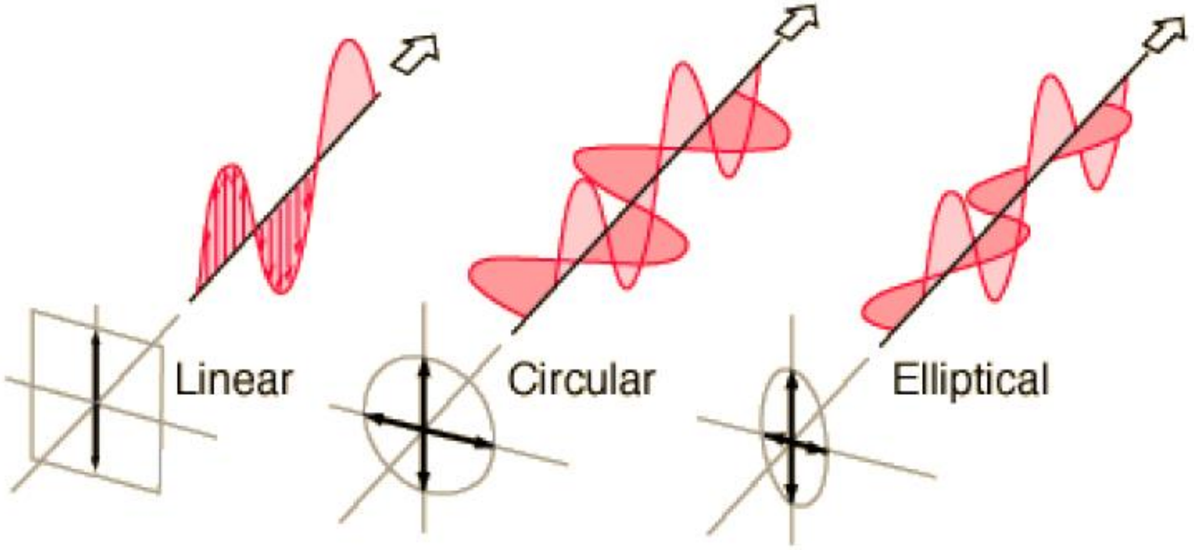
Light polarization



Jones vectors and matrices

Since light is composed of oscillating electric and magnetic fields, Jones reasoned that the most natural way to represent light is in terms of the electric field vector. When written as a column vector, this vector is known as a Jones vector and has the form:

$$\vec{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix}$$

These values can be complex numbers, so both amplitude and phase information is present. Oftentimes, however, it is not necessary to know the exact amplitudes and phases of the vector components. Therefore Jones vectors can be normalized and common phase factors can be neglected.

$$\begin{bmatrix} E_0 e^{i\phi} \\ E_0 e^{i\psi} \end{bmatrix} \rightarrow \begin{bmatrix} e^{i\phi} \\ e^{i\psi} \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ e^{i(\psi - \phi)} \end{bmatrix}$$

TM or p TE or s

Horizontal and vertical linear polarization states (reflection plane xz).

$$\vec{E}_h = \begin{bmatrix} E_x(t) \\ 0 \end{bmatrix} \text{ and } \vec{E}_v = \begin{bmatrix} 0 \\ E_y(t) \end{bmatrix} \Rightarrow \vec{E}_h = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ and } \vec{E}_v = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

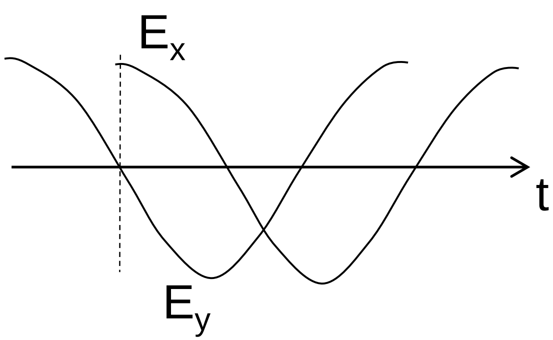
Normalized representation

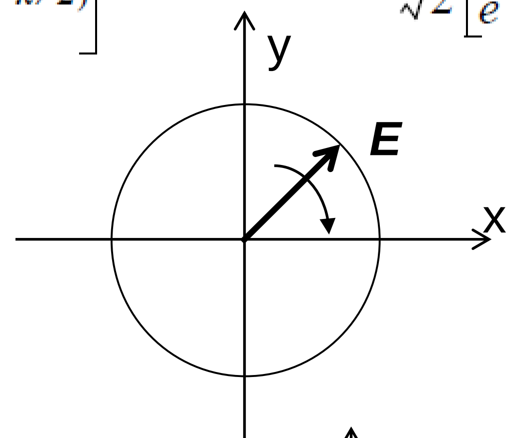
$$\vec{E}_{45^\circ} = \begin{bmatrix} E_x(t) \\ E_x(t) \end{bmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Linearly polarized light at 45°

Right-circular polarized light

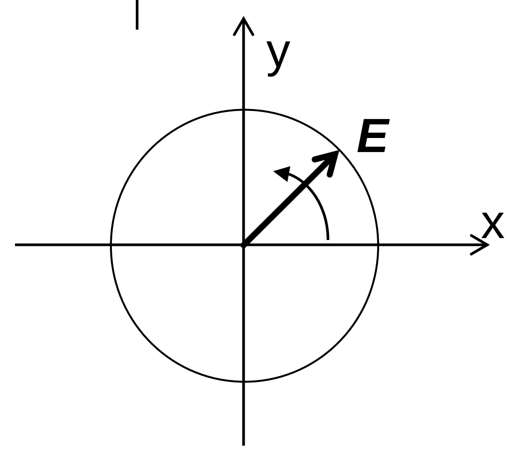
$$\vec{E}_R = \begin{bmatrix} E_0 e^{i\phi} \\ E_0 e^{i(\phi - \pi/2)} \end{bmatrix} \Rightarrow \vec{E}_R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ e^{-i\pi/2} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$$





Left-circular polarized light

$$\vec{E}_L = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}.$$



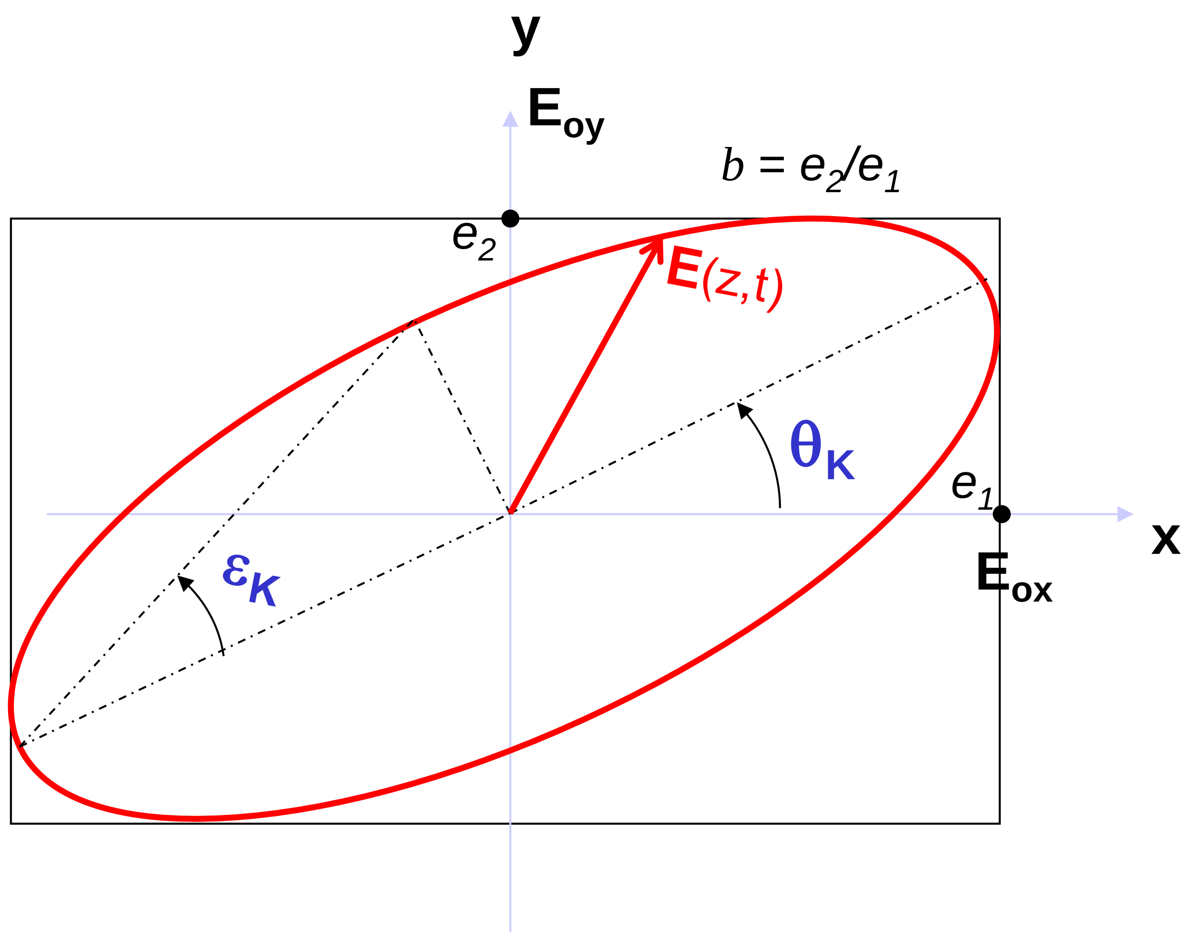
Normalized representation

Elliptically polarized light

$$\tilde{E}_o = \begin{bmatrix} E_{ox} \\ E_{oy} e^{i\delta} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{E_{oy}}{E_{ox}} e^{i\delta} \end{bmatrix} = \begin{bmatrix} 1 \\ b e^{i\delta} \end{bmatrix}$$

$$\tan 2\theta_K = \frac{2E_{ox}E_{oy}\cos\delta}{E_{ox}^2 - E_{oy}^2} = \frac{2b\cos\delta}{1-b^2}$$

$$\sin 2\varepsilon_K = \frac{2E_{ox}E_{oy}\sin\delta}{E_{ox}^2 + E_{oy}^2} = \frac{2b\sin\delta}{1+b^2}$$



To model the effect of a medium on light's polarization state, we use Jones matrices.

Since we can write a polarization state as a (Jones) vector, we use matrices, $\mathbf{A}$, to transform them from the input polarization, E_0, to the output polarization, E_1.

$$\underset{\sim}{E}_1 = \underset{\sim}{\mathbf{A}}\underset{\sim}{E}_0$$

This yields:

$$\underset{\sim}{E}_{1x} = \underset{\sim}{a}_{11}\underset{\sim}{E}_{0x} + \underset{\sim}{a}_{12}\underset{\sim}{E}_{0y}$$

$$\underset{\sim}{E}_{1y} = \underset{\sim}{a}_{21}\underset{\sim}{E}_{0x} + \underset{\sim}{a}_{22}\underset{\sim}{E}_{0y}$$

$$\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

For example, an x-polarizer can be written:

$$\mathbf{A}_x = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

So:

$$\underset{\sim}{E}_1 = \mathbf{A}_x\underset{\sim}{E}_0 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \underset{\sim}{E}_{0x} \\ \underset{\sim}{E}_{0y} \end{bmatrix} = \begin{bmatrix} \underset{\sim}{E}_{0x} \\ 0 \end{bmatrix}$$

Other Jones matrices

A y-polarizer:

$$\mathbf{A}_y = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

A half-wave plate:

$$\mathbf{A}_{HWP} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

A half-wave plate rotates 45-degree-polarization to -45-degree, and vice versa.

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

A quarter-wave plate:

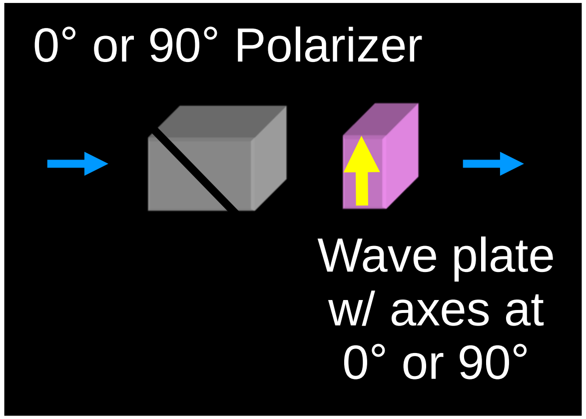
$$\underset{\sim}{\mathbf{A}}_{QWP} = \begin{bmatrix} 1 & 0 \\ 0 & \pm i \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & \pm i \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \pm i \end{bmatrix}$$

A wave plate is not a wave plate if it's oriented wrong.

Remember that a wave plate wants ±45° (or circular) polarization.

If it sees, say, x polarization, nothing happens.



$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

A_{HWP}

So use Jones matrices until you're really on top of this!!!

Summary

TABLE 1. Jones Matrices of Common Optical Devices

Vertical Linear Polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	Right Circular Polarizer	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Horizontal Linear Polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$	Left Circular Polar-izer	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$
Linear Polarizer at 45°	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	Quarter-wave plate, fast axis vertical	$\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$ Retardation $\lambda/4$
Lossless fiber trans-mission	$\begin{bmatrix} e^{i\phi}\cos\theta & -e^{-i\psi}\sin\theta \\ e^{i\psi}\sin\theta & e^{-i\phi}\cos\theta \end{bmatrix}$	Quarter-wave plate, fast axis horizontal	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ Retardation $\lambda/4$

Half-wave plate, fast axis horizontal $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ Retardation $\lambda/2$

Half-wave plate, fast axis vertical $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ Retardation $\lambda/2$

General retarder, fast axis horizontal $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$

In terms of waves (wavelength λ), this is a retarder $\lambda^*\varphi/2\pi$

Rotated Jones matrices

Okay, so $E_1 = \mathrm{A}\, E_0$. What about when the polarizer or wave plate responsible for A is rotated by some angle, θ?

Rotation of a vector by an angle θ means multiplication by a rotation matrix:

$$E_0' = R(\theta) E_0 \quad \text{and} \quad E_1' = R(\theta) E_1$$

where:

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Rotating E_1 by θ and inserting the identity matrix $R(\theta)^{-1} R(\theta)$, we have:

$$E_1' = R(\theta) E_1 = R(\theta) \mathbf{A} E_0 = R(\theta) \mathbf{A} \left[R(\theta)^{-1} R(\theta) \right] E_0$$

$$= \left[R(\theta) \mathbf{A} R(\theta)^{-1} \right] \left[R(\theta) E_0 \right] = \left[R(\theta) \mathbf{A} R(\theta)^{-1} \right] E_0' = \mathbf{A}' E_0'$$

Thus:

$$\boxed{\mathbf{A}' = R(\theta) \mathbf{A} R(\theta)^{-1}}$$

Rotated Jones matrix for a polarizer

Applying this result to an x-polarizer: $\mathbf{A}' = R(\theta)\,\mathbf{A}R(\theta)^{-1}$

$$A_x(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$A_x(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ 0 & 0 \end{bmatrix}$$

$$A_x(\theta) = \begin{bmatrix} \cos^2(\theta) & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin^2(\theta) \end{bmatrix}$$

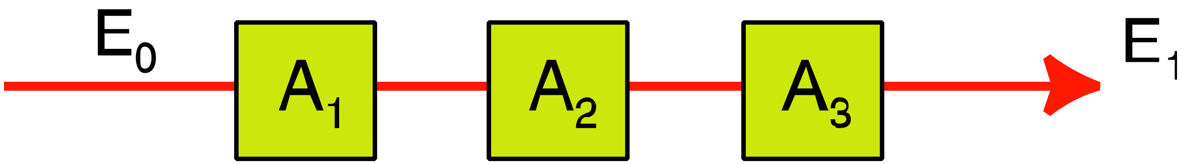
$$A_x(45^\circ) = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \qquad A_x(\varepsilon) \approx \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 0 \end{bmatrix}$$

Jones Matrices for standard components

Component	Symbol	Jones matrix
Horizontal linear polarizer	↔	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
Vertical linear polarizer	↕	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
Linear polarizer at +45°	↗	$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
Linear polarizer at −45°	↖	$\frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
Quarter-wave plate, fast axis vertical		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
Quarter-wave plate, fast axis horizontal		$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$
Homogeneous circular polarizer right	↻	$\frac{1}{2}\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$
Homogeneous circular polarizer left	↺	$\frac{1}{2}\begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$

To model the effect of many media on light's polarization state, we use many Jones matrices.

To model the effects of more than one component on the polarization state, just multiply the input polarization Jones vector by all of the Jones matrices:

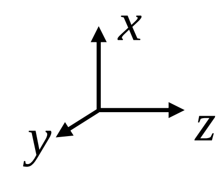


$$E_1 = \mathbf{A}_3 \mathbf{A}_2 \mathbf{A}_1 E_0$$

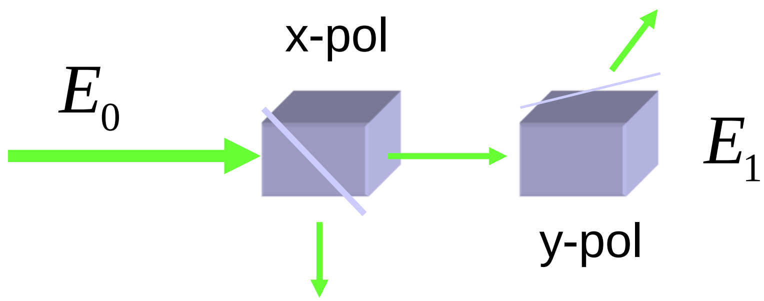
Remember to use the correct order!

A single Jones matrix (the product of the individual Jones matrices) can describe the combination of several components.

Multiplying Jones Matrices



Crossed polarizers:

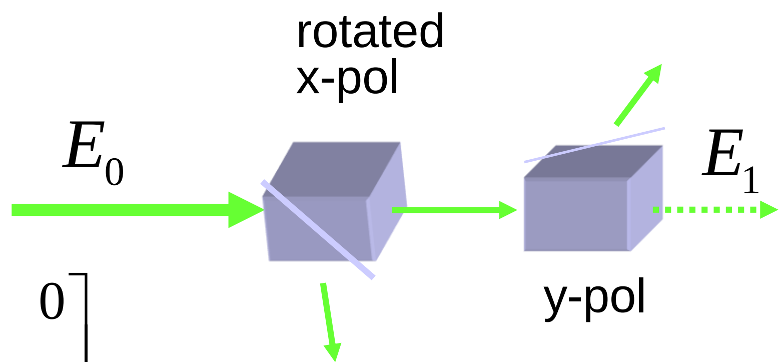


$$\underset{\sim}{E}_1 = \mathbf{A}_y \mathbf{A}_x \underset{\sim}{E}_0$$

$$\mathbf{A_y}\ \mathbf{A_x} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

so no light leaks through.

Uncrossed polarizers (slightly):

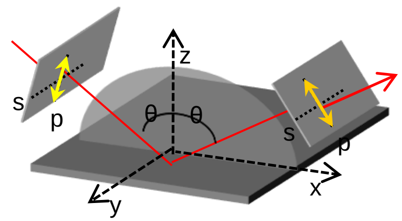


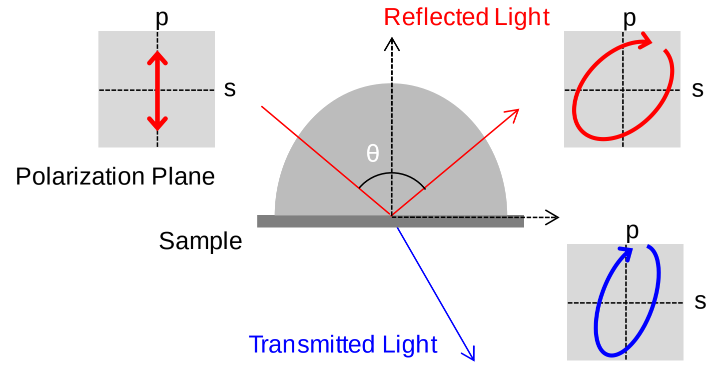
$$\mathbf{A_y}\ \mathbf{A_x}(\varepsilon) = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \varepsilon & 0 \end{bmatrix}$$

$$\mathbf{A_y}\ \mathbf{A_x}(\varepsilon) \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \varepsilon & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 \\ \varepsilon E_x \end{bmatrix}$$

So $I_{out} \approx \varepsilon^2\, I_{in,x}$

The MagnetoOptical Effect





Dielectric tensor

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_0 & 0 & 0 \\ 0 & \varepsilon_0 & 0 \\ 0 & 0 & \varepsilon_0 \end{bmatrix} \quad \xrightarrow{\mathbf{M}} \quad \hat{\varepsilon} = \begin{bmatrix} \varepsilon_0 & i\varepsilon_z & -i\varepsilon_y \\ -i\varepsilon_z & \varepsilon_0 & i\varepsilon_x \\ i\varepsilon_y & -i\varepsilon_x & \varepsilon_0 \end{bmatrix}$$

$\varepsilon_x = \varepsilon_0 Q\, m_x$
$\varepsilon_y = \varepsilon_0 Q\, m_y$
$\varepsilon_z = \varepsilon_0 Q\, m_z$;

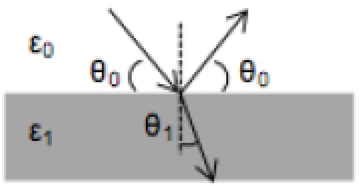
Fresnell reflection coefficients

$$\begin{pmatrix} E_{rp} \\ E_{rs} \end{pmatrix} = \begin{pmatrix} r_{pp} & 0 \\ 0 & r_{ss} \end{pmatrix} \begin{pmatrix} E_{ip} \\ E_{is} \end{pmatrix} \quad \xrightarrow{\mathbf{M}} \quad \begin{pmatrix} E_{rp} \\ E_{rs} \end{pmatrix} = \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \begin{pmatrix} E_{ip} \\ E_{is} \end{pmatrix}$$

$$r_{pp} = \frac{E_{rTM}}{E_{iTM}} \quad r_{ss} = \frac{E_{rTE}}{E_{iTE}} \qquad\qquad r_{ps} = \frac{E_{rTM}}{E_{iTE}} \quad r_{sp} = \frac{E_{rTE}}{E_{iTM}}$$

The MagnetoOptical Effect general case:

Oblique incidence and arbitrary direction of M



$$r_{pp} = \frac{n_1 \cos\theta_0 - n_0 \cos\theta_1}{n_1 \cos\theta_0 + n_0 \cos\theta_1} - \frac{2 n_0 n_1^{-1} \cos\theta_0 \sin\theta_1 \epsilon_{xz}}{(n_1 \cos\theta_0 + n_0 \cos\theta_1)^2}$$

$$\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \Rightarrow$$

$$r_{sp} = \frac{-n_0 n_1^{-1} \cos\theta_0 (\epsilon_{xy} \cos\theta_1 + \epsilon_{yz} \sin\theta_1)}{(n_1 \cos\theta_0 + n_0 \cos\theta_1)(n_0 \cos\theta_0 + n_1 \cos\theta_1) \cos\theta_1}$$

$$r_{ps} = \frac{-n_0 n_1^{-1} \cos\theta_0 (\epsilon_{xy} \cos\theta_1 - \epsilon_{yz} \sin\theta_1)}{(n_1 \cos\theta_0 + n_0 \cos\theta_1)(n_0 \cos\theta_0 + n_1 \cos\theta_1) \cos\theta_1}$$

$$r_{ss} = \frac{n_0 \cos\theta_0 - n_1 \cos\theta_1}{n_0 \cos\theta_0 + n_1 \cos\theta_1}$$

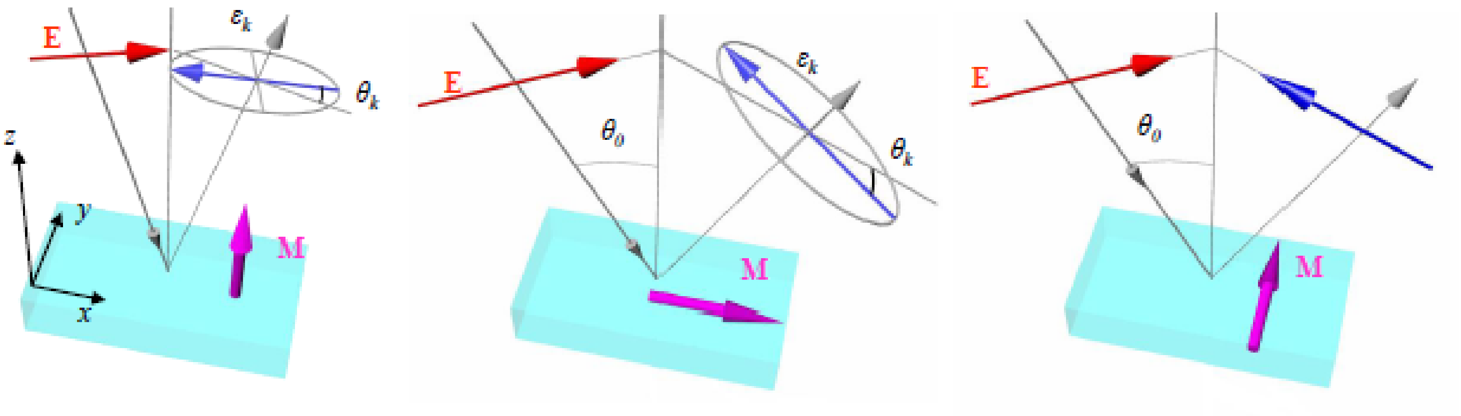
$$n_i = \sqrt{\epsilon_i} \qquad \cos\theta_1 = \sqrt{1 - \sin^2\theta_1} = \sqrt{1 - \frac{n_0^2}{n_1^2}\sin^2\theta} = \frac{\sqrt{n_1^2 - n_0^2 \sin^2\theta}}{n_1}$$

$\varepsilon_{xy} = i\varepsilon_1 Q\, m_z$; $\varepsilon_{xz} = -i\varepsilon_1 Q\, m_y$; $\varepsilon_{yz} = i\varepsilon_1 Q\, m_x$;
$\varepsilon_{xy} = -\varepsilon_{yx}$; $\varepsilon_{zx} = -\varepsilon_{xz}$; $\varepsilon_{zy} = -\varepsilon_{yz}$;

$\Rightarrow$

$r_{pp} = r^0_{pp} + r_{pp}{}^M \propto m_y$
$r_{ps} \propto - m_x - m_z$
$r_{sp} \propto m_x - m_z$

MOKE configurations



Polar | Longitudinal | Transverse

$$\theta + i\varepsilon = f(M_Z)$$

$$\theta + i\varepsilon = f(M_X)$$

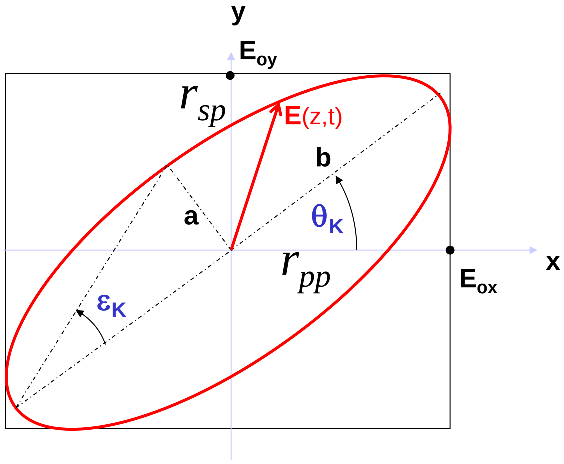
$$R_{pp} = f(M_Y)$$

$$\begin{pmatrix} \varepsilon & aM_z & 0 \\ -aM_z & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{pmatrix} \qquad \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & aM_x \\ 0 & -aM_x & \varepsilon \end{pmatrix} \qquad \begin{pmatrix} \varepsilon & 0 & -aM_y \\ 0 & \varepsilon & 0 \\ aM_y & 0 & \varepsilon \end{pmatrix}$$

Normalized representation

Elliptically polarized light

$$\tilde{E}_r = \begin{bmatrix} r_{pp} \\ r_{sp} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{|r_{sp}|}{|r_{pp}|} e^{i\delta} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{|r_{sp}|}{|r_{pp}|} (\cos\delta + i\sin\delta) \end{bmatrix} = \begin{pmatrix} 1 \\ be^{i\delta} \end{pmatrix}$$



$$\tan 2\theta_K = \frac{2|r_{pp}||r_{sp}|\cos\delta}{|r_{pp}|^2 - |r_{sp}|^2} = \frac{2b\cos\delta}{1-b^2}$$

$$\sin 2\varepsilon_K = \frac{2|r_{pp}||r_{sp}|\sin\delta}{|r_{pp}|^2 + |r_{sp}|^2} = \frac{2b\sin\delta}{1+b^2}$$

$r_{sp} << r_{pp}$

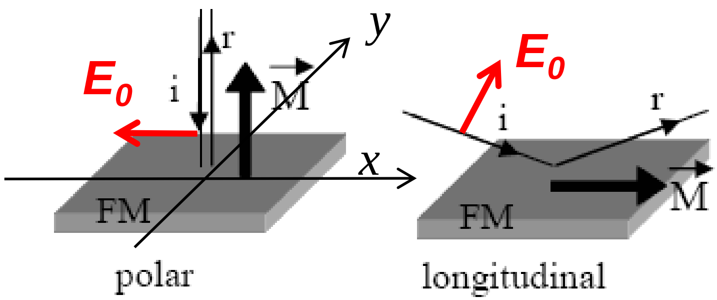
$$\tan 2\theta_K \approx 2\theta_K \approx \frac{2|r_{sp}|\cos\delta}{|r_{pp}|}$$

$$\sin 2\varepsilon_K \approx 2\varepsilon_K \approx \frac{2|r_{sp}|\sin\delta}{|r_{pp}|}$$

$$\mathrm{Re}\left[\frac{r_{sp}}{r_{pp}}\right] = \frac{|r_{sp}|}{|r_{pp}|}\cos\delta \qquad \mathrm{Im}\left[\frac{r_{sp}}{r_{pp}}\right] = \frac{|r_{sp}|}{|r_{pp}|}\sin\delta$$

$$\theta_K \quad \mathrm{Re}\left[\frac{r_{sp}}{r_{pp}}\right] \quad \mathrm{Re}\left[\frac{r_{ps}}{r_{ss}}\right] \qquad \varepsilon_K \quad \mathrm{Im}\left[\frac{r_{sp}}{r_{pp}}\right] \quad \mathrm{Im}\left[\frac{r_{ps}}{r_{ss}}\right]$$

Polar & Longitudinal MOKE

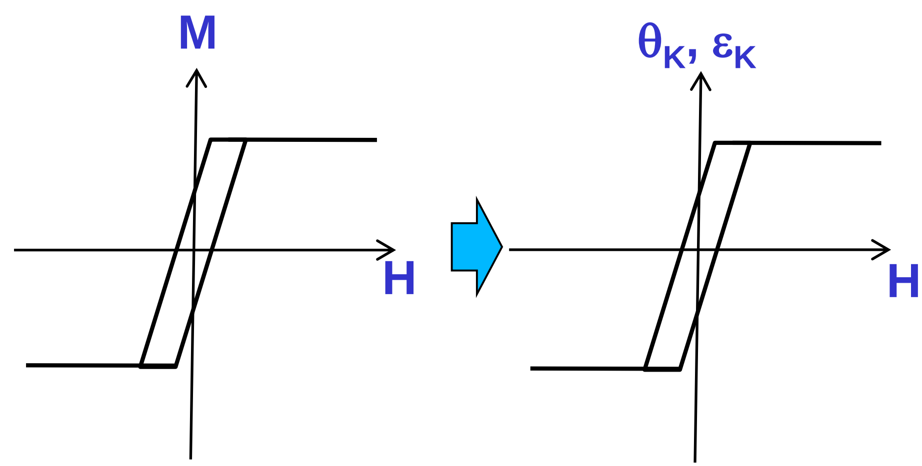


$$\tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ -\varepsilon_{xy} & \varepsilon_{xx} & 0 \\ 0 & 0 & \varepsilon_{xx} \end{pmatrix} \qquad \tilde{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{xx} & \varepsilon_{yz} \\ 0 & -\varepsilon_{yz} & \varepsilon_{xx} \end{pmatrix}$$

$\varepsilon_{xy} = i\varepsilon_1 Q\, m_z;\ \varepsilon_{yz} = i\varepsilon_1 Q\, m_x;$

$$\theta_K \rightarrow \mathrm{Re}\left[\frac{r_{sp}}{r_{pp}}\right] \qquad \mathrm{Re}\left[\frac{r_{ps}}{r_{ss}}\right]$$

$$\varepsilon_K \rightarrow \mathrm{Im}\left[\frac{r_{sp}}{r_{pp}}\right] \qquad \mathrm{Im}\left[\frac{r_{ps}}{r_{ss}}\right]$$



P-MOKE:
eigenmodes are LCP and RCP polarized EMs

Birifringence

θ_K *related to different propagation velocity (rotation)*

ε_K *related to differential absorption (ellipticity)*

Dichroism

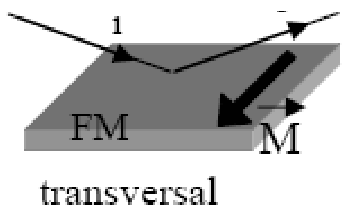
$\theta_K, \varepsilon_K (H) \propto M(H)$

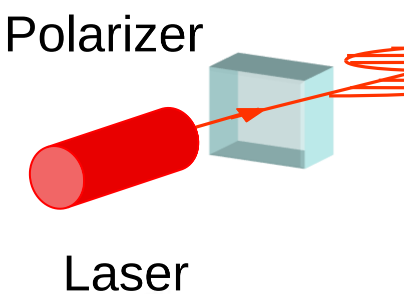
Materials	rotation	Photon energy	temperature	field	literature
	(deg)	(eV)	(K)	(T)	
Fe	0.87	0.75	RT		1.21)
Co	0.85	0.62	〃		1.21)
Ni	0.19	3.1	〃		1.21)
Gd	0.16	4.3	〃		1.22)
Fe_3O_4	0.32	1	〃		1.23)
MnBi	0.7	1.9	〃		1.24)
PtMnSb	2.0	1.75	〃	1.7	1.8)
CoS_2	1.1	0.8	4.2	0.4	1.25)
$CrBr_3$	3.5	2.9	4.2		1.26)
EuO	6	2.1	12		1.27)
$USb_{0.8}Te$	9.0	0.8	10	4.0	1.28)
$CoCr_2S_4$	4.5	0.7	80		1.29)
a-GdCo	0.3	1.9	RT		1.30)
Ce*Sb	90		2		1.31)

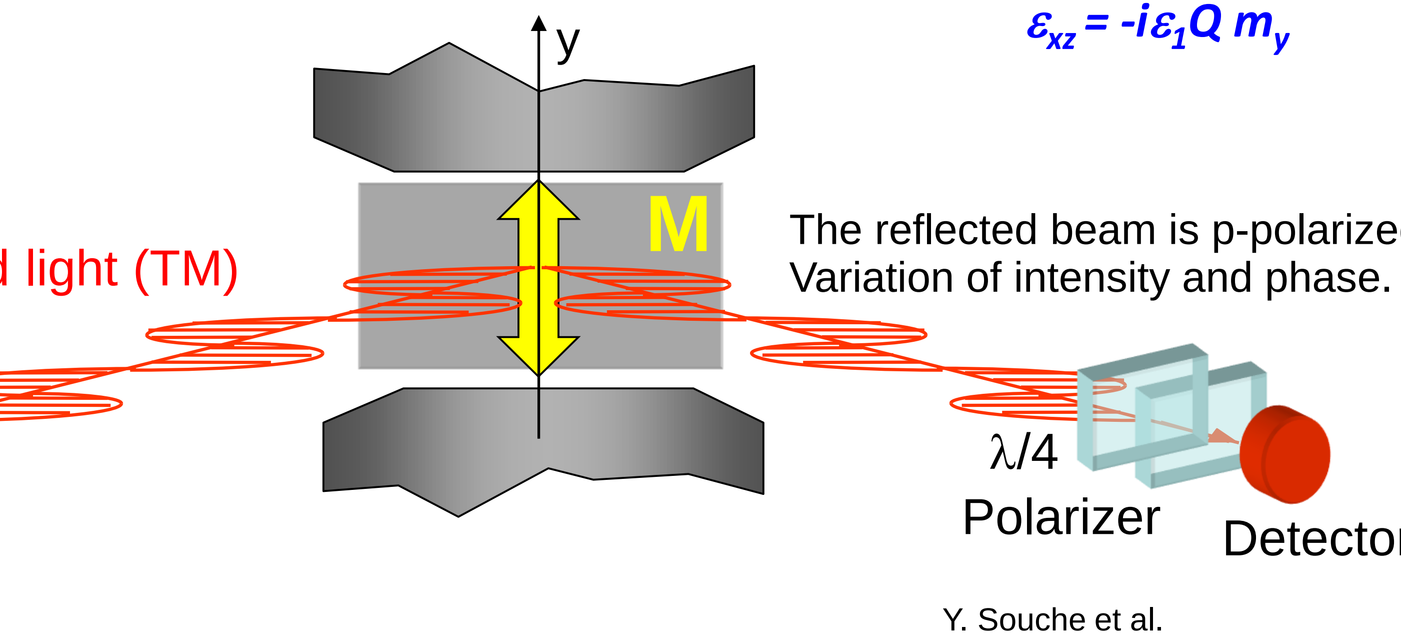
Transverse Kerr effect

$$r_{pp} = \underbrace{\frac{n_1\cos\theta_0 - n_0\cos\theta_1}{n_1\cos\theta_0 + n_0\cos\theta_1}}_{r^o_{pp}} - \underbrace{\frac{2n_0 n_1^{-1}\cos\theta_0\sin\theta_1\epsilon_{xz}}{(n_1\cos\theta_0 + n_0\cos\theta_1)^2}}_{r^m_{pp} m_y}$$

$$\varepsilon_{xz} = -i\varepsilon_1 Q\, m_y$$

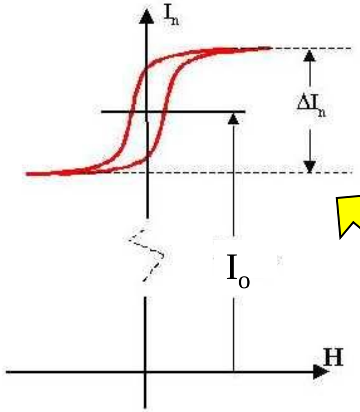






The reflected beam is p-polarized.
Variation of intensity and phase.

Y. Souche et al.
JMMM 226-230, 1686 (2001);
JMMM 242-245, 964 (2002).

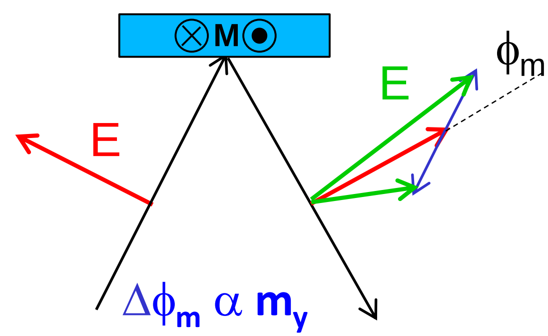


$$E_r = r_{pp} E_o$$

$$r_{pp} = r^o_{pp} + r^m_{pp} m_y$$

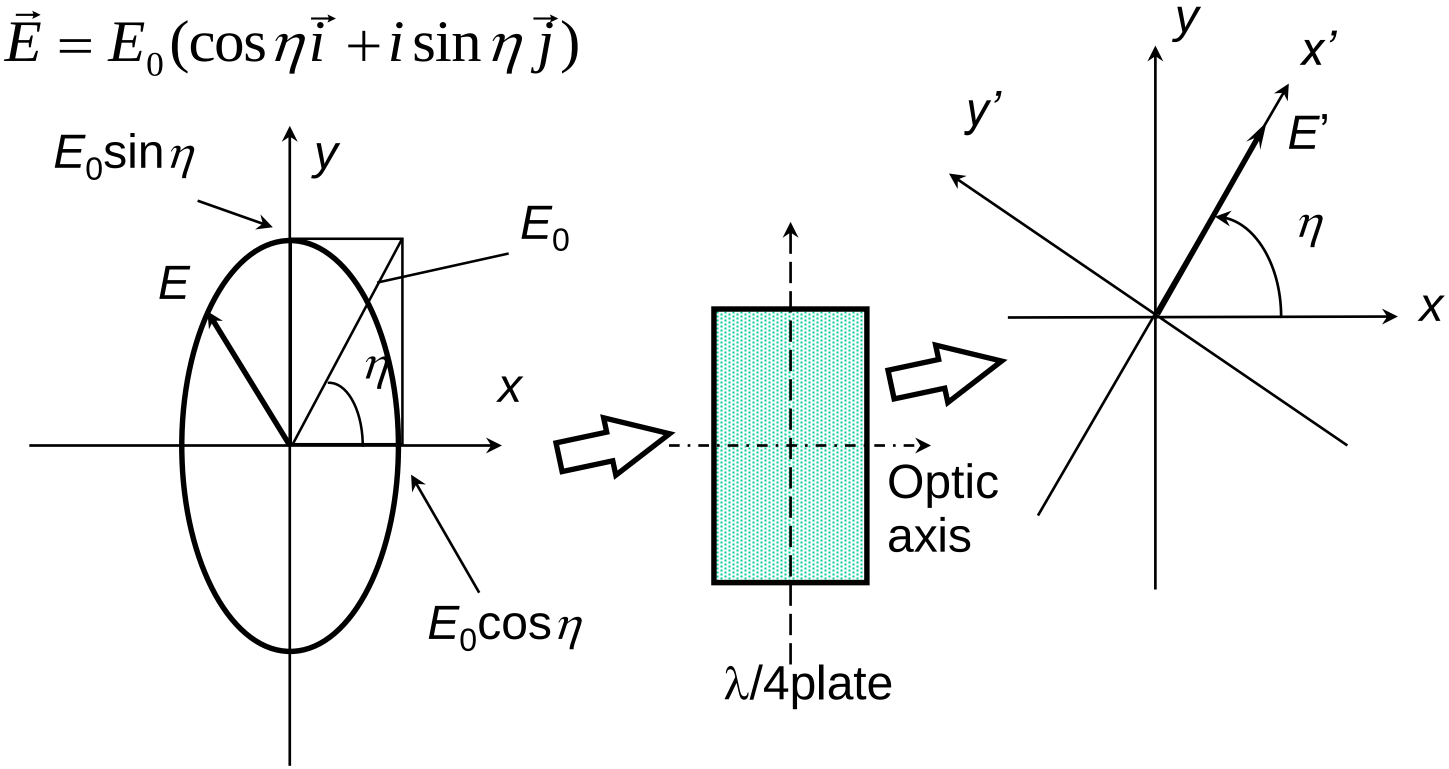
$$I_r = E_r (E_r)^*$$

$$I_r = I_o + \Delta I_m \qquad \Delta I_m / I_o \propto m_y$$



Measurement of ellipticity

$$\vec{E} = E_0(\cos\eta\,\vec{i} + i\sin\eta\,\vec{j})$$

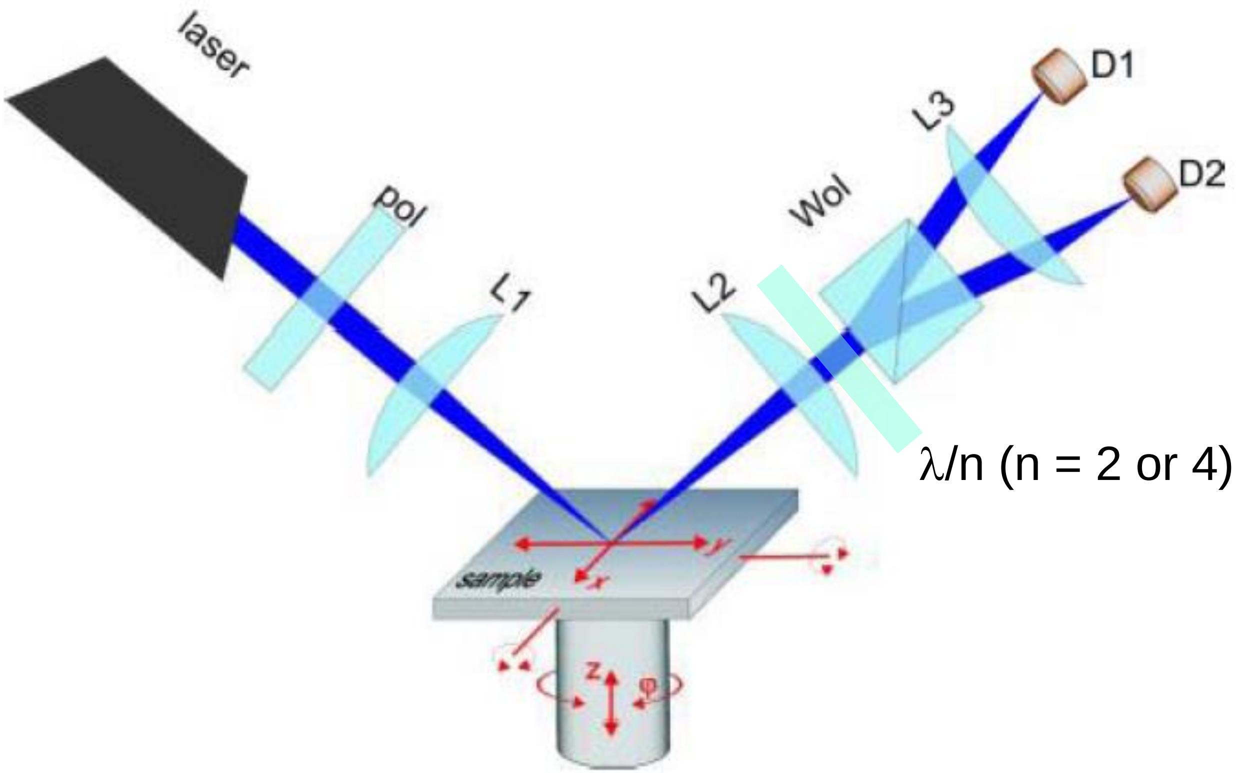


$$\tilde{E}_o = \begin{bmatrix} 1 \\ -bi \end{bmatrix} \qquad \tilde{A}_{QWP} = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \qquad \tilde{E}_0' = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ -bi \end{bmatrix} = \begin{bmatrix} 1 \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{\sin\eta}{\cos\eta} \end{bmatrix} = \begin{bmatrix} \cos\eta \\ \sin\eta \end{bmatrix}$$

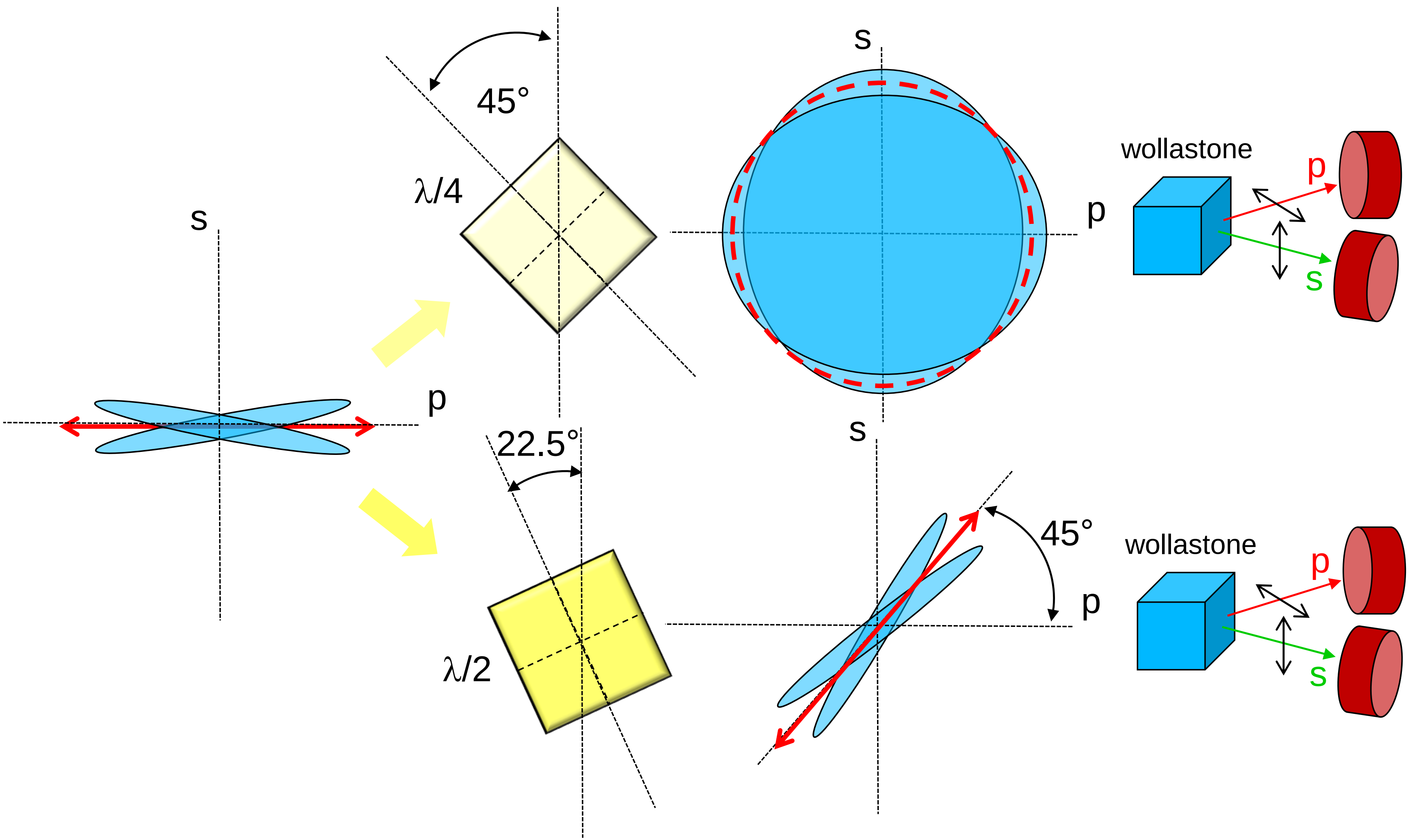
Adding an analyzed (i.e. A polarazier in front of the detector I get η (Malus law)

$$\tilde{E}_0'' = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \cos\eta \\ \sin\eta \end{bmatrix} = \begin{bmatrix} \cos\eta \\ 0 \end{bmatrix} \qquad I = (\tilde{E}_0'')^2 = \cos^2\eta$$

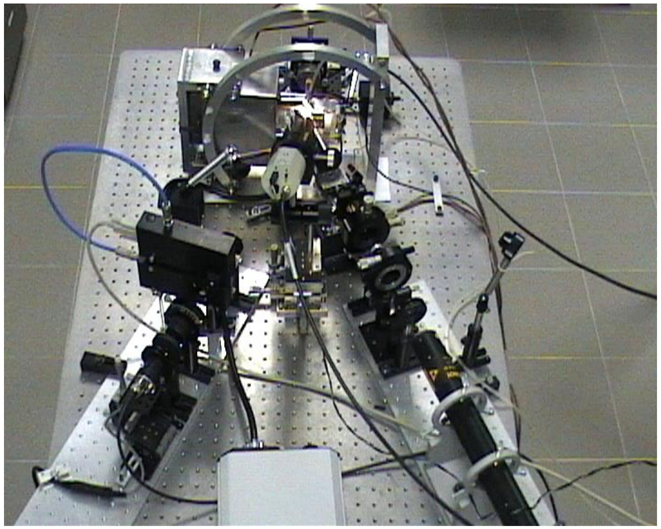
Measurement of ellipticity & rotation: high sensitivity



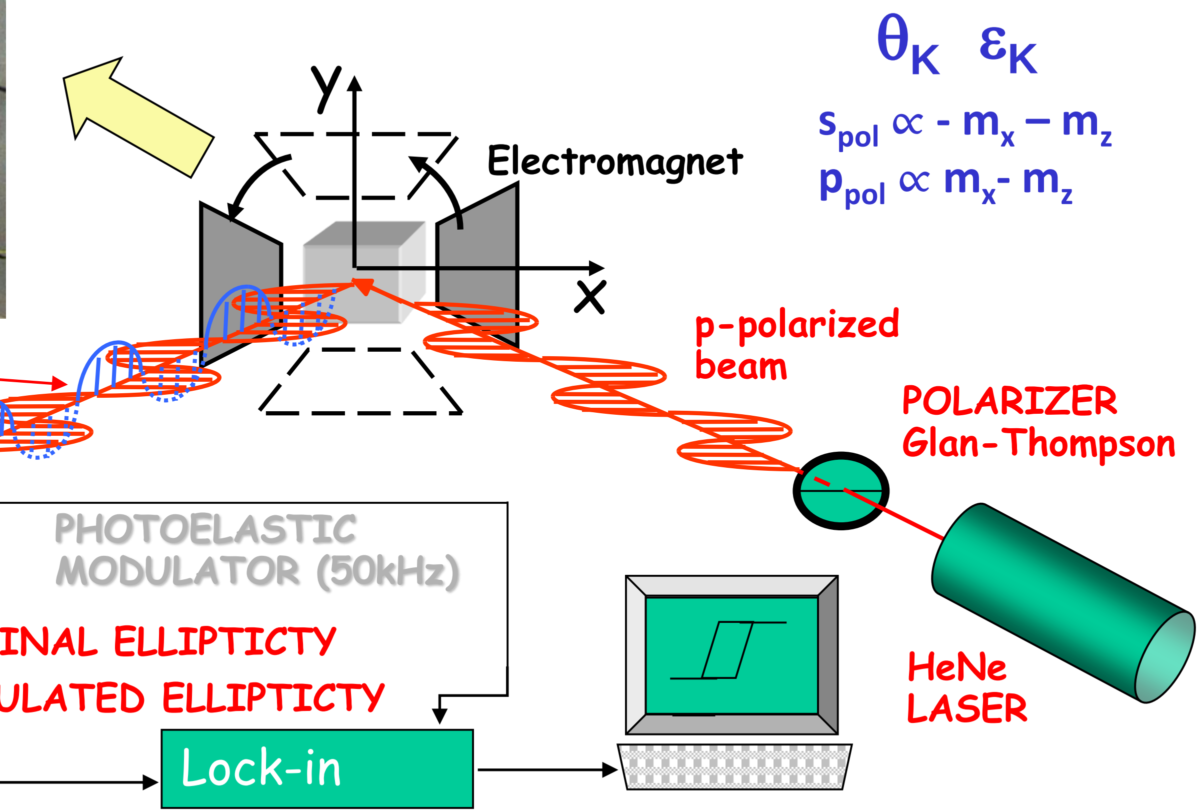
Measurement of ellipticity & rotation: high sensitivity



Measuring θ_K and ε_K

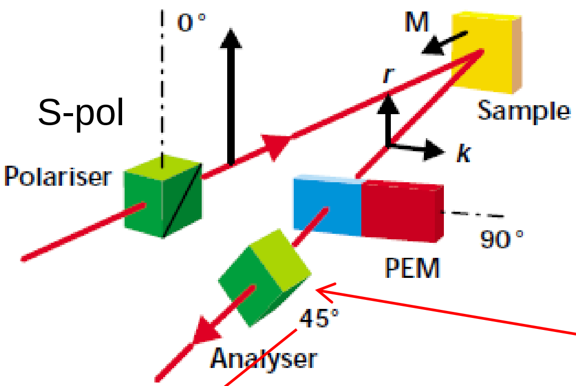


Modulation polarization technique for recording the longitudinal and polar Kerr effects, which are proportional to the magnetization components m_x m_z..



More details in: P. Vavassori, Appl. Phys. Lett. **77**, 1605 (2000)

Measuring θ_K and ε_K polar and longitudinal



PEM

$$P = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$$\varphi = 2\pi \Delta n l/\lambda = \varphi_0 \cos\omega t$$
$$\Delta n = A\cos\omega t$$
$$\varphi_0 = 2\pi l A/\lambda$$

$$I = \left|\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}\begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right|^2$$

$$I = \tfrac{1}{2}|r_{ps} + r_{ss}e^{i\phi}|^2 = \tfrac{1}{2}(|r_{ps}|^2 + r_{ps}r_{ss}^* e^{-i\phi} + r_{ps}^* r_{ss} e^{i\phi}),$$

Now:

$$e^{i\varphi} = e^{i\frac{2\pi A l}{\lambda}\cos\omega t} = J_0(\varphi_m) + 2iJ_1(\varphi_m)\sin\omega t + 2J_2(\varphi_m)\cos 2\omega t + \ldots$$

$$\varphi_m = 137.8^\circ$$
$$J_1 = 0.519 \text{ and } J_2 = 0.432$$

$$I_{DC} = |r_{ss}|^2 = r_{ss}r_{ss}^* \qquad r_{ps}r_{ss}^* + r_{ss}r_{ps}^* = 2\,\mathrm{Re}[r_{ps}r_{ss}^*] \qquad r_{ps}r_{ss}^* - r_{ss}r_{ps}^* = 2i\,\mathrm{Im}[r_{ps}r_{ss}^*]$$

$$\frac{I}{I_{DC}} = 1 + 2\times 2J_2\cos 2\omega t\,\mathrm{Re}(r_{ps}/r_{ss}) - 2\times 2J_1\sin\omega t\,\mathrm{Im}(r_{ps}/r_{ss}) + \ldots$$

If I can measure the normalized photodiode intensity at ω and 2ω

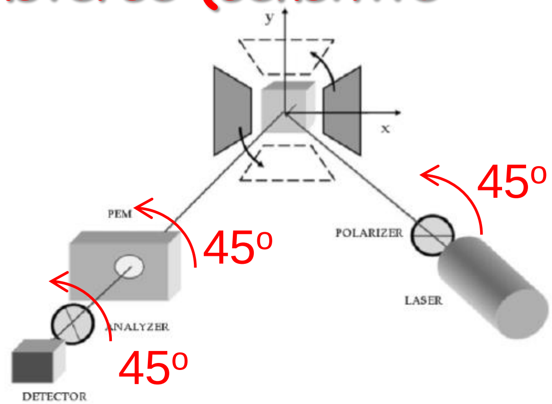
$$i_{\omega}^{p(s)} = \frac{-4J_1}{\sqrt{2}}\tan\beta\,\mathrm{Im}\left[\frac{r_{sp(ps)}}{r_{pp(ss)}}\right] \Rightarrow \varepsilon_K$$

$$i_{2\omega}^{p(s)} = \frac{4J_2}{\sqrt{2}}\tan\beta\,\mathrm{Re}\left[\frac{r_{sp(ps)}}{r_{pp(ss)}}\right] \Rightarrow \theta_K$$

(I considered here the general case of an analyzer at b respect to extinction with the initial polarizer)

$$A = \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}$$

Measuring θ_K and ε_K transverse (sensitive to m_y along y-direction)



PEM rotated 45°

$$\frac{1}{2}\begin{pmatrix} 1+e^{i\phi} & 1-e^{i\phi} \\ 1-e^{i\phi} & 1+e^{i\phi} \end{pmatrix}$$

$$I = \left| \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \frac{1}{2}\begin{pmatrix} 1+e^{i\phi} & 1-e^{i\phi} \\ 1-e^{i\phi} & 1+e^{i\phi} \end{pmatrix} \begin{pmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1+\beta \\ 1-\beta \end{pmatrix} \right|^2,$$

$$I = \tfrac{1}{8}|A + Be^{i\phi}|^2,$$

$$A = (r_{pp} + r_{sp})(1+\beta) + (r_{ps} + r_{ss})(1-\beta) \text{ and } B = (r_{pp} - r_{sp})(1+\beta) + (r_{ps} - r_{ss})(1-\beta).$$

$$\theta_k(45^\circ) = 2J_2(|r_{pp}|^2\{1+\beta\}^2 - |r_{ss}|^2\{1-\beta\}^2 + \{1-\beta^2\}\{r_{pp}r_{ps}^* + r_{pp}^*r_{ps} + r_{ss}r_{ps}^* + r_{ss}^*r_{ps}\})/(|r_{pp}|^2\{1+\beta\}^2$$

$$+ |r_{ss}|^2\{1-\beta\}^2 + 2|r_{sp}|^2\{1+\beta\} + \{1-\beta^2\}\{r_{pp}r_{ps}^* + r_{pp}^*r_{ps} - r_{ss}r_{ps}^* - r_{ss}^*r_{ps}\}).$$