THE EUROPEAN SCHOOL ON MAGNETISM

Brno 2019

Lecture 16

General and torque magnetometry

Measurement of magnetic materials and magnetic fields

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Outline

- Introduction: Maxwell equations
- Generation and measurement of magnetic fields
- Measurement of soft and hard magnetic materials

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- Introduction: Maxwell equations
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Maxwell equations

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$

Gauss's law (1835)

$$\nabla \cdot \boldsymbol{B} = 0$$

absence of magnetic charges

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

Faraday's law (1831)

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$

Ampère's law (1826)

+ Maxwell (1861)

conservation of the charge

$$\frac{d\rho}{dt} + \nabla \cdot \vec{j} = 0$$

$$\epsilon_0 = 1/(c^2 \mu_0)$$

Maxwell equations

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}$$ Gauss's law (1835)

$$\nabla \cdot \boldsymbol{B} = 0$$ absence of magnetic charges

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$ Faraday's law (1831)

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{j} + \mu_0 \epsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$$ Ampère's law (1826)
+ Maxwell (1861)

conservation of the charge

$$\frac{d\rho}{dt} + \nabla \cdot \vec{j} = 0$$

old SI before *20 May 2019* $\mu_0 = 4\pi \cdot 10^{-7}\ \mathrm{H/m}$

new SI: *20 May 2019*

$$\mu_0 = 4\pi[1 + 2.0(2.3) \times 10^{-10}] \times 10^{-7}\mathrm{H/m}$$

$$\epsilon_0 = 1/(c^2\mu_0) = 8.85 \cdot 10^{-12}\ \mathrm{F/m}$$

Maxwell equations in matter

$$
\begin{aligned}
\nabla \cdot \boldsymbol{D} &= \rho \\
\nabla \cdot \boldsymbol{B} &= 0 \\
\nabla \times \boldsymbol{E} &= -\frac{\partial \boldsymbol{B}}{\partial t} \\
\nabla \times \boldsymbol{H} &= \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}
\end{aligned}
$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$$

$$\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M}$$

constitutive relations

$$\boldsymbol{P}(\boldsymbol{E})$$

$$\boldsymbol{M}(\boldsymbol{H}) \qquad \boldsymbol{J} = \mu_0 \boldsymbol{M}$$

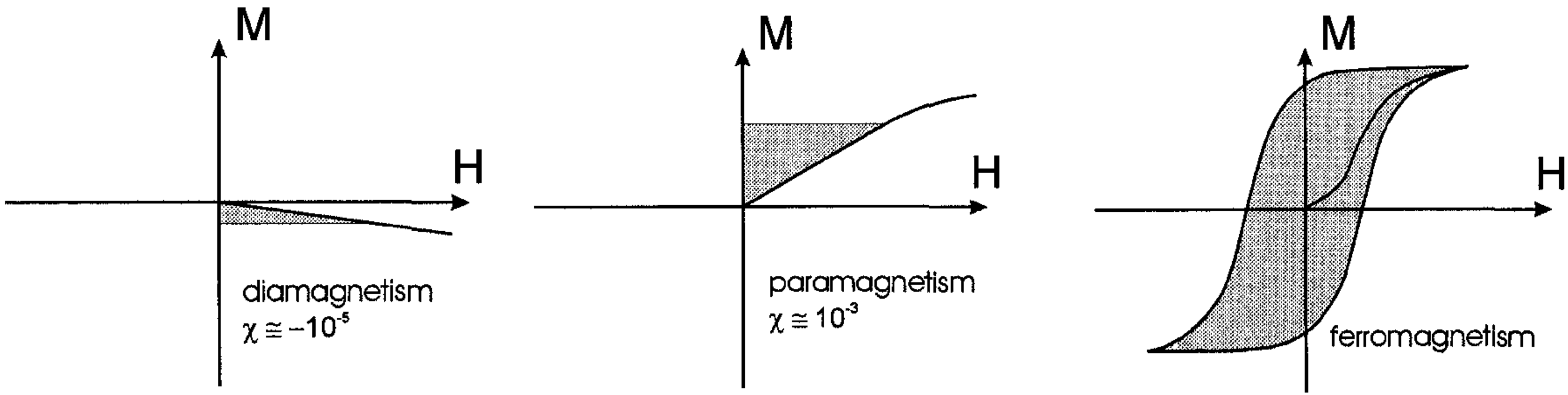
all quantities are "local averages"

$$F(\boldsymbol{r}) = \frac{1}{\Delta V} \int_{\Delta V} F_{micro} d^3 r$$

$$du = \mu_0 \boldsymbol{H} \cdot d\boldsymbol{M} + \boldsymbol{E} \cdot d\boldsymbol{P}$$

constitutive relations

$$M(H)$$



Magnetostatic field

magnetostatic approximation

$$\begin{cases} \nabla \cdot \boldsymbol{H} & = & -\nabla \cdot \boldsymbol{M} \\ \nabla \times \boldsymbol{H} & = & \boldsymbol{j} \end{cases}$$

Magnetostatic field

magnetostatic approximation

$$\begin{cases} \nabla \cdot \boldsymbol{H} &= -\nabla \cdot \boldsymbol{M} \\ \nabla \times \boldsymbol{H} &= \boldsymbol{j} \end{cases}$$

$$\boxed{\begin{cases} \nabla \cdot \boldsymbol{H}_M &= -\nabla \cdot \boldsymbol{M} \\ \nabla \times \boldsymbol{H}_M &= 0 \end{cases}}$$

magnetic scalar potential

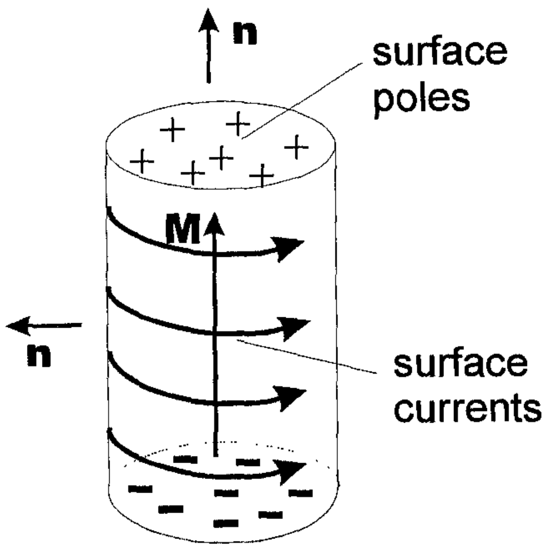
$$\boldsymbol{H}_M = -\nabla \phi_M$$

Poisson equation

$$\nabla^2 \phi_M = -\rho_M \qquad \rho_M = -\nabla \cdot \boldsymbol{M}$$

solution

$$\phi_M = -\frac{1}{4\pi}\int_V \frac{\nabla \cdot \boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} d^3 r' + \frac{1}{4\pi}\int_\Sigma \frac{\boldsymbol{n} \cdot \boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|} d^2 r'$$



Demagnetizing field

with uniform magnetization $\phi_M = -\frac{1}{4\pi}\int_V \frac{\nabla\cdot\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3r' + \frac{1}{4\pi}\int_\Sigma \frac{\boldsymbol{n}\cdot\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}d^2r'$

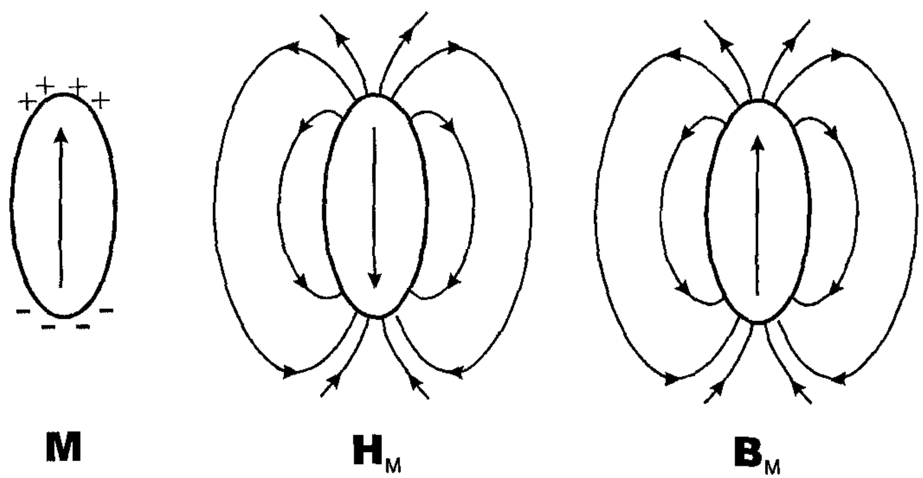
$$\phi_M = \frac{1}{4\pi}\boldsymbol{M}\cdot\int_\Sigma \frac{\boldsymbol{n}}{|\boldsymbol{r}-\boldsymbol{r}'|}d^2r'$$

the magnetostatic field

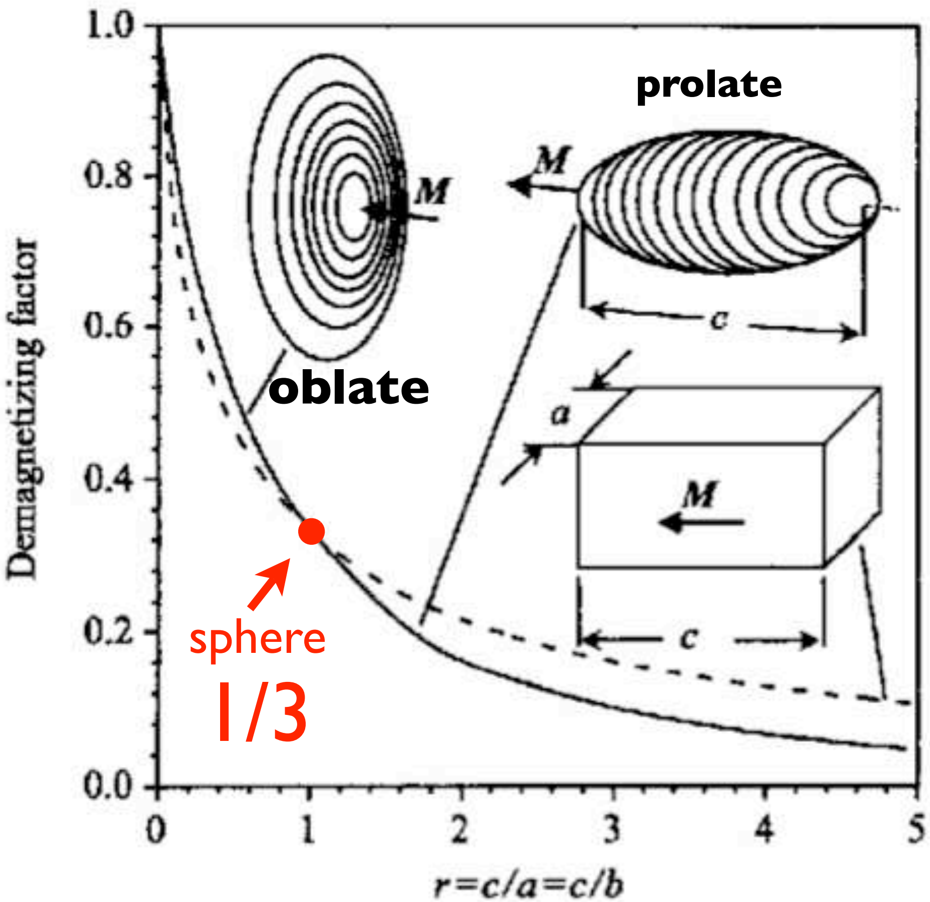
$$\boldsymbol{H}_M = -\nabla\phi_M$$

is **uniform** inside the body for ellipsoids

$$\boldsymbol{H}_M = -N\boldsymbol{M}$$



Demagnetizing factor



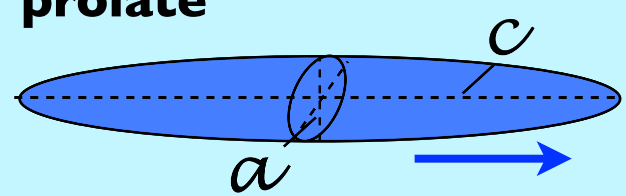
$$N_a + N_b + N_c = 1$$

Demagnetizing coefficient

$$\mu_0 H = -N_d J$$

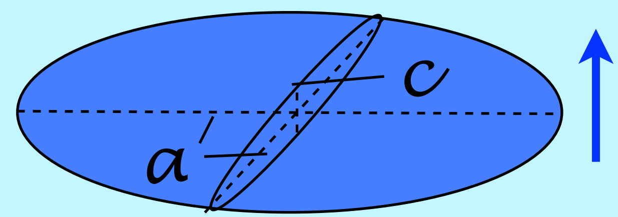
prolate

$r = c/a$



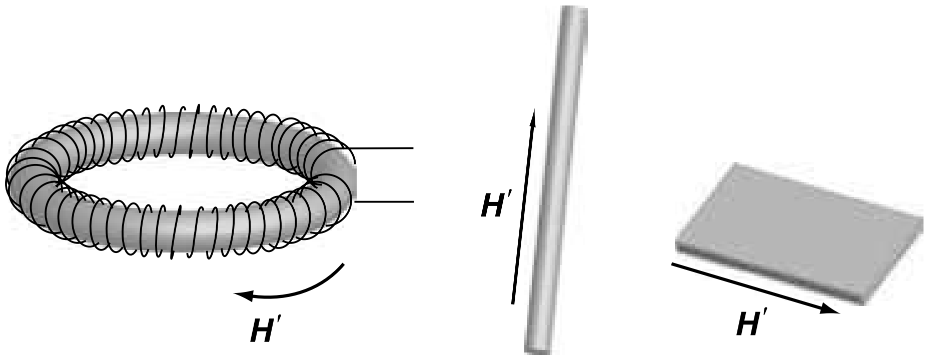
$$N_{d_c} = \frac{1}{r^2 - 1}\left[\frac{r}{\sqrt{r^2-1}}\ln\left(r + \sqrt{r^2 - 1}\right) - 1\right]$$

oblate



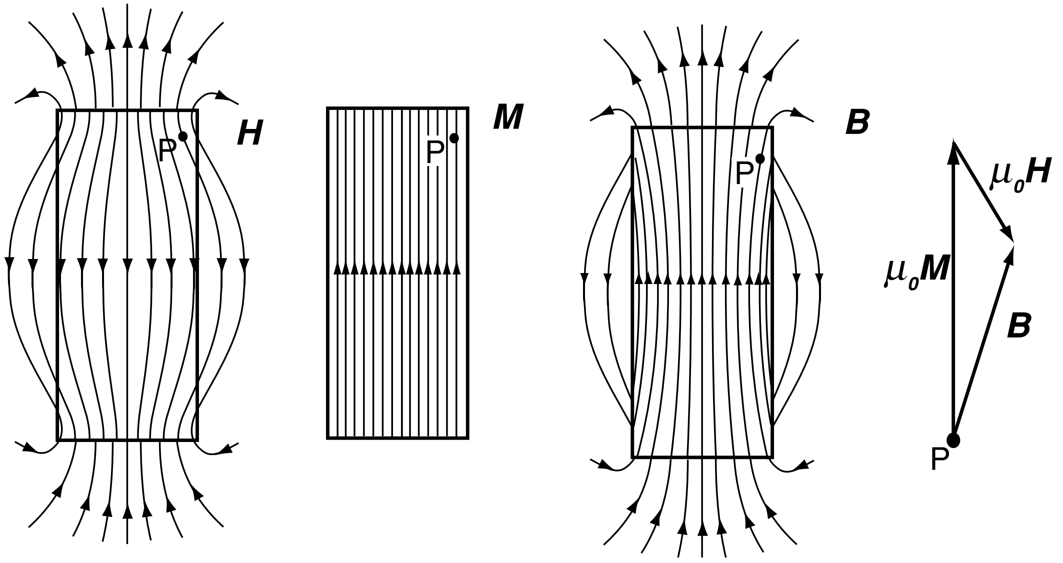
$$N_{d_c} = \frac{1}{1 - r^2}\left[1 - \frac{r}{\sqrt{1-r^2}}\arcsin\left(\sqrt{1 - r^2}\right)\right]$$

Demagnetizing field



shapes with no or small demagnetizing effects

non ellipsoidal shapes...



Outline

- Introduction: Maxwell equations
- Generation and measurement of magnetic fields
- Measurement of soft and hard magnetic materials

Generation of magnetic fields

- Solenoids, Helmholtz coils,
- electromagnets,
- with permanent magnets,
- large magnetic fields with currents

Generation of magnetic fields

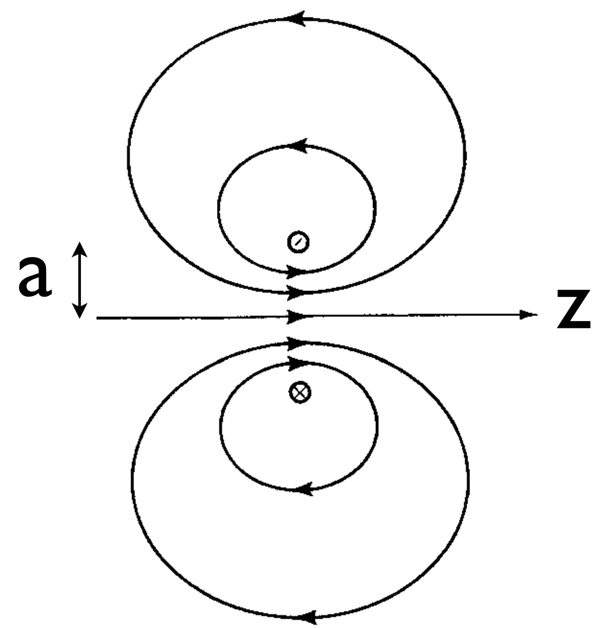
current loop

Biot-Savart law

$$\boldsymbol{H} = \frac{I}{4\pi} \oint \frac{d\boldsymbol{l}(r') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} d^3 r'$$

$$H_z = \frac{a^2 I}{2(a^2 + z^2)^{3/2}}$$

$m = \pi a^2 I$



B.D. Cullity, Introduction to magnetic materials, 2nd editions, IEEE Press, Wiley, 2009

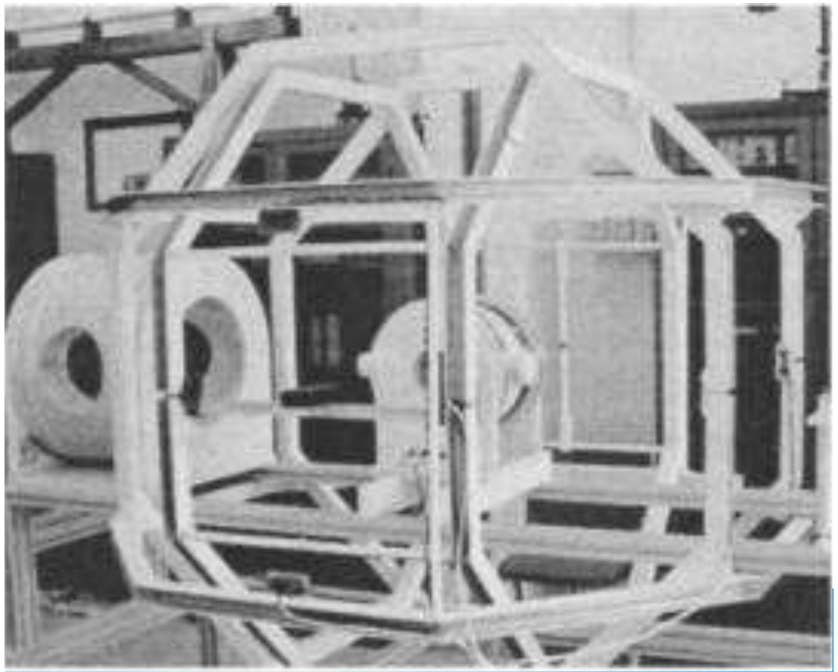
Generation of magnetic fields

Helmholtz coil

< 8 kA/m

$$H_x(x,0) = 4Nia^2\left[\frac{1}{(4a^2+(2x+a)^2)^{3/2}} + \frac{1}{(4a^2+(2x-a)^2)^{3/2}}\right],$$

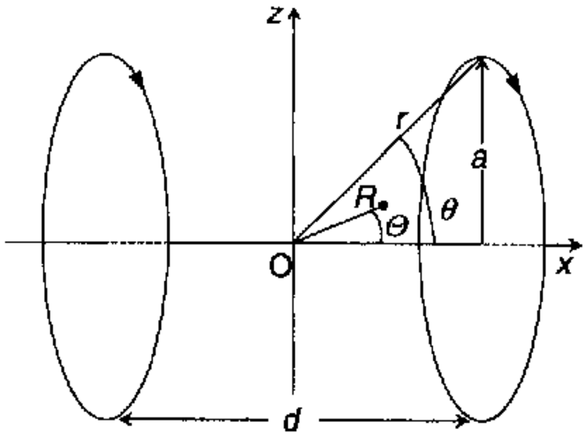
$$H_x(0,0) = 0.7155\,\frac{Ni}{a}$$

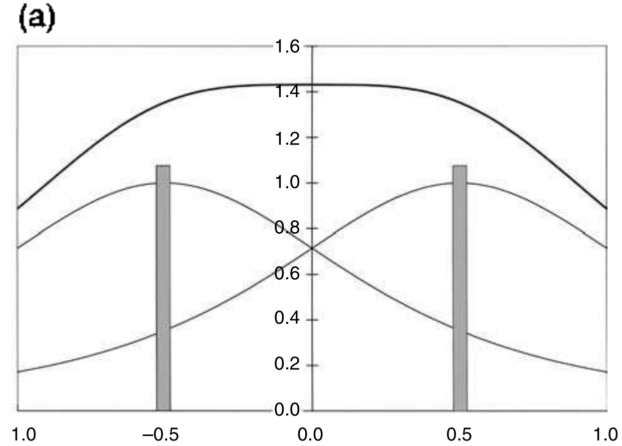


d=a

www.inrim.it

www.laboratorio.elettrofisico.com

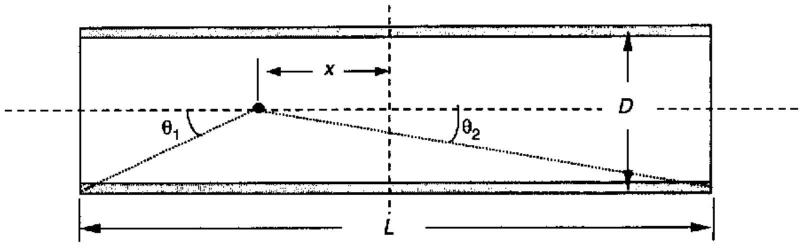




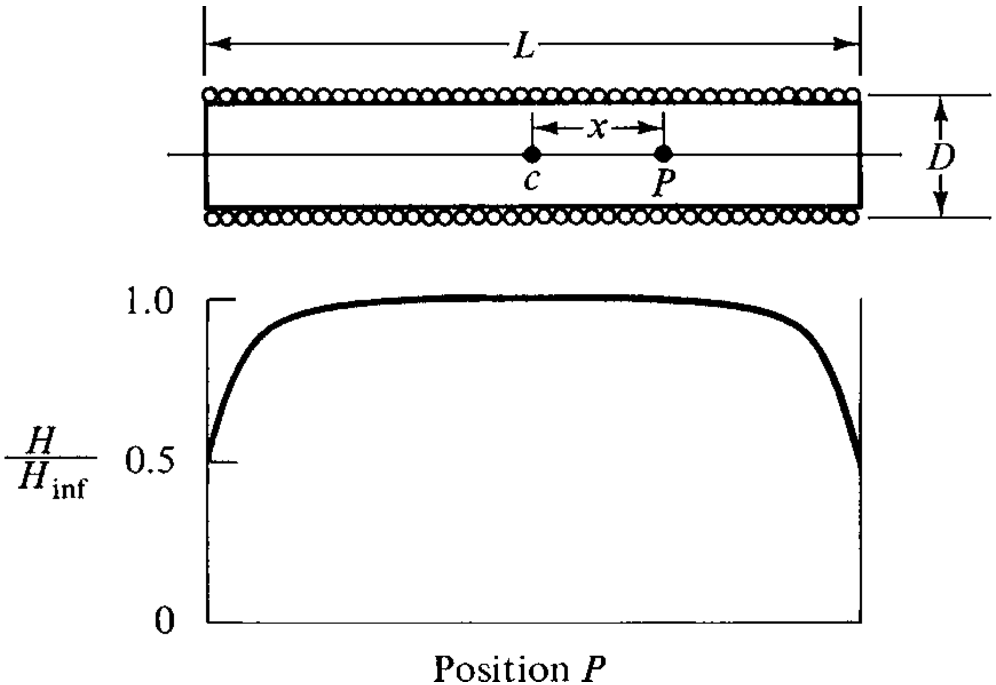
Generation of magnetic fields

thin, long solenoid

< 80 kA/m



$$H = C_1 \frac{ni}{L}\left[\frac{L+2x}{2\sqrt{D^2+(L+2x)^2}} + \frac{L-2x}{2\sqrt{D^2+(L-2x)^2}}\right],$$

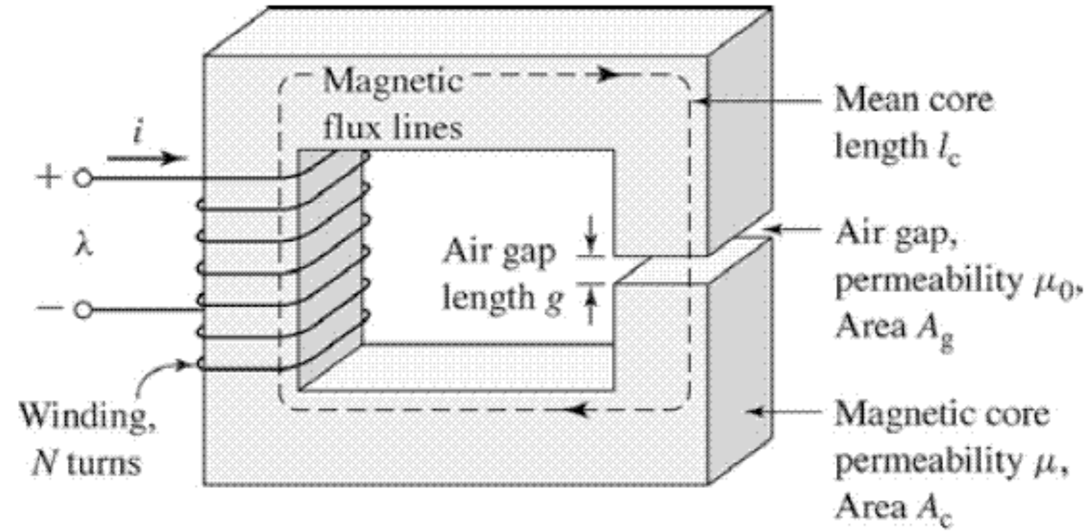


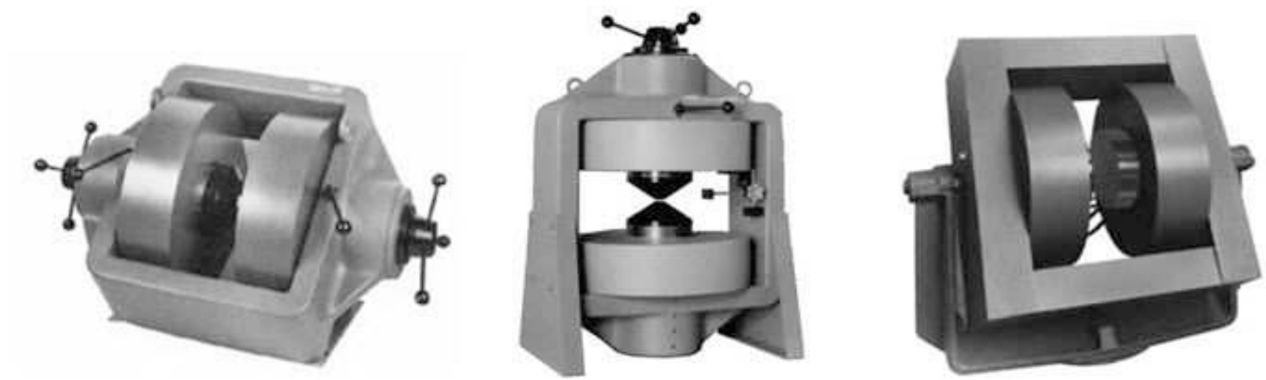
B.D. Cullity, Introduction to magnetic materials, 2nd editions, IEEE Press, Wiley, 2009

Generation of magnetic fields

Electromagnets < 2.5 - 3.0 T

$$H_g = \frac{N\,I}{l_g + \frac{S_g}{S_m}\frac{\mu_0}{\mu_m} l_m}$$





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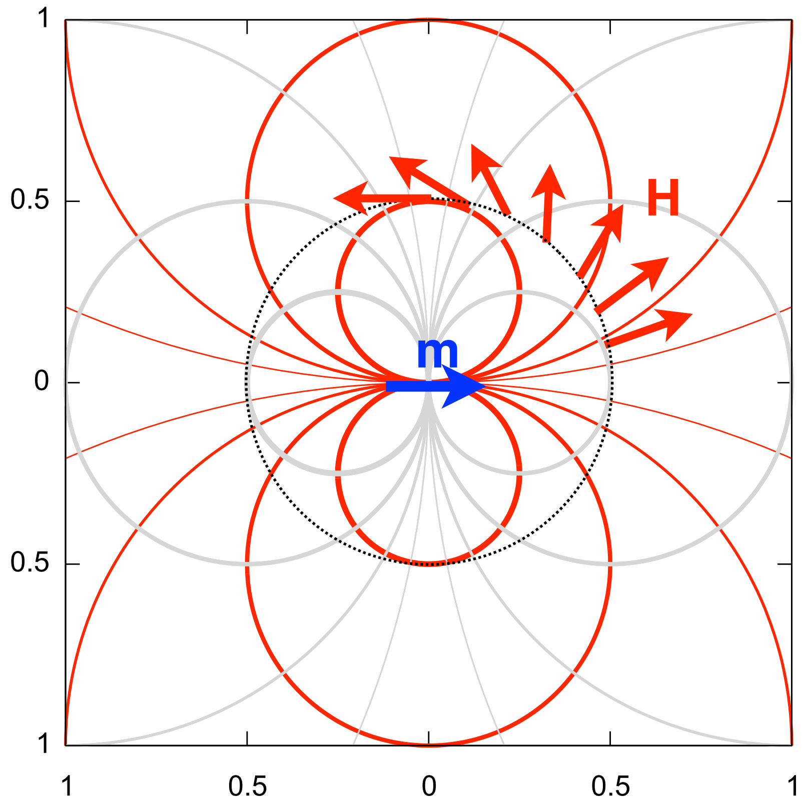
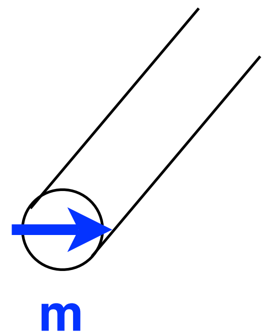
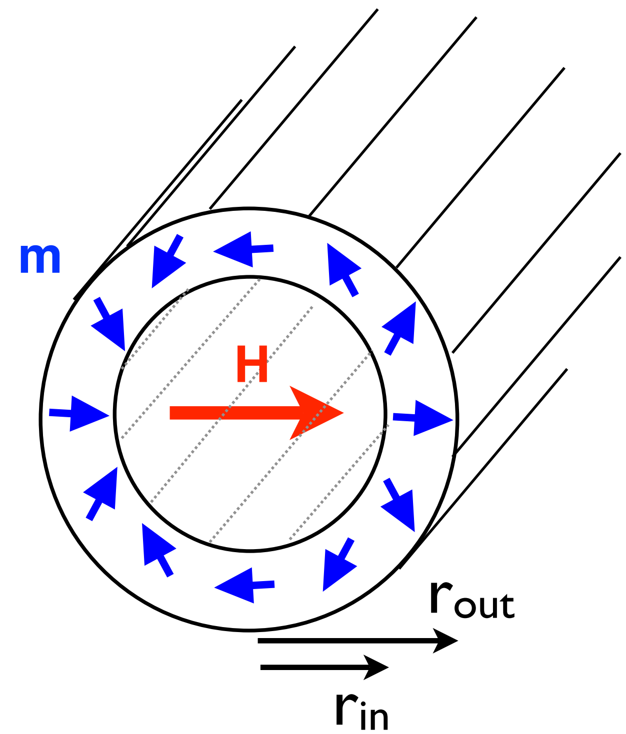
Generation of magnetic fields

with permanent magnets: the Halbach's cylinder

< 2 T







$$H = M_r \ln\left(\frac{r_{out}}{r_{in}}\right)$$

Point dipole

$$H_r = \frac{m}{4\pi}\frac{2\cos\theta}{r^3}$$

$$H_\theta = \frac{m}{4\pi}\frac{\sin\theta}{r^3}$$

Line dipole

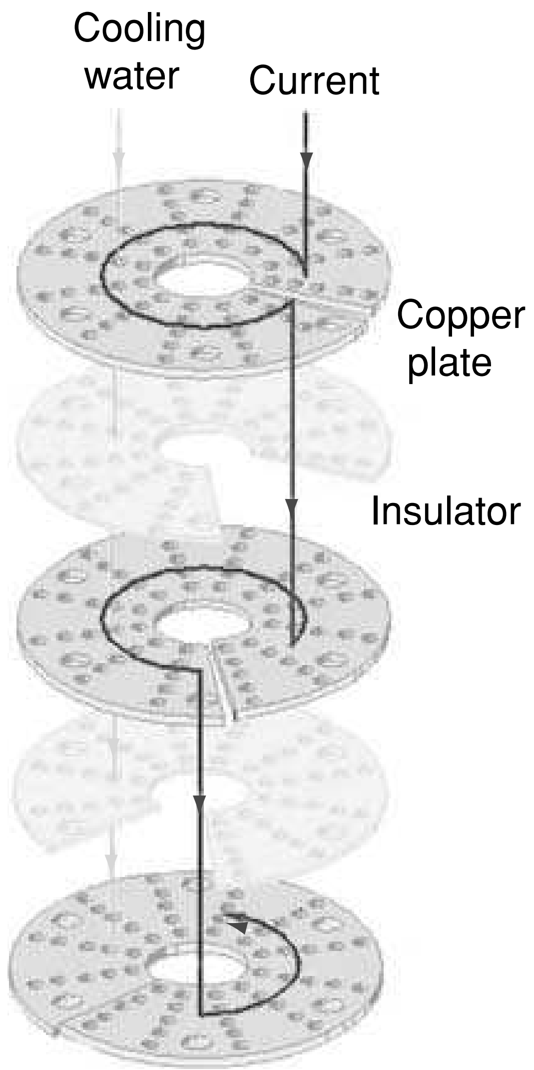
$$H_r = \frac{m}{2\pi d}\frac{\cos\theta}{r^2}$$

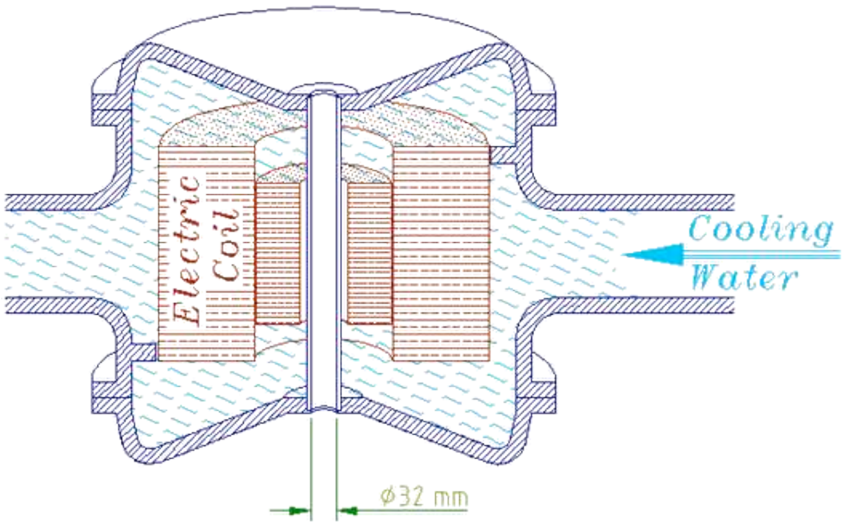
$$H_\theta = \frac{m}{2\pi d}\frac{\sin\theta}{r^2}$$

Generation of magnetic fields

Bitter's coils

< 20 - 30 T





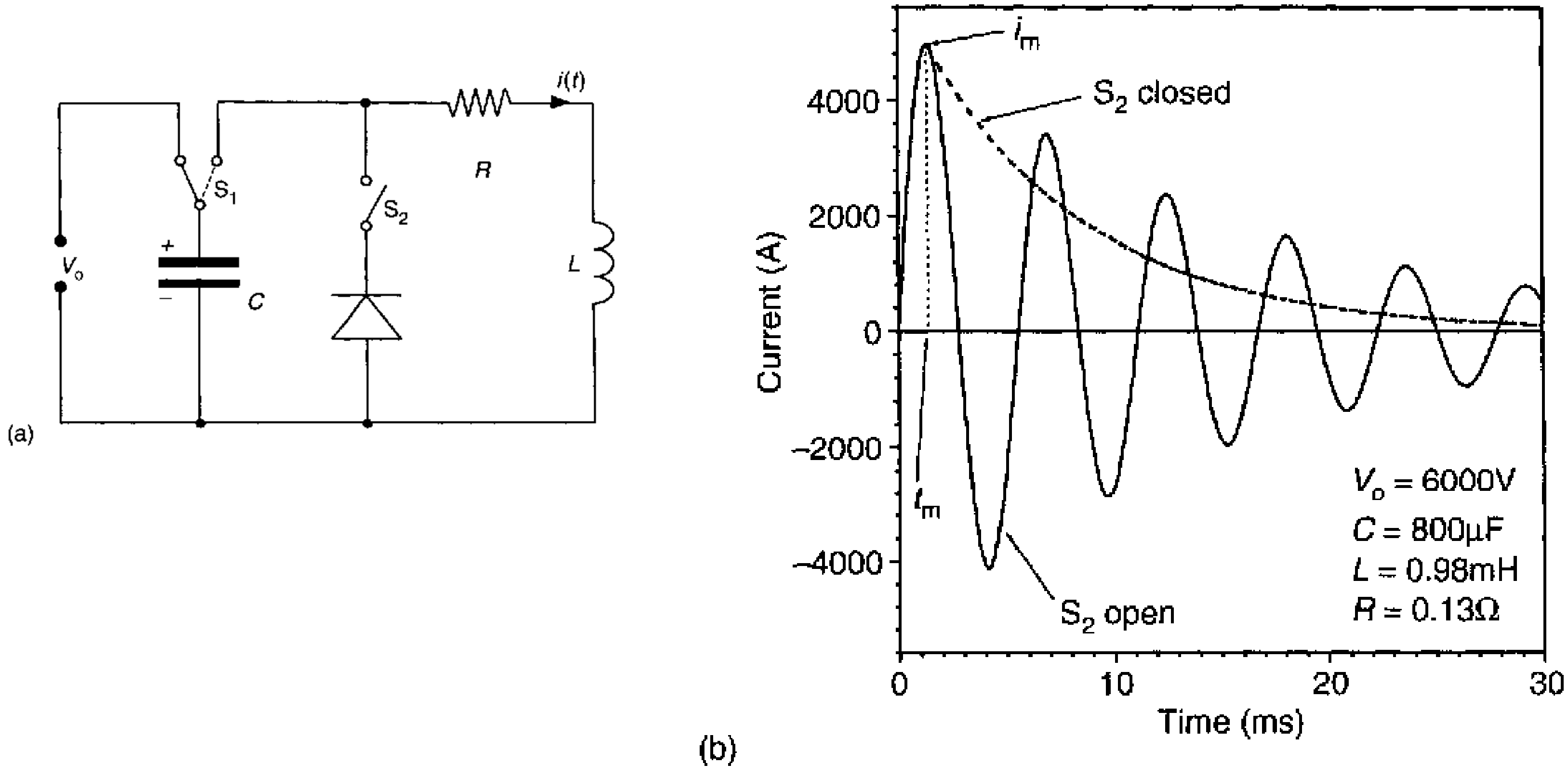
www.ru.nl/hfml

A little frog (alive !) and a water ball levitate inside a Ø32mm vertical bore of a Bitter solenoid in a magnetic field of about 16 Tesla at the Nijmegen High Field Magnet Laboratory.

Generation of magnetic fields

Current pulse

< 100 T



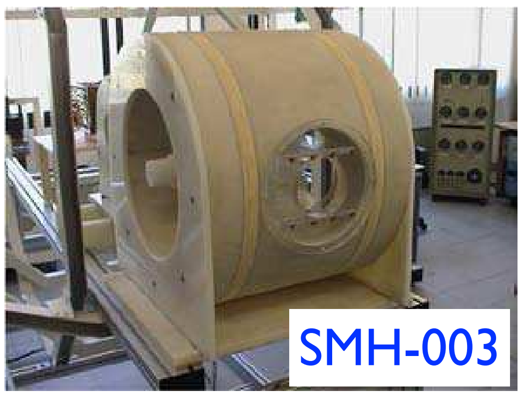
typical 7 T in labs

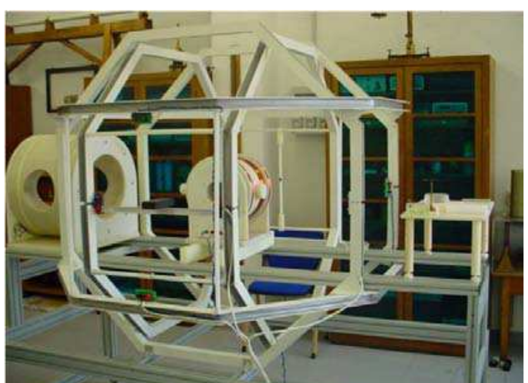
Measurement of magnetic fields

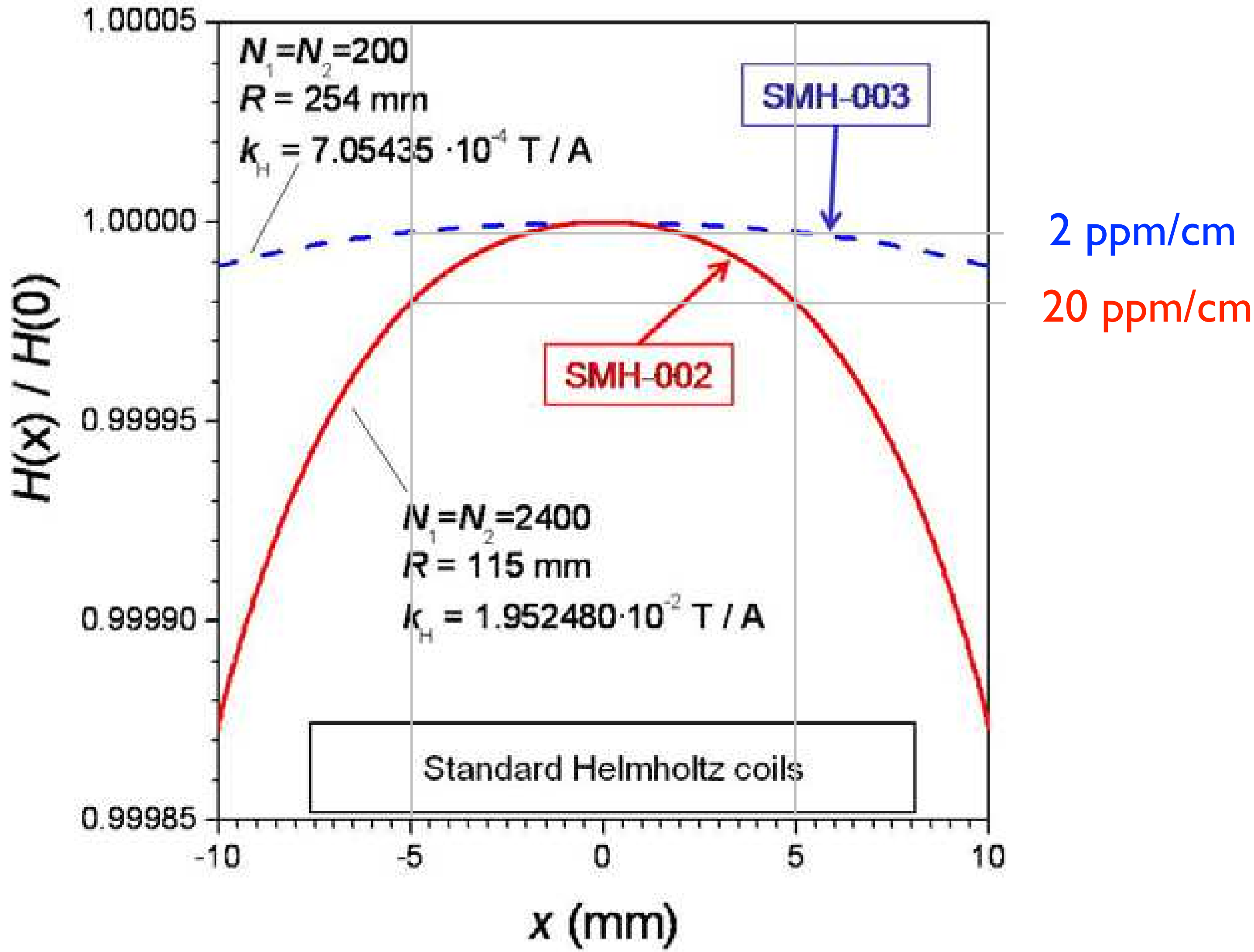
- standard coils
- H-coils
- Hall effect
- flux gate
- nuclear magnetic resonance

Measurement of magnetic fields

Standard coil **B =** ***k*** **I**

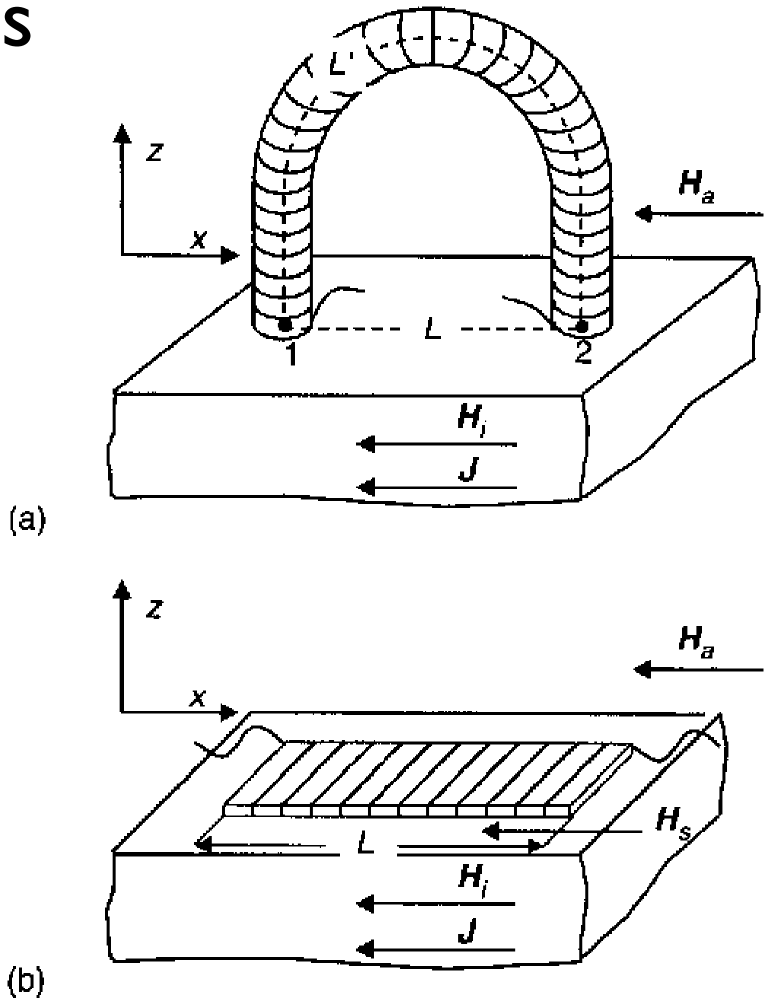






Measurement of magnetic fields

H coils



$$e = -\frac{d\Phi}{dt}$$

$$\Phi(t_2) - \Phi(t_1) = -\int_{t_1}^{t_2} e \, dt.$$

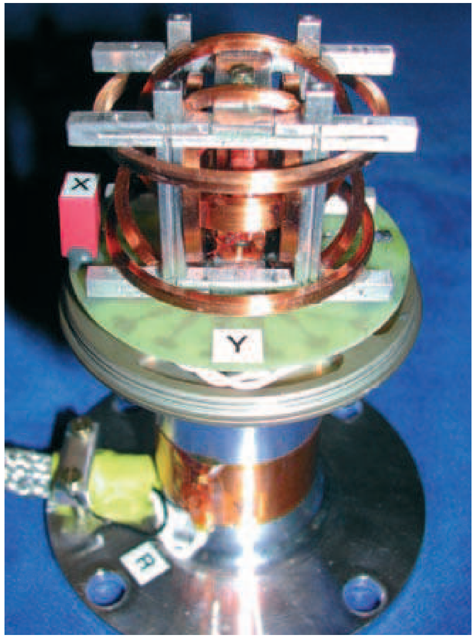
10^{-7} - 50 T

depends on frequency

FIGURE 5.3 A Chattock coil is linked with a flux proportional to the difference of the magnetic potential between its ends. When it is placed over a magnetized sheet and its end surfaces are at a distance L, it provides the quantity $V(L) = \int_L \mathbf{H}_s \cdot d\mathbf{x}$, with $\mathbf{H}_s$ the effective field at the sheet surface. This quantity can equally be measured by means of a uniformly wound flat coil.

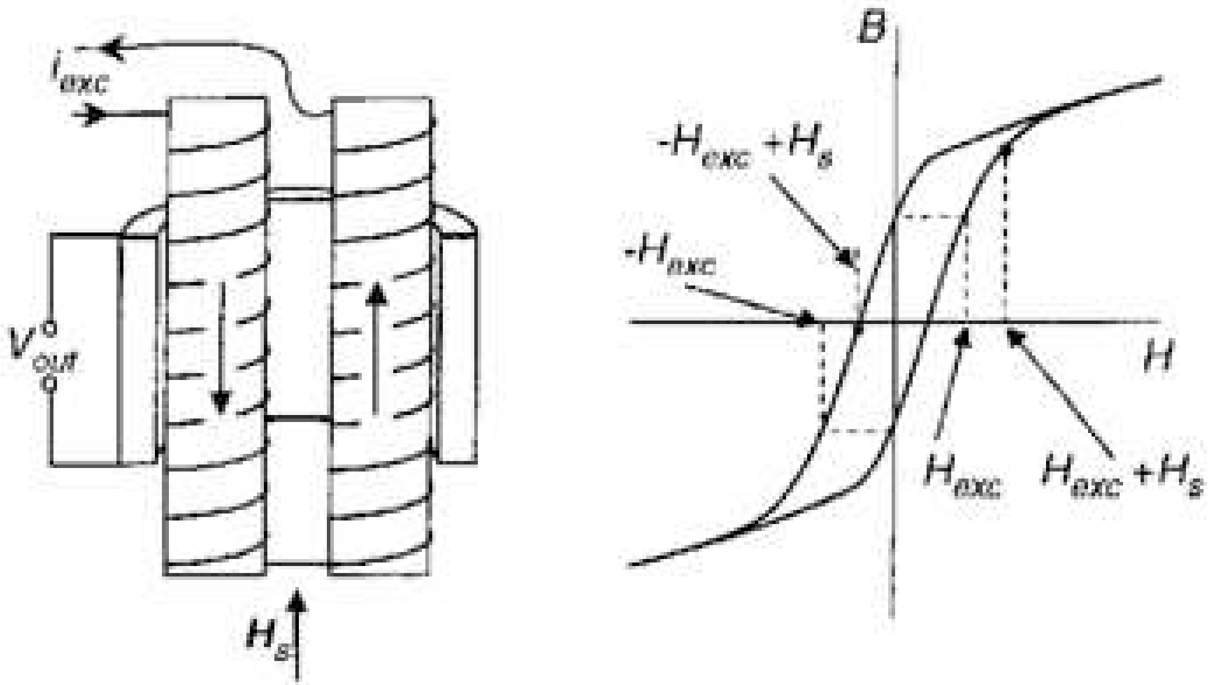
Measurement of magnetic fields

Fluxgate



7X7X4 cm

http://sci.esa.int/
MAG: The Fluxgate Magnetometer of Venus Express



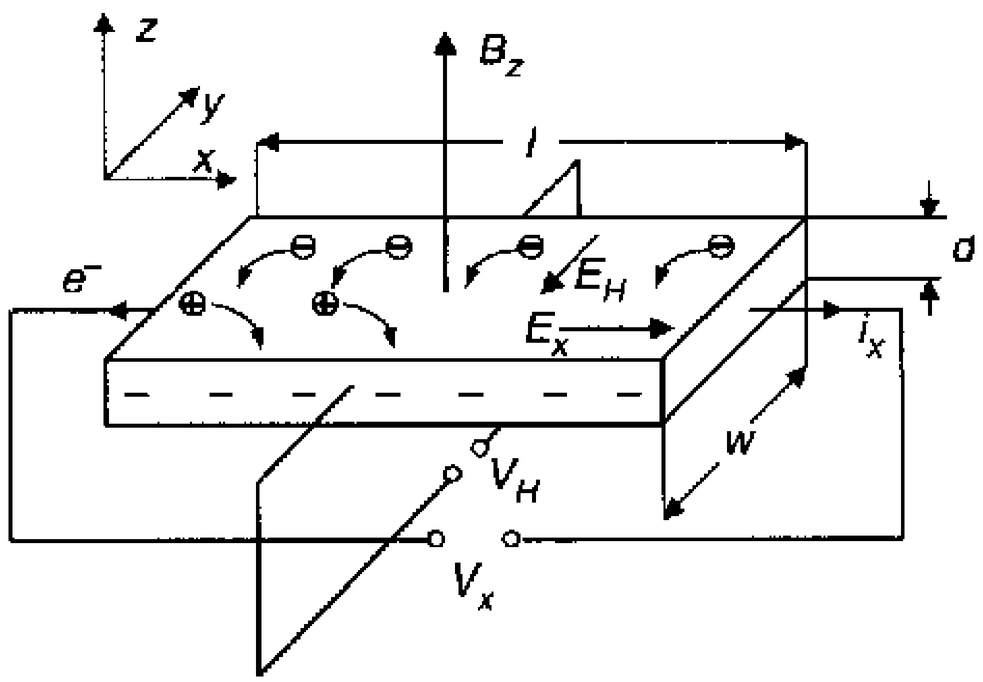
down to 10^{-9} T

Two-core fluxgate sensor (Vacquier type).

The two identical cores are subjected to the same DC field but the driving AC excitation is in opposite directions. This implies that the in voltage V_{out} detected by a secondary winding the odd order harmonics are subtracted and compensated while the even order harmonics sum up.

Measurement of magnetic fields

Hall effect gaussmeters



www.brockhaus.com

10^{-6} - 50 T

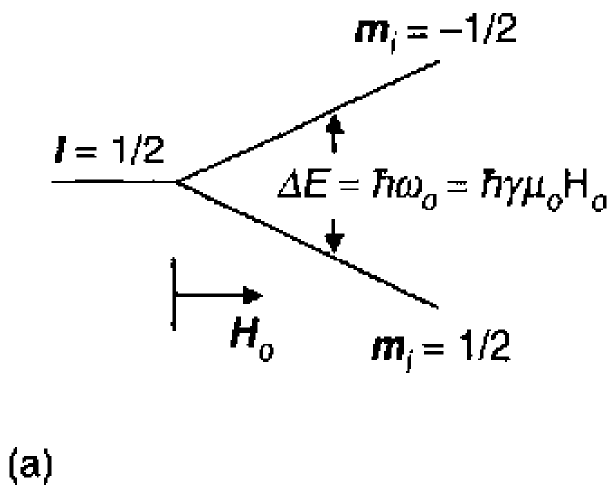
static, 0.1% accuracy

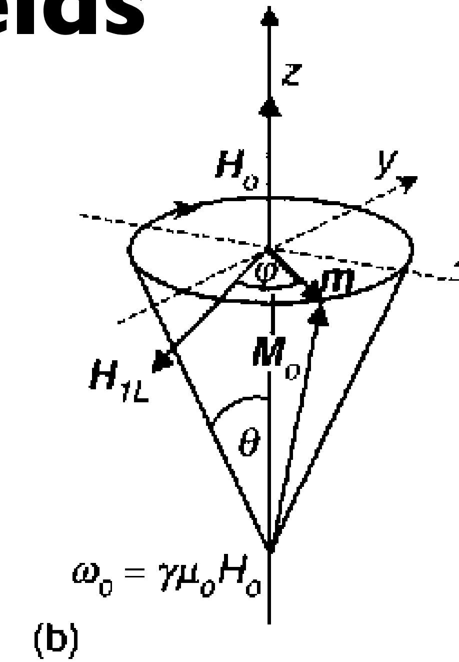
need appropriate temperature compensation

Measurement of magnetic fields

magnetic resonance

$$\mu_i = \gamma_i I_i$$





	μ_i	$\gamma_i/(2\pi)$
	$\times 10^{-26}$ Am^2	(MHz/T)
electron (1e)	-928.476 4620(57)	28 024.951 64(17)
nuclear magneton	0.5050 783 699(31)	
proton (1p)	1.410 606 7873(97)	42.577 478 92(29)
shielded proton (1p)	1.410 570 547(18)	42.576 385 07(53)
neutron (1n)	-0.966 236 50(23)	29.164 6933(69)
deuteron (1p,1n)	0.433 073 5040(36)	
trition (1p,2n)	1.504 609 503(12)	
helion (2p,1n)	-1.074 617 522(14)	
shielded helion (2p,1n)	-1.074 553 080(14)	32.434 099 66(43)
muon	-4.490 448 26(10)	

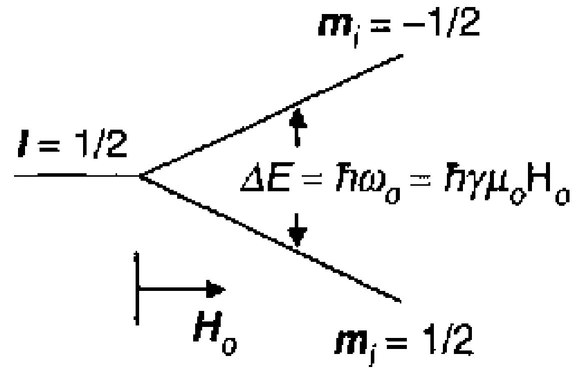
Element	μ_i/μ_N	$I_i/\hbar$	$\gamma_i/(2\pi)$	abundance
			(MHz/T)	%
^{1}H (1p)	2.793	1/2	42.577	99.98
^{2}H (1p,1n)	0.857	1	6.532	0.02
^{3}He (2p,1n)	-2.127	1/2	-32.434	0.000137
^{7}Li (3p,4n)	3.256	3/2	16.546	92.60
^{13}C (6p,7n)	-0.702	1/2	-10.702	1.11
^{14}N (7p,7n)	0.404	1	3.079	99.63
^{19}F (9p,10n)	2.629	1/2	40.079	100.00
^{23}Na (11p,12n)	2.217	3/2	11.266	100.00
^{31}P (15p,16n)	1.132	1/2	17.257	100.00
^{39}K (19p,20n)	0.392	3/2	1.992	93.10
^{57}Fe (26p,31n)	-0.090	1/2	-1.378	2.19
^{133}Cs (55p,78n)	5.615	7/2	12.229	100.00
^{209}Bi (83p,126n)	4.080	9/2	6.911	100.00

Measurement of magnetic fields

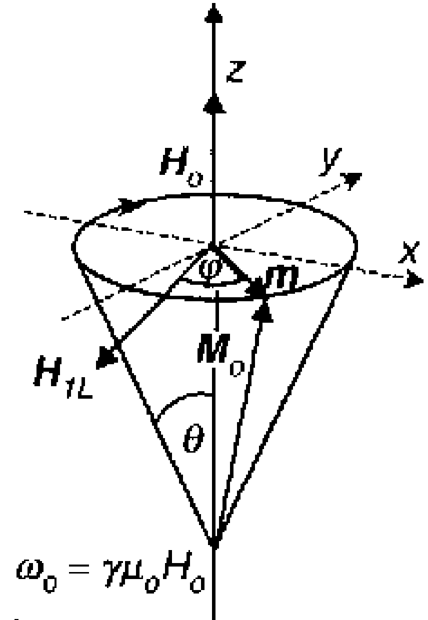
NMR

$$\mu_N = \frac{e\hbar}{2m_p},$$

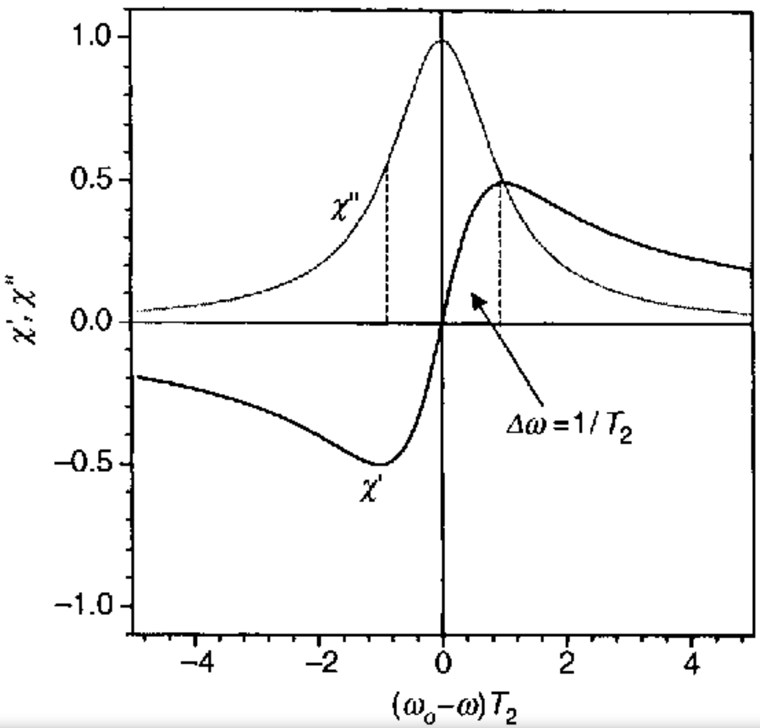
$$\Delta E = 2g\mu_N I \mu_0 H_0 = \hbar\gamma\mu_0 H_0,$$



(a)



(b)



commercial setups

50 mT - 14 T

100 / 1000 ppm/cm

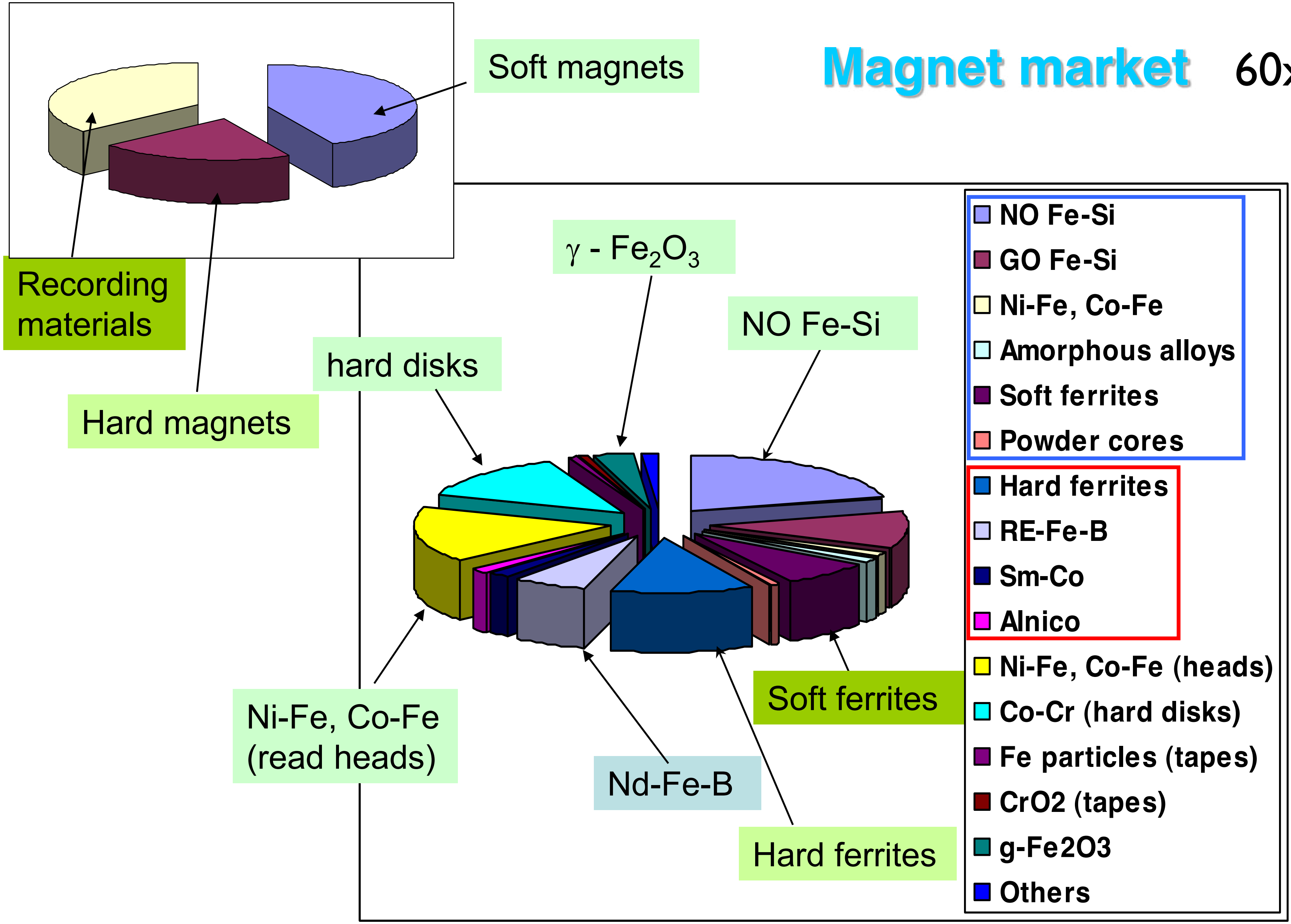
Outline

- Introduction: Maxwell equations
- Generation and measurement of magnetic fields
- Measurement of soft and hard magnetic materials

Source: F. Fiorillo, *Measurement and characterization of magnetic materials*, Elsevier, 2004.

Measurement of magnetic materials

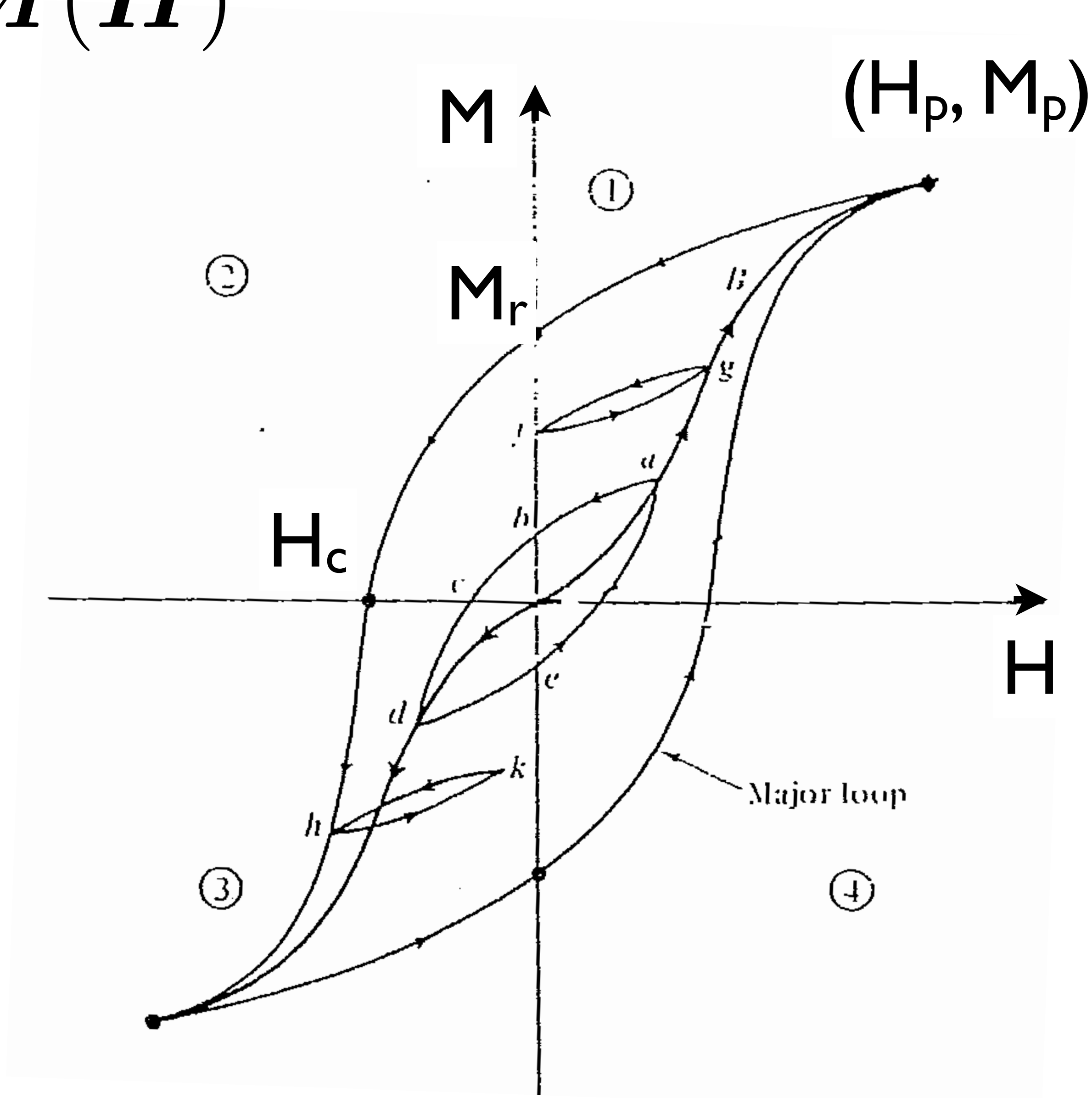
Magnet market 60×10^{9}€

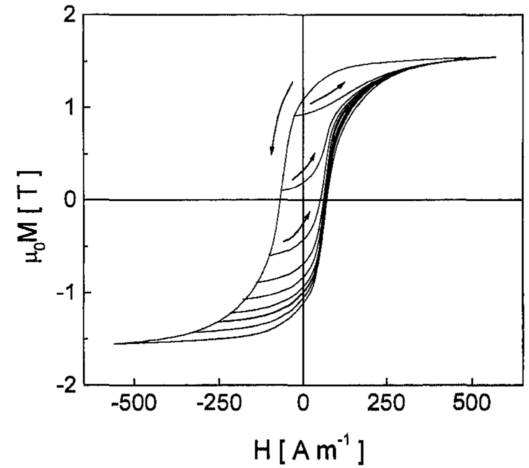


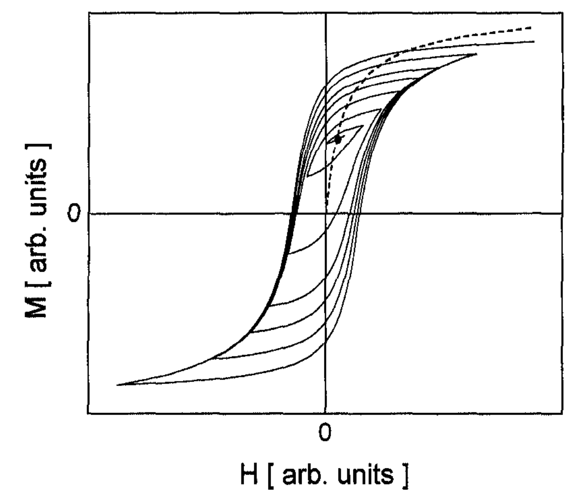
Hysteresis loop

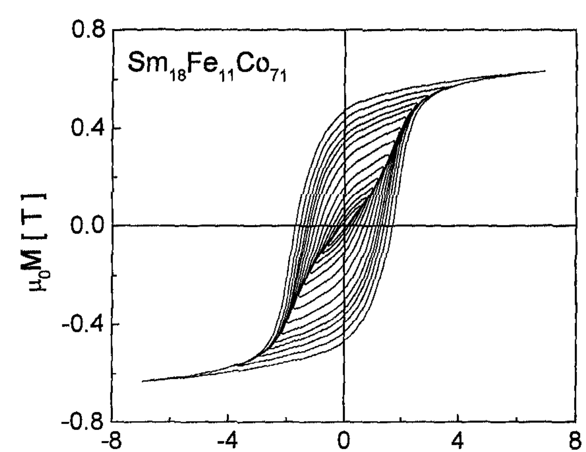
constitutive relation

$$M(H)$$



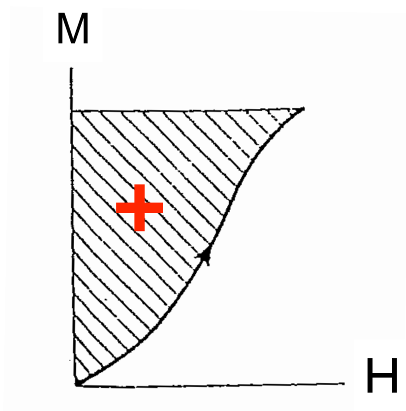


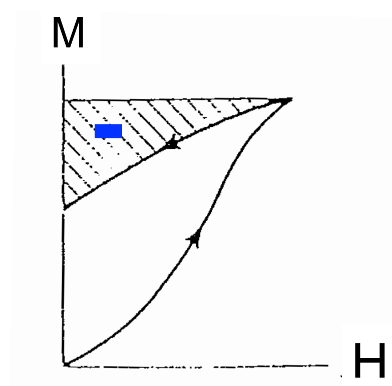


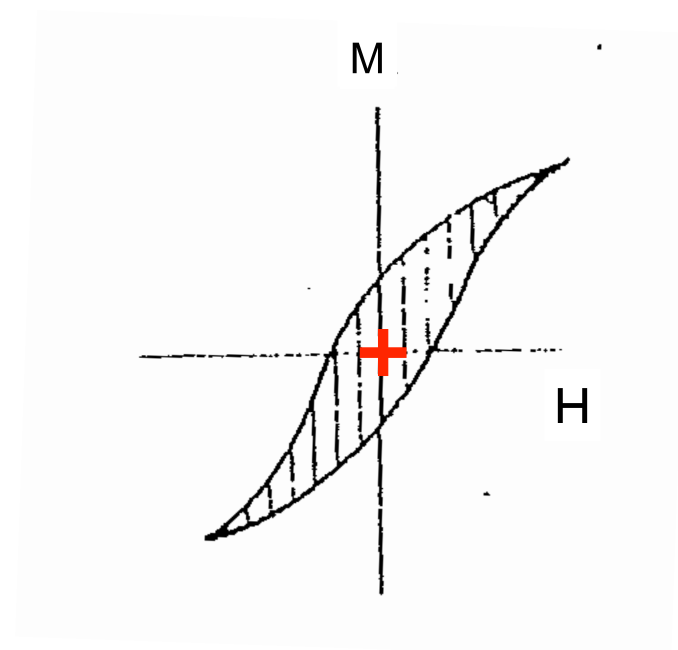


Hysteresis loop

$$du = \mu_0 H dM - T d_i s$$



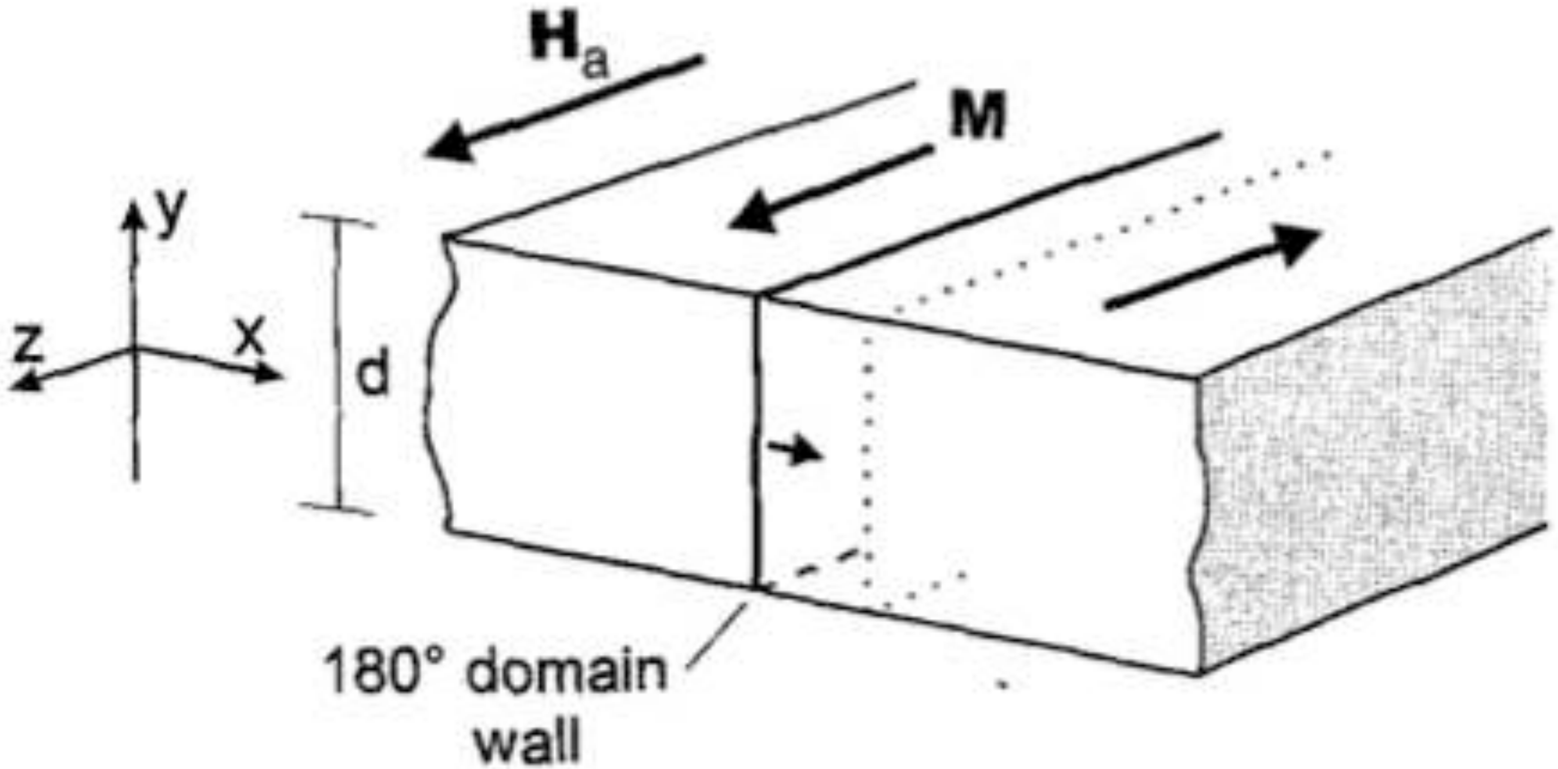




energy losses

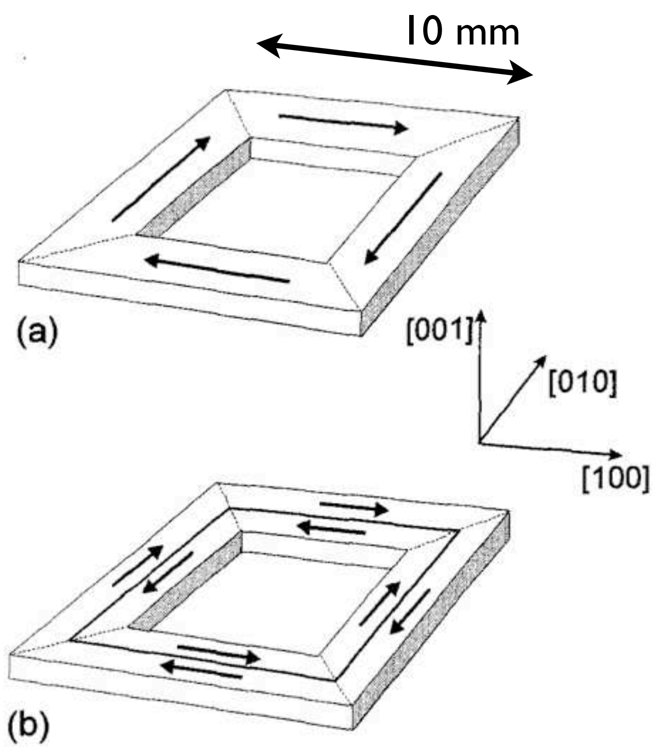
Domain wall motion

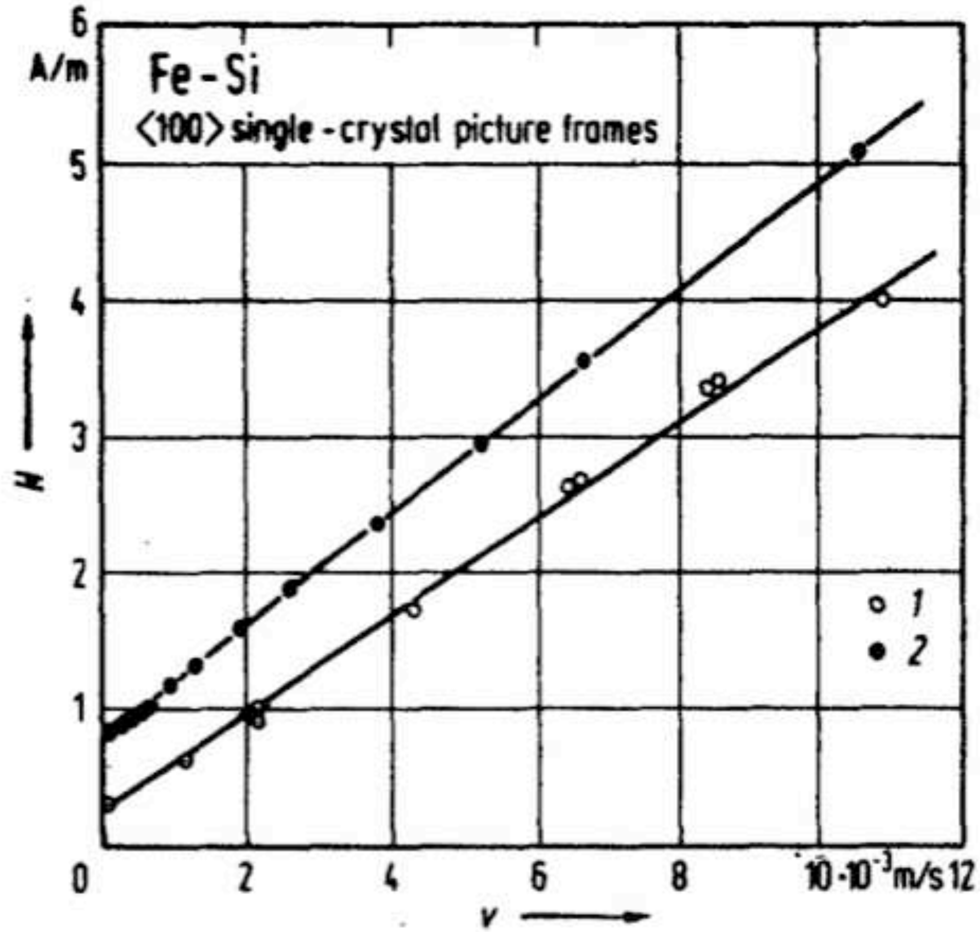
slab



Domain wall motion

picture-frame single crystal





for a single domain wall

$$\sigma G \frac{d\phi}{dt} = H_a - H_0$$

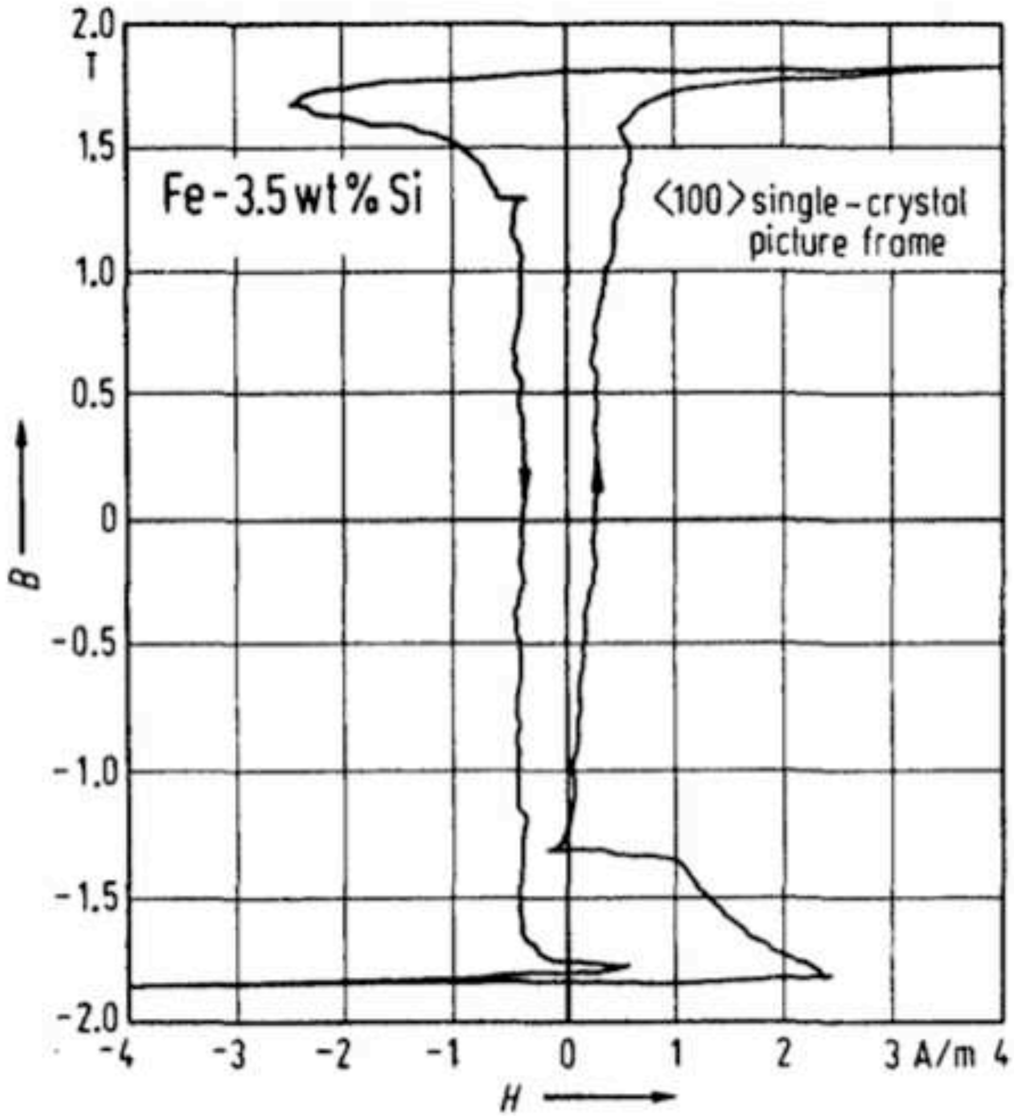
G=0.1356 slab

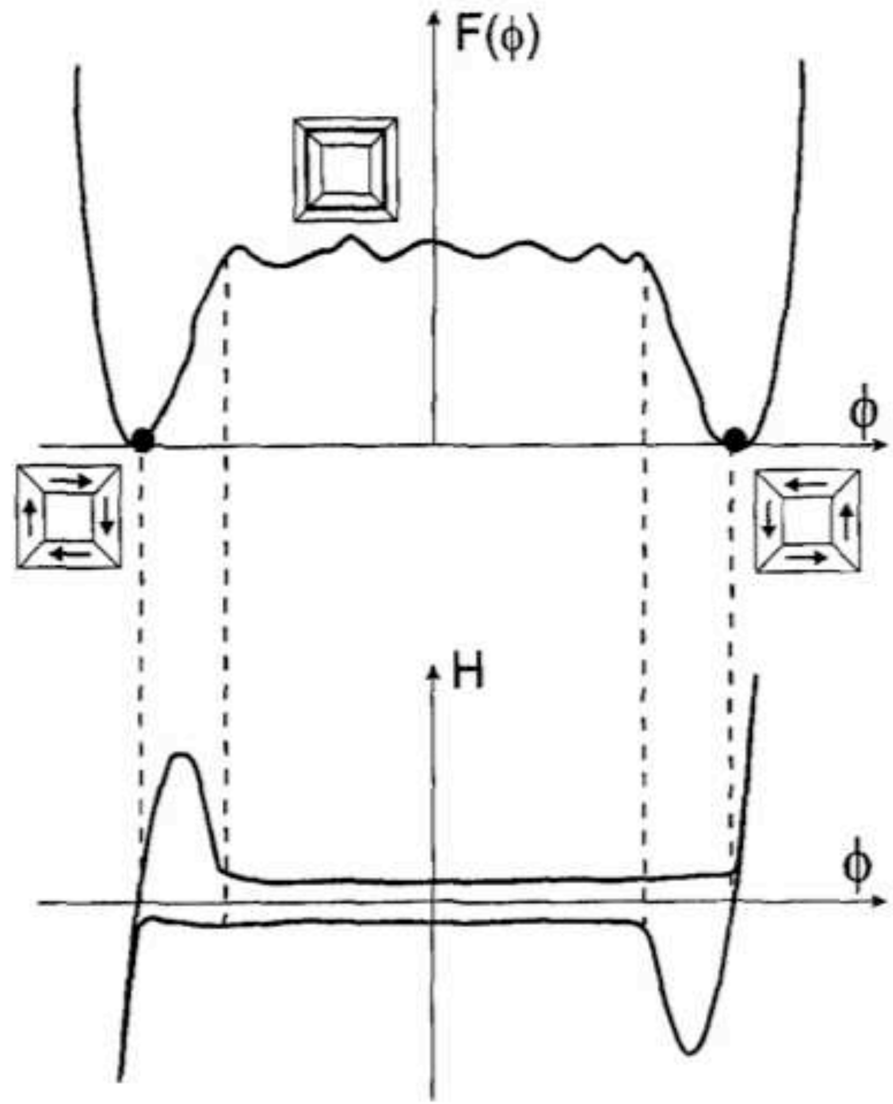
G. Bertotti, *Hysteresis in Magnetism*. Academic Press, (1998)

Hellmiss and Storm, IEEE Trans. Magn. **10** (1974)

Domain wall motion

of picture frame

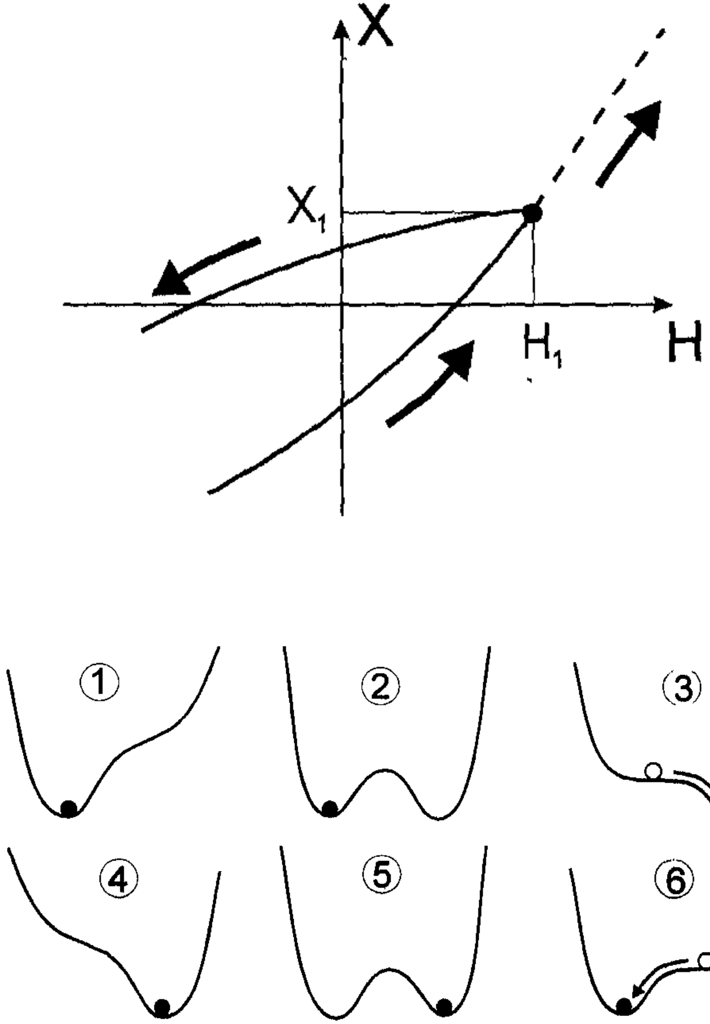


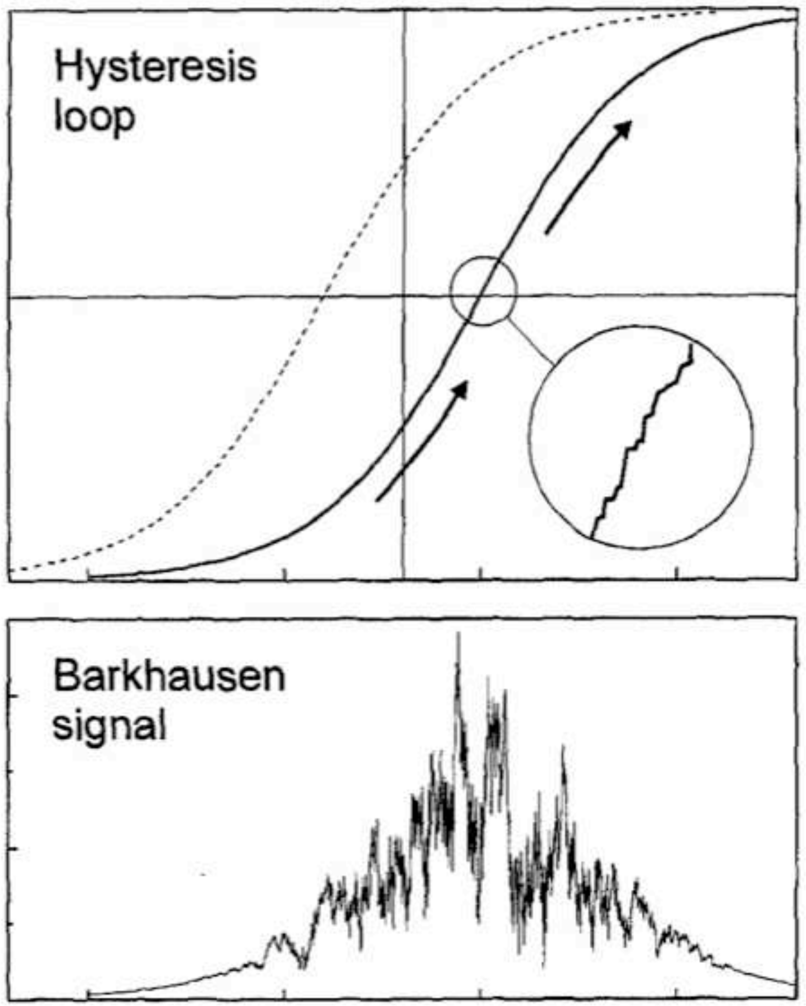


G. Bertotti, *Hysteresis in Magnetism.* Academic Press, (1998)

Hysteresis loop

branching and metastability

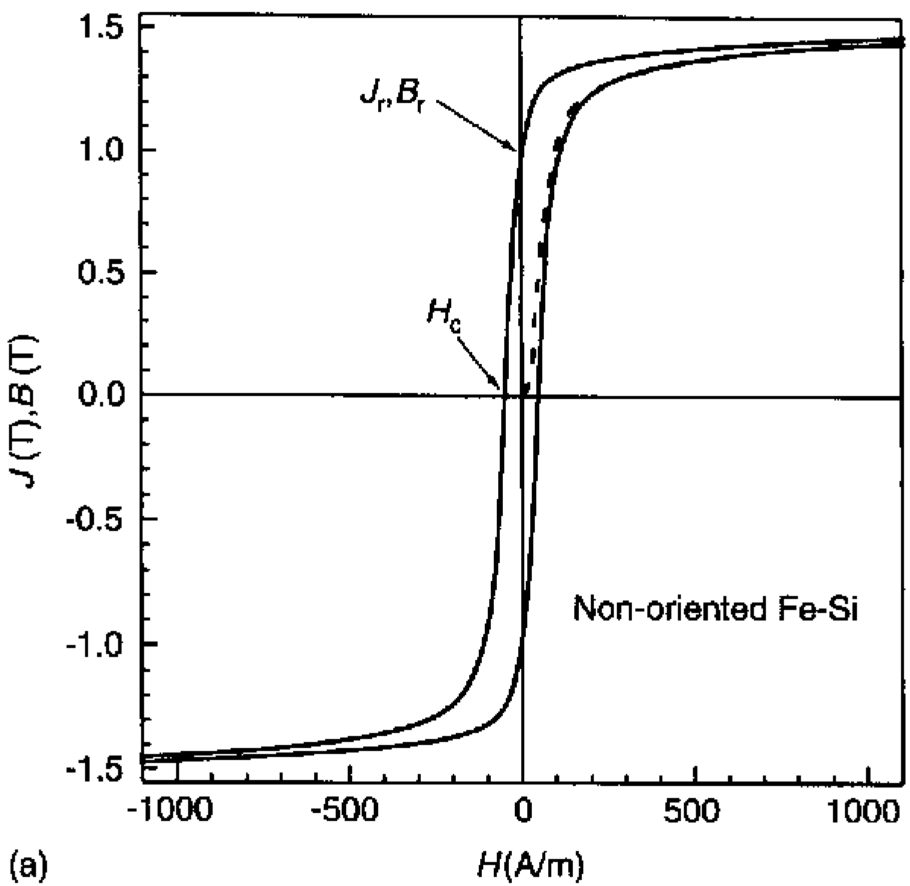




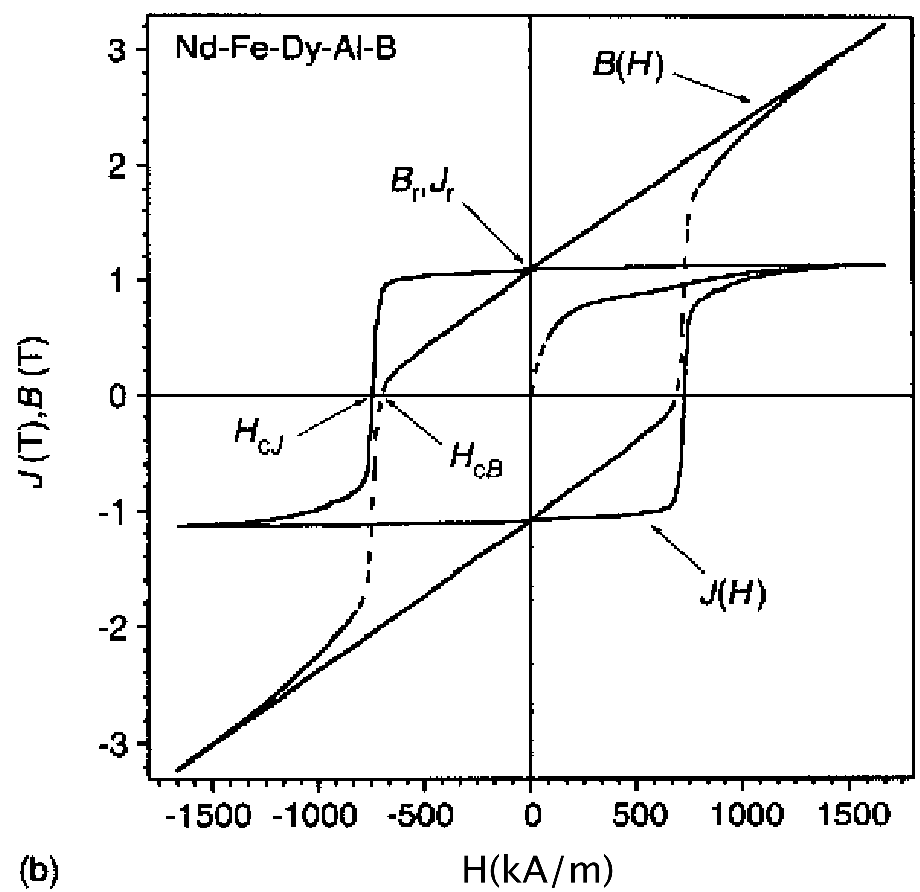
G. Bertotti, *Hysteresis in Magnetism.* Academic Press, (1998)

Measurement of magnetic materials

Soft



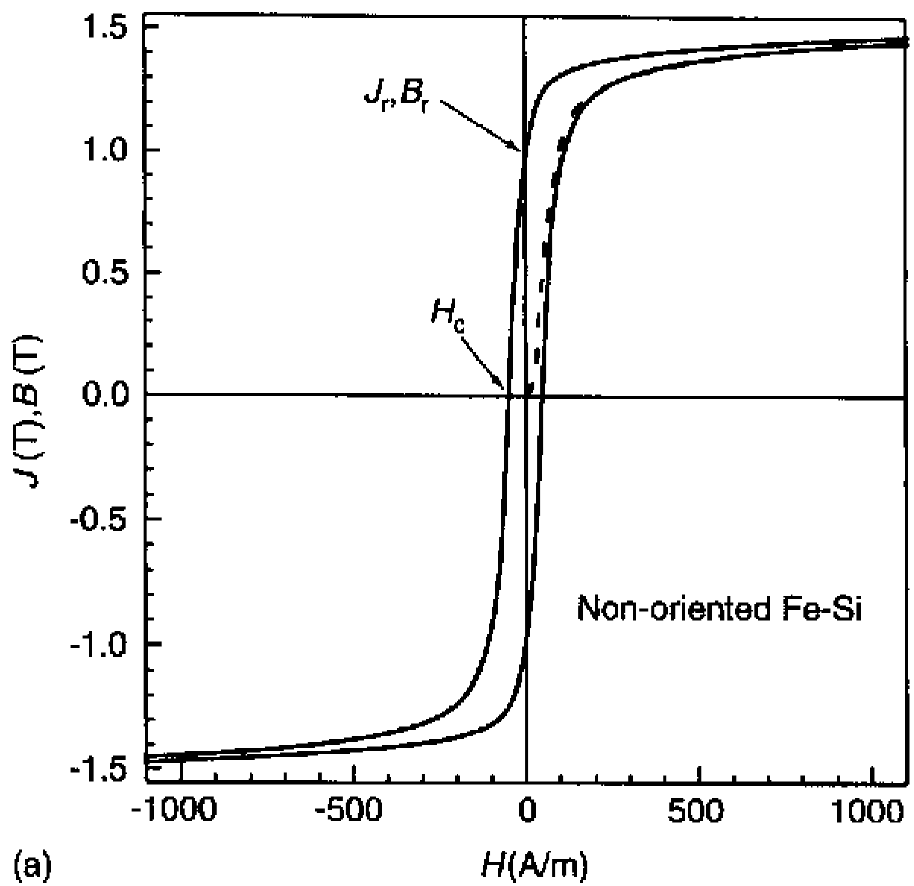
Hard



$\mu_0 H$ (T) 800 kA/m => 1T

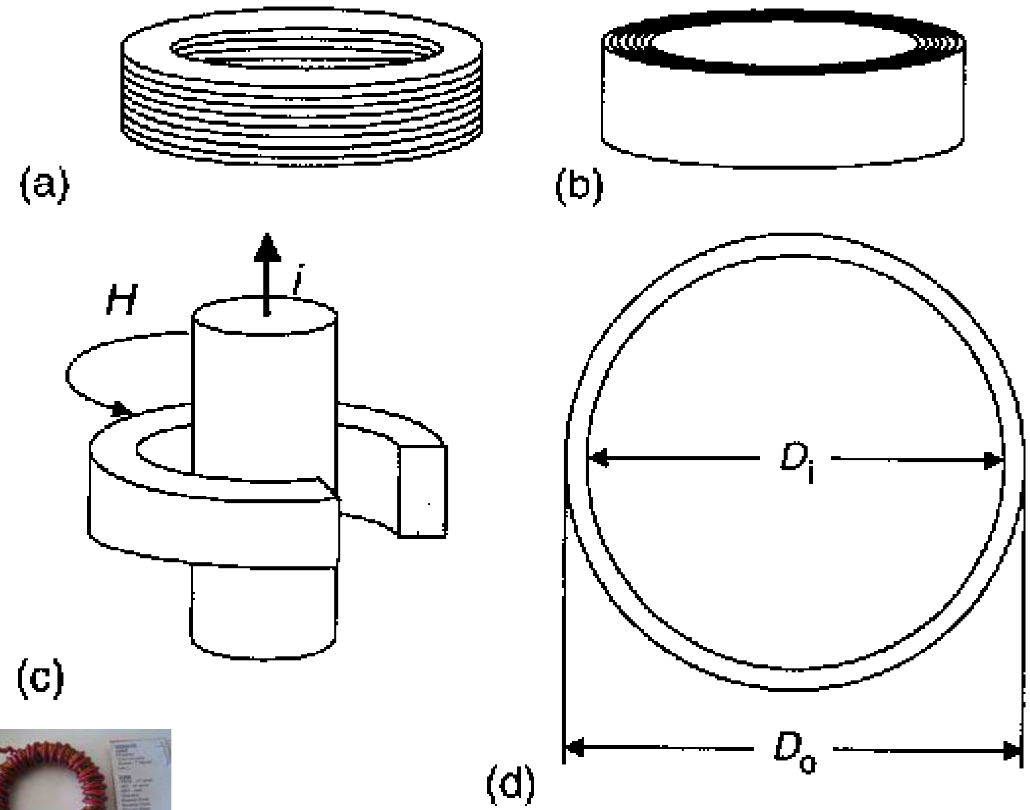
Measurement of magnetic materials

Soft



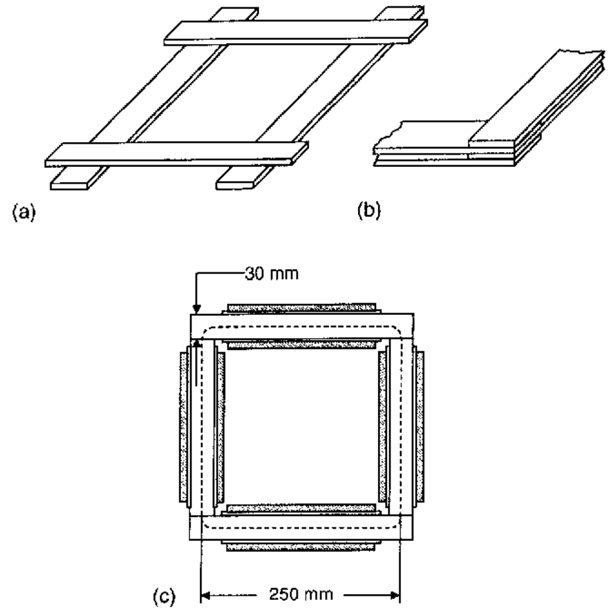
Soft magnetic materials

toroids

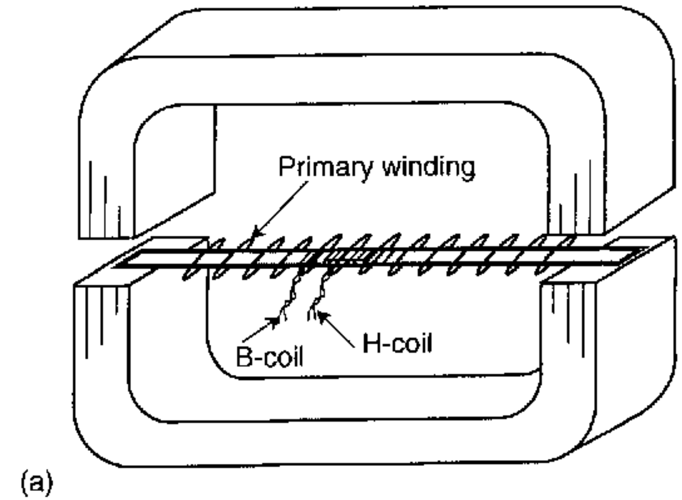


Soft magnetic materials

Epstein frame

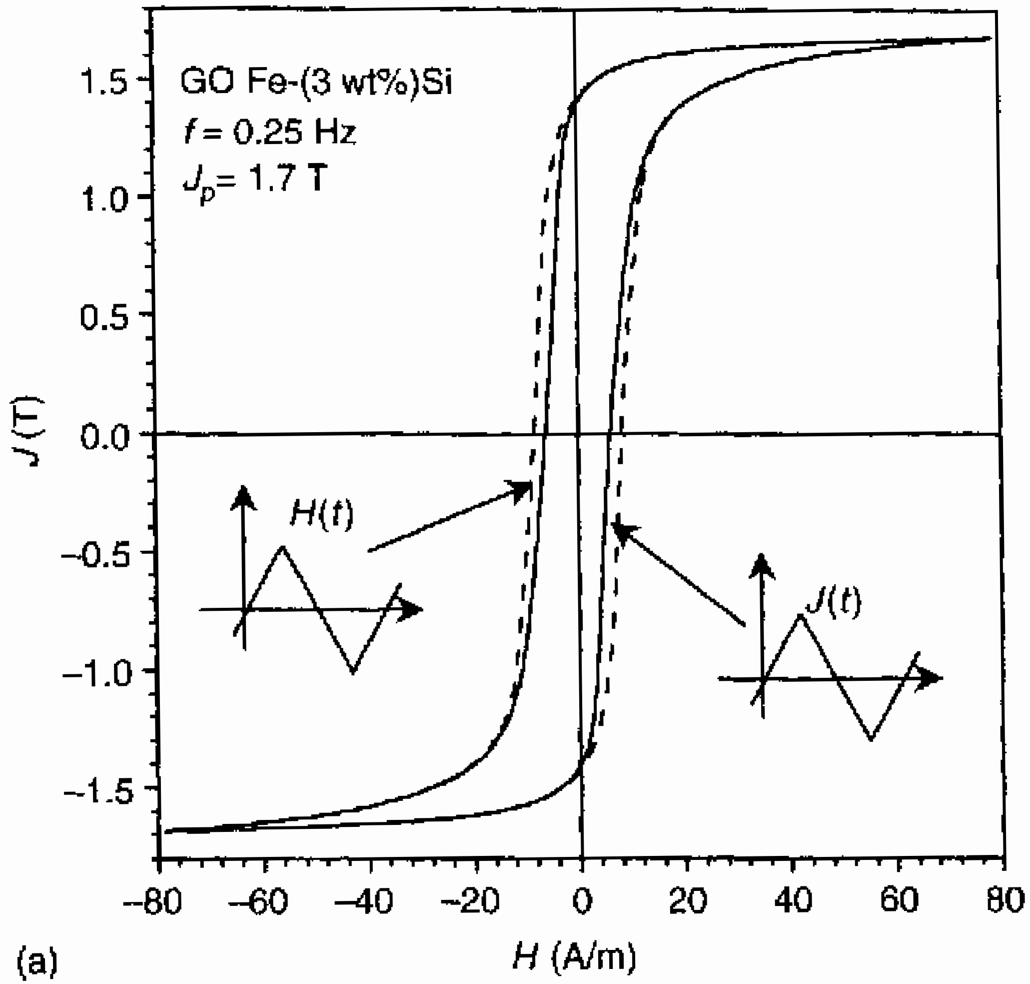


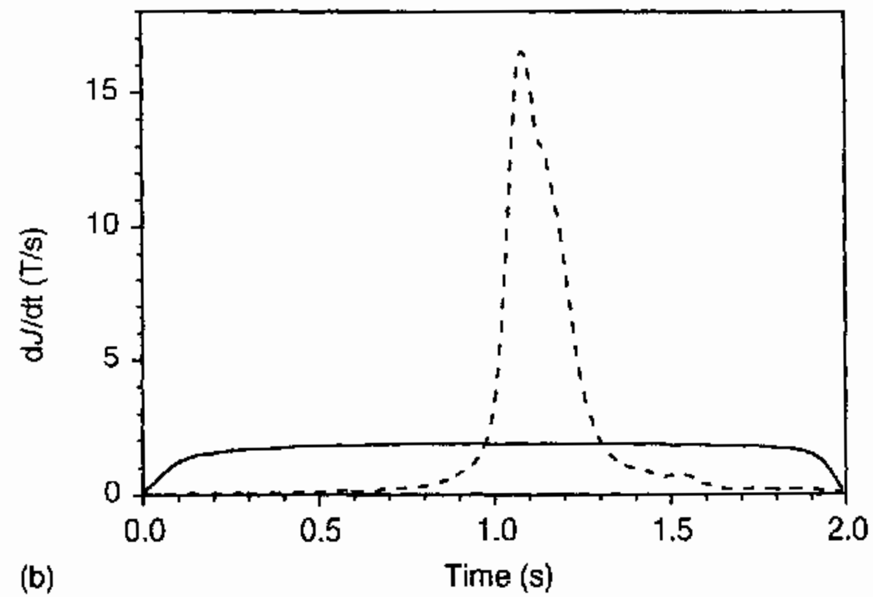
single sheet tester



Soft magnetic materials

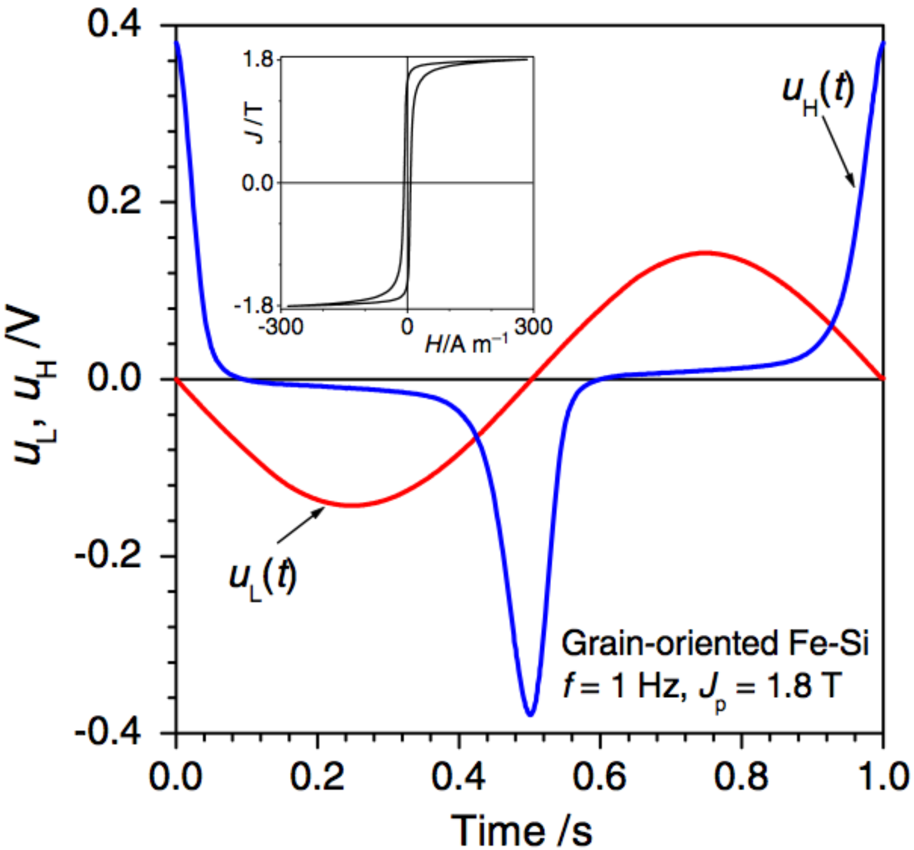
rate effects

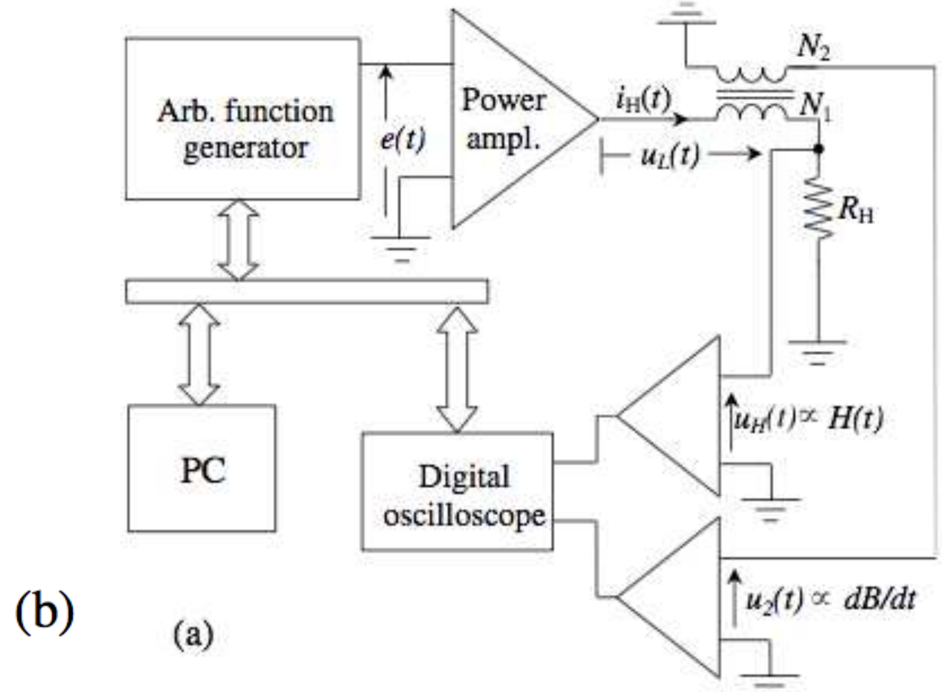


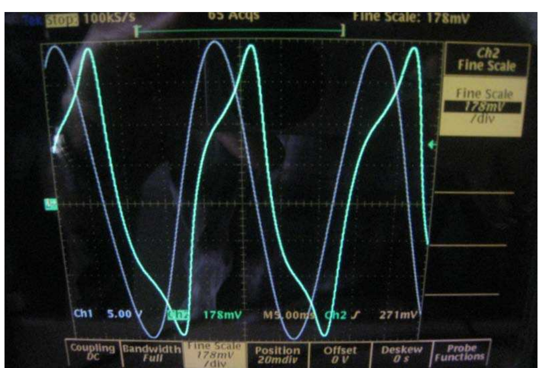


Soft magnetic materials

Hysteresigraph-Wattmeter

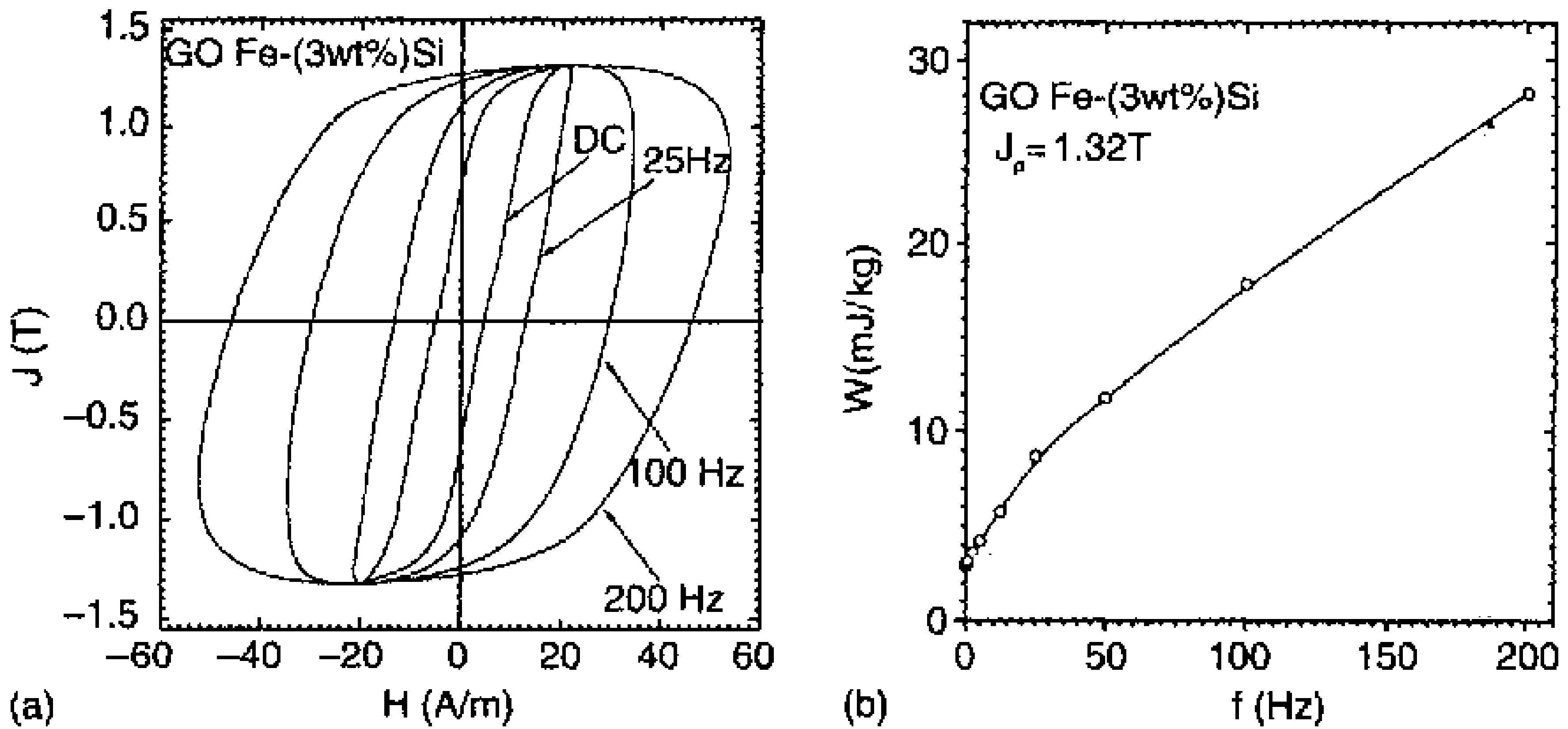






Soft magnetic materials

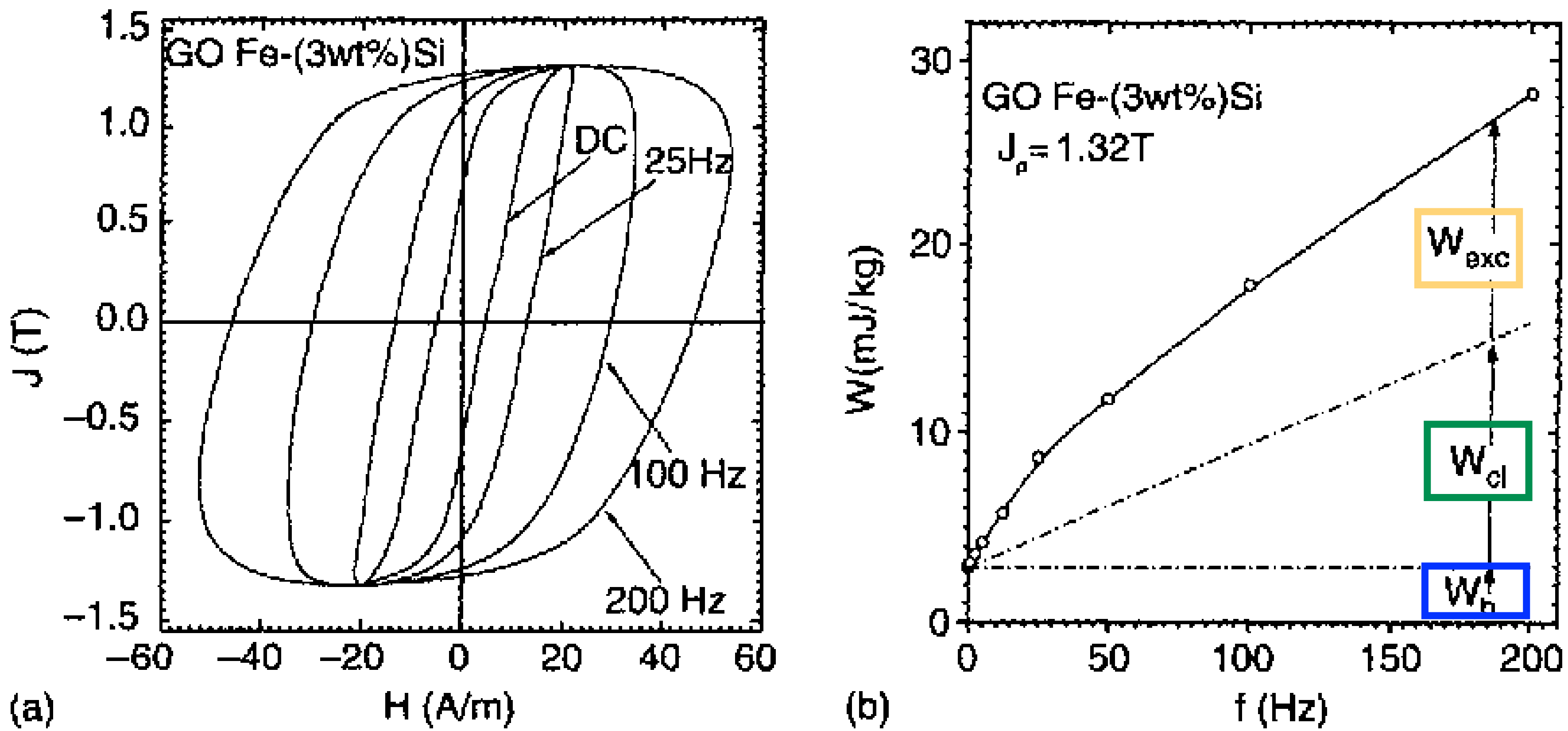
Dynamic hysteresis loop



Soft magnetic materials

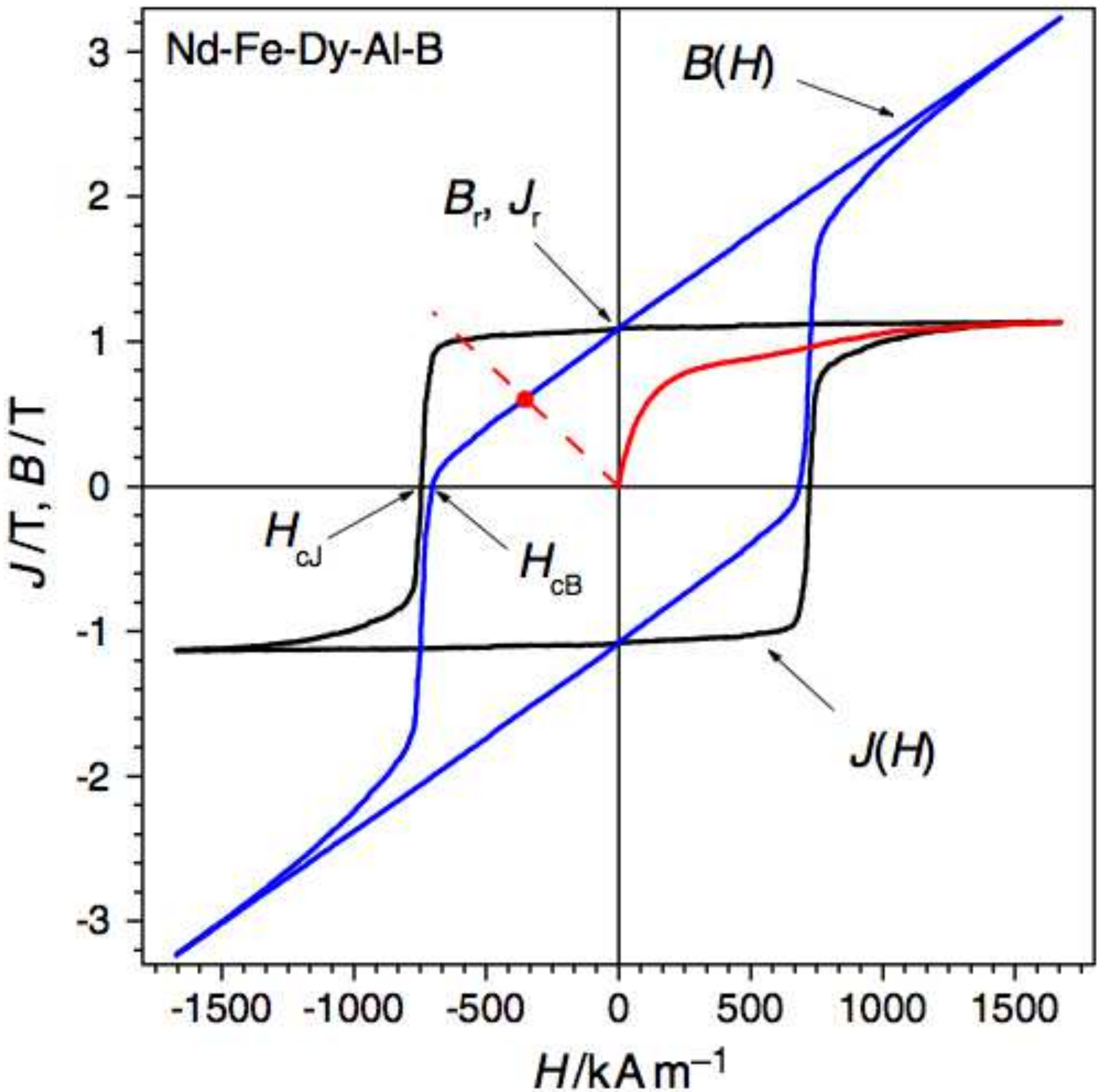
Dynamic hysteresis loop

loss separation



$$W = \boxed{W_{hyst}(J_p)} + \boxed{\frac{\pi^2}{6}\sigma_e d^2 J_p^2 f} + \boxed{8\sqrt{\sigma_e G S V_0} J_p^{3/2} f^{1/2}}$$

Hard magnetic materials



Hard magnetic materials

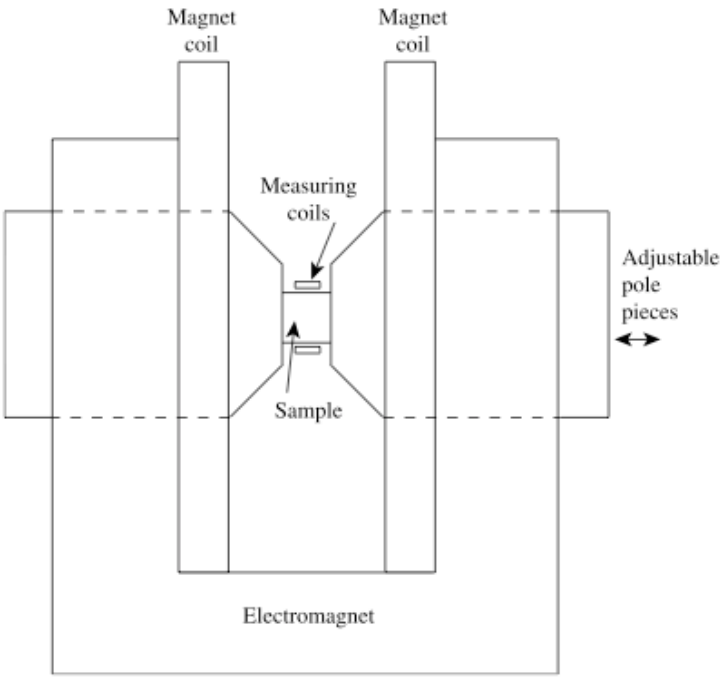
Measurement techniques

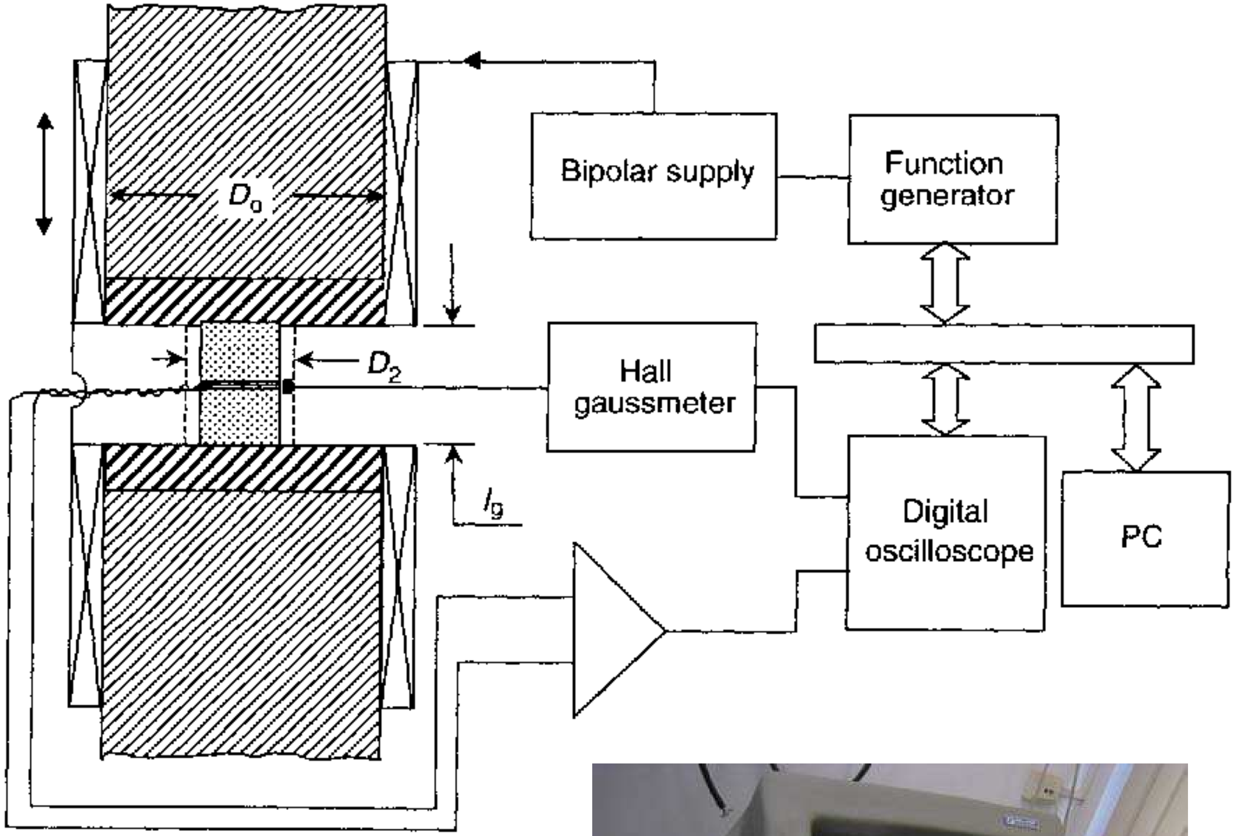
- measurement of permanent magnets
- measurement of magnetic moment: VSM, AGFM
- Torque measurements

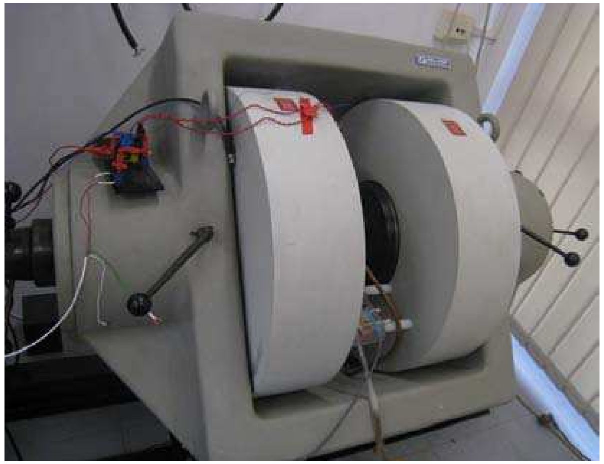
Source: F. Fiorillo, *Measurement and characterization of magnetic materials*, Elsevier, 2004.

Hard magnetic materials

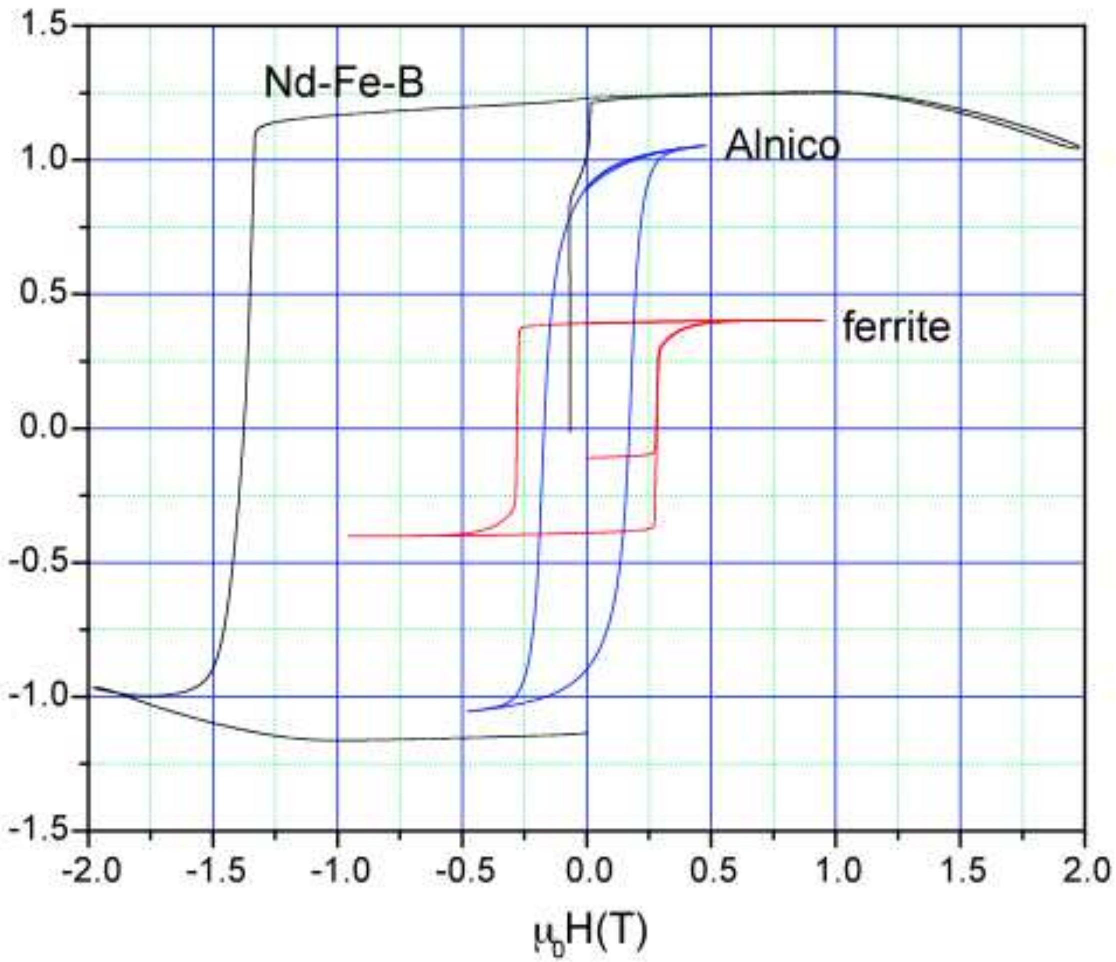
Electromagnet

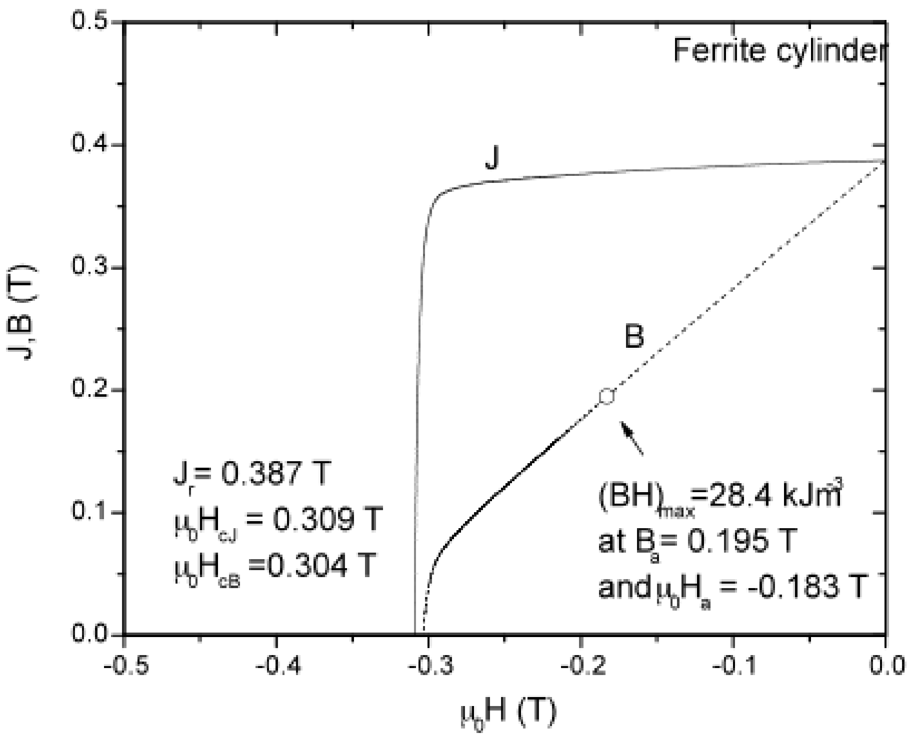






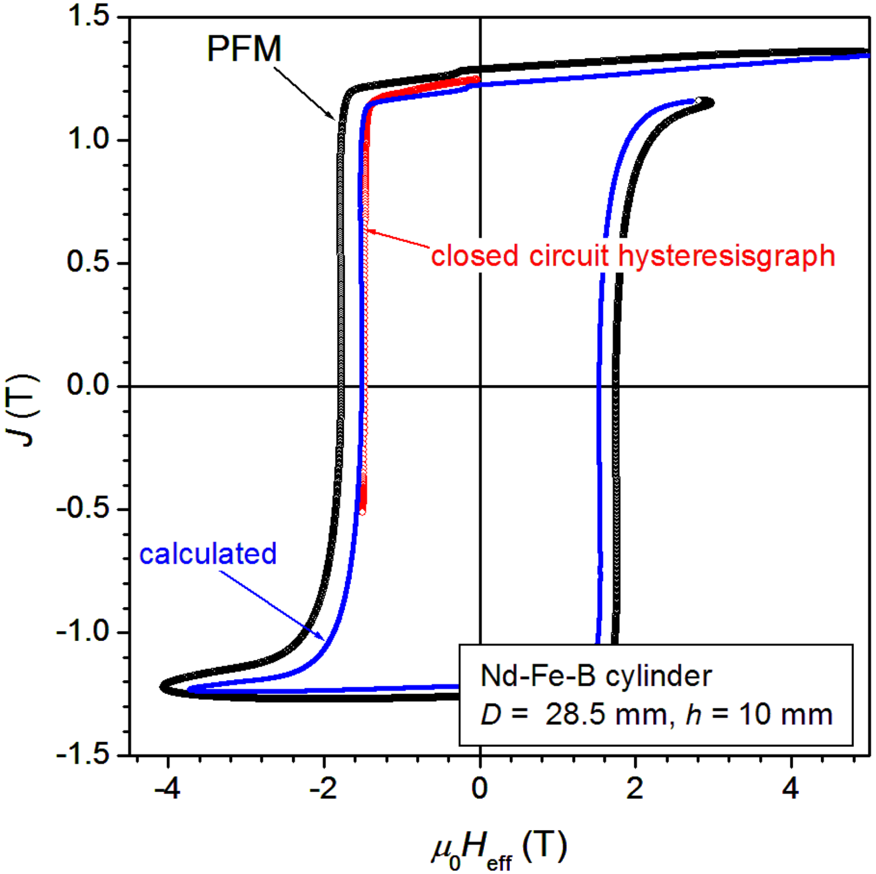
Hard magnetic materials

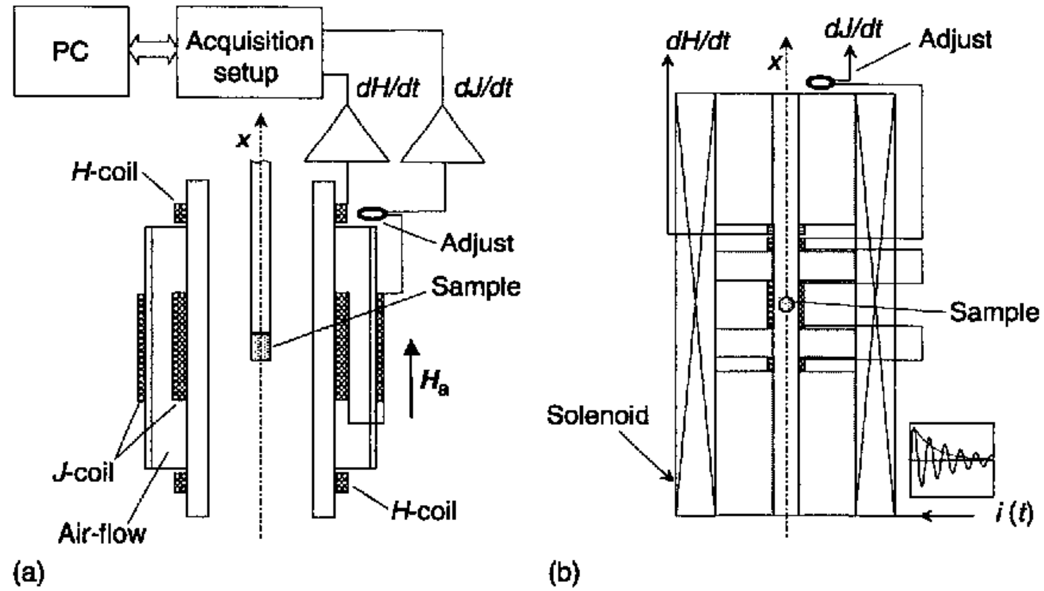


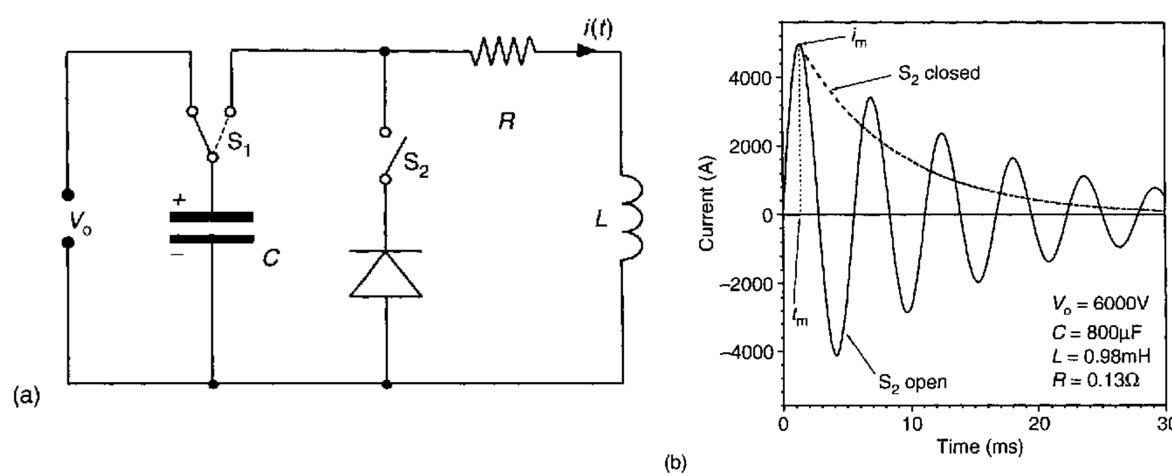


Hard magnetic materials

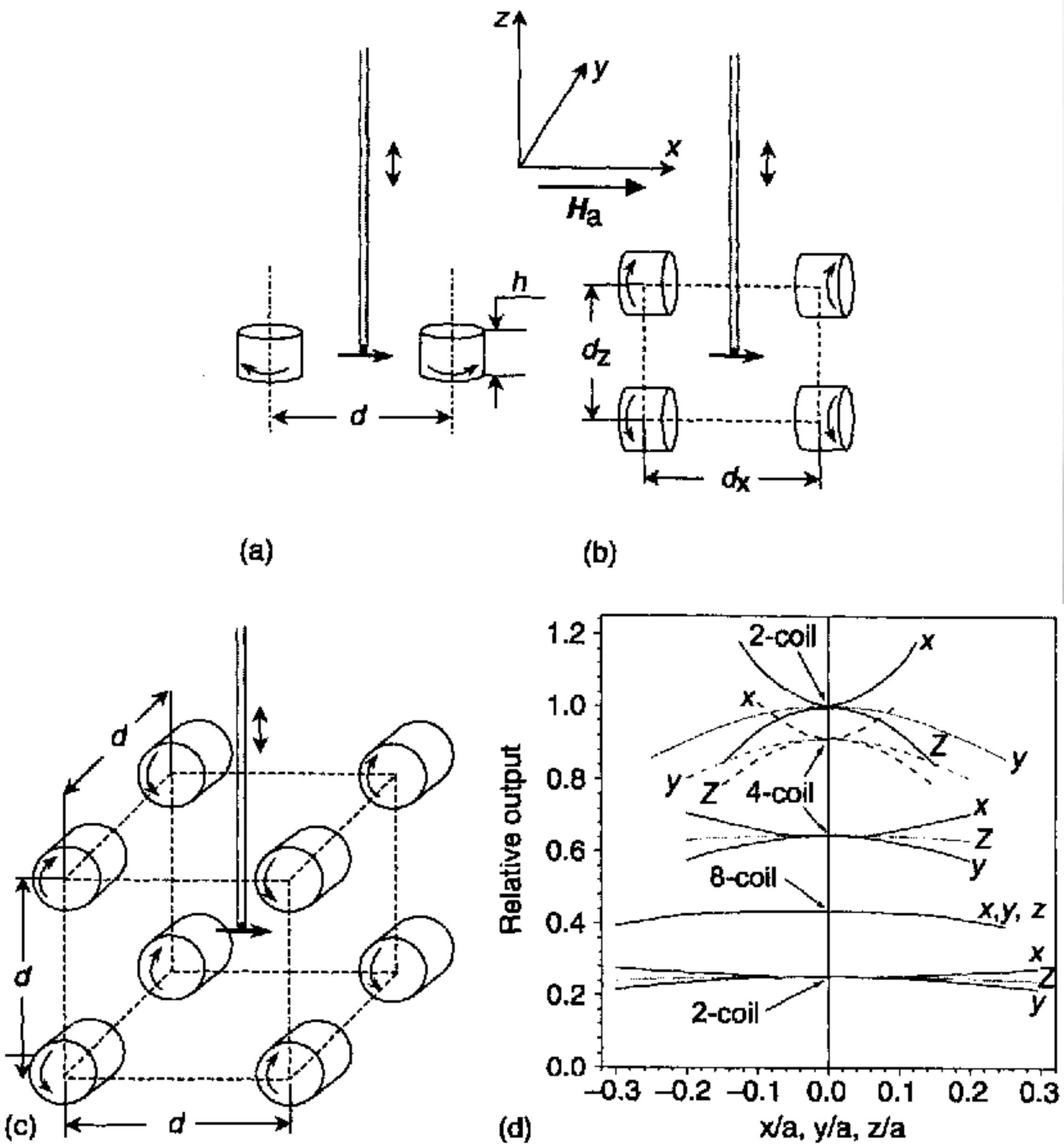
Pulsed field magnetometer







Vibrating Sample Magnetometer (VSM)

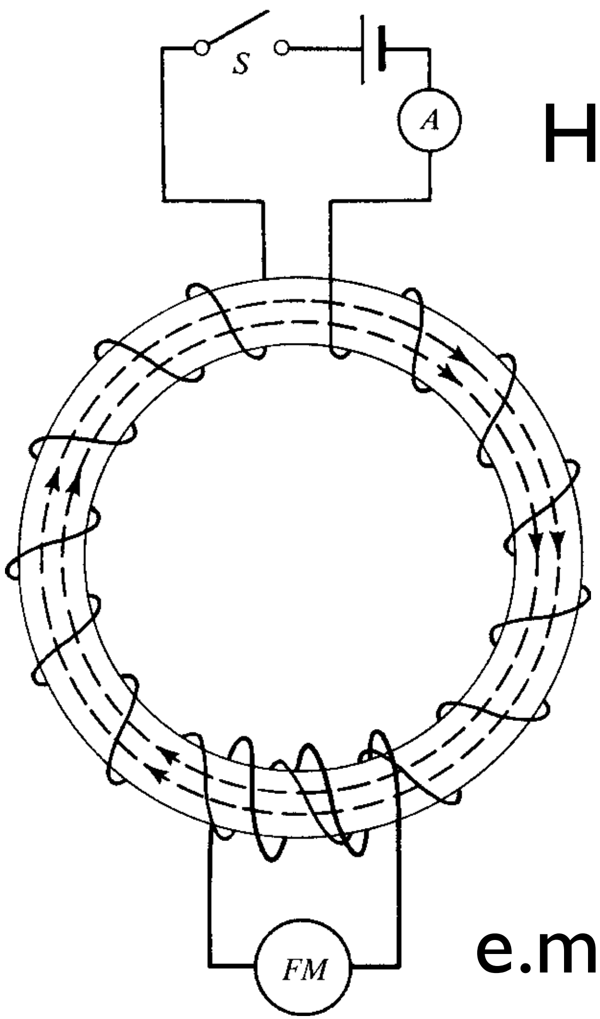


Reciprocity

$\mathbf{B} = k\ \mathbf{i}$

$\Phi = k\ \mathbf{m}$

Measurement of magnetic flux



$H = (N_1/l)\ I$

e.m.f. = $- N_2 S(dB/dt)$

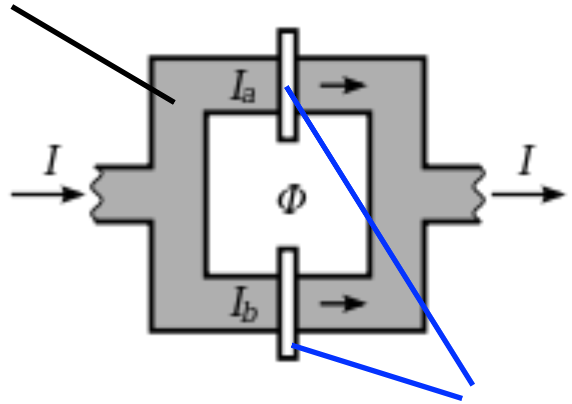
flux meter

resolution 0.5 10^{-6} Vs

Measurement of magnetic flux

SQUID

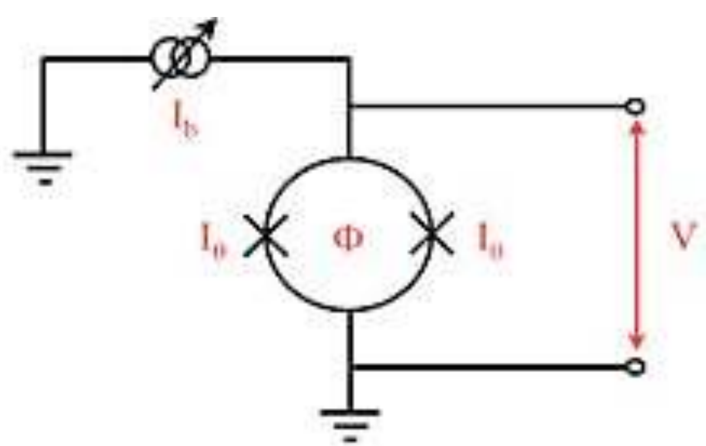
Superconducting QUantum Interference Device



$$\Phi_B = n\boxed{\frac{h}{2e}}$$

magnetic flux quantum

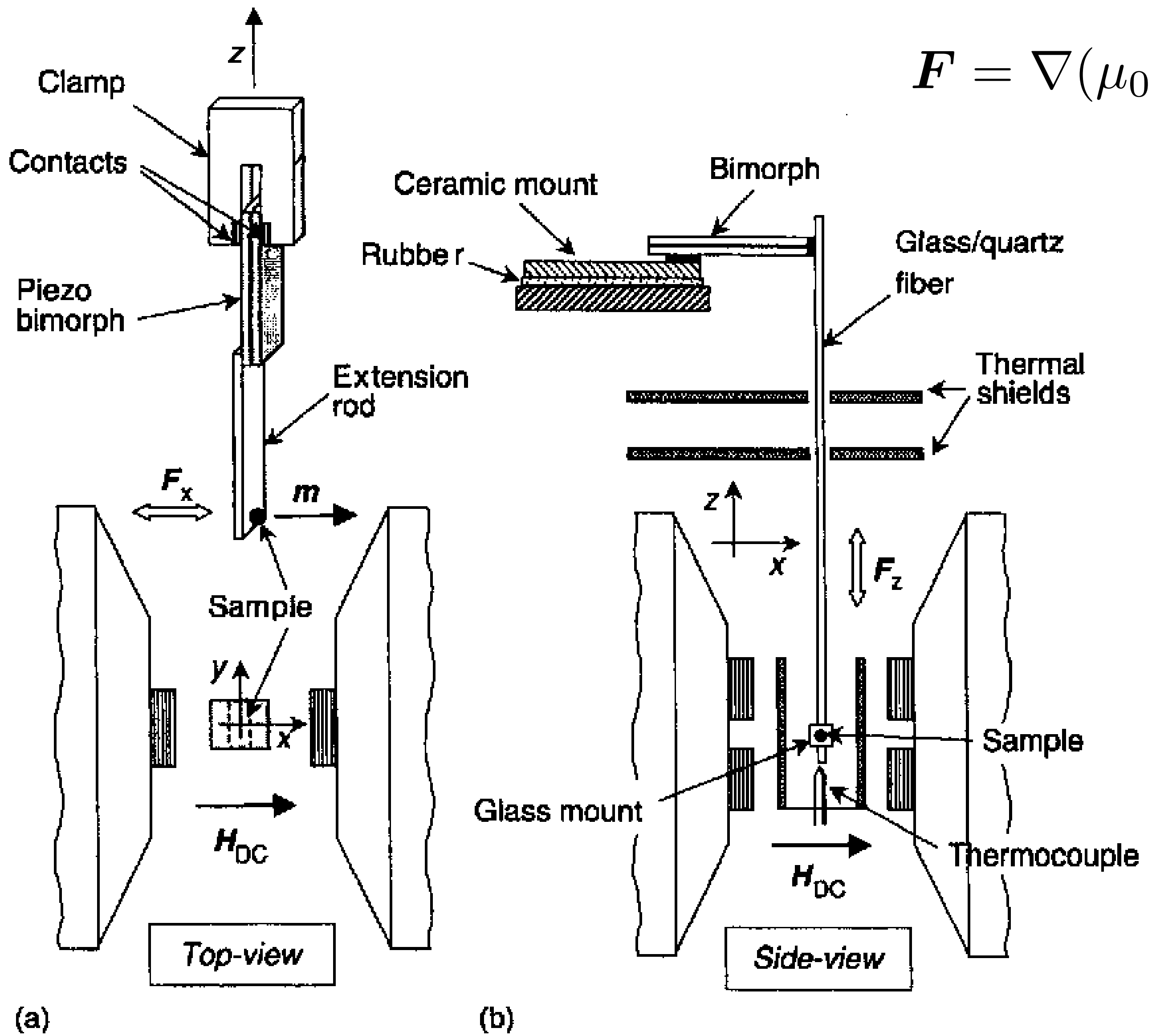
2.067833848...×10^{-15} Wb



www.qdusa.com MPMS

Alternating Gradient Force Magnetometer AGFM

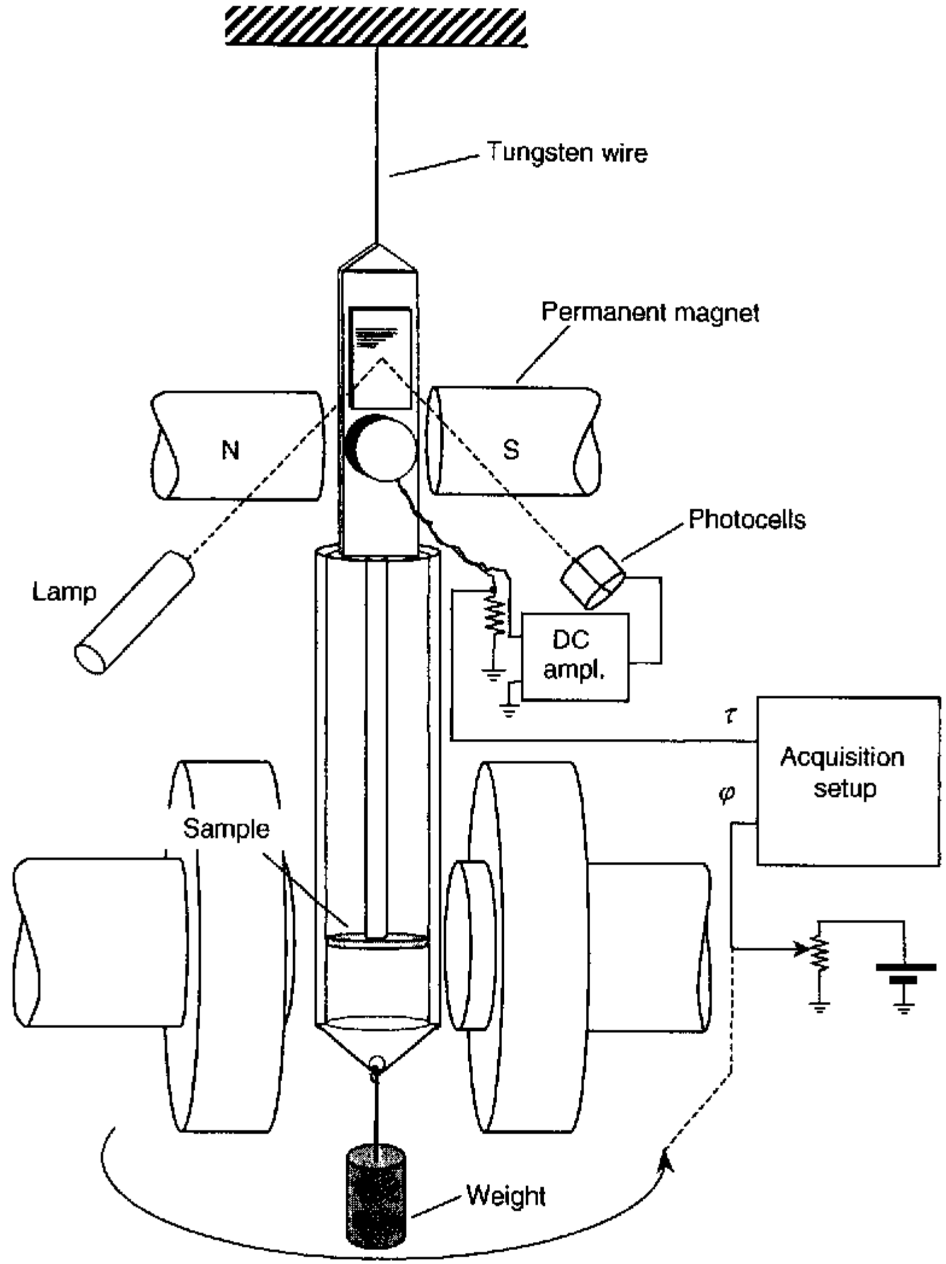
$$\boldsymbol{F} = \nabla(\mu_0 \boldsymbol{m} \cdot \boldsymbol{H})$$



Anisotropy - torque

torque

mx***H***



IEC standards

IEC 60404-2
Magnetic materials - Part 2: Methods of measurement of the magnetic properties of electrical steel sheet and strip by means of an Epstein frame
IEC 60404-3
Magnetic materials - Part 3: Methods of measurement of the magnetic properties of magnetic sheet and strip by means of a single sheet tester
IEC 60404-4
Magnetic materials - Part 4: Methods of measurement of d.c. magnetic properties of magnetically soft materials
IEC 60404-5
Magnetic materials - Part 5: Permanent magnet (magnetically hard) materials - Methods of measurement of magnetic properties
IEC 60404-6
Magnetic materials - Part 6: Methods of measurement of the magnetic properties of magnetically soft metallic and powder materials at frequencies in the range 20 Hz to 200 kHz by the use of ring specimens
IEC 60404-10
Magnetic materials - Part 10: Methods of measurement of magnetic properties of magnetic sheet and strip at medium frequencies
IEC 60404-13
Magnetic materials - Part 13: Methods of measurement of density, resistivity and stacking factor of electrical steel sheet and strip

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High Sensitivity Magnetometers, Springer (2017).