ESM 2018
Krakow

MP2 - Precessional dynamics, dissipation processes, elementary and soliton excitations

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MP2: Precessional dynamics

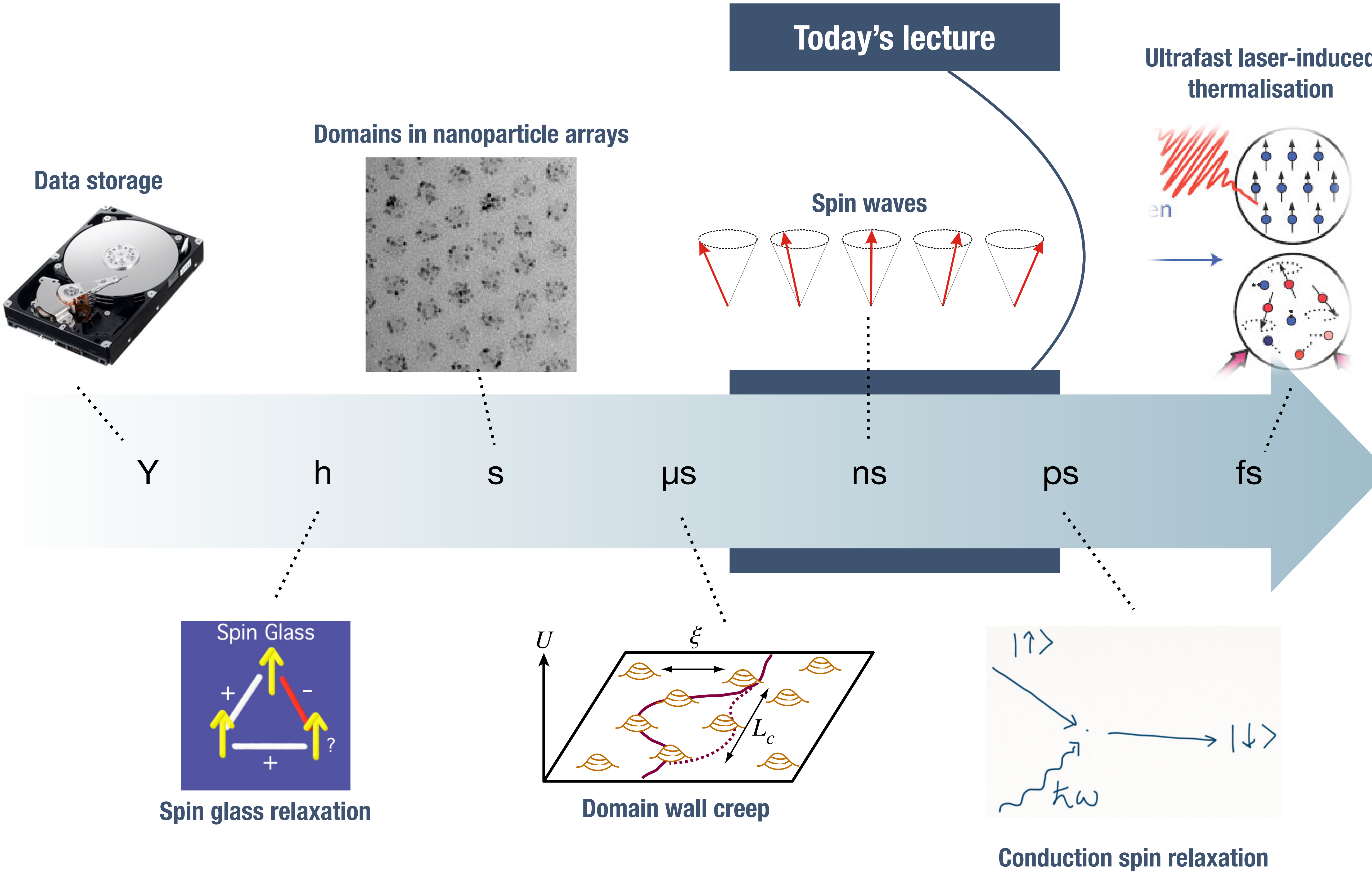
- Overarching theme: **Landau-Lifshitz equation**

- Linear excitations – spin waves
  Dispersion relations, applications in information processing

- Dissipation processes
  Intrinsic and extrinsic contributions, Gilbert damping

- Dynamics of topological solitons
  Lagrangian formulation, domain wall motion, vortex gyration

Time scales


Today's lecture
Ultrafast laser-induced thermalisation
Domains in nanoparticle arrays
Data storage
Spin waves
Y
h
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Spin Glass
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+
Spin glass relaxation
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Domain wall creep
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Conduction spin relaxation

Magnetisation dynamics

- In MP1, we saw how magnetic moments couple to each other and to their environment (e.g., exchange, dipole-dipole interactions).

- But how do they evolve in time? Consider Heisenberg picture in quantum mechanics,

$$i\hbar \frac{d}{dt} \langle \mathbf{S}(t) \rangle = \big\langle \left[\mathbf{S}, \mathcal{H} \right] \big\rangle$$

- Consider a *single spin* in an applied magnetic field ***H***. The Zeeman Hamiltonian is

$$\mathcal{H} = -g\mu_0\mu_B \mathbf{S} \cdot \mathbf{H}$$

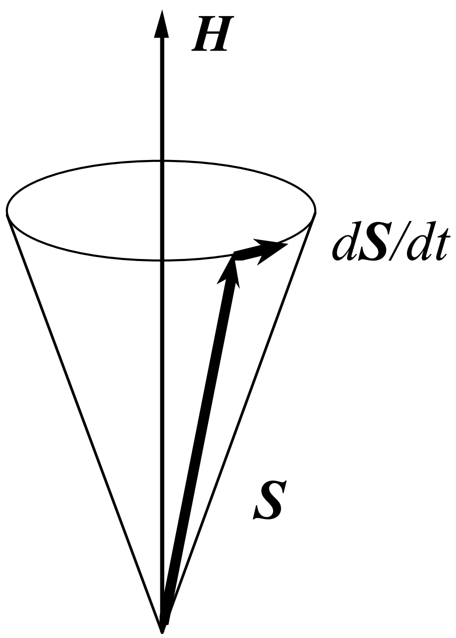
- To see how this works, expand out the S_x term:

$$\begin{aligned} [S_x, \mathcal{H}] &= -g\mu_0\mu_B [S_x, S_x H_x + S_y H_y + S_z H_z] \\ &= -g\mu_0\mu_B \left(H_y [S_x, S_y] + H_z [S_x, S_z] \right) \end{aligned}$$

Magnetisation dynamics

- By applying the usual commutation rules for the spin operators

$$[S_x, S_y] = iS_z \qquad [S_y, S_z] = iS_x \qquad [S_z, S_x] = iS_y$$



we obtain

$$[S_x, \mathcal{H}] = -g\mu_0\mu_B i\left(H_y S_z - H_z S_y\right)$$

- Combining with the other spin components, we find

$$\frac{d\langle \mathbf{S}(t)\rangle}{dt} = \frac{g\mu_0\mu_B}{\hbar}\langle \mathbf{S}\rangle \times \mathbf{H}$$

- This describes the precession of a spin in a magnetic field. With the definition of the gyromagnetic constant

$$\gamma = \frac{gq_e}{2m} = \frac{g\mu_B}{\hbar} < 0 \qquad \gamma_0 = \mu_0 \frac{g|\mu_B|}{\hbar} = -\mu_0\gamma$$

~28 GHz/T

Magnetisation dynamics

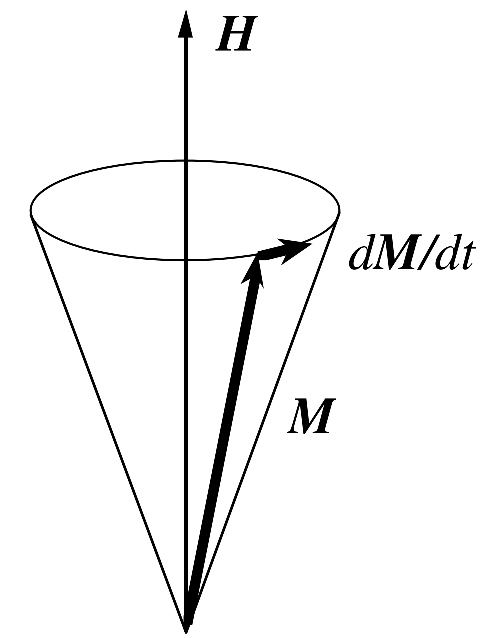
- By averaging over the spins in the Bloch equation,

$$\frac{d\langle \mathbf{S}(t)\rangle}{dt} = \frac{g\mu_0\mu_B}{\hbar}\langle \mathbf{S}\rangle \times \mathbf{H}$$

we can express the torque equation for a general magnetisation field ***M*** as

$$\boxed{\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}}$$

$$\mathbf{M} = g\mu_B N\langle \mathbf{S}\rangle / V$$



- The micromagnetics approach allows for a classical description of the magnetisation dynamics by treating the magnetisation as a continuous field ***M*** subject to torques applied by magnetic fields ***H***.

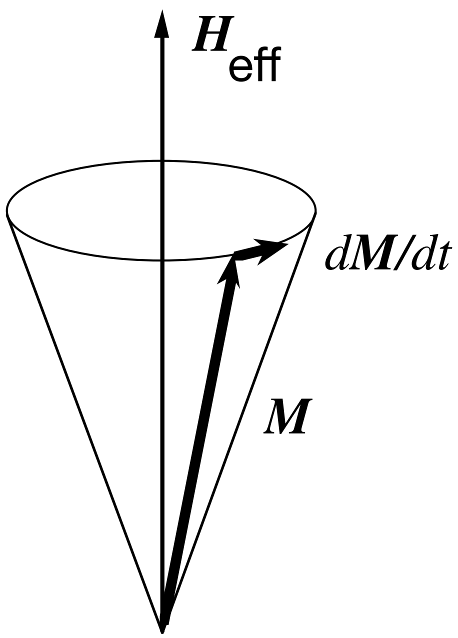
Precessional dynamics

- Generalise torque equation to any magnetic energy by replacing ***H*** with the <u>effective field</u> $\boldsymbol{H}_{\text{eff}}$

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}}$$

where

$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0}\frac{\delta E}{\delta \mathbf{M}}$$



- The energy density accounts for *all* relevant contributions to the magnetic Hamiltonian (see MP1)

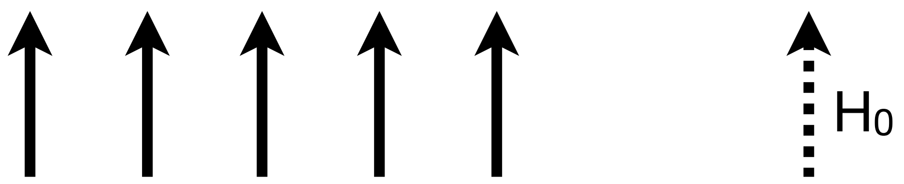
- Magnetisation precesses about its *local effective field*

- Note that this torque equation conserves the norm of the magnetisation vector and describes dynamics at constant energy

$$\frac{d}{dt}\|\mathbf{M}\|^2 = 0 \qquad \frac{d}{dt}\left(\mathbf{M}\cdot\mathbf{H}_{\text{eff}}\right) = 0$$

Linear excitations - Spin waves

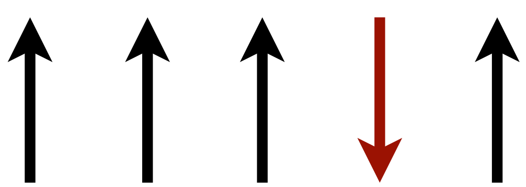
- Small amplitude (linear) excitations of magnetisation are described by **spin waves**
- Consider a chain of spins uniformly aligned along an applied field H_0



$$E = -N(g\mu_B S)H_0 \equiv E_0$$

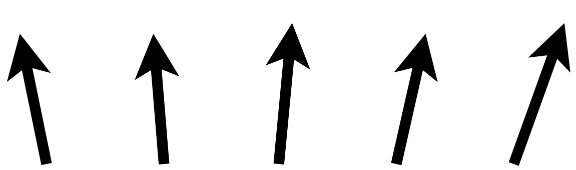
- What is the smallest excitation possible? **One spin reversal**. There are two ways to accomplish this:

1) Flip one spin along the chain



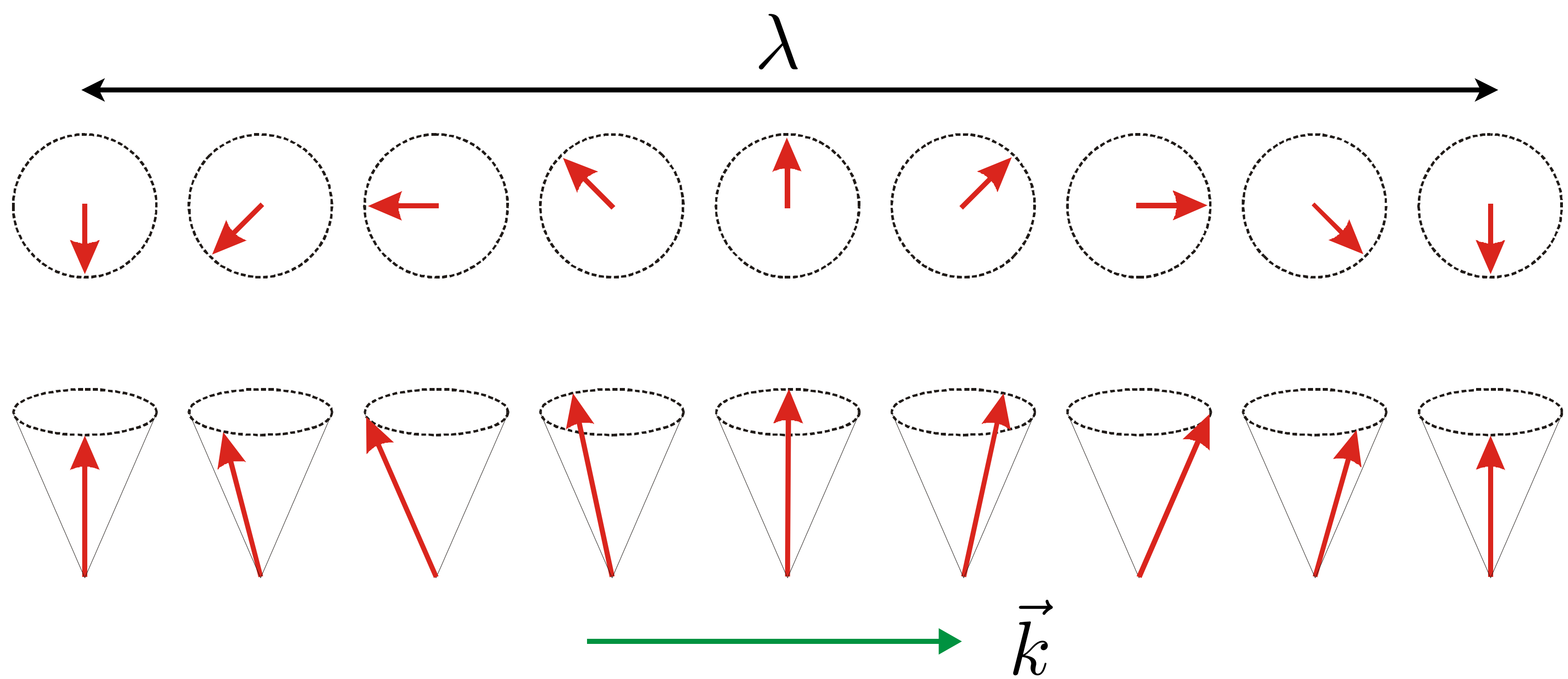
$$E - E_0 = 2J$$

2) Distribute the spin reversal by canting all spins



$$E - E_0 = \hbar\omega \ll 2J$$

Spin waves



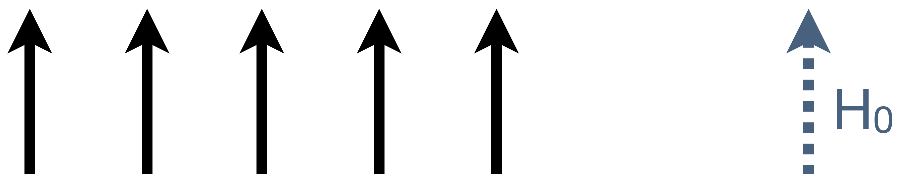
- Spin waves are elementary excitations of a magnetic system
- Quantised spin-wave: **magnon** (*cf* phonons for elastic waves)
- It is more favourable energetically to distribute flipped spin over all lattice sites, rather than to have it localised to one lattice site.

(NB. Such excitations do exist - Stoner excitations - and these are important at high energies)

$$\hbar\omega(\vec{k})$$

Spin wave dispersion relations

- Consider a uniformly magnetised system along the positive *z* axis. Suppose there is an applied external field H_0 along the positive *z* direction:



- Let

$$\mathbf{M} = M_s \mathbf{m} \qquad \qquad \|\mathbf{m}\| = 1$$

- If we allow for spatial variations in ***m***, we need to also include exchange,

$$E = E_Z + E_{\mathrm{ex}} = -\mu_0 M_s H_0 m_z + A\left[(\nabla m_x)^2 + \cdots\right]$$

- From this expression, we can derive an expression for the effective field

$$\mathbf{H}_{\mathrm{eff}} = -\frac{1}{\mu_0 M_s}\frac{\partial E}{\partial \mathbf{m}} = H_0\hat{\mathbf{z}} + \frac{2A}{\mu_0 M_s}\nabla^2 \mathbf{m}$$

Linearising the equations of motion

- Study small amplitude fluctuations of the magnetisation by *linearising* the equations of motion

- Write the magnetisation in terms of static and dynamic components. Assume the ground state consists of uniform magnetic state along +z:

$$\mathbf{m}(\mathbf{r},t) = \mathbf{m}_0 + \delta\mathbf{m}(\mathbf{r},t) = (0,0,1) + (m_x(\mathbf{r},t), m_y(\mathbf{r},t), 0)$$

static **dynamic**

- Similarly, decompose the effective field into static and dynamic components:

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_{\text{eff},0} + \mathbf{h}_{\text{eff}}(\mathbf{r},t)$$

Terms that depend on $\mathbf{m}_0$

Terms that depend on $\delta\mathbf{m}$

Linearising the equations of motion

- Rewrite the precession term in the Landau-Lifshitz equation in terms of static and dynamic parts, retain only linear terms in the dynamic components:

$$\frac{d\mathbf{m}}{dt} = -\gamma_0 \mathbf{m} \times \mathbf{H}_{\text{eff}}$$

$$\frac{d\mathbf{m}}{dt} = -\gamma_0 \left(\underbrace{\delta\mathbf{m}}_{\text{dynamic}} \times \mathbf{H}_{\text{eff},0} + \mathbf{m}_0 \times \underbrace{\mathbf{h}_{\text{eff}}}_{\text{dynamic}} \right)$$

- Assume plane wave solutions for the dynamic part

$$m_{x,y}(\mathbf{r}, t) = c_0 e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$$

- Left-hand side of the torque equation becomes

$$\frac{d\delta\mathbf{m}}{dt} = -i\omega \begin{bmatrix} m_x \\ m_y \end{bmatrix}$$

$$\delta\mathbf{m} = (m_x, m_y, 0)$$

dynamic magnetisation

Linearising the equations of motion

- In a similar way, the terms on the right-hand side (RHS) of the equation become

$$\frac{d\mathbf{m}}{dt} = -\gamma_0 \left(\delta\mathbf{m} \times \mathbf{H}_{\mathrm{eff},0} + \mathbf{m}_0 \times \mathbf{h}_{\mathrm{eff}}\right)$$

$$-i\omega \begin{bmatrix} m_x \\ m_y \end{bmatrix} \qquad \delta\mathbf{m} \times H_0 \hat{\mathbf{z}} \qquad \hat{\mathbf{z}} \times \left(\frac{2A}{\mu_0 M_s} \nabla^2 \delta\mathbf{m} \right)$$

which leads to the matrix equation

$$-i\omega \begin{bmatrix} m_x \\ m_y \end{bmatrix} = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} \qquad \omega_k = \gamma_0 \left(H_0 + \frac{2A}{\mu_0 M_s} k^2 \right)$$

$$\begin{bmatrix} i\omega & -\omega_k \\ \omega_k & i\omega \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} = 0$$

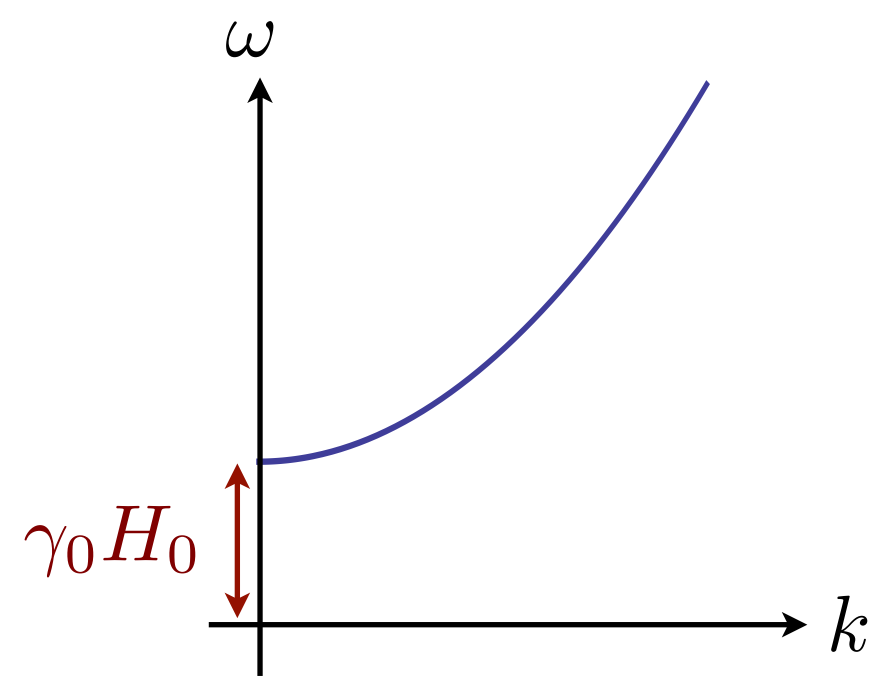
Spin wave dispersion relation

- Condition of vanishing determinant of the 2x2 matrix gives the ***dispersion relation*** for the spin waves:

$$-\omega^2 + \omega_k^2 = 0$$
$$\Rightarrow \omega = \omega_k = \gamma_0 H_0 + Dk^2$$

where we have defined a ***spin-wave stiffness***

$$D \equiv \frac{2\gamma A}{M_s}$$



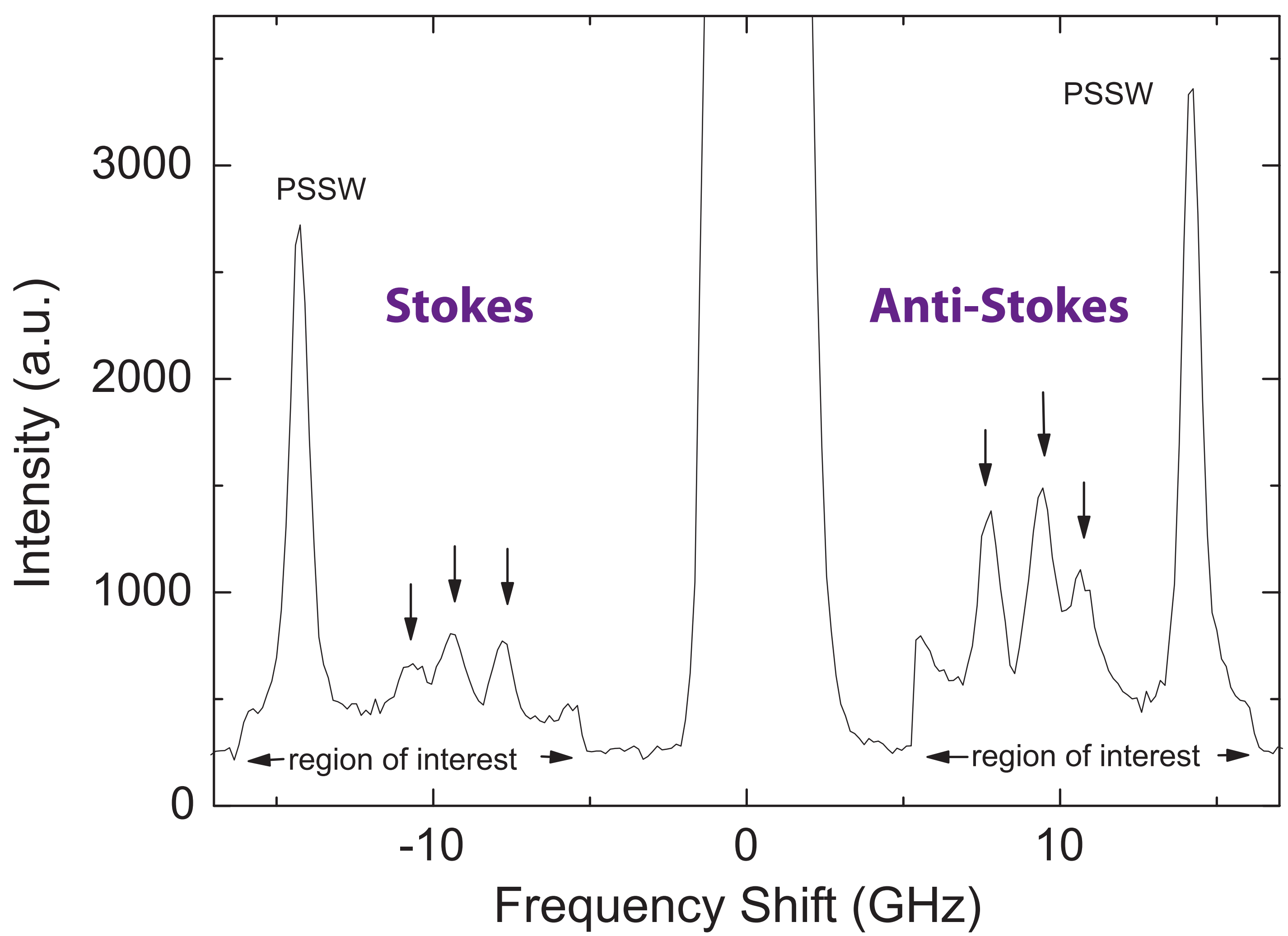
- Spin waves in ferromagnets are *dispersive* with a “band gap” due to applied and anisotropy fields

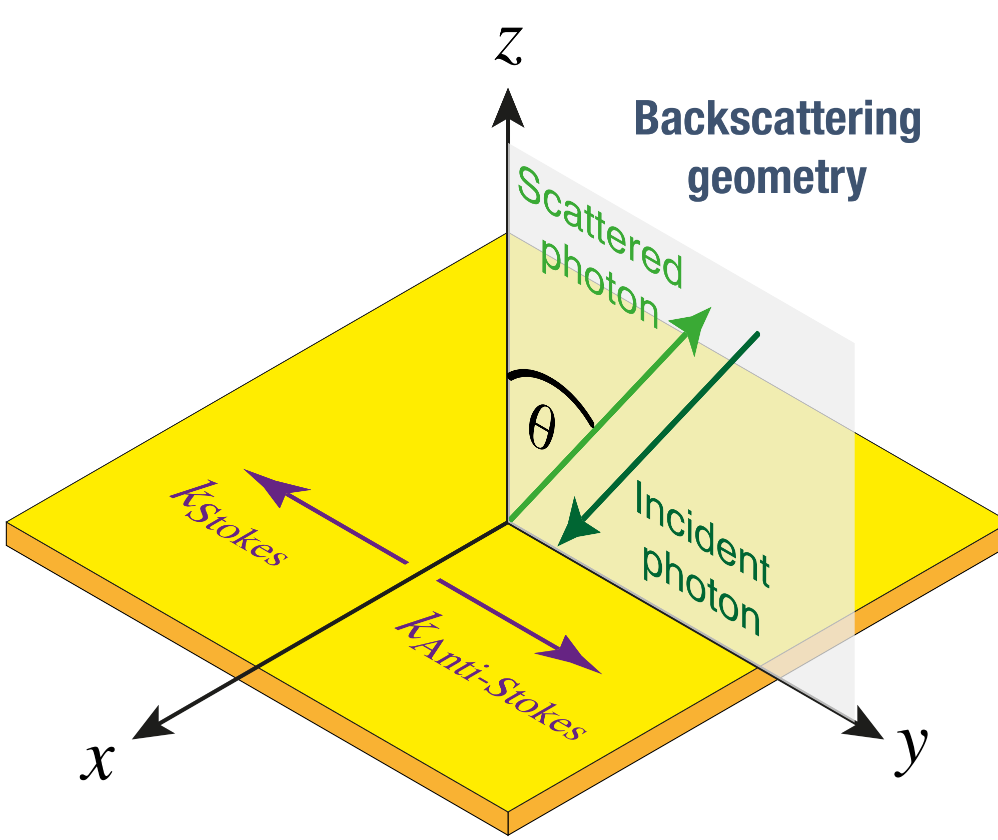
$$\frac{\omega}{k} \neq \frac{\partial \omega}{\partial k}$$

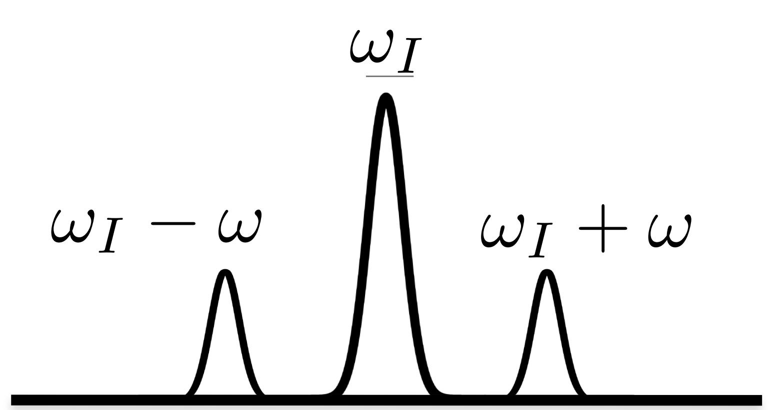
- Other energy contributions will bring supplementary terms to the dispersion relation

Brillouin light scattering spectroscopy

- Probe spin wave spectra by scattering light off surfaces
- Reflected photons give information about spin waves that are created (Stokes) or annihilated (anti-Stokes)

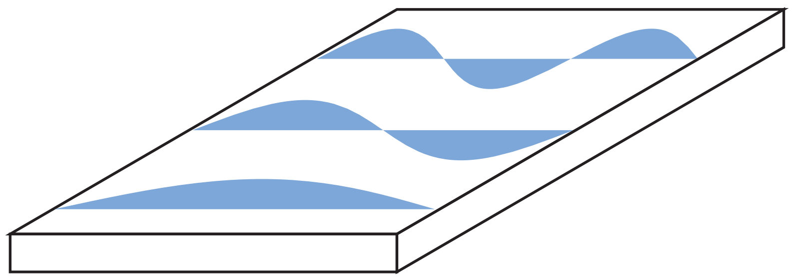






Mode confinement in nanostructures

- Translational invariance is broken in nanostructured magnetic elements
- Boundary conditions determine the quantisation conditions



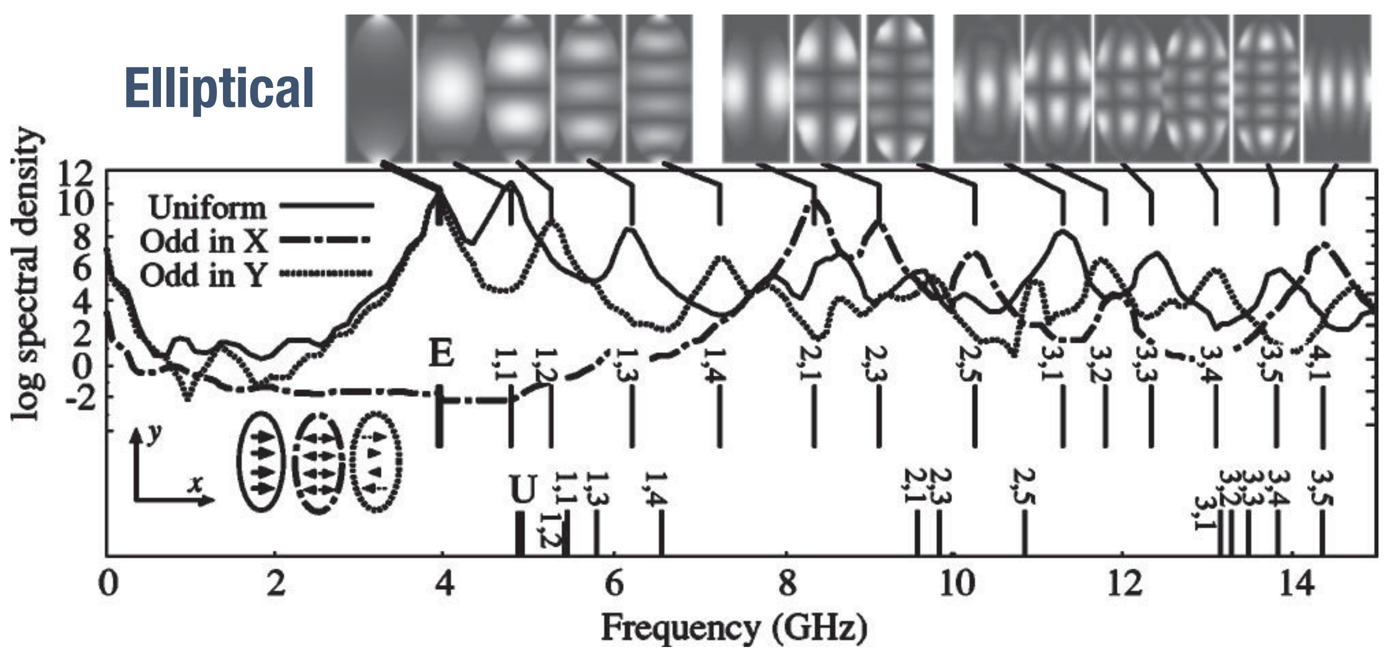
Boundary condition for magnetisation

$$\left.\frac{\partial \vec{M}}{\partial \vec{n}}\right|_{S} = \kappa \vec{M}$$

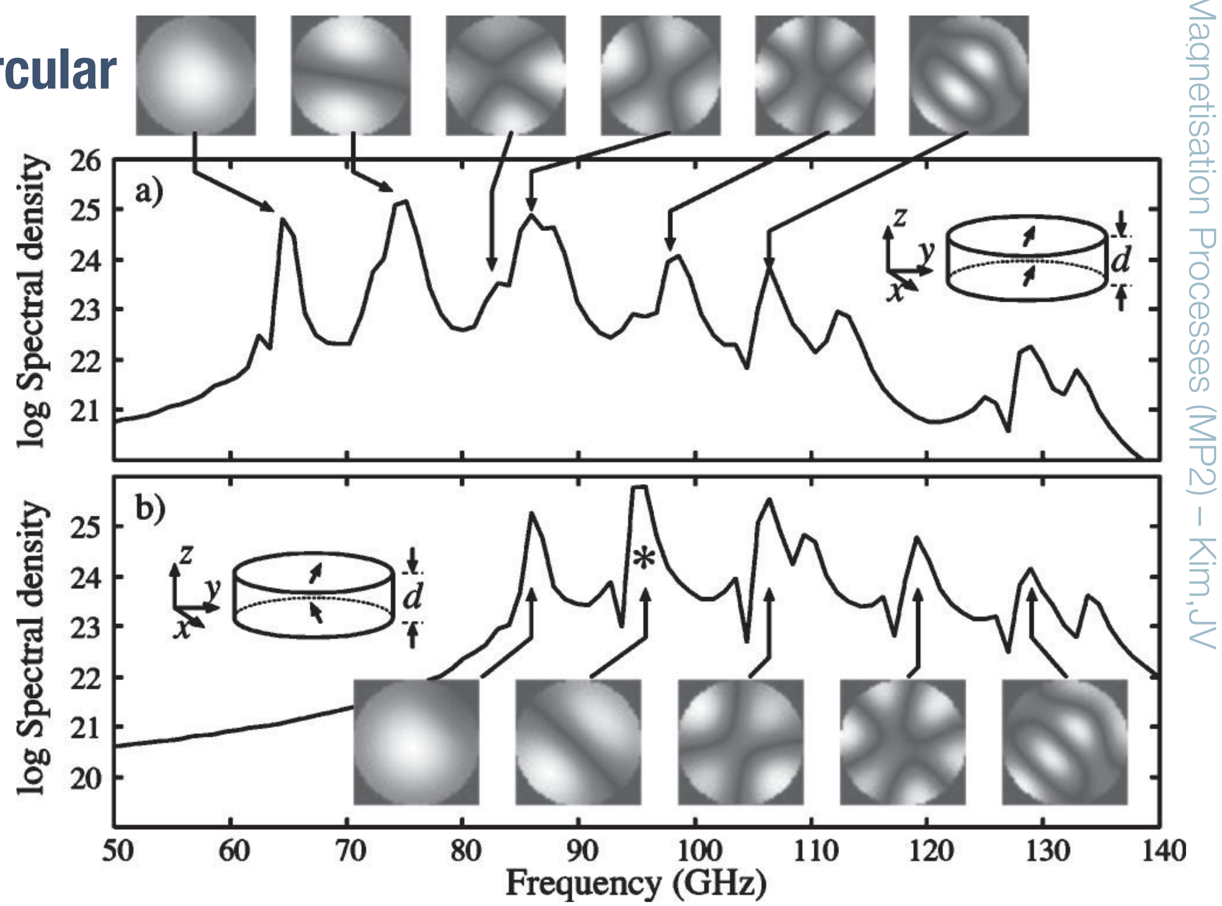
Micromagnetics

R D McMichael & M D Stiles, *J Appl Phys* **97**, 10J901 (2005)

Elliptical



Circular

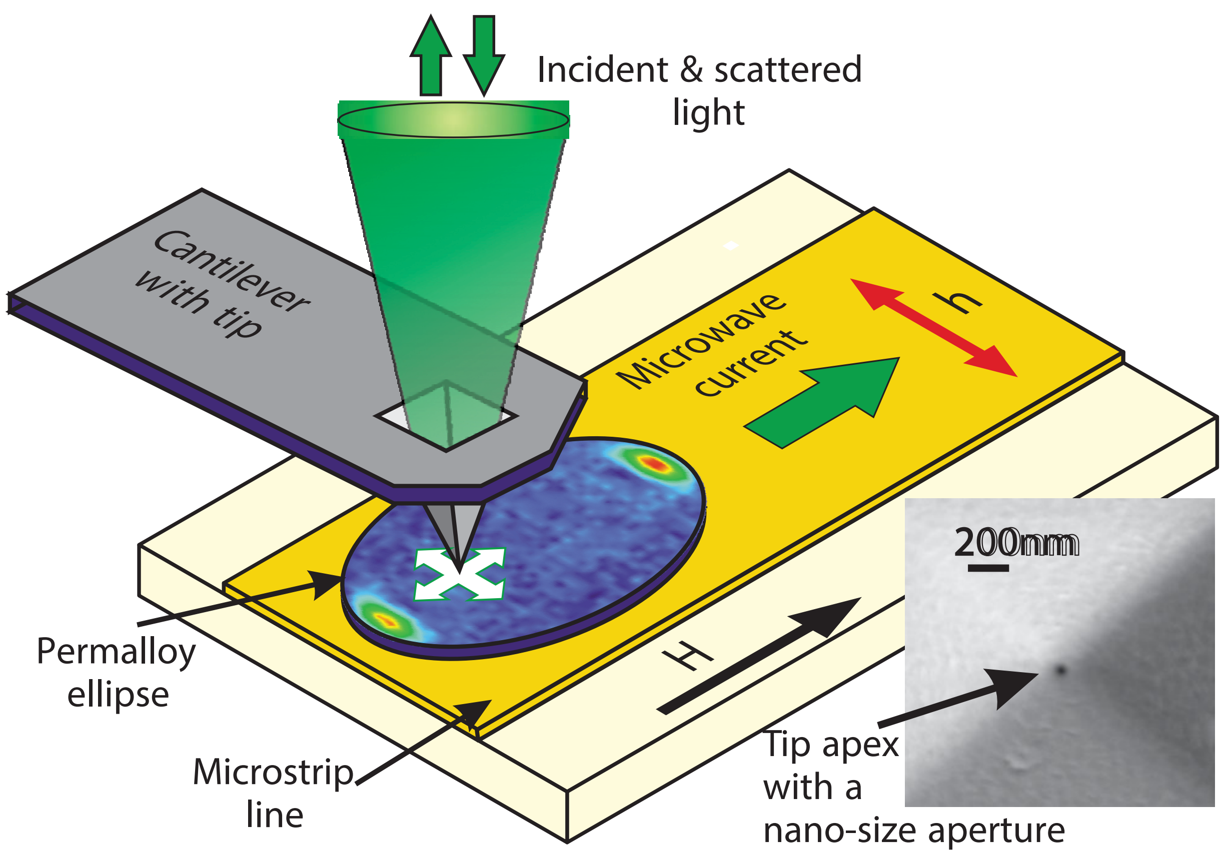


Mode confinement in nanostructures

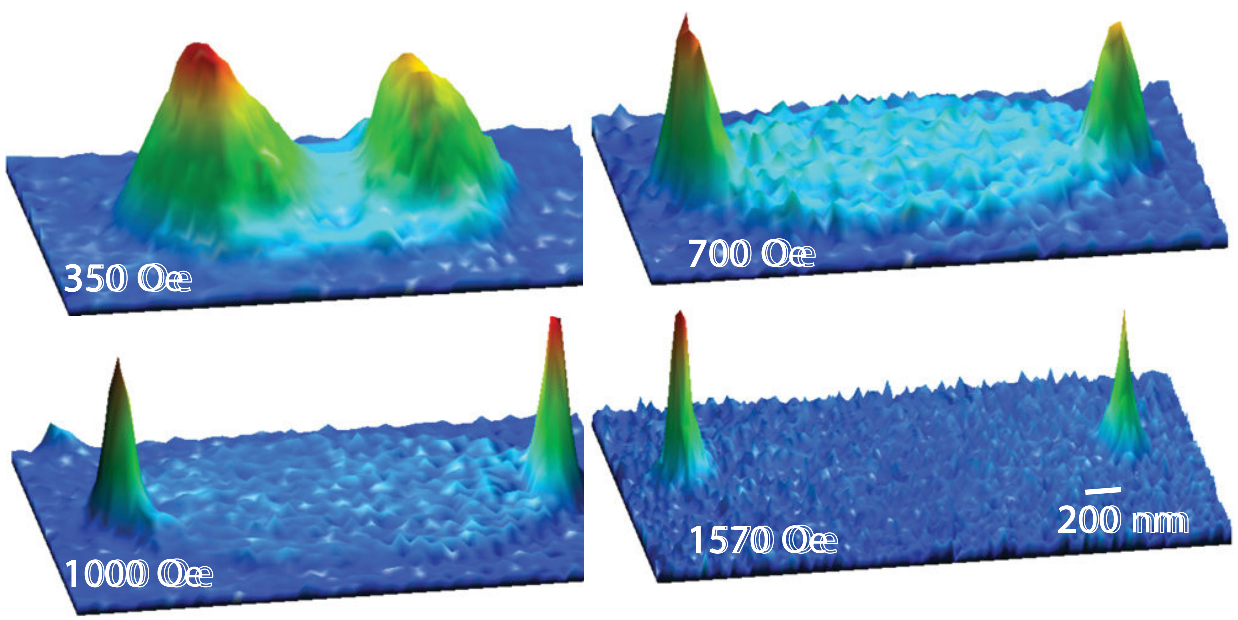
Experiments

- Brillouin light scattering with nano-sized apertures and near-field imaging allows confined modes to be probed

Microfocus BLS setup



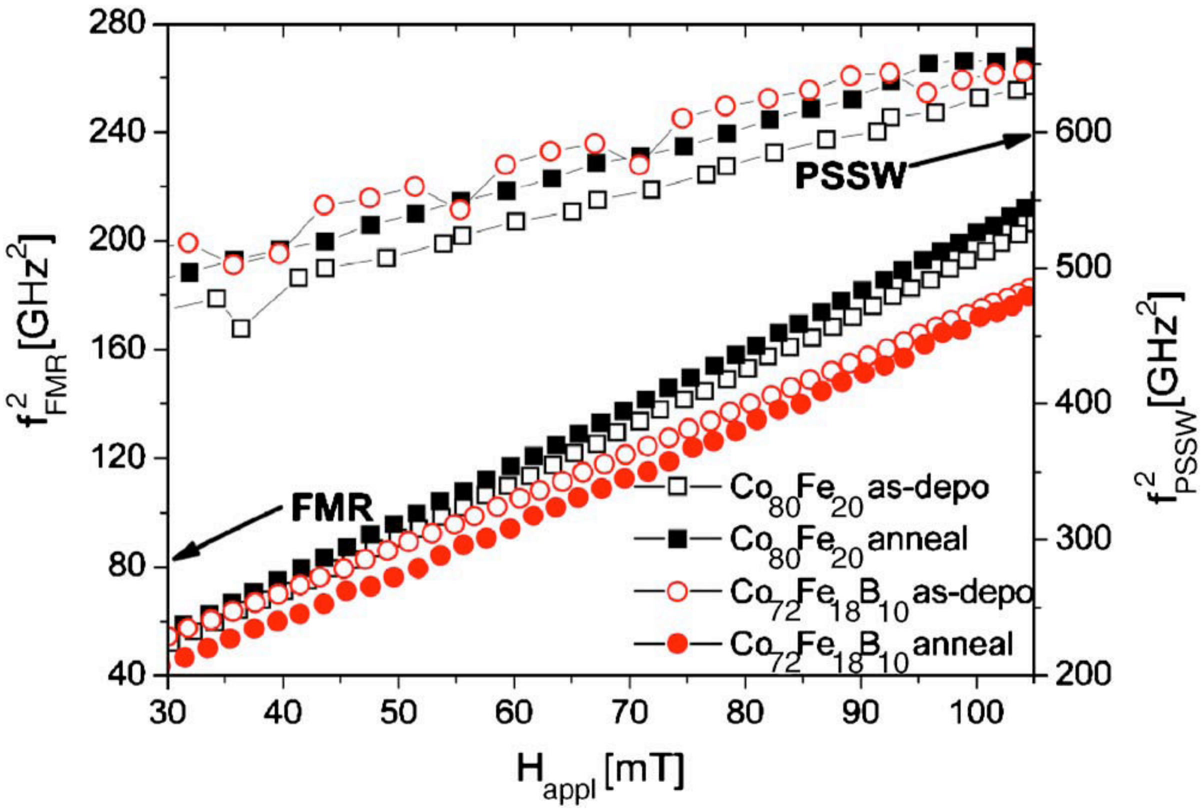
Edge modes in a ferromagnetic ellipse

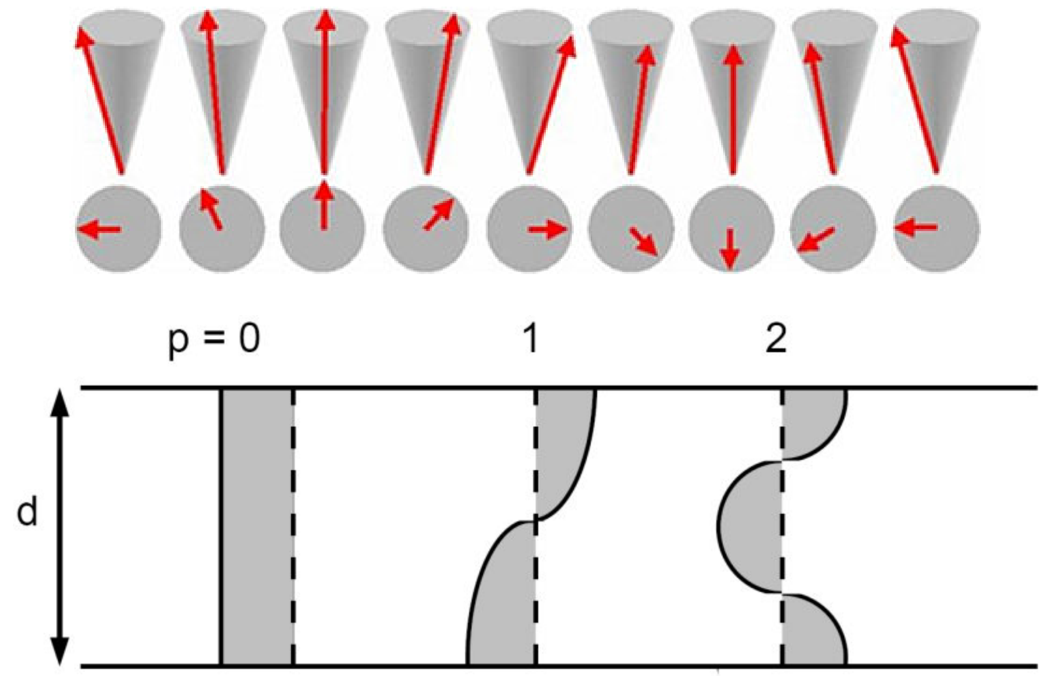


J Jersch et al, *Appl Phys Lett* **97**, 152502 (2010)

Spin waves as probes of magnetic properties

- Example: Determine exchange constant *A* from frequencies of perpendicular standing spin waves (PSSW)





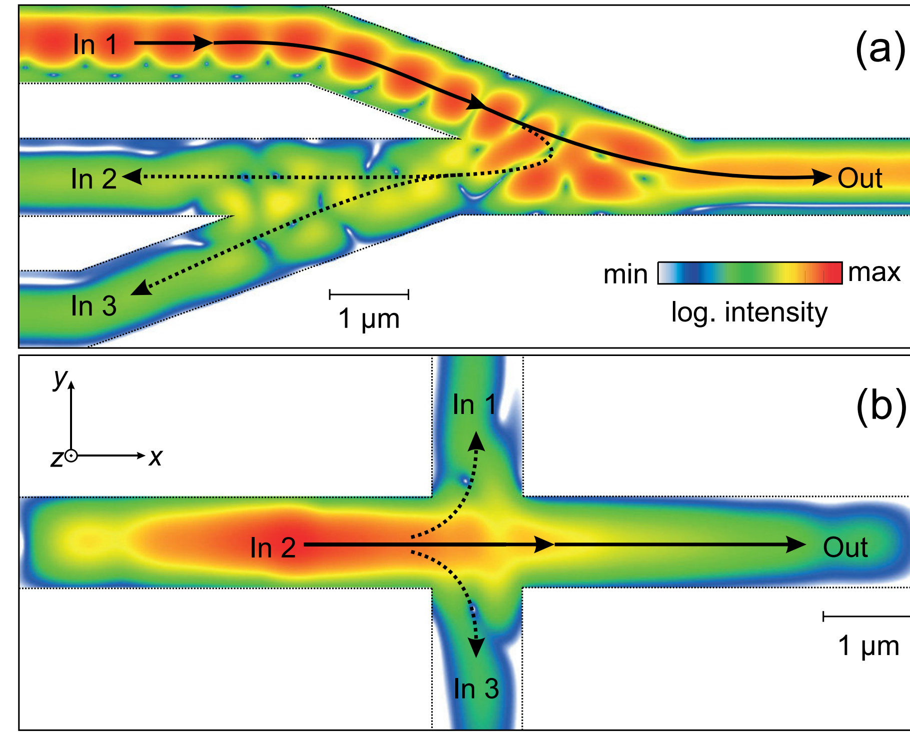
$$\omega_{\rm PSSW}^2 = \left[\omega_H + \omega_{\rm eff} + \gamma \frac{2A}{M_{\rm s}} \left(\frac{\pi p}{d}\right)^2\right] \left[\omega_H + \gamma \frac{2A}{M_{\rm s}} \left(\frac{\pi p}{d}\right)^2\right]$$

C Bilzer et al, *J Appl Phys* **100**, 053903 (2008)

Information technologies with spin waves

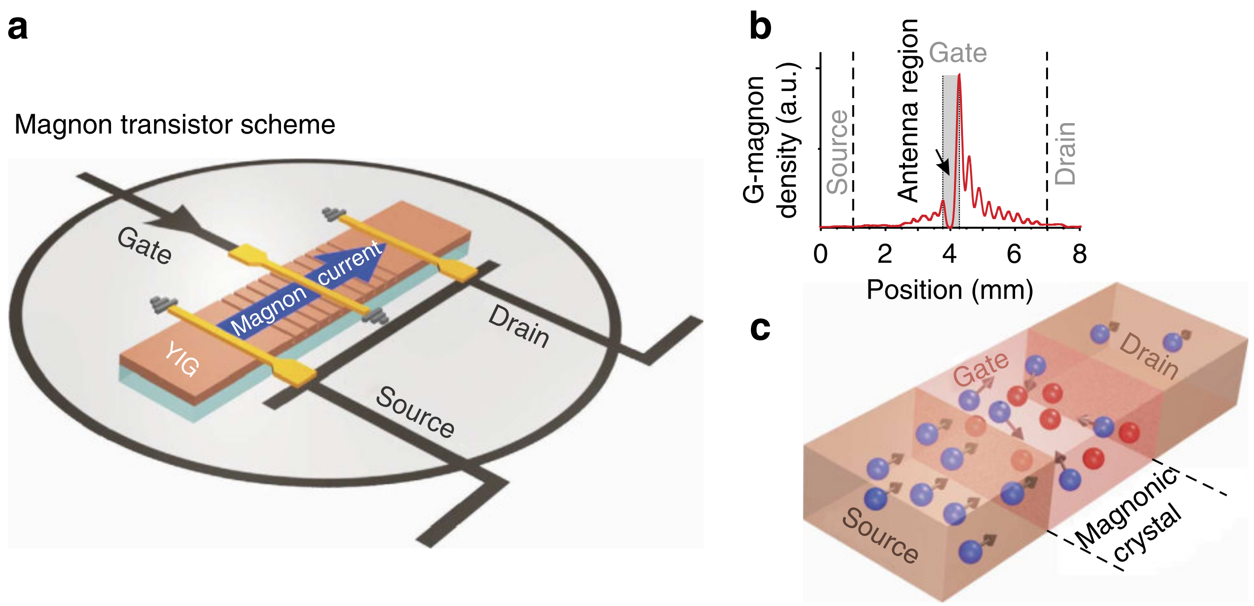
Spin wave majority gates

S Klingler et al, *Appl Phys Lett* **106**, 212406 (2015)



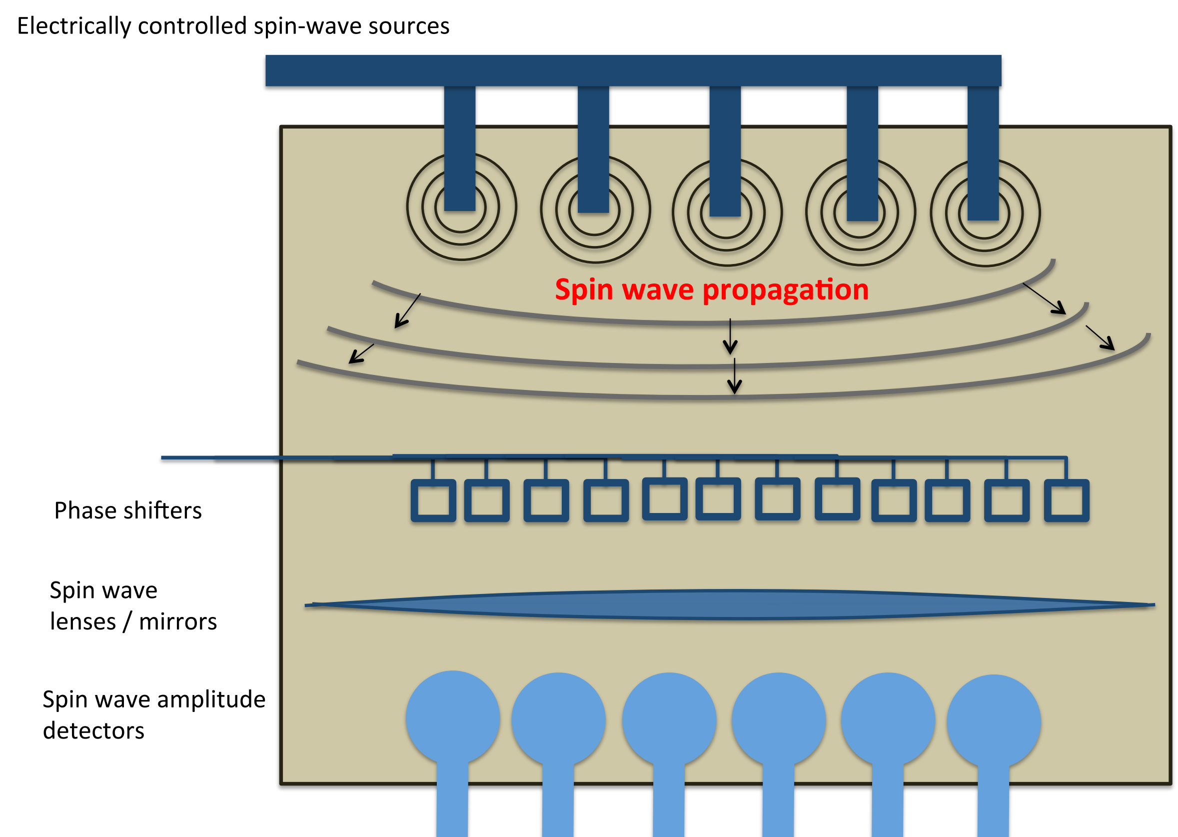
Magnon transistor

A A Serga et al, Nat Commun **5**, 4700 (2014)



Non-Boolean computing

A Papp et al, *IWCE* (2015)



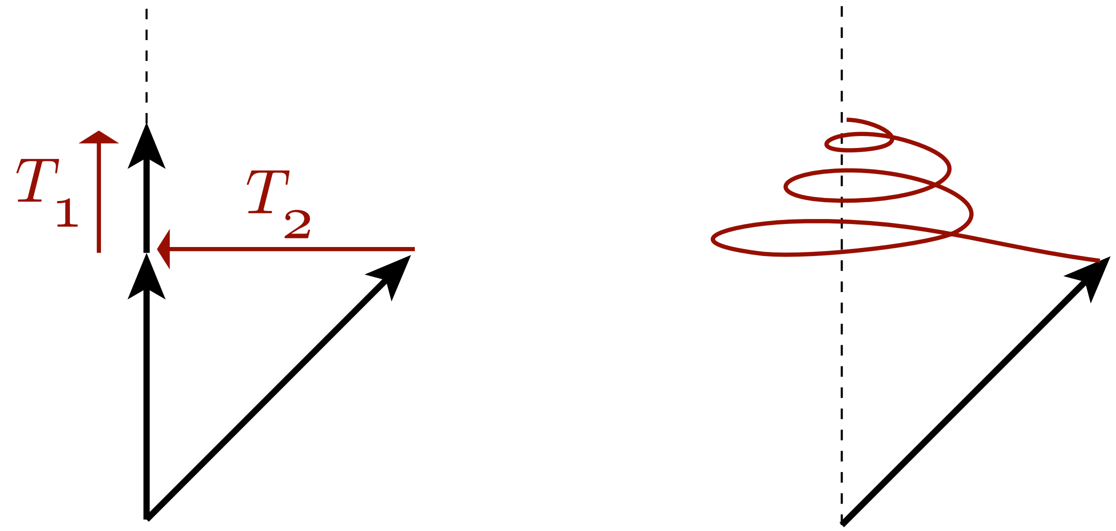
Magnetic relaxation

- How does magnetisation reach equilibrium?

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} = 0 \quad \text{at equilibrium}$$

Relaxation times

T_1: longitudinal
T_2: transverse



Overall result:

M spirals to equilibrium

Two possibilities:

(i) **Two-step process** ($T_2 << T_1$) ||***M***|| is <u>not</u> conserved

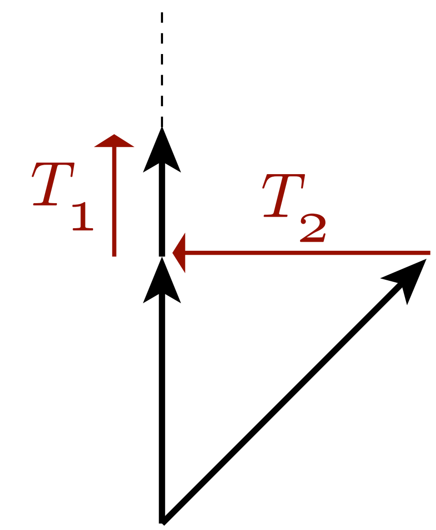
(ii) **Viscous damping** ($2T_2 = T_1$) ||***M***|| is conserved

Phenomenology

(i) Two-step processes: **Bloch-Bloembergen** terms – ||***M***|| is *not* conserved

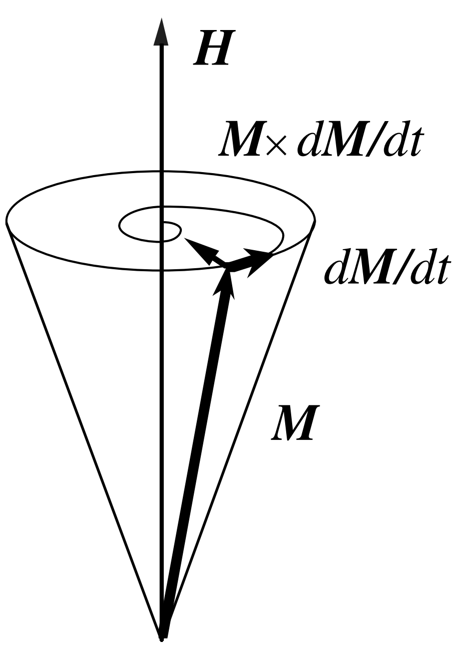
$$\frac{dM_z}{dt} = -\gamma_0 \left(\mathbf{M} \times \mathbf{H}_{\text{eff}}\right)_z - \frac{M_z - M_s}{T_1}$$

$$\frac{dM_{x,y}}{dt} = -\gamma_0 \left(\mathbf{M} \times \mathbf{H}_{\text{eff}}\right)_{x,y} - \frac{M_{x,y}}{T_2}$$



(ii) Viscous damping: **Gilbert** term – ||***M***|| is conserved

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{d\mathbf{M}}{dt}$$



Only the Gilbert term is compatible with the basic assumption of micromagnetics

Gilbert vs Landau-Lifshitz

The **Gilbert term** can be rewritten in the following way to make the physics more transparent

$$(1+\alpha^2)\,\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\alpha\gamma_0}{M_s}\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

directed along precession trajectory

directed towards instantaneous effective field

This is referred to as the **Landau-Lifshitz equation**.

Note that α – the damping constant – determines the rate at which **energy dissipation** can occur:

- Governs magnetisation reversal times
- Governs switching fields, currents

The Landau-Lifshitz equation gives a good description of the damped magnetisation dynamics in strong ferromagnets (on the ~ns time scale).

Spin wave damping

- With the inclusion of Gilbert damping, linearised equations give

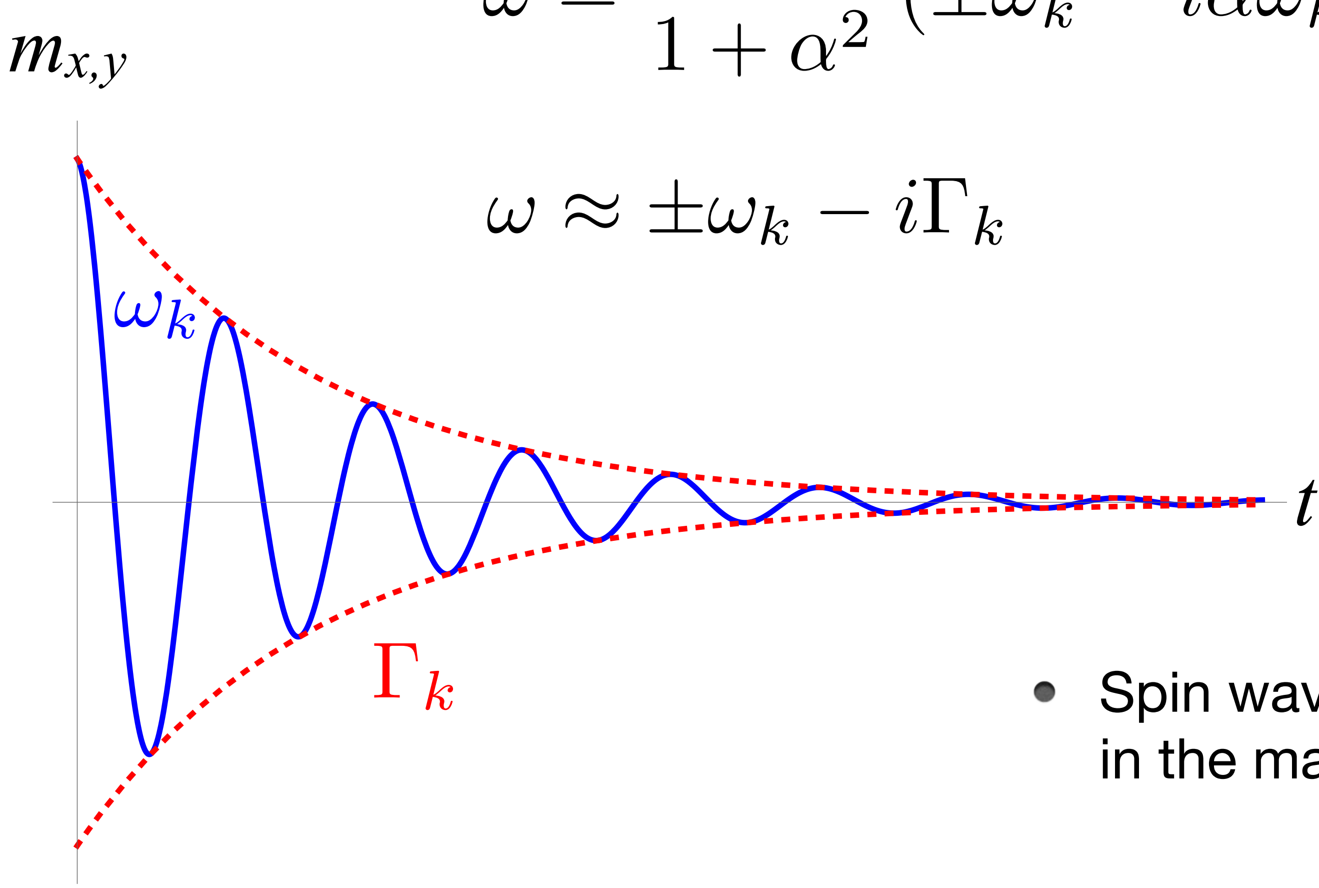
$$-i\omega \begin{bmatrix} 1 & \alpha \\ -\alpha & 1 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix} = \begin{bmatrix} 0 & -\omega_k \\ \omega_k & 0 \end{bmatrix} \begin{bmatrix} m_x \\ m_y \end{bmatrix}$$

- This leads to the complex frequencies

$$\omega = \frac{1}{1+\alpha^2}\left(\pm\omega_k - i\alpha\omega_k\right)$$

$$\omega \approx \pm\omega_k - i\Gamma_k$$

$\alpha \ll 1$ **Weak damping**



- Spin waves represent damped oscillations in the magnetisation

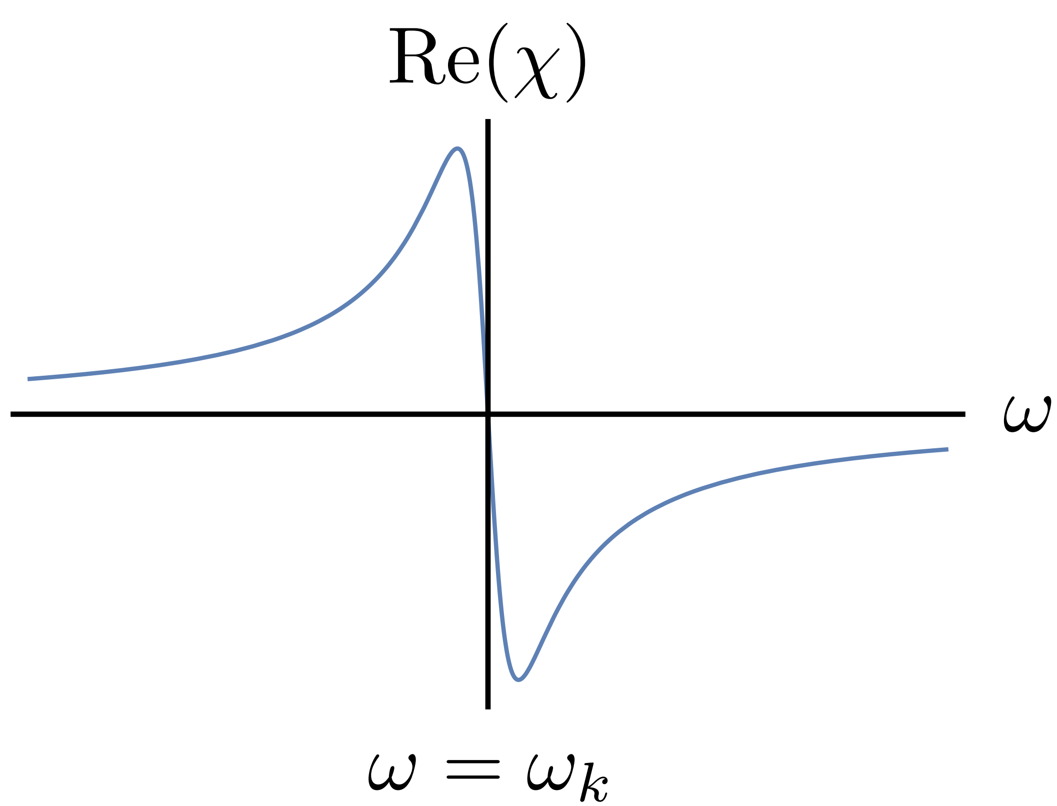
Spin wave susceptibilities

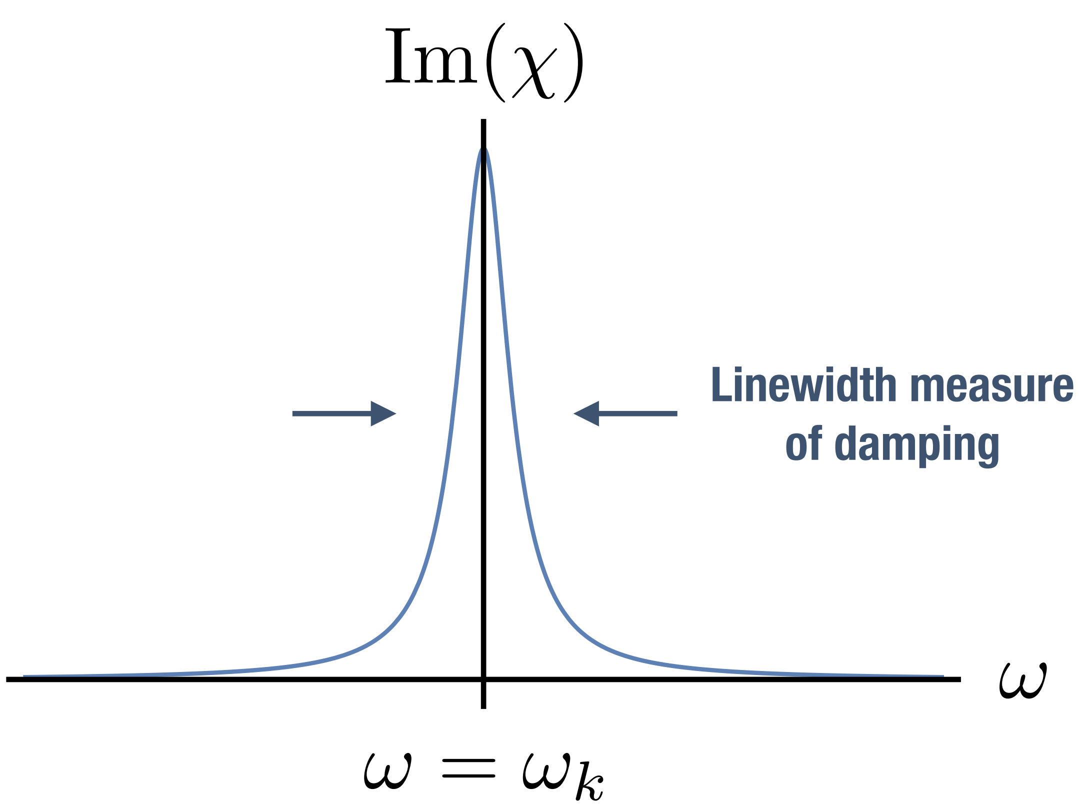
- From linear response theory, it can be shown that the frequency-dependent magnetic susceptibility can be written as

$$\chi(\omega) = \sum_k \frac{1}{\omega - \omega_k + i\Gamma_k}$$

- The susceptibility is a complex-valued Green's function and describes the magnetic response to a driving field

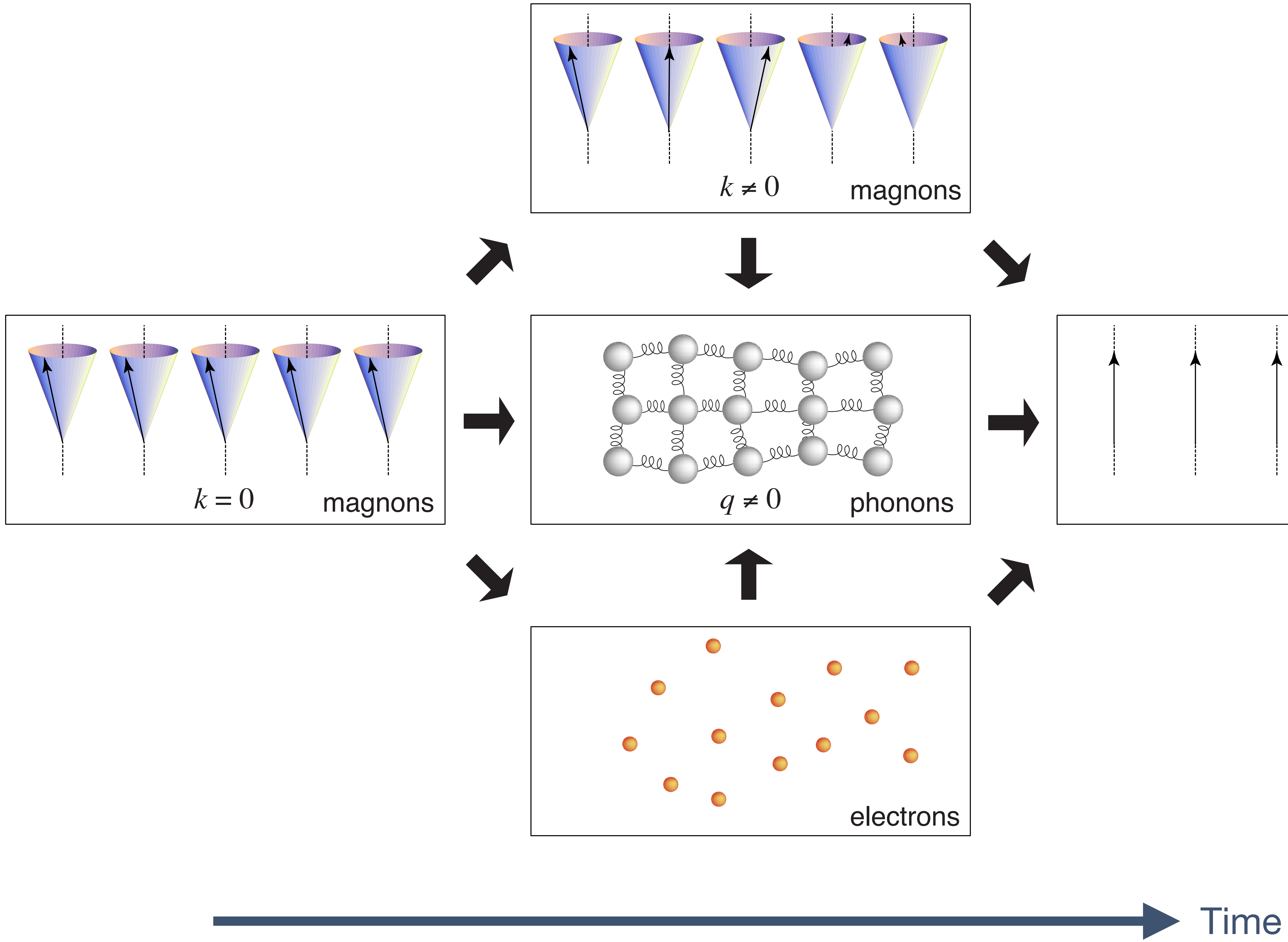
$$m(\omega) = \chi(\omega) h(\omega)$$





Relaxation processes

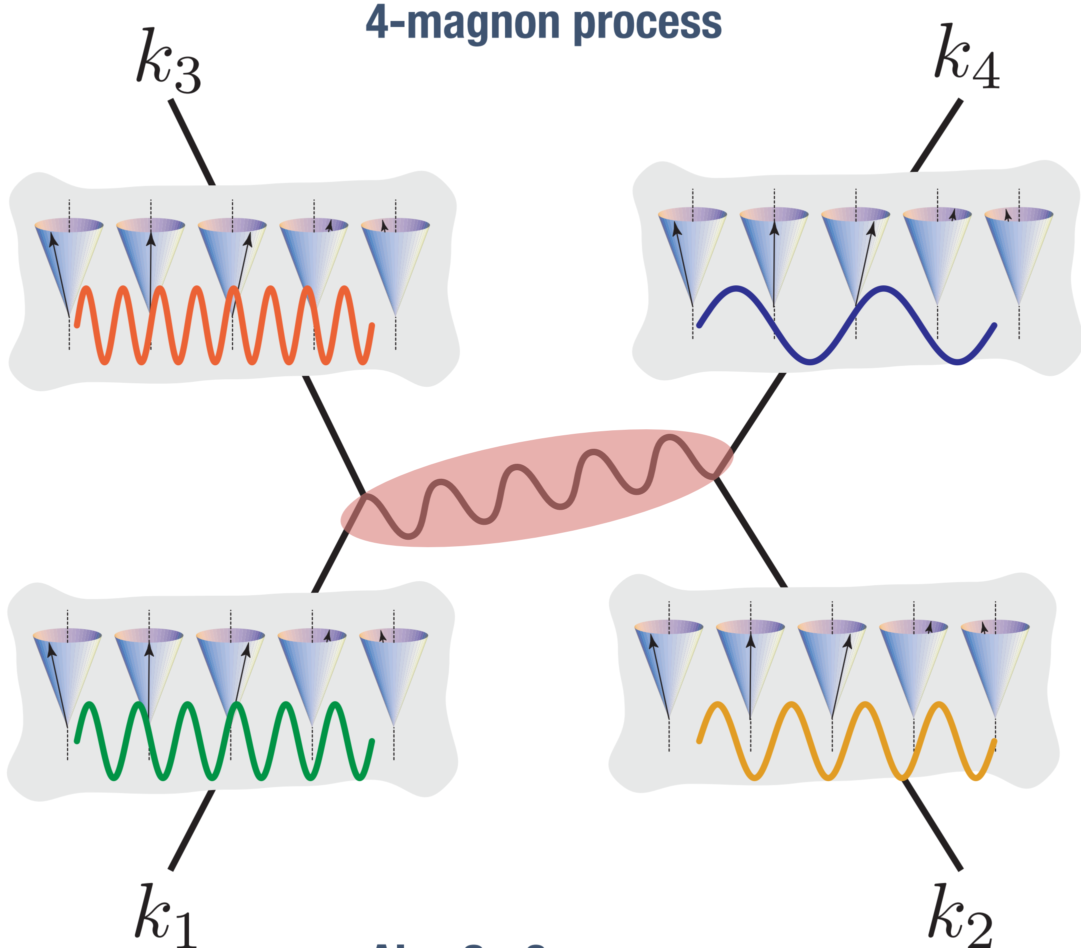
$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\mathrm{eff}} + \frac{\alpha}{M_s}\mathbf{M} \times \frac{d\mathbf{M}}{dt}$$



Relaxation processes (intrinsic)

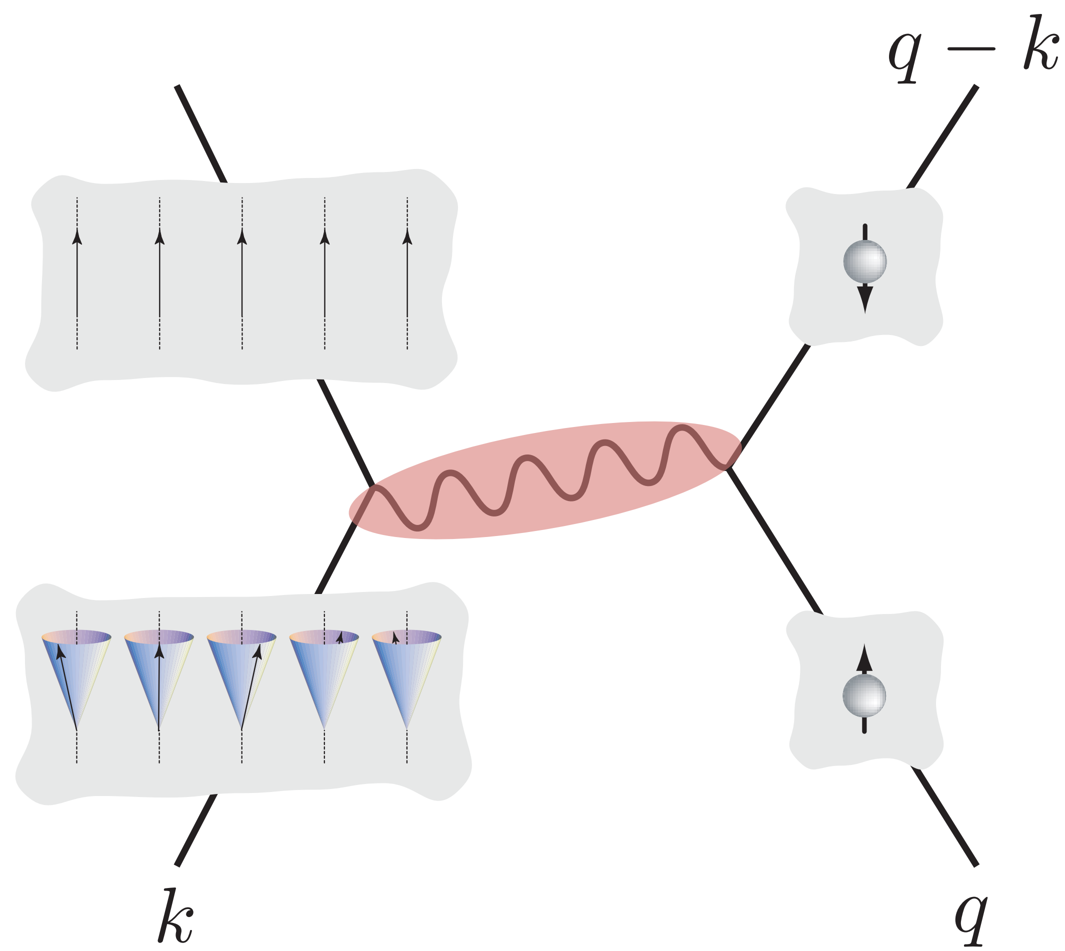
Magnon-magnon

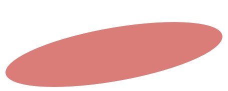
4-magnon process



Also 2-, 3-magnon processes

Magnon-electron



 Exchange, anisotropy, …

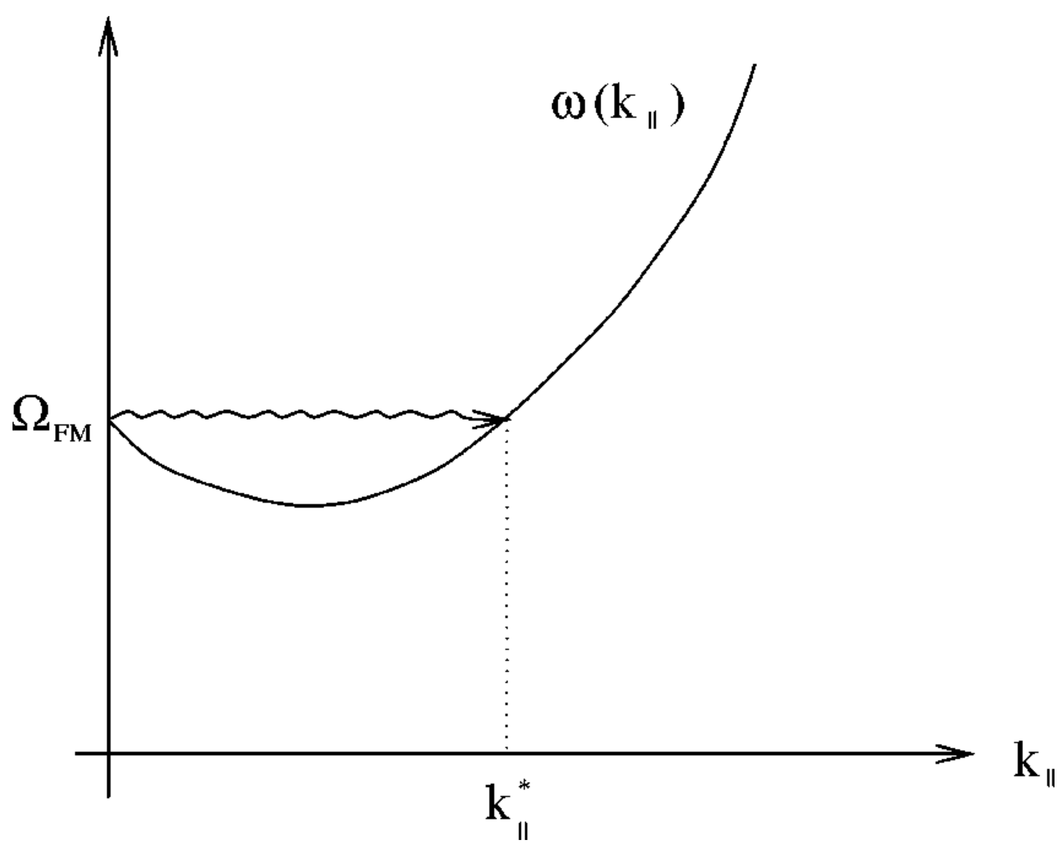
 *sd* coupling, spin-orbit

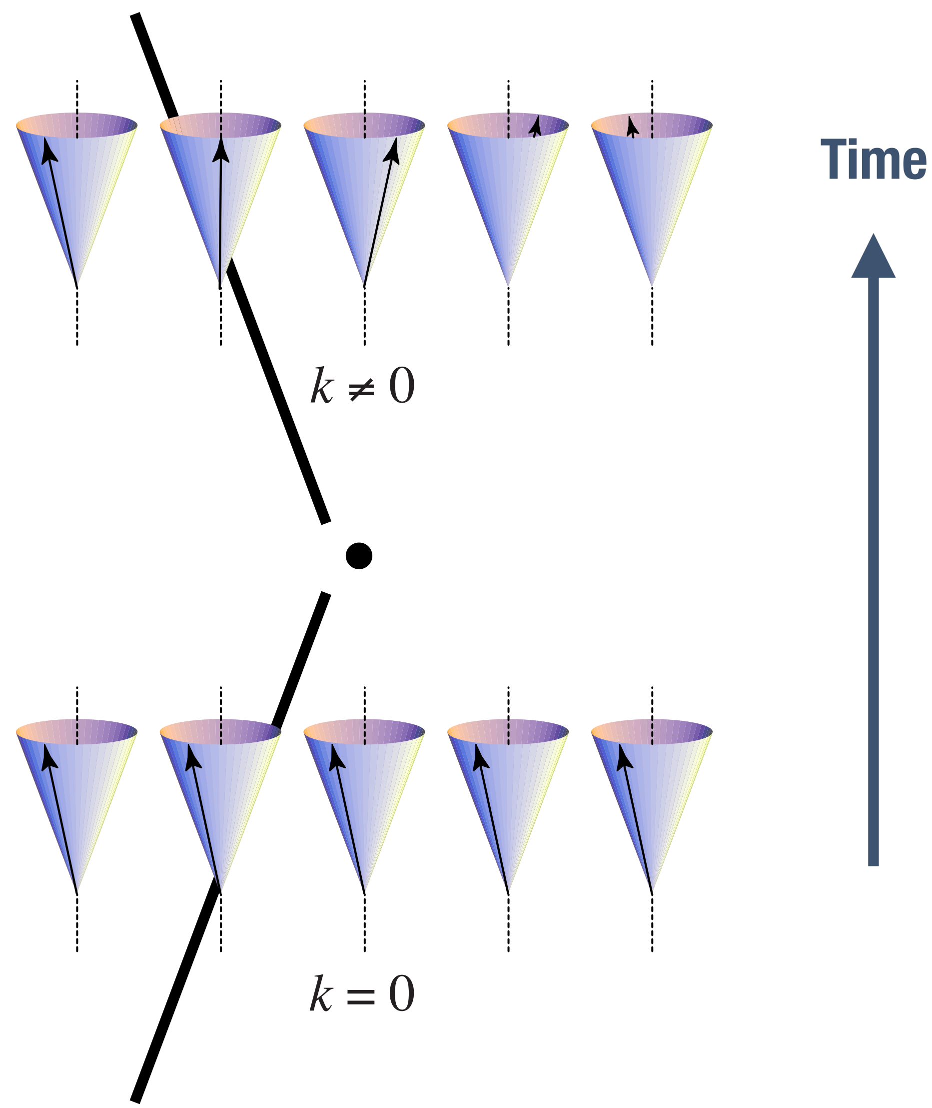
Magnon-phonon Similar to pictures above

Relaxation processes (extrinsic)

Two-magnon scattering

- Uniform (FMR) mode is damped by scattering to finite *k* spin wave





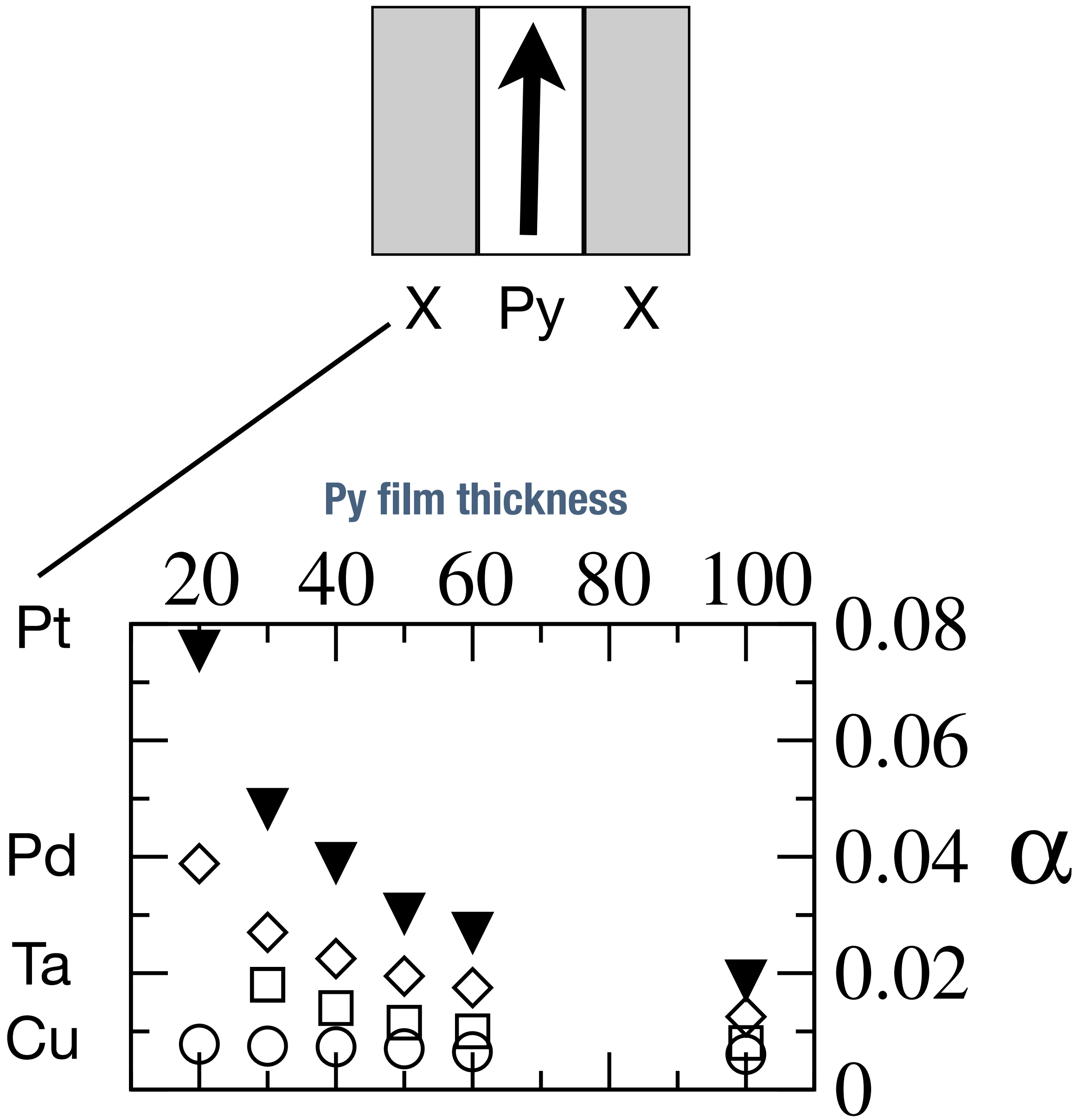
- Note that linear momentum is not conserved in this process

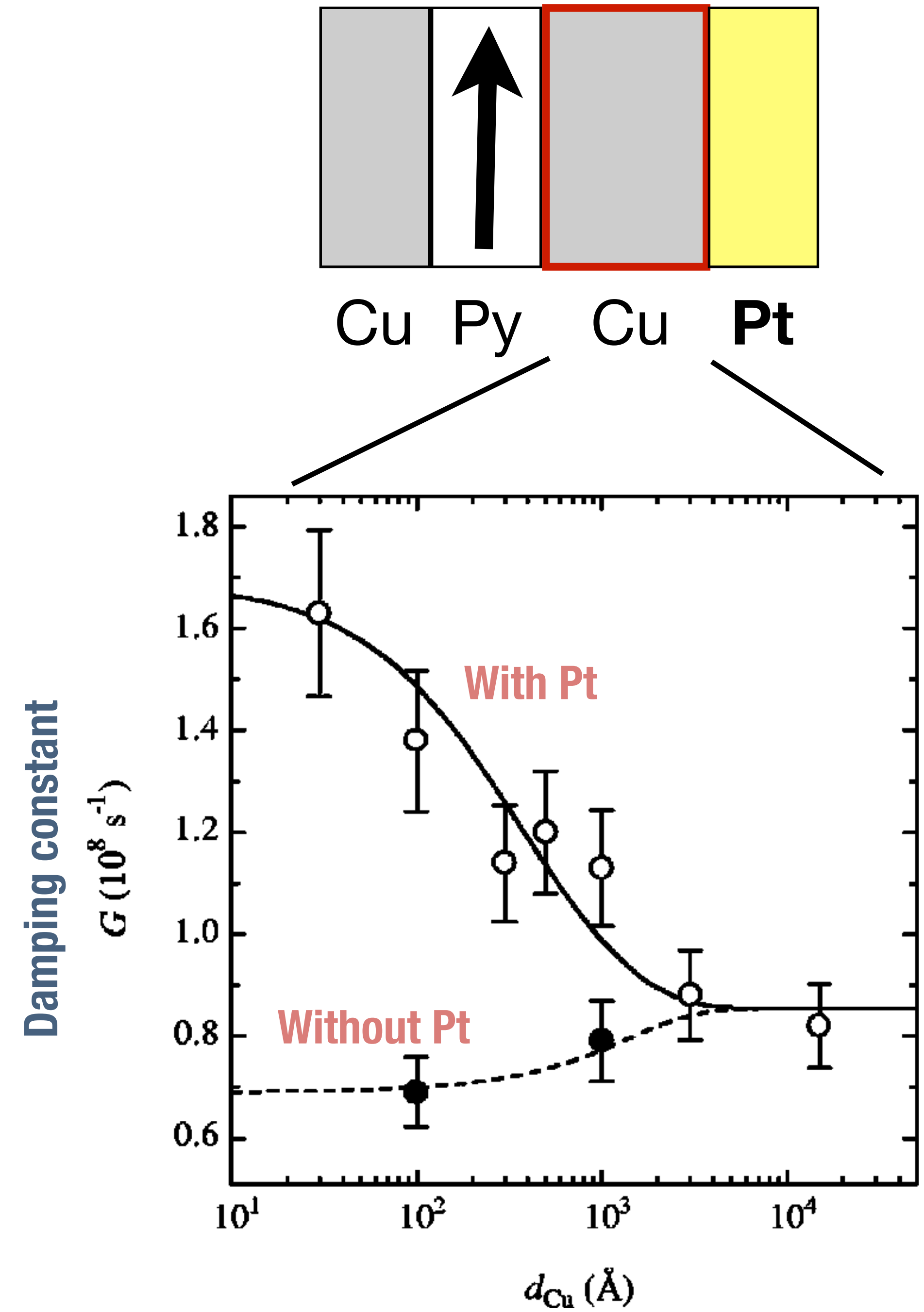
  Question: How might this occur?

Relaxation processes (extrinsic)

Spin pumping

- Example of non-local damping. Spin flips occur in neighbouring films.



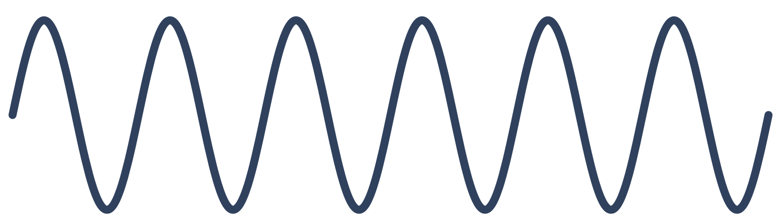


S Mizukami et al, *Jpn J Appl Phys* **40**, 580 (2001)
S Mizukami et al, *Phys Rev B* **66**, 104413 (2002)

Dynamics of solitons

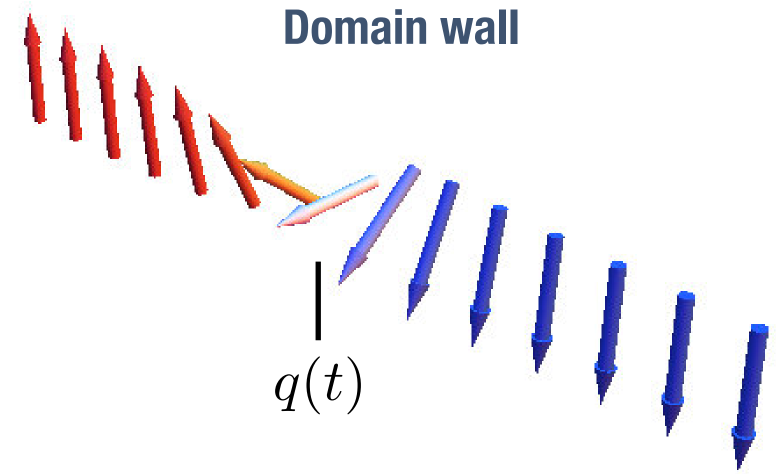
- We've seen that domain walls, vortices and skyrmions are nonuniform, nontrivial spin configurations – *topological solitons*
- By knowing their static profiles, how can we describe their motion (at velocity *v*)?

Plane wave



$$e^{ikx} \to e^{i(kx-vt)}$$

Domain wall



$$\mathbf{m}(x) \to \mathbf{m}(x-vt), \mathbf{m}[x - X_0(t)]?$$

- Unlike plane waves, in general it is not possible to translate static solution to obtain moving solution. **Need to satisfy Landau-Lifshitz!**
- Need to use method of collective coordinates, Lagrangian formulation

Lagrangian formulation

- In order to describe domain wall motion, it is convenient to use a slight different approach to describe the magnetisation dynamics
- Instead of trying to solve the Landau-Lifshitz equation, we can use another formulation in terms of the ***Lagrangian***

$$\mathcal{L} = \frac{M_s}{\gamma}\dot{\phi}(1-\cos\theta) - \mathcal{E}$$

Lagrangian density

$$L = \int dV \; \mathcal{L}$$

Lagrangian

- The idea is that if we can describe the domain wall in terms of its position *X* and conjugate momentum *P*, then we can derive its dynamics directly from the Lagrangian:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{X}} - \frac{\partial L}{\partial X} = 0$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{P}} - \frac{\partial L}{\partial P} = 0$$

Dissipation - Gilbert damping

- To describe the full dynamics, we need to include the dissipation term

  Gilbert damping can be accounted for through a *Rayleigh dissipation function* of the form:

$$\mathcal{F} = \frac{1}{2}\frac{\alpha M_s}{\gamma}\left(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2\right)$$

  which appears in the equations of motion as

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} + \frac{\partial F}{\partial \dot{q}} = 0$$

Equations of motion with dissipation

  where

$$F = \int dV\,\mathcal{F}$$

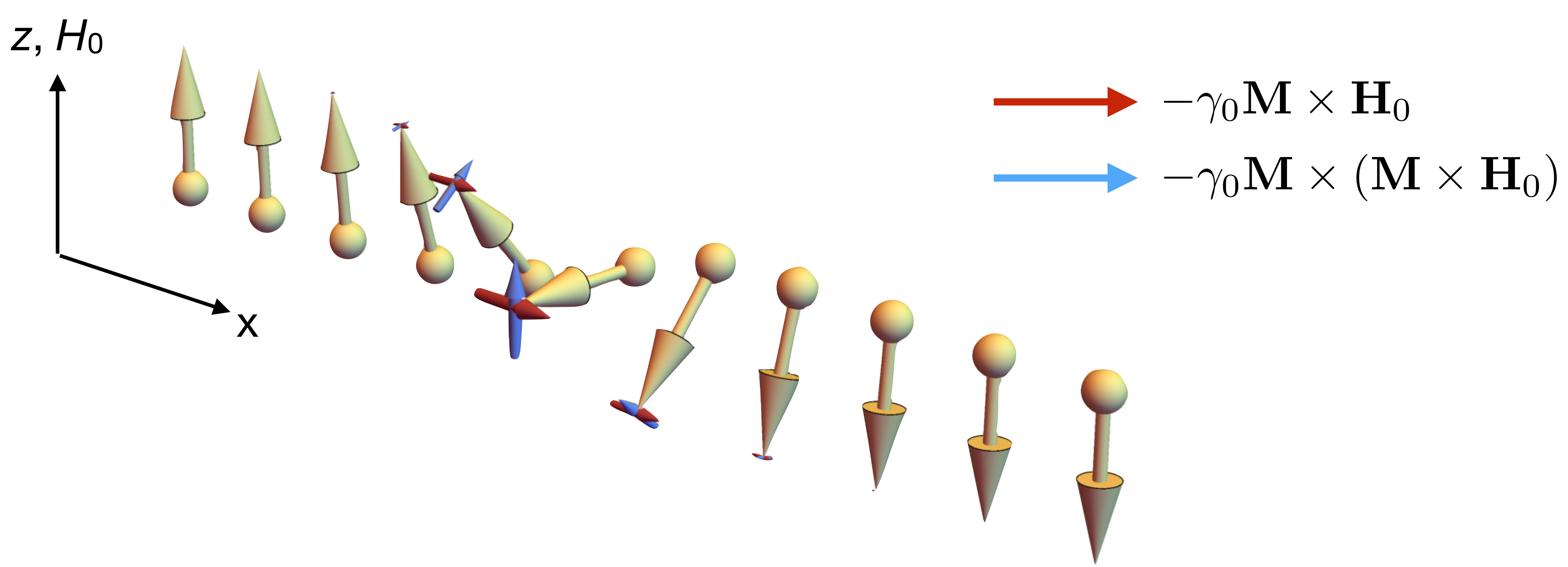
  and the *q*'s are generalised coordinates.

Domain wall dynamics

- How does a domain wall move in response to applied fields and currents?
- Recall Landau-Lifshitz equation

$$\frac{d\mathbf{M}}{dt} = -\gamma_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\alpha\gamma_0}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

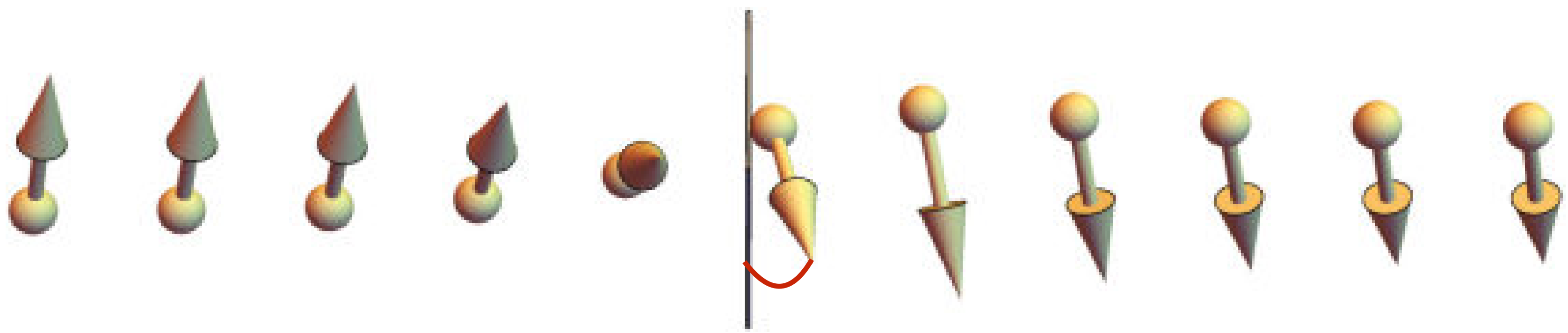
- At equilibrium, the magnetisation is aligned along the direction of $\boldsymbol{H}_{\text{eff}}$.
- Consider torques due to an applied field, $\boldsymbol{H}_0$, along +z direction (i.e., left domain)



Domain wall dynamics

- Motion of the domain wall can be described by a one-dimensional model with two variables:

$X_0(t)$ **position of domain wall centre**



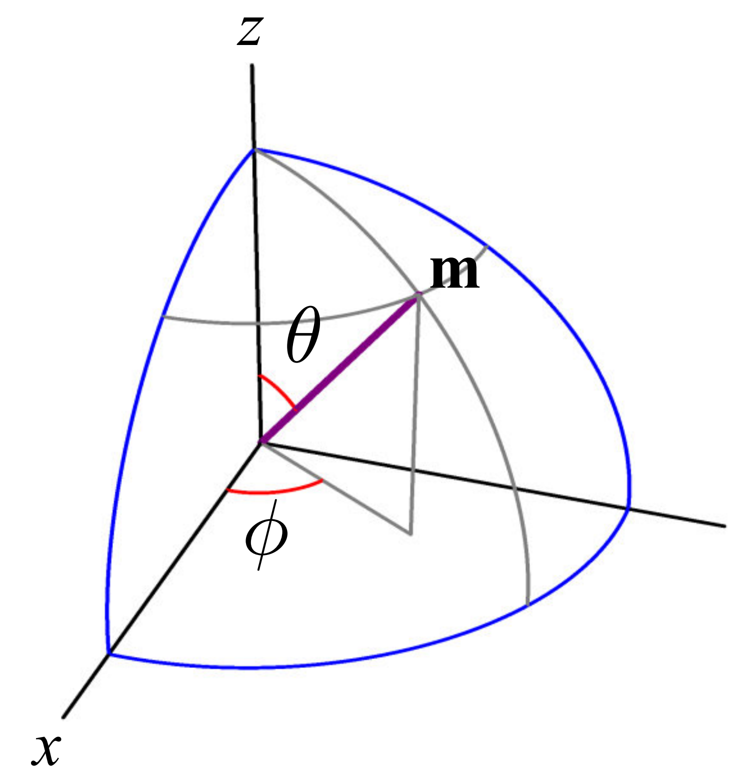
$\phi_0(t)$ **"tilt" angle, measured from xz plane**

Collective coordinates

- $X_0(t)$ translates wall profile along x (direction of propagation), $\phi_0(t)$ ensures that Landau-Lifshitz is satisfied (not Galilean invariant):

$$\theta(x,t) = 2\tan^{-1}\left[\exp\left(-\frac{x - X_0(t)}{\Delta}\right)\right]$$

$$\phi(x,t) = \phi_0(t)$$



$$\mathbf{m} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$$

Domain wall Lagrangian

- Take energy terms from MP1 (exchange, anisotropy, dipolar, Zeeman …) and integrate out the spatial degrees of freedom using trial solution to obtain Lagrangian

$$\theta(x,t) = 2\tan^{-1}\left[\exp\left(-\frac{x - X_0(t)}{\Delta}\right)\right] \qquad \phi(x,t) = \phi_0(t)$$

$$\mathbf{m} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$$

Trial solution

Berry phase ("Kinetic energy")

$$L_B = \frac{M_s}{\gamma}\int dV \dot{\phi}\,(1 - \cos\theta)$$

(Potential) Energy

$$U(X_0, \phi_0) = \int dV \quad \begin{aligned} &\mathcal{E}_{\mathrm{ex}} = A\,(\nabla \mathbf{m})^2 \\ +\; &\mathcal{E}_K = -K\,(\mathbf{m}\cdot\hat{\mathbf{e}})^2 \\ +\; &\mathcal{E}_d = -\frac{1}{2}\mu_0 \mathbf{M}\cdot\mathbf{H}_d \\ +\; &\mathcal{E}_Z = -\mu_0 \mathbf{M}\cdot\mathbf{H}_0 \end{aligned}$$

Integrate out spatial variables

$$L = L_B - U$$

(Domain wall) Lagrangian

Domain wall equations of motion

- From the Lagrangian and the dissipation function, derive the equations of motion for the domain wall:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{X}_0} - \frac{\partial L}{\partial X_0} + \frac{\partial F}{\partial \dot{X}_0} = 0$$

$$\Rightarrow \quad -\dot{\phi}_0 + \frac{\alpha \dot{X}_0}{\Delta} = -\frac{\gamma}{2M_s}\frac{\partial U}{\partial X_0}$$

Generalised forces

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\phi}_0} - \frac{\partial L}{\partial \phi_0} + \frac{\partial F}{\partial \dot{\phi}_0} = 0$$

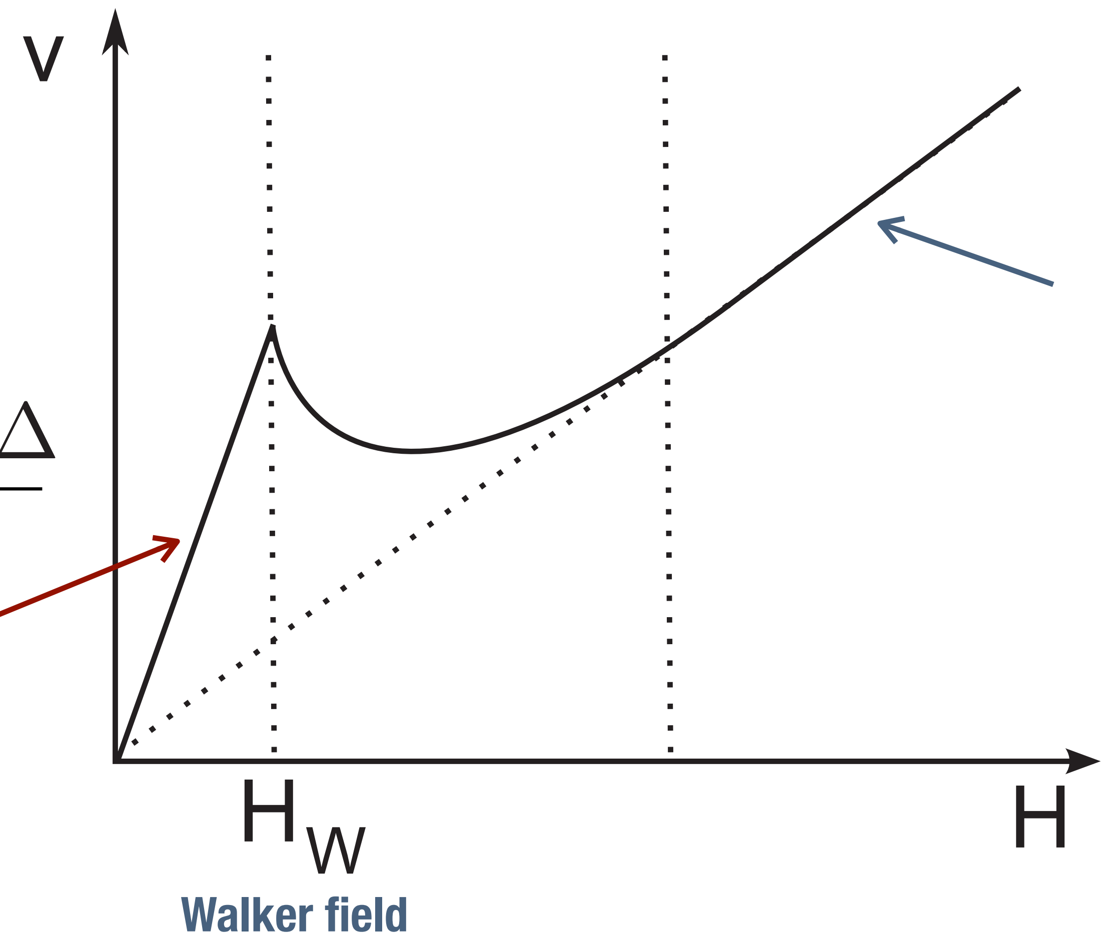
$$\Rightarrow \quad \frac{\dot{X}_0}{\Delta} + \alpha\dot{\phi}_0 = -\frac{1}{2}\gamma_0 M_s \sin 2\phi_0 - \frac{\gamma}{2M_s\Delta}\frac{\partial U}{\partial \phi_0}$$

Generalised forces

Domain wall motion under applied field

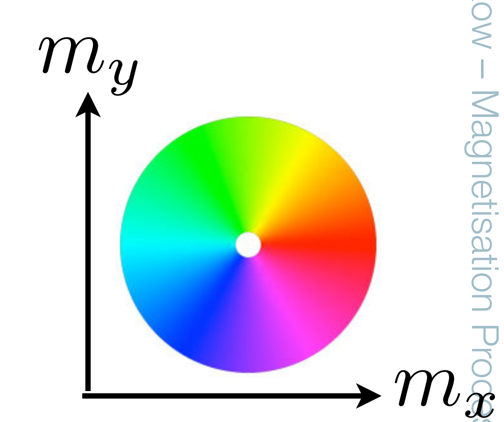
$$\dot{X}_0 = \frac{\gamma_0 H_0 \Delta}{\alpha}$$

Steady state



$$\langle \dot{X}_0 \rangle = \frac{\alpha \gamma_0 H_0 \Delta}{1+\alpha^2}$$

Precessional



- More complicated things can occur in realistic systems

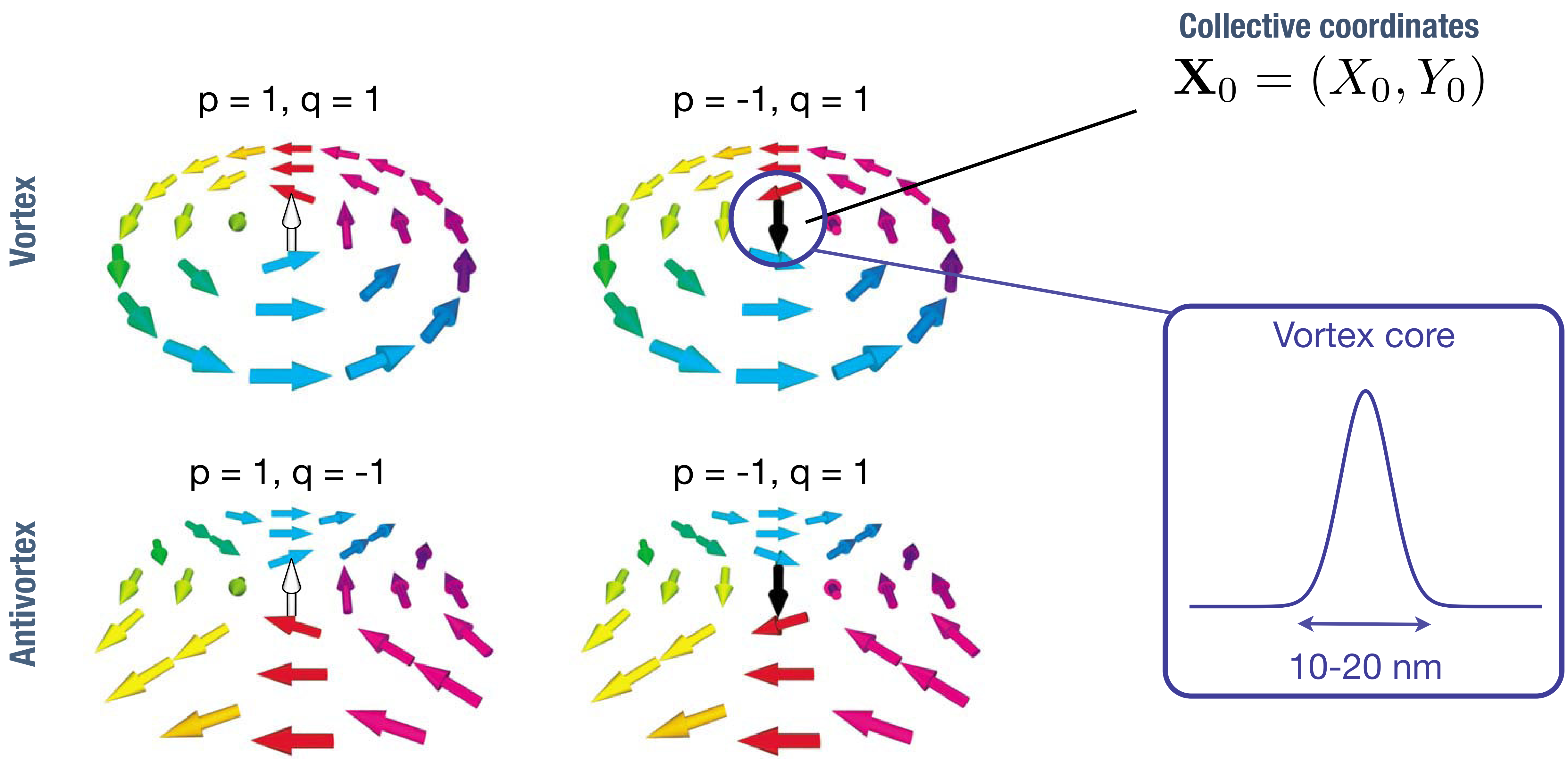
Steady state

(Animation)

Walker breakdown showing vortex nucleation at edges

Vortex dynamics

- The Lagrangian approach can be used to derive the equations of motion for a vortex
- Parametrise with the core position in the film plane (**X**$_0$), topological charge (*q*), and polarisation (*p*).



Vortex dynamics

- Vortex Lagrangian with Gilbert damping leads to "Thiele" equation, which describes the dynamics of the vortex core position

$$\mathbf{G} \times \dot{\mathbf{X}}_0 + \alpha \mathbf{D} \cdot \dot{\mathbf{X}}_0 = -\frac{\partial U}{\partial \mathbf{X}_0}$$

where

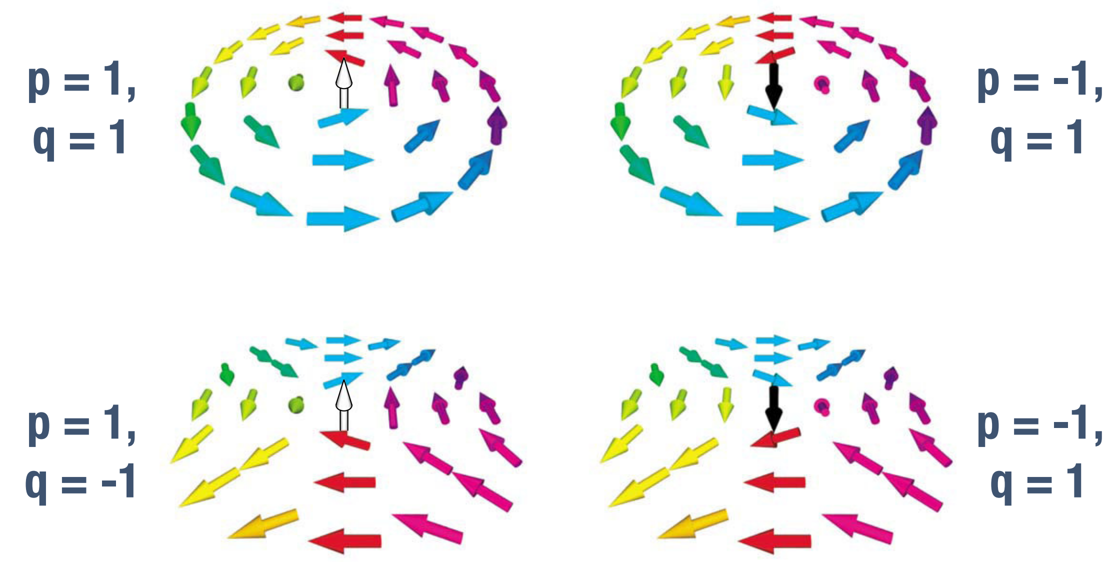
$$\mathbf{G} = \frac{M_s}{\gamma} \int dV \, \sin\theta \, (\nabla\phi \times \nabla\theta)$$ **Gyrovector**

$$\mathbf{D} = \frac{M_s}{\gamma} \int dV \, \left(\nabla\theta \otimes \nabla\theta + \sin^2\theta \, \nabla\phi \otimes \nabla\phi \right)$$ **Damping tensor**

- The gyrovector is

$$\mathbf{G} = \frac{2\pi M_s d p q}{\gamma} \hat{\mathbf{z}}$$

d: film thickness



Vortex dynamics

- The natural motion for a magnetic vortex is *gyrotropic.* In fact, the motion is intrinsically non-Newtonian. Consider the conservative case without damping:

$$\mathbf{G} \times \dot{\mathbf{X}}_0 = -\frac{\partial U}{\partial \mathbf{X}_0}$$

With the definition of the gyrovector:

$$-G\dot{Y}_0 = -\frac{\partial U}{\partial X_0}$$

$$G\dot{X}_0 = -\frac{\partial U}{\partial Y_0}$$

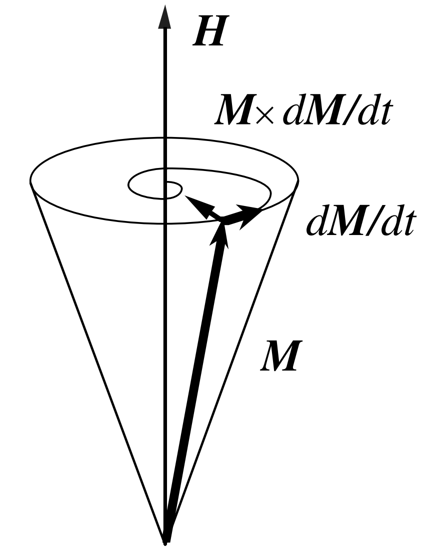
$$G = \frac{2\pi M_s dpq}{\gamma}$$

For a Newtonian system, we have (for comparison)

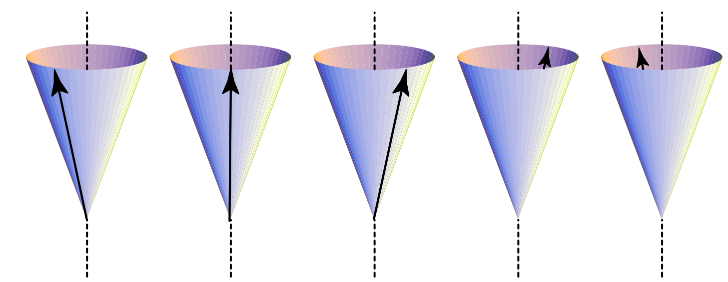
$$m\frac{d^2\mathbf{X}_0}{dt^2} = -\frac{\partial U}{\partial \mathbf{X}_0}$$

mass

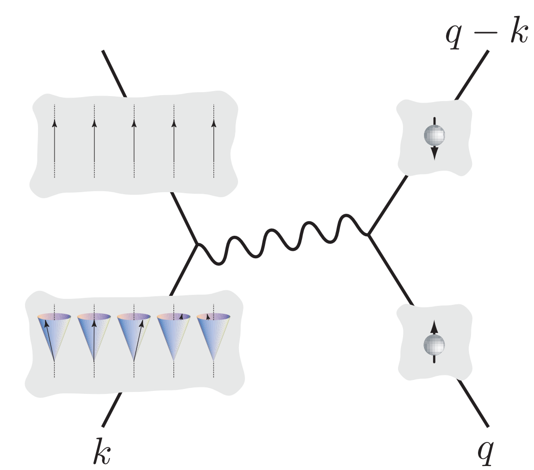
Summary



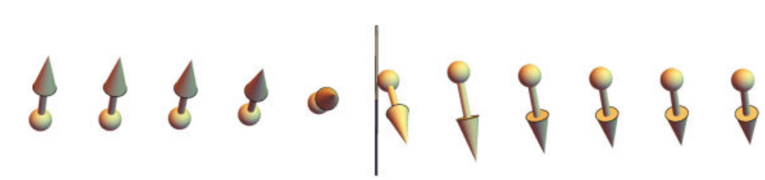
- Landau-Lifshitz equation provides framework to describe damped precessional dynamics



- Spin waves
  Linear (small amplitude) excitations, useful probes



- Relaxation processes
  Gilbert, Bloch-Bloembergen; intrinsic and extrinsic processes



- Domain wall and vortex dynamics
  Lagrangian formulation, collective coordinates