

Magnetic ordering, magnetic anisotropy and the mean-field theory



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Ferromagnets

Mean-field approximation

Curie temperature and critical exponents

Magnetic susceptibility

Temperature dependence of magnetization and Bloch law

Antiferromagnets

Neel temperature

Susceptibility

Magnetic anisotropy

Magnetic-dipole contribution

Magneto-crystalline anisotropy and its temperature dependence

Ferro- and antiferromagnets in an external field

Ferromagnets

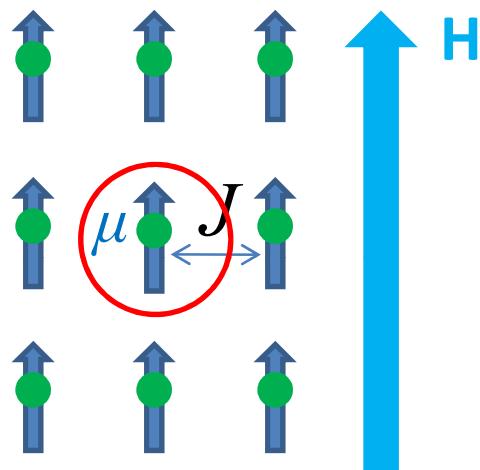
Mean-field approximation

Curie temperature and critical exponents

Magnetic susceptibility

Temperature dependence of magnetization and Bloch law

Mean-field approximation: Weiss molecular field

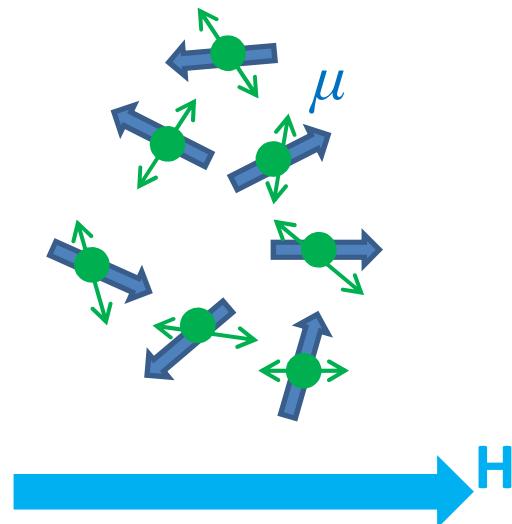


$$H_m = wM \quad \text{Weiss molecular field}$$

- field created by neighbor magnetic moments

Alignment of a magnetic moment μ
in the total field $H+wM$?

A brief reminder: independent magnetic moments in external field



➤ Thermal energy:

$$U_T = kT$$

At room temperature: $4.1 \cdot 10^{-21} J$

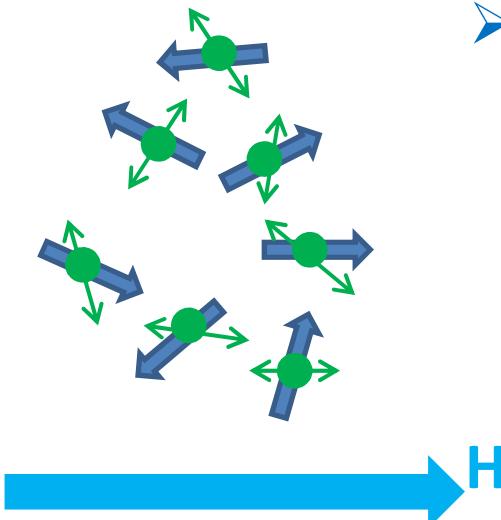
➤ Potential energy of a magnetic moment in the field H:

$$U_H = -\mu H \cos \theta$$

In a field of 1MA/m: $1.2 \cdot 10^{-23} J$

$$U_H / U_T \sim 0.003$$

Paramagnetism: independent magnetic moments in external field



➤ Induced magnetization

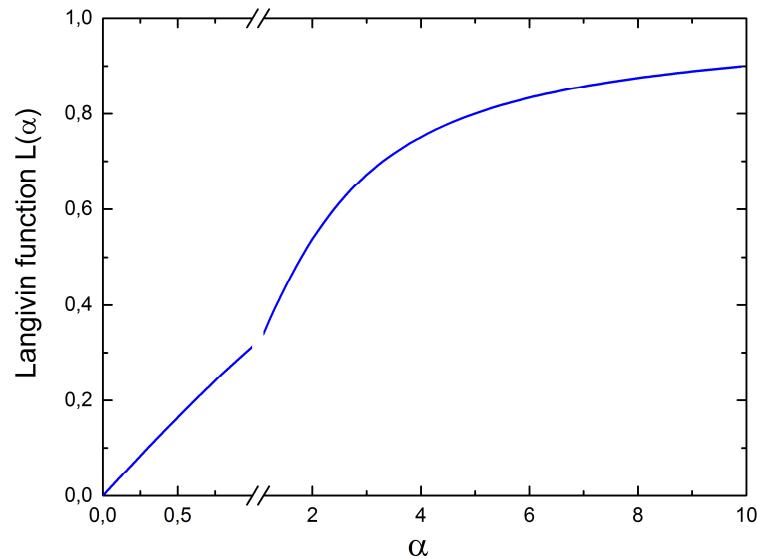
Probability for a magnetic moment to orient at the angle θ :

$$\exp\left(-\frac{U_H}{U_T}\right) = \exp\left(\frac{\mu H}{kT} \cos \theta\right)$$

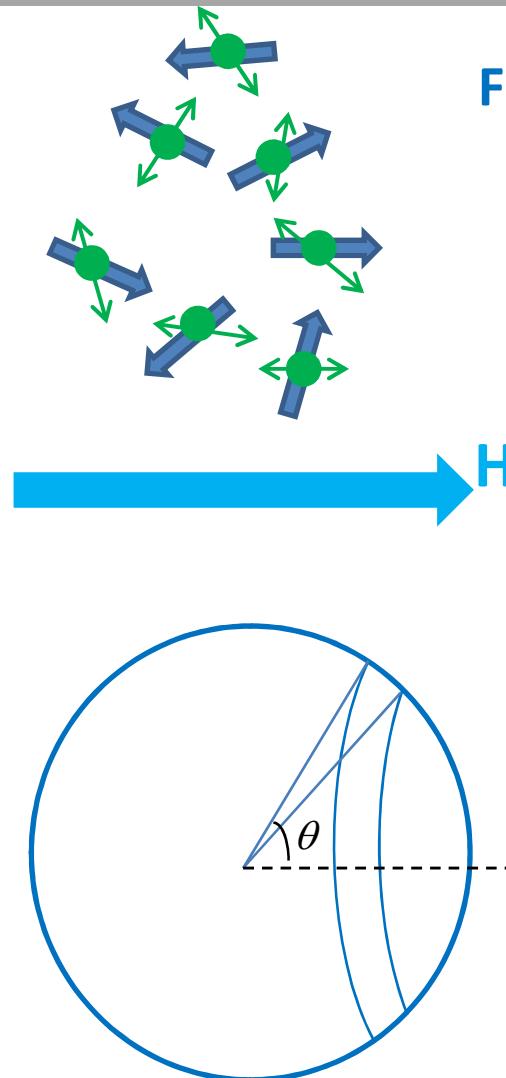
Full magnetization along the field:

$$M = N\mu \left(\coth \alpha - \frac{1}{\alpha} \right) \quad \alpha = \frac{\mu H}{kT}$$

Langevin function $L(\alpha)$

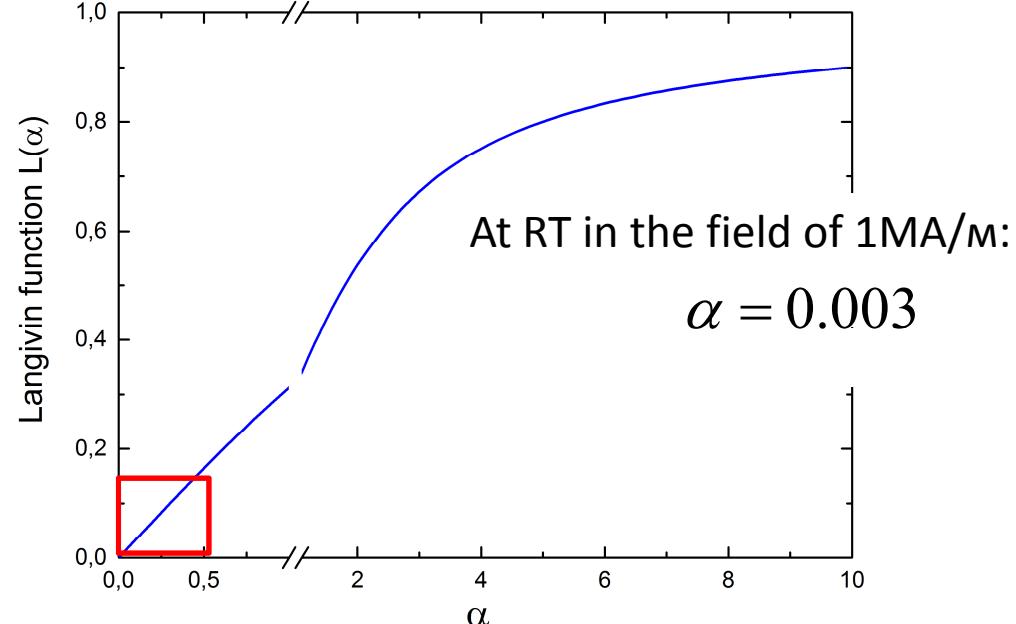


Paramagnetism: independent magnetic moments in external field



Full magnetization along the field:

$$M = N\mu \left(\coth \alpha - \frac{1}{\alpha} \right) \quad \alpha = \frac{MH}{kT}$$



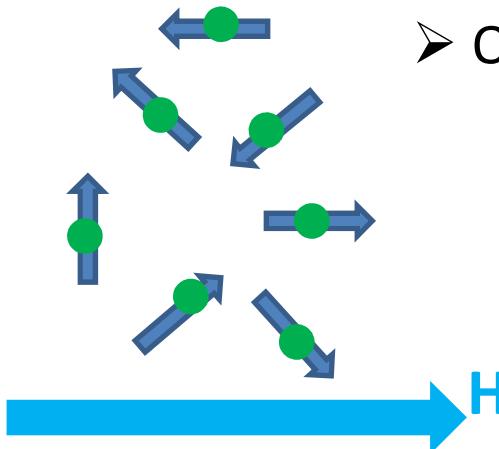
$$M = N\mu L(\alpha) = N\mu(\alpha / 3 - \dots) = \frac{N\mu^2}{3kT} H$$

Paramagnetic susceptibility:

$$\chi_{para} = \frac{\partial M}{\partial H} = \frac{N\mu^2}{3kT}$$

Curie law

Paramagnetism: independent magnetic moments in external field



➤ Quantization of the magnetic moment

$$\mu_z = g\mu_B J_z$$

$$J_z = J, J-1, \dots, -J$$

$$\alpha = \frac{gJ\mu_B H}{kT}$$

Resulting magnetization along the field:

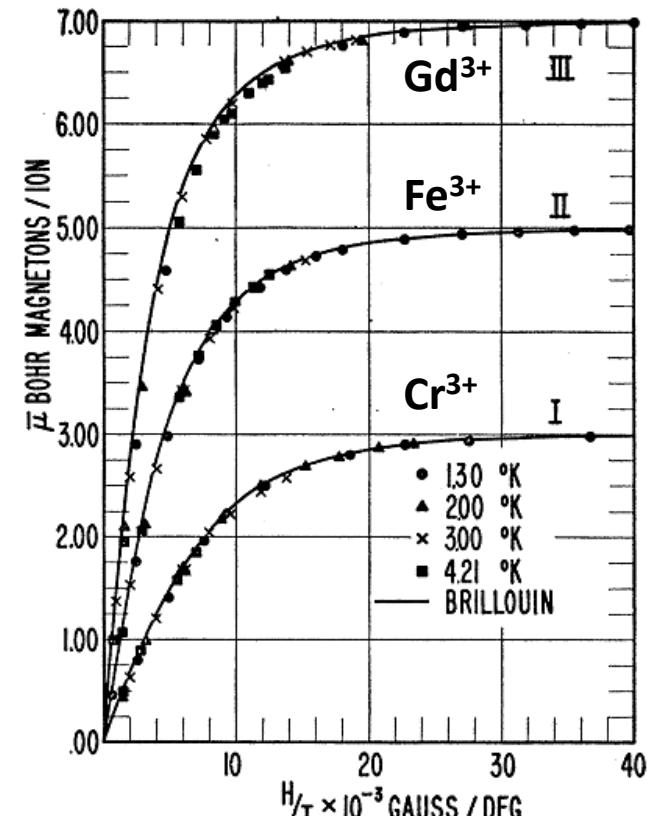
$$M = N g J \mu_B B_J(\alpha)$$

Brillouin function

At small α : $M = N g J \mu_B \left(\frac{J+1}{3J} \alpha - \dots \right)$

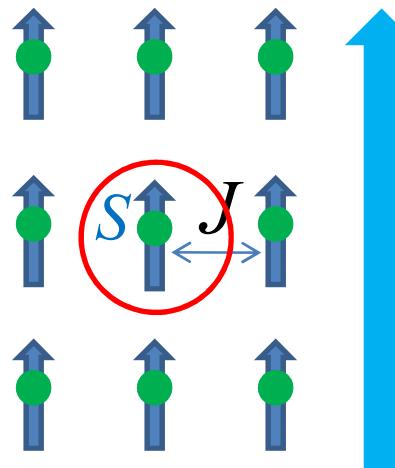
Curie law

$$\chi_{para} = \frac{Ng^2 J(J+1)\mu_B^2}{3kT}$$



[Henry, PR 88, 559 (1952)]

Ferromagnets: Weiss molecular field



H Averaged projected magnetic moment:

$$\langle \mu_z \rangle = g\mu_B S B_S(\alpha)$$



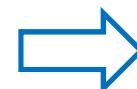
$$\langle S_z \rangle = S B_S(\alpha)$$

$$\alpha = \frac{g\mu_B S(H + H_m)}{kT}$$

Weiss field due to exchange interactions J with N nearest neighbors:

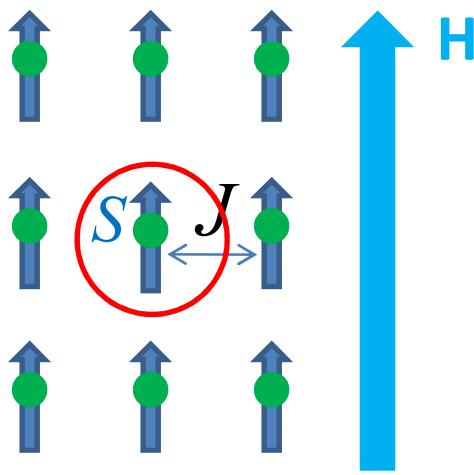
$$H_m = \frac{NJ\langle S_z \rangle}{g\mu_B}$$

$$\langle S_z \rangle = S B_S \left(\frac{g\mu_B S \left(H + \frac{NJ\langle S_z \rangle}{g\mu_B} \right)}{kT} \right)$$



Self-consistent solution

Back to ferromagnets: Weiss molecular field

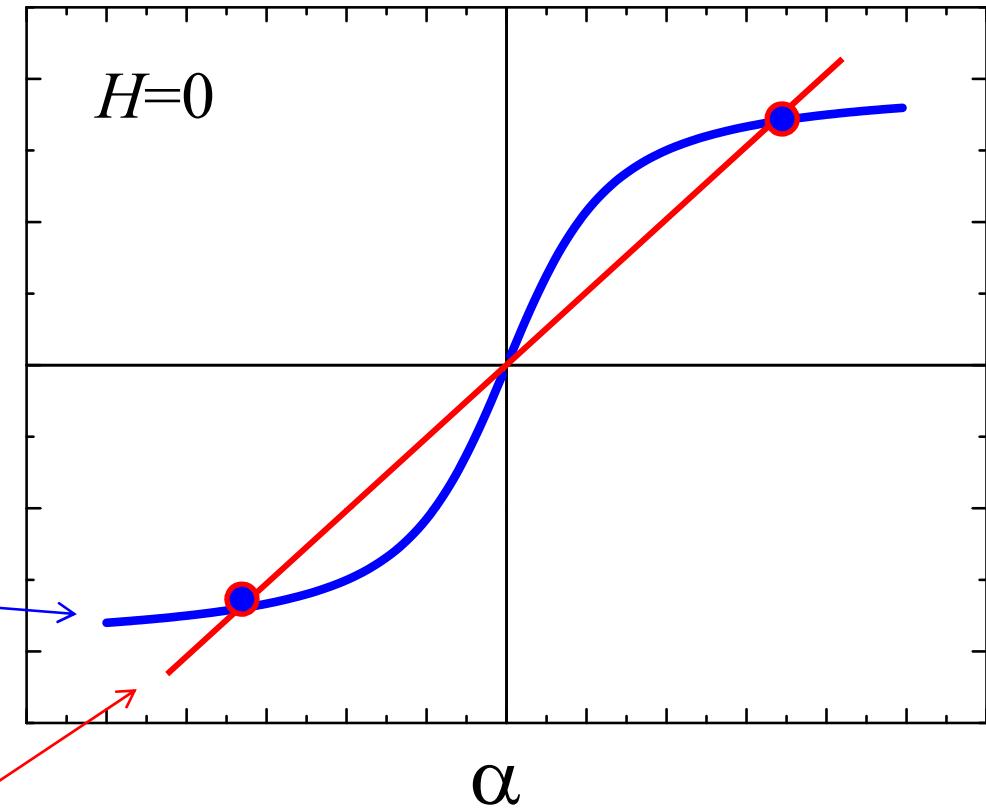


$$\langle S_z \rangle = SB_S(\alpha)$$

$$\alpha = \frac{g\mu_B S(H + H_m)}{kT}$$

$$\langle S_z \rangle = \frac{kT}{NJS} \alpha - \frac{g\mu_B}{NJ} H = 0$$

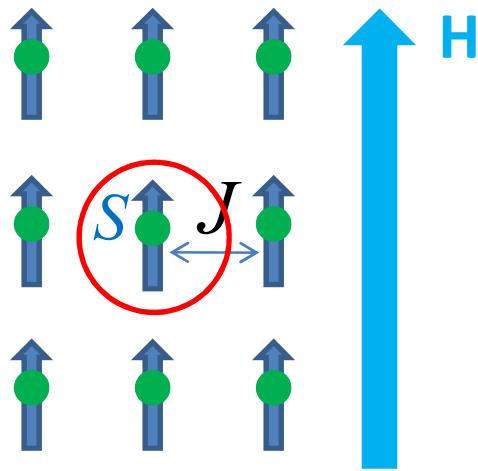
$$S_z(\alpha) > 0$$



2 non-zero solutions: $\pm \langle S_z \rangle$

**2 opposite orientations
of the order parameter**

Ferromagnets ($S=1/2$): Curie temperature



Variations of these curves with T:

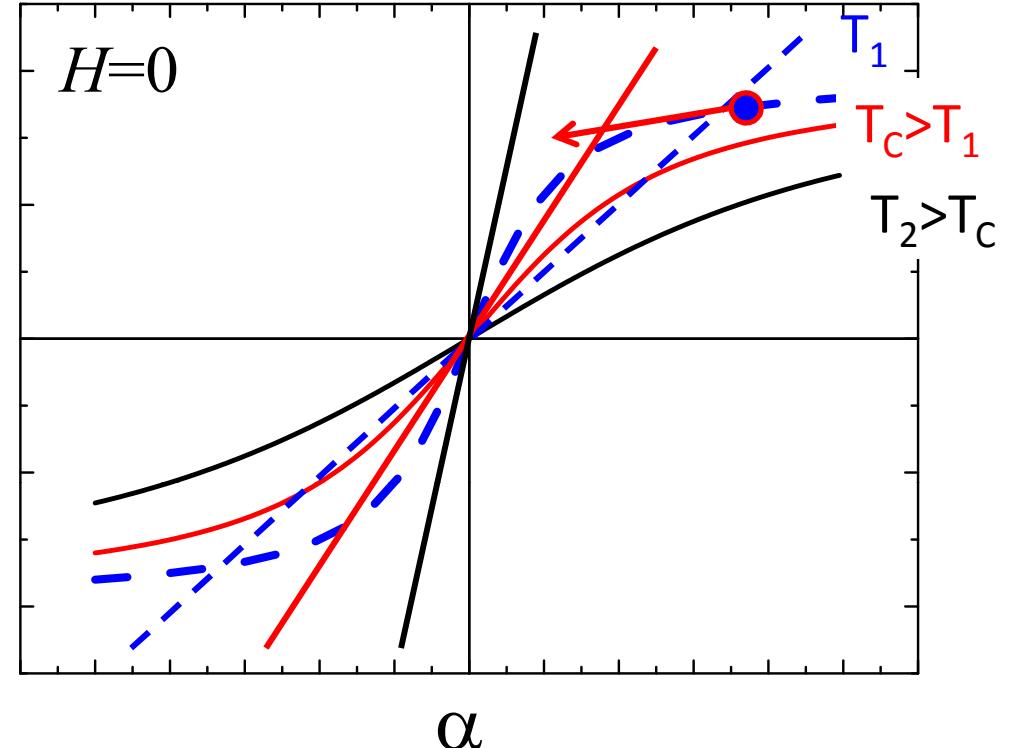
$$\langle S_z \rangle = S B_S(\alpha)$$

$$\langle S_z \rangle = \frac{kT}{NJS} \alpha - \frac{g\mu_B}{NJ} H$$

$\underset{=0}{\textcircled{d}}$

$$\text{At } T \geq T_c \quad \langle S_z \rangle = 0$$

$$B_S(\alpha)$$

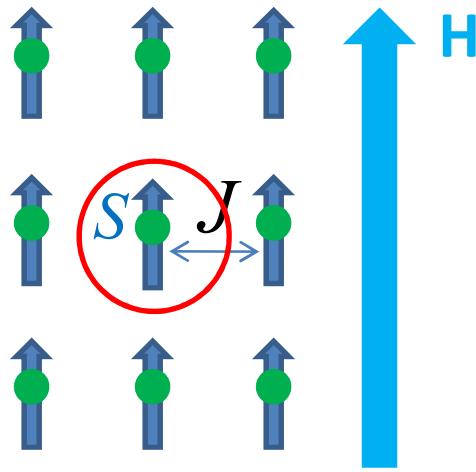


$$\text{Ordering temperature: } T_c = \frac{JN}{k}$$

$\langle S_z \rangle$ changes continuously with T

2nd order phase transition

Ferromagnets ($S=1/2$): Magnetization in a vicinity of T_c

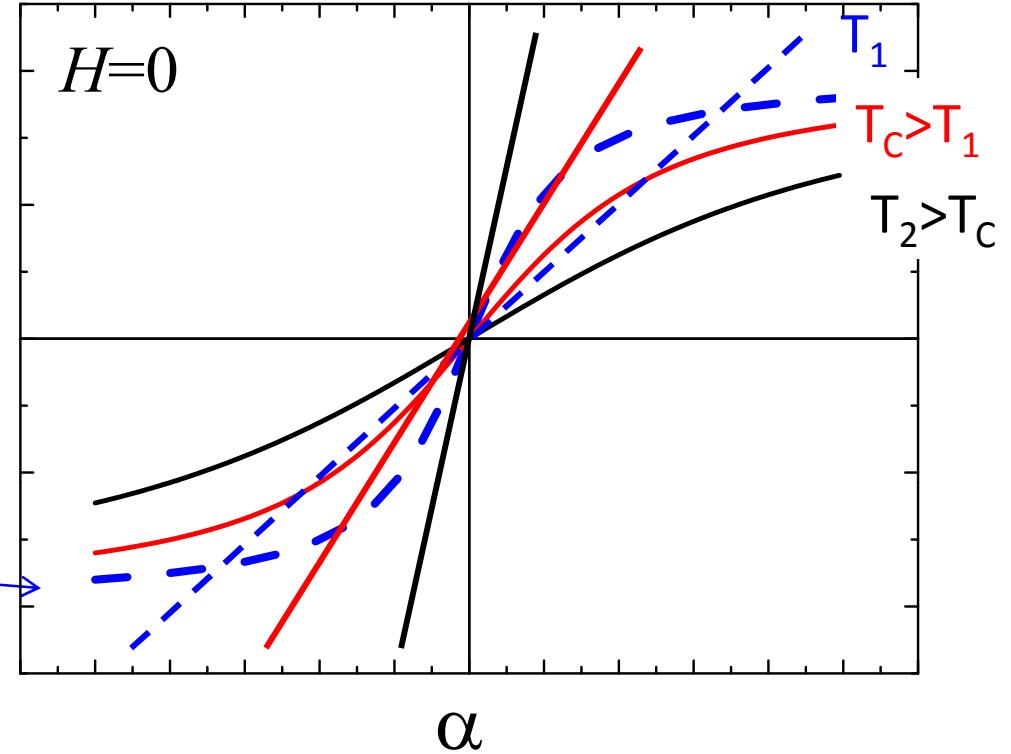


$$\langle S_z \rangle = SB_S(\alpha)$$

$$\langle S_z \rangle = \frac{kT}{NJS} \alpha - \frac{g\mu_B}{NJ} H \quad _{=0}$$

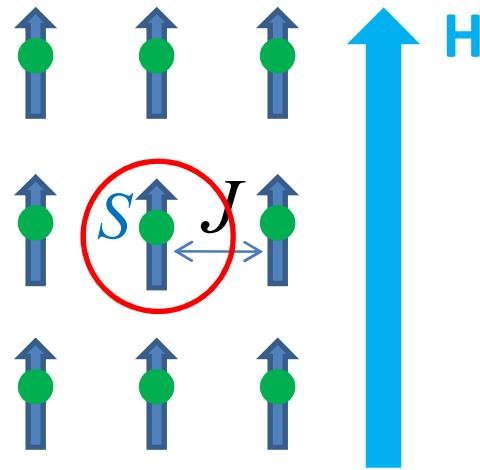
At $T \rightarrow T_c$ $\langle S_z \rangle \rightarrow 0$
 $\alpha \rightarrow 0$

$$B_S(\alpha)$$



$$\langle S_z \rangle = \frac{3}{4} \left(1 - \frac{T_c}{T} \right)^{1/2}$$

Ferromagnets ($S=1/2$): Curie temperature



Ordering temperature: $T_C = \frac{JN}{k}$

2nd order phase transition

Mean-field approximation
predicts:

1D case: $T_C = 2\frac{J}{k} > 0$

2D case: $T_C = 4\frac{J}{k}$

3D case: $T_C = 6\frac{J}{k}$

Reality:

No magnetic ordering

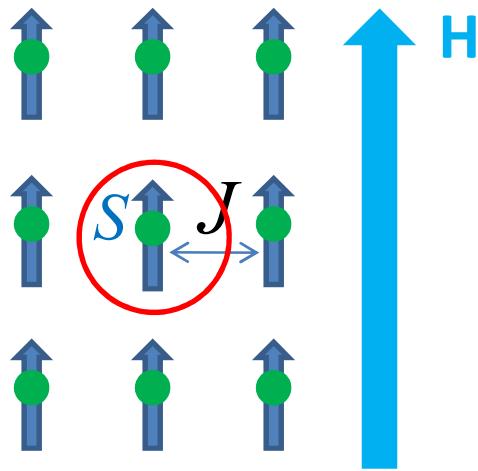
$$T_C = 2.269 \frac{J}{k}$$

$$T_C = 4.511 \frac{J}{k}$$

Agreement
gets better
for the 3D case

Reason:
fluctuations
were neglected

Ferromagnets ($S=1/2$): at low temperatures

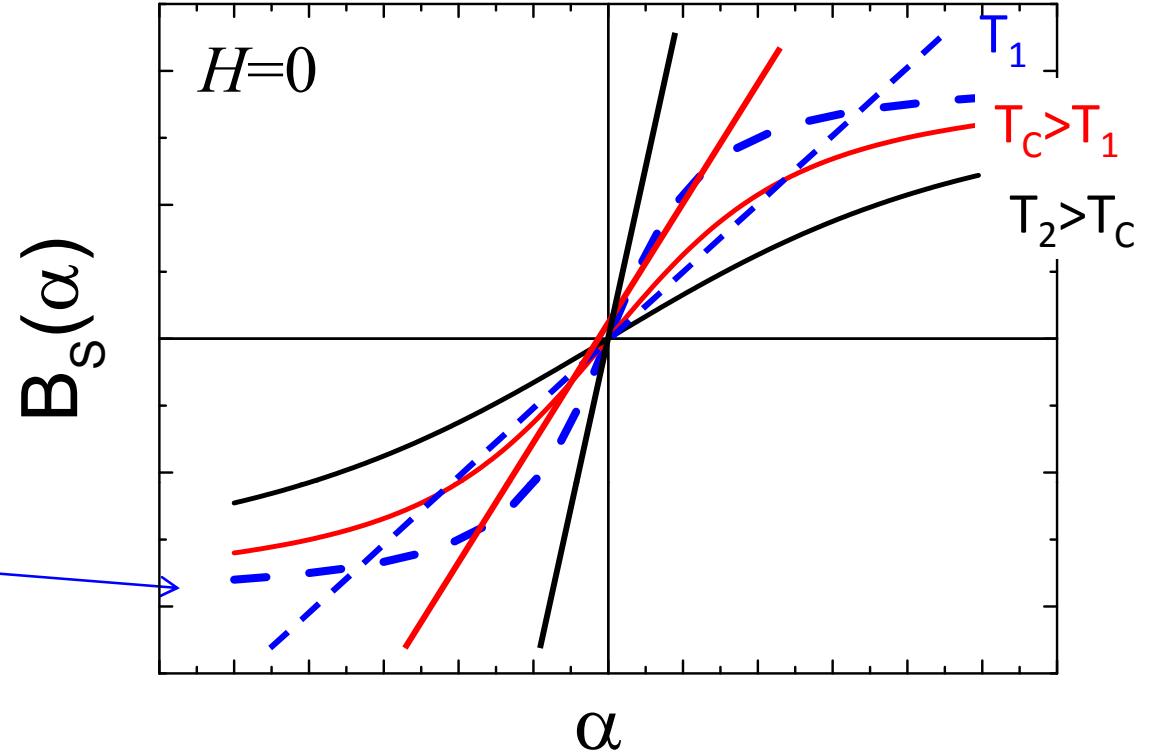


$$\langle S_z \rangle = S B_S(\alpha)$$

$$\alpha = \frac{g\mu_B S(H + H_m)}{kT}$$

At $T \rightarrow 0$ $\langle S_z \rangle \rightarrow S$

$$\alpha \rightarrow \infty$$



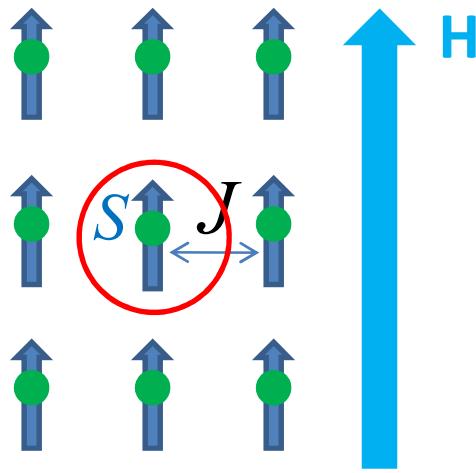
Prediction:

$$\langle S_z \rangle = \frac{1}{2} \left(1 - 2e^{-2T_C/T} \right)$$

Reality:

$$M(T) = M_0 \left(1 - (T/T_C)^{3/2} \right)$$

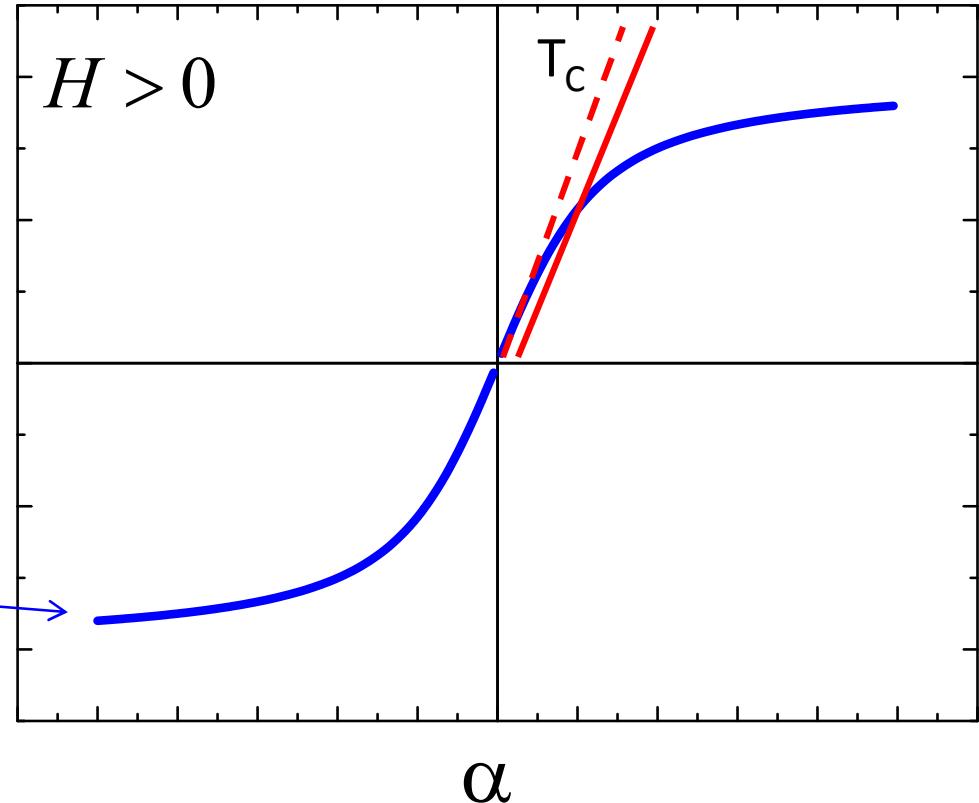
Ferromagnets ($S=1/2$) in applied field



$$\langle S_z \rangle = SB_S(\alpha)$$

$$\alpha = \frac{g\mu_B S(H + H_m)}{kT}$$

$$S_z(\alpha) \wedge v$$



Susceptibility

Above T_C
in small fields

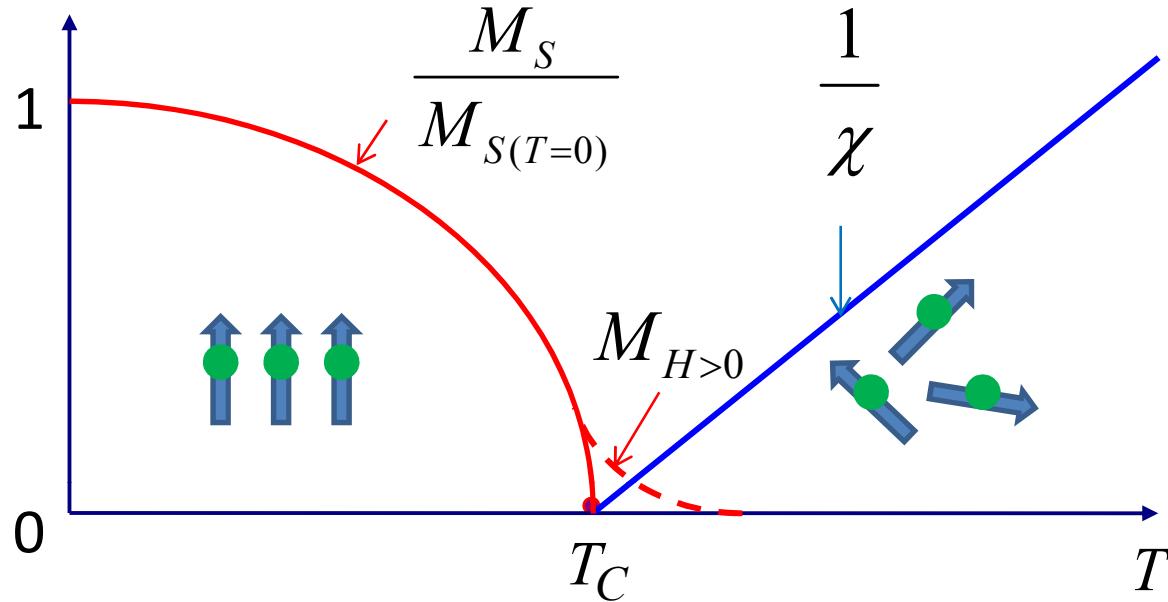
$$\langle S_z \rangle \approx \frac{\mu_B H}{k(T - T_C)}$$

$$\chi_{T>T_C} \approx \frac{1}{k(T - T_C)}$$

**Curie-Weiss
law**

Ferromagnets

Weiss molecular field theory:



Temperature Curie

$$T_C = \frac{JN}{k}$$

$$\langle S_z \rangle \propto \left(1 - \frac{T_C}{T}\right)^{1/2}$$

Critical exponent
 $\beta = 1/2$

Curie-Weiss law

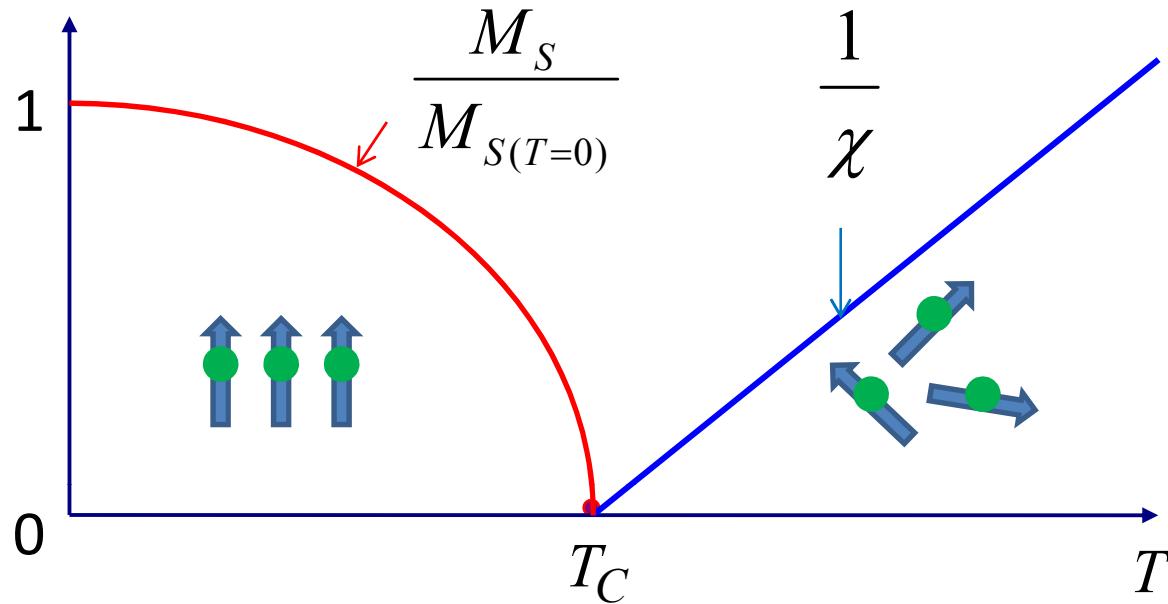
$$\chi_{T>T_C} \propto \frac{1}{k(T - T_C)}$$

Critical exponent
 $\gamma = -1$

Landau theory of phase transitions gives the same values of critical exponents

Ferromagnets

Weiss molecular field theory:



$$M_S \propto (T_C - T)^\beta$$

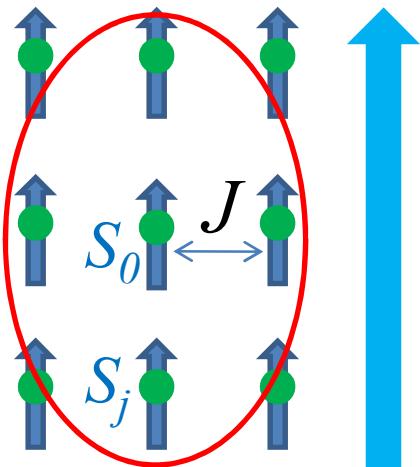
$$\chi \propto (T_C - T)^{-\gamma}$$

$$\xi \propto (T_C - T)^{-\nu}$$

Spin-spin correlations

System	β	γ	ν
2D Ising	0.125	1.75	1
2D XY system	0.231 ^a	—	—
2D Heisenberg	—	—	—
3D Ising	0.325	1.241	0.630
3D XY system	0.345	1.316	0.669
3D Heisenberg	0.365	1.386	0.705
Landau theory and MF	0.5	1	0.5

Mean-field approximation: Bethe mean-field theory



From uncorrelated spins
to uncorrelated clusters of spins

- S_0 interaction with its 4 neighbors is treated exactly (cluster)
- S_j are subject to effective (Weiss) field

$$T_C = \frac{2J}{k \ln(N/(N-2))}$$

Approximation predicts:

1D case: $T_C = 0$

2D case: $T_C = 2.885 \frac{J}{k}$

3D case: $T_C = 4.993 \frac{J}{k}$

Reality:

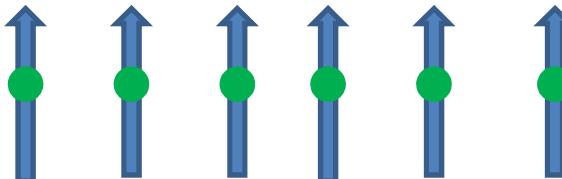
No magnetic ordering

$$T_C = 2.269 \frac{J}{k}$$

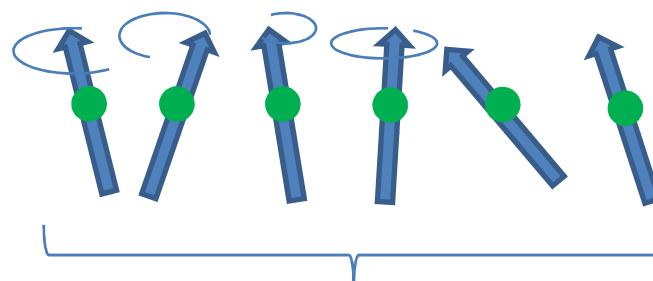
$$T_C = 4.511 \frac{J}{k}$$

Agreement
is improved

Temperature dependence of magnetization

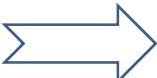
At T=0 
 $\langle S_z \rangle \rightarrow S$ $M(r) = M_0$
Minimum of the exchange energy

At T>0 Thermal fluctuations



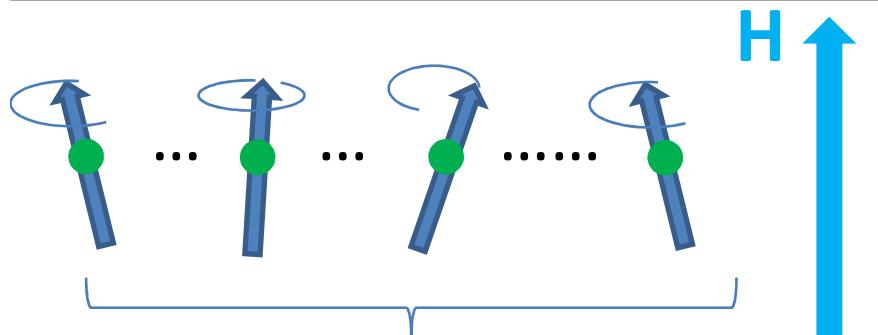
$$M(r)$$

Superposition of harmonic waves – **thermal spin waves**
(introduced by Bloch)

Nonparallel S_i  Increase of the exchange energy

The magnitude of the gradient of M is important!

Magnetization at T>0



Characteristic period
of magnetization variation λ

$\lambda \gg a$ (inter-atomic distance)

$\lambda \ll L$ (sample size)



$$\mathbf{M}(\mathbf{r}) = \mathbf{M}_0 + \Delta\mathbf{M}(\mathbf{r})$$

$$|\Delta\mathbf{M}(\mathbf{r})| \ll \mathbf{M}_0$$

Small deviation of \mathbf{M} from \mathbf{M}_0

$$(\mathbf{M}(\mathbf{r}))^2 = \mathbf{M}_0^2 = \text{const}$$

Length of \mathbf{M} is conserved

Plane waves in a continuous medium
(some analogy with sound waves)

**Energy of a ferromagnet as a function
of the spatial distribution of magnetization?**

Increase of the Zeeman energy:

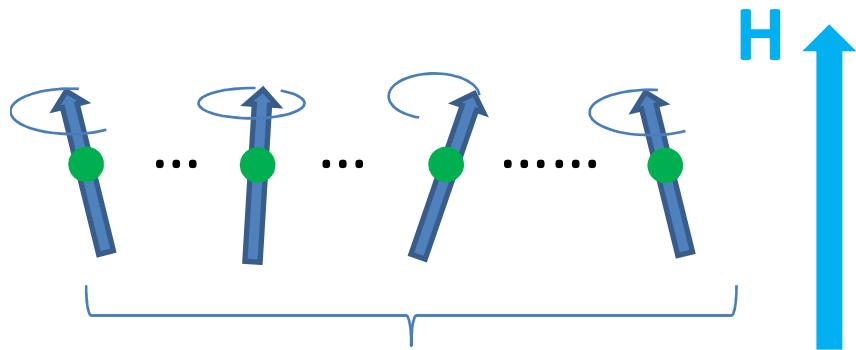
$$-(\mathbf{M}(\mathbf{r}) - \mathbf{M}_0)\mathbf{H}$$

Increase of the exchange energy:

Exchange constant

$$\left(\frac{A}{M_0^2} \right) \left[(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2 \right]$$

Magnetization at T>0



Energy of a ferromagnet as a function
of the spatial distribution of magnetization:

$$E = \int \left[\left(\frac{A}{M_0^2} \right) [(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2] - (M - M_0) H_0 \right] dr$$

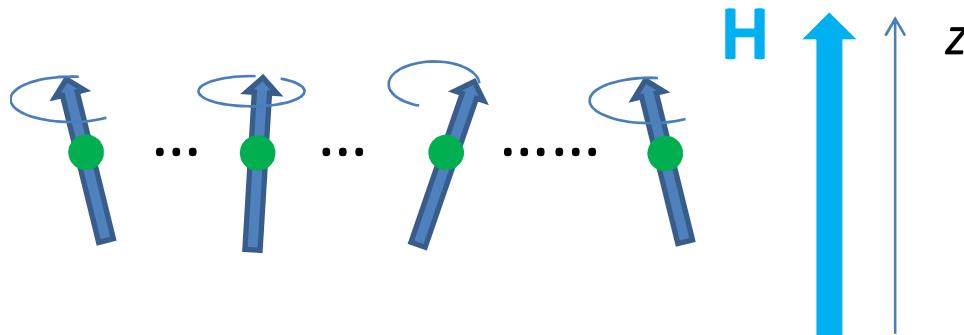
Increase of the Zeeman energy

$$-(\mathbf{M(r)} - \mathbf{M}_0) \mathbf{H}$$

Increase of the exchange energy

$$\left(\frac{A}{M_0^2} \right) [(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2]$$

Magnetization at T>0



$$E = \int \left[\left(\frac{A}{M_0^2} \right) [(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2] - (M - M_0) H_0 \right] dr$$

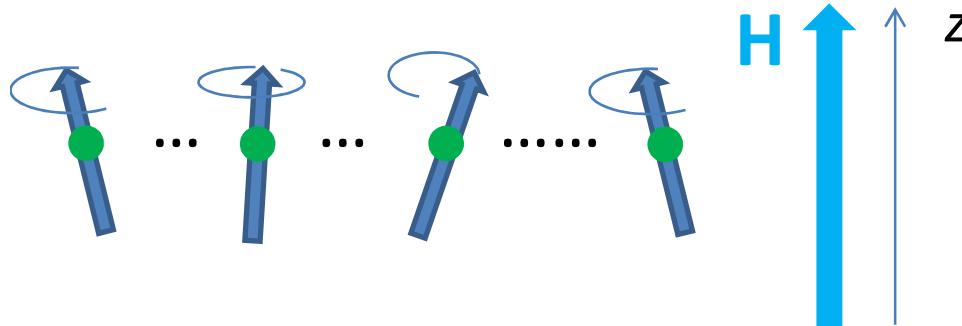
$$M_x, M_y \ll M_0$$

Small deviations of \mathbf{M} :

$$M_z \approx \left(1 - \frac{M_x^2 + M_y^2}{2M_0^2} \right) M_0$$

$$E = \int \left[\left(\frac{A}{M_0^2} \right) [(\nabla M_x)^2 + (\nabla M_y)^2] + \frac{H_0(M_x^2 + M_y^2)}{2M_0} \right] dr$$

Magnetization at T>0

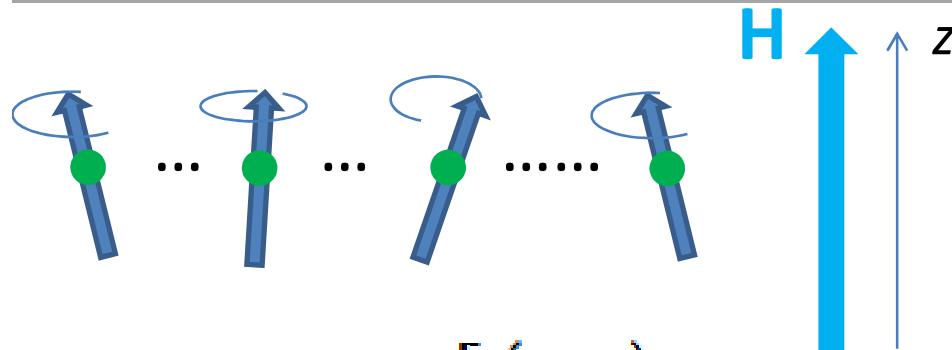


$$E = \int \left[\left(\frac{A}{M_0^2} \right) [(\nabla M_x)^2 + (\nabla M_y)^2] + \frac{H_0(M_x^2 + M_y^2)}{2M_0} \right] dr$$

Introduce complex combinations: $M^\pm = M_x \pm iM_y$

$$E = \int \left[\left(\frac{A}{M_0^2} \right) (\nabla M^+ \nabla M^-) + \left(\frac{H_0}{2M_0} \right) M^+ M^- \right] dr$$

Magnetization at T>0: Exchange waves



$$E = \int \left[\left(\frac{A}{M_0^2} \right) (\nabla M^+ \nabla M^-) + \left(\frac{H_0}{2M_0} \right) M^+ M^- \right] d\mathbf{r}$$

$$M^\pm = M_x \pm i M_y \quad \rightarrow$$

$$M^-(\mathbf{r}) = \left(\frac{2\mu M_0}{V} \right) \sum_k b_k e^{i\mathbf{k}\cdot\mathbf{r}}$$

$$M^+(\mathbf{r}) = \left(\frac{2\mu M_0}{V} \right) \sum_k b_k^* e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$E = \sum_k \left(\frac{2\mu A}{M_0} k^2 + \mu H_0 \right) b_k^* b_k$$

A sum of the energies
of plane waves –
spin waves

Magnetization at T>0: magnons

Energy of a ferromagnet:

$$E = \sum_k \left(\frac{2\mu A}{M_0} k^2 + \mu H_0 \right) b_k^* b_k$$

$$n_k = b_k^* b_k$$

number of quasiparticles - **magnons**
in the state with an energy:

$$\varepsilon_k = \frac{2\mu A}{M_0} k^2 + \mu H_0$$

Quasi-momentum of magnon

$$\mathbf{P} = \hbar \mathbf{k}$$

Effective mass of a magnon

$$m^* = \frac{M_0 \hbar^2}{4\mu A} = \frac{M_0 \hbar}{4\gamma A}$$

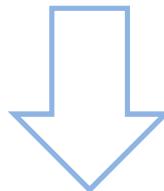
Bose-Einstein distribution
of thermal magnons

$$n_k = \frac{1}{e^{\varepsilon_k/k_B T} - 1}$$

Magnetization at T>0: magnons

$$\sum_k n_k = \frac{1}{\mu} \left(VM_0 - \int M_z d\mathbf{r} \right)$$

Each magnon reduces the total magnetic moment of the sample by μ



Quadratic dispersion
Bose-Einstein distribution

$$M(T) = M_0 - \frac{5.157 \cdot 10^{-5}}{(\gamma A M_0)^{3/2}} T^{3/2}$$

Bloch (3/2-) law: correct temperature dependence of magnetization at low T

Magnetic ordering, magnetic anisotropy and the mean-field theory

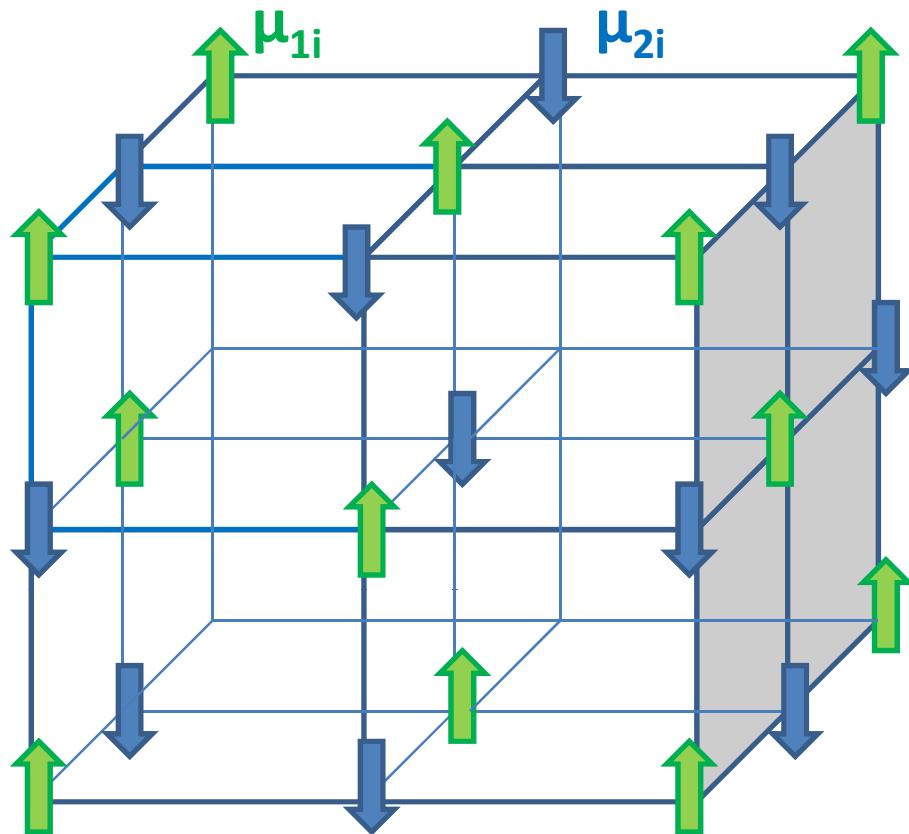


Antiferromagnets

Neel temperature

Susceptibility

Antiferromagnets



Magnetic sublattices:

$$\mathbf{M}_1 = \sum \mu_{1i} / V$$

$$\mathbf{M}_2 = \sum \mu_{2i} / V$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

Magnetization
(ferromagnetic vector)

$$\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$$

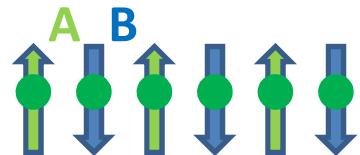
Antiferromagnetic vector)

Equivalent magnetic sublattices

- same type of magnetic ions in the same crystallographic positions

Number of magnetic sublattices is equivalent to the number of magnetic ions in the magnetic unit cell

Antiferromagnets: Neél temperature



$$H=0$$

Equivalent sublattices:

Molecular (Weiss) fields for the case of two sublattices:

$$w_{AA}=w_{BB}=w_J$$

$$H_A = w_{AA}M_A + w_{AB}M_B$$

$$H_B = w_{BB}M_B + w_{BA}M_A$$

$$w_{AB} = w_{BA} = w_2$$

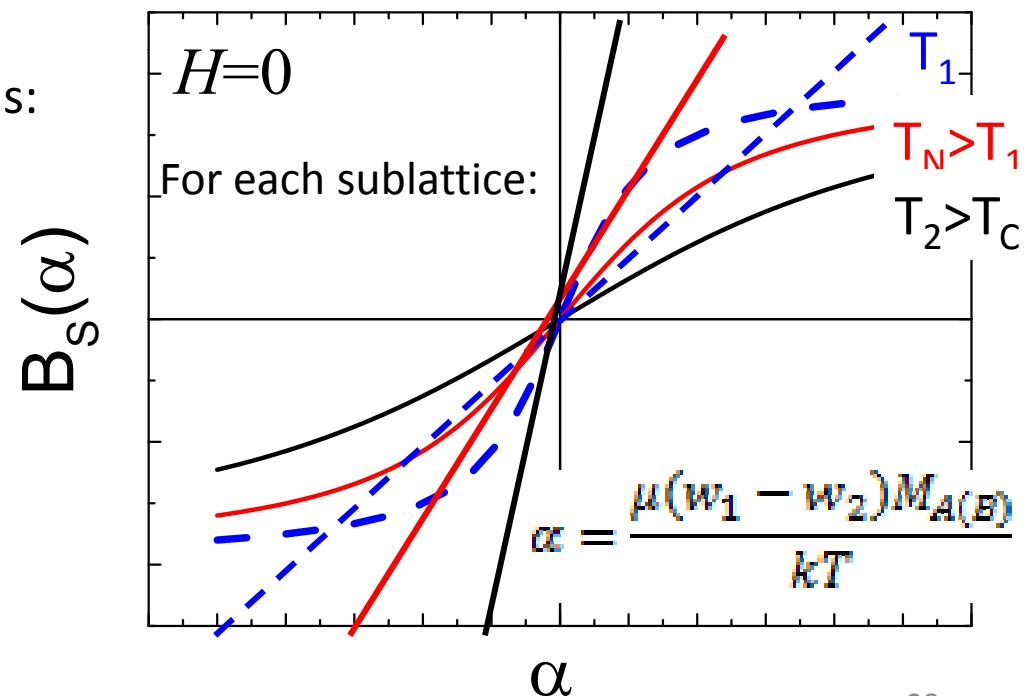
$$M_A = -M_B$$

Magnetization of a sublattice in the presence of the molecular fields:

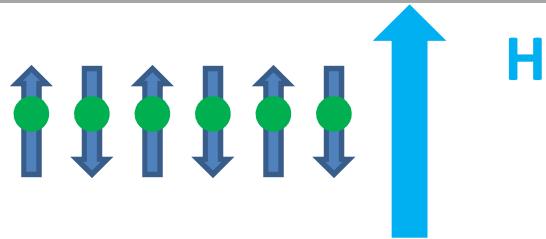
$$M_{A(B)} = \frac{N\mu}{2} L \left(\frac{\mu(w_1 - w_2) M_{A(B)}}{kT} \right)$$

Neél temperature:

$$T_N = \frac{N\mu^2}{6k} (w_1 - w_2)$$



Antiferromagnets: susceptibility



$$\chi_{afm} \leq \chi_{par}$$

$$H_A = w_{AA}M_A + w_{AB}M_B + H$$

$$H_B = w_{BB}M_B + w_{BA}M_A + H$$

Equivalent sublattices:

$$w_{AA} = w_{BB} = w_1 > 0$$

$$w_{AB} = w_{BA} = w_2 < 0$$

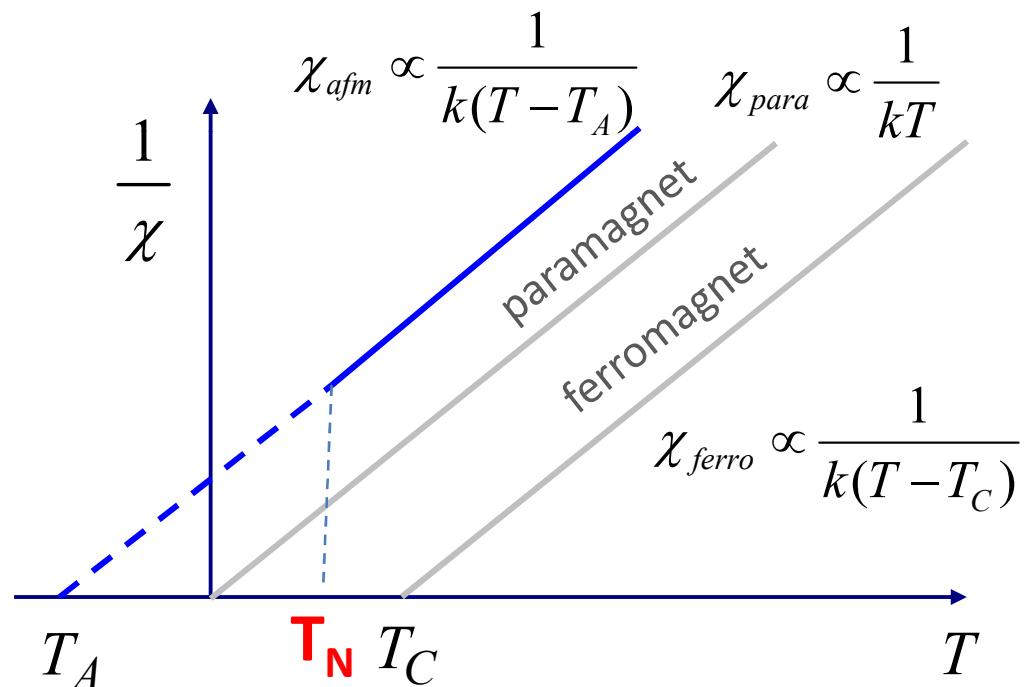
$$\cancel{M_A = -M_B}$$

Asymptotic Curie temperature:

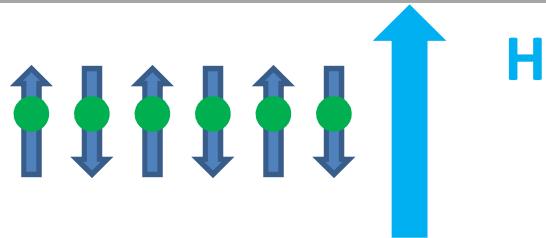
$$T_A = \frac{N\mu^2}{6k} (w_1 + w_2)$$

Susceptibility above T_N :

$$\chi = \frac{N\mu^2}{3k} \frac{1}{T - T_A}$$



Antiferromagnets: susceptibility



$$\chi_{afm} \leq \chi_{par}$$

Equivalent sublattices:

$$w_{AA} = w_{BB} = w_I > 0$$

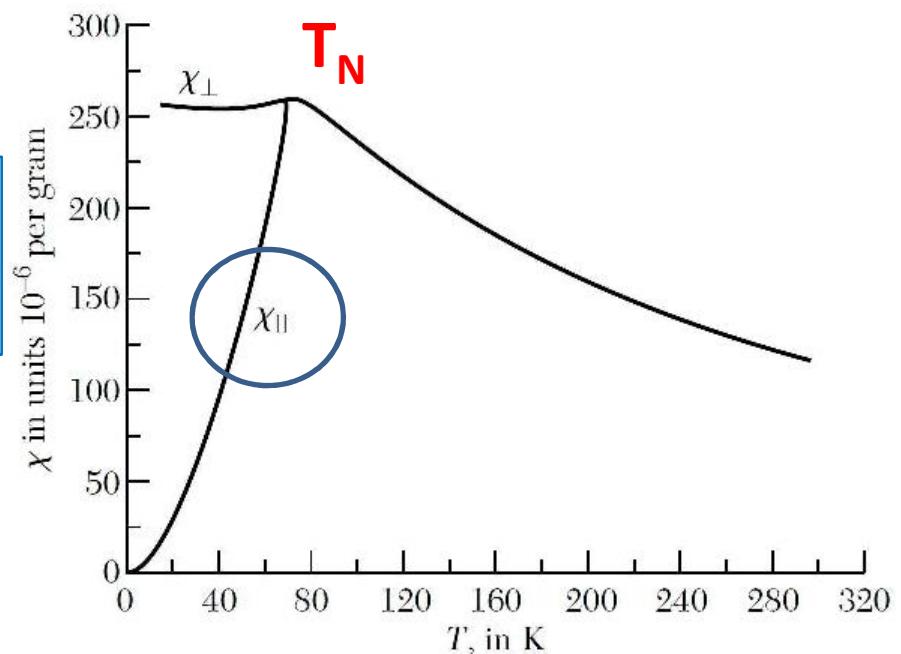
$$w_{AB} = w_{BA} = w_2 < 0$$

~~$$M_A = -M_B$$~~

Susceptibility in the Néel point:

$$\chi_{max} = \frac{N\mu^2}{3k} \frac{1}{T_N - T_A} = \frac{1}{w_2}$$

Susceptibility at T=0: $\chi=0$



Antiferromagnets: some examples

Compound	Crystal structure	Paramagnetic lattice structure	No. of sublattices	T_N
MnF_2	Rutile	Body-centered rectangular	2	72°
FeF_2	Rutile	Body-centered rectangular	2	79°
MnO	$NaCl$	f.c.c.	4	122°
FeO	$NaCl$	f.c.c.	4	198°
MnS	$NaCl$	f.c.c.	4	165°
$MnSe^*$	$NaCl$	f.c.c.	4	$\sim 150^\circ$
$FeCl_2$	$CdCl_2$	Hexagonal layer structure	3	23.5°
$CoCl_2$	$CdCl_2$	Hexagonal layer structure	3	24.9°
$NiCl_2$	$CdCl_2$	Hexagonal layer structure	3	49.6°

[P. W. Anderson, Phys. Rev. 79, 705 (1950)]

Magnetic ordering, magnetic anisotropy and the mean-field theory



Magnetic anisotropy

Magnetic-dipole contribution

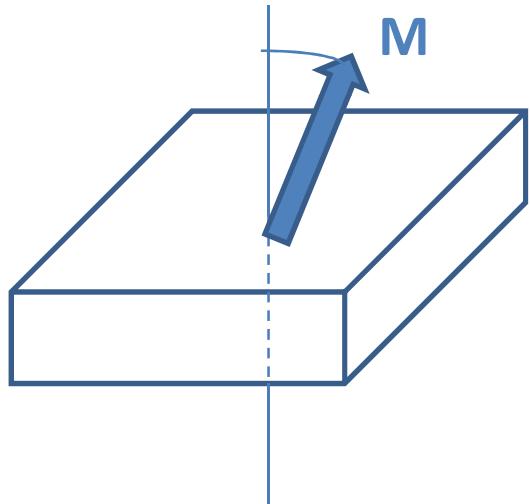
Magneto-crystalline anisotropy and its temperature dependence

Magnetic anisotropy

Magnetic anisotropy energy – energy required to rotate magnetization from an “easy” to a “hard” direction

Without anisotropy net magnetization of 3D solids would be weak;
in 2D systems it would be absent

[Mermin and Wagner, PRL 17, 1133 (1966)]



Magnetization – axial vector

Sources of anisotropy:

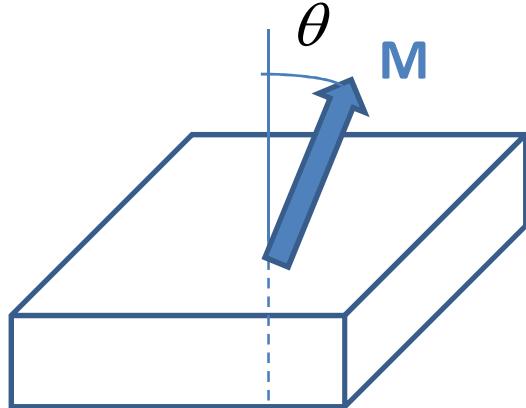
- Deformation
- Intrinsic electrical fields
- Shape etc....

- polar impacts

Magnetic anisotropy sets an axis, not a direction

Magnetic anisotropy: phenomenological consideration

Uniaxial anisotropy



$$E_a = K_{u1} \sin^2 \theta + K_{u2} \sin^4 \theta + \dots$$

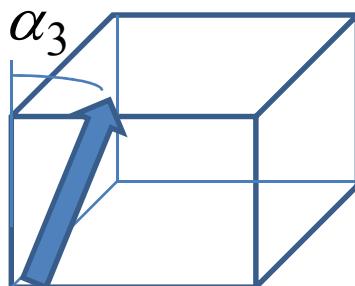
$K_{u1} > 0$ (001) – easy plane

$K_{u1} < 0$ [001] – easy axis

Effective anisotropy field

$$\mathbf{H}_a = -\frac{\partial E_a}{\partial \mathbf{M}} = -\frac{2K_{u1}}{M} \cos \theta \mathbf{n} + \dots$$

Cubic anisotropy



$$w_a = K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_1^2 \alpha_3^2) + K_2 \alpha_1^2 \alpha_2^2 \alpha_3^2 + \dots$$

$K_1 > 0$ easy axes- {100}

$K_1 < 0$ easy axes- {111}

Magnetic anisotropy

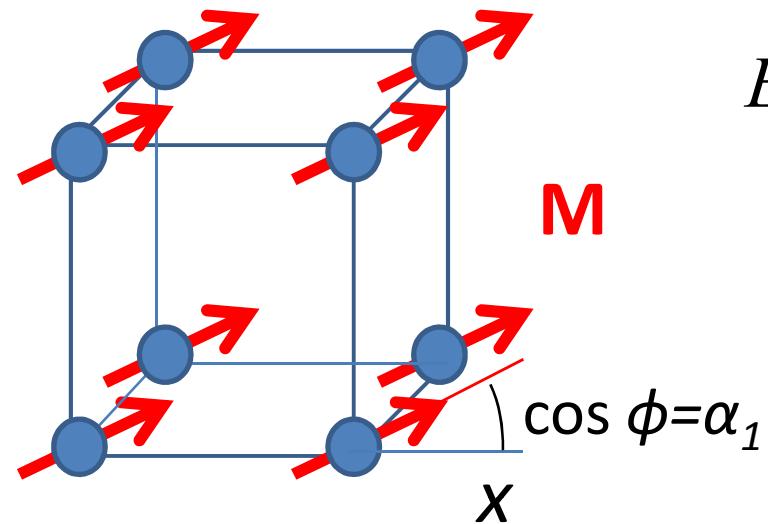
Compound	k_1 , erg cm ⁻³	k_2 , erg cm ⁻³
Fe	$4,6 \cdot 10^5$	$1,5 \cdot 10^5$
Ni	$-5 \cdot 10^4$	$2,3 \cdot 10^4$
Co	$4,1 \cdot 10^6$	$1,0 \cdot 10^6$
$\text{Y}_3\text{Fe}_5\text{O}_{12}$	$6,5 \cdot 10^5$	
NiFe_2O_4	$-6,2 \cdot 10^4$	
$\text{BaFe}_{12}\text{O}_{19}$	$3,3 \cdot 10^6$	

Origins of magnetic anisotropy

- Single-ion anisotropy
 - Anisotropic exchange
 - Magnetic-dipolar interactions
- } Spin-orbit interaction
- } Interactions between pairs of magnetic ions

Magnetic-dipolar contribution to the anisotropy

Cubic crystal



For a pair of neighboring ions:

$$E(\phi) = w_{ex}$$

$$-l\left(\alpha_1^2 - \frac{1}{3}\right)$$

$$+ q\left(\alpha_1^4 - \frac{6}{7}\alpha_1^2 + \frac{3}{35}\right) + \dots$$

Dipolar interaction

Quadrupolar interaction

$$E_a = -2Nq(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_1^2\alpha_3^2) + \text{const}$$

$$K_1 = -2Nq$$

N – number of atoms per volume unit

– cubic anisotropy constant

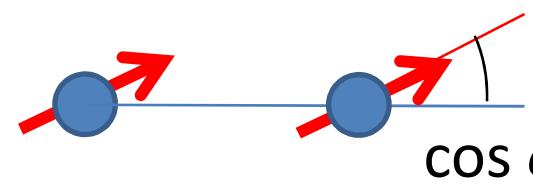
fcc:

$$K_1 = 16/9Nq$$

vcc:

$$K_1 = Nq$$

Magnetic-dipolar contribution to the anisotropy

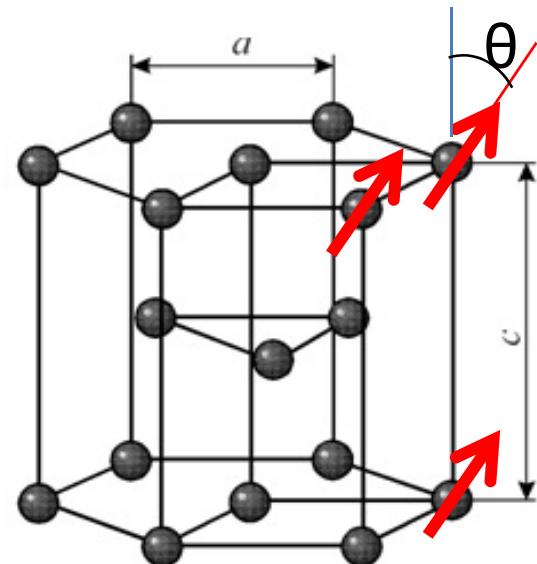


$$E(\phi) = l \left(\alpha_1^2 - \frac{1}{3} \right) + q \left(\alpha_1^4 - \frac{6}{7} \alpha_1^2 + \frac{3}{35} \right) + \dots$$

Cubic crystal

$$E_a = K_1 (\alpha_1^2 \alpha_2^2 + \alpha_2^2 \alpha_3^2 + \alpha_1^2 \alpha_3^2) + \dots$$

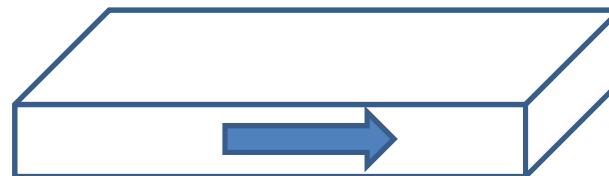
Hexagonal crystal



$$E_a = K_{u1} \sin^2 \theta + K_{u2} \sin^4 \theta + \dots$$

$$K_{u1} \sim l > q$$

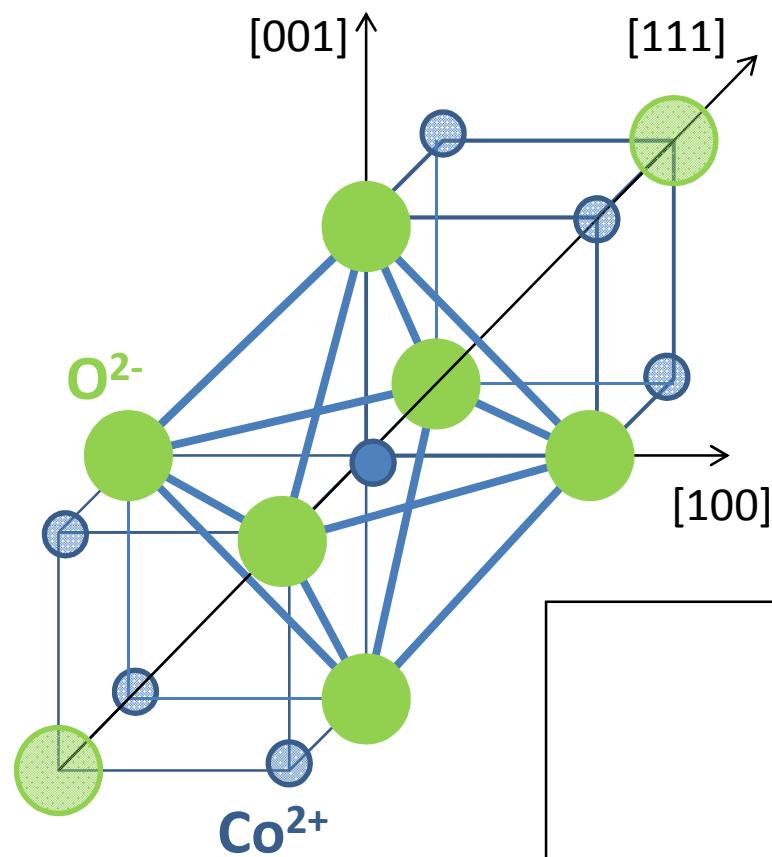
Contributes to the shape anisotropy



This films demonstrate in-plane anisotropy

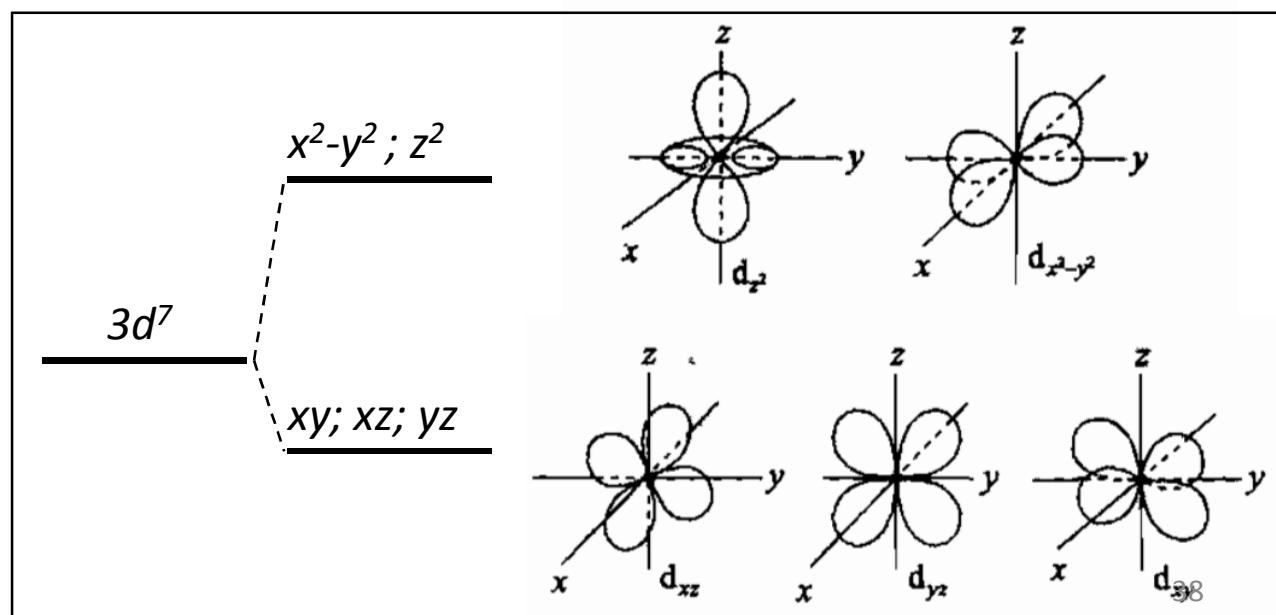
Single-ion anisotropy (crystal-field theory)

Spinel CoFe_2O_4

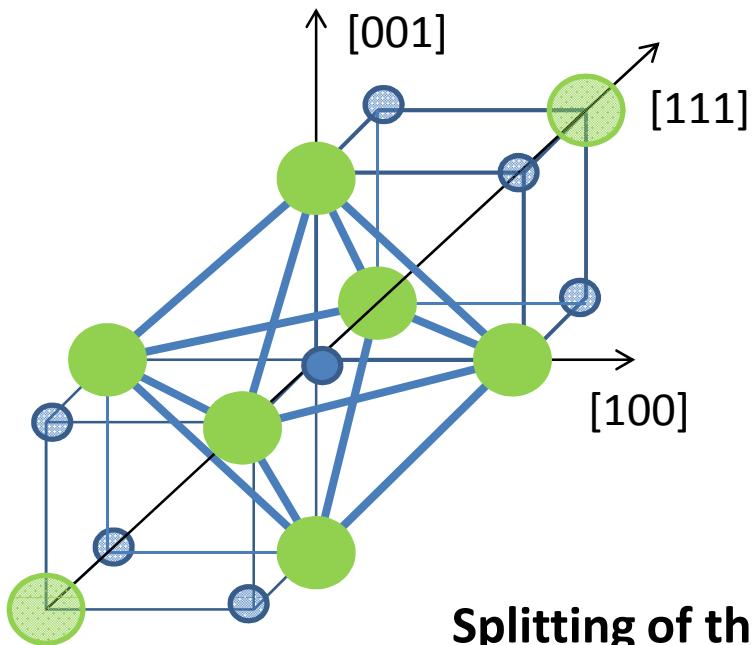


Co^{2+} in the octahedral coordination
+ trigonal distortion along [111]

**Splitting of the 3d shell
In the ligand field:**

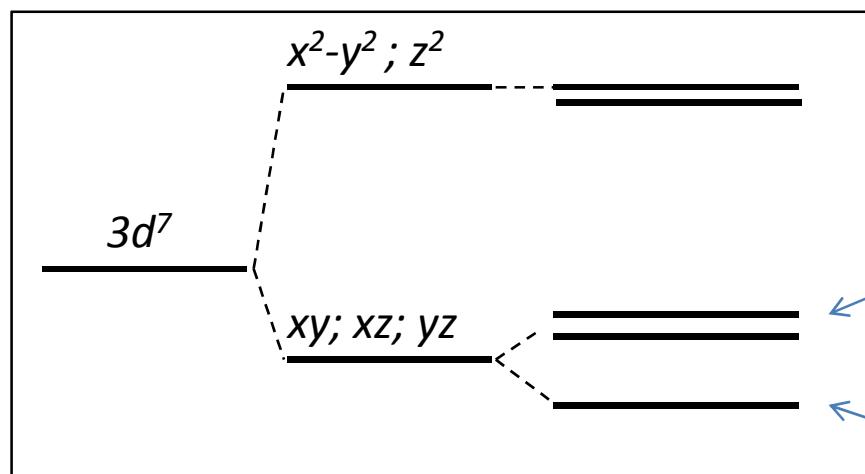


Single-ion anisotropy (crystal-field theory)



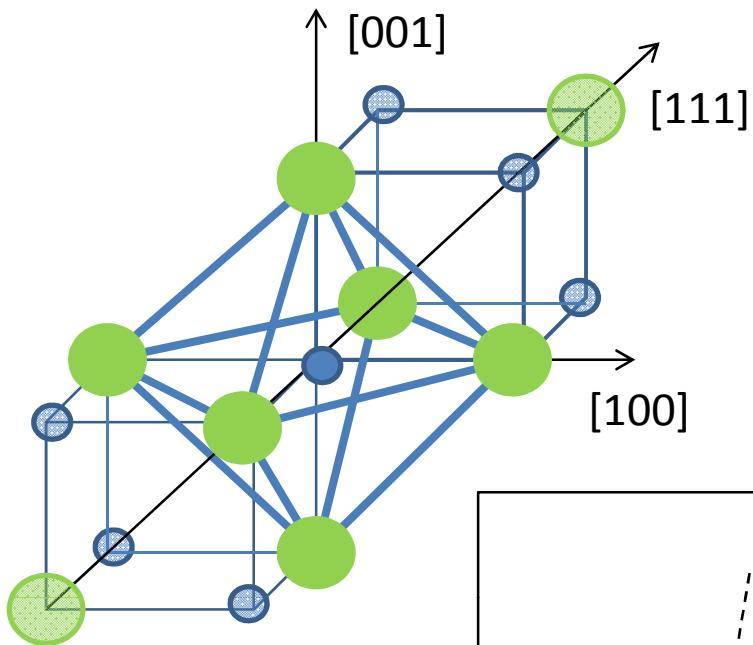
Co^{2+} in the octahedral coordination
+ trigonal distortion along [111]

**Splitting of the 3d shell
In the ligand field + trigonal distortion**



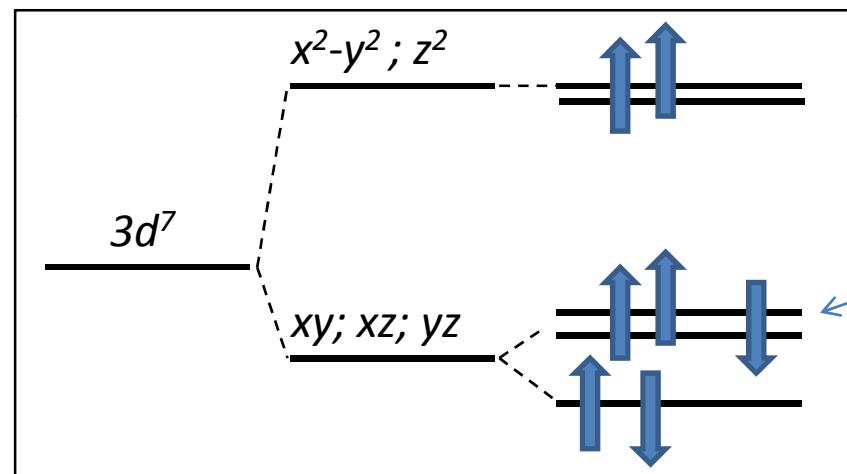
- Maxima of electronic density
 - in the plane (111)
- Maxima of electronic density
 - along the axis [111]

Single-ion anisotropy (crystal-field theory)



Co²⁺ in the octahedral coordination
+ trigonal distortion along [111]

Splitting of the 3d shell
In the ligand field + trigonal distortion



Spin-orbit interaction

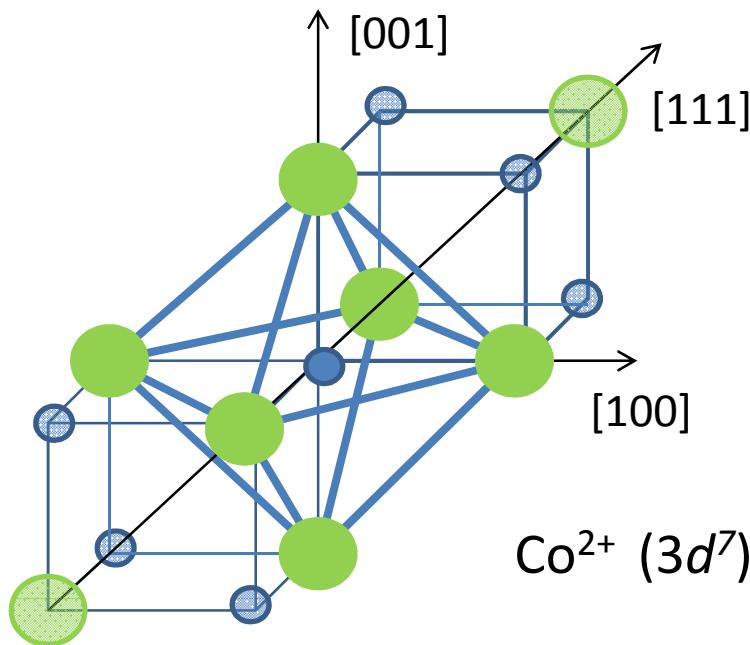
$$w_{SO} = \lambda \mathbf{L} \cdot \mathbf{S} = \lambda LS |\cos \theta|$$

θ -angle between \mathbf{S} и [111]

3rd Hunds rule:

$$\text{Co}^{2+} (3d^7) \rightarrow \lambda < 0$$

Single-ion anisotropy (crystal-field theory)



Co^{2+} in the octahedral coordination
+ trigonal distortion along [111]

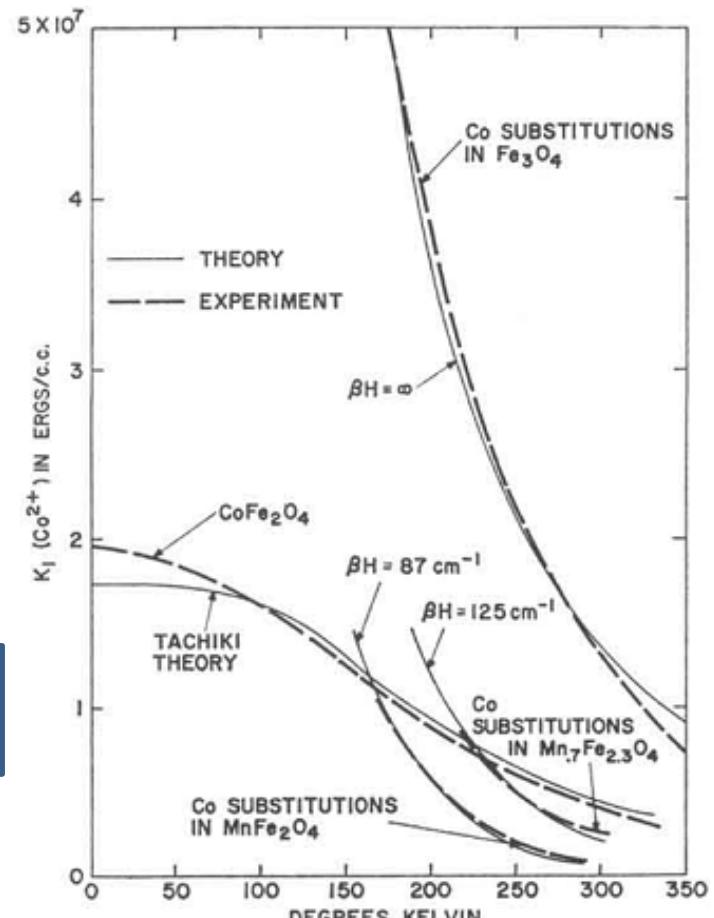
$$w_{SO} = \lambda \mathbf{L} \cdot \mathbf{S} = \lambda LS |\cos \theta|$$

θ -angle between \mathbf{S} и [111]

Averaging over 4 types of Co^{2+} positions
with distortions along different $\{111\}$ axes:

$$w_a = -N\lambda LS(\alpha_x^2\alpha_y^2 + \alpha_y^2\alpha_z^2 + \alpha_z^2\alpha_x^2)$$

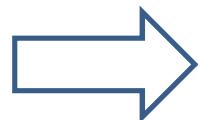
$$K_1 > 0$$



Temperature dependence of magneto-crystalline anisotropy

Classical theory gives:

$$E_a = K_n \alpha^n$$



$$\frac{K_n(T)}{K_n(0)} = \left(\frac{M_S(T)}{M_S(0)} \right)^{\frac{n(n+1)}{2}}$$

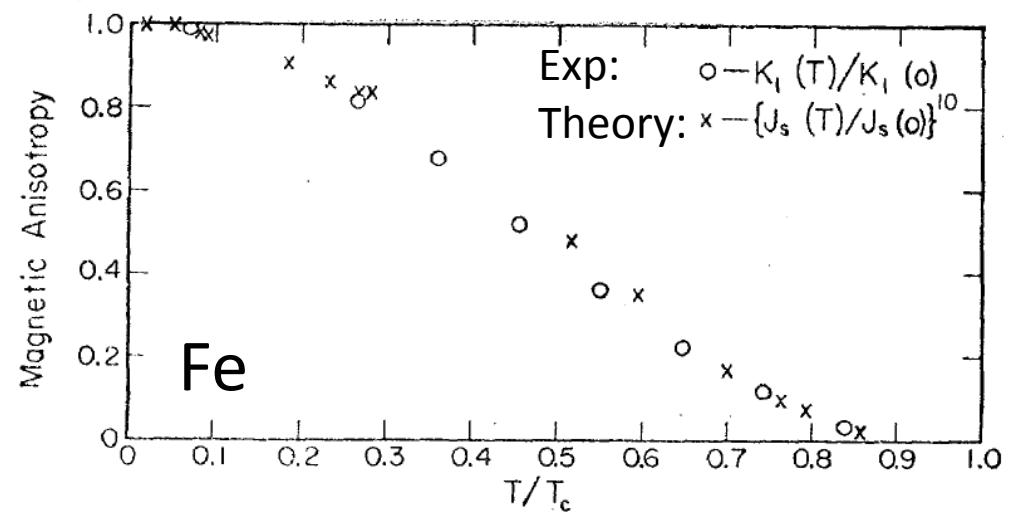
Cubic anisotropy

$$\frac{K_n(T)}{K_n(0)} = \left(\frac{M_S(T)}{M_S(0)} \right)^{10}$$

Uniaxial anisotropy

$$\frac{K_n(T)}{K_n(0)} = \left(\frac{M_S(T)}{M_S(0)} \right)^3$$

[Zener, Phys. Rev. 96, 1335 (1954)]



Competition between different anisotropies
gives rise to spin-reorientation transitions

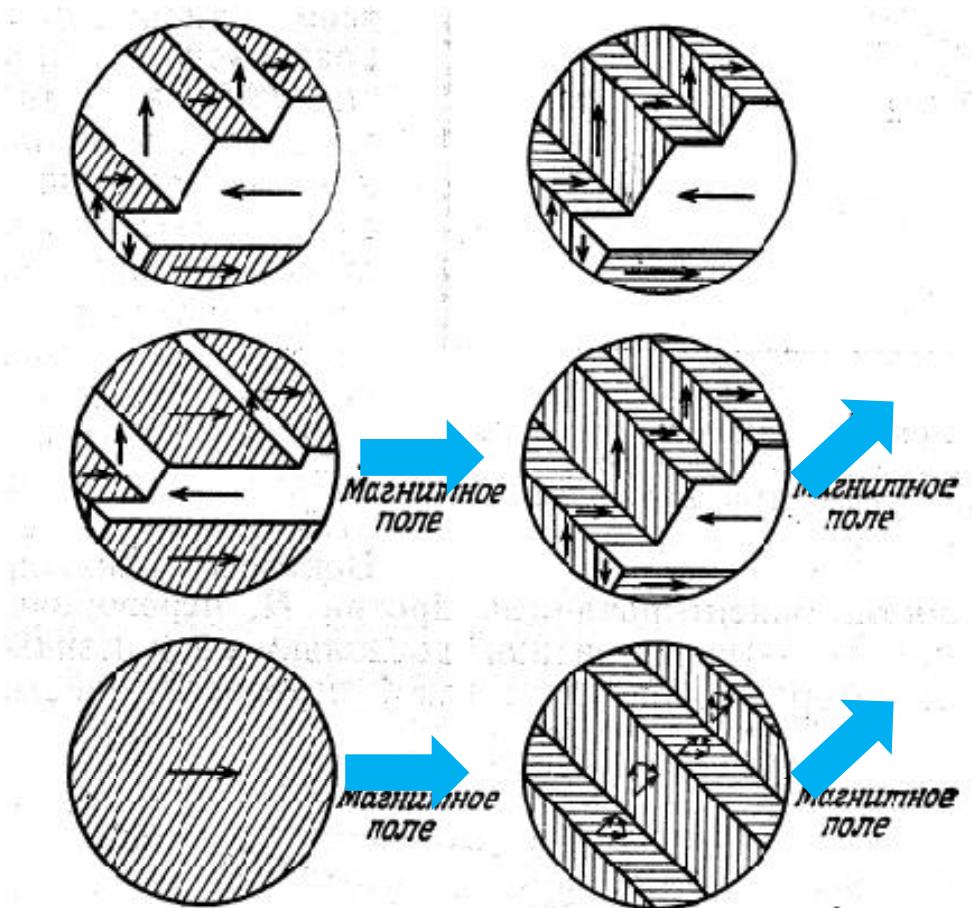
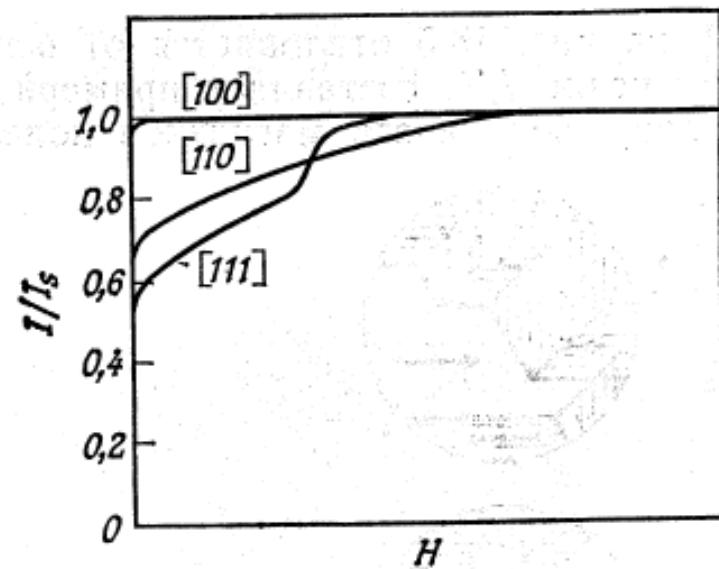
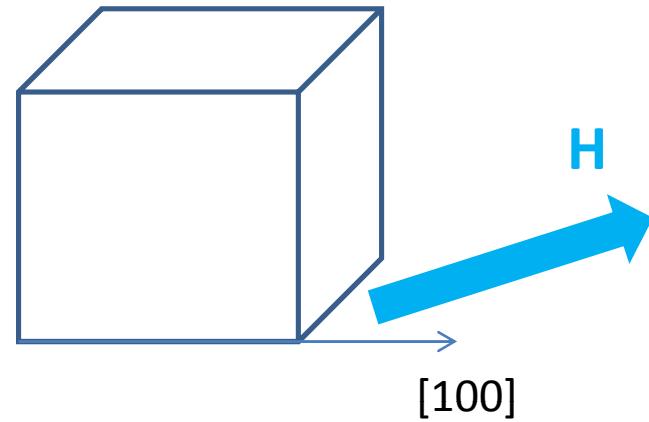
Magnetic ordering, magnetic anisotropy and the mean-field theory



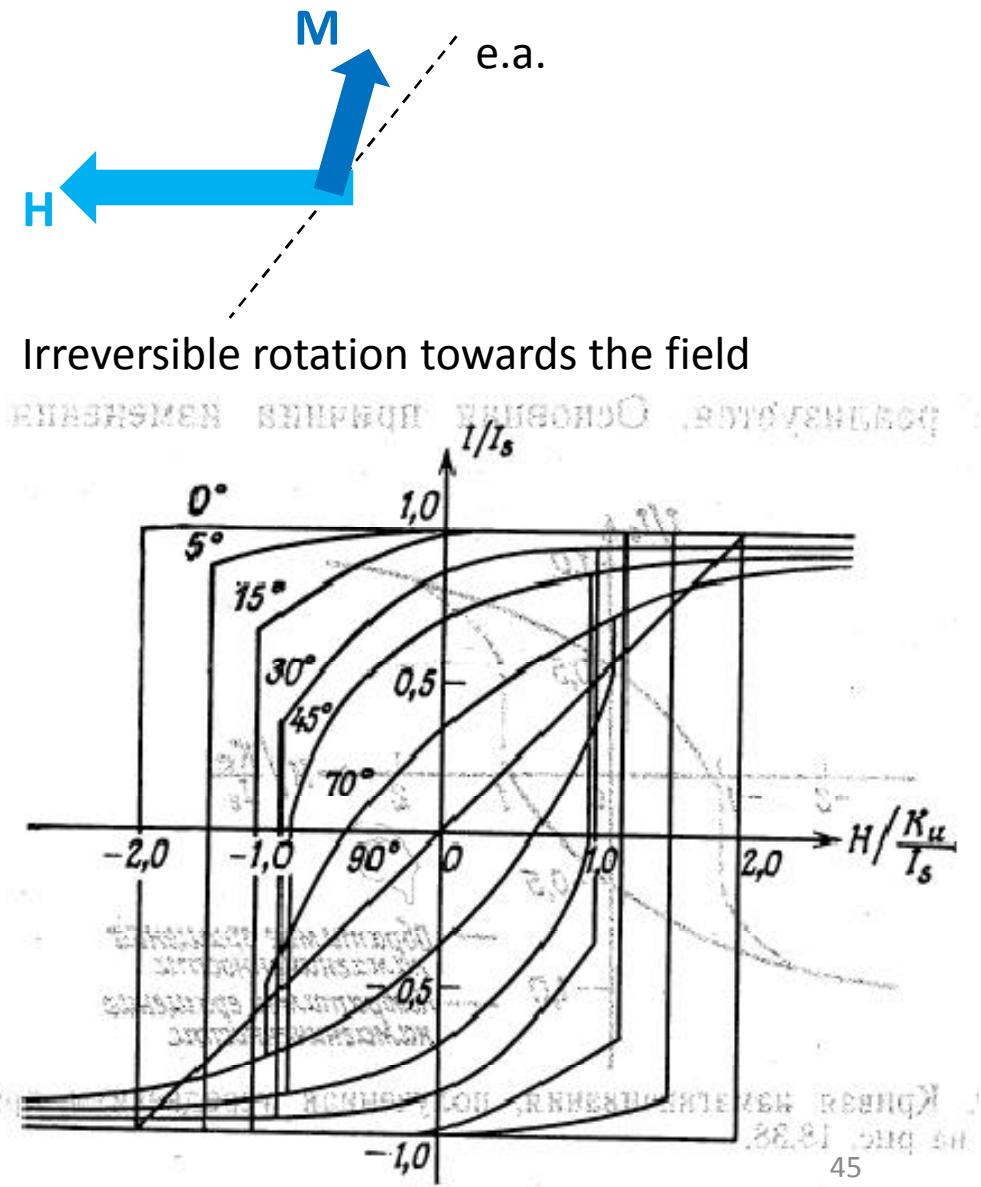
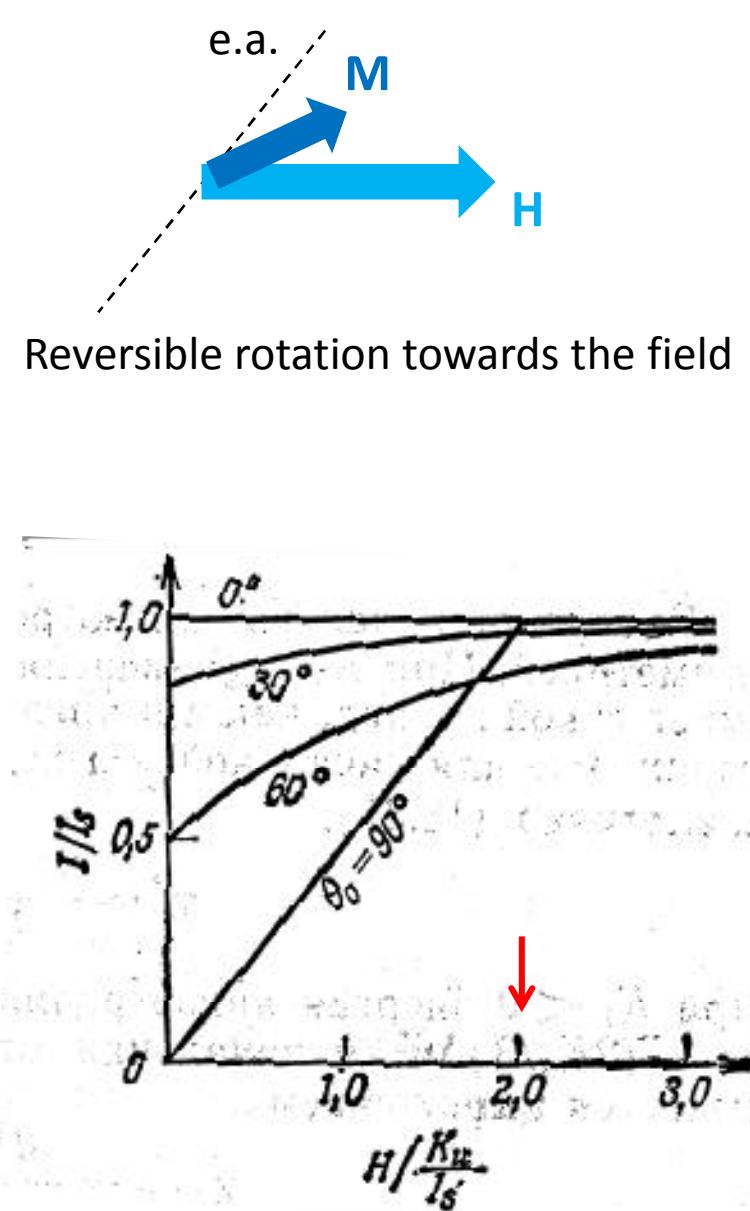
Ferro- and antiferromagnets in an external field

Ferromagnets in applied magnetic field: domain walls displacement

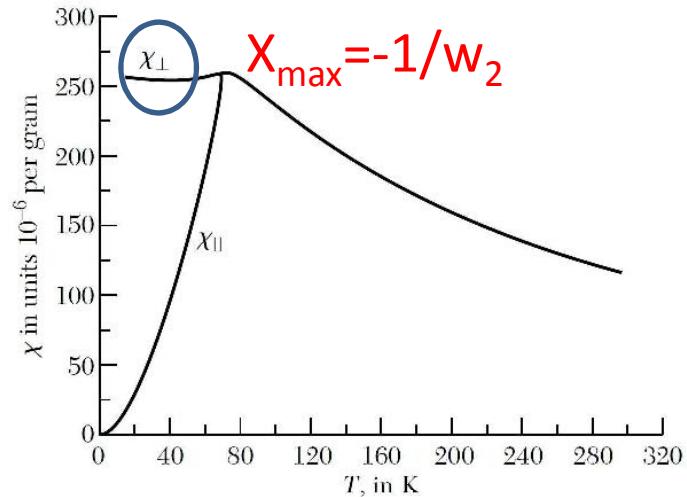
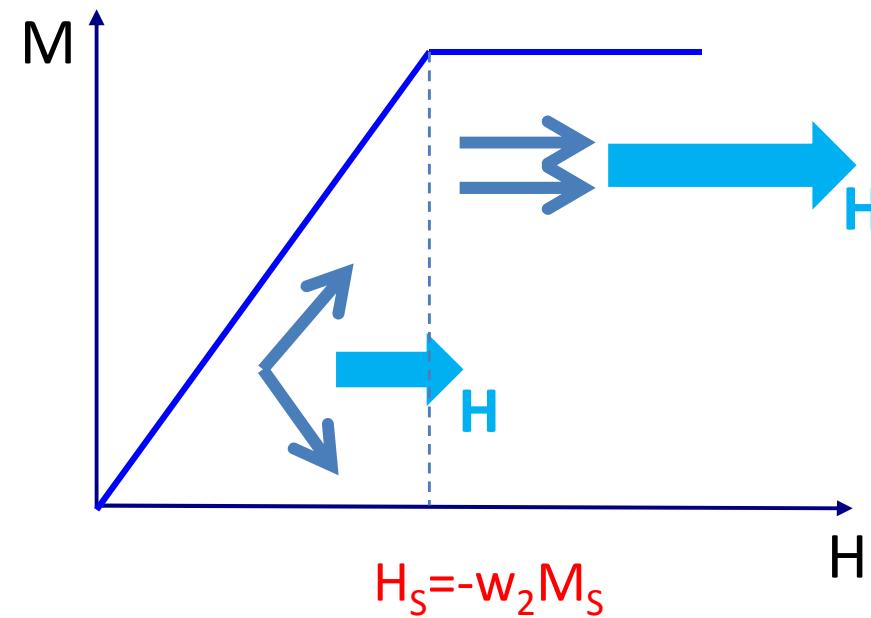
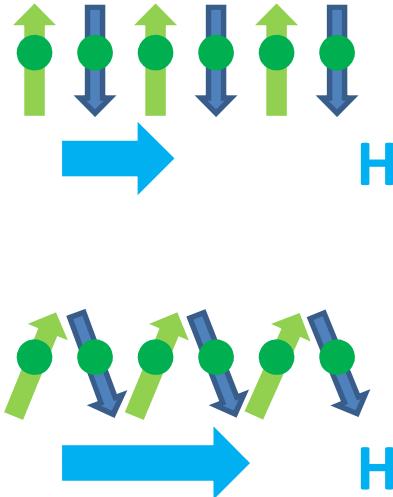
Cubic anisotropy ($K_1 > 0$)



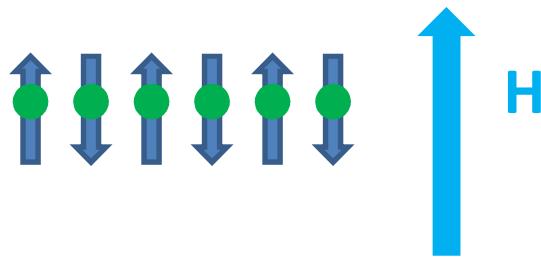
Ferromagnets in applied magnetic field: rotation of magnetization



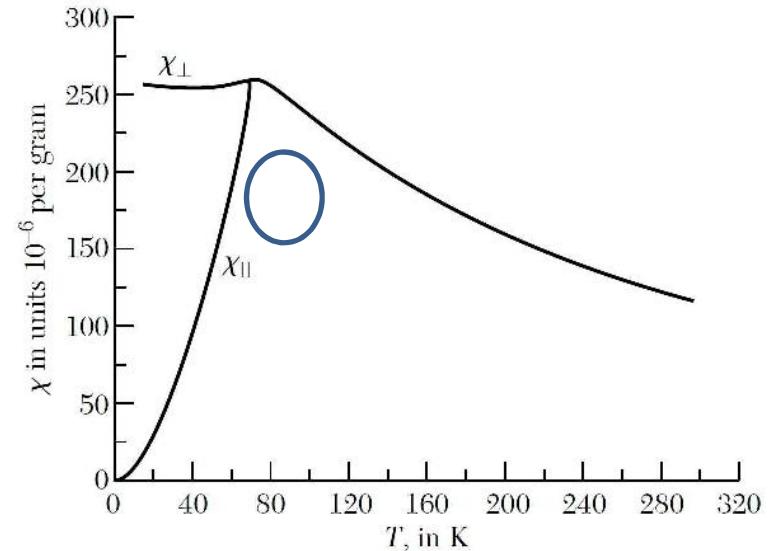
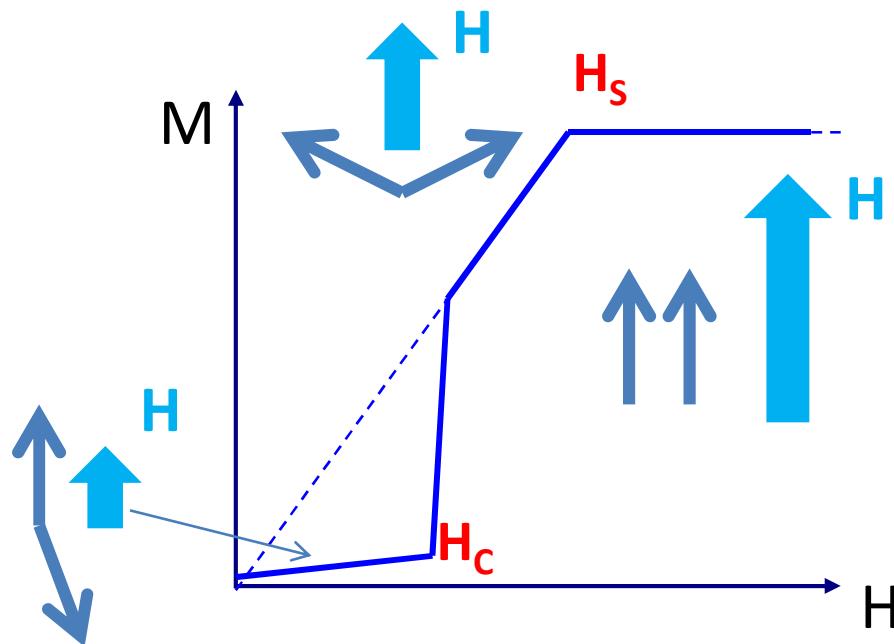
Magnetization process in a multisublattice medium: spin-flop and spin-flip transitions in an antiferromagnet



Magnetization process in a multisublattice medium: spin-flop and spin-flip transitions in an antiferromagnet



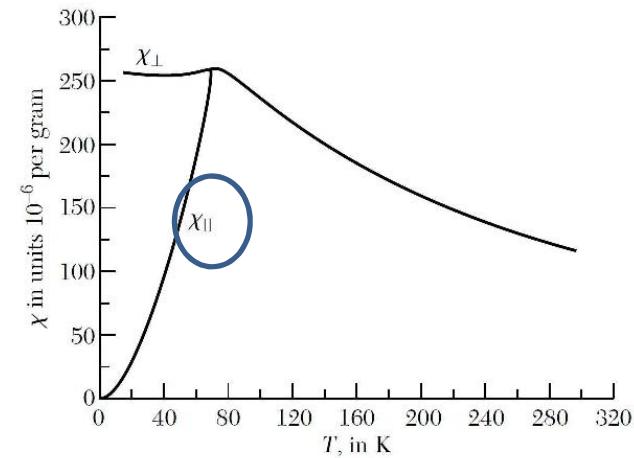
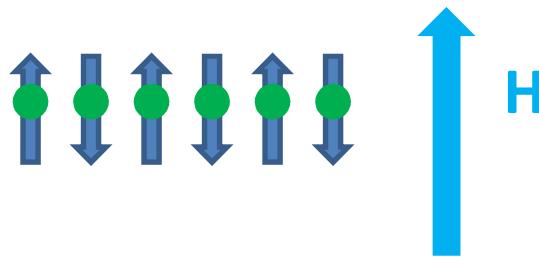
If the anisotropy is weak:



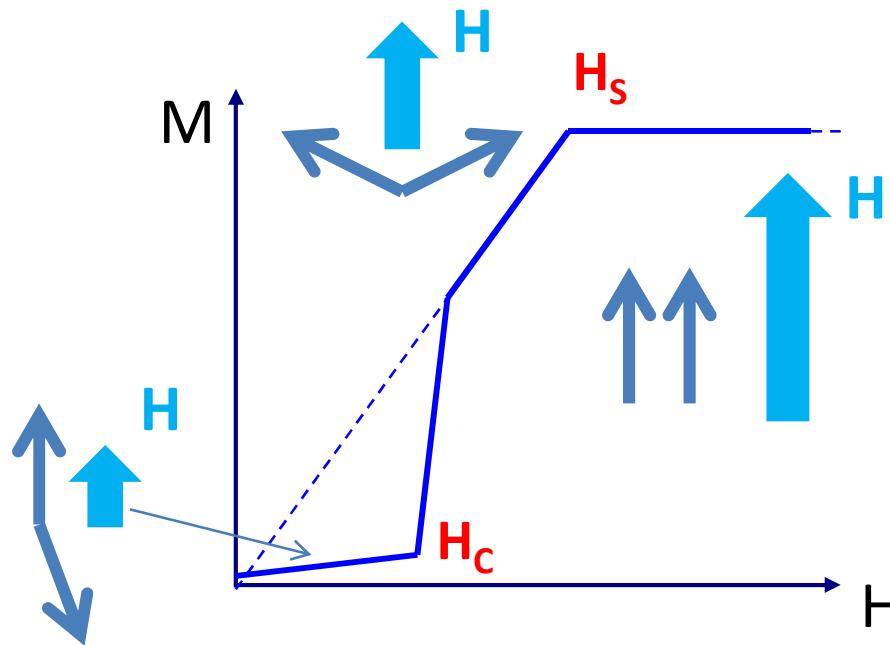
Condition for the **spin-flop**
(spin-reorientation transition)

$$H \geq \sqrt{\frac{2K_u}{\chi_{\perp} - \chi_{\parallel}}} = H_c$$

Magnetization process in a multisublattice medium: spin-flop and spin-flip transitions in an antiferromagnet



If the anisotropy is weak:



If the anisotropy is strong:

