Magnetic moments, dipoles and fields

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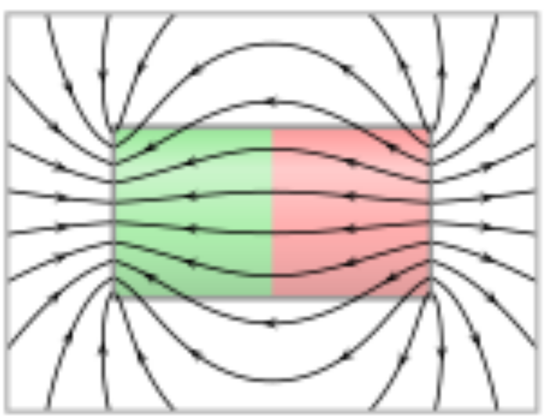
ESM 2018

Overview

Origin of magnetic moments



Magnetic fields and demagnetising factors

Units in magnetism

Useful References

- J. M. D. Coey; Magnetism and Magnetic Magnetic Materials. Cambridge University Press (2010) 614 pp

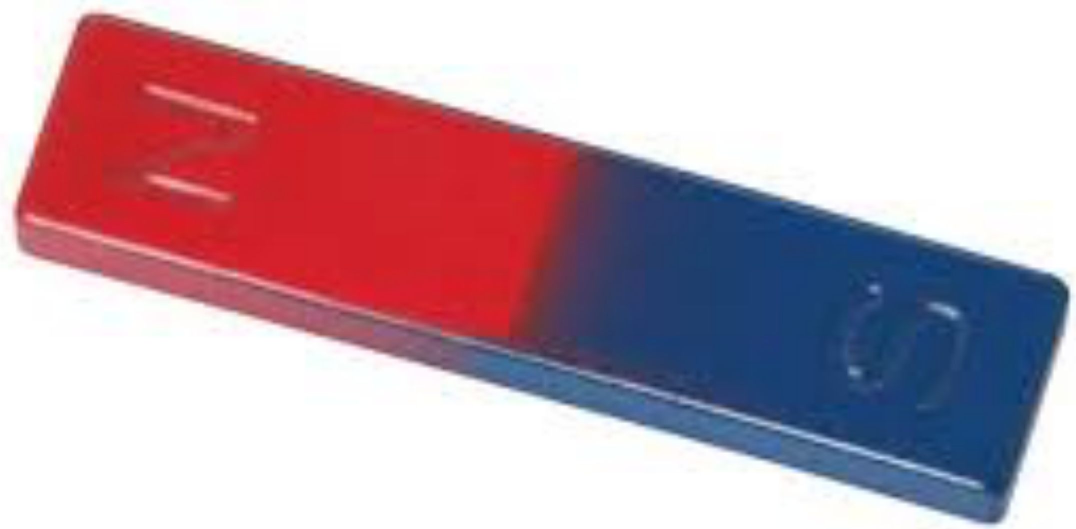
- Stephen Blundell Magnetism in Condensed Matter, Oxford 2001

- D. C. Jiles An Introduction to Magnetism and Magnetic Magnetic Materials, CRC Press 480 pp

- J. D. Jackson Classical Electrodynamics 3rd ed, Wiley, New York 1998

Magnetic moments

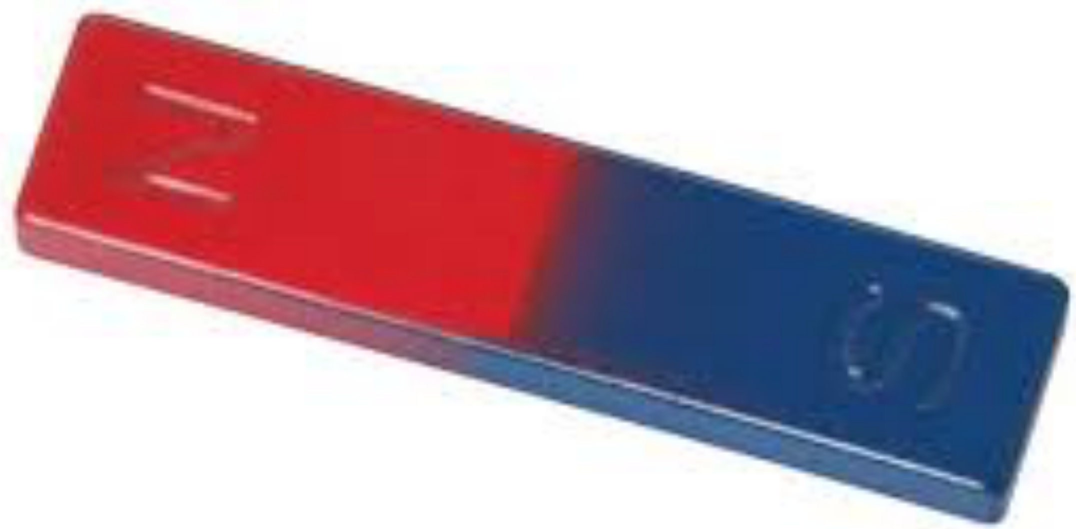
What is a magnet?



"A magnet is a material or object that produces a magnetic field"

Wikipedia

What is a magnet?

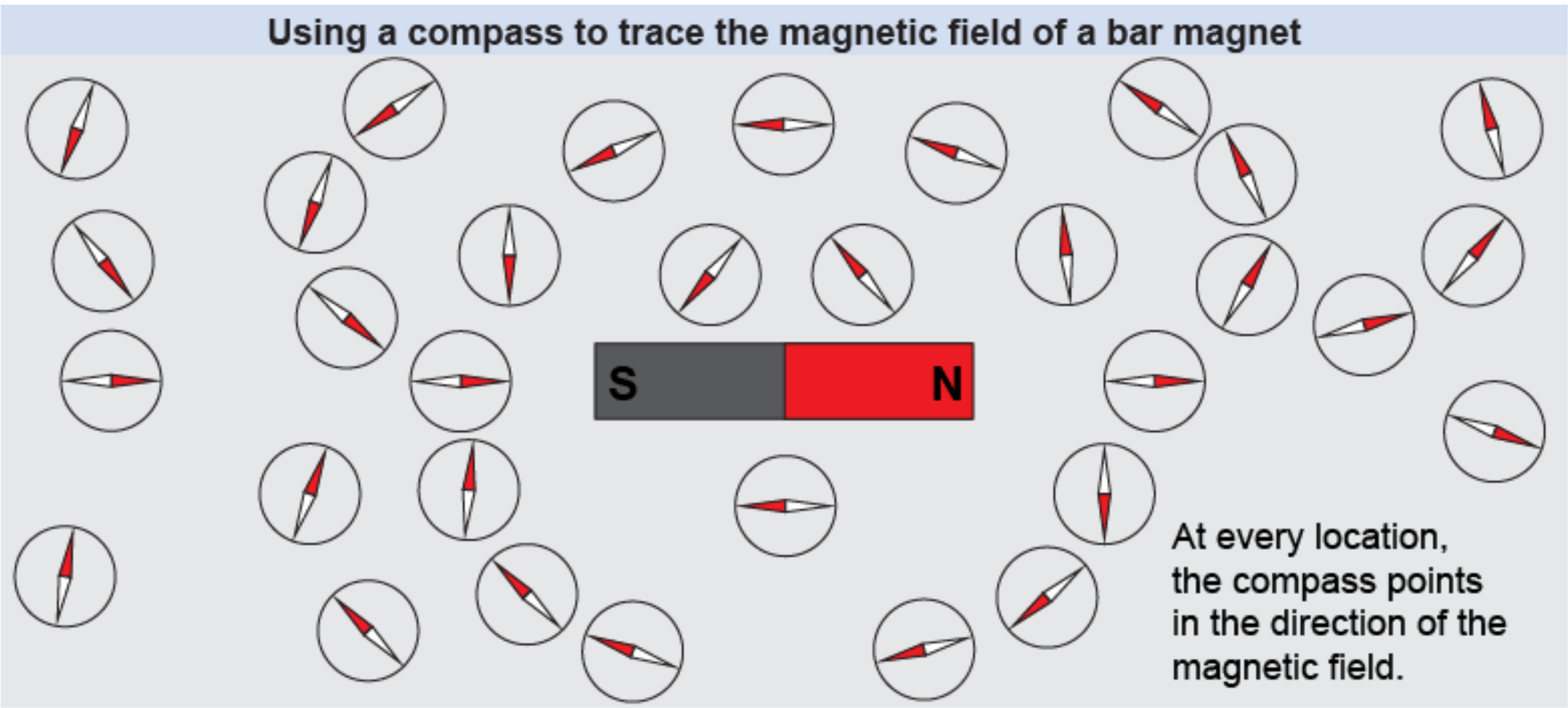


*"A magnet is a material or **object** that produces a magnetic field"*

Wikipedia

What is a magnetic field?

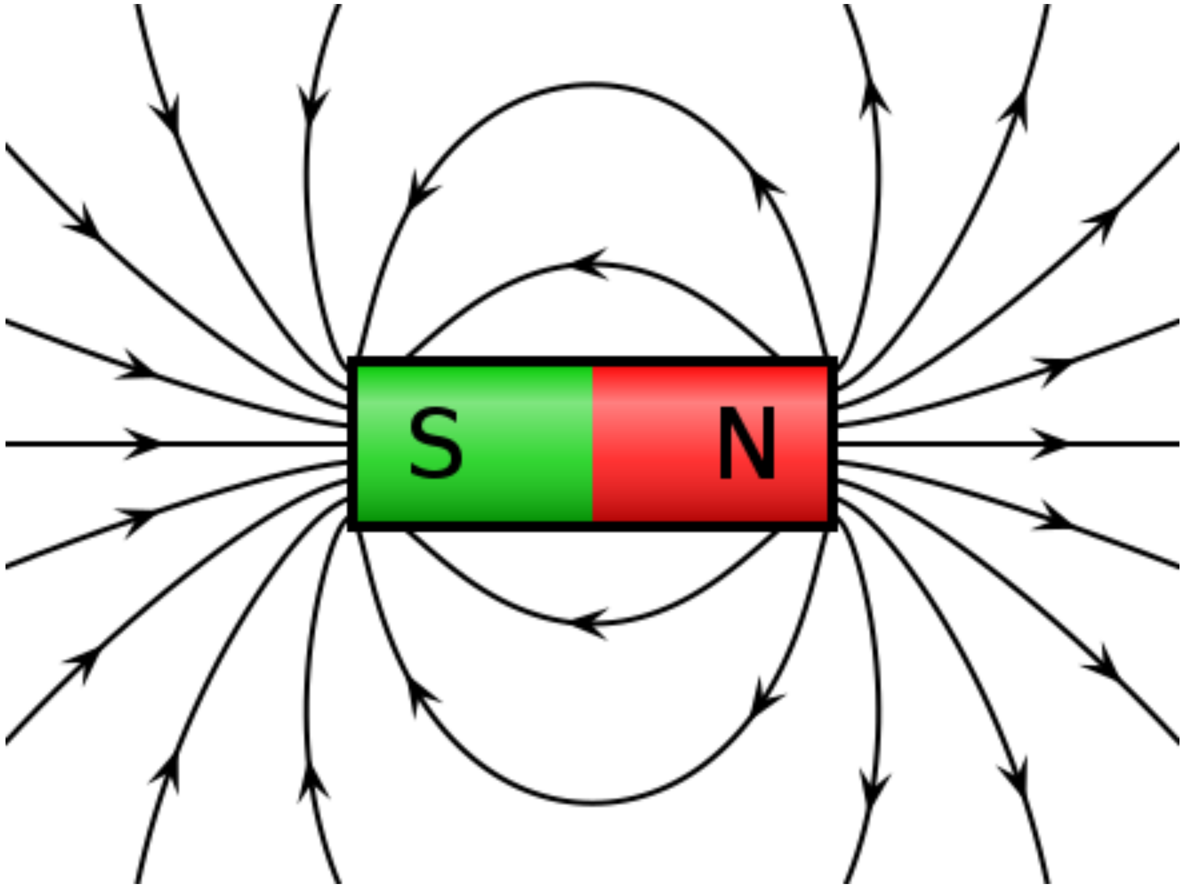
- An invisible vector field that interacts with other **magnets**



https://education.pasco.com/epub/PhysicsNGSS/BookInd-515.html

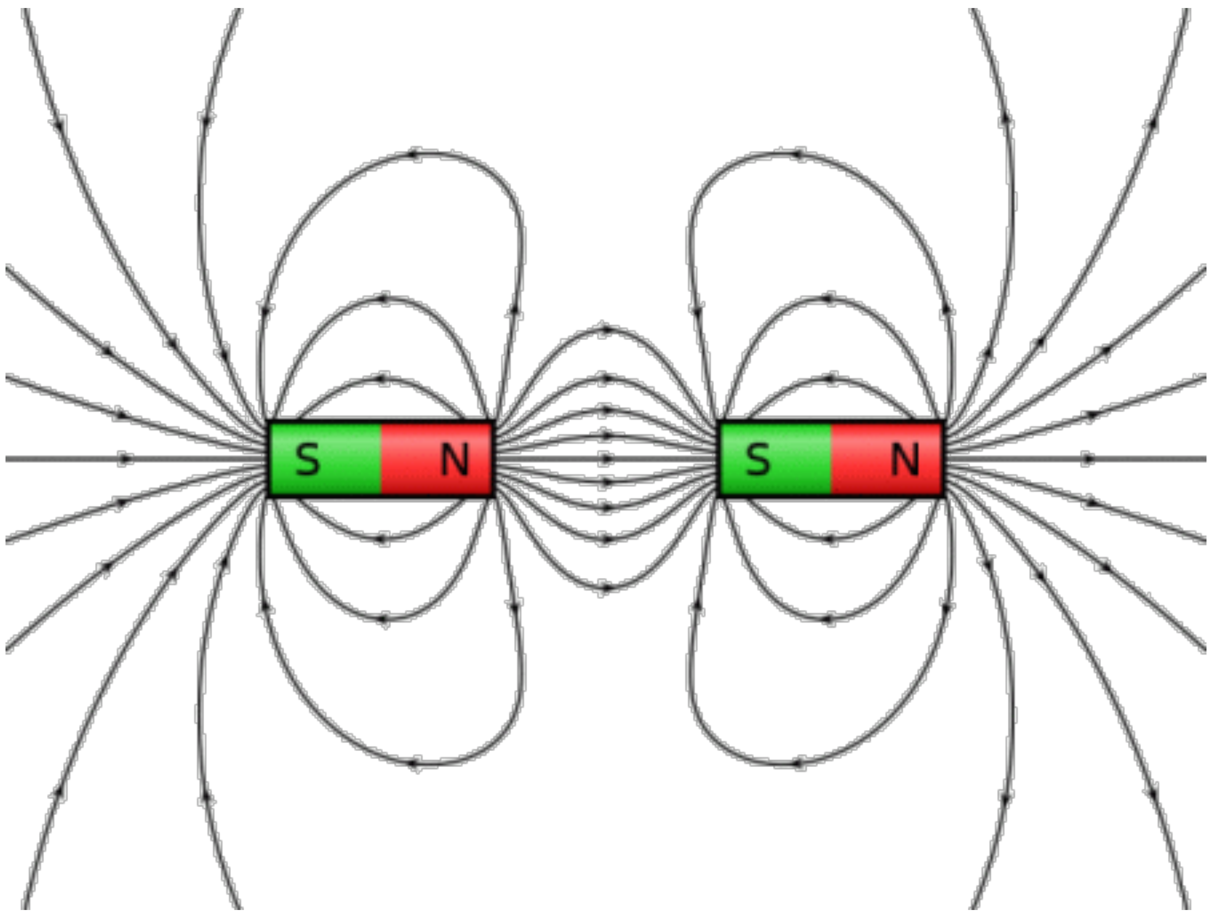
What is a magnetic field?

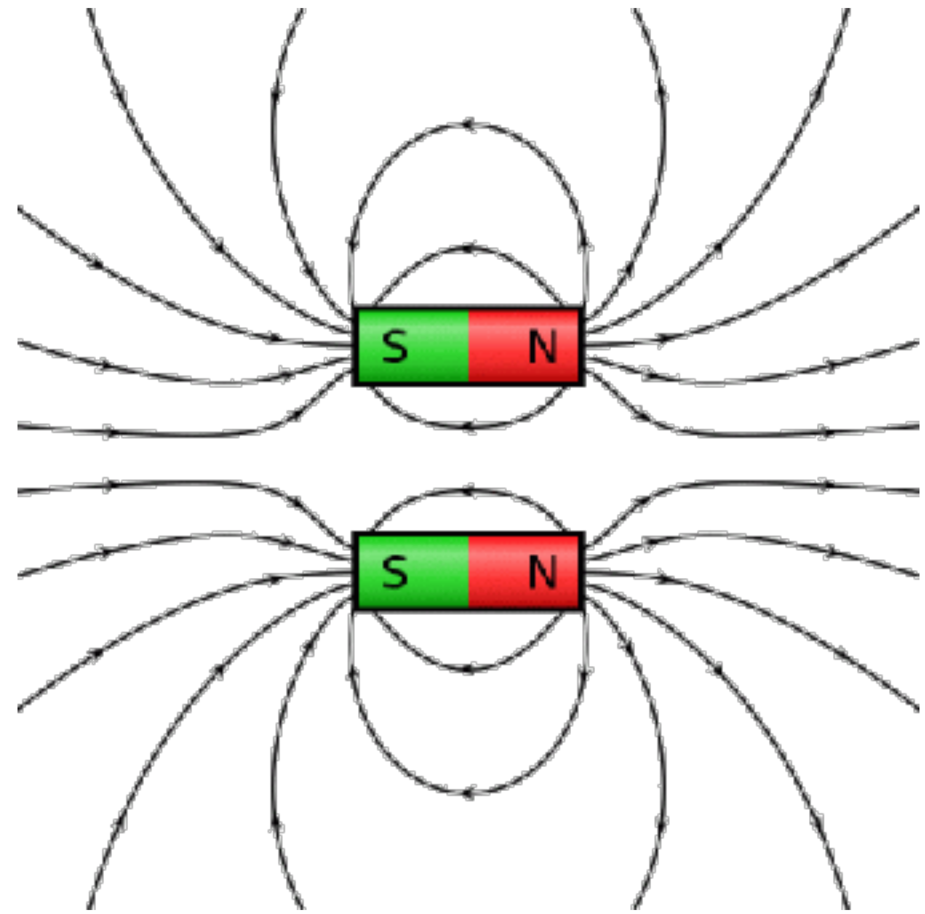
- An invisible vector field that interacts with other **magnets**



What is a magnetic field?

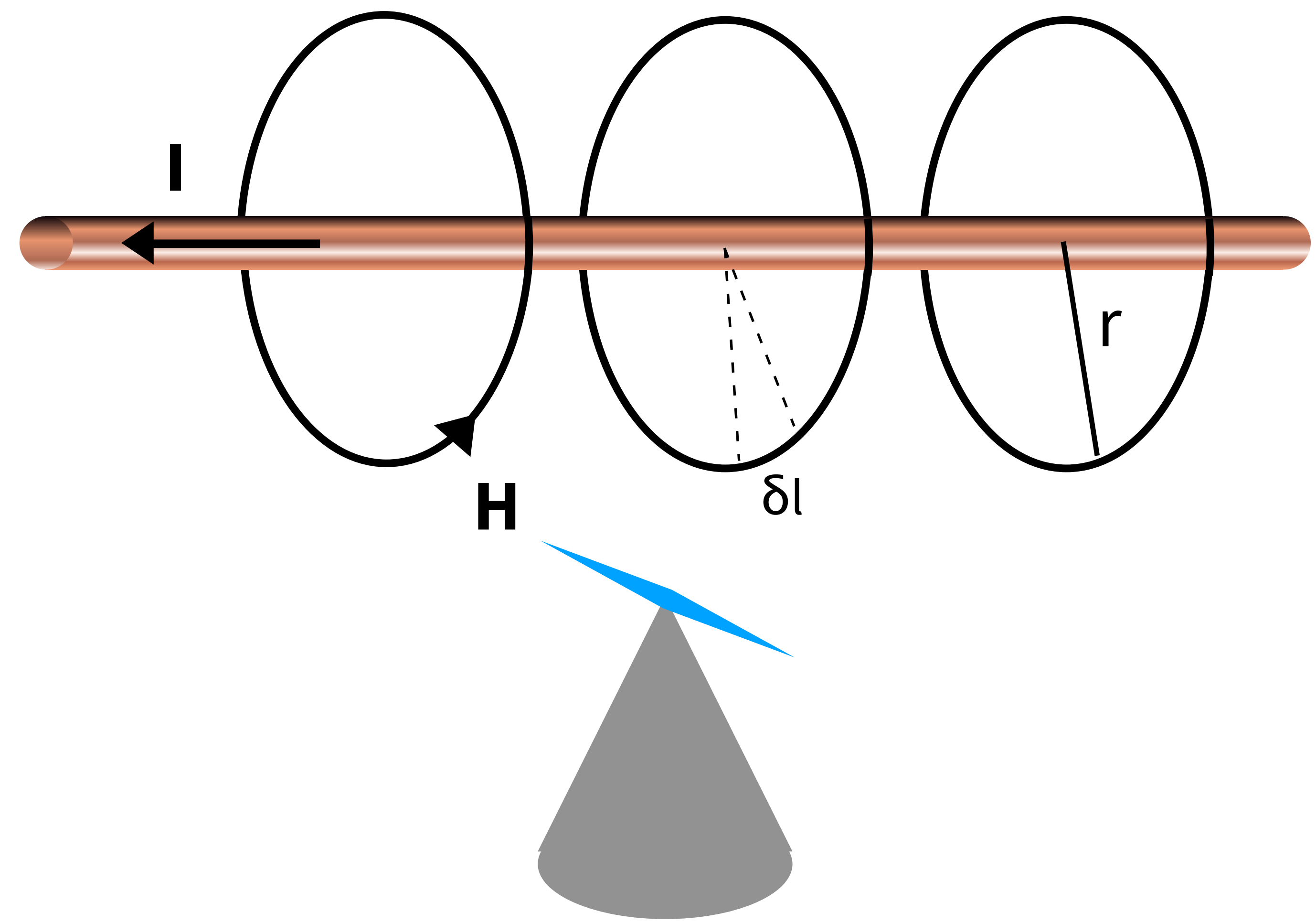
- An invisible vector field that interacts with other **magnets**





Magnetic field, Øersted 1820

- Oersted discovered in 1820 that a current carrying wire was able to rotate a compass needle



- Current and field are related by Ampere's Law

$$I = \oint \mathbf{H} dl$$

- Example for I = 1A, integral around the loop is 2πr, r = 2 mm
  H ~ 80 A/m

- Earth's magnetic field ~ 40 A/m

Interaction of two current-carrying wires, Ampere 1825

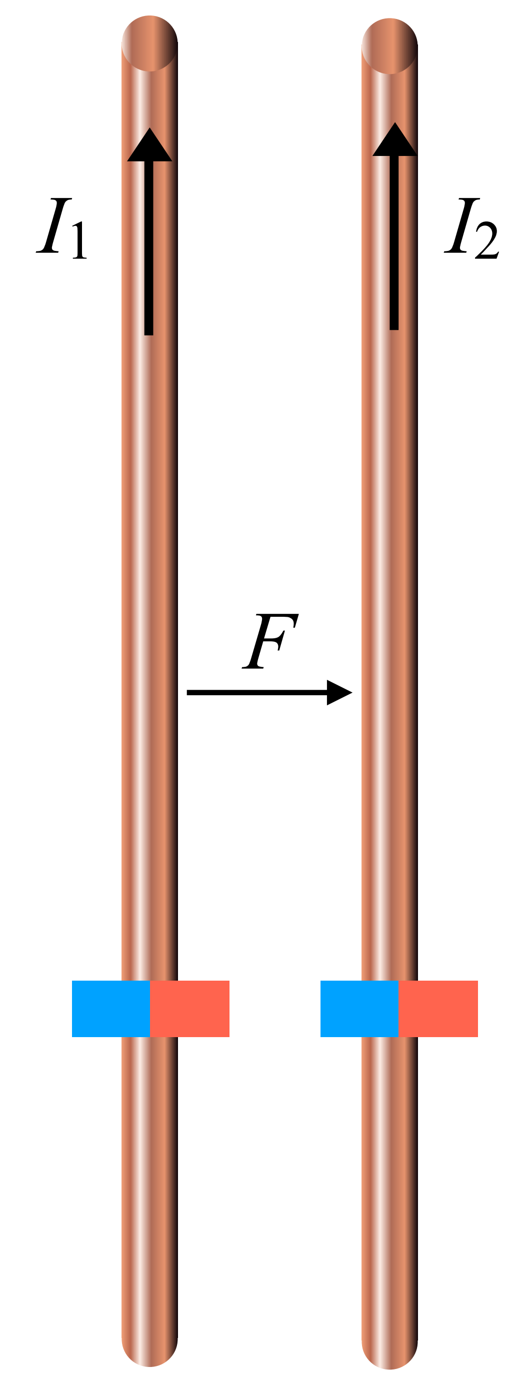
- Two current carrying wires (one longer than the other) are attracted to each other for parallel current, and repel for anti-parallel current.

$$\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

- The parallel wires “look like” magnets in the **perpendicular** direction

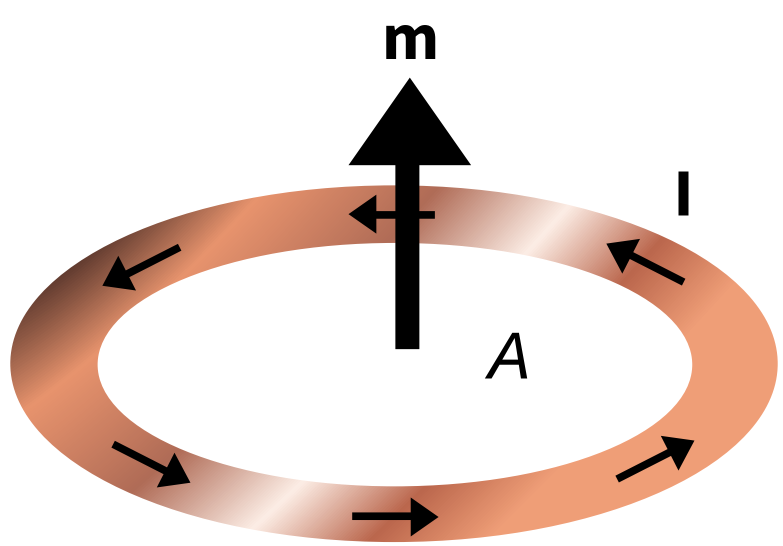
- Weird but central to electromagnetism (E and B fields in light)

- Different from electrostatics as this is a **dynamic** effect from the motion of charge



Equivalence of currents and magnetic moments

- So currents look like magnets… do magnets look like currents?



$$\mathbf{m} = I_{\perp} A$$

- Can express a current loop as an effective moment, ie a source of magnetic field

- What kind of currents do we need compared to typical magnetic fields?

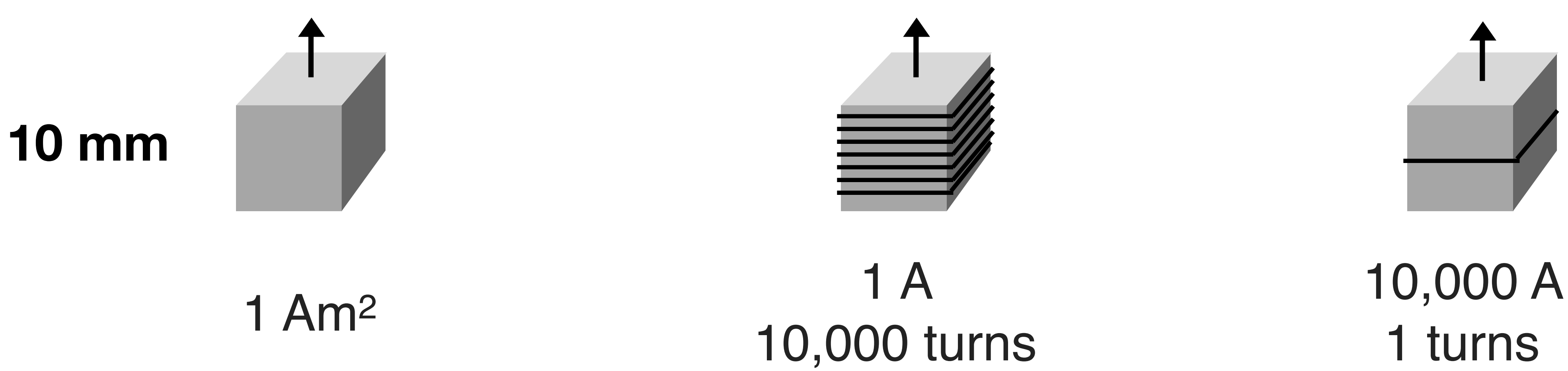
Comparison of current magnitudes and magnets

- Using the equivalence of current loops and magnetic moments we can compare the effective currents for a typical small magnet

- Moment given by for a single loop and a solenoid respectively, where *n* is the number of turns of the coil

$$\mathbf{m} = I_{\perp}A \qquad \mathbf{m} = nI_{\perp}A$$

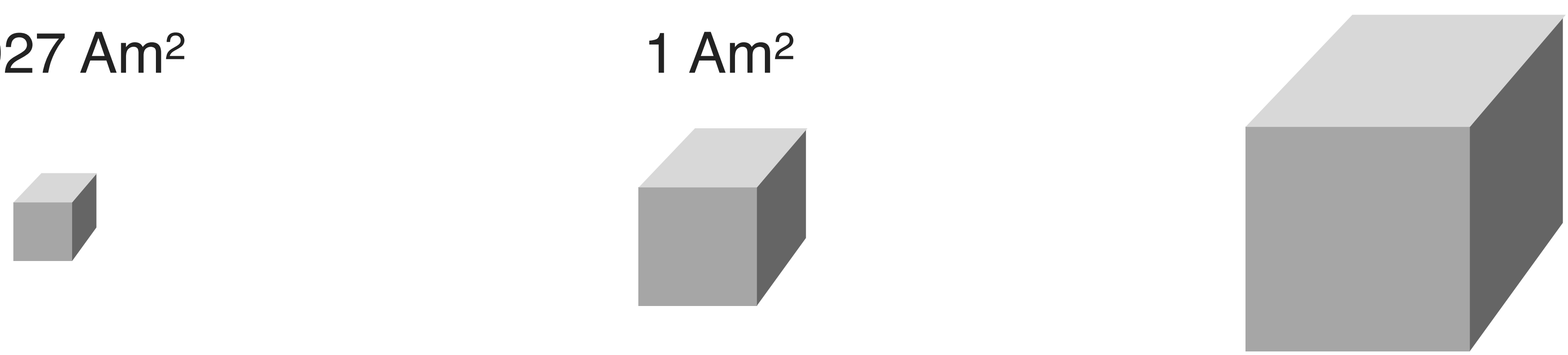
- For a small magnet



- At small sizes, magnets generate much larger fields -> applications in motors

Difference between magnetic moment and magnetisation

- Magnetic moment is specific to the sample (bigger magnet, bigger field)



0.027 Am^2 — **3 mm**

1 Am^2 — **10 mm**

15.6 Am^2 — **25 mm**

- Magnetization is the **moment density**

Assume NdFeB
$M_s \sim 1$ MA/m

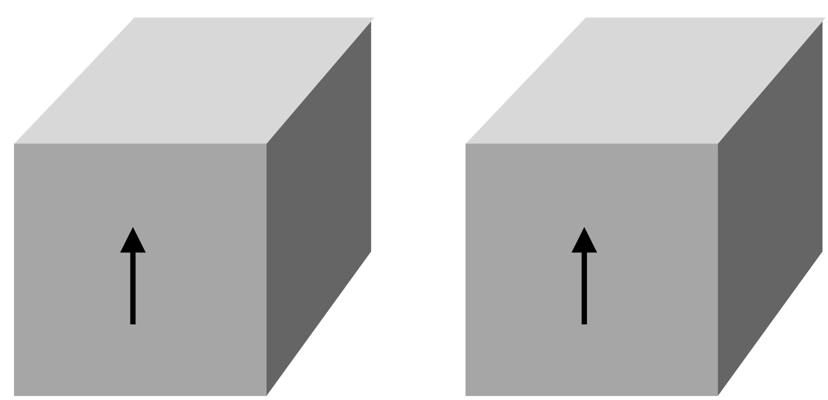
$$\mathbf{m} = \mathbf{M}V$$

  - Magnetisation is a property of the **material**
  - Moment is a property of a **magnet**

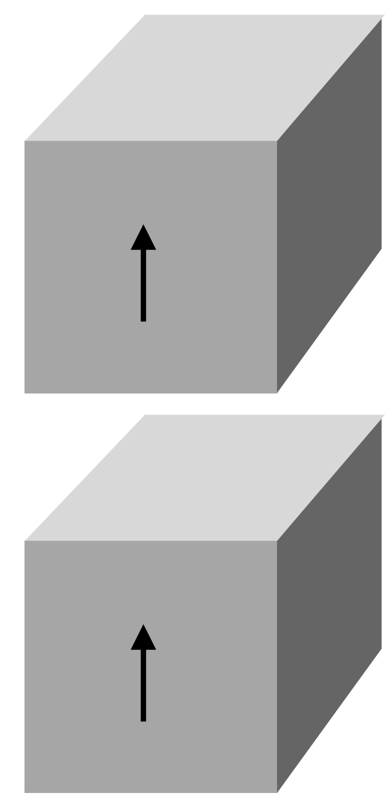
- Magnetisation is scale independent

Vectorial nature of magnetic moments

- A magnetic moment generates a field around it
  - Interaction with non-magnets is weak
  - Interaction with magnets is stronger but orientation dependent



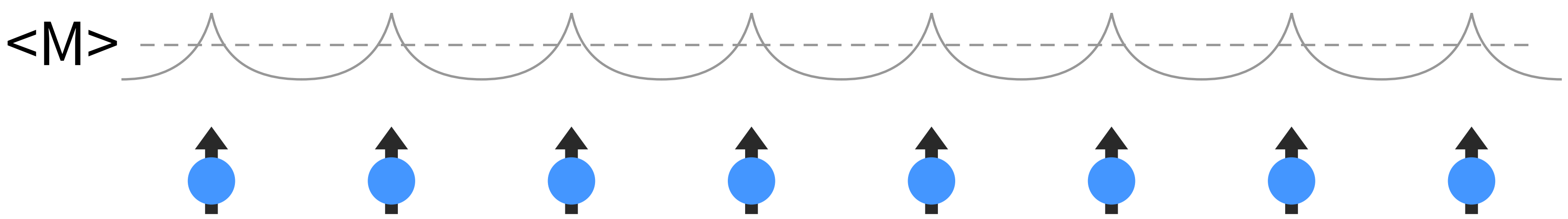
Weak repulsion



Strong attraction

Physical origin of magnetization and magnetic moment

- At the atomic scale the magnetic moments fluctuate strongly in time and space due to the electrons ‘orbiting’ nuclei

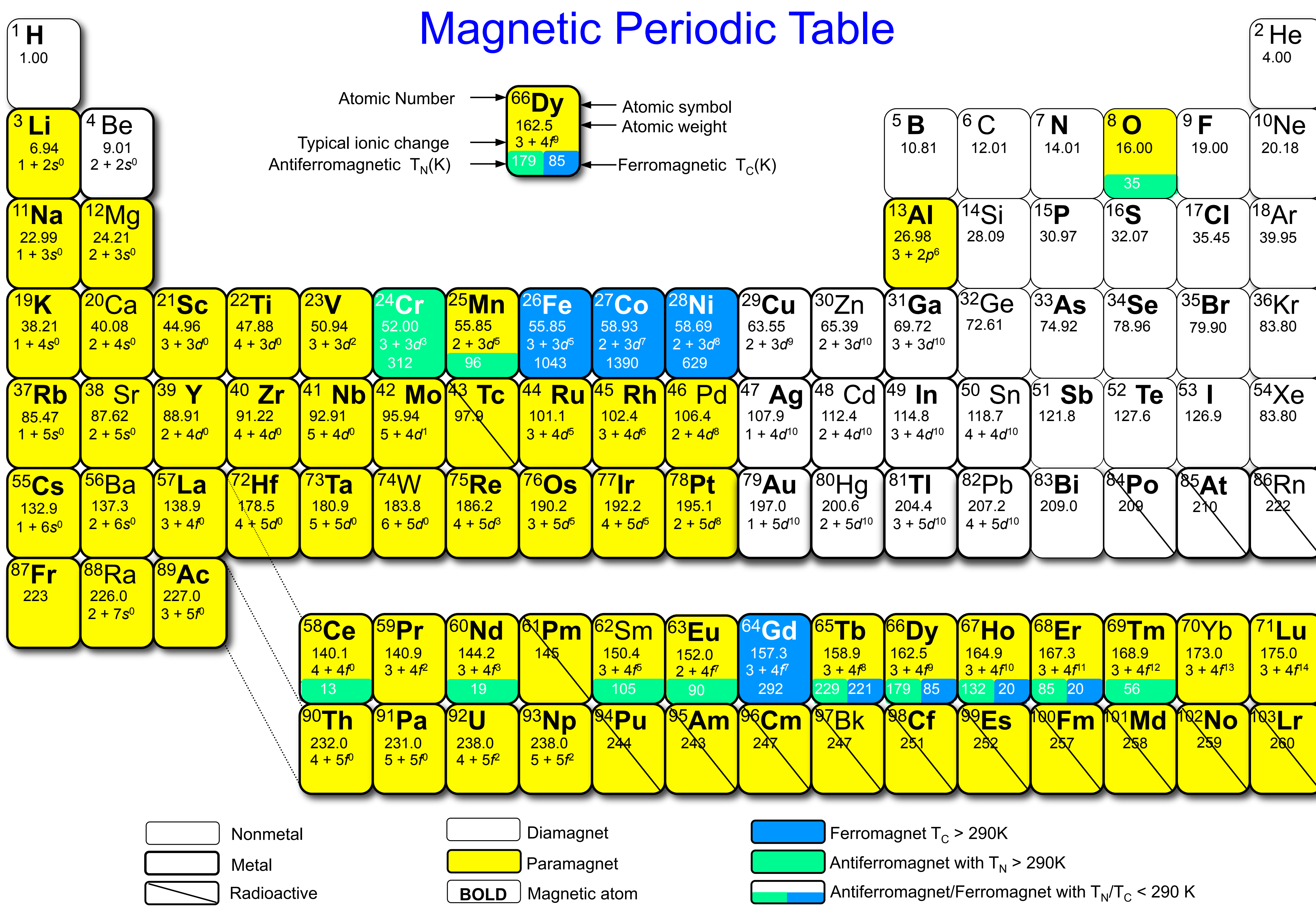


- Use a continuous medium approximation to calculate an average magnetisation $<M>$ (moment/volume)

- Avoids all the horrible details of fluctuating moments and can treat magnetism on a **continuum** level

- Good approximation for ferromagnets for volumes much larger than the atomic volume

Which elements are magnetic

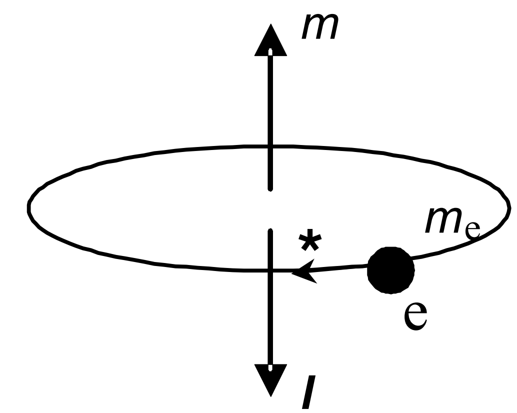


From Coey

Bohr magneton

- Can consider an electron 'orbiting' an atom
- A moving charge looks like a 'current', generating an effective magnetic moment
- In Bohr's quantum theory, orbital angular momentum ***l*** is quantized in units of $\hbar$; *h* is Planck's constant, 6.6226 10^{-34} Js;$\hbar$=h/2π=1.05510^{-34} Js
- The orbital angular momentum is ***l*** = $m_e \mathbf{r} \wedge \mathbf{v}$
- It is the z-component of $\boldsymbol{l}_z$ that is quantized in units of $\hbar$, taking a value m_l

  m_l is a quantum number, an *integer* with no units. Eliminating r in the expression for m
- μ_B is the Bohr magneton, the basic unit of atomic magnetism



$$m = IA = -\frac{evr}{2}$$

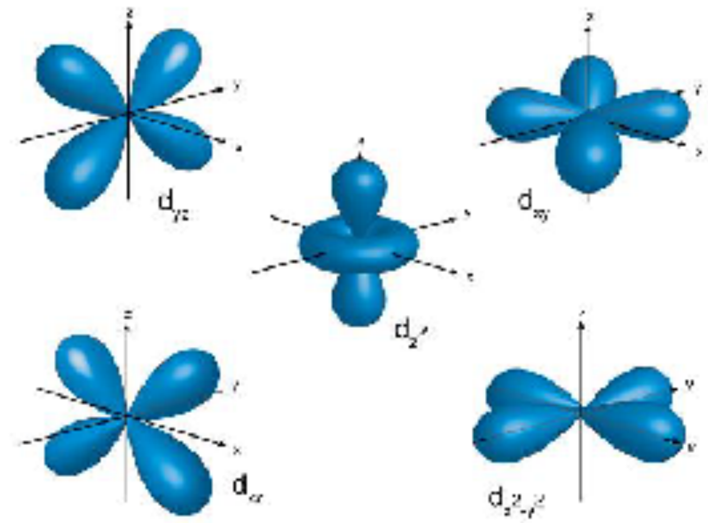
$$m = -\frac{e}{2m_e}\mathbf{l} = \frac{e\hbar}{2m_e}m_l = m_l\mu_B$$

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24}\mathrm{Am}^2 \,|\, \mathrm{JT}^{-1}$$

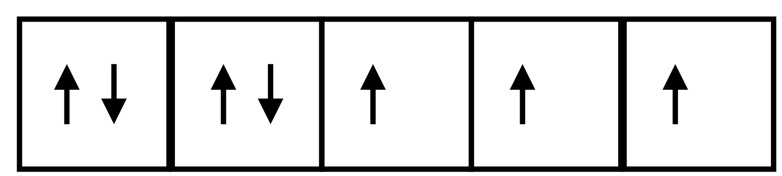
* electrons travel in the opposite direction to currents

Non-integer magnetic moments

- Transition metal magnets tend to have non-integer magnetic moments, eg Fe ~ 2.2 μ_B, Co ~ 1.72 μ_B, Ni ~ 0.6 μ_B
- If electrons carry quanta of angular momentum, how is this possible?
- Classic explanation is itinerant magnetism: electrons are delocalised and form bands
- First principles calculations reveal a non-integer magnetic moment quite localised to the atom
- Effect due to electrons hopping between different *d*-orbitals



		Fe (BCC)
Fe s	↑	0.31
	↓	0.32
p	↑	0.36
	↓	0.40
d	↑	4.37
	↓	2.19
Total	↑	5.07
	↓	2.93



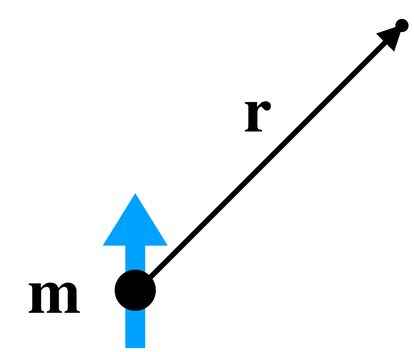
K Schwarz *et al* 1984 *J. Phys. F: Met. Phys.* **14** 2659

Field from a dipole

- The magnetic induction (field) from a point dipole can be derived classically (see Jackson) and is given by

$$\mathbf{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{|\mathbf{r}|^3}\right)$$

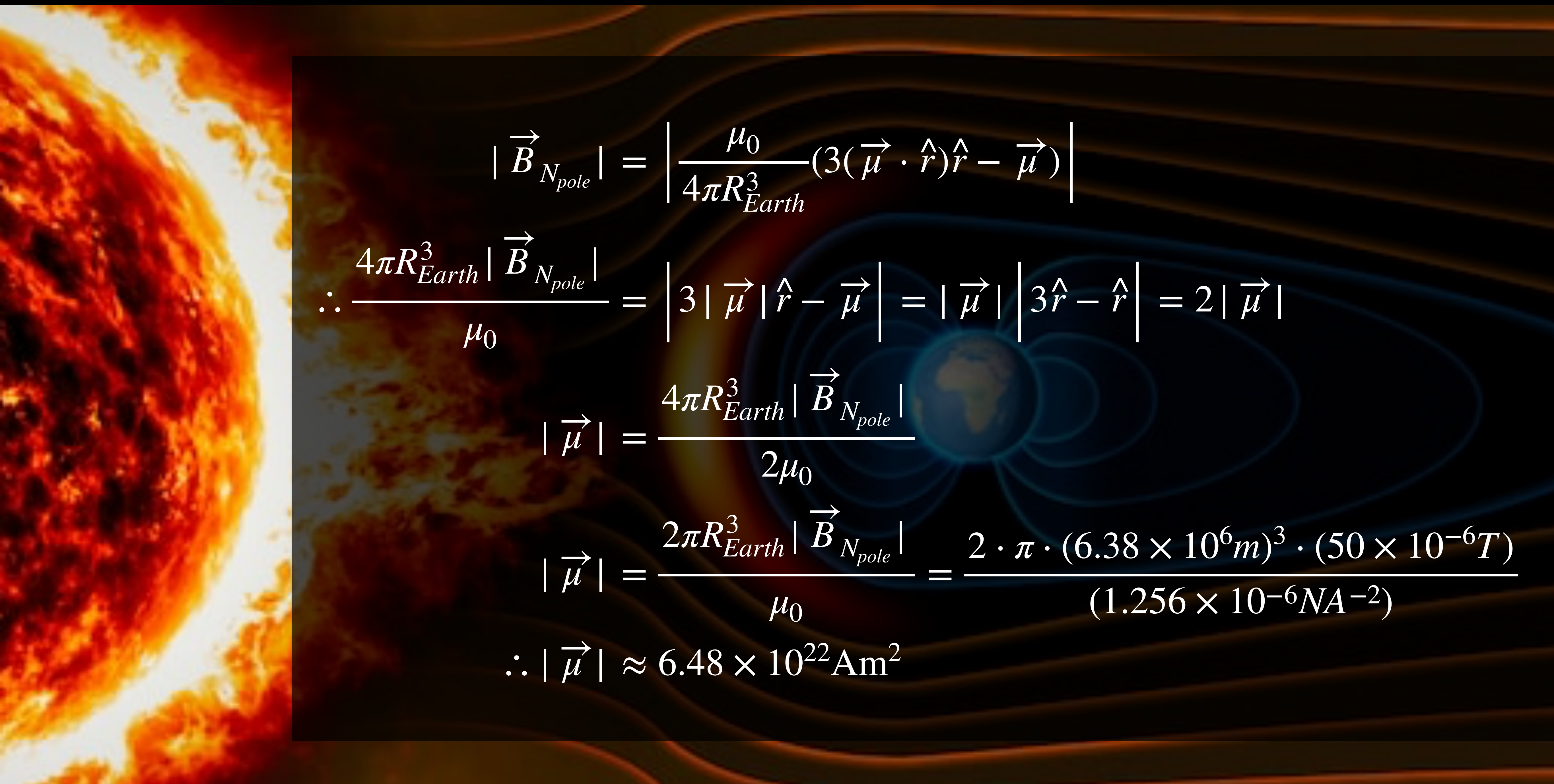
B-field at any point from a point dipole



- Ignores any distribution of magnetic ‘charge’ at the dipole (need a multipole description)

J. D. Jackson, Classical electrodynamics (2nd ed.). New York: Wiley. (1975)

Question: What is the size of the Earth's magnetic moment?



$$|\vec{B}_{N_{pole}}| = \left| \frac{\mu_0}{4\pi R_{Earth}^3}(3(\vec{\mu}\cdot\hat{r})\hat{r} - \vec{\mu}) \right|$$

$$\therefore \frac{4\pi R_{Earth}^3 |\vec{B}_{N_{pole}}|}{\mu_0} = \left|3|\vec{\mu}|\hat{r} - \vec{\mu}\right| = |\vec{\mu}|\left|3\hat{r} - \hat{r}\right| = 2|\vec{\mu}|$$

$$|\vec{\mu}| = \frac{4\pi R_{Earth}^3 |\vec{B}_{N_{pole}}|}{2\mu_0}$$

$$|\vec{\mu}| = \frac{2\pi R_{Earth}^3 |\vec{B}_{N_{pole}}|}{\mu_0} = \frac{2\cdot\pi\cdot(6.38\times10^6 m)^3\cdot(50\times10^{-6}T)}{(1.256\times10^{-6}NA^{-2})}$$

$$\therefore |\vec{\mu}| \approx 6.48\times10^{22}\,\mathrm{Am}^2$$

- Assume an effective dipole at the centre of the Earth and a magnetic flux density at the North Pole of 50 μT and R_{Earth} = 6.36 x 10^6 m

Question 2: What is the *magnetization* of the Earth?

$$M_{Earth} = \frac{m_{Earth}}{V_{Earth}}$$

$$M_{Earth} = \frac{m_{Earth}}{\frac{4\pi}{3}R_{Earth}^3}$$

$$M_{Earth} = \frac{6.48 \times 10^{22}}{\frac{4\pi}{3}(6.38 \times 10^6)^3} \approx 60\mathrm{A/m}^{-1}$$

Question 3: If the source of the magnetic field is an electrical current at the equator, what is its size?

$$m = IA$$

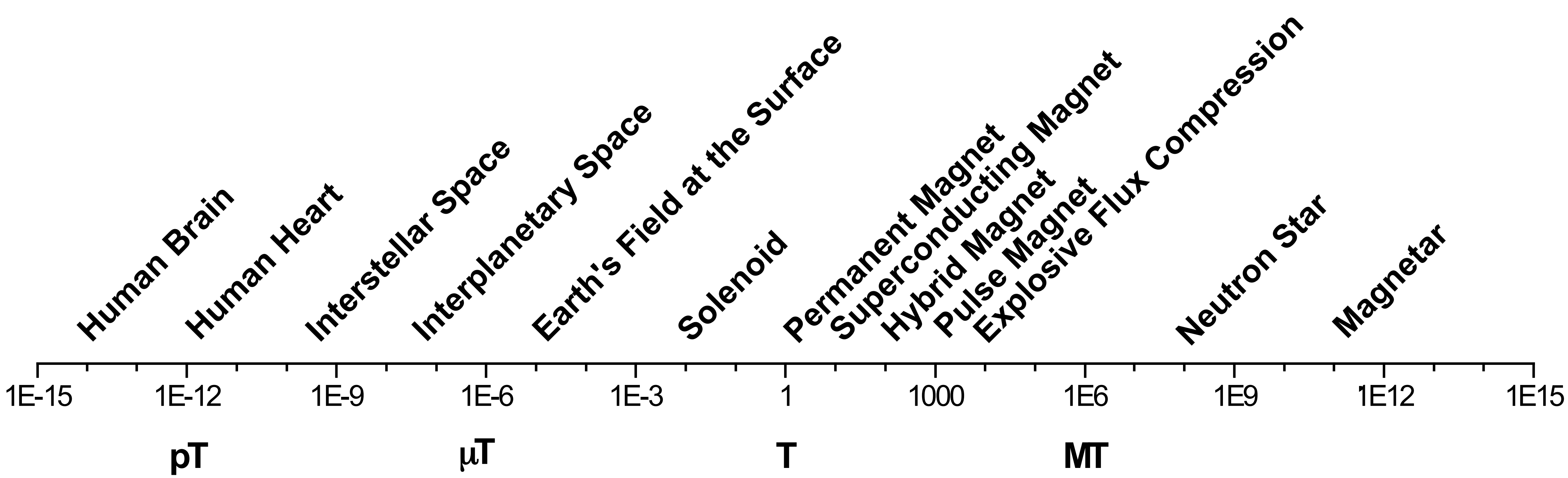
$$I_{Equator} = \frac{m_{Earth}}{\pi R_{Earth}^2}$$

$$I_{Equator} = \frac{6.48 \times 10^{22}}{\pi (6.38 \times 10^6)^2} \approx 5 \times 10^8 \mathrm{A}$$

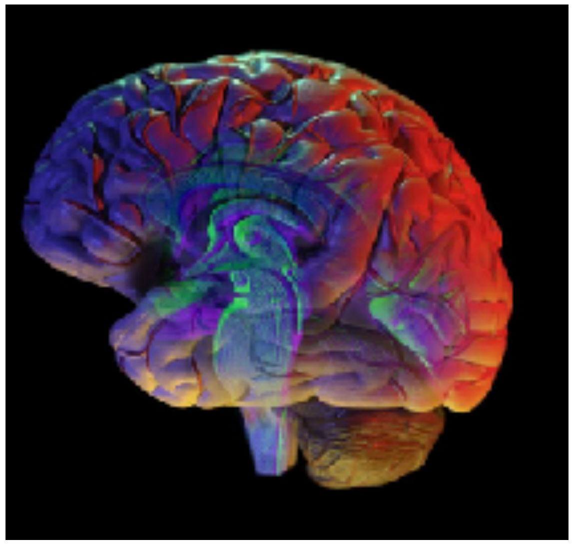
Magnetic fields and demagnetising factors

What ranges of magnetic fields exist?

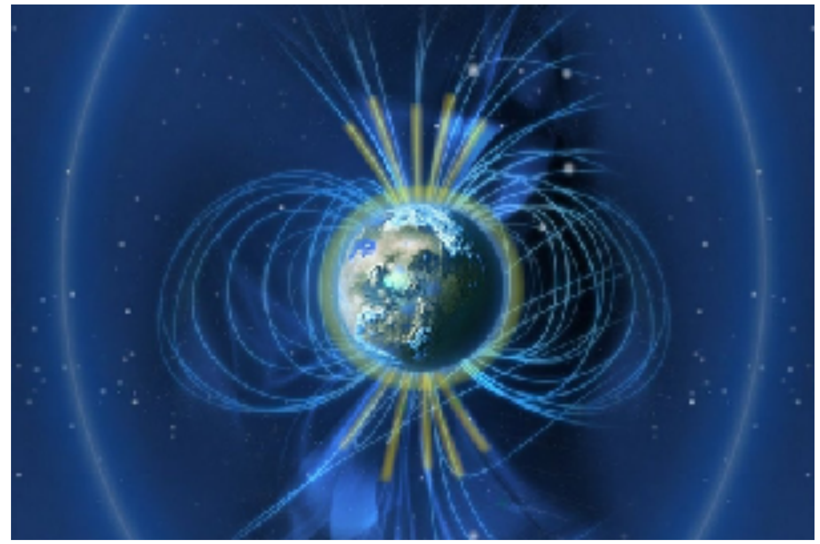
- Historically a terrestrial 1T field was considered ‘large’
- Today that is not generally true
  - Recording Media coercivity ~1T
  - MRI ~ 5T



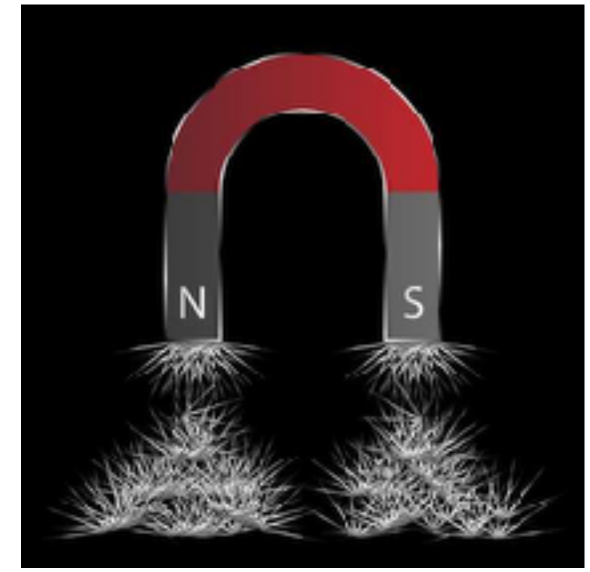
Typical values of magnetic fields



Human Brain 1 fT

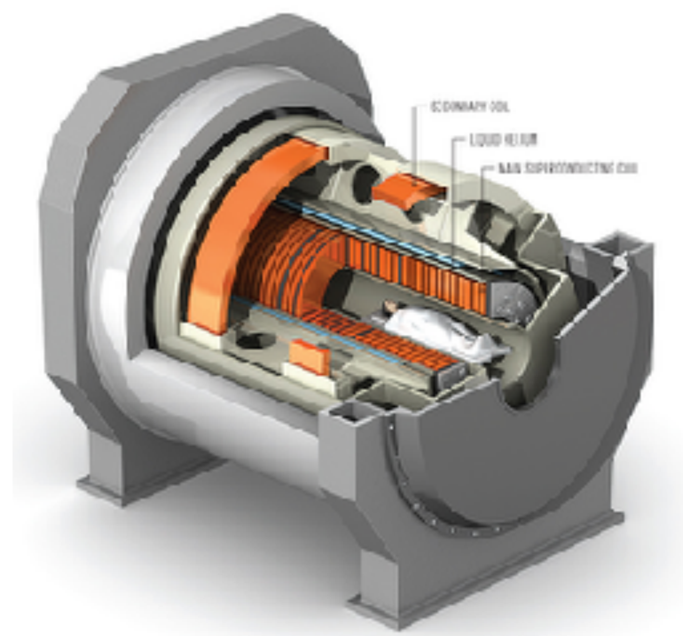


Earth 50 μT

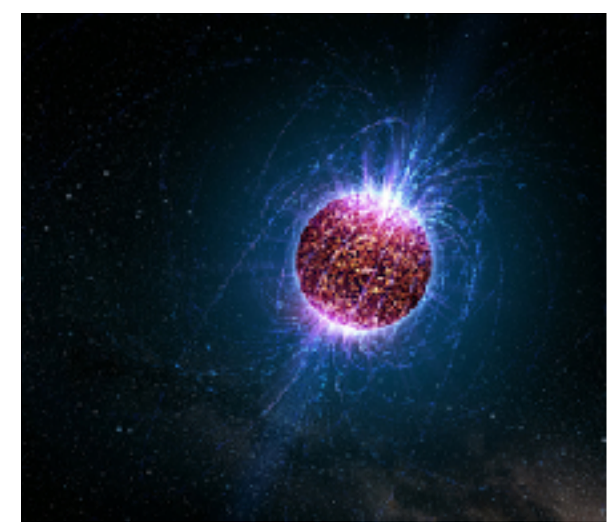


Permanent Magnet 0.5-1T

Electromagnet 1T



Superconducting magnet 10 T



Magnetar 10^{12} T

Magnetic fields in free space

- Two definitions of magnetic field

  Magnetic Field **H** [A/m] Magnetic flux density **B** [T]

- When talking about generated magnetic fields in free space, they express the exact same physical phenomenon, and are related by

$$\mathbf{B} = \mu_0 \mathbf{H}$$

- $\mu_0 = 4\pi\ 10^{-7}$ H/m is the permeability of free space

- The difference between H-field and B-field is a common point of confusion, but only when considering a magnetic medium $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$

- **B**-field component arising from **applied H**-field is exactly $\mathbf{B} = \mu_0 \mathbf{H}$

Magnetic fields in media

- The actual B-field in response to media is generally more complex

$$\mathbf{B} = \mu_0 \left(\mathbf{M} + \mathbf{H}\right)$$

- Or alternatively in terms of a relative permeability or susceptibility

$$\mathbf{B} = \mu_0 \mu_\mathrm{r} \mathbf{H} = \mu_0 (1 + \chi) \mathbf{H}$$

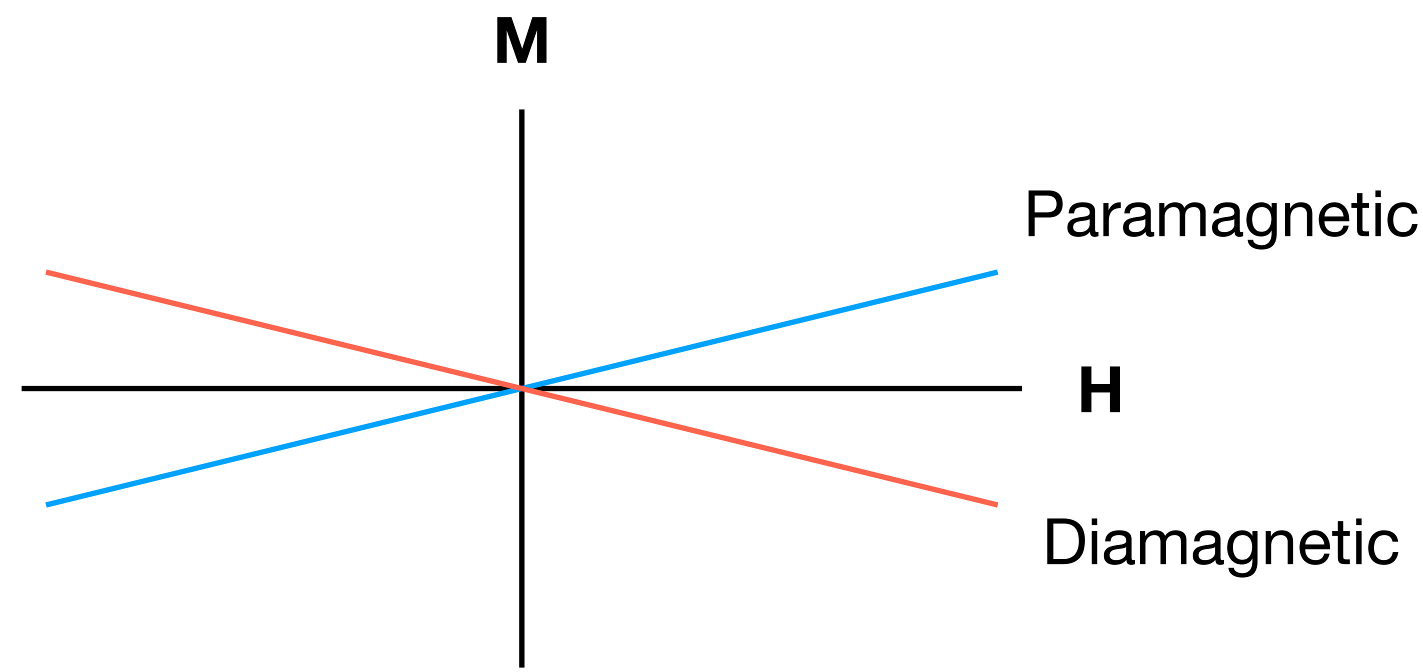
- where the susceptibility gives the full magnetic response, or limit of small fields (initial susceptibility)

$$\chi = \mathbf{M}(\mathbf{H}) = \left.\frac{d\mathbf{M}}{d\mathbf{H}}\right|_{H \to 0}$$

- Different media ave very different responses, ferromagnets highly non-linear

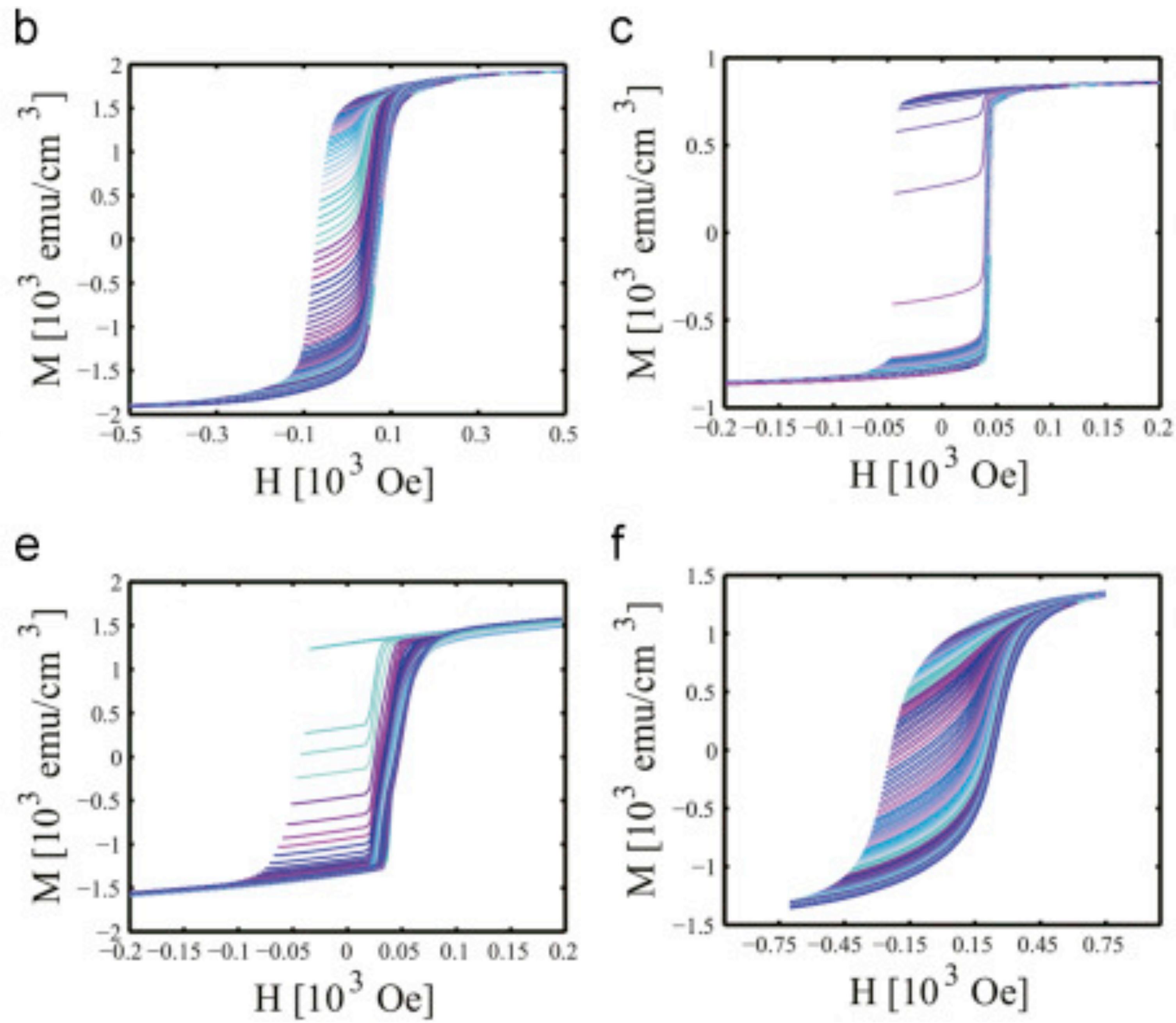
Diamagnetism and Paramagnetism

- Diamagnets and paramagnets have a weak magnetic response ($\chi << 1$), ~ 10^{-4} - 10^{-6}
- Response typically isotropic with respect to the field
- Diamagnets repel external magnetic fields due to Larmor precession of bound electrons that induces a moment opposite to the applied field
- Paramagnets weakly align with an external field overcoming thermal fluctuations



Ferromagnetism

- Complex and anisotropic behaviour of **M**(**H**)
- Definition of χ = M/H is not very sensible in most cases
- Saturated case easier to deal with!



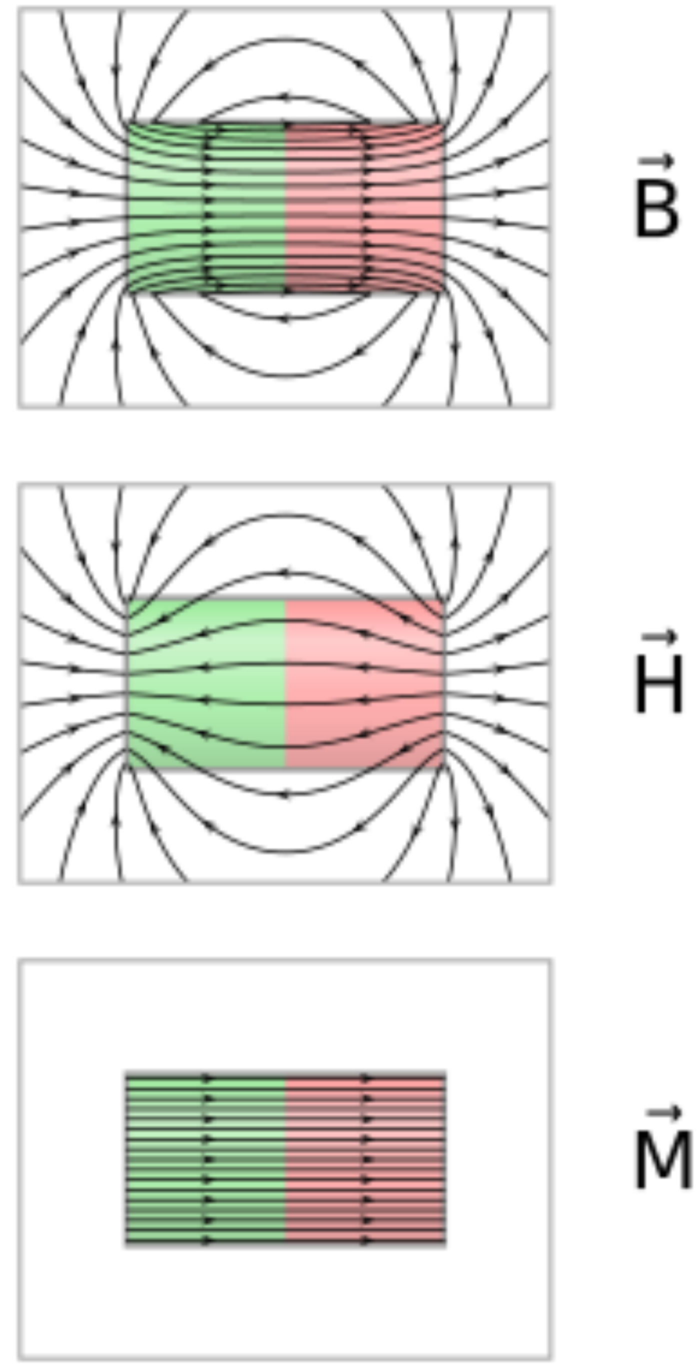
Yue Cao et al, JMMM 395, 361-375 (2015)

Relation between B and H in a saturated material

- Magnetic field around a saturated magnet simple **B** = μ_0**H**
- What about inside the magnet?

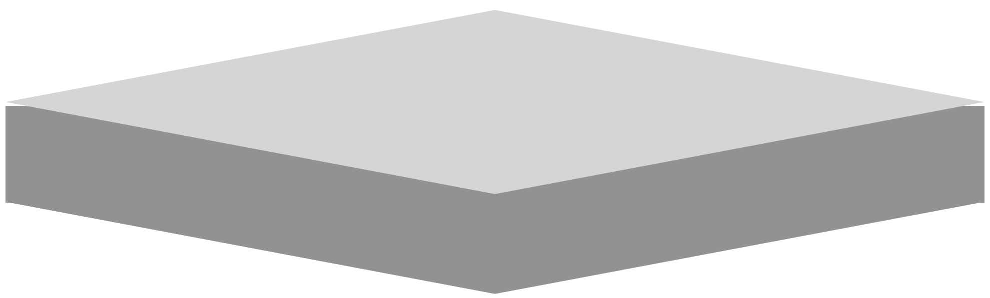
$$\mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H})$$

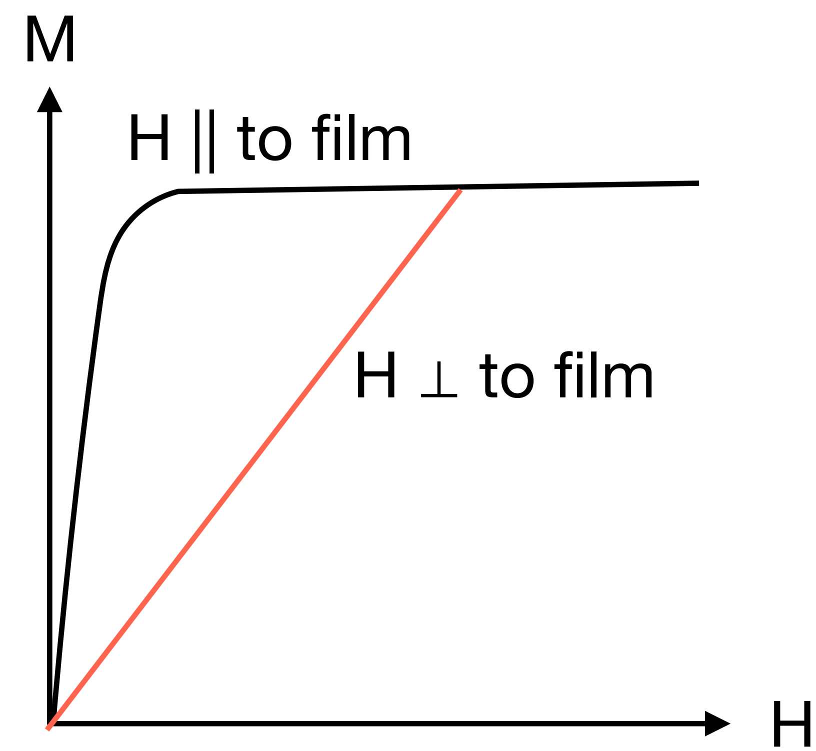
- Why do we care?
- In general magnetization processes are anisotropic and depend on sample shape



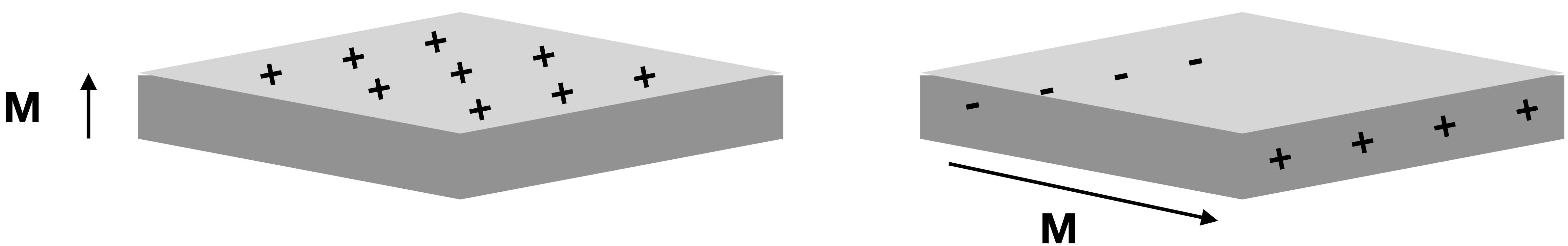
Example: thin magnetic film

- Much easier to magnetise in the plane than out-of-plane





- Origin is *demagnetising* field - aims to minimise surface charges



Demagnetizing fields

- Local effective field inside the magnet depends on surface

$$\boldsymbol{H}(\boldsymbol{r}) = \frac{1}{4\pi}\left\{-\int_V \frac{(\nabla.\boldsymbol{M})(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} d^3r' + \int_S \frac{\boldsymbol{M}.\mathbf{e}_n(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} d^2r'\right\}$$

- Since M is uniform, first (bulk) term is zero

- For surface term, $\mathbf{M}.\mathbf{e}_n$ determines surface charge density, larger surface leads to larger field opposing magnetisation

- Leads to concept of a *demagnetising field*

$$\boxed{\mathbf{H} = \mathbf{H}_{app} - \mathbf{H}_d}$$

Demagnetization Factor

- Calculating demagnetisation field is tedious (lots of boring and complicated integrals)

- Simplify - invent a “demagnetising factor” or “shape factor” *N*

$$\mathbf{H}_d = -N\mathbf{M}$$

- Shape factor gives a constant of proportionality between the demagnetising field and shape

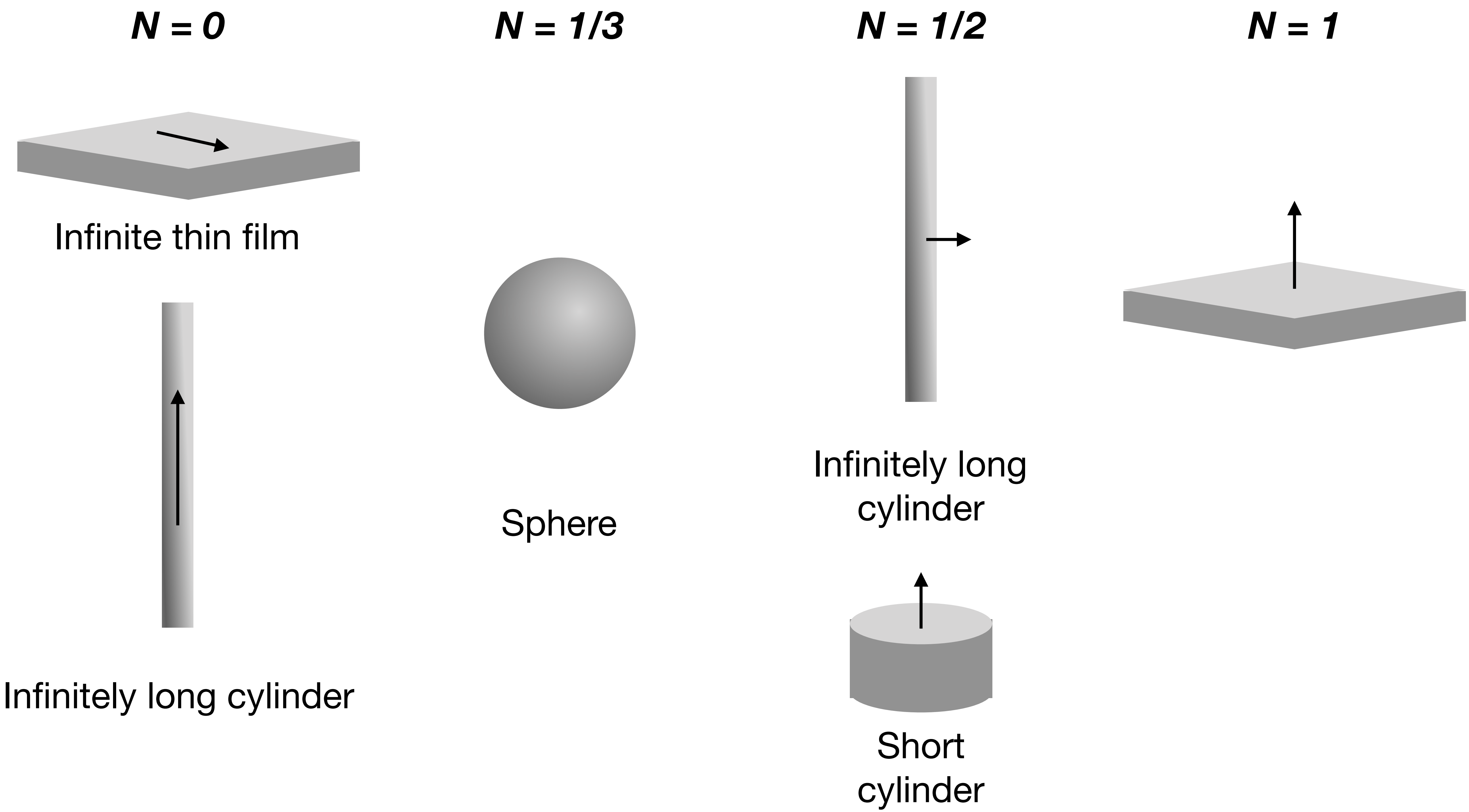
- Always between 0-1 and in general a tensor with trace 1

$$N_x + N_y + N_z = 1$$

- Known for simple geometric shapes (spheres, ellipsoids, rectangular prisms)

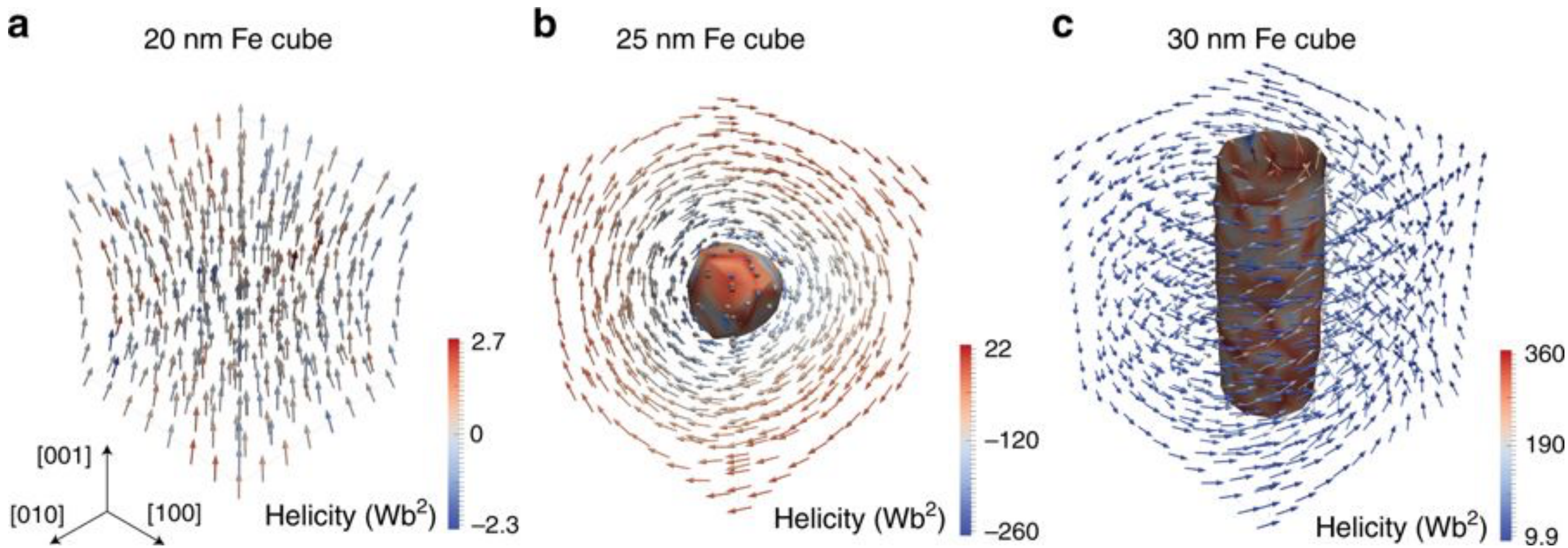
- Is usually calculated numerically for anything complicated

Demagnetization factors for different shapes



Beware of non-uniformities

- In general magnetization is not uniform for other shapes



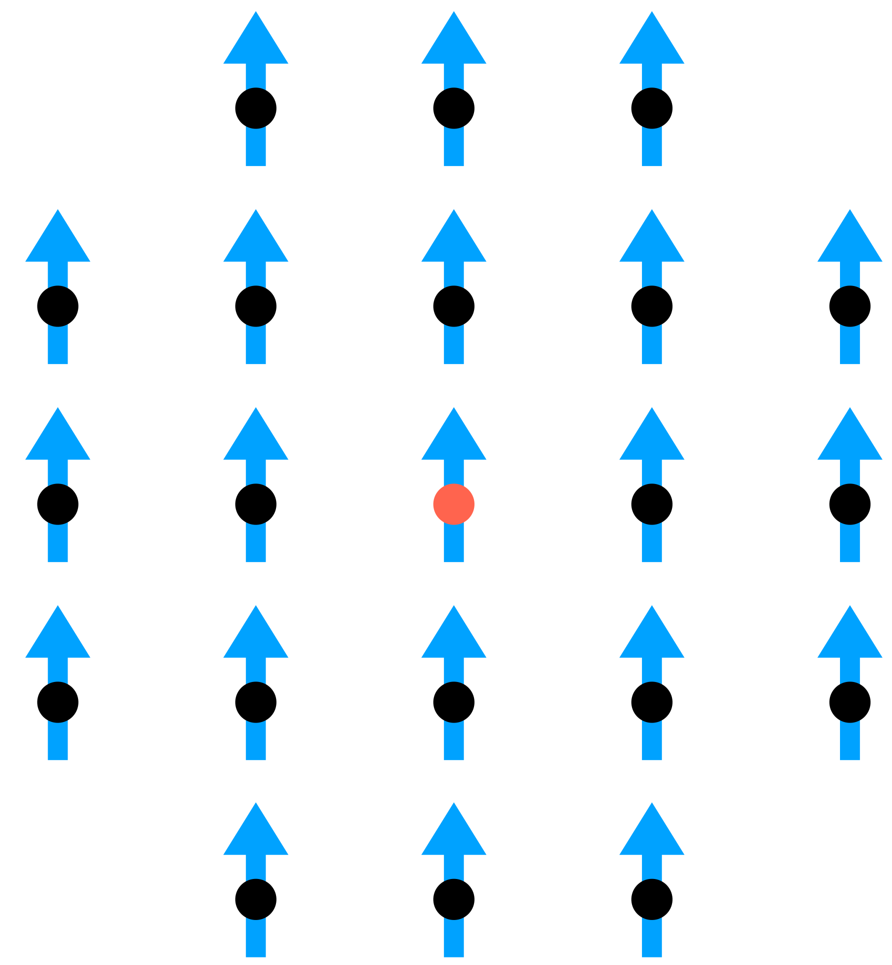
Jay Shah et al, Nature Communications 9 1173 (2018)

Dipole fields and magnetostatics

- Assume a lattice of dipoles in shape of a sphere
- Total dipole field at a point in the centre summing over all other dipoles is zero

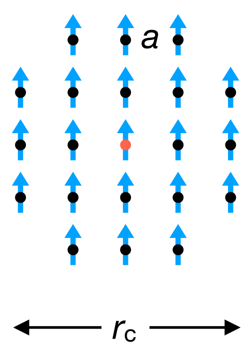
$$\mathbf{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{|\mathbf{r}|^3}\right)$$

- Where does the demagnetising field come from?

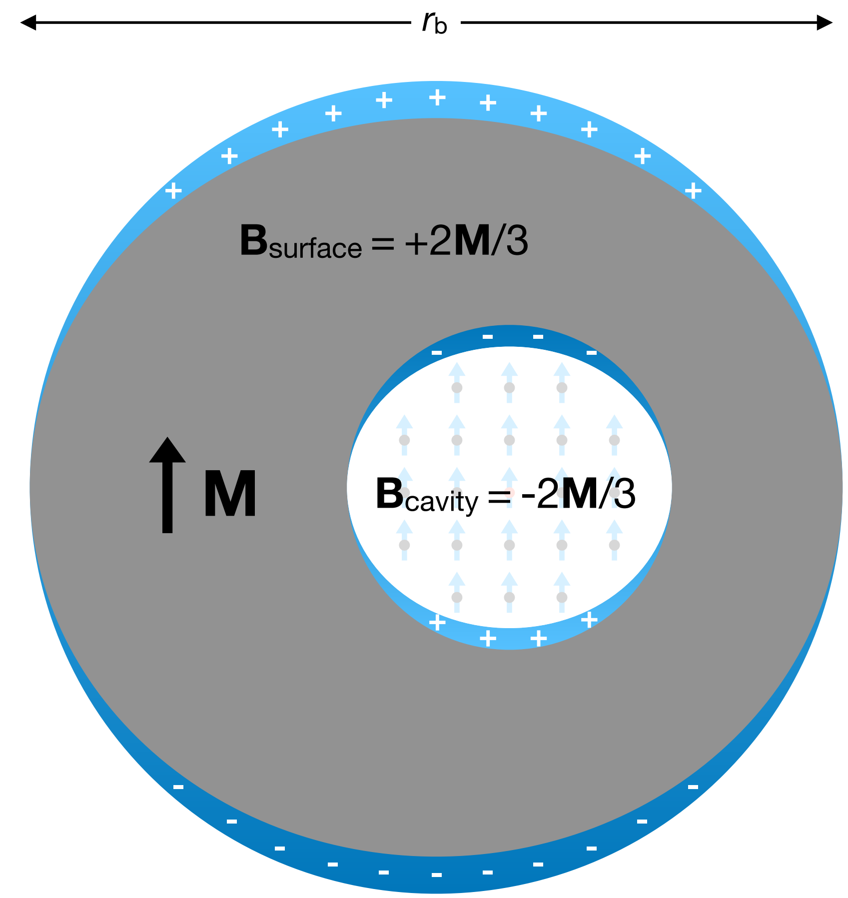


Classical solution: Lorentz cavity field

- Divide the problem into local and macroscopic fields $a << r_c << r_b$



$\mathbf{B}_{loc} = 0$



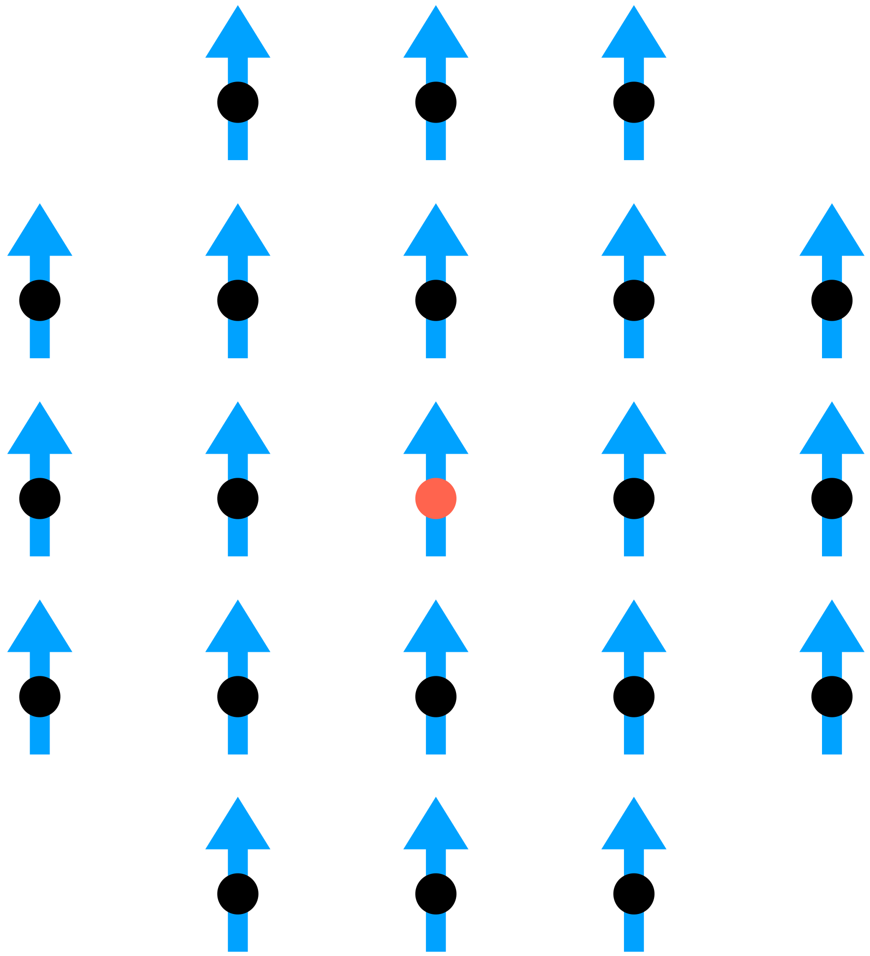
$$\mathbf{B}_{loc} = \mathbf{B}_{surface} + \mathbf{B}_{cavity}$$

$$= +2\mathbf{M}/3 - 2\mathbf{M}/3 = 0$$

- Suggests the local field at an atom is zero, despite global “demagnetising field”

What about nanoparticles and clusters?

- Small system where Lorentz approximation is not true ($a \ll r_c \ll r_b$)
- Average field for a sphere of dipoles is zero
- Where did the demagnetising field go?



Field inside a dipole

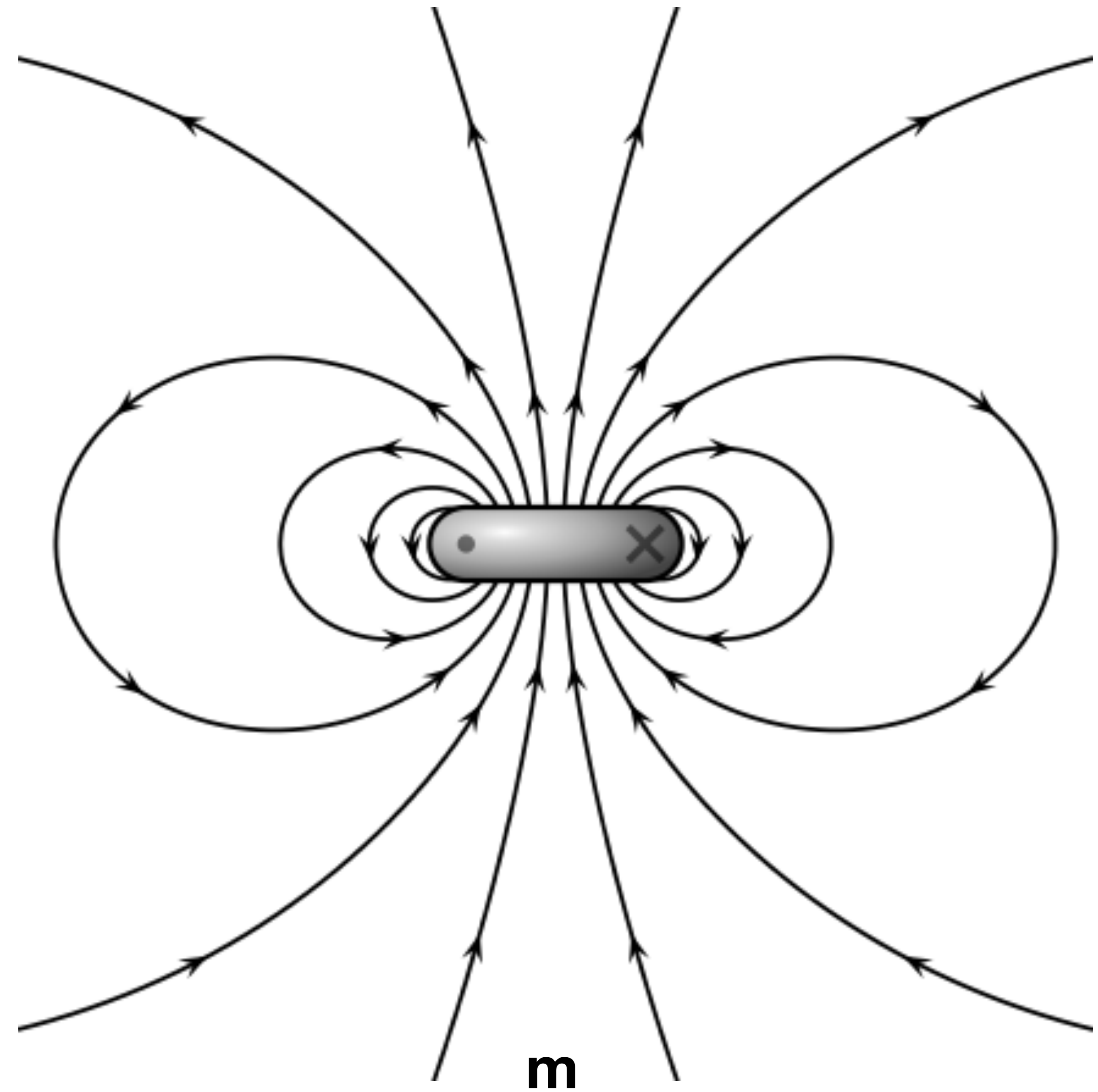
- Inside the current loop

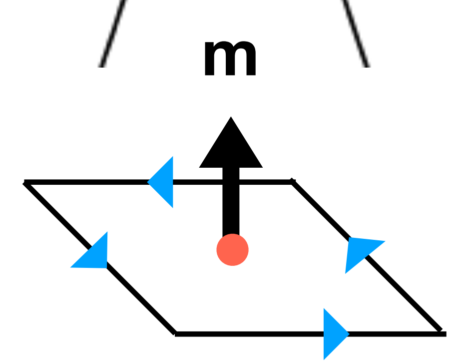
$$\mathbf{B} = \frac{\mu_0}{4\pi}\left[\frac{3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}}{|\mathbf{r}|^3} + \frac{8\pi}{3}\mathbf{m}\delta(x)\right]$$

- Second term comes from treating limiting field at origin over volume $\delta(x)$

- Field at centre of current loop *looks like* macroscopic field

- BUT averaged over the volume encompassed by the loop



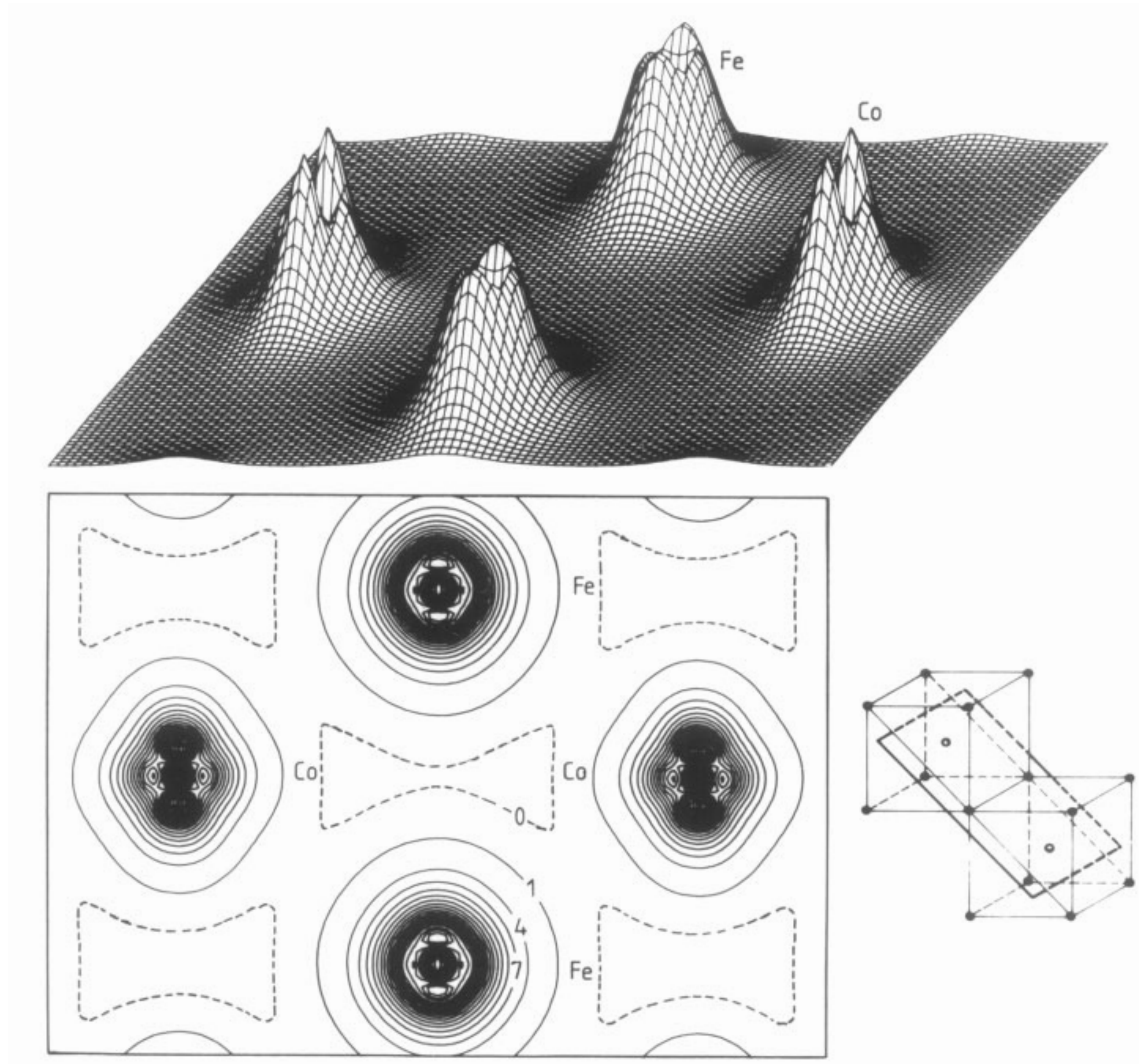


J. D. Jackson, Classical electrodynamics (2nd ed.). New York: Wiley. (1975)

Reality: much more complicated

- Dipole approximation is not terrible
- But large local deviations from the average at atomic sites
- Which field is needed for spin dynamics for an atom?
    - A problem for both atomistic and micromagnetic simulations
- In the end its a moot point, only sample symmetry matters since **M** x **H** = 0 for **M** || **H**

Calculated electron spin density in CoFe alloy



K Schwarz *et al* 1984 *J. Phys. F: Met. Phys.* **14** 2659

Magnetic units

Magnetism units

- The older Gaussian/cgs units are still common in the literature
- (Some) conversion factors between the different systems

Quantity	Symbol	Gaussian & cgs emu	Conversion factor	SI
Magnetic flux density	B	gauss (G)	10^{-4}	tesla (T)
Magnetic field strength	H	oersted (Oe)	$10^3/4\pi$	A/m
Magnetization	M	emu/cc	10^3	A/m, J/T/m^3
Magnetic Moment	m	emu	10^{-3}	Am^2, J/T
Permeability of free space	μ_0	dimensionless	$4\pi \times 10^{-7}$	H/m, T^2 J^{-1} m^3

http://www.ieeemagnetics.org/images/stories/magnetic_units.pdf

Old units

- Redefinition of SI system in 2018 now makes the speed of light *c* and electronic charge *e* fixed constants.

- Now μ_0 is in principle a measurable quantity, defined from the fine structure constant ~ 1/137

$$\left(h/e^2\right)_{\text{exp}} = (\mu_0 c/2)_{\text{fixed}} \cdot (1/\alpha)_{\text{exp}}$$

$$(\mu_0)_{\text{exp}} = \left(2h/ce^2\right)_{\text{fixed}} \cdot (\alpha)_{\text{exp}}$$

- This breaks the previous convention fixing μ_0 as $4\pi\ 10^{-7}$ H/m and thus compatibility between the SI units and old CGS units

Magnetics has been one of the scientific disciplines most resistant to adoption of the SI. With the revised SI, the “peaceful coexistence” of two systems of units [Silsbee 1962] is no longer feasible. The following recommendations warrant consideration.

1) Scholarly journals that publish articles in magnetics should require use of the SI and disallow EMU such as oersted, gauss, and “emu per cubic centimeter.” Authors who find the expression of magnetic field strength H in units of ampere per meter to be inconvenient could instead refer to $\mu_0 H$ in units of tesla (or milli-, micro-, nano-, or picotesla). Similarly, magnetization M could be expressed as $\mu_0 M$ or as magnetic polarization J in units of tesla or millitesla.
2) For the benefit of future generations of magneticians, professors should use SI in classroom instruction. Commercial instruments and magnetometers should be programmed to report measurement results in SI.
3) In writing equations, it is adequate to use phrases such as “where μ_0 is the permeability of vacuum” (or “the vacuum magnetic permeability” or “the permeability of free space” or “the magnetic constant”) without giving a numerical value. This follows typical usage when referring to the speed of light c, the Boltzmann constant k, or the Bohr magneton μ_B.

Ronald B. Goldfarb IEEE Magn. Lett. 8 1110003 (2017)

A recent trend to using teslas for everything

- **B**, μ_0**M** and μ_0**H** are all defined in terms of magnetic field (intensity) in teslas (T)

- Started with Superconducting and Permanent magnet communities, probably due to avoidance of odd numerical conversions, dimensions and units

- Now common in the literature, theoretical and experimental

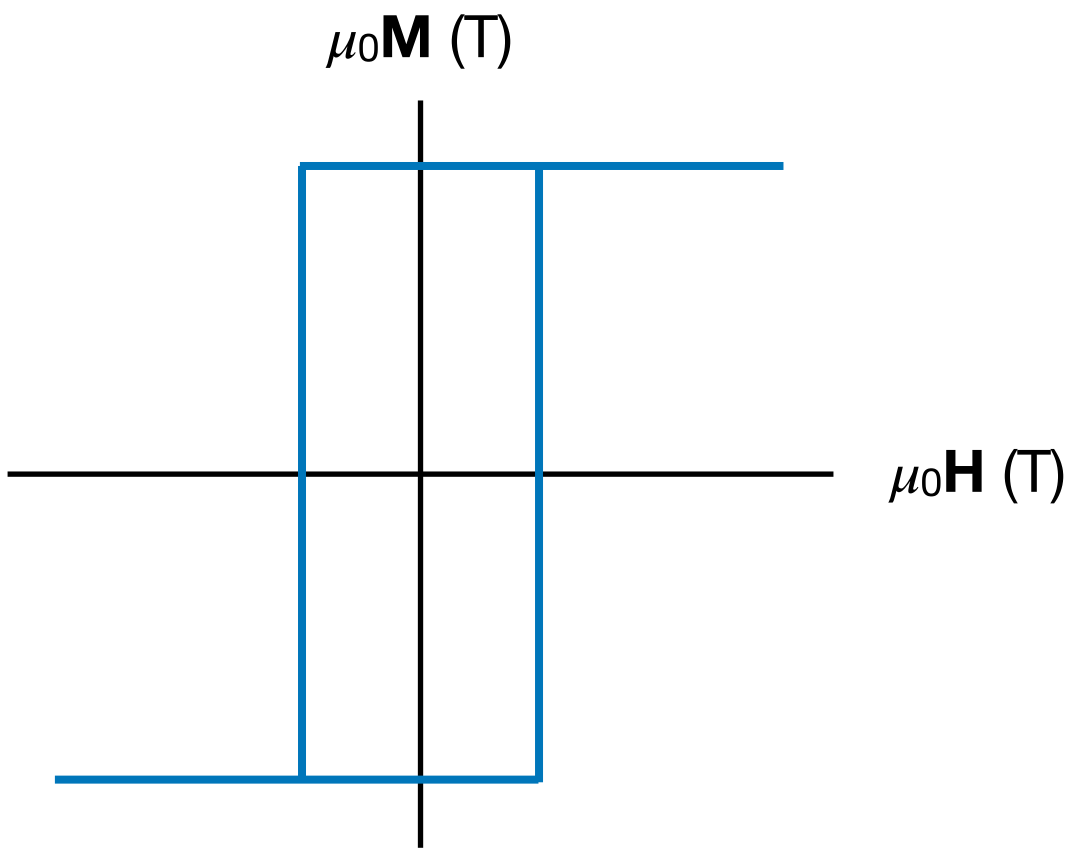
- Best way is to think about everything as current loop ‘sources’ of flux μ_0**M** and μ_0**H**

$$\mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H})$$

$$\mathbf{B}_{\text{total}} = \mathbf{B}_{\text{magnetization}} + \mathbf{B}_{\text{applied}}$$

- This convention leads to oddities in hysteresis - what are the units of **M.B**, both in Tesla??

Making sense of M-B loops



- Not immediately obvious that this is useful - a single loop cycle should give units of energy (density)

- BUT - can easily extract the magnetization in sensible dimensions by dividing by μ_0

$$\mathbf{M}(\mathrm{JT^{-1}m^{-3}}) \equiv \frac{\mu_0\mathbf{M}}{\mu_0} \frac{(\mathrm{T})}{(\mathrm{T^2J^{-1}m^3})}$$

- In this case, a hysteresis cycle Int (M.B) has units of J/m^3

- Same is true of $\mathbf{B}_{tot}$ ($\mathbf{H}$) loops but with inverted units

Summary

- Magnetic moments and current loops behave equivalently
- Quantum mechanical origin of magnetic moments not too far from a classical current loop
- Magnetic fields are different inside and outside magnetic media
- Internal magnetic fields in magnets are generally complicated
- Units in magnetism are generally horrible, but always use SI
- Remembering that μ_0 has units of $T^2\ J^{-1}\ m^3$ will make you happy