The European School on Magnetism 2019 – Practical on domain walls

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1 Simple description

In this tutorial we derive the profile and energy of domain walls in simple cases. The domain wall will be modeled as a one-dimensional object, describing the spatial variation of magnetization with the function $\theta(x)$. Let us assume the following free boundary conditions, mimicking two extended domains with opposite magnetization vectors separated by a domain wall whose profile we propose to derive here: $\theta(-\infty) = 0$ and $\theta(+\infty) = \pi$. We assume the simplest form of volume density of magnetic anisotropy, uniaxial of second order: $E(\theta) = K_\mathrm{u} \sin^2 \theta$. We recall that the volume density of exchange energy reads, in its micromagnetism form: $E_\mathrm{ex} = A(\mathrm{d}\theta/\mathrm{d}x)^2$, with A the exchange stiffness.

1.1 Dimensional analysis

Based on a dimensional analysis, exhibit approximate expressions for both the domain wall width δ and the domain wall energy $\mathcal{E}$. What are the SI units for $\mathcal{E}$? Discuss the form of these quantities in relation with the meaning and effects of exchange and anisotropy.

1.2 Simple variational model

We assume the following solution for a wall with width ℓ: $\theta = 0$ for $x < -\ell/2$, $\theta = \pi(x/\ell + 1/2)$ for $x \in [-\ell/2; \ell/2]$ and $\theta = \pi$ for $x > \ell/2$. Plot this profile. Calculate the total anisotropy and exchange energy of the system, $\mathcal{E}$. Provide the value of ℓ minimizing energy, and the resulting energy. Discuss both.

2 Euler-Lagrange equation

We will seek to exhibit a magnetization configuration that minimizes an energy density integrated over an entire system. Finding the minimum of a continuous quantity integrated over space is a common problem solved through Euler-Lagrange equation, which we will deal with in a textbook one-dimensional framework here. Let us consider a microscopic variable defined as $E(\theta, \mathrm{d}\theta/\mathrm{d}x)$, where x is the spatial coordinate and θ a quantity defined at each point. In the case of micromagnetism we will have:

$$E\left[\theta(x), \frac{\mathrm{d}\theta}{\mathrm{d}x}(x)\right] = A\left[\frac{\mathrm{d}\theta}{\mathrm{d}x}(x)\right]^2 + E_\mathrm{a}[\theta(x)] \tag{1}$$

When applied to micromagnetism $E_\mathrm{a}(\theta)$ may contain anisotropy, Zeeman and dipolar terms (the latter taken as local through the hypothesis of demagnetizing coefficients or other approximations). We define the integrated quantity:

$$\mathcal{E}[\theta] = \int_{x_A}^{x_B} E\left[\theta(x), \frac{\mathrm{d}\theta}{\mathrm{d}x}(x)\right] \mathrm{d}x + \mathcal{E}_A\left[\theta(x_A)\right] + \mathcal{E}_B\left[\theta(x_B)\right]. \tag{2}$$

A and B are the boundaries of the system, while $\mathcal{E}_A(\theta)$ and $\mathcal{E}_B(\theta)$ are surface energy terms. These may stem from,*e.g.*, surface magnetic anisotropy, or the Dzyaloshinskii-Moriya interaction. Let us now consider an infinitesimal function variation $\delta\theta(x)$ for θ. Show that extrema of $\mathcal{E}$ are determined by the following local relationships:

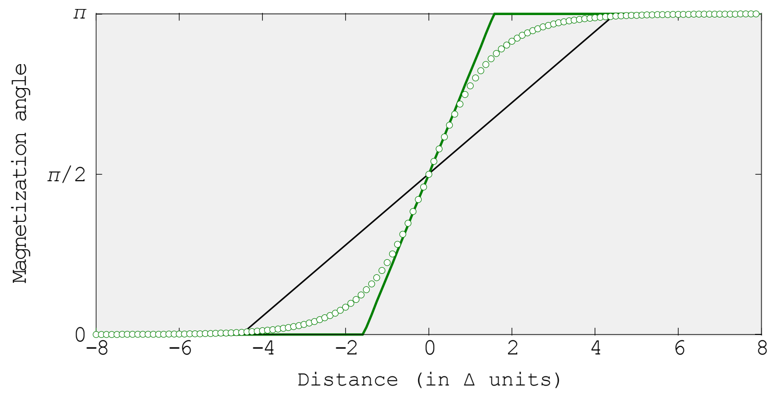

Figure 1: Bloch domain wall profile: the exact solution (green dots) versus the asymptotic profile (red line). The solution with linear ersatz is shown as a dark line.

$$\frac{\partial E}{\partial \theta} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial E}{\partial \frac{\mathrm{d}\theta}{\mathrm{d}x}}\right) \equiv 0 \tag{3}$$

$$\frac{\mathrm{d}\mathcal{E}_A}{\mathrm{d}\theta} - \left.\frac{\partial E}{\partial \frac{\mathrm{d}\theta}{\mathrm{d}x}}\right|_A = 0 \tag{4}$$

$$\frac{\mathrm{d}\mathcal{E}_B}{\mathrm{d}\theta} + \left.\frac{\partial E}{\partial \frac{\mathrm{d}\theta}{\mathrm{d}x}}\right|_B = 0 \tag{5}$$

Note that equations Eq.(4) and Eq.(5) differ in sign because a surface quantity should be defined with respect to the unit vector normal to the surface, with a unique convention for the sense, such as the outwards normal. Here the abscissa x is outwards for point B however inwards at point A. An alternative microscopic explanation would be that for a given sign of $\mathrm{d}\theta/\mathrm{d}x$ the exchange torque exerted on a moment to the right (at point B) is opposite to that exerted to the left (at point A), whereas the torque exerted by a surface anisotropy energy solely depends on θ.

3 Micromagnetic Euler equation

Apply the above equations to the case of micromagnetism [Eq.(1)]. Starting from Eq.(3), exhibit a differential equation linking $E_\mathrm{a}(\theta)$ with $\mathrm{d}\theta/\mathrm{d}x$. Equations 4-5 are called Brown equations. $\mathcal{E}_A(\theta)$ and $\mathcal{E}_B(\theta)$ may be surface magnetic anisotropy, for instance. Discuss the microscopic meaning of these equations. Comment the special case of free boundary conditions (all bulk and surface energy terms vanish at A and B), in terms of energy partition. Show that $\mathcal{E}$ can be expressed as:

$$\mathcal{E}[\theta] = 2\int_{\theta(x_A)}^{\theta(x_B)} \sqrt{AE_\mathrm{a}(\theta)}\,\mathrm{d}\theta \tag{6}$$

4 The Bloch domain wall

By integrating the equations exhibited in the previous section, derive now the exact profile of the domain wall:

$$\theta(x) = 2\arctan\left(\exp\frac{x}{\Delta_\mathrm{u}}\right) \tag{7}$$

and its total energy $\mathcal{E}$. $\Delta_\mathrm{u} = \sqrt{A/K_\mathrm{u}}$ is the anisotropy exchange length.

The most common way to define the Bloch domain wall width δ_Bl is by replacing the exact $\theta(x)$ by its linear asymptotes (red line on Figure 1). Derive δ_Bl as a function of Δ_u.

Let us stress several issues:

- The model of the Bloch wall was named after D. Bloch who published this model in 1932[1].
- As often in physics we have seen in this simple example that a dimensional analysis yields a good insight into a micromagnetic situation. It is always worthwhile starting with such an analysis before undertaking complex analytical or numerical approaches, which especially for the latter may hide the physics at play.
- We have exhibited here a characteristic length scale in magnetism. Other length scales may occur, depending on the energy terms in balance. The physics at play will often depend on the dimensions of your system with respect to the length scales relevant in your case. Starting with such an analysis is also wise.
- When the system has a finite size the anisotropy and exchange energy do not cancel at the boundaries. The integration of Euler's equations is more tedious, involving elliptical functions.

References

[1] F. Bloch, *Zur Theorie der Austauschproblems und der Remanenzerscheinung der Feromagnetika*, Z. Phys. 74, 295 (1932).