

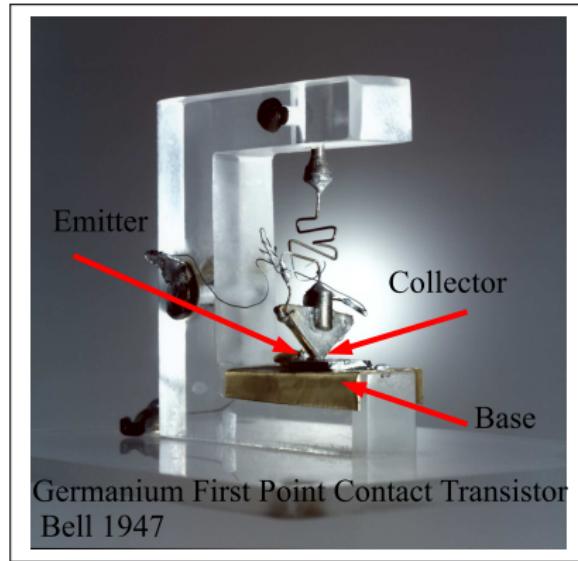
# Introduction to (Electron) Transport

## ESM-Cargese 2017

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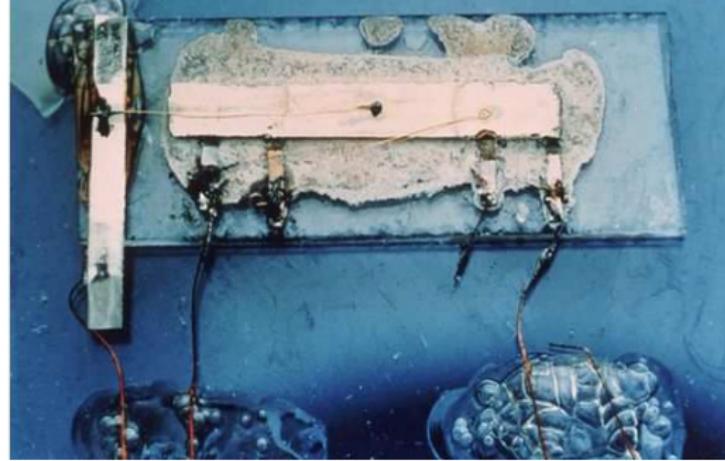
Institut Néel - Université Grenoble-Alpes

15 octobre 2017

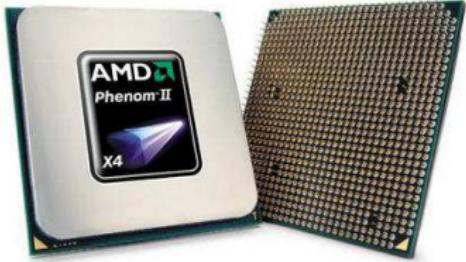
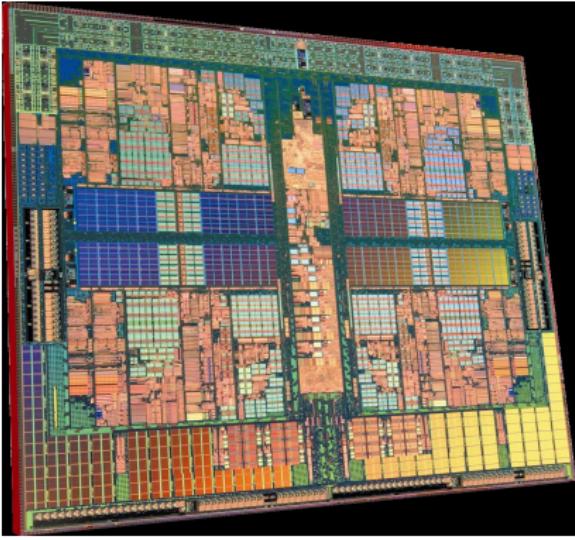


Modern electronics was created in 1947 when the **transistor** was invented in Bell laboratories by Bardeen, Shockley and Brattain (Nobel prize 1955).

First Integrated circuit \_ Kilby 1958



With the possibility to integrate components on a single chip (Kilby 1958, also Nobel winner in 2000), it allows to densify circuits : **integrated circuits (IC)**.



45 nm transistors  
800 million transistors  
3.5 Ghz  
125 Watt

Recent **microprocessors** contain more than a billion transistors,  
transistor channel length decreasing  
from 45 nm to 32 nm to 22 nm, now(2014) 14 nm technology,  
next is 10 nm ... **5 nm ?**

# Spintronics

Semiconductor based electronics does not take the **electron spin** into account (only x2 in calculations).

When the spin of the electron is explicitly taken into account, it becomes an **extra degree of freedom**

Charge transport + spin = **magnetic electronics** =**spintronics**

New structures, New physics effects, Giant MagnetoResistance  
**(Nobel prize 2007)**

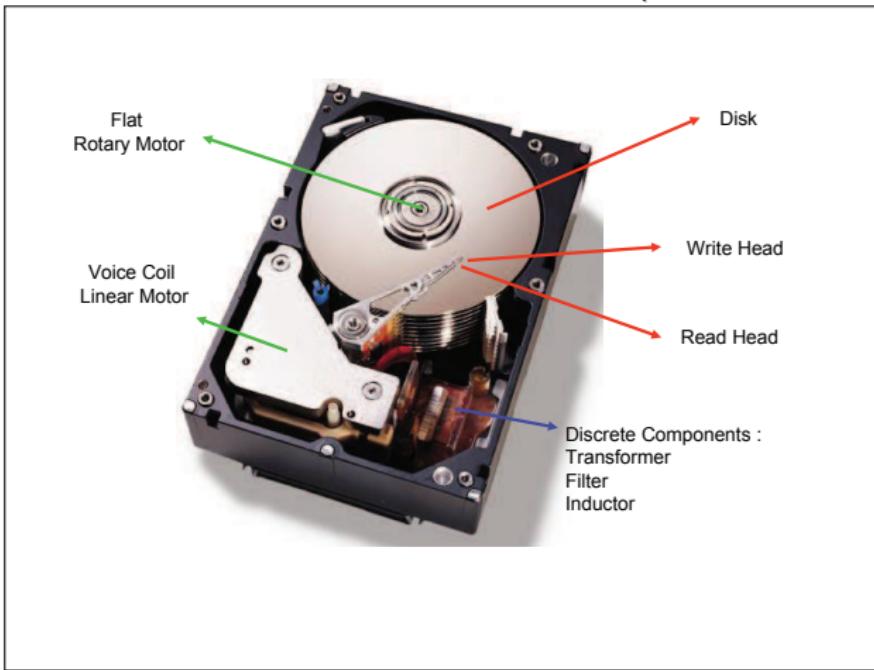
# Physics Nobel 2007



Albert Fert (Orsay) + Peter Grünberg (Jülich)  
**Giant Magnetoresistance** (discovery 1987-1988)

# The famous application

Spin electronics (spintronics) at the nanoscale has allowed a revolution in recording consumer electronics (Hard Disk Drive).



# Outline

Introduction to electron transport

Part 1 : Electron transport and spin transport

Part 2 : What happens at the nanoscale ?

# Outline

Introduction to electron transport

Part 1 : Electron transport and spin transport

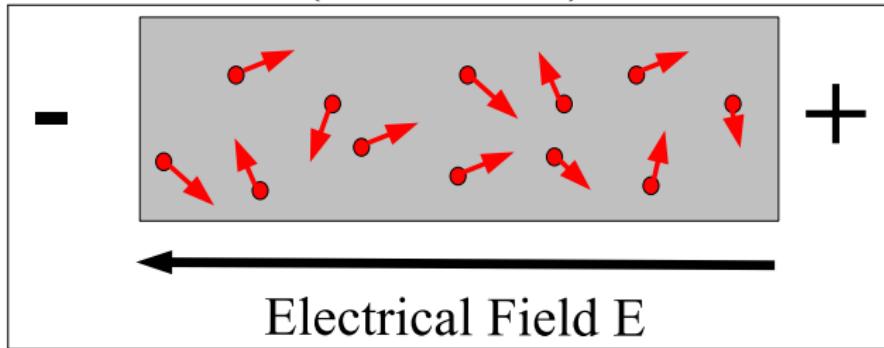
Part 2 : What happens at the nanoscale ?

# Outline Part 1

- Ohm's law (classical)
- Boltzmann equation (semi-classical)
- Temperature dependence
- Field dependence

# Drude's model

In a conducting material (silicon, copper) :



Carrier charge :  $q$  (Coulomb)

Carrier density :  $n$  ( $/m^3$ )

Current density :  $\vec{j} = qn\vec{v}$  ( $A/m^2$ )

Applying an electric field :  $\vec{E}$  (Volt/m)

No collisions  $\rightarrow$  constant acceleration !

The carriers are accelerated between collisions  
which redistribute the momenta

# Drude's model

Average time between Collisions :  $\tau$  (s)

Momentum acquired during  $\tau$  is  $qE\tau$

Average momentum of carriers  $p = qE\tau$

classical mechanics  $\vec{p} = m\vec{v} = q\vec{E}\tau$

So the **drift velocity**  $\vec{v} = \frac{q\vec{E}\tau}{m}$

The current  $\vec{j} = qn\vec{v} = \frac{q^2 n \tau}{m} \vec{E}$

The current is proportional to the applied electric field

$$\vec{j} = \sigma \vec{E} \text{ Ohm's law}$$

# Conductivity

$$\vec{j} = \sigma \vec{E}$$

$\sigma$  is the **conductivity** (in Siemens per meter (S/m))

Its inverse  $\rho = 1/\sigma$  is the **resistivity** in Ohm.meter( $\Omega \cdot m$ )

$$\sigma = \frac{q^2 n \tau}{m}$$

High conductivity means :

**large density** of carriers

**long collision time**

**small carrier mass**

it does **not** depend on the **sign** of the charge

$$\vec{j} = \sigma \vec{E} = q \cdot n \cdot \vec{v}$$

One defines the **mobility**  $\mu$  :  $\vec{v} = \mu \vec{E}$

$$\mu = \frac{\sigma}{nq} = \frac{q\tau}{m} (\text{m}^2/\text{Vs})$$

# Example : Numbers for Cu

Copper :



## Example : Numbers for Cu



Assuming one conduction electron per atom

Density of carriers :  $8.47 \cdot 10^{28} / \text{m}^3$

electron charge :  $-1.6 \cdot 10^{-19} \text{ C}$

electron mass :  $9.11 \cdot 10^{-31} \text{ kg}$

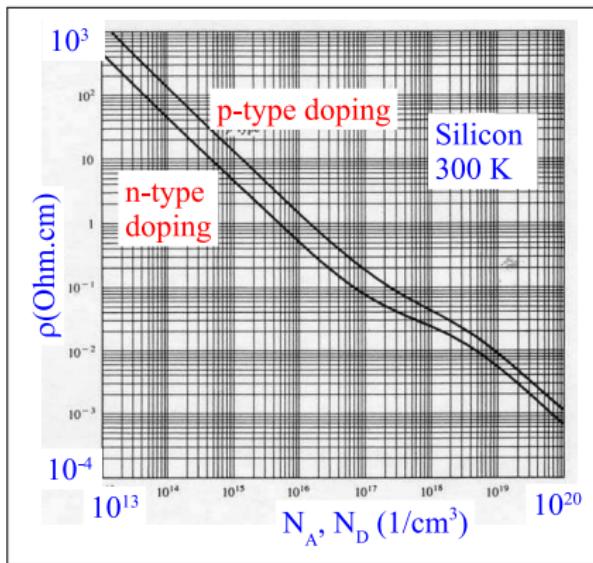
**resistivity** at 300 K :  $1.7 \mu\Omega \cdot \text{cm}$

mobility at 300 K :  $43 \text{ cm}^2/\text{Vs}$

collision time =  $2.4 \cdot 10^{-14} \text{ s}$

**drift velocity** ( $E=1 \text{ V/mm}$ ) =  $4.3 \text{ m/s}$

# Density of carriers



In semiconductors, since  $\rho \propto 1/n$  (assuming  $\tau$  is constant),  
doping allows  $n$  and  $\rho$  to span 7 orders of magnitude  
low carrier density can be modified by an electrical field  
Field effect transistor MOSFET, p-n junction (diode),  
Ohm's law does not hold anymore

# Improving the classical image

The classical image of the carriers is rapidly unable to explain transport phenomena :

- gap, bands (insulators / semiconductors / metals)
- - (effective) mass different from electron mass (high mobility semicond., heavy fermions)
- - spin ...

Electron is a fermion and there are correlation effects (not free electrons). It is more correct to use quantum mechanics in these solid state materials (and it becomes a bit more complicated !)

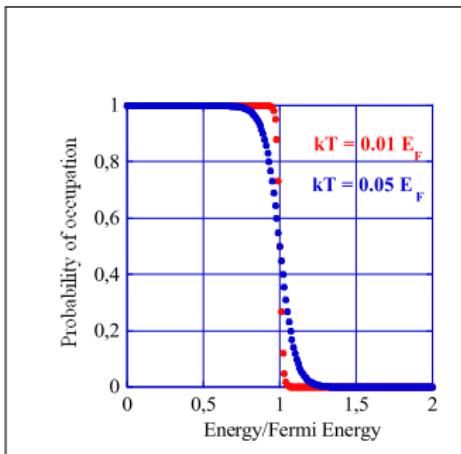
Lets add some QM (but not too much).

# Fermi-Dirac statistics

Electrons are **fermions** so they follow **Pauli principle** and abide by the **Fermi-Dirac statistics**

$$f(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}$$

# Fermi-Dirac statistics



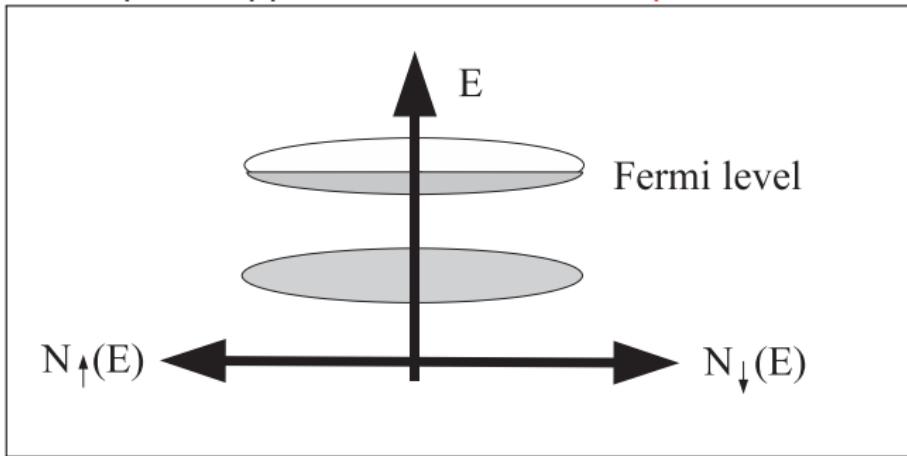
$E_F = 2\text{-}5 \text{ eV}$  and  $kT = 25 \text{ meV}$  at  $300 \text{ K}$   
so  $kT$  is  $1\%$   $E_F$

Only electrons in the energy range  $E_F - kT$ ,  $E_F + kT$  participate to transport.

(Out of equilibrium (non thermal) transport is possible in extreme cases : one talks about hot electron injection)

# Band structure

Electron transport happens in a more or less **periodic atomic lattice**

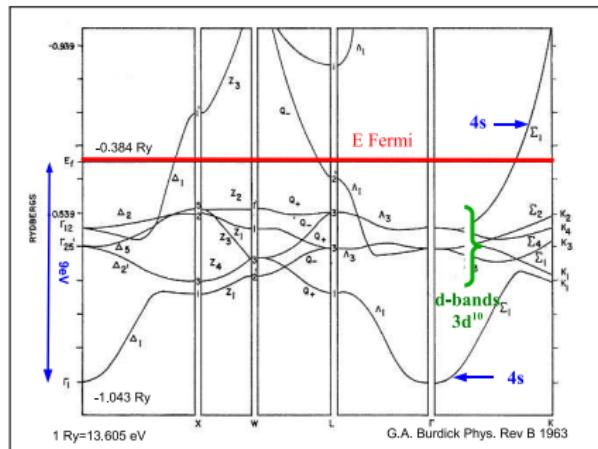


Such a band structure leads to a first definition of metals and insulators

Metals have a **finite density of states** at the Fermi level

For **insulators**, the Fermi energy is in a band gap,  
so **no carriers** at 0 Kelvin

# Effective mass



Cu band structure. Cu=[Ar]3d<sup>10</sup>4s<sup>1</sup>

The effect of interactions can be represented by free electrons with  
an effective mass.

$$E = E_0 + \frac{\hbar^2 k^2}{2m^*} \text{ i.e. } m^* = \frac{\hbar^2}{\frac{\partial^2 E}{\partial k^2}}$$

4s electrons are **light** (free, delocalised),  
3d electrons are **heavier** (more localised)

# Electron velocity

The classical electron has a velocity  $\frac{mv^2}{2} = \frac{3}{2}kT$   
for Cu it gives  $v=1.1 \cdot 10^5 \text{ m/s}$

Since the kinetic energy is not anymore  $kT$  but  $E_F$ ,

$$E = \frac{\hbar^2 k^2}{2m} = E_F$$

a quantum electron has a velocity  $v_F = \frac{\hbar k_F}{m} = 10^6 \text{ m/s}$

The distance between scattering events is the mean free path  $\lambda$

For Cu at 300 K :  $v_F = 10^6 \text{ m/s}$  and  $\tau = 2 \cdot 10^{-14} \text{ s}$  gives  $\lambda = 20 \text{ nm}$

# Boltzmann Equation

Electrons should be treated as **interacting particles** :

- not the free electron mass but an **effective mass**

Density should be taken from **band structure** calculations

Carriers could be **holes**

Drude → **Semi-classical model**

# Boltzmann Equation

How to proceed when an electric field is applied ?

Considering  $f(\vec{r}, \vec{v}, t)$  to be the distribution of electrons

= probability to find one electron at  $\vec{r}$  with velocity  $\vec{v}$  at time t

$$f(\vec{r}, \vec{v}, t) = f_0 + g(\vec{r}, \vec{v}, t)$$

with  $f_0(\vec{r}, \vec{v})$  the stationary distribution (no  $\vec{E}$ )

Without collisions :  $f(\vec{r} + \vec{v}dt, \vec{v} + \frac{\vec{F}}{m}dt, t + dt) = f(\vec{r}, \vec{v}, t)$

i.e.  $df=0$

$$\text{i.e. } \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \frac{\partial f}{\partial \vec{v}} = 0$$

$$\text{With collisions : } \frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \frac{\partial f}{\partial \vec{v}} = \left( \frac{\partial f}{\partial t} \right)_{coll}$$

# Boltzmann Equation

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + \frac{\vec{F}}{m} \frac{\partial f}{\partial \vec{v}} = (\frac{\partial f}{\partial t})_{coll}$$

Relaxation time approximation :  $(\frac{\partial f}{\partial t})_{coll} = -\frac{g}{\tau}$

$\tau$  is the relaxation time

If the electrical field goes to zero, the distribution comes back to equilibrium with characteristic time  $\tau$ .

# Conductivity

Looking for  $\sigma$

Applying an electric field along x

$$j_x = \sigma E_x = q \int v_x f(v) dv = q \int v_x g(v) dv$$

Using  $\frac{-g}{\tau} = \frac{q\vec{E}}{m} \frac{\partial f}{\partial \vec{v}}$  one gets

$$\sigma = \frac{-q^2}{4\pi^3} \int v \tau \frac{\partial f}{\partial E} d^3 k$$

$$\sigma = \frac{q^2}{12\pi^3 \hbar} \langle \lambda \rangle S_F$$

$\langle \lambda \rangle$  is the average mean free path on the Fermi surface

# Back to Drude

Free electron band :  $E = \frac{\hbar^2 k^2}{2m}$

Fermi surface is a sphere

$$S_F = 4\pi k_F^2$$

$$\text{Density of states in k-space} = \frac{2V}{(2\pi)^3}$$

$$N = \frac{4\pi k_F^3}{3} \cdot \frac{2V}{(2\pi)^3}$$

$$k_F = (3n\pi^2)^{1/3}$$

$$\sigma = \frac{e^2 n \tau}{m}$$
 it is Drude's result

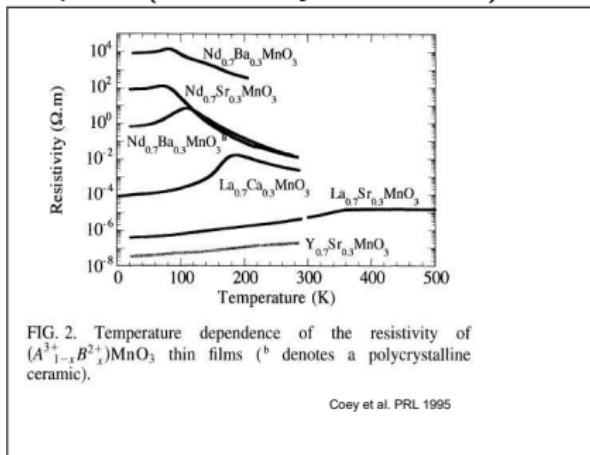
# Mean free path

$$\lambda = v\tau$$

To be compared to characteristic lengthscales

→ :

- $\lambda \approx$  lattice parameter  $a$  : electron localisation (insulation character)  
→ hopping transport (thermally activated)



# Mean free path

$$\lambda = v\tau$$

To be compared to characteristic lenghtscales

→ :

- $\lambda \approx$  lattice parameter  $a$  : electron localisation (insulation character)
- $a < \lambda <$  sample size : diffusive regime
- $\lambda >$  sample size : ballistic behaviour (full quantum treatment required, including contacts)

$\lambda$  can be limited by the sample surface contribution : Fuchs Sondheimer correction for finite size.

$g(r,v)$  depends on  $z$  (non uniform)

**Thin films resistivity is larger than bulk resistivity**

# Relaxation time

Several microscopic scattering events may happen in a conductor :

Assuming the **scattering rates** are **independent** and using Drude's

$\sigma = \frac{q^2 n \tau}{m}$  one gets :

Matthiesen rule :  $\frac{1}{\tau} = \sum \frac{1}{\tau_i}$

One adds the resistivities due to the different scattering mechanisms :

$$\rho = \rho_1 + \rho_2 + \rho_3$$

# Relaxation time

## Microscopic relaxation mechanisms

In a periodic potential, a plane wave is a solution : the conductivity is infinite

But the sample is never periodic (=mathematically periodic)

**Temperature-independent scattering :**

defects, impurities, surface-interface

**Temperature-dependent scattering :**

lattice excitations (phonons)

magnetic excitations (spin waves = magnons)

electron-electron collisions

Scattering can be **elastic** ( $E$  conserved) or inelastic

The relaxation time for wavevector  $\vec{k}$  is different from the relaxation time for spin

→ spin-flip and non spin-flip relaxation times

# Mean free path : localisation limit

Ioffe-Regel limit (shortest mean free path for a metal)

$$\langle \lambda \rangle = a$$

with  $\alpha$  electron per site and  $1/a^3$  density of sites

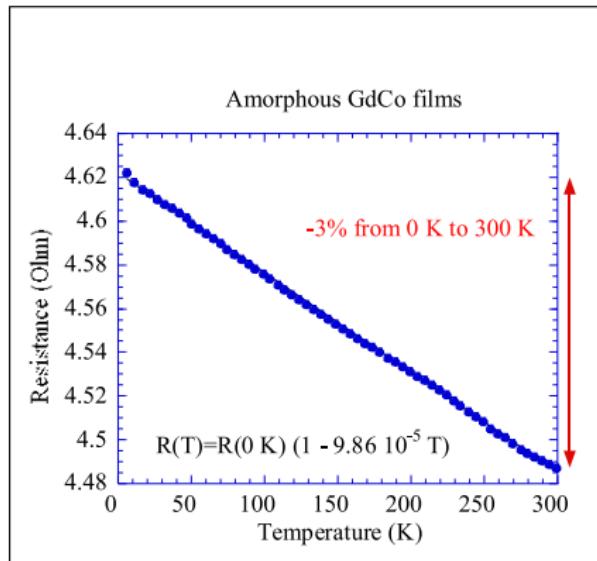
$$\sigma = \frac{e^2 \langle \lambda \rangle}{12\pi^3 \hbar} S_F$$

and  $S_F = 4\pi k_F^2$  with  $n = \frac{\alpha}{a^3} = \frac{k_F^3}{3\pi^2}$

$$\sigma = 0.33 \frac{\alpha^{2/3} e^2}{\hbar a}$$

This gives a **maximum resistivity** for a **metal** :  $100-300 \mu\Omega.cm$

# Conductivity : Amorphous Film



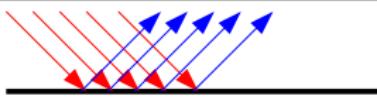
Temperature-dependence of an **amorphous** film  
**Amorphous = Maximum atomic disorder**

# Finite size effect

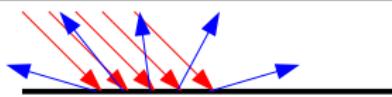
The Fuchs-Sondheimer model takes into account the z dependence of the electron distribution in a thin film (thickness t).

$$-\frac{g}{\tau} = \frac{\hbar k_z}{m} \cdot \frac{\partial g}{\partial z} + \frac{q\vec{E}}{m} \frac{\partial g}{\partial \vec{v}}$$

Depending on the **boundary conditions** at the film interfaces :



Specular : probability p



Diffusive : probability 1-p

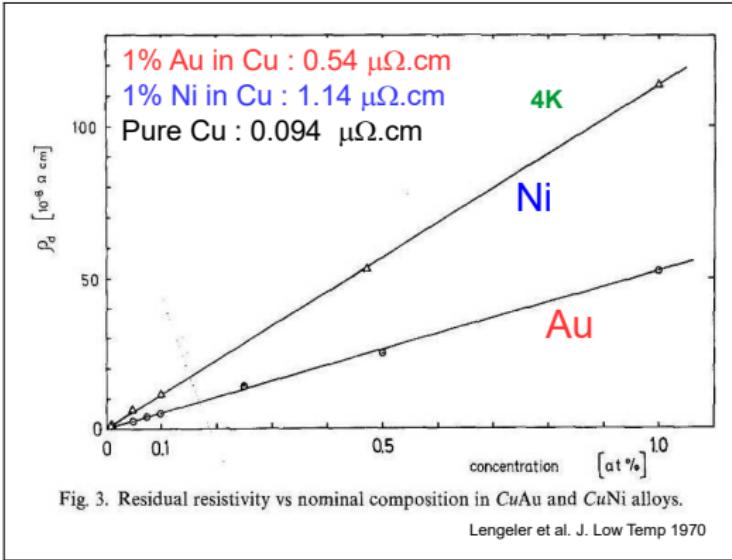
**specular reflexion** at the interfaces :  $g(v_z > 0) = g(v_z < 0)$  at the interface

**diffusive** interfaces  $g(z=0) = g(z=t) = 0$

$$\rho = \rho_{bulk} \left(1 + \frac{3\lambda}{8t}\right)$$

Fuchs-Sondheimer approximation (diffusive +  $\lambda \ll$  thickness)

# Impurities contribution

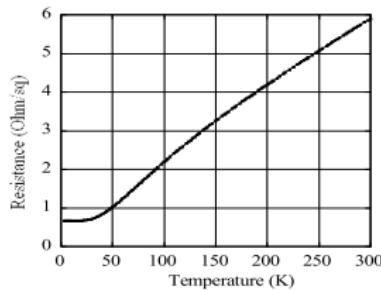


The residual resistivity ratio RRR can be used to measure the purity of a sample

$$RRR = \frac{\rho(300K)}{\rho(4.2K)}$$

# Temperature independent defect contribution

Iridium seuil 9nm



Ir layer (9 nm thick) on Sapphire

$$R_{sq} (300K) = 5.91 \text{ Ohm/sq}$$

$$R_{sq} (3K) = 0.67 \text{ Ohm/sq}$$

$$RRR = 8.8$$

Quality control of thin films  
Ordered / disordered alloys  
Mixed interface (multilayers)  
Annealing effect

# Crystallographic contribution

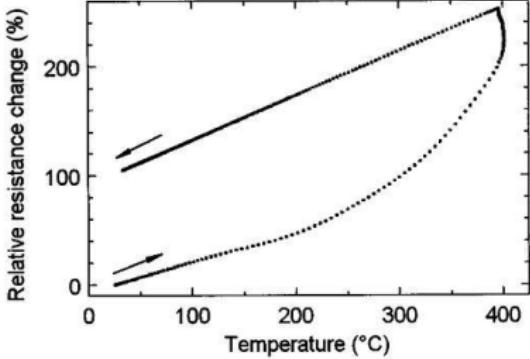


FIG. 1. Relative resistance change vs temperature of NiFe/Cu/NiFe trilayers during a thermal cycle at 4 K/min and isothermal annealing at 400 °C for 2 h in N<sub>2</sub>/H<sub>2</sub>(5 vol %) gas mixture. The irreversible resistance change originates mainly from interdiffusion.

Appl. Phys. Lett., Vol. 77, No. 3, 17 July 2000

Annealing effect  
Intermixing at the NiFe/Cu interfaces

# Phonon Contribution

Static defects (impurities, crystallographic) do not depend on T  
Lattice vibrations (phonons) do

The number of phonons in a mode varies as  $n(\hbar\omega) = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$

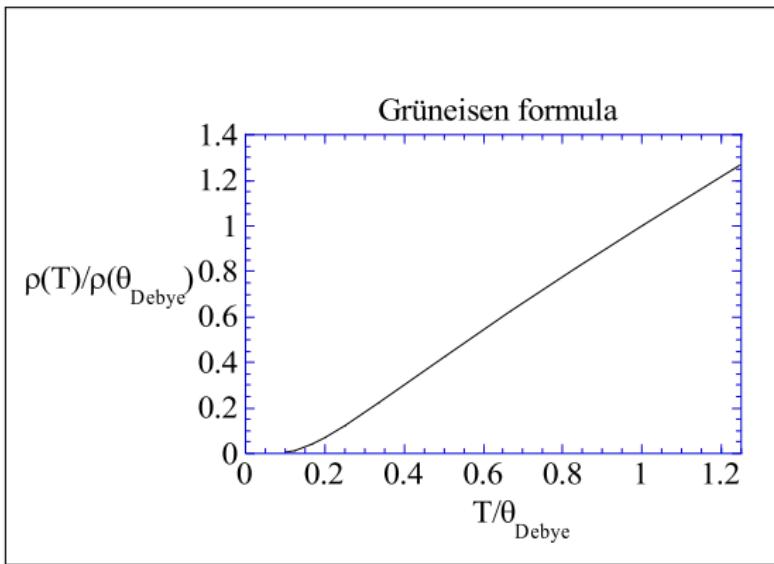
The density of modes  $D(\omega) = \frac{V\omega^2}{2\pi^2 c_s^3}$  (Debye model)

The total number of phonons /m<sup>3</sup> is  $\langle n \rangle = \frac{3}{2\pi^2 c_s^3} \int_0^{\omega_D} \frac{\omega^2}{e^{\frac{\hbar\omega}{kT}} - 1}$

When T ≪ θ<sub>D</sub>,  $\langle n \rangle \propto T^3$

When T ≫ θ<sub>D</sub>,  $\langle n \rangle \propto T$

# Phonon Contribution



$R(T)$  is **linear** at high temperature and  $\propto T^3$  at low temperature  
→ metallic resistance can be used as **temperature sensor**  
(platinum  $\text{Pt}_{100}$  and  $\text{Pt}_{1000}$ )

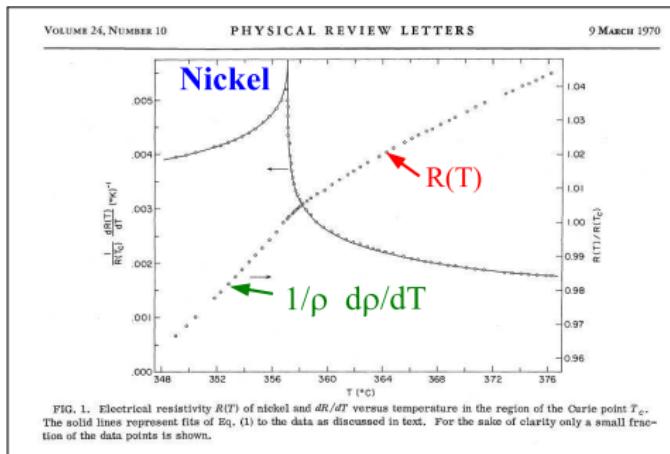
# Magnetic contribution

For magnetic materials, magnetic excitations also cause scattering  
spin waves are similar to lattice vibrations but :

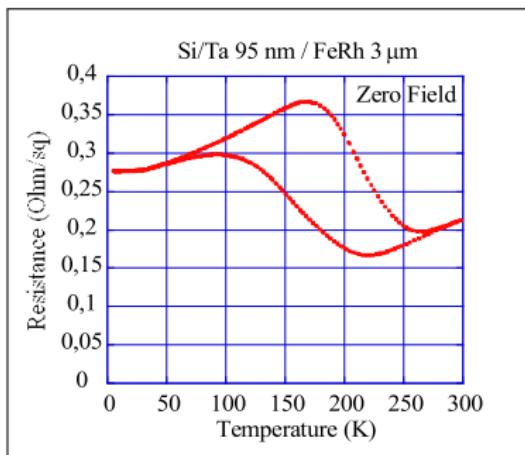
$$E = \hbar\omega = Dq^2 \text{ for spin waves and } E = \hbar\omega = Aq \text{ for phonons}$$

Example : At the Curie temperature, ferromagnetic order disappears.

The magnetic susceptibility diverges at  $T_c$ , magnetic fluctuations diverge.



# Phase transition



$\text{Fe}_{50}\text{Rh}_{50}$  antiferromagnetic to ferromagnetic transition can be evidenced on the  $R(T)$  curve

AF period is twice the F period in real space, half in k-space. An half-filled band may split.

(For measurement : No need to apply some magnetic field to study a magnetic transition)

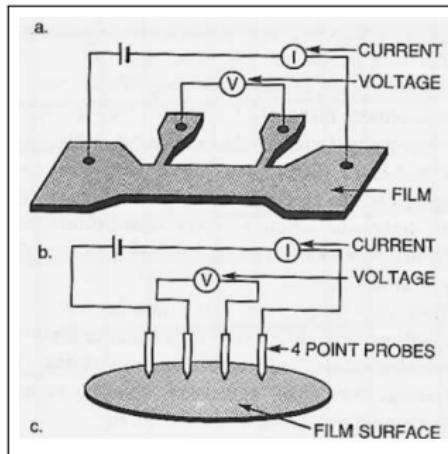
# electron - electron

Material	Cu	Ni	Au	Pt
Resistivity 300 K ( $10^{-8}\Omega.m$ )	1.7	7	2.2	10

Ni  $3d^94s^1 = \text{Cu} \text{ minus 1 electron}$

Pt  $5d^96s^1$ , Au  $5d^{10}4s^1$

# How to measure a Resistance ?

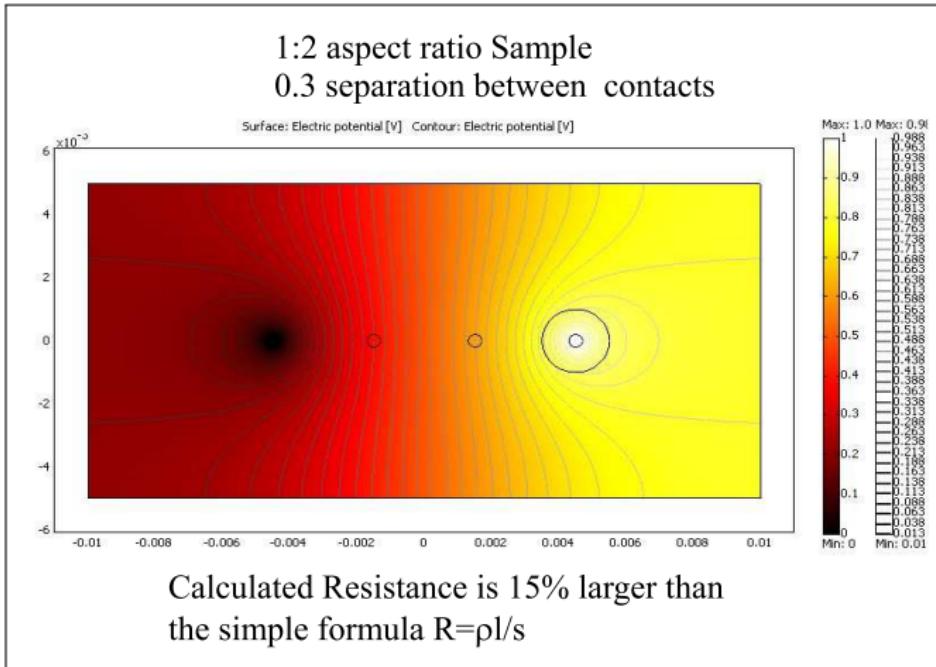


4 wire measurement of resistance

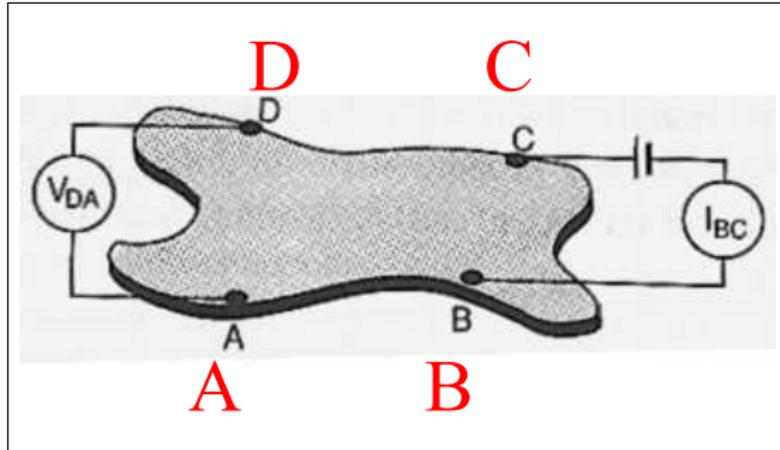
$$R = \rho \frac{\text{length}}{\text{width} \cdot \text{thickness}}$$

2 wire measurement includes the **cable** resistance and the **contact** resistance

# Resistance measurement



# Van der Pauw protocol



- if the resistivity is homogeneous
- if the sample is simply connex
- if the 4 contacts are small and on the edge

$$e^{-\frac{\pi R_1}{R_\square}} + e^{-\frac{\pi R_2}{R_\square}} = 1$$

$R_\square = \frac{\rho}{\text{thickness}}$  is the resistance per square or **sheet resistance**  
Similar trick for Hall effect (spinning current protocol).

## Introduction to electron transport

Part 1 : Electron transport and **spin transport - Magn. field effects**

Part 2 : What happens at the nanoscale ?

# Cyclotron

Electron trajectories under an applied magnetic field will become helicoïdal

classical Lorentz force :  $\vec{f} = q\vec{E} + q\vec{v} \wedge \vec{B}$

longer trajectories to go from A to B → increased resistivity

Metals obey **Kohler's scaling** :

$$\frac{\Delta\rho}{\rho} = f(\omega_c\tau)$$

and  $\omega_c = \frac{qB}{m}$  (cyclotron pulsation) so  $\frac{\Delta\rho}{\rho} = f\left(\frac{B}{\rho_0}\right)$

$\rho_0$  is the resistivity at B=0

large effect when the resistivity is small (single crystal at low T)

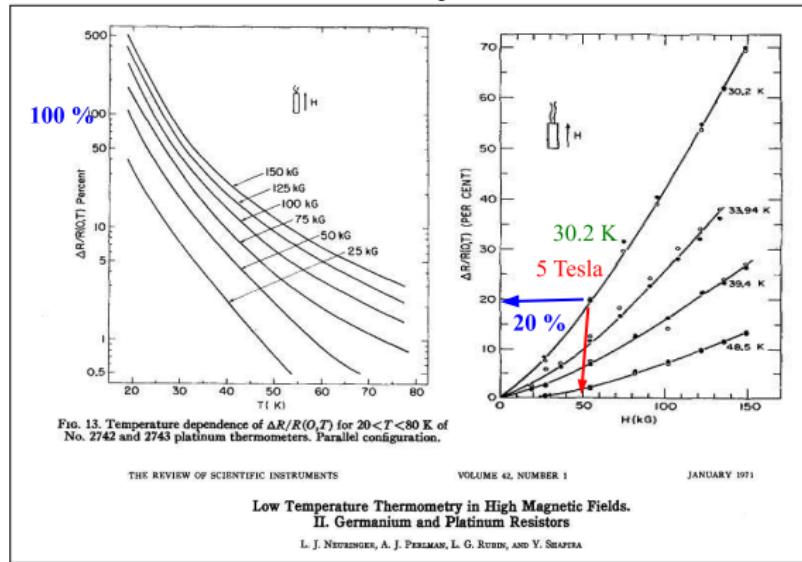
**cyclotron magnetoresistance**

# Kohler scaling

It is most often a  $B^2$  law and  $\frac{\delta\rho}{\rho} = 0.1\%$  in 1 Tesla at usual metals at room temperature

# Metal Normal Magnetoresistance

The cyclotron MR and thermometry : Pt sensor



At low temp, magnetoresistance has to be taken into account

# Ferromagnetic metals

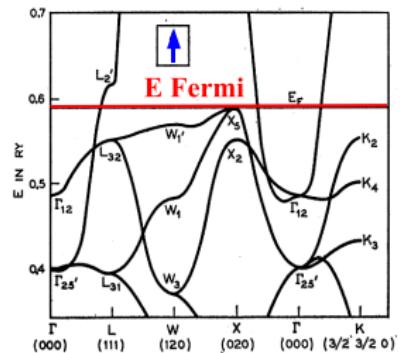


FIG. 6. The  $\uparrow$  spin band structure of Ni in model (c).

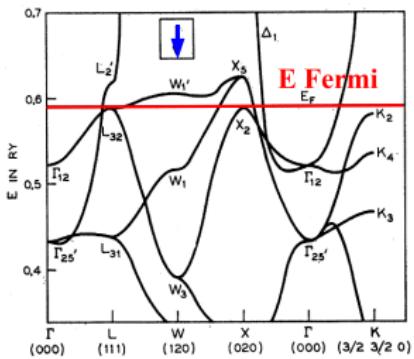


FIG. 7. The  $\downarrow$  spin band structure of Ni in model (c).

J.C. Phillips Phys. Rev. B 1964

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VOLUME 133, NUMBER 4A

17 FEBRUARY 1964

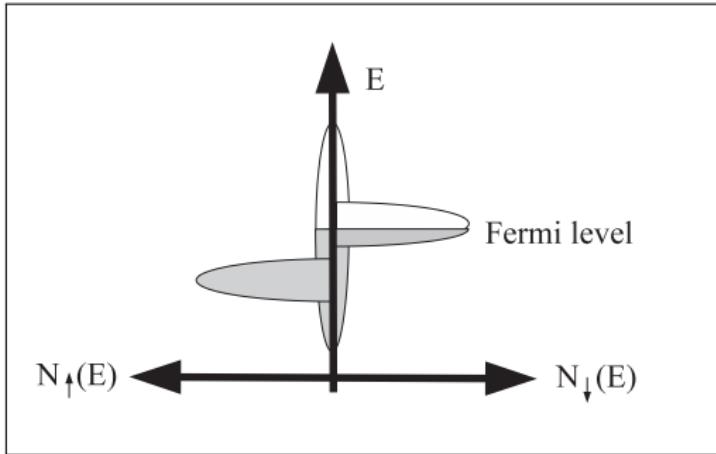
## Fermi Surface of Ferromagnetic Nickel\*

J. C. PHILLIPS†

Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois

# Magnetic conductors

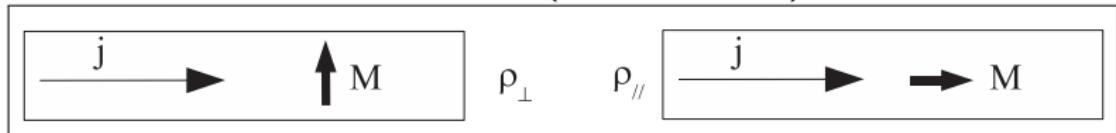
In a ferromagnetic metal



density of state, effective mass, mean free path ...  
become **spin-dependent**  
 $3d^{\uparrow}, 3d^{\downarrow}$  (heavy),  $4s^{\uparrow}, 4s^{\downarrow}$  (light) electrons  
**s-d scattering**

# Magnetoresistance

## Anisotropic Magnetoresistance (volume effect)

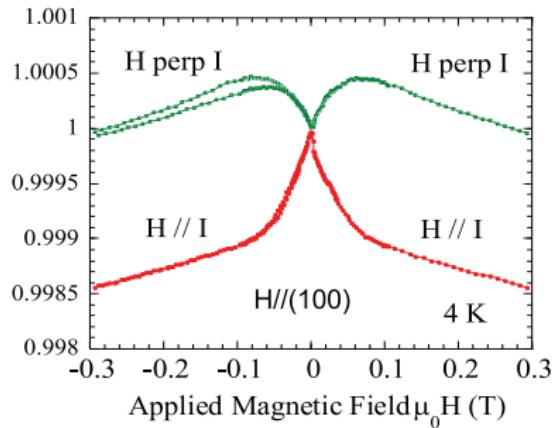
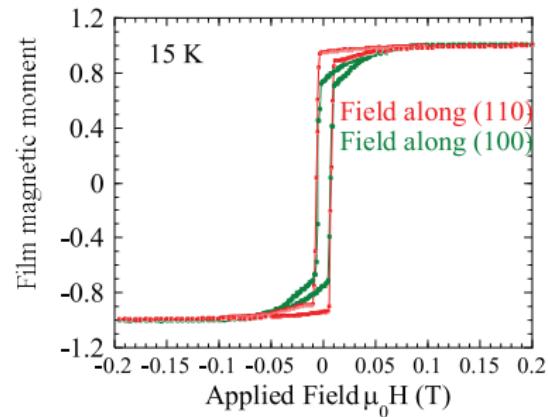


Phenomenological angle dependence :

$$\rho = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp}) \cos^2(\mathbf{k}, \mathbf{M}) \quad (1)$$

Value : a few percents in FeNi

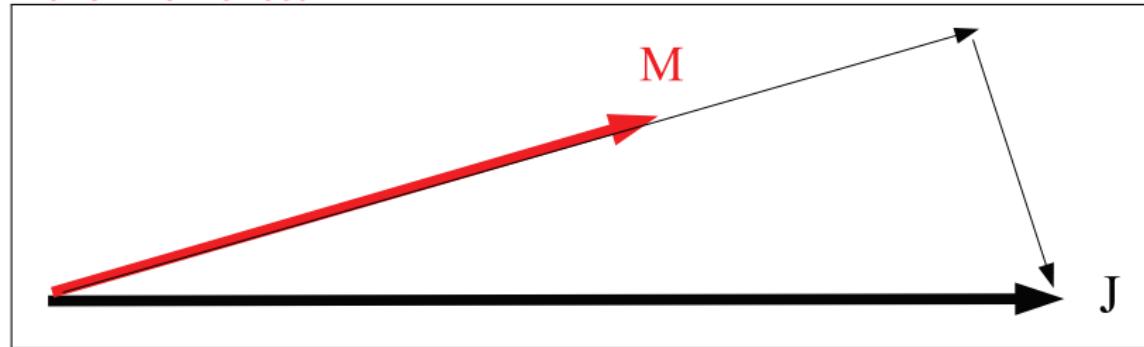
# Anisotropic Magnetoresistance



example of AMR (LaSrMnO manganite epitaxial film)

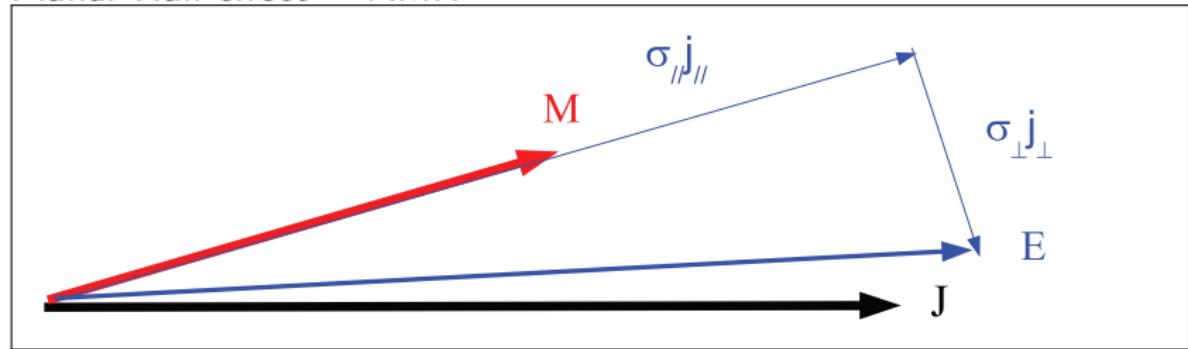
# Anisotropic Magnetoresistance

Planar Hall effect = AMR



# Anisotropic Magnetoresistance

Planar Hall effect = AMR



There is a transverse E-field  $\Rightarrow$  similar to Hall

# Anisotropic Magnetoresistance

## Physical Origin :

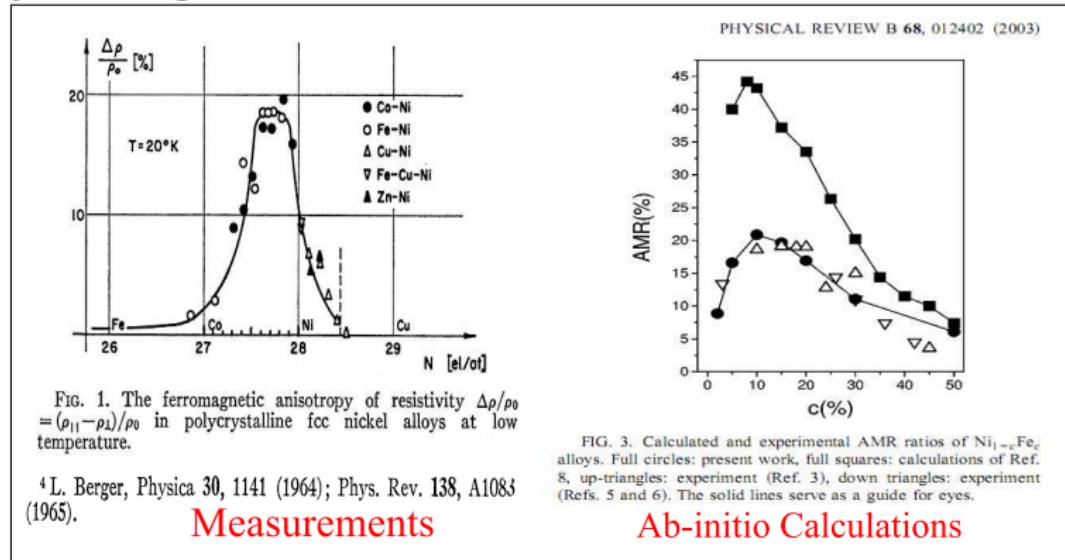


FIG. 1. The ferromagnetic anisotropy of resistivity  $\Delta\rho/\rho_0 = (\rho_{||} - \rho_{\perp})/\rho_0$  in polycrystalline fcc nickel alloys at low temperature.

<sup>4</sup>L. Berger, Physica 30, 1141 (1964); Phys. Rev. 138, A1083 (1965).

## Measurements

FIG. 3. Calculated and experimental AMR ratios of Ni<sub>1-x</sub>Fe<sub>x</sub> alloys. Full circles: present work, full squares: calculations of Ref. 8, up-triangles: experiment (Ref. 3), down triangles: experiment (Refs. 5 and 6). The solid lines serve as a guide for eyes.

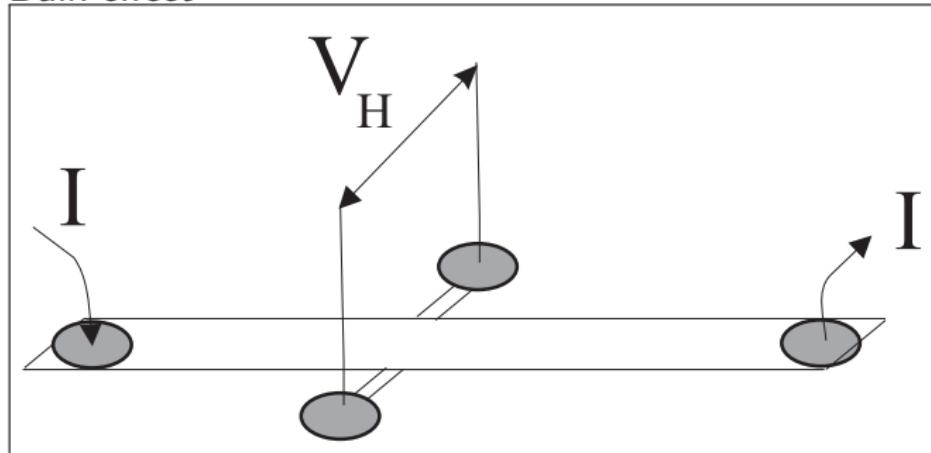
## Ab-initio Calculations

$$\frac{\Delta\rho}{\rho} = \left( \frac{\lambda_{spin-orbit} \hbar^2}{\Delta E} \right)^2$$

Spin-orbit interaction  $\lambda_{SO} \vec{L} \vec{S}$  mixes 3d up and down states, nearly ( $\Delta E$  degenerate at  $E_F$ ).

# Field-effect : Hall

## Bulk effect



$$\vec{f} = q\vec{E} + q\vec{v} \wedge \vec{B}$$

Normal(ordinary) Hall effect  $V_H = R_H I B_z$

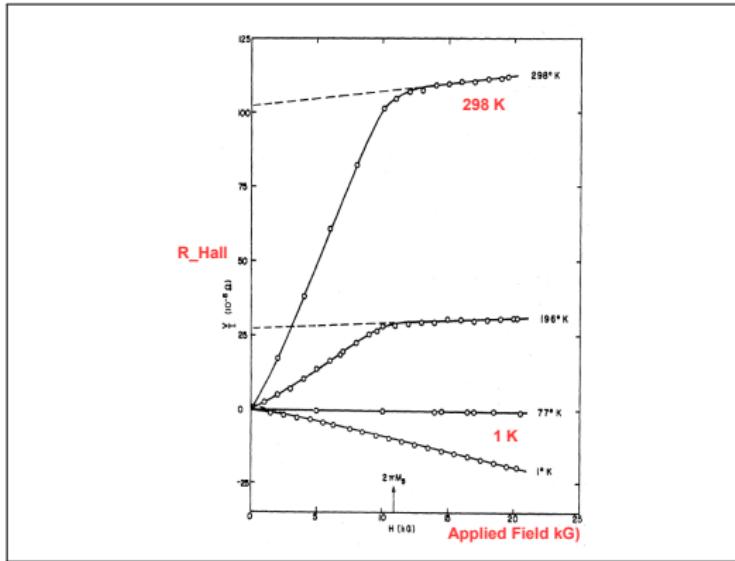
$$R_H = \frac{1}{n.q}$$

If you know  $n$  : **magnetic field sensor**

If you know  $B_z$  : **doping characterisation**

# Field-effect : EHE

In a ferromagnetic sample, a new contribution to Hall effect appears



Strange temperature-dependence  
Dheer, P.R. (1967)

Review : Nagaosa et al. Rev. Mod. Phys. 2010

# Field-effect : EHE

Extraordinary Hall effect  $V_H = R_e I M_z$

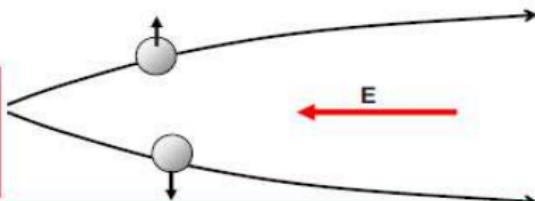
Also called Anomalous Hall effect

Due to spin-orbit coupling, scattering of carriers on magnetic moments is not left-right symmetric

# EHE : mechanisms

## a) Intrinsic deflection

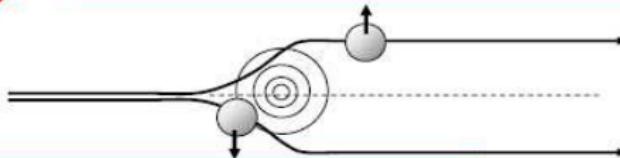
Interband coherence induced by an external electric field gives rise to a velocity contribution perpendicular to the field direction. These currents do not sum to zero in ferromagnets.



$$\frac{d\langle \vec{r} \rangle}{dt} = \frac{\partial E}{\hbar \partial k} + \frac{e}{\hbar} \mathbf{E} \times \mathbf{b}_n$$

Electrons have an anomalous velocity perpendicular to the electric field related to their Berry's phase curvature

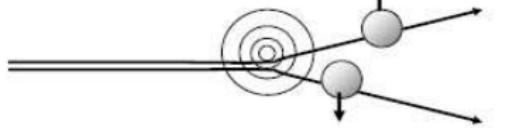
## b) Side jump



The electron velocity is deflected in opposite directions by the opposite electric fields experienced upon approaching and leaving an impurity.  
The time-integrated velocity deflection is the side jump.

## c) Skew scattering

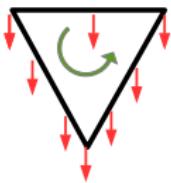
Asymmetric scattering due to the effective spin-orbit coupling of the electron or the impurity.



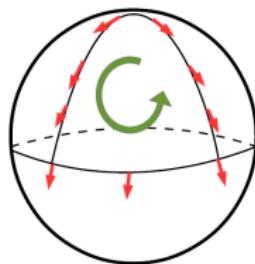
Nagaosa et al. Rev. Mod. Phys. 2010

$$R_e = \alpha_{skew} \rho + \beta_{side, Berry} \rho^2$$

# Berry Phase



Parallel Transport  
(no rotation)



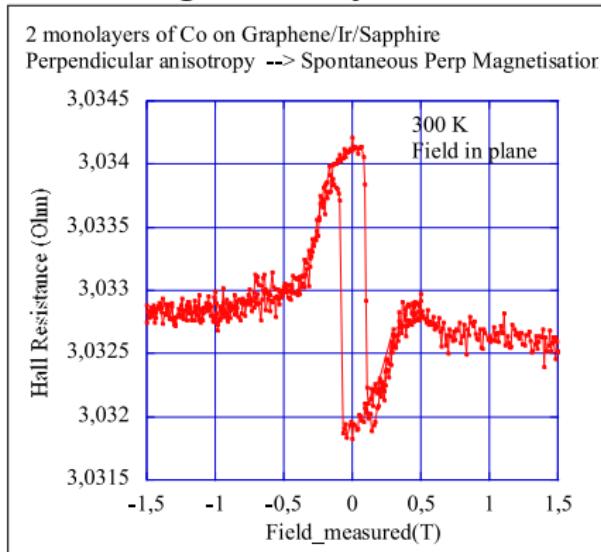
Geometrical Berry Phase

Usual Dynamical Phase  $e^{\frac{-iEt}{\hbar}}$

Geometrical Phase :  $\gamma_n(C) = i \int_C < n(R) | \nabla_R n(R) > .dR$   
M. Berry, Proc. R. Soc. Lond. A392,(1984)

# EHE : magnetometry

EHE can be used as a magnetometry tool



2 monolayers of Cobalt

# Field-effect : EHE

4158 Appl. Phys. Lett., Vol. 80, No. 22, 3 June 2002

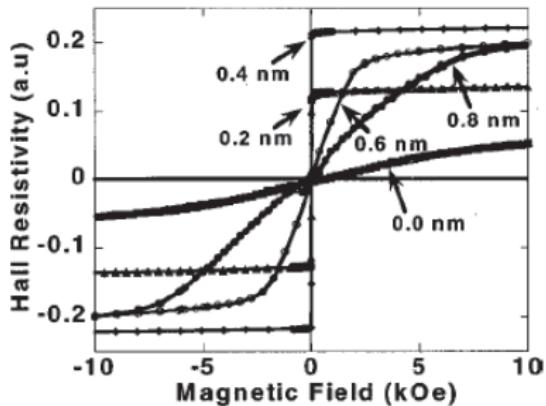


FIG. 1. Extraordinary Hall effect as a function of applied field for a series of samples of the composition Pt 3 nm/Co<sub>90</sub>Fe<sub>10</sub> 0.6 nm/Al  $t_{\text{Al}}$  with  $0 < t_{\text{Al}} < 1.2$  nm. Samples were naturally oxidized in air for 24 h.

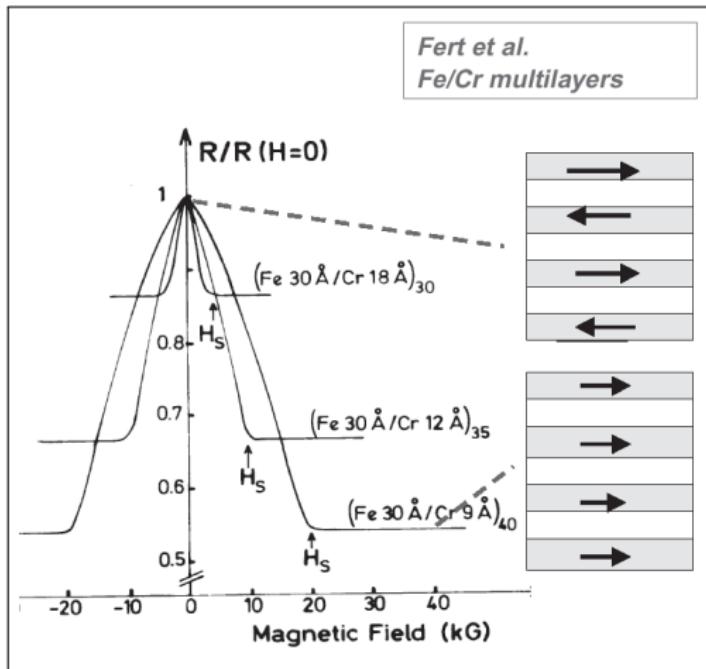
# Lecture 1

Introduction to electron transport

Part 1 : Electron transport and spin transport

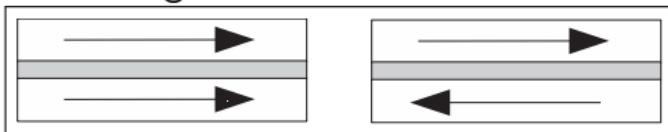
Part 2 : What happens at the nanoscale ?

# Giant Magnetoresistance



# Magnetoresistance : GMR

## Giant magnetoresistance

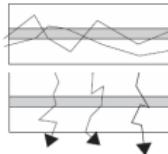


➤ CIP



(current in plane)

➤ CPP



(current perpendicular to plane)

lengthscales :

current-in-plane : mean free path

current-perpendicular to plane : spin diffusion length

# Two-current model

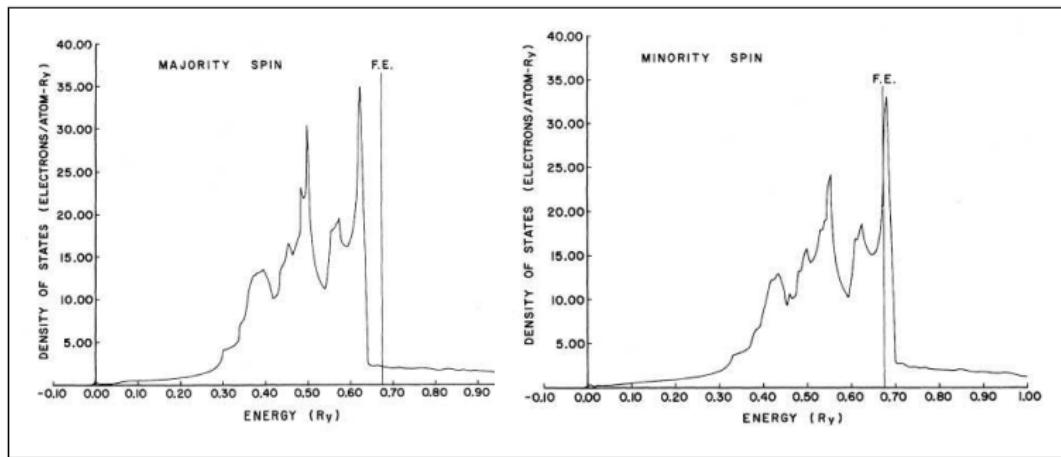
Mott 1930

If spin flip can be neglected, the total current is the sum of the current carried by  $\uparrow$  and  $\downarrow$

The material is equivalent to 2 resistors in parallel

$$\frac{1}{\rho} = \frac{1}{\rho_{\uparrow}} + \frac{1}{\rho_{\downarrow}}$$

# Two-current model



The 4s electrons are lighter than the 3d ones

The 4s electrons are mainly responsible for carrying the charge current

In a **strong ferromagnet** like Ni, at  $E_F$  there are  $4s_{\uparrow}$  and  $4s_{\downarrow}$  electrons and only  $3d_{\downarrow}$  ones.

No possibility for  $4s_{\uparrow}$ - $3d_{\uparrow}$  scattering

Mean free path  $\lambda_{\uparrow}$  is longer than mean free path  $\lambda_{\downarrow}$

Mean free path  $\lambda_{\uparrow}$  is longer than mean free path  $\lambda_{\downarrow}$

Example : Cobalt :  $\lambda_{\uparrow}=10$  nm and  $\lambda_{\downarrow}=1$  nm

Introducing  $\alpha = \frac{\rho_{\uparrow}}{\rho_{\downarrow}}$  the **bulk resistivity assymetry**

## Vocabulary

Magnetisation  $\uparrow$  / magnetisation  $\downarrow$

Defines the quantification axis

In quantum mechanics the magnetic moment is opposite to the angular momentum

orbital angular momentum :  $\vec{\sigma}_l = \vec{r} \wedge \vec{p}$  (or  $L = \frac{\hbar}{i} \wedge \vec{\nabla}$ )

and magnetic (orbital) moment  $\vec{\mu}_{orb} = -\frac{e}{2m}\vec{\sigma}_l$

Spin $\uparrow$  / spin $\downarrow$  carriers

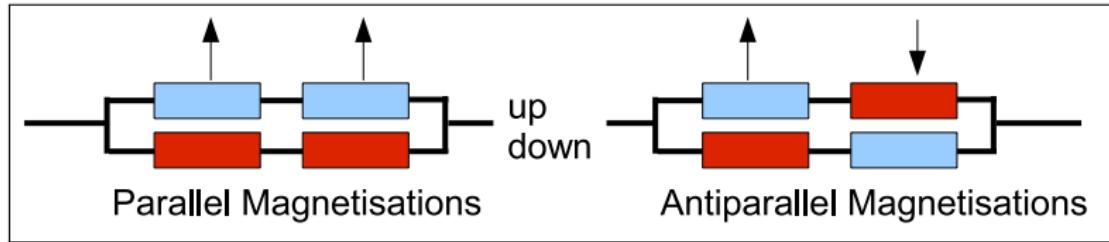
in transport, spin $\uparrow$  is (wrongly) said to be parallel to magnetic moment  $\uparrow$

## Majority/minority carriers

In a magnetic multilayer, the magnetisation may vary (antiparallel configuration)

A spin  $\uparrow$  electron may be majority carrier in one layer and minority carrier in the next one.

# GMR Resistor model



$$\rho_{parallel} = \frac{2\rho_{\uparrow}\rho_{\downarrow}}{\rho_{\uparrow}+\rho_{\downarrow}} \text{ and } \rho_{antiparallel} = \frac{\rho_{\uparrow}+\rho_{\downarrow}}{2}$$

# GMR current-in-plane

2-current-model applied to the multilayer  
spin-dependent Boltzmann eq. (similar to Fuchs-Sondheimer  
treatment (thickness dependence))

$$-\frac{g \uparrow}{\tau \uparrow} = \frac{\hbar k_z \uparrow}{m \uparrow} \cdot \frac{\partial g \uparrow}{\partial z} + \frac{q \vec{E}}{m \uparrow} \frac{\partial g \uparrow}{\partial \vec{v} \uparrow}$$

$$-\frac{g \downarrow}{\tau \downarrow} = \frac{\hbar k_z \downarrow}{m \downarrow} \cdot \frac{\partial g \downarrow}{\partial z} + \frac{q \vec{E}}{m \downarrow} \frac{\partial g \downarrow}{\partial \vec{v} \downarrow}$$

The **boundaries conditions** are :

Spin dependent reflection/transmission/diffusion **at each interface**

A lot of material parameters required to calculate cip-GMR :  
systematic study as a function of all thicknesses

or use litterature values (same crystallinity, texture, interface  
roughness ...)

# GMR perpendicular-to-plane

ingredients for Valet-Fert model (1993)

Spin-dependent electrochemical potential  $\mu_\uparrow$

Spin-dependent currents  $\vec{j}^\uparrow = \sigma^\uparrow \frac{\partial \mu^\uparrow}{\partial z}$

In a bulk :  $\mu = \mu^\uparrow = \mu^\downarrow = E_F + q.Potential$

Far from the spacer : Polarised charge current

Need for spin-flip near the spacer region

# spin diffusion length

In a non magnetic metal most scattering events do **not flip** the spin of the electrons

Scattering on a magnetic impurities or absorption/emission of a magnetic excitation (magnon) can flip the spin.

**Spin-flip** scattering is an inelastic event

⇒ vanishingly small at low temperature, not common at higher T  
(1 event out of 1000 in a non magnetic metal), 5 nm in a ferromagnetic metal

# spin diffusion length

$$D \frac{\partial^2 \Delta\mu_i}{\partial x^2} = \frac{\Delta\mu_i}{\tau_{sf}}$$

at the interface between 2 conductors, the spin polarisation of the current cannot change discontinuously.

$$l_{sf} = \sqrt{\frac{v_F \tau_{sf} \lambda}{3}}$$

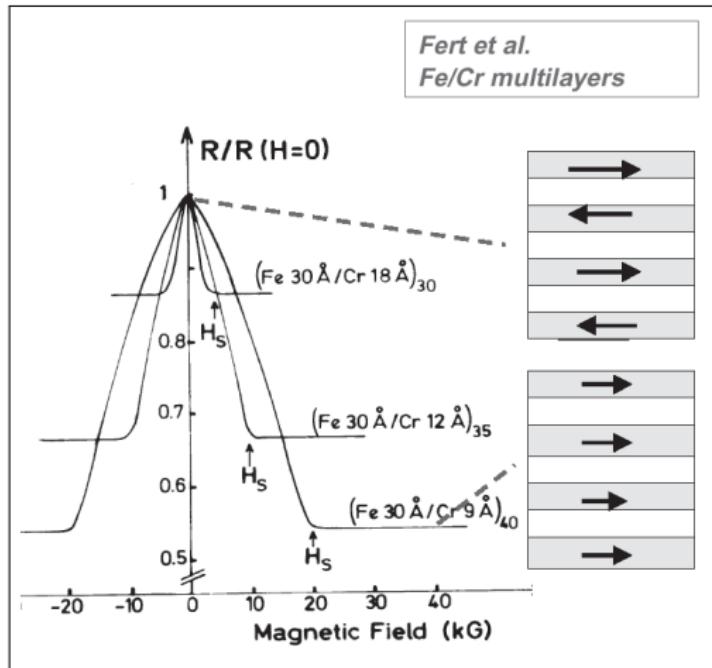
(proof :  $l_{sf} = \sqrt{\frac{N}{3}}\lambda$  random walk and  $\tau_{sf} = N.\tau$ )

Close to the interface (lengthscale  $l_{sf}$ ), an out-of-equilibrium spin population exists :

spin accumulation effect

spin injection from a ferromagnetic electrode to a semiconductor  
(Datta-Das transistor)

# Magnetoresistance : GMR



How to use this Giant Magnetoresistance effect :  
**Room temperature, smaller fields**

# Magnetoresistance : oscillating GMR

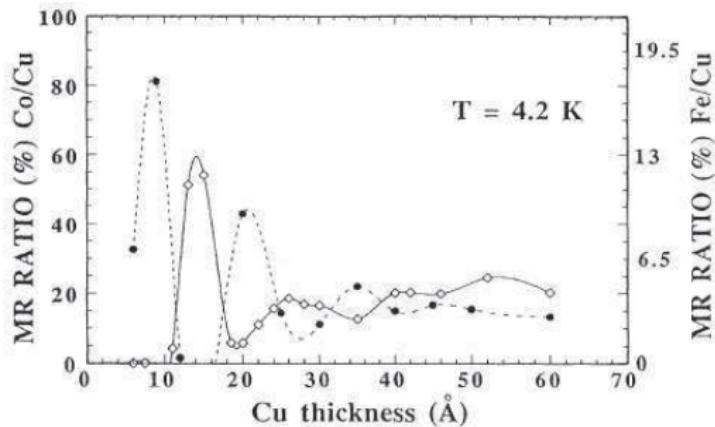


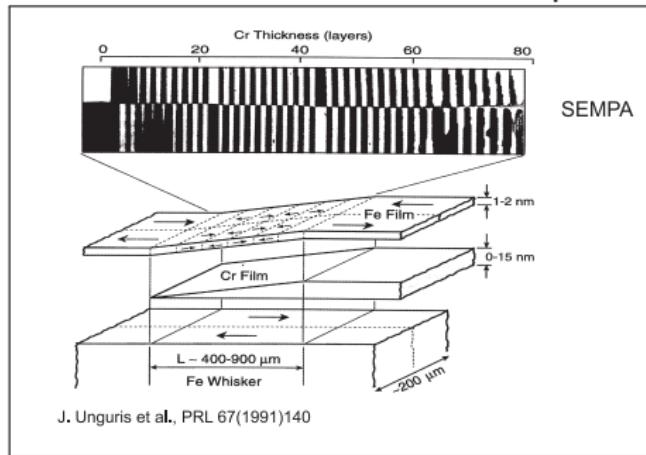
FIG. 3. Variation of the MR ratio as a function of the thickness of copper  $t_{\text{Cu}}$  for  $60 \times (15\text{-}\text{\AA} \text{ Fe}/t_{\text{Cu}} \text{ Cu})$  (open symbols) and  $30 \times (15\text{-}\text{\AA} \text{ Co}/t_{\text{Cu}} \text{ Cu})$  (black dots) multilayers. The solid and the dashed lines are guides for the eye. Notice the different vertical scale for Fe/Cu and Co/Cu.

F. Petroff et al., Phys. Rev. B 44, 5355 (1991)

# Interlayer coupling : RKKY

$$J = \frac{\cos(2.k_F.r)}{r^3}$$

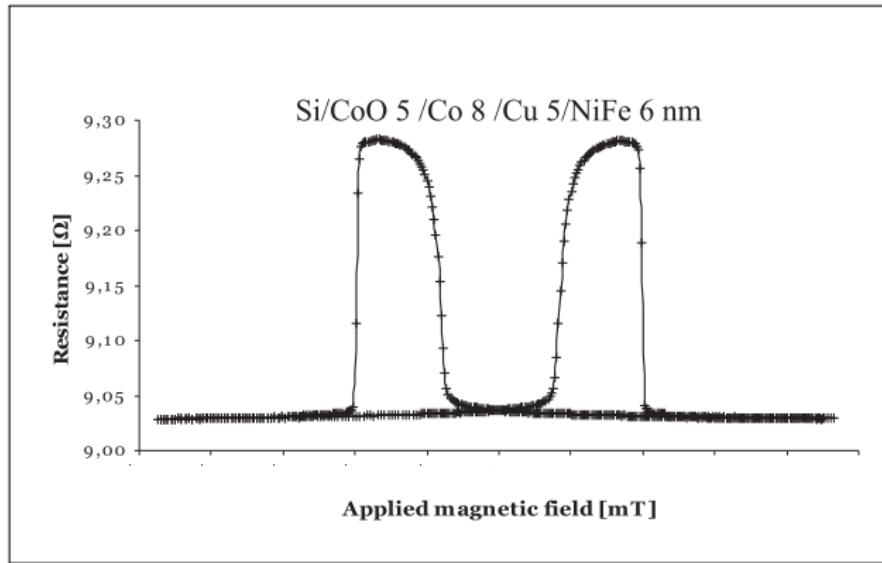
oscillating coupling through a metallic spacer. lengthscale  $k_F$  finite, discrete thickness effect, period becomes a few monolayers.



Use also to create **artifical antiferromagnet** : Co/Ru/Co

Now, **difference in coercive fields** or **exchange bias** is prefered to **pin** one of the layer and keep the other one free.

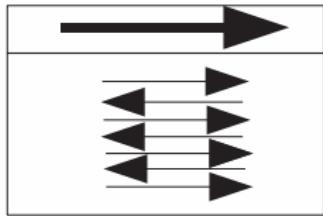
# Magnetoresistance : GMR



GMR junction using a soft material (FeNi) and a harder one (Cobalt)

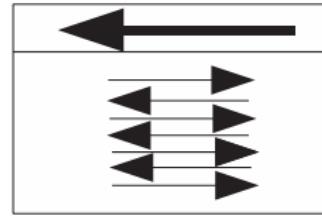
# Interface effect

## Exchange bias



Favorable AF-F alignment

F layer  
AF layer

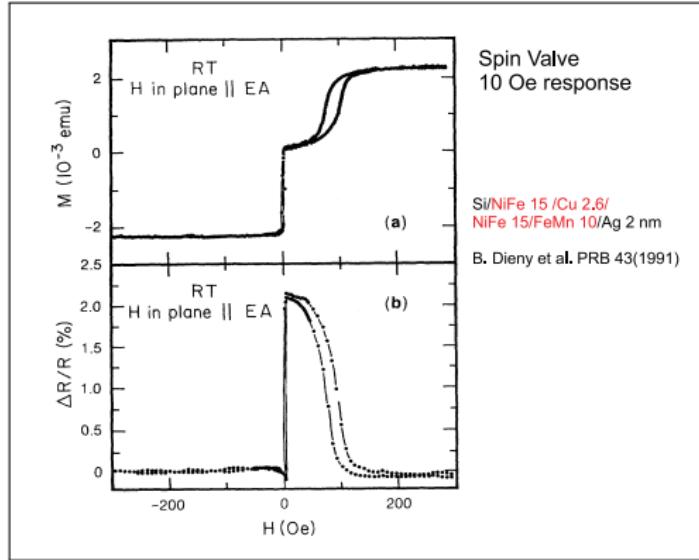


Unfavorable AF-F alignment

It increases the coercive field (**coercivity enhancement**)

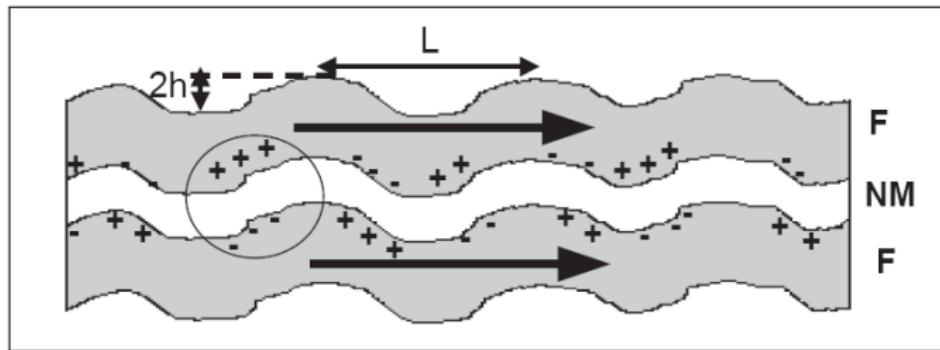
It **biases the ferromagnetic layer** if the system has been cooled under field

# Magnetoresistance : Spin Valve



**Pinning** of one layer using a FeMn antiferromagnetic layer  
(exchange bias)

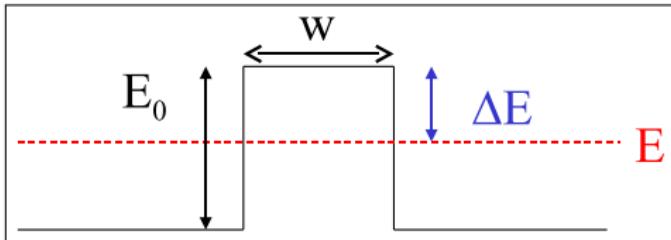
# Interlayer coupling : Orange peel



Coupling may still exist in “uncoupled” system

orange peel=magnetostatic, pinholes

# Tunnel Effect



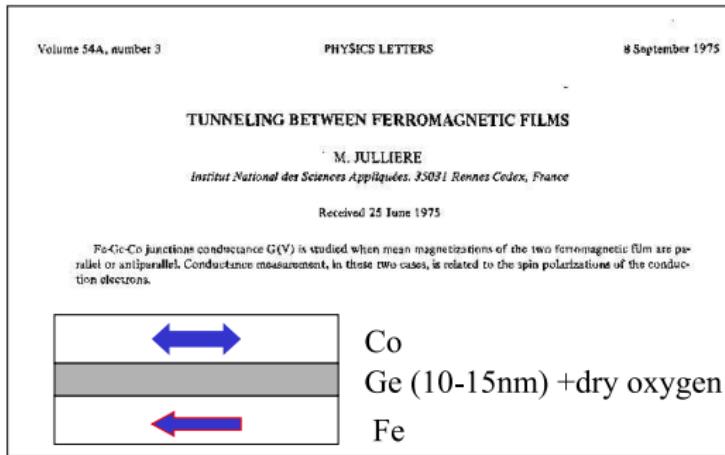
A thin insulating layer ( $\text{Al}_2\text{O}_3$ ,  $\text{MgO}$ ) is inserted between 2 metallic electrodes

Classical transport can not happen if the electron energy is smaller than the barrier height.

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} |\psi\rangle + V(x) |\psi\rangle = E |\psi\rangle \text{ Schroedinger 1D}$$

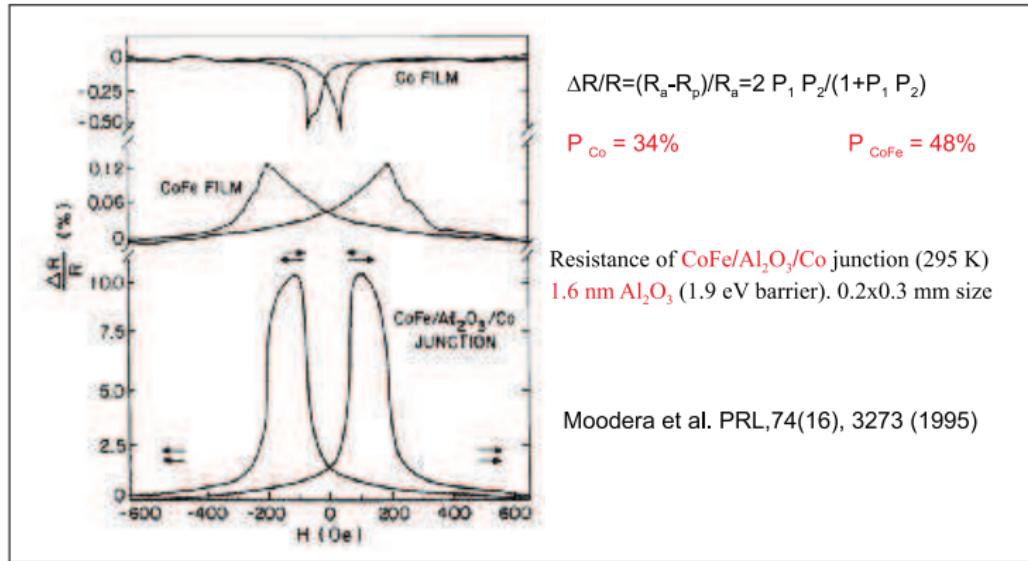
Quantum mechanics allows for propagation of an evanescent wave inside the barrier. If the barrier is thin enough, the probability to tunnel through the barrier is non zero.

# Magnetic Tunnel Resistance



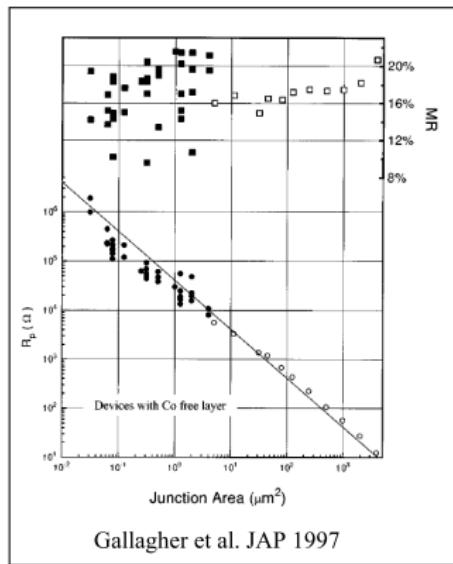
When electrodes are ferromagnetic, tunneling probabilities depend on the spin-dependent density of states at  $E_F$   
 So on the **relative magnetic configuration** : parallel /antiparallel

# Magnetoresistance : TMR



$$TMR(\text{Julliere}) = \frac{R_{\text{antiparallel}} - R_{\text{parallel}}}{R_{\text{parallel}}} = \frac{2P_1 P_2}{1 - P_1 P_2} \quad (2)$$

# Magnetoresistance : Barrier quality



Junction resistance should scale as  $\frac{1}{S}$

Difficult to obtain since any thickness fluctuation, pinhole will short-circuit the barrier

One uses the **Area Resistance**  $RA = R_{junction} \cdot S$

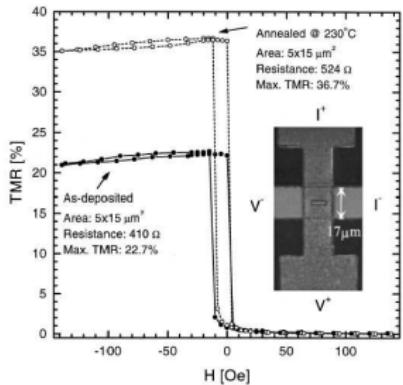
It depends mainly on the barrier (thickness,height)

# Magnetoresistance : Barrier quality

NiFe(100 Å)/CoFe(20 Å)/Al<sub>2</sub>O<sub>3</sub>/CoFe(40 Å)/MnRh(170 Å)

Appl. Phys. Lett., Vol. 73, No. 22, 30 November 1998

Sousa et al. (Lisbon)



Barrier (Simmons' fit)  
0.87 to 0.77 nm wide  
1.8 eV to 2.5 eV

FIG. 1. Tunneling magnetoresistance vs field for an as-deposited spin tunnel junction, and for the same junction after consecutive anneals up to 230 °C. In the inset, the four-probe measuring scheme is illustrated using an optical microscope picture of the junction.

Annealing can repair (improve) the barrier quality

# Magnetoresistance : Barrier quality

RBS measurement of Al and O distributions

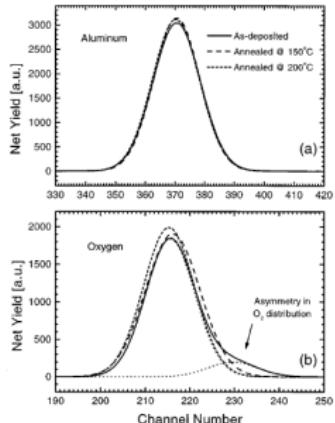
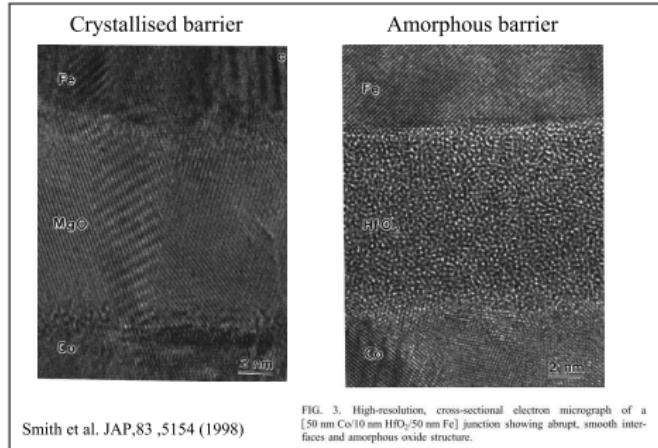


FIG. 5. Gaussian curve fits to the aluminum (a) and oxygen (b) peaks obtained by Rutherford backscattering analysis, for as-deposited junctions, and for junctions annealed at 150 and 200 °C.

Sousa et al. APL 98

For example, Oxygen diffusion out of the barrier is cured

# Magnetoresistance : Barrier quality

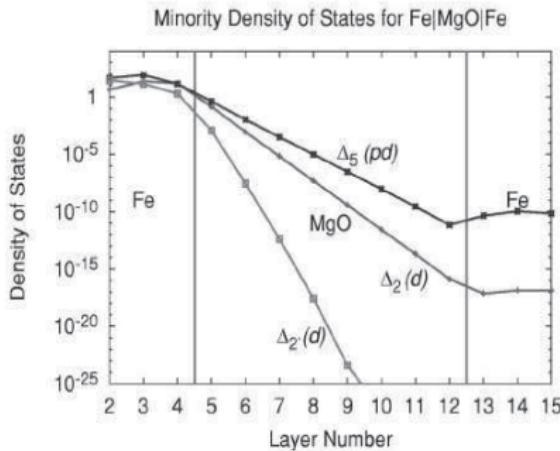
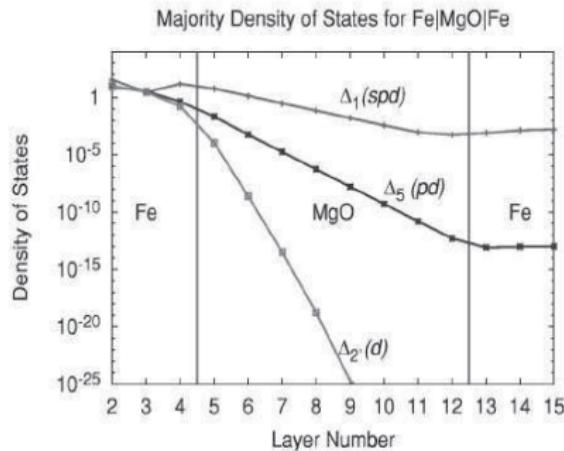


The barrier can be amorphous ( $\text{Al}_2\text{O}_3$ ) but a crystalline barrier ( $\text{MgO}$ ) will bring new effects  
selective tunnelling according to symmetry of the wavefunction  
→ **spin filtering**

# Magnetoresistance : TMR spin filtering

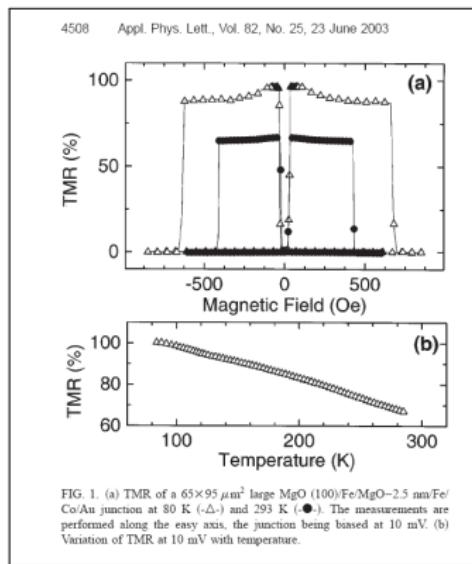
BUTLER, ZHANG, SCHULTHESS, AND MacLAREN

PHYSICAL REVIEW B 63 05



electron wavefunction's propagation through the tunnel barrier depends on symmetry

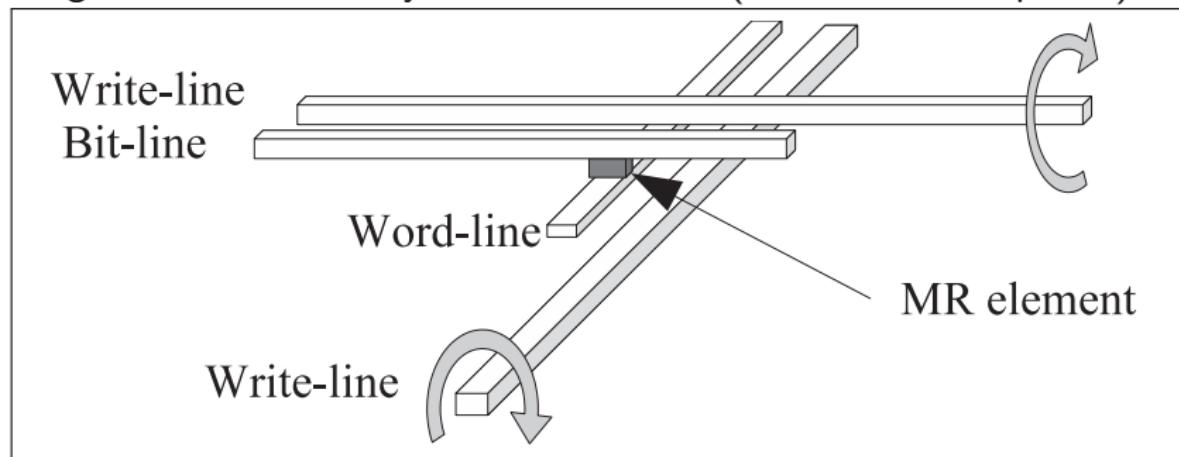
# Magnetoresistance : Giant TMR



TMR ratio larger than 100 % can be achieved using epitaxial Fe/MgO/Fe (Nancy group)  
MBE(epitaxial) or sputtering  
thick barrier (2-3 nm)  
Use as a **memory element** or **sensor**

# Magnetoresistance : MRAM

Magnetic RAM memory, it is commercial (Freescale, Everspin ...)



second generation : **heat-assisted**

⇒ reduces  $H_c$ , helps select cell

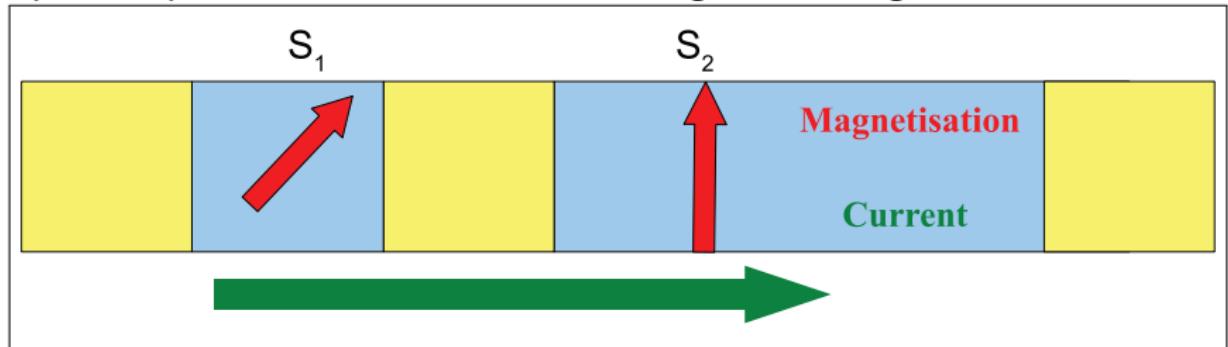
third generation : no more field lines : **spin torque** assisted

# Spin torque

what is Spin torque?

GMR = effect of magnetic configuration on currents

Spin Torque = effect of currents on magnetic configuration



# Spin torque

Appl. Phys. Lett. **88**, 232507 (2006)

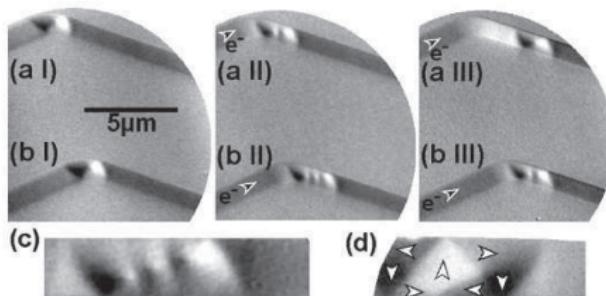
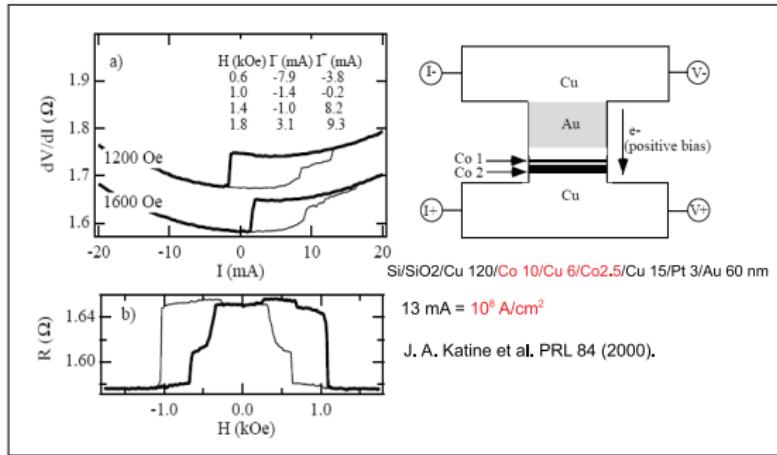


FIG. 4. XMCDPEEM images of two domain walls in adjacent lines after consecutive pulse injections with  $j=8.7 \times 10^{11} \text{ A/m}^2$ . Top wall: (a I) vortex wall after remagnetization which transforms to a double vortex (a II) and back (a III) during consecutive injections. Bottom wall: (b I) a vortex wall that transforms to a triple vortex (b II) and back to a double vortex spin structure (b III). (c) High resolution image of the spin structure of a triple vortex wall. (d) High resolution spin structure of the double vortex wall with two vortices that have opposite sense of rotation.

Domain wall propagation **without applied field**  
Magnetisation reversal of a nanoparticle  
Spin torque oscillator (current-tunable GHz emission)

# Spin torque



Magnetic reversal of a patterned electrode

# GHz dynamics

## Landau-Lifschitz-Gilbert equation

$$\frac{d\mathbf{M}}{dt} = \gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}) + \text{Spin Torque} \quad (3)$$

$\gamma$  the gyromagnetic ratio

$\alpha$  the damping constant

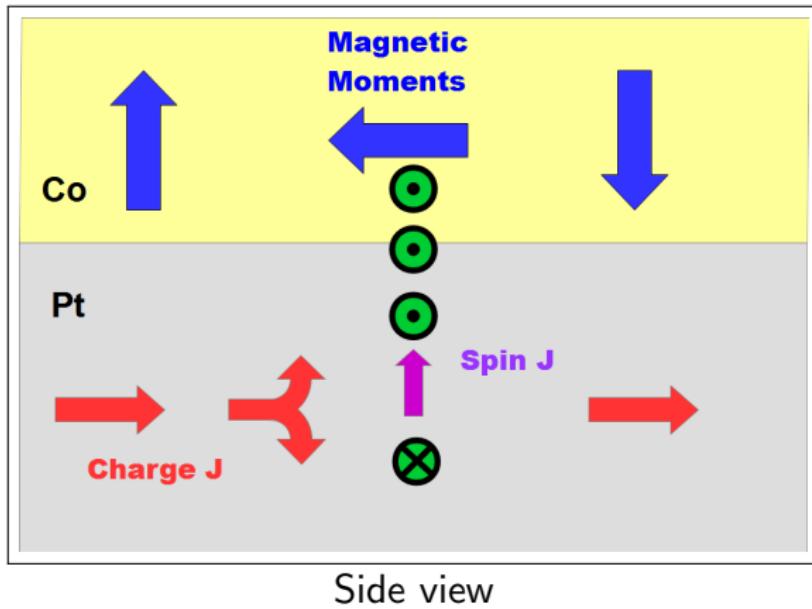
$\mathbf{H}_{\text{eff}}$  the effective field :

$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0} \frac{\partial E}{\partial \mathbf{M}}$$

The effective field includes contributions from the applied field (Zeeman energy), the demagnetizing field (shape anisotropy), magnetocrystalline and exchange energies.

The STT may induce an antidamping (STT oscillator)

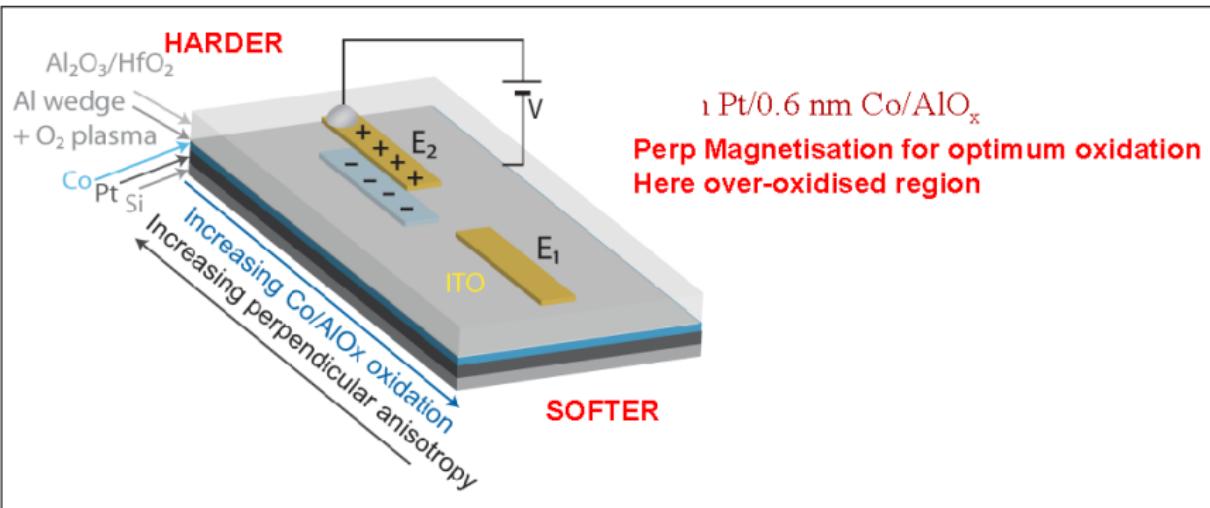
# Spin Hall Effect



$$\text{Spin current } J_s = \theta_{SHE} J_c$$

Pt and Ta : Injection with Opposite Spin Signs

# Voltage control of magnetisation reversal



Sputtering Pt/Co/AlOx  
ALD dielectrics + ITO Sputtering (Institut Néel, see  
Bernard-Mantel et al.)