Magnetic interactions

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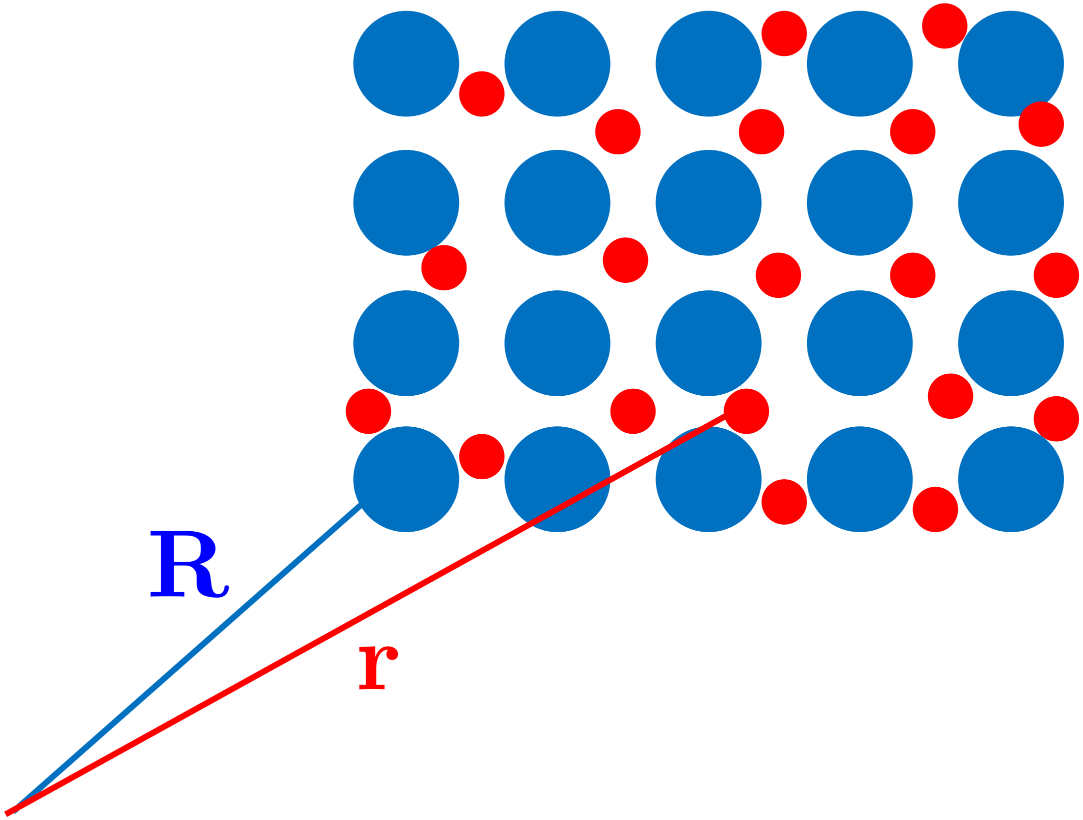
Outline

- Introduction
- Interactions
- Models
- STONER model
- HEISENBERG model

Introduction

Quantum mechanical description of solids



$$\hat{H} = \hat{T}_I + \hat{T}_e + V_{II} + V_{ee} + +V_{eI}$$

Quantum mechanical description of solids

$$\hat{H} = \hat{T}_I + \hat{T}_e + V_{II} + V_{ee} + +V_{eI}$$

Adiabatic approximation

Electrons:

$$\hat{H}(\mathbf{R}) = \hat{T}_e(\mathbf{R}) + V_{ee}(\mathbf{R}) + +V_{eI}(\mathbf{R})$$

Ions:

$$\hat{H} = \hat{T}_I + V_{II} + E(\mathbf{R})$$

$$\mathbf{R} = \{\mathbf{R_1}, \mathbf{R_2}, \mathbf{R_3}, \ldots\}$$

Solution of the electron problem

$$\hat{H}(\mathbf{R}) = \hat{T}_e(\mathbf{R}) + V_{ee}(\mathbf{R}) + +V_{eI}(\mathbf{R})$$

Many-electron Schrödinger equation:

$$\hat{H}(\mathbf{R})\Phi(\mathbf{r},\mathbf{R}) = E(\mathbf{R})\Phi(\mathbf{r},\mathbf{R})$$

Electron coordinates:

$$\mathbf{r} = \{\mathbf{r_1},\mathbf{r_2},\mathbf{r_3},\ldots\}$$

Fixed ion coordinates:

$$\mathbf{R} = \{\mathbf{R_1},\mathbf{R_2},\mathbf{R_3},\ldots\}$$

Solution of the electron problem

Many-electron Schrödinger equation:

$$\hat{H}(\mathbf{R})\Phi(\mathbf{r},\mathbf{R}) = E(\mathbf{R})\Phi(\mathbf{r},\mathbf{R})$$

- Free electrons
- Hartree approximation
- Hartree-Fock approximation
- Density functional theory

One-electron Schrödinger equation:

$$\hat{H}\varphi(\mathbf{r}) = (-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V(\mathbf{r}))\varphi(\mathbf{r}) = \varepsilon\varphi(\mathbf{r})$$

Magnetic interactions

Interactions

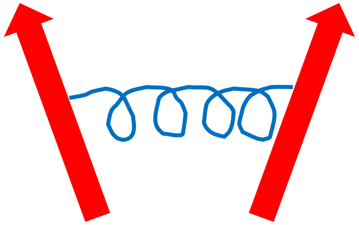
There is no elementary magnetic interaction!

Dipol-dipol interaction between magnetic moments:

$$E_{DD}(\mathbf{R}) = \frac{1}{R^3}(\mathbf{M_1} \cdot \mathbf{M_2} - 3(\mathbf{M_1} \cdot \hat{\mathbf{R}})(\mathbf{M_2} \cdot \hat{\mathbf{R}}))$$

$$\mathbf{M} \sim 1\mu_B$$

$$E_{DD} \sim 10^{-5} eV$$



$$\mu_B = \frac{e\hbar}{2mc}$$

Hartree-Fock approximation

Exchange interaction caused by Pauli principle:

Ansatz for the wave function:

$$\Phi_{HF}(\mathbf{r}_1 \ldots \mathbf{r}_i \ldots \mathbf{r}_N) = \frac{1}{\sqrt{N!}} \det |\varphi_{\alpha_i}(\mathbf{r}_i)|$$

Hartee-Fock energy:

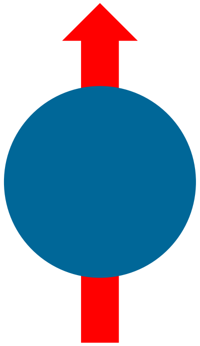
$$E_{HF}[\varphi_\alpha] = \sum_i^N \int d^3r \varphi^*_{\alpha_i}(\mathbf{r})\hat{H}(\mathbf{r})\varphi_{\alpha_i}(\mathbf{r})$$

$$+\frac{1}{2}\sum_{i\neq j}\int d^3r d^3r' \frac{\epsilon^2}{|\mathbf{r}-\mathbf{r}'|}[\varphi^*_{\alpha_i}(\mathbf{r})\varphi_{\alpha_i}(\mathbf{r})\varphi^*_{\alpha_j}(\mathbf{r}')\varphi_{\alpha_j}(\mathbf{r}')$$

$$-\varphi^*_{\boxed{\alpha_j}}(\mathbf{r})\varphi_{\alpha_i}(\mathbf{r})\varphi^*_{\boxed{\alpha_i}}(\mathbf{r}')\varphi_{\alpha_j}(\mathbf{r}')]$$

Exchange of two electrons!

Limits of magnetic phenomena

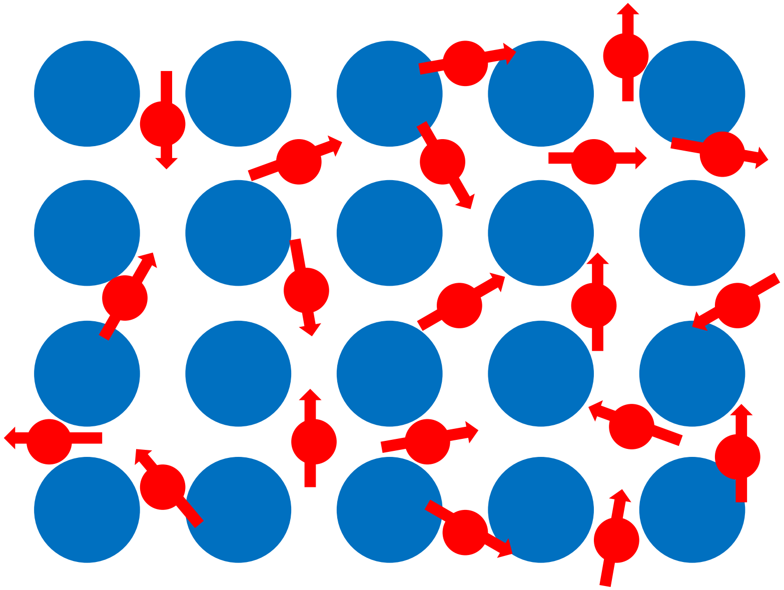


Electrons in isolated atoms:

Mostly magnetic, Hund‘s rule

Electrons in an ideal Fermi gas:

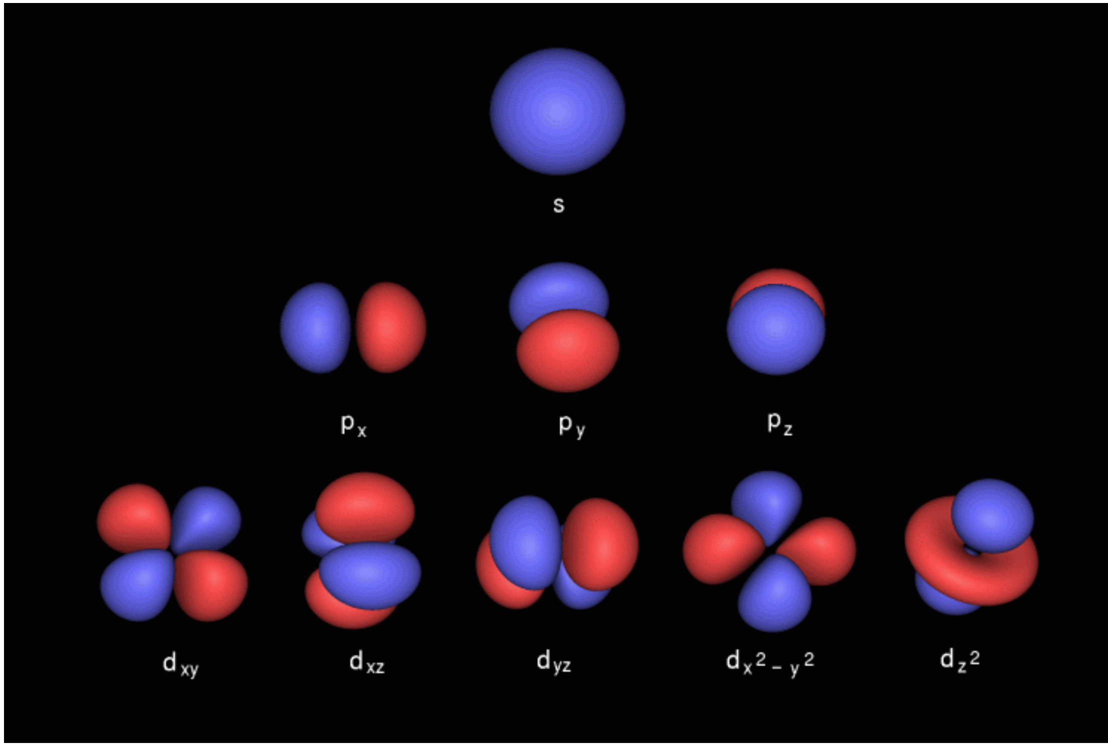
Mostly non-magnetic



Localisation of the electrons

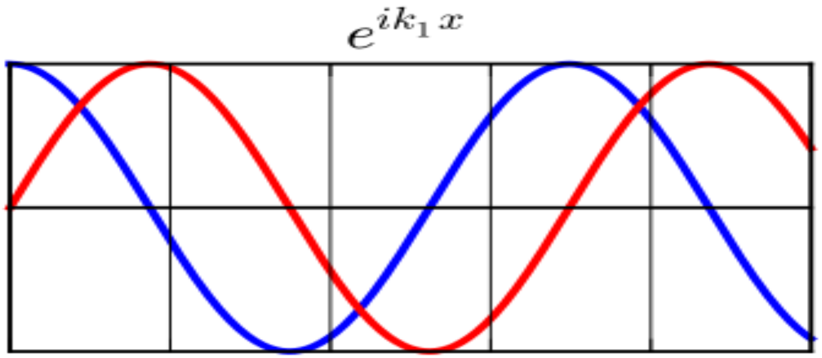
Atomic orbitals:

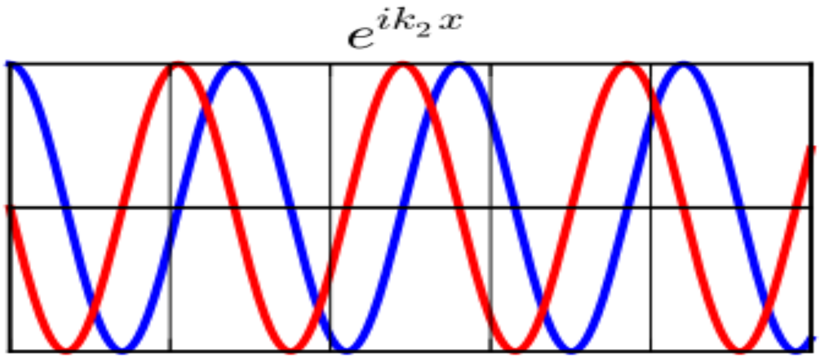
localised



Bloch waves:

delocalised





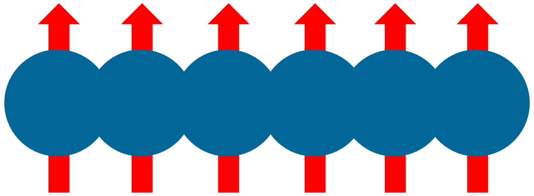
Localisation of the electrons

Degree of electron localisation causes magnetism or not!

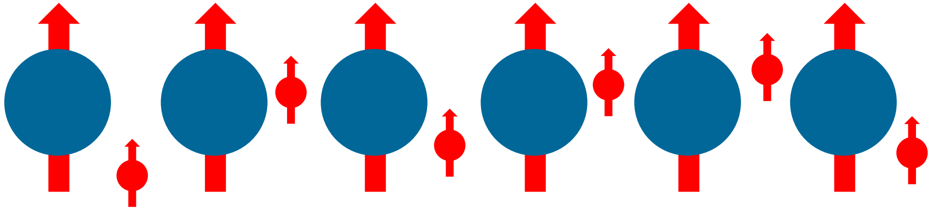
- Simple metals and semiconductors:
  non-magnetic
- Rare earth atoms:
  atomic magnetic moments
- Transition metals and actinide:
  weakly localised electrons

Interatomic exchange

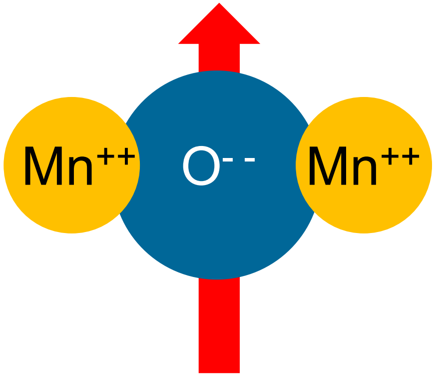
Direct exchange:



Indirect exchange:



Superexchange:



Itinerant exchange: magnetism of delocalised electrons

Models

Models

Magnetic insulators:

EuO, EuS, MnO, …

Magnetic metals:

Fe, Co, Ni, …

ISING

HEISENBERG

HUBBARD

$$\hat{H} = -\sum_{ij} I_{ij}\, \mathbf{s_i} \cdot \mathbf{s_j}$$

$$\hat{H} = \sum_{ij\sigma} t_{ij} a^{+}_{i\sigma} a_{j\sigma} + \frac{1}{2} U \sum_{i\sigma} n_{i\sigma} n_{i-\sigma}$$

$$\hat{H} = -\sum_{ij} J_{ij} s_i s_j$$

$$s_i = \pm 1$$

$$n_{i\sigma} = a^{+}_{i\sigma} a_{i\sigma}$$

$$\sigma = \pm\frac{1}{2}$$

Mean field approximation

$$< \hat{A}\hat{B} >= \hat{A} < \hat{B} > + < \hat{A} > \hat{B} - < \hat{A} >< \hat{B} >$$

WEISS

STONER

STONER model

STONER model

One-electron Schrödinger equation for spin-dependent potential:

$$(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} + V^{\pm}(\mathbf{r}))\varphi_m^{\pm}(\mathbf{r}) = \varepsilon_m^{\pm}\varphi_m^{\pm}(\mathbf{r})$$

Charge density:

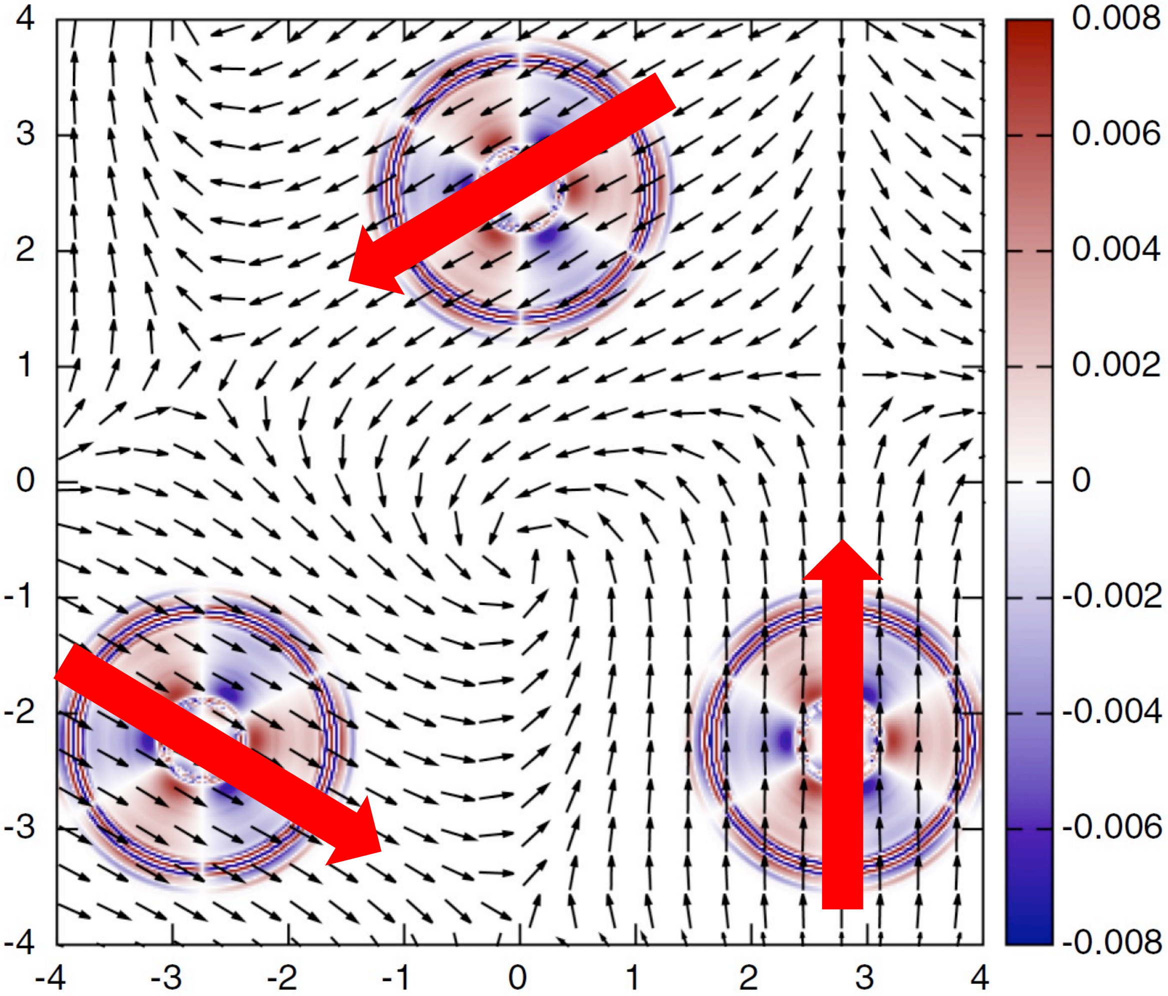
$$n(\mathbf{r}) = n^{+}(\mathbf{r}) + n^{-}(\mathbf{r}) = \sum_m |\varphi_m^{+}(\mathbf{r})|^2 + \sum_m |\varphi_m^{-}(\mathbf{r})|^2$$

Magnetization density:

$$m(\mathbf{r}) = n^{+}(\mathbf{r}) - n^{-}(\mathbf{r}) = \sum_m |\varphi_m^{+}(\mathbf{r})|^2 - \sum_m |\varphi_m^{-}(\mathbf{r})|^2$$

Magnetization density and magnetization

$$m(\mathbf{r}) = n^+(\mathbf{r}) - n^-(\mathbf{r}) = \sum_m |\varphi_m^+(\mathbf{r})|^2 - \sum_m |\varphi_m^-(\mathbf{r})|^2$$



Local magnetic moment per unit cell **M**

$$M = \int_{V_Z} d^3r \; m(\mathbf{r})$$

STONER model

One-electron Schrödinger equation for spin-dependent potential:

$$(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2}+V^{\pm}(\mathbf{r}))\varphi_m^{\pm}(\mathbf{r})=\varepsilon_m^{\pm}\varphi_m^{\pm}(\mathbf{r})$$

Spin-dependent potential:

$$V^{\pm}(\mathbf{r})=V(\mathbf{r})\mp\frac{1}{2}IM \qquad M=\int_{V_Z} d^3r\; m(\mathbf{r})$$

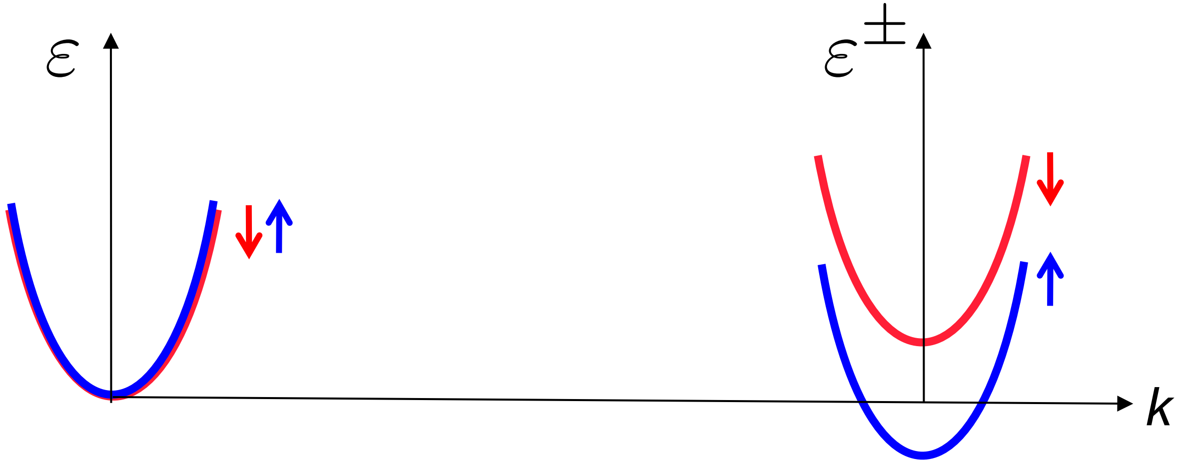
Spin-polarized band structure

Wave function unchanged by spin polarization, constant potential:

$$\varphi_m^{\pm}(\mathbf{r}) = \varphi_m(\mathbf{r})$$

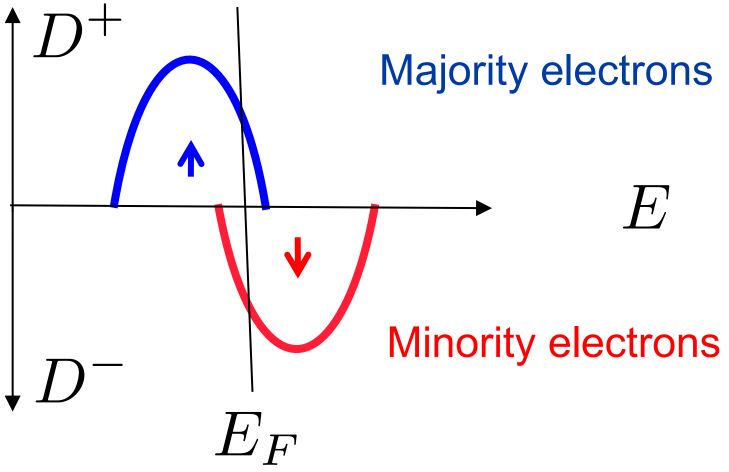
Splitting of the eigenvalues:

$$\varepsilon_m^{\pm} = \varepsilon_m \mp \frac{1}{2} I M$$



Spin-polarized density of states

$$D^{\pm}(E) = D_0(E \pm \frac{1}{2} I M)$$



D^+

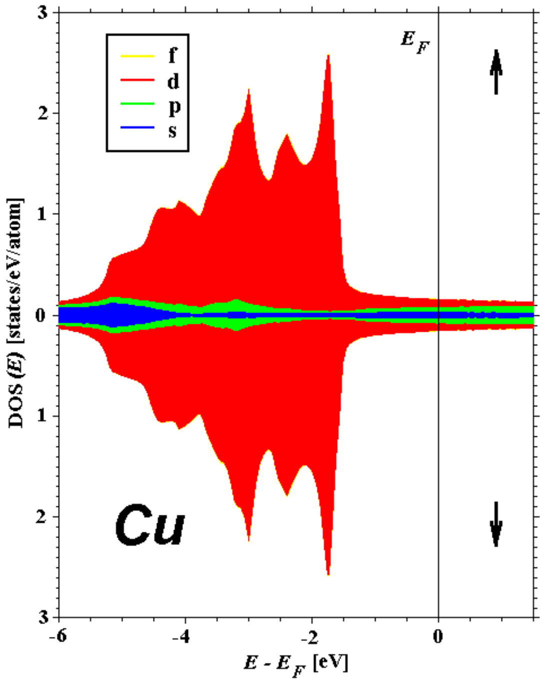
Majority electrons

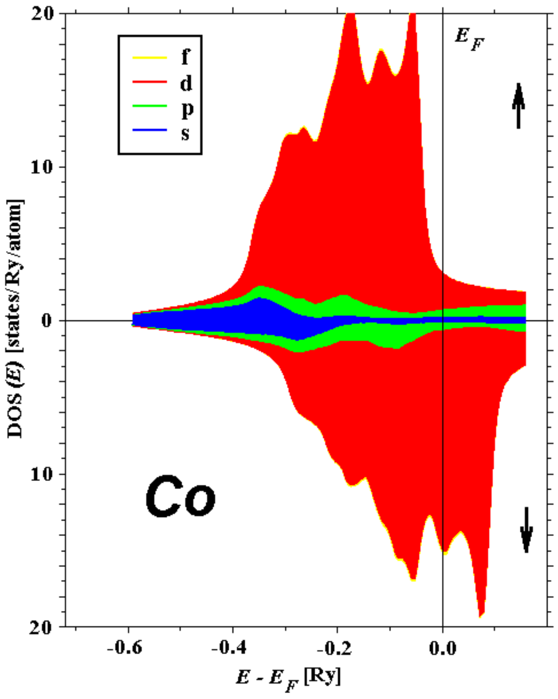
E

D^-

Minority electrons

E_F





STONER model

Number of electrons:

$$N = \int^{E_F} dE\{D_0(E + IM/2) + D_0(E - IM/2)\}$$

Magnetic moment:

$$M = \int^{E_F} dE\{D_0(E + IM/2) - D_0(E - IM/2)\}$$

Fixed: $N, D_0(E)$ To be determined: E_F, M

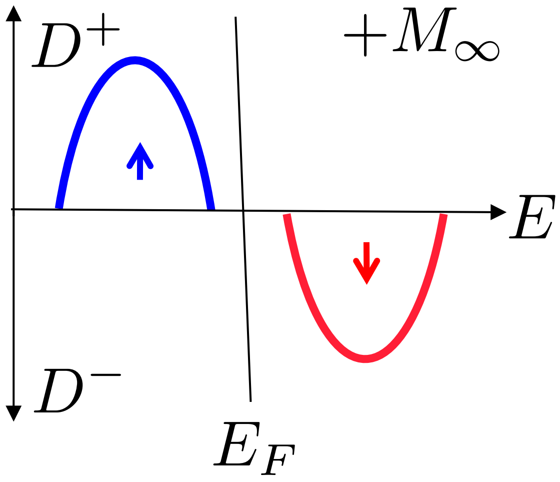
$$F(M) = \int^{E_F(M)} dE\{D_0(E + IM/2) - D_0(E - IM/2)\}$$

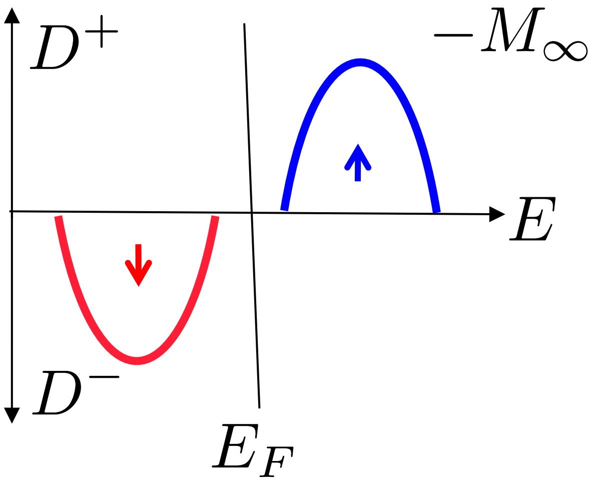
Self-consistent solution

STONER model

Properties of F(M):

- $F(0) = 0$
- $F(-M) = -F(M)$ bzw. $E_F(-M) = E_F(M)$
- $F(\pm\infty) = \pm M_\infty$ and $-M_\infty \leq F(M) \leq M_\infty$
- $F'(M) \geq 1$ monotonically increasing





STONER model

$$F(M) = \int^{E_F(M)} dE\{D_0(E + IM/2) - D_0(E - IM/2)\}$$

$$\frac{dF}{dM} = \int^{E_F(M)} dE[\frac{d}{dM}\{D_0(E + IM/2) - D_0(E - IM/2)\} \\ +\{D_0(E + IM/2) - D_0(E - IM/2)\}\frac{dE_F}{dM}]$$

$$F'(M) = \int^{E_F(M)} dE[\{D_0(E + IM/2) + D_0(E - IM/2)\} \\ +\{D_0(E + IM/2) - D_0(E - IM/2)\}\frac{dE_F}{dM}]$$

STONER model

Calculation of $\frac{dE_F}{dM}$ **from** $dN = 0$

$$dN = \frac{dN}{dE_F} dE_F + \frac{dN}{dM} dM = 0$$

$$N = \int^{E_F} dE \{ D_0(E + IM/2) + D_0(E - IM/2) \}$$

$$0 = (D_0^+ + D_0^-) dE_F + \frac{I}{2} (D_0^+ - D_0^-) dM$$

$$\frac{dE_F}{dM} = \frac{I}{2} \frac{(D_0^+ - D_0^-)}{(D_0^+ + D_0^-)}$$

STONER model

$$F'(M) = \int^{E_F(M)} dE[\{D_0(E + IM/2) + D_0(E - IM/2)\}$$
$$+\{D_0(E + IM/2) - D_0(E - IM/2)\}\frac{dE_F}{dM}]$$

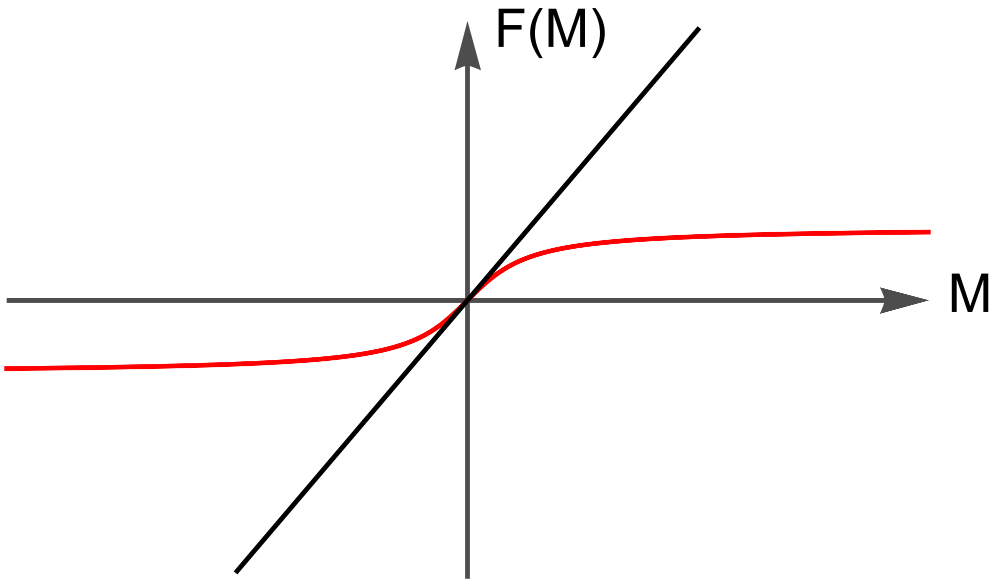
$$\frac{dE_F}{dM} = \frac{I}{2}\frac{(D_0^+ - D_0^-)}{(D_0^+ + D_0^-)}$$

$$F'(M) = \frac{I}{2}(D_0^+ + D_0^-)\{1 - \frac{(D_0^+ - D_0^-)^2}{(D_0^+ + D_0^-)^2}\} \geq 0$$

STONER model

Paramagnetic solution:

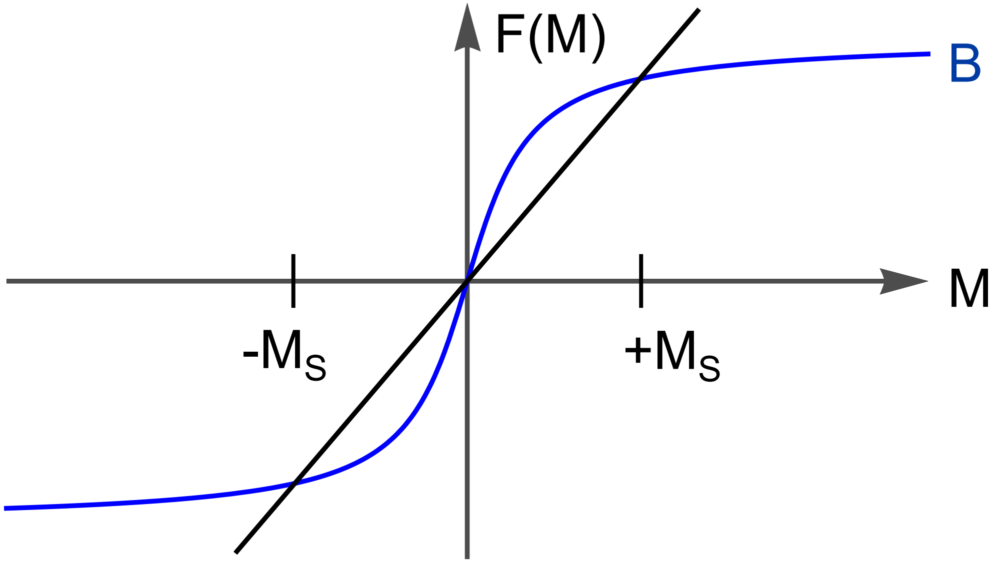
- trivial solution M=0



STONER model

Ferromagnetic solution:

- trivial solution M=0
- two solutions with spontaneous magnetization $\pm M_S$



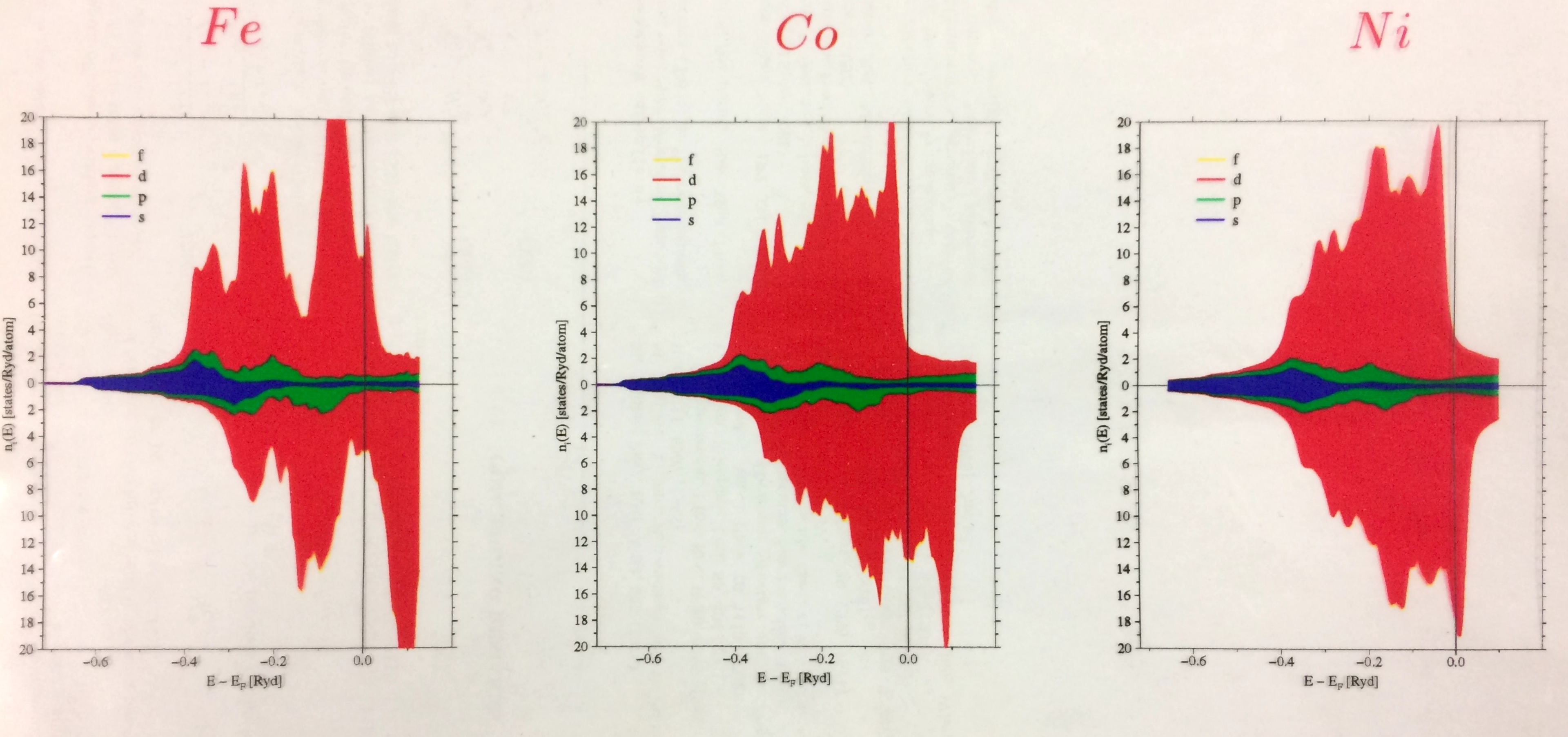
STONER criterion: $F'(0) = ID_0(E_F) > 1$

STONER model

STONER criterion: $F'(0) = ID_0(E_F) > 1$

	$D_0(E_F)\ [eV^{-1}]$	$I\ [eV]$	$ID_0(E_F)$	$M\ [\mu_B/atom]$
Na	0.23	1.82	0.41	
Al	0.21	1.22	0.25	
Cr	0.35	0.76	0.27	
Mn	0.77	0.82	0.63	
Fe	1.54	0.93	1.434	2.22
Co	1.72	0.99	1.70	1.71
Ni	2.02	1.01	2.04	0.61
Cu	0.14	0.73	0.11	
Pd	1.14	0.68	0.78	
Pt	0.79	0.63	0.5	

Density of states for bulk ferromagnets



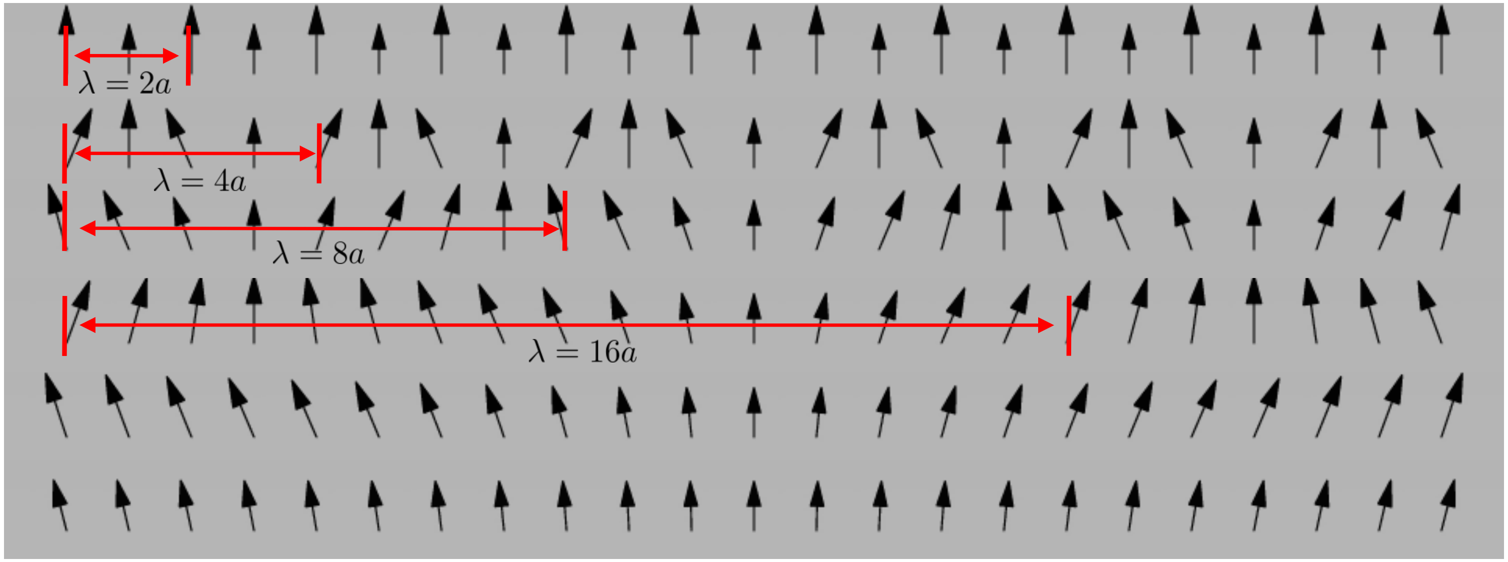
HEISENBERG model

Magnons and second quantization

$$H = -J \sum_{\langle ij \rangle} \boldsymbol{s}_i \cdot \boldsymbol{s}_j$$

$k = \dfrac{\pi}{a}$

$\lambda = 2a$



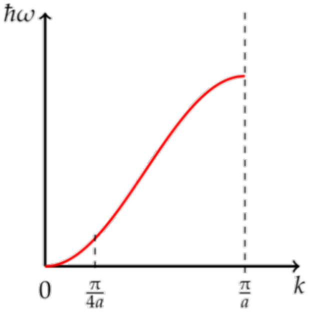
$k \to 0$

$\lambda \to \infty$

Dispersion relation of spin waves in ferromagnets:

$\varepsilon(\boldsymbol{k}) = 2Js\,(1 - \cos(ka))$ (only one basis atom)

n basis atoms lead to n magnon branches $\varepsilon_i(\boldsymbol{k})$.



Magnons in second quantization

Hamiltonian: $H = -J \sum_{\langle ij \rangle} \boldsymbol{s}_i \cdot \boldsymbol{s}_j = -J \sum_{\langle ij \rangle} \left[s_i^z s_j^z + \frac{1}{2} \left(s_i^- s_j^+ + s_i^+ s_j^- \right) \right]$

$$s_i^{\pm} = s_i^x \pm \mathrm{i} s_i^y$$

$$\left[s_i^z, s_j^{\pm}\right] = \pm s_i^{\pm} \delta_{i,j}, \quad \left[s_i^+, s_j^-\right] = 2 s_i^z \delta_{i,j}$$

lowers *z* component raises *z* component

Bosonization: $|0\rangle$ is the ground state (magnon vacuum); analyze **small** fluctuations

Holstein-Primakoff transformation

$$s_i^- = 2 s a_i^\dagger \sqrt{1 - \frac{a_i^\dagger a_i}{2s}}$$

$$s_i^+ = \sqrt{1 - \frac{a_i^\dagger a_i}{2s}}\, 2 s a_i$$

$$s_i^z = s - a_i^\dagger a_i$$

$$\left[a_i, a_j^\dagger\right] = \delta_{i,j}, \quad \left[a_i, a_j\right] = \left[a_i^\dagger, a_j^\dagger\right] = 0$$

"cold large spins"

$a_i^\dagger a_i / (2s) \ll 1$

Semiclassic approximation (linear spin-wave theory)

$s_i^- = 2 s a_i^\dagger$ creates a boson

$s_i^+ = 2 s a_i$ destroys a boson

$s_i^z = s - a_i^\dagger a_i$ counts bosons

Plug this into Hamiltonian and ignore non-bilinear terms.

Magnons in second quantization

Free-boson Hamiltonian: $H = -JNs^2 - Js \sum_{\langle ij \rangle} \left(a_j^\dagger - a_i^\dagger \right) (a_j - a_i)$

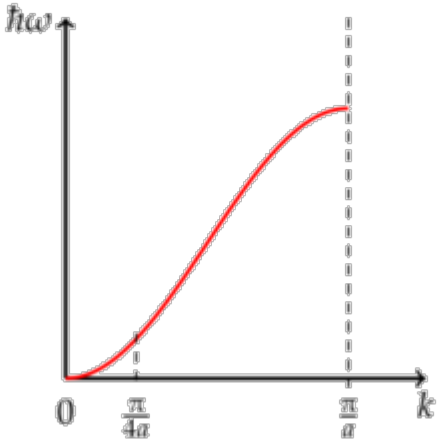
Fourier transformation: $a_i = \frac{1}{\sqrt{N}} \sum_{\boldsymbol{k}} \exp(-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{r}_i) a_{\boldsymbol{k}}$

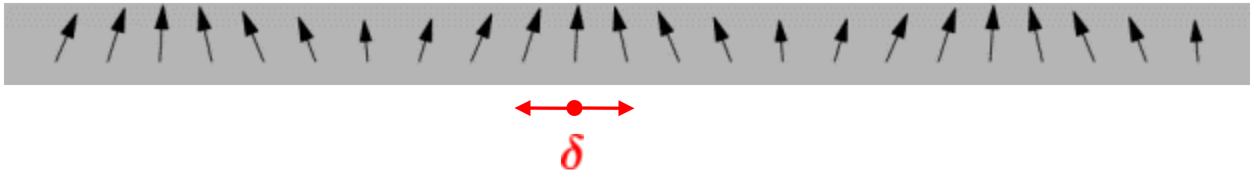
$a_{\boldsymbol{k}}^\dagger$ creates a magnon

$a_{\boldsymbol{k}}$ destroys a magnon

k-space Hamiltonian: $H = \sum_{\boldsymbol{k}} \varepsilon_n(\boldsymbol{k}) a_{\boldsymbol{k}}^\dagger a_{\boldsymbol{k}}$

energy $\varepsilon_n(\boldsymbol{k}) = s(J(0) - J(\boldsymbol{k}))$ with $J(\boldsymbol{k}) = J \sum_{\boldsymbol{\delta}} \exp(-\mathrm{i}\boldsymbol{k} \cdot \boldsymbol{\delta})$





$\varepsilon(\boldsymbol{k}) = 2Js\,(1 - \cos(ka))$ (only one basis atom)

n basis atoms lead to n magnon branches $\varepsilon_n(\boldsymbol{k})$.

Topological states and magnetism

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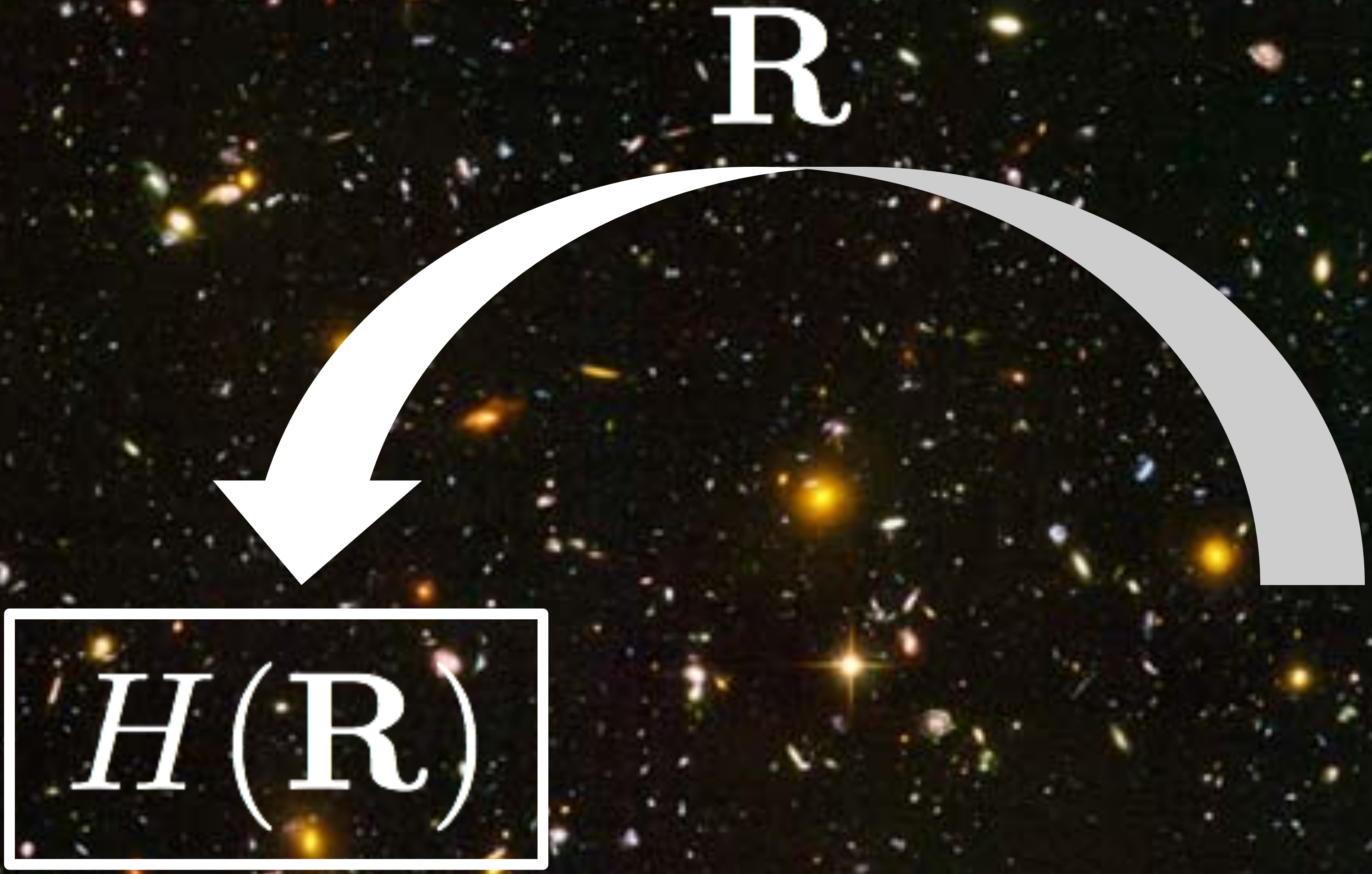
Outline

- Introduction
- Topological electron states
- The quantum Hall effects
- The topological Hall effect
- Summary

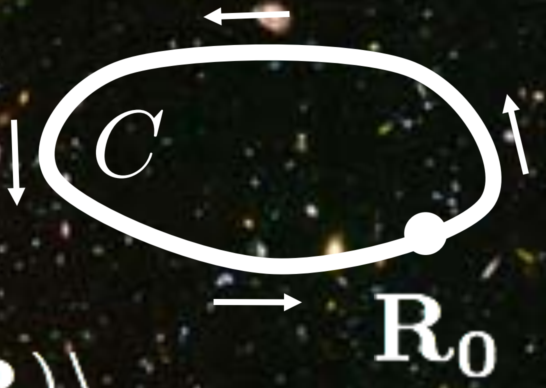
What is a Berry phase?



$\mathbf{R}$

$$\boxed{H(\mathbf{R})}$$

Schrödinger equation and adiabatic evolution



$$H(\mathbf{R})|\varphi_n(\mathbf{R})\rangle = E_n(\mathbf{R})|\varphi_n(\mathbf{R})\rangle$$

$$|\varphi_n(\mathbf{R}_0)\rangle = \boxed{\exp(i\gamma_n(C))}\,|\varphi_n(\mathbf{R}_0)\rangle$$

M. V. Berry, Proc. R. Soc. A **392**, 1802 (1984)

What is a Berry curvature?

Berry phase:

$$\gamma_n(C) = i \oint_c dR \langle \varphi_n(\mathbf{R}) | \nabla_{\mathbf{R}} | \varphi_n(\mathbf{R}) \rangle$$

Berry connection:

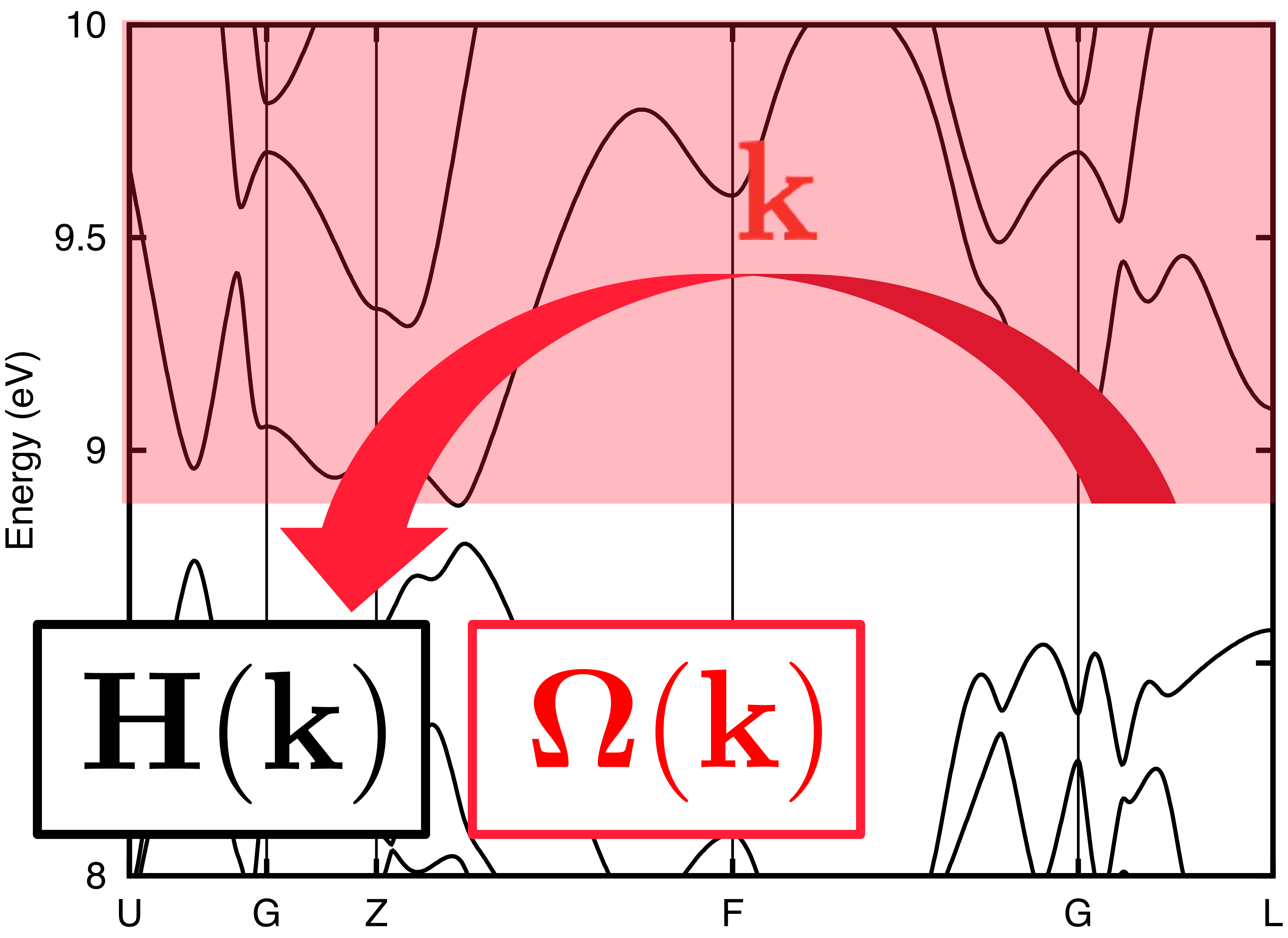
$$\mathbf{A}_n(\mathbf{R}) = i \langle \varphi_n(\mathbf{R}) | \nabla_{\mathbf{R}} | \varphi_n(\mathbf{R}) \rangle$$

Berry curvature:

$$\begin{aligned} \mathbf{\Omega}_n(\mathbf{R}) &= \nabla \times \mathbf{A}_n(\mathbf{R}) \\ &= i \langle \nabla_{\mathbf{R}} \varphi_n(\mathbf{R}) | \times | \nabla_{\mathbf{R}} \varphi_n(\mathbf{R}) \rangle \end{aligned}$$

M. V. Berry, Proc. R. Soc. A **392**, 1802 (1984)

Berry curvature of Bloch states



Semiclassical equation of motion

Change of momentum:

$$\hbar\dot{\mathbf{k}} = -e\mathbf{E} \qquad - \; e\dot{\mathbf{r}} \times \mathbf{B}(\mathbf{r})$$

Lorentz force

Change of position:

$$\dot{\mathbf{r}} \; = \frac{\partial E_n(\mathbf{k})}{\hbar\partial\mathbf{k}} \boxed{- \; \dot{\mathbf{k}} \times \boldsymbol{\Omega}_{\mathbf{n}}(\mathbf{k})}$$

Anomalous velocity

M.-C. Chang and Q. Niu, Phys. Rev. B **53**, 7010 (1996)

Transversal transport coefficients

Ohm’s law and conductivity tensor

$$\mathbf{j} = \hat{\sigma} \cdot \mathbf{E} = \begin{pmatrix} \sigma_{xx} & \enclose{circle}{\sigma_{xy}} & 0 \\ \enclose{circle}{\sigma_{yx}} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \mathbf{E}$$

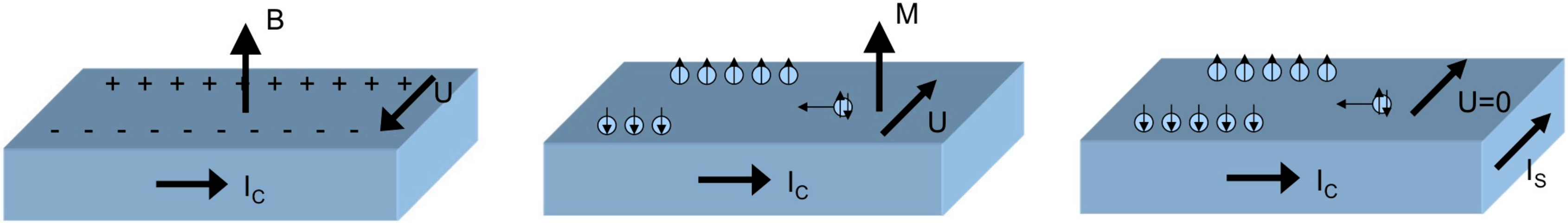
The Hall trio

The Hall trio

Hall effect
1879

Anomalous Hall effect
1881

Spin Hall effect
2004



Lorentz force

Berry curvature

$$\mathbf{\Omega}_n(\mathbf{k})$$

spin-orbit interaction:

$$\mathbf{s} \cdot \mathbf{L}$$

Nagaosa, Sinova et al., Rev. Mod. Phys. **82**, 1539 (2010)

Intrinsic anomalous and spin Hall conductivity

$$\sigma_{xy}^{\pm} = \frac{e^2}{\hbar(2\pi)^3} \sum_n \int_{BZ} d^3k f_n(\mathbf{k}) \Omega_z^n(\mathbf{k})$$

$$\sigma_{xy}^{\pm} = \frac{e^2}{\hbar(2\pi)^3} \sum_n \int^{E_F} dE \Omega_z^n(E)$$

Anomalous Hall effect:

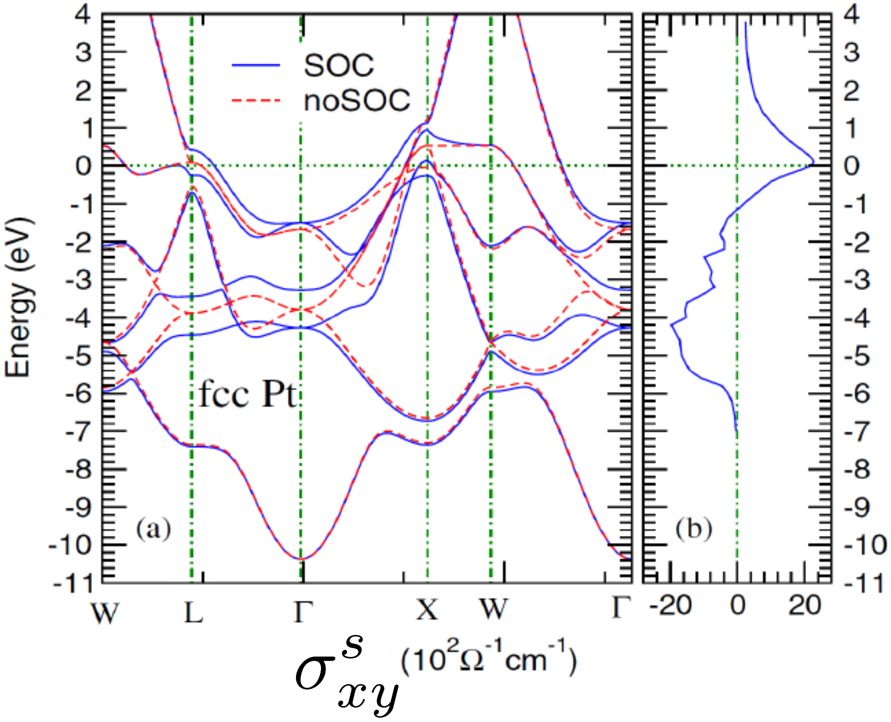
$$\sigma_{xy} = \sigma_{xy}^{+} + \sigma_{xy}^{-}$$

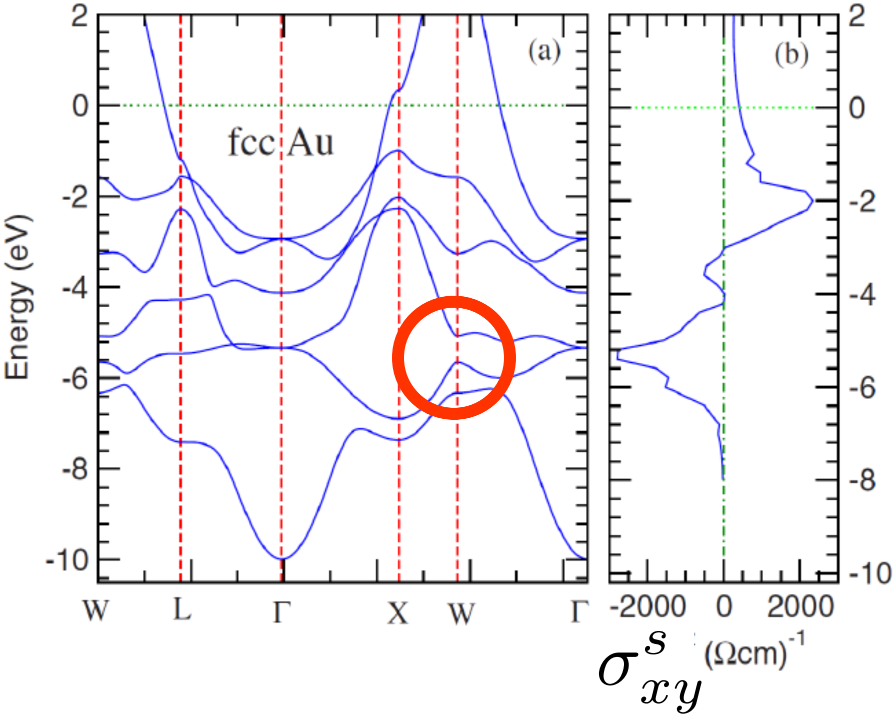
Spin Hall effect:

$$\sigma_{xy}^{s} = \sigma_{xy}^{+} - \sigma_{xy}^{-}$$

Intrinsic spin Hall conductivity

$$\sigma^s_{xy} = \frac{e^2}{\hbar(2\pi)^3}\int^{E_F} dE\,\Omega_z(E)$$





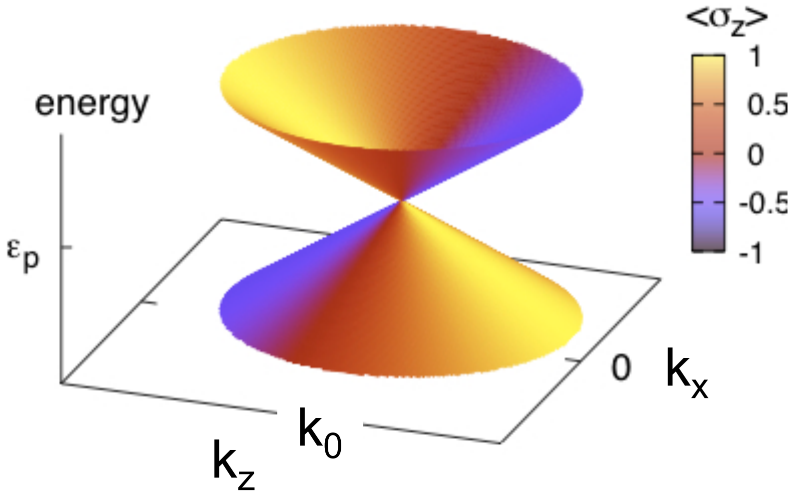
Pt: **2000** $\frac{\hbar}{e}(\Omega\ cm)^{-1}$

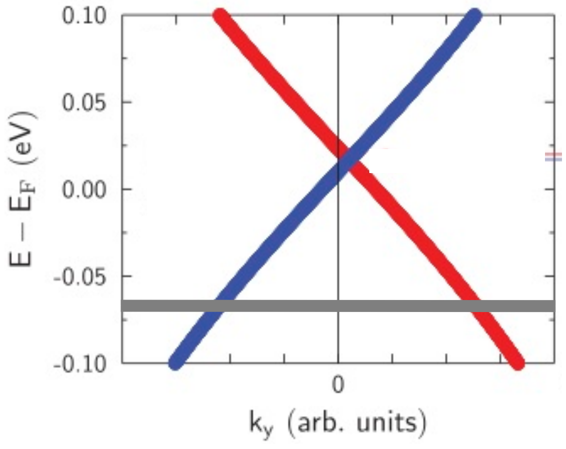
Au: **400** $\frac{\hbar}{e}(\Omega\ cm)^{-1}$

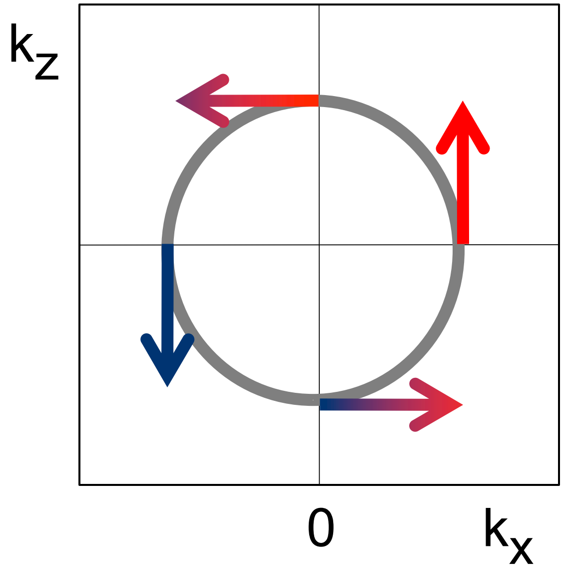
Guo et al., PRL **100**, 096401 (2008); J. Appl. Phys. **105**, 07C701 (2009)

Diabolic points

Band crossing and diabolic points





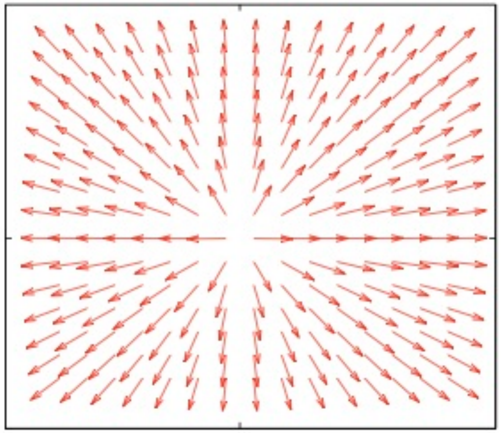


J. von Neumann, E. Wigner, Phys. Z. 1928

Point charge and magnetic monopole

Point charge field:

$$\mathbf{E}_{\pm}(\mathbf{r}) = \pm Q \frac{\mathbf{r}-\mathbf{r_0}}{|\mathbf{r}-\mathbf{r_0}|^3}$$



Magnetic monopole:

$$\mathbf{B}_{\pm}(\mathbf{r}) = \pm g \frac{\mathbf{r}-\mathbf{r_0}}{|\mathbf{r}-\mathbf{r_0}|^3}$$

Berry curvature monopole:

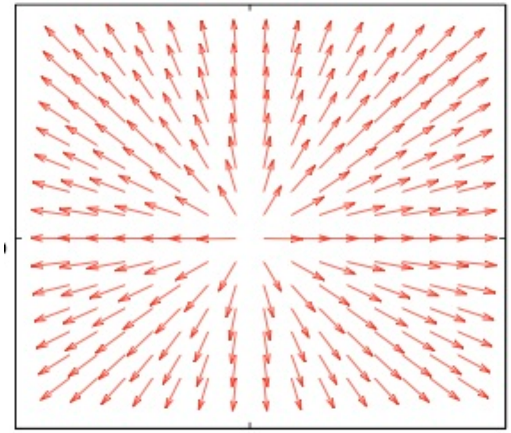
$$\boldsymbol{\Omega}_{\pm}(\mathbf{k}) = \pm g \frac{\mathbf{k}-\mathbf{k_0}}{|\mathbf{k}-\mathbf{k_0}|^3}$$

P.A.M. Dirac, Phys. Rev. 1948

Dirac quantization

Monopole field:

$$\mathbf{B}_{\pm}(\mathbf{r}) = \pm g \frac{\mathbf{r}-\mathbf{r_0}}{|\mathbf{r}-\mathbf{r_0}|^{\mathbf{3}}}$$



Dirac‘s quantization of the monopole field:

$$\mathbf{B}(\mathbf{r}) = \nabla \times \mathbf{A}(\mathbf{r}) + \sum_j g_j \frac{\mathbf{r}-\mathbf{r_j}}{|\mathbf{r}-\mathbf{r_j}|^{\mathbf{3}}}$$

$$g_j = \pm\frac{1}{2}$$

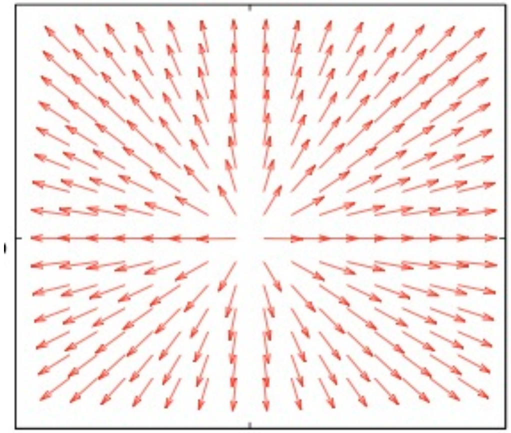
$$\frac{1}{2\pi}\int_V d\mathbf{r}\; \nabla\cdot\mathbf{B}(\mathbf{r}) \;=\; \frac{1}{2\pi}\int_{\partial V} d\sigma\; \mathbf{n}\cdot\mathbf{B}(\mathbf{r}) \;=\; C \quad C \in \mathbf{Z}$$

P.A.M. Dirac, Phys. Rev. 1948

Berry curvature monopoles

Monopole field:

$$\mathbf{\Omega}_{\pm}(\mathbf{k}) = \pm\frac{1}{2}\frac{\mathbf{k}-\mathbf{k_0}}{|\mathbf{k}-\mathbf{k_0}|^{\mathbf{3}}}$$



Dirac‘s quantization of the monopole field:

$$\mathbf{\Omega}(\mathbf{k}) = \nabla\times\mathbf{A}(\mathbf{k}) + \sum_j g_j \frac{\mathbf{k}-\mathbf{k_j}}{|\mathbf{k}-\mathbf{k_j}|^{\mathbf{3}}}$$

$$g_j = \pm\frac{1}{2}$$

$$\frac{1}{2\pi}\int_V d\mathbf{k}\,\nabla\cdot\mathbf{\Omega}(\mathbf{k}) = \frac{1}{2\pi}\int_{\partial V} d\sigma\,\mathbf{n}\cdot\mathbf{\Omega}(\mathbf{k}) = C \quad C\in\mathbf{Z}$$

P.A.M. Dirac, Phys. Rev. 1948

Börge Göbel

Tomáš Rauch

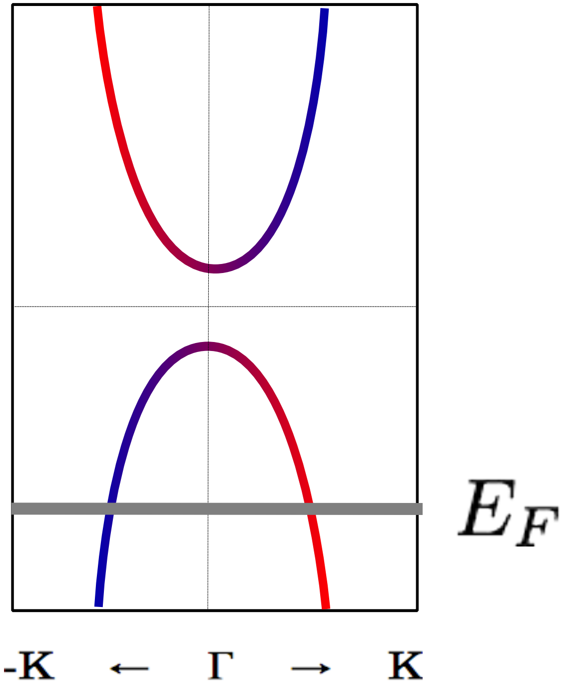
Alexander Mook

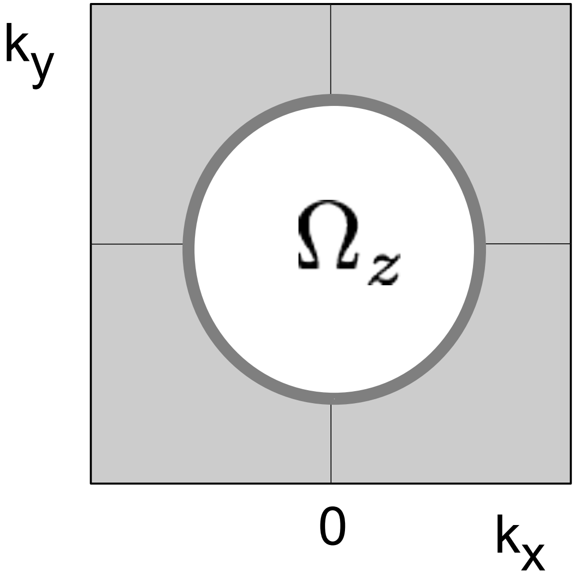
Jürgen Henk

Topological states

Intrinsic spin Hall conductivity

$$\sigma^{s}_{xy} = \frac{e^2}{\hbar(2\pi)^3}\int^{E_F} dE\,\Omega_z(E)$$

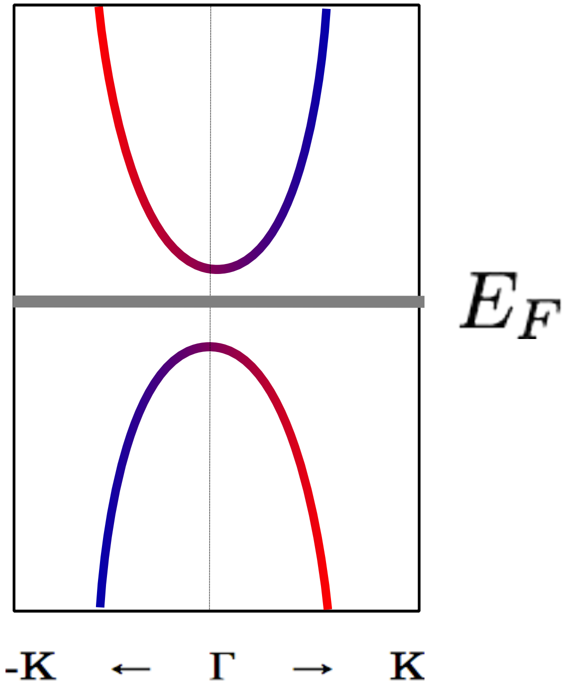


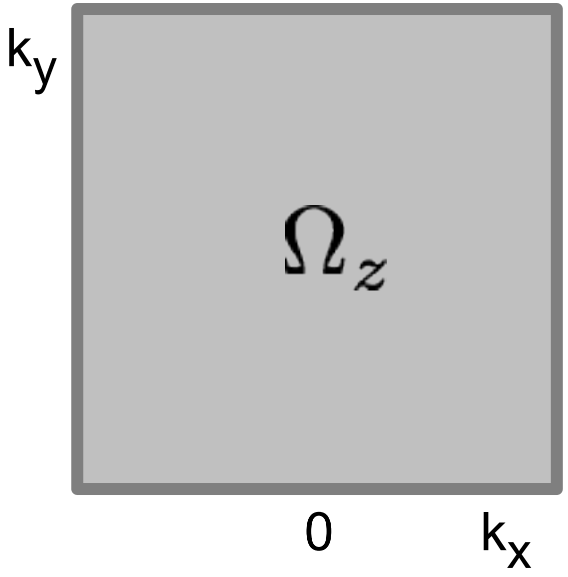


Spin Hall effect of an insulator and Chern number

$$\sigma_{xy}^{s} = \frac{e^2}{\hbar} C$$

$$C = \int_{BZ} d\mathbf{k}\, \Omega_z(\mathbf{k})$$

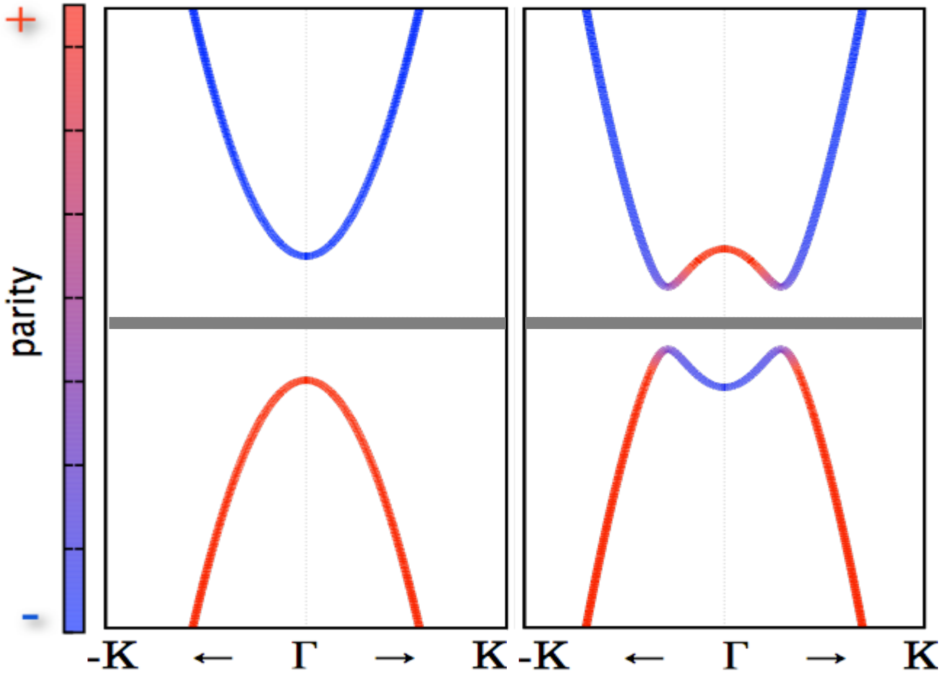




Chern number

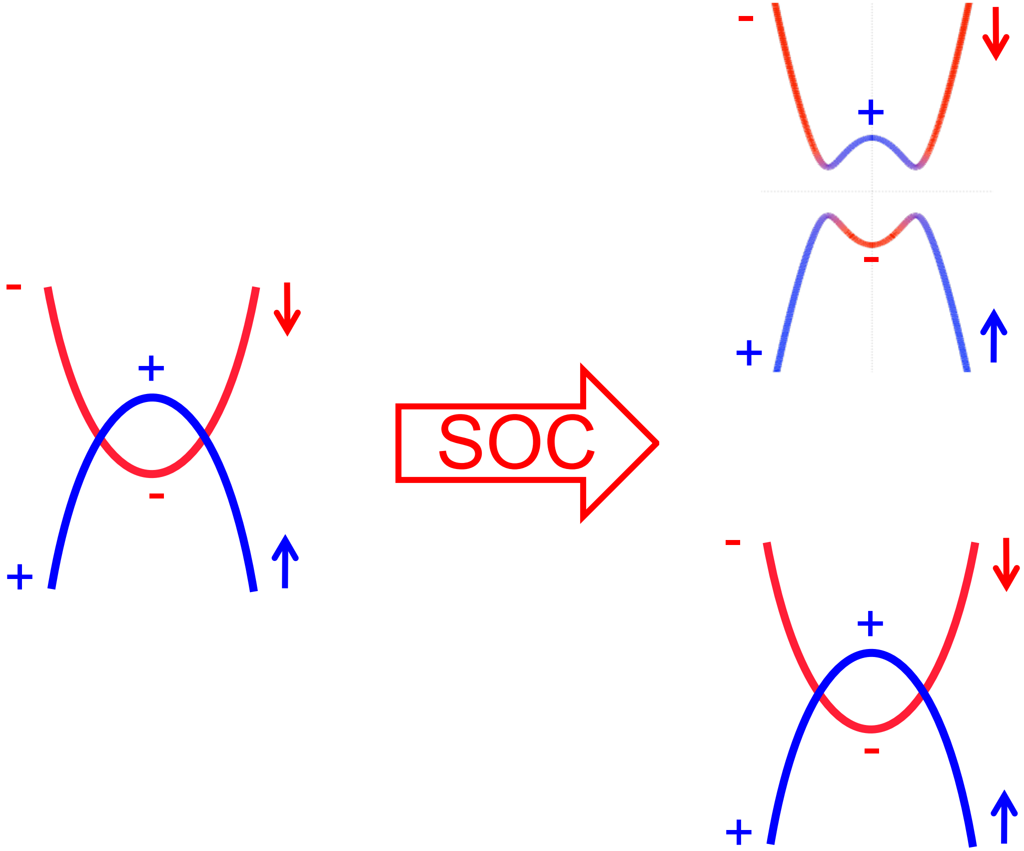
$$C = \int_{BZ} d\mathbf{k}\, \Omega_z(\mathbf{k})$$

$$C = 0$$



$$C = m$$

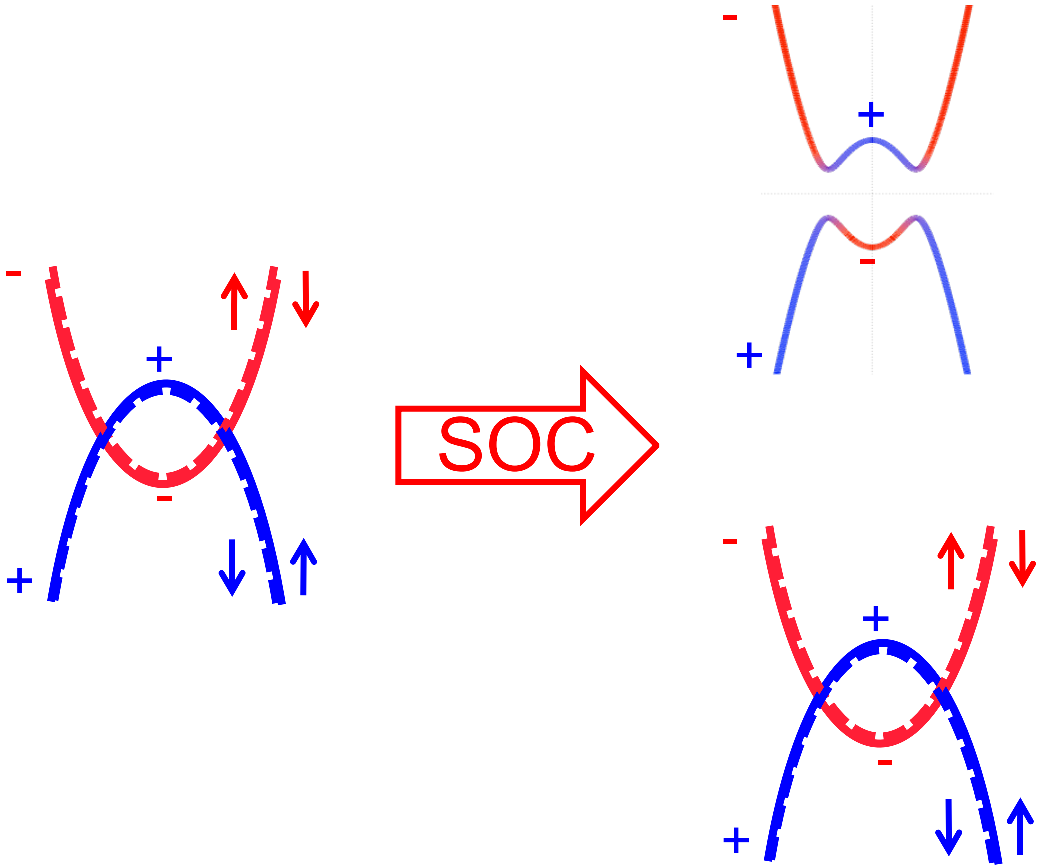
Band inversion without TRS



CHERN insulator:
Gap in 2d

WEYL semimetal:
Gapless in 3d

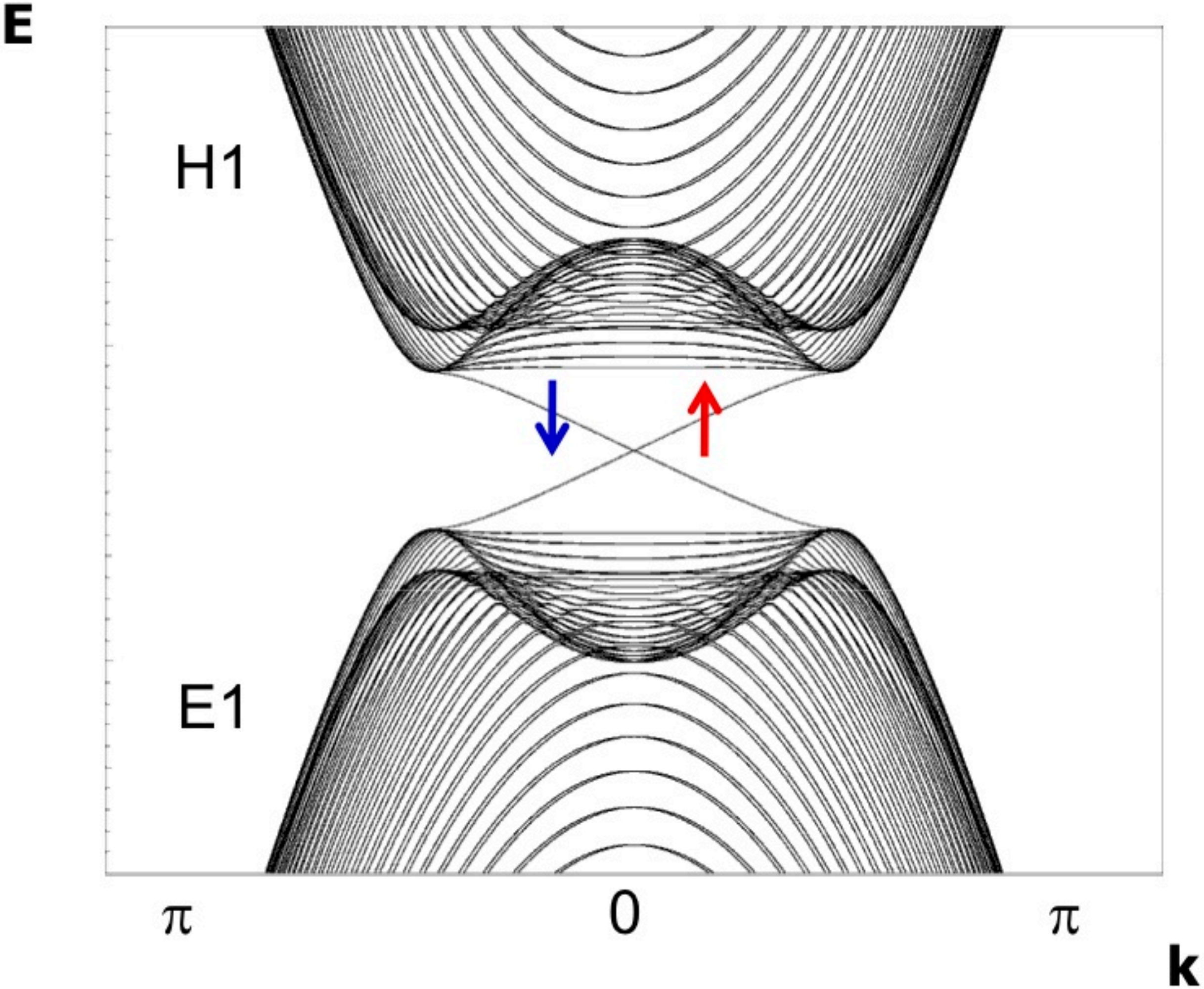
Band inversion with TRS



Z2 TI:
gap in 2d and 3d
Kramers
degeneracy

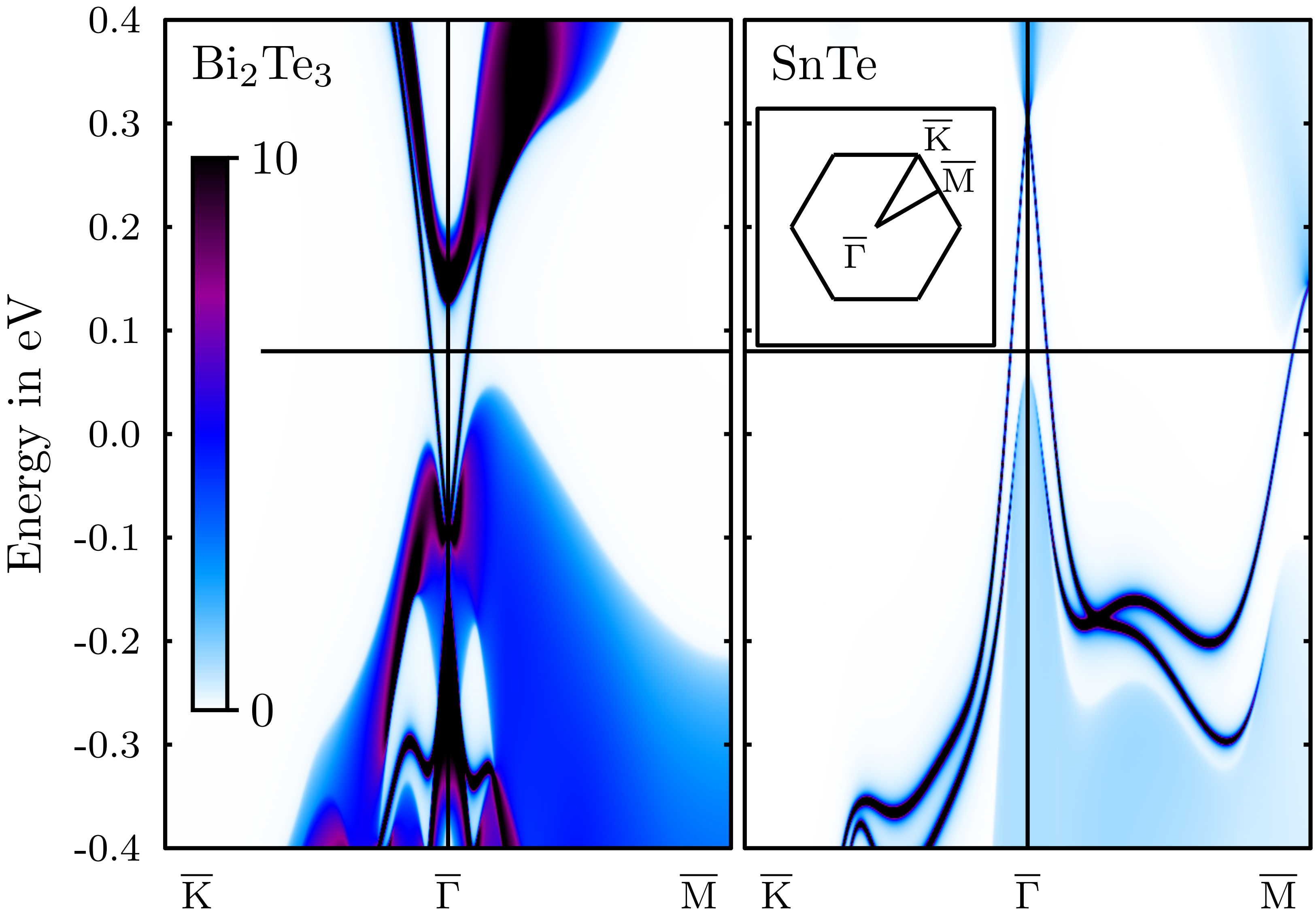
DIRAC semimetal:
no gap in 3d
+ crystal symmetry

Topological surface state of a Z2 TI



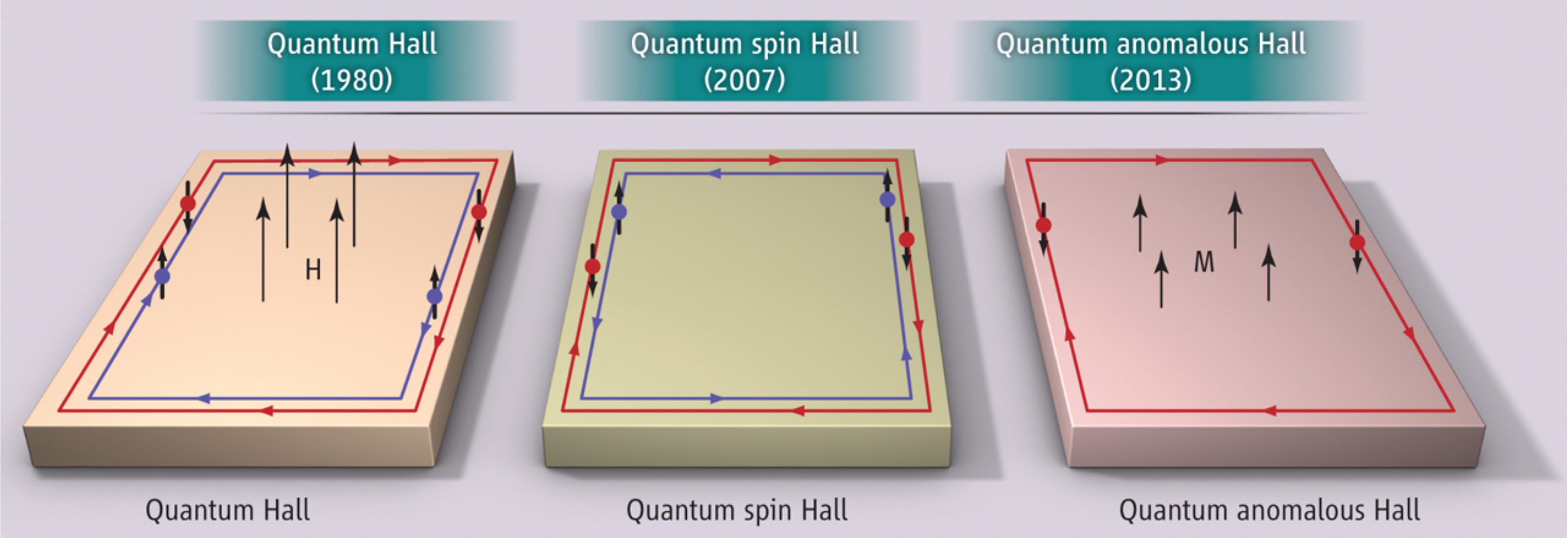
B. A. Bernevig, T. L. Hughes, S. C. Zhang, Science **314**, 1757 (2006)

Topological surface state in Bi_2Te_3



T. Rauch, M. Flieger, J. Henk, and I. M., Phys. Rev. B **88**, 245120 (2013)

The quantum Hall trio



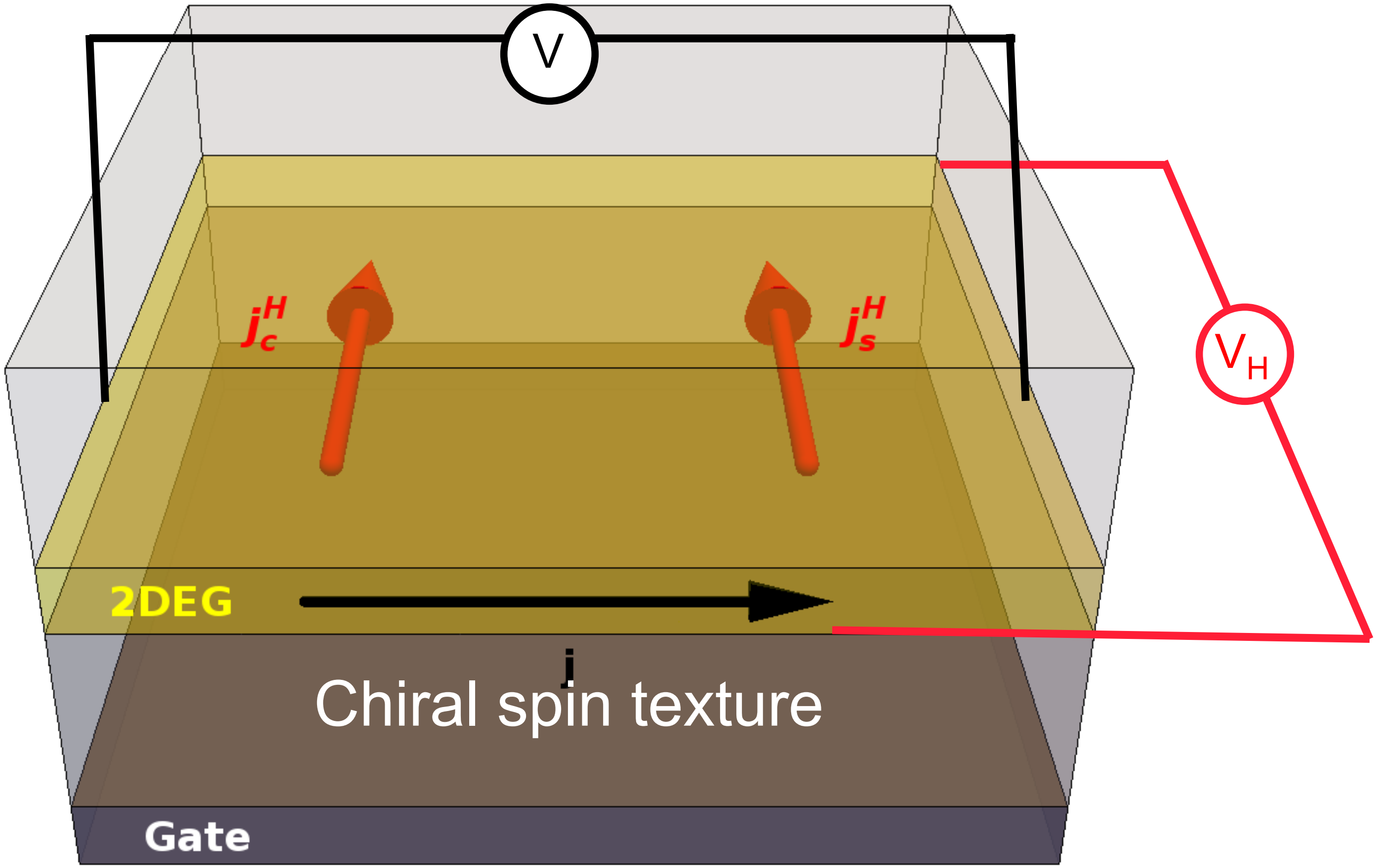
The conductance is quantized!

S Oh Science 2013;340:153-154

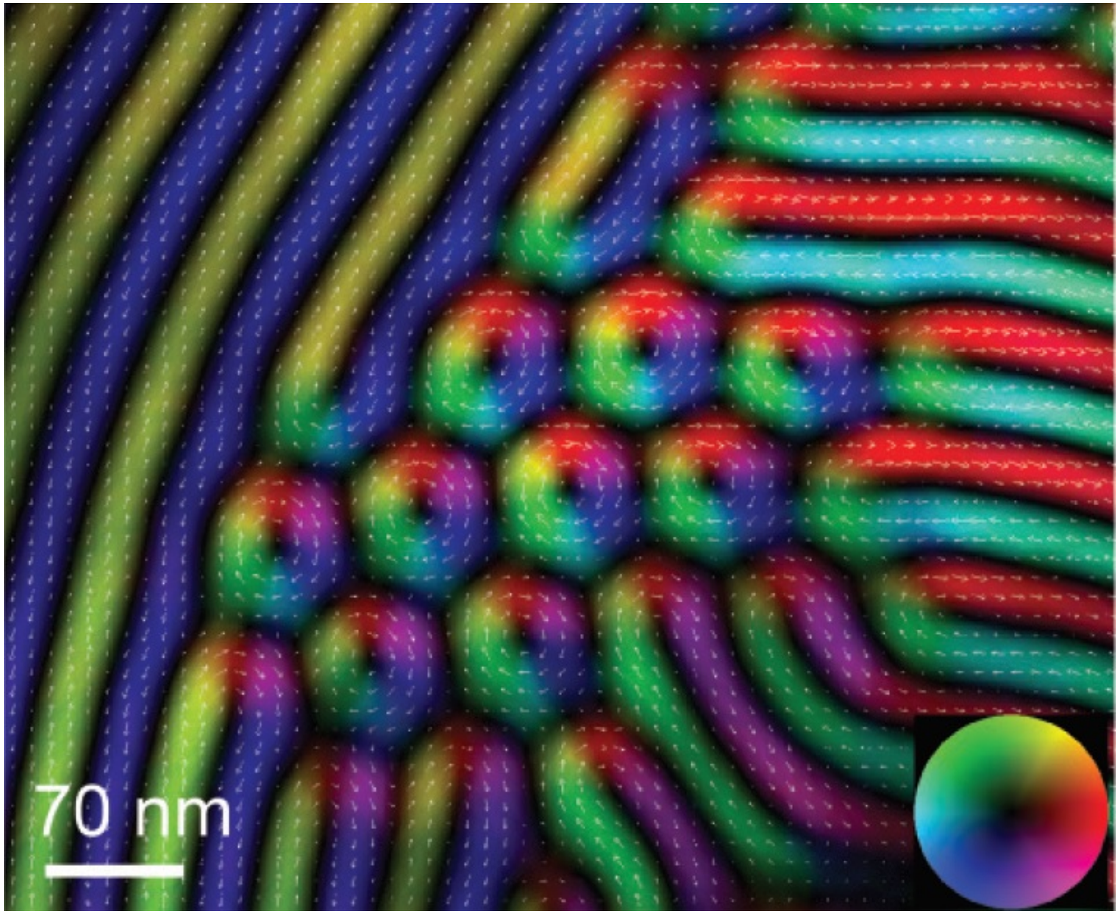
Topological Hall effect

B. Göbel, A. Mook, J. Henk, and I. M., Phys. Rev. B **95**, 094413 (2017)
B. Göbel, A. Mook, J. Henk, and I. M., New Journ. Phys., accepted (2017)

Experiment to measure the THE

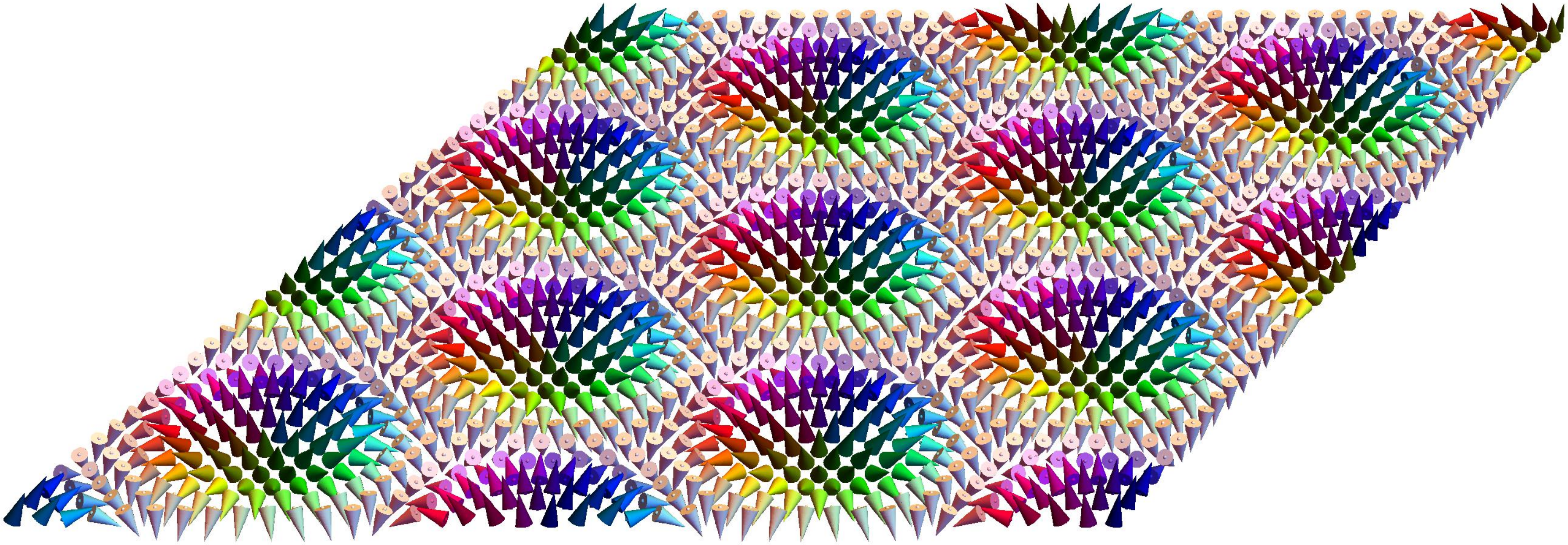


Skyrmions



M. Nagao et al., Experimental observation of multiple-q states for
the magnetic skyrmion lattice and skyrmion excitations under a zero magnetic field.
Phys. Rev. B 92, 140415 (2015)

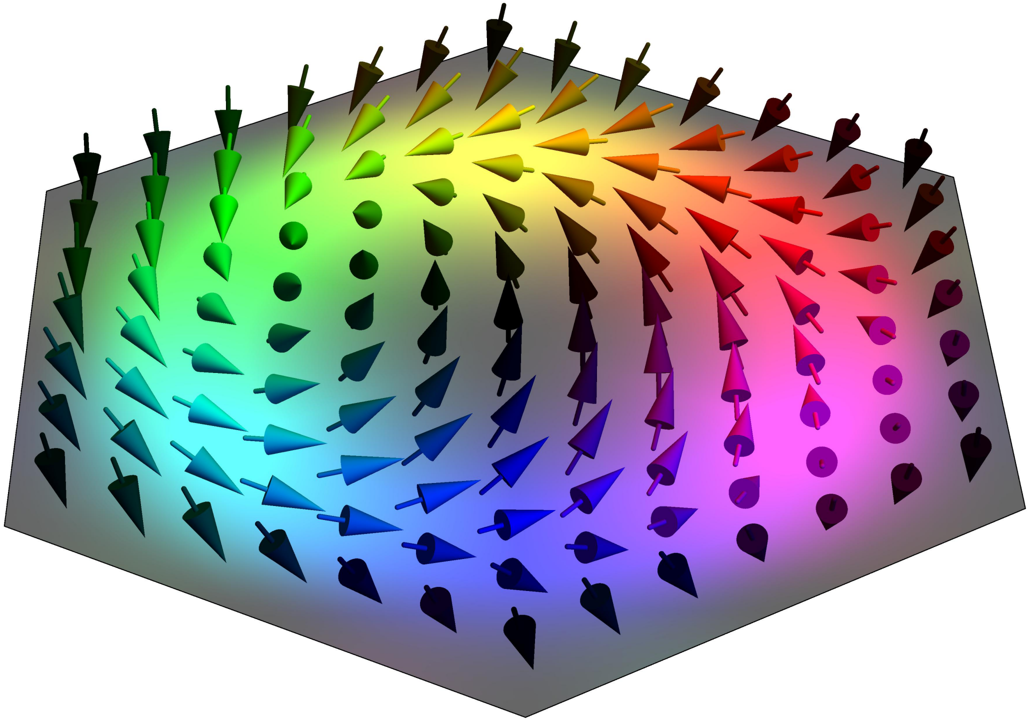
Skyrmion lattice



B. Göbel, A. Mook, J. Henk, and I.M., Phys. Rev. B **95**, 094413 (2017)

Skyrmion – background spin texture

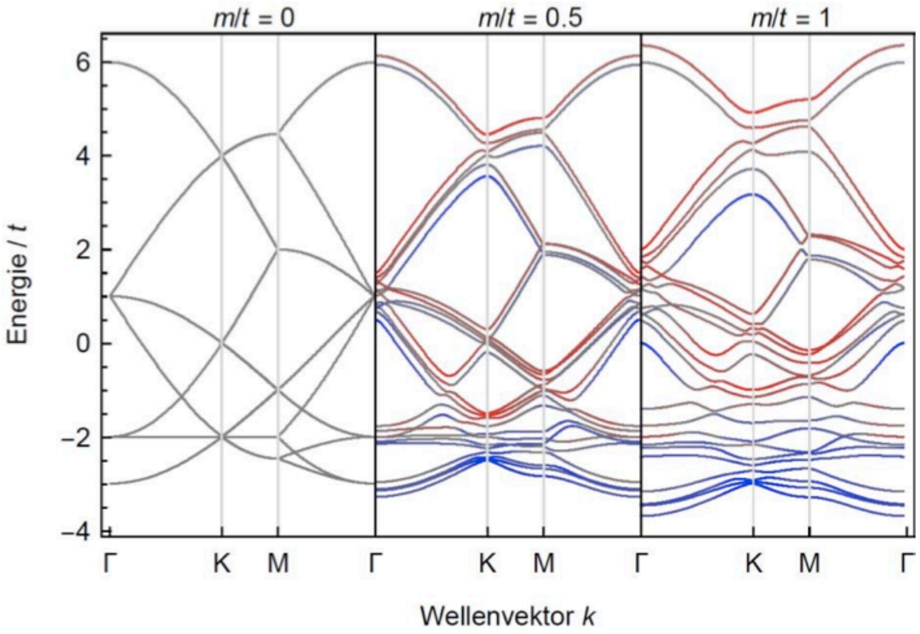
$$\mathbf{n}_i$$

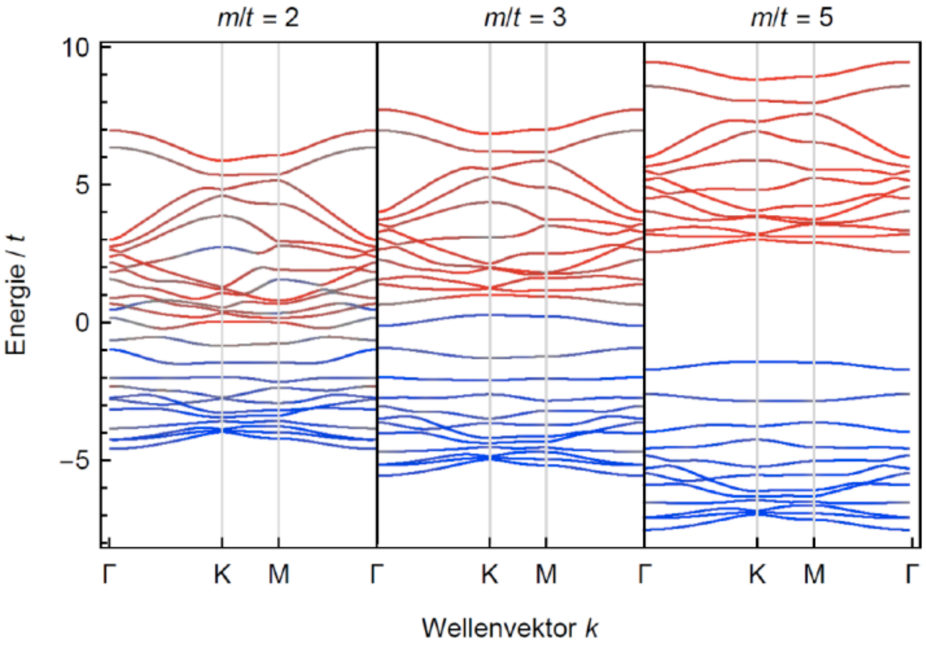


$$\hat{H} = \sum_{ij} t_{ij} c_i^{+} c_j - J \sum_i c_i^{+} \hat{\sigma} c_i \cdot \mathbf{n}_i$$

Electron bandstructure in background spin texture

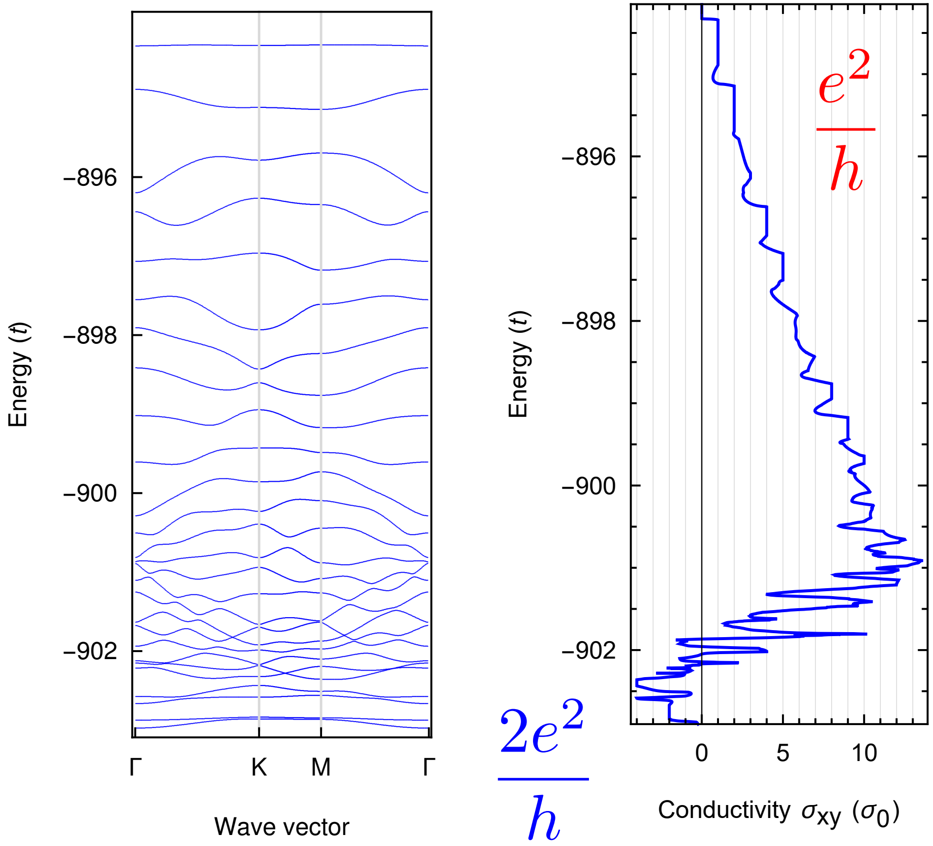
$$\hat{H} = \sum_{ij} t_{ij} c_i^+ c_j - J \sum_i c_i^+ \hat{\sigma} c_i \cdot \mathbf{n}_i$$





Electrons in the skyrmion field and THE

$$\sigma_{xy}^{\pm} = \boxed{\frac{e^2}{h}} \frac{1}{2\pi} \sum_n \int_{BZ} d^2k f_n(\mathbf{k}) \Omega_z^n(\mathbf{k})$$

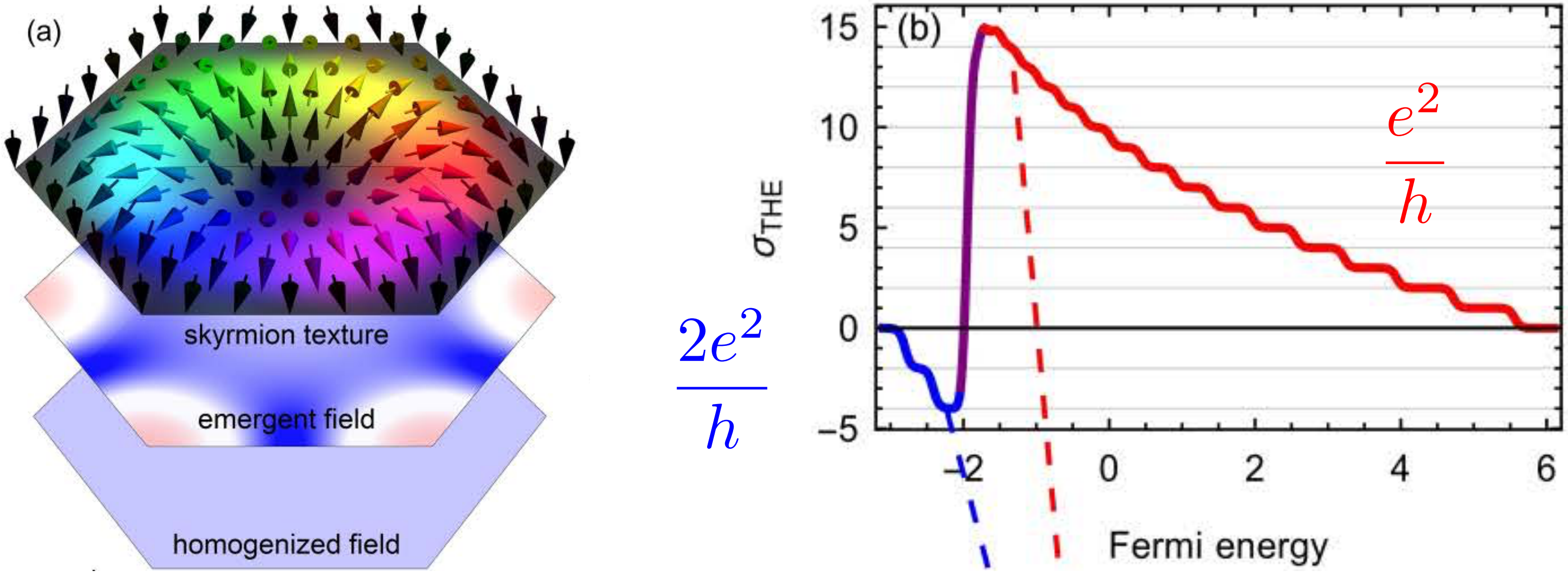


$\frac{e^2}{h}$

$\frac{2e^2}{h}$

THE from Berry curvature of the electrons

$$\sigma_{xy}^{\pm} = \frac{e^2}{h}\,\frac{1}{2\pi}\sum_n \int_{BZ} d^2k\, f_n(\mathbf{k})\Omega_z^n(\mathbf{k})$$

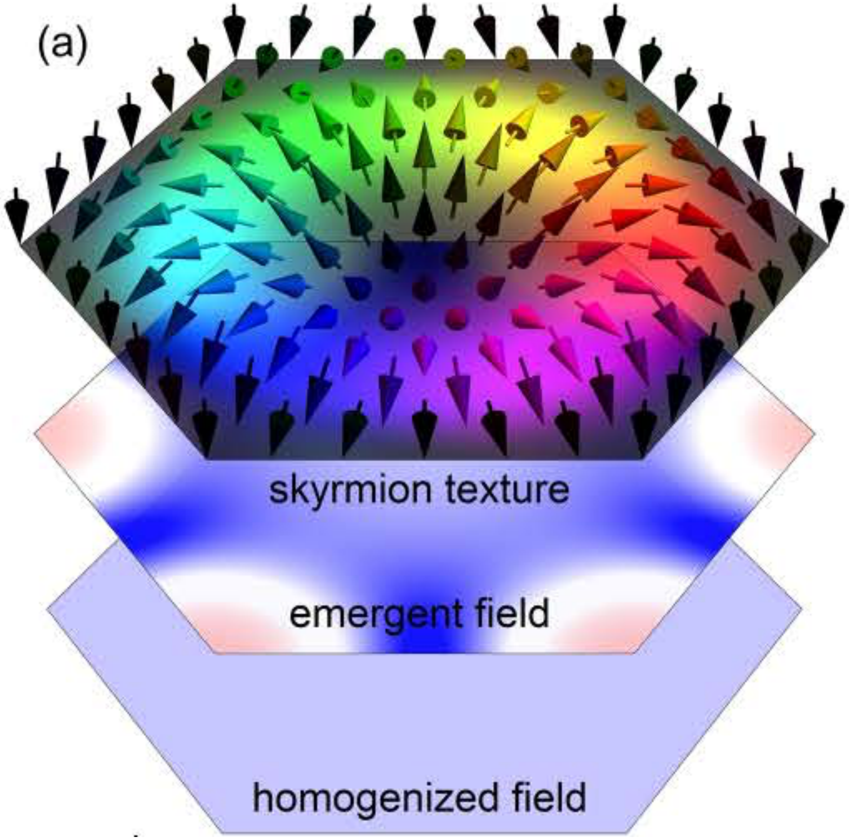


Börge Göbel, Alexander Mook, Jürgen Henk and Ingrid Mertig, Phys. Rev. B **95,** 094413 (2017)

From THE to QHE

Spin texture, skyrmion number and emergent field

$$N_{sk} = \frac{1}{4\pi} \int_{xy} d^2 r \; \mathbf{n}(\mathbf{r}) \cdot [\frac{\partial \mathbf{n}(\mathbf{r})}{\partial \mathbf{x}} \times \frac{\partial \mathbf{n}(\mathbf{r})}{\partial \mathbf{y}}]$$

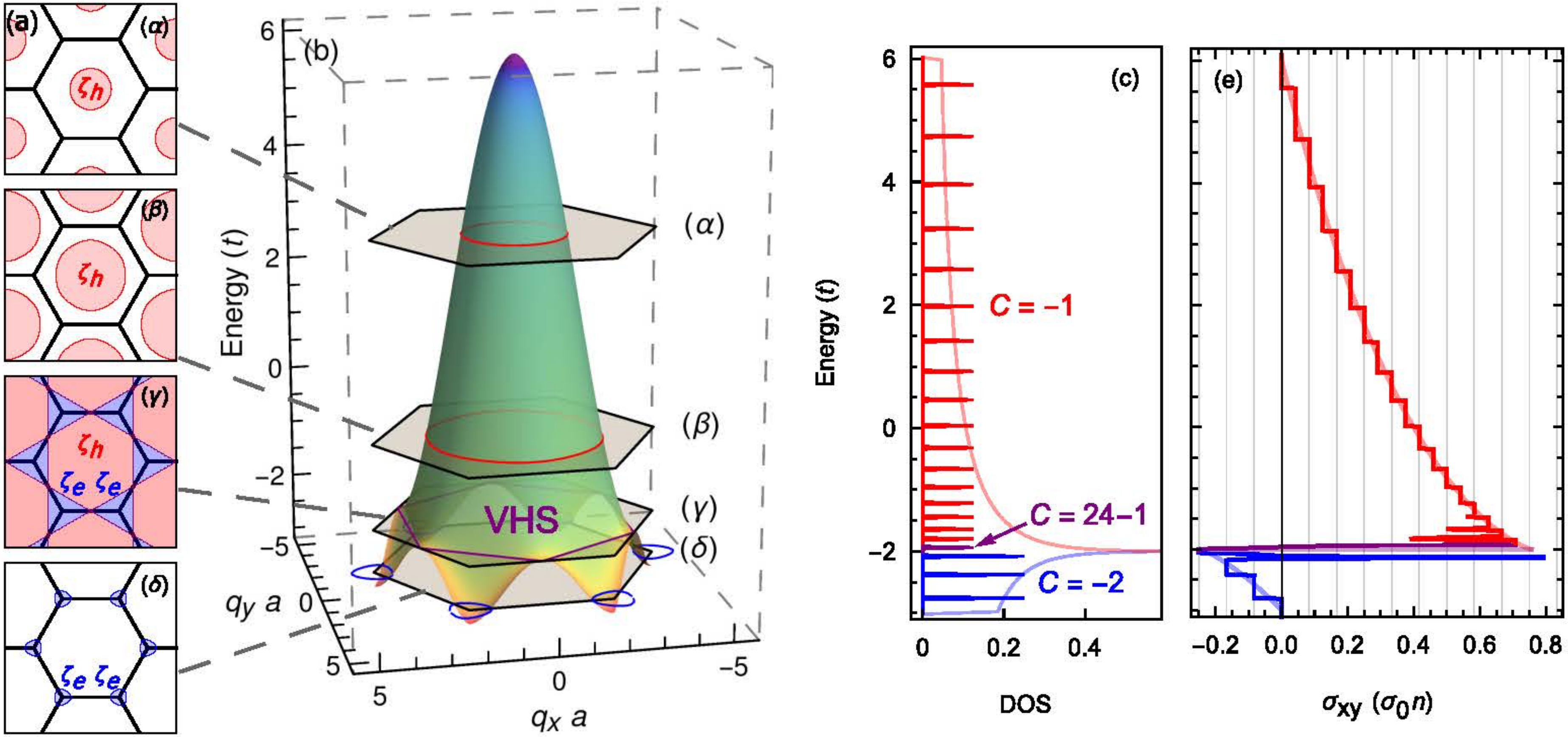


Börge Göbel, Alexander Mook, Jürgen Henk and Ingrid Mertig, Phys. Rev. B **95,** 094413 (2017)

Keita Hamamoto, Motohiko Ezawa, and Naoto Nagaosa, Phys. Rev. B **92,** 115417 (2015)

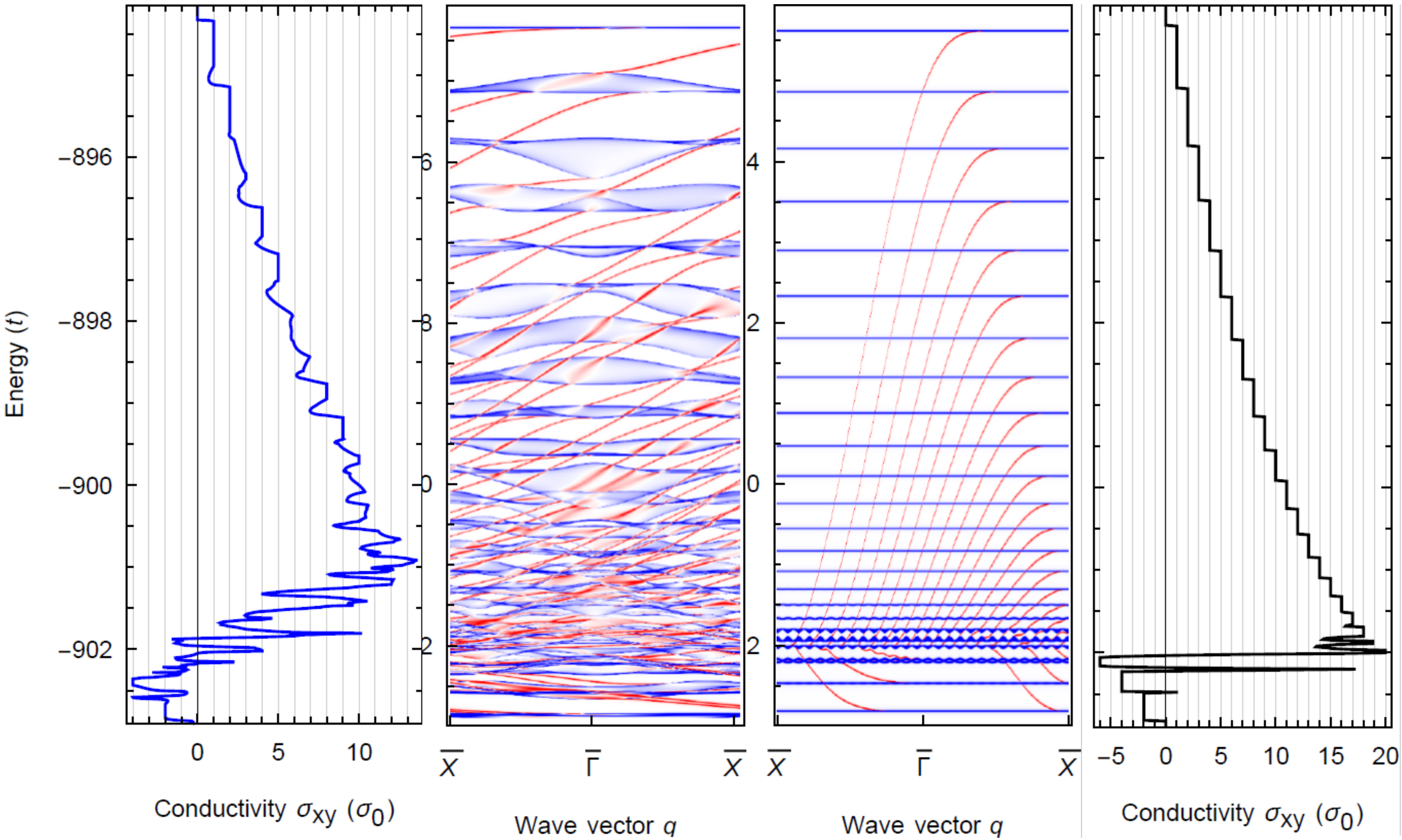
Free electrons in a triangular lattice

$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

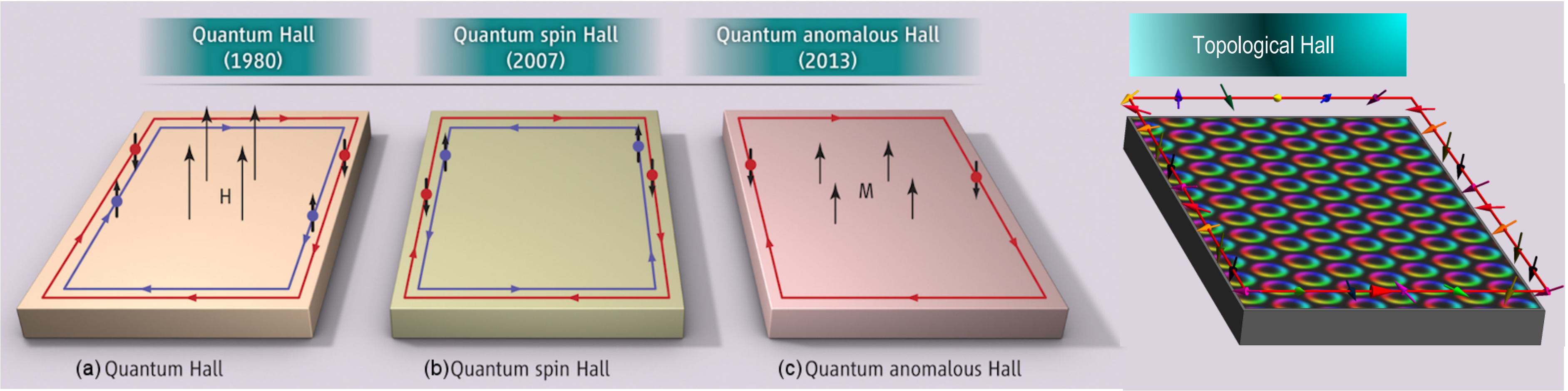


Börge Göbel, Alexander Mook, Jürgen Henk and Ingrid Mertig, Phys. Rev. B **95,** 094413 (2017)

Comparison of THE and QHE



The quantum Hall trio and THE



The conductance is quantized!

S. Oh, Science **340**,153-154 (2013)

Summary

- Chern number – topological invariant
- Berry curvature acts like a magnetic field and causes anomalous veolcity!
- Anomalous velocity is the origin of the transversal transport coefficients: spin and anomalous Hall effect and quantum spin and anomalous Hall effect, as well as the topological Hall and quantum topological Hall effect!