Magneto-Optics

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Leibniz Institute for Solid State and Materials Research (IFW) Dresden, Germany

THE EUROPEAN SCHOOL ON MAGNETISM

Magneto-Optics

Magneto-Optics

Describes:

The influence of magnetic field or spontaneous magnetization
on the emission and propagation
of light in matter

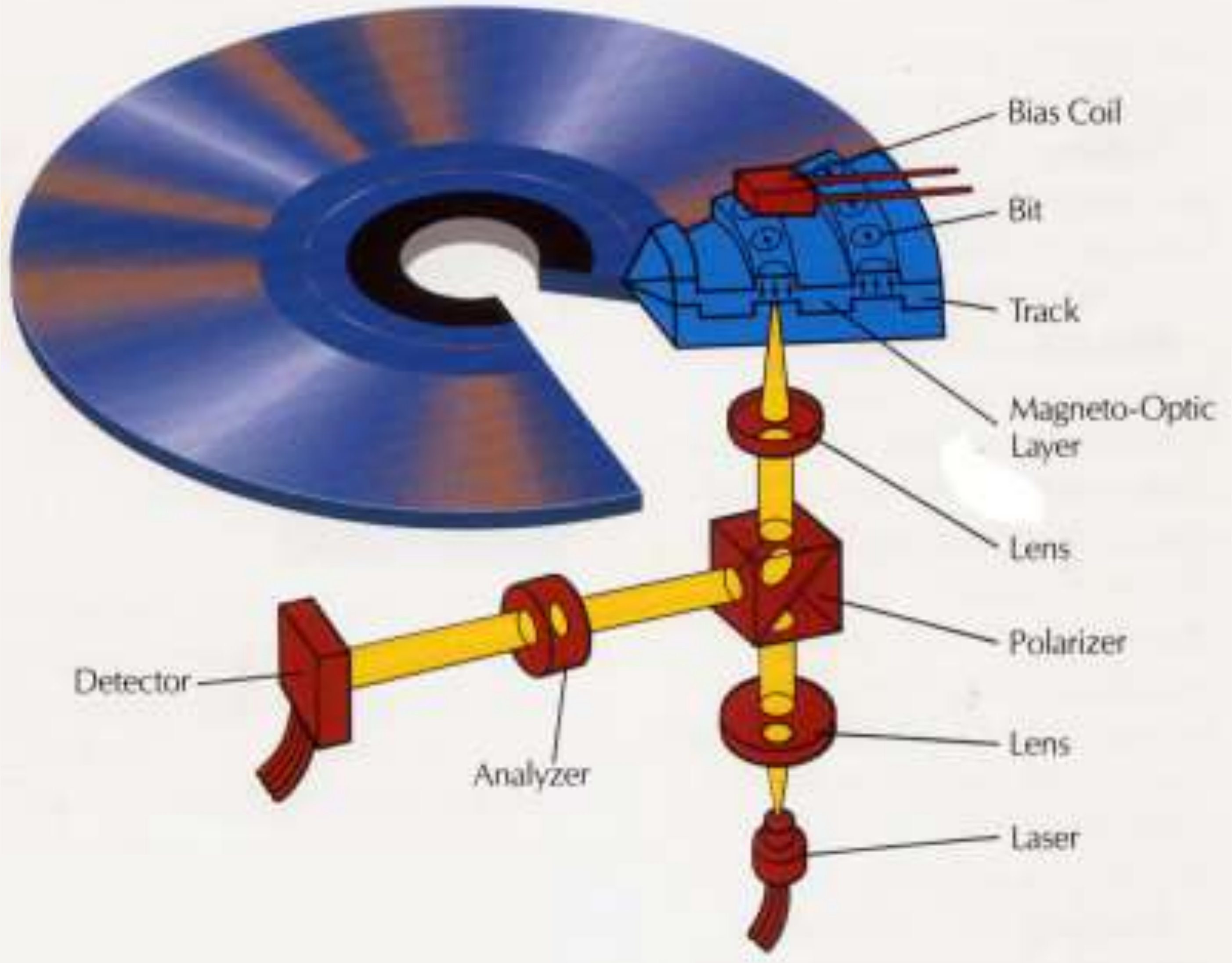
Magneto-Optics

Magneto-Optics

Can be used for:

Magneto-Optics

MO Recording



http://magnetic-recording.blogspot.com

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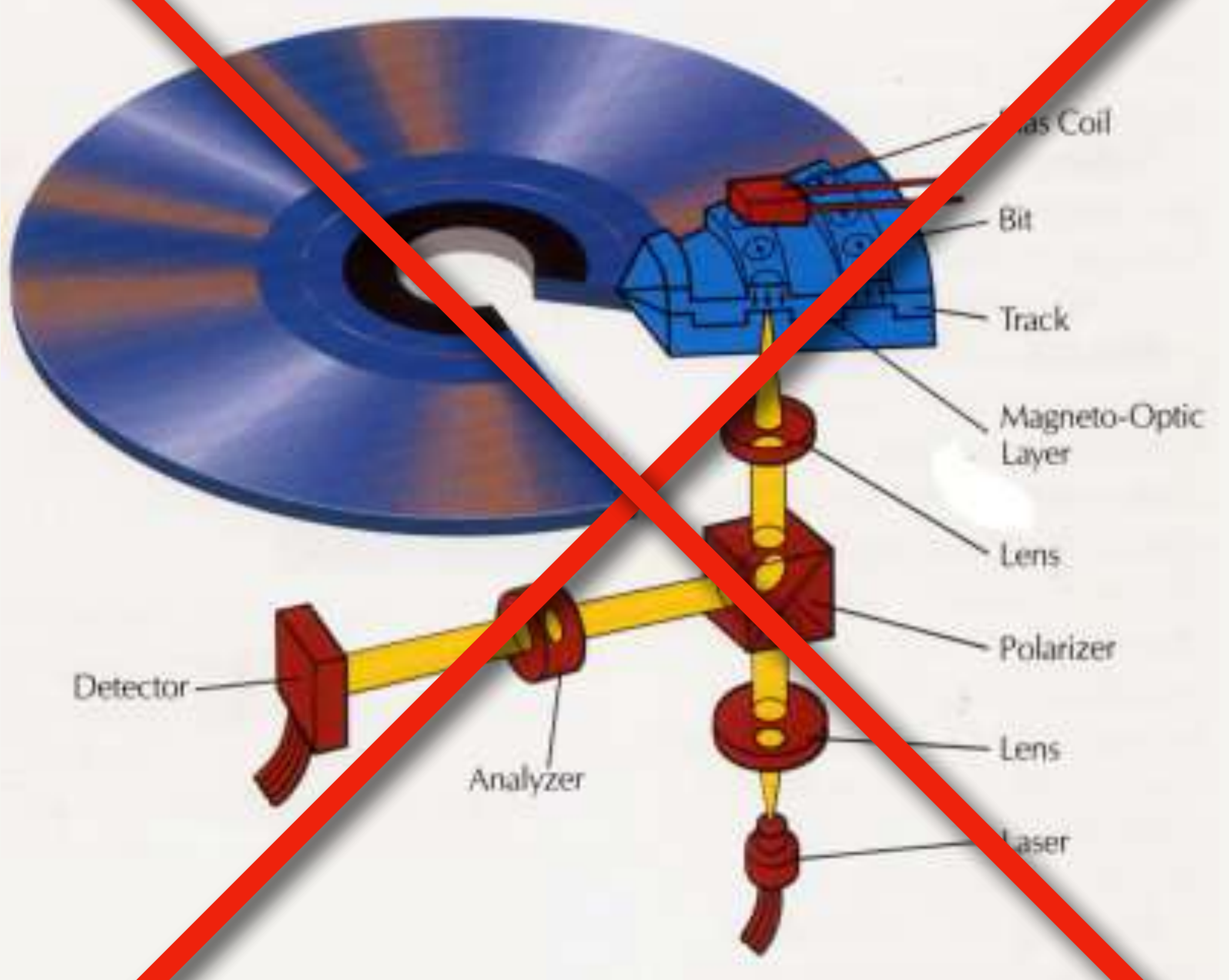

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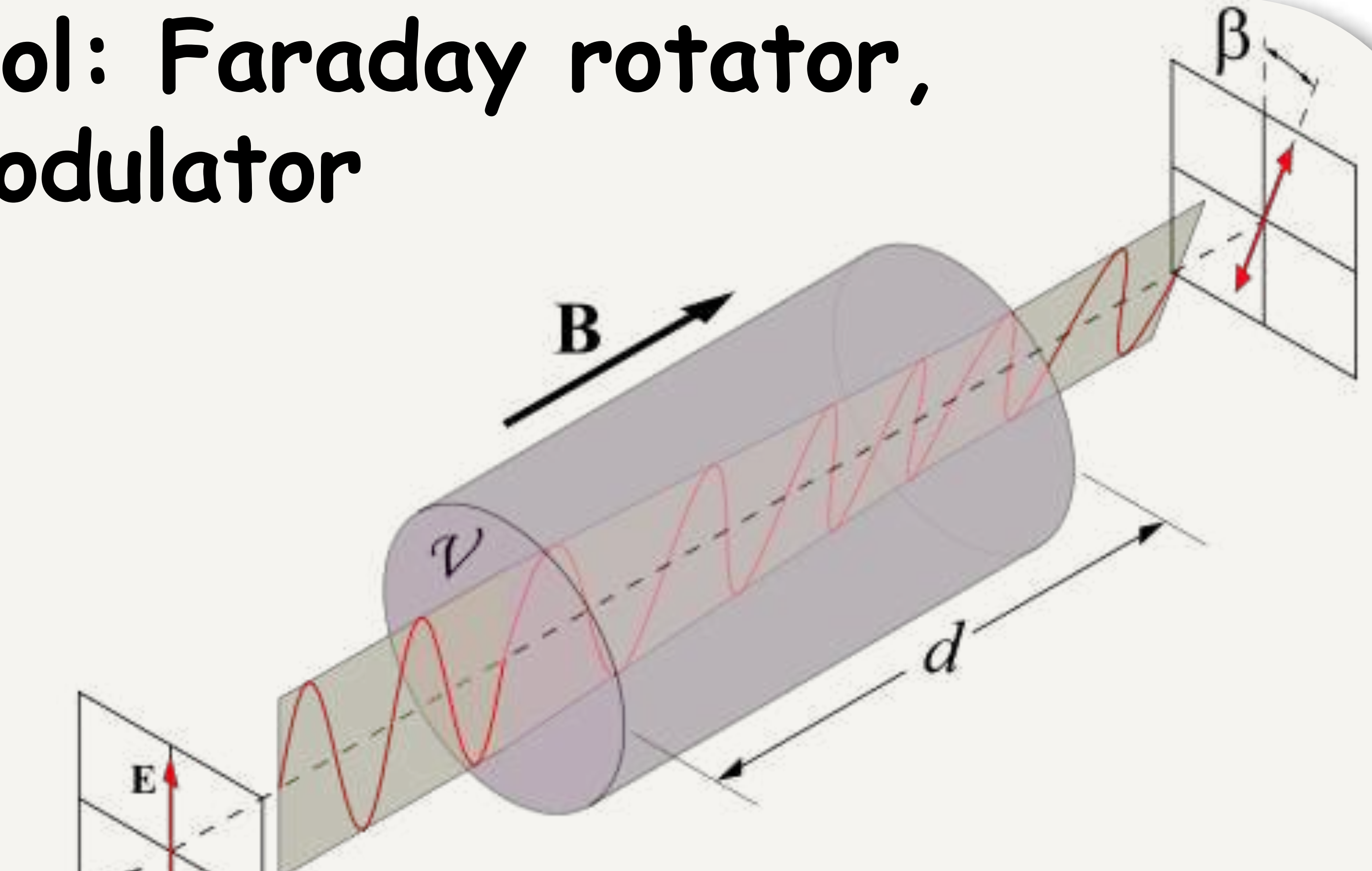
Light control: Faraday rotator, isolator, modulator

www.holmarc.com



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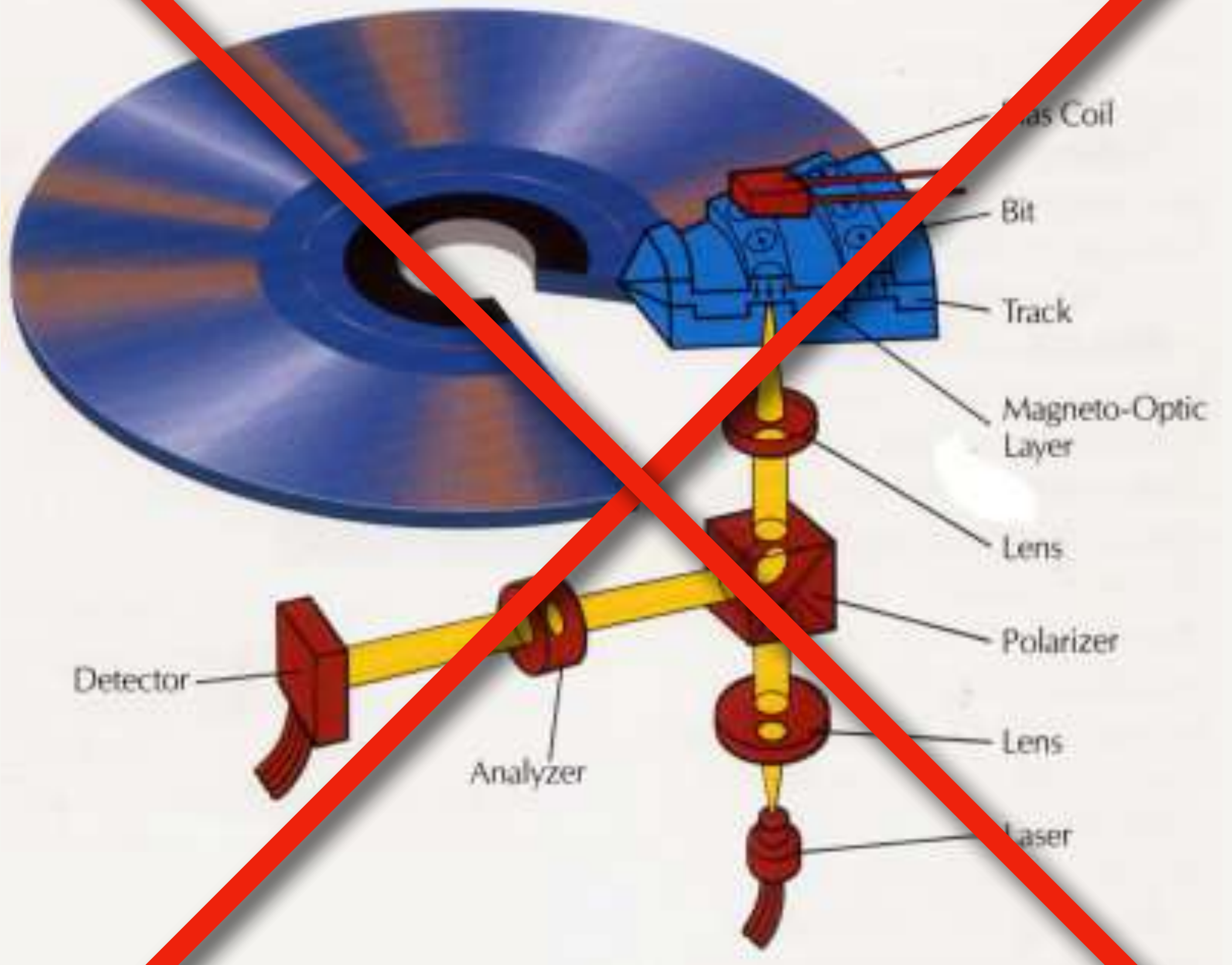

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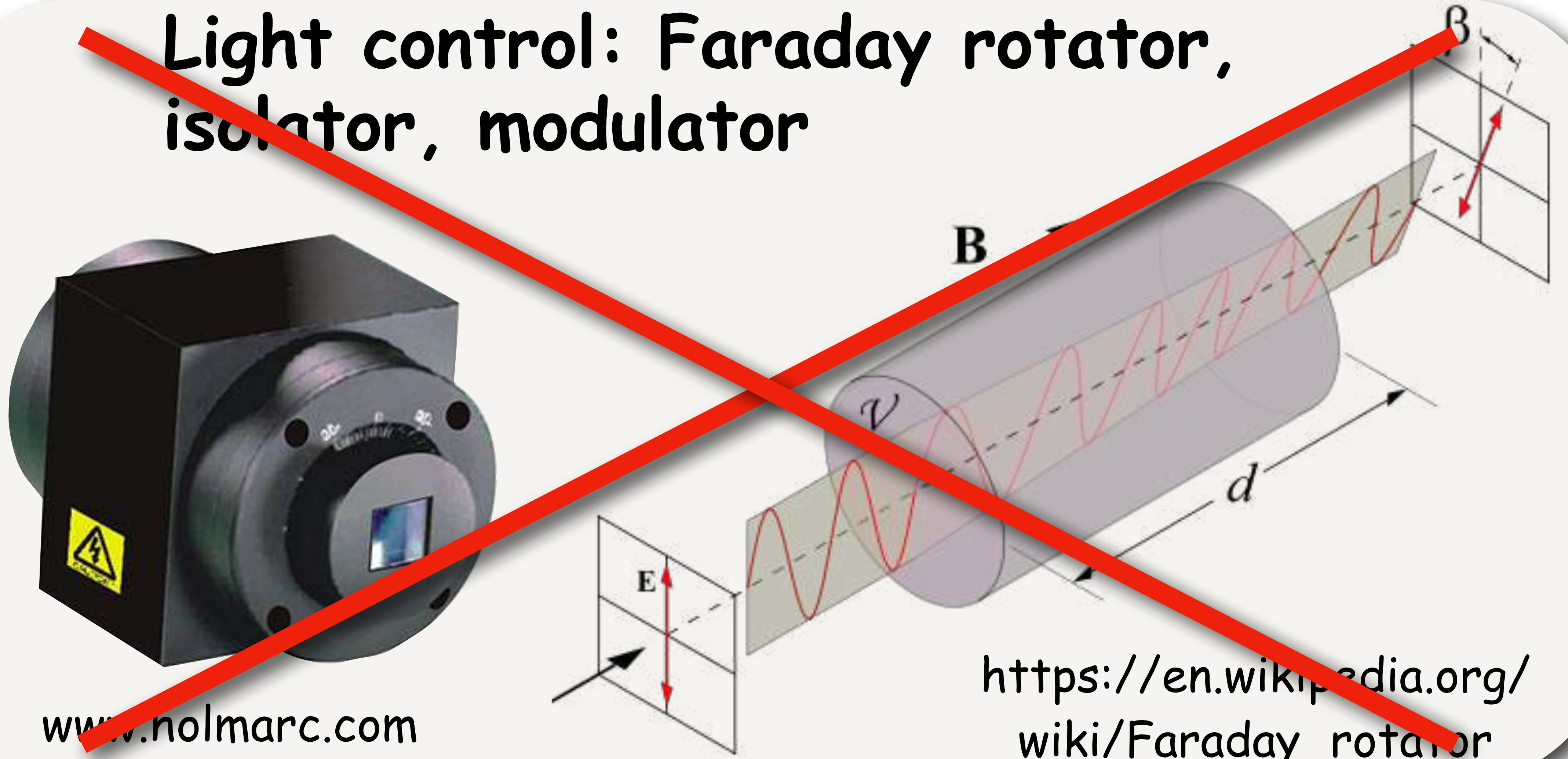

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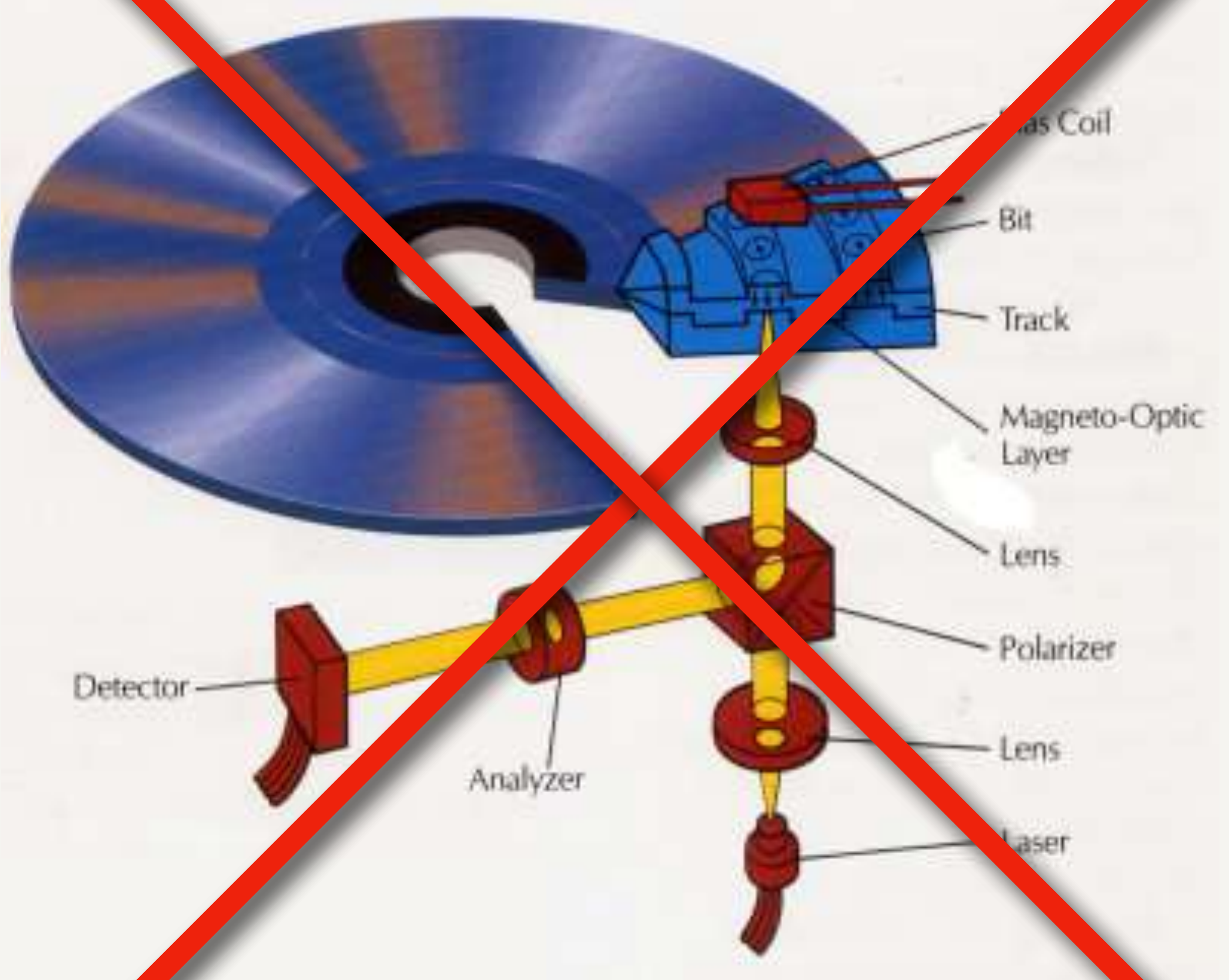

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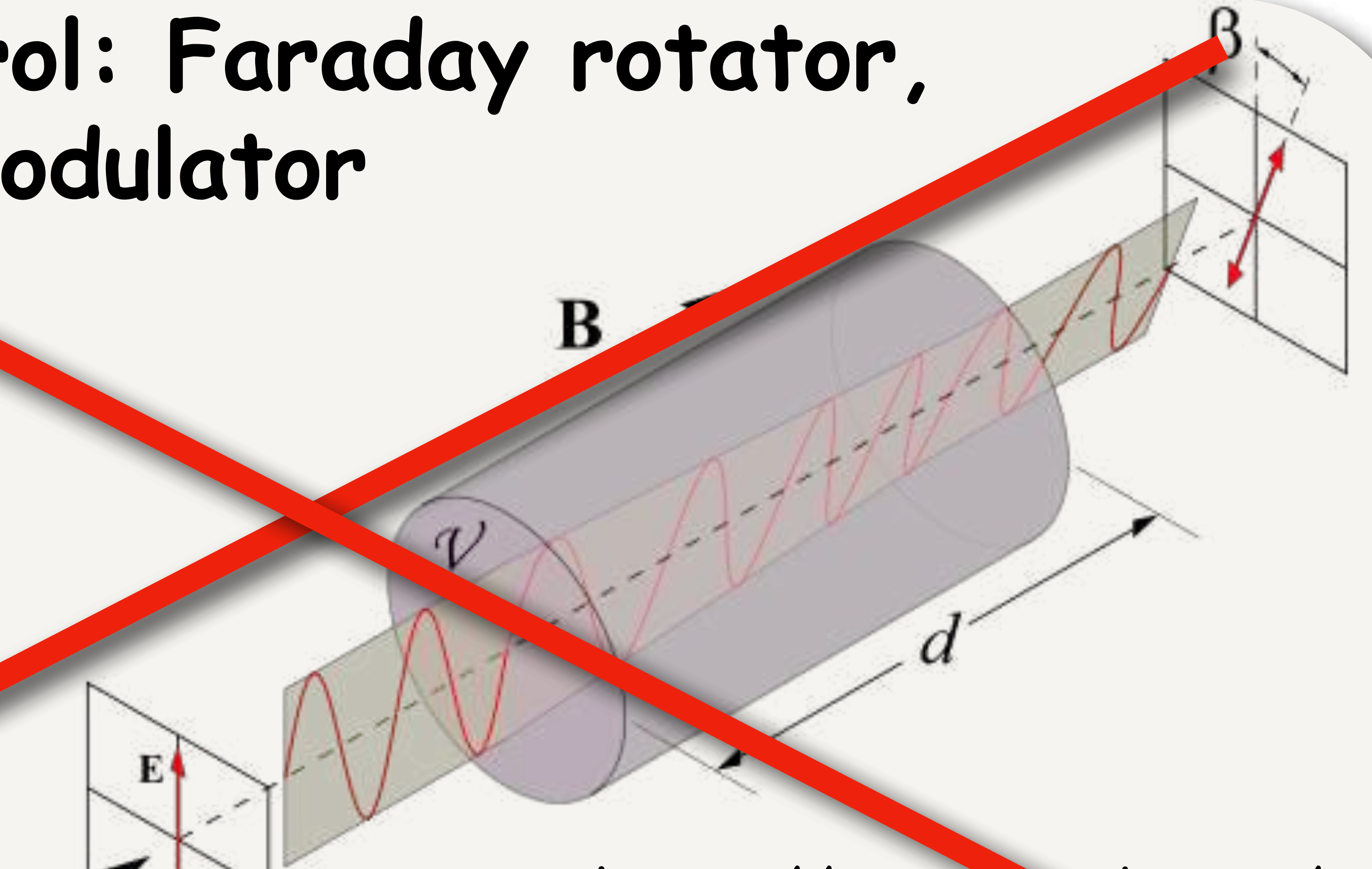

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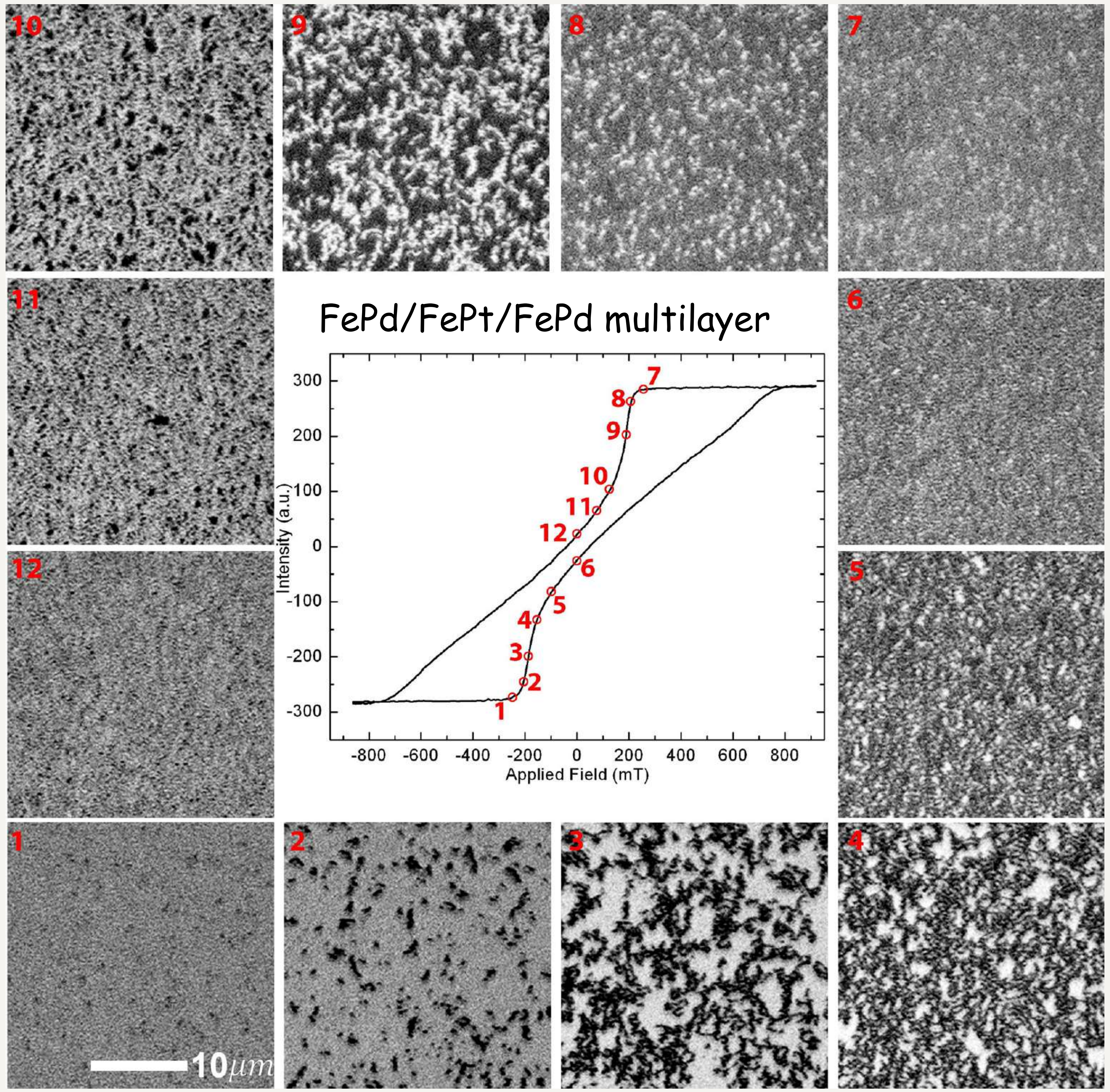
Magnetometry and domain imaging



From: I. Soldatov, R.S., J. Appl. Phys. 122, 153906 (2017)

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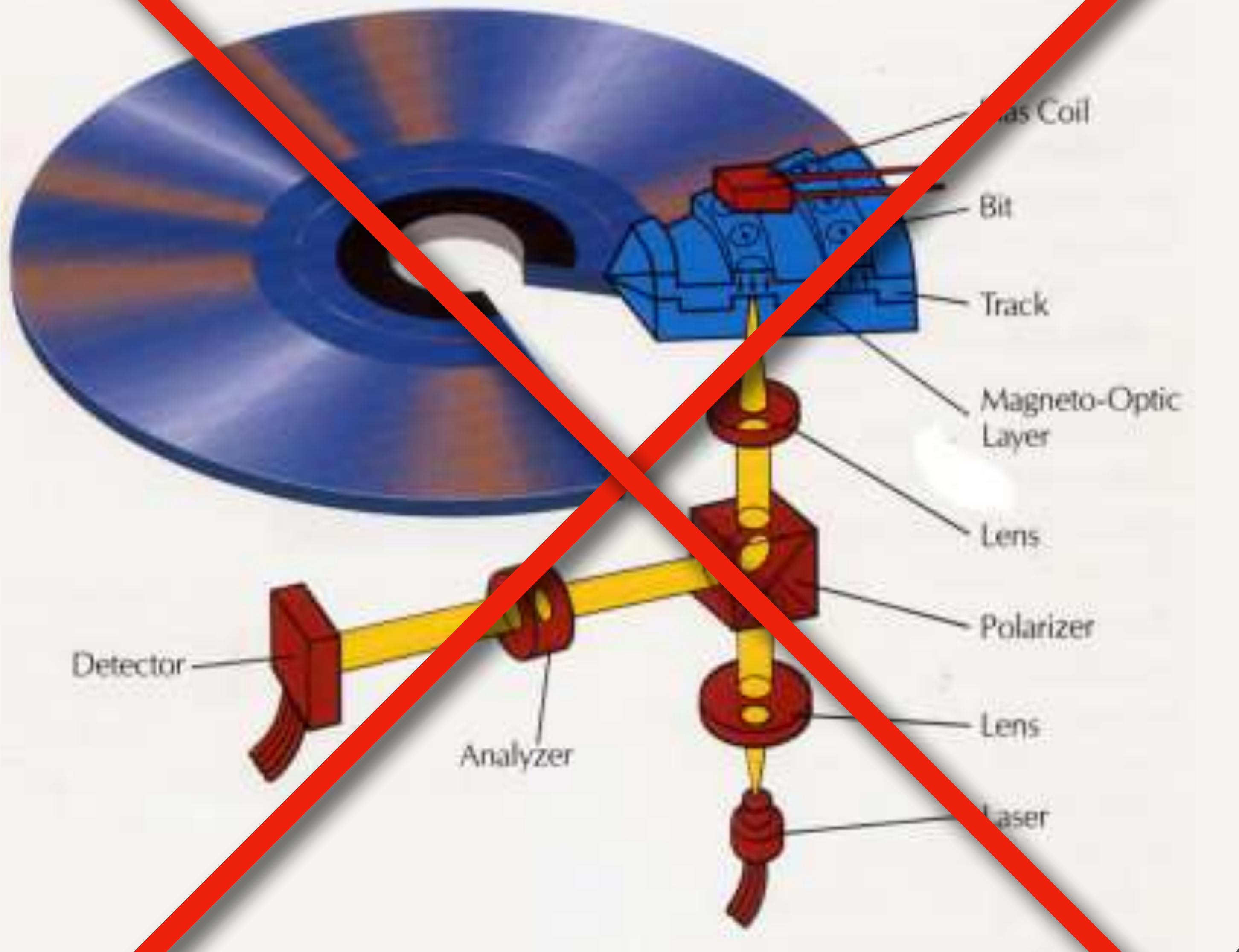

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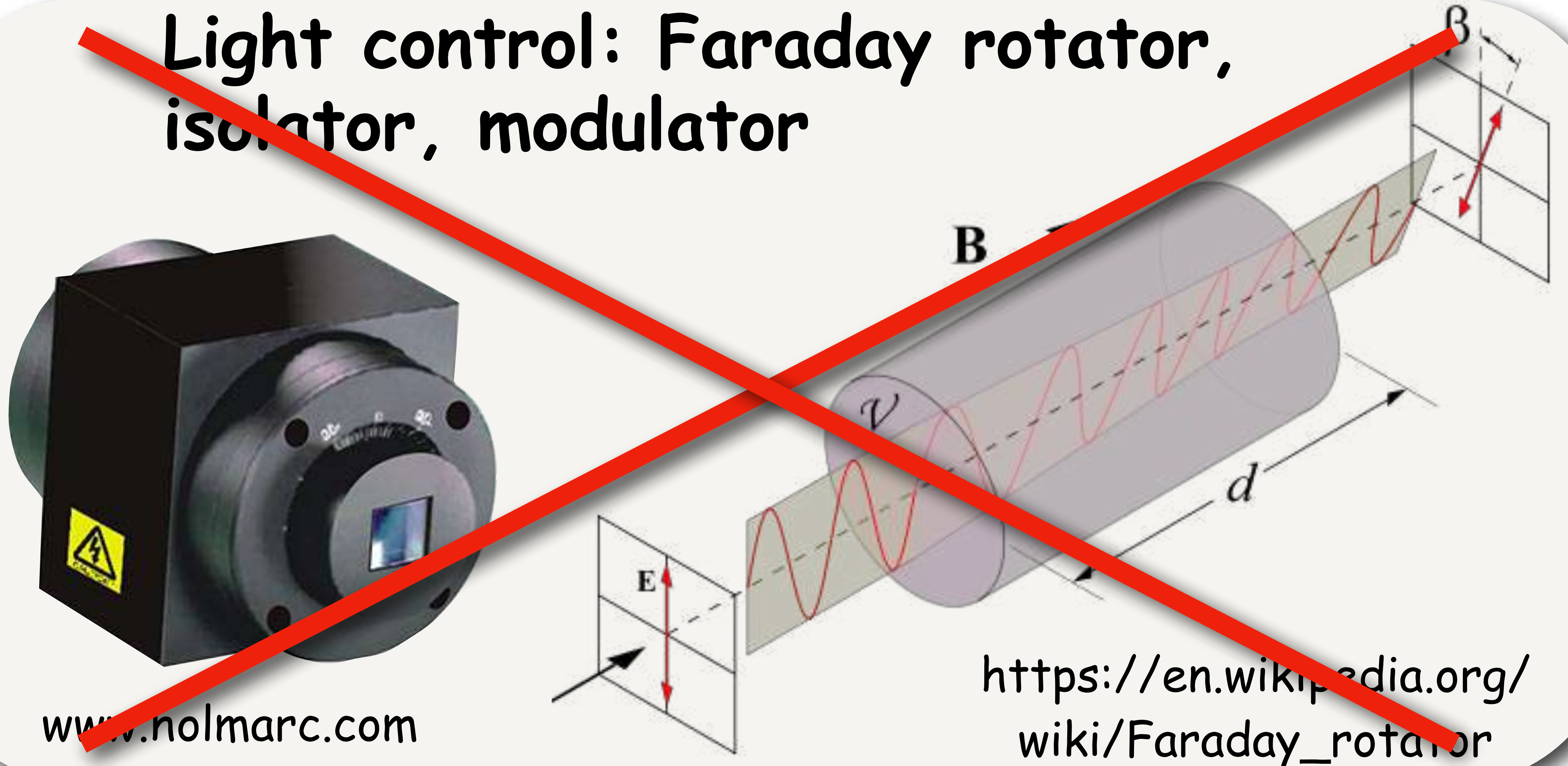

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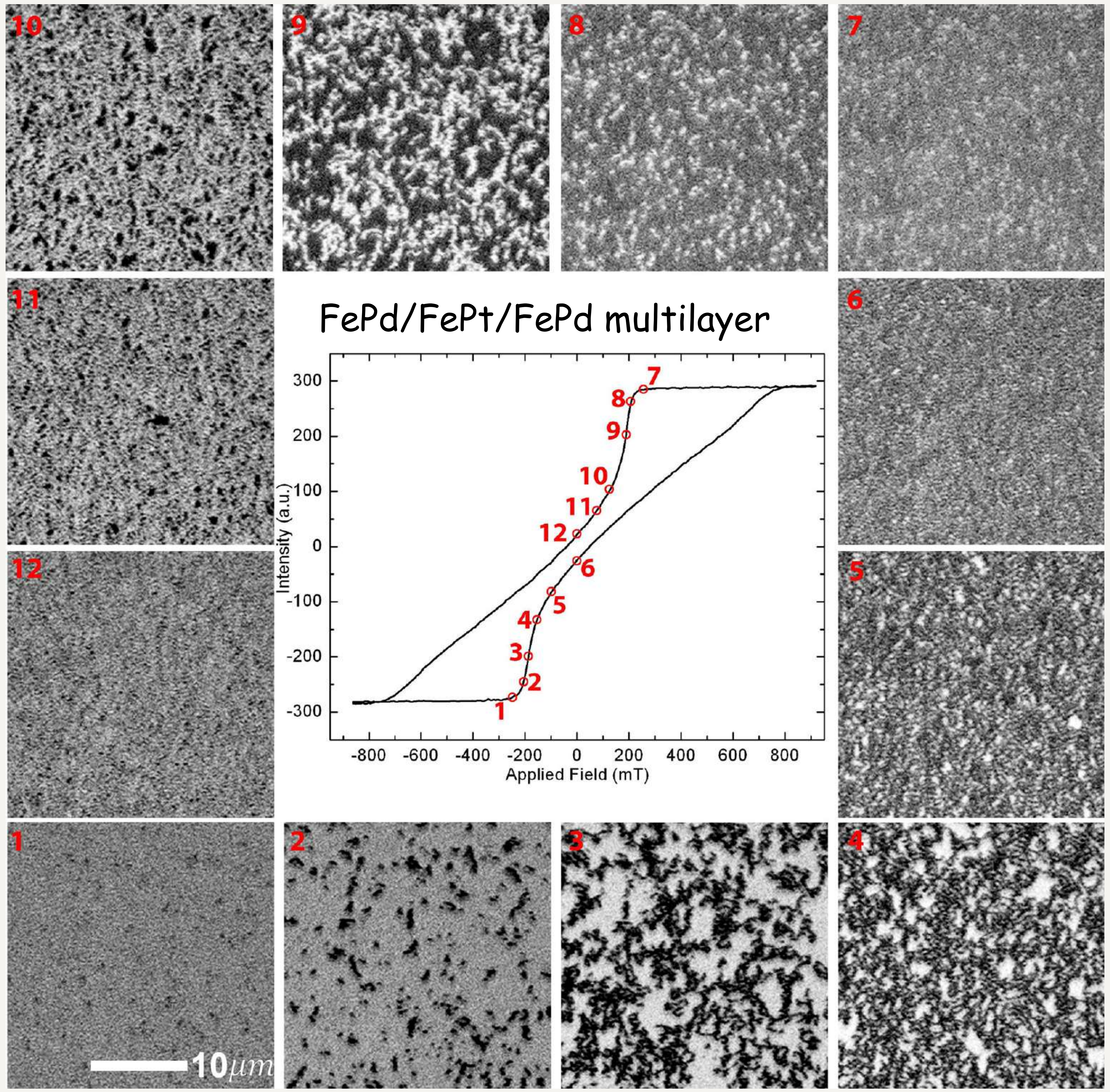

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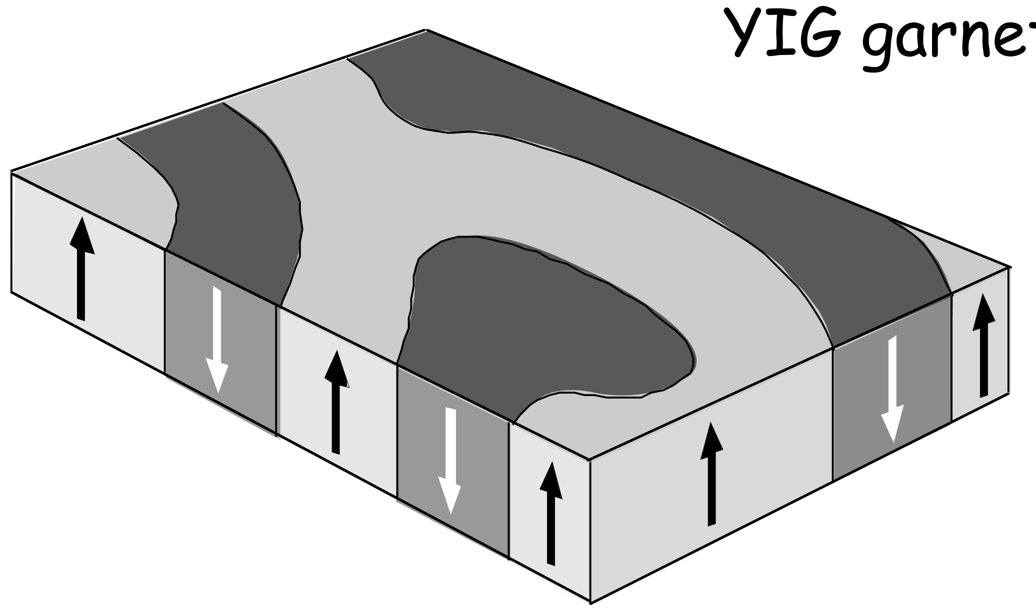
Magneto-Optical effects (for domain imaging)

Transmission:

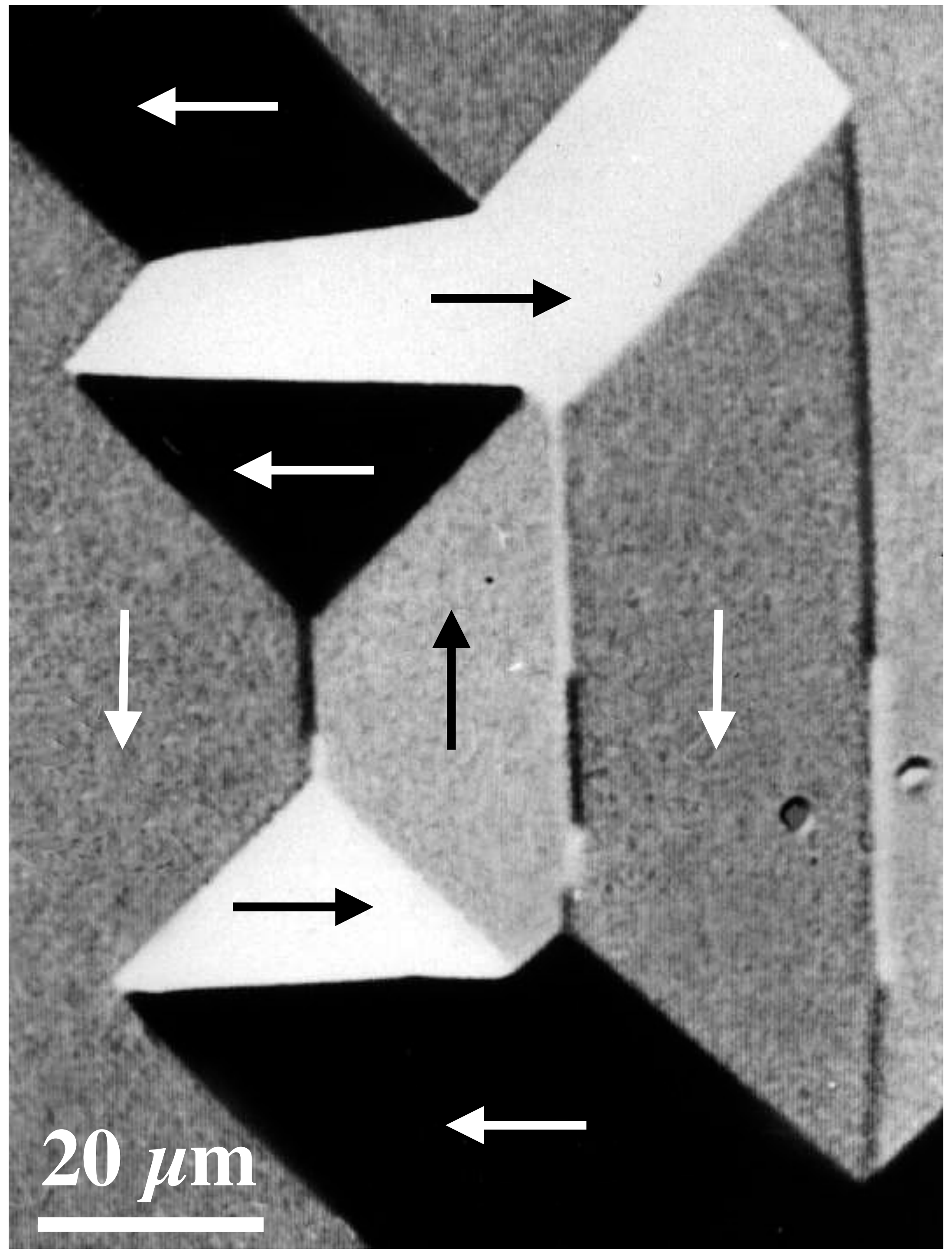
Faraday effect

YIG garnet

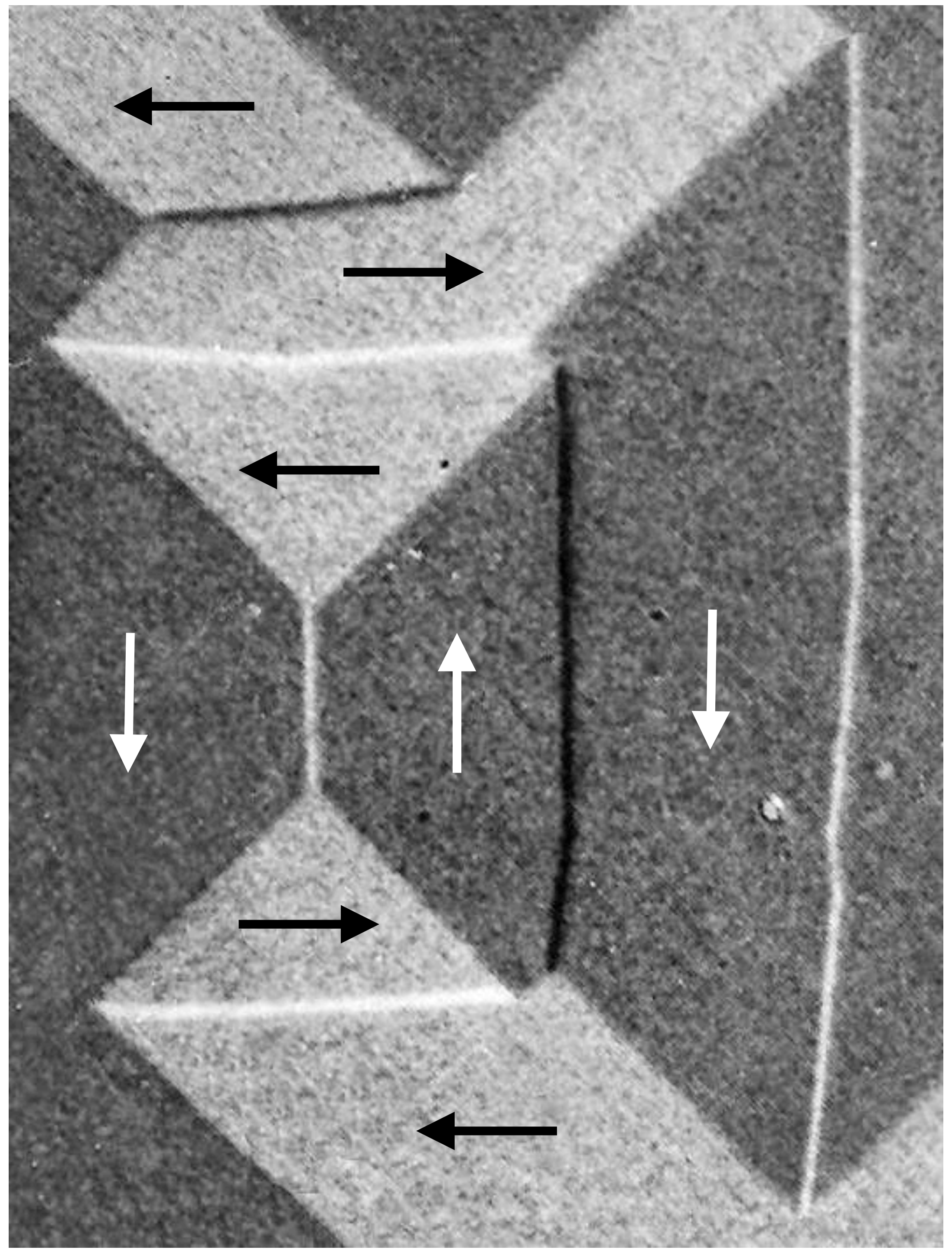


Reflection:

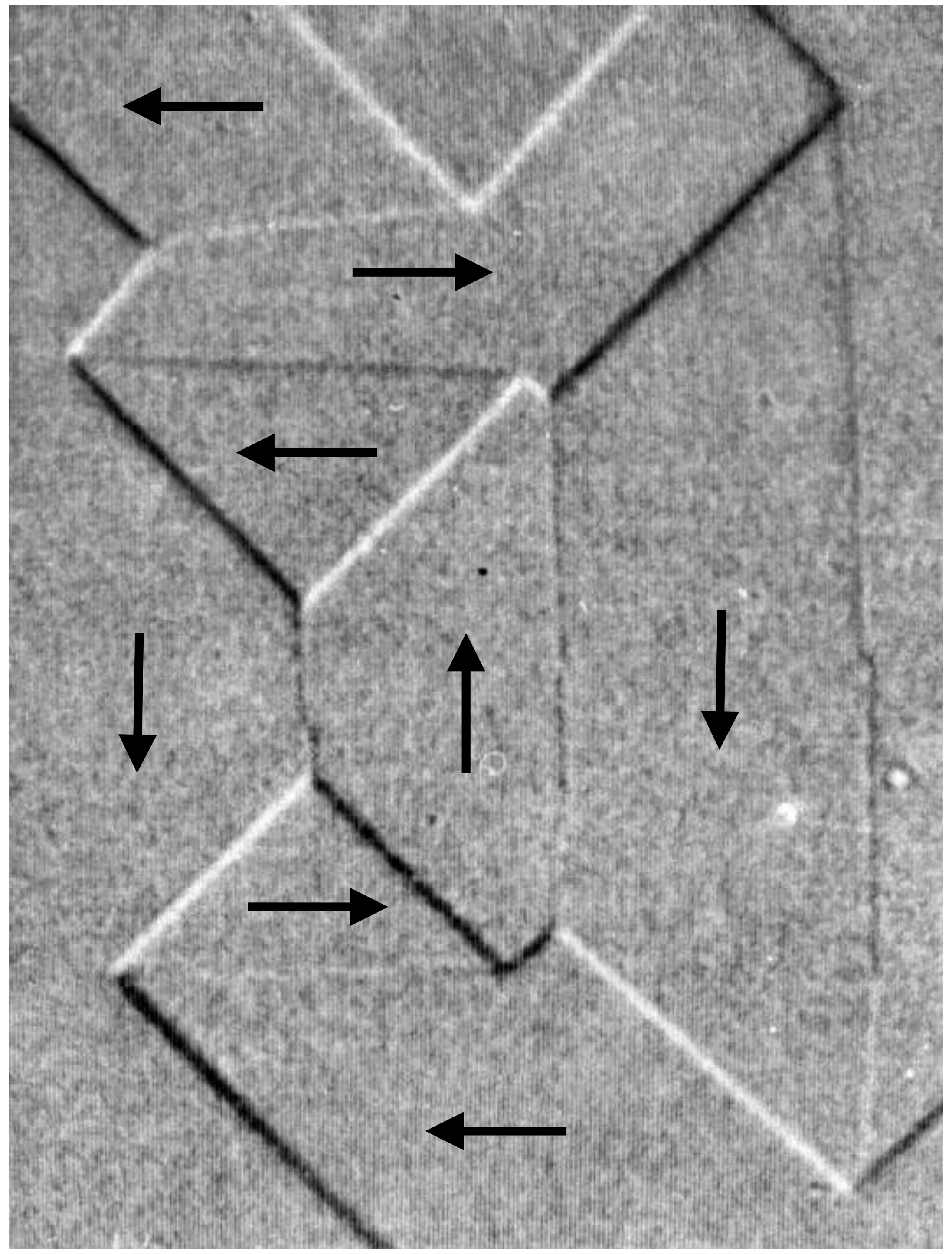
Kerr effect



Voigt & Gradient effect



Gradient effect



Iron sheet, (100) surface

From: W. Kuch, R.S., P. Fischer and U. Hillebrecht:
Magnetic Microscopy of Layered Structures. Springer (2015)

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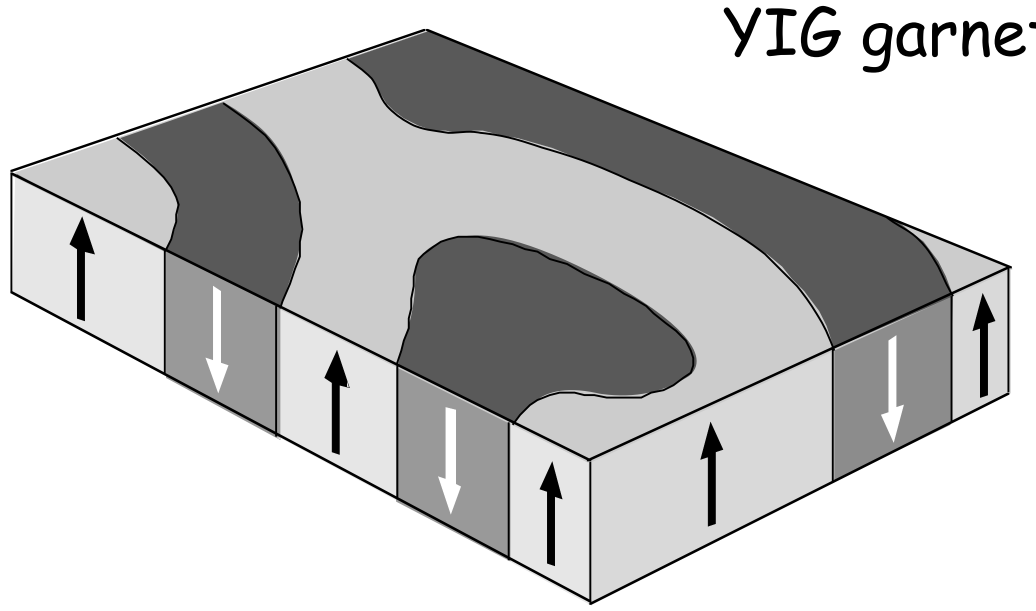

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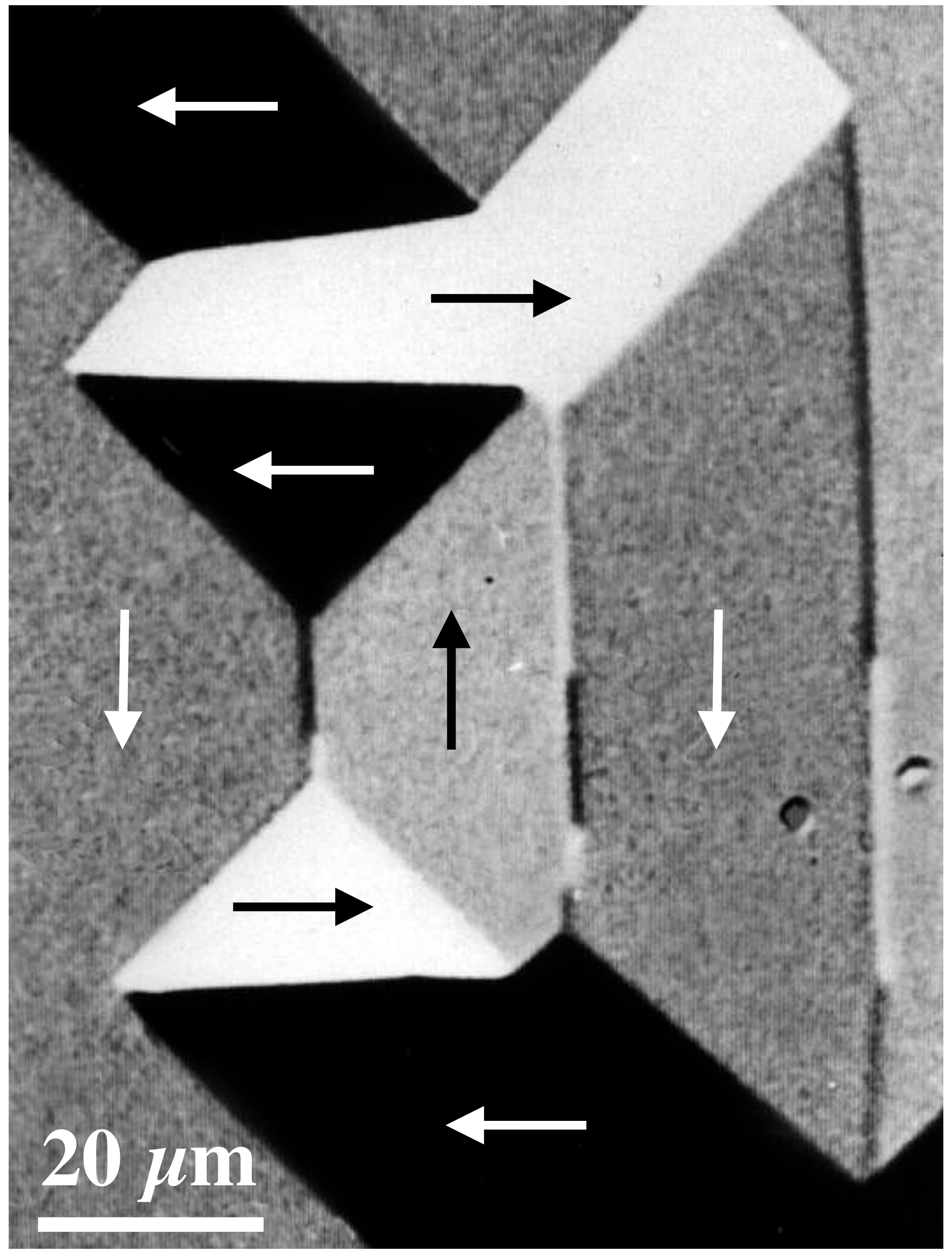

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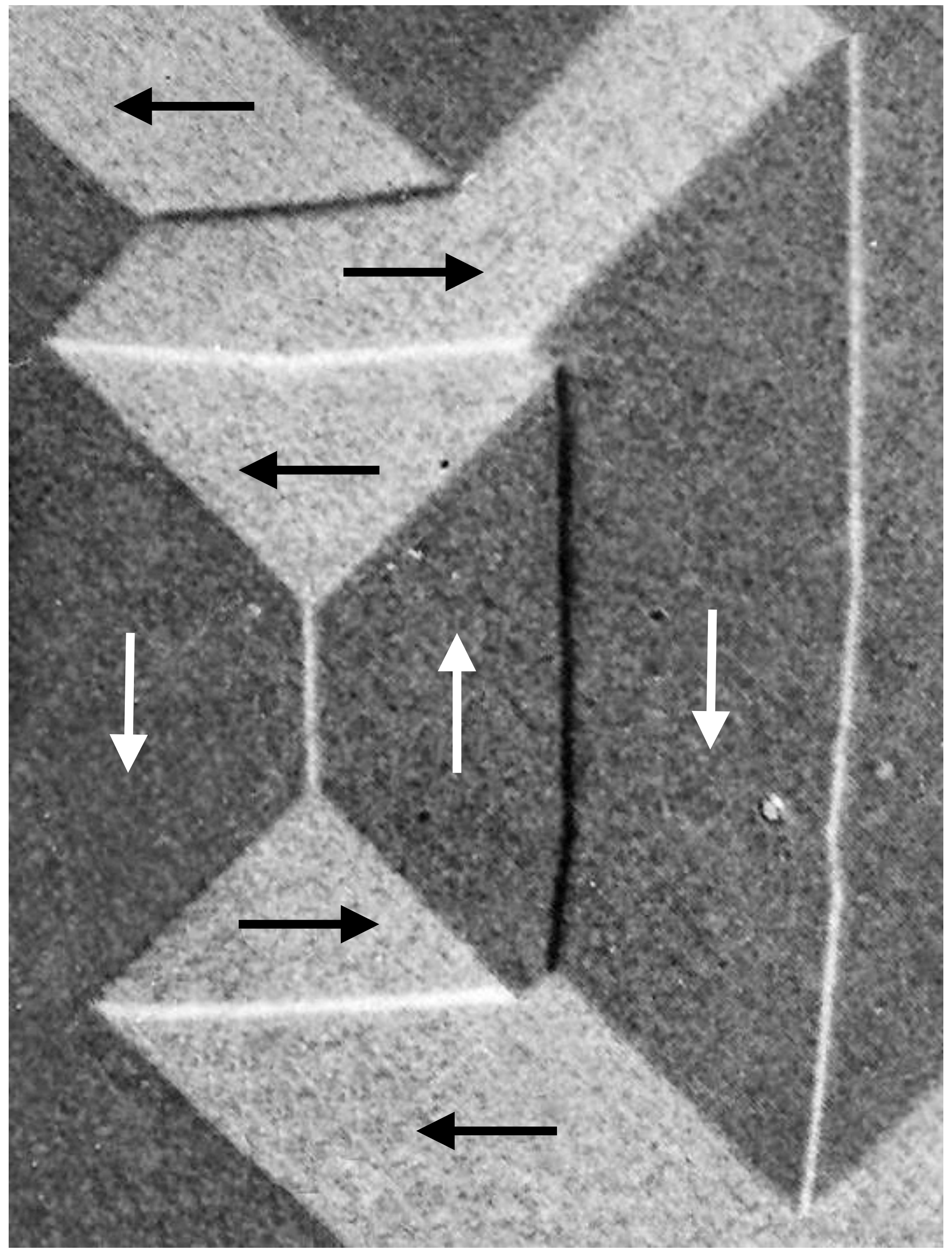

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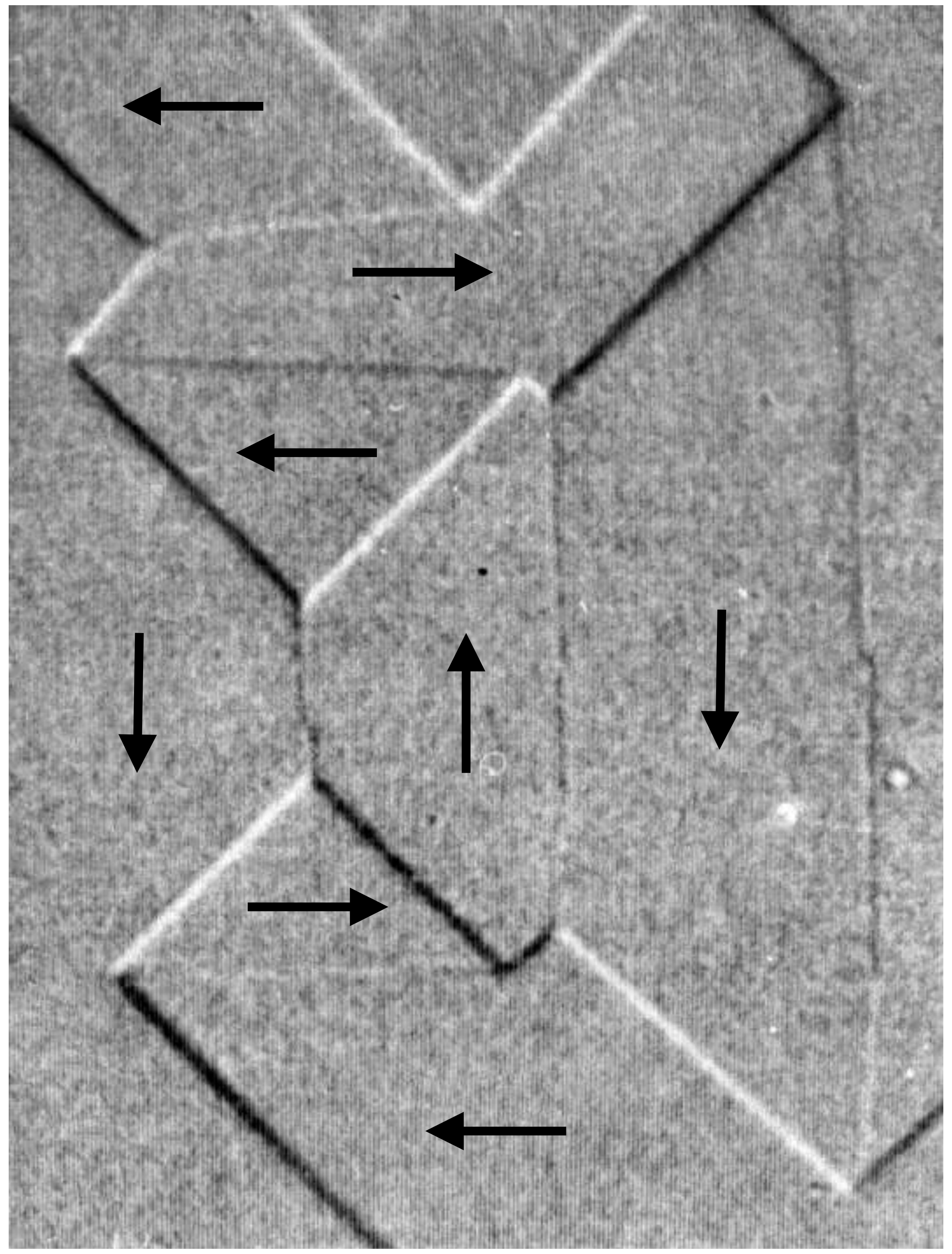

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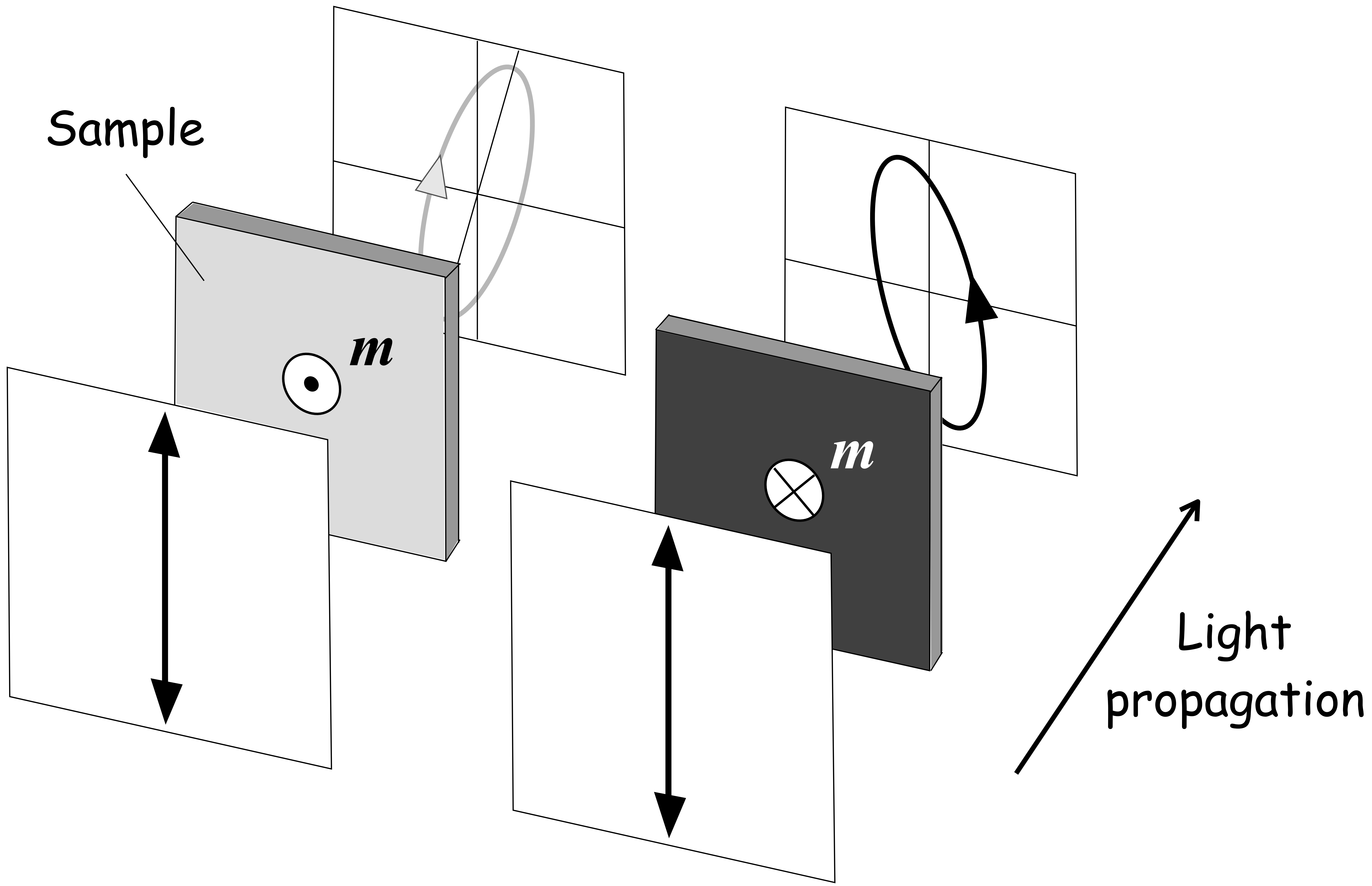
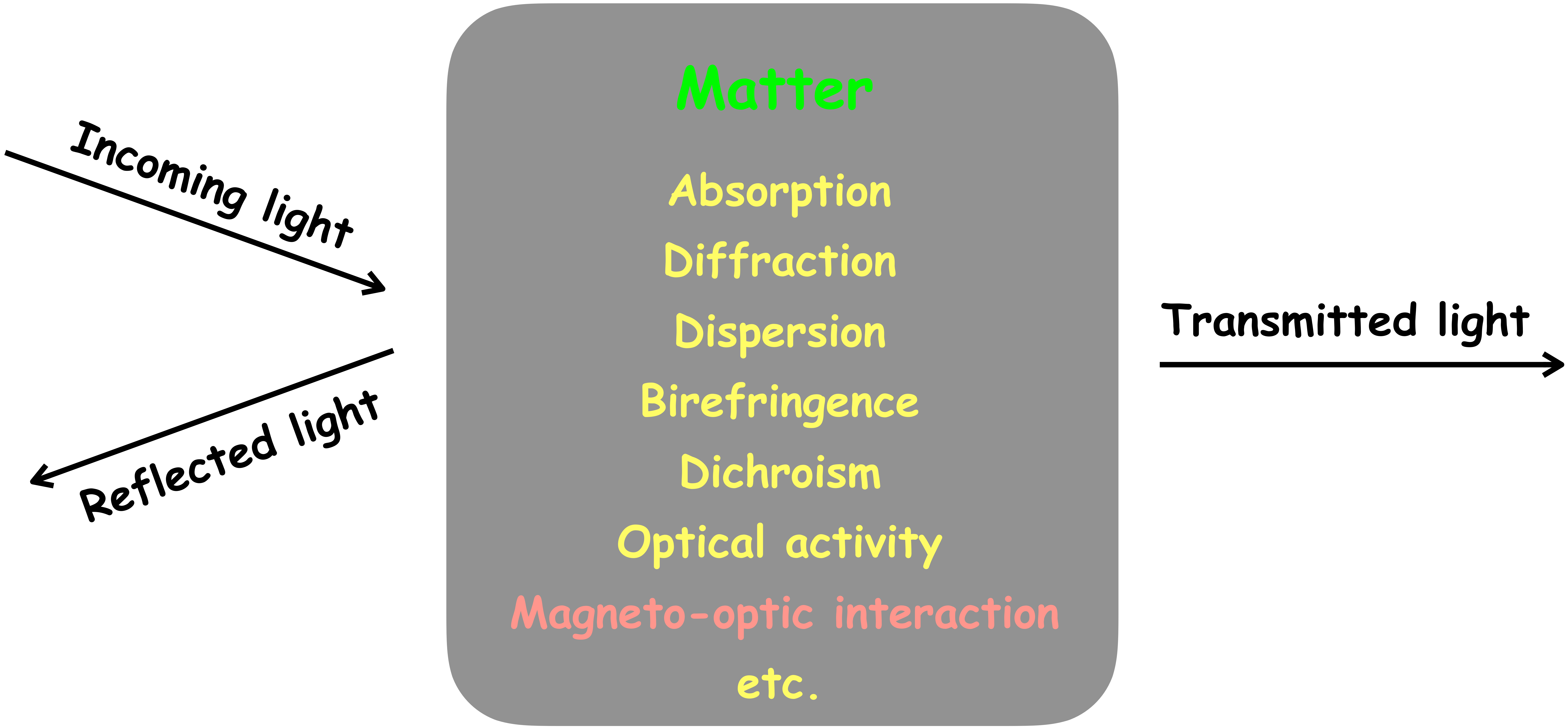
Magneto-Optical effects

Commonality of all those mo effects:
they lead to transformation of linearly polarized light into rotated, elliptically polarized light in dependence on magnetization direction



Light-matter interaction



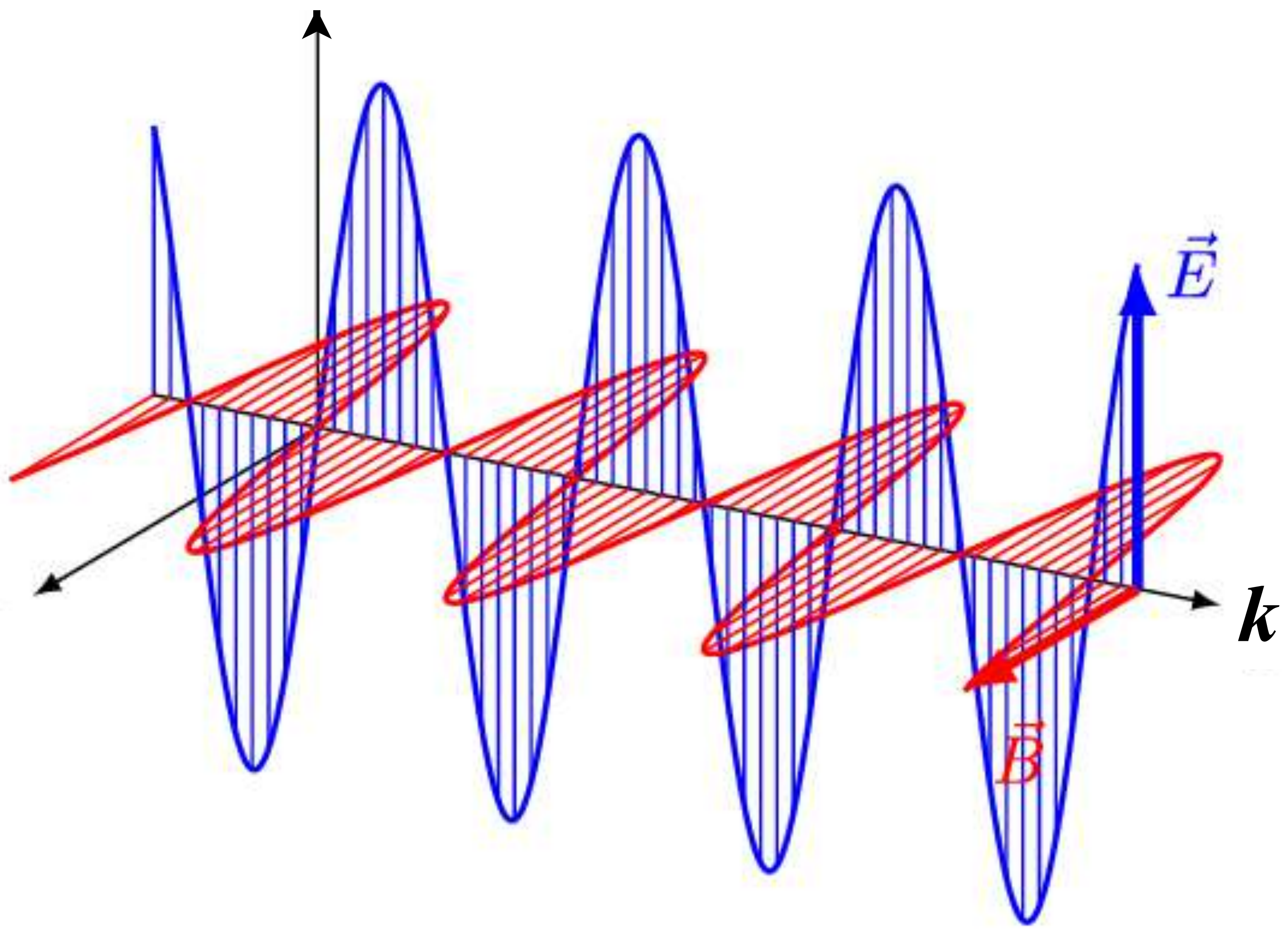
Contents

Theoretical

Practical

1.
Optical Basics

Light is a transverse electromagnetic wave



https://de.wikipedia.org/wiki/Datei:EM-Wave.gif

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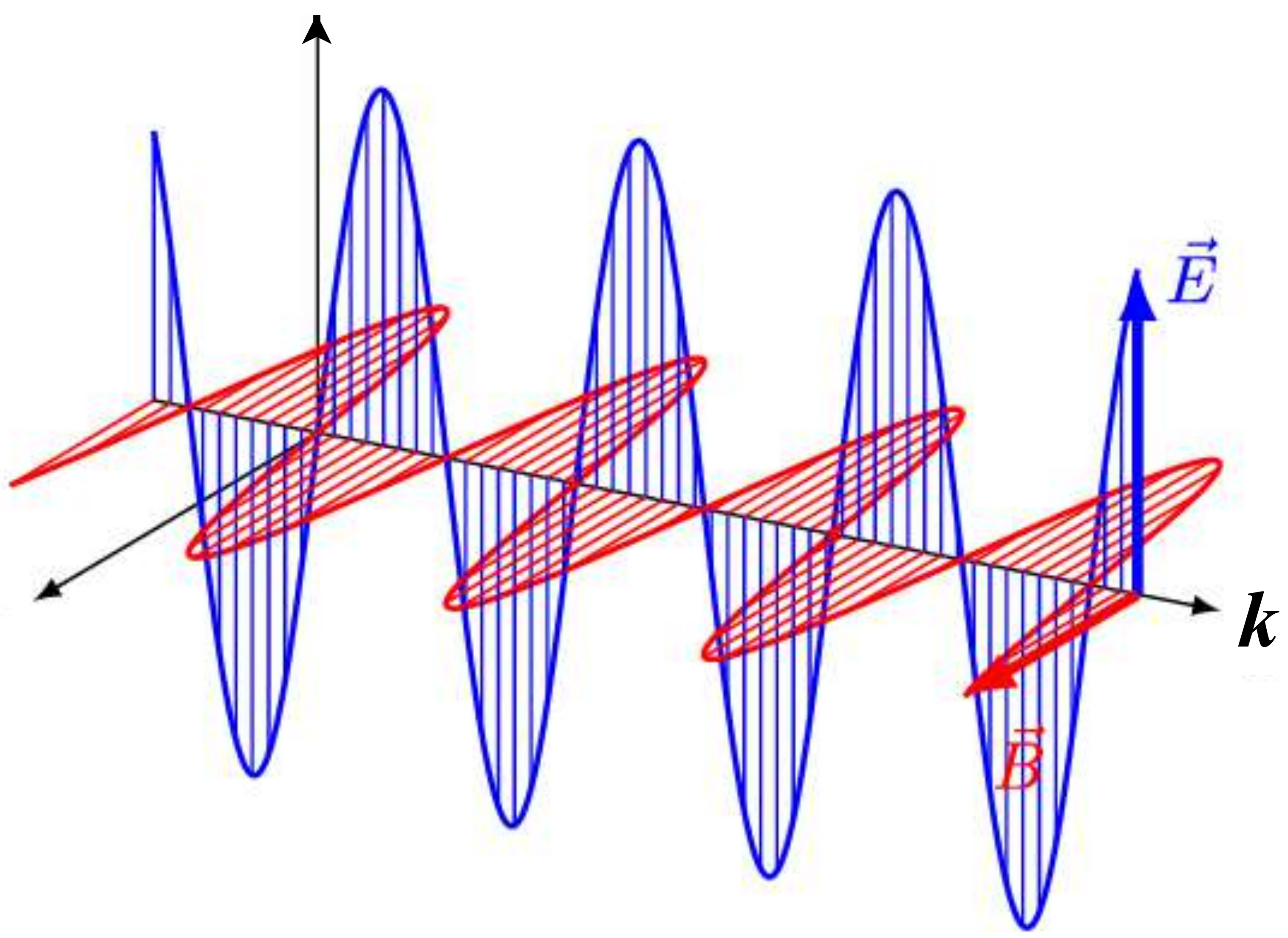

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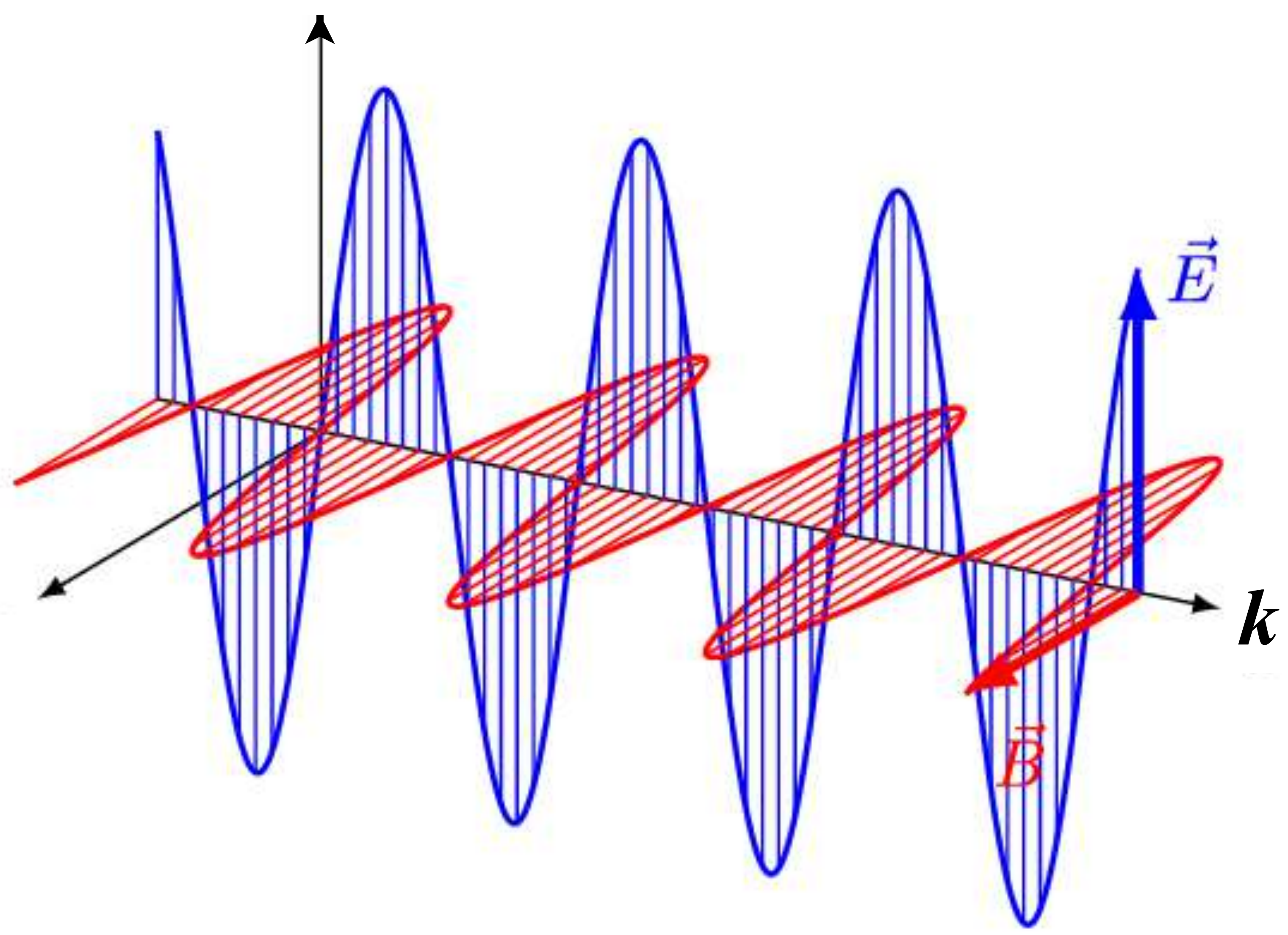

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Electrodynamic theory

Maxwell equations

$$\mathrm{div}\boldsymbol{D} = \nabla\cdot\boldsymbol{D} = \rho$$
$$\mathrm{div}\boldsymbol{B} = \nabla\cdot\boldsymbol{B} = 0$$
$$\mathrm{rot}\boldsymbol{E} = \nabla\times\boldsymbol{E} = -\dot{\boldsymbol{B}}$$
$$\mathrm{rot}\boldsymbol{H} = \nabla\times\boldsymbol{H} = \boldsymbol{j} + \dot{\boldsymbol{D}}$$

Material equations

$$\boldsymbol{D} = \epsilon_0\epsilon_r\boldsymbol{E} = \epsilon_0\boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{B} = \mu_0\mu_r\boldsymbol{H} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$
$$\boldsymbol{j} = \sigma\boldsymbol{E}$$

∇ denotes 3-dimensional gradient operator,

$\nabla\cdot$ denotes divergence operator,

$\nabla\times$ denotes curl operator

$\boldsymbol{E}$: electric field
$\boldsymbol{D}$: displacement field
$\boldsymbol{H}$: magnetic field
$\boldsymbol{B}$: magnetic induction
$\boldsymbol{M}$: magnetization
$\boldsymbol{P}$: electric polarization
$\boldsymbol{j}$: electric current density
ρ : electric charge density
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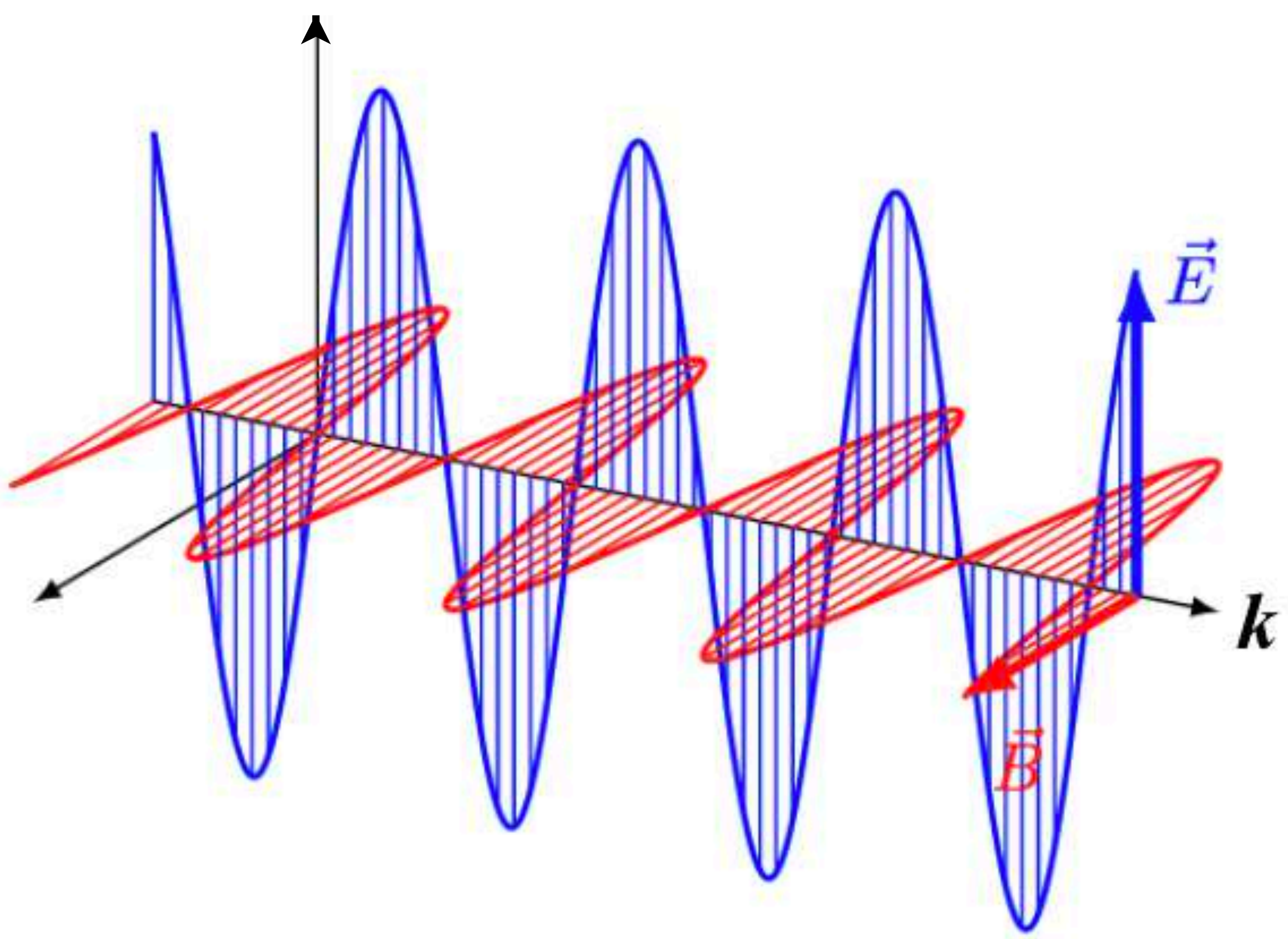

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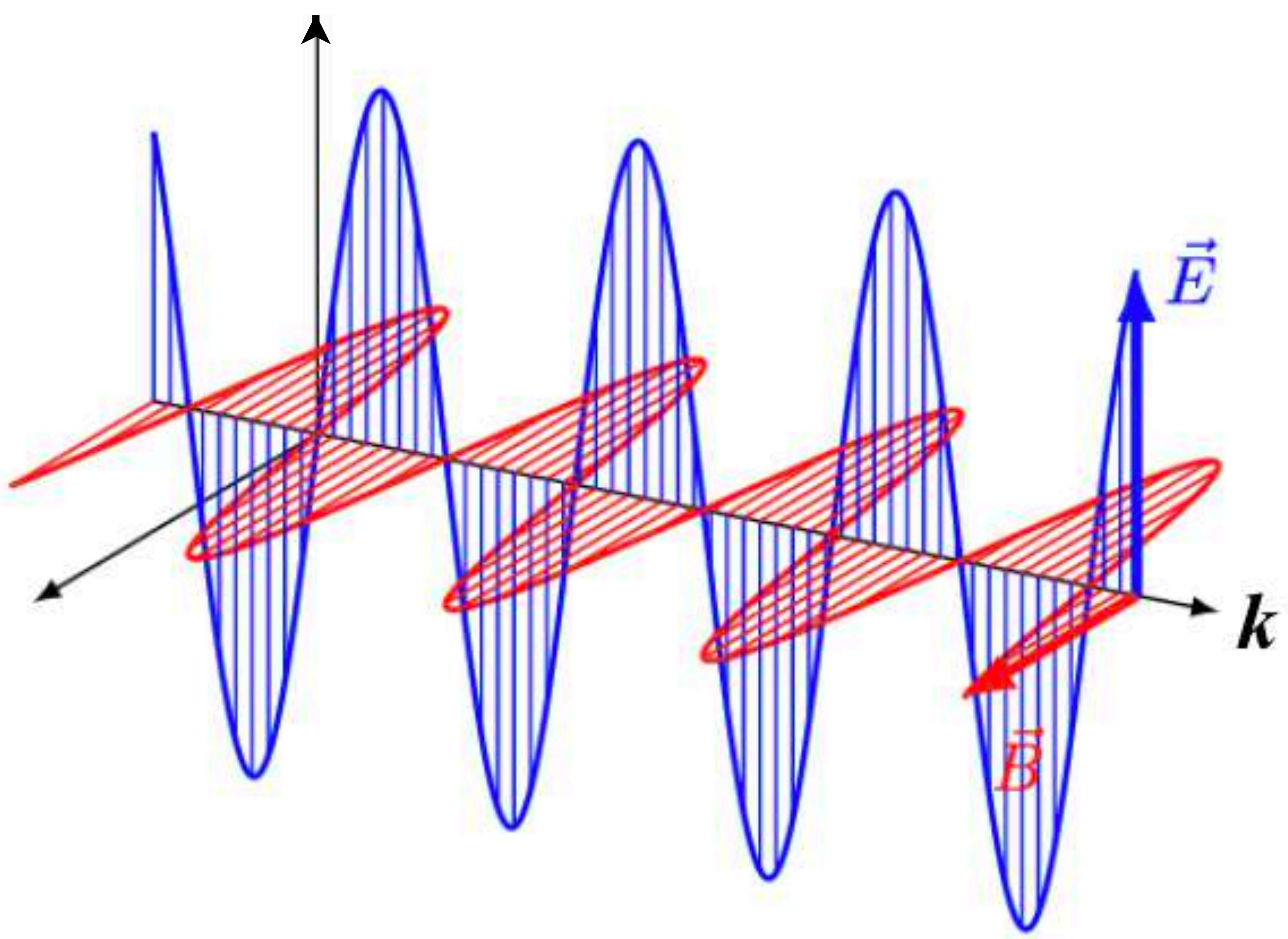

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$$\Downarrow$$

$$\nabla^2\boldsymbol{E} = -\mu_0\mu_r\left(\sigma\frac{\partial\boldsymbol{E}}{\partial t} + \epsilon_0\epsilon_r\frac{\partial^2\boldsymbol{E}}{\partial t^2}\right) \Longrightarrow \boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}^0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)]$$

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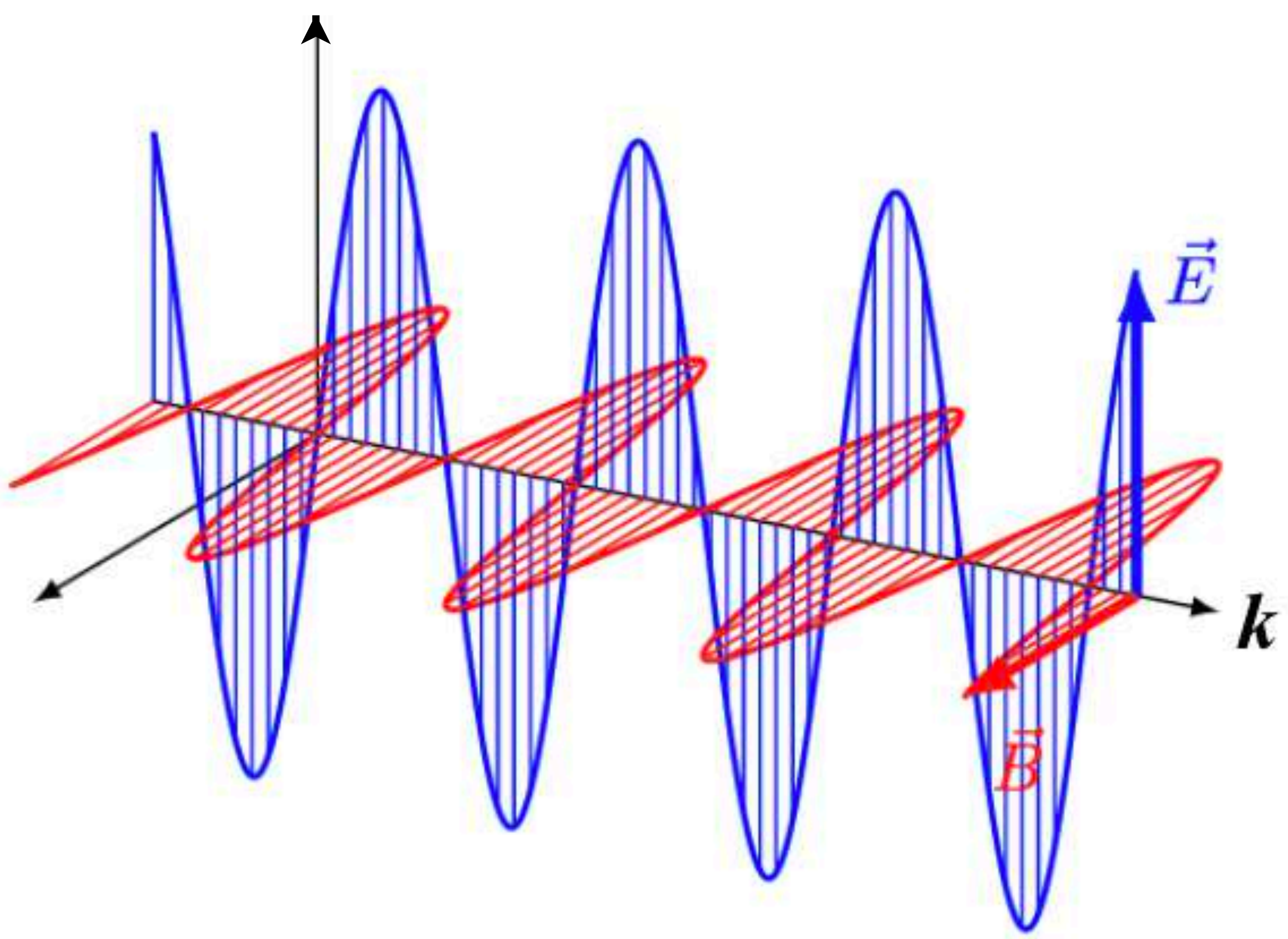

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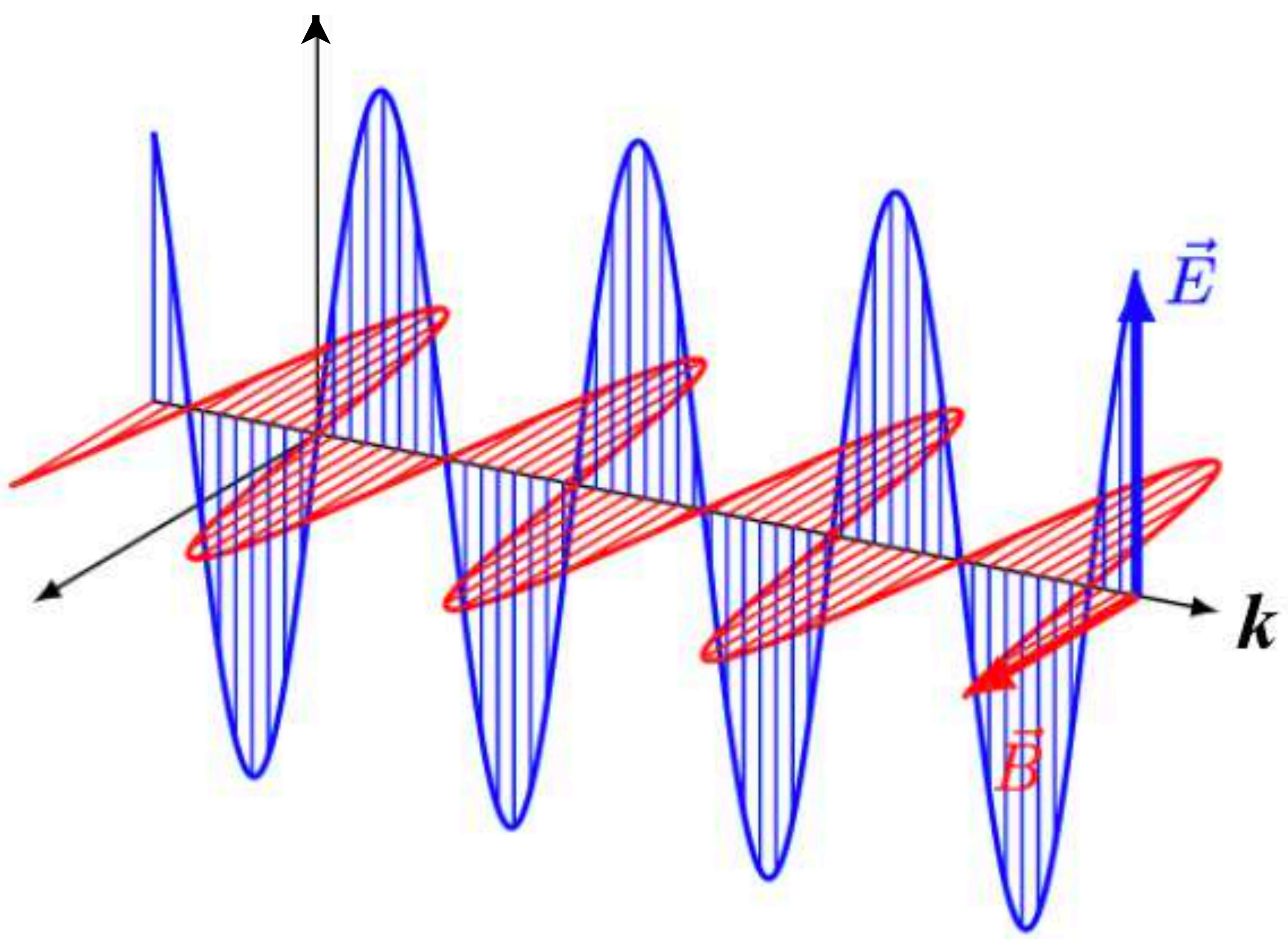

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Wave equation

Plane-wave solution
(harmonic in time and space)

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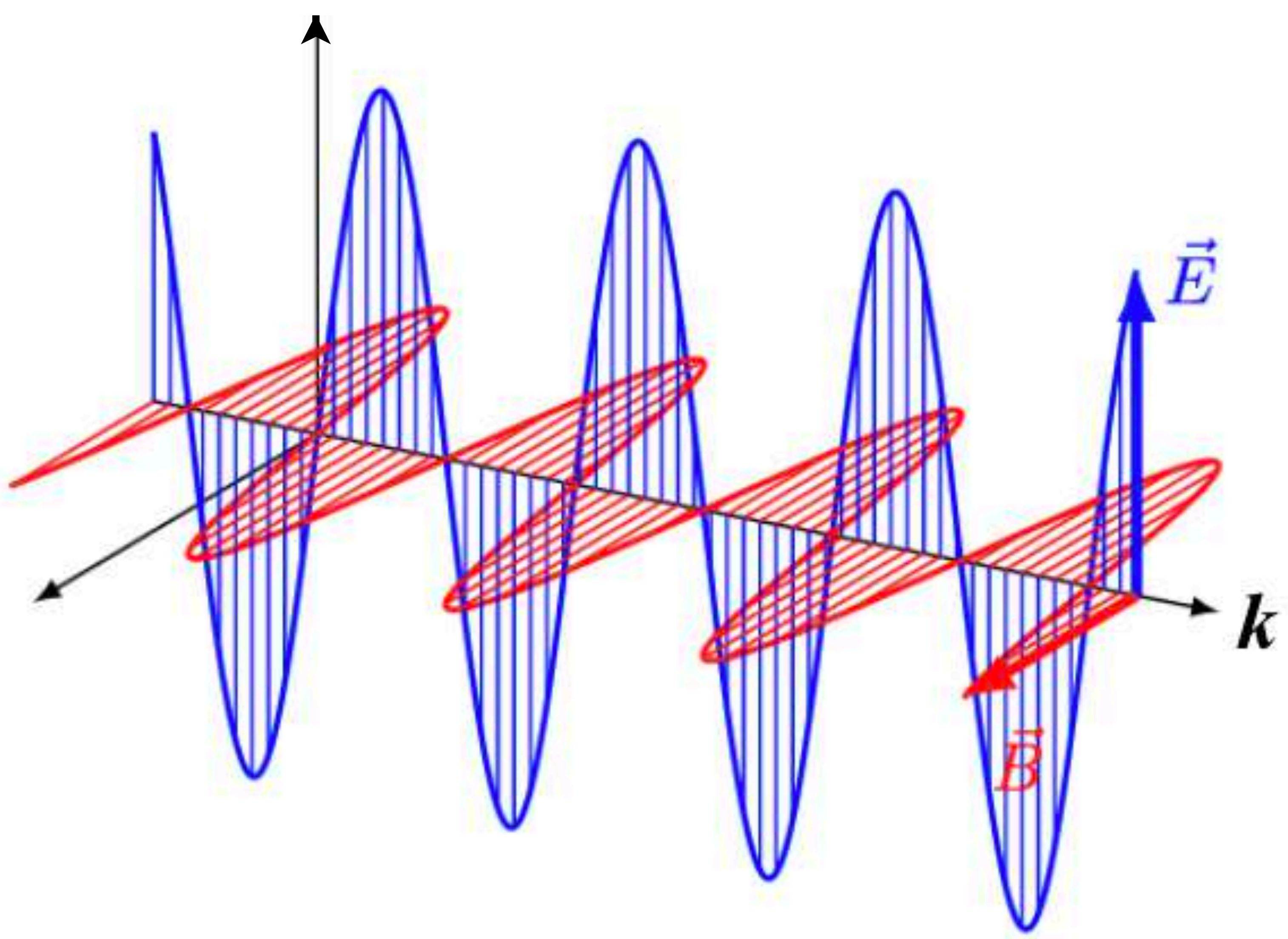

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$$\boxed{\exp(\mathrm{i}\Theta) = \cos\Theta + \mathrm{i}\sin\Theta}$$

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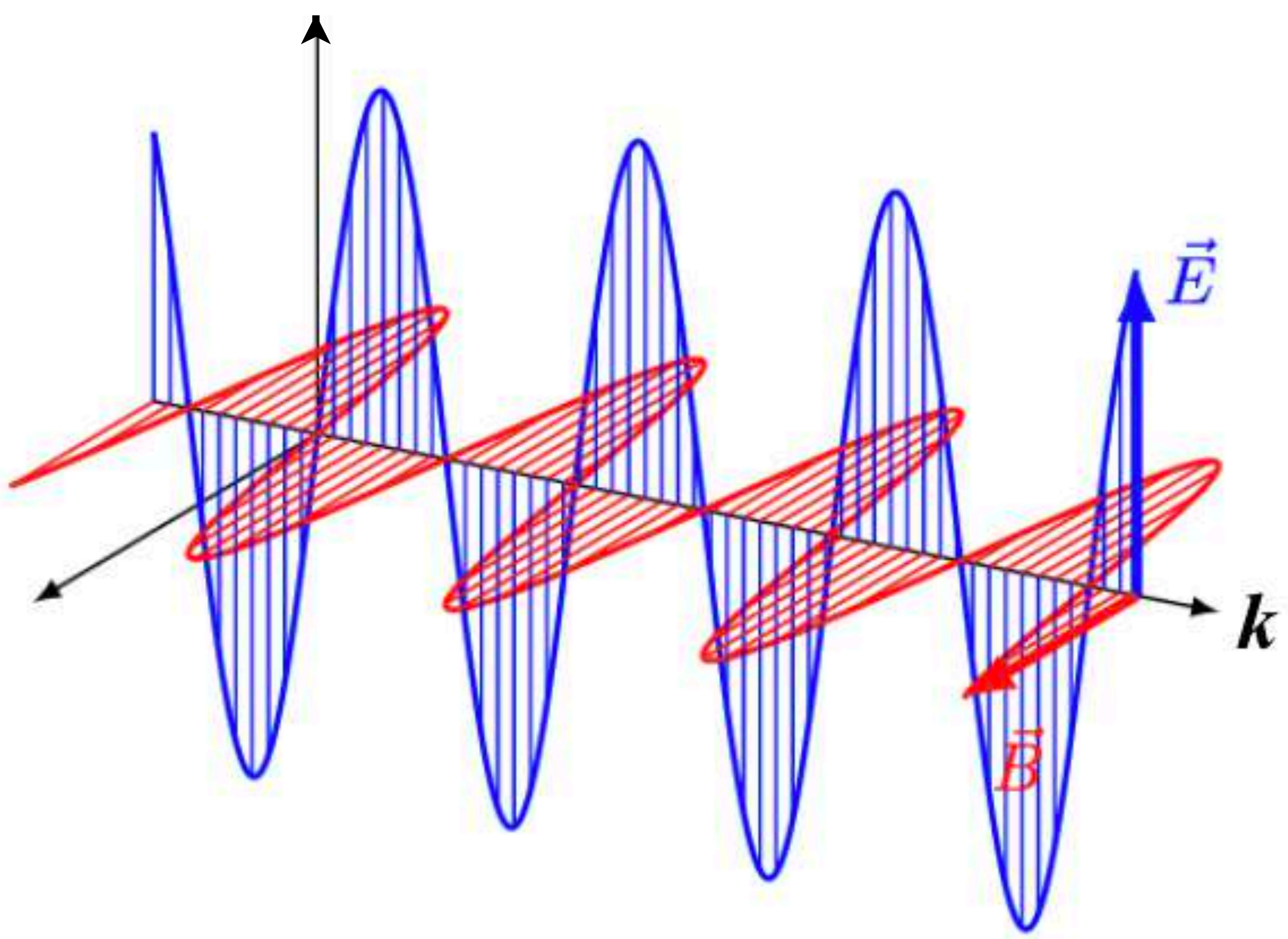

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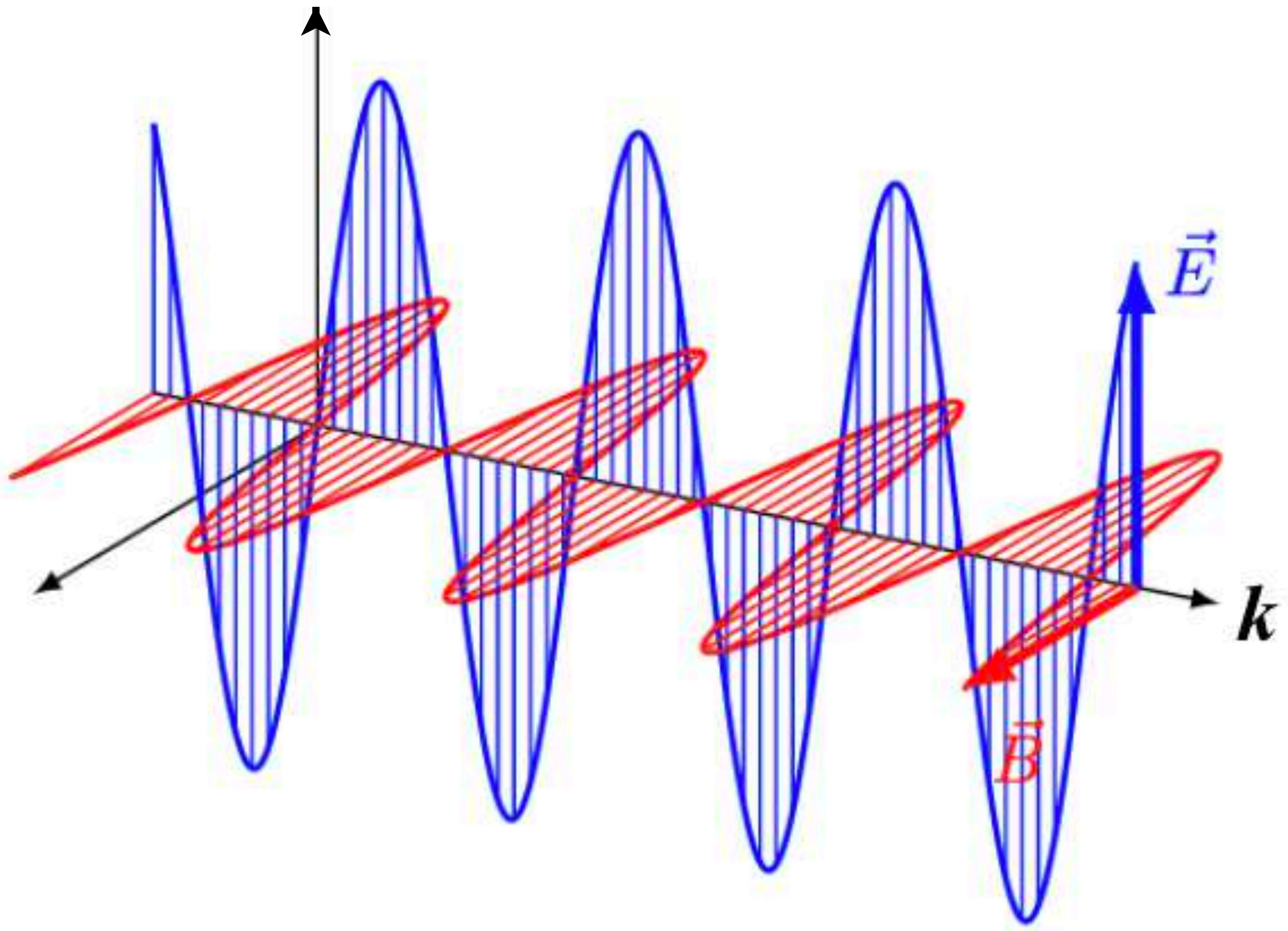
$$\nabla^2\boldsymbol{E} = -\mu_0\mu_r\left(\sigma\frac{\partial\boldsymbol{E}}{\partial t} + \epsilon_0\epsilon_r\frac{\partial^2\boldsymbol{E}}{\partial t^2}\right) \implies \boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}^0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)]$$

$$\underbrace{\nabla\times(\nabla\times\boldsymbol{B})}_{\cancel{\nabla(\nabla\cdot\boldsymbol{B})} - \nabla^2\boldsymbol{B}} = \mu_0\mu_r\nabla\times(\nabla\times\boldsymbol{H}) = \mu_0\mu_r\left[\sigma(\nabla\times\boldsymbol{E}) + \epsilon_0\epsilon_r\frac{\partial}{\partial t}(\nabla\times\boldsymbol{E})\right]$$

$$\Downarrow$$

$$\nabla^2\boldsymbol{B} = -\mu_0\mu_r\left(\sigma\frac{\partial\boldsymbol{B}}{\partial t} + \epsilon_0\epsilon_r\frac{\partial^2\boldsymbol{B}}{\partial t^2}\right) \implies \boldsymbol{B}(\boldsymbol{r},t) = \boldsymbol{B}^0\exp[\mathrm{i}(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)]$$

Light is a transverse electromagnetic wave



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Differential operators $\Longrightarrow$ algebraic fcts:

$$\nabla\cdot \Longrightarrow \mathrm{i}\boldsymbol{k}\cdot \;;\; \nabla\times \Longrightarrow \mathrm{i}\boldsymbol{k}\times \;;\; \frac{\partial}{\partial t} \Longrightarrow -\mathrm{i}\omega$$

$$\nabla\cdot\boldsymbol{E} = 0 \Longrightarrow \boldsymbol{k}\cdot\boldsymbol{E} = 0 \Longrightarrow \boldsymbol{k}\perp\boldsymbol{E}$$

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$$\nabla\times\boldsymbol{E} = -\dot{\boldsymbol{B}} \Longrightarrow \boldsymbol{k}\times\boldsymbol{E} = \omega\boldsymbol{B} \Longrightarrow \boldsymbol{E}\perp\boldsymbol{B}$$

Electrodynamic theory

Maxwell equations

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Assumption: Electrically neutral media, i.e. $\rho(\boldsymbol{r},t) = 0$

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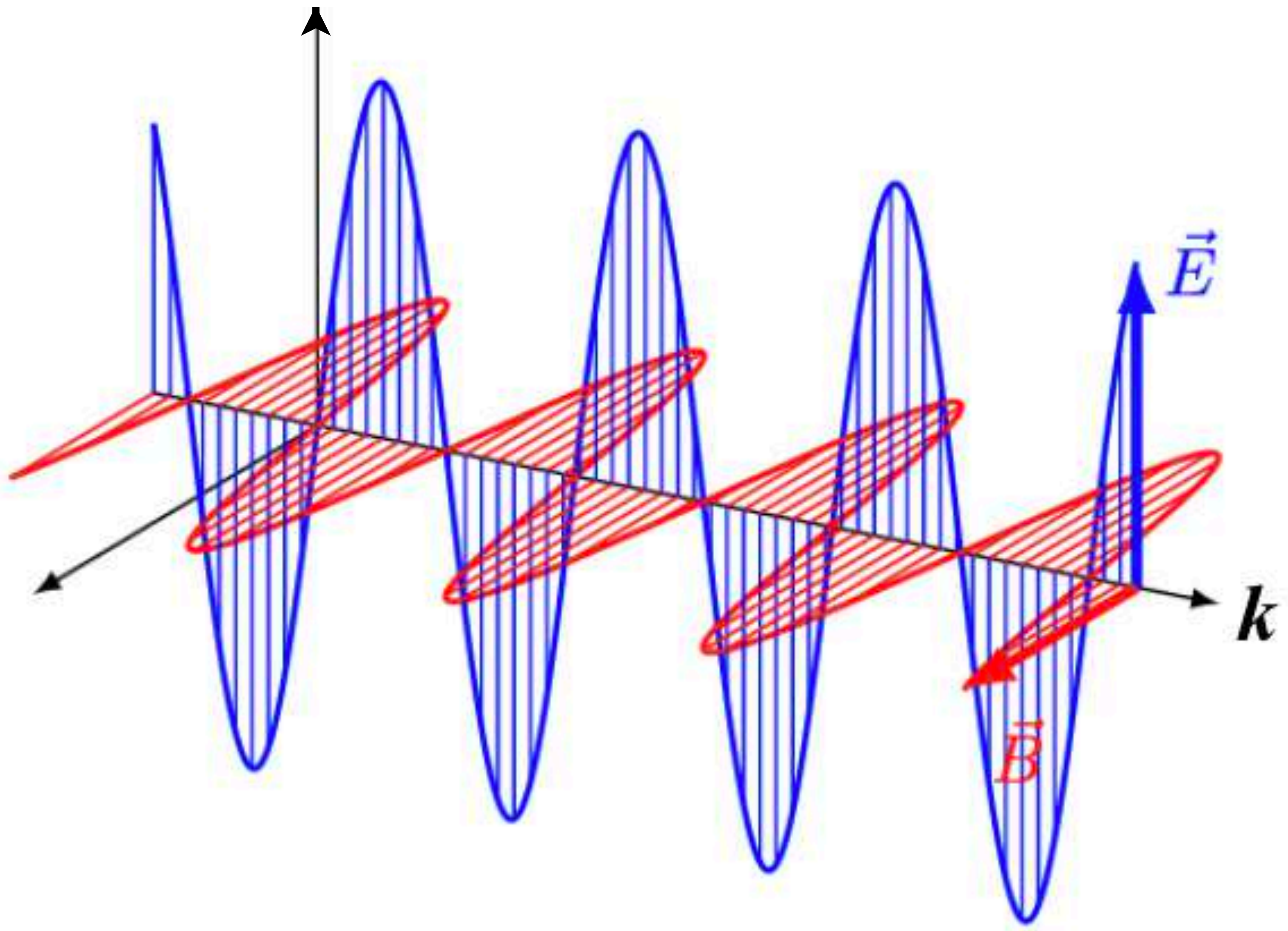

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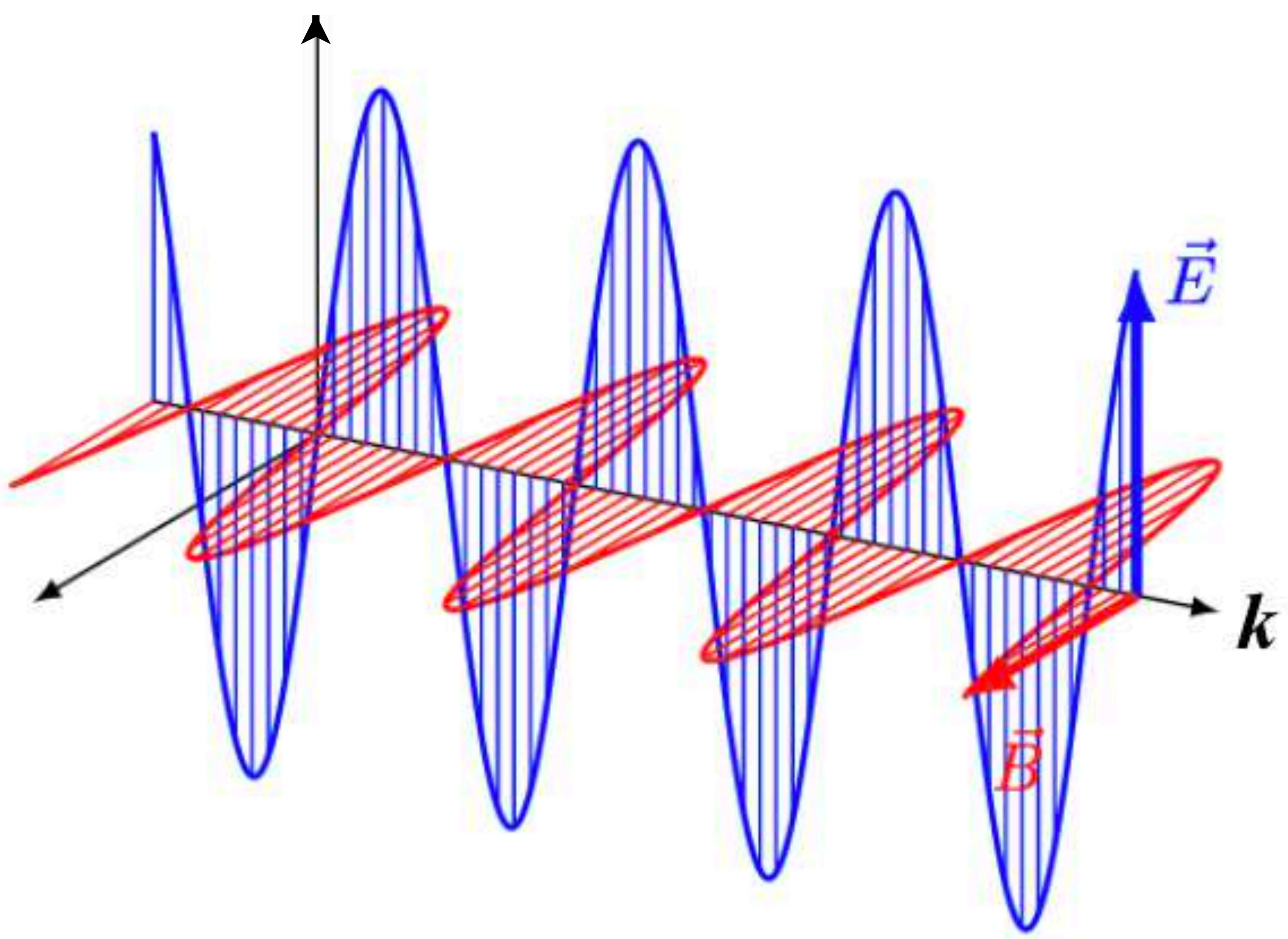

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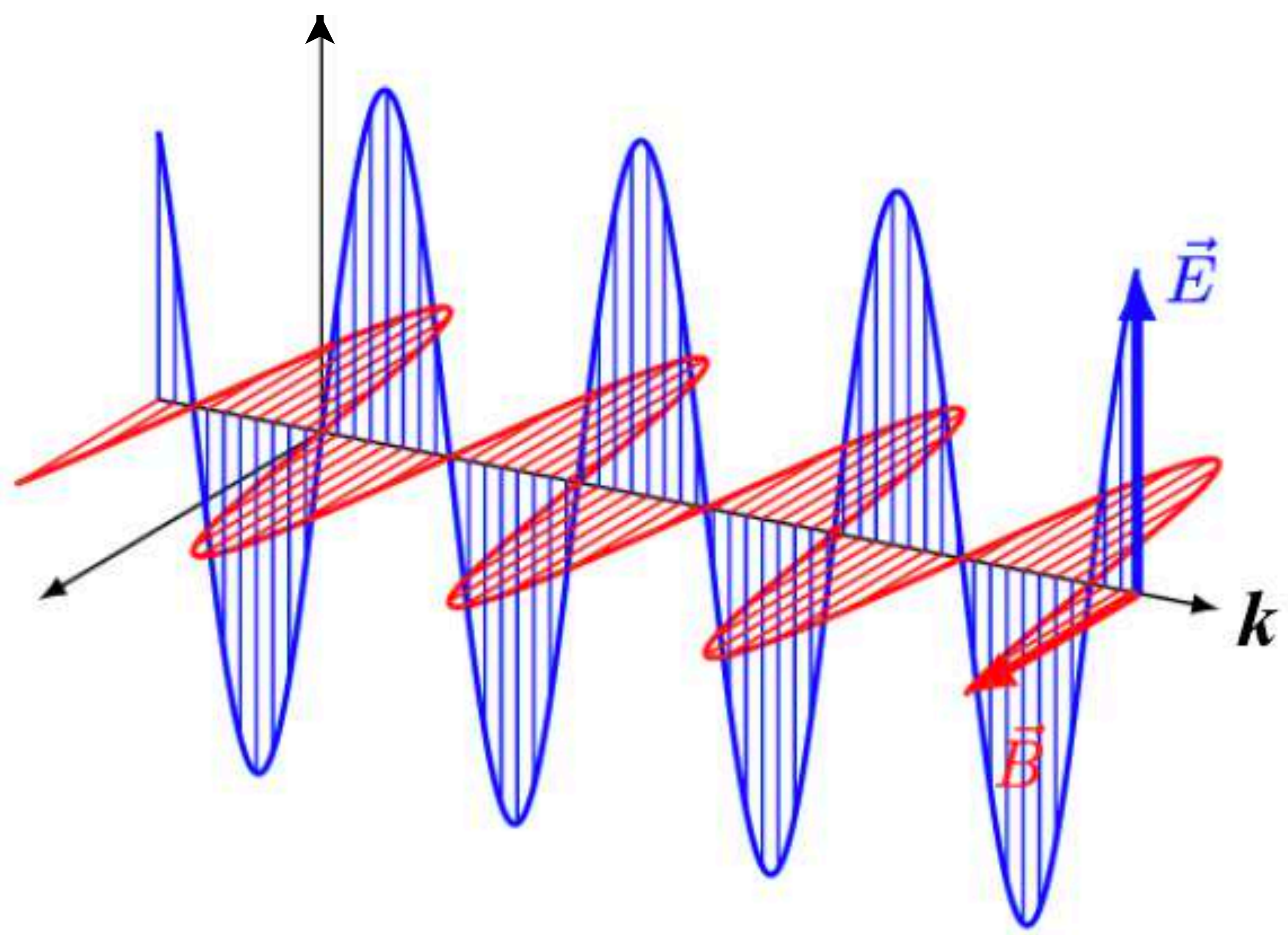

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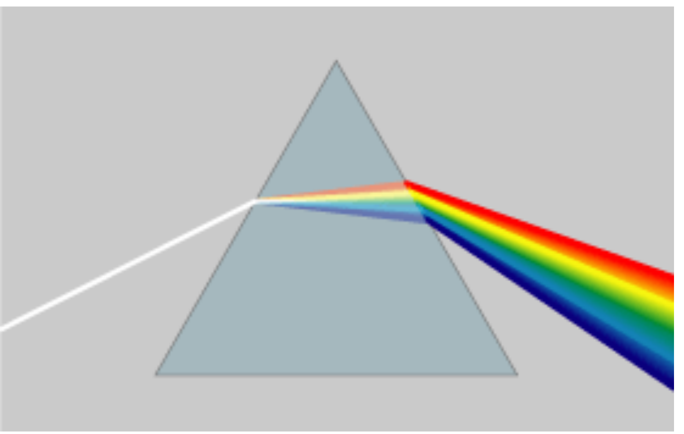

Dispersion relation (with $|\boldsymbol{k}| = \frac{2\pi}{\lambda}$)

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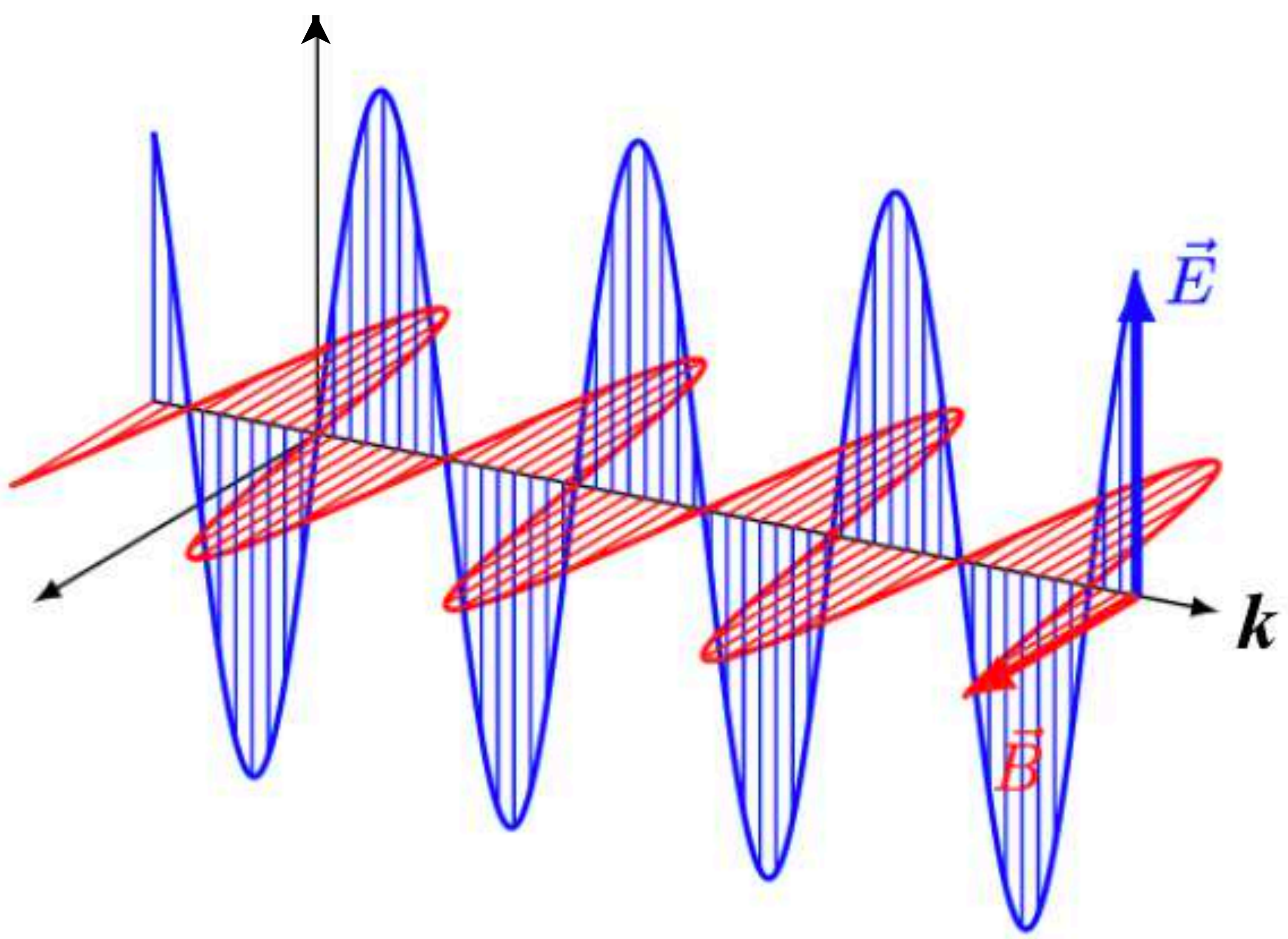

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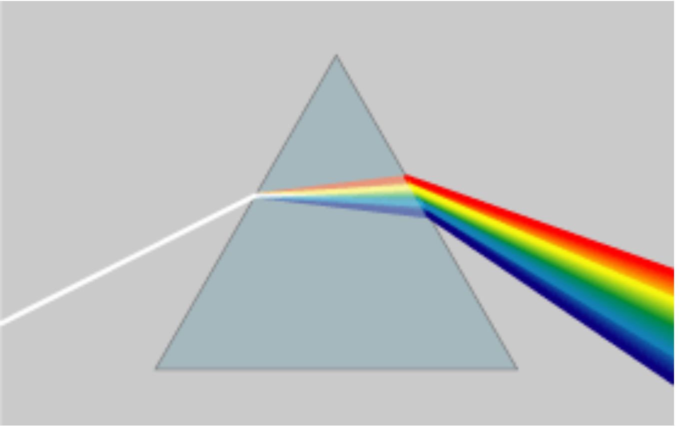

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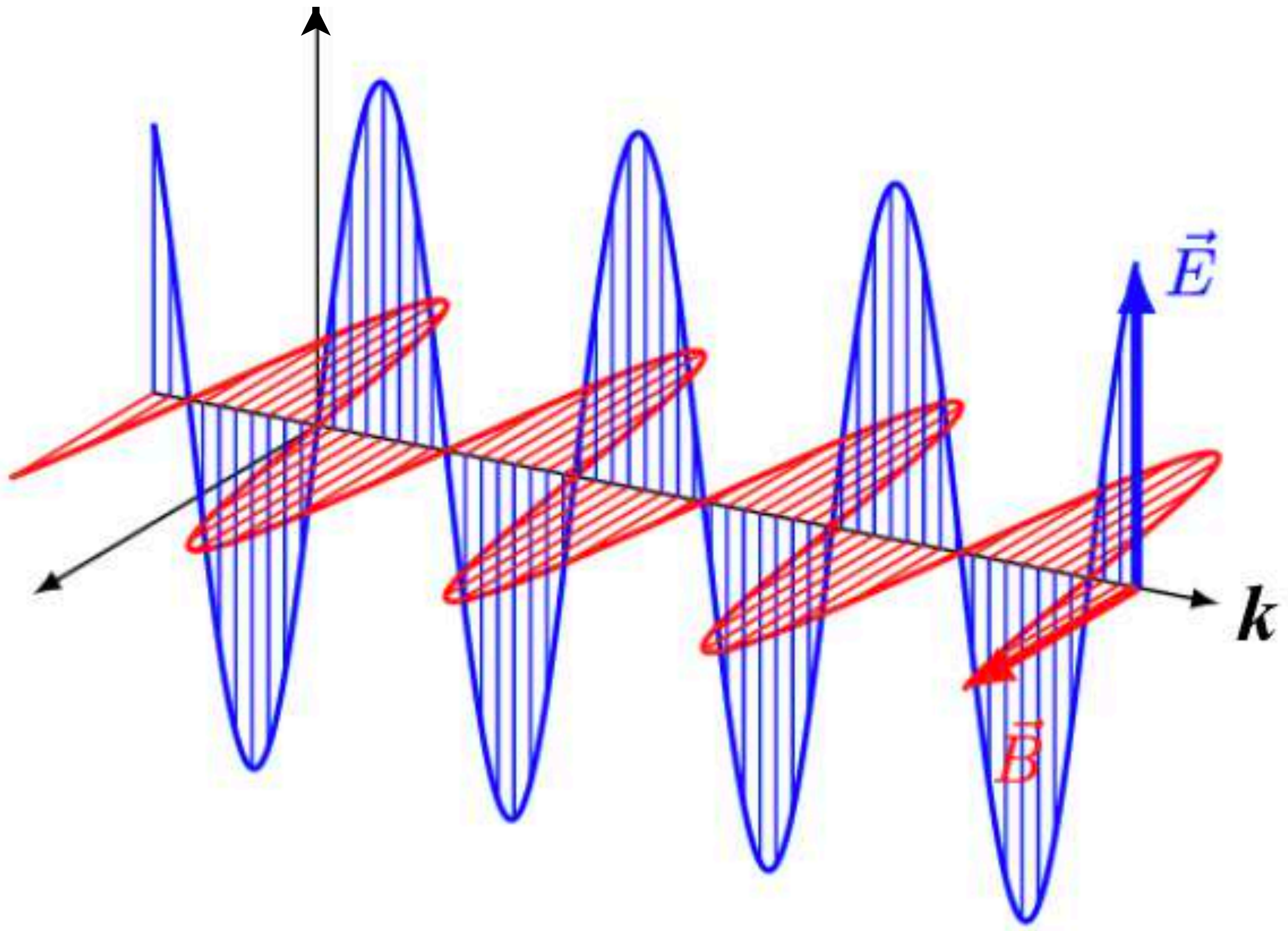
$\widetilde{\epsilon}_r$ effective permittivity, is complex in case of metals

Light is a transverse electromagnetic wave



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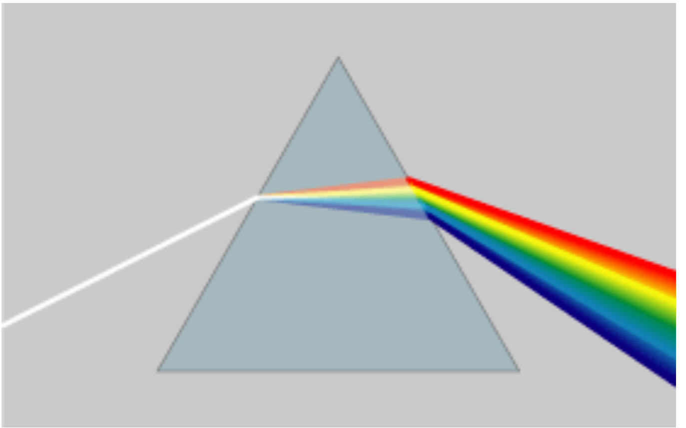

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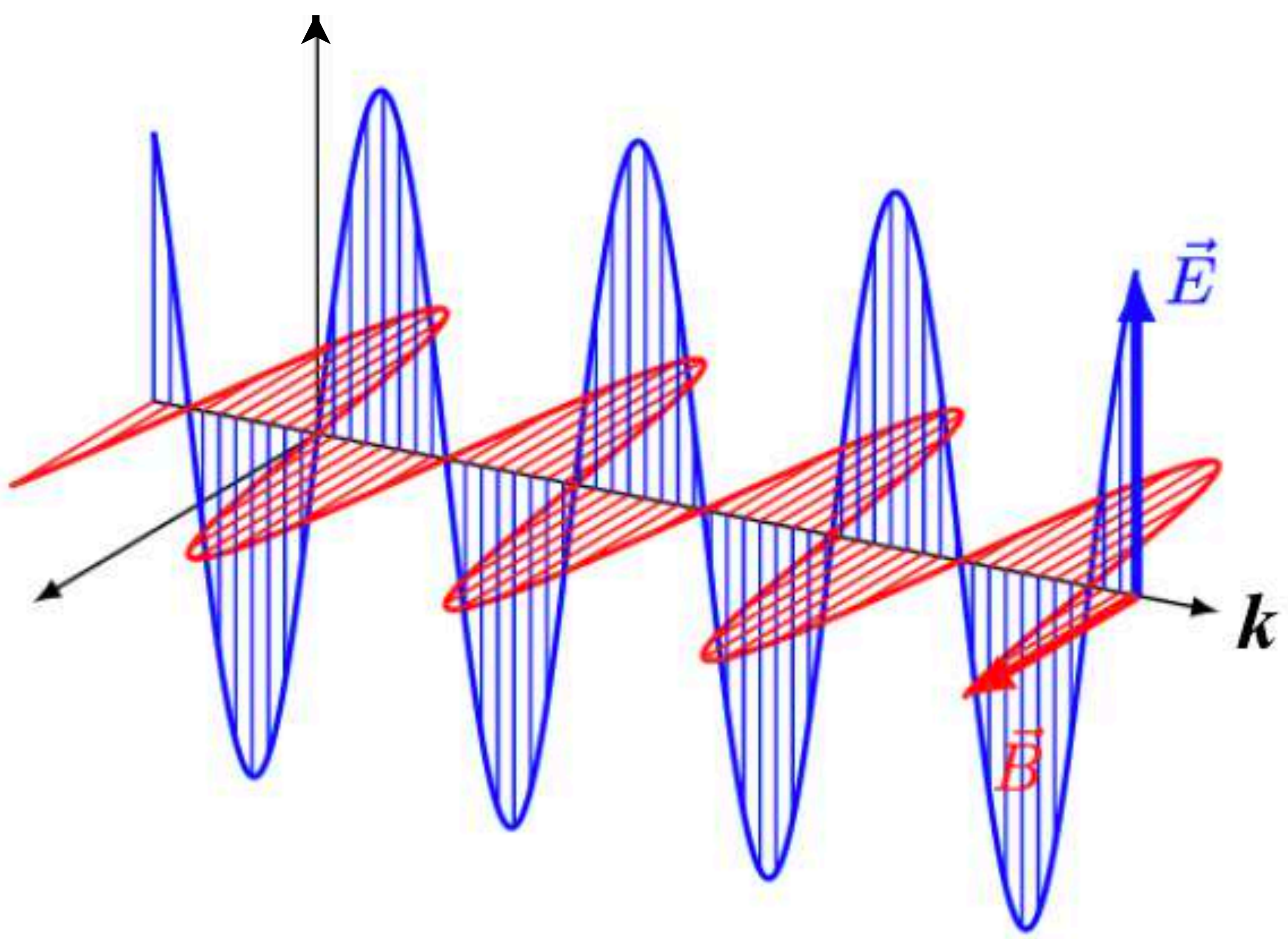

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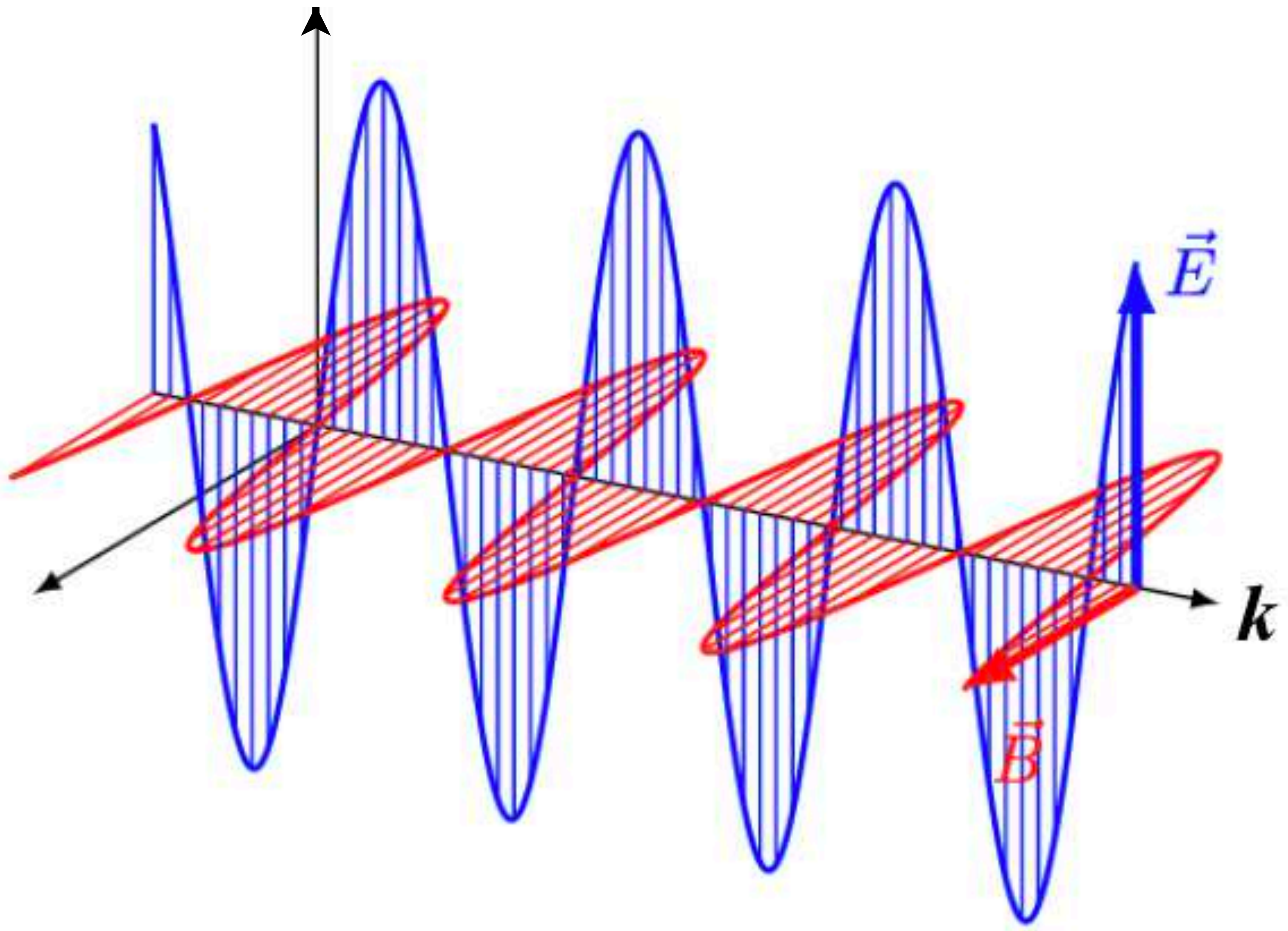

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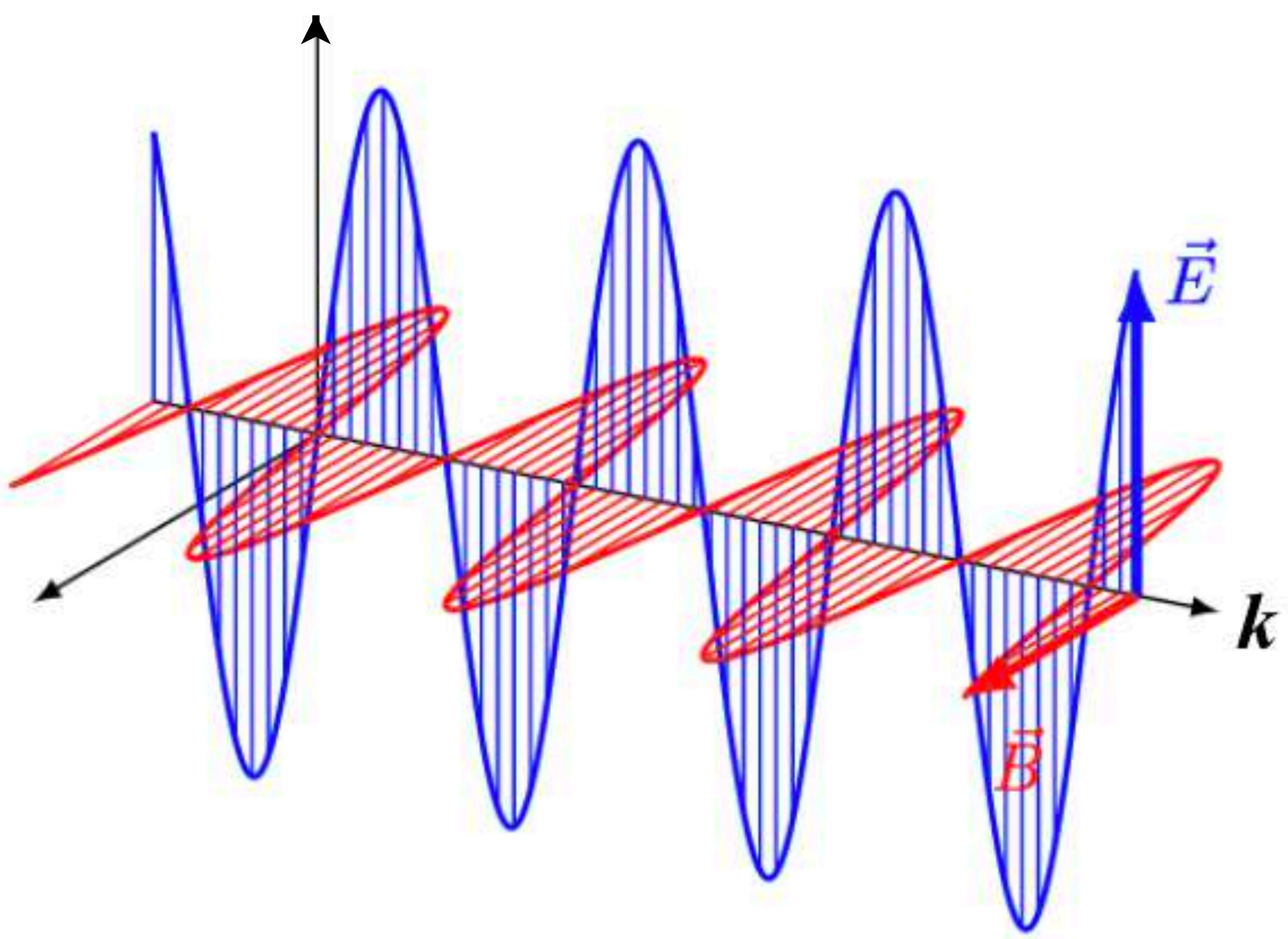

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$\boldsymbol{j}$: electric current density
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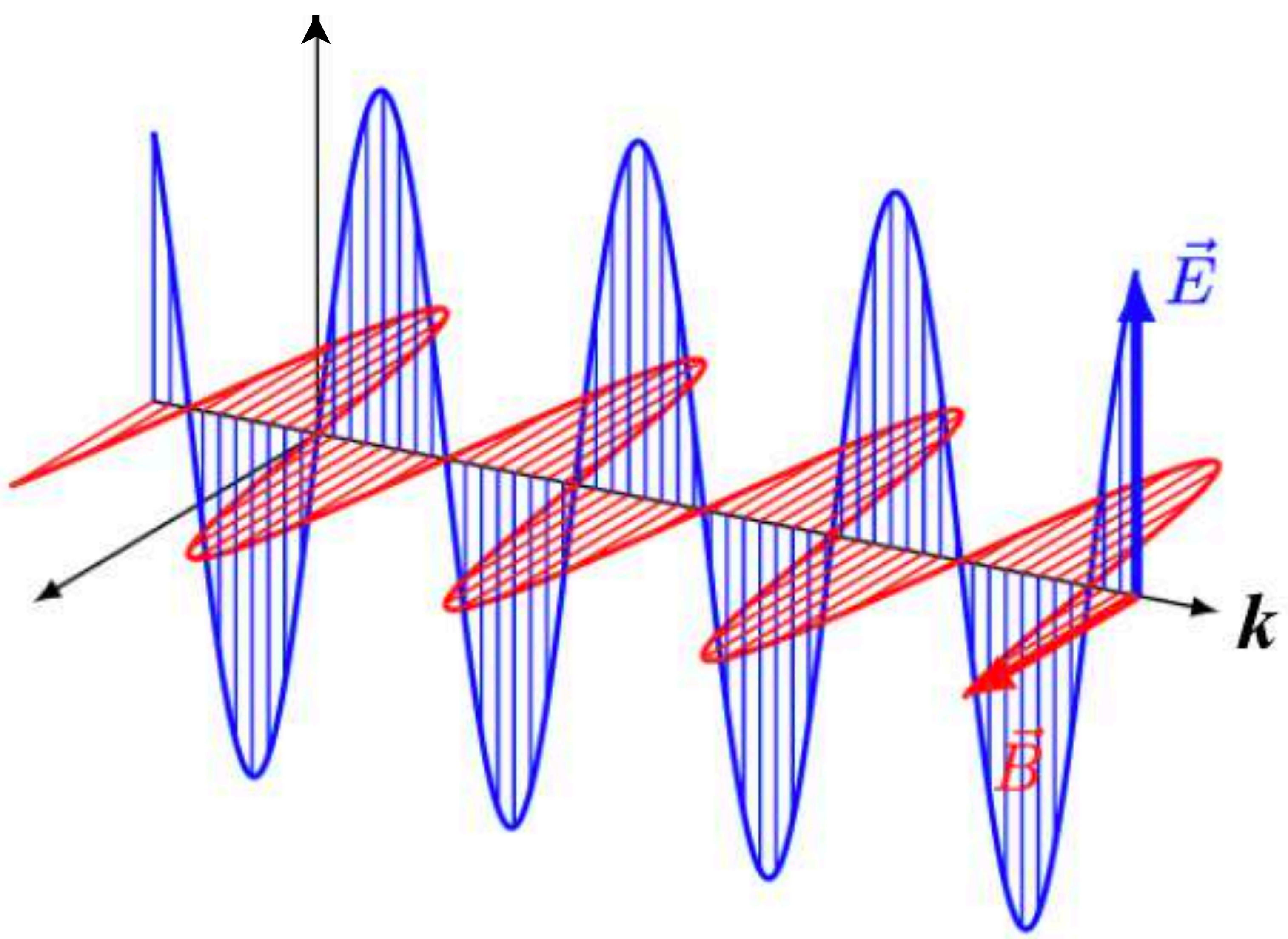
ω : angular frequency $= 2\pi f$

Light is a transverse electromagnetic wave



https://de.wikipedia.org/wiki/Datei:EM-Wave.gif

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Electrodynamic theory

Maxwell equations

$$\mathrm{div}\boldsymbol{D} = \nabla\cdot\boldsymbol{D} = \rho$$
$$\mathrm{div}\boldsymbol{B} = \nabla\cdot\boldsymbol{B} = 0$$
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$$\boldsymbol{D} = \epsilon_0\epsilon_\mathrm{r}\boldsymbol{E} = \epsilon_0\boldsymbol{E} + \boldsymbol{P}$$
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Assumptions: isotropic medium and $\rho(\boldsymbol{r},t) = 0$

$\widetilde{\boldsymbol{k}} = \boldsymbol{k}' + \mathrm{i}\boldsymbol{k}''$ wave vector is complex for metals

Refraction index n:

Vacuum: $|\boldsymbol{k}| = k_0 = \dfrac{\omega}{c_0}$, with $c_0 = \dfrac{1}{\sqrt{\epsilon_0\,\mu_0}}$

$$\boxed{\boldsymbol{k}^2 = \mu_0\cancel{\mu_\mathrm{r}}\epsilon_0\cancel{\widetilde{\epsilon}_\mathrm{r}}\,\omega^2}$$

Dispersion relation

$\boldsymbol{E}$: electric field
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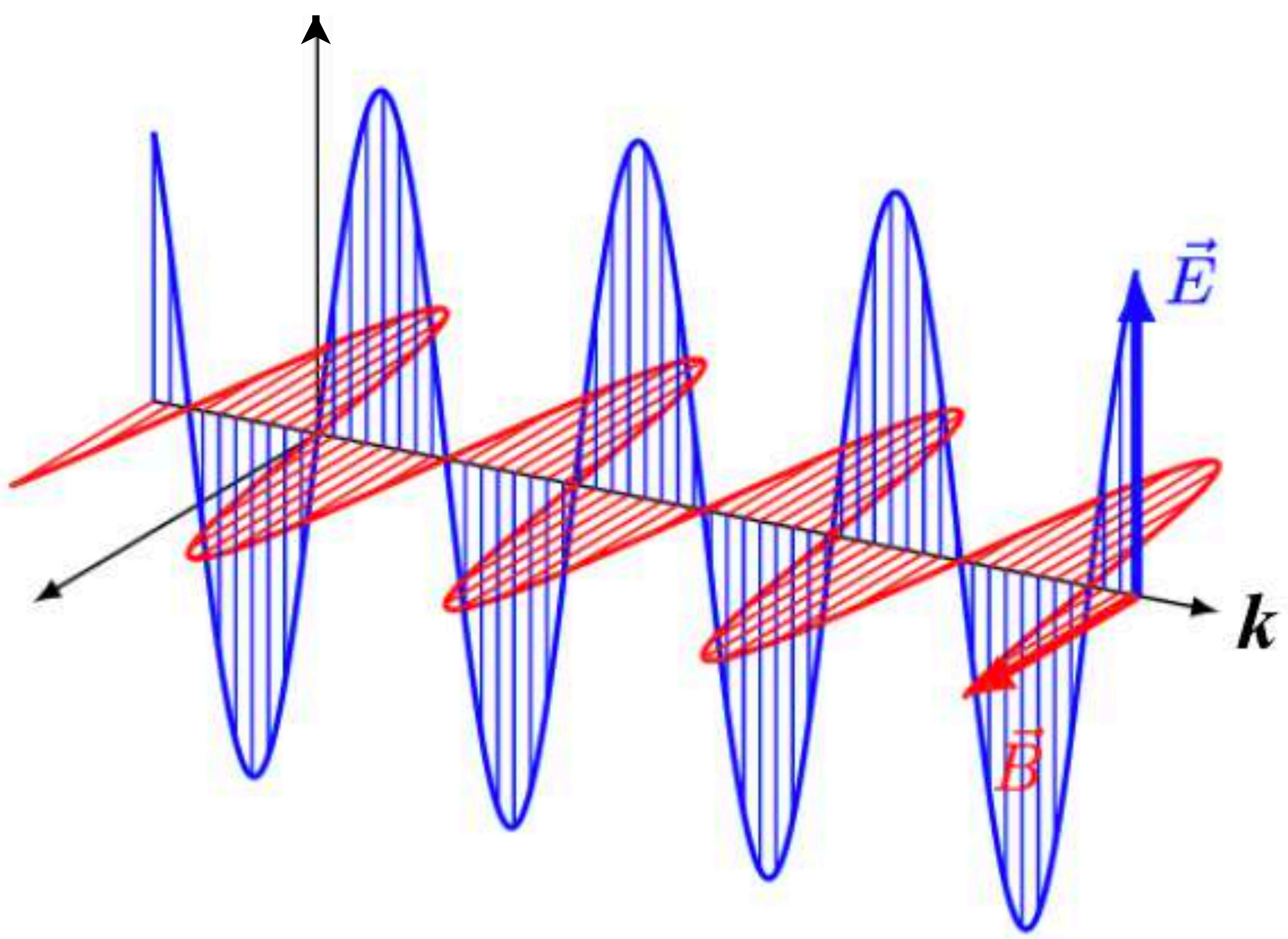

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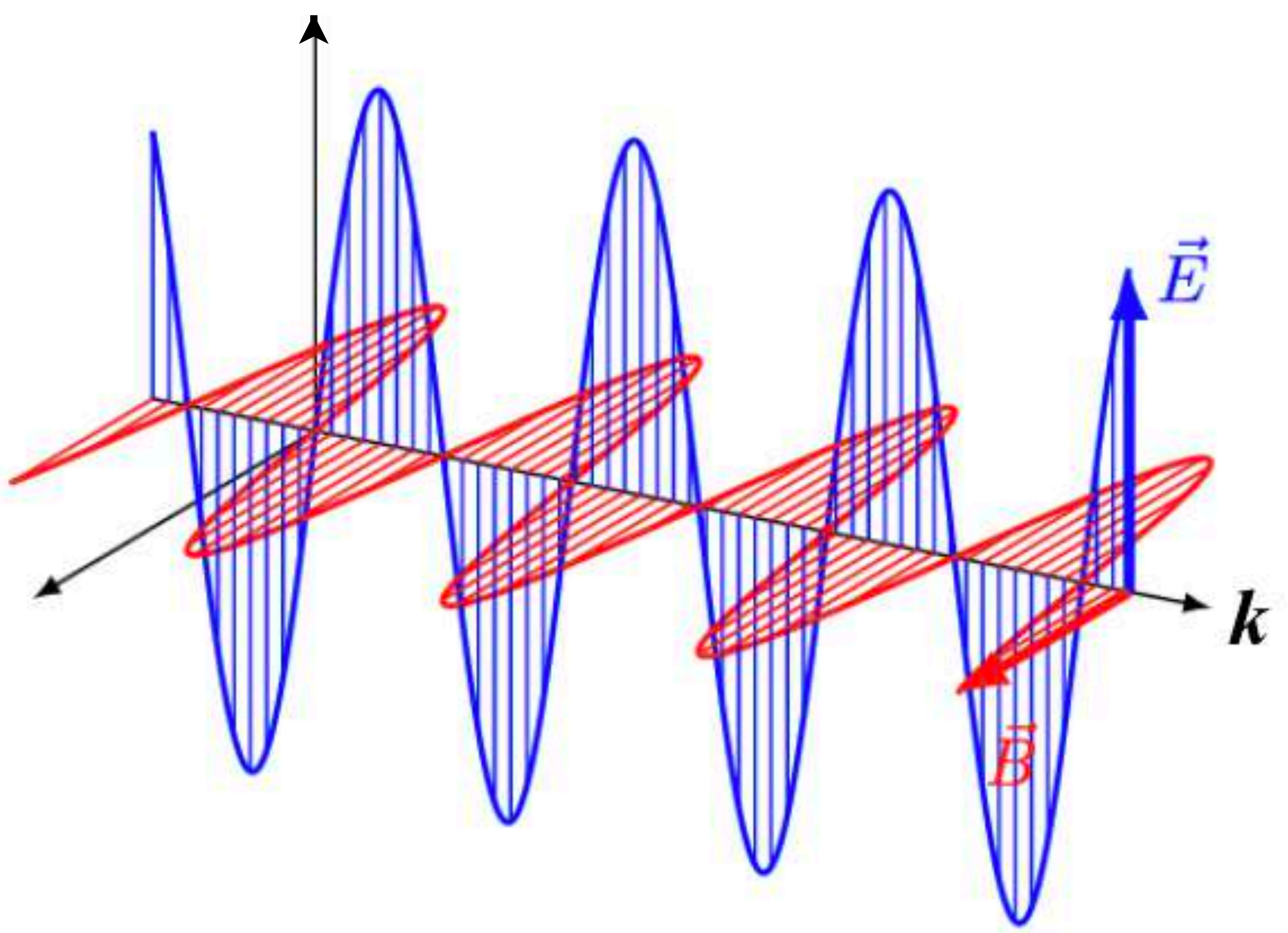

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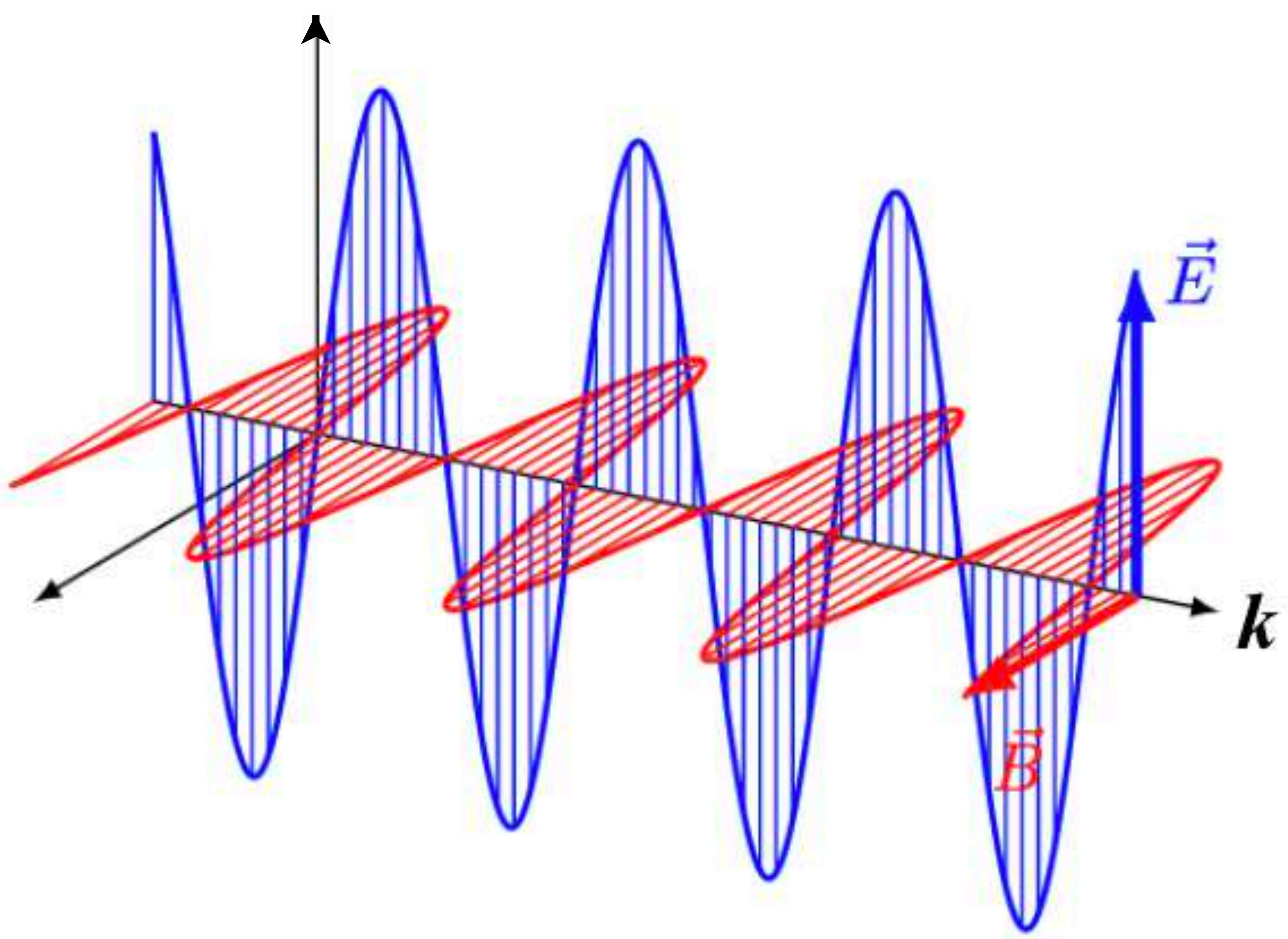

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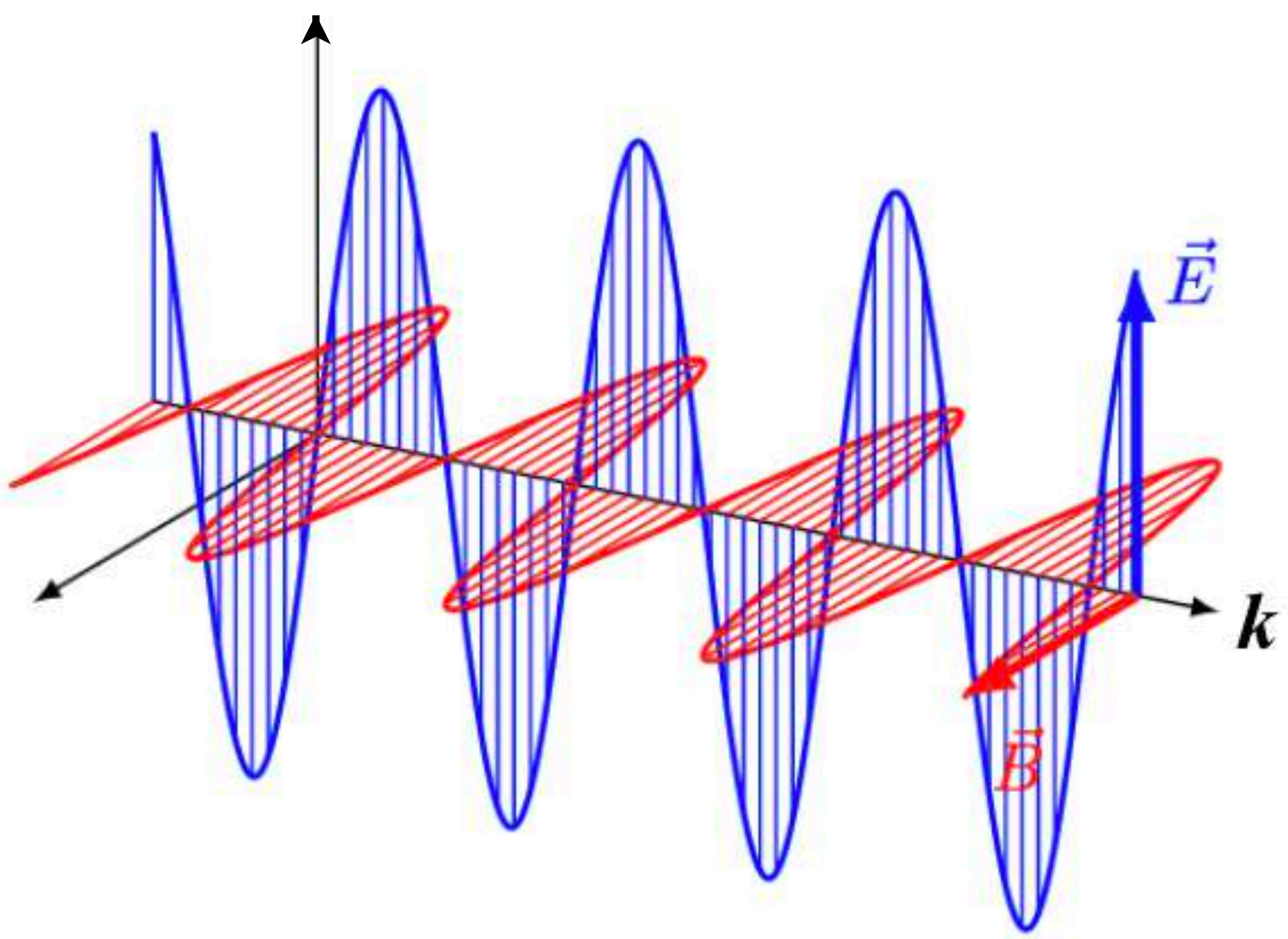

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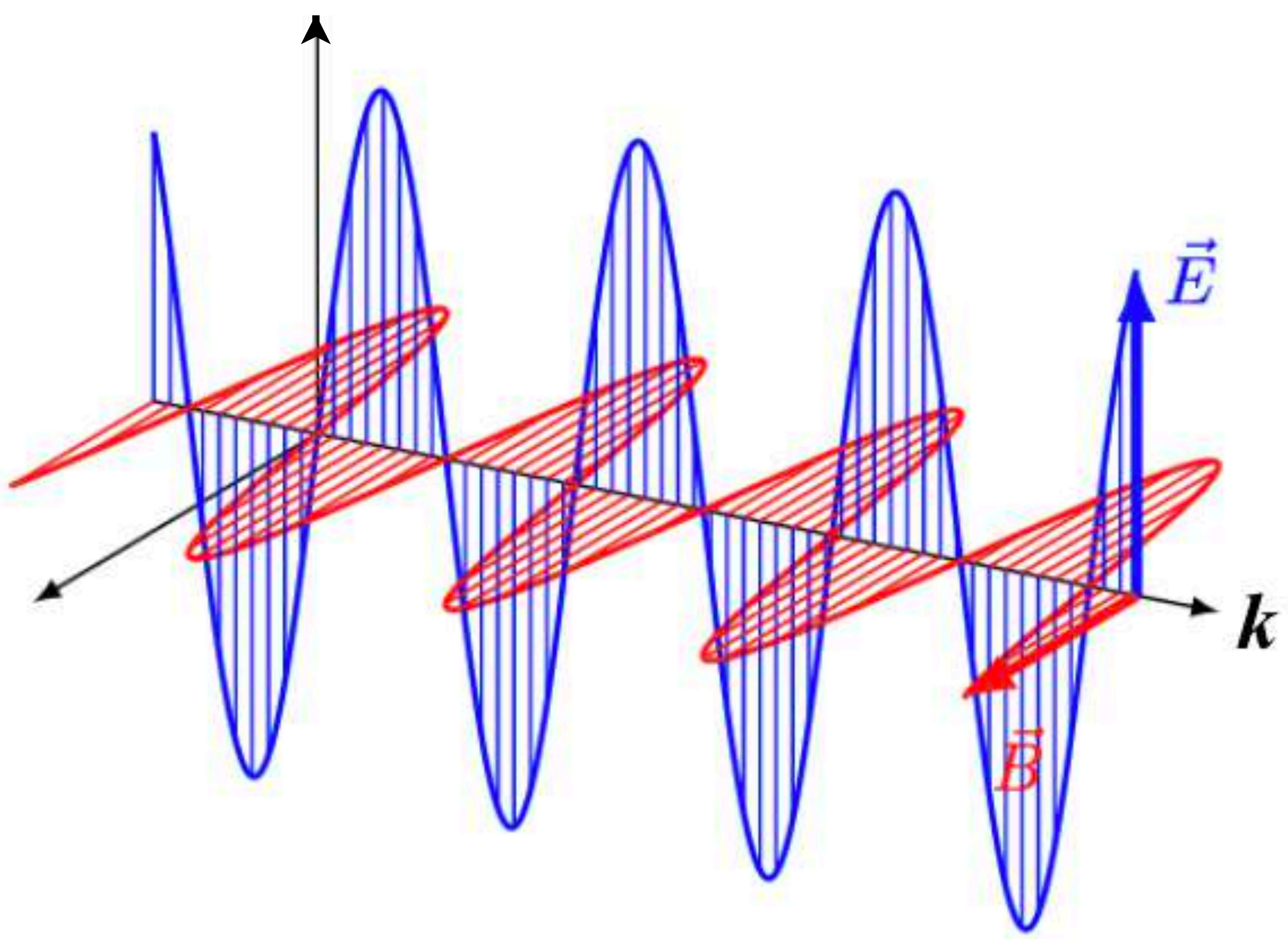

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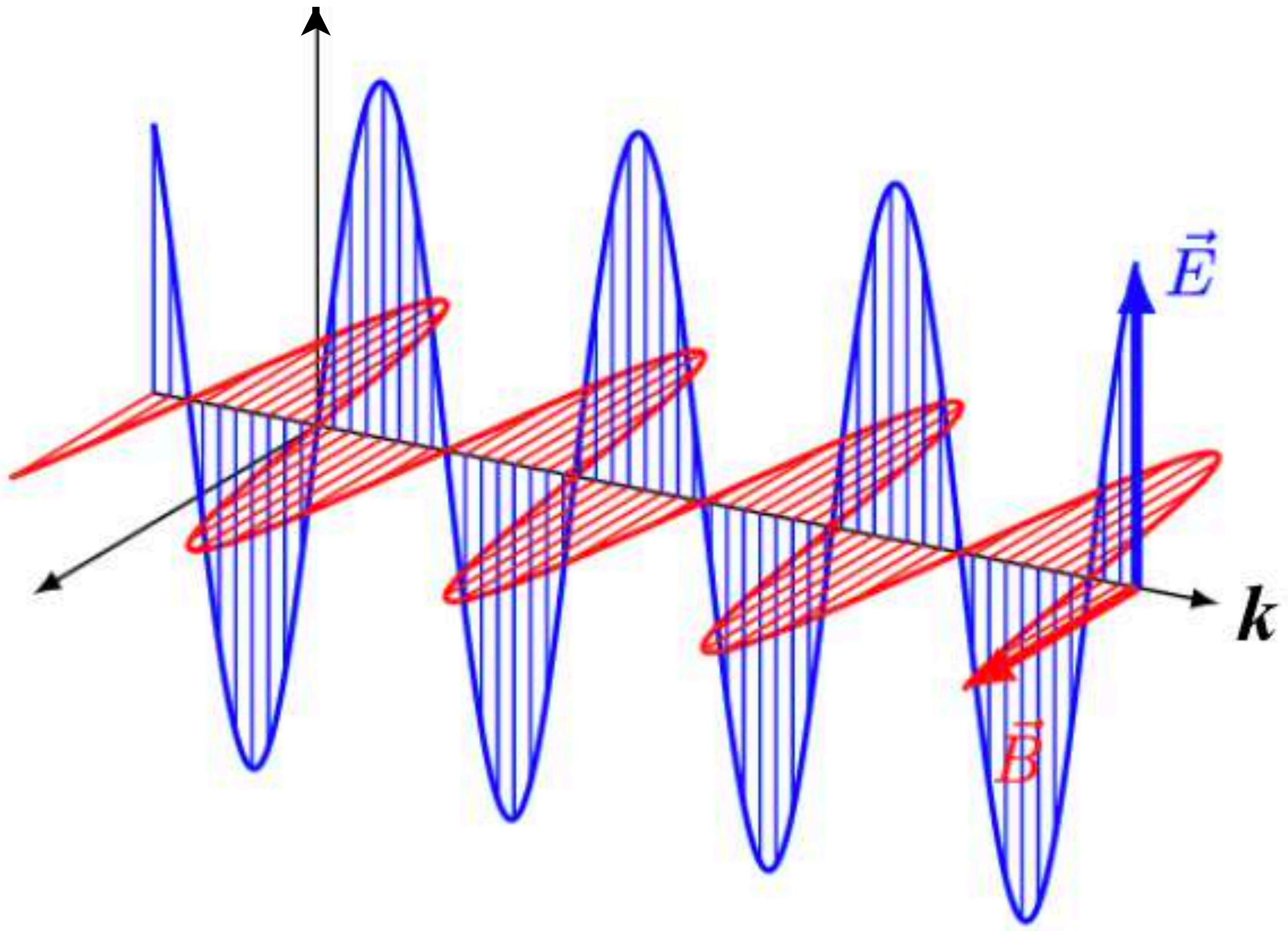

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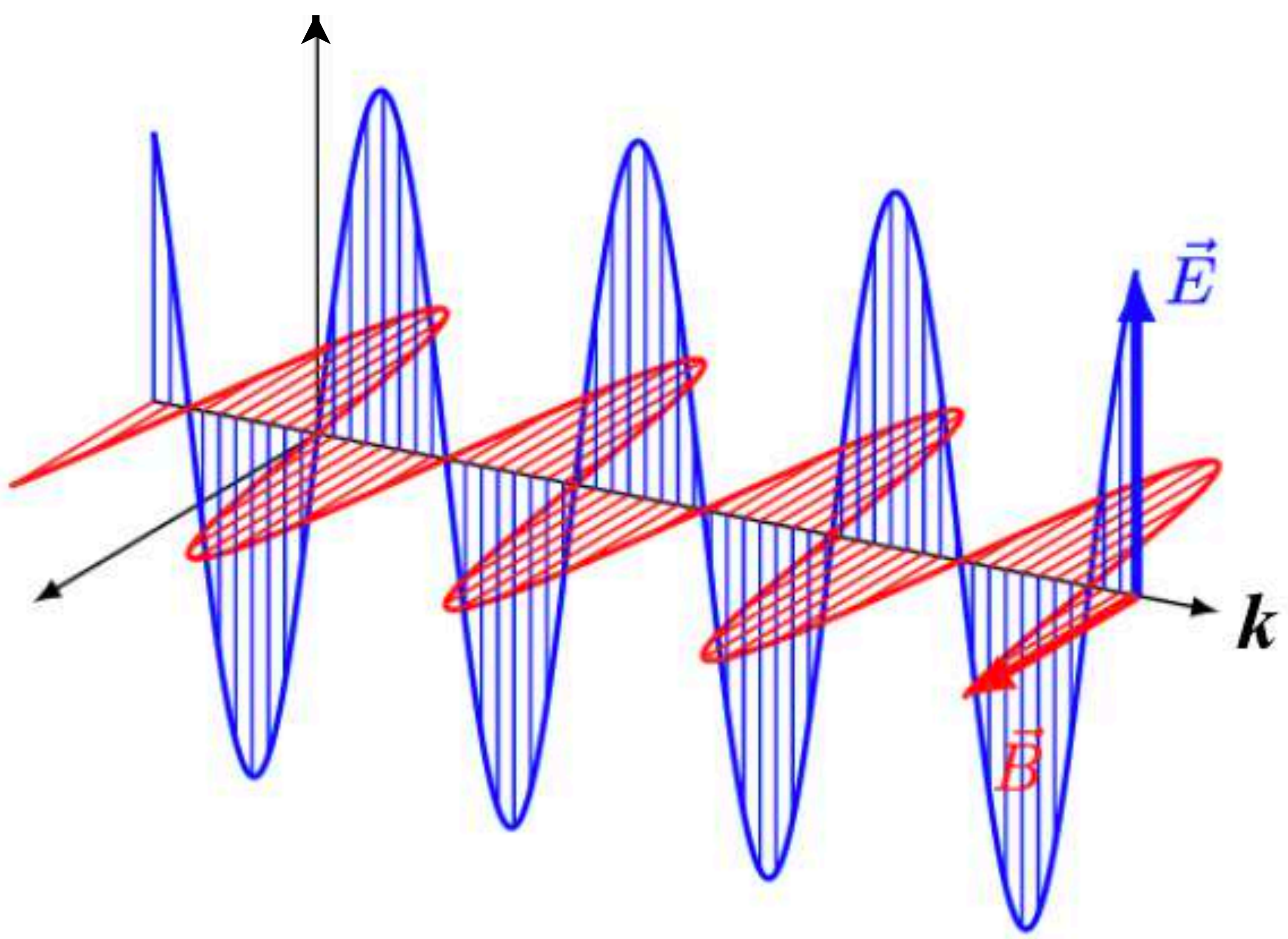

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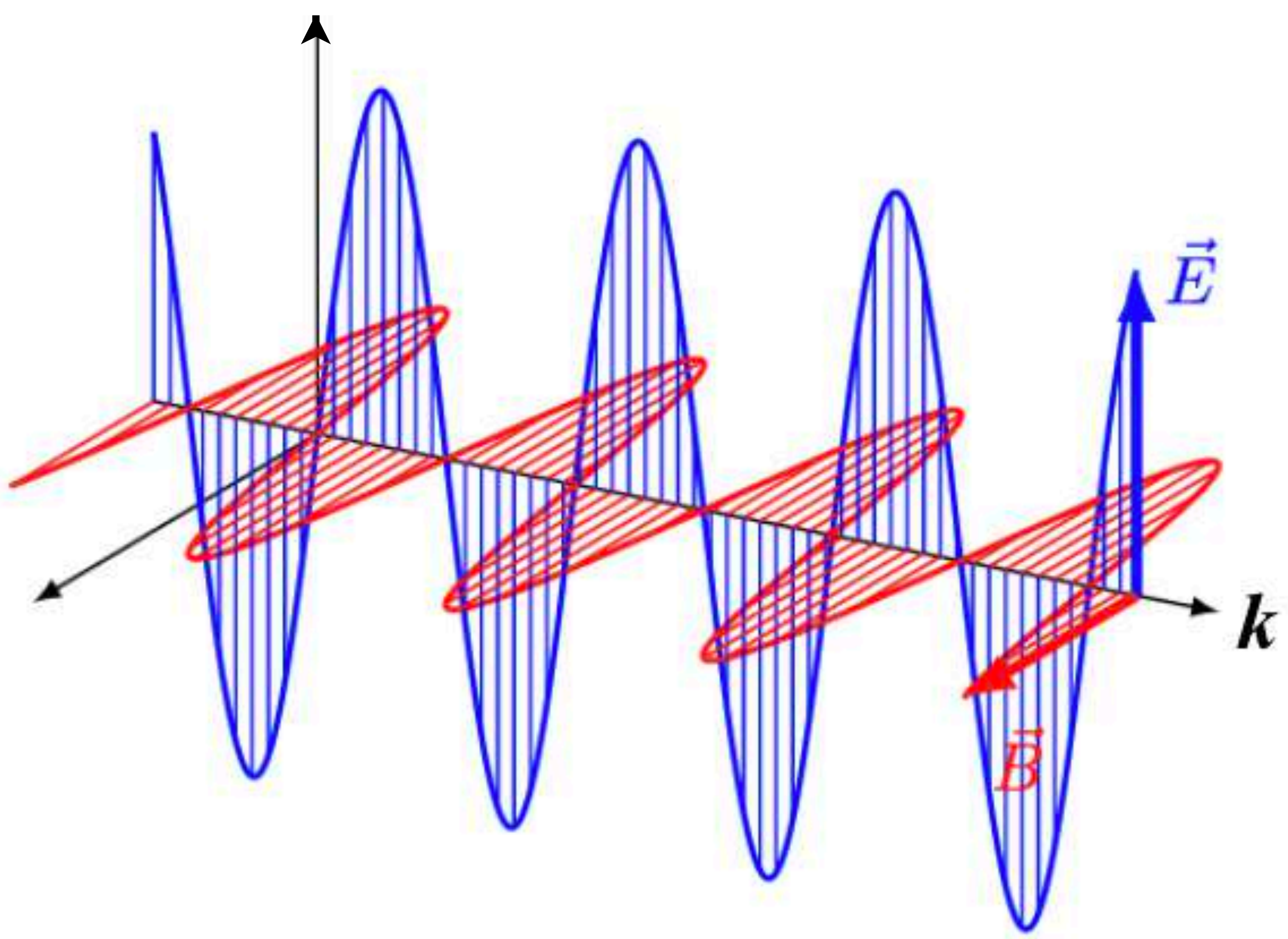
Electrodynamic theory

$\tilde{n} = n' + \mathrm{i}n''$ refractive index is complex for metals

n' : true refraction index

n'' : extinction coefficient

Light is a transverse electromagnetic wave



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Electrodynamic theory

$\tilde{n} = n' + in''$ refractive index is complex for metals

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$\boldsymbol{E} = \boldsymbol{E}^0 e^{i(\boldsymbol{k}\cdot z - \omega t)}$ plane wave in z-direction

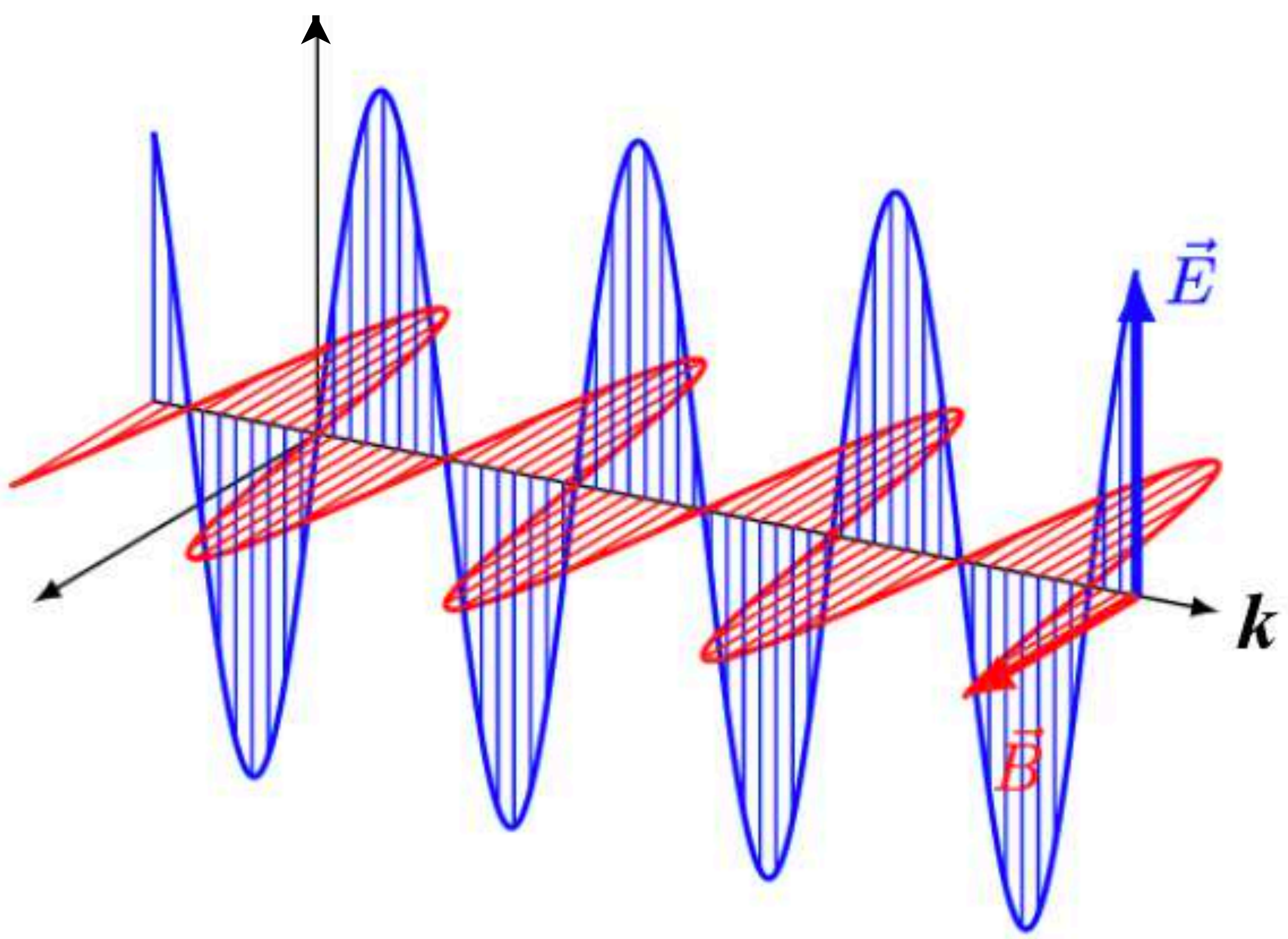
$\Downarrow$ $k_0 = \frac{\omega}{c_0}$ and $|\tilde{\boldsymbol{k}}| = n k_0 = \frac{\omega}{\boldsymbol{v}}$

$$\boldsymbol{E} = \boldsymbol{E}^0 e^{i[\frac{\omega}{c_0}(n'+in'')z - \omega t]} = \underbrace{\boldsymbol{E}^0 e^{-\frac{\omega}{c_0}n''z}}_{\text{amplitude, is exponentially attenuated as wave progresses in conductor}} \underbrace{e^{i[\frac{\omega}{c_0}n'z - \omega t]}}_{\text{wave advances in } z\text{-direction with speed } c_0/n' \text{ as if } n' \text{ were usual index of refraction}}$$

$\Downarrow$

energy of wave is absorbed in case of **conductive** medium

Light is a transverse electromagnetic wave



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amplitude, is exponentially attenuated as wave progresses in conductor

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$\Downarrow$

energy of wave is absorbed in case of **conductive** medium

(remember: $\boldsymbol{k}^2 = \mu_0 \mu_r \epsilon_0 \underbrace{(\epsilon_r + \boxed{\frac{\mathrm{i}\sigma}{\epsilon_0\, \omega}})}_{\tilde{\epsilon}_r} \omega^2$ Dispersion relation)

$$\tilde{n} = \frac{c_0}{v} = \sqrt{\tilde{\epsilon}_r \tilde{\mu}_r} = n' + \mathrm{i}n''$$

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Irradiance: $\boldsymbol{I}(\boldsymbol{r}) = \mathrm{amplitude}^2 = \boldsymbol{I}_0 \mathrm{e}^{-\alpha z}$ with $\alpha = \frac{2\omega n''}{c_0}$ absorption coefficient

$\uparrow$

Power of electromagnetic radiation (radiative flux)

Electrodynamic theory

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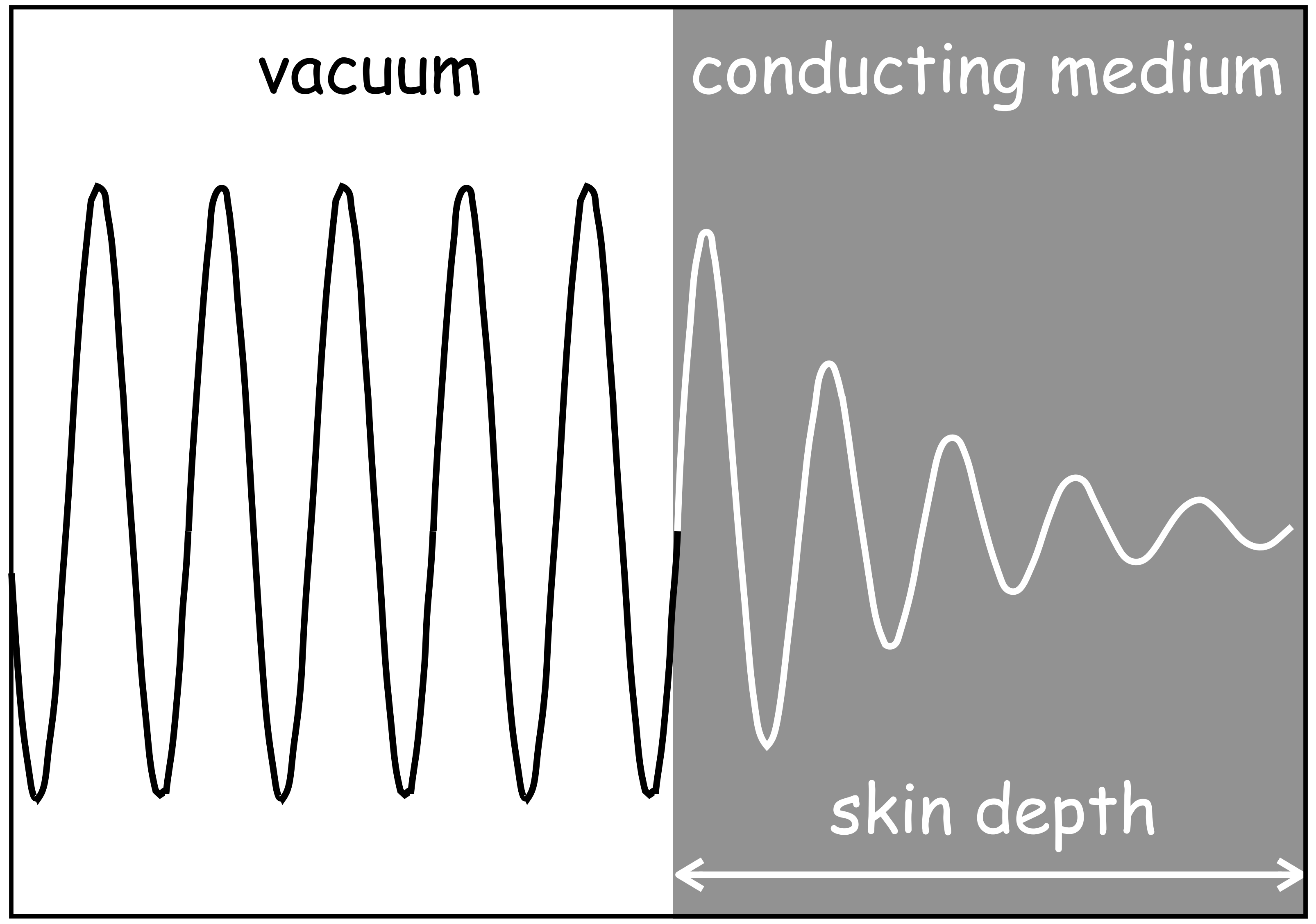
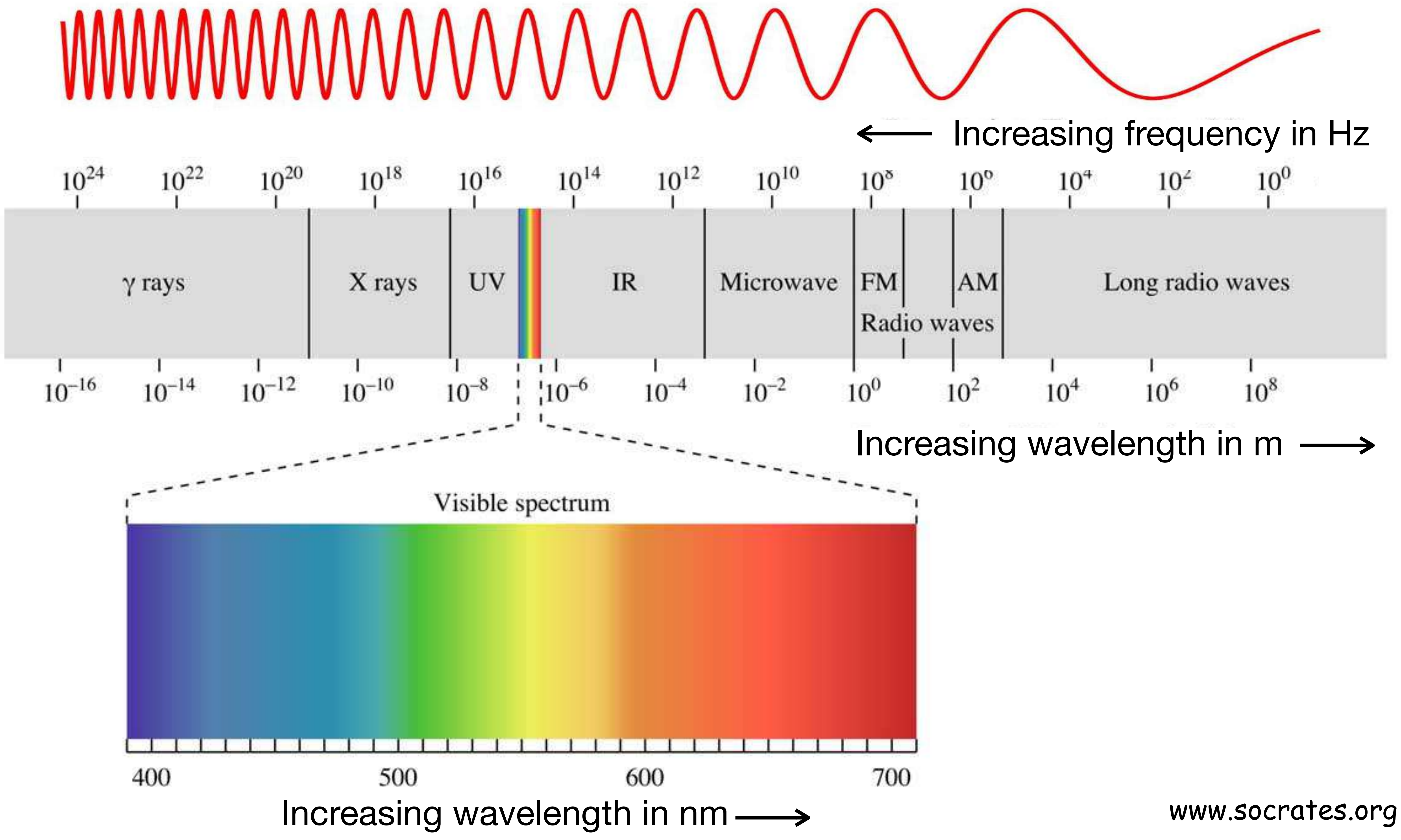
$\Longrightarrow$ radiative flux drops by factor $\mathrm{e}^{-1} = 1/3$ after wave has propagated $\underbrace{\text{distance of } 1/\alpha}_{\text{penetration depth or skin depth, around 40 nm}}$



Electrodynamic theory

Remarks:

1) Conventional Magnet-Optics: Visible light



www.socrates.org

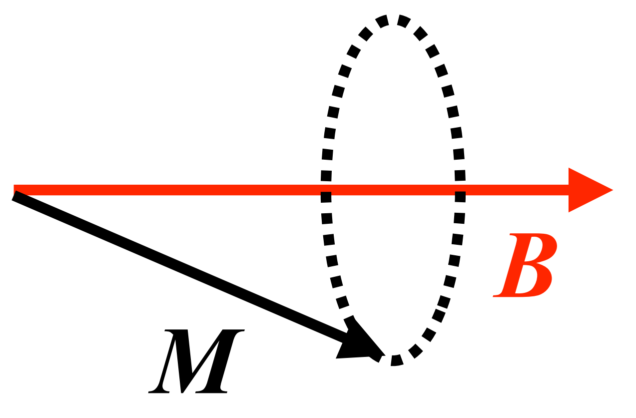
Electrodynamic theory

Remarks:

1) Conventional Magnet-Optics: Visible light

Frequency of visible light (~ 500 THz) >> Larmor frequency (~ 100 GHz)

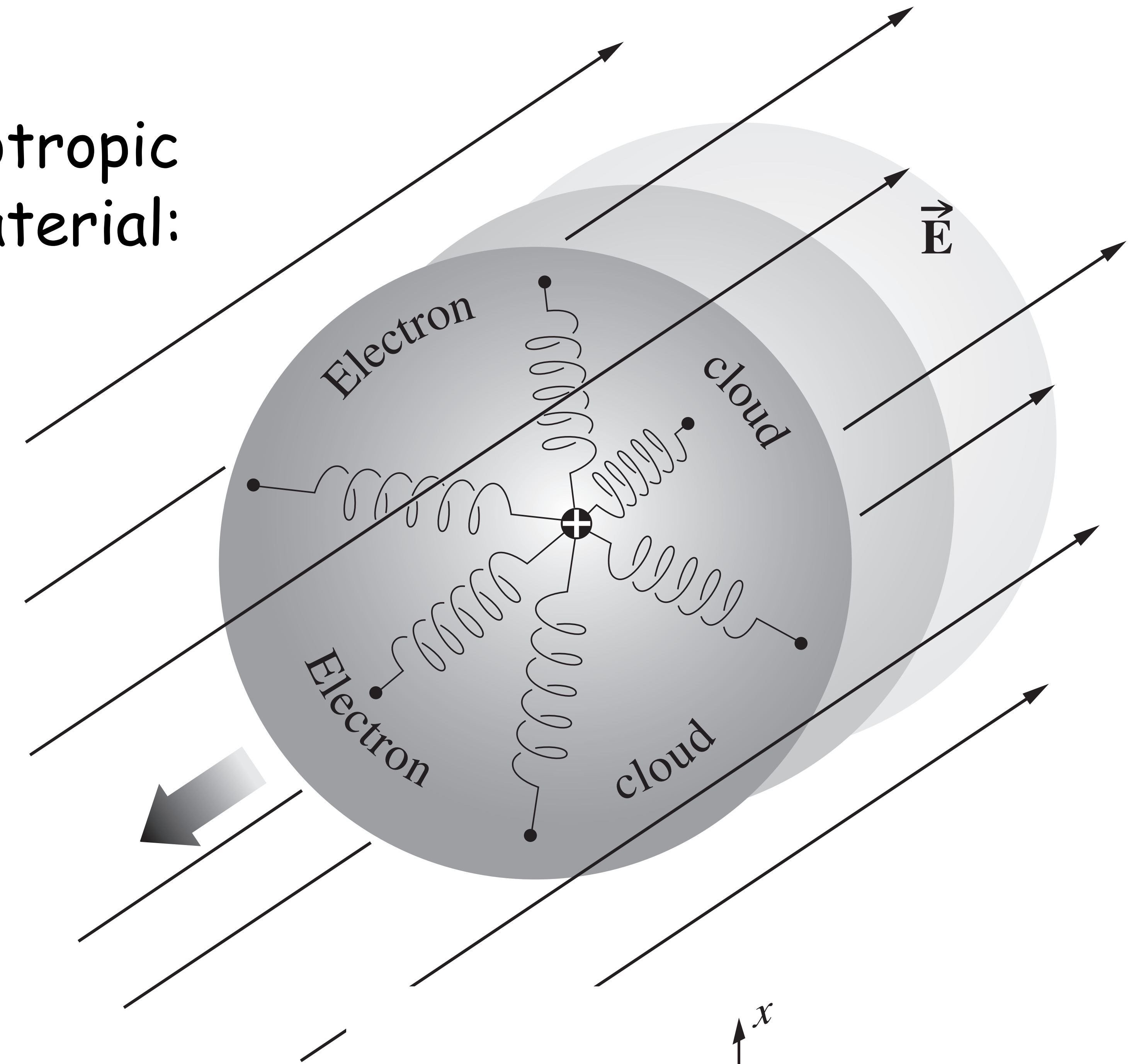
$\Longrightarrow$ Magnetic moments cannot follow the alternating magnetic field of light wave



$\Longrightarrow$ $\mu_r \approx 1$, only electric field component relevant

$\Longrightarrow$ $\boxed{\boldsymbol{D} = \epsilon_0 \widetilde{\epsilon}_r \boldsymbol{E}}$ is relevant (not $\boldsymbol{B} = \mu_0 \mu_r \boldsymbol{H}$)

Nonetheless, all magnetic information is acounted for (see later) isotropic material:



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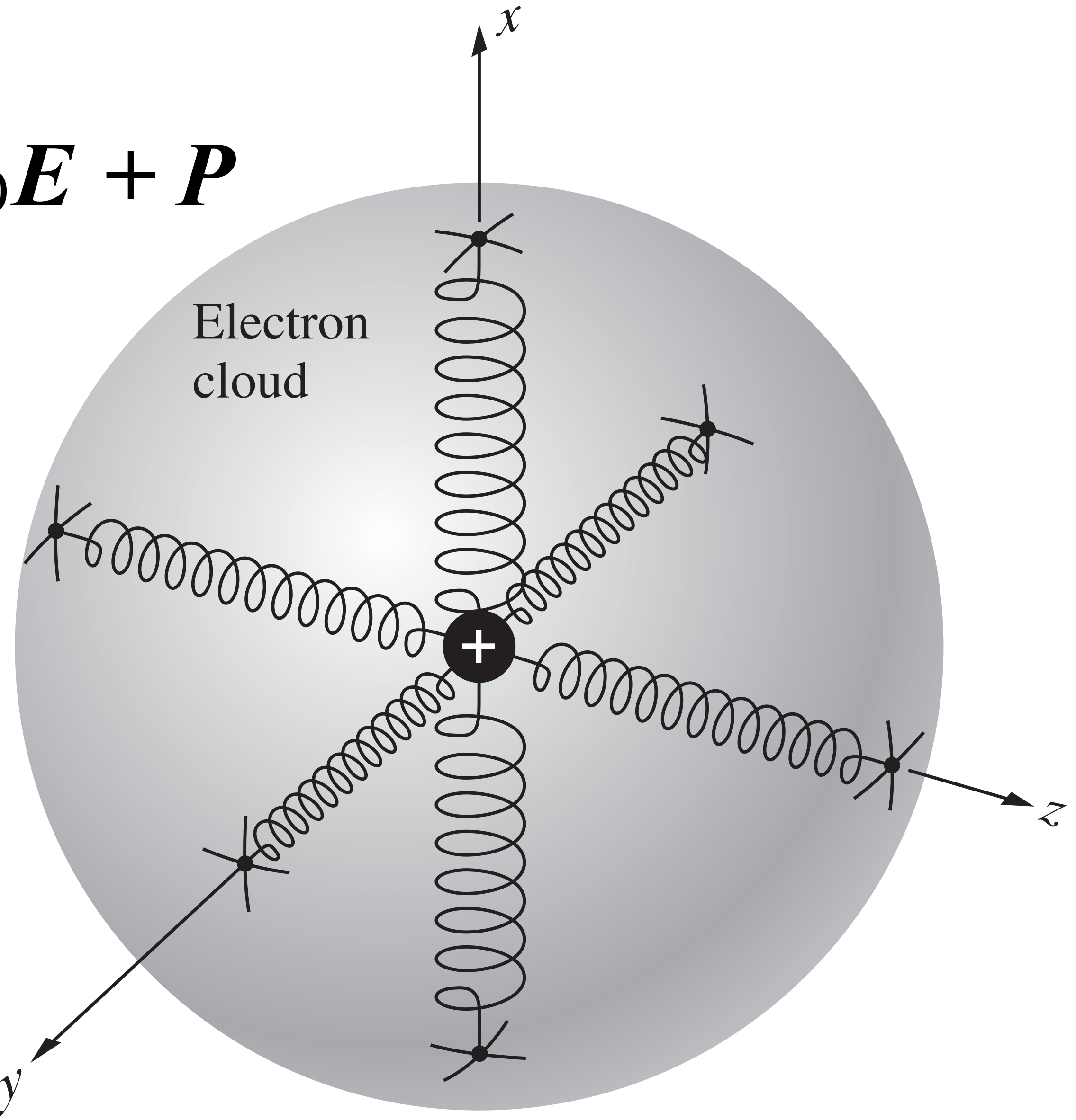
anisotropic material:



E. Hecht: Optics. Pearson (2017)

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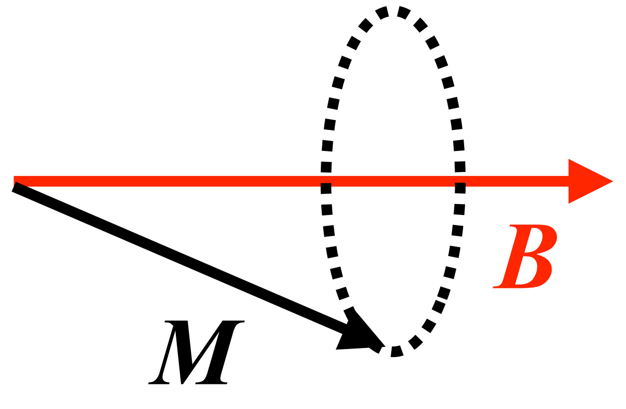

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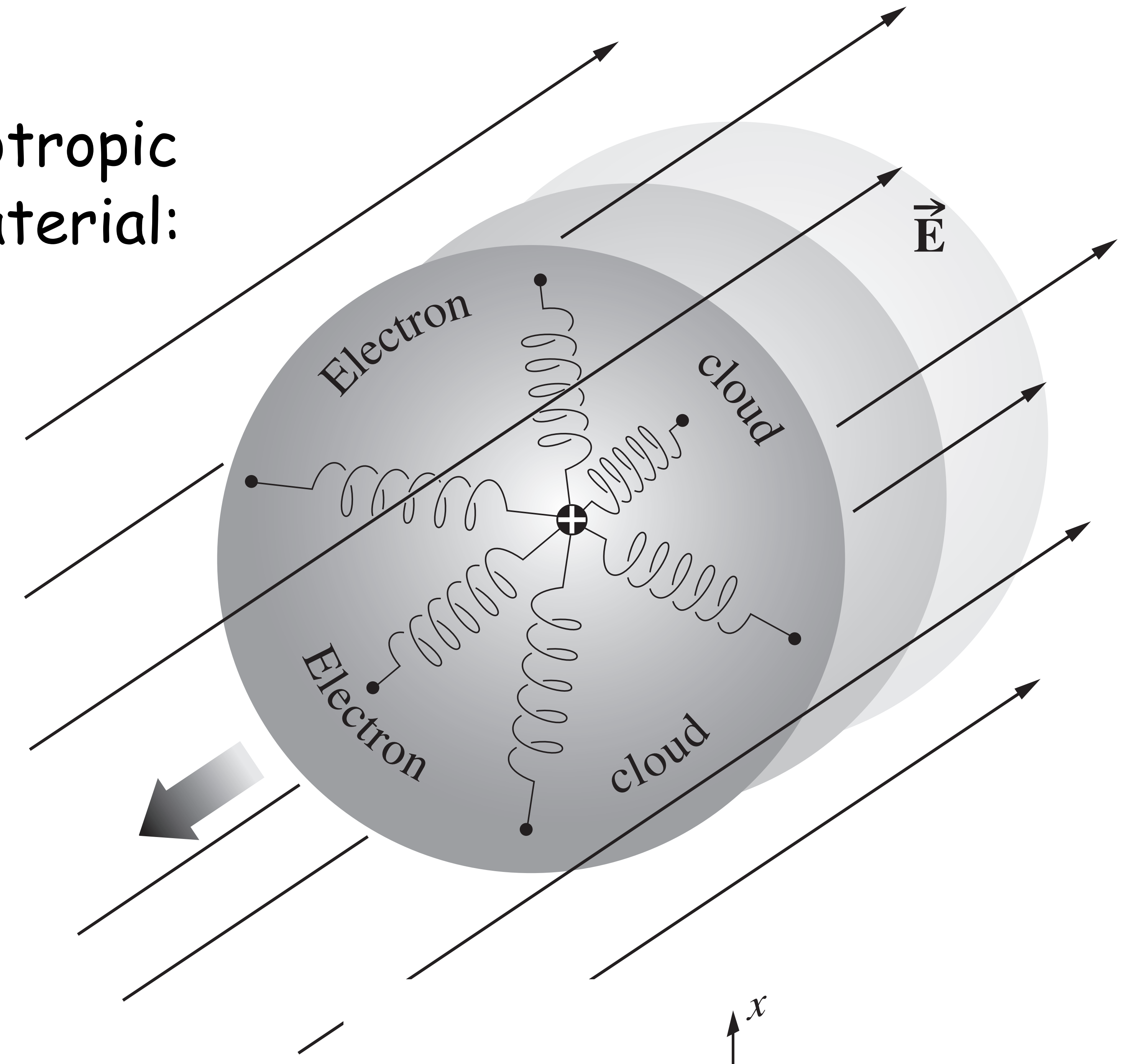
2) ϵ_r is tensor

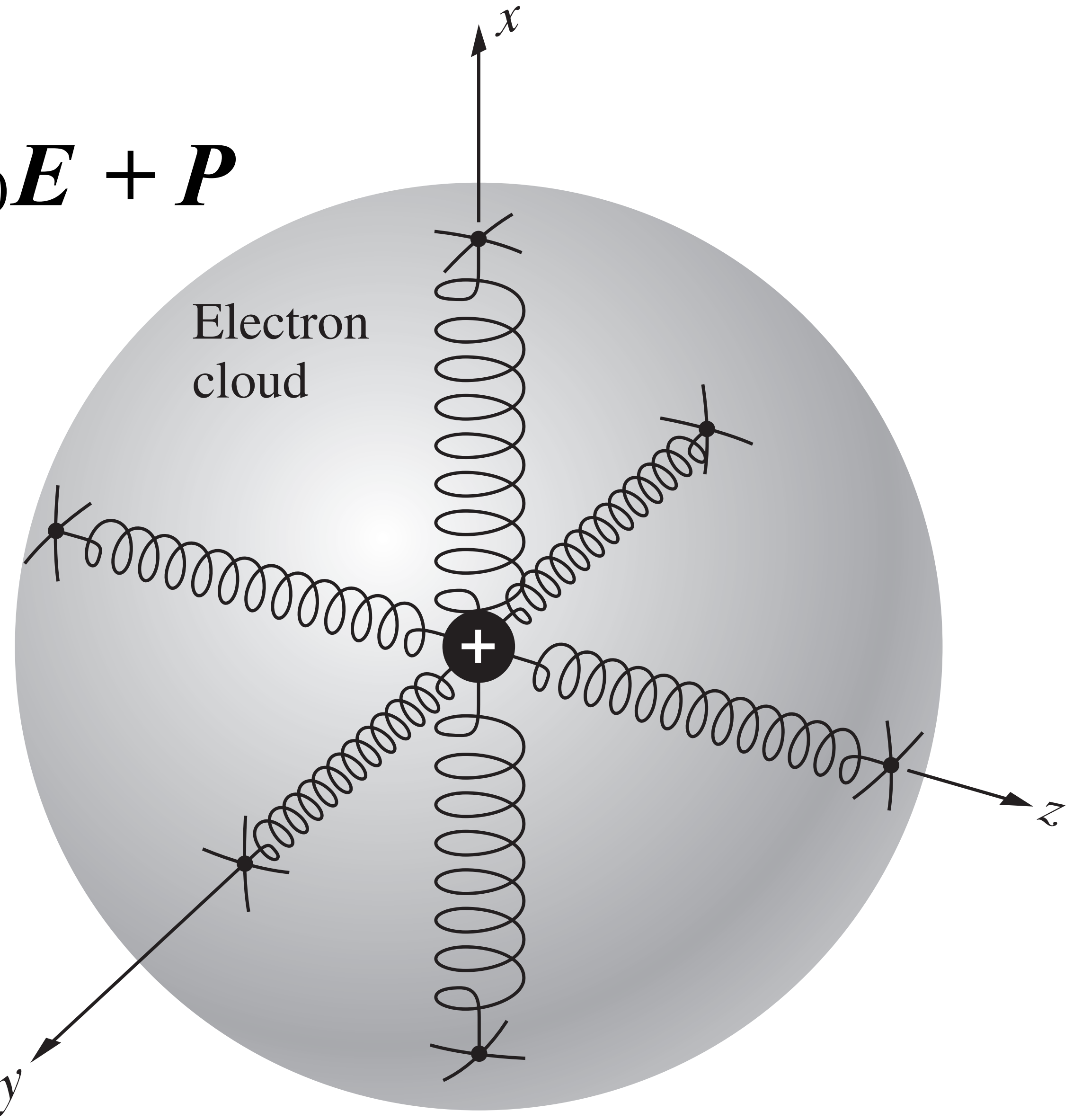
$\Longrightarrow$ $\boldsymbol{D}$ (= elect. field in material) must not be in direction of incoming $\boldsymbol{E}$

isotropic material:



$D = \epsilon_0 \epsilon_r E = \epsilon_0 E + P$

anisotropic material:



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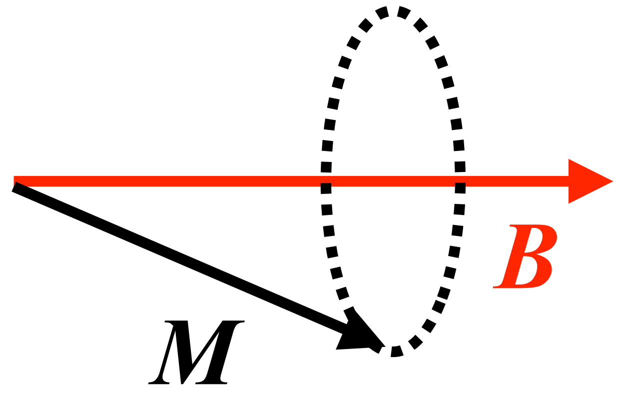
$\Longrightarrow$ $\mu_r \approx 1$, only electric field component relevant



$\Longrightarrow$ $\boxed{D = \epsilon_0 \tilde{\epsilon}_r E}$ is relevant (not $B = \mu_0 \mu_r H$)

Nonetheless, all magnetic information is acounted for (see later)

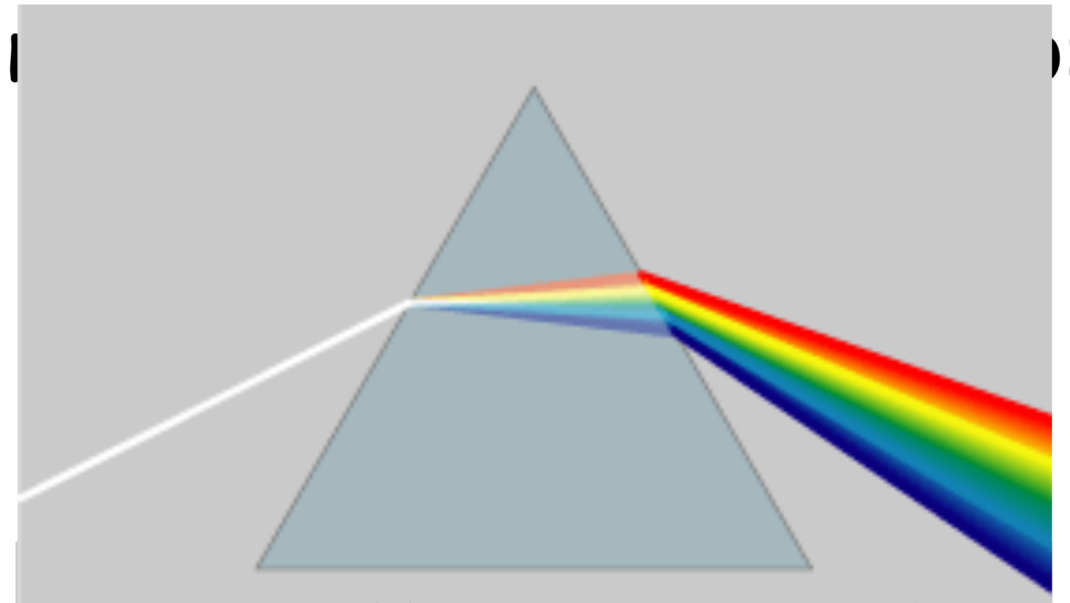
2) ϵ_r is tensor

$\Longrightarrow$ D (= elect. field in material) must r ming E

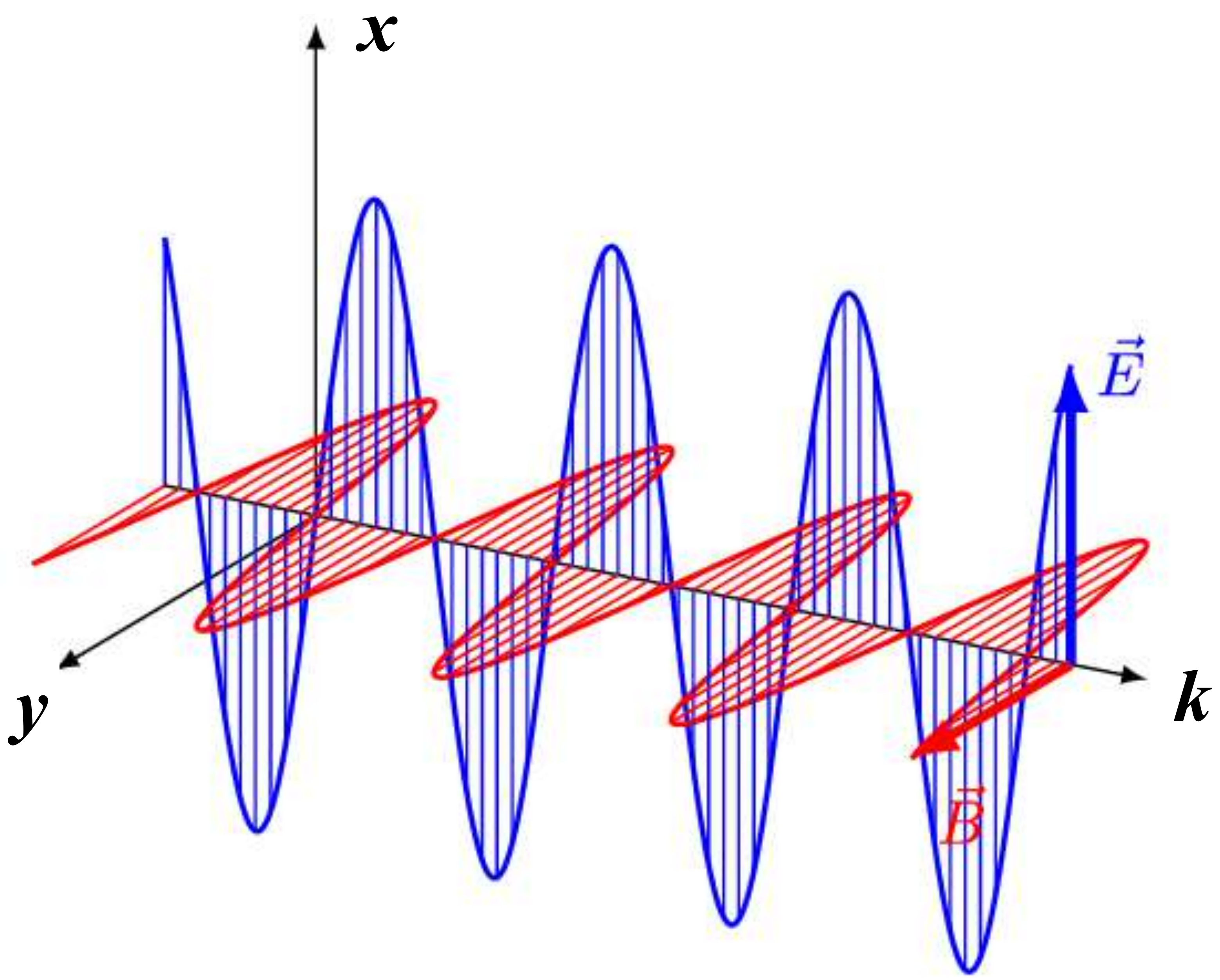
3) ϵ_r and n are frequency-dependent

$$n(\omega) \approx \sqrt{\epsilon_r(\omega)}$$

$\Longrightarrow$ propagation of wave is dispersive



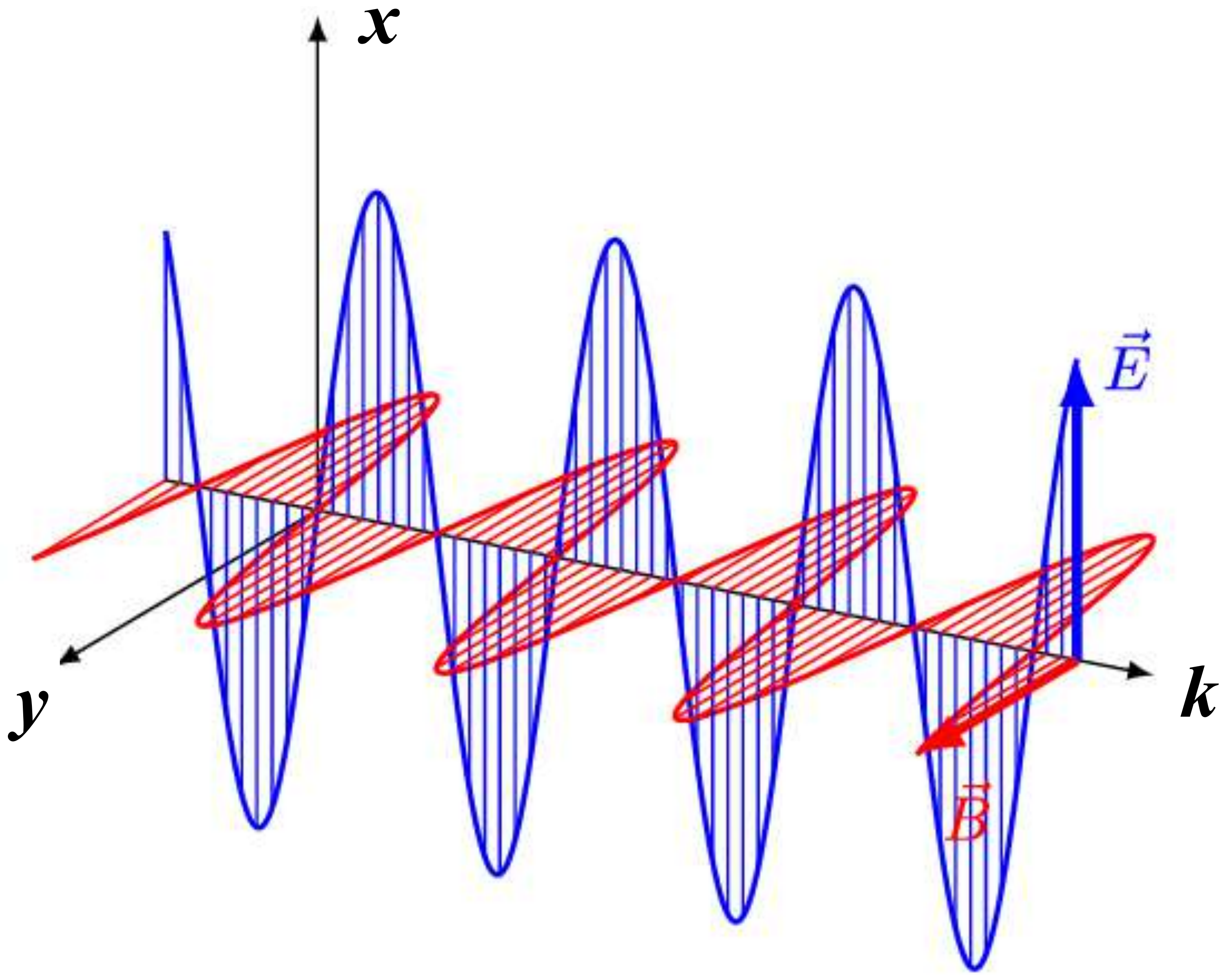
https://de.wikipedia.org/wiki/Dispersion_(Physik)



Electrodynamic theory

↓

- Light is transverse electromagnetic wave, described by oscillating electric and magnetic fields



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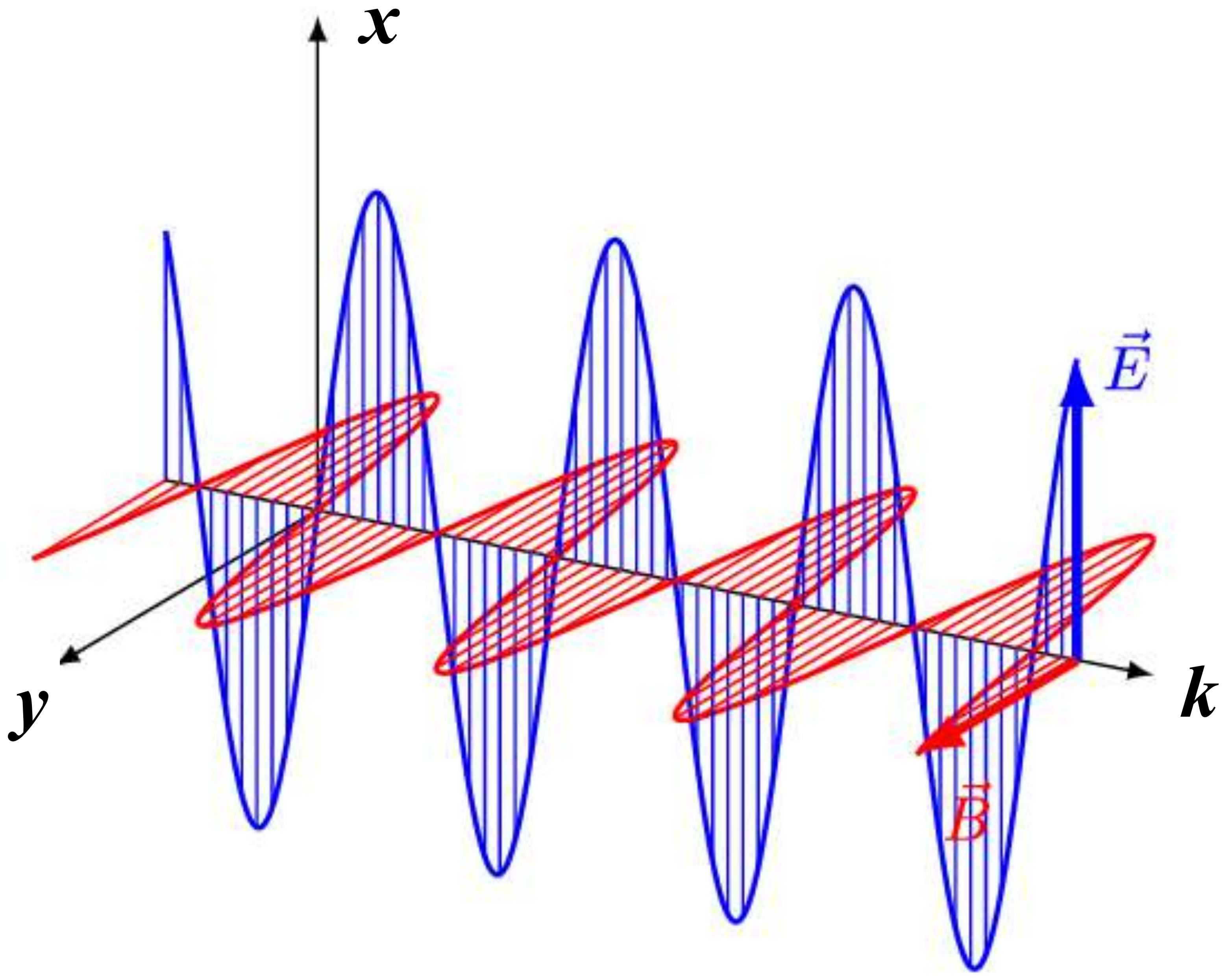

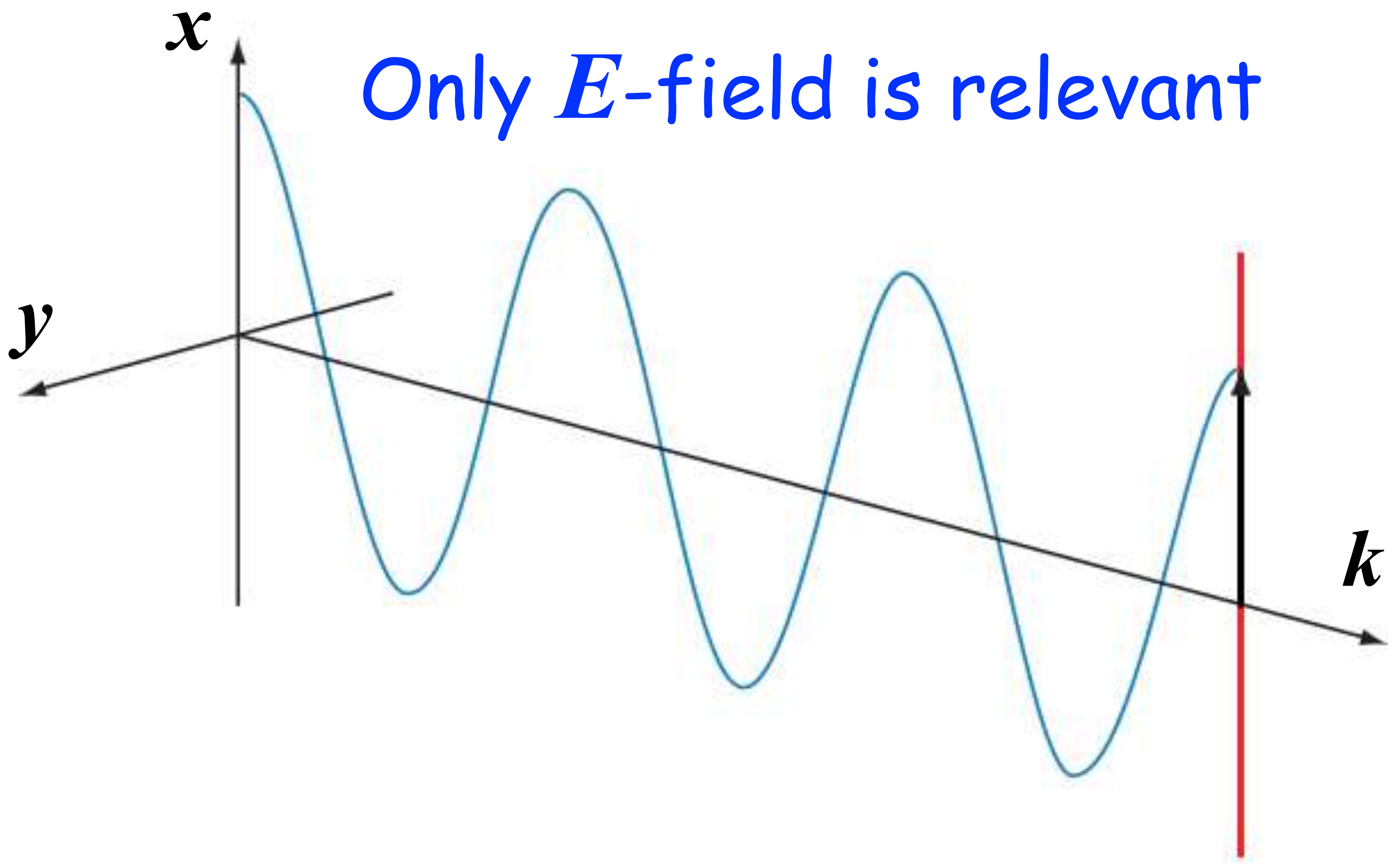

Electrodynamic theory

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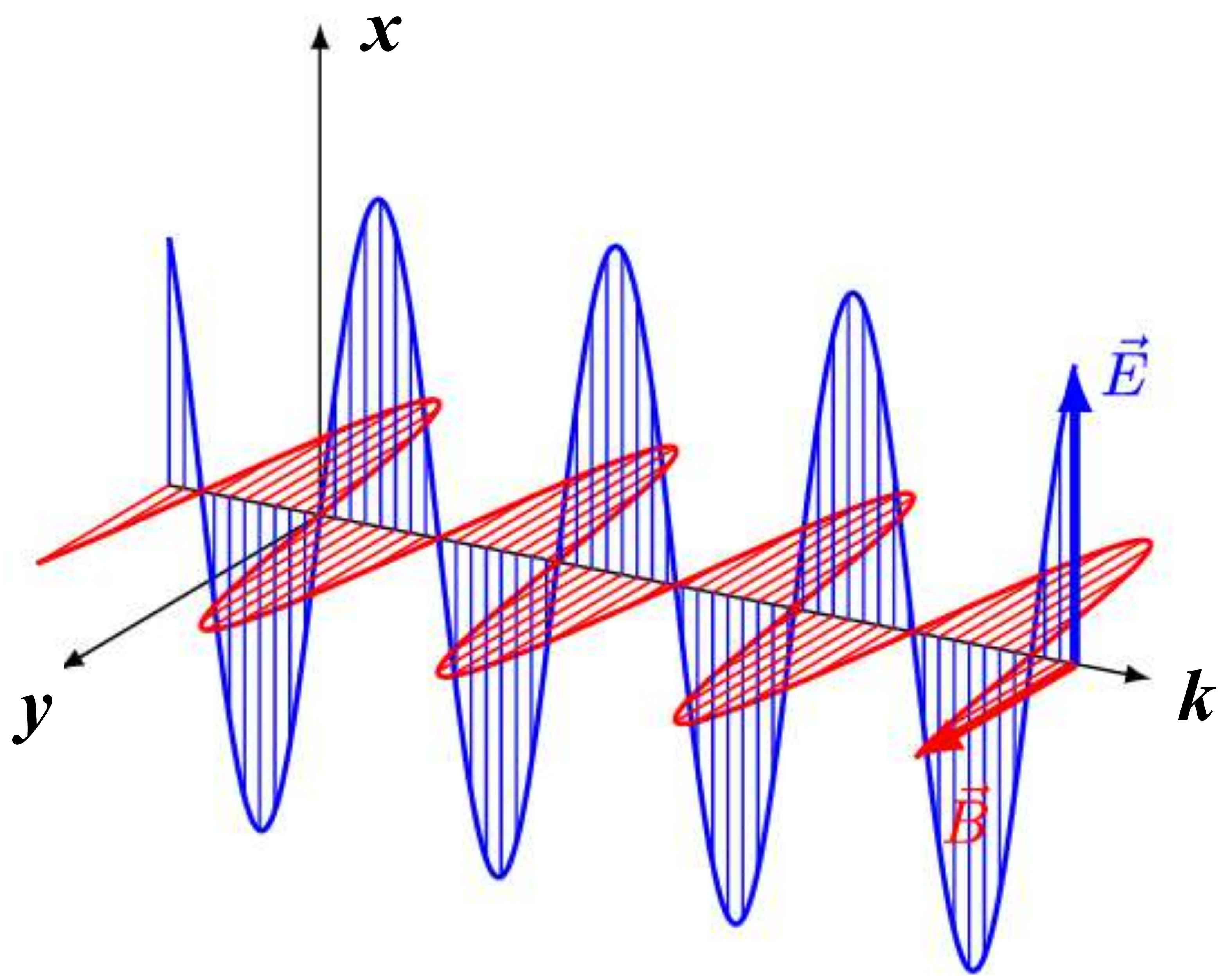
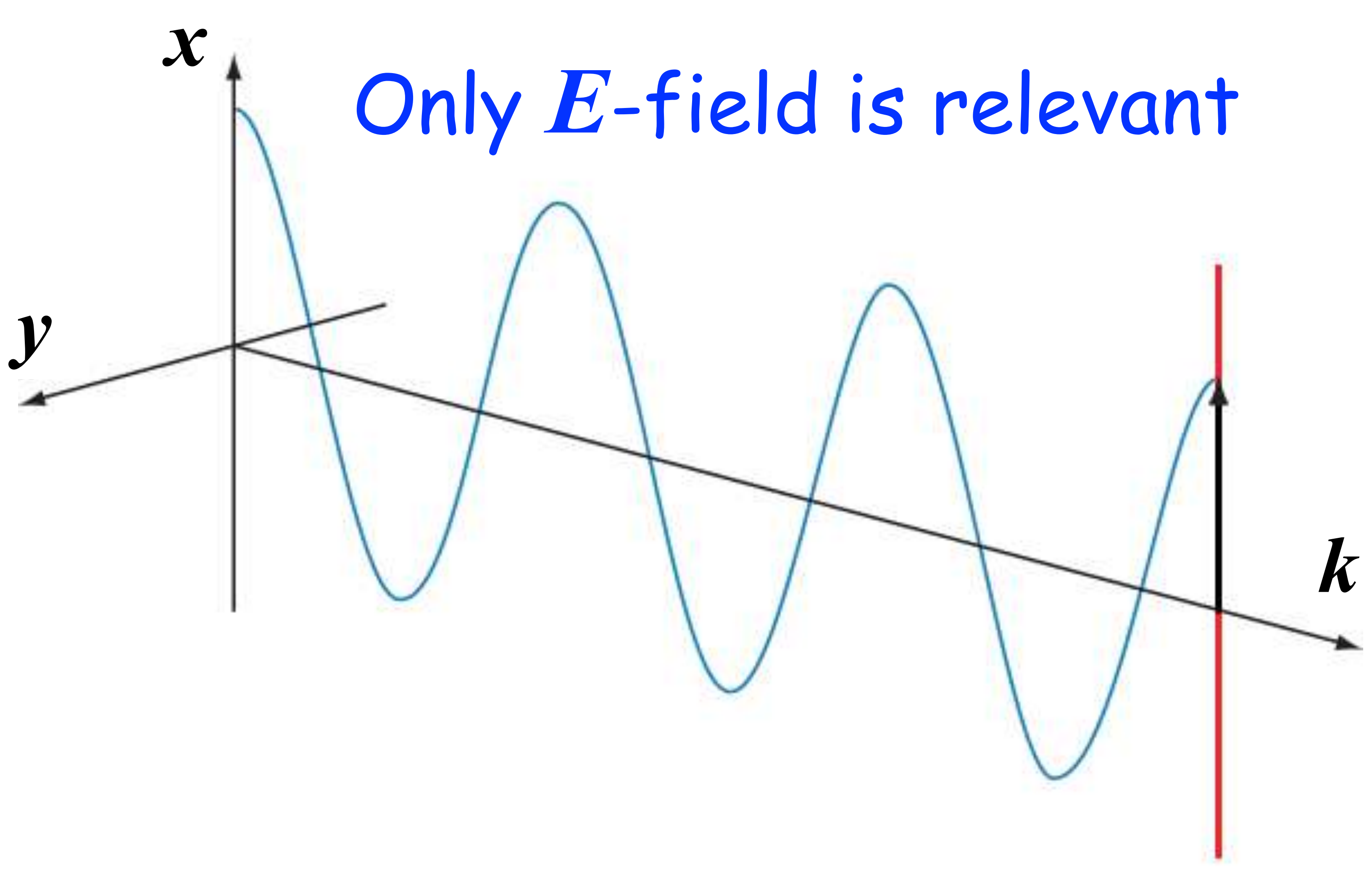
- Light is transverse electromagnetic wave, described by oscillating electric and magnetic fields
- Electric field acts much stronger with matter $\Longrightarrow$ polarization direction of light wave is conventionally described by its $\boldsymbol{E}$-vector or by its $\boldsymbol{D}$-vector in case of **anisotropic** media
- All relations, derived by electrodynmaic theory, are valid for both, transparent (dielectric) media as well as absorbing (conductive) materials
- Conductivity is simply taken into account by introducing complex dielectric constant and refraction index
- Due to transverse nature: variation of $\boldsymbol{E}$-vector is confined to plane perpendicular to $\boldsymbol{k} \Longrightarrow$ express wave in 2D-basis with x-and y-directions as unit vectors ...





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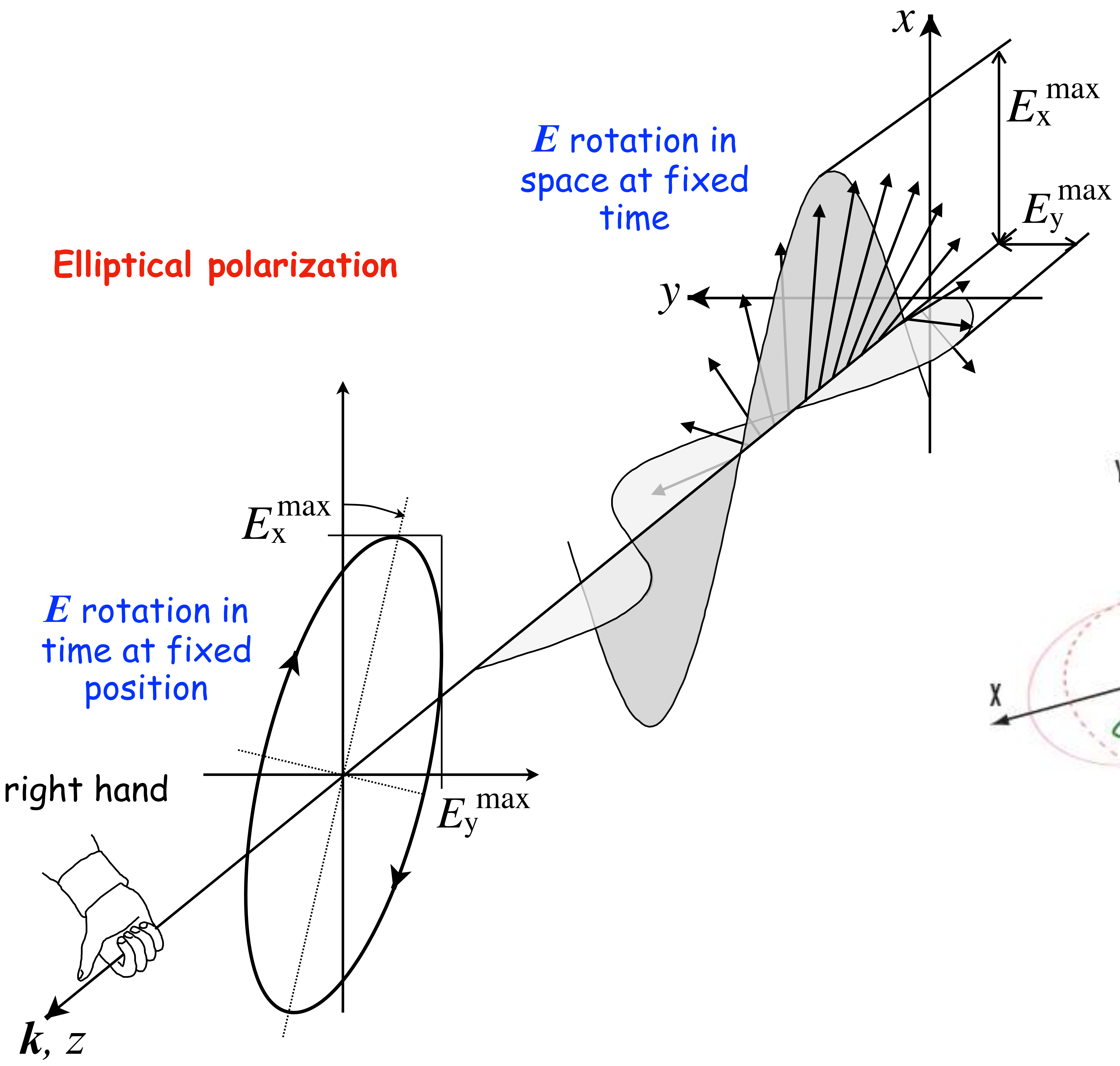


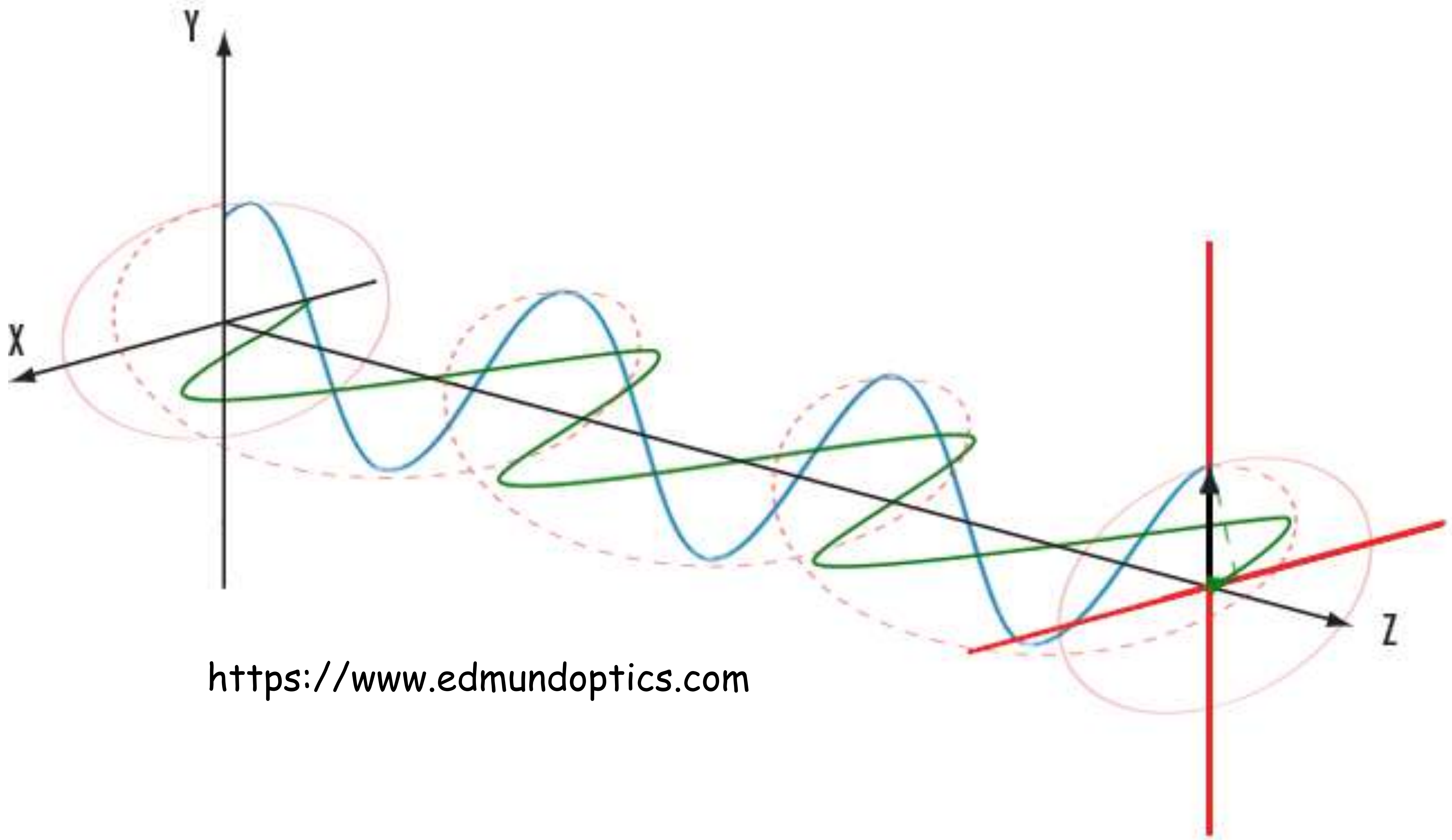
Polarized light (general)

$$E_j(z,t) = \mathrm{Re}\left(e_j E_j^0 \exp[\mathrm{i}(\boldsymbol{k}_z z - \omega t)]\right) \qquad j = \{x,y\}$$

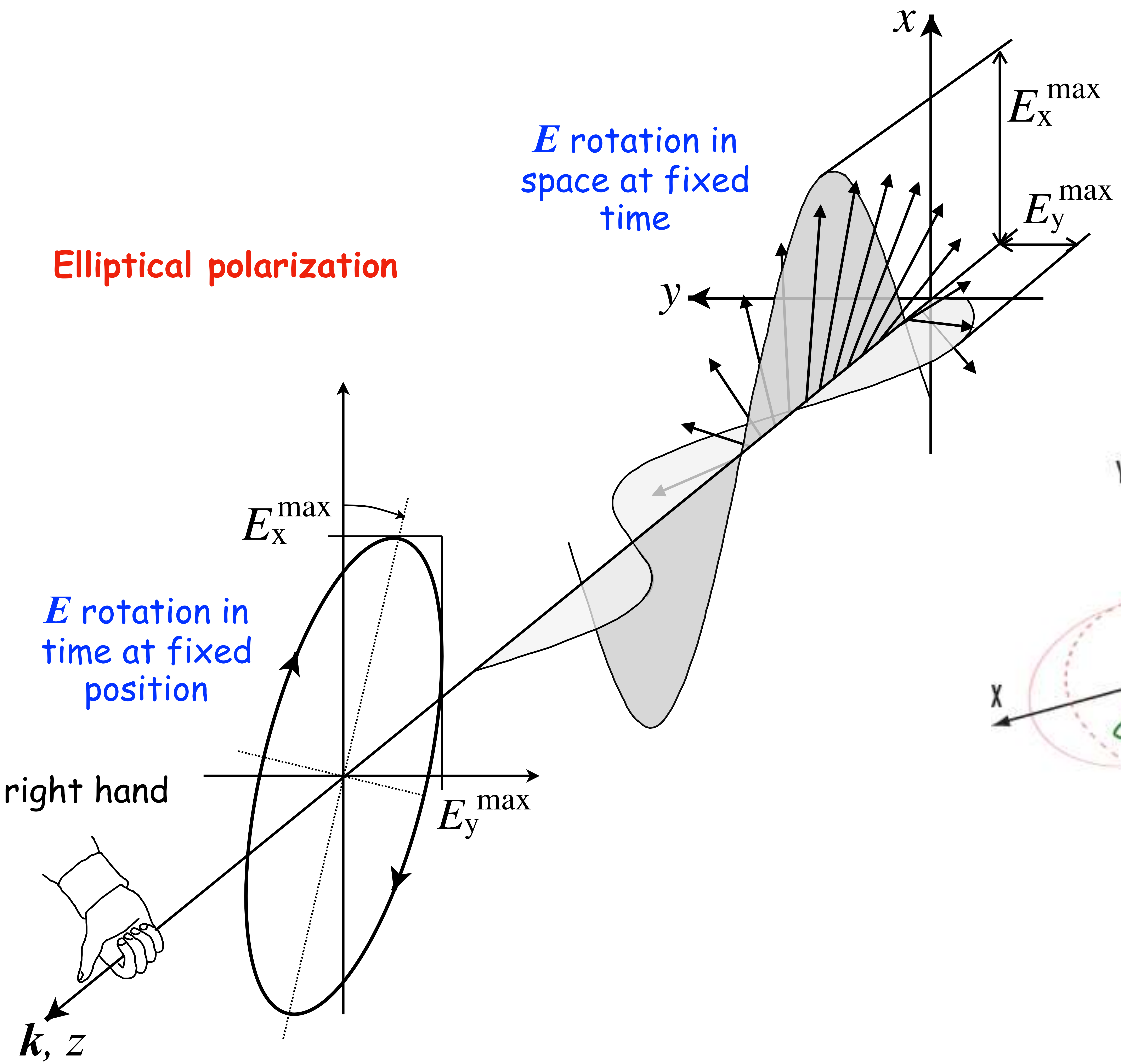
with $E_j^0 = E_j^{max} \exp[\mathrm{i}\,\delta_j]\big)$

e_x, e_y: unit vectors along x- and y

δ_j: phase retardations



https://www.edmundoptics.com



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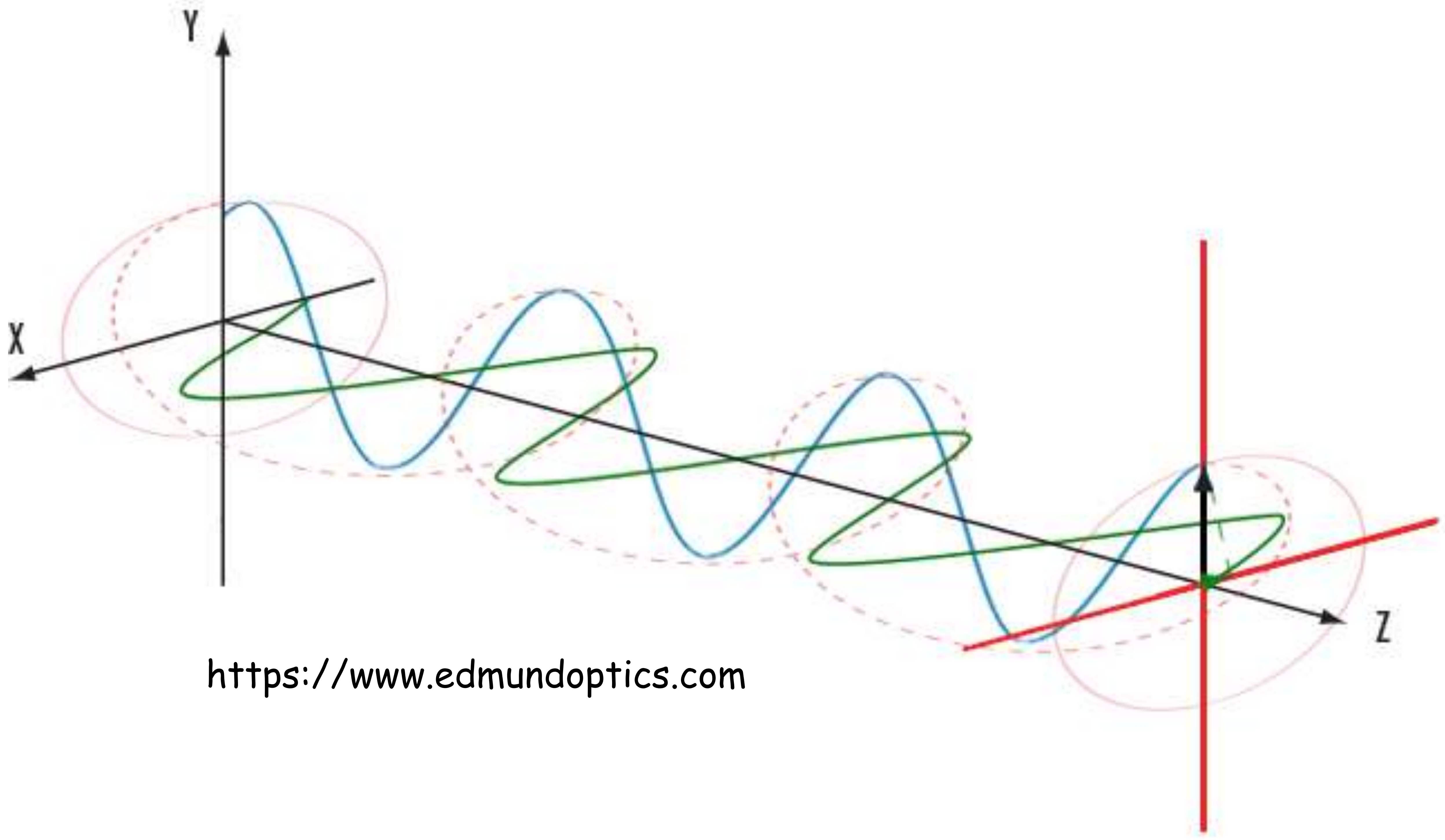

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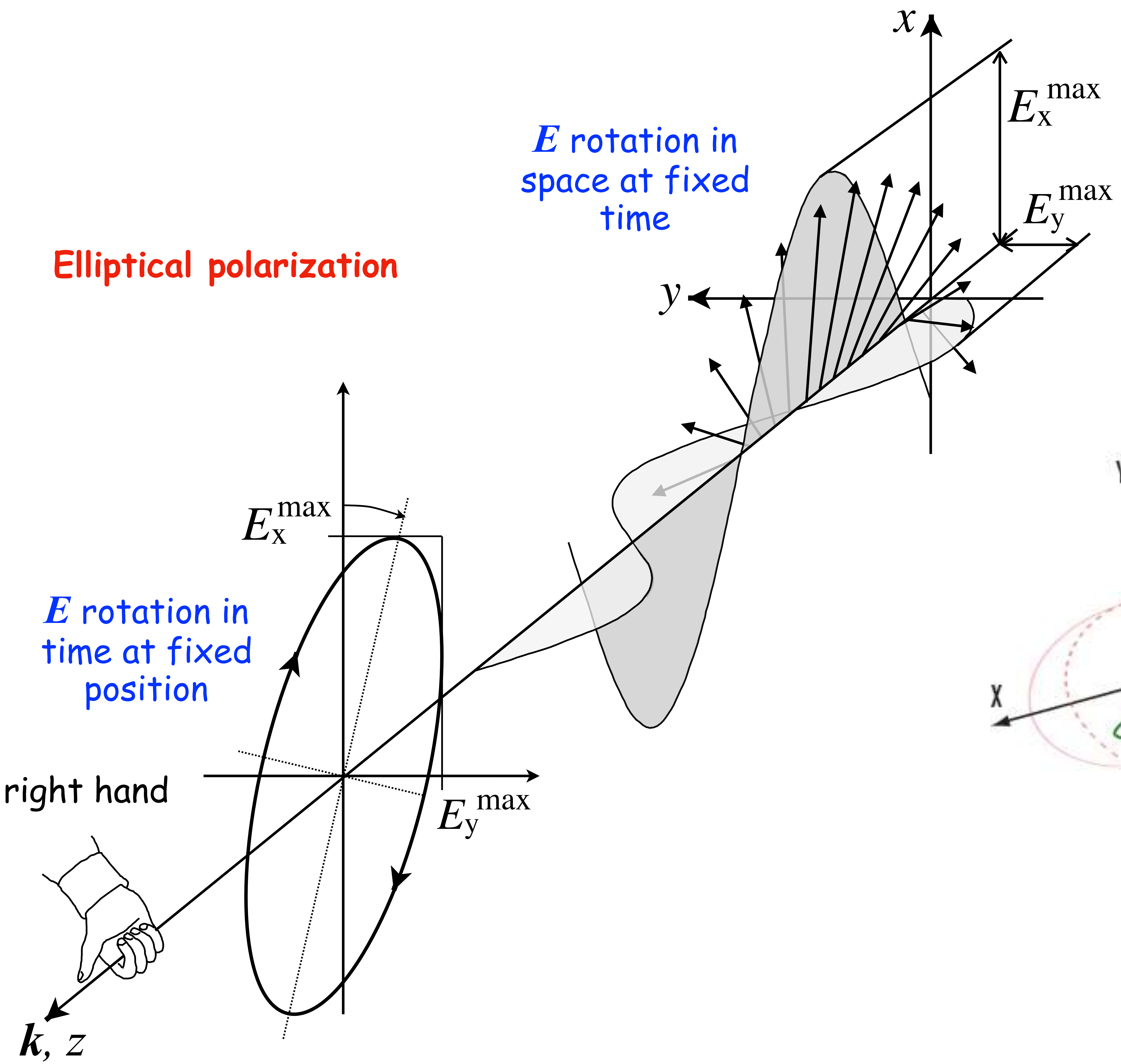
Elliptical polarization



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δ_j: phase retardations

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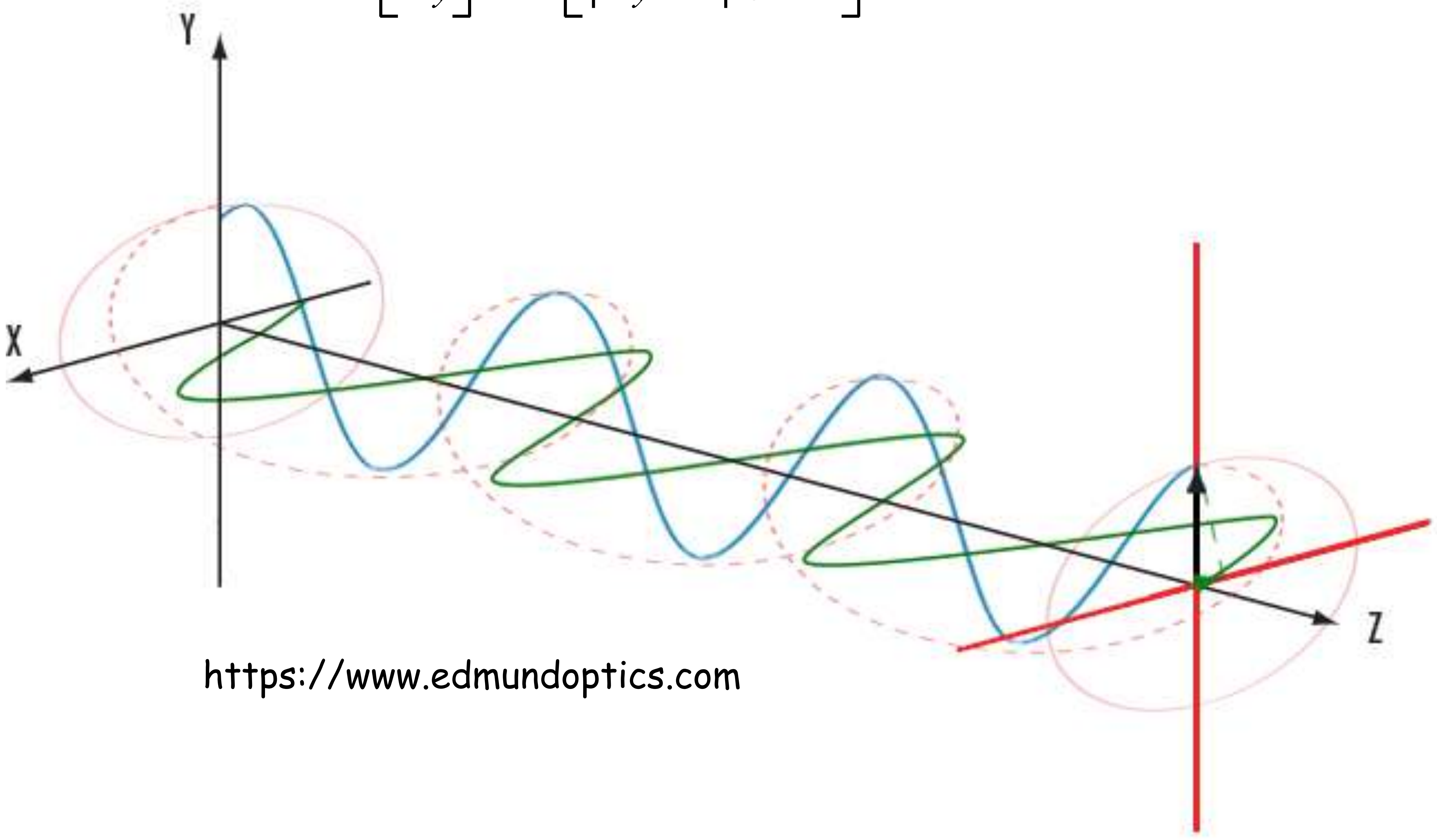

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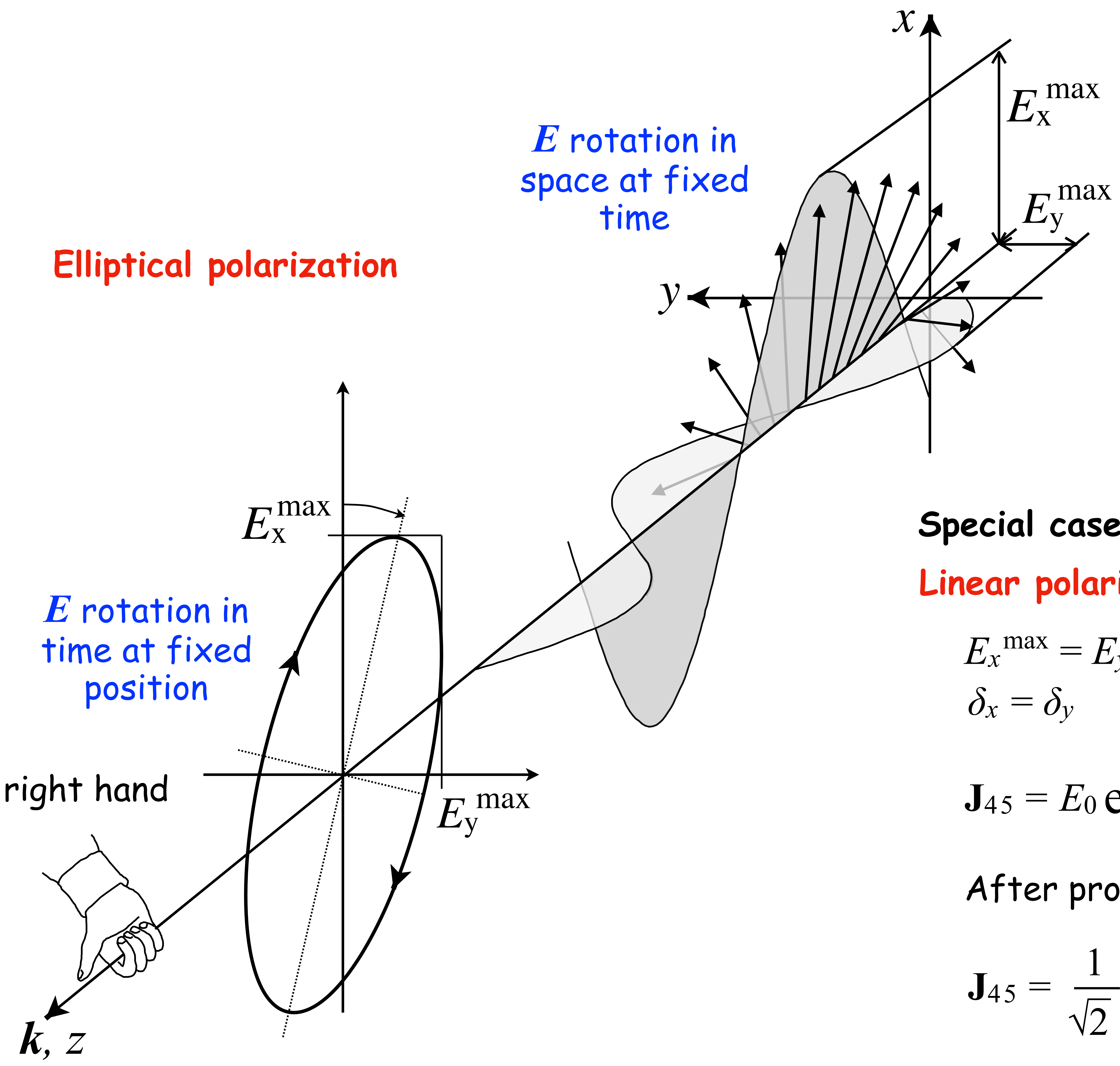

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Special cases:

Linear polarization

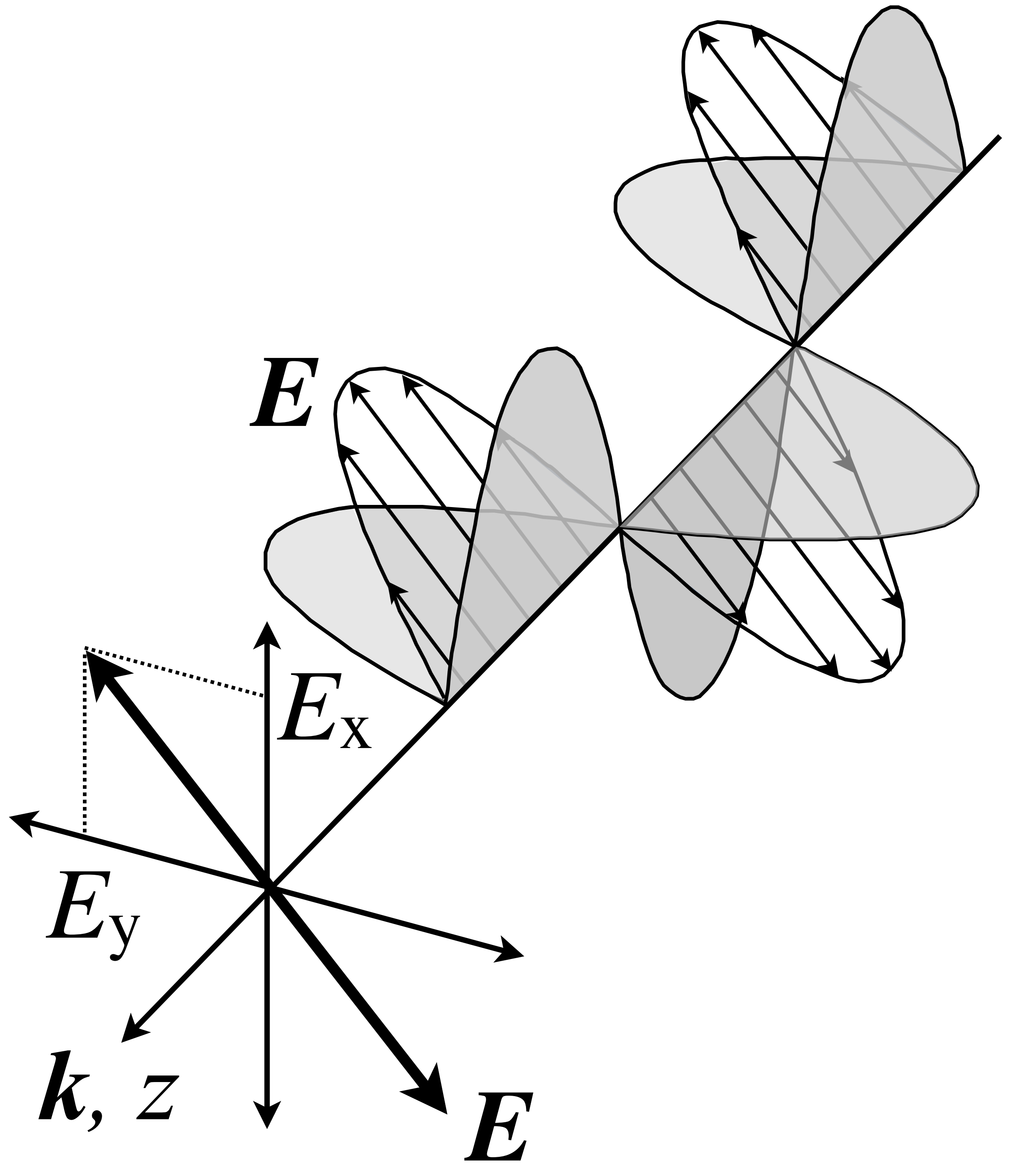
$$E_x{}^{max} = E_y{}^{max} = E_0$$

$$\delta_x = \delta_y$$

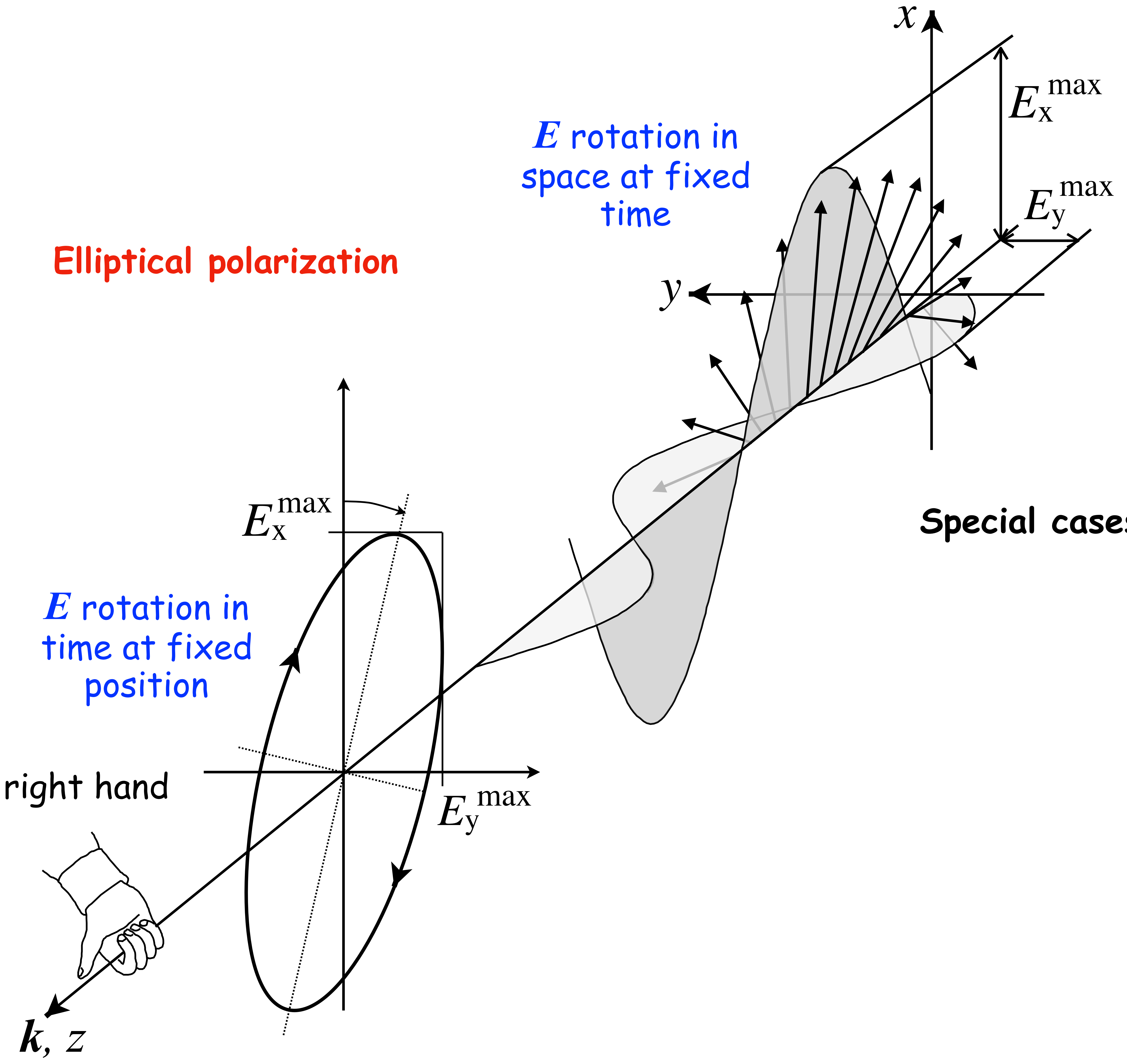
$$\mathbf{J}_{45} = E_0 e^{\mathrm{i}\delta_x} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

After proper normalization:

$$\mathbf{J}_{45} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Elliptical polarization



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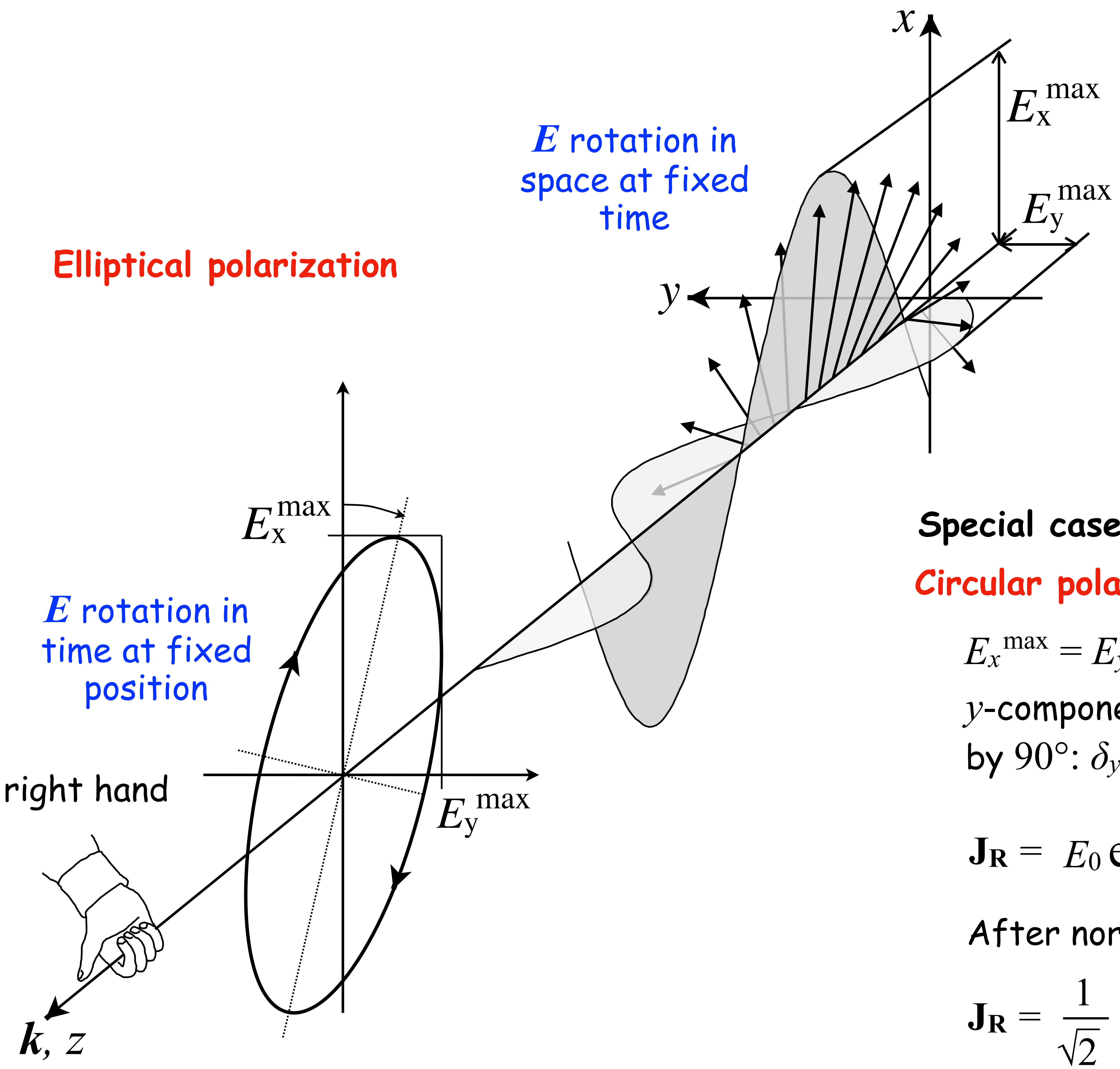

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Special cases:

Circular polarization

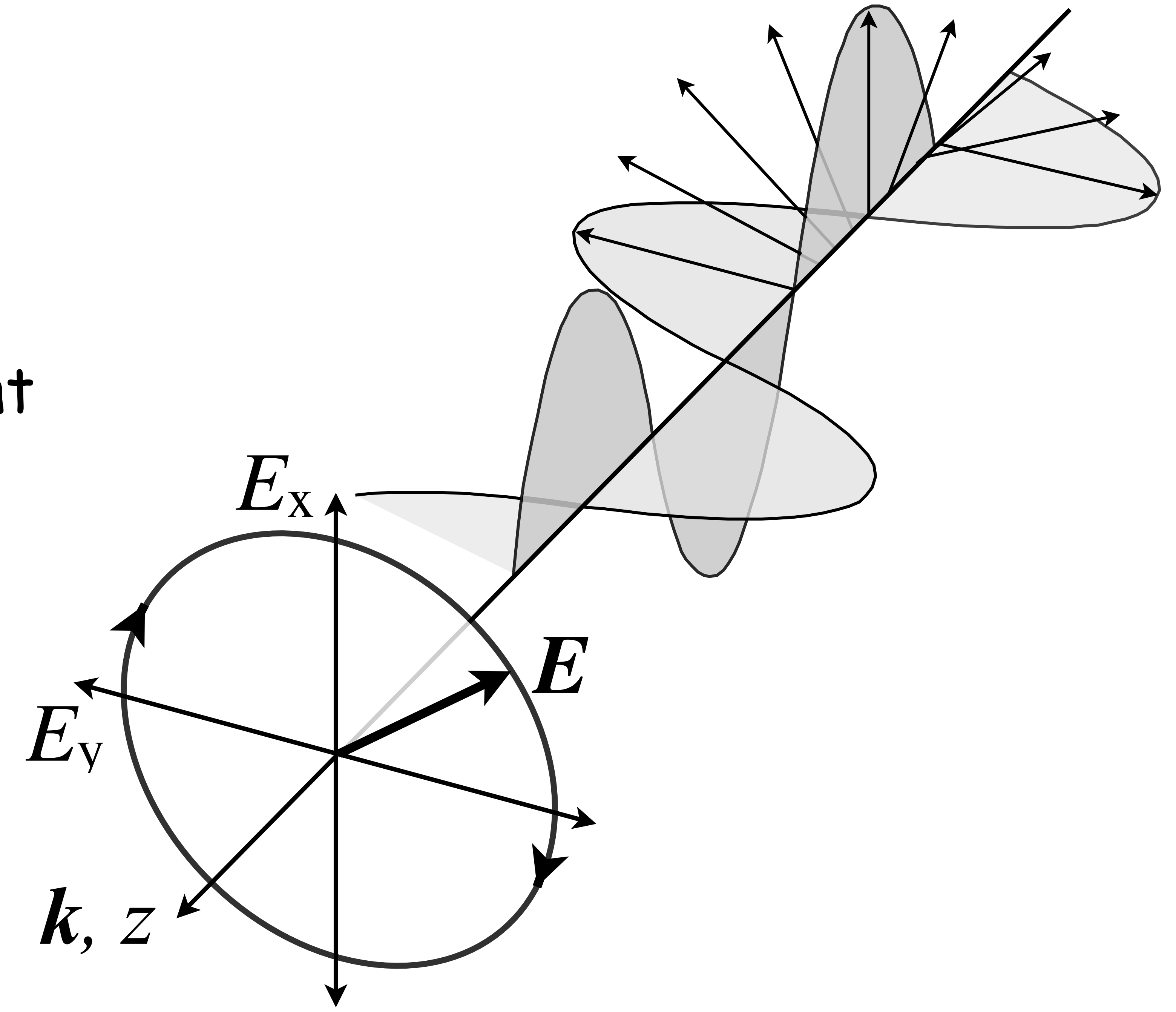
$E_x{}^{max} = E_y{}^{max} = E_0$

y-component leads x-component by 90°: $\delta_y = \delta_x - \pi/2$

$$\mathbf{J_R} = E_0\,\mathrm{e}^{\mathrm{i}\,\delta_x} \begin{bmatrix} 1 \\ \mathrm{e}^{-\mathrm{i}\pi/2} \end{bmatrix}$$

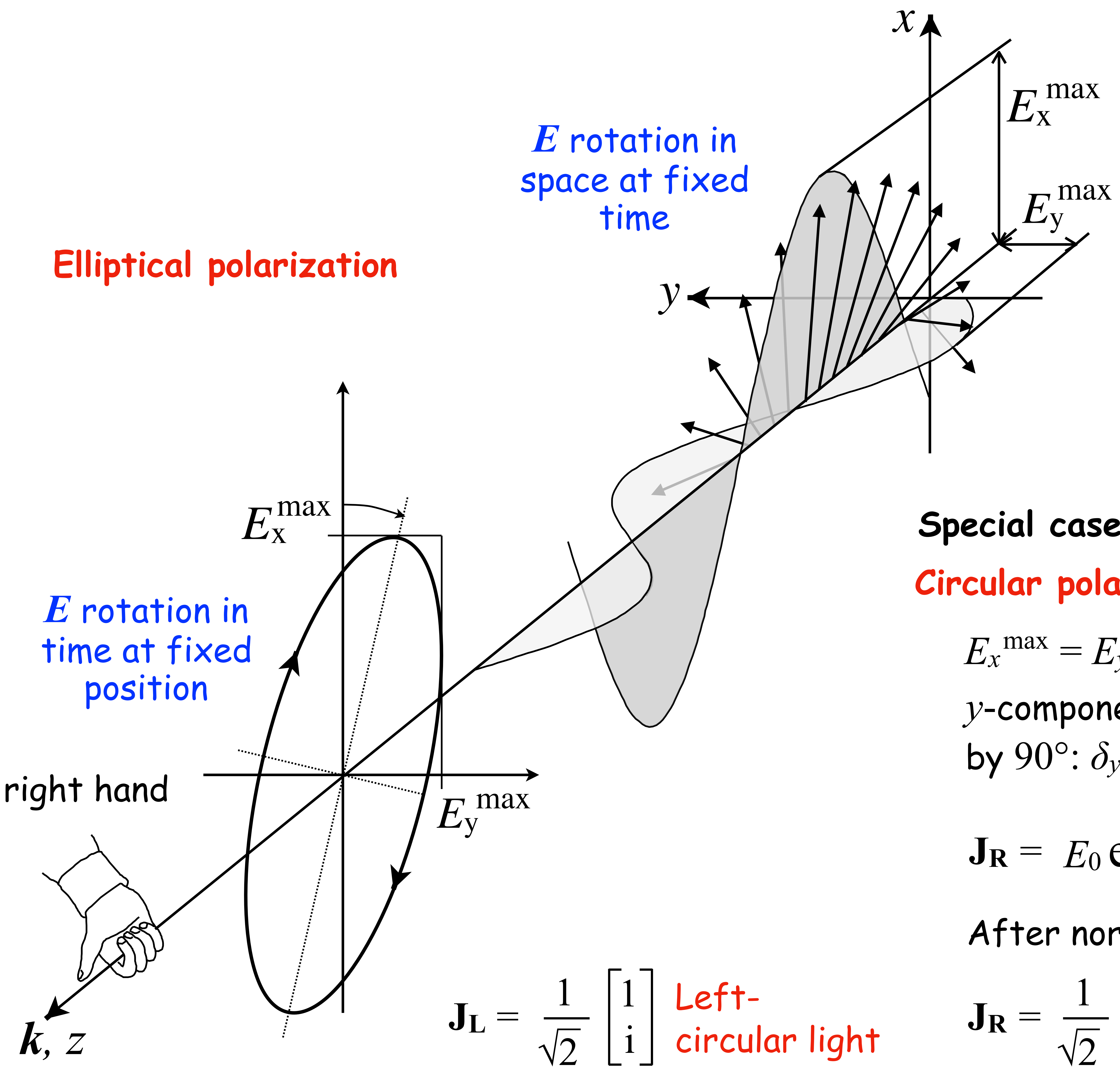
After normalization:

$$\mathbf{J_R} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -\mathrm{i} \end{bmatrix}$$ Right-circular light



1. Optical Basics – 1.2 Polarized Light

Elliptical polarization



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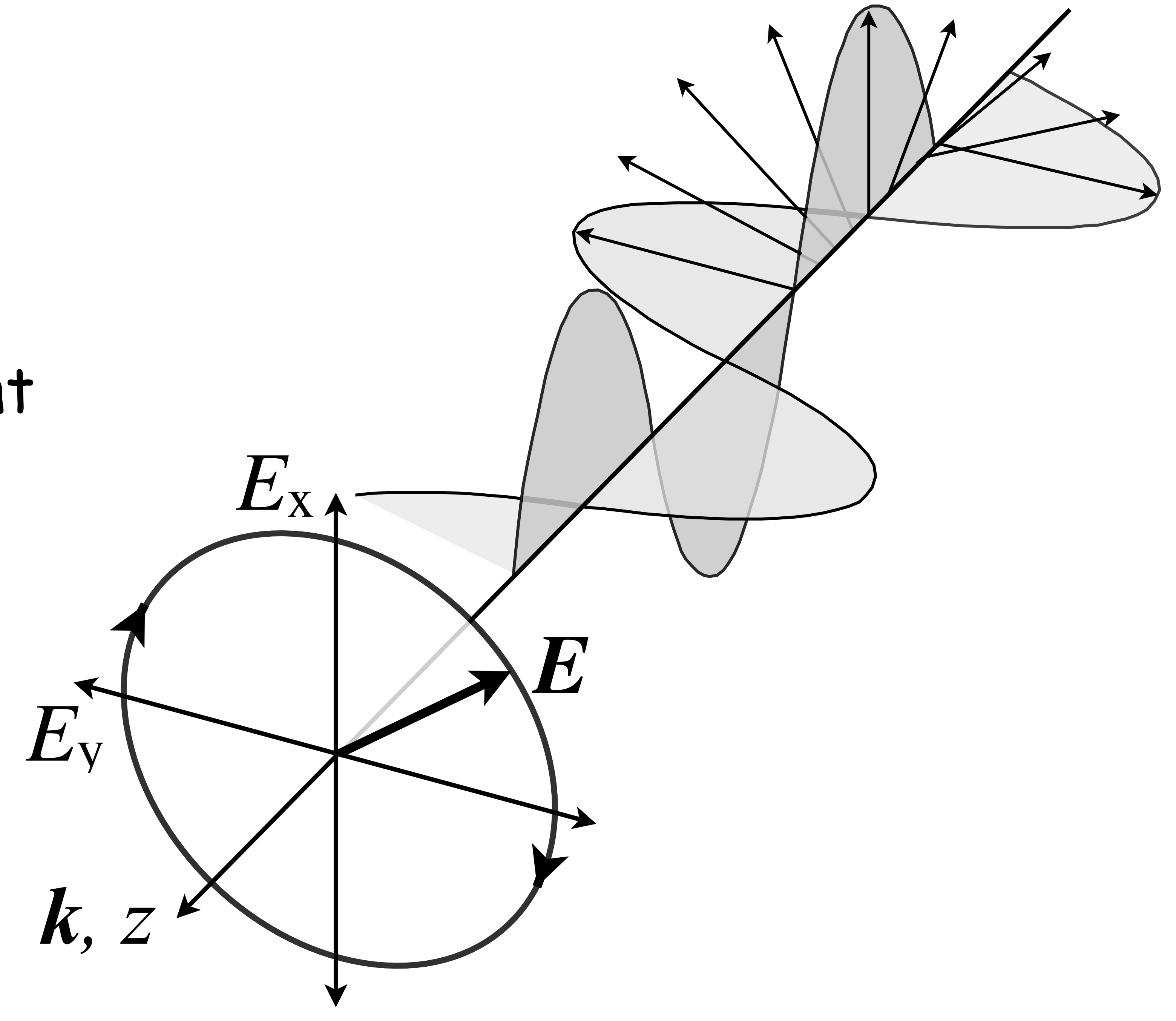

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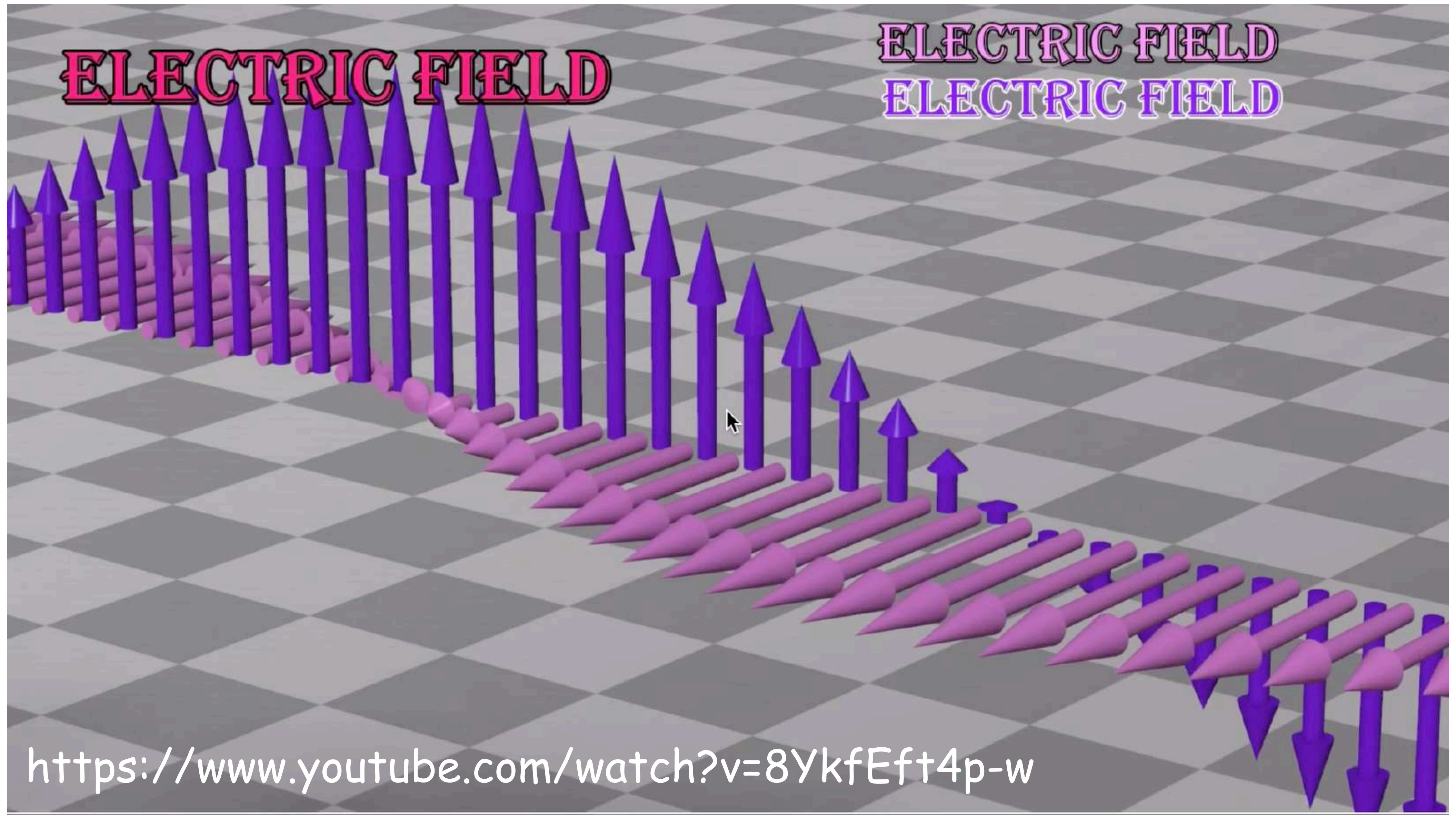

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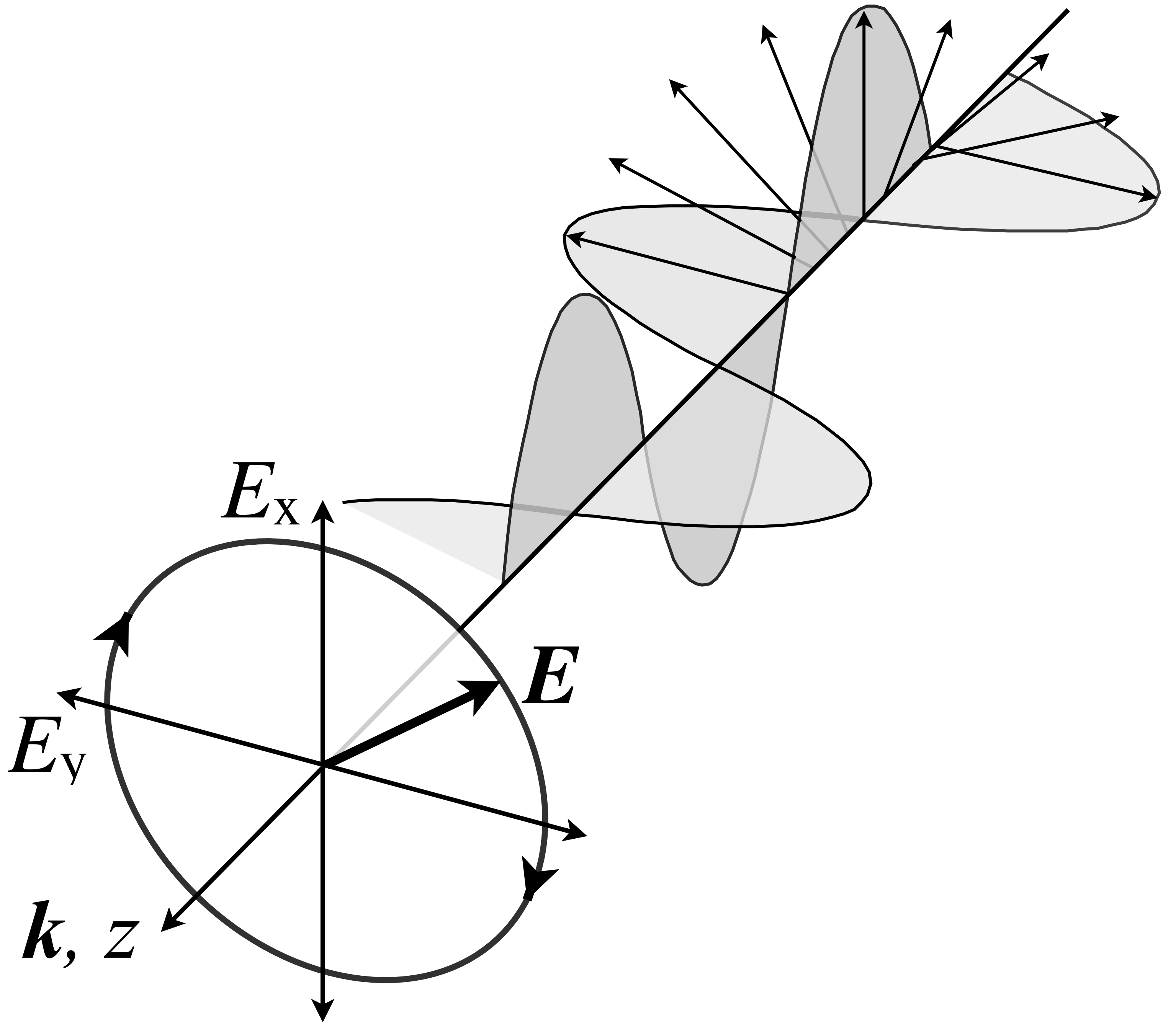

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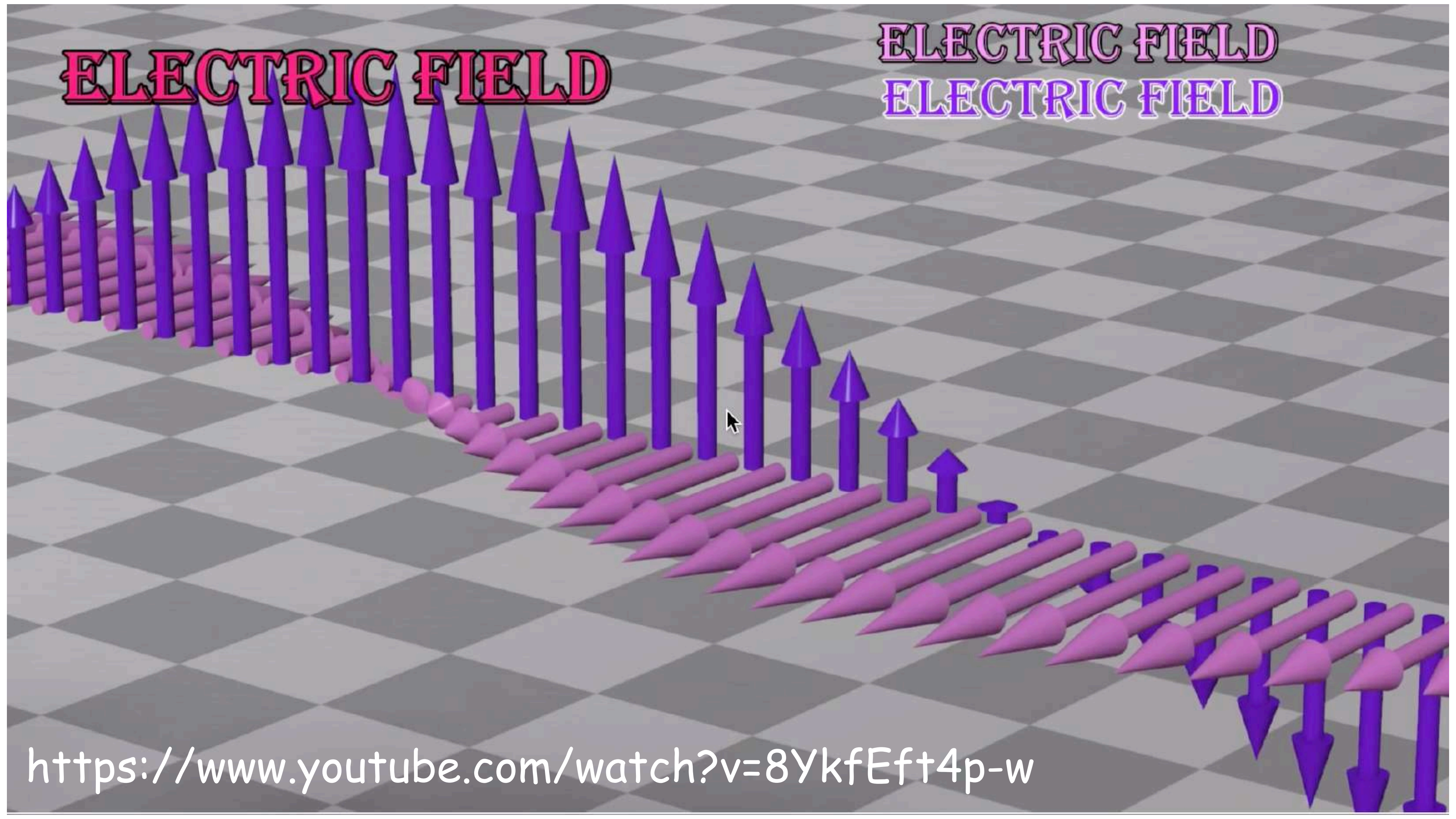

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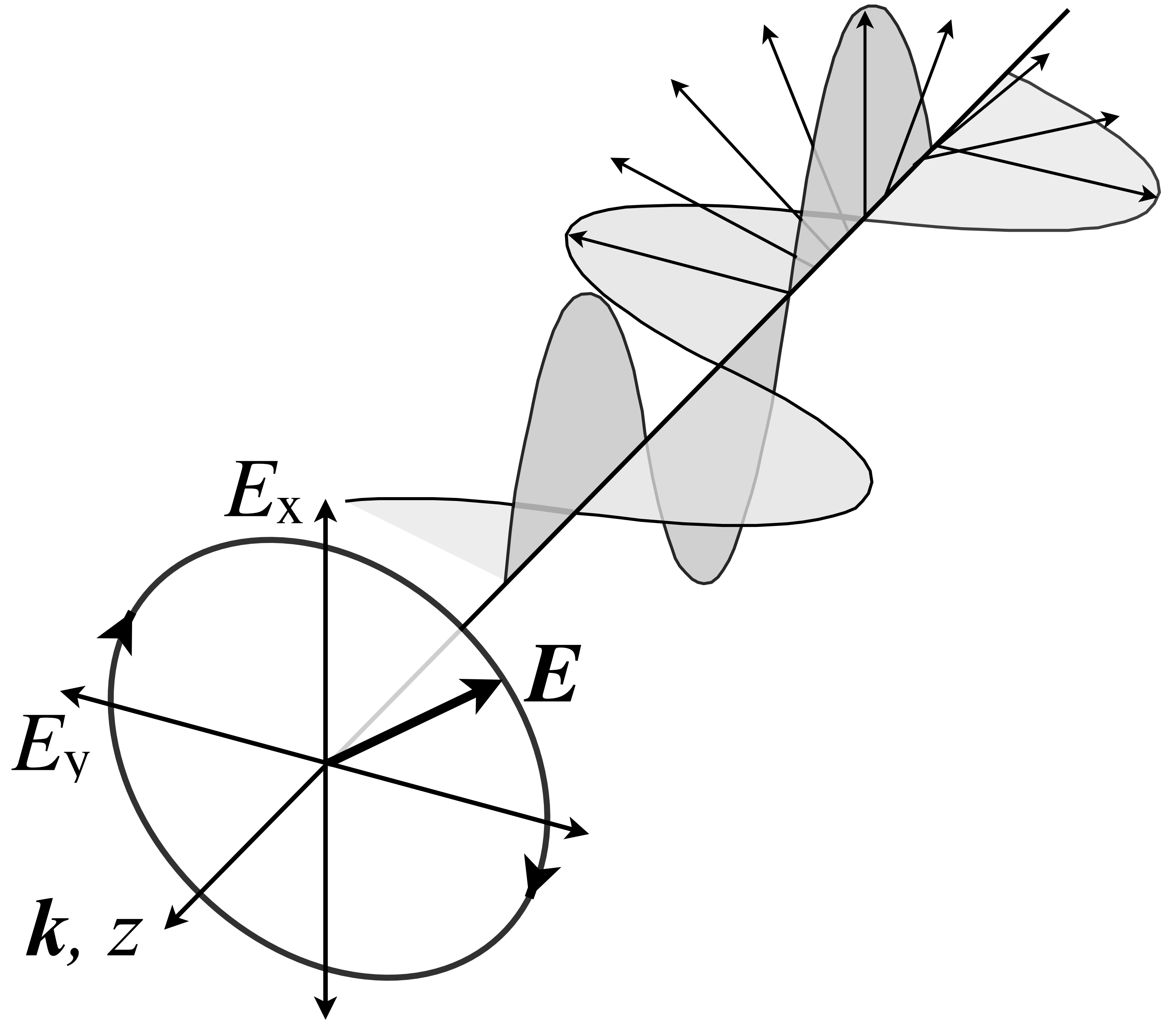
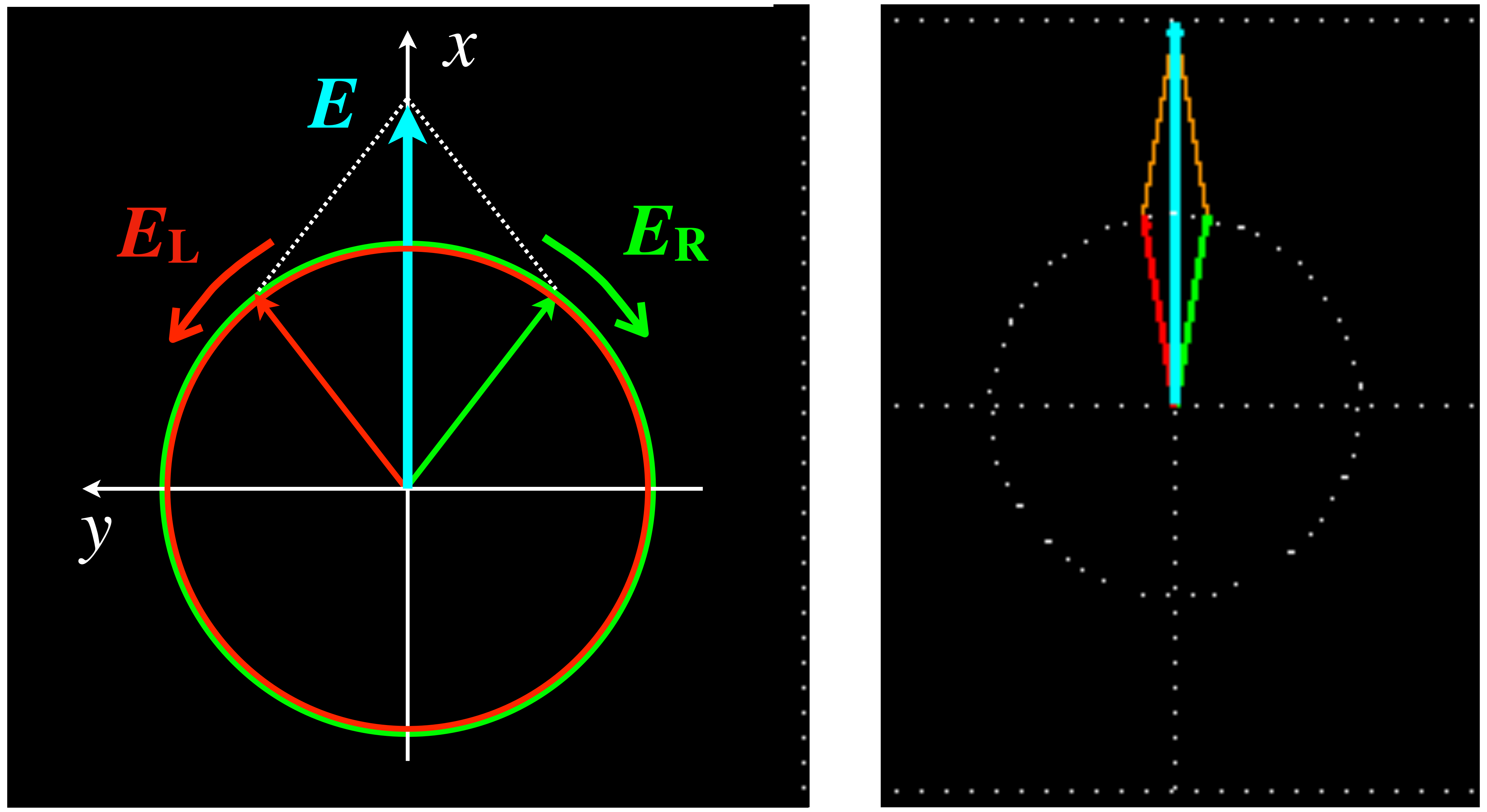
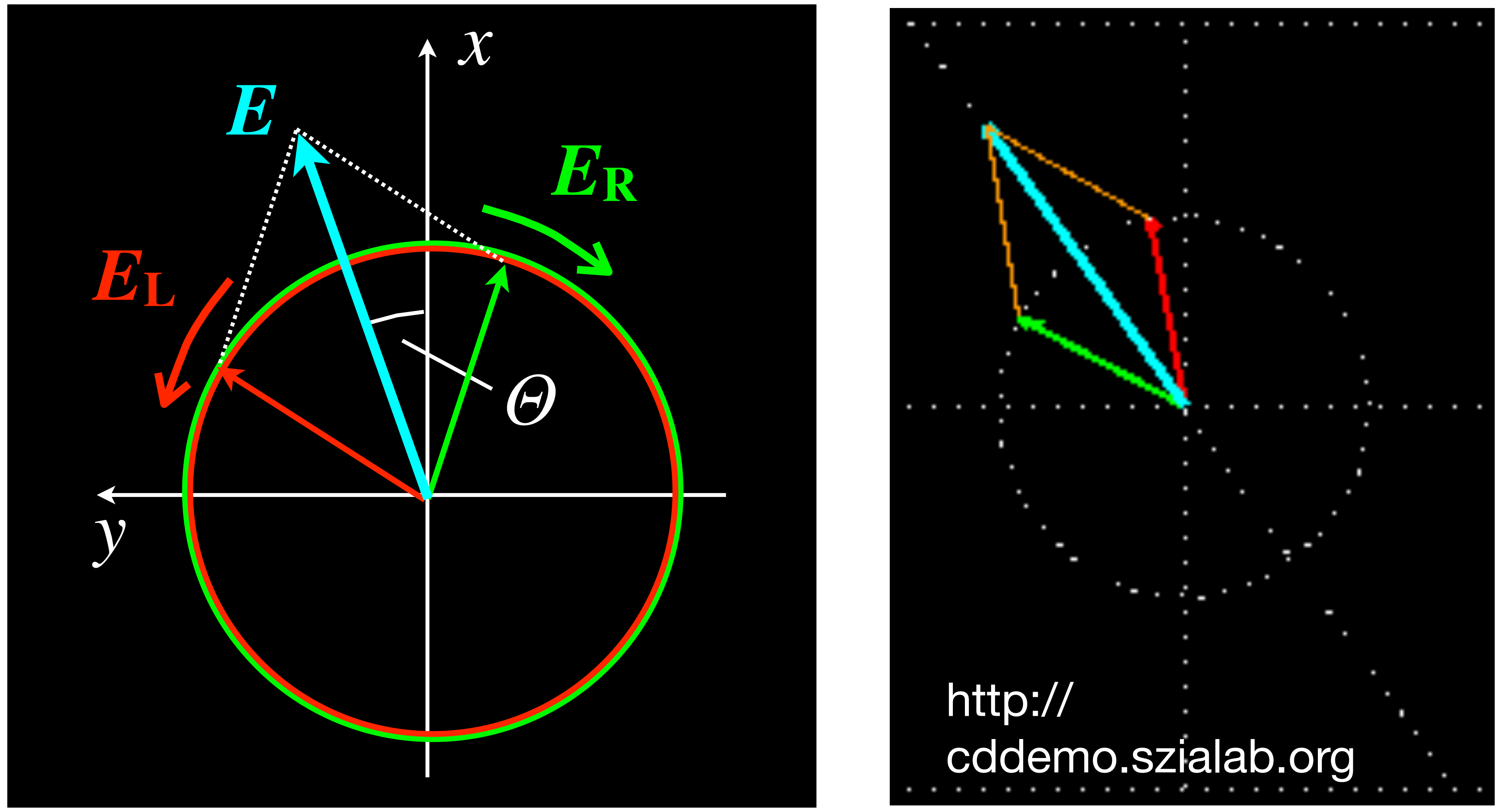


1. Optical Basics – 1.2 Polarized Light

Interpretation of polarized light: superposition of right- and left-handed circularly polarized waves



Equal amplitude and equal phase:
⟹ linearly polarized wave along x-axis

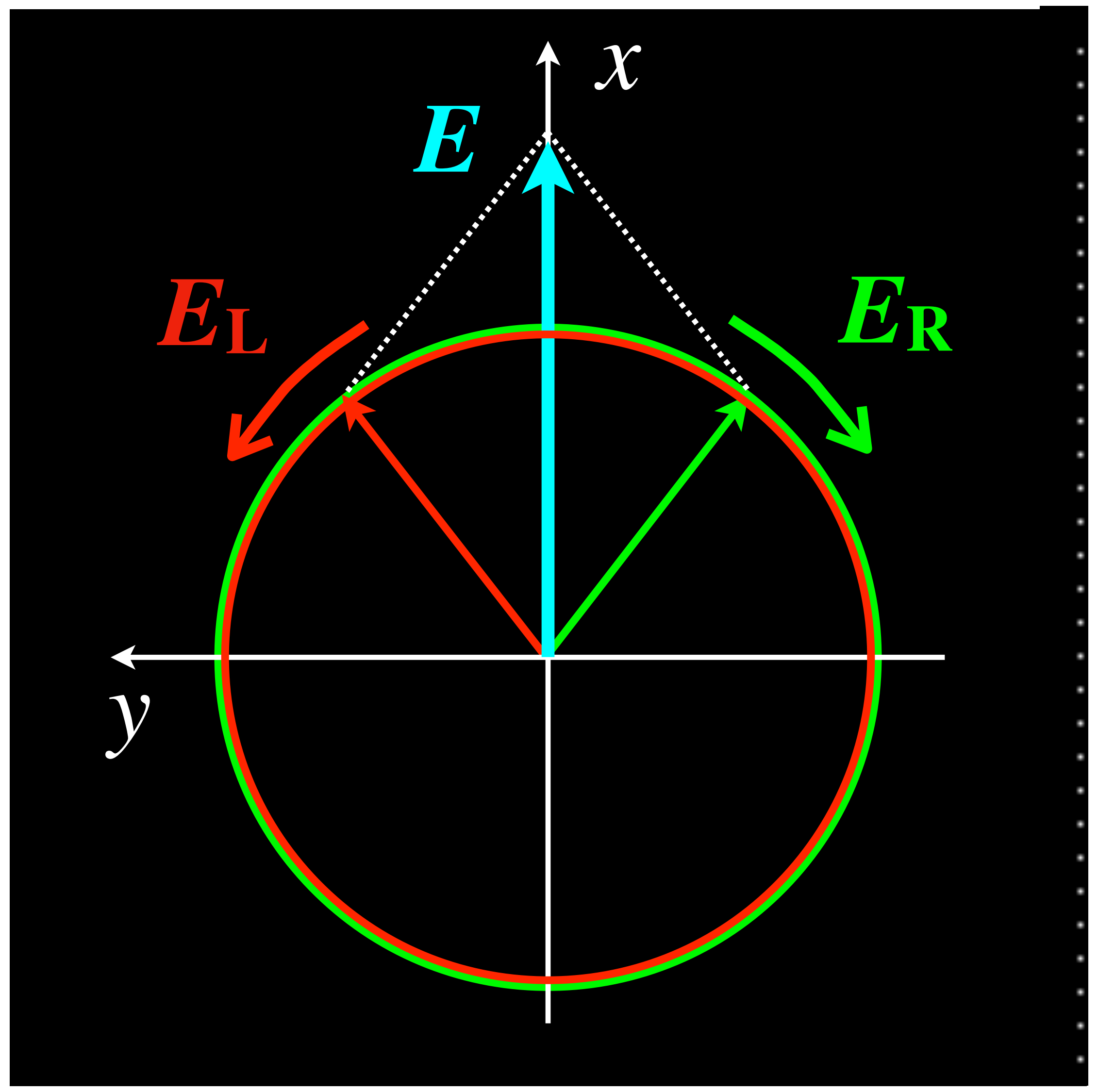


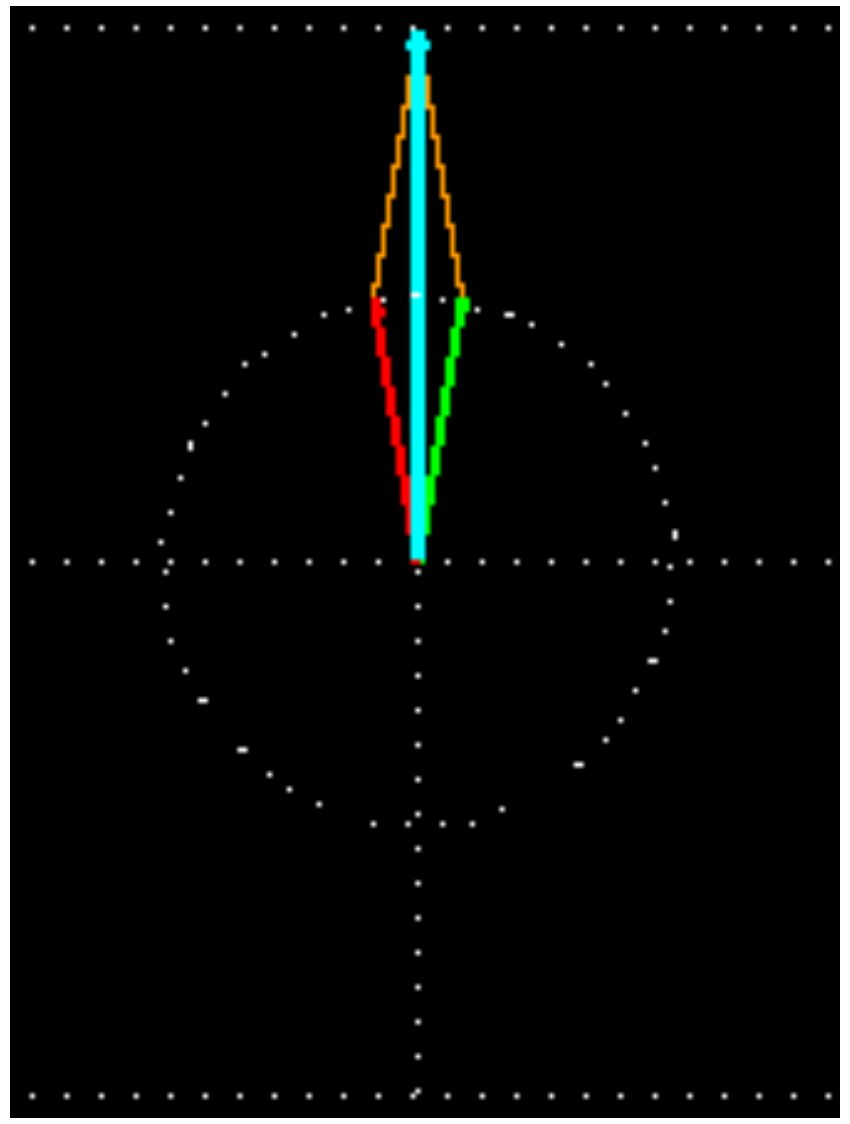
Equal amplitude, but different phase:
⟹ linearly polarized wave along tilted axis

Phase difference: caused by different refraction indices for partial waves
⟹ different velocities

In general, a material that displays two different indices of refraction is said to be **birefringent**

Interpretation of polarized light: superposition of right- and left-handed circularly polarized waves





Equal amplitude and equal phase:

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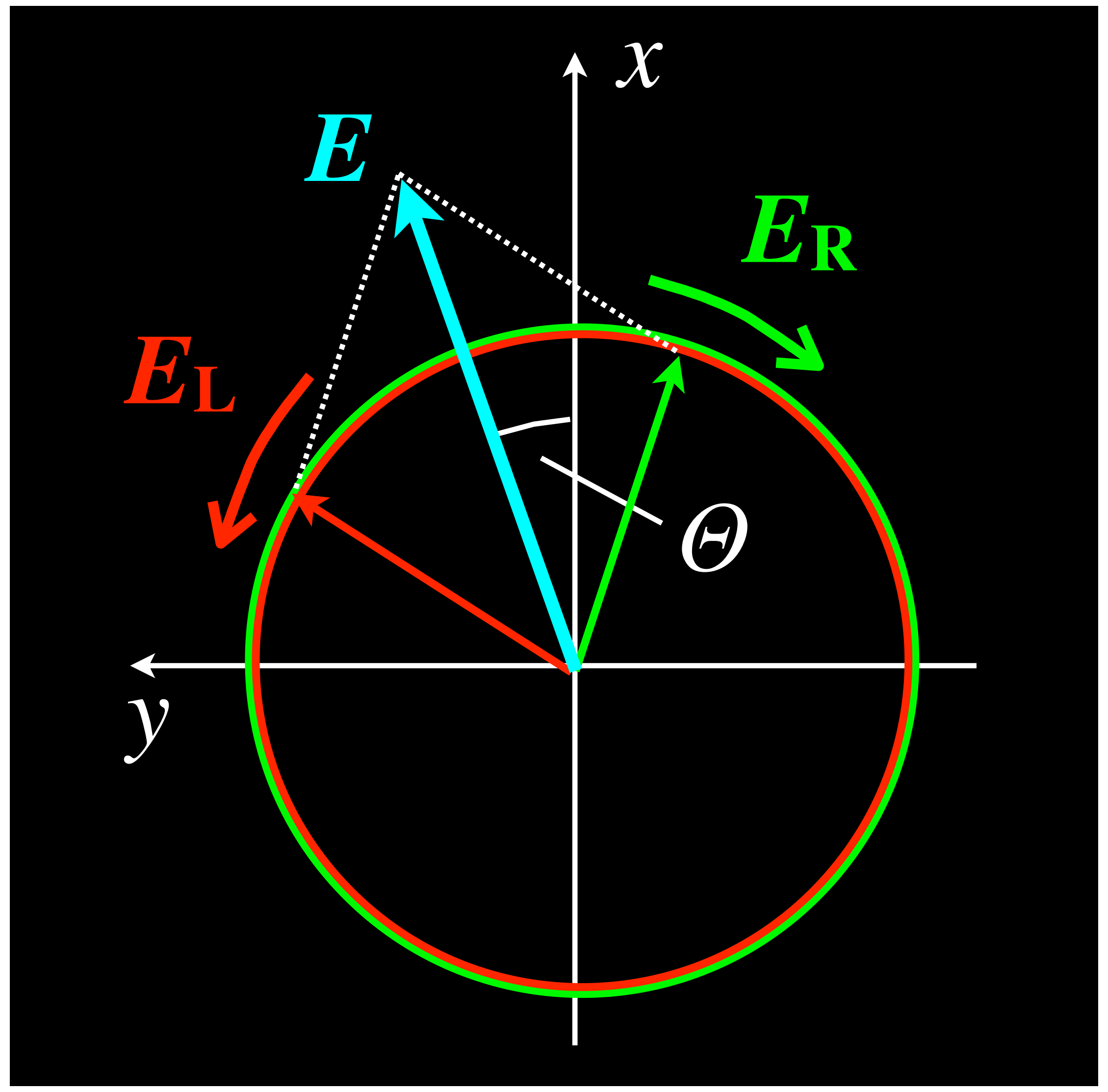

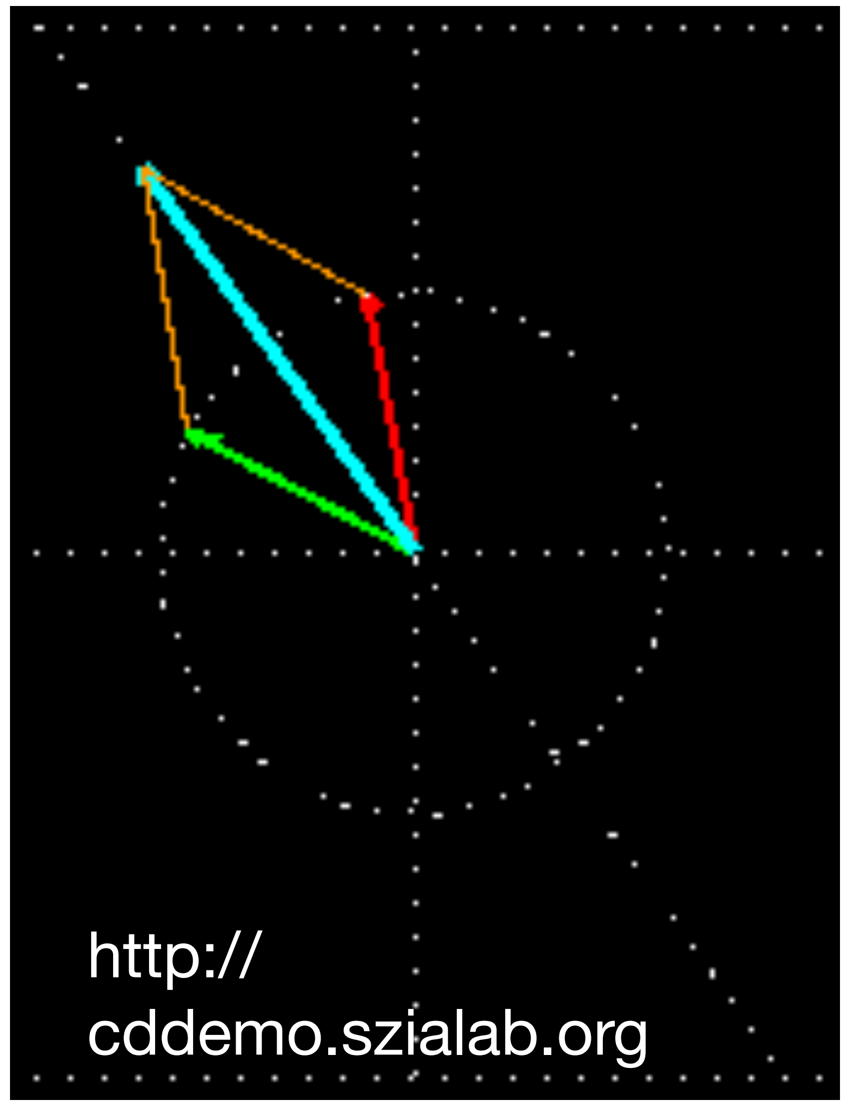

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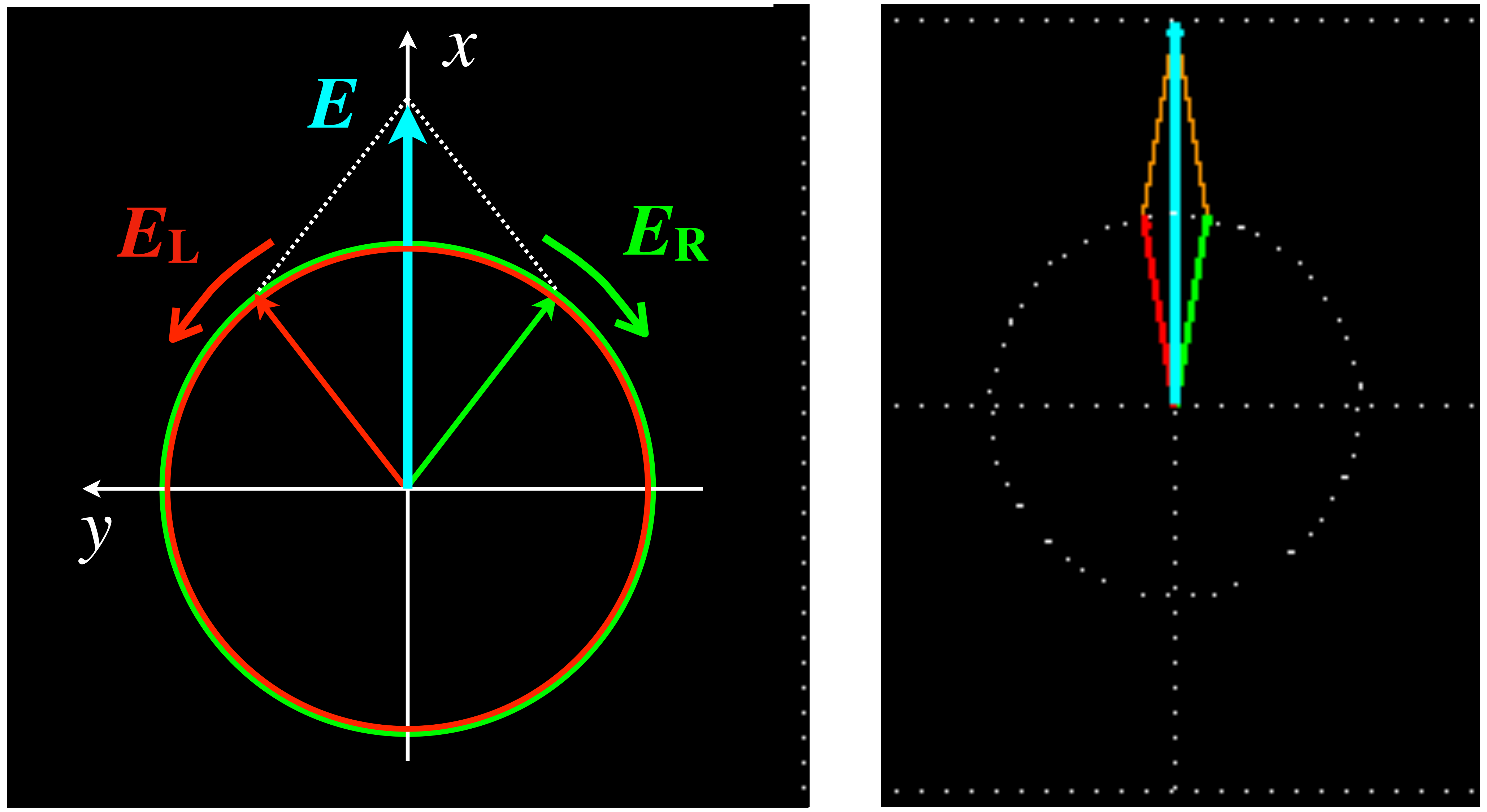

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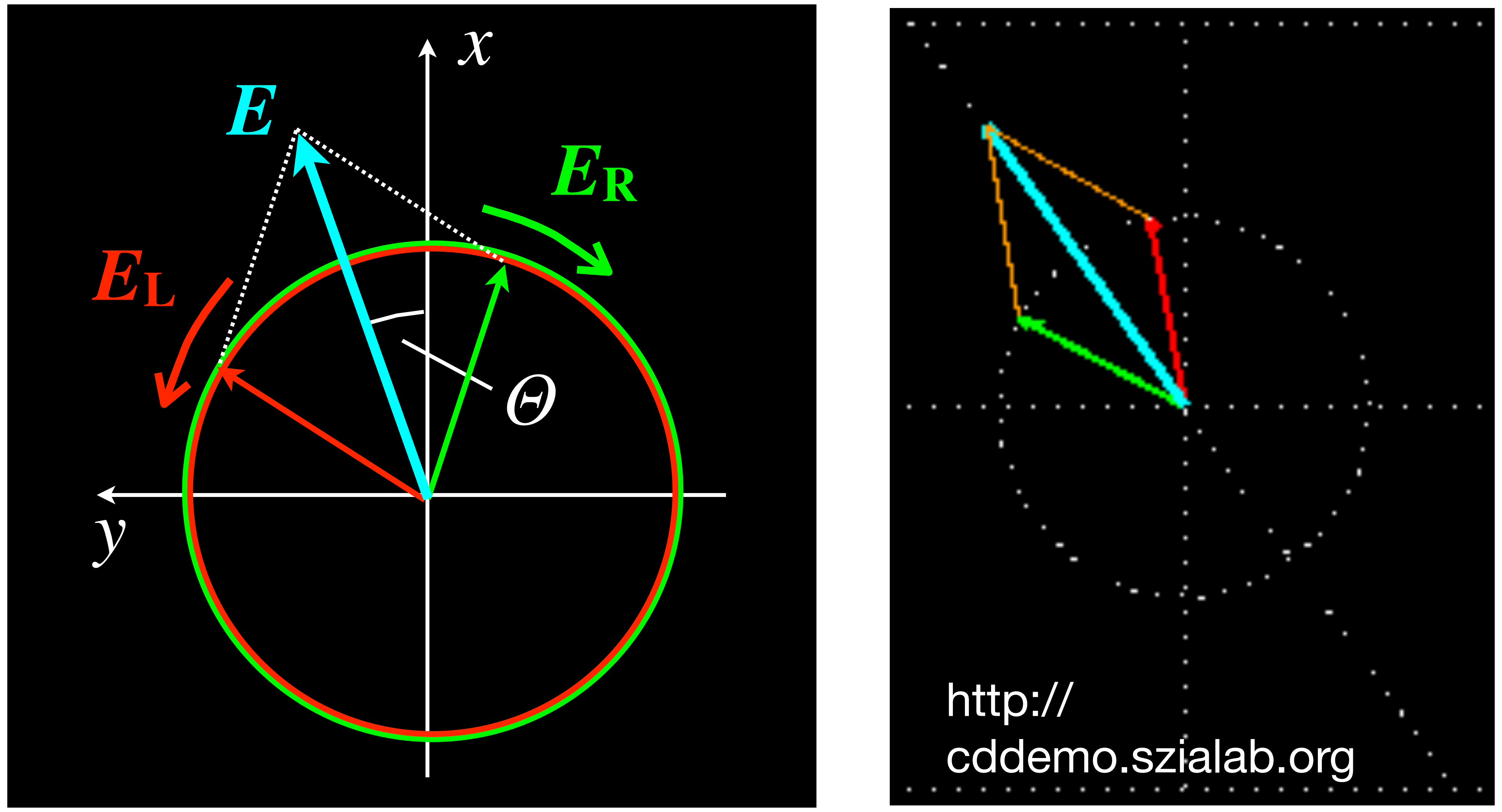

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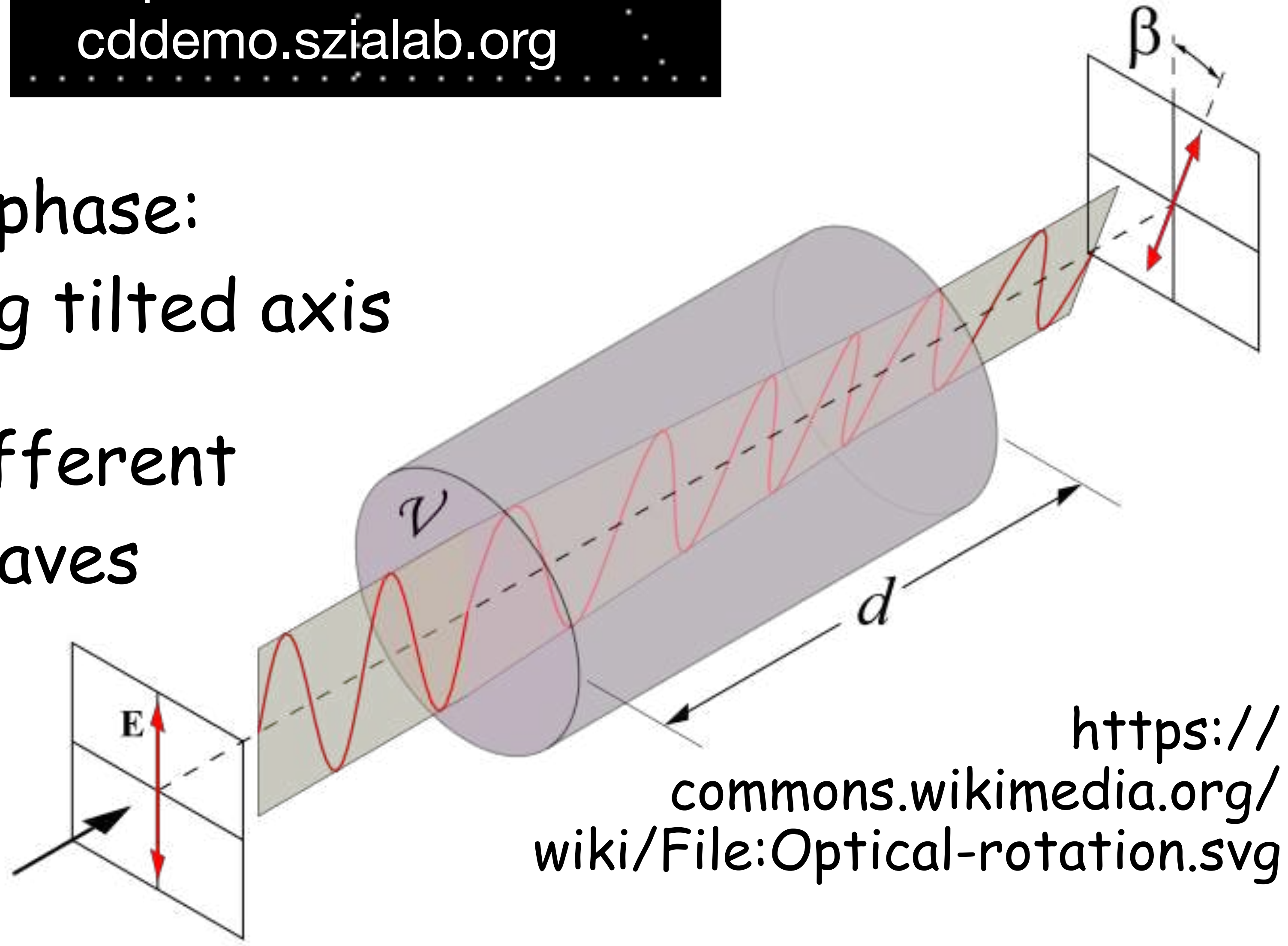
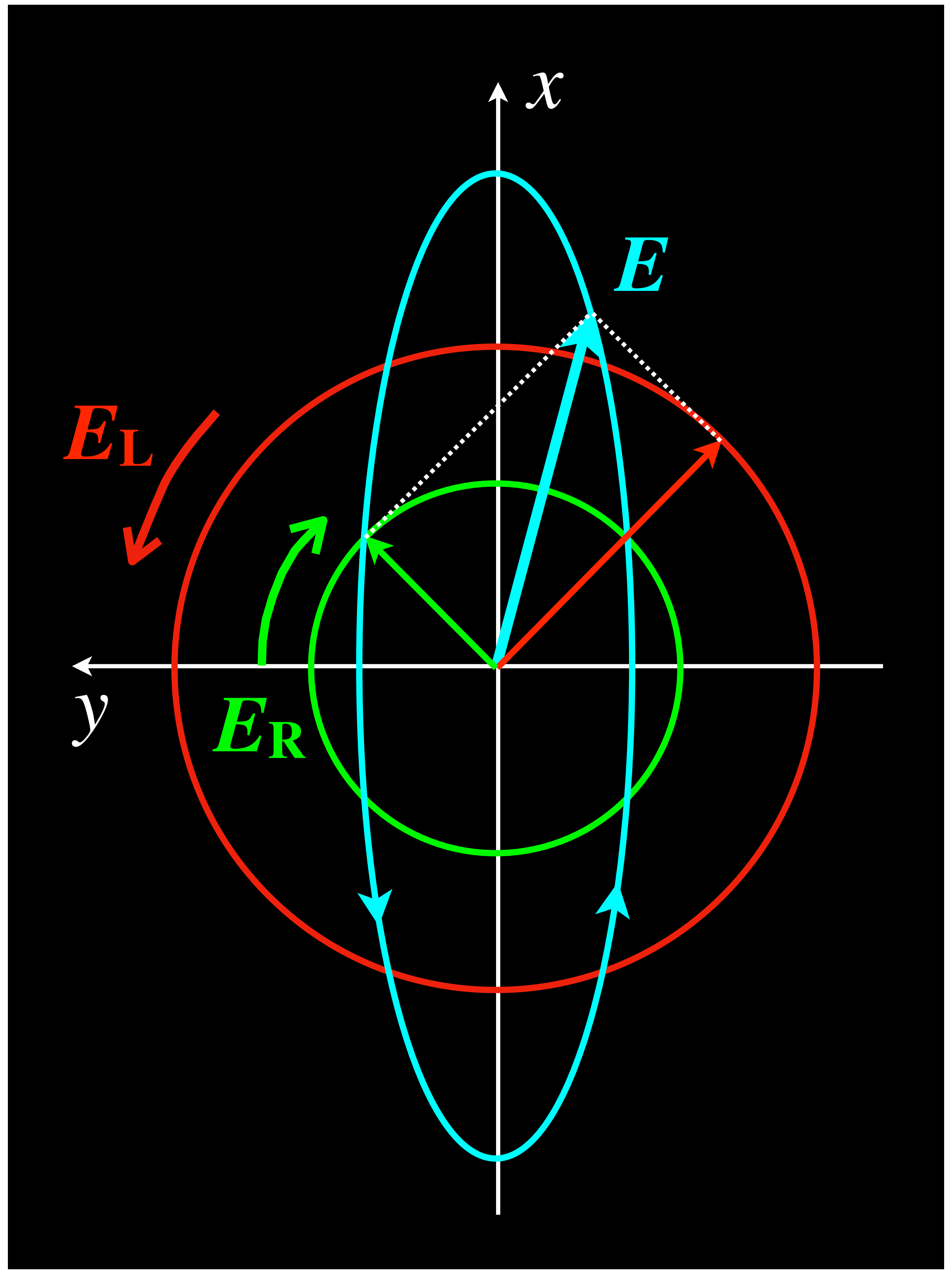


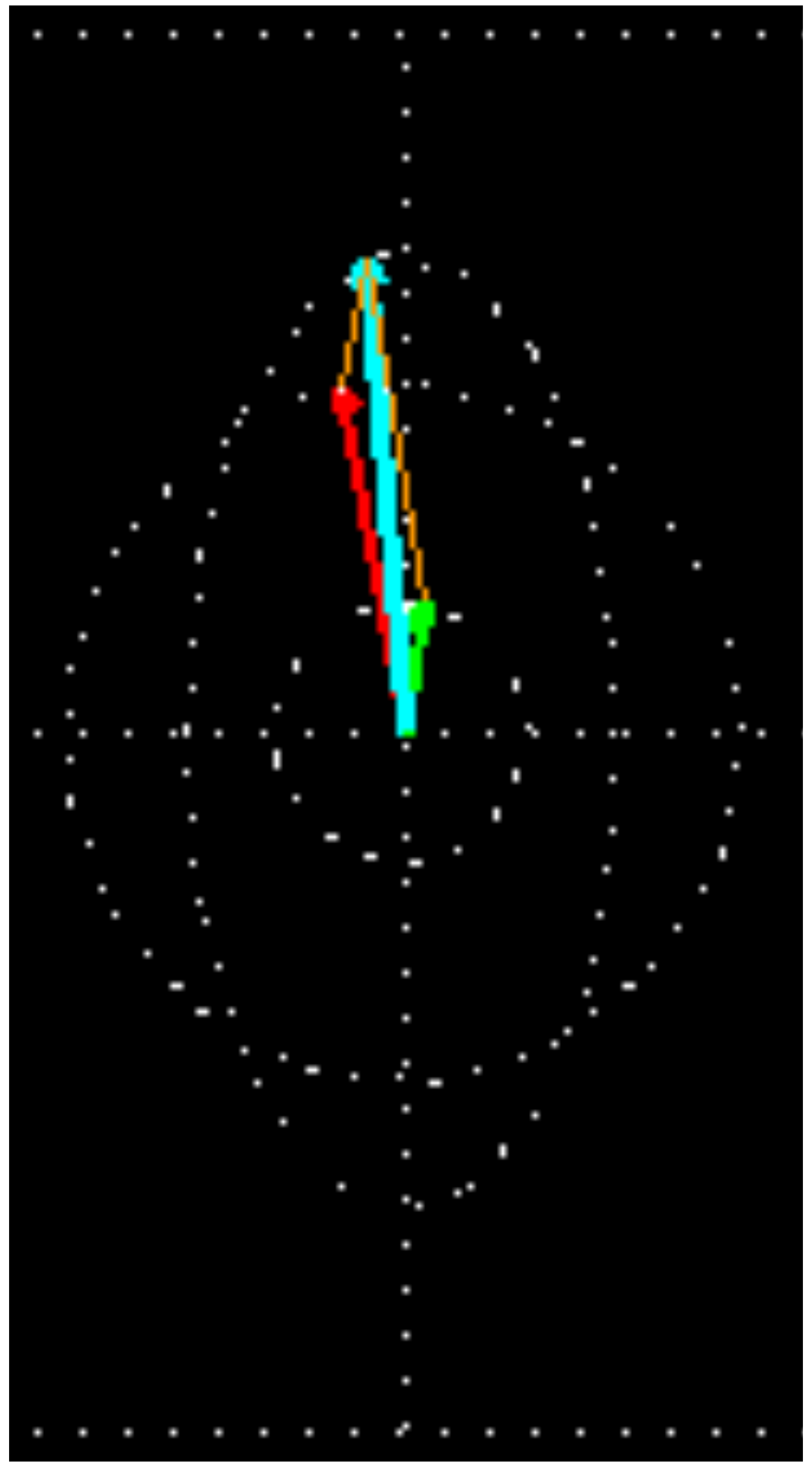
https://commons.wikimedia.org/wiki/File:Optical-rotation.svg

Optical activity

On traveling through material: continuous increase of rotation

Interpretation of polarized light: superposition of right- and left-handed circularly polarized waves



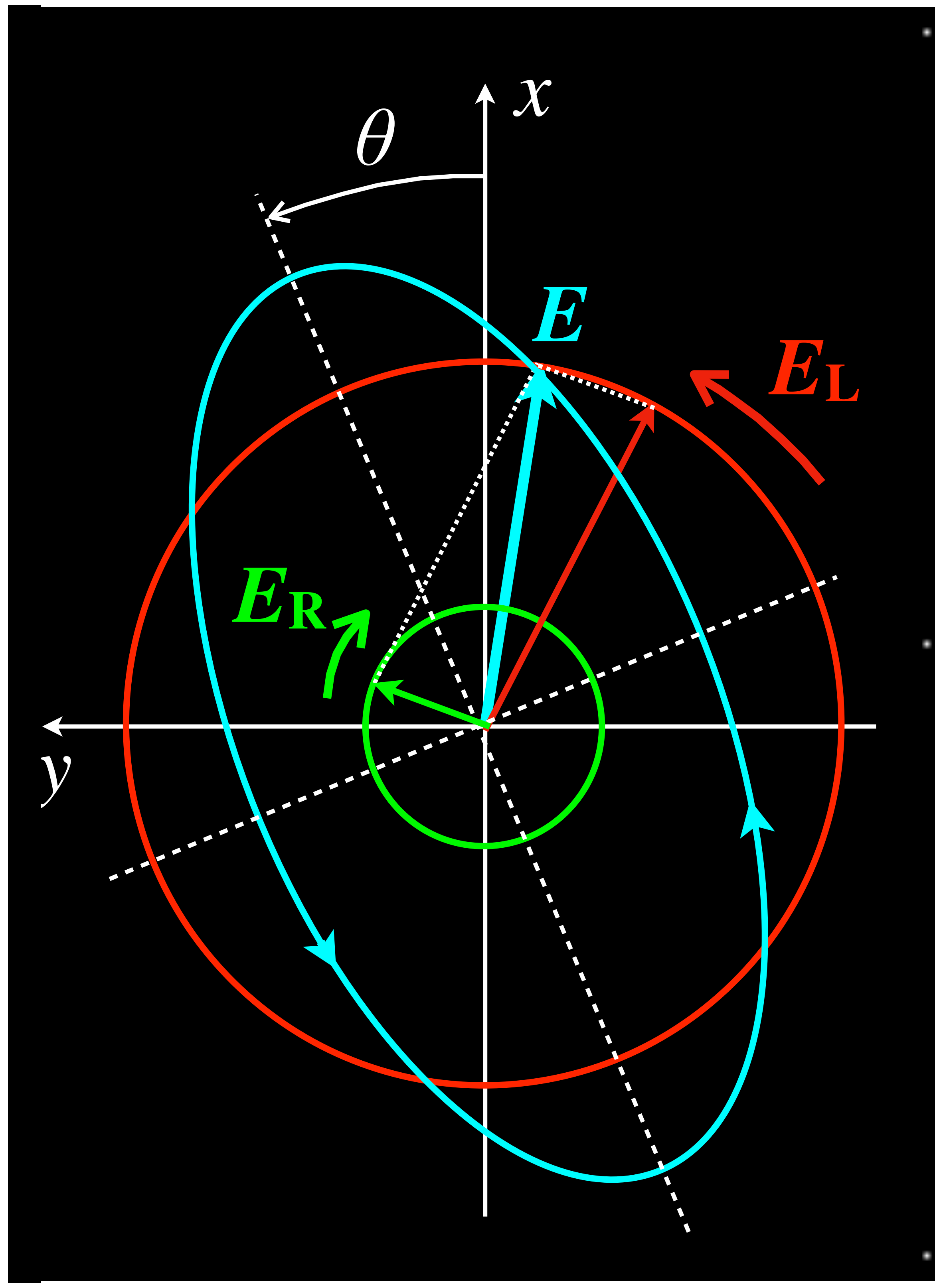


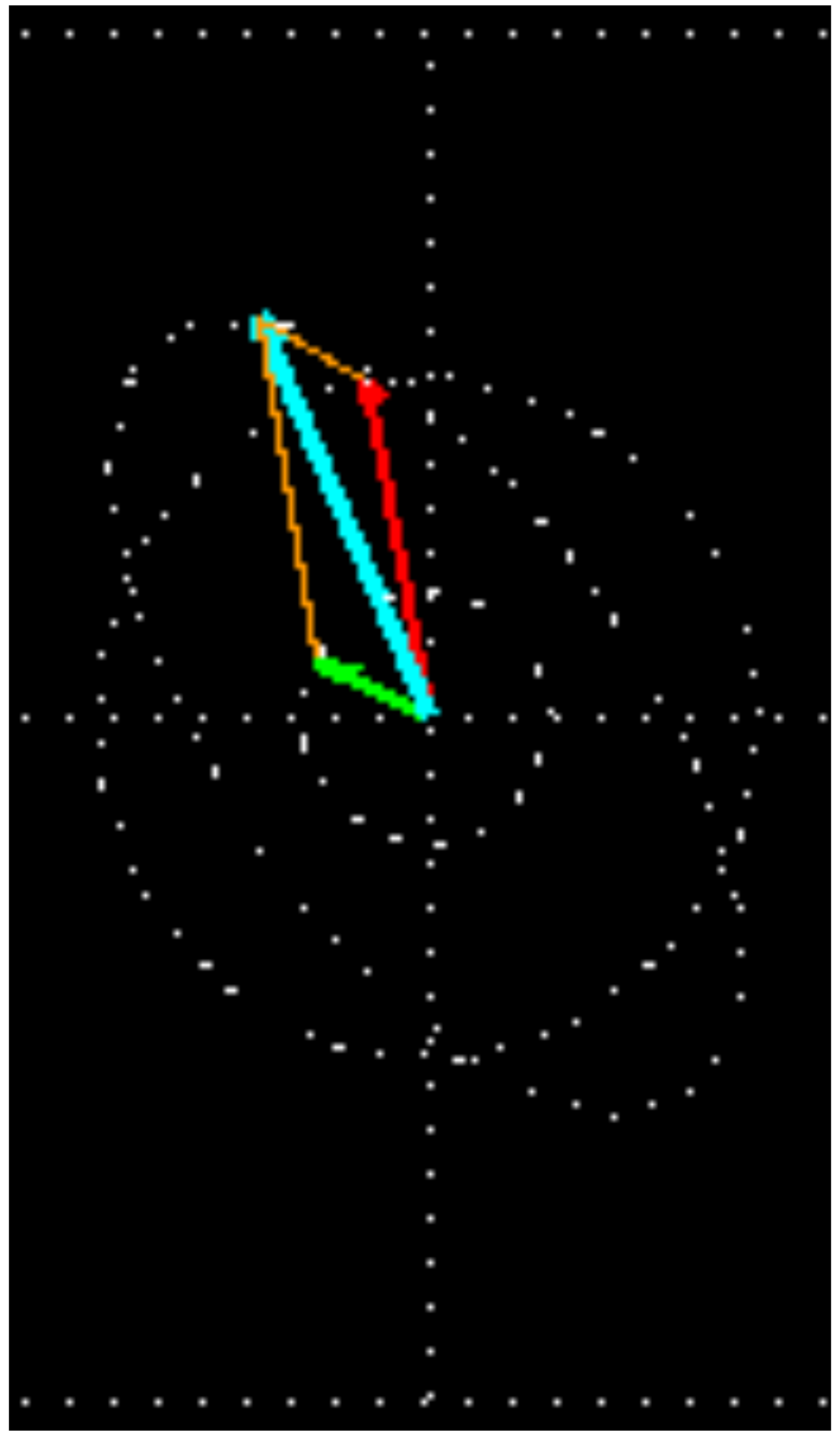
Same phase, but different amplitude:

$\Longrightarrow$ elliptically polarized wave along x-axis

Amplitude difference:
caused by different absorption of circular partial waves

In general, a material that displays different absorption of partial waves is said to be **dichroic**



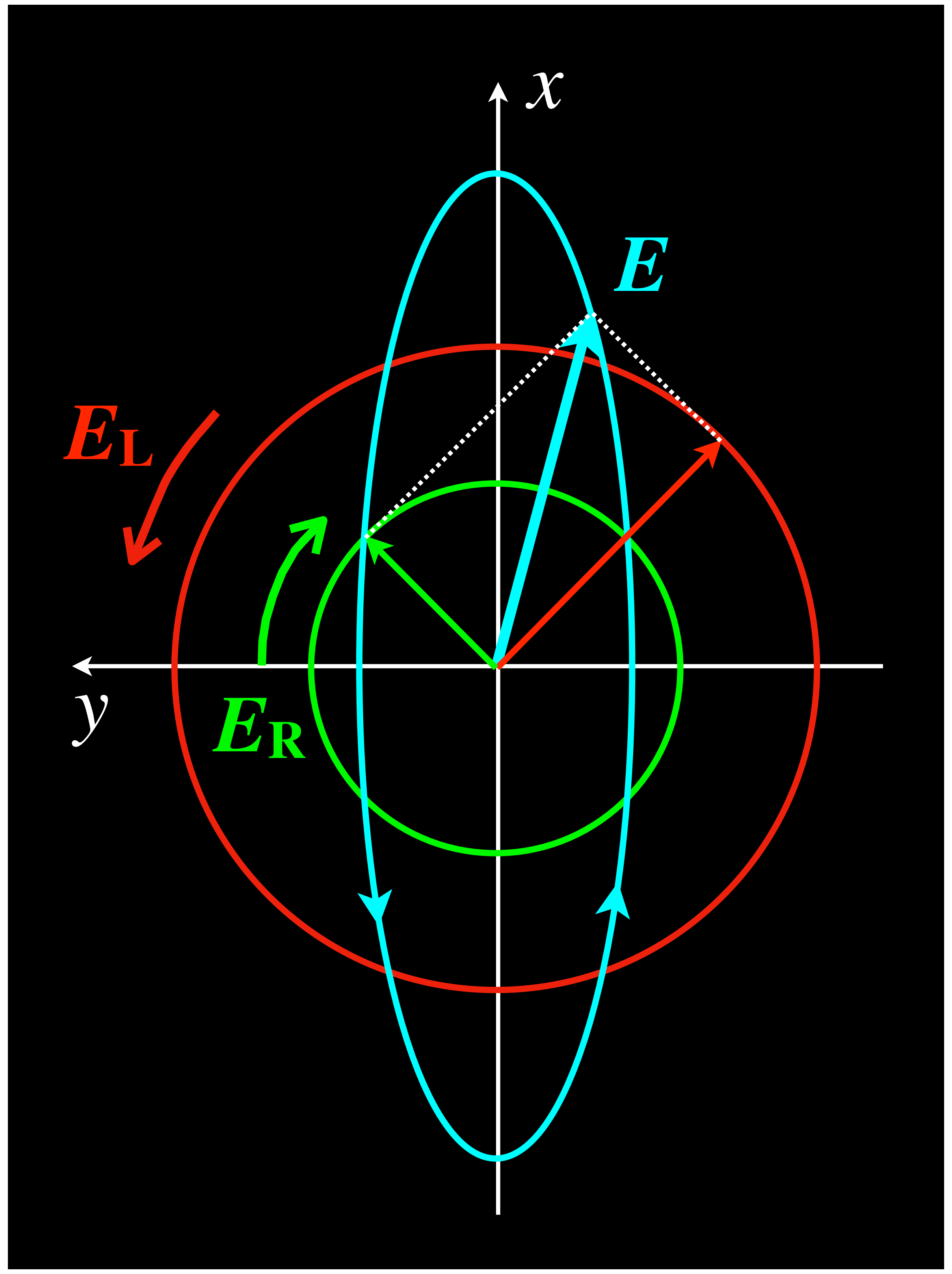


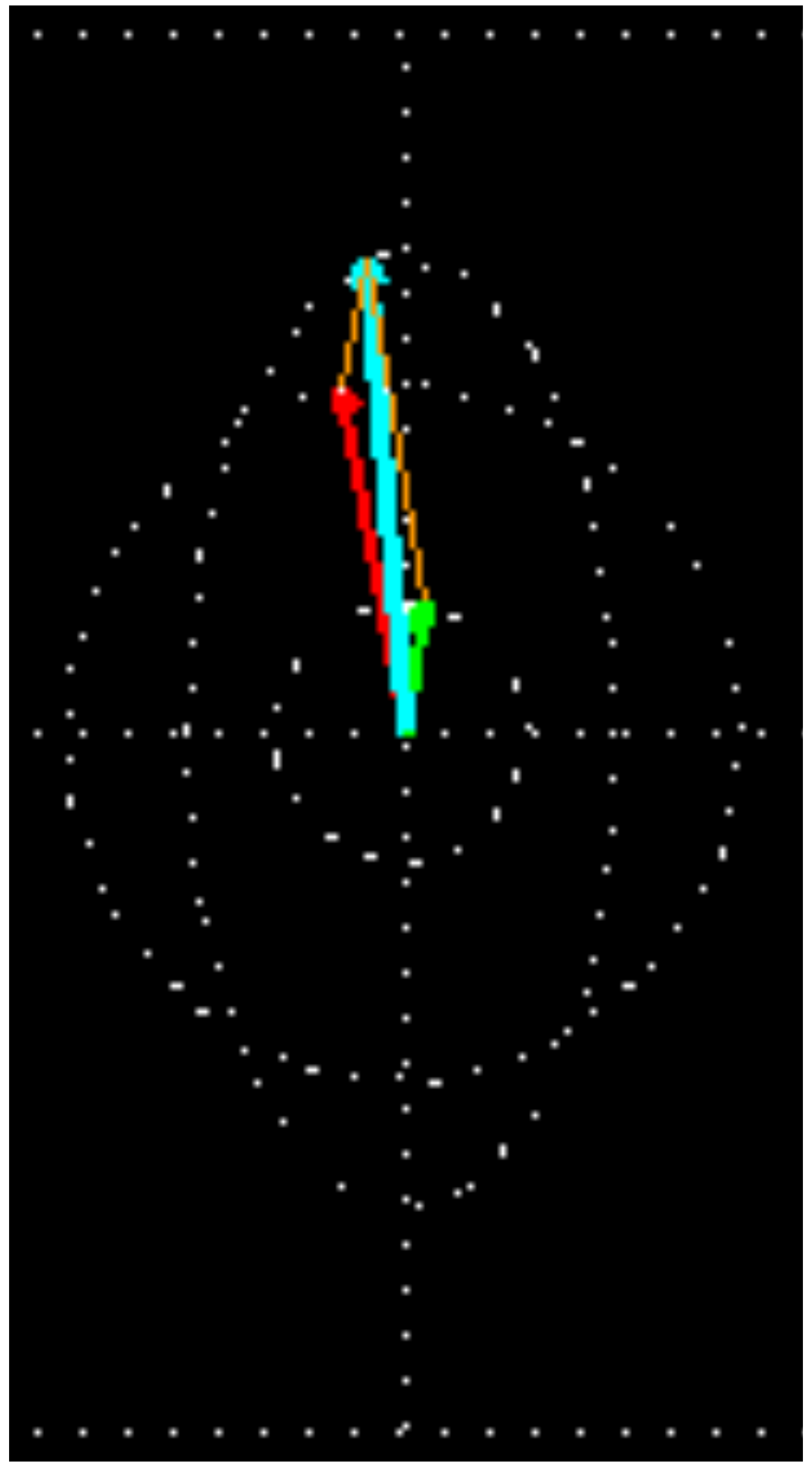
Different phase **and** different amplitude:

$\Longrightarrow$ rotated elliptically polarized wave

$\Longrightarrow$ **dichroism** and **birefringence**

Interpretation of polarized light: superposition of right- and left-handed circularly polarized waves





Same phase, but different amplitude:

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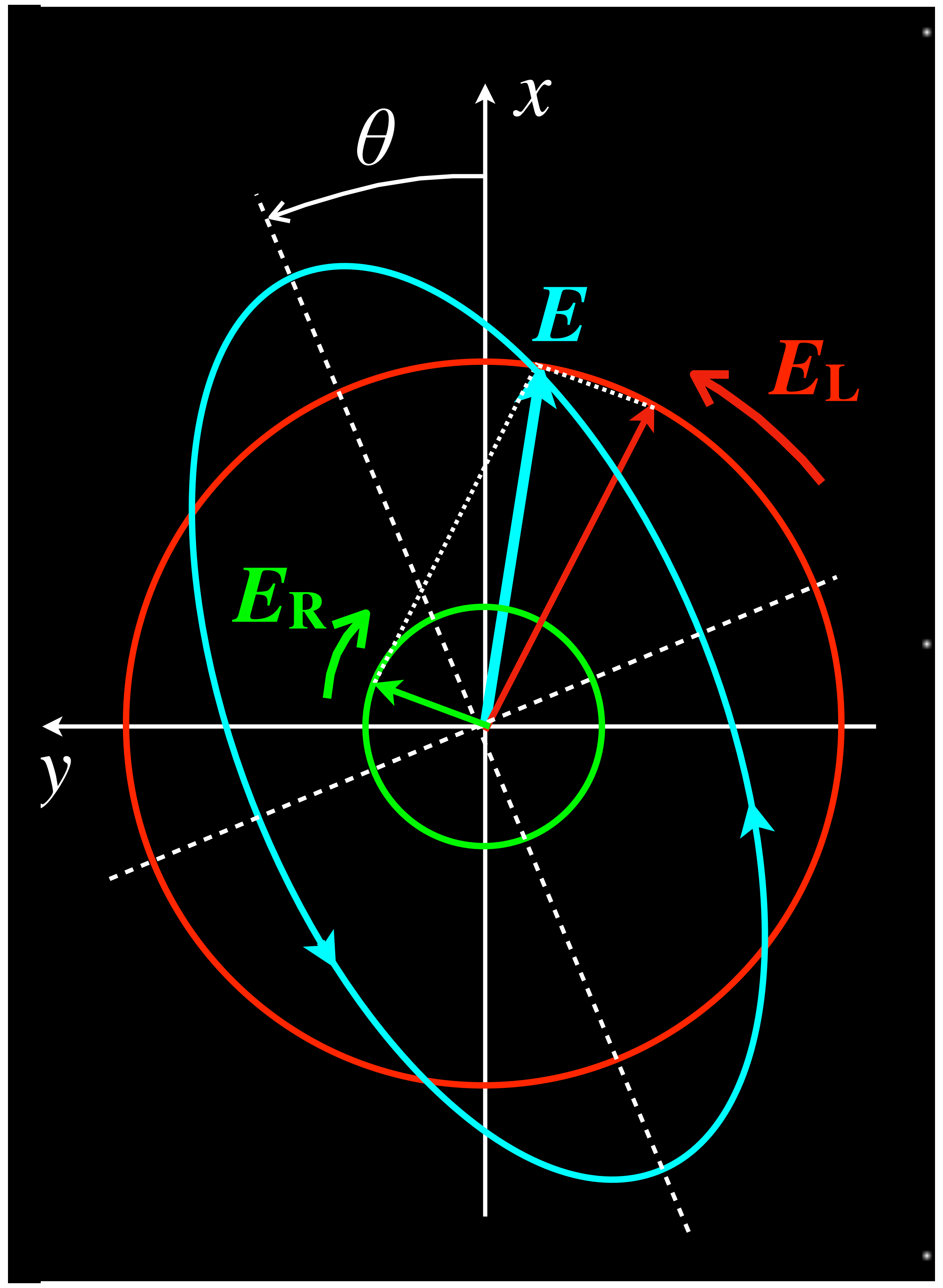
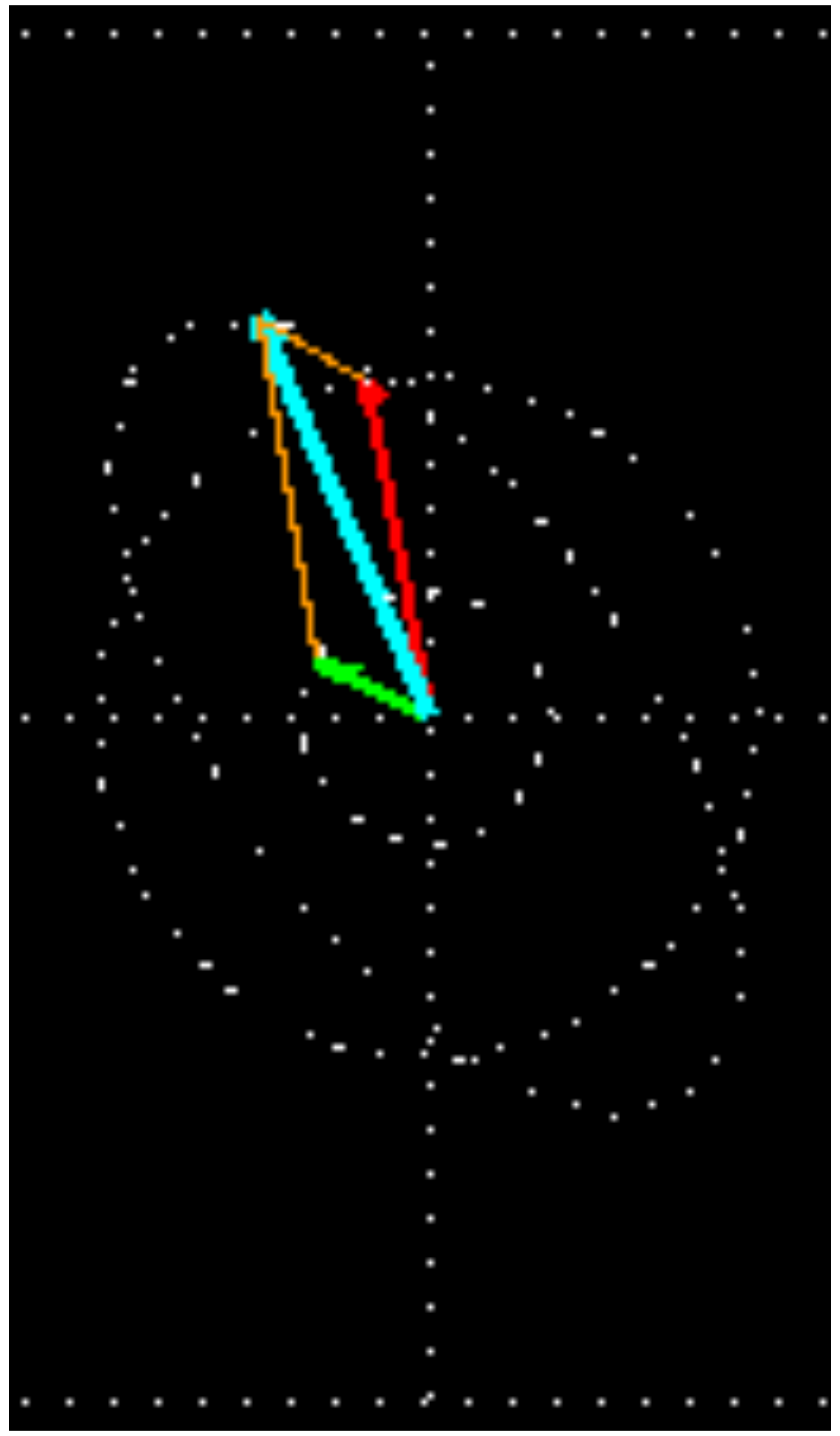
Amplitude difference:
caused by different absorption of circular partial waves

In general, a material that displays different absorption of partial waves is said to be **dichroic**



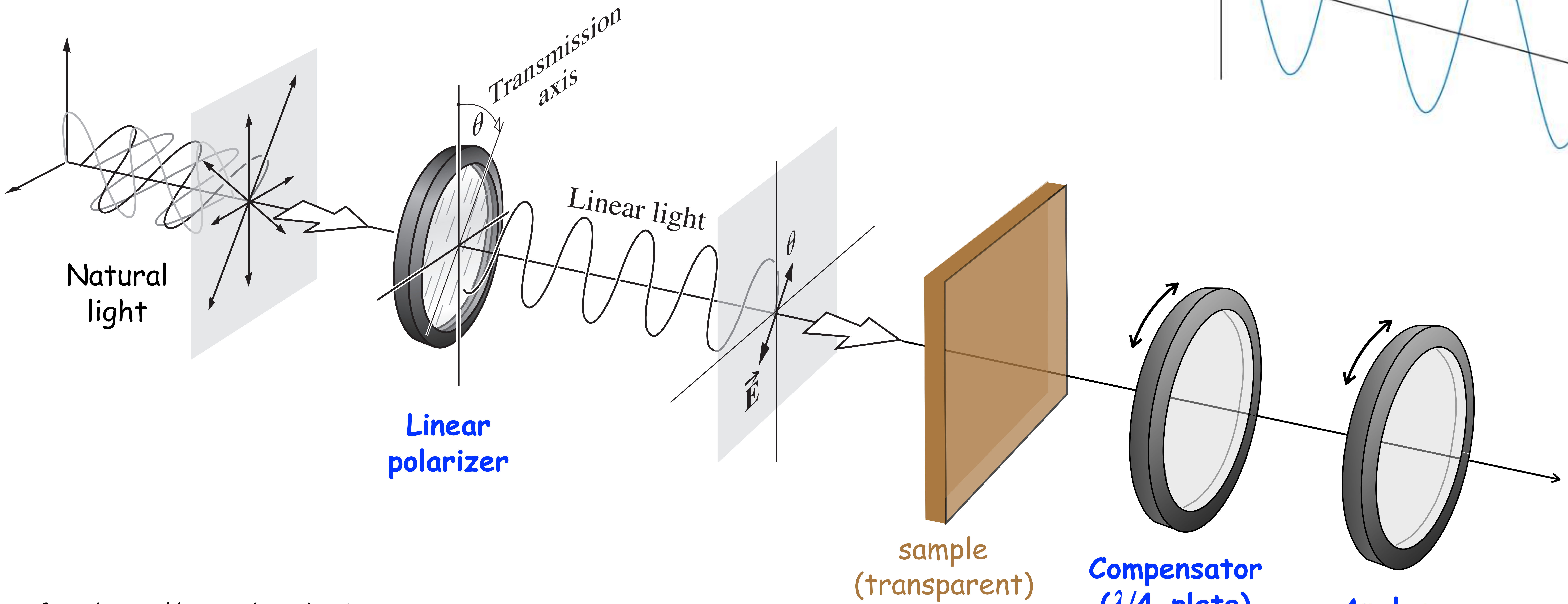


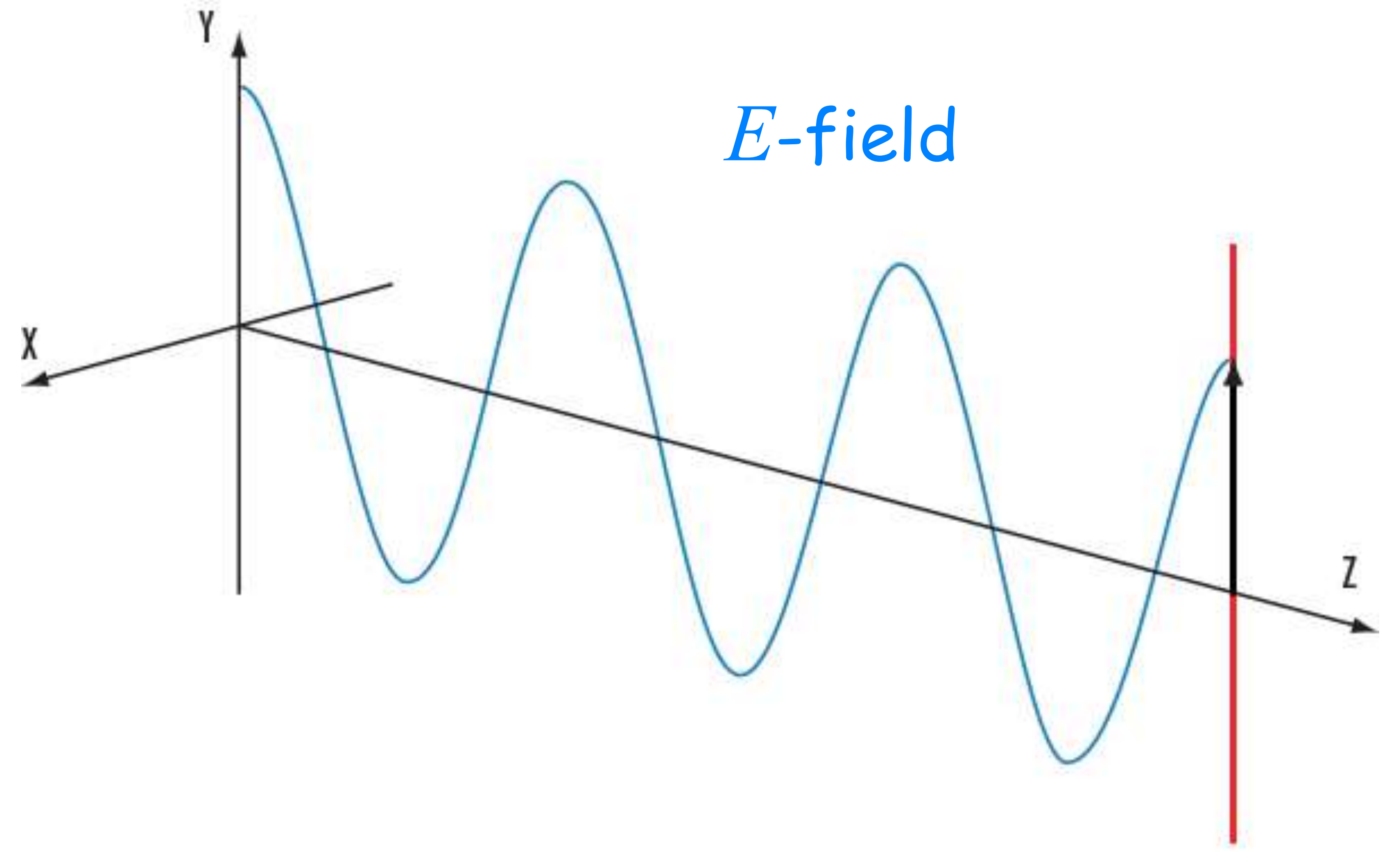
Different phase **and** different amplitude:

$\Longrightarrow$ rotated elliptically polarized wave

$\Longrightarrow$ **dichroism** and **birefringence**

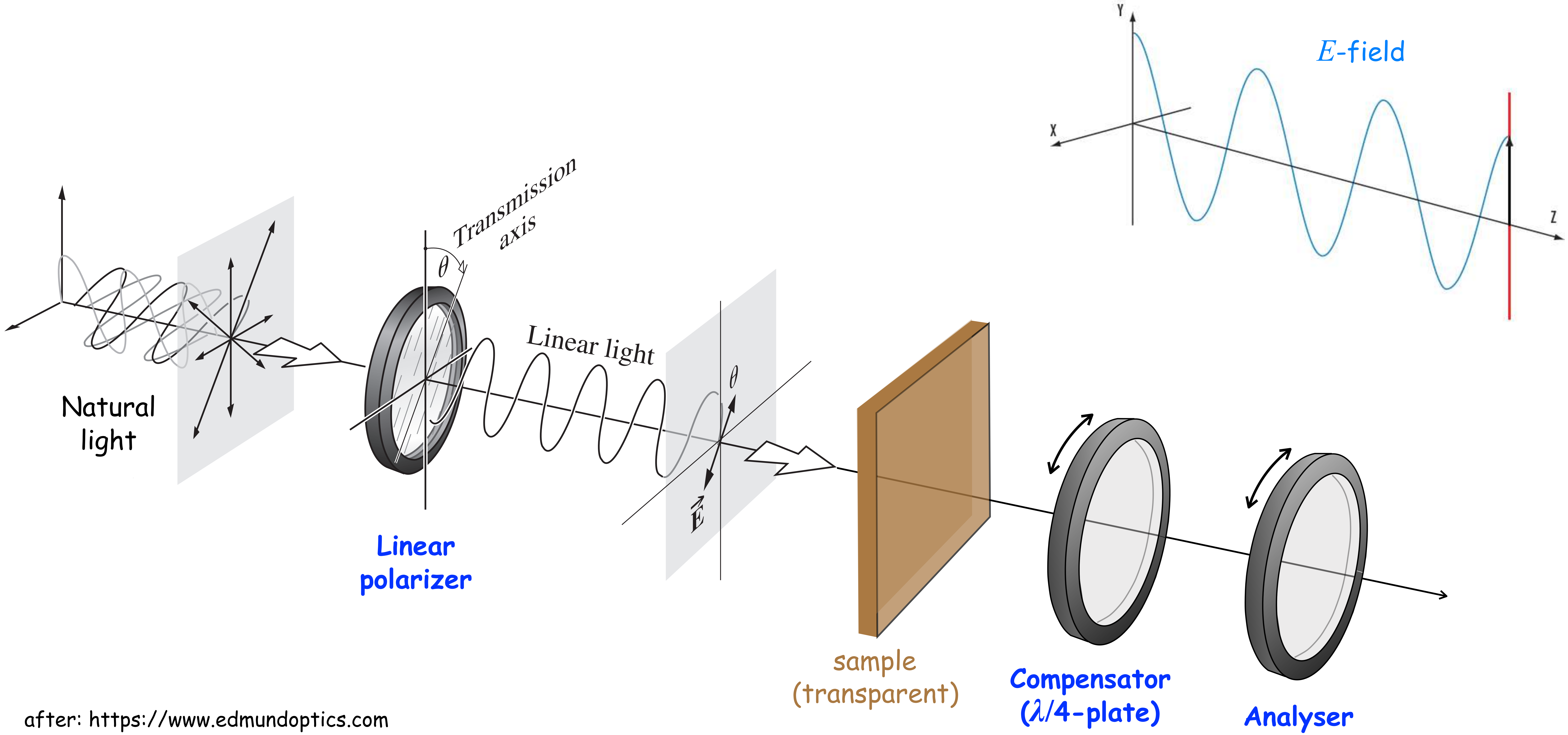
Components for magneto-optical experiment





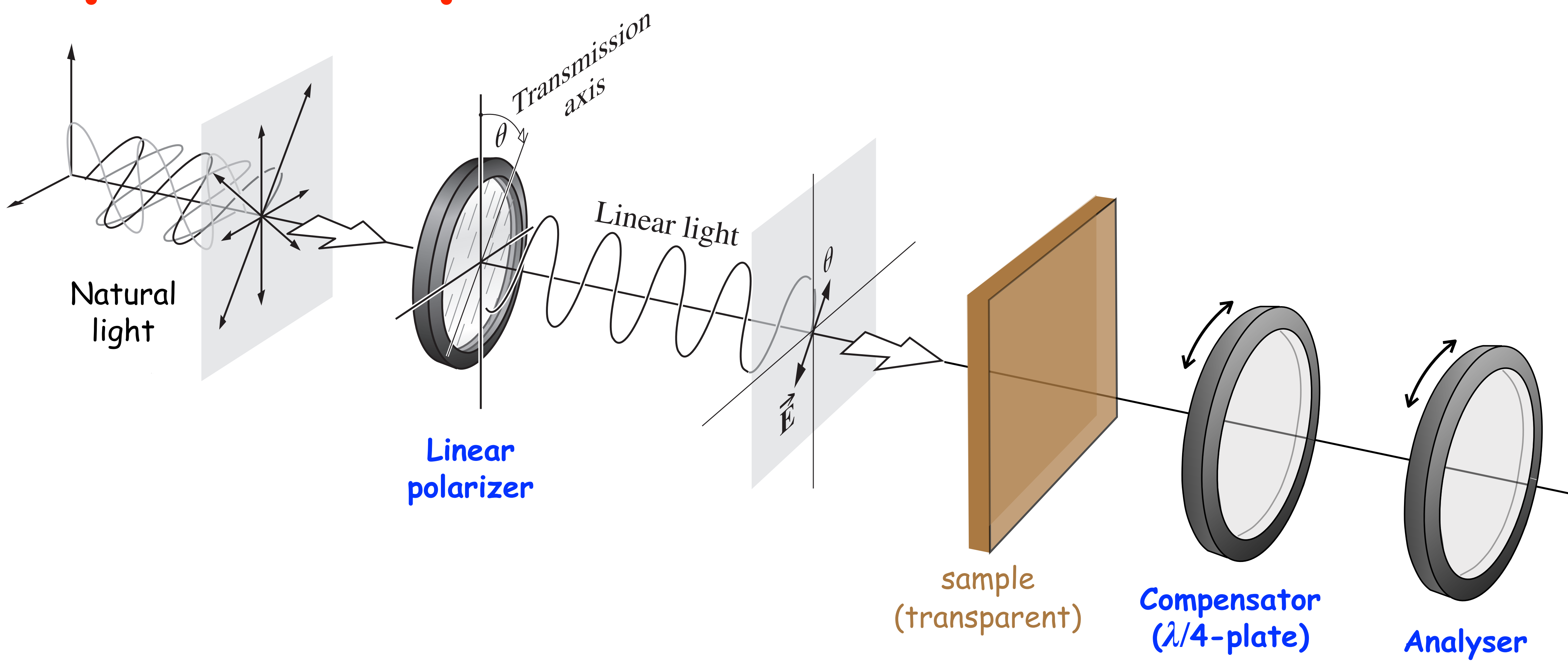
after: https://www.edmundoptics.com

Components for magneto-optical experiment

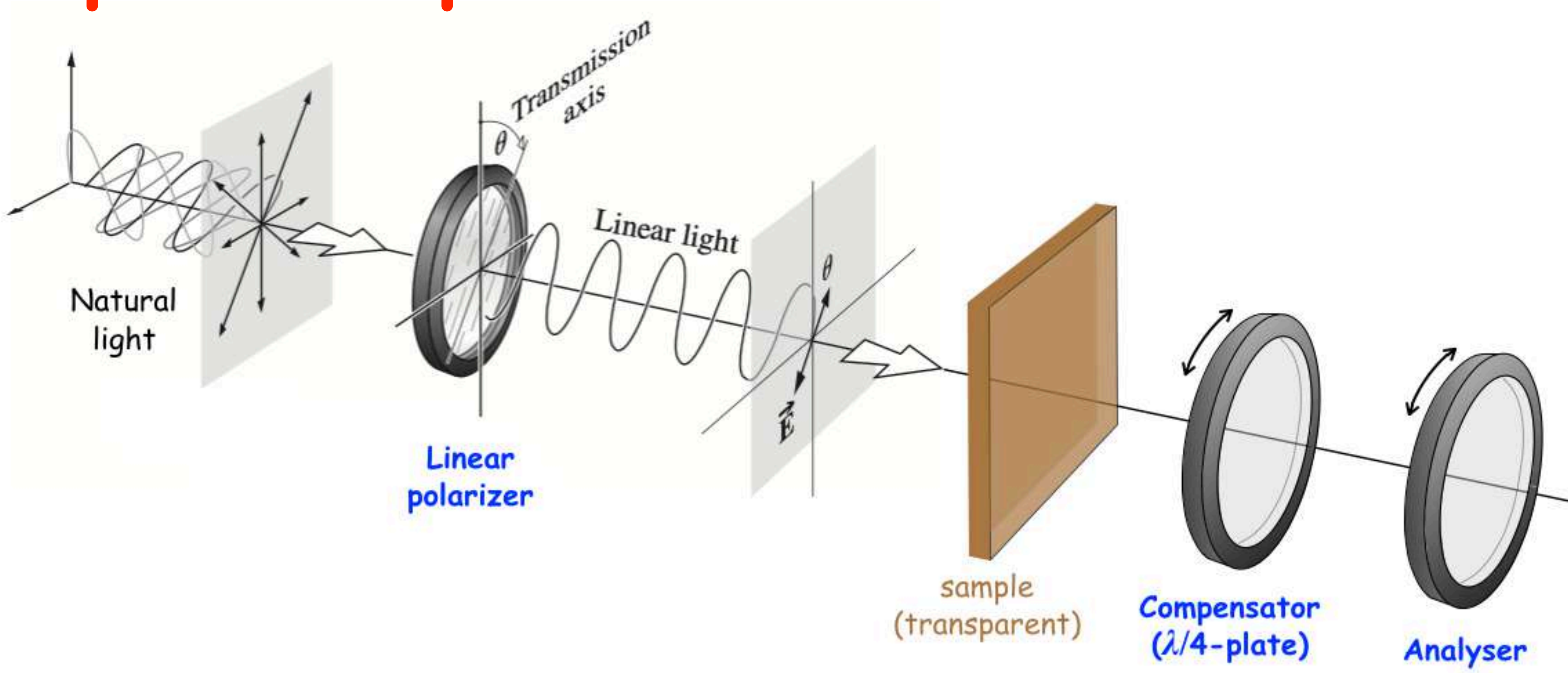


after: https://www.edmundoptics.com

Components for magneto-optical experiment

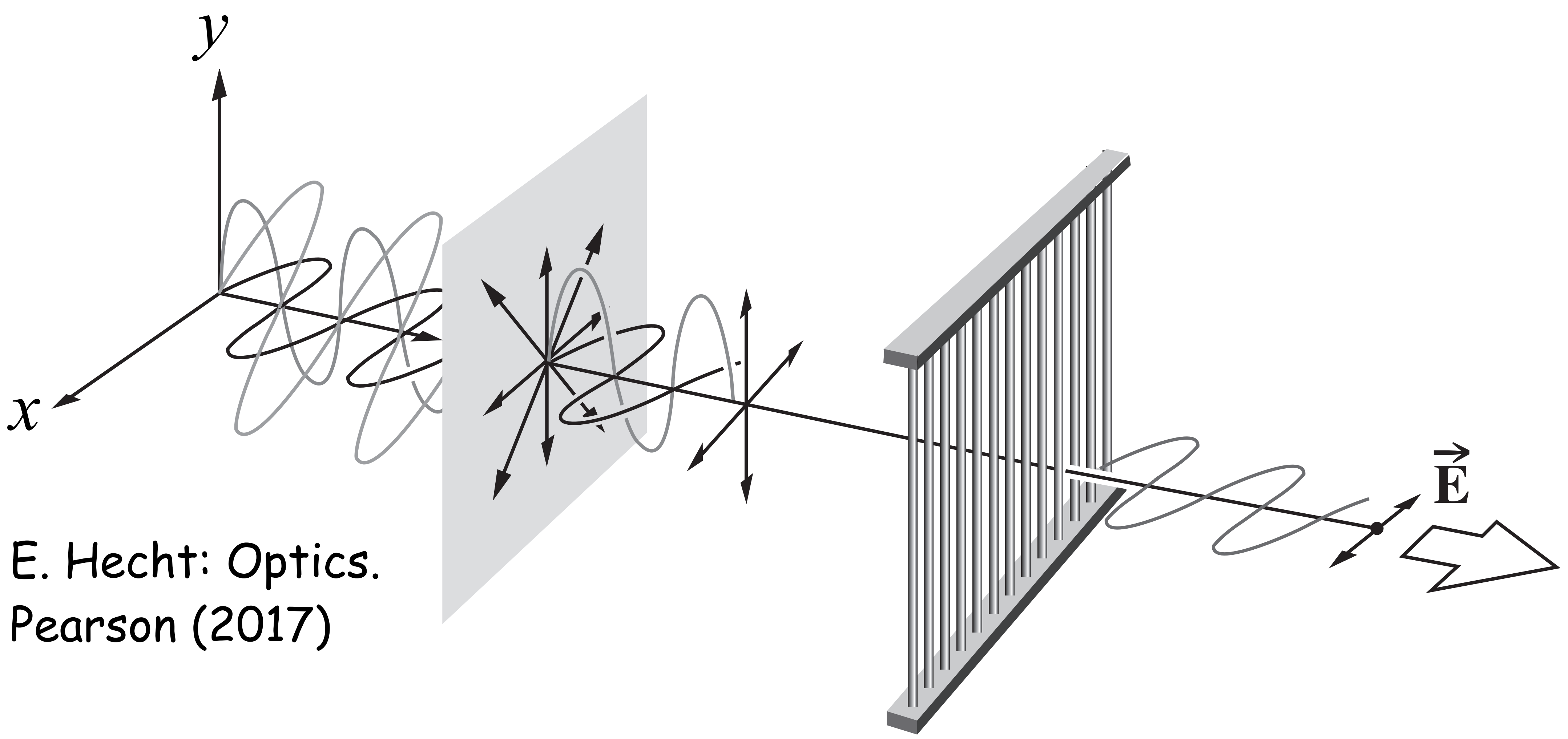


Components for magneto-optical experiment



Linear polarizer:

Example: wire-grid polarizer



E. Hecht: Optics. Pearson (2017)

- grid of parallel conducting wires
- resolve E-field into 2 orthogonal components ($\parallel$ and $\perp$ to wires)
- E_y drives conduction electrons along wire length $\Longrightarrow$ E_y-component is reflected back
- little electron motion along E_x $\Longrightarrow$ E_x is unaltered and goes trough

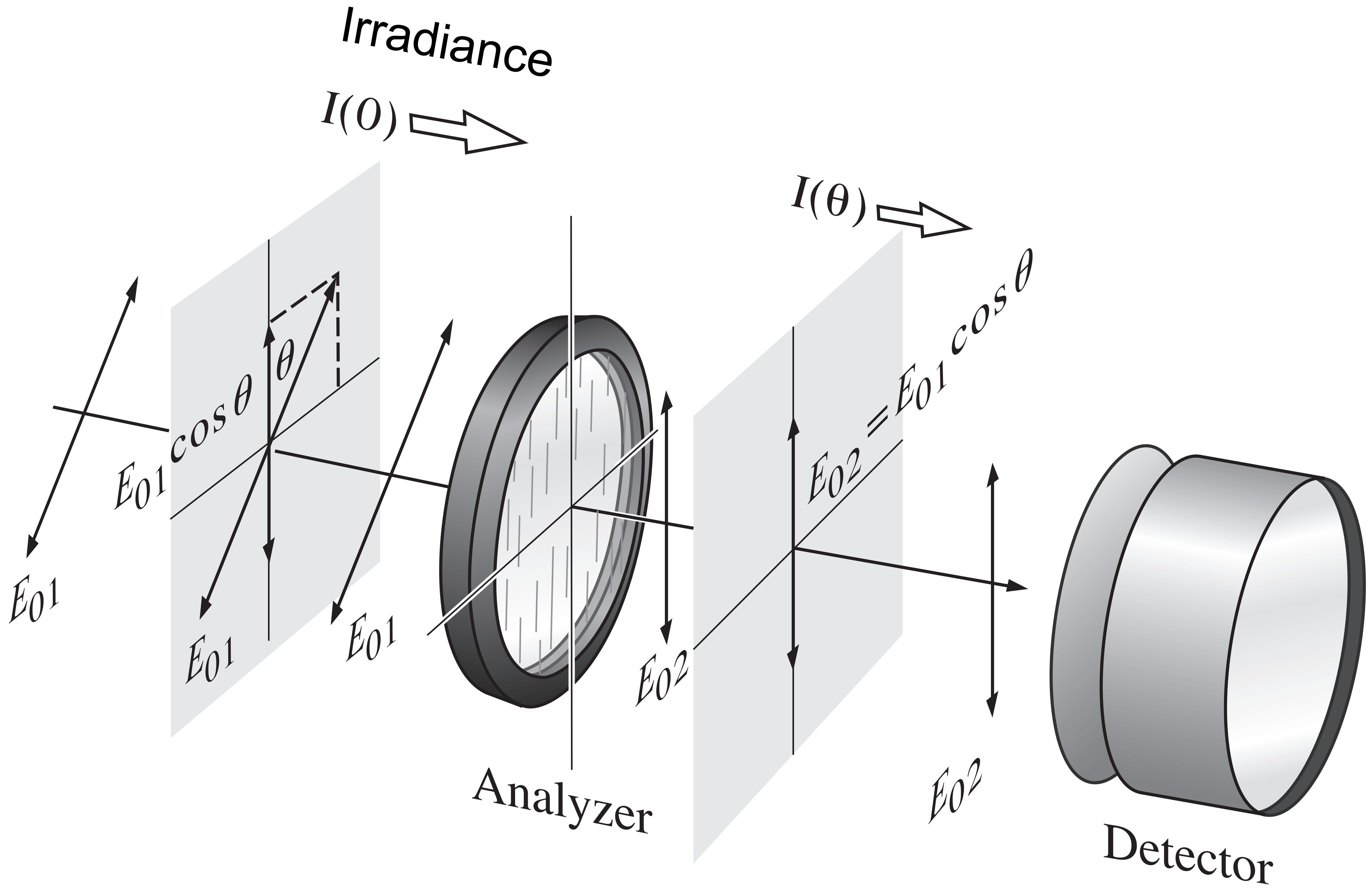
  $\Longrightarrow$ Transmission axis $\perp$ to wires

- Similar principle for **Absorptive Polarizers** (stretched plastic foils with aligned molecule chains)

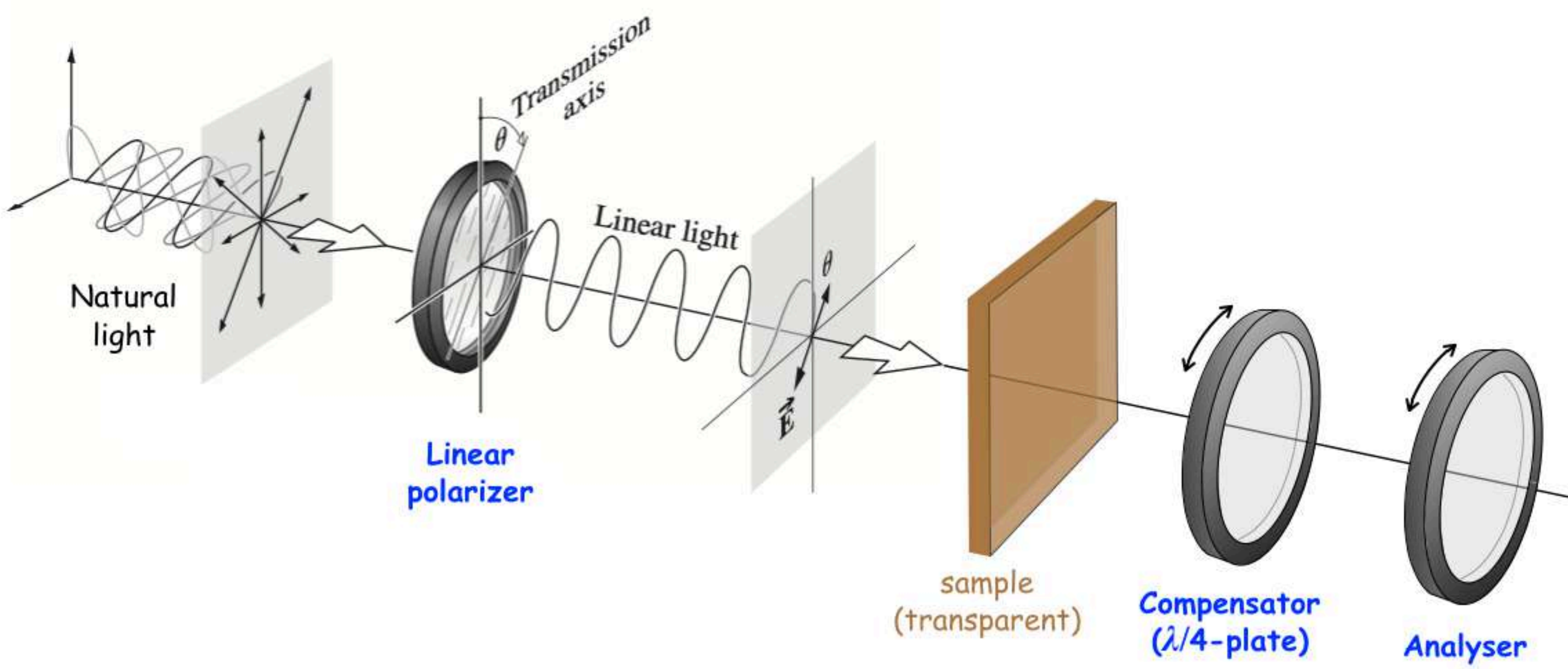
Components for magneto-optical experiment

Analyser:

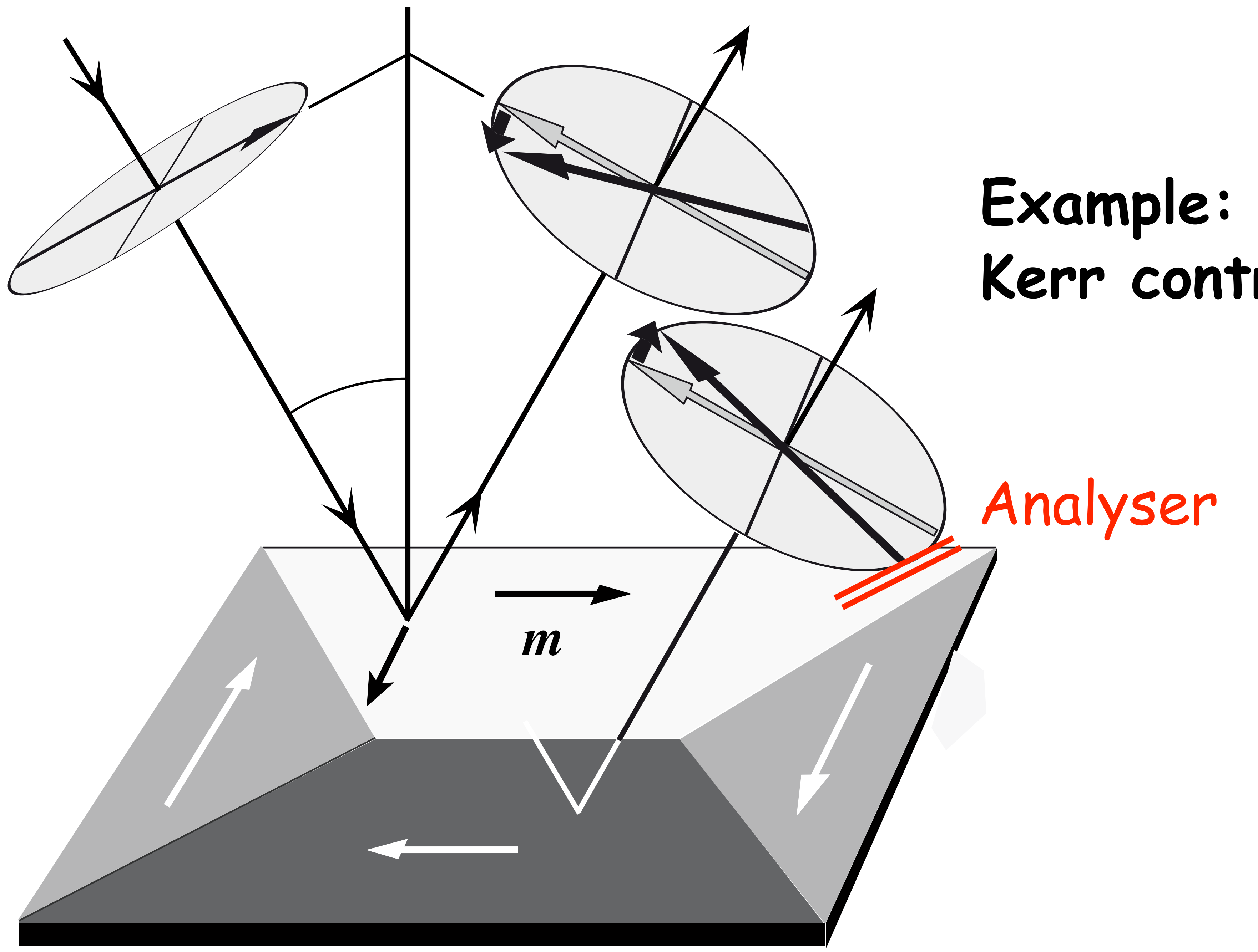
like polarizer



Malus' law: $I(\Theta) = I(\Theta)\cos^2\Theta$

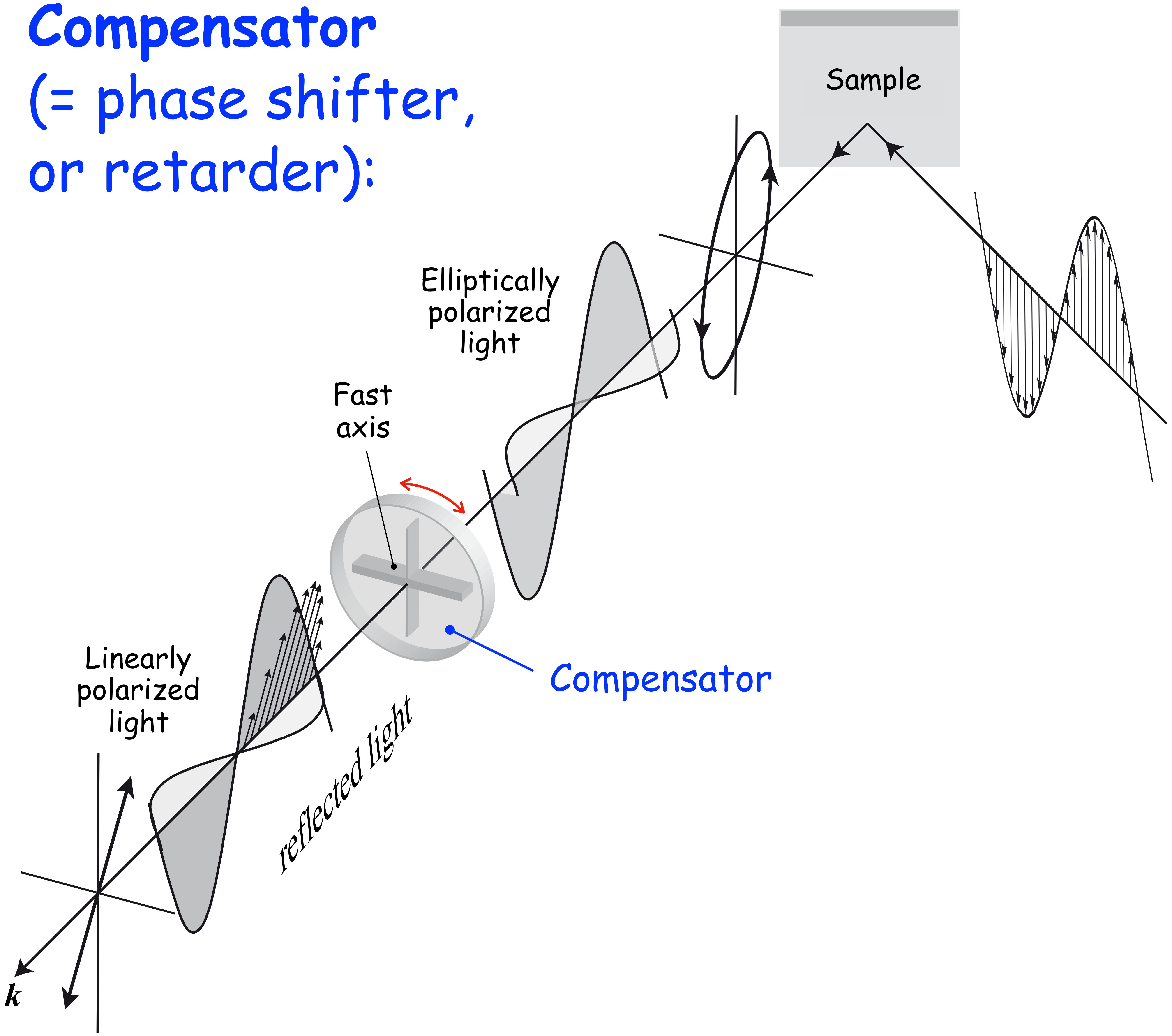


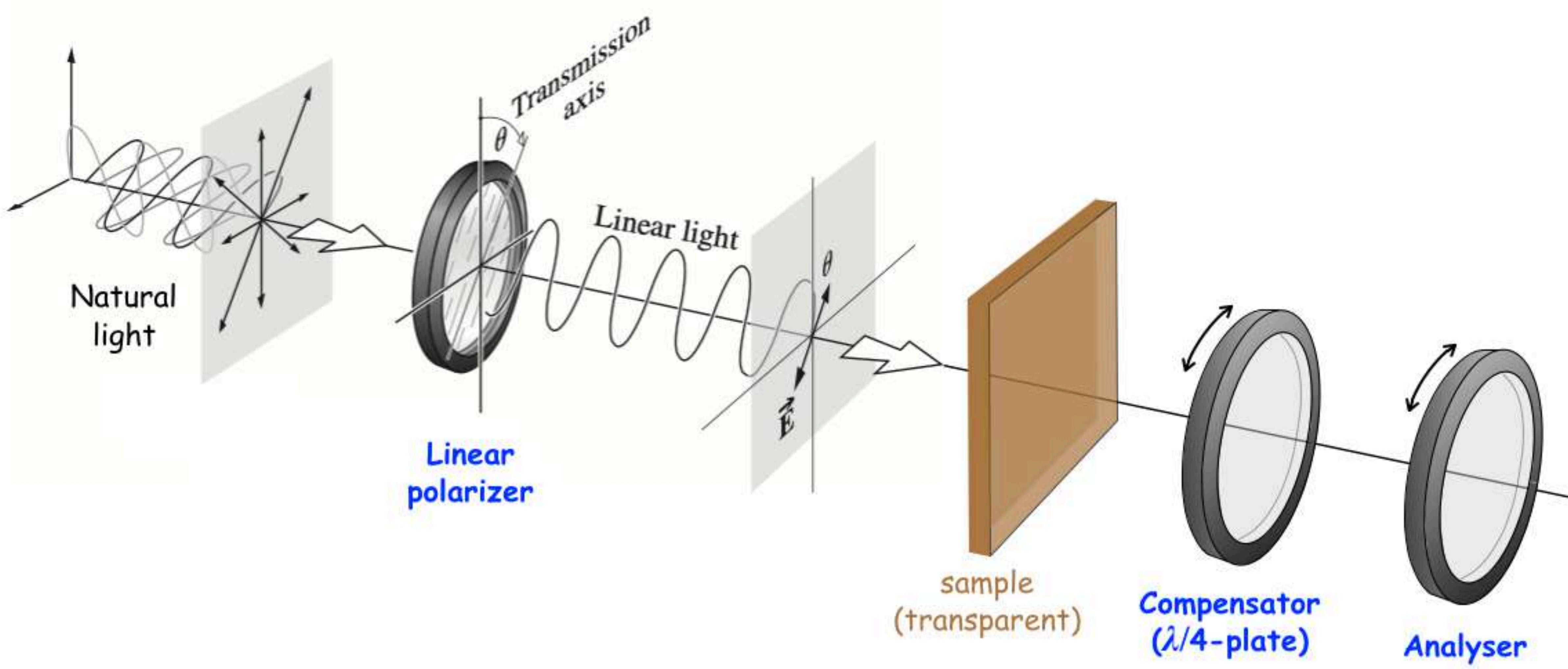
Example: Kerr contrast



Components for magneto-optical experiment

Compensator
(= phase shifter, or retarder):





Components for magneto-optical experiment

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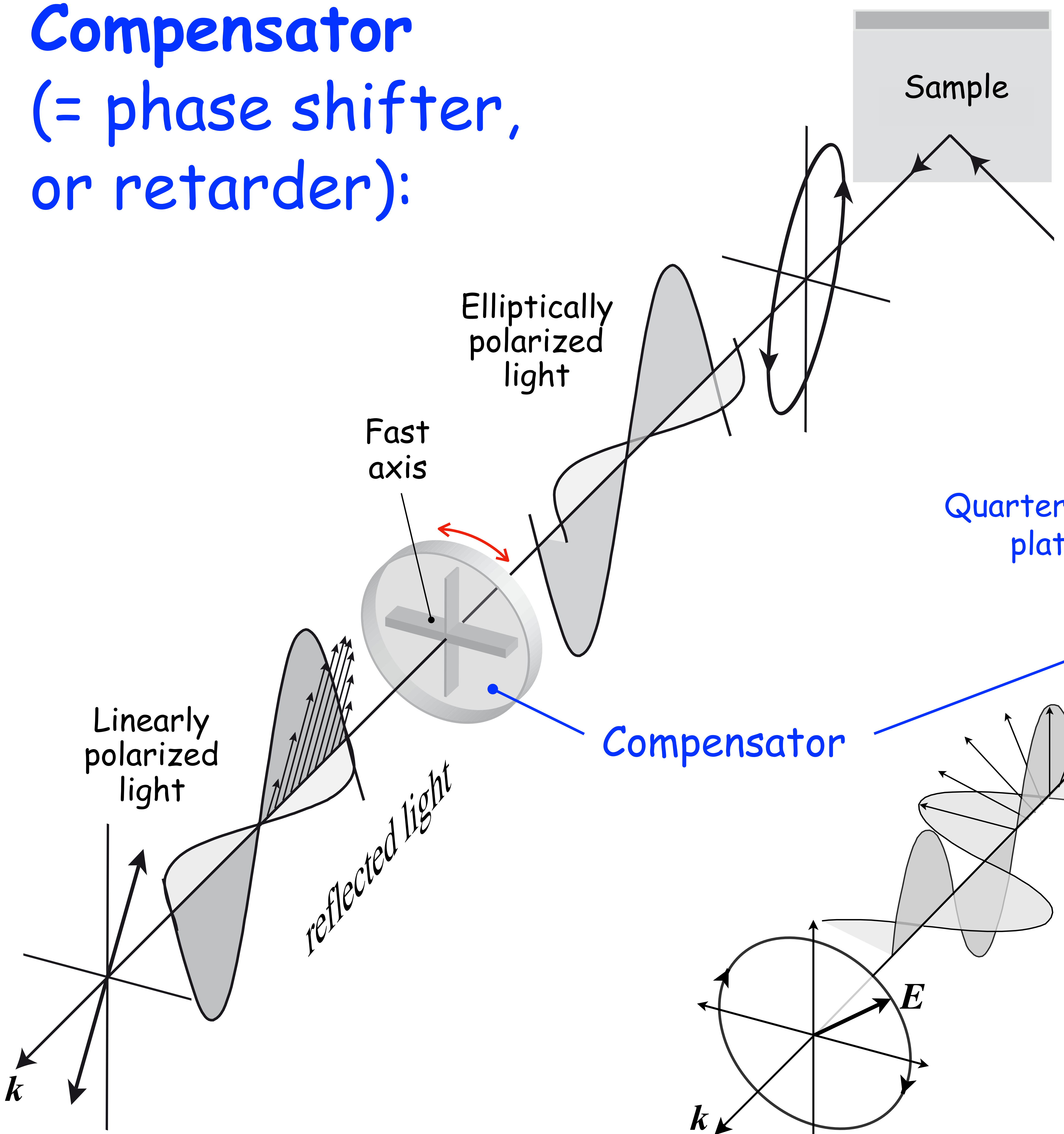
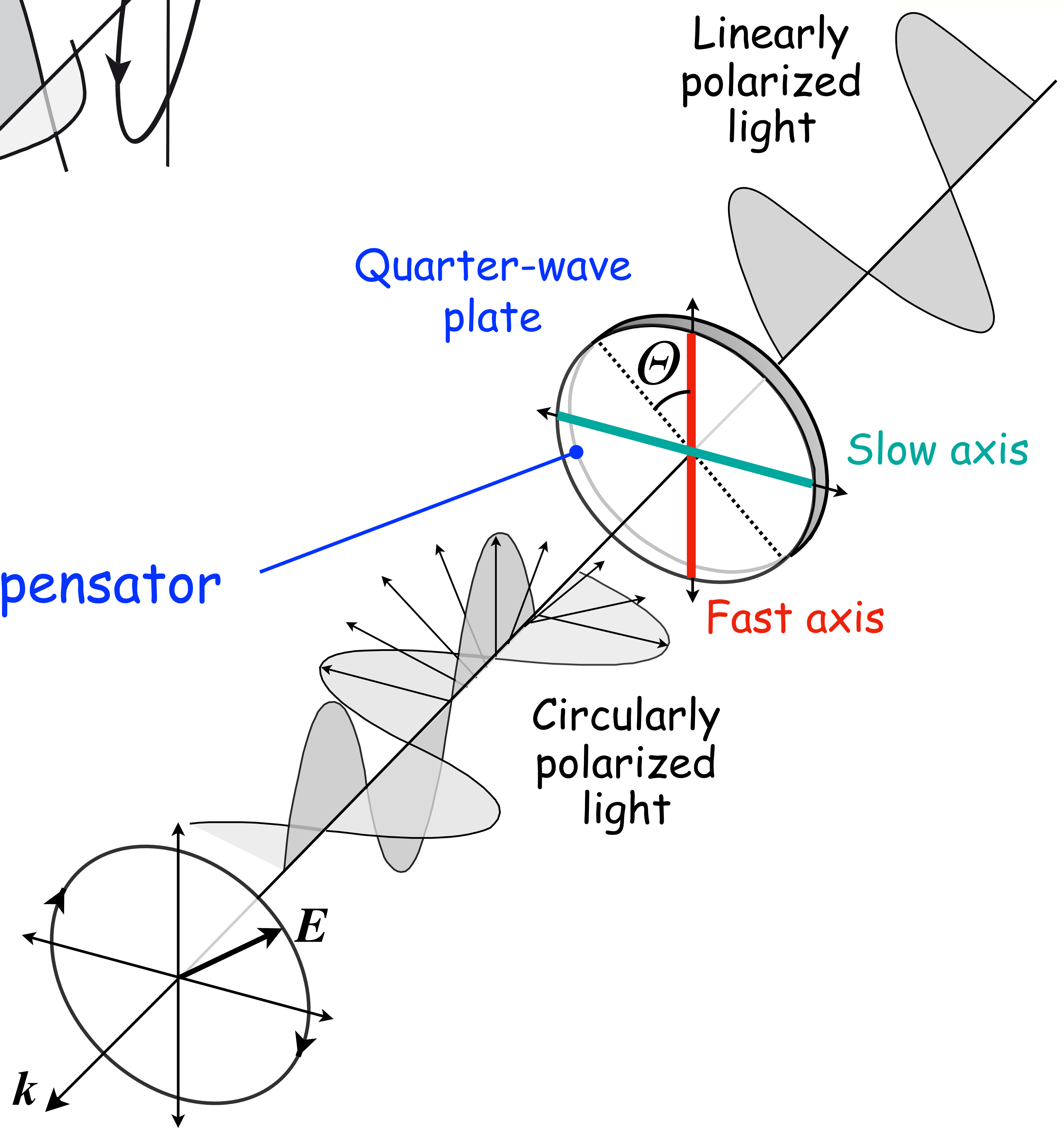
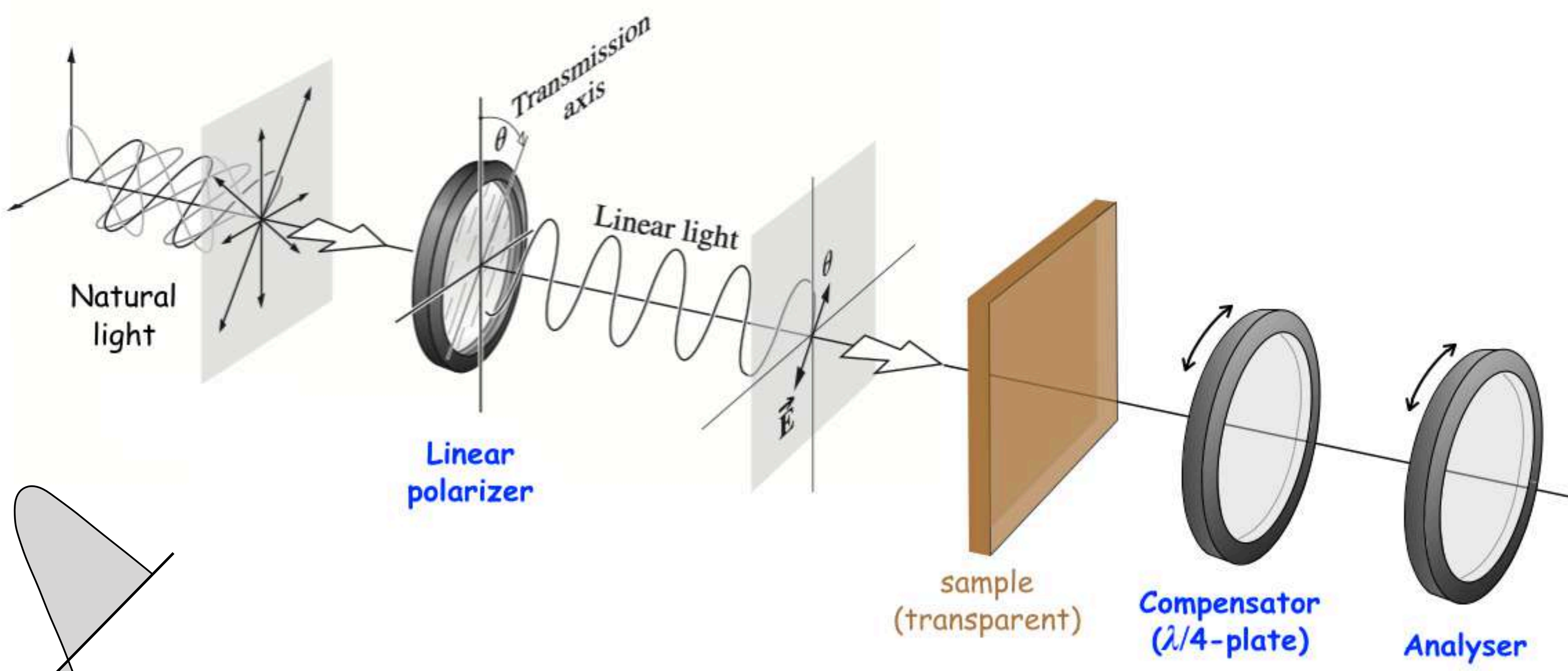
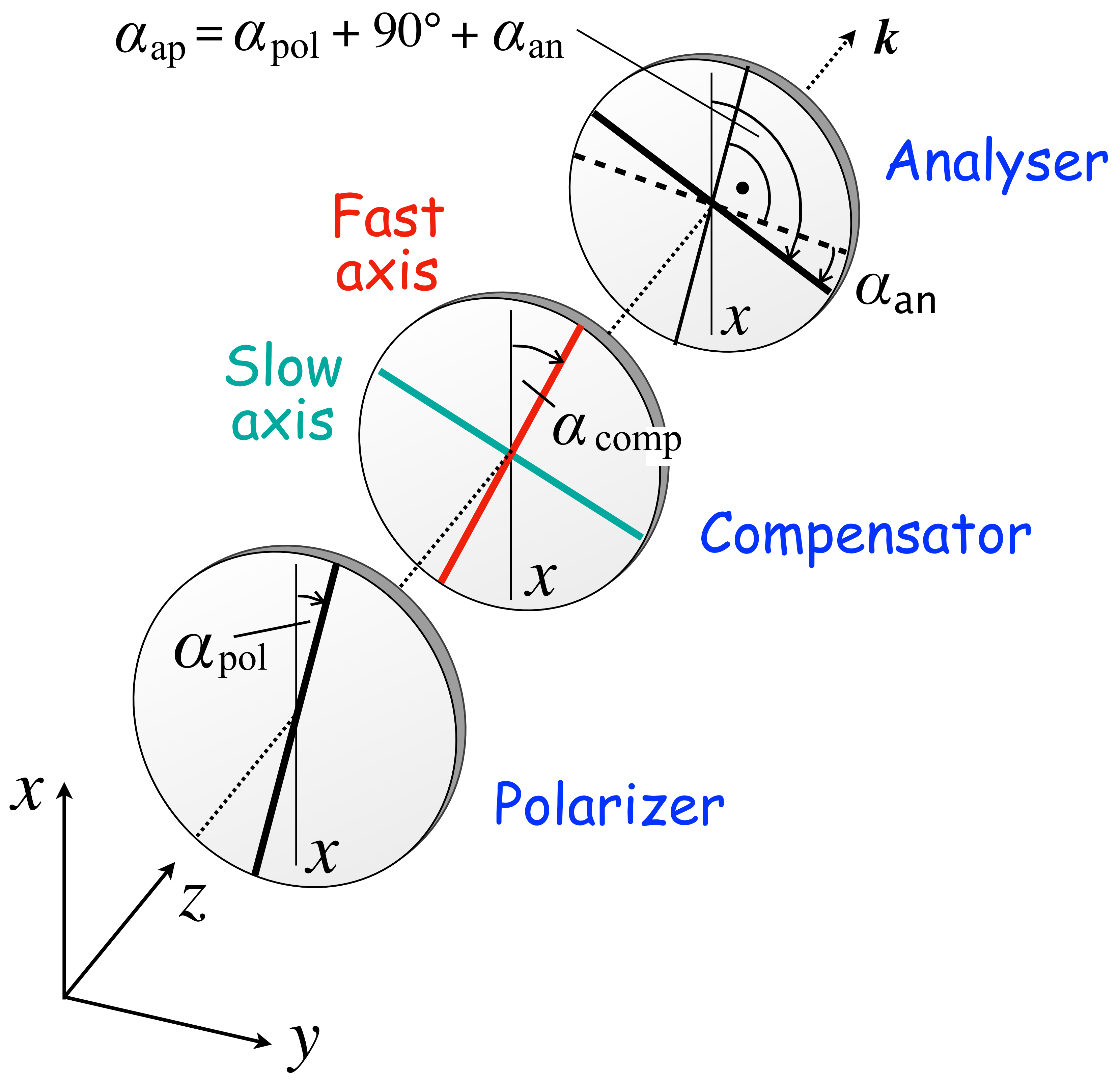






Example: Quarter-wave plate

- birefringent, uniaxial crystal (like quartz, mica)
- incident light is decomposed into 2 rays that are mutually perpendicularly polarized (ordinary and extra-ordinary rays)
- both rays feel different refraction indices $\Longrightarrow$ fast and slow axes
- retardation $R(\Theta) = \frac{\pi}{2}\sin(2\Theta)$
- example: $\Theta = 45° \Longrightarrow R(\Theta) = 90° \Longrightarrow$ circular light
- inversely: any phase shift can be linearized or otherwise adjusted



Description by Jones matrix algebra

$$\begin{pmatrix} E_x' \\ E_y' \end{pmatrix} = \boldsymbol{\Gamma}_{comp}\,\boldsymbol{\Gamma}_{an} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$

Emerging light — Jones matrices of optical elements — Incident light

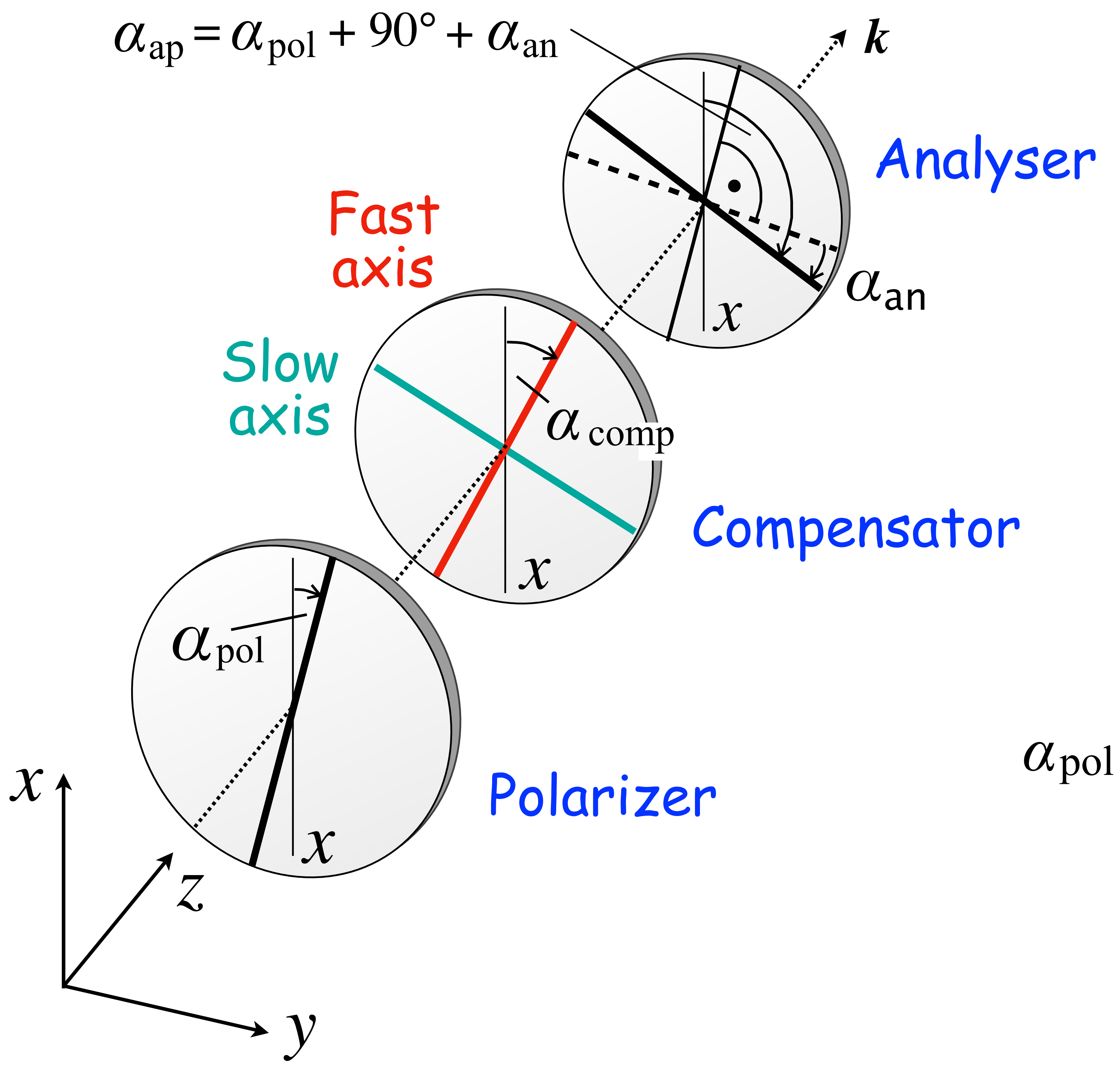
Quarter-wave plate:

$$\boldsymbol{\Gamma}_{comp} = \begin{pmatrix} \cos(-\alpha_{comp}) & \sin(-\alpha_{comp}) \\ -\sin(-\alpha_{comp}) & \cos(-\alpha_{comp}) \end{pmatrix} \begin{pmatrix} e^{-\frac{i\pi}{4}} & 0 \\ 0 & e^{+\frac{i\pi}{4}} \end{pmatrix} \begin{pmatrix} \cos\alpha_{comp} & \sin\alpha_{comp} \\ -\sin\alpha_{comp} & \cos\alpha_{comp} \end{pmatrix}$$

Coordinate transformation from xy-coordinates to eo-coordinates of plate — Jones matrix of retartation plate — Coordinate transformation back to xy-coordinates

Analyser:

$$\boldsymbol{\Gamma}_{an} = \begin{pmatrix} \cos\alpha_{ap} & -\sin\alpha_{ap} \\ \sin\alpha_{ap} & \cos\alpha_{ap} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\alpha_{ap} & \sin\alpha_{ap} \\ -\sin\alpha_{ap} & \cos\alpha_{ap} \end{pmatrix}$$



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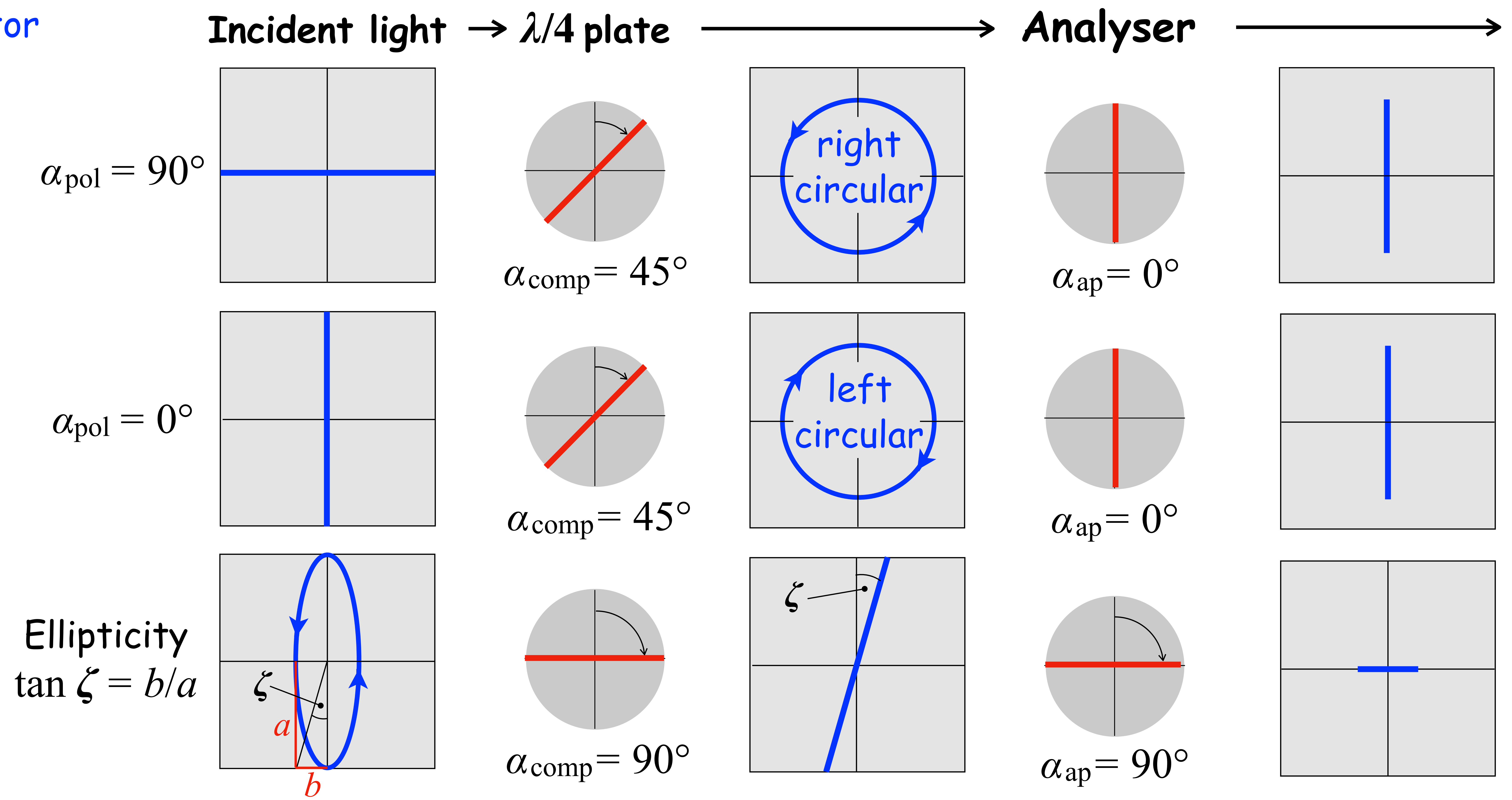
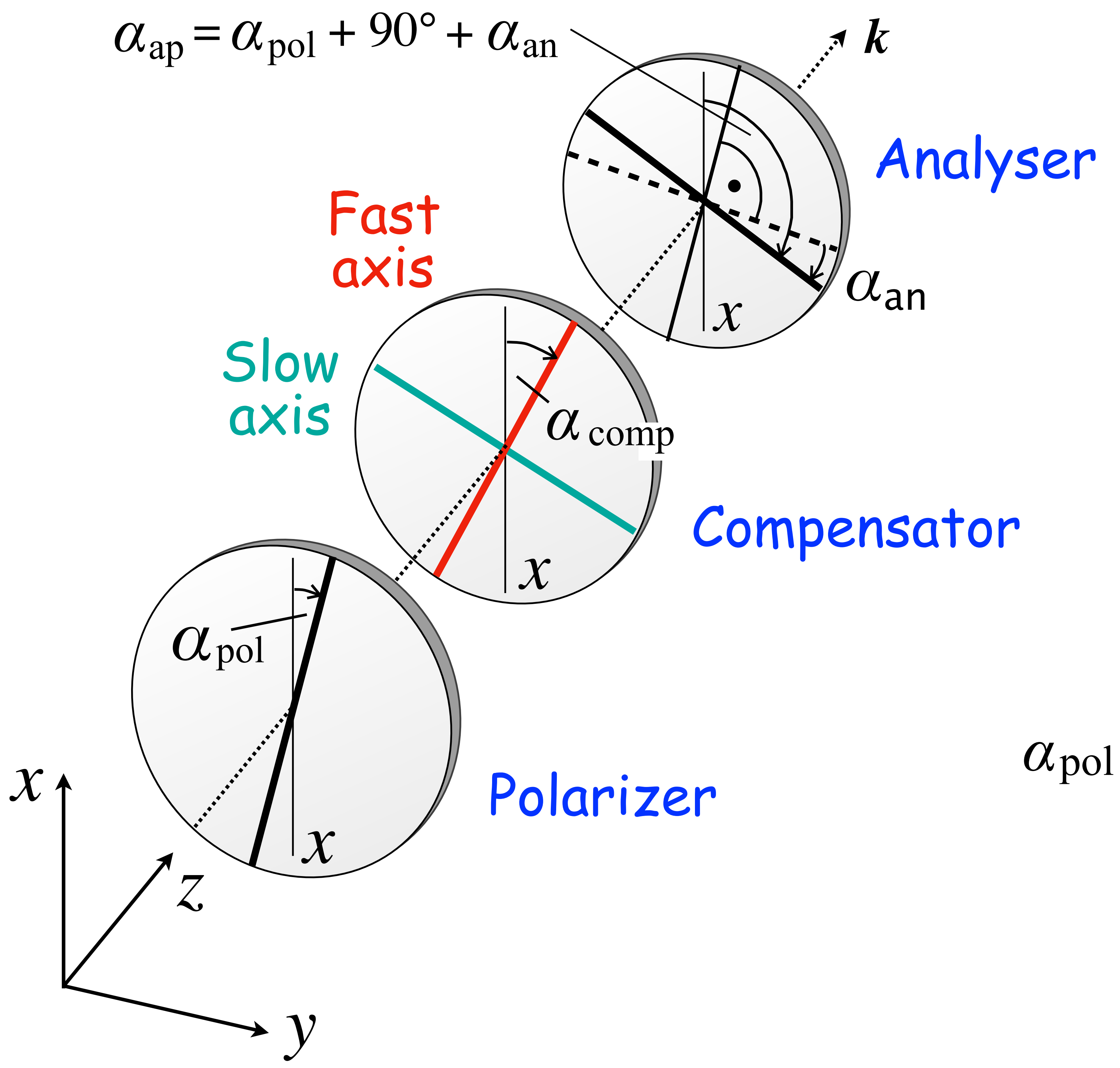
Emerging light — Jones matrices of optical elements — Incident light





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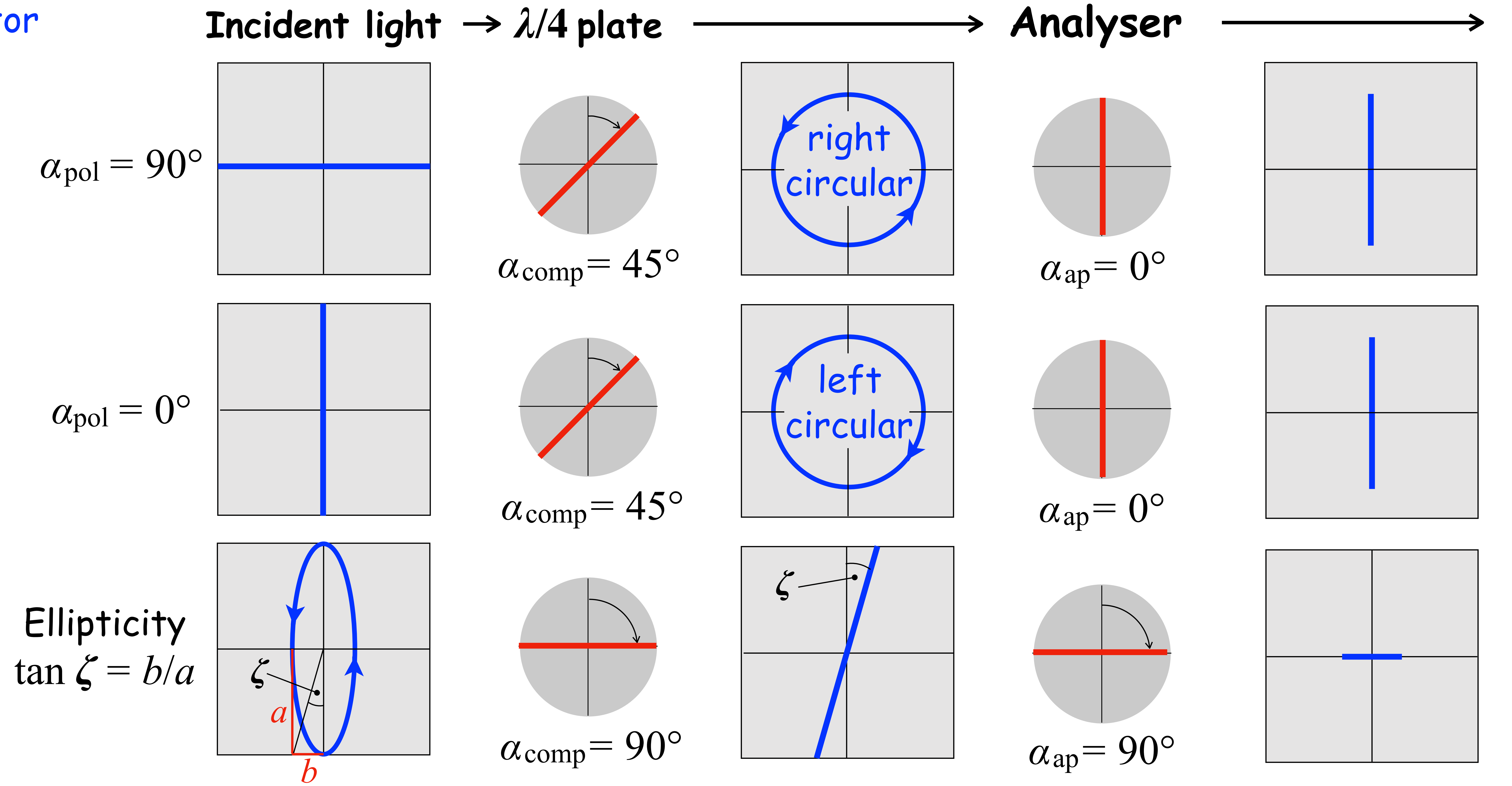

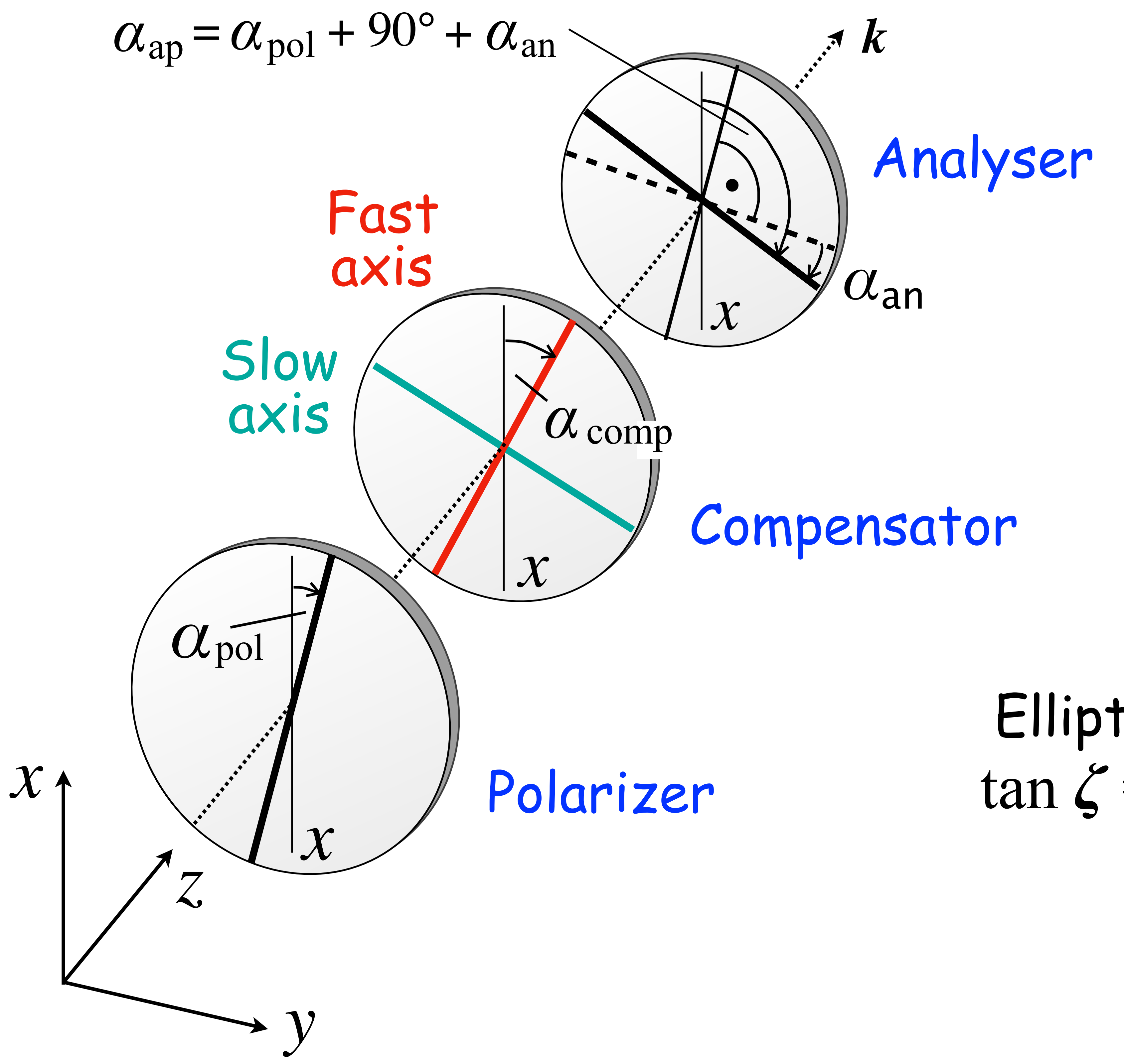

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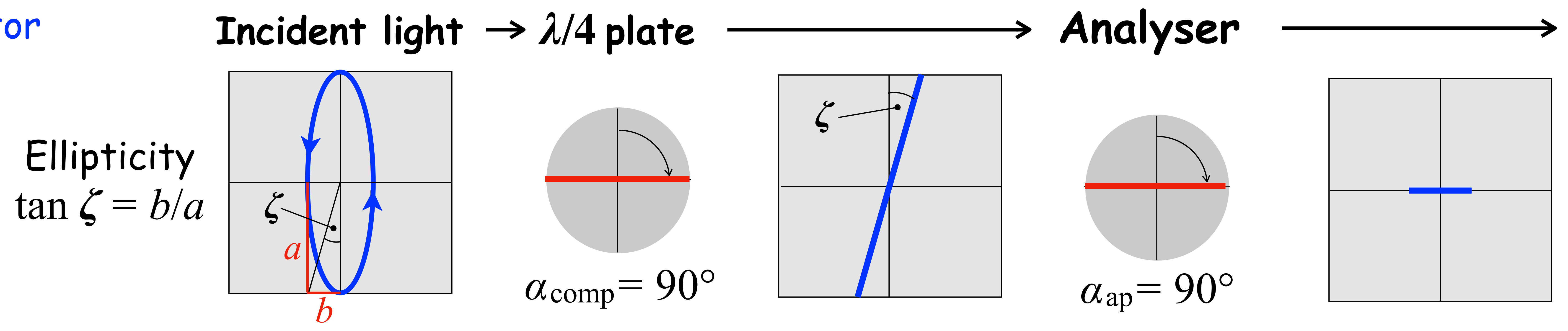

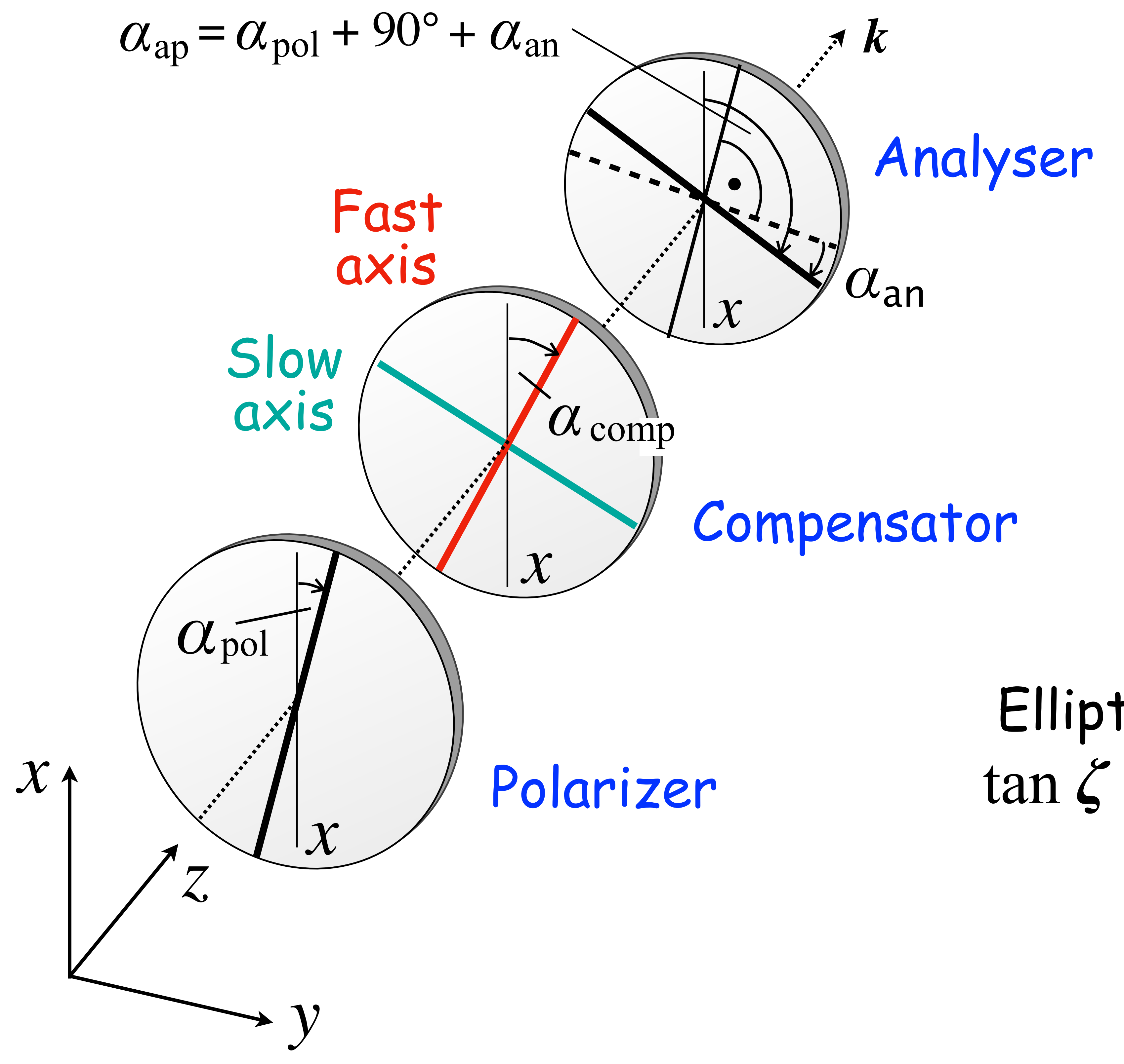

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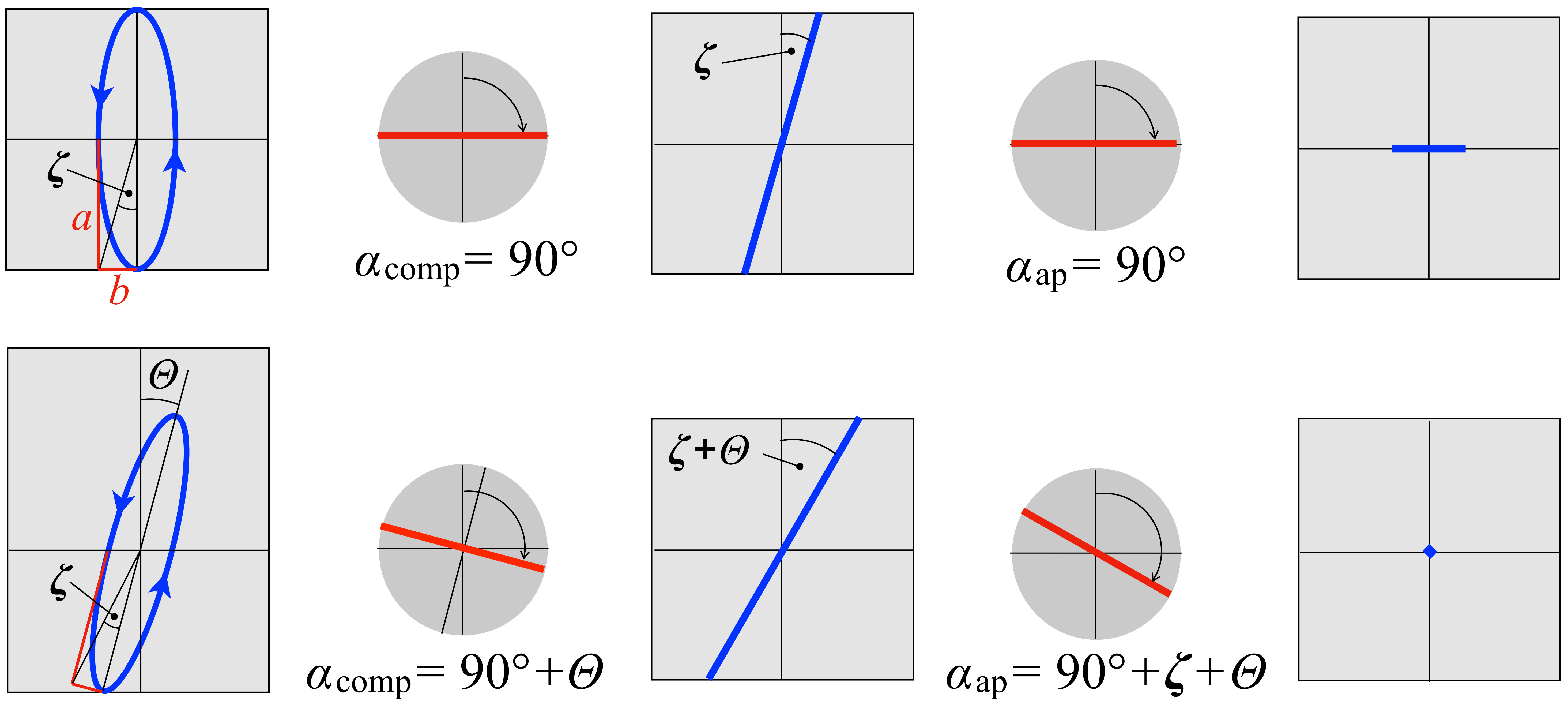
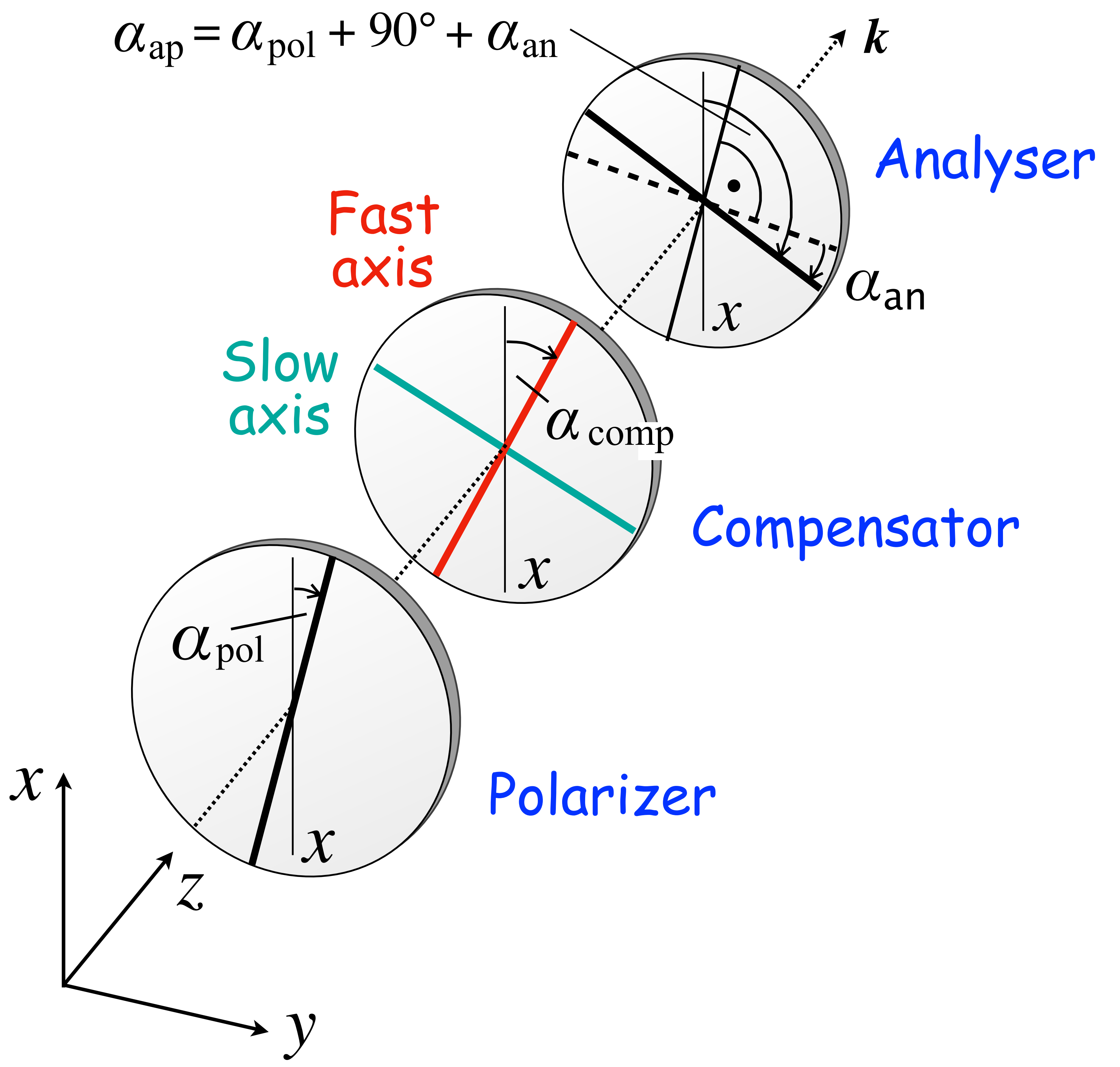
Emerging light — Jones matrices of optical elements — Incident light

Incident light → $\lambda/4$ plate → Analyser →

Ellipticity
$\tan\zeta = b/a$





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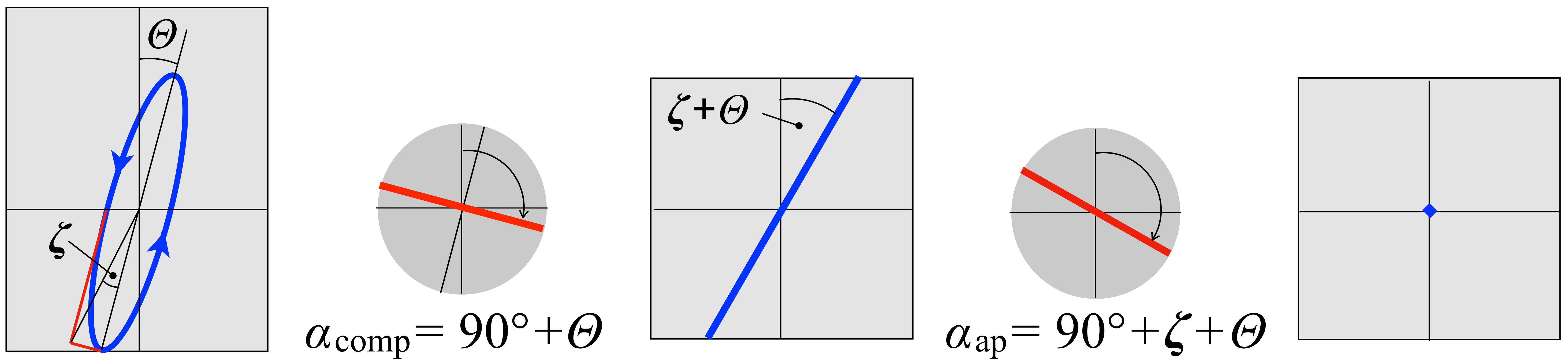

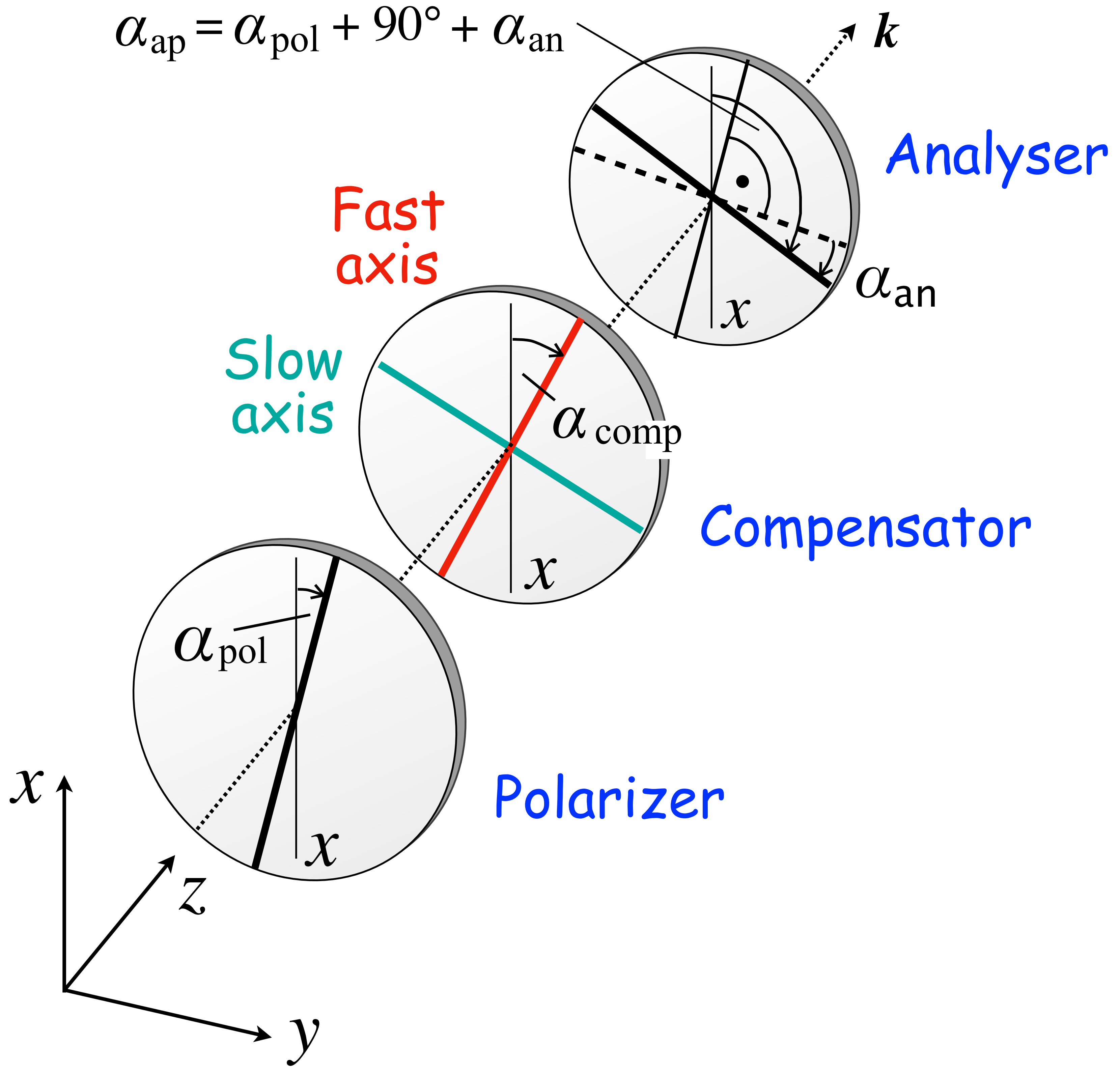

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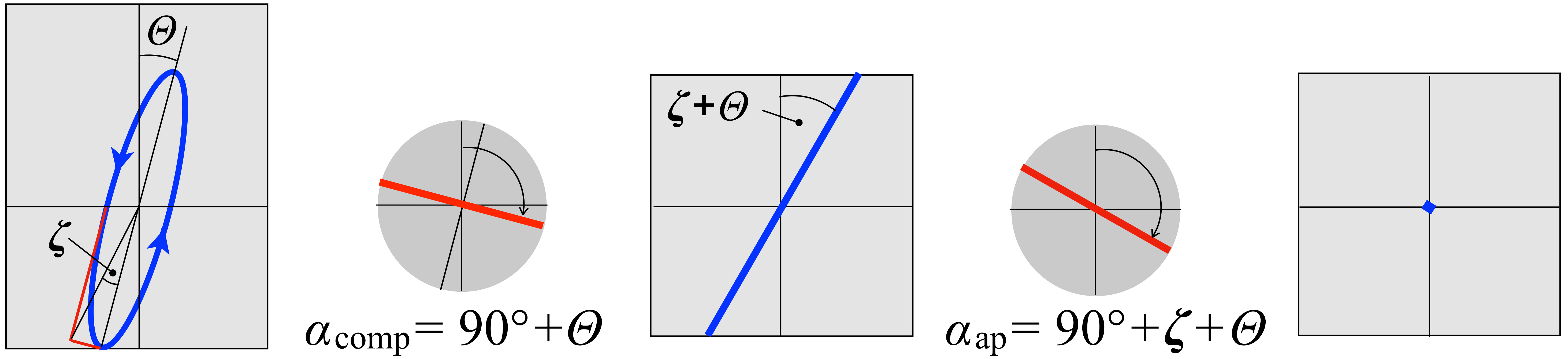

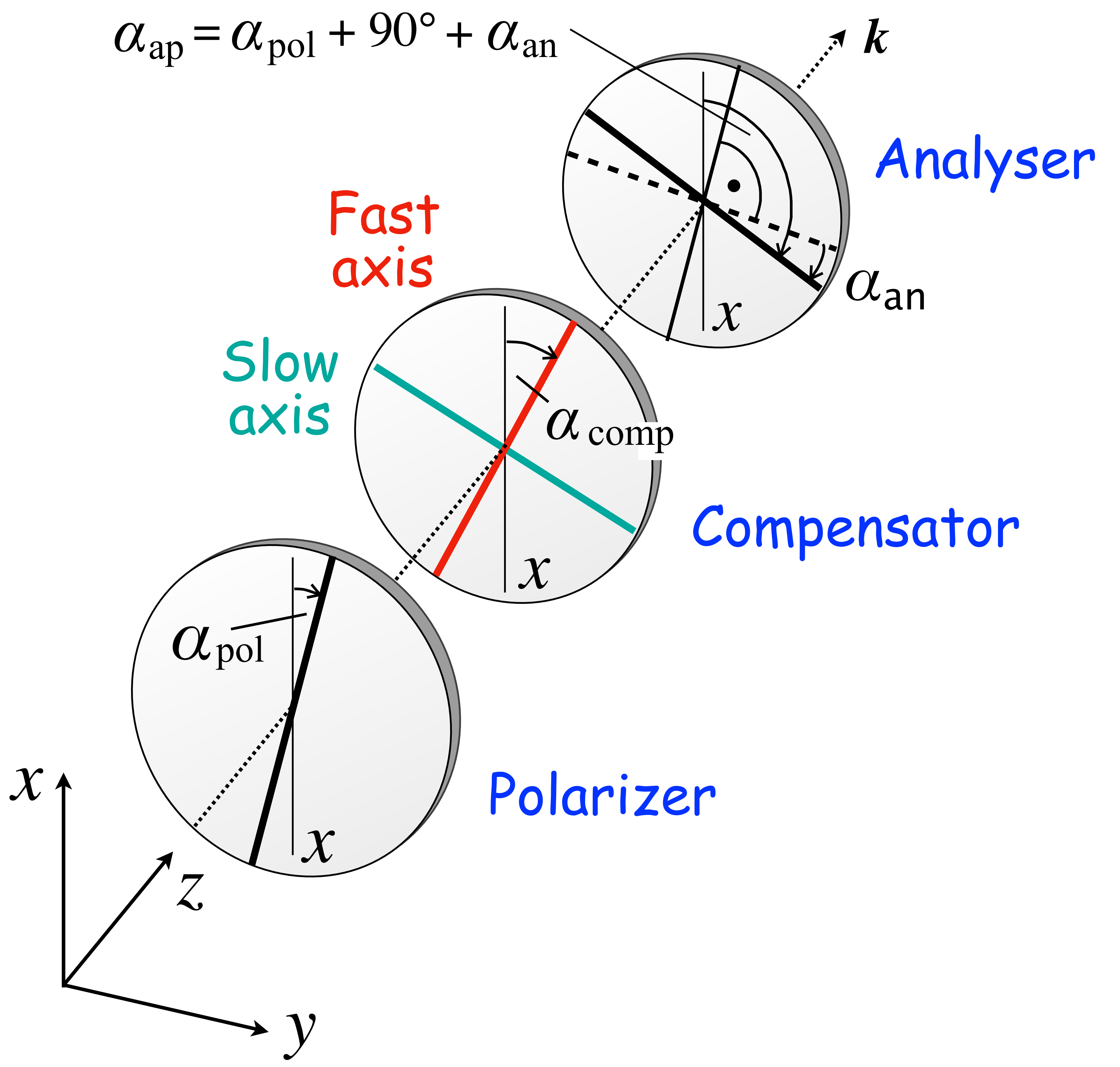

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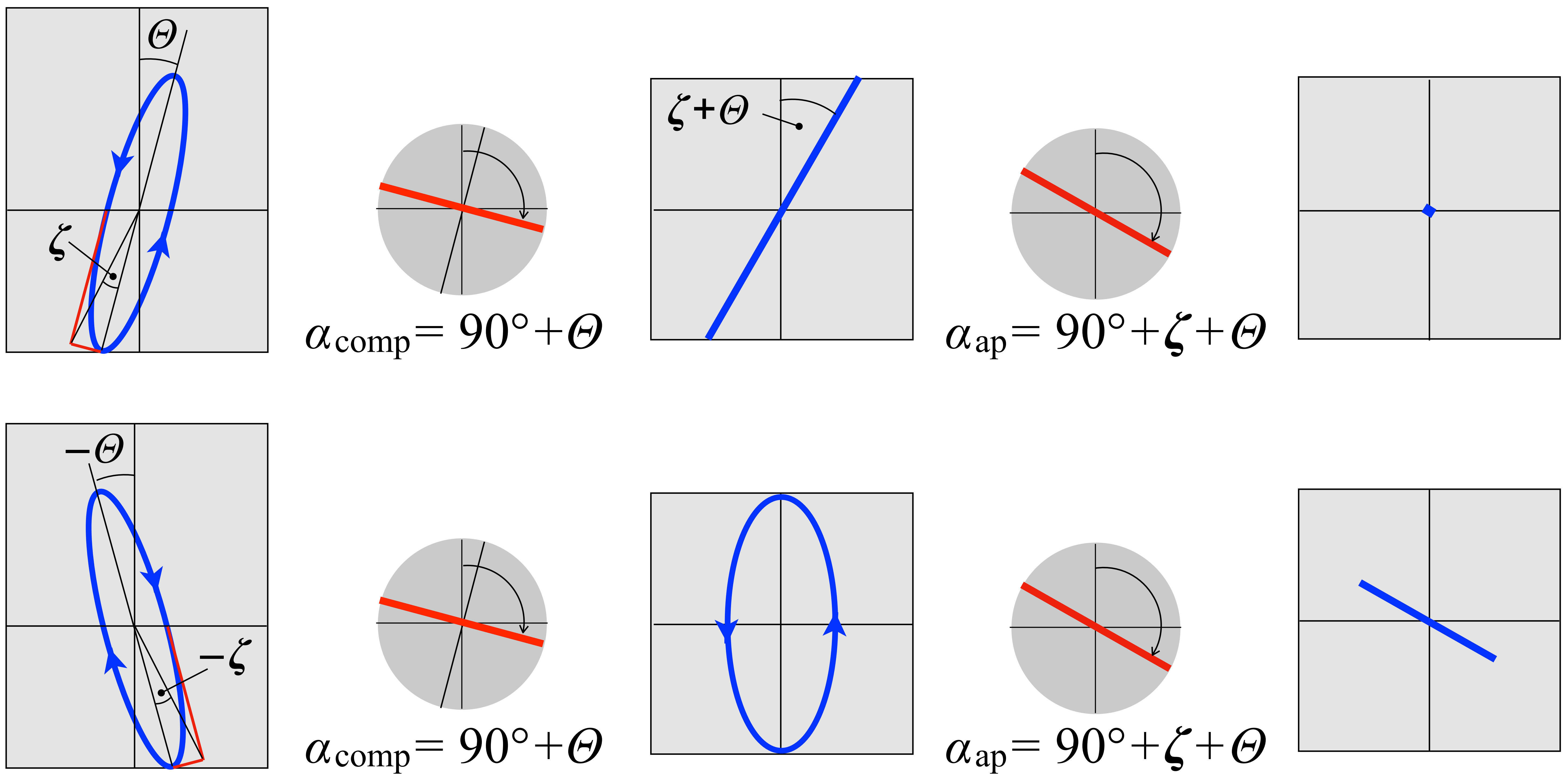
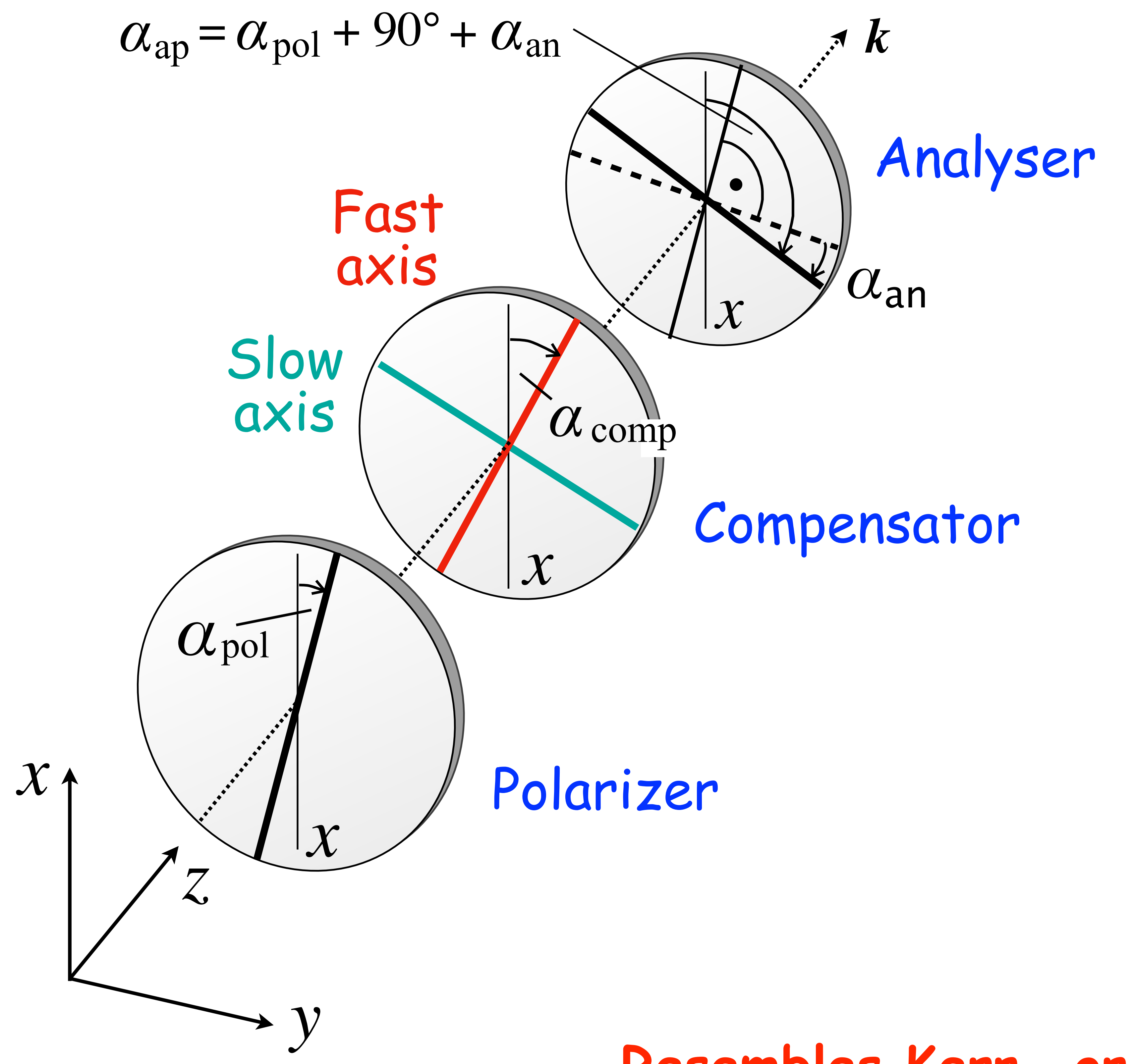


1. Optical Basics – 1.2 Polarized Light



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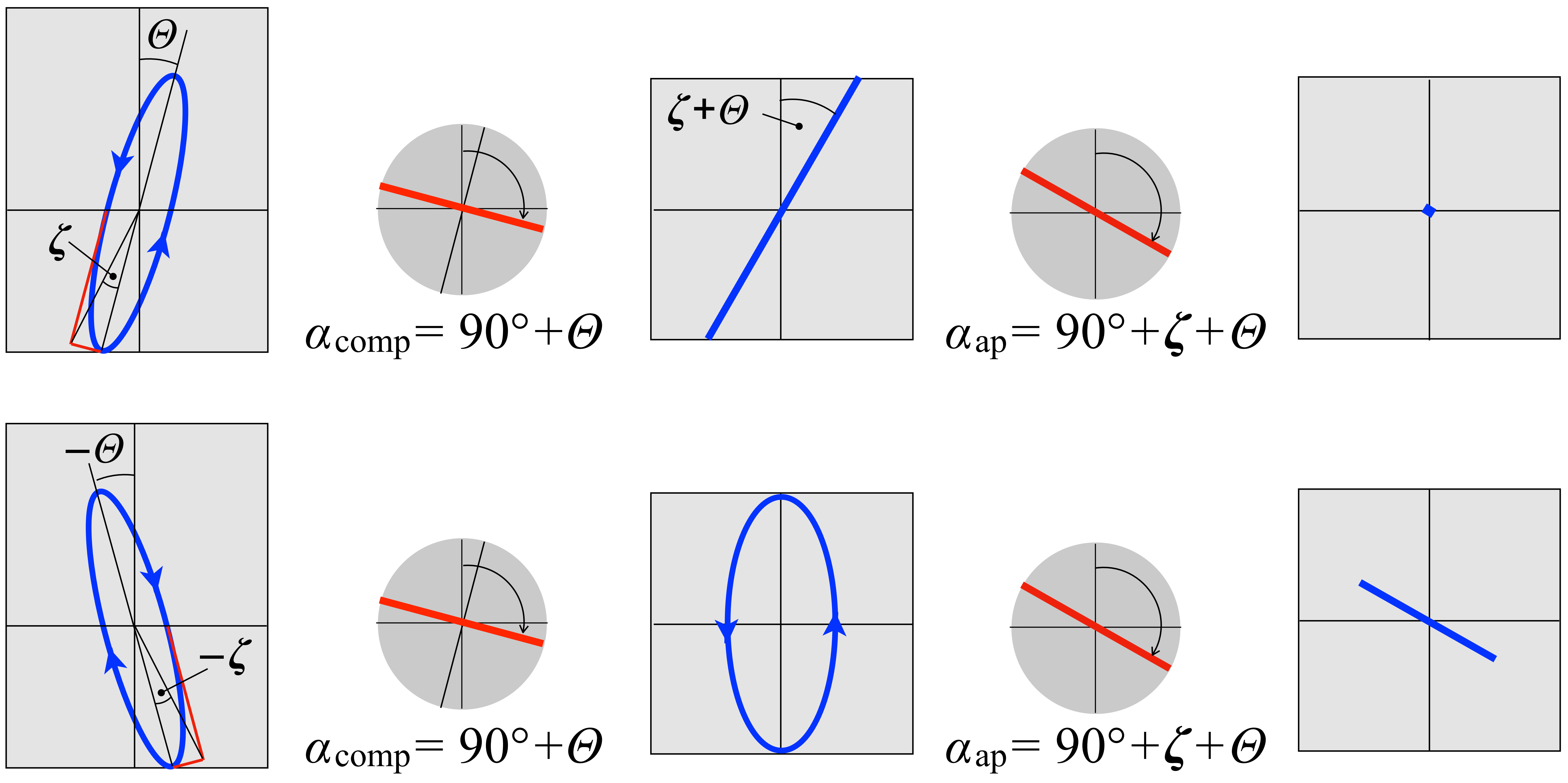
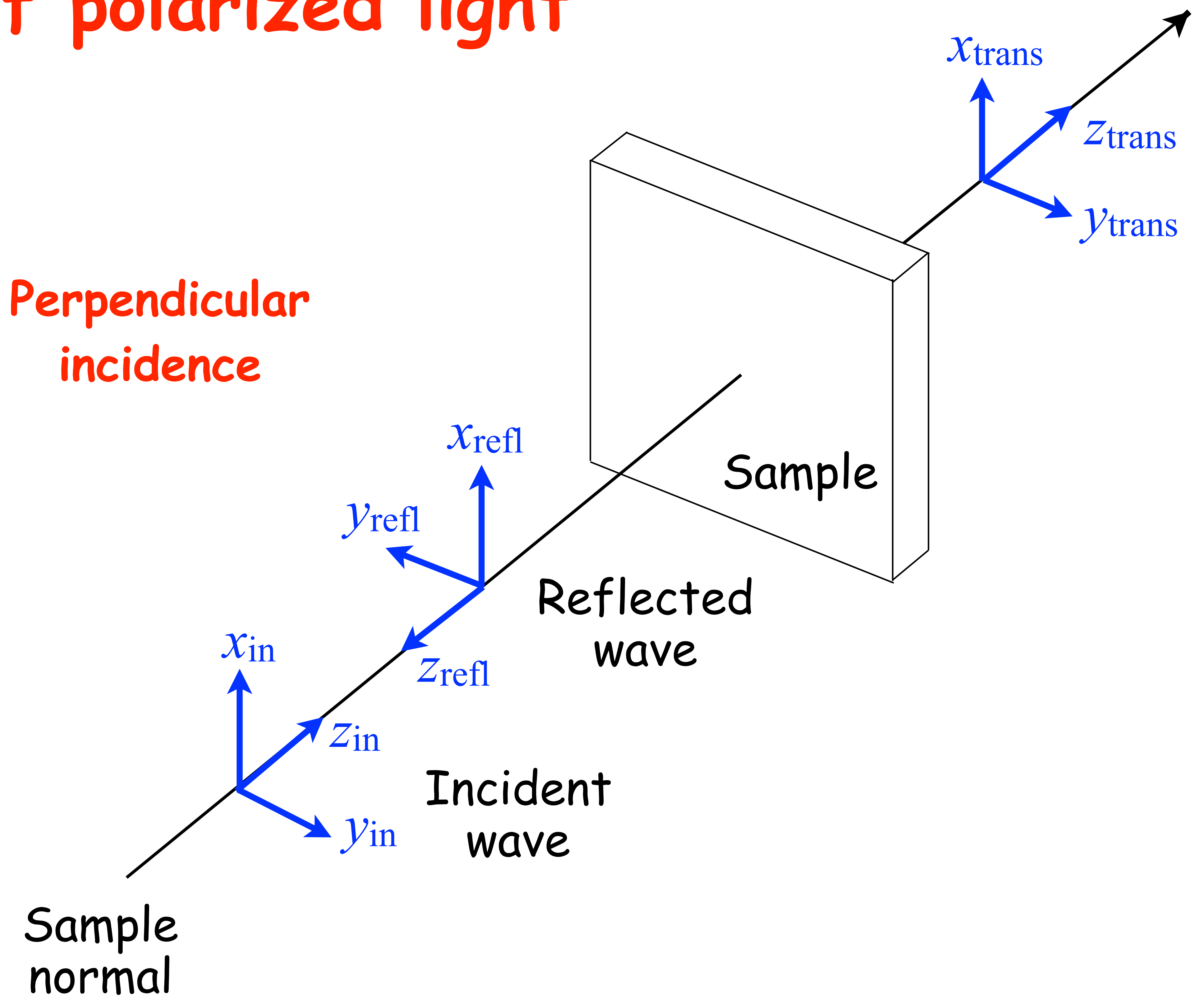
Emerging light — Jones matrices of optical elements — Incident light

Incident light → $\lambda/4$ plate ⟶ Analyser ⟶

Resembles Kerr- or Faraday effect (strongly exaggerated)



Reflection & transmission of polarized light



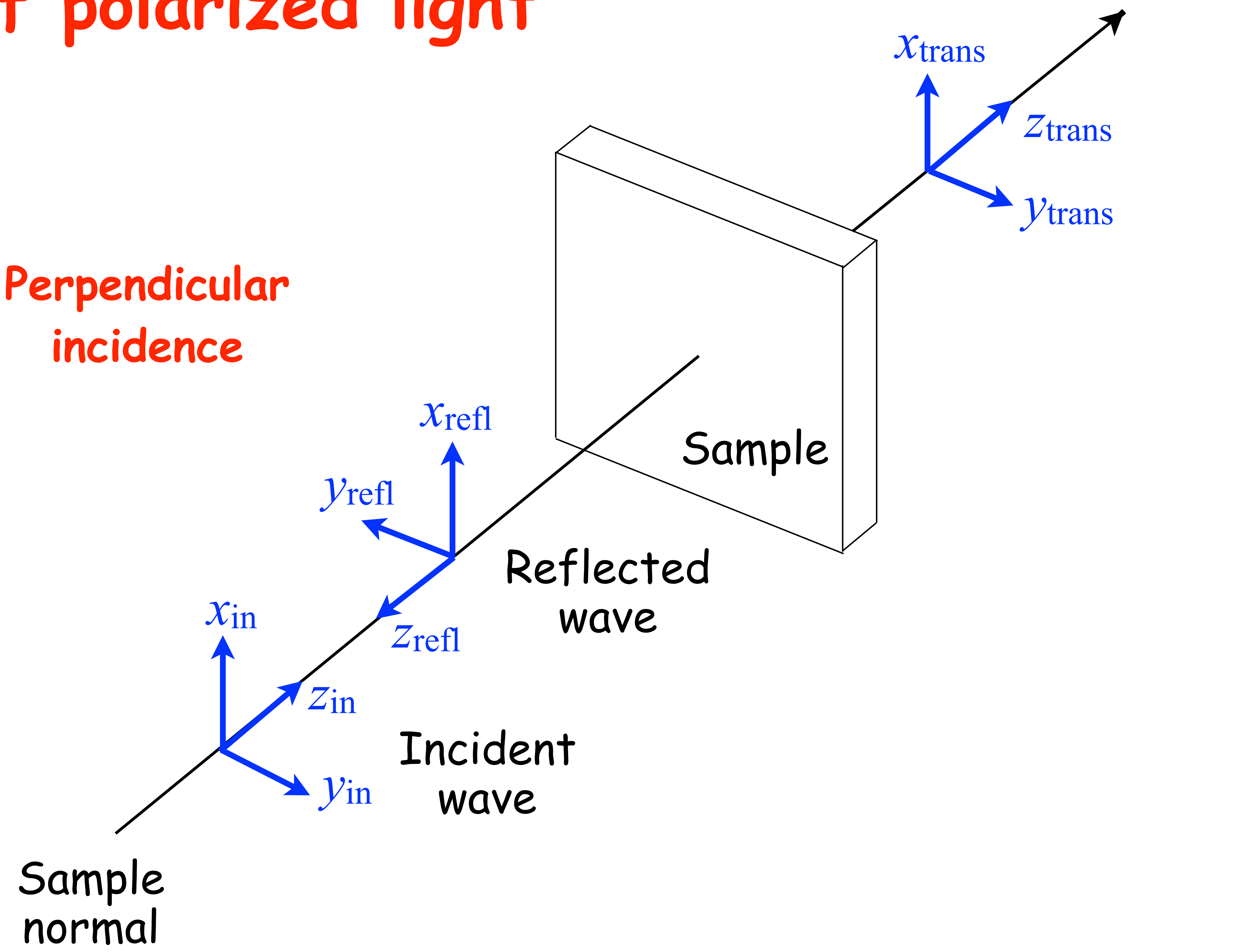
Jones matrixes for reflection and transmission

$$\begin{pmatrix} E_x^{refl} \\ E_y^{refl} \end{pmatrix} = \begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} \begin{pmatrix} E_x^{in} \\ E_y^{in} \end{pmatrix} \; ; \quad \begin{pmatrix} E_x^{trans} \\ E_y^{trans} \end{pmatrix} = \begin{pmatrix} t_{xx} & t_{xy} \\ t_{yx} & t_{yy} \end{pmatrix} \begin{pmatrix} E_x^{in} \\ E_y^{in} \end{pmatrix}$$

out in

Reflection & transmission of polarized light

Perpendicular incidence



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out in

Example: incident wave, polarized along x

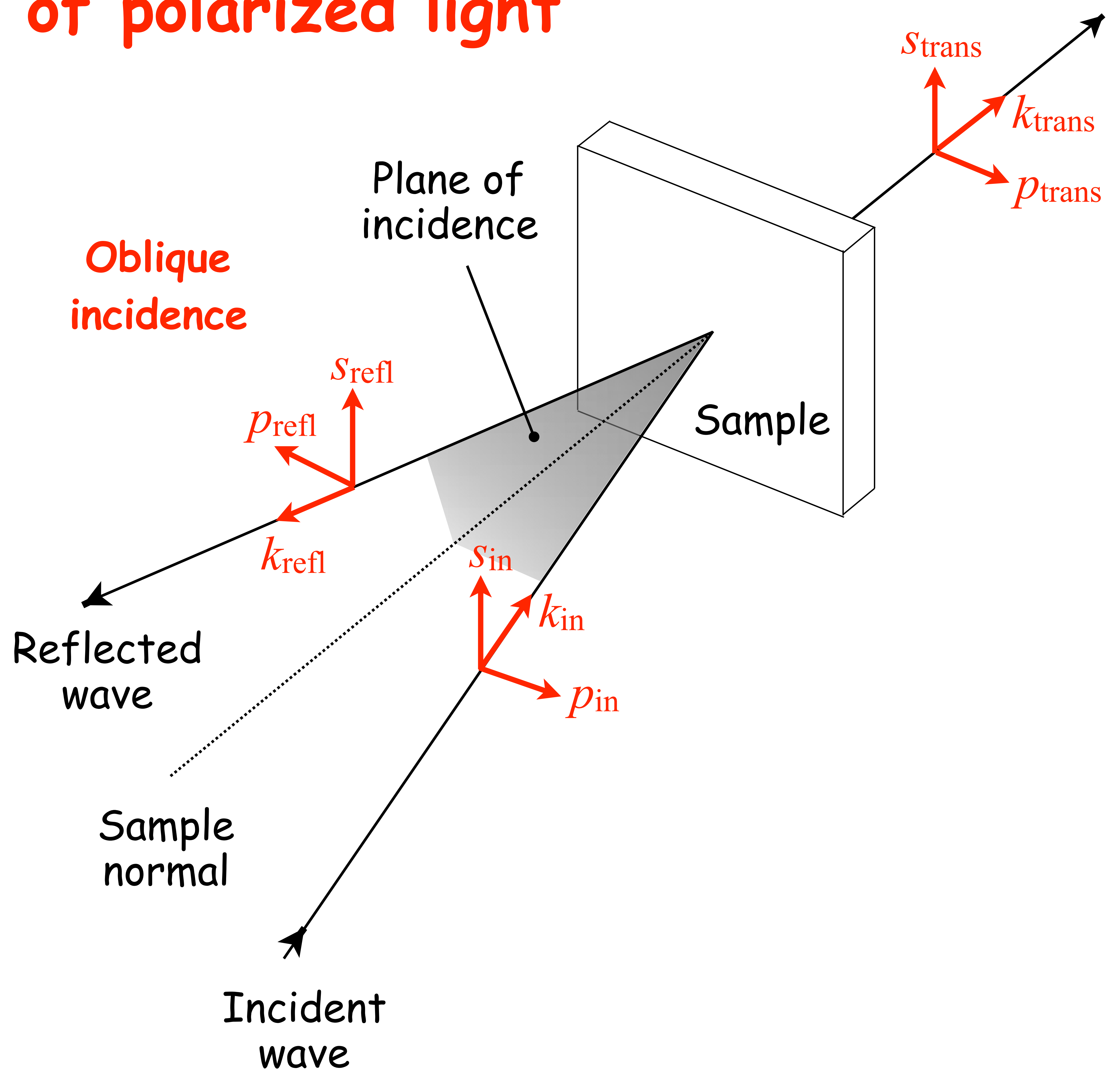
$$\begin{pmatrix} E_{\mathrm{x}}^{\mathrm{refl}} \\ E_{\mathrm{y}}^{\mathrm{refl}} \end{pmatrix} = \begin{pmatrix} r_{\mathrm{xx}} & r_{\mathrm{xy}} \\ r_{\mathrm{yx}} & r_{\mathrm{yy}} \end{pmatrix} \begin{pmatrix} E_{\mathrm{x}}^{\mathrm{in}} \\ 0 \end{pmatrix} = \begin{pmatrix} r_{\mathrm{xx}} E_{\mathrm{x}}^{\mathrm{in}} \\ r_{\mathrm{yx}} E_{\mathrm{x}}^{\mathrm{in}} \end{pmatrix}$$

If $r_{\mathrm{yx}} = 0$: $\Longrightarrow$ Reflected wave is polarized along incident wave

If $r_{\mathrm{yx}} \neq 0$: $\Longrightarrow E_{\mathrm{y}}^{\mathrm{refl}} \neq 0$

$\Longrightarrow$ Reflected wave is rotated

Reflection & transmission of polarized light



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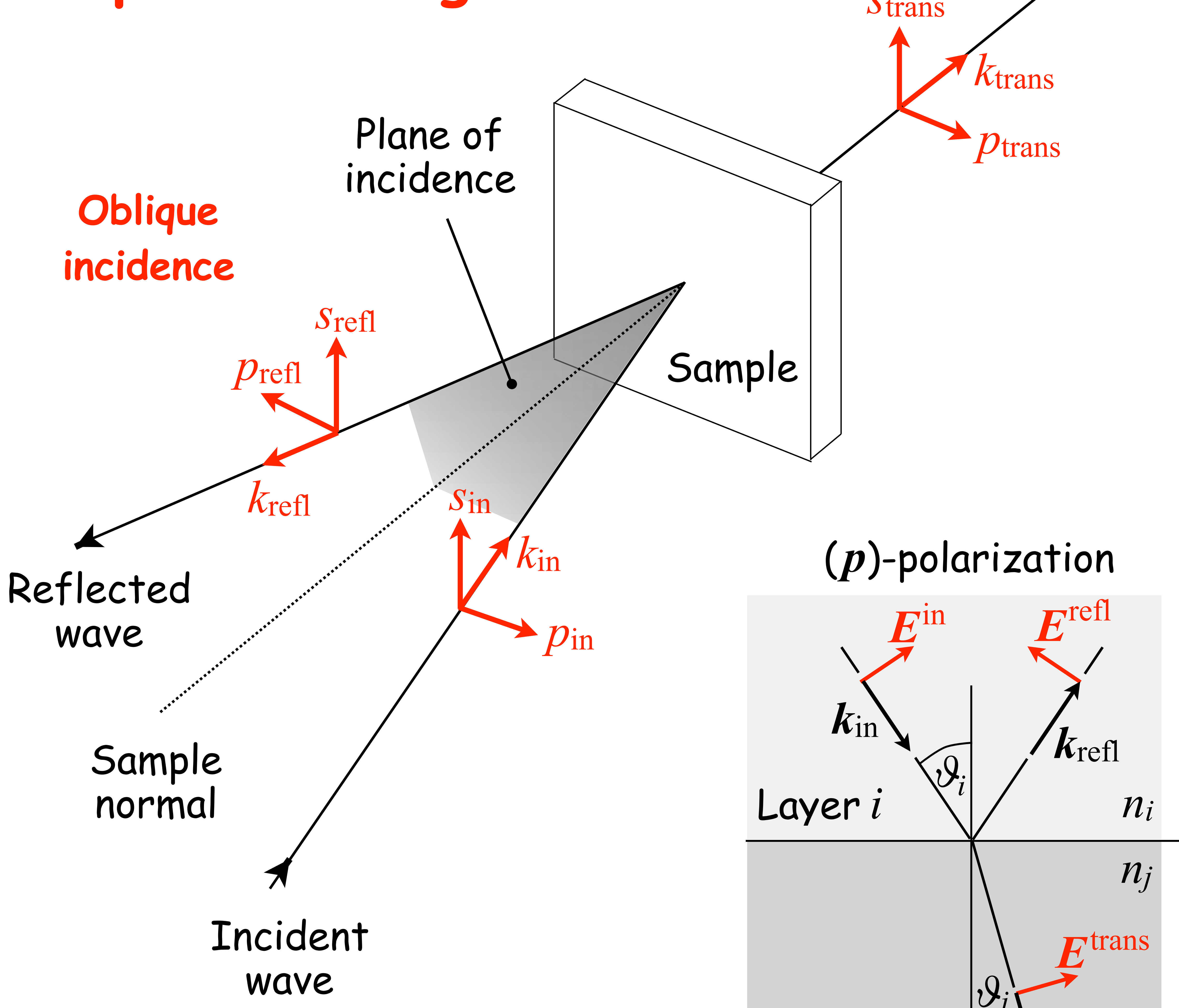
out in

Reflection & transmission of polarized light



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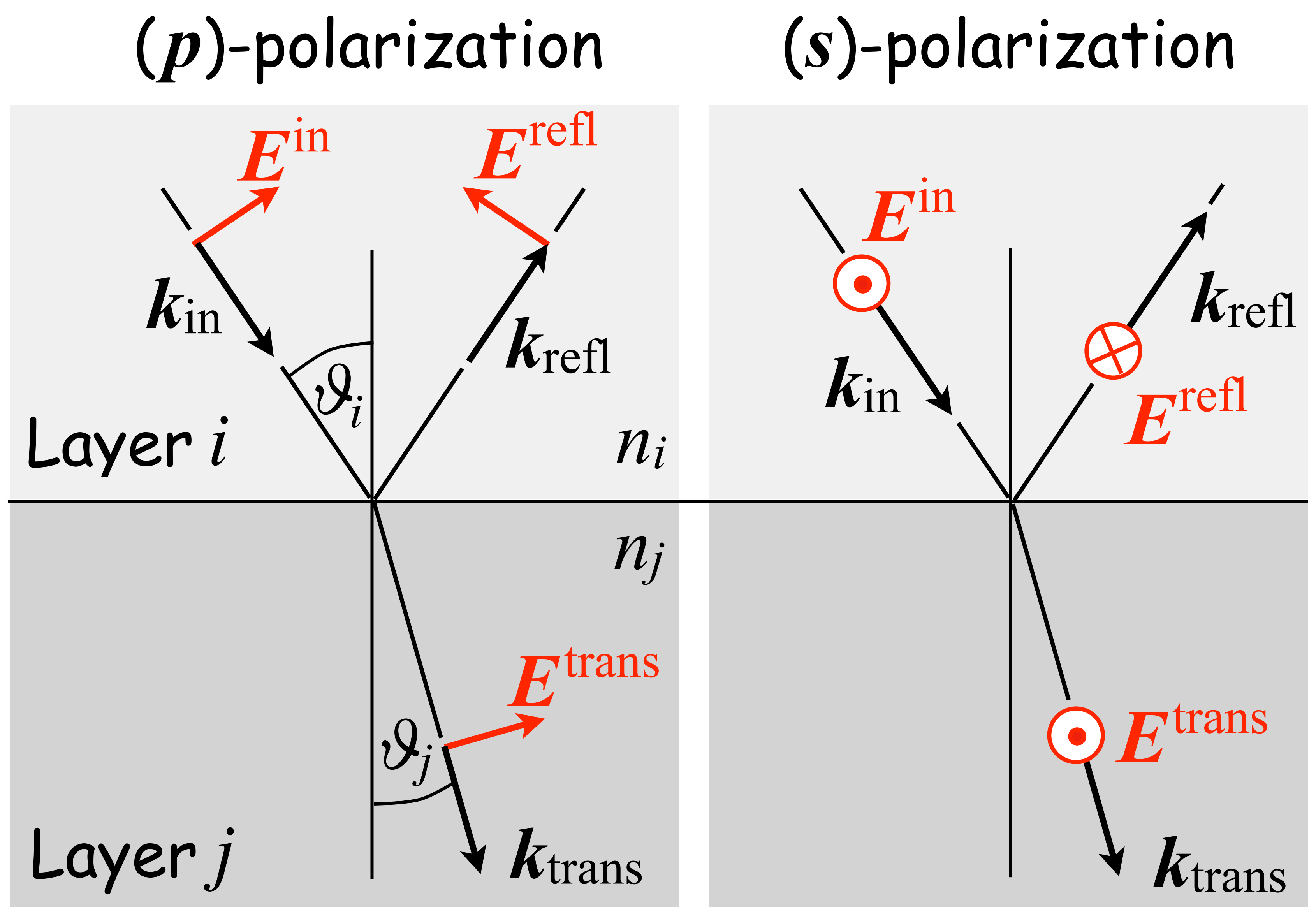
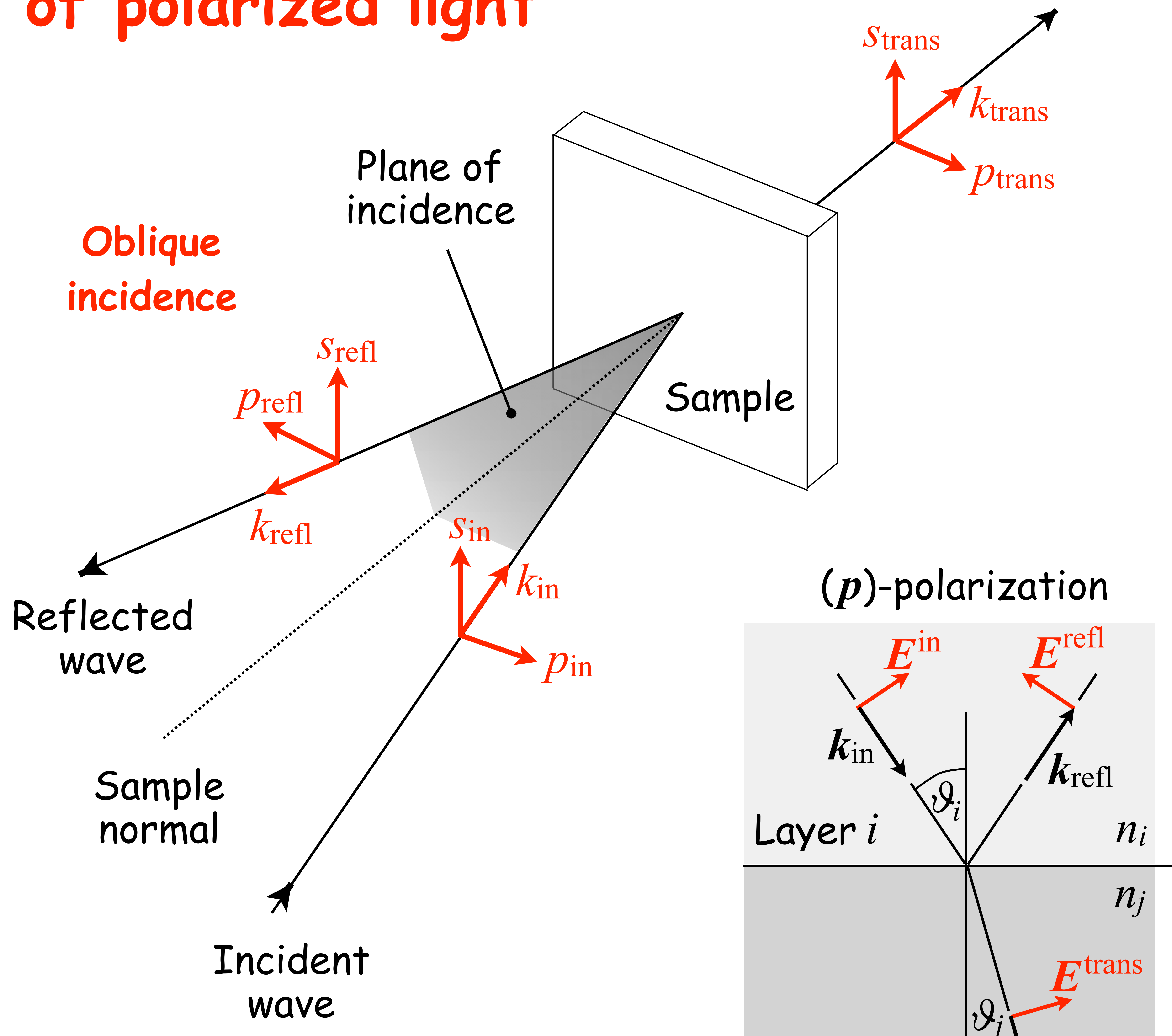
out in



Reflection & transmission of polarized light



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out in

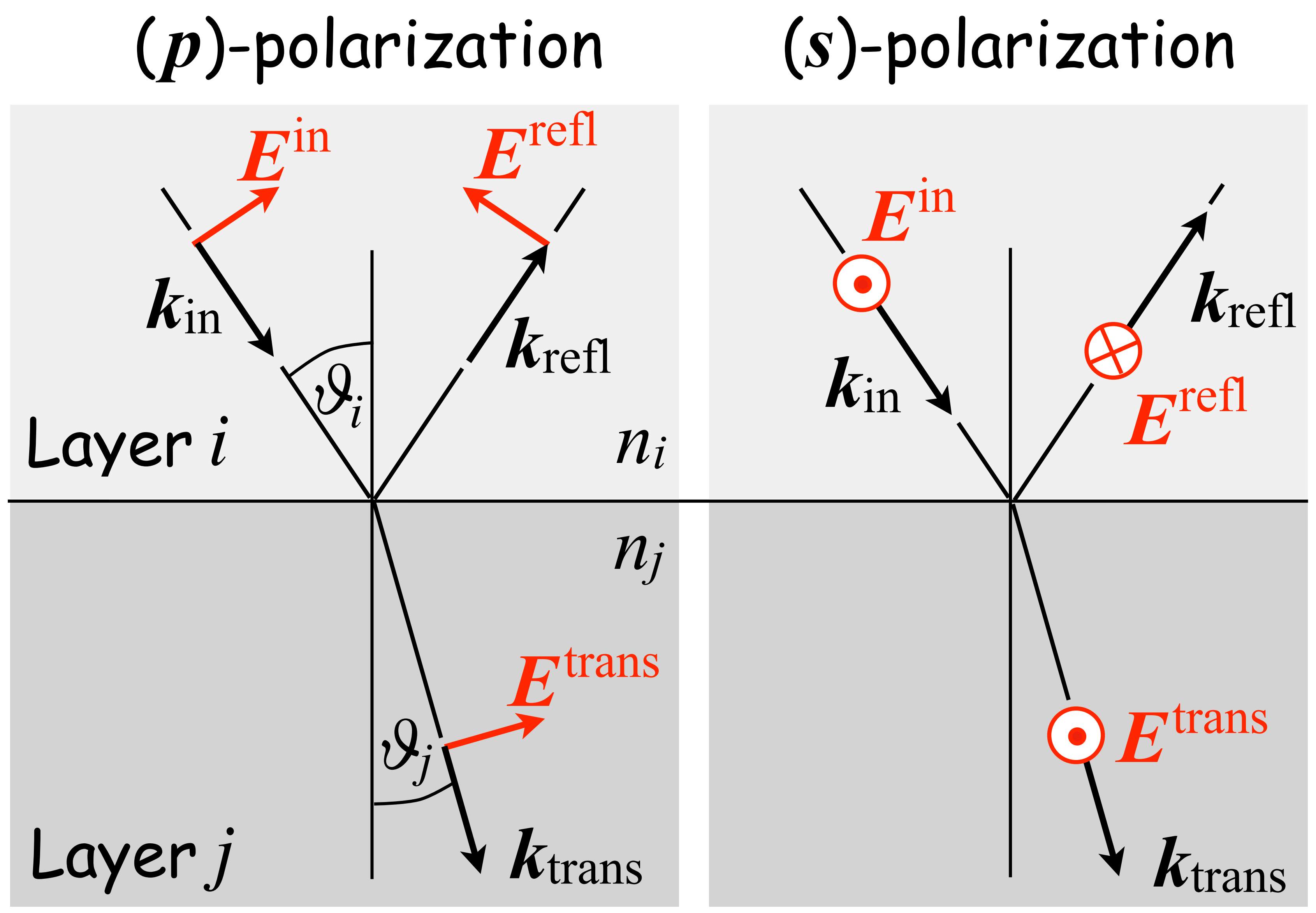
Fresnel equations:

$$r_{pp}^{ij} = \left(\frac{E^{refl}}{E^{in}}\right)_p = \frac{n_i \cos\vartheta_j - n_j \cos\vartheta_i}{n_i \cos\vartheta_j + n_j \cos\vartheta_i}$$

$$r_{ss}^{ij} = \left(\frac{E^{refl}}{E^{in}}\right)_s = \frac{n_i \cos\vartheta_i - n_j \cos\vartheta_j}{n_i \cos\vartheta_i + n_j \cos\vartheta_j}$$

$$t_{pp}^{ij} = \left(\frac{E^{trans}}{E^{in}}\right)_p = \frac{2 n_i \cos\vartheta_i}{n_i \cos\vartheta_j + n_j \cos\vartheta_i}$$

$$t_{ss}^{ij} = \left(\frac{E^{trans}}{E^{in}}\right)_s = \frac{2 n_i \cos\vartheta_i}{n_i \cos\vartheta_i + n_j \cos\vartheta_j}$$



Snell's law: $n_i \sin\vartheta_i = n_j \sin\vartheta_j$

Summary:

Maxwell and material equations

$$\downarrow$$

General wave equation,
solutions of which describe propagable light waves in medium

$$\downarrow$$

All material-specific properties of light propagation
(including absorption, dispersion, optical anisotropy and optical activity,
as well as the magneto-optical effects)
are completely included in the complex dielectric ε-tensor

$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$

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$$D = \epsilon E$$

2.

Magneto-Optical Effects

$$\boldsymbol{D} = \boldsymbol{\epsilon}\boldsymbol{E}$$

For materials with **cubic** crystal symmetry:

$$\boldsymbol{\epsilon} = \epsilon_{\text{iso}} \begin{pmatrix} 1 & -\mathrm{i}Q_{\mathrm{V}}m_3 & \mathrm{i}Q_{\mathrm{V}}m_2 \\ \mathrm{i}Q_{\mathrm{V}}m_3 & 1 & -\mathrm{i}Q_{\mathrm{V}}m_1 \\ -\mathrm{i}Q_{\mathrm{V}}m_2 & \mathrm{i}Q_{\mathrm{V}}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1 m_1^2 & B_2 m_1 m_2 & B_2 m_1 m_3 \\ B_2 m_1 m_2 & B_1 m_2^2 & B_2 m_2 m_3 \\ B_2 m_1 m_3 & B_2 m_2 m_3 & B_1 m_3^2 \end{pmatrix}$$

Faraday and Kerr effects

(Intrinsic) Voigt effect

Q_{V} and B_{i} : complex (i.e. $Q_{\mathrm{V}} = Q'_{\mathrm{V}} + \mathrm{i}Q''_{\mathrm{V}}$ etc.) and frequency-dependent parameters

2. Magneto-Optical Effects – 2.1 Dielectric Tensor

$$\boldsymbol{D} = \boldsymbol{\epsilon}\boldsymbol{E}$$

For materials with **cubic** crystal symmetry:

$$\boldsymbol{\epsilon} = \epsilon_{\text{iso}} \begin{pmatrix} 1 & -\mathrm{i}Q_{\text{V}}m_3 & \mathrm{i}Q_{\text{V}}m_2 \\ \mathrm{i}Q_{\text{V}}m_3 & 1 & -\mathrm{i}Q_{\text{V}}m_1 \\ -\mathrm{i}Q_{\text{V}}m_2 & \mathrm{i}Q_{\text{V}}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1 m_1^2 & B_2 m_1 m_2 & B_2 m_1 m_3 \\ B_2 m_1 m_2 & B_1 m_2^2 & B_2 m_2 m_3 \\ B_2 m_1 m_3 & B_2 m_2 m_3 & B_1 m_3^2 \end{pmatrix}$$

Faraday and Kerr effects

(Intrinsic) Voigt effect

Q_{V} and B_{i} : complex (i.e. $Q_{\text{V}} = Q'_{\text{V}} + \mathrm{i}Q''_{\text{V}}$ etc.) and frequency-dependent parameters

Assumption: no magneto-optical effects ($B_{\text{i}} = Q_{\text{V}} = 0$, or $\boldsymbol{m} = 0$)

$$\boldsymbol{\epsilon} = \epsilon_{\text{iso}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Longrightarrow \begin{pmatrix} D_{\text{x}} \\ D_{\text{y}} \\ D_{\text{z}} \end{pmatrix} = \epsilon_{\text{iso}} \begin{pmatrix} E_{\text{x}} \\ E_{\text{y}} \\ E_{\text{z}} \end{pmatrix}$$

$$\boldsymbol{D} = \epsilon \boldsymbol{E}$$

For materials with **cubic** crystal symmetry:

$$\epsilon = \epsilon_{\mathrm{iso}} \begin{pmatrix} 1 & -\mathrm{i}Q_{\mathrm{V}}m_3 & \mathrm{i}Q_{\mathrm{V}}m_2 \\ \mathrm{i}Q_{\mathrm{V}}m_3 & 1 & -\mathrm{i}Q_{\mathrm{V}}m_1 \\ -\mathrm{i}Q_{\mathrm{V}}m_2 & \mathrm{i}Q_{\mathrm{V}}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1 m_1^2 & B_2 m_1 m_2 & B_2 m_1 m_3 \\ B_2 m_1 m_2 & B_1 m_2^2 & B_2 m_2 m_3 \\ B_2 m_1 m_3 & B_2 m_2 m_3 & B_1 m_3^2 \end{pmatrix}$$

Faraday and Kerr effects

(Intrinsic) Voigt effect

Q_{V} and B_{i} : complex (i.e. $Q_{\mathrm{V}} = Q_{\mathrm{V}}' + \mathrm{i}Q_{\mathrm{V}}''$ etc.) and frequency-dependent parameters

Assumption: no magneto-optical effects ($B_{\mathrm{i}} = Q_{\mathrm{V}} = 0$, or $\boldsymbol{m} = 0$)

$$\epsilon = \epsilon_{\mathrm{iso}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \implies \begin{pmatrix} D_{\mathrm{x}} \\ D_{\mathrm{y}} \\ D_{\mathrm{z}} \end{pmatrix} = \epsilon_{\mathrm{iso}} \begin{pmatrix} E_{\mathrm{x}} \\ E_{\mathrm{y}} \\ E_{\mathrm{z}} \end{pmatrix}$$

$\implies$ magneto-optical effects (rotation and ellipticity) require off-diagonal elements

Wave equation: $$-\nabla^2 \boldsymbol{E} + \mu_0 \boldsymbol{\mu}(\boldsymbol{\sigma} \frac{\partial \boldsymbol{E}}{\partial t} + \epsilon_0 \boldsymbol{\epsilon} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}) = 0$$

Wave equation: $$-\nabla^2 \boldsymbol{E} + \mu_0 \boldsymbol{\mu} (\boldsymbol{\sigma} \frac{\partial \boldsymbol{E}}{\partial t} + \epsilon_0 \boldsymbol{\epsilon} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}) = 0$$

$\Downarrow$ with $\nabla \cdot \Rightarrow \mathrm{i}\boldsymbol{k}\cdot$, $\nabla \times \Rightarrow \mathrm{i}\boldsymbol{k}\times$, $\frac{\partial}{\partial t} \Rightarrow -\mathrm{i}\omega$

$$-\widetilde{\boldsymbol{k}}^2 \boldsymbol{E} + \mu_0 \epsilon_0 \boldsymbol{\mu}\, \omega^2 \widetilde{\boldsymbol{\epsilon}}\, \boldsymbol{E} = 0$$ with $\boldsymbol{\epsilon} + \mathrm{i}\boldsymbol{\sigma}/(\epsilon_0 \omega) = \widetilde{\boldsymbol{\epsilon}}$

2. Magneto-Optical Effects – 2.2 Solutions

Wave equation: $$-\nabla^2\boldsymbol{E}+\mu_0\boldsymbol{\mu}(\boldsymbol{\sigma}\frac{\partial\boldsymbol{E}}{\partial t}+\epsilon_0\boldsymbol{\epsilon}\,\frac{\partial^2\boldsymbol{E}}{\partial t^2})=0$$

$\Downarrow$ with $\nabla\cdot\Rightarrow \mathrm{i}\boldsymbol{k}\cdot$, $\nabla\times\Rightarrow \mathrm{i}\boldsymbol{k}\times$, $\frac{\partial}{\partial t}\Rightarrow -\mathrm{i}\omega$

$$-\widetilde{\boldsymbol{k}}^2\boldsymbol{E}+\mu_0\epsilon_0\boldsymbol{\mu}\,\omega^2\,\widetilde{\boldsymbol{\epsilon}}\,\boldsymbol{E}=0$$ with $\boldsymbol{\epsilon}+\mathrm{i}\boldsymbol{\sigma}/(\epsilon_0\omega)=\widetilde{\boldsymbol{\epsilon}}$

$\Downarrow$ with $\boldsymbol{\mu}=1$ and $k_0=\omega/c_0$

$$-\widetilde{\boldsymbol{k}}^2E_\mathrm{i}+\sum_{j=1}^{3}k_0^2\,\widetilde{\epsilon}_\mathrm{ij}\,E_\mathrm{j}=0$$ System of 3 linear equations for 3 vector components E_i

2. Magneto-Optical Effects – 2.2 Solutions

Wave equation: $$-\nabla^2 \boldsymbol{E} + \mu_0 \boldsymbol{\mu}\left(\boldsymbol{\sigma}\frac{\partial \boldsymbol{E}}{\partial t} + \epsilon_0 \boldsymbol{\epsilon}\,\frac{\partial^2 \boldsymbol{E}}{\partial t^2}\right) = 0$$

$\Downarrow$ with $\nabla\cdot \Rightarrow \mathrm{i}\boldsymbol{k}\cdot$, $\nabla\times \Rightarrow \mathrm{i}\boldsymbol{k}\times$, $\frac{\partial}{\partial t} \Rightarrow -\mathrm{i}\omega$

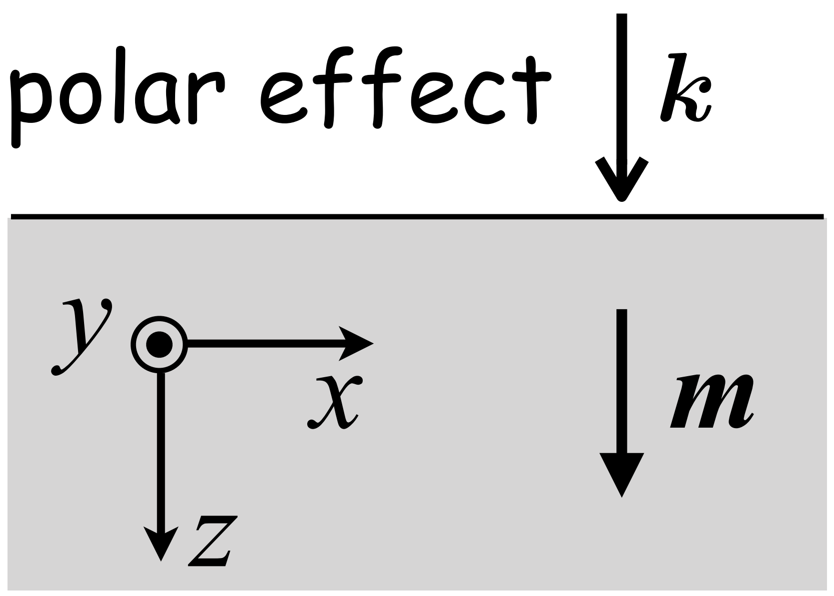
$$-\widetilde{\boldsymbol{k}}^2 \boldsymbol{E} + \mu_0 \epsilon_0 \boldsymbol{\mu}\,\omega^2\,\widetilde{\boldsymbol{\epsilon}}\,\boldsymbol{E} = 0$$ with $\boldsymbol{\epsilon} + \mathrm{i}\boldsymbol{\sigma}/(\epsilon_0\omega) = \widetilde{\boldsymbol{\epsilon}}$

$\Downarrow$ with $\boldsymbol{\mu} = 1$

$$-\widetilde{\boldsymbol{k}}^2 E_\mathrm{i} + \sum_{j=1}^{3} k_0^2\,\widetilde{\epsilon}_\mathrm{ij}\,E_\mathrm{j} = 0$$

Wave equation: $$-\widetilde{\boldsymbol{k}}^2 E_{\mathrm{i}} + \sum_{j=1}^{3} k_0^2\, \widetilde{\epsilon}_{\mathrm{ij}}\, E_{\mathrm{j}} = 0$$

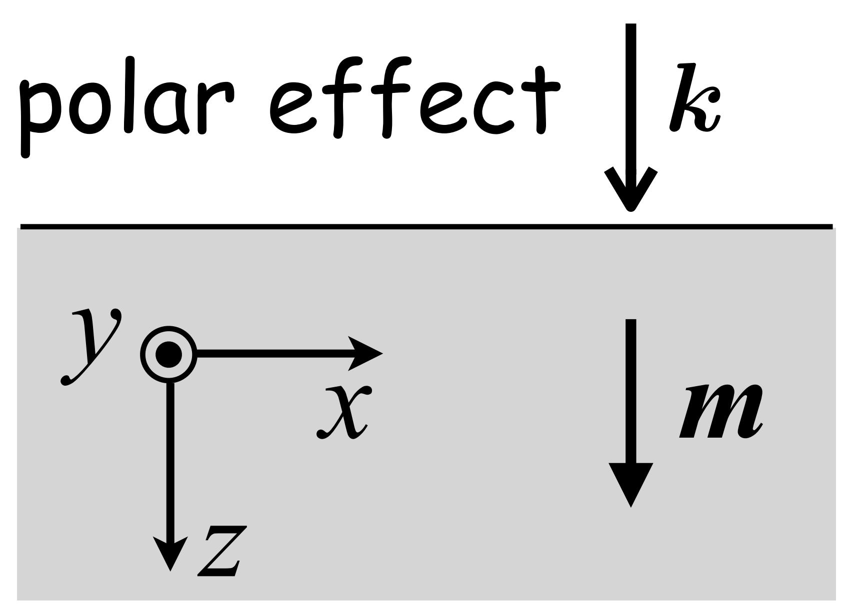
2. Magneto-Optical Effects – 2.2 Solutions



Wave equation: $$-\widetilde{\boldsymbol{k}}^2 E_{\mathrm{i}} + \sum_{j=1}^{3} k_0^2\, \widetilde{\epsilon}_{\mathrm{ij}}\, E_{\mathrm{j}} = 0$$

Assumption: light incidence, $\boldsymbol{m}$ and surface normal || z-axis

$$m_3 = m_{\mathrm{z}} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_{\mathrm{z}} = |\boldsymbol{k}|$$



Wave equation: $-\widetilde{\boldsymbol{k}}^2 E_\mathrm{i} + \sum_{j=1}^{3} k_0^2\, \widetilde{\epsilon}_\mathrm{ij}\, E_\mathrm{j} = 0$

Assumption: light incidence, $\boldsymbol{m}$ and surface normal || z-axis

$$m_3 = m_\mathrm{z} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_\mathrm{z} = |\boldsymbol{k}|$$

$\Downarrow$ with magneto-optical tensors $\boldsymbol{\epsilon} = \epsilon_\mathrm{iso} \begin{pmatrix} 1 & -\mathrm{i}Q_\mathrm{V}m_3 & \mathrm{i}Q_\mathrm{V}m_2 \\ \mathrm{i}Q_\mathrm{V}m_3 & 1 & -\mathrm{i}Q_\mathrm{V}m_1 \\ -\mathrm{i}Q_\mathrm{V}m_2 & \mathrm{i}Q_\mathrm{V}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1m_1^2 & B_2m_1m_2 & B_2m_1m_3 \\ B_2m_1m_2 & B_1m_2^2 & B_2m_2m_3 \\ B_2m_1m_3 & B_2m_2m_3 & B_1m_3^2 \end{pmatrix}$

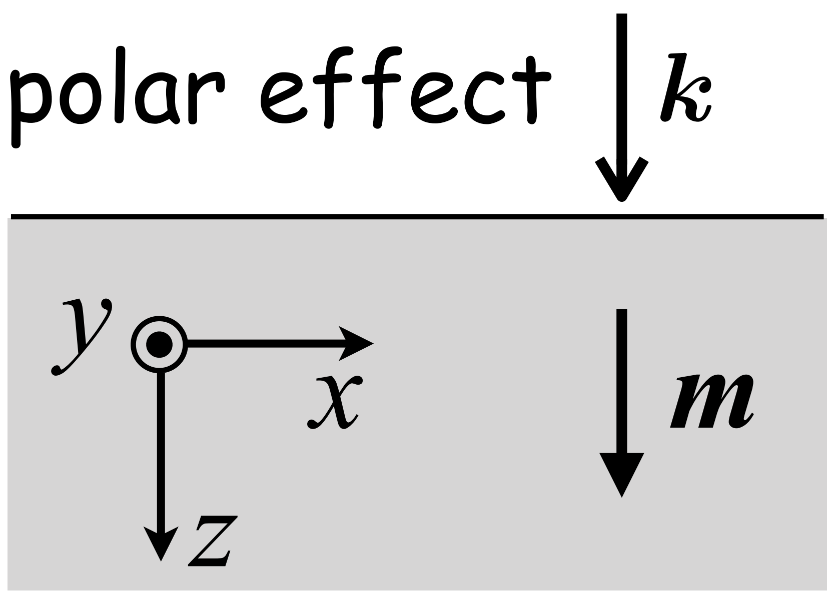
$$(k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{x} - k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{y} + 0 \cdot E_\mathrm{z} = 0$$

$$k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{x} + (k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{y} + 0 \cdot E_\mathrm{z} = 0$$

$$0 \cdot E_\mathrm{x} + 0 \cdot E_\mathrm{y} + (k_0^2\epsilon_\mathrm{iso} + B_1)E_\mathrm{z} = 0$$

2. Magneto-Optical Effects – 2.2 Solutions

Wave equation: $$-\tilde{\boldsymbol{k}}^2 E_\mathrm{i} + \sum_{j=1}^{3} k_0^2\, \tilde{\epsilon}_\mathrm{ij}\, E_\mathrm{j} = 0$$



Assumption: light incidence, $\boldsymbol{m}$ and surface normal || z-axis

$$m_3 = m_\mathrm{z} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_\mathrm{z} = |\boldsymbol{k}|$$

$\Downarrow$ with magneto-optical tensors $\boldsymbol{\epsilon} = \epsilon_\mathrm{iso} \begin{pmatrix} 1 & -\mathrm{i}Q_\mathrm{V}m_3 & \mathrm{i}Q_\mathrm{V}m_2 \\ \mathrm{i}Q_\mathrm{V}m_3 & 1 & -\mathrm{i}Q_\mathrm{V}m_1 \\ -\mathrm{i}Q_\mathrm{V}m_2 & \mathrm{i}Q_\mathrm{V}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1m_1^2 & B_2m_1m_2 & B_2m_1m_3 \\ B_2m_1m_2 & B_1m_2^2 & B_2m_2m_3 \\ B_2m_1m_3 & B_2m_2m_3 & B_1m_3^2 \end{pmatrix}$

$$(k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{x} - k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{y} + 0\cdot E_\mathrm{z} = 0$$

$$k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{x} + (k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{y} + 0\cdot E_\mathrm{z} = 0$$

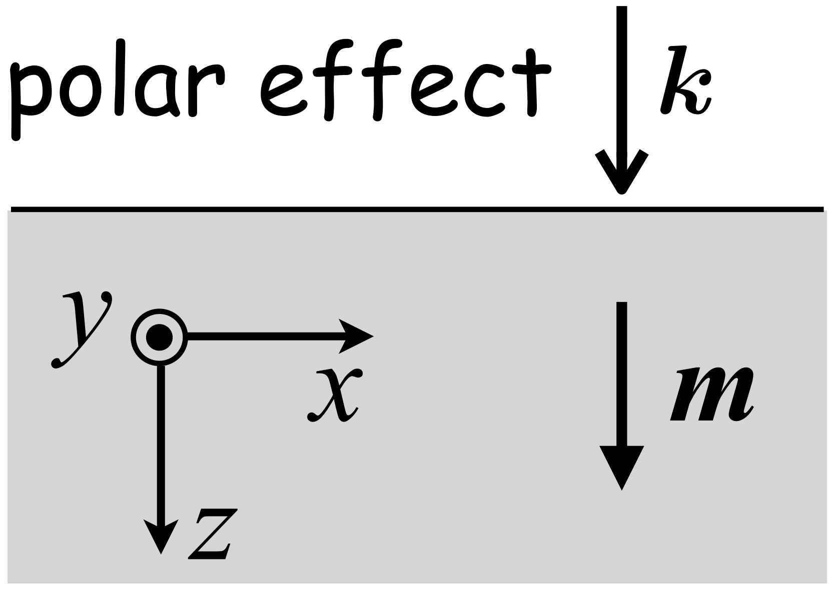
$$0\cdot E_\mathrm{x} + 0\cdot E_\mathrm{y} + (k_0^2\epsilon_\mathrm{iso} + B_1)E_\mathrm{z} = 0$$

$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_\mathrm{iso} + B_1)\left[(\epsilon_\mathrm{iso} - k_\mathrm{z}^2/k_0^2)^2 - (Q_\mathrm{V}\epsilon_\mathrm{iso})^2\right] = 0$$ (characteristic equation for k_z)

2. Magneto-Optical Effects – 2.2 Solutions

Wave equation: $$-\tilde{\boldsymbol{k}}^2 E_\mathrm{i} + \sum_{j=1}^{3} k_0^2\, \tilde{\epsilon}_\mathrm{ij}\, E_\mathrm{j} = 0$$



Assumption: light incidence, $\boldsymbol{m}$ and surface normal || z-axis (polar effect)

$$m_3 = m_\mathrm{z} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_\mathrm{z} = |\boldsymbol{k}|$$

$\Downarrow$ with magneto-optical tensors $\boldsymbol{\epsilon} = \epsilon_\mathrm{iso}\begin{pmatrix} 1 & -\mathrm{i}Q_\mathrm{V}m_3 & \mathrm{i}Q_\mathrm{V}m_2 \\ \mathrm{i}Q_\mathrm{V}m_3 & 1 & -\mathrm{i}Q_\mathrm{V}m_1 \\ -\mathrm{i}Q_\mathrm{V}m_2 & \mathrm{i}Q_\mathrm{V}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1m_1^2 & B_2m_1m_2 & B_2m_1m_3 \\ B_2m_1m_2 & B_1m_2^2 & B_2m_2m_3 \\ B_2m_1m_3 & B_2m_2m_3 & B_1m_3^2 \end{pmatrix}$

$$(k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{x} - k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{y} + 0\cdot E_\mathrm{z} = 0$$

$$k_0^2\epsilon_\mathrm{iso}\mathrm{i}Q_\mathrm{V}E_\mathrm{x} + (k_0^2\epsilon_\mathrm{iso} - k_\mathrm{z}^2)E_\mathrm{y} + 0\cdot E_\mathrm{z} = 0$$

$$0\cdot E_\mathrm{x} + 0\cdot E_\mathrm{y} + (k_0^2\epsilon_\mathrm{iso} + B_1)E_\mathrm{z} = 0$$

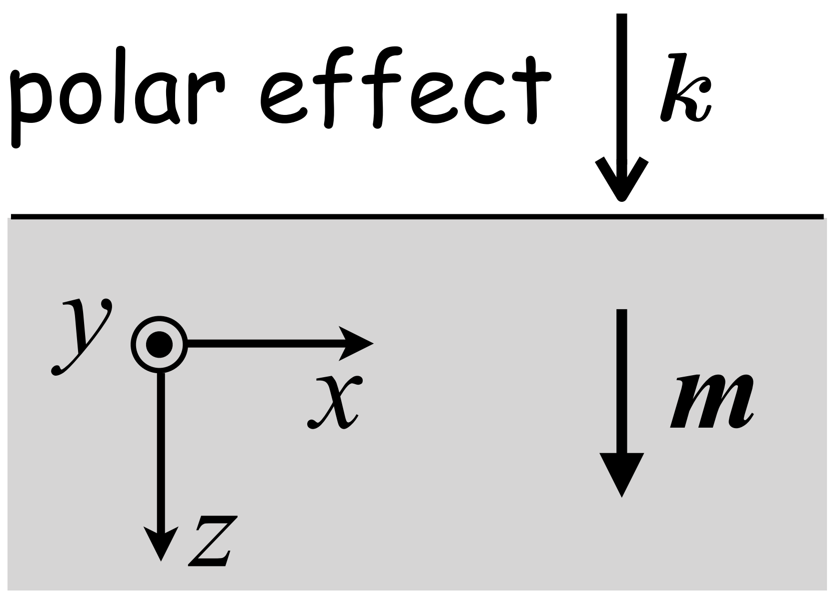
$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_\mathrm{iso} + B_1)\left[(\epsilon_\mathrm{iso} - k_\mathrm{z}^2/k_0^2)^2 - (Q_\mathrm{V}\epsilon_\mathrm{iso})^2\right] = 0$$ (characteristic equation for k_z)

$$(k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm x} - k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm y} + 0\cdot E_{\rm z} = 0$$

$$k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm x} + (k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm y} + 0\cdot E_{\rm z} = 0$$

$$0\cdot E_{\rm x} + 0\cdot E_{\rm y} + (k_0^2\epsilon_{\rm iso} + B_1)E_{\rm z} = 0$$



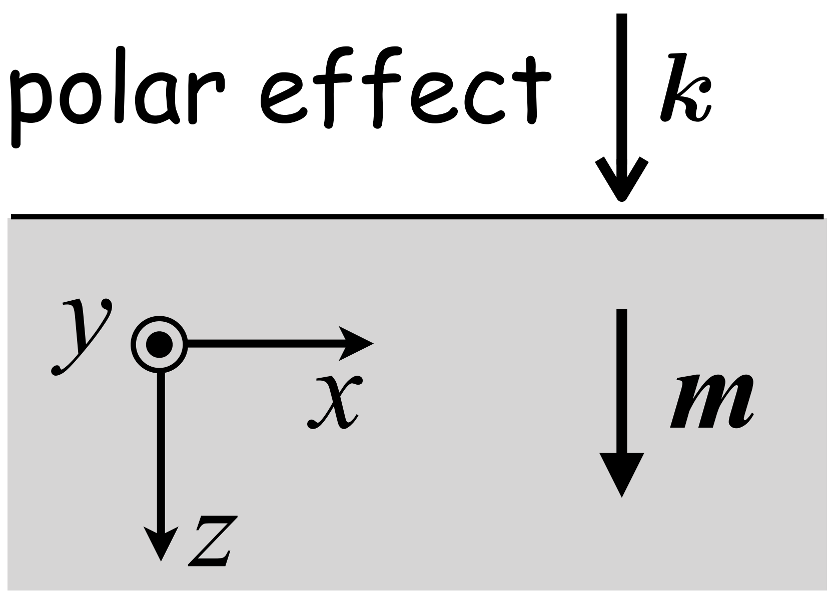
$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_{\rm iso} + B_1)\left[(\epsilon_{\rm iso} - k_{\rm z}^2/k_0^2)^2 - (Q_{\rm V}\epsilon_{\rm iso})^2\right] = 0$$

(characteristic equation for $k_{\rm z}$)

$$(k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm x} - k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm x} + (k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$0\cdot E_{\rm x} + 0\cdot E_{\rm y} + (k_0^2\epsilon_{\rm iso} + B_1)E_{\rm z} = 0$$

polar effect



$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_{\rm iso} + B_1)\left[(\epsilon_{\rm iso} - k_{\rm z}^2/k_0^2)^2 - (Q_{\rm V}\epsilon_{\rm iso})^2\right] = 0$$ (characteristic equation for $k_{\rm z}$)

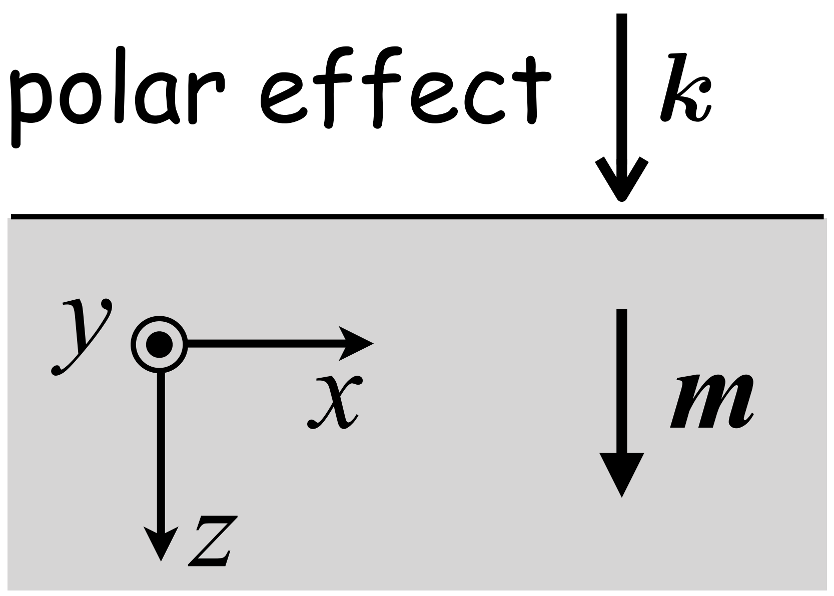
$\Downarrow$ solution ($|Q_{\rm V}| \ll 1$)

Allowed propagation directions of wave (eigenmodes):

$$k_{\rm z} = \pm k_0\sqrt{\epsilon_{\rm iso}}(1 \pm Q_{\rm V}/2)$$

2. Magneto-Optical Effects – 2.2 Solutions

$$(k_0^2\epsilon_{\mathrm{iso}} - k_{\mathrm{z}}^2)E_{\mathrm{x}} - k_0^2\epsilon_{\mathrm{iso}}\mathrm{i}Q_{\mathrm{V}}E_{\mathrm{y}} + 0\cdot E_{\mathrm{z}} = 0$$
$$k_0^2\epsilon_{\mathrm{iso}}\mathrm{i}Q_{\mathrm{V}}E_{\mathrm{x}} + (k_0^2\epsilon_{\mathrm{iso}} - k_{\mathrm{z}}^2)E_{\mathrm{y}} + 0\cdot E_{\mathrm{z}} = 0$$
$$0\cdot E_{\mathrm{x}} + 0\cdot E_{\mathrm{y}} + (k_0^2\epsilon_{\mathrm{iso}} + B_1)E_{\mathrm{z}} = 0$$



$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

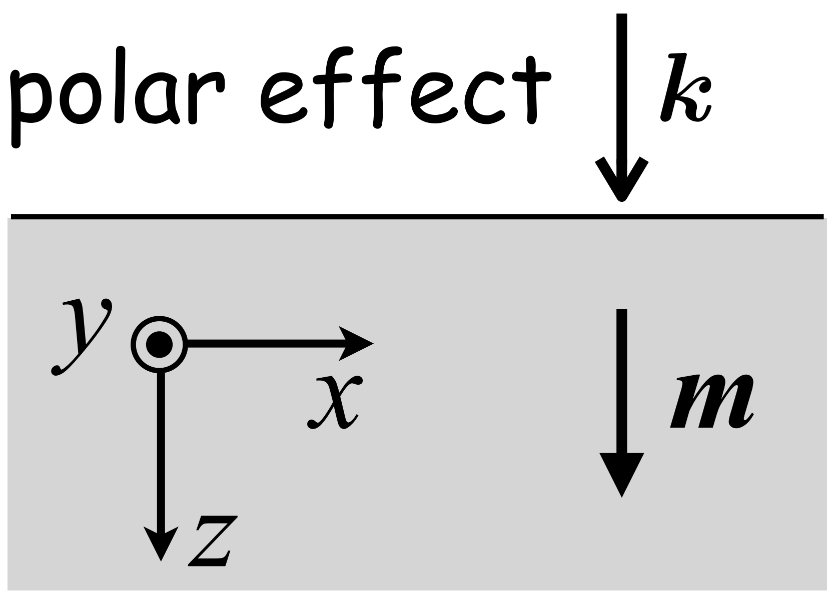
$$(\epsilon_{\mathrm{iso}} + B_1)\left[(\epsilon_{\mathrm{iso}} - k_{\mathrm{z}}^2/k_0^2)^2 - (Q_{\mathrm{V}}\epsilon_{\mathrm{iso}})^2\right] = 0$$ (characteristic equation for k_{z})

$\Downarrow$ solution ($|Q_{\mathrm{V}}| \ll 1$)

Allowed propagation directions of wave (eigenmodes):

$$k_{\mathrm{z}} = \pm k_0\sqrt{\epsilon_{\mathrm{iso}}}(1 \pm Q_{\mathrm{V}}/2) \quad \underset{k_{\mathrm{z}} = nk_0}{\Longrightarrow} \quad n_{\pm} = \sqrt{\epsilon_{\mathrm{iso}}}(1 \pm Q_{\mathrm{V}}/2)$$

2. Magneto-Optical Effects – 2.2 Solutions



$$(k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm x} - k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm x} + (k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$0\cdot E_{\rm x} + 0\cdot E_{\rm y} + (k_0^2\epsilon_{\rm iso} + B_1)E_{\rm z} = 0$$

$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_{\rm iso} + B_1)\left[(\epsilon_{\rm iso} - k_{\rm z}^2/k_0^2)^2 - (Q_{\rm V}\epsilon_{\rm iso})^2\right] = 0$$ (characteristic equation for $k_{\rm z}$)

$\Downarrow$ solution ($|Q_{\rm V}| \ll 1$)

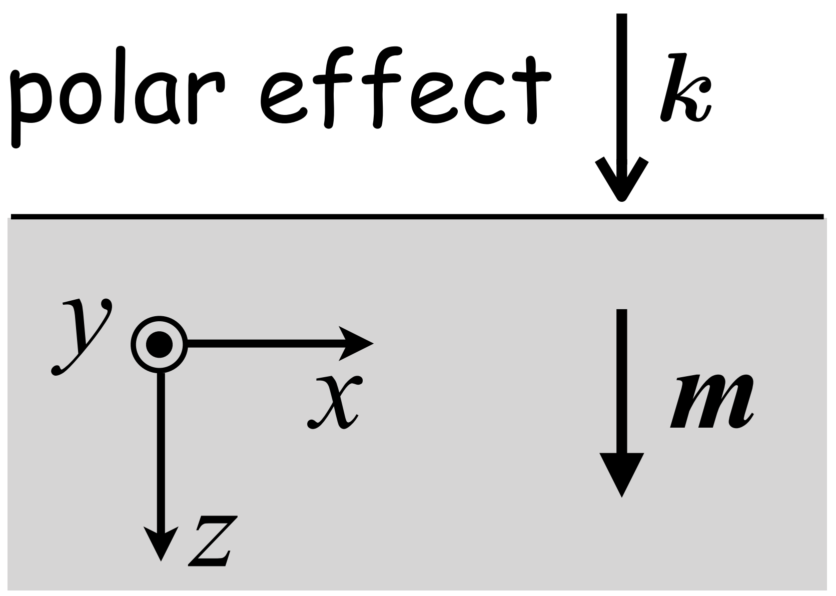
Allowed propagation directions of wave (eigenmodes):

$$k_{\rm z} = \pm k_0\sqrt{\epsilon_{\rm iso}}(1 \pm Q_{\rm V}/2) \quad \underset{k_{\rm z} = nk_0}{\Longrightarrow} \quad n_\pm = \sqrt{\epsilon_{\rm iso}}(1 \pm Q_{\rm V}/2)$$

$\Downarrow$

$$E_{\rm y} = \pm\mathrm{i}E_{\rm x}$$

$$(k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm x} - k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$k_0^2\epsilon_{\rm iso}\mathrm{i}Q_{\rm V}E_{\rm x} + (k_0^2\epsilon_{\rm iso} - k_{\rm z}^2)E_{\rm y} + 0\cdot E_{\rm z} = 0$$
$$0\cdot E_{\rm x} + 0\cdot E_{\rm y} + (k_0^2\epsilon_{\rm iso} + B_1)E_{\rm z} = 0$$



$\Downarrow$ non-trivial solutions if determinant of coefficient matrix vanishes

$$(\epsilon_{\rm iso} + B_1)\left[(\epsilon_{\rm iso} - k_{\rm z}^2/k_0^2)^2 - (Q_{\rm V}\epsilon_{\rm iso})^2\right] = 0$$ (characteristic equation for $k_{\rm z}$)

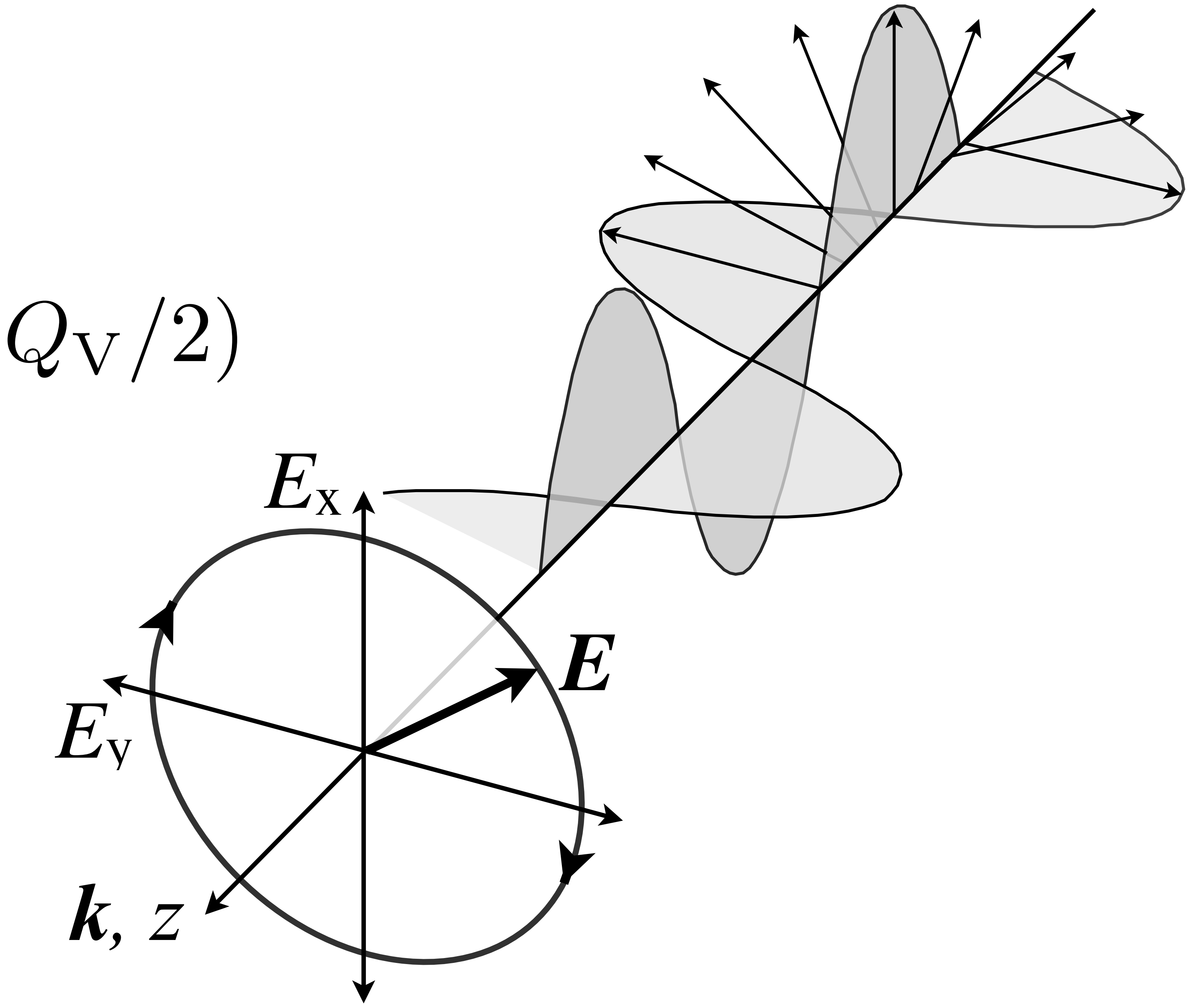
$\Downarrow$ solution ($|Q_{\rm V}| \ll 1$)

Allowed propagation directions of wave (eigenmodes):

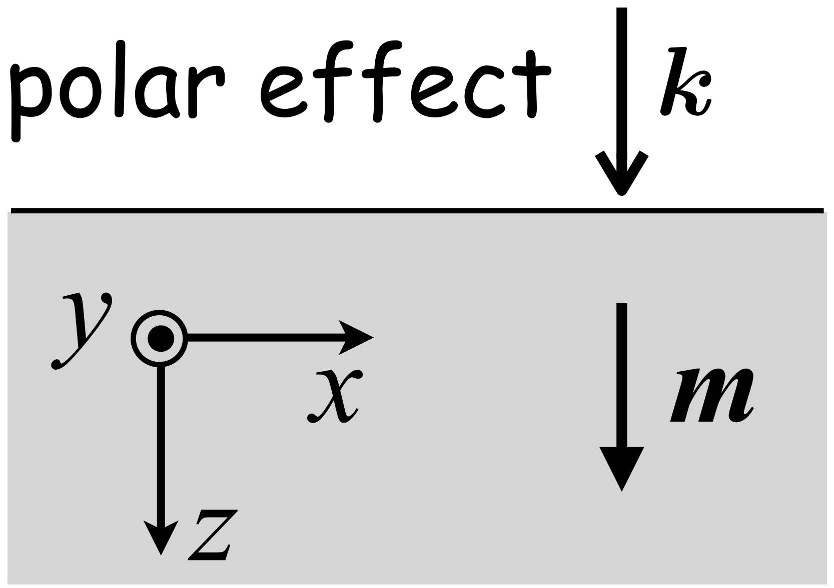
$$k_{\rm z} = \pm k_0\sqrt{\epsilon_{\rm iso}}(1 \pm Q_{\rm V}/2) \quad \underset{k_{\rm z} = nk_0}{\Longrightarrow} \quad n_\pm = \sqrt{\epsilon_{\rm iso}}(1 \pm Q_{\rm V}/2)$$

$\Downarrow$

$$E_{\rm y} = \pm\mathrm{i}E_{\rm x}$$



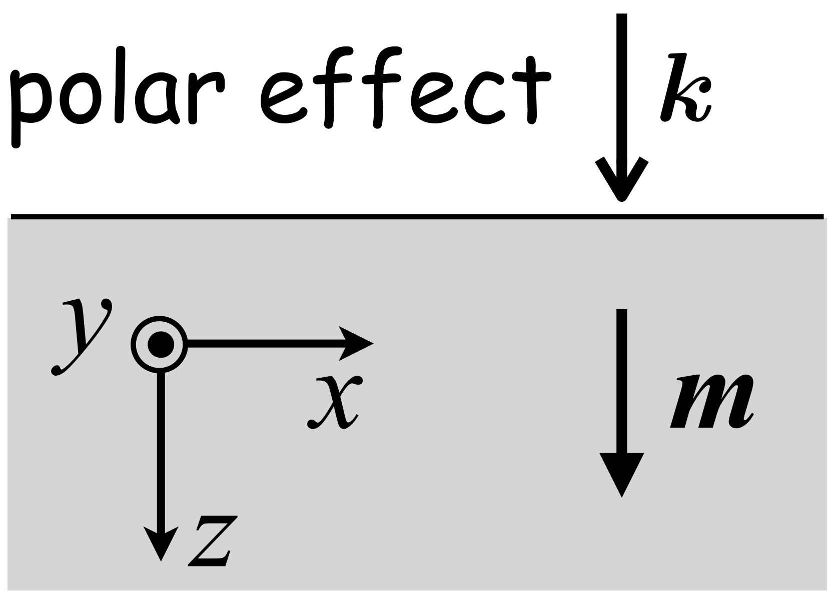
remember Jones vector: $\mathbf{J} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$: $\quad \mathbf{J_L} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ \mathrm{i} \end{bmatrix}$ Left-circular light | $\mathbf{J_R} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -\mathrm{i} \end{bmatrix}$ Right-circular light



$$n_{\pm} = \sqrt{\epsilon_{\mathrm{iso}}}(1 \pm Q_{\mathrm{V}}/2)$$

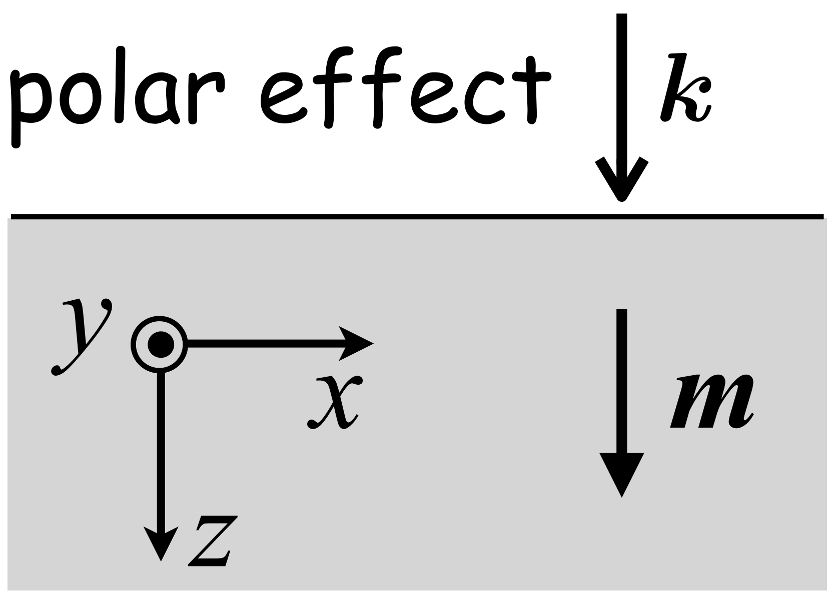
$$E_{\mathrm{y}} = \pm \mathrm{i} E_{\mathrm{x}}$$

2. Magneto-Optical Effects – 2.2 Solutions



$$\boxed{E_\mathrm{y} = \pm \mathrm{i} E_\mathrm{x}}$$

$$\boxed{n_\pm = \sqrt{\epsilon_\mathrm{iso}}(1 \pm Q_\mathrm{V}/2)}$$

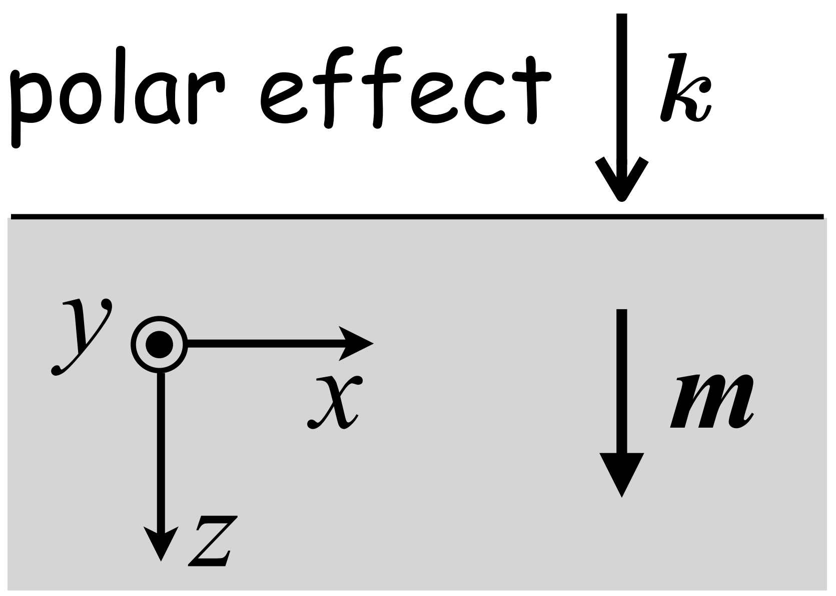


$$E_y = \pm \mathrm{i} E_x$$

$$n_{\pm} = \sqrt{\epsilon_{\mathrm{iso}}}(1 \pm Q_V/2)$$

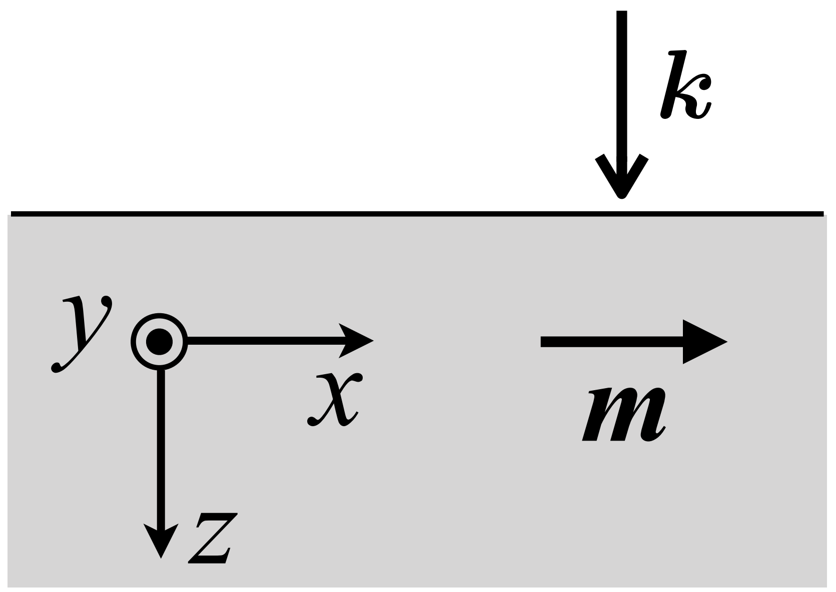
Summary:

- For light propagating along magnetization: normal modes are left- and right circularly polarized waves
- They are propagating as though the magnetic material has refractive indices n_+ for left- and n_- for right-circularly polarized radiation


polar effect
k
y
x
m
z

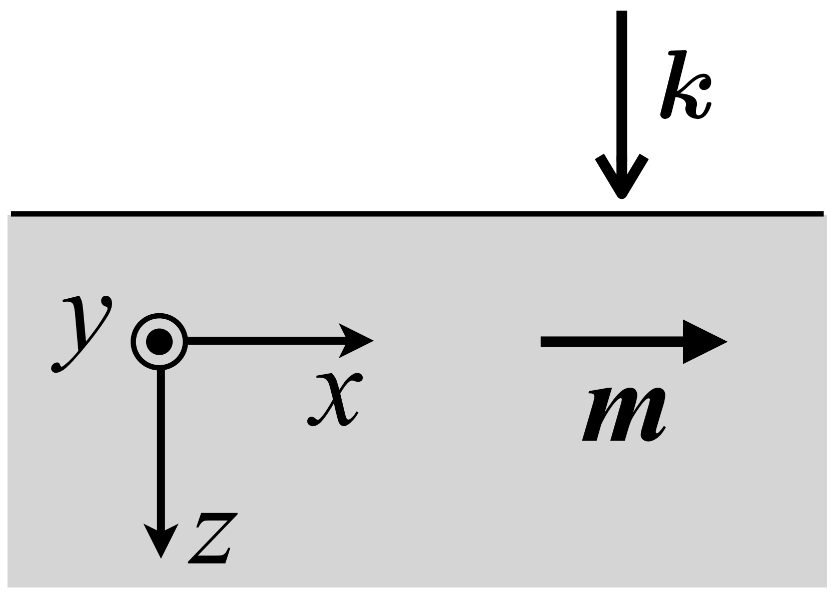
Assumption: light propagates perpendicular to $\boldsymbol{m}$

$$m_1 = m_{\mathrm{x}} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_{\mathrm{z}} = |\boldsymbol{k}| = nk_0$$



2. Magneto-Optical Effects – 2.2 Solutions

Assumption: light propagates perpendicular to $\boldsymbol{m}$



$$m_1 = m_\mathrm{x} = |\boldsymbol{m}| = 1 \quad \text{and} \quad k_3 = k_\mathrm{z} = |\boldsymbol{k}| = nk_0$$

$\Downarrow$ with magneto-optical tensors

$$(\epsilon_\mathrm{iso} + B_1 - n^2)E_\mathrm{x} + 0 \cdot E_\mathrm{y} + 0 \cdot E_\mathrm{z} = 0$$
$$0 \cdot E_\mathrm{x} + (\epsilon_\mathrm{iso} - n^2)E_\mathrm{y} - \epsilon_\mathrm{iso}\,\mathrm{i}\,Q_\mathrm{V} E_\mathrm{z} = 0$$
$$0 \cdot E_\mathrm{x} + \epsilon_\mathrm{iso}\,\mathrm{i}\,Q_\mathrm{V} E_\mathrm{y} + \epsilon_\mathrm{iso} E_\mathrm{z} = 0\,,$$

$\Downarrow$

$$(\epsilon_\mathrm{iso} + B_1 - n^2)\left[\epsilon_\mathrm{iso}(1 - Q_\mathrm{V}^2) - n^2\right] = 0 \qquad \text{(characteristic equation)}$$

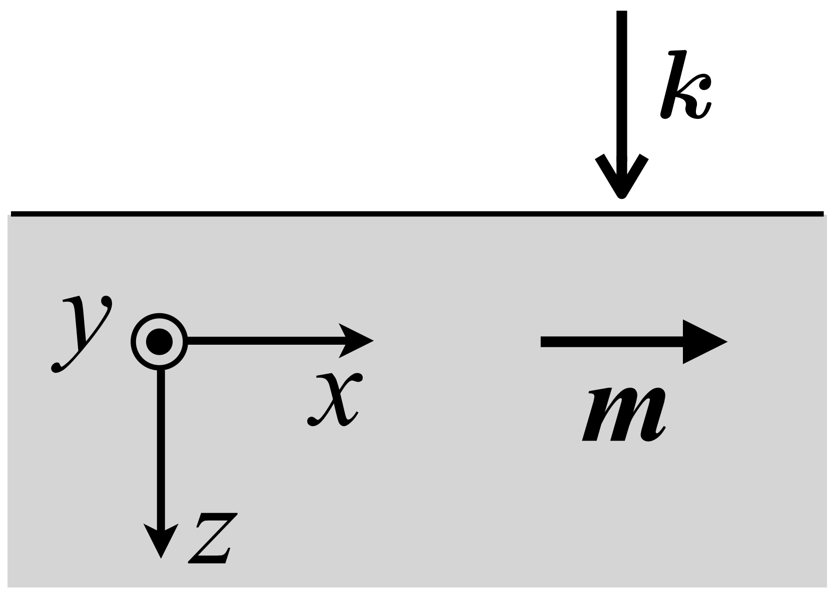
$\Downarrow$

$$n_\parallel^2 = \epsilon_\mathrm{iso} + B_1\,, \quad \boldsymbol{E} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \text{and} \qquad n_\perp^2 = \epsilon_\mathrm{iso}(1 - Q_\mathrm{V}^2)\,, \quad \boldsymbol{E} = \begin{pmatrix} 0 \\ 1 \\ -\mathrm{i}Q_\mathrm{V} \end{pmatrix}$$

$$|\mathrm{i}Q_\mathrm{V}| \ll 1$$

2. Magneto-Optical Effects – 2.2 Solutions

Assumption: light propagates perpendicular to $\boldsymbol{m}$



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$$\begin{aligned}
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\end{aligned}$$

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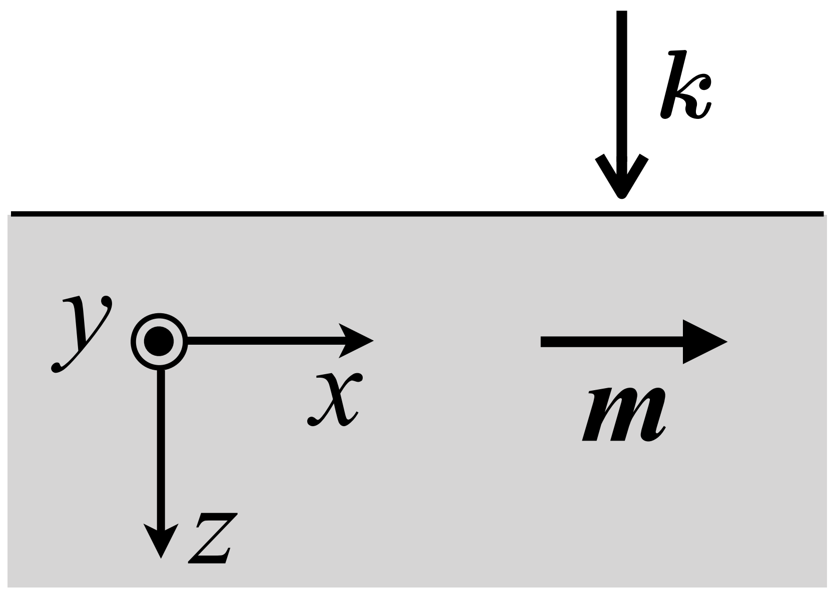

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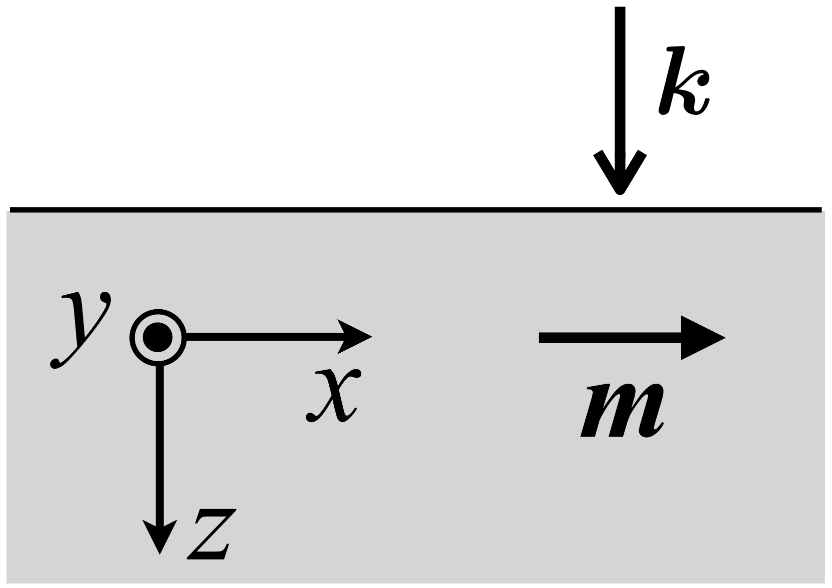

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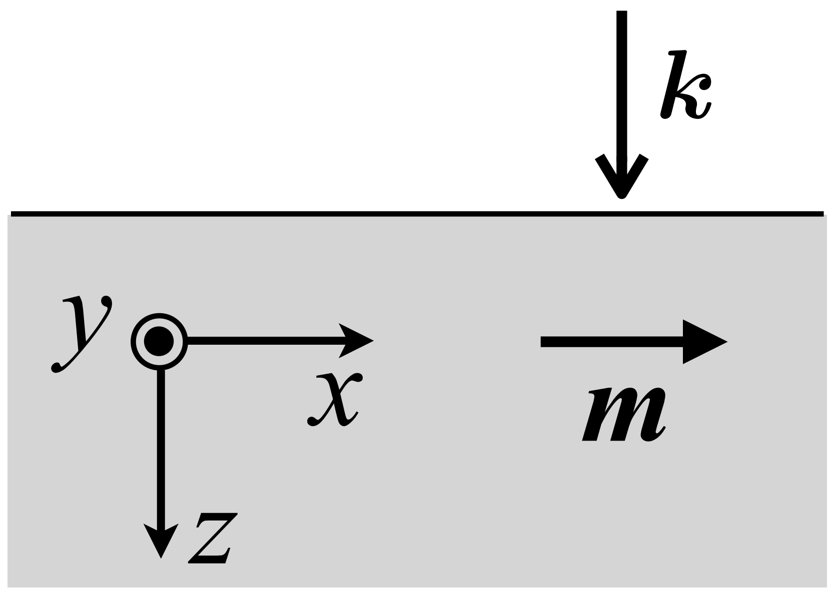
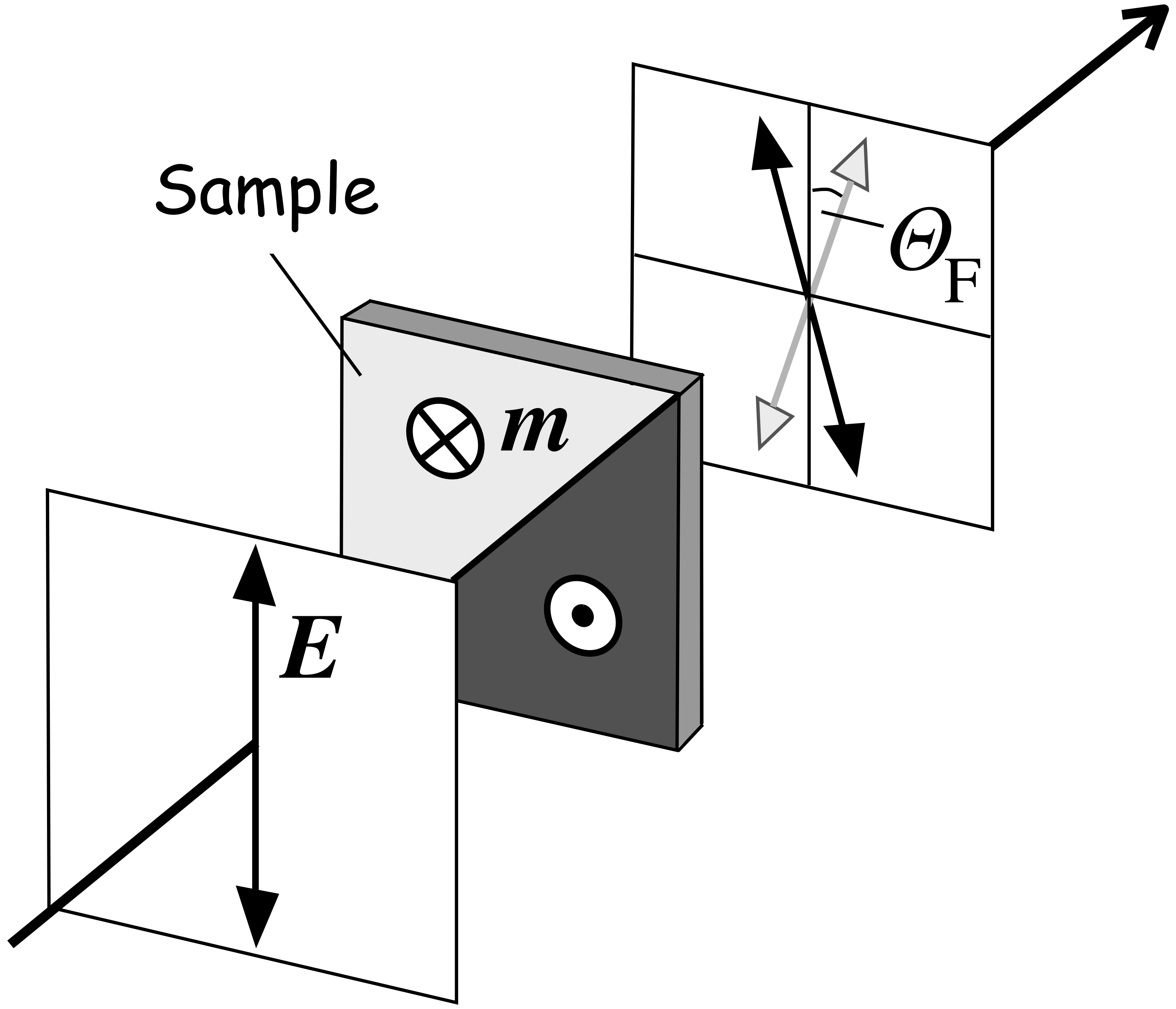


Summary:

- For light propagating perpendicular to magnetization: normal modes are linearly polarized waves with polarization planes parallel and perpendicular to magnetization direction
- They are propagating as though the magnetic material has refractive indices $n_{\parallel}$ and $n_{\perp}$

Polar configuration



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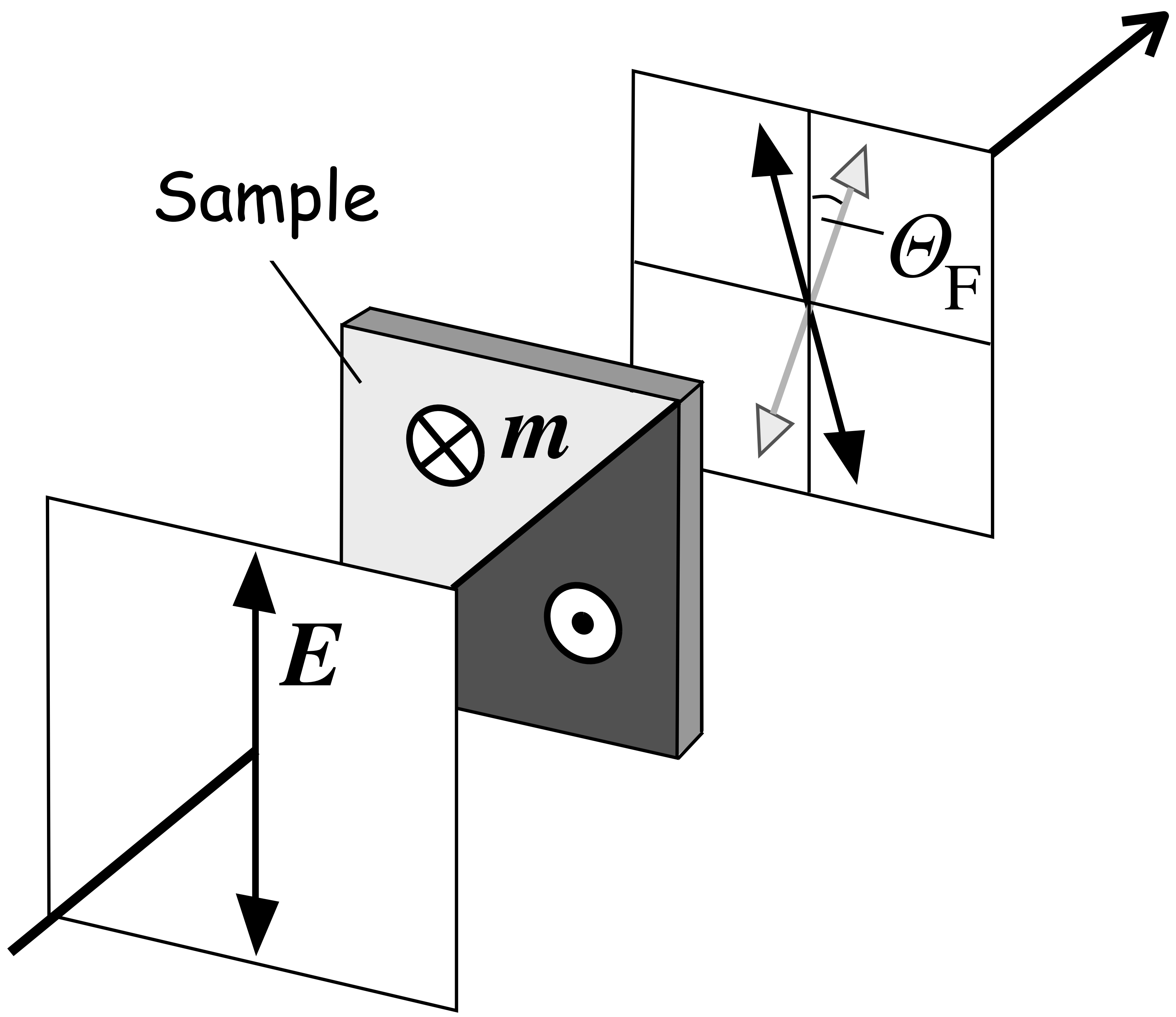

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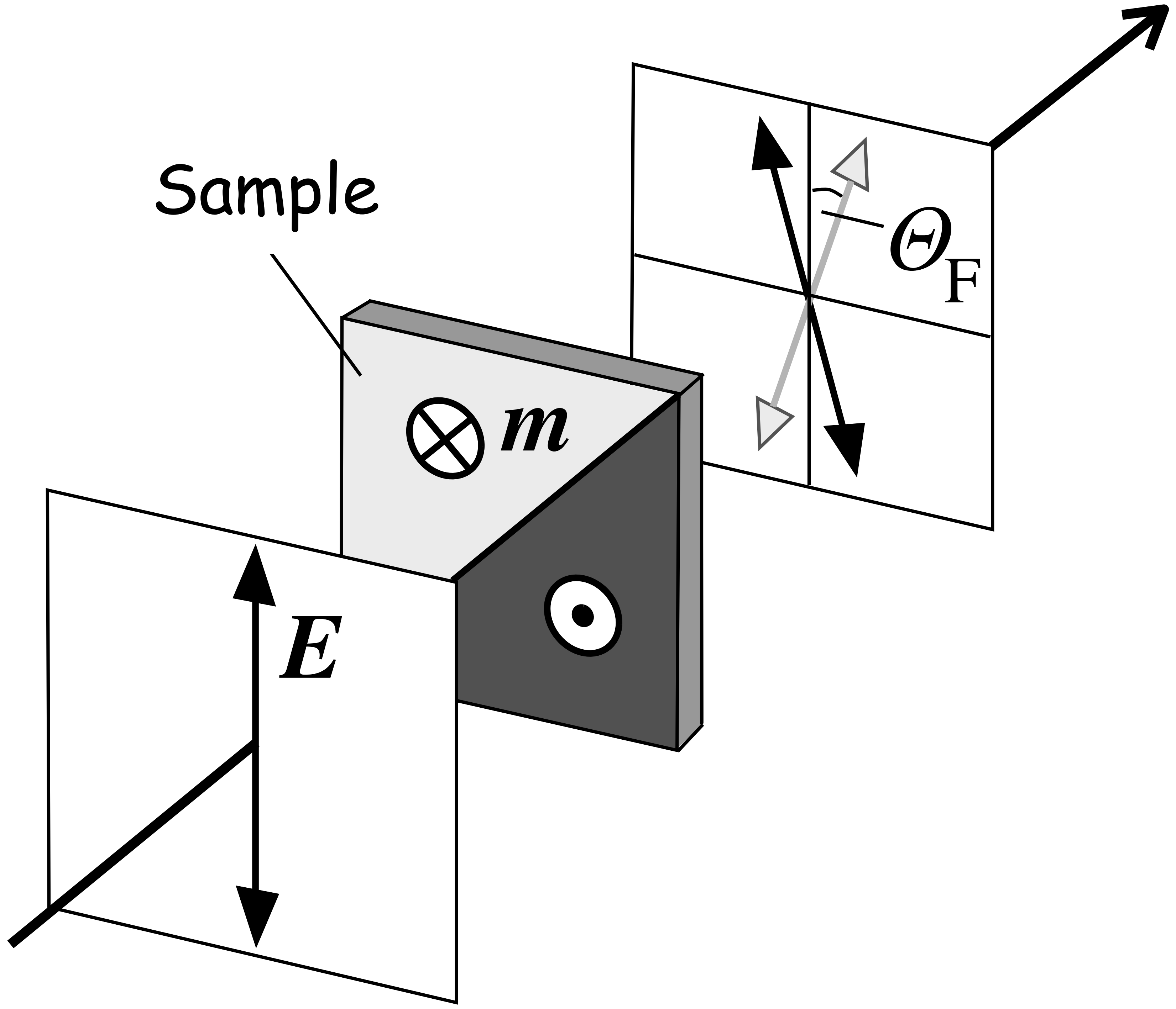
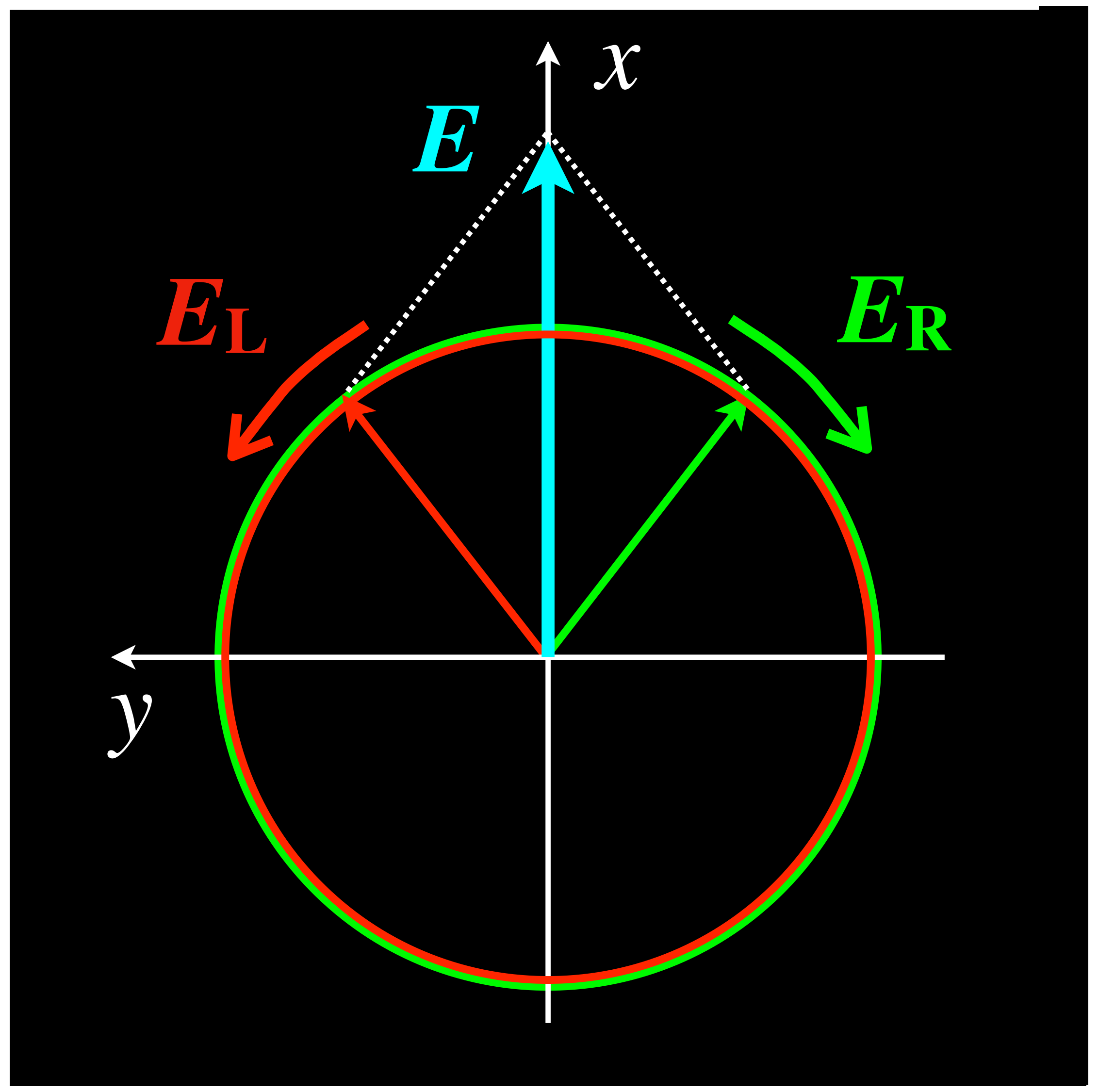
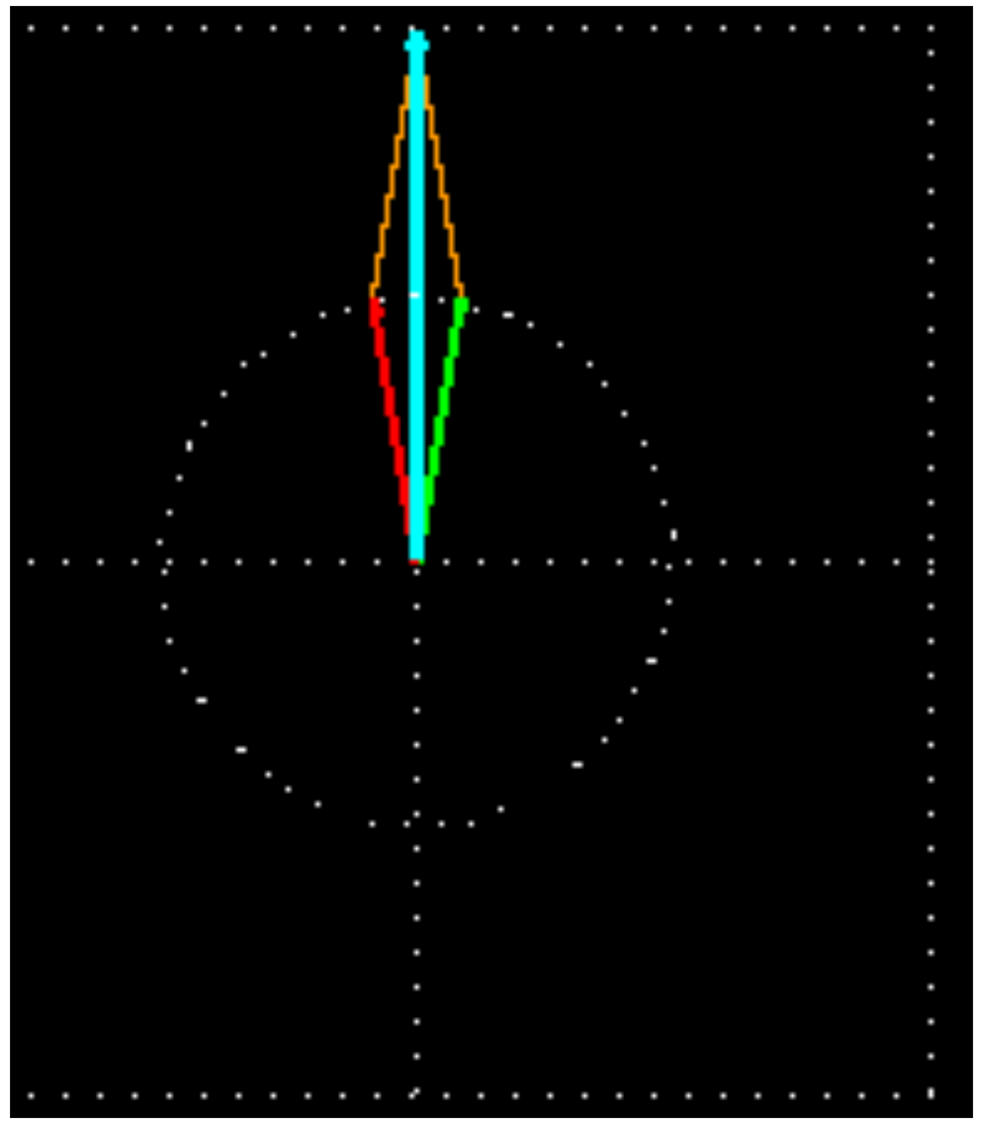


- Remember: linearly polarized light = superposition of right- and left-handed circularly polarized waves with equal amplitude and phase





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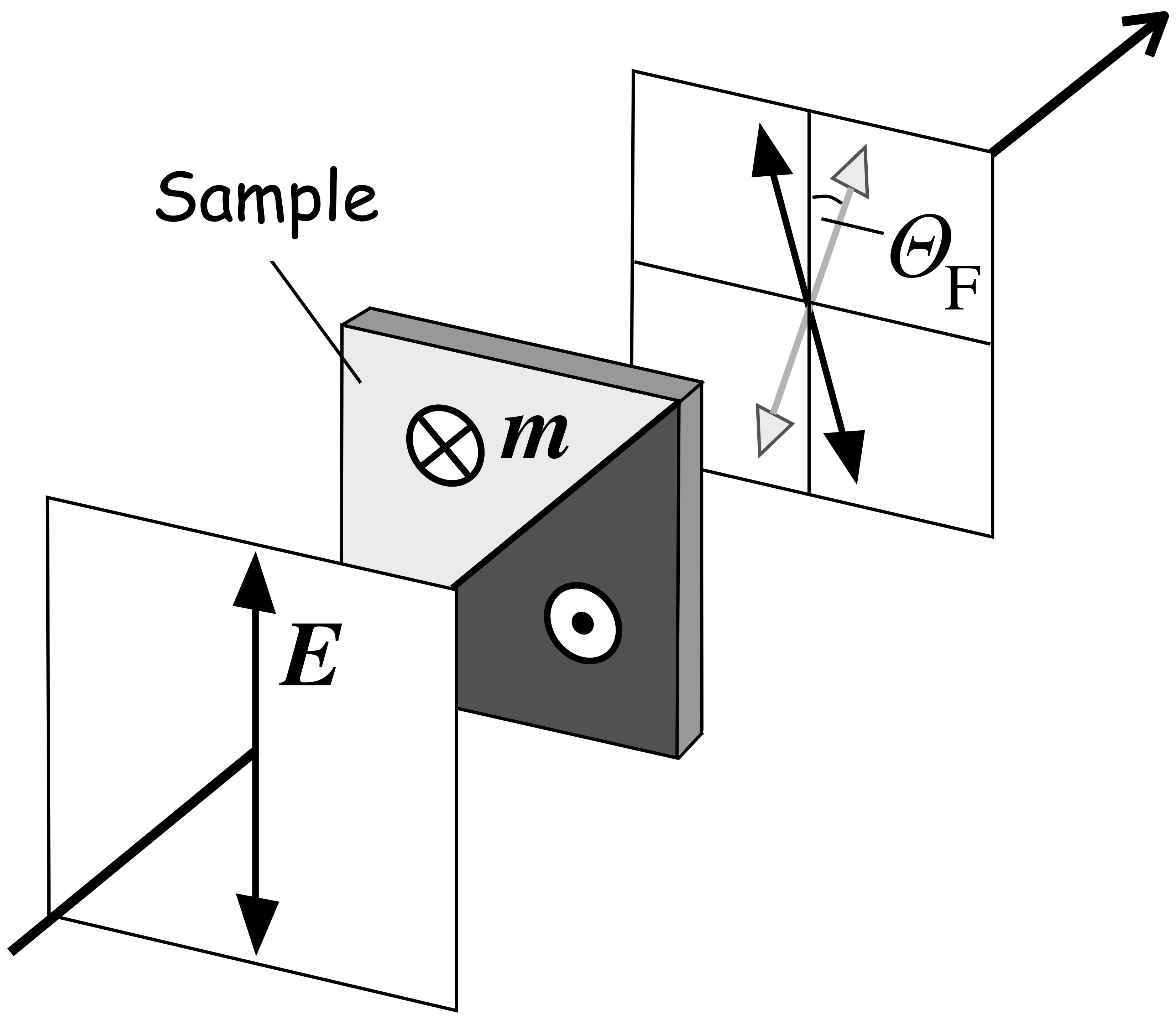

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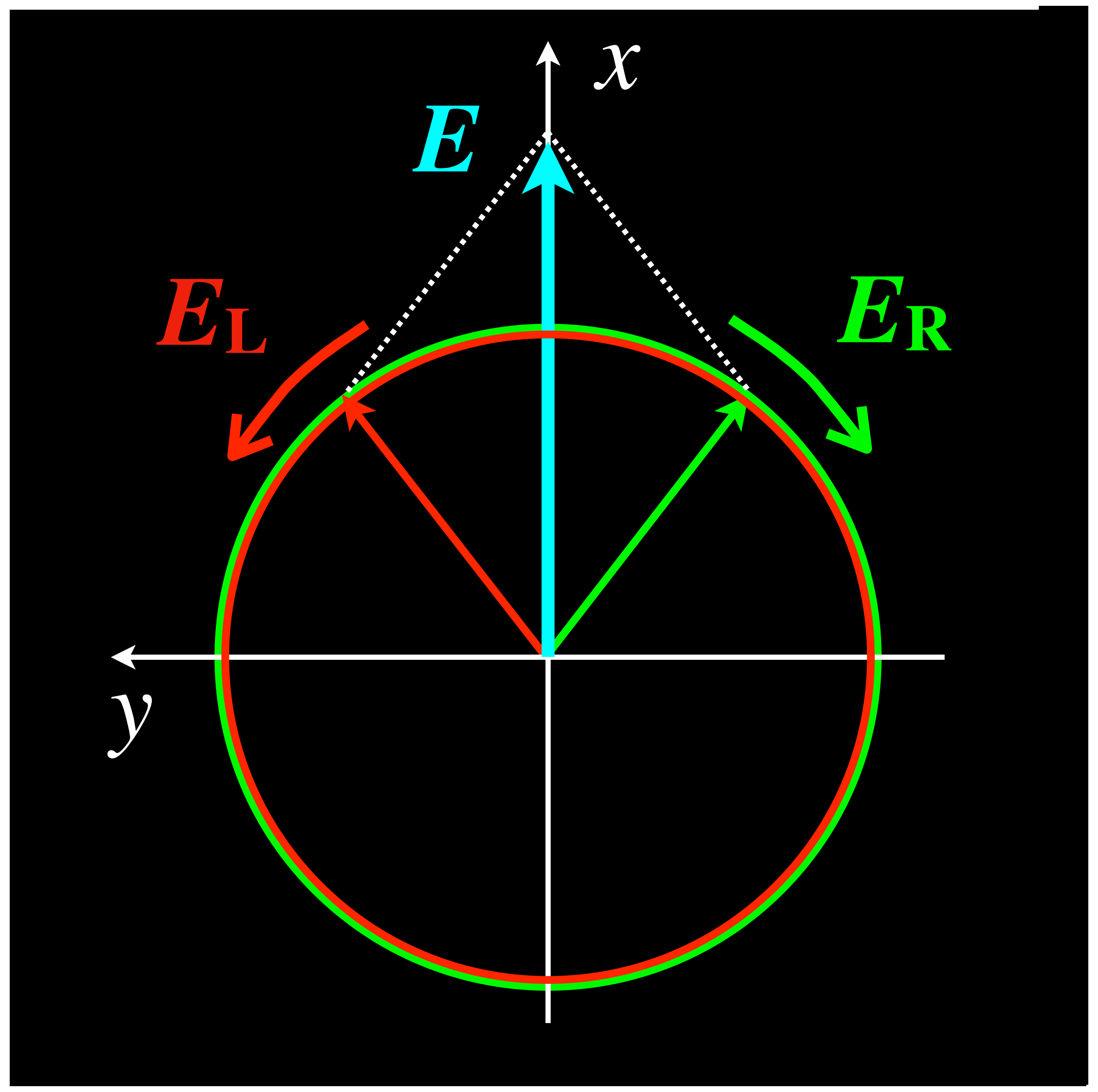
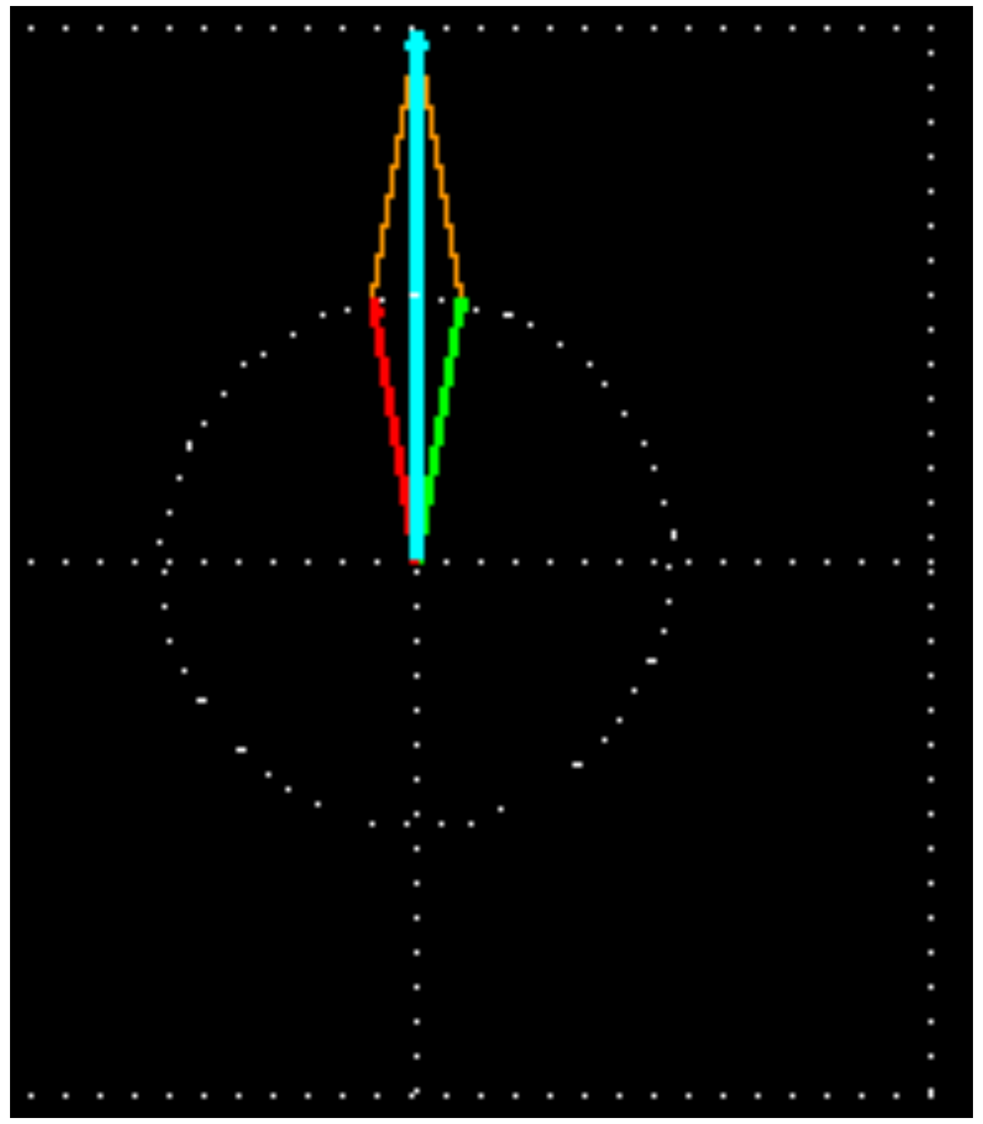
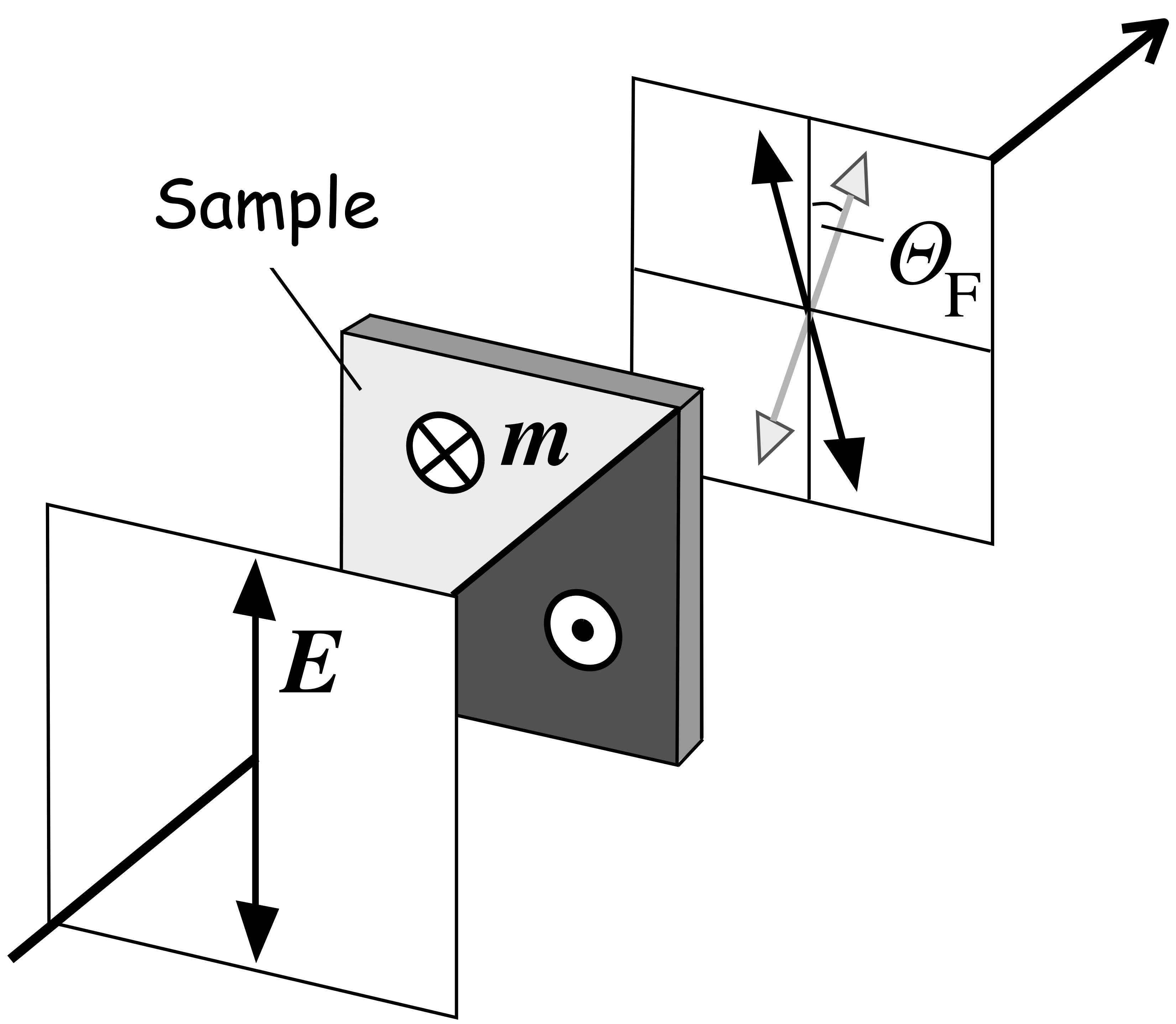




- For light propagating along magnetization: normal modes are left- and right circularly polarized waves, i.e. linear light entering medium is resolved into these 2 modes which travel without interaction

2. Magneto-Optical Effects – 2.3 Faraday Effect

Polar configuration



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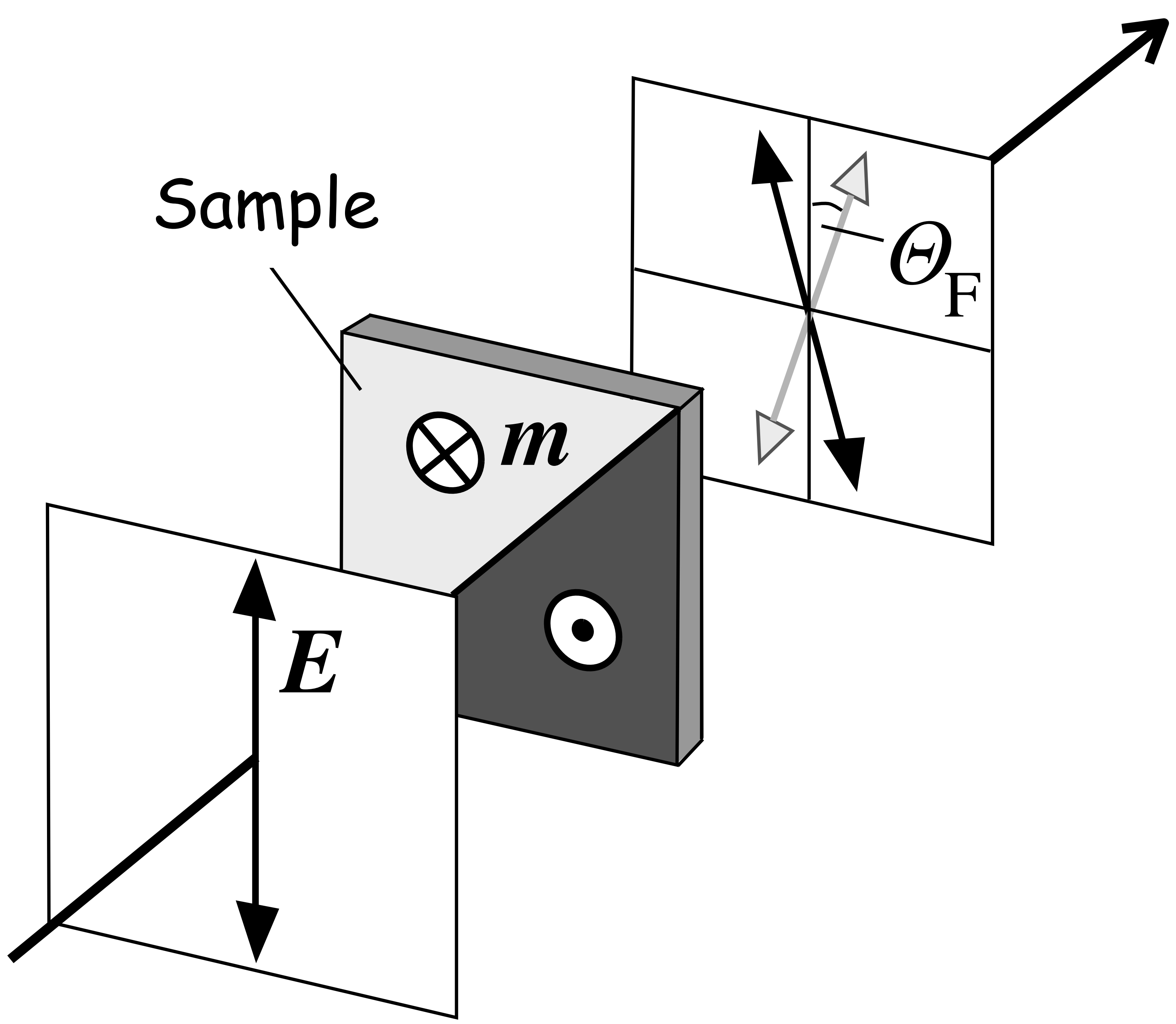

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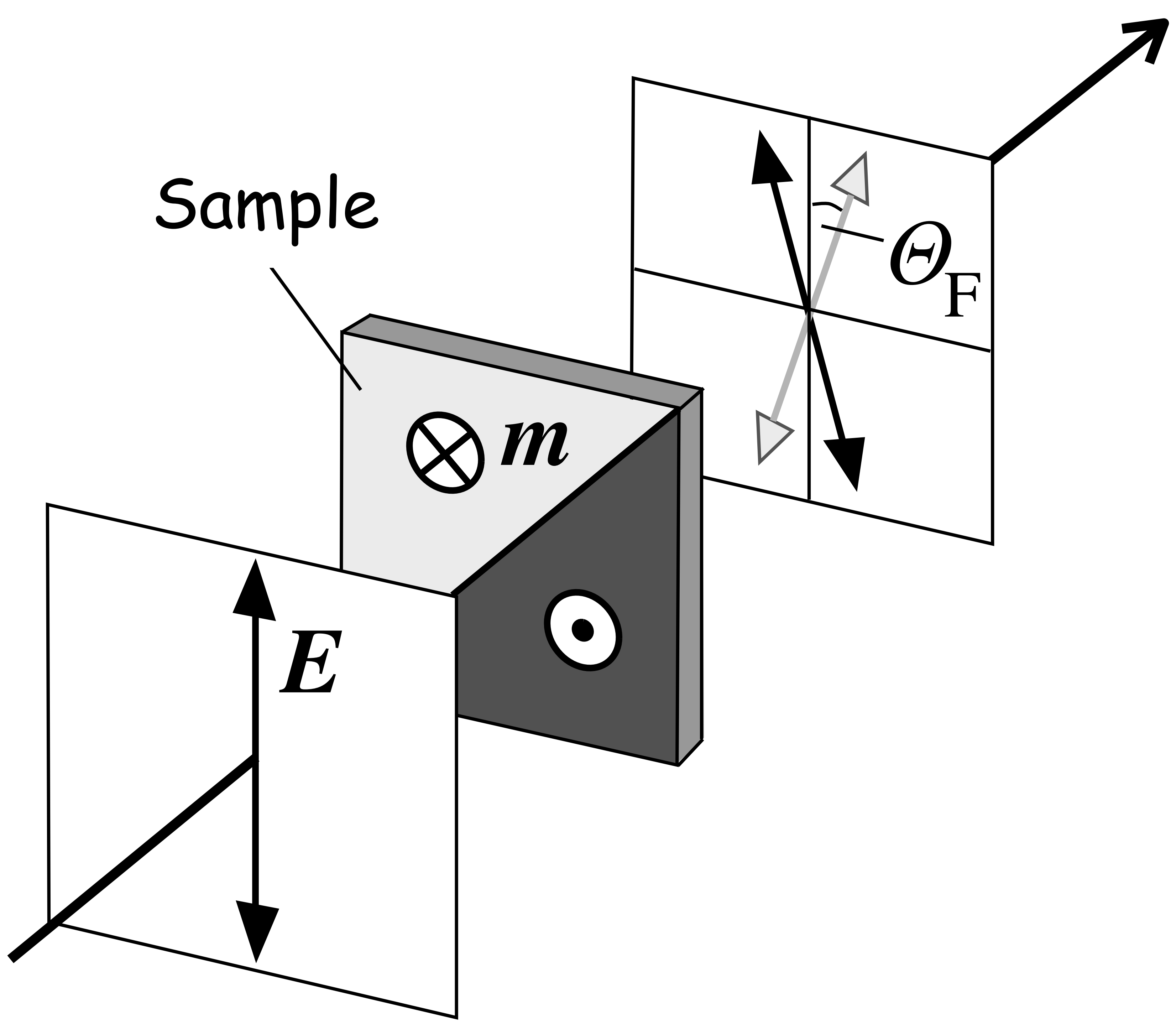

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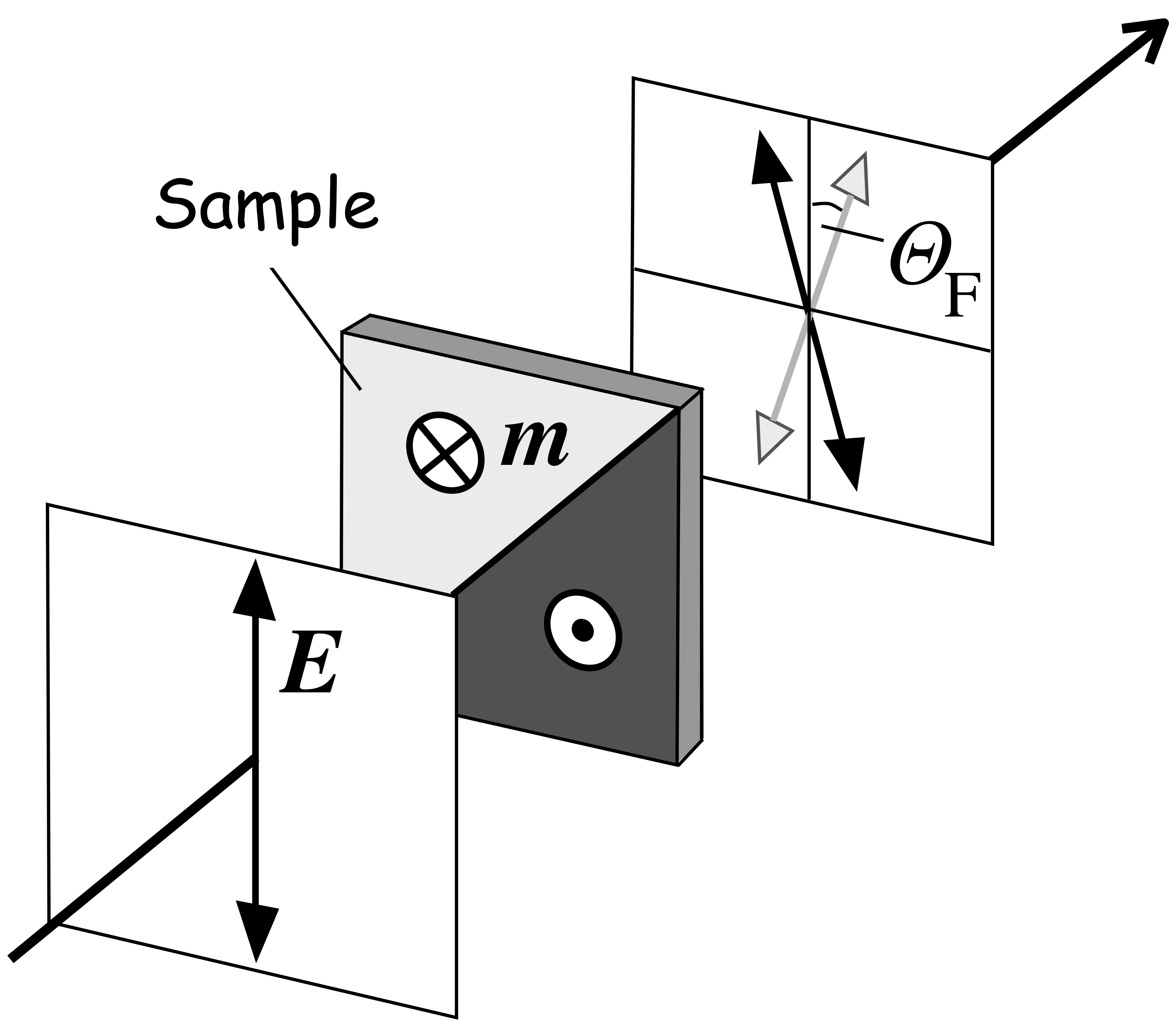

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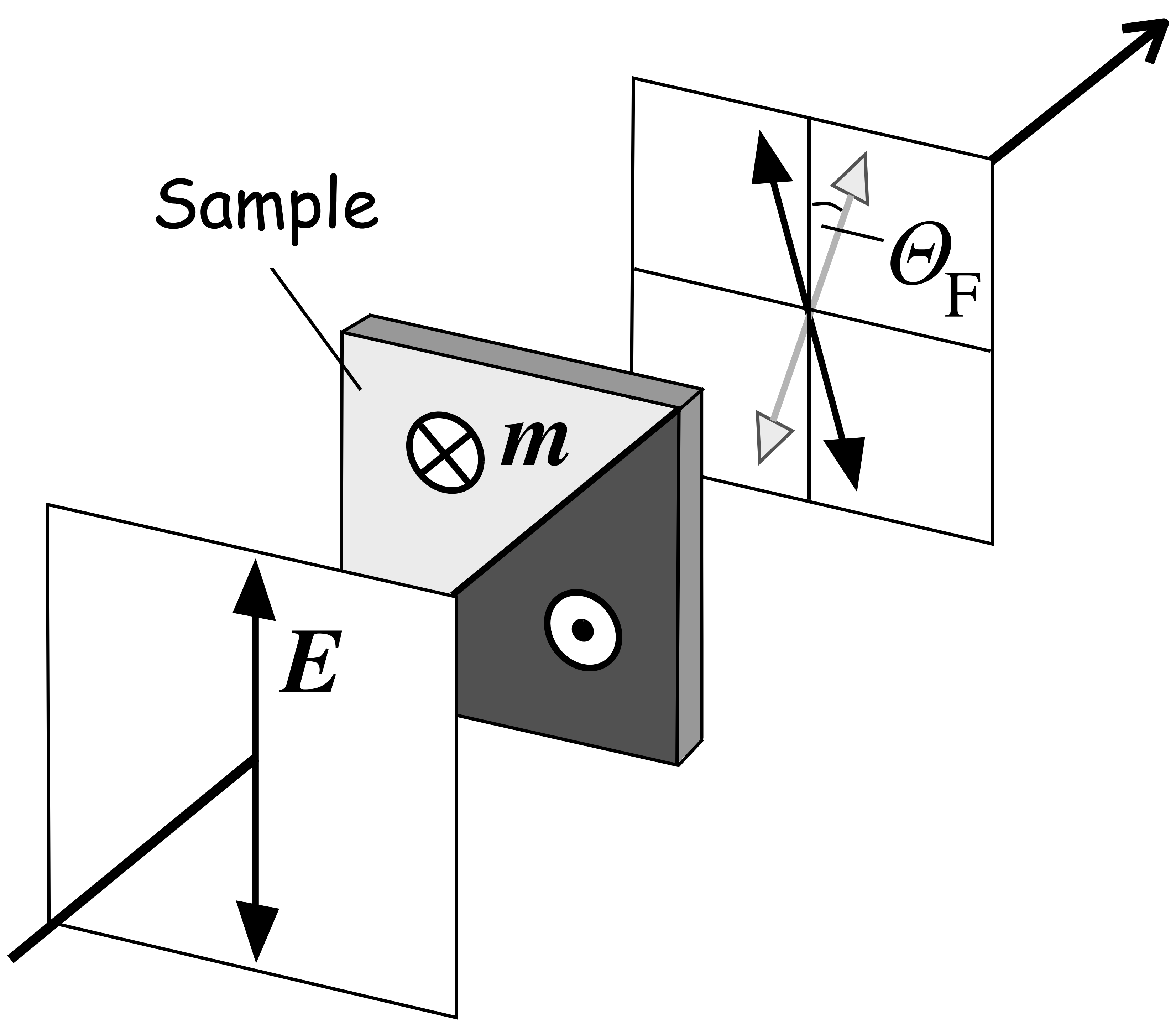

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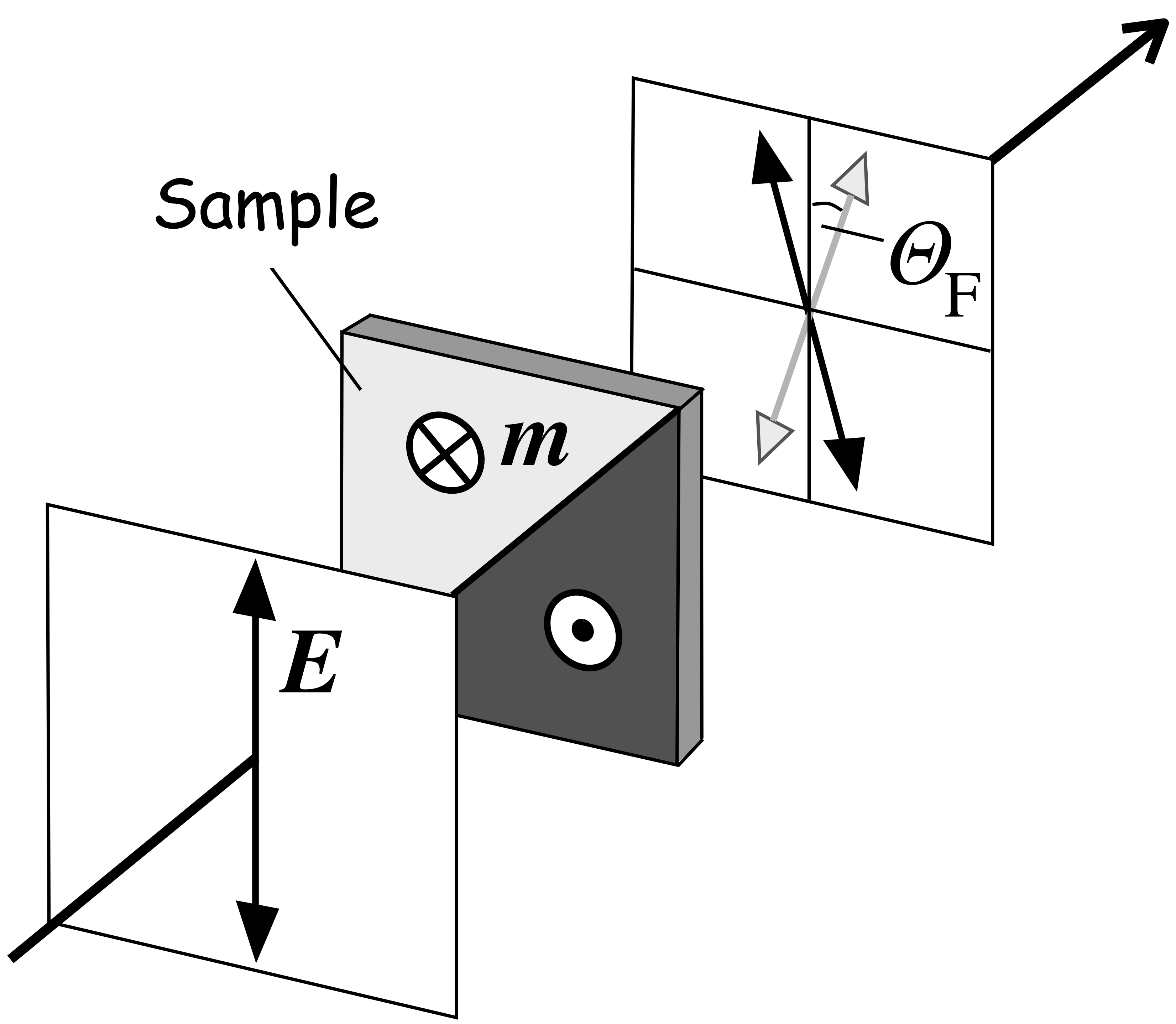

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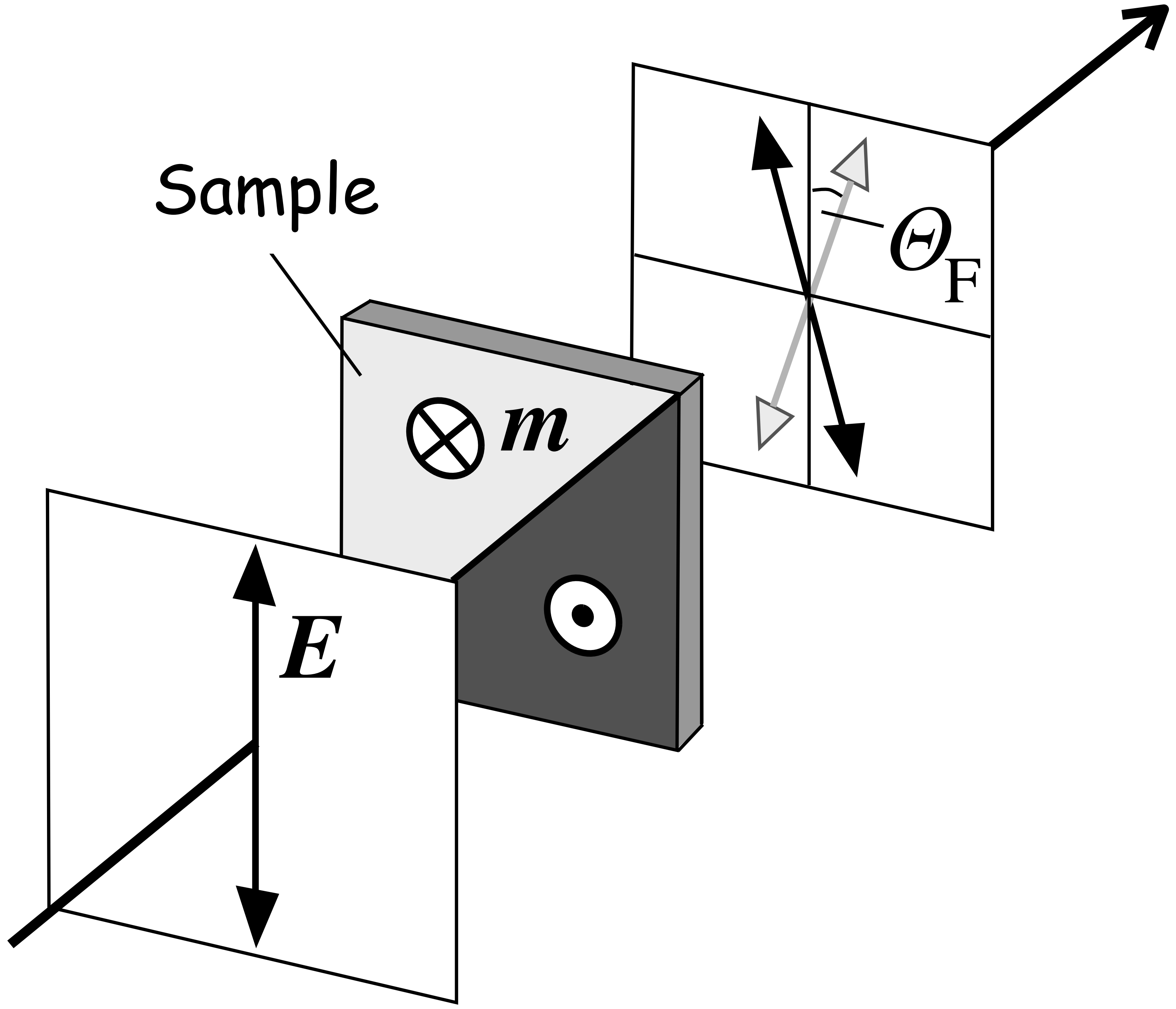

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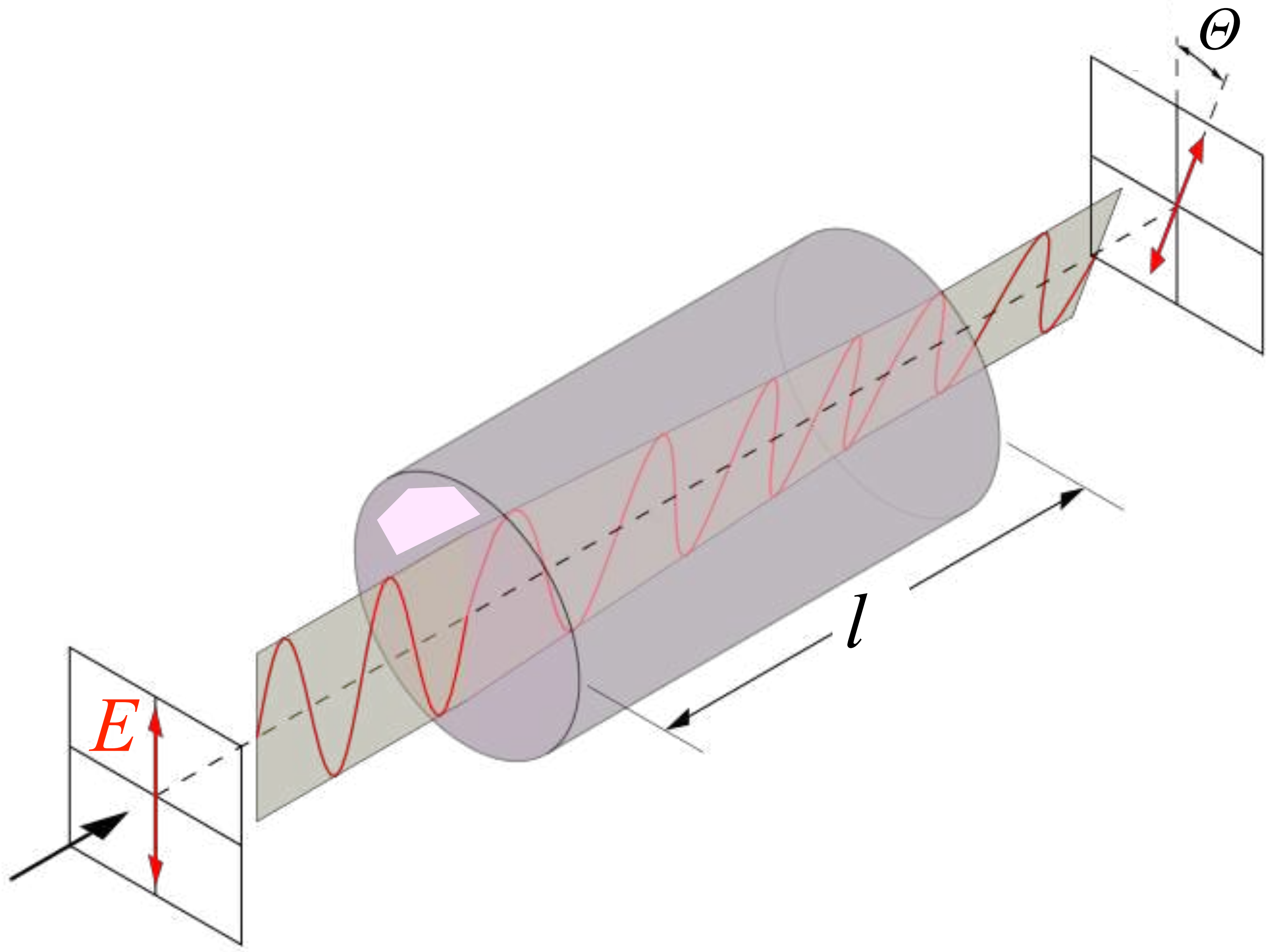

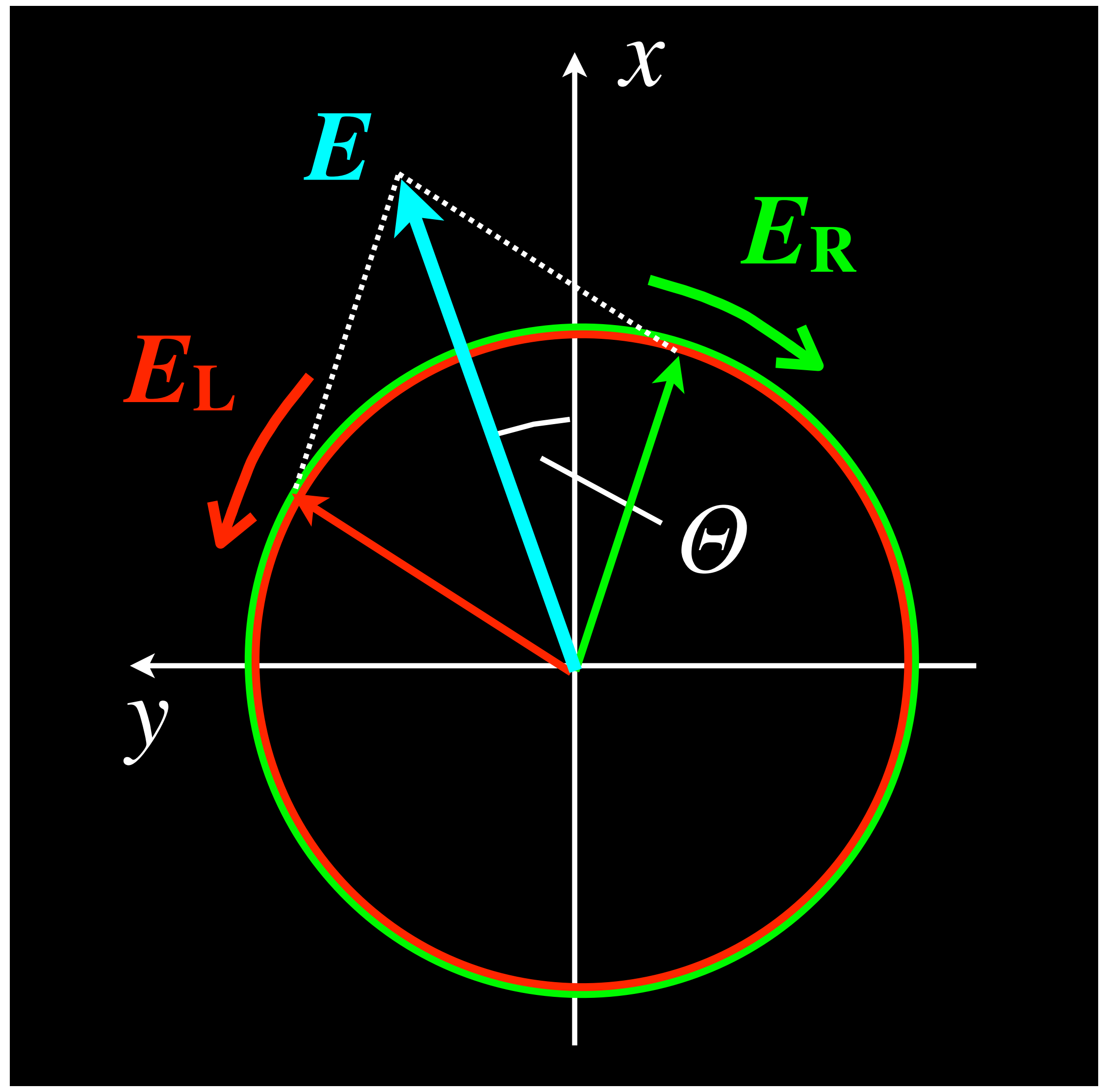

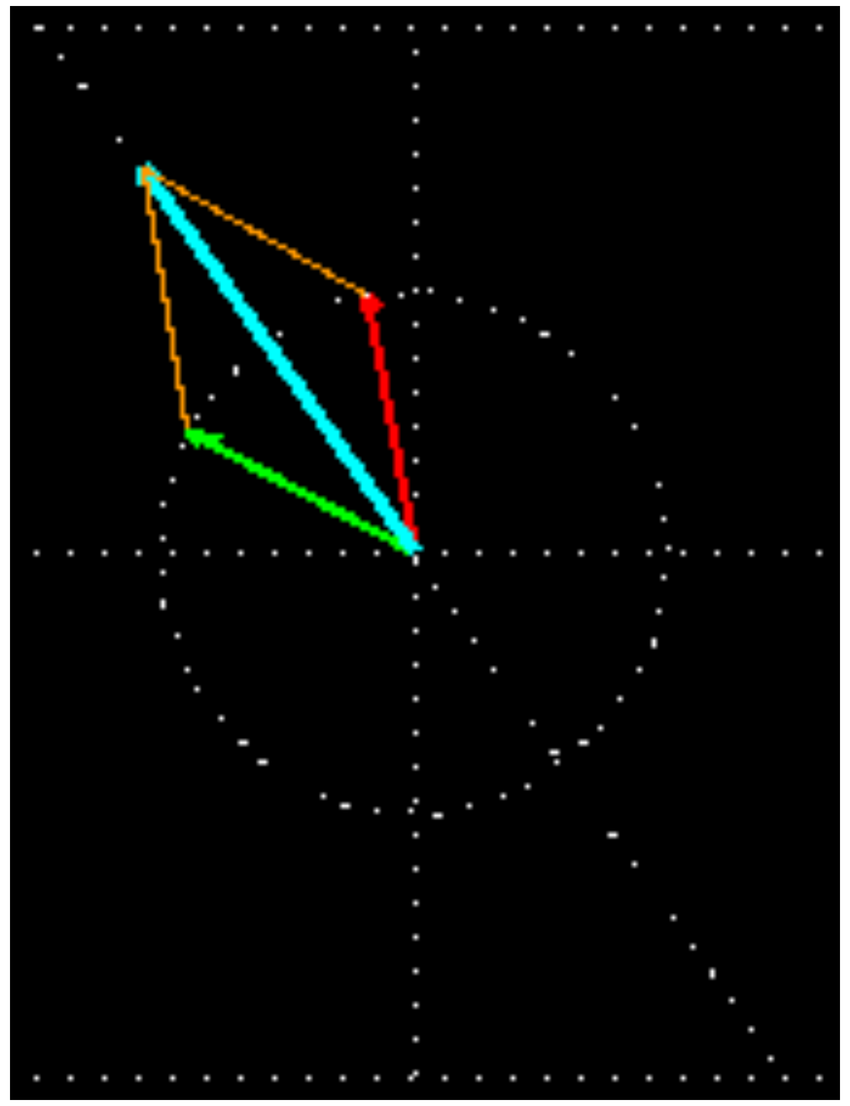

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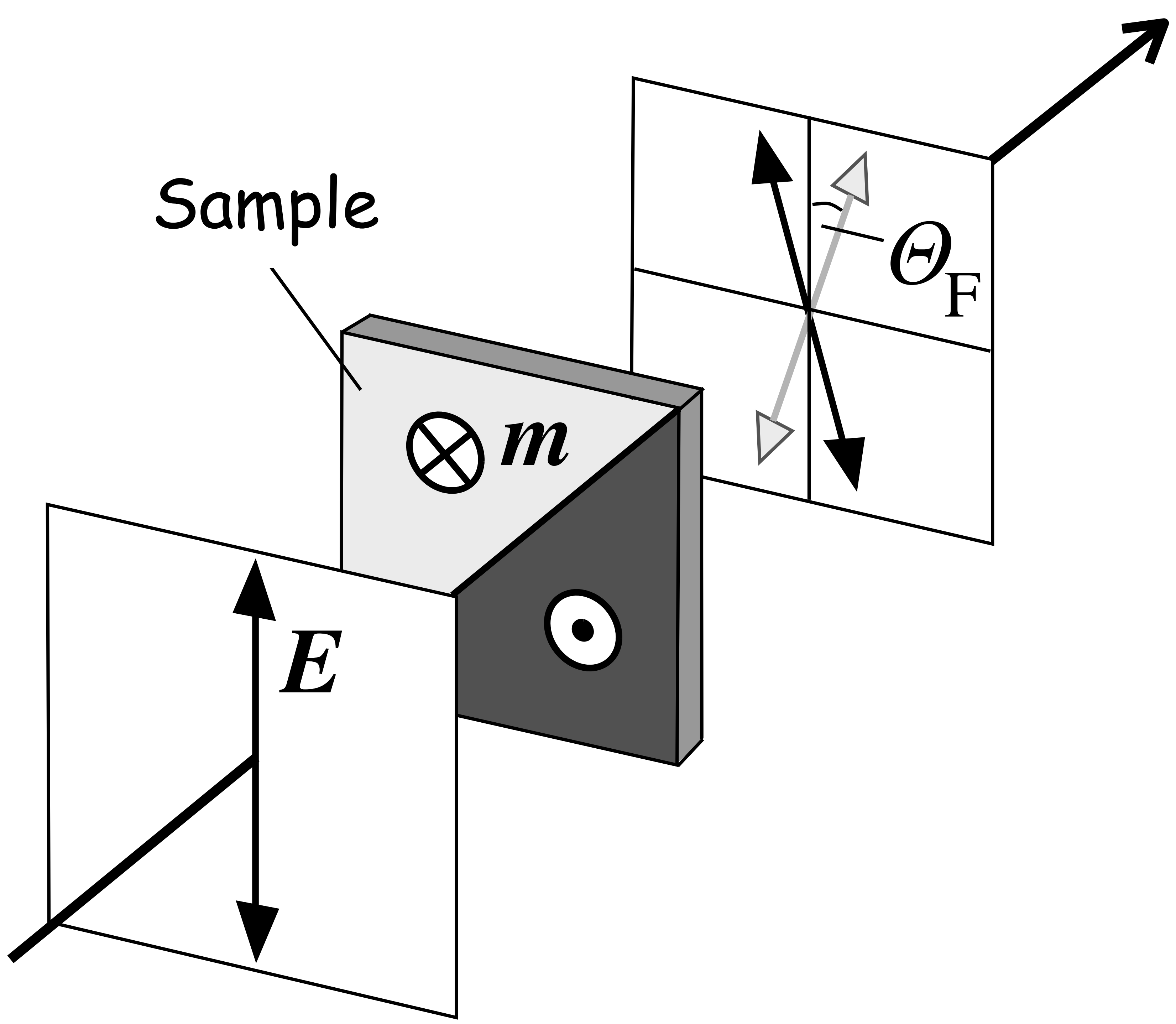

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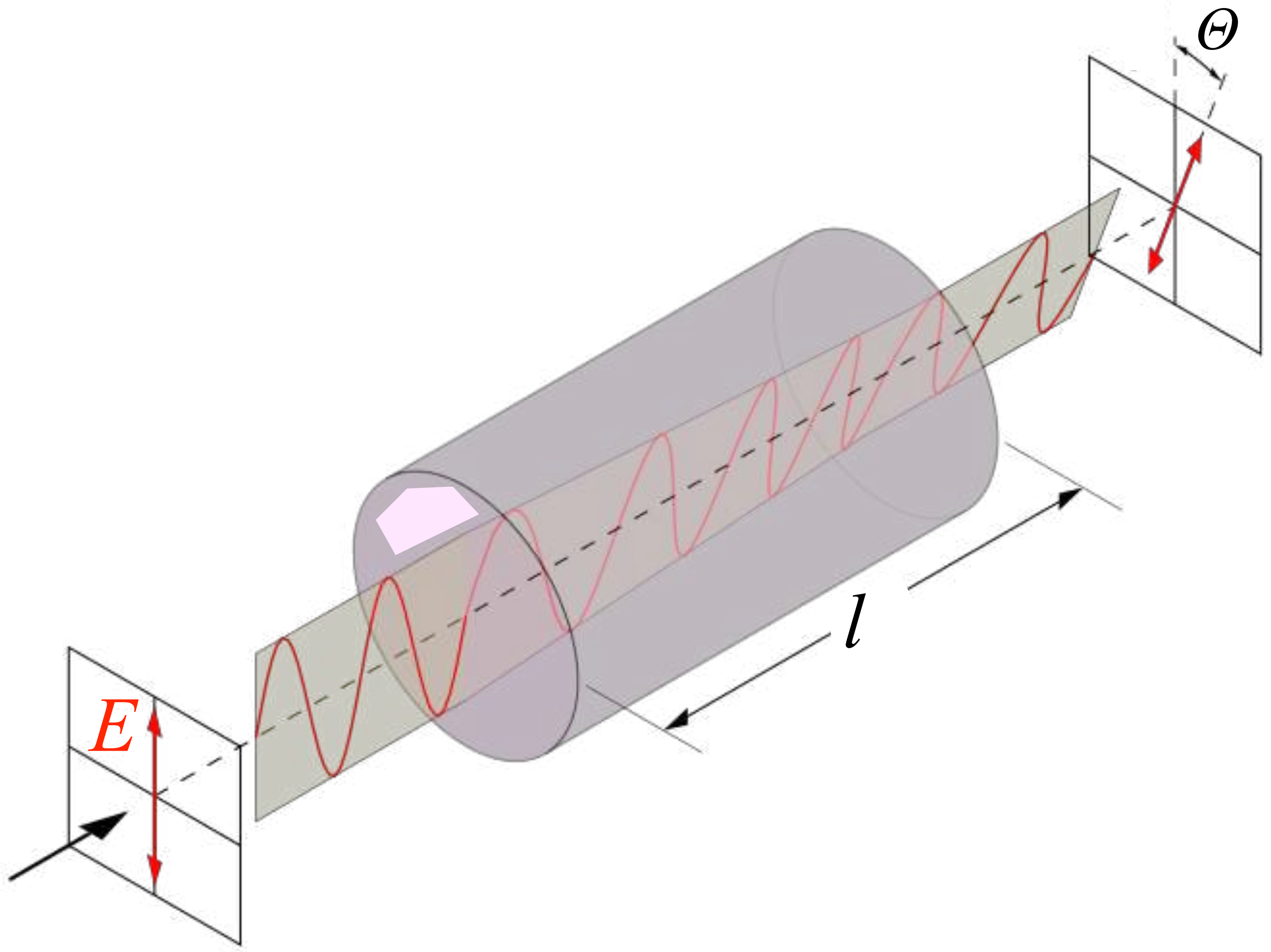
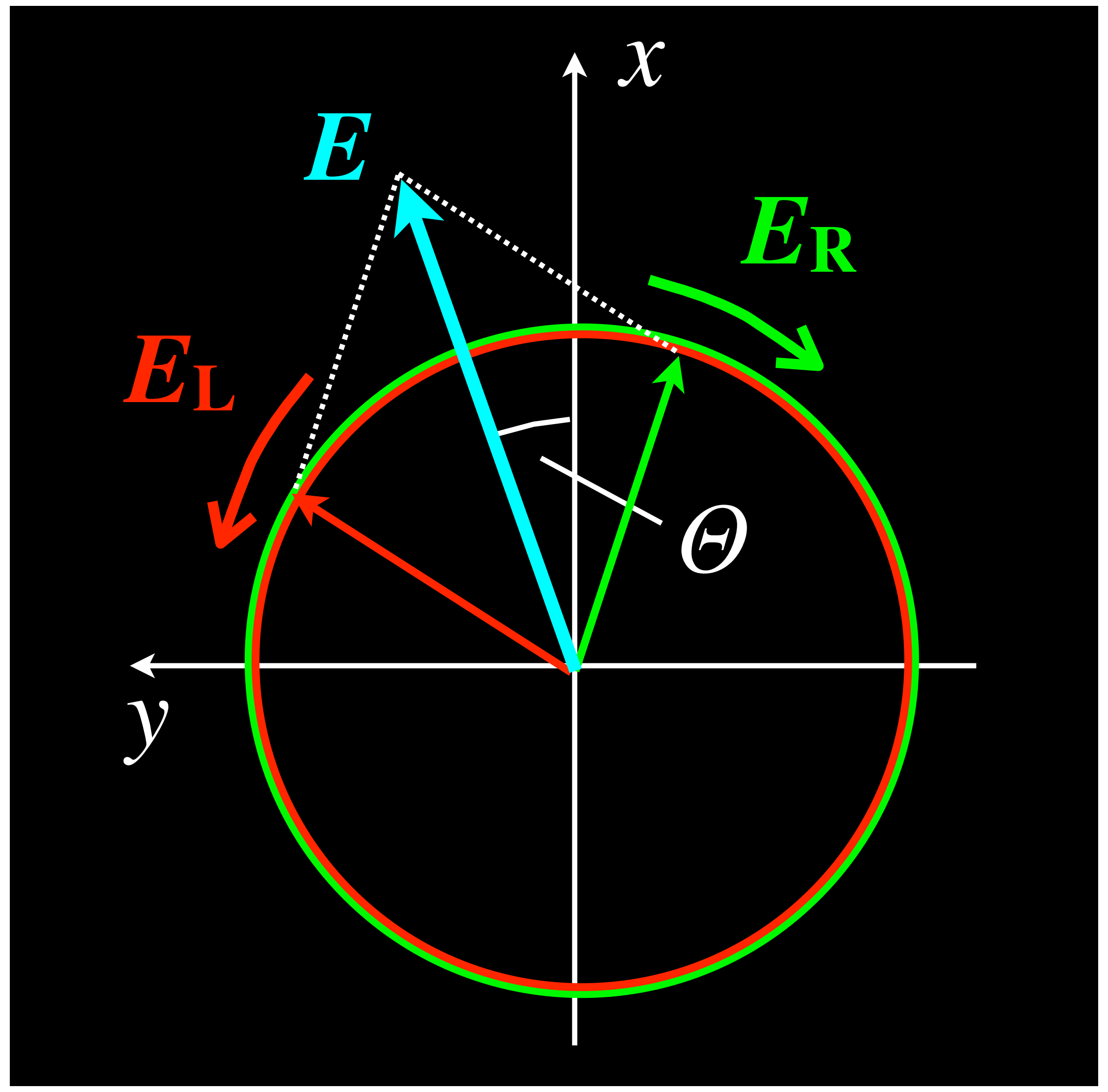
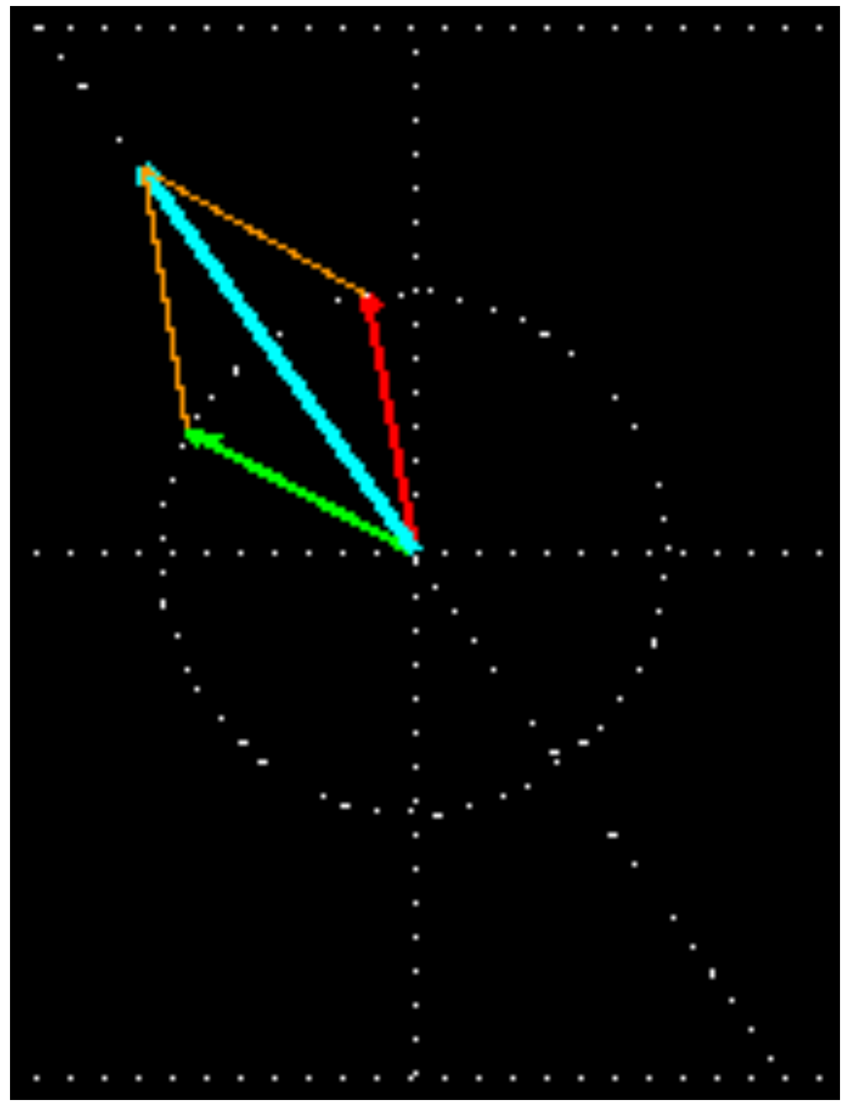
Faraday rotation:
Circular birefringence

$$\theta_F = -\frac{\pi}{\lambda_0}\left(n'_+ - n'_-\right) l$$

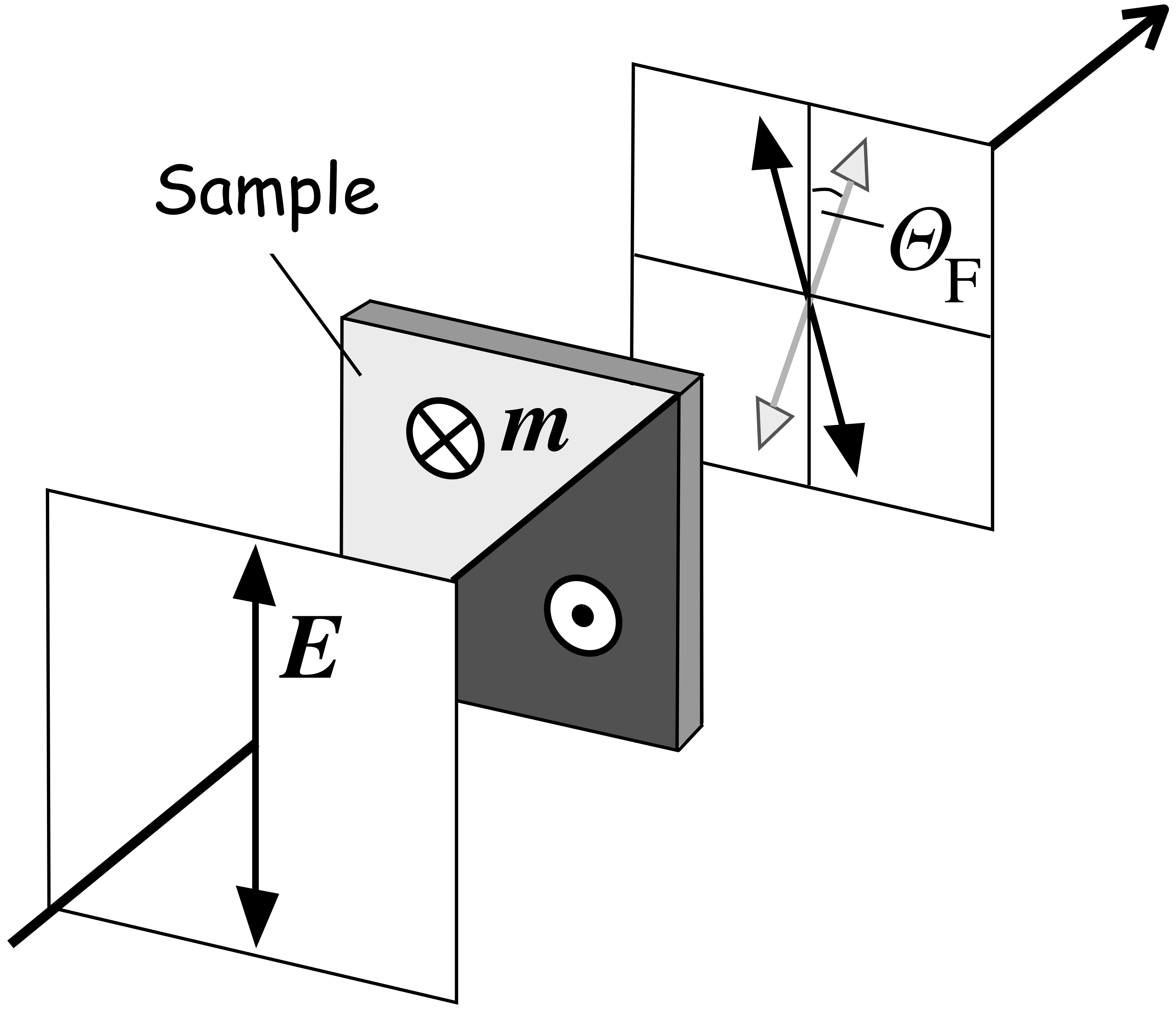






2. Magneto-Optical Effects – 2.3 Faraday Effect

Polar configuration



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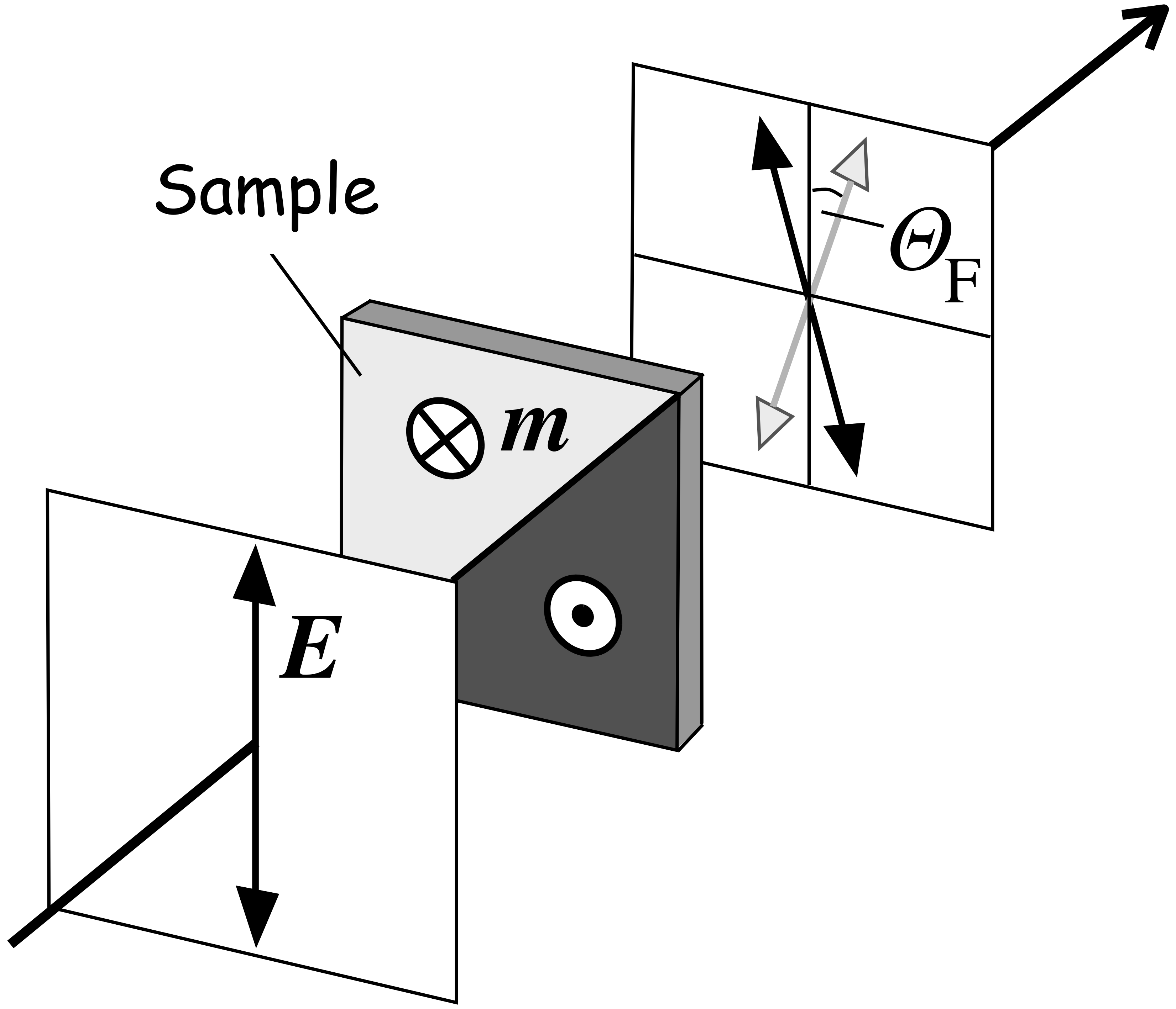

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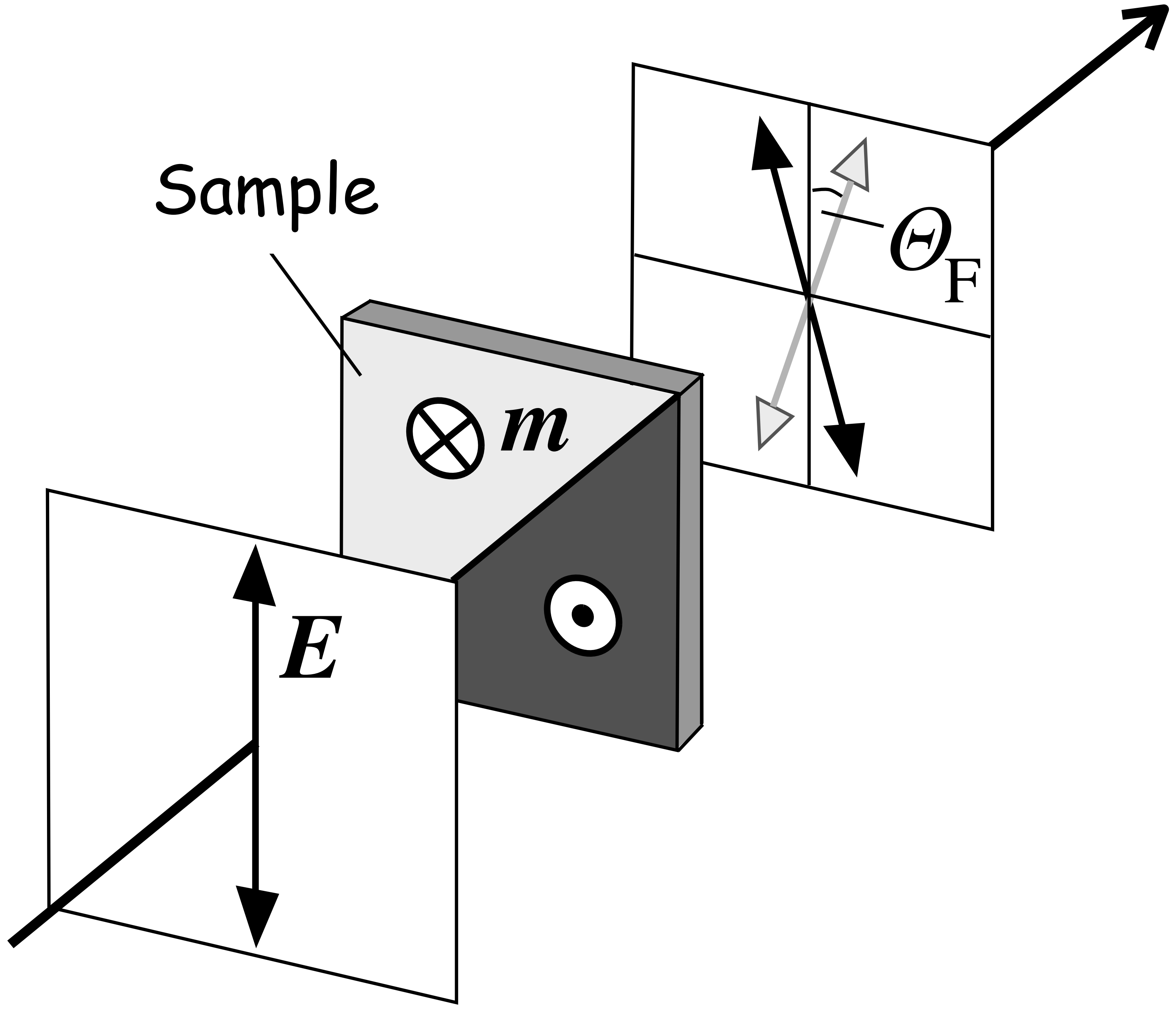

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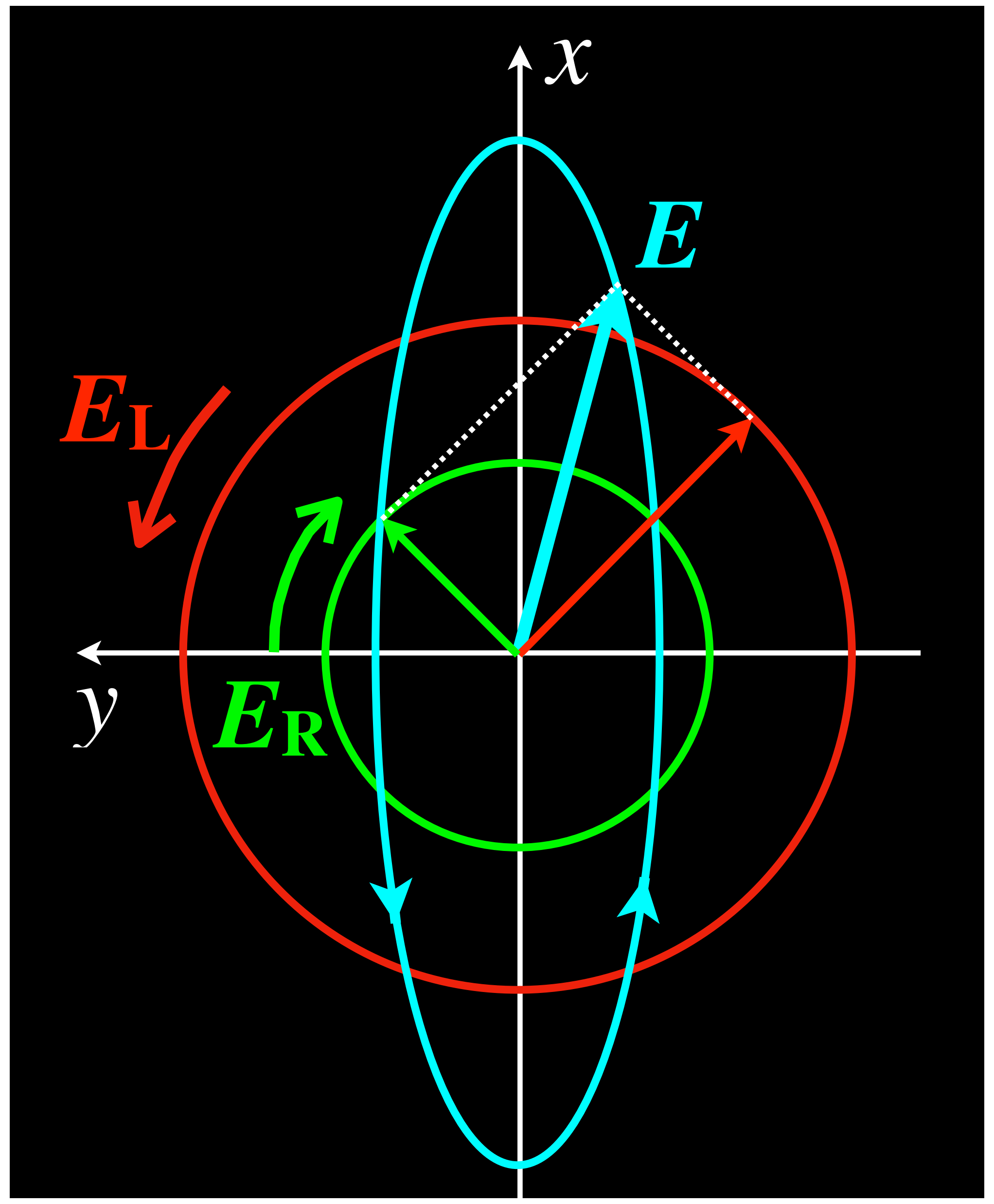

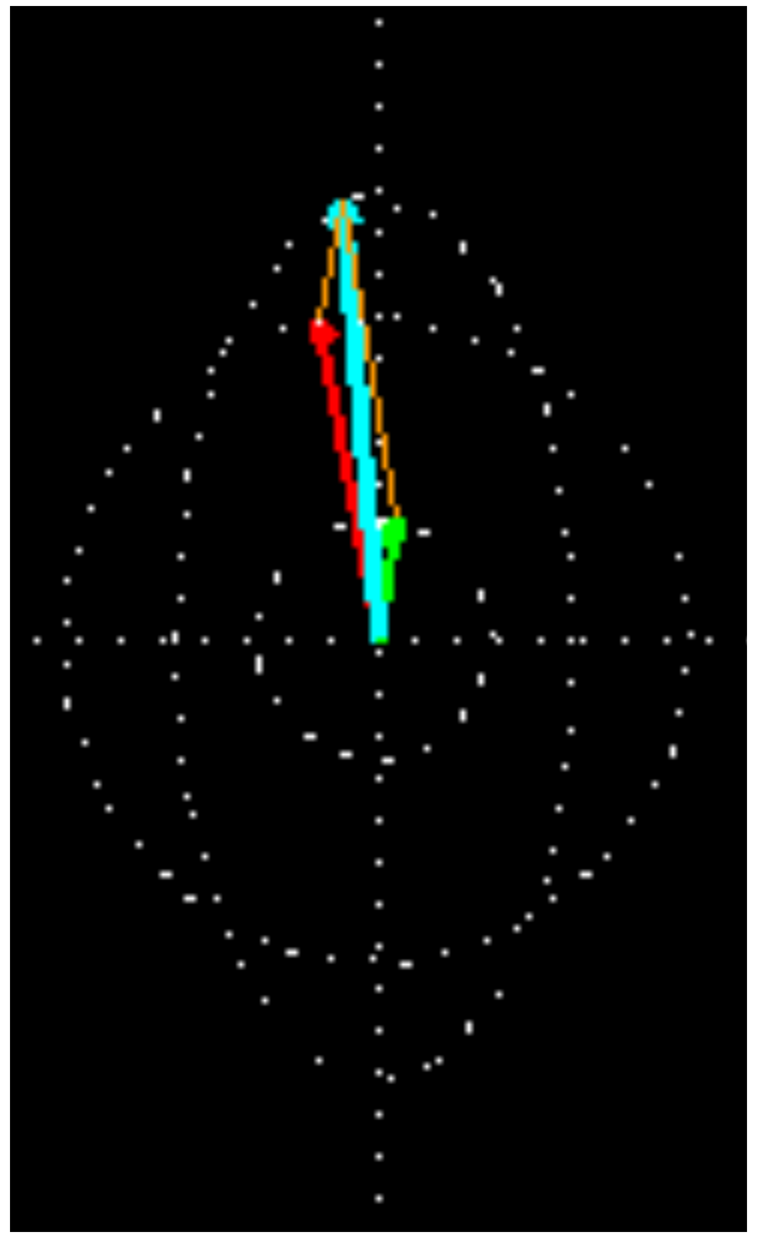

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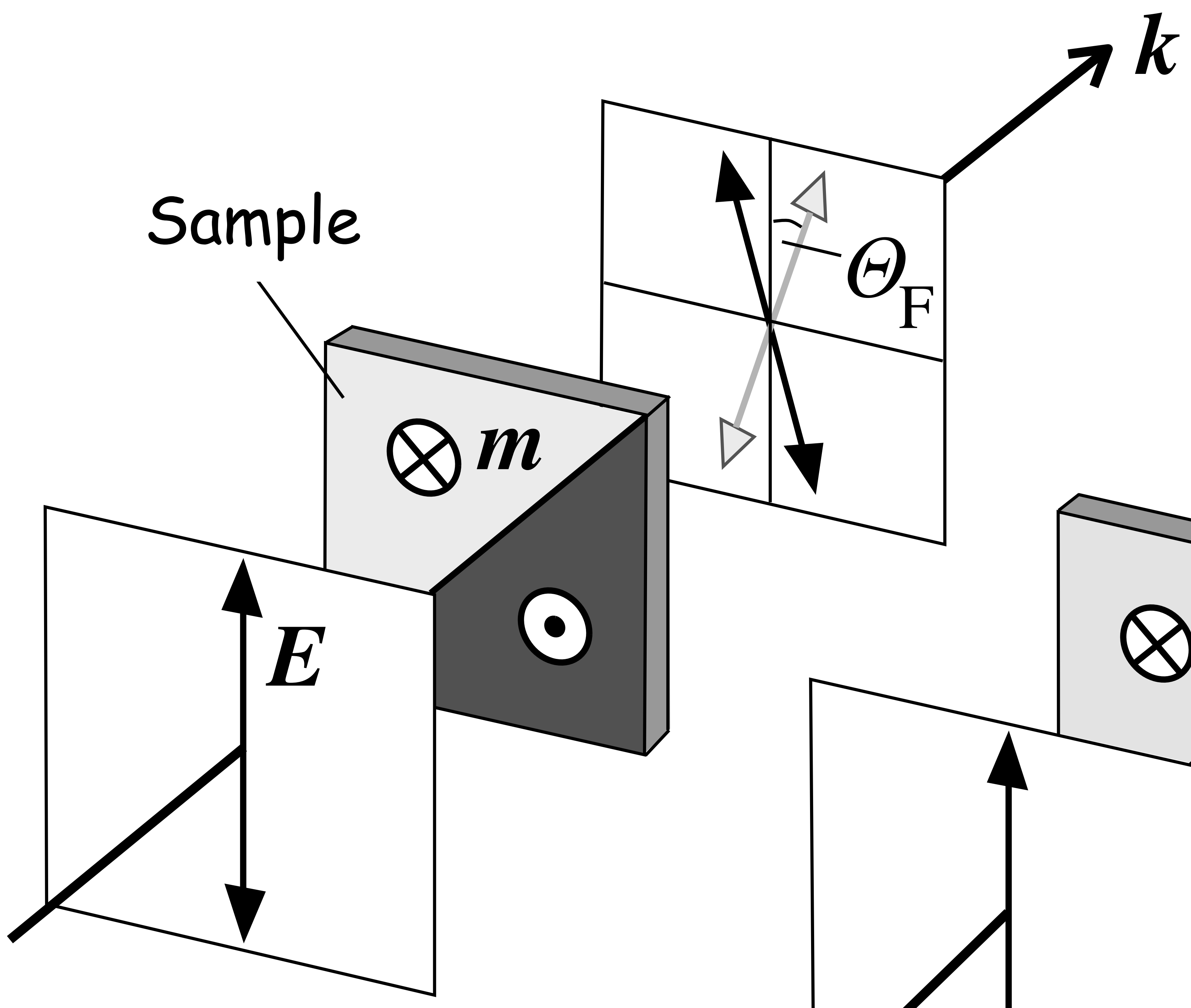

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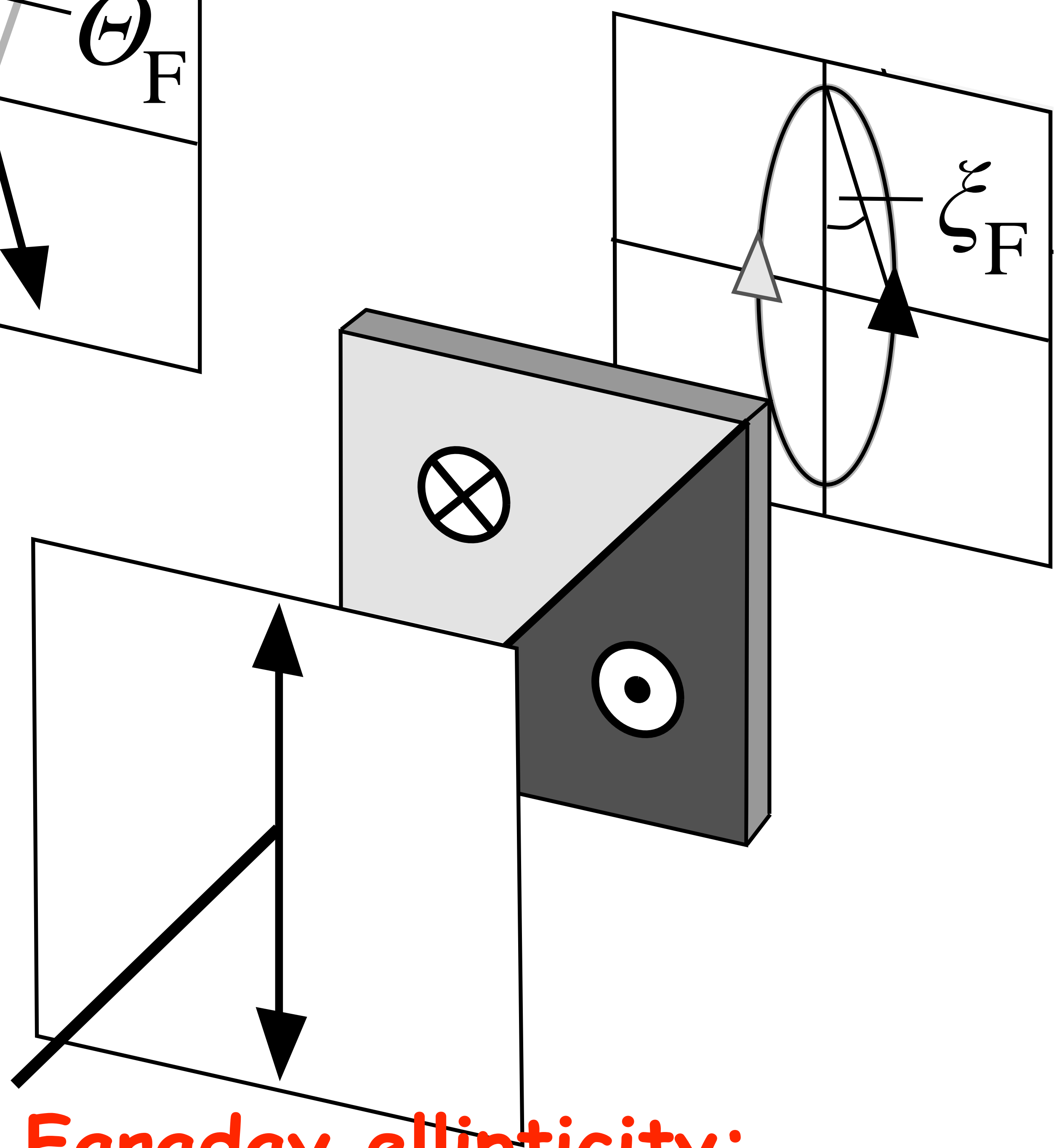
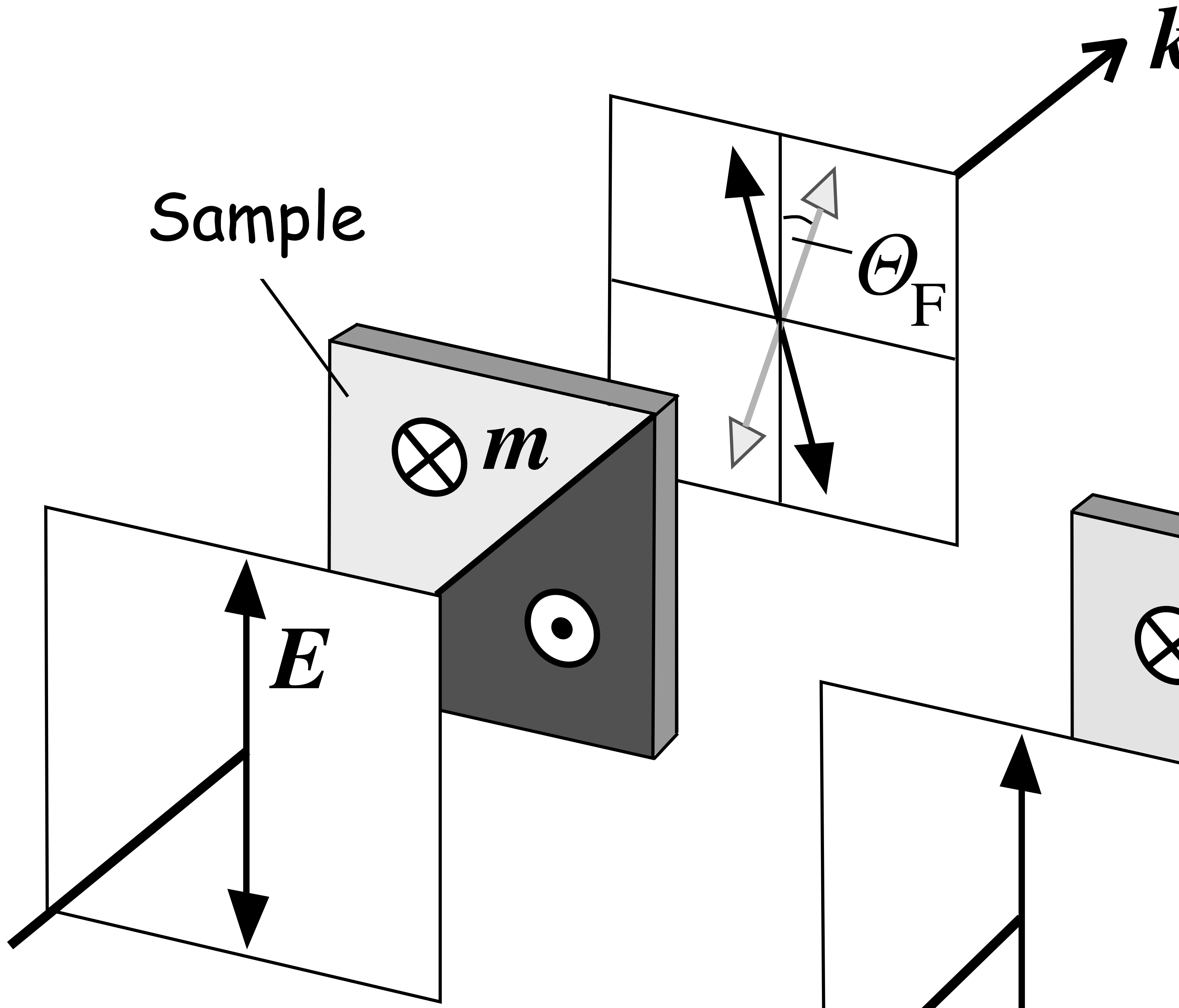


Faraday ellipticity:
Circular dichroism

$$\xi_F = -\frac{\pi}{\lambda_0}\left(n''_+ - n''_-\right) l$$
$$= \frac{1}{4}\left(\alpha''_- - \alpha''_+\right) l$$

Polar configuration



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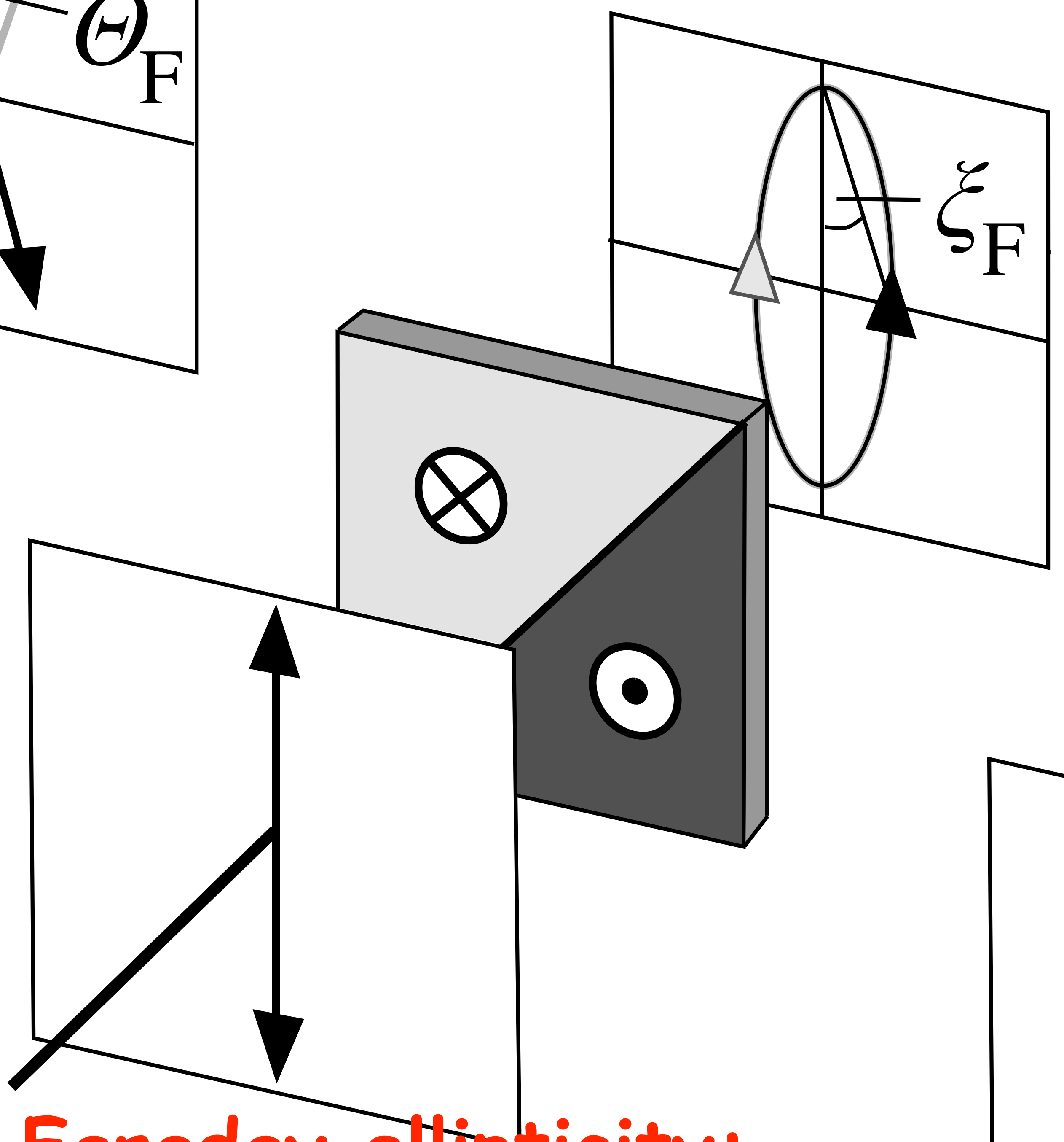

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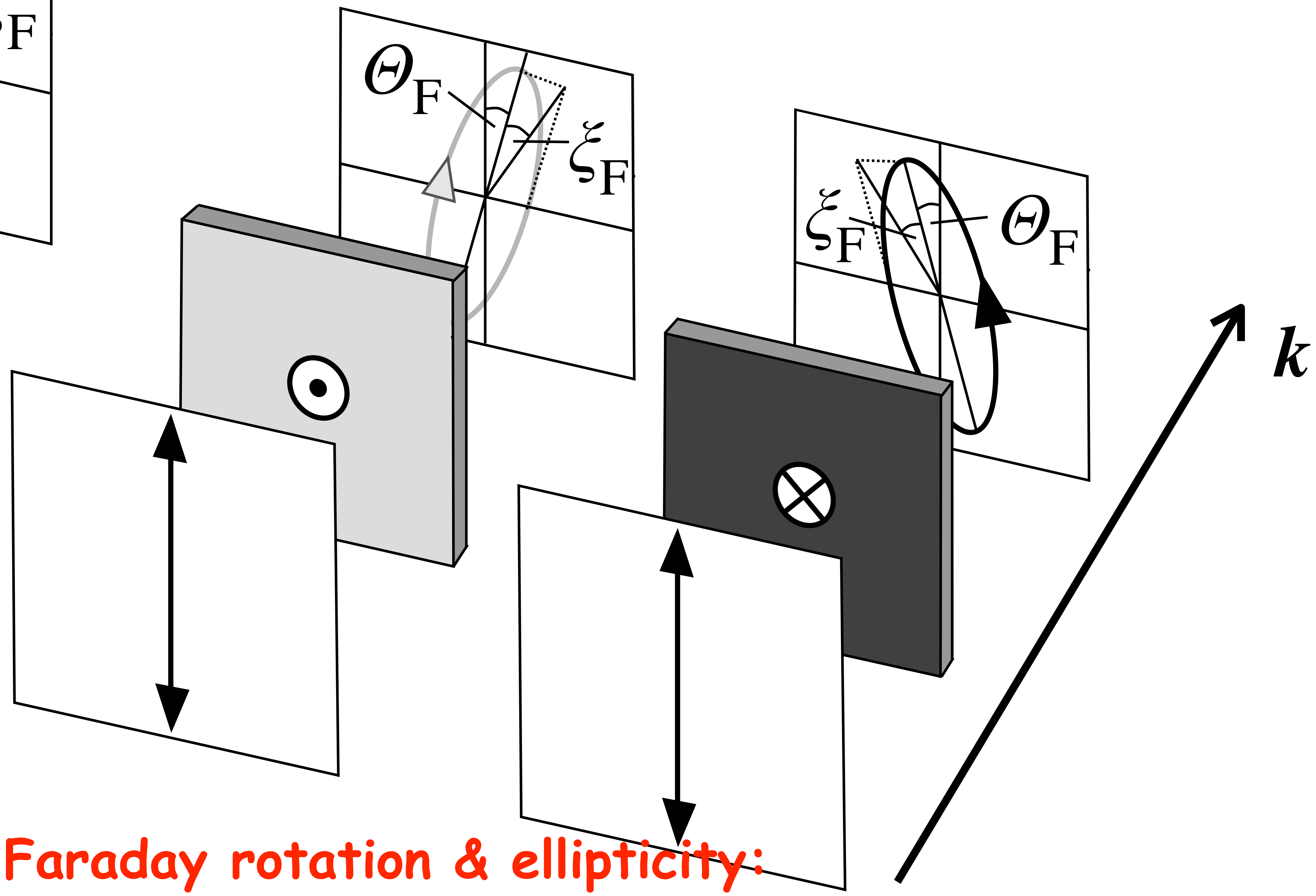
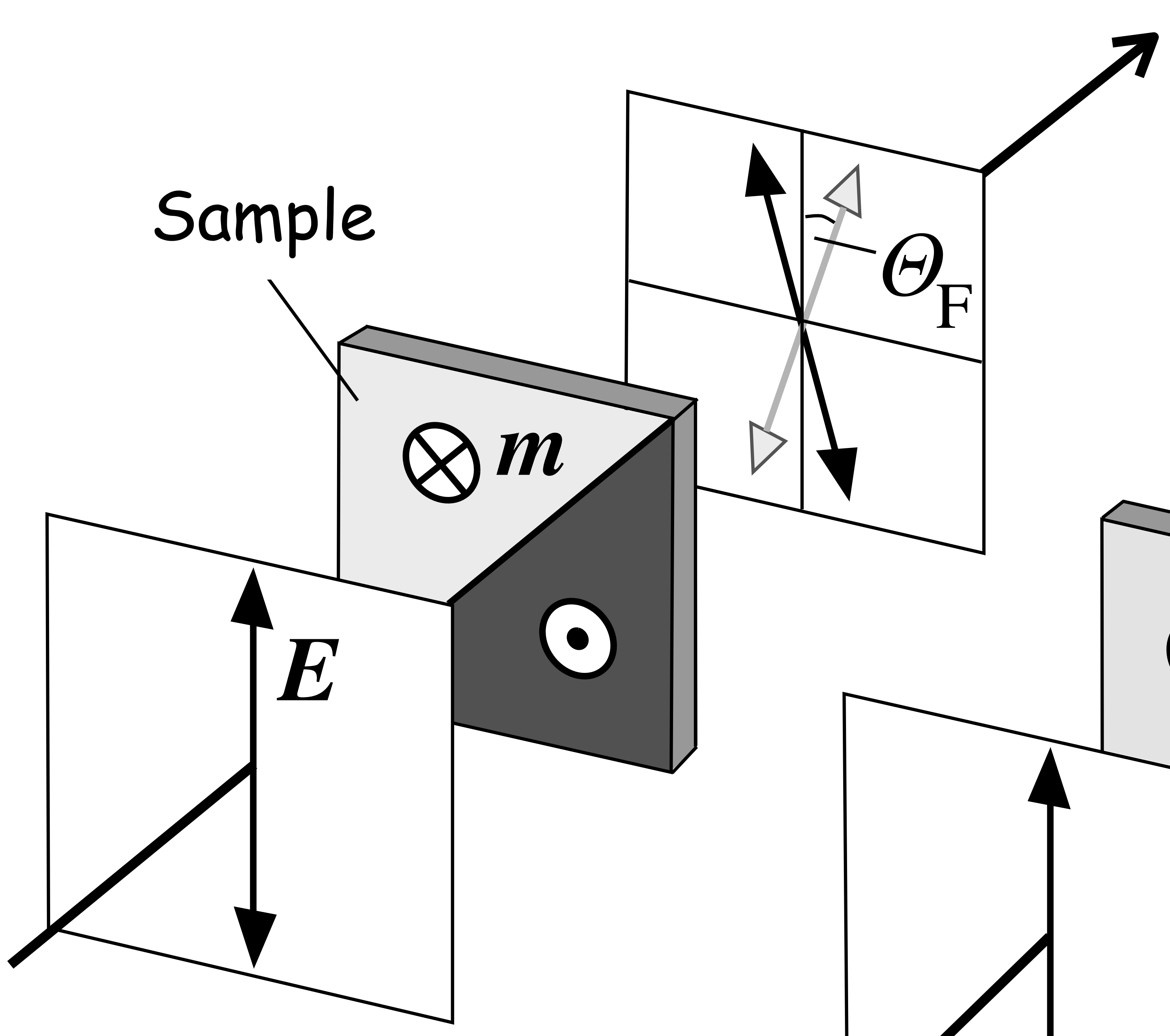


Faraday rotation & ellipticity:
Circular birefringence & dichroism



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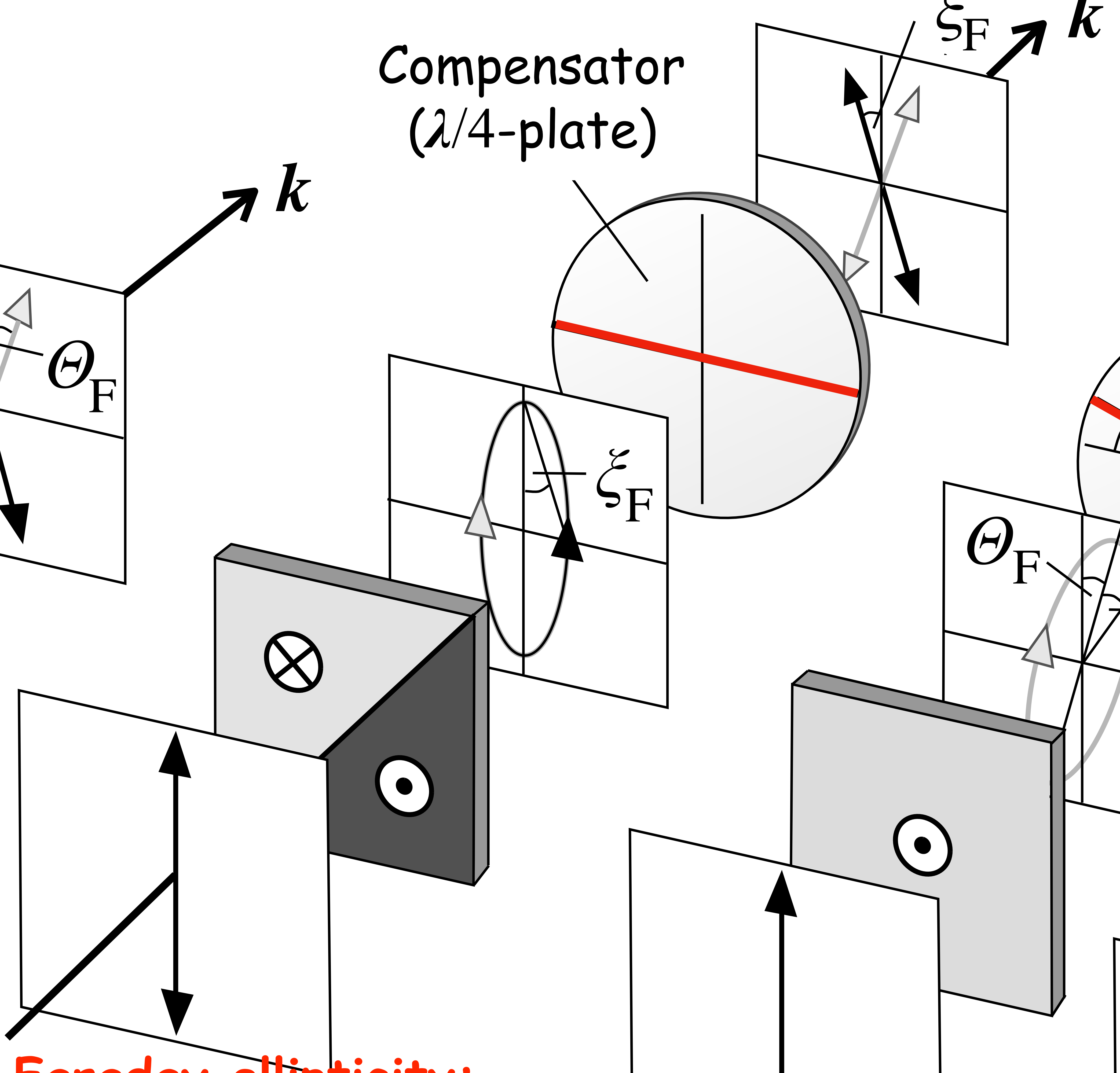

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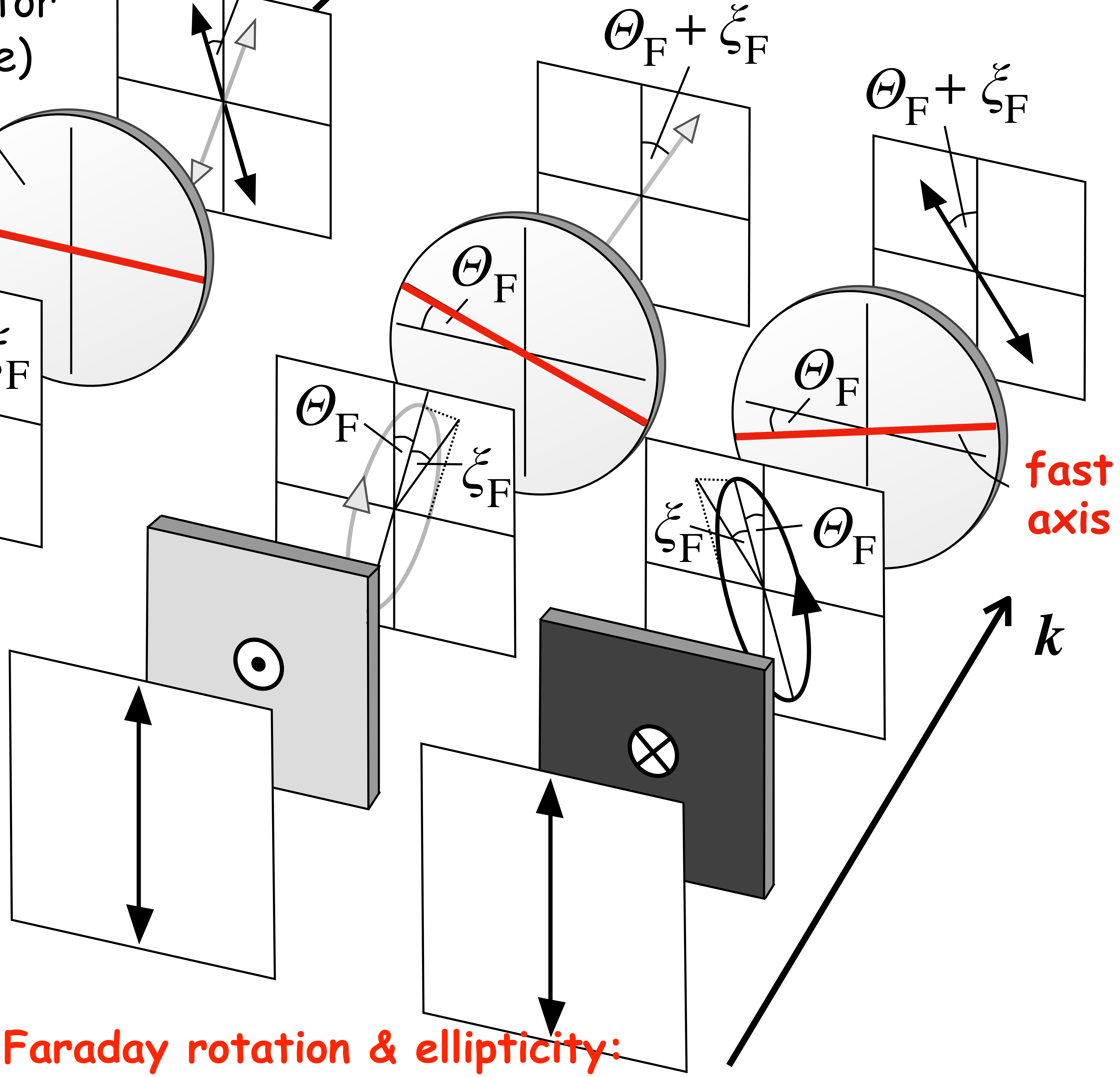

Faraday rotation & ellipticity:
Circular birefringence & dichroism

2. Magneto-Optical Effects – 2.4 Kerr Effect

- Like Faraday effect, but in **reflection** on absorbing media and much **weaker** as light only interacts with magnetization within thin surface layer given by penetration depth of light

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$$\begin{pmatrix} E_{\mathrm{x}}^{\mathrm{refl}} \\ E_{\mathrm{y}}^{\mathrm{refl}} \end{pmatrix} = \begin{pmatrix} r_{\mathrm{xx}} & r_{\mathrm{xy}} \\ r_{\mathrm{yx}} & r_{\mathrm{yy}} \end{pmatrix} \begin{pmatrix} E_{\mathrm{x}}^{\mathrm{in}} \\ E_{\mathrm{y}}^{\mathrm{in}} \end{pmatrix}$$

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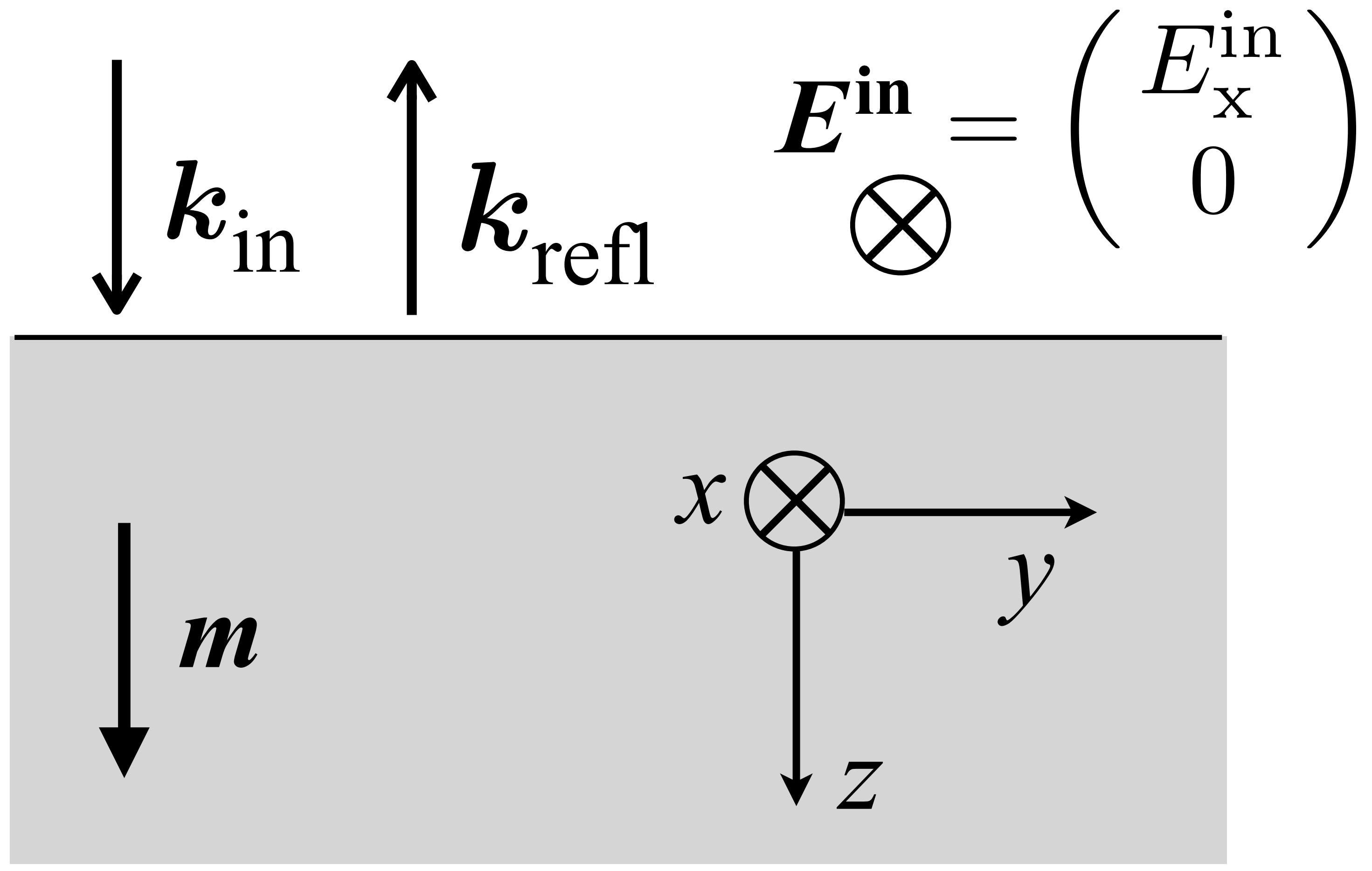

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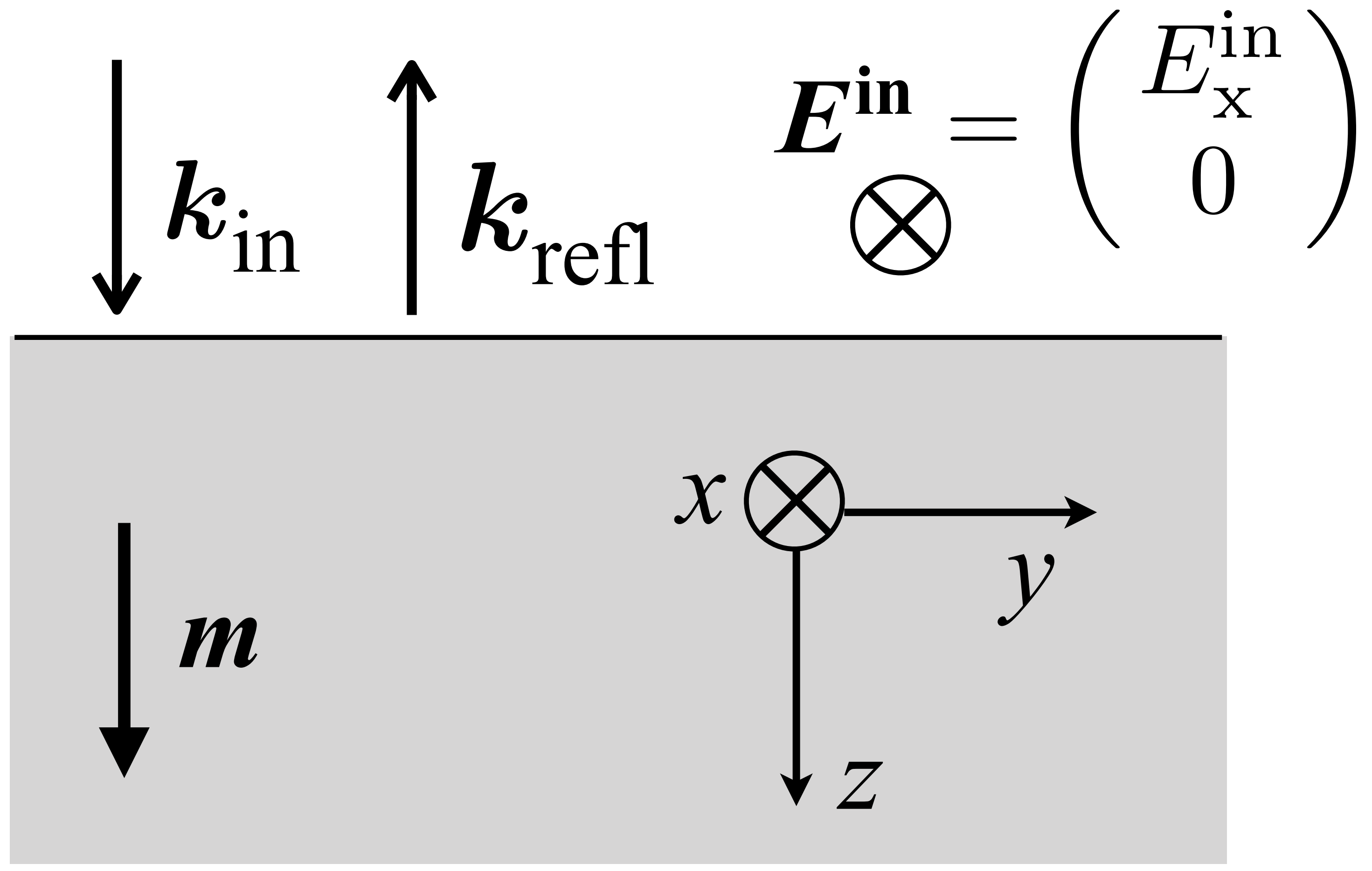

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2. Magneto-Optical Effects – 2.4 Kerr Effect

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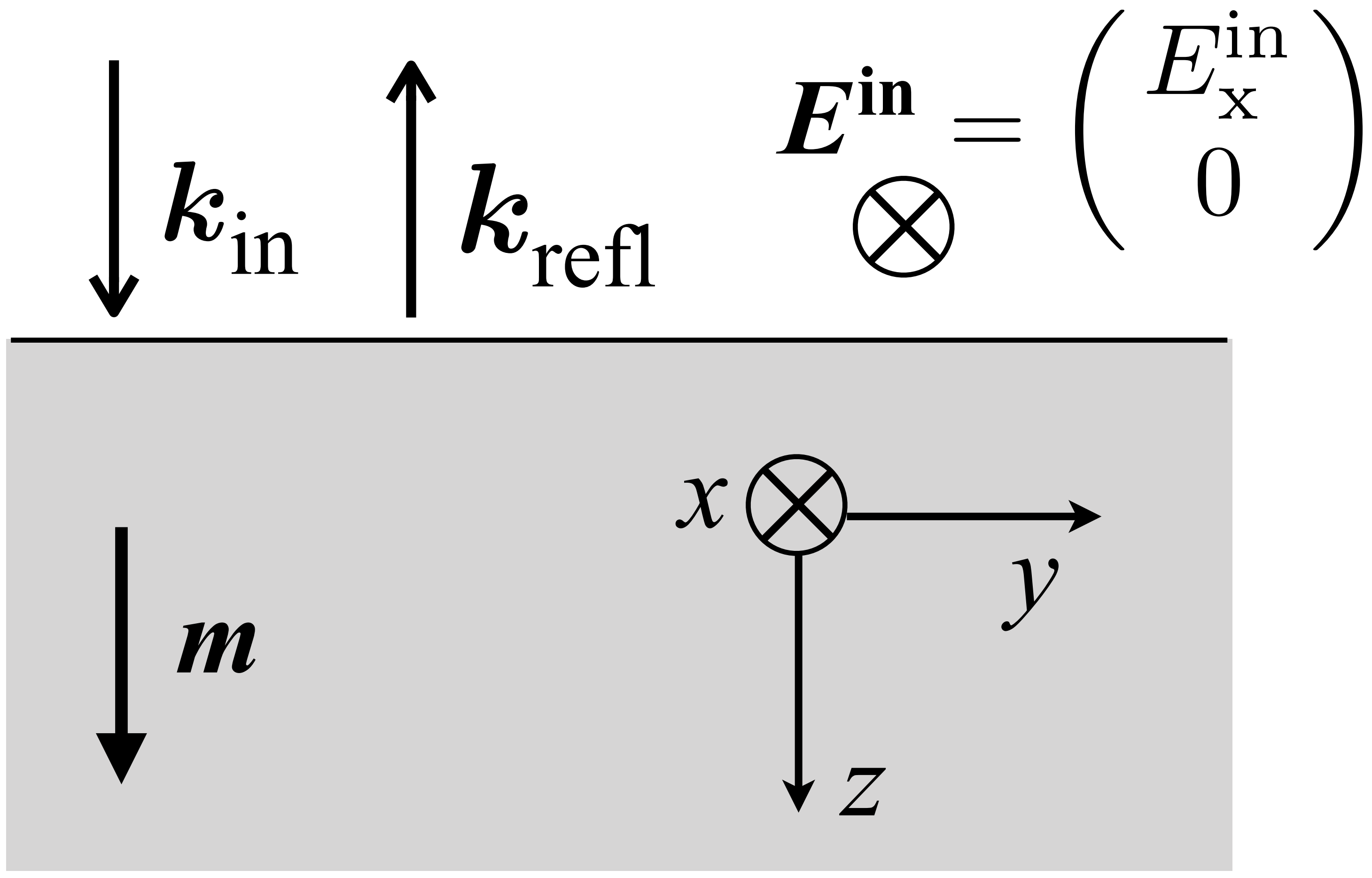

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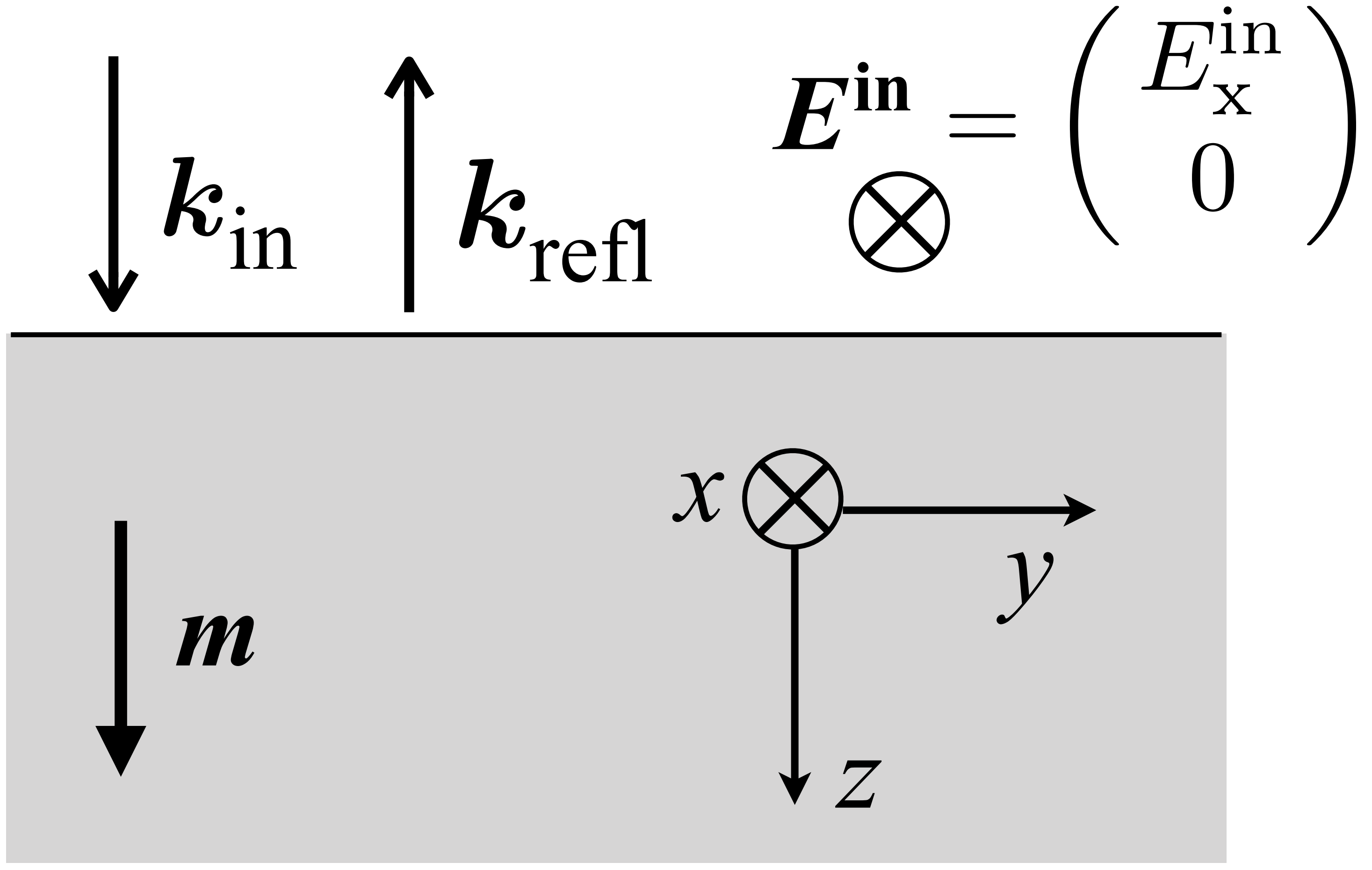

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with Fresnel equations,

$$n_\pm = \sqrt{\epsilon_{\mathrm{iso}}}\,(1 \pm Q_{\mathrm{V}}/2)$$

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$$\Longrightarrow$$

$$r_{\mathrm{xx}} = \frac{1-\bar{n}}{1+\bar{n}} \equiv N$$ Coefficient of normally reflected light

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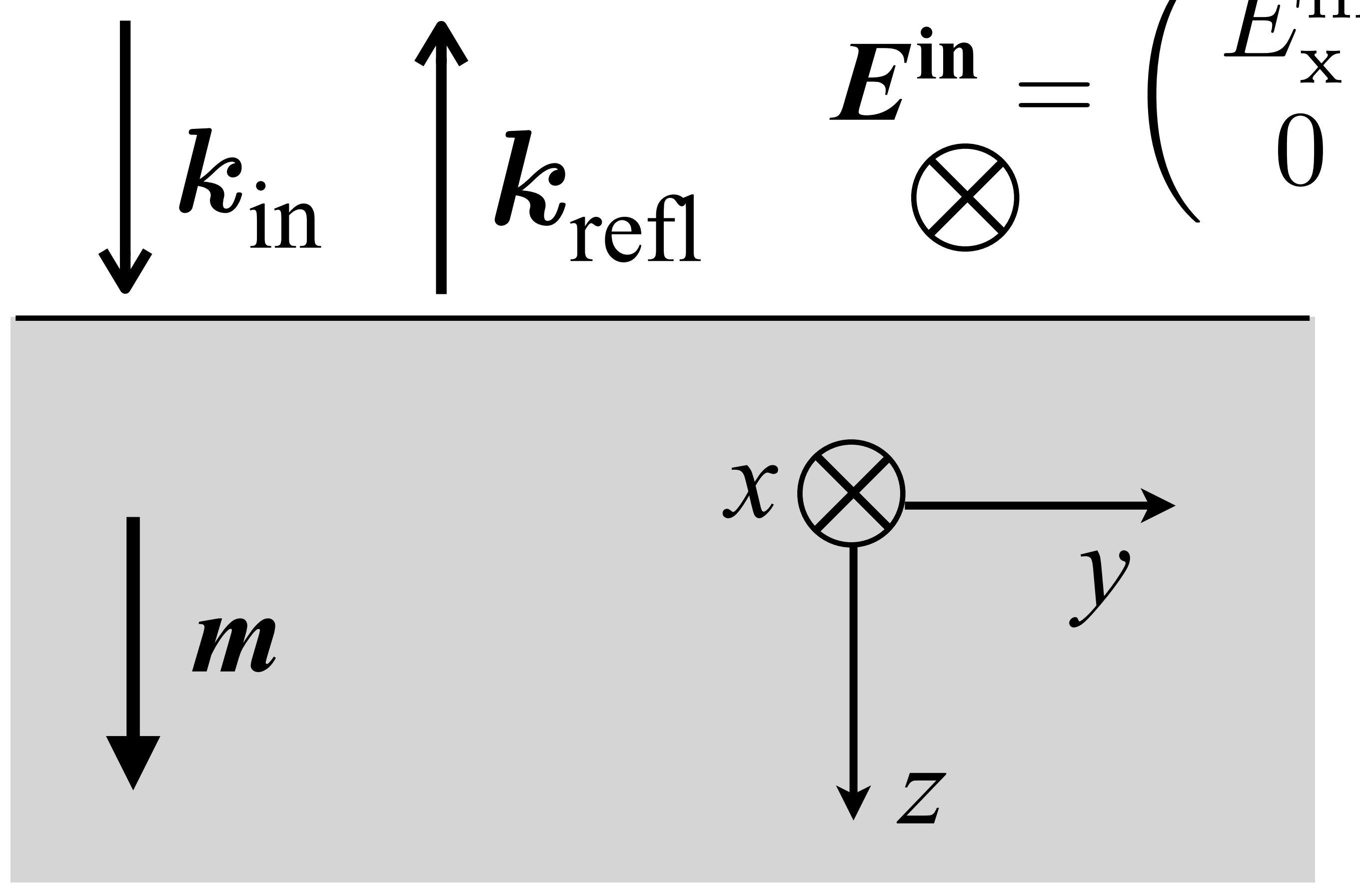

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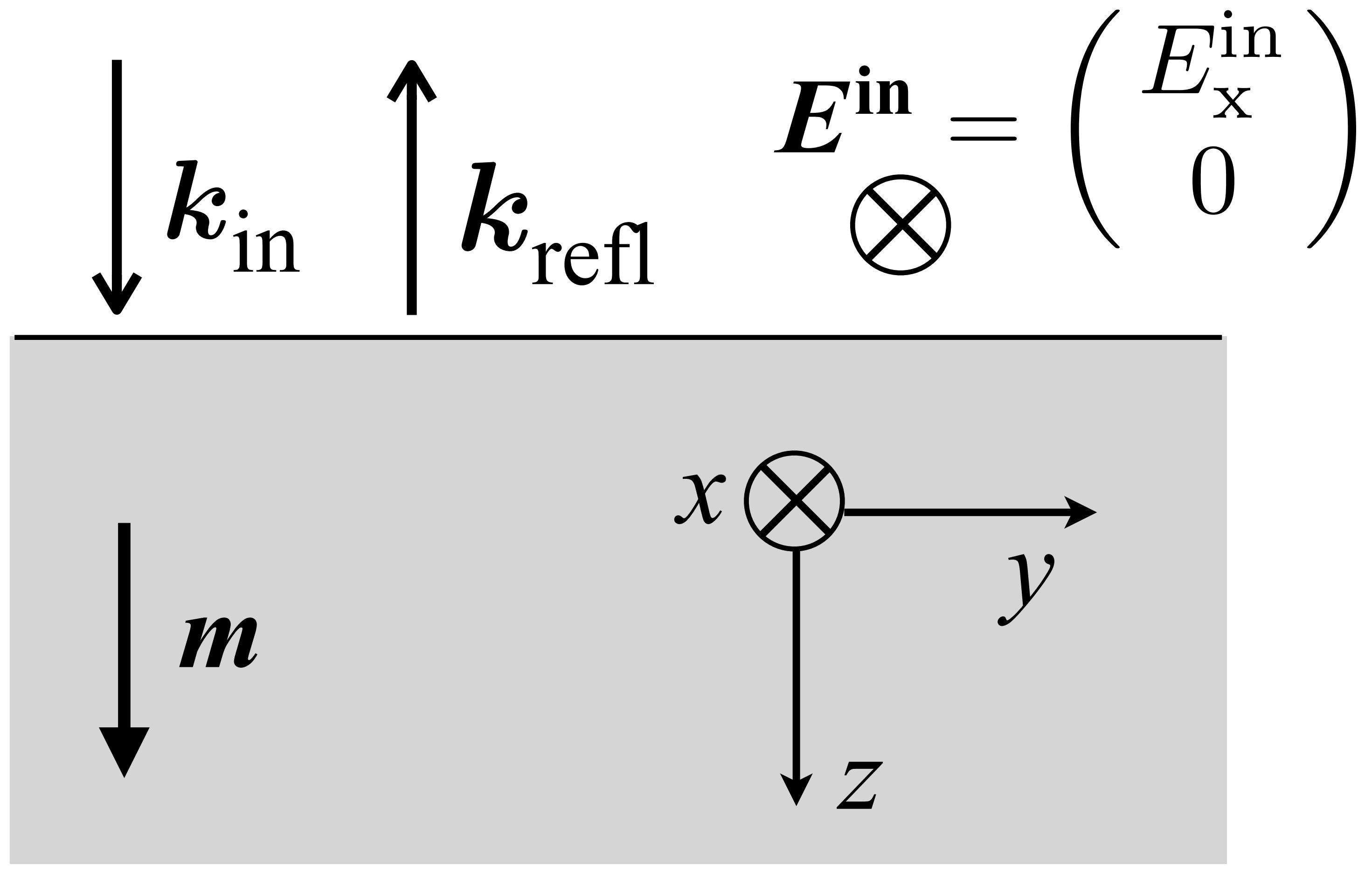

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- Summary:
  - impinging linear wave, polarized along $x \Longrightarrow$ generates reflected wave that has small y-component
  - x-component $=$ regularly reflected amplitude R_N | y-component $=$ Kerr amplitude R_K

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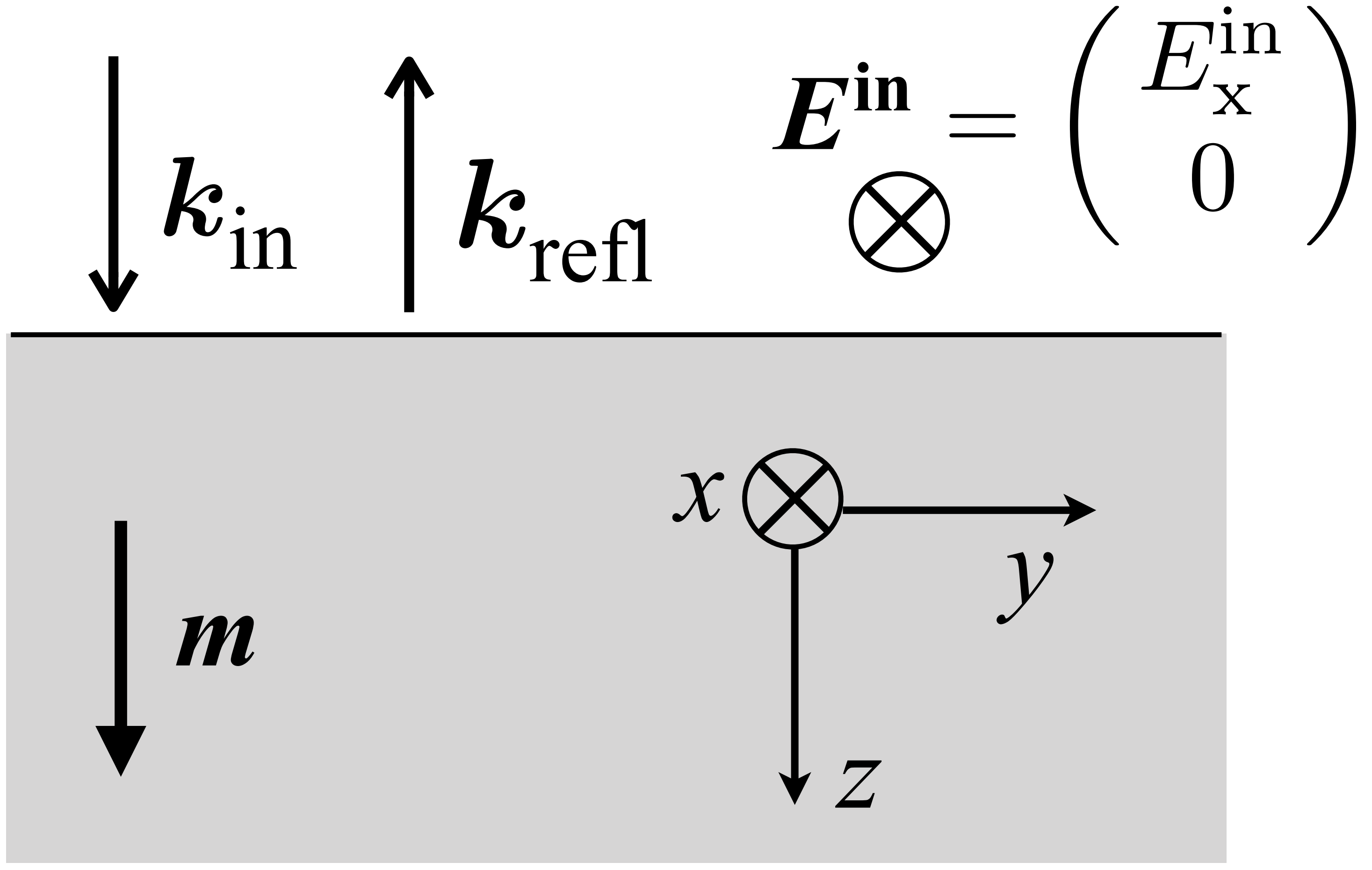

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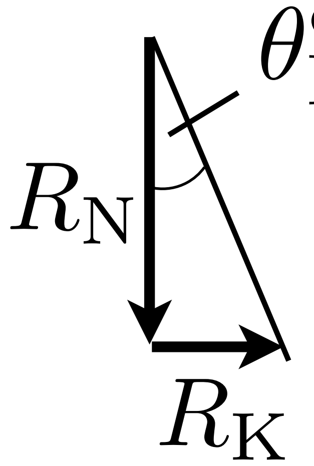
- Complex Kerr rotation:



$$\theta_{\mathrm{K}}^{\mathrm{c}} = \theta_{\mathrm{K}} + \mathrm{i}\,\xi_{\mathrm{K}} = \frac{R_{\mathrm{K}}}{R_{\mathrm{N}}} = \frac{K}{N}$$

with

$\theta_{\mathrm{K}} = \mathrm{Re}(\theta_{\mathrm{K}}^{\mathrm{c}})$ Kerr rotation

$\xi_{\mathrm{K}} = \mathrm{Im}(\theta_{\mathrm{K}}^{\mathrm{c}})$ Kerr ellipticity

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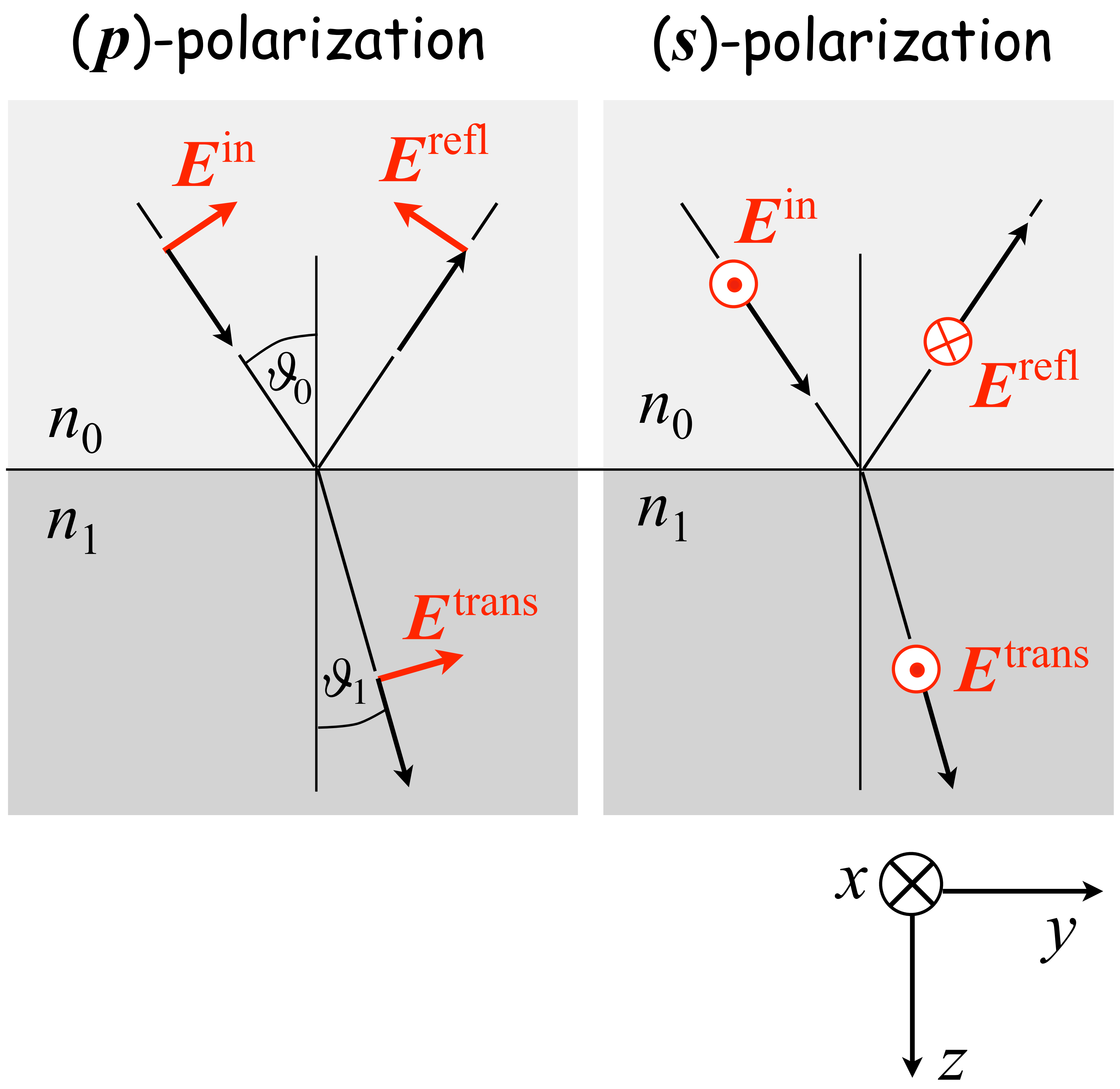
2. Magneto-Optical Effects – 2.4 Kerr Effect

- General case of oblique incidence: better use sp-coordinates

$$\begin{pmatrix} E_{\mathrm{p}}^{\mathrm{refl}} \\ E_{\mathrm{s}}^{\mathrm{refl}} \end{pmatrix} = \begin{pmatrix} r_{\mathrm{pp}} & r_{\mathrm{ps}} \\ r_{\mathrm{sp}} & r_{\mathrm{ss}} \end{pmatrix} \begin{pmatrix} E_{\mathrm{p}}^{\mathrm{in}} \\ E_{\mathrm{s}}^{\mathrm{in}} \end{pmatrix}$$



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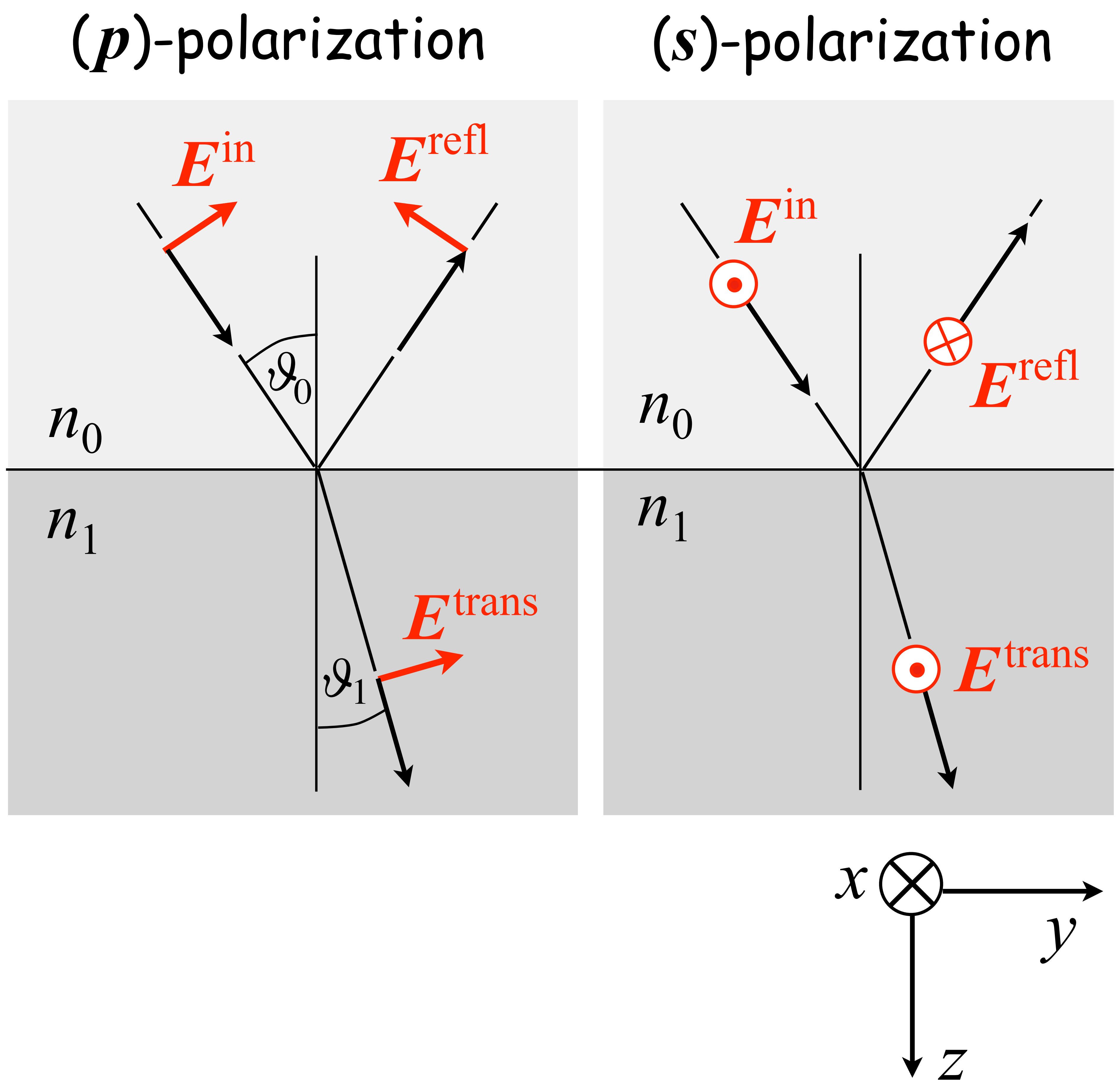

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- Reflection coefficients:

$$r_\mathrm{pp} = \frac{n_0\cos\vartheta_1 - n_1\cos\vartheta_0}{n_1\cos\vartheta_0 + n_0\cos\vartheta_1} - \frac{\mathrm{i}\,2n_0n_1\cos\vartheta_0\sin\vartheta_1 m_\mathrm{x}Q_\mathrm{V}}{(n_1\cos\vartheta_0 + n_0\cos\vartheta_1)^2},$$

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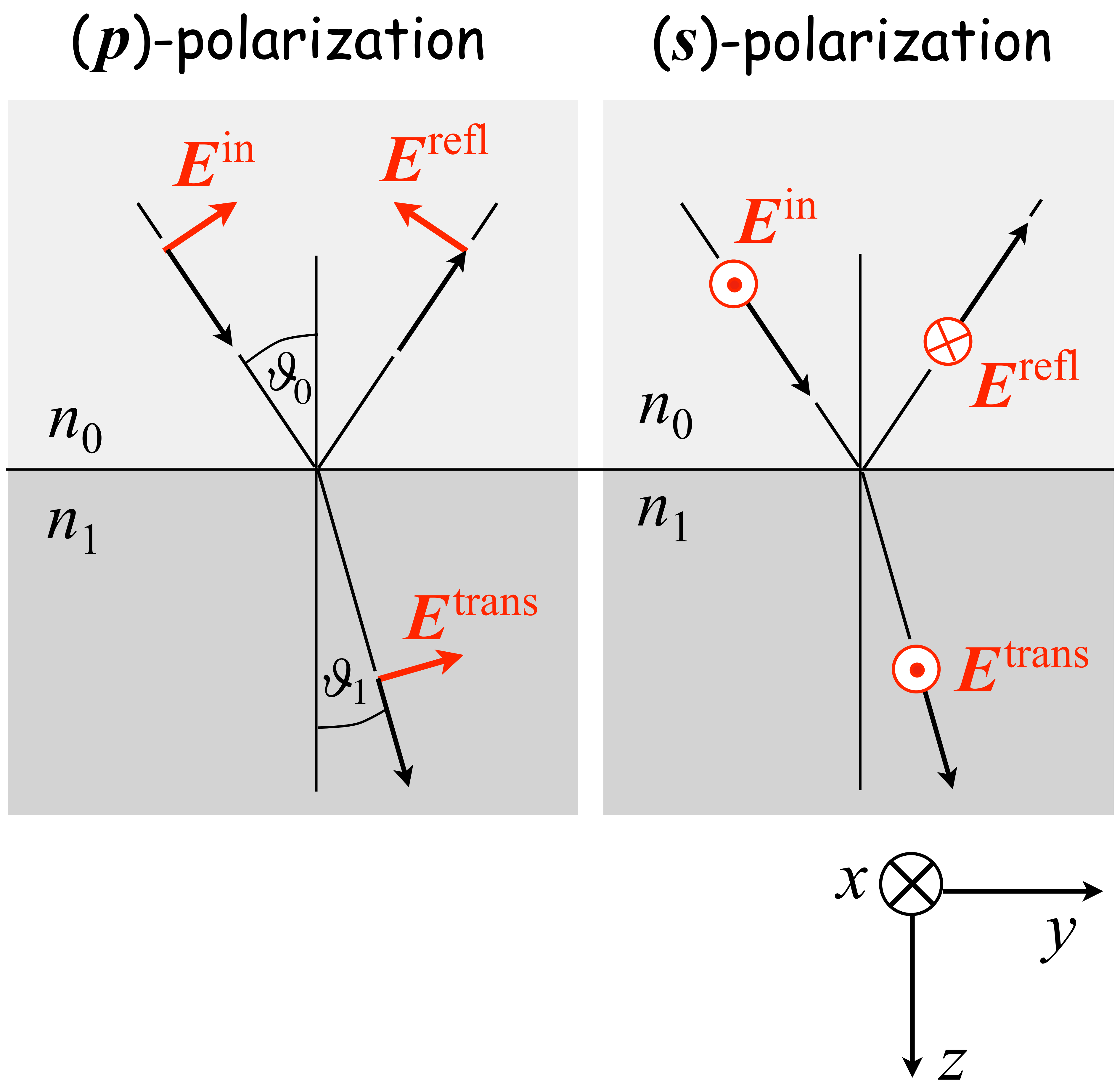

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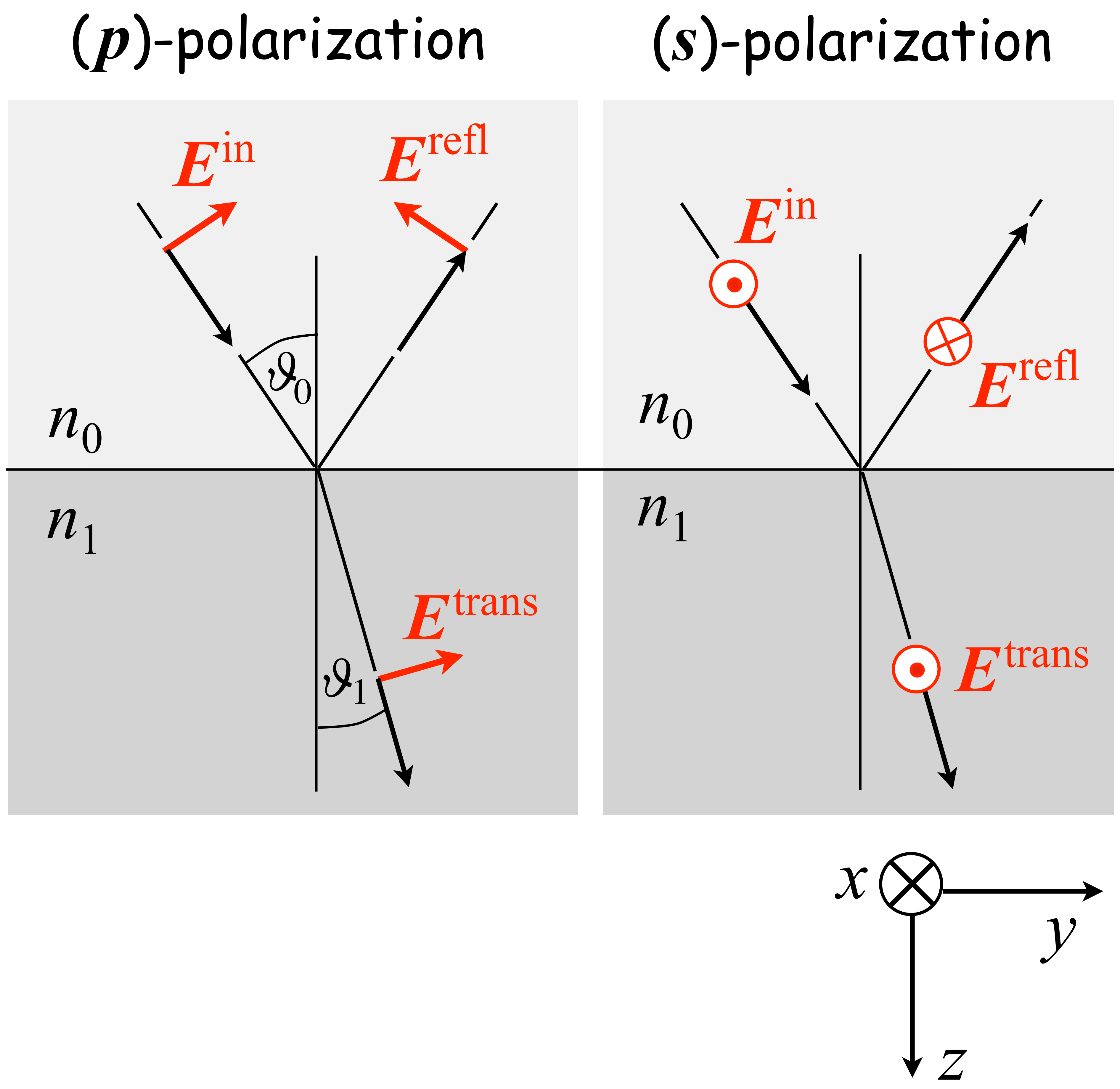

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- Complex Kerr rotation = ratio of off-diagonal to diagonal elements of reflection matrix:

$$\boxed{\theta_\mathrm{K}^\mathrm{s} \equiv \frac{r_\mathrm{ps}}{r_\mathrm{ss}}} \quad \boxed{\theta_\mathrm{K}^\mathrm{p} \equiv \frac{r_\mathrm{sp}}{r_\mathrm{pp}}}$$

Kerr coefficient (depends on $\boldsymbol{m}$)

Coefficient of normally reflected light (does not depend on $\boldsymbol{m}$)

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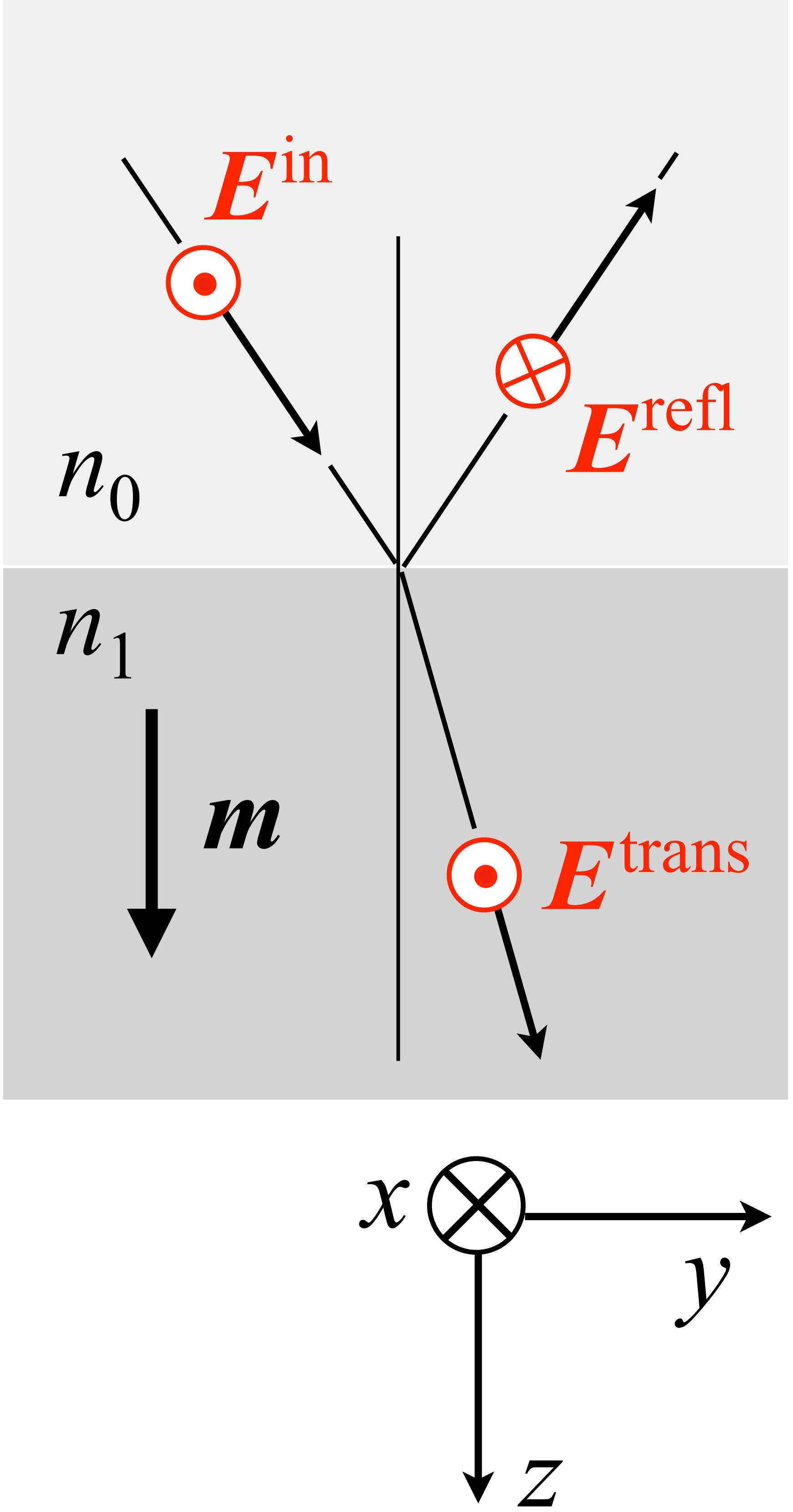
(*s*)-polarization



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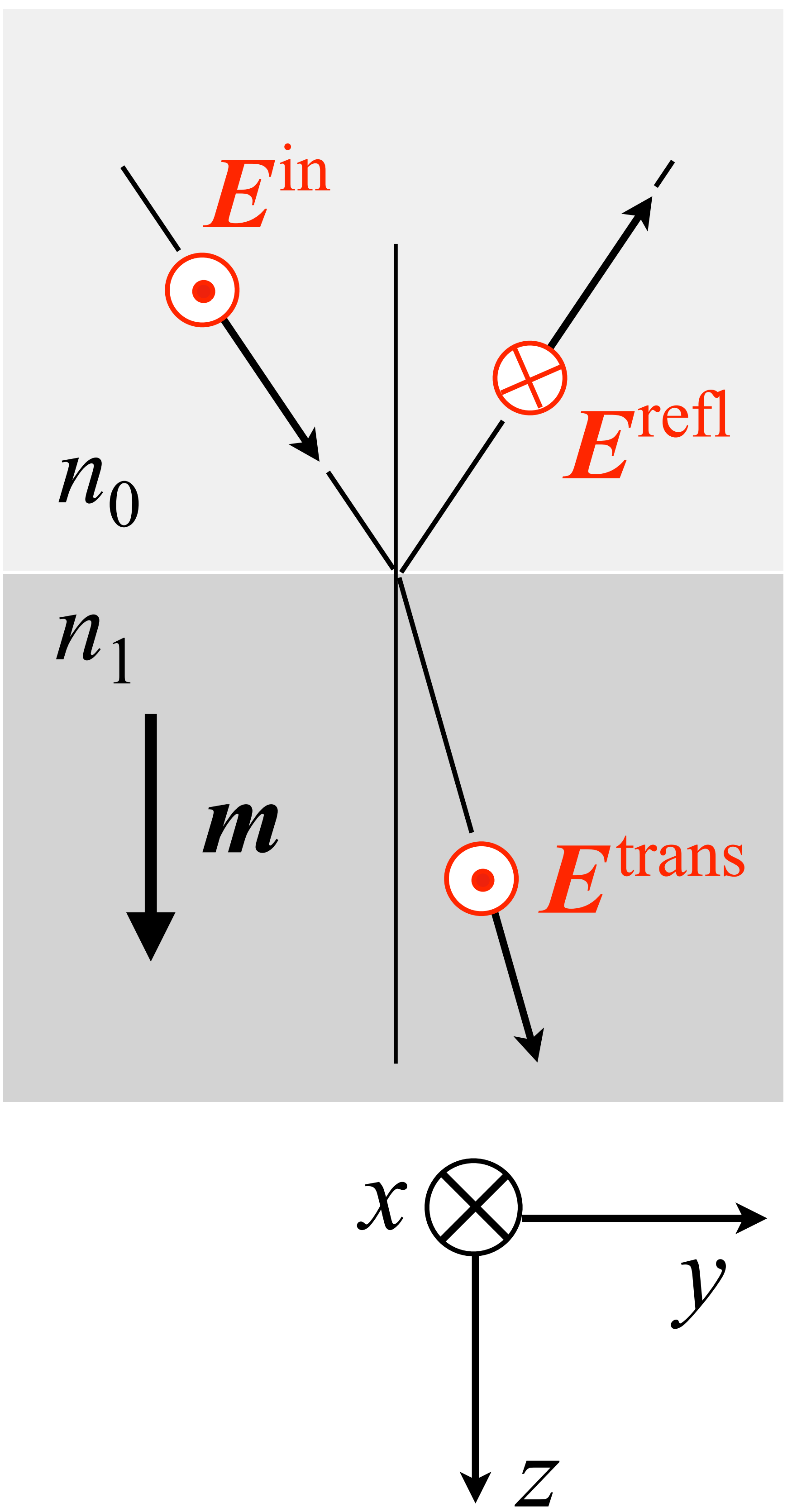

- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\mathrm{z}} = 1$, $m_{\mathrm{x}} = m_{\mathrm{y}} = 0$) $\Longrightarrow$ Polar Kerr effect

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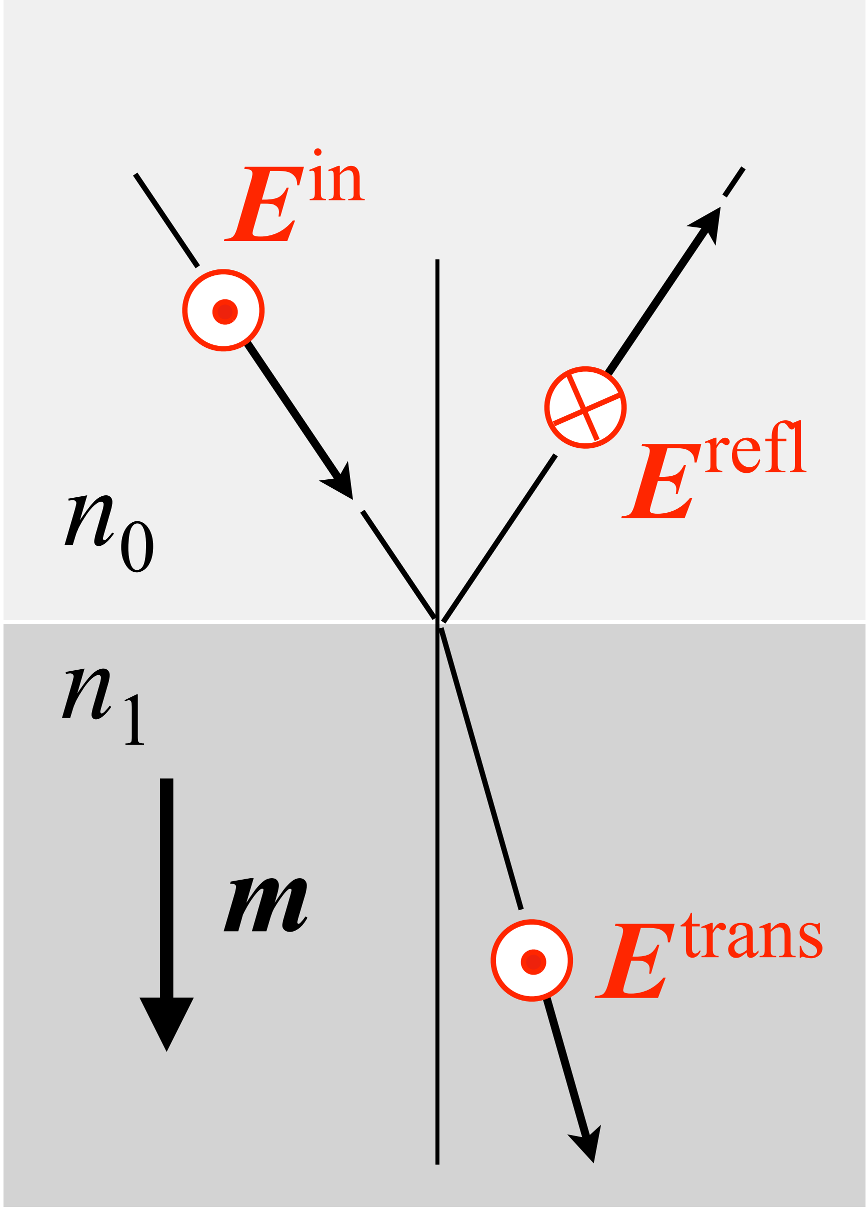
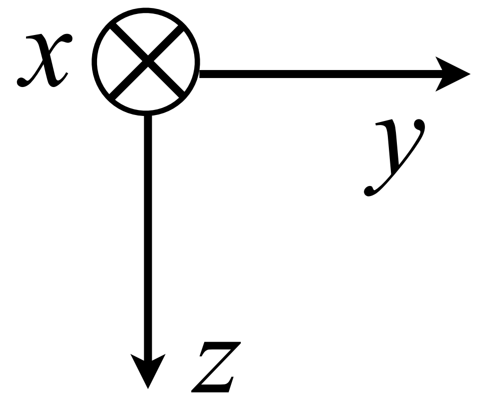
(*s*)-polarization





- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_z = 1$, $m_x = m_y = 0$) $\Longrightarrow$ **Polar Kerr effect**

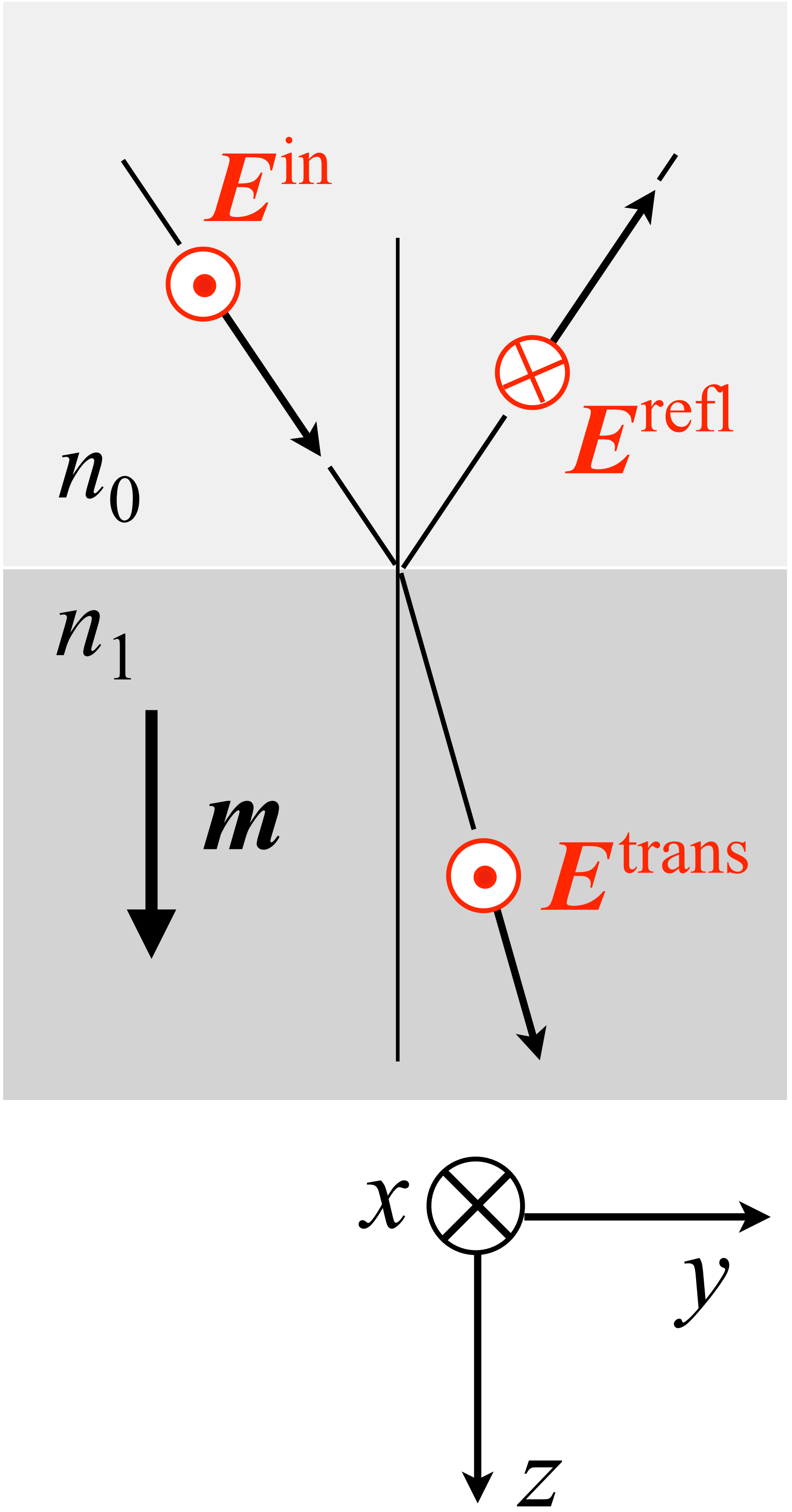
$$(\theta_K^s)^{pol} = \left(\frac{r_{ps}}{r_{ss}}\right)^{pol} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_V}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

$$r_{\mathrm{ss}} = \frac{n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1} ,$$

$$r_{\mathrm{ps}} = -\frac{\mathrm{i}\, n_0 n_1 \cos\vartheta_0 (m_{\mathrm{z}} \cos\vartheta_1 - m_{\mathrm{y}} \sin\vartheta_1) Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1) \cos\vartheta_1}$$

$$\boxed{\theta_{\mathrm{K}}^{\mathrm{s}} \equiv \frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}}$$

(*s*)-polarization



- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\mathrm{z}} = 1$, $m_{\mathrm{x}} = m_{\mathrm{y}} = 0$) $\Longrightarrow$ Polar Kerr effect

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \left(\frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}\right)^{\mathrm{pol}} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

$\Downarrow$ Fresnel equations

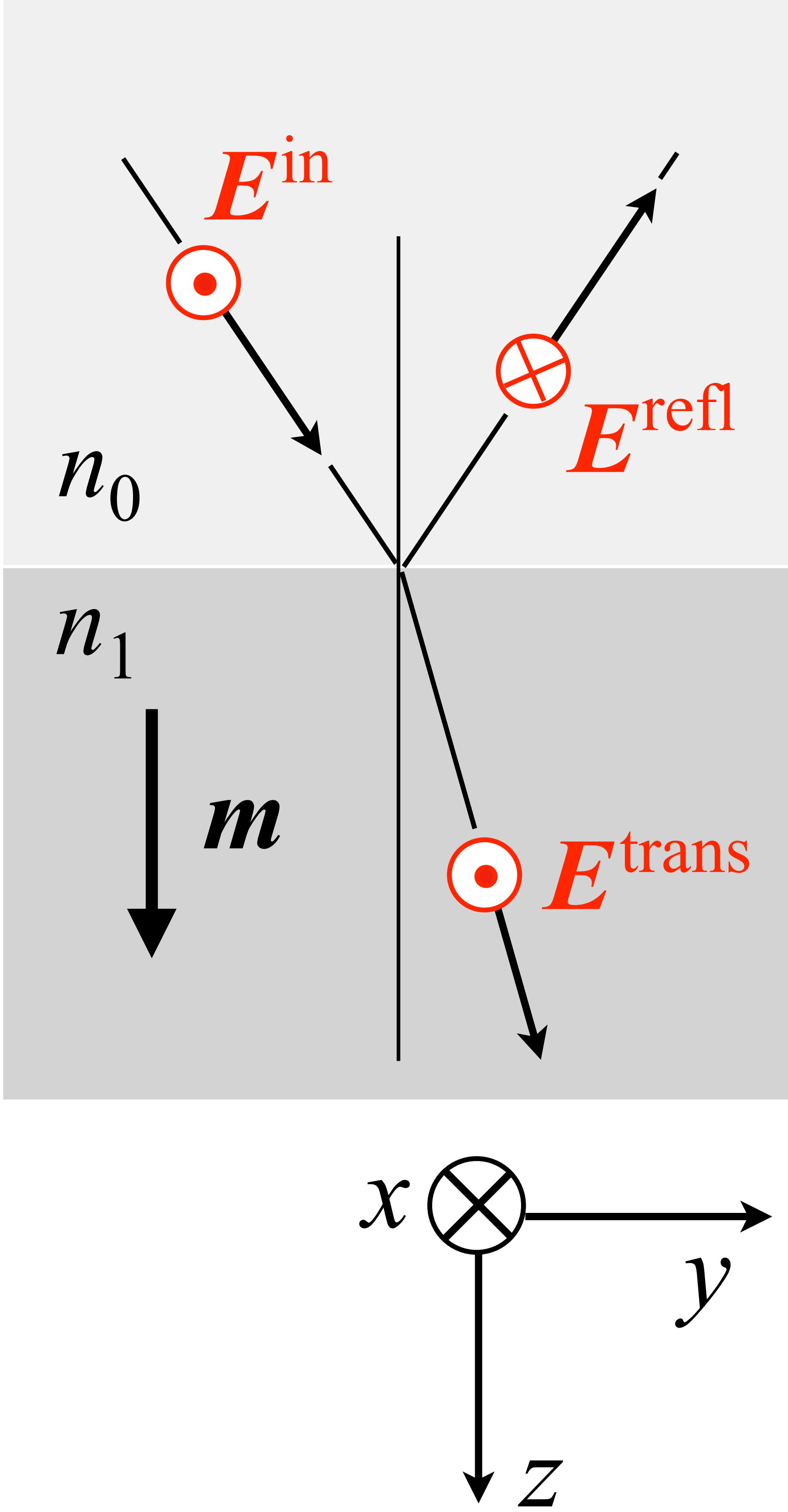
$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \frac{-\mathrm{i}\, n_1 Q_{\mathrm{V}} t_{\mathrm{ss}}^{01} t_{\mathrm{pp}}^{01}}{4\, n_0 \cos\vartheta_0} \frac{1}{r_{\mathrm{ss}}^{01}}$$

$$r_{\rm ss} = \frac{n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1}\,,$$

$$r_{\rm ps} = -\frac{\mathrm{i}\, n_0 n_1 \cos\vartheta_0 (m_{\rm z} \cos\vartheta_1 - m_{\rm y} \sin\vartheta_1) Q_{\rm V}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1) \cos\vartheta_1}$$

$$\boxed{\theta_{\rm K}^{\rm s} \equiv \frac{r_{\rm ps}}{r_{\rm ss}}}$$

(*s*)-polarization



- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\rm z} = 1$, $m_{\rm x} = m_{\rm y} = 0$) $\Longrightarrow$ Polar Kerr effect

$$(\theta_{\rm K}^{\rm s})^{\rm pol} = \left(\frac{r_{\rm ps}}{r_{\rm ss}}\right)^{\rm pol} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_{\rm V}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

$\Downarrow$ Fresnel equations

$$(\theta_{\rm K}^{\rm s})^{\rm pol} = \boxed{\frac{-\mathrm{i}\, n_1 Q_{\rm V} t_{\rm ss}^{01} t_{\rm pp}^{01}}{4\, n_0 \cos\vartheta_0}} \frac{1}{r_{\rm ss}^{01}}$$

Kerr coefficient $K_{\rm ps}$

out

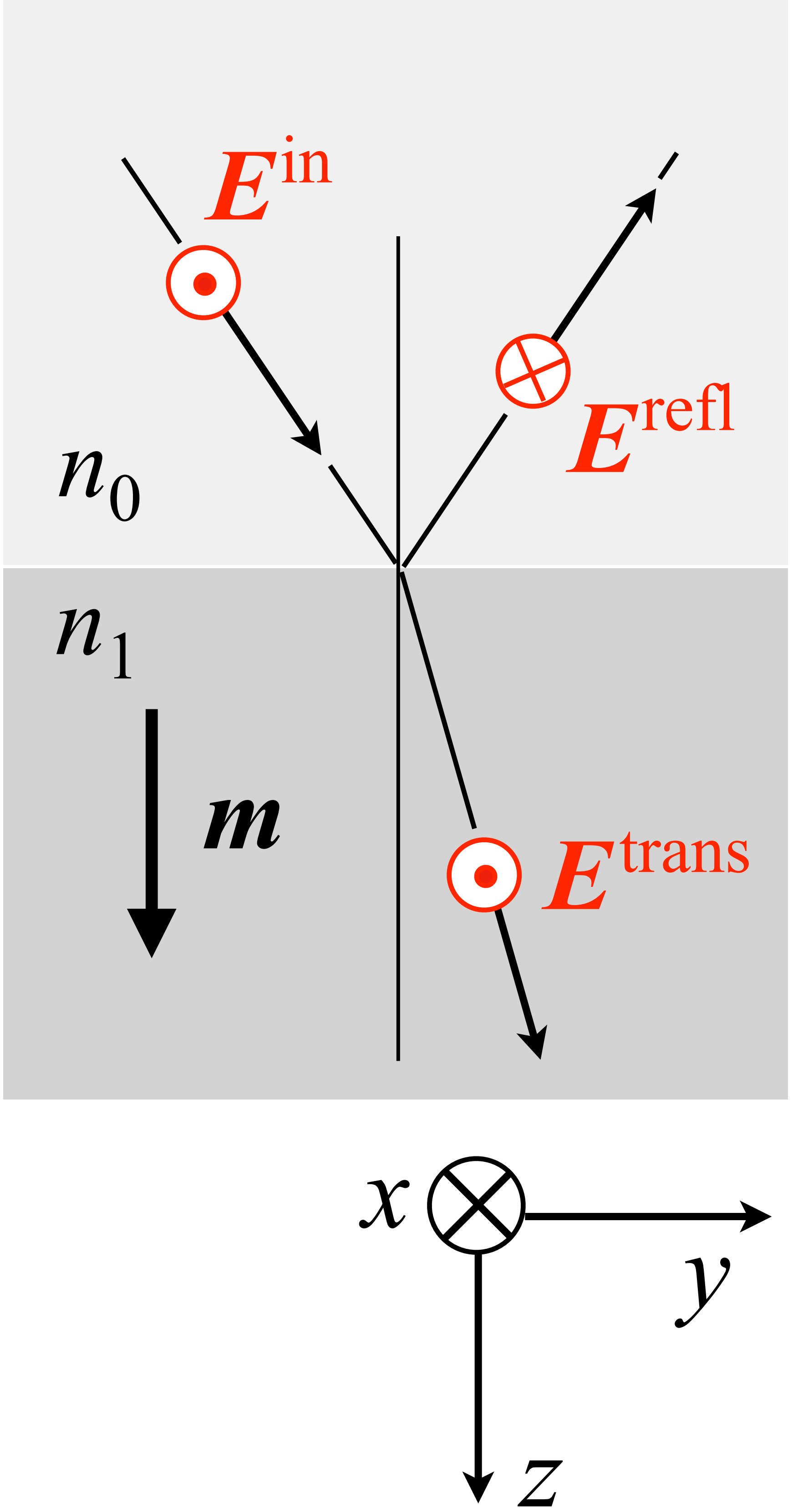
in

$$r_{\mathrm{ss}} = \frac{n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1}\,,$$

$$r_{\mathrm{ps}} = -\frac{\mathrm{i}\, n_0 n_1 \cos\vartheta_0 (m_{\mathrm{z}} \cos\vartheta_1 - m_{\mathrm{y}} \sin\vartheta_1) Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1)\cos\vartheta_1}$$

$$\boxed{\theta_{\mathrm{K}}^{\mathrm{s}} \equiv \frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}}$$

(*s*)-polarization



- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\mathrm{z}} = 1$, $m_{\mathrm{x}} = m_{\mathrm{y}} = 0$) $\Longrightarrow$ Polar Kerr effect

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \left(\frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}\right)^{\mathrm{pol}} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

$\Downarrow$ Fresnel equations

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \boxed{\frac{-\mathrm{i}\, n_1 Q_{\mathrm{V}} t_{\mathrm{ss}}^{01} t_{\mathrm{pp}}^{01}}{4\, n_0 \cos\vartheta_0}}\,\frac{1}{r_{\mathrm{ss}}^{01}}$$

Kerr coefficient K_{ps}

out

in

Fresnel equations

$$t_{\mathrm{pp}}^{ij} = \left(\frac{E^{\mathrm{trans}}}{E^{\mathrm{in}}}\right)_{\mathrm{p}} = \frac{2n_0 \cos\vartheta_0}{n_0 \cos\vartheta_1 + n_1 \cos\vartheta_0}$$

$$t_{\mathrm{ss}}^{ij} = \left(\frac{E^{\mathrm{trans}}}{E^{\mathrm{in}}}\right)_{\mathrm{s}} = \frac{2n_0 \cos\vartheta_0}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1}$$

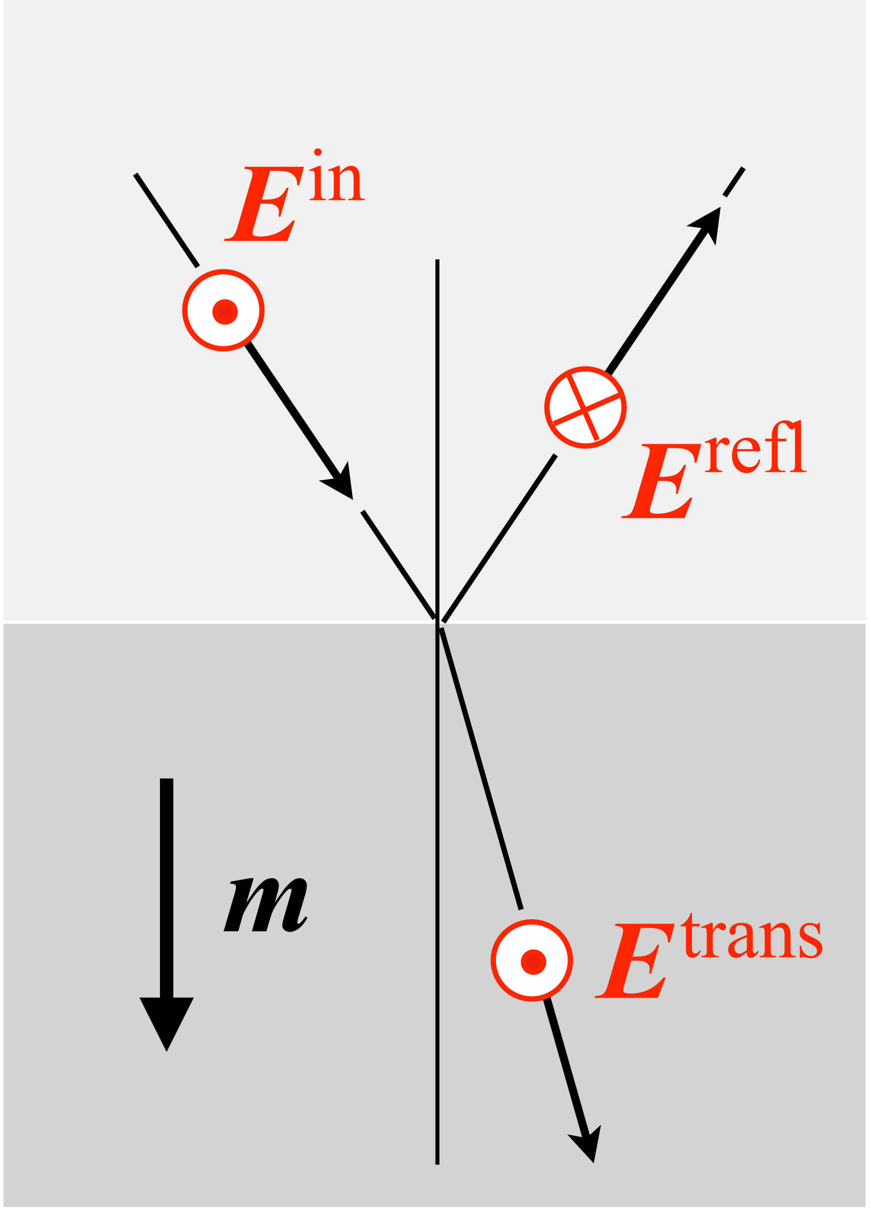
$\Longrightarrow$ Polar Kerr effect is strongest at perpendicular incidence

$$r_{\mathrm{ss}} = \frac{n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1}\,,$$

$$r_{\mathrm{ps}} = -\frac{\mathrm{i}\, n_0 n_1 \cos\vartheta_0 (m_{\mathrm{z}} \cos\vartheta_1 - m_{\mathrm{y}} \sin\vartheta_1) Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1)\cos\vartheta_1}$$

$$\boxed{\theta_{\mathrm{K}}^{\mathrm{s}} \equiv \frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}}$$

(s)-polarization



- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\mathrm{z}} = 1$, $m_{\mathrm{x}} = m_{\mathrm{y}} = 0$)

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \left(\frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}\right)^{\mathrm{pol}} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

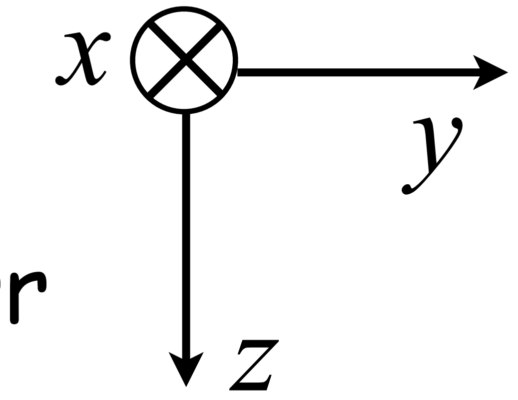
$\Downarrow$ Fresnel equations

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \boxed{\frac{-\mathrm{i}\, n_1 Q_{\mathrm{V}} t_{\mathrm{ss}}^{01} t_{\mathrm{pp}}^{01}}{4\, n_0 \cos\vartheta_0}}\, \frac{1}{r_{\mathrm{ss}}^{01}}$$

Kerr coefficient K_{ps}

out

in



Interpretation:

- Incident light is s-polarized, resulting in p-polarized Kerr amplitude
- Portion of light, which penetrates into metal (t_{ss}^{01}), generates Kerr amplitude
- Size of Kerr amplitude depends on Q_{V} and incidence angle ϑ_0
- p-polarized Kerr amplitude has to leave metal, described by transmission coefficient (t_{pp}^{01})

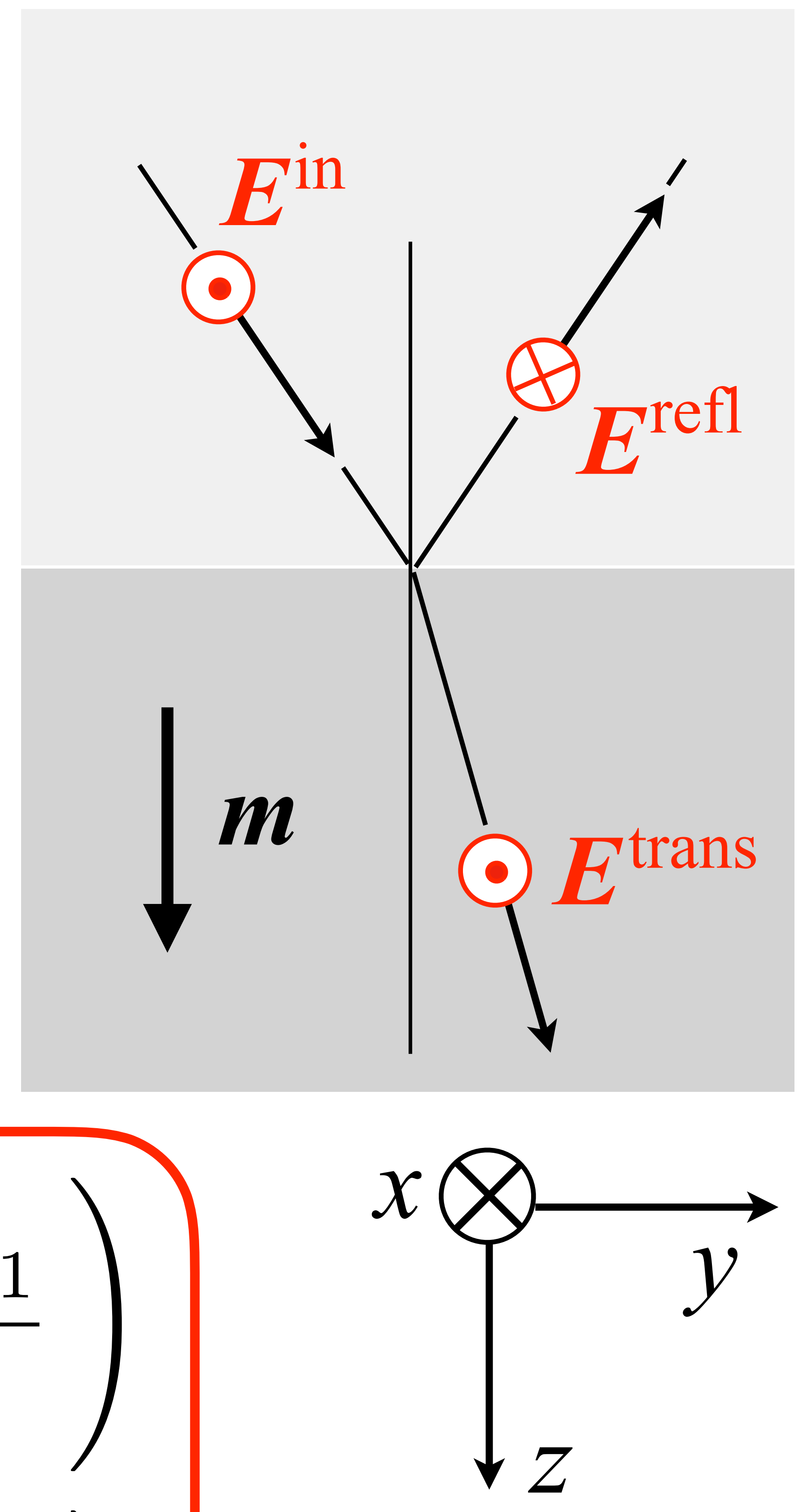
2. Magneto-Optical Effects - 2.4 Kerr Effect

$$r_{\mathrm{ss}} = \frac{n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1}{n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1}\,,$$

$$r_{\mathrm{ps}} = -\frac{\mathrm{i}\, n_0 n_1 \cos\vartheta_0 (m_{\mathrm{z}} \cos\vartheta_1 - m_{\mathrm{y}} \sin\vartheta_1) Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 + n_1 \cos\vartheta_1) \cos\vartheta_1}$$

$$\boxed{\theta_{\mathrm{K}}^{\mathrm{s}} \equiv \frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}}$$

(*s*)-polarization



- Assumption: s-polarized, oblique incidence, perpendicular magnetization ($m_{\mathrm{z}} = 1$, $m_{\mathrm{x}} = m_{\mathrm{y}} = 0$)

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \left(\frac{r_{\mathrm{ps}}}{r_{\mathrm{ss}}}\right)^{\mathrm{pol}} = \frac{-\mathrm{i}\, n_0 n_1 \cos\vartheta_0 Q_{\mathrm{V}}}{(n_1 \cos\vartheta_0 + n_0 \cos\vartheta_1)(n_0 \cos\vartheta_0 - n_1 \cos\vartheta_1)}$$

$\Downarrow$ Fresnel equations

$$(\theta_{\mathrm{K}}^{\mathrm{s}})^{\mathrm{pol}} = \boxed{\frac{-\mathrm{i}\, n_1 Q_{\mathrm{V}} t_{\mathrm{ss}}^{01} t_{\mathrm{pp}}^{01}}{4\, n_0 \cos\vartheta_0}} \frac{1}{r_{\mathrm{ss}}^{01}}$$

Kerr coefficient K_{ps}

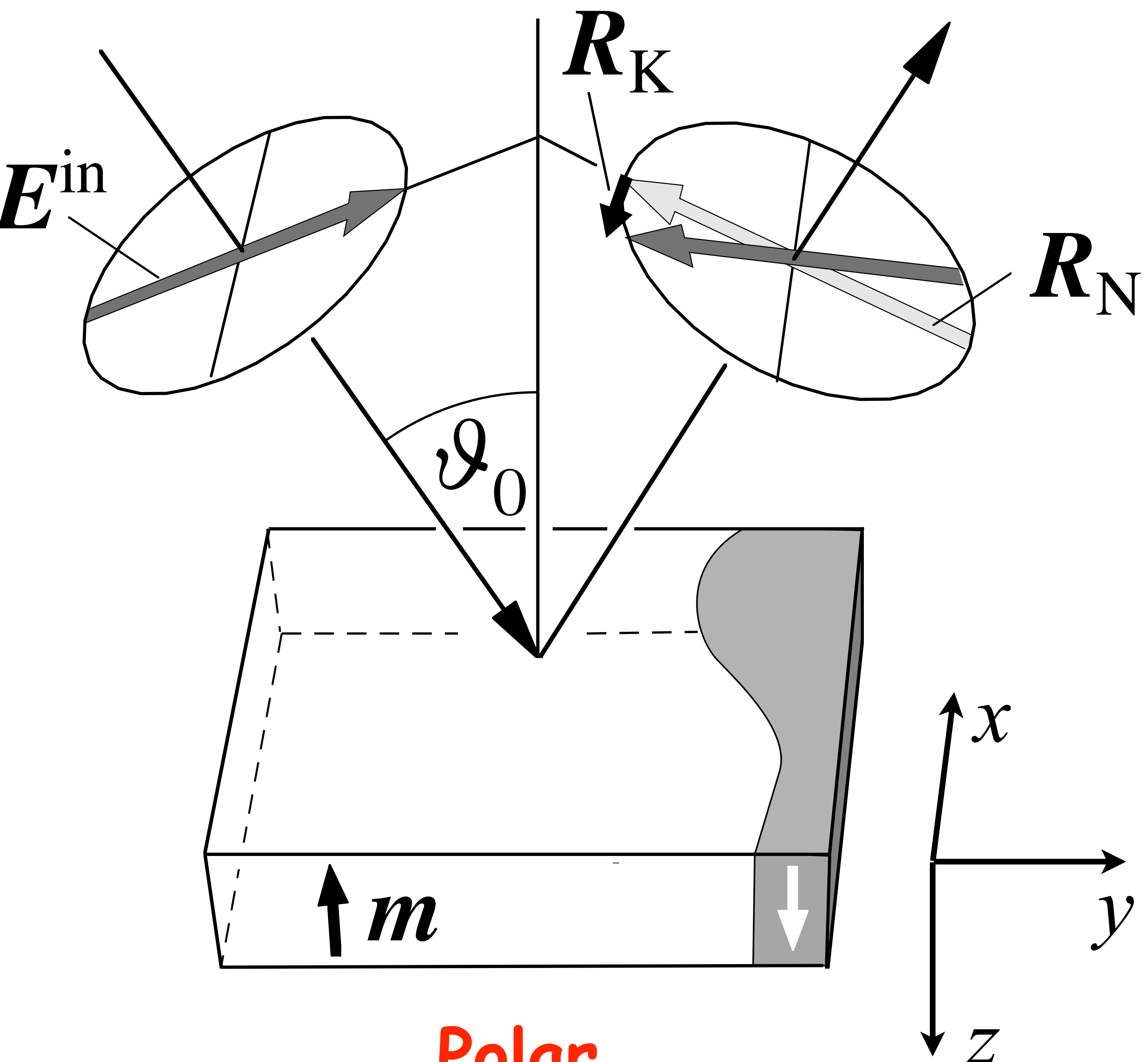
out

in

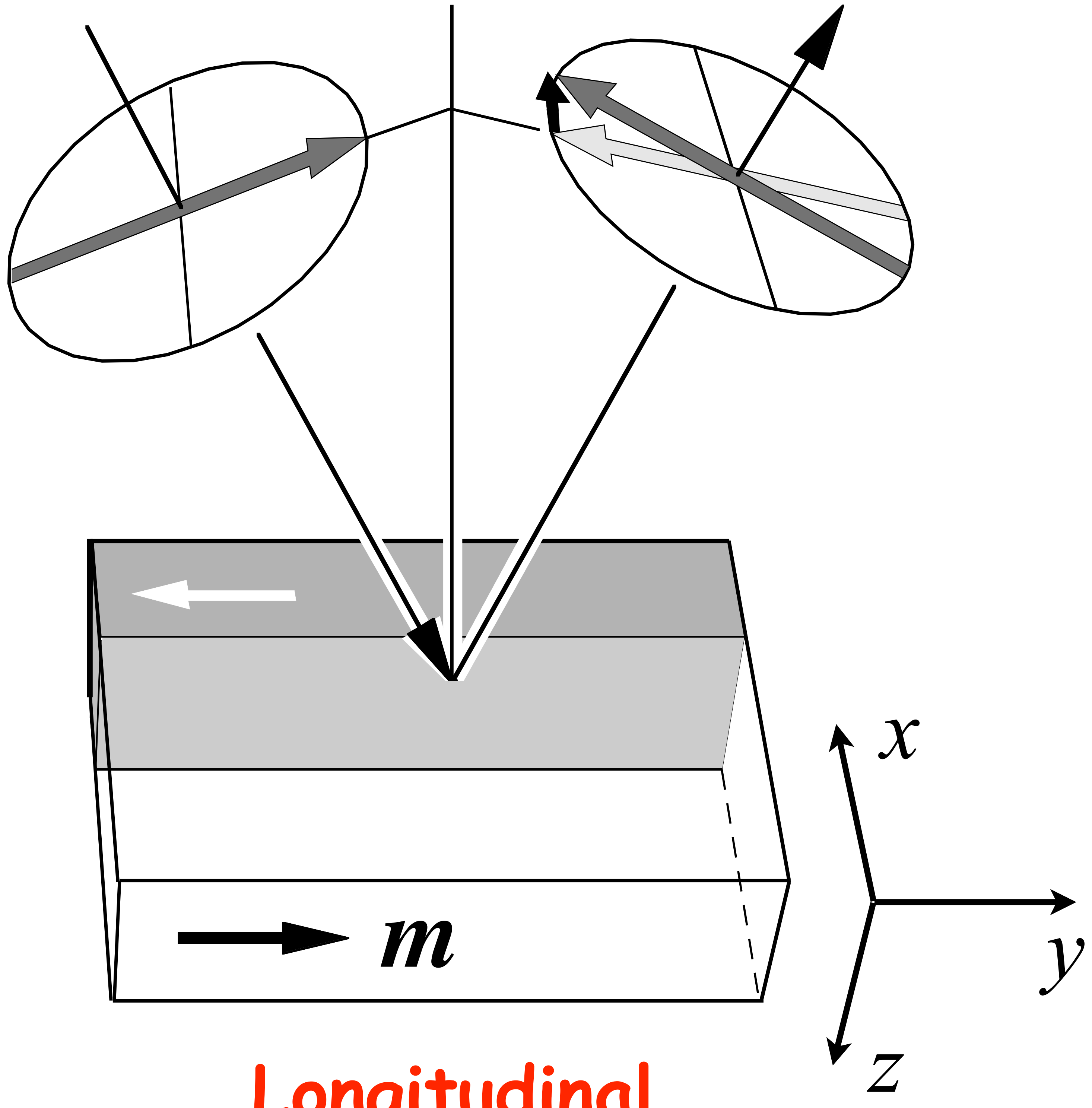
General:

$$\boxed{\begin{aligned}
\boldsymbol{K}_{\mathrm{ps}} &= \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{ss}}^{01}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{-n_1}{n_0}\right)\\
\boldsymbol{K}_{\mathrm{sp}} &= \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{ss}}^{01}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{+n_1}{n_0}\right)\\
\boldsymbol{K}_{\mathrm{pp}} &= \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{pp}}^{01}}{4\cos\vartheta_0} \left(-2\sin\vartheta_0\,,\ 0\,,\ 0\right)
\end{aligned}}$$

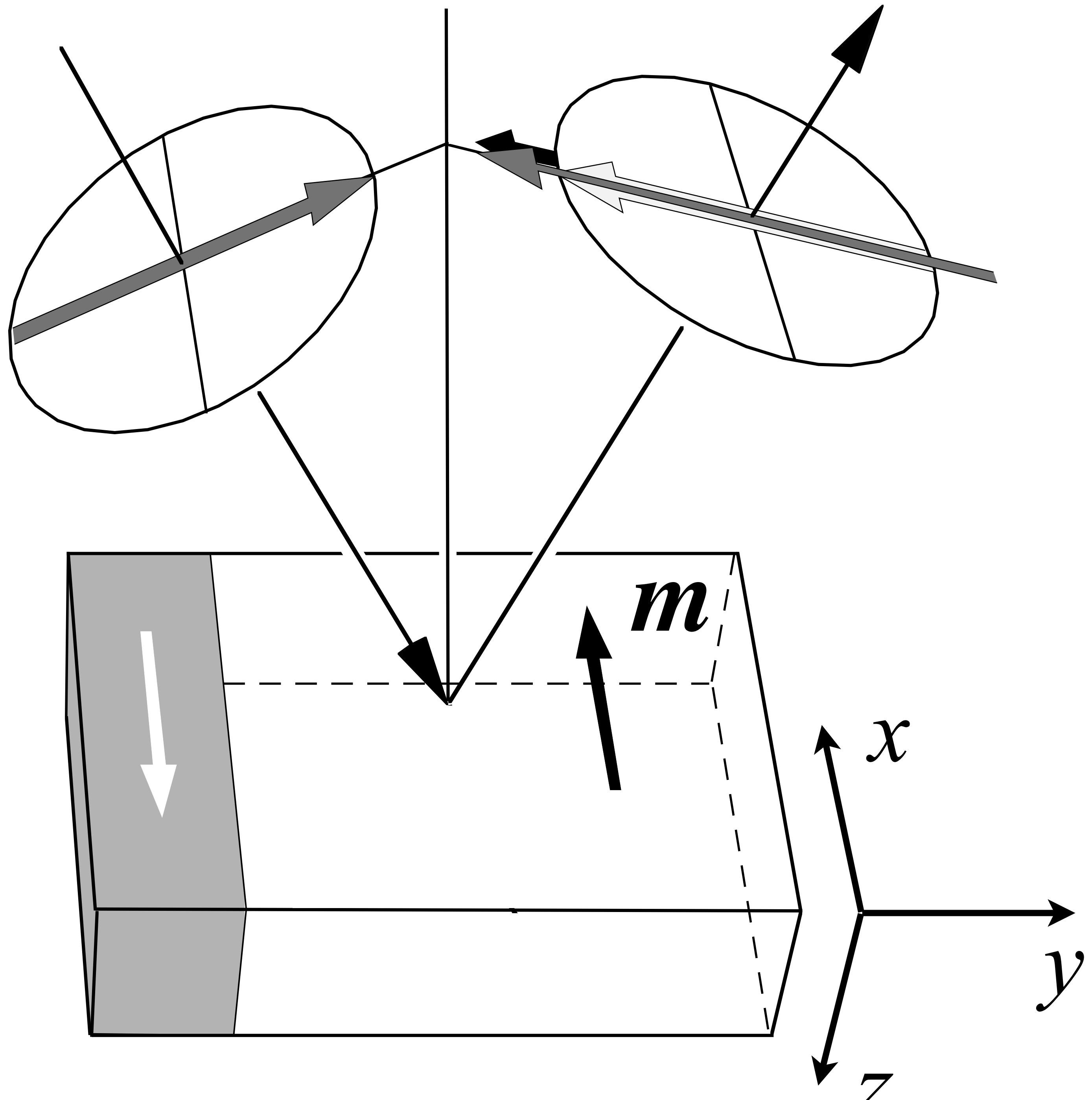
Three basic Kerr configurations



Polar



Longitudinal



Transverse

$$K_{sp}^{pol} = \frac{iQ_V n_1}{4 n_0 \cos\vartheta_0} t_{ss}^{01} t_{pp}^{01}$$

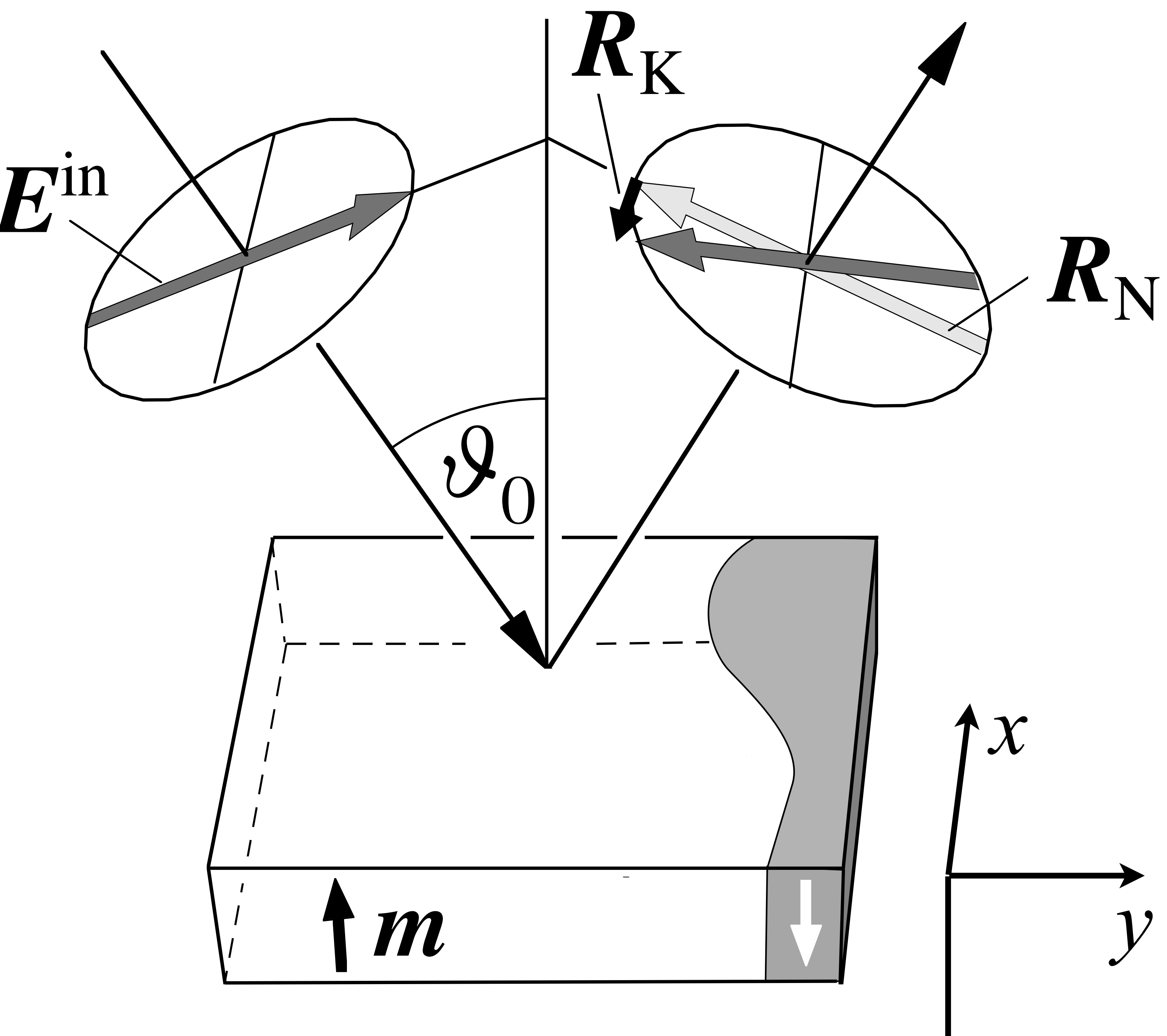
$$K_{ps}^{pol} = -K_{sp}^{pol}$$

$$K_{sp}^{lon} = \frac{iQ_V \sin\vartheta_0}{4\cos\vartheta_0 \cos\vartheta_1} t_{ss}^{01} t_{pp}^{01}$$

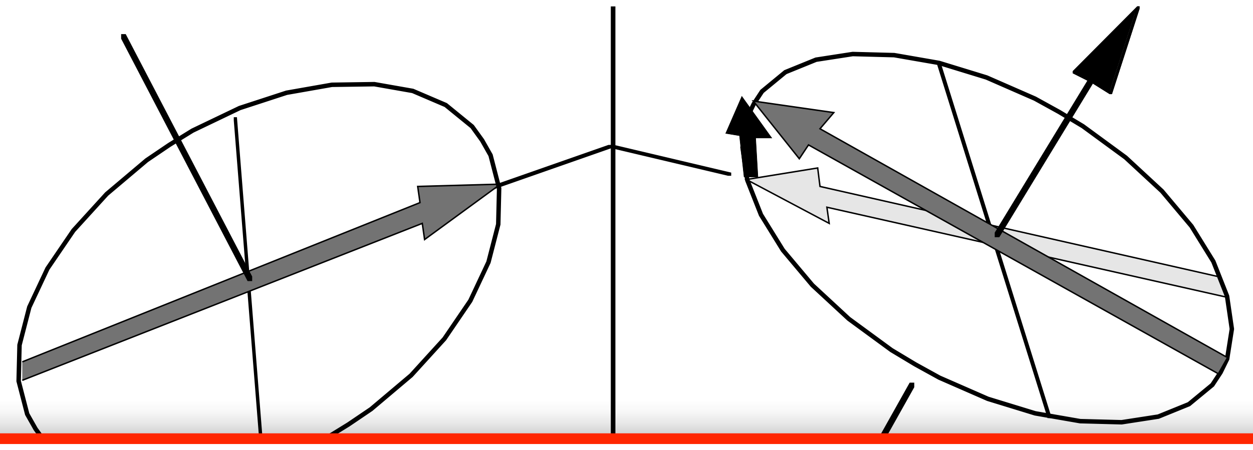
$$K_{ps}^{lon} = K_{sp}^{lon}$$

$$K_{pp}^{tra} = -\frac{iQ_V \sin\vartheta_0}{2\cos\vartheta_0} t_{pp}^{01} t_{pp}^{01}$$

Three basic Kerr configurations



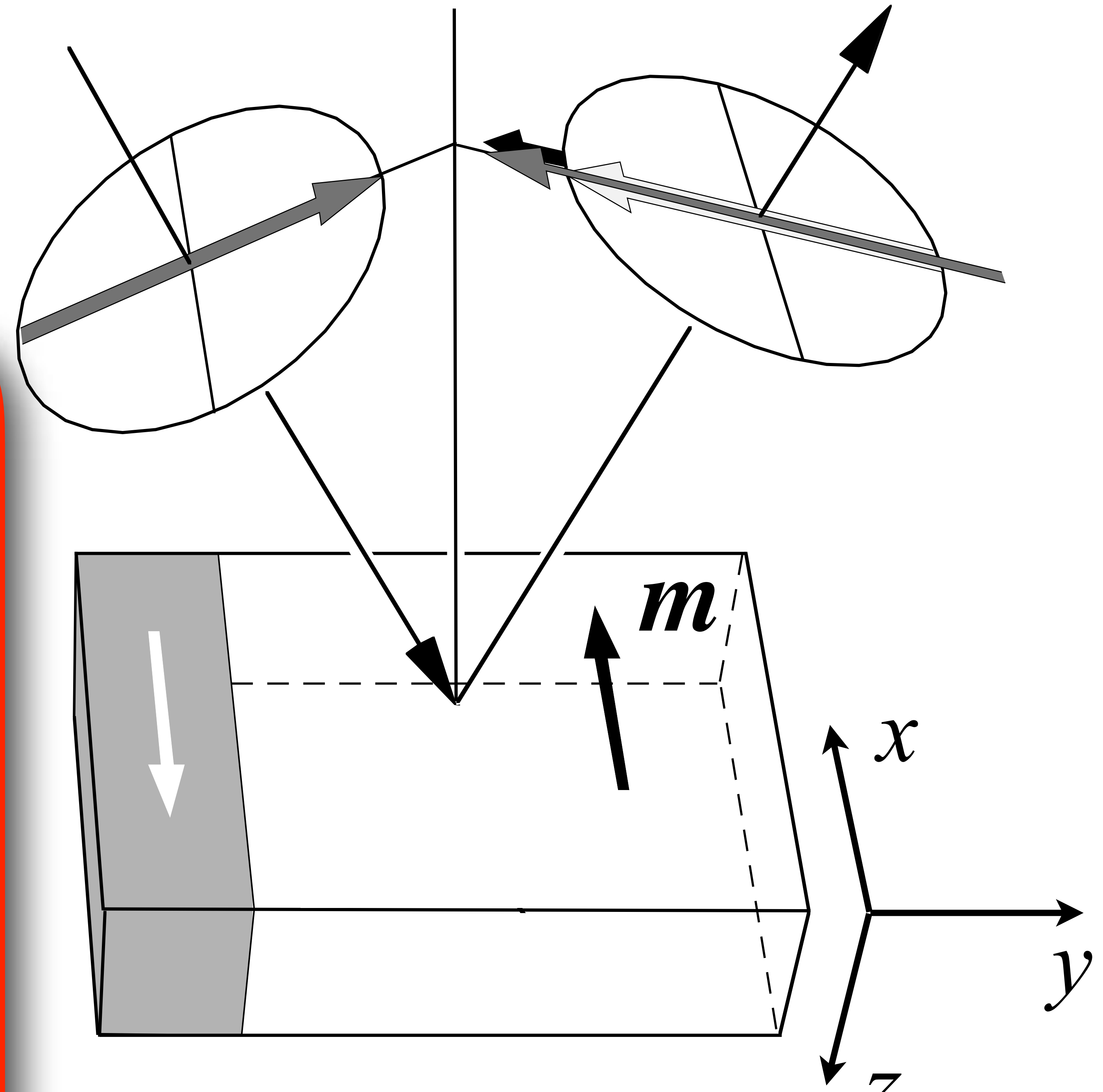
Polar



$$\boldsymbol{K}_{\mathrm{ps}} = \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{ss}}^{01}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{-n_1}{n_0}\right)$$

$$\boldsymbol{K}_{\mathrm{sp}} = \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{ss}}^{01}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{+n_1}{n_0}\right)$$

$$\boldsymbol{K}_{\mathrm{pp}} = \frac{\mathrm{i}Q_{\mathrm{V}} t_{\mathrm{pp}}^{01} t_{\mathrm{pp}}^{01}}{4\cos\vartheta_0} \left(-2\sin\vartheta_0\,,\ 0\,,\ 0\right)$$



Transverse

$$K_{\mathrm{sp}}^{\mathrm{pol}} = \frac{\mathrm{i}Q_{\mathrm{V}} n_1}{4 n_0 \cos\vartheta_0}\, t_{\mathrm{ss}}^{01}\, t_{\mathrm{pp}}^{01}$$

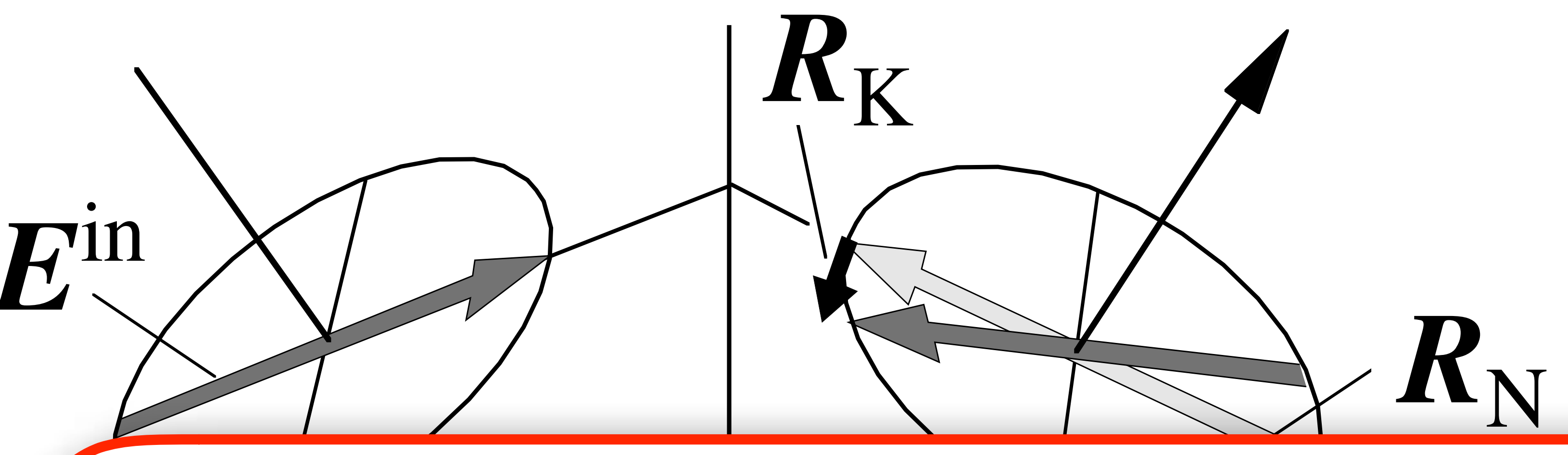
$$K_{\mathrm{ps}}^{\mathrm{pol}} = -K_{\mathrm{sp}}^{\mathrm{pol}}$$

$$K_{\mathrm{sp}}^{\mathrm{lon}} = \frac{\mathrm{i}Q_{\mathrm{V}} \sin\vartheta_0}{4\cos\vartheta_0\cos\vartheta_1}\, t_{\mathrm{ss}}^{01}\, t_{\mathrm{pp}}^{01}$$

$$K_{\mathrm{ps}}^{\mathrm{lon}} = K_{\mathrm{sp}}^{\mathrm{lon}}$$

$$K_{\mathrm{pp}}^{\mathrm{tra}} = -\frac{\mathrm{i}Q_{\mathrm{V}} \sin\vartheta_0}{2\cos\vartheta_0}\, t_{\mathrm{pp}}^{01}\, t_{\mathrm{pp}}^{01}$$

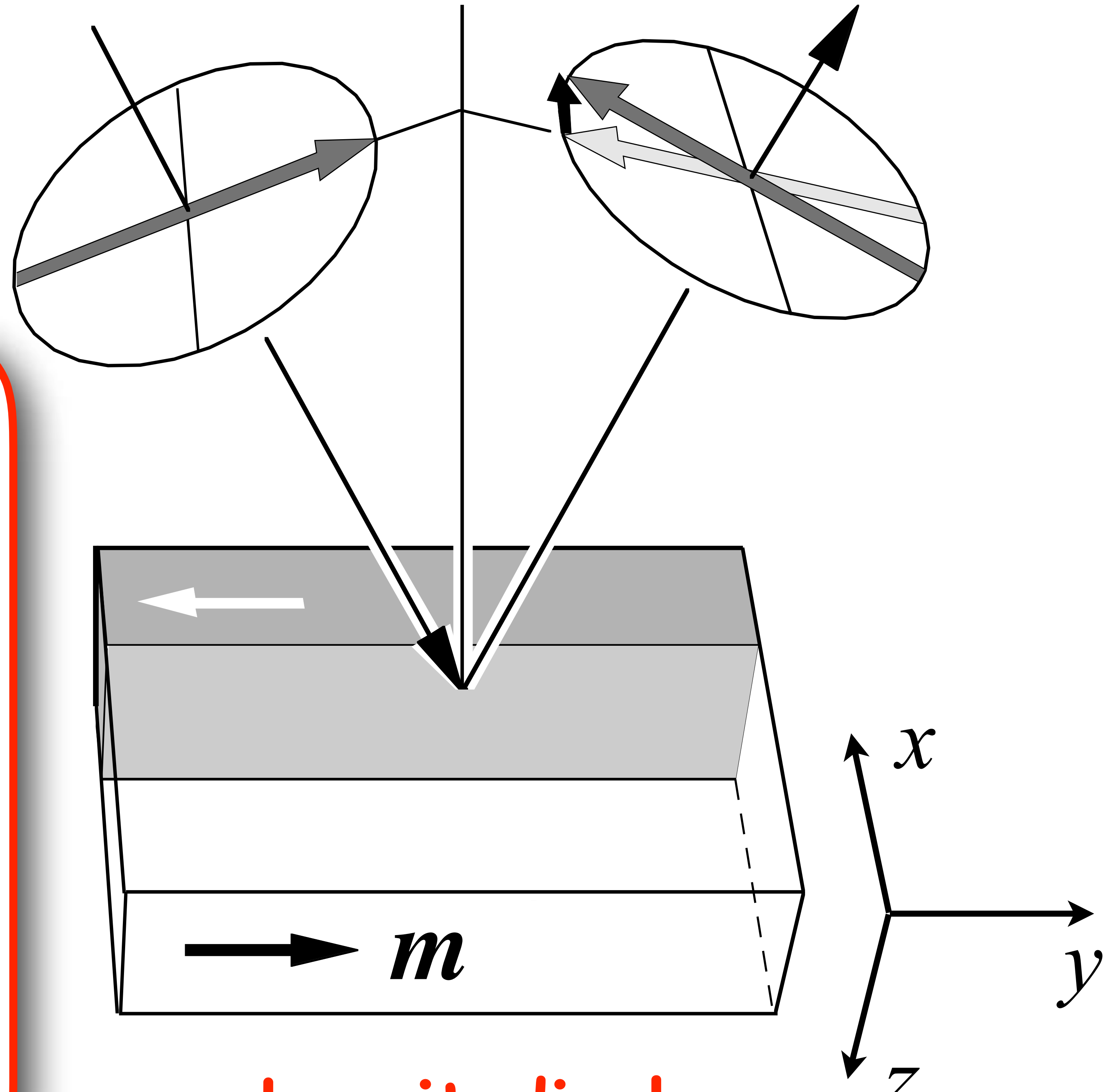
Three basic Kerr configurations



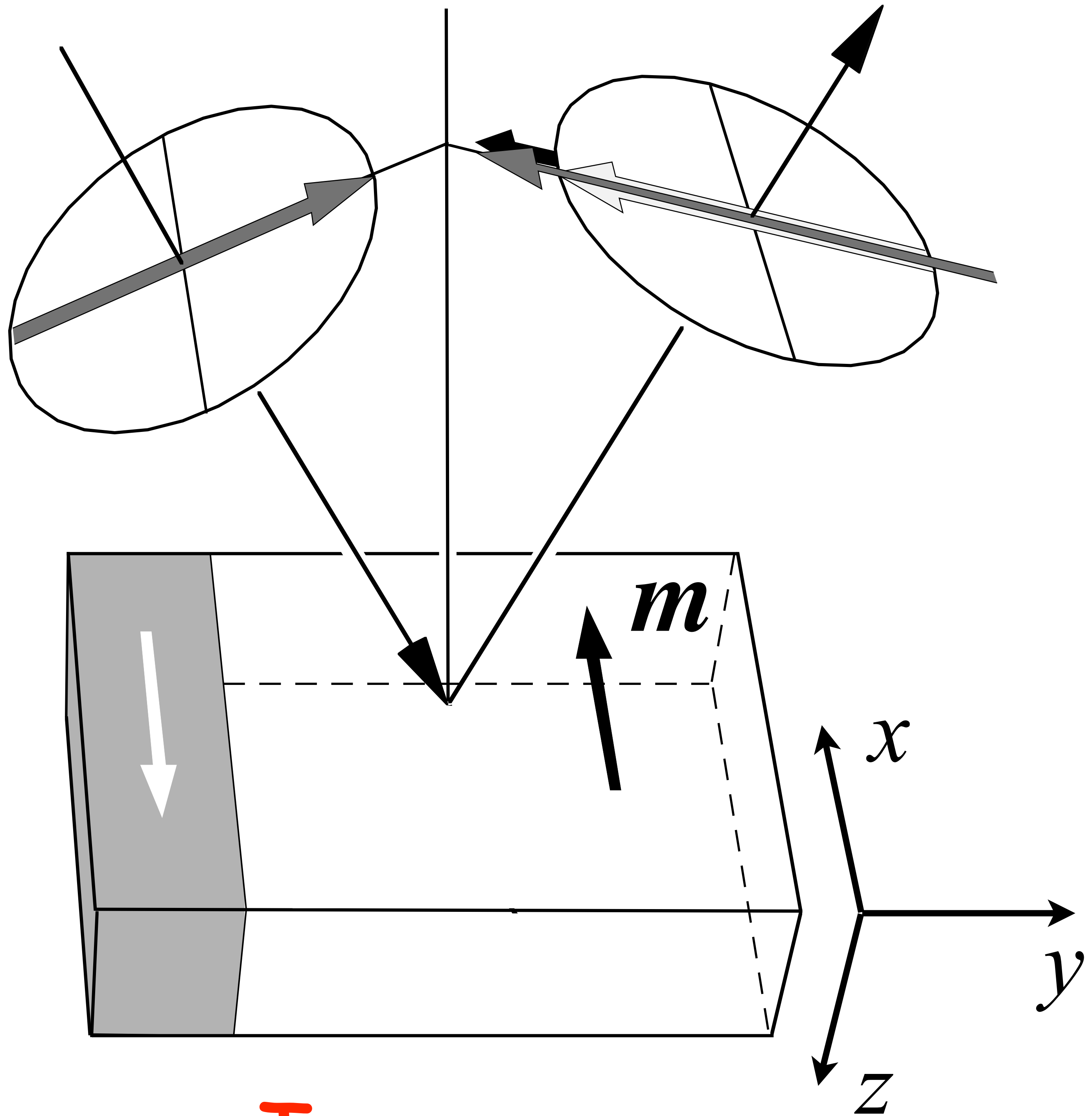
$$\boldsymbol{K}_{\mathrm{ps}} = \frac{\mathrm{i}Q_{\mathrm{V}} t^{01}_{\mathrm{pp}} t^{01}_{\mathrm{ss}}}{4\cos\vartheta_0}\left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{-n_1}{n_0}\right)$$

$$\boldsymbol{K}_{\mathrm{sp}} = \frac{\mathrm{i}Q_{\mathrm{V}} t^{01}_{\mathrm{pp}} t^{01}_{\mathrm{ss}}}{4\cos\vartheta_0}\left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{+n_1}{n_0}\right)$$

$$\boldsymbol{K}_{\mathrm{pp}} = \frac{\mathrm{i}Q_{\mathrm{V}} t^{01}_{\mathrm{pp}} t^{01}_{\mathrm{pp}}}{4\cos\vartheta_0}\left(-2\sin\vartheta_0\,,\ 0\,,\ 0\right)$$



Longitudinal



Transverse

$$K^{\mathrm{pol}}_{\mathrm{sp}} = \frac{\mathrm{i}Q_{\mathrm{V}} n_1}{4n_0\cos\vartheta_0}\, t^{01}_{\mathrm{ss}}\, t^{01}_{\mathrm{pp}}$$

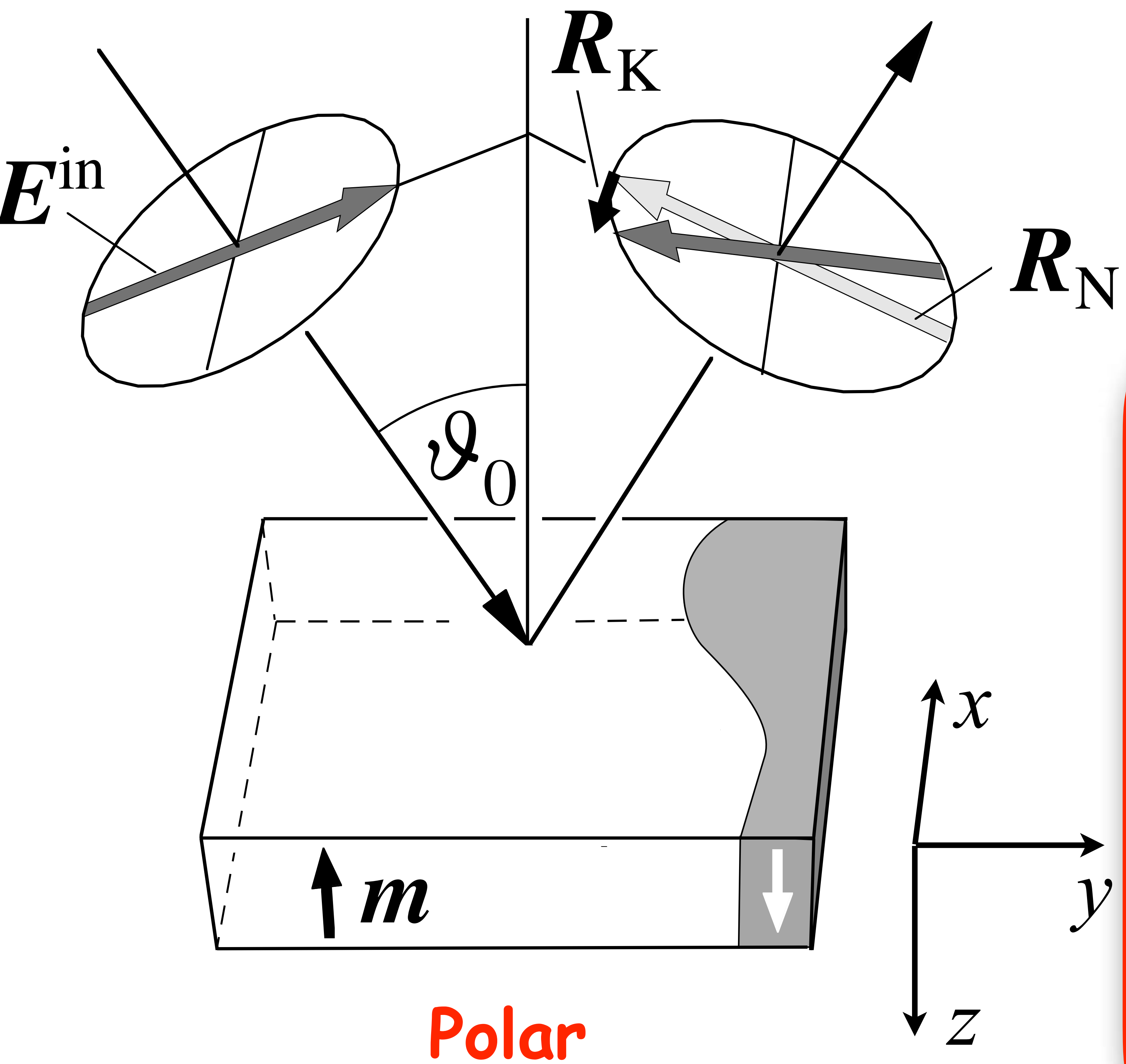
$$K^{\mathrm{pol}}_{\mathrm{ps}} = -K^{\mathrm{pol}}_{\mathrm{sp}}$$

$$K^{\mathrm{lon}}_{\mathrm{sp}} = \frac{\mathrm{i}Q_{\mathrm{V}}\sin\vartheta_0}{4\cos\vartheta_0\cos\vartheta_1}\, t^{01}_{\mathrm{ss}}\, t^{01}_{\mathrm{pp}}$$

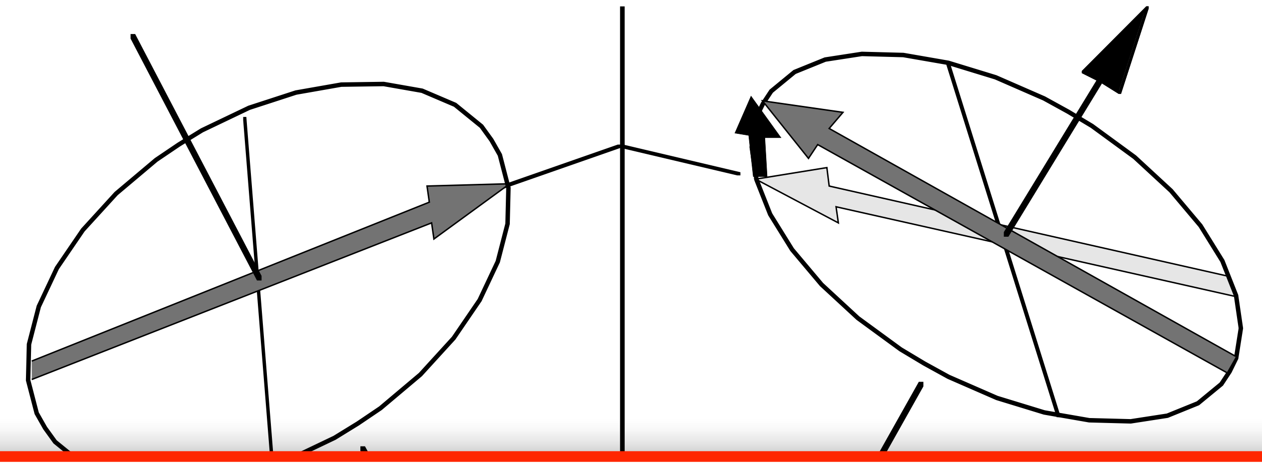
$$K^{\mathrm{lon}}_{\mathrm{ps}} = K^{\mathrm{lon}}_{\mathrm{sp}}$$

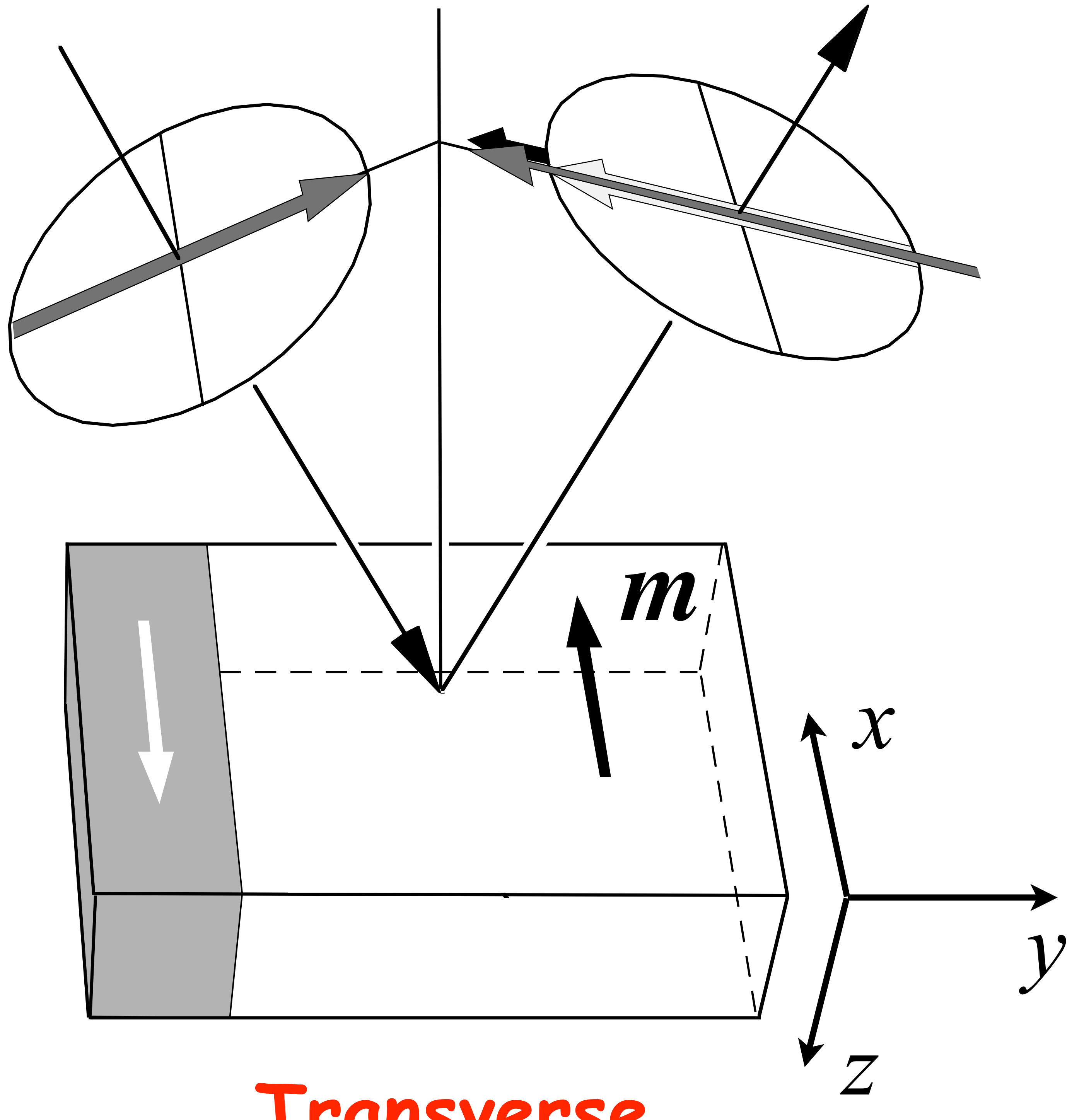
$$K^{\mathrm{tra}}_{\mathrm{pp}} = -\frac{\mathrm{i}Q_{\mathrm{V}}\sin\vartheta_0}{2\cos\vartheta_0}\, t^{01}_{\mathrm{pp}}\, t^{01}_{\mathrm{pp}}$$

Three basic Kerr configurations



Polar





Transverse

$$\boldsymbol{K}_\mathrm{ps} = \frac{\mathrm{i}Q_\mathrm{V} t^{01}_\mathrm{pp} t^{01}_\mathrm{ss}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{-n_1}{n_0}\right)$$

$$\boldsymbol{K}_\mathrm{sp} = \frac{\mathrm{i}Q_\mathrm{V} t^{01}_\mathrm{pp} t^{01}_\mathrm{ss}}{4\cos\vartheta_0} \left(0\,,\ \frac{\sin\vartheta_0}{\cos\vartheta_1}\,,\ \frac{+n_1}{n_0}\right)$$

$$\boldsymbol{K}_\mathrm{pp} = \frac{\mathrm{i}Q_\mathrm{V} t^{01}_\mathrm{pp} t^{01}_\mathrm{pp}}{4\cos\vartheta_0} \left(-2\sin\vartheta_0\,,\ 0\,,\ 0\right)$$

$$K^\mathrm{pol}_\mathrm{sp} = \frac{\mathrm{i}Q_\mathrm{V} n_1}{4 n_0 \cos\vartheta_0}\, t^{01}_\mathrm{ss}\, t^{01}_\mathrm{pp}$$

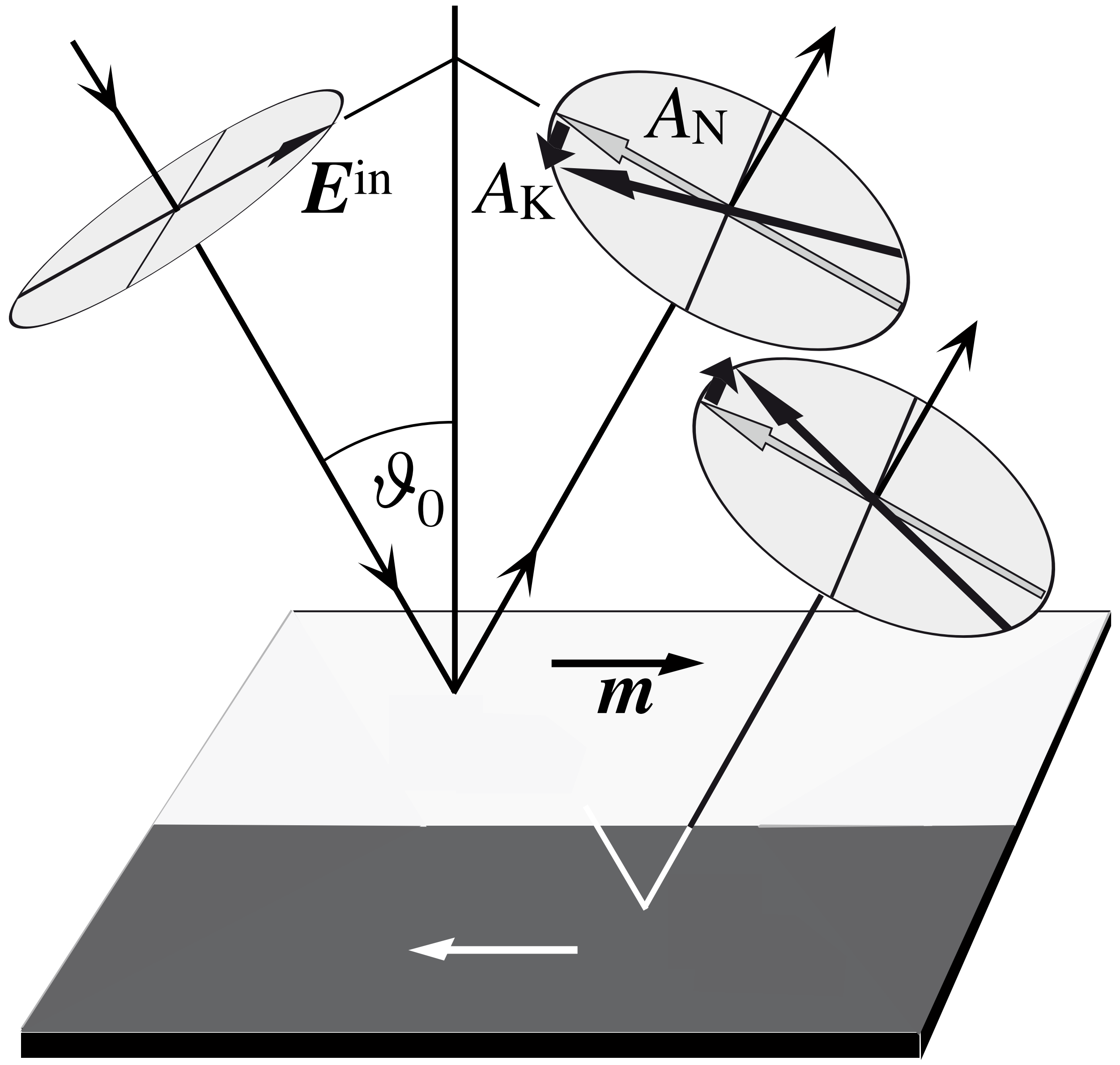
$$K^\mathrm{pol}_\mathrm{ps} = -K^\mathrm{pol}_\mathrm{sp}$$

$$K^\mathrm{lon}_\mathrm{sp} = \frac{\mathrm{i}Q_\mathrm{V} \sin\vartheta_0}{4\cos\vartheta_0\cos\vartheta_1}\, t^{01}_\mathrm{ss}\, t^{01}_\mathrm{pp}$$

$$K^\mathrm{lon}_\mathrm{ps} = K^\mathrm{lon}_\mathrm{sp}$$

$$K^\mathrm{tra}_\mathrm{pp} = -\frac{\mathrm{i}Q_\mathrm{V} \sin\vartheta_0}{2\cos\vartheta_0}\, t^{01}_\mathrm{pp}\, t^{01}_\mathrm{pp}$$

Kerr contrast



Generalized amplitudes:

$$A_{\text{N}} = R_{\text{N}}^{\text{p}} \sin\alpha_{\text{pol}} \cos\alpha_{\text{an}} - R_{\text{N}}^{\text{s}} \cos\alpha_{\text{pol}} \sin\alpha_{\text{an}} \,,$$

$$A_{\text{K}} = R_{\text{K}}^{\text{pol}} \cos\left(\alpha_{\text{an}} - \alpha_{\text{pol}}\right) m_{\text{pol}} - R_{\text{K}}^{\text{lon}} \cos\left(\alpha_{\text{an}} + \alpha_{\text{pol}}\right) m_{\text{lon}} + R_{\text{K}}^{\text{tra}} \sin\alpha_{\text{pol}} \cos\alpha_{\text{an}}\, m_{\text{tra}} \,.$$

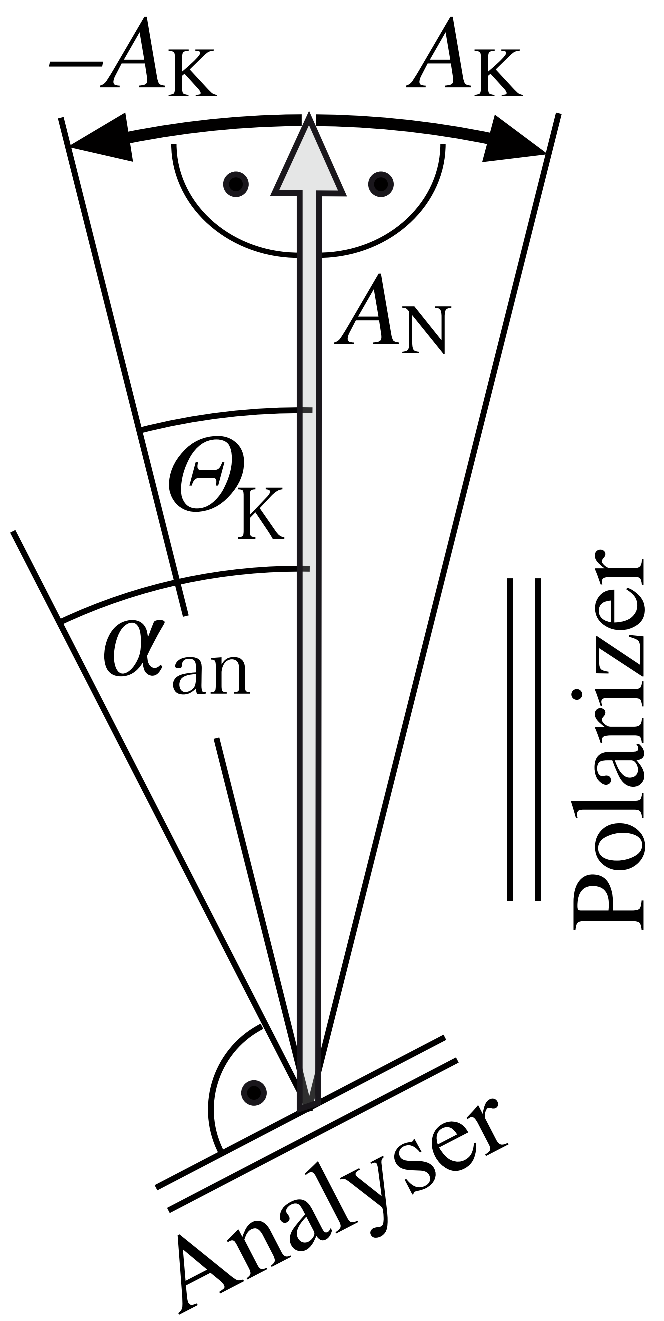
Domain Intensities:

$$I_1 = \left[A_{\text{N}} \sin(\alpha_{\text{an}}) - A_{\text{K}} \cos(\alpha_{\text{an}})\right]^2 + I_0$$

$$I_2 = \left[A_{\text{N}} \sin(\alpha_{\text{an}}) + A_{\text{K}} \cos(\alpha_{\text{an}})\right]^2 + I_0$$

$\Downarrow$ $C = (I_2 - I_1)/(I_2 + I_1)$, optimization

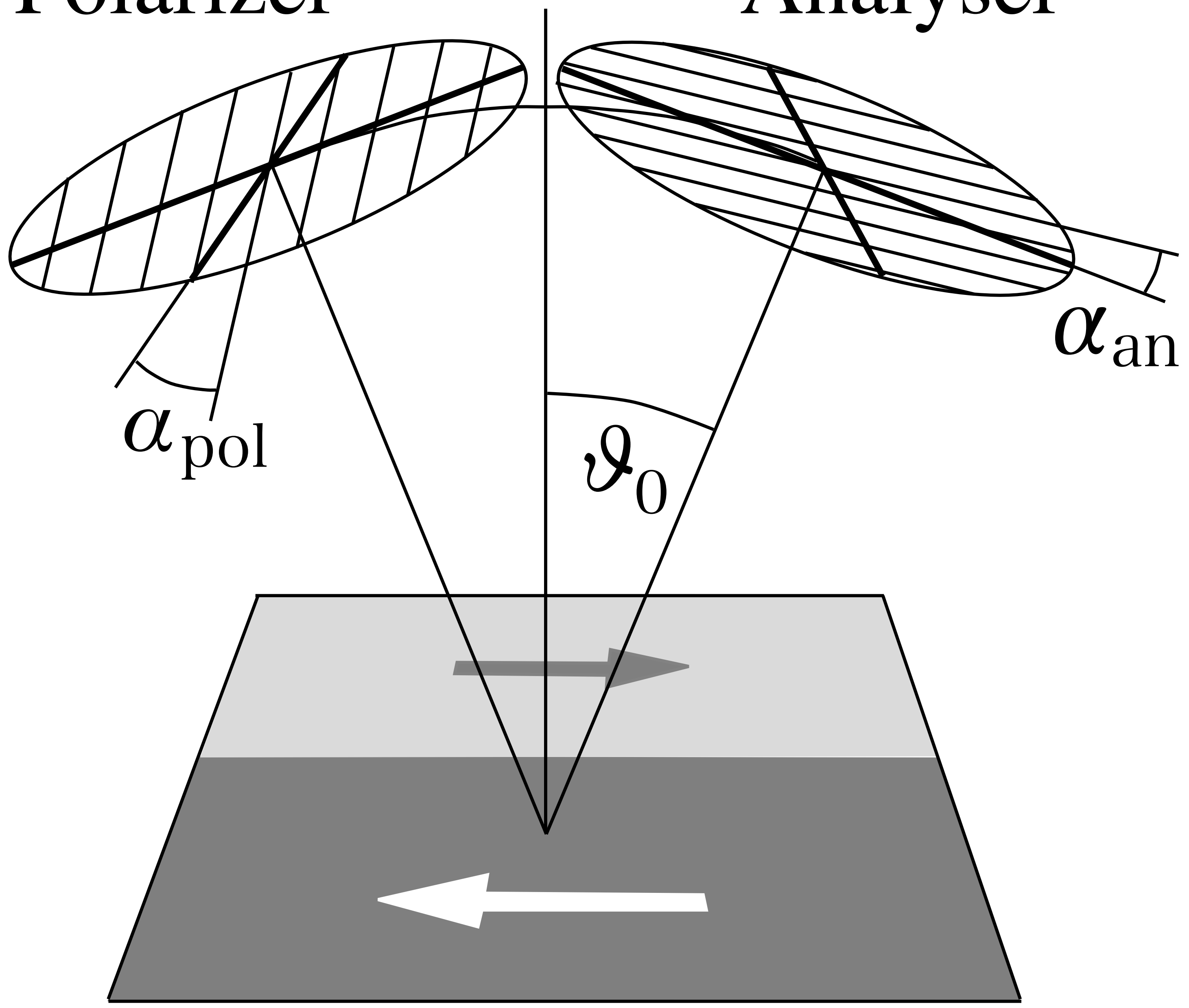
$$C_{\text{opt}} \approx \frac{A_{\text{K}}}{\sqrt{(A_{\text{K}}^2 + I_0)}}$$ with $\alpha_{\text{an}} >> \theta_{\text{K}}$



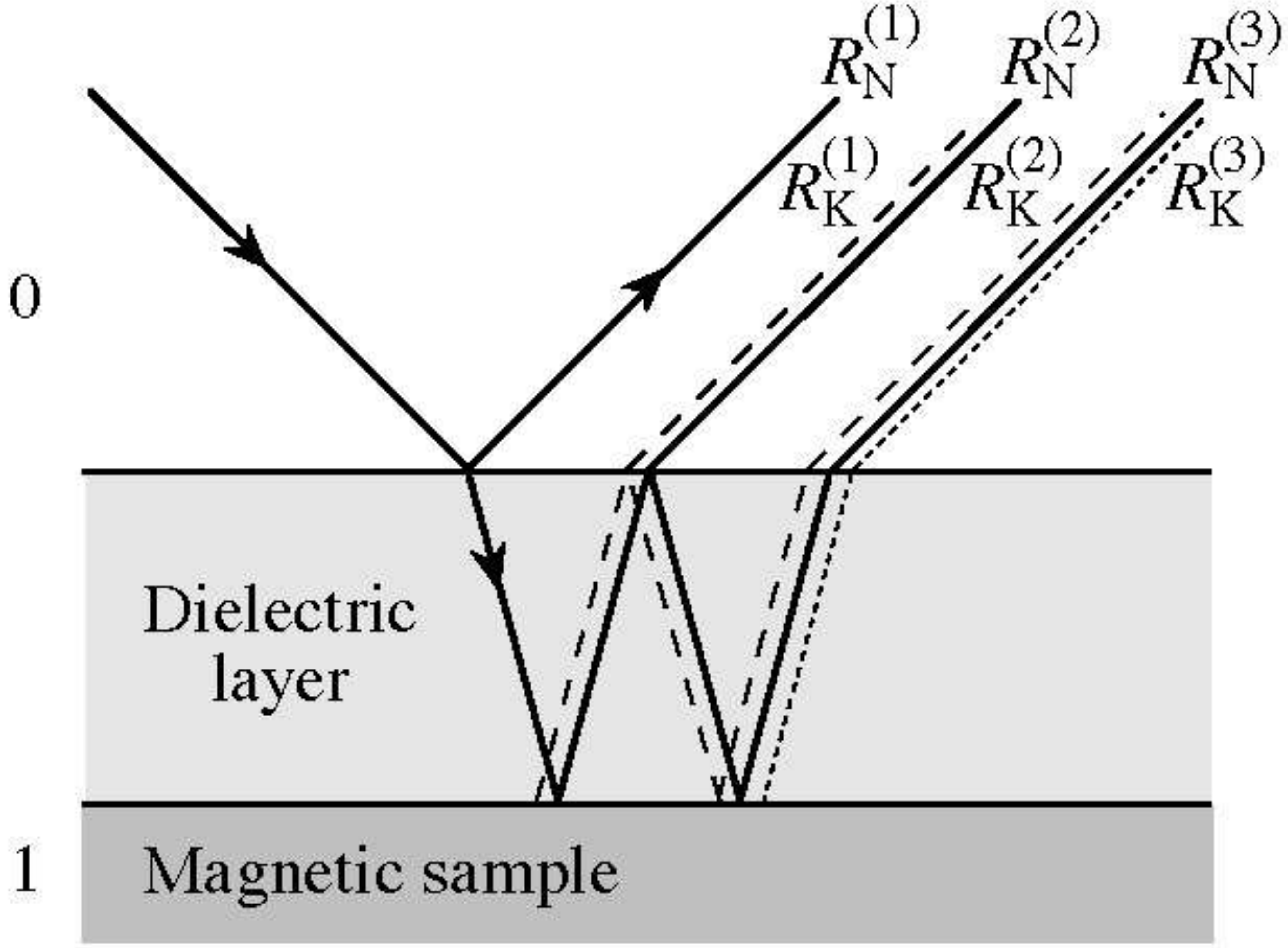
Optimum contrast depends on:

- background intensity I_0 and Kerr amplitude A_{K},
- not on Kerr rotation

$\Longrightarrow$ Figure of merit = Kerr amplitude (not Kerr rotation)



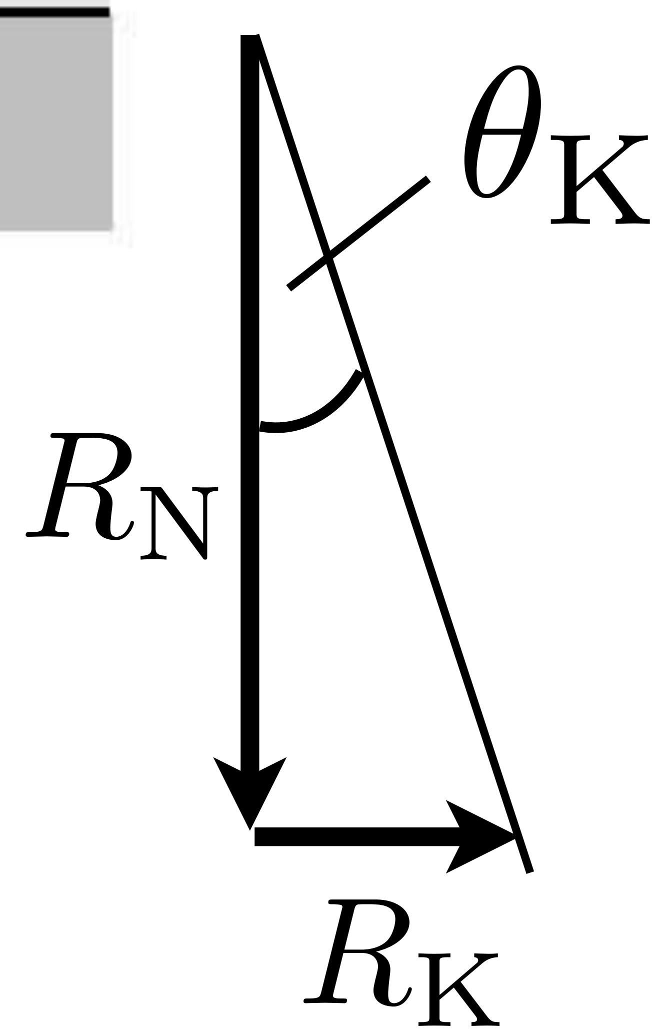
Contrast enhancement in Kerr microscopy by Antireflection Coating



Dielectric anti-reflection coating:

→ regular amplitude goes to zero (destructive interference)

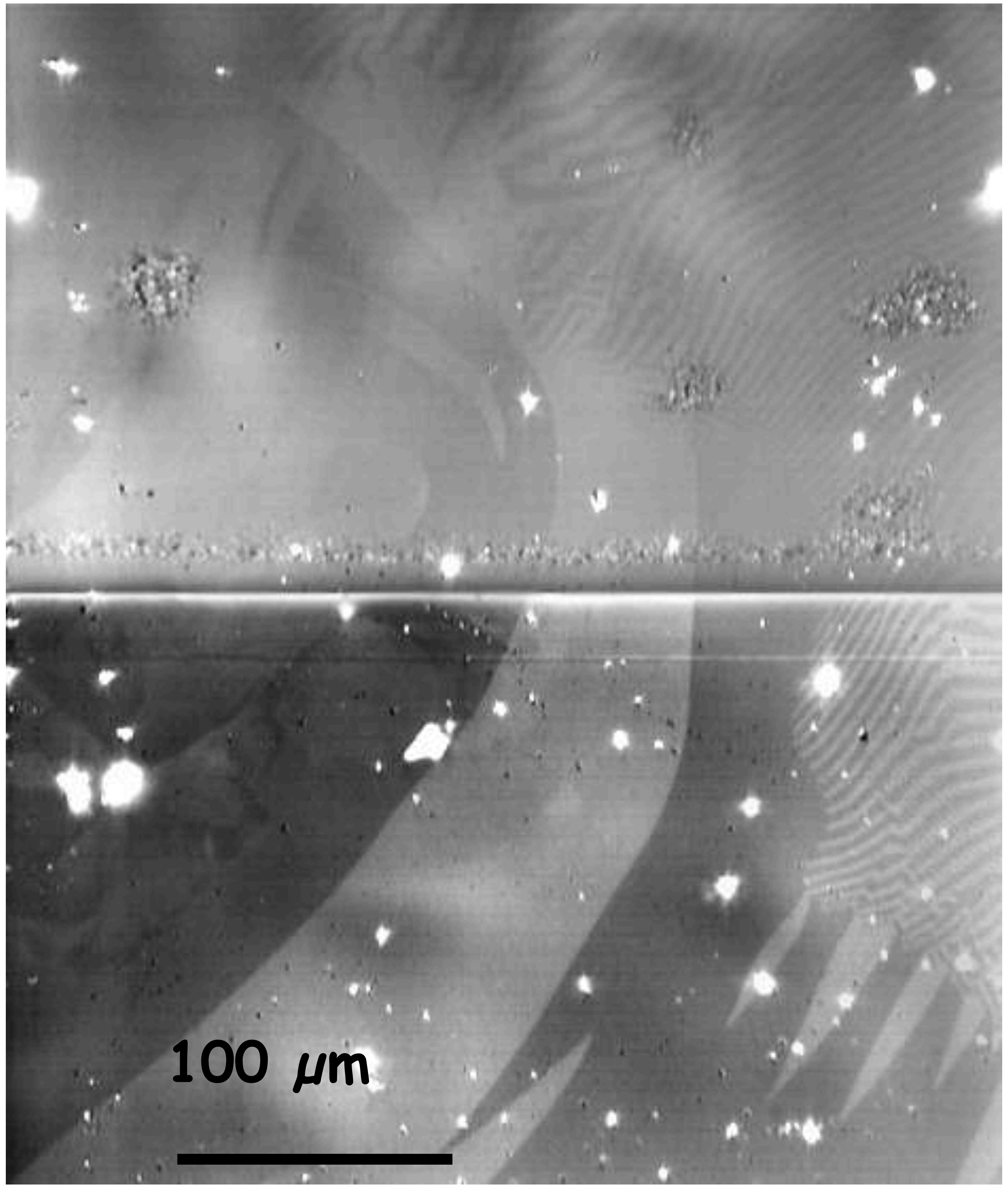
→ Kerr amplitude is enhanced (by constructive interference)



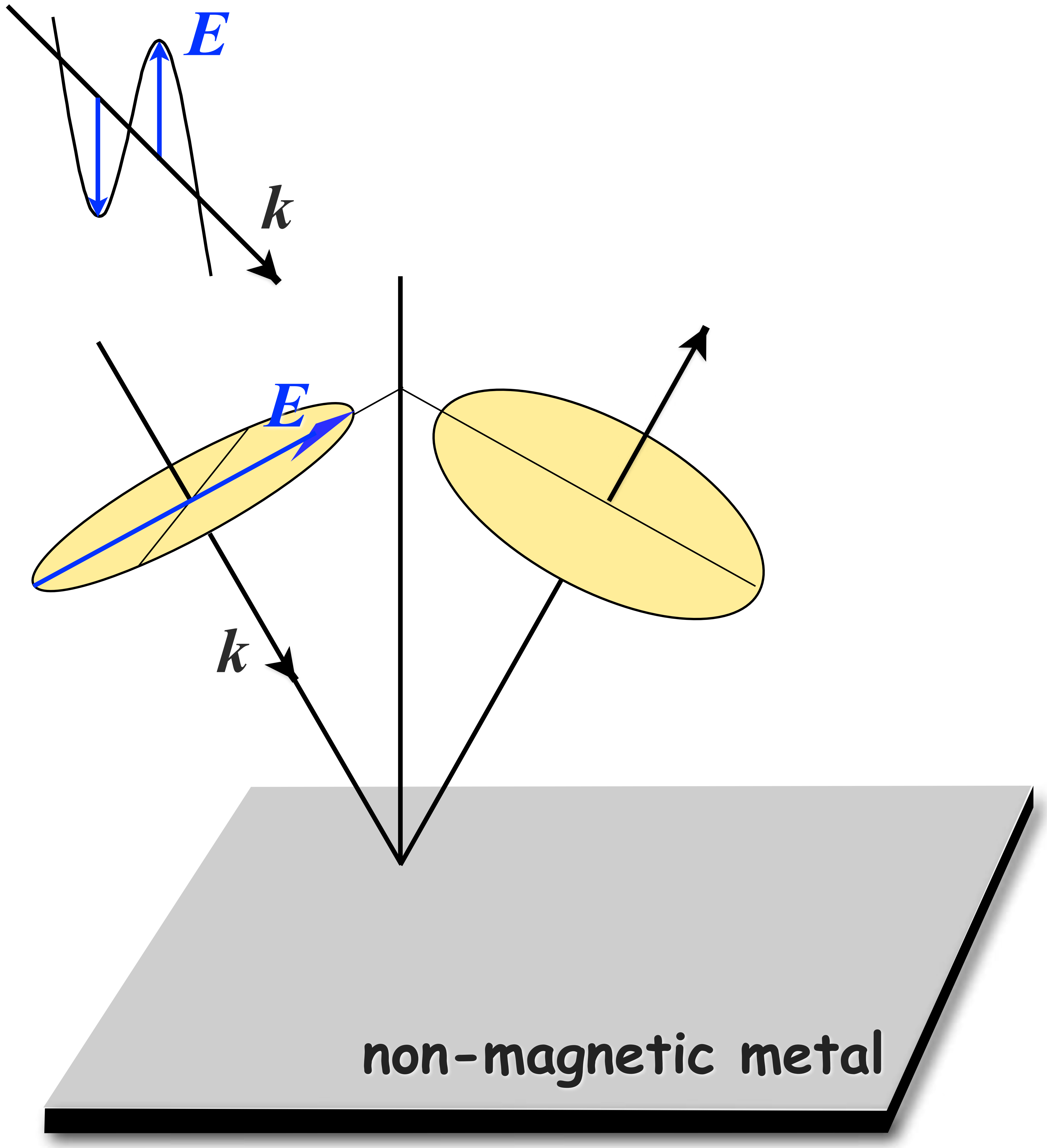
Amorphous ribbon

without interference layer

with ZnS interference layer



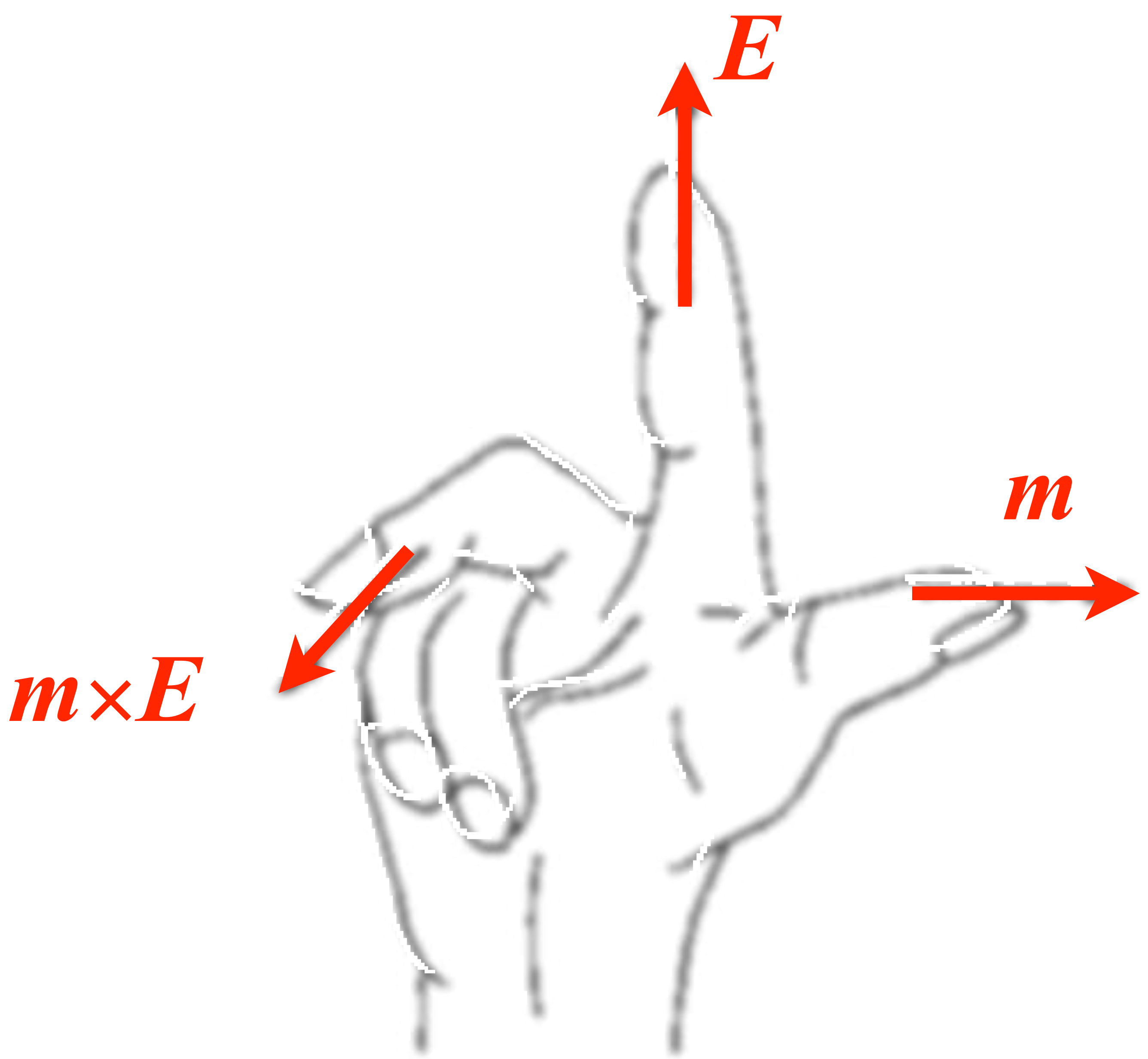
Interpretation in terms of Lorentz concept



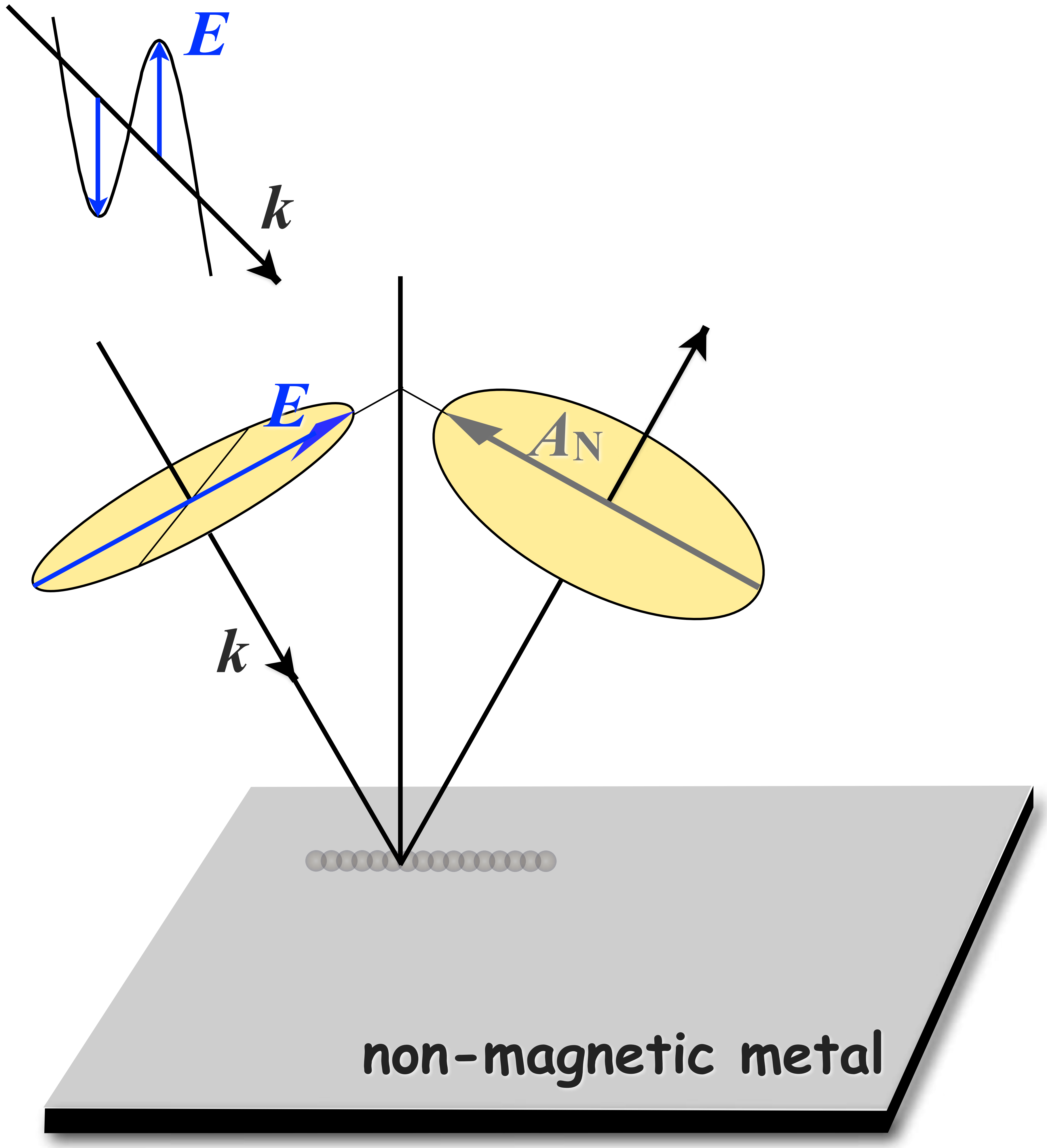
$$\boldsymbol{D} = \epsilon_0 \boldsymbol{\epsilon} \boldsymbol{E} = \varepsilon_0 n^2 \begin{pmatrix} 1 & -\mathrm{i}Q_\mathrm{V} m_3 & \mathrm{i}Q_\mathrm{V} m_2 \\ \mathrm{i}Q_\mathrm{V} m_3 & 1 & -\mathrm{i}Q_\mathrm{V} m_1 \\ -\mathrm{i}Q_\mathrm{V} m_2 & \mathrm{i}Q_\mathrm{V} m_1 & 1 \end{pmatrix} \boldsymbol{E}$$

$$= \varepsilon_0 n^2 [\boldsymbol{E} + \mathrm{i}Q_\mathrm{V} (\boldsymbol{m} \times \boldsymbol{E})]$$

Lorentz force



Interpretation in terms of Lorentz concept



$$\boldsymbol{D} = \epsilon_0 \boldsymbol{\epsilon} \boldsymbol{E} = \varepsilon_0 n^2 \begin{pmatrix} 1 & -\mathrm{i}Q_\mathrm{V} m_3 & \mathrm{i}Q_\mathrm{V} m_2 \\ \mathrm{i}Q_\mathrm{V} m_3 & 1 & -\mathrm{i}Q_\mathrm{V} m_1 \\ -\mathrm{i}Q_\mathrm{V} m_2 & \mathrm{i}Q_\mathrm{V} m_1 & 1 \end{pmatrix} \boldsymbol{E}$$

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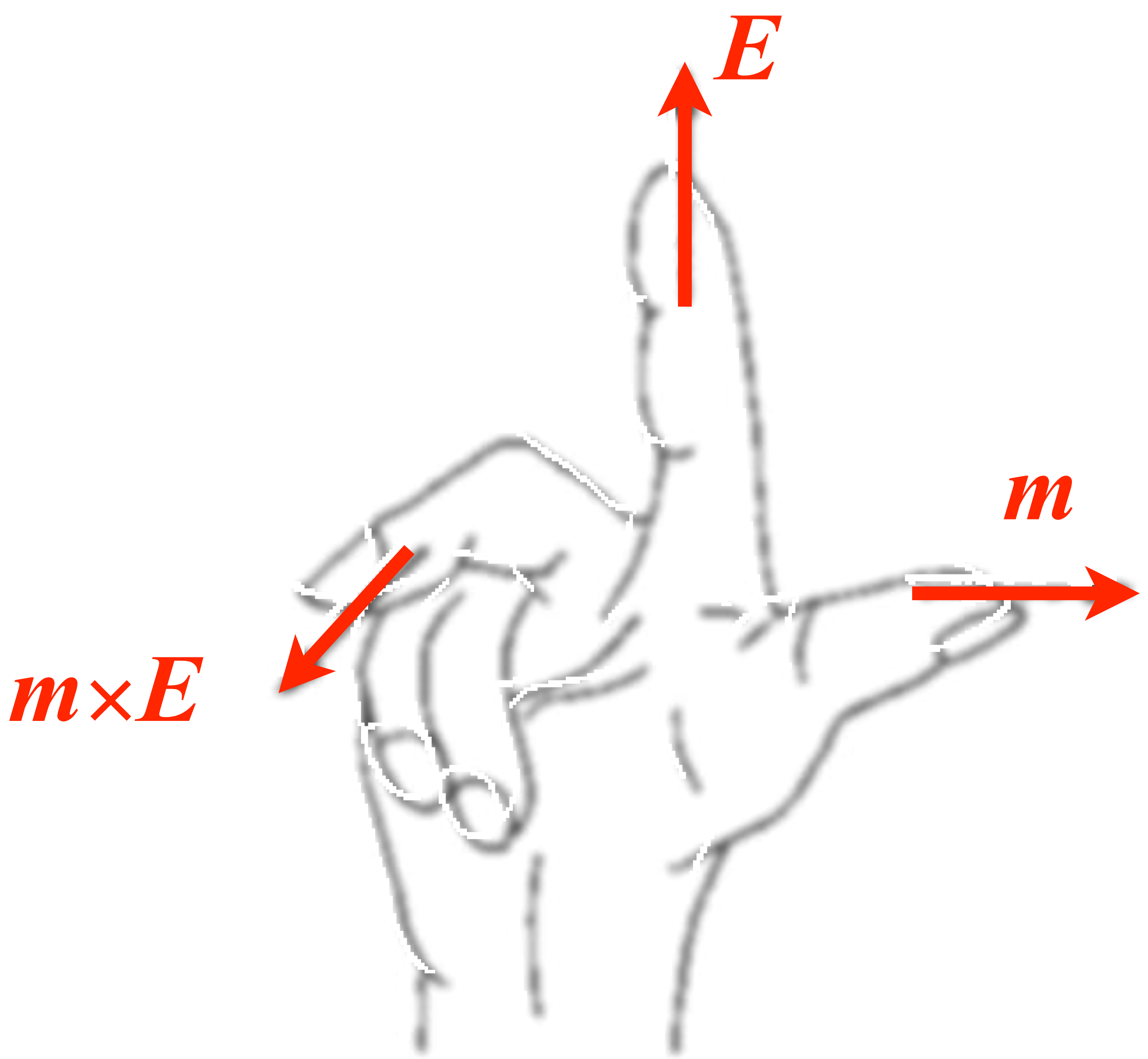

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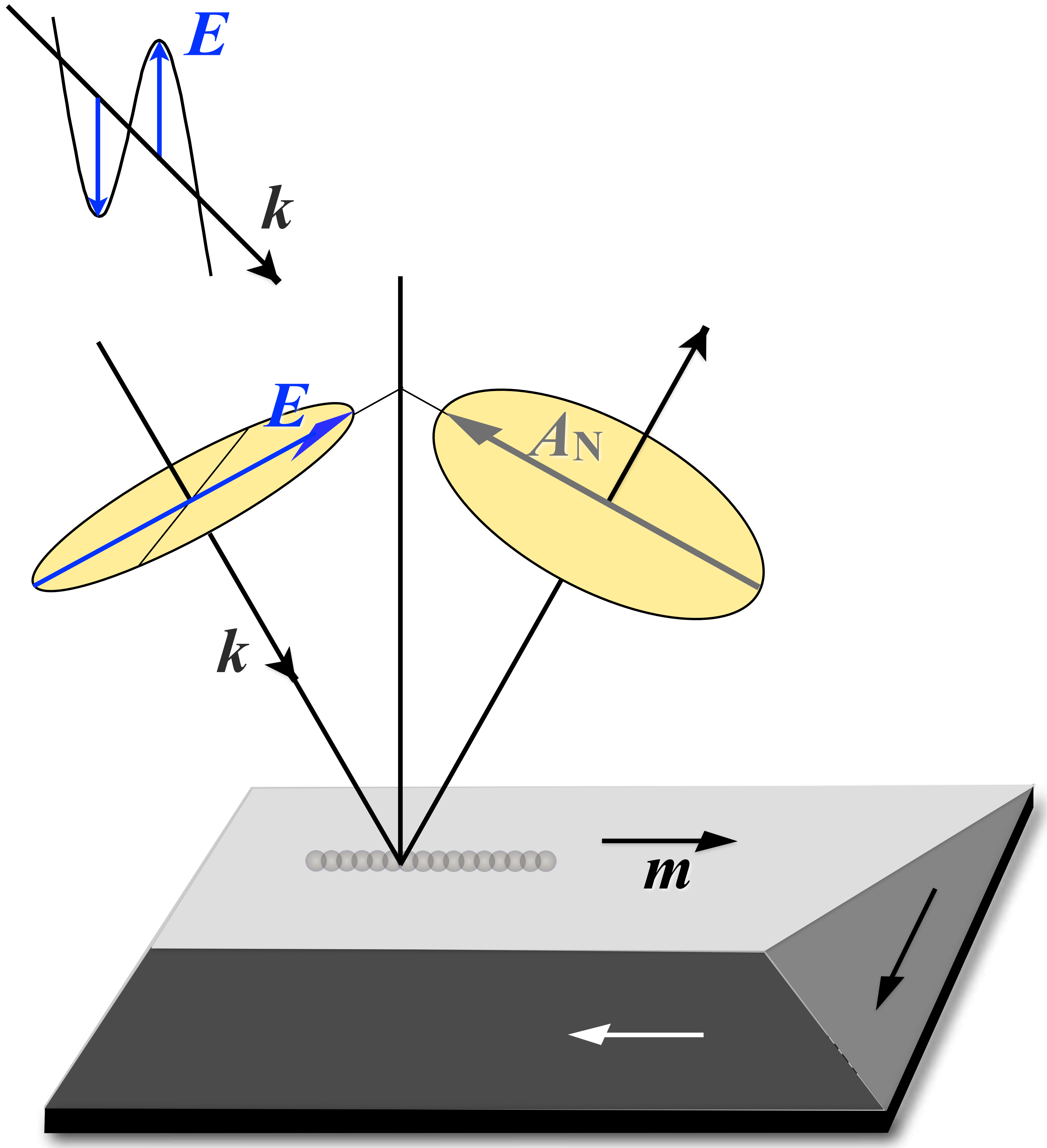

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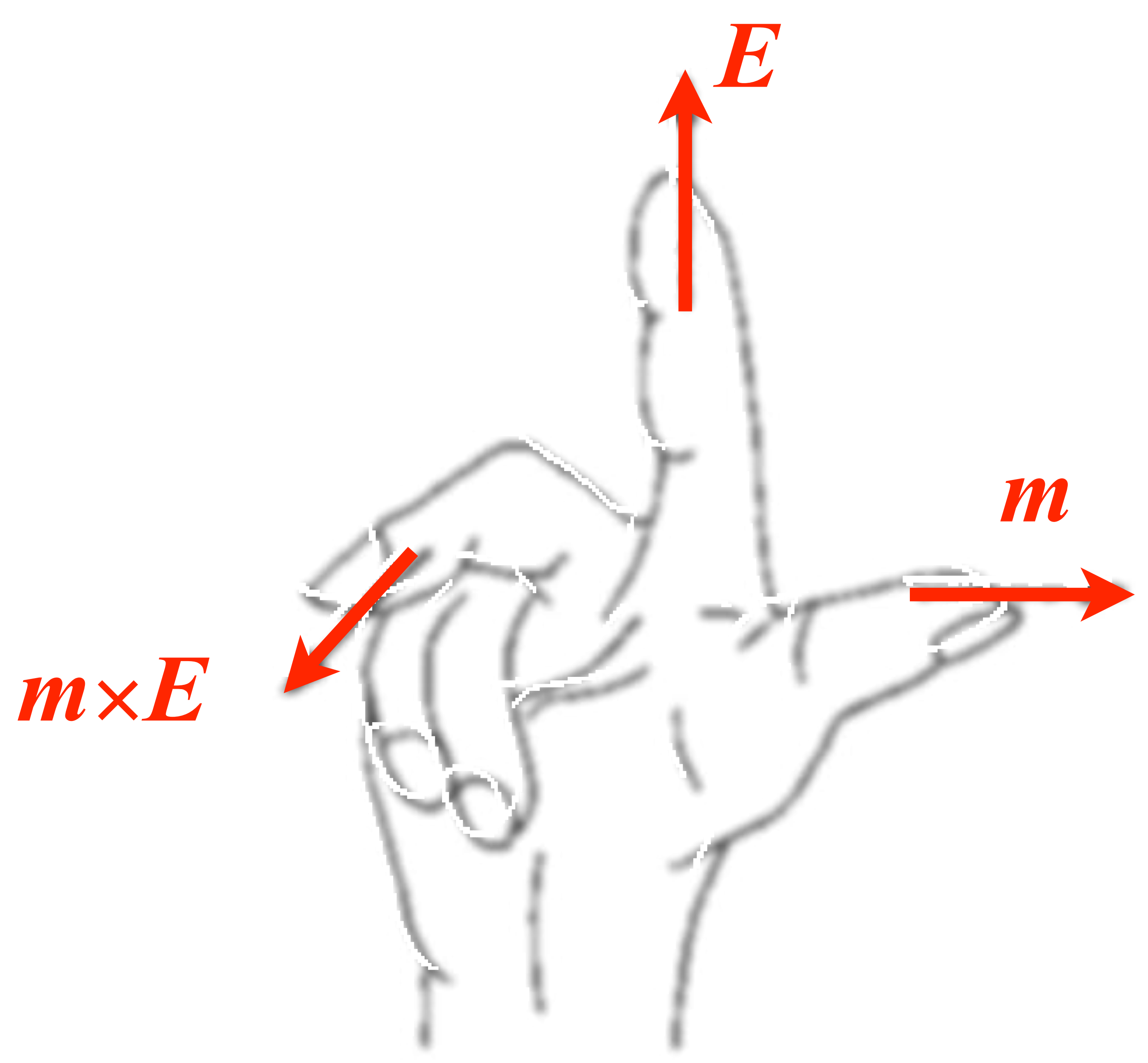

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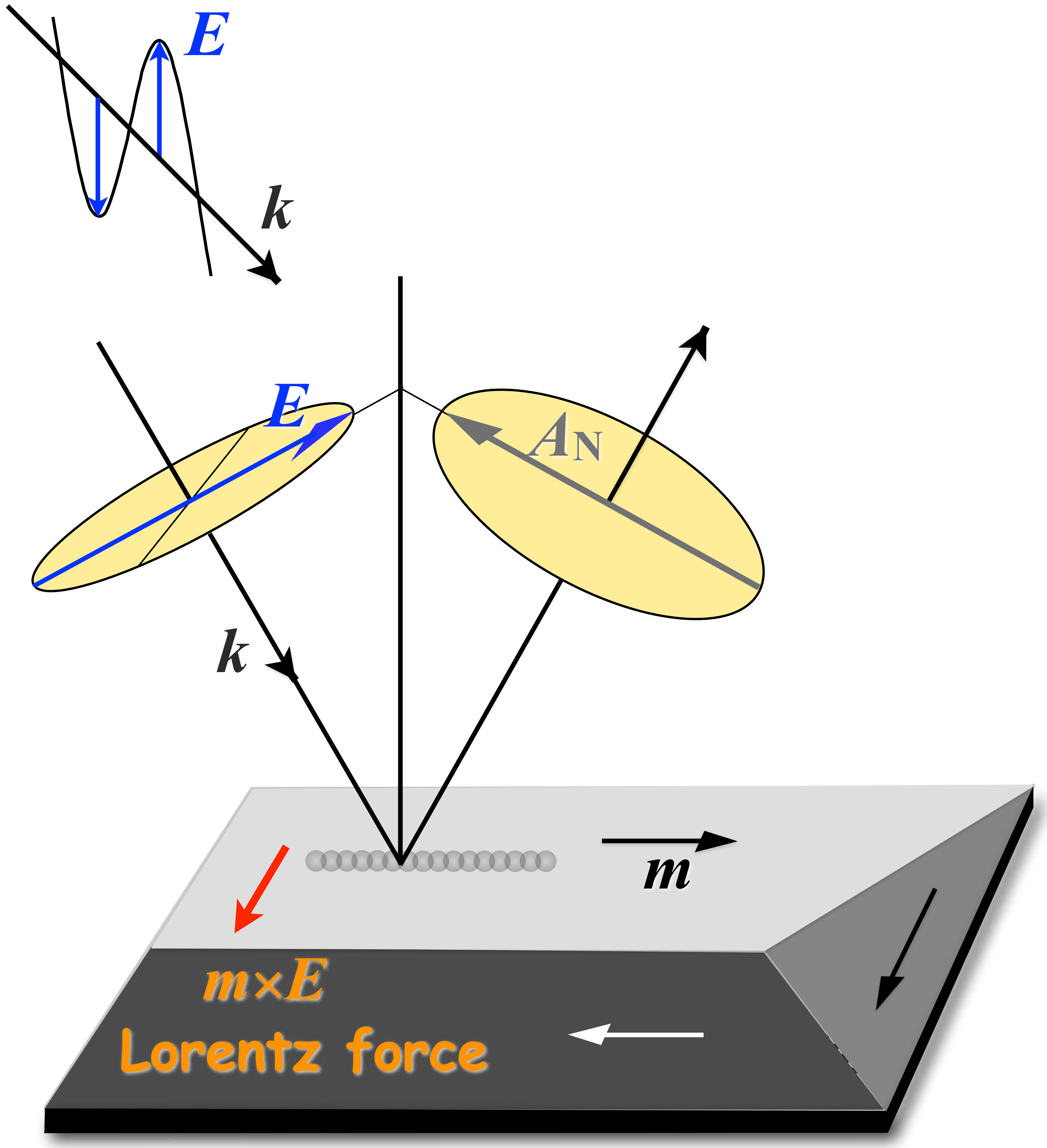

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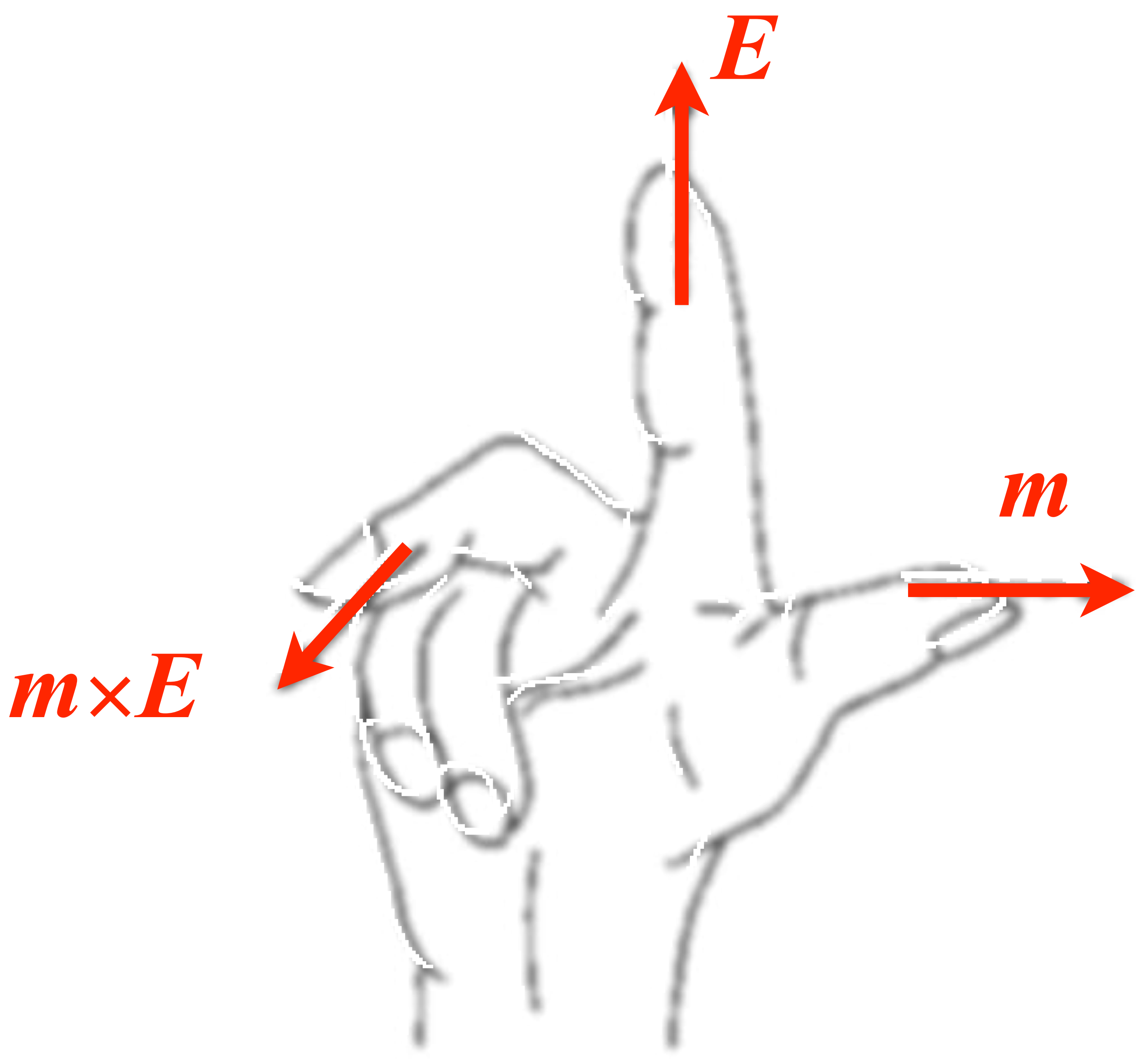

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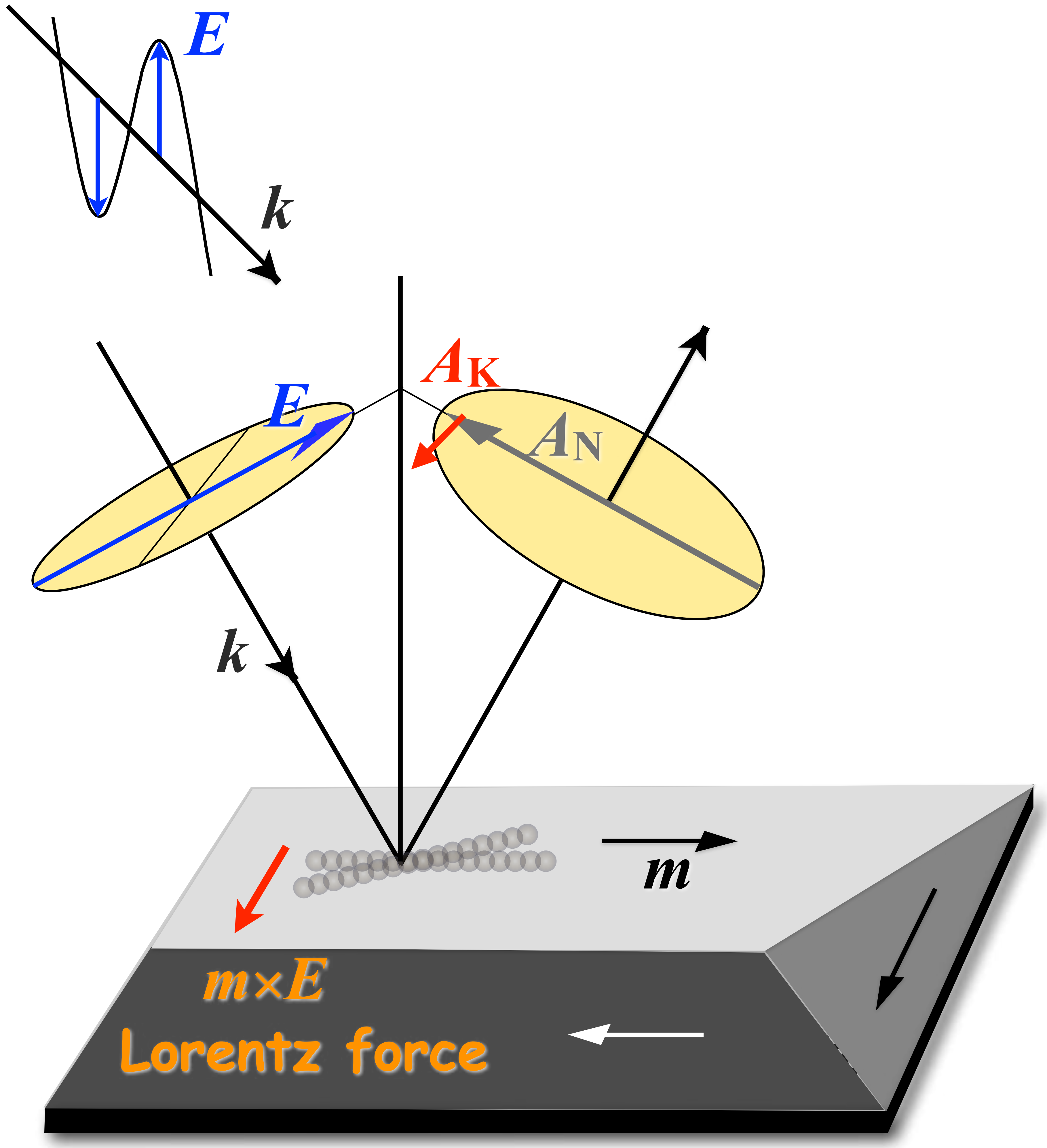

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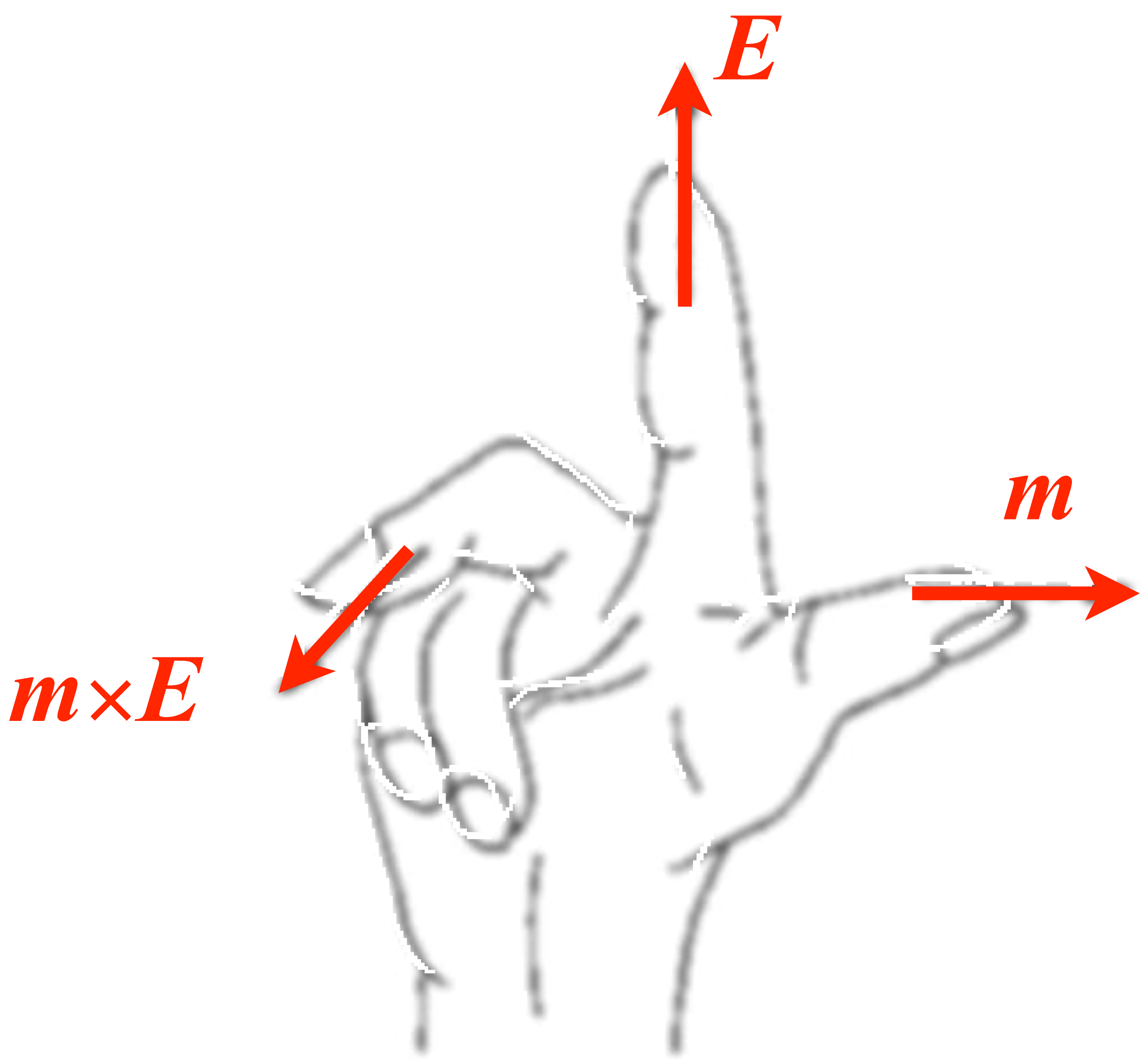

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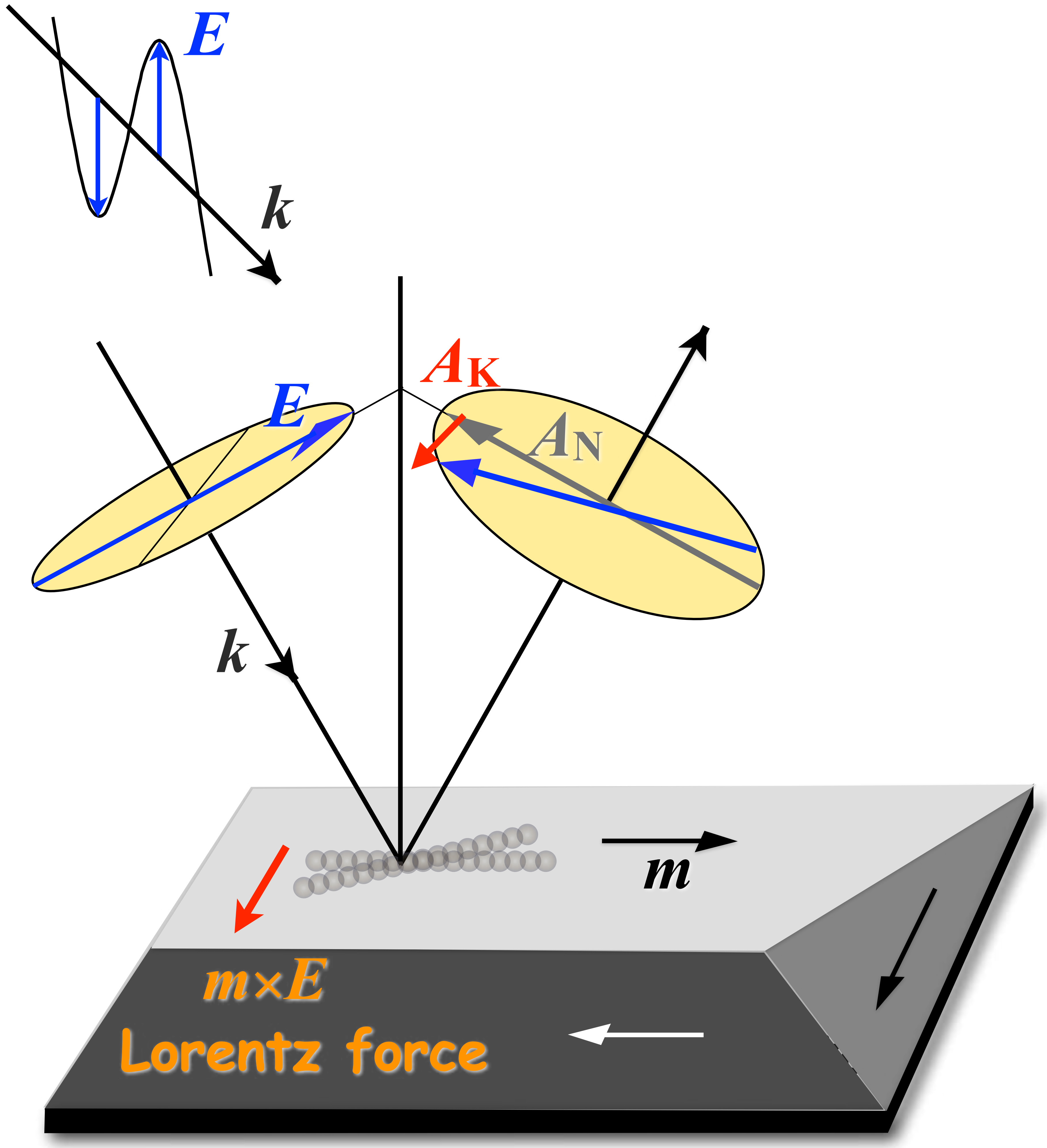
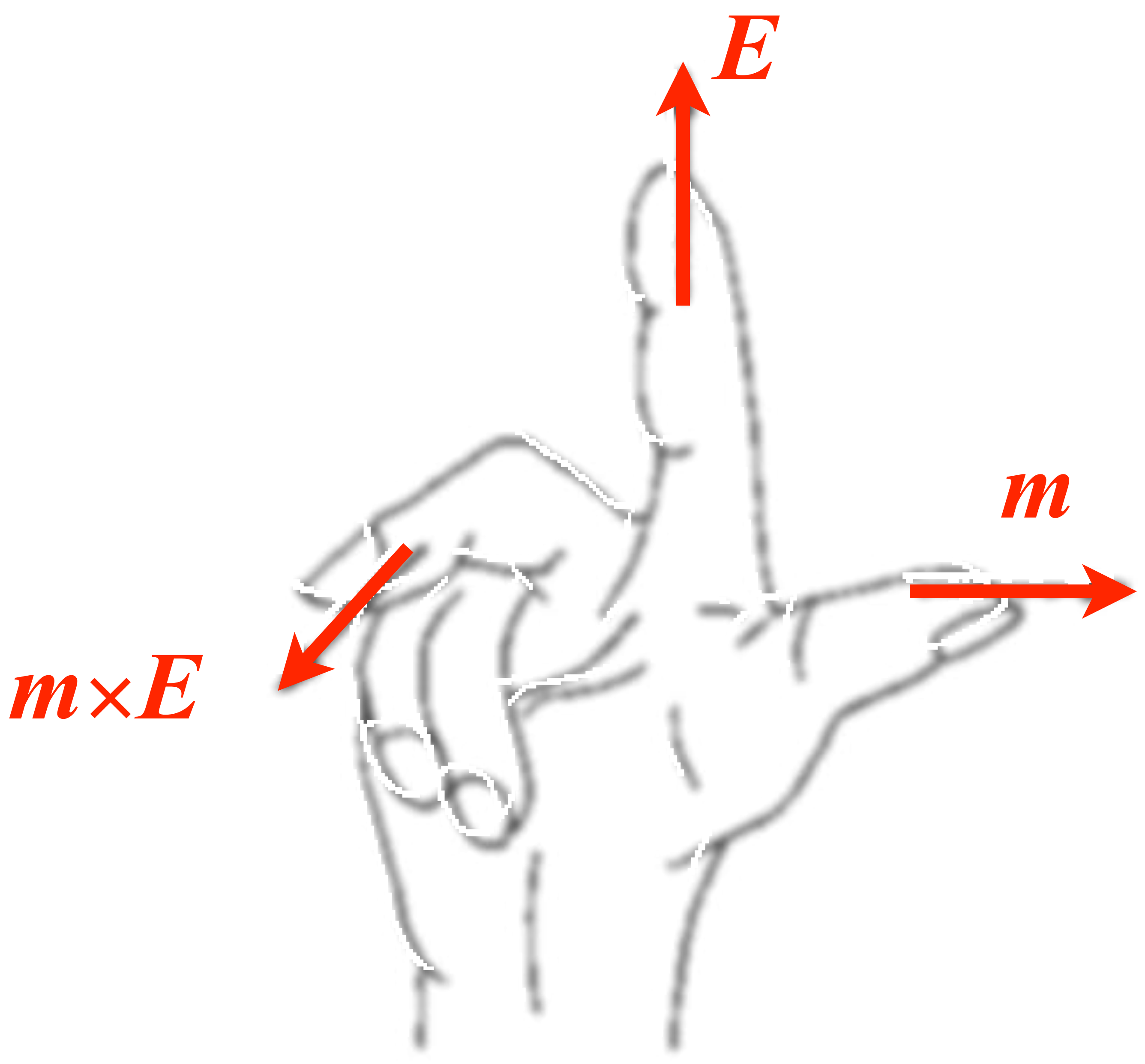


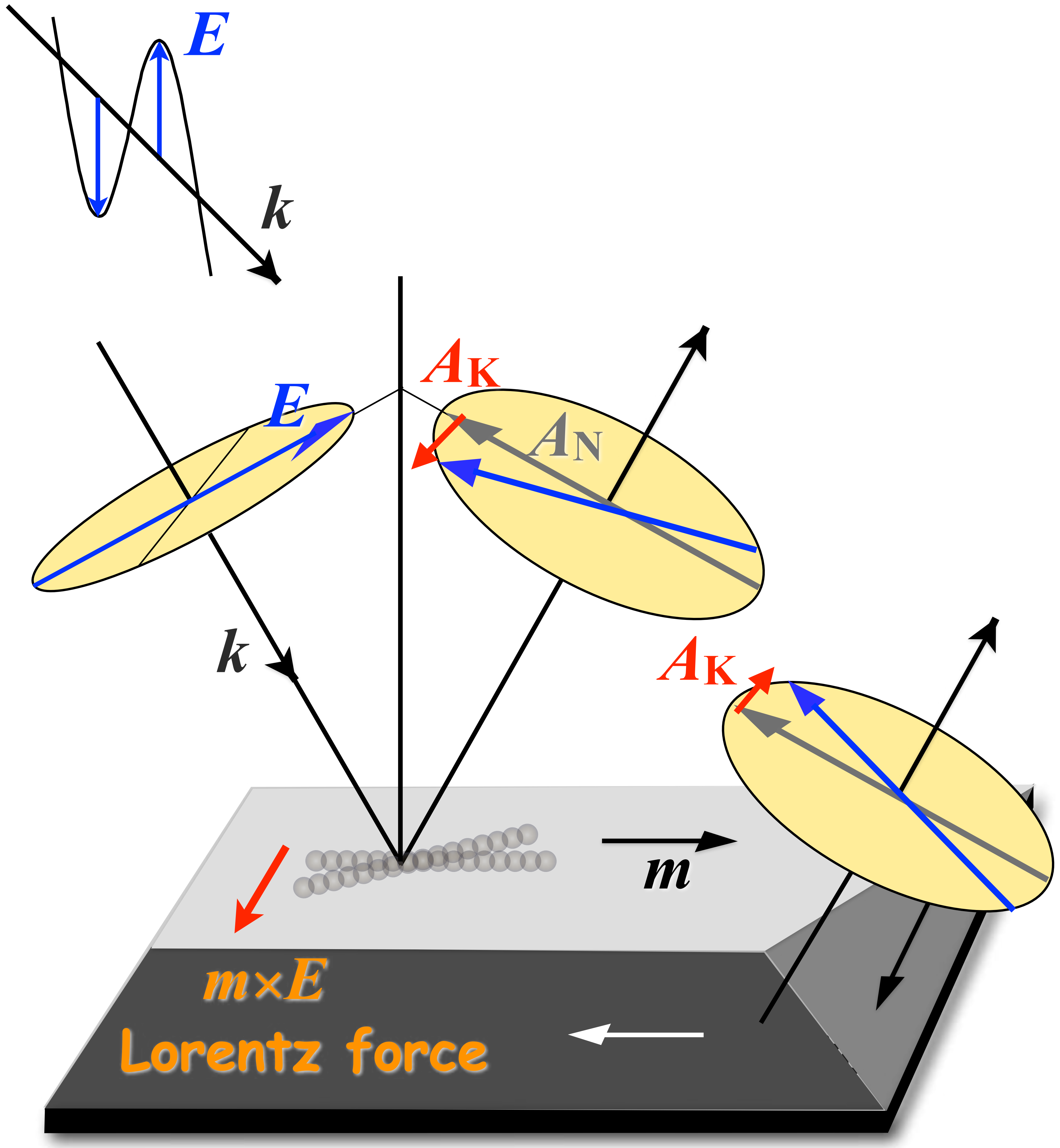
$$D = \epsilon_0 \boldsymbol{\epsilon} E = \varepsilon_0 n^2 \begin{pmatrix} 1 & -iQ_V m_3 & iQ_V m_2 \\ iQ_V m_3 & 1 & -iQ_V m_1 \\ -iQ_V m_2 & iQ_V m_1 & 1 \end{pmatrix} E$$

$$= \varepsilon_0 n^2 [E + iQ_V (m \times E)]$$

→ Lorentz force



Interpretation in terms of Lorentz concept



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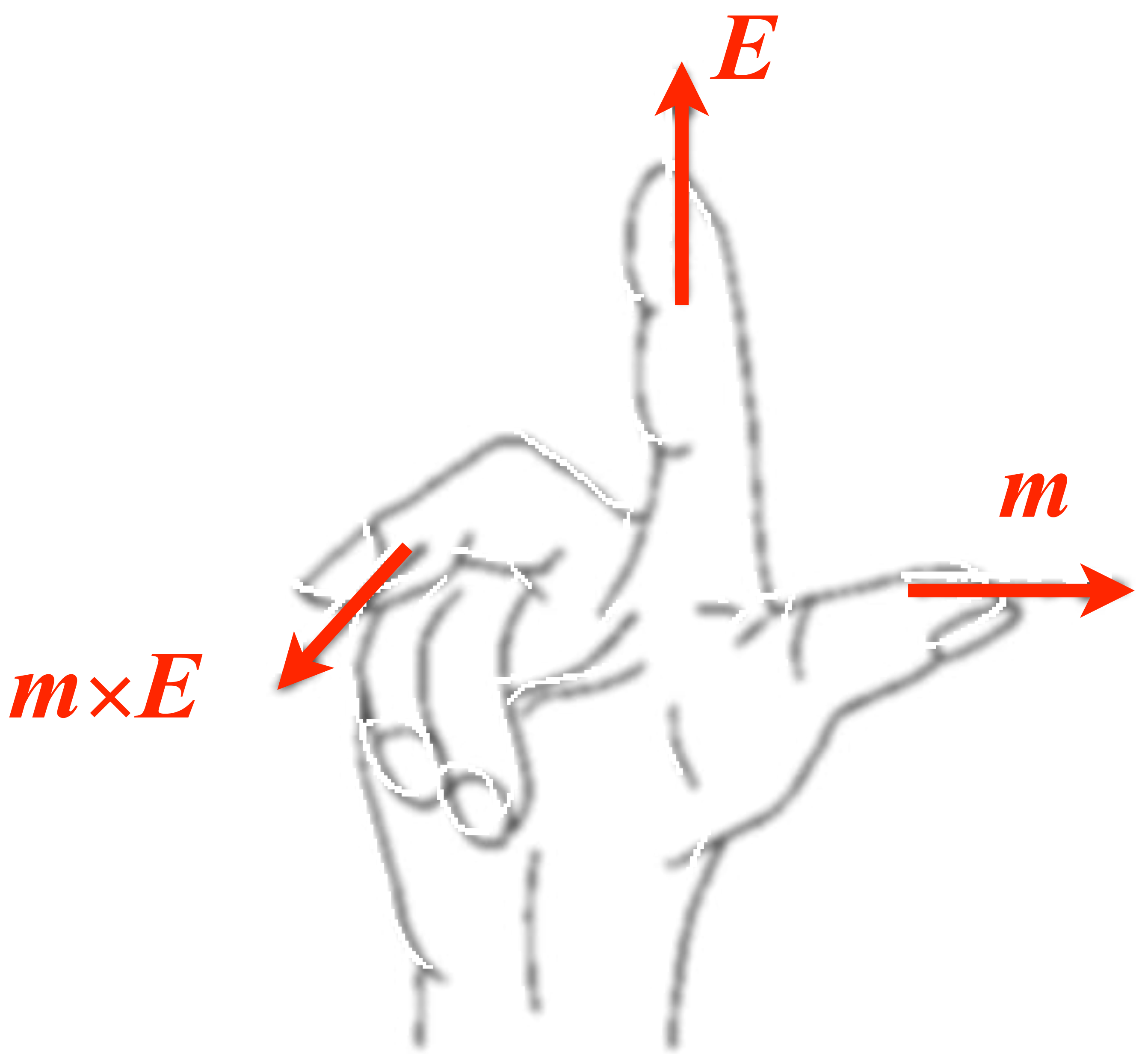

2. Magneto-Optical Effects – 2.4 Kerr Effect

Interpretation in terms of Lorentz concept

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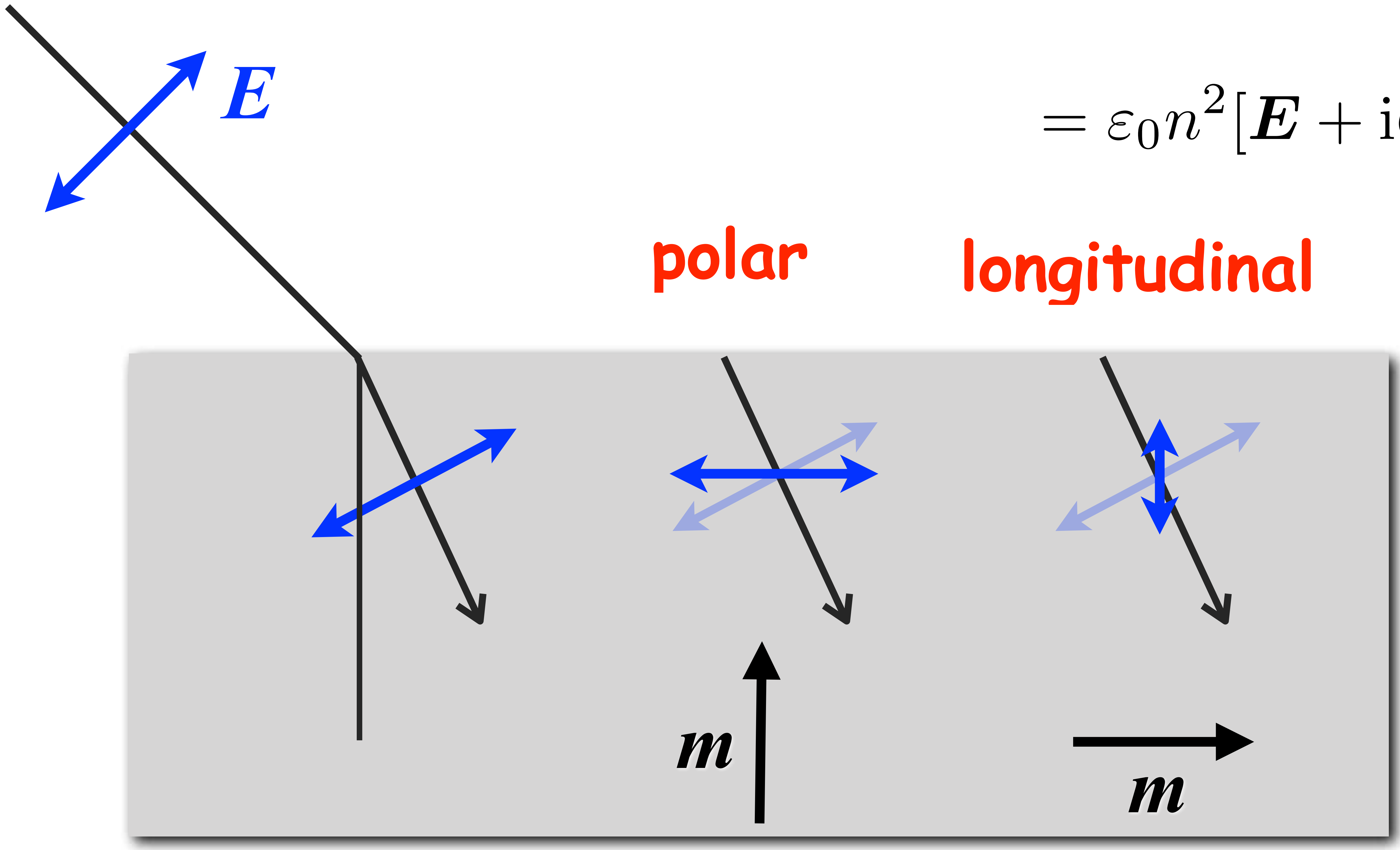
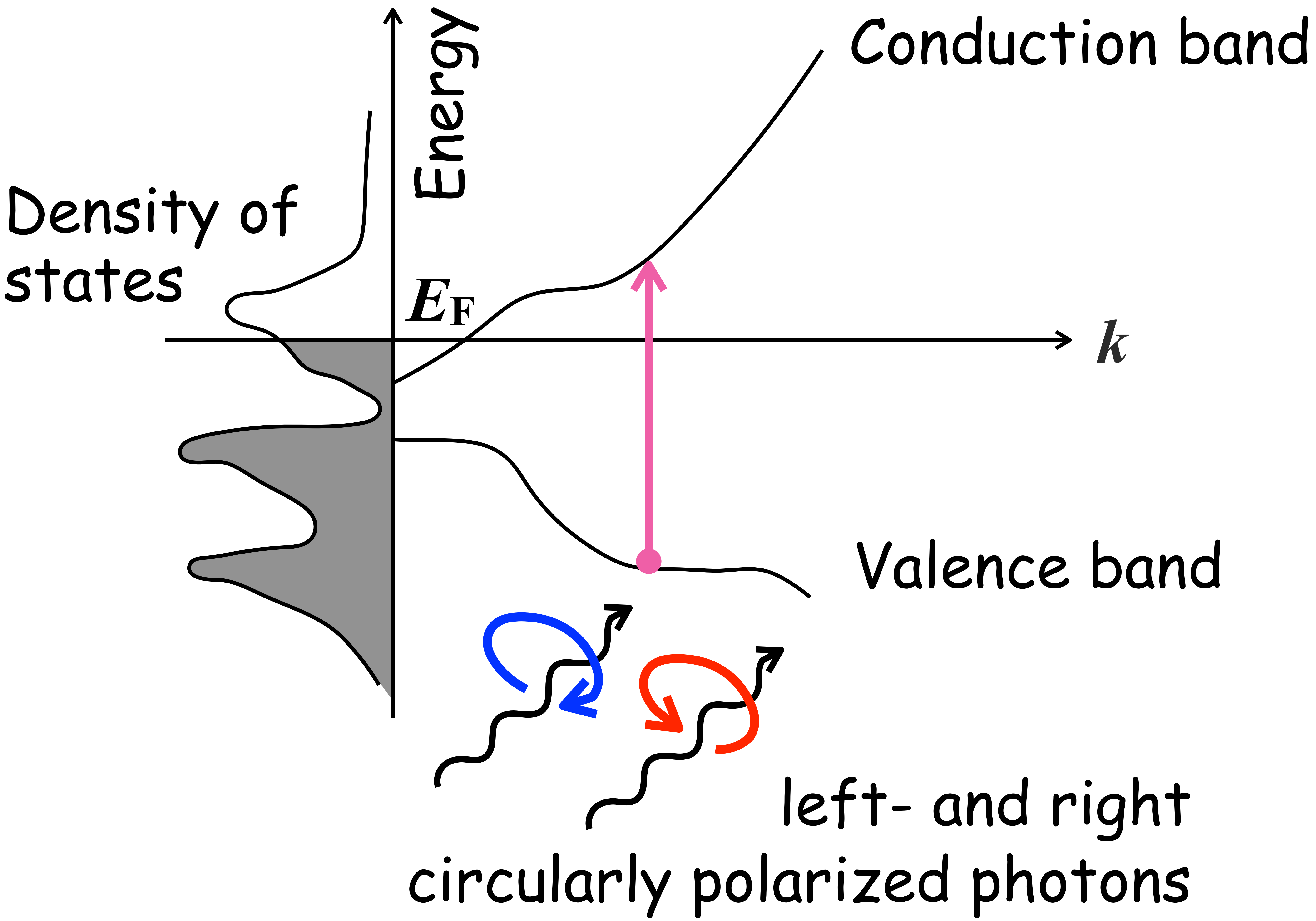


- diffraction index $n_\mathrm{metal} \approx 3$ (Fe)
- $\boldsymbol{m}_\mathrm{pol} \times \boldsymbol{E} \;>>\; \boldsymbol{m}_\mathrm{long} \times \boldsymbol{E}$
- component of $\boldsymbol{E}$ perpendicular to $\boldsymbol{m}$ counts

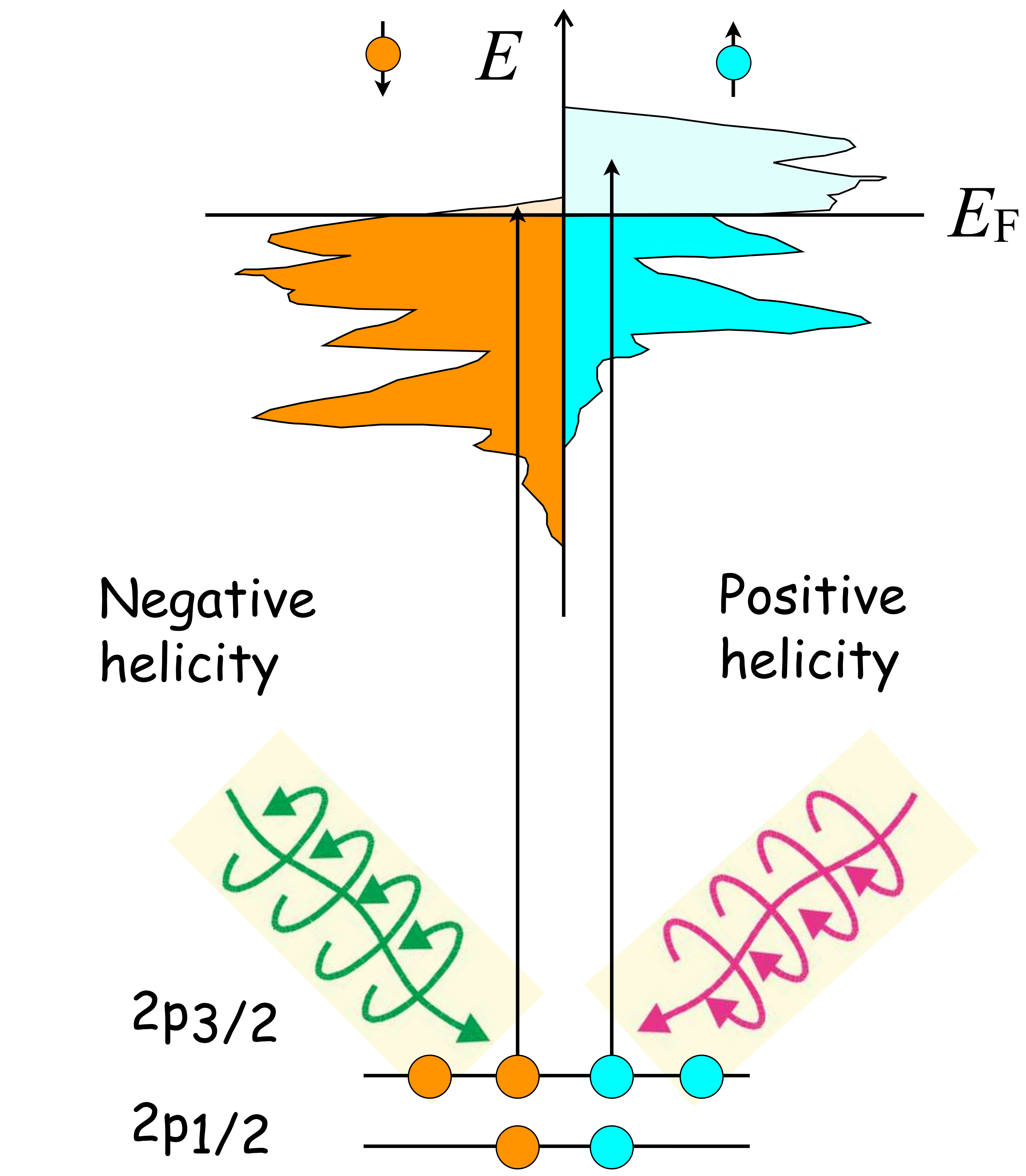
$\Longrightarrow$ polar magnetization much easier to measure

Microscopic description

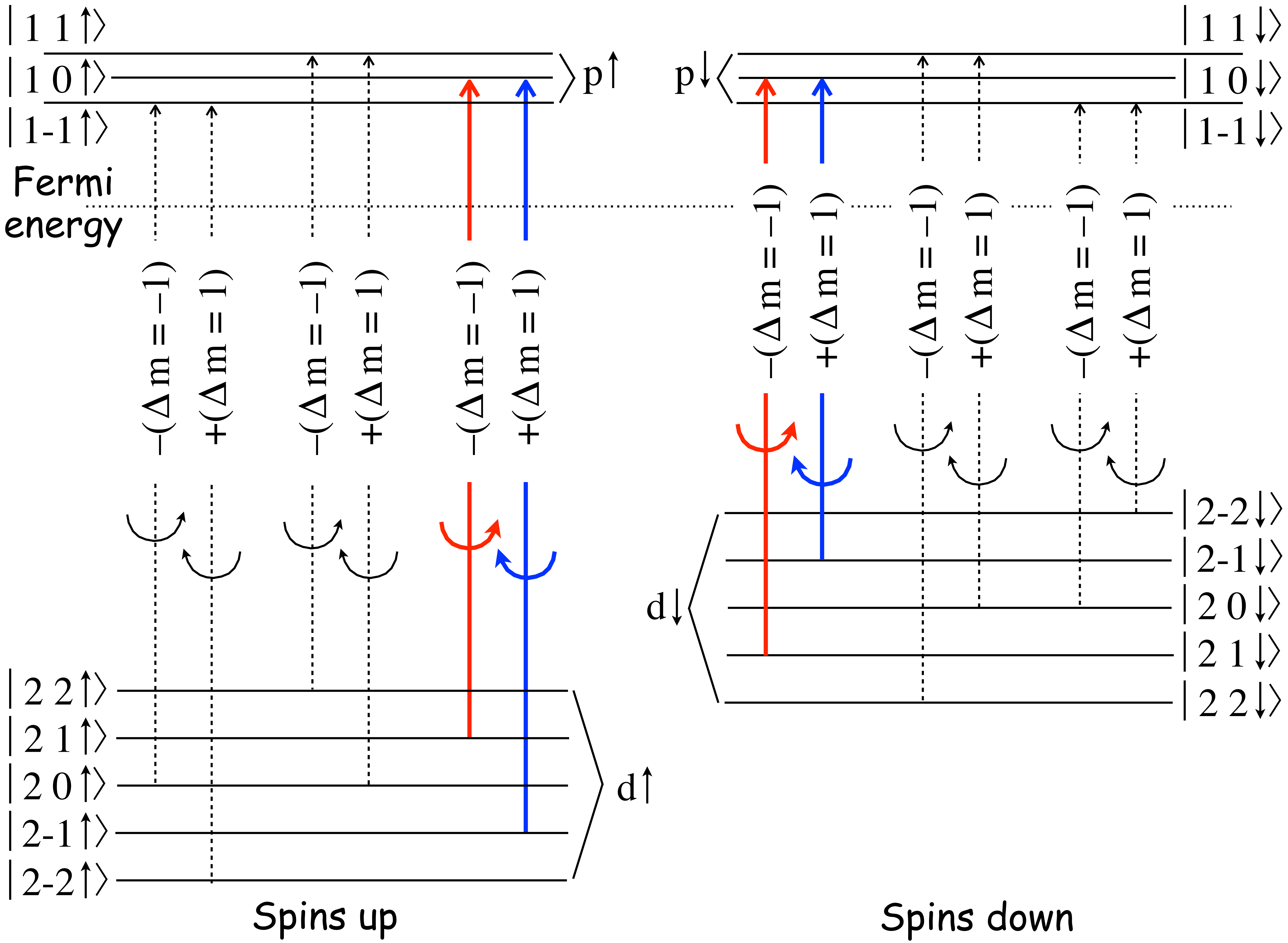
Conventional magneto-optical effects



Compare: X-ray Magnetic Circular Dichroism



Microscopic description



$|l\ m \uparrow\rangle$

Orbital quantum number (p-states: l = 1 , d-states: l = 2)

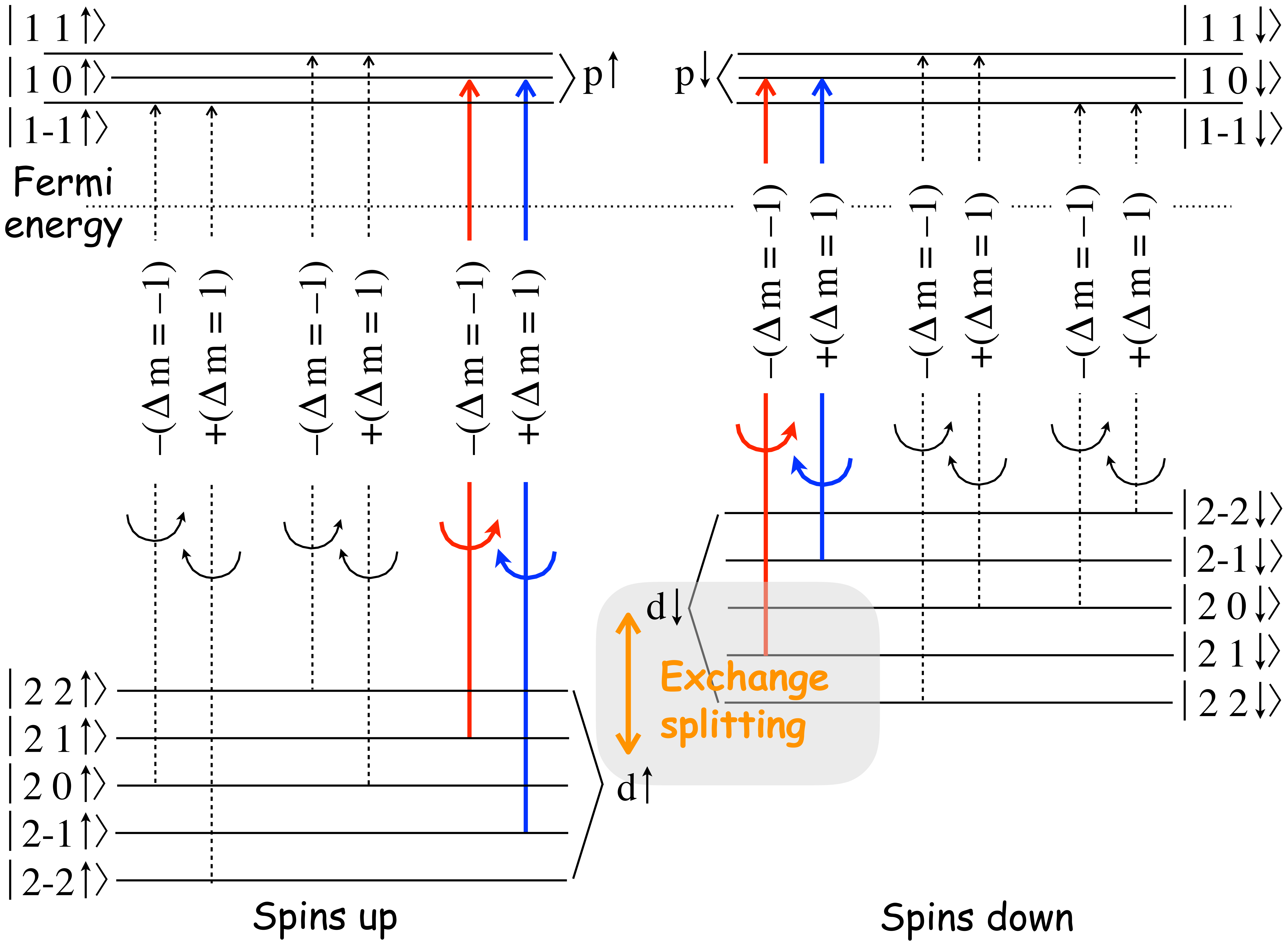
Magnetic quantum number

Spin orientation

Selection rules for electric dipole transitions, excited by circular photons:

- Electron spin is conserved:
  $\Delta s = 0$
- Total orbital momentum is conserved:
  $\Delta l = \pm 1$
- Total orbital momentum along quantization axis is conserved:
  - right circular: $\Delta m = +1$
  - left circular: $\Delta m = -1$

Microscopic description



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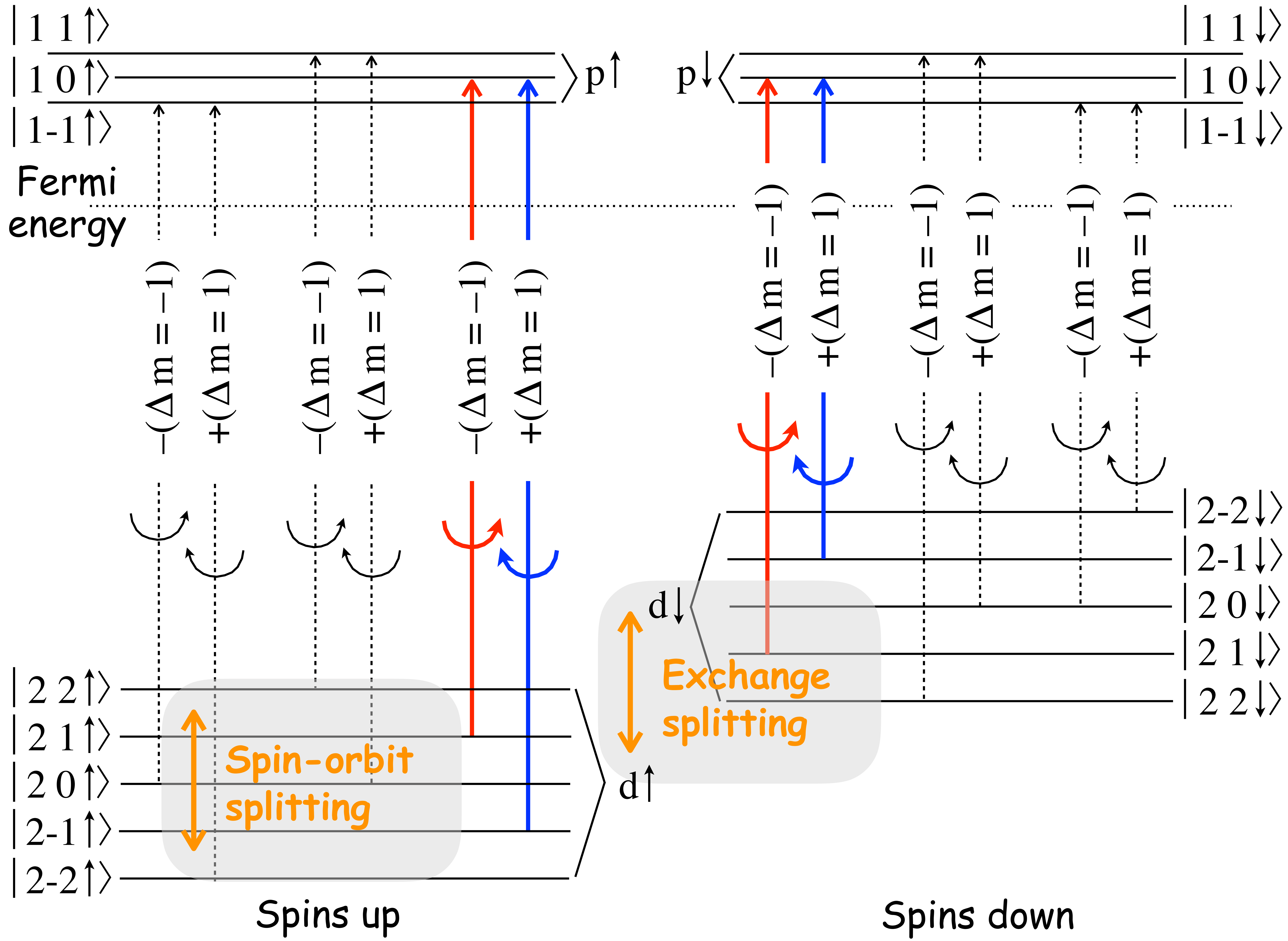

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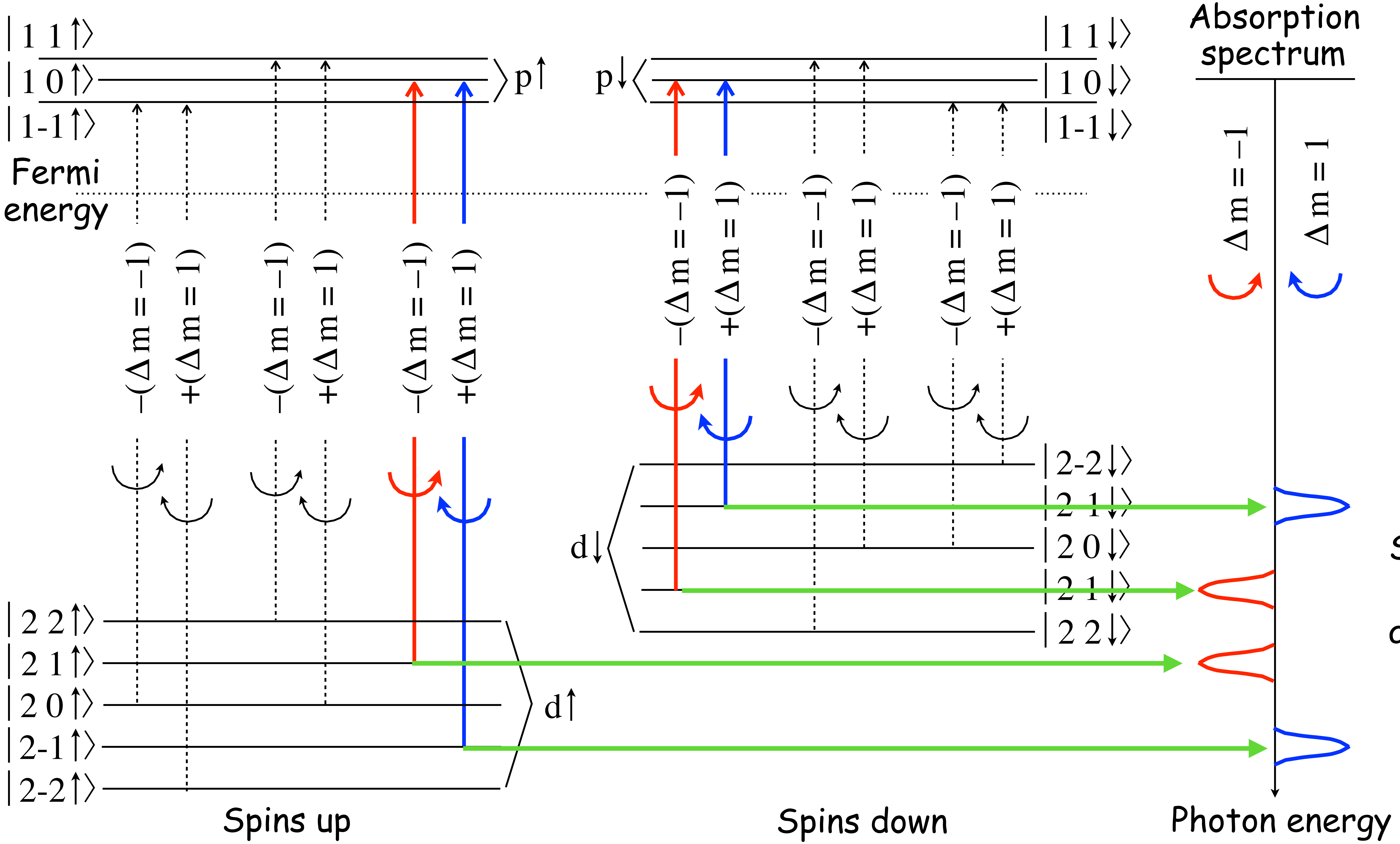
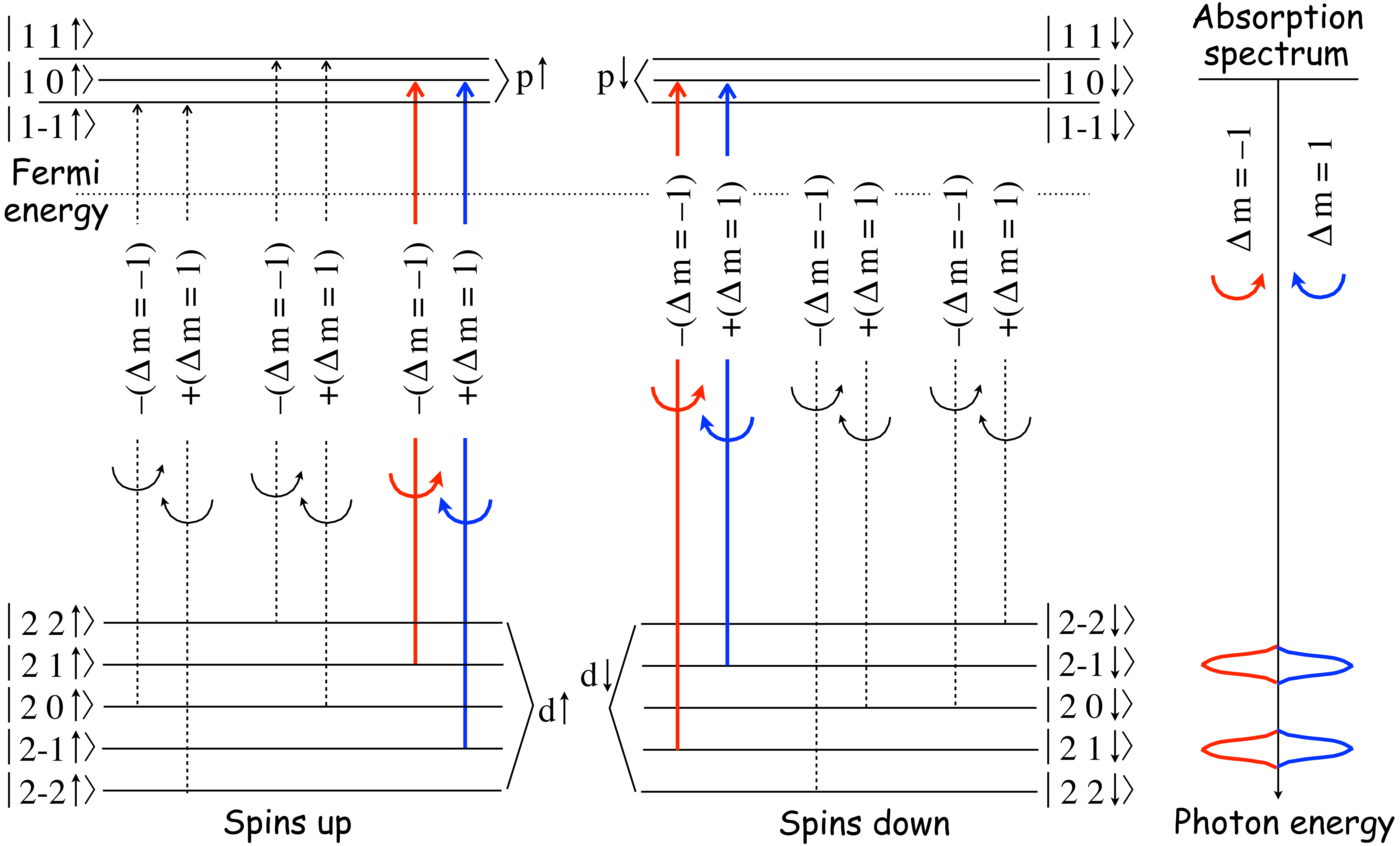
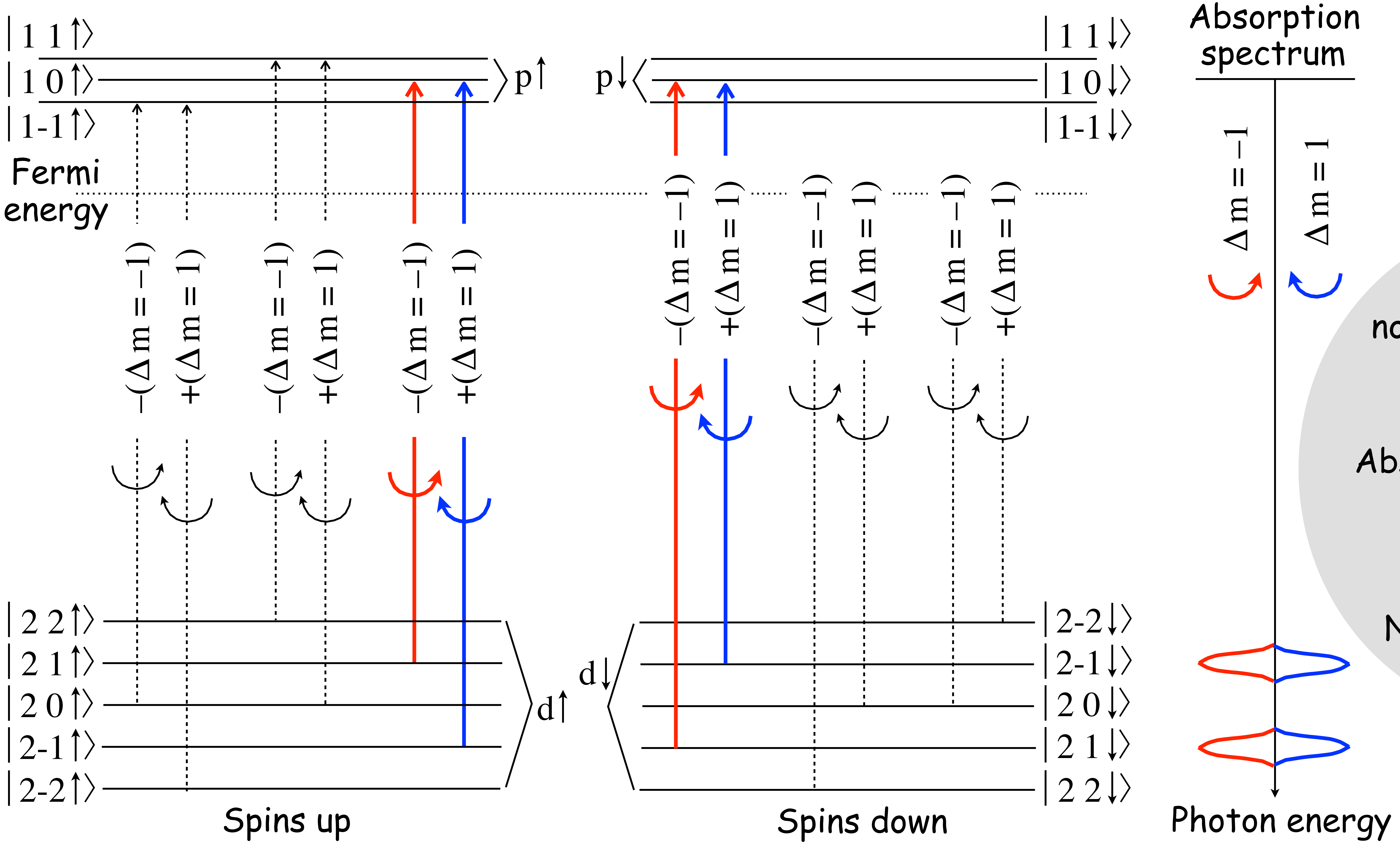


Strong dichroism = difference in absorption for left and right-handed polarization

Microscopic description



Microscopic description



Assumption: no exchange splitting

⇓

Absorption spectra are the same

⇓

No magneto-optical activity

Microscopic description

- Imaginary part of the complex dielectric constant can be related to the electronic transitions by

$$\epsilon''_{\pm}(\omega) \propto \frac{1}{\omega^2} \sum_{i,f} f(\mathcal{E}_i)\,[1 - f(\mathcal{E}_f)]\, |< i|p_{\pm}|f >|^2\, \delta(\omega_f - \omega_i - \omega)$$

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Summation overall initial and final states in k-space

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Occupancy of initial and final states (Fermi-Dirac distribution)

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Transition probabilities for left and right polarized light

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Summation overall initial and final states in k-space	Occupancy of initial and final states (Fermi-Dirac distribution)	Transition probabilities for left and right polarized light	Energy conservation

after: P. Vavassori, ESM 2018

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Summation overall initial and final states in k-space	Occupancy of initial and final states (Fermi-Dirac distribution)	Transition probabilities for left and right polarized light	Energy conservation

after: P. Vavassori, ESM 2018

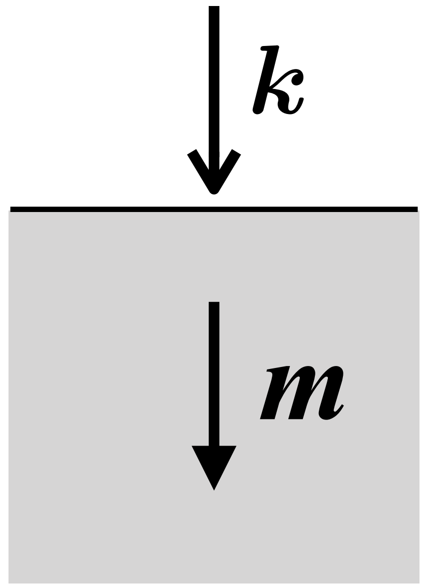
- Connection of microscopic to classical description:

$$(n'_{\pm} + \mathrm{i}n''_{\pm})^2 = \epsilon'_{\pm} + \mathrm{i}\epsilon''_{\pm}$$

$\Downarrow$ with $n_{\pm} = \sqrt{\epsilon_{\mathrm{iso}}}(1 \pm Q_{\mathrm{V}}/2)$ and $\bar{n} = \frac{1}{2}(n_+ + n_-)$

$$Q_V = 2\frac{n_+ - n_-}{n_+ + n_-}$$

2. Magneto-Optical Effects – 2.4 Kerr Effect



- For light propagating along magnetization: normal modes are left- and right circularly polarized waves
- They are propagating as though the magnetic material has refractive indices n_+ for left- and n_- for right-circularly polarized radiation

→ **Circular magnetic birefringence and dichroism (Kerr- and Faraday effects)**

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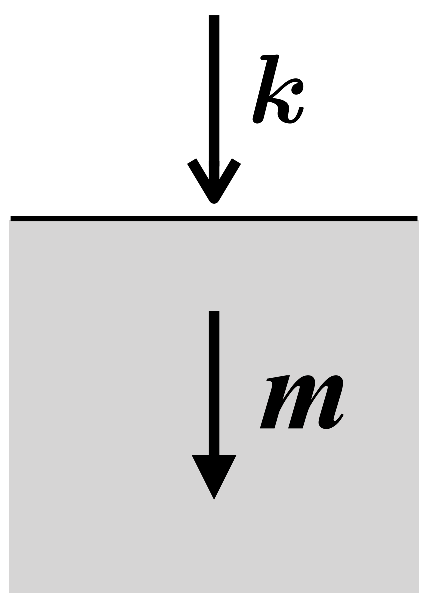


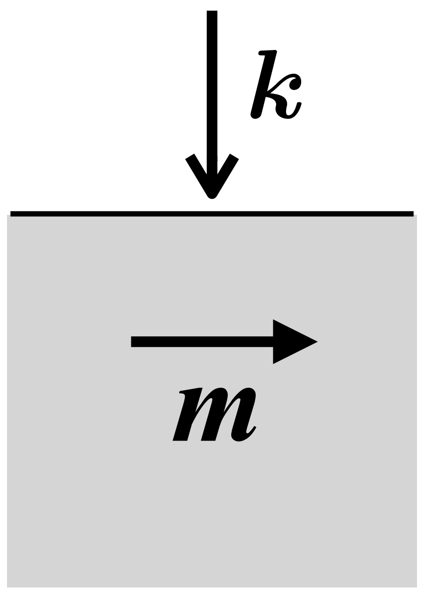
- For light propagating perpendicular to magnetization: normal modes are linearly polarized waves with polarization planes parallel and perpendicular to magnetization direction
- They are propagating as though the magnetic material has refractive indices $n_{||}$ and $n_\perp$

⟶ Both modes propagate with different velocities

⟶ modes are shifted in phase

⟶ after leaving the sample they unify to elliptical wave

→ **Linear magnetic birefringence (Voigt effect)**



2. Magneto-Optical Effects – 2.5 Voigt Effect

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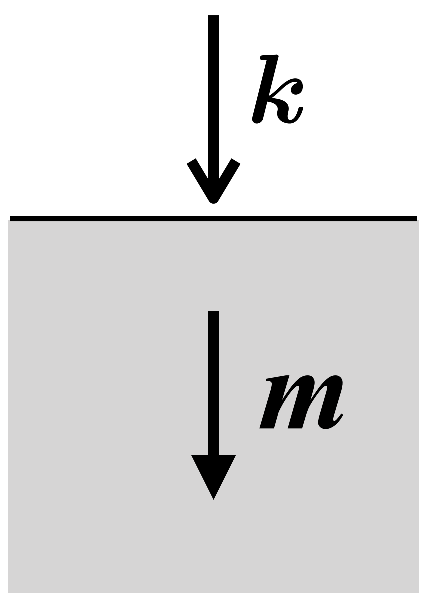

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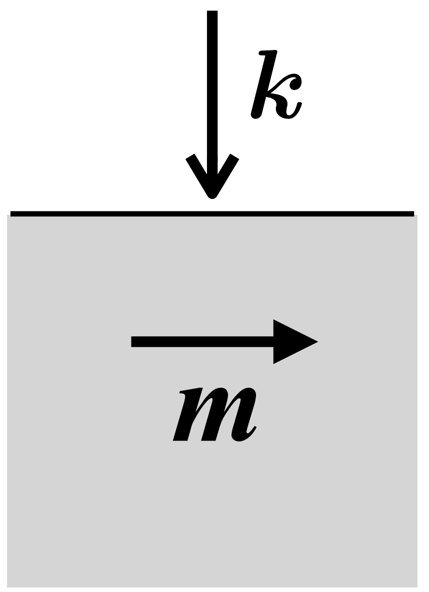

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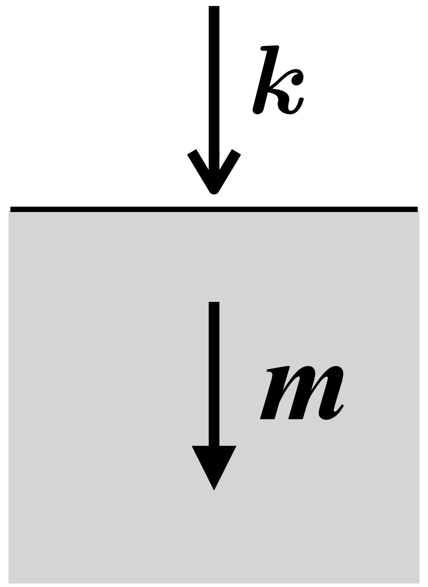

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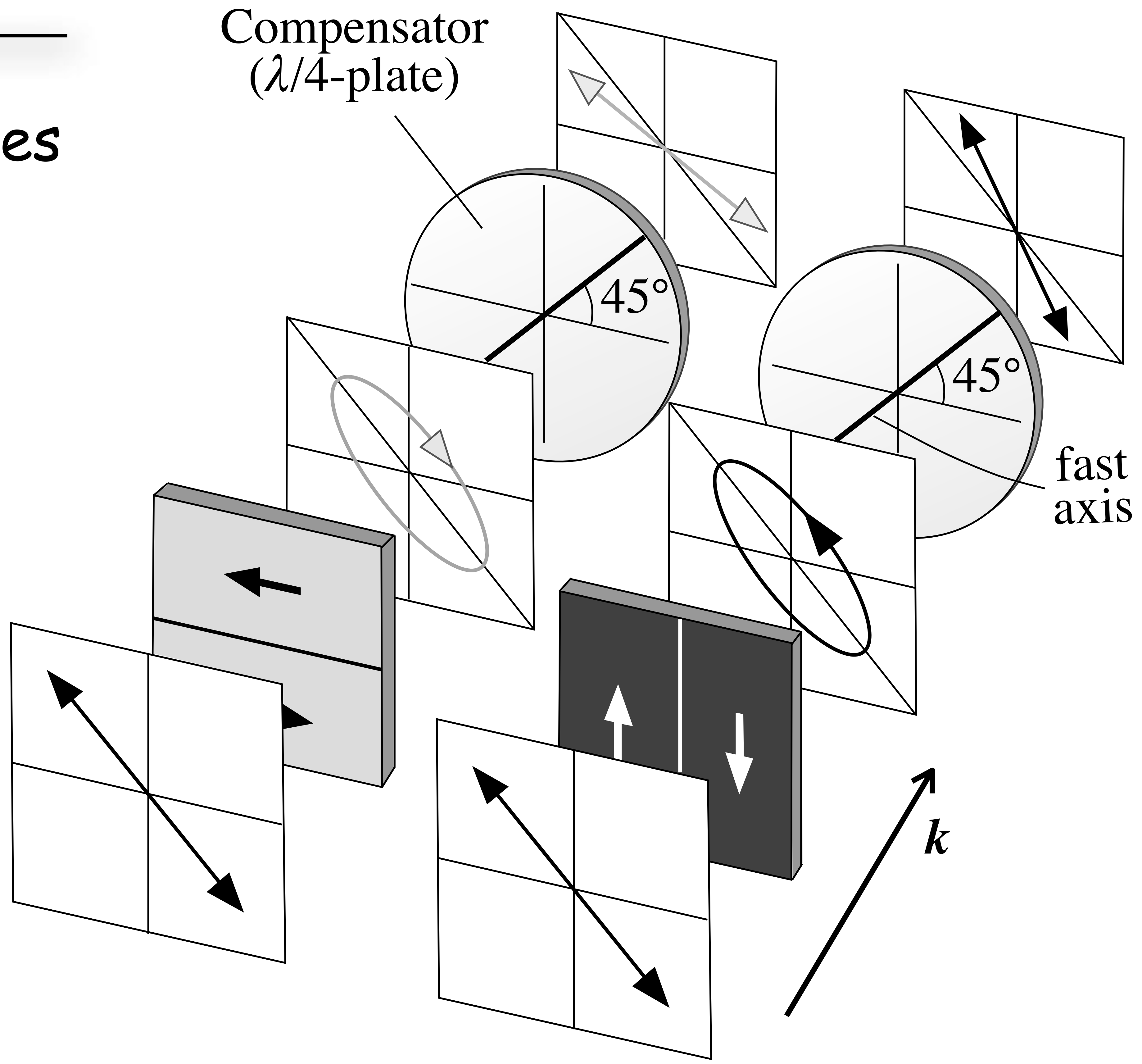


2. Magneto-Optical Effects - 2.5 Voigt Effect

- Dielectric tensors, for materials with **cubic** crystal symmetry:

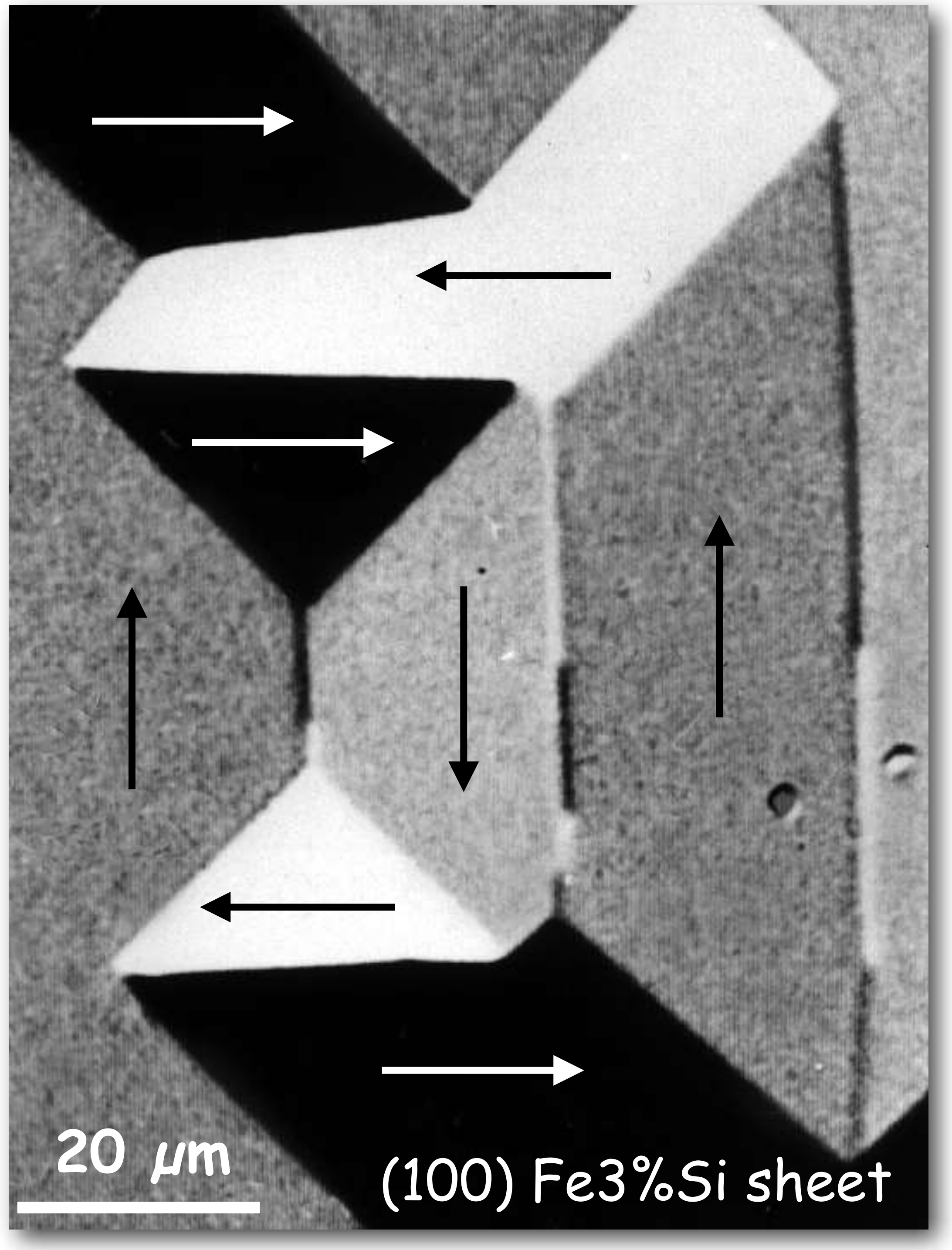
$$\epsilon = \epsilon_{\text{iso}} \begin{pmatrix} 1 & -\mathrm{i}Q_{\mathrm{V}}m_3 & \mathrm{i}Q_{\mathrm{V}}m_2 \\ \mathrm{i}Q_{\mathrm{V}}m_3 & 1 & -\mathrm{i}Q_{\mathrm{V}}m_1 \\ -\mathrm{i}Q_{\mathrm{V}}m_2 & \mathrm{i}Q_{\mathrm{V}}m_1 & 1 \end{pmatrix} + \begin{pmatrix} B_1 m_1^2 & B_2 m_1 m_2 & B_2 m_1 m_3 \\ B_2 m_1 m_2 & B_1 m_2^2 & B_2 m_2 m_3 \\ B_2 m_1 m_3 & B_2 m_2 m_3 & B_1 m_3^2 \end{pmatrix}$$

Kerr effect

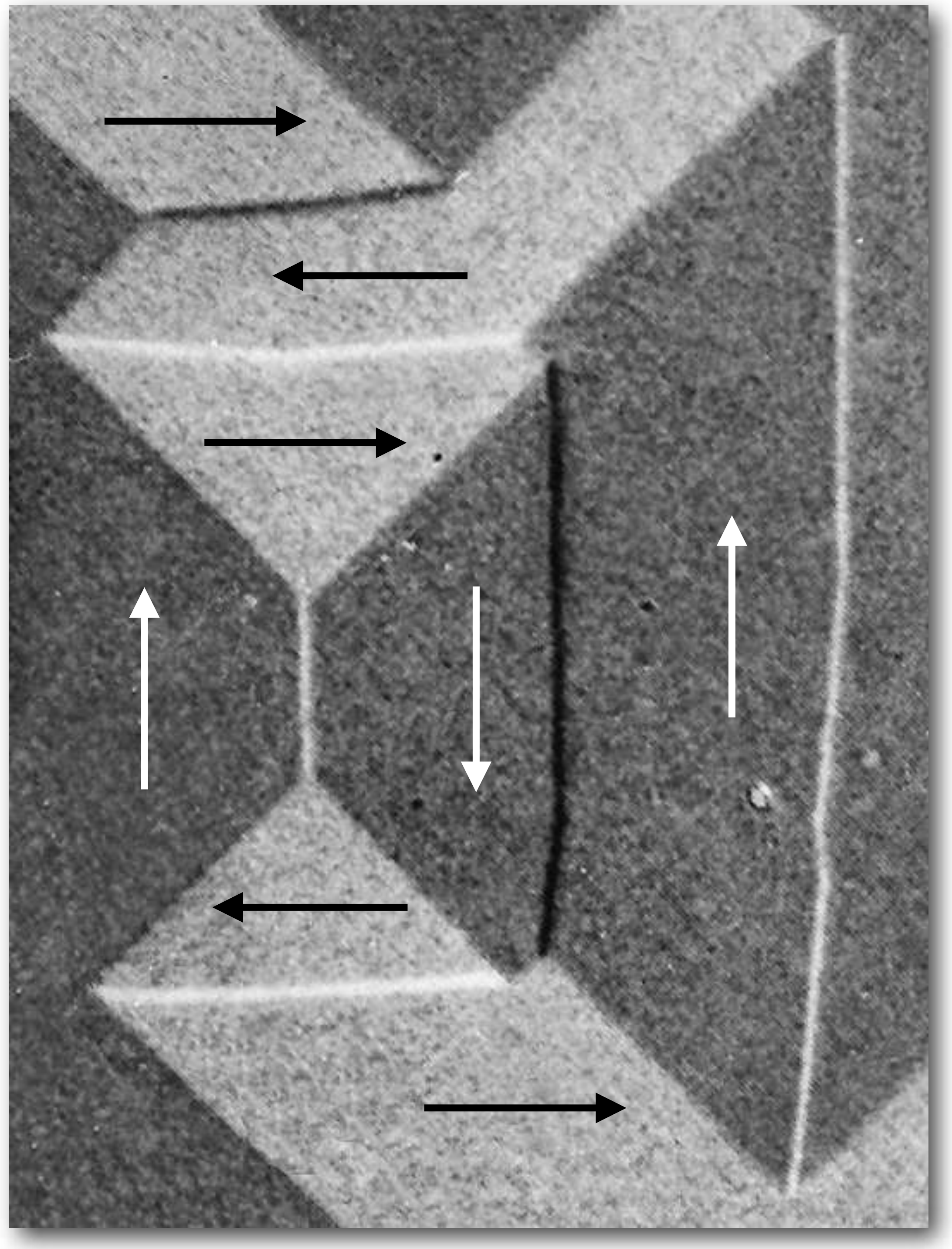
(Intrinsic) Voigt effect

Voigt effect is quadratic in magnetization

- polarizer $\parallel$
- oblique incidence



Kerr Effect



Voigt Effect

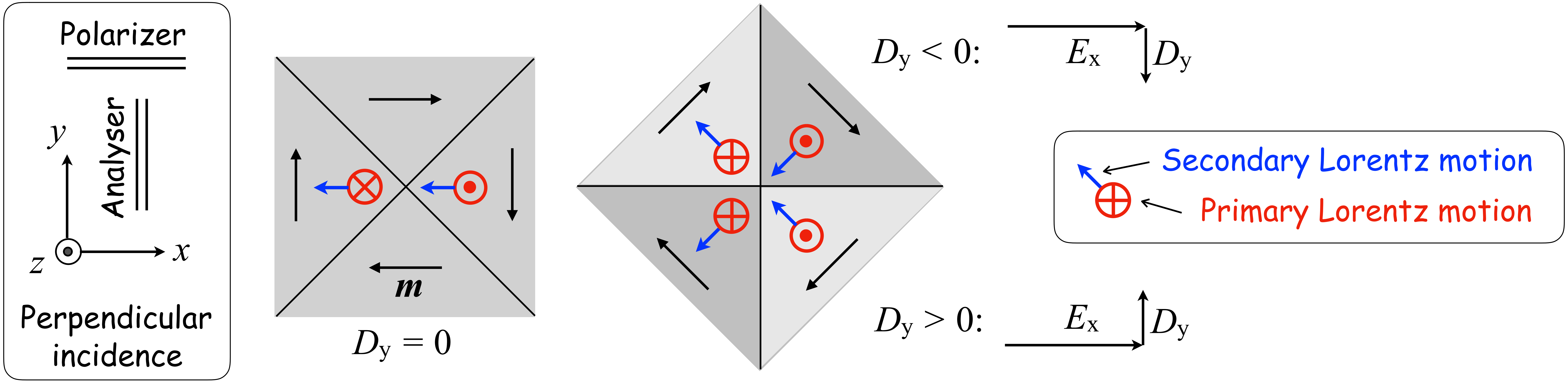
- polarizer: 45° to $\boldsymbol{m}$ $//$
- perpendicular incidence, compensator

- Dielectric tensors, for materials with **cubic** crystal symmetry:

$$\boldsymbol{\epsilon} = \epsilon_{\mathrm{iso}} \begin{pmatrix} 1 & -\mathrm{i}Q_{\mathrm{V}}m_3 & \mathrm{i}Q_{\mathrm{V}}m_2 \\ \mathrm{i}Q_{\mathrm{V}}m_3 & 1 & -\mathrm{i}Q_{\mathrm{V}}m_1 \\ -\mathrm{i}Q_{\mathrm{V}}m_2 & \mathrm{i}Q_{\mathrm{V}}m_1 & 1 \end{pmatrix}$$

Kerr effect

- Quadratic effect, derived from gyrotropic interaction $\boldsymbol{m} \times \boldsymbol{E}$ (like Kerr effect):



$$\Big(-iQ_V m_2 \quad iQ_V m_1 \quad 1 \Big)$$

Kerr effect

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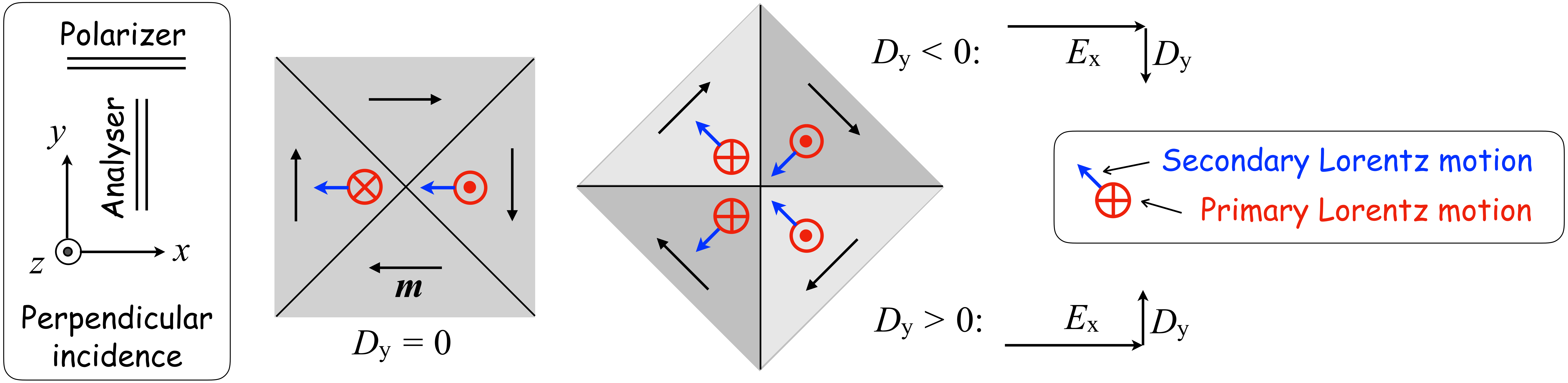
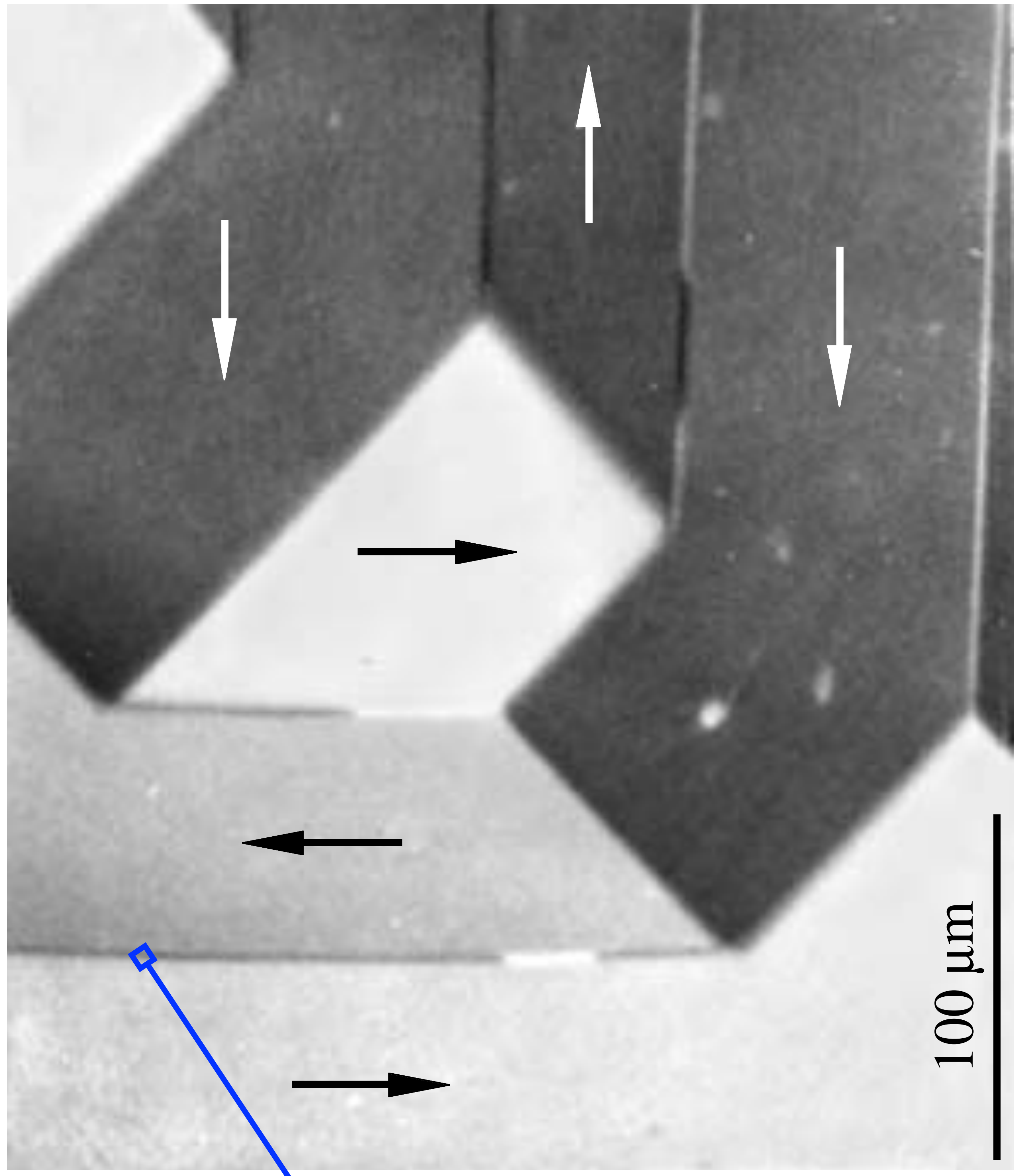


Reflection: $\uparrow E_y$ and $\downarrow E_y$

Matching of phase of one E_y component with E_x by compensator

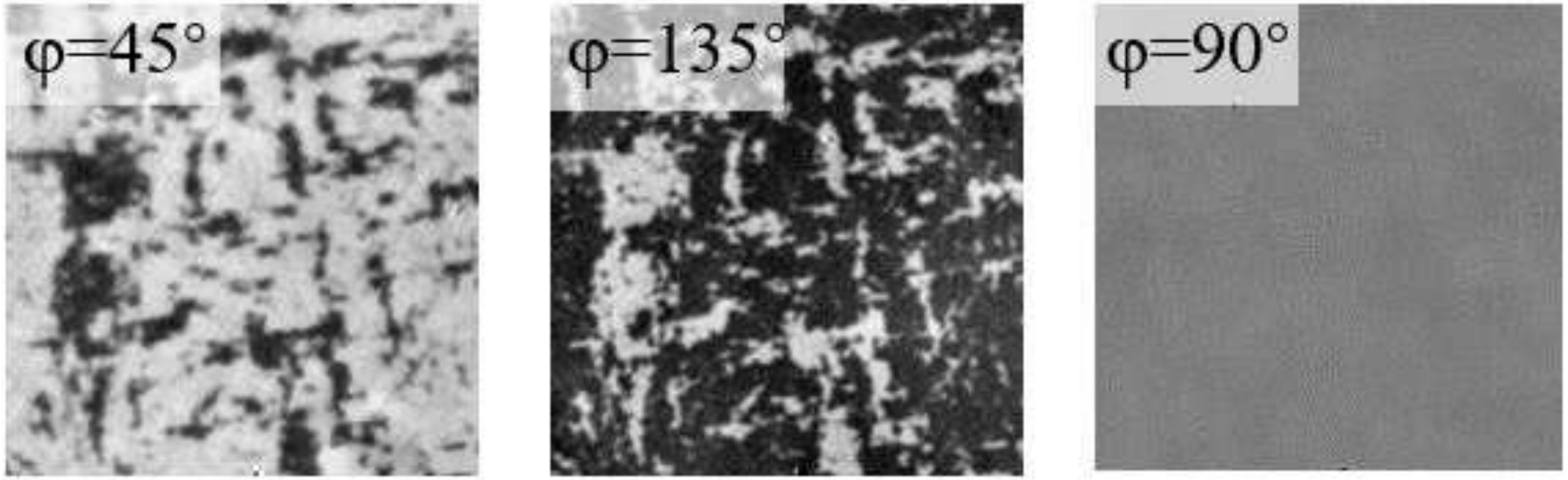
$\Longrightarrow$ black-white contrast of domain boundary

Application: Imaging of domains in transparent garnet crystals



Wall contrast due to polar Faraday effect

Imaging of domains in antiferromagnets

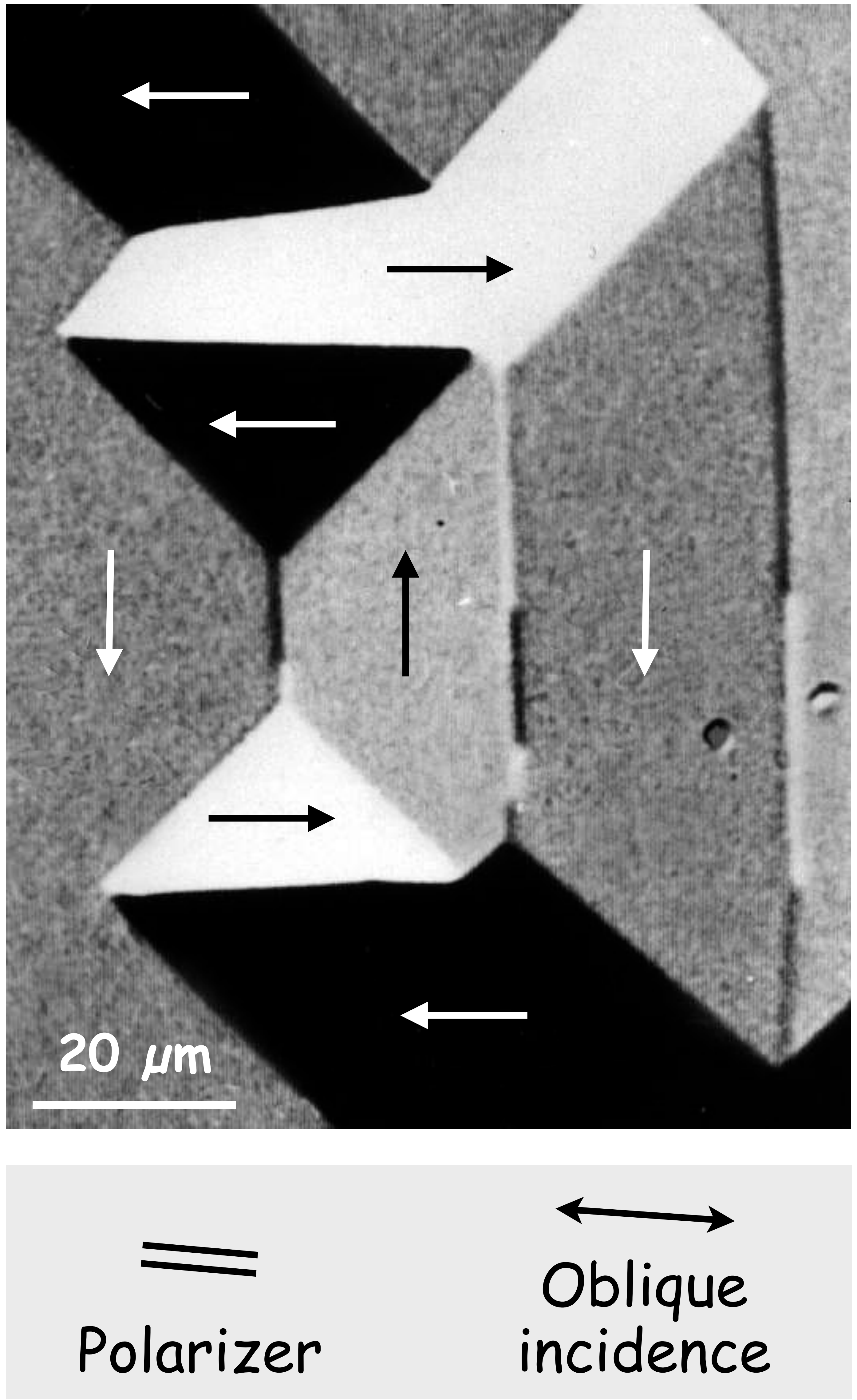


Imaging Antiferromagnetic Domains in Nickel-oxide Thin Films by Magneto-optical Voigt Effect

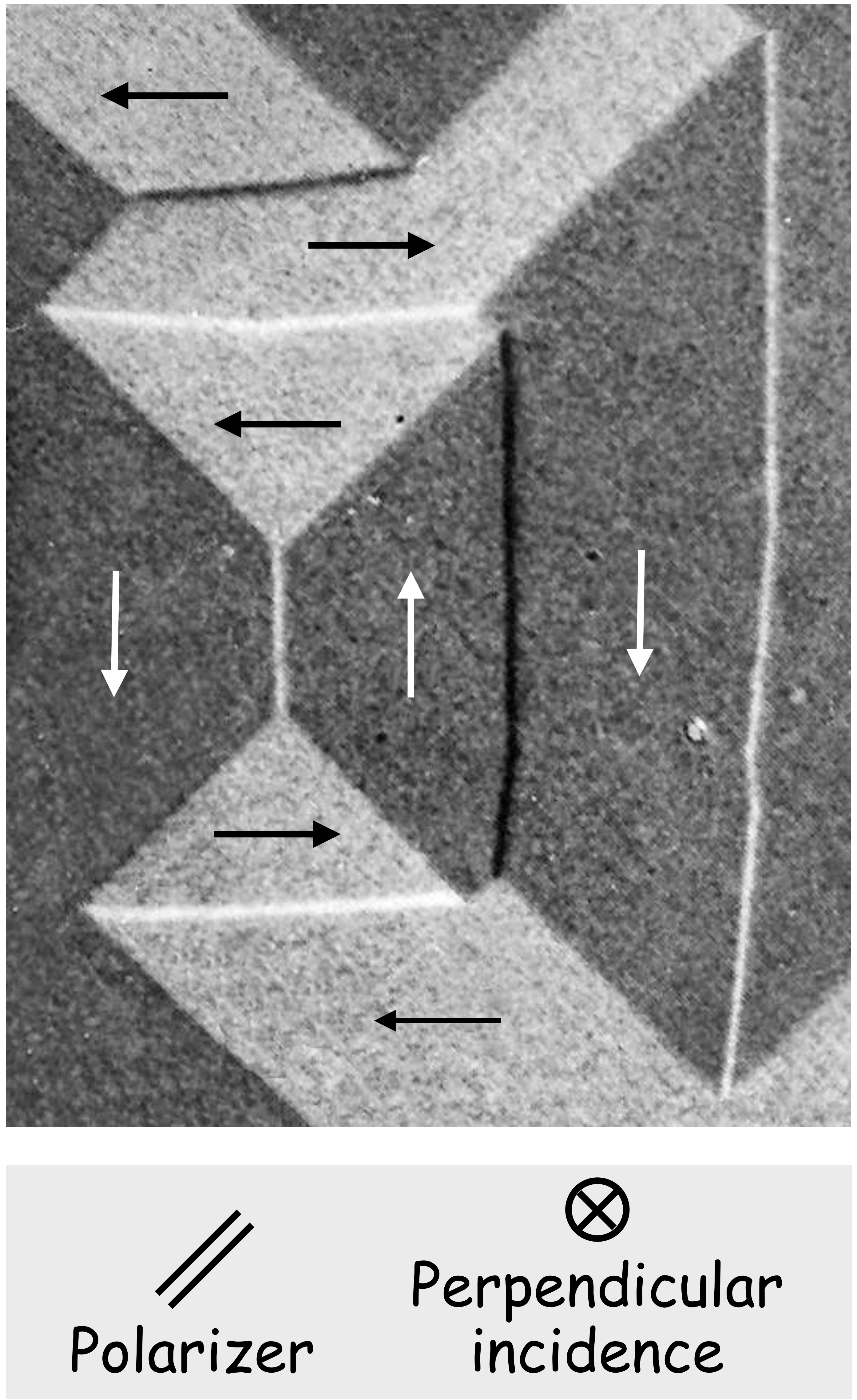
Jia Xu[1], Chao Zhou[1], Mengwen Jia[1], Dong Shi[1], Changqing Liu[1], Haoran Chen[1], Gong Chen[2], Guanhua Zhang[3], Yu Liang[3], Junqin Li[4], Wei Zhang[5]*, Yizheng Wu[1,6]*

https://arxiv.org/pdf/1906.06844.pdf

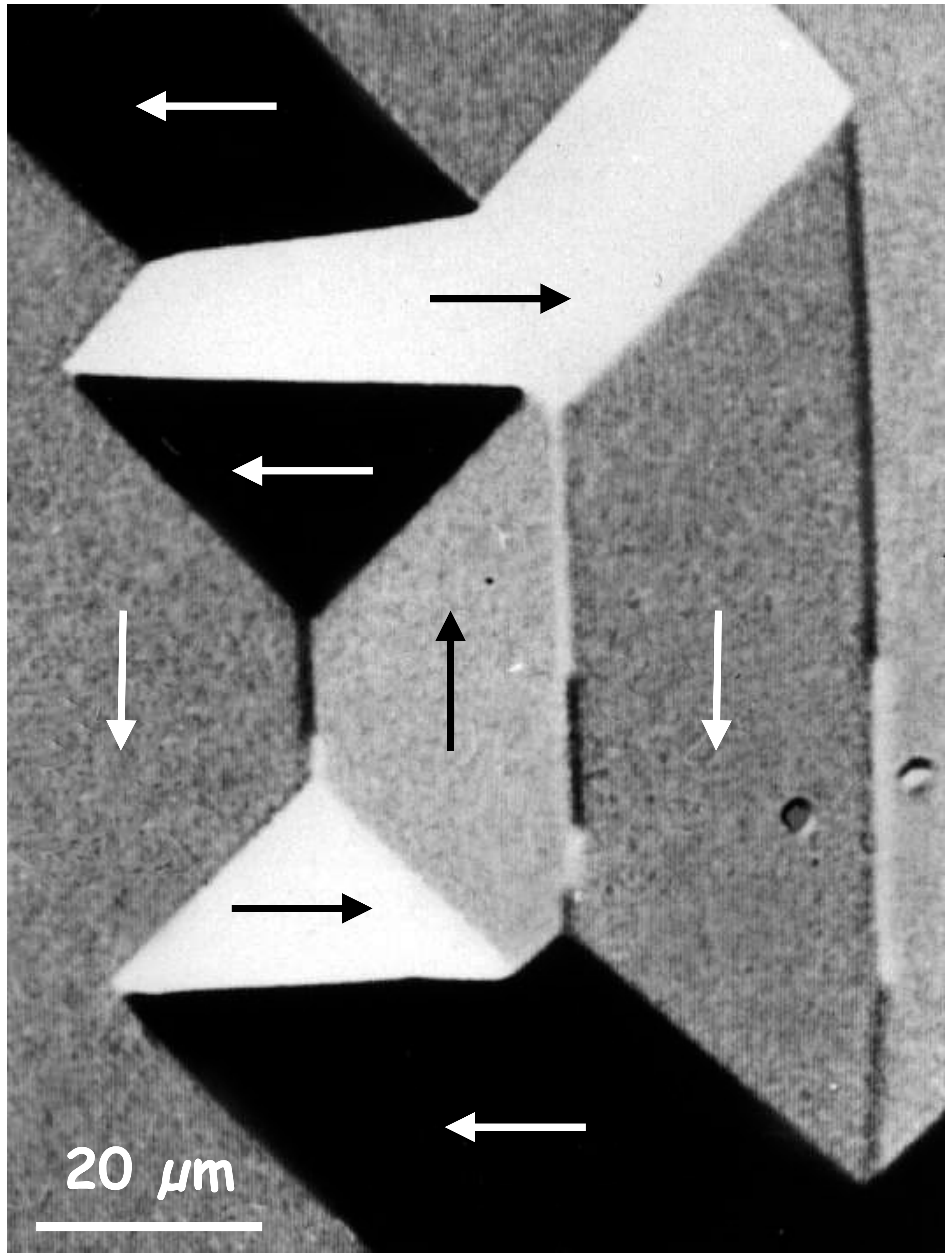
(Longitudinal) Kerr effect



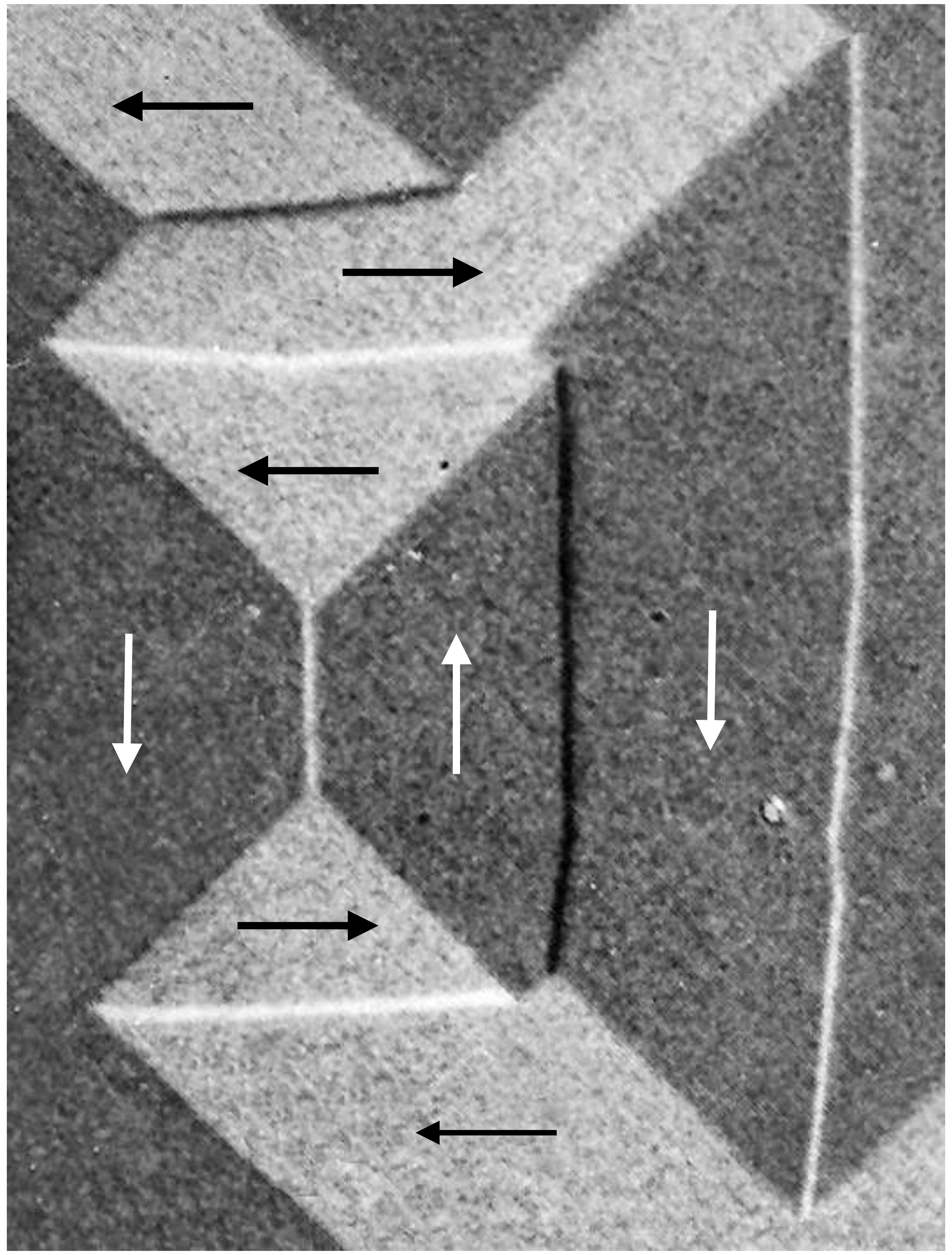
Voigt-

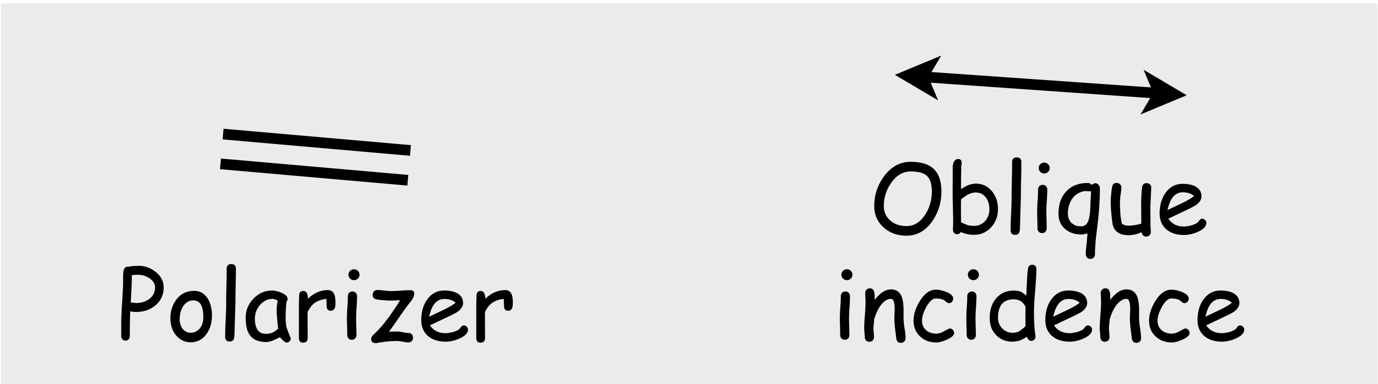


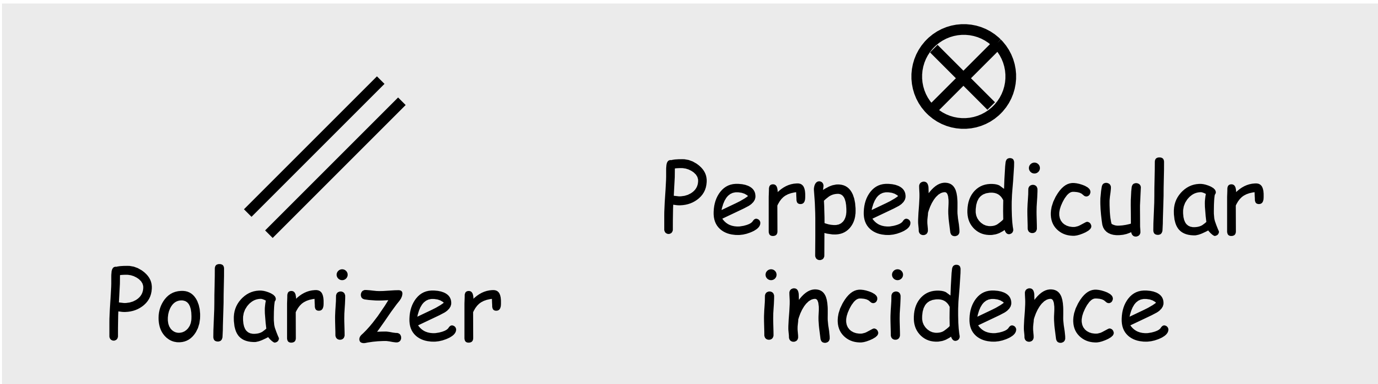
(Longitudinal) Kerr effect



Voigt- and Gradient effect







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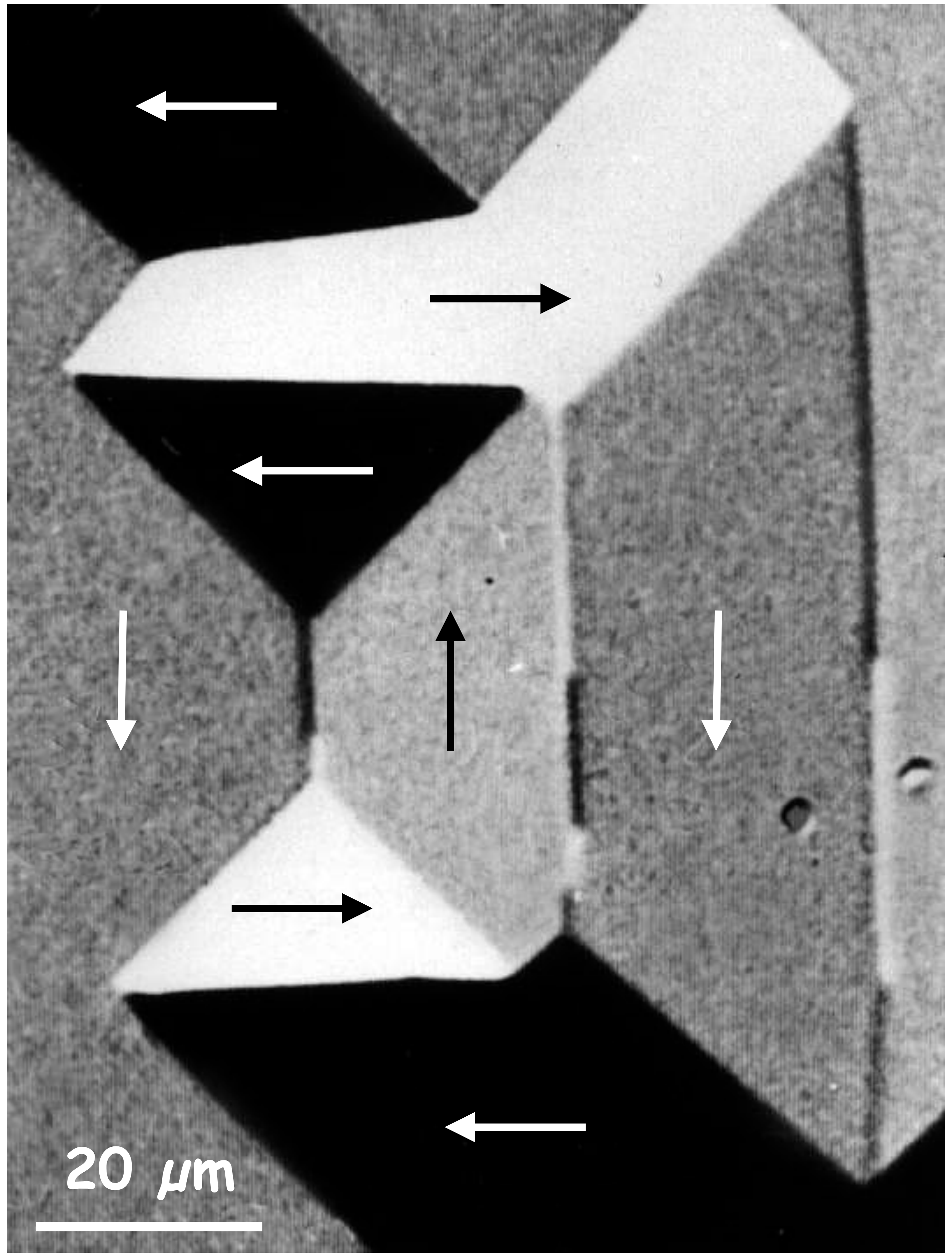
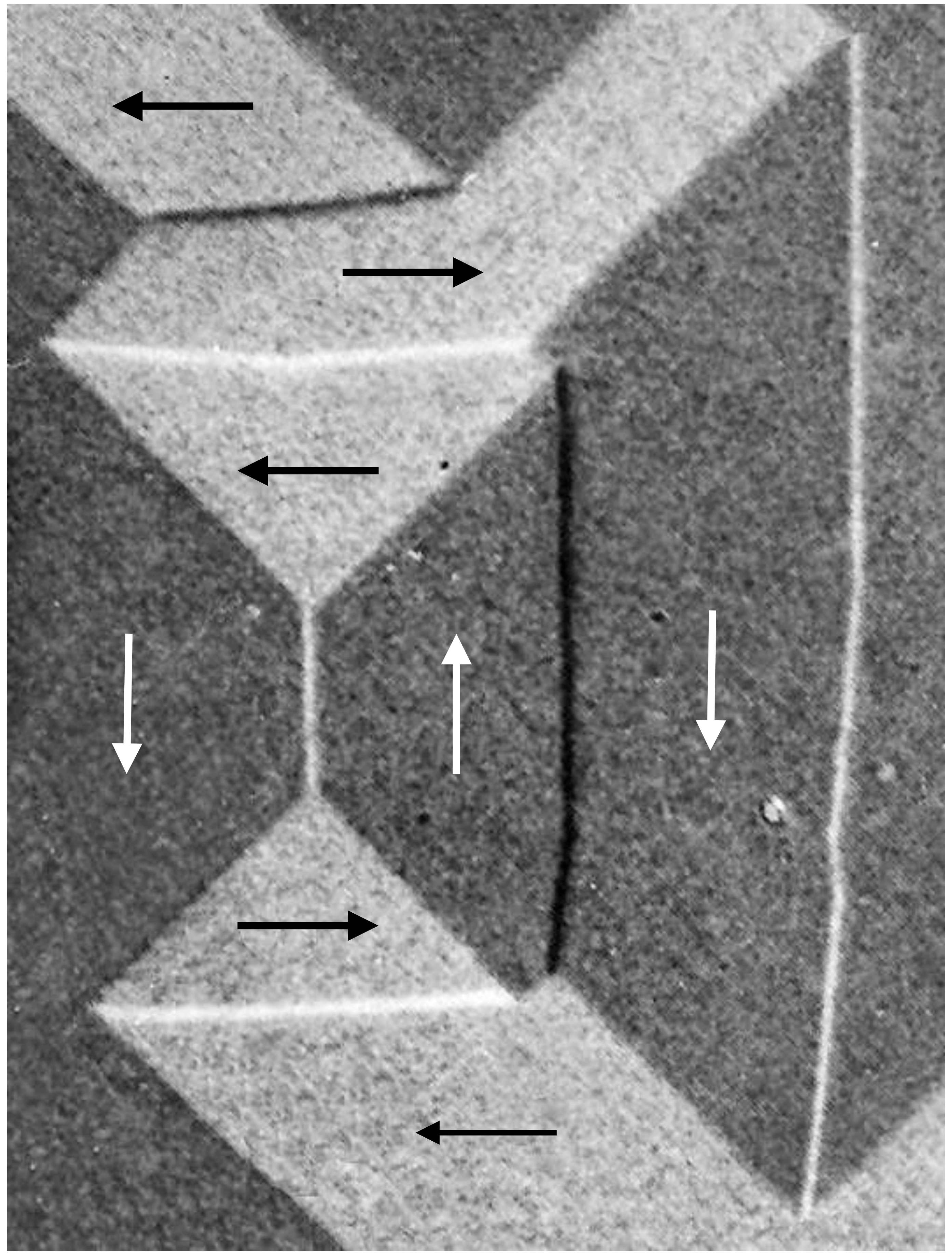


Polarizer

Oblique incidence

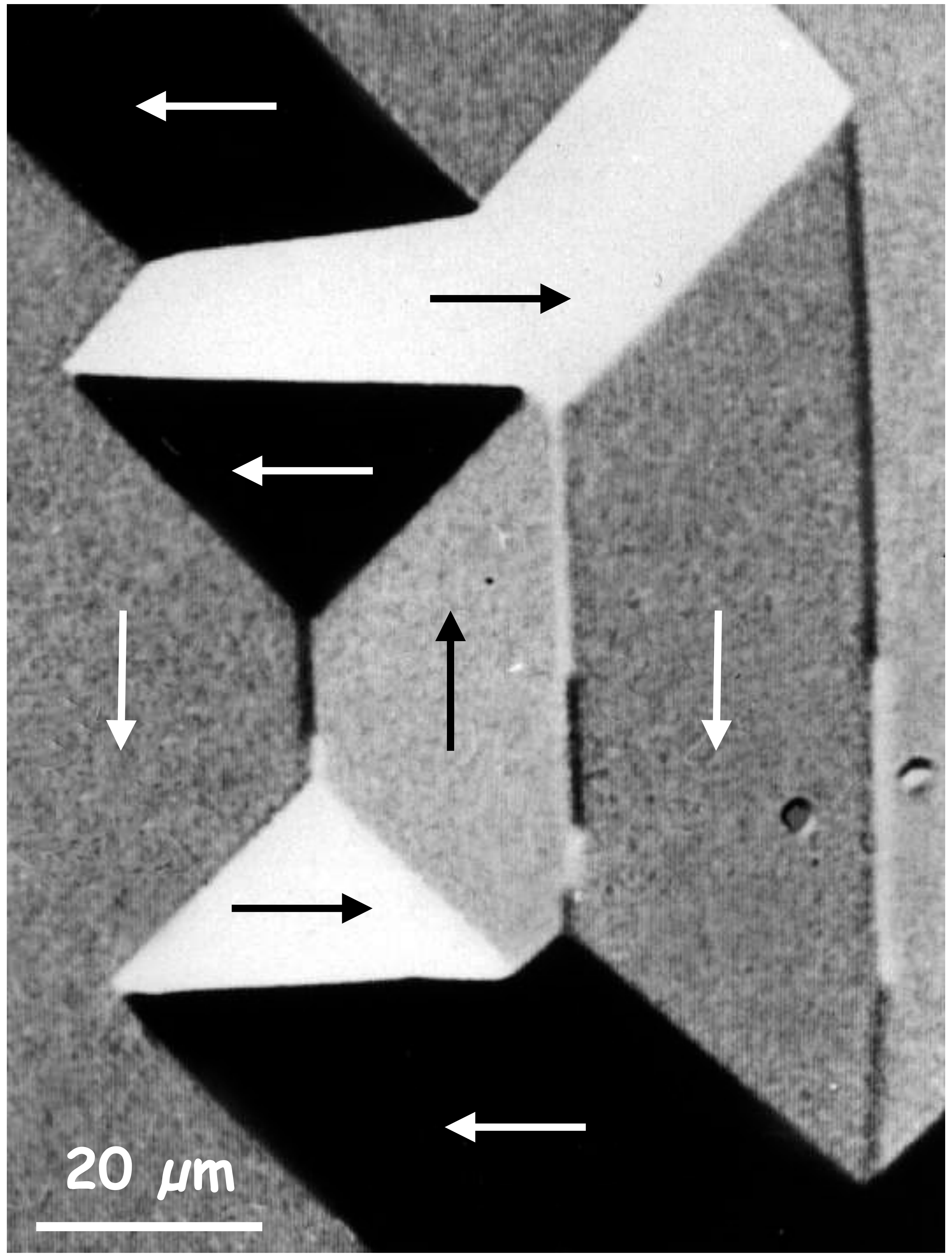
Voigt- and Gradient effect



Polarizer

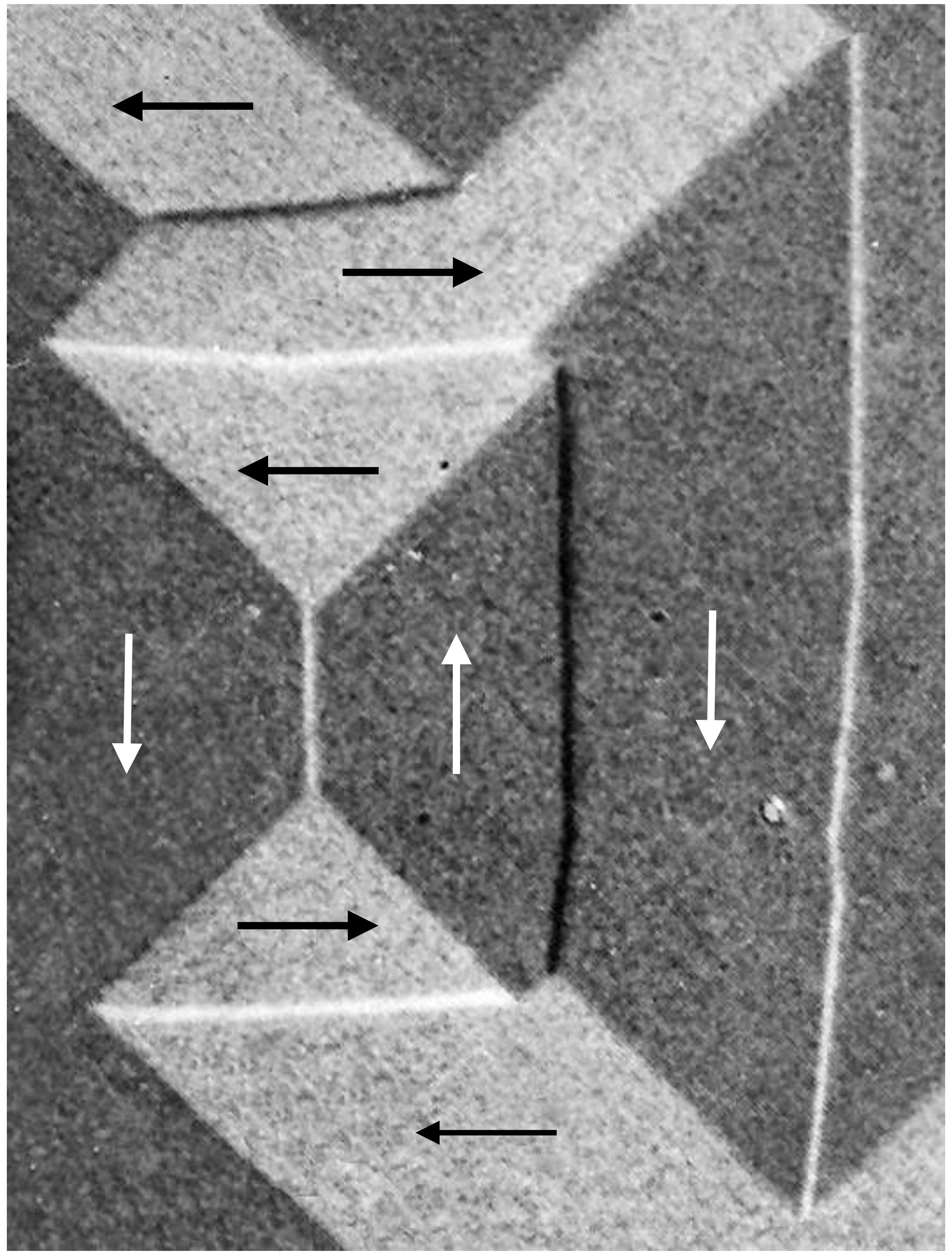
Perpendicular incidence

(Longitudinal) Kerr effect



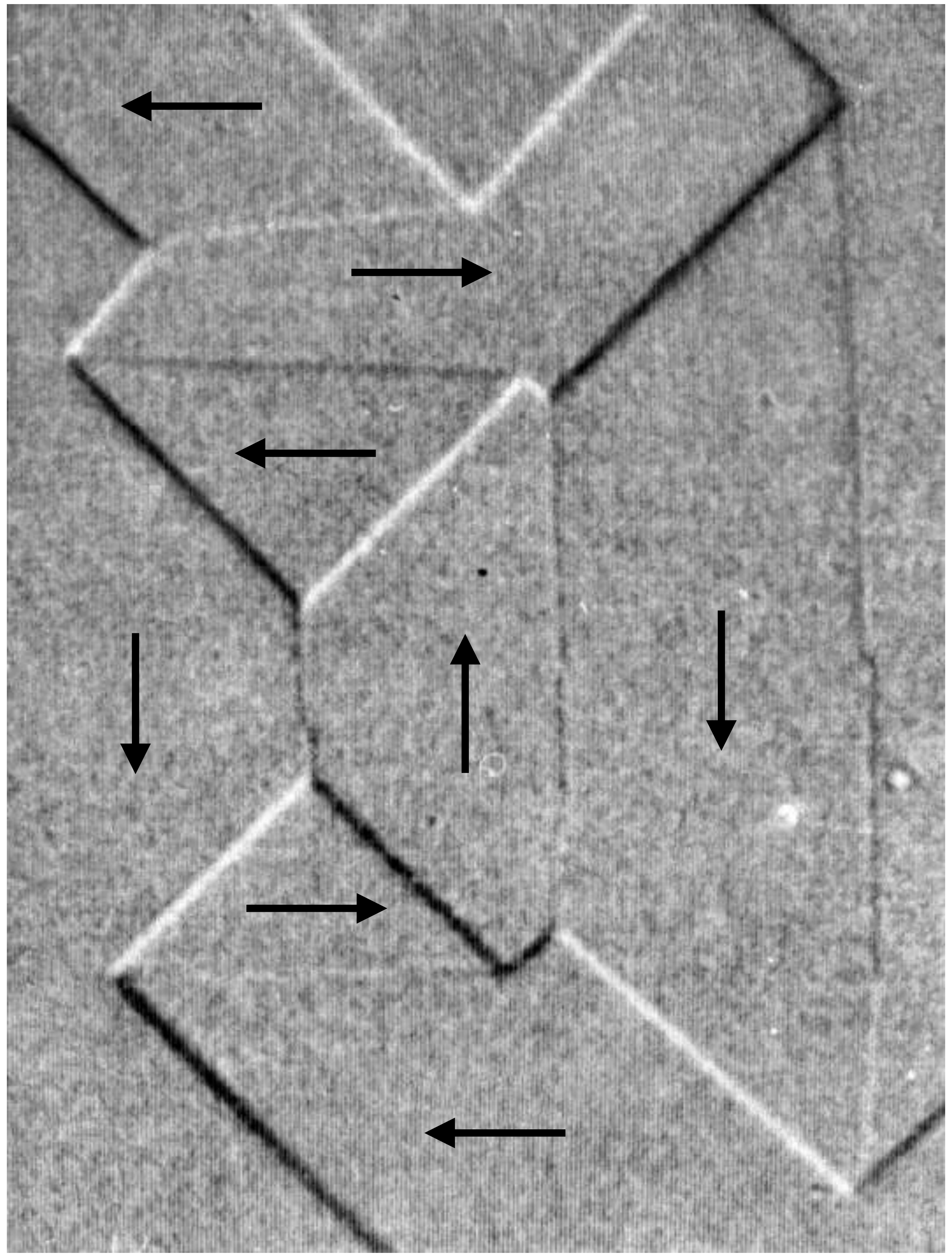
Polarizer | Oblique incidence

Voigt- and Gradient effect



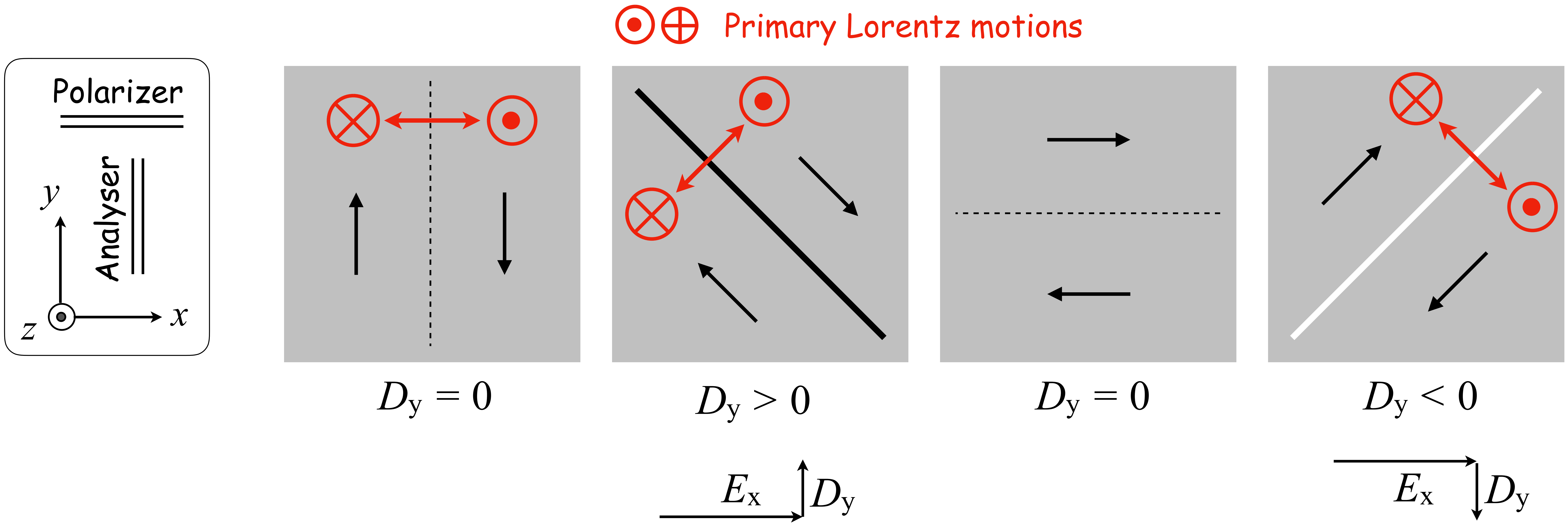
Polarizer | Perpendicular incidence

Gradient effect



Polarizer | Perpendicular incidence

- Component of dielectric tensor: $D_y = P(-\partial m_y/\partial y)E_x$



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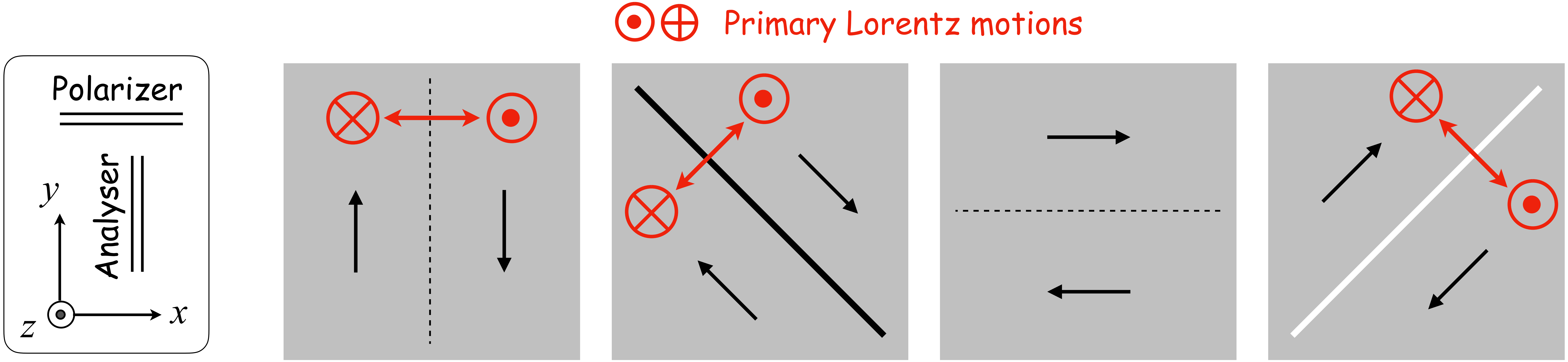


$D_y = 0$ | $D_y > 0$ | $D_y = 0$ | $D_y < 0$

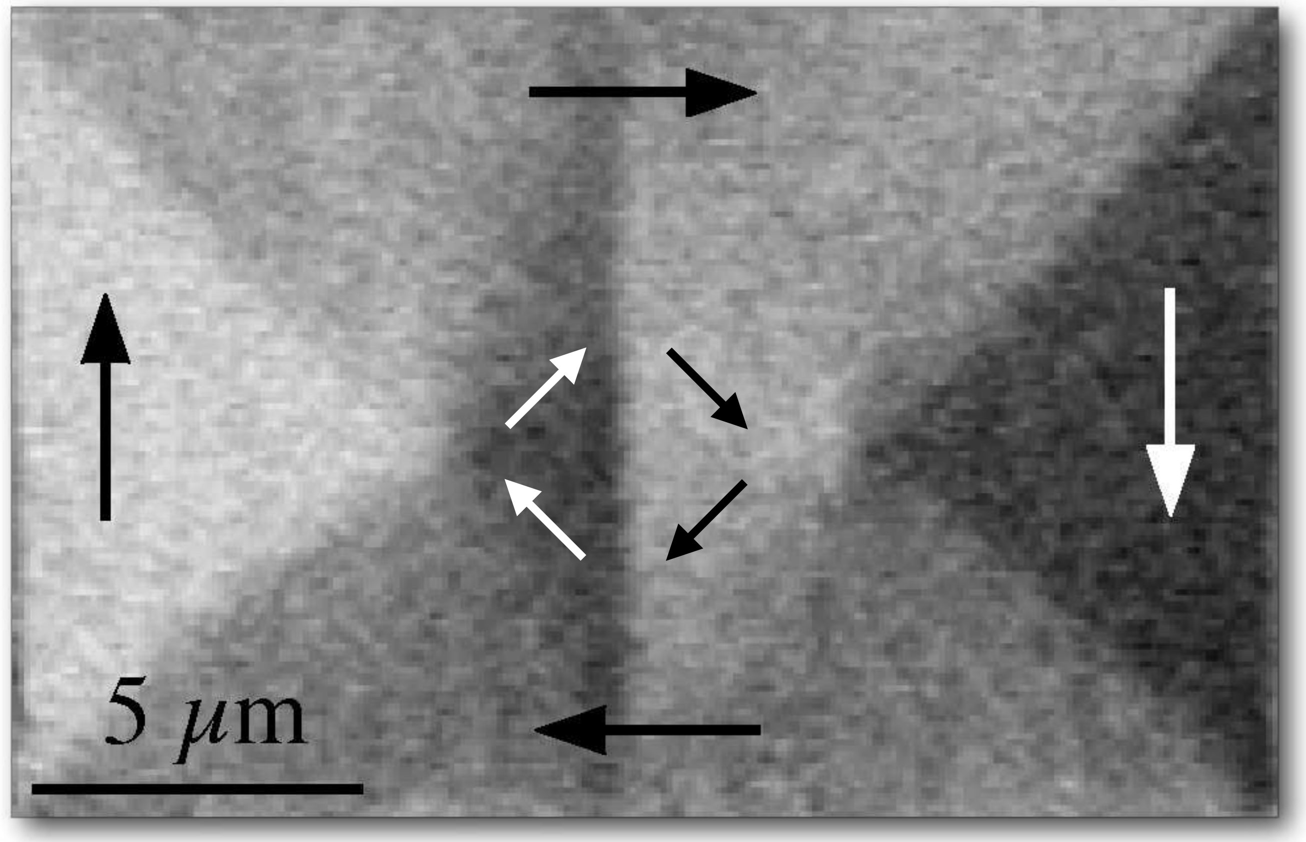
E_x $\uparrow D_y$ | E_x $\downarrow D_y$

Reflection: $\uparrow E_y$ and $\downarrow E_y$

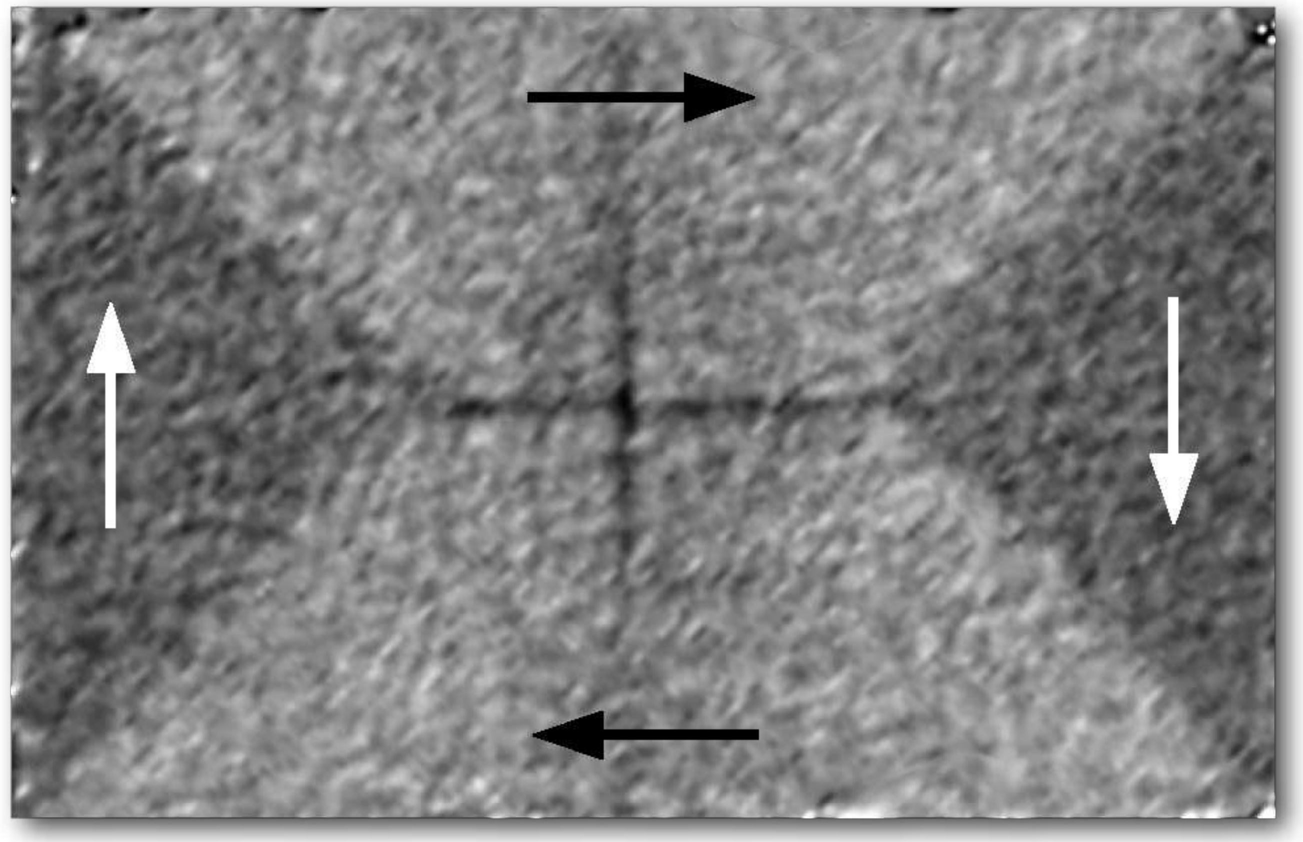
Matching of phase of one E_y component with E_x by compensator
$\Longrightarrow$ black-white contrast of domain boundary

Application:

Crosstie wall in NiFe-film (50 nm thick)

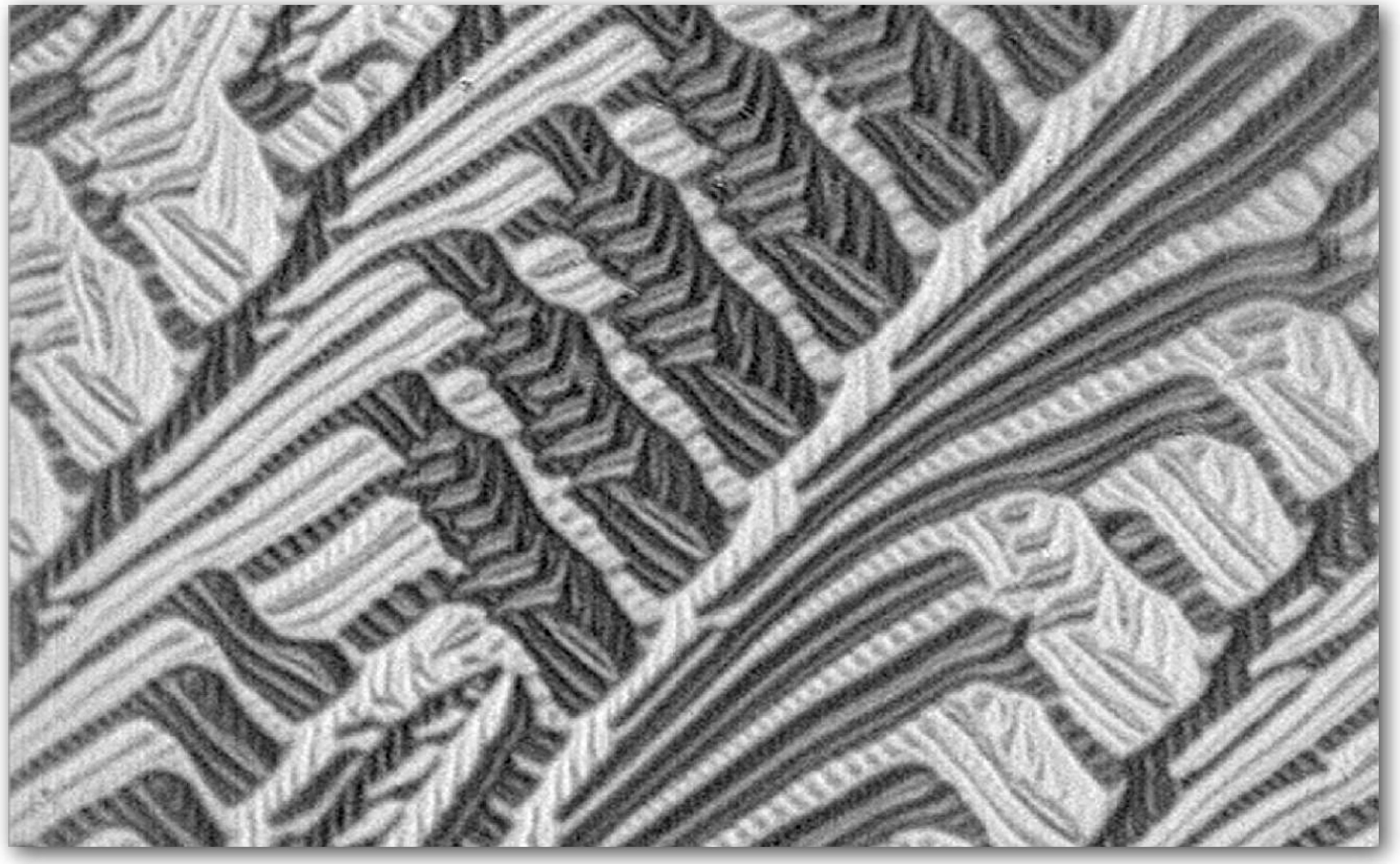


Kerr-contrast

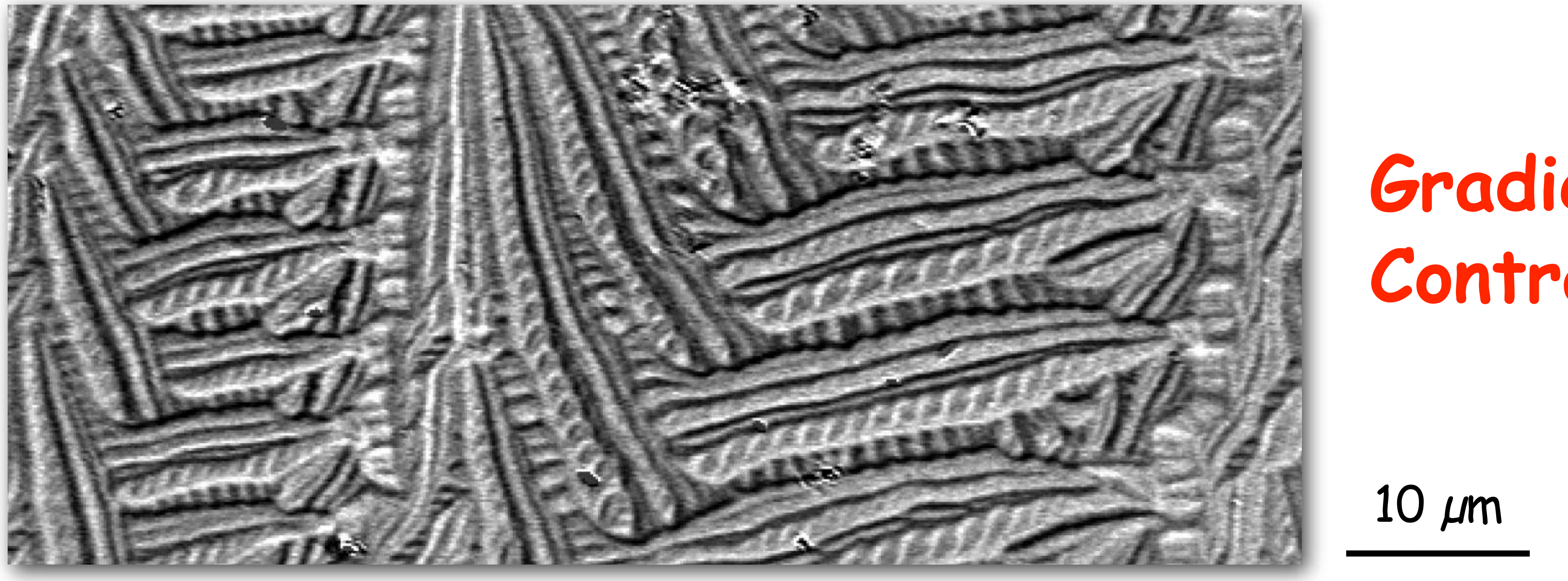


Voigt- and Gradient-Contrast

FeSi (111) surface



Kerr-contrast



Gradient-Contrast

Michael Faraday
(1791-1867)

1849: small change of polarization plane due to magneto-optic interaction in transmission, circular birefringence & dichroism, ~ M

Fowler and Fryer, 1956

John Kerr
(1824-1907)

1877: small change of polarization plane due to magneto-optic interaction in reflection, circular birefringence, ~ M

Williams et al., 1951;

Fowler and Fryer, 1952

Woldemar Voigt
(1850-1919)

1898: small change of polarization state due to magneto-optic interaction in reflection, linear birefringence, ~ M^2

Transmission:
Dillon, 1958

Reflection,
together with Gradient effect:
R.S. and Hubert, 1990

2. Magneto-Optical Effects – Summary

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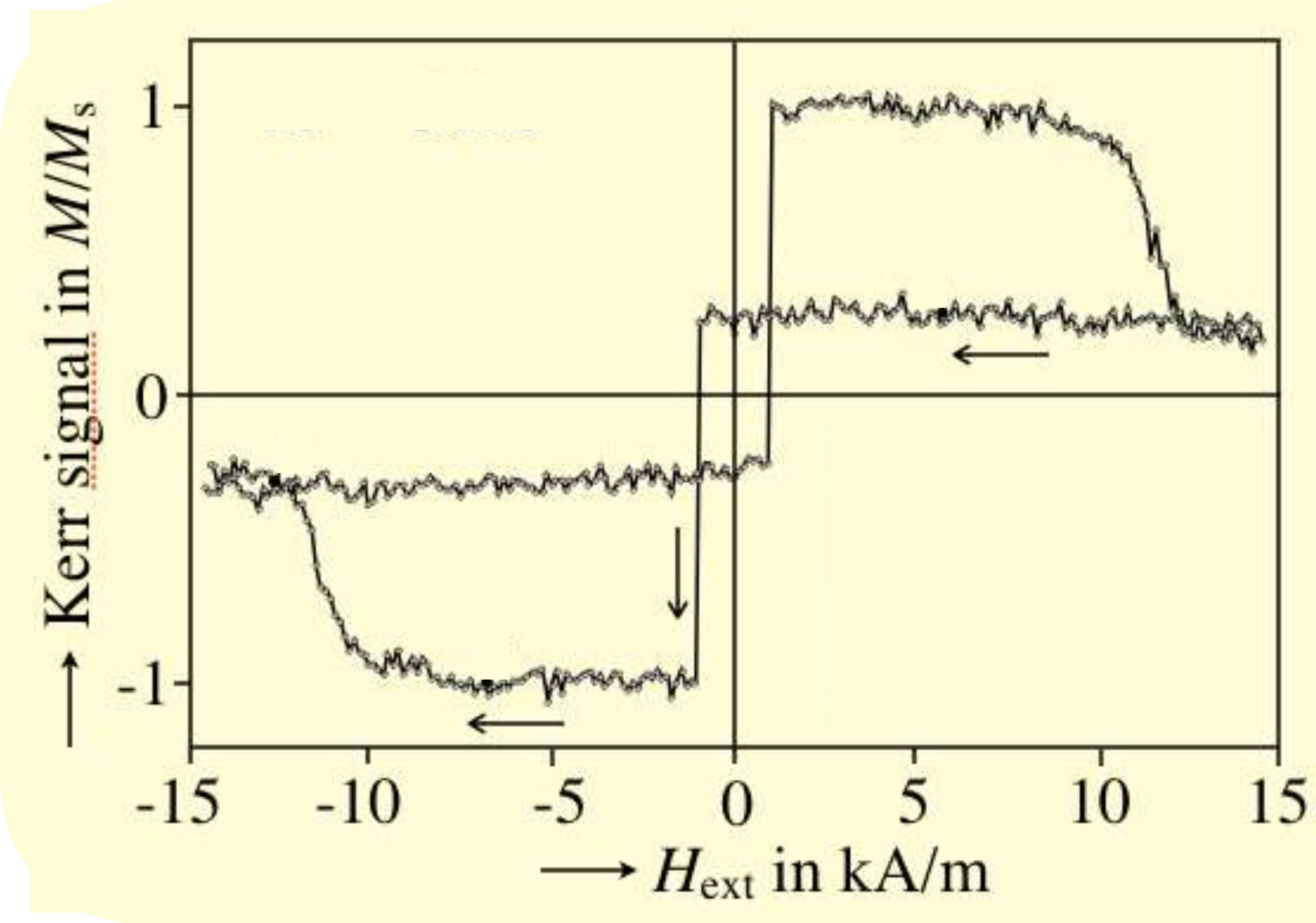
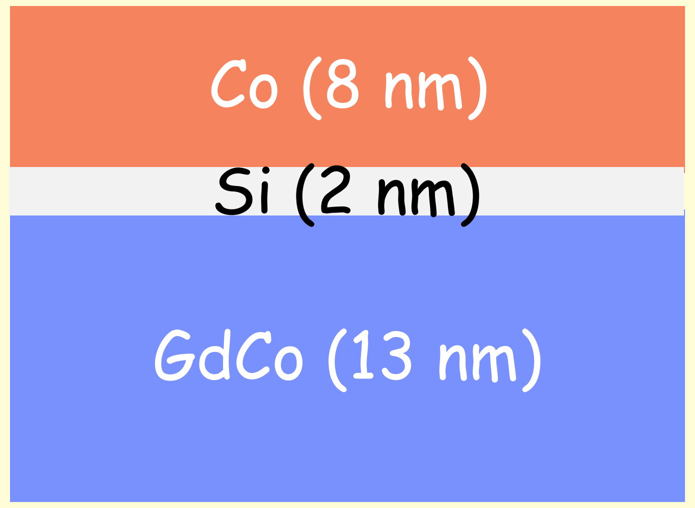
3.
MOKE Magnetometry

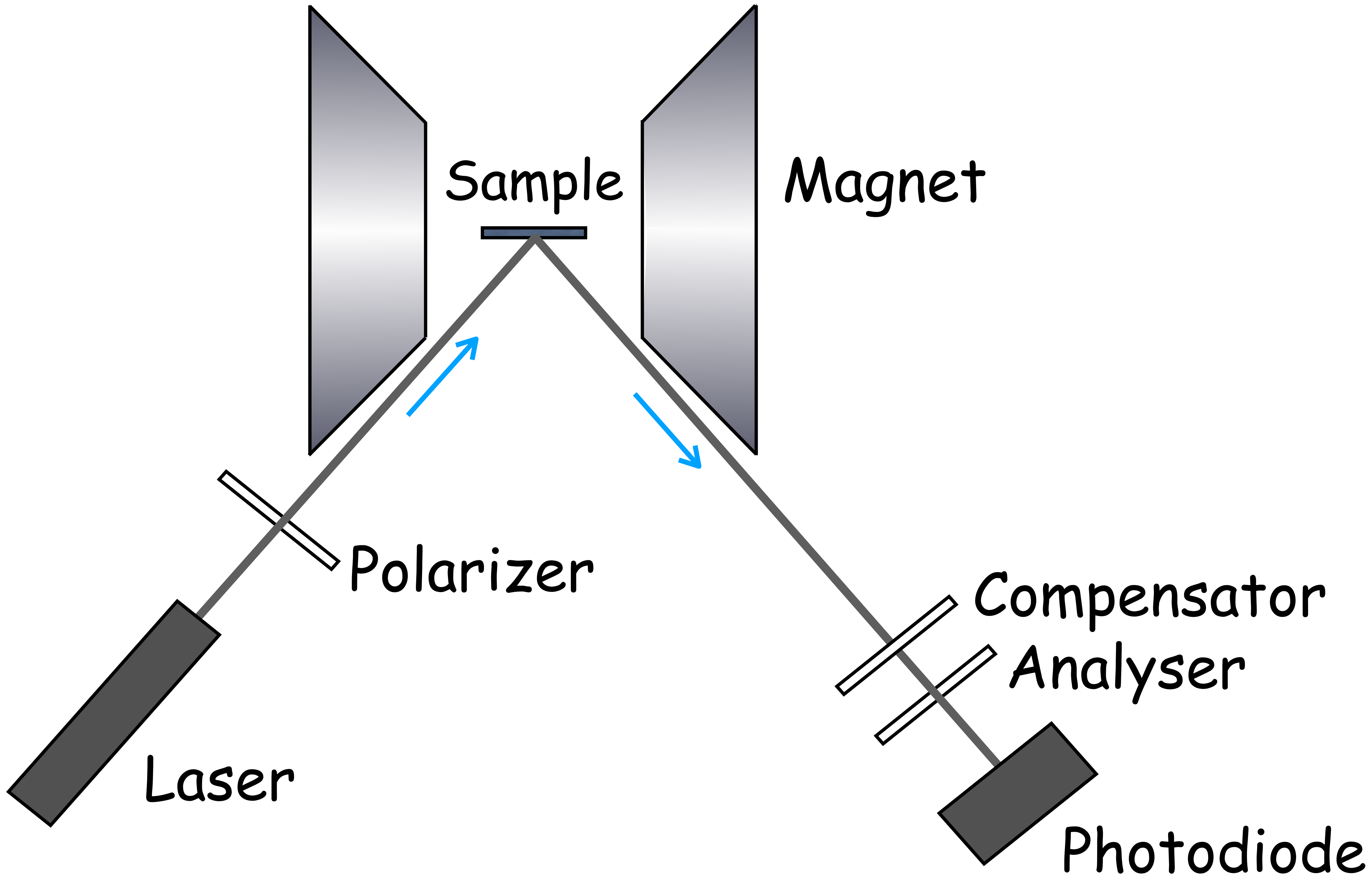
3. MOKE Magnetometry

- Principle: measure and plot Kerr intensity as function of $\boldsymbol{H}$





A. Svalov et al., J. Alloys and Compounds 615 (2014)



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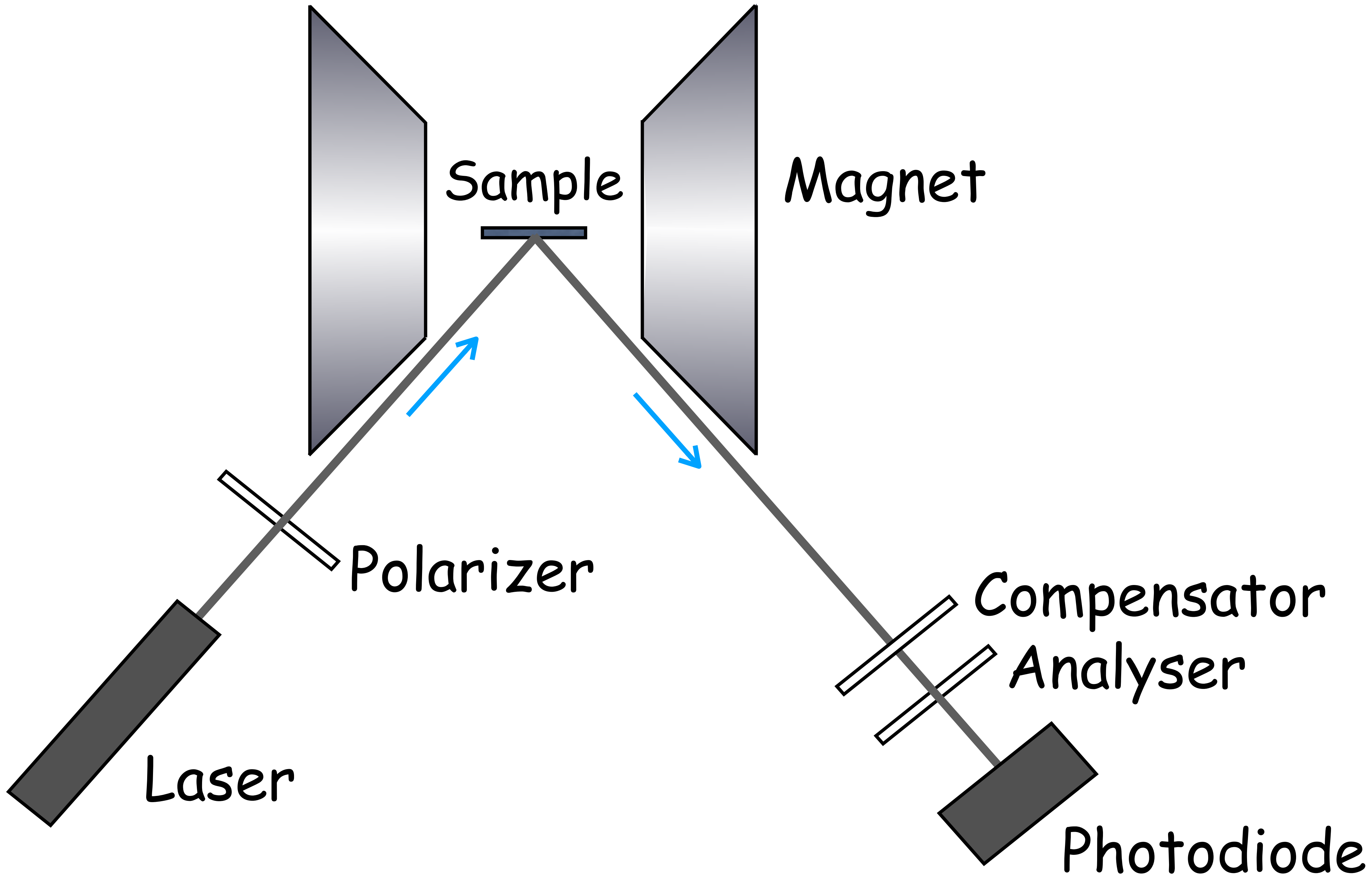

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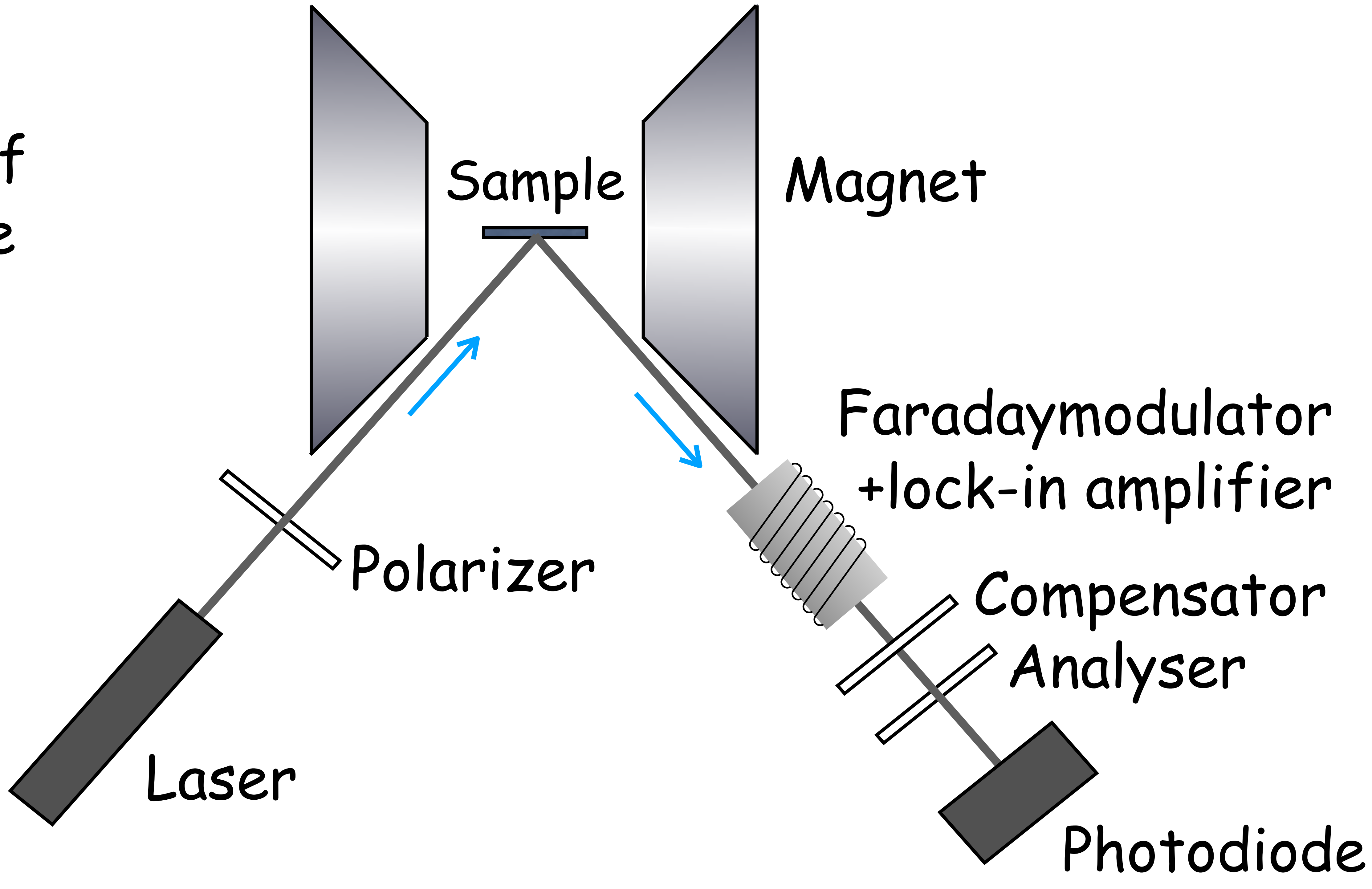

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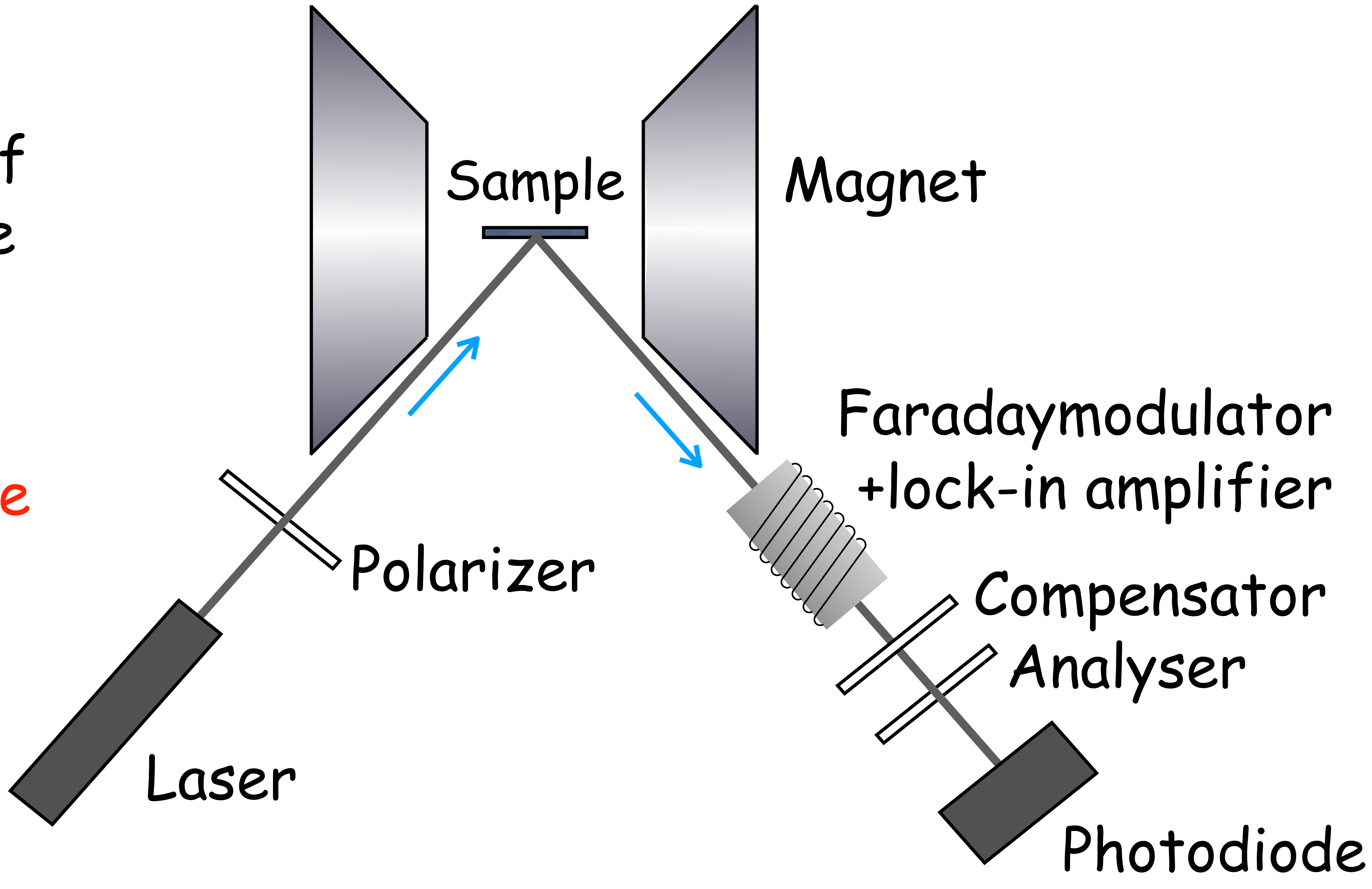

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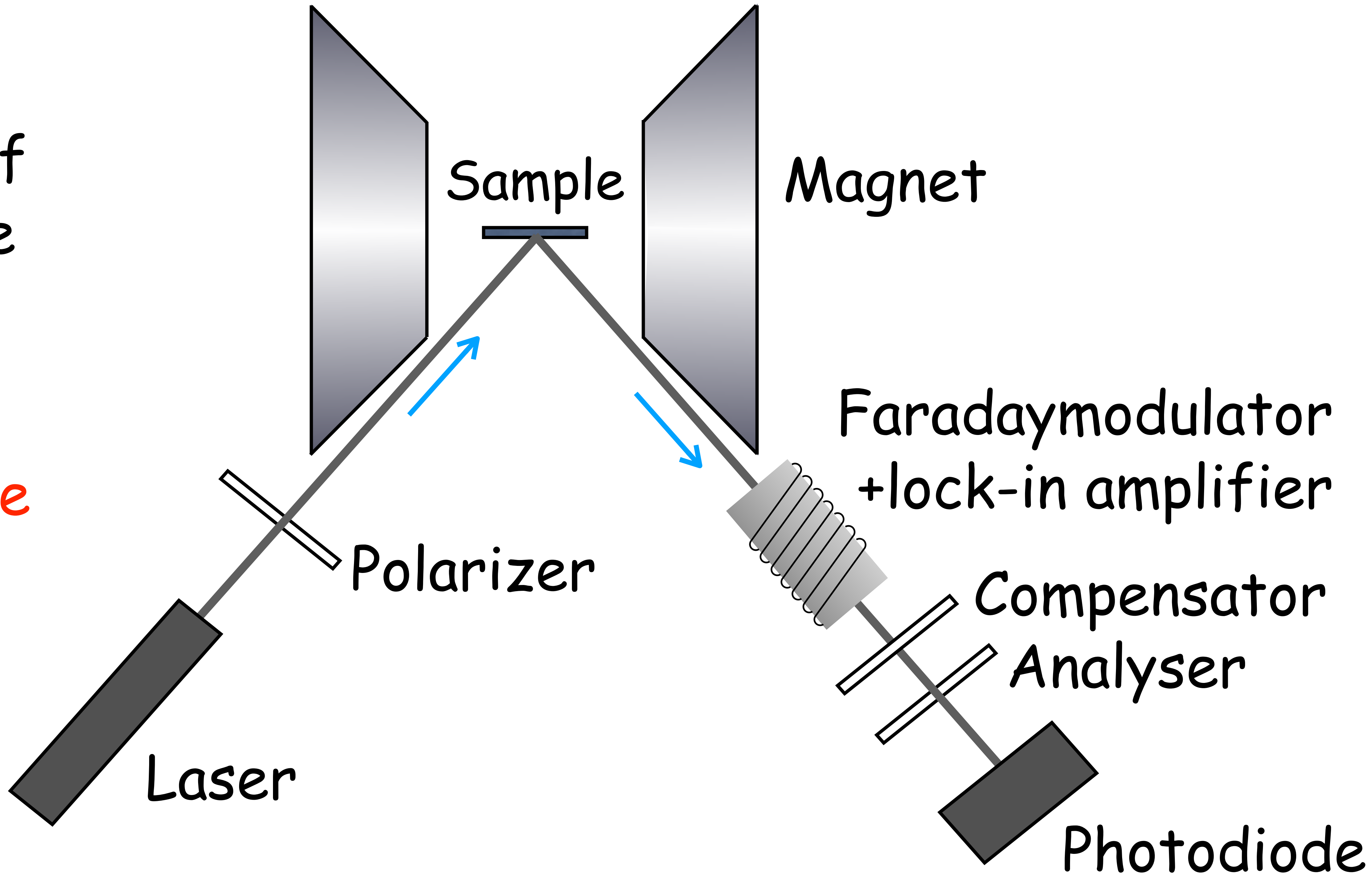

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  - Space-resolved measurements are possible by scanning over the surface
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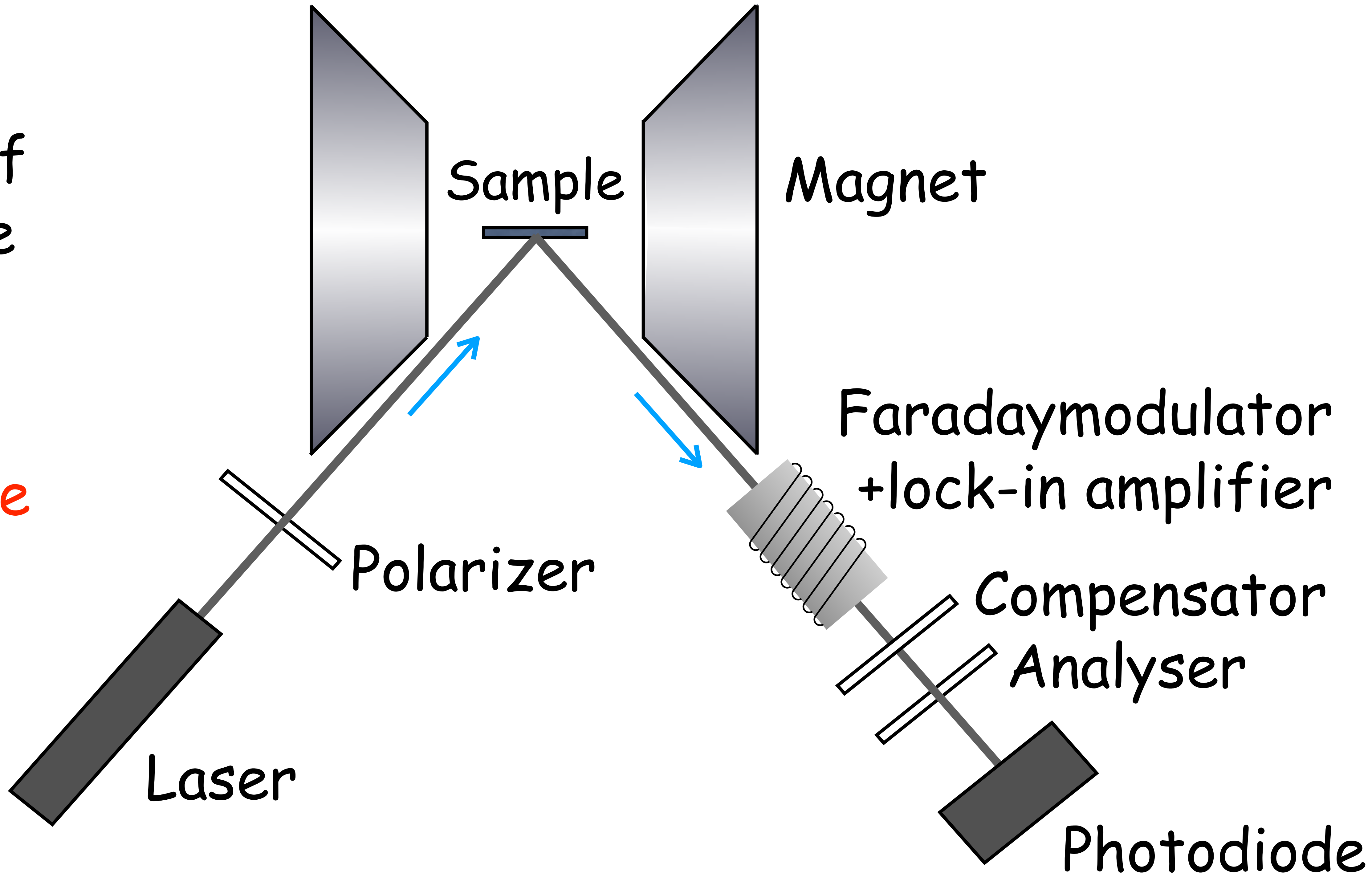
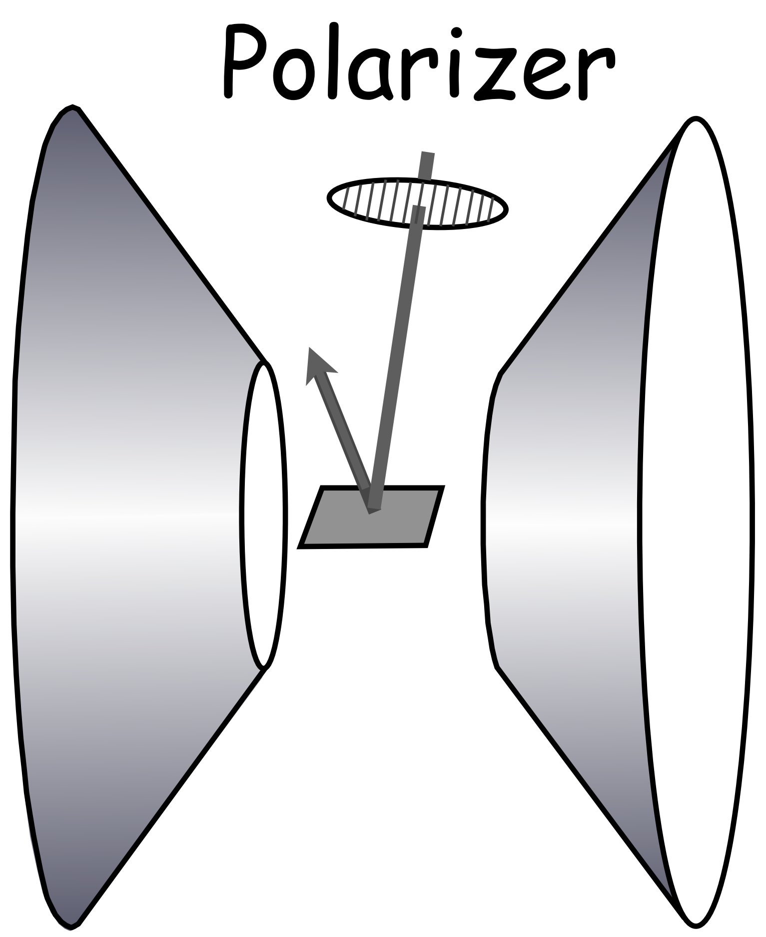
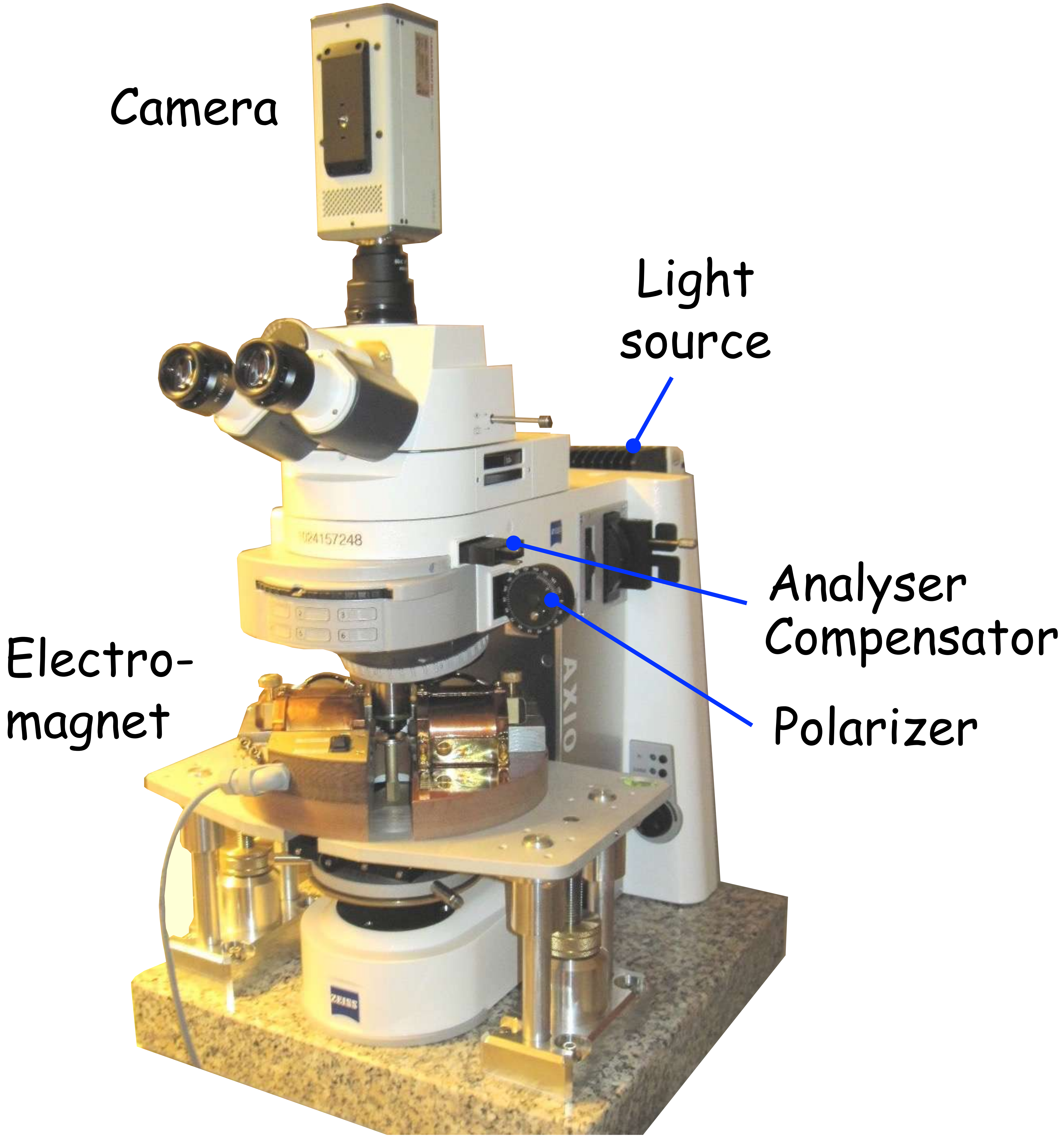


- Use of transverse Kerr effect (T-MOKE)
  - Polarizer set parallel to the plane of incidence and analyser omitted
  - M-component perpendicular to the plane of incidence causes variation of the reflected intensity, which can be detected electronically
  - Fits nicely into electromagnet



4.
Magneto-optical Kerr microscopy

Wide-field Kerr microscope



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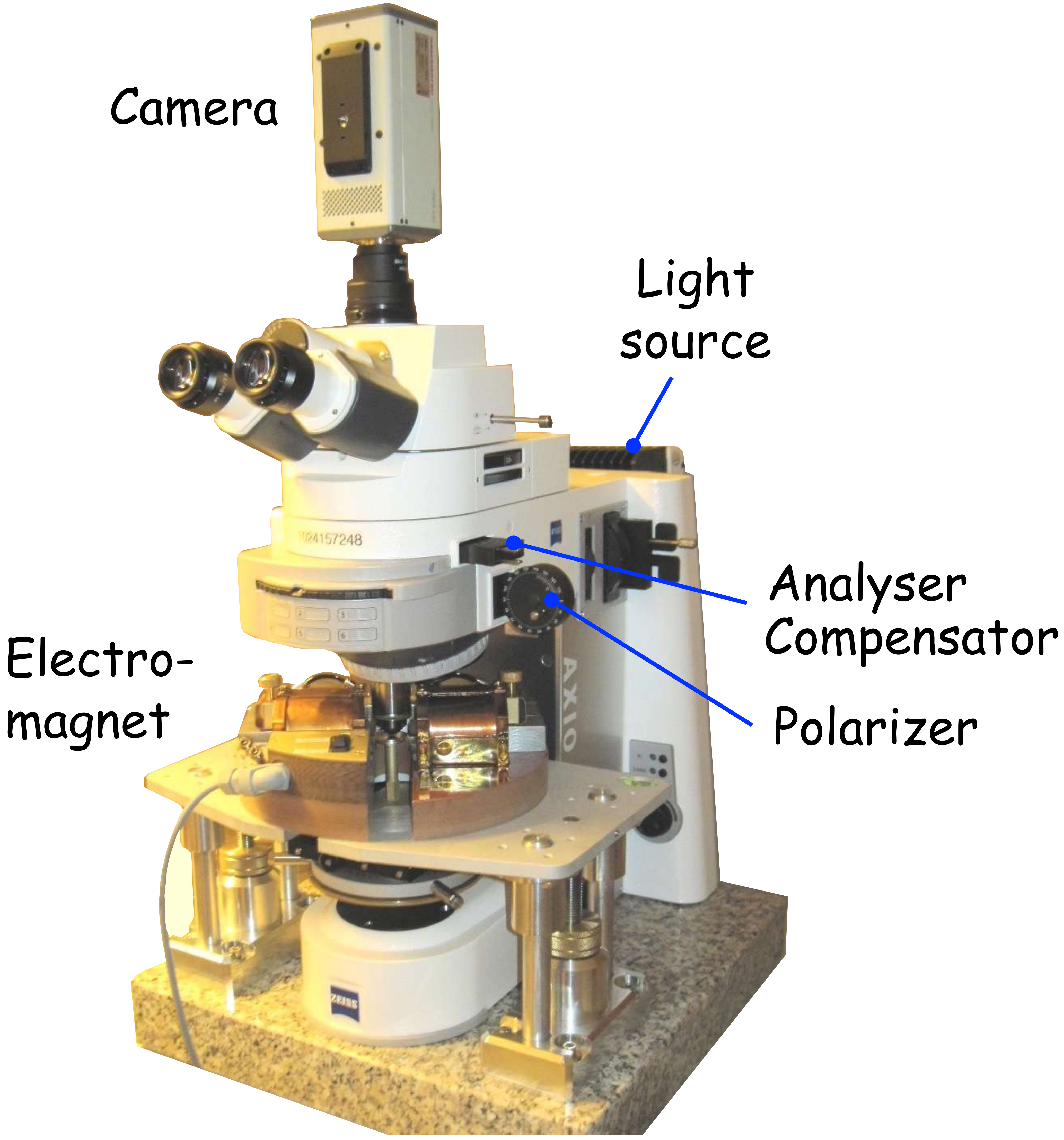


First Kerr images

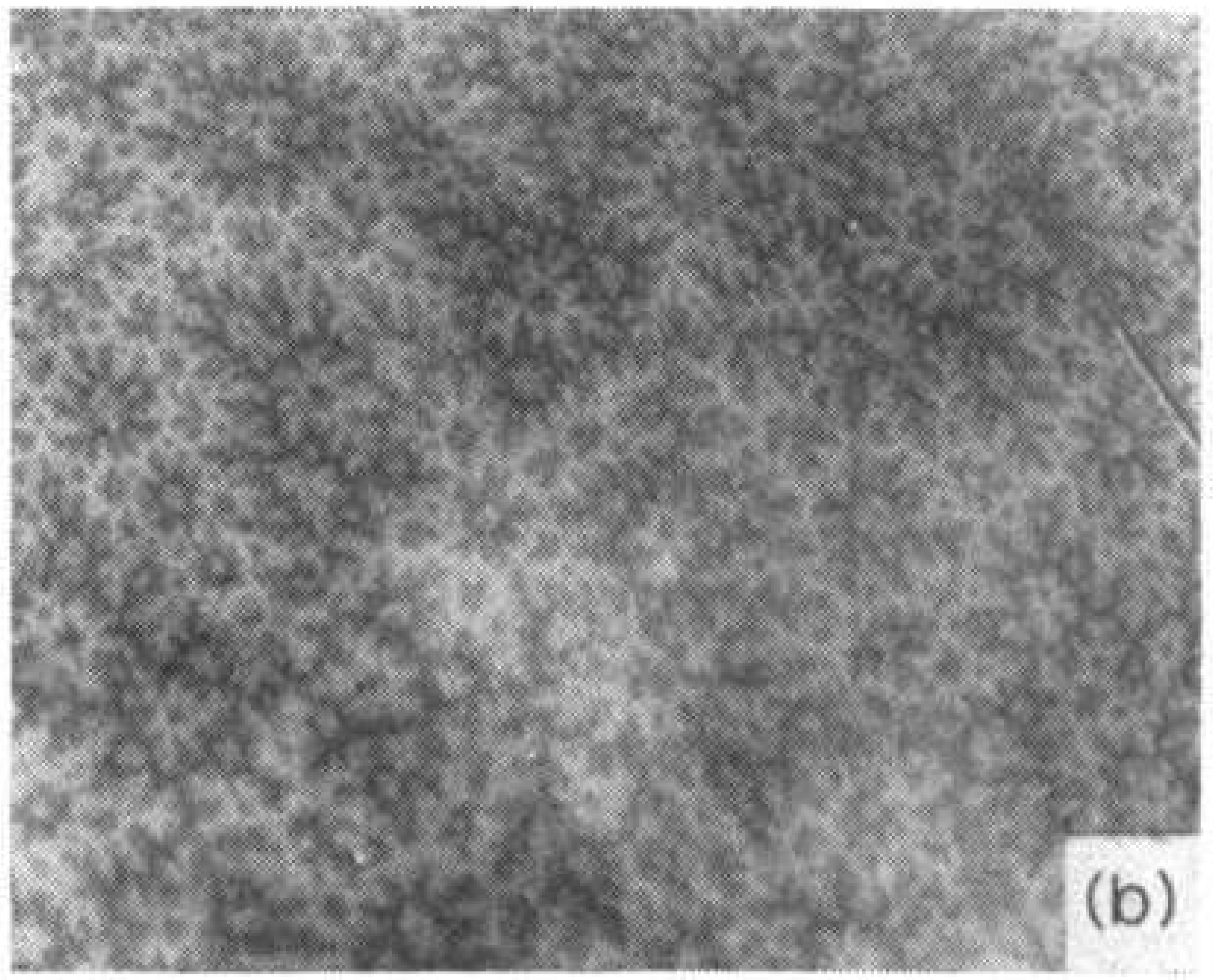


FIG. 1. Polarized light patterns showing dependence of rotation upon polarity of domains in cobalt single crystal. (Surface approximately normal to c axis.) (a) Rotation of $+\frac{1}{2}°$ optically compensated. (b) Rotation of $-\frac{1}{2}°$ optically compensated. (c) No rotational compensation.

Magnetic Domains on Silicon Iron by the Longitudinal Kerr Effect*

C. A. FOWLER, JR., AND E. M. FRYER
Department of Physics, Pomona College, Claremont, California
(Received March 14, 1952)

USE of the normal Kerr magneto-optic effect to observe domain patterns in ferromagnetic substances having free surface poles has been described by Williams, Foster, and Wood.[1] Since plane polarized light reflected normally from a polished magnetic surface is not affected by magnetization in the plane of the surface, oblique reflection using the longitudinal Kerr magneto-optic effect[2] has been investigated as a means of observing

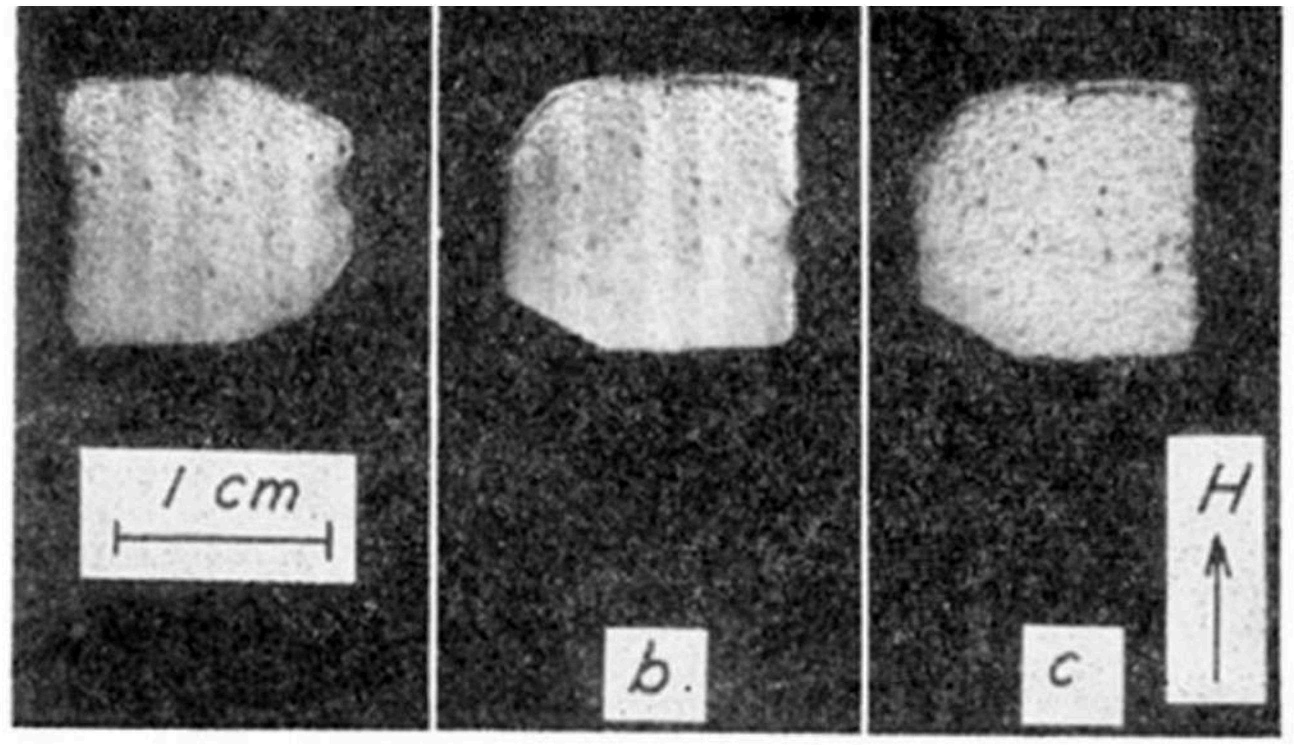


FIG. 2. Oblique photographs of the (100) surface of the crystal. In (a) and (b) the crystal is unmagnetized. An external field in the direction shown was applied in (c).

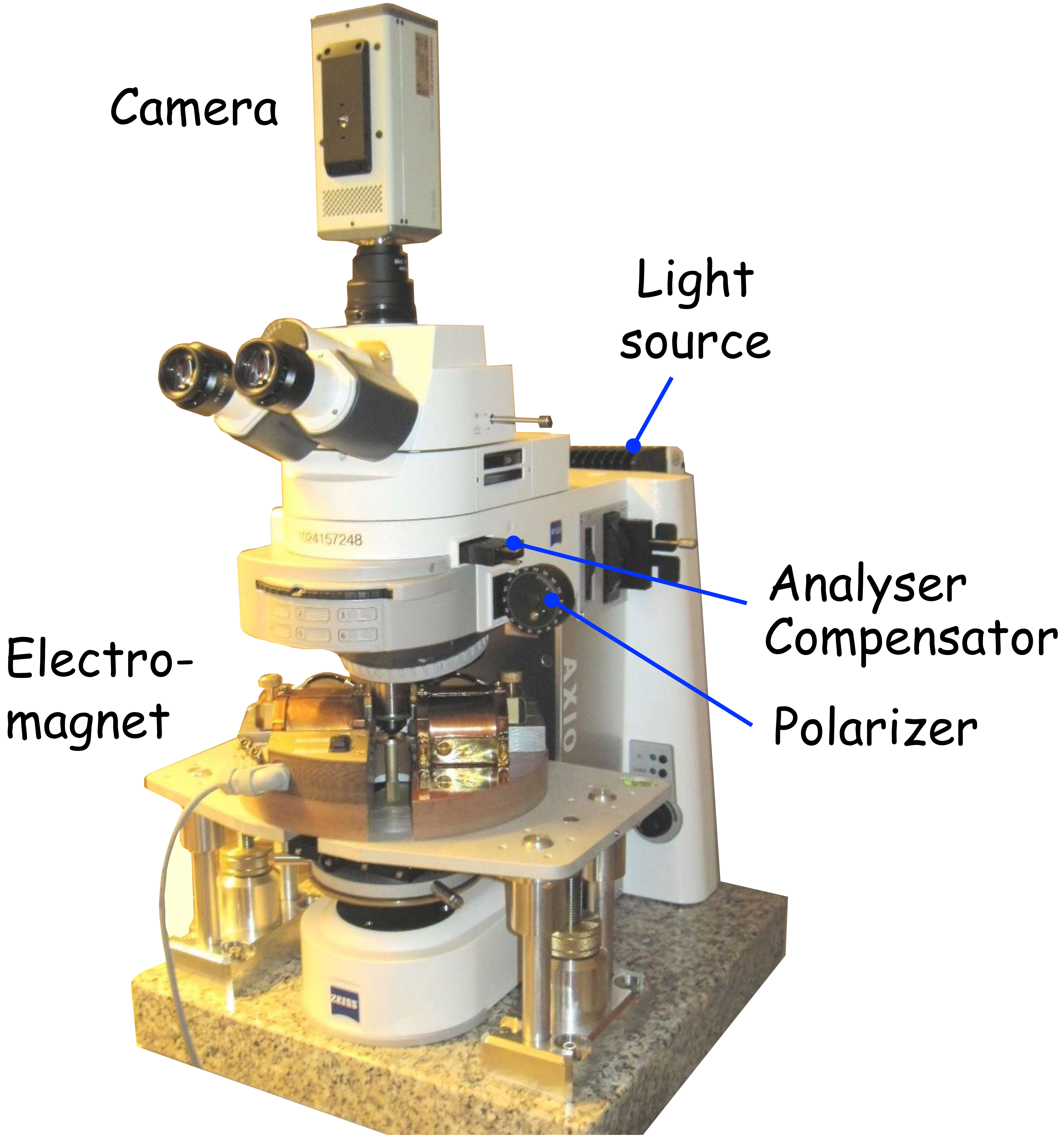
Observation of Magnetic Domains by the Kerr Effect

H. J. WILLIAMS, F. G. FOSTER, AND E. A. WOOD
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received January 18, 1951)

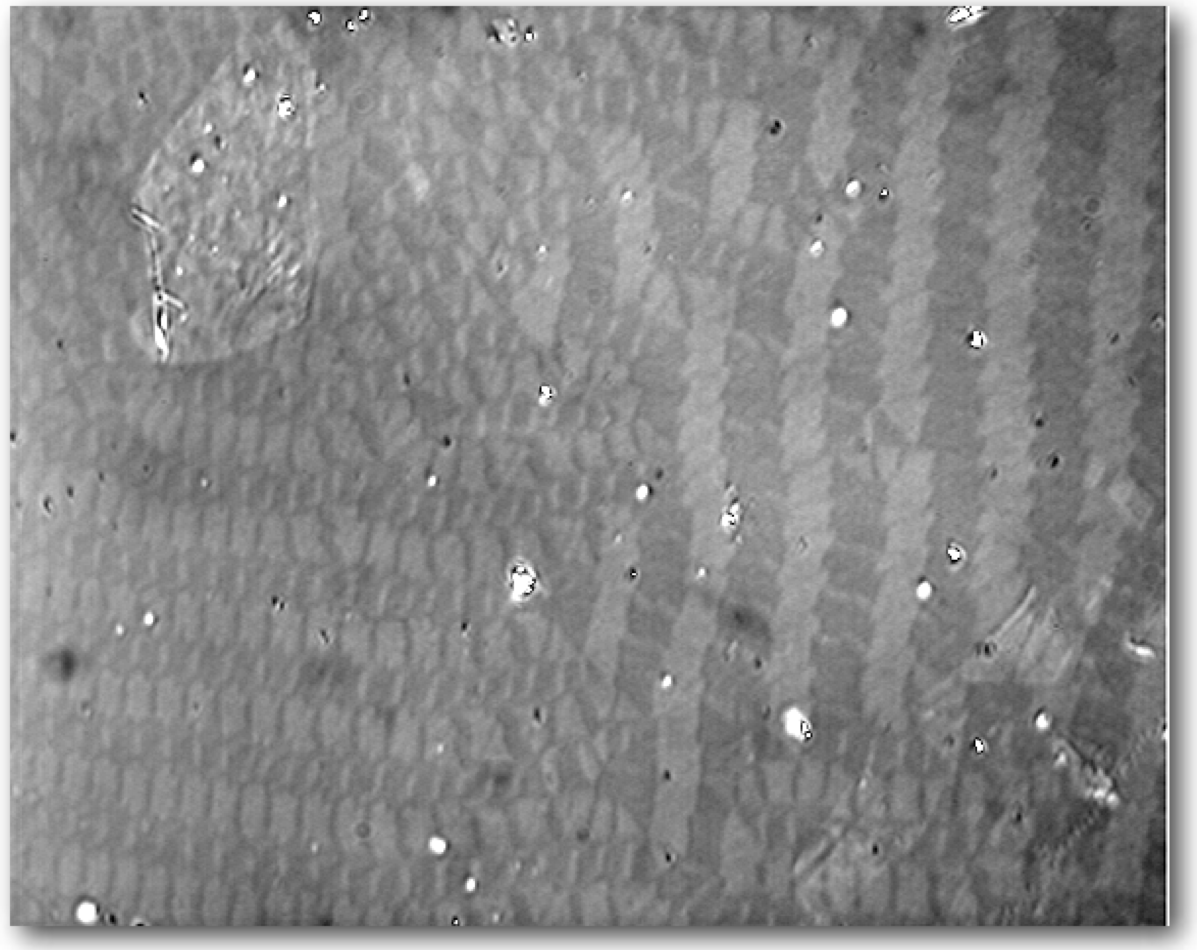
MAGNETIC domains have been observed by means of the Kerr magneto-optic effect on surfaces perpendicular and inclined to the c axis of hexagonal cobalt. The direction of easy magnetization in cobalt is along the c axis, so the domains are

4. Magneto-Optical Kerr Microscopy

Wide-field Kerr microscope

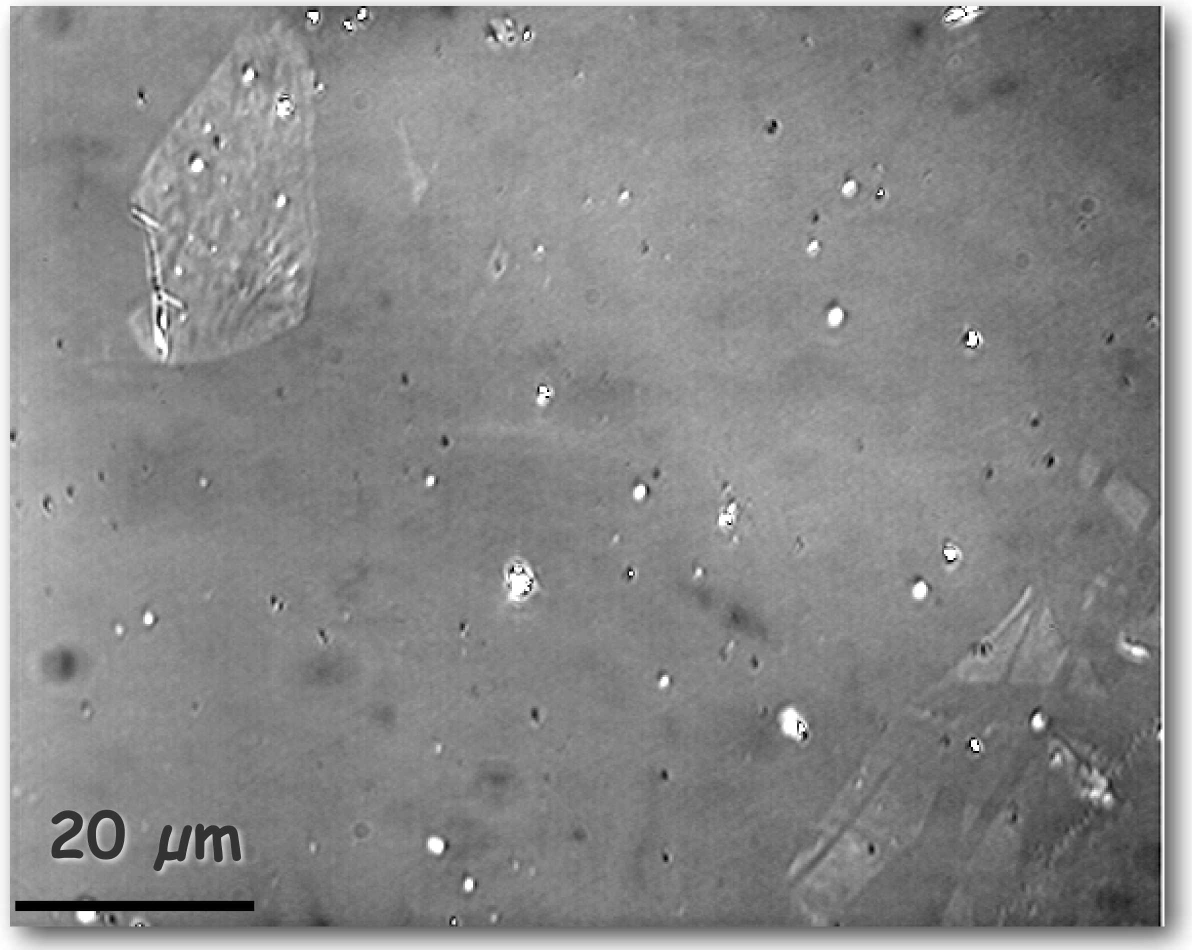


Today: Digital contrast enhancement (difference image technique)



Original image

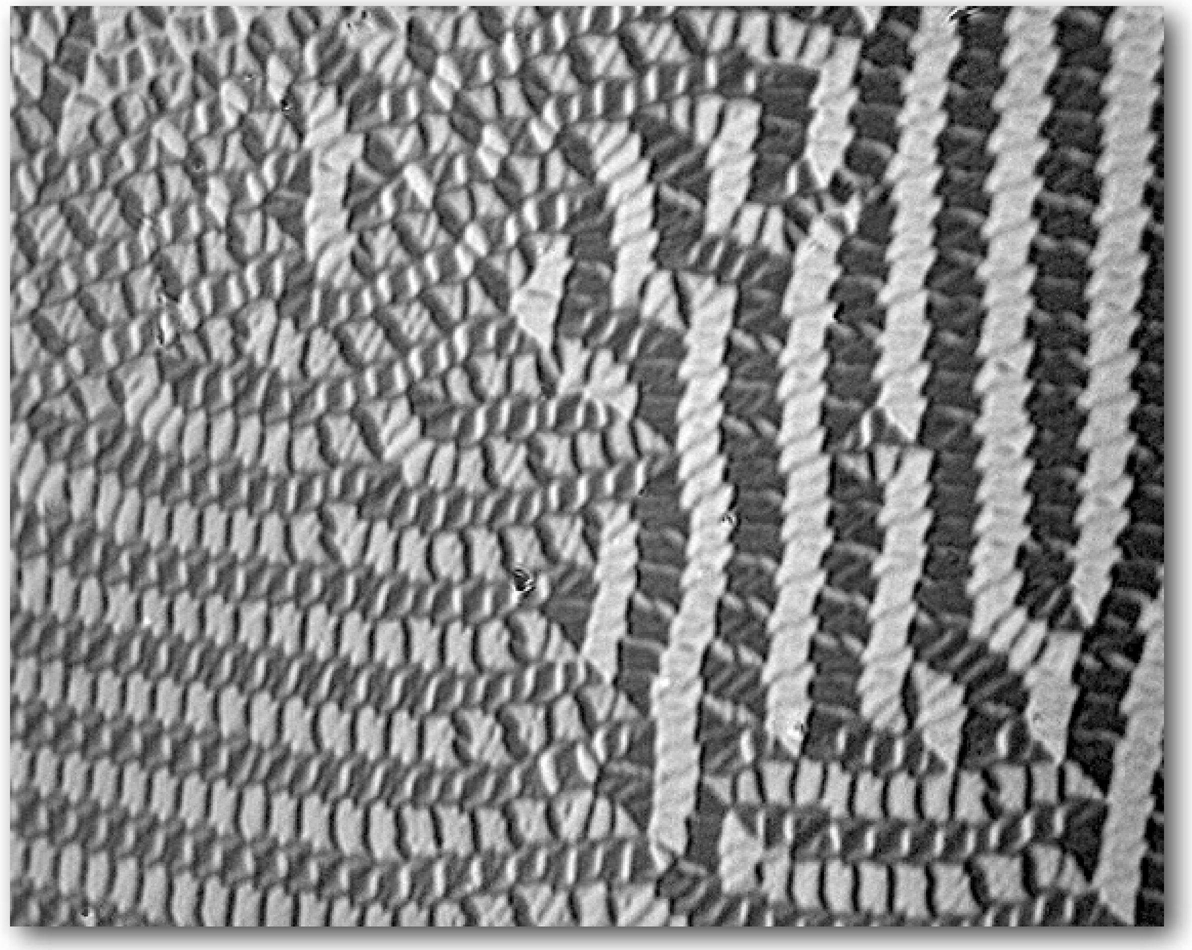
–



Reference image

↓

Amorphous ribbon



Schmidt et al. (1985)

Difference image

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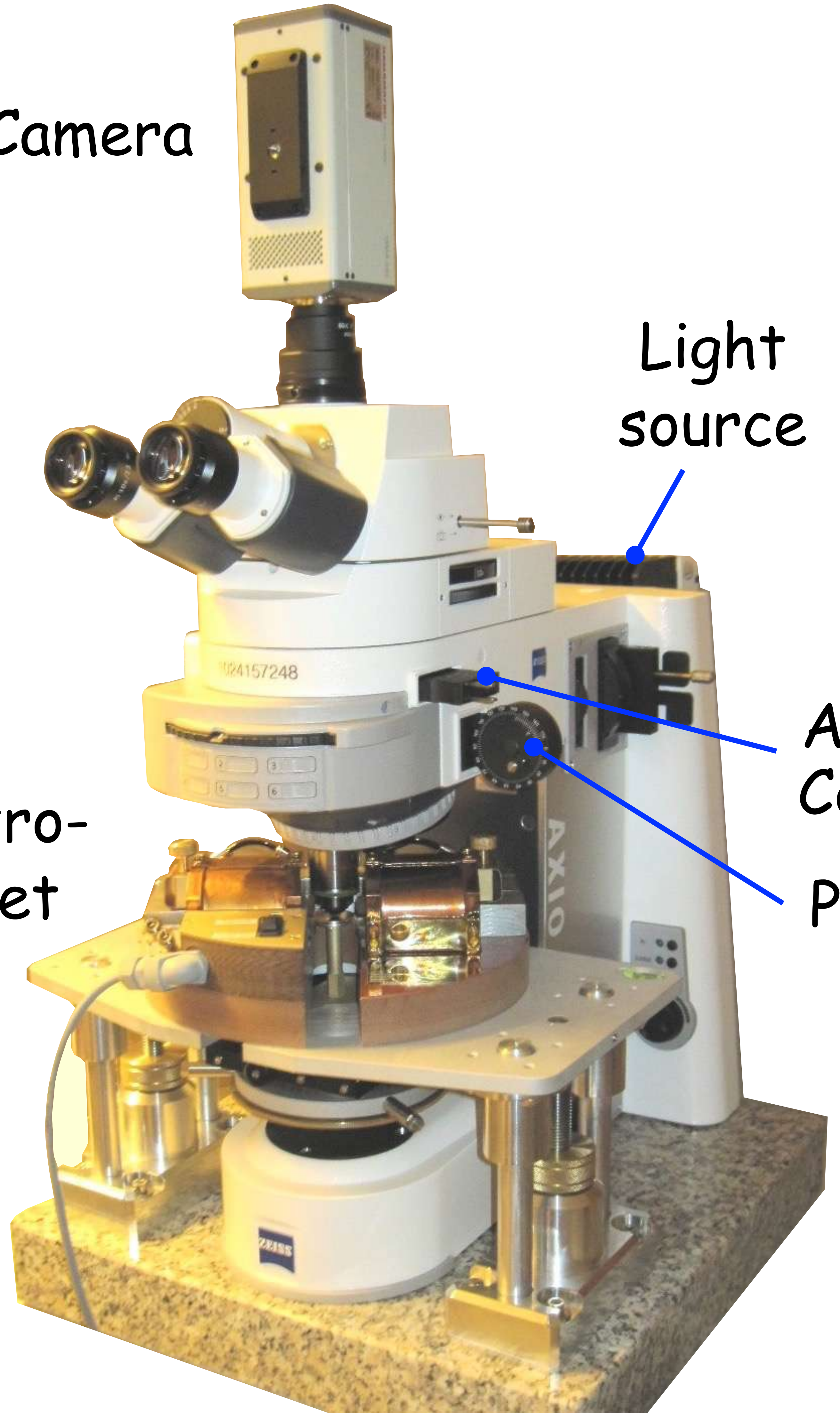

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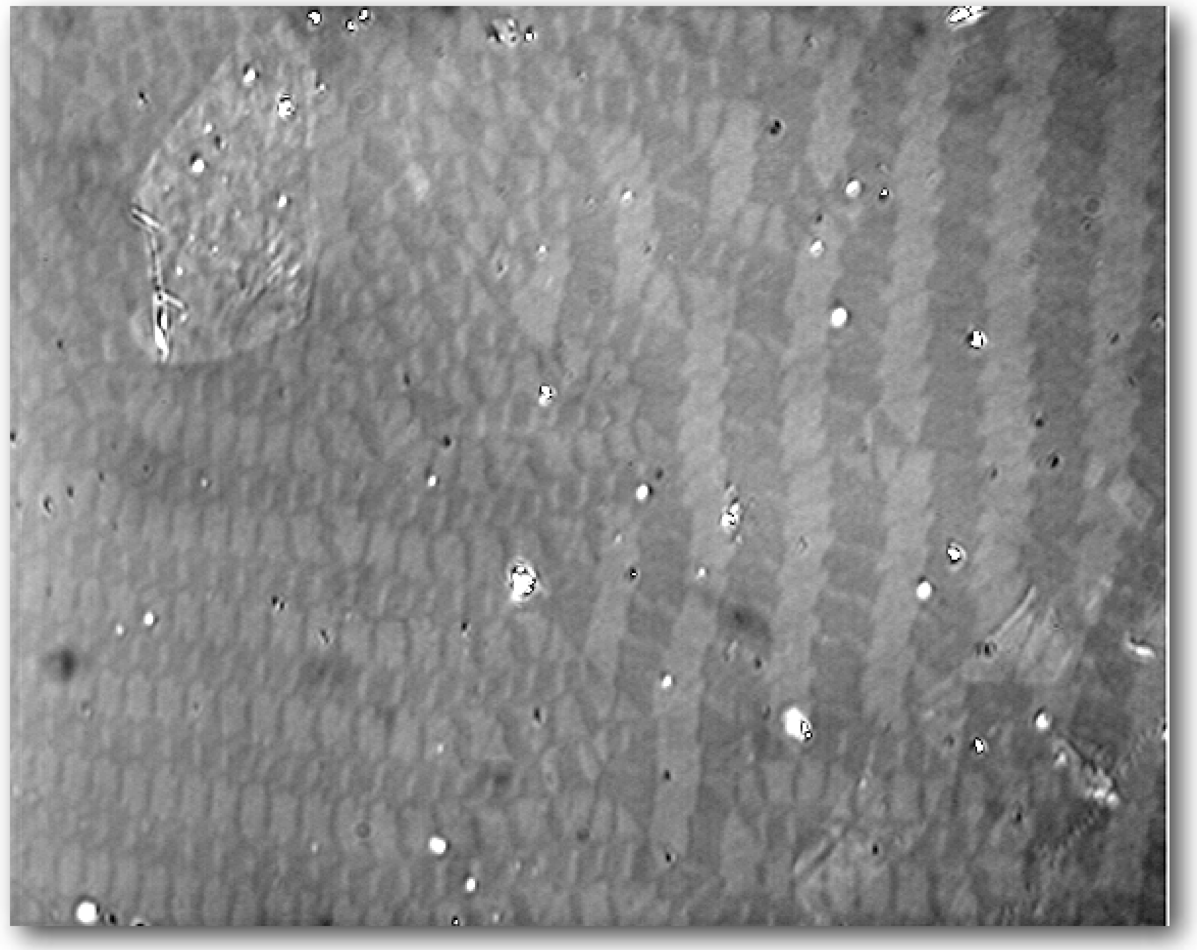
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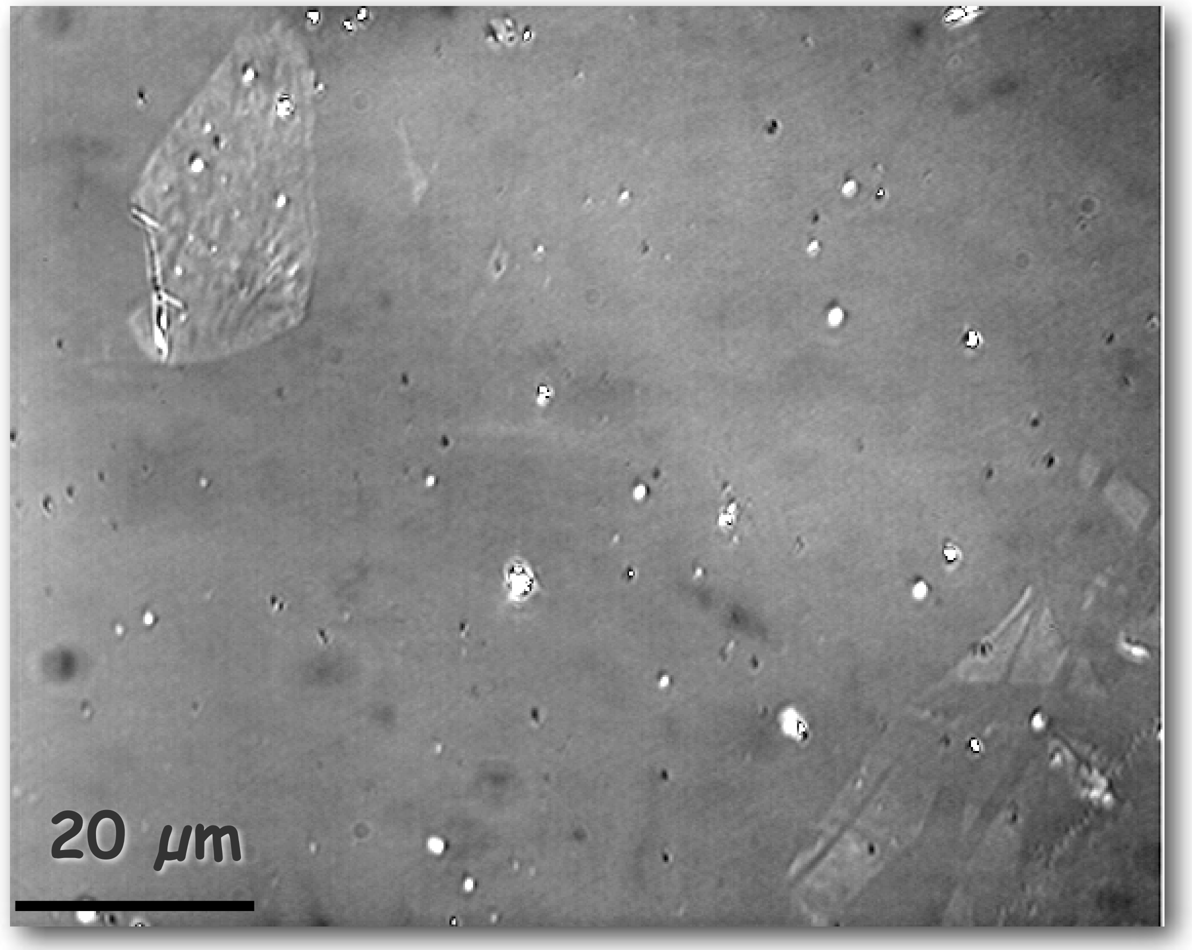

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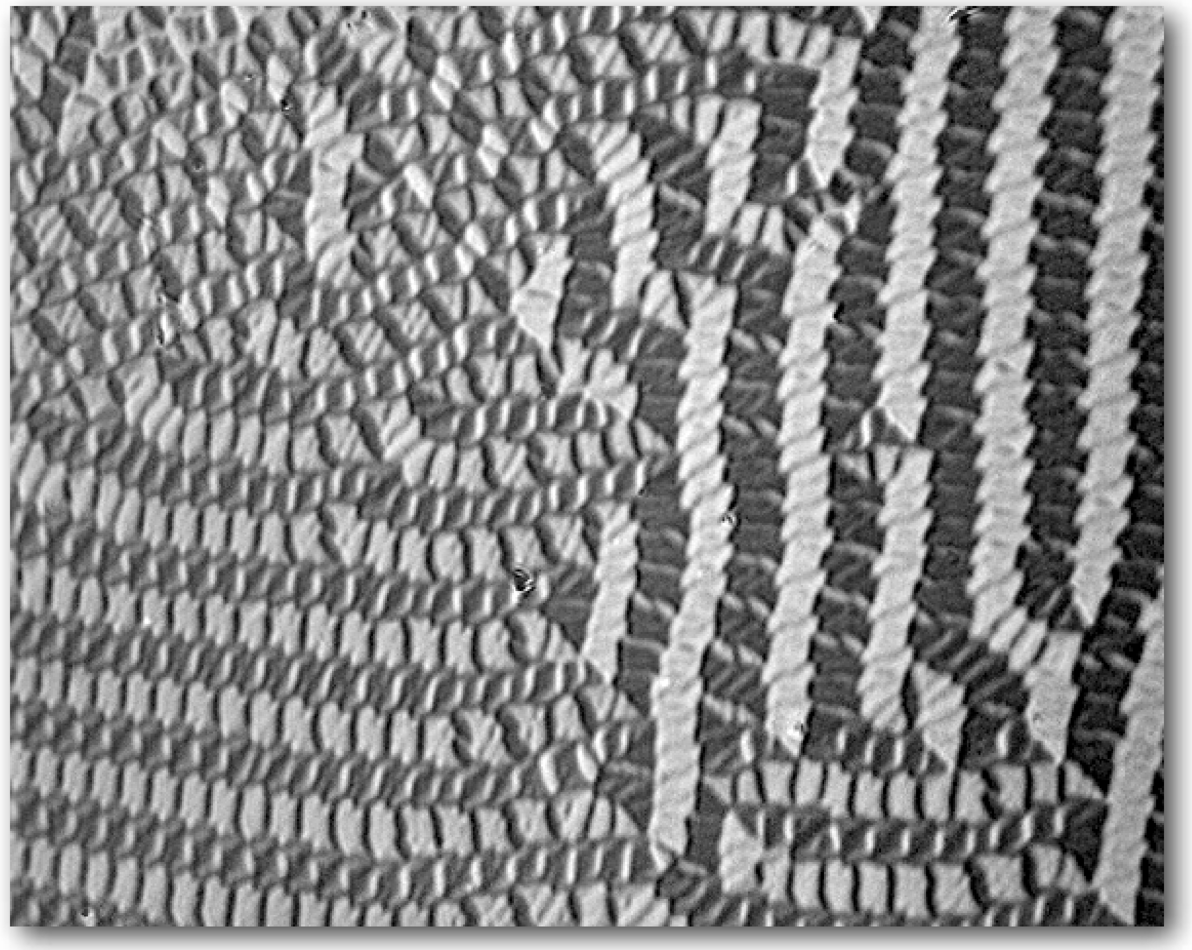
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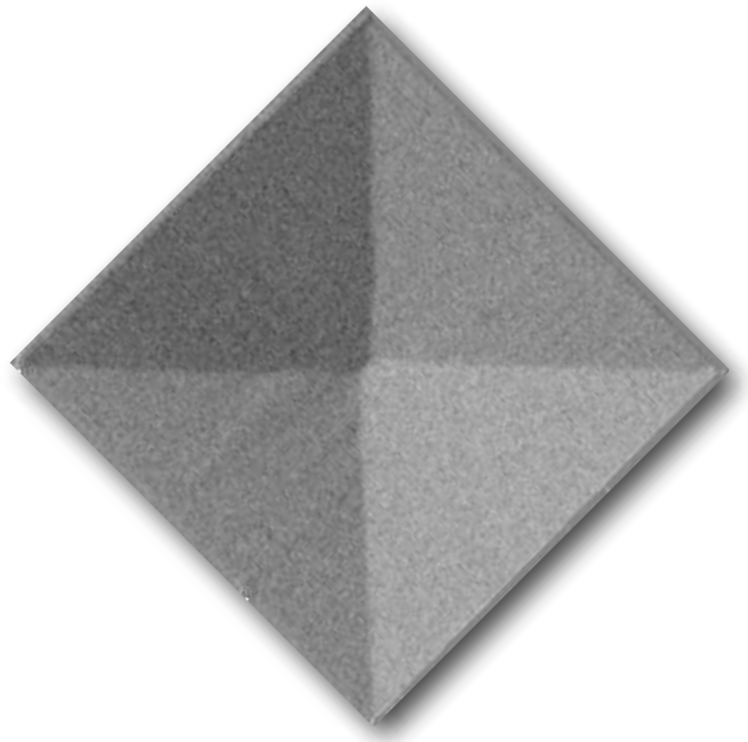
Important: Difference Imaging in real time !

Digitally enhanced wide-field Kerr microscope: flexible technique

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- Application of in-plane and perpendicular magnetic fields up to Tesla range

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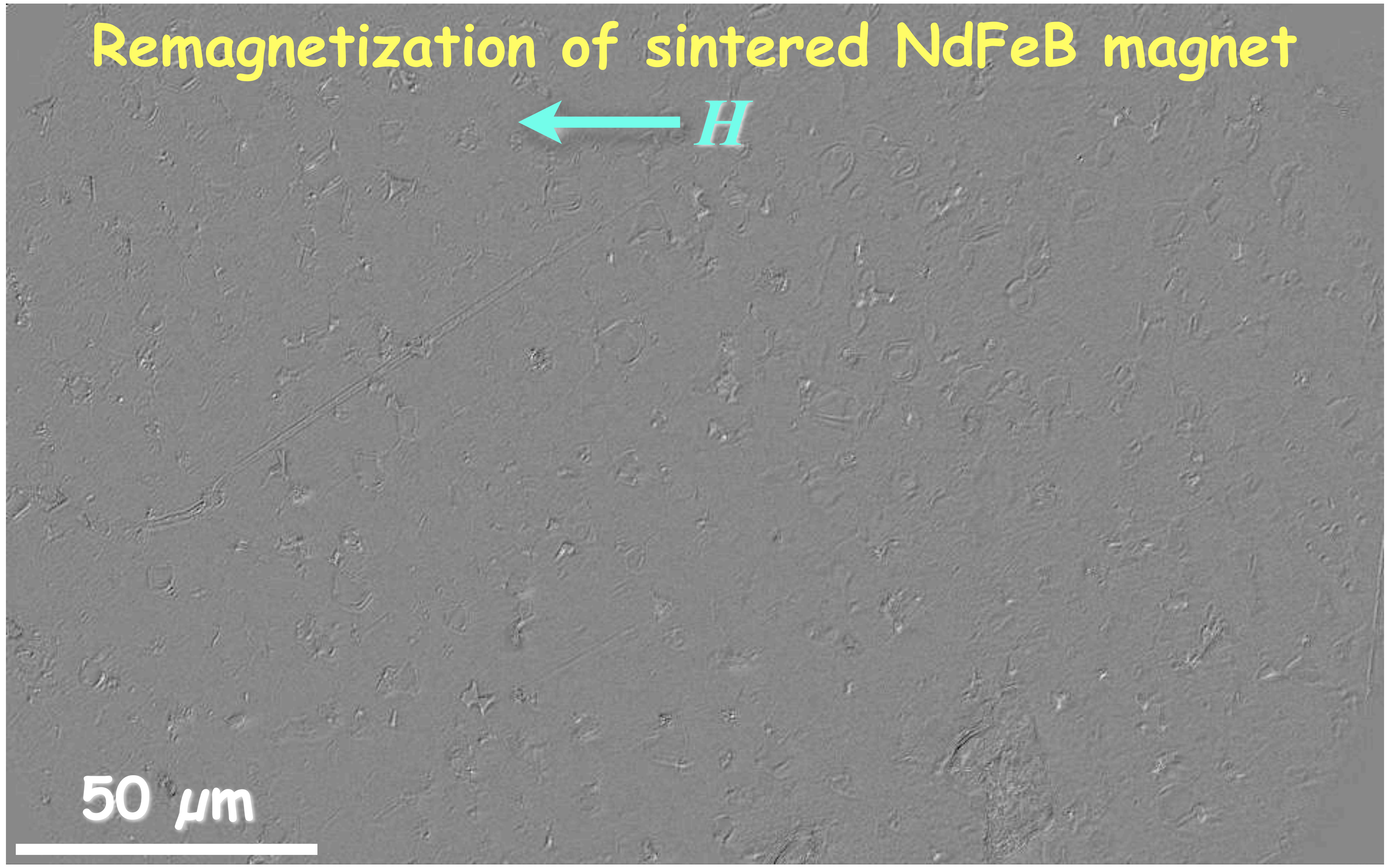

Imaging courtesy Ivan Soldatov, IFW

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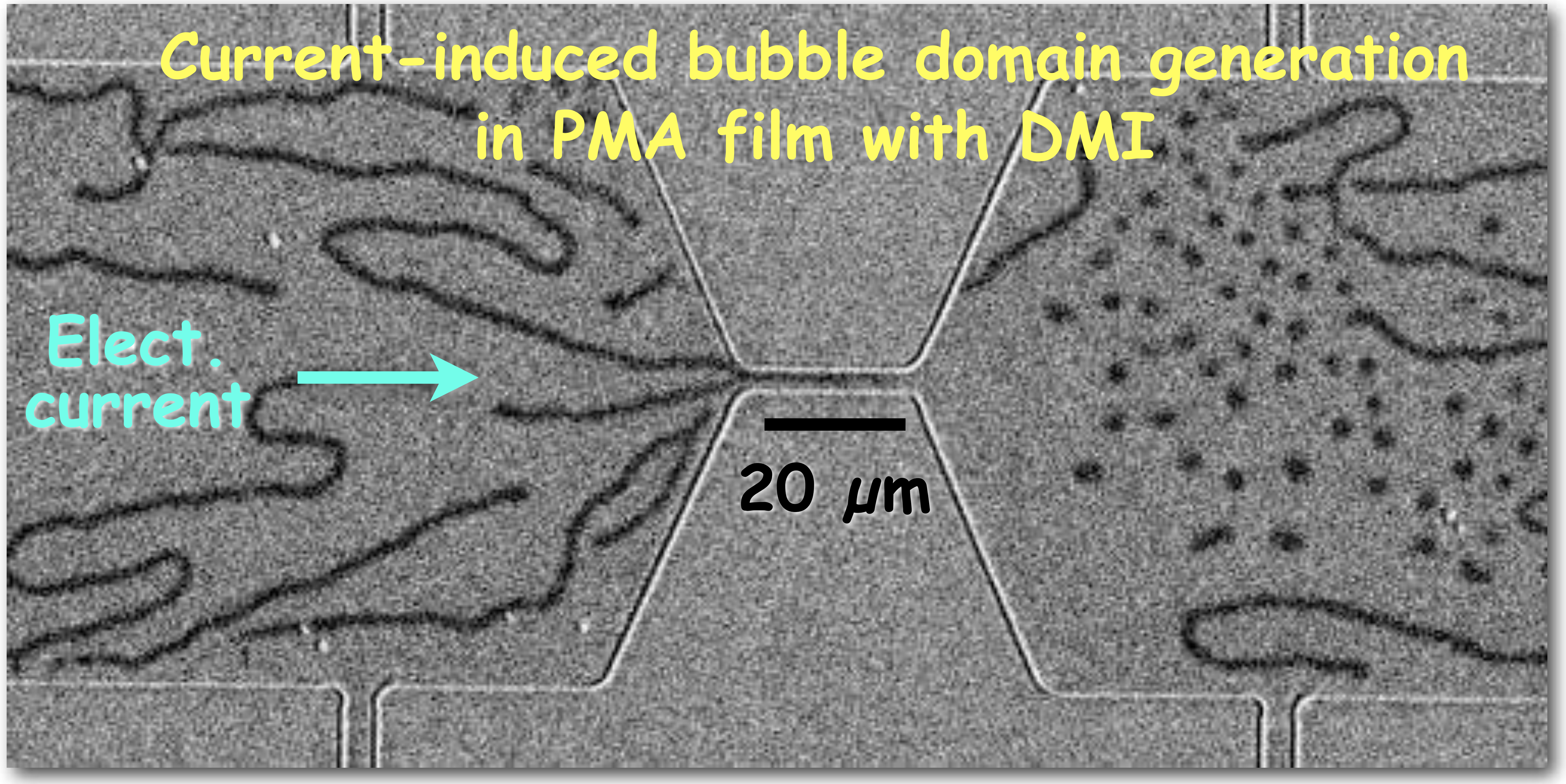

Courtesy: Wanjun Jiang, Suzanne Velthuis, Axel Hoffmann, Argonne National Lab.

4. Magneto-Optical Kerr Microscopy

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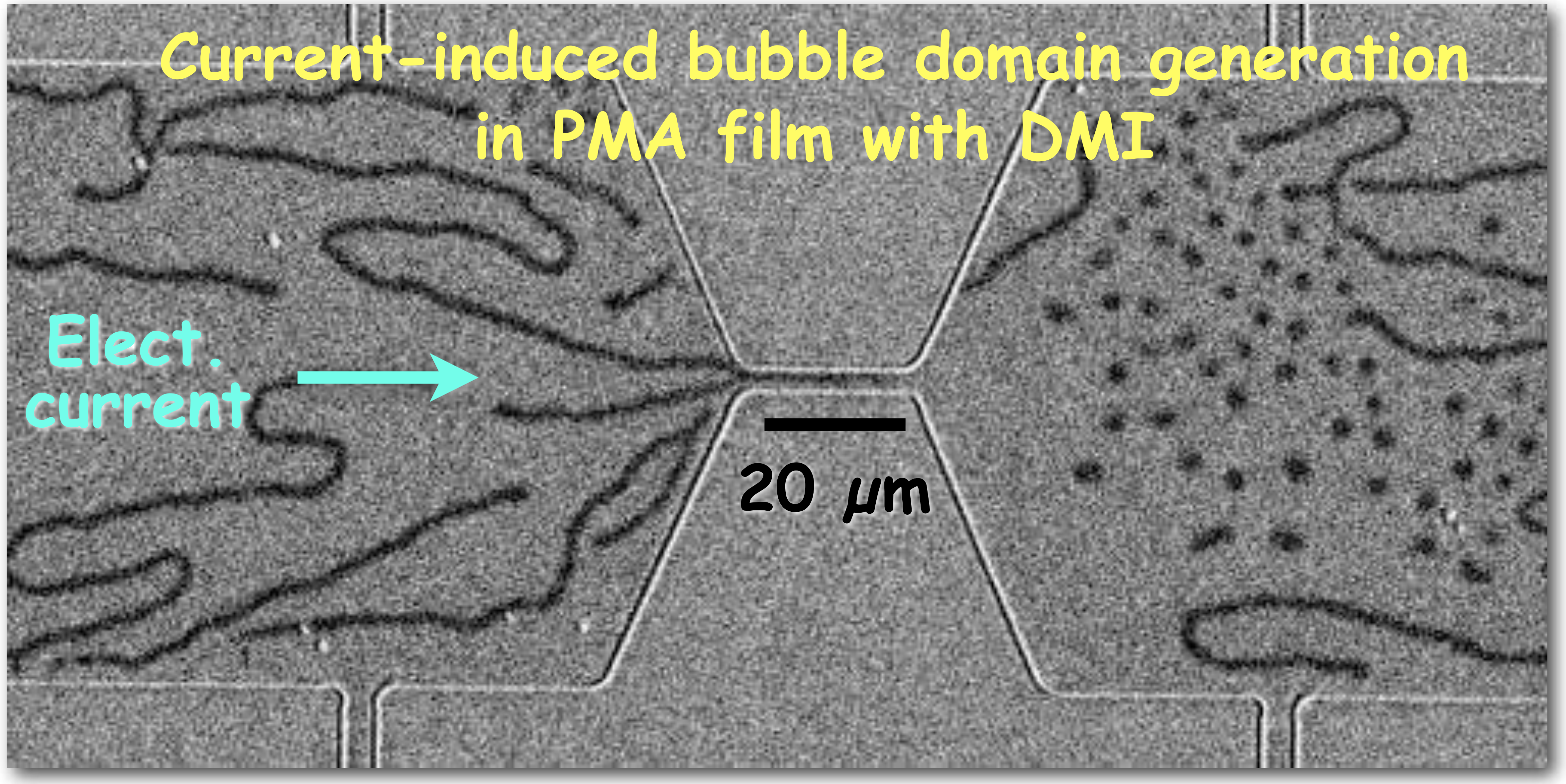

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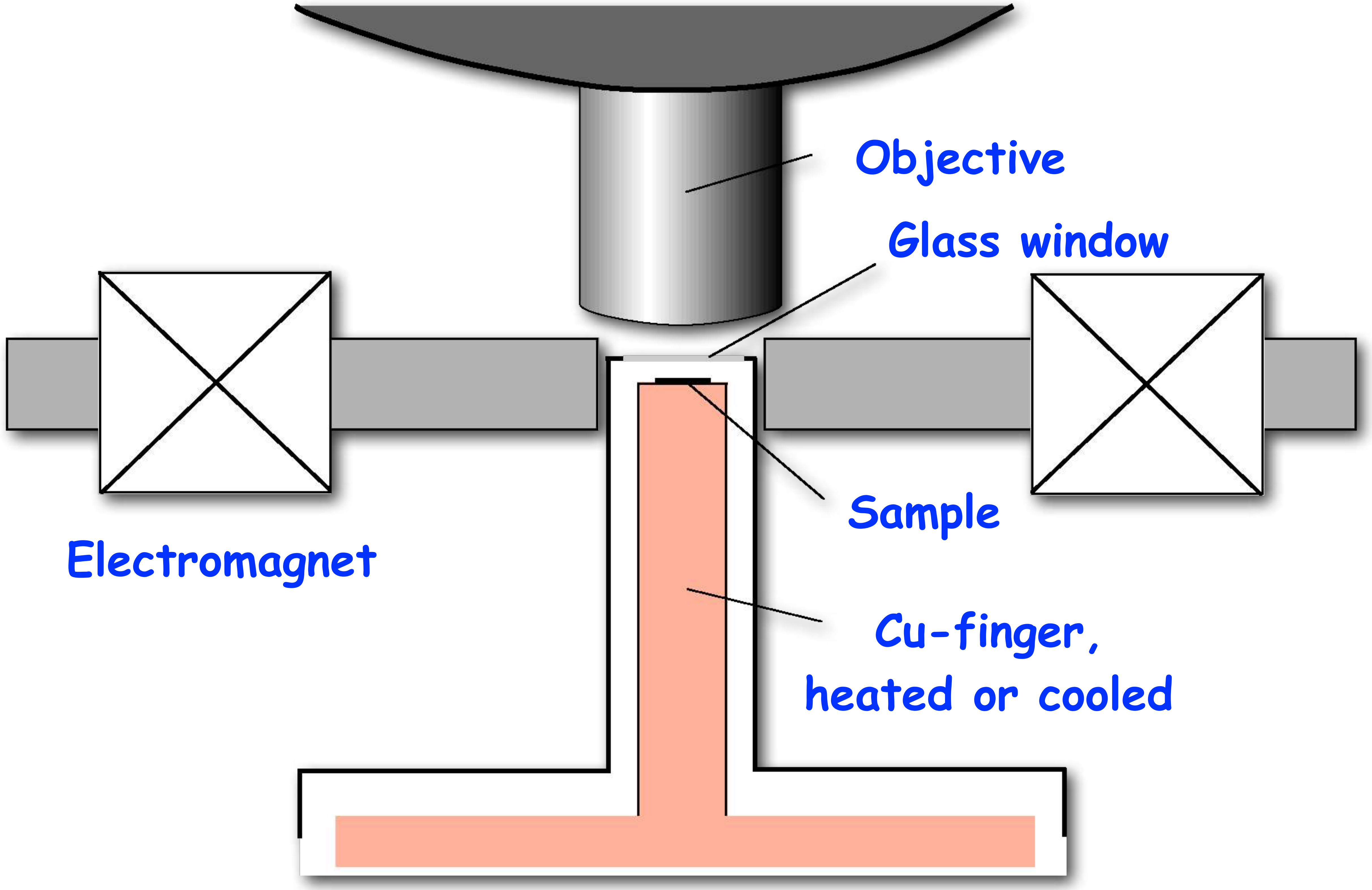
- Application of high- and low temperature (4 K up to 800 K)



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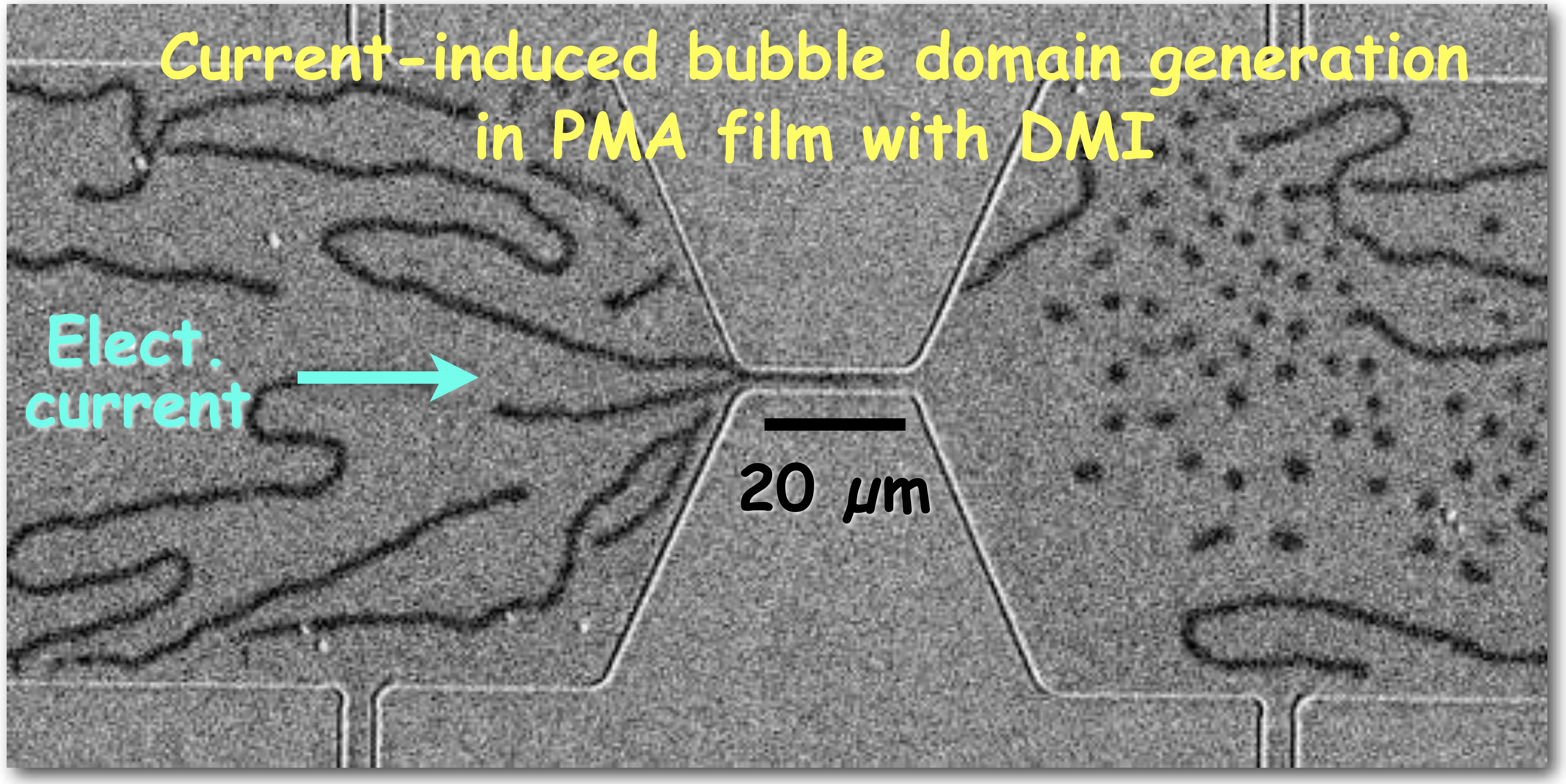

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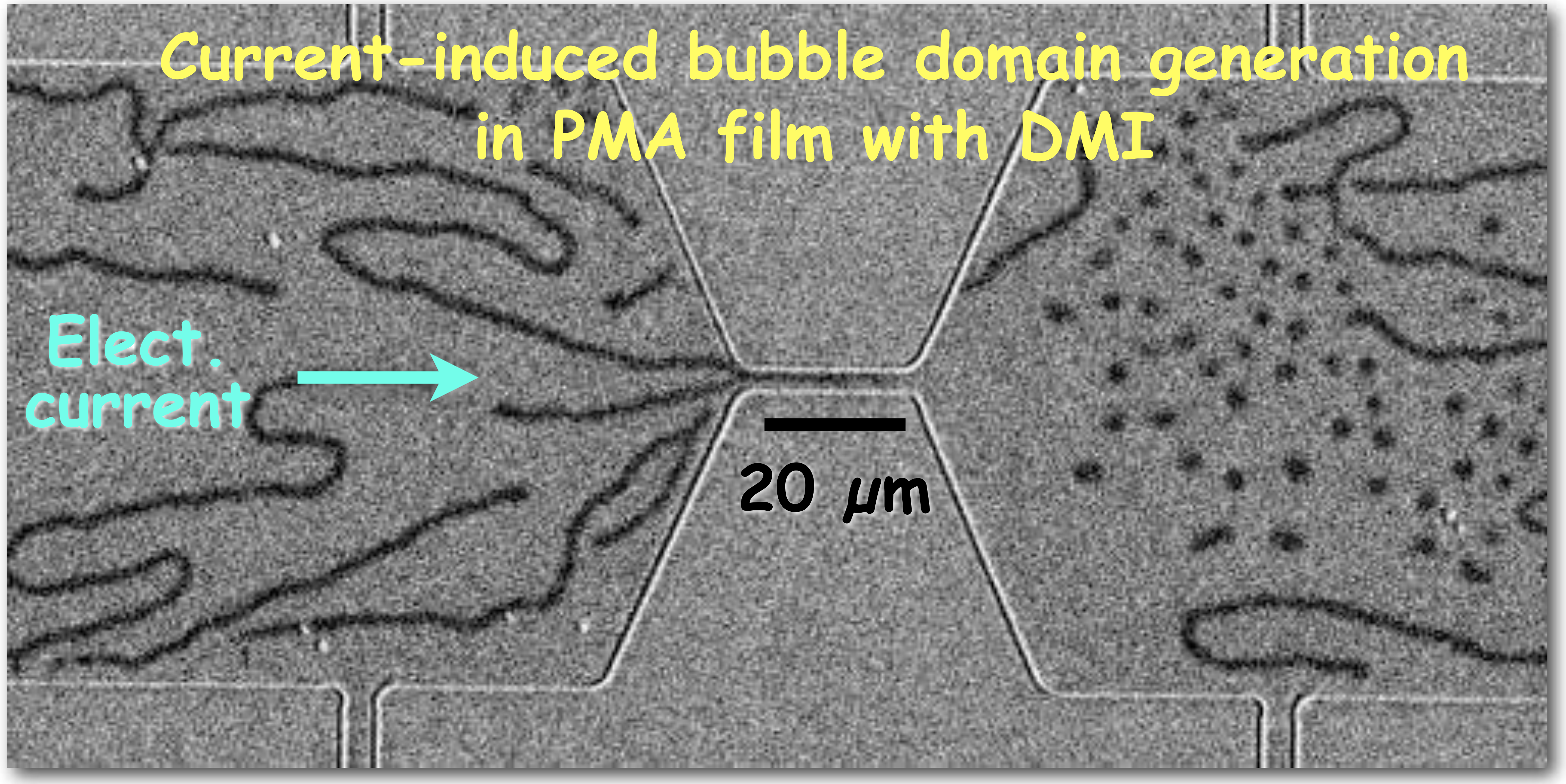

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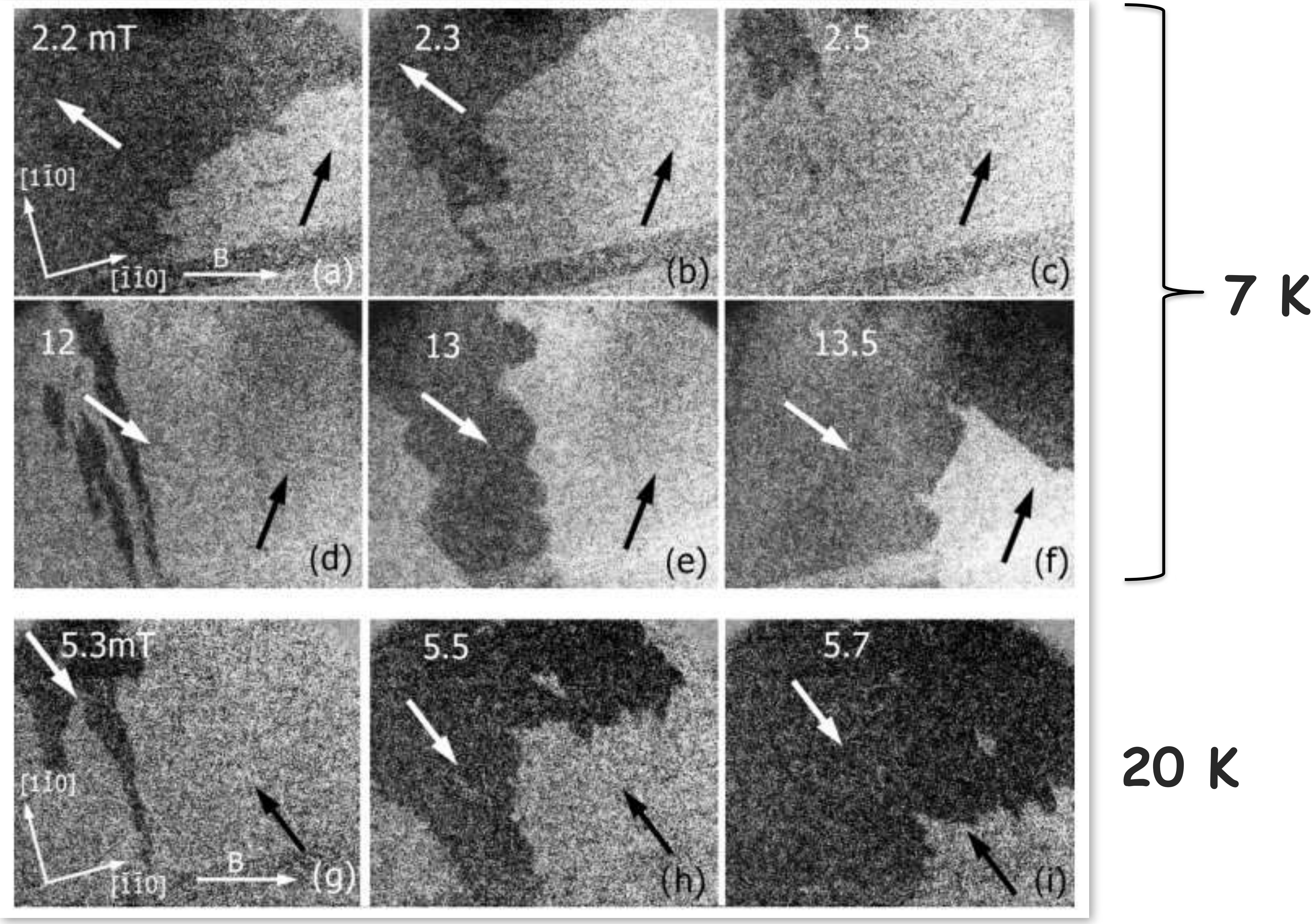
Diluted magnetic semiconductor MnGaAs



I. V. Soldatov, et al., Phys. Rev. B 90, 104423 (2014)

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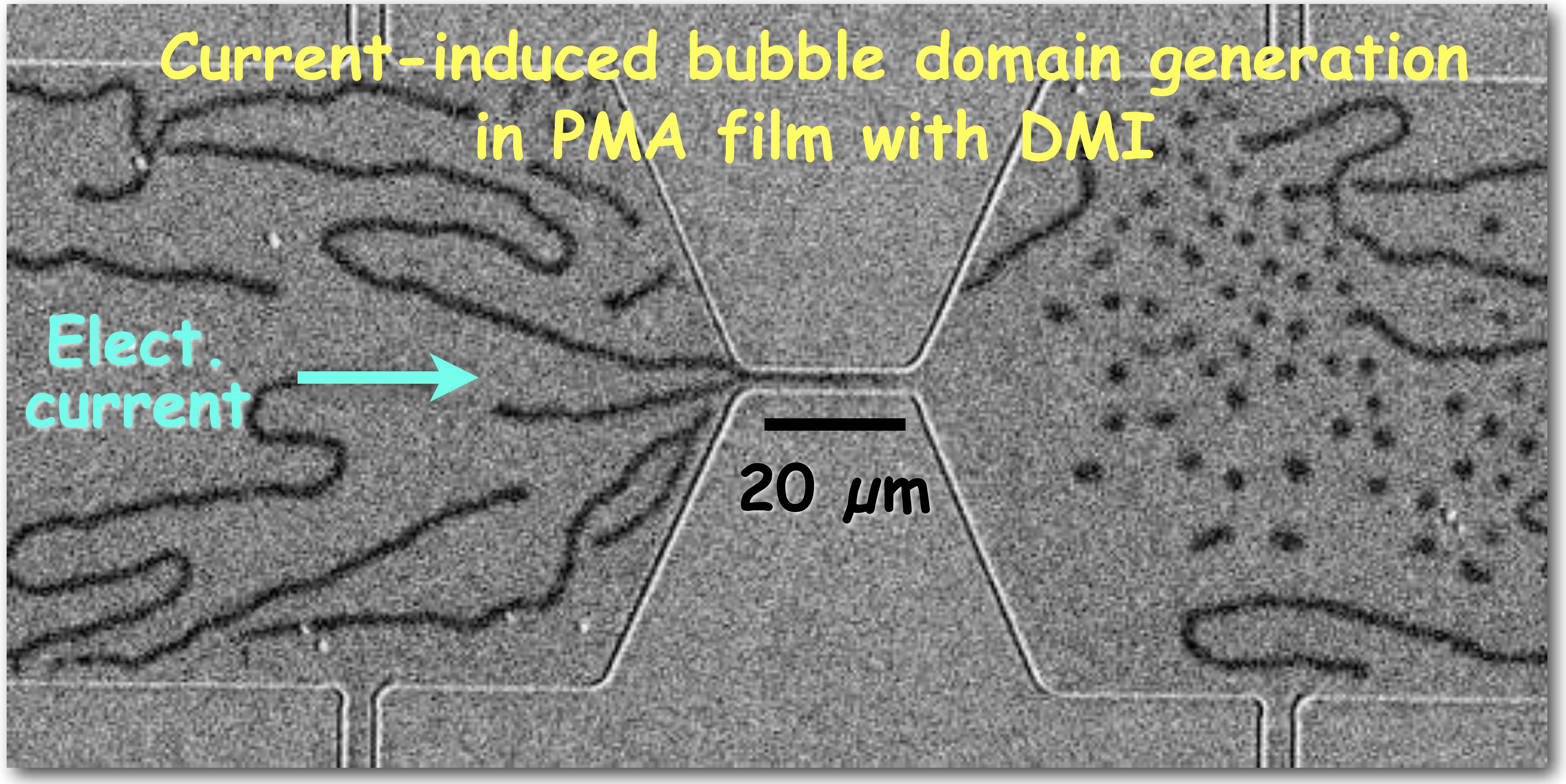

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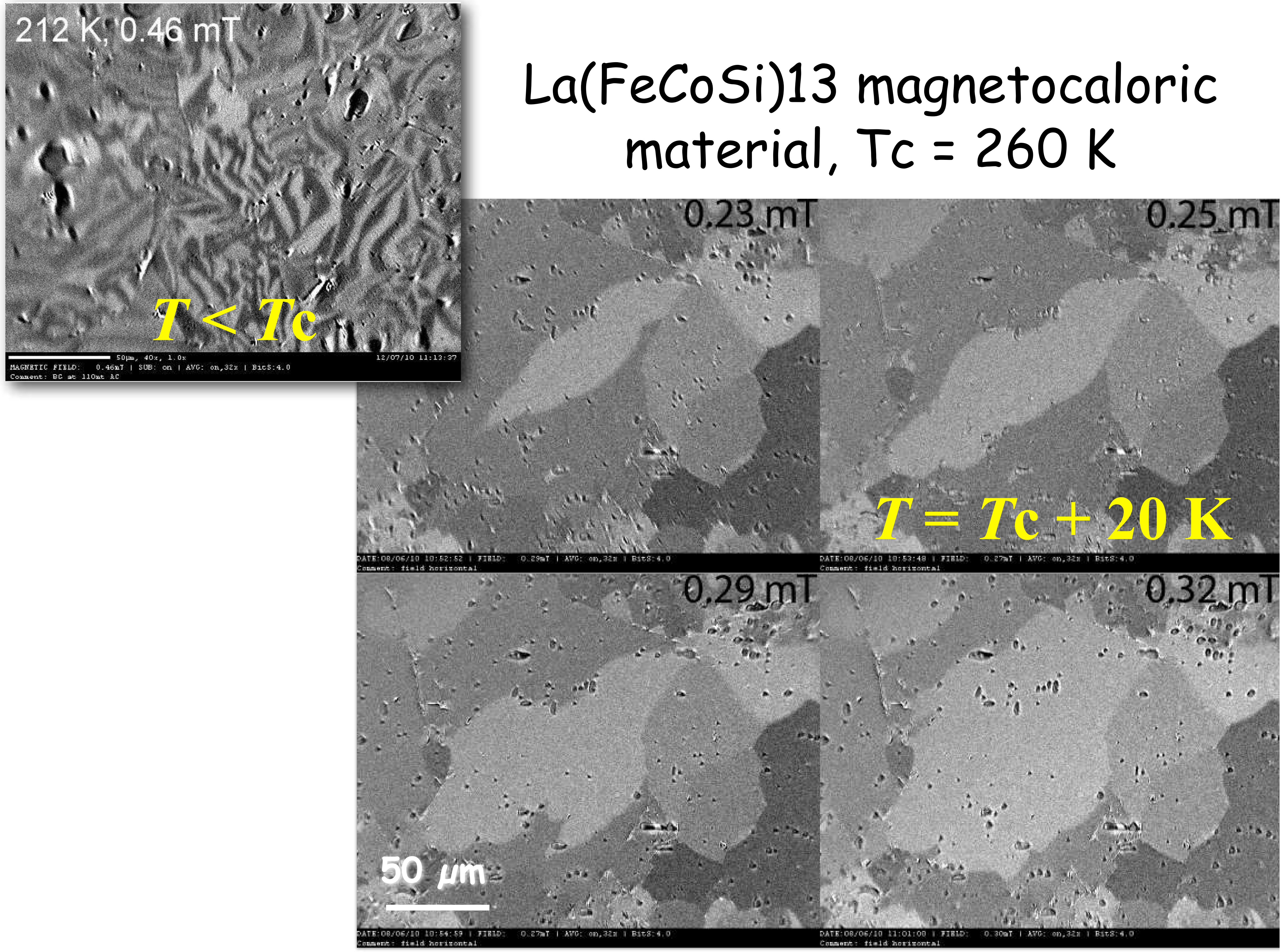
- Application of high- and low temperature (4 K up to 800 K)

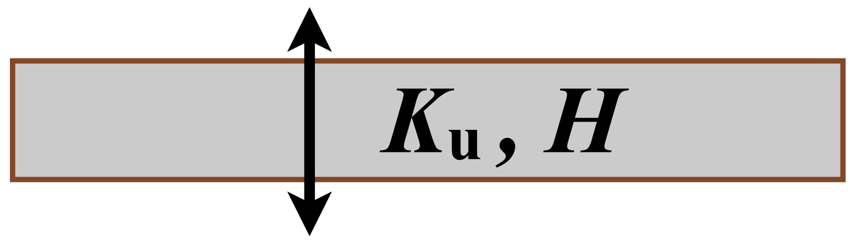
La(FeCoSi)13 magnetocaloric material, Tc = 260 K



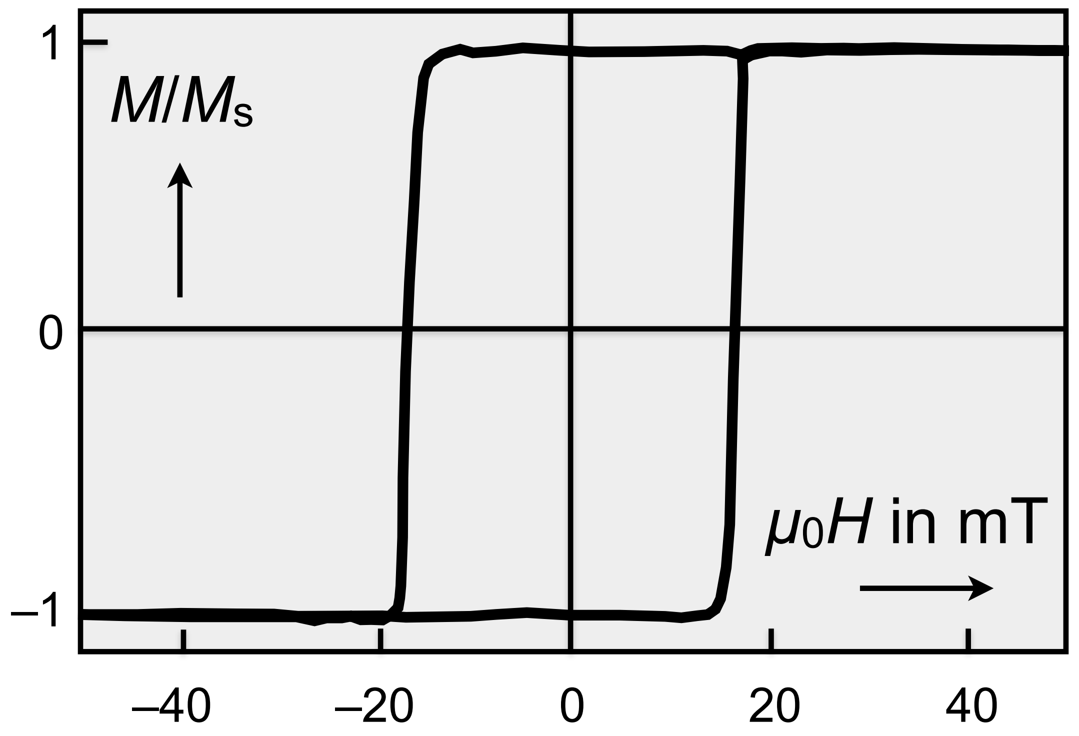
Together with K. Skokov and O. Gutfleisch, Darmstadt University, (unpublished)

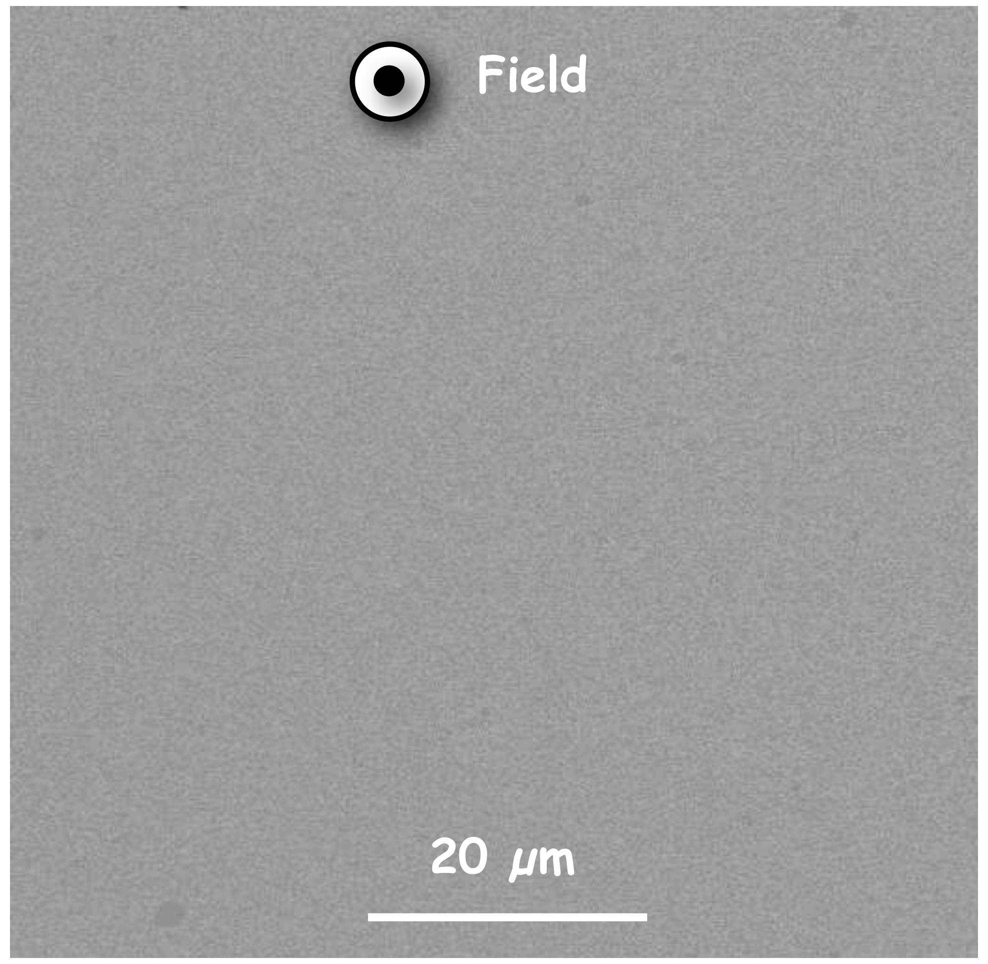
Combination with MOKE magnetometry

Sputtered metallic films with ⊥ anisotropy



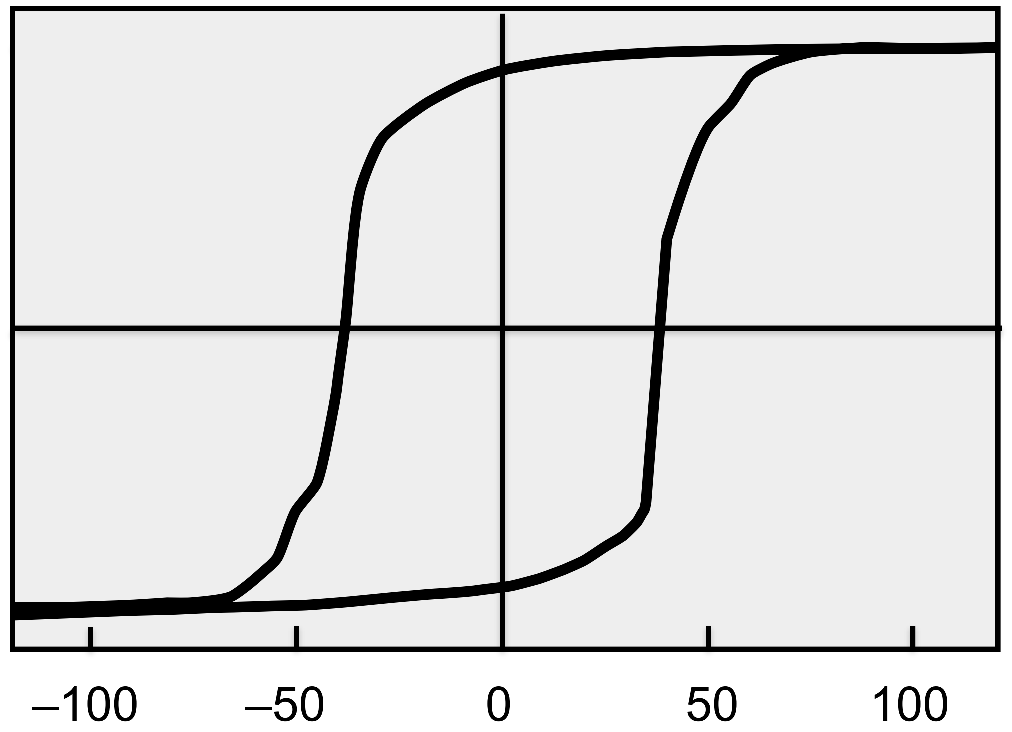
CoPt (3 nm)

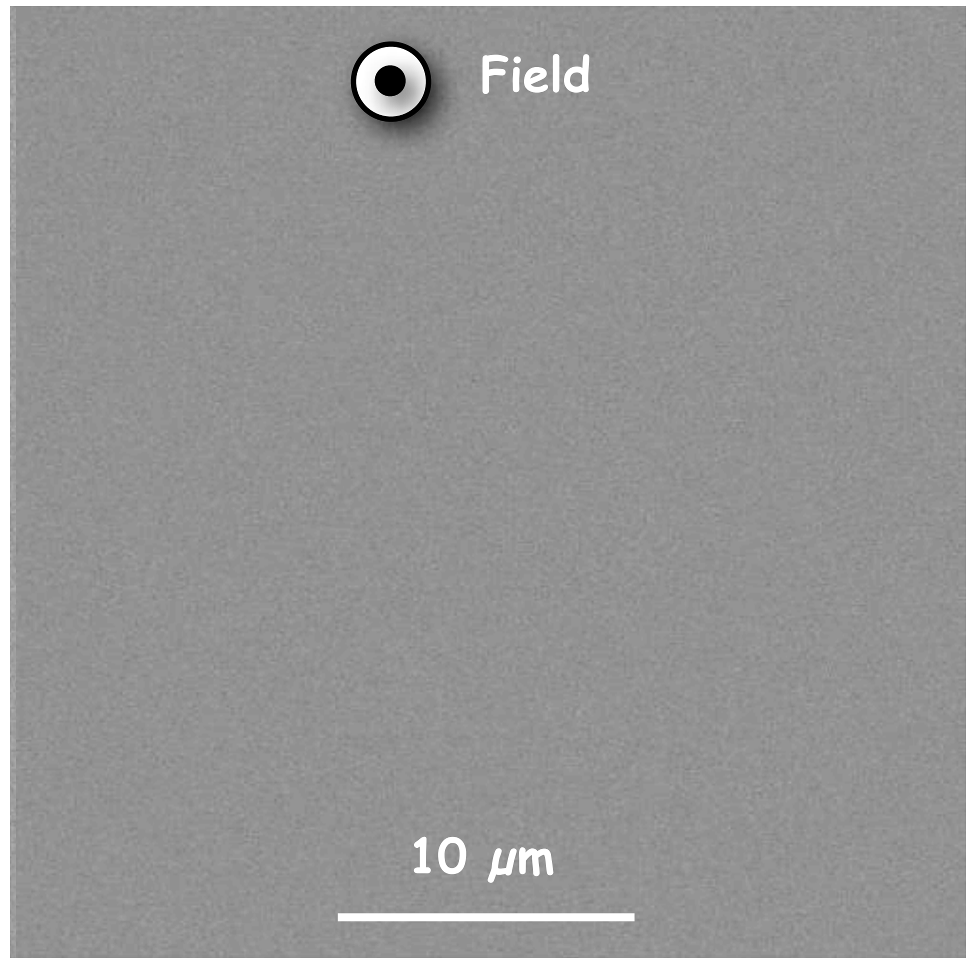




Sample courtesy
D. Makarov, IFW Dresden

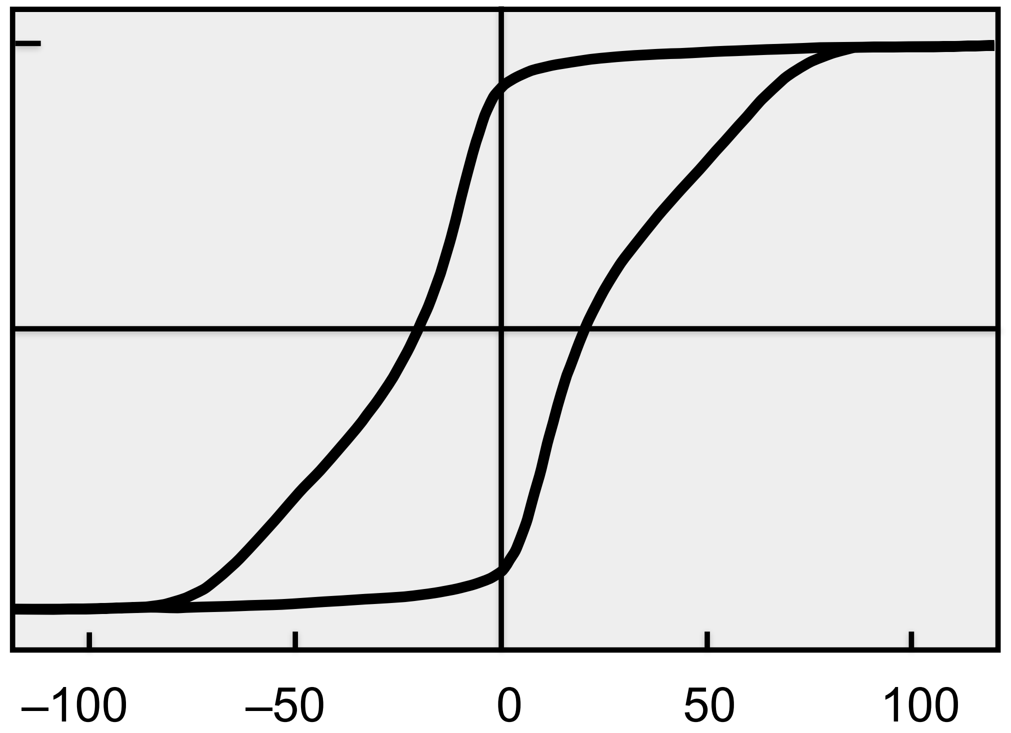
FePt (16 nm)

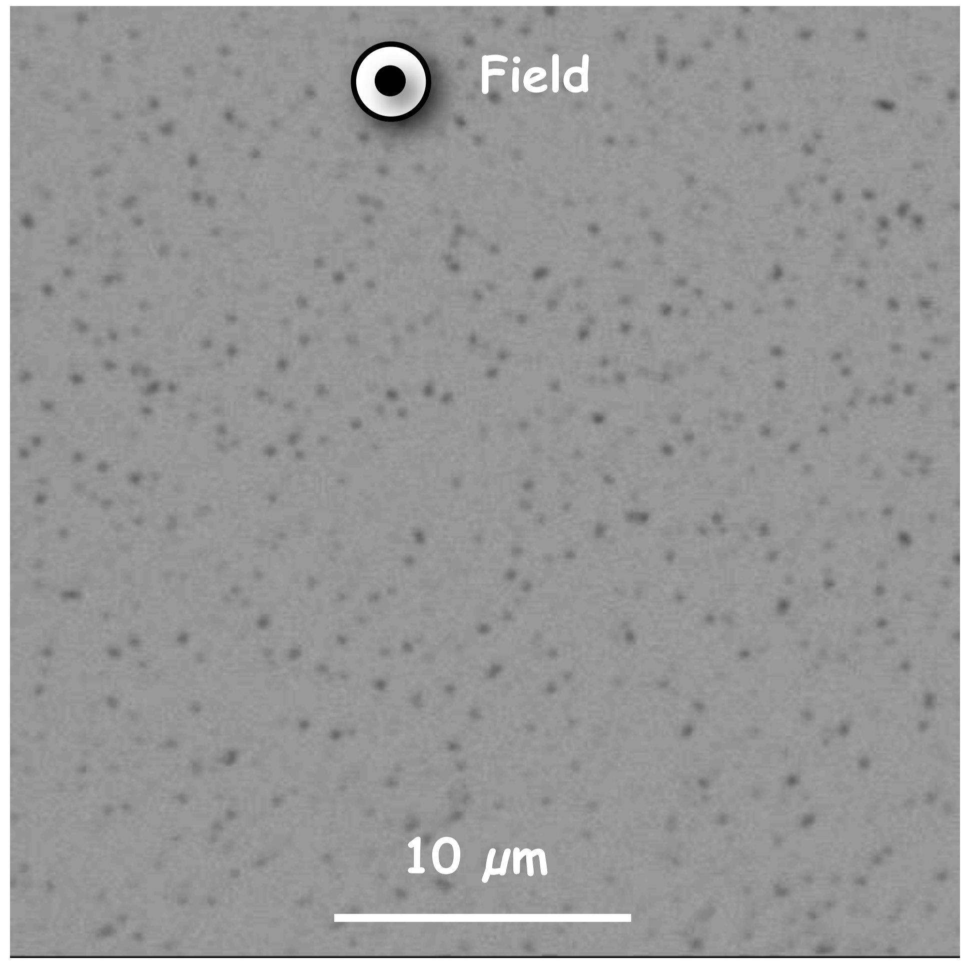




Sample courtesy
P. He and S.M. Zhou, Fudan

FePd (15 nm)/FePt (20 nm)

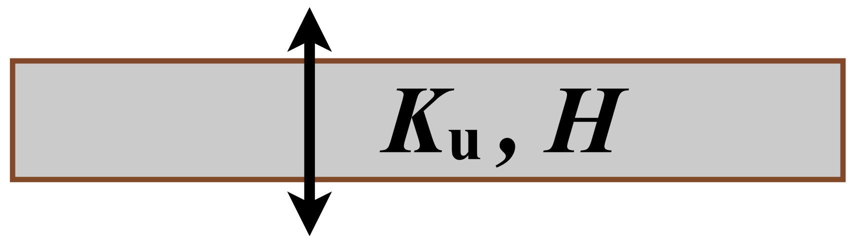




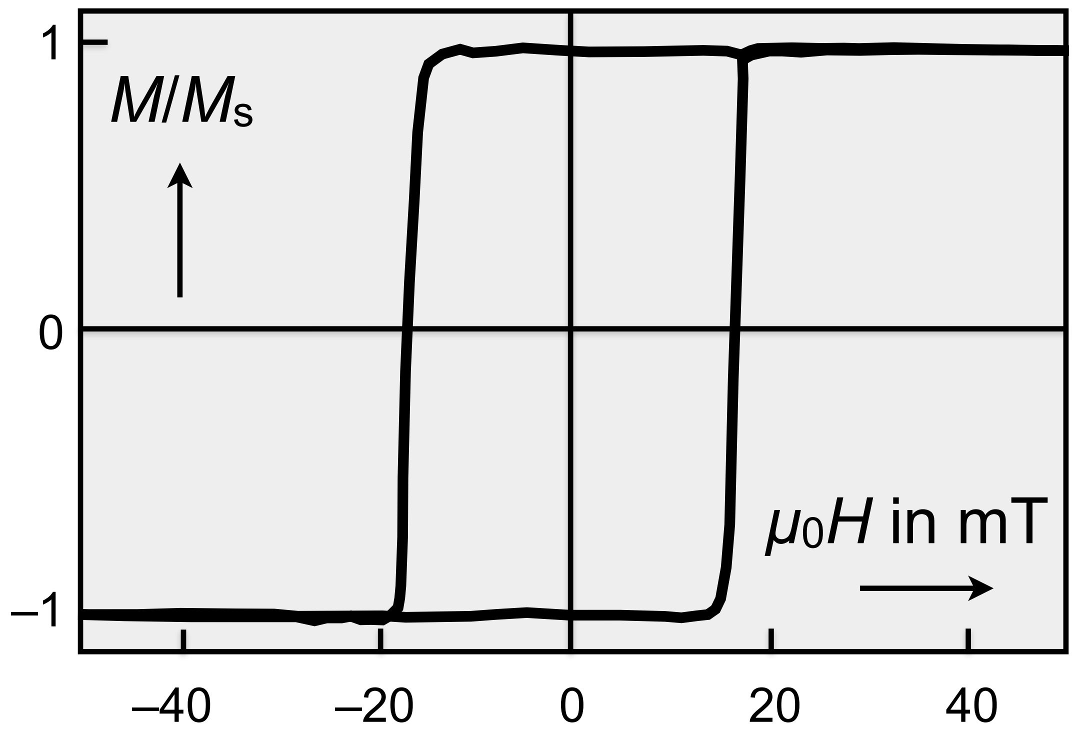
Sample courtesy
M. Shibihan and S.M. Zhou, Tongji

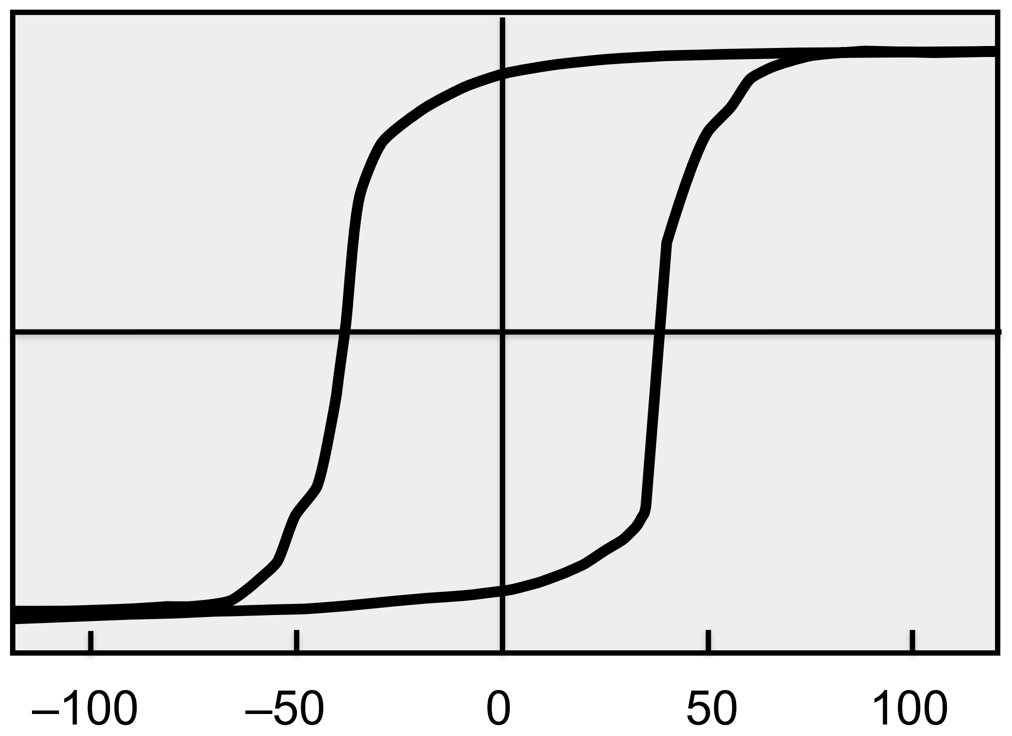
Combination with MOKE magnetometry

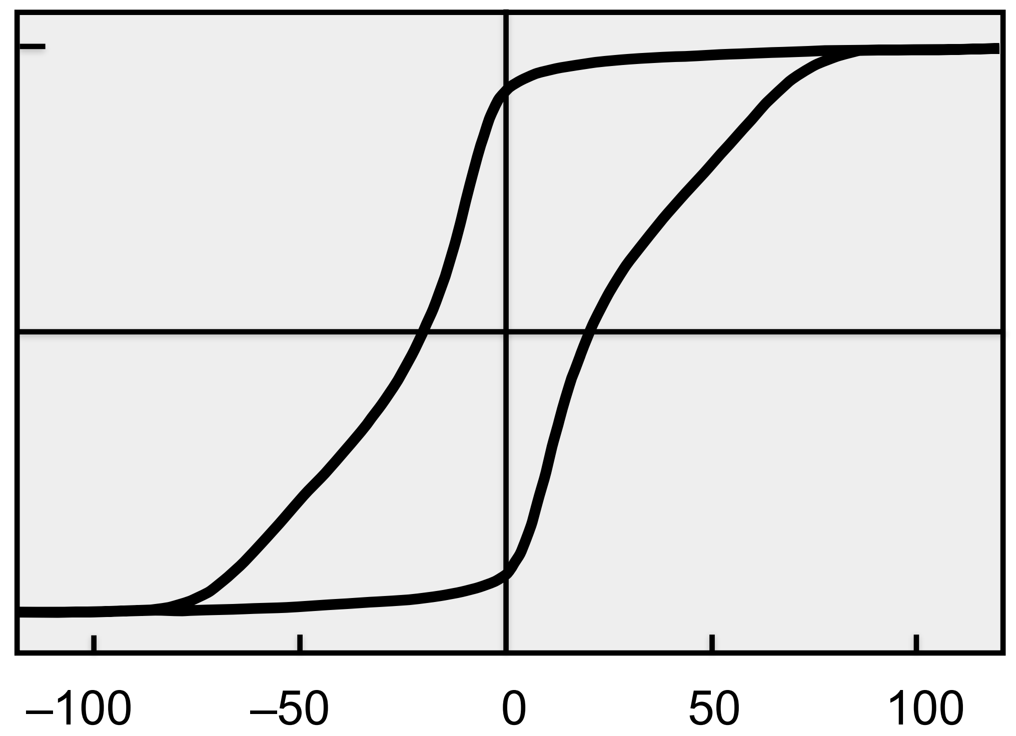
Sputtered metallic films with ⊥ anisotropy

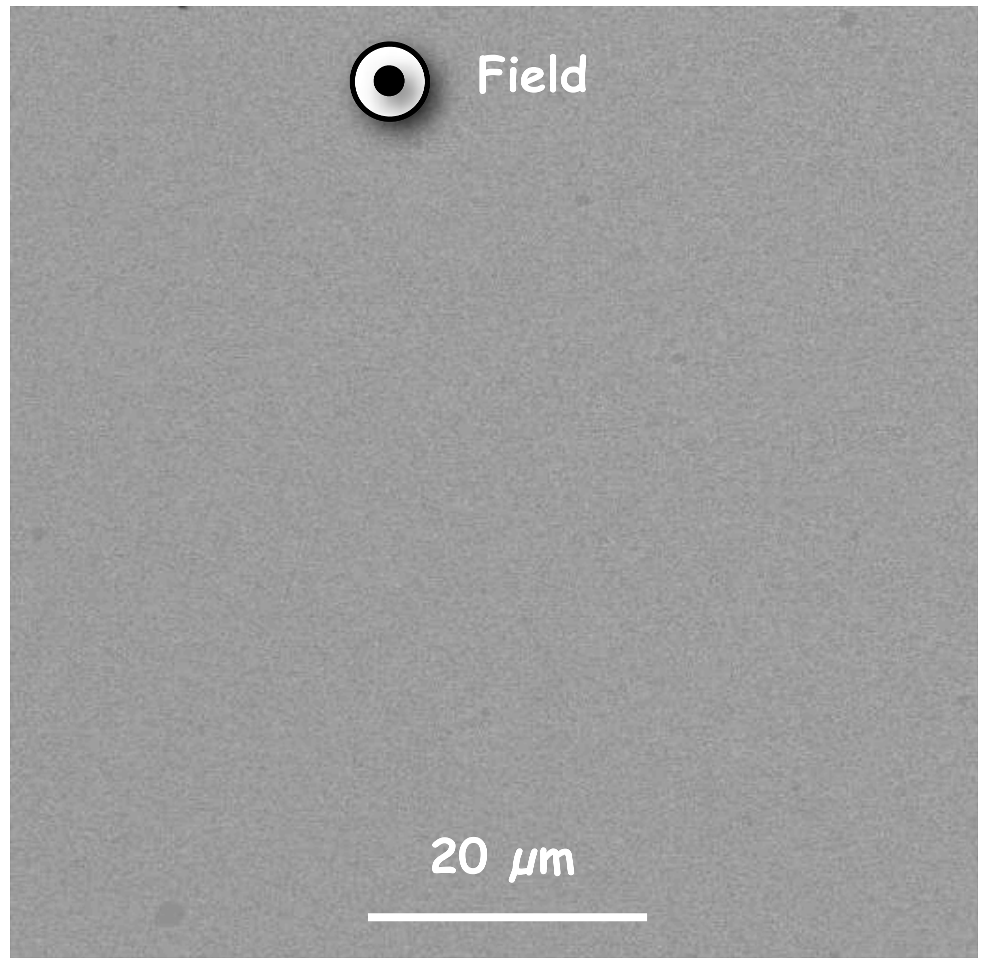


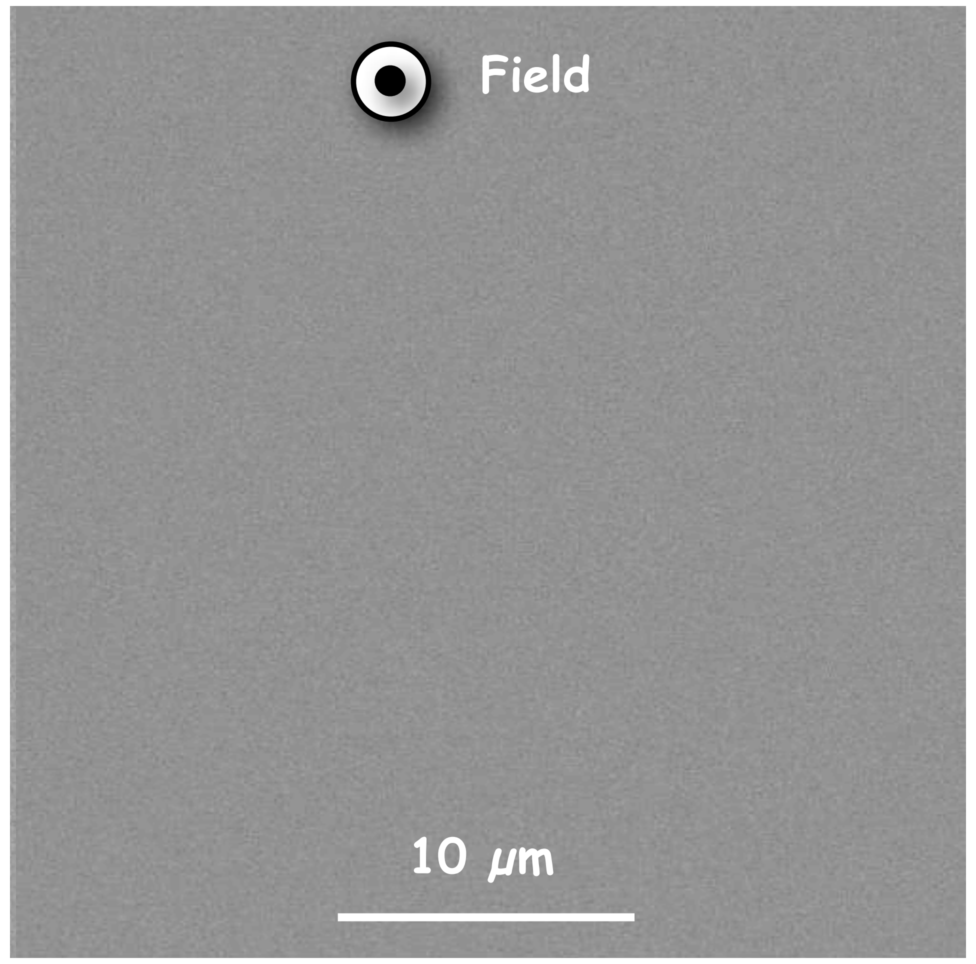
CoPt (3 nm)	FePt (16 nm)	FePd (15 nm)/FePt (20 nm)

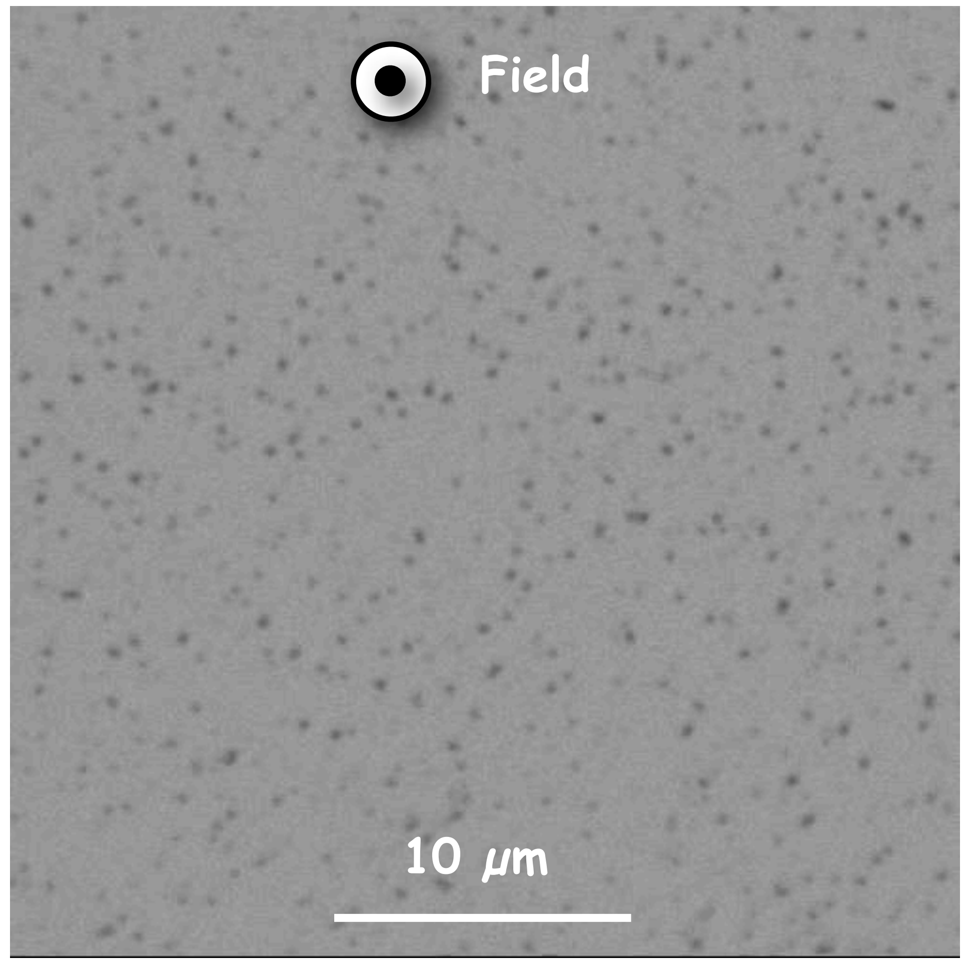












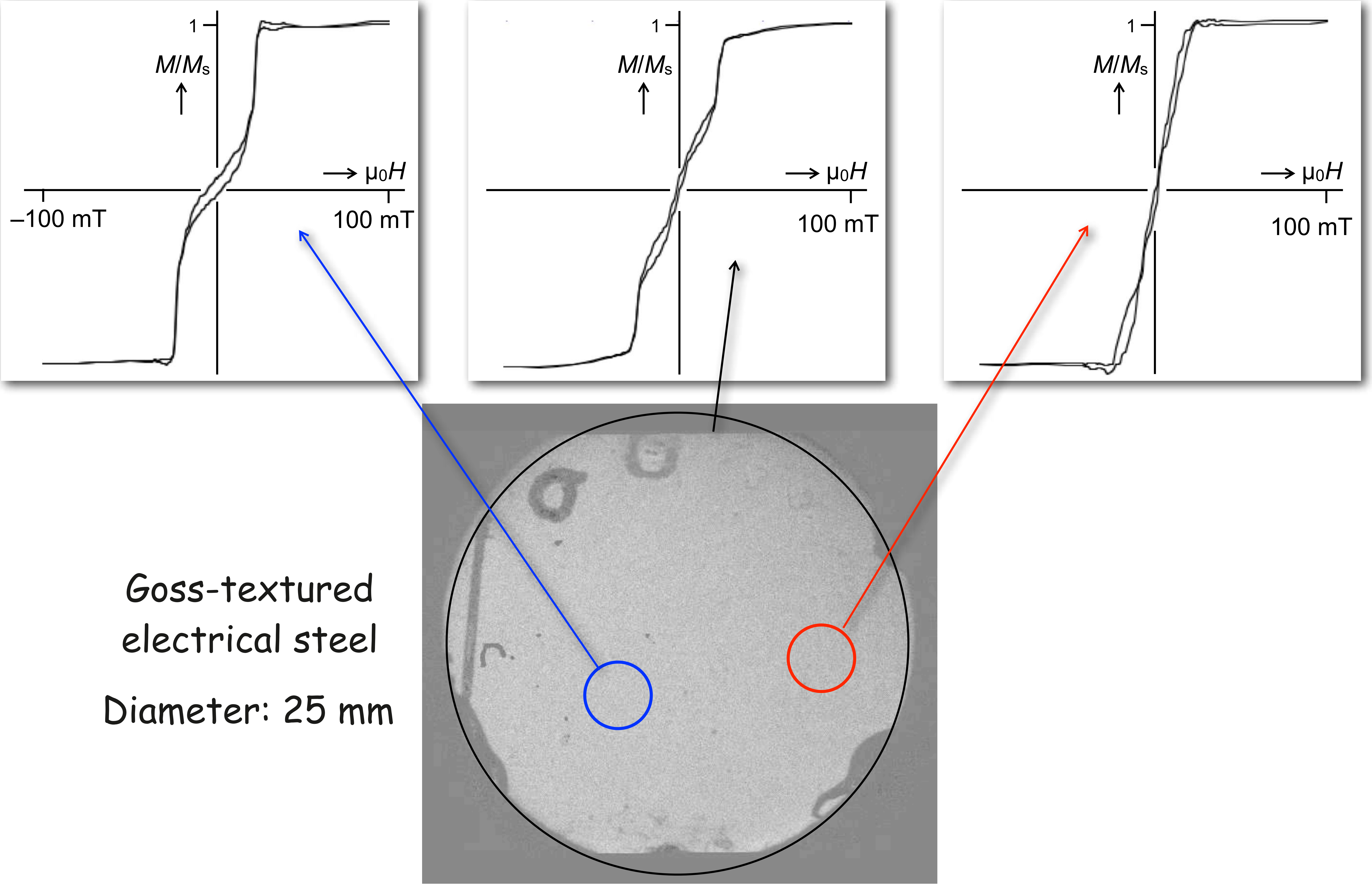
Sample courtesy
D. Makarov, IFW Dresden

Sample courtesy
P. He and S.M. Zhou, Fudan

Sample courtesy
M. Shibihan and S.M. Zhou, Tongji

4. Magneto-Optical Kerr Microscopy

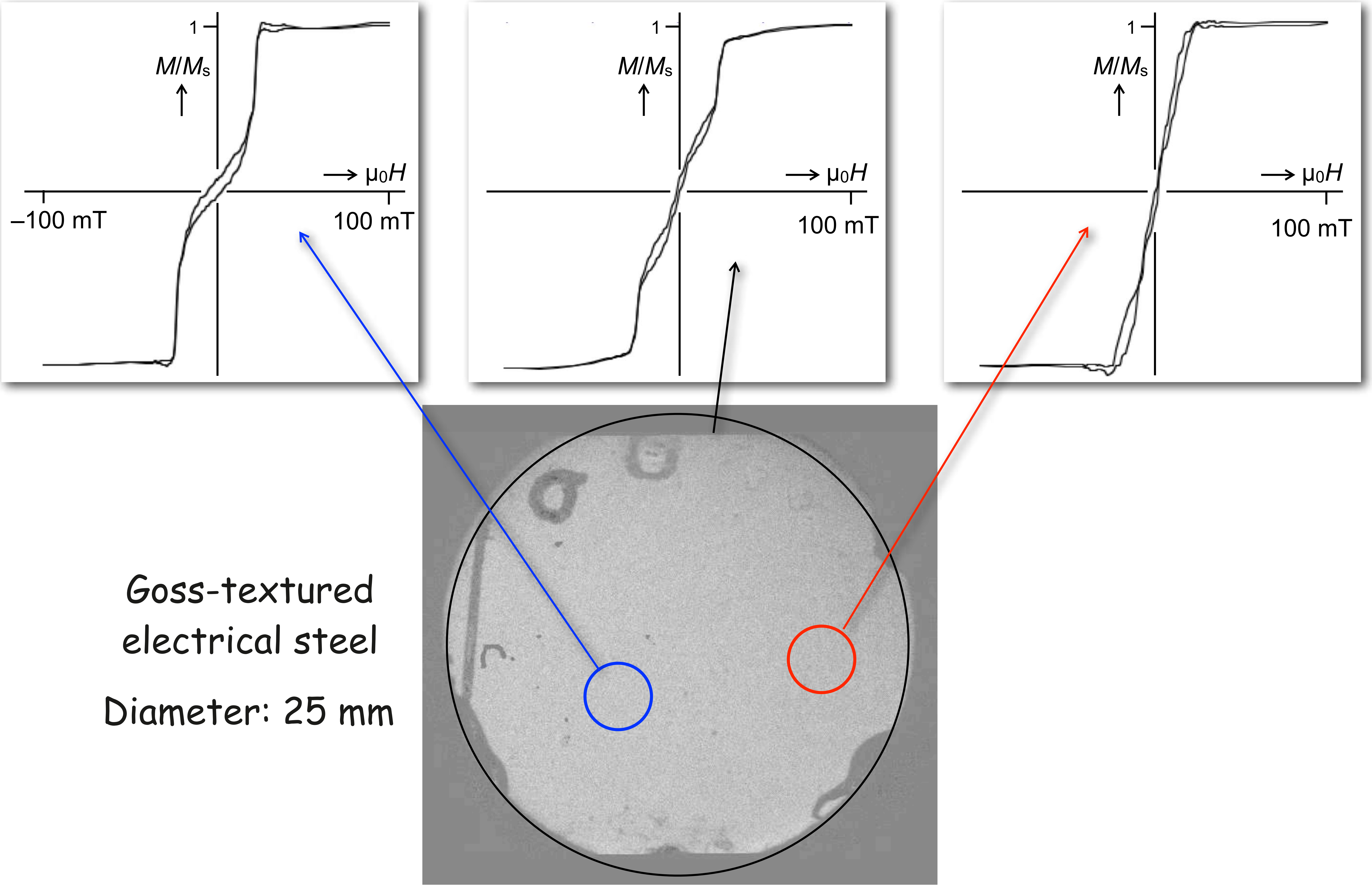
Combination with MOKE magnetometry



Goss-textured electrical steel

Diameter: 25 mm

Combination with MOKE magnetometry



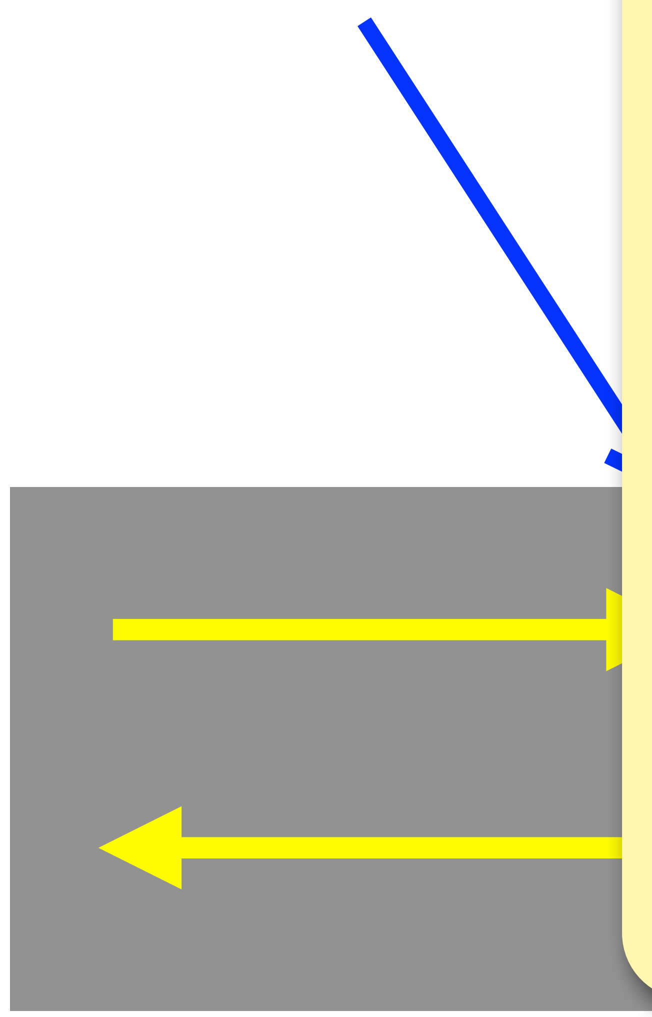
Goss-textured electrical steel

Diameter: 25 mm

4. Magneto-Optical Kerr Microscopy

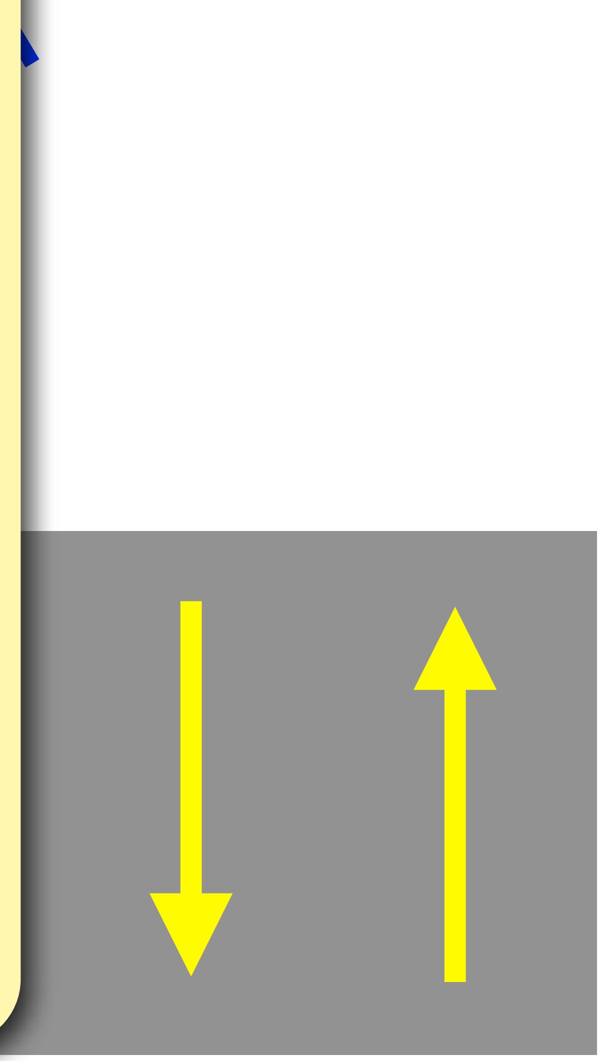
Kerr-contrast law

Kerr contrast is proportional
to magnetization component along
propagation direction of reflected light beam



$$\boldsymbol{D} = \varepsilon \boldsymbol{E} = \varepsilon \begin{pmatrix} 1 & -\mathrm{i}\,Q\,m_3 & \mathrm{i}\,Q\,m_2 \\ \mathrm{i}\,Q\,m_3 & 1 & -\mathrm{i}\,Q\,m_1 \\ -\mathrm{i}\,Q\,m_2 & \mathrm{i}\,Q\,m_1 & 1 \end{pmatrix} \boldsymbol{E}$$

$$= \varepsilon \boldsymbol{E} + \mathrm{i}\,\varepsilon\,Q\,\boldsymbol{m} \times \boldsymbol{E}$$



Longitudinal Kerr effect

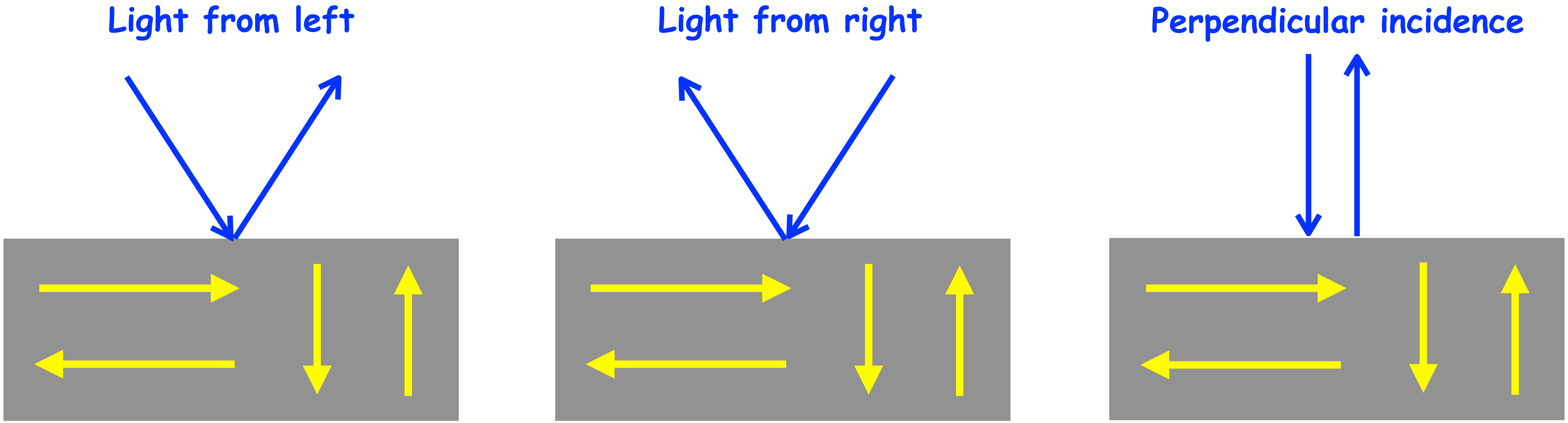
(sensitive to
in-plane and out-of-plane magnetization)

Polar Kerr effect

(only sensitive to
out-of-plane magnetization)

Kerr-contrast law

Kerr contrast is proportional
to magnetization component along
propagation direction of reflected light beam



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4. Magneto-Optical Kerr Microscopy

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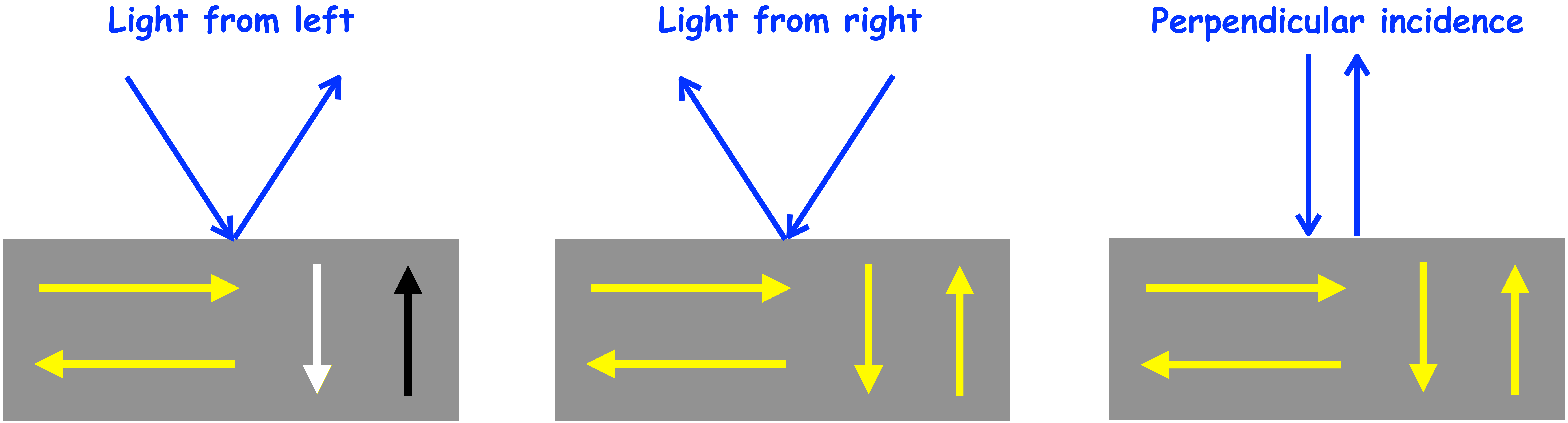

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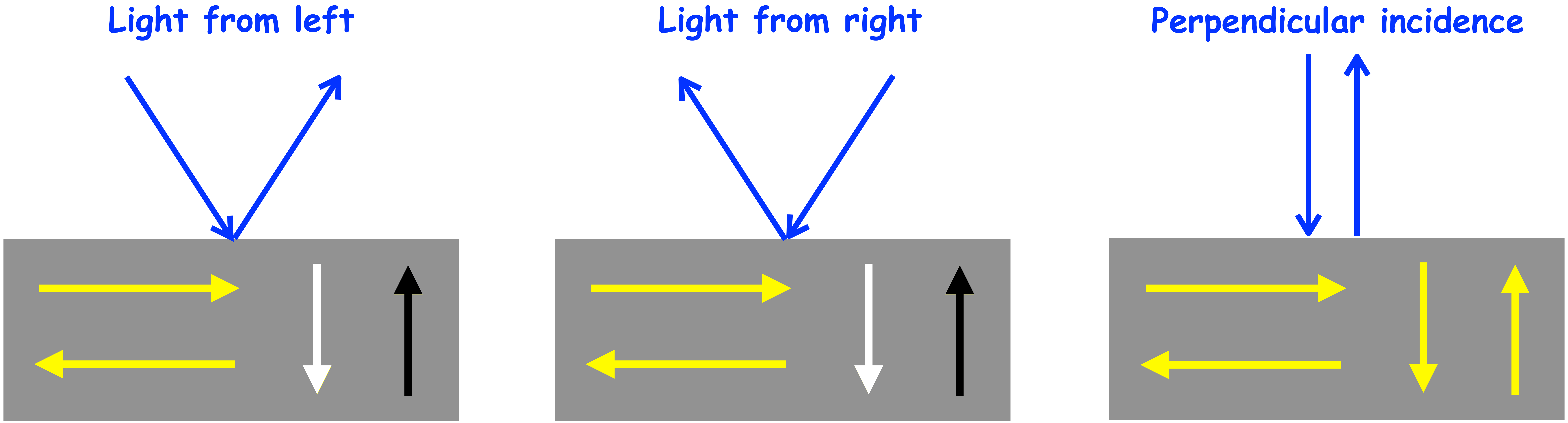

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4. Magneto-Optical Kerr Microscopy

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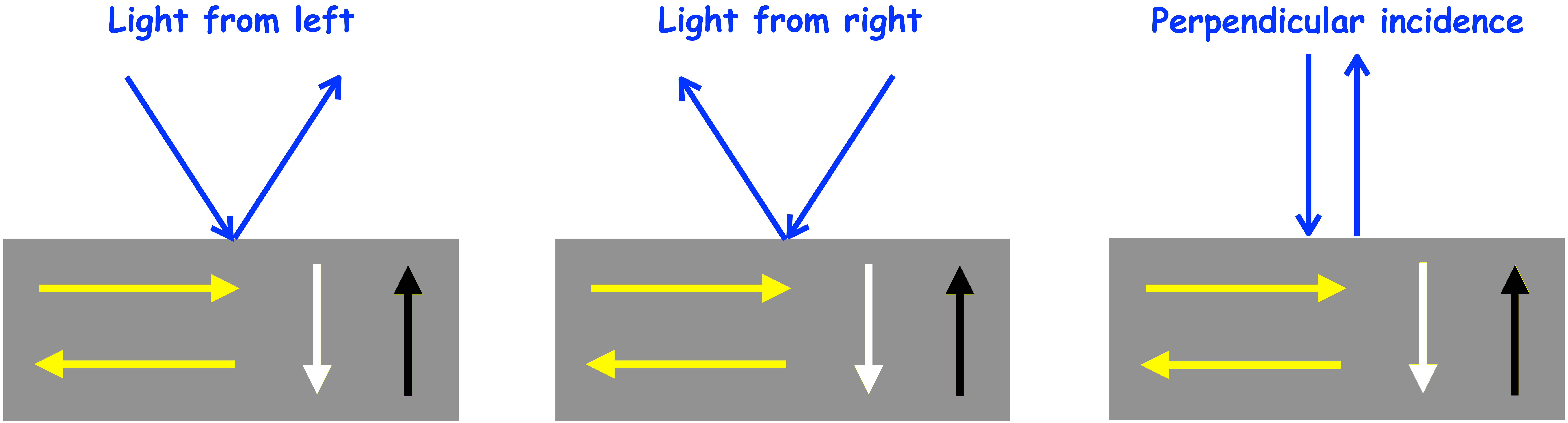

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Polar Kerr effect

(only sensitive to
out-of-plane magnetization)

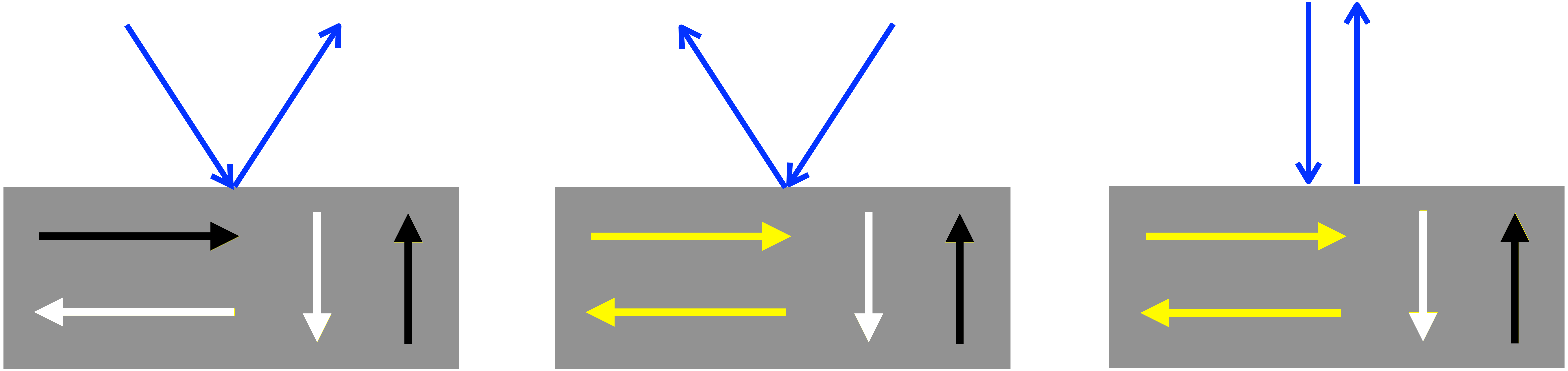
Kerr-contrast law

Kerr contrast is proportional
to magnetization component along
propagation direction of reflected light beam

Light from left

Light from right

Perpendicular incidence



Longitudinal Kerr effect

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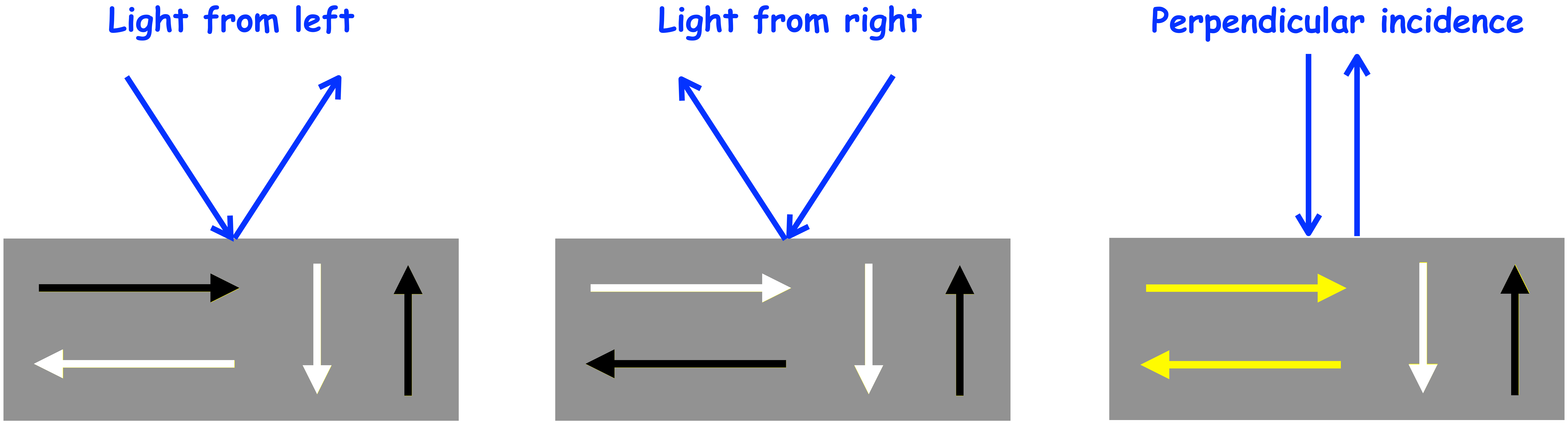

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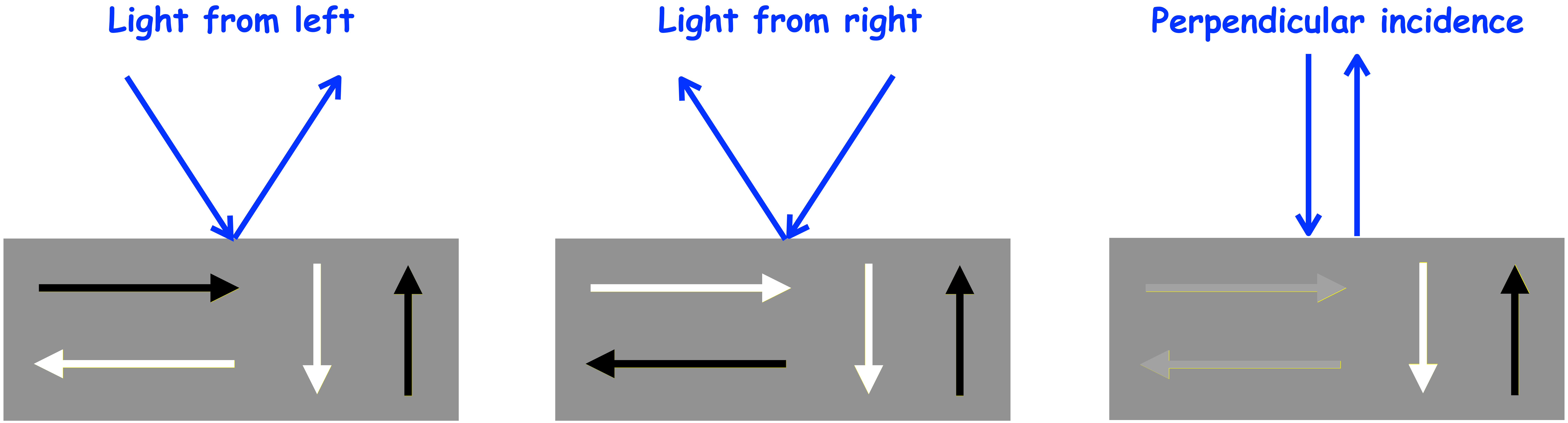
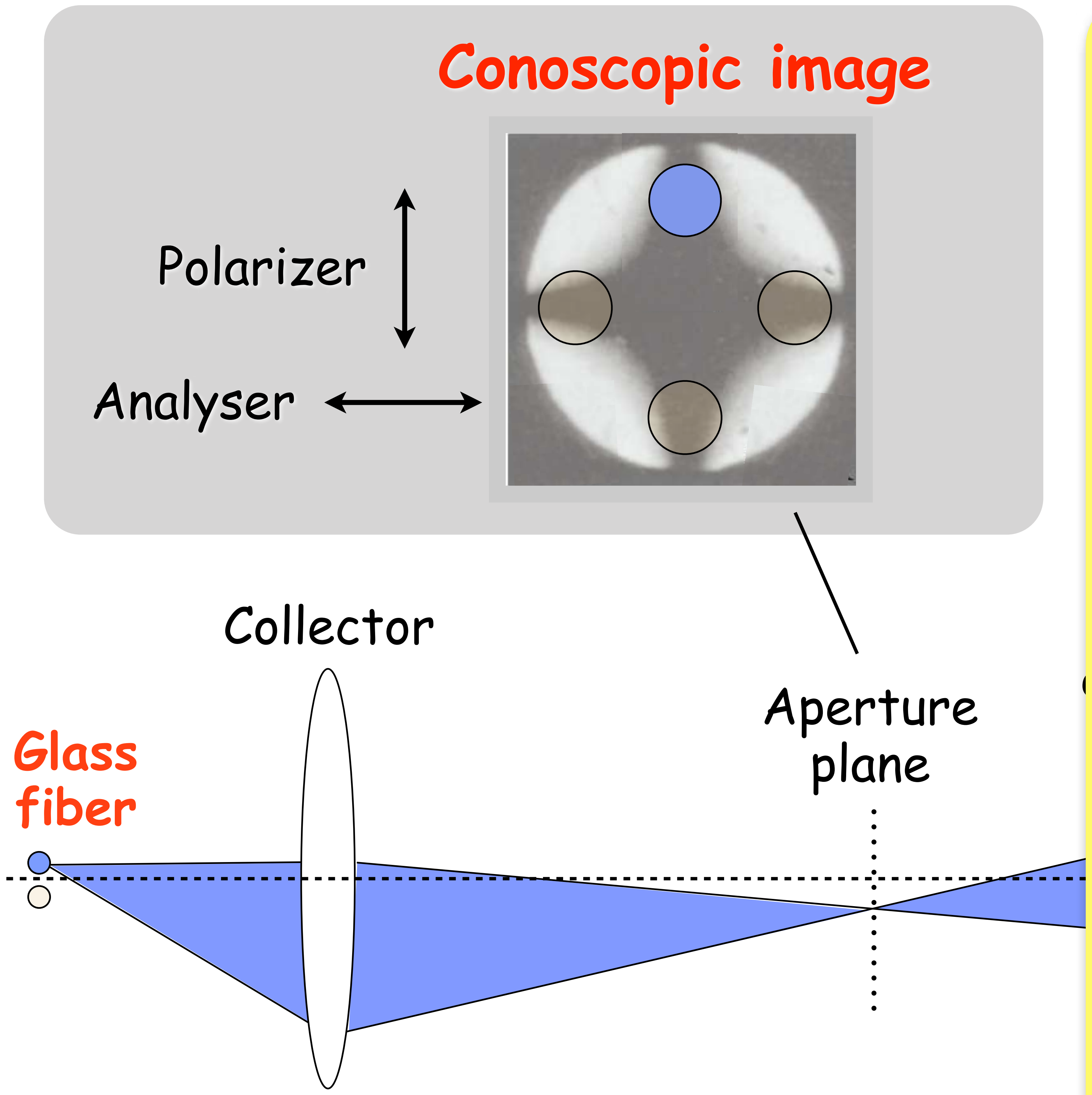


Longitudinal Kerr effect
(sensitive to
in-plane and out-of-plane magnetization)

Polar Kerr effect
(only sensitive to
out-of-plane magnetization)

4. Magneto-Optical Kerr Microscopy

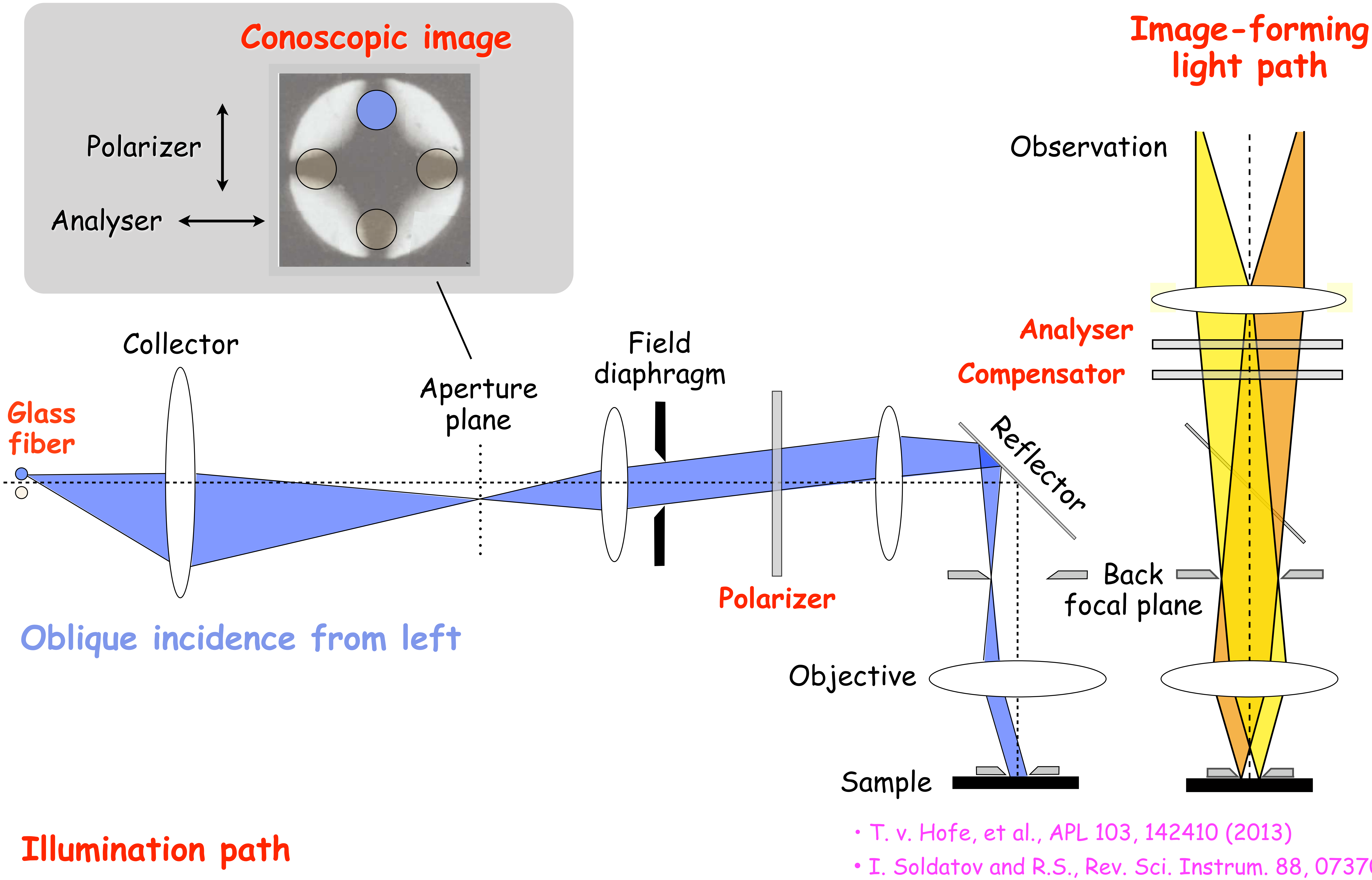


Oblique incidence from left

Illumination path

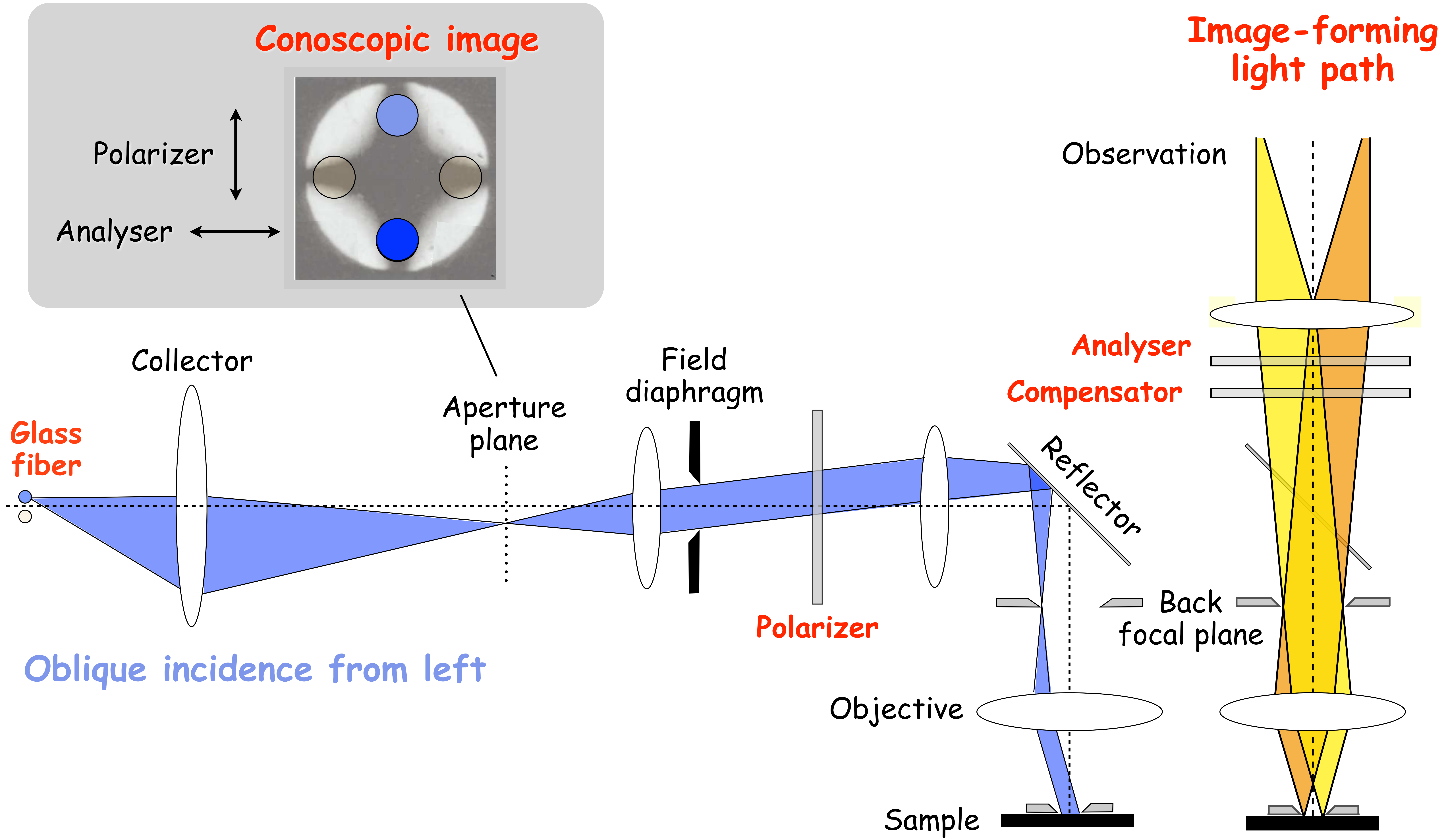
LED light source with glass fibers

4. Magneto-Optical Kerr Microscopy



- T. v. Hofe, et al., APL 103, 142410 (2013)
- I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

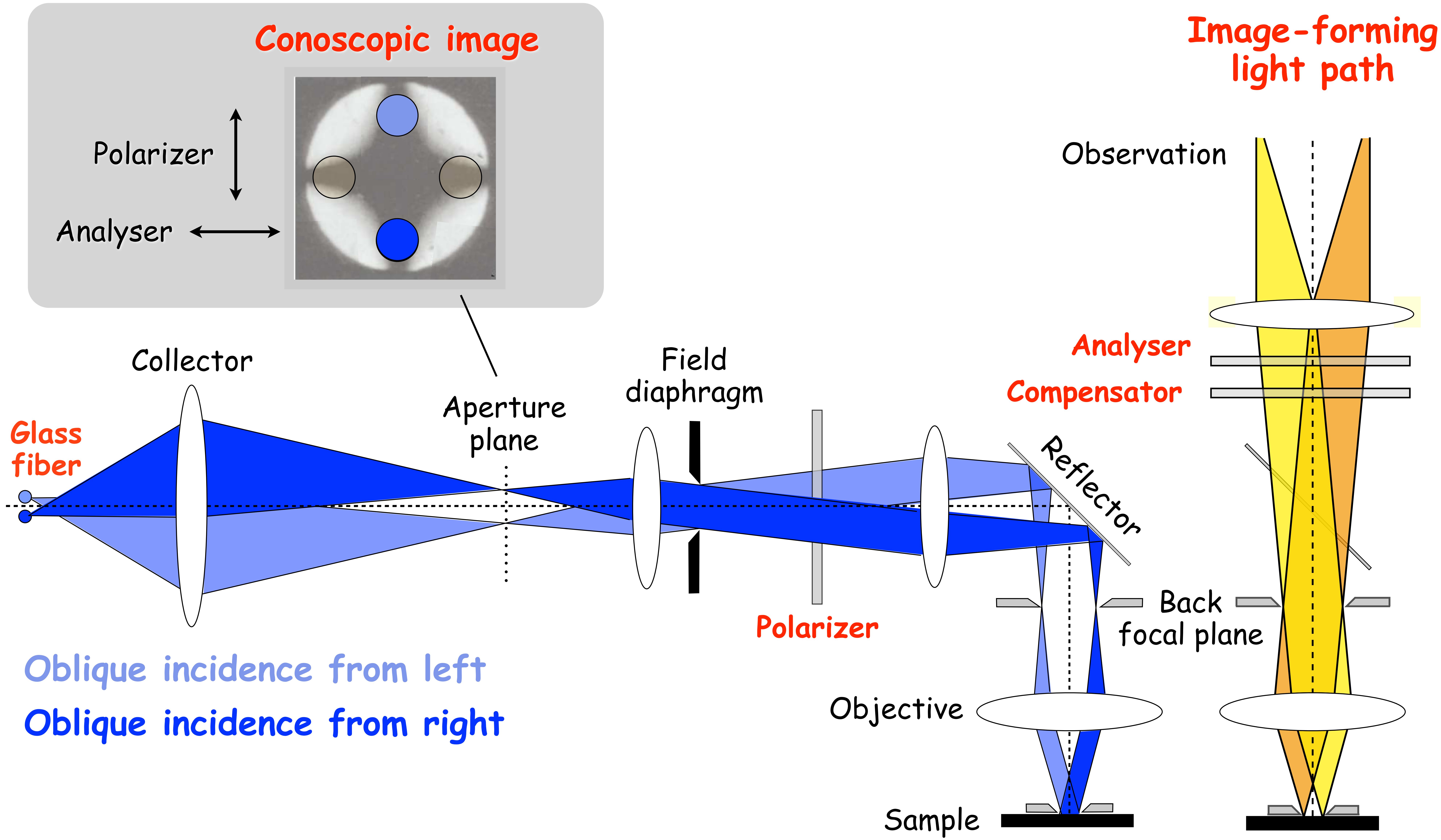
4. Magneto-Optical Kerr Microscopy



Illumination path

- T. v. Hofe, et al., APL 103, 142410 (2013)
- I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy



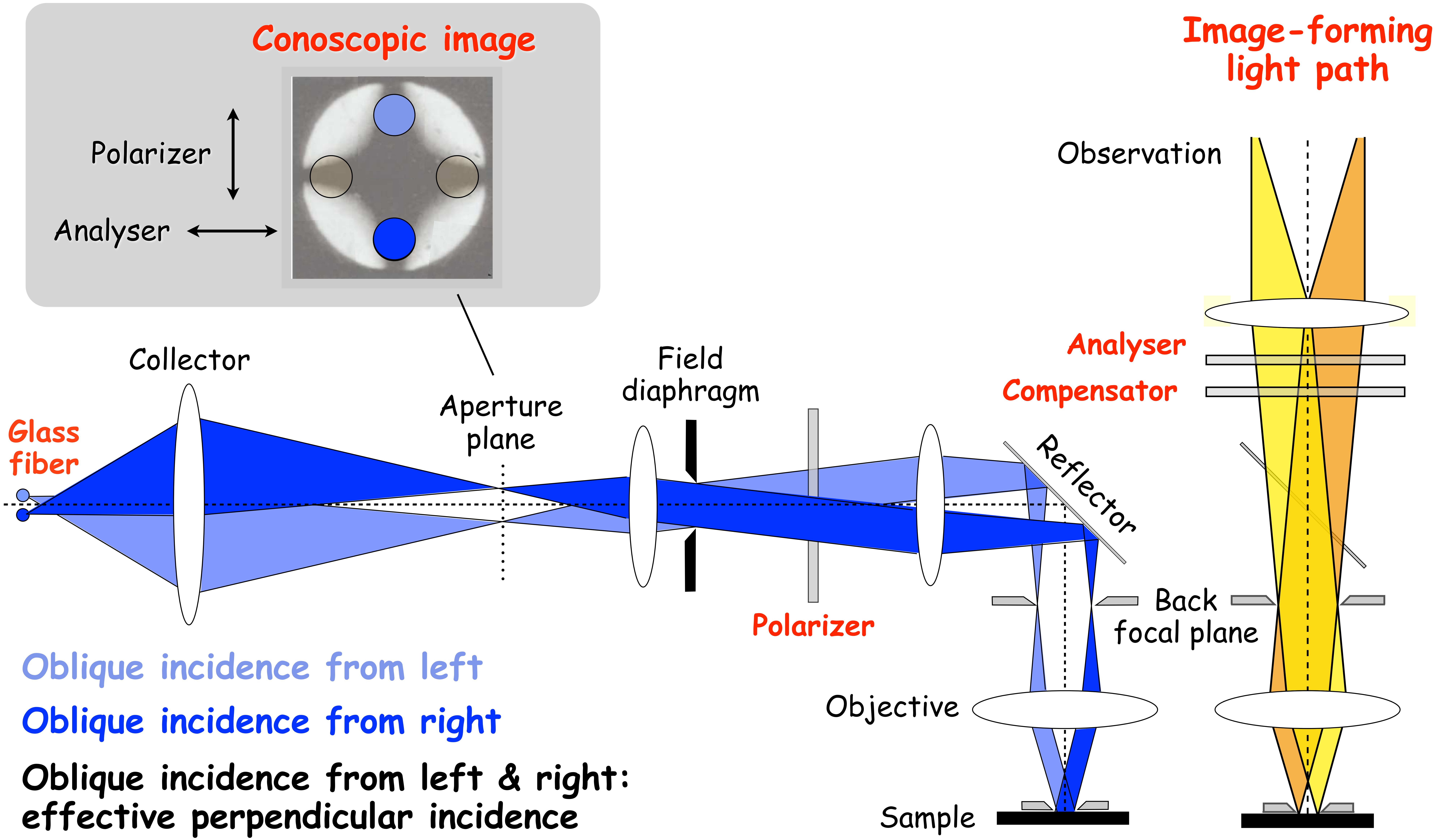
Oblique incidence from left

Oblique incidence from right

Illumination path

- T. v. Hofe, et al., APL 103, 142410 (2013)
- I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy



Oblique incidence from left

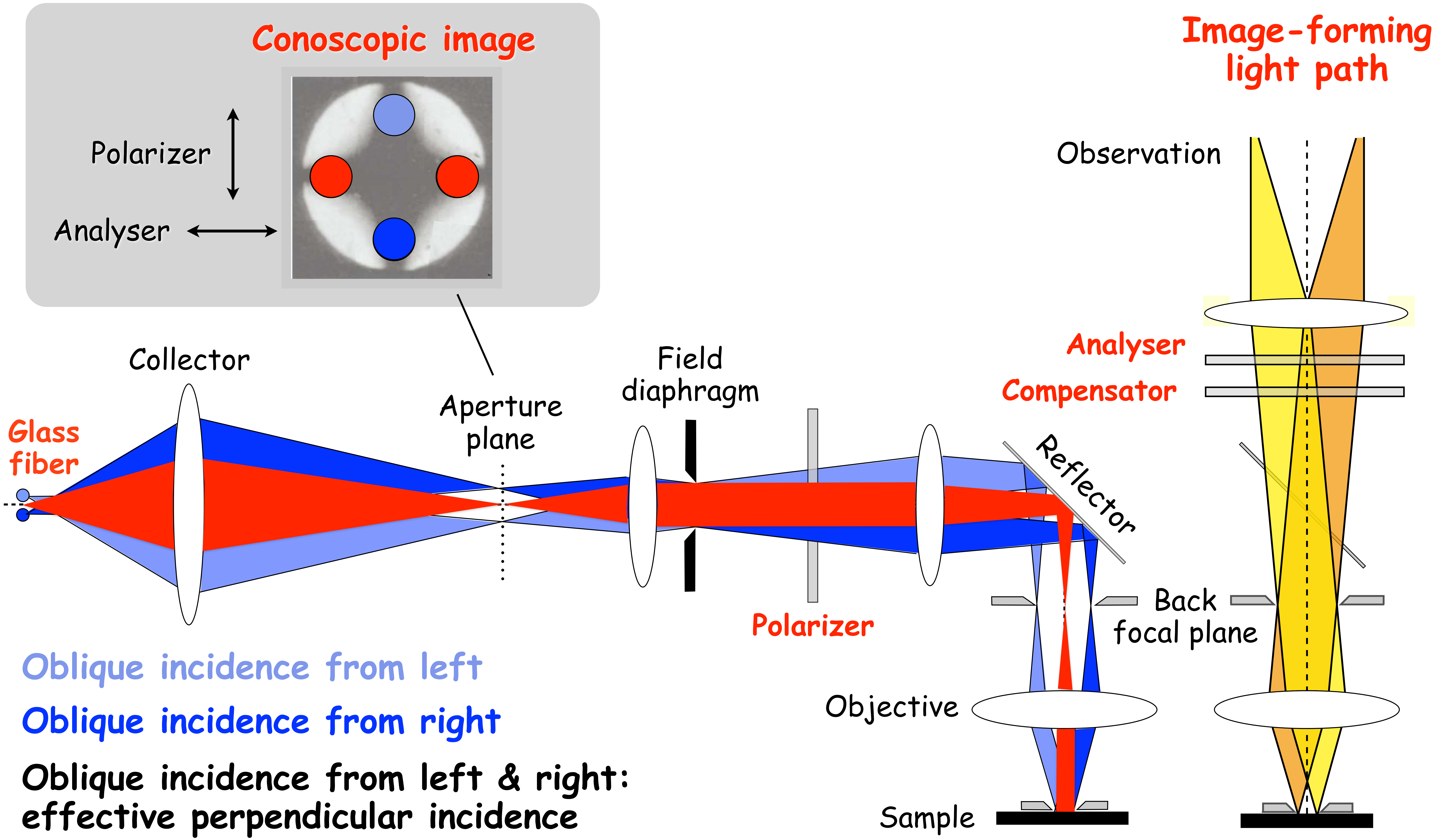
Oblique incidence from right

Oblique incidence from left & right: effective perpendicular incidence

Illumination path

- T. v. Hofe, et al., APL 103, 142410 (2013)
- I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy



Oblique incidence from left

Oblique incidence from right

Oblique incidence from left & right: effective perpendicular incidence

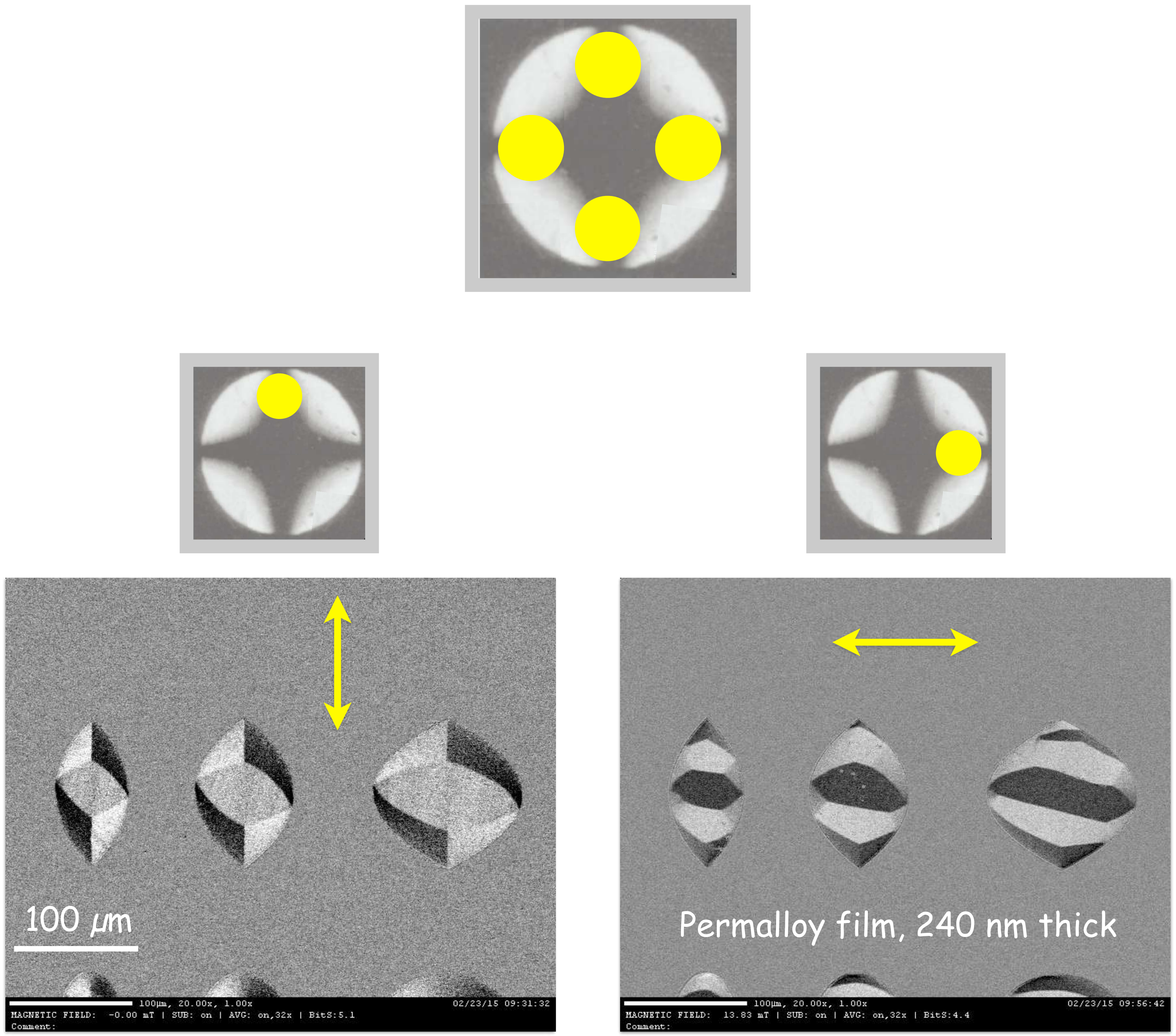
Oblique incidence from front or back

- T. v. Hofe, et al., APL 103, 142410 (2013)
- I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy

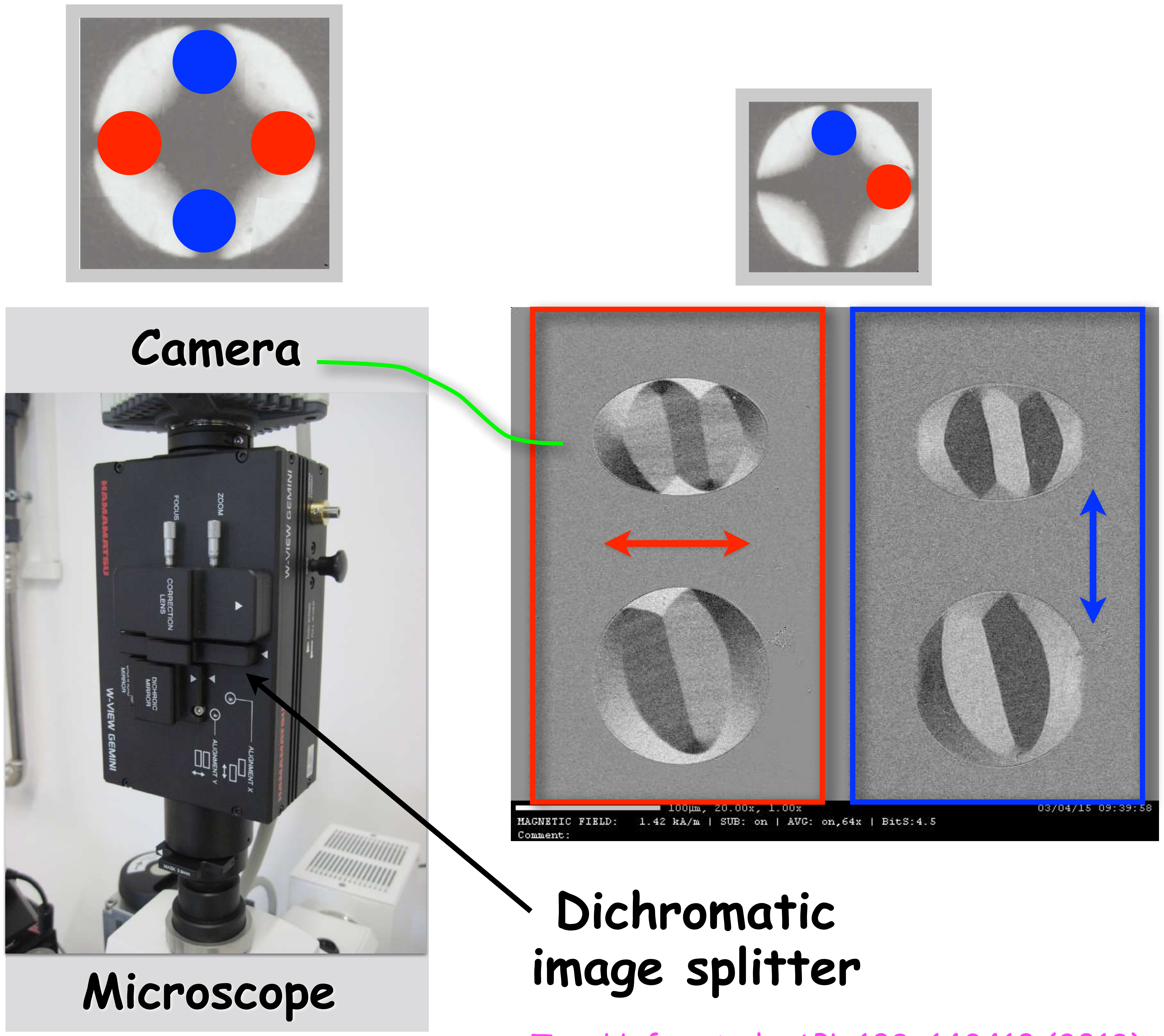
Wide-field Kerr-microscope: sensitivity selection

Monochromatic



I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

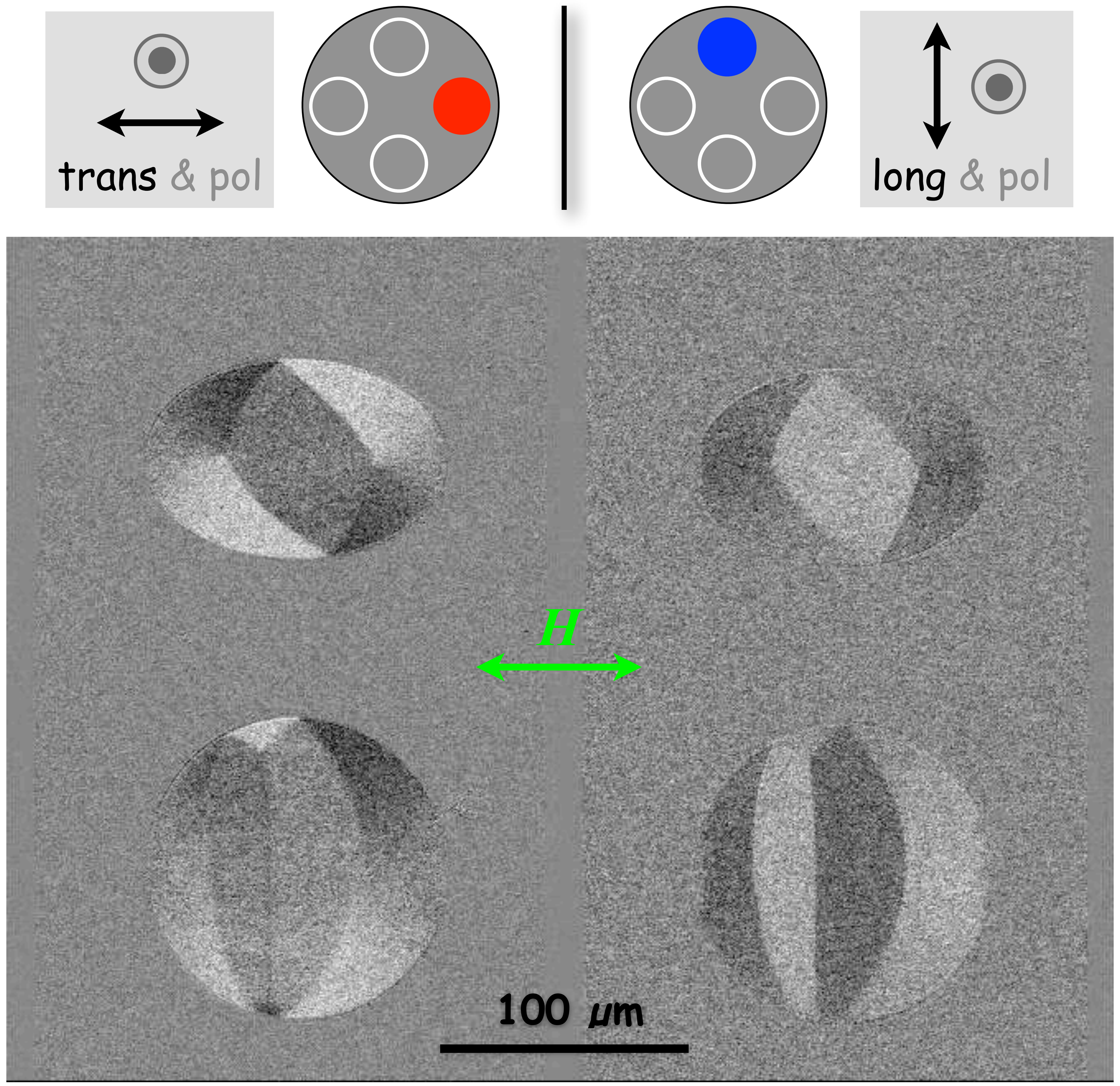
Dichromatic



T. v. Hofe, et al., APL 103, 142410 (2013)

Wide-field Kerr-microscope: sensitivity selection

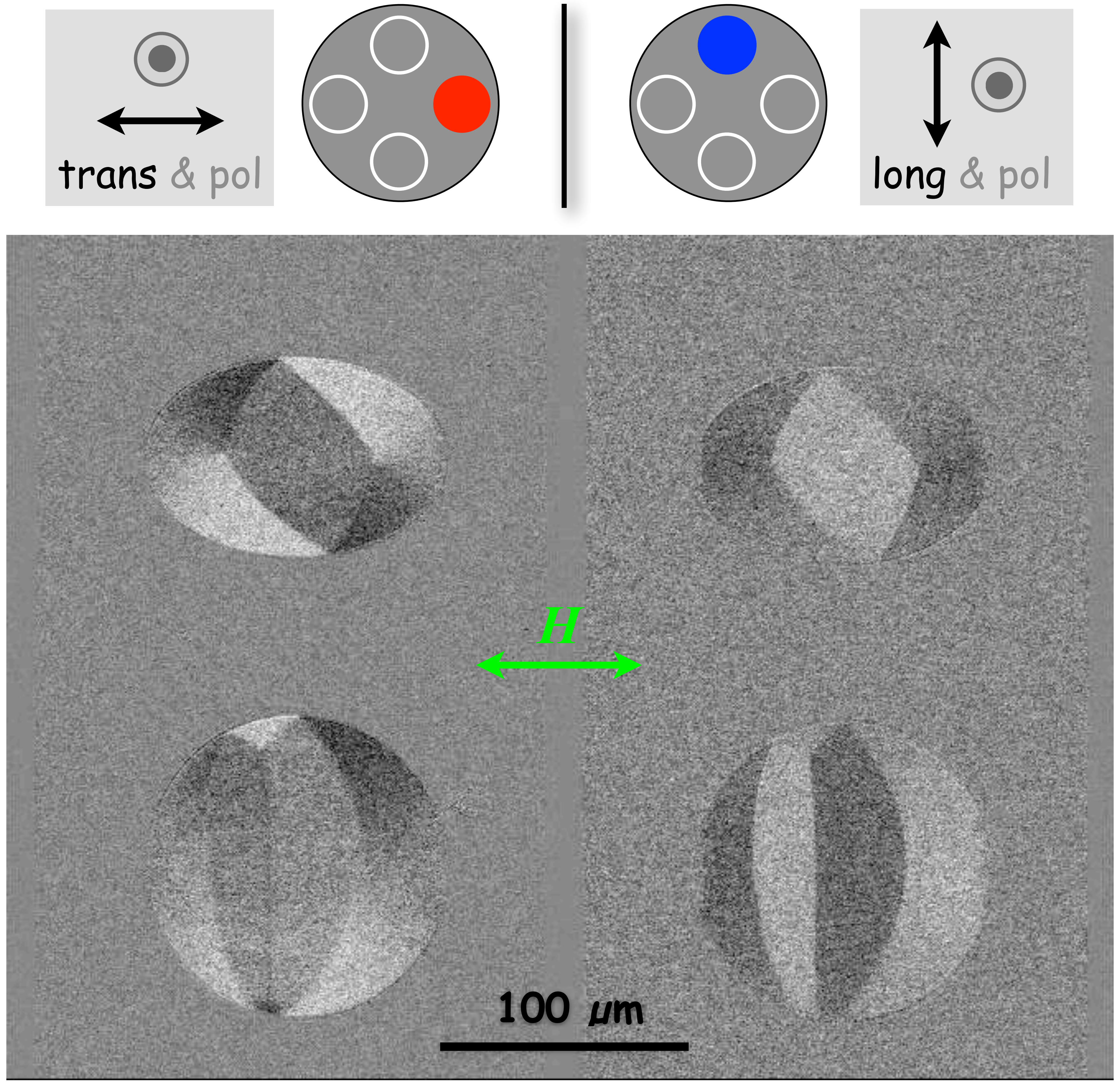
Dichromatic



Sample: Permalloy film, 240 nm thick

Wide-field Kerr-microscope: sensitivity selection

Dichromatic

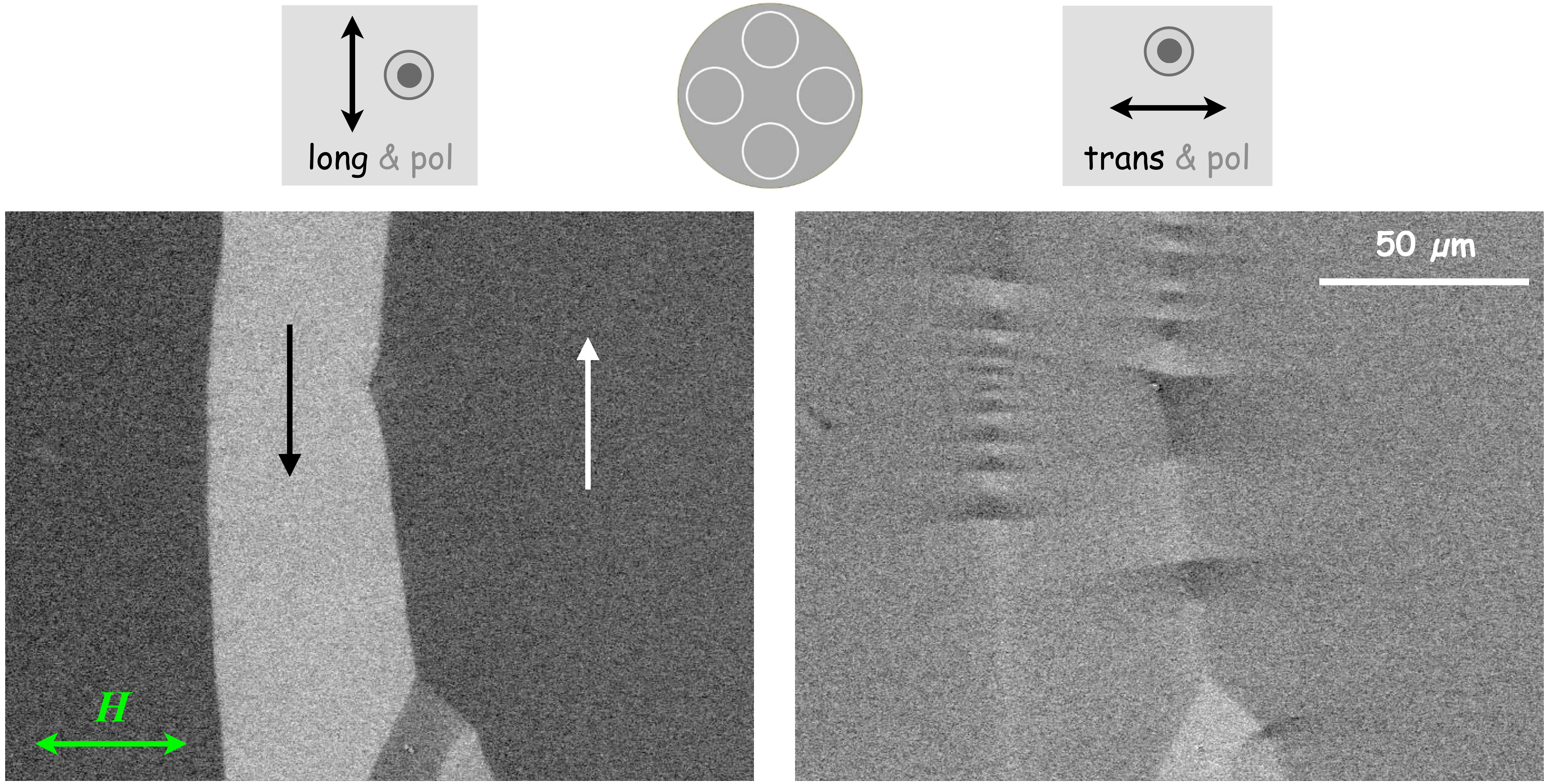


Sample: Permalloy film, 240 nm thick

Wide-field Kerr-microscope: sensitivity selection

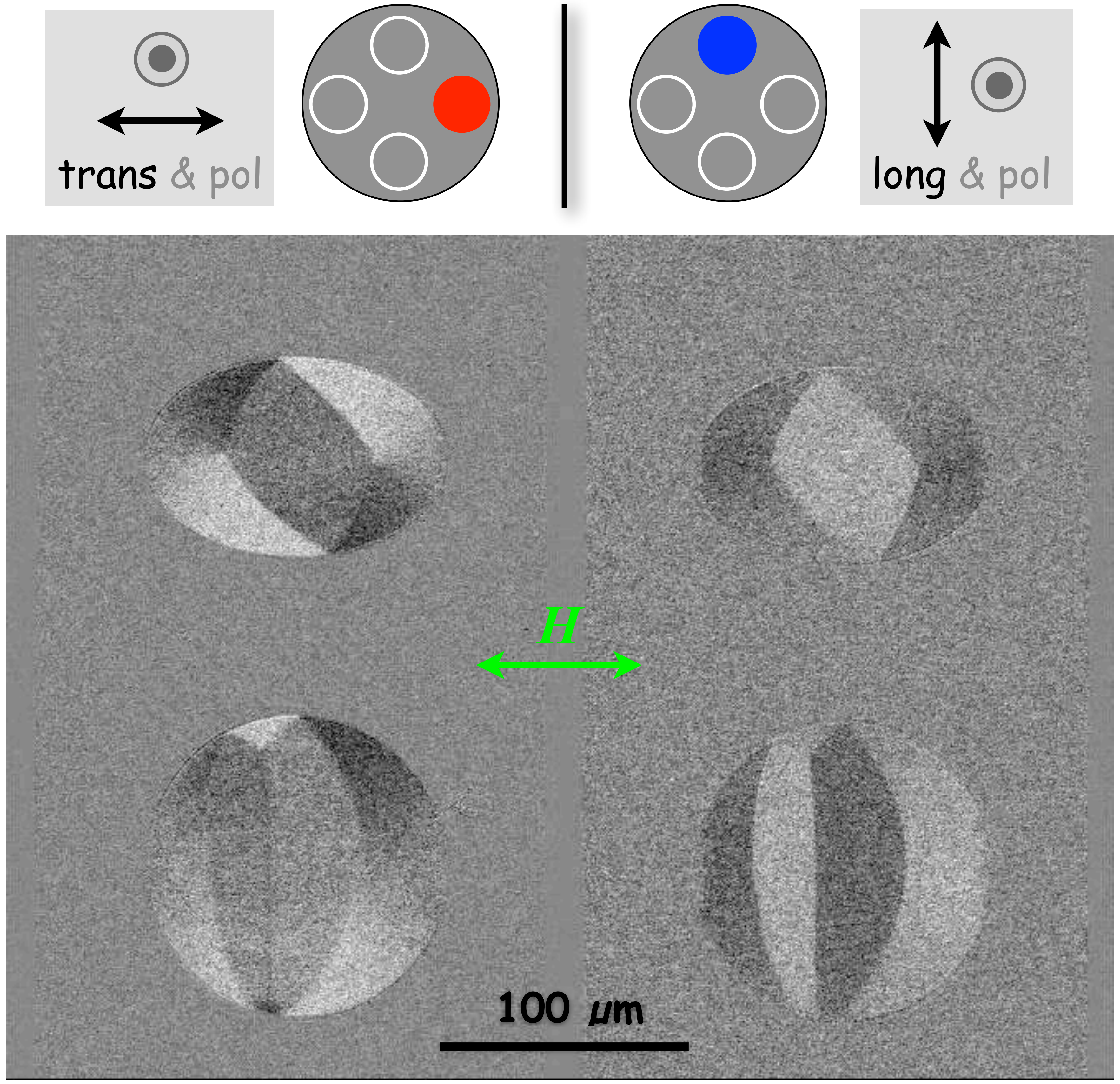
Monochromatic

Pulsing of LEDs,
synchronous with camera



Sample: Permalloy film, 40 nm thick,
courtesy Peter Savin, Ural Federal University, Russia

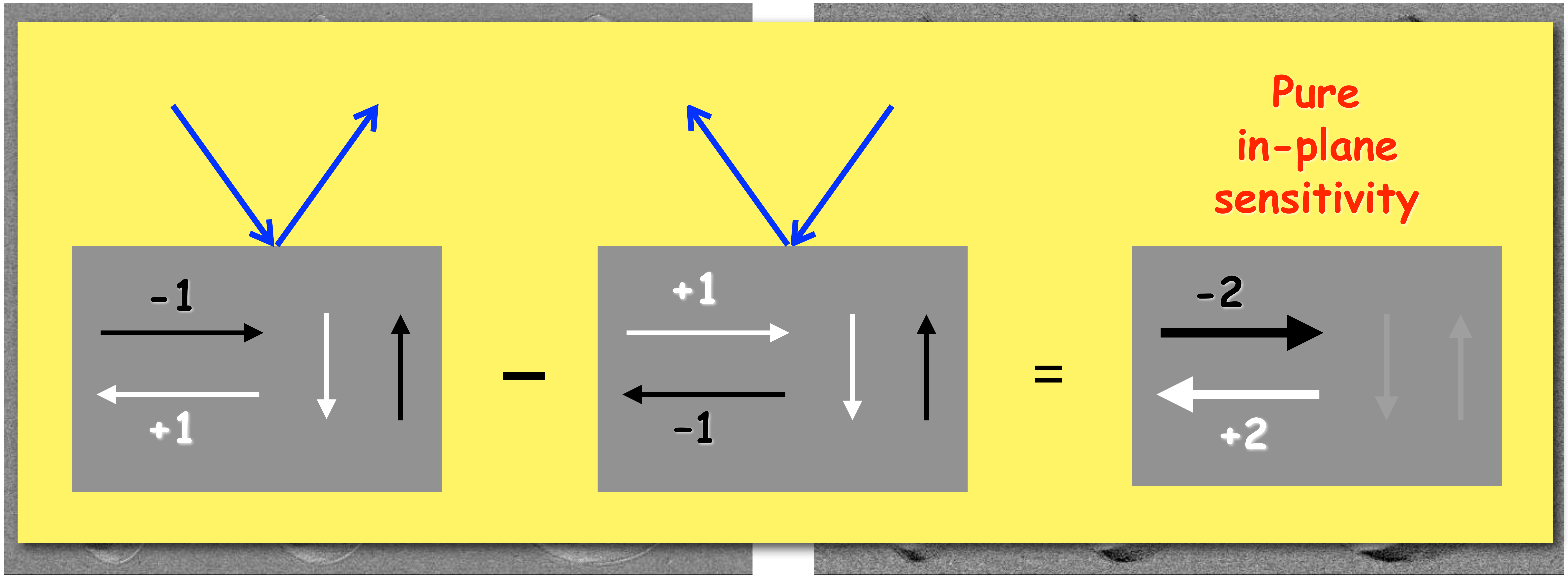
Dichromatic



Sample: Permalloy film, 240 nm thick

Wide-field Kerr-microscope: sensitivity selection & Sensitivity enhancement

Pulsing of LEDs, synchronous with camera



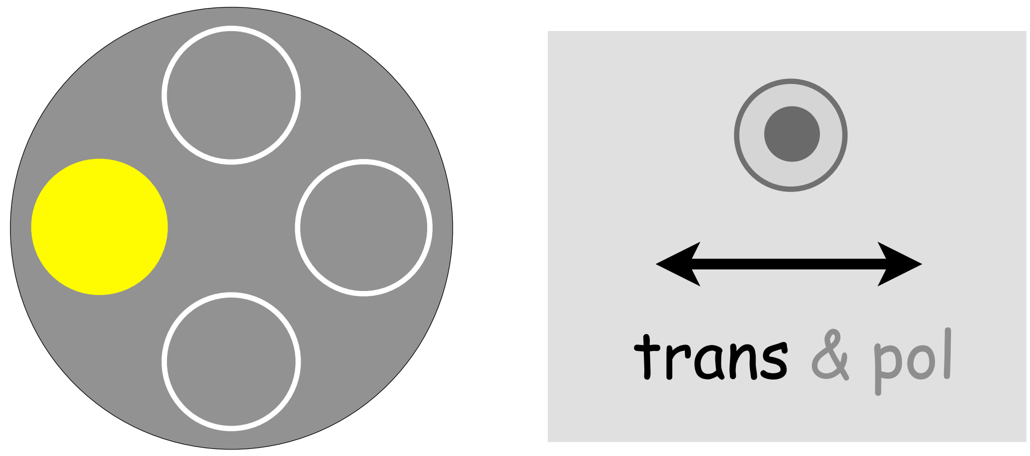
Sample: Permalloy film, 240 nm thick

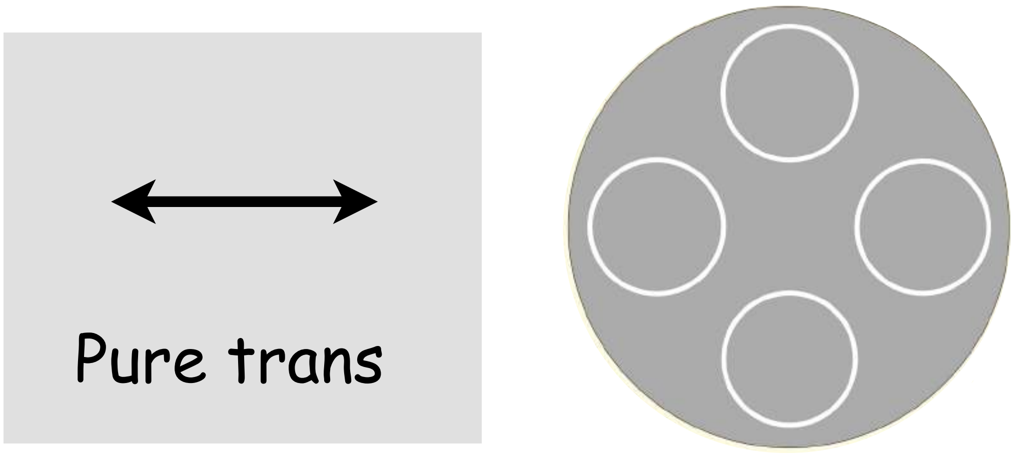
I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy

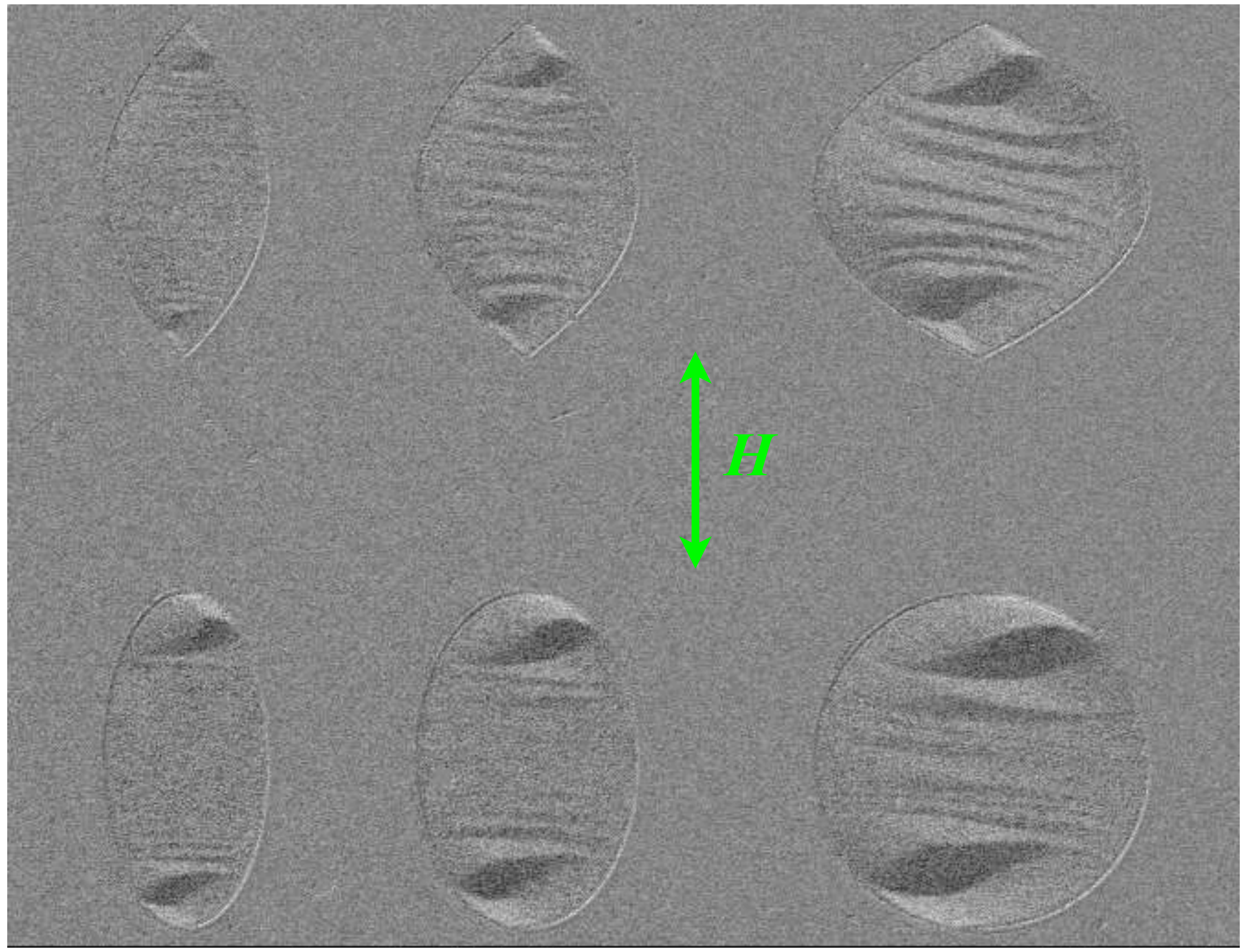
Wide-field Kerr-microscope: sensitivity selection & Sensitivity enhancement

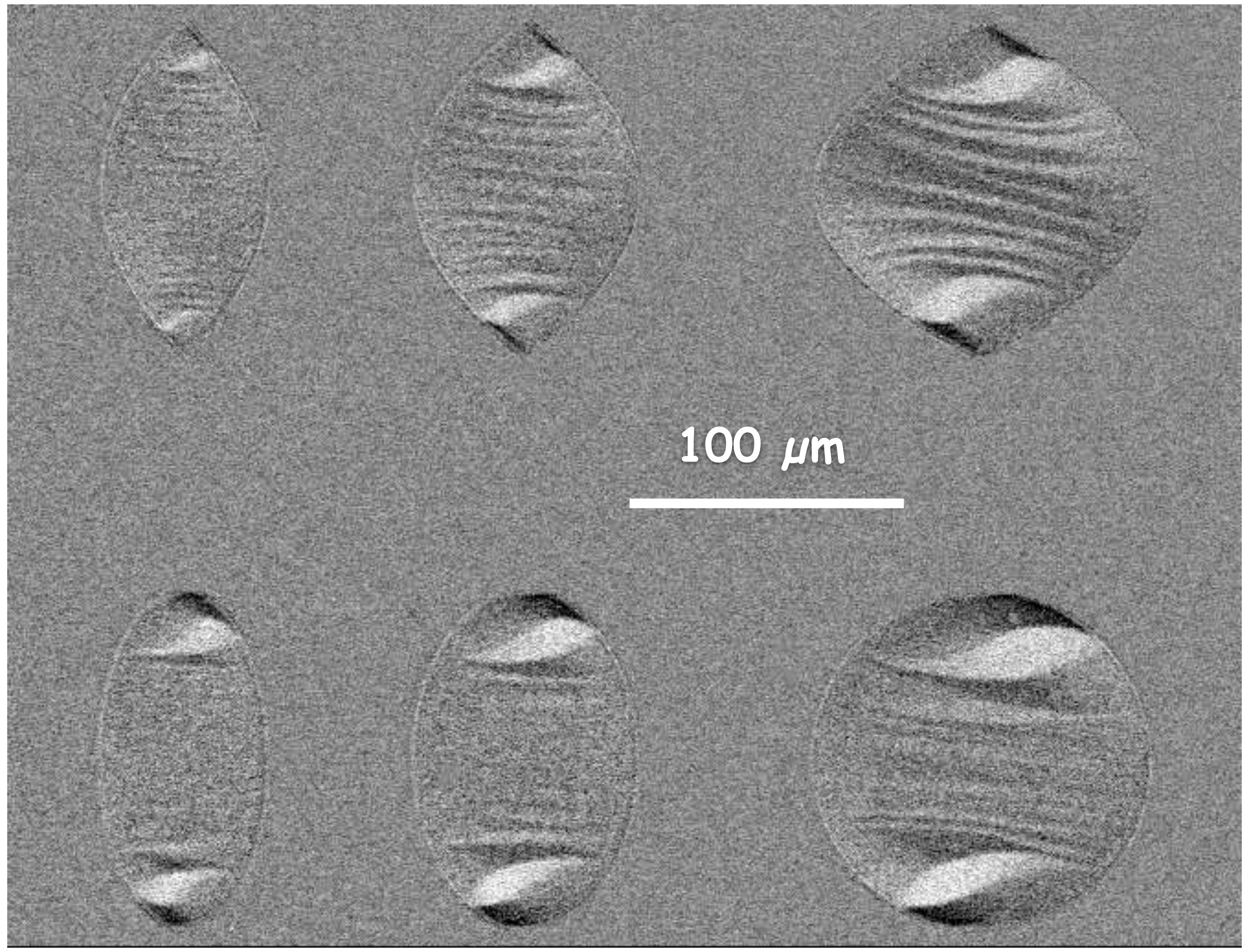
Pulsing of LEDs, synchronous with camera





Subtraction





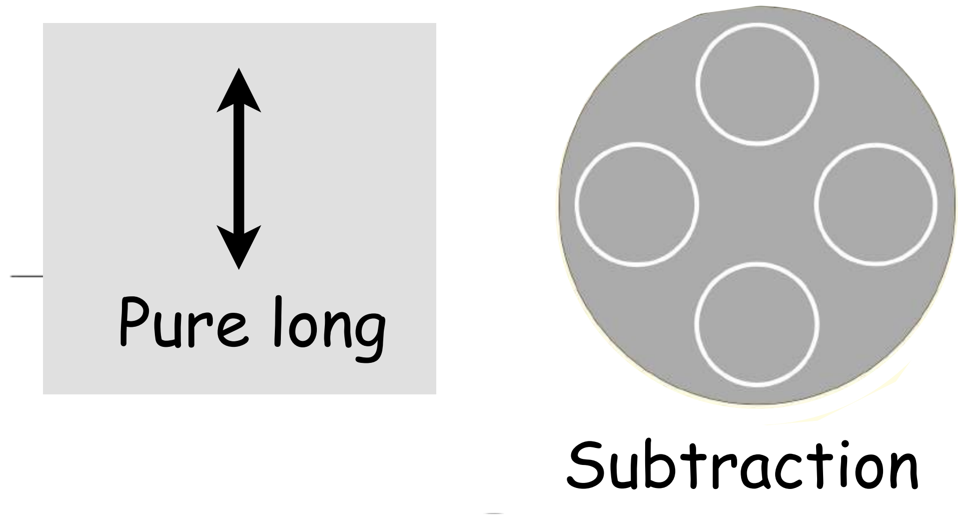
Sample: Permalloy film, 240 nm thick

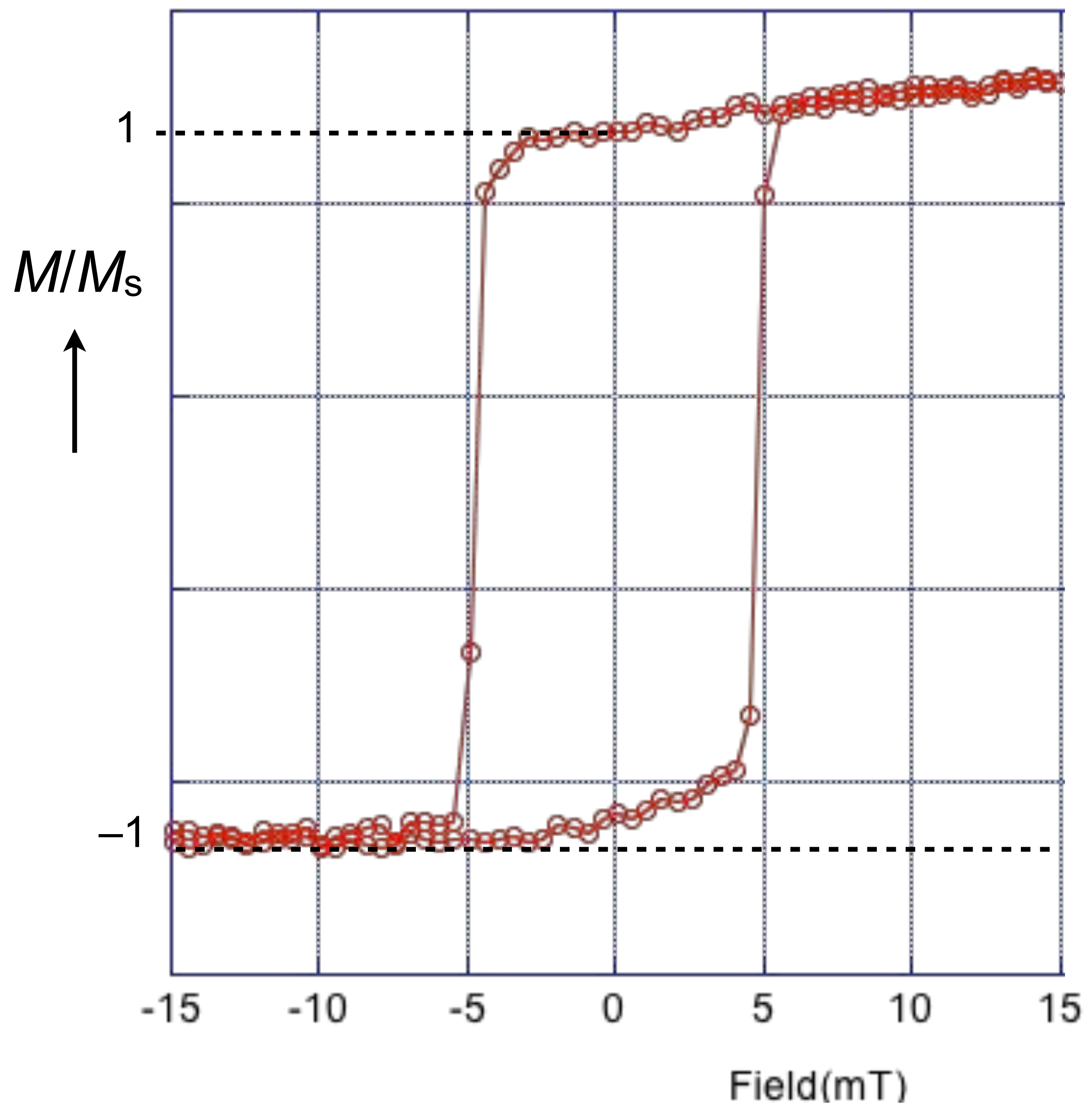
I. Soldatov and R.S., Rev. Sci. Instrum. 88, 073701 (2017)

4. Magneto-Optical Kerr Microscopy

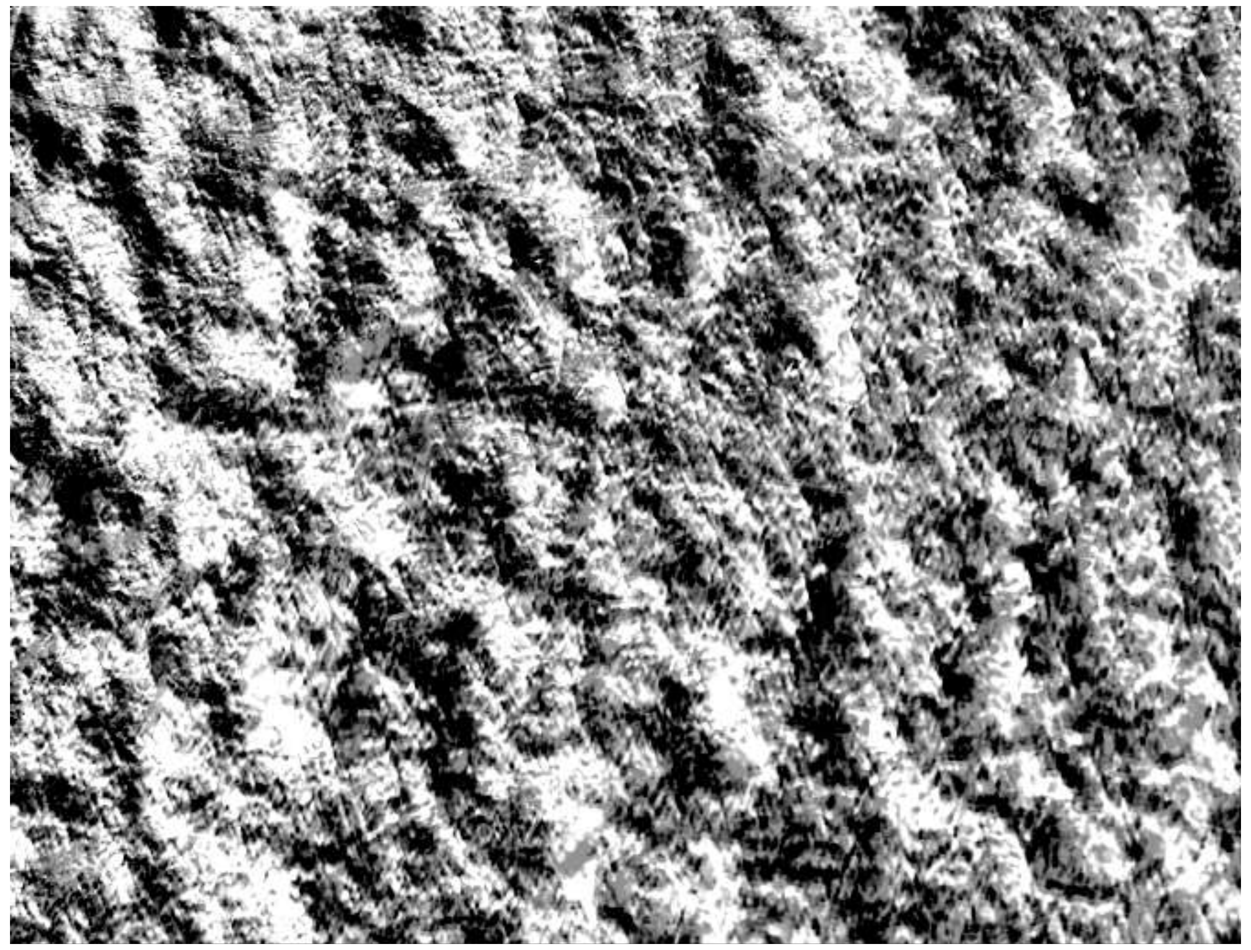
Wide-field Kerr-microscope: sensitivity selection & Sensitivity enhancement

MOKE loop and domains on 150 nm Permalloy on paper

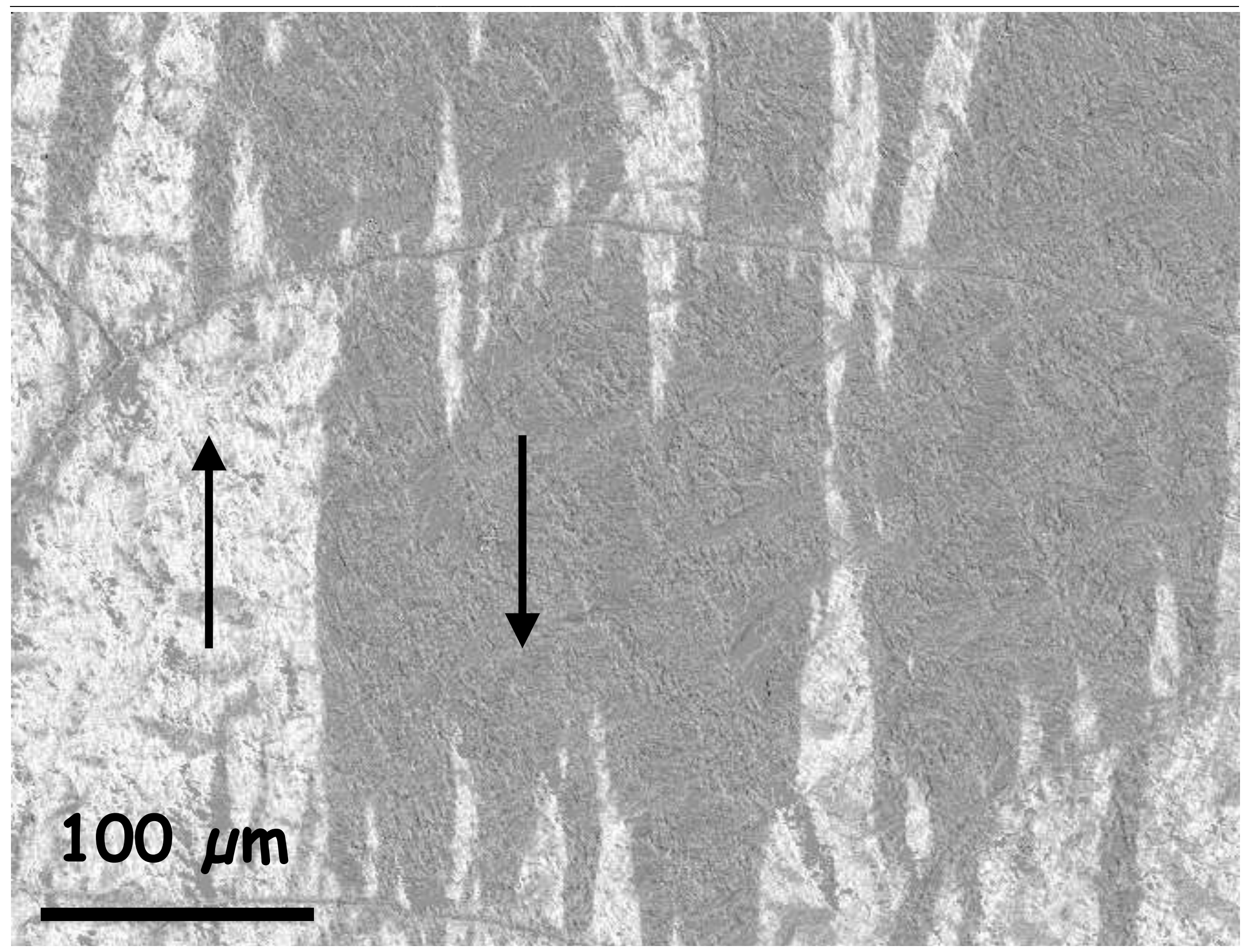




Together with M. Akin, Hannover (unpublished)



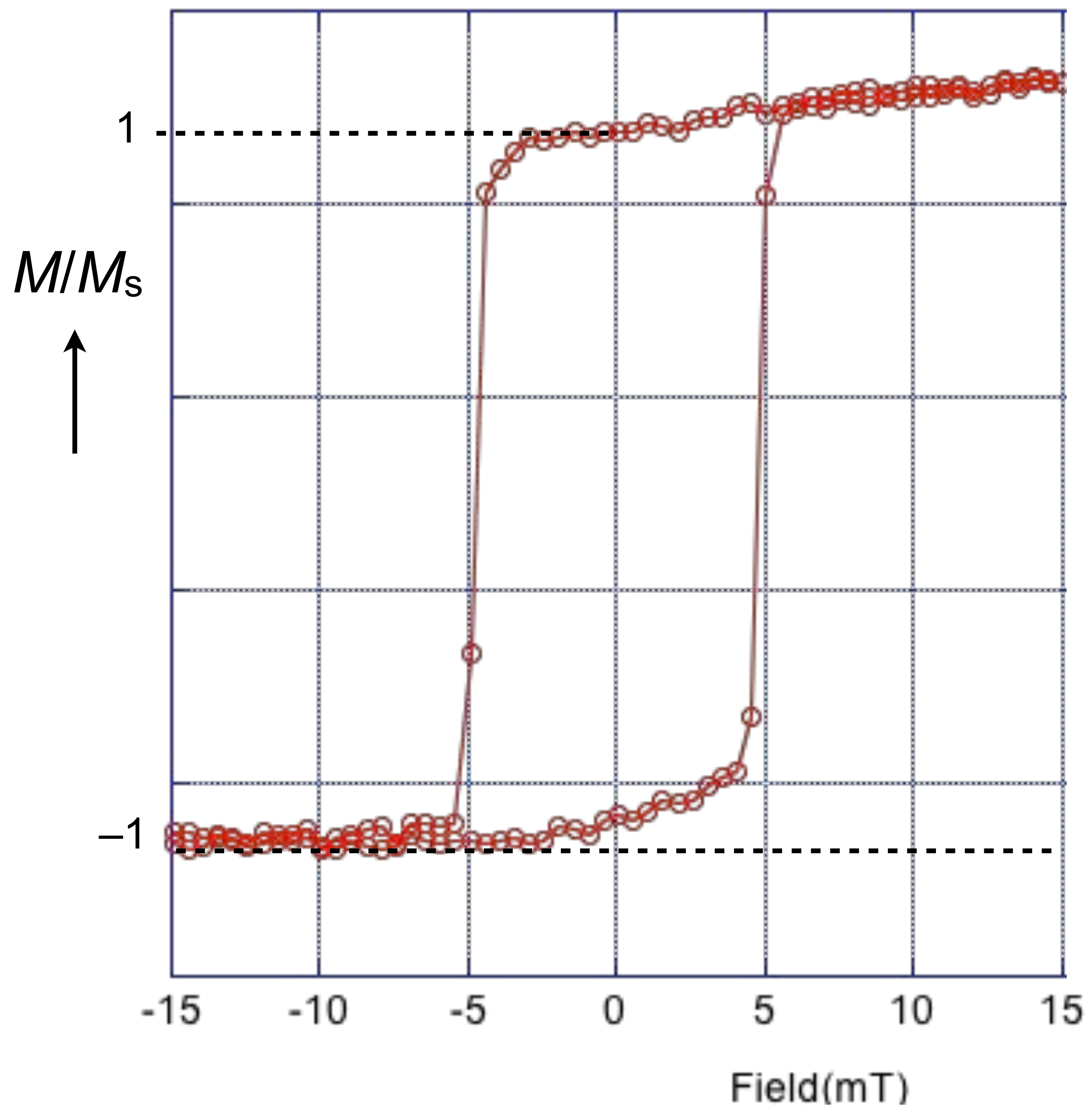
Topography



Domains

Wide-field Kerr-microscope: sensitivity selection & Sensitivity enhancement

MOKE loop and domains on 150 nm Permalloy on paper



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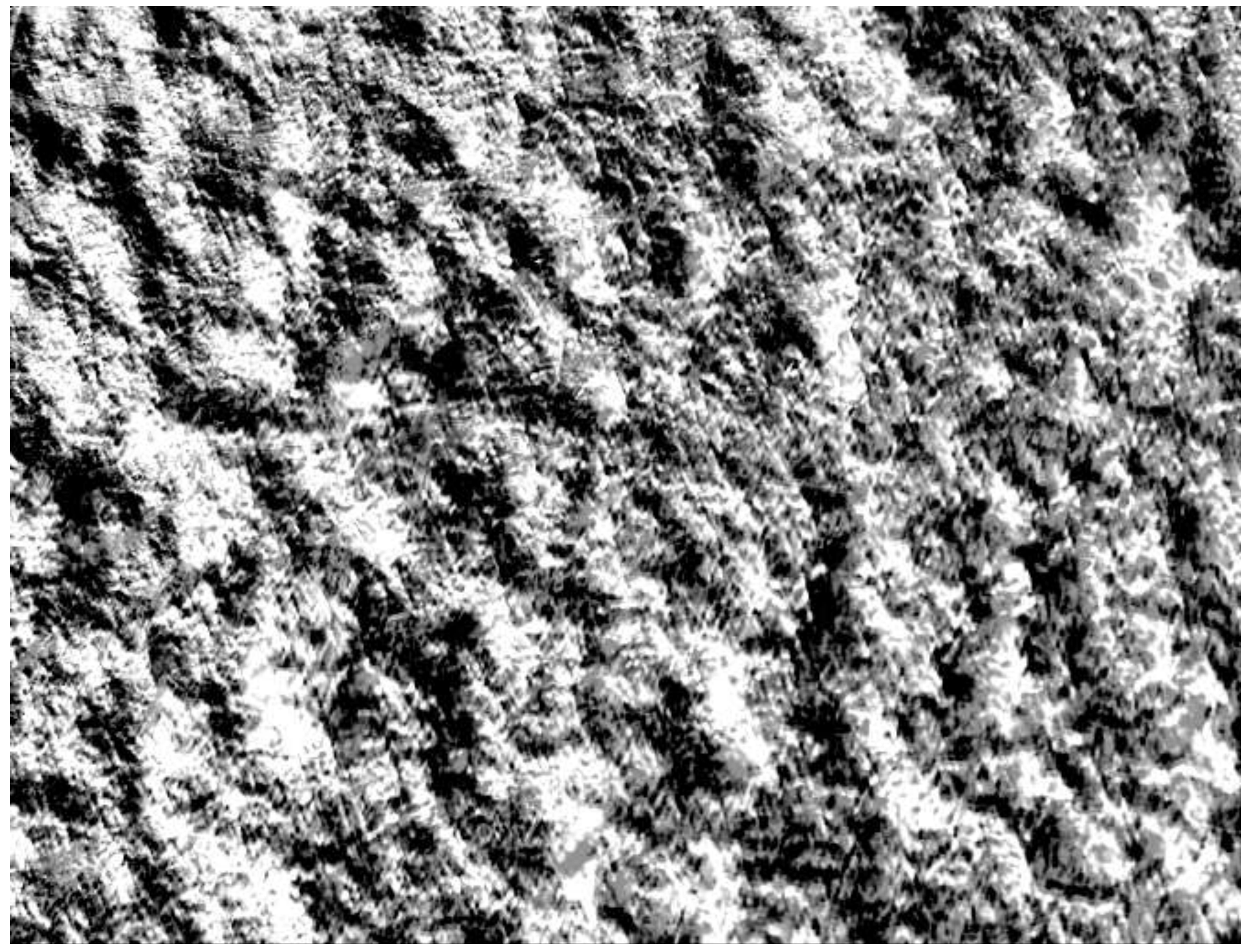

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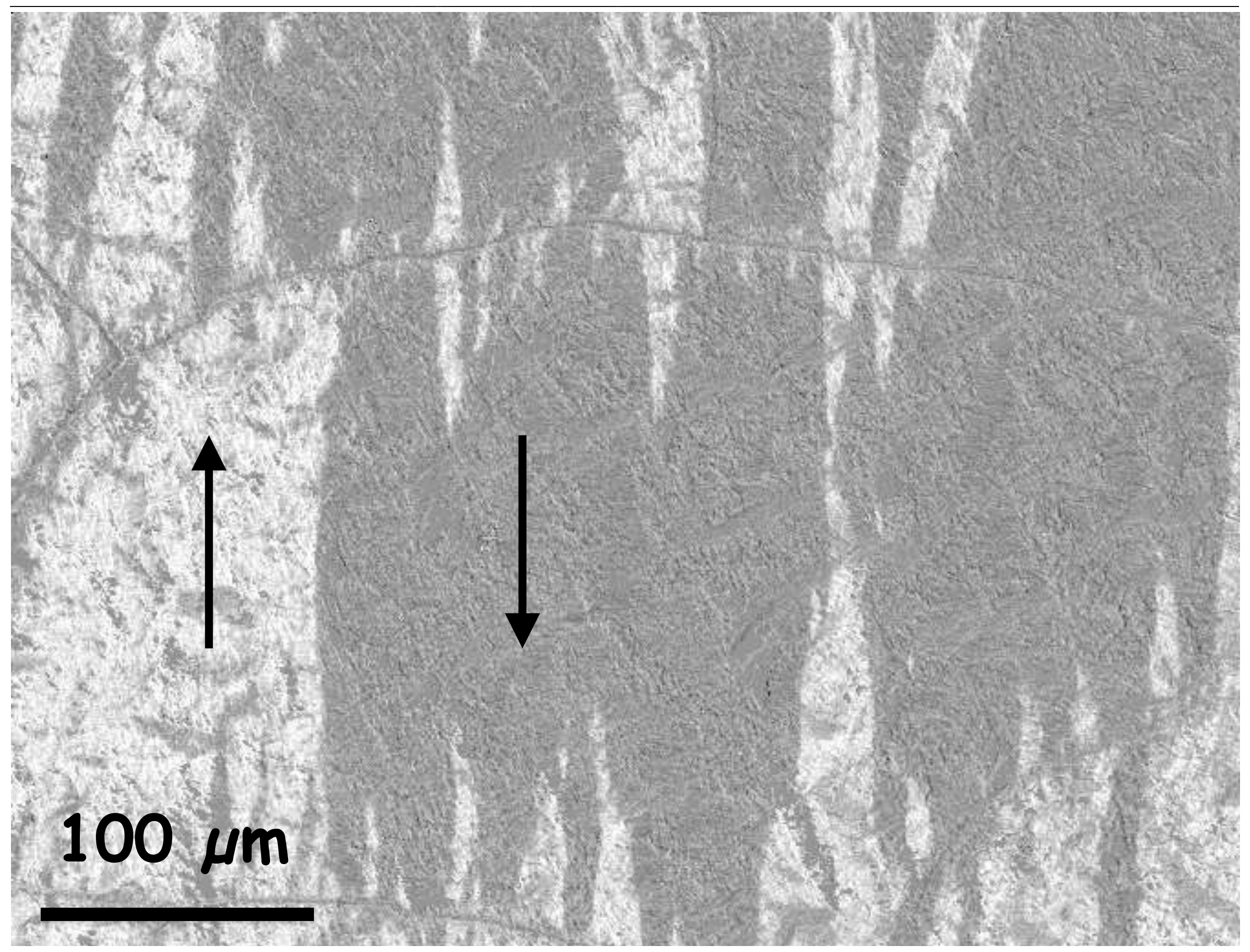
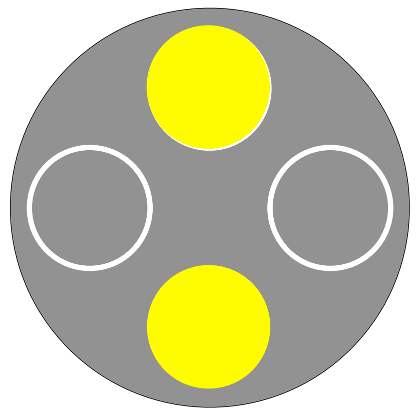


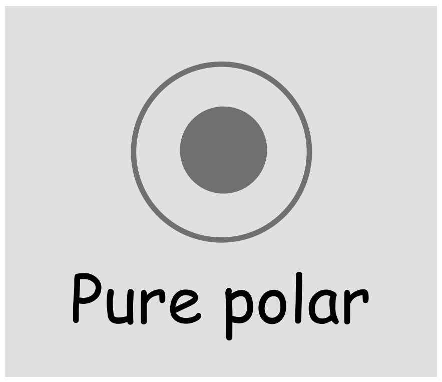
Domains

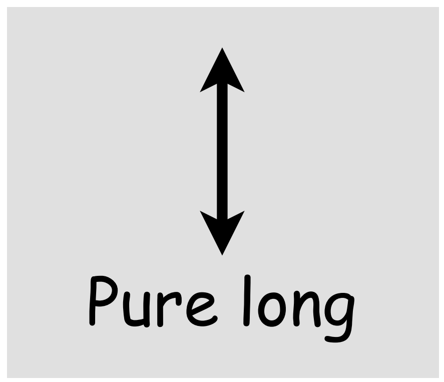
4. Magneto-Optical Kerr Microscopy

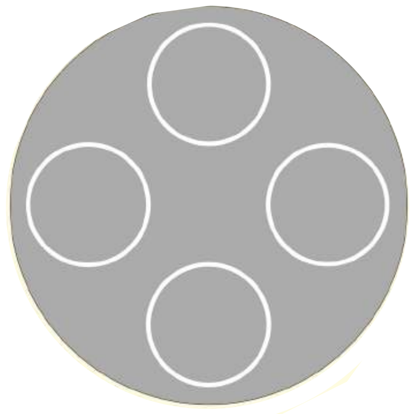
Wide-field Kerr-microscope: sensitivity selection
Separation of polar and planar magnetization

Addition

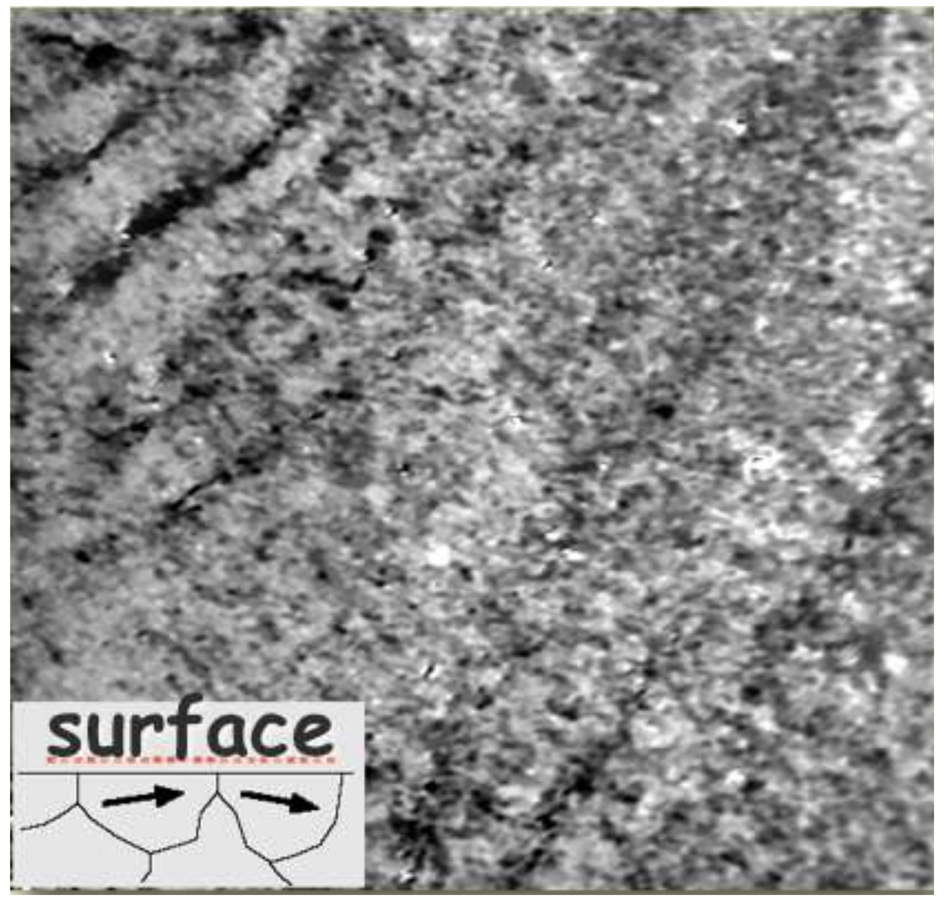




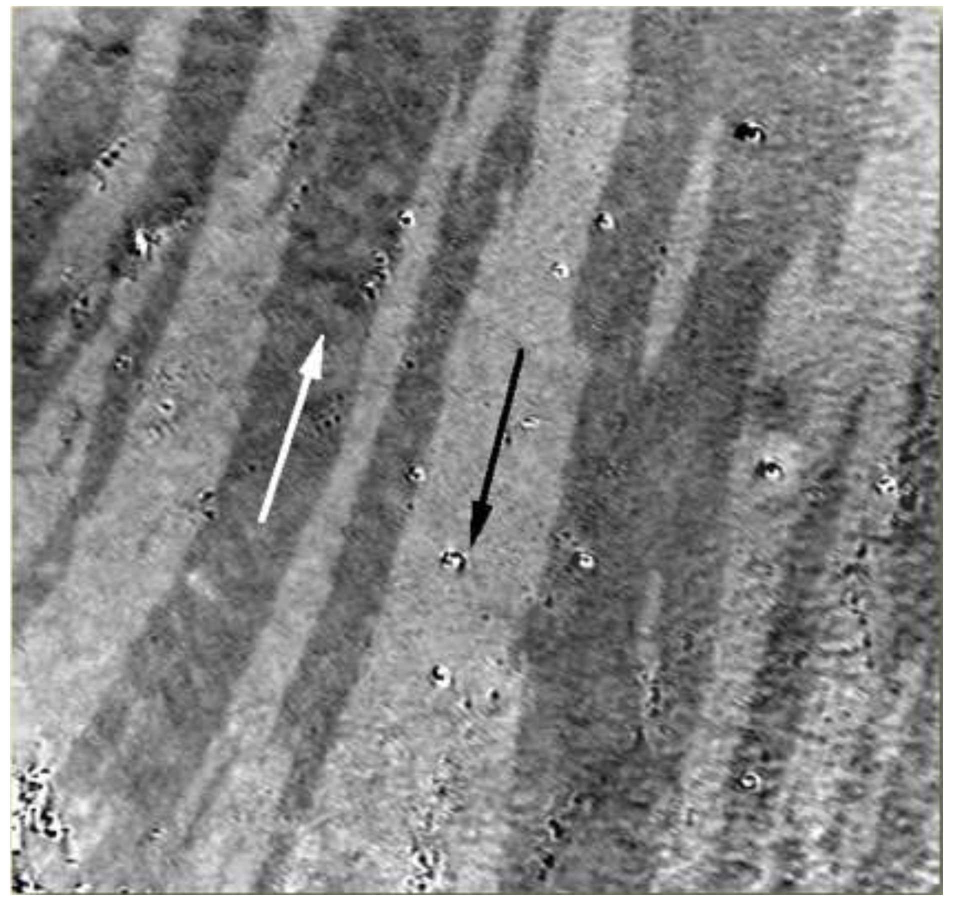


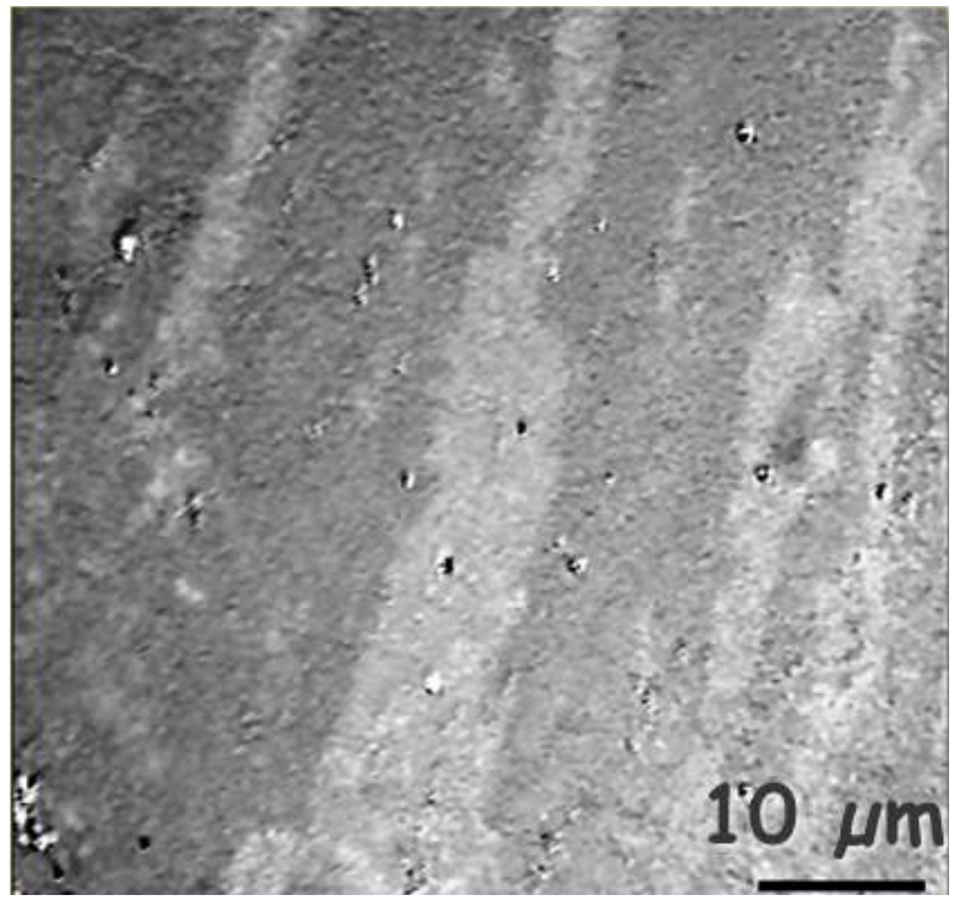


Subtraction

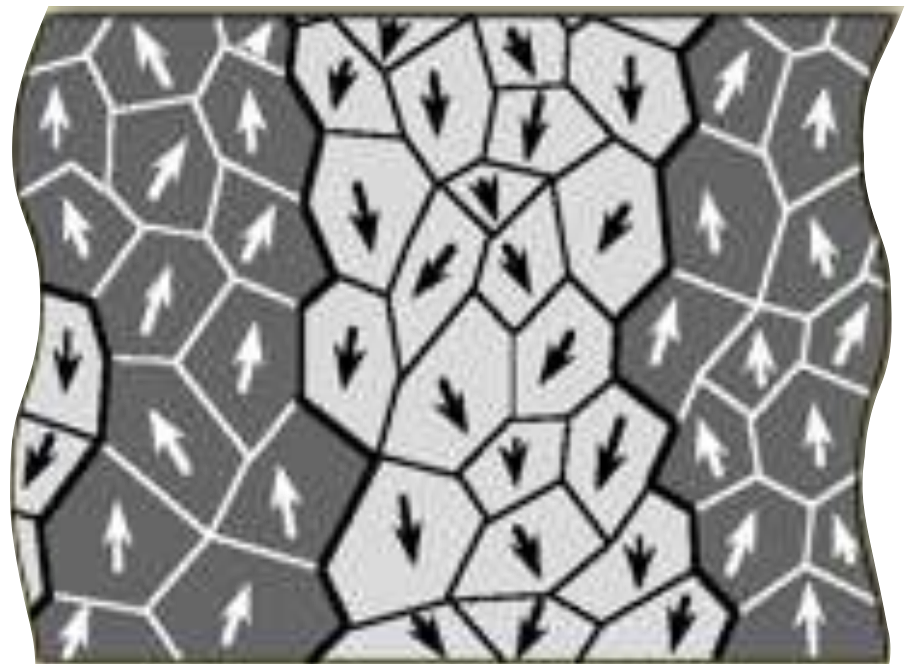


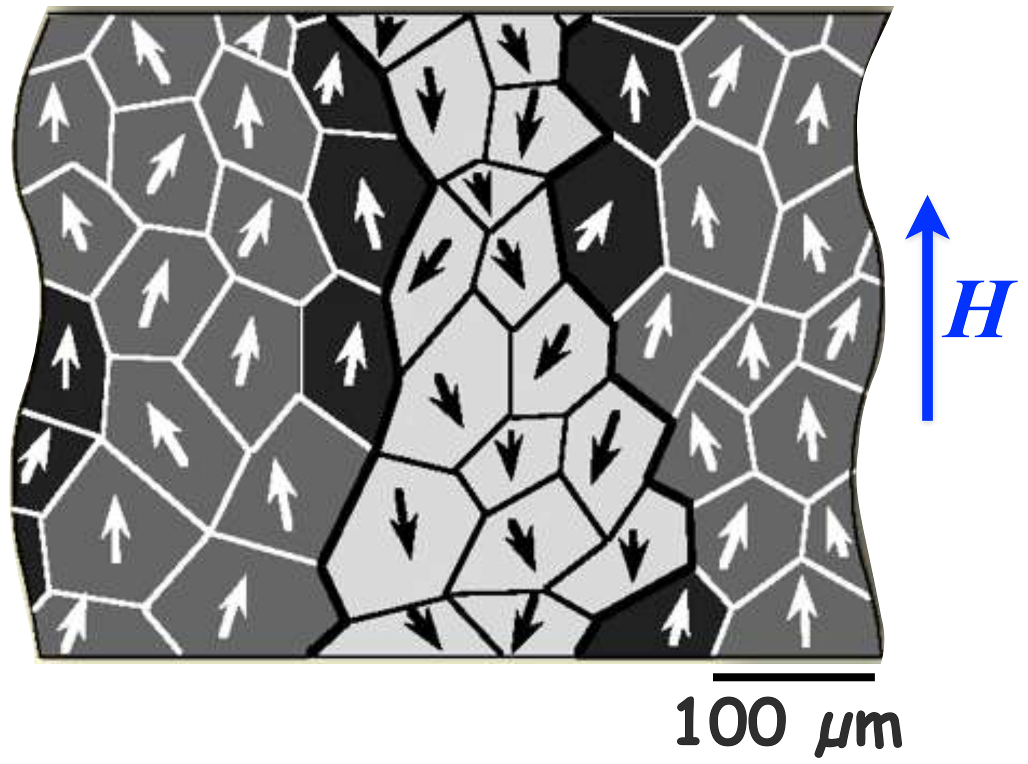
surface || texture axis





Interaction domains in fine-grained NdFeB material



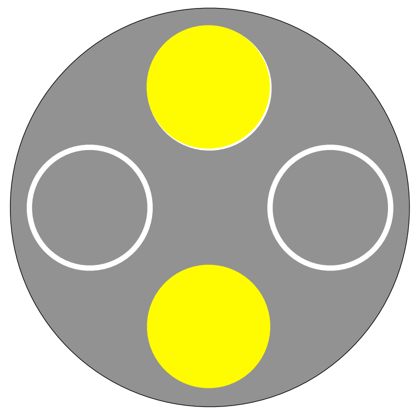


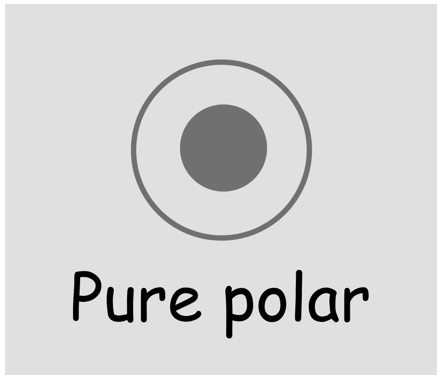
4. Magneto-Optical Kerr Microscopy

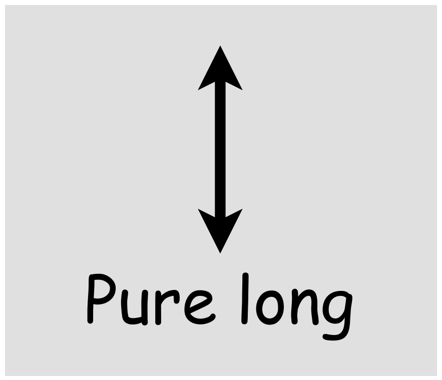
Wide-field Kerr-microscope: sensitivity selection

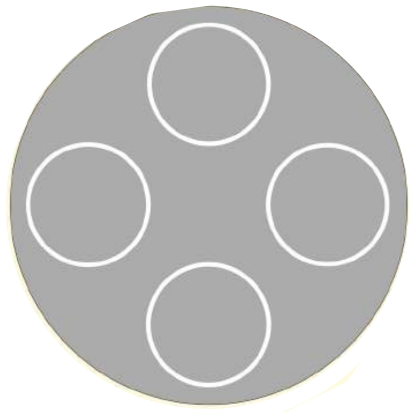
Separation of polar and planar magnetization

Addition

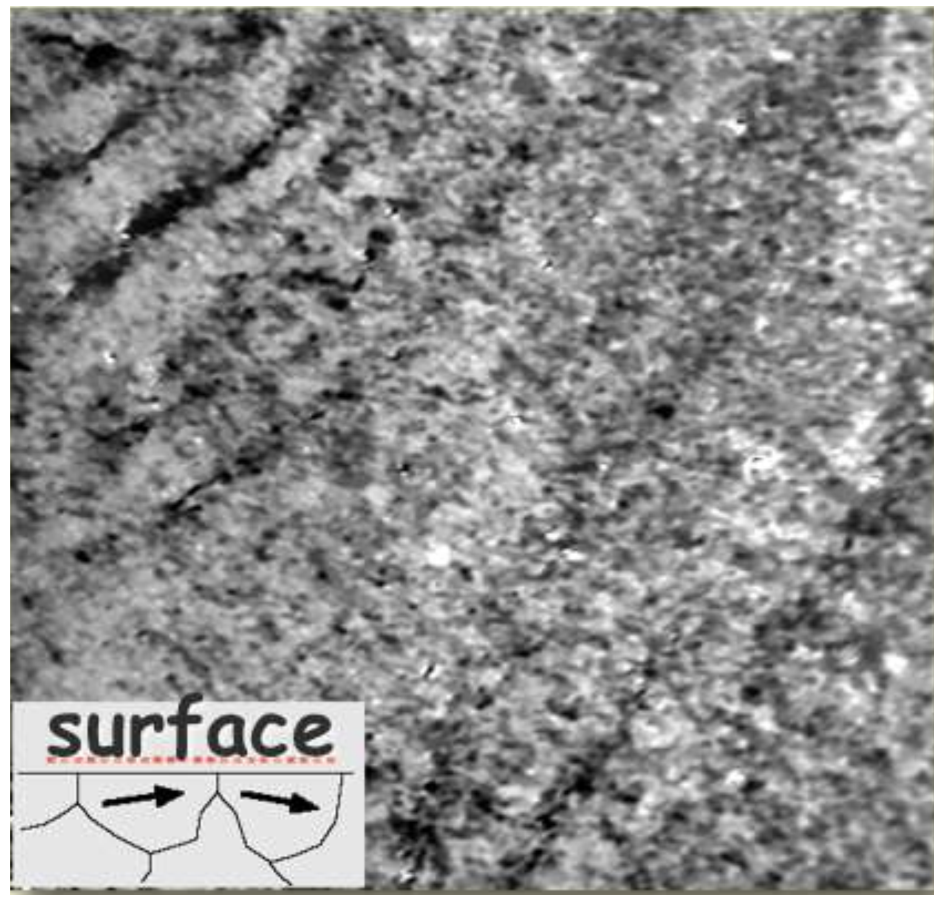




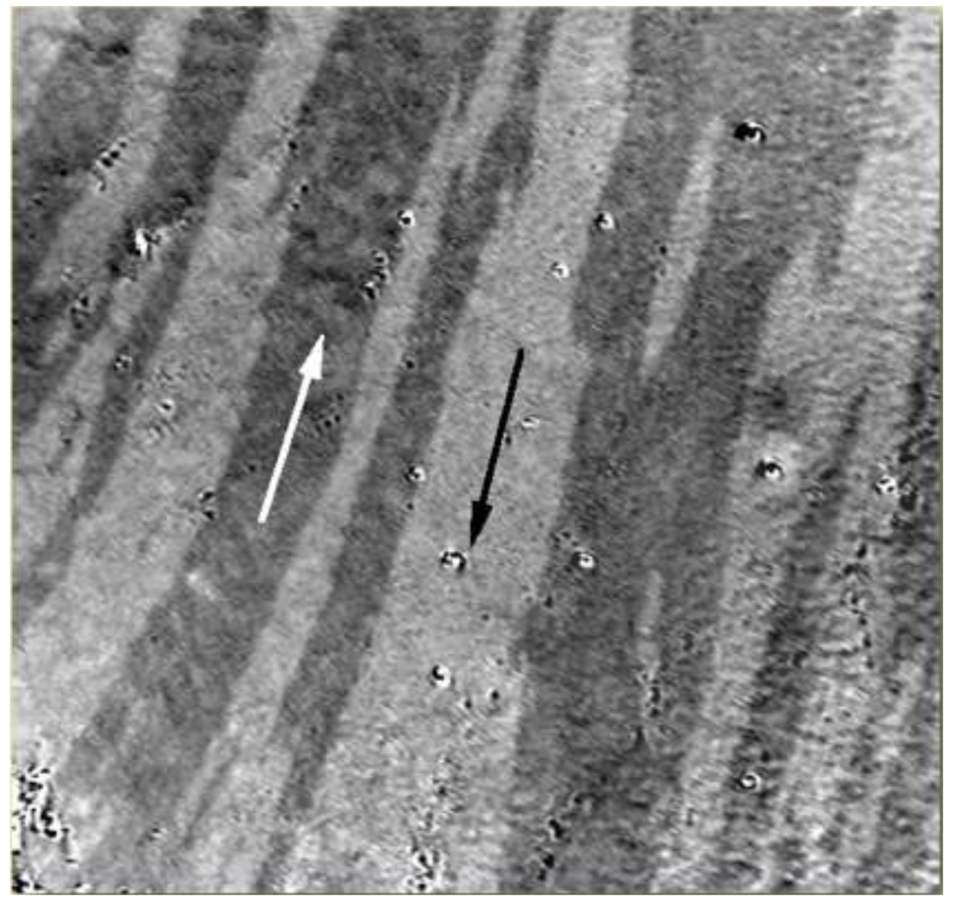


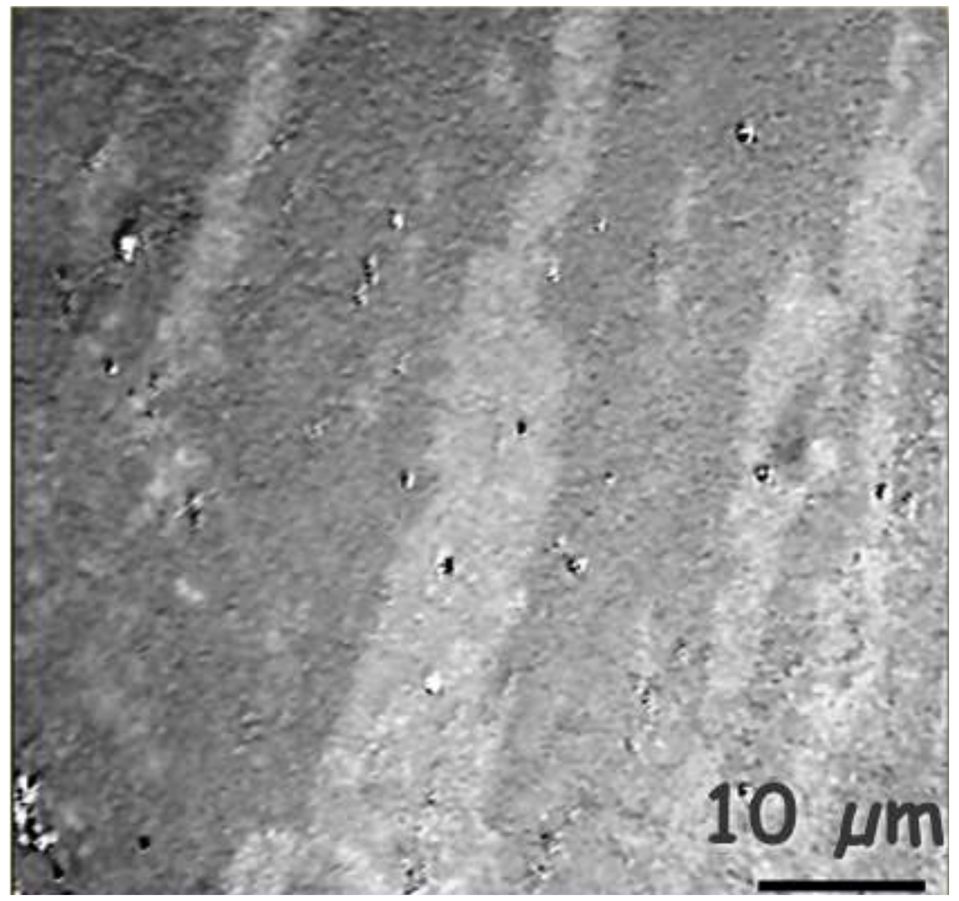


Subtraction

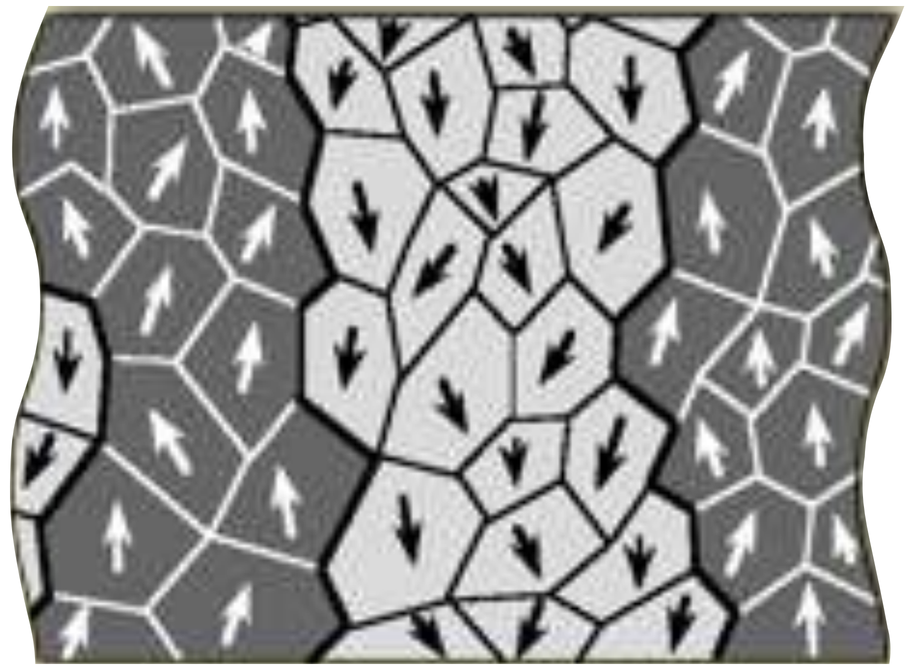


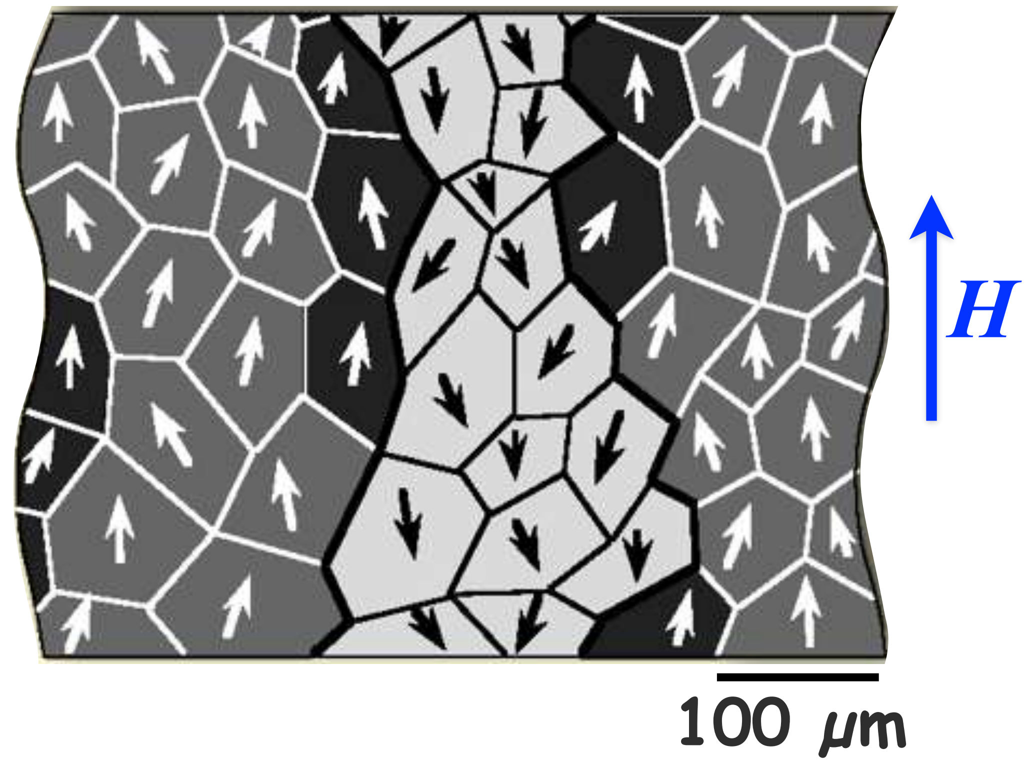
surface || texture axis





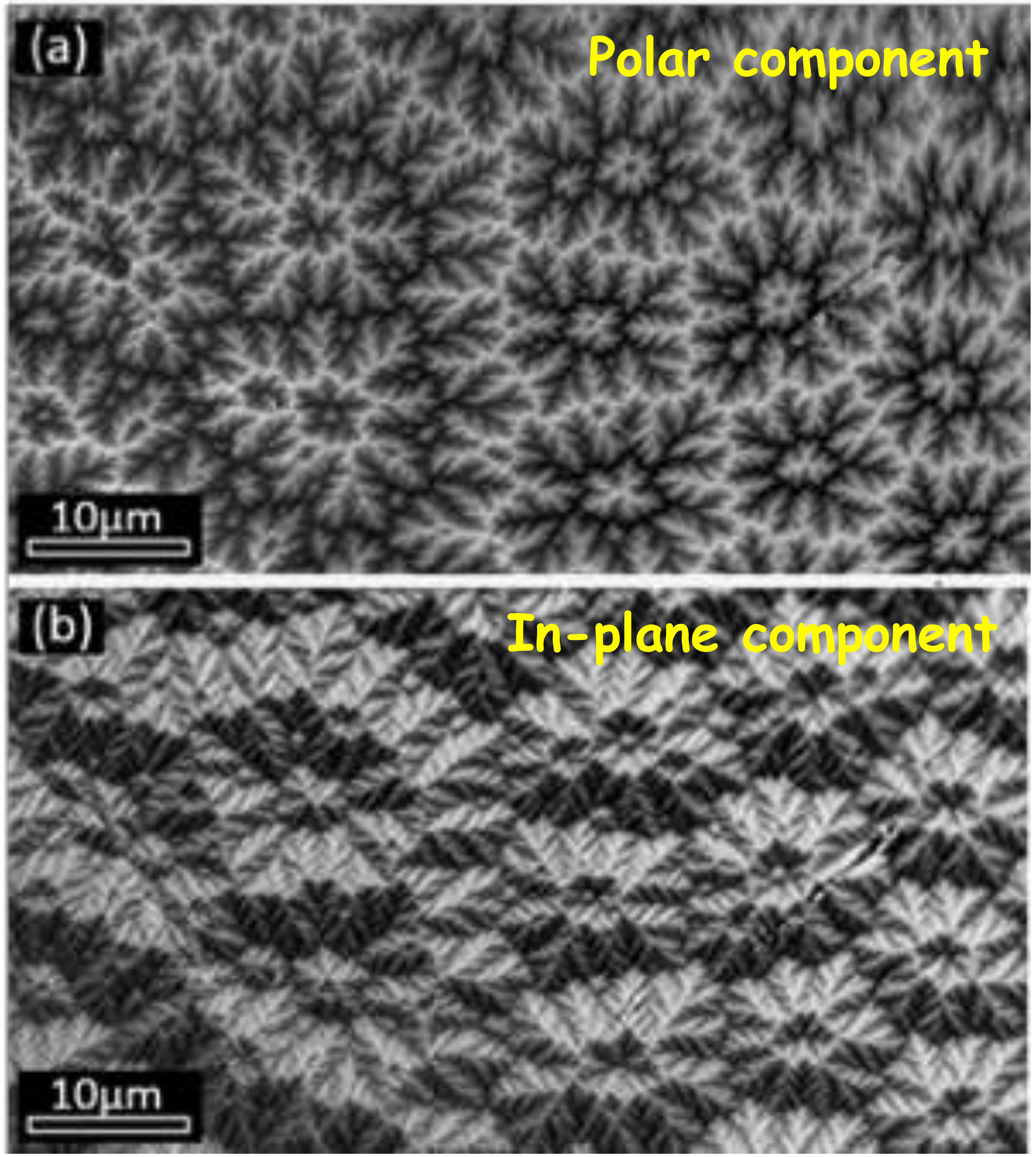
Interaction domains in fine-grained NdFeB material



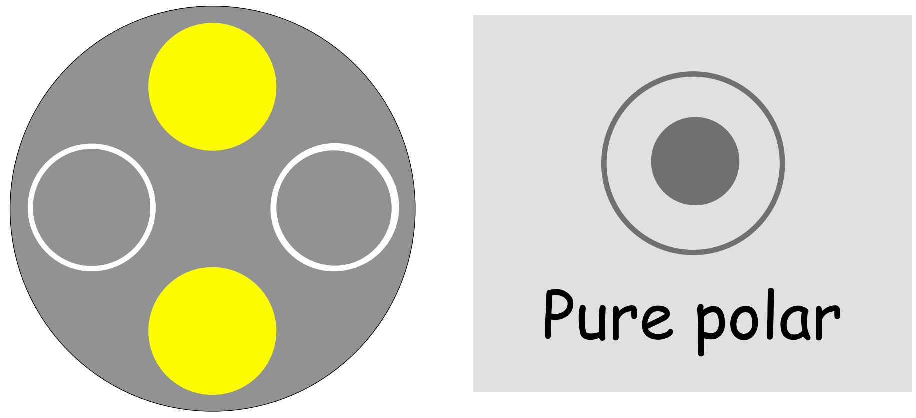


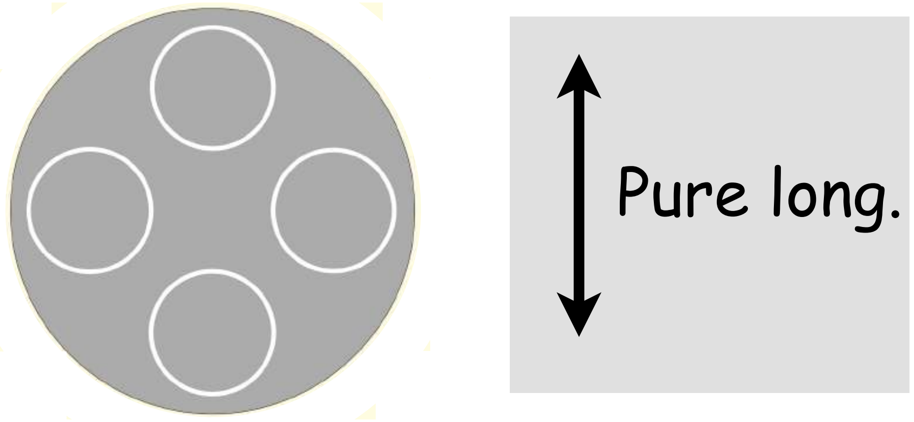
Wide-field Kerr-microscope: sensitivity selection

Separation of polar and planar magnetization



Fe_3Sn_2 crystal,
basal plane of
rhombohedral structure



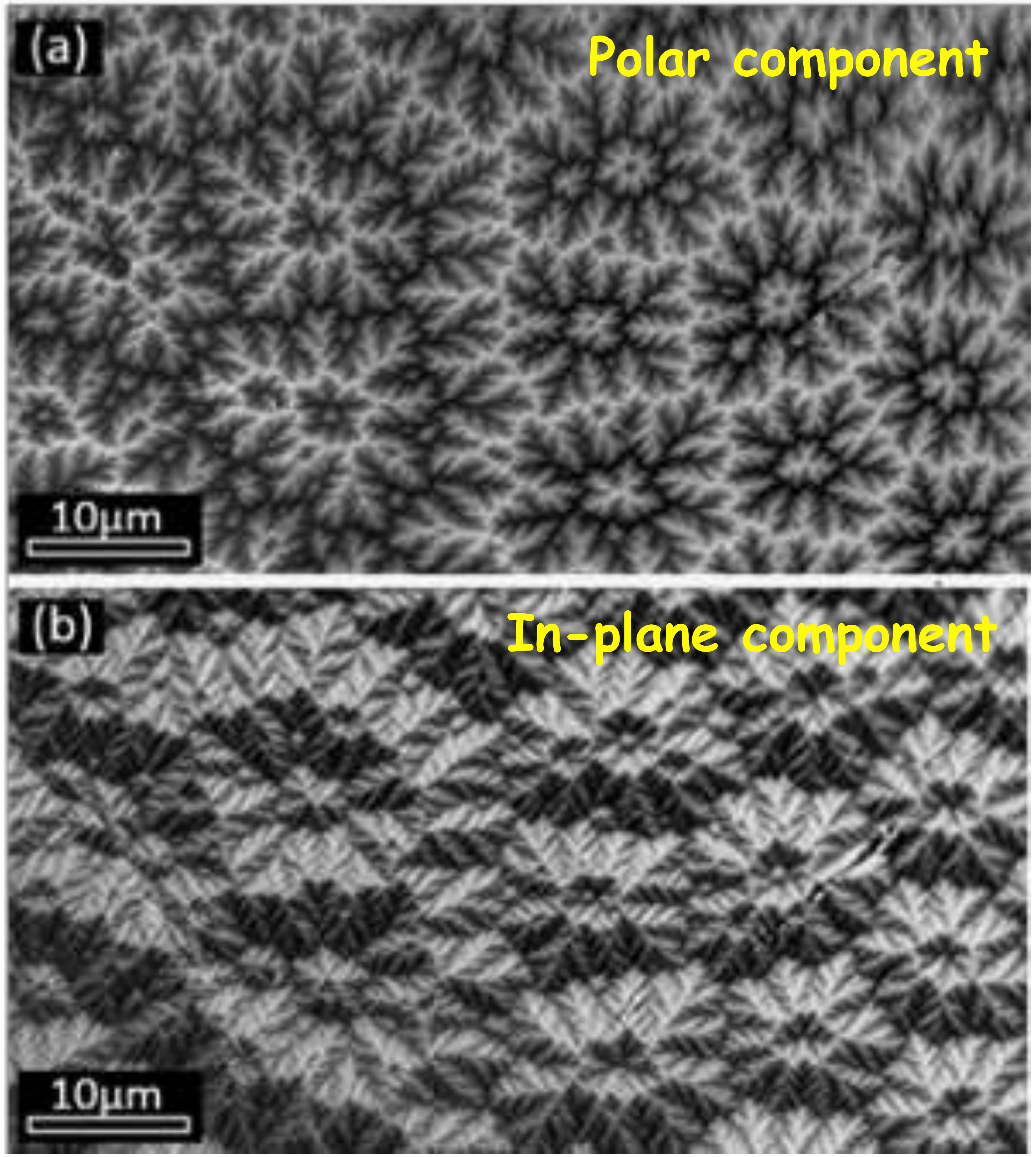


Together with B. Fayyazi and
O. Gutfleisch, Darmstadt
and I. Soldatov, IFW
(accepted for Acta Mat)

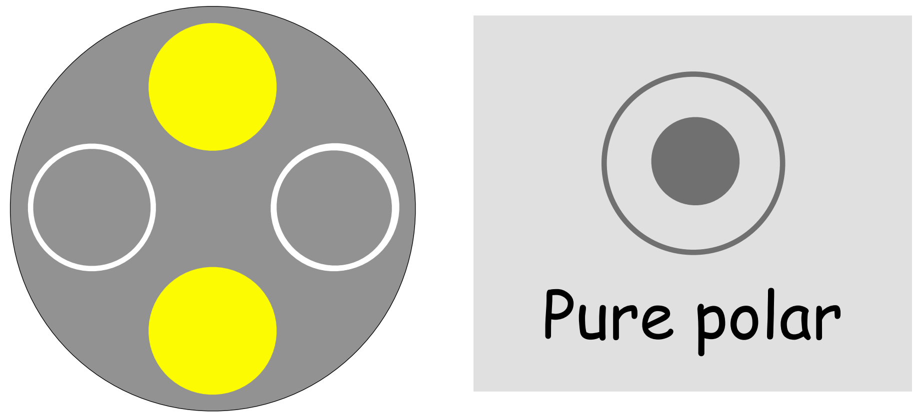
4. Magneto-Optical Kerr Microscopy

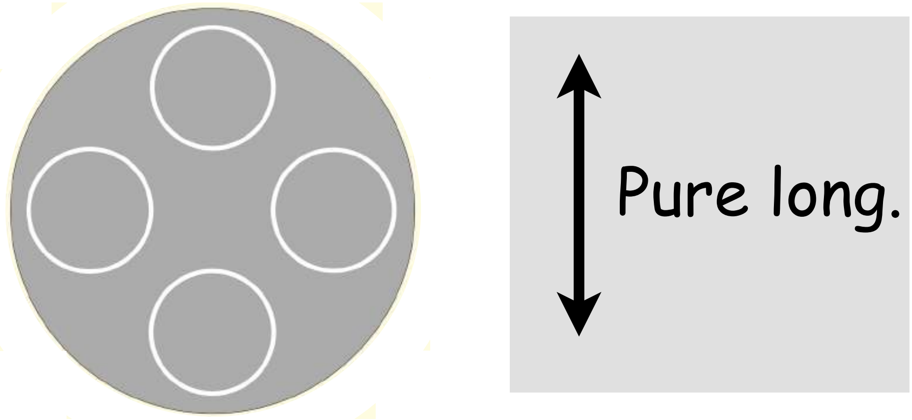
Wide-field Kerr-microscope: sensitivity selection

Separation of polar and planar magnetization



Fe_3Sn_2 crystal,
basal plane of
rhombohedral structure





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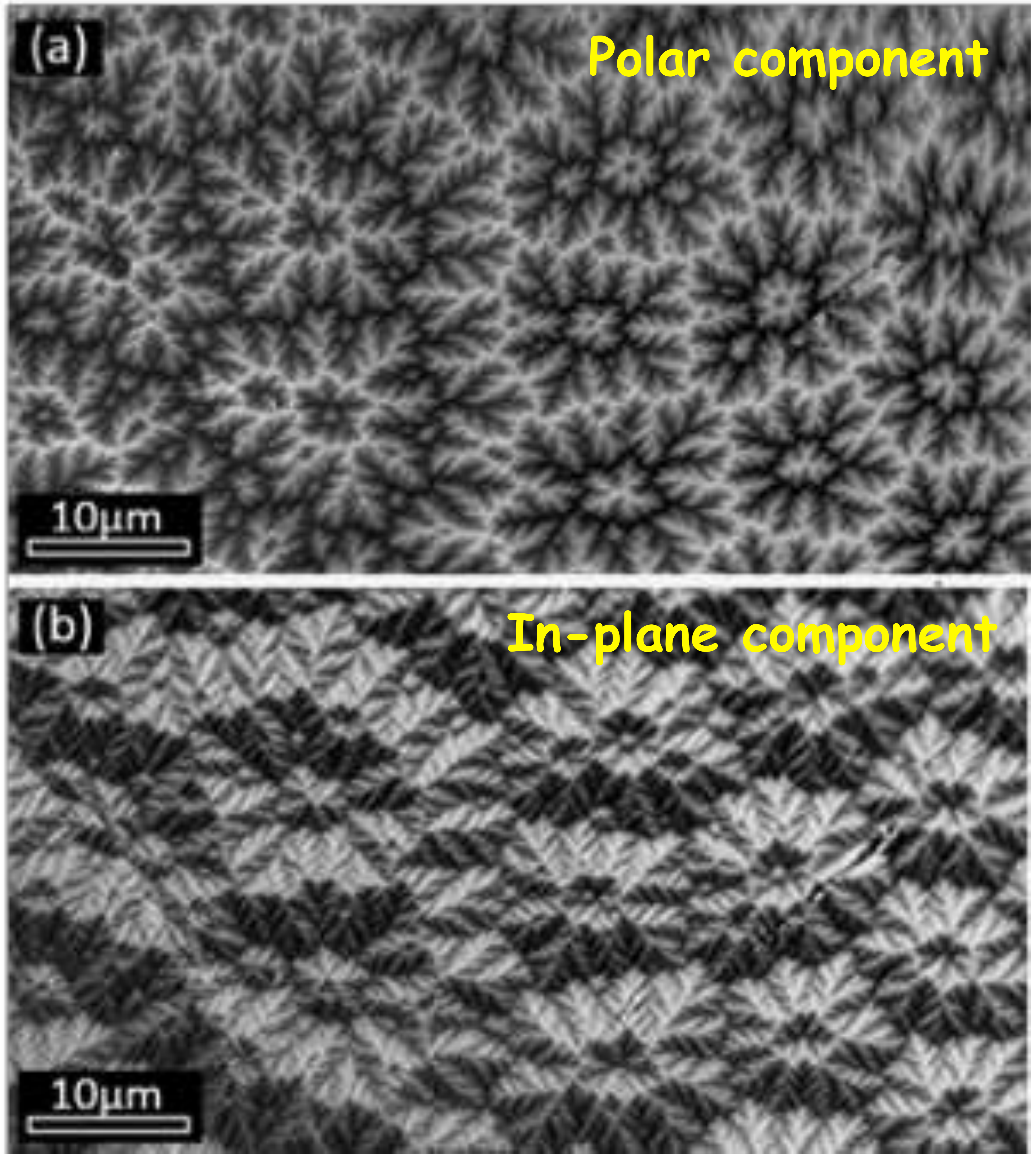
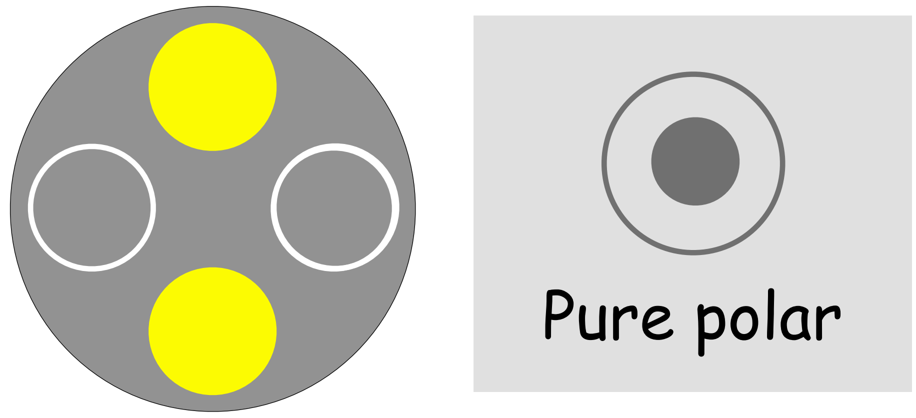
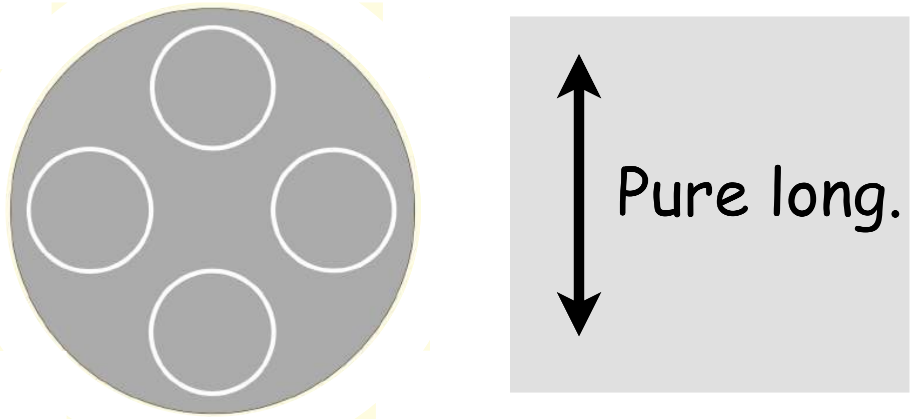
Wide-field Kerr-microscope: sensitivity selection

Separation of polar and planar magnetization

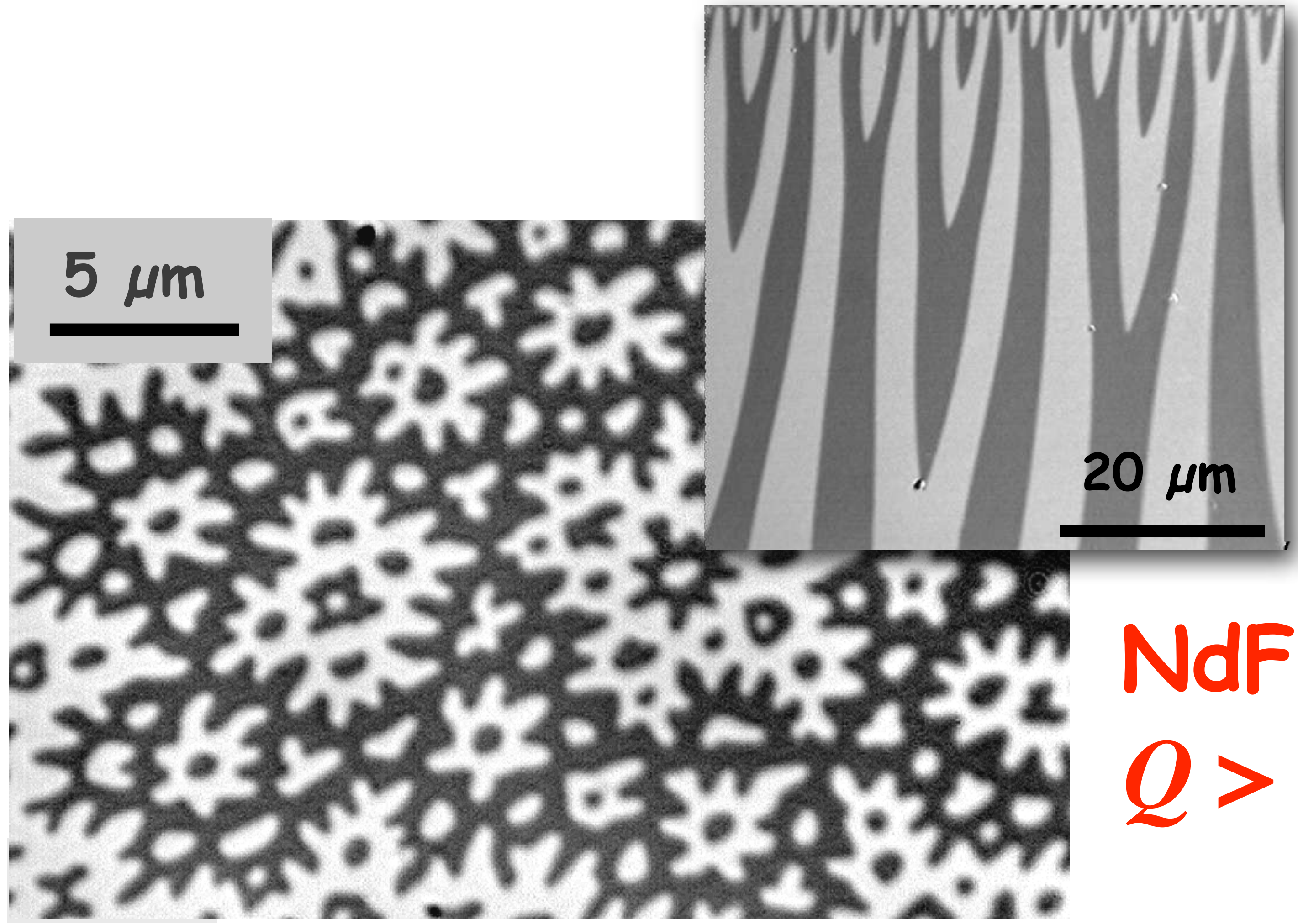


Fe_3Sn_2 crystal,
basal plane of
rhombohedral structure



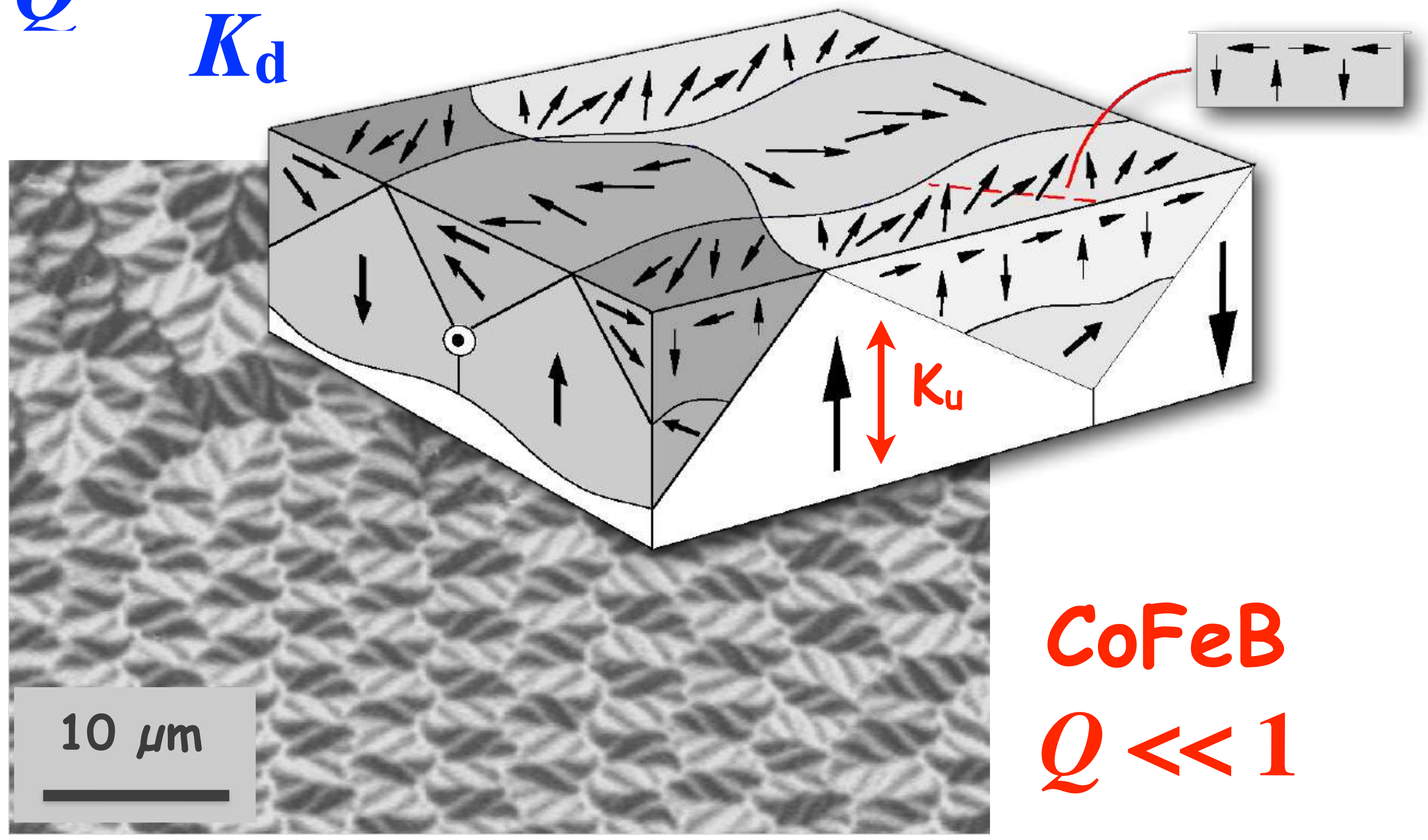


Together with B. Fayyazi and O. Gutfleisch, Darmstadt and I. Soldatov, IFW (accepted for Acta Mat)



NdFeB
$Q > 1$

$$Q = \frac{K_u}{K_d}$$

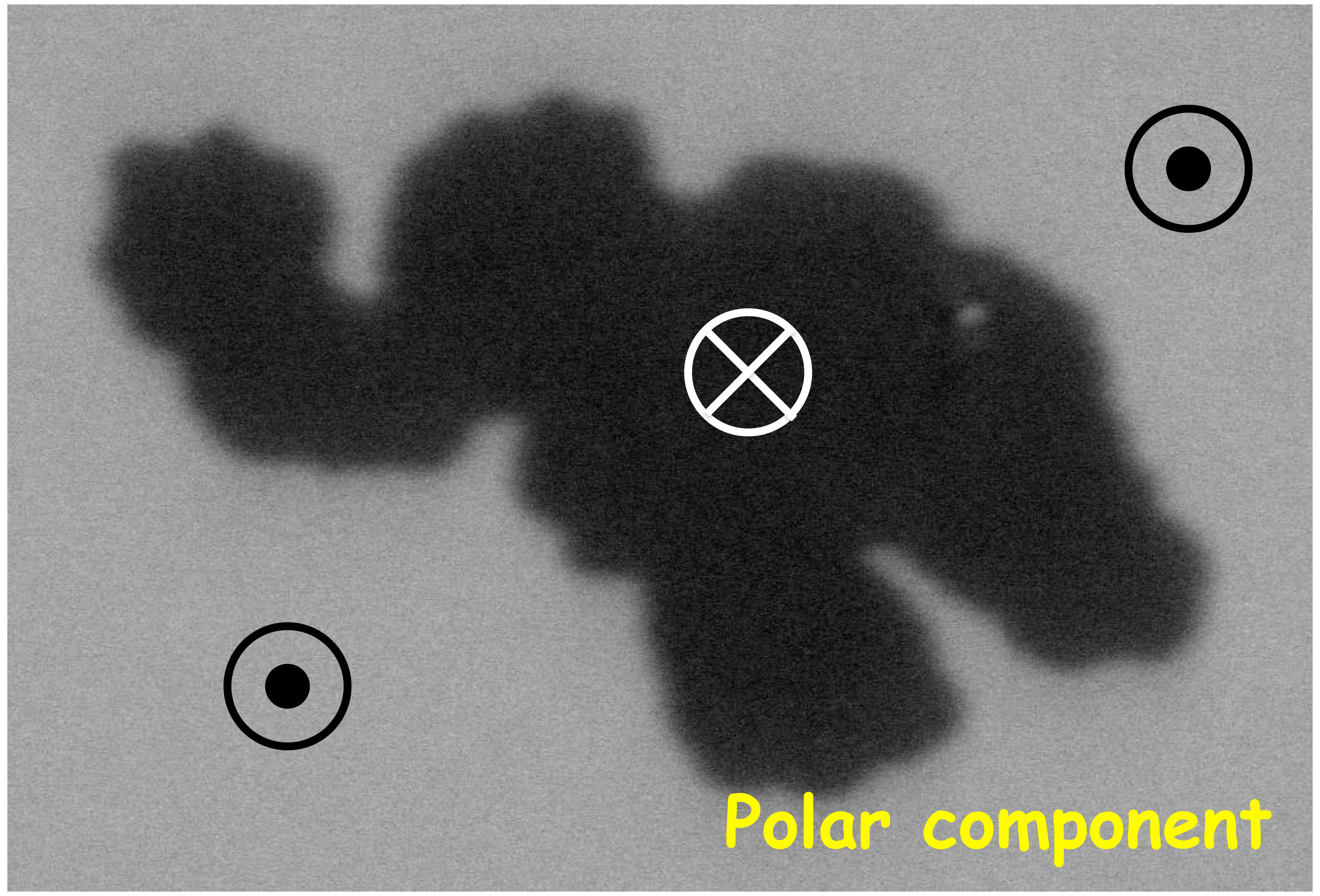


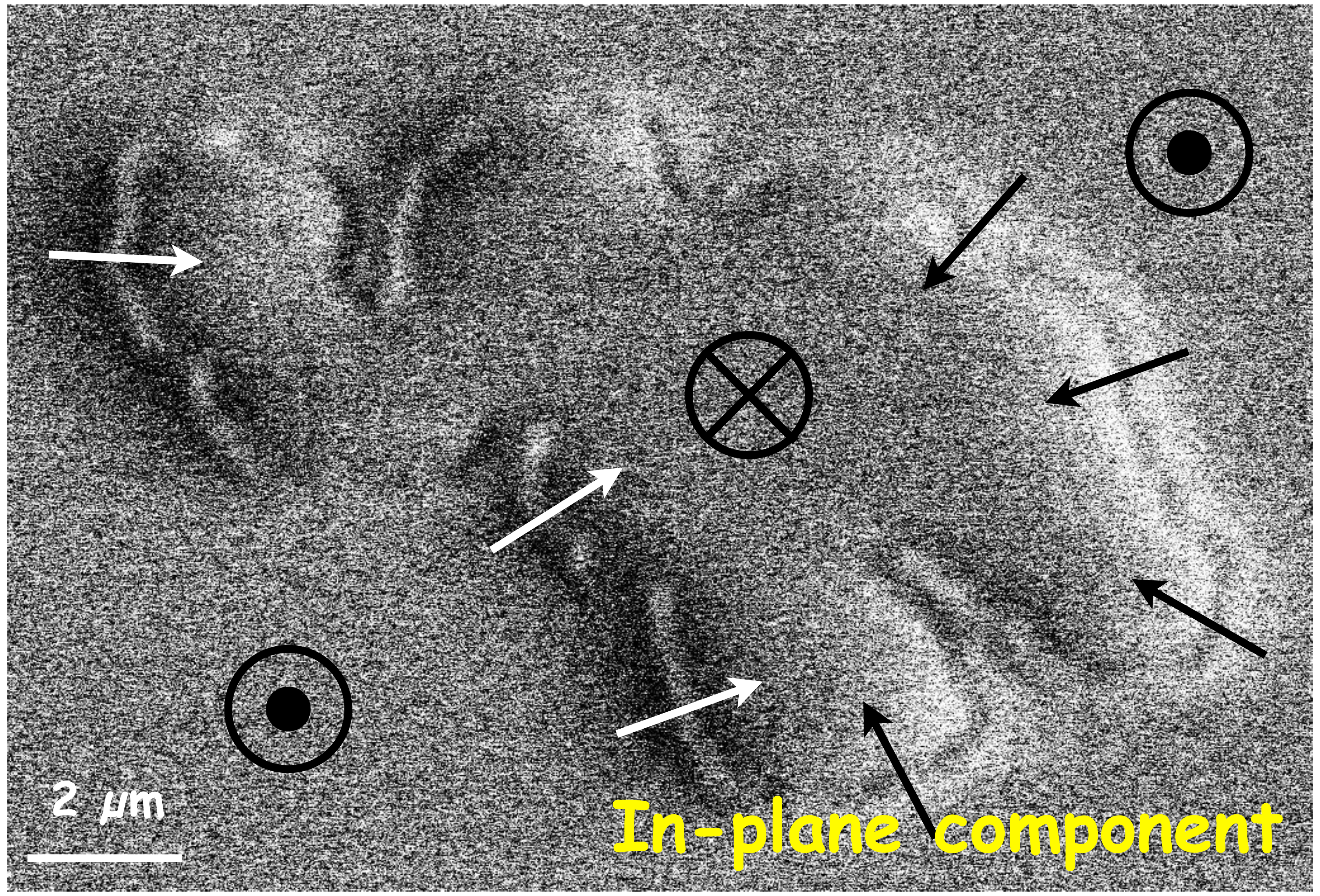
CoFeB
$Q << 1$

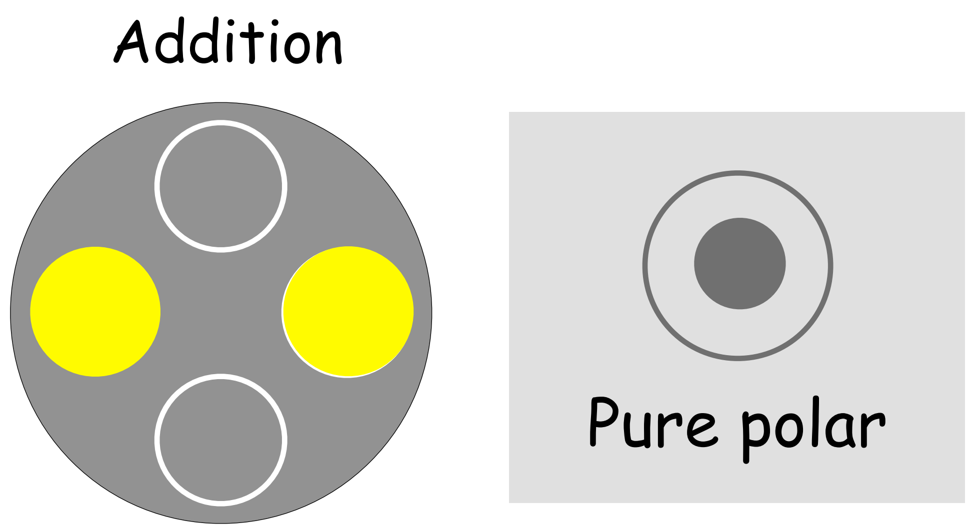
Wide-field Kerr-microscope: sensitivity selection

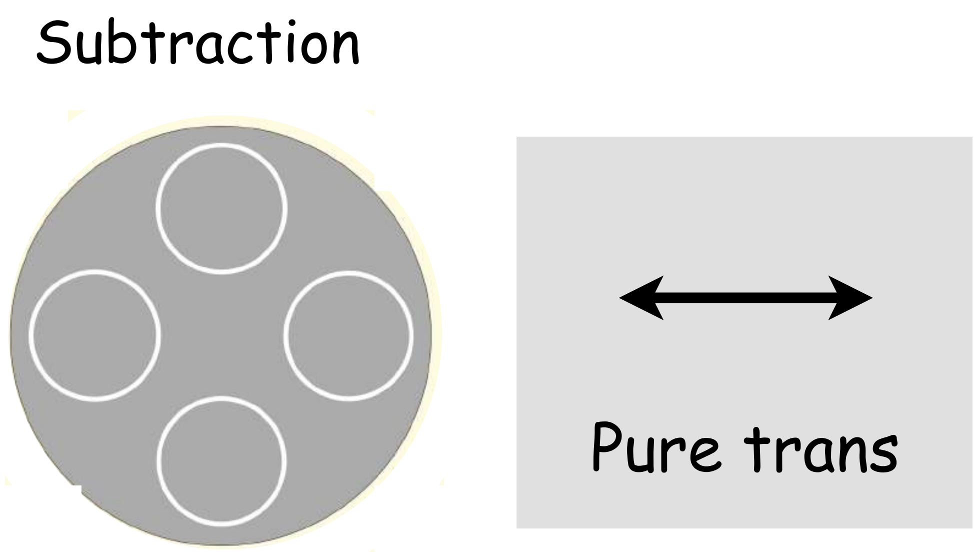
Separation of polar and planar magnetization

Pt(5 nm) [Co(0.3 nm)/Pt(0.7 nm)]$_3$







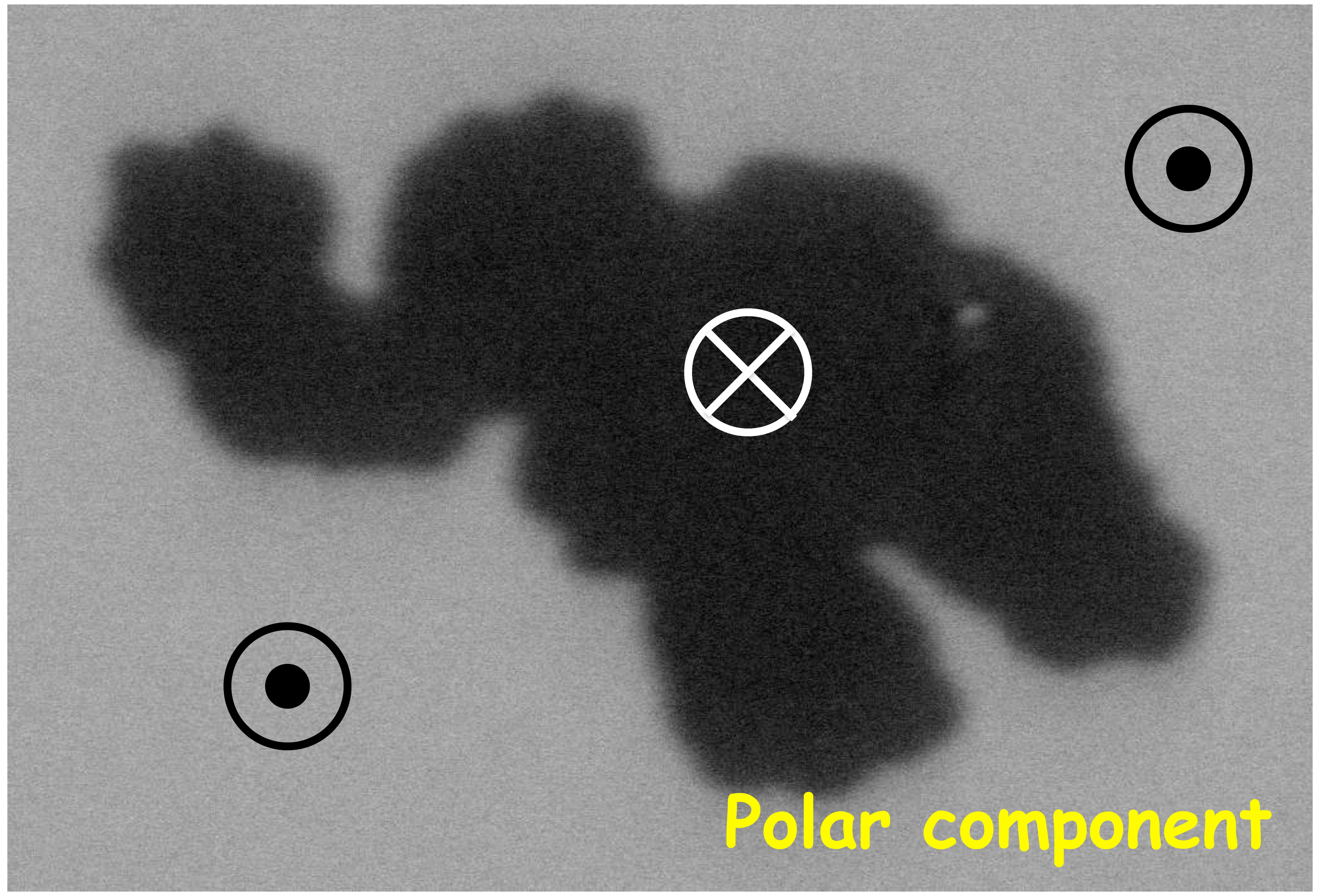


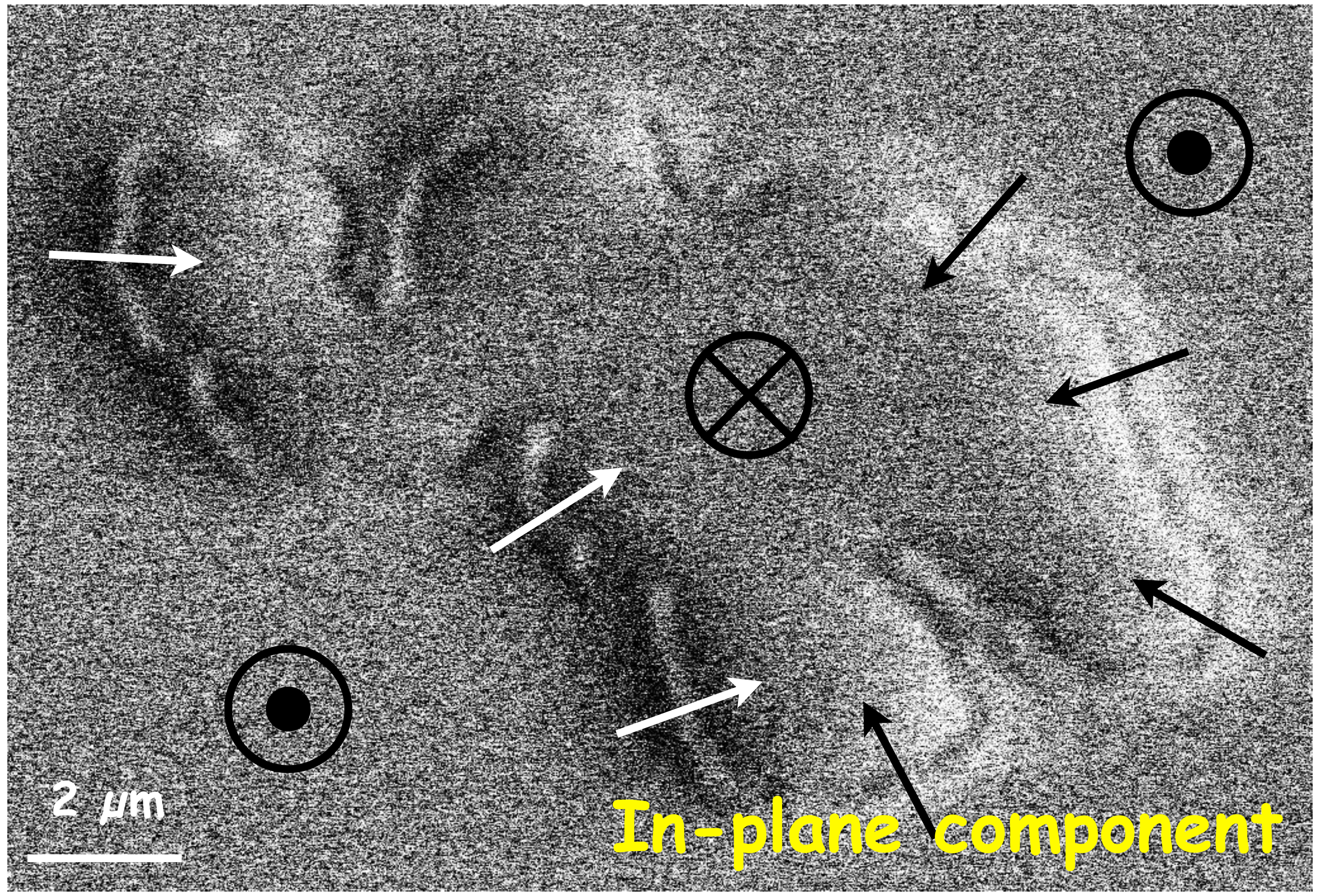
(sample: D. Makarov, IFW)

Wide-field Kerr-microscope: sensitivity selection

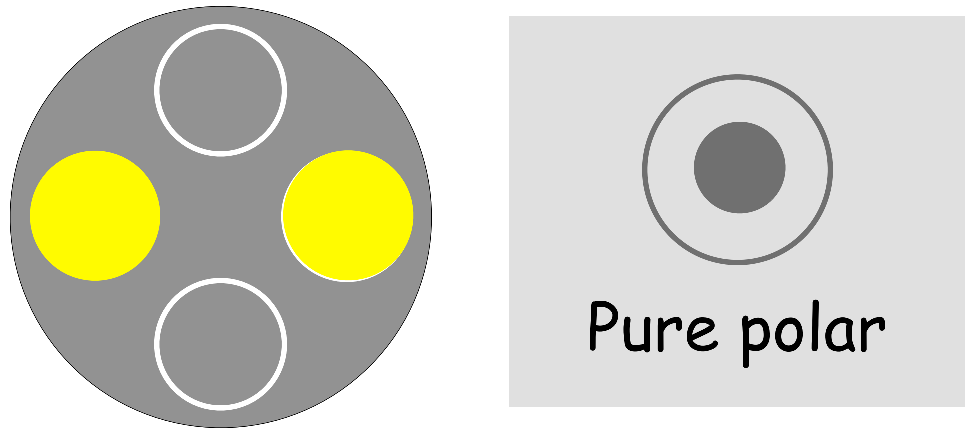
Separation of polar and planar magnetization

Pt(5 nm) [Co(0.3 nm)/Pt(0.7 nm)]$_3$

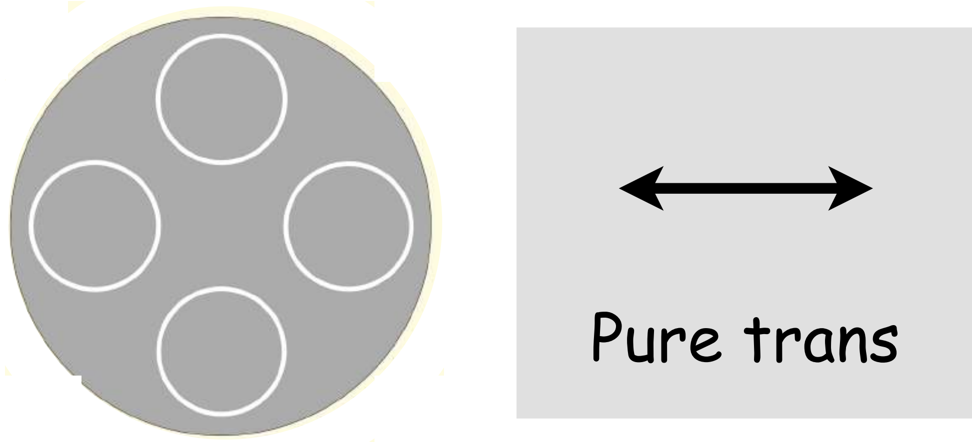




Addition



Subtraction

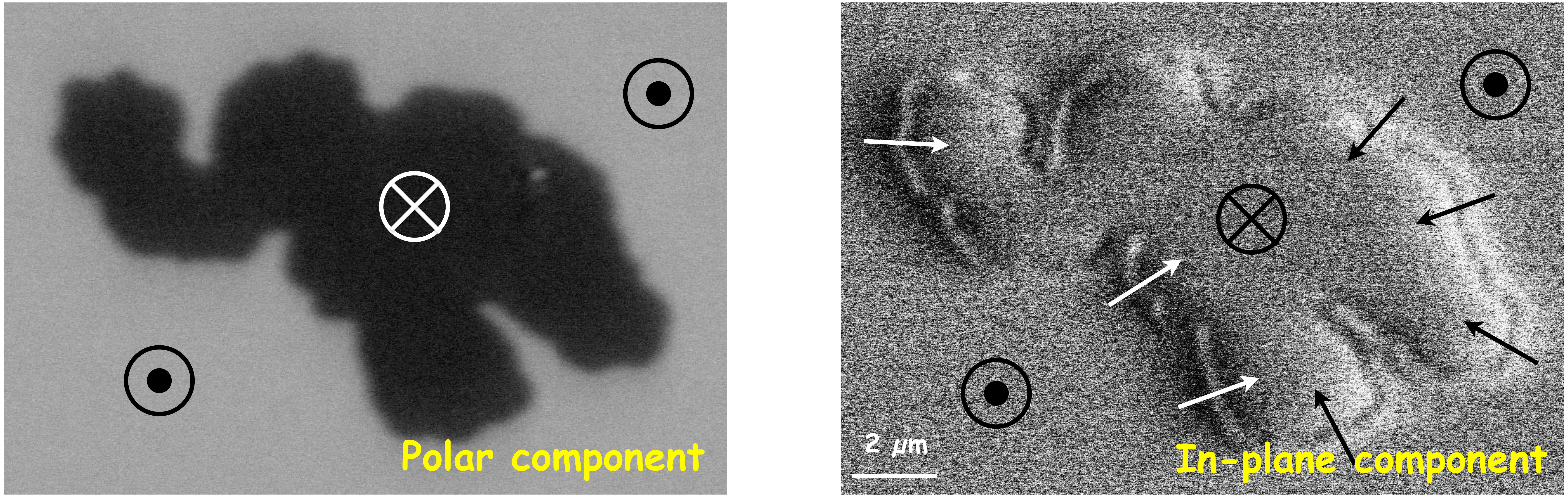


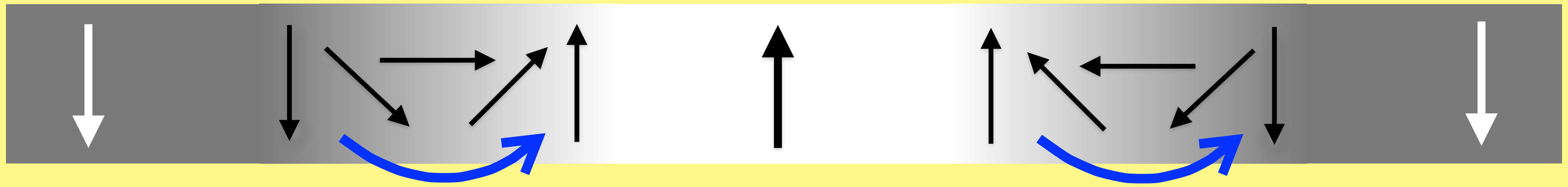
(sample: D. Makarov, IFW)

4. Magneto-Optical Kerr Microscopy

Wide-field Kerr-microscope: sensitivity selection
Separation of polar and planar magnetization

Pt(5 nm) [Co(0.3 nm)/Pt(0.7 nm)]$_3$





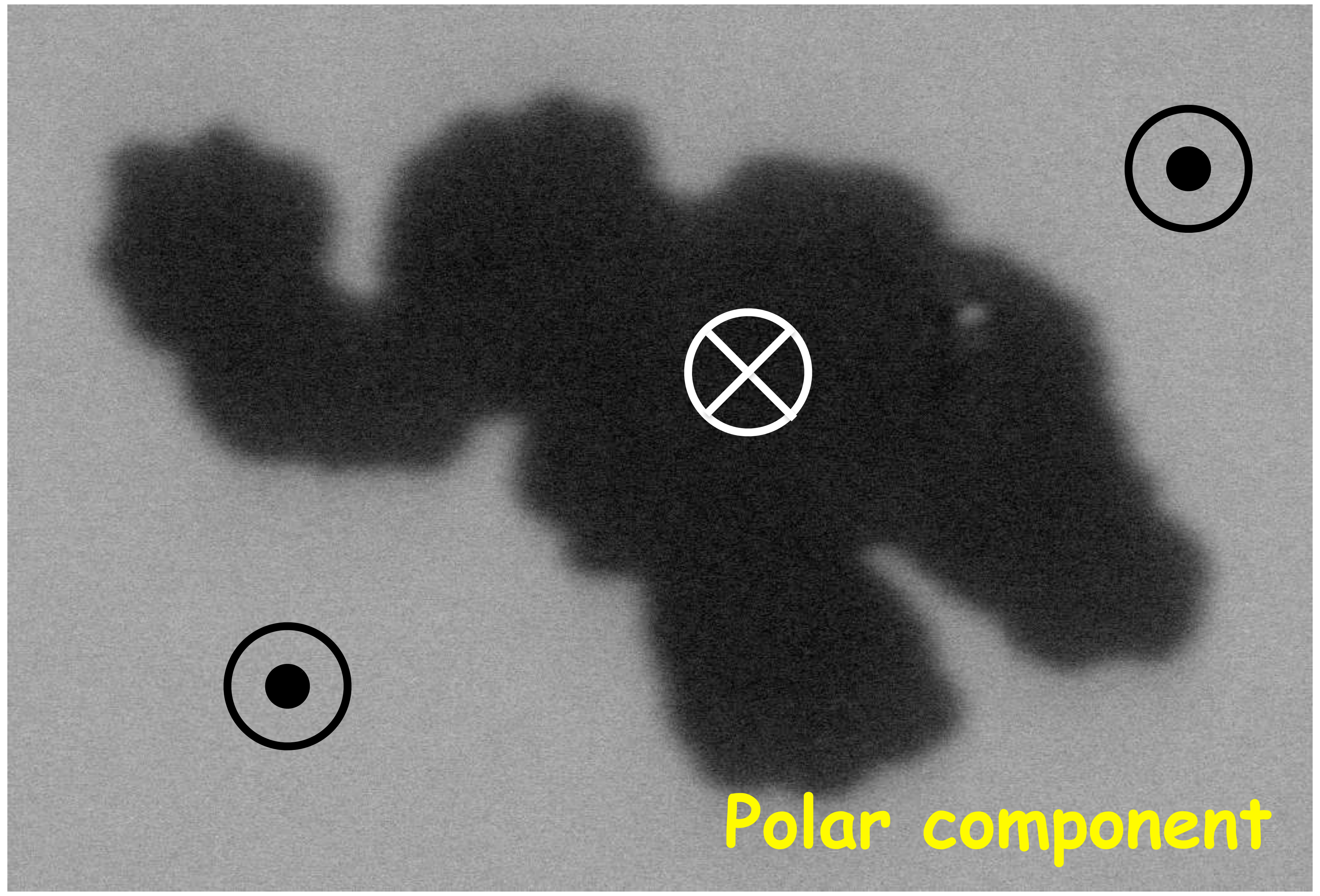
Side view

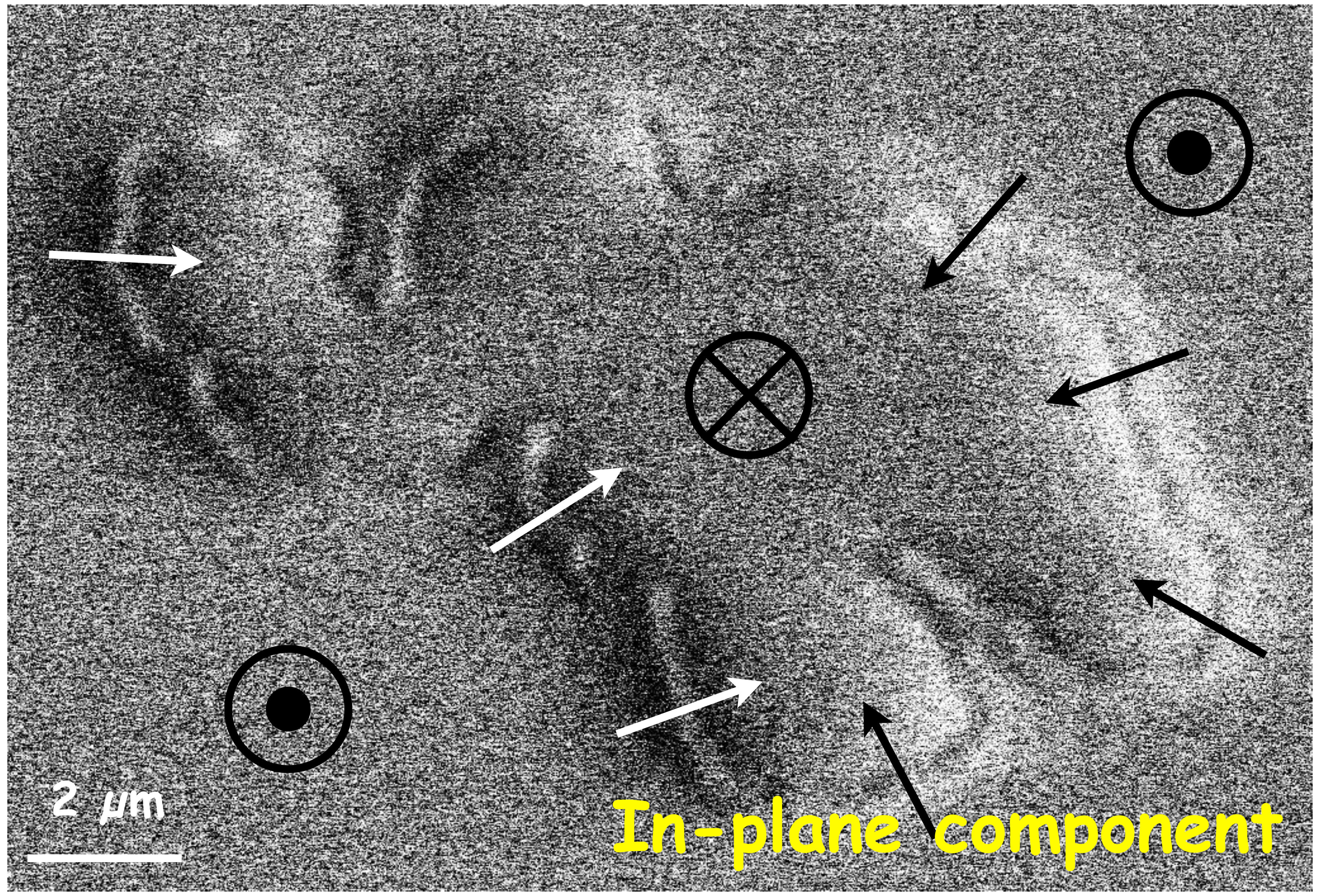
(sample: D. Makarov, IFW)

Wide-field Kerr-microscope: sensitivity selection

Separation of polar and planar magnetization

Pt(5 nm) [Co(0.3 nm)/Pt(0.7 nm)]$_3$





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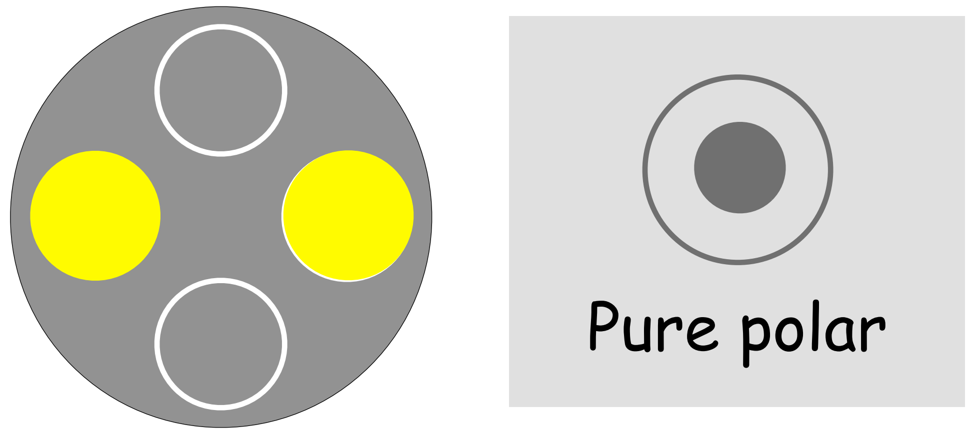
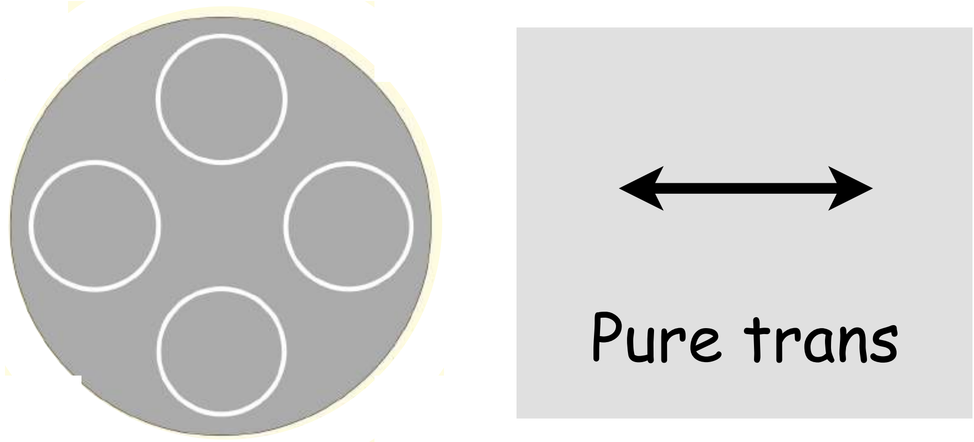


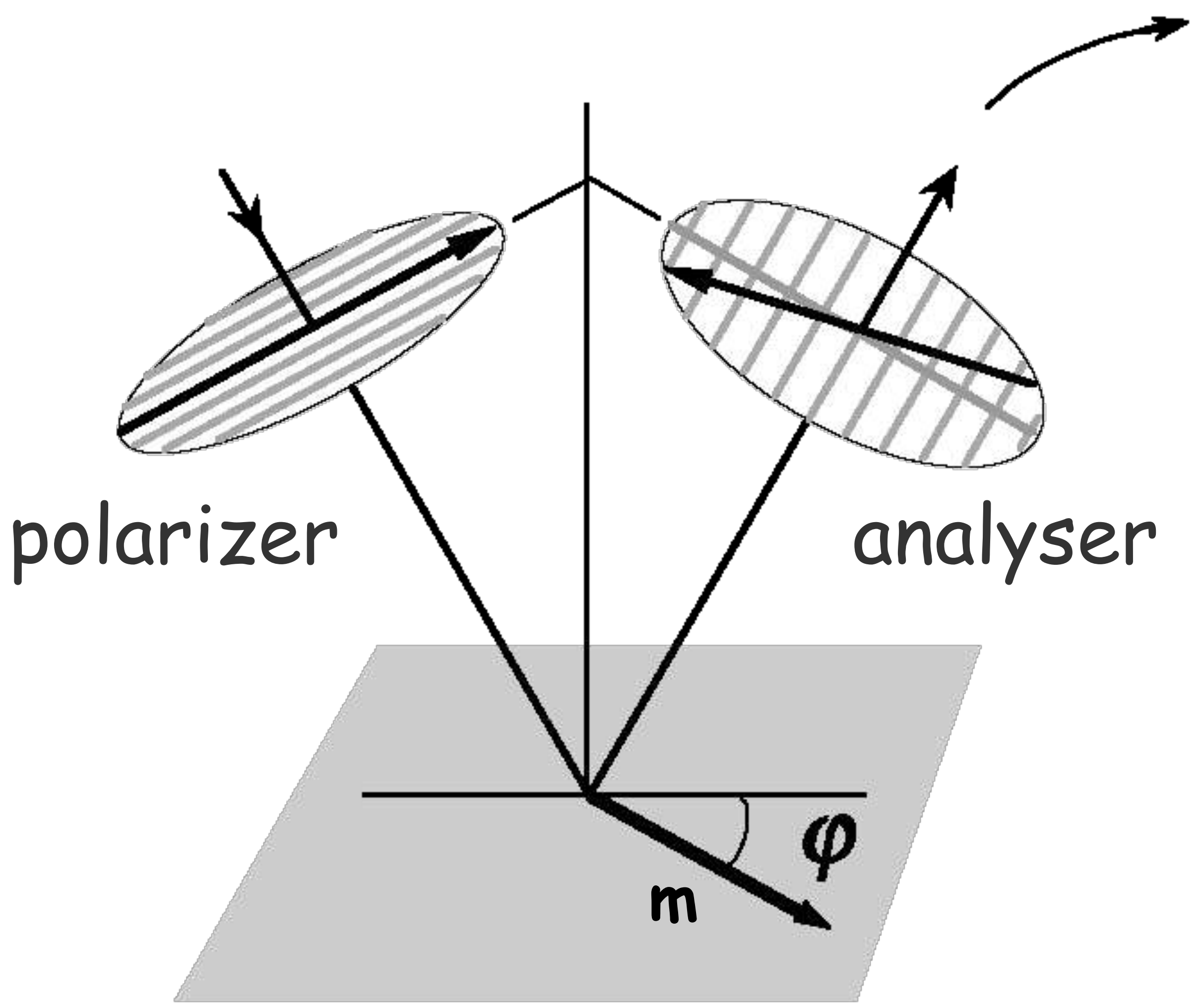
Subtraction

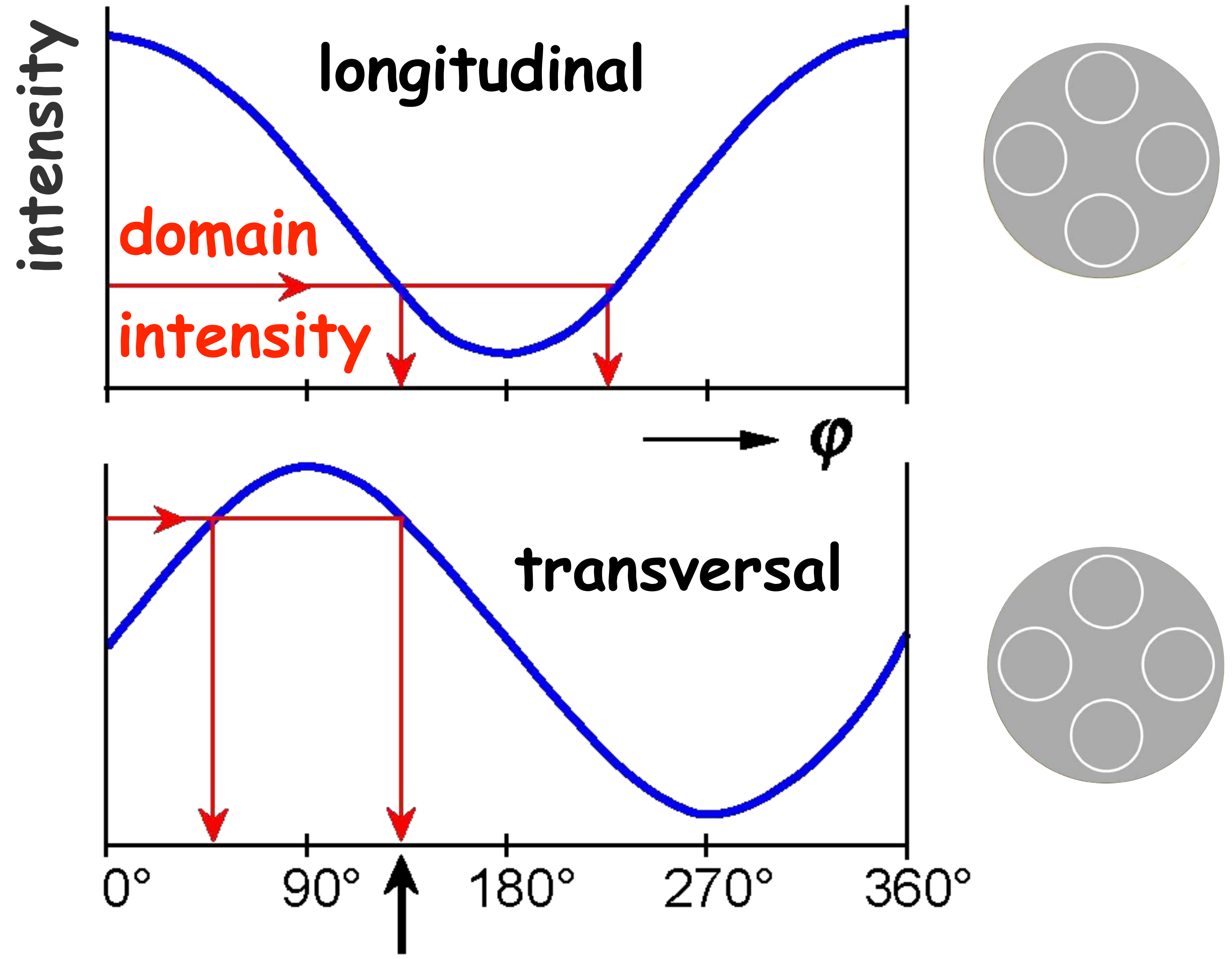


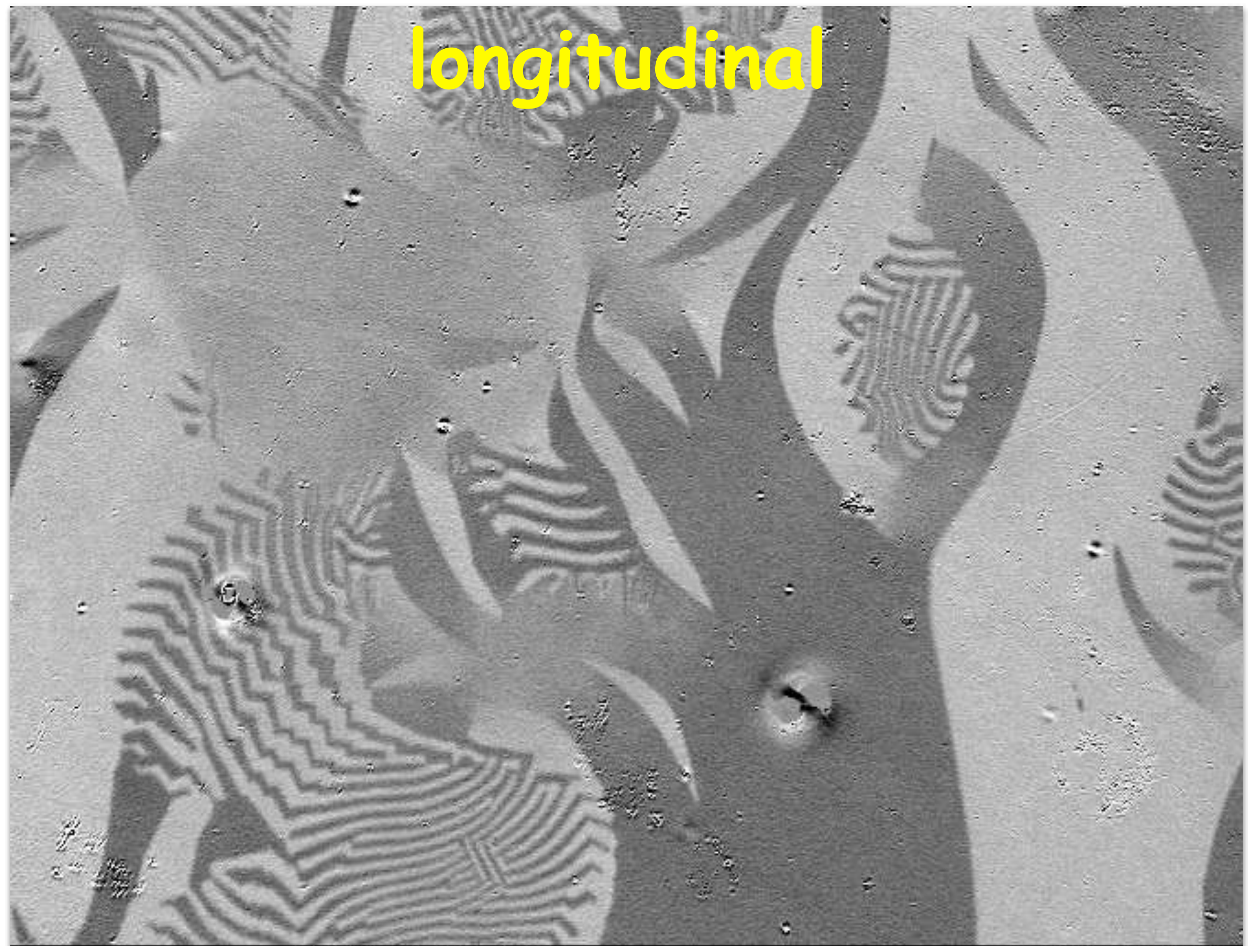
(sample: D. Makarov, IFW)

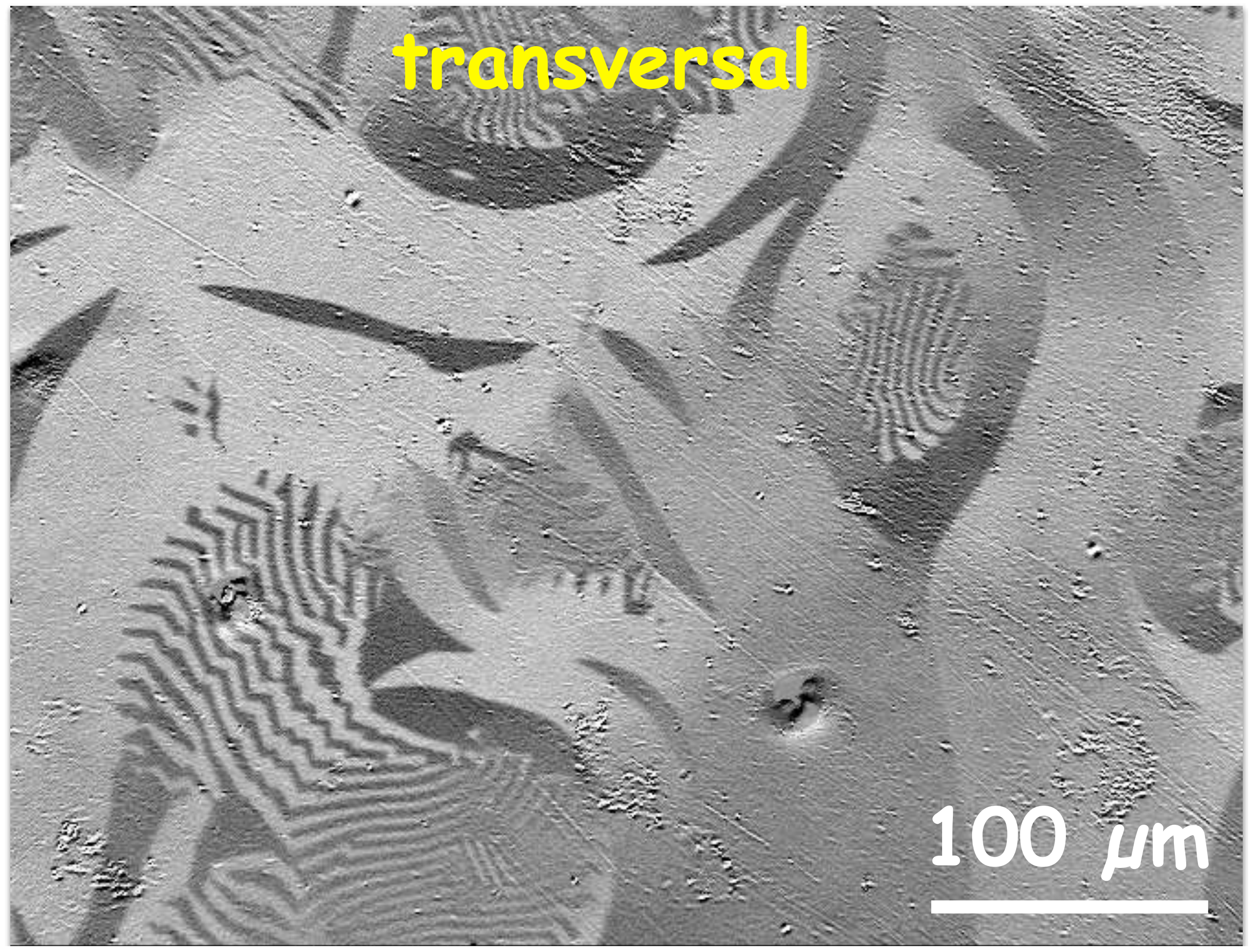
Quantitative Kerr-microscopy

Magnetostrictive amorphous ribbon



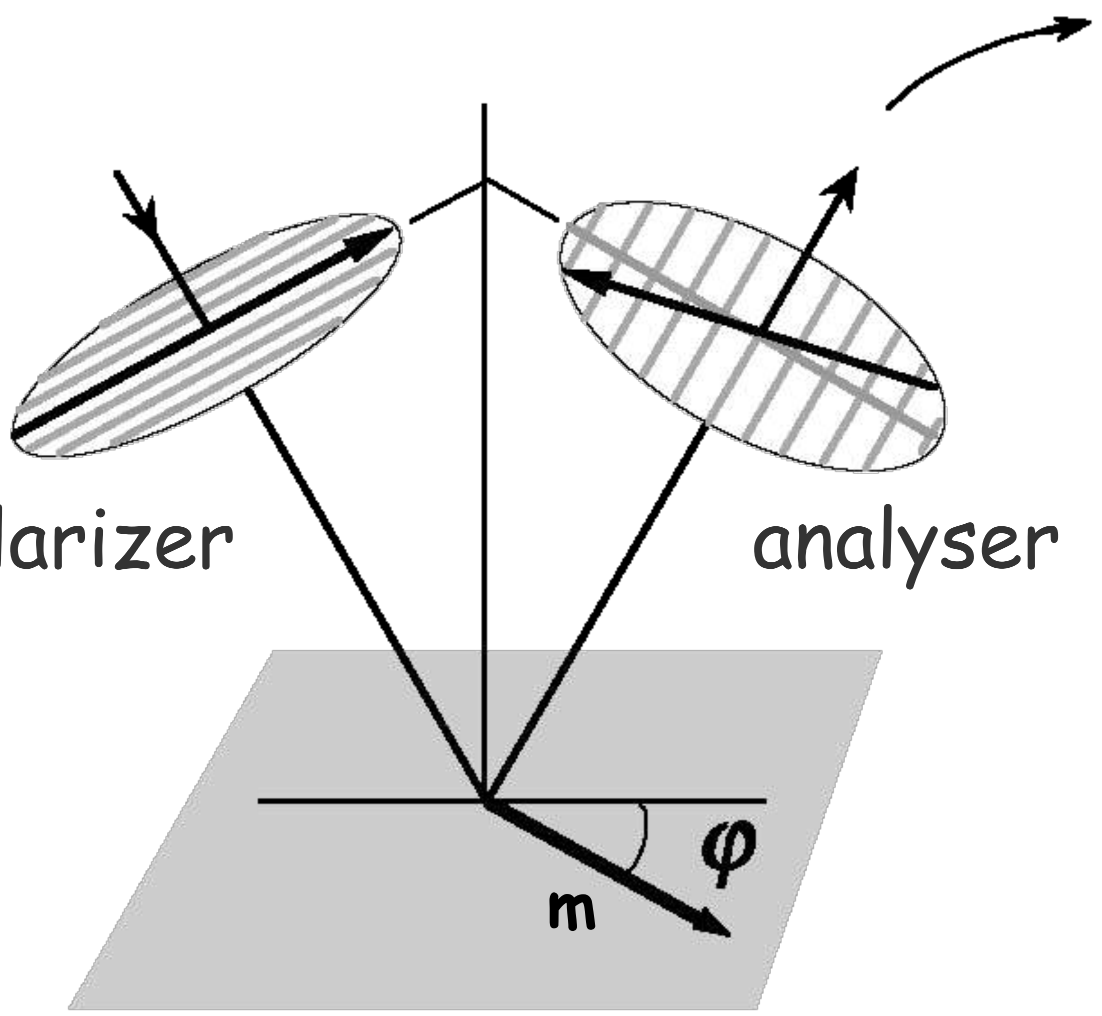


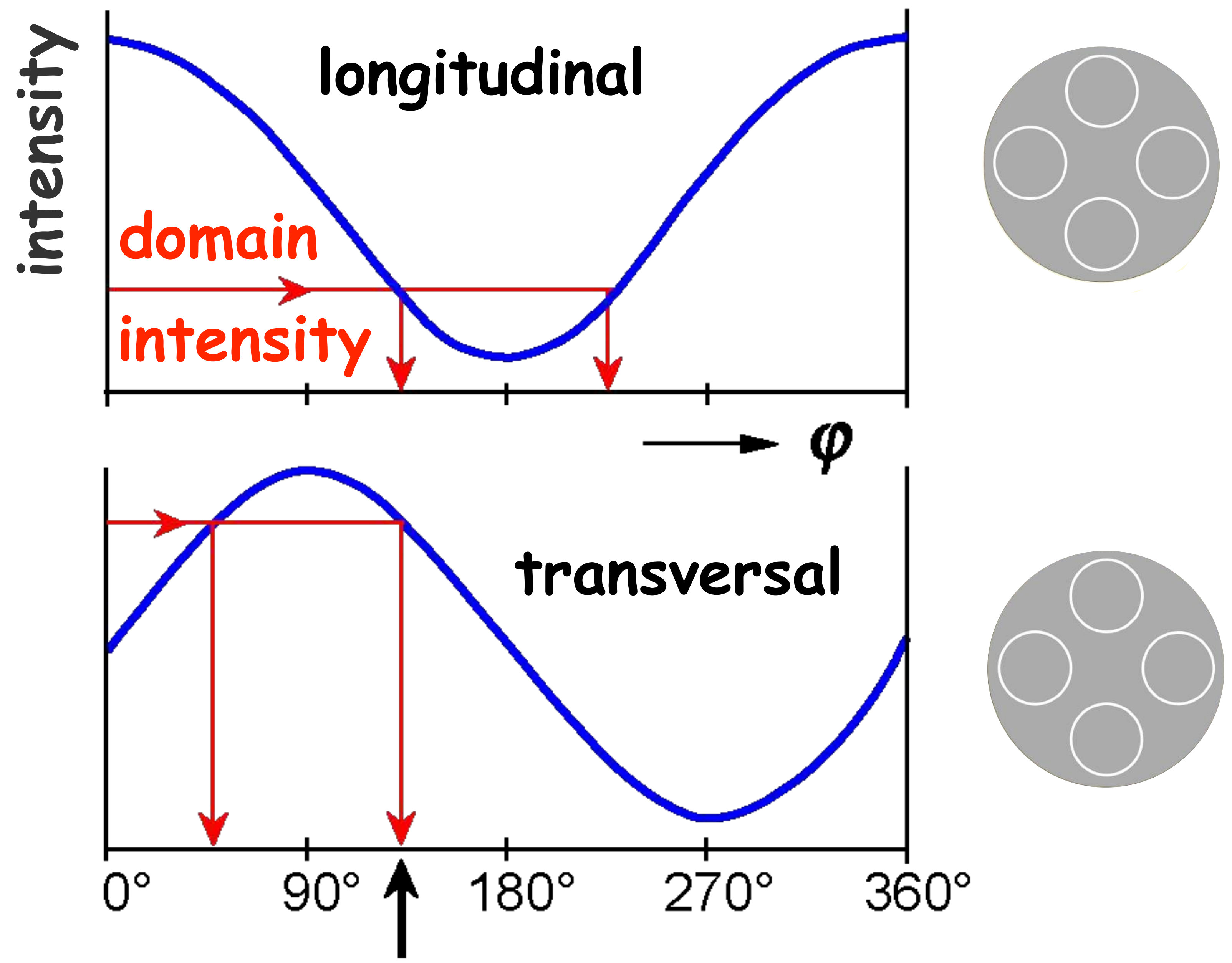


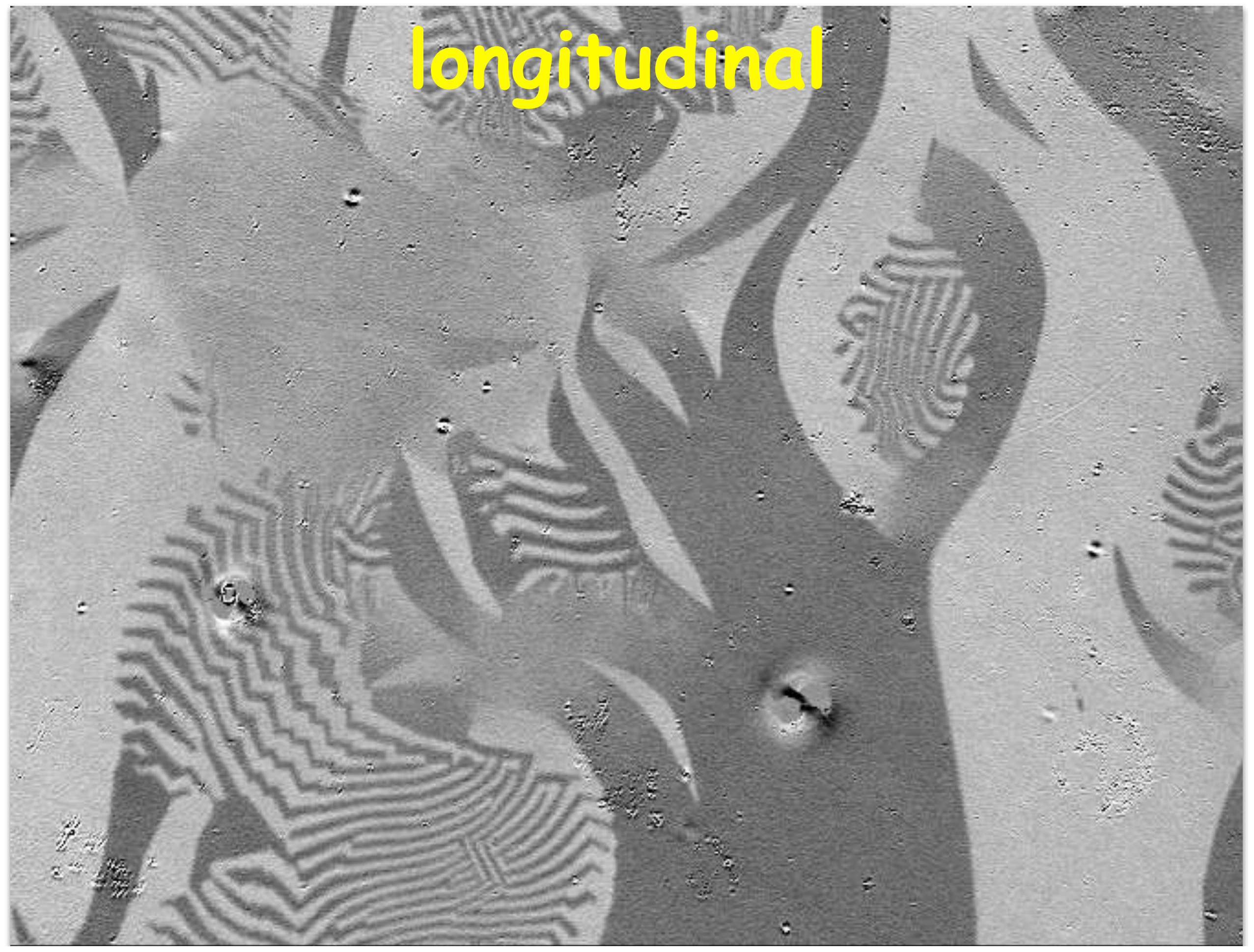


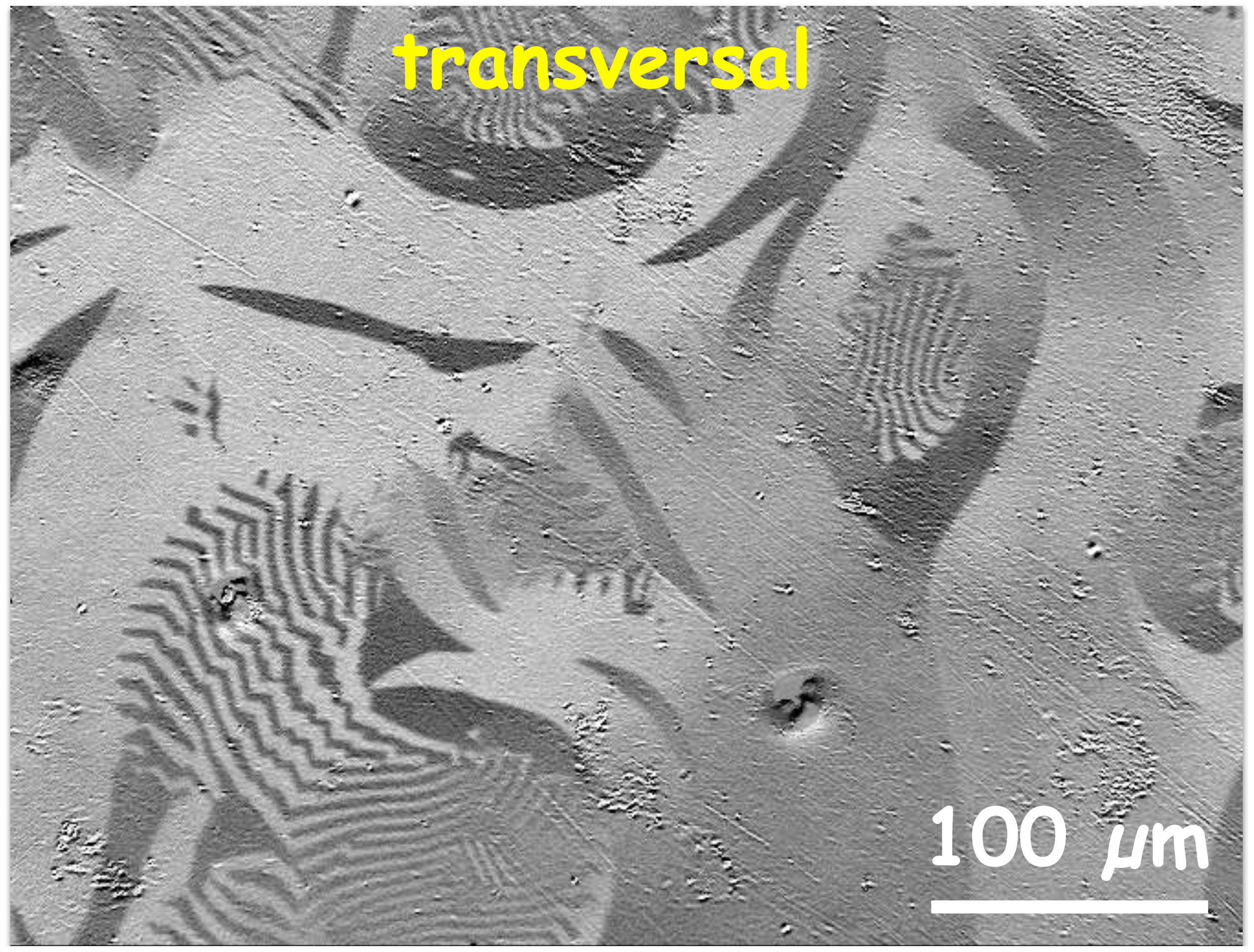
Quantitative Kerr-microscopy

Magnetostrictive amorphous ribbon





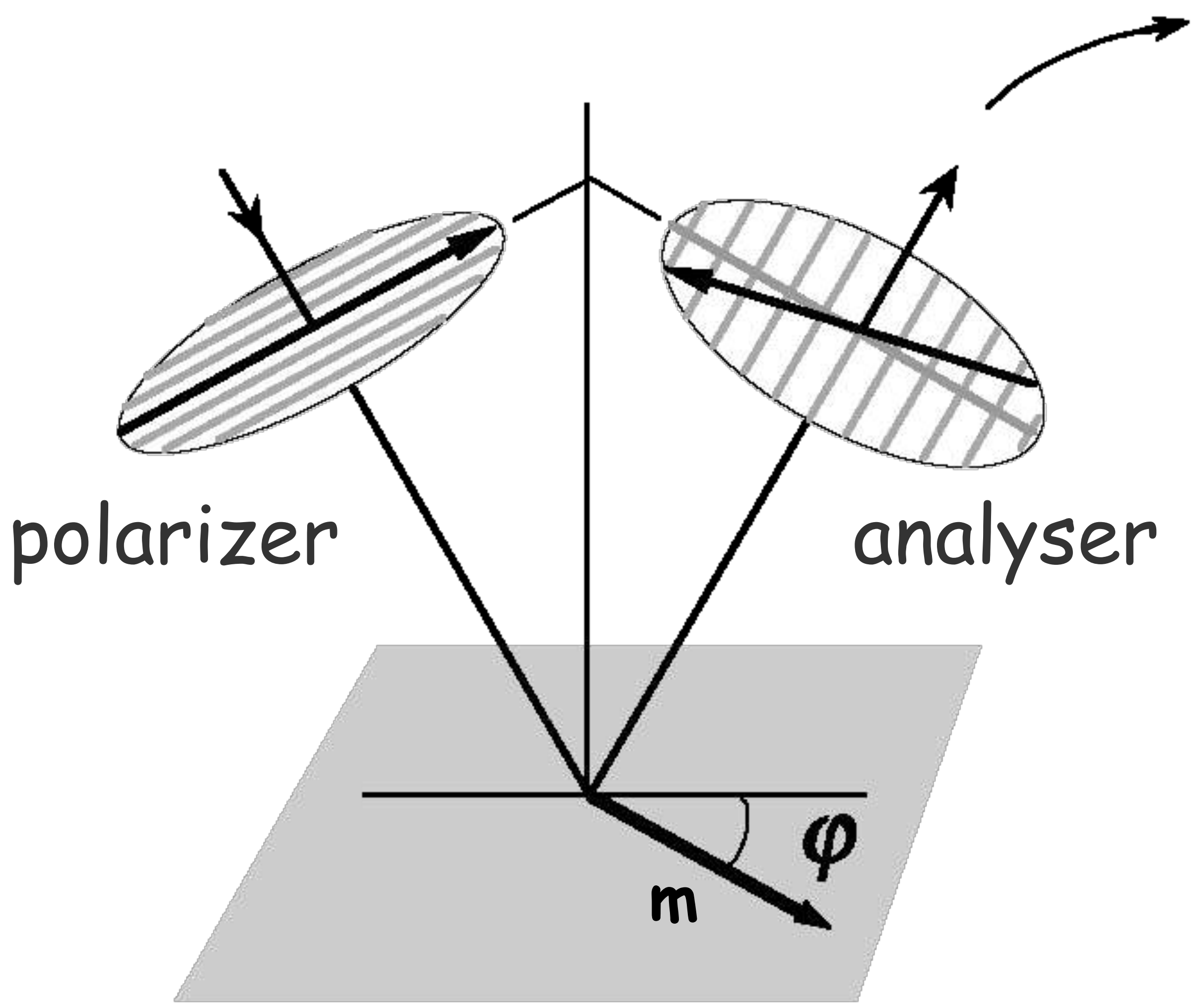


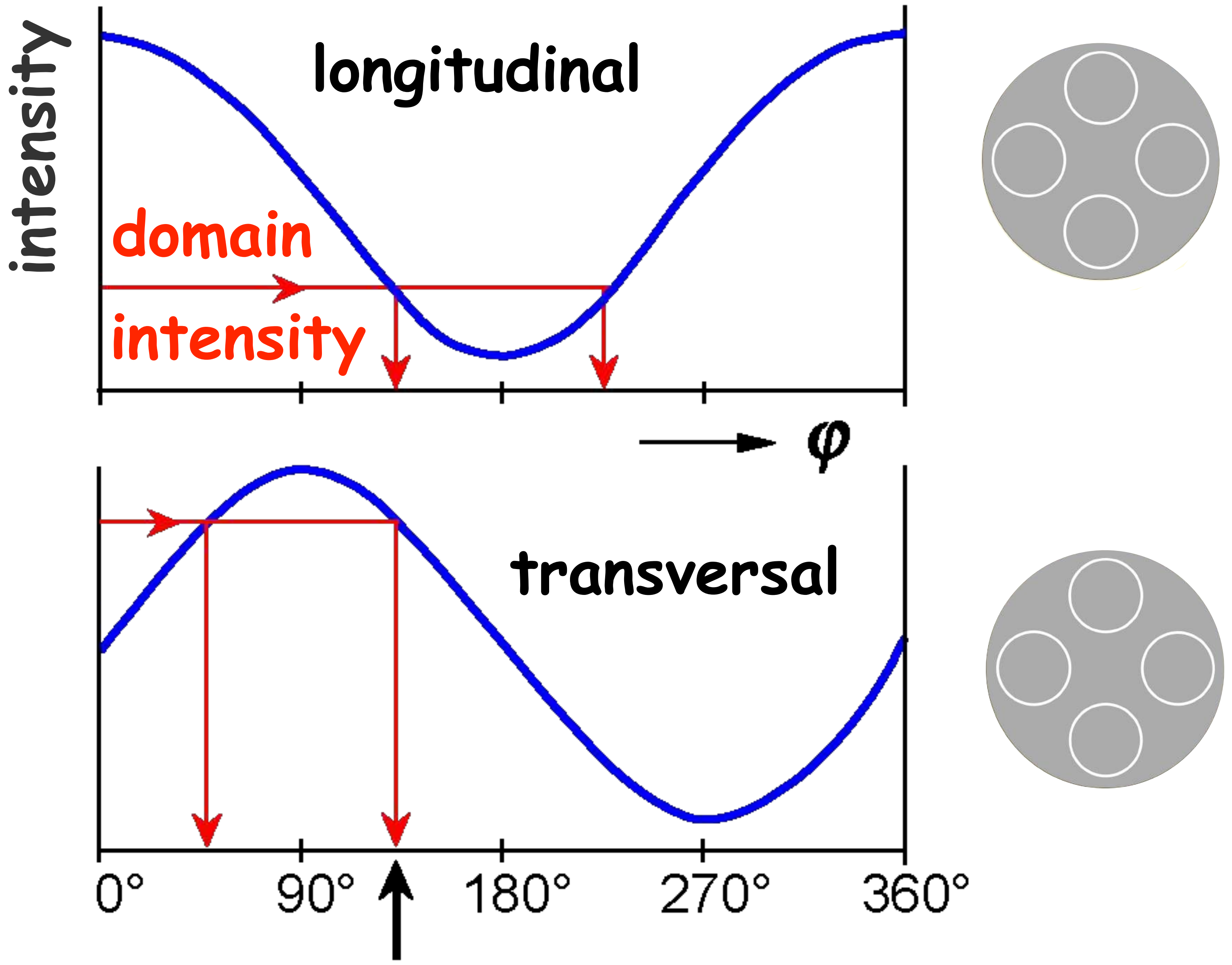


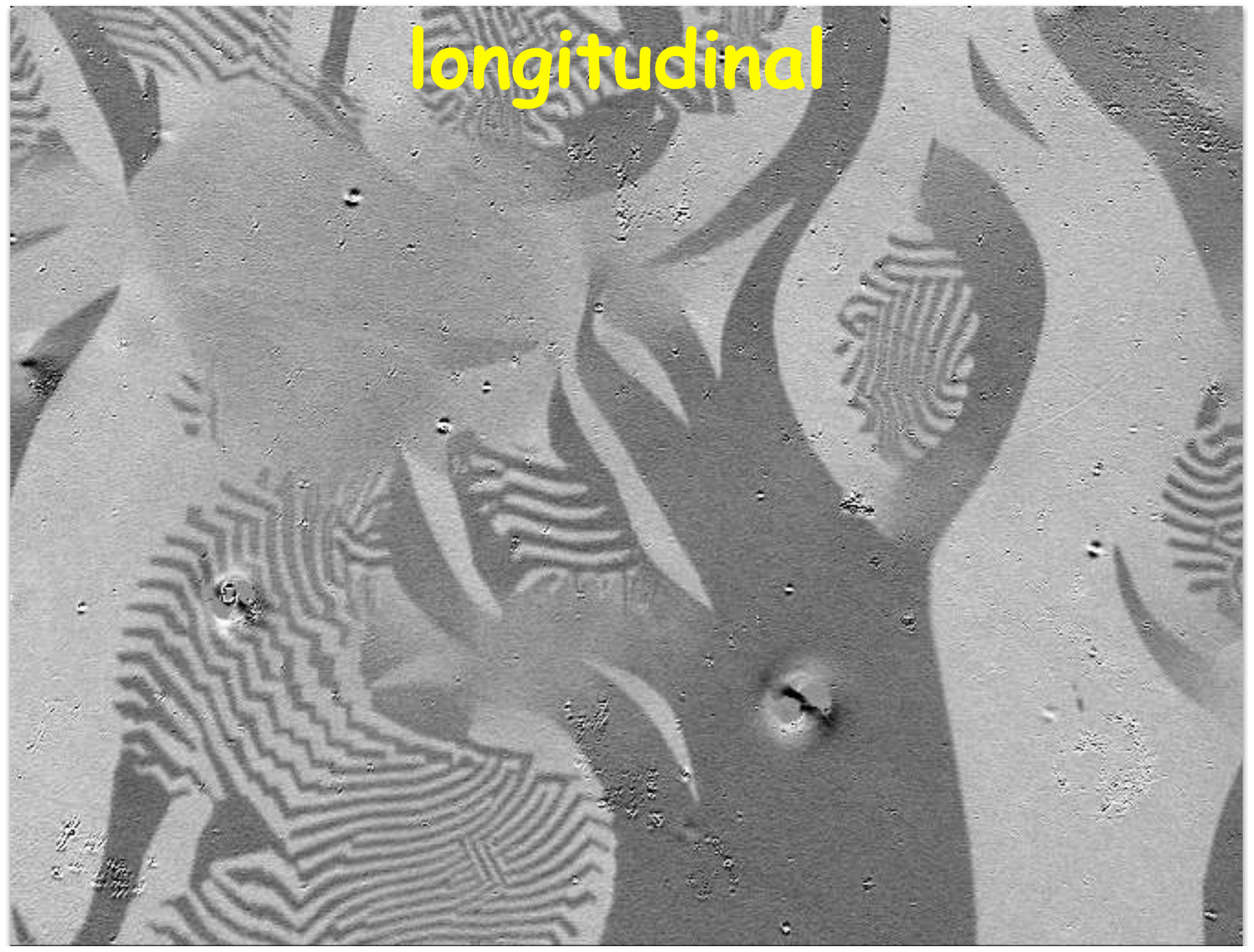
4. Magneto-Optical Kerr Microscopy

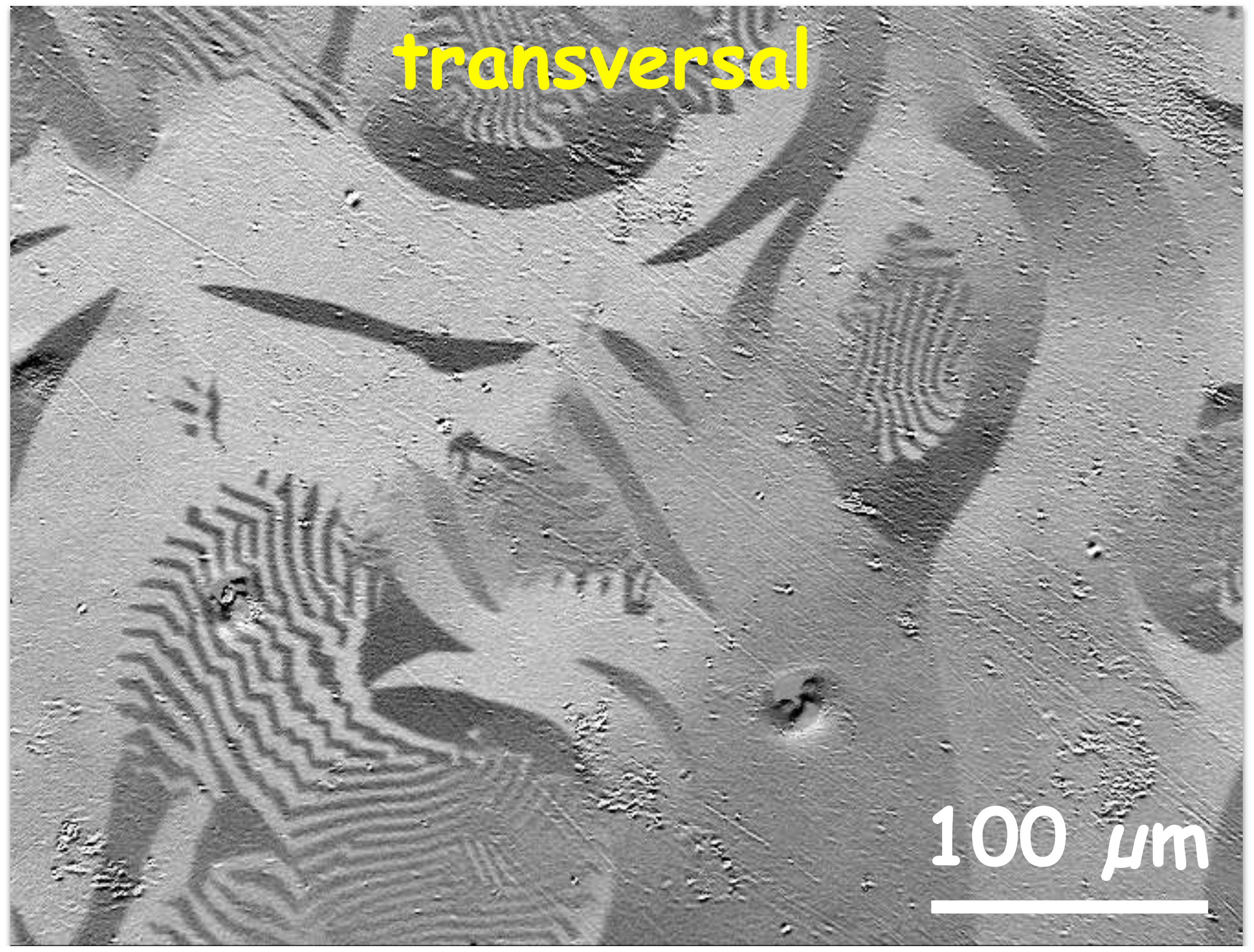
Quantitative Kerr-microscopy

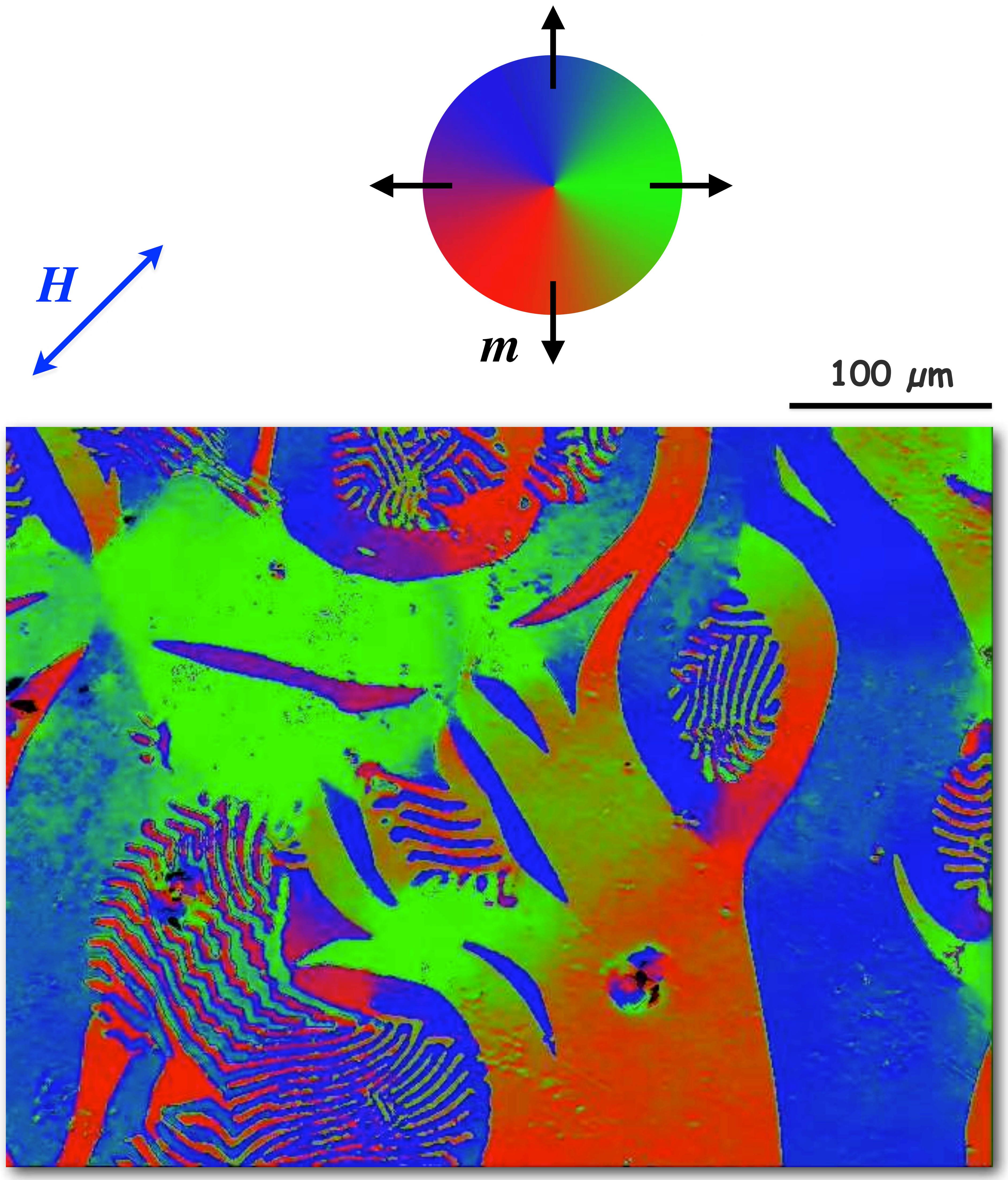
Magnetostrictive amorphous ribbon







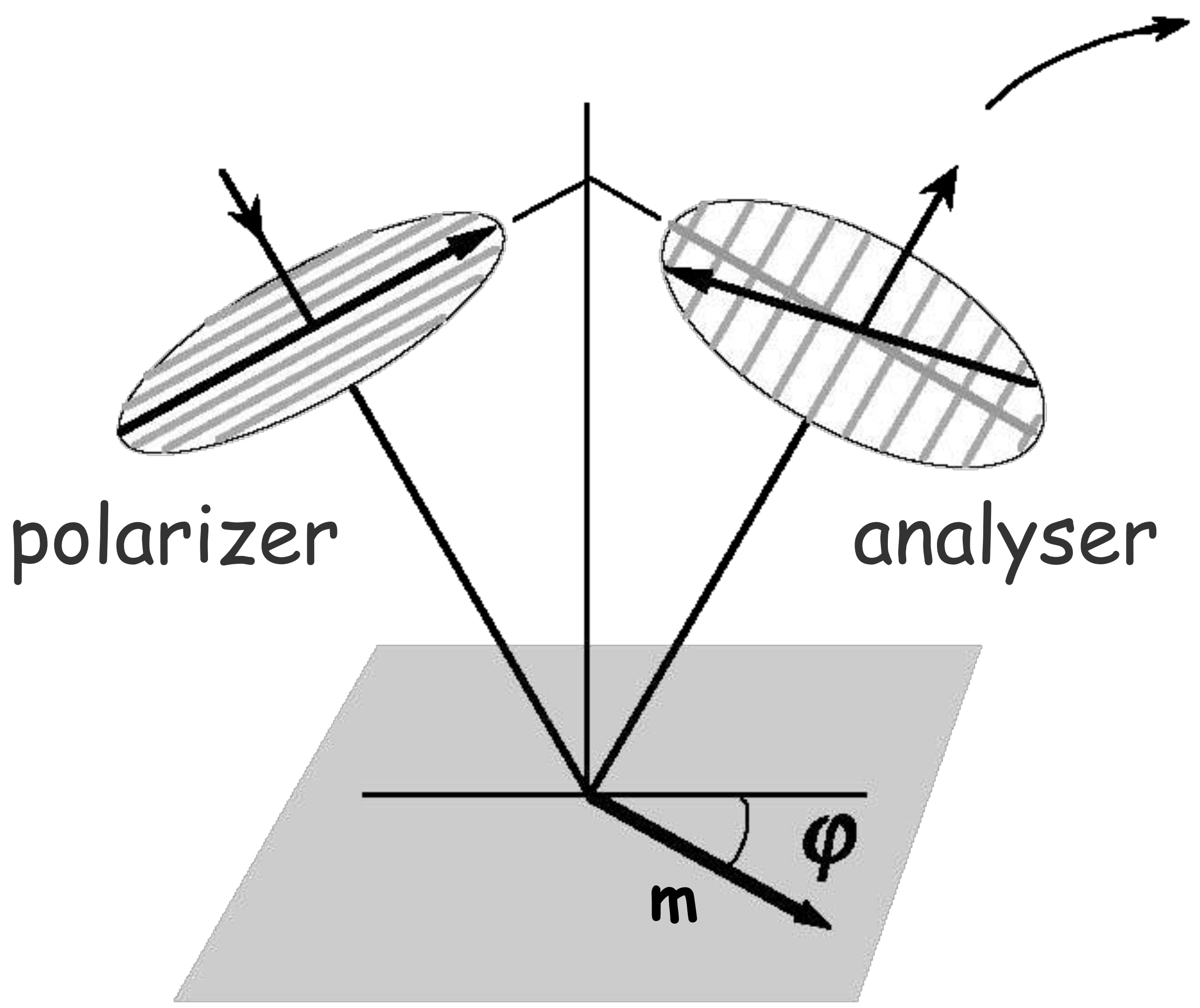


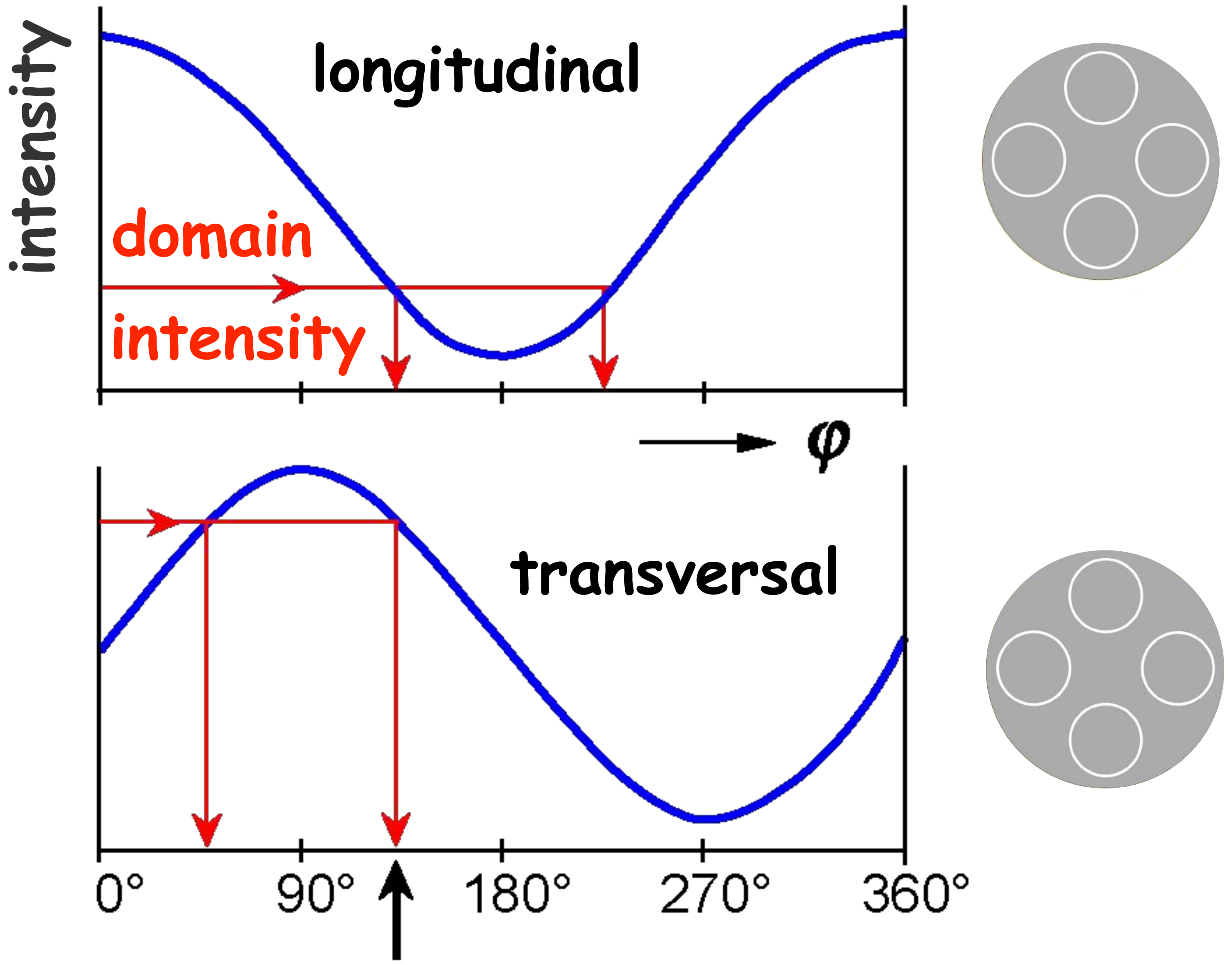


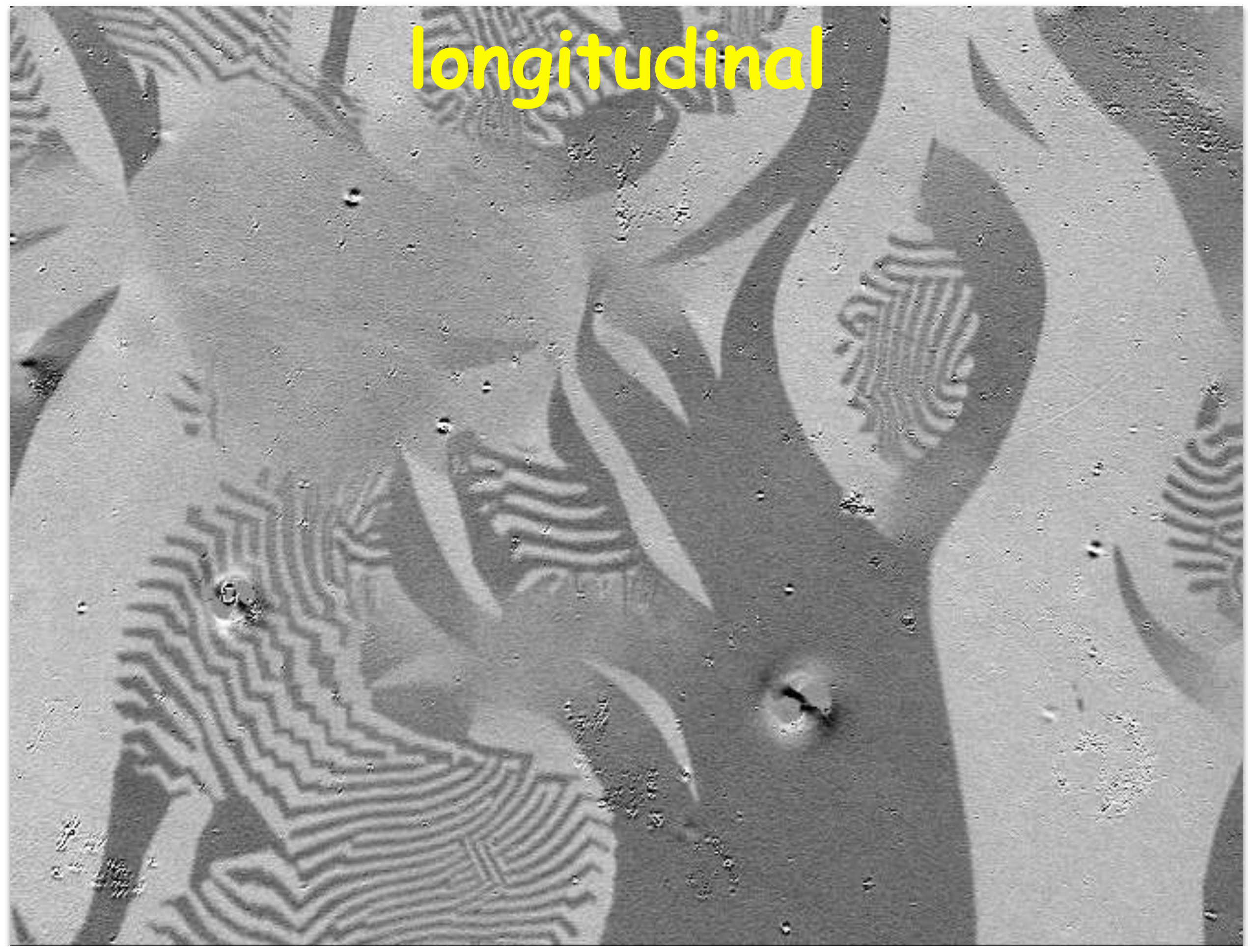
I. Soldatov and R.S., Phys. Rev. B. 95, 014426 (2017)

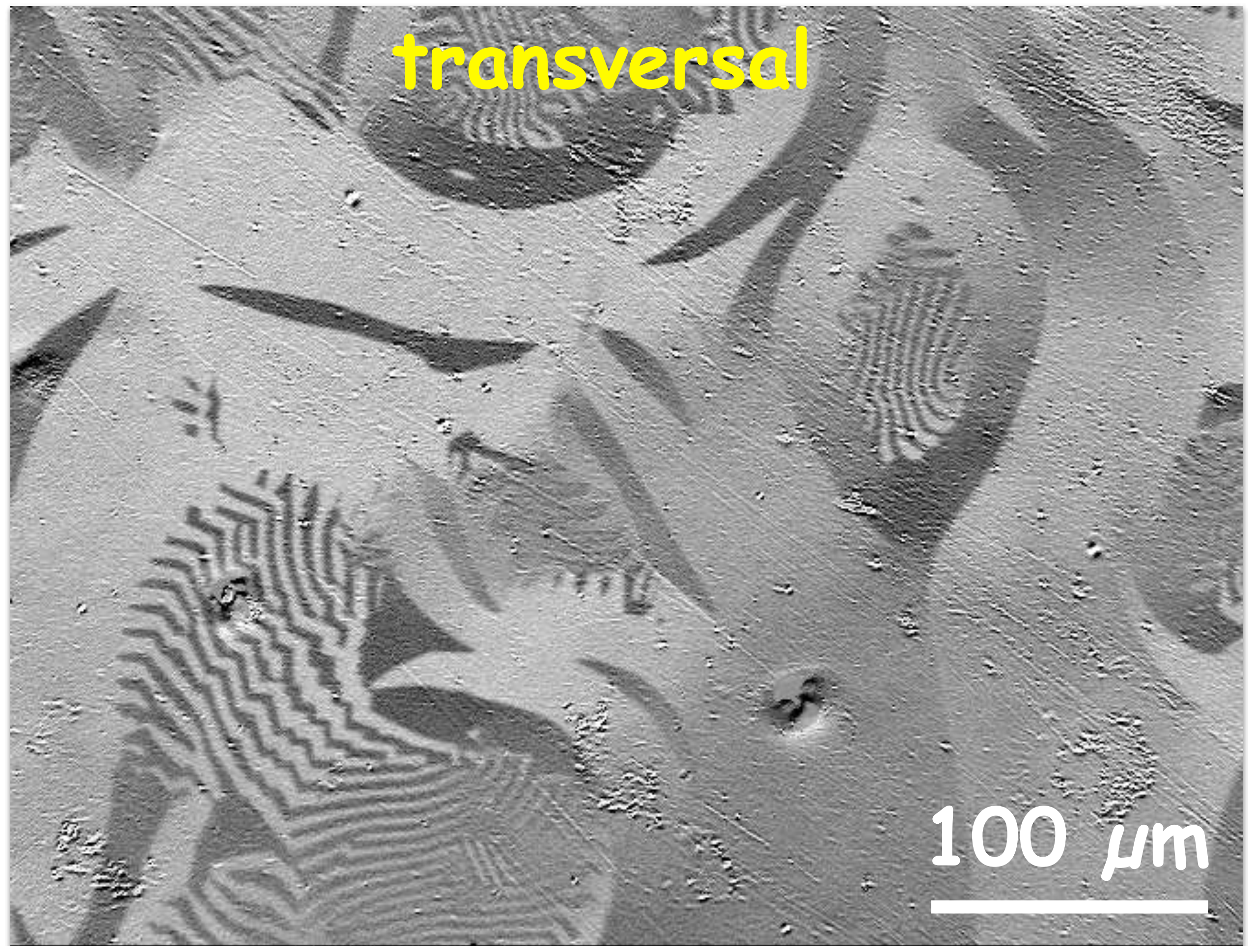
Magnetostrictive amorphous ribbon

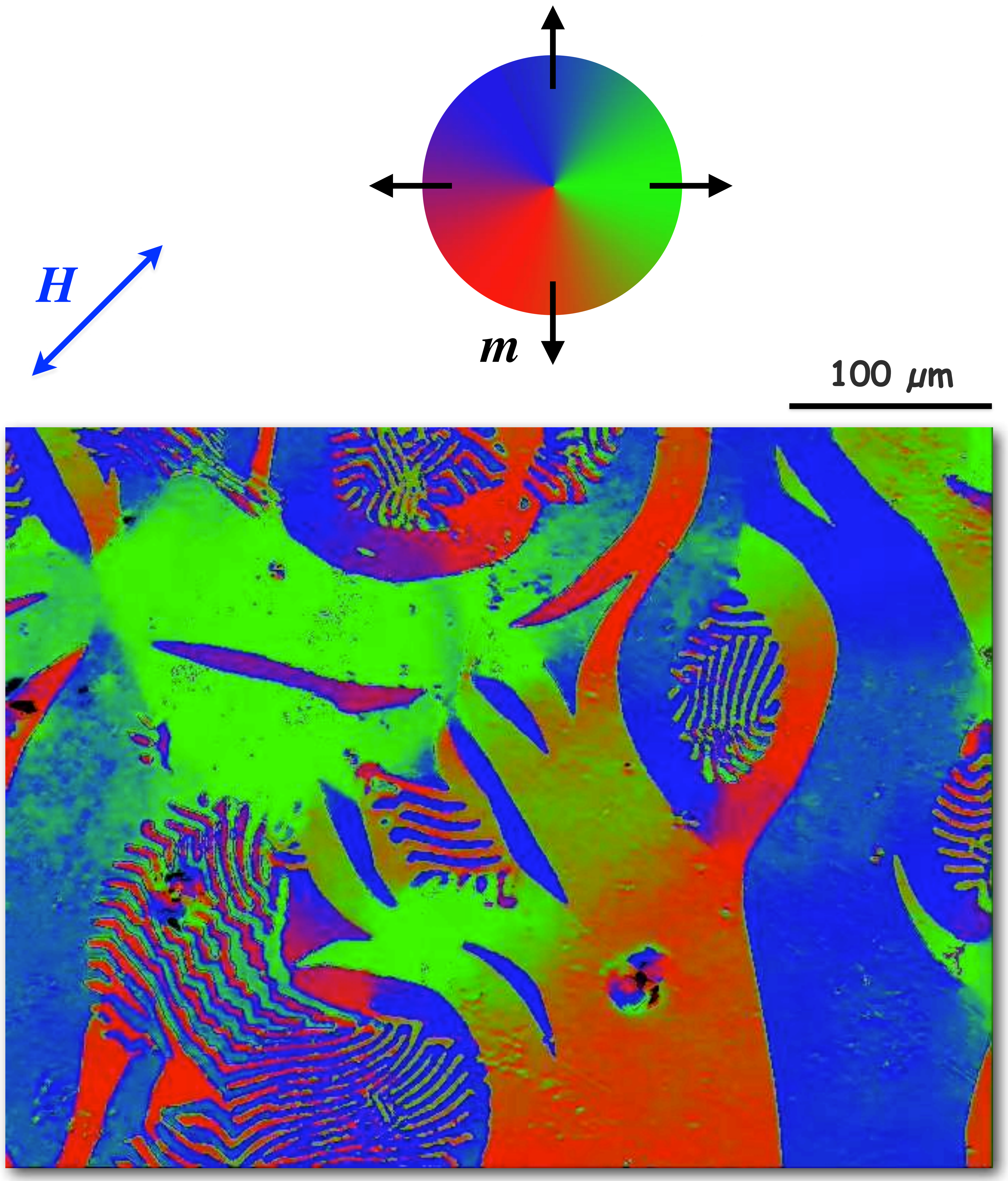
Quantitative Kerr-microscopy











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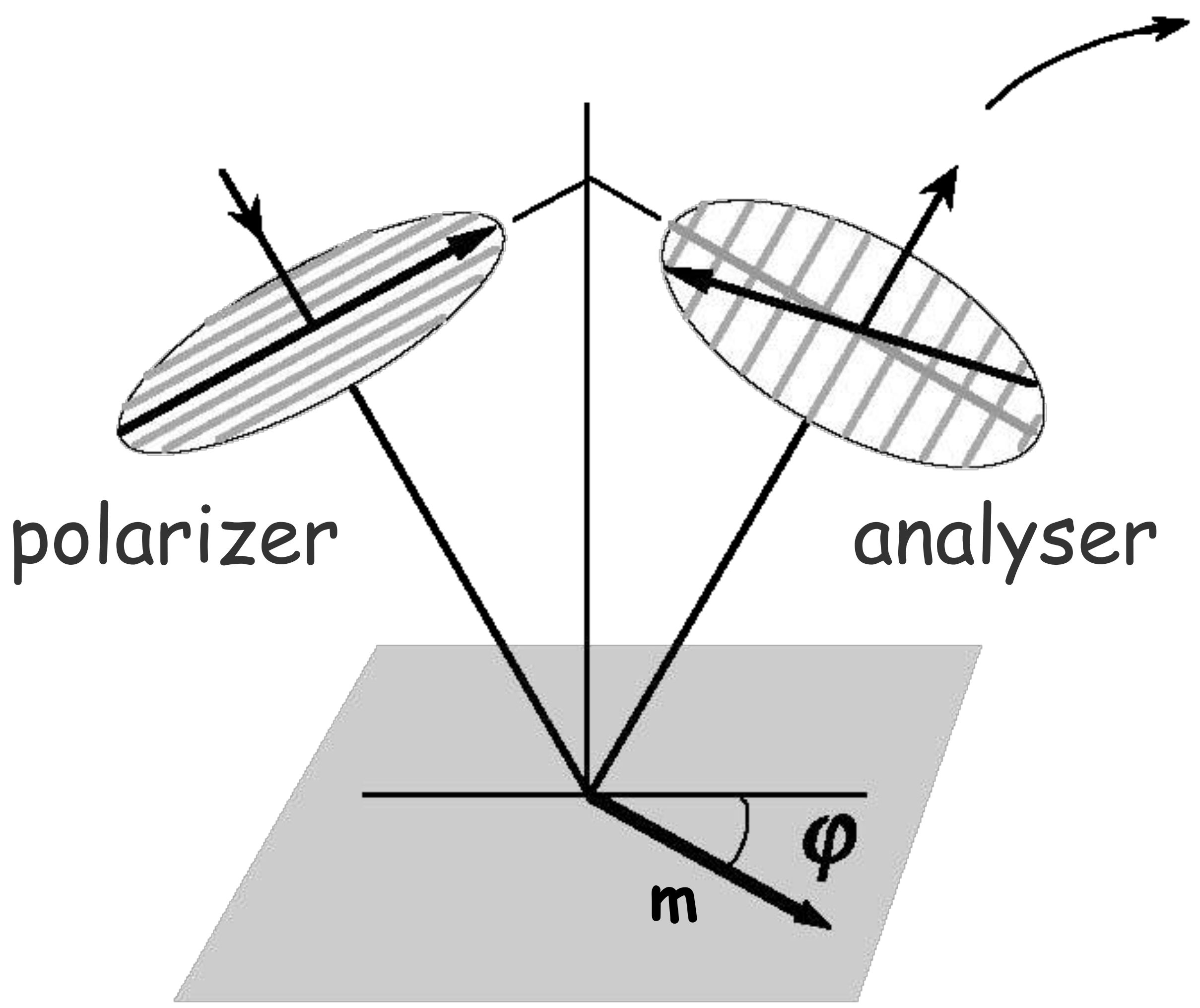
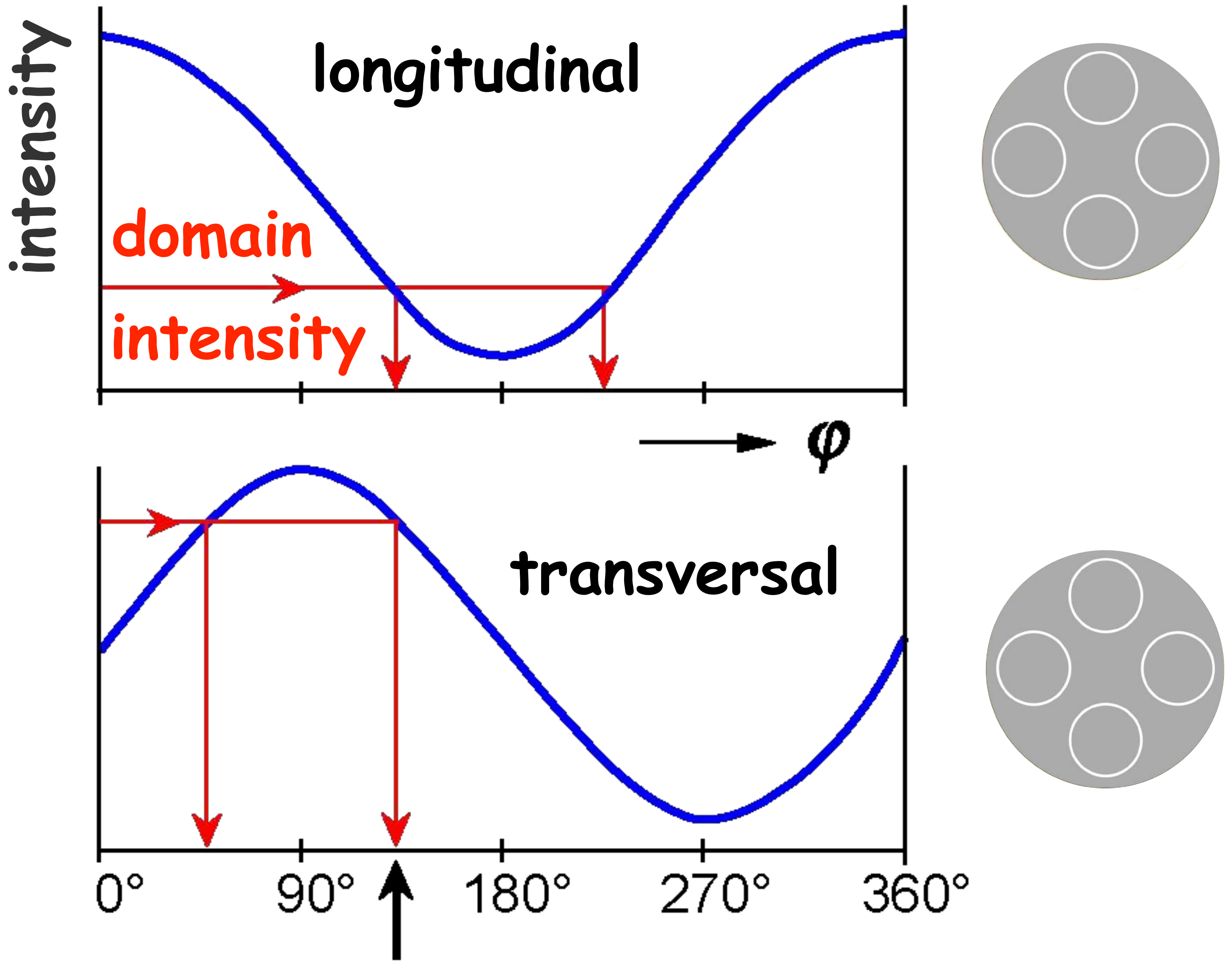
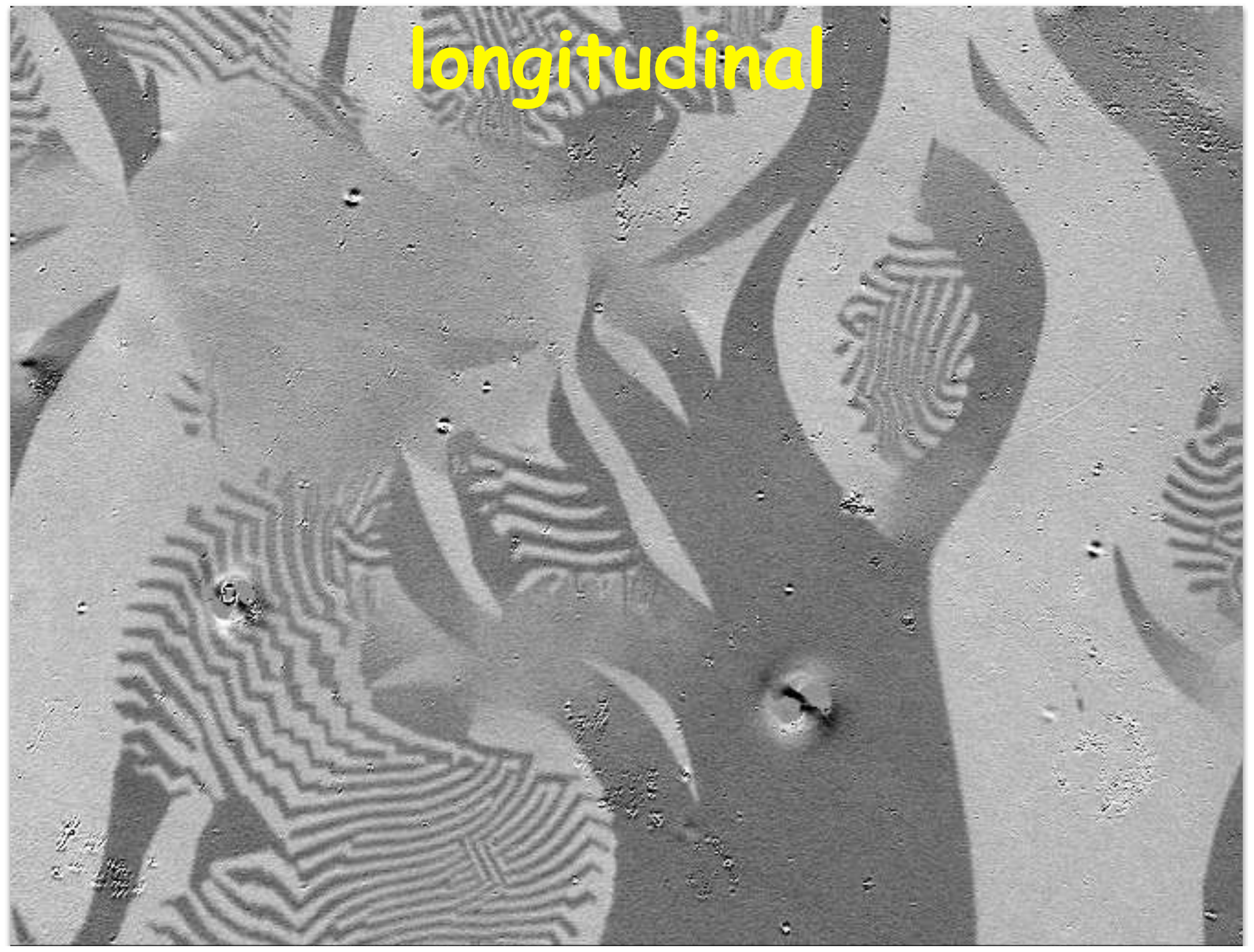
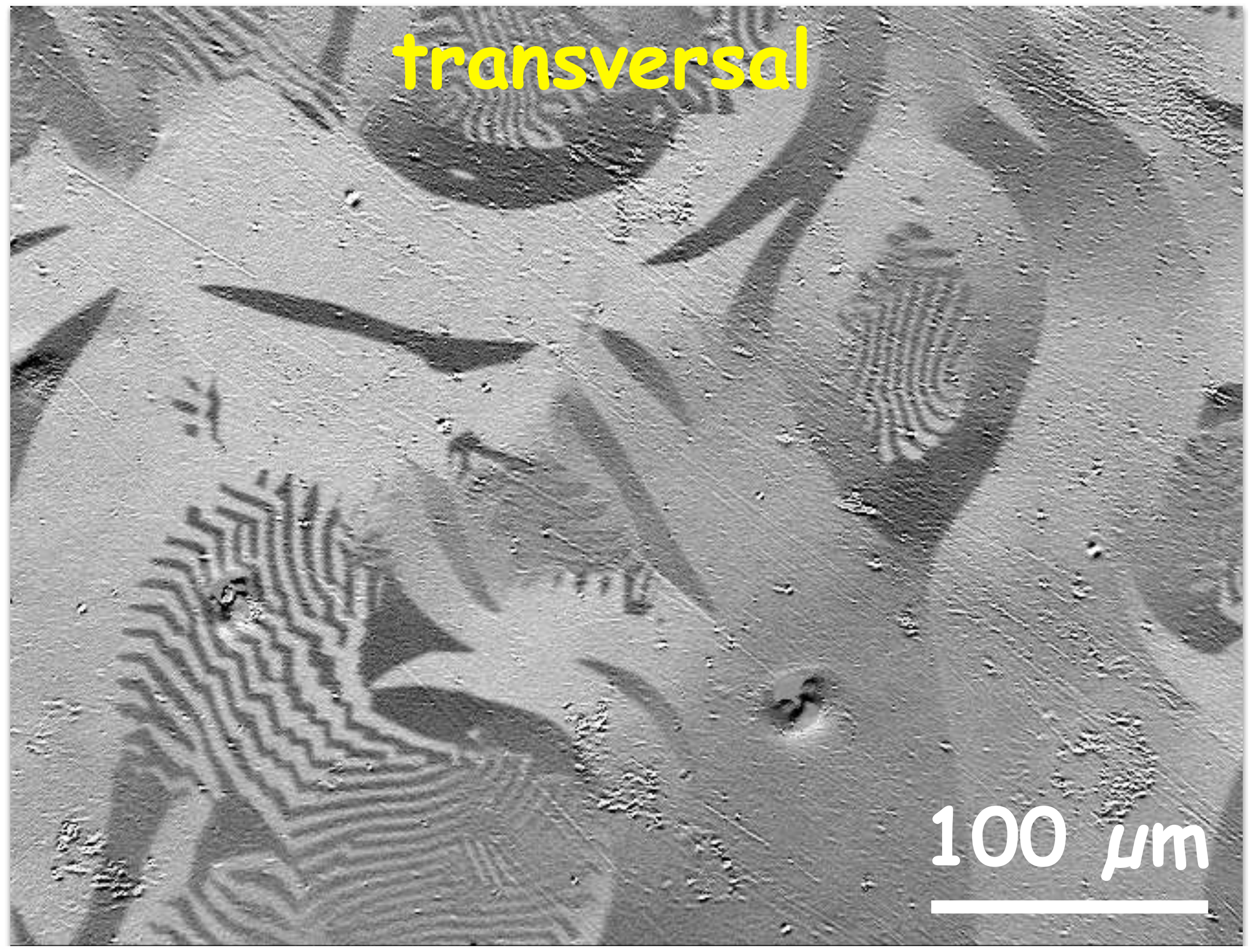
Quantitative Kerr-microscopy

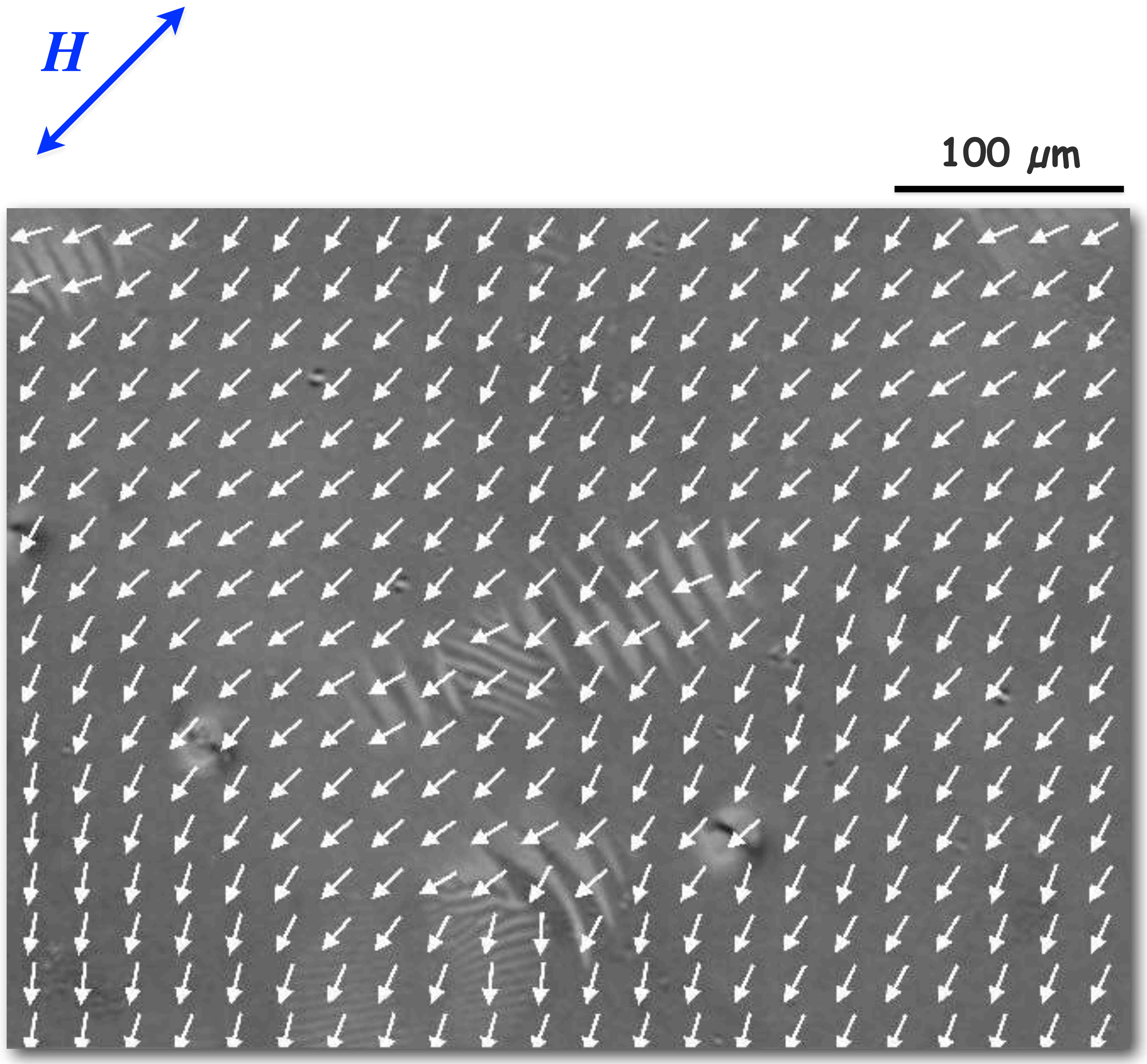
Magnetostrictive amorphous ribbon







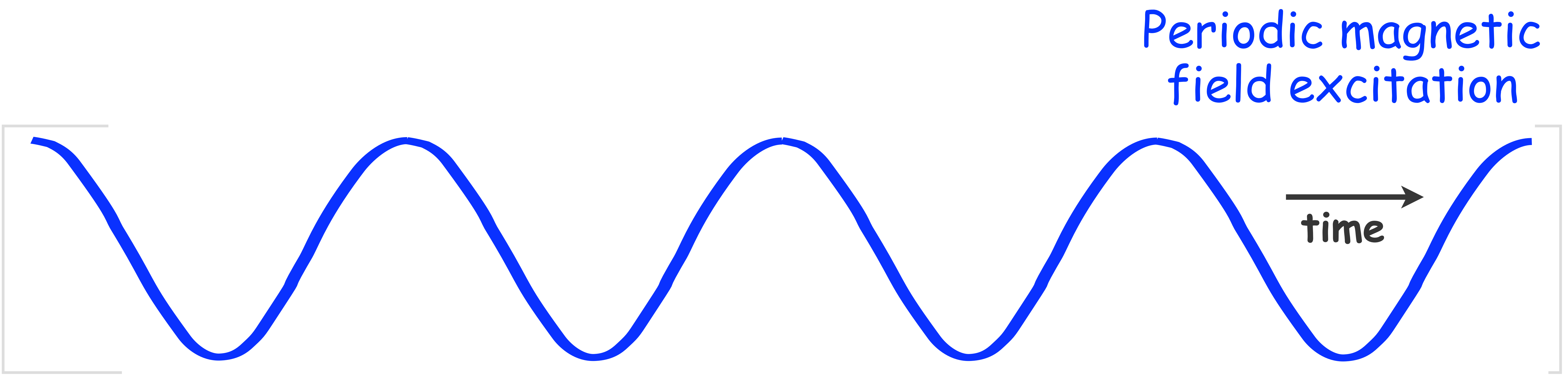




I. Soldatov and R.S., Phys. Rev. B. 95, 014426 (2017)

Time-resolved Kerr-microscopy

Single-shot mode

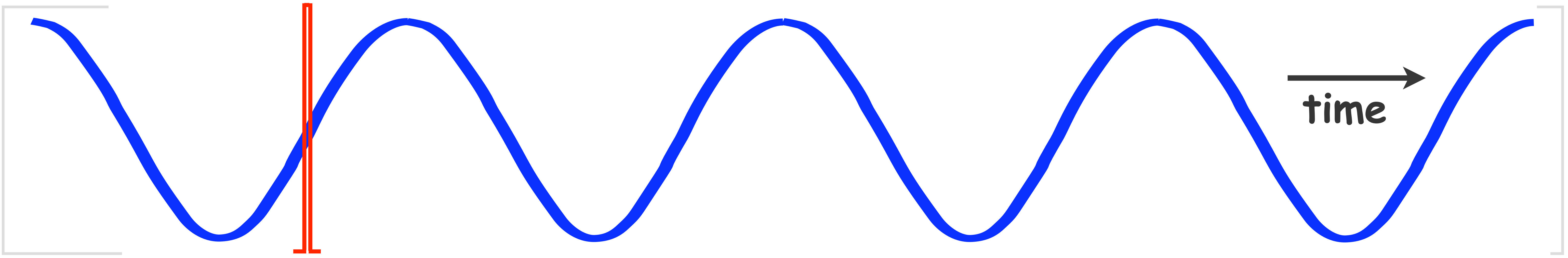


Time-resolved Kerr-microscopy

Single-shot mode

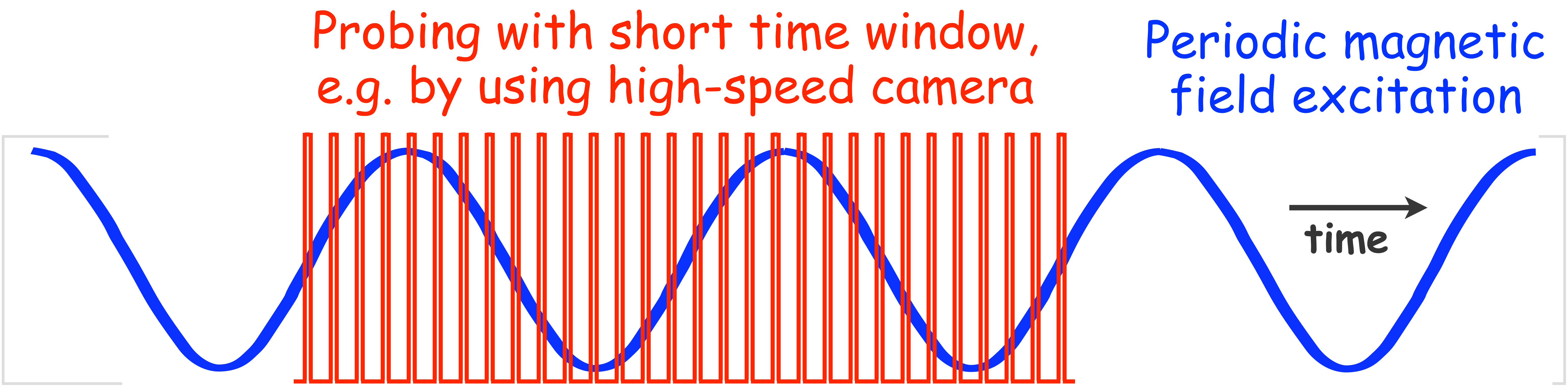
Probing with short time window, e.g. by using high-speed camera

Periodic magnetic field excitation



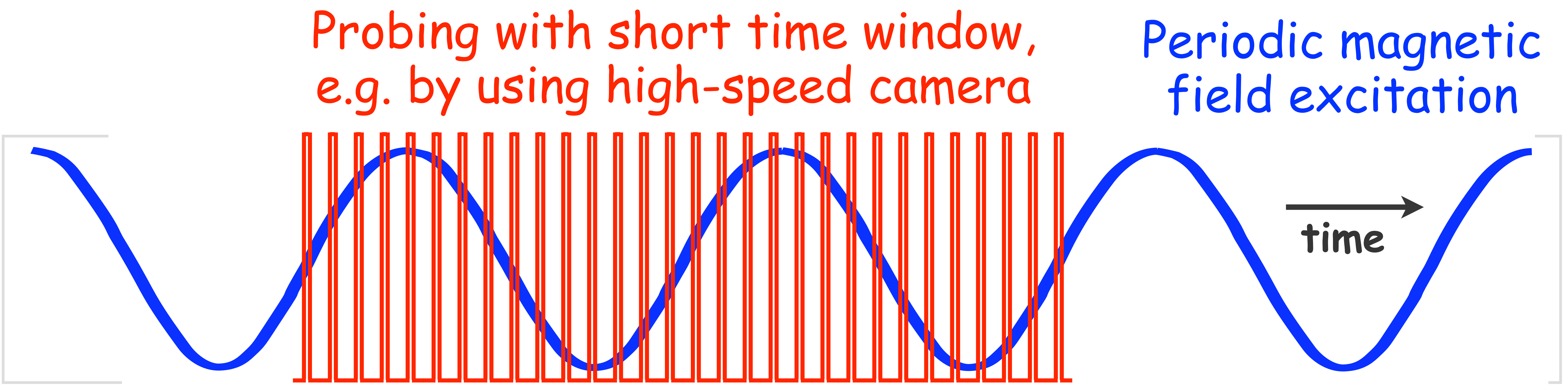
Time-resolved Kerr-microscopy

Single-shot mode



Time-resolved Kerr-microscopy

Single-shot mode



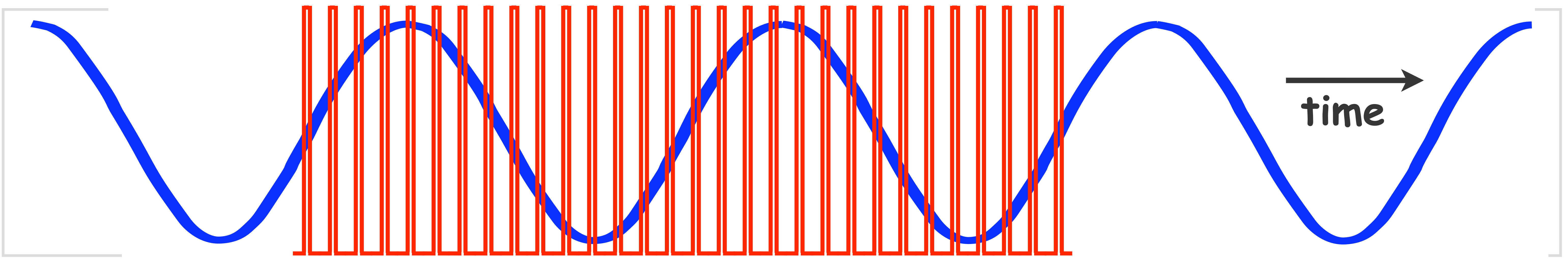
Requires **sufficient repetition rate** and **sufficient signal**

Time-resolved Kerr-microscopy

Single-shot mode

Probing with short time window, e.g. by using high-speed camera

Periodic magnetic field excitation



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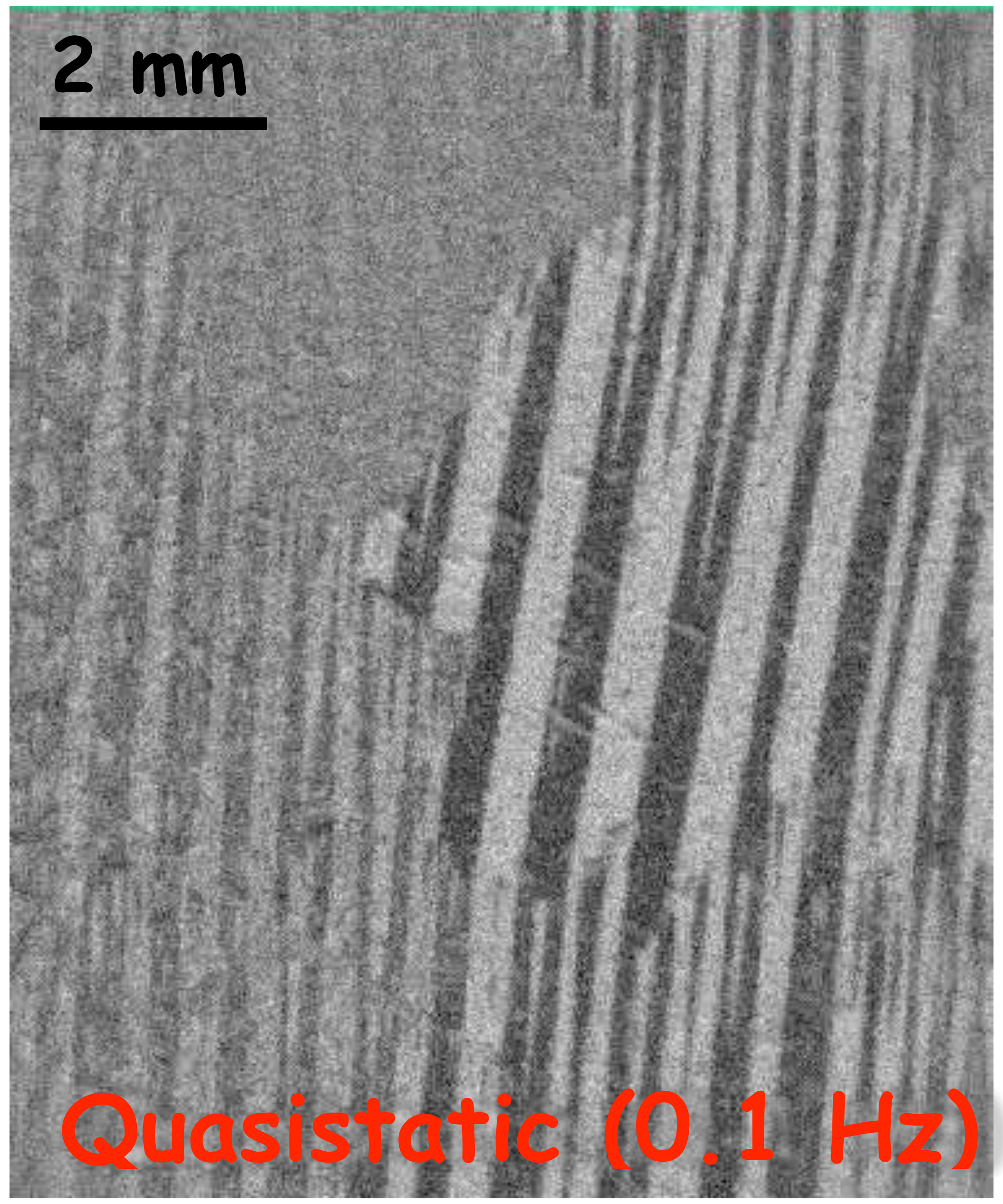
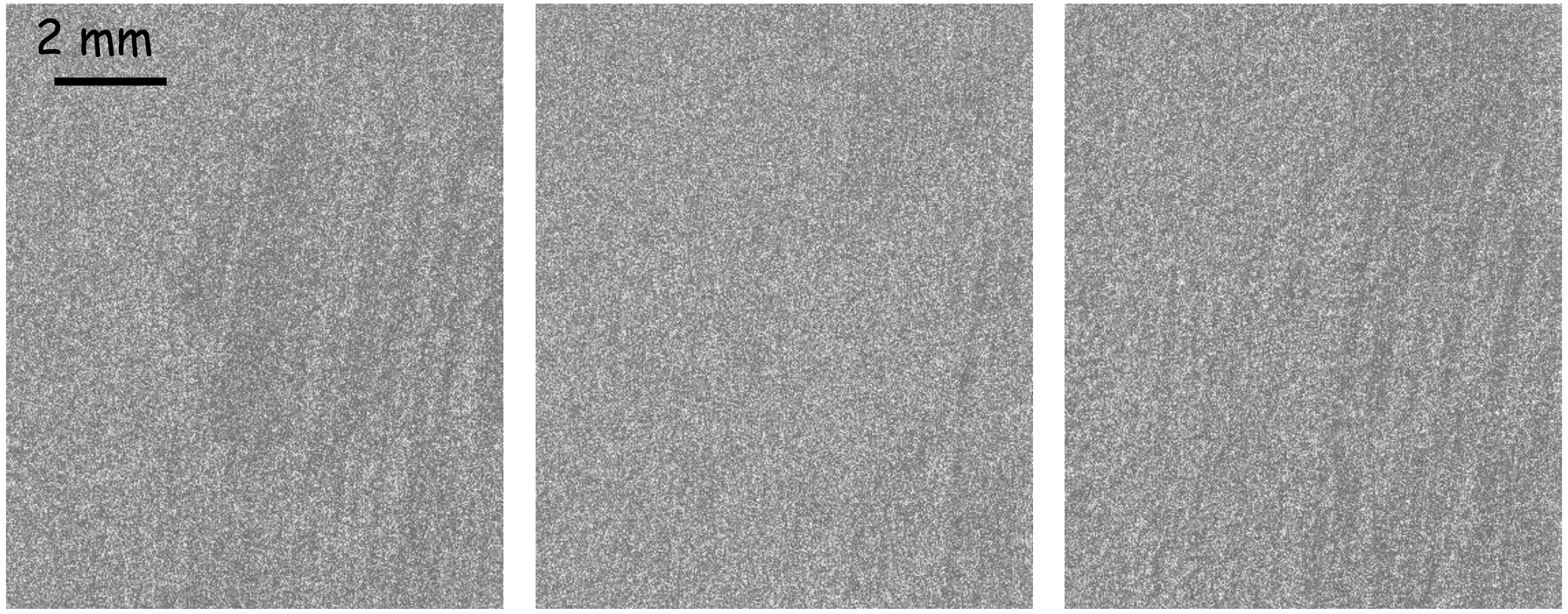
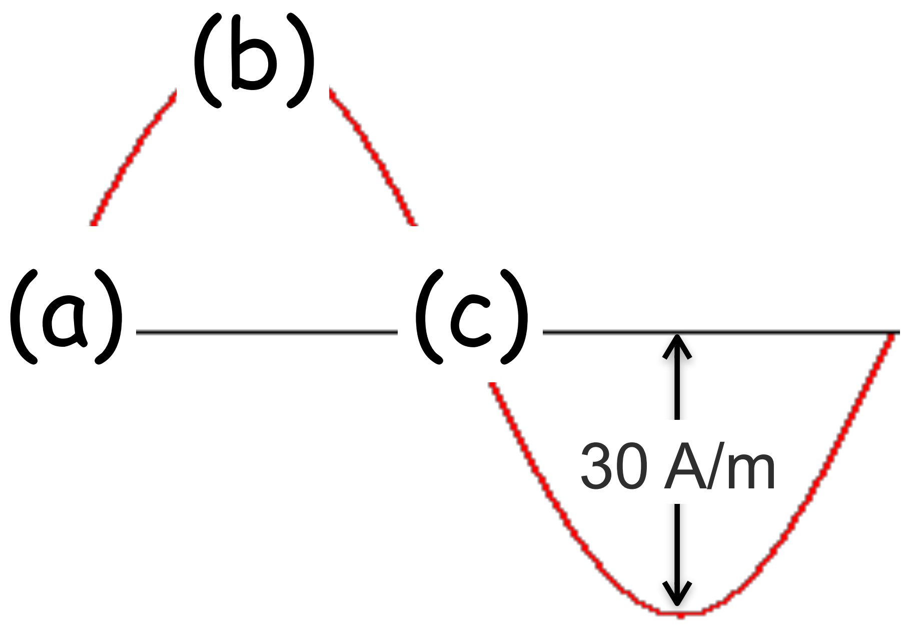




(a) (b) (c)

Single shot (50 Hz)

FeSi Goss sheet



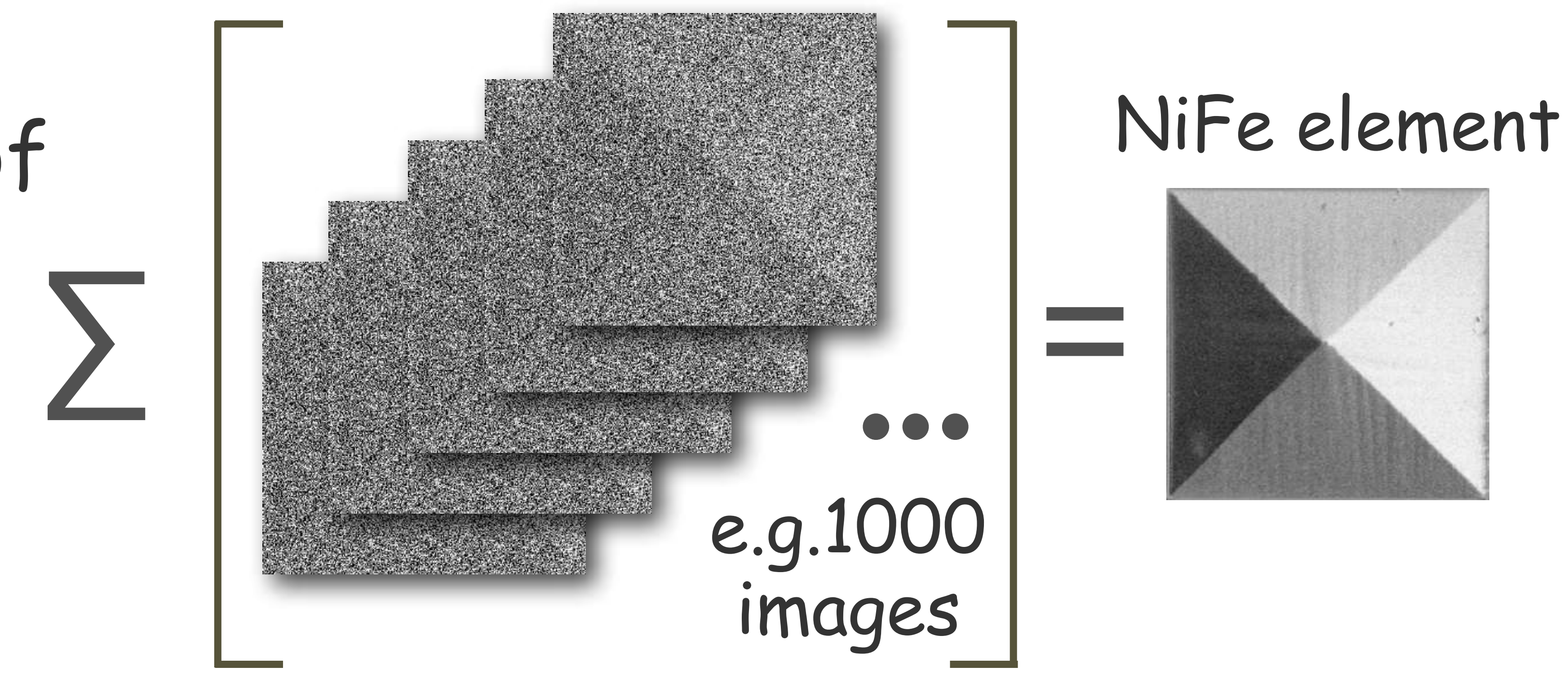
Time-resolved Kerr-microscopy

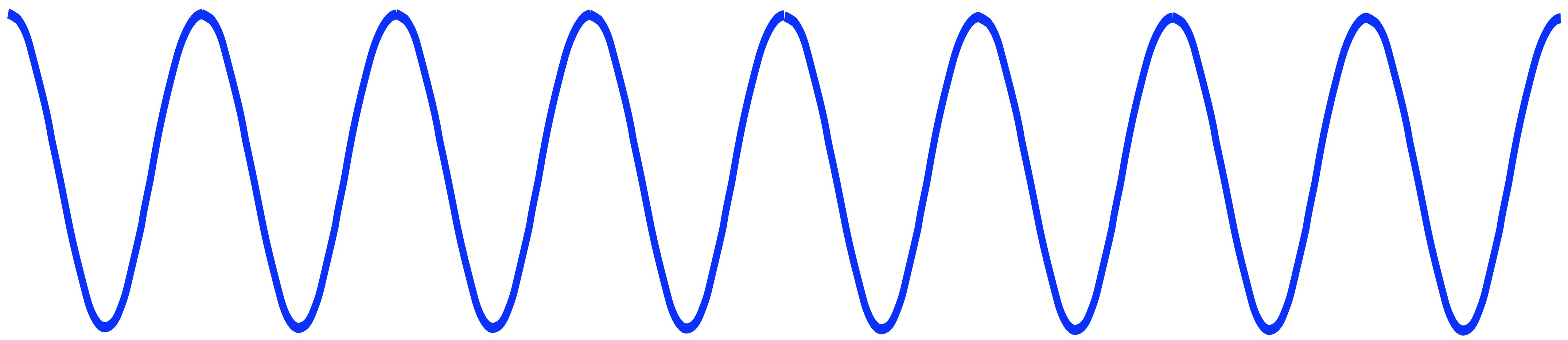
Stroboscopic mode

If signal and repetition rate are limited:

→ no single-shot imaging possible

→ accumulation of large number of independent events necessary (at fixed time delay)





Periodic magnetic field excitation

Time-resolved Kerr-microscopy

Stroboscopic mode

If signal and repetition rate are limited:

→ no single-shot imaging possible

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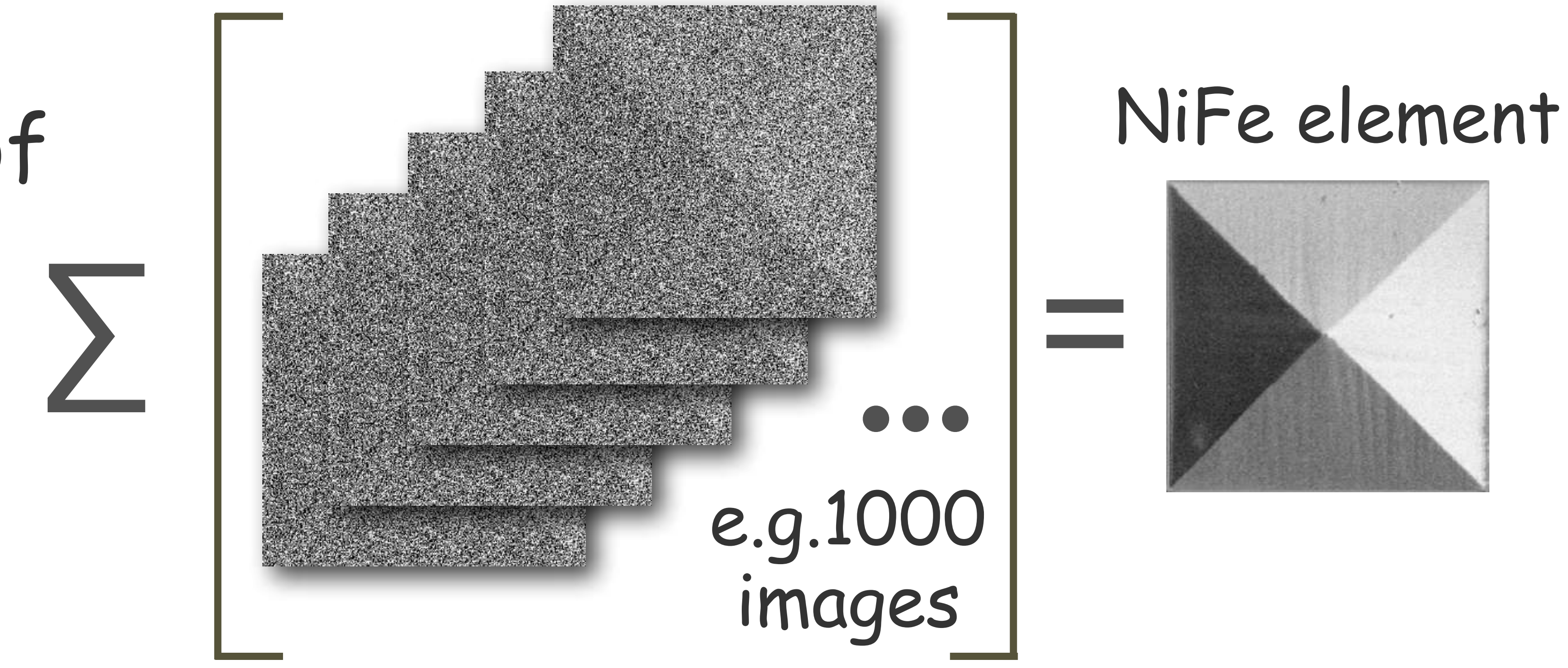
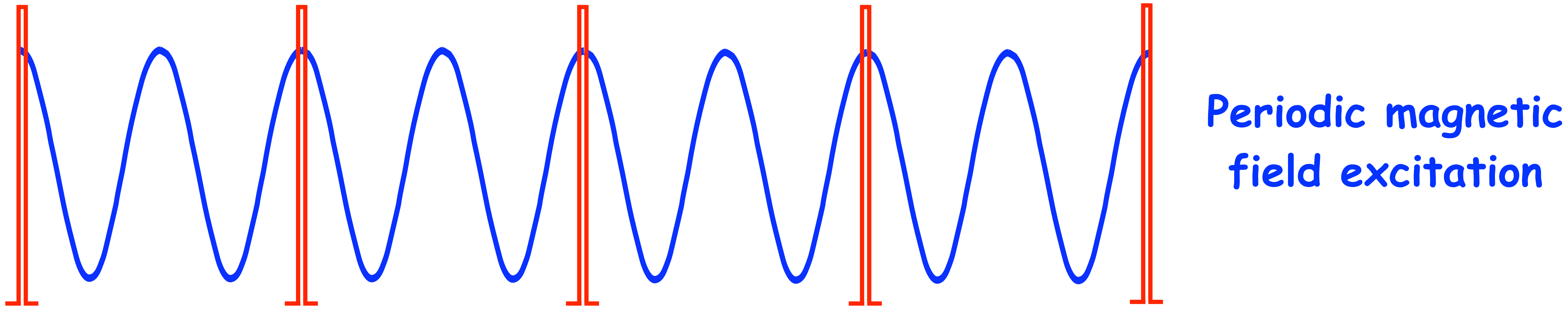


Probing with defined time-delay



Time-resolved Kerr-microscopy

Stroboscopic mode

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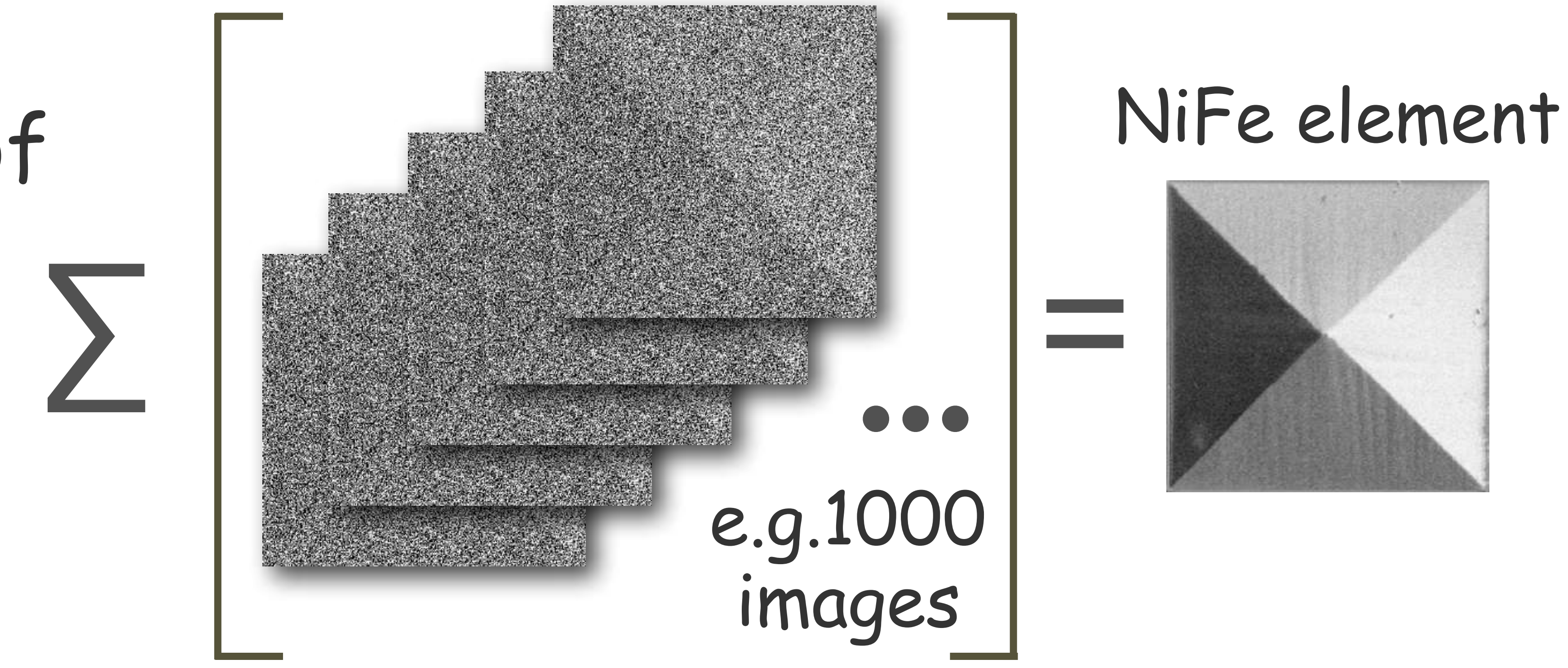

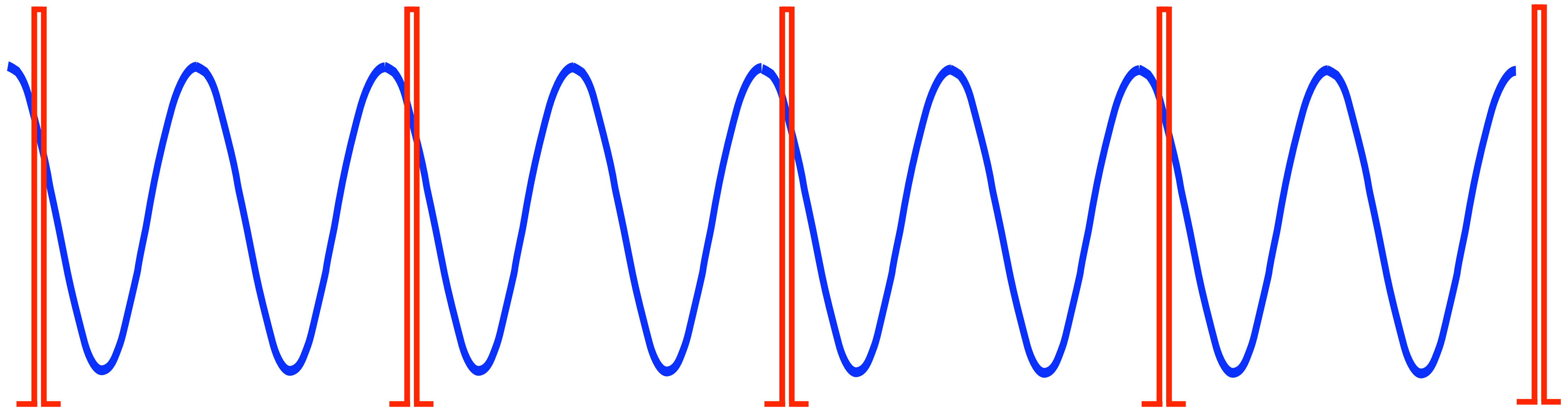

Time-resolved Kerr-microscopy

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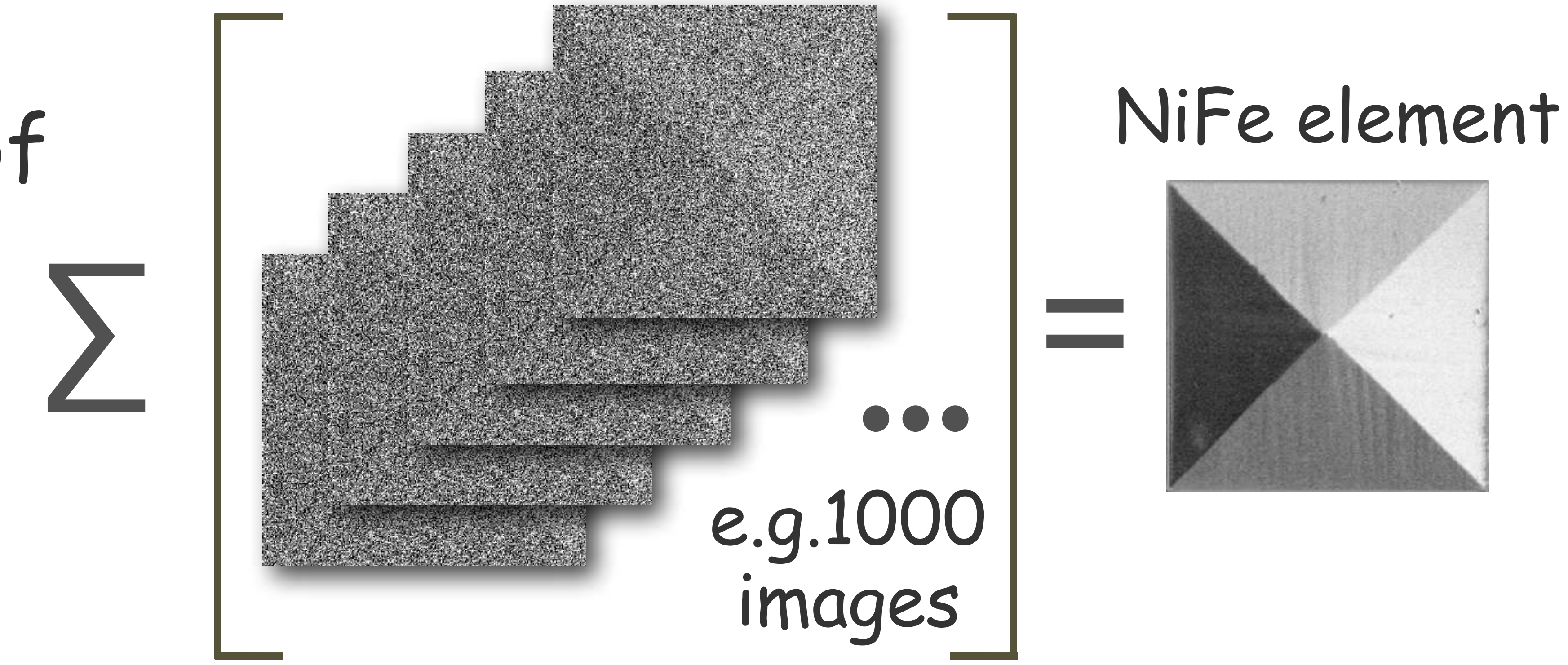
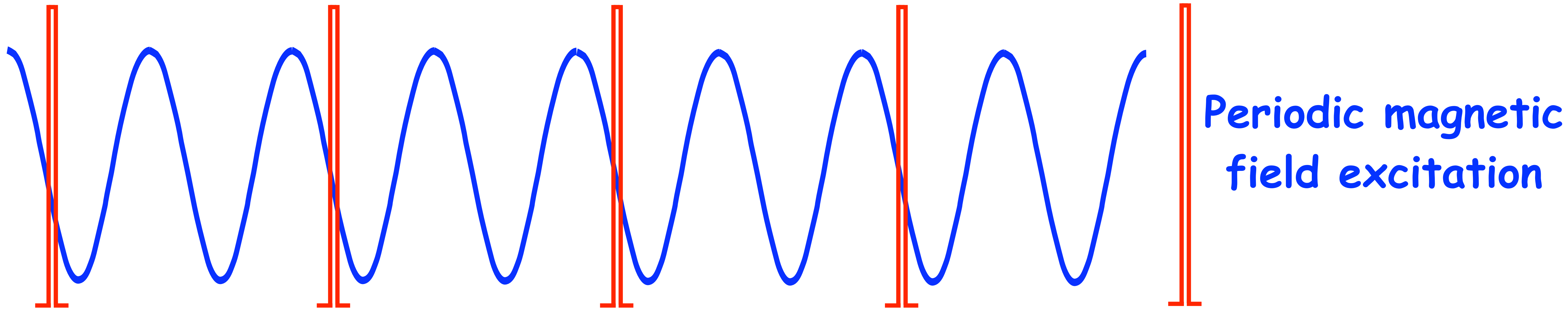
→ no single-shot imaging possible

→ accumulation of large number of independent events necessary (at fixed time delay)



Probing with defined time-delay



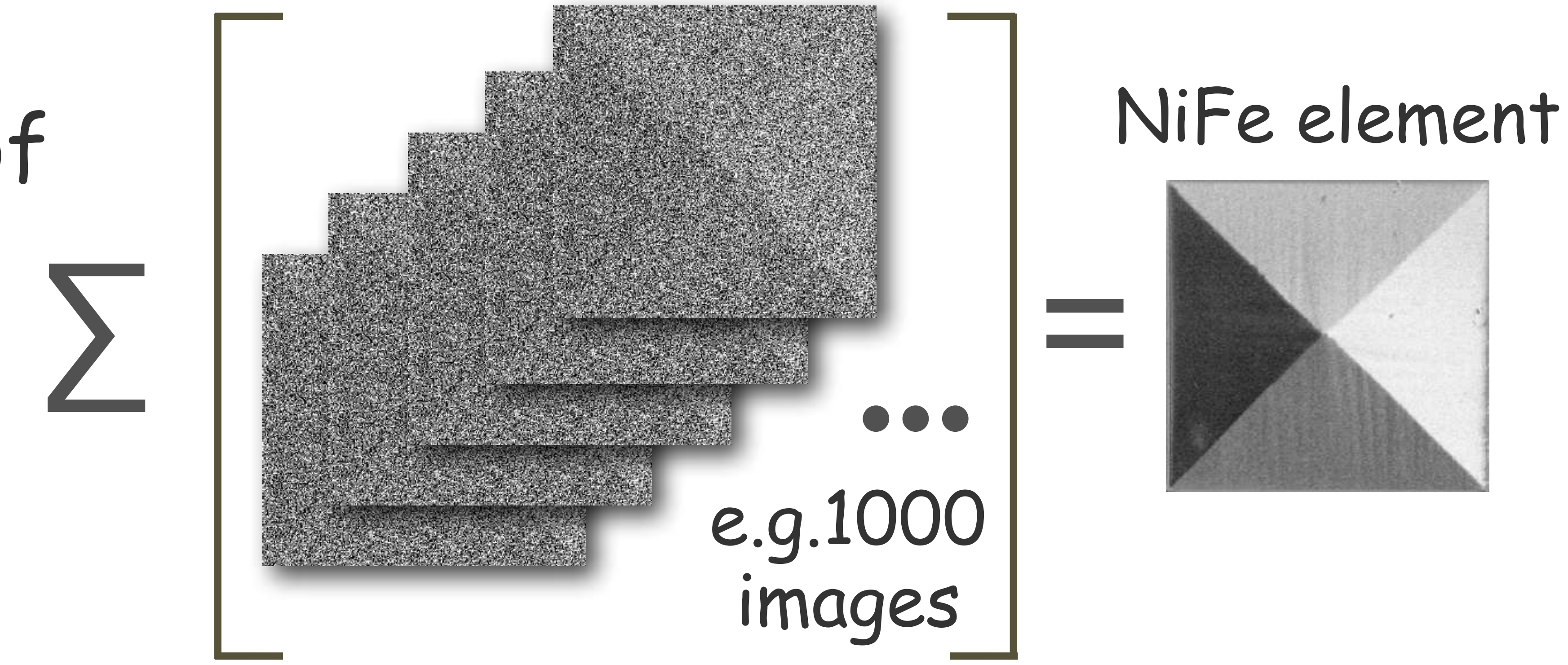
Time-resolved Kerr-microscopy

Stroboscopic mode

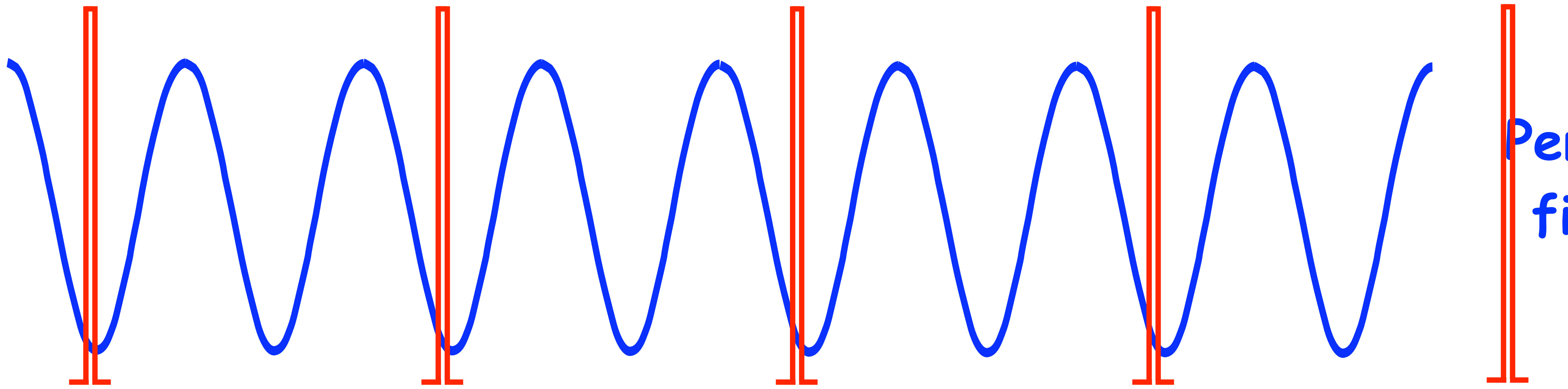
If signal and repetition rate are limited:

→ no single-shot imaging possible

→ accumulation of large number of independent events necessary (at fixed time delay)



Probing with defined time-delay



Time-resolved Kerr-microscopy

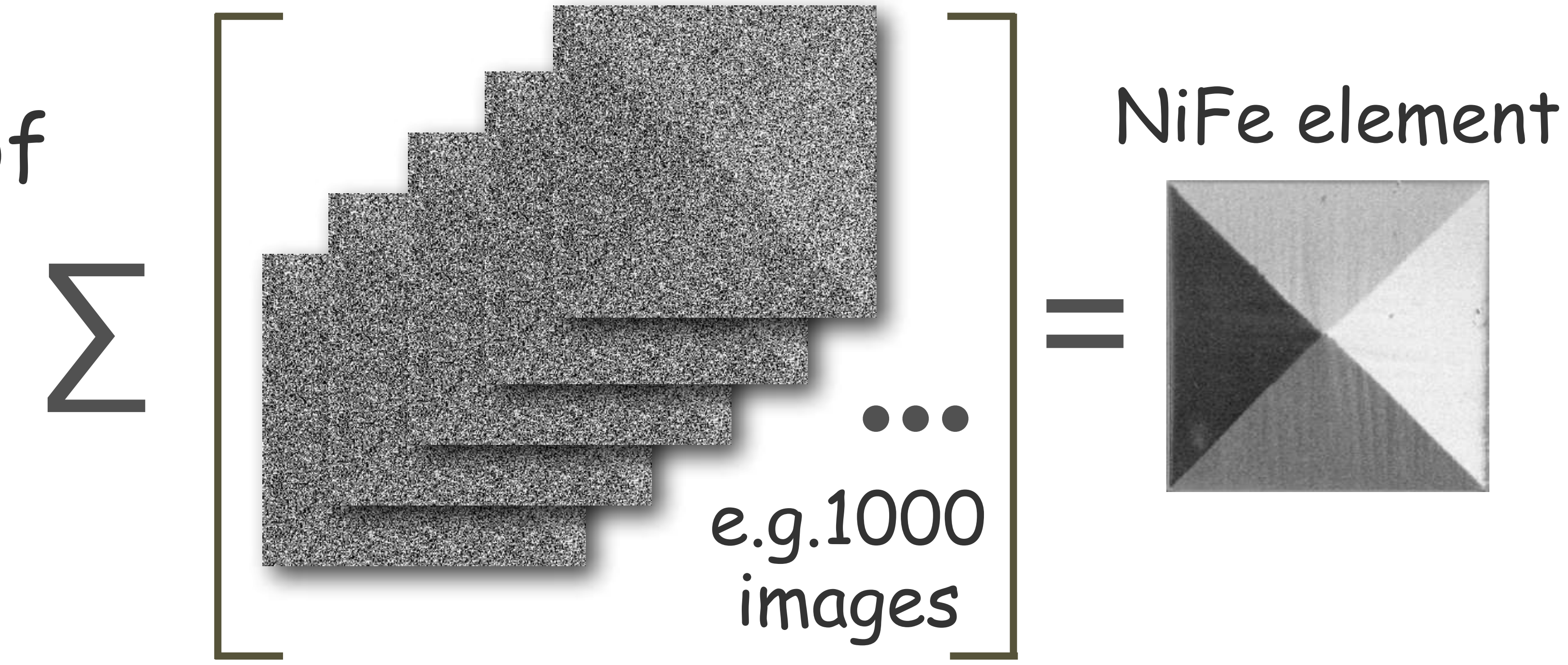
Stroboscopic mode

If signal and repetition rate are limited:

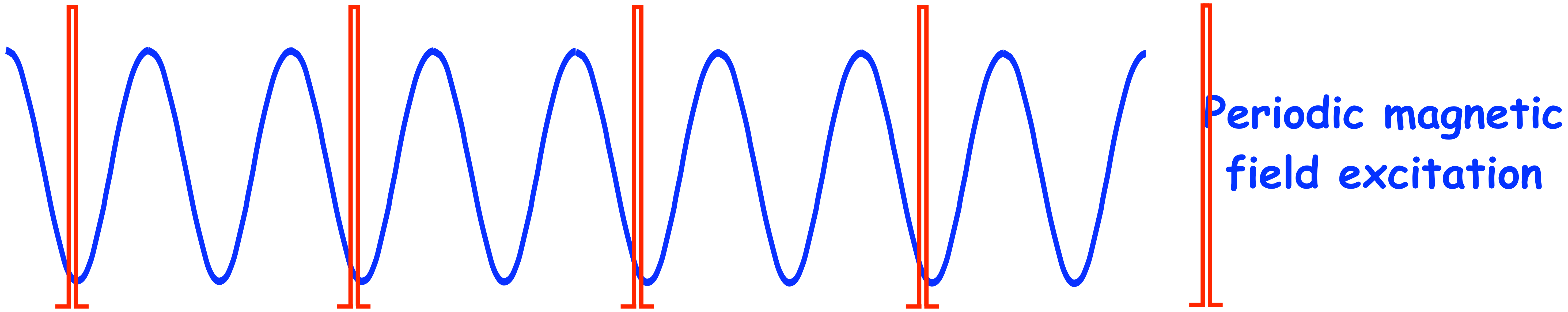
→ no single-shot imaging possible

→ accumulation of large number of independent events necessary (at fixed time delay)

→ Requires **repetitive process**



Probing with defined time-delay



Time-resolved Kerr-microscopy

Stroboscopic mode

If signal and repetition rate are limited:

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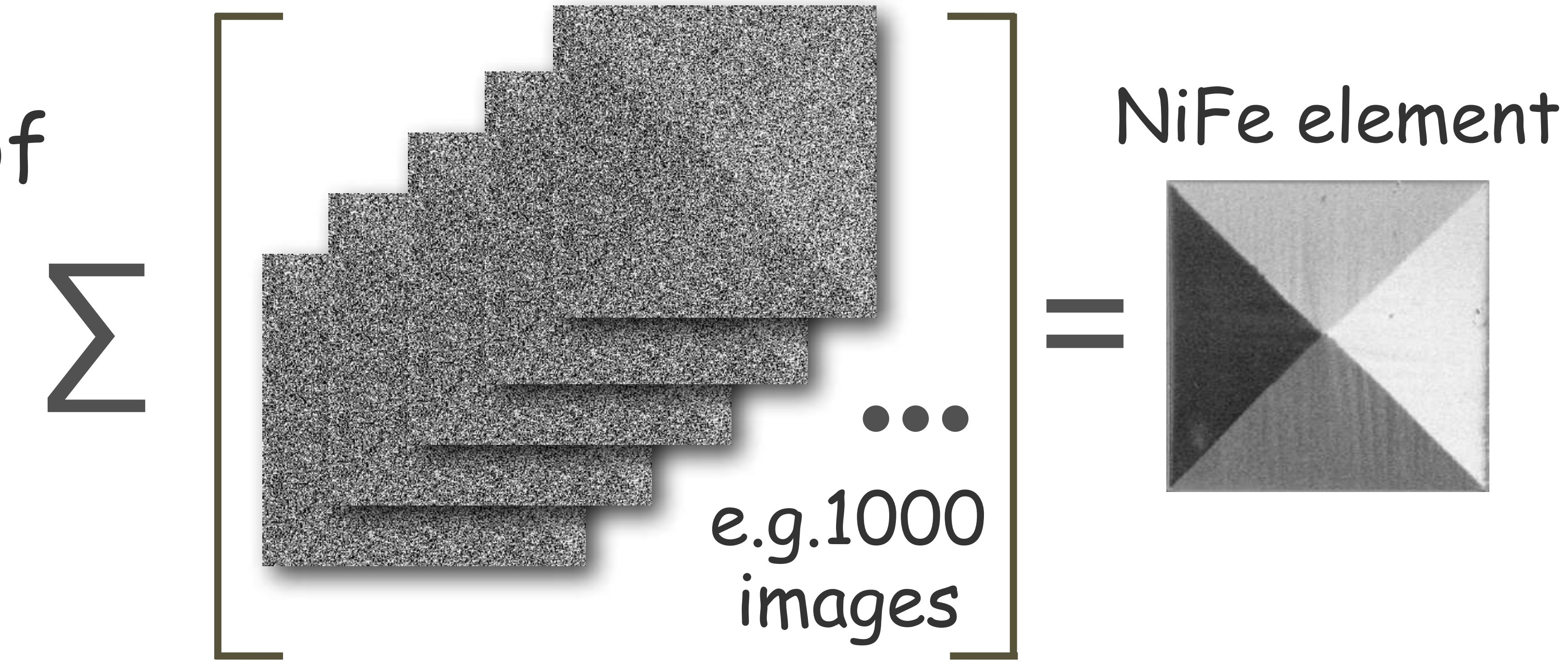
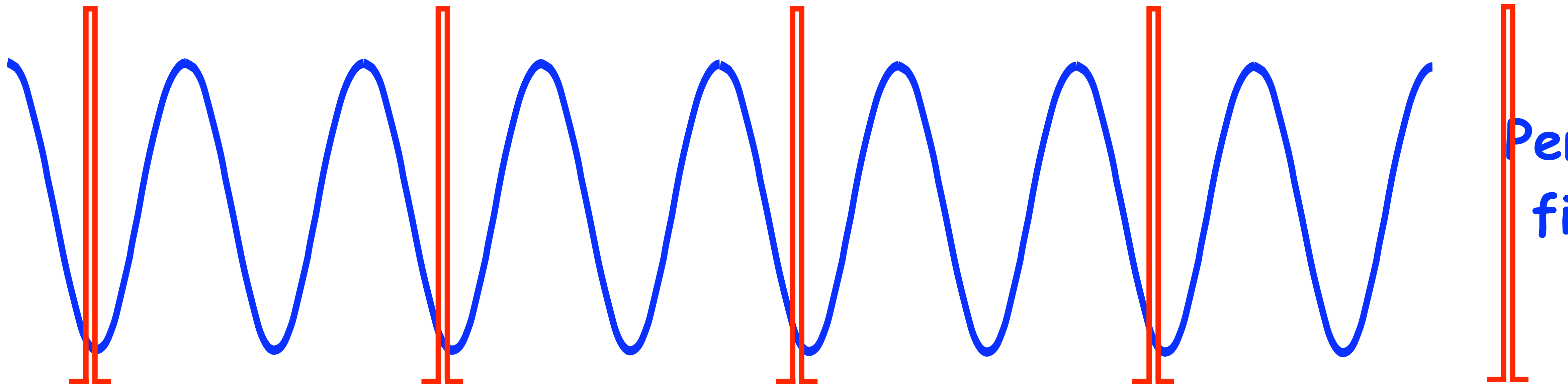
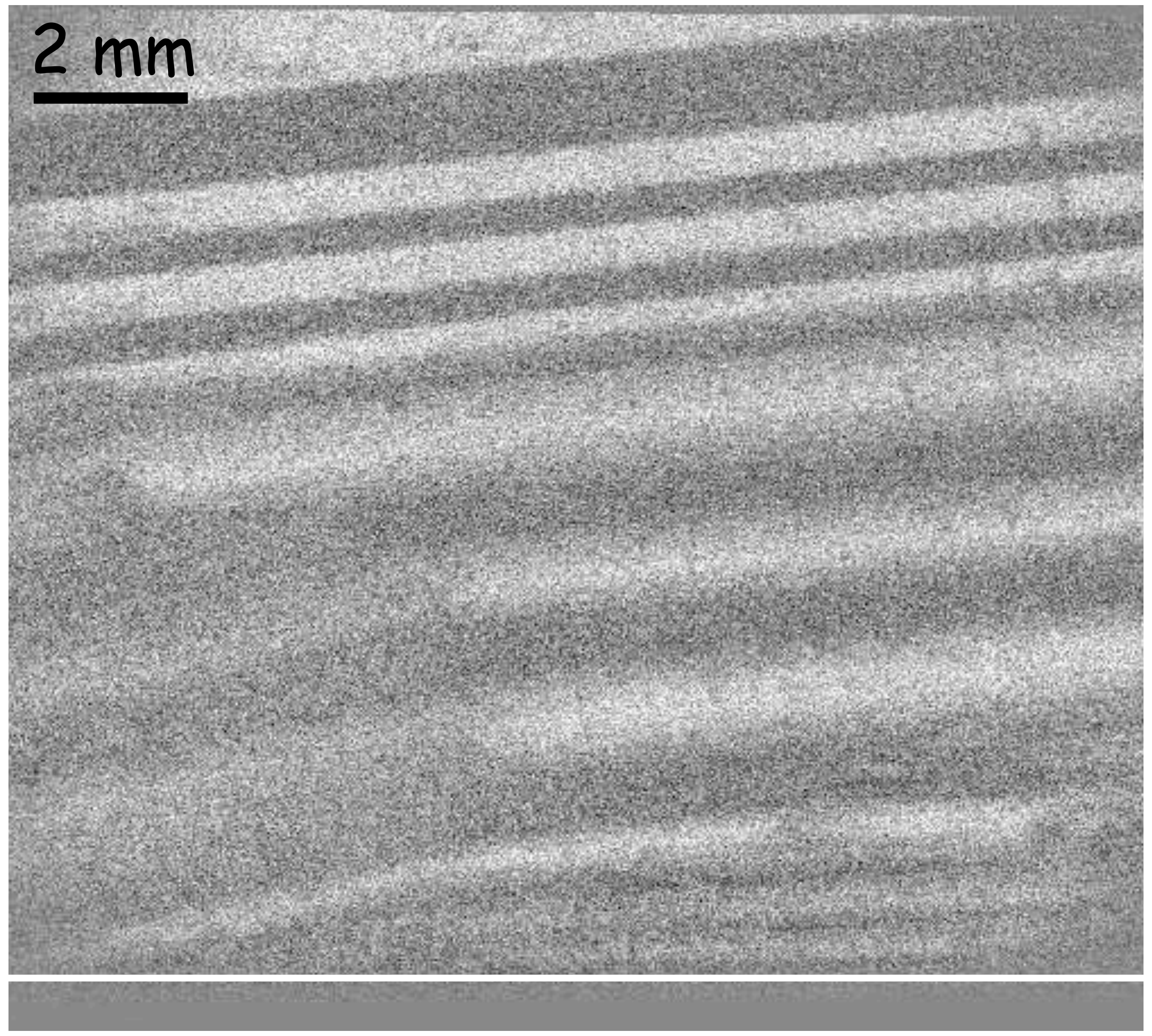


Probing with defined time-delay

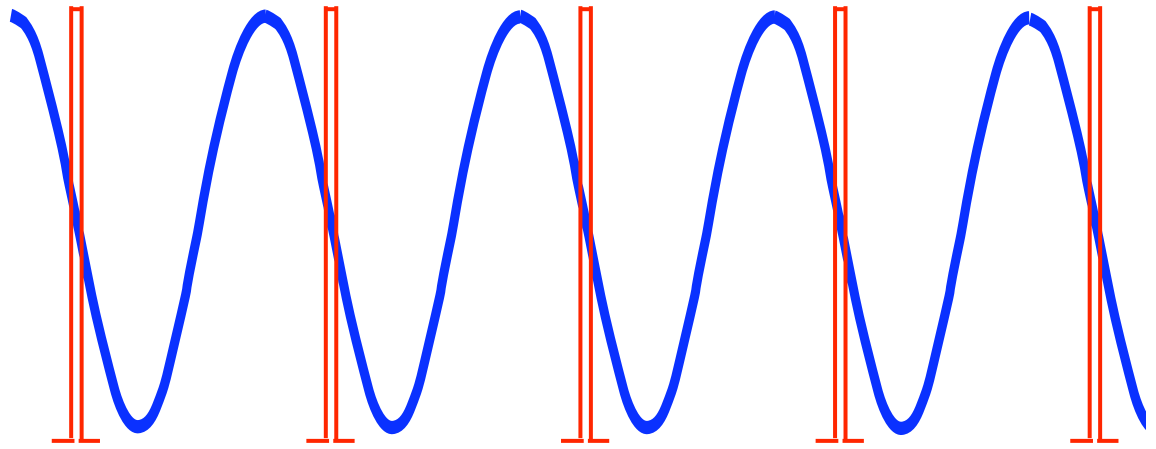


Periodic magnetic field excitation

FeSi Goss sheet



Sheet edge



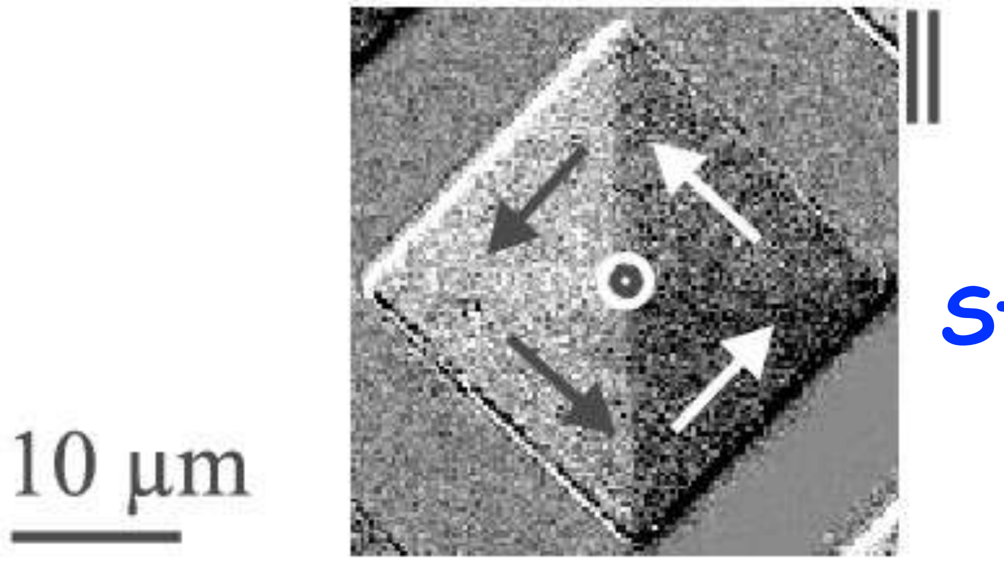
50 Hz excitation

Probing at 0° delay, averaging of 100 images

Time-resolved Kerr-microscopy

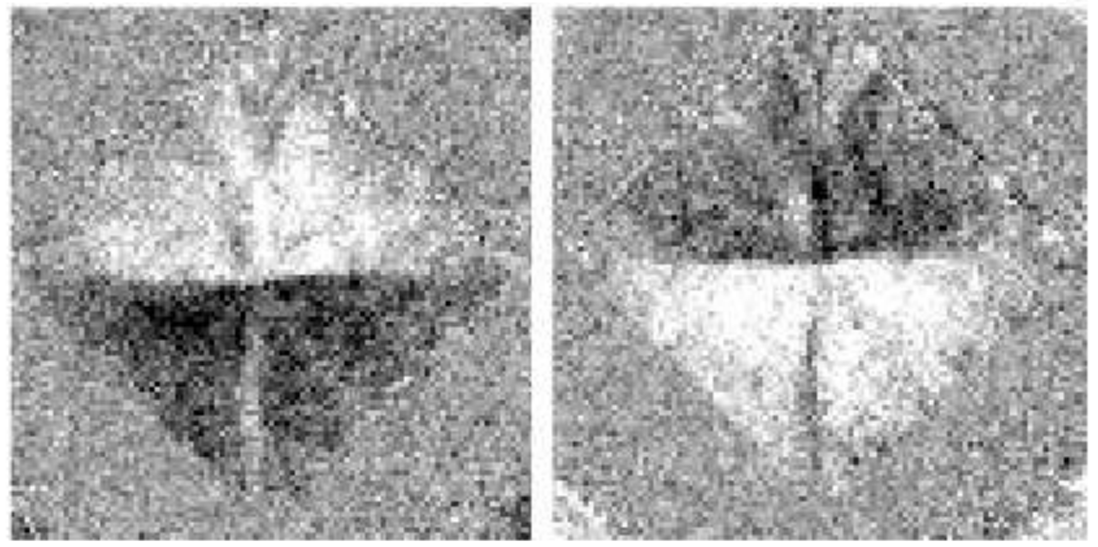
Stroboscopic mode

Spin waves in NiFe square, stroboscopically imaged

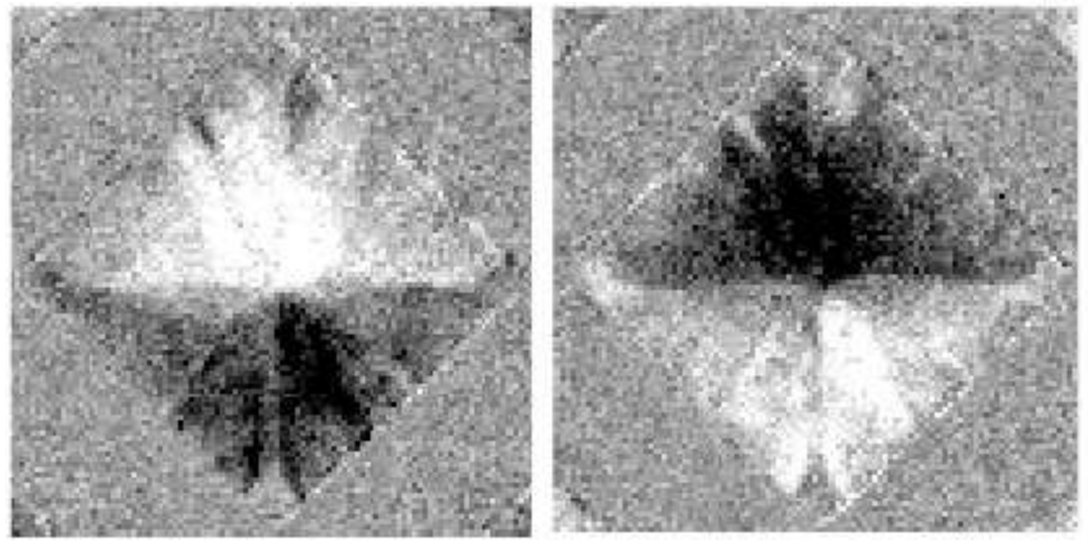


Static

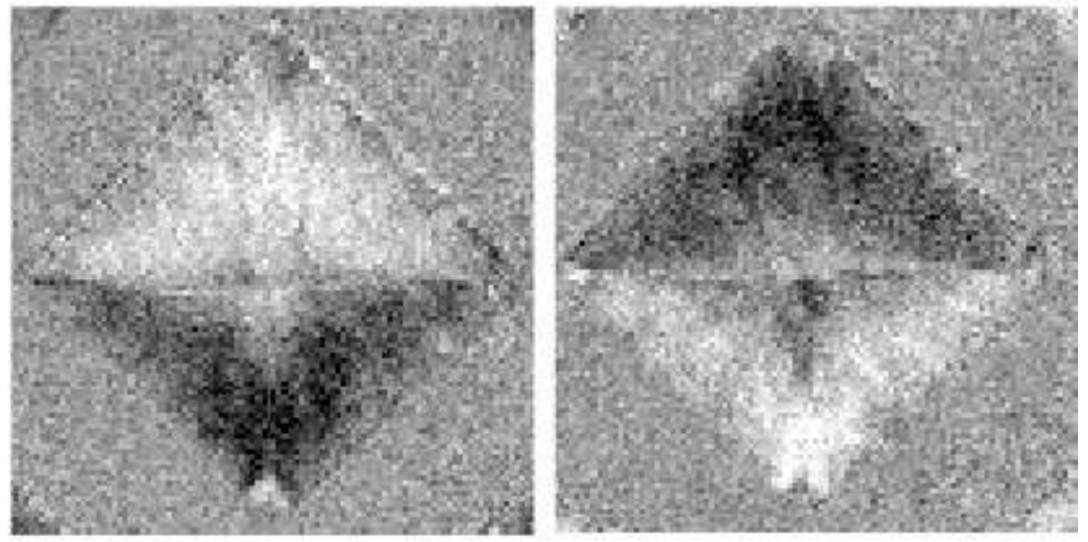
1 GHz



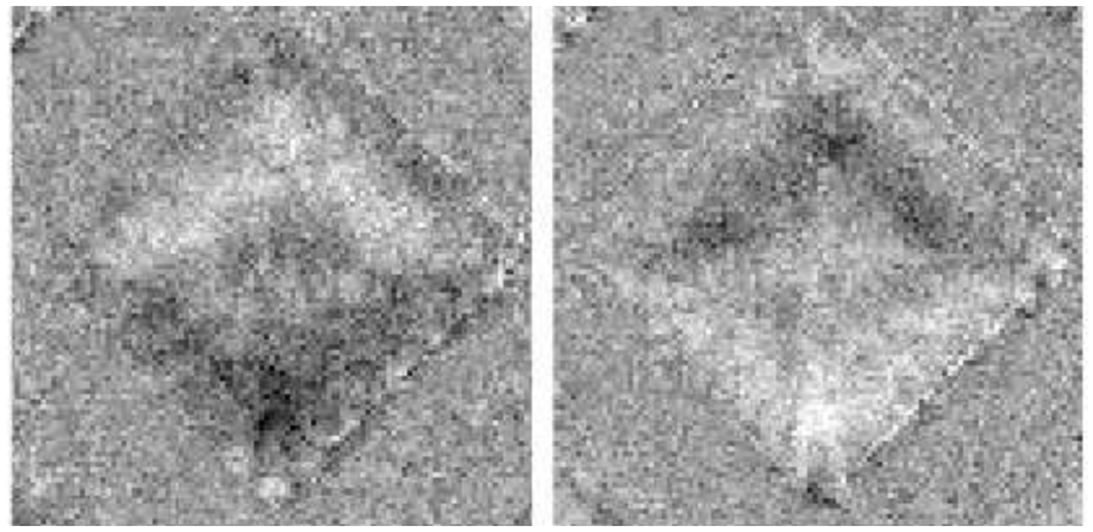
2 GHz



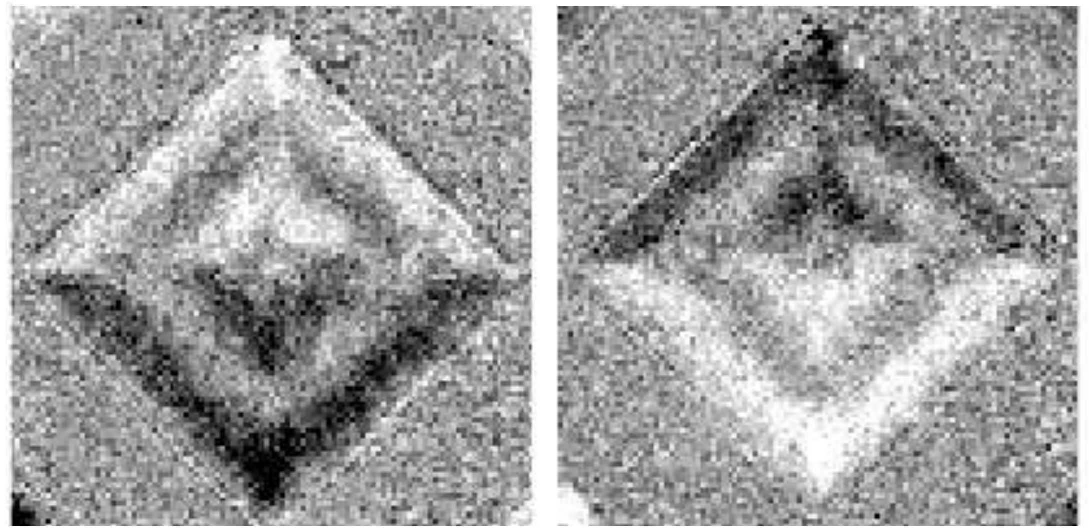
2.5 GHz



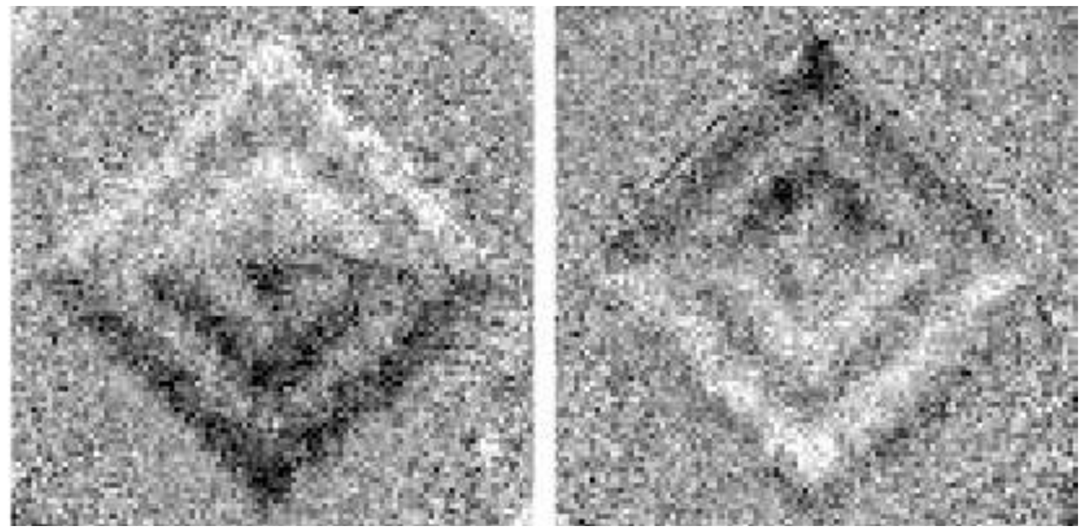
3 GHz



3.5 GHz



4 GHz

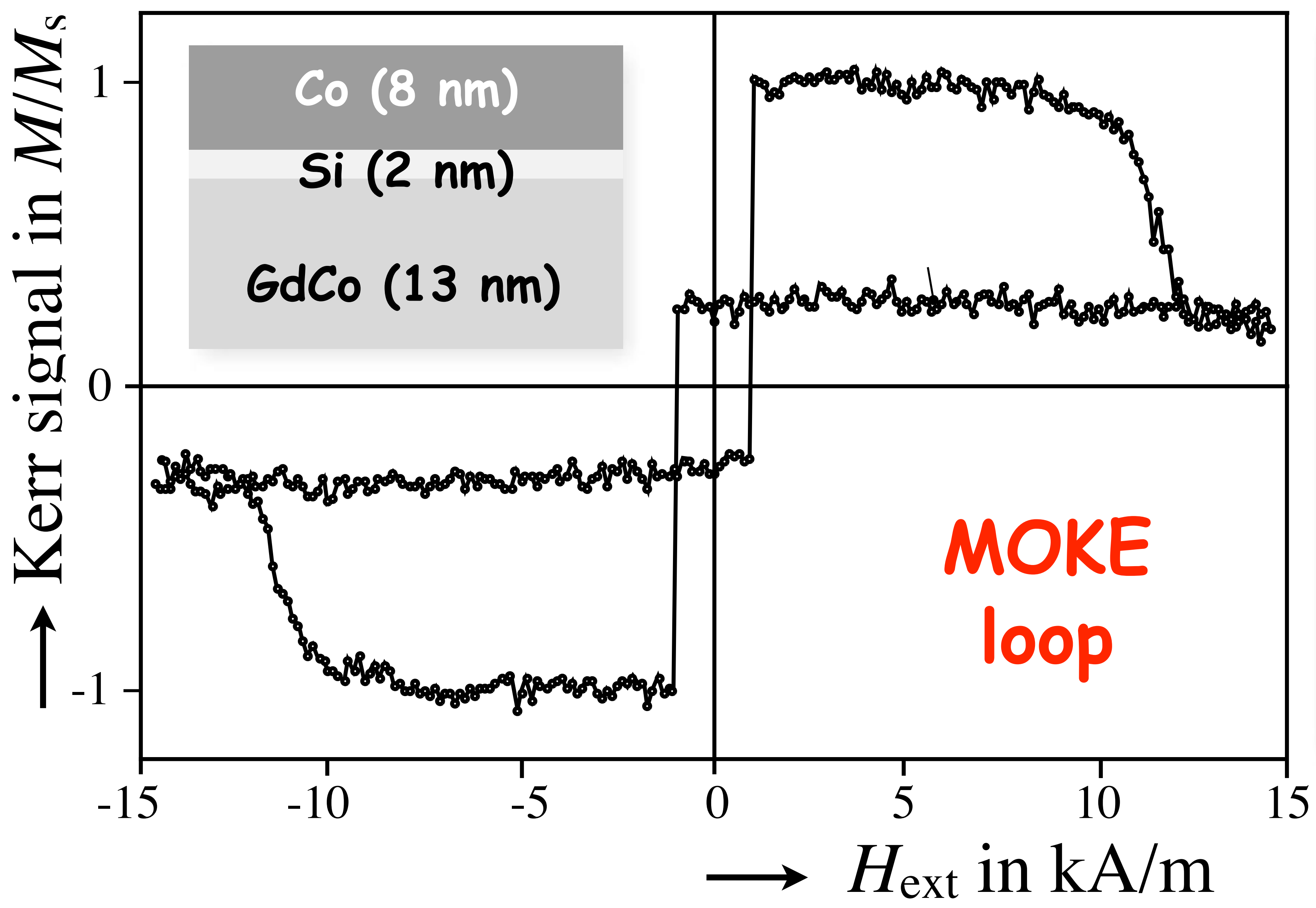


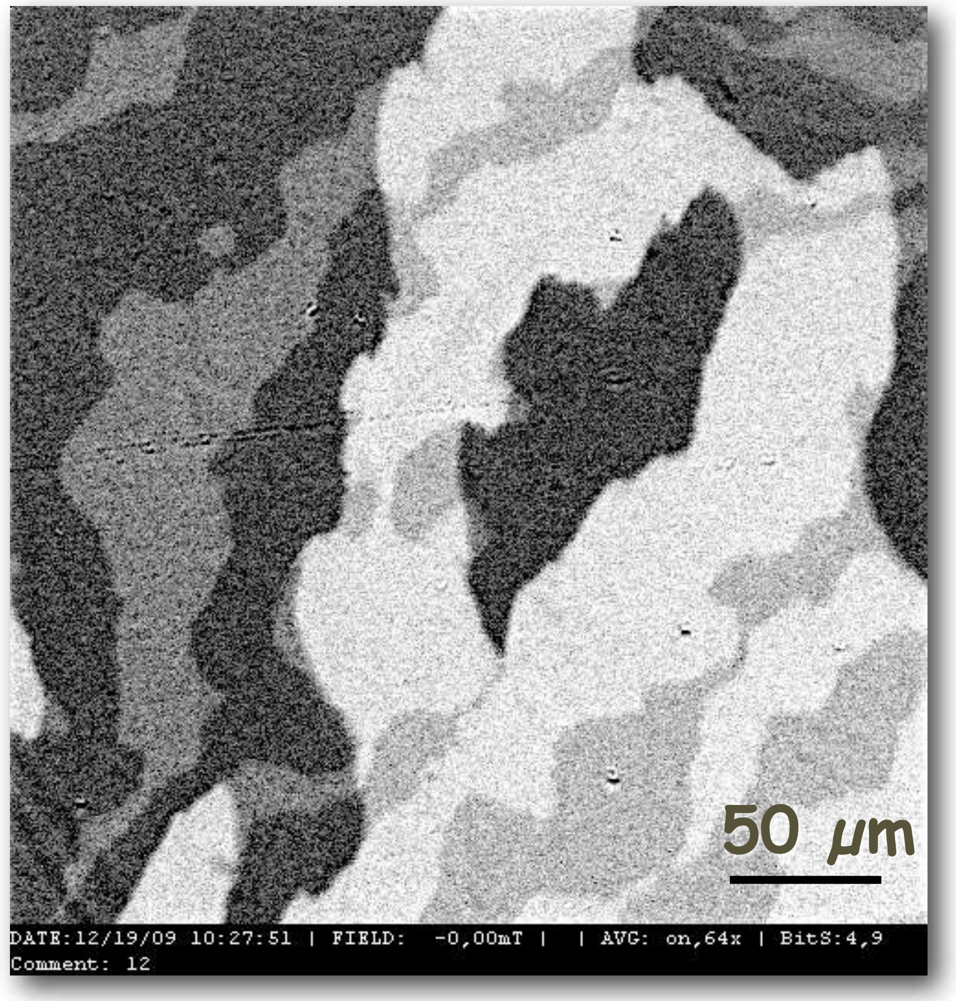
From: N.O.Urs et al., AIP Advances 6, 055605 (2016)

Depth selective Kerr-microscopy

Domain analysis in multilayers

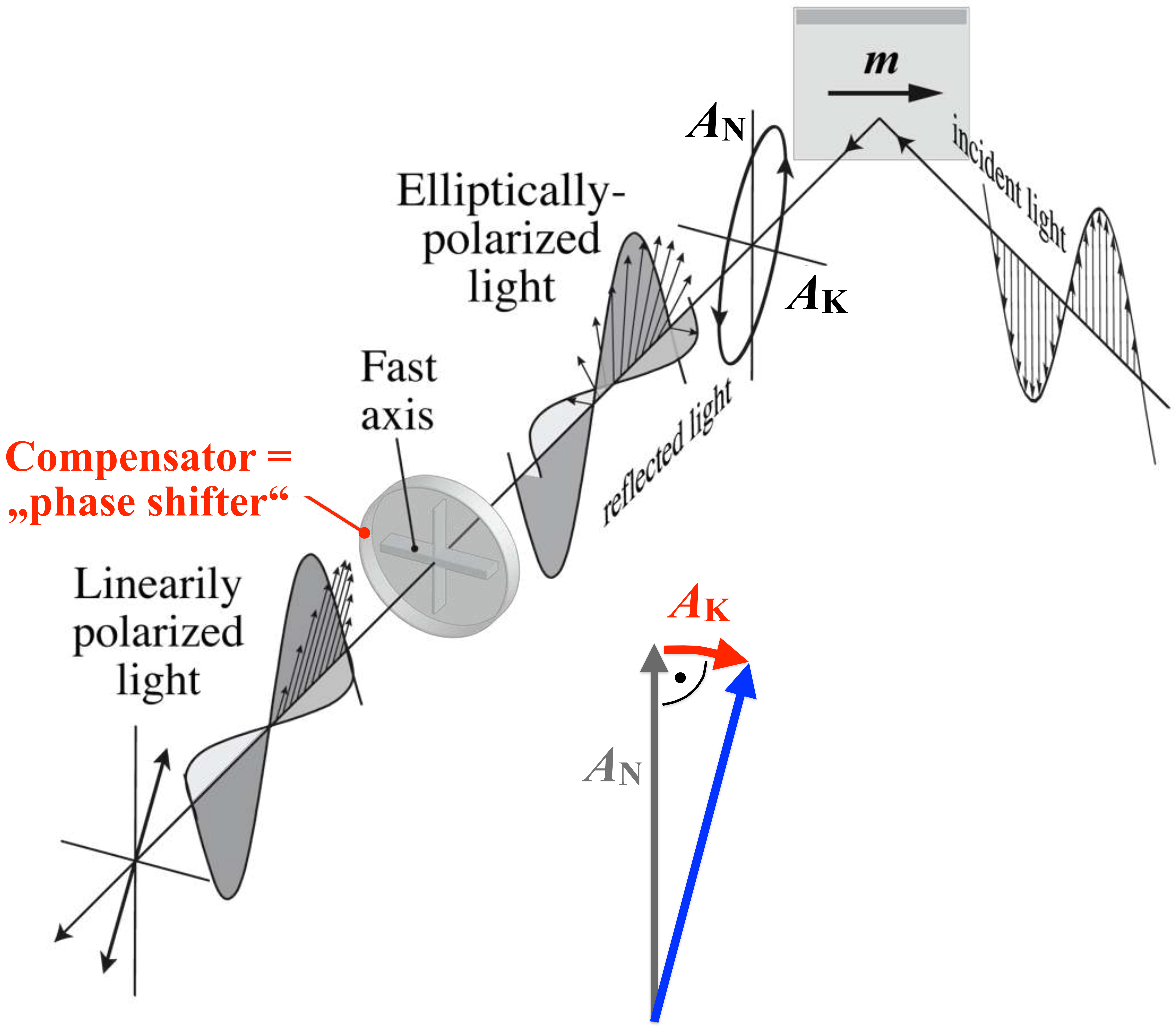
Co/Si/GdCo trilayer



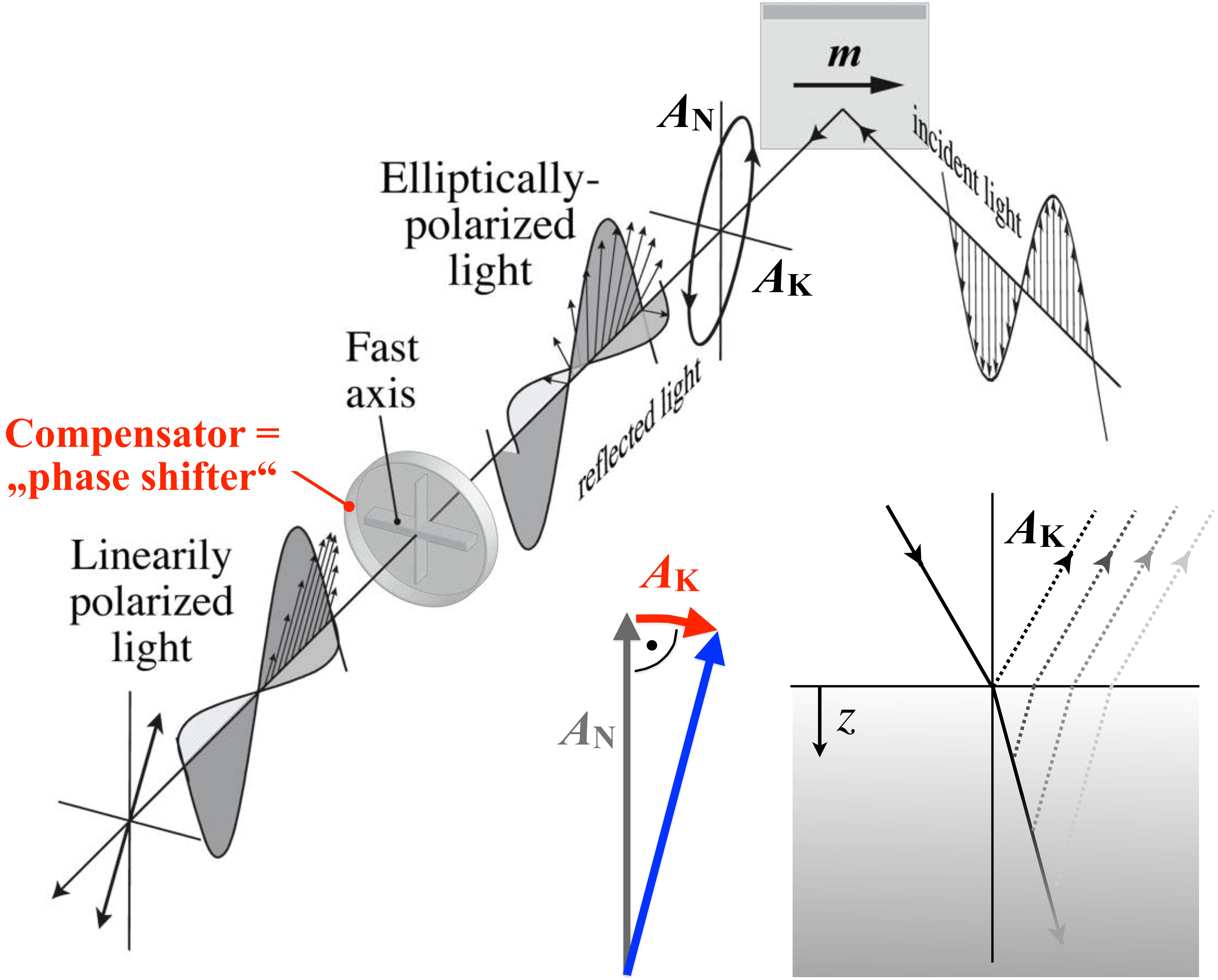


Together with A. Svalov and G. Kurlyandskaya, Ekaterinburg
A. Svalov et al., J. Alloys and Compounds 615 (2014)

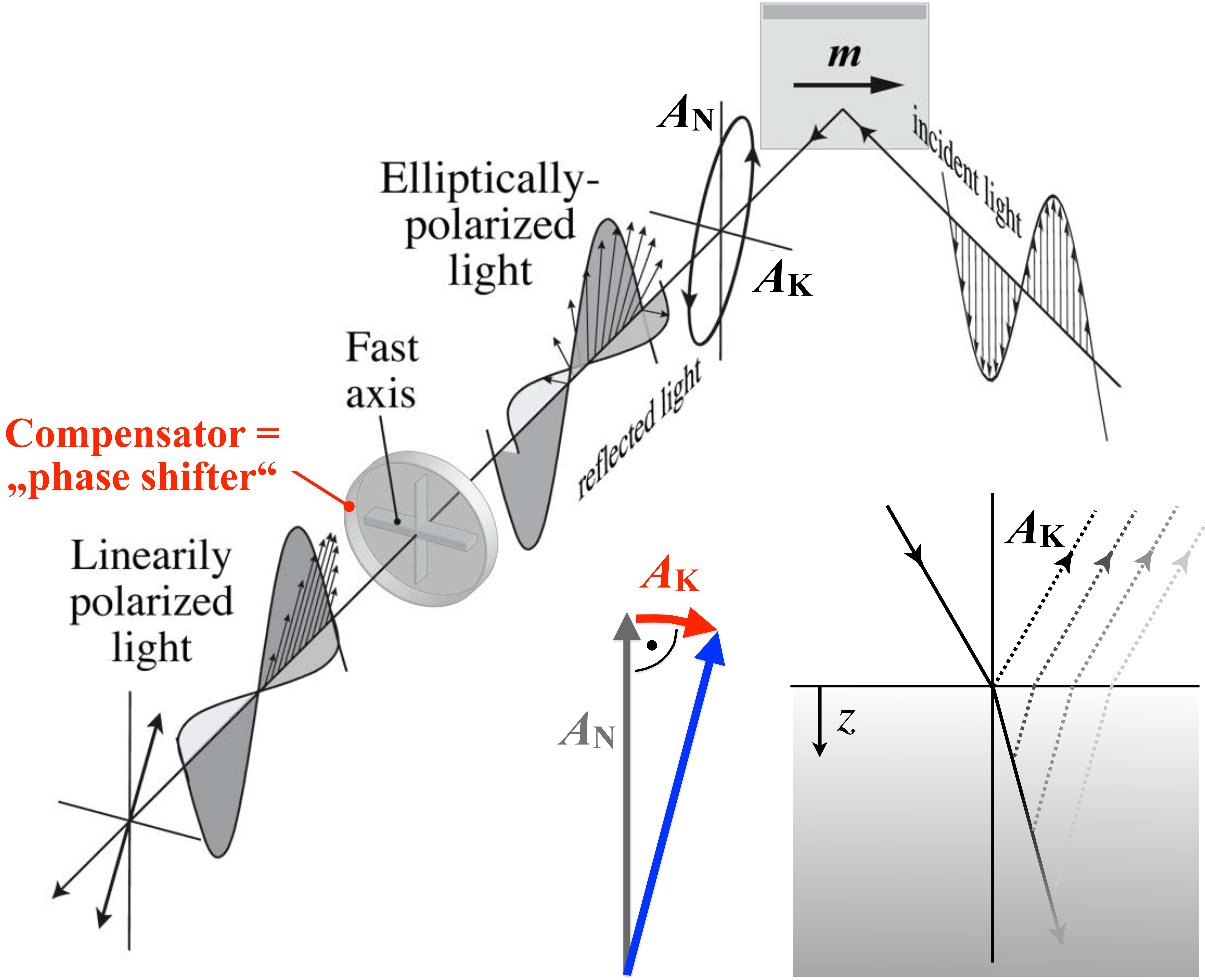
Depth selective Kerr-microscopy



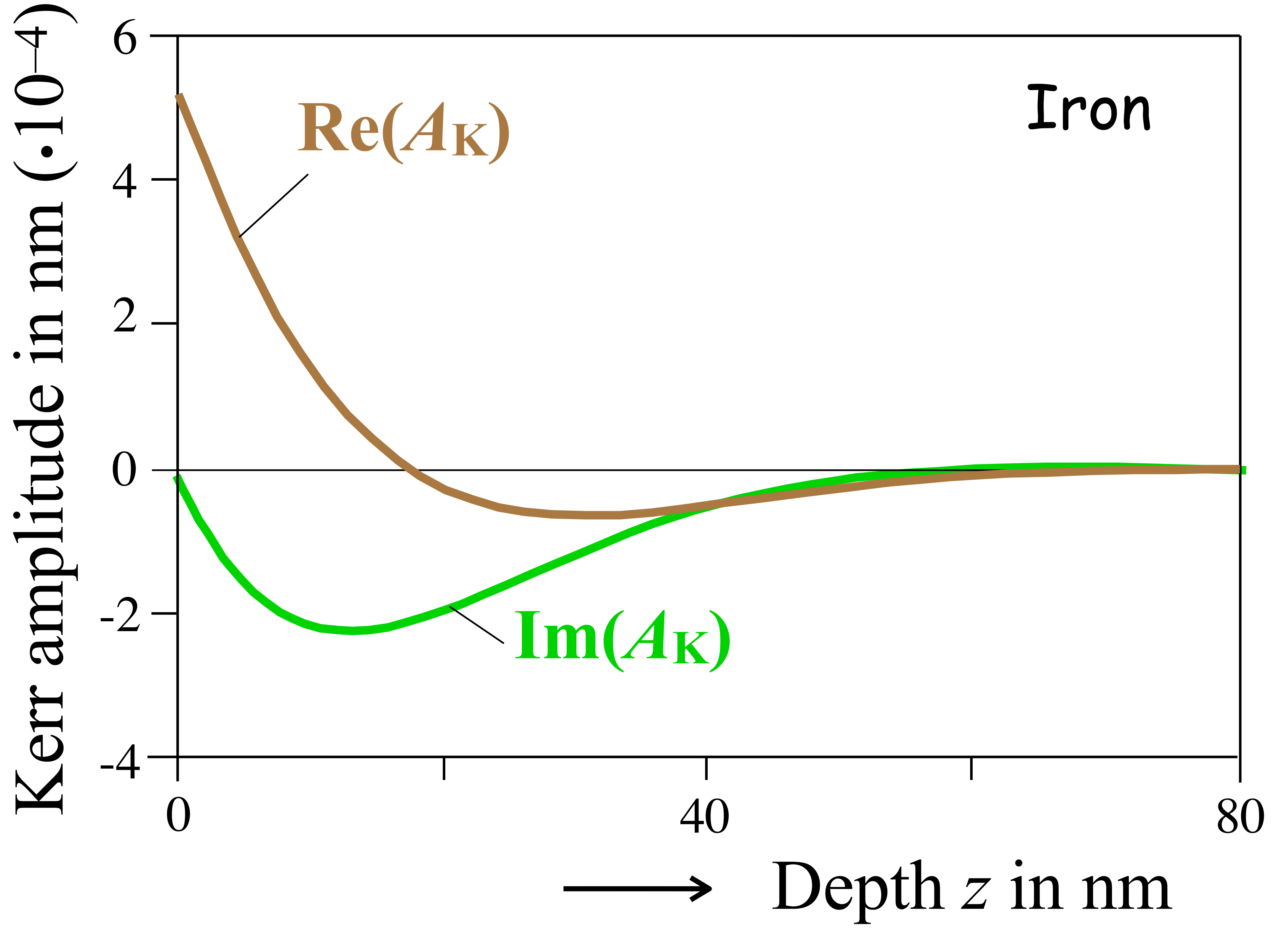
Depth selective Kerr-microscopy



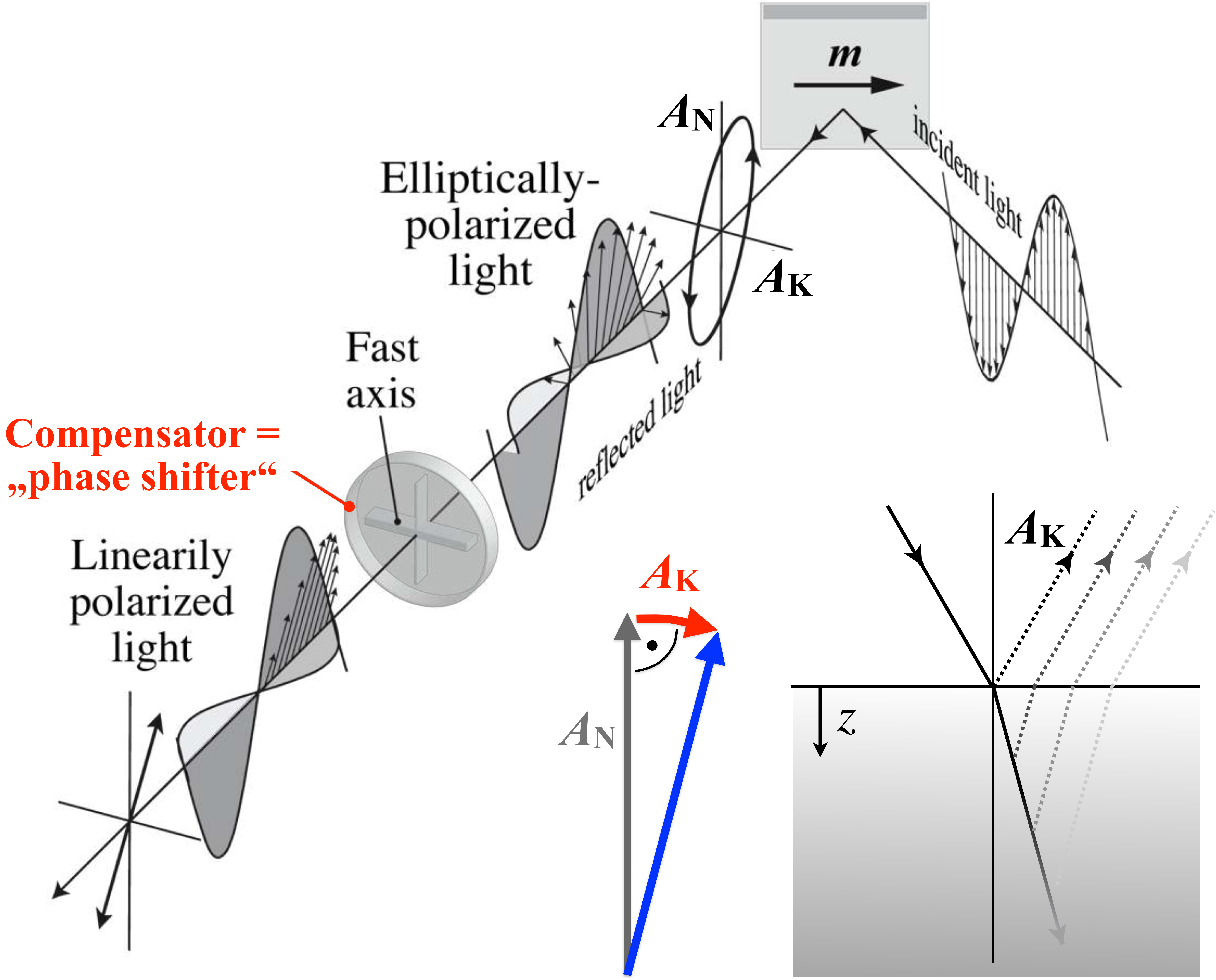
Depth selective Kerr-microscopy



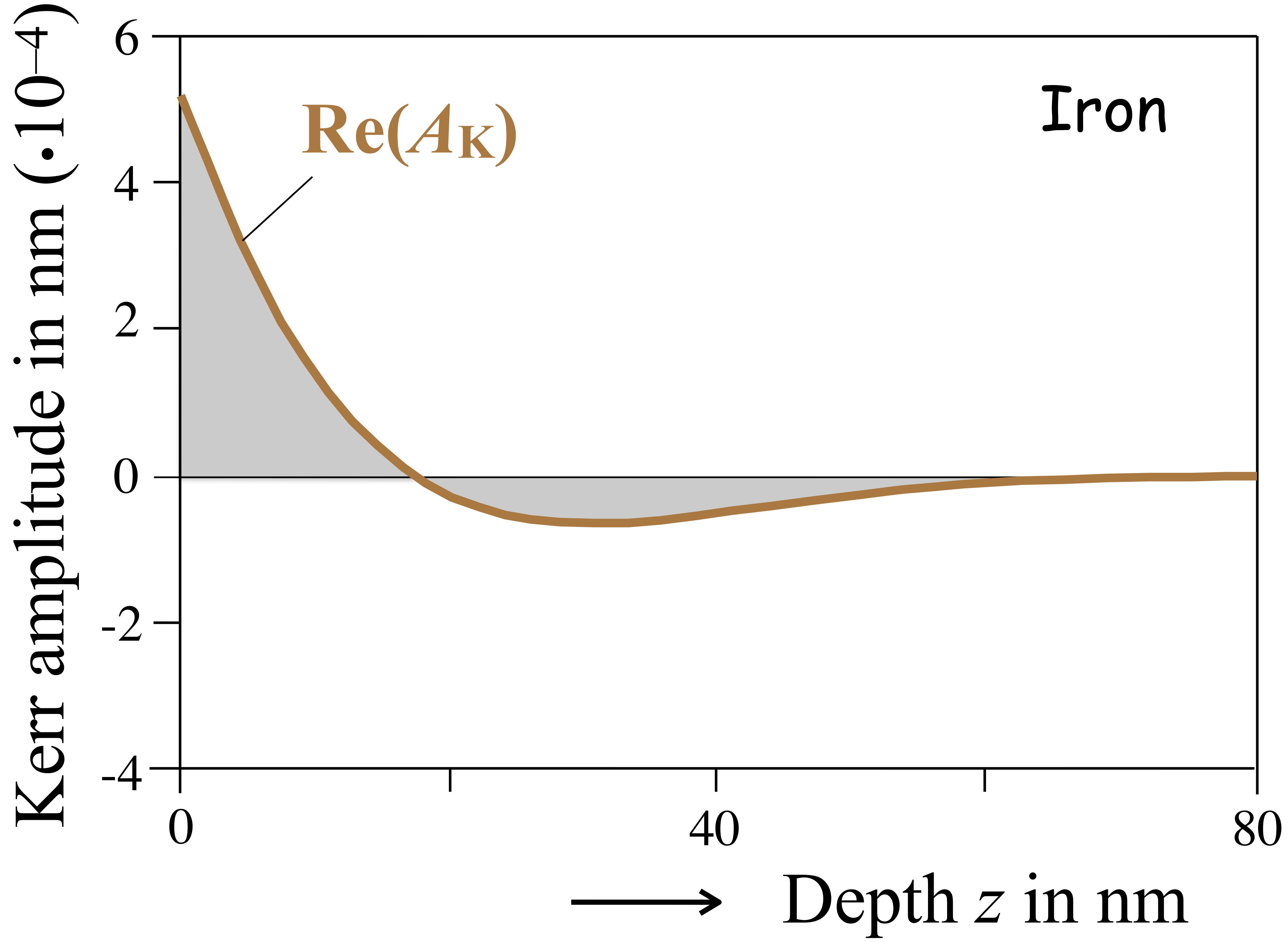
Hubert, Kambersky, Träger, Wenzel in 1990ies



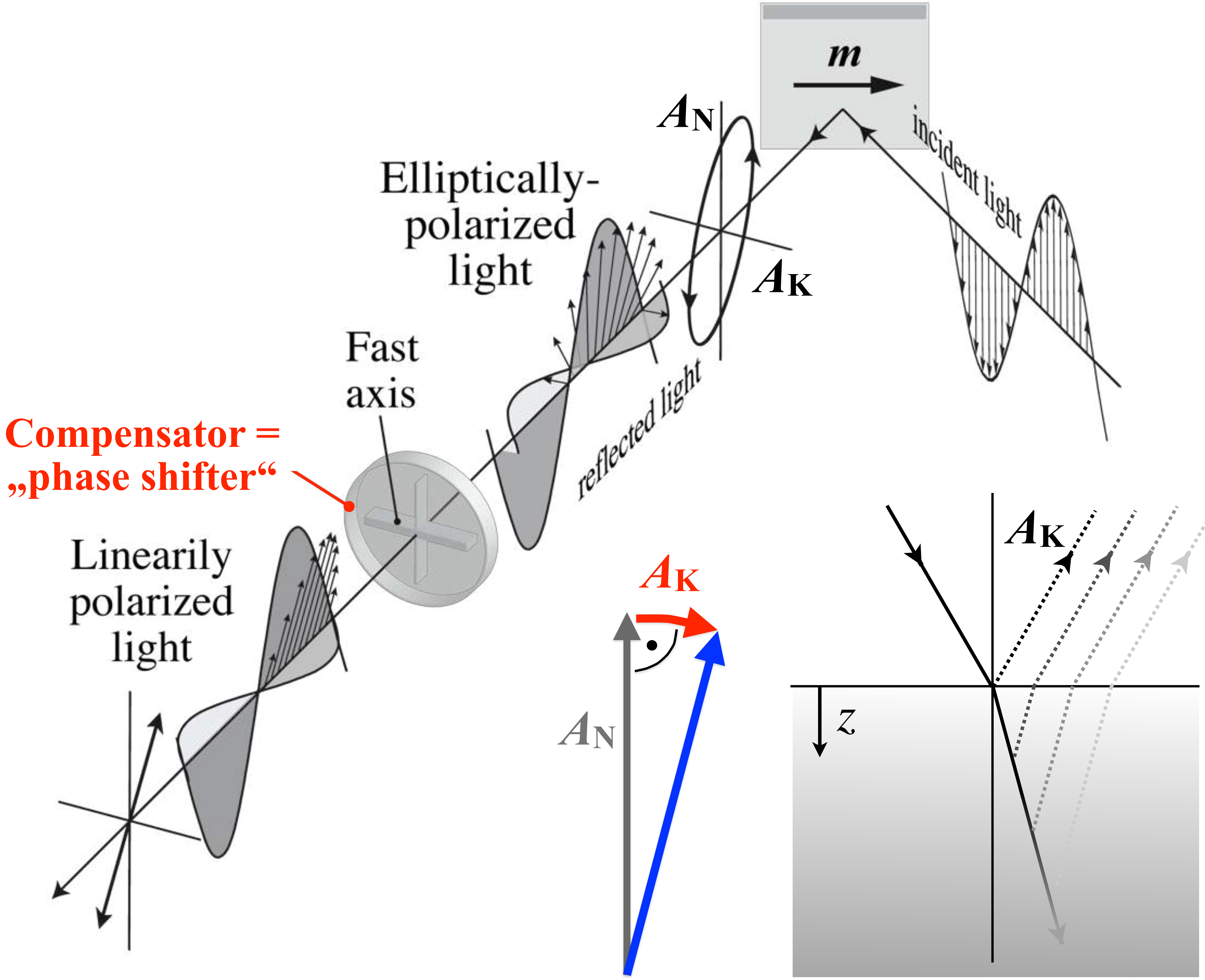
Depth selective Kerr-microscopy



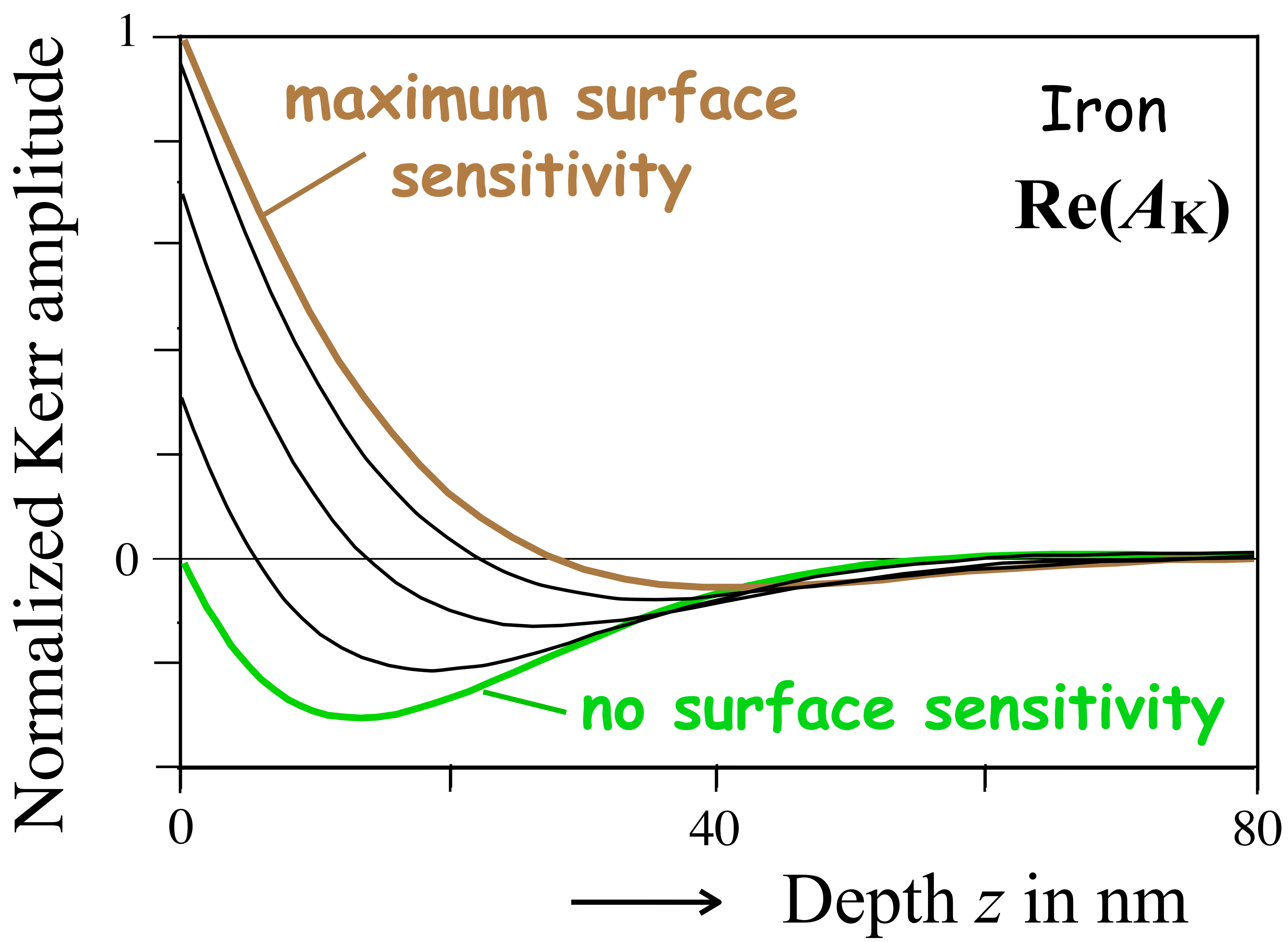
Hubert, Kambersky, Träger, Wenzel in 1990ies



Depth selective Kerr-microscopy

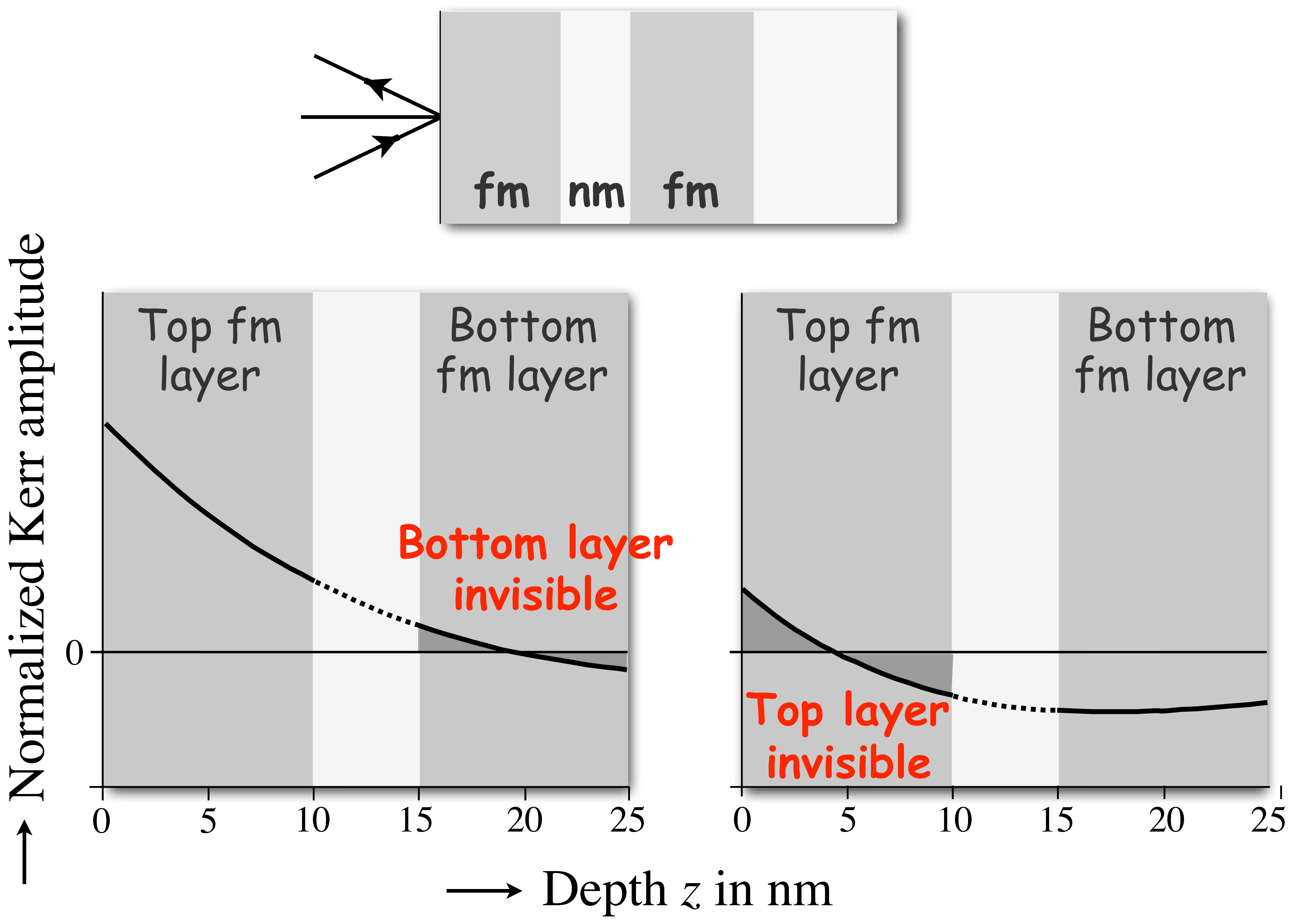


Hubert, Kambersky, Träger, Wenzel in 1990ies

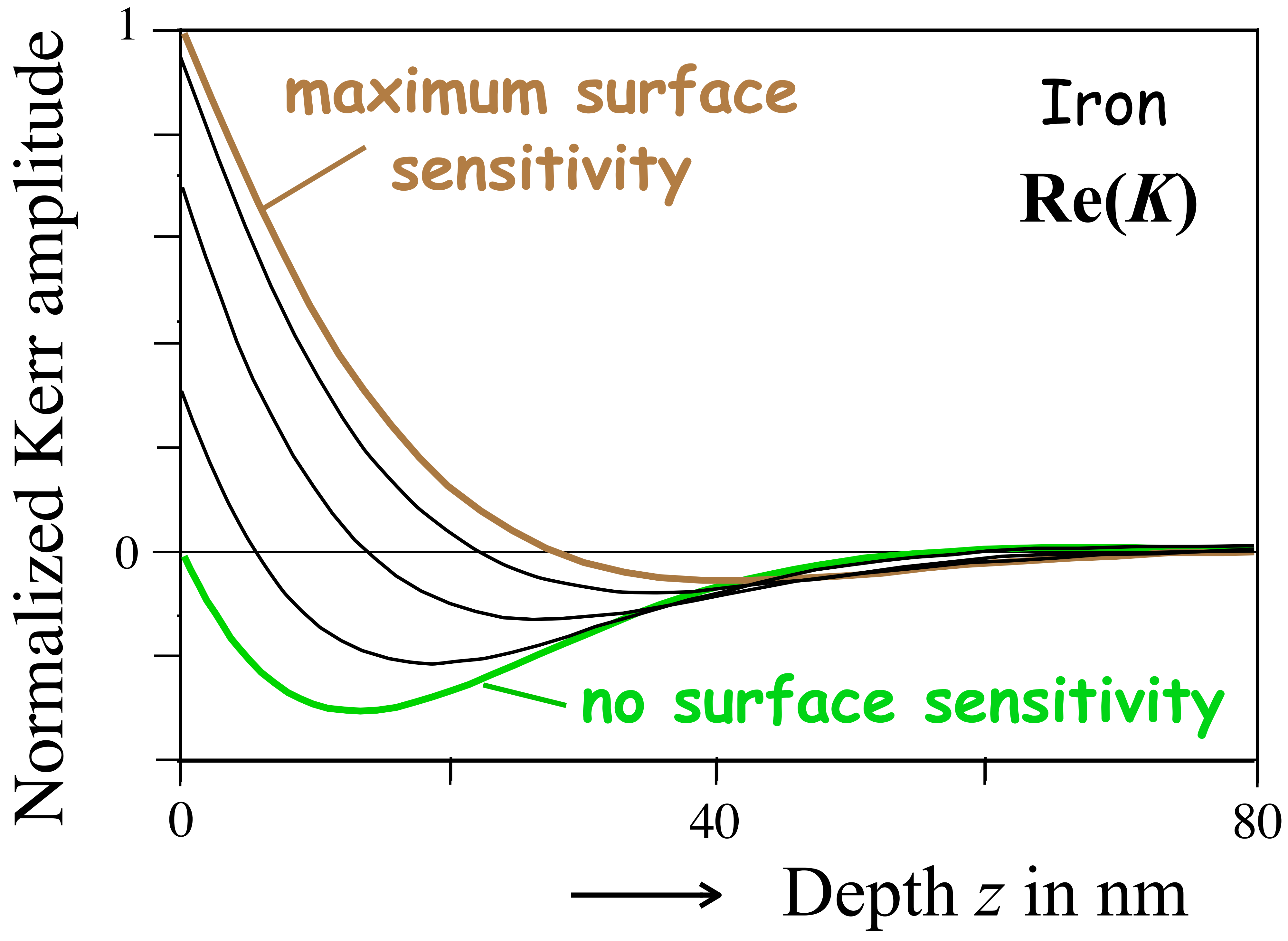


4. Magneto-Optical Kerr Microscopy

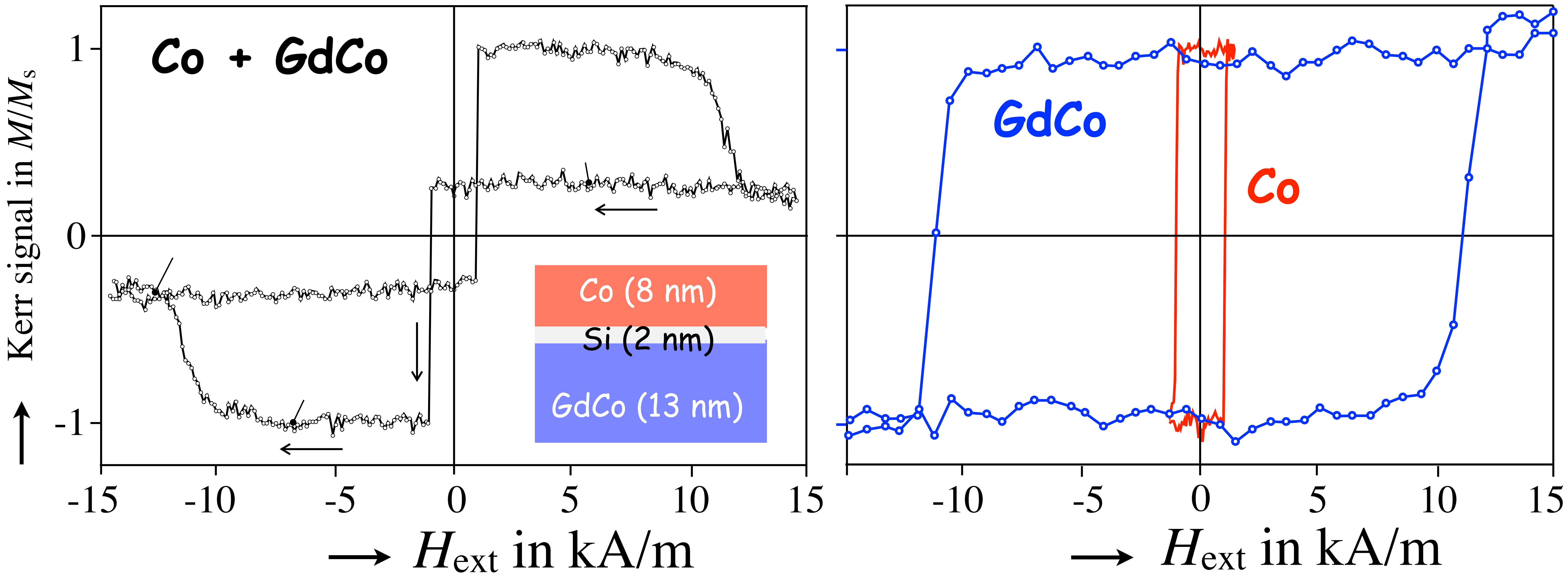
Depth selective Kerr-microscopy

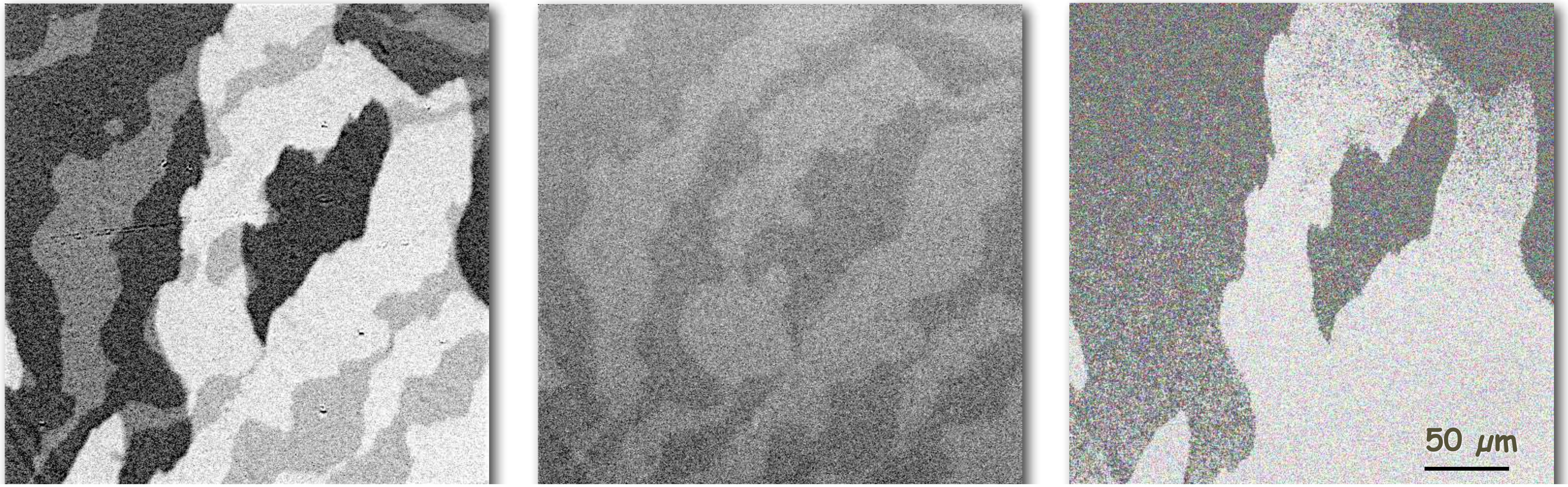


ubert, Kambersky, Träger, Wenzel in 1990ies



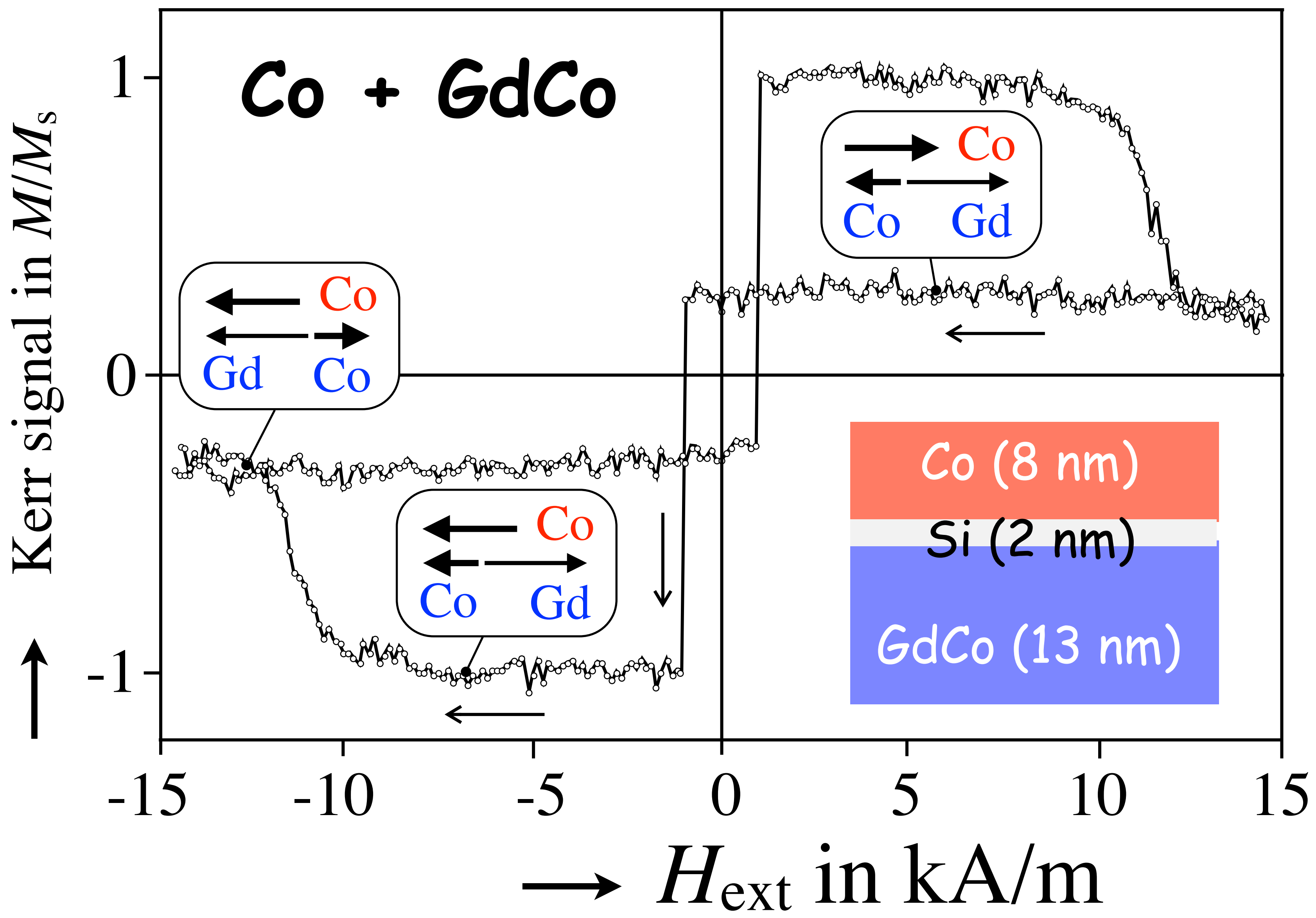
Depth selective Kerr-microscopy

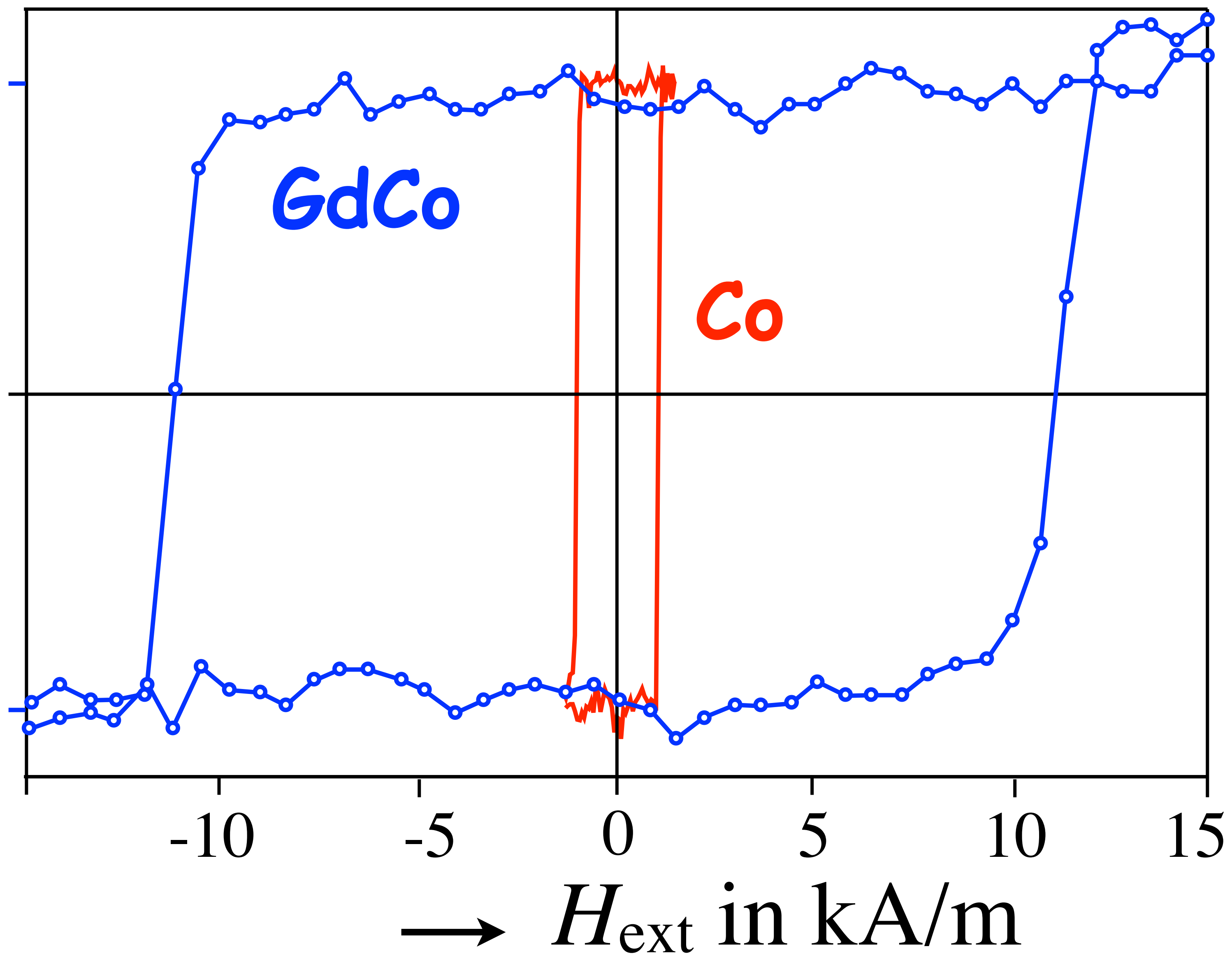


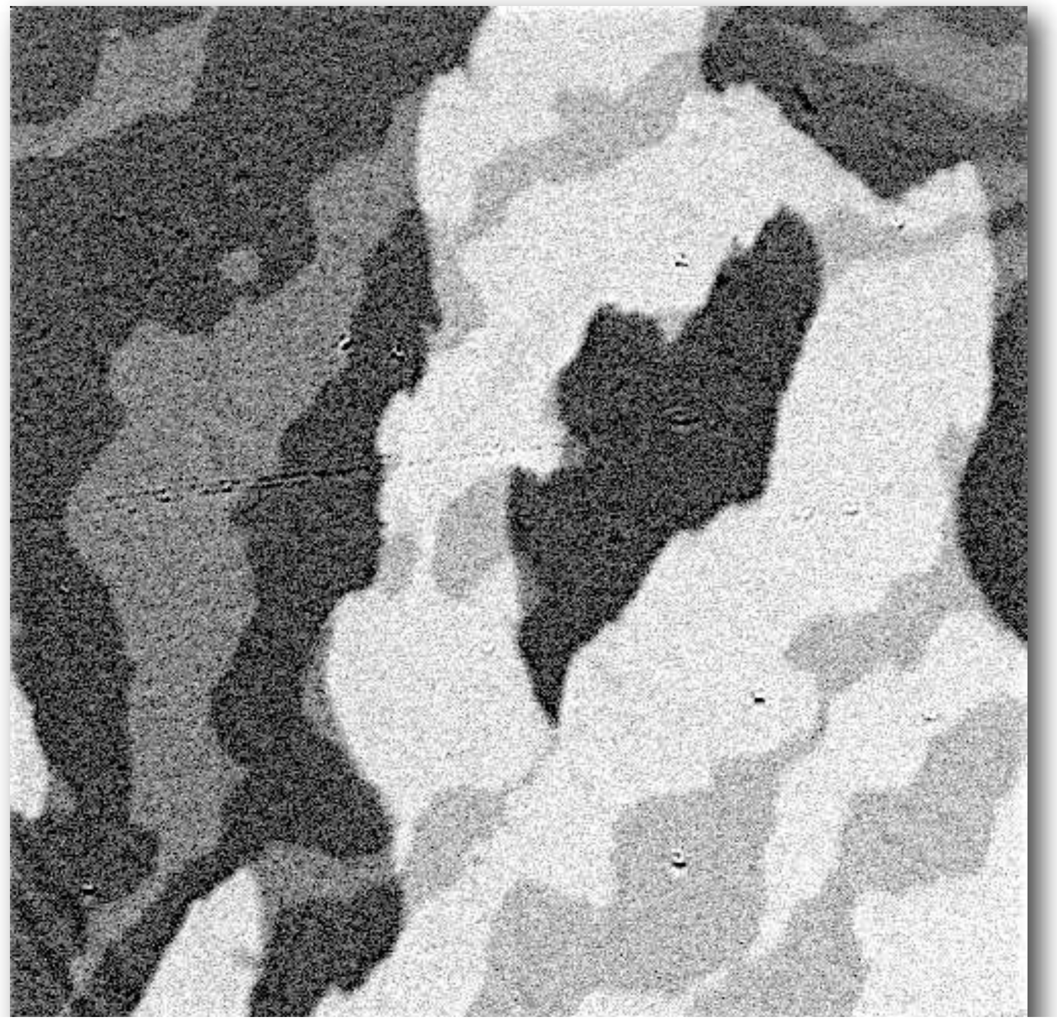


Mixed Kerr signal | GdCo layer | Co layer

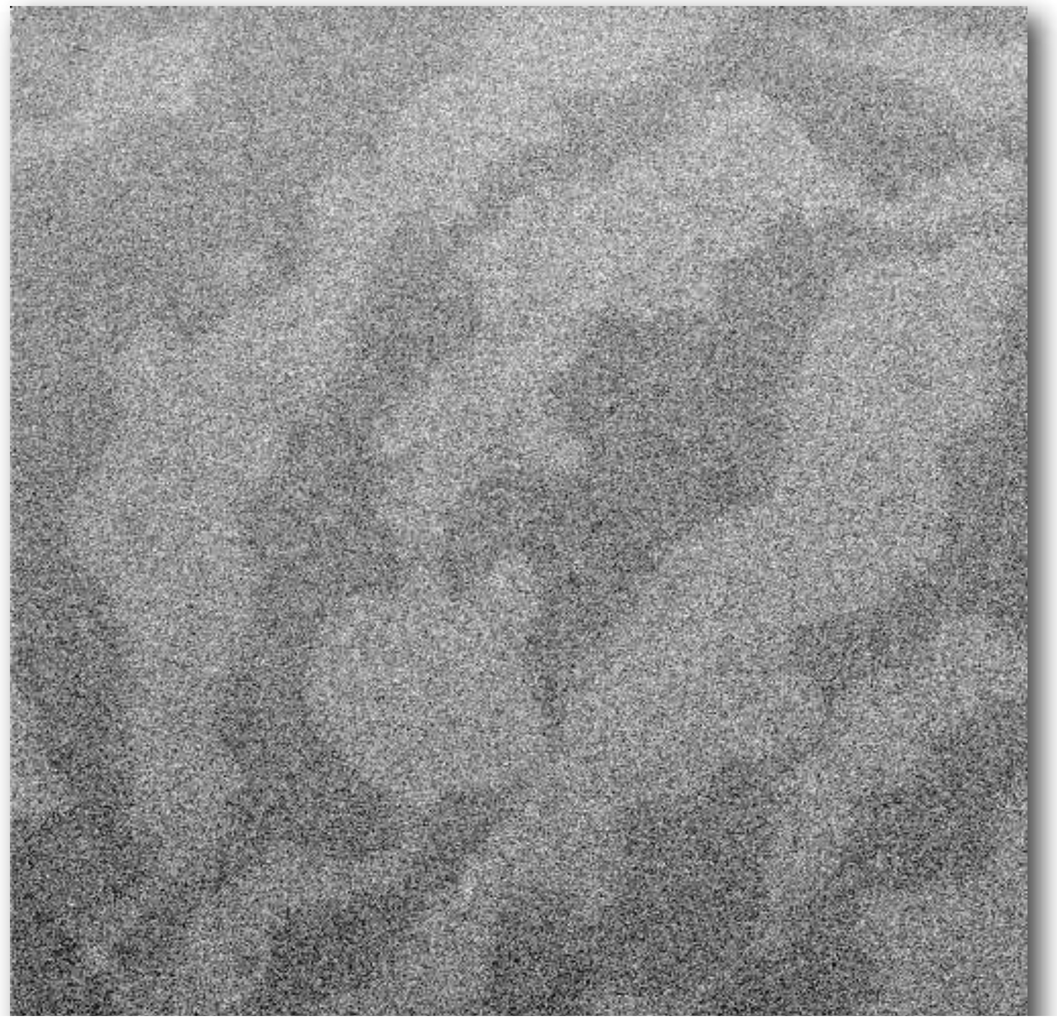
Depth selective Kerr-microscopy



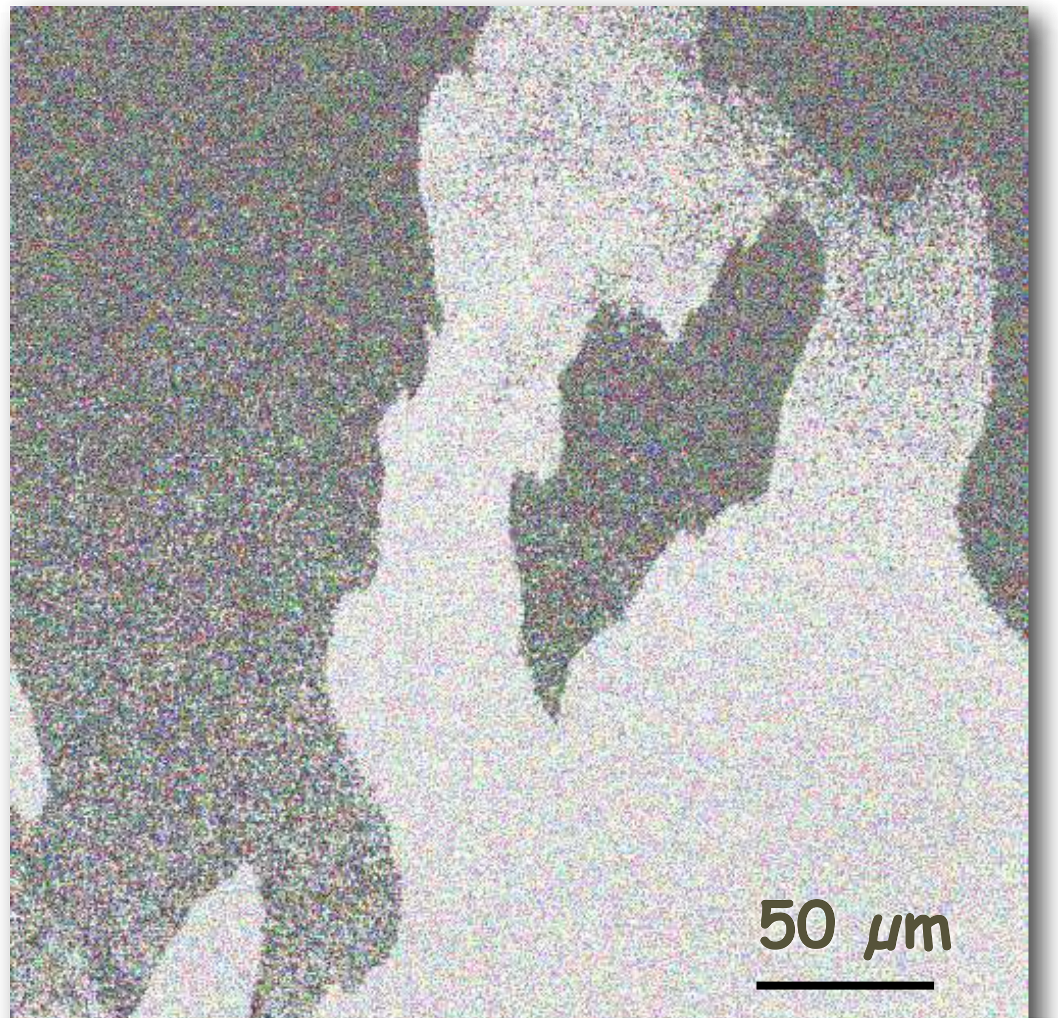




Mixed Kerr signal

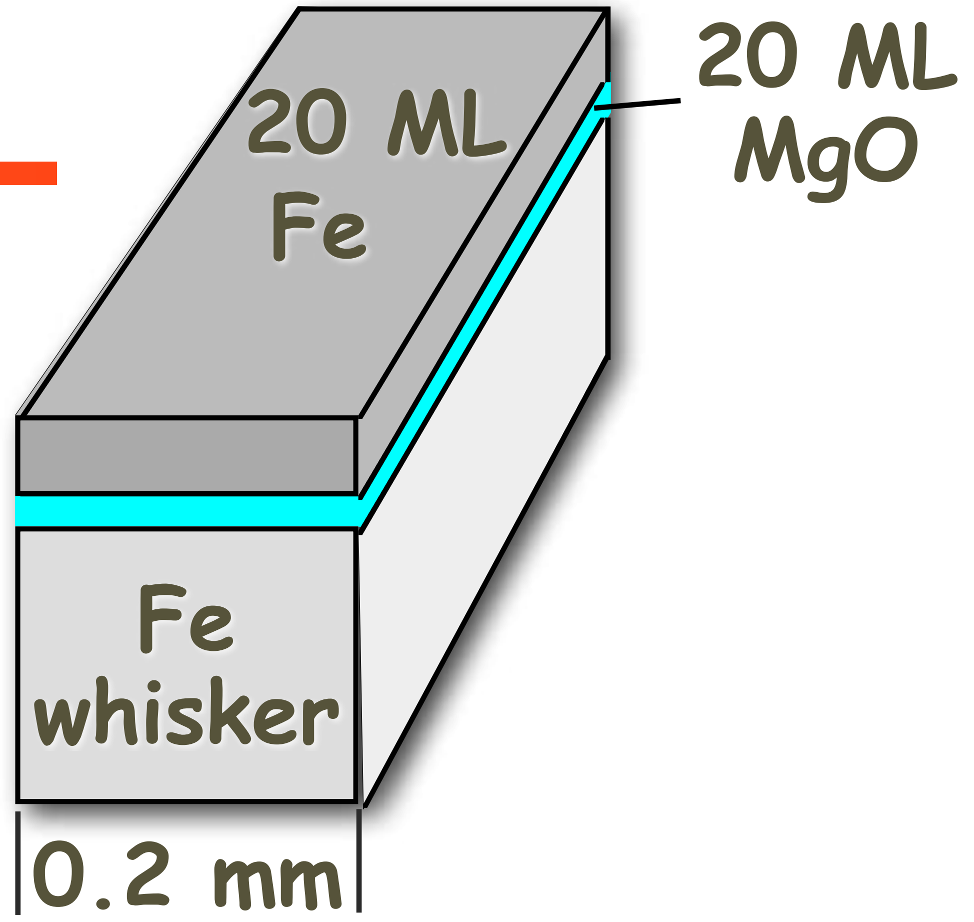


GdCo layer



Co layer

4. Magneto-Optical Kerr Microscopy



Depth selective Kerr-microscopy

Advantage of layer-selective Kerr microscopy:
different composition of layers not required

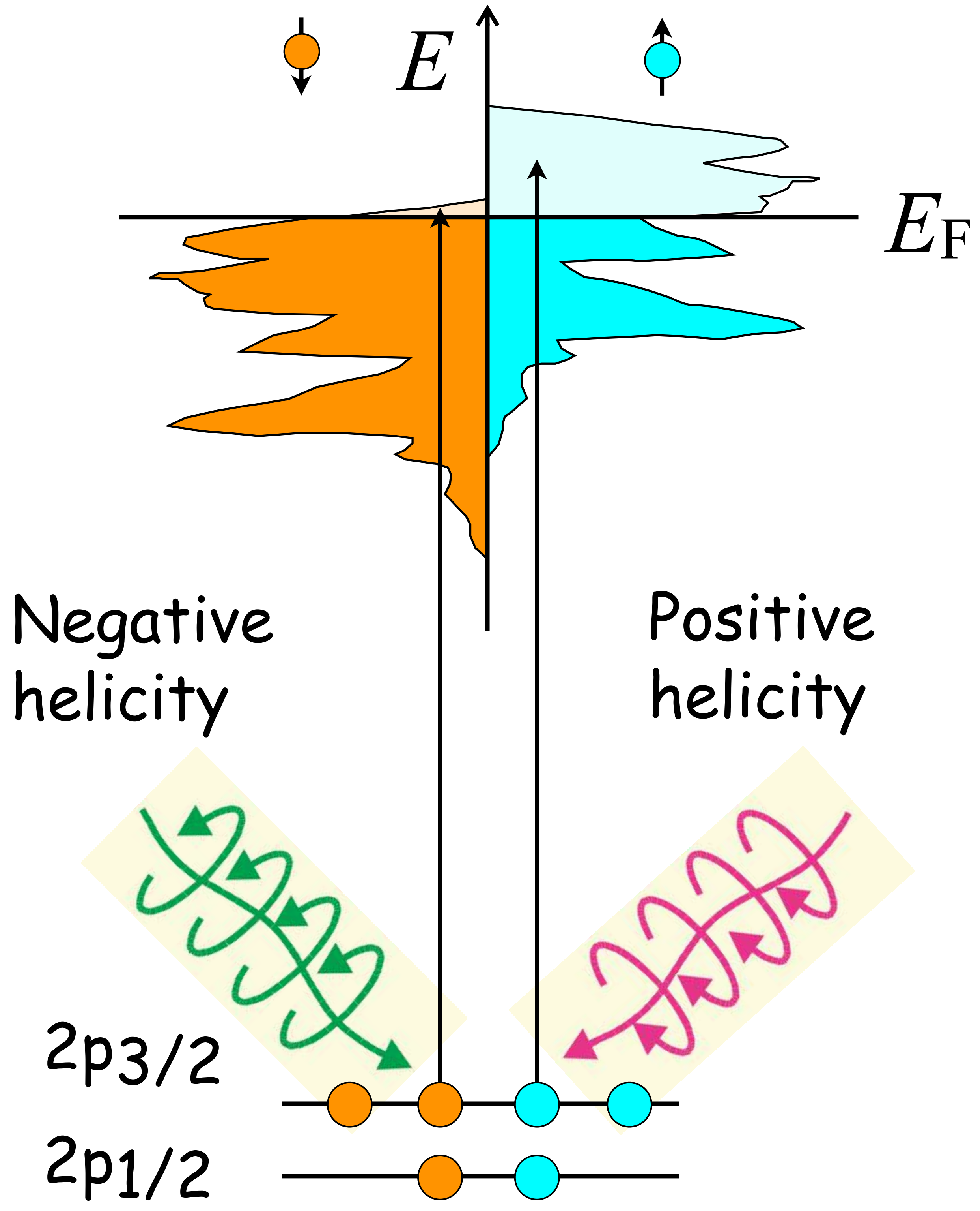
X-ray Magnetic Circular Dichroism

Absorption of circularly polarized x-rays depends on orientation of M-direction with respect to helicity of the X-rays, change of sign by reversing M

Initial states are well defined inner-core levels

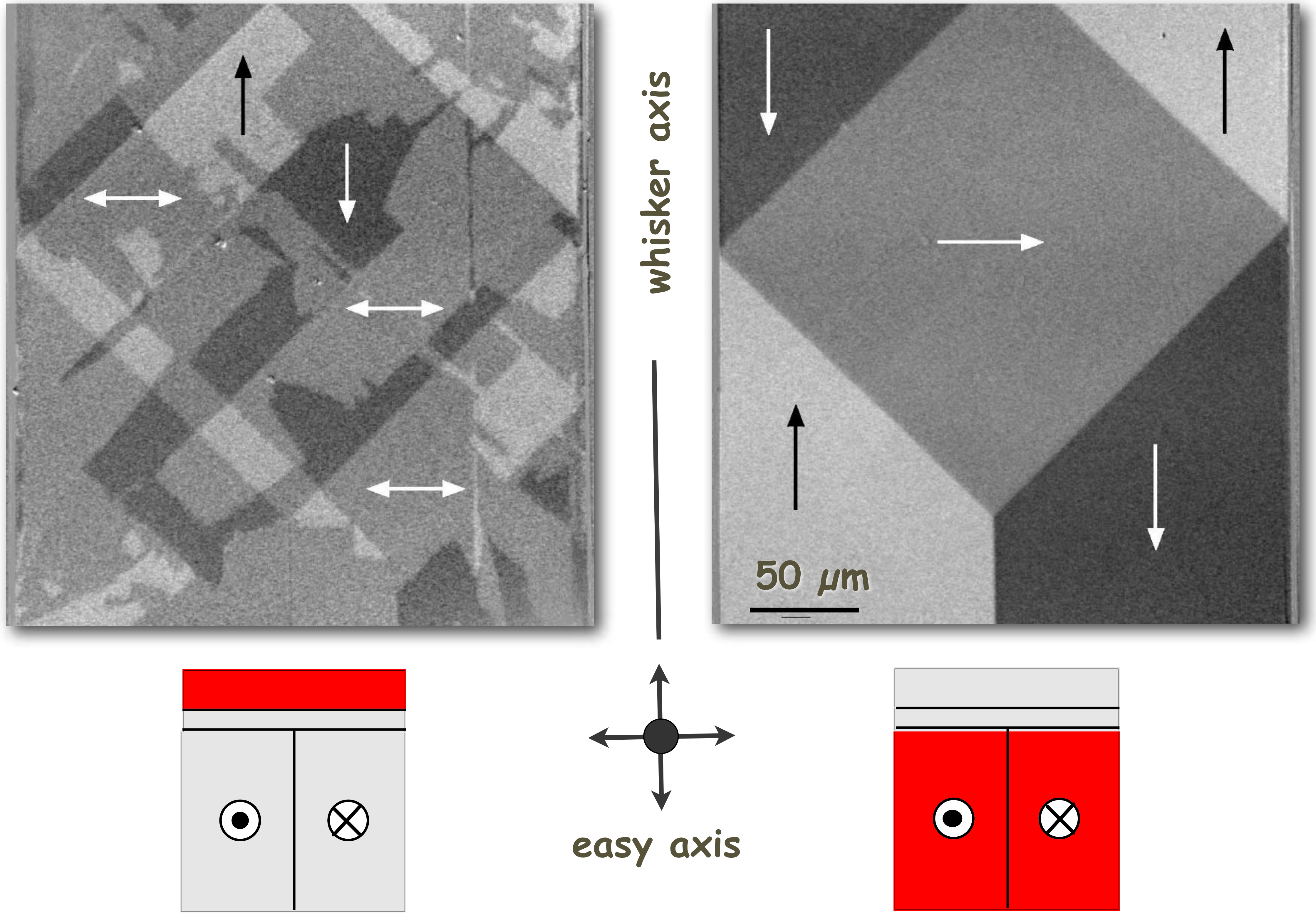
→ XMCD is element selective

→ layer selective imaging requires different compositions of layers



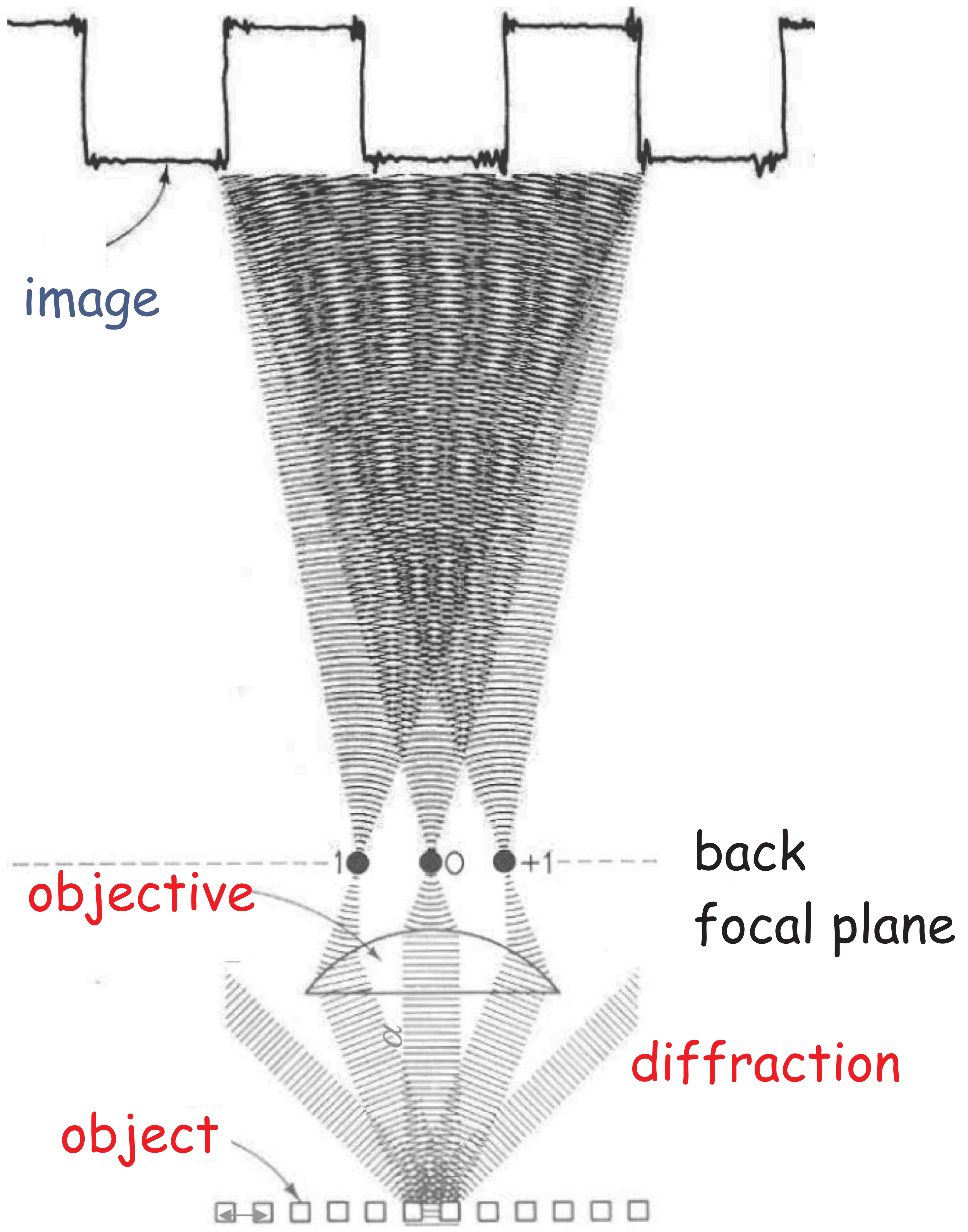
Domains in Fe-film

Whisker domains



R. S., et al., PRB 65, 144405 (2003)

Resolution of Kerr-microscopy



Resolution of optical microscopy is determined by constructive interference

Diffraction limited image formation

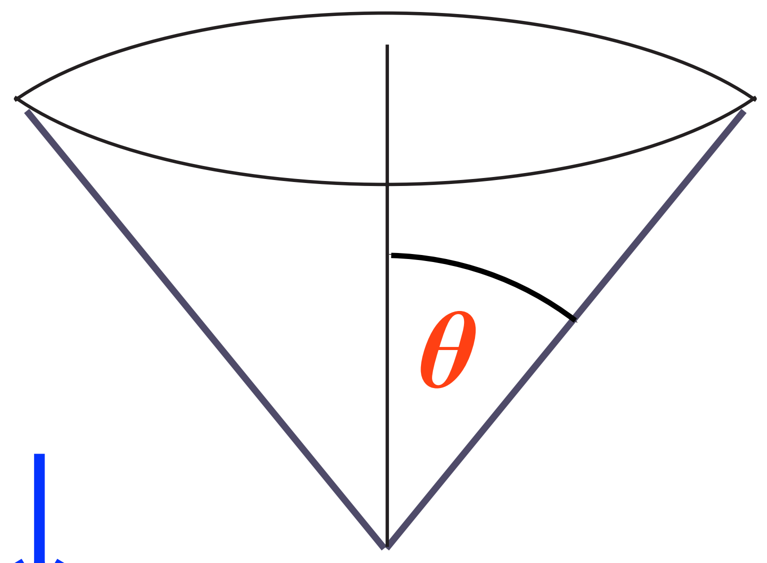
Abbe limit: $d = \dfrac{\lambda}{NA}$

d = separation between particles, still allowing to see them

λ = wavelength

NA = numerical aperture of objective

Numerical aperture: $NA = n \sin\theta$



θ = half the cone angle of light accepted by objective

n = refraction index of medium between sample and objective

↓

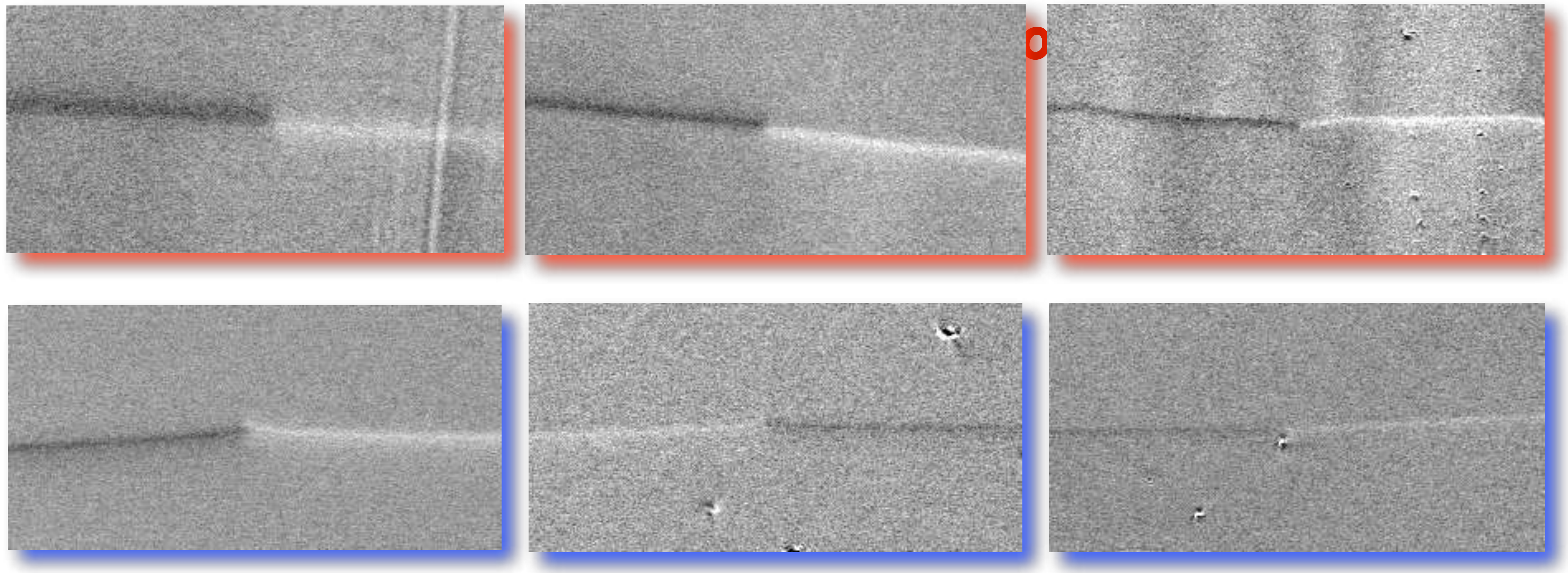
best around 350 nm (Abbe)
best around 220 nm (Rayleigh)
best around 170 nm (Sparrow)

Resolution of Kerr-microscopy

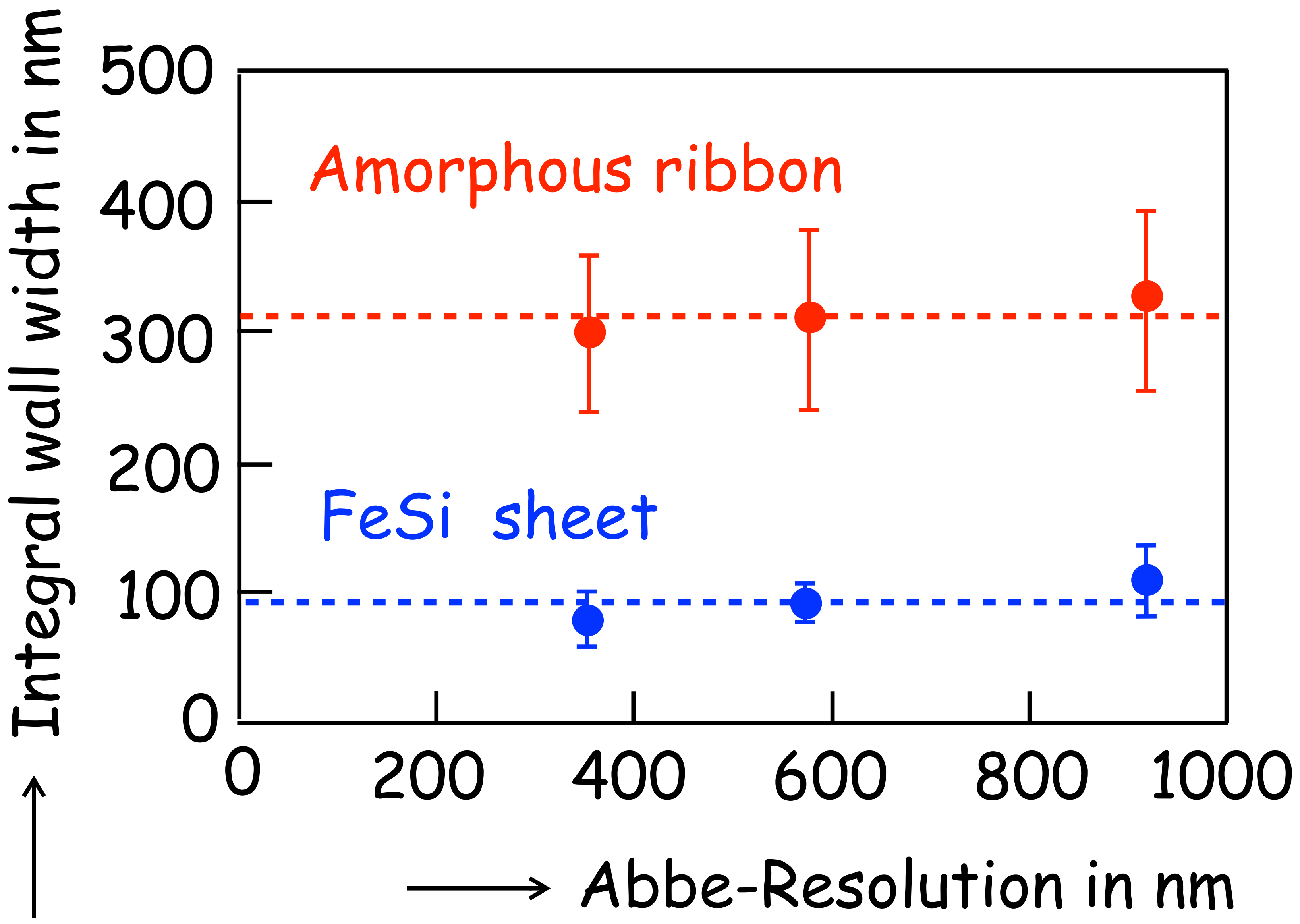
	100/1.3 oil 350 nm	50x/0.8 575 nm	20x/0.5 920 nm

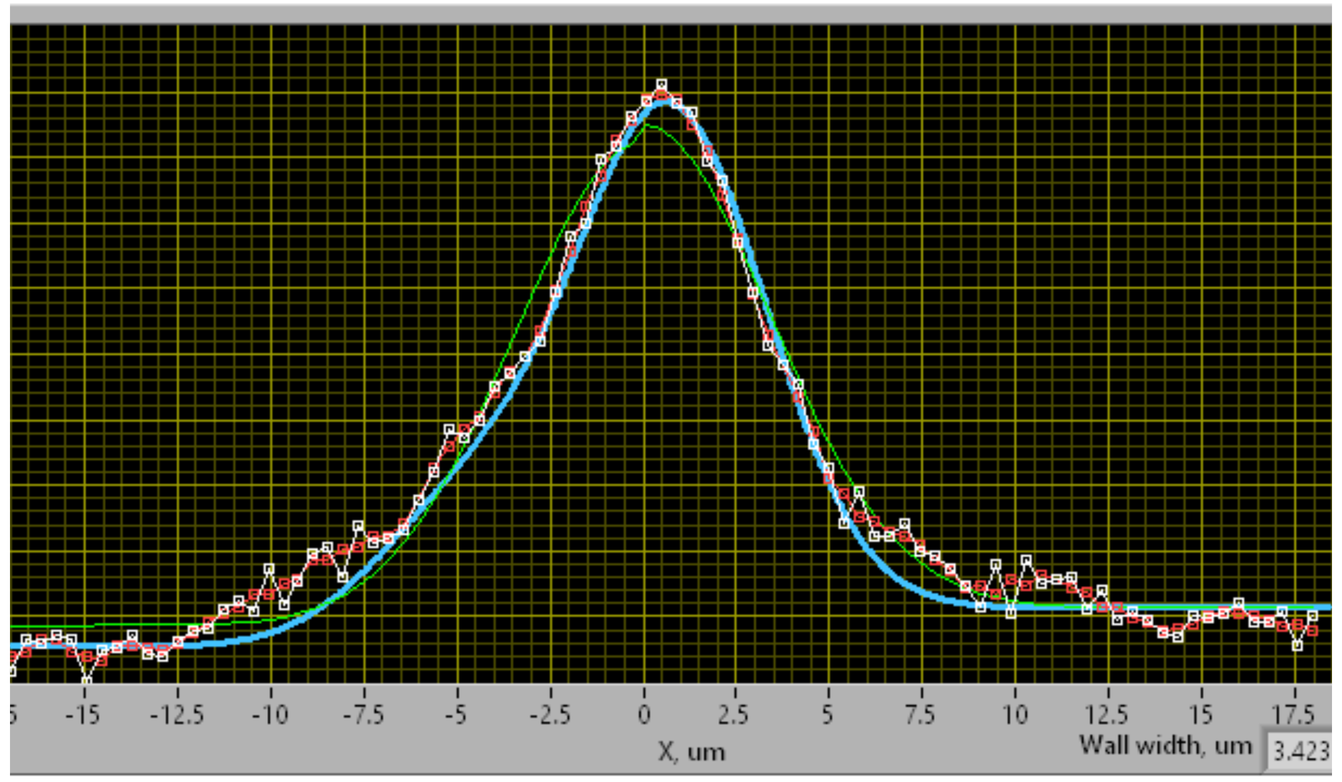
Amorphous ribbon

FeSi sheet



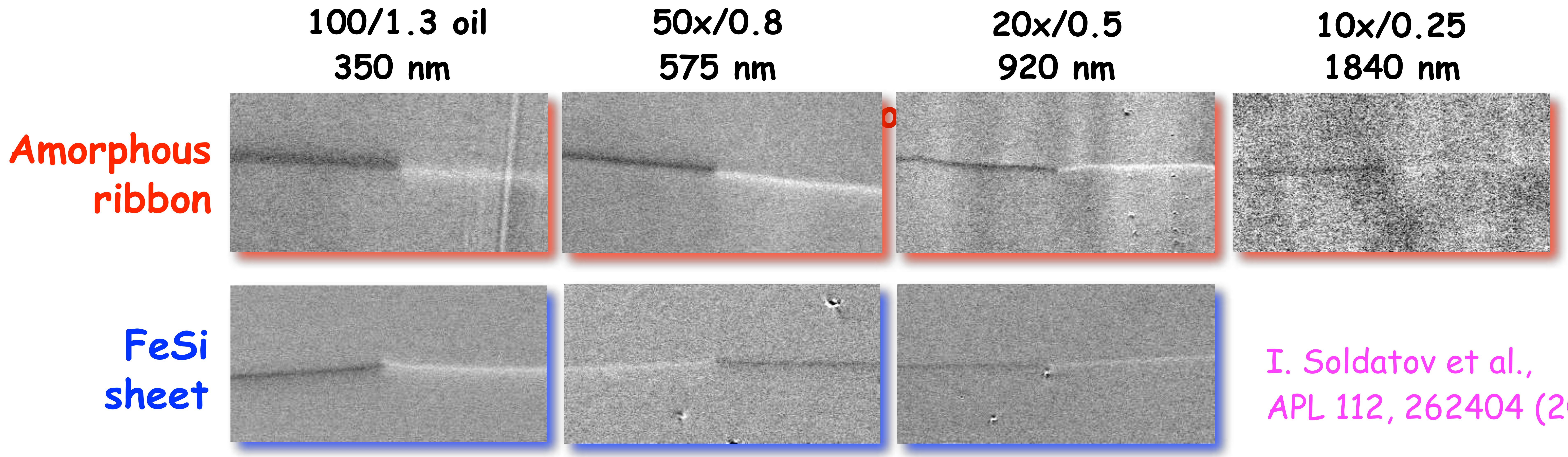
I. Soldatov et al., APL 112, 262404 (2018)



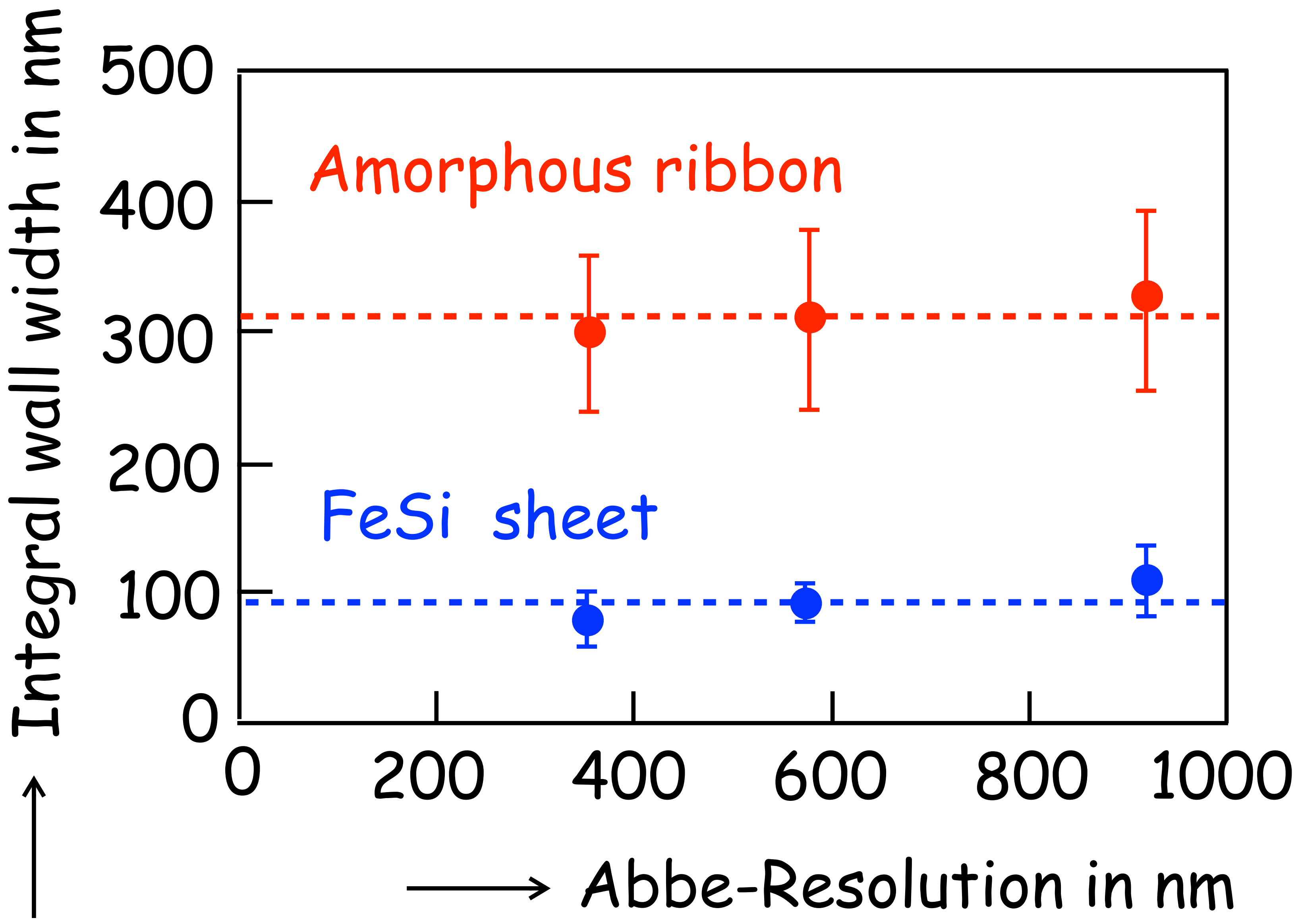


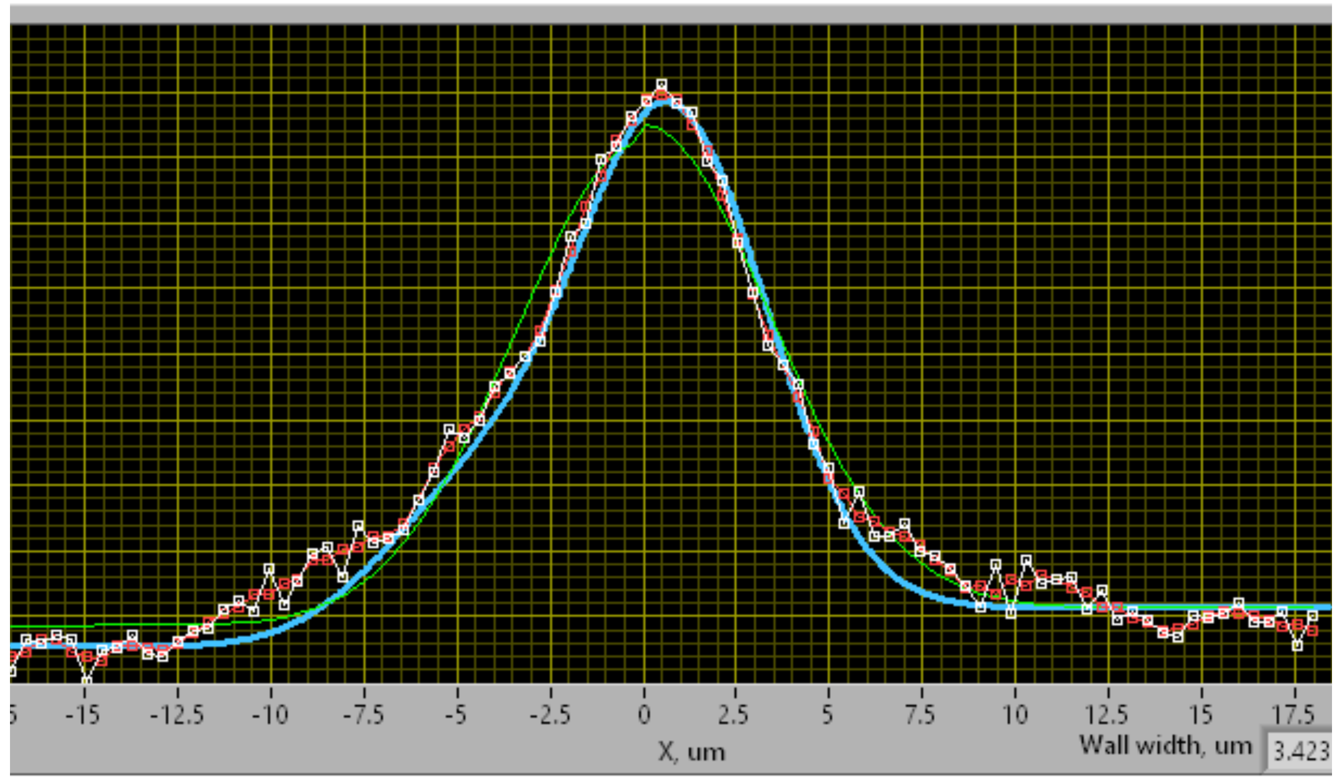
$$\text{Wall width} = \frac{\text{Integrated wall intensity}}{\text{Maximum domain intensity}}$$

Resolution of Kerr-microscopy



I. Soldatov et al., APL 112, 262404 (2018)





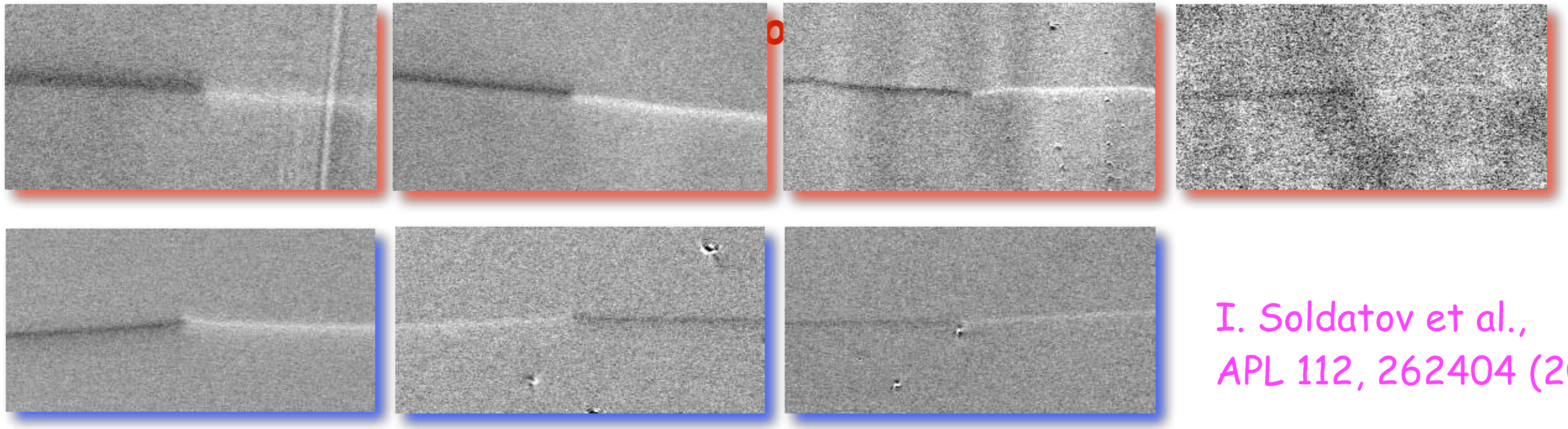
$$\text{Wall width} = \frac{\text{Integrated wall intensity}}{\text{Maximum domain intensity}}$$

Resolution of Kerr-microscopy

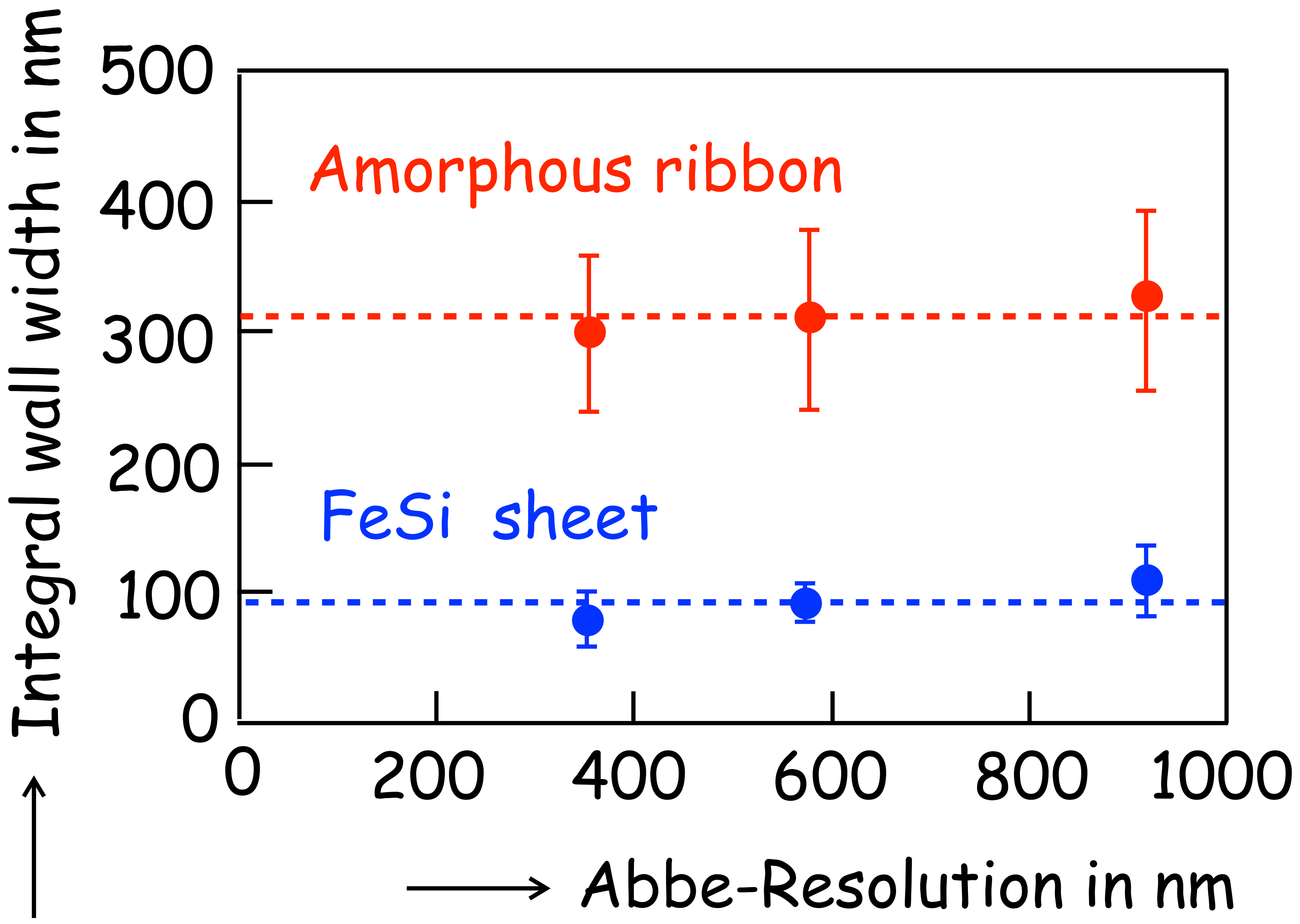
	100/1.3 oil 350 nm	50x/0.8 575 nm	20x/0.5 920 nm	10x/0.25 1840 nm

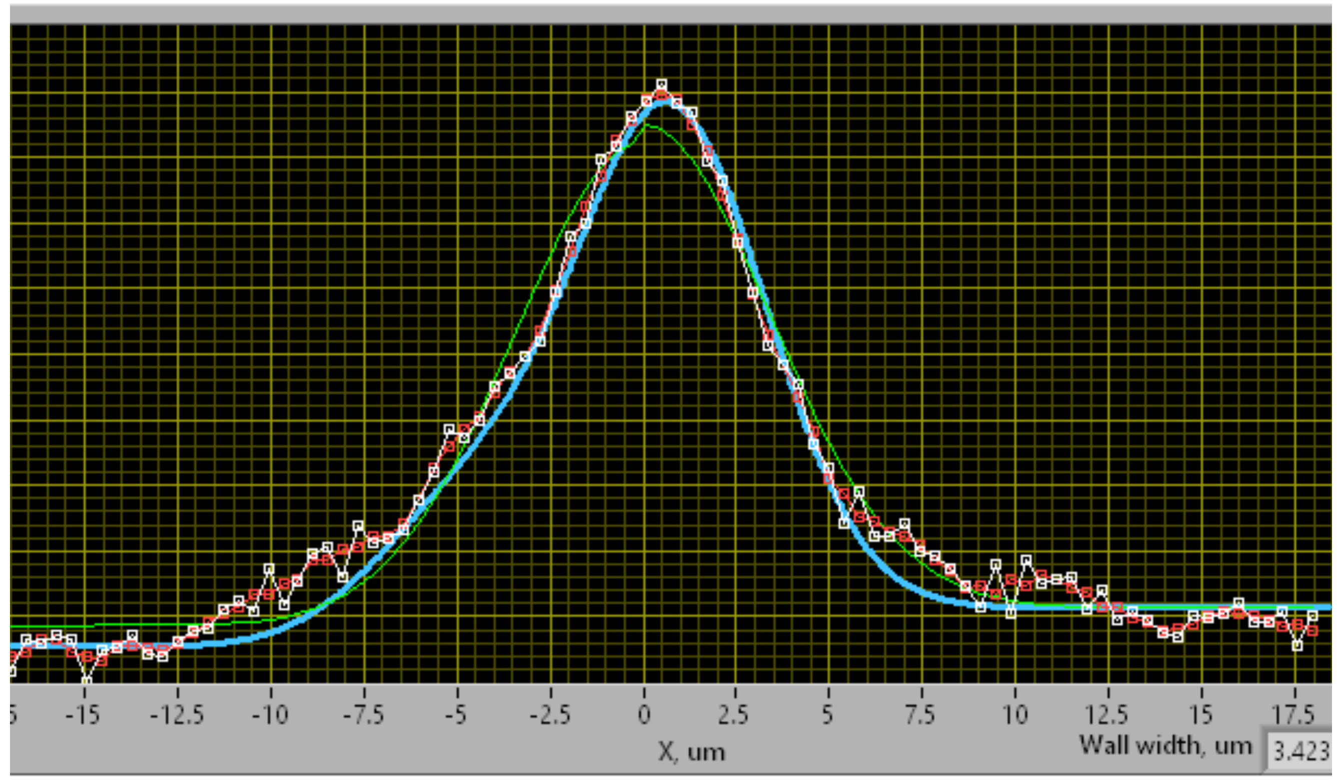
Amorphous ribbon

FeSi sheet



I. Soldatov et al., APL 112, 262404 (2018)





$$\text{Wall width} = \frac{\text{Integrated wall intensity}}{\text{Maximum domain intensity}}$$

Amorphous ribbon:

wall width = 310 nm

→ 15% of resolution still visible

⇓

100x/1.3 oil objective with 350 nm resolution

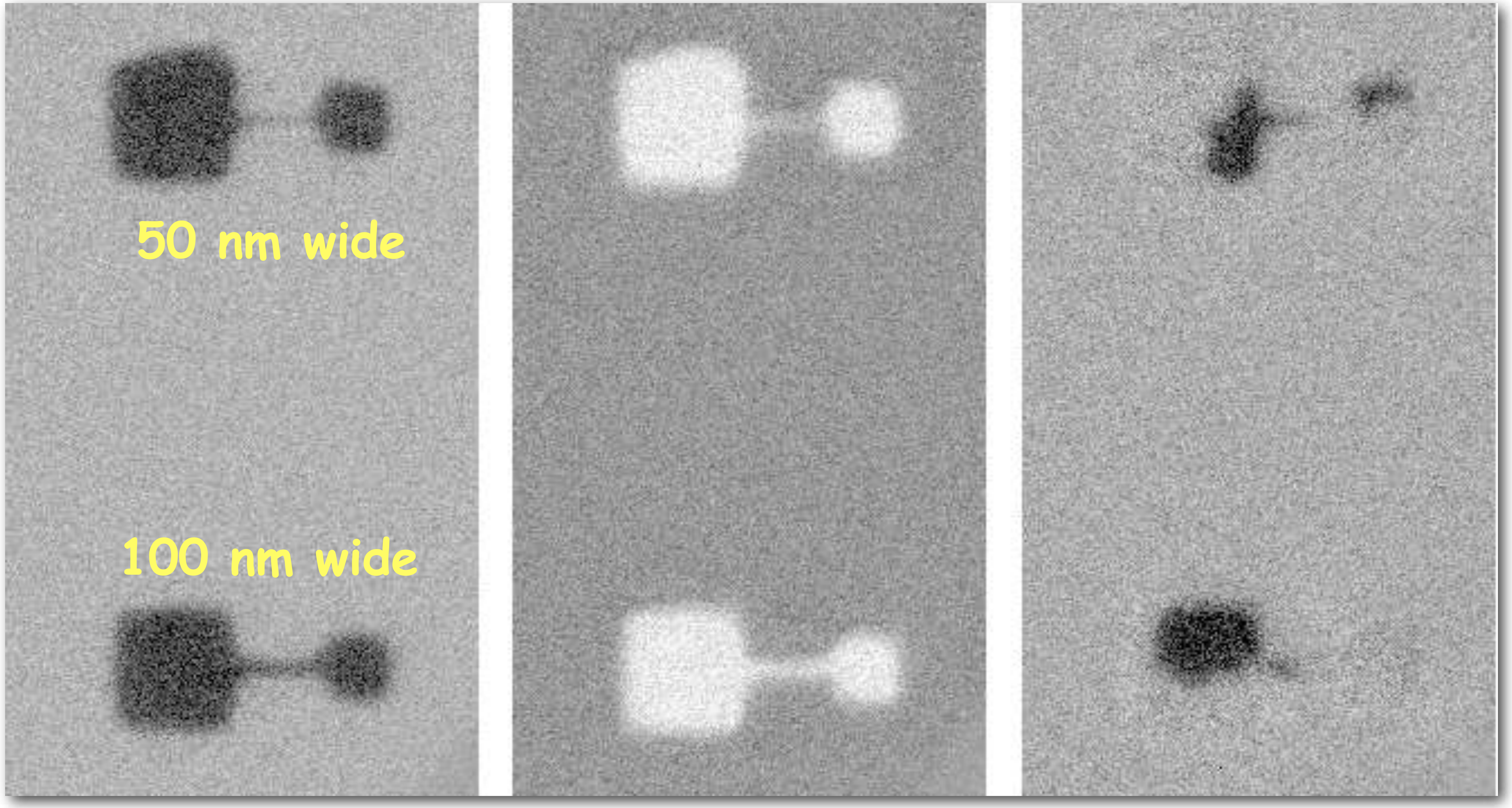
⇓

50 nm wide walls should be visible

Resolution of Kerr-microscopy

High-resolution observations

Positive remanence | Negative remanence | Demagnetized

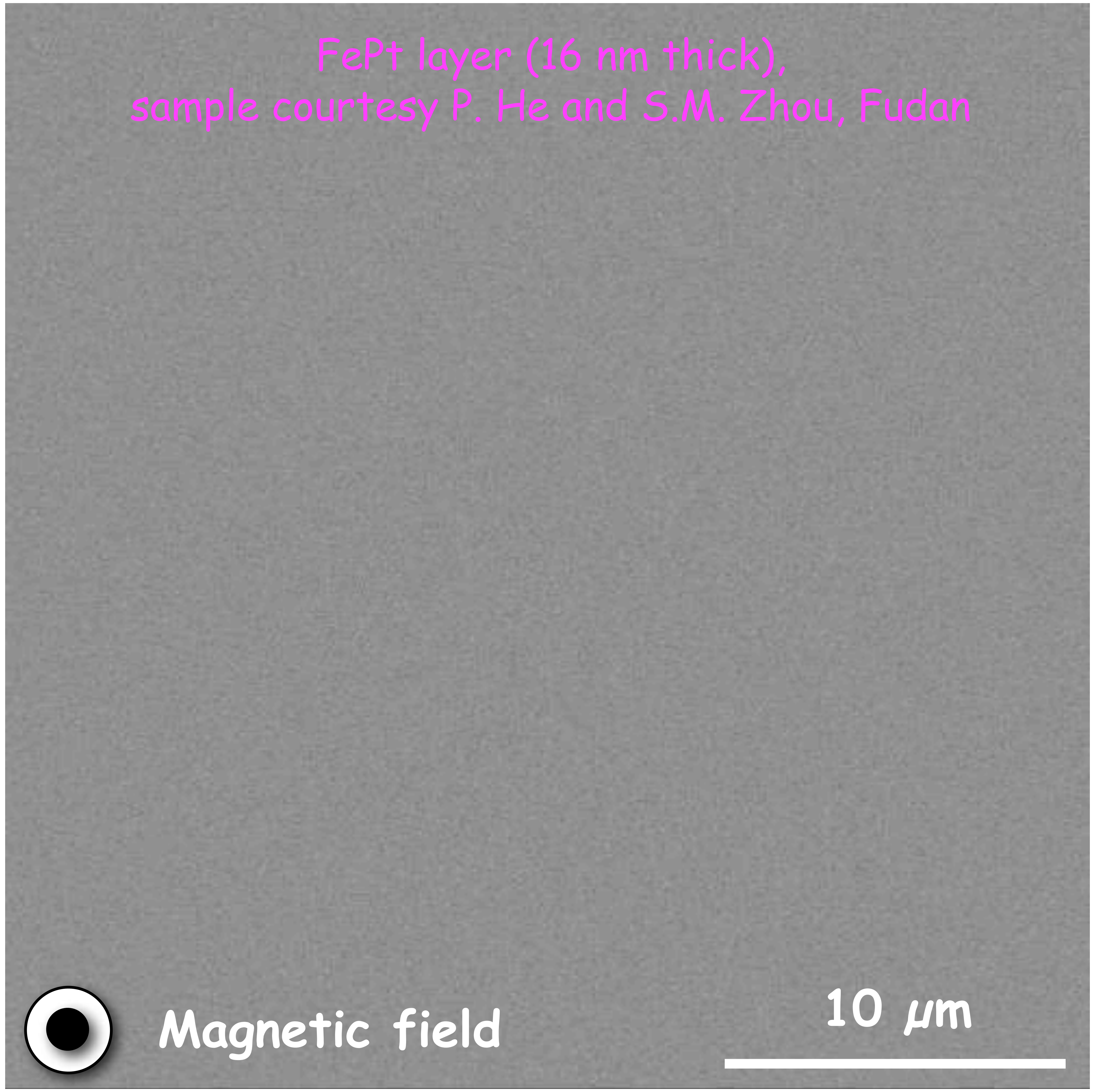


Nanowires (50 and 100 nm wide, 2 μm long)
of magnetic film system with perpendicular anisotropy

(sample courtesy Jimmy Zhu and Matt Moneck, Carnegie Mellon University, Pittsburgh)

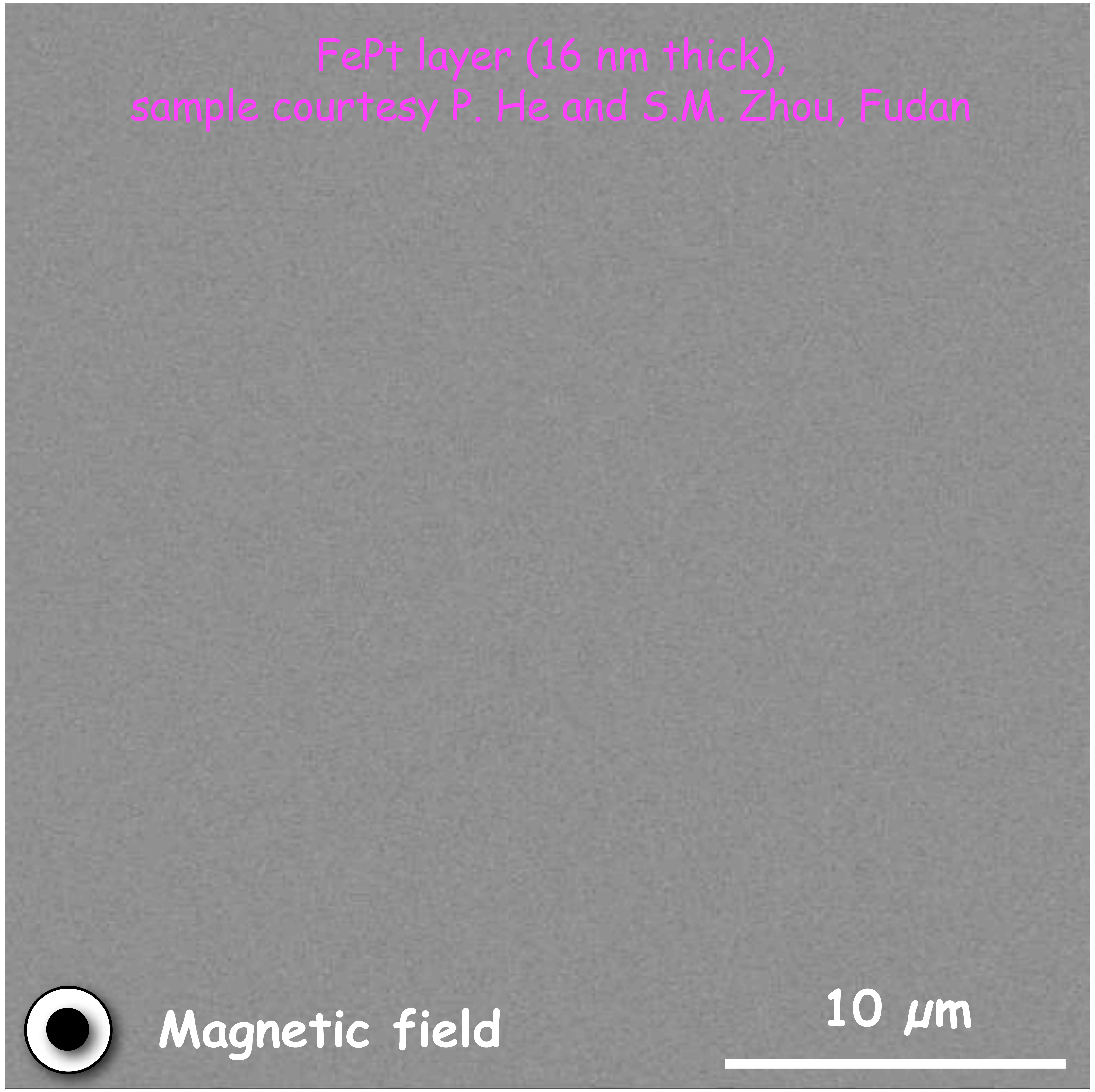
Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy



Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy



Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy

FePt layer (16 nm thick),
sample courtesy P. He and S.M. Zhou, Fudan

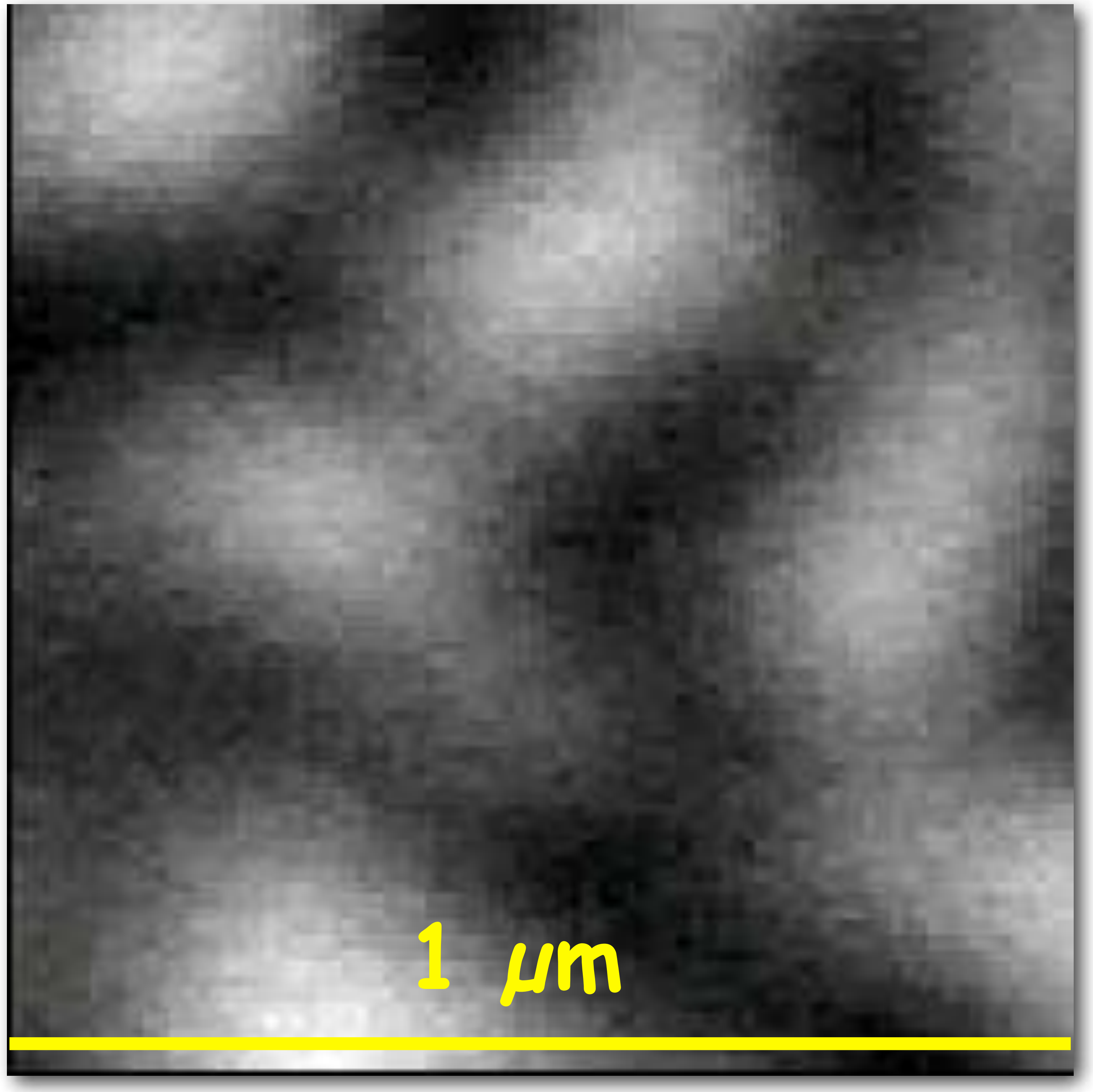


Image is folded by point-spread-function of microscope

→ loss of information

→ recovery of lost information by mathematical deconvolution

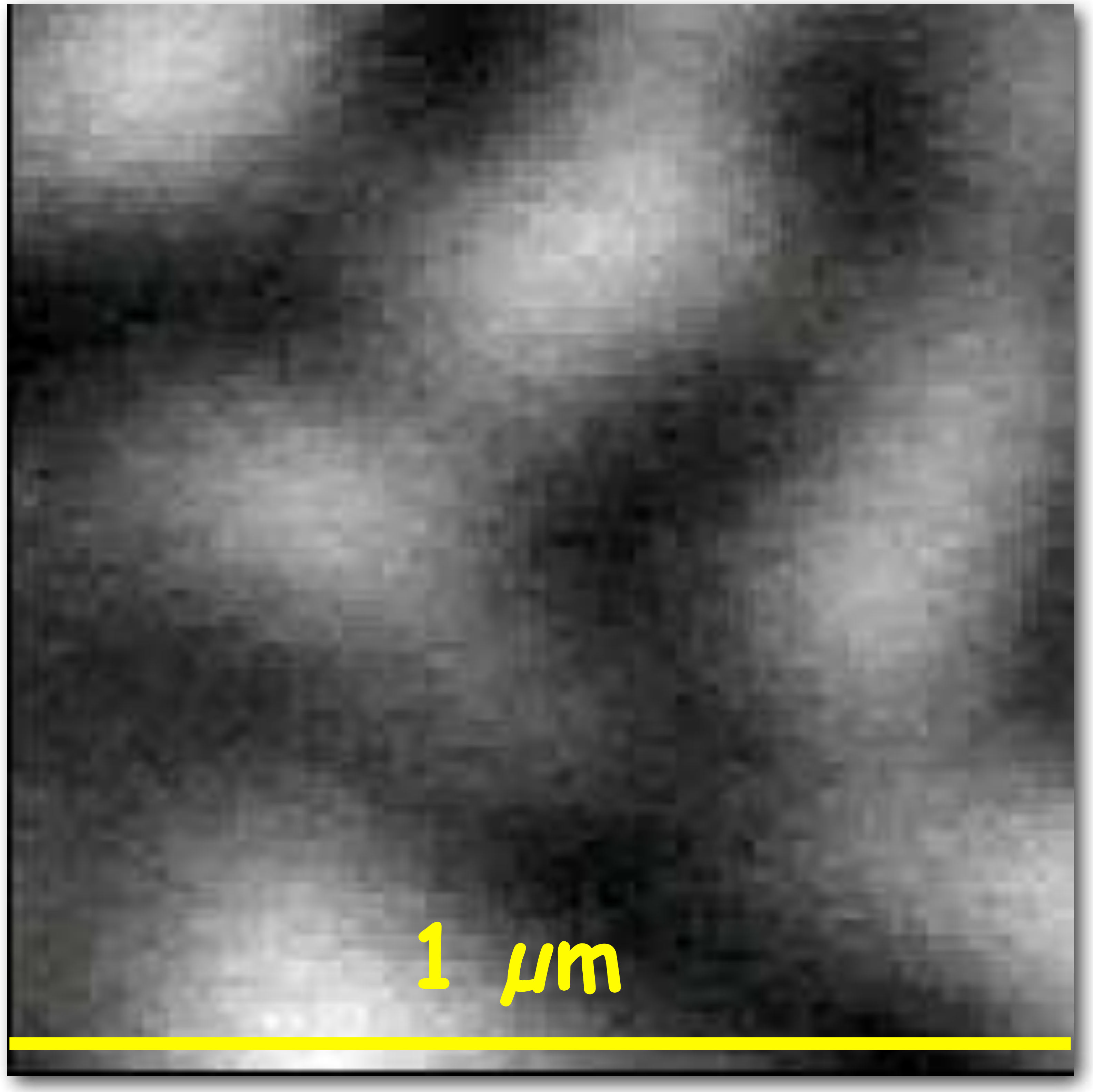
→ enhancement of resolution down to 50 nm regime

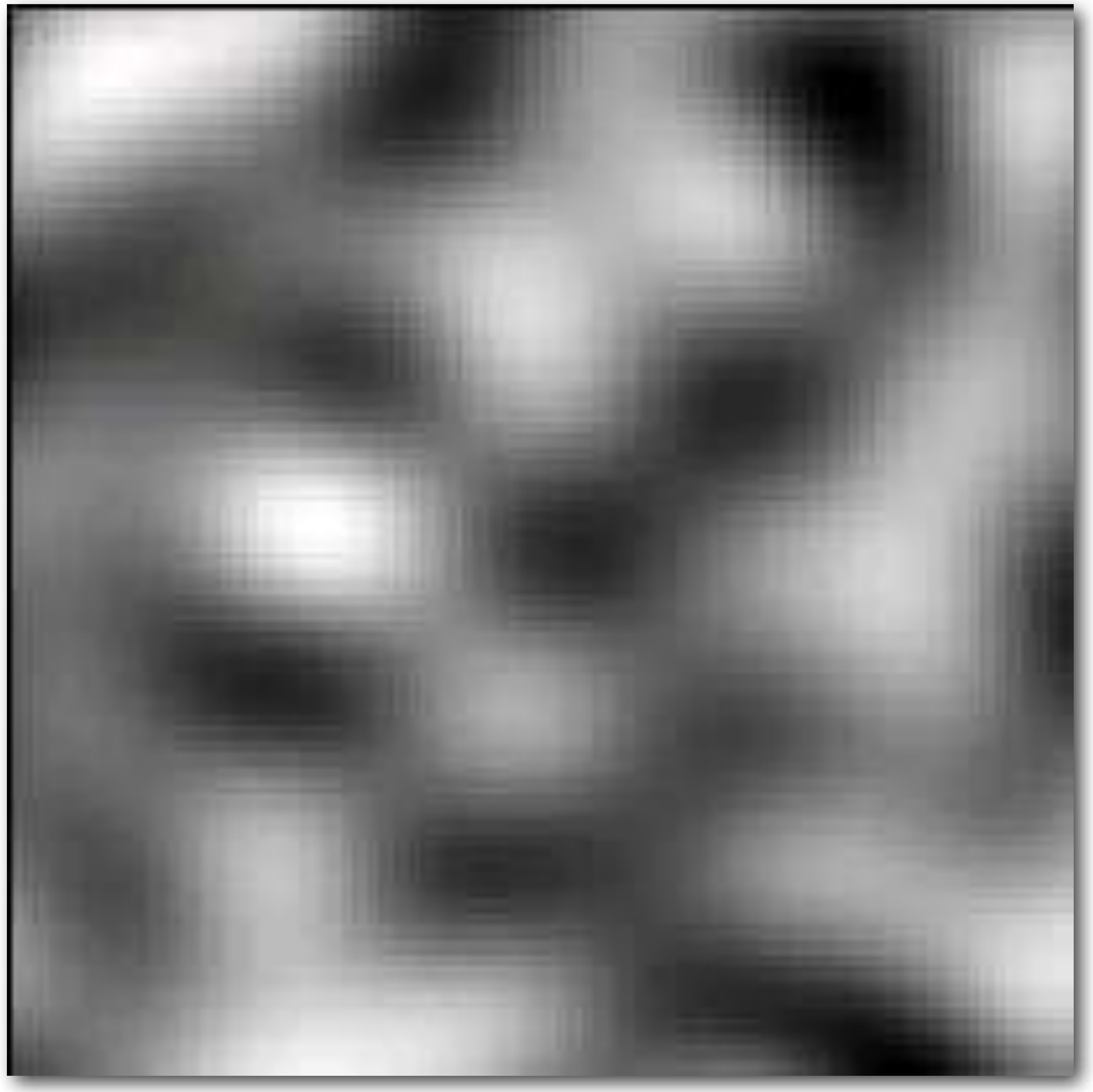
together with
N.Gorn & D.Berkov, Innovent Jena
(under development)

Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy

FePt layer (16 nm thick),
sample courtesy P. He and S.M. Zhou, Fudan



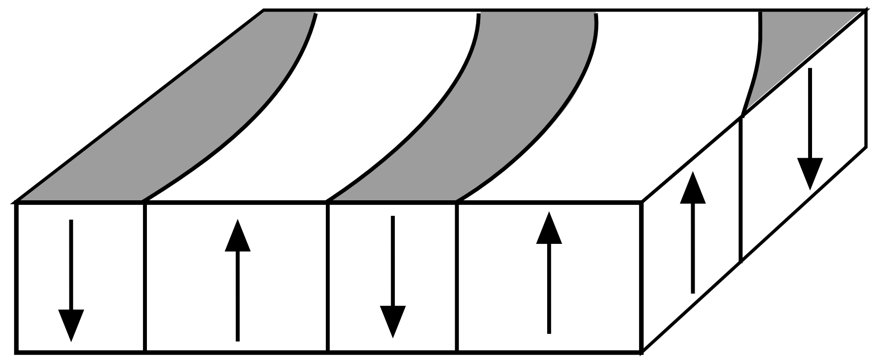


Deconvoluted

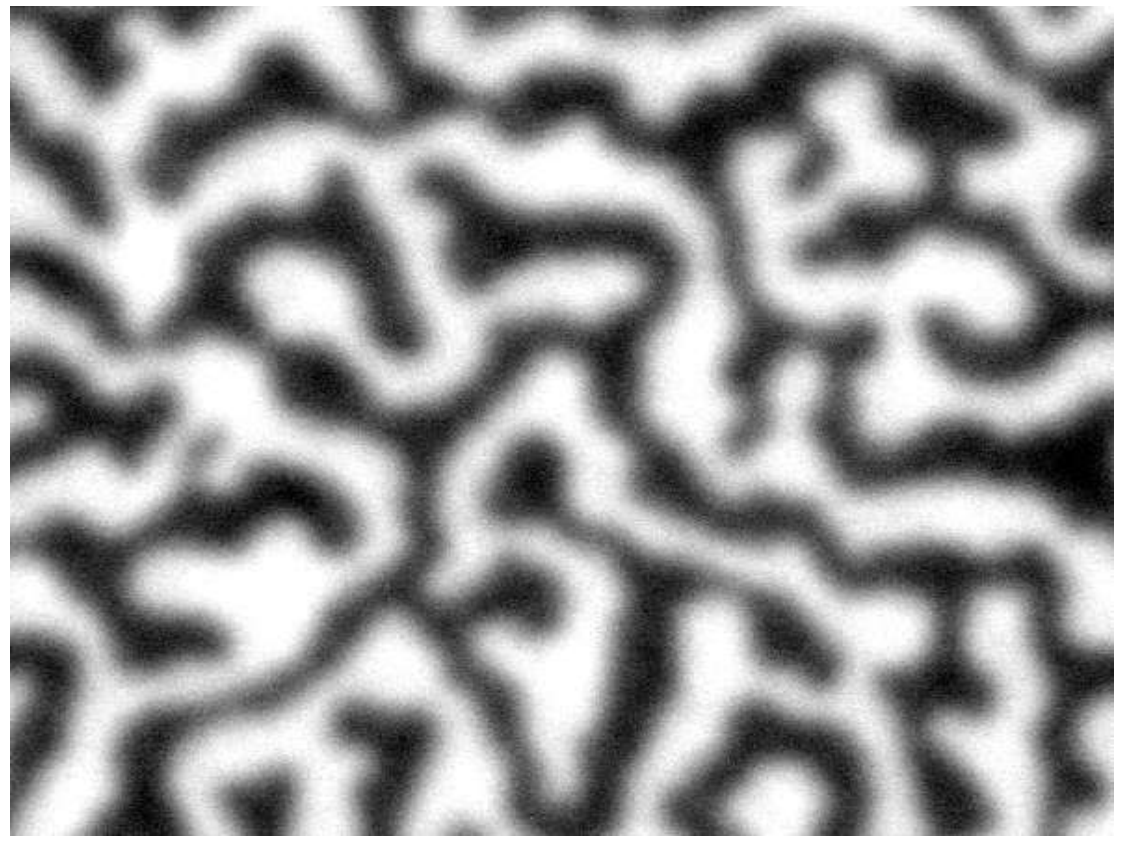
4. Magneto-Optical Kerr Microscopy

Resolution of Kerr-microscopy

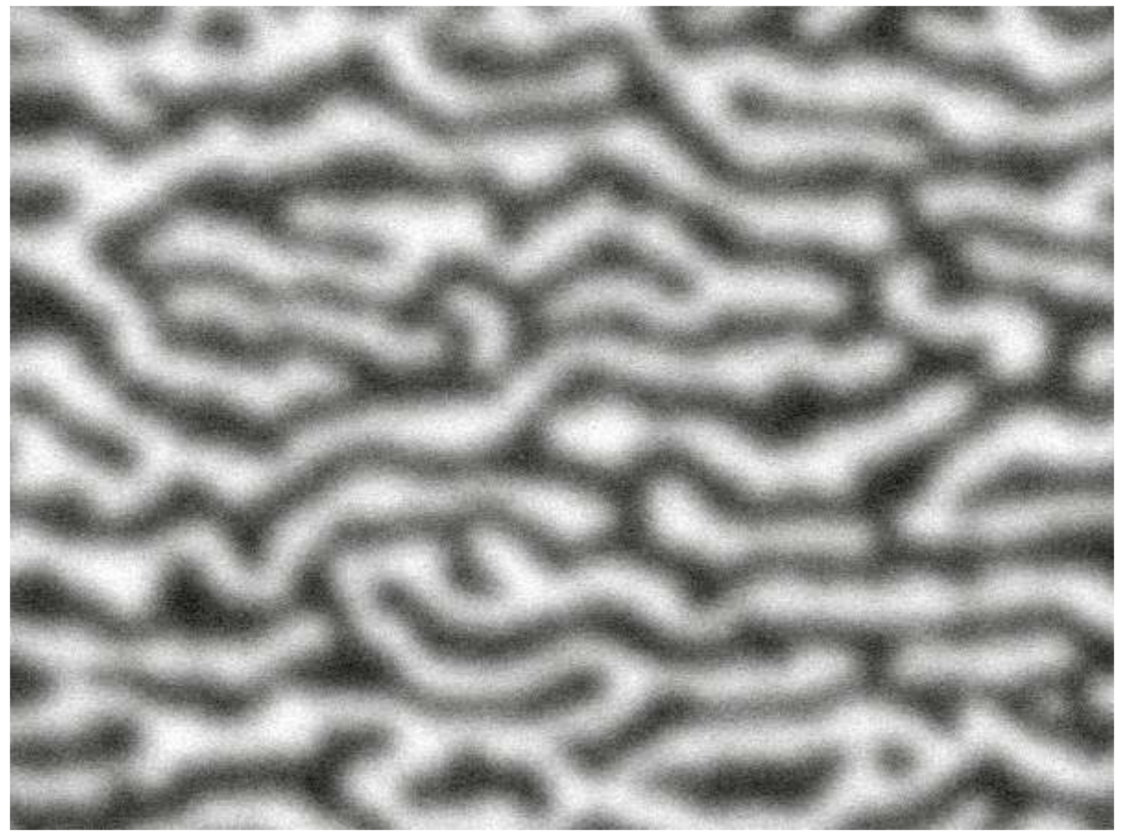
Band-, Bubble domains and Skyrmions



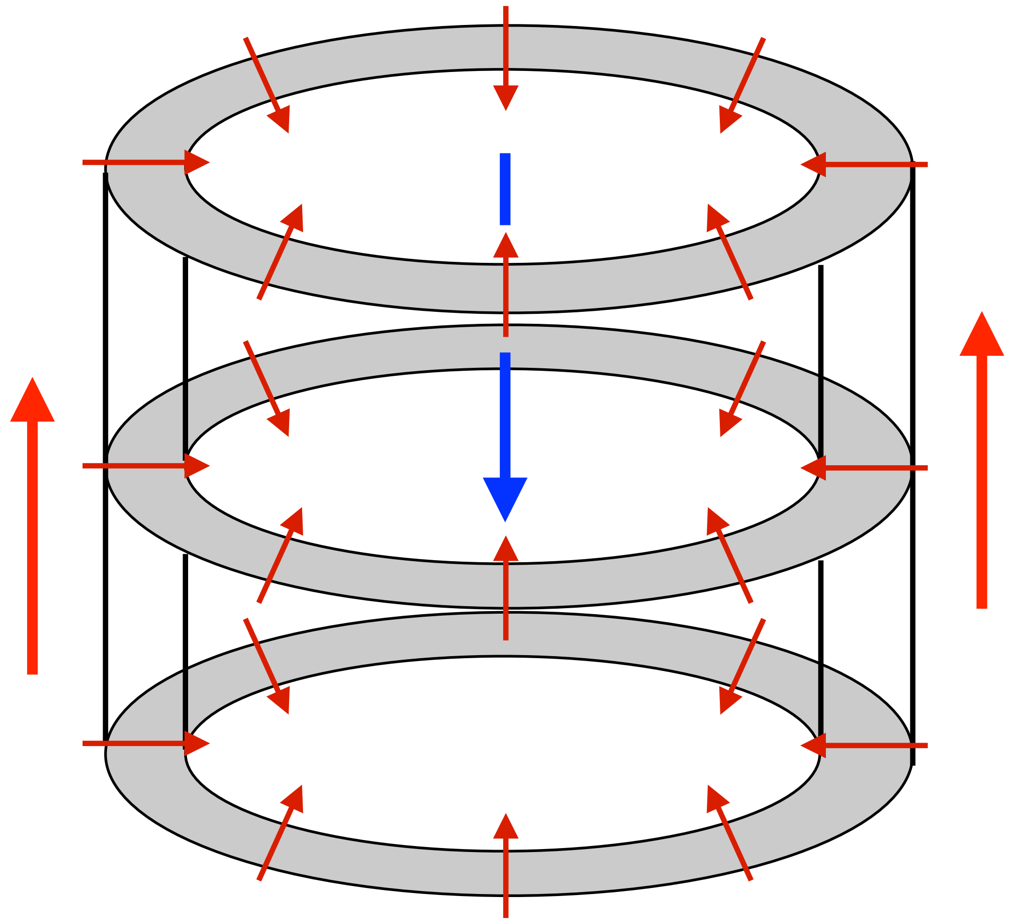
Demagnetized in perpendicular field



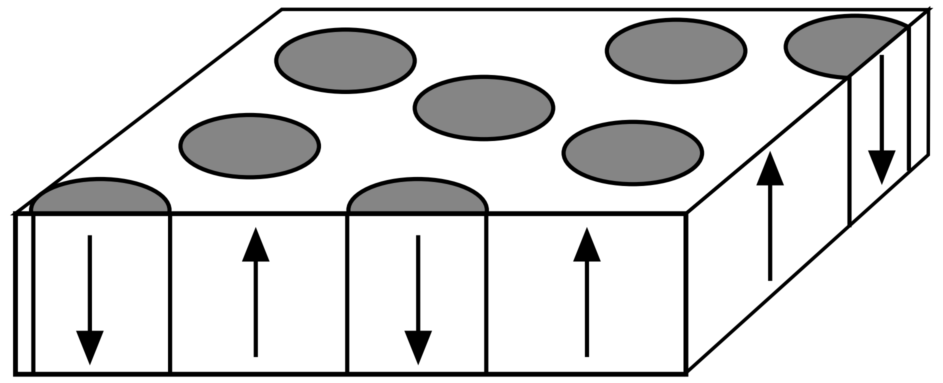
Demagnetized in in-plane field



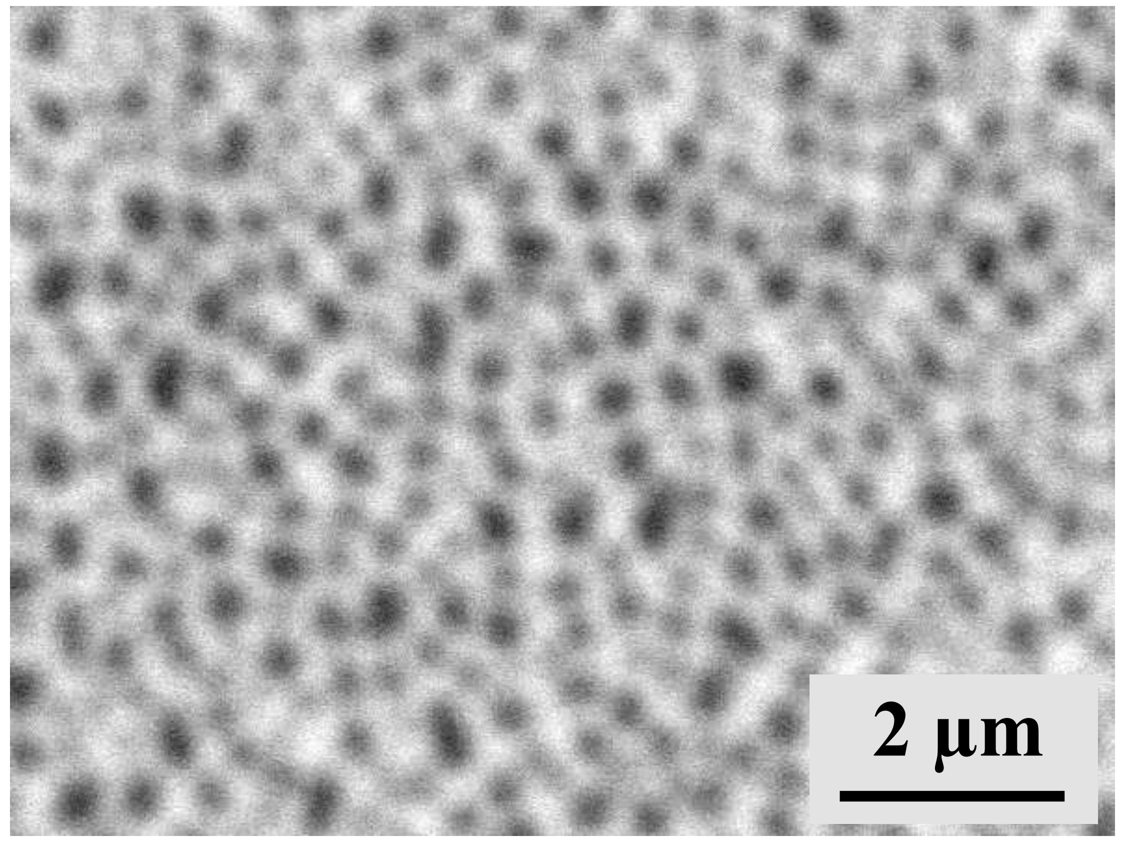
Néel bubble



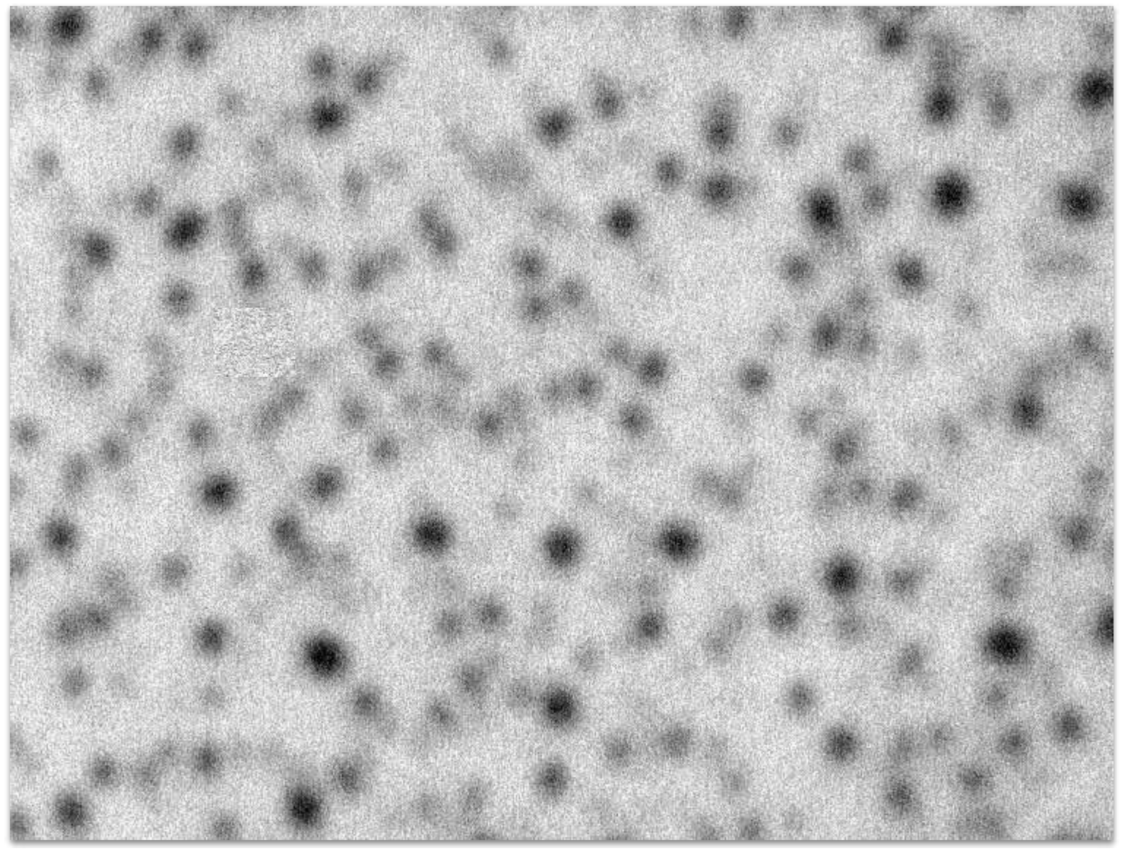
[Pt(1.5 nm)/Co(1 nm)/Ir(1 nm)]$_5$Pt(15 nm)
(sample: A. Hoffmann & group, Argonne)



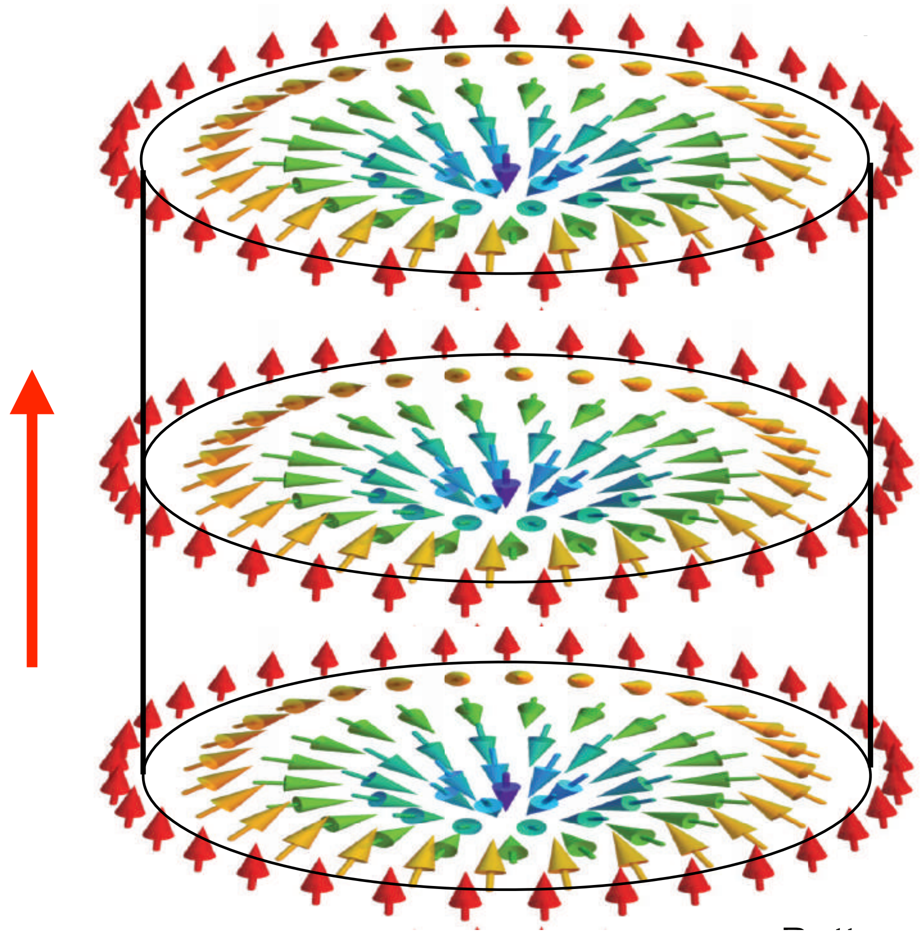
After in-plane field pulse



Bubble collapse in perpendicular field



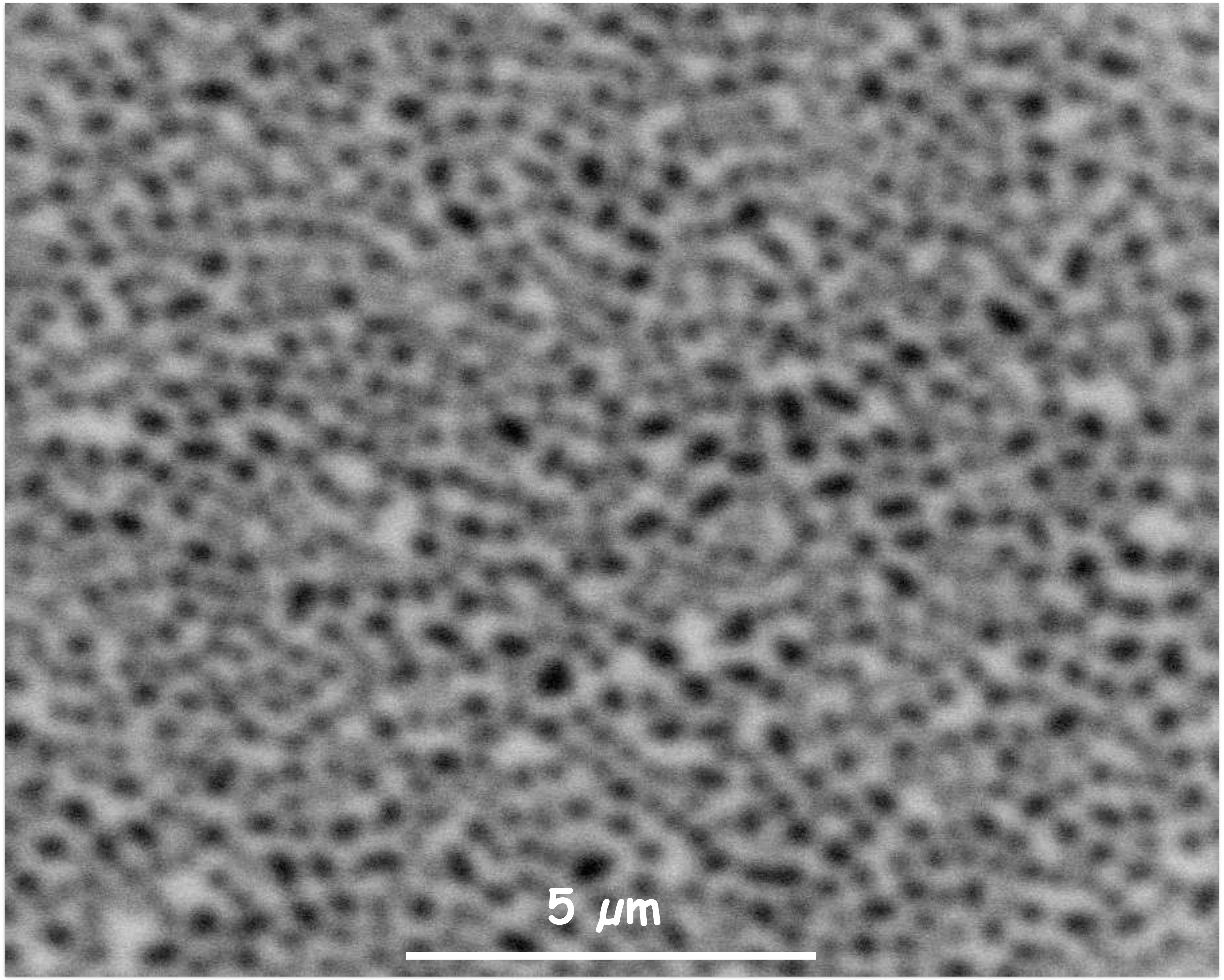
Skyrmion



Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy

[Pt(1.5 nm)/Co(1 nm)/Ir(1 nm)]$_5$Pt(15 nm)
(sample: A. Hoffmann & group, Argonne)



Resolution of Kerr-microscopy

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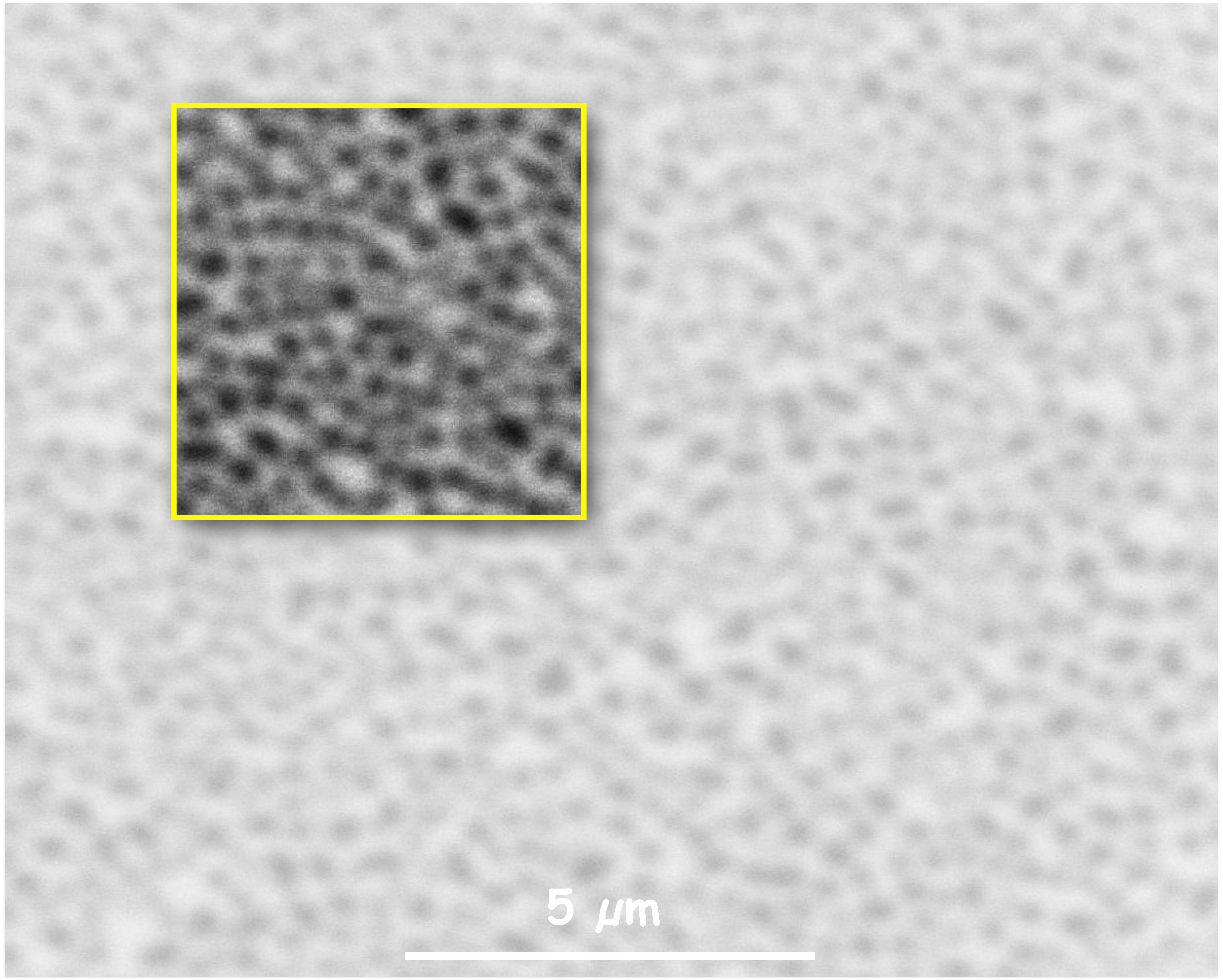

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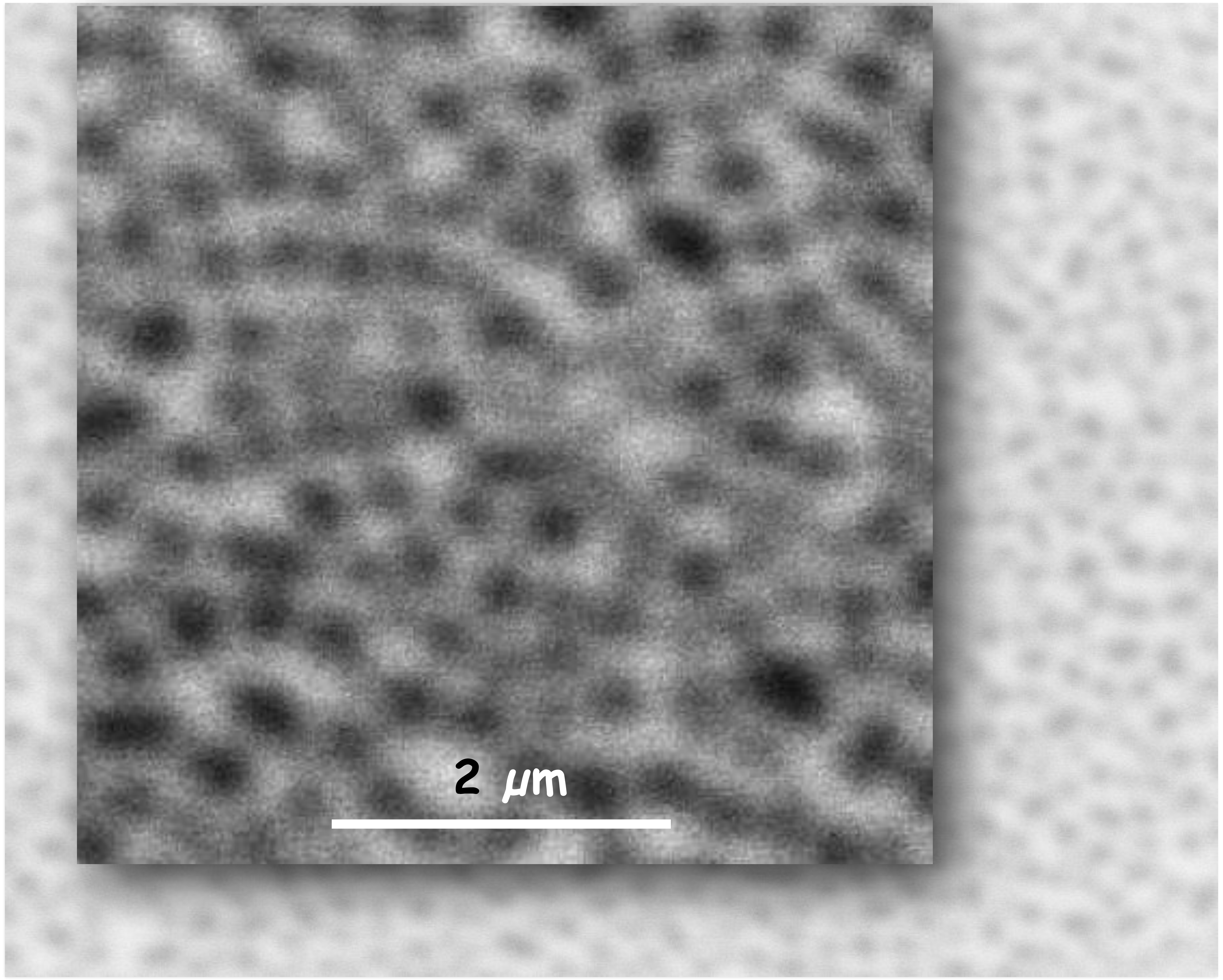

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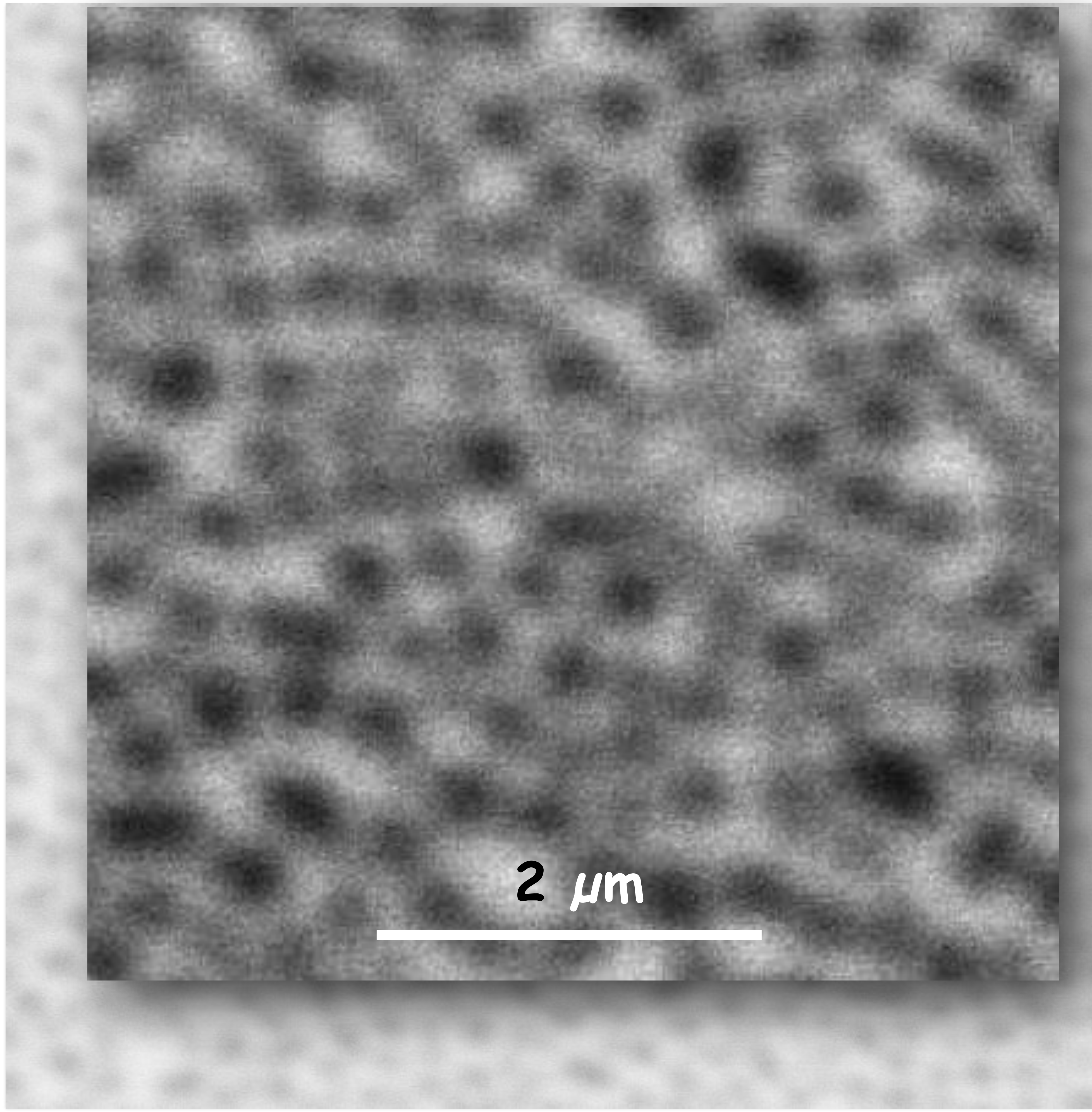
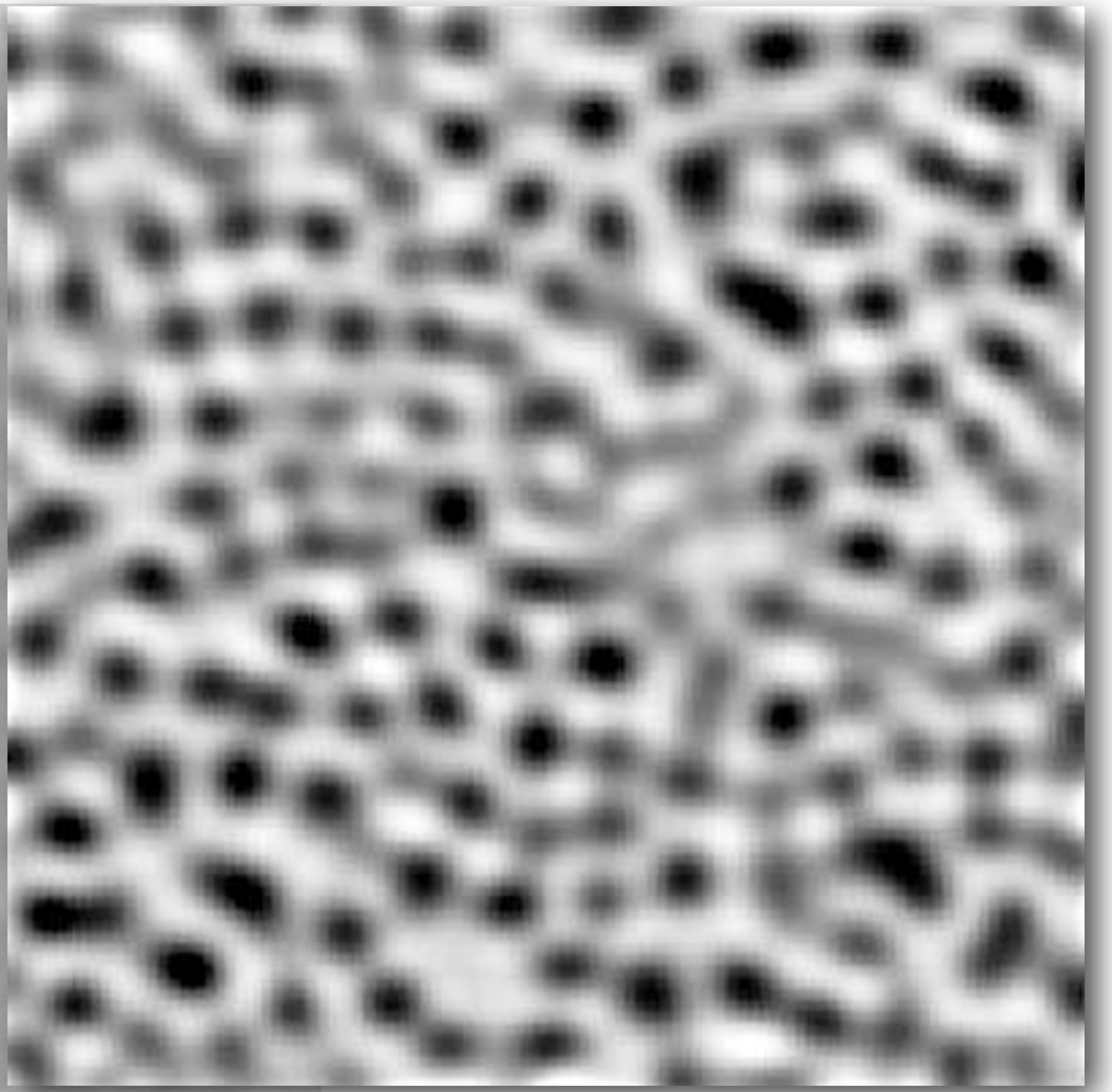


Deconvoluted



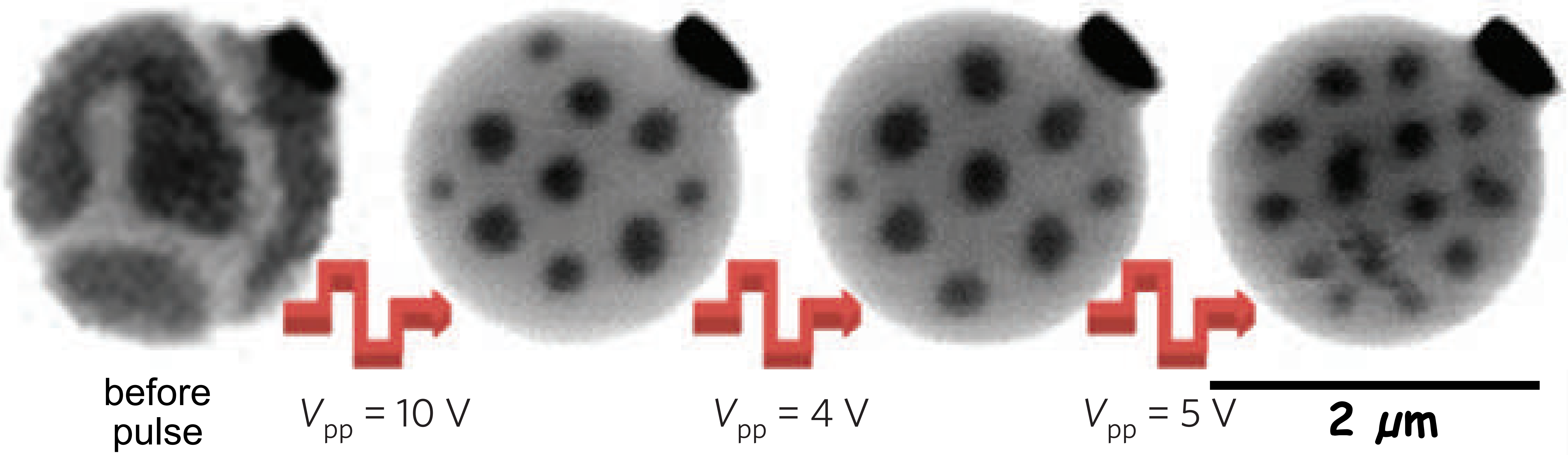
Deconvolution:
N. Gorn & D. Berkov, Jena

Resolution of Kerr-microscopy

Ultra-high-resolution Kerr microscopy

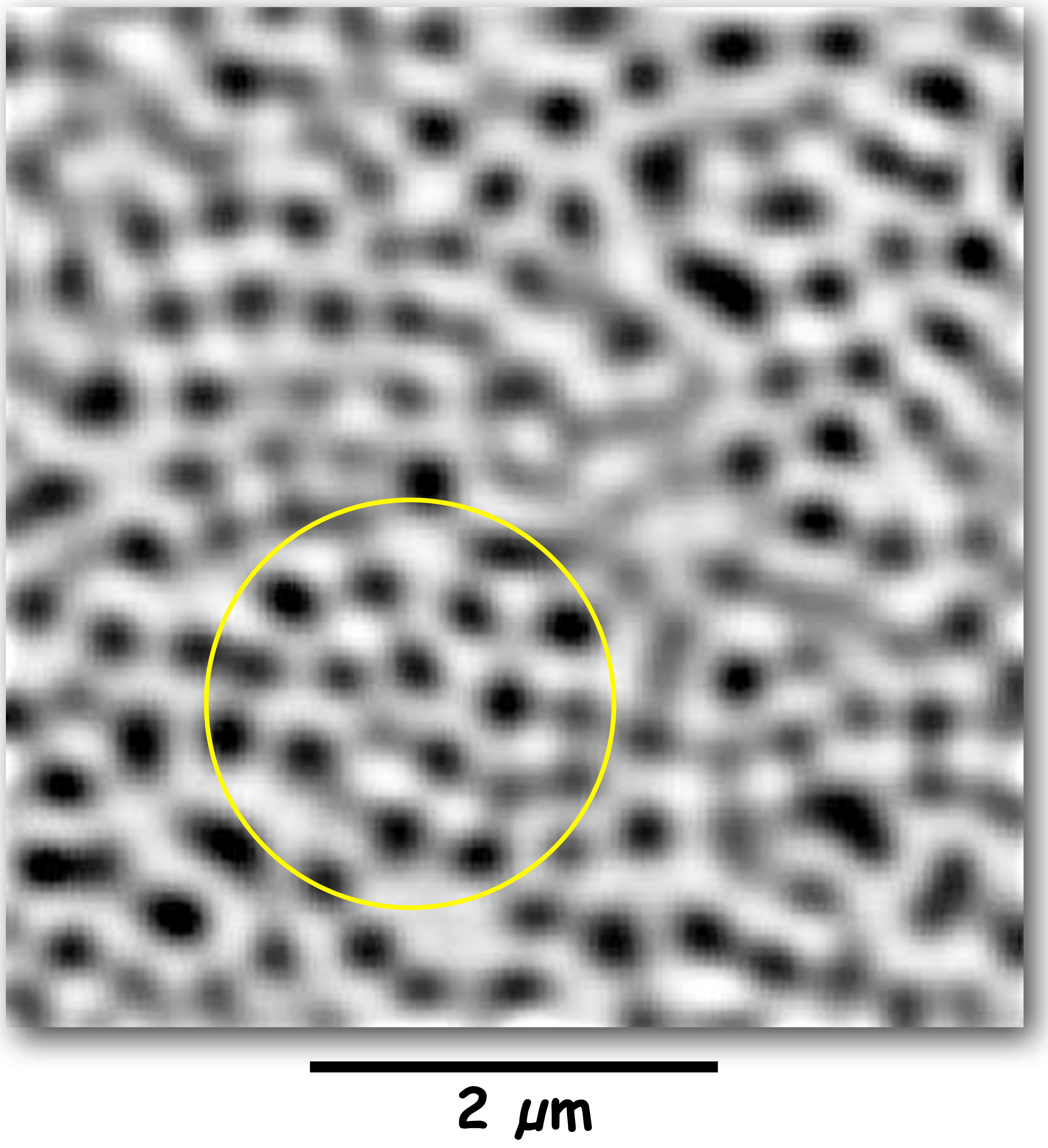
[Pt(3 nm)/Co(0.9 nm)/Ta(4 nm)]$_{15}$

Imaged by STXM (Scanning Transmission X-ray Microscopy), based on XMCD



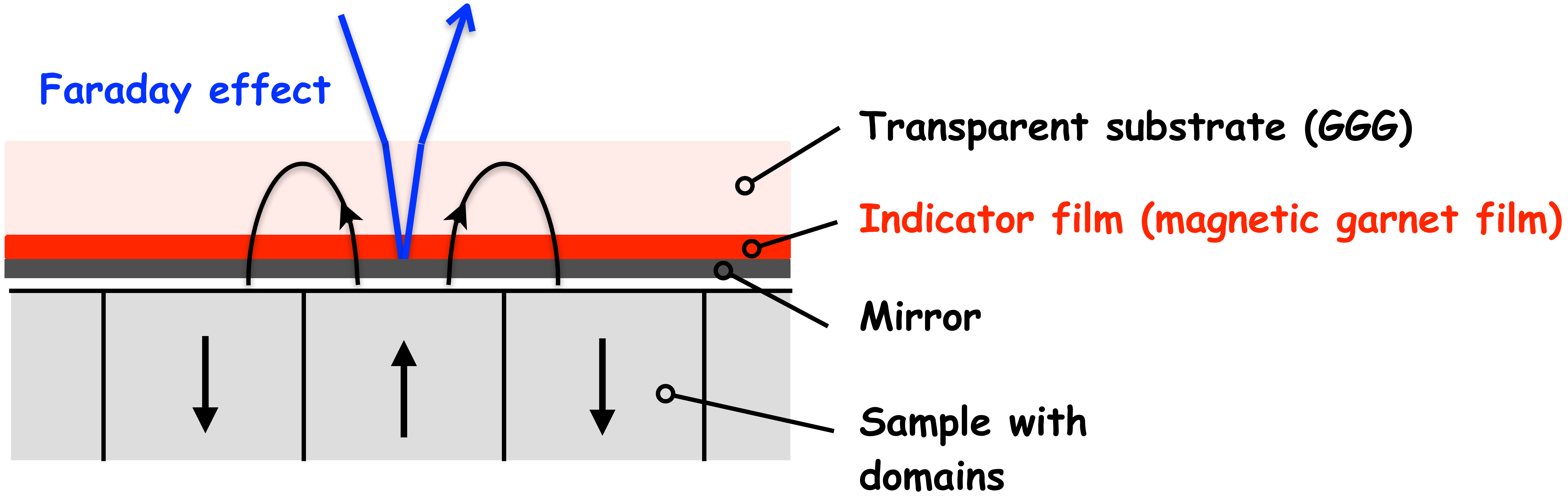
S. Woo, et al., Nature Materials 15, 501 (2016)

Deconvoluted

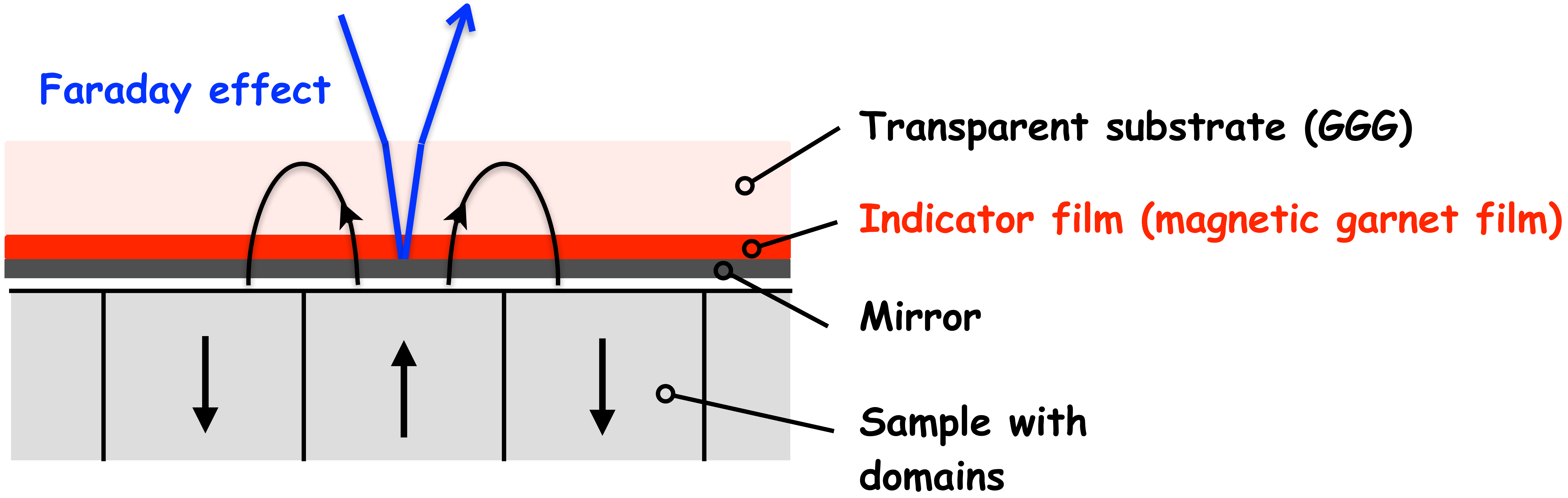


5.
MOIF microscopy

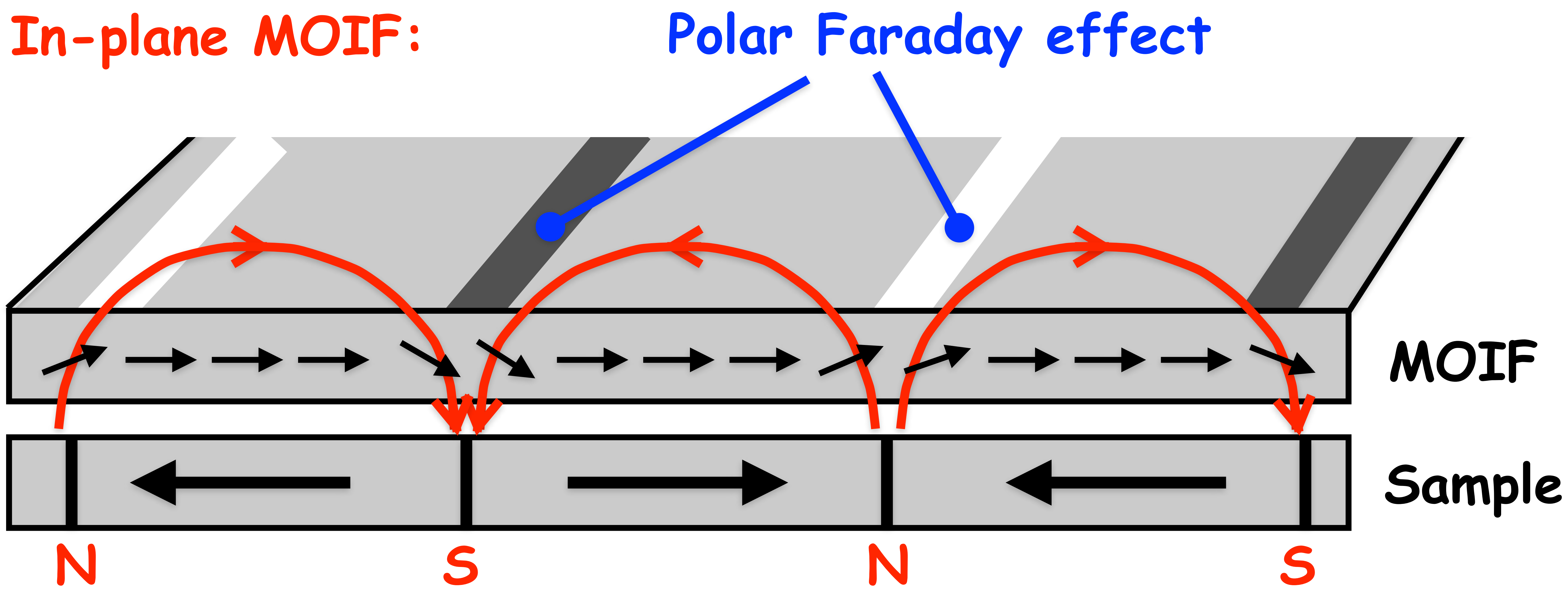
5. MOIF Microscopy

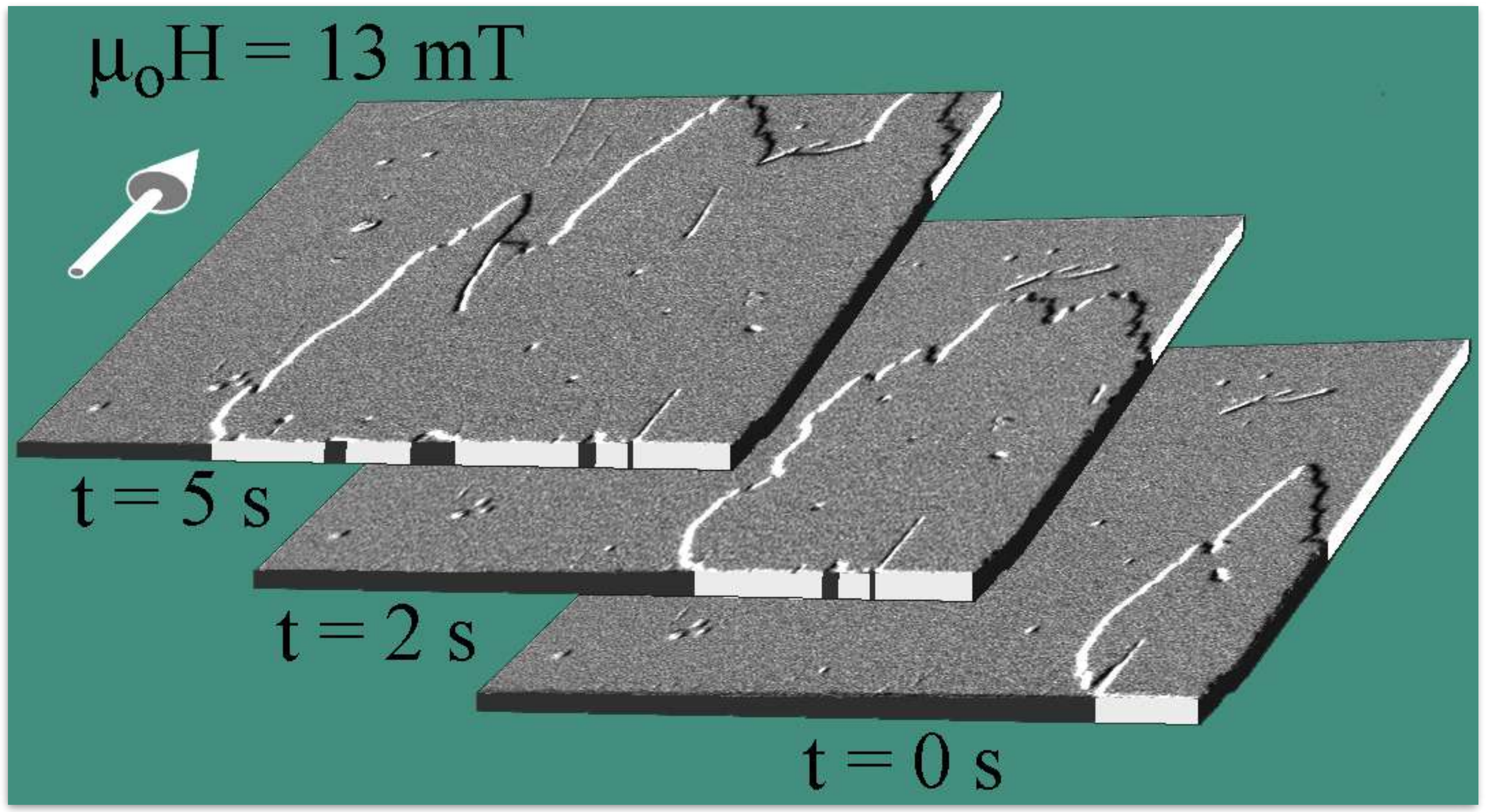


5. MOIF Microscopy

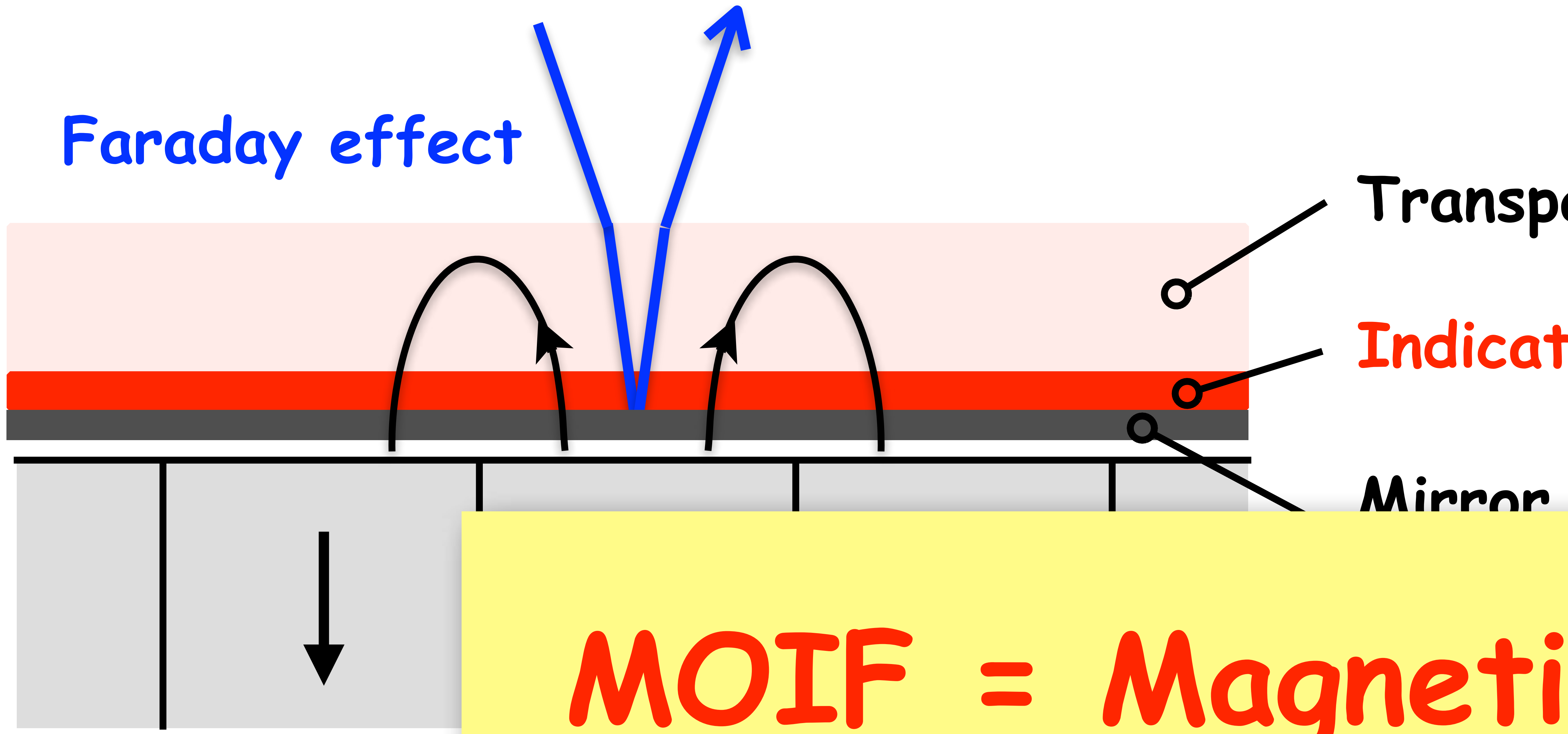


Co/Ru/Co trilayer
(courtesy R. Shull, NIST)





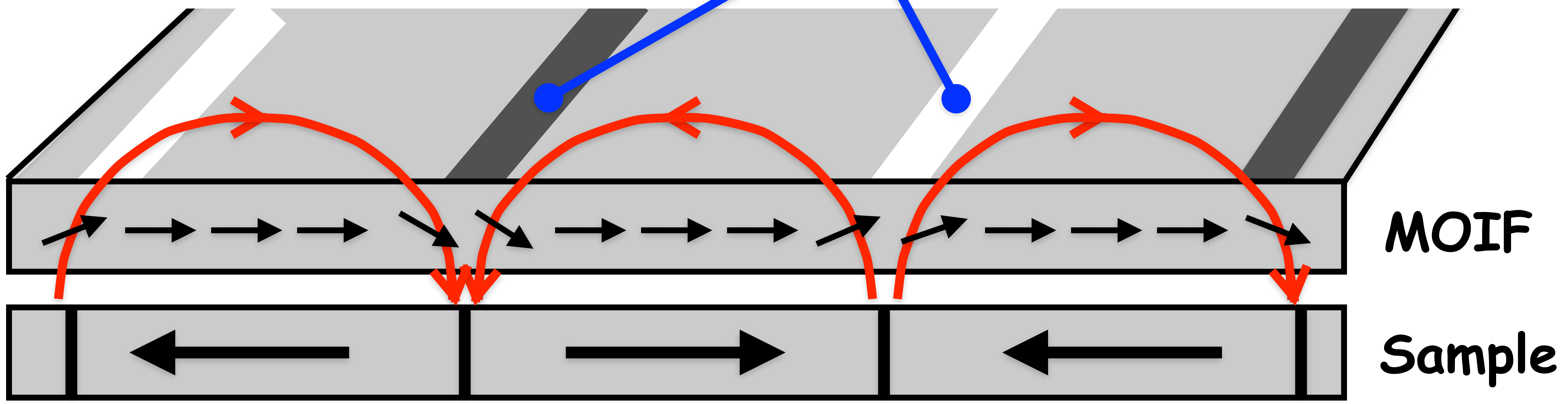
5. MOIF Microscopy

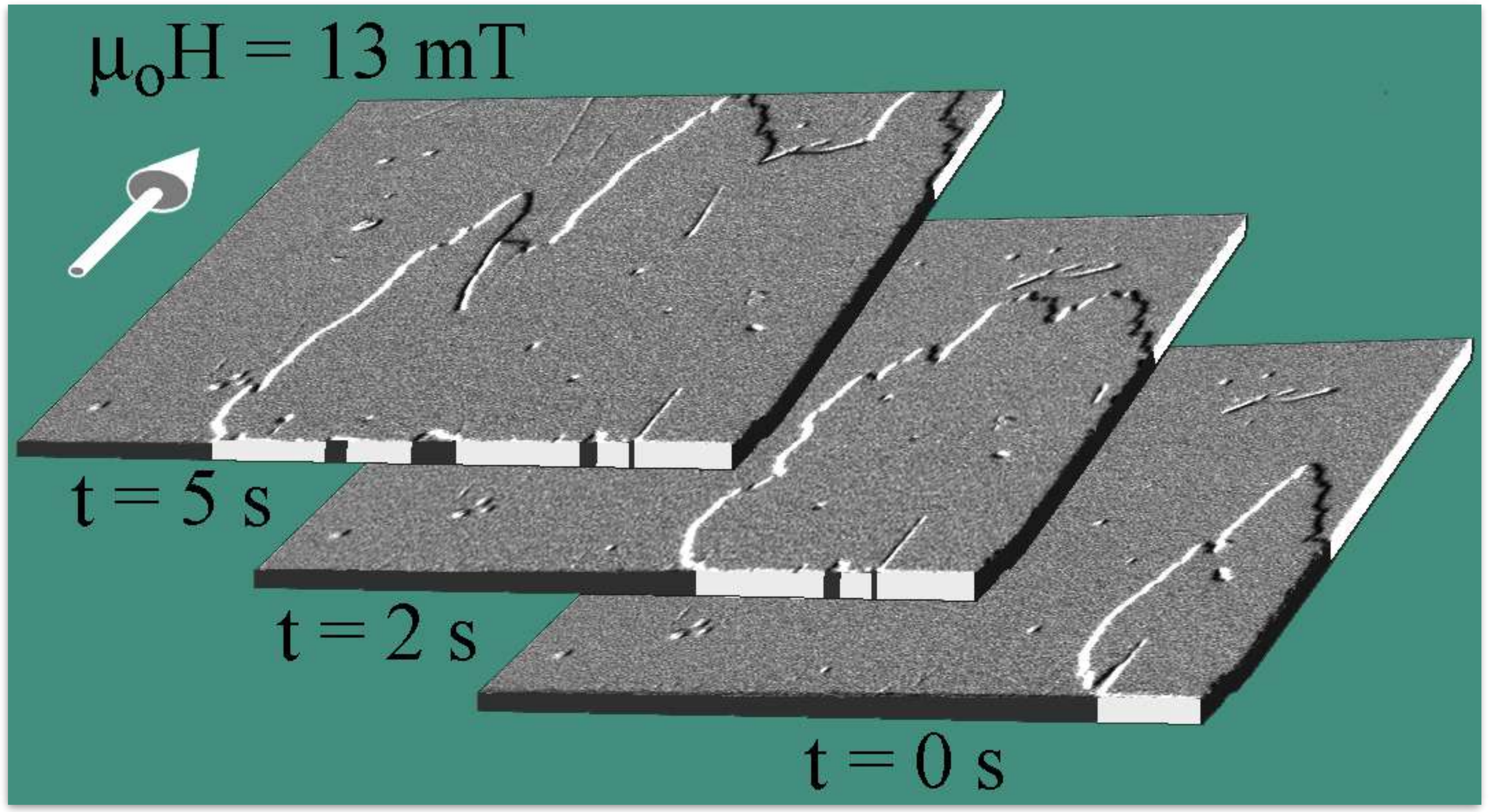


MOIF = Magnetic Pole Microscopy

In-plane MOIF:

Polar Faraday effect

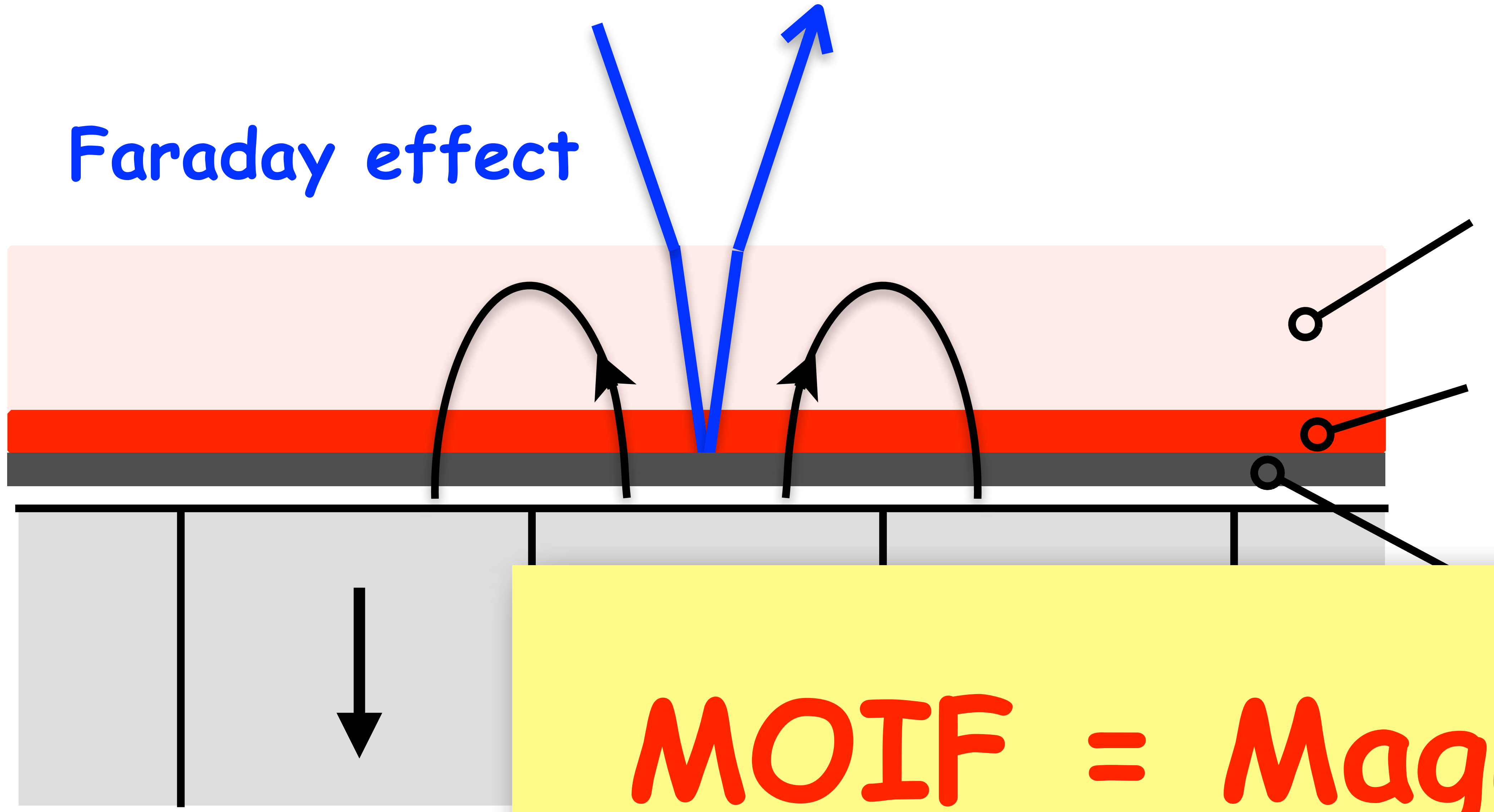




o/Ru/Co trilayer
(courtesy R. Shull, NIST)

5. MOIF Microscopy

Faraday effect

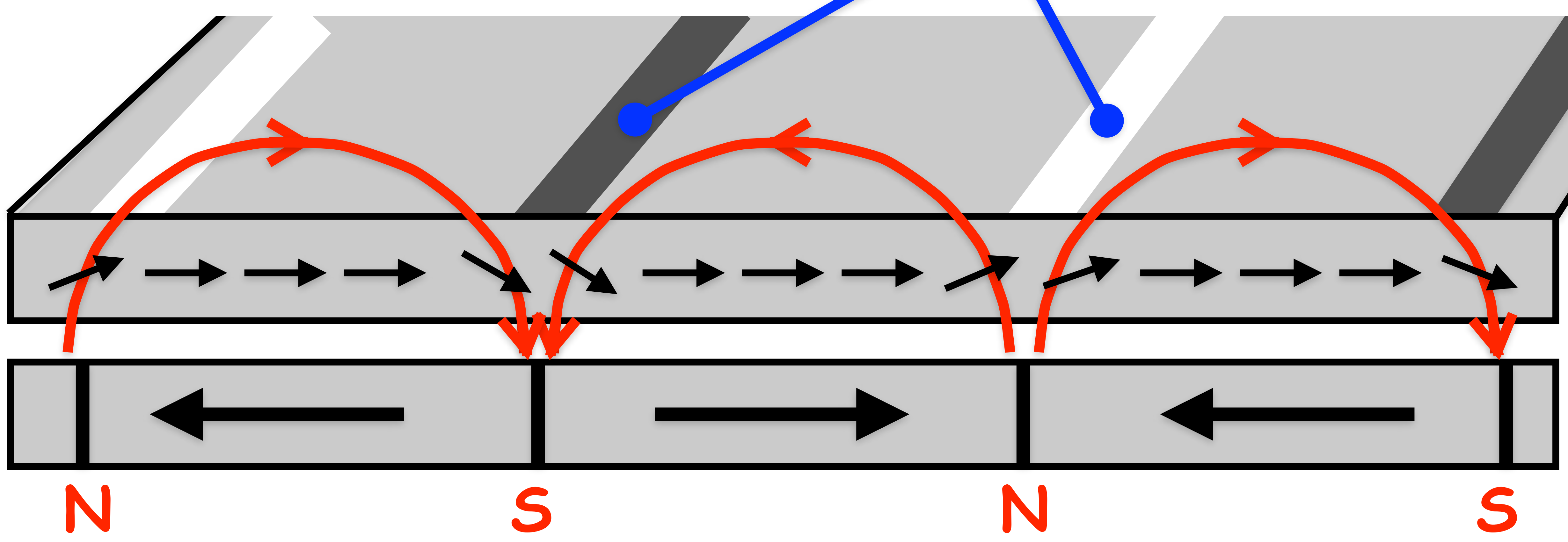


MOIF = Magnetic Pole Microscopy

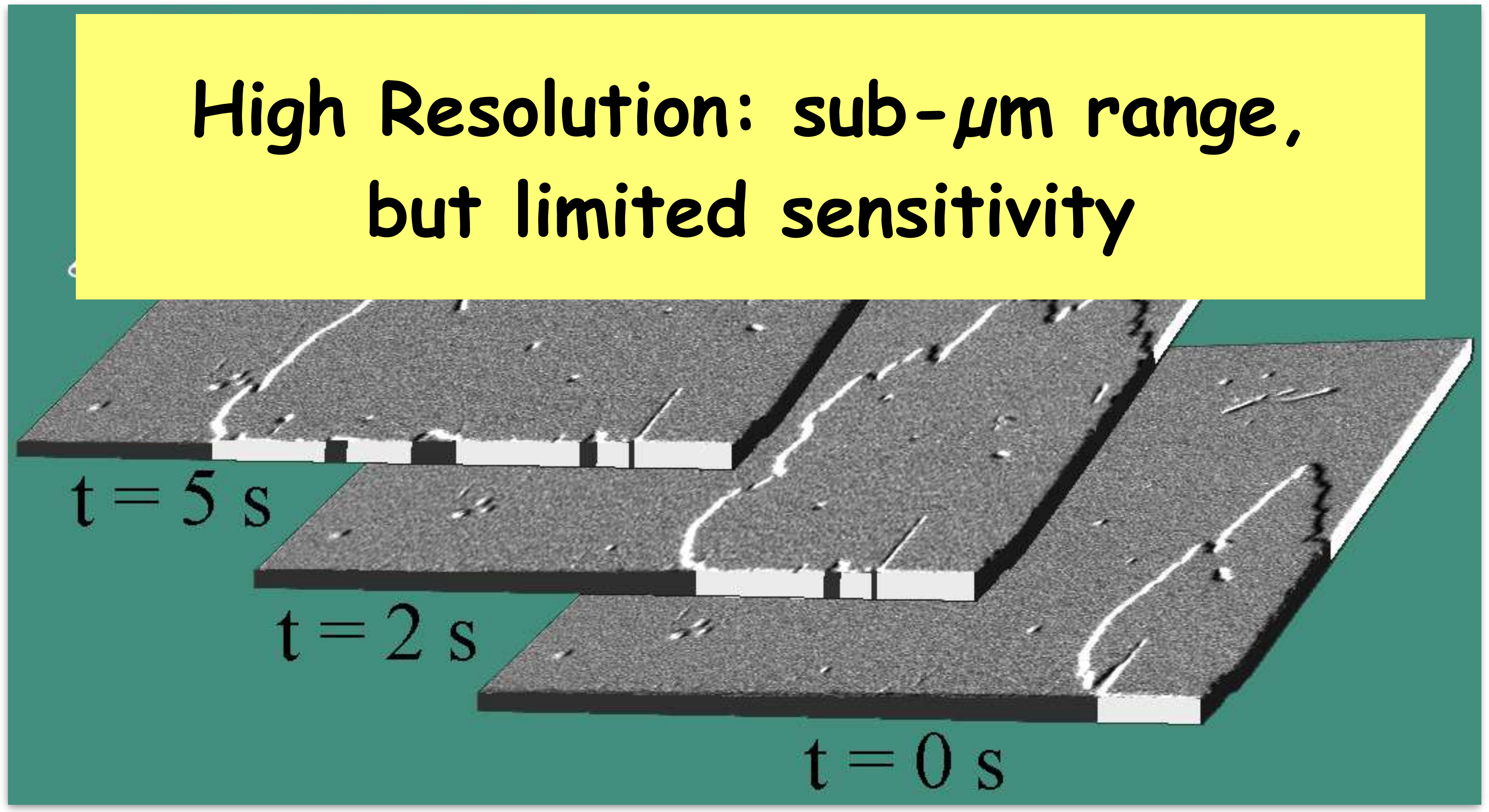
Co/Ru/Co trilayer
(courtesy R. Shull, NIST)

In-plane MOIF:

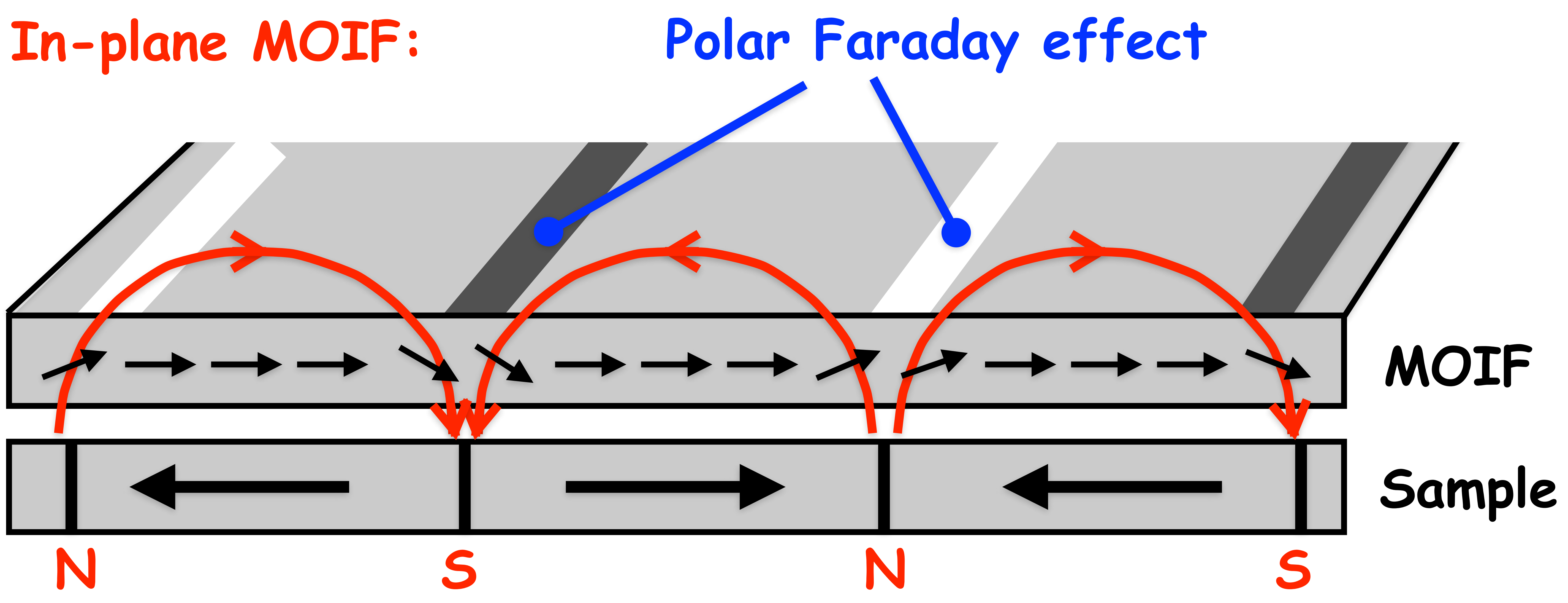
Polar Faraday effect

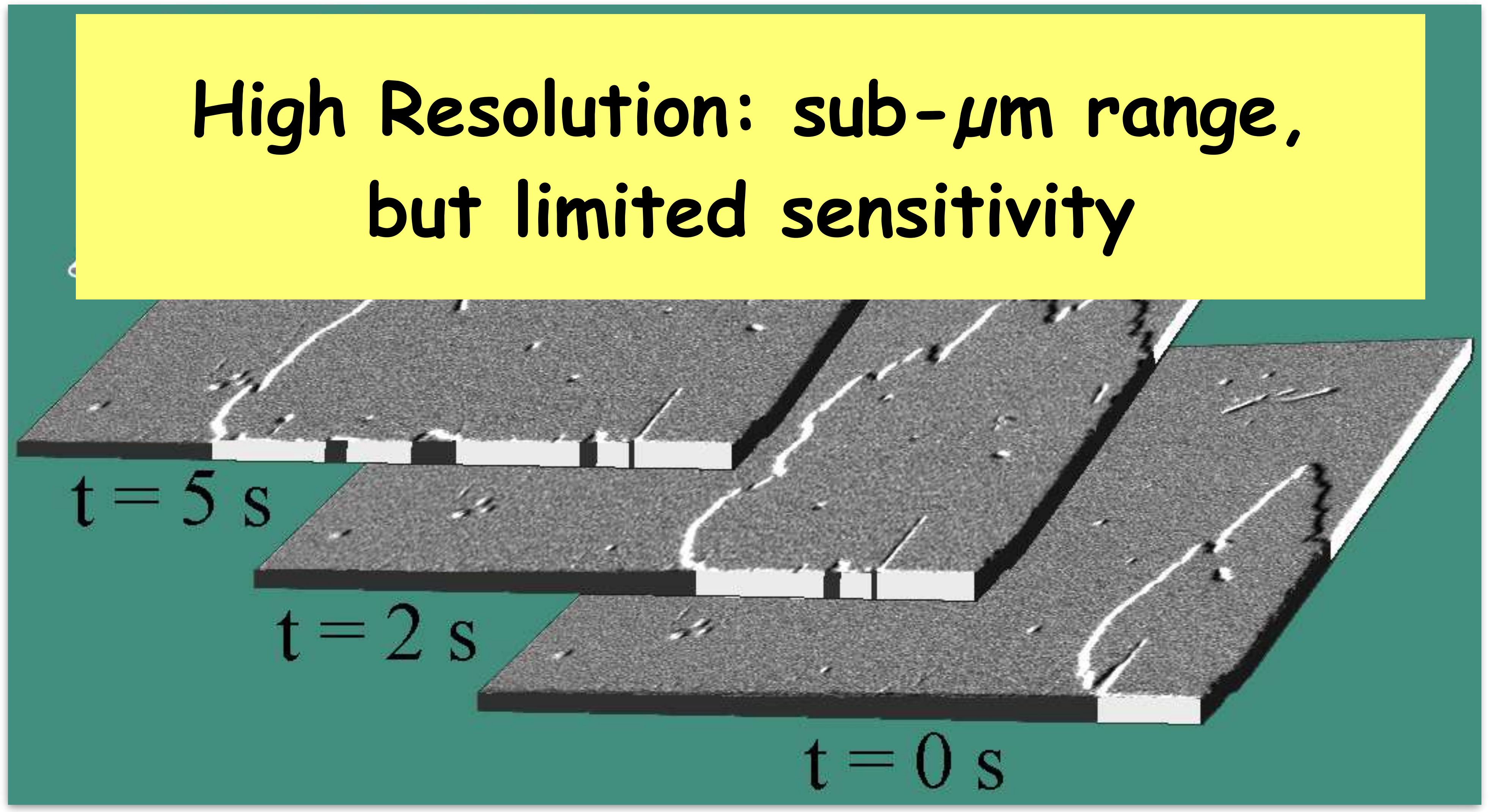


High Resolution: sub-μm range, but limited sensitivity



5. MOIF Microscopy

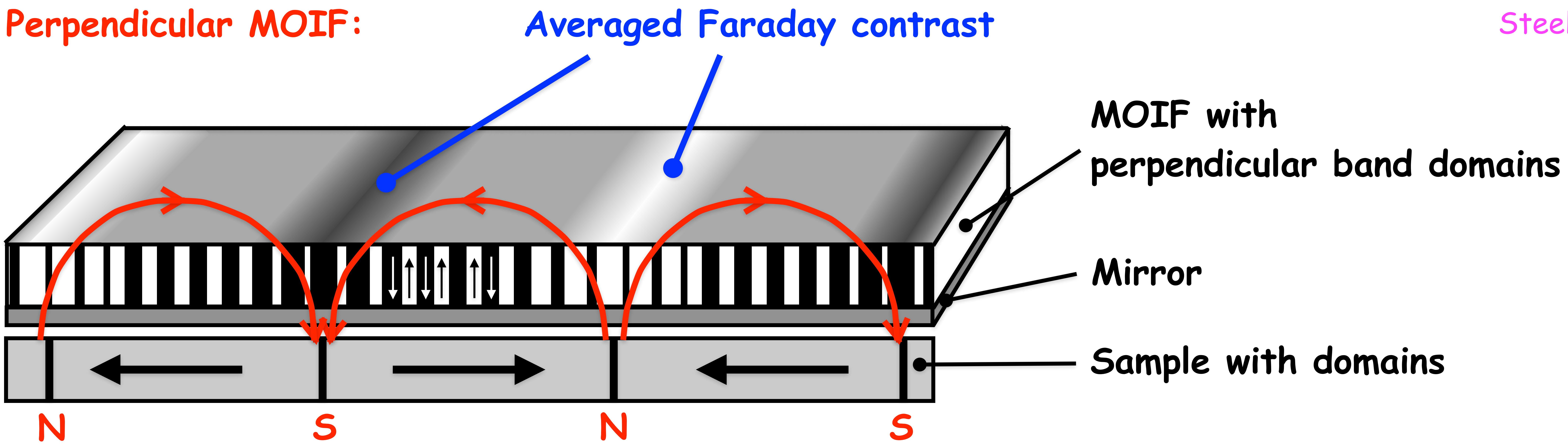




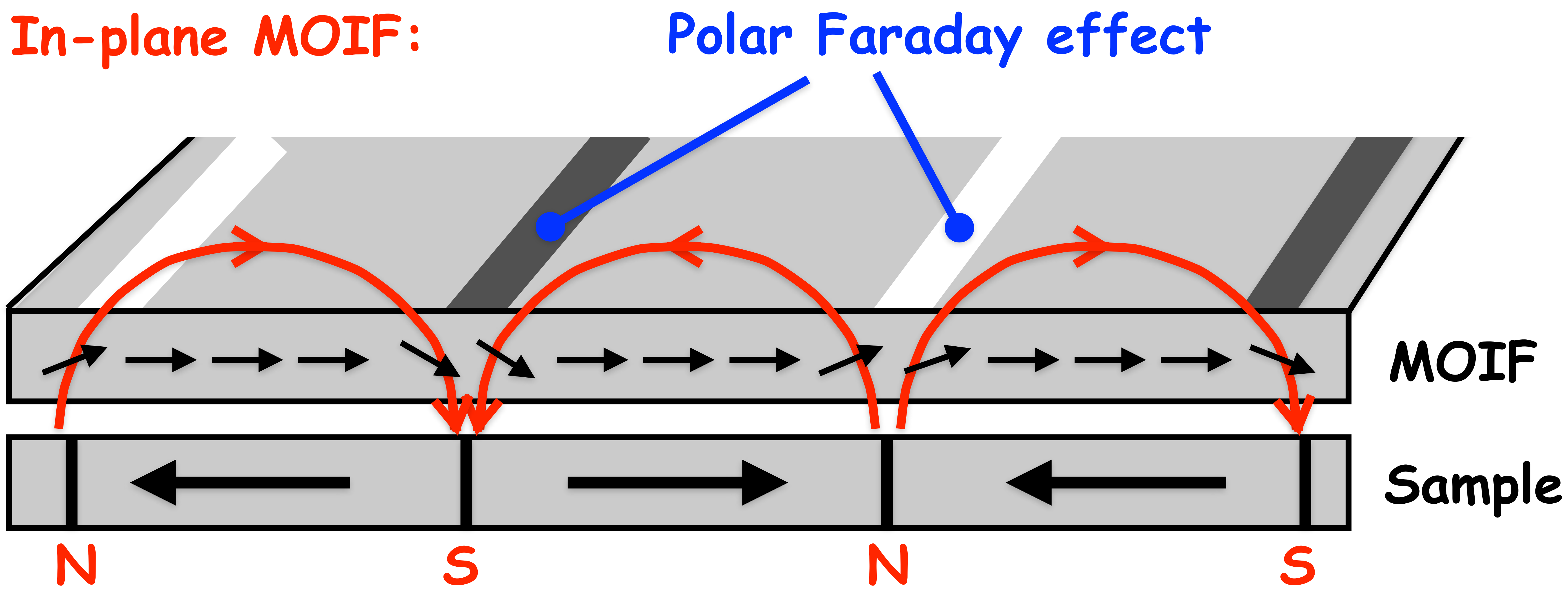
5. MOIF Microscopy

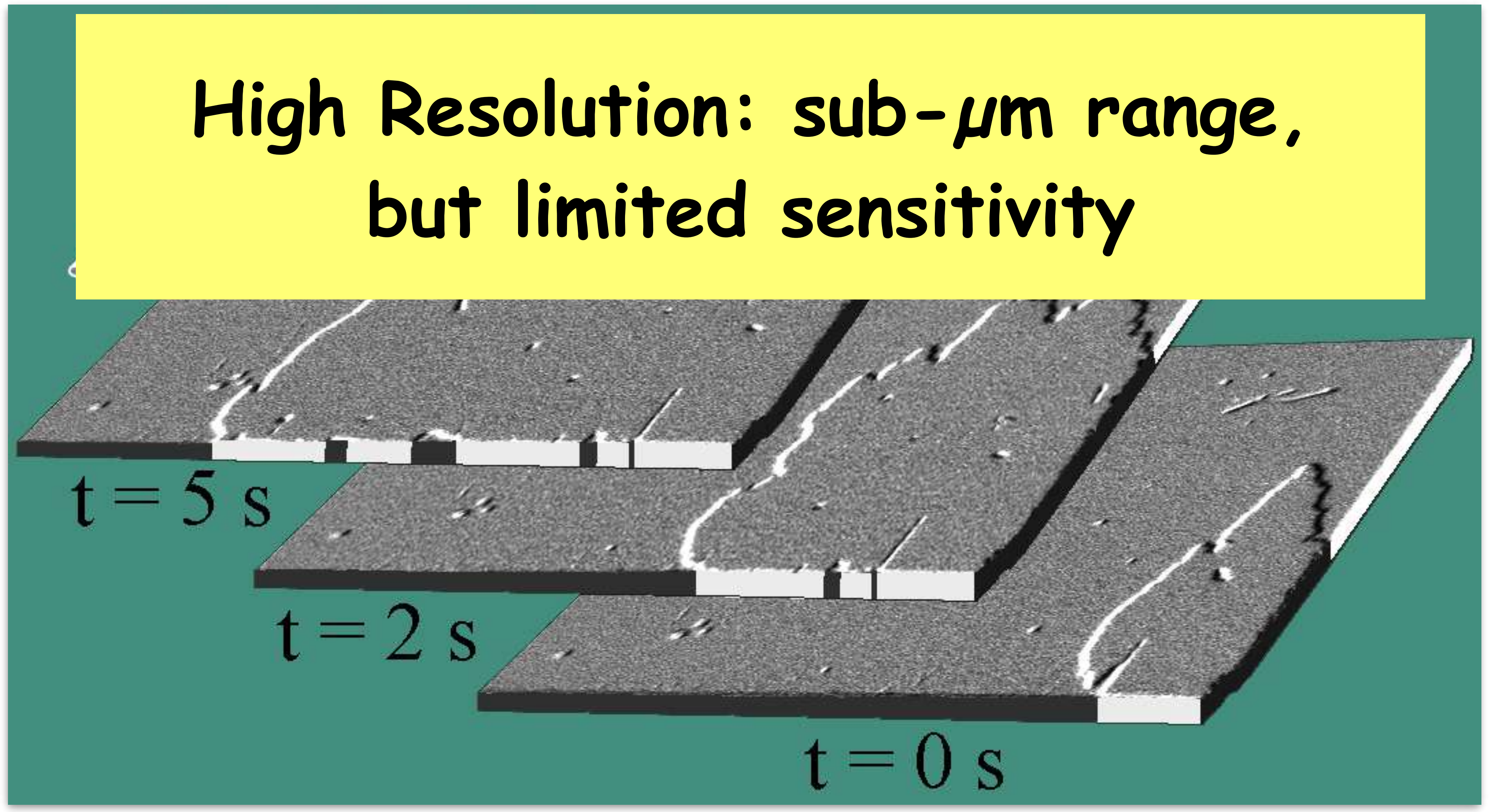
H. Richert et al.,
Steel Res. Int. 87 (2015)

Perpendicular MOIF:



In-plane MOIF:

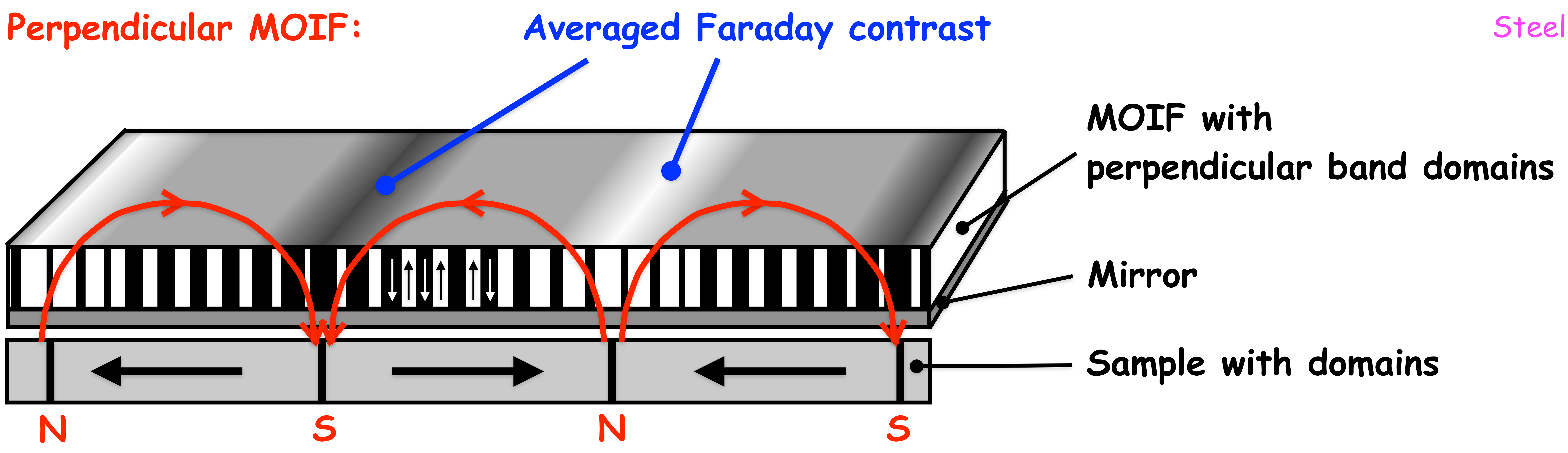


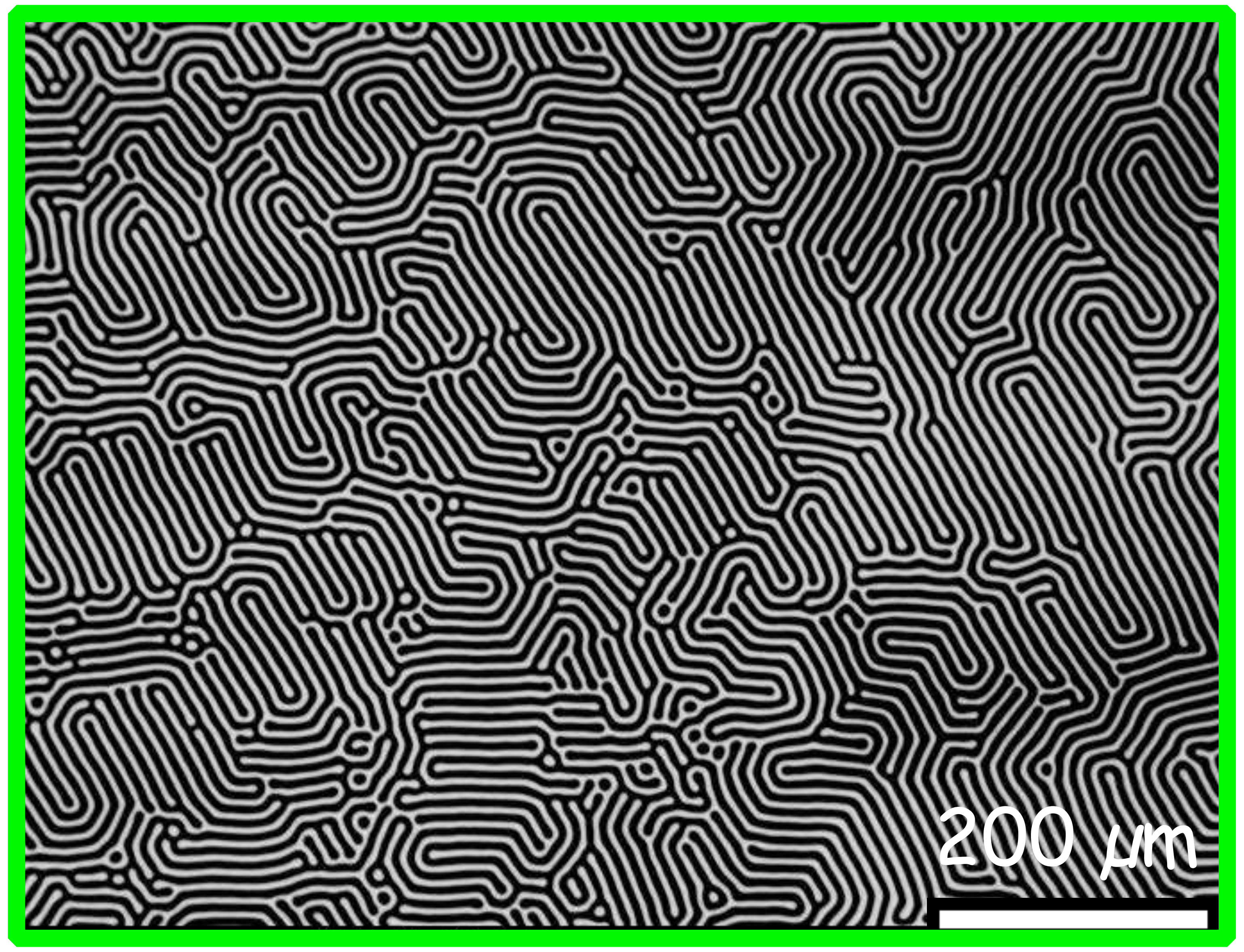


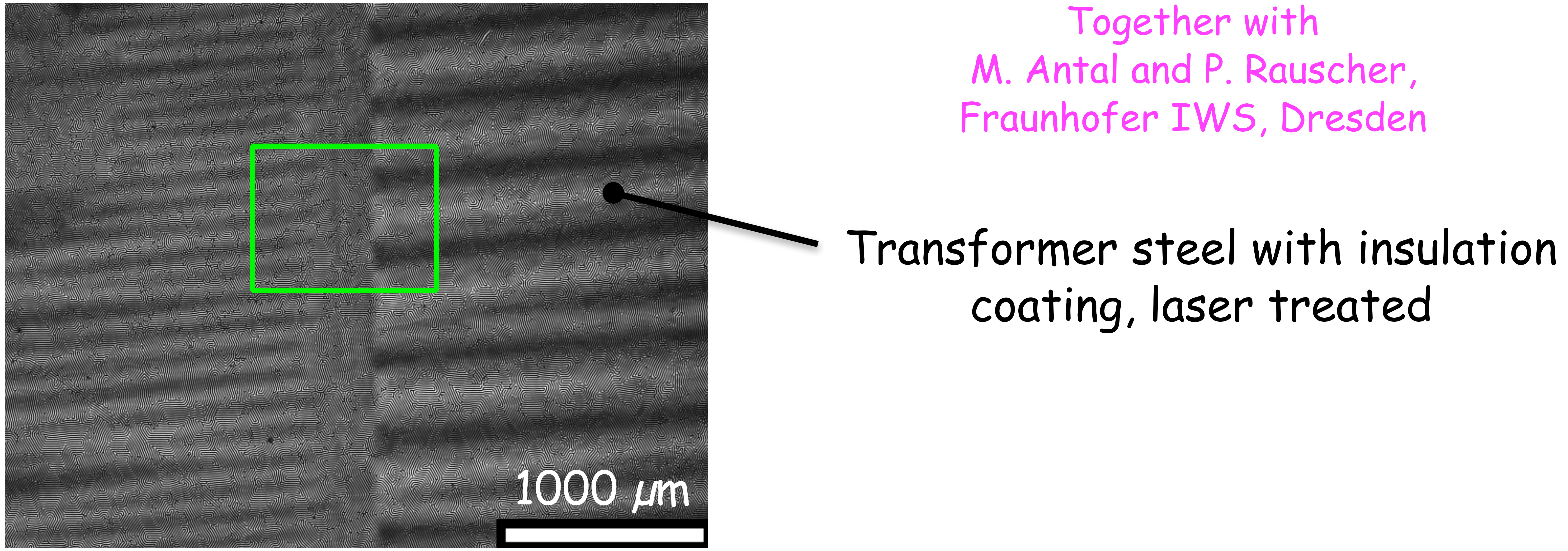
5. MOIF Microscopy

H. Richert et al.,
Steel Res. Int. 87 (2015)

Perpendicular MOIF:





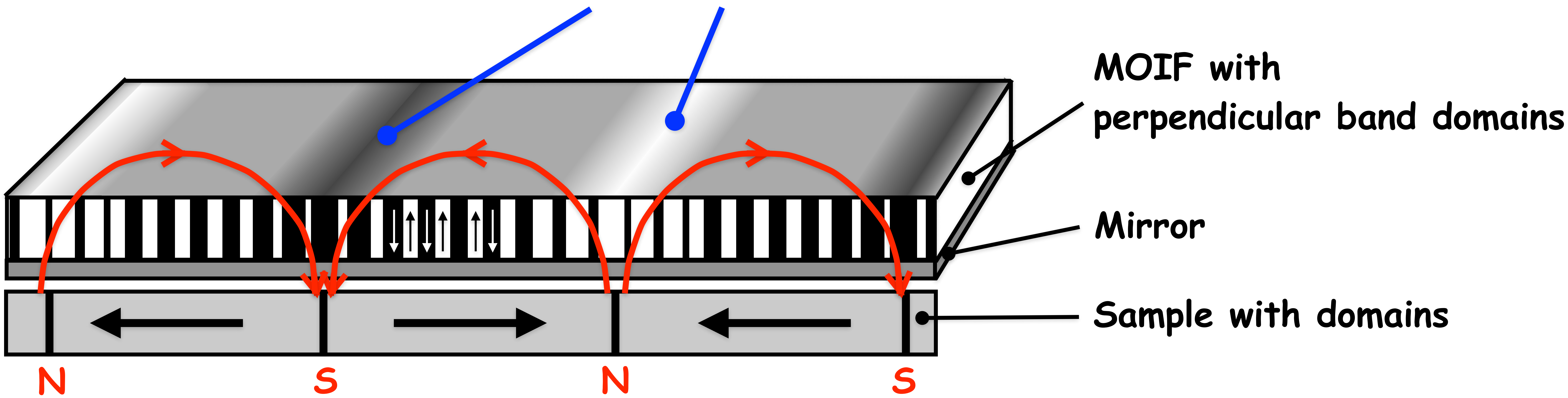


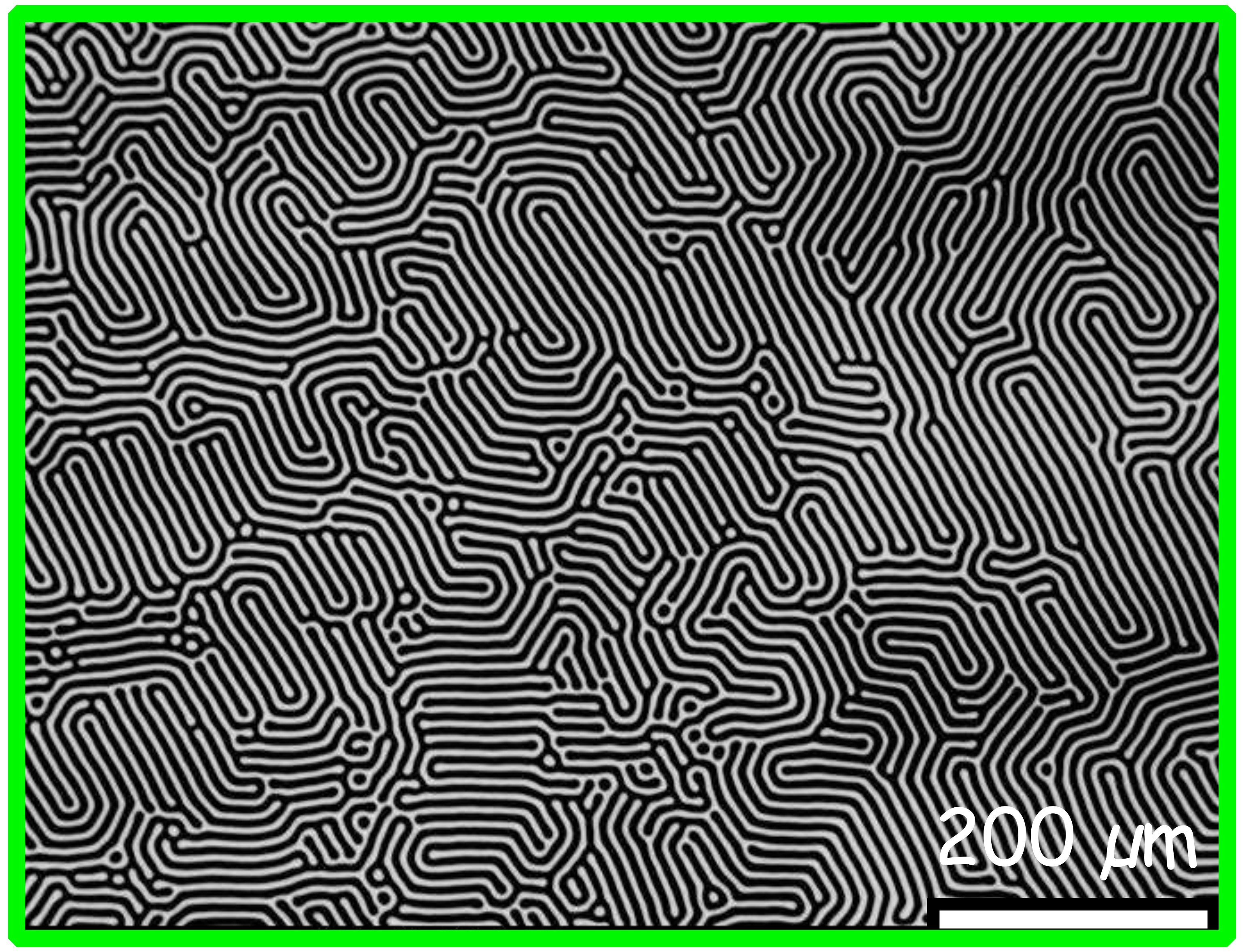
Together with
M. Antal and P. Rauscher,
Fraunhofer IWS, Dresden

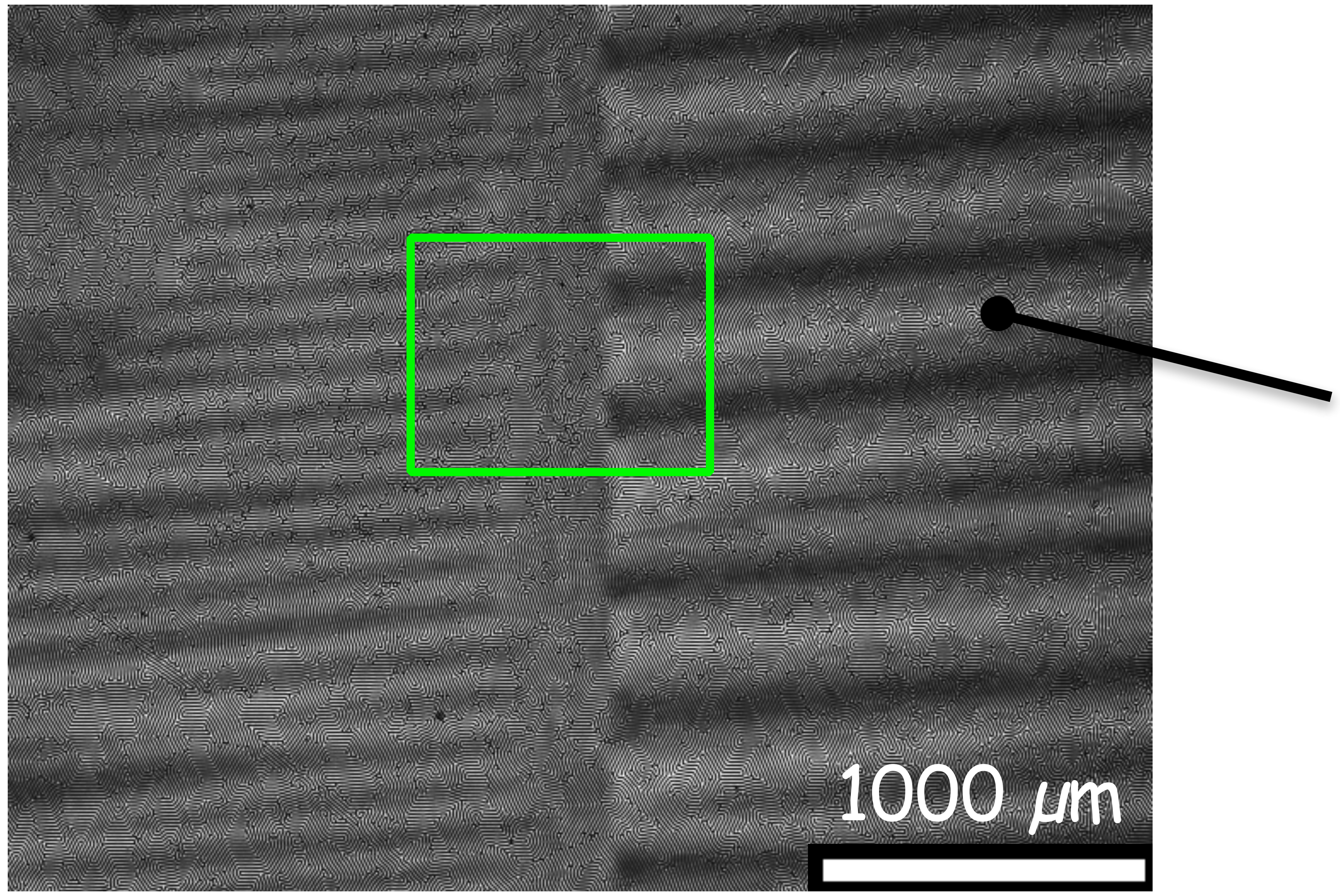
5. MOIF Microscopy

H. Richert et al.,
Steel Res. Int. 87 (2015)

Perpendicular MOIF:





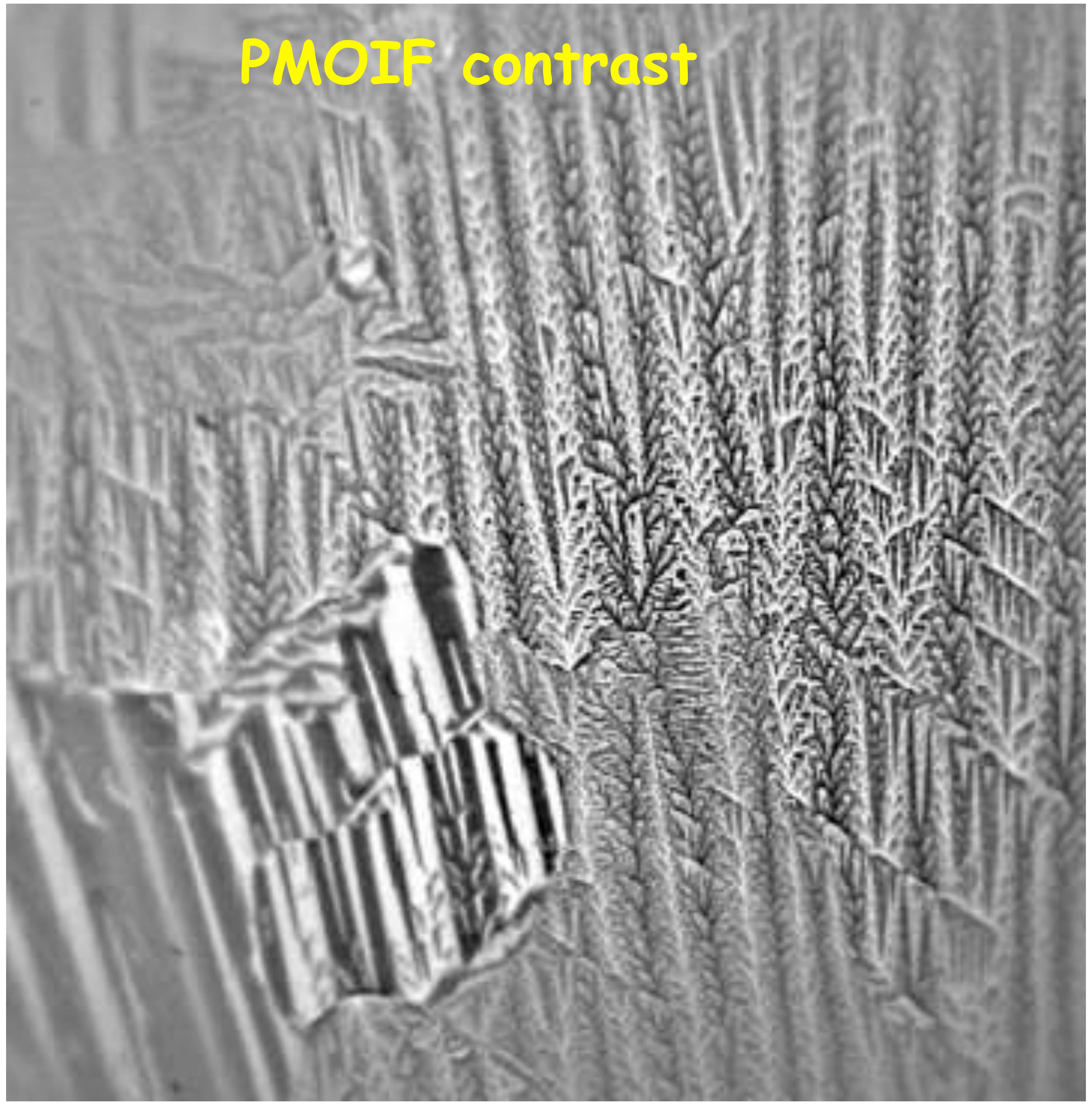


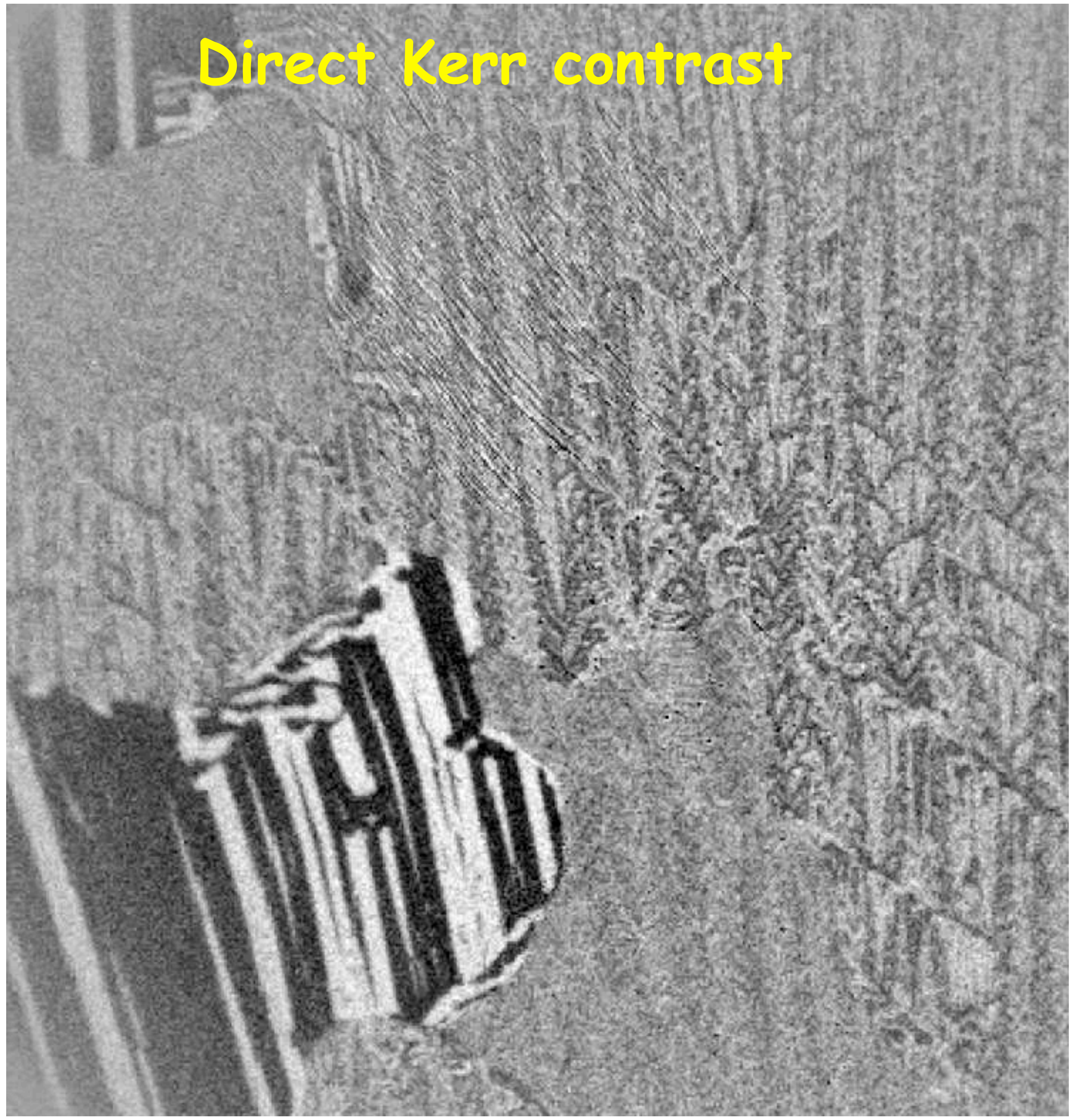
Together with
M. Antal and P. Rauscher,
Fraunhofer IWS, Dresden

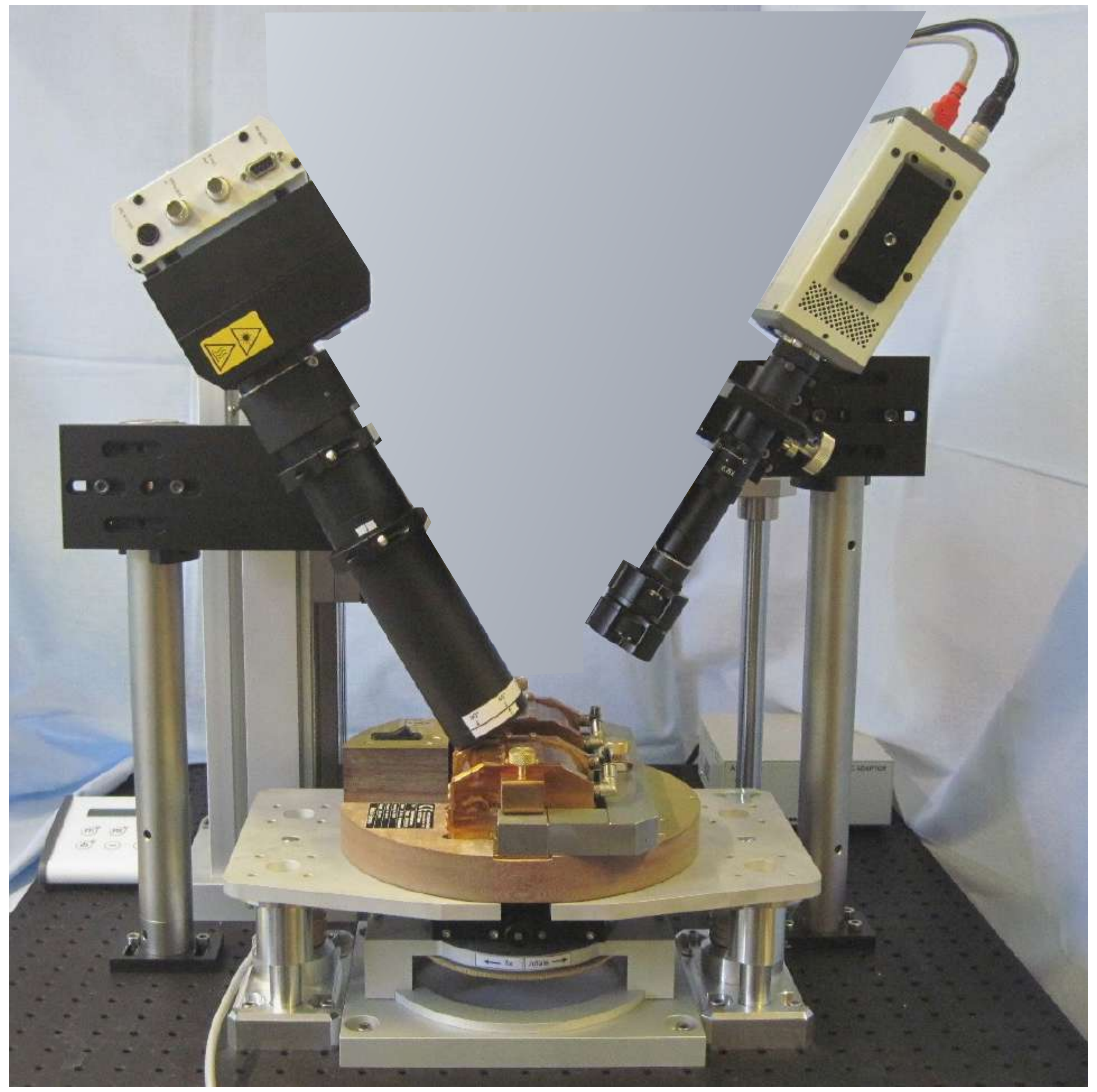
Resolution: order of µm,
high sensitivity

5. MOIF Microscopy

Transformer steel with polished surface, overview microscope



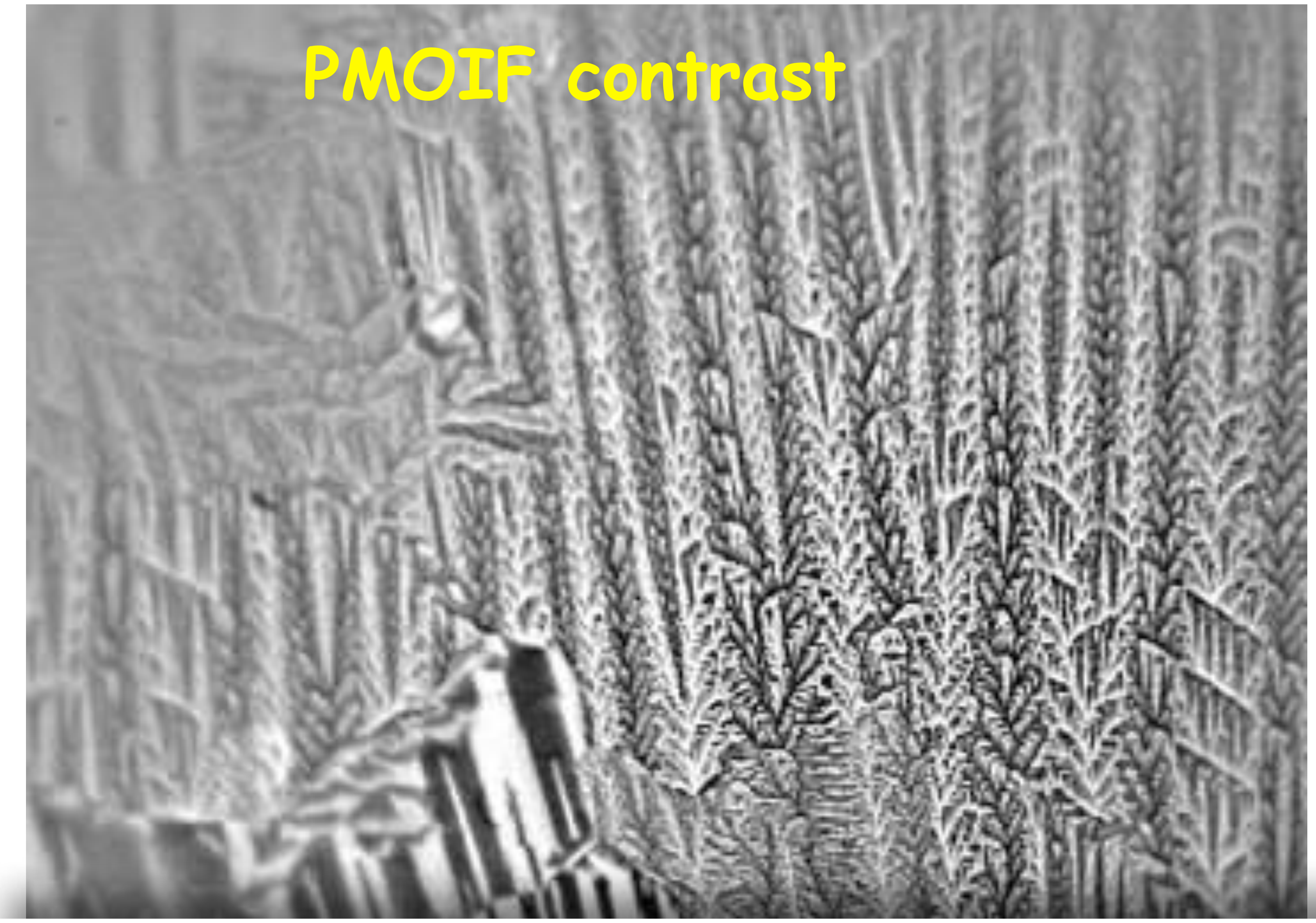


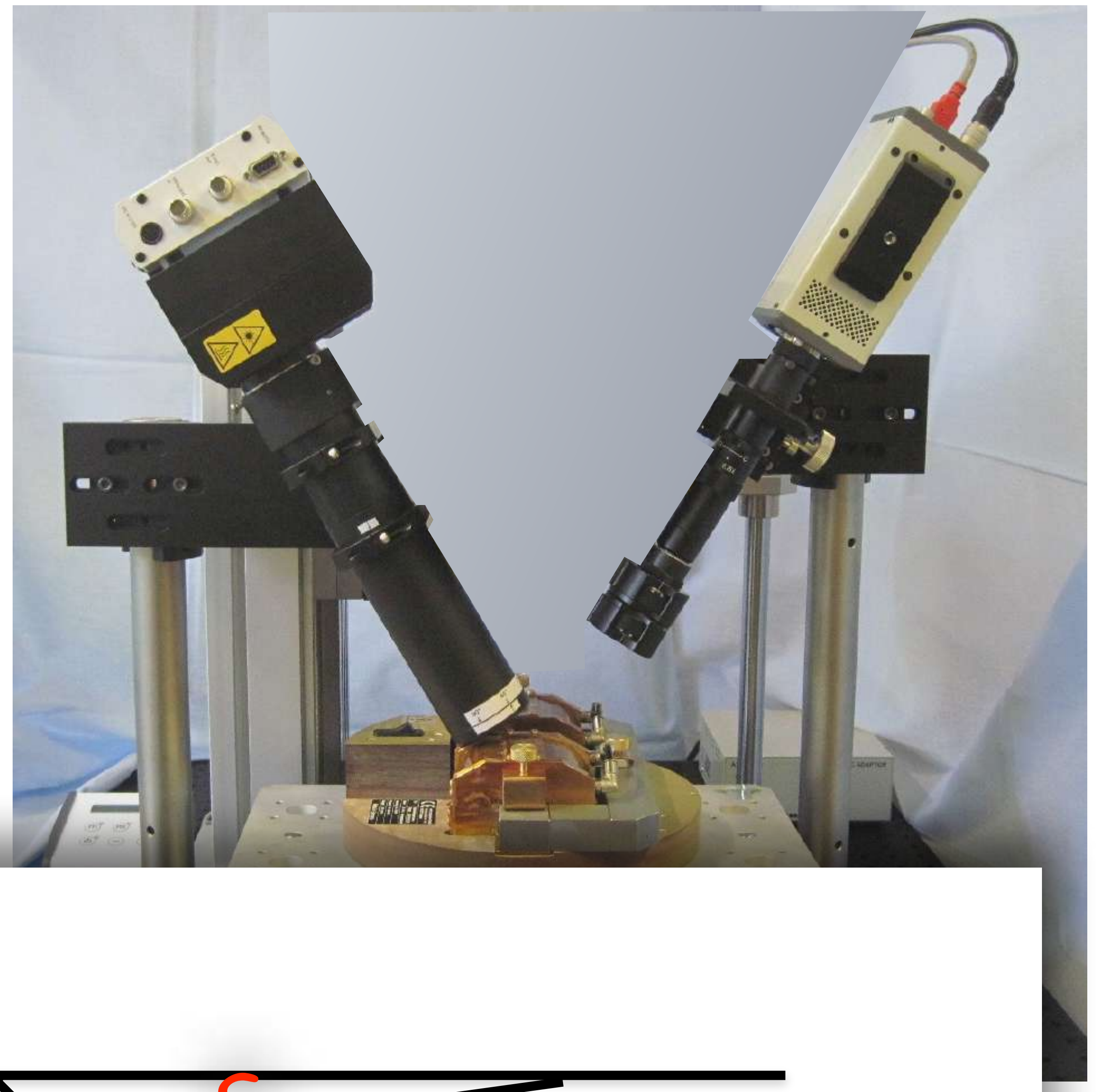


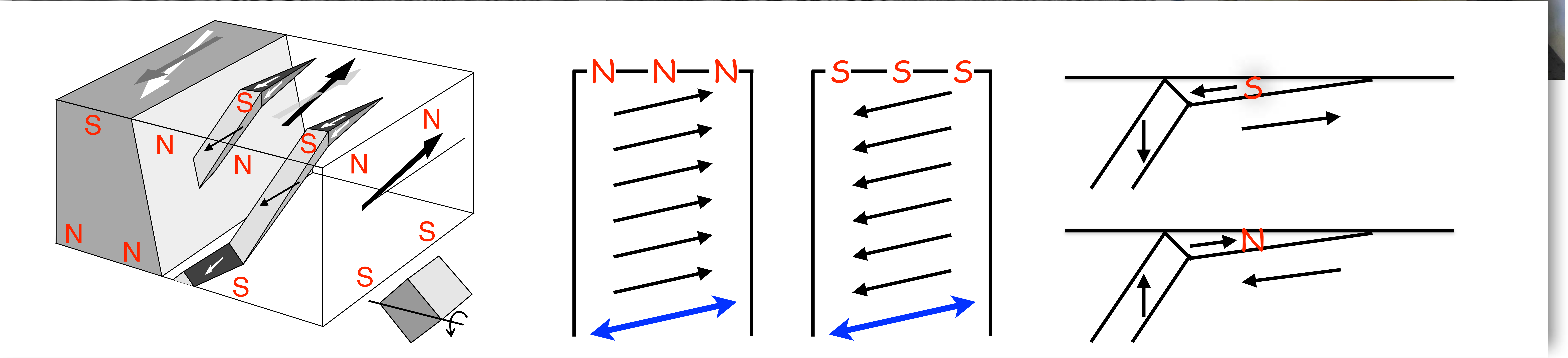
10 mm

5. MOIF Microscopy

Transformer steel with polished surface, overview microscope

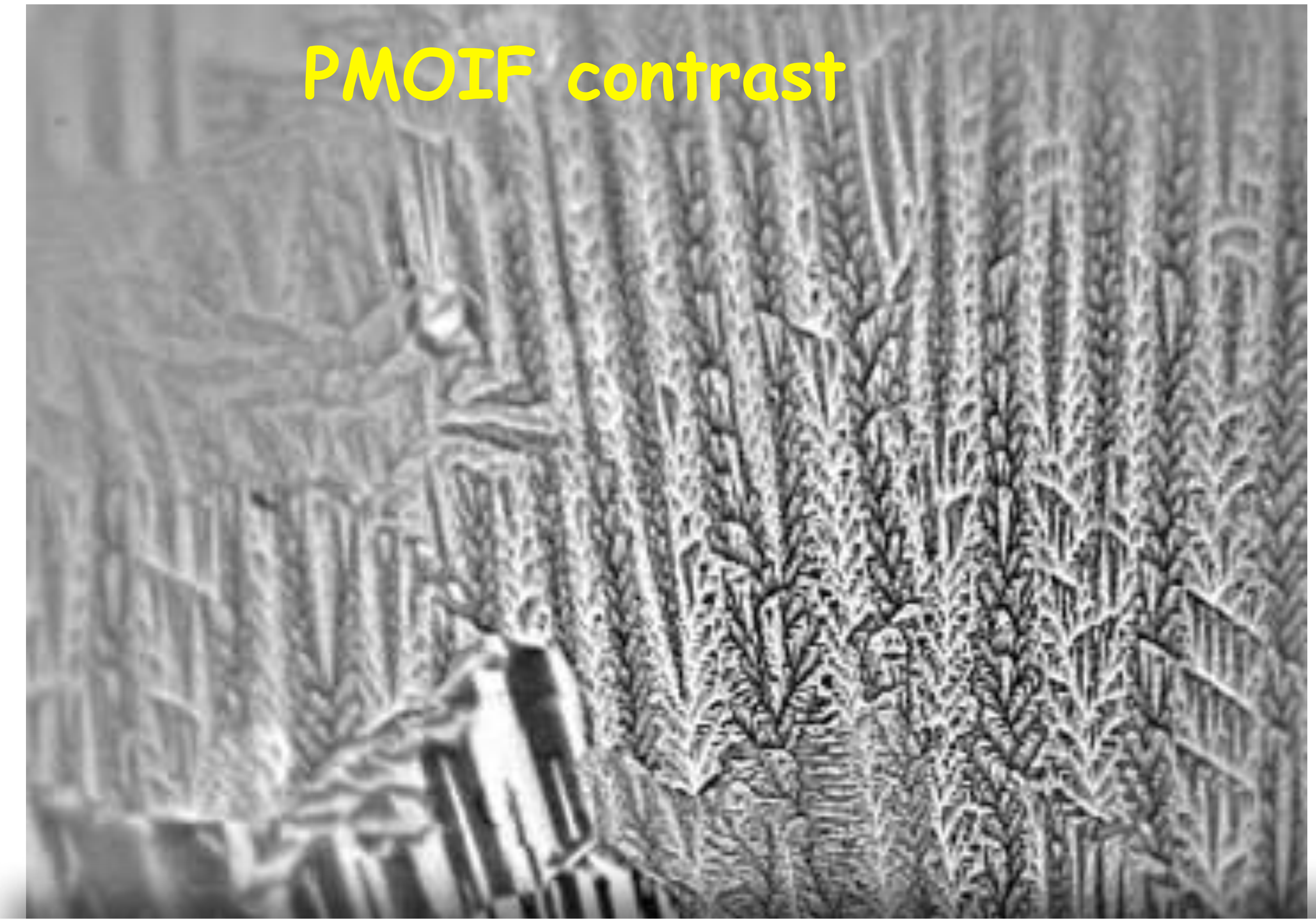


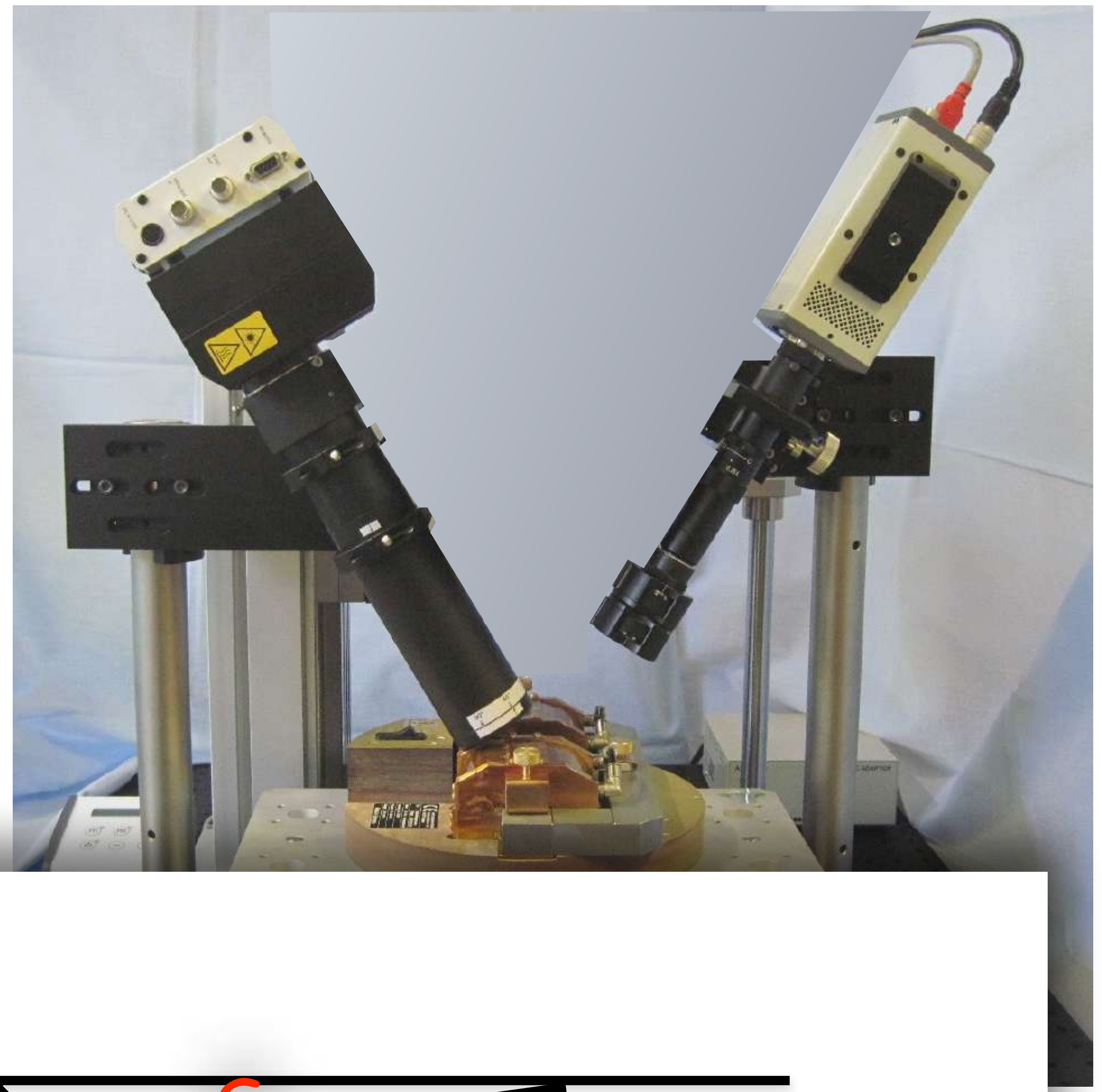


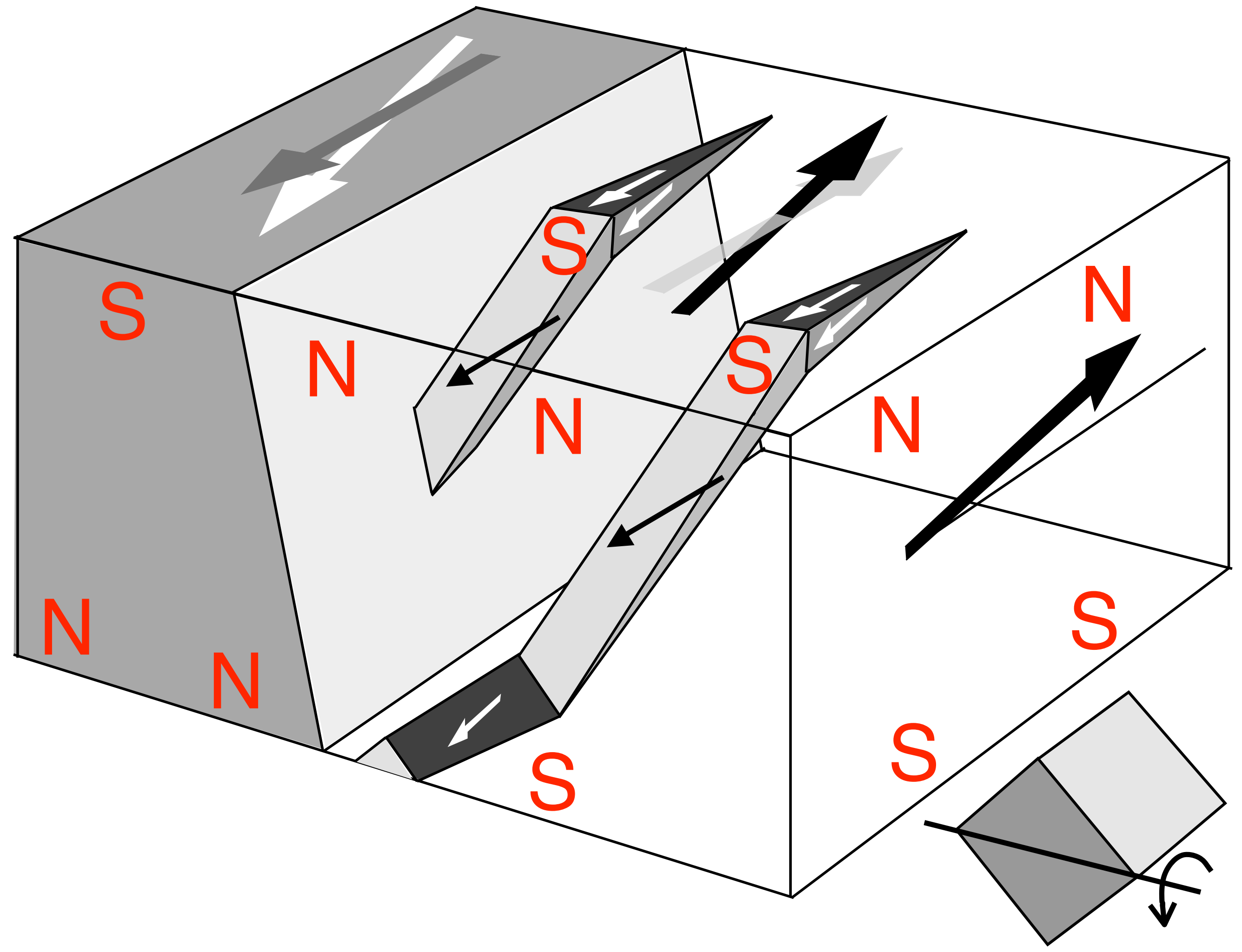


5. MOIF Microscopy

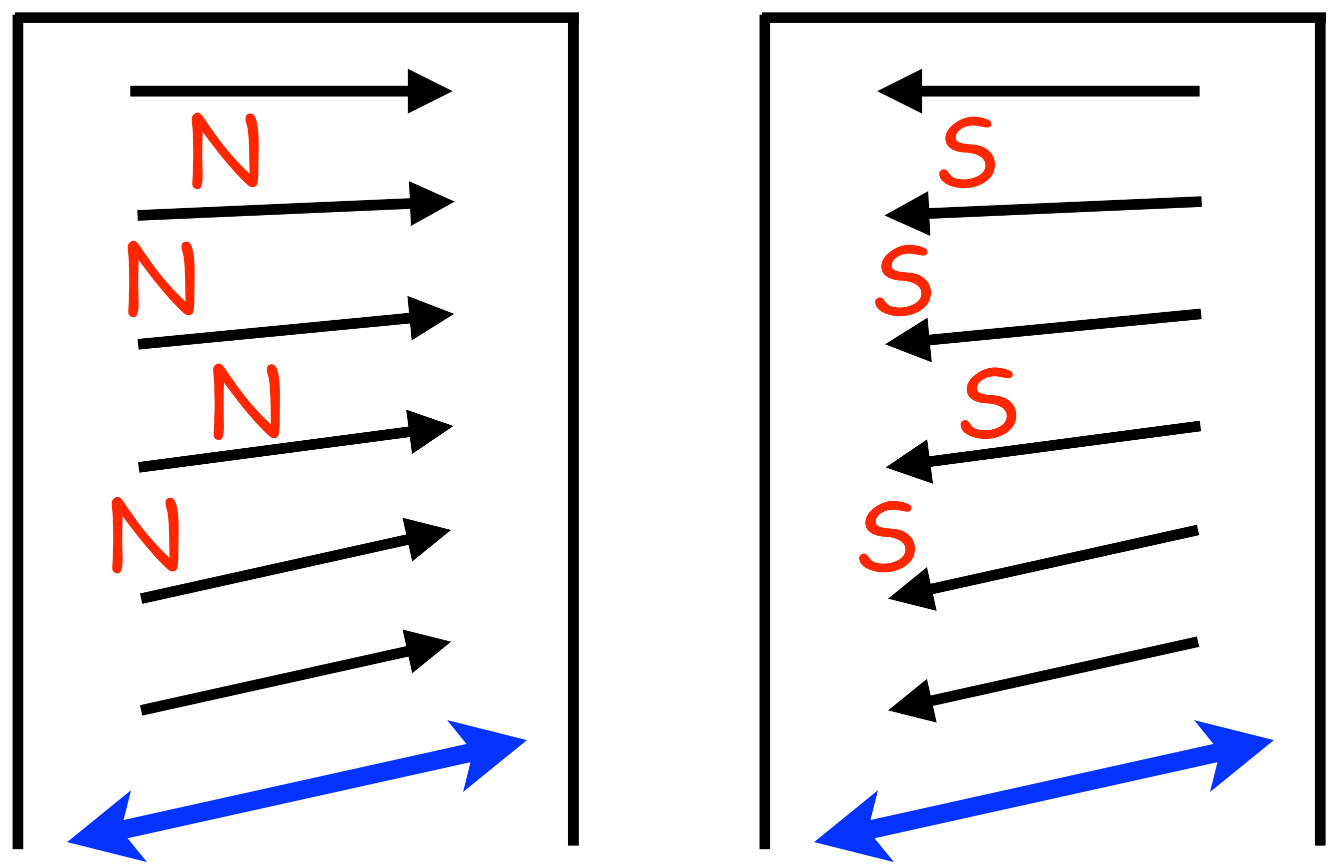
Transformer steel with polished surface, overview microscope

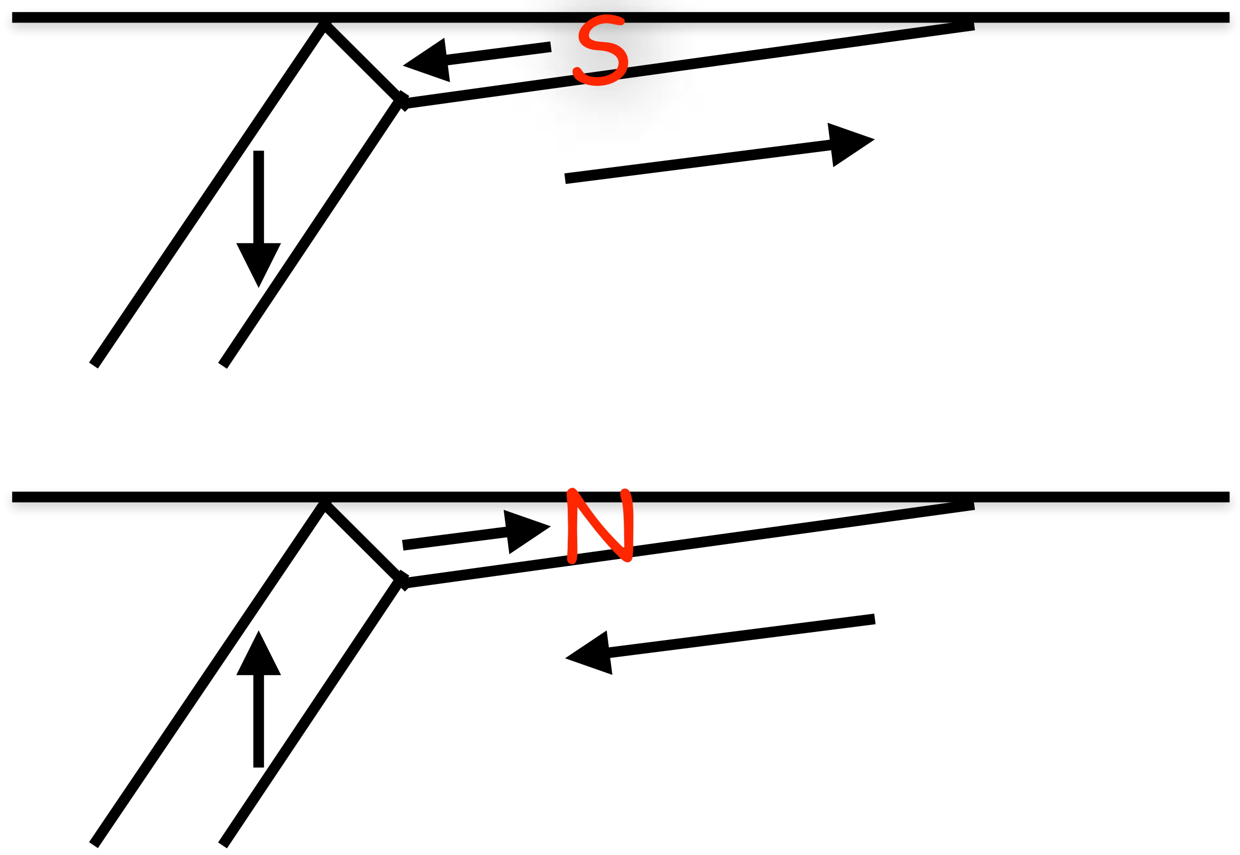




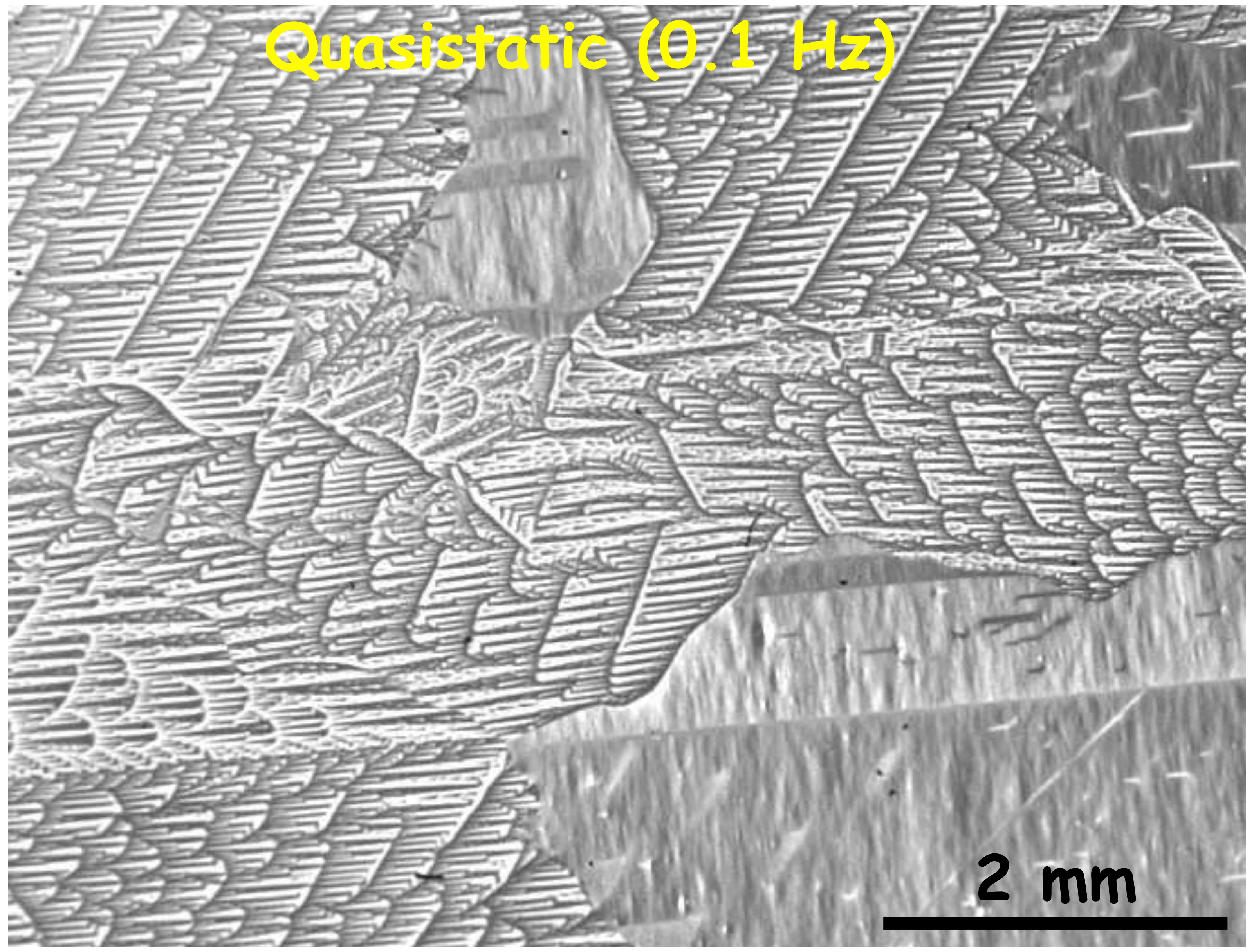


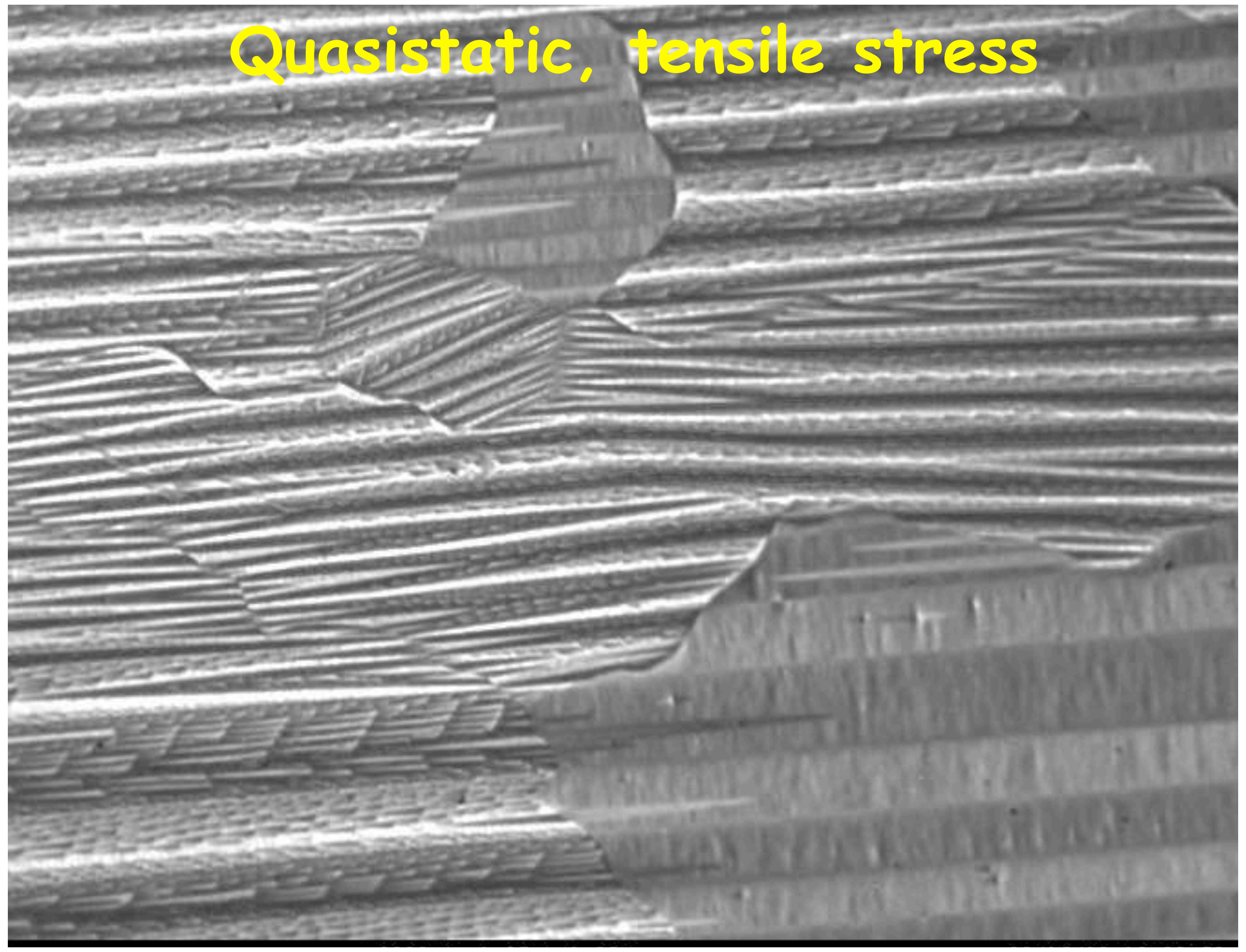
μ*-corrected





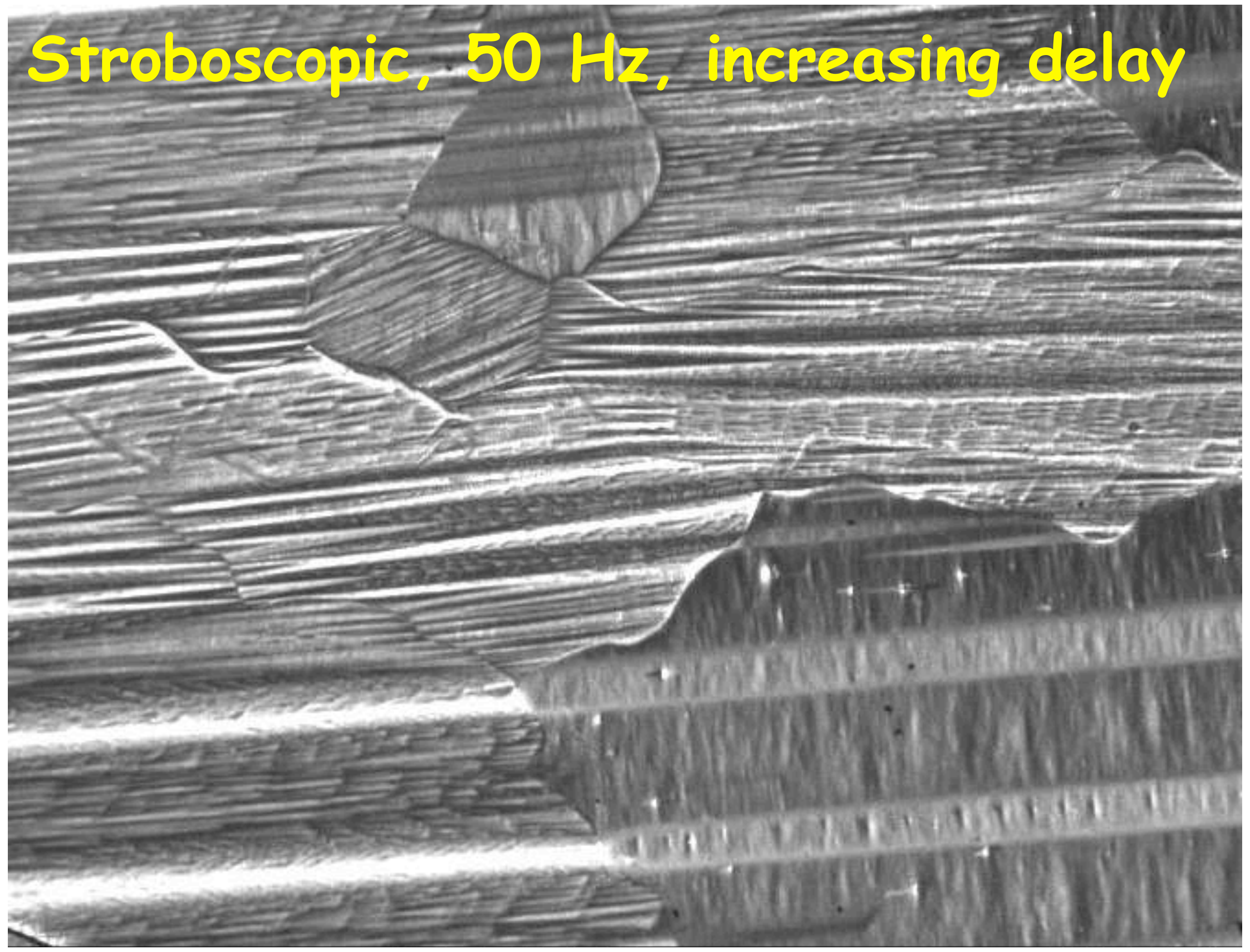
5. MOIF Microscopy



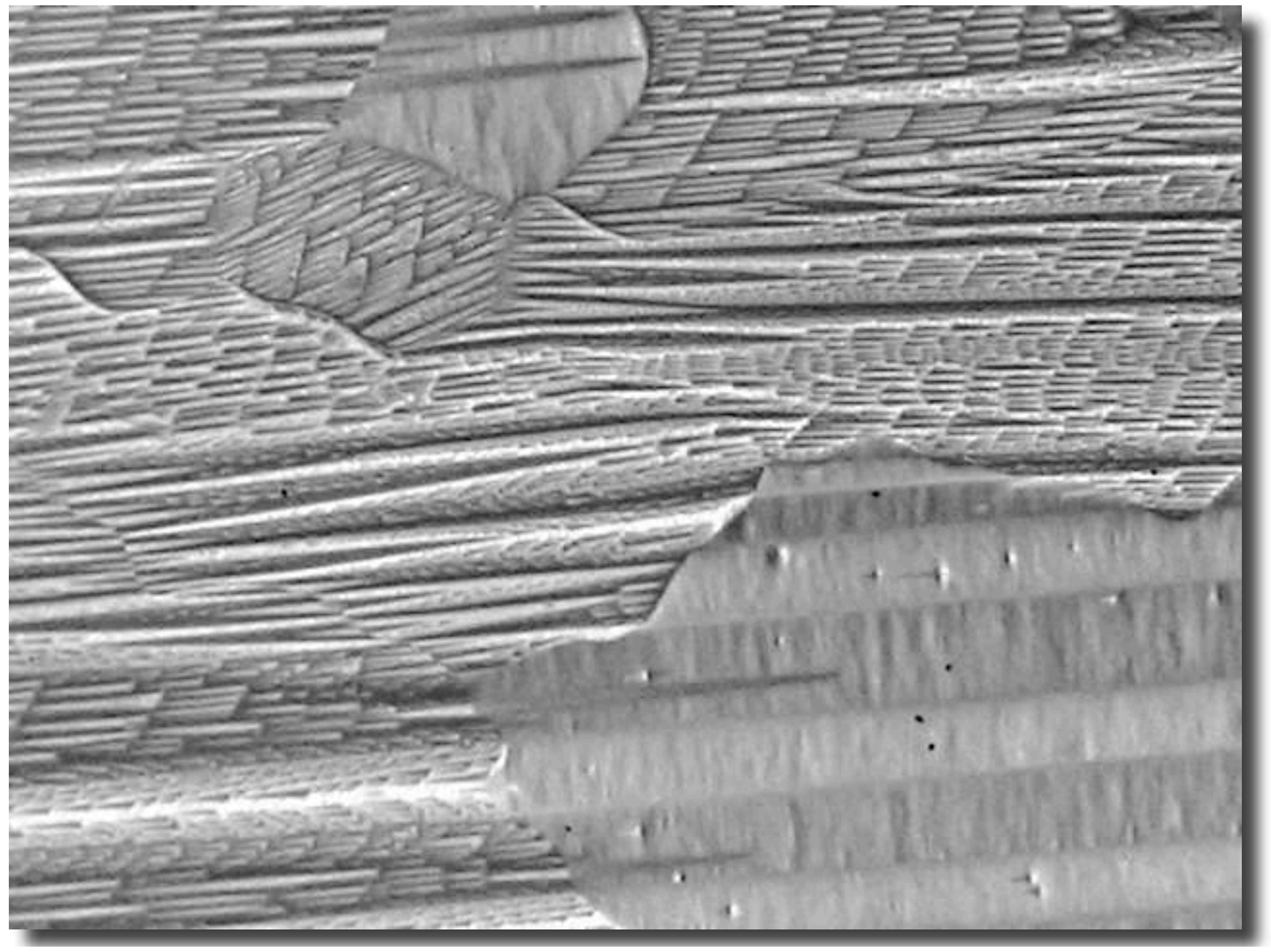


FeSi Goss
sheet, polished

R.S. et al.,
JMMM 474 (2019)



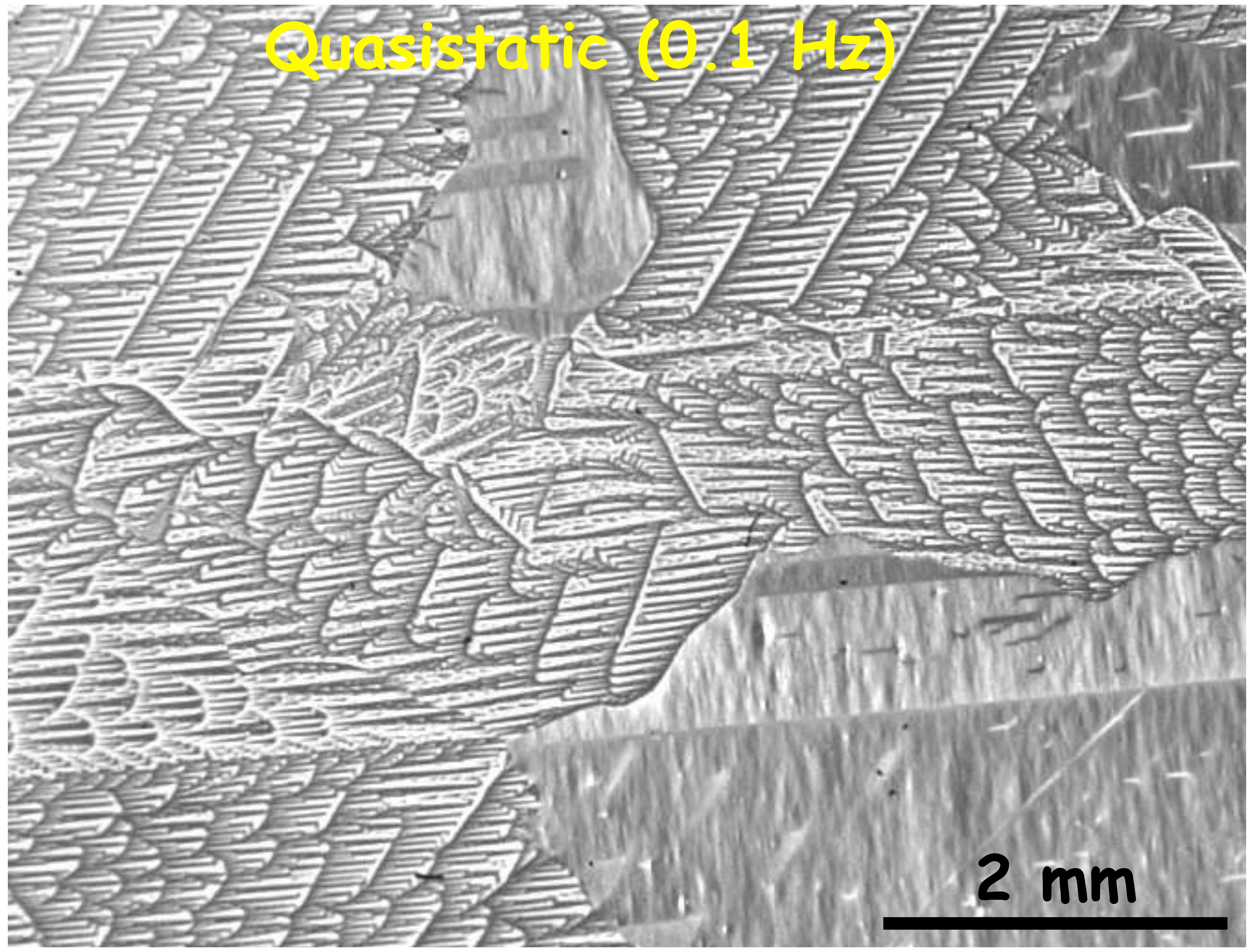
Single shot, 50 Hz, 30 A/m

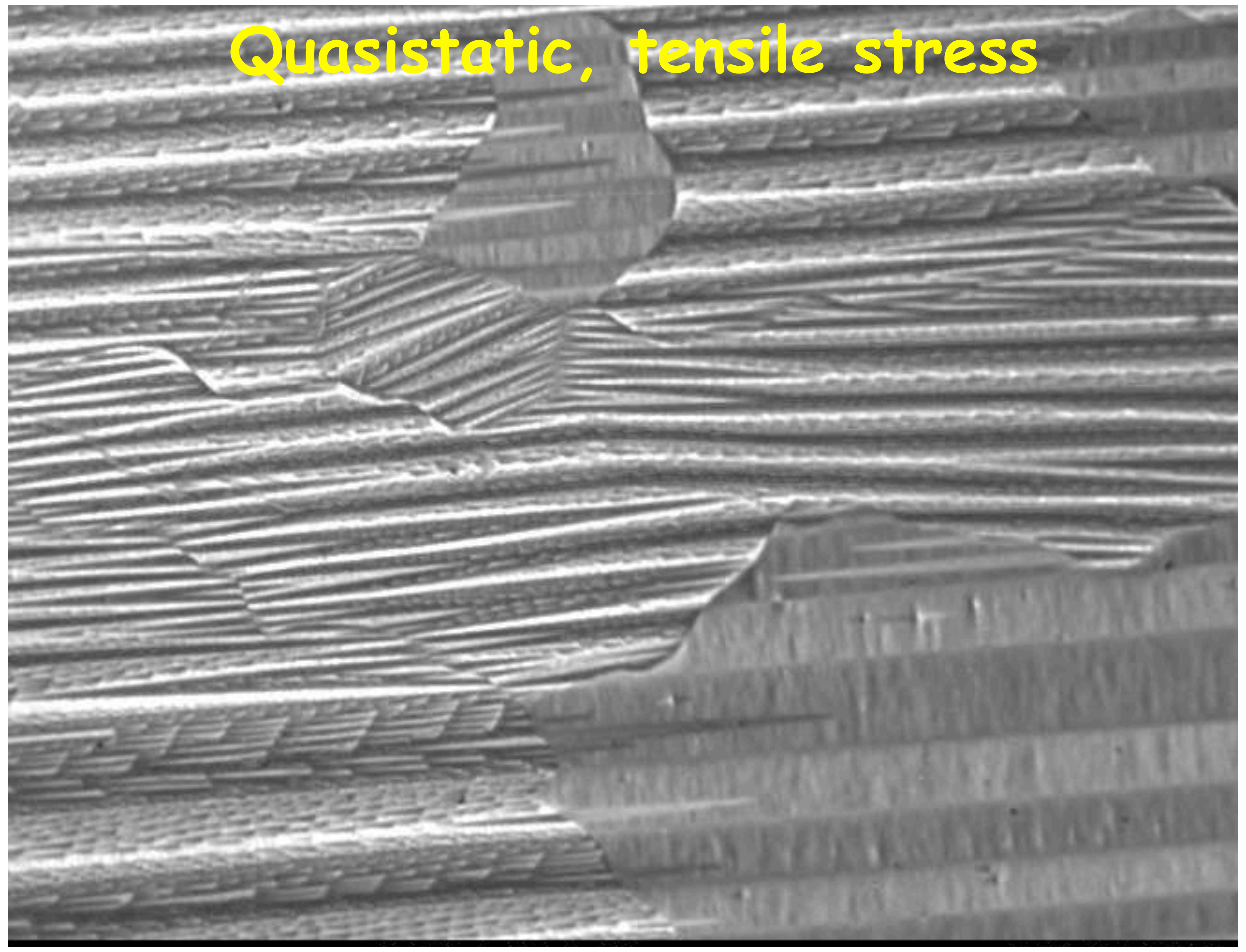


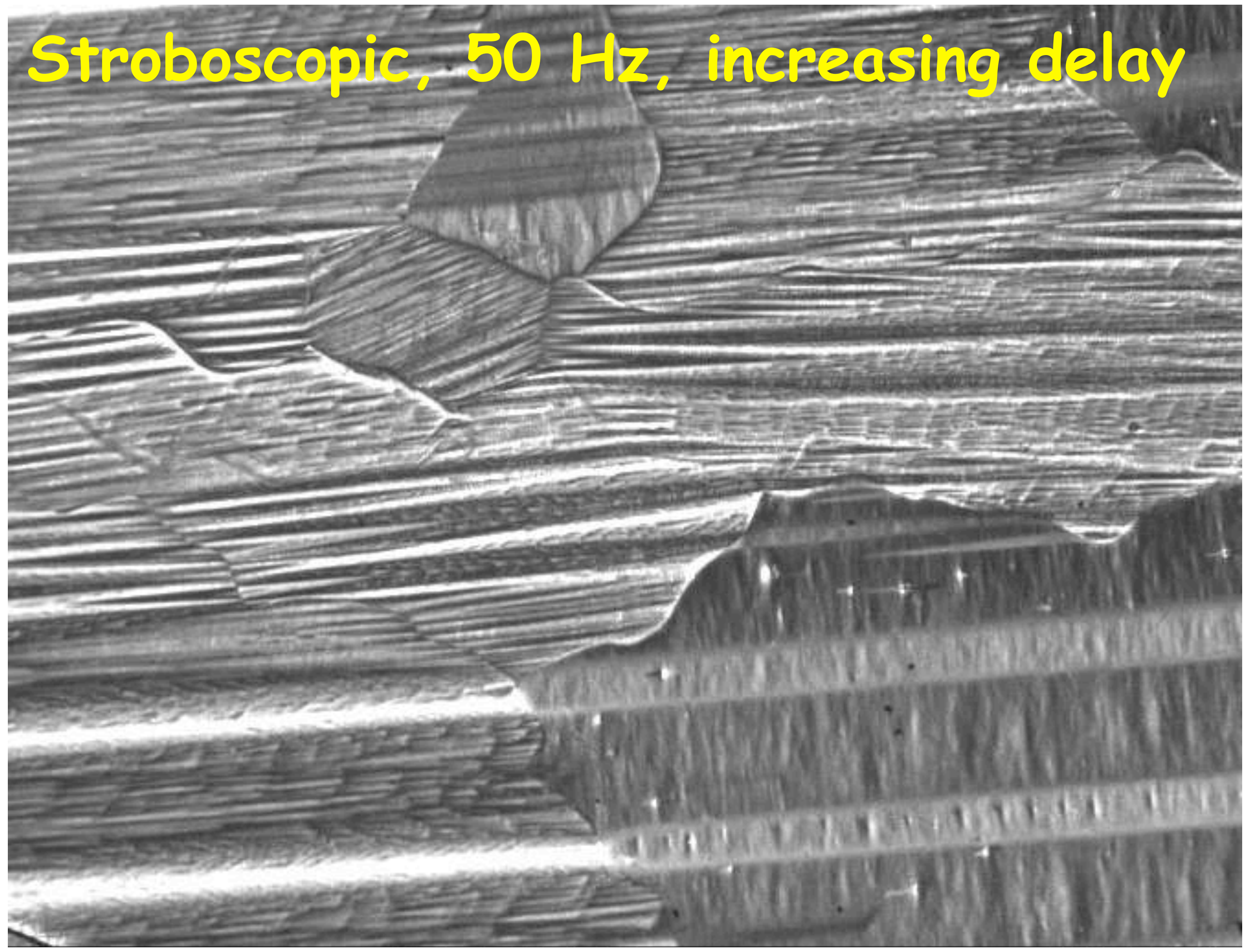
5. MOIF Microscopy

FeSi Goss sheet, polished

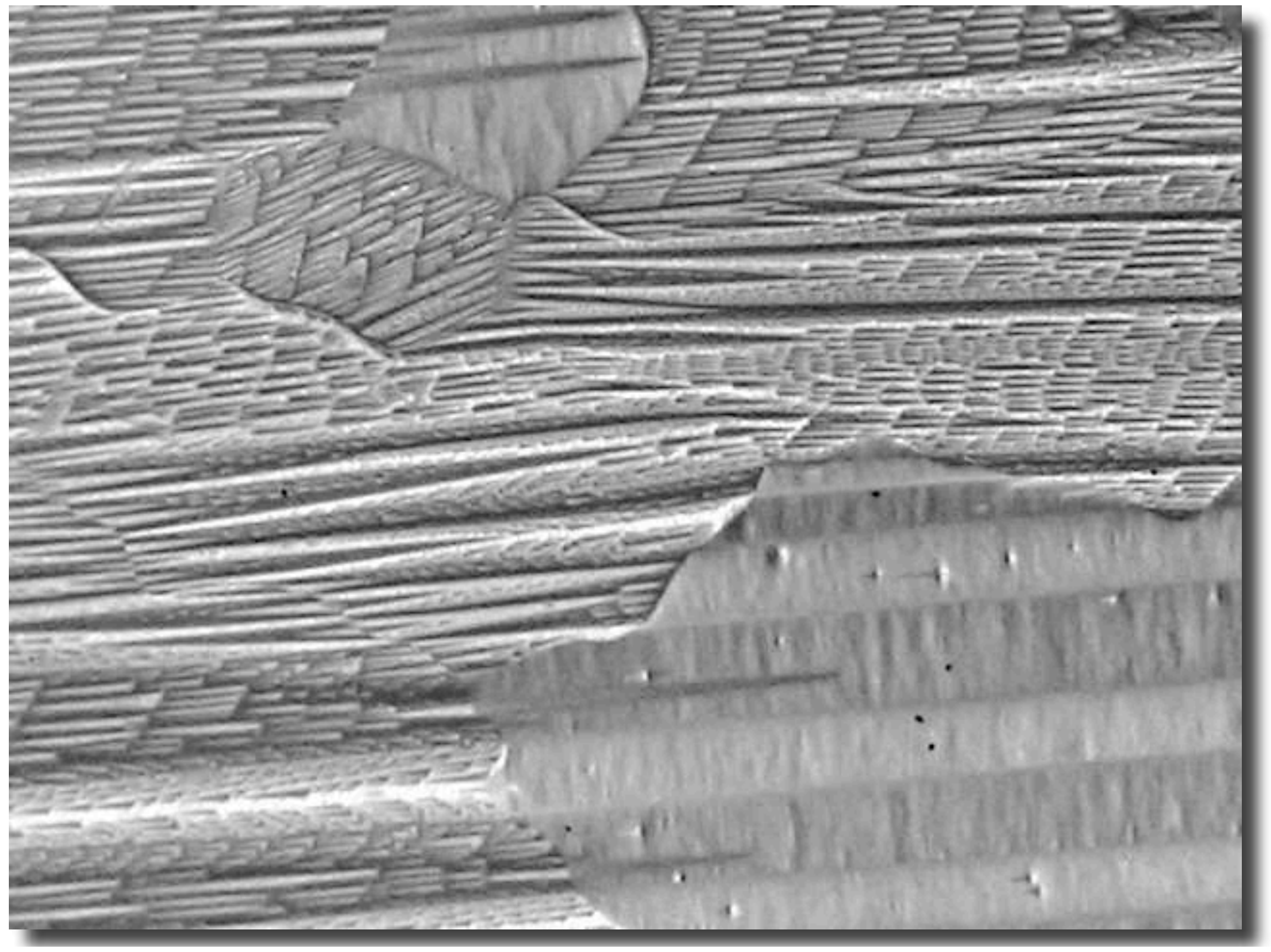
R.S. et al., JMMM 474 (2019)







Single shot, 50 Hz, 30 A/m



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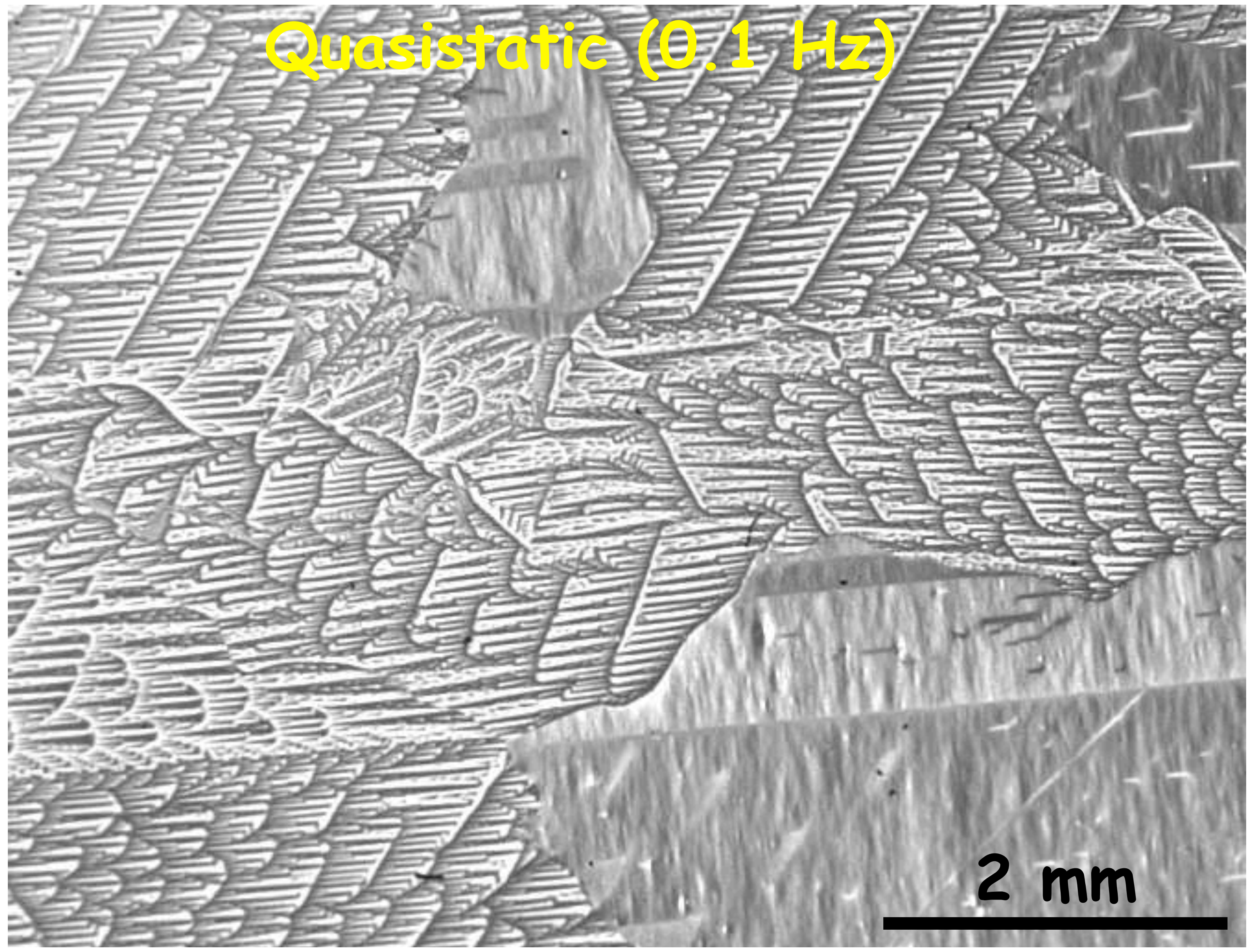

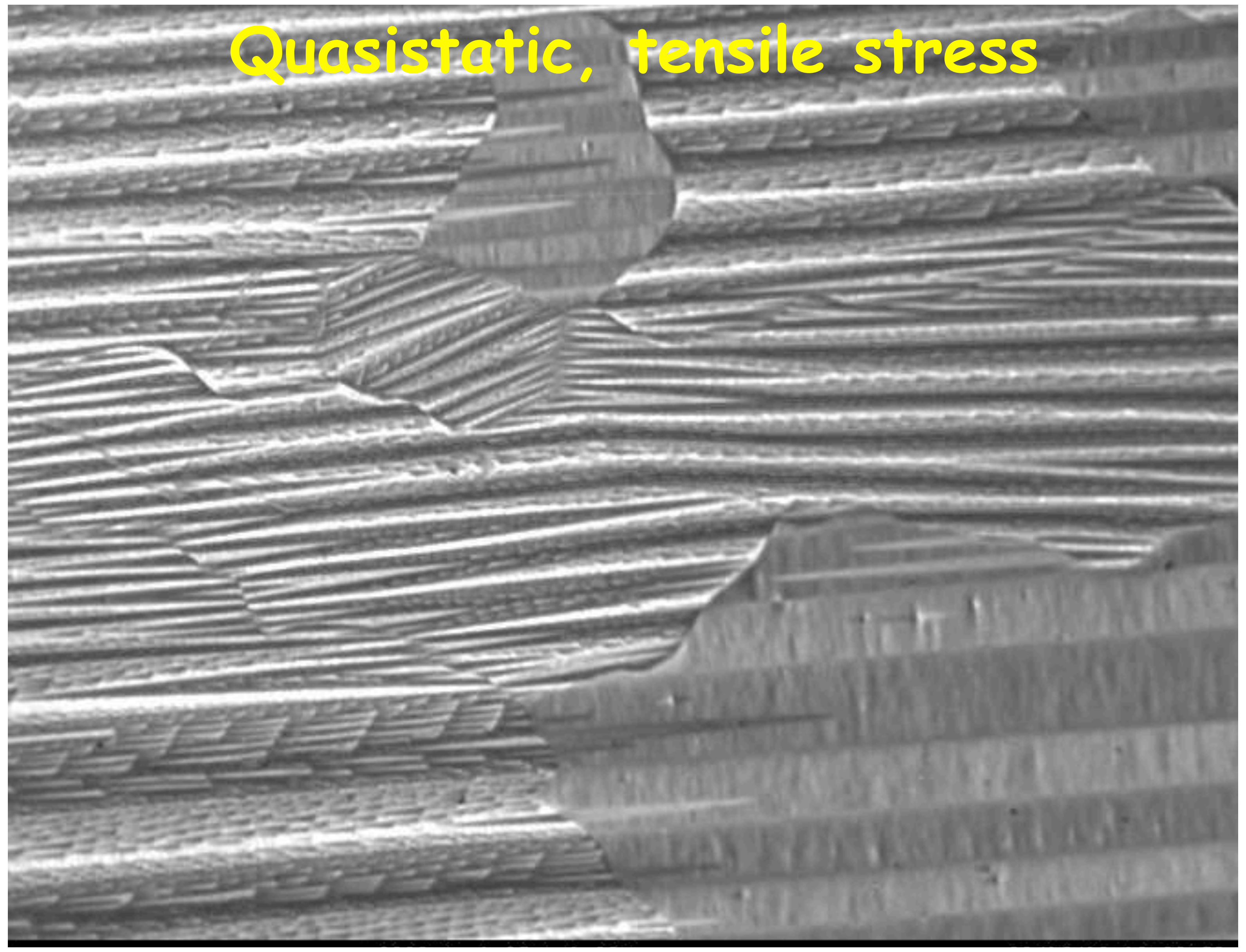

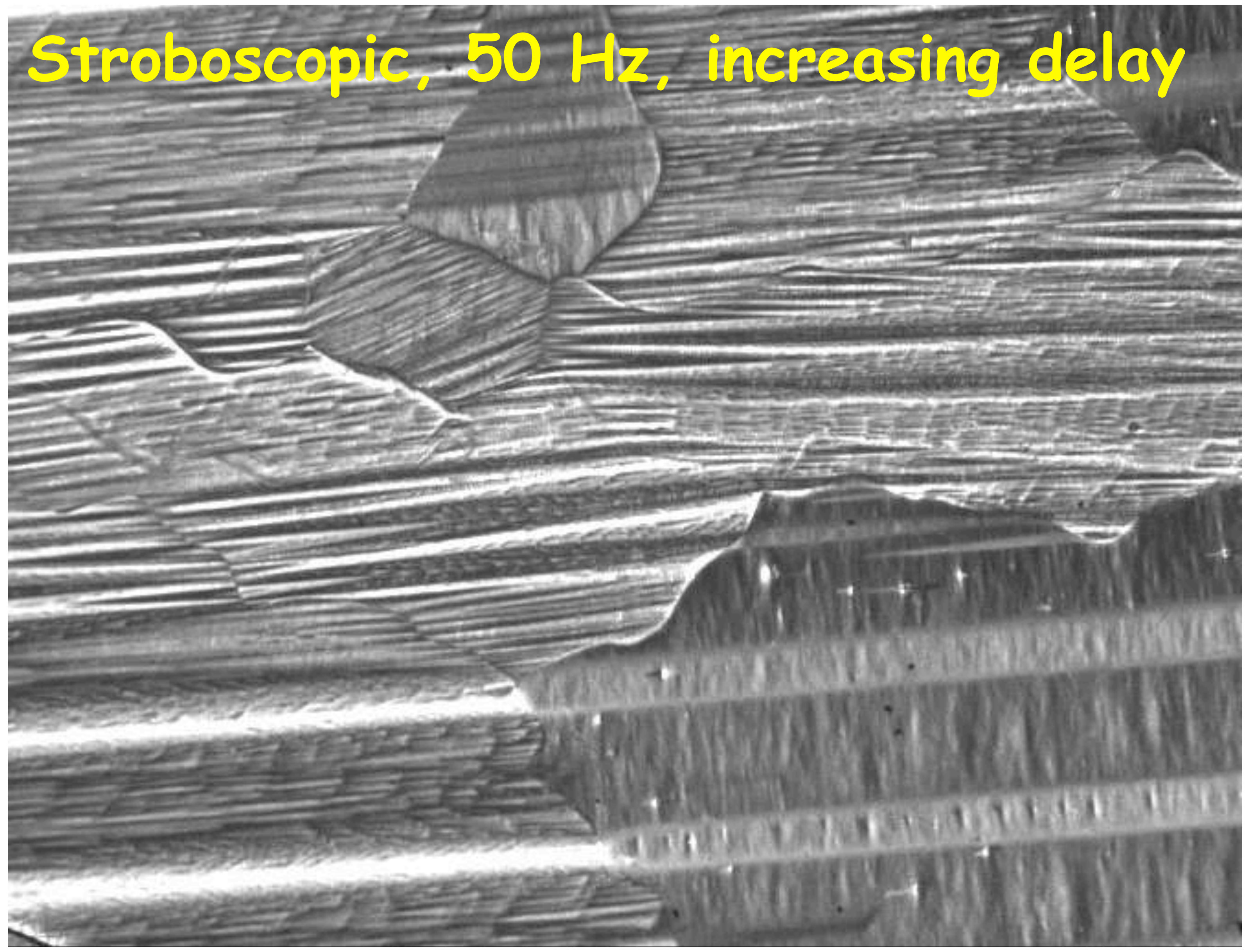

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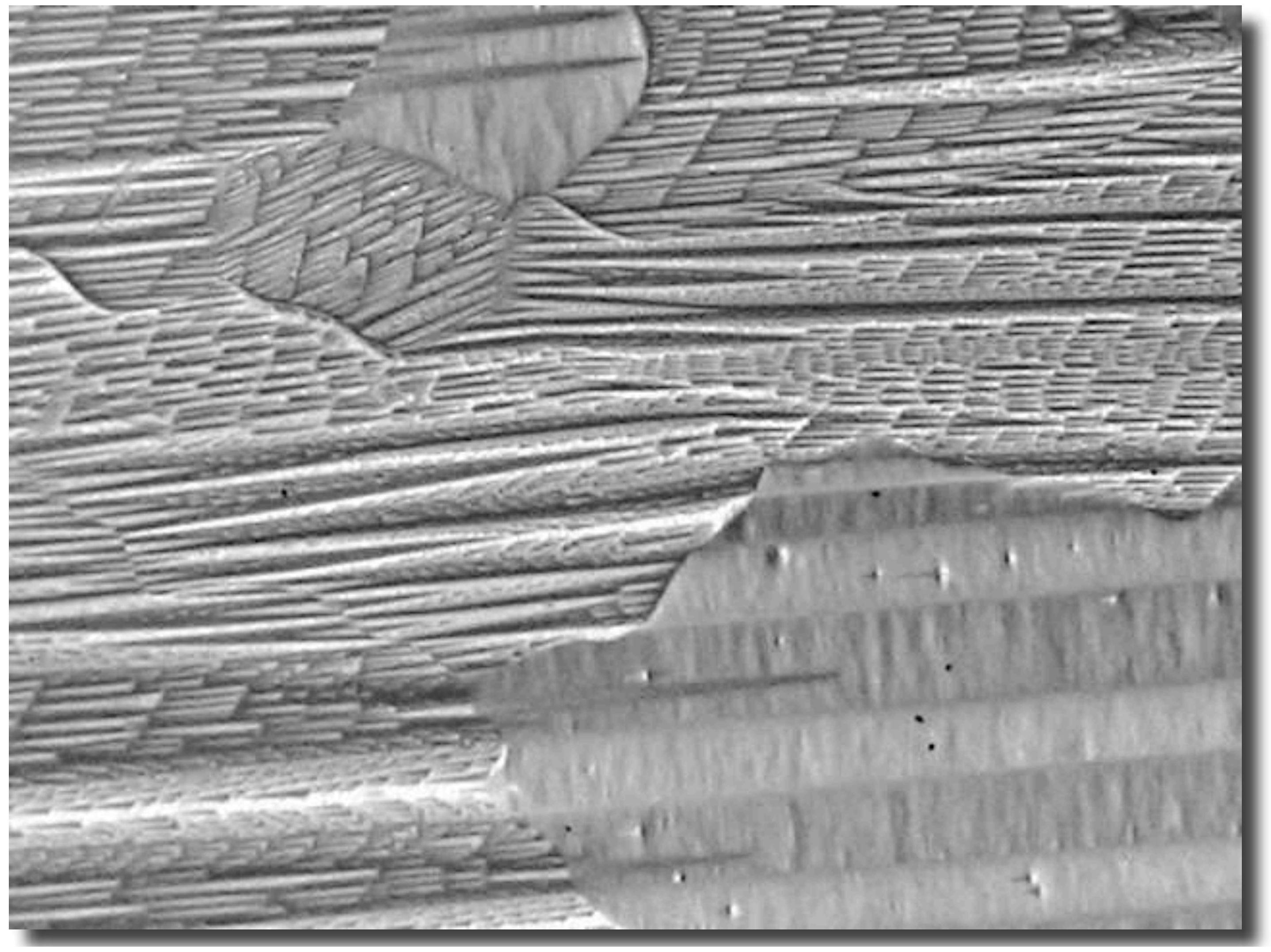
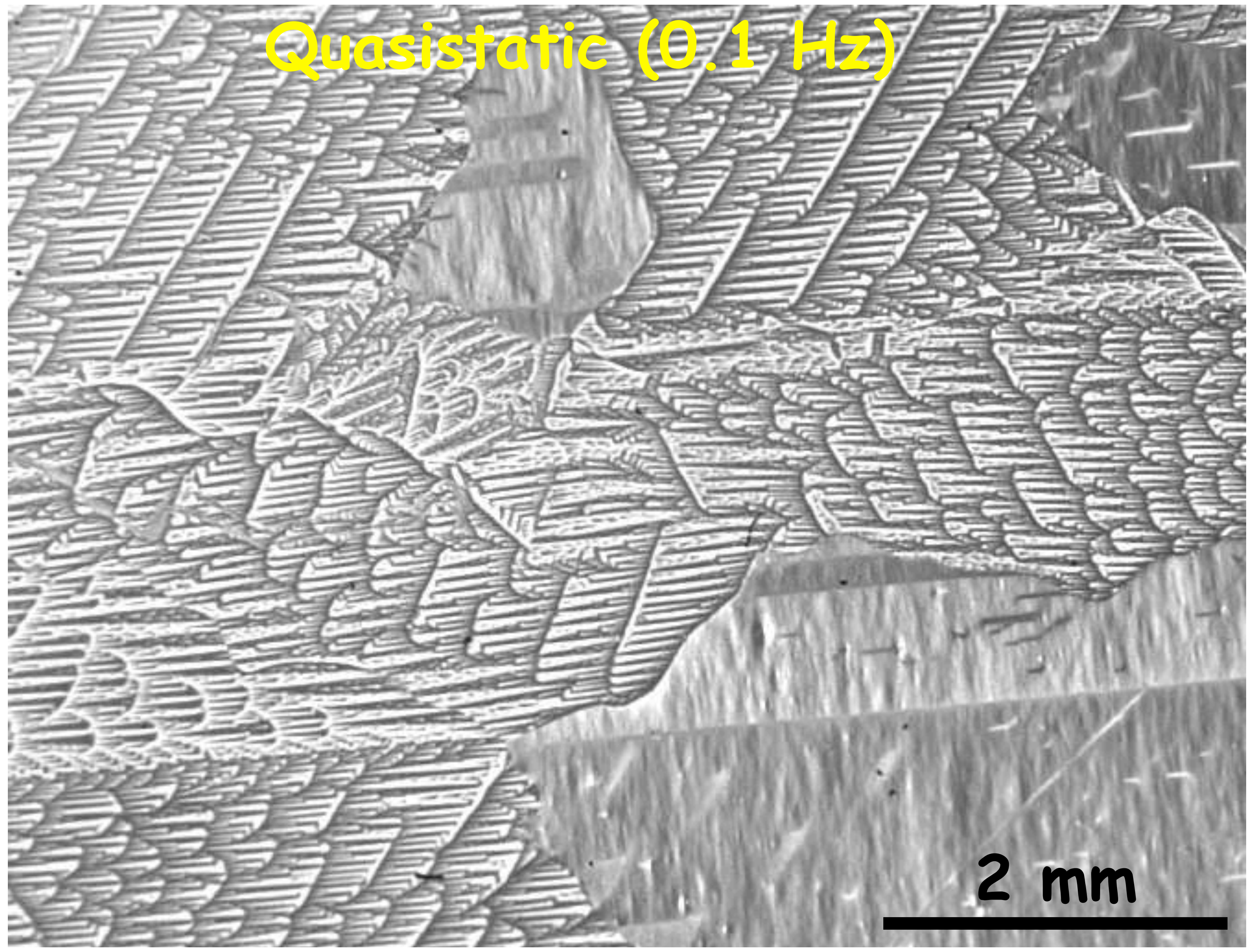
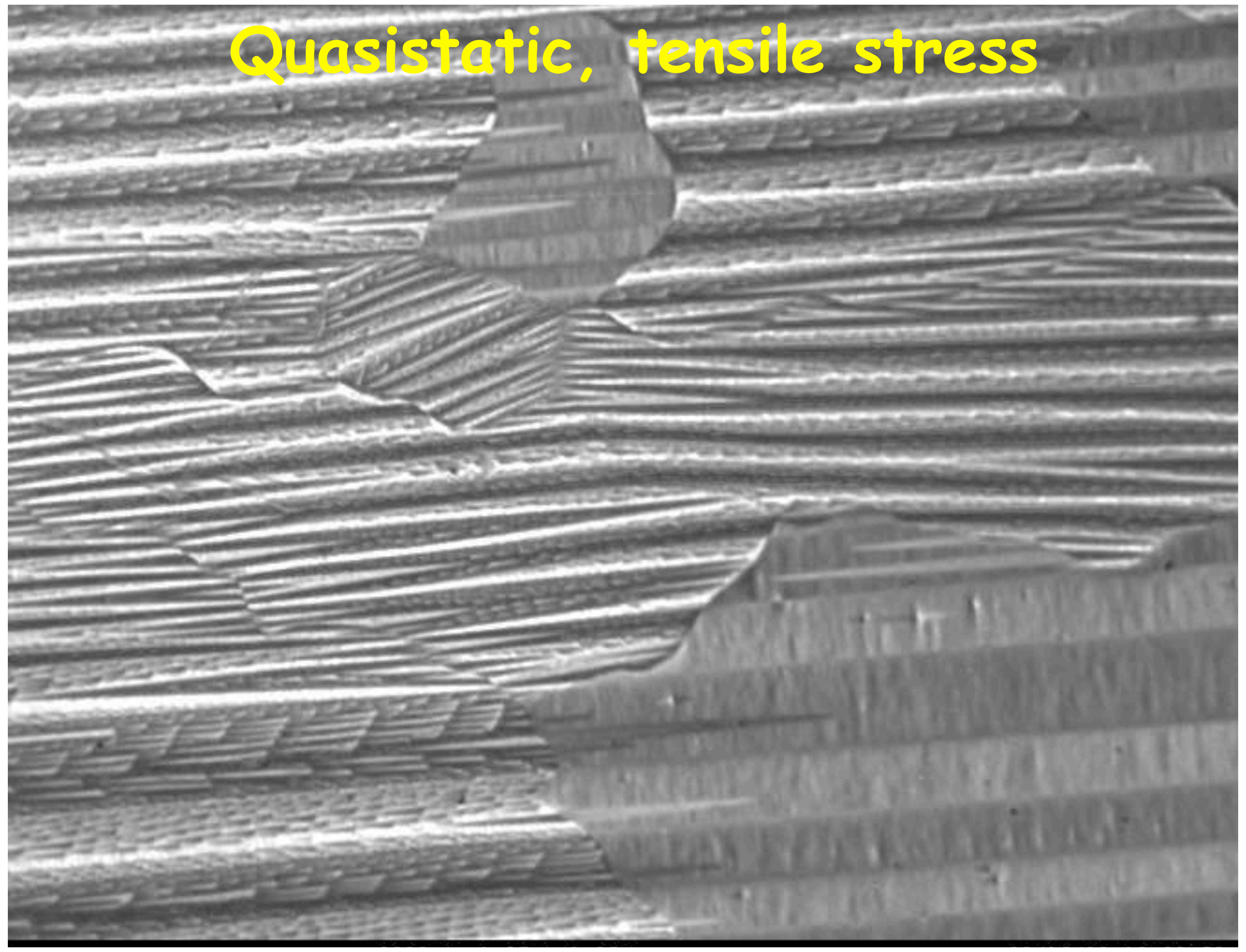


FeSi Goss
sheet, polished

R.S. et al.,
JMMM 474 (2019)

5. MOIF Microscopy





FeSi Goss
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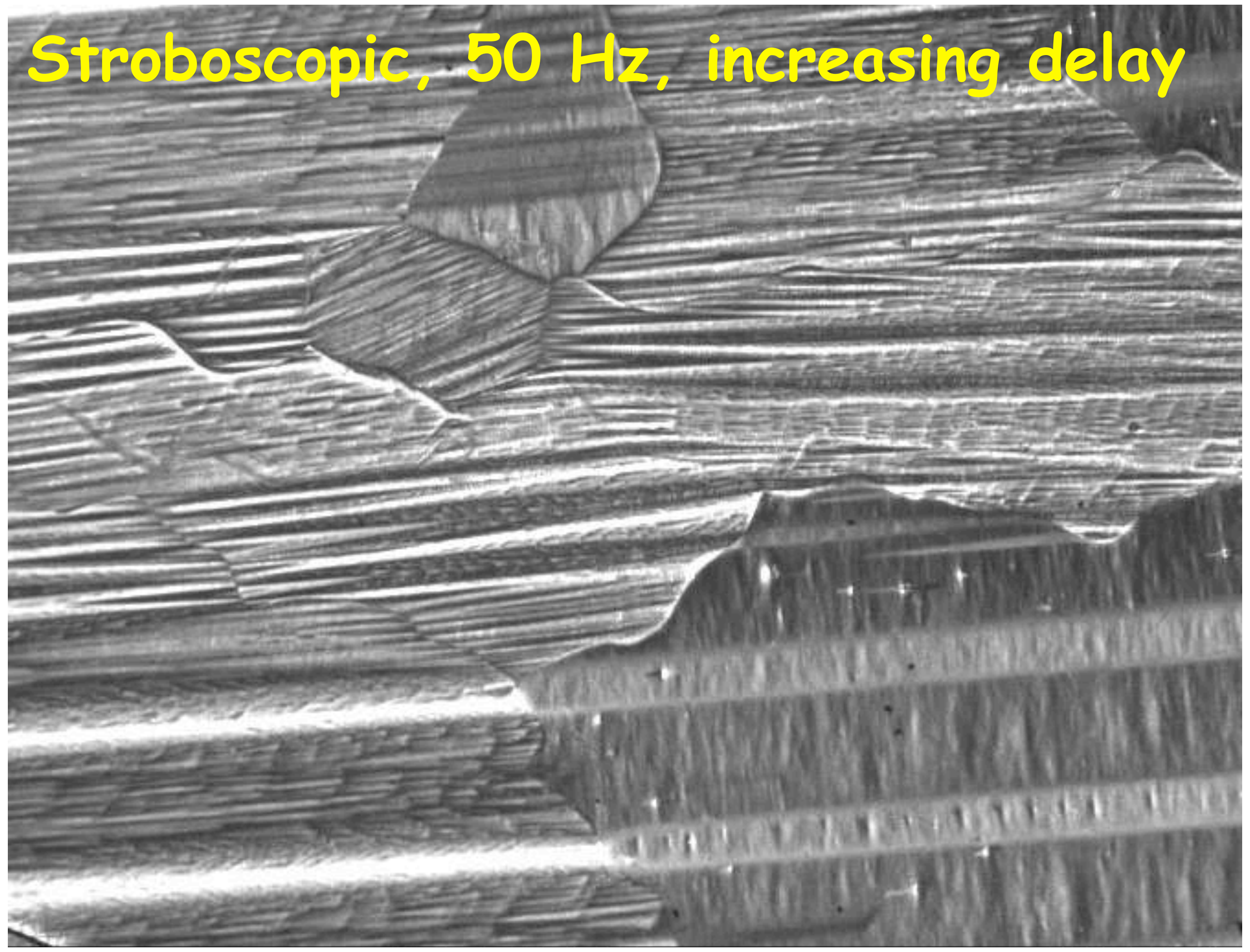
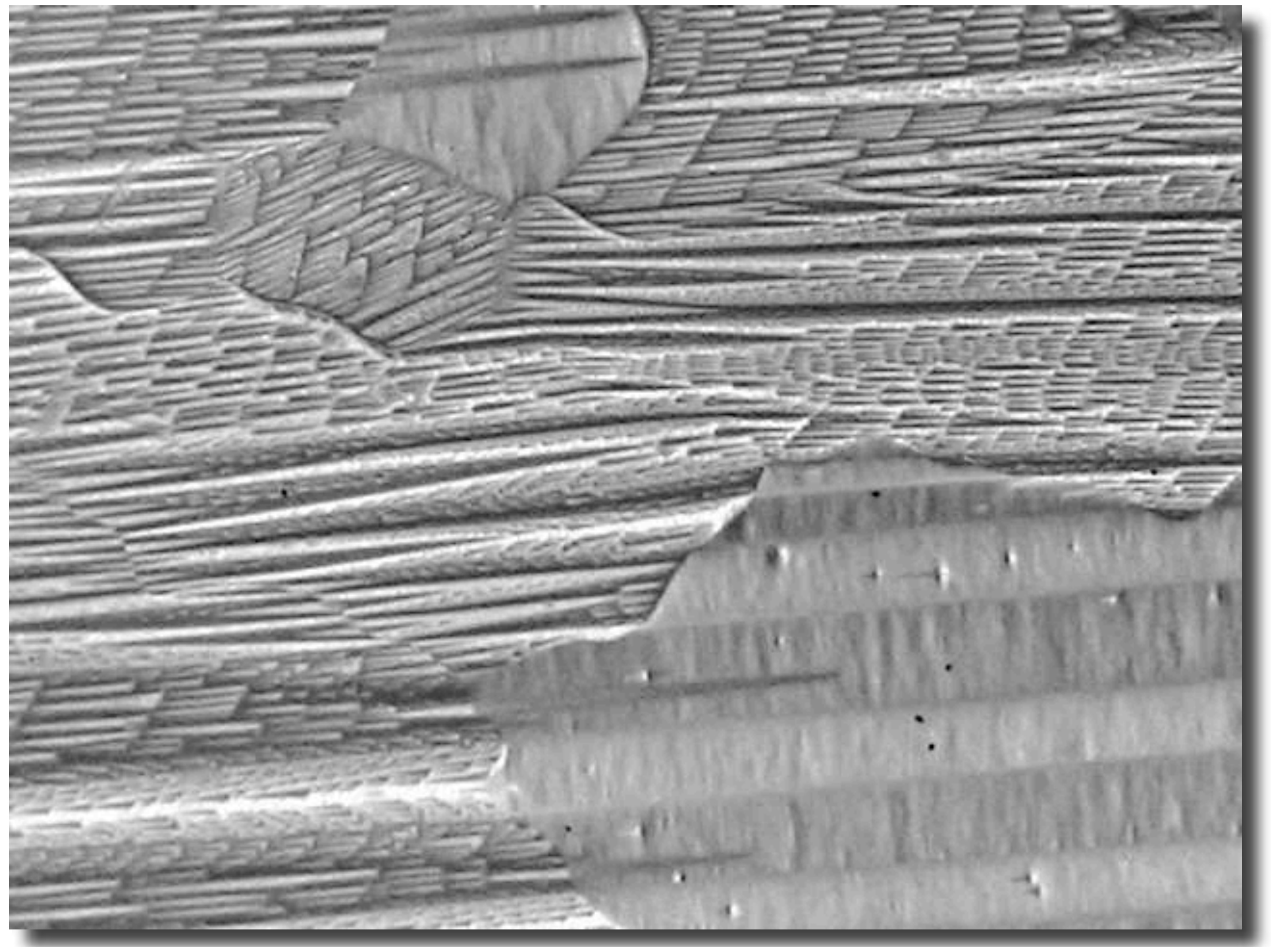
R.S. et al.,
JMMM 474 (2019)



Single shot, 50 Hz, 30 A/m



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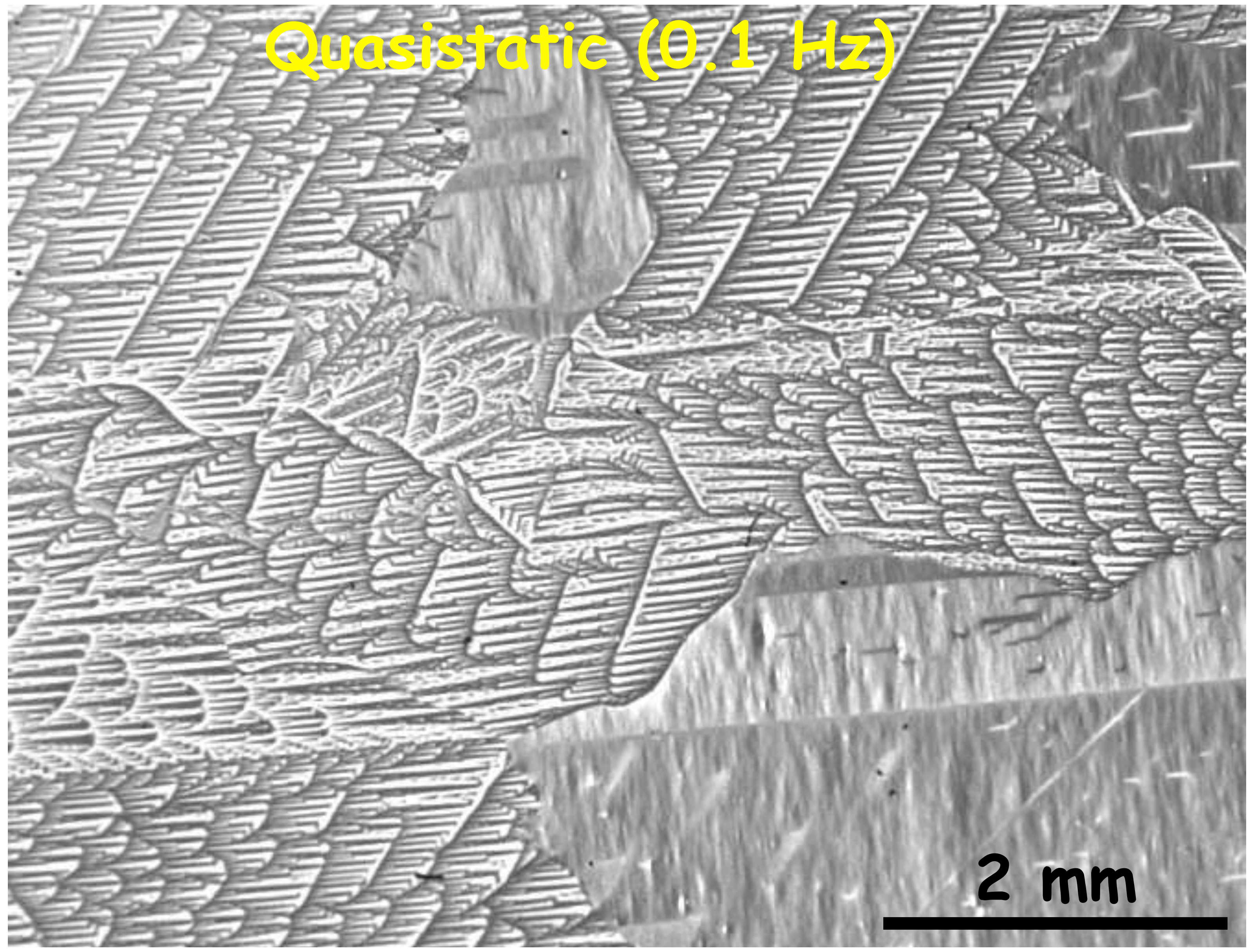
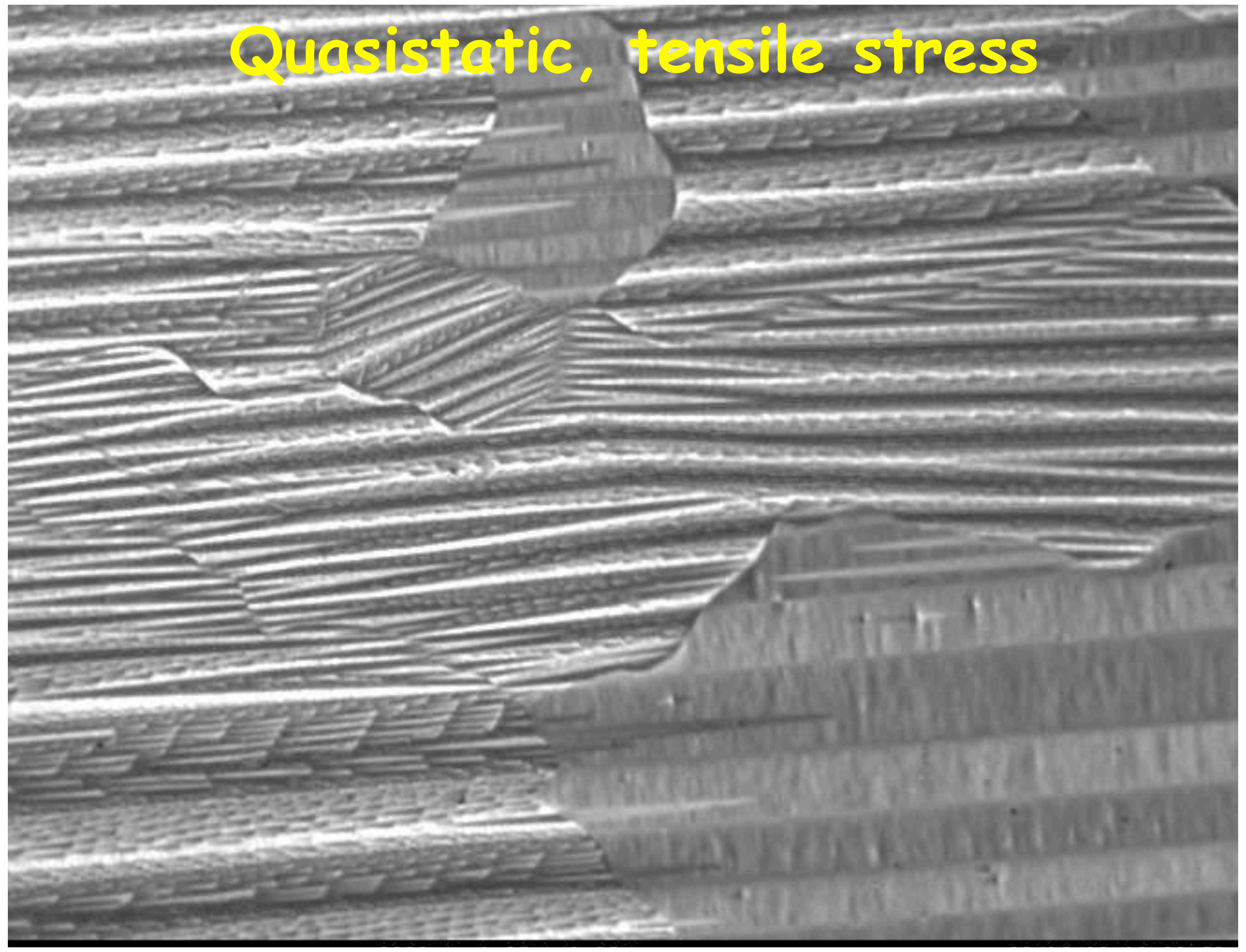
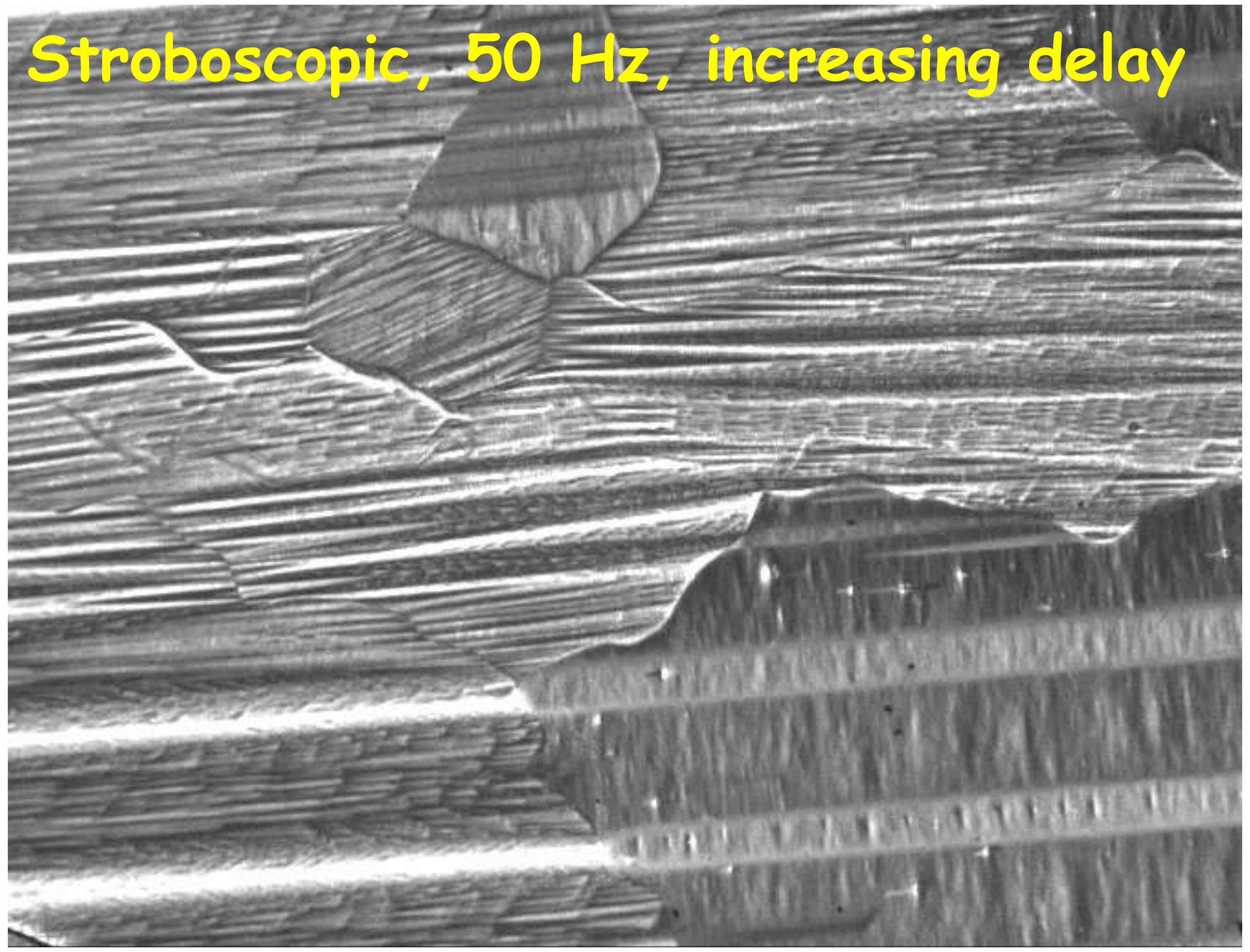
FeSi Goss
sheet, polished

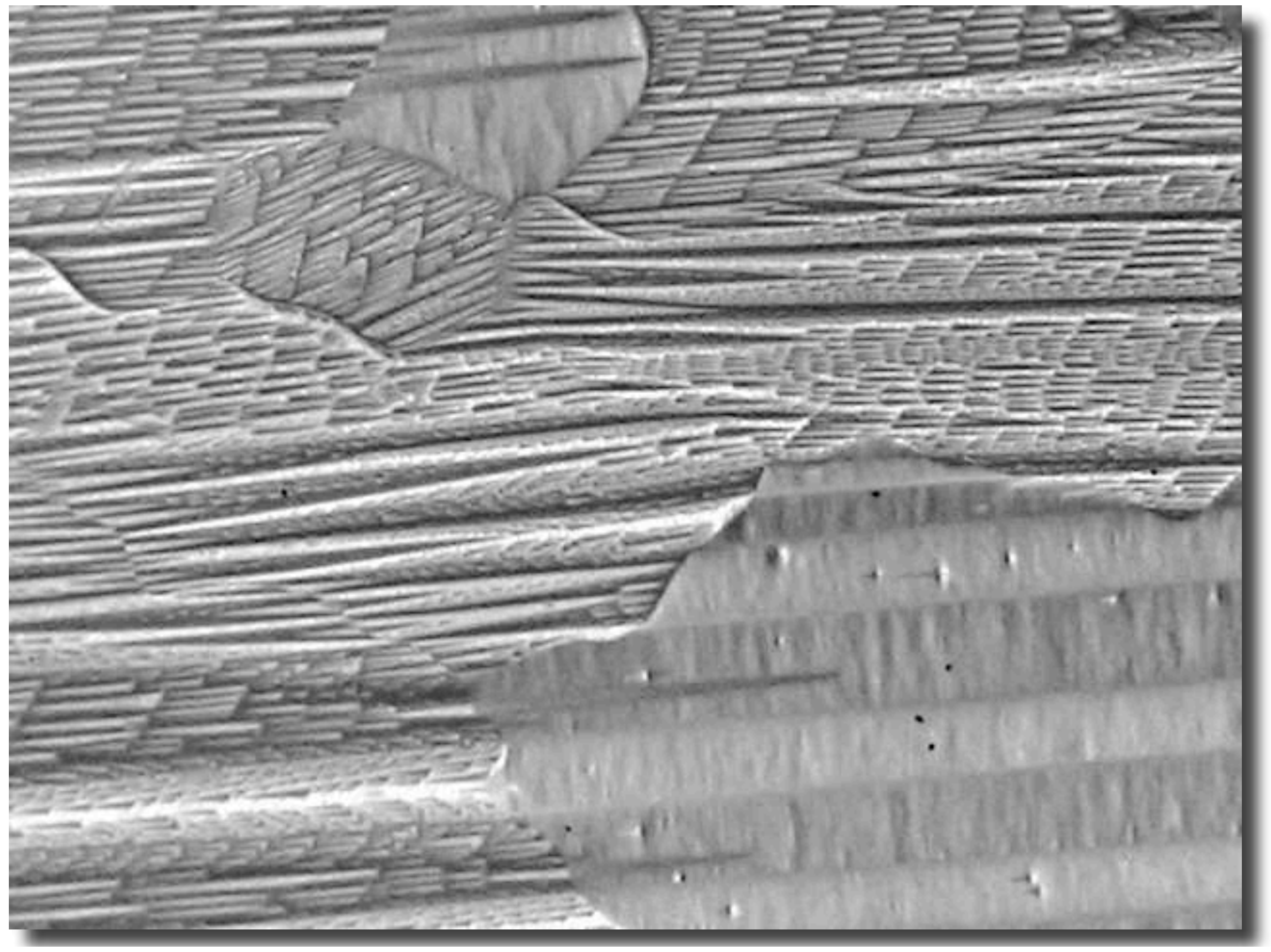
R.S. et al.,
JMMM 474 (2019)







Single shot, 50 Hz, 30 A/m



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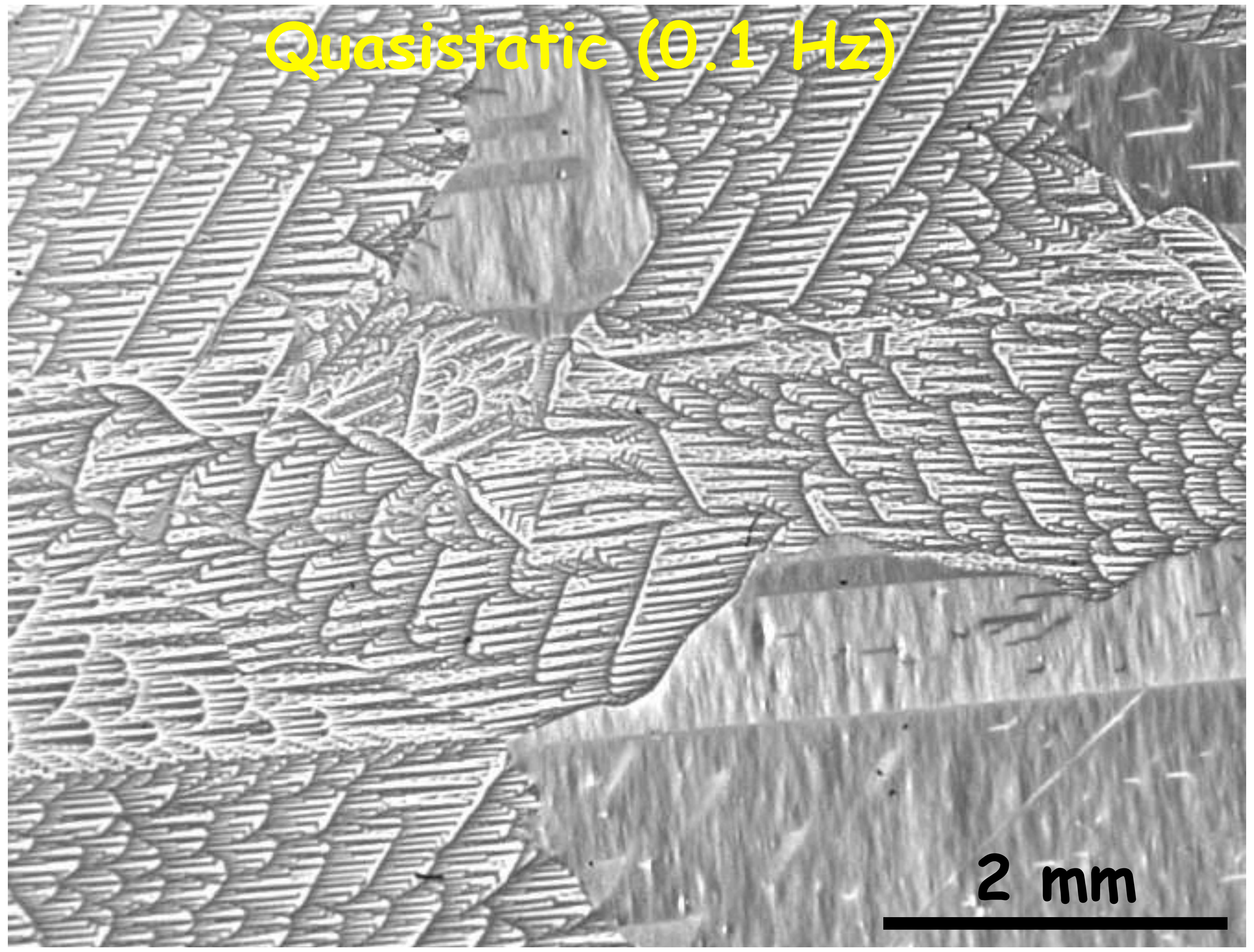
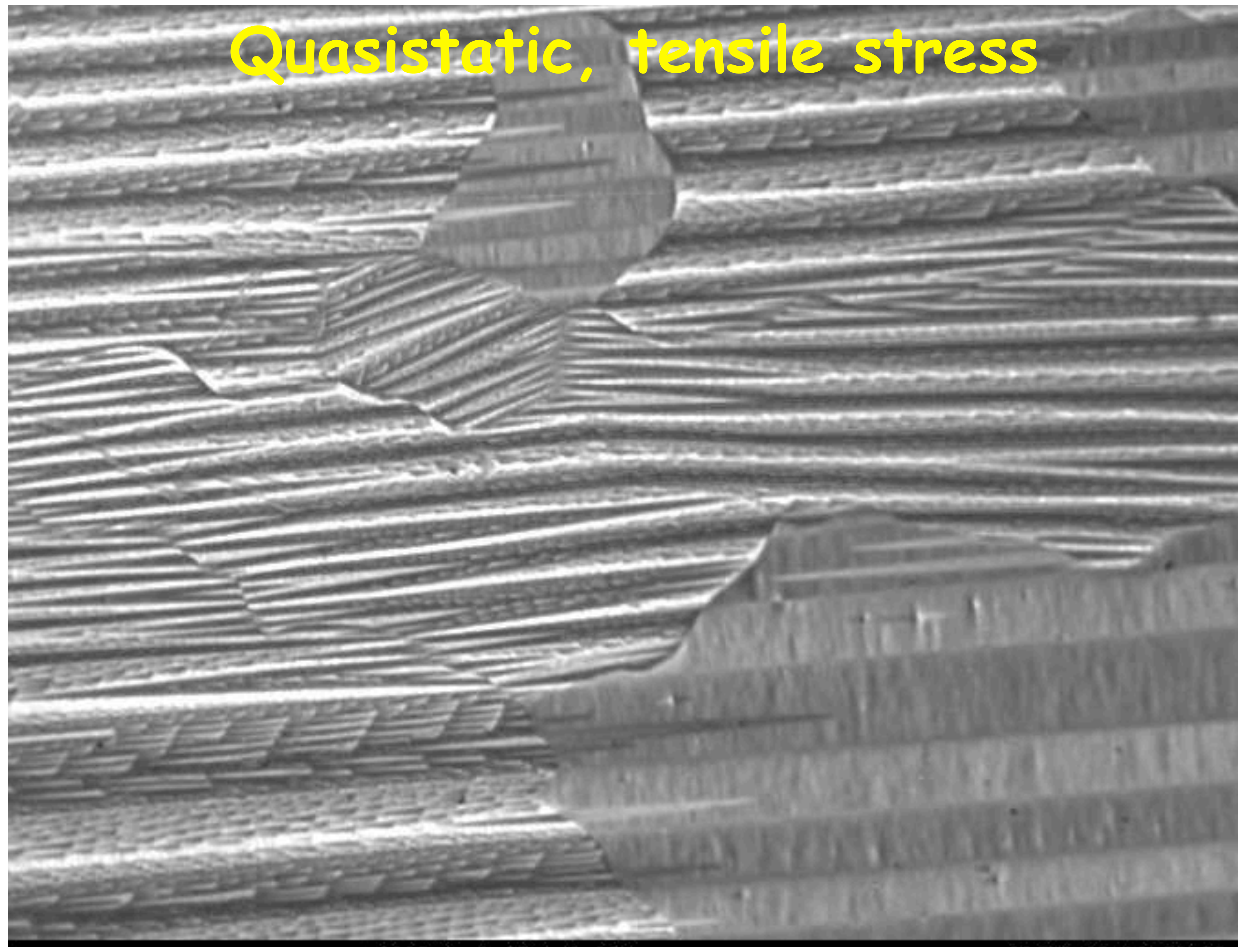
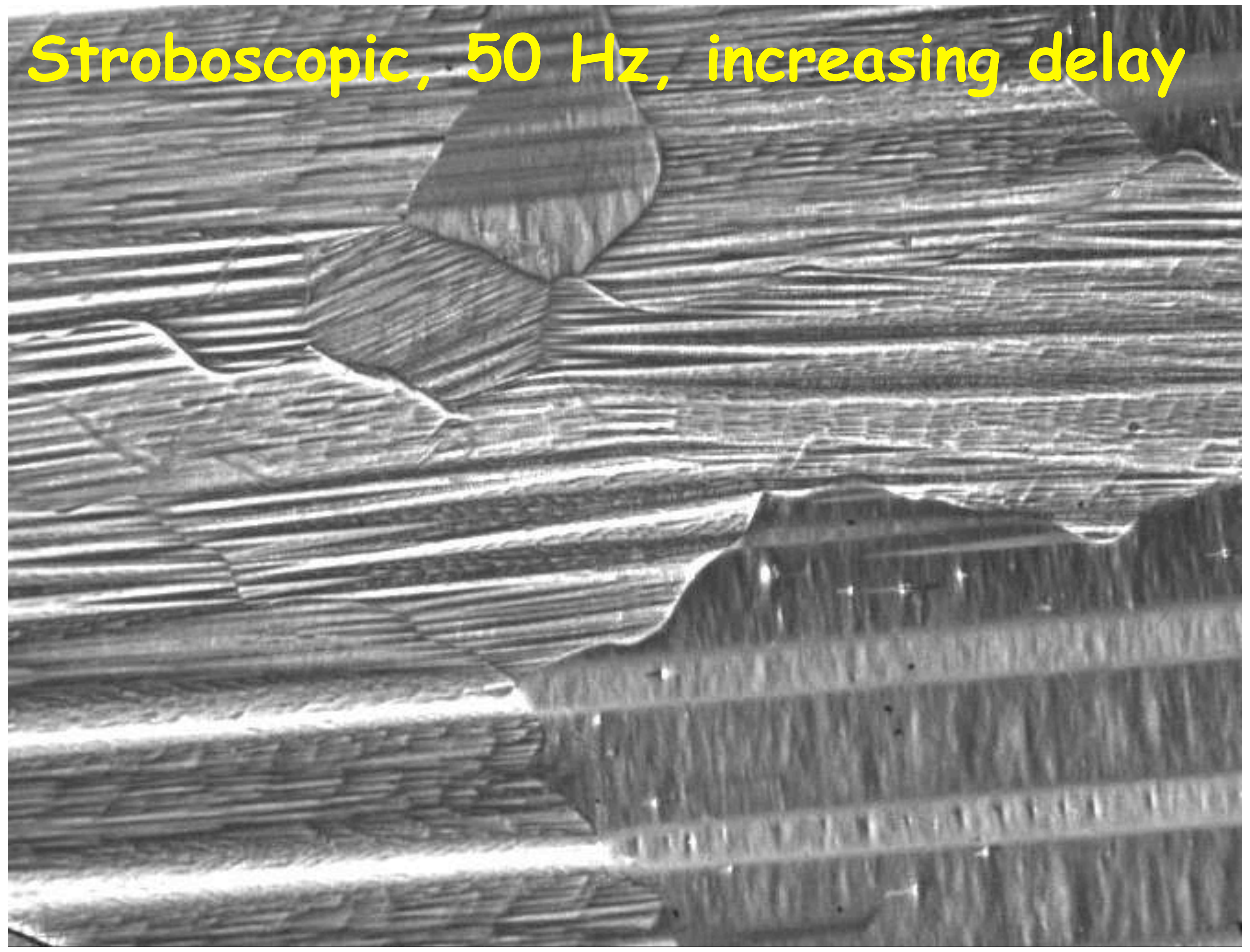


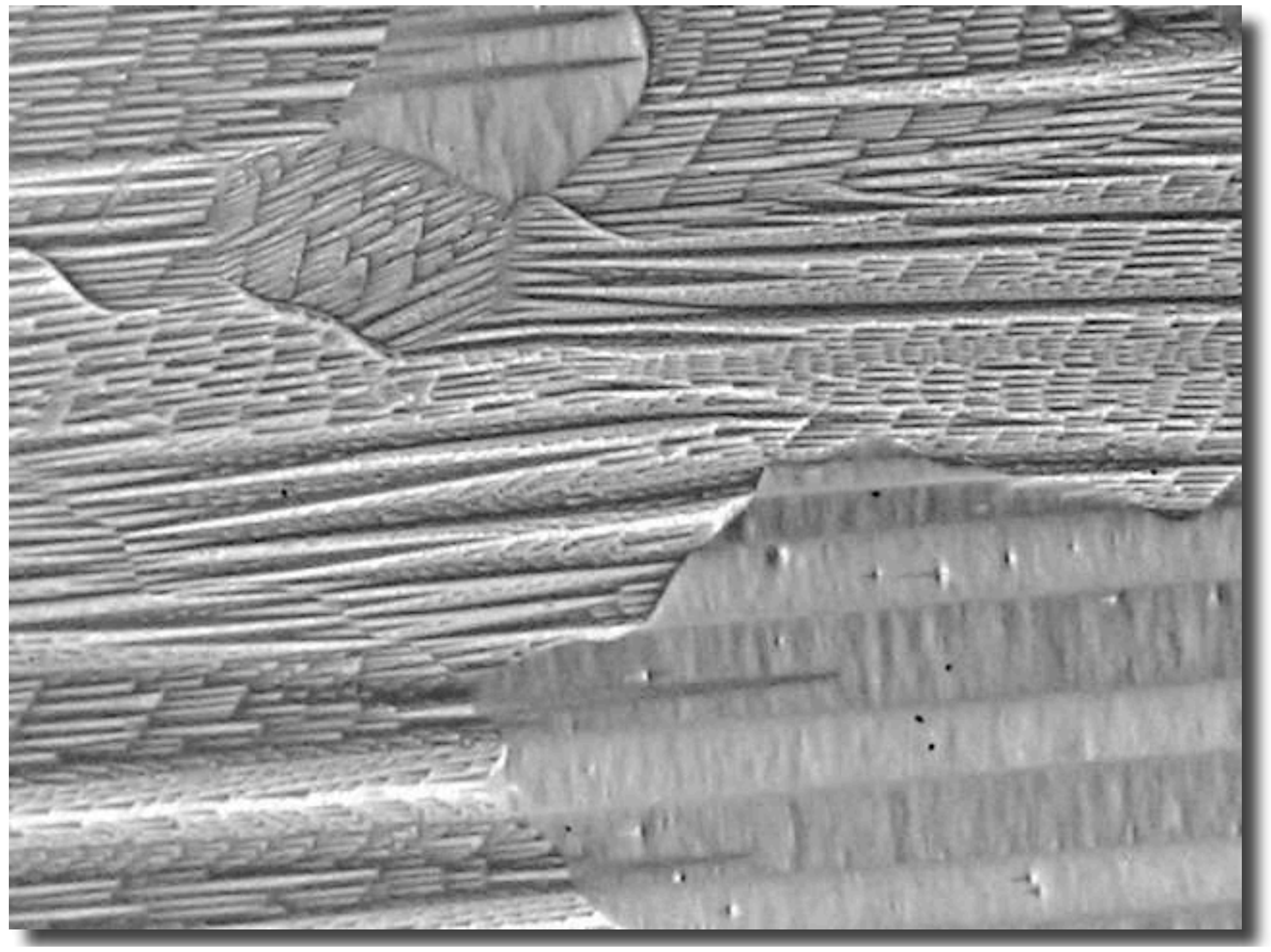


FeSi Goss sheet, polished

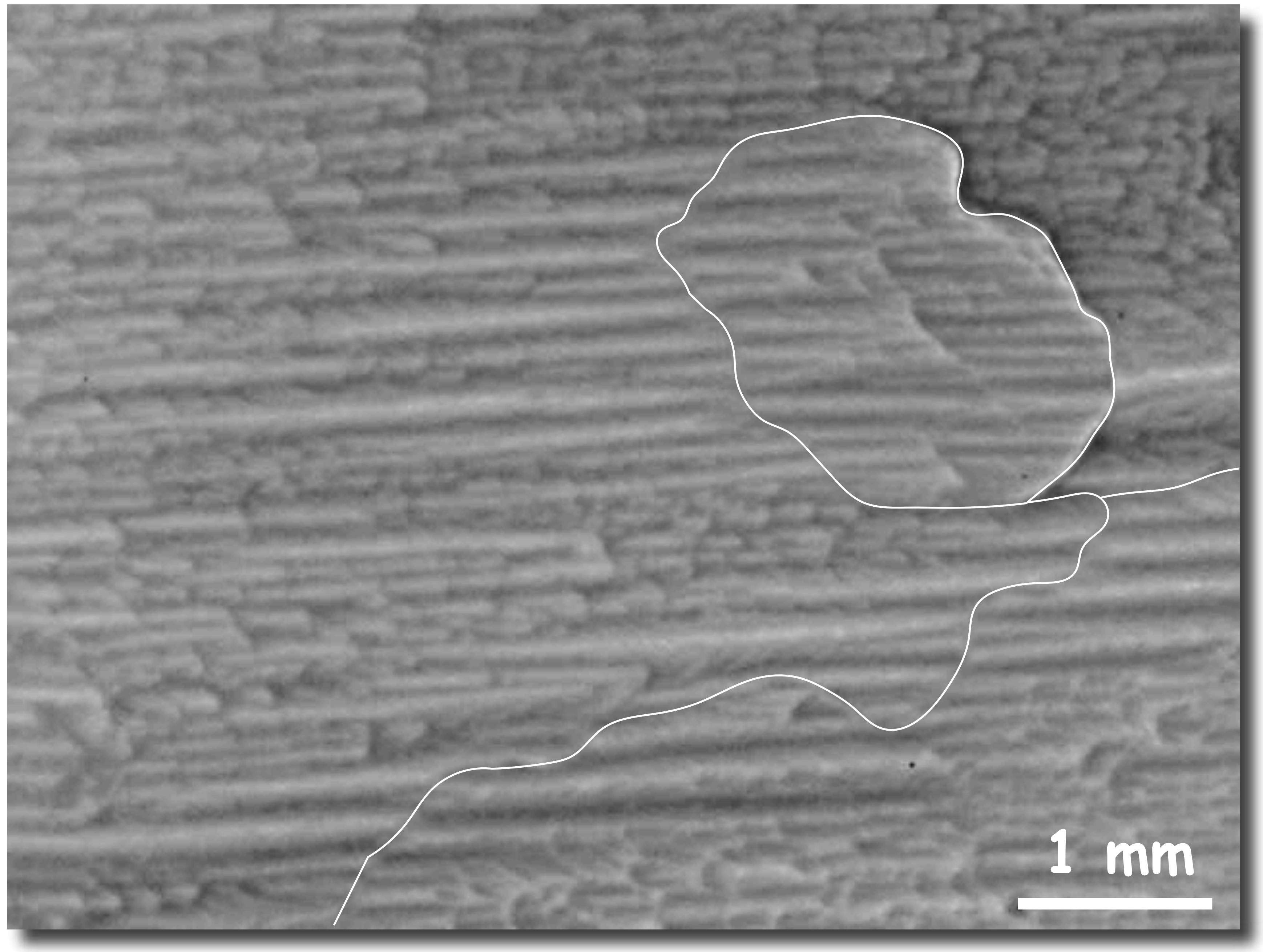
R.S. et al., JMMM 474 (2019)



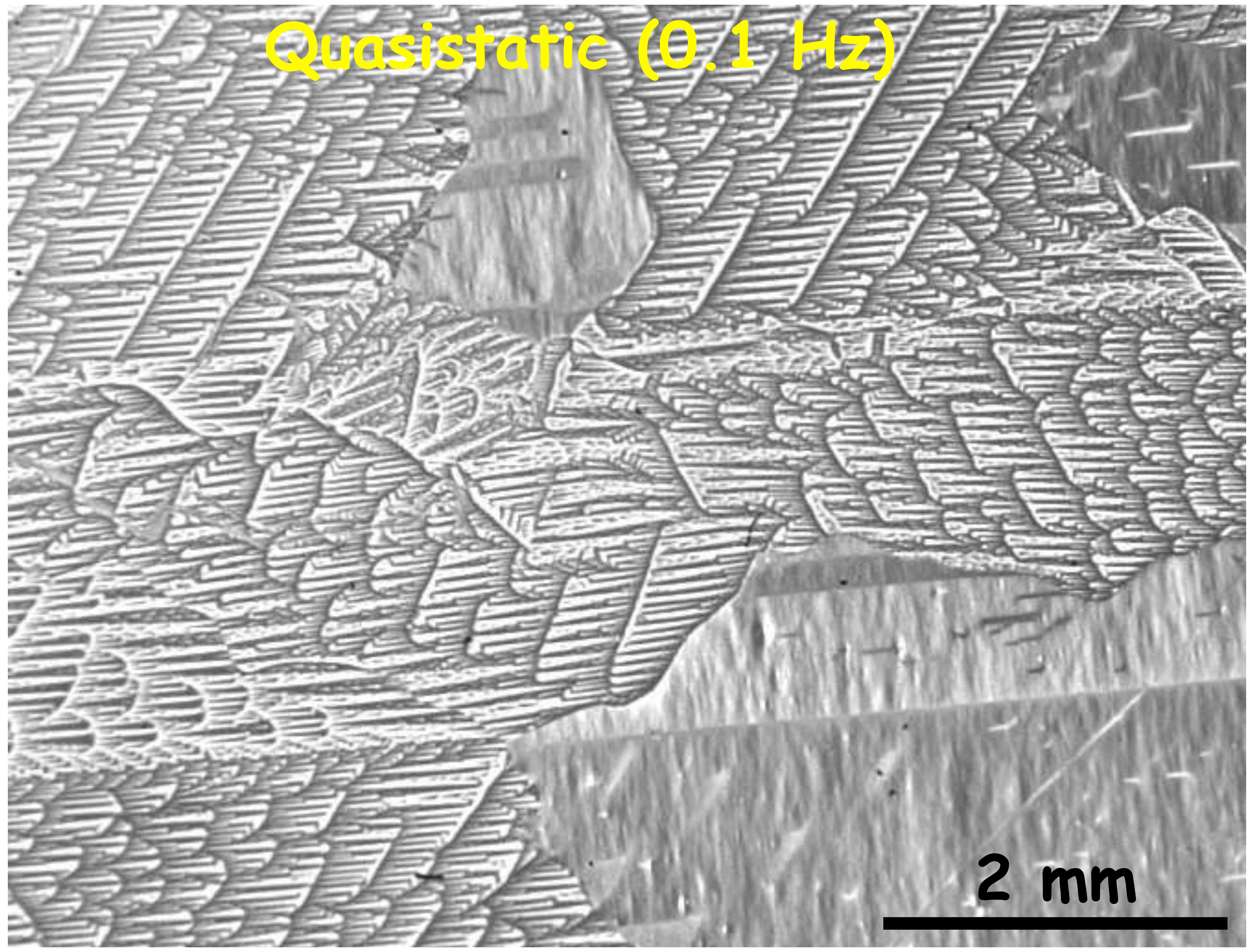
Single shot, 50 Hz, 30 A/m

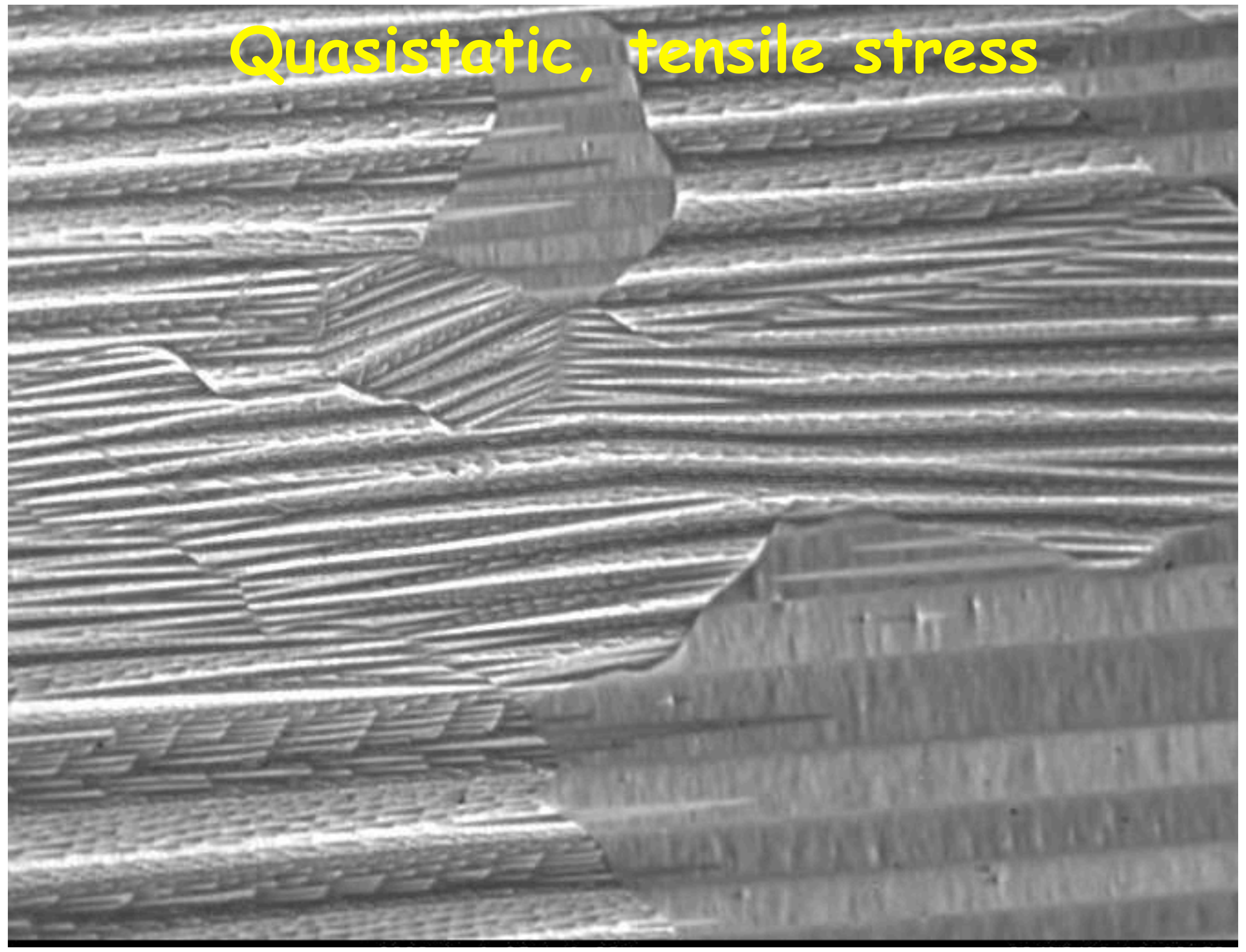


FeSi electrical steel **with coating**, 50 Hz single-shot imaging:



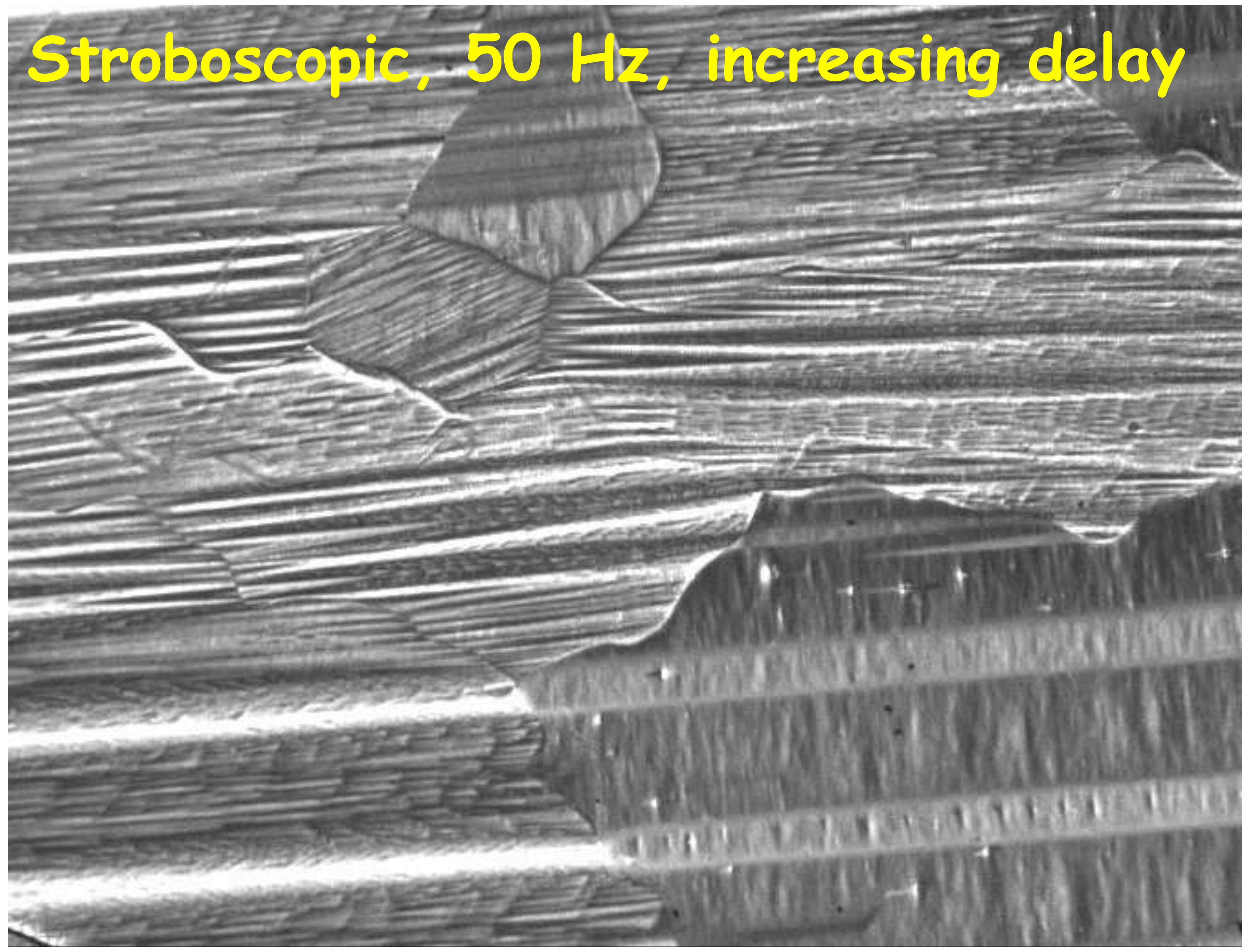
5. MOIF Microscopy



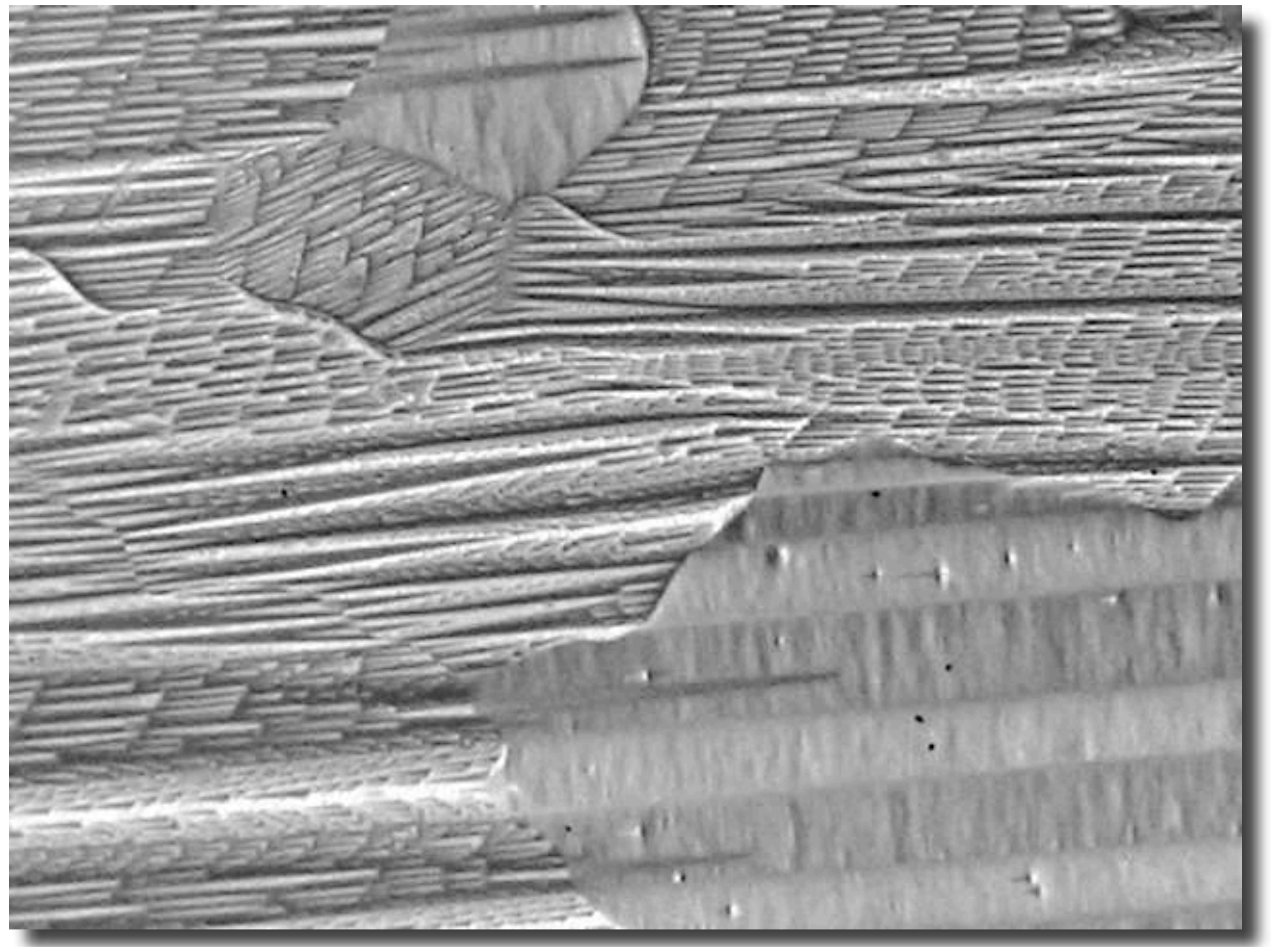


FeSi Goss sheet, polished

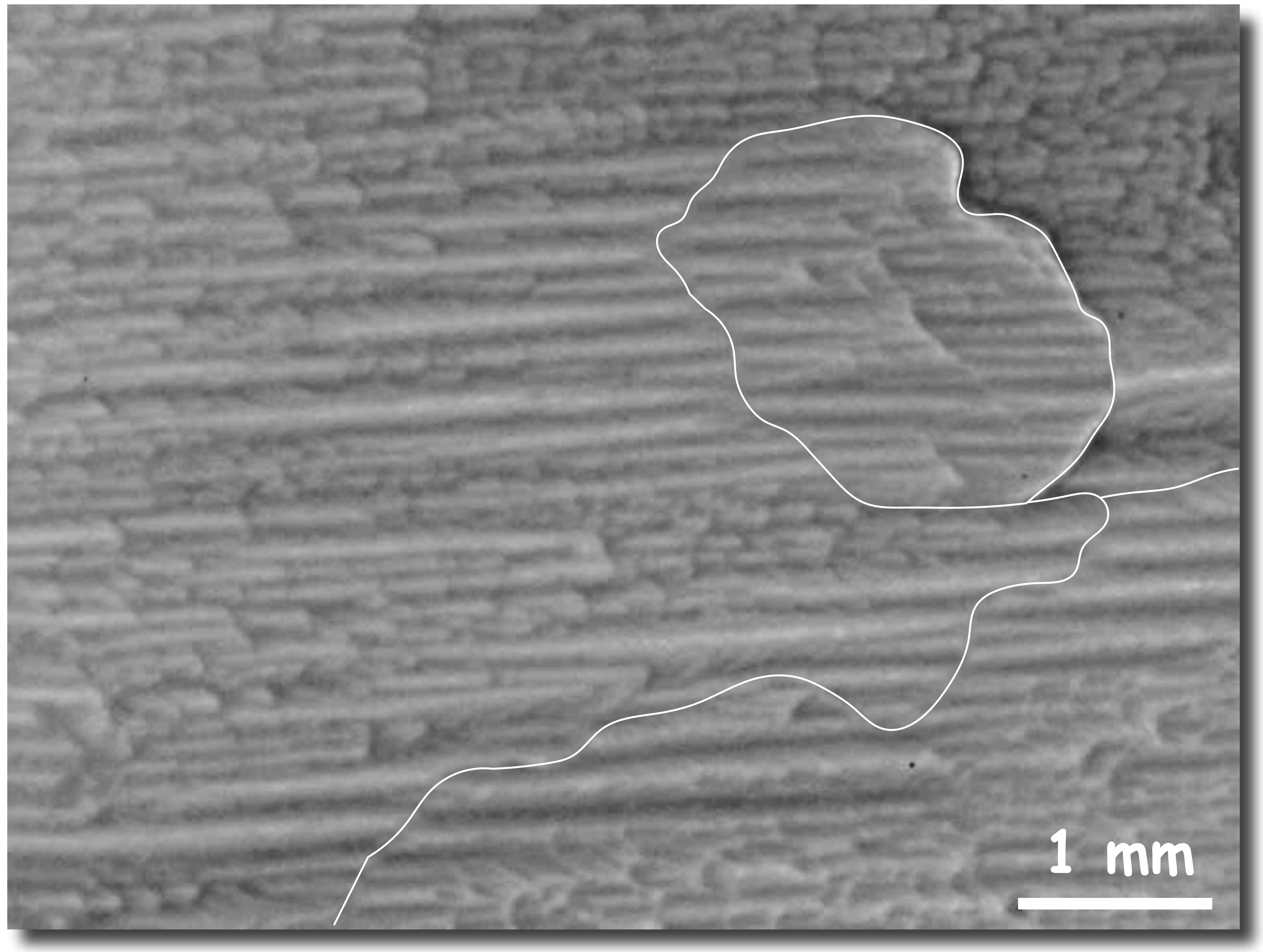
R.S. et al., JMMM 474 (2019)



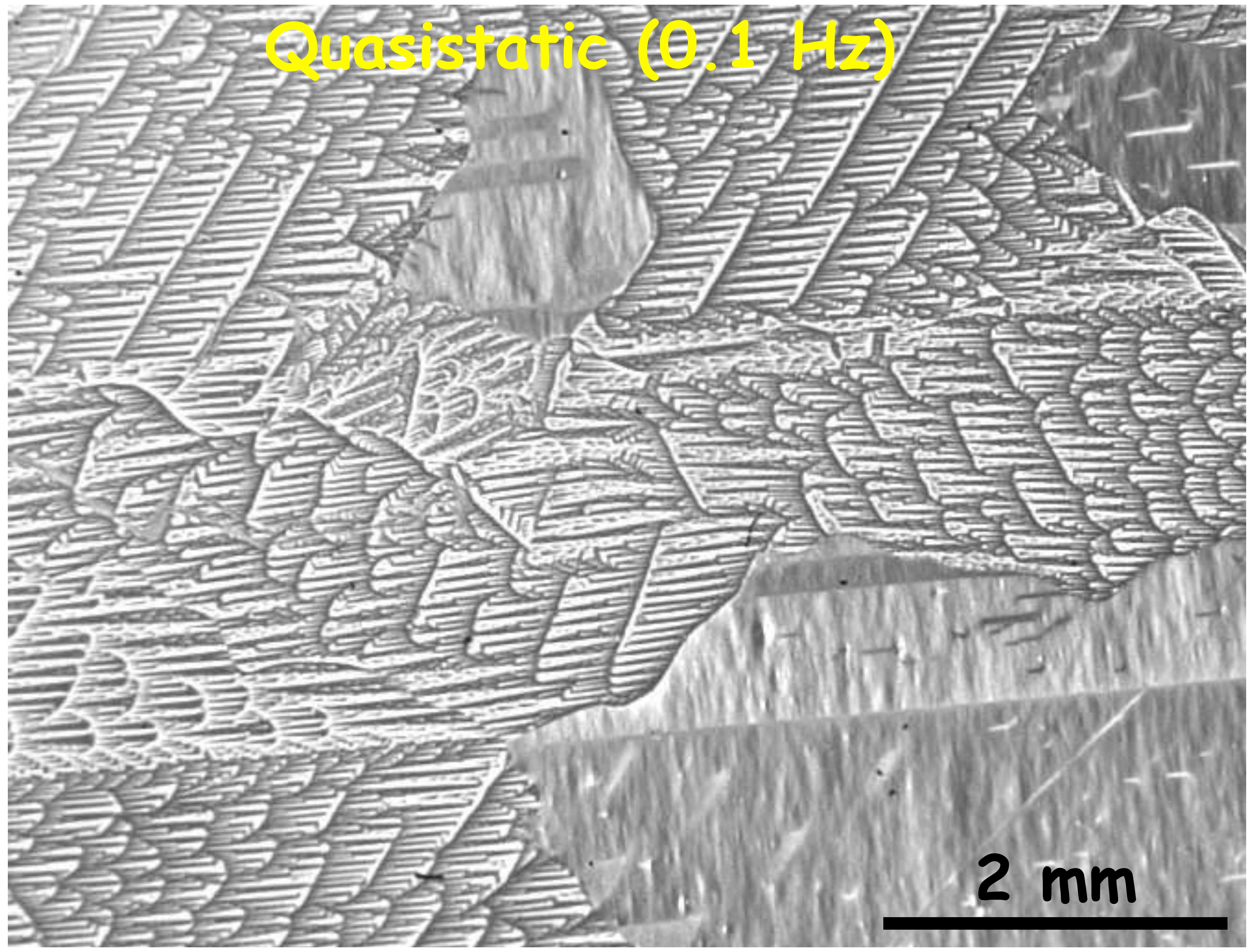
Single shot, 50 Hz, 30 A/m

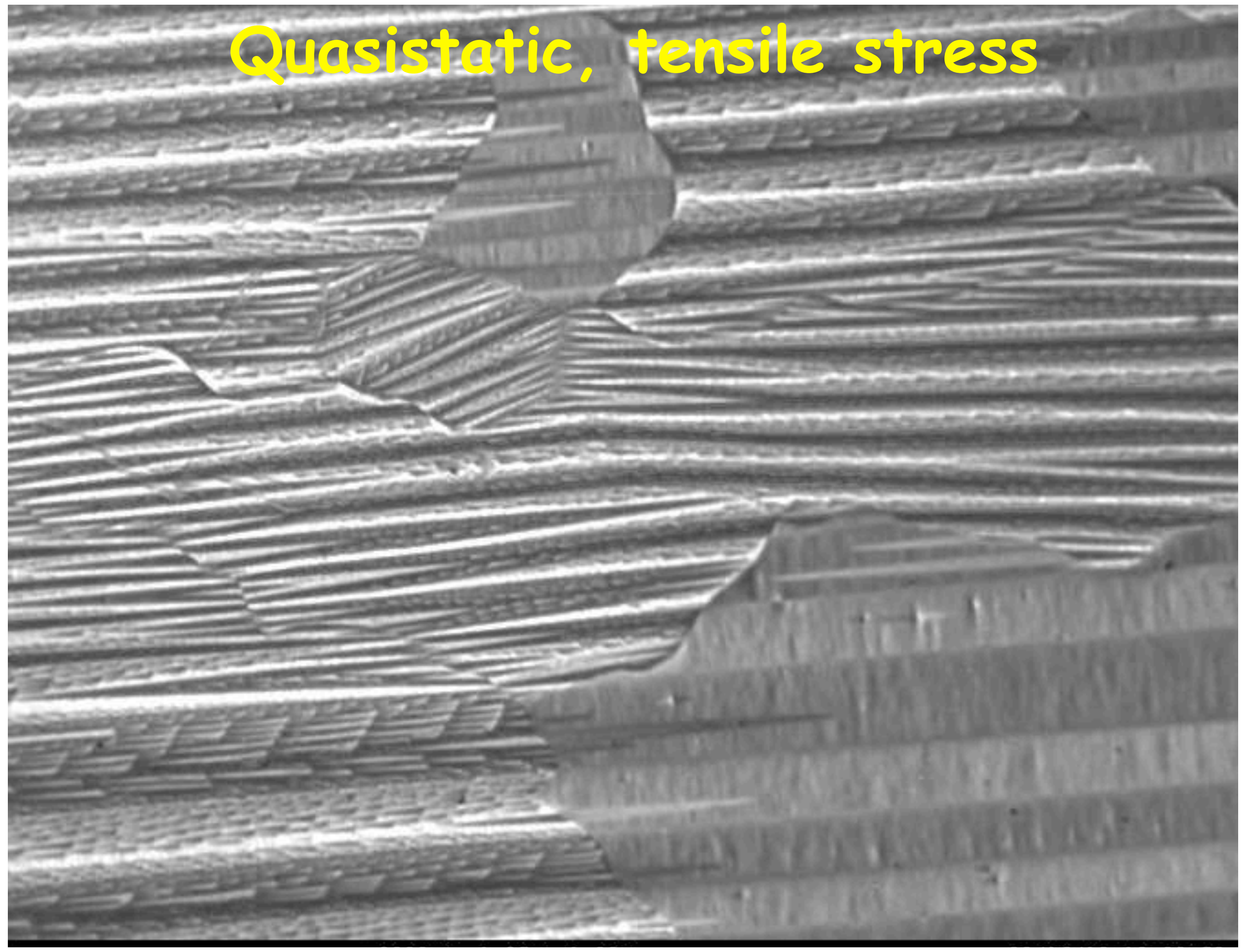


FeSi electrical steel **with coating**, 50 Hz single-shot imaging:



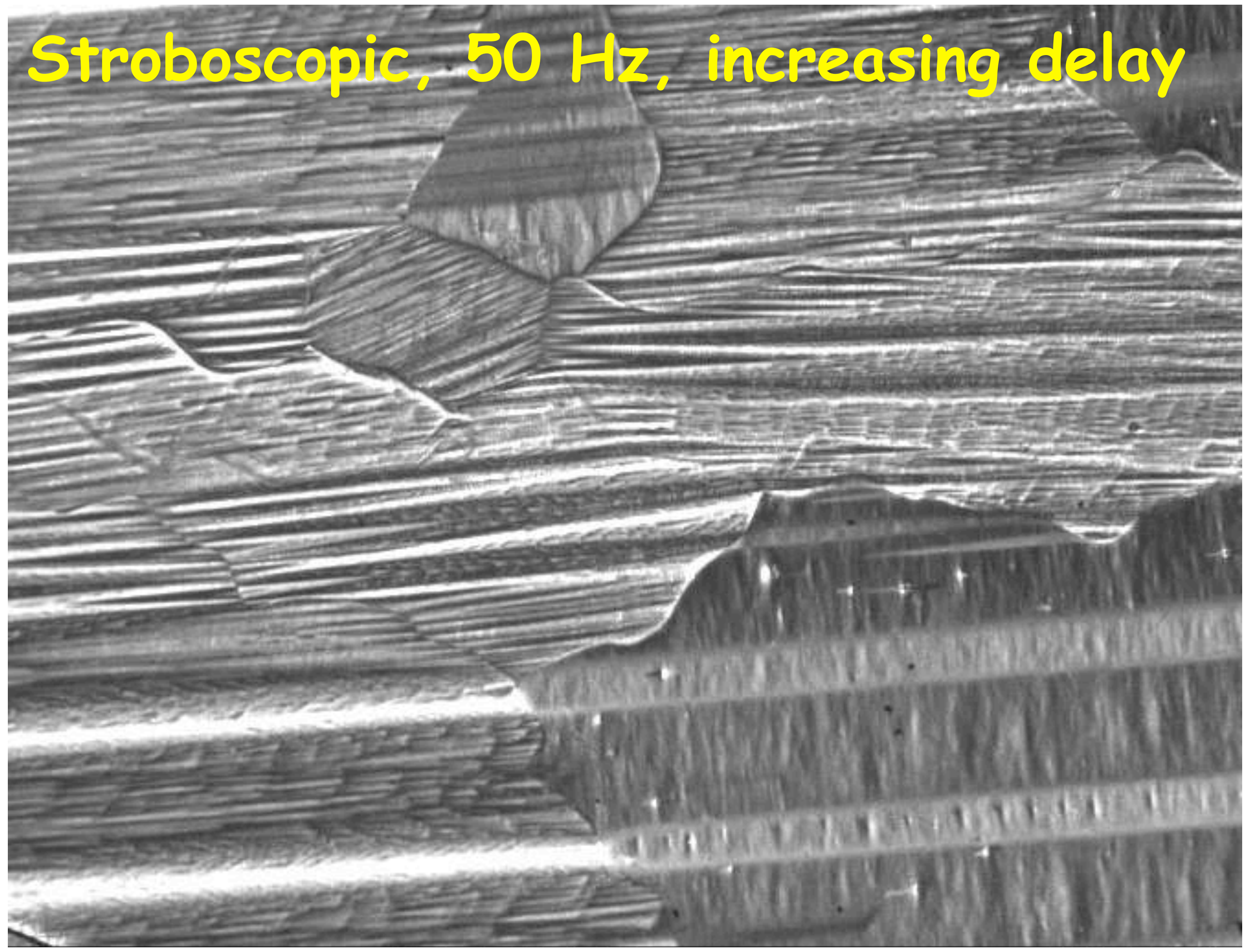
5. MOIF Microscopy



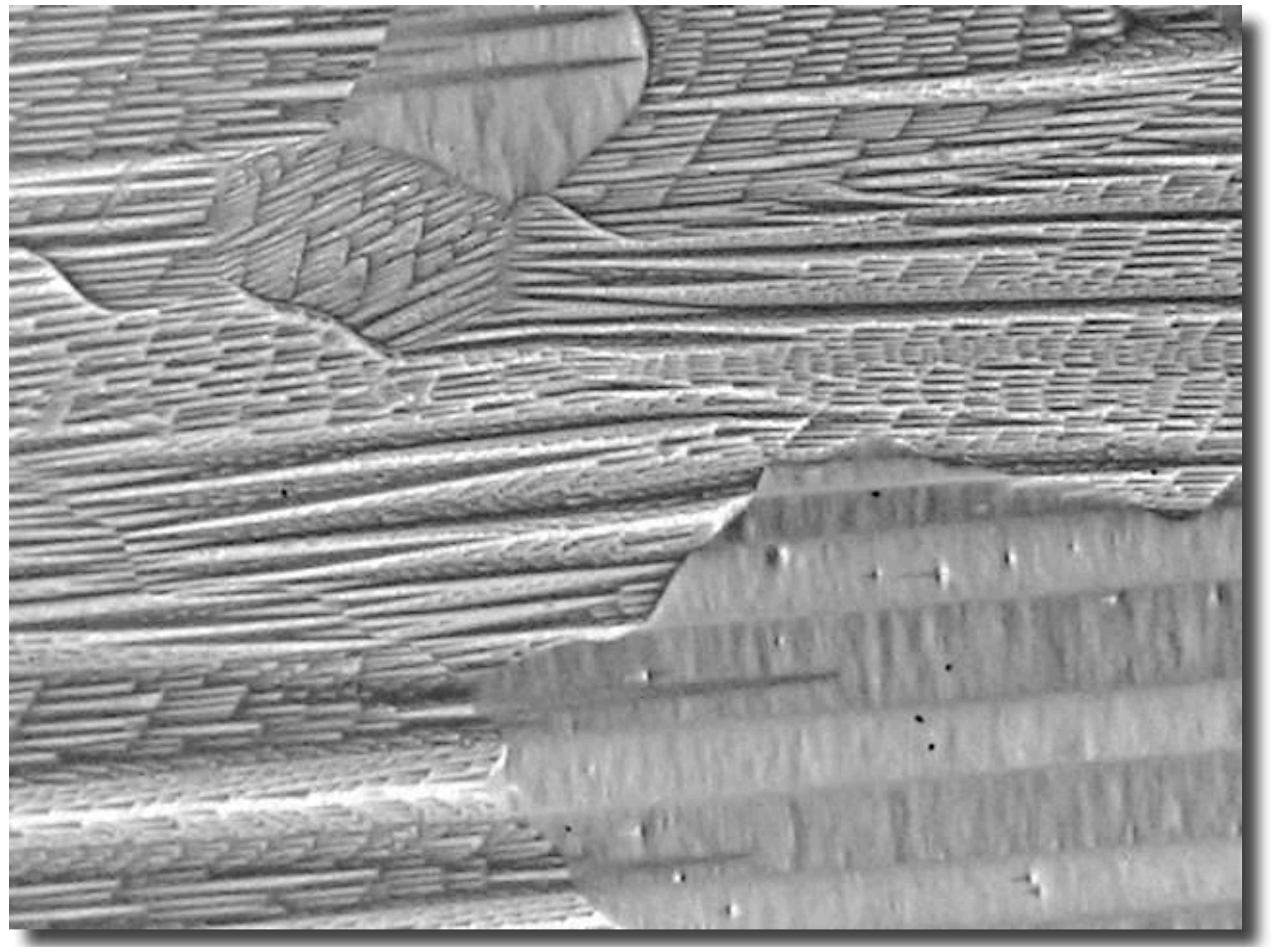


FeSi Goss sheet, polished

R.S. et al., JMMM 474 (2019)



Single shot, 50 Hz, 30 A/m



FeSi electrical steel **with coating**, 50 Hz single-shot imaging: