Multiferroics

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References

Multiferroics: a magnetic twist for ferroelectricity
S.-W. Cheong and M. Mostovoy
Nature Mat. 6, 13 (2007)

Multiferroics of spin origin
Y. Tokura, S. Seki and N. Nagaosa
Rep. Prog. Phys. 77, 076501 (2014)

Non-collinear magnetism in multiferroic perovskites
E. Bousquet and A. Cano
J. Phys.: Condens. Matter 28, 123001 (2016)

The evolution of multiferroics
M. Fiebig, Th. Lottermoser, D. Meier, and M. Trassin
Nature Reviews Materials 1, 16046 (2016)

... and the original papers therein!

Outline

1. Definitions & motivation

2. Type I

3. Type II

4. Dynamics

Ferroic

Generic name given to *(anti-)ferromagnets*, *ferroelectrics* and *ferroelastics*.

Ferromagnetism: spontaneous magnetization $\mathbf{M} \neq 0$ ↑ ↑ ↑ ↑

Antiferromagnetism: spontaneous magnetization $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2 \neq 0$ ↑ ↓ ↑ ↓

Ferroelectricity: spontaneous electric polarization $\mathbf{P} \neq 0$ – + – +

Ferroelasticity: spontaneous strain $u_{ij} \neq 0$

Multiferroics

Materials that exhibit more than one ferroic property...

...**and preferably a strong coupling!**

Ferromagnetism: spontaneous magnetization $\mathbf{M} \neq 0$ $\uparrow \uparrow \uparrow \uparrow$

Antiferromagnetism: spontaneous magnetization $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2 \neq 0$ $\uparrow \downarrow \uparrow \downarrow$

Ferroelectricity: spontaneous electric polarization $\mathbf{P} \neq 0$ $- + \; - +$

Ferroelasticity: spontaneous strain $u_{ij} \neq 0$ | | |

Maxwell's equations (electromagnetism)

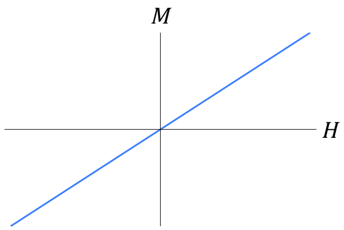
$$\nabla \cdot \mathbf{D} = \rho_f, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}, \quad \nabla \times \mathbf{H} = \mathbf{J}_f + \partial_t \mathbf{D}.$$

Constitutive equations (matter)

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \qquad \mathbf{H} = \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

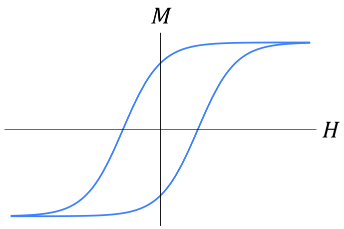
Standard dielectrics & paramagnets

$$\frac{1}{\varepsilon_0 \chi_e} P = E \qquad \frac{1}{\chi_m} M = H$$



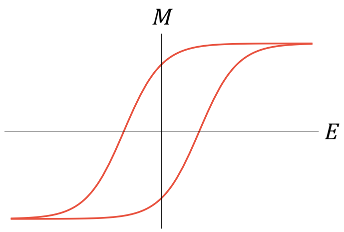
Ferroelectrics & ferromagnets $\longrightarrow$ multiferroics

$$\left(\frac{1}{\varepsilon_0 \chi_e} + \beta_e P^2 + \ldots\right) P = E$$
$$\left(\frac{1}{\chi_m} + \beta_m M^2 + \ldots\right) M = H$$



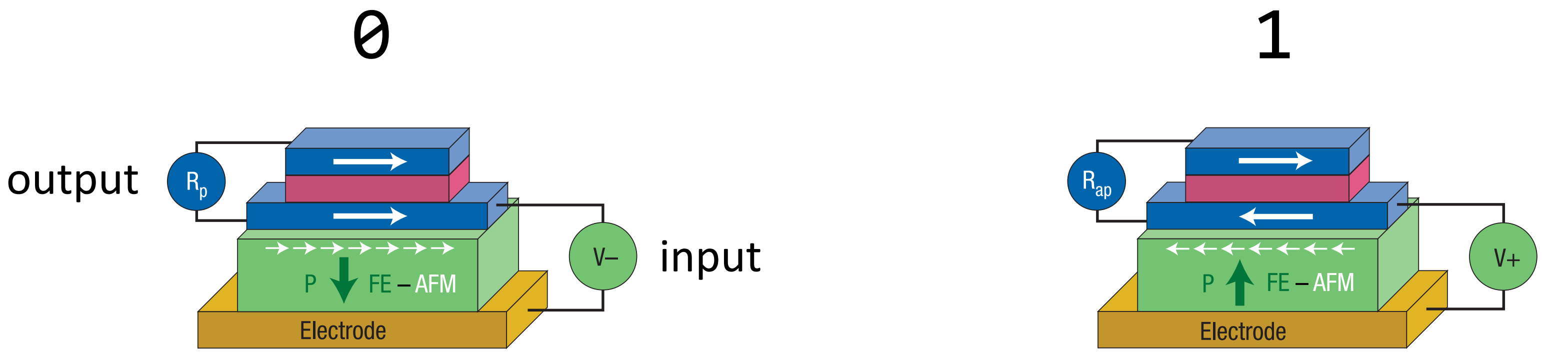
Magnetoelectric multiferroics

$$\left(\frac{1}{\varepsilon_0 \chi_e} + \beta_e P^2 + \ldots\right) P + \alpha_{\mathrm{ME}} M = E$$
$$\left(\frac{1}{\chi_m} + \beta_m M^2 + \ldots\right) M + \alpha_{\mathrm{ME}} P = H$$

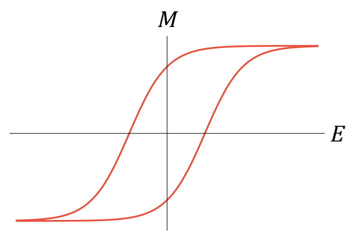


$$F_{\mathrm{ME}} = \alpha_{\mathrm{ME}} P M + \ldots$$

Multiferroic RAM



electric-field writing of magnetic information:
faster and more energy-efficient



Types of multiferroics

Type I

$$T_c^{\text{ferroelectricity}} \neq T_c^{\text{(anti)-ferromagnetism}}$$

Independent phenomena that just superpose each other (weak coupling)

Type II

$$T_c^{\text{ferroelectricity}} = T_c^{\text{(anti)-ferromagnetism}}$$

Interrelated & interacting phenomena (strong coupling)

Outline

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Type-I multiferroics (alias *ferroelectromagnets* before 1993)

Compound	Type of electric order	Type of magnetic order	Θ_E, K	Θ_M, K	Magnetoelectric measurements
1. Pb $(Fe_{2/3}W_{1/3})O_3$	FE	AFM	178	363	Measurement of internal magnetic field at the FE transition
2. Pb $(Fe_{1/2}Nb_{1/2})O_3$	FE	AFM	387	143	$M_c(\tilde{E})$
3. $BiFeO_3$	AFE? FE	AFM	1123 ?	≈ 650	$\varepsilon(\Theta_M)$, induction of P in spin-flop
4. $Eu_{1/2}Ba_{1/2}TiO_3$	FE	FM	165[156]	4,2[156]	
5. Pb $(Mn_{2/3}W_{1/3})O_3$	AFE?	AFM	473[144]	203[144]	$\varepsilon(\Theta_M)$
6. Pb $(Mn_{1/2}Re_{1/2})O_3$	AFE?	FIM	393	103	
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	FE	WFM	68	9	
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15. $Bi_2Bi_4Fe_2Ti_3O_{18}$	FE	FM?	Θ_{EI}-1171 Θ_{EII}-1025[158]	723? [159]	χ^{ME}? [158]
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25. $Cr_3B_7O_{13}Br$	FE	AFM	4	50	
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28. $Mn_3B_7O_{13}Br$	FE	AFM	566	≈ 6[21]	
29. $Mn_3B_7O_{13}I$	FE	WFM	412	26[115]	$m(\dot{E})$
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33. $Co_3B_7O_{13}Cl$	FE	WFM	623	11,5, 15 22	χ^{ME}, $m(\tilde{E})$
34. $Co_3B_7O_{13}Br$	FE	WFM	458	20	
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36. $Cu_3B_7O_{13}Cl$	FE	AFM WFM	365	20 8,4[111]	χ^{ME}
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40. $Ni_3B_7O_{13}I$	FE	WFM	64	64	χ^{ME}, $\varepsilon(H)$, $P(H)$, $m(\tilde{E})$
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48. $Co_{1.75}Mn_{1.25}O_4$	FE	FIM	170[136]	170[136]	χ^{ME}
49. Cu $(HCOO)_2 \cdot 4H_2O$	AFE	WFM	≈ 235	17	
50. β-$Tb_2(MoO_4)_3$	FE	AFM		2[161]	
51. β-$NaFeO_2$	FE	WFM	723[162]	723[162]	
52. FeS	FE	AFM	411?	≈ 593	

Notes: The data marked with a question mark require more precise determination. Notation: Fe—ferroelectric, AFE—antiferroelectric, PE—pyroelectric, FM—ferromagnet, AFM—antiferromagnet, WFM—antiferromagnet with weak ferromagnetism, FIM—ferrimagnet, χ^{ME}—static ME susceptibility, M_c—spontaneous magnetization, $\tilde{E}$—amplitude of the *alternating electric field*, ε—*dielectric permittivity*.

Type-I multiferroics: perovskites

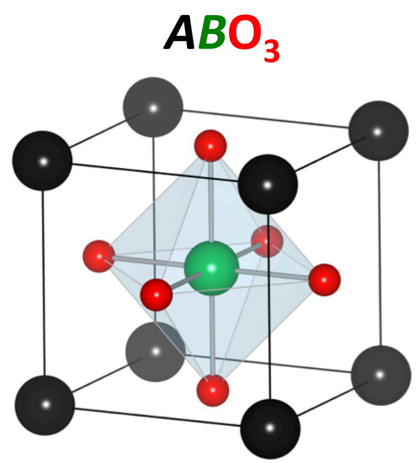
Strategy

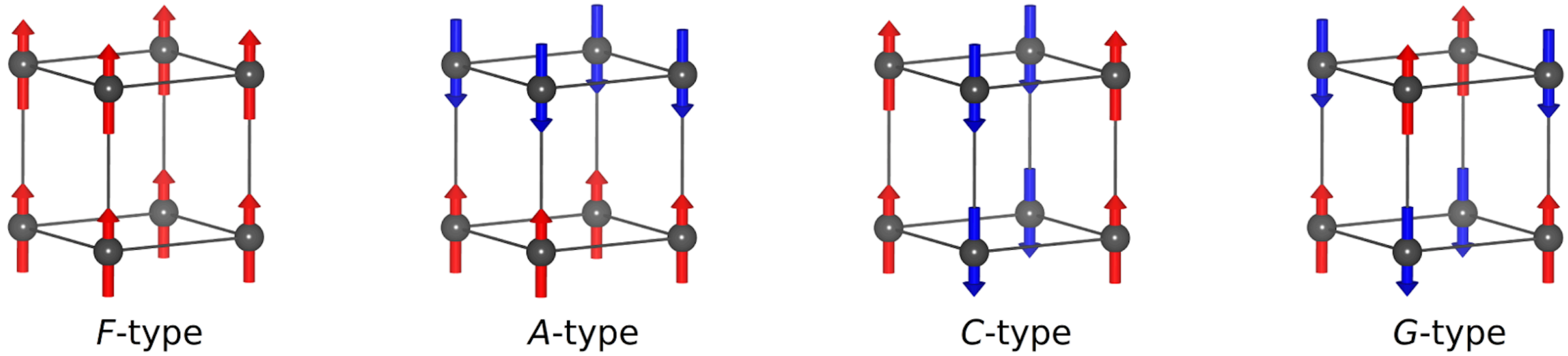
Proper ferroelectricity

due to the relative ***A*-O** (or ***B*-O**) displacements

+

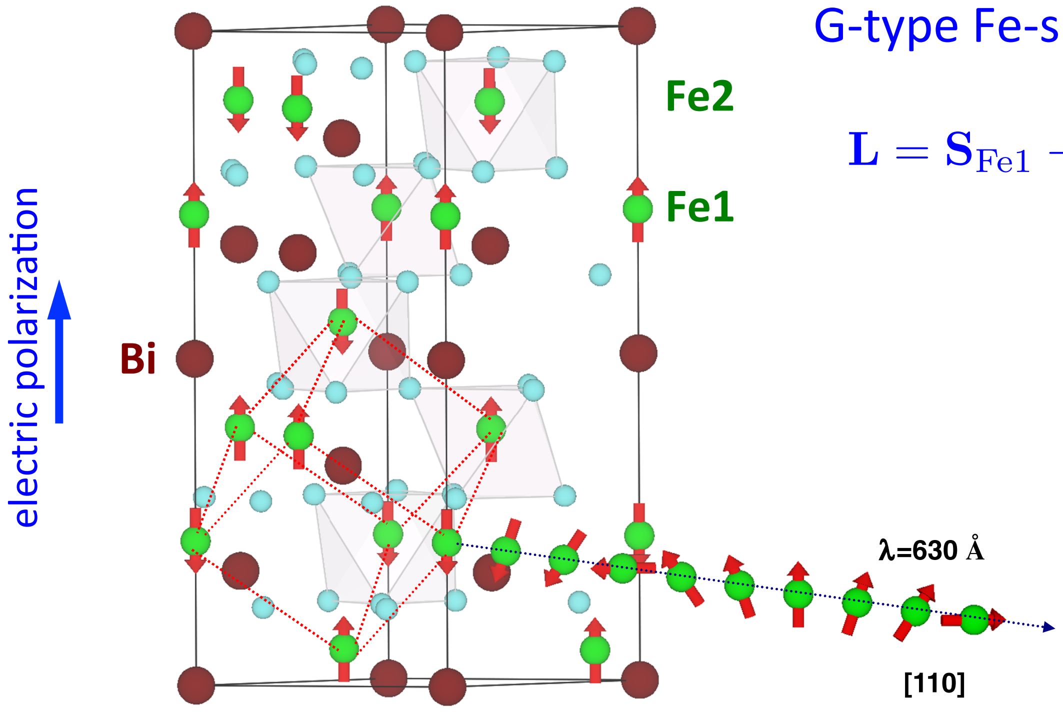
Long-range order of ***B***-atom magnetic moments





Type-I multiferroics: perovskites

$BiFeO_3$ ($T_c = 1083$ K, $T_N = 643$ K)



G-type Fe-spin order

$$\mathbf{L} = \mathbf{S}_{\mathrm{Fe1}} - \mathbf{S}_{\mathrm{Fe2}}$$

Spontaneous polarization $P \longrightarrow$ spiral modulation of the primordial antiferromagnetic order

$$H_L = -KL_z^2 + A(\nabla \mathbf{L})^2 + \underbrace{gP[\mathbf{L}(\nabla \cdot \mathbf{L}) - (\mathbf{L} \cdot \nabla)\mathbf{L}]_z}_{\text{ME coupling}}$$

Kadomtseva et al., JETP Lett. 79, 571 (04)

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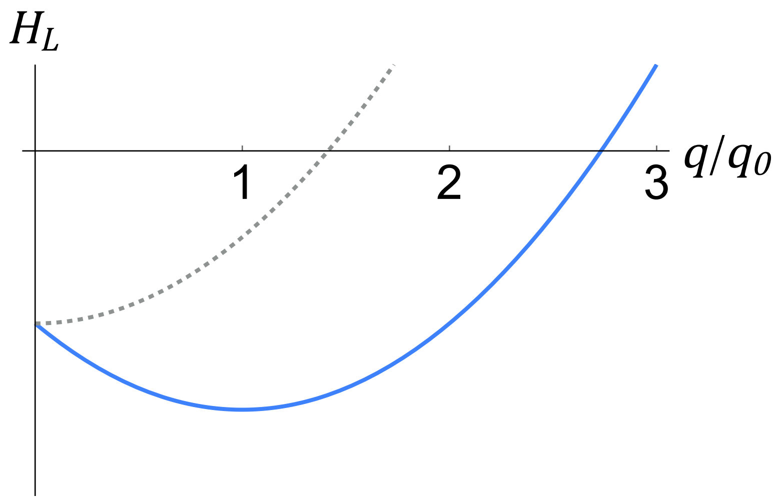
$$\mathbf{L} = L_0\big(\cos\phi\sin(\mathbf{q}\cdot\mathbf{r}),\, \sin\phi\sin(\mathbf{q}\cdot\mathbf{r}),\, \cos(\mathbf{q}\cdot\mathbf{r})\big)$$

$$\mathbf{q} = (q_x,\, q_y,\, 0) \qquad \phi = \arctan(q_y/q_x)$$

$$H_L \sim L_0^2[-\tfrac{K}{2} + Aq^2 - gPq]$$

$$q_0 = gP/(2A) \sim (63\ \mathrm{nm})^{-1}$$



...but otherwise ferroelectricity and antiferromagnetism emerge in $BiFeO_3$ as independent phenomena

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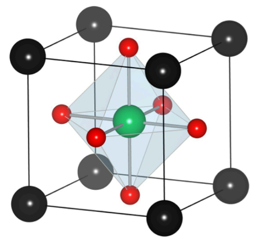

Generalization to other *AB*O_3 perovskites?

$Bi^{3+} \longrightarrow 6s^2$ lone pair (not involved in the *sp* hybridization) $\longrightarrow$ *A*-site ferroelectricity

But in general ferroelectricity is *B*-site with empty $3d^0$ shells (e.g. $BaTiO_3$)...

... while 3*d* electrons are necessary for the magnetic ordering of transition-metal elements

No-Go Theorem

Hill, J. Phys. Chem. B **104**, 6694 (00)

Smolenski & Chupis, Sov. Phys. Usp. **25** 475 (82)

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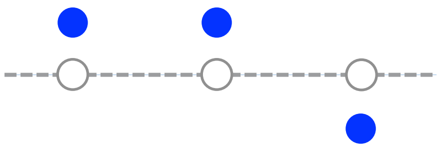
Type-I multiferroics: hexagonal manganites

h-*R*MnO_3 ($T_c \geq 1200$ K, $T_N \leq 120$ K)

Improper ferroelecticity

$$P_z \propto (Q_1^2 - 3Q_2^2)Q_1 = Q^3 \cos 3\Phi$$

R

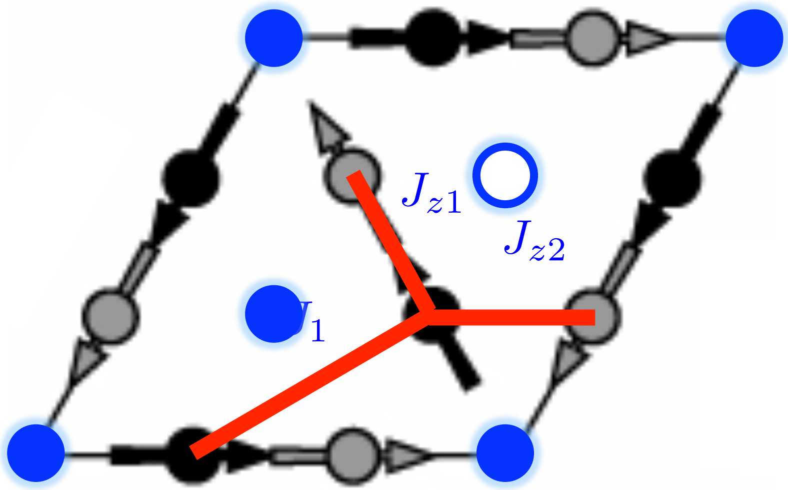


$$u(\mathbf{r}_n) = Q_1 \cos(\mathbf{q}\cdot\mathbf{r}_n) + Q_2 \sin(\mathbf{q}\cdot\mathbf{r}_n) = Q\cos(\mathbf{q}\cdot\mathbf{r}_n - \Phi)$$

$$\mathbf{q} = (1/3, 1/3, 0)$$

Ferroelectricity is the by-product of the unit-cell trimerization (not a primary order parameter)

Non-collinear Mn-spin order



$$H = \sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j$$

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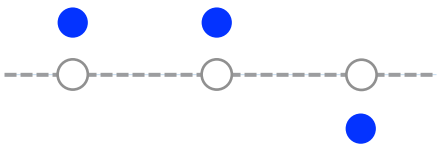

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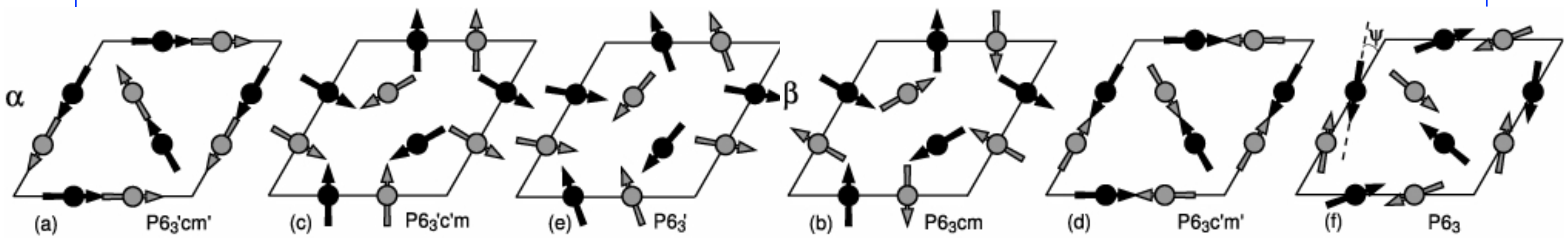

... but even more decoupled from the ferroelectric order!

Outline

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$$T_c^{\text{ferroelectricity}} \neq T_c^{\text{(anti)-ferromagnetism}}$$

Independent phenomena that just superpose each other (weak coupling)

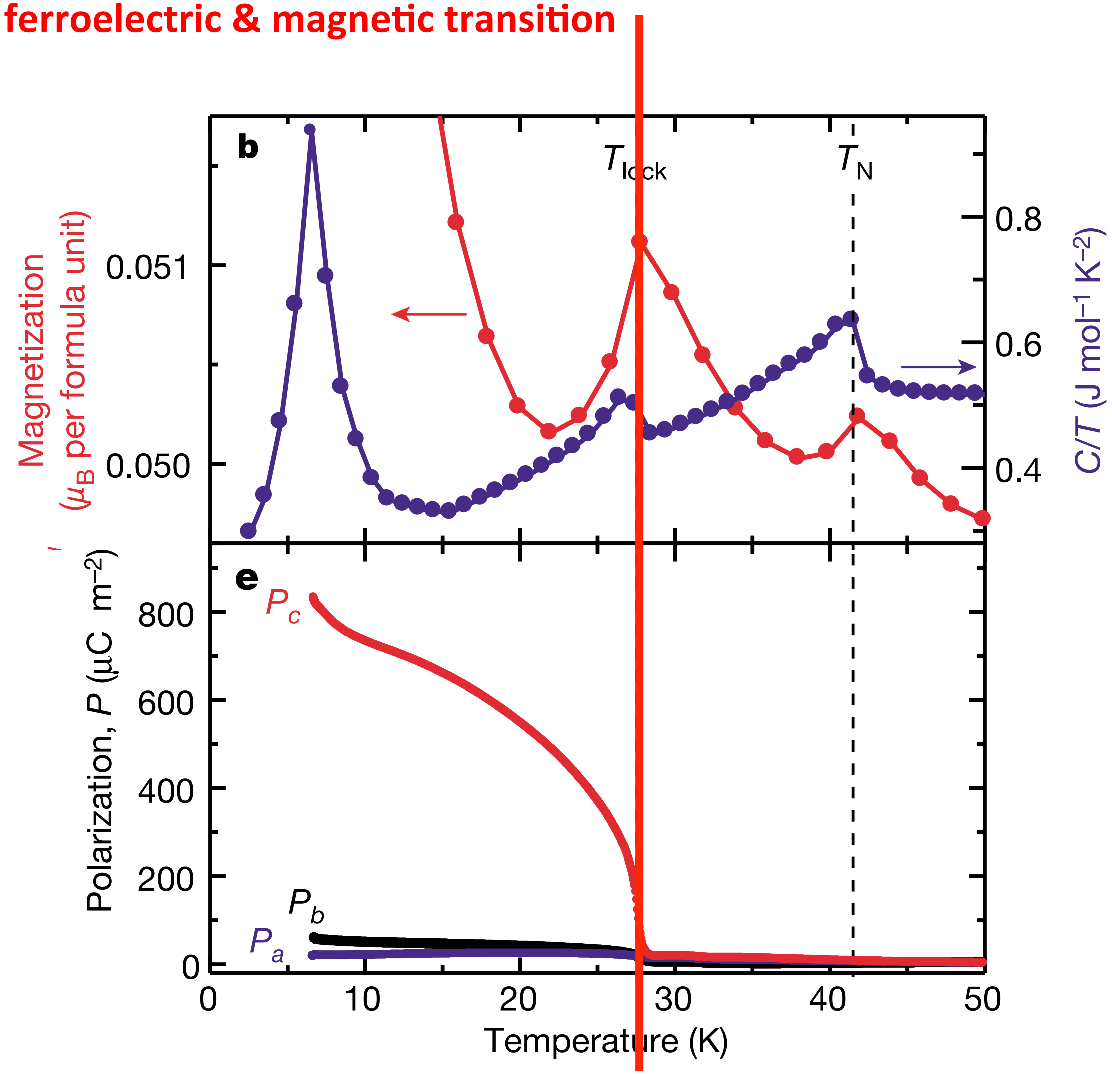
Type II

$$T_c^{\text{ferroelectricity}} = T_c^{\text{(anti)-ferromagnetism}}$$

Interrelated & interacting phenomena (strong coupling)

Type-II multiferroics: $TbMnO_3$

Simultaneous ferroelectric & magnetic transition



Kimura et al., Nature **426**, 55 (03)

Type-II multiferroics: $TbMnO_3$

Can magnetic order be the direct consequence of ferroelectricity?

No

(P preserves time-reversal symmetry)

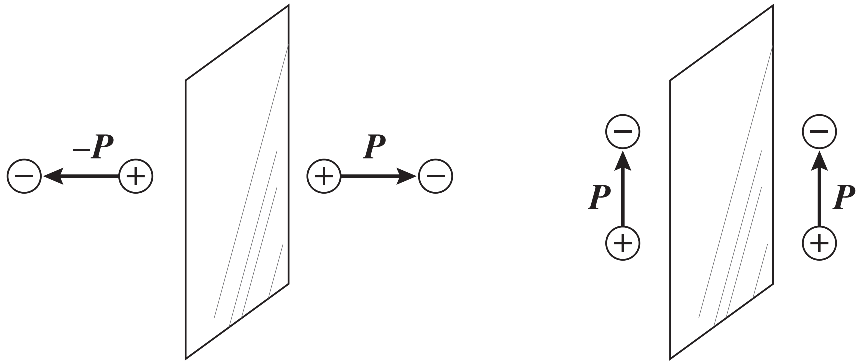
Can ferroelectricity be the direct consequence of magnetic order?

?

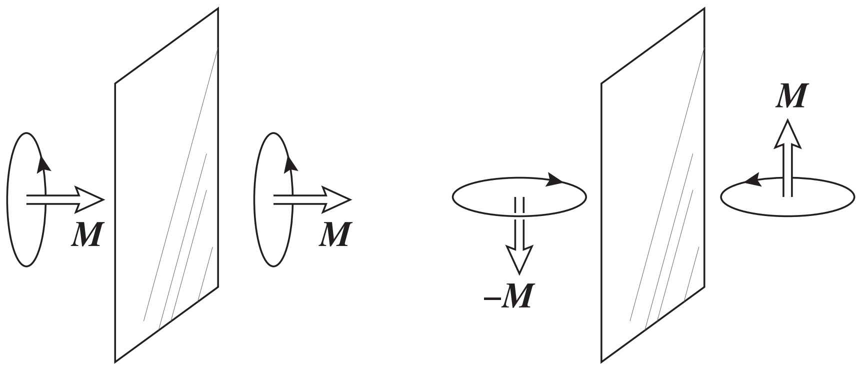
Type-II multiferroics

Can ferroelectricity be the direct consequence of magnetic order?

polar vector



axial vector

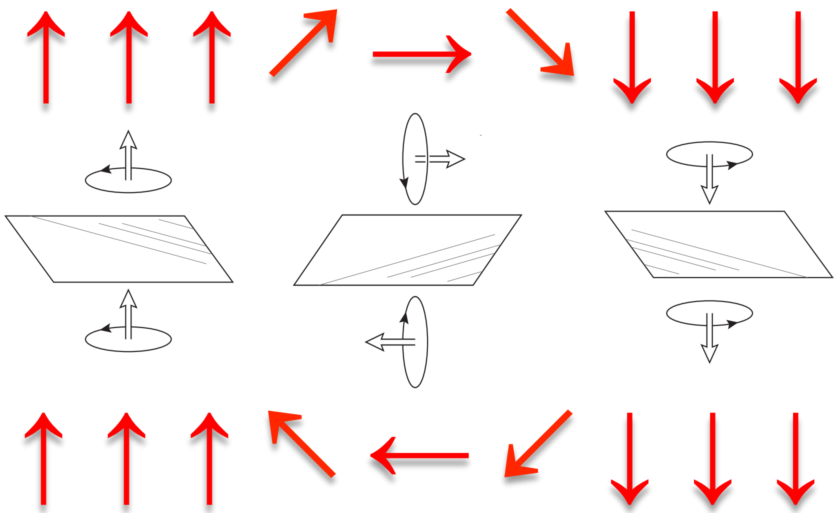


Type-II multiferroics

Can ferroelectricity be the direct consequence of magnetic order?

Néel wall

clockwise



anti-clockwise

Néel walls are not invariant under inversion symmetry → local polar axis

Idem for Bloch walls in e.g. D_{2h} systems

Bar'yakhtar et al. JETP Lett. **37**, 673 (83)

Type-II multiferroics

Can ferroelectricity be the direct consequence of magnetic order? YES!

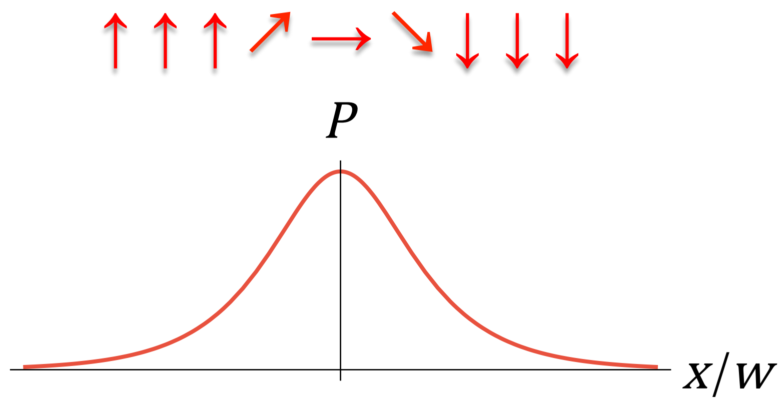
How?

$$F_{ME} = -g\mathbf{P} \cdot [(\mathbf{M} \cdot \nabla)\mathbf{M} - \mathbf{M}(\nabla \cdot \mathbf{M})]$$

Inhomogeneous magnetoelectric effect (universal)

Néel wall

$$\mathbf{M} = M_0\left(\frac{\pm 1}{\cosh(x/w)},\, 0,\, -\tanh(x/w)\right)$$



$$F_{ME} = -g\mathbf{P} \cdot [(M_x\partial_x)\mathbf{M} - \mathbf{M}(\partial_x M_x)]$$

$$= -gP_z[(M_x\partial_x)M_z - M_z(\partial_x M_x)] = \pm gP_z\frac{M_0^2}{\cosh(x/w)}$$

The direction of the (local) polarization is linked to the chirality of the wall

Bloch walls: It does not work in this simple form

Bar'yakhtar et al. JETP Lett. 37, 673 (83)

Type-II multiferroics

Can ferroelectricity be the direct consequence of magnetic order? **YES!**

Can the magnetically-induced polarization be global?

$$F_{ME} = -g\mathbf{P}\cdot[(\mathbf{M}\cdot\nabla)\mathbf{M} - \mathbf{M}(\nabla\cdot\mathbf{M})]$$

Néel wall

$$\uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \uparrow\ \nearrow\ \rightarrow\ \searrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$$

Long-range cycloidal order

$$\uparrow\ \nearrow\ \rightarrow\ \searrow\ \downarrow\ \swarrow\ \leftarrow\ \nwarrow\ \uparrow\ \nearrow\ \rightarrow\ \searrow\ \downarrow\ \swarrow\ \leftarrow\ \nwarrow\ \uparrow\ \nearrow\ \rightarrow\ \searrow\ \downarrow$$

$$\mathbf{M} = M_0[\sin(\mathbf{q}\cdot\mathbf{r})\hat{\mathbf{e}}_1 + \cos(\mathbf{q}\cdot\mathbf{r})\hat{\mathbf{e}}_2]$$

Ferroelectricity in spiral magnets

$$F_{ME} \quad \rightarrow \quad \mathbf{P} \propto (\hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2) \times \mathbf{q}$$

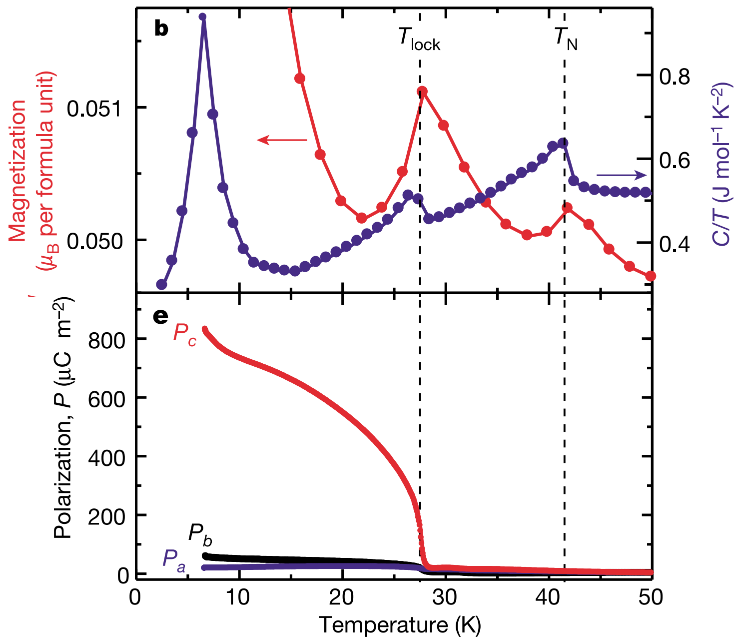
Mostovoy, PRL 96, 067601 (06)

Type-II multiferroics

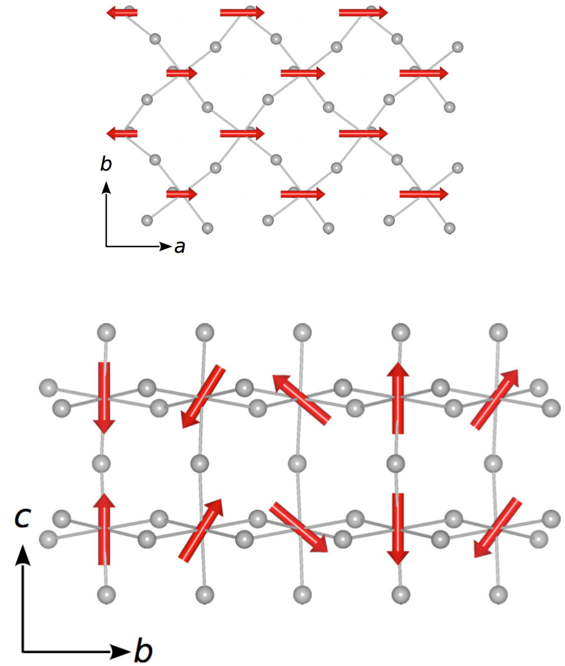
Can ferroelectricity be the direct consequence of magnetic order? YES!

Can the magnetically-induced polarization be global? YES!

Does $TbMnO_3$ display cycloidal order? YES!



Kimura et al., Nature **426**, 55 (03)



Kenzelmann et al., PRL **95**, 087206 (05)

The spontaneous electric polarization in $TbMnO_3$ has a magnetic origin

previous example: Cr_2BeO_4, Newnham et al., J. Appl. Phys. **49**, 6088 (78)

Type-II multiferroics

Can ferroelectricity be the direct consequence of magnetic order? **YES!**

Can the magnetically-induced polarization be global? **YES!**

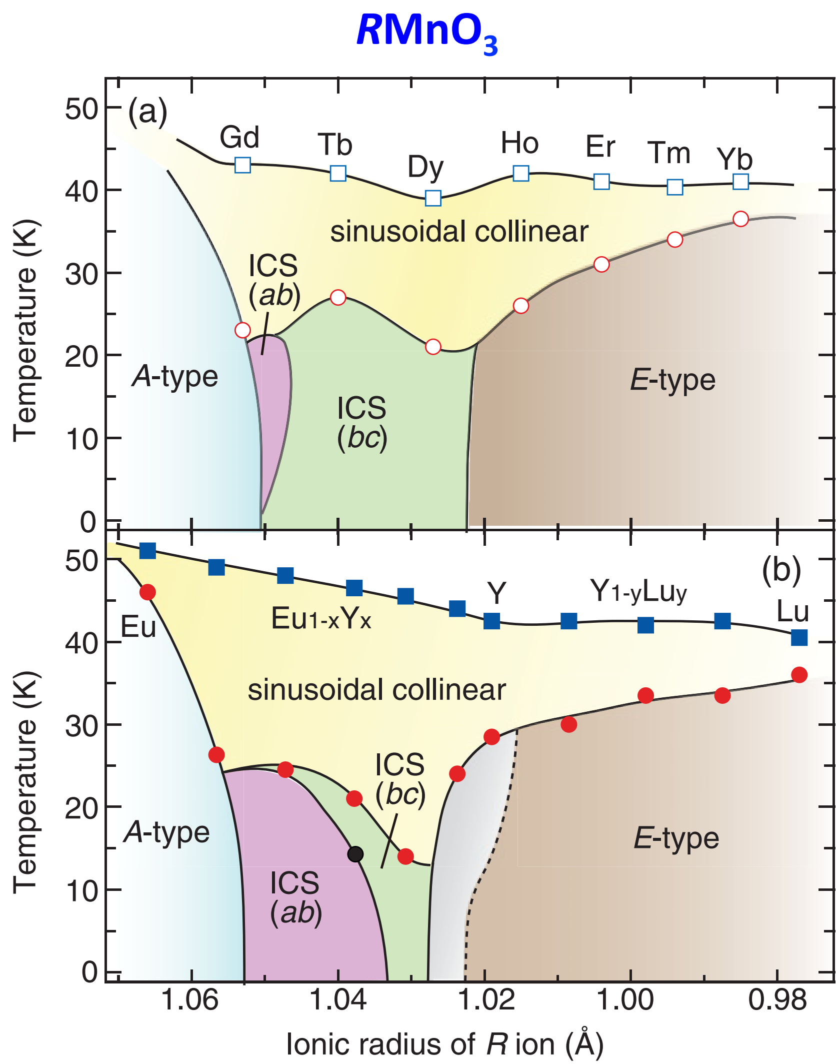
Does $TbMnO_3$ display cycloidal order? **YES!**

- Why does $TbMnO_3$ display cycloidal order?

- What is the microscopic version of $F_{ME} = -g\mathbf{P} \cdot [(\mathbf{M} \cdot \nabla)\mathbf{M} - \mathbf{M}(\nabla \cdot \mathbf{M})]$?

Type-II multiferroics

Why does $TbMnO_3$ display cycloidal order?



Type-II multiferroics

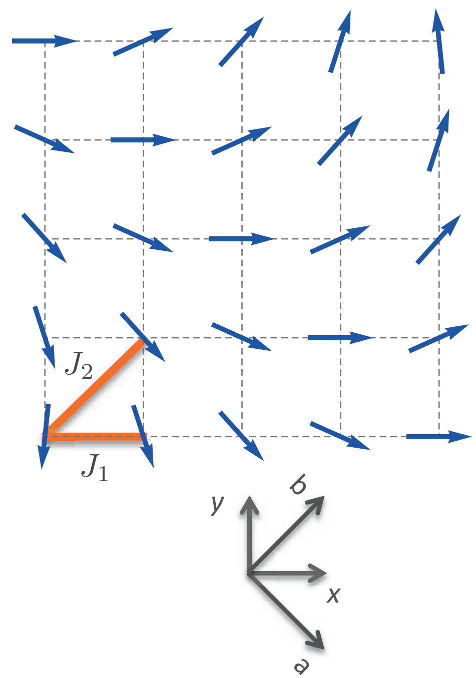
Why does $TbMnO_3$ display cycloidal order?

FM nearest- **vs.** AFM next-nearest–neighbor exchange interactions in the *ab* plane

$$H = -J_1^{\mathrm{FM}} \sum_i \mathbf{S}_i \cdot (\mathbf{S}_{i-x} + \mathbf{S}_{i+x} + \mathbf{S}_{i-y} + \mathbf{S}_{i+y}) + J_2^{\mathrm{AFM}} \sum_i \mathbf{S}_i \cdot (\mathbf{S}_{i-x-y} + \mathbf{S}_{i+x+y} + \mathbf{S}_{i+x-y} + \mathbf{S}_{i-x+y})$$

$$\mathbf{Q} = \frac{Q}{\sqrt{2}}(1,\,1,\,0)$$

$$\mathbf{S}_i = S\cos(\mathbf{Q}\cdot\mathbf{r}_i)\hat{\mathbf{x}} + S\sin(\mathbf{Q}\cdot\mathbf{r}_i)\hat{\mathbf{y}}$$



FM state

$$E_{\mathrm{FM}} = -4S^2 J_1(1 - J_2/J_1)$$

Cycloidal state

$$E_{\mathrm{cycloidal}} = 2S^2[-J_1\cos\frac{Q}{\sqrt{2}} + J_2(1+\cos\sqrt{2}Q)] \underset{\cos\frac{Q_0}{\sqrt{2}} = \frac{J_1}{2J_2}}{=} -2S^2 J_1^2/J_2$$

$J_2 > 2J_1 \quad \rightarrow \quad$ The cycloidal wins

Type-II multiferroics

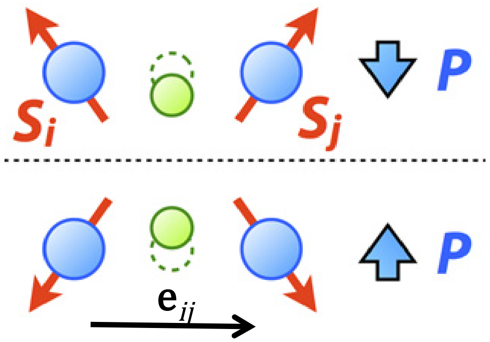
What is the microscopic version of $F_{ME} = -g\mathbf{P}\cdot[(\mathbf{M}\cdot\nabla)\mathbf{M} - \mathbf{M}(\nabla\cdot\mathbf{M})]$?

If P is mostly due to:

Ionic displacements ($\mathrm{P} \sim Z^* \mathrm{u}$)

Dzyaloshinskii-Moriya interaction

$$H_{\mathrm{DM}} = \mathbf{D}_{ij}\cdot(\mathbf{S}_i \times \mathbf{S}_j) = \underbrace{d(\mathbf{u}\times\hat{\mathbf{e}}_{ij})}_{\mathbf{D}_{ij}}\cdot(\mathbf{S}_i \times \mathbf{S}_j)$$



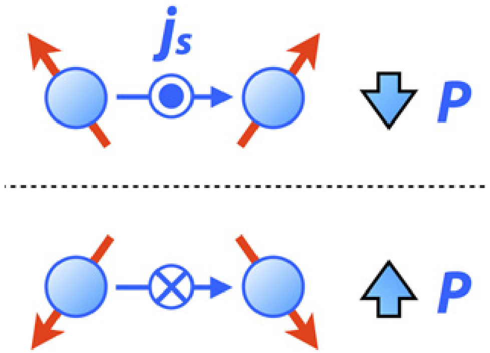
Inverse Dzyaloshinskii-Moriya mechanism

$$\mathbf{P} \propto \mathbf{u} \propto \hat{\mathbf{e}}_{ij} \times (\mathbf{S}_i \times \mathbf{S}_j)$$

Sergienko & Dagotto, PRB 73, 094434 (06)

Electrons

$$\mathbf{j}_s \propto (\mathbf{S}_i \times \mathbf{S}_j)$$



Spin-current mechanism

$$\mathbf{P} \propto \hat{\mathbf{e}}_{ij} \times \mathbf{j}_s$$

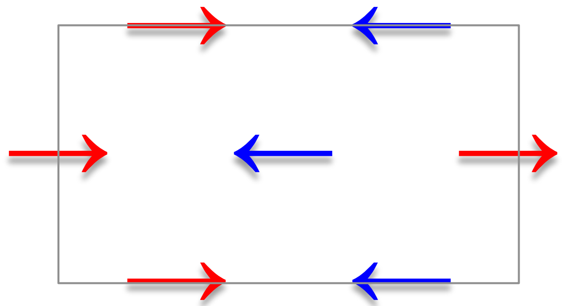
Katsura, Nagaosa, & Balatsky, PRL 95, 057205 (05)

Type-II multiferroics

Is spin-spiral magnetism the only type of magnetism that induces ferroelectricity?

$HoMnO_3$

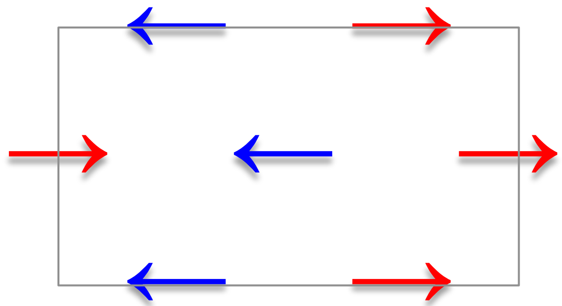
E-type AFM order



E_1

inversion ($\mathbf{r} \rightarrow -\mathbf{r}$)

two different magnetic domains



E_2

Type-II multiferroics

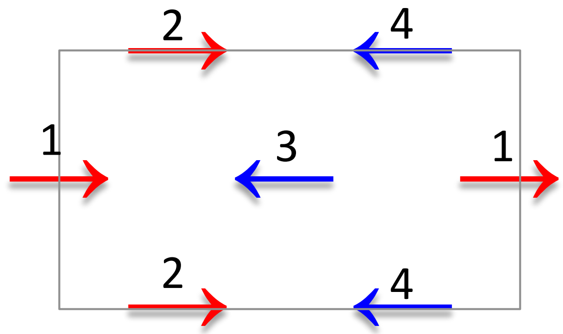
Is spin-spiral magnetism the only type of magnetism that induces ferroelectricity?

$HoMnO_3$

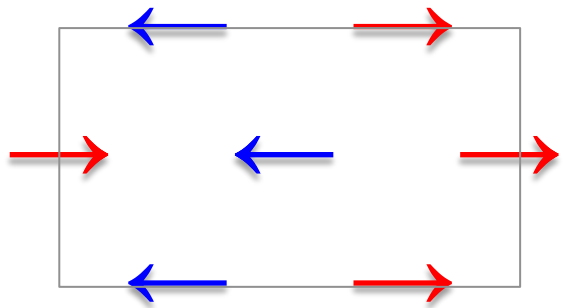
E-type AFM order

$$\mathbf{E}_1 = \mathbf{S}_1 + \mathbf{S}_2 - \mathbf{S}_3 - \mathbf{S}_4$$
$$\mathbf{E}_2 = \mathbf{S}_1 - \mathbf{S}_2 - \mathbf{S}_3 + \mathbf{S}_4$$

	$(2_a\vert\frac{1}{2}\,\frac{1}{2}0)$	$(2_c\vert00\frac{1}{2})$, $(I\vert000)$	$(1\vert010)$, $(1'\vert000)$
E_1	-1 0	0 1	-1 0
E_2	0 1	1 0	0 -1
P_a	1	-1	1



$\mathbf{E} = (\mathbf{E}_1,\, 0)$



$\mathbf{E} = (0,\, \mathbf{E}_2)$

$$F_{ME} = -gP_a(E_1^2 - E_2^2)$$
$$= -4gP_a(\mathbf{S}_1\cdot\mathbf{S}_2 - \mathbf{S}_1\cdot\mathbf{S}_4 - \mathbf{S}_2\cdot\mathbf{S}_3 + \mathbf{S}_3\cdot\mathbf{S}_4)$$
$$= -P_a \sum_{\langle i,j\rangle} J'_{ij}\ \mathbf{S}_i\cdot\mathbf{S}_j$$

Exchange striction

Sergienko, Sen & Dagotto, PRL 97, 227204 (2006)

Type-II multiferroics

$$H = \sum_{i,j} J_{ij}\, \mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{D}_{ij} \cdot (\mathbf{S}_i \times \mathbf{S}_j)$$

in a dielectric (= polarizable lattice):

$$J_{ij} = J_{ij}(\mathbf{P}_{ij}) = J_{ij}^{(0)} + \mathbf{J}'_{ij} \cdot \mathbf{P}_{ij} + \ldots$$

$$\mathbf{D}_{ij} = \mathbf{D}_{ij}(\mathbf{P}_{ij}) = \mathbf{D}_{ij}^{(0)} + \underline{\mathbf{D}_{ij}} \cdot \mathbf{P}_{ij} + \ldots$$

Mechanisms for multiferroicity

Symmetric exchange striction $(\mathbf{J}'_{ij})$: works for collinear orders but requires special symmetry

Antisymmetric exchange striction $(\underline{\mathbf{D}'_{ij}})$: works for any symmetry but needs non-collinearity

+ single-site mechanisms

$$H_{\mathrm{sia}} = -\sum_i K(\mathbf{S}_i \cdot \hat{\mathbf{n}}_i)^2$$

Outline

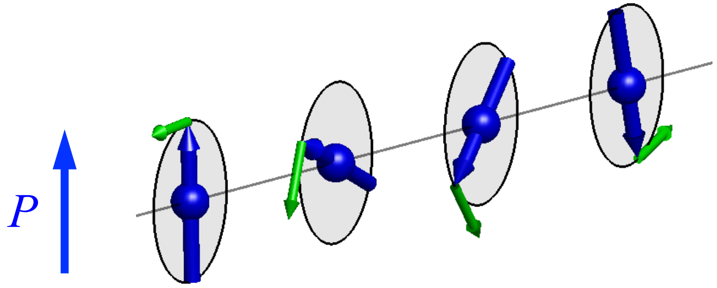
1. Definitions & classification

2. Type I

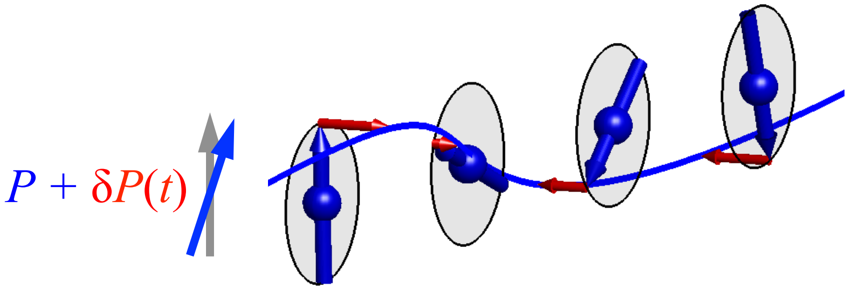
3. Type II

4. Dynamics

Spin-waves



magnon



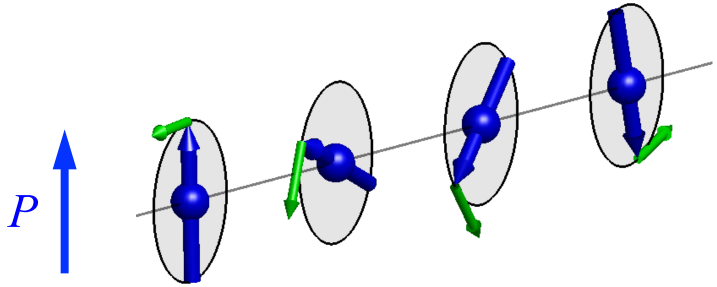
electro-active magnon
= **electromagnon**

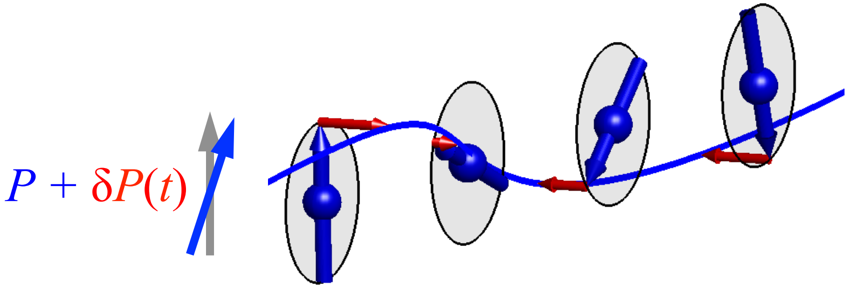
In $BiFeO_3$

$$H_{ME} = gP[\mathbf{L}(\nabla\cdot\mathbf{L}) - (\mathbf{L}\cdot\nabla)\mathbf{L}]_z \quad \rightarrow \quad g\delta\mathbf{P}(t)\cdot[\delta\mathbf{L}(t)(\nabla\cdot\mathbf{L}) + \mathbf{L}(\nabla\cdot\delta\mathbf{L}(t)) - (\delta\mathbf{L}(t)\cdot\nabla)\mathbf{L} - (\mathbf{L}\cdot\nabla)\delta\mathbf{L}(t)]$$

Cazayous et al. PRL 101, 037601 (2008)

Spin-waves



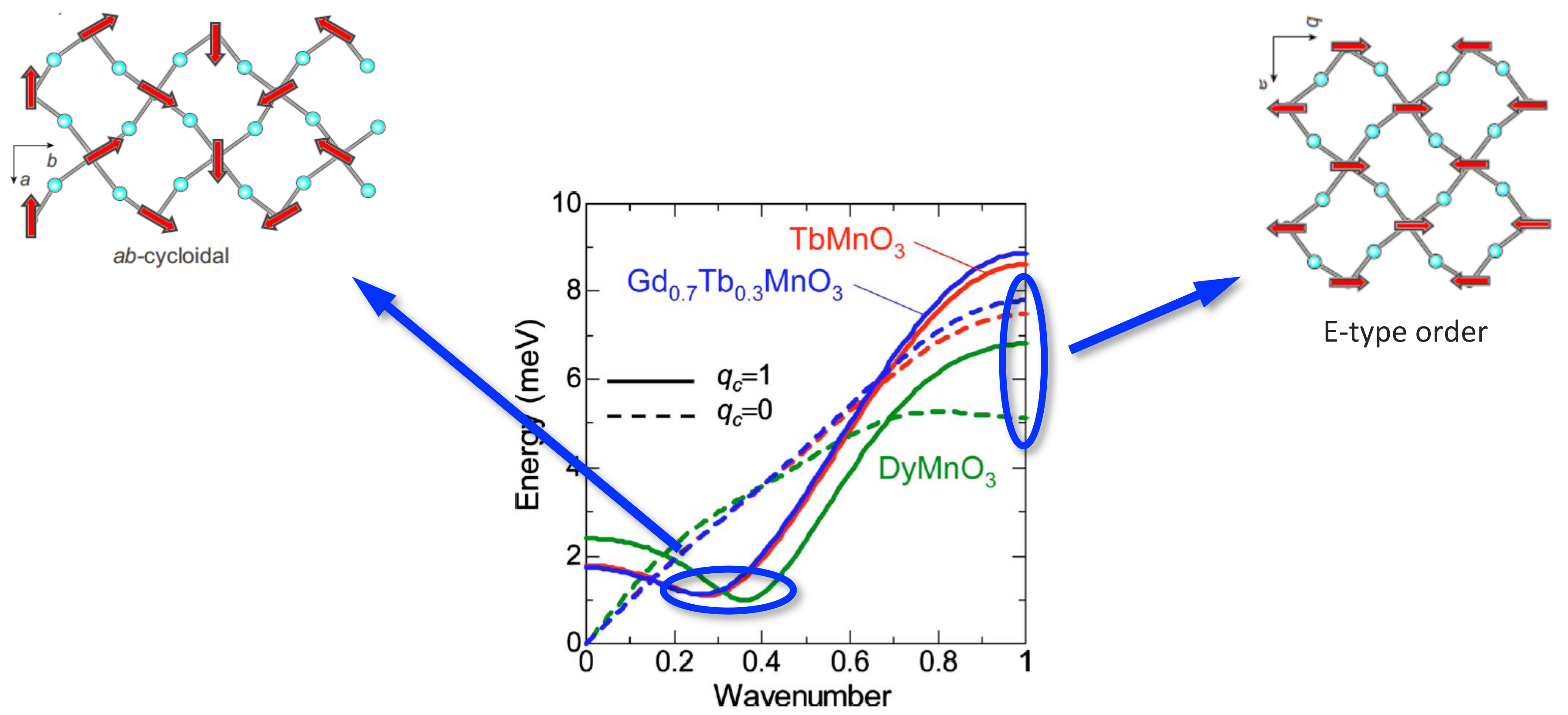


In $R\mathrm{MnO}_3$

$$H_{ME} = g\mathbf{P}\cdot[\mathbf{M}(\nabla\cdot\mathbf{M}) - (\mathbf{M}\cdot\nabla)\mathbf{M}] \quad \rightarrow \quad g\delta\mathbf{P}(t)\cdot[\delta\mathbf{M}(t)(\nabla\cdot\mathbf{M}) + \mathbf{M}(\nabla\cdot\delta\mathbf{M}(t)) - (\delta\mathbf{M}(t)\cdot\nabla)\mathbf{M} - (\mathbf{M}\cdot\nabla)\delta\mathbf{M}(t)]$$

Pimenov et al. Nature Phys. 2, 97 (06)
Katsura et al., PRL 98, 027203 (07); Cano & Kats, PRB 78, 012104 (08)

Spin-waves



In *R*MnO$_3$

The spectrum displays two types of **electromagnons**

Valdes-Aguilar et al. PRL 102, 047203 (09)

Dynamical ME effect

$$H = \underbrace{\sum_{i,j}[J_{ij}(\mathbf{P})\,\mathbf{S}_i \cdot \mathbf{S}_j + \mathbf{D}_{ij}(\mathbf{P}) \cdot (\mathbf{S}_i \times \mathbf{S}_j)]}_{\text{spin + striction}} + \frac{1}{2\chi_e}P^2 - \mathbf{E}\cdot\mathbf{P} - \mathbf{H}\cdot\mathbf{M}$$

Equations of motion $\longrightarrow$ constitutive equations

$$\begin{pmatrix} \mathbf{P} \\ \mathbf{M} \end{pmatrix}_\omega = \begin{pmatrix} \hat{\chi}_e(\omega) & \hat{\alpha}_{ME}(\omega) \\ \hat{\alpha}^T_{ME}(\omega) & \hat{\chi}_m(\omega) \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}_\omega$$

The dynamical magnetoelectric coupling $\hat{\alpha}_{ME}(\omega)$ can emerge even if:

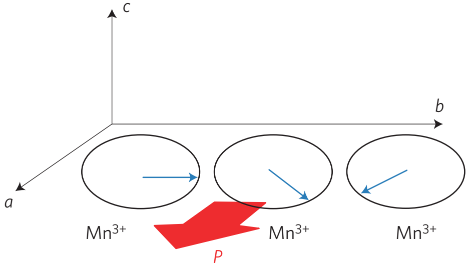
- there is no static counterpart

- there is no multiferroicity

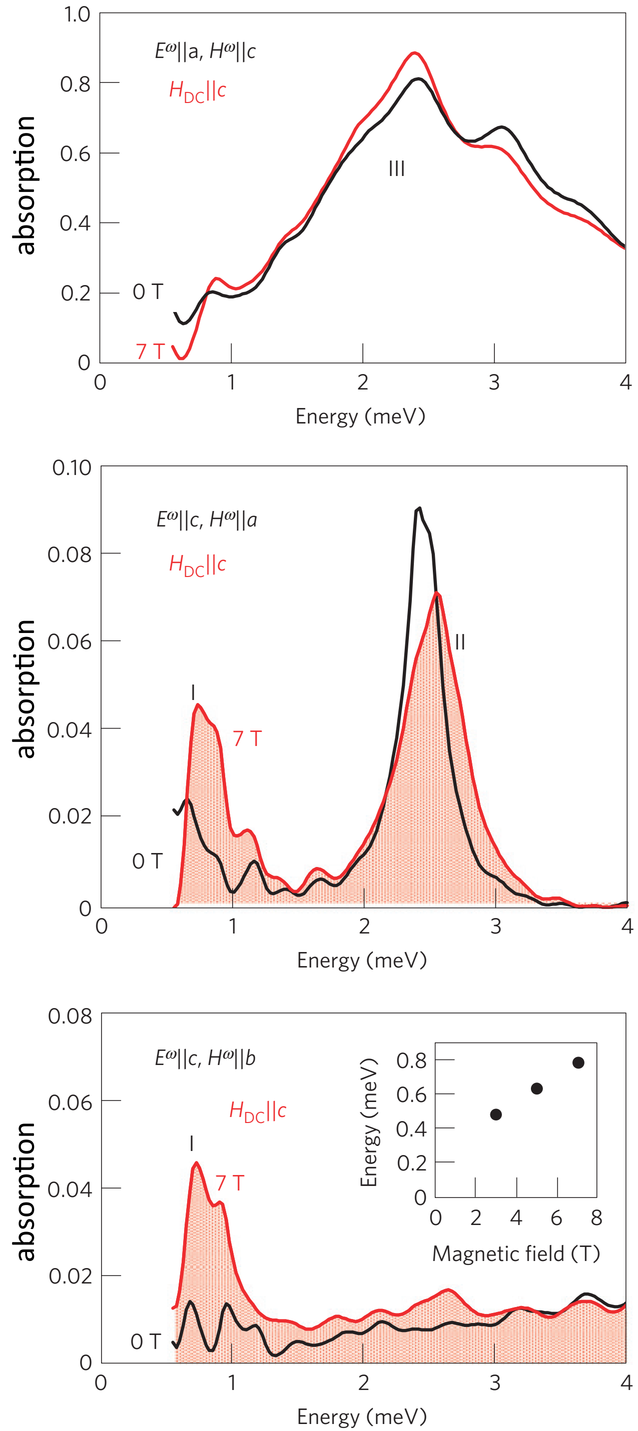
The propagation of light is affected by phonon, magnon, and electromagnon excitations

Dynamical ME effect

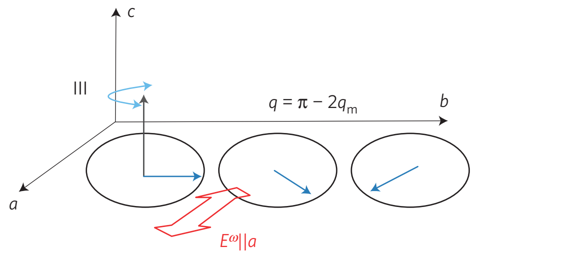
$Eu_{0.55}Y_{0.45}MnO_3$



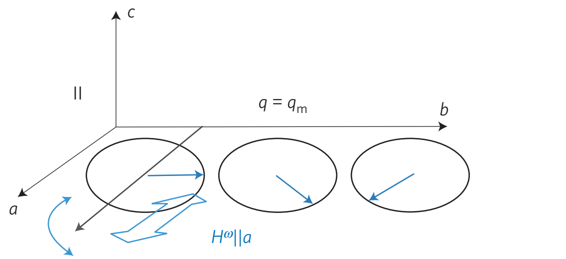
ab-cycloidal (*P* // *a*)



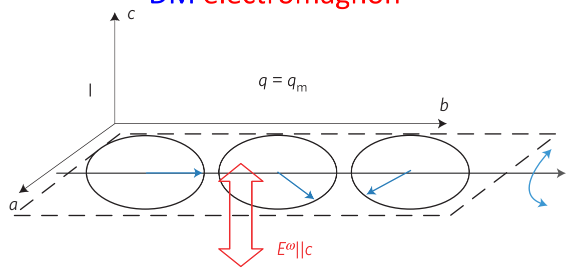
Heisenberg electromagnon



magnon

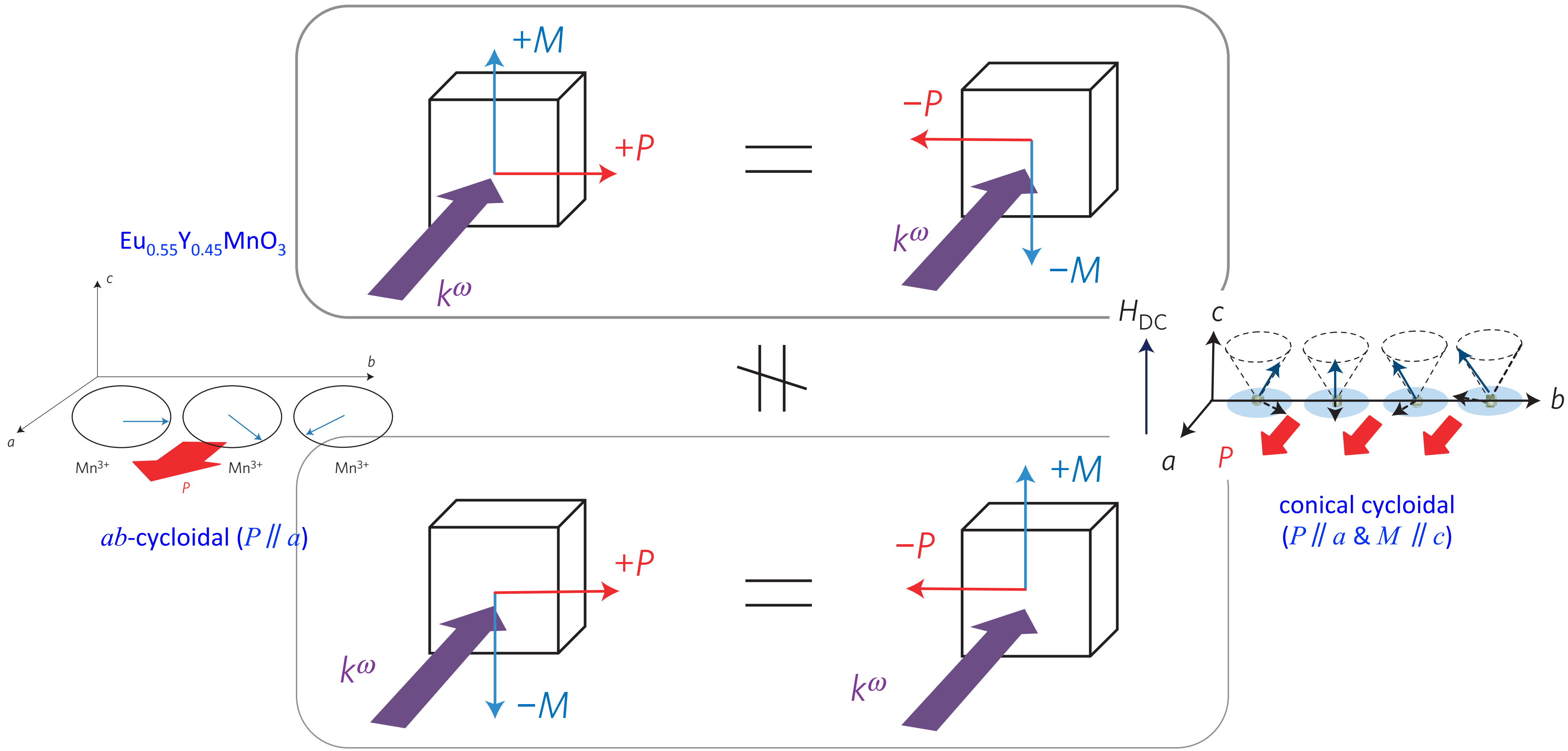


DM electromagnon



Takahashi et al., Nature Phys. 8, 121 (11)

Dynamical ME effect & directional dichroism



The propagation of light depends on $\mathbf{P} \times \mathbf{M}$ via the dynamical ME effect