Fields, Units, Magnetostatics

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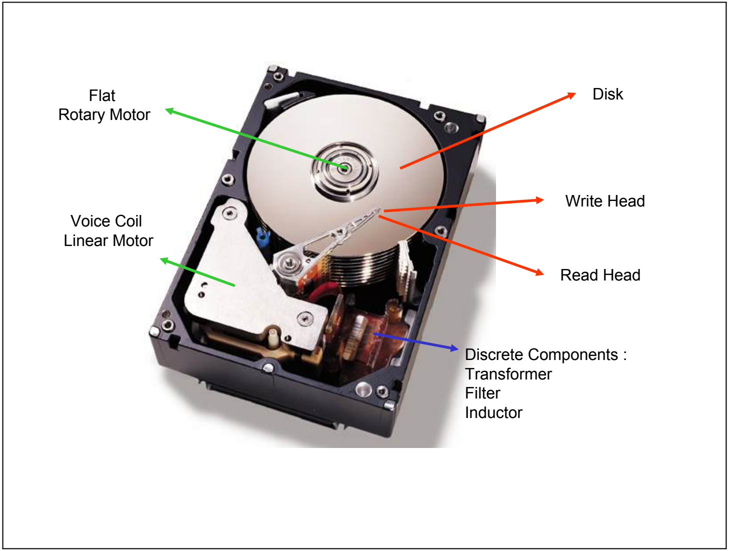
Motivation

Magnetism is around us and magnetic materials are widely used

- Magnet Attraction (coins, fridge)
- Contactless Force (hand)
- Repulsive Force : Levitation
- Magnetic Energy - Mechanical Energy (Magnetic Gun)
- Magnetic Energy - Electrical Energy (Induction)
- Magnetic Liquids

A device full of magnetic materials : the Hard Disk drive


Flat
Rotary Motor
Disk
Voice Coil
Linear Motor
Write Head
Read Head
Discrete Components :
Transformer
Filter
Inductor

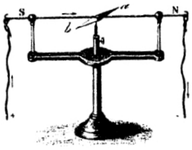
- How to describe Magnetic Matter ?
- How Magnetic Materials impact field maps, forces ?
- How to model them ?
- Here macroscopic, continous model
- Next lectures :
  Atomic magnetism, microscopic details (exchange mechanisms, spin-orbit, crystal field ...)

Magnetostatics w/o magnets : Reminder

Up to 1820, magnetism and electricity were two subjects not experimentally connected

H.C. Oersted experiment (1820 - Copenhagen)

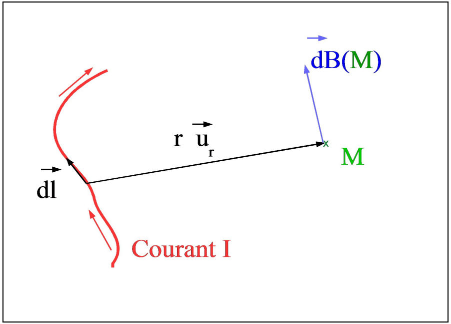
Hans Christian Ørsted
(1777-1851)



Magnetostatics induction field B

Looking for a mathematical expression

Fields and forces created by an electrical circuit (C1, I)



Elementary $\vec{dB}$ induction field created at M

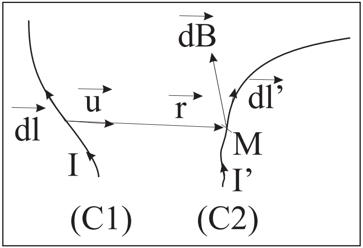
Biot and Savart law (1820) $\vec{dB}(M) = \frac{\mu_0 I \vec{dl} \wedge \vec{u}}{4\pi r^2}$

Magnetostatics : Vocabulary

$$d\vec{B}(M) = \frac{\mu_0 I \vec{dl} \wedge \vec{u}}{4\pi r^2}$$

$\vec{B}$ is the magnetic induction field
$\vec{B}$ is a long-range vector field ($\frac{1}{r^2}$ becomes $\frac{1}{r^3}$ for a closed circuit).

Force created by (C1, I) on (C2, I')



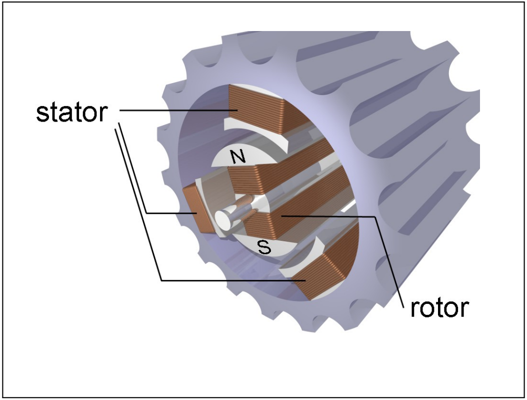
Laplace Law $\vec{dF}(M) = I'\vec{dl'} \wedge \vec{B}(M)$

What is the Force between 2 parallel wires carrying the same current I : attractive/repulsive ?

definition for Ampère :

1 A if 2 parallel wires 1m apart and force is f=2 10^{-7}N/m.

Origin of the electric-mechanical transducer = motors (linear and rotary motors)



Synchronous Motor (dc current rotor, ac current stator).
Downsizing, Mechanical Torque, Energy Yield,
Move to permanent magnet rotors.

Magnetostatics : units

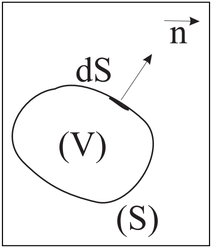
$$d\vec{F}(M) = I' d\vec{l}' \wedge \vec{B}(M)$$

Using SI units :

Force F	Newton(N)
Intensity	Ampère (A)
Magnetic Induction B	Tesla (T)

so 1 T $=$ 1 $\mathrm{NA^{-1}m^{-1}}$ and $\mu_o = 4\pi 10^{-7}$ $\mathrm{NA^{-2}}$ exact value

Some magnetic induction $\vec{B}$ properties



$$\iint_S \vec{B} \cdot \vec{dS} = 0$$

$\vec{B}$ flux is conservative
B lines never stop (closed B loops) !

B flux is conserved. It is a relevant quantity with a name :
Wb(Weber) = T.m^2
(B-field is sometimes called the magnetic flux density)

$\vec{B}$ flux conservation is equivalent to one of the local Maxwell equation :

$$\vec{\nabla} \cdot \vec{B} = 0$$

$\vec{B}$ can be derived from a vector potential $\vec{A}$ so that $\vec{B} = \vec{\nabla} \times \vec{A}$

For the preceding circuit :

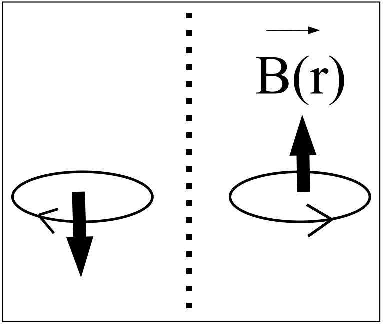
$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_{(C1)} \frac{\vec{dl}}{r}$$

applying the curl operator one comes back to $\vec{B}$
Note : $\vec{A}$ is not unique. $\vec{A}(\vec{r}) + \vec{\text{grad}}\phi(\vec{r})$ is also solution
A gauge can be chosen (i.e. $\vec{\nabla} \cdot \vec{A} = 0$, Coulomb gauge)

Magnetostatics : $\vec{A}$

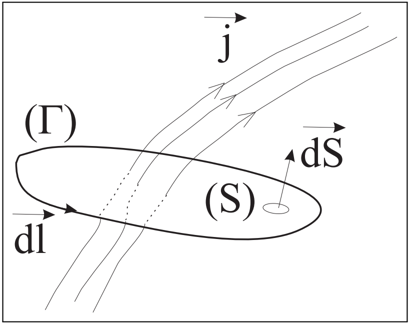
This is equivalent to the role of the electric potential V in electrostatics with $\vec{E} = -\vec{\mathrm{grad}} V$
(numerical simulation interest)

Mirror symmetry for a current loop :



$\vec{B}$ is a axial vector.

$\vec{B}$ is NOT time-reversal invariant, unlike electrostatics.



Ampere Theorem

$$\int_{(\Gamma)} \vec{B} \cdot \vec{dl} = \mu_0 I \text{ no magnet}$$

Note : with magnetic materials it becomes : $\int_{(\Gamma)} \vec{H} \cdot \vec{dl} = I$

Magnetostatics : Ampère theorem

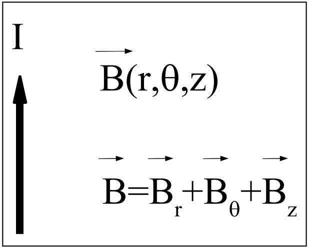
Similar to $\vec{B}$ flux conservation
Ampère theorem has a local equivalent (Maxwell)

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$$

where $\vec{j}$ is the volume current density (A/m^2 !)

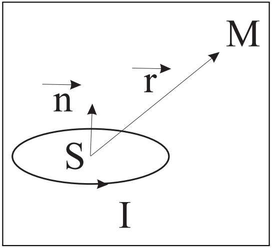
Application to the infinite straight wire

$$\int_{(\Gamma)} \vec{B} \cdot \vec{dl} = \mu_0 I$$



$$\vec{B} = \frac{\mu_0 I}{2\pi r} \vec{u_\theta}$$

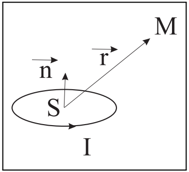
Current Carrying Loop Magnetic Moment



Circular Loop (radius R), carrying current I, oriented surface $\vec{S}$

Its magnetic moment is $\vec{m} = \vec{S} \cdot I$ unit A.m^2

Magnetostatics : Dipolar Approximation



When $r >> R$, $\vec{B}$ created by the loop becomes

$$\vec{B} = \frac{\mu_0}{4\pi r^3}(2mcos\theta\vec{u}_r + msin\theta\vec{u}_\theta)$$

Magnetostatics : Dipolar Approximation

$$\vec{B} = \frac{\mu_0}{4\pi r^3}(2mcos\theta\vec{u}_r + msin\theta\vec{u}_\theta)$$

can also be written along $\vec{r}$ and $\vec{m}$:

$$\vec{B} = \frac{\mu_0}{4\pi}(\frac{3(\vec{m}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3})$$

Earth Field = Dipolar Field (good approximation).

Magnetostatics : Earth Field

Model: WMM (2014-2019) IGRF (1590-2019)

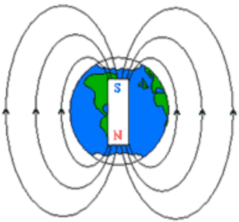
Country: -Choose a country-

City: -Choose a city-

Magnetic Field

Model Used:	WMM2015
Latitude:	42.1363° N
Longitude:	8.5959° E
Elevation:	0.0 km Mean Sea Level

Date	Declination (+ E | - W)	Inclination (+ D | - U)	Horizontal Intensity	North Comp (+ N | - S)	East Comp (+ E | - W)	Vertical Comp (+ D | - U)	Total Field
2017-10-07	2.2720°	58.0333°	24,488.3 nT	24,469.1 nT	970.8 nT	39,240.2 nT	46,254.5 nT
Change/year	0.1082°/yr	-0.0062°/yr	19.1 nT/yr	17.2 nT/yr	47.0 nT/yr	21.1 nT/yr	28.0 nT/yr
Uncertainty	0.33°	0.22°	133 nT	138 nT	89 nT	165 nT	152 nT

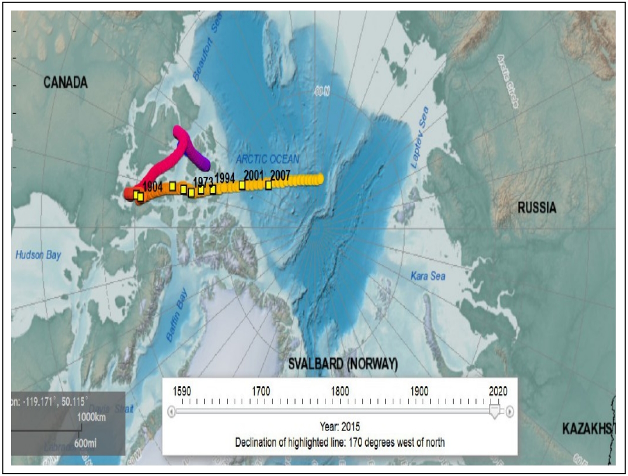


Geographic North Pole

is

Magnetic South Pole

online model : www.ngdc.noaa.gov



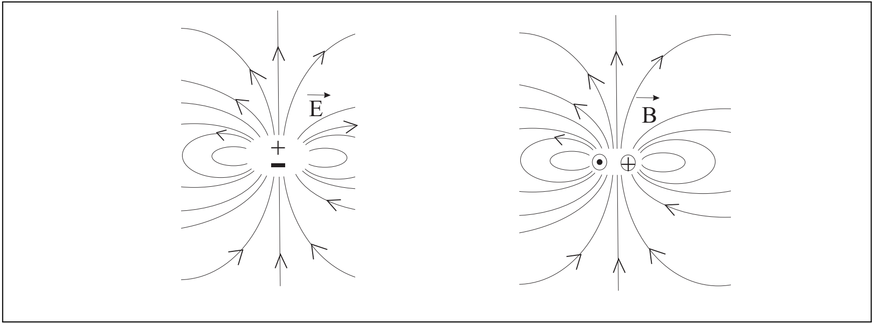
The magnetic pole moves up toward Russia. Presently (86°N, 159°W), its speed is 55 km/year to N-NW.

The magnetic dipolar field is equivalent to the electric dipolar field
One defines an electric dipole $\vec{p} = q\vec{l}$ and

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3}(2pcos\theta\vec{u}_r + psin\theta\vec{u}_\theta)$$

For an elementary loop $\vec{m}$ is the loop magnetic dipole .

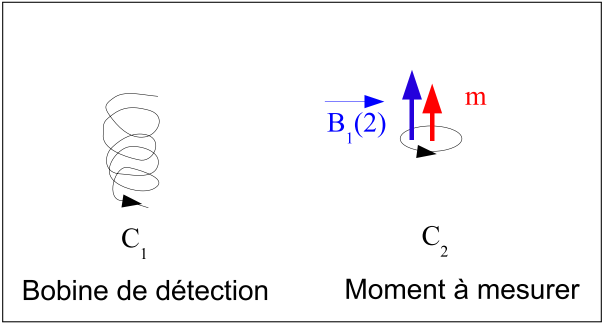
Magnetostatics : Field lines



Fields around an electric dipole and a magnetic dipole

Reciprocity Theorem

How to optimise the signal sensed by a coil close to the sample ?



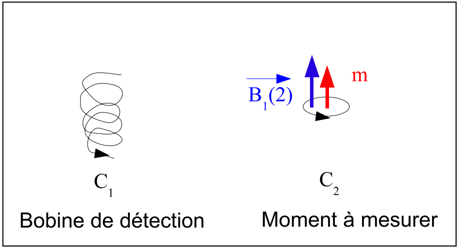
$$\vec{m} = I_2.\vec{S_2}$$

Signal $=$ flux of induction created by sample $\vec{m}$ through C_1

$$\phi_{21} = \vec{B_2}(1).\vec{S_1}$$

Mutual inductance M_{12} equals M_{21}

Reciprocity Theorem



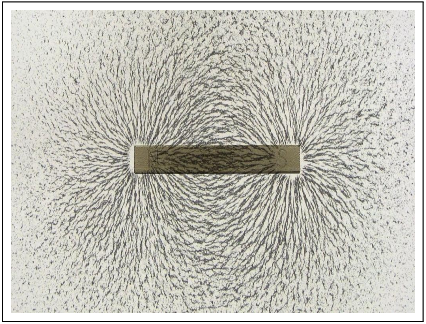
$$\phi_{21} = \vec{B}_2(1).\vec{S}_1$$

$$\phi_{21} = M.I_2 \text{ et } \phi_{12} = M.I_1$$

$$\phi_{21} = \phi_{12}.I_2/I_1 = \vec{B}_1(2).\vec{S}_2.I_2/I_1 = \vec{B}_1(2).\vec{m}/I_1$$

The sample $\vec{m}$ creates a B-flux in the detection coil equal to the scalar product $\vec{m}$ and $\vec{B}$ at $\vec{m}$ assuming 1 A in the detection coil.

Experimental Facts :
So-called magnetic materials produce effects similar to the ones created by electric circuits.



Iron filings + magnet equivalent to Iron filing (or compass) and solenoid

A magnetic material will be modeled as a set of magnetic dipoles.

$$\Delta\vec{m} = \sum_i \vec{m}_i$$

Magnetisation $\vec{M}$ is the magnetic moment per unit volume :

$$\vec{M} = \frac{\Delta\vec{m}}{\Delta V}$$

Average over 1 nm to smoothen the atomic contributions (continuous model).

Magnetic Moment	$\vec{m} = I \cdot \vec{S}$	unit is $A \cdot m^2$
Magnetisation	$M = \frac{\Delta m}{\Delta V}$	unit is $A \cdot m^{-1}$

Magnetostatics : Current Analogy

Summing all atomic dipole contributions :

- OK for atomistic model AND small volume.
  For large sample OR continuous model
- Equivalent Current Distribution
  Amperian Approach for magnetisation.
- Equivalent Charge Distribution
  Coulombian Approach for magnetisation.

Magnetostatics with Magnets : Amperian View Point

When $\vec{M} = \vec{M}(\vec{r})$,
determining $\vec{B}$ field $\vec{A}$ vector field everywhere is mathematically equivalent to a magnetostatics w/o magnets problem, where beside the real currents ones adds :

volume current density due to M : $\vec{j}_V = \vec{\nabla} \times \vec{M}$

and a surface current density due to M $\vec{j}_S = \vec{M} \times \vec{n}$
(uniform M, no volume current)

Proof for Amperian approach

From Biot-Savart :

$$\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{u}}{r^2}$$

$$\vec{A}(Q) = \frac{\mu_0}{4\pi} \iiint_{(\mathfrak{D})} \frac{\vec{M}(P) \times \vec{u}}{r^2} \, \mathrm{d}v$$

$$\vec{A}(Q) = \frac{\mu_0}{4\pi} \iiint_{(\mathfrak{D})} \vec{M}(P) \times \vec{\mathrm{grad}}_P(\frac{1}{r}) \, \mathrm{d}v$$

Since we have $\vec{\nabla} \times (f \cdot \vec{g}) = f \cdot \vec{\nabla} \times \vec{g} + \vec{\mathrm{grad}} f \times \vec{g}$ thnn

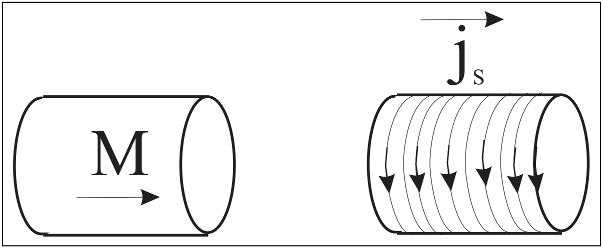
$$\vec{A}(Q) = -\frac{\mu_0}{4\pi} \iiint_{(\mathfrak{D})} \vec{\nabla} \times (\frac{\vec{M}(P)}{r}) \, \mathrm{d}v + \frac{\mu_0}{4\pi} \iiint_{(\mathfrak{D})} \frac{\vec{\nabla} \times \vec{M}}{r} \, \mathrm{d}v$$

Since

$$\iiint_{(V)} \vec{\nabla} \times \vec{g} \, \mathrm{d}v = \iint_{(S)} \vec{n} \times \vec{g} \, \mathrm{d}S$$

$$\vec{A}(Q) = \frac{\mu_0}{4\pi} \iint_{(\mathfrak{S})} \frac{\vec{M} \times \vec{n}}{r} \, \mathrm{d}S + \frac{\mu_0}{4\pi} \iiint_{(\mathfrak{D})} \frac{\vec{\nabla} \times \vec{M}}{r} \, \mathrm{d}v$$

A uniformly magnetised cylindrical magnet is equivalent to ?



surface currents $\vec{j}_S$	$\vec{M} \wedge \vec{n}$	$A.m^{-1}$
volume currents $\vec{j}_V$	$\vec{\nabla} \times \vec{M}$	$\frac{A.m^{-1}}{m} \equiv A.m^{-2}$

Magnetostatics : Magnetic Field H

In vacuum : $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$

When Magnetic material is present : $\vec{j} = \vec{j_0} + \vec{j_v}$
with $\vec{j_0}$ the real current density and $\vec{j_v} = \vec{\nabla} \times \vec{M}$

$$\Rightarrow \vec{\nabla} \times \vec{B} = \mu_0 \vec{j_0} + \mu_0 \vec{j_v}$$

$$\Rightarrow \vec{\nabla} \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{j_0}$$

One defines $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$
one gets : $\vec{\nabla} \times \vec{H} = \vec{j_0}$
With equation $\vec{B} = \mu_0(\vec{H} + \vec{M})$

Magnetostatics : Magnetic Field H

$\vec{H}$ is named Magnetic Field
$\vec{B} = \mu_0(\vec{H} + \vec{M})$

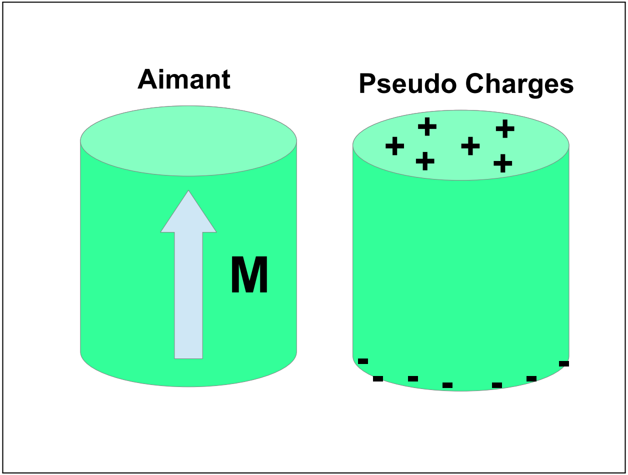
- Replacing $\vec{M}$ by $\vec{j_s}$, $\vec{J_v}$
  allows to calculate $\vec{B}$ everywhere
- In the absence of $\vec{j_0}$
  $\vec{\nabla} \times \vec{H} = \vec{0}$, whatever $\vec{M}$
  looks like $\vec{\nabla} \times \vec{E} = \vec{0}$ for electrostatics

  A magnetic scalar potential ϕ can be introduced :
  $\vec{H} = -\vec{\text{grad}}\phi$
  Good for calculations

Magnetostatics : Coulomb Point of View

- There is no magnetic charge.
- No magnetic monopole
- Using an electrostatic analogy, magnetic matter is represented by a distribution of virtual magnetic charges, which allows to calculate the H-field created by magnetisation.

2nd point of view : Coulomb Analogy



Magnetostatics : Coulombian approach

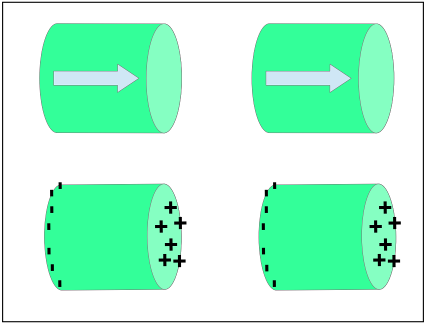
Magnetostatics	Electrostatics
magnetic dipole	Electric dipole
$\vec{m} = I \cdot \vec{S} = q_m \cdot \vec{l}$	$\vec{p} = q \cdot \vec{l}$
$\vec{H}_m = -\vec{\text{grad}} V_m$	$\vec{E} = -\vec{\text{grad}} V$
$\vec{H}_m = \frac{-1}{4\pi} \vec{\text{grad}} \frac{\vec{m} \cdot \vec{u}}{r^2}$	$\vec{E} = \frac{-1}{4\pi\epsilon_0} \vec{\text{grad}} \frac{\vec{p} \cdot \vec{u}}{r^2}$
$V_m = \frac{1}{4\pi} \frac{\vec{m} \cdot \vec{u}}{r^2}$	$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \vec{u}}{r^2}$

magnetic charges are called also magnetic poles or magnetic masses

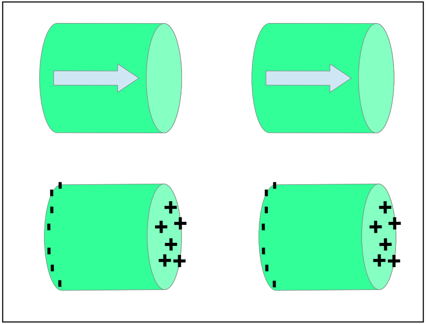
Magnetostatics : Coulombian approach

$\vec{H}$ created by a magnetic charge q_m is :

$$\vec{H} = \frac{1}{4\pi}\frac{q_m}{r^2}\vec{u} \text{ avec } \vec{u} = \frac{\vec{r}}{|\vec{r}|}$$



Magnetostatics : Coulombian approach



the force between two magnetic charges :

$$\vec{f} = \mu_0 {q'}_m \vec{H} = \frac{1}{4\pi} \frac{{q'}_m q_m}{r^2} \vec{u}$$

Coulomb Approach

To get the mathematics right :
$\vec{H}$ created by $\vec{M}(\vec{r})$ is correct if we use :

Volume charge density $\rho = -\vec{\nabla} \cdot \vec{M}$
Surface charge density $\sigma = \vec{M} \cdot \vec{n}$

Amperian approach gives $\vec{B}$
Coulombian gives $\vec{H}$.
We use only one approach since $\vec{B} = \mu_0(\vec{H} + \vec{M})$ True Everywhere

SI system of units : Gaussian cgs system

Please only use the S.I. system of units : M.K.S.A

In the past centuries, several subjects were developed independently and then several ways to rationalise units were proposed.

In magnetism, cgs units are still found (some modern equipment, litterature).
c.g.s : no μ_0, no ϵ_0. c and 4π appear in Maxwell equations.

c.g.s. and S.I. equivalent quantities do not always have the same dimension !
$\vec{f}(Newton) = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$ is $\vec{f}(dyne) = \frac{q_1 q_2}{r^2}$
Charge unit is directly related to mechanical units in cgs. Need for A in S.I.

SI system of units : Gaussian cgs system

In c.g.s B and H have the same dimension and in vacuum the same numerical value.

1 Gauss (B) = 1 Oersted (H). It prevents their rapid disapearance !

Conversion :
1 Tesla = 10 000 Gauss
$\frac{10^3}{4\pi}$ A/m = 1 Oersted

SI-cgs : moment and susceptibility

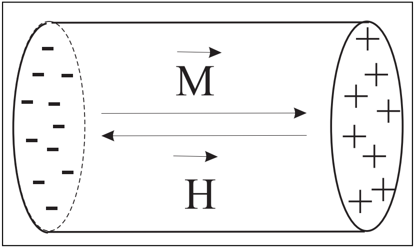
$$m = I.S = 1cgsA.1cm^2$$

$$m = 10A.10^{-4}m^2 = 10^{-3}A.m^2$$

1000 emu = 1 A.m^2 (1 e.m.u./g = 1 Am2/kg)

cgs : $\vec{B} = \vec{H} + 4\pi\vec{M}$
cgs susceptibility is 4π larger
The sum of the demag coefficient is not 1 but 4π in cgs

Let s look at a cylindrical magnet with zero applied H-field



$\vec{H}$ inside the magnetic material is not zero, is antiparallel to $\vec{M}$
$\vec{H}$ is called the demagnetising field $\vec{H}_d$

$$\vec{H} = \vec{H}_0 + \vec{H}_d$$

Demagnetising field

Application to Material Characterisation :
$\vec{M} = f(\vec{H})$ is a characteristic curve for a material.
Most measurements give $\vec{M} = f(\vec{H}_0)$
Mathematical result
For an ellipsoïd, magnetised uniformly ($\vec{M}(\vec{r}) =$ constant, $\forall \vec{r}$)
$\vec{B}$ and $\vec{H}_d$ are uniform and :

$$\vec{H}_d = -[D]\vec{M}$$

$[D]$ is the demagnetising coefficient tensor
(named [N] in some texts)

Demagnetising Coefficients

Choosing the symmetry axes, the tensor can be represented as a 3x3 matrix :

$$[D] = \begin{pmatrix} D_x & 0 & 0 \\ 0 & D_y & 0 \\ 0 & 0 & D_z \end{pmatrix}$$

and the following relation is true :
The matrix trace is 1 i.e.
$D_x + D_y + D_z = 1$.

Demagnetising Coefficients

For a sphere, $D_x = D_y = D_z = \frac{1}{3}$

For a very flat disk (axis Oz), $D_x = D_y = 0$ et $D_z = 1$

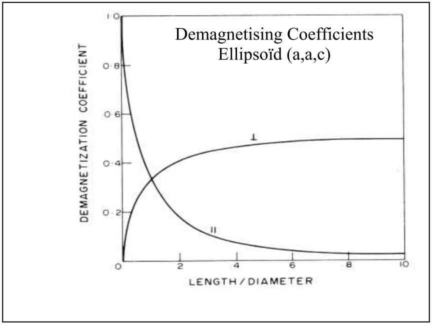
For an elongated wire, $D_x = D_y = \frac{1}{2}$ and $D_z = 0$

For a less symmetrical shape, an educated guess is to consider the ellipsoid with the same aspect ratio.
However uniformly magnetised BUT not ellipsoidal shapes produce non uniform $\vec{H}_d$!

It is the time consuming step for micromagnetics.

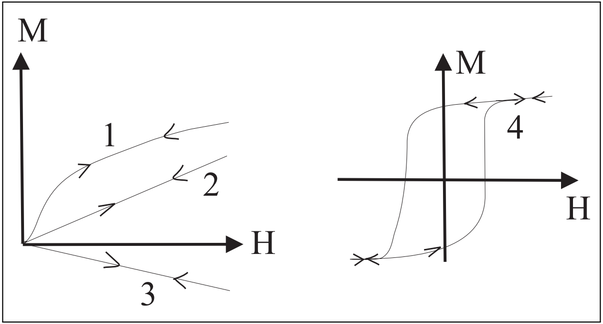
Demagnetising Coefficients

For ellipsoïds, there are analytical expressions for Demag Coefficients.



Magnetic Behaviours

Magnetic behaviours under field :
To characterise a material : $\vec{M} = f(\vec{H})$ or sometimes $\vec{M} = f(\vec{B})$
Usually the measurement gives M_z
For anisotropic materials (films, single crystals) $\vec{M} = f(\vec{H})$ is measured along different crystallographic axes (see magnetic anisotropy lecture)



Susceptibility

For linear responses (2 et 3) one can define $\vec{M} = \chi \vec{H}$.
$\vec{M}$ and $\vec{H}$ are parallel.
χ is the magnetic susceptibility .
Unitless scalar in S.I. for an isotropic material.

Susceptibility

Since $\vec{B} = \mu_0(\vec{H} + \vec{M})$

$$\vec{B} = \mu_0(1 + \chi)\vec{H} = \mu\vec{H}$$

where $\mu = \mu_0(1 + \chi)$ is the permeability
and $\mu_r = \frac{\mu}{\mu_0} = 1 + \chi$ the relative permeability
$\chi > 0$ for paramagnetism
$\chi < 0$ for diamagnetism
χ ranges from -10^{-5} to 10^6

Susceptibility : demag correction

One measures : $M = \chi_0 H_0$
However $H = H_0 + H_d = H_0 - DM$
So : $M = \chi_0(H + D.M)$
Finally : $M = \frac{\chi_0}{1-D\chi_0} H$
Or : $M = \frac{\chi}{1+D\chi} H_0$
What happens for very soft materials ?
(χ_0 is limited to $\frac{1}{D}$, need for closed circuit (D=0) to measure large χ)

Magnetic Susceptibility

For nonlinear materials (1) a differential susceptibility at a specific field $\vec{H}_0$.

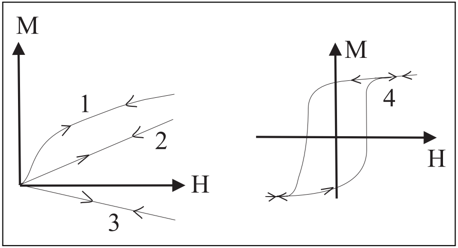
$$\chi = (\frac{dM}{dH})_{H_0}$$

in particular initial susceptibility χ_i

$$\chi_i = (\frac{dM}{dH})_{H_0=0}$$

and High field susceptibility (residual after saturation)

Permanent Magnets



For hysteretic materials (4) there is a remanent magnetisation.

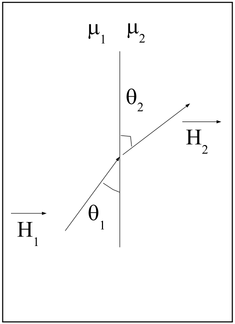
Family of permanent magnets

Field lines (B and H)

B-lines are closed
H-lines start from positive pseudo-charges and finish at negative pseudo-charges.
H same as E

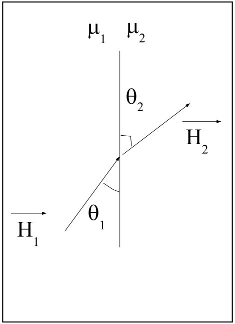
Field lines across interfaces

For a linear, homogeneous isotropic material its permeability μ can be defined.
Interface between μ_1 and μ_2
Continuities of field components.



$$\vec{\nabla} \cdot \vec{B} = 0 \text{ gives } B_{normal} \text{ continuity}$$
$$\vec{\nabla} \times \vec{H} = \vec{0} \text{ gives } H_{tangent} \text{ continuity}$$

Field lines across interfaces

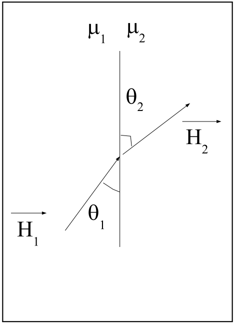


$$B_{n1} = B_{n2} \text{ so } \mu_1 H_{n1} = \mu_2 H_{n2}$$

$$H_{t1} = H_{t2} \text{ so } \frac{\mu_1 H_{n1}}{H_{t1}} = \frac{\mu_2 H_{n2}}{H_{t2}}$$

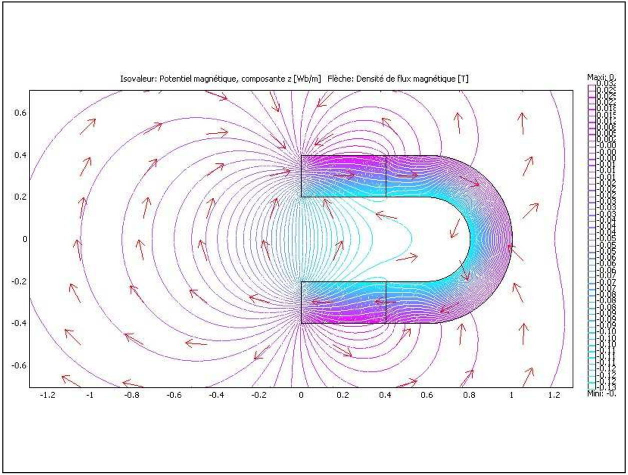
$$\text{so } \mu_1 \tan\theta_1 = \mu_2 \tan\theta_2$$

Flux Guide



$$\mu_1 \tan\theta_1 = \mu_2 \tan\theta_2$$

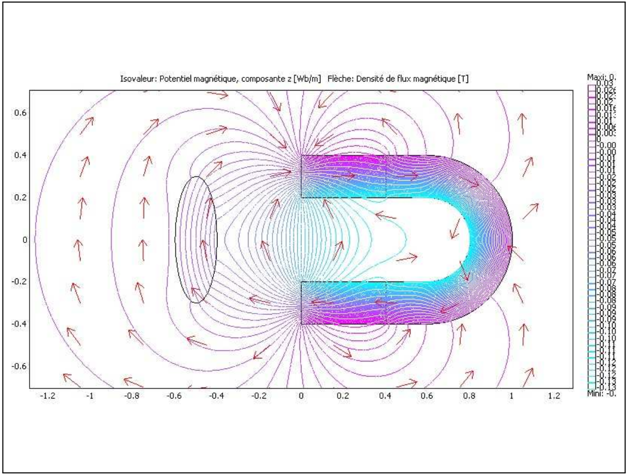
If (2) very soft ($\mu_2 >> \mu_1$) then
$\tan\theta_2$ much smaller than $\tan\theta_1$
Field lines are parallel to interface in the soft
It is the principle for Flux Guidance (soft iron cores).



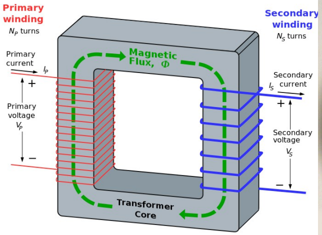
Field Map for a U-shaped Magnet.

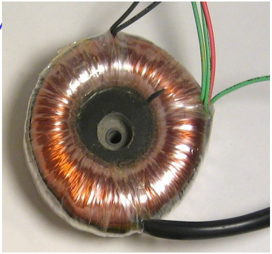
Flux Guidance

inserting a soft material (χ=10 ellipse)

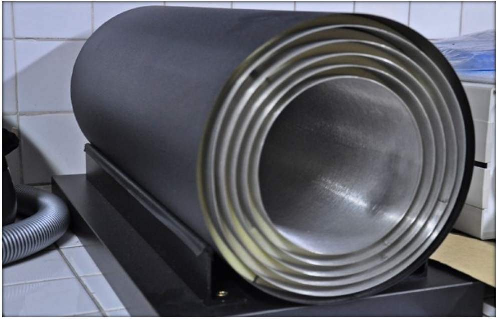


Flux Guides


Primary winding
N_P turns
Primary current
I_P
Primary voltage
V_P
Magnetic Flux, Φ
Transformer Core
Secondary winding
N_S turns
Secondary current
I_S
Secondary voltage
V_S



Magnetic Shield



Mu metal shielding for sensitive electronics.
Available volume with residual smaller than 1 nanoTesla.
Mumétal HiMu80 = Ni 80, Mo 5, Si 0.5, Cu 0.02, + Fe.

Magnetic Energy

Energy for a fixed moment $\vec{m}$ in applied field $\vec{B} = \mu_0 \vec{H}$

$$W = -\vec{m} \cdot \vec{B}$$

Stable position ?

Force

Force on a magnetic moment
Calculating on one elementary loop :

$$F_z = m\frac{\partial B_{0z}}{\partial z} \text{ for a loop m in applied field } B_{0z}$$

More generally :

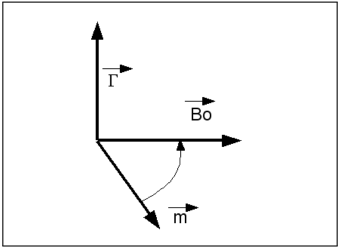
$$\vec{F} = -\vec{\text{grad}}\,W = \vec{\text{grad}}(\vec{m} \cdot \vec{B}_0)$$

Force created by a uniform field ?

Torque

$\vec{m}$ in applied field $\vec{B_0}$ experiences a torque $\vec{\Gamma}$:

$$\vec{\Gamma} = \vec{m} \times \vec{B_0}$$



The torque tends to align $\vec{m}$ parallel to $\vec{B_0}$

Energies

magnetic moments experience 2 sources of field :

- applied fields

- demagnetising fields

Both should be considered.

Zeeman Energy

In applied field $\vec{H}_0$ one gets :

$$E_{zeeman} = -\vec{M}_0 \cdot \mu_0 \vec{H}_0 V = -\vec{M}_0 \cdot \vec{B}_0 V$$

$$E_{zeeman}/volume = -\vec{M}_0 \cdot \mu_0 \vec{H}_0$$

Demagnetising Energy

$\vec{H}_d$ created by the material :

$$E_d = -\frac{\mu_0}{2}\vec{M}_0 \cdot \vec{H}_d$$

Do not forget the 1/2 ! ! !

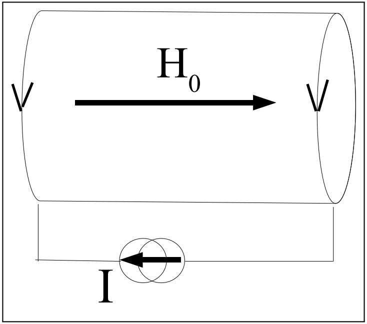
Magnetostatic Energy

The volume magnetostatic Energy is the sum :
Zeeman Energy + Demagnetising Energy.

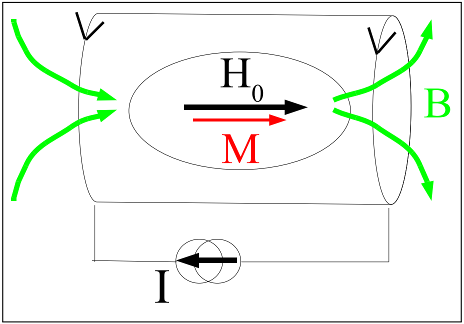
$$E_m = -\frac{\mu_0}{2}\vec{M}_0 \cdot \vec{H}_d - \mu_0 \vec{M}_0 \cdot \vec{H}_0$$

Calculating the Work to magnetise a sample

Using a solenoid with constant current,



Insert the sample



Since I is constant, if M varies then B-flux varies.

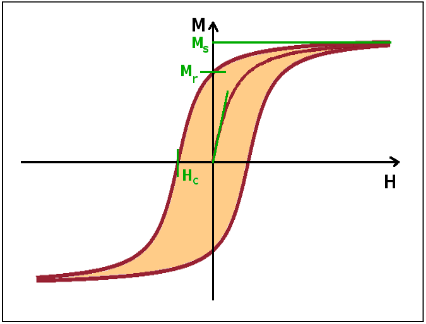
The current generator must work :

$$P = I.\frac{d\phi}{dt} = I\frac{NSdB}{dt} = \mu_0 INS\frac{dH+dM}{dt}$$
$$dW = (\mu_0 HdH + \mu_0 HdM)V$$

Magnetising Work

The energy stored in the field is $\mu_0 H^2 V/2 = LI^2/2$
$=$ the long solenoid inductance : $L = \mu_0 N^2 S/l$
The energy to magnetise the sample varies as $\mu_0 H.dM$

Magnetic Losses



When the loop M(H) is not reversible, what represents its area ?

The energy losses per loop.

Questions

Questions

Lunch time