

MICROMAGNETISM I higher-education Magnetism GROUND STATES, HYSTERESIS AND DYNAMICS

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What is micromagnetism?

The aim of micromagnetism is calculation of magnetic states and their dynamics for a given ferromagnetic system, i.e.

> magnetic moment configuration in the classical (semi-classical) approximation

given: >intrinsic parameters >microstructure >morphology (geometry) >applied field >temperature >External stimuli (current, voltage, laser



Micromagnetic simulations need as an input experimental information About microstructure
About local intrinsic parameters

This information does not alway exist with sufficient details



1963

324 × 499

Number 18

Interscience Tracts on Physics and Astronomy

Analytical theory for magnetization reversal

A.Aharoni,

A.Hubert,

H.Kronmuller



Springer.





Going to numerics

H. Kronmuller, J.Fidler, J.Miltat etc.

Numerical micromagnetics

T.Shrefl (1993), M.Donahue (NIST oomf1998)

Principle lines in micromagnetism:

Classical micromagnetism

Total energy minimization Hysteresis processes:coercivity and remanence. Thin films, multilayers, magnetic elements: dots, antidots, stripes, wires etc.

Magnetisation dynamics Integration of the LLG equation (up to 100 ns) Fast magnetisation switching of thin films and magnetic elements

Temperature effects Langevin dynamics simulations (LLG equation + white noise)

Spin-torque dynamics current-induce magnetisaiton dynamics, Slonczewki equation

Long-time dynamics Magnetic viscocity and thermal stability Kinetic Monte Carlo and energy barrier calculations

High-temperature dynamics
 Femto-second dynamics and HAMR
 Landau-Lifshitz-Bloch equation

MICROMAGNETIC PROGRAMS

OOMMF (OBJECT-ORIENTED MICROMAGNETIC FRAMEWORK)

HTTP://MATH.NIST.GOV/OOMMF

- ➢ FINITE DIFFERENCES
- ➢ FAST FOURIER TRANSFORM FOR MAGNETOSTATIC INTERACTIONS

• MUMAX3

HTTP://MUMAX.GITHUB.IO/

- ➢ FINITE DIFFERENCES
- GPU-BASED
- BORIS

FOR SPINTRONICS AND ULTRAFAST DYNAMICS HTTPS://WWW.BORIS-SPINTRONICS.UK/

NMAG (FIDIMAG FROM THE SAME GROUP), JOOMF
 FINITE ELEMENTS (MULTISCALE ATOMISTIC/MICROMAG CODE)
 HTTP://NMAG.SOTON.AC.UK/NMAG
 HTTPS://COMPUTATIONALMODELLING.GITHUB.IO/FIDIMAG/





Classical micromagnetic sumulations:

W.F.Brown "Micromagnetics" 1963

Classical approximation of continuous magnetic media Consists of the minimization of total magnetic energy:

$$\begin{split} E_{ext} &= -\int_{V} \vec{H}_{ext}(\vec{r}) \vec{M}(\vec{r}) dV \quad \text{External field} \\ E_{ani} &= -\int_{V} K_{ani}(\vec{r}) [\vec{m}(\vec{r}) \vec{e}(\vec{r})]^2 dV \quad \text{Anisotropy} \\ E_{ex} &= \int_{V} A_{ex}(\vec{r}) [(\nabla m_x)^2 + (\nabla m_y)^2 + (\nabla m_z)^2] dV \quad \text{Exchange} \\ E_{magn} &= -\frac{1}{2} \int_{V} \vec{M}(\vec{r}) \vec{H}_{magn}(\vec{r}) dV \quad \text{Magnetostatic} \end{split}$$

Magnetostatic energy:

$$\vec{H}_{magn} = -grad \ U_{magn}$$
$$\Delta U_{magn} = \begin{cases} 4\pi \nabla \vec{M} & inside \\ 0 & outside \end{cases}$$

 $\begin{array}{l} U_{magn}^{outside} \Big|_{\Sigma} = U_{magn}^{inside} \Big|_{\Sigma} \\ \left(\frac{\partial U_{magn}^{inside}}{\partial \vec{n}} - \frac{\partial U_{magn}^{outside}}{\partial \vec{n}} \right)_{\Sigma} = 4\pi \vec{M} \vec{n} \end{array}$

Boundary conditions:



Finite elements:

DISCRETIZATION IN FINITE DIFFERENCES:

Sample Dx = 312 Dy = 24-194 Dz = 4 Empty space around the system Dx = 15 Dy = 5-10 Dz = 3 Unit = discretization length = 1.288 nm



System geometry and dimensions



Defines magnetisation, e.g. on nodes, and uses continuous interpolation inside element. Constant magnetisation inside each cell

 $E_Z = -\int \vec{M} \vec{H} dV \rightarrow -H \sum \vec{M}_i \Delta^3$

Exchange field





Minimization of surface charges



Closure domains: in magnetic hard directions problem: magnetostriction!

POLE AVOIDANCE PRINCIPLE

The minimization of the magnetostatic energy leads to the "avoidance" of surface charges = Magnetisation parallel to surfaces



SATURATED CUBE



•The magnetostatic fields are stronger at the corners.

•This expression will not be valid if the magnetisation process is not homogeneous

M-MAG STANDARD PROBLEM (FROM OOMMF WEBPAGE)





See also M.A.Schabes abd H.N.Bertram "Magnetisation processes in ferromagnetic cubes", J.Appl. Phys. 64 (1988) 1347.



Important length scales:

Néel Domain wall width (Exchange correlation length) $\delta_{\rm o} = (A/K_1)^{1/2}$

> Exchange correlation length ("magnetostatic correlation length") $l_o = (A/\mu_o M_s{}^2)^{1/2}$

Critical domain size

 $R_{SD} = 36 (AK_1)^{1/2} / \mu_o M_s^2$

Dynamics: the Landau-Lifshitz-Gilbert equation of motion





Slonszewski Spin-transfer torque (two ferromagnets, one with fixed polarization p with non-Magnetic spacer)

also for spin-orbit torques (thin film coupled to another layer with high SO coupling)

$$\frac{dm}{dt} = -\frac{\gamma}{1+\alpha^2} ([m \times H_{eff}] + \alpha [m \times [m \times H_{eff}]] - \frac{\hbar j}{eM_S d} g(\Theta) \left(\beta [m \times p] - [m \times [m \times p]]\right)).$$

Zhang-Li model (current through magnetic materials, For domain walls

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times H_{eff} + \alpha \vec{M} \times \frac{\partial \vec{M}}{\partial t} + -u \frac{\partial \vec{M}}{\partial x} + \beta u \vec{M} \times \frac{\partial \vec{M}}{\partial x}$$

THERMAL MICROMAGNETICS

Langevin dynamics approach

$$\frac{d\vec{M}}{dt} = -\frac{\gamma}{1+\alpha^2}\vec{M} \times \vec{H} - \frac{\gamma\alpha}{M_s(1+\alpha^2)}\vec{M} \times (\vec{M} \times \vec{H})$$

$$= \vec{H}_{Zeeman} + \vec{H}_{aniso} + \vec{H}_{exch} + \vec{H}_{magnetost} + \vec{H}_{there}$$

$$< H_{therm,i}(t) >= 0, \quad < H_{therm,i}(t)H_{therm,j}(t') >= \frac{2ck_BT}{M_sV}\delta_{ij}\delta(t-t')$$

Initially introduced for nanoparticlesThis was brought to micromagnetics.

No correlations In time and different particles!!

W.F.Brown, Phys Rev 130 (1963) 1677.

MICROMAGNETIC MODELS OF NANOSTRUCTURED MATERIALS







Models need nanostructure and micromagnetic parameters from experiment

Nanotecnology

Magnetsotatic fields strongly depend on the shape of the nanoelement



Magnetic nanostructures have at least one of the dimensions comparable to magnetic correlation lengths: D ~ Lex

This leads to new finte-size fenomena related to competition between the exchange and magnetostatic energies

- ✓ Shape anisotropy
- ✓ Configurational anisotropy
- Magnetisation deviation at the nanostructure boarders
- ✓ Stabilisation of different states (e.g.vortex or skyrmion states)
- ✓ Confinement and quantisation of spinwaves





LITOGRAPHED STRIPES WITH PERPENDICULAR ANISOTROPY





 $w = 2 \mu m$

- J. Jorzick et al., Phys. Rev. Lett.
- J. P. Park et al., Phys. Rev. Lett.
- · C. Bayer et al., Appl. Phys. Lett.



Magnetic Vortex State in circular ferromagnetic dot



Vortex translation mode in a circular dot

Vortex core trajectory



The "*translational mode*" of the vortex excitations corresponds to the spiral vortex core rotation around the dot center. Its direction (counter-clockwise or clockwise) is defined by the combination of the vortex polarization and chirality.

VORTEX CORE REVERSAL



FIG. 1 (color online). Schematics of a field-pulse driven vorte; core switching. The vortex core magnetization can be switched by a short magnetic field pulse applied in the film plane. Thi switching process requires only 40-50 ps.



R.Hertel et al PRL <u>98</u> (2007) 117201



Experimental observation with synchrotron: B.Van Wayenberge, Nature 444 (2006)

TRIANGULAR DOTS: SIMULTANIOUS CONTROL OF POLARITY AND VORTICITY



DivM <0

(b)

(e)

(a)

(d)

111

(c)

(f)

.

DivM >0



*Small fields + small duration -> core reversal * Larger fields + larger duration -> chirality reversal

TOPOLOGICALLY NON-TRIVIAL MAGNETIC STRUCTURES (SYSTEMS WITH DMI)



Non-trivial skyrmion

Single magnetic (baby-)skyrmion

"hedgehog" Neel skyrmion







- Spins down at core, up everywhere else, homogeneous in *z*-direction
- Topological excitation,

characterized by winding number: $4\pi W = \int d\vec{x} \,\Omega \cdot \frac{\partial\Omega}{\partial x} \times \frac{\partial\Omega}{\partial y}$

Pictures from Fert et al., Nature Nanotechnology (2013)

Topological effects in skymions



ANTIDOTS



a) Diluted regime D<<L:
nucleation
b) Pinned regime (D ~ L)
Nucleation +pinning
c) Creap regime (L-D <<D)
Multiple nucleation
d) chess-like structure



TWO MAIN PROPAGATION MECHANISMS



MAGNETIC CYLINDRICAL NANOWIRES





Naturally perpendicular anisotropy

Naturally chiral systems

No Walker breakdown

Velocities up to

Charolou et al PRL(2019)

DIFFERENT REVERSAL MODES IN NANOWIRES



0



REMANENT STATES AND REVERSAL MODES

Small diameters <40nm Single domain state Transverse domain wall



The MOST typical situation Intermediate diameters: Single domain state Vortex (BLOCH-point) domain Wall



C.A. Ferguson et al. , J. Magn. Magn. Mater. 381 (2015) 457–462 Vortex (vortex-antivortex) DW



Large diameters >100nm or large Ms : Vortex (skyrmions) tube dynamical or at the remamence



DEMAGNETISATION PROCESES



SOME ADVICES

- ✓ Use always small discretization length (1-2 nm), vortices or Bloch points may be there
- ✓ Remember that this is a continuous approximation (not valid at atomic scale or rapidly varying parameters)
- ✓ Magnetic systems can have many metastable states (different programs not always give the same result)
- ✓ Dynamic integration may be better than direct energy minimization (takes the system our of plane)
- ✓ Temperature fluctuations are only valid for low temperatures only, they are good as a mean to "shake" the system
- ✓ Do no expect coincidence with the experiment, especially in the value of the coercive field
- ✓ Be critical and check against analytical calculations

MICROMAGNETISM II (NON-STANDARD), TEMPERATURE, MULTISCALE DESCRIPTION AND ULTRAFAST DYNAMICS

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THERMAL FLUCTUATIONS PLAY VERY IMPORTANT ROLE IN MAGNETISATION DYNAMICS:

At the microscopic level:

- At the equilibrium they are responsible for <u>thermally excited</u> <u>spinwaves</u>.
- Spinwaves are responsible for temperature-dependence of macroscopic properties and for thermal magnetisation reversal via the spinwave instabilities and energy transfer to main reversal mode.

At more macroscopic level

•Thermal fluctuations are responsible for random walk in a complex energy landscape

•Eventually <u>energy barriers</u> could be overcome with the help of thermal fluctuations leading to magnetisation decay.

LONG-TIME DYNAMICS: ENERGY BARRIER EVALUATION

Slow processes: ^B k T <<∆E (Energy barrier)

 $t_i = t_0 \exp(\Delta E / k_B T)$

Energy barrier calculation is essential part for determination of long-time thermal stability and slow thermal relaxation

>This is important from the point of view of magnetic recording applications.

Evalulation of energy barriers should be done in a multidimensional space and is a difficult problem

- In systems with many minima, the energy barriers should be constantly re-evaluated
- The result may depend on the initial guess due to the motion on a sphere



Elastic nudged band method



ENERGY BARRIERS FOR SKYRMIONS



D.Cortes-Ortuño et al SREP 7 (2017)





Transition state theory

- Connection to (higher-frequency) modes through the Arrhenius prefactor (attempt frequency)
- Example: Langer's theory of transition rates

$$\Gamma \equiv \frac{1}{\tau} = \frac{\lambda_+}{2\pi} \Omega_0 \exp\left(-\frac{E_a}{k_B T}\right)$$

Dynamical prefactor

Linearised dynamics at *S*, rate of growth of unstable mode





Ratio of curvatures

J S Langer, Ann Phys 54, 258 (1969)

$$H = \left\{ \frac{\partial^2 E}{\partial \eta_i \partial \eta_j} \right\} \qquad \text{Hessian matrix}$$

$$\Omega_0 = \sqrt{\frac{\det H^A}{|\det H^S|}} = \sqrt{\frac{\Pi_i \lambda_i^A}{\Pi_j |\lambda_j^S|}}$$

Ratio of products of eigenvalues of H

KINETIC MONTE CARLO

- >Evaluate all energy barriers in multidimensional space
- Evaluate all transition rates, according to the Arrhenius law

 $f_i = f_0 \exp(-\Delta E / k_B T)$ $f = \sum f_i$

Choose a particle (cluster) with the probability proportional to its transition rate and invert it

>Approximate the waiting time from the exponential distribution

Recalculate all the energy barriers $D(t)dt = f \exp(-ft)dt$

Solution of the Master equations (not too many barriers)

$$\frac{dm_i}{dt} = -\sum_j p_{ij}m_i + \sum_j (1-p_{ji})m_j$$

Leaving well Returning to well

CoCrPt magnetic recording media



DYNAMICS: ATOMISTIC VS MICROMAGNETIC APPROACH

structure



Generalized Heisenberg $H = -\frac{1}{2} \sum_{ij} J_{ij}^{\alpha\beta} S_i^{\alpha} S_j^{\beta} \rightarrow E = A \int_V \left| \left(\frac{\partial m_x}{\partial x} \right)^2 + \left(\frac{\partial m_y}{\partial y} \right)^2 + \left(\frac{\partial m_z}{\partial x} \right)^2 \right| dV$

Takes into account crystal

Can be long-range

Micromagnetic exchange

 $A_{\nu}(0 K) = \frac{1}{V_{at}} \sum_{i} J_{0j}^{\nu} \left(a_{0j}^{\nu} \right)^{2}$

Spins, S

(macrospins), m

The theory of thermal magnetization fluctuations of single domain, noninteracting particles was introduced by W.F.Brown (W.F.Brown Phys Rev **130** (1963) 1677)

"We now suppose that in the presence of thermal agitation, the dissipative "the effective field" describes only statistical (ensemble) average of rapidly fluctuating random forces, and that for individual particle this expression must be augmented by a term h(t) whose statistical average is zero"

 $< h_i(t) >= 0, \quad < h_i(t)h_j(t+\tau) >= \mu \delta_{ij}\delta(\tau), \quad i,j = x, y, z$

"The random-field components are formal concepts, introduced for convenience, to produce the fluctuations δM "

W.F.Brown outlined two methods:

-Based on the fluctuation-dissipation theorem -Imposing the condition that the equilibrium solution of the Fokker-Plank equation is the Boltzman distribution $\mu = \frac{2\alpha k_{B}T}{M_{V}(1+\alpha^{2})}$

O.Chubykalo et al JMMM 272 (2004) 251 For colored noise see U.Atxitia, O.C.-F. PRL 102 (2009) 057203

DYNAMICS: ATOMISTIC/MICROMAGNETIC APPROACH:

• Dynamic behaviour of the magnetisation is based on the Landau-Lifshitz-Gilbert equation

$$\vec{S}_i = -\frac{\gamma}{1+\alpha^2}\vec{S}_i \times H_i(t) - \frac{\alpha\gamma}{1+\alpha^2}\vec{S}_i \times (\vec{S}_i \times \vec{H}_i(t))$$

- where γ_0 is the gyromagnetic ratio and α is a intrinsic damping (not to be confused with atomistic coupling-to-the bath parameter damping)
- additional thermal field converts it to stochastic differential equation (Stratonovich sense)-> Langevin dyanmics

$$< h_i(t) >= 0$$
 $< h_i(0)h_j(t) >= \delta(t)\delta_{ij}2\alpha k_bT/\gamma$

THERMAL SPIN WAVES FROM MAGNETIZATION FLUCTUATIONS



K.Yu Guslienko et al JMMM 272 (2004)



$$Max I(\omega, k) = \frac{\gamma k_{B}T(1+\alpha^{2})}{\alpha V_{0}M_{s}} \frac{1}{\omega_{k}^{2}}$$





THERMAL LANGEVIN DYNAMICS: ATOMISTIC VERSUS MICROMAGNETICS APPROACH





Temp (K)

G.Grinstein and R.H.Koch PRL 90 (2003) 207201



N.Kazantseva et al PRB 77 (2008)

Langevin dynamics for the micromagnetics does not correctly describe spinwaves spectrum (high k are cut): The spectrum is cut, DOS is not correct and Tc is largely over estimated

SPINWAVE ROLE DURING THE MAGNETISATION REVERSAL

In the vicinity of the nucleation the spinwave instabilities occur.
 Spinwaves generation becomes chaotic and the system thermalizes through the distribution of energy over all degrees of freedom

><u>At the nucleation reversal</u> the chaos is suppressed and the energy is transferred to the main eigenmode



HIERARCHICAL MULTI-SCALE APPROACH



TEMPERATURE-DEPENDENT SPIN WAVE SPECTRUM

Exchange stiffness calculation 2x10⁹ $\left|\mathsf{F}(\omega, \mathbf{k})\right|^2$ =0K theory ω(1/s **k**=(0,0,8) T=10 K 1.2x10 T=200 K -o- T=350 K 1x10⁹ 8.0x10¹³⁻ k=(0,0,16) 4.0x10¹³] **k**=(0,0,32) 0.0 ω (1/s)^{1.0x10¹⁴} 5.0x10¹³ 25 30 0.0 15 20 35 1.5x10¹⁴ 10 **k**=(0,0,k)

$$\omega \propto A(T) k^2$$
 $A(T) = A(0) m^{2-arepsilon}$ due to spin-spin correlations

U.Atxitia.... O.C.-F... PRB 82 (2010) 054415

Material-specific, depends on number of neighbors, cristal structure etc.

TEMPERATURE-DEPENDENT DOMAIN WALL WIDTH (HCP CO)

- Domain wall width increases with Temperature
- Slightly different width in x and z directions (< 1 nm difference)
 24-25nm
 - $A(T) \propto m^{1.8}$
- Independently evaluated from the Classical spectral density method (CSDM)

Moreno et al PRB 94 (2016)



DZYALOSHINSKI-MORIYA INTERACTION

$$E_{DMI} = -\frac{1}{2} \sum_{i,j} \mathbf{D}_{ij} \left[\mathbf{S}_i \times \mathbf{S} \right]$$
$$\mathbf{D}_{ij} = d \left[\mathbf{u}_{ij} \times \mathbf{z} \right]$$

$$\Rightarrow E_{DMI} = 2\widehat{D} \int [m_z \nabla \mathbf{m} - (\mathbf{m} \cdot \nabla)m_z] dS$$

Interfacial FMI

$$\widehat{D}(T) = \widehat{D}_0 m^{\beta}$$





D.Cortés-Ortuño New J.Phys.(2018)

$$\omega(k) = \gamma_0 (H + \frac{D}{M}k\cos(ak) + \frac{2A}{M}\sin^2(ak))$$

Long wave part: asymmetric in k part gives DMI, Symmetric in k part gives exchange

TEMPERATURE-DEPENDENT DMI FROM THE SW SPECTRUM

T = 50.0K :: m = 0.965430

$$A(T) \propto m^{1.5}; \quad D(T) \propto m^{1.5}$$

Not universal; simple cubic lattice!

R.Tomasello et al PRB 97 (2018)

D/J=0.1 Simple cubic lattice $\mathbf{D}_{ij} = d\left[\mathbf{u}_{ij} \times \mathbf{z}\right]$ A(T)/A(0), D(T)/D(0)Exchange stiffness DMI 0 0.6 0.7 0.8 0.9 1 0.3 0.4 0.5 0.2 M(T)/M(0)

SKYRMIONS IN CO/Pt DOT: MOVING IN THE PARAMETER SPACE WITH TEMPERATURE



Temperature-dependent skyrmion radius R.Tomasello, --O.C.F. et al PRB 97 (2018)

MICROMAGNETIC MODELING AT HIGH TEMPERATURES:

$\textbf{T} \neq 0 \implies |M| \neq \textbf{CONSt} \quad \textbf{LLG micromagnetics is not valid}$

Landau-Lifshitz-Bloch micromagnetic equation



COMPARISON: ATOMISTIC VS MACROSCPIN

solid line - one spin LLB





Timescale is defined by exchange interactions

Critical slowing down phenomena near phase transition

O.Chubykalo-Fesenko et al PRB 74 (2006)

Transverse relaxation



Tmescale is defined by external field



THE LANDAU-LIFHITZ-BLOCH (LLB) EQUATION

$$\frac{d\vec{m}}{dt} = \gamma \left[\vec{m} \times \vec{H}\right] + \gamma \alpha_{\parallel} \frac{\left(\vec{m} \vec{H}_{eff}\right)}{m^2} - \gamma \alpha_{\perp} \frac{\left[\vec{m} \times \left[\vec{m} \times \vec{H}_{eff}\right]\right]}{m^2}$$

D.Garanin PRB 55 (1997)

MICROMAGNETIC MODELING BASED ON THE LANDAU-LIFSHITZ-BLOCH EQUATION



Exchange-coupled macrospins Thermal fields can be included

Large scale modeling

U. Atxitia et al. Applied Physics Letters 91, 232507 (2007)

MODELING OF LASER-INDUCED DEMAGNETISATION WITHIN THE LLB APPROACH: COMPARISON WITH EXPERIMENTS

Schematics: Laser excitation.. in a thermal macrospin model



LLB micromagnetics is coupled to electronic temperature from laser heat

Multiple comparison with experiments confirm heat mechanism

U.Atxitia et al PRB 81 (2010)



MICROMAGNETIC MODELING OF FEPT DYNAMICS UNDER CIRCULARLY POLARISED LASER PULSE :

P.Nieves, O.C.-F. Phys Rev Appl 5 (2016) 014006



Inverse Faraday effect+

heat

There is an optimum T_e dynamics where the IFE is more efficient

MODELING OF LASER-INDUCED DYNAMICS IN FEPT ACCUMULATIVE SUPER-PARAMAGNETIC EFFECT

Ab-initio effect -> LLB for siwtching probability -> rate equations



Inverse Faraday effect: ΔM=-7.1% Ms (σ+)
-3.5% (Ms) (σ -)
MCD effect
Δ T=32 K(for Tmax=1100K)
for spin up and spin down

> R.John et al Sci. Rep.. 7 (2017) 4114.

MESSAGES:

- Necessity to include temperature unto modelling in many cases
- Atomistic Langevin dynamics simulations are consistent with thermal spinwave theory
- Micromagnetic Lngevin dynamics simulations can be used in the range T < Tc/3
- Thermal spin wave spectrum –softening with tempeture
- Allows to evaluate Exchange stiffness and DMI parameters, They scale equally with temperature as m^{2-e}
- The scaling relation K(m), A(m), D(m) provides the tool to evaluate many important characteristics
- The example: skyrmion radius increases with temperature
- Importance of spinwaves during laser-induced magnetization dynamics:: excitation of two spinwave modes at percolation length as a criterion for switching