Domains and domain walls

Andrés Cano
ICMCB-CNRS, University of Bordeaux, France
Department of Materials, ETH Zürich, Switzerland

References

On the theory of the dispersion of magnetic permeability in ferromagnetic bodies
L.D. Landau and E. Lifshitz
Phys. Z. Sowjet. 8, 153 (1935); Collected papers of L.D. Landau, pp.101-114

Physical theory of ferromagnetic domains
C. Kittel
Rev. Mod. Phys. 21, 541 (1949)

Magnetic domains: the analysis of magnetic microstrctures
A. Hubert and R. Schafer (Springer, Berlin, 1998)

Microscopic approach to current-driven domain wall dynamics
G. Tatara. H. Kohno, J. Shibata
Physics Reports 468, 213 (2008)

Outline

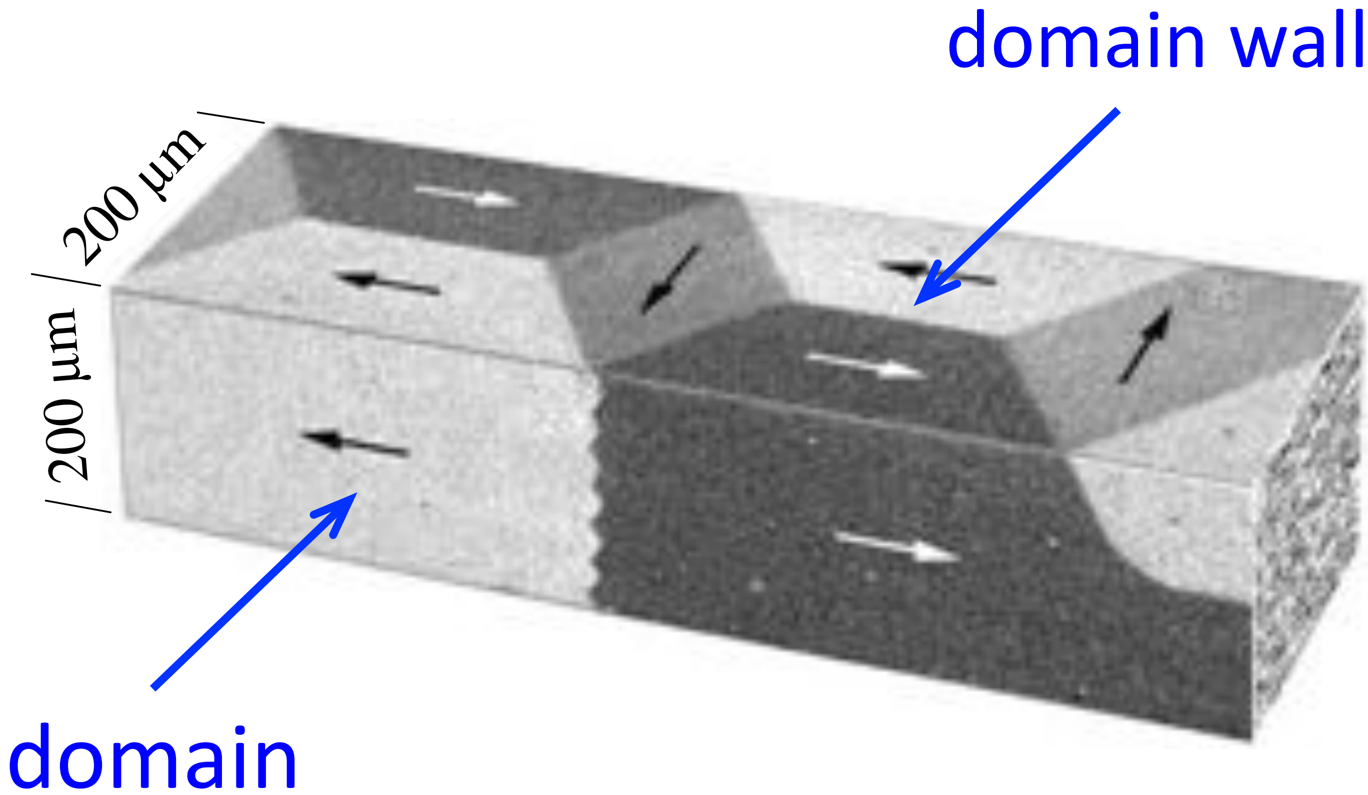
1. History & motivation

2. Observation techniques

3. The origin of domains

4. Domain walls

5. Domain wall motion

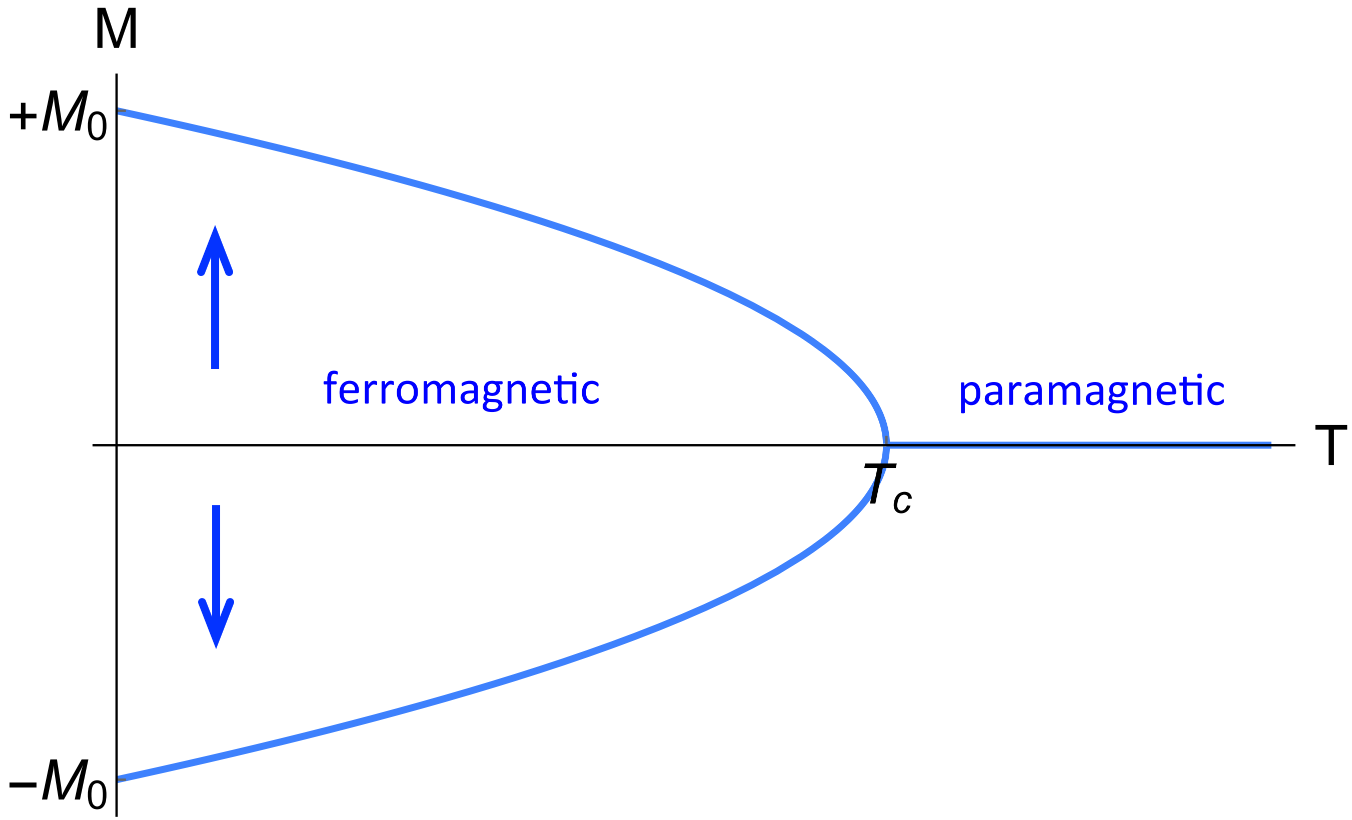

domain wall
200 µm
200 µm
domain

The concept of **domain:**

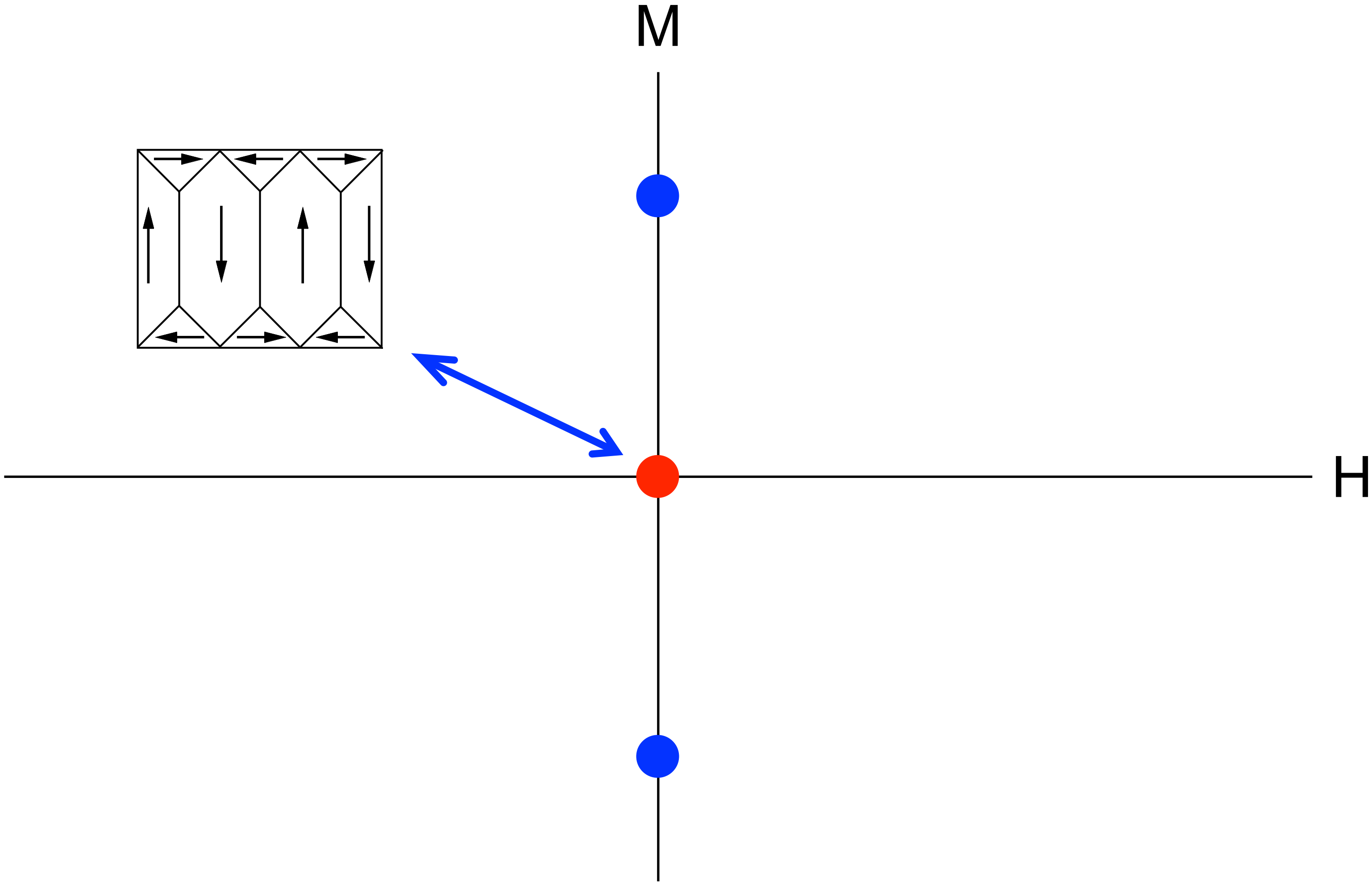
Postulated by Pierre Weiss in 1907 to explain why ferromagnetic bodies can appear non-magnetic.

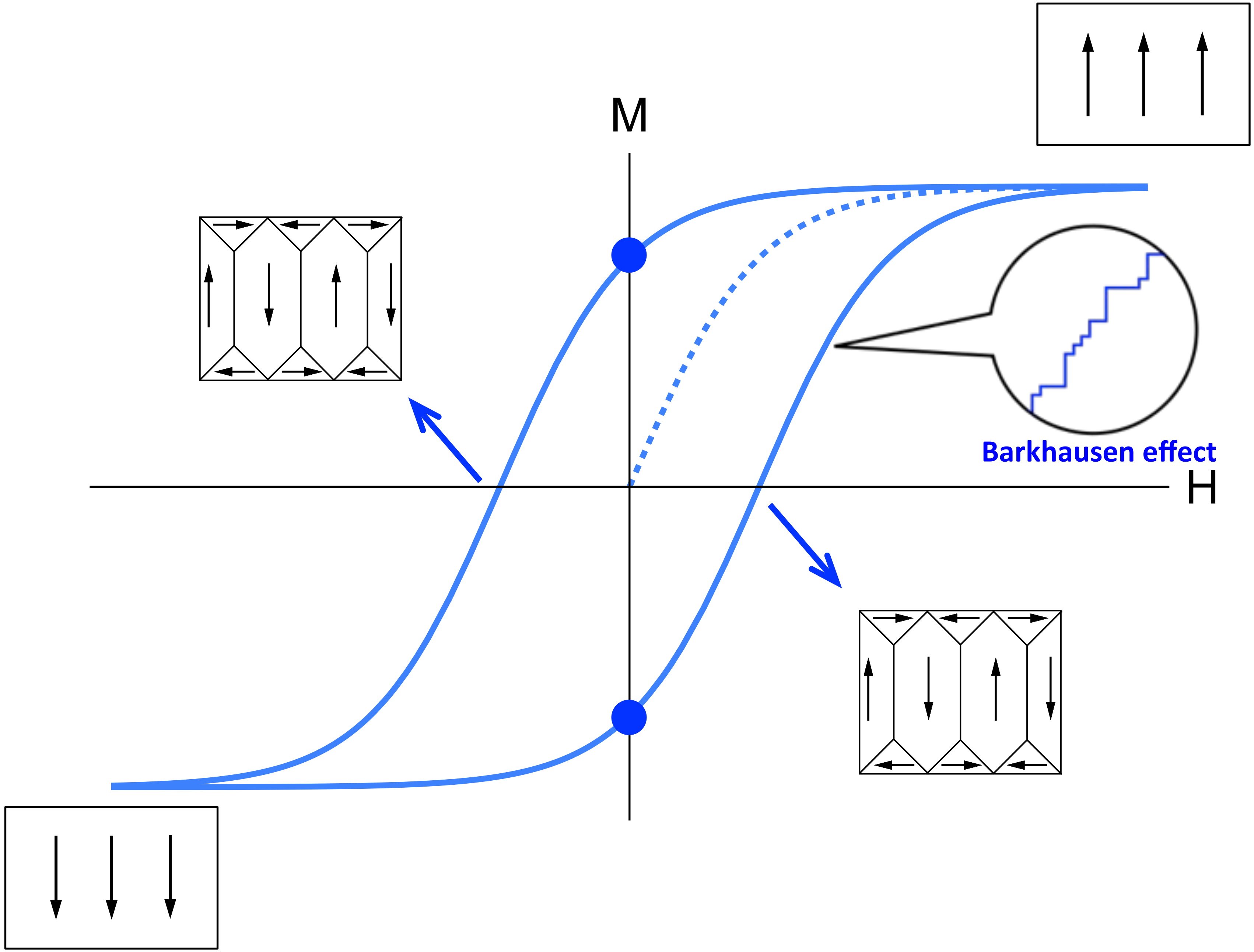
110 aniversary!

$$F = a(T - T_c)M^2 + bM^4$$

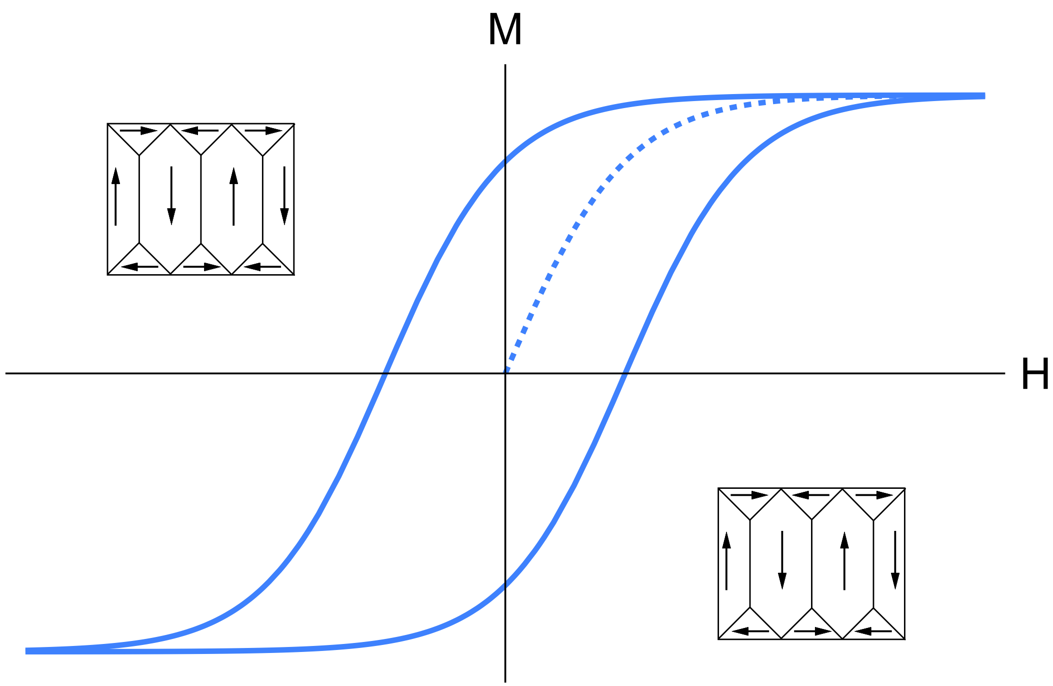


Two possible states below T_c


M
H


M
H
Barkhausen effect

The distinct response of ferromagnets is inherently related to domains (and domain walls)

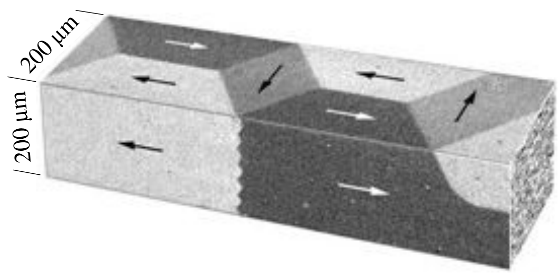


Outline

1. History & motivation
2. **Observation techniques**
3. The origin of domains
4. Domain walls
5. Domain wall motion

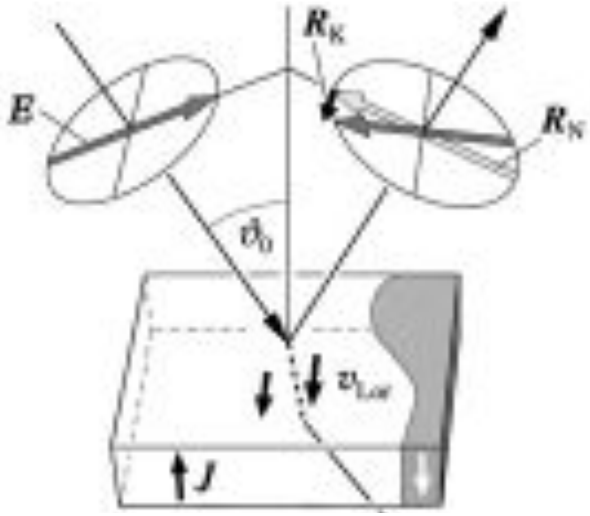
Observation techniques

Magneto-optical Kerr effect (MOKE)



$$\mathbf{D} = \varepsilon(\mathbf{E} + iQ\mathbf{M} \times \mathbf{E})$$

weak (but detectable) dependence on the magnetization of the optical constants

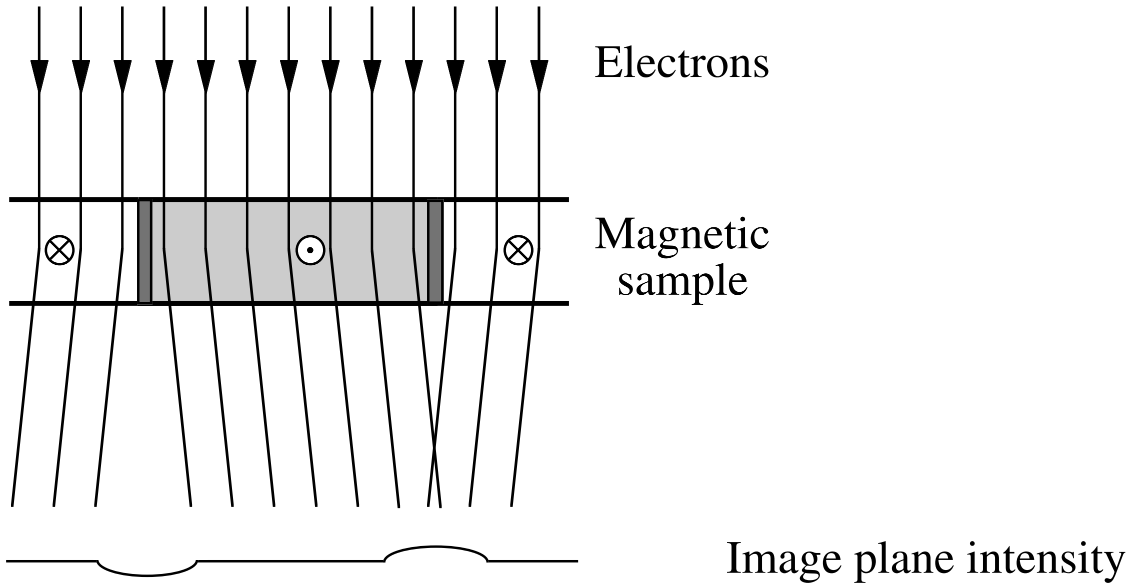


Observation techniques

Transmission Electron Microscopy (TEM)

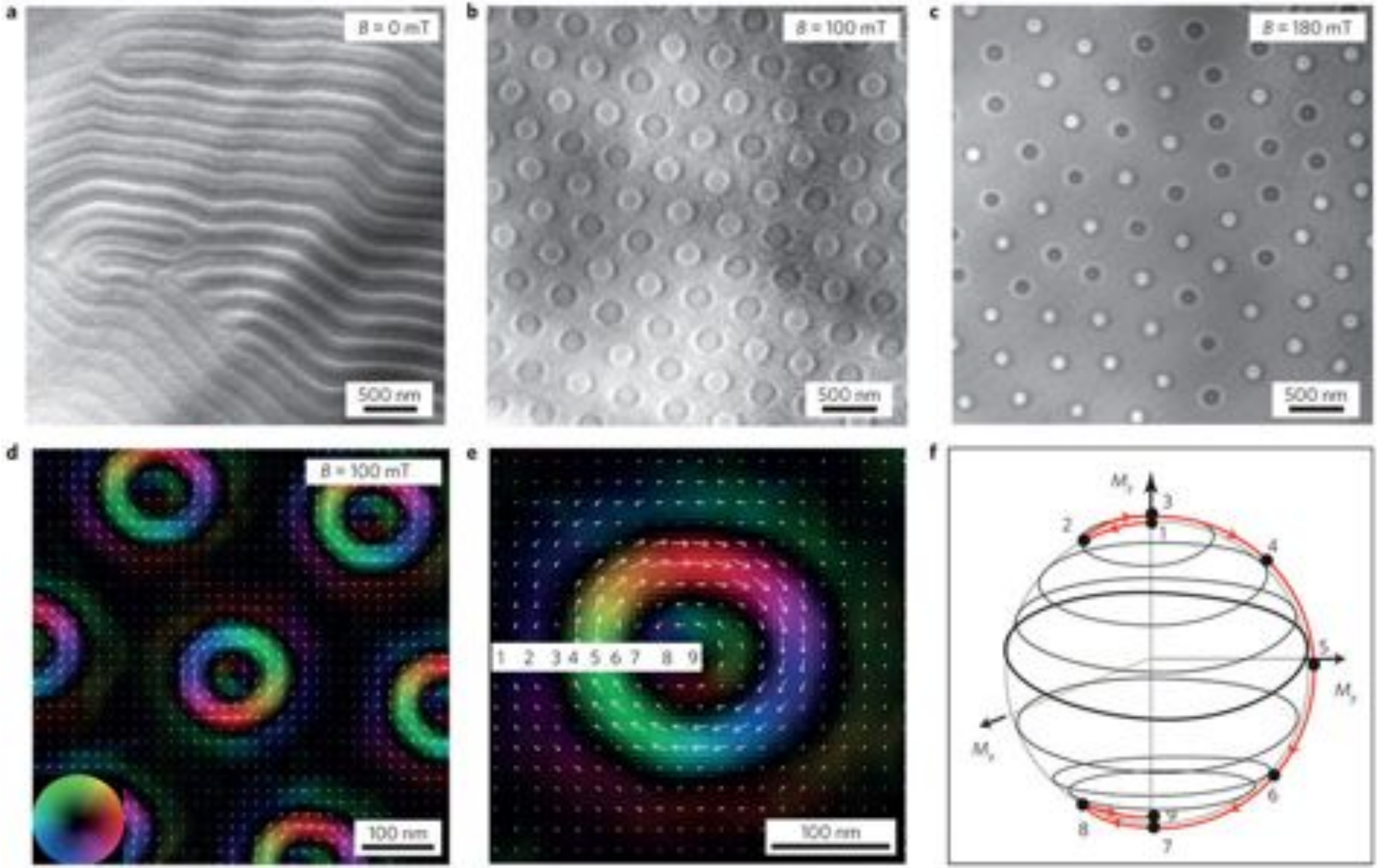
$$\mathbf{F}_{\text{Lorentz}} = q(\mathbf{v} \times \mathbf{B})$$

electrons are deflected by the Lorentz force



Observation techniques

Transmission Electron Microscopy (TEM)



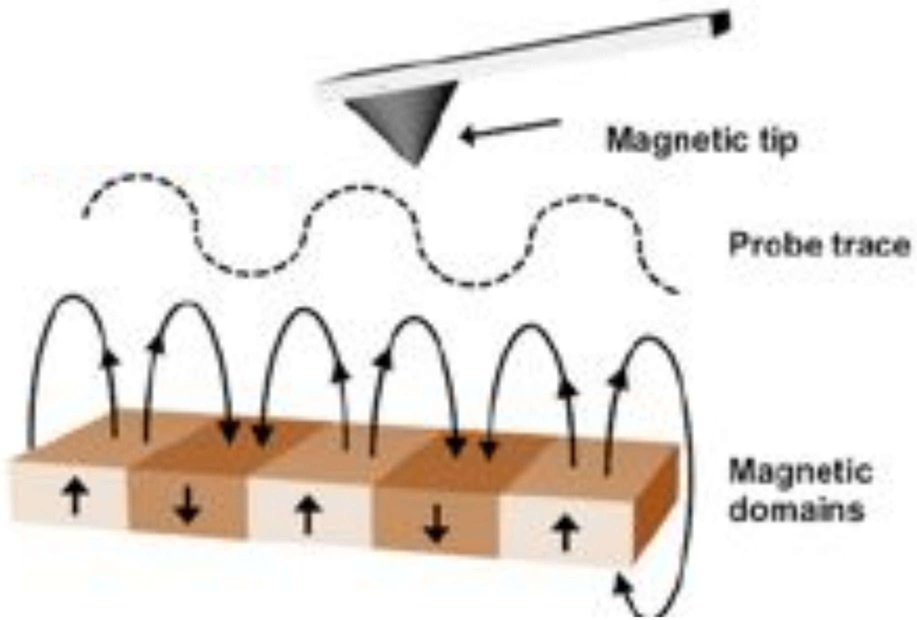
Observation techniques

$$\overbrace{\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}}^{\nabla \cdot \mathbf{B} = 0}$$

longitudinal variations of $\mathbf{M}$ are a source of magnetic field (stray field)

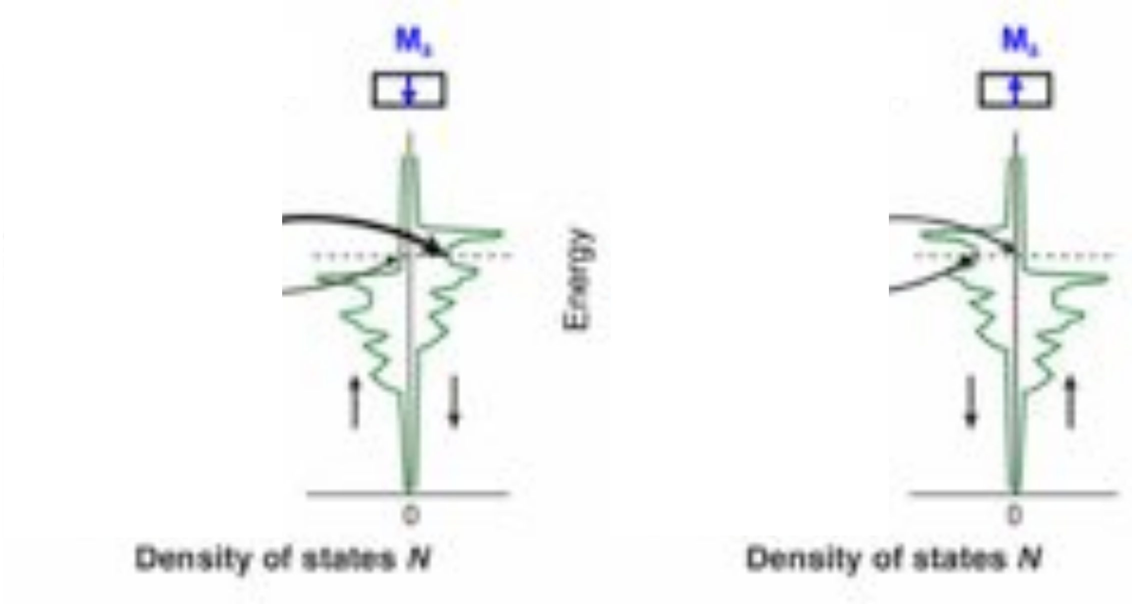
Magnetic force microscopy (MFM)

$$\mathbf{F} = \mu_0(\mathbf{m}_{\mathrm{tip}} \cdot \nabla)\mathbf{H}_{\mathrm{stray}}$$

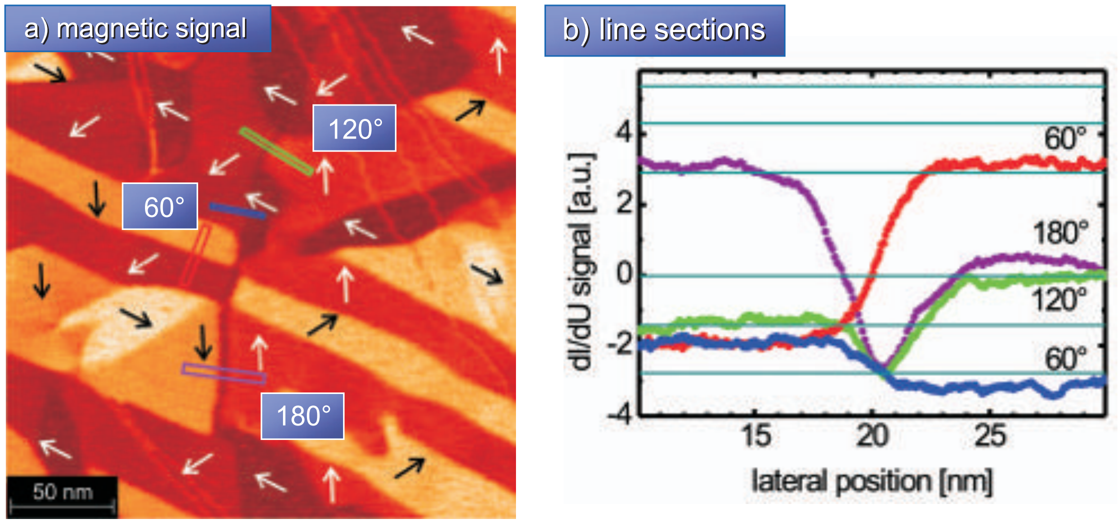


Observation techniques

Spin-polarized scanning-tunneling microscopy (SP-STM)

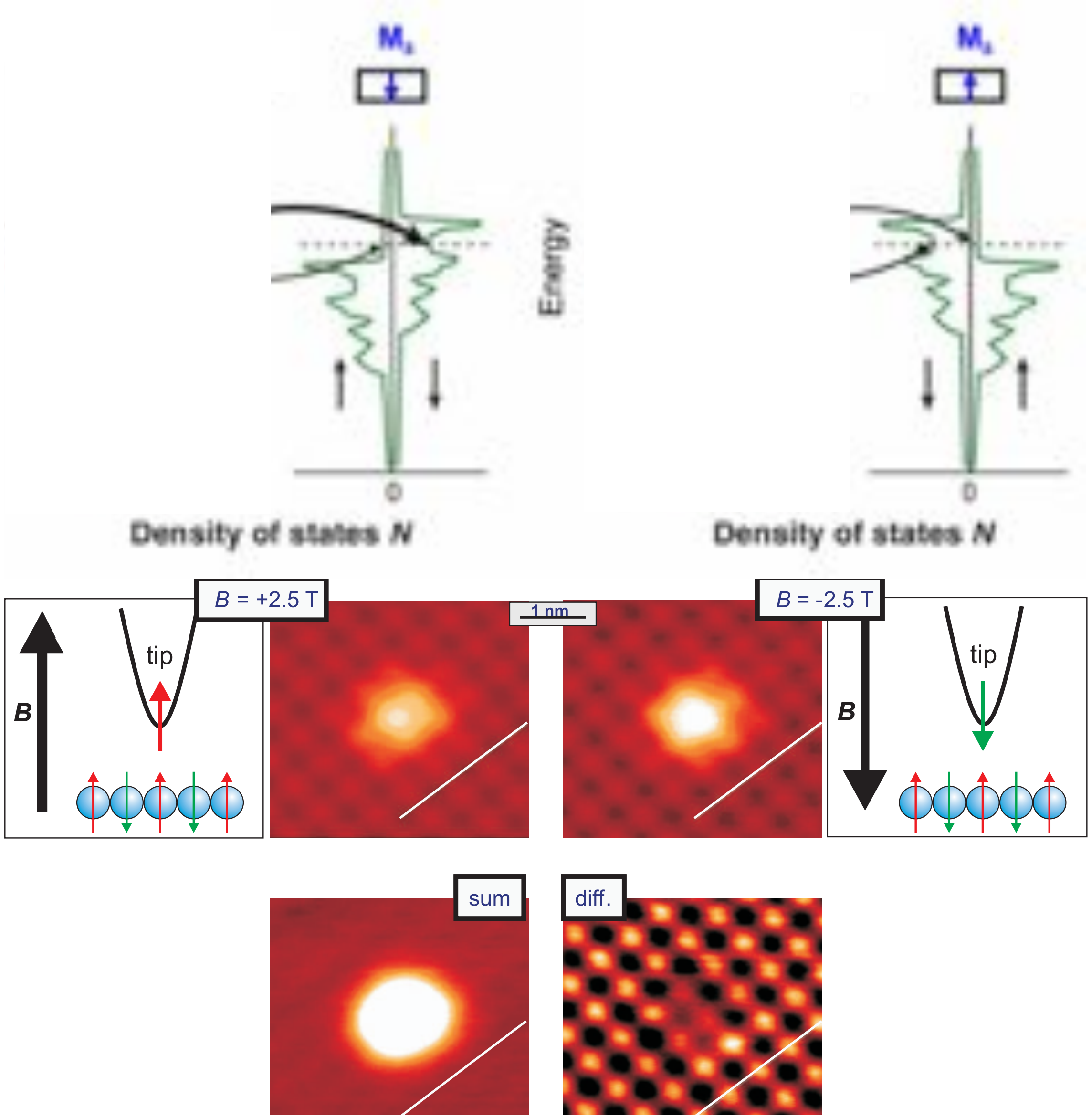


90ML Dy/W(110)



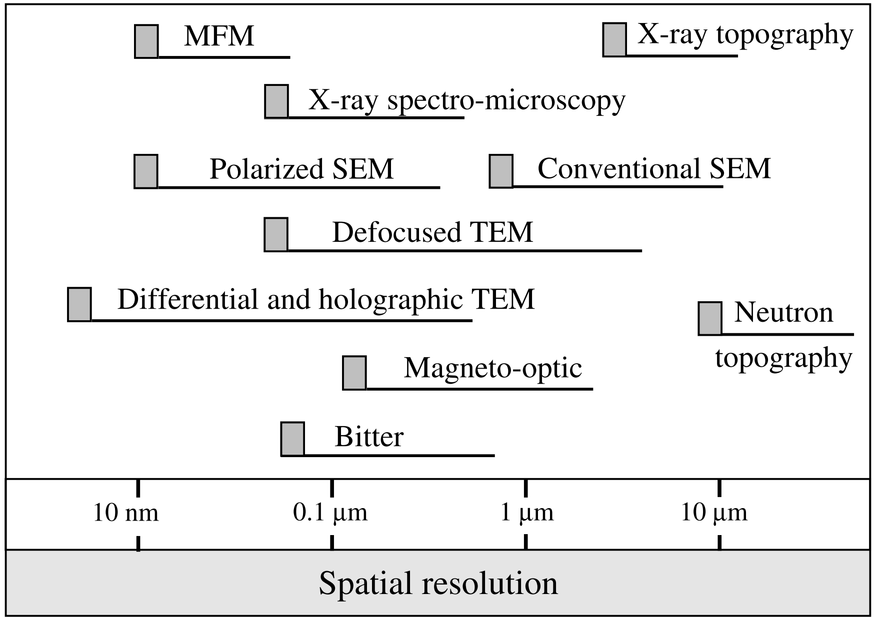
Observation techniques

Spin-polarized scanning-tunneling microscopy (SP-STM)



1ML Fe/W(110)

Observation techniques



Method of domain observation	**Sensitivity to small variations in magnetization**	**Evaluation of the magnetization vector**	**Allowed magnetic field range**	**Sample preparation quality requirements**	**Necessary capital investment**
Bitter	very good	indirect	100 A/cm	moderate–low	low
Magneto-optic	fair	direct	any	high	moderate
Digital MO	good	quantitative	any	moderate	high
Defocused TEM	very good	indirect	3000 A/cm	high	high
Differential TEM	good	quantitative	1000 A/cm	high	very high
Holograph. TEM	good	quantitative	100 A/cm	very high	very high
Secondary SEM	poor	indirect	100 A/cm	low	high
Backscatt. SEM	poor	rather direct	300 A/cm	moderate–low	high
Pol. SEM	good	quantitative	100 A/cm	very high	very high
X-Ray topography	poor	indirect	any	moderate	extremely high
Neutron	poor	indirect	any	low	extremely high
MFM	good	indirect	3000 A/cm	low	moderate

Outline

1. History & motivation

2. Observation techniques

3. The origin of domains

4. Domain walls

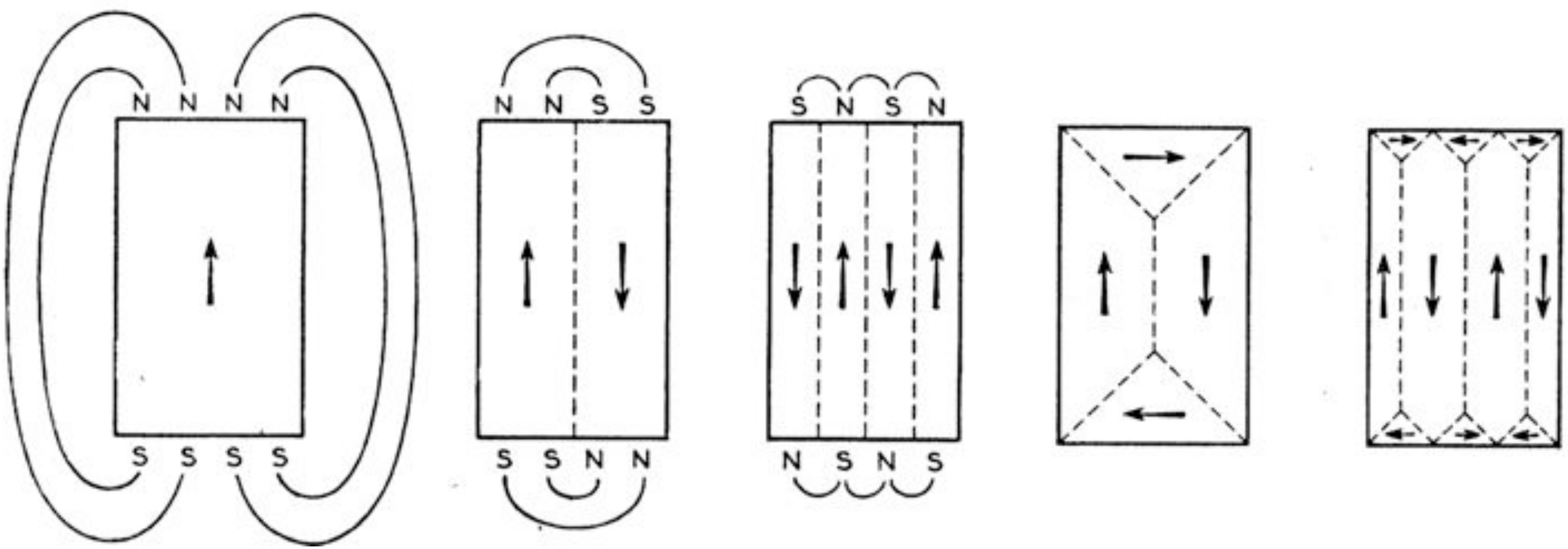
5. Domain wall motion

The origin of domains

On the theory of the dispersion of magnetic permeability in ferromagnetic bodies
Landau & Lifshitz, Phys. Z. Sowjet. 8, 153 (1935)

The origin of domains

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M} \quad \longrightarrow \quad E_{\text{magnetostatic}} = \frac{\mu_0}{2} \int H_d^2(\mathbf{M}) dV$$



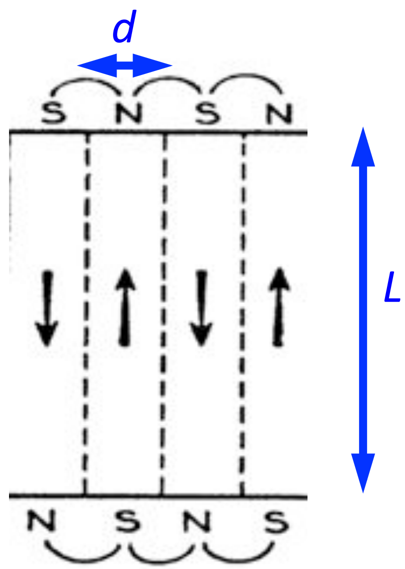
domains form to minimize the magnetic energy

$$\nabla \cdot \mathbf{M} = 0$$

$$\hat{\mathbf{n}} \cdot \mathbf{M}|_{\text{surface}} = 0$$

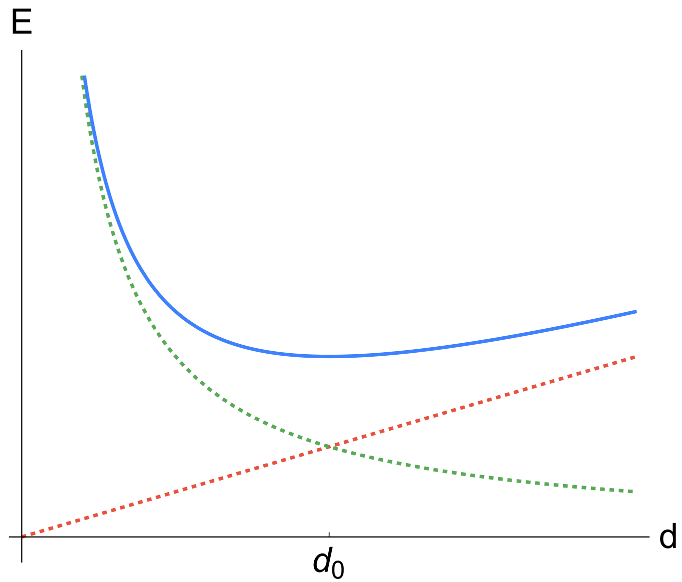
Size of domains (*d*)

$$E(d,L) \Longrightarrow \left.\frac{\partial E(d,L)}{\partial d}\right|_{d_0} = 0$$



strategy: compute the total magnetic energy of the system and determine *d* from the principle of minimum energy

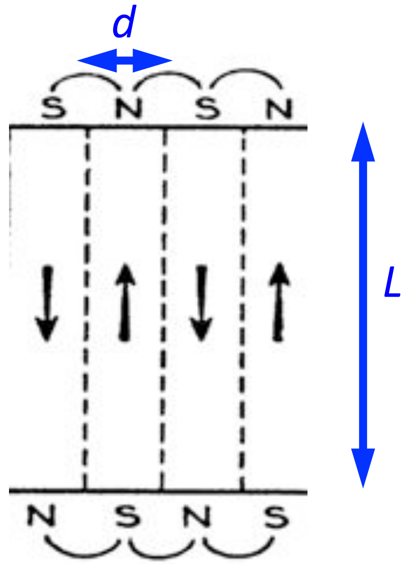
$$E(d,L) = \underbrace{1.7M^2 d}_{\text{magnetostatic}} + \underbrace{\varepsilon_{\rm dw}(L/d)}_{\text{domain wall}}$$



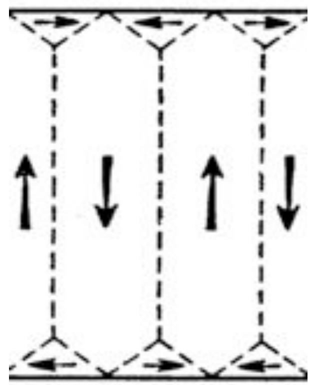
$d_0 \sim L^{1/2}$ Kittel's law

Size of domains (*d*)

$$E_{\rm stripe}(d,L) = \underbrace{1.7 M^2 d}_{\text{magnetostatic}} + \underbrace{\varepsilon_{\rm dw}(L/d)}_{\text{domain wall}}$$



$$E_{\text{flux clousure}}(d,L) = \underbrace{\frac{K}{2} M^2 d}_{\text{anisotropy}} + \underbrace{\varepsilon_{\rm dw}(L/d)}_{\text{domain wall}}$$



$d_0 \sim L^{1/2}$ Kittel's law

To know the actual size of the domains we need
to determine the **energy of the domain walls**

Outline

1. History & motivation

2. Observation techniques

3. The origin of domains

4. **Domain walls**

5. Domain wall motion

Domain walls

Micromagnetic formalism

$$\overbrace{\mathbf{S}_i = \mathbf{S}(\mathbf{r}_i)}^{\text{discrete}} \quad \longrightarrow \quad \overbrace{\mathbf{S}(\mathbf{r}) = \mathbf{M}(\mathbf{r})/M_s}^{\text{continuous}}$$

$$\mathbf{S}(\mathbf{r}_i) \cdot \mathbf{S}(\mathbf{r}_i + \Delta\mathbf{r}_i) \quad \longrightarrow \quad 1 - \frac{1}{2}(\Delta\mathbf{r}_i \cdot \nabla\mathbf{S}|_{\mathbf{r}=\mathbf{r}_i})^2$$

$$H_{\text{ex}} = -\sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \quad \longrightarrow \quad \int A\Big[\nabla\Big(\frac{\mathbf{M}(\mathbf{r})}{M_s}\Big)\Big]^2 dv \quad (A \sim J/a)$$

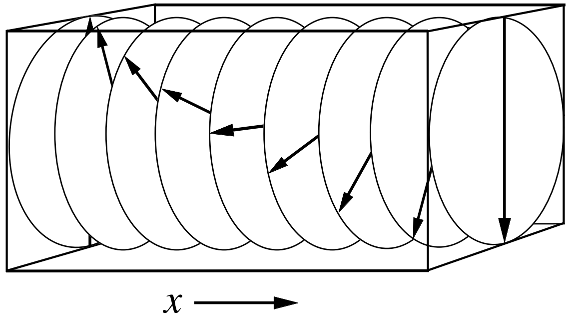
Domain walls

Uniaxial ferromagnet (*m* = *M*/*M*$_s$)

$$E = \int \Big\{ \underbrace{A(\nabla \mathbf{m})^2}_{\text{exchange}} + \underbrace{K(m_x^2 + m_y^2)}_{\text{anisotropy}} - \underbrace{\frac{\mu_0}{2}\mathbf{M}\cdot\mathbf{H}_d(\mathbf{M})}_{\text{stray field}} \Big\} dv$$

$$\mathbf{m} = \big(0,\, \sin\theta(x),\, \cos\theta(x)\big)$$

Bloch wall $(\nabla \cdot \mathbf{M} = 0)$



$$\varepsilon_{dw} = \int_{-\infty}^{\infty} (A\theta'^2 + K\sin^2\theta)dx$$

$$A\theta'' - K\sin\theta\cos\theta = 0$$

$$\tfrac{d}{dx}\left[A(\tfrac{d\theta}{dx})^2 + K\cos^2\theta\right] = 0$$

$$\ddot\theta - \tfrac{g}{l}\sin\theta\cos\theta = 0$$

$$\tfrac{A}{K}(\tfrac{d\theta}{dx})^2 + \cos^2\theta = 1$$

$$\pm\sqrt{\tfrac{A}{K}}\tfrac{d\theta}{dx} = \sqrt{1-\cos^2\theta} = \sin\theta$$

$$\tfrac{d\theta}{\sin\theta} = \pm\sqrt{\tfrac{K}{A}}dx \quad \rightarrow \quad \ln\tan\tfrac{\theta}{2} = \sqrt{\tfrac{K}{A}}(x - X)$$

Domain walls

Uniaxial ferromagnet (***m*** = ***M***/M_s)

$$E = \int \Big\{ \underbrace{A(\nabla \mathbf{m})^2}_{\text{exchange}} + \underbrace{K(m_x^2 + m_y^2)}_{\text{anisotropy}} - \underbrace{\frac{\mu_0}{2}\mathbf{M}\cdot\mathbf{H}_d(\mathbf{M})}_{\text{stray field}} \Big\} dv$$

$$\mathbf{m} = \big(0,\ \sin\theta(x),\ \cos\theta(x)\big)$$

$$\varepsilon_{dw} = \int_{-\infty}^{\infty} (A\theta'^2 + K\sin^2\theta)dx$$

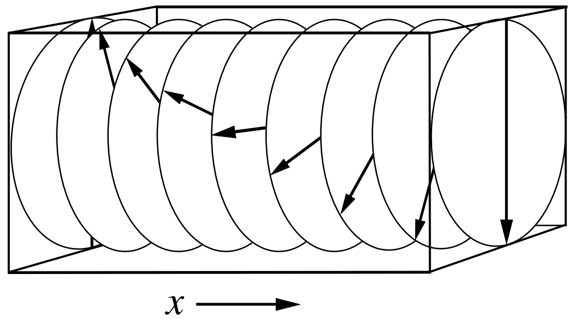
$$A\theta'' - K\sin\theta\cos\theta = 0$$

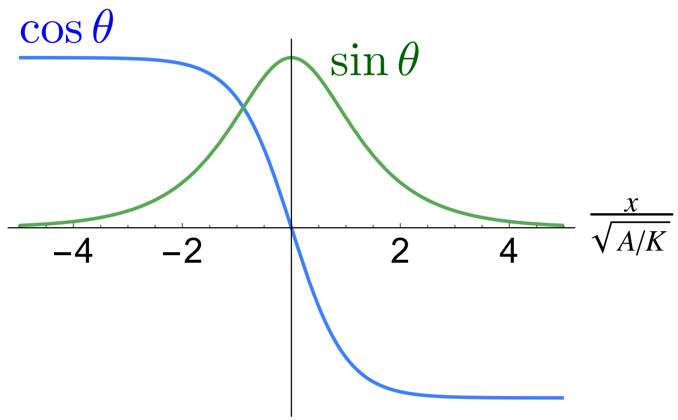
$$\theta(x) = 2\arctan[\exp(x/w)]$$

$$w = \sqrt{A/K}$$

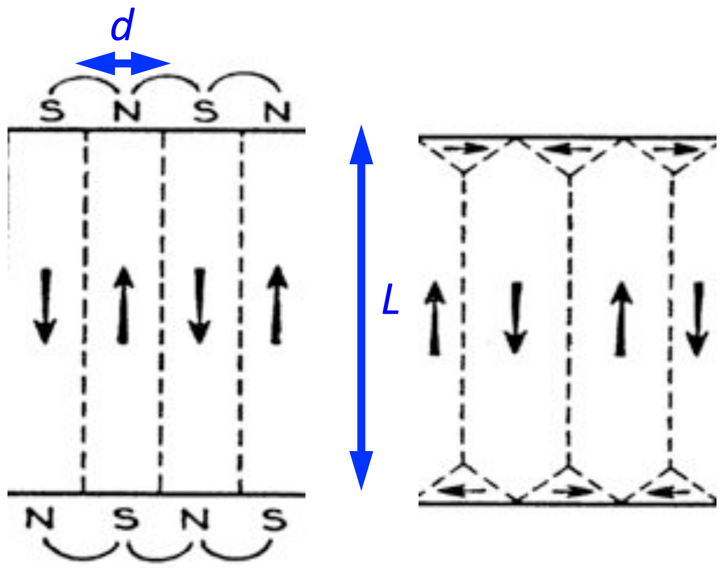
$$\varepsilon_{dw} = 4\sqrt{AK}$$

Bloch wall $(\nabla\cdot\mathbf{M} = 0)$



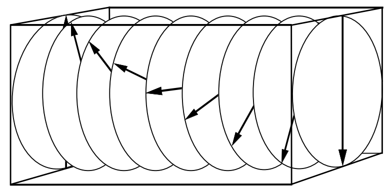


Domains & domain walls



Fe, Ni

$$w \sim 10 \text{ nm}$$
$$d_0(L = 1 \text{ cm}) \sim 10 \ \mu\text{m}$$

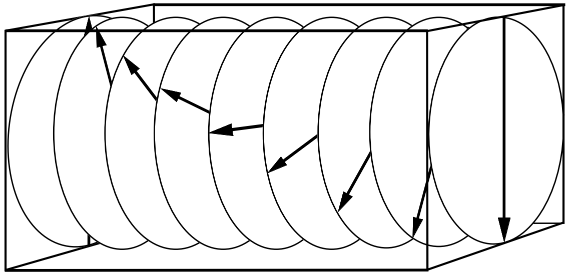


$$E_{dw}^{d=d_0} \sim E_{\text{magnetostatic}}^{d=d_0} \ll E_{\text{magnetostatic}}^{d\to\infty}$$

$d_0 \sim (wL)^{1/2} \quad \longrightarrow$ **single-domain state** if $\quad L \lesssim w$

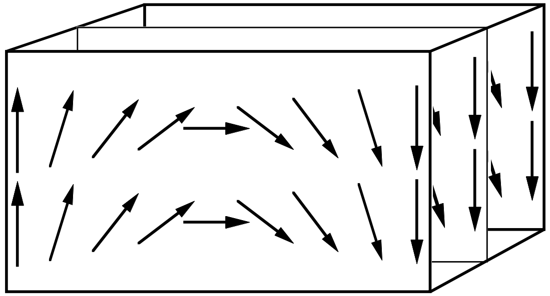
Domain walls in thin films

Bloch wall



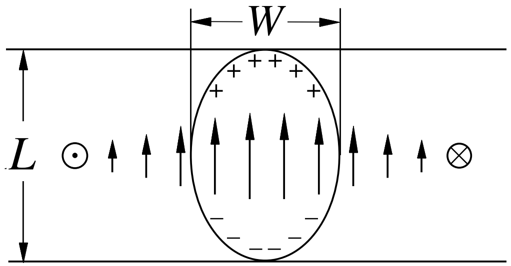
$$\nabla \cdot \mathbf{M} = 0$$

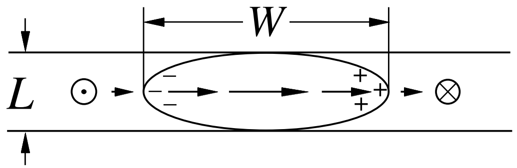
Neel wall



$$\nabla \cdot \mathbf{M} \neq 0$$

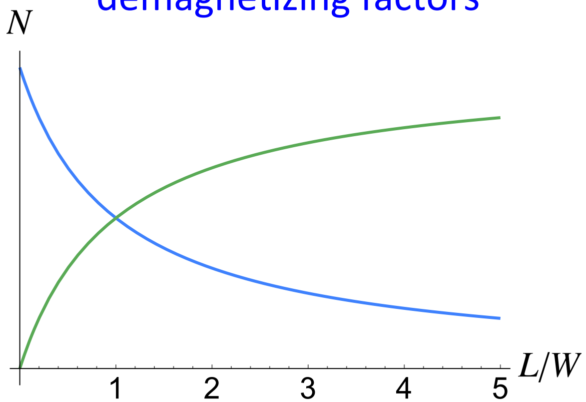
top view





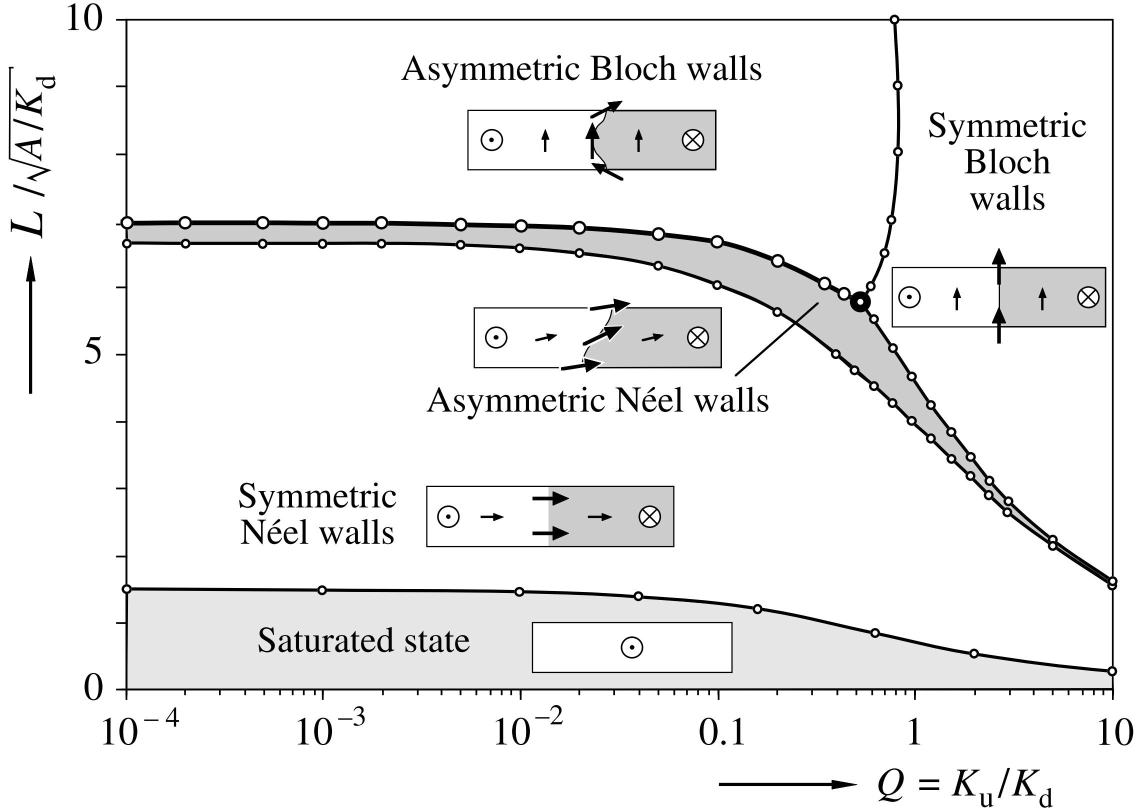
$$N_{\text{Bloch}} = \frac{W}{W + L}$$

demagnetizing factors



$$N_{\text{Néel}} = \frac{L}{W + L}$$

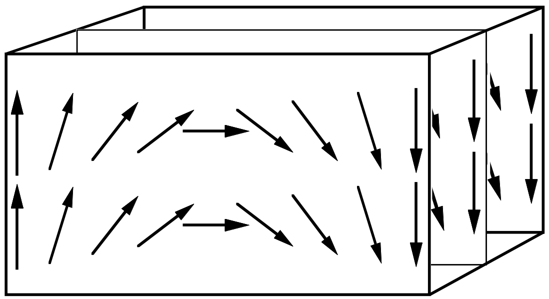
Domain walls in thin films



- Multi-dimensional description due to the stray fields
- Additional length scales
- Analytical -> numerical calculations & ansatzs + variational procedures

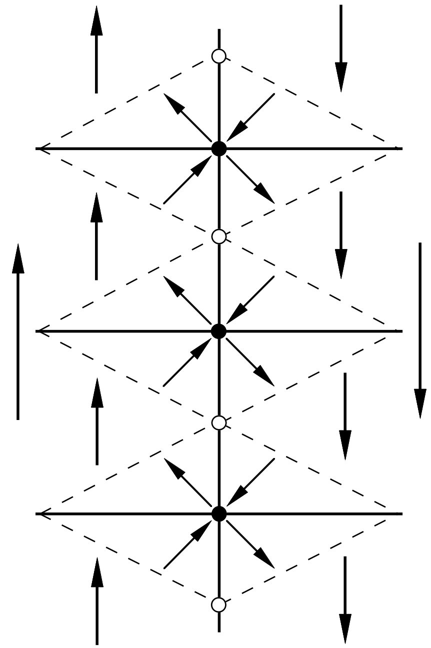
Domain walls in thin films

Neel wall

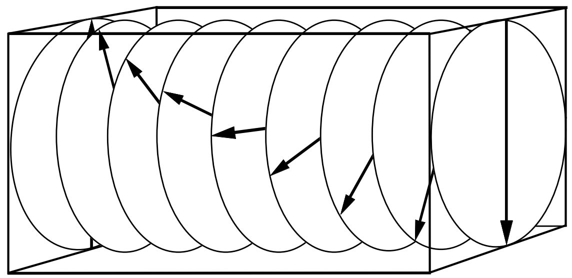


$$\nabla \cdot \mathbf{M} \neq 0$$

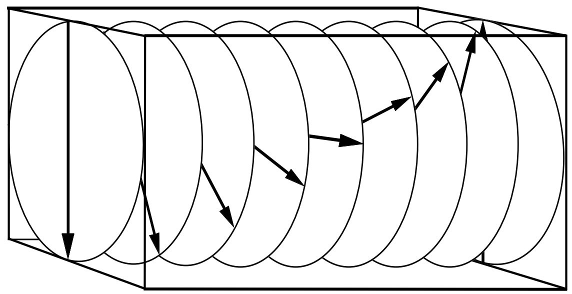
cross-tie wall



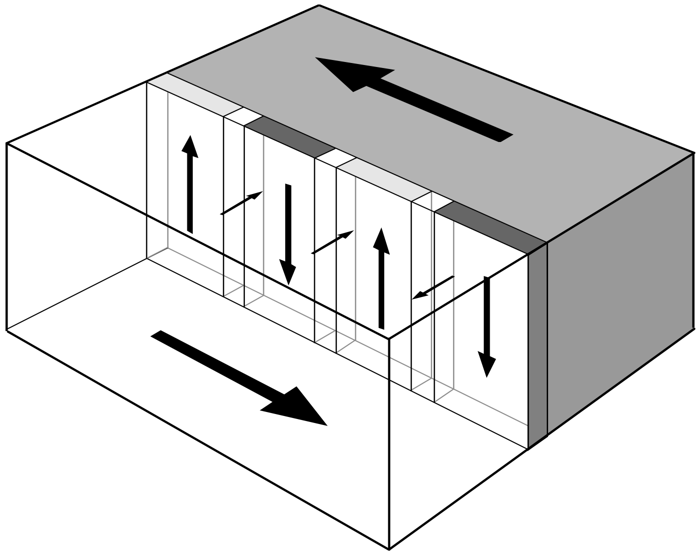
Bloch walls



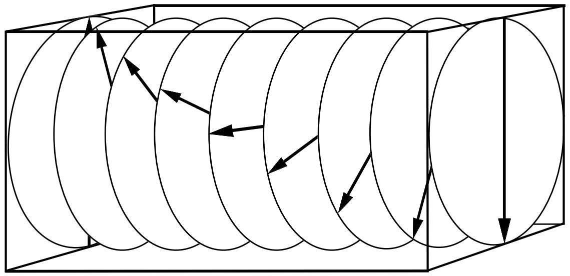
two (equivalent)
rotation senses



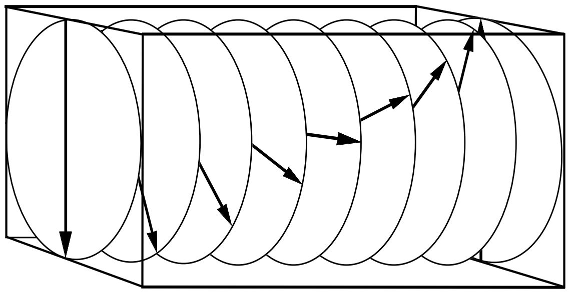
Bloch lines & Bloch points



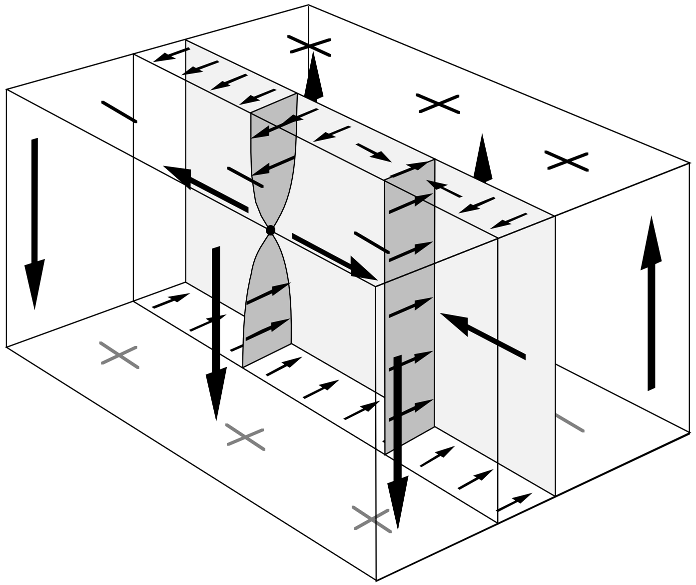
Bloch walls



two (equivalent)
rotation senses



Bloch lines & Bloch points



Outline

1. History & motivation

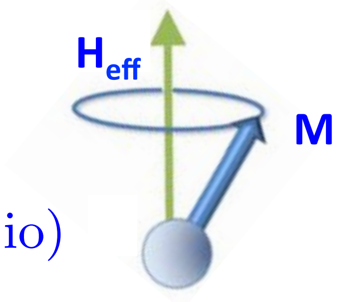
2. Observation techniques

3. The origin of domains

4. Domain walls

5. Domain wall motion

Domain wall motion



$$\dot{\mathbf{M}} = -\gamma \underbrace{\mathbf{M} \times \mathbf{H}_{\text{eff}}}_{\text{torque}} \qquad \gamma = \frac{\mu_0 g e}{2m_e} \text{ (gyromagnetic ratio)}$$

$$E = \int [A(\nabla \mathbf{M})^2 - K M_z^2 - \mathbf{M} \cdot \mathbf{H}_{\text{tot}}] dv \qquad \mathbf{H}_{\text{tot}} = \mathbf{H}_{\text{ext}} + \mathbf{H}_{\text{d}}$$

$$\delta E = \int [\underbrace{(A\nabla^2 \mathbf{M} + K M_z \hat{\mathbf{e}}_z + \mathbf{H}_{\text{tot}})}_{\mathbf{H}_{\text{eff}}} \cdot \delta \mathbf{M}] dv$$

Landau-Lifshitz-Gilbert equation:

$$\dot{\mathbf{M}} = -\gamma \underbrace{\mathbf{M} \times \mathbf{H}_{\text{eff}}}_{\text{torque}} - \underbrace{\alpha \mathbf{M} \times \dot{\mathbf{M}}}_{\text{damping}}$$

Domain wall motion

$$\dot{\mathbf{m}} = -\gamma\, \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha \mathbf{m} \times \dot{\mathbf{m}}$$

$$E = \int [A(\nabla \mathbf{m})^2 - K m_z^2 + \underbrace{K_d m_y^2}_{K_d = \frac{\mu_0}{2} M_s^2} - \mathbf{M} \cdot \mathbf{H}_{\text{ext}}] d^3 x$$

$m_y = 0 \;\rightarrow\;$ stray-field-free wall

$$\mathbf{m} = -\gamma(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\dot{\theta} - \alpha\dot{\phi}\sin\theta = \frac{2\gamma}{M_s}[-\frac{A}{\sin\theta}\nabla\cdot(\sin^2\theta\nabla\phi) + \frac{K_d}{2}\sin\theta\sin 2\phi]$$

$$\dot{\phi}\sin\theta + \alpha\dot{\theta} = \frac{2\gamma}{M_s}\left\{A\left[\nabla^2\theta - \frac{1}{2}\sin 2\theta(\nabla\phi)^2\right] - \frac{K + K_d\sin^2\phi}{2}\sin 2\theta\right\} + \gamma H\sin\theta$$

Domain wall motion

$$\dot{\mathbf{m}} = -\gamma\, \mathbf{m} \times \mathbf{H}_{\text{eff}} - \alpha \mathbf{m} \times \dot{\mathbf{m}}$$

$$E = \int [A(\nabla \mathbf{m})^2 - K m_z^2 + \underbrace{K_d m_y^2}_{K_d = \frac{\mu_0}{2} M_s^2} - \mathbf{M} \cdot \mathbf{H}_{\text{ext}}] d^3x$$

$m_y = 0 \rightarrow$ stray-field-free wall

$$\mathbf{m} = -\gamma(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\dot{\theta} - \xcancel{\alpha\dot{\phi}\sin\theta} = \frac{2\gamma}{M_s}[-\frac{A}{\sin\theta}\xcancel{\nabla\cdot(\sin^2}\theta\nabla\phi) + \frac{K_d}{2}\sin\theta\sin 2\phi]$$

$$\xcancel{\dot{\phi}\sin\theta} + \alpha\dot{\theta} = \frac{2\gamma}{M_s}\left\{A\left[\nabla^2\theta - \frac{1}{2}\sin\xcancel{2\theta(\nabla}\phi)^2\right] - \frac{K + K_d\sin^2\phi}{2}\sin 2\theta\right\} + \gamma H\sin\theta$$

$\theta = \theta(x, t)$ and $\phi =$ conts.

Domain wall motion

$$\dot{\mathbf{m}} = -\gamma\,\mathbf{m}\times\mathbf{H}_{\text{eff}} - \alpha\mathbf{m}\times\dot{\mathbf{m}}$$

$$E = \int [A(\nabla\mathbf{m})^2 - Km_z^2 + \underbrace{K_d m_y^2}_{K_d = \frac{\mu_0}{2}M_s^2} - \mathbf{M}\cdot\mathbf{H}_{\text{ext}}]d^3x$$

$m_y = 0 \rightarrow$ stray-field-free wall

$$\mathbf{m} = -\gamma(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$

$$\dot{\theta} - \cancel{\alpha\dot{\phi}\sin\theta} = \frac{2\gamma}{M_s}[-\frac{A}{\sin\theta}\cancel{\nabla\cdot(\sin^2\theta\nabla\phi)} + \frac{K_d}{2}\sin\theta\sin 2\phi]$$

$$\cancel{\dot{\phi}\sin\theta} + \alpha\dot{\theta} = \frac{2\gamma}{M_s}\{A[\nabla^2\theta - \frac{1}{2}\cancel{\sin 2\theta(\nabla\phi)^2}] - \frac{K+K_d\sin^2\phi}{2}\sin 2\theta\} + \gamma H\sin\theta$$

$\theta = \theta(x,t)$ and $\phi =$ conts.

$$\gamma\underbrace{\left(\frac{\alpha K_d}{M_s}\sin 2\phi - H\right)}_{=0}\sin\theta = \frac{2\gamma}{M_s}\underbrace{\left(A\partial_x^2\theta - \frac{K+K_d\sin^2\phi}{2}\sin 2\theta\right)}_{=0}$$

Walker's solution

$$\theta(x,t) = 2\arctan\{\exp[\pm(x\pm vt)/w_*]\}, \quad \sin 2\phi = H/H_c$$

$$w_* = \sqrt{A/(K+K_d\sin^2\phi)}, \quad H_c = \frac{\alpha}{2}M_s, \quad v = \frac{\gamma}{\alpha}w_* H$$

$$\mathbf{m} = \left(\frac{\cos\phi}{\cosh[(x-vt)/w_*]}, \frac{\sin\phi}{\cosh[(x-vt)/w_*]}, \pm\tanh[(x-vt)/w_*]\right)$$

Domain wall motion

The wall moves at a constant speed (~ *H* for low fields).

If the speed increases the angle increases -> stray field & wall narrowing.

There is a maximum velocity.

There is a critical field above which this solution is not valid.

Walker's solution

$$\theta(x,t) = 2\arctan\{\exp[\pm(x \pm vt)/w_*]\}, \quad \sin 2\phi = H/H_c$$

$$w_* = \sqrt{A/(K + K_d \sin^2\phi)}, \quad H_c = \frac{\alpha}{2} M_s, \quad v = \frac{\gamma}{\alpha} w_* H$$

$$\mathbf{m} = \left(\frac{\cos\phi}{\cosh[(x-vt)/w_*]}, \frac{\sin\phi}{\cosh[(x-vt)/w_*]}, \pm\tanh[(x-vt)/w_*]\right)$$

Domain wall motion

Longitudinal susceptibility

$$v = \frac{\gamma}{\alpha} w_* H_\omega e^{i\omega t} \to \Delta x = \int_0^t v dt = \frac{\frac{\gamma}{\alpha} w_*}{i\omega} H_\omega e^{i\omega t} \to \Delta M_\omega = \frac{\frac{\gamma}{\alpha} w_*}{i\omega} H_\omega e^{i\omega t} \frac{L_z}{d} \times \text{Surface}$$

$$\chi_l(\omega) \equiv \frac{1}{V} \frac{\Delta M_\omega}{H_\omega e^{i\omega t}} = \frac{\gamma w_*}{i\omega\, \alpha d} \quad \text{(relaxation behavior with no resonance)}$$

Walker's solution

$$\theta(x,t) = 2 \arctan\{\exp[\pm(x \pm vt)/w_*]\}, \quad \sin 2\phi = H/H_c$$

$$w_* = \sqrt{A/(K + K_d \sin^2 \phi)}, \quad H_c = \frac{\alpha}{2} M_s, \quad v = \frac{\gamma}{\alpha} w_* H$$

$$\mathbf{m} = \left(\frac{\cos \phi}{\cosh[(x - vt)/w_*]}, \frac{\sin \phi}{\cosh[(x - vt)/w_*]}, \pm \tanh[(x - vt)/w_*] \right)$$

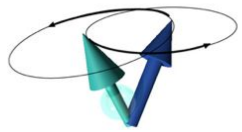
Domain wall motion

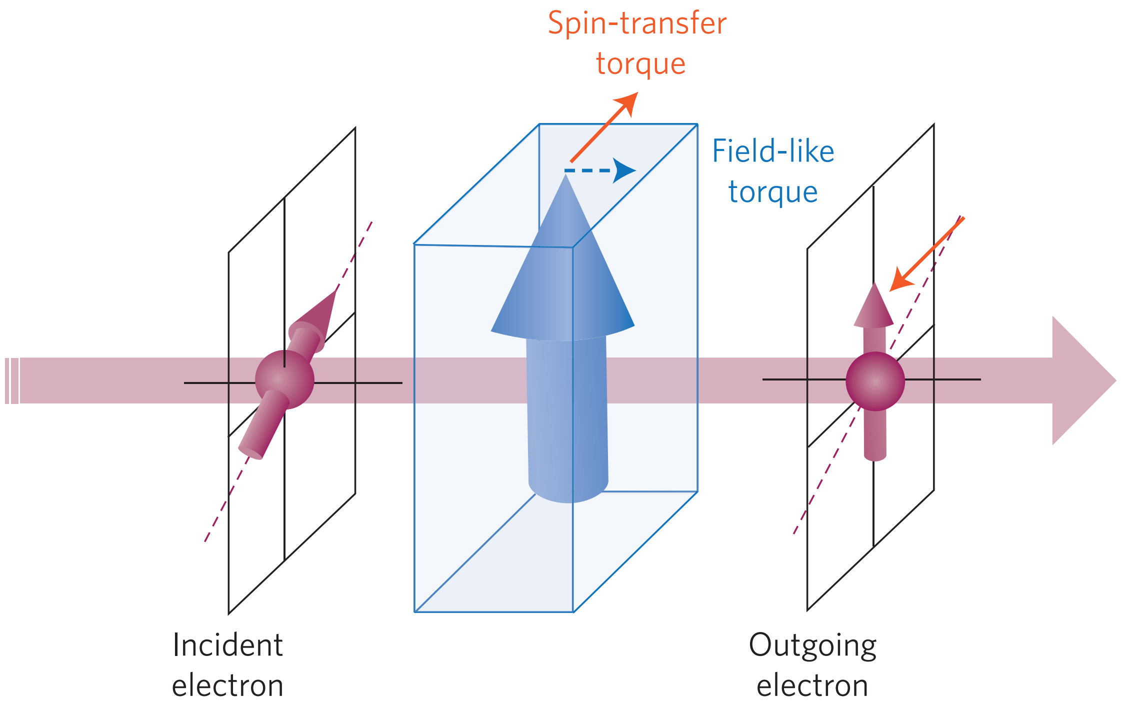
Field- vs. current-induced motion

$$\mathbf{M} = -\frac{g\mu_B}{a^3}\mathbf{S} = -\frac{\hbar\gamma}{a^3}\mathbf{S}$$

$$\dot{\mathbf{S}} = -\gamma\,\mathbf{S}\times(\mathbf{H}_{\text{eff}}+\mathbf{H}_0) - \frac{\alpha}{S}\mathbf{S}\times\dot{\mathbf{S}} - \underbrace{\frac{a^3}{2eS}(\mathbf{j}_s\cdot\nabla)\mathbf{S}}_{\text{spin-transfer torque}} - \underbrace{\frac{a^3\beta}{2eS^2}[\mathbf{S}\times(\mathbf{j}_s\cdot\nabla)\mathbf{S}]}_{\text{field-like torque}}$$

Exchange interaction between localized (3*d*) and itinerant spins $H_{\text{int}} = -J_{s\text{-}d}\int d^3x\,\mathbf{S}\cdot\underbrace{(c^\dagger\boldsymbol{\sigma}c)}_{\mathbf{s}}$

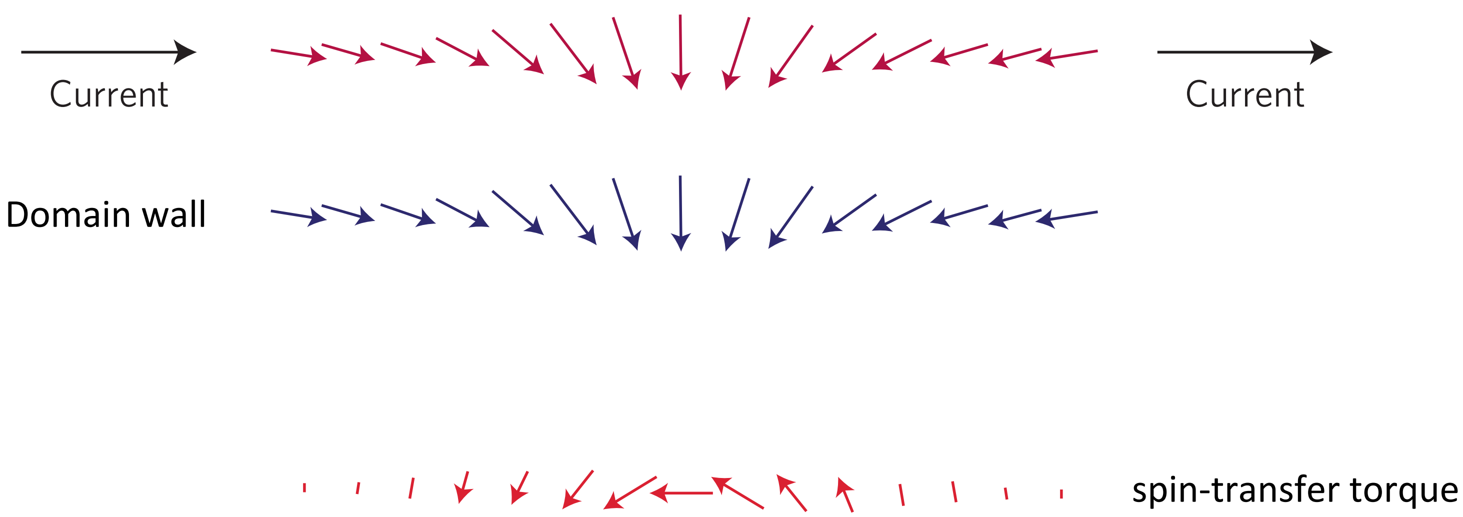




Domain wall motion

Field- vs. current-induced motion

$$\dot{\mathbf{S}} = -\gamma\,\mathbf{S}\times(\mathbf{H}_{\text{eff}}+\mathbf{H}_0) - \frac{\alpha}{S}\mathbf{S}\times\dot{\mathbf{S}} - \underbrace{\frac{a^3}{2eS}(\mathbf{j}_s\cdot\nabla)\mathbf{S}}_{\text{spin-transfer torque}} - \underbrace{\frac{a^3\beta}{2eS^2}[\mathbf{S}\times(\mathbf{j}_s\cdot\nabla)\mathbf{S}]}_{\text{field-like torque}}$$



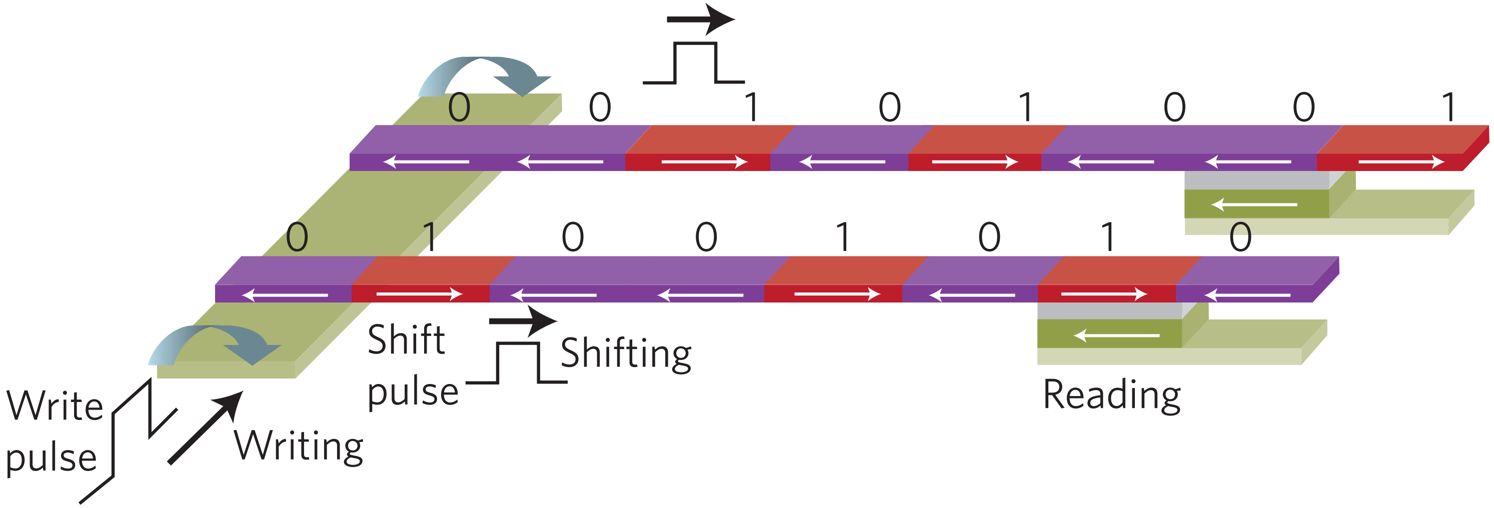
the angular moment lost by the electrons is transferred to the domain wall

Domain wall motion

Field- vs. current-induced motion

$$\dot{\mathbf{S}} = -\gamma\,\mathbf{S}\times(\mathbf{H}_{\rm eff}+\mathbf{H}_0) - \frac{\alpha}{S}\mathbf{S}\times\dot{\mathbf{S}} - \underbrace{\frac{a^3}{2eS}(\mathbf{j}_s\cdot\nabla)\mathbf{S}}_{\text{spin-transfer torque}} - \underbrace{\frac{a^3\beta}{2eS^2}[\mathbf{S}\times(\mathbf{j}_s\cdot\nabla)\mathbf{S}]}_{\text{field-like torque}}$$

racetrack-memory concept



Domain wall motion

Walker's solution

$$\theta = 2\arctan\left[\exp\left(\pm\frac{x \pm X(t)}{w_*}\right)\right]$$

$$\phi_0 = \text{constant}$$

$X(t)$ can be understood as the position of the wall

What is the conjugate momentum?

Landau-Lifshitz-Gilbert equation ⟷ Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L_S}{\partial \dot{q}} + \nabla\cdot\frac{\partial L_S}{\partial \nabla q} - \frac{\partial L_S}{\partial q} = -\frac{\partial W_S}{\partial \dot{q}} \qquad q = (\theta, \phi)$$

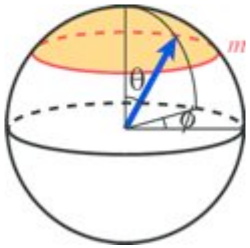
Spin Lagrangian

$$L_S = L_B - H_S \quad \left(\mathbf{M} = -\gamma\frac{\hbar S}{a^3}\mathbf{n}\right)$$

$$L_B = \int \frac{d^3x}{a^3}\hbar S\dot{\phi}(\cos\theta - 1)$$

L_B is a spin Berry phase

$$H_S = \frac{S^2}{2}\int\frac{d^3x}{a^3}\left[J(\nabla\mathbf{n})^2 - Kn_z^2 + K_\perp n_y^2 + \tfrac{2\gamma\hbar}{S}\mathbf{n}\cdot\mathbf{H}\right]$$



Dissipation function

$$W_S = \frac{\alpha\hbar S}{2}\int\frac{d^3x}{a^3}\dot{\mathbf{n}} = \frac{\alpha\hbar S}{2}\int\frac{d^3x}{a^3}(\dot{\theta}^2 + \dot{\phi}^2\sin\theta)$$

Domain wall motion

Walker's solution

$$\theta = 2\arctan\left[\exp\left(\pm\frac{x \pm X(t)}{w_*}\right)\right]$$

$$\phi_0 = \text{constant}$$

$X(t)$ can be understood as the position of the wall

What is the conjugate momentum?

Spin Lagrangian & dissipation function

$$L_S = -\frac{\hbar N S}{w_*}\left(\dot{\phi}_0 X + \tfrac{K_\perp S w}{2\hbar}\sin^2\phi_0 - \gamma X H\right)$$

$$W_S = \frac{\hbar N S}{w_*}\frac{\alpha w_*}{2}\left[\left(\frac{\dot{X}}{w_*}\right)^2 + \dot{\phi}_0^2\right]$$

X and ϕ_0 are conjugate variables

non-linear relation due to internal ϕ_0 degree of freedom (even if the wall is rigid)

Domain wall motion

Spin Lagrangian & dissipation function

$$L_S = -\frac{\hbar N S}{w_*}\left(\dot{\phi}_0 X + \tfrac{K_\perp S w}{2\hbar}\sin^2\phi_0 - \gamma X H\right)$$

$$W_S = \frac{\hbar N S}{w_*}\frac{\alpha w_*}{2}\left[\left(\frac{\dot{X}}{w_*}\right)^2 + \dot{\phi}_0^2\right]$$

Equations of motion for the rigid wall

$$\tfrac{1}{w_*}\dot{X} - \alpha\dot{\phi}_0 = \kappa_\perp \sin 2\phi_0$$

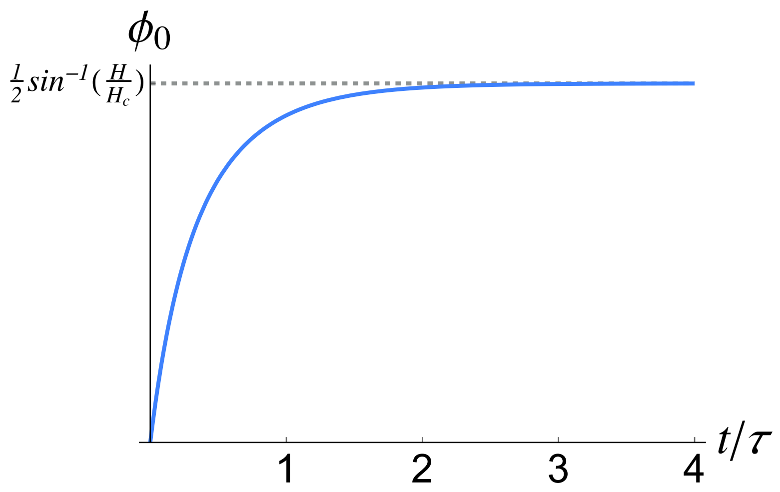
$$\dot{\phi}_0 + \tfrac{\alpha}{w_*}\dot{X} = \gamma H$$

Transient behavior

$$\phi_0(t) = \arctan\left(\frac{D\tanh(t/\tau)}{\tau^{-1} + C\tanh(t/\tau)}\right)$$

$$\tau^{-1} = \tfrac{1}{1+\alpha^2}\sqrt{[\alpha K_\perp S/(2\hbar)]^2 - (\gamma H)^2}$$

$$D = \frac{\gamma H}{1+\alpha^2}, \qquad C = \frac{\alpha K_\perp S/(2\hbar)}{1+\alpha^2}$$



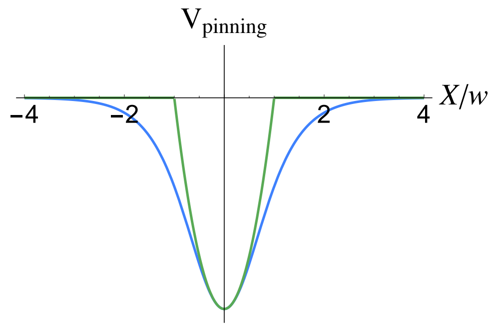
Domain wall motion

Pinning

$$V_{\text{pinning}} = -\int \frac{d^3x}{a^3} \Delta K \frac{(Sa)^3}{2} \delta(\mathbf{r}) \sin^2\theta$$

local change of the easy-axis anisotropy

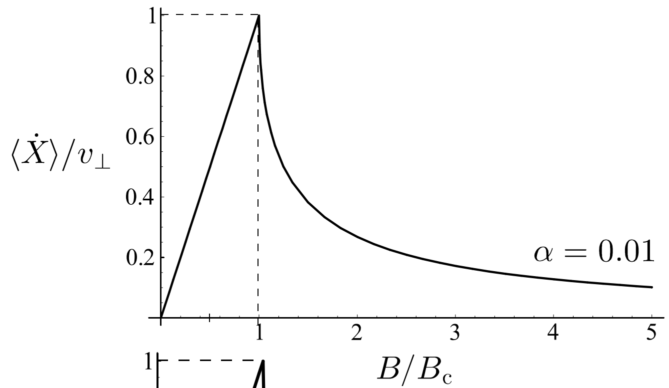
$$V_{\text{pinning}} = \frac{\Delta K S^2}{2} \frac{1}{\cosh^2(X/w)} \quad \rightarrow \quad \frac{M}{2}\Omega^2(X^2 - w^2)\Theta(w - |X|)$$

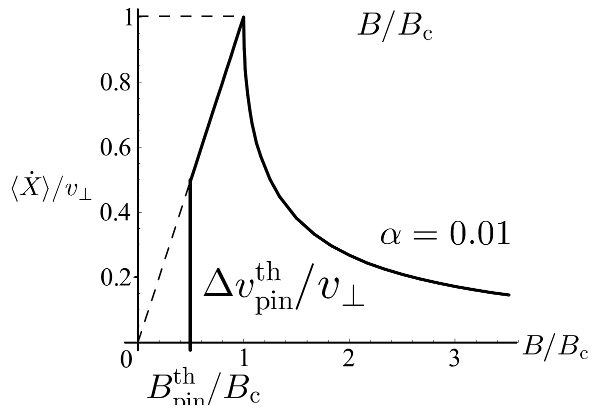


Equations of motion for the rigid wall

$$\frac{1}{w_*}\dot{X} - \alpha\dot{\phi}_0 = \kappa_\perp \sin 2\phi_0$$

$$\dot{\phi}_0 + \frac{\alpha}{w_*}\dot{X} = \gamma H - \underbrace{\nu_{\text{pin}} \frac{X}{w_*}\Theta(w - |X|)}_{F_{\text{pinning}}}$$





Domain wall motion

Equations of motion

$$\frac{1}{w_*}\dot{X} - \alpha\dot{\phi}_0 = \kappa_\perp \sin 2\phi_0$$

$$\dot{\phi}_0 + \frac{\alpha}{w_*}\dot{X} = \gamma H - \underbrace{\nu_{\rm pin}\frac{X}{w_*}\Theta(w - |X|)}_{F_{\rm pinning}}$$

Linear regime

$$\ddot{X} + 2\alpha\kappa_\perp\dot{X} + 2\nu_{\rm pin}\kappa_\perp X = 2\gamma\kappa_\perp w_* H$$

domain wall motion $\longrightarrow$ motion of an effective point particle