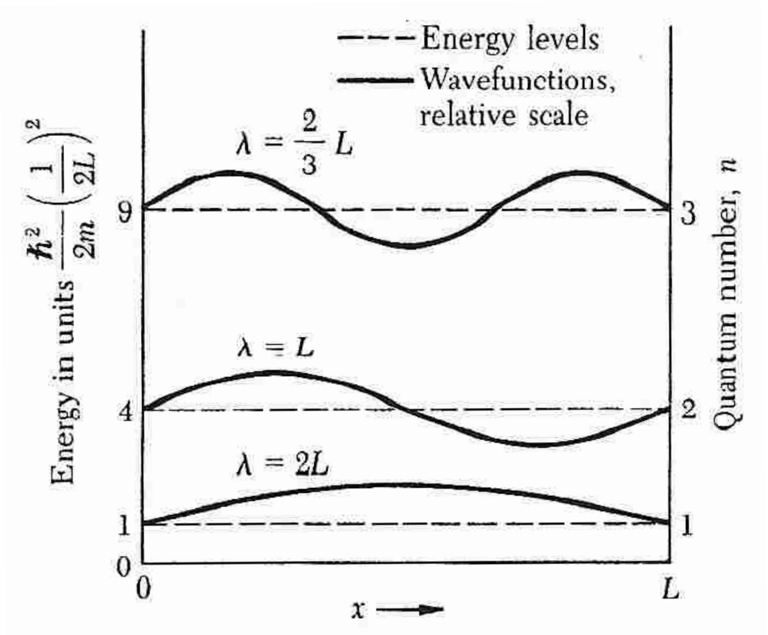
Logic



Transport in metals: Free electron model

- Conduction electrons:
  - valence electrons that move freely through volume of metal
  - do not feel metallic ions, form uniform gas
  - Subject to Pauli exclusion principle (Fermi statistics)
- Consider **1D model for *N* electron gas**

Box, length *L*



Infinite potential energy barriers, standing wave solutions

$$\mathcal{H} = \frac{p^2}{2m} \qquad p = -i\hbar\frac{d}{dx}$$

Free particle Hamiltonian

$$\mathcal{H}\psi_n = -\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} = \varepsilon_n\psi_n$$

1D Schrödinger eq.

$$\psi_n = A\sin\left(\frac{2\pi}{\lambda_n}x\right)$$

$$\psi_n(0) = \psi_n(L) = 0$$

$$\varepsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \Rightarrow \varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2$$

Fermi energy

$$2n_F = N$$

Spin degeneracy

Free electron model

- Simple generalization to **3D**

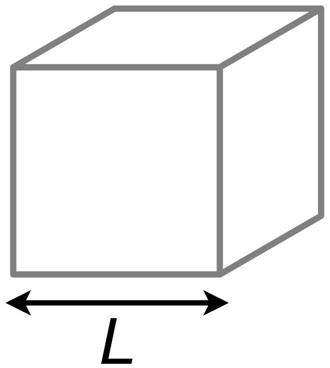
$$\mathbf{p} = -i\hbar\nabla \qquad \mathcal{H} = \frac{\mathbf{p}^2}{2m}$$

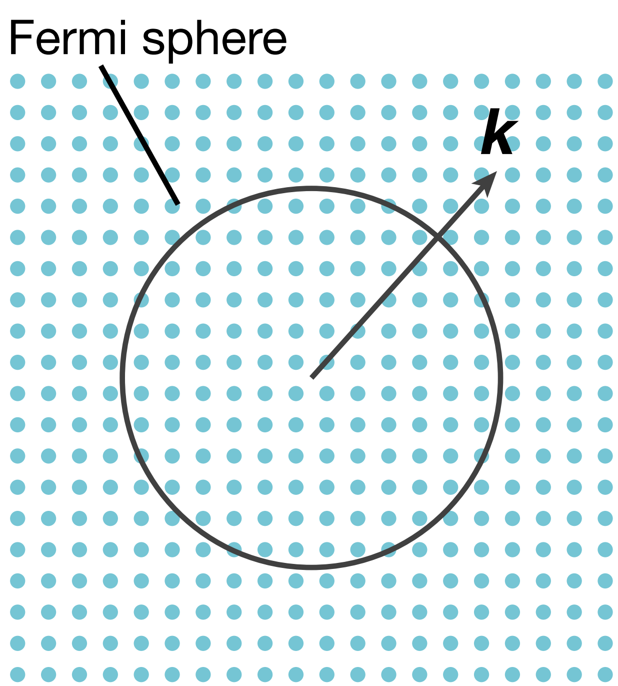
$$\mathcal{H}\psi_n = -\frac{\hbar^2}{2m}\nabla^2\psi_n = \varepsilon_n\psi_n$$

$$\varepsilon_F = \frac{\hbar^2}{2m}k_F^2 \qquad k = 2\pi/\lambda$$

- One k state (k_x, k_y, k_z) in volume element $(2\pi/L)^3$

$$k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3} \Rightarrow \varepsilon_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$





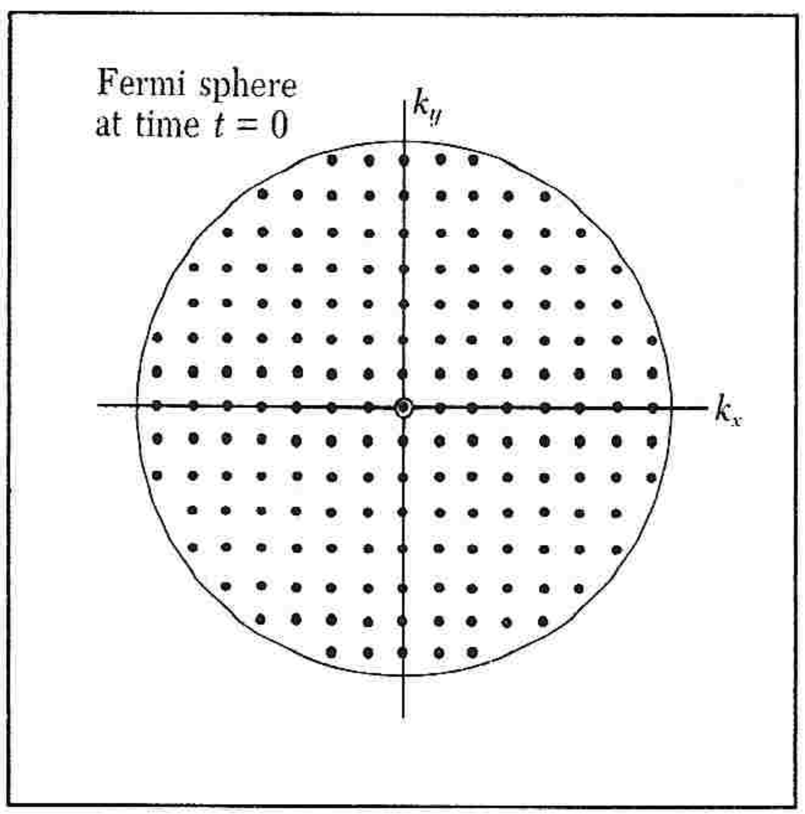
Fermi energy defined by Fermi sphere

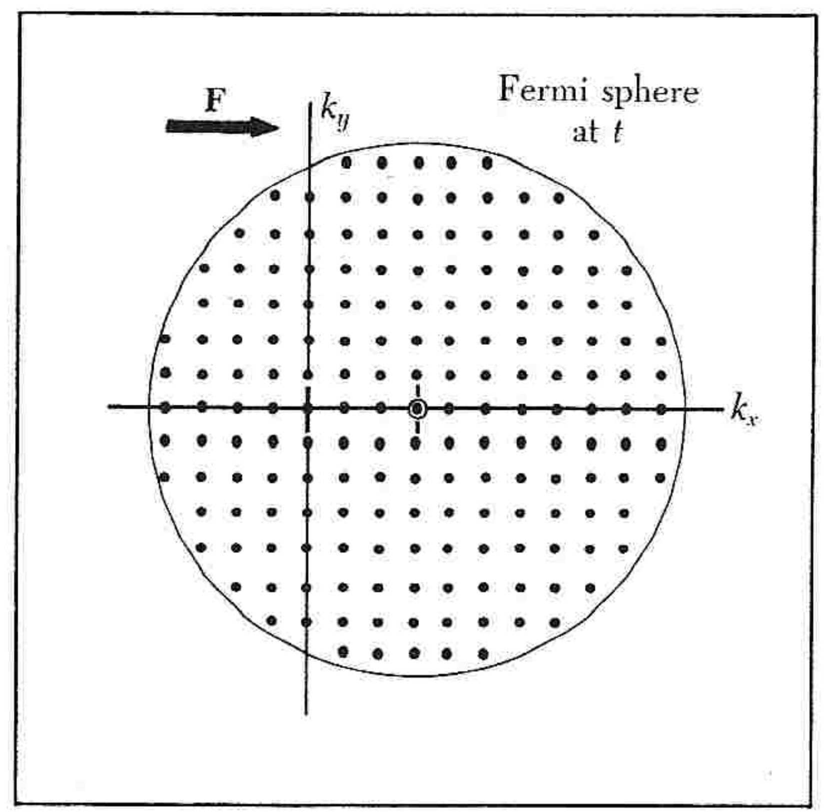
Electrical conductivity, Ohm's law

- Momentum of free electron is given by $m\mathbf{v} = \hbar\mathbf{k}$
- In an electric field ***E*** and a magnetic field ***B***, force acting on charge is

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = \hbar\frac{d\mathbf{k}}{dt} = -e\left(\mathbf{E} + \mathbf{v}\times\mathbf{B}\right)$$

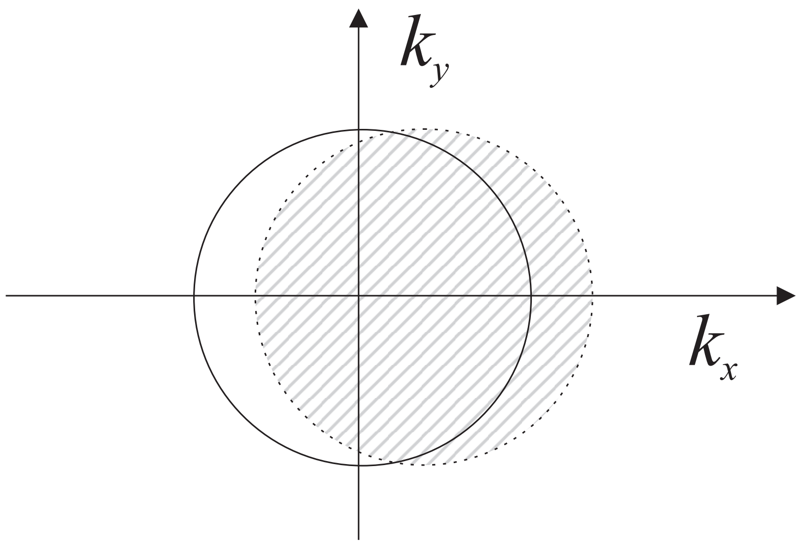
- In ***E*** field alone, Fermi surface is displaced at a rate of $\delta\mathbf{k} = -e\mathbf{E}\delta t/\hbar$





Electrical conductivity, Ohm's law

- Displacement of Fermi surface can be maintained at a constant value at steady state because of collisions
  e.g. scattering with phonons, lattice impurities



- For a mean collision time, τ

$$\mathbf{v} = -\frac{e\mathbf{E}\tau}{m}$$

- The electric current density then is

Ohm's law

$$\boxed{\mathbf{j} = nq\mathbf{v} = \frac{ne^2\tau\mathbf{E}}{m} \qquad \mathbf{j} = \sigma\mathbf{E}}$$

$$\sigma = \frac{ne^2\tau}{m} \qquad \rho = 1/\sigma$$

Drude

- Define mean-free path as $l = v_F\tau$

	τ (fs) @ 273 K	v_F (x 10^6 ms^{-1})	l (nm) @ 273 K
Cu	27	1.57	42
Au	30	1.4	56
Fe	2.4	1.98	4.8

Nearly-free electron gas

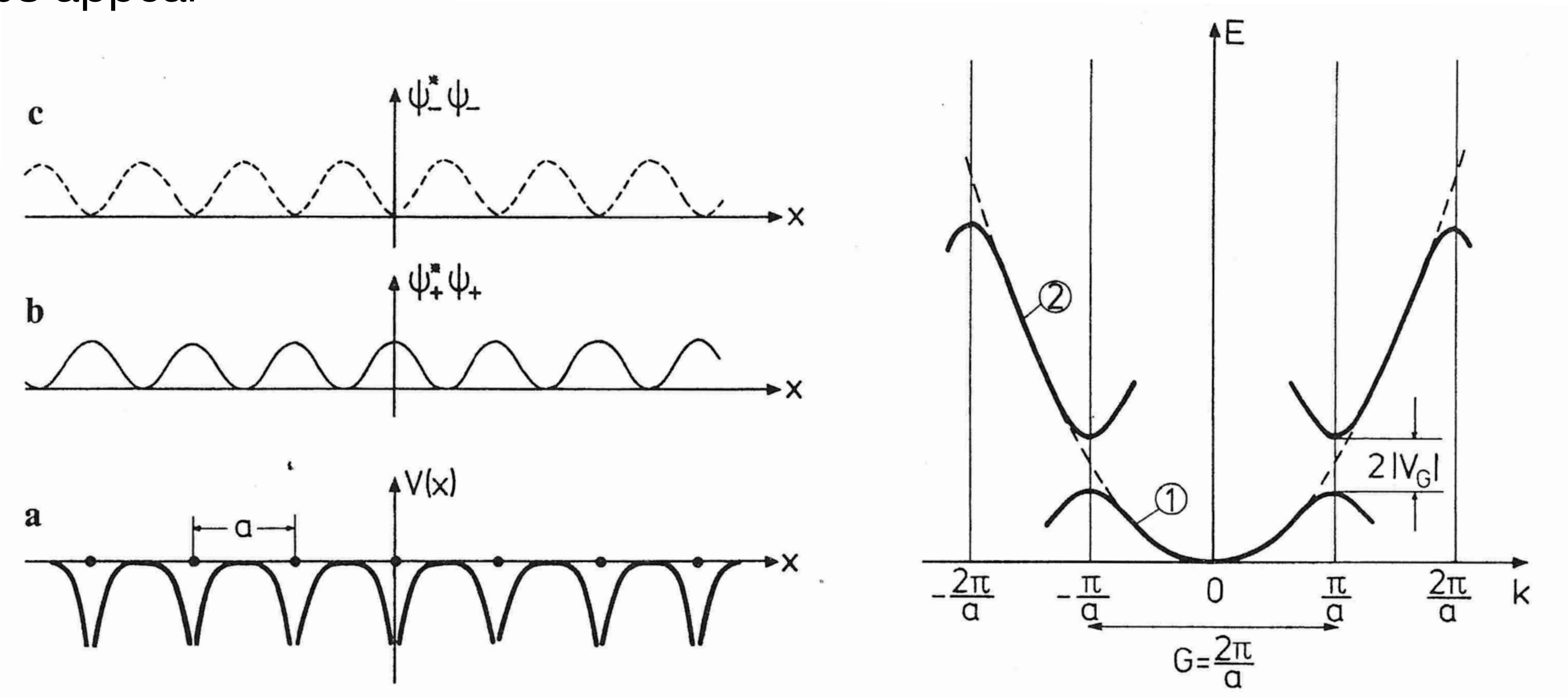
- Account for periodic lattice of background metallic ions - **periodic potential**
- Electrons in weak periodic potential are *nearly free*
- Solution to Schrödinger equation are **Bloch functions**

$$\psi_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r}) \exp(i\mathbf{k} \cdot \mathbf{r})$$

describes lattice periodicity

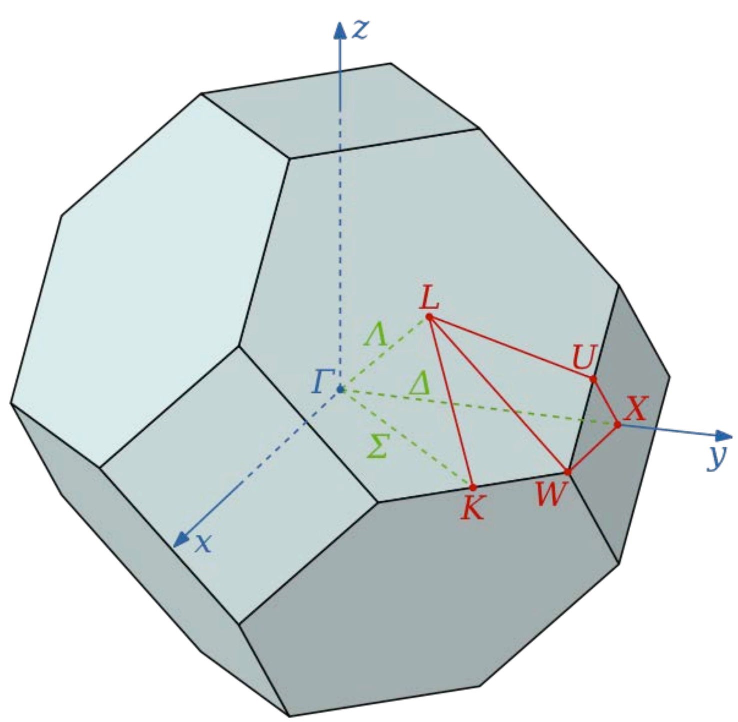
plane waves

- Degeneracies at the Bragg plane are lifted by the perturbations due to potential: **band gaps** appear

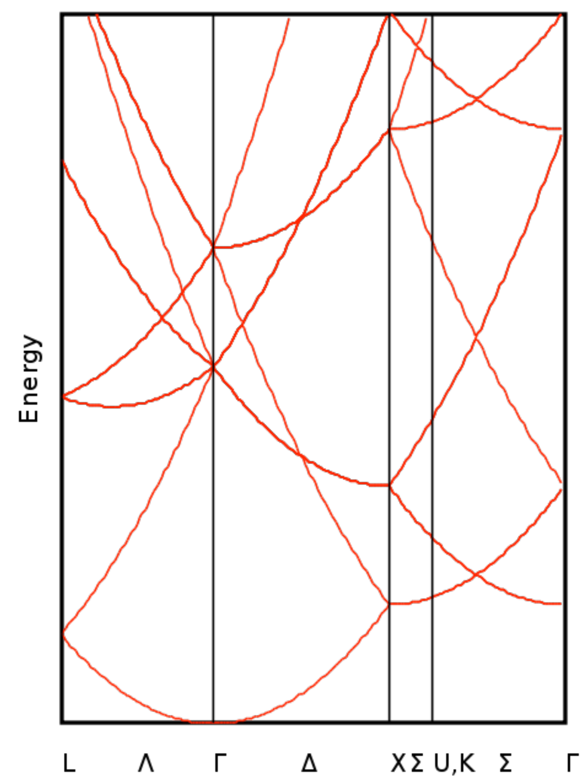


Fermi surfaces

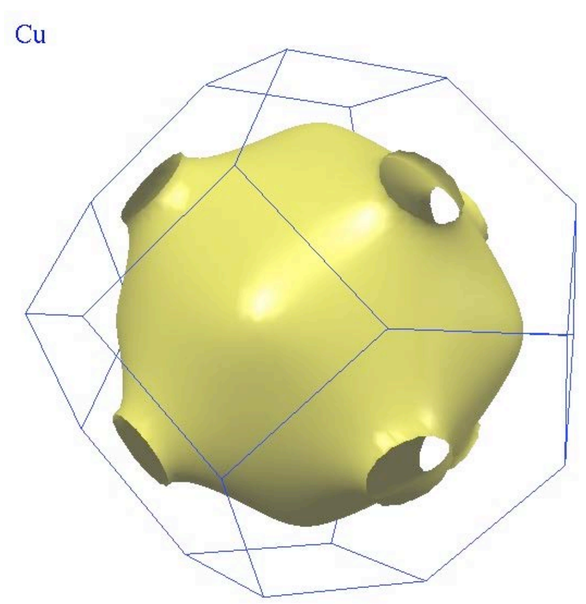
- Fermi surface is surface in *k* space of constant energy equal to the **Fermi energy**
- It separates filled states from empty states at absolute zero
- Shape of Fermi surface determines electrical properties,
  i.e., currents are due to changes in occupancy near this surface
- Account for crystal symmetry (*cf* band structure, nearly-free electron model)



fcc Brillouin zone



fcc free electron bands



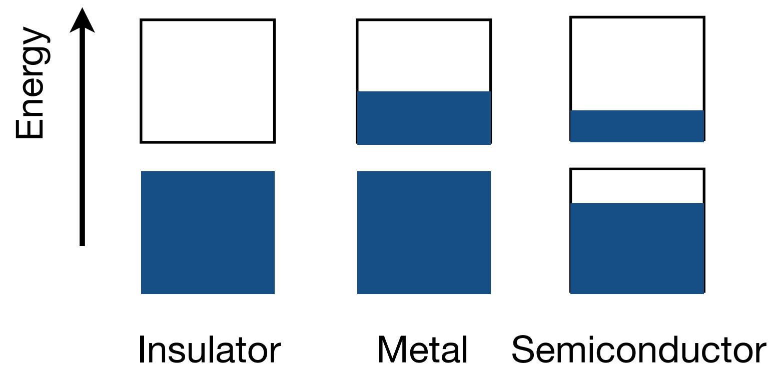
calculated surface for Cu

Fermi surfaces and conductivity

- Electrical conductivity is determined by how electrons respond to electric fields

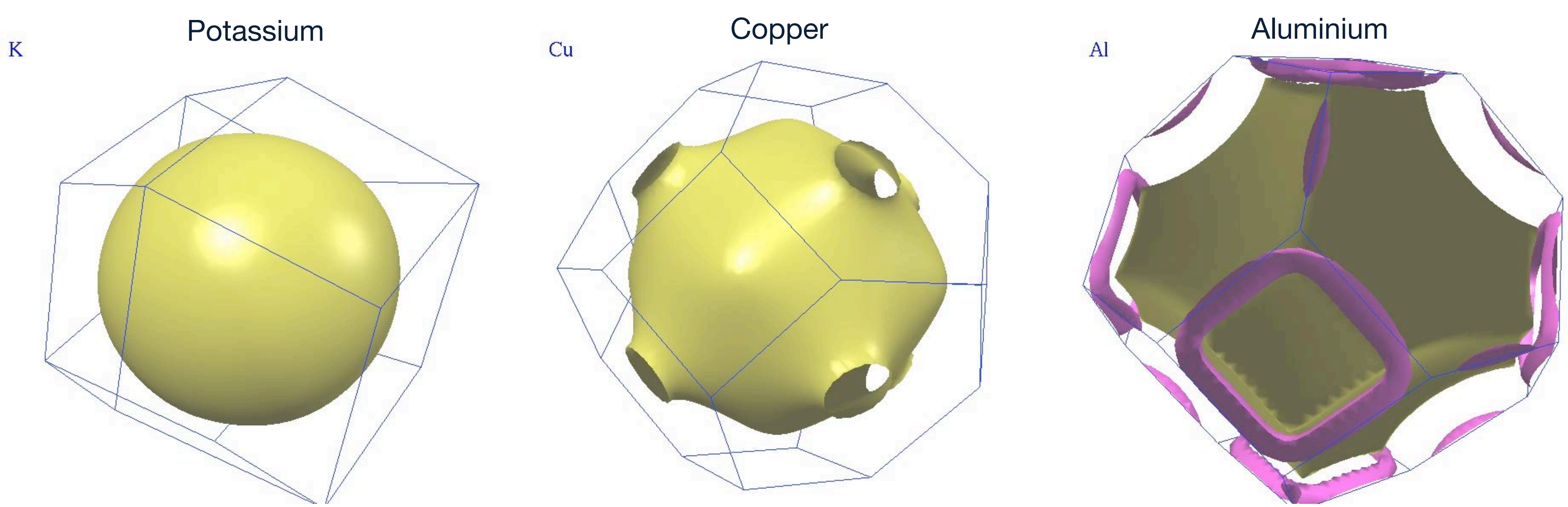
  Insulator: allowed energy bands are either empty or full

  Metal: One or more bands are partially filled



- Metals: Conduction processes occur at Fermi surface

http://www.phys.ufl.edu/fermisurface/



$4s$ $3d^{10}\,4s$ $3s^2\,3p$

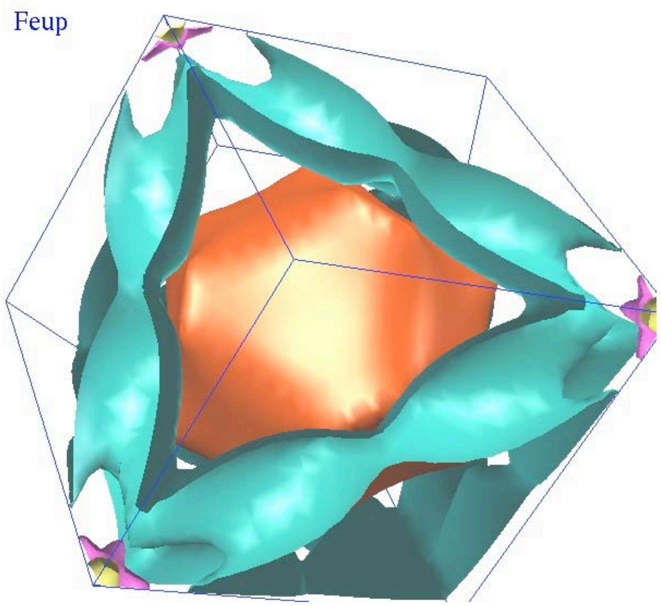
Fermi surface - ferromagnetic metals

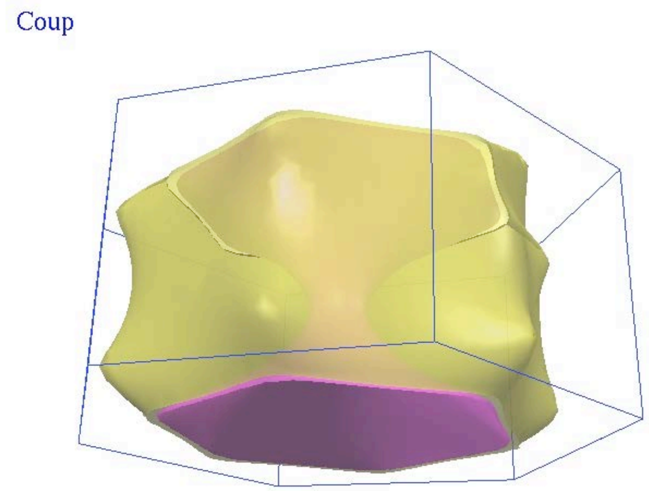
- Different Fermi surface for spin up and spin down electrons

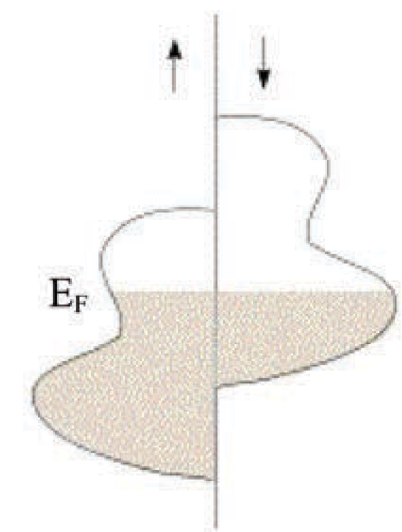
bcc iron

hcp cobalt

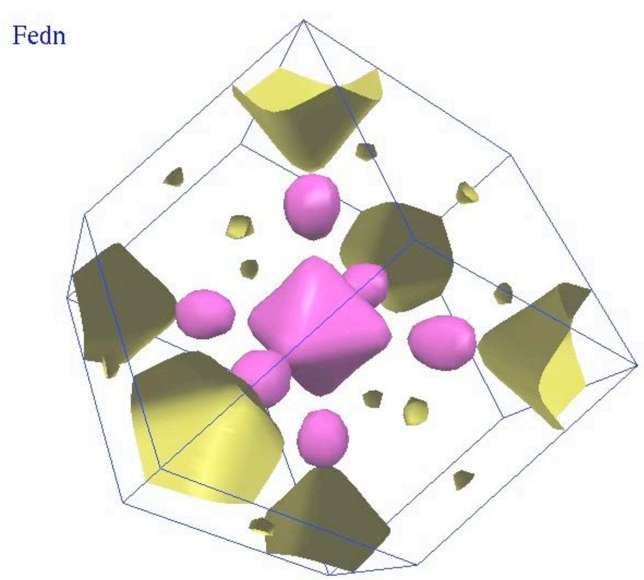
majority

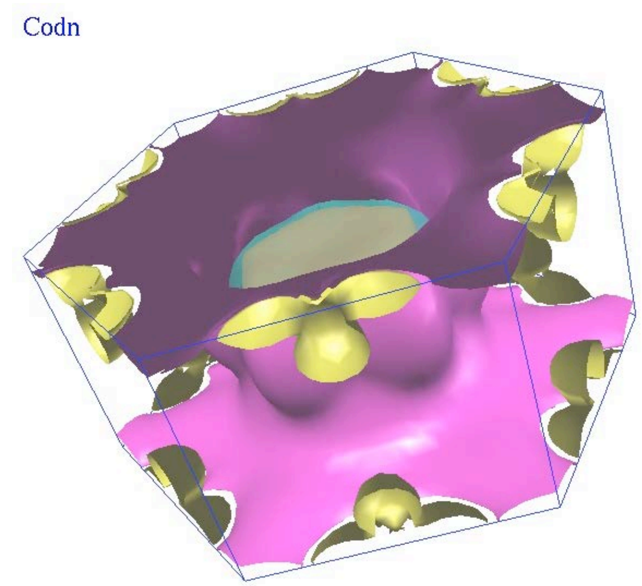






minority



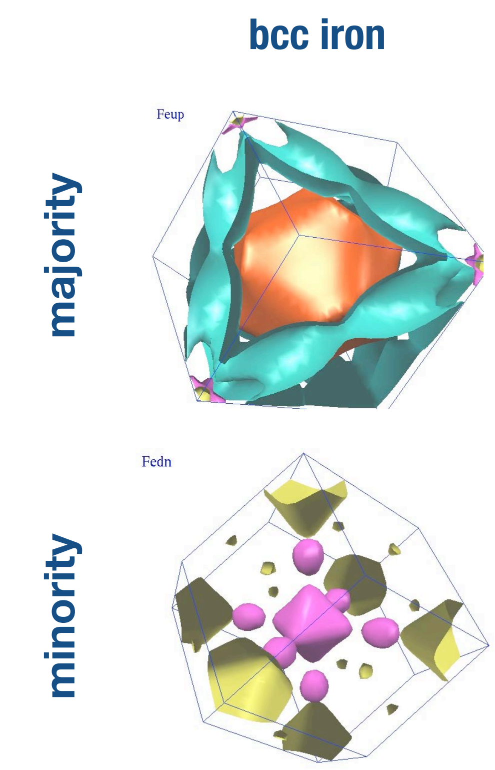


- Conduction not same for spin-up/down electrons
- **Spin-dependent** transport processes

$3d^6\,4s^2$

$3d^7\,4s^2$

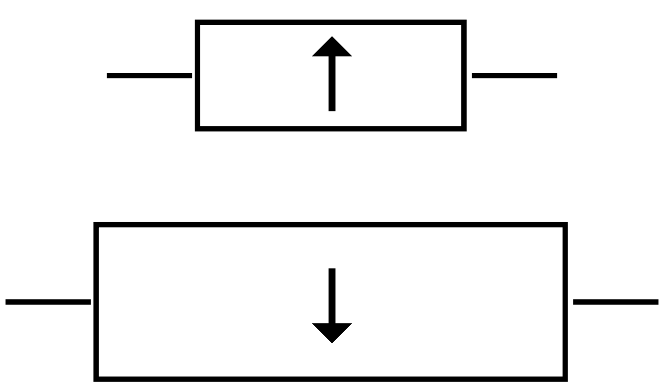
Two-current model (Mott)

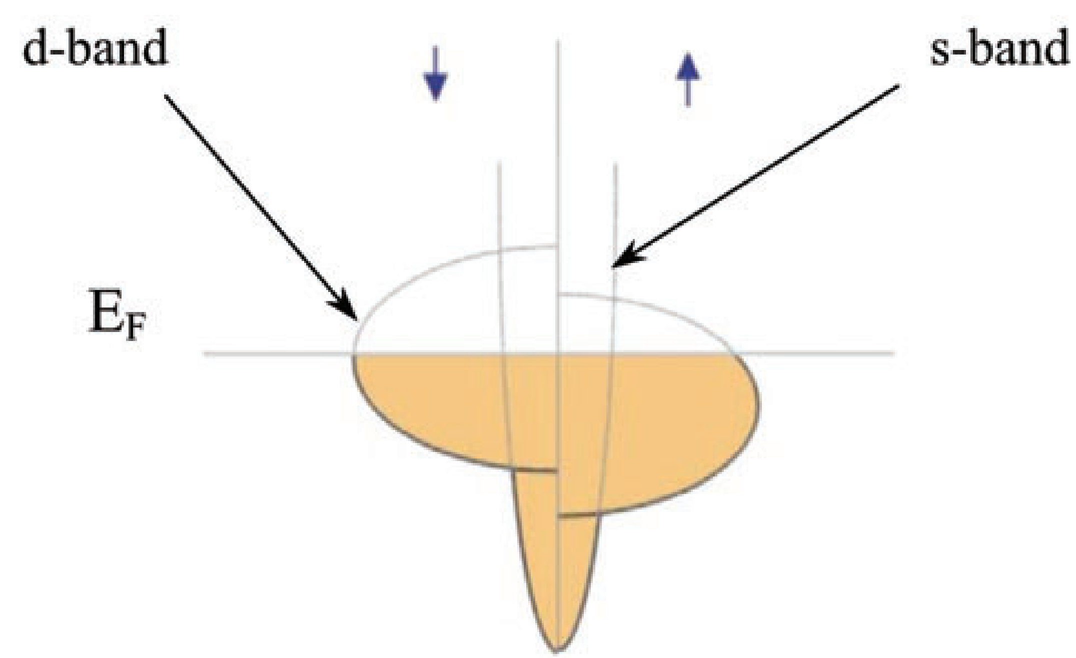


- Electrical resistance depends on Fermi velocities, density of states @ Fermi surface, etc.
- Nonmagnetic metals: Fermi surface is identical for spin-up and spin-down
- Ferromagnetic metals: different resistances for spin-up and spin-down channels
- Electrical conduction approximated as **two independent spin-channels**

$$\mathbf{j}_{\uparrow,\downarrow} = \sigma_{\uparrow,\downarrow}\mathbf{E} \qquad \sigma_{\uparrow,\downarrow} = \frac{ne^2\tau_{\uparrow,\downarrow}}{m}$$

$$R_\uparrow \neq R_\downarrow$$





Two-current model

- Spin-mixing can occur, but rare (at low T) compared with spin-conserving scattering processes

- Spin-orbit interaction and electron-magnon scattering can lead to spin flips
  (Question: Why? Which symmetry principles underlie these processes?)

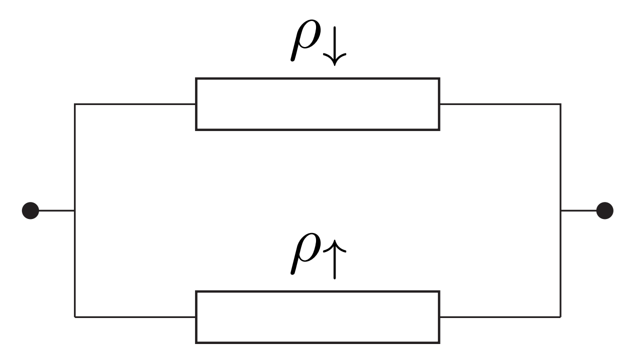
$$\mathcal{H}_{\mathrm{so}} = \frac{\hbar^2}{2m^2c^2r}\frac{dV}{dr}\mathbf{L}\cdot\mathbf{S}$$

$$\boxed{\frac{1}{\rho} = \frac{1}{\rho_1} + \frac{1}{\rho_2} + \cdots}$$

- Deviations to <u>Matthiessen's rule</u> in such cases

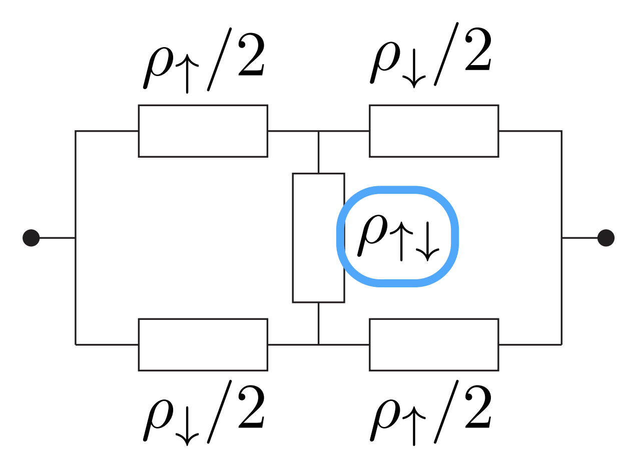
no mixing

$$\rho = \frac{\rho_\uparrow \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}$$



with mixing

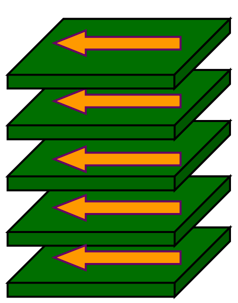
$$\rho = \frac{\rho_\uparrow \rho_\downarrow + \rho_{\uparrow\downarrow}(\rho_\uparrow + \rho_\downarrow)}{\rho_\uparrow + \rho_\downarrow + 4\rho_{\uparrow\downarrow}}$$



Giant magnetoresistance

- Giant magnetoresistance effect is an important manifestation of spin-dependent transport
- Electrical resistance of a metallic magnetic multilayer depends on the **relative orientation** of the constituent layer magnetizations

Antiferromagnetically coupled layers



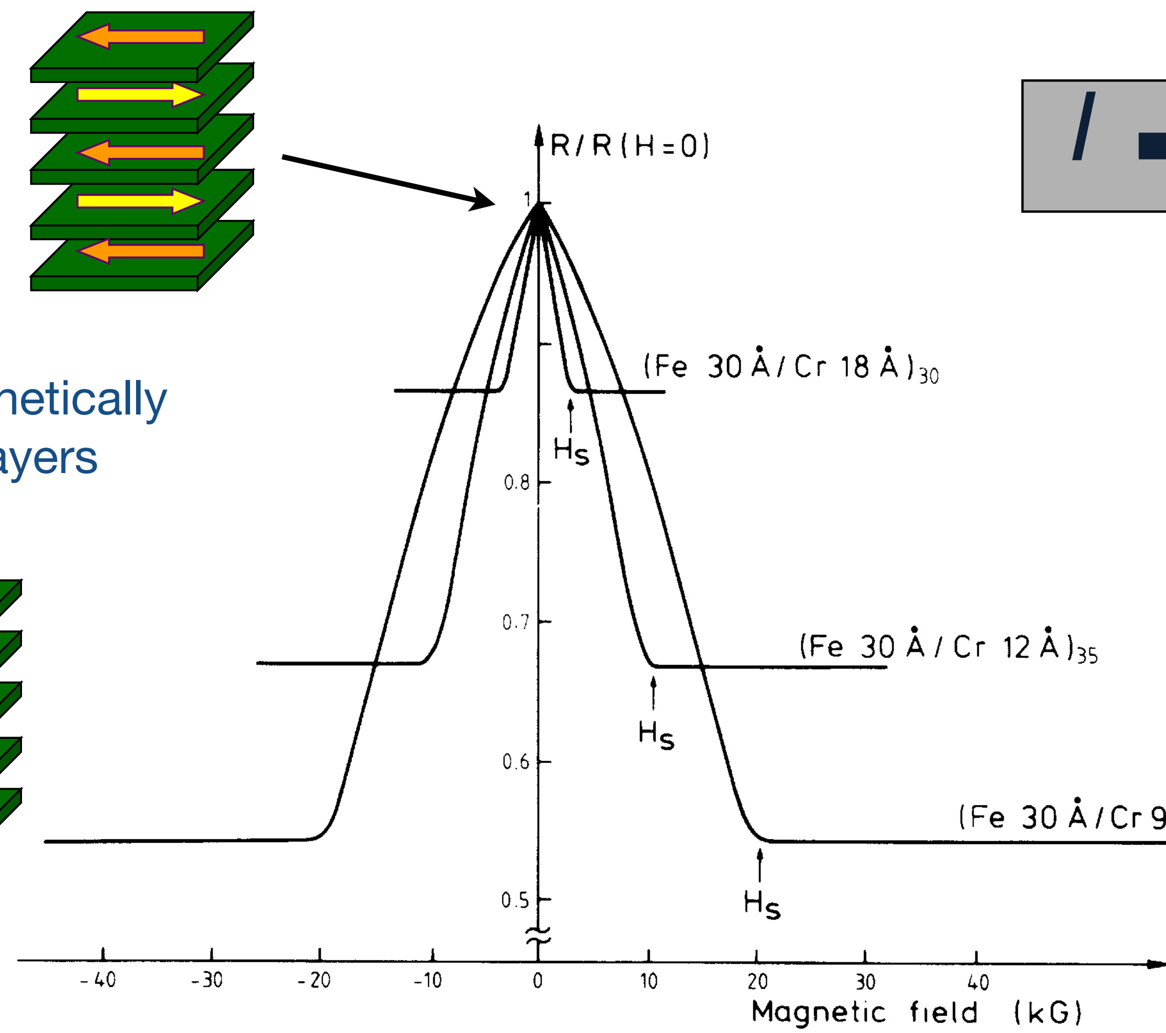
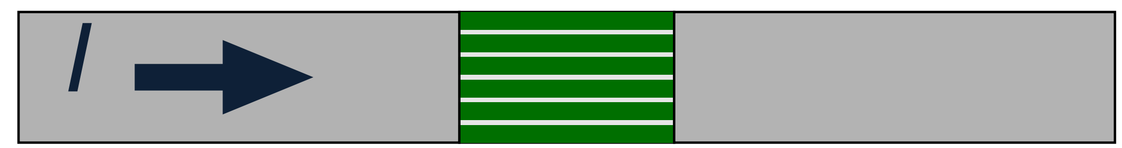
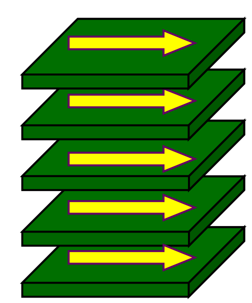


FIG. 3 Magnetoresistance of three Fe/Cr superlattices at 4.2 K. The current and the applied field are along the same [110] axis in the plane of the layers.



Current applied in the film plane: "CIP"-GMR

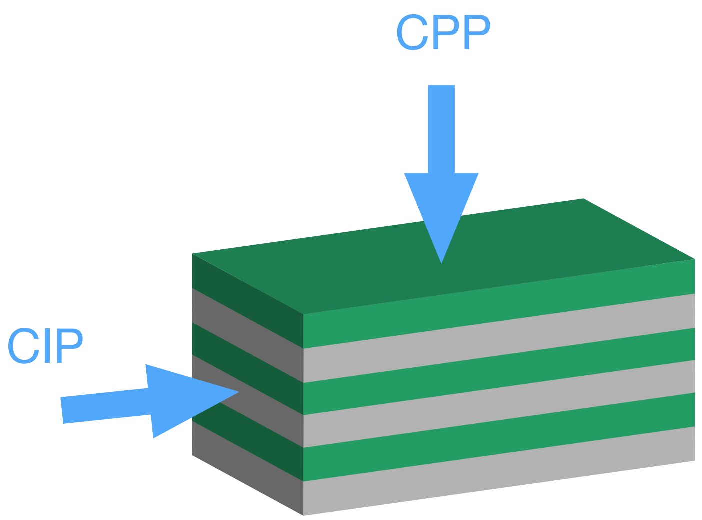


$$R_{AP} > R_P$$

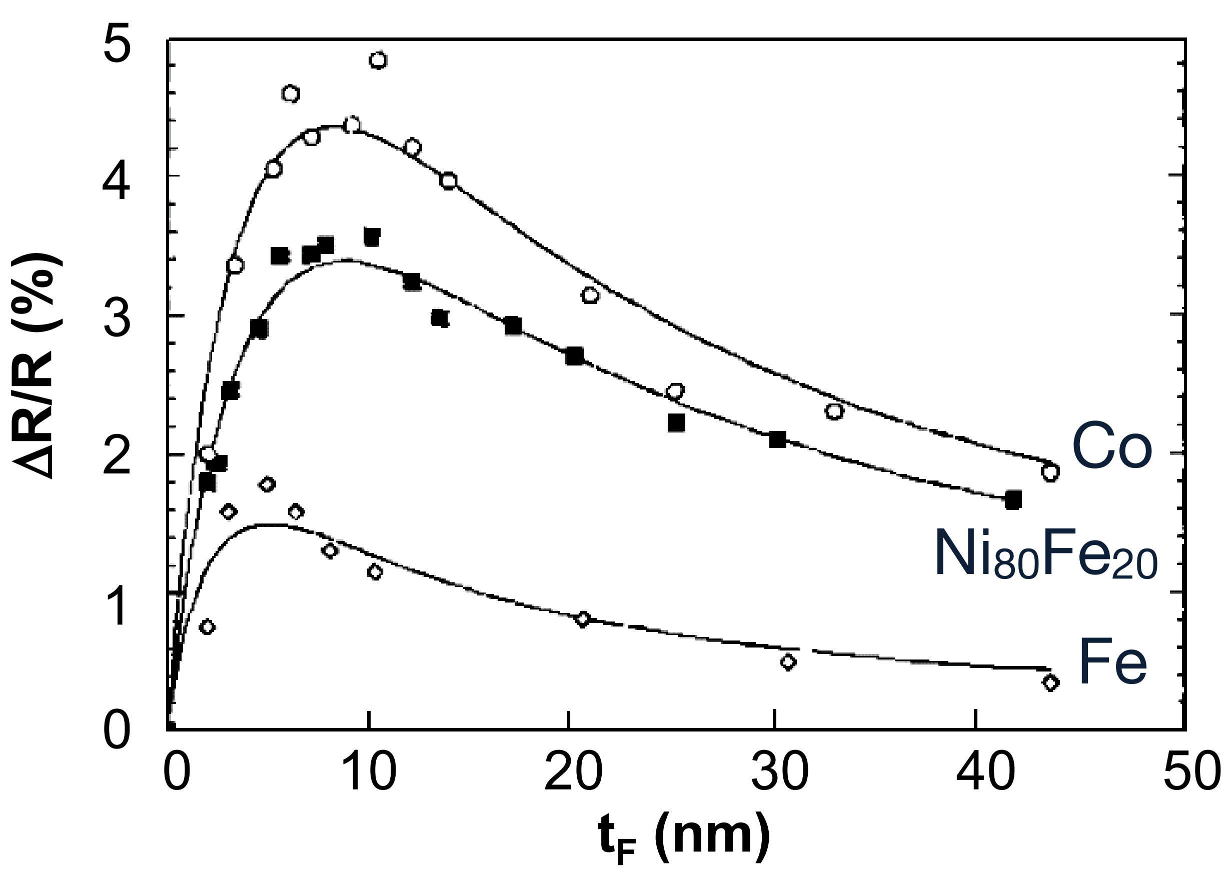
From *Baibich et al*, Phys Rev Lett **61**, 2472 (1988)

Giant magnetoresistance

- GMR is an interface effect (thin films important)
- Appears in both current-in-plane (CIP) and current-perpendicular-to-plane (CPP) geometries
- Resistance variation of order of a few percent



$$\mathrm{GMR} = \frac{\mathrm{R_{AP}} - \mathrm{R_P}}{\mathrm{R_P}} \times 100\%$$

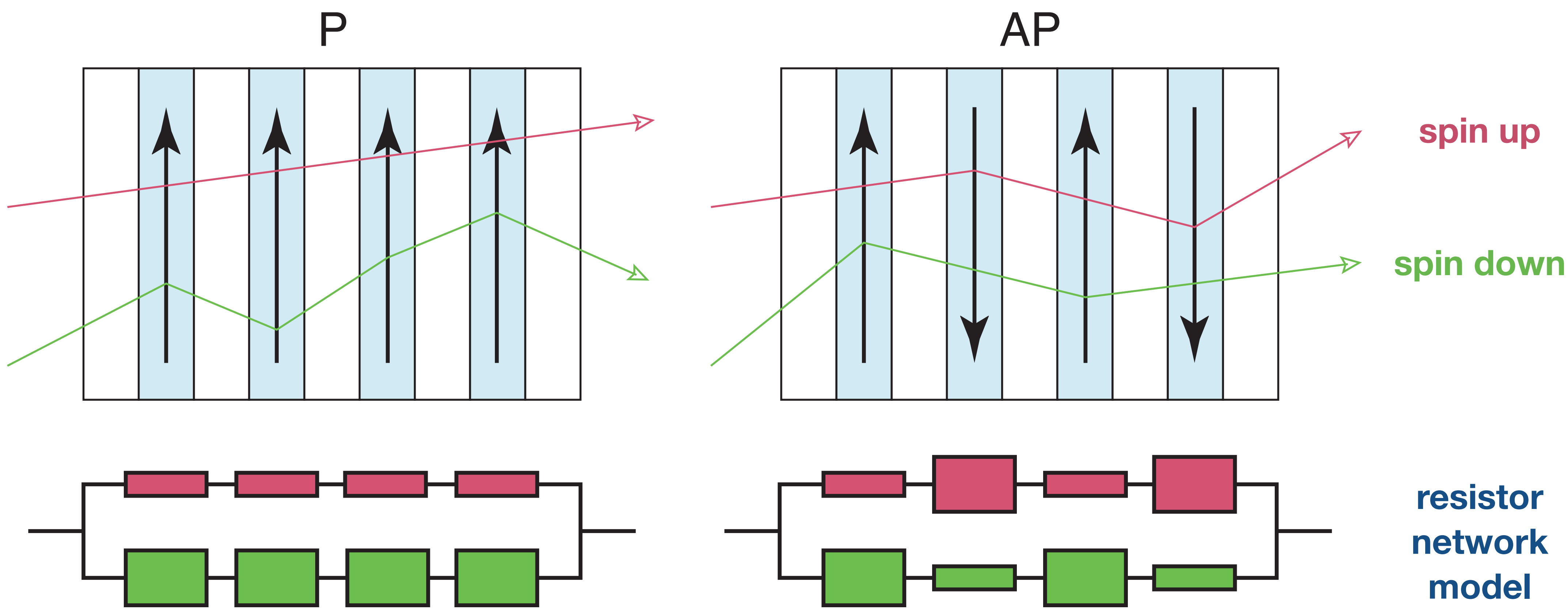


F t_F / Cu (2.5 nm) / NiFe (5 nm)

(CIP GMR, Dieny 1994)

Phenomenological model

- How can we understand the giant magnetoresistance based on what we've learnt about spin-dependent transport?
- Consider how electrons propagate through parallel and antiparallel alignment of magnetization in a superlattice structure



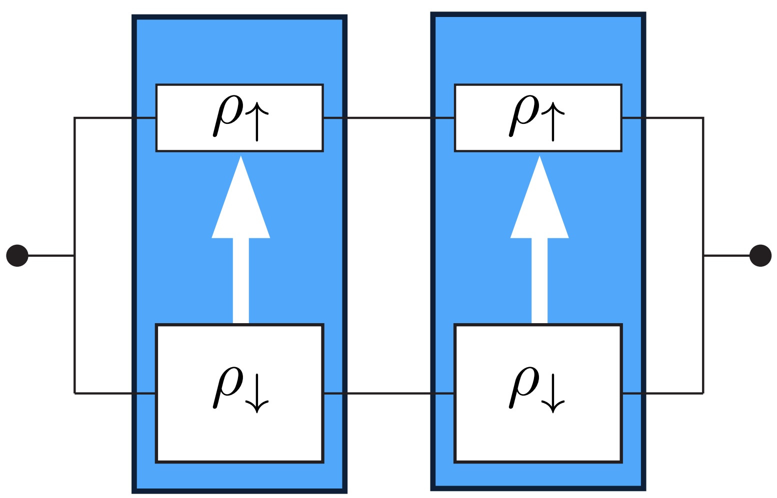
- Basic resistor model tells us that there ought to be a difference in the overall resistance of the two configurations

Phenomenological model

- An equivalent resistor model is a good starting point. Let's naively suppose that we can just combine spin-up and spin-down resistances.

e.g., CPP GMR in trilayer

$$\frac{\Delta\rho}{\rho_{\mathrm{AP}}} = \left(\frac{\rho_\uparrow - \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}\right)^2 = \left(\frac{\alpha - 1}{\alpha + 1}\right)^2$$

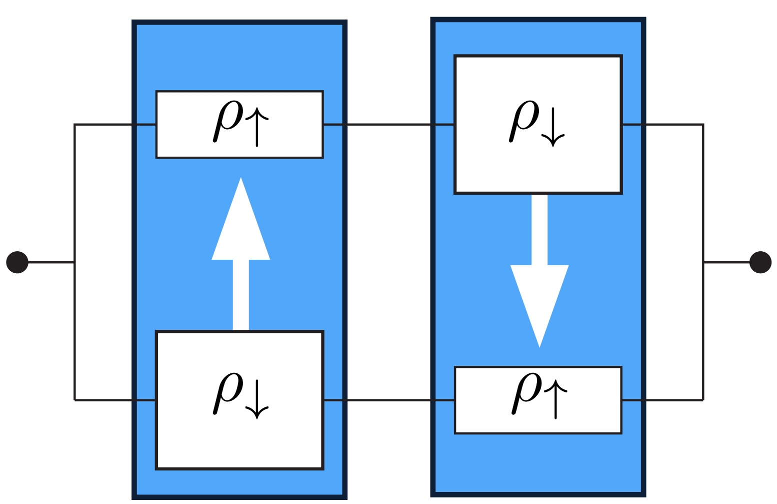


P state

$$\rho_{\mathrm{P}} = \frac{2\rho_\uparrow\rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}$$

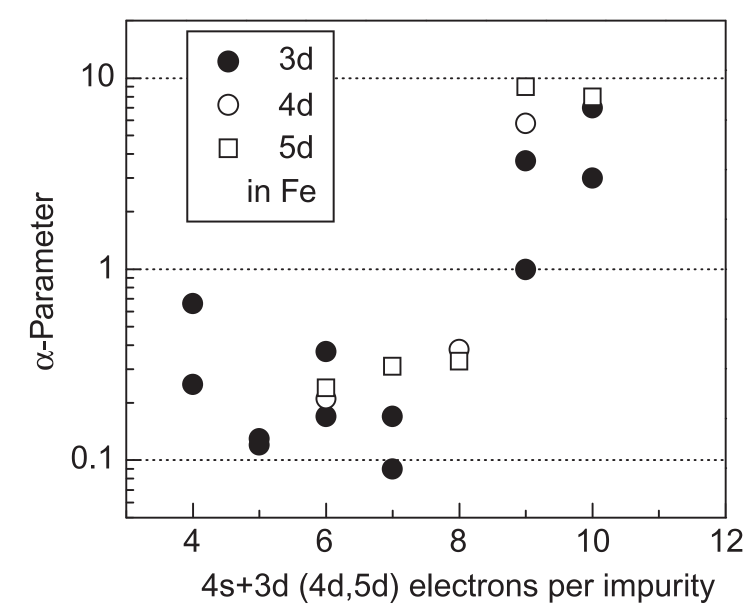
$$\alpha = \frac{\rho_\uparrow}{\rho_\downarrow}$$

spin asymmetry parameter

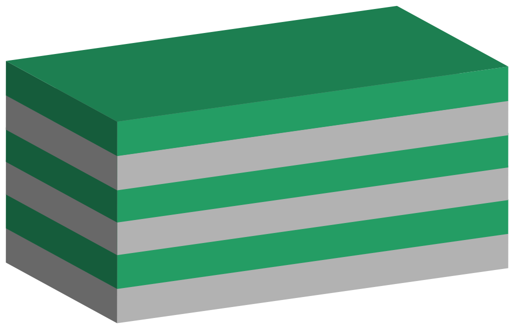


AP state

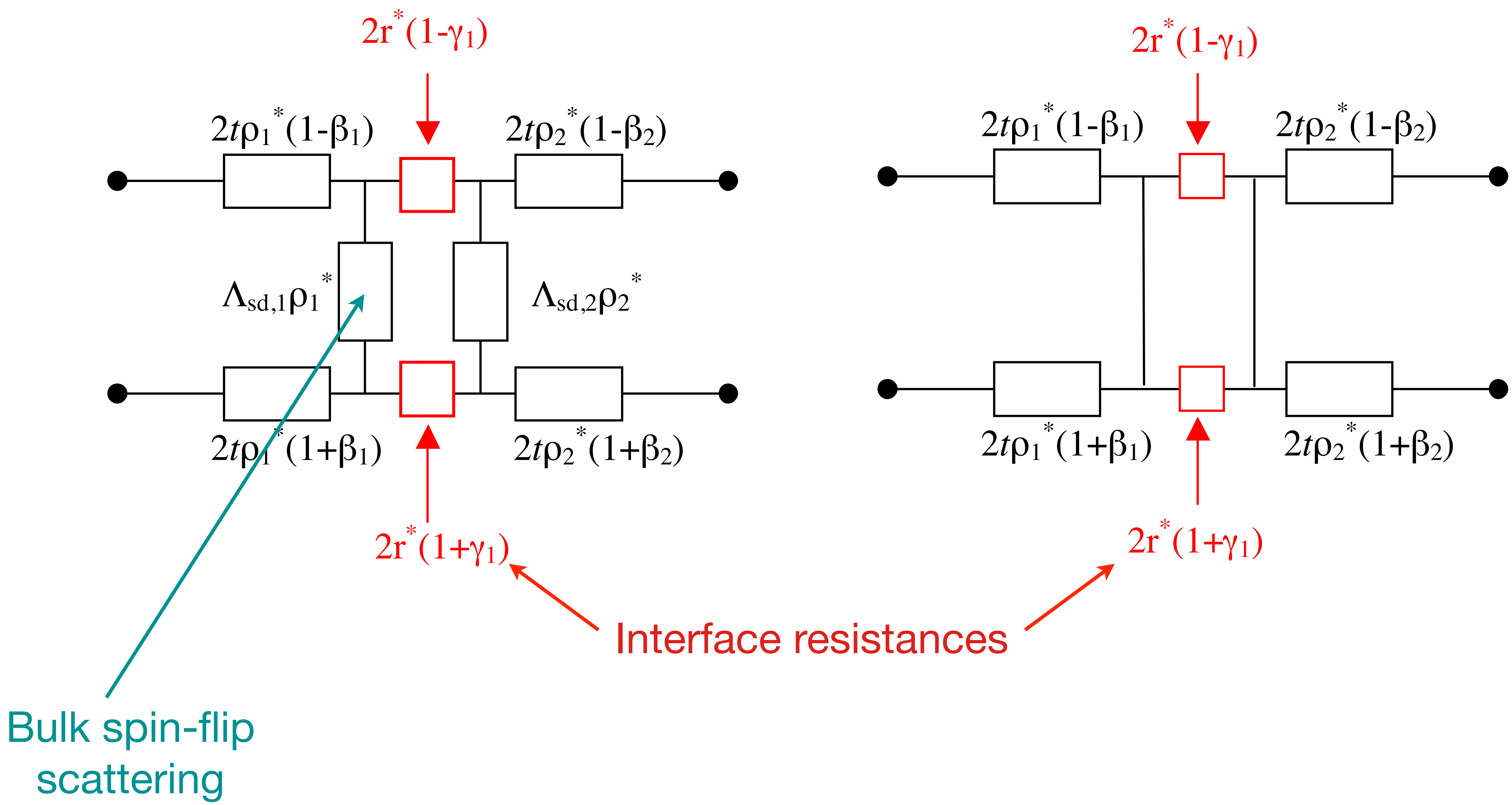
$$\rho_{\mathrm{AP}} = \frac{\rho_\uparrow + \rho_\downarrow}{2}$$



Towards a better resistor network



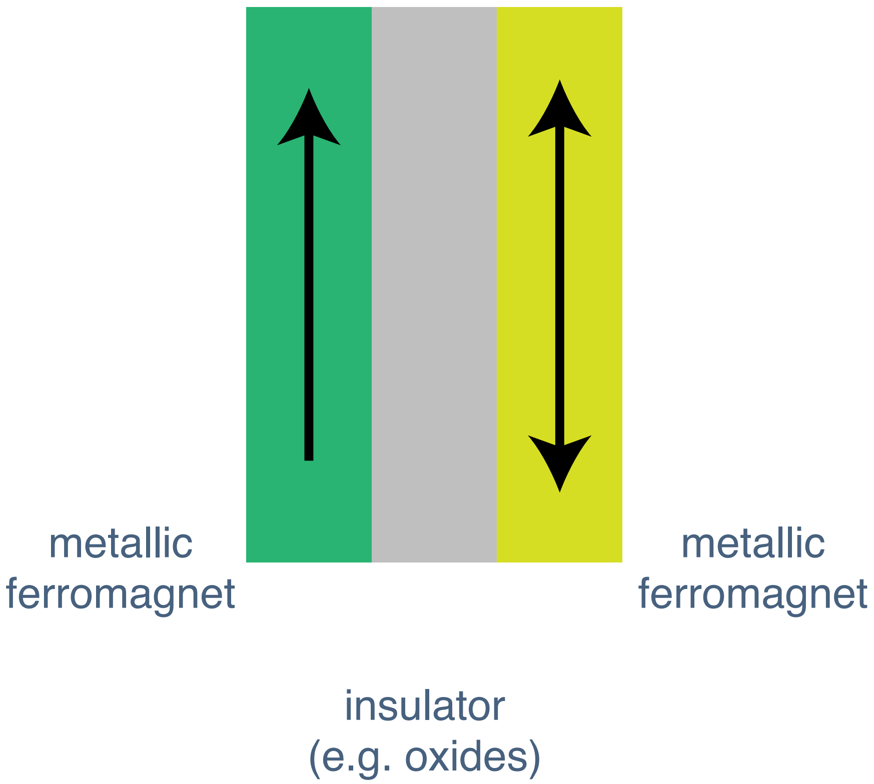
- Take into account bulk spin-flip scattering and interface resistances with additional elements in the circuit

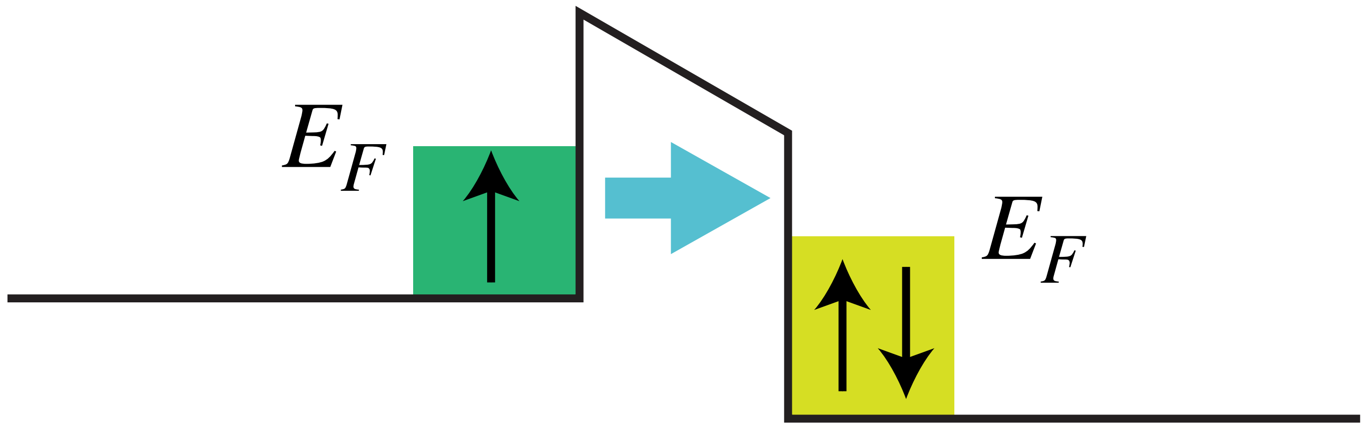


Valet & Fert, Phys Rev B **48**, 7099 (1993)

Tunnel magnetoresistance

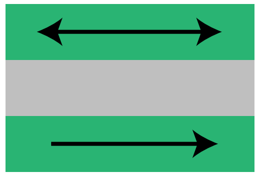
- Recall: spin-up and spin-down electrons do not have the same Fermi surface in ferromagnetic metals
  e.g. asymmetry leads to different scattering rates at interfaces
- In magnetic tunnel junctions, a thin insulating layer separates ferromagnetic electrodes
- Transport through this insulating layer is by quantum tunnelling
- Tunnelling for spin-up and spin-down electrons is also asymmetric!





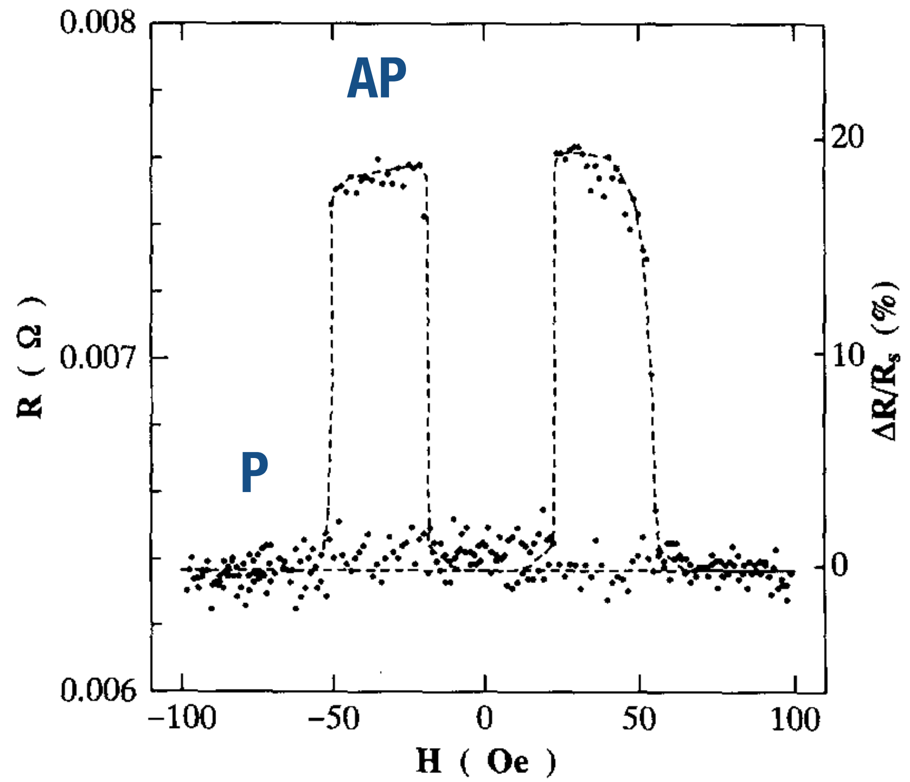
Tunnel magnetoresistance

- Similarly to all metallic case, transport through insulator layer is spin-dependent
  i.e. spin-up and spin-down electrons do not see the same barrier height



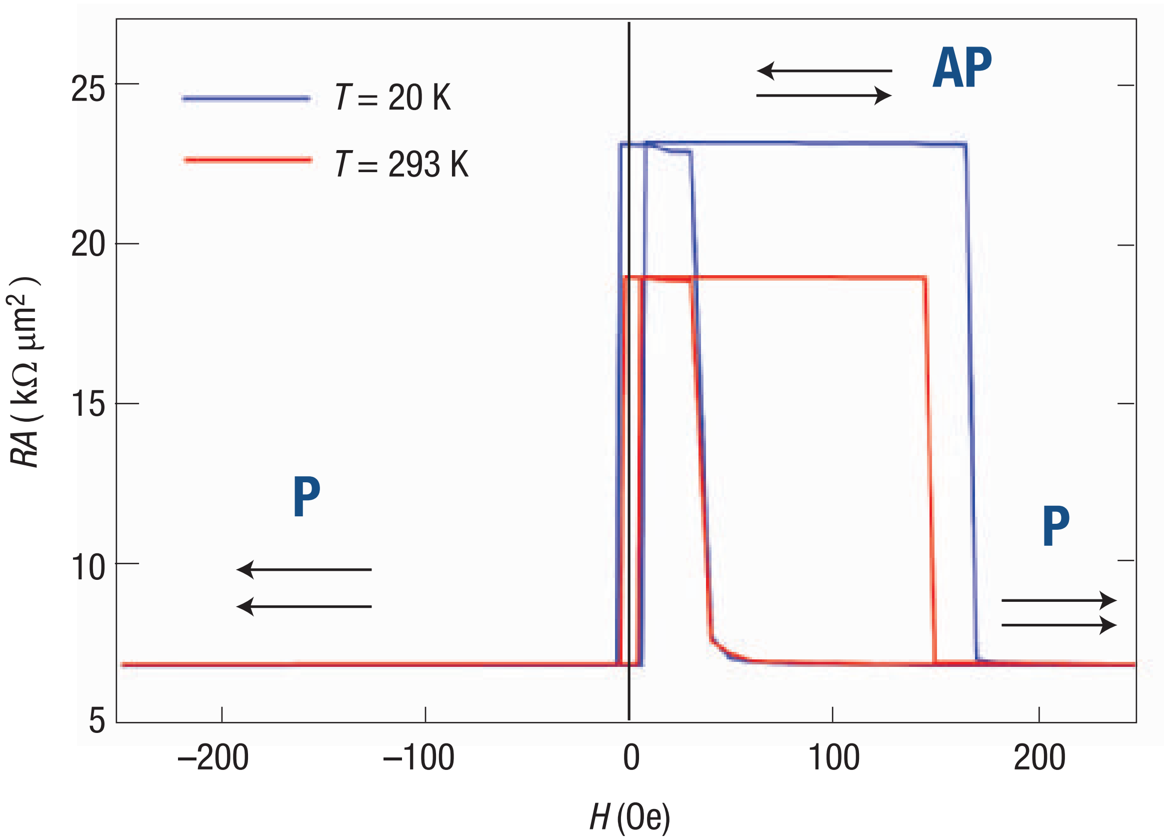
- **Tunnel magnetoresistance (TMR)** describes tunnelling resistance that depends on relative magnetization orientation (e.g., of a trilayer system)

Fe/Al_2O_3/Fe



Miyazaki et al, J Magn Magn Mater **139**, L231 (1995)

Fe/MgO/Fe



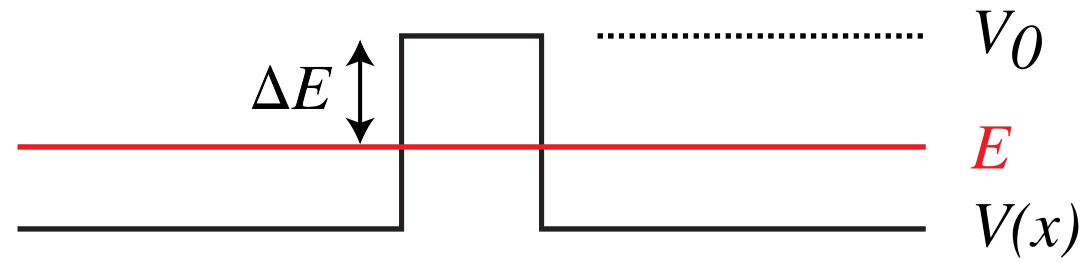
Yuasa et al, Nat Mater **3**, 868 (2004)

Quantum mechanical tunnelling

- In quantum physics, a particle can exist in or tunnel through a region where $E < V_0$

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right]|\psi(x)\rangle = E|\psi(x)\rangle$$

1D Schrödinger equation



$$E - V_0 = -\Delta E < 0$$

- Outside the barrier region, we have propagating plane waves

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}|\psi(x)\rangle = E|\psi(x)\rangle \qquad |\psi(x)\rangle = e^{ikx-\omega t} \qquad k = \pm\sqrt{\frac{2mE}{\hbar^2}}$$

Plane waves

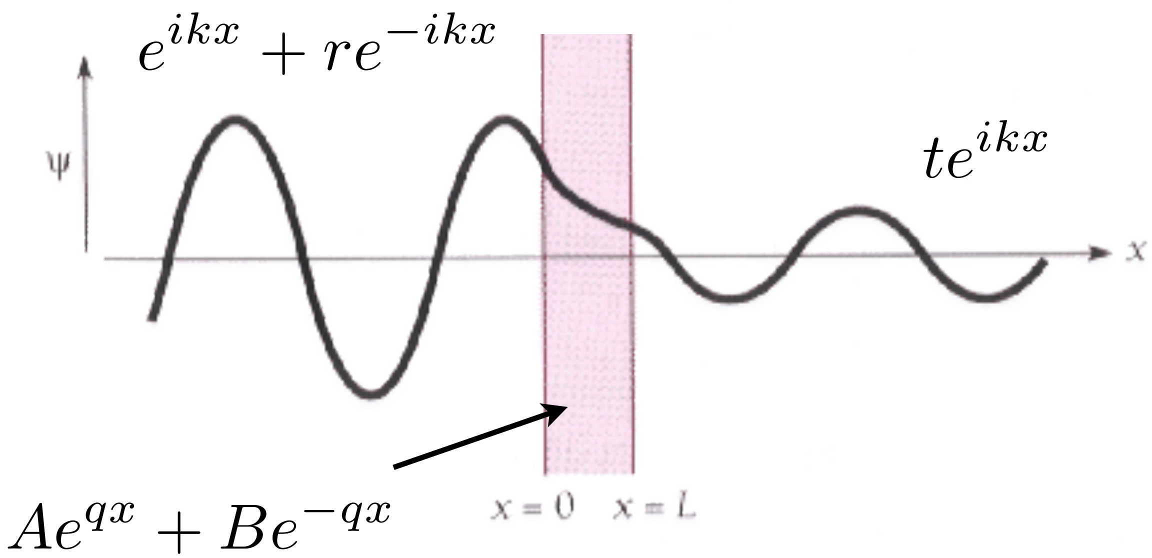
- Inside the barrier region, we have evanescent waves

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}|\psi(x)\rangle = (E - V_0)|\psi(x)\rangle \qquad |\psi(x)\rangle_b = e^{qx-i\omega t} \qquad q = \pm\sqrt{\frac{2m\Delta E}{\hbar^2}}$$

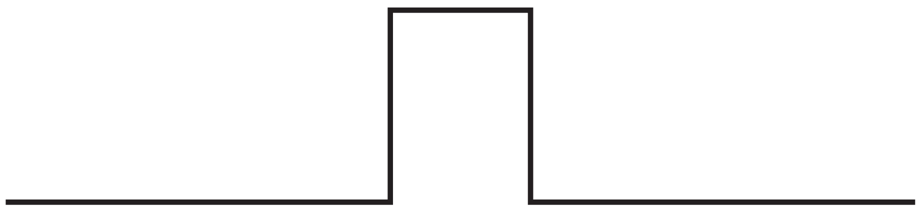
Evanescent waves

Quantum mechanical tunnelling

- Need to ensure continuity of wavefunction and its derivative at the interfaces



r: reflection coefficient
t: transmission coefficient



- Tunneling probability is given by probability of finding the particle in region 2

$$P = \psi_2^* \psi_2 = t^* t = |t|^2$$

$$P \simeq \frac{16k^2q^2}{(k^2+q^2)^2} e^{-2qL}$$

Probability decreases exponentially as a function of the barrier width *L*

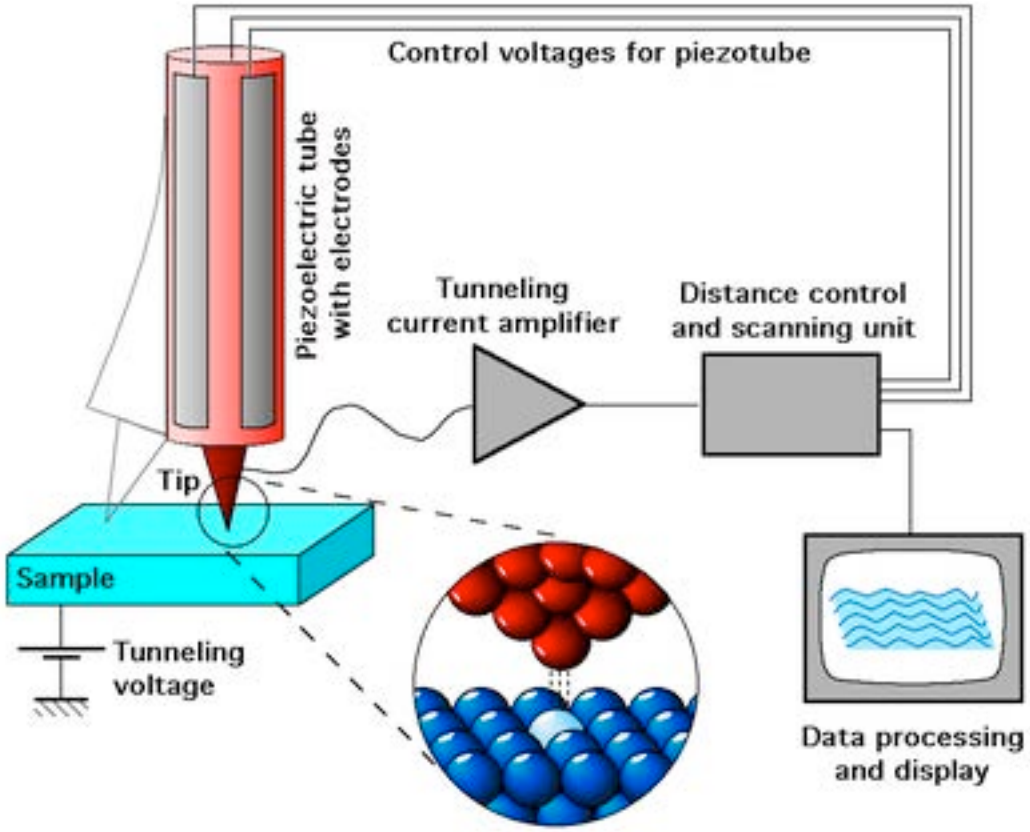
Quantum mechanical tunnelling

- What is the typical "penetration depth" into the barrier?

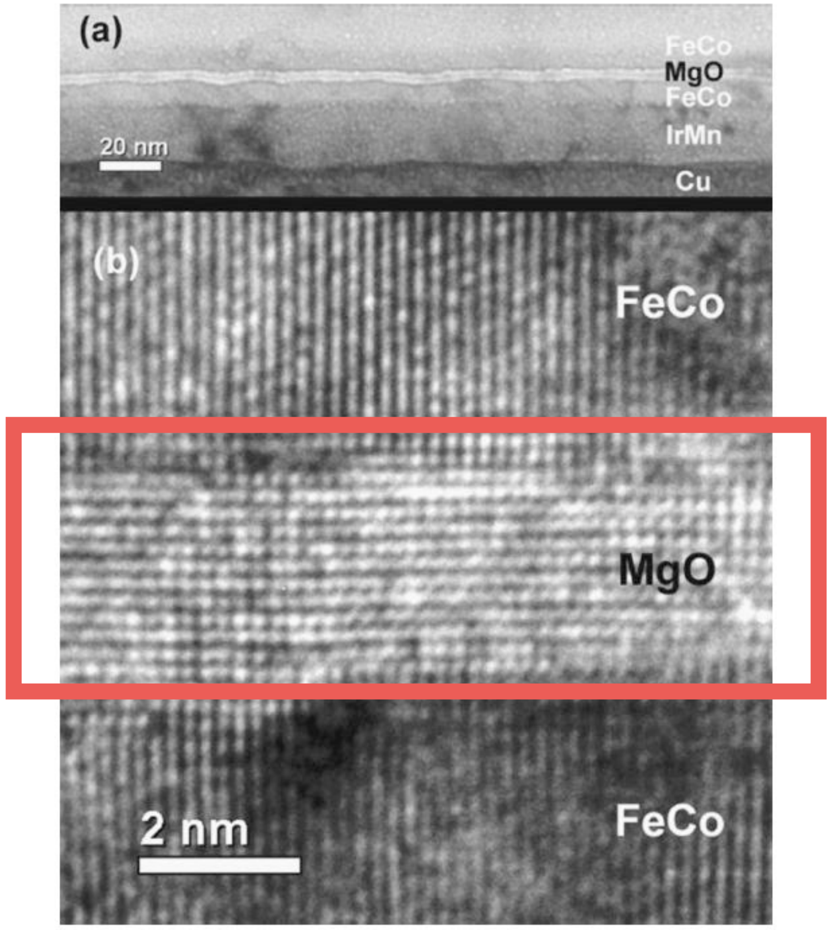
$$q = \pm\sqrt{\frac{2m\Delta E}{\hbar^2}}$$

$$\boxed{\begin{array}{c}\Delta E = 1\ \mathrm{eV} \\ m = m_0 = 9.1\times10^{-31}\ \mathrm{kg}\end{array} \Rightarrow \frac{1}{q} \simeq 0.2\ \mathrm{nm}}$$

- Therefore, to observe tunneling effects, one requires barrier widths to be on the **nanometre** scale.



STM



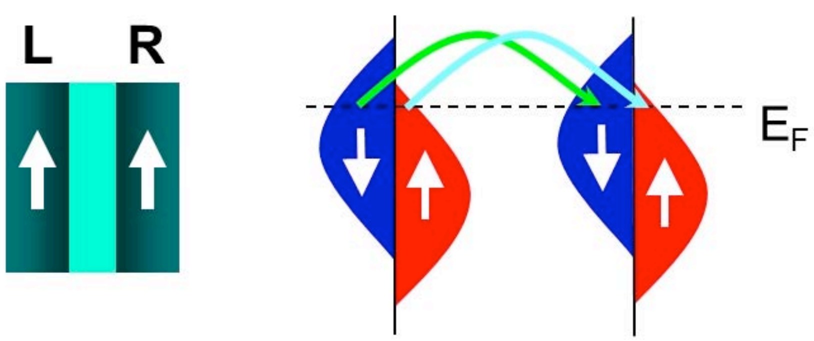
insulator/ tunnel barrier

magnetic tunnel junction

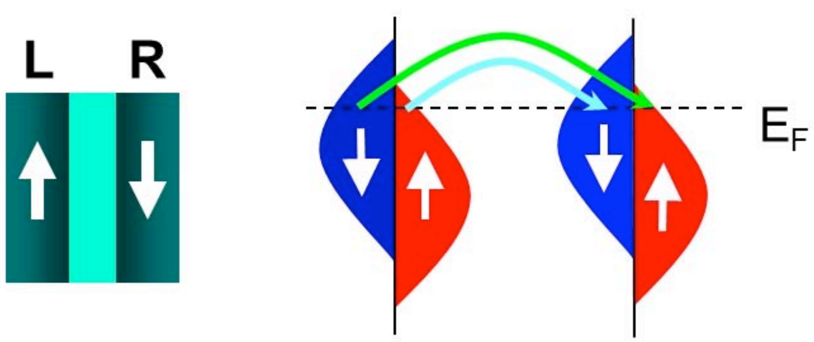
Jullière model

Jullière, Phys. Lett. **54A**, 225 (1975)

- 1. Assume no spin flips when electrons tunnel through barrier
  - Two independent conduction channels for spin-up and spin-down
  - Tunneling of spin state from first film into second film is determined by unfilled states of same spin in second film
- 2. Assume conductance *G* for a spin channel is given by the product of the effective density of states *n* of the two ferromagnetic electrodes



$G_P \propto n_L^{\uparrow} n_R^{\uparrow} + n_L^{\downarrow} n_R^{\downarrow}$



$G_{AP} \propto n_L^{\uparrow} n_R^{\downarrow} + n_L^{\downarrow} n_R^{\uparrow}$

$$G^\sigma = n_L^\sigma n_R^\sigma \qquad G = G^\uparrow + G^\downarrow$$

conductances

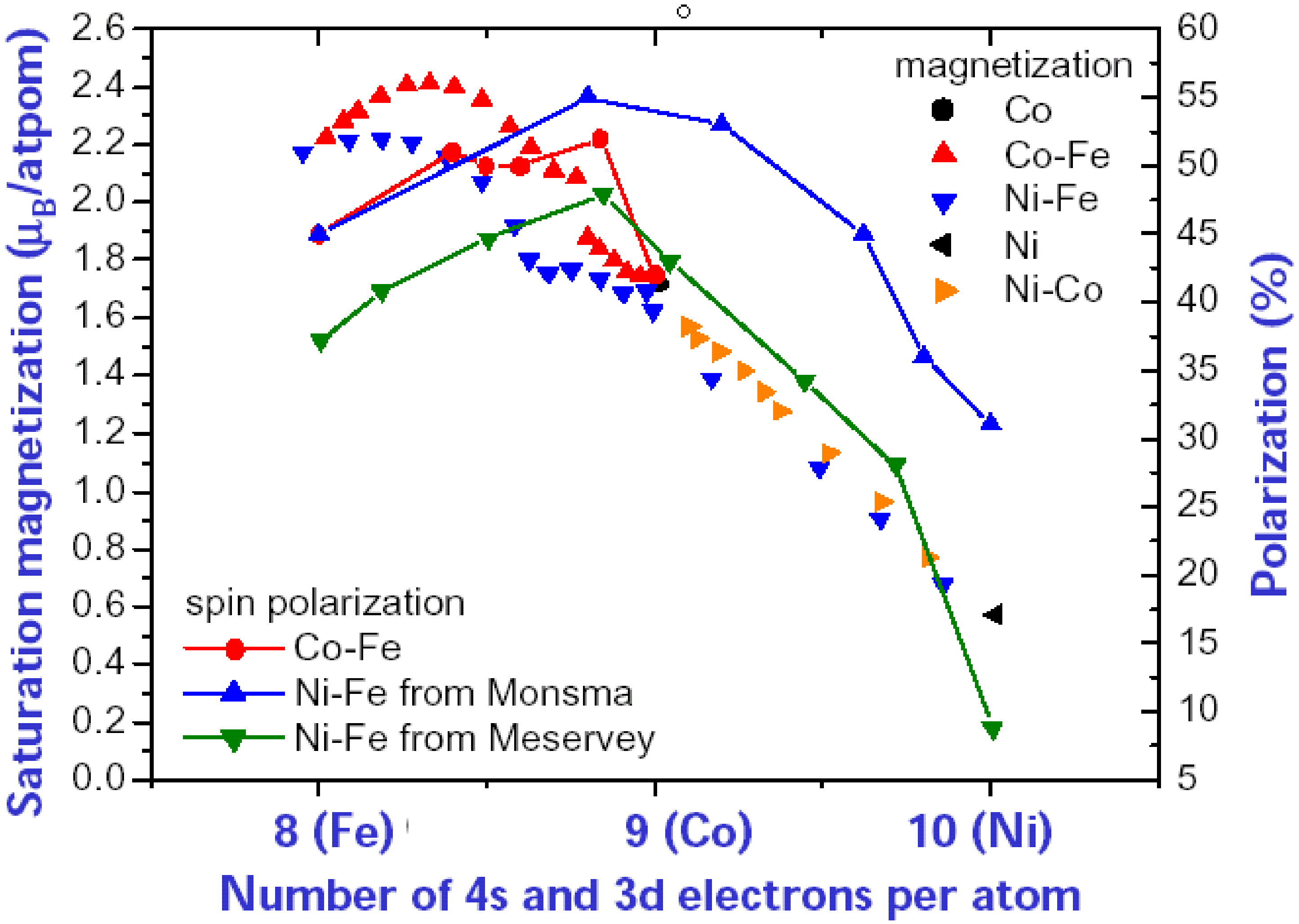
$$P = \frac{n^\uparrow - n^\downarrow}{n^\uparrow + n^\downarrow}$$

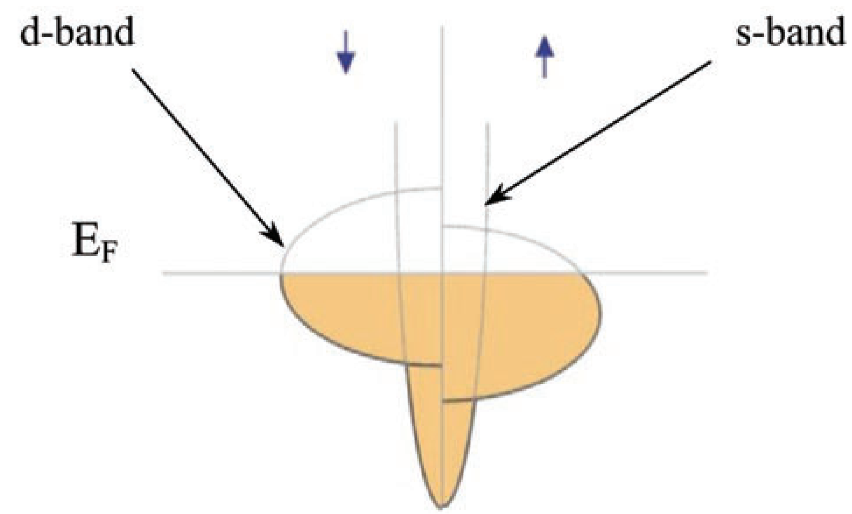
spin polarization

$$\boxed{\mathrm{TMR} \equiv \frac{R_{\mathrm{AP}} - R_{\mathrm{P}}}{R_{\mathrm{P}}} = \frac{2P_L P_R}{1 - P_L P_R}}$$

Jullière model

- Large spin-polarization is needed for large TMR



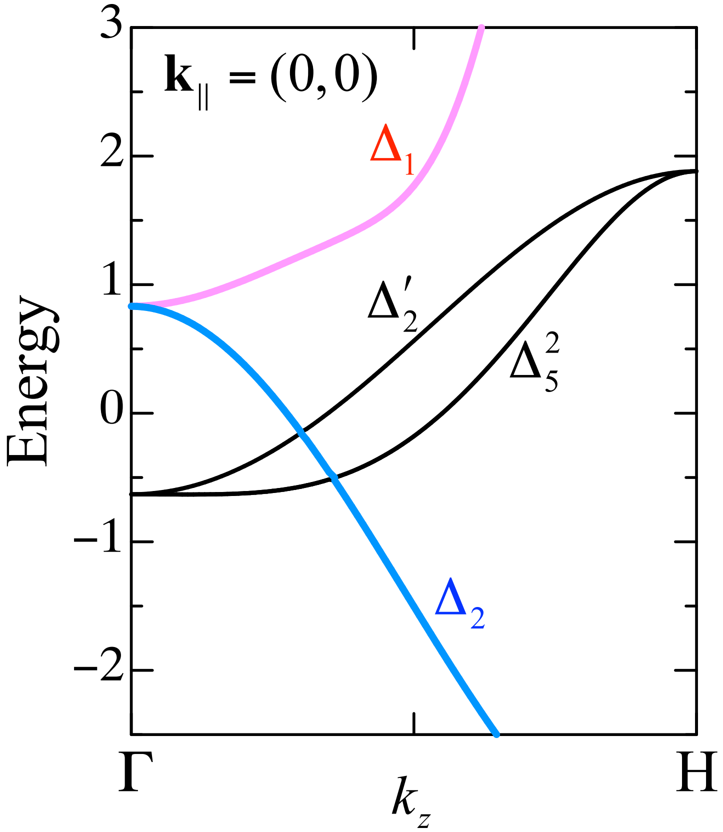


$$\mathrm{TMR} \equiv \frac{R_{\mathrm{AP}} - R_{\mathrm{P}}}{R_{\mathrm{P}}} = \frac{2P_L P_R}{1 - P_L P_R}$$

- Assuming 50% spin polarization, Jullière's model predicts TMR of ~67%

Fe/MgO/Fe: Example of symmetry filtering

- High quality crystalline Fe/MgO/Fe: spin filtering based on band symmetry

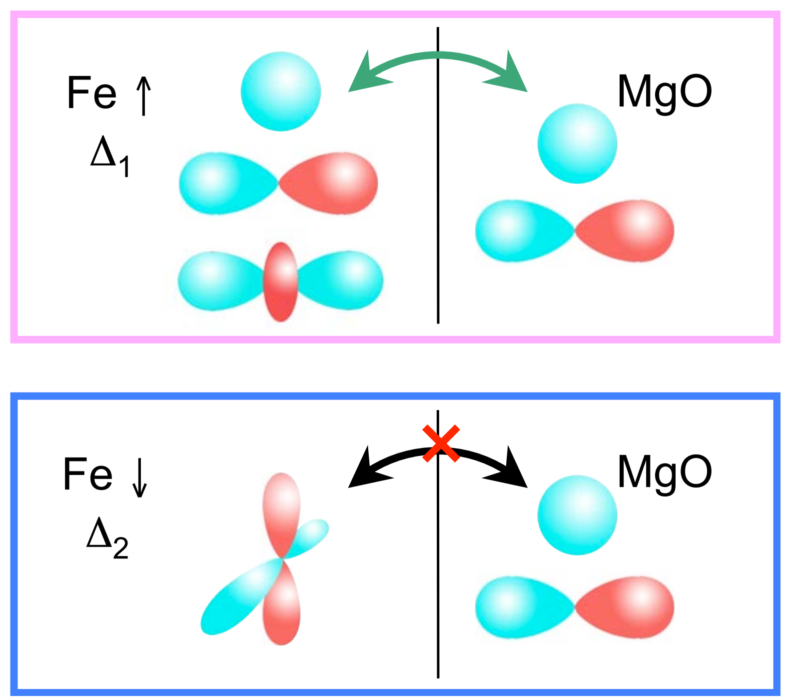


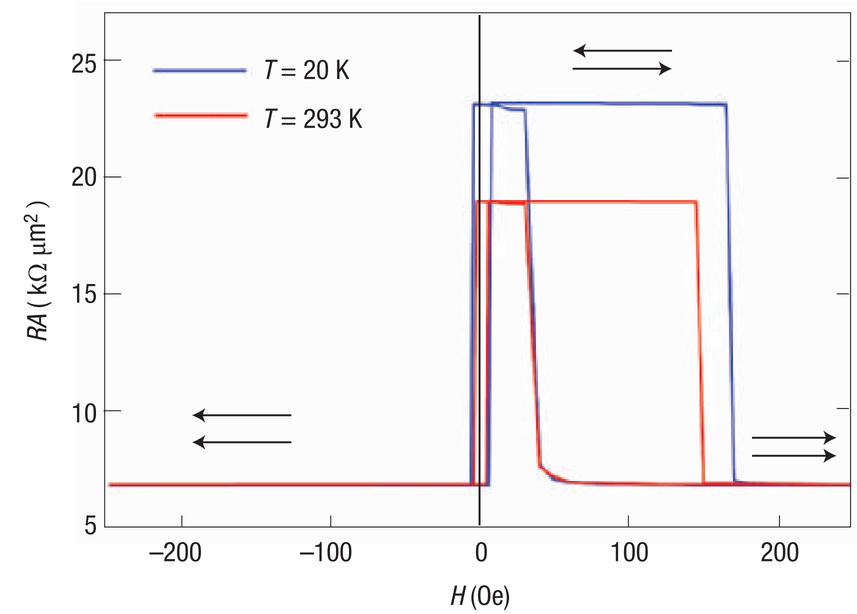
Δ_1 $s, p_z, d_{3z^2-r^2}$

Δ_2 $d_{x^2-y^2}$

Δ_2' d_{xy}

Δ_5^2 p_x, p_y, d_{yz}, d_{zx}





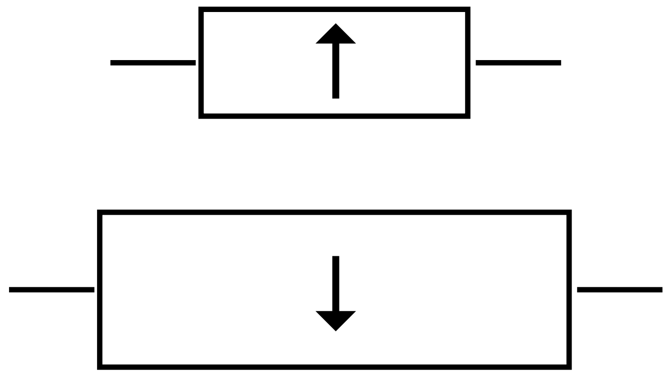
- MgO is also good tunnel barrier for Co-based alloys

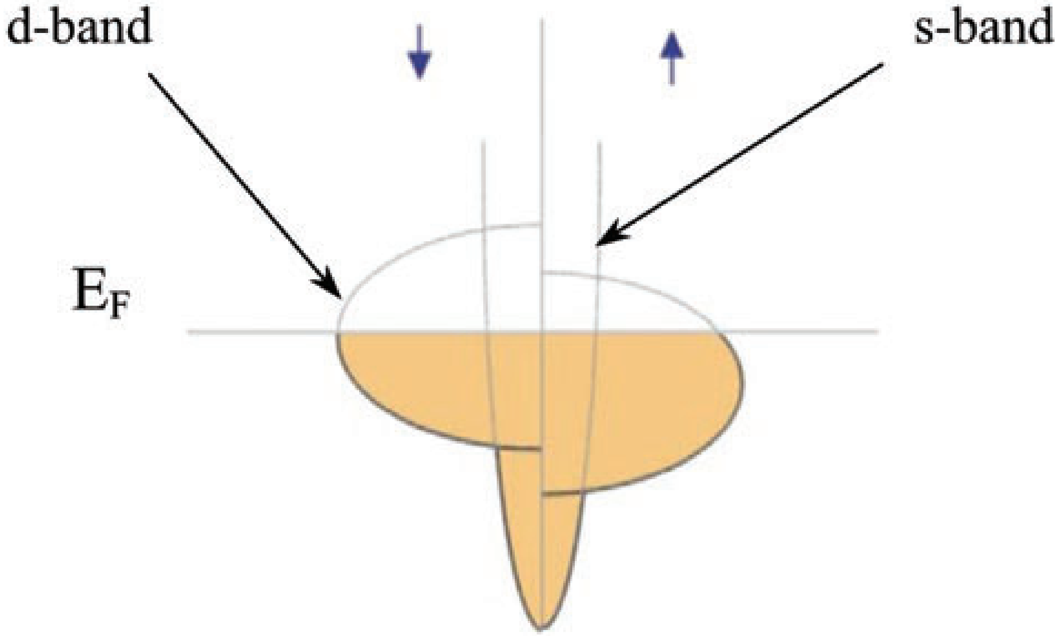
Junctions	MR Ratios (%)	
	LT	RT
CoFeB/MgO/CoFeB	1010	500
$Co_2Cr_{0.6}Fe_{0.4}$/MgO/$Co_{50}Fe_{50}$	317	109
$Co_2FeAl_{0.5}Si_{0.5}$/MgO/$Co_2FeAl_{0.5}Si_{0.5}$	390	220
Co_2MnGe/MgO/$Co_{50}Fe_{50}$	376	160

Spin diffusion

- Let's try to go a little further beyond the two-current model. Recall:

$$\mathbf{j}_{\uparrow,\downarrow} = \sigma_{\uparrow,\downarrow}\mathbf{E} \qquad \sigma_{\uparrow,\downarrow} = \frac{ne^2\tau_{\uparrow,\downarrow}}{m}$$



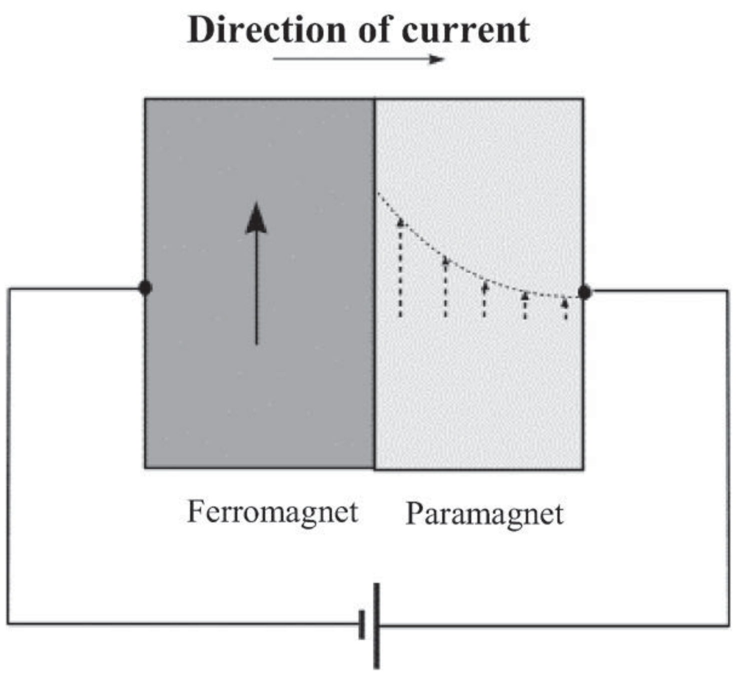


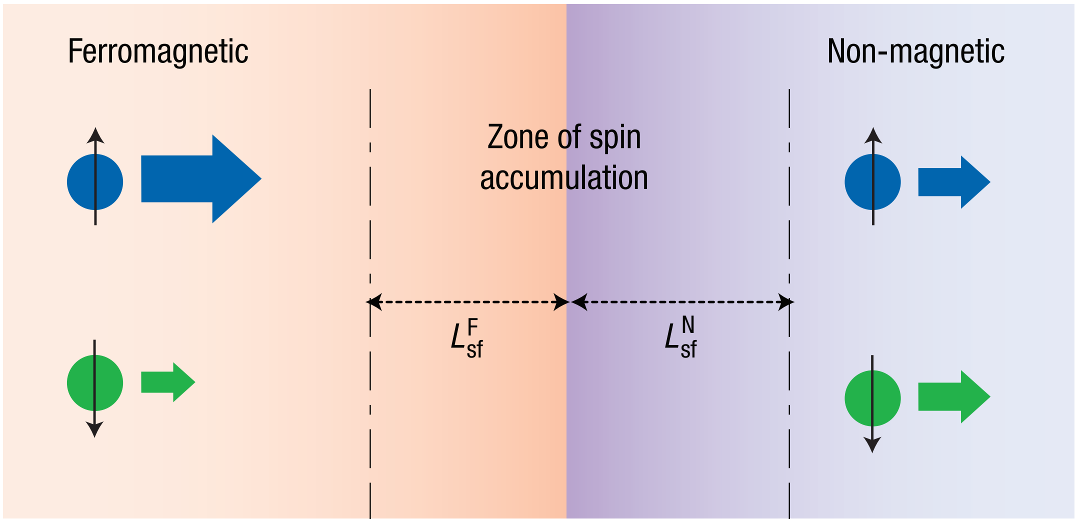
- Express current in terms of spin-dependent conductivity and electrochemical potential:

$$j_{\uparrow,\downarrow}(x) = \frac{\sigma_{\uparrow,\downarrow}}{e}\frac{\partial\mu_{\uparrow,\downarrow}(x)}{\partial x} \qquad \tau_{\uparrow\downarrow} \gg \tau_{\uparrow,\downarrow}$$

Microscopic picture at F/N interface

- Let's take a closer look at spin transport near ferromagnet/normal metal interface





- Transition from spin polarized current in **F** to unpolarized current in **NM** occurs over finite distance around interface

- Concept of spin electrochemical potential, μ

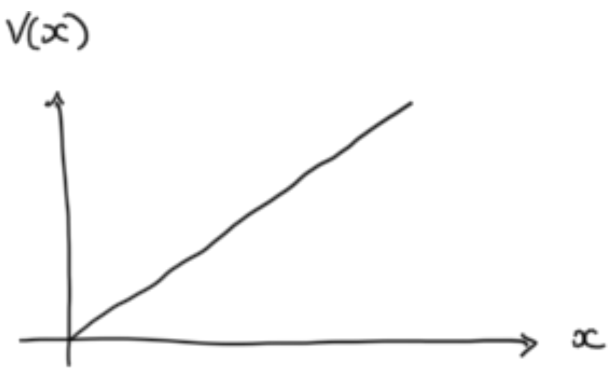
$$\mu(x) = -eV(x) + \frac{e^2 D}{\sigma} n(x) \qquad j(x) = \frac{\sigma}{e} \frac{\partial \mu(x)}{\partial x}$$

Electrochemical potential

$$\mu(x) = -\,eV(x) + \frac{e^2 D}{\sigma} n(x)$$

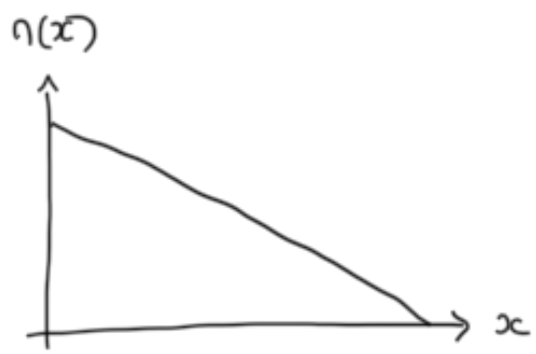
$$j(x) = \frac{\sigma}{e} \frac{\partial \mu(x)}{\partial x}$$

Electric potential



$(j = \sigma E)$

Particle density



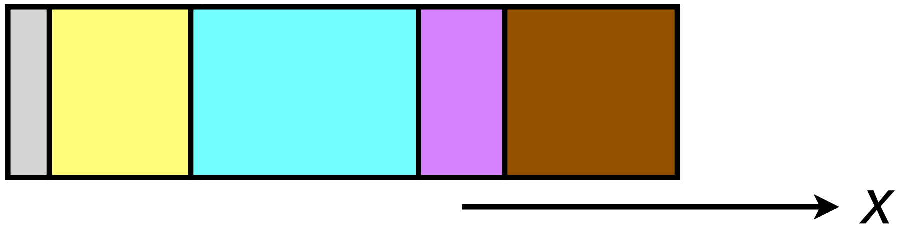
Both potentials lead to current flow toward the right

- Assume spin-dependent conductivity and electrochemical potential

$$j_{\uparrow,\downarrow}(x) = \frac{\sigma_{\uparrow,\downarrow}}{e} \frac{\partial \mu_{\uparrow,\downarrow}(x)}{\partial x}$$

$$\tau_{\uparrow\downarrow} \gg \tau_{\uparrow,\downarrow}$$

Diffusion equations



- Transport across arbitrary multilayer can obtained using conservation conditions:

1. Conservation of current

$$\frac{\partial j}{\partial x} = \frac{\partial j_\uparrow}{\partial x} + \frac{\partial j_\downarrow}{\partial x} = 0 \qquad \Rightarrow \frac{\sigma_\uparrow}{e}\frac{\partial^2 \mu_\uparrow}{\partial x^2} + \frac{\sigma_\downarrow}{e}\frac{\partial^2 \mu_\downarrow}{\partial x^2} = 0$$

2. Conservation of spin

$$\frac{\partial j_\uparrow}{\partial x} - \frac{\partial j_\downarrow}{\partial x} = e\left(\frac{n_\uparrow - n_\downarrow}{\tau_{\uparrow\downarrow}}\right) \qquad \frac{\partial^2 \mu_\uparrow}{\partial x^2} = \frac{e^2}{2\sigma_\uparrow}\left(\frac{n_\uparrow - n_\downarrow}{\boxed{\tau_{\uparrow\downarrow}}}\right)$$

spin relaxation time

3. Charge neutrality (screen is very efficient in metals)

$$n_c = n_\uparrow + n_\downarrow = 0$$

Combining all these leads to equation for spin diffusion

$$\boxed{D_F \frac{\partial^2(\mu_\uparrow - \mu_\downarrow)}{\partial x^2} = \frac{\mu_\uparrow - \mu_\downarrow}{\tau_{\uparrow\downarrow}}} \qquad D_F = \frac{\sigma_\uparrow D_\downarrow + \sigma_\downarrow D_\uparrow}{\sigma_\uparrow + \sigma_\downarrow}$$

Spin diffusion and spin accumulation

- The previous result allows us to write the diffusion in a compact form

$$\frac{\partial^2(\mu_\uparrow - \mu_\downarrow)}{\partial x^2} = \frac{\mu_\uparrow - \mu_\downarrow}{l_{\mathrm{sf}}^2}$$

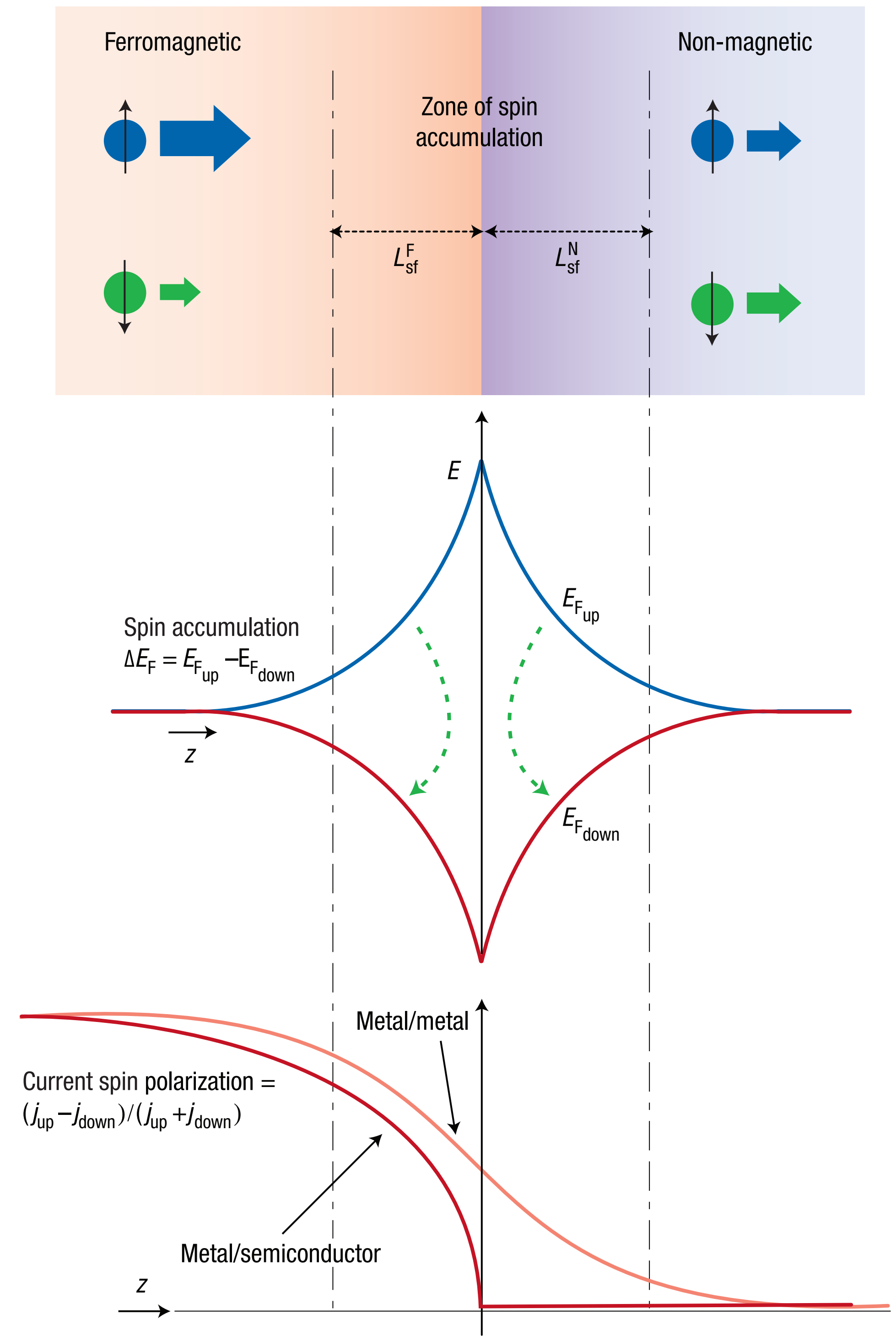
Define spin accumulation as

$$\Delta\mu = \mu_\uparrow - \mu_\downarrow$$

Thus, spin accumulation is governed by a simple diffusive process

$$\frac{\partial^2 \Delta\mu}{\partial x^2} = \frac{\Delta\mu}{l_{\mathrm{sf}}^2} \qquad \Delta\mu \sim e^{\pm x/l_{\mathrm{sf}}}$$

Chappert et al, Nat Mater **6**, 813 (2007)

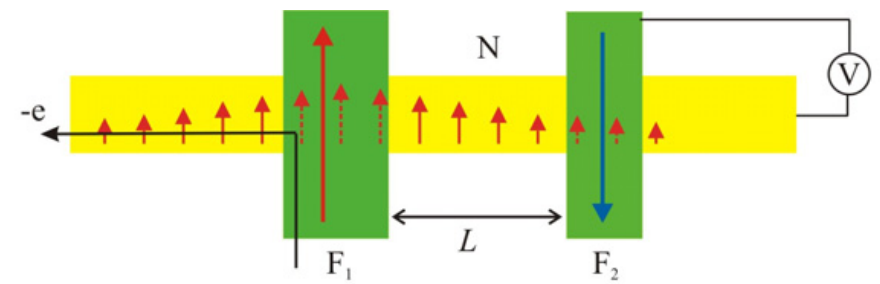


Metal (300 K)	l_{sf} (nm)
Cu	10^2-10^3
Au	60
Co	~38
Permalloy	~3

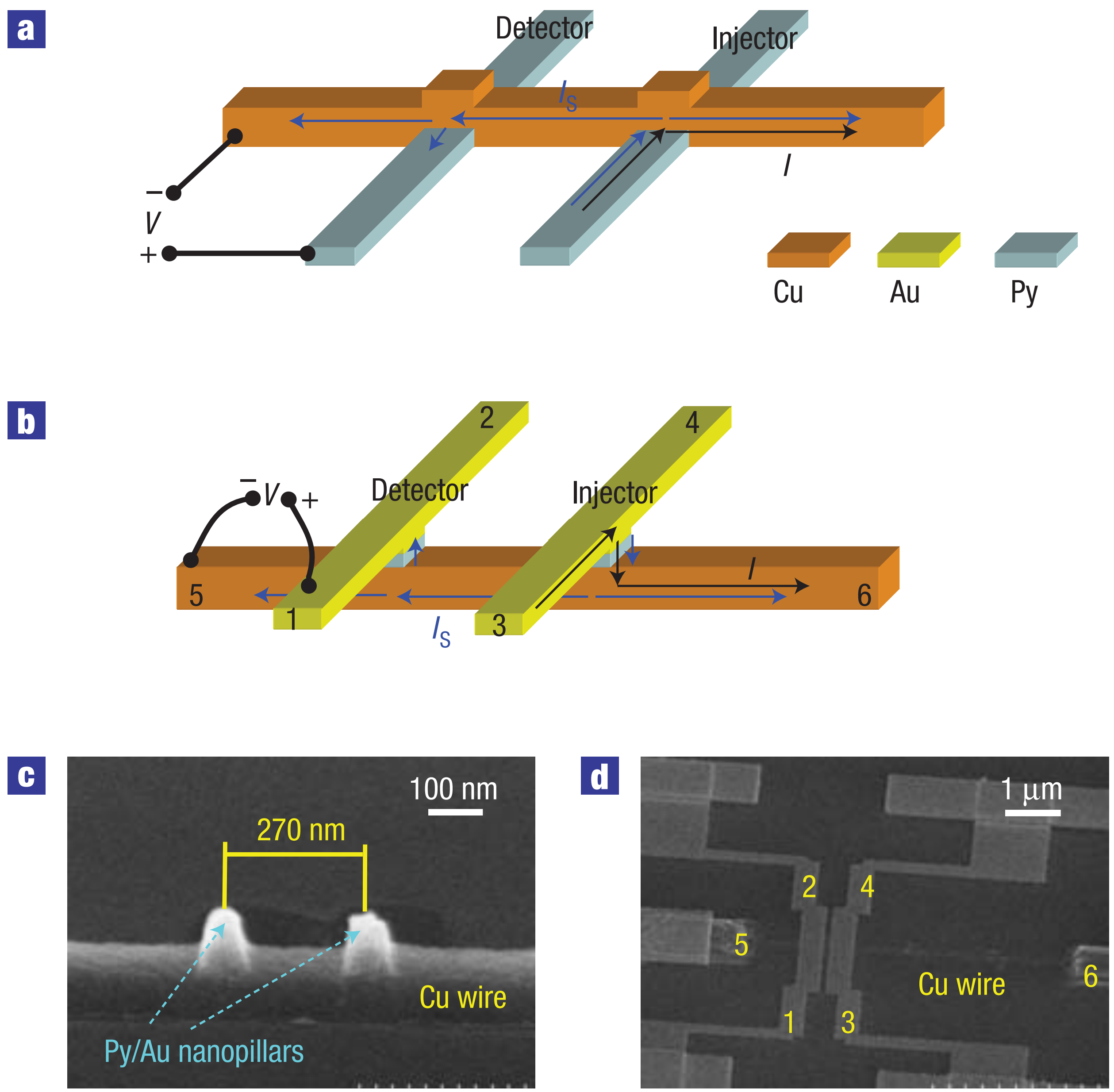
Bass & Pratt, J Phys: Condens Matter **9**, 183201 (2007)
(Check for updated values in the literature!)

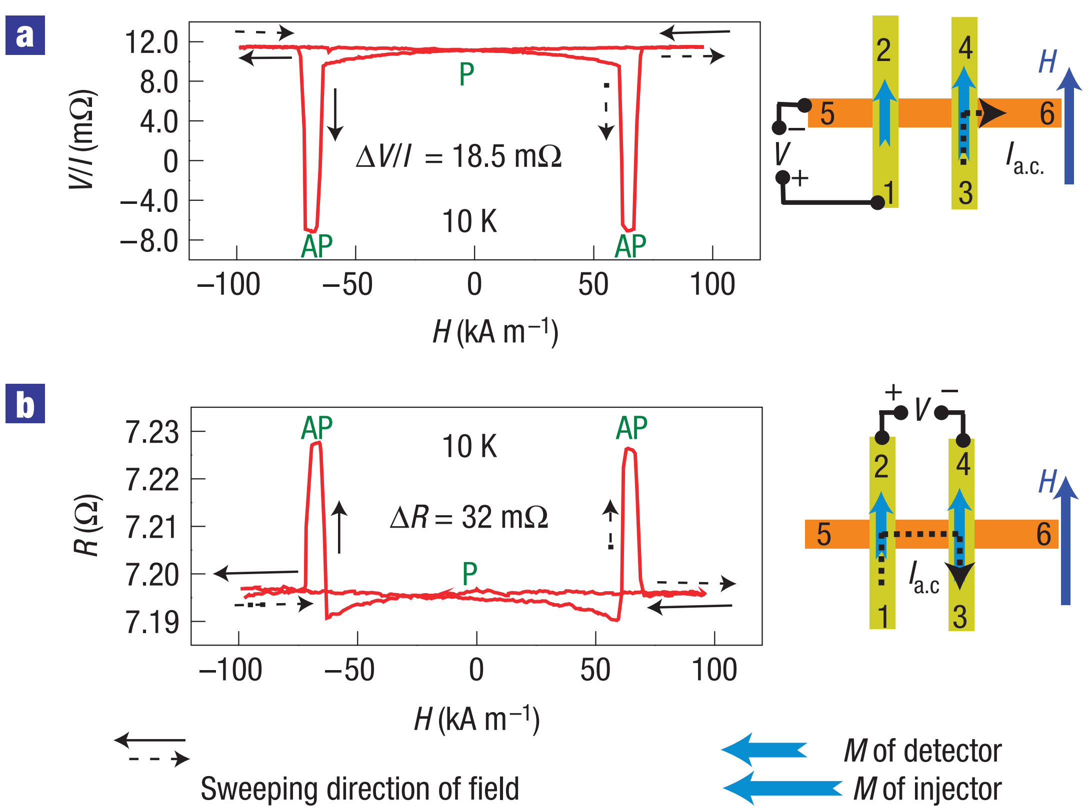
Lateral spin valves

- Spin diffusion can be exploited in lateral geometries with spin valve effect



Ji *et al*, J Phys D: Appl Phys **40**, 1280 (2007)



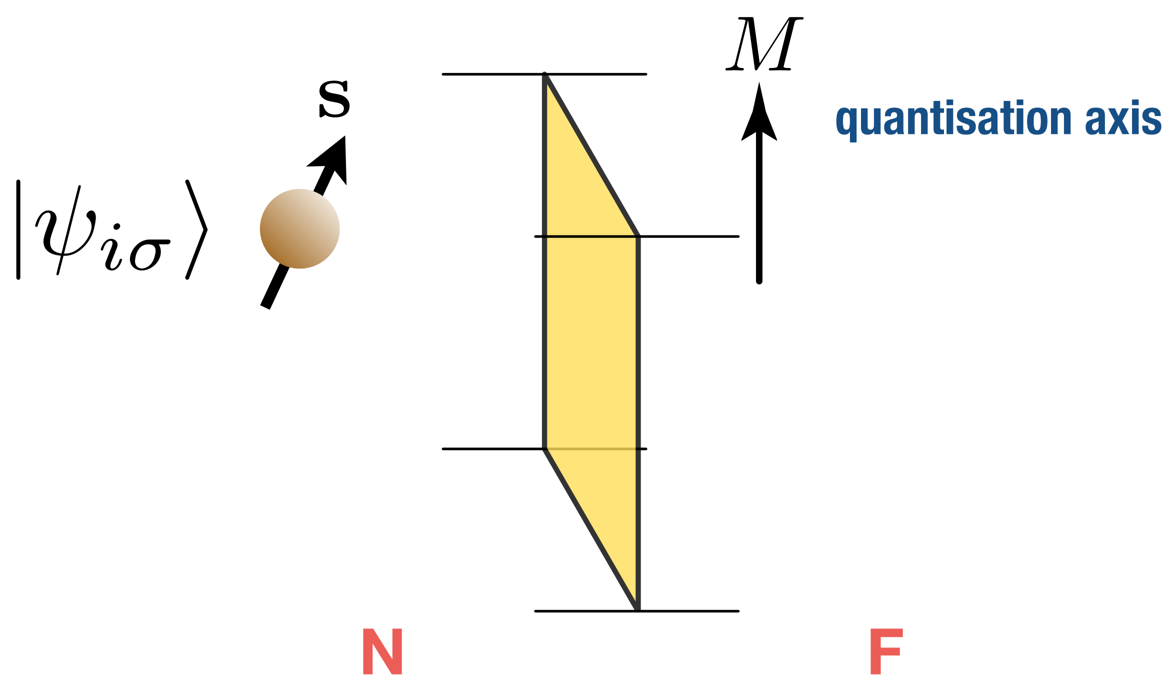


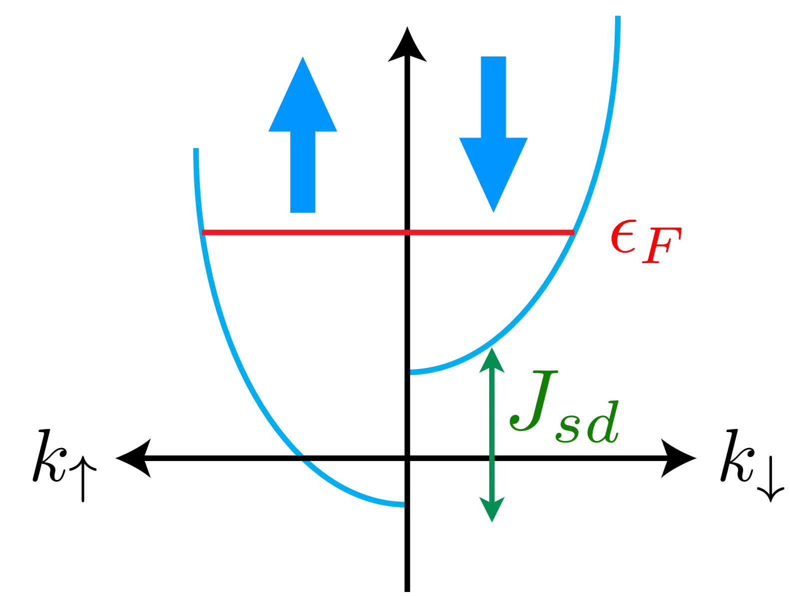
Yang *et al*, Nat Phys **4**, 851 (2008)

Single electron at N/F interface

Stiles & Zangwill, Phys Rev B **66**, 014407 (2002)

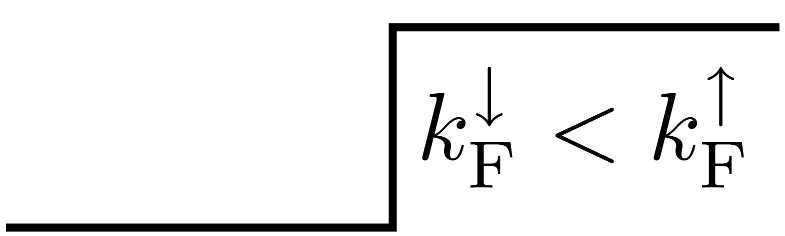
- Consider a free electron in the normal metal arriving at the normal metal (N)/ ferromagnet (F) interface. Solve 1D Schrödinger equation





$$\epsilon_{\mathrm{F}} = \frac{\hbar^2 k_{\mathrm{F}}^2}{2m} \qquad \epsilon_{\mathrm{F}} = \frac{\hbar^2 (k_{\mathrm{F}}^{\uparrow,\downarrow})^2}{2m} \mp \frac{J_{sd}}{2}$$

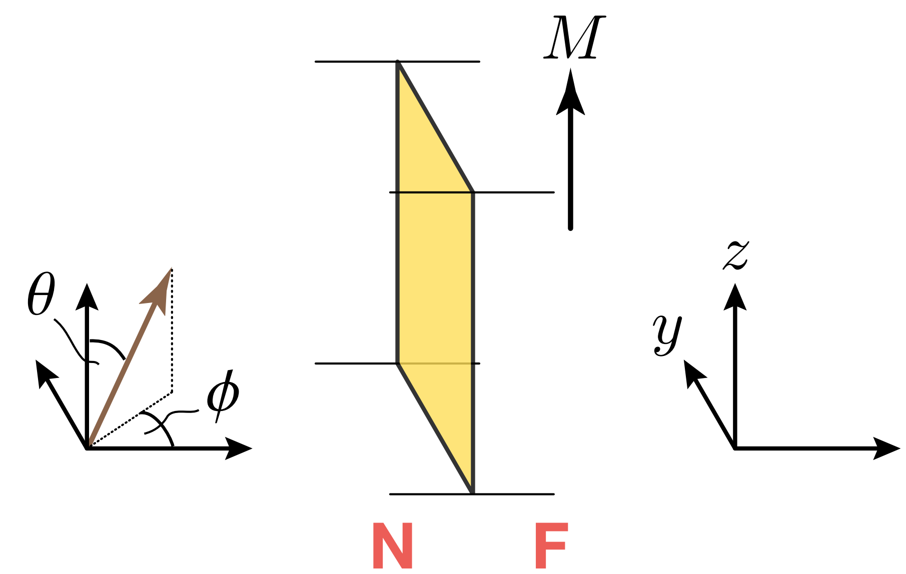
- Because the bands in the ferromagnet are spin-split, there is a spin-dependent step potential at the interface



Single electron at N/F interface

$$\psi^{\mathrm{in}} = e^{ik_x x} e^{i\mathbf{k}_{||}\cdot\mathbf{r}_{||}} \begin{pmatrix} \cos(\theta/2)e^{-i\phi/2}|\uparrow\rangle \\ \sin(\theta/2)e^{i\phi/2}|\downarrow\rangle \end{pmatrix}$$

incident wavefunction



$$\psi^{\mathrm{ref}} = e^{-ik_x x} e^{i\mathbf{k}_{||}\cdot\mathbf{r}_{||}} \begin{pmatrix} R_\uparrow \cos(\theta/2)e^{-i\phi/2}|\uparrow\rangle \\ R_\downarrow \sin(\theta/2)e^{i\phi/2}|\downarrow\rangle \end{pmatrix}$$

reflected wavefunction

$$\psi^{\mathrm{tr}} = e^{i\mathbf{k}_{||}\cdot\mathbf{r}_{||}} \begin{pmatrix} T_\uparrow e^{ik_x^\uparrow x} \cos(\theta/2)e^{-i\phi/2}|\uparrow\rangle \\ T_\downarrow e^{ik_x^\downarrow x} \sin(\theta/2)e^{i\phi/2}|\downarrow\rangle \end{pmatrix}$$

transmitted wavefunction

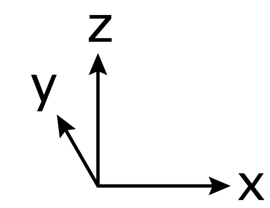
- Assume constant effective mass
- Apply usual quantum mechanical matching conditions across interface to obtain reflection and transmission coefficients

Spin currents

Stiles & Zangwill, Phys Rev B **66**, 014407 (2002)

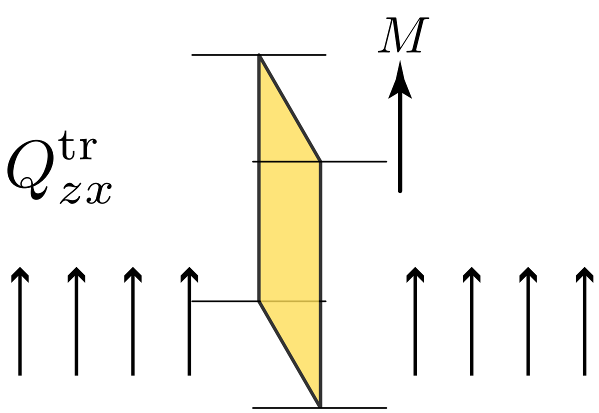
- Let's look at the spin current through this interface. What is conserved?

$$\overleftrightarrow{\mathbf{Q}}(\mathbf{r}) = Re \sum_{i\sigma\sigma'} \psi^*_{i\sigma}(\mathbf{r})\ \hat{\mathbf{s}} \otimes \hat{\mathbf{v}}\ \psi_{i\sigma'}(\mathbf{r})$$



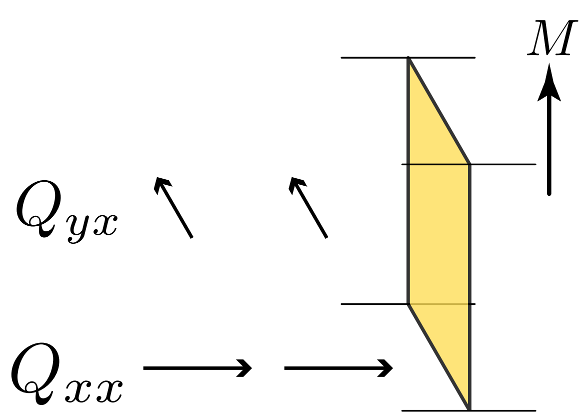
Conserved

$$Q^{\mathrm{in}}_{zx} + Q^{\mathrm{ref}}_{zx} = Q^{\mathrm{tr}}_{zx}$$



longitudinal spin current

NOT conserved



$$Q^{\mathrm{in}}_{\perp x} + Q^{\mathrm{ref}}_{\perp x} \neq Q^{\mathrm{tr}}_{\perp x}$$

transverse spin current

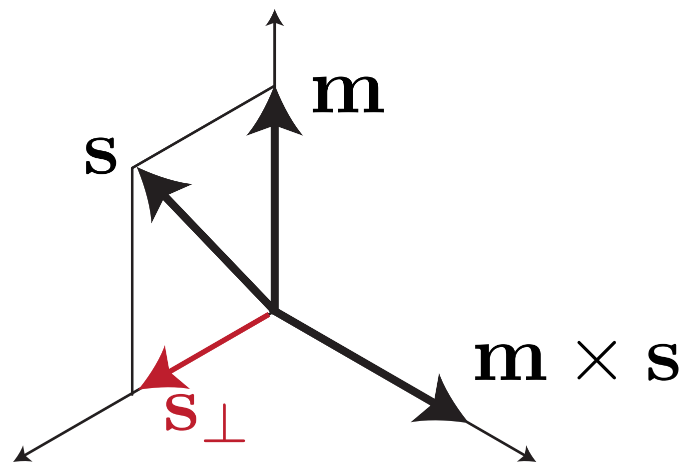
- From conservation of spin angular momentum, argue that missing transverse spin current is transferred to ferromagnet *M*

$$\left[\frac{\partial \mathbf{m}}{\partial t}\right]_{\mathrm{STT}} \propto \mathbf{s}_{\perp}$$

Spin transfer torques

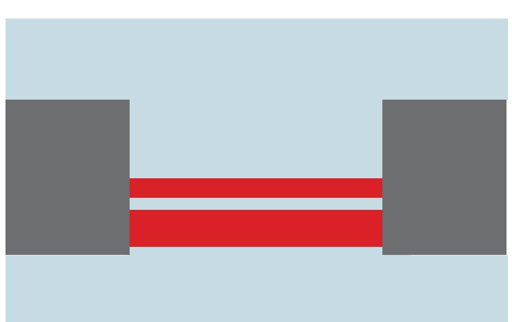
- Express transverse spin component in terms of vector products

$$\mathbf{s}_\perp \propto (\mathbf{m} \times \mathbf{s}) \times \mathbf{m}$$

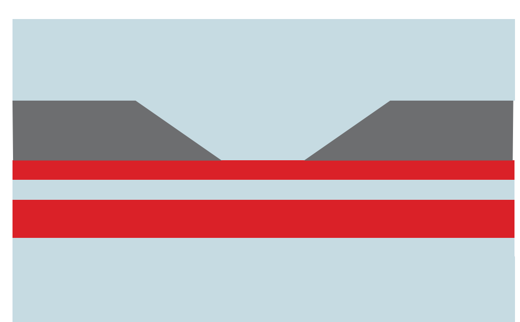


$$\left[\frac{\partial \mathbf{m}}{\partial t}\right]_{\mathrm{STT}} \propto (\mathbf{m} \times \mathbf{s}) \times \mathbf{m}$$

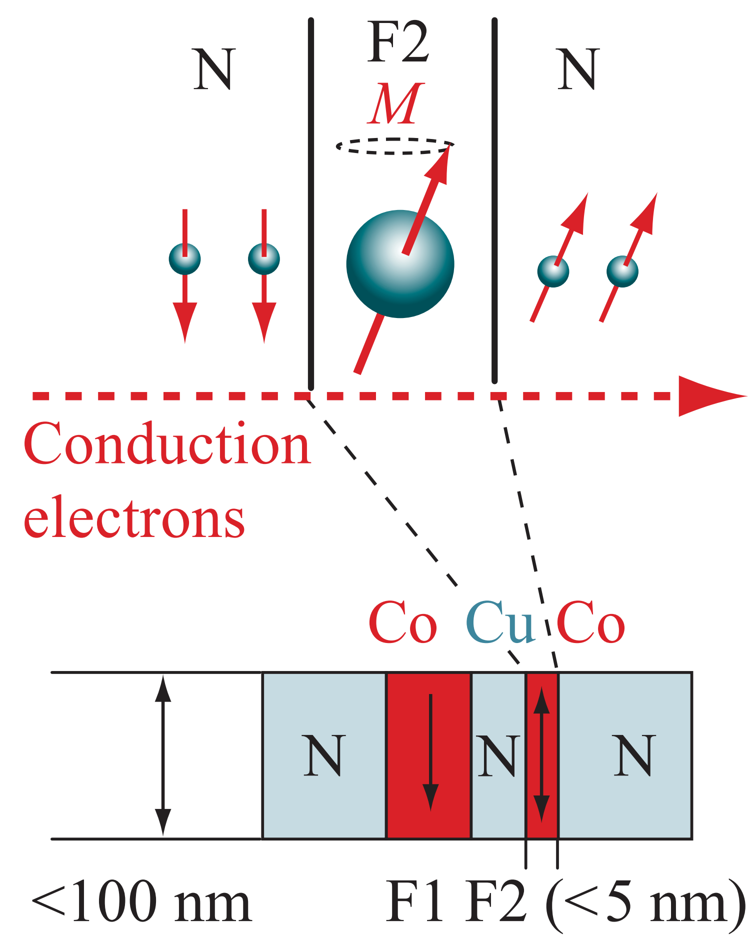
- Typical realisations involve the CPP geometry where ***s*** is related to the magnetisation of a second (reference) layer



Nanopillars



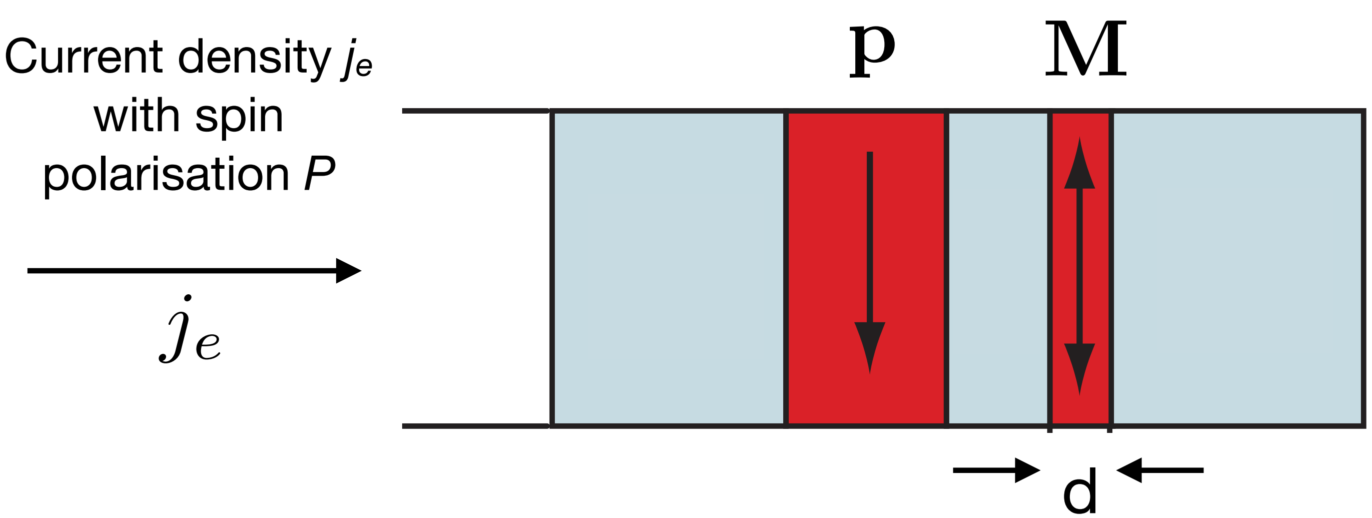
Nanocontacts



Slonczewski model of CPP torques

- Accounting for transport properties, obtain Slonczewski term for spin-transfer torques

$$\frac{\partial \mathbf{M}}{\partial t} = \underbrace{-\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}}}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t}}_{\text{Damping}} + \underbrace{\boxed{\sigma j_e \mathbf{M} \times (\mathbf{p} \times \mathbf{M})}}_{\text{Spin-transfer torque (Slonczewski)}}$$



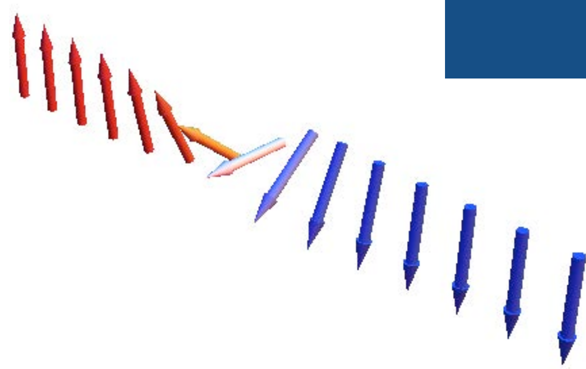
$$\sigma = \frac{g \mu_B}{2e} \frac{1}{M_s^2 d} P$$

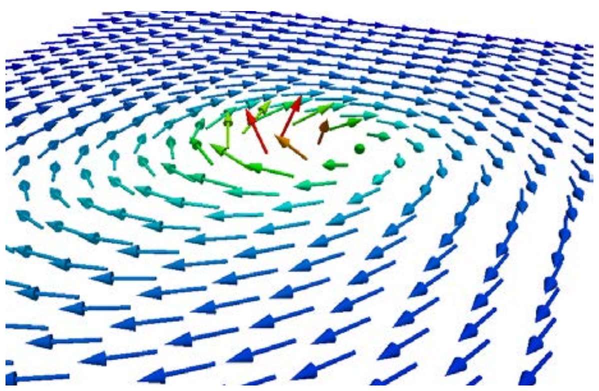
efficiency factor

- Current density matters, not currents. We did not observe STT before the advent of nanofabrication
- Need typical densities of **10^{12} A/m²**: 1 mA for 1000 nm², 1 000 000 A for 1 mm²

J C Slonczewski, J Magn Magn Mater **159**, L1 (1996)

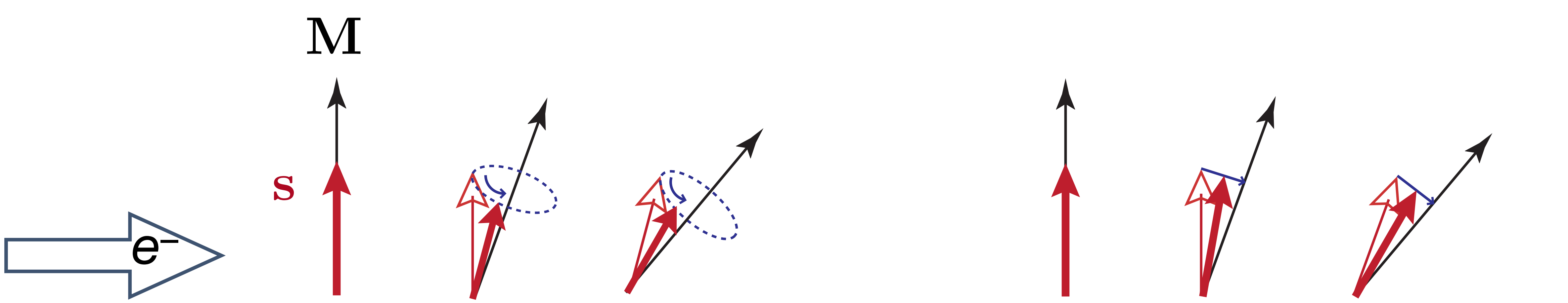
Current-in-plane torques





- Spin-transfer torques also occur in continuous systems in which there are gradients in the magnetisation
- Important for micromagnetic states like domain walls, vortices, skyrmions
- Torques determined by how well the conduction electron spin tracks the local magnetisation
- Like CPP case, spin transfer involves the absorption of transverse component of spin current

Adiabatic | **Nonadiabatic**



Conduction electron spin precesses about *sd* field

Conduction electron spin relaxes toward *sd* field

Zhang-Li model of CIP torques

S Zhang & Z Li
Phys Rev Lett **93**, 127204 (2004)

- In the drift-diffusion limit

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \boxed{+ \mathbf{T}_{\text{CIP}}}$$

$$\mathbf{T}_{\text{CIP}} = -\frac{b_J}{\mu_0 M_s^2} \mathbf{M} \times [\mathbf{M} \times (\mathbf{j}_e \cdot \nabla)\, \mathbf{M}] - \frac{c_J}{\mu_0 M_s} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla)\, \mathbf{M}$$

adiabatic **nonadiabatic**

$$b_J = \frac{P \mu_B}{e M_s (1 + \xi^2)} \qquad c_J = \frac{P \mu_B \xi}{e M_s (1 + \xi^2)}$$

P: spin polarisation

- In this model, nonadiabaticity is a ratio between *sd*-exchange and spin flip time scales

$$\xi = \frac{\tau_{ex}}{\tau_{sf}} \qquad \tau_{sf} \sim 10^{-12} \text{ s} \qquad \tau_{ex} \sim 10^{-15} \text{ s}$$

- Many other theories have been proposed to describe this parameter

Re-interpreting Zhang-Li

Thiaville *et al*, Europhys Lett **69**, 990 (2005)

- By recognising that the pre-factors in the CIP torques and the current density $\mathbf{j}_e$ can be expressed in terms of an effective spin-drift velocity **u**

$$\mathbf{u} = P\frac{g\mu_B}{2e}\frac{1}{M_s}\mathbf{j}_e = P\frac{\hbar}{2e}\frac{1}{M_s}\mathbf{j}_e \qquad [\mathbf{u}] = \mathrm{m/s}$$

$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma_0\mathbf{M}\times\mathbf{H}_{\mathrm{eff}}}_{\text{precession}} + \underbrace{\frac{\alpha}{M_s}\mathbf{M}\times\frac{d\mathbf{M}}{dt}}_{\text{damping}} \boxed{\underbrace{-\left(\mathbf{u}\cdot\nabla\right)\mathbf{M}}_{\text{adiabatic}} + \underbrace{\frac{\beta}{M_s}\mathbf{M}\times\left[\left(\mathbf{u}\cdot\nabla\right)\mathbf{M}\right]}_{\text{nonadiabatic}}}$$

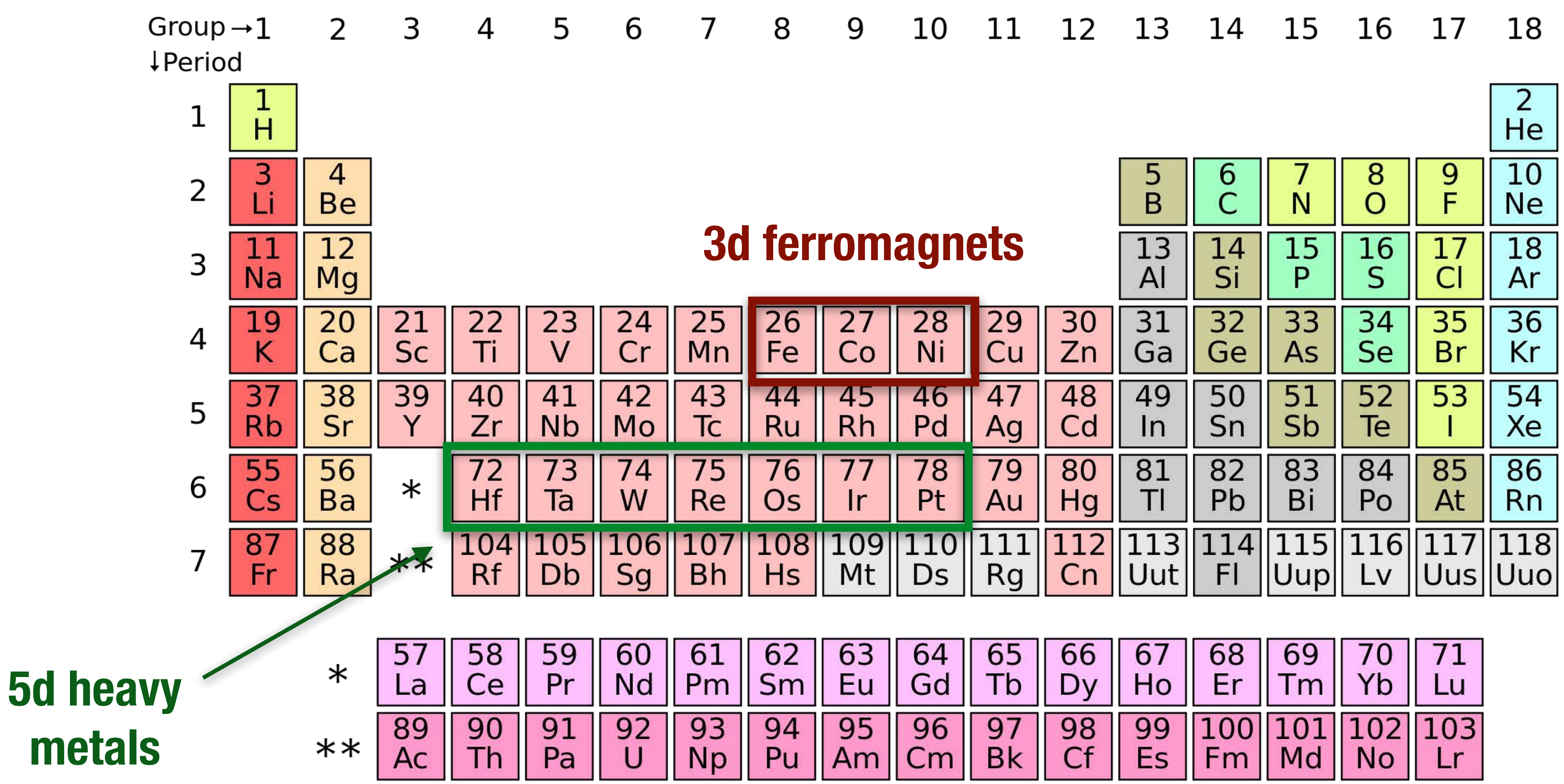
- Rearranging into a more suggestive form:

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{M} = -\gamma_0\mathbf{M}\times\mathbf{H}_{\mathrm{eff}} + \frac{\alpha}{M_s}\mathbf{M}\times\left(\frac{\partial}{\partial t} + \frac{\beta}{\alpha}\mathbf{u}\cdot\nabla\right)\mathbf{M}$$

Convective derivative

Spin-orbit coupling

- In magnetic multilayered structures, metallic ferromagnets in contact with 5d transition metals (“heavy metals”) exhibit strong effects due to **spin-orbit coupling**



$$E_{\mathrm{SO}} \sim \mathbf{L} \cdot \mathbf{S}$$

Spin-orbit coupling

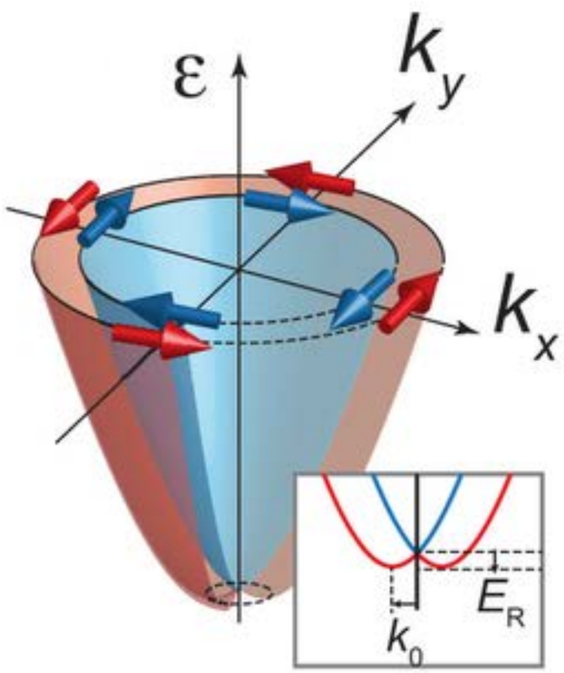
- Examples (often interesting for inducing perpendicular magnetic anisotropy – PMA):

  Pt | Co (0.4 - 1 nm) | AlOx

  (**Ta, W, Hf**) | CoFeB (1 nm) | MgO

  Pt | [Co (0.4 nm) | Ni (0.6 nm)]$_n$

- Lack of inversion symmetry, allows for a class of spin-orbit interactions seen in two-dimensional systems, e.g. Rashba interaction



Wave vector dependent effective Rashba field

$$\mathscr{H}_R = \alpha_R \left(\sigma \times \mathbf{p}\right) \cdot \hat{\mathbf{z}}$$

Rashba Hamiltonian

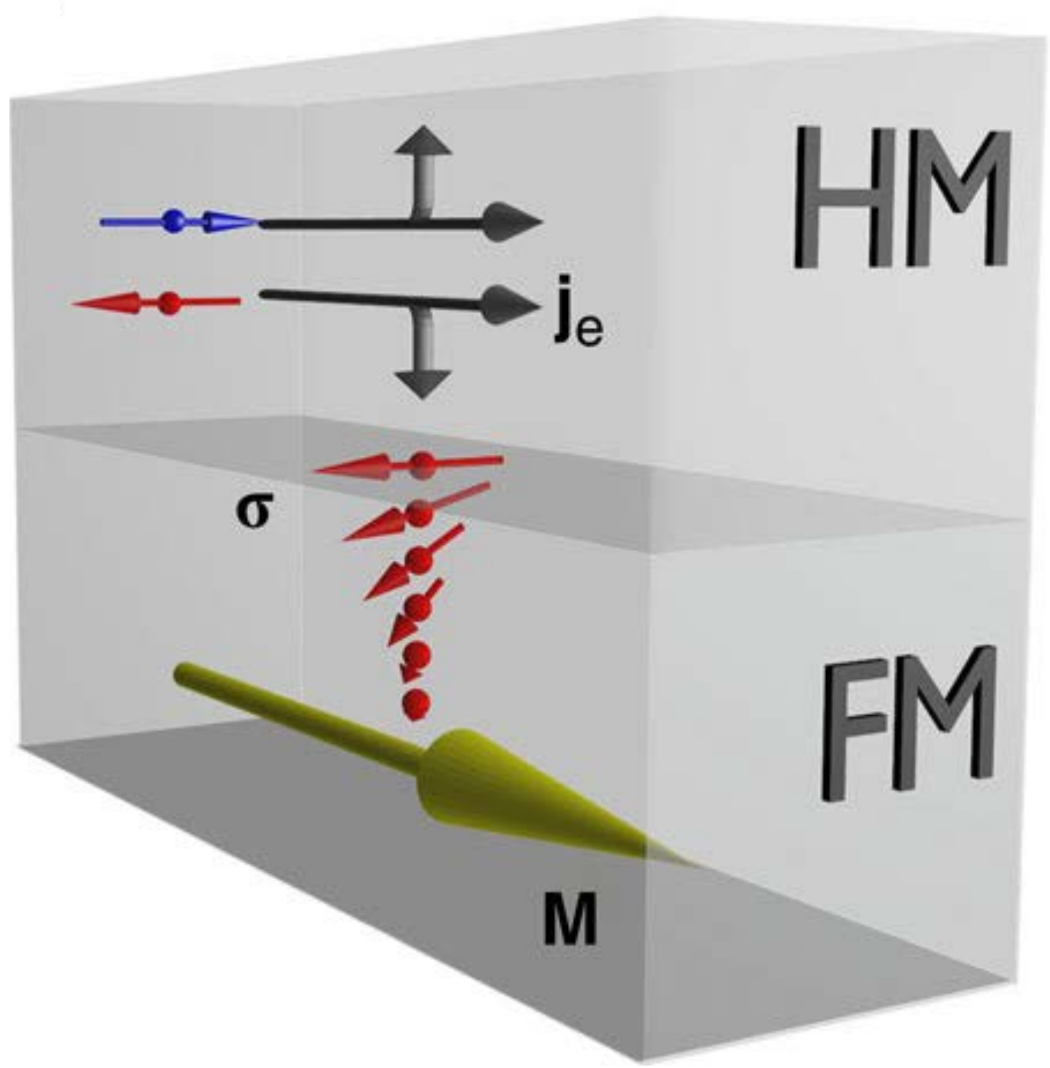
$$\mathscr{H}_{\mathrm{el}} = \frac{\mathbf{p}^2}{2m} + \mathscr{H}_R$$

Free electron + Rashba

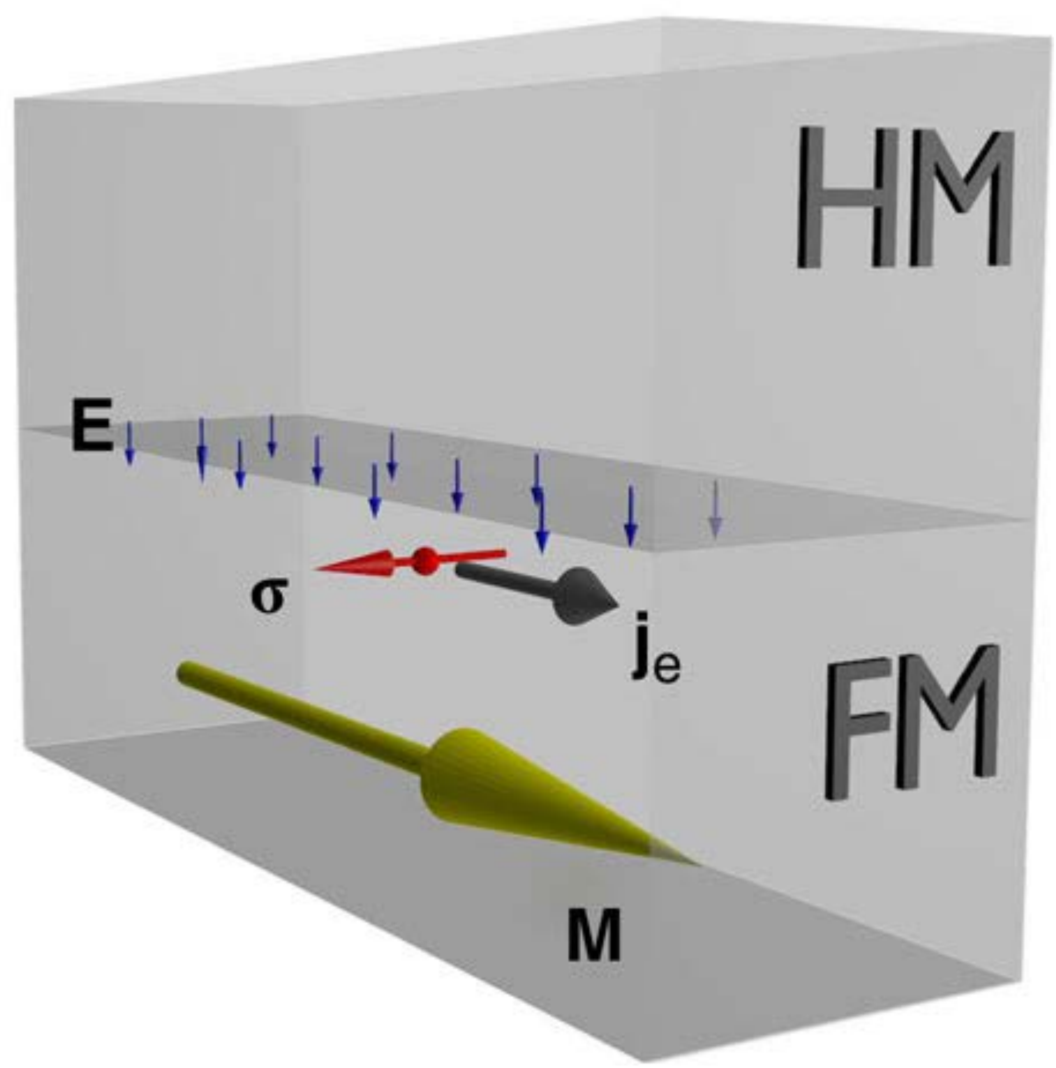
Spin-orbit torques

- Such spin-orbit effects due to the heavy metal (HM) give rise to ***spin-orbit torques*** on the ferromagnet (FM)

Spin Hall effect

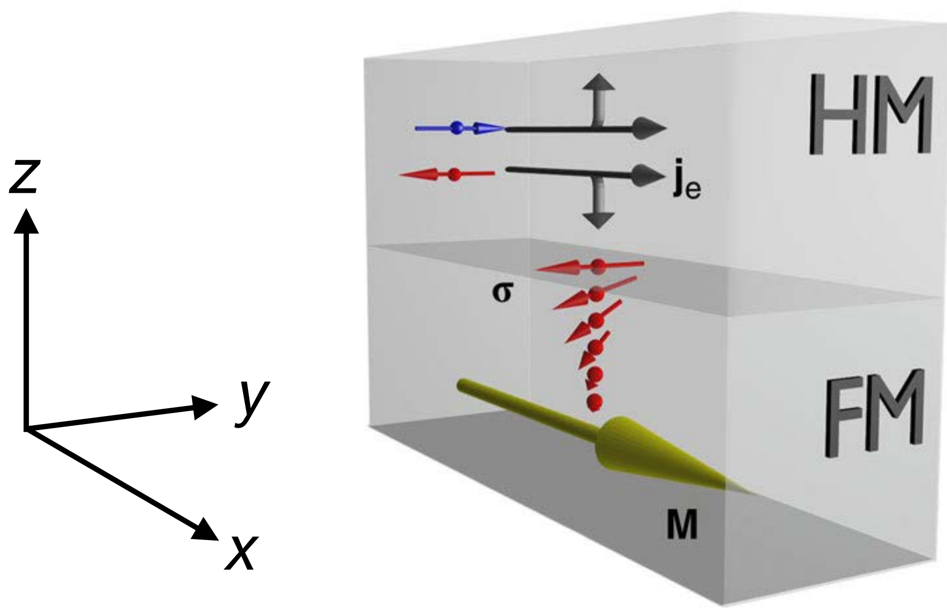


Rashba torques



Spin-orbit torques

- Torques due to the spin Hall effect can be described using the Slonczewski form



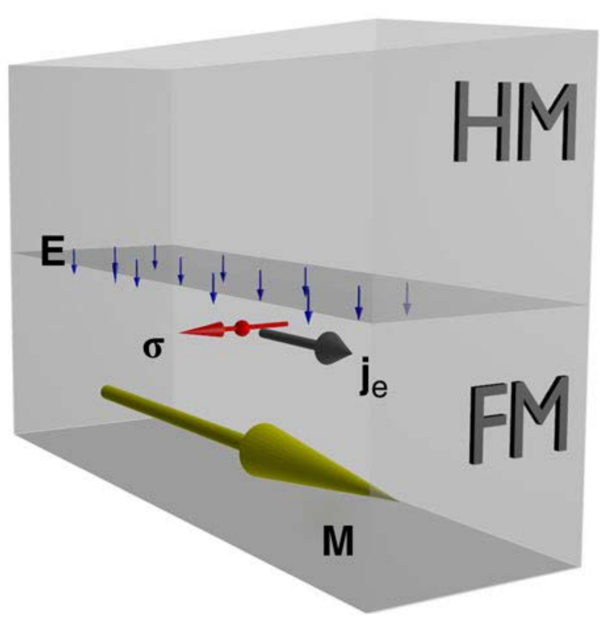
$$\mathbf{T}_{\mathrm{SH}} = \sigma_{\mathrm{SH}} j_e\, \mathbf{M} \times (\hat{\mathbf{y}} \times \mathbf{M})$$

$$\sigma_{\mathrm{SH}} = \frac{g\mu_B}{2e} \frac{1}{M_s^2 d} \theta_{\mathrm{SH}}$$

efficiency

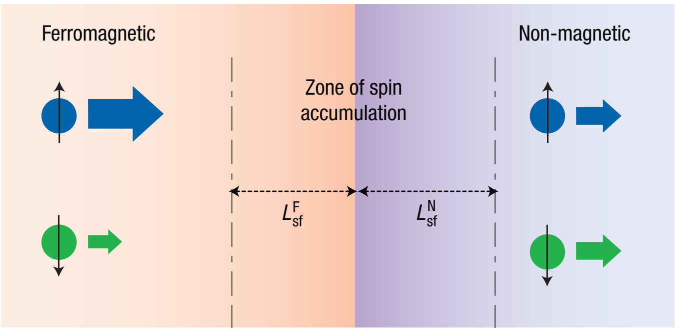
spin Hall angle

- Torques due to the Rashba effect can be assimilated to an effective field



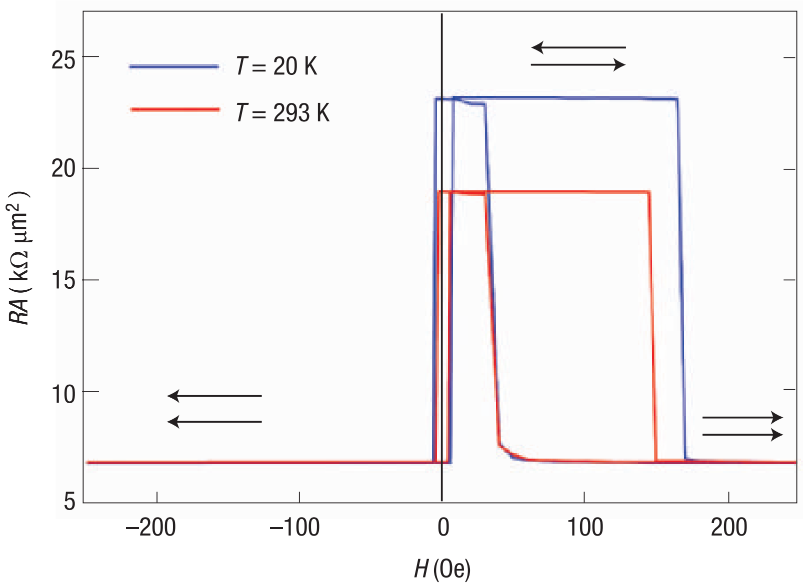
$$\mathbf{T}_{\mathrm{R}} = -\gamma_0 \mathbf{M} \times (H_R\, \hat{\mathbf{y}})$$

Summary

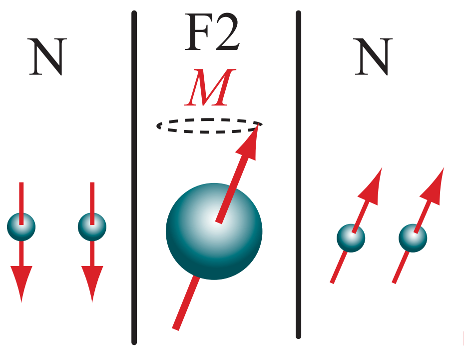


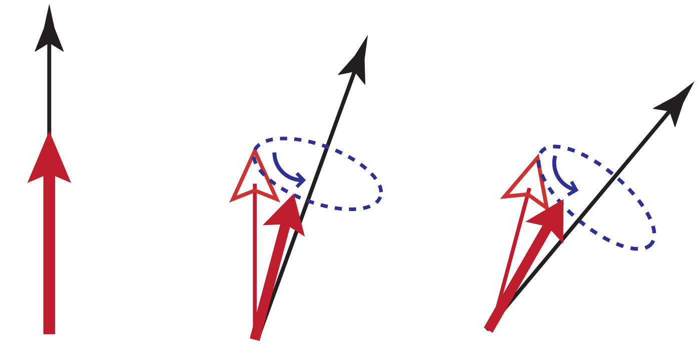
- Magnetism affects transport and vice versa

- Magnetoresistance and spin diffusion
  Spin polarized currents within two-current model
  Giant and tunnel magnetoresistance
  Lateral spin diffusion allows for "nonlocal" effects



- Spin transport torques
  Spin filtering at ferromagnet/normal metal interfaces
  Slonczewski model (CPP)
  Zhang-Li model (CIP)
  Spin-orbit torques





Further reading

Books

- *Nanomagnetism and Spintronics*, edited by T Shinjo (Elsevier, 2014), 2nd ed.

- *Spin Current*, edited by S Maekawa, S O Valenzuela, E Saitoh & T Kimura (Oxford Univ. Press, 2017), 2nd ed.

- *Quantum Theory of Magnetism*, R M White (Springer, 2006), 3rd ed.

Review papers, book chapters

- J F Gregg *et al*, *Spin electronics - A review*, J Phys D: Appl Phys **35**, R121 (2002)

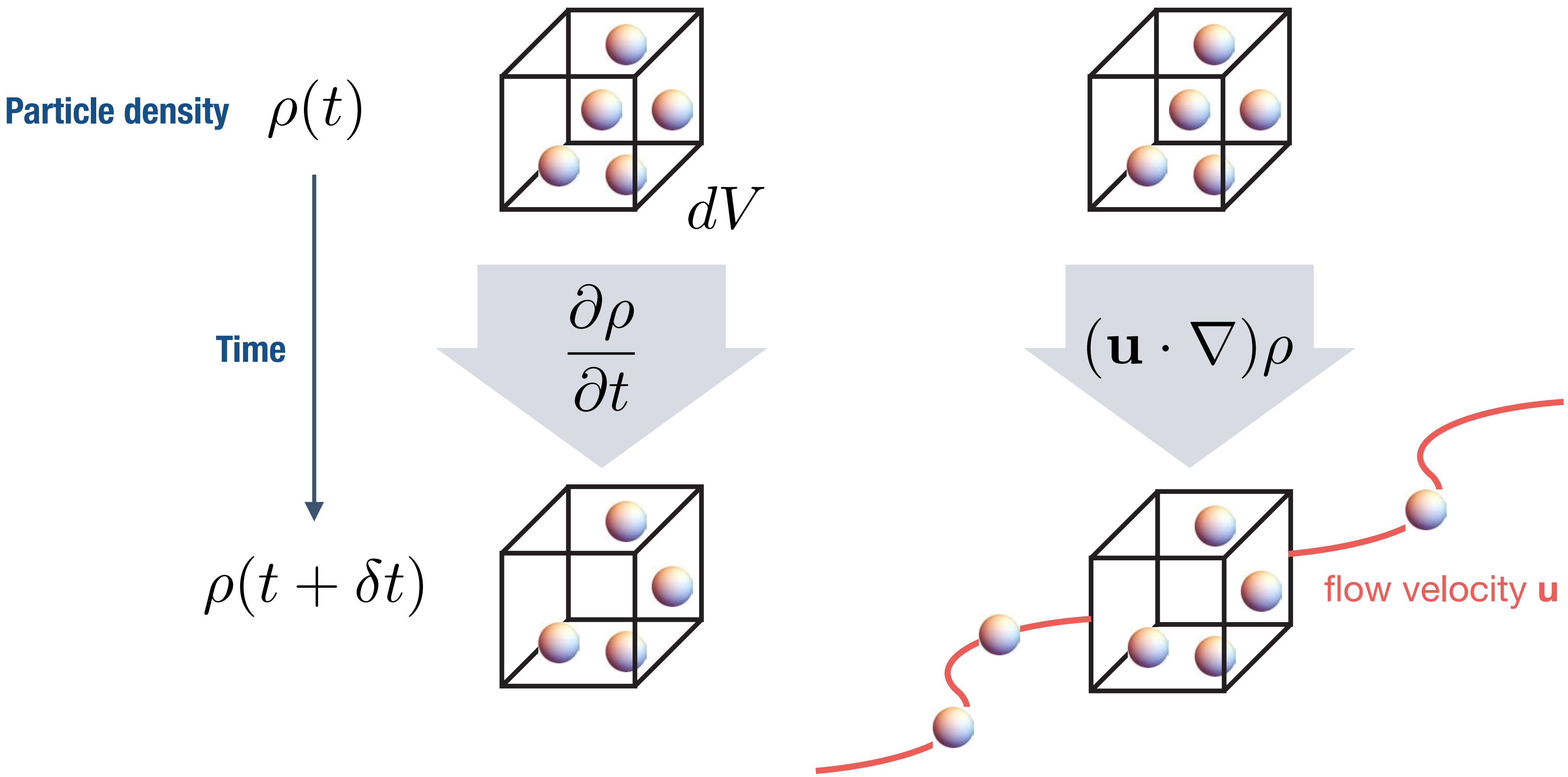
- C Chappert et al, *The emergence of spin electronics in data storage*, Nat Mater **6**, 813 (2007)

- W H Butler and X-G Zhang, *Electron Transport in Magnetic Multilayers*, in *Ultrathin Magnetic Structures III*, edited by J A C Bland and B Heinrich (Springer, 2005)

Convective derivatives

- Consider time evolution of an element *dV* of a fluid
- Convective derivative *D* accounts for local variations and particle flow

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{u} \cdot \nabla)$$



Analogy with fluid dynamics?

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{M} = -\gamma_0 \mathbf{M}\times\mathbf{H}_{\mathrm{eff}} + \frac{\alpha}{M_s}\mathbf{M}\times\left(\frac{\partial}{\partial t} + \frac{\beta}{\alpha}\mathbf{u}\cdot\nabla\right)\mathbf{M}$$

- This form can almost be obtained by replacing the time derivative of the usual Landau-Lifshitz equation

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma_0 \mathbf{M}\times\mathbf{H}_{\mathrm{eff}} + \frac{\alpha}{M_s}\mathbf{M}\times\frac{\partial \mathbf{M}}{\partial t}$$

with the *convective derivative*

$$\frac{\partial}{\partial t} \to \left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)$$

It almost works *except* for the β/α term. $\mathbf{u}$ therefore represents the average drift velocity of the magnetisation (under applied currents), which for ferromagnetic metals makes some sense.

- No consensus (theoretically and experimentally) over the ratio β/α, which can vary between 0.1 and 10