



FT-2: Magneto-optics and Magneto-plasmonics Part 1

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THE EUROPEAN SCHOOL ON
MAGNETISM 2018
MAGNETISM BY LIGHT



Outline

Magneto-optics

Brief overview of the Magneto-optical Kerr effects (MOKE)

Advanced MOKE: vector magnetometry

Magnetic nanostructures

micro-MOKE

Approach 1: focused beam

Approach 2: microscopy

MOKE from diffracted beams:

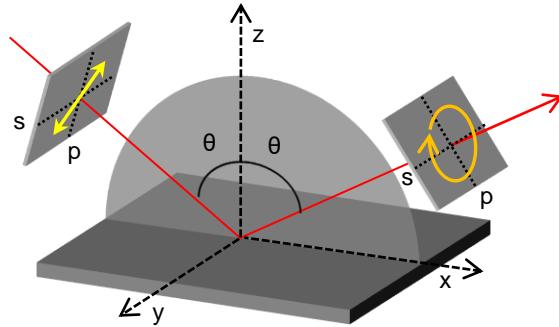
simple theory of diffracted MOKE

in conjunction with micromagnetics and MFM

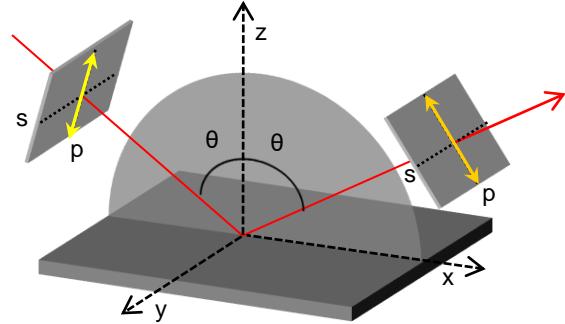
from magnetometry to magnetic imaging



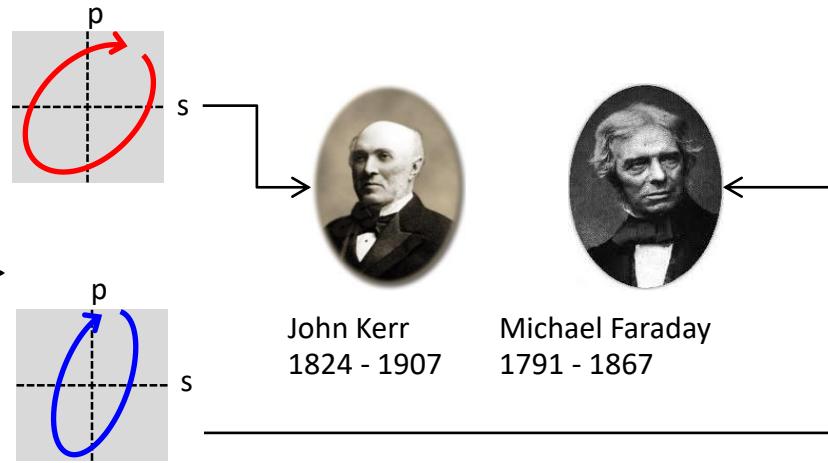
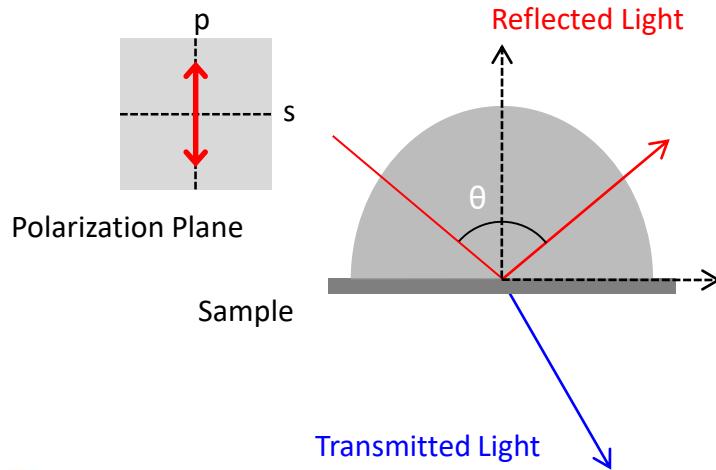
The MagnetoOptical Effect



$$R = \begin{pmatrix} r_{s \rightarrow s} \\ r_{p \rightarrow p} \end{pmatrix}$$



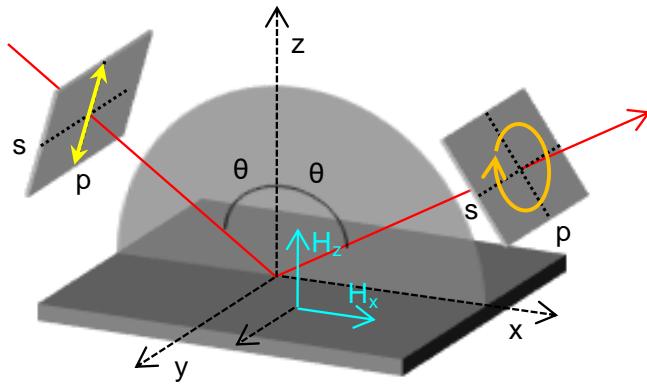
But, what happens if we applied a magnetic field??



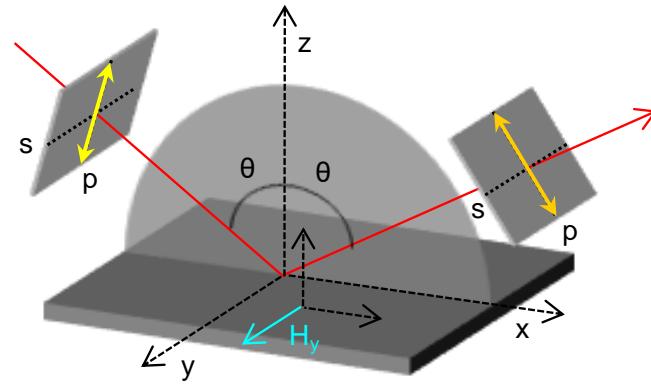


The Magneto-Optical Effect

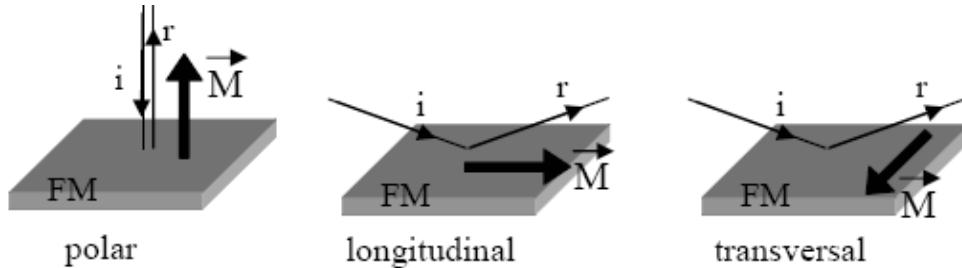
Polar and Longitudinal Configuration



Transverse Configuration



<p>Dielectric Tensor</p> $\begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{pmatrix}$	<p>Reflectivity matrix</p> $R = \begin{pmatrix} r_{s \rightarrow s} & r_{p \rightarrow s} \\ r_{s \rightarrow p} & r_{p \rightarrow p} \end{pmatrix}$
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$$\hat{\epsilon} = \begin{bmatrix} \epsilon_0 & 0 & 0 \\ 0 & \epsilon_0 & 0 \\ 0 & 0 & \epsilon_0 \end{bmatrix} \quad \Rightarrow \quad \hat{\epsilon} = \begin{bmatrix} \epsilon_0 & i\epsilon_z & -i\epsilon_y \\ -i\epsilon_z & \epsilon_0 & i\epsilon_x \\ i\epsilon_y & -i\epsilon_x & \epsilon_0 \end{bmatrix}$$

$$\epsilon_x = \epsilon_0 Q m_x; \epsilon_y = \epsilon_0 Q m_y; \epsilon_z = \epsilon_0 Q m_z;$$

- Non-destructive;
- High sensitivity;
- Finite penetration depth (~ 10 nm);
- Fast (time resolved measurements);
- Laterally resolved (microscopy);
- Can be easily used in vacuum and cryogenic systems;

J. Kerr, Philosophical Magazine 3 321 (1877)

Z. Q. Qui and S. D. Bader, Rev. Sci. Instrum. 71, 1243 (2000)

The magneto-optic Kerr effect (MOKE) is widely used in studying technologically relevant magnetic materials.

It relies on small, magnetization induced changes in the optical properties which modify the polarization or the intensity of the reflected light.

Macroscopically, magneto-optic effects arise from the antisymmetric, off-diagonal elements in the dielectric tensor.

Fresnell reflection coefficients

Sample
$$\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix}$$

$$r_{pp} = r_{pp}^0 + r_{pp}^M \propto m_y$$

$$r_{ps} \propto -m_x - m_z$$

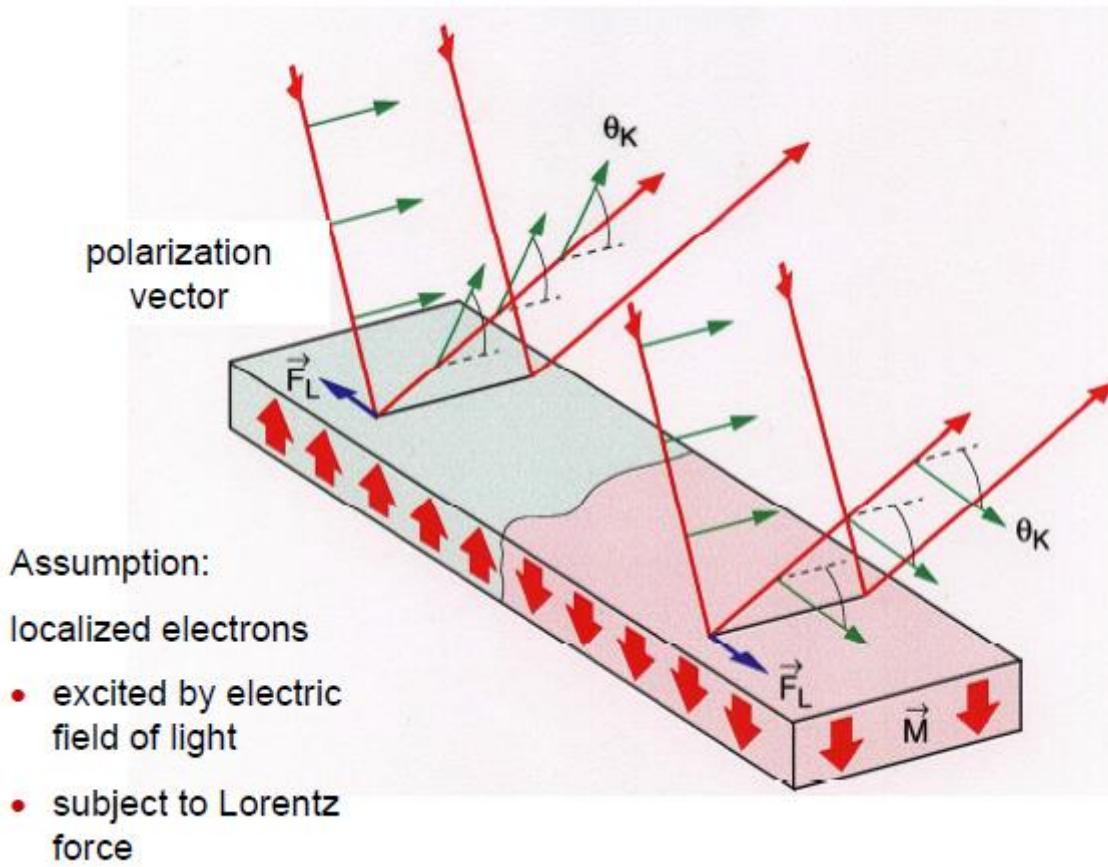
$$r_{sp} \propto m_x - m_z$$

$$r_{pp} = \frac{E_{rTM}}{E_{iTMM}} \quad r_{ps} = \frac{E_{rTM}}{E_{iTTE}} \quad r_{sp} = \frac{E_{rTE}}{E_{iTMM}} \quad r_{ss} = \frac{E_{rTE}}{E_{iTTE}}$$

P. Vavassori, APL 77 1605 (2000)



MOKE origin: classical picture Lorentz force





Electron theory of Magneto-Optics

Microscopically, the coupling between the electric field of the propagating light and the electron spin in a magnetic medium occurs through the spin-orbit interaction splitting of optical absorption lines (Zeeman effect).

$$E_{SO} = \xi(r) \mathbf{S} \cdot \mathbf{L}$$

Here, $\xi(r)$ is the spin-orbit parameter or coupling constant, which depends on the gradient of the electrostatic potential of the nuclear charges.

Its values are of the order of **10-100meV** and, thus, the spin-orbit interaction is much weaker than the exchange interaction ($\approx 1\text{eV}$).



Electron theory of Magneto-Optics

Different electronic structure
for up and down electrons

Conductivity tensor of each layer

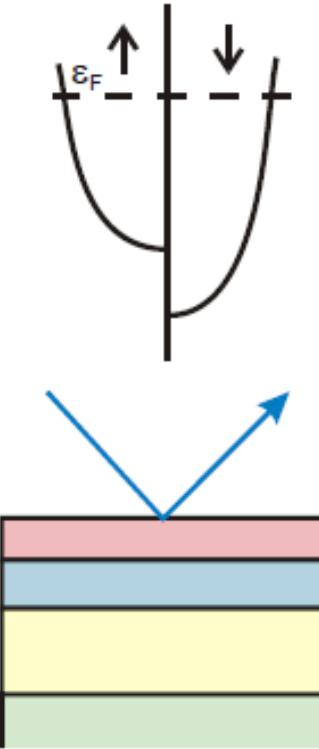
$$\vec{\sigma}(\vec{M}) = \begin{bmatrix} \sigma_0 & -\sigma_1 m_z & \sigma_1 m_y \\ \sigma_1 m_z & \sigma_0 & -\sigma_1 m_x \\ -\sigma_1 m_y & \sigma_1 m_x & \sigma_0 \end{bmatrix}$$

Reflectivity matrix of whole sample

$$R = \begin{bmatrix} r_{ss} & r_{ps} \\ r_{sp} & r_{pp} \end{bmatrix}$$

Measured Kerr effect

$$\text{e.g.: } \theta_s = \Re \left(\frac{r_{ps}}{r_{ss}} \right)$$



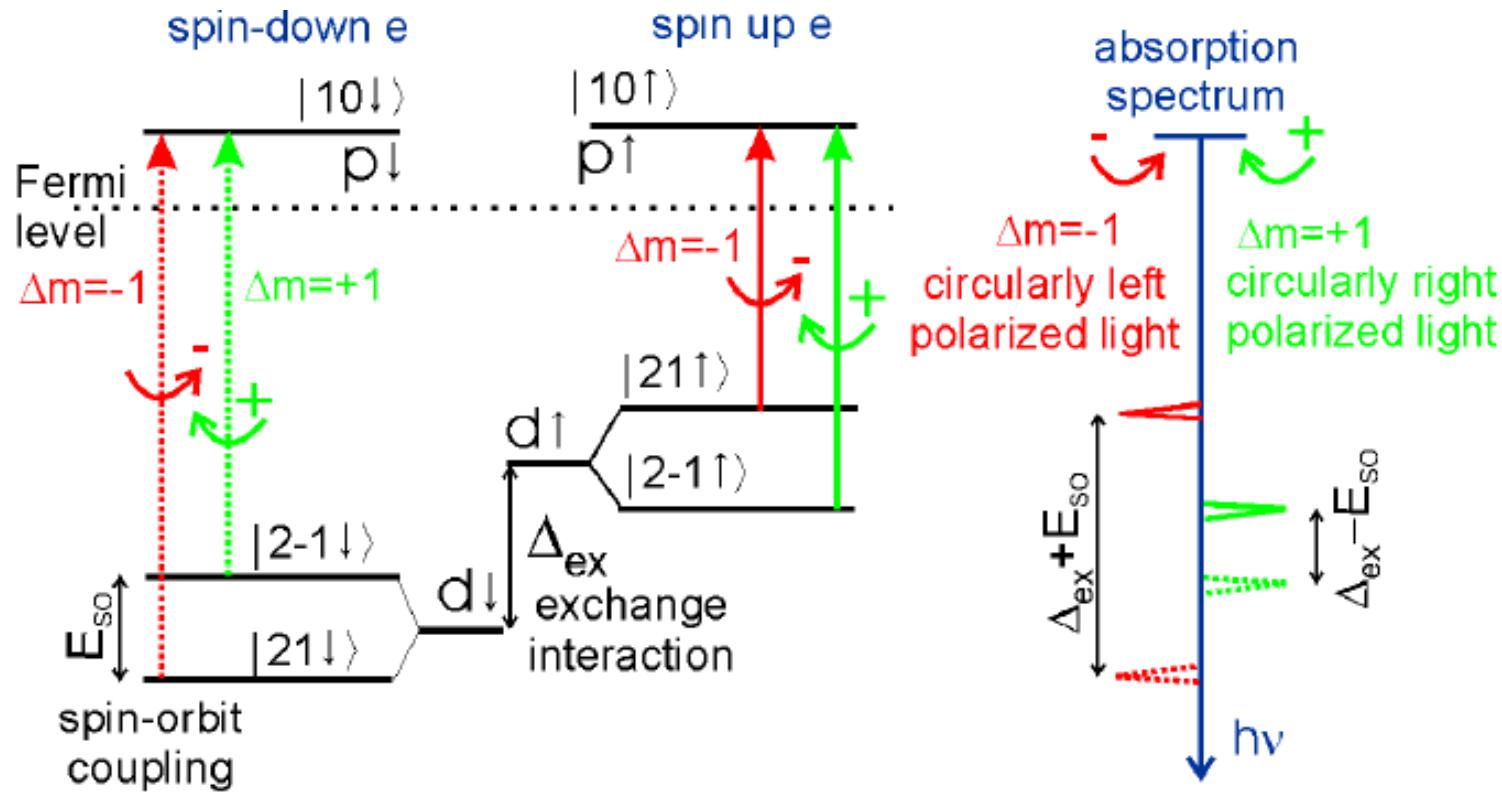


Microscopic origin of Magneto-Optics

Optical transitions between the d-orbitals and the p-orbitals

σ^+ and σ^- Selection rules $\Delta l = \pm 1$, $\Delta m_l = \pm 1$, $\Delta s = 0$

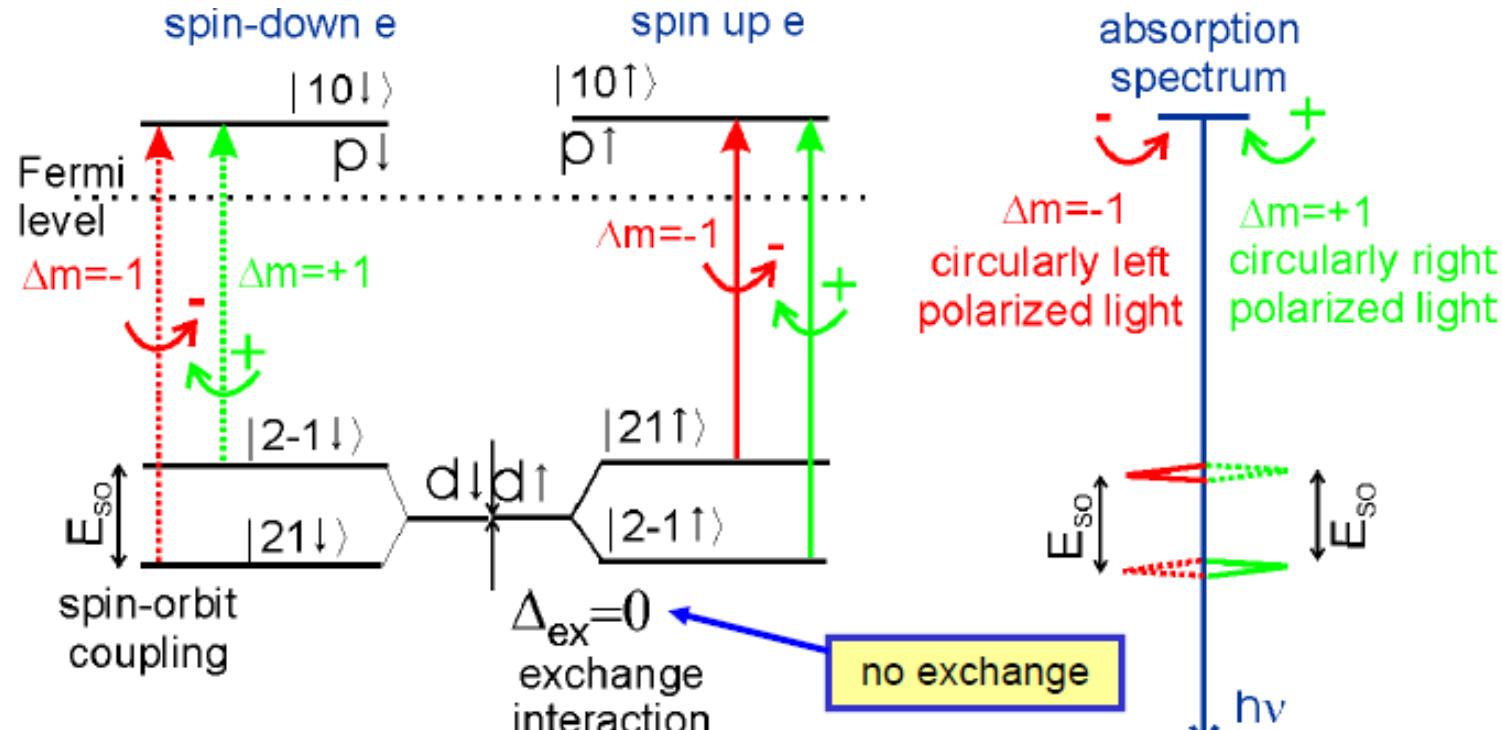
Only transition from $m = \pm 1$ to $m = 0$ are considered for simplicity



⇒ Both spin orbit coupling and exchange are necessary for MO activity



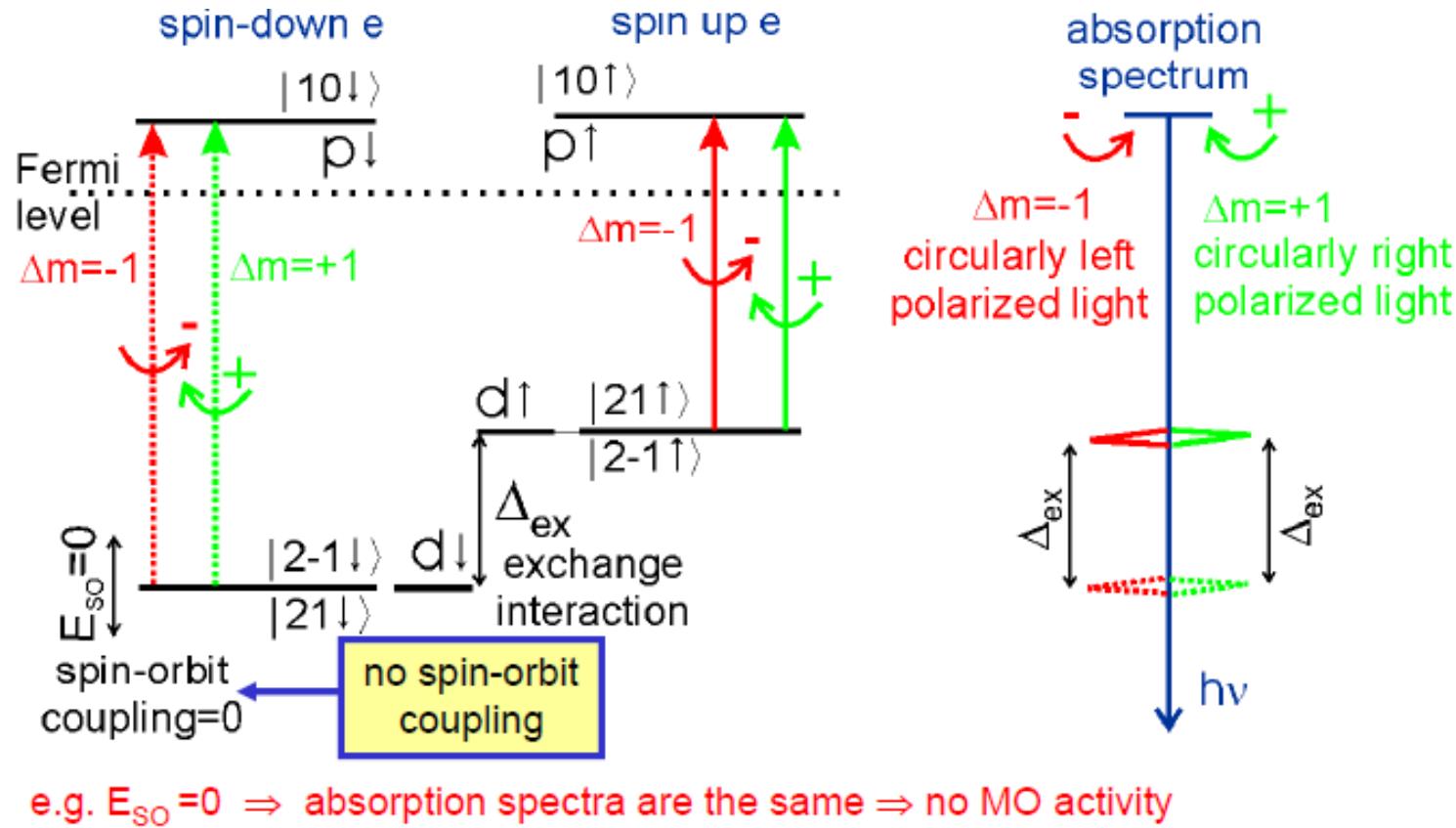
Microscopic origin of Magneto-Optics



e.g. $\Delta_{ex}=0 \Rightarrow$ absorption spectra are the same \Rightarrow no MO activity

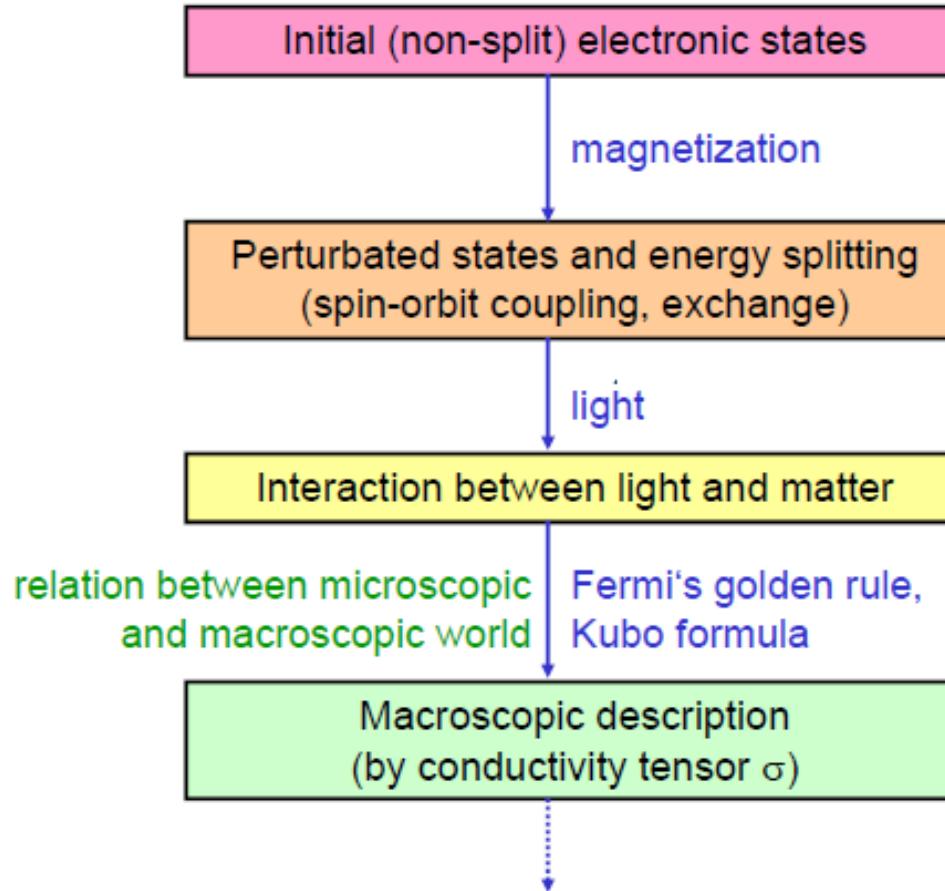


Microscopic origin of Magneto-Optics





Microscopic origin of Magneto-Optics





Microscopic origin of Magneto-Optics

⇒ Fermi's golden rule, Kubo formula:
(calculation of conductivity tensor)

Diagonal conductivity:

$$\Re[\sigma_{xx}] \sim \sum_{i,f} f(E_i) [1 - f(E_f)] \times [|\langle i | p_- | f \rangle|^2 + |\langle i | p_+ | f \rangle|^2] \times \delta(E_f - E_i - \hbar\omega)$$

Non-diagonal conductivity:

$$\Re[\sigma_{xy}] \sim \sum_{i,f} f(E_i) [1 - f(E_f)] \times [|\langle i | p_- | f \rangle|^2 - |\langle i | p_+ | f \rangle|^2] \times \delta(E_f - E_i - \hbar\omega)$$

Summation over
all initial and
final states in
k-space

Occupancy of
initial and final
states (Fermi-Dirac
distribution)

Transitions probabilities
for left and right
polarized light (non-zero
only if selection rules
are fulfilled)

Energy must
be conserved

- $\langle i |, | f \rangle$ - initial, final states
- $p_{\pm} = p_x \pm ip_y, \quad p_x = i\hbar\partial/\partial x$
linear momentum operator

Bruno, PRB 53, 9214 (1996)



Microscopic origin of Magneto-Optics

⇒ Selection rules of electric dipole transitions:

- Energy : $E_f - E_i = \hbar\omega$
(absorbed photon energy = difference between final and initial state)
- Linear momentum : $\hbar\omega/c \approx 0$
(photon has negligible linear momentum
⇒ vertical transitions)
- Electron spin : $\Delta s=0$
(spin of electron is preserved for electric dipole transitions)
- Orbitum momentum : $\Delta l=\pm 1$
(photon has angular momentum $1\hbar$).
Therefore only $s \leftrightarrow p$, $p \leftrightarrow d$ etc. are allowed
- Orbital momentum along z-axis : $\Delta m=\pm 1$
(determines if photon is right/left polarized)

$$\langle i | p_+ | f \rangle \neq 0$$

- $|i\rangle, |f\rangle$ - initial, final states
- $p_{\pm} = p_x \pm ip_y$, $p_x = i\hbar\partial/\partial x$
linear momentum operator



Electron theory of Magneto-Optics

MOKE results from a lifting of orbital degeneracy due to spin-orbit interaction (SOI) in the presence of spontaneous spin polarization.

- Magnetization → Splitting of spin-states (Exchange)
 - No direct cause of difference of optical response between LCP and RCP
- Spin-orbit interaction → Splitting of orbital states
 - Absorption of circular polarization → Induction of circular motion of electrons
- Condition for large magneto-optical response
 - Presence of strong (allowed) transitions
 - Involving elements with large spin-orbit interaction
 - Not directly related with Magnetization



A simple case: $M \parallel z$

$$\tilde{\epsilon} = \begin{pmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} & 0 \\ 0 & 0 & \epsilon_{xx} \end{pmatrix} \quad \epsilon_{xy} = i\epsilon_0 Q m_z;$$

Maxwell Equation $\text{rot rot } E(\omega) + \frac{\tilde{\epsilon}(\omega)}{c^2} \frac{\partial^2}{\partial t^2} E(\omega) = 0$

$$E(\omega) = E_0 e^{i(nk_0 z - \omega t)} \rightarrow n^2 \mathbf{E} - \mathbf{n}(\mathbf{n} \cdot \mathbf{E}) - \hat{\epsilon} \mathbf{E} = 0$$

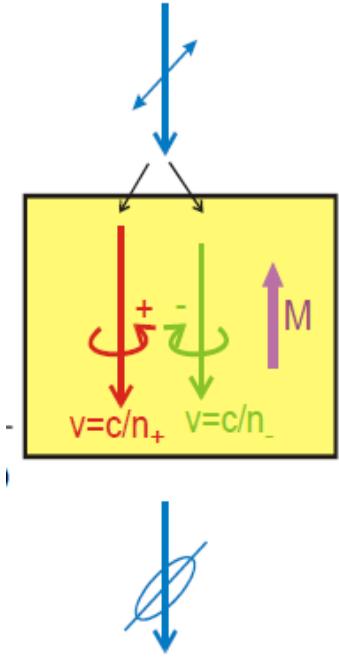
Eigenequation

$$\begin{pmatrix} n^2 - \epsilon_{xx} & -\epsilon_{xy} & 0 \\ \epsilon_{xy} & n^2 - \epsilon_{xx} & 0 \\ 0 & 0 & -\epsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0$$

Eigenvalue $n_{\pm}^2 = \epsilon_{xx} \pm i\epsilon_{xy}$ \rightarrow Eigenmodes: LCP and RCP

Different modes : different speed and attenuation

Without off-diagonal terms : No difference between LCP & RCP



Therefore, incident light becomes elliptically polarized after propagation in a MO active material



Phenomenology of MO effect

Linearly polarized light can be decomposed to LCP and RCP

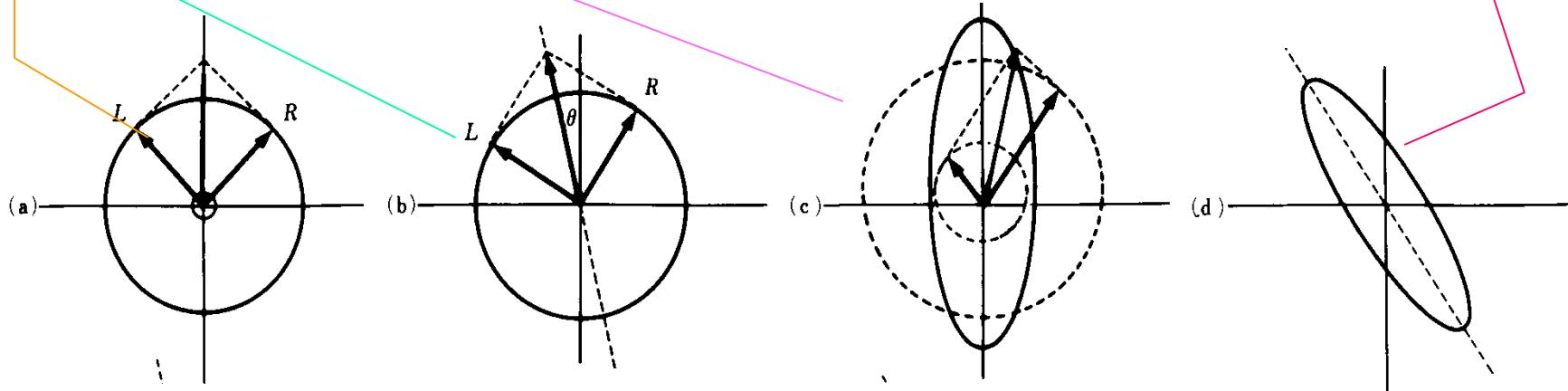
Difference in phase causes rotation of the direction of Linear polarization

Different speed
(phase lag)

Difference in amplitudes makes Elliptically polarized light

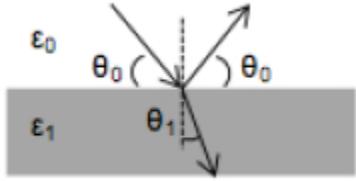
Different attenuation

In general, elliptically polarized light
With the principal axis rotated





General case: Oblique incidence and arbitrary direction of M



$$r_{pp} = \frac{n_1 \cos \theta_0 - n_0 \cos \theta_1}{n_1 \cos \theta_0 + n_0 \cos \theta_1} - \frac{2n_0 n_1^{-1} \cos \theta_0 \sin \theta_1 \epsilon_{xz}}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)^2}$$

$$\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \rightarrow$$

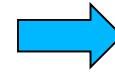
$$r_{sp} = \frac{-n_0 n_1^{-1} \cos \theta_0 (\epsilon_{xy} \cos \theta_1 + \epsilon_{yz} \sin \theta_1)}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)(n_0 \cos \theta_0 + n_1 \cos \theta_1) \cos \theta_1}$$

$$r_{ps} = \frac{-n_0 n_1^{-1} \cos \theta_0 (\epsilon_{xy} \cos \theta_1 - \epsilon_{yz} \sin \theta_1)}{(n_1 \cos \theta_0 + n_0 \cos \theta_1)(n_0 \cos \theta_0 + n_1 \cos \theta_1) \cos \theta_1}$$

$$r_{ss} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{n_0 \cos \theta_0 + n_1 \cos \theta_1}$$

$$n_i = \sqrt{\epsilon_i} \quad \cos \theta_1 = \sqrt{1 - \sin^2 \theta_1} = \sqrt{1 - \frac{n_0^2}{n_1^2} \sin^2 \theta} = \frac{\sqrt{n_1^2 - n_0^2 \sin^2 \theta}}{n_1}$$

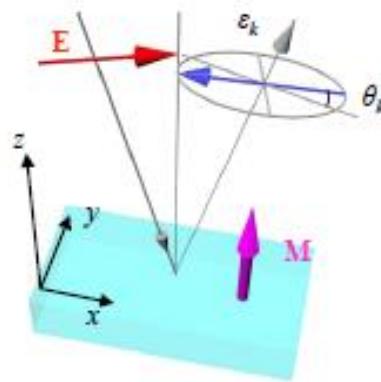
$$\begin{aligned} \epsilon_{xy} &= i \epsilon_1 Q m_z; \quad \epsilon_{xz} = -i \epsilon_1 Q m_y; \quad \epsilon_{yz} = i \epsilon_1 Q m_x; \\ \epsilon_{xy} &= -\epsilon_{yx}; \quad \epsilon_{zx} = -\epsilon_{xz}; \quad \epsilon_{zy} = -\epsilon_{yz}; \end{aligned}$$



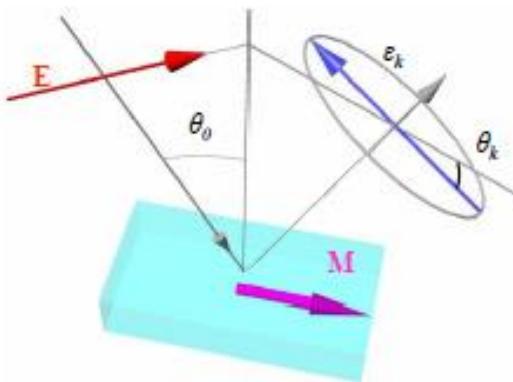
$$\begin{aligned} r_{pp} &= r_{pp}^0 + r_{pp}^M \propto m_y \\ r_{ps} &\propto -m_x - m_z \\ r_{sp} &\propto m_x - m_z \end{aligned}$$



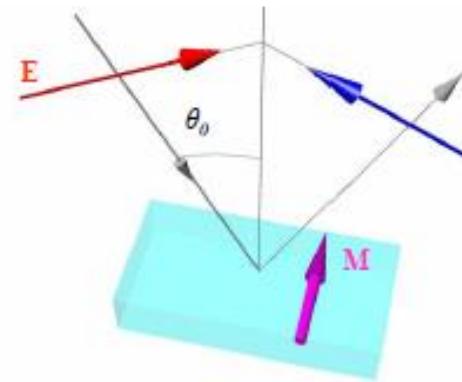
Summary of phenomenology



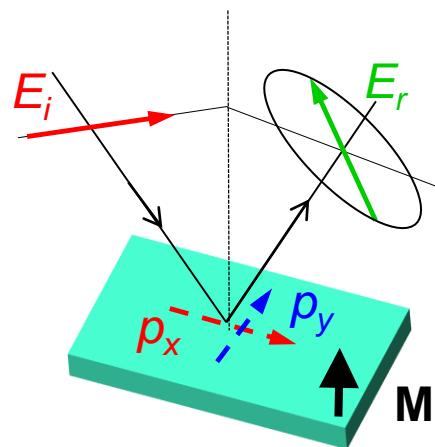
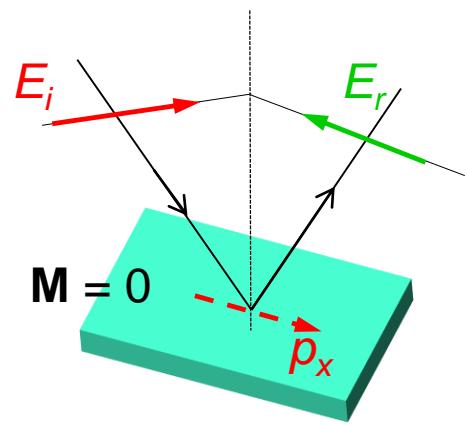
Polar



Longitudinal

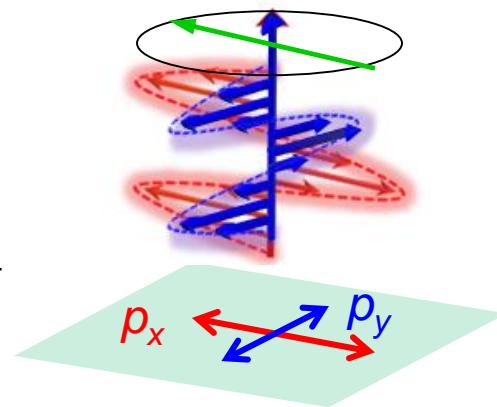


Transverse



Polarization conversion

$$p_y = \frac{\epsilon_{yx}}{(\epsilon_{xx} - \epsilon_0)}$$



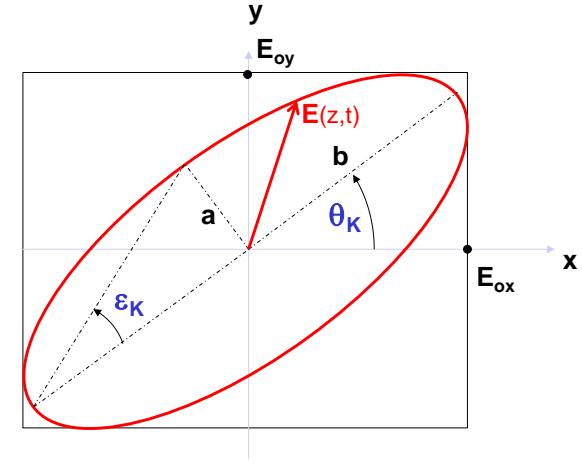


Longitudinal and polar Kerr effect

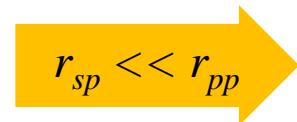
Normalized representation

Elliptically polarized light

$$\tilde{E}_r = \begin{bmatrix} r_{pp} \\ r_{sp} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{|r_{sp}|}{|r_{pp}|} e^{i\delta} \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{|r_{sp}|}{|r_{pp}|} (\cos \delta + i \sin \delta) \end{bmatrix}$$



$$\tan 2\theta_K = \frac{2|r_{pp}||r_{sp}|\cos\delta}{|r_{pp}|^2 - |r_{sp}|^2} = \frac{2b\cos\delta}{1-b^2}$$



$$\tan 2\theta_K \approx 2\theta_K \approx \frac{2|r_{sp}|\cos\delta}{|r_{pp}|}$$

$$\sin 2\varepsilon_K = \frac{2|r_{pp}||r_{sp}|\sin\delta}{|r_{pp}|^2 + |r_{sp}|^2} = \frac{2b\sin\delta}{1+b^2}$$

$$\sin 2\varepsilon_K \approx 2\varepsilon_K \approx \frac{2|r_{sp}|\sin\delta}{|r_{pp}|}$$

$$\text{Re} \left[\frac{r_{sp}}{r_{pp}} \right] = \frac{|r_{sp}|}{|r_{pp}|} \cos \delta \quad \text{Im} \left[\frac{r_{sp}}{r_{pp}} \right] = \frac{|r_{sp}|}{|r_{pp}|} \sin \delta$$

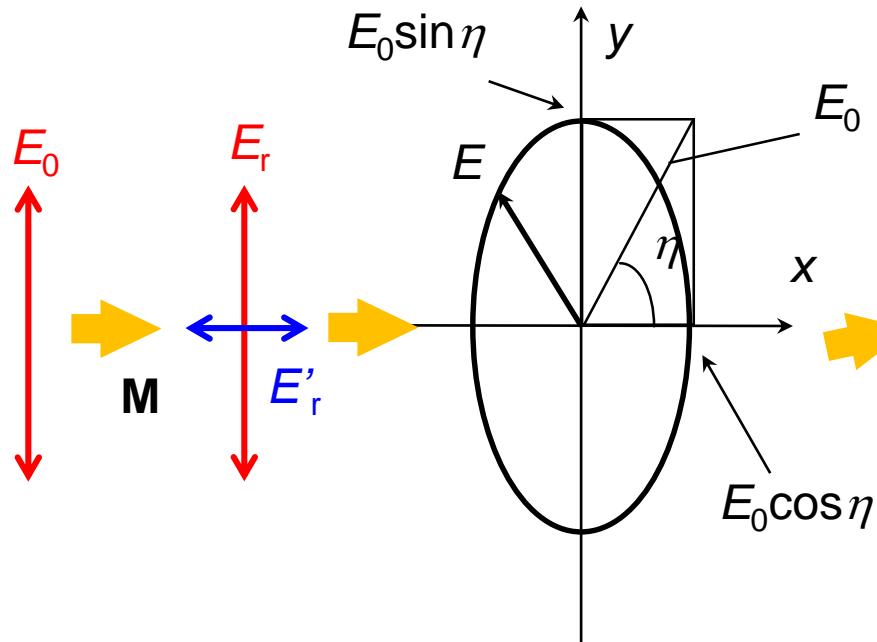


$$\theta_K \quad \text{Re} \left[\frac{r_{sp}}{r_{pp}} \right] \quad \text{Re} \left[\frac{r_{ps}}{r_{ss}} \right]$$

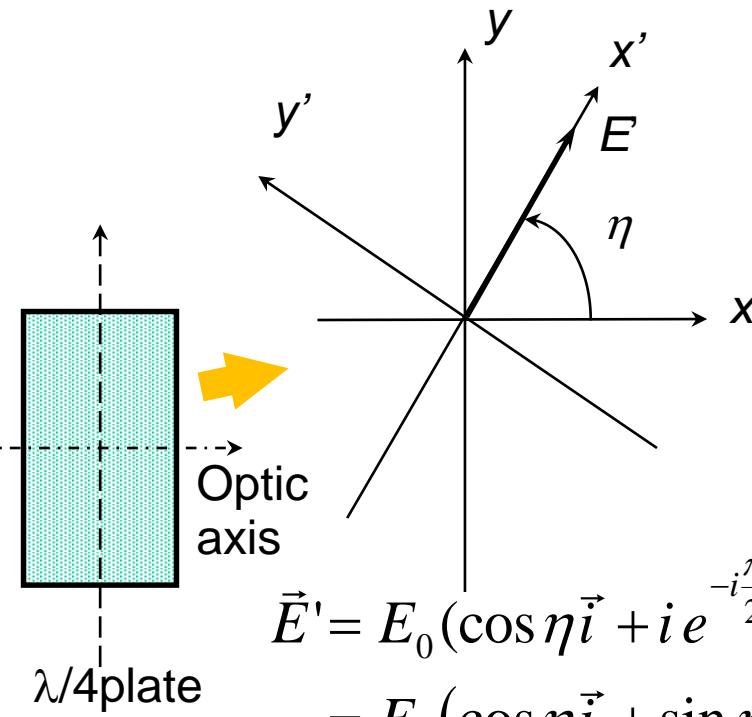
$$\varepsilon_K \quad \text{Im} \left[\frac{r_{sp}}{r_{pp}} \right] \quad \text{Im} \left[\frac{r_{ps}}{r_{ss}} \right]$$



Measurement of ellipticity



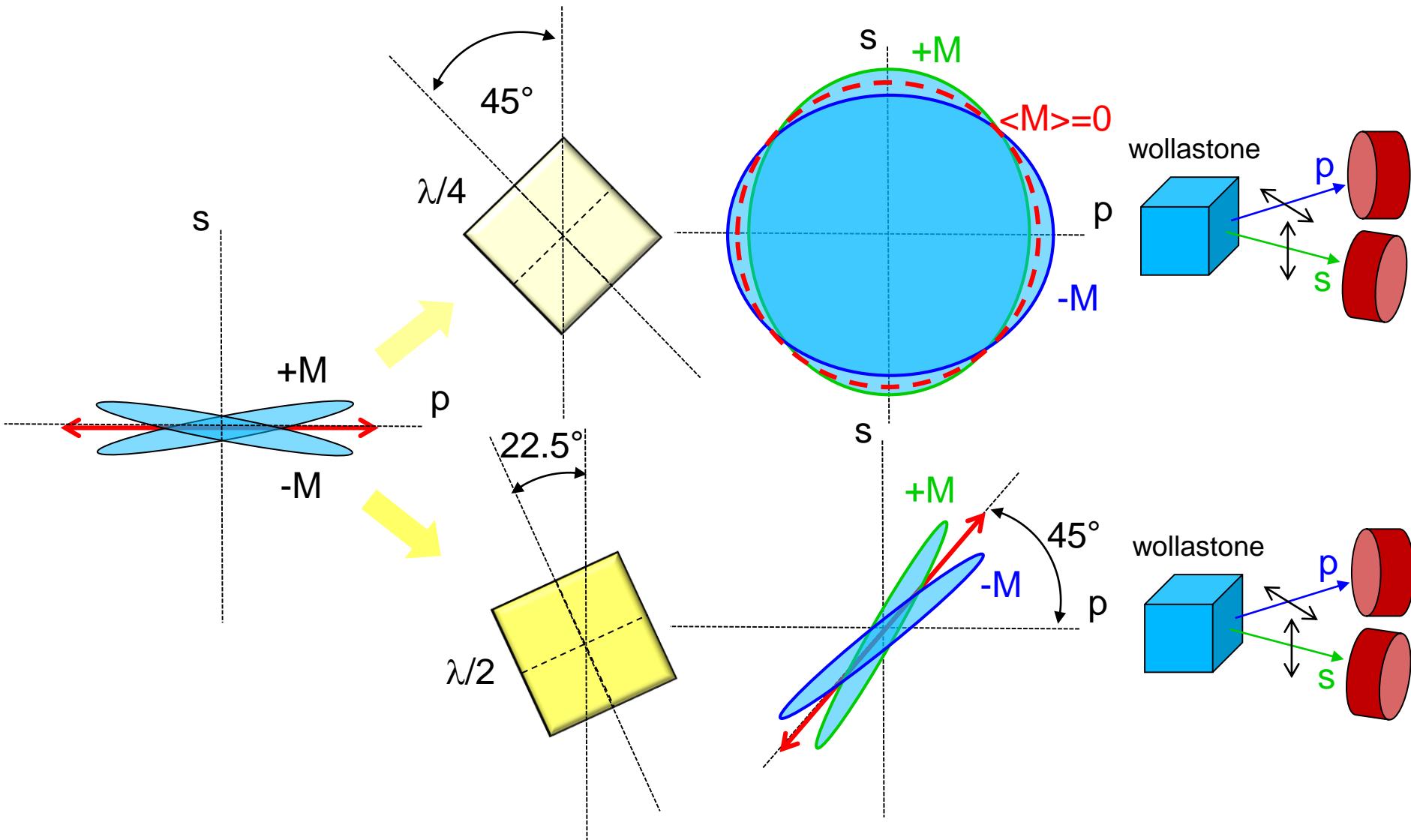
$$\vec{E} = E_0(\cos \eta \vec{i} + i \sin \eta \vec{j})$$



$$\begin{aligned}\vec{E}' &= E_0(\cos \eta \vec{i} + i e^{-i\frac{\pi}{2}} \sin \eta \vec{j}) \\ &= E_0(\cos \eta \vec{i} + \sin \eta \vec{j}) \\ &= E_0 \vec{i}'\end{aligned}$$

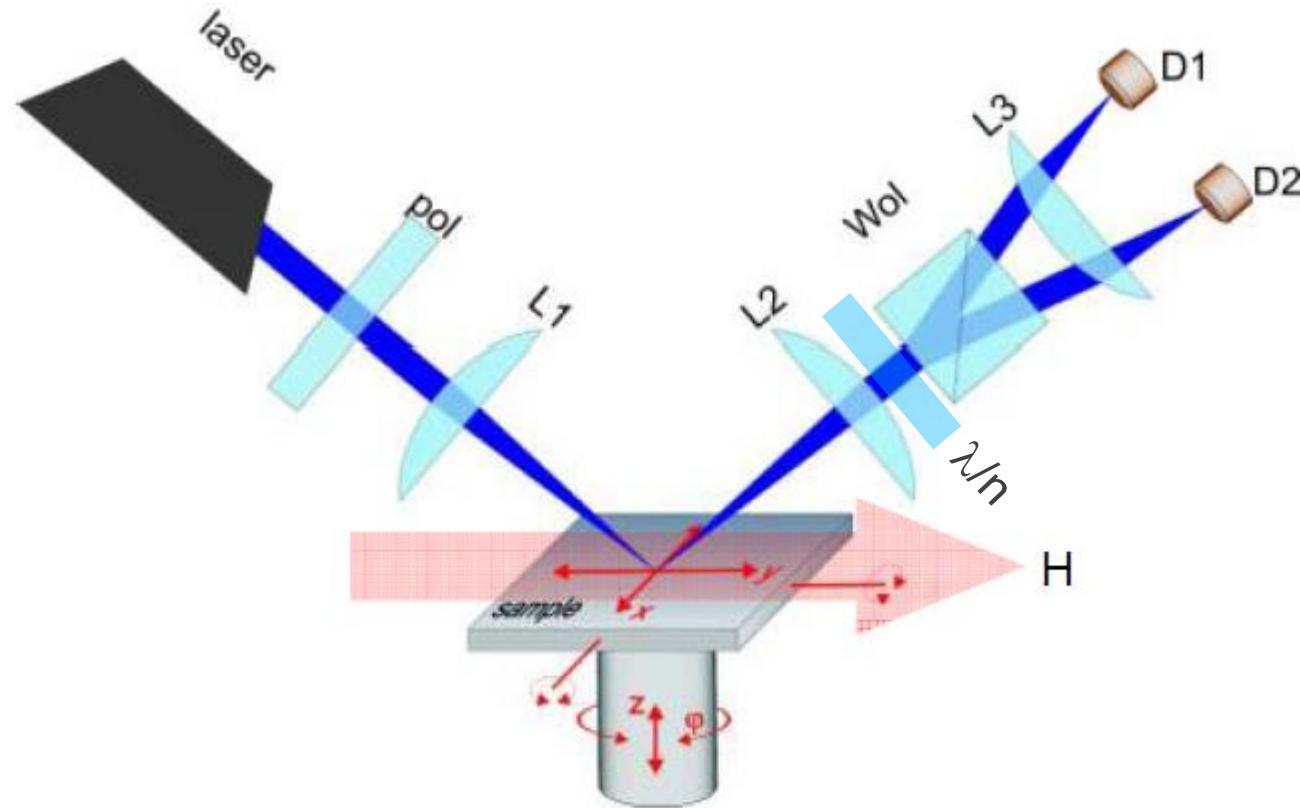


Measurement of ellipticity & rotation: high sensitivity



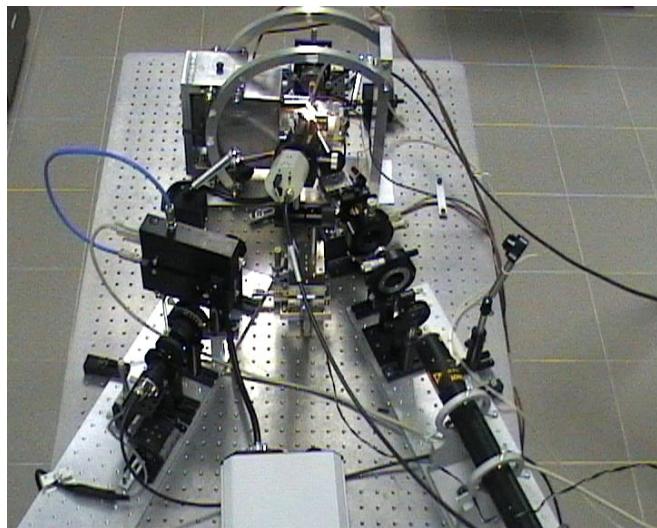


Measurement of ellipticity & rotation: high sensitivity



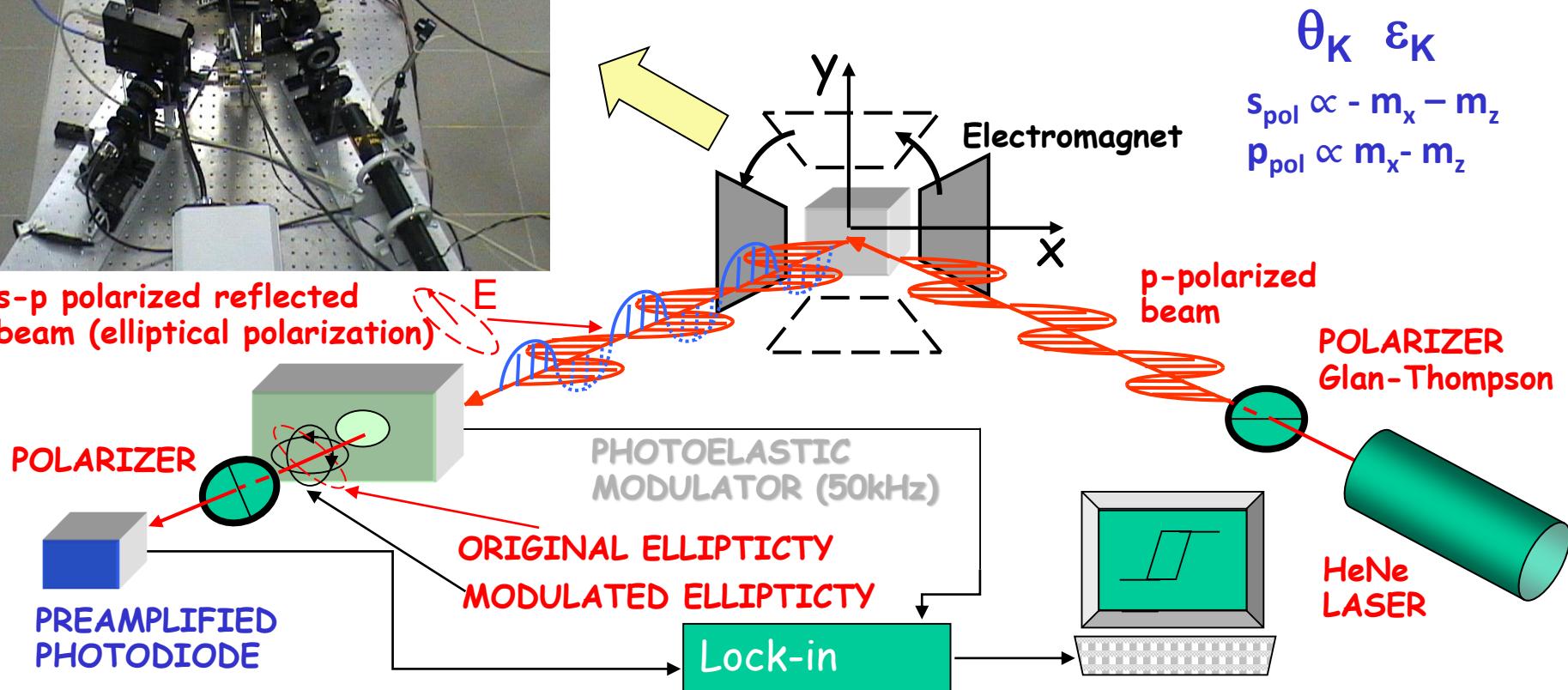


Measuring θ_K and ε_K



s-p polarized reflected beam (elliptical polarization)

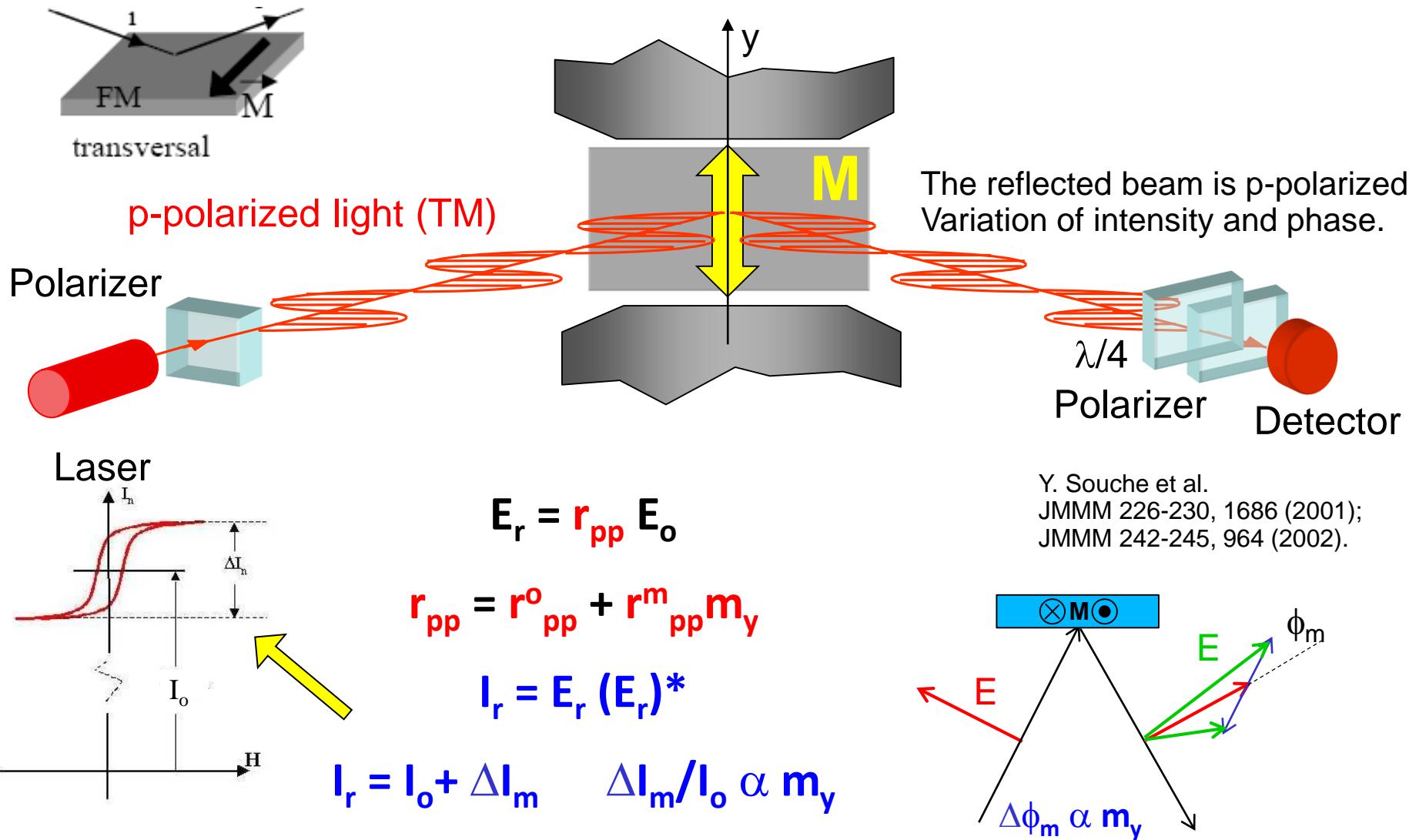
Modulation polarization technique for recording the longitudinal and polar Kerr effects, which are proportional to the magnetization components m_x m_z ..



More details in: P. Vavassori, Appl. Phys. Lett. 77, 1605 (2000)



Transverse Kerr effect





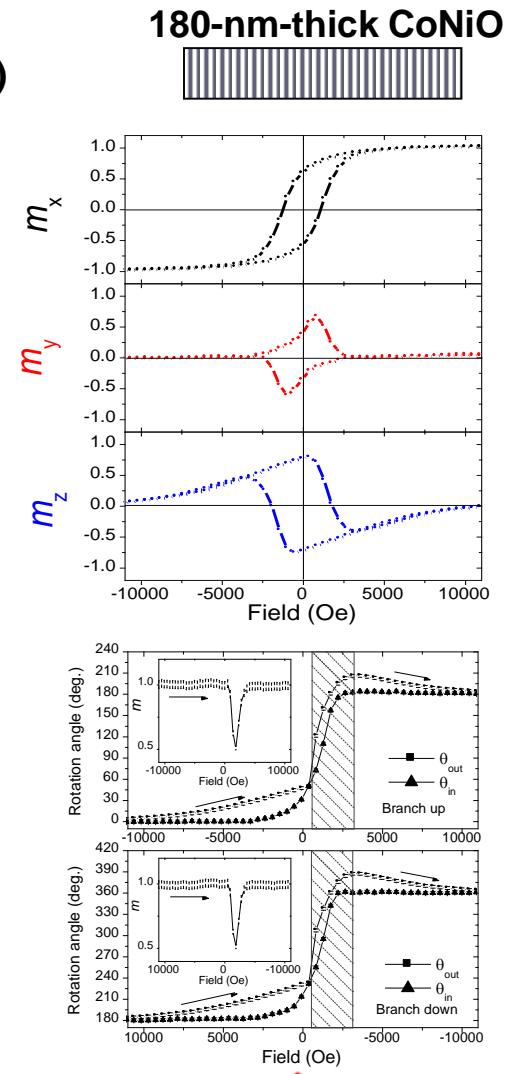
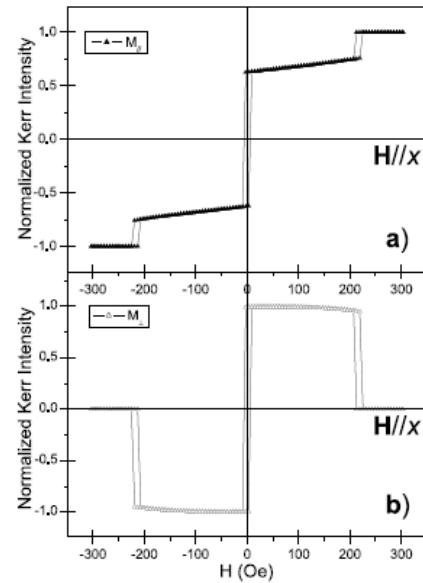
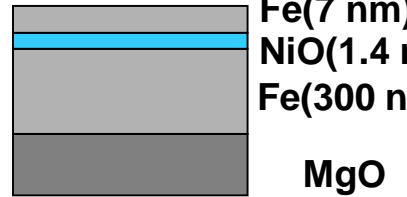
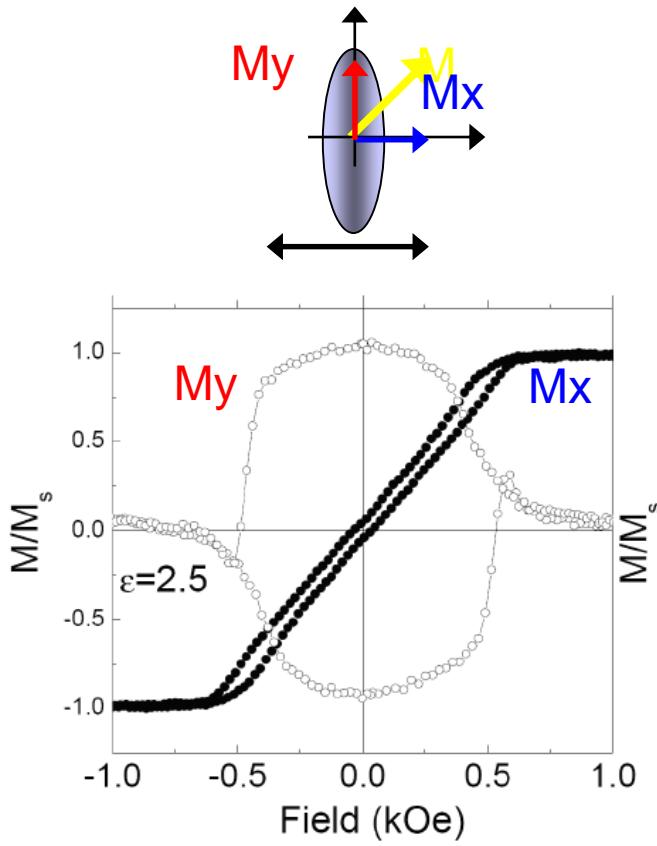
Supplementary information

Examples of application of MOKE for magnetic characterization of materials and nanosctructures



Vector analysis of reversal

Reconstruction of the magnetization vector during the reversal



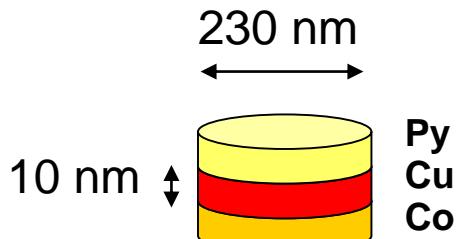
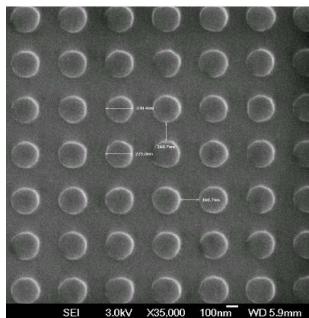
- F. Carace, P. Vavassori, G. Gubbiotti, S. Tacchi, M. Madami, G. Carlotti, and T. Okuno, Thin Solid Films **515/2**, 727 (2006).

- A. Brambilla, P. Biagioni, M. Portalupi, P. Vavassori, M. Zani, M. Finazzi, R. Bertacco, L. Duò, and F. Ciccacci, Phys. Rev. B. **72**, 174402 (2005).

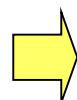
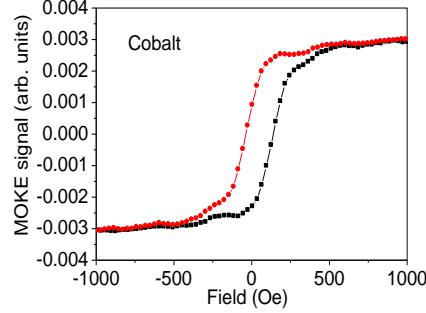
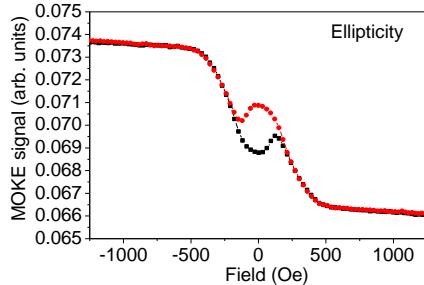
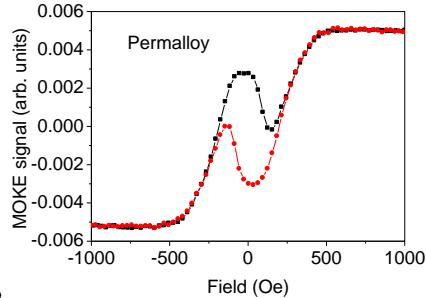
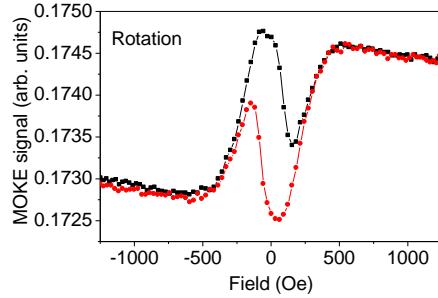
- P. Vavassori, Appl. Phys. Lett. **77**, 1605 (2000).



Element sensitivity (and layer sensitivity)

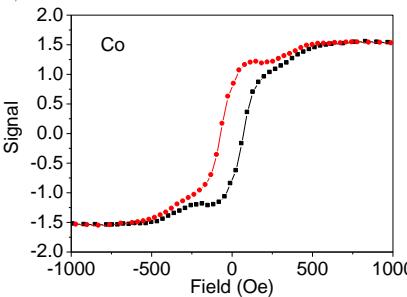
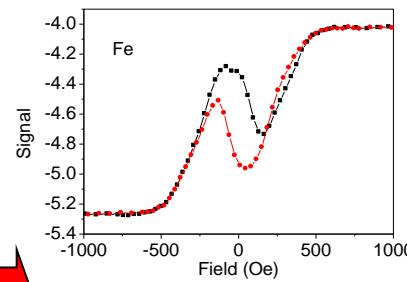


MOKE

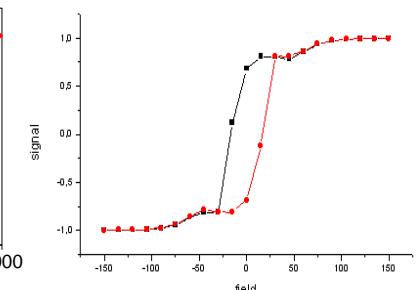
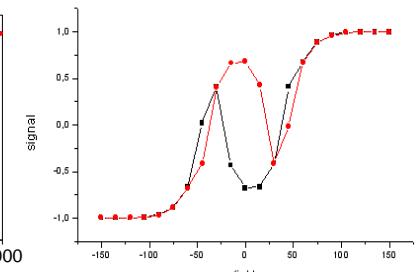


XMCD x-ray magnetic circular dichroism (FeL₃ Py hysteresis loop and CoL₃ thresholds), → chemical and magnetic sensitivity

XMCD



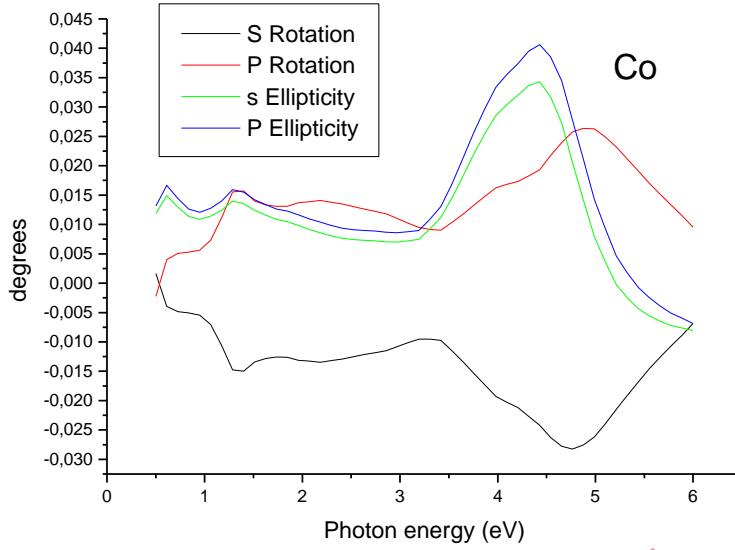
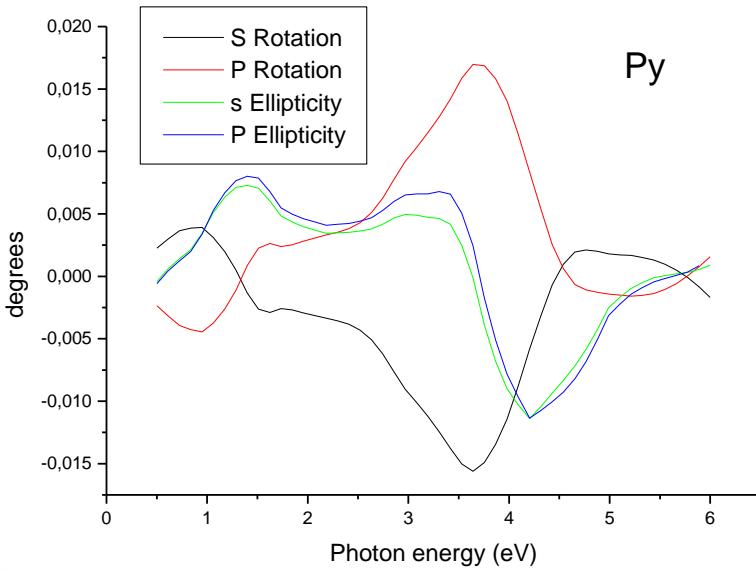
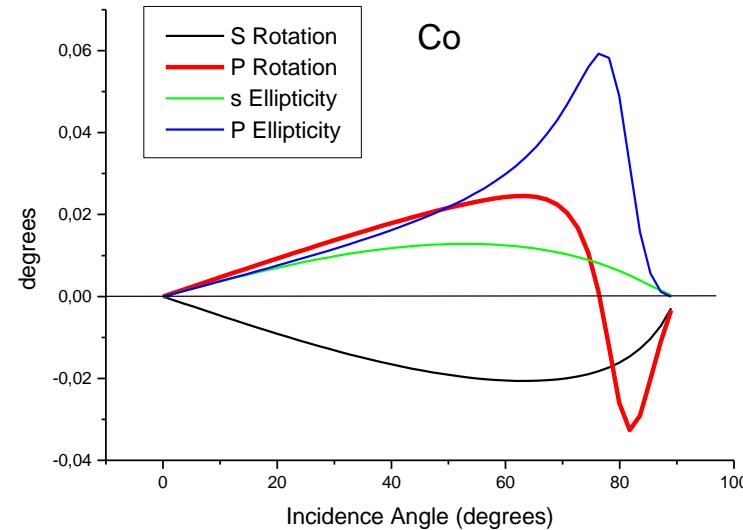
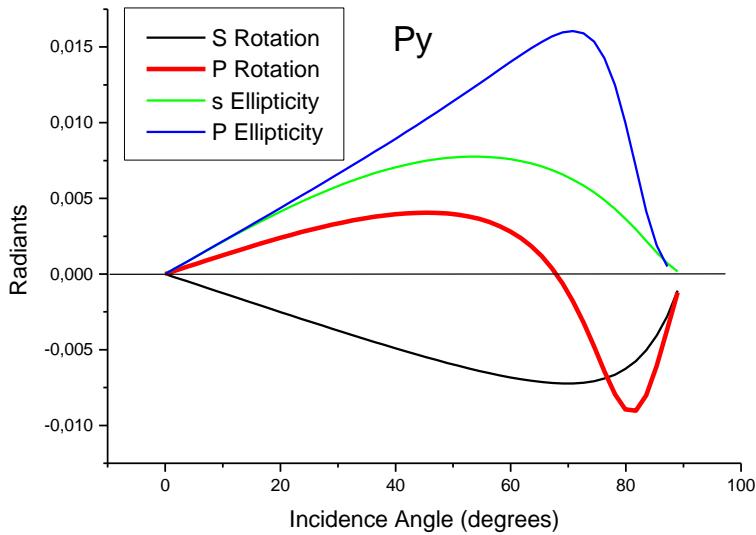
Simulations



JOURNAL OF PHYSICS D-APPLIED PHYSICS 41, 134014 (2008)

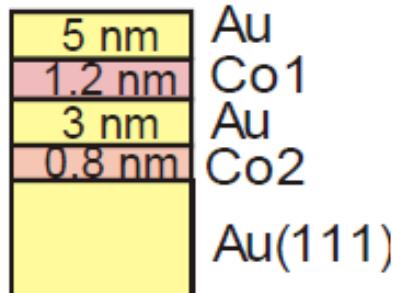


Element sensitivity (thin layer regime)





Layer sensitivity

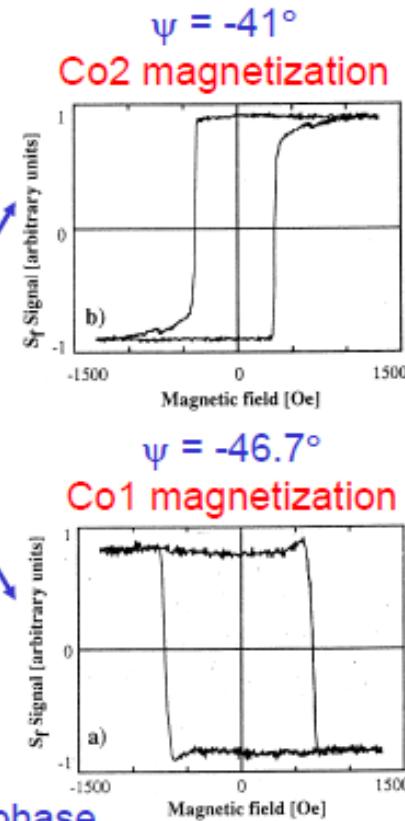
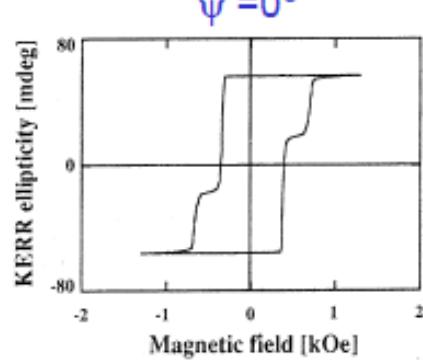


Measured MOKE is:

$$\theta_s = \Re \left(\frac{r_{ps}}{r_{ss}} \right)$$

When Babinet-Soleil compensator of retardation ψ added between sample and detector:

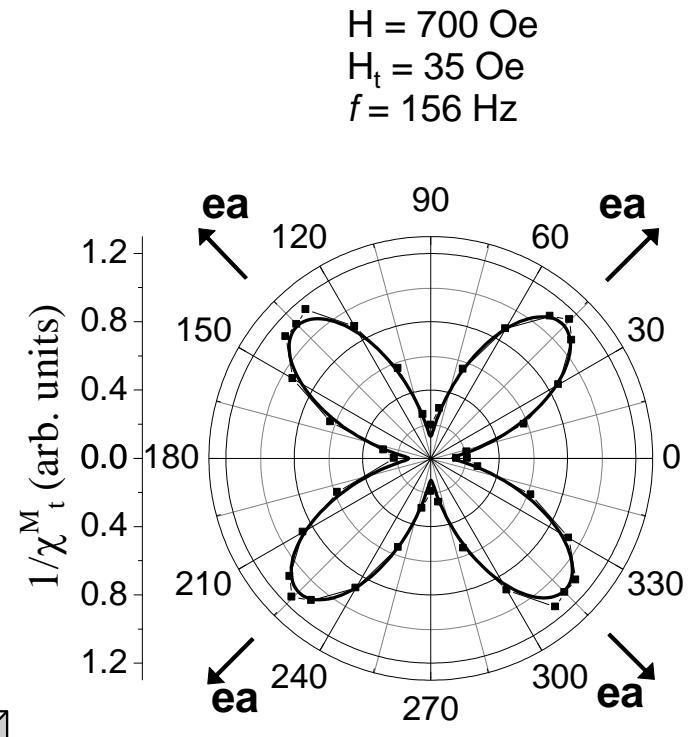
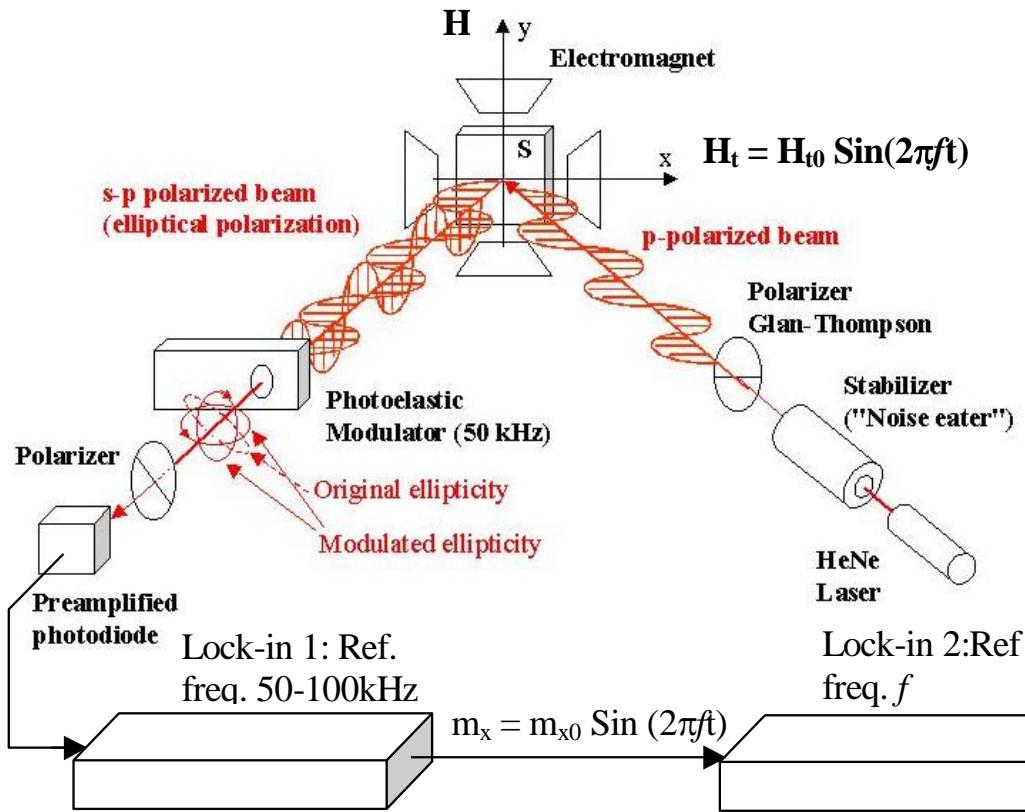
$$\theta_s = \Re \left(\frac{r_{ps}}{r_{ss}} e^{i\psi} \right) \Rightarrow$$



As Co1 and Co2 have different phase, different ψ may provide sensitivity solely to Co1 or Co2 layers



MOKE transverse susceptibility: anisotropy



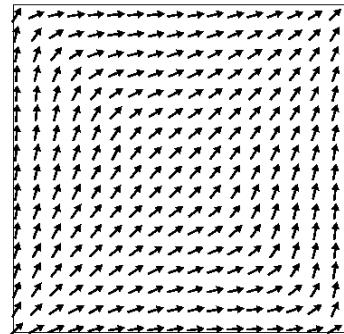
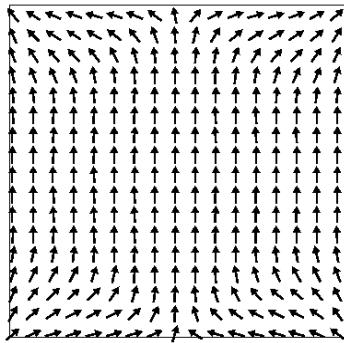
The quantity measured with the Lock-in2 is proportional to the transverse susceptibility $\chi_t = \Delta\theta_0 / H_{t0}$.

It can be shown that : $1/\chi_t = (E_o''(\theta_{eq}) / \langle M \rangle_{eq})$ where $E_o(\theta_{eq})$ is the total free energy and $\langle M \rangle_{eq}$ is the average magnetization, which makes an angle θ_{eq} with the EA.

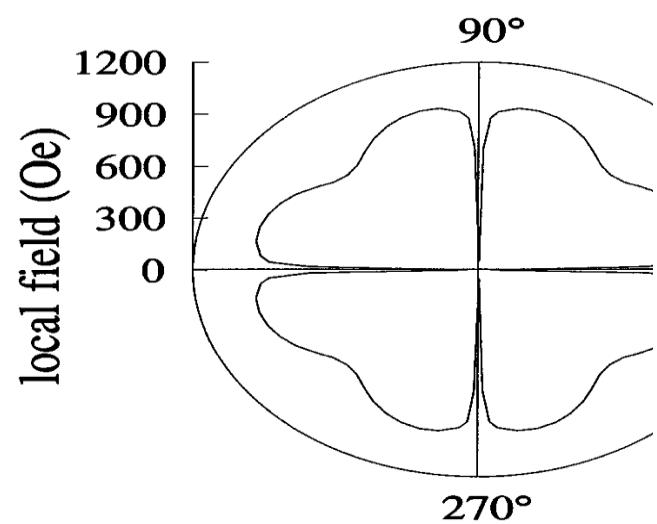
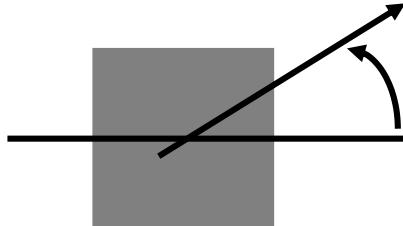


Configurational anisotropy

Even small perturbations from uniform magnetization which must exist in any nonellipsoidal magnet give rise to a (strong) angular dependence of magnetostatic dipolar energy in symmetric squared particles, which should be magnetically isotropic in the approximation of M uniform. This anisotropy is called configurational anisotropy



Py squares
 $150 \times 150 \times 15 \text{ nm}^3$

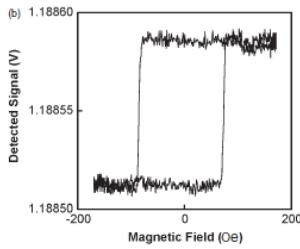
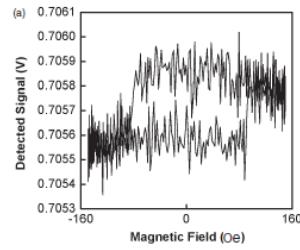
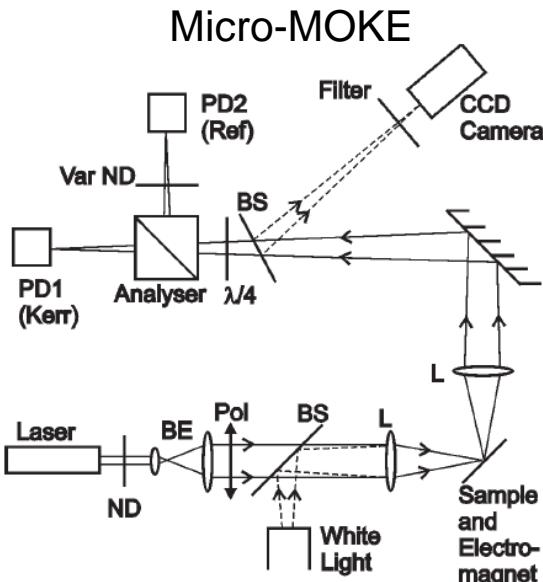


fourfold symmetry
eightfold symmetry

P. Vavassori, et al., Phys. Rev. B **72**, 054405 (2005)

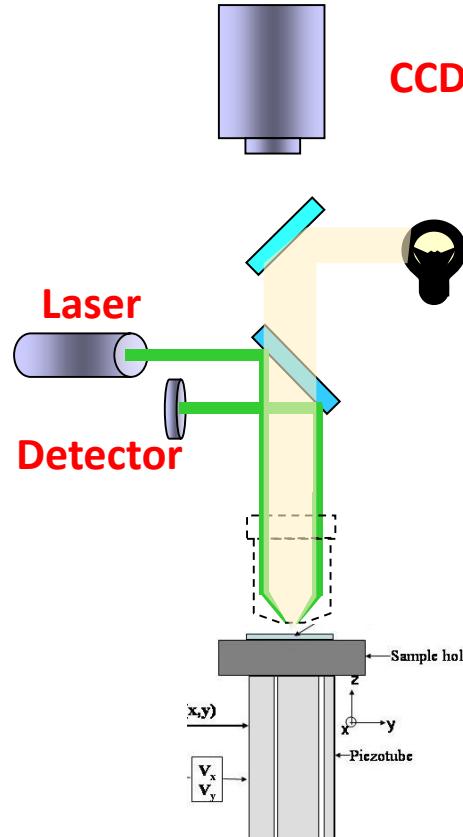


Kerr microscopy: focused laser beam

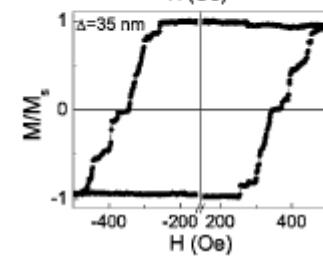
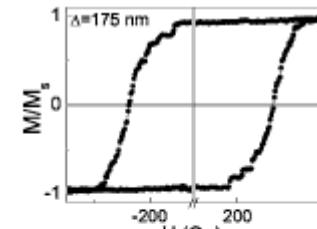
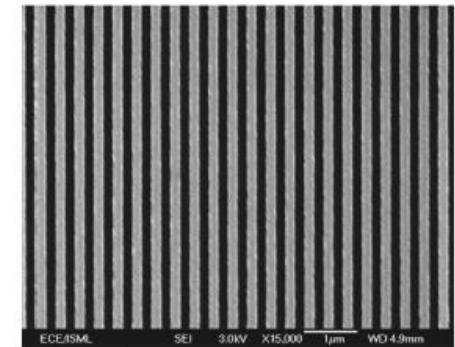


Py wire 200 nm wide
Single loop

Py wire 100 nm wide
1000 loops



Py wires 175 nm wide

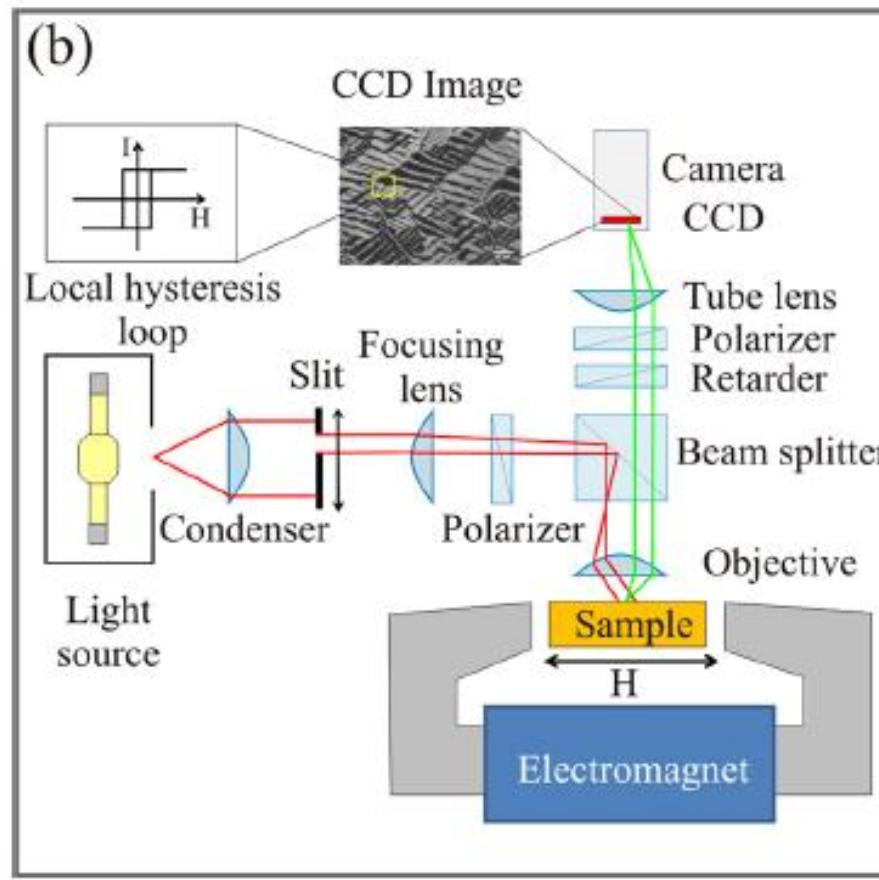
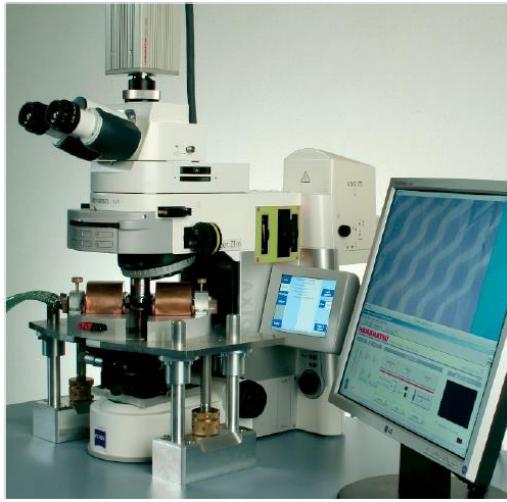


PHYSICAL REVIEW 72, 224413 (2005)

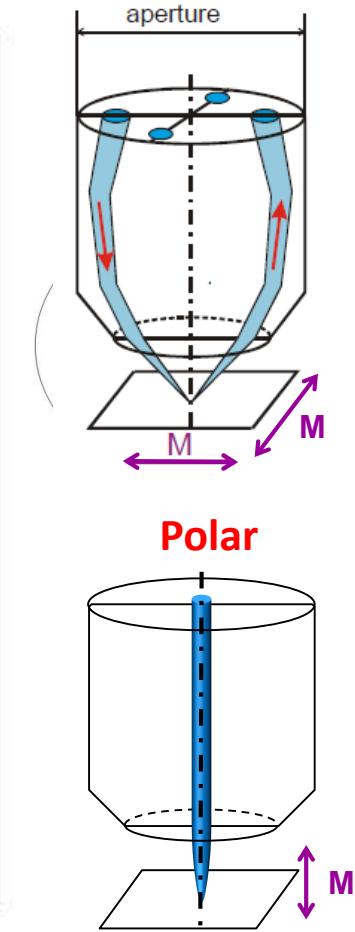
D. A. Allwood, et al., J. Phys. D: Appl. Phys. 36, 2175 (2003)



Kerr microscopy: imaging



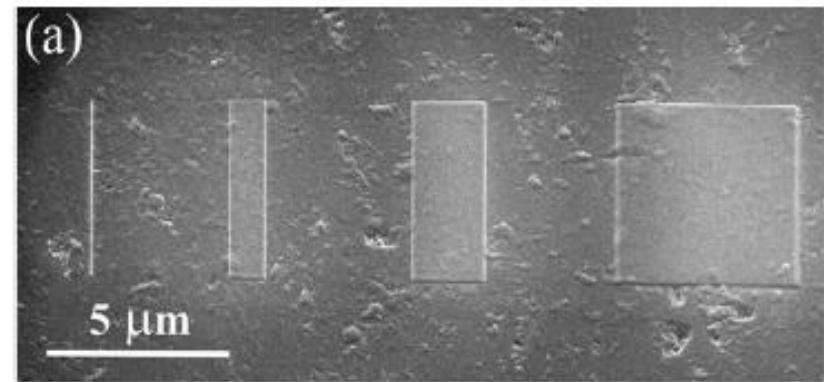
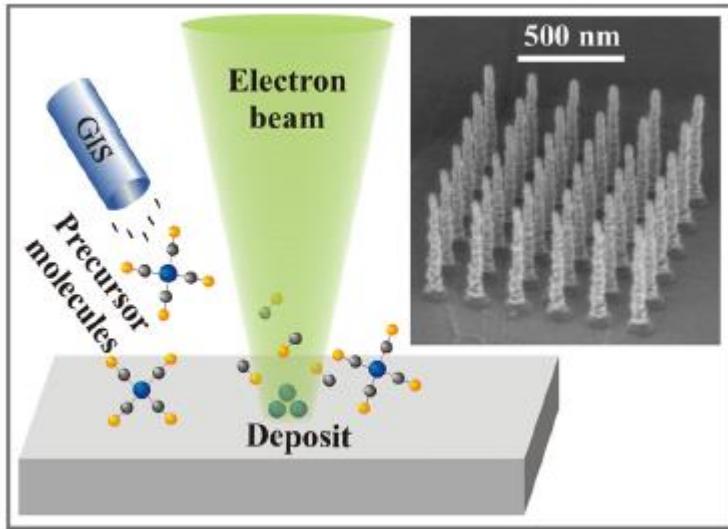
Longitudinal
and transverse





Magnetometry of ultra-small nanostructure

EBID
electron beam induced deposition

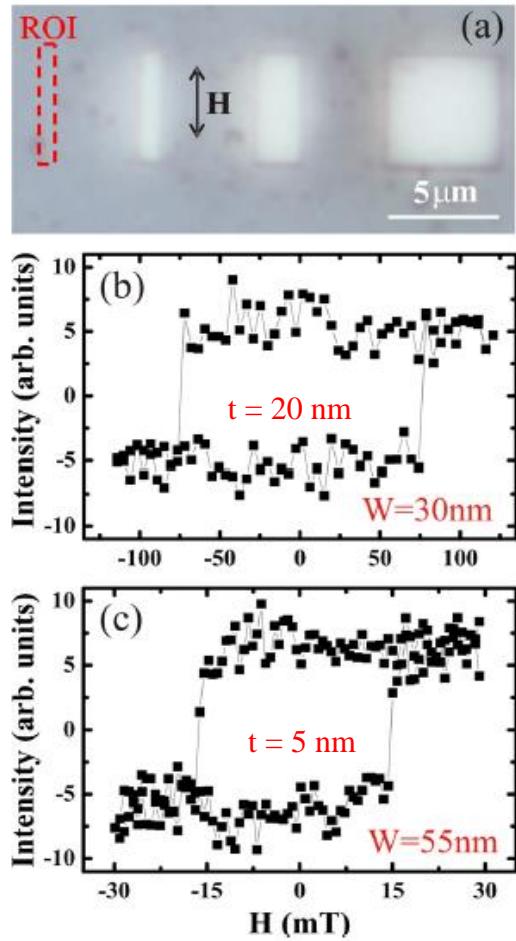


Scanning electron microscopy image of the set of EBID cobalt structures

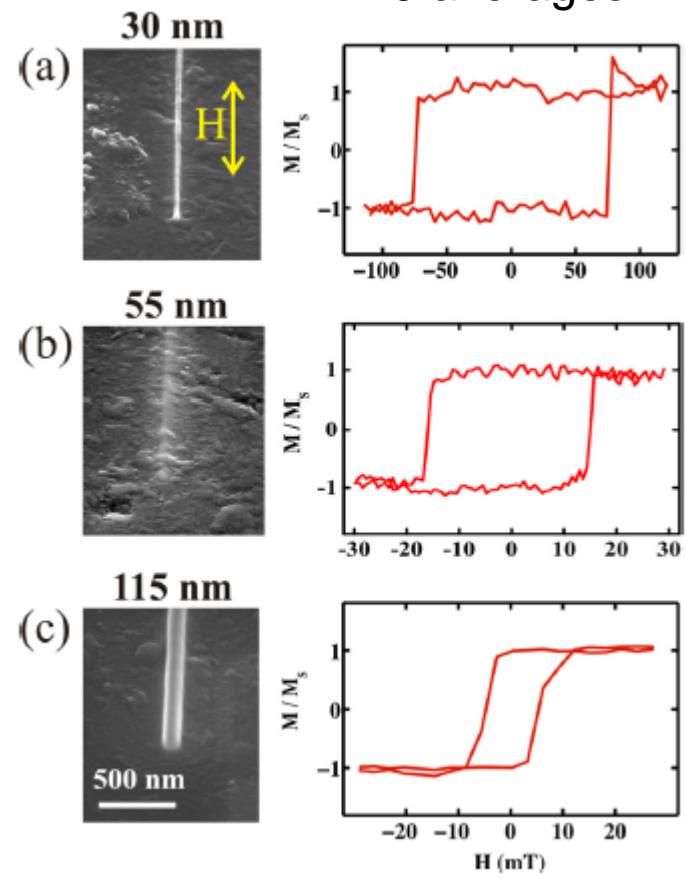


Magnetometry of ultra-small nanostructure

Single sweep



9 averages



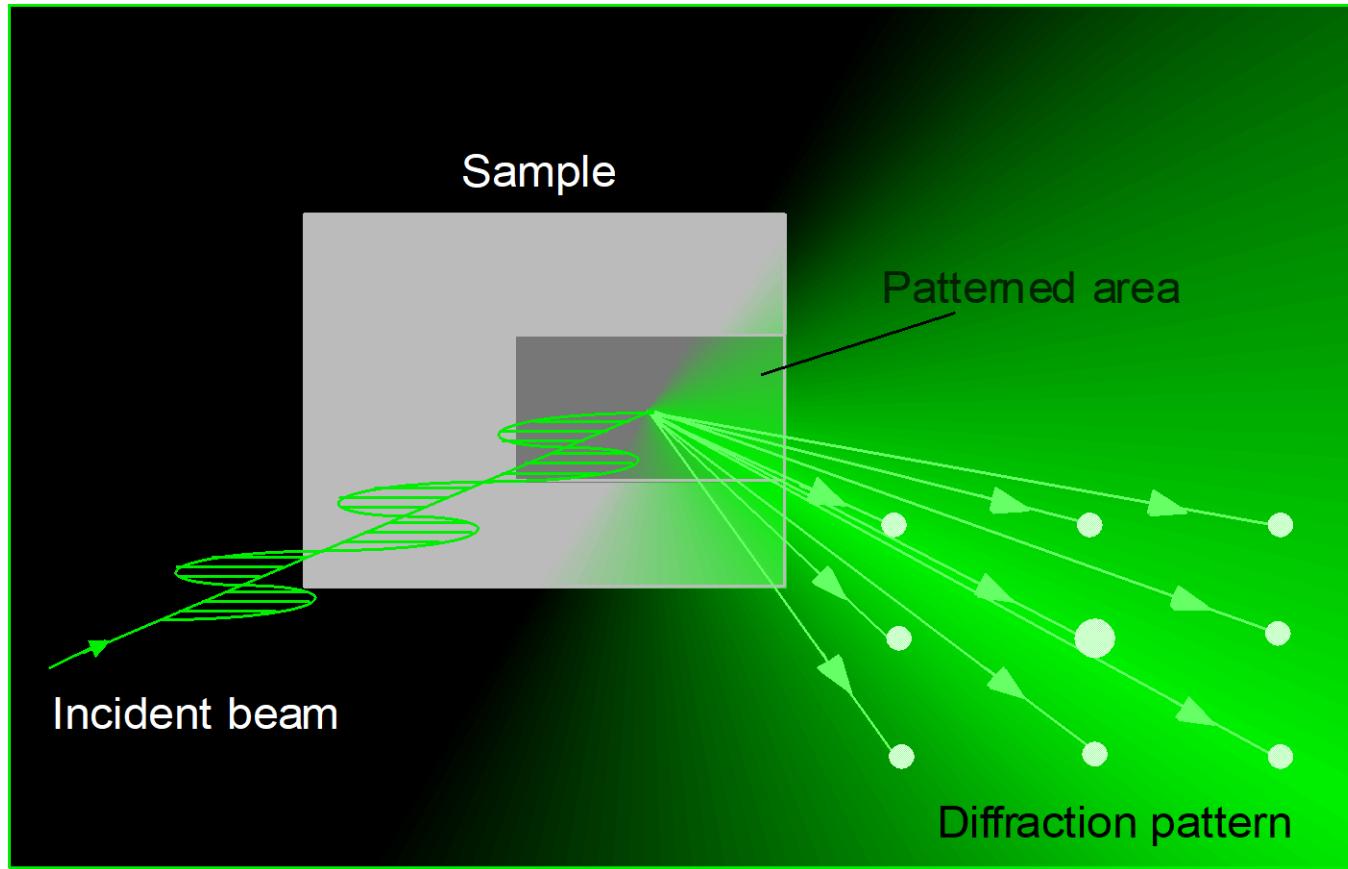
single sweep measurement sensitivity of approximately $1 \times 10^{-15} \text{ Am}^2$
sensitivity of 10^{-12} to 10^{-13} Am^2 for the latest generation of SQUID magnetometer

APPLIED PHYSICS LETTERS 100, 142401 (2012)



Diffraction of light by an array

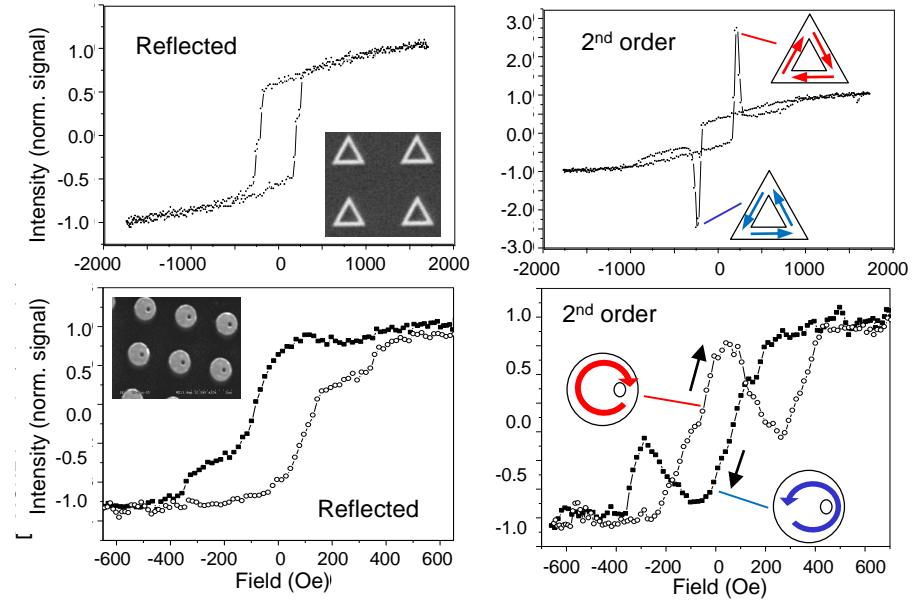
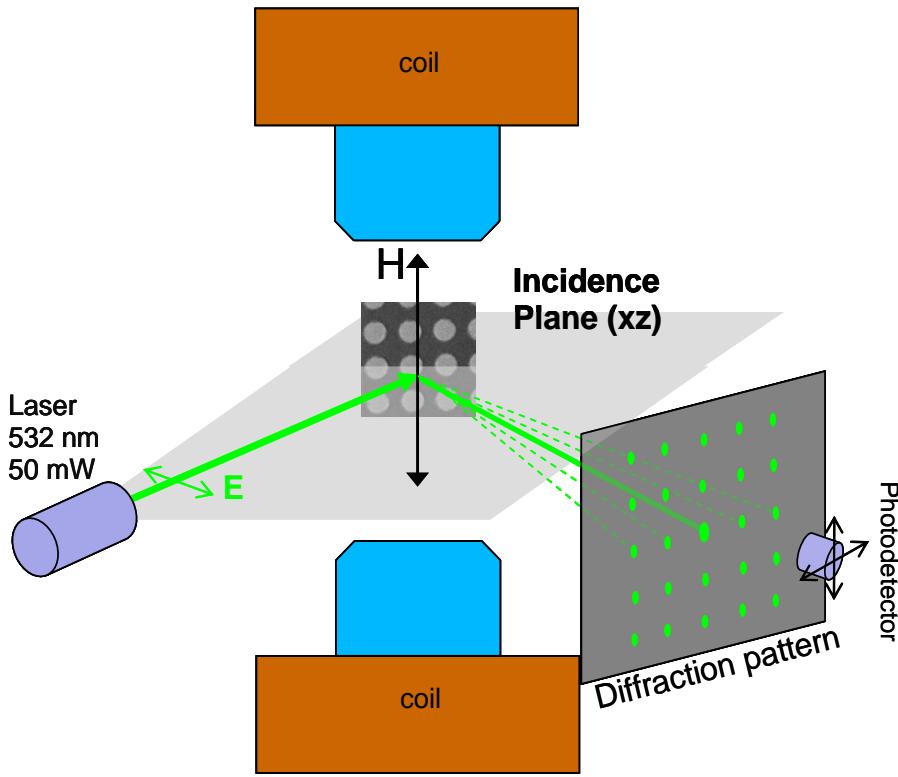
As is well known for optical gratings, when a beam of light is incident upon a sample that has a structure comparable to the wavelength of radiation, the beam is not only reflected but is also diffracted. If the material is magnetic, one may ask whether the diffracted beams also carry information about the magnetic structure.



"Diffracted-MOKE: What does it tell you?",
M. Grimsditch and P. Vavassori J. Phys.: Condensed Matter **16**, R275 - R294 (2004).



Examples of D-MOKE loops



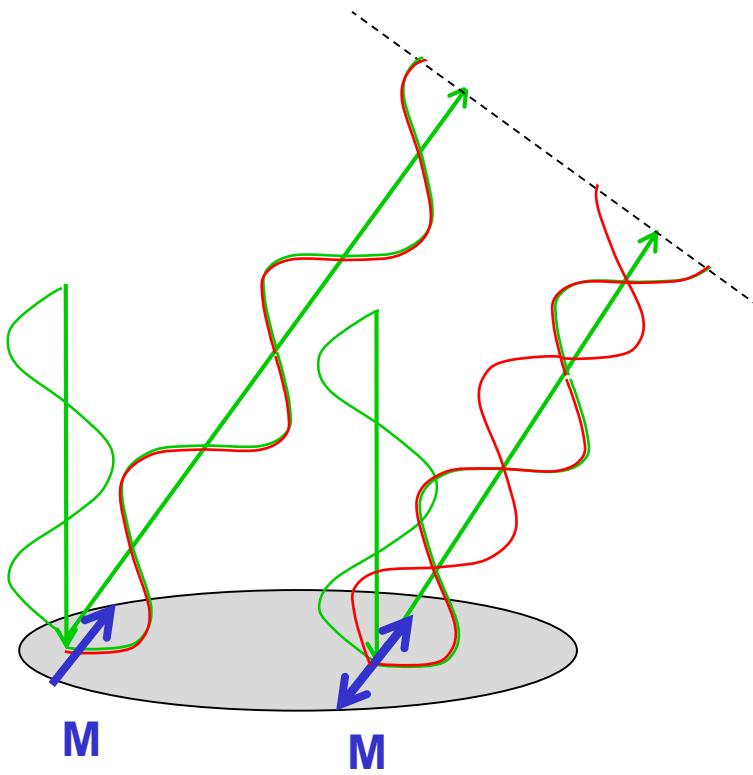
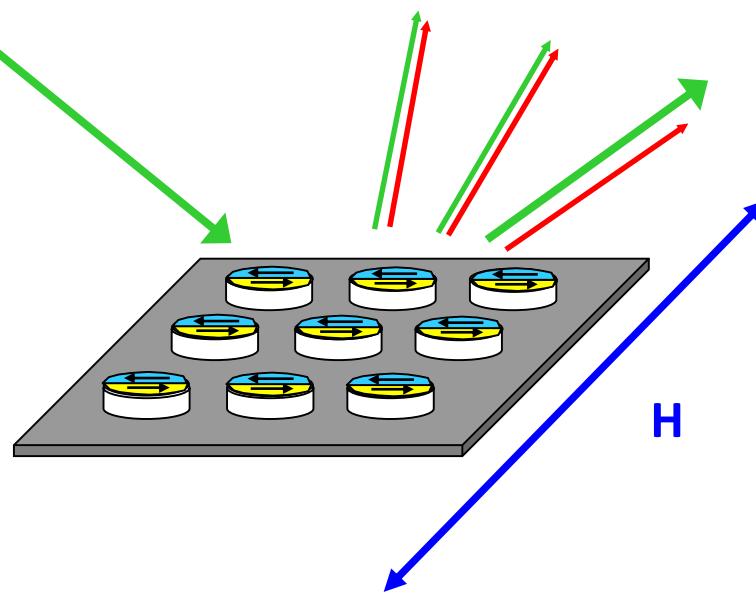
Peculiar structures due to
- Collective properties
- Interference effects

- P. Vavassori, et al., Phys. Rev. B **59** 6337 (1999)
M. Grimsditch, P. Vavassori, et al., Phys. Rev. B **65**, 172419 (2002)
P. Vavassori, et al., Phys. Rev. B **67**, 134429 (2003)
P. Vavassori, et al., Phys. Rev. B **69**, 214404 (2004)
P. Vavassori, et al., J Appl. Phys. **99**, 053902 (2006)
P. Vavassori, et al., J. Appl. Phys. **101**, 023902 (2007)
P. Vavassori, et al., Phys. Rev. B **78**, 174403 (2008)



Intuitive explanation of D-MOKE loops

$$r_{pp} = r_{pp}^o + r_{pp}^m m_y(x, y)$$



O. Geoffroy et al., J. Magn. Magn. Mat. **121** (1993) 516
Y. Souche et al., J. Magn. Magn. Mat., **140-144** (1995) 2179

Peculiar structures due to
- Interference effects
- Collective properties



Simple theory of diffracted-MOKE

Physical-optics approximation provides a very simple and physically transparent description.

The electric field in the n^{th} order diffracted beam, due to the periodic modulation of the “effective” reflectivity r'_{pp} is:

$$E_n^d = E_o f_n$$

$$f_n = \int_S r'_{\text{pp}} \exp\{i n \mathbf{G} \cdot \mathbf{r}\} dS$$

where n integer, \mathbf{G} reciprocal lattice vector and S is the unit cell.

$$r'_{\text{pp}} = r_{\text{pp}}^o + r_{\text{pp}}^m m_y(x, y, H)$$

$$E_n^d = E_o (r_{\text{pp}}^o f_n^{nm} + r_{\text{pp}}^m f_n^m) \text{ with } r_{\text{pp}}^o(\theta_i, \theta_n, \varepsilon_{\text{dots}}, \varepsilon_{\text{subst}}), r_{\text{pp}}^m(\theta_i, \theta_n, \varepsilon_{\text{dots}}, Q)$$

$$f_n^{nm} = \int_S \exp\{i n \mathbf{G} \cdot \mathbf{r}\} dx dy \quad \leftarrow \text{non-magnetic form factor}$$

$$f_n^m(H) = \int_{\text{Dot}} m_y(x, y, H) \exp\{i n \mathbf{G} \cdot \mathbf{r}\} dx dy \quad \leftarrow \text{magnetic form factor}$$

$$I_n^d = E_n^d (E_n^d)^*$$

$$\Delta I_n^m (m_y) = A_n \text{Re}[f_n^m] + B_n \text{Im}[f_n^m]$$



What are the differences due to?

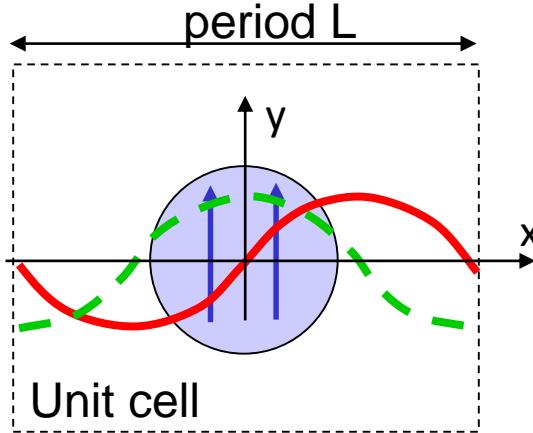
Diffracted spots in the scattering plane

$\mathbf{G}_x = 2\pi/L$ reciprocal lattice vector

$$\Delta I_n^m \propto A_n \operatorname{Re}[f_n^m] + B_n \operatorname{Im}[f_n^m]$$

$$\operatorname{Re}[f_n^m] = \int_{\text{Dot}} m_y \cos(n \mathbf{G}_x \cdot \mathbf{x}) dS$$

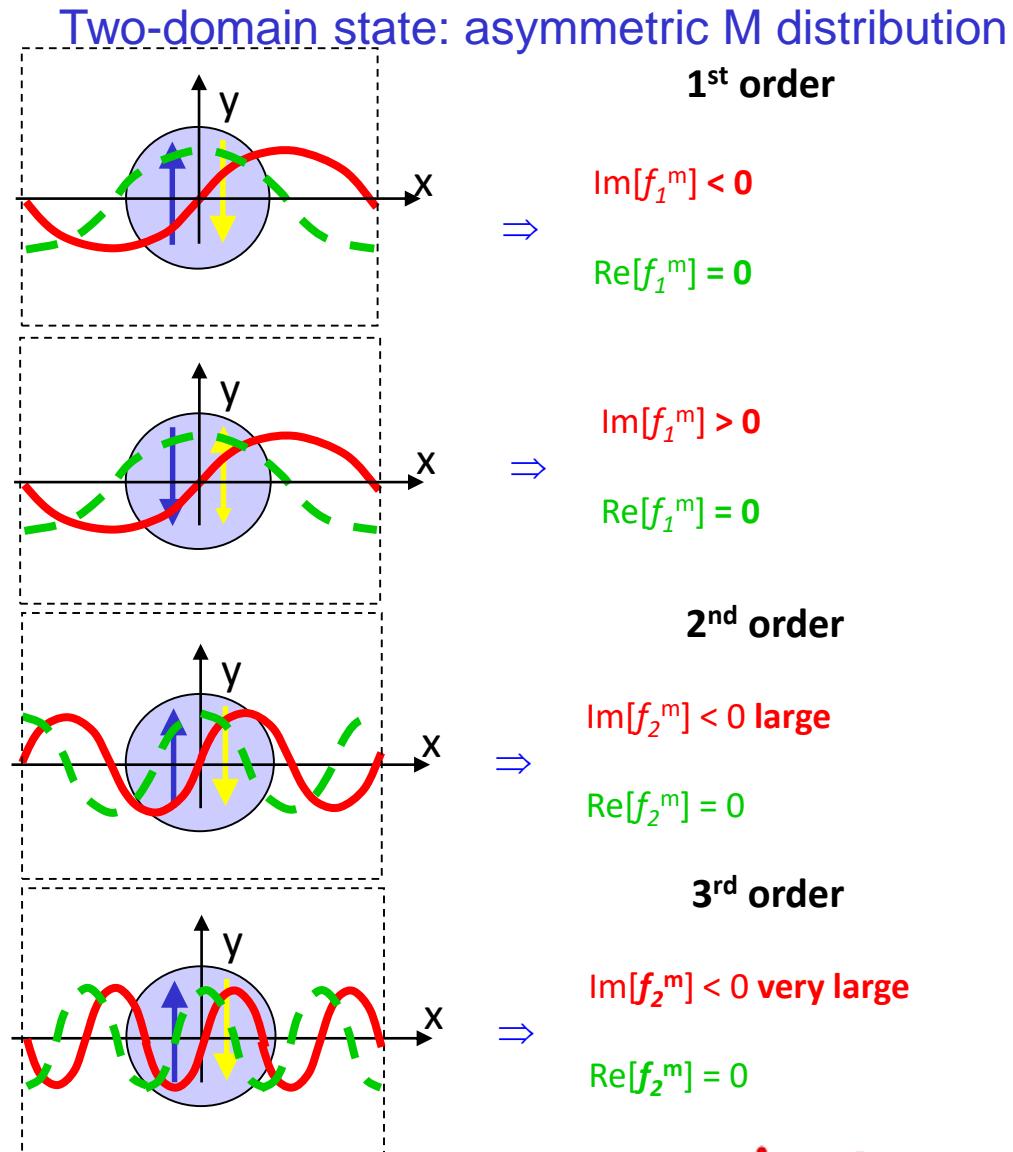
$$\operatorname{Im}[f_n^m] = \int_{\text{Dot}} m_y \sin(n \mathbf{G}_x \cdot \mathbf{x}) dS$$



Saturated state \Rightarrow

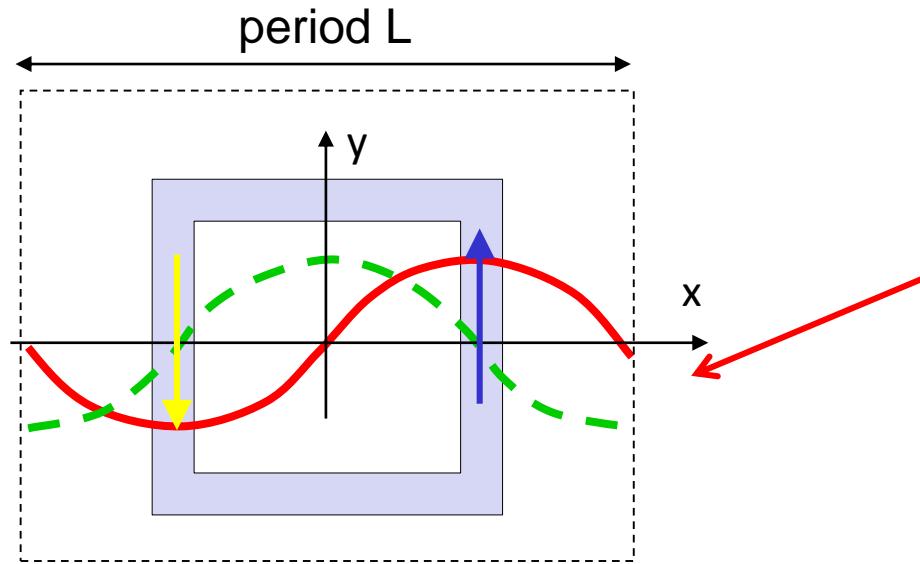
$$\operatorname{Im}[f_1^m] = 0$$

$$\operatorname{Re}[f_1^m] > 0$$

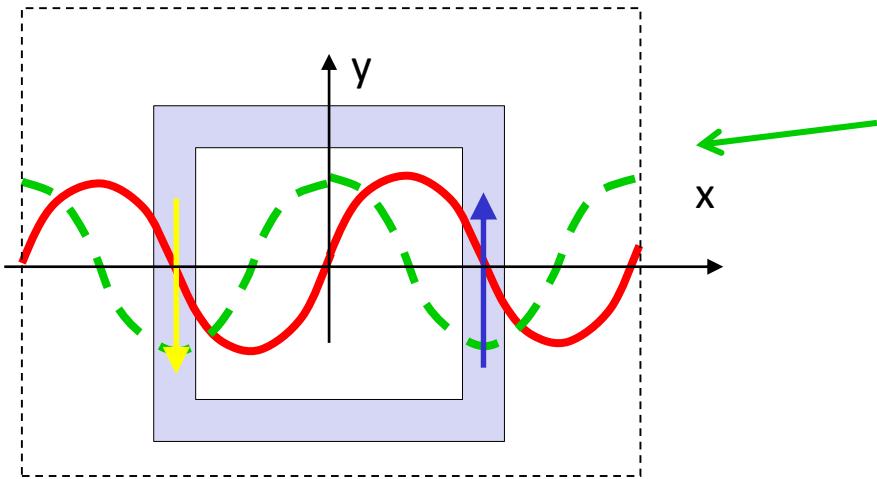




Tuning the sensitivity to selected portions of the dot!

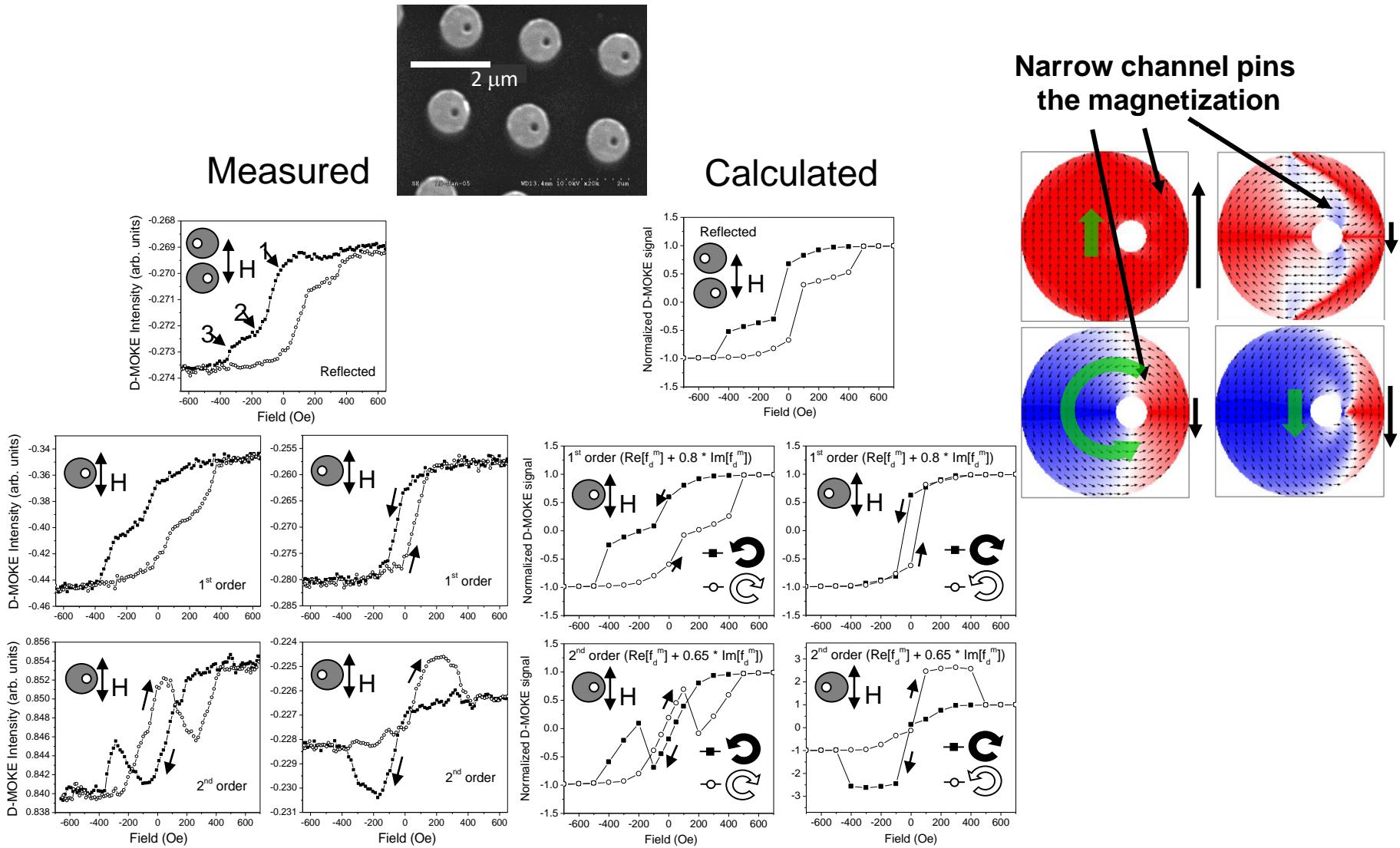


Immaginary contribution:
highlight any asymmetric
(y-mirror symmetry breaking)
magnetic (m_y) behaviour



Real contribution:
y-mirror symmetry m_y

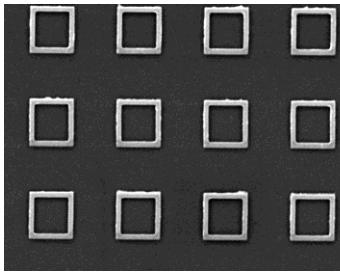
Asymmetry to induce the desired vortex rotation



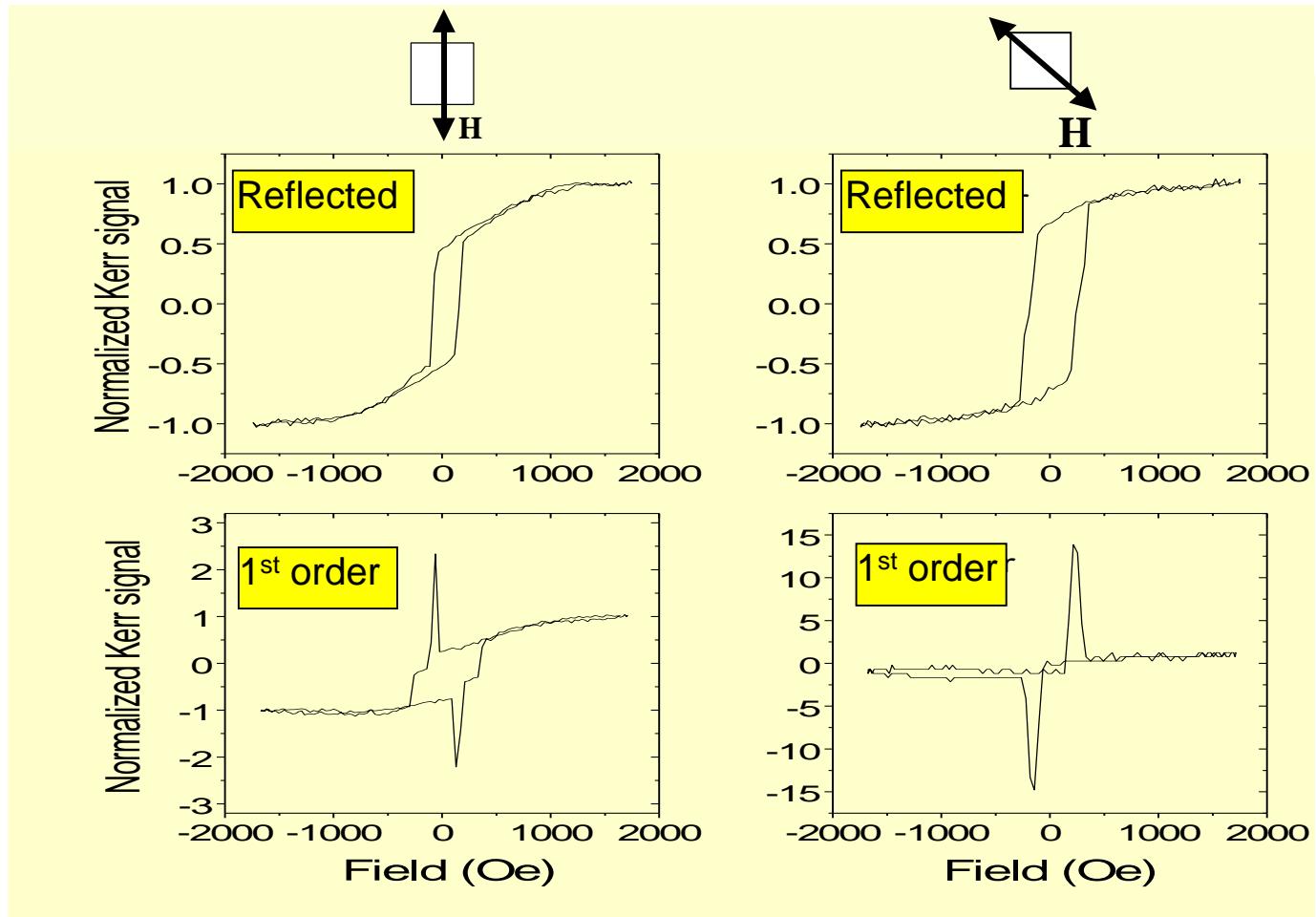
P. Vavassori, R. Bovolenta, V. Metlushko, and B. Ilic, J Appl. Phys. **99**, 053902 (2006)



Measured D-MOKE loops from square rings



Square lattice ($4.1 \times 4.1 \mu\text{m}^2$) of Permalloy square rings ($2.1 \mu\text{m}$ side). Nominal width 250 nm. Thickness 30 nm.

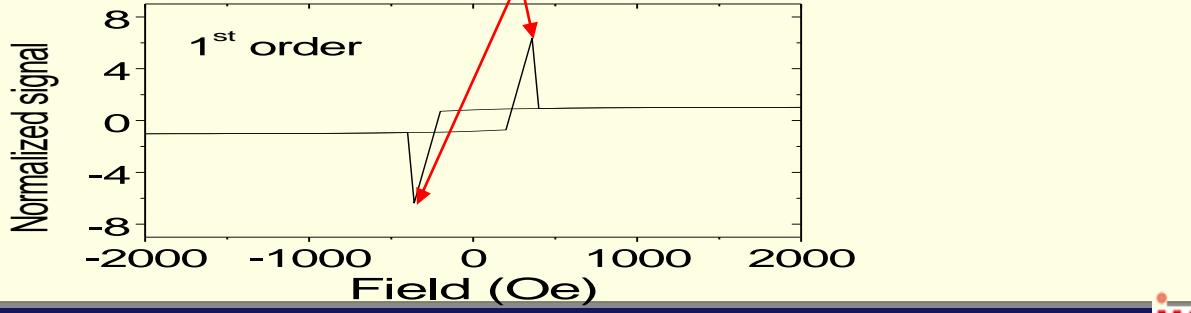
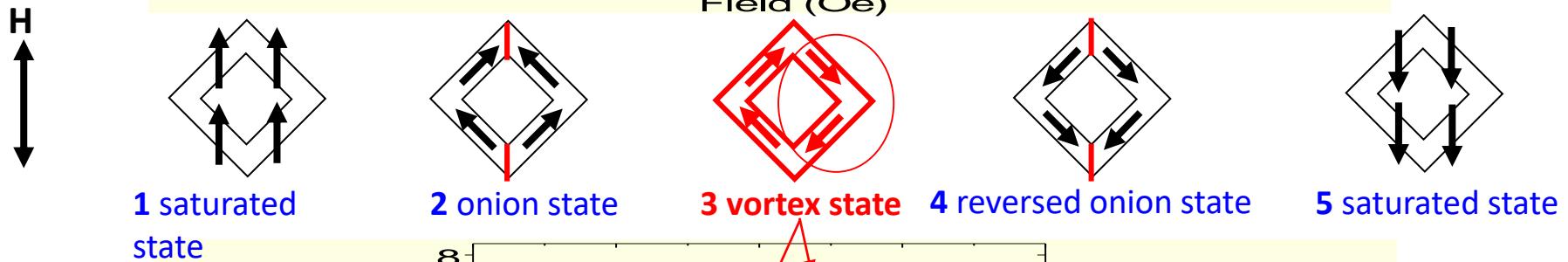
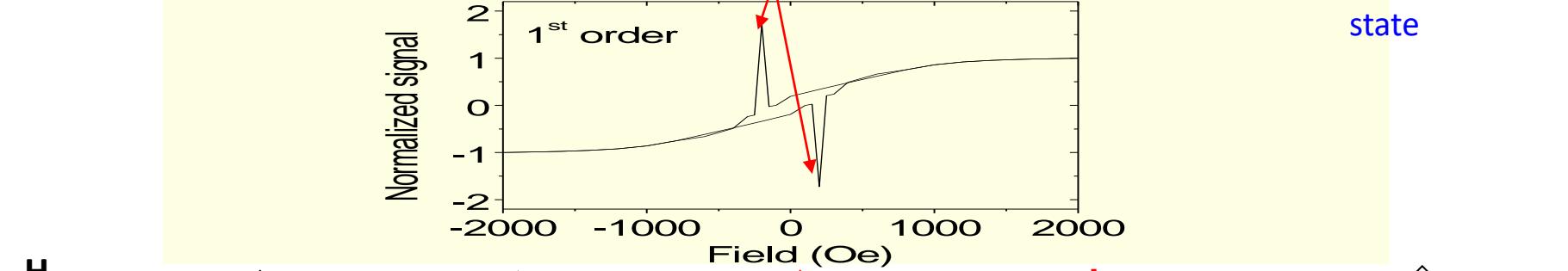


P. Vavassori, et al., Phys. Rev. B **67**, 134429 (2003)

Note intense peaks in the diffracted loops

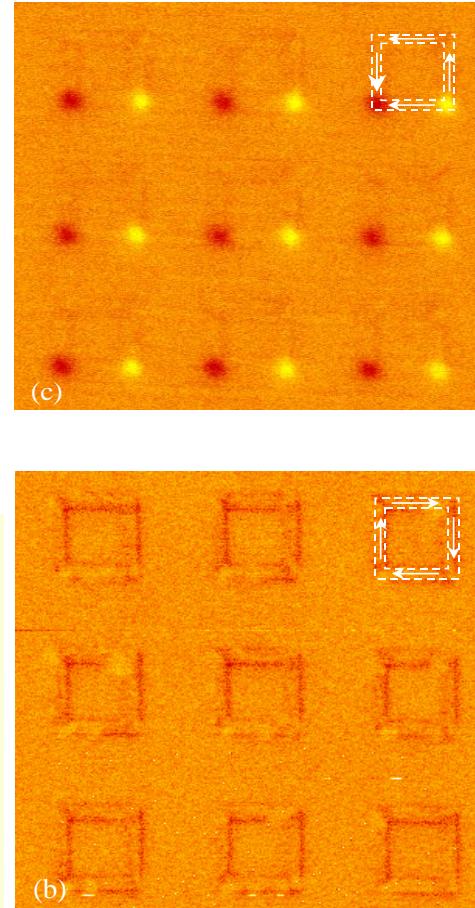
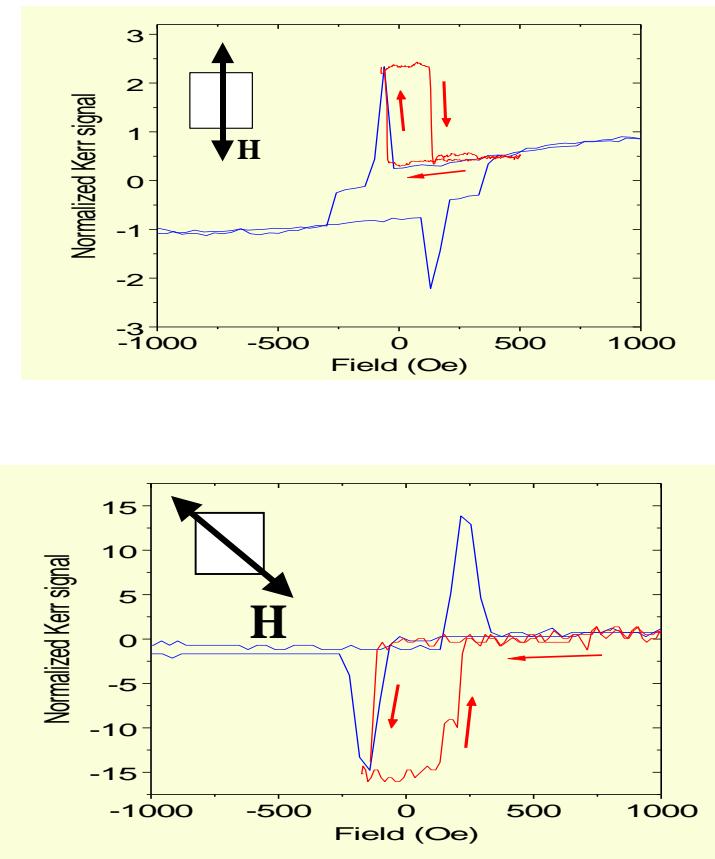


Square ring structures





Quenching structures in intermediate states and image them with MFM



P. Vavassori, M. Grimsditch, V. Novosad, V. Metlushko, B. Ilic, P. Neuzil, and R. Kumar, Phys. Rev. B **67**, 134429 (2003)



About A_n and B_n

$$\Delta I_n^m (m_y) = 2 f_n^{nm} \{ A_n \operatorname{Re}[f_n^m] + B_n \operatorname{Im}[f_n^m] \}$$

A_n and B_n (θ_i , θ_n , $\varepsilon_{\text{dots}}$, ε_{sub} , Q)

$$A_n = \operatorname{Re}[r_{pp}^o * r_{pp}^m]$$

$$B_n = \operatorname{Im}[r_{pp}^o * r_{pp}^m]$$

$$r'_{pp} = r_{pp}^o + r_{pp}^m \quad \text{"effective" reflectivity}$$

Y. Suzuki, C. Chappert, P. Bruno, and P. Veillet, *J. Magn. Magn. Mater.* **165** 516 (1997) only for size $\gg \lambda$

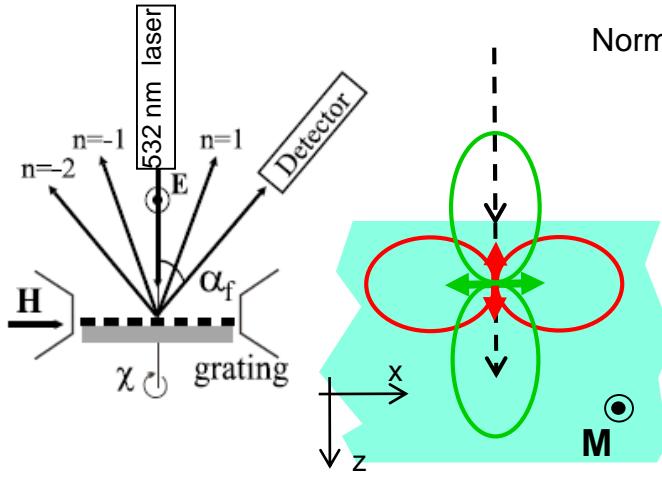
For inhomogeneous gratings $r_{pp}^o = r_{pp, \text{dot}}^o + r_{pp, \text{sub}}^o$

An interesting characteristic of D-MOKE related to this : the absolute value of $(\Delta I/I_o)_n$ is increased up to several times the specular value. Effect due to the compensation of the non-magnetic component of the light diffracted by the magnetic dots and the light diffracted by the (non-magnetic) complementary part of the substrate.

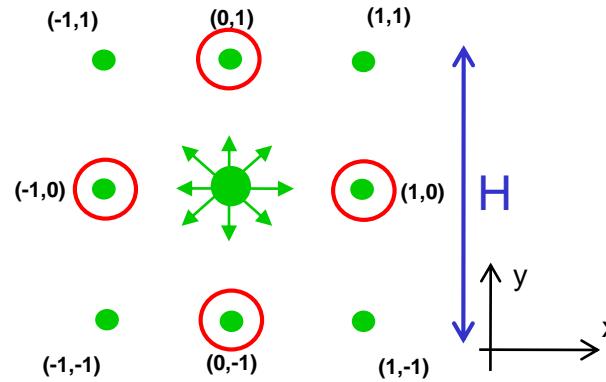
$$I_{o,n} = |r_{pp, \text{dot}}^o|^2 f_n^{nm} + |r_{pp, \text{sub}}^o|^2 f_n^{nm} = (|r_{pp, \text{dot}}^o|^2 - |r_{pp, \text{sub}}^o|^2) f_n^{nm}$$



Different approach: towards Fourier imaging? 1st step



Normal incidence: the scattering plane is defined by the selected spot

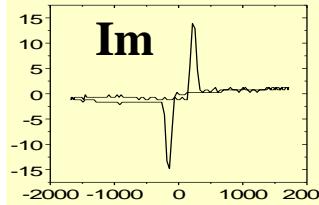
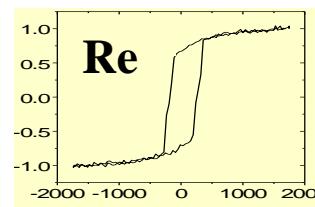
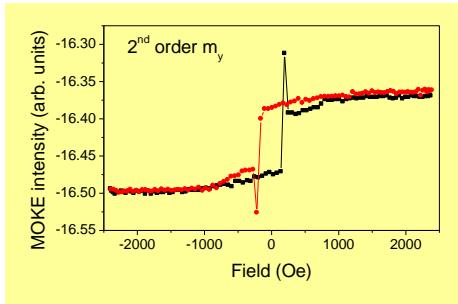


Sensitivity to (m_x, m_y)

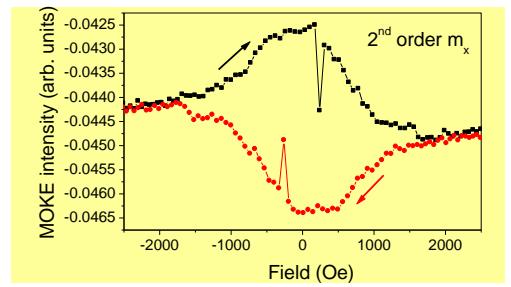
Sensitivity to m_y

$$(\Delta I_n^m)_{\text{norm}} = A_n \operatorname{Re}[f_n^m] - B_n \operatorname{Im}[f_n^m]$$

$$(\Delta I_{-n}^m)_{\text{norm}} = -A_n \operatorname{Re}[f_n^m] - B_n \operatorname{Im}[f_n^m]$$



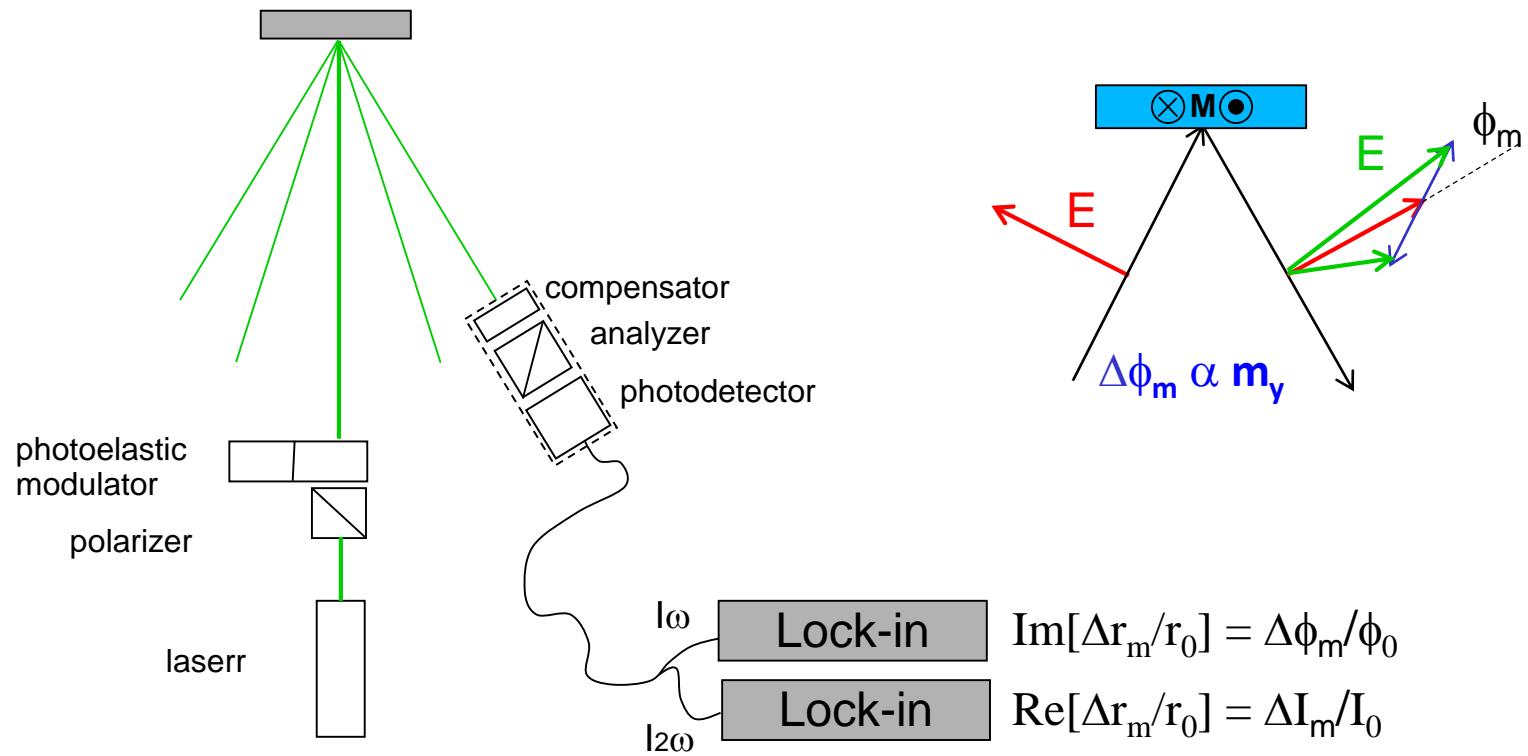
Sensitivity to m_x



Vectorial D-MOKE



D-MOKE problem fully solved



$$\Delta I_m^n / I_o^n = A_n \text{Re}[f_n^m] - B_n \text{Im}[f_n^m]$$

$$\Delta \phi_m^n / \phi_o^n = B_n \text{Re}[f_n^m] - A_n \text{Im}[f_n^m]$$

$$\Delta I_m^{-n} / I_o^{-n} = -A_n \text{Re}[f_n^m] - B_n \text{Im}[f_n^m]$$

$$\Delta \phi_m^{-n} / \phi_o^{-n} = -B_n \text{Re}[f_n^m] - A_n \text{Im}[f_n^m]$$

$\text{Re}[f_n^m]$
 $\text{Im}[f_n^m]$

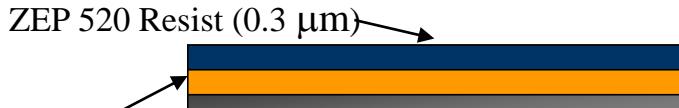
K. Postava et al. "Null ellipsometer with phase modulation," Opt. Express **12**, 6040 (2004)



Samples: arrays of NiFe triangular rings

Electron beam lithography

Double Layer Resist Spin-coating



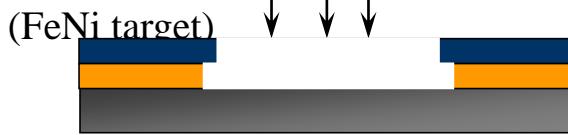
EB Patterning



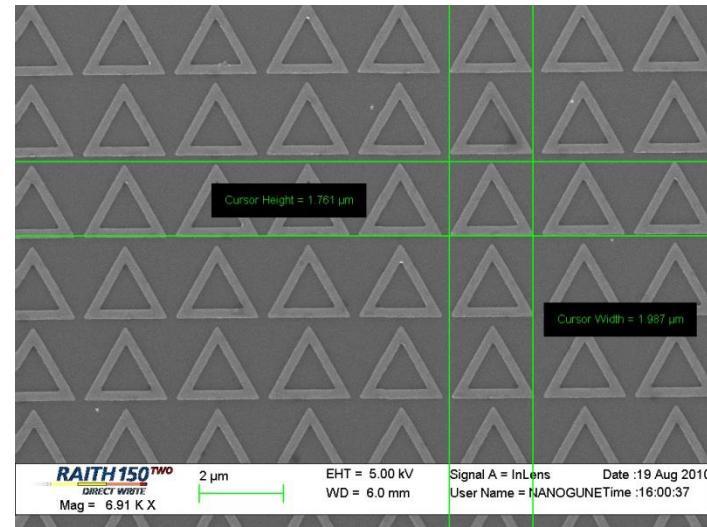
Resist Development



EB Deposition



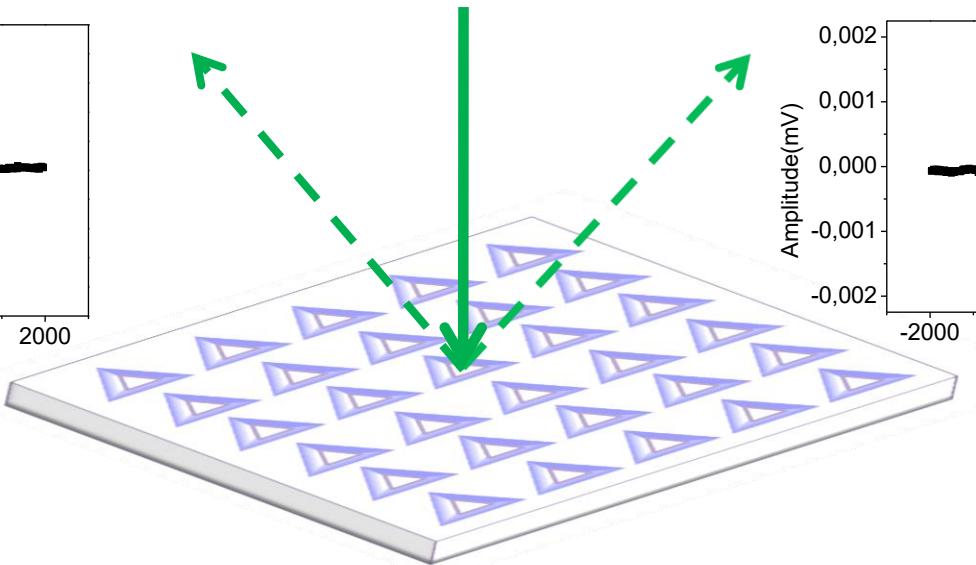
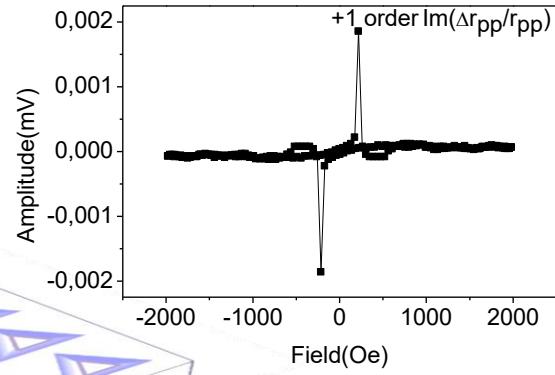
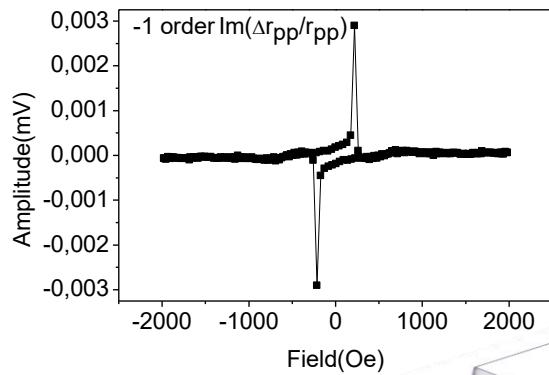
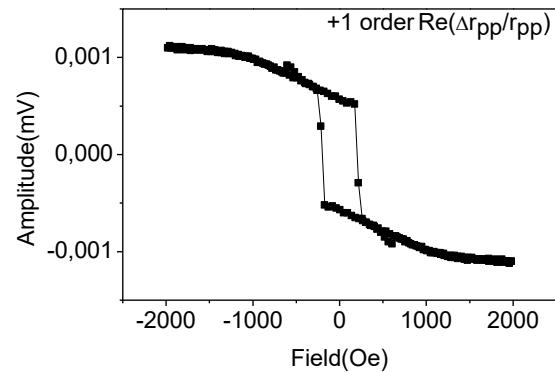
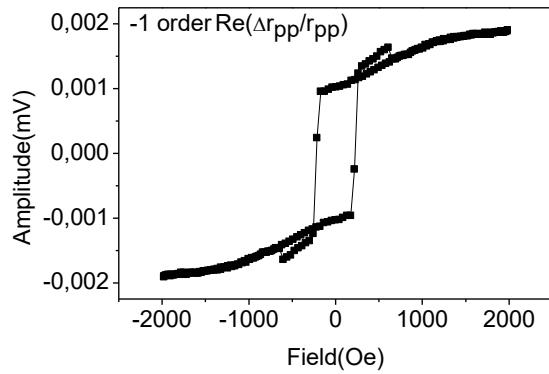
Lift-off process



Triangular rings (2.1 μm side).
Nominal width 250 nm.
Nominal thickness 30 nm.



Normal incidence: extraction of magnetic form factors



$$\Delta I_m^n / I_o^n = A_n \text{Re}[f_n^m] - B_n \text{Im}[f_n^m]$$

$$\Delta \phi_m^n / \phi_o^n = B_n \text{Re}[f_n^m] - A_n \text{Im}[f_n^m]$$

$$\Delta I_m^{-n} / I_o^{-n} = -A_n \text{Re}[f_n^m] - B_n \text{Im}[f_n^m]$$

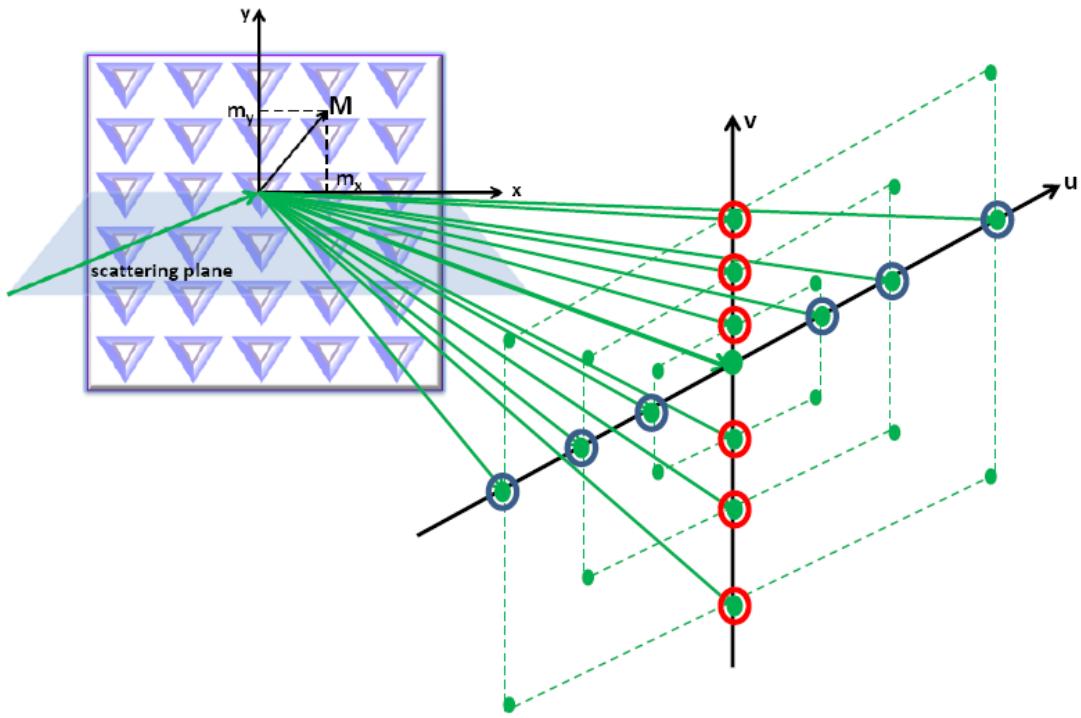
$$\Delta \phi_m^{-n} / \phi_o^{-n} = -B_n \text{Re}[f_n^m] - A_n \text{Im}[f_n^m]$$

$\text{Re}[f_n^m]$

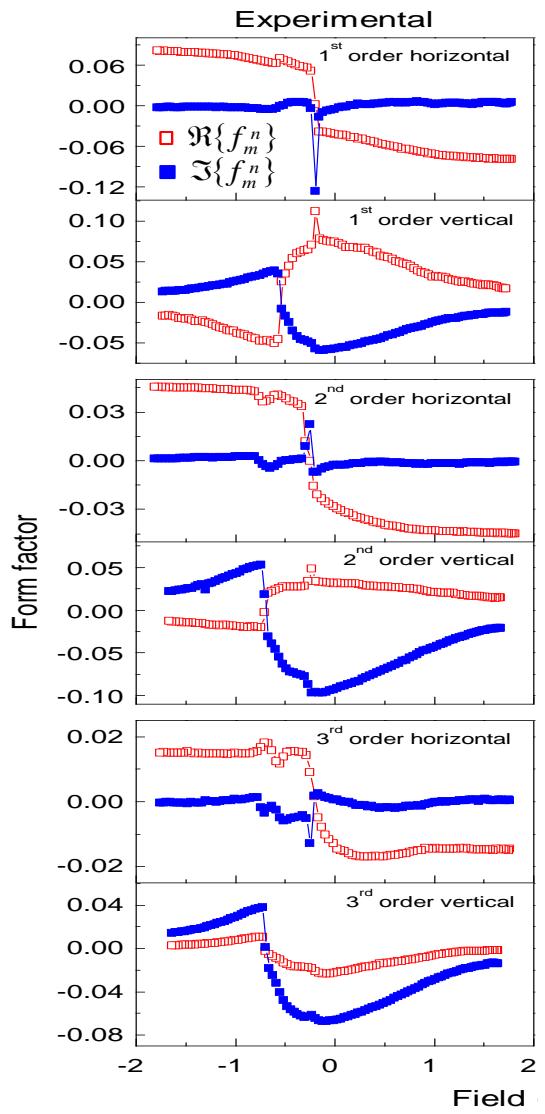
$\text{Im}[f_n^m]$



Exp and calculated $\text{Re}[f_m]$ and $\text{Im}[f_m]$ – horizontal and vertical plane

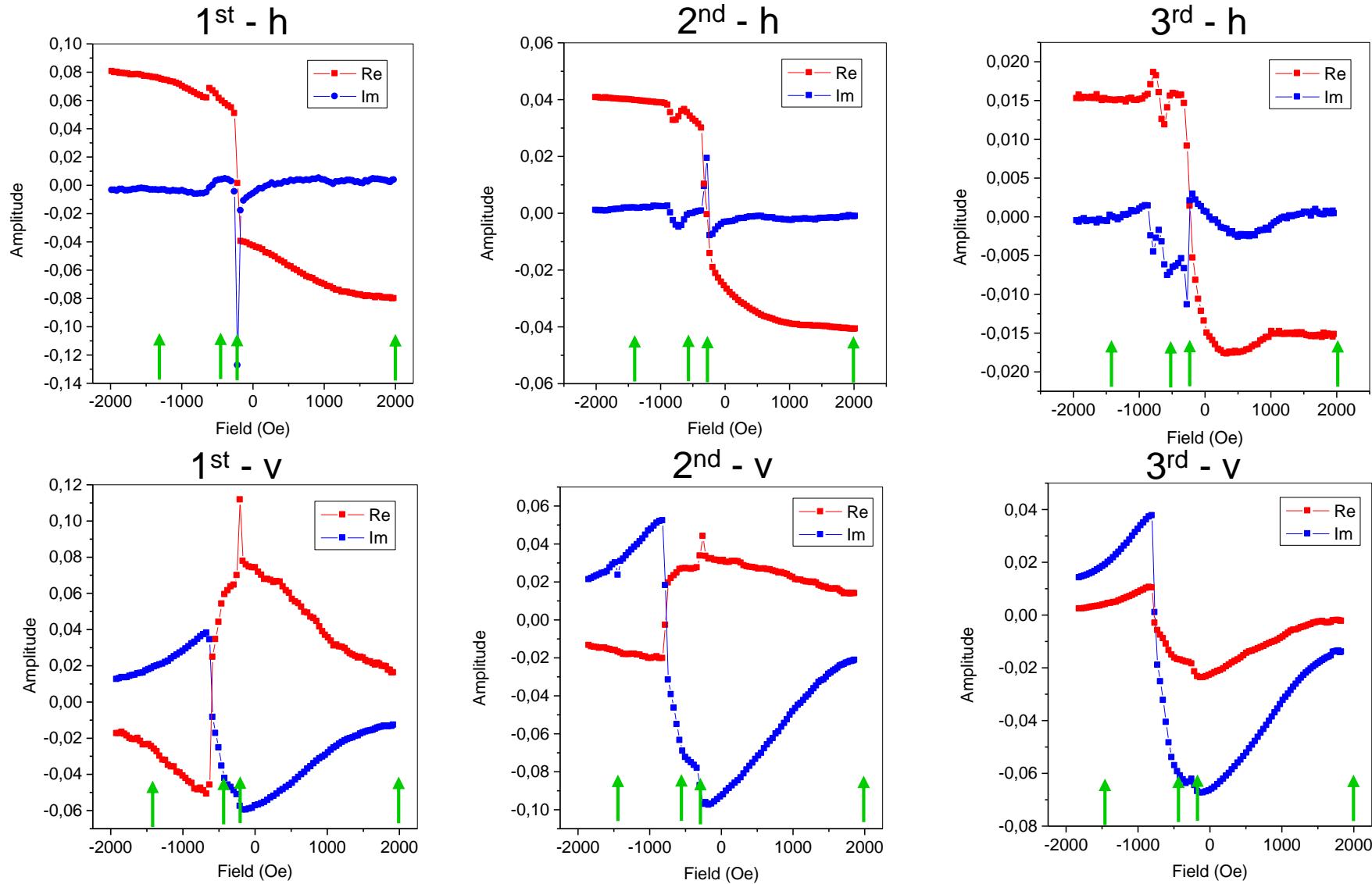


$$m_{\perp}(\vec{r}) = \sum_{n=-3}^3 f_n^m e^{-i(n\vec{G}\cdot\vec{r})} = \sum_{n=-3}^3 \text{Im}[f_n^m] \sin(n\vec{G} \cdot \vec{r}) + \text{Re}[f_n^m] \cos(n\vec{G} \cdot \vec{r})$$



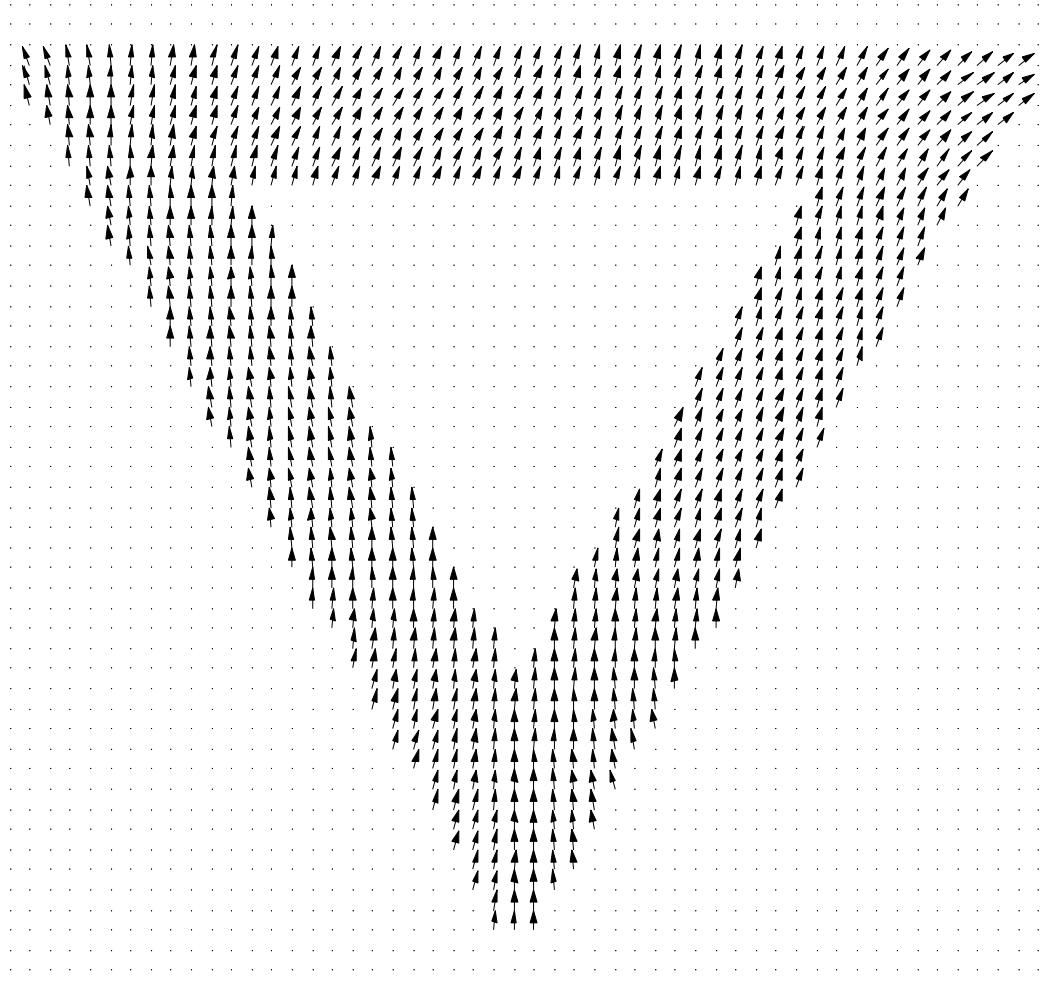


Re and Im parts of the magnetic form factors



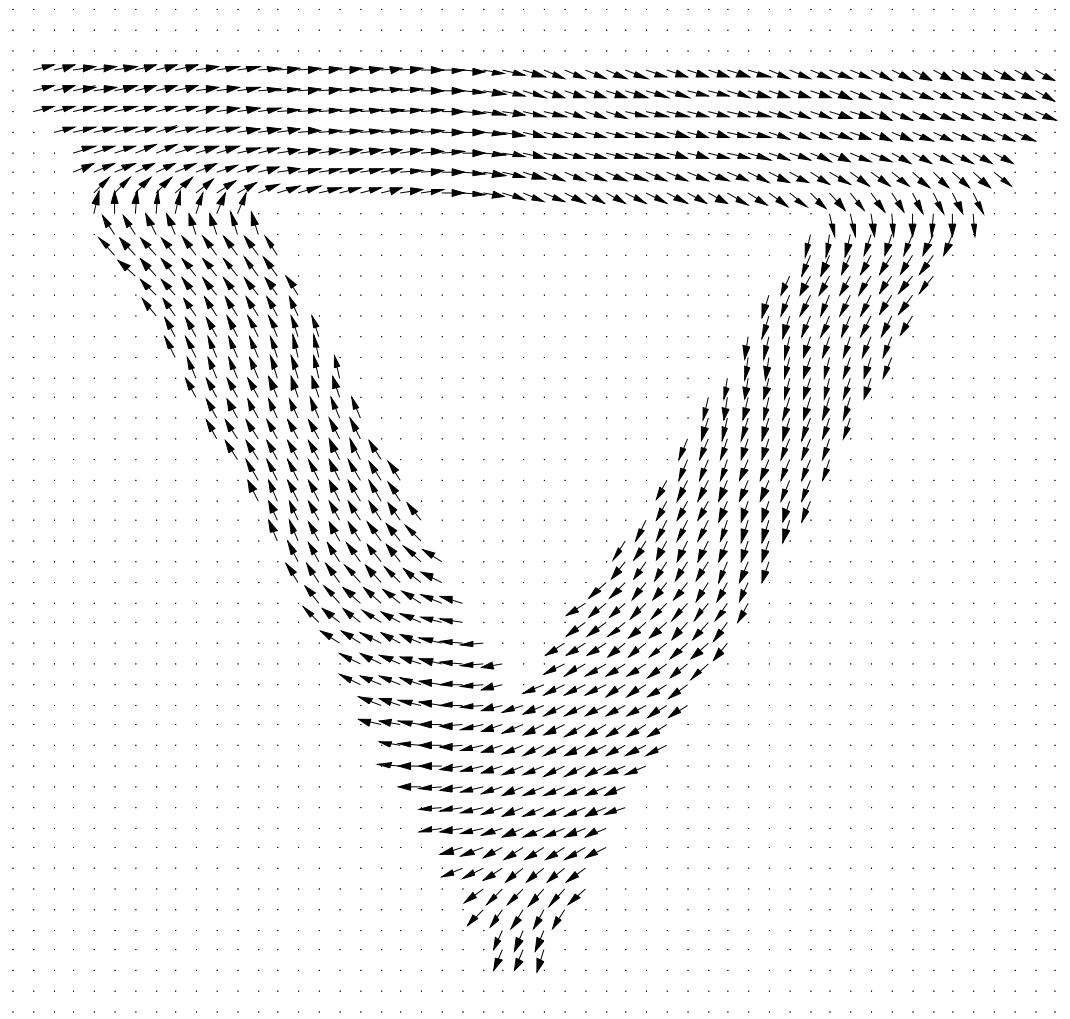


Saturated state



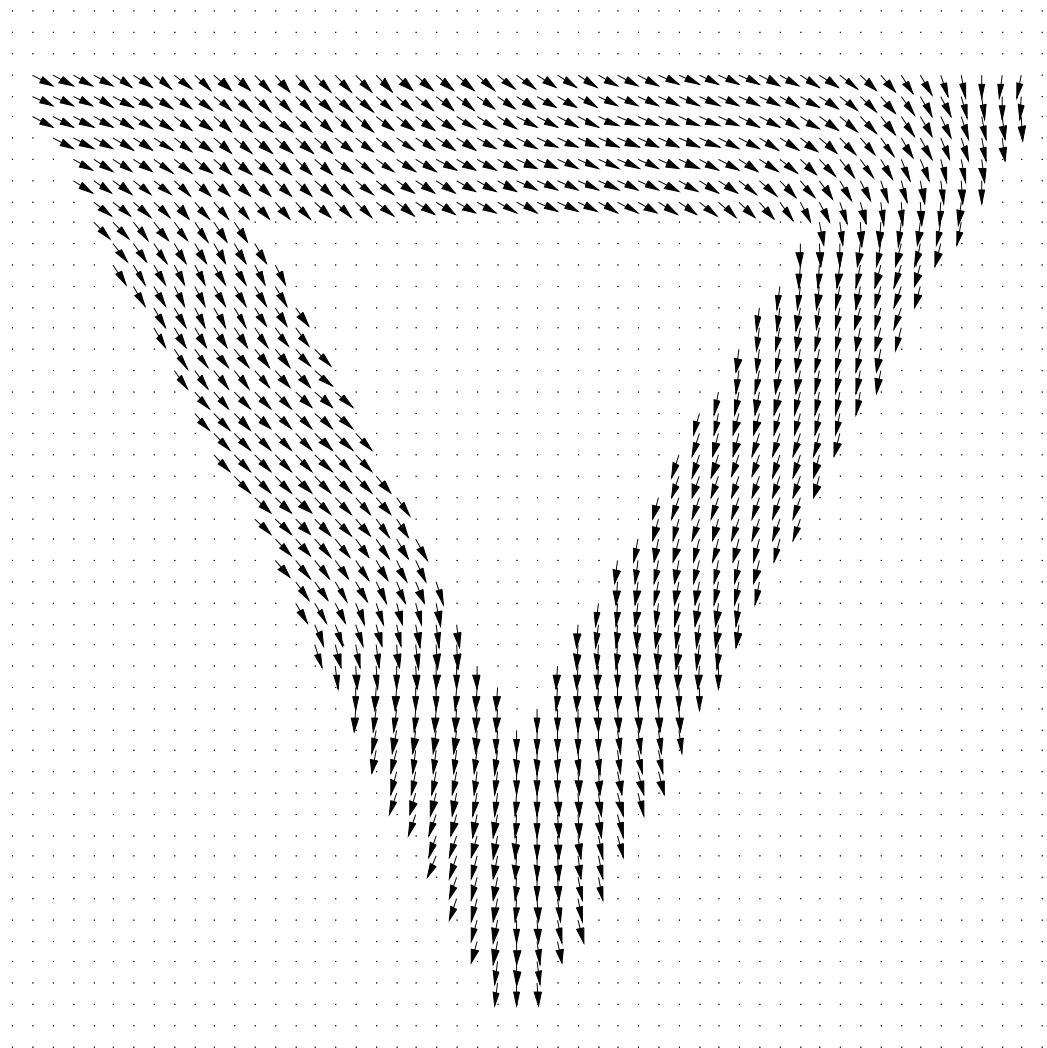


“Peak” state



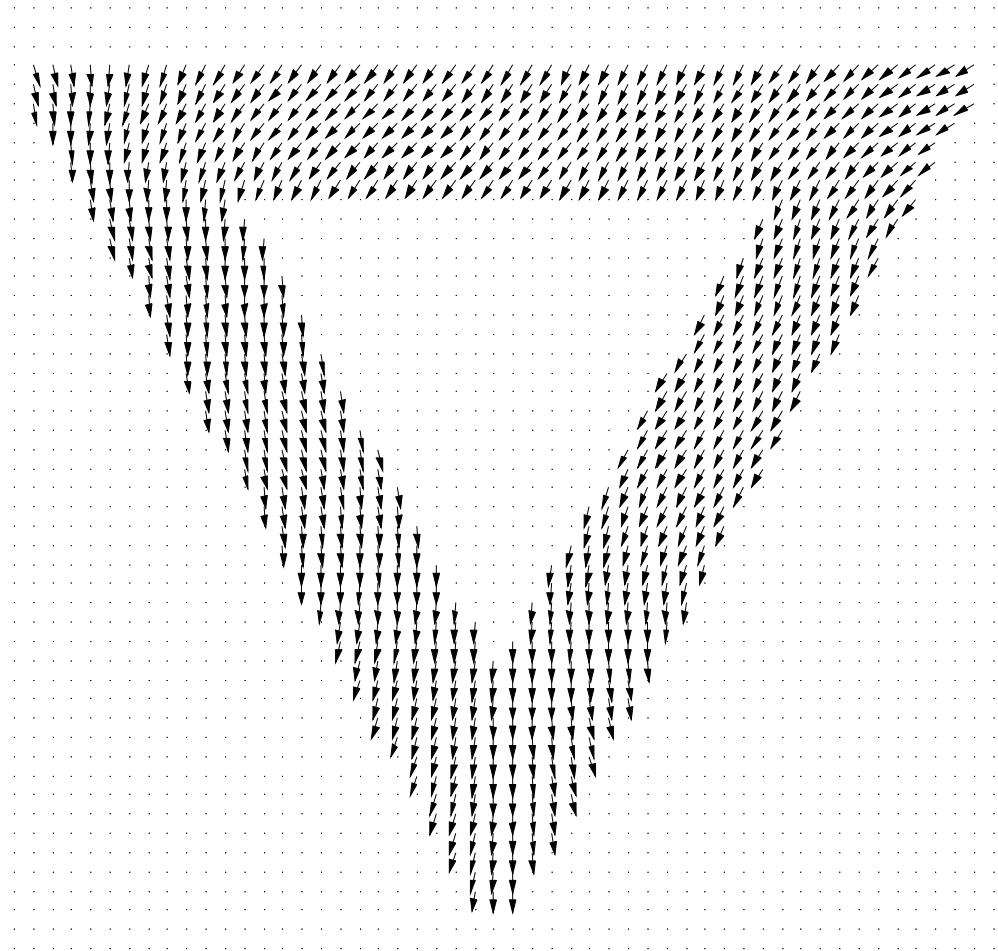


After “peak”



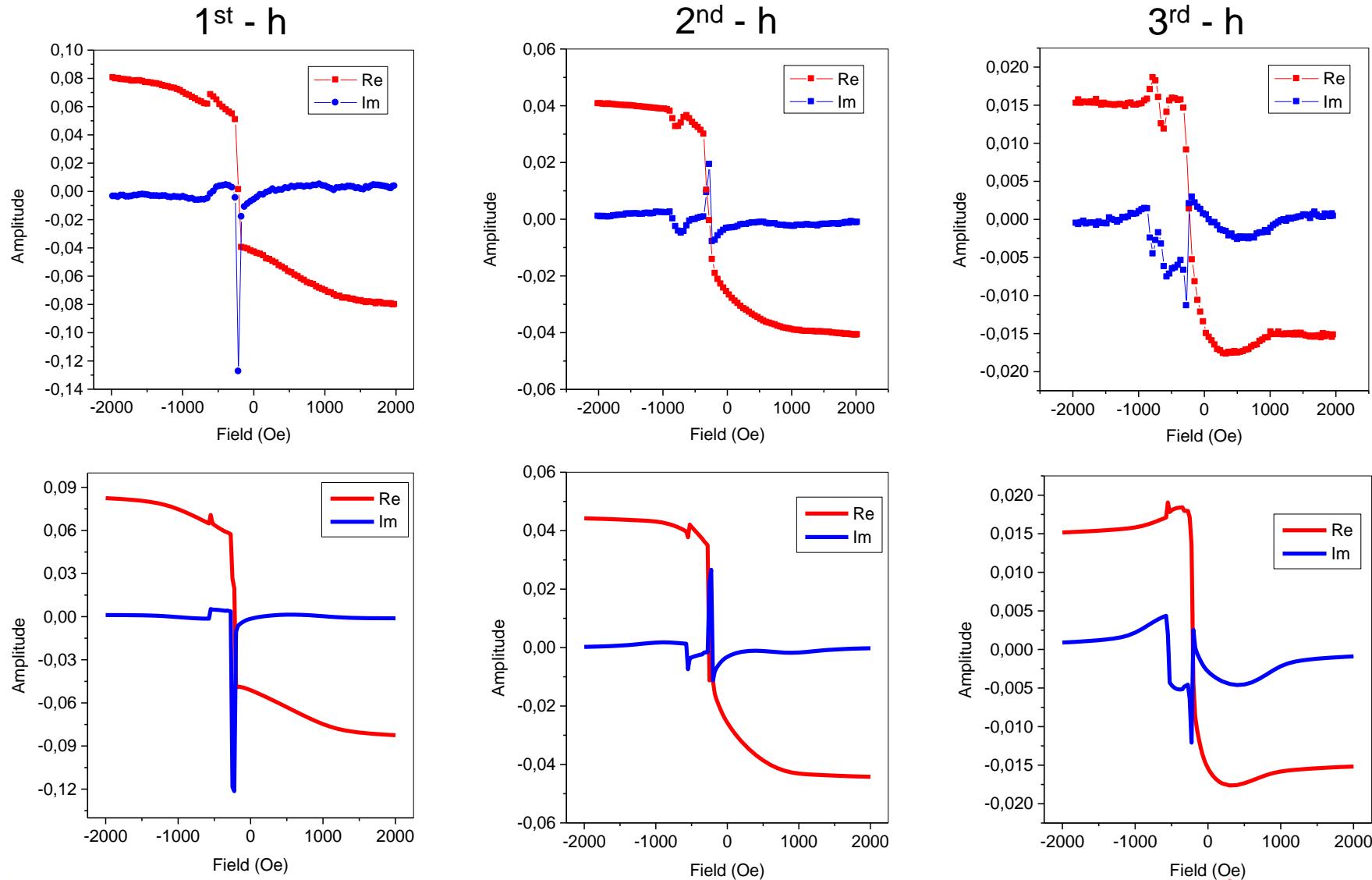


Towards negative saturation



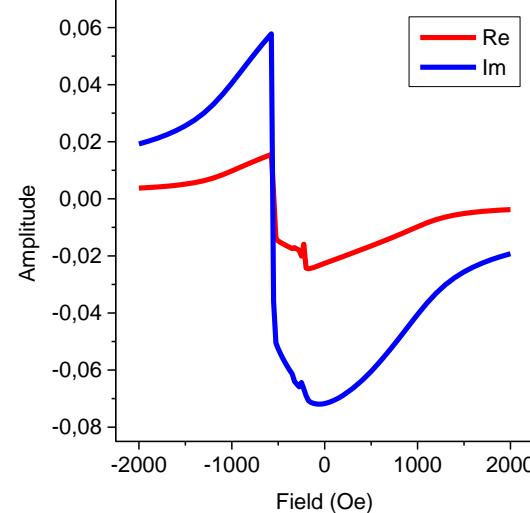
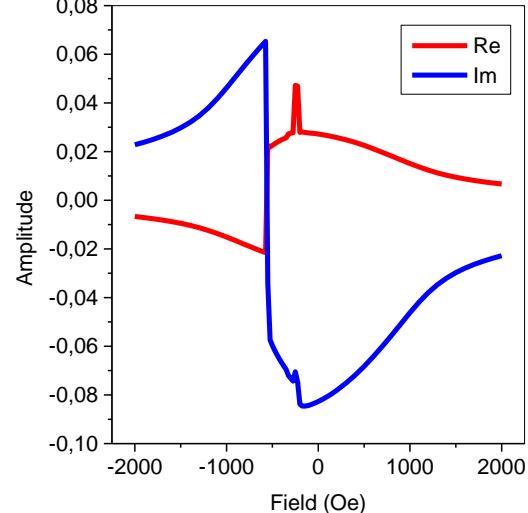
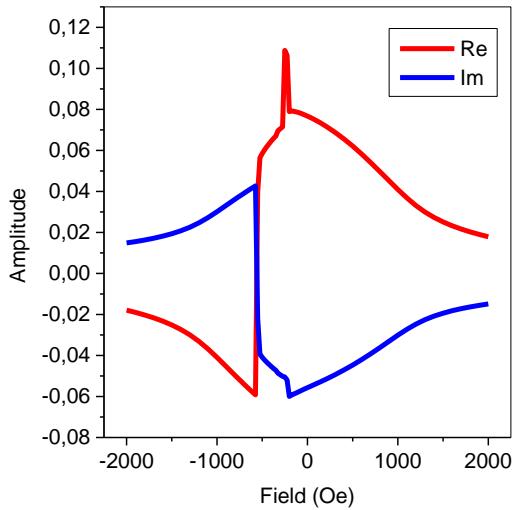
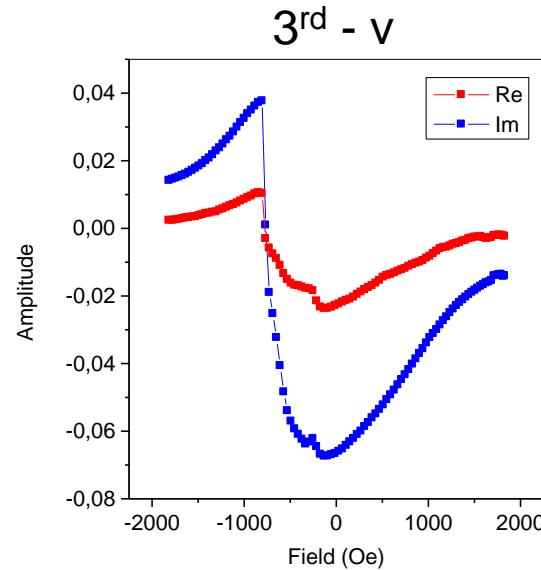
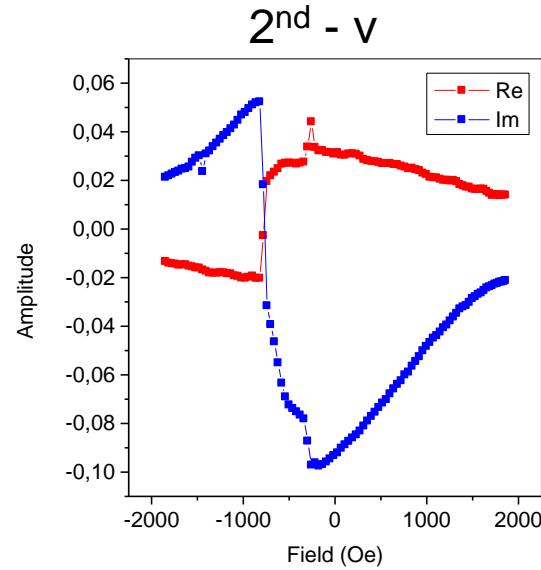
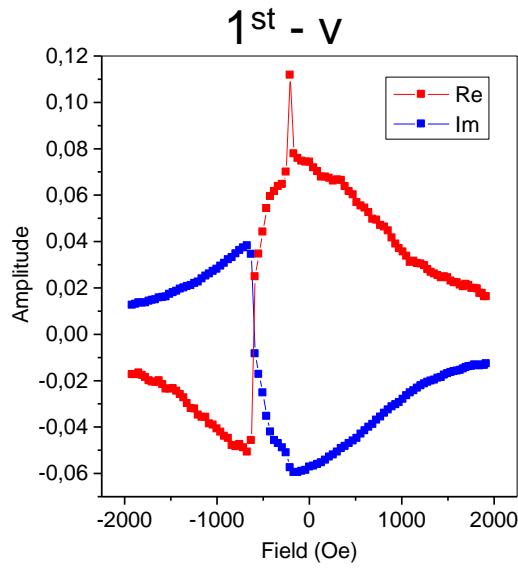


Exp and calculated $\text{Re}[f_m]$ and $\text{Im}[f_m]$ – horizontal plane





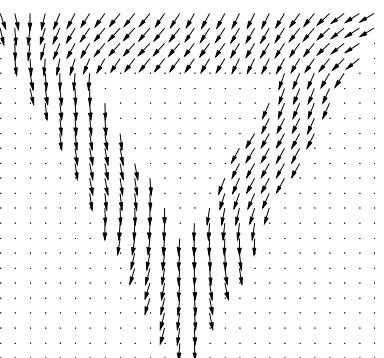
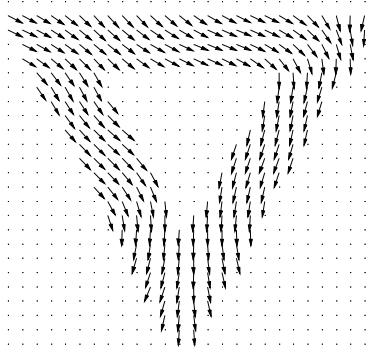
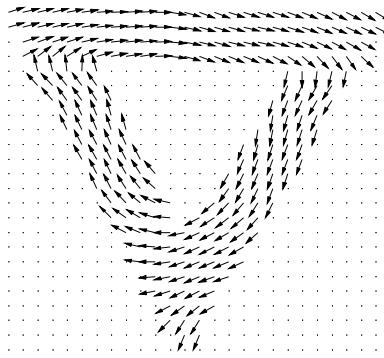
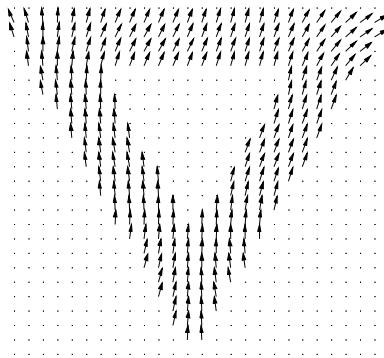
Exp and calculated $\text{Re}[f_m]$ and $\text{Im}[f_m]$ – vertical plane



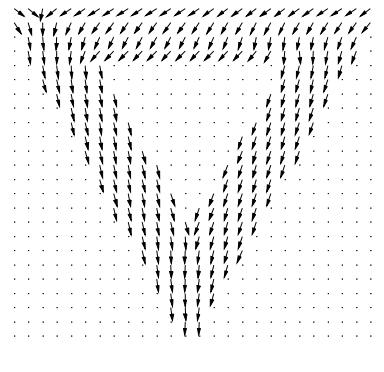
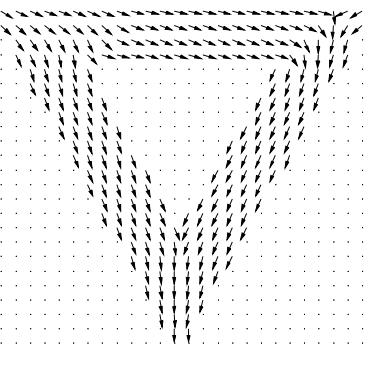
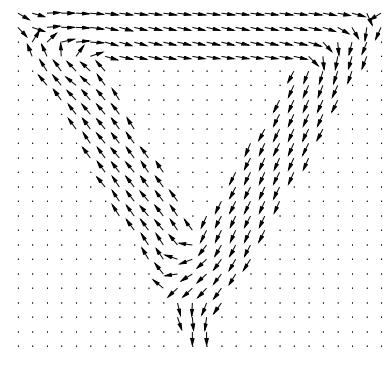
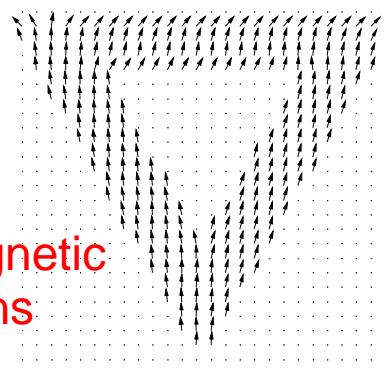


Magnetic imaging proved

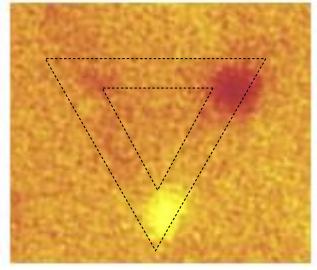
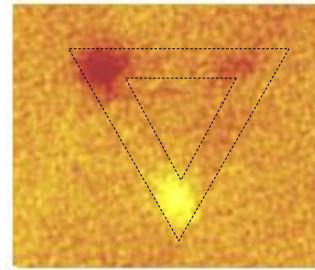
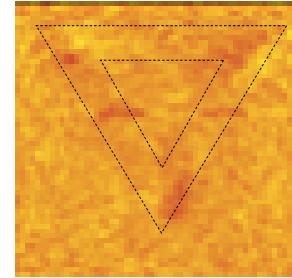
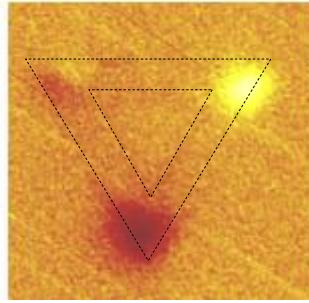
D-MOKE



Micromagnetic simulations



MFM
(quenched
To 0 field)



APPLIED PHYSICS LETTERS 99, 092501 (2011)



Concluding remarks

MOKE is a powerful technique for studying technologically relevant magnetic materials.

MOKE magnetometry based on microscopy provides a noninvasive probe of magnetization reversal for individual ultra-small nano-structures.

D-MOKE is a powerful technique to investigate the collective behavior of magnetic nano-arrays.

Next lecture:

Interplay between plasma excitations and MO-activity (magnetoplasmonics).