Introduction to Magnetic Frustration

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Outline of the lecture

@ On the route to frustration: ordering and time/dynamics issues of ordered magnets
- classical case
- quantum case
- stability of Néel states

@ Historical point of view
- A first example of frustration
- Condensed matter and statistical mechanics eventually meet
- Entropy is interesting

@ Phylogeny of frustration
- Study of a simple case
- What can we play with
- Well, it's not that simple...
- But frustration helps deconfinement (fractionalization)

@ Emergence in frustration
- Back to spin ice
- From spin to (magnetic) charge, and deconfinement
- Emergent gauge structure

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Fred brings home one too many cute refrigerator magnets.

Ferromagnetic material, at T<<Tc

« We » all expect it to stick to the fridge. Well, it should not...

Unless we break time reversal symmetry: $E_\uparrow = E_\downarrow$

At T=0, we have $p(\uparrow) = \frac{e^{-E_\uparrow/kT}}{Z} = p(\downarrow) = \frac{e^{-E_\downarrow/kT}}{Z} = \frac{1}{2}$

So,
$$\begin{aligned}\langle M\rangle &= m_\uparrow p(\uparrow) + m_\downarrow p(\downarrow)\\ &= \frac{1}{2}(m_\uparrow + m_\downarrow) = 0\end{aligned}$$

Statistical physics tells us that there is no such thing as a sticking fridge magnet...

Still, they do stick! Why? Why does stat. phys. fail at describing real life?

Note: $\langle M\rangle$ is NOT an order parameter

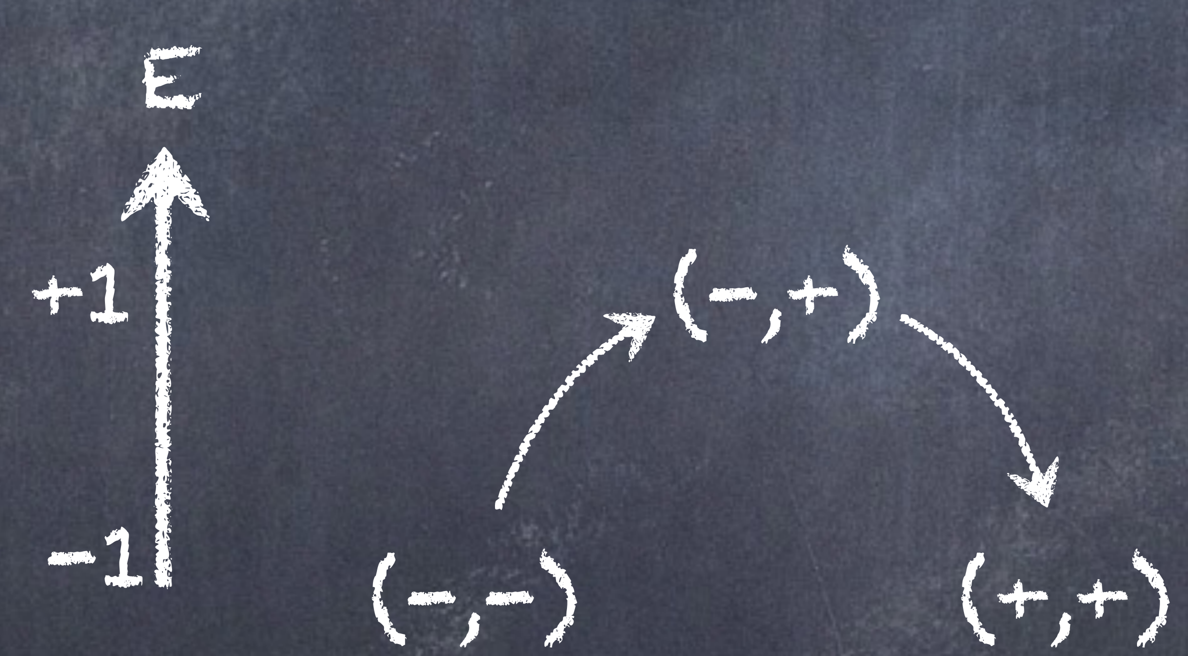
Take $\mathcal{H} = -\sum_{\langle i,j \rangle} \sigma_i \sigma_j$ with $\sigma_i = \pm 1$ (Ising spins)

1 - spin: States : + / − $E_+ = E_- = 0$ obviously, <M> = 0

2 - spins: States : ++, +−, −+, −− $E_{++} = E_{--} = -1$, $E_{+-} = E_{-+} = +1$ also, <M> = 0

But! (−,−) -> (+,+): two paths: (−,−) -> (−,+) -> (+,+)
(−,−) -> (+,+) -> (+,+)

Let's consider one path



so, there is a time issue.

Boltzman tells us that $p_{--\to-+} \propto e^{-\Delta E/kT} = e^{-2/T}$

$p_{-+\to++} \propto 1$

So, $p_{--\to++} \propto e^{-2/T}$

4 - spins: States :

```
++++
+++-
++-+
+-++
-+++
++--
+-+-
-++-
-+-+
--++
+--+
---+
--+-
-+--
+---
----
```

(1)	++++, ----	E=-3
(2)	+++-, -+++, ++--, --++, ---+, +---	E=-1
(3)	++-+, +-++, -++-, +--+, --+-, -+--	E=+1
(4)	+-+-, -+-+	E=+3

idem, <M> = 0

Let's consider one flipping path:

(1) (2) (2) (2) (1)

(-,-,-,-) -> (-,-,-,+) -> (-,-,+,+) -> (-,+,+,+) -> (+,+,+,+)

$e^{-2/T}$ 1 1 1

$p \propto e^{-2/T}$

Let's consider another flipping path:

(1) (2) (4) (2) (1)

(-,-,-,-) -> (-,-,-,+) -> (-,+,-,+) -> (-,+,+,+) -> (+,+,+,+)

$e^{-2/T}$ $e^{-4/T}$ 1 1

$p \propto e^{-6/T}$

N - spins: how to go from -- ... - to ++ ... + ? [again, we know that <M>=0]

a) we nucleate from one side and propagate to the other.
then, $p_{(--...-)\to(++...+)} \propto e^{-2/T}$

Note: we need equations of motion to do that.
A stochastic process would not!

b) we nucleate n « defects »

- - ... - -> ------+-- ... -+- ... -+- ... -+- ... -+-...---

n

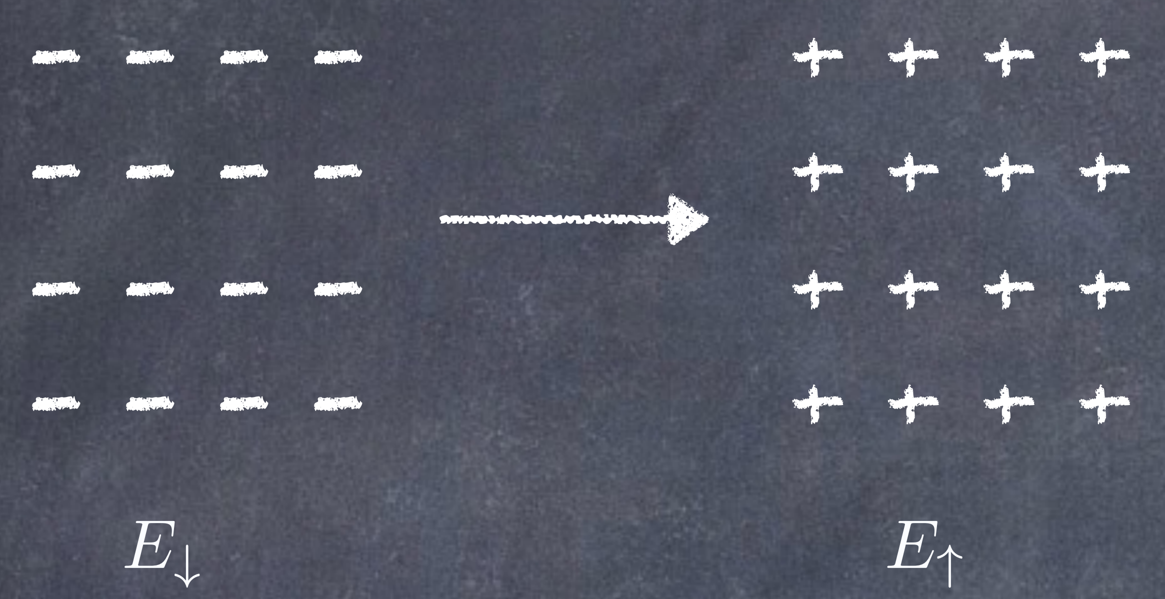
« propagation » of defects is, energy-wise, costless.

E(-+- ... -) = E(-++++++- ... -)

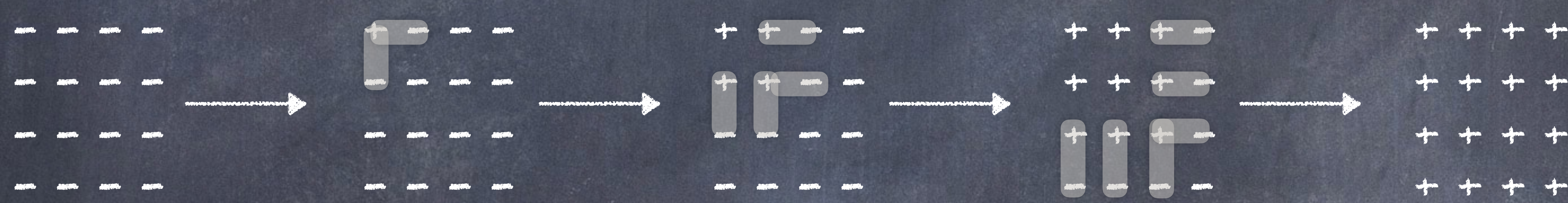
So, $p \propto \left(e^{-2/T}\right)^n$

It looks like you must be very « lucky » to reverse everyone at small cost; still, it's possible.
Thanks to dimensionality!

In higher dimensions d, i.e d>1, it's worse.
Let's try d=2.



Of course, <M>=0.



So, $p \propto e^{-4/T} \times e^{-8/T} \times e^{-12/T} = e^{-24/T}$

Whatever the way you try (luck is no longer at play here), probability collapse.
In other words, time durations diverge!

Statistical physics tells us there are no permanent magnets, but stochastic dynamics explains why, actually, we do observe them.

A duration longer than the age of the universe.

-> There are « permanent » magnets, and spontaneous broken symmetry (in CM) is a fancy way for describing lack of patience.

-> Collateral statement: in high dimensional ordered magnets, fluctuations can only marginally modify the magnetic texture they are built on.

The path we have followed in statistical physics hast its quantum counterpart. Antiferromagnets do not exist!

$$\mathcal{H} = \sum_{\langle i,j \rangle} \vec{S}_i . \vec{S}_j$$

S_i are quantum « objects », i.e operators, like Pauli matrices for instance.

Let's try S=1/2.

2 spins.

$$\begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}$$

Eigenvalues: -0.75, 0.25, 0.25, 0.25

S_{total} (GS) = 0

4 spins.

$$\left(\begin{array}{cccccccccccccccc}
\frac{3}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{2} & -\frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{3}{4} & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{4} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & -\frac{3}{4} & 0 & \frac{1}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{4} & 0 & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & -\frac{1}{4} & \frac{1}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{4}
\end{array}\right)$$

Eigenvalues:-1.61603, -0.957107, -0.957107, -0.957107, 0.75, ...

S_{total} (GS) = 0

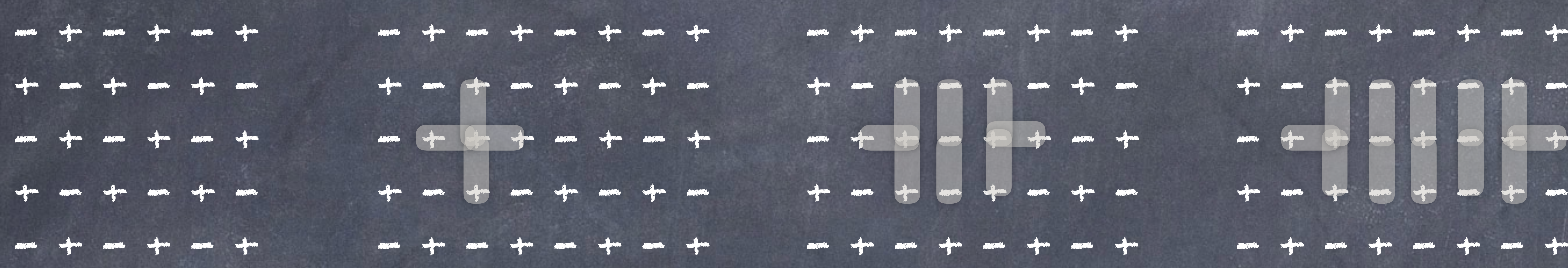
We can go on like this, but there's better.
Marshall W. 1955 Proc. R. Soc. A 232 48 (also an argument of Landau and Görter)
$S_{GS}=0$
So, there are no antiferromagnets!

Again, dynamics is crucial. Anderson 52, Bernu 92
-> concept of « tower of states »
A quantum (canonical) antiferromagnet is a symmetric top whose moment of inertia diverges with N, the number of spins

Here again, it's a matter of time i.e. dynamics.
-> It is too slow to be observed

And here also, fluctuations (or excitations) marginally modify the Ground State (in high dimensions)

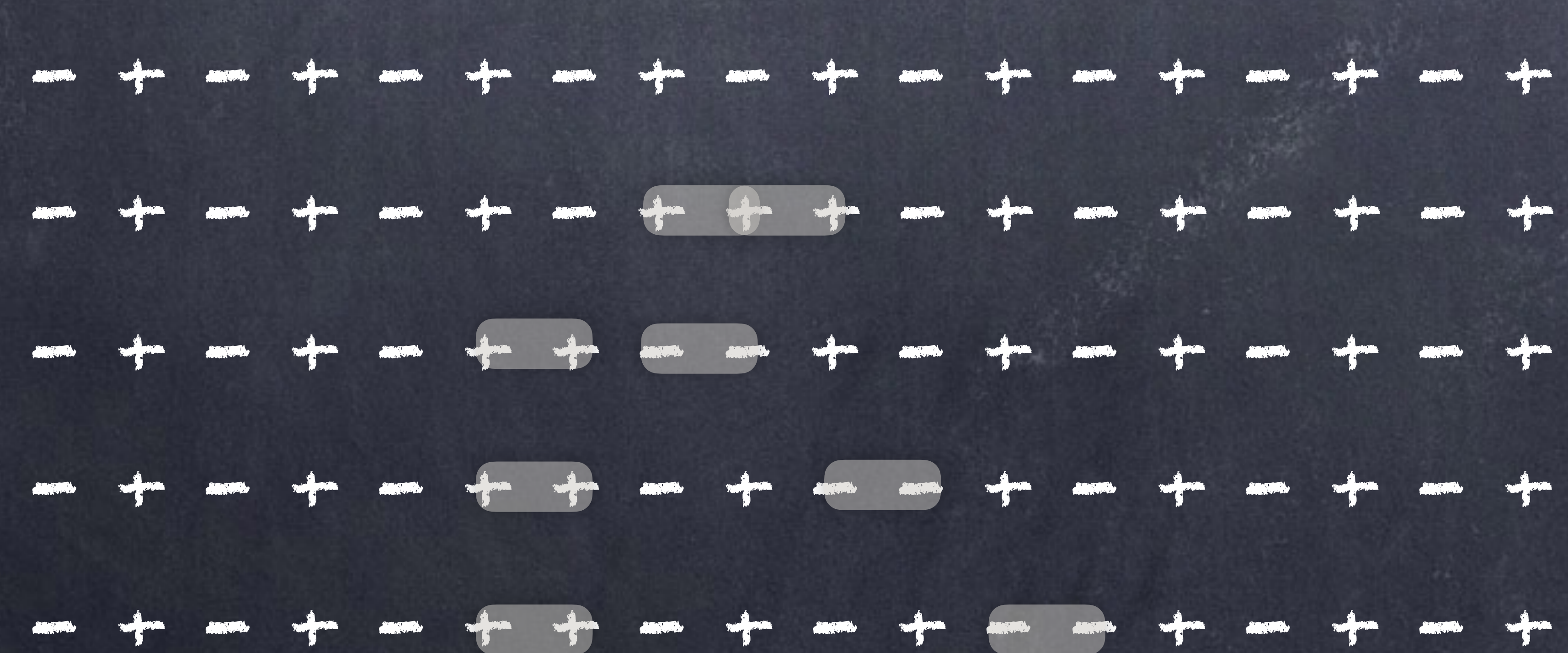
As we did for the ferromagnets, what about injecting a « defect » in the texture? Available m



$S_i^+.S_j^- + S_i^-.S_j^+$

There is a confining potential, proportional to the length of the motion of the defect. Too
energy.
It is not possible to « split » the defect in high dimensions.

But it is possible in low dimension.



This kind of excitation is called a spinon (it's a domain wall). Such an excitation is called <u>fractionalized</u>.
The crucial point is that deconfinement is provided by dimension!

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Summary:

- we observe F and AF because of time/dynamics issues.
- For 1D, excitations are very peculiar
- For d≧2, ground states are t-disconnected and excitations do not (marginally) modify states

Stability: take $\mathcal{H} = \sum_{\langle i,j \rangle} \vec{S}_i . \vec{S}_j$ with quantum S, on a cubic-like lattice (« high » d).

The ground state, for all the reasons we mentioned is:

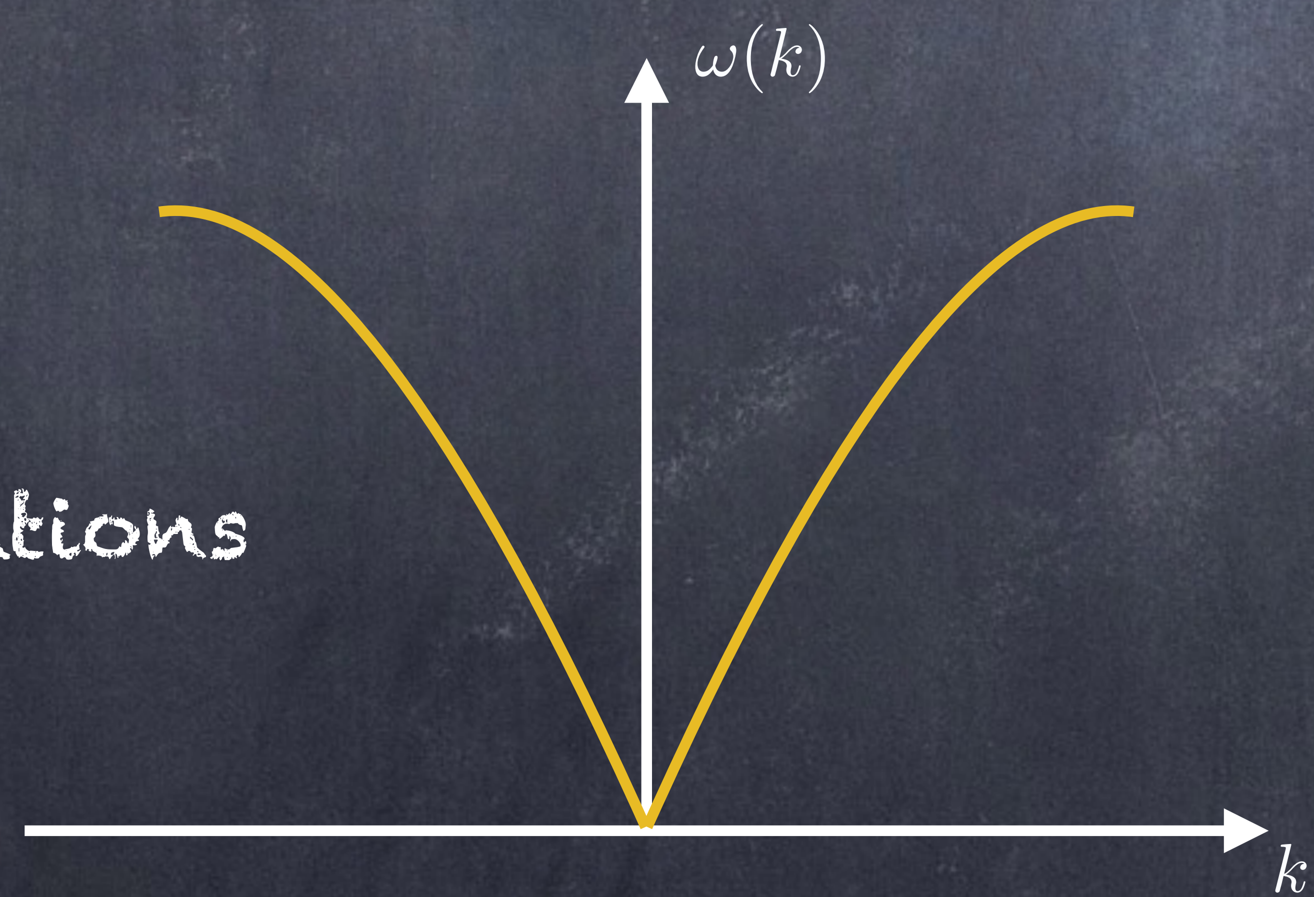
↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓ ↑ ↓

Excitations -> bound spinous (magnons)

Semi-classical approach: at each site i, $\langle S_i^z \rangle = S - \delta S$

where $\delta S \propto \sum_k \frac{1}{\omega_k} \left(n_B(\omega_k) + \frac{1}{2} \right)$

Spectrum of fluctuations



Hence, small S and « flat » ω_k are interesting directions to look for disordered/destabilized ground states.

Frustration is a nice way to dress the 2nd issue.
Disordered but strongly correlated!

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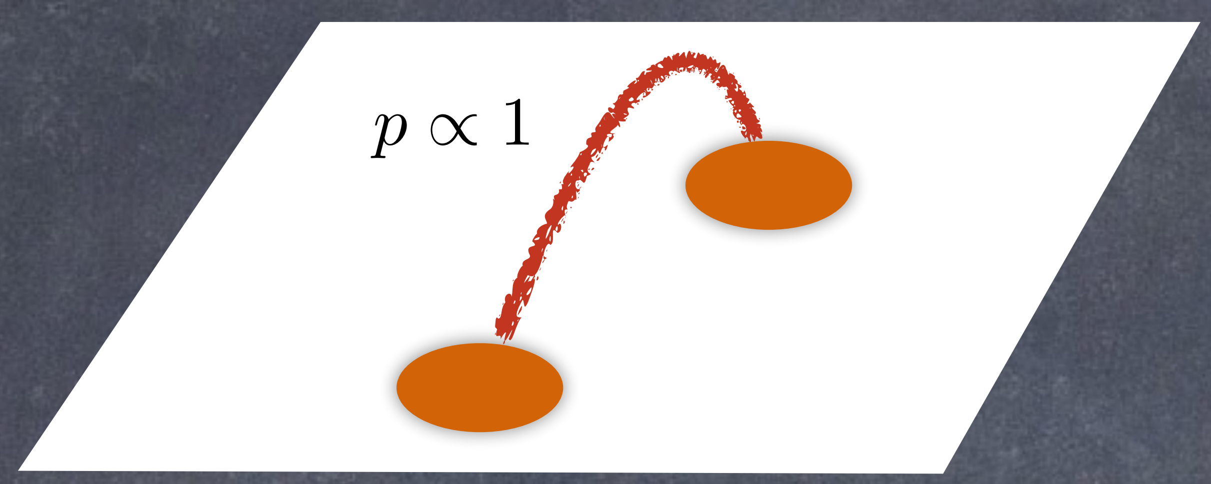
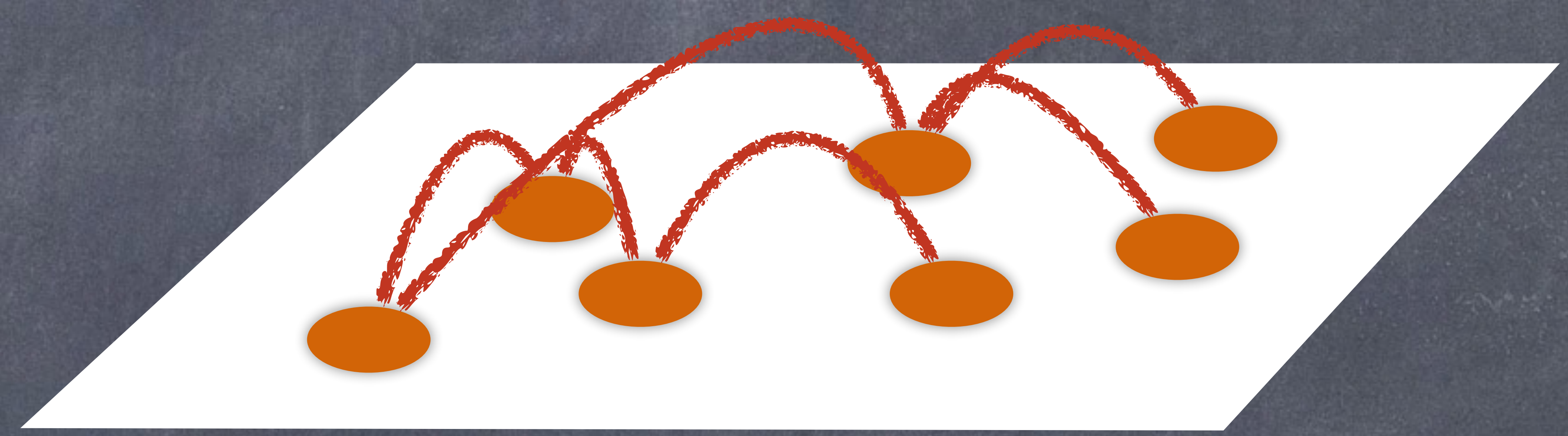
Somehow, we are looking for





Natural question, seen from the reverse point of view. What about the consequences?
If a system is correlated, but never orders, what about it's degeneracy at low temperatures? What about the 3rd principle of thermodynamics? [bottom up question]

Let's have a look at it from the historical point of view - top down approach. :-)

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Historical point of view

First started with thermal engines: convert heat into mechanical work

Thermodynamics time line

XVII

Francis Bacon, Denis Papin, Robert Boyle, Émilie du Châtelet, Antoine Lavoisier

XVIII

Joseph Fourier, James Joule, Sadi Carnot, Rudolf Clausius, Robert Brown, William Thomson, James Clerk Maxwell, Ludwig Boltzmann, Walter Nernst

XIX

Max Planck, Albert Einstein,

Emergence of laws

1 - The internal energy of an isolated system is constant.
[energy is conserved, internal energy is defined]
explicit statement - Rudolf Clausius (1850)

2 - Heat cannot spontaneously flow from a colder location to a hotter location
[entropy increases, principle of evolution]
1824 - Sadi Carnot

3 - As a system approaches absolute zero, all processes cease and the entropy of the system approaches a minimum value
[our point...]
Walter Nersnt (1906/1912), Max Planck (1911), Albert Einstein (1907)

Historical point of view

Emergence of laws

1 - ...

2 - ...

3 - [our point...]

William Nernst (1906/1912), Max Planck (1911), Albert Einstein (1907)

At absolute zero, one cannot extract heat anymore.
(Guillaume Amontons, 1702, Lord Kelvin, 1848)

$$\Delta Q = T\Delta S$$

William Nernst (1906/1912),

Max Planck (1911),

Albert Einstein (1907),

W. Nernst, Weber die berechnung chemischer gleichgewichte aus thermischen messungen, Nachr. Kgl. Ges. Wiss. Gott., no 1, pp. 1-40, 1906

A. Einstein, Die Plancksche theorie der strahlung und die theorie der spezifischen warme , Annalen der Physik , vole. 22, pp. 180-190, 1907.

W. Nernst, Thermodynamik und spezifische warme , Preussische Academie der Wissenschaften (Berlin). Sitzungsberichte , no 1, p. 134140, 1912.

M. Planck, Thermodynamik (3rd edition) . Berlin : De Gruyter, 1911.

Historical point of view

Emergence of laws

1 - ...

2 - ...

3 - [our point...]

William Nernst (1906/1912), Max Planck (1911), Albert Einstein (1907)

At absolute zero, one cannot extract heat anymore.
(Guillaume Amontons, 1702, Lord Kelvin, 1848)

$$\Delta Q = T \Delta S$$

William Nernst (1906/1912), $\lim_{T \to 0} \frac{\delta Q}{T} = 0$ unattainability principle

Max Planck (1911), $S(T) \underset{T \to 0}{\longrightarrow} 0$

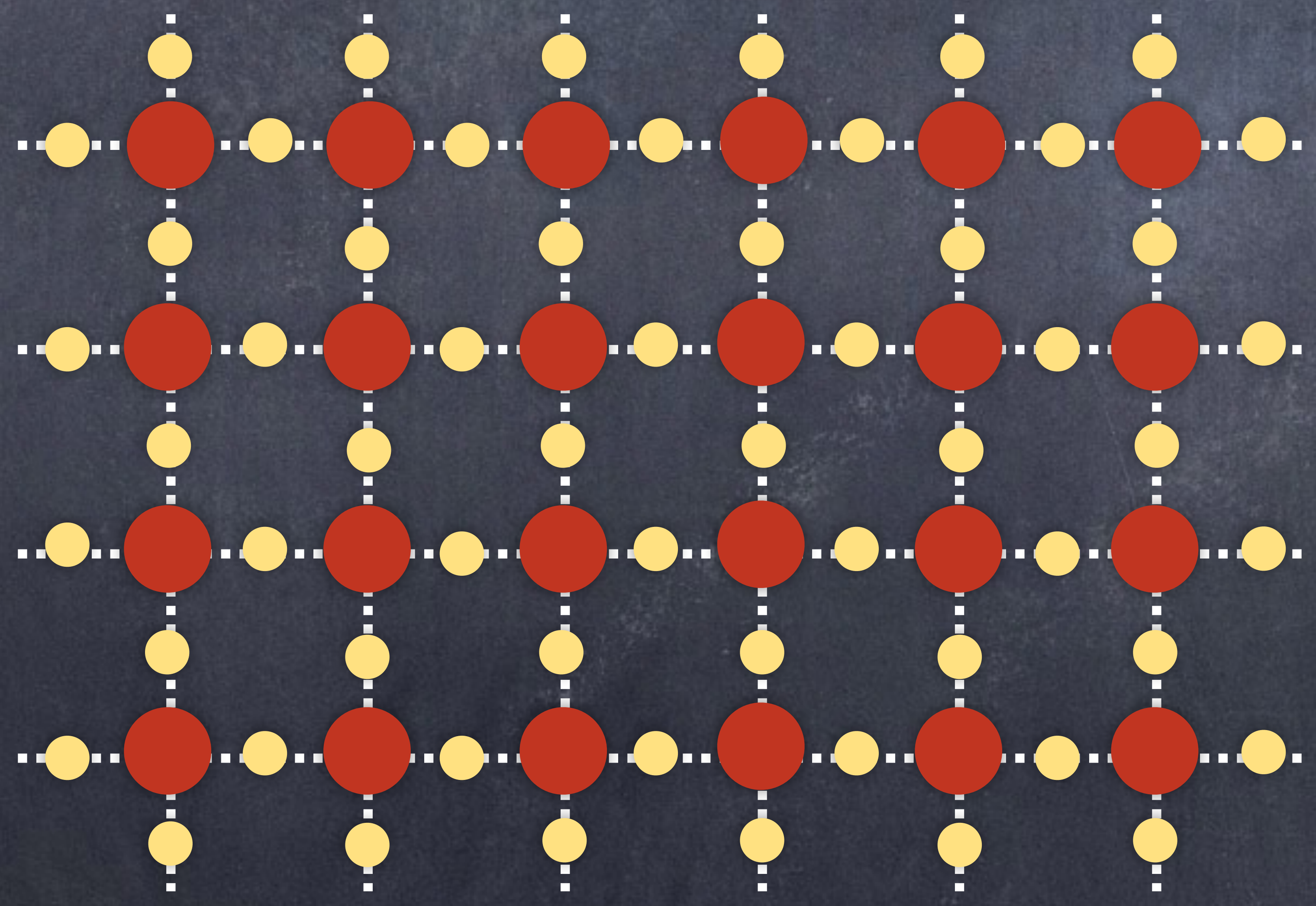
Albert Einstein (1907), $S(T) \underset{T \to 0}{\longrightarrow} S_0 < \infty$

Reaching the lowest possible temperatures is worth the challenge.
Early 20th century - William Giauque

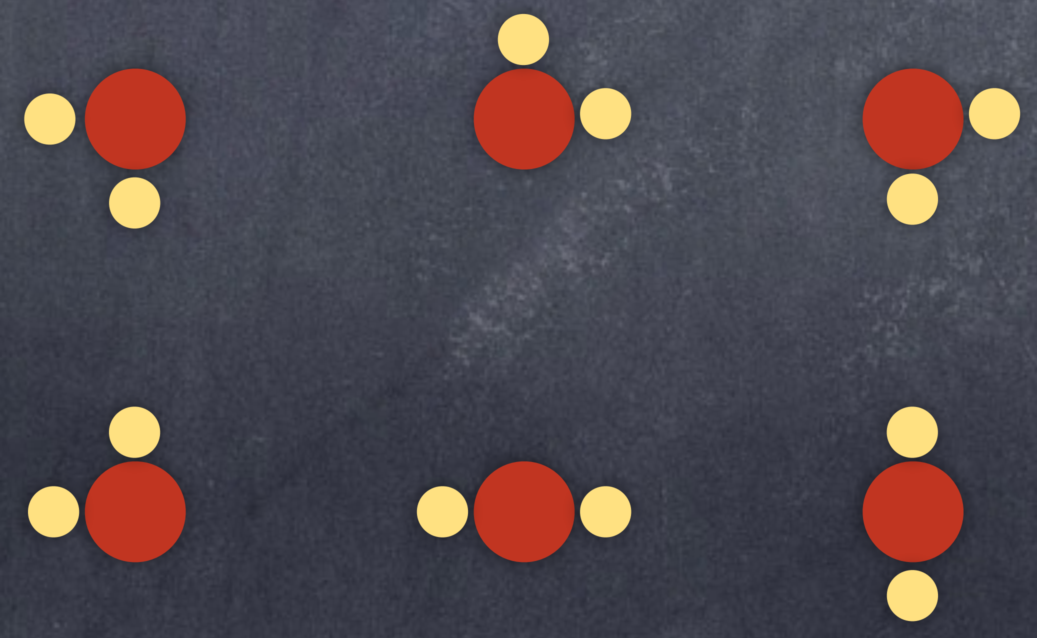
Historical point of view

William Giauque: common water ice, I_h, has a residual entropy
W. F. Giauque, M. F. Ashley, Phys. Rev. 43, 81 (1933)
W. F. Giauque, J. W. Stout, J. Am. Chem. Soc. 58, 1144 (1936)

Linus Pauling (explanation): configurational proton disorder
L. Pauling, J. Am. Chem. Soc. 57, 2680 (1935)
based on Bernal-Fowler ice rules
J. D. Bernal, R. H. Fowler, J. Chem. Phys. 1, 515 (1933)

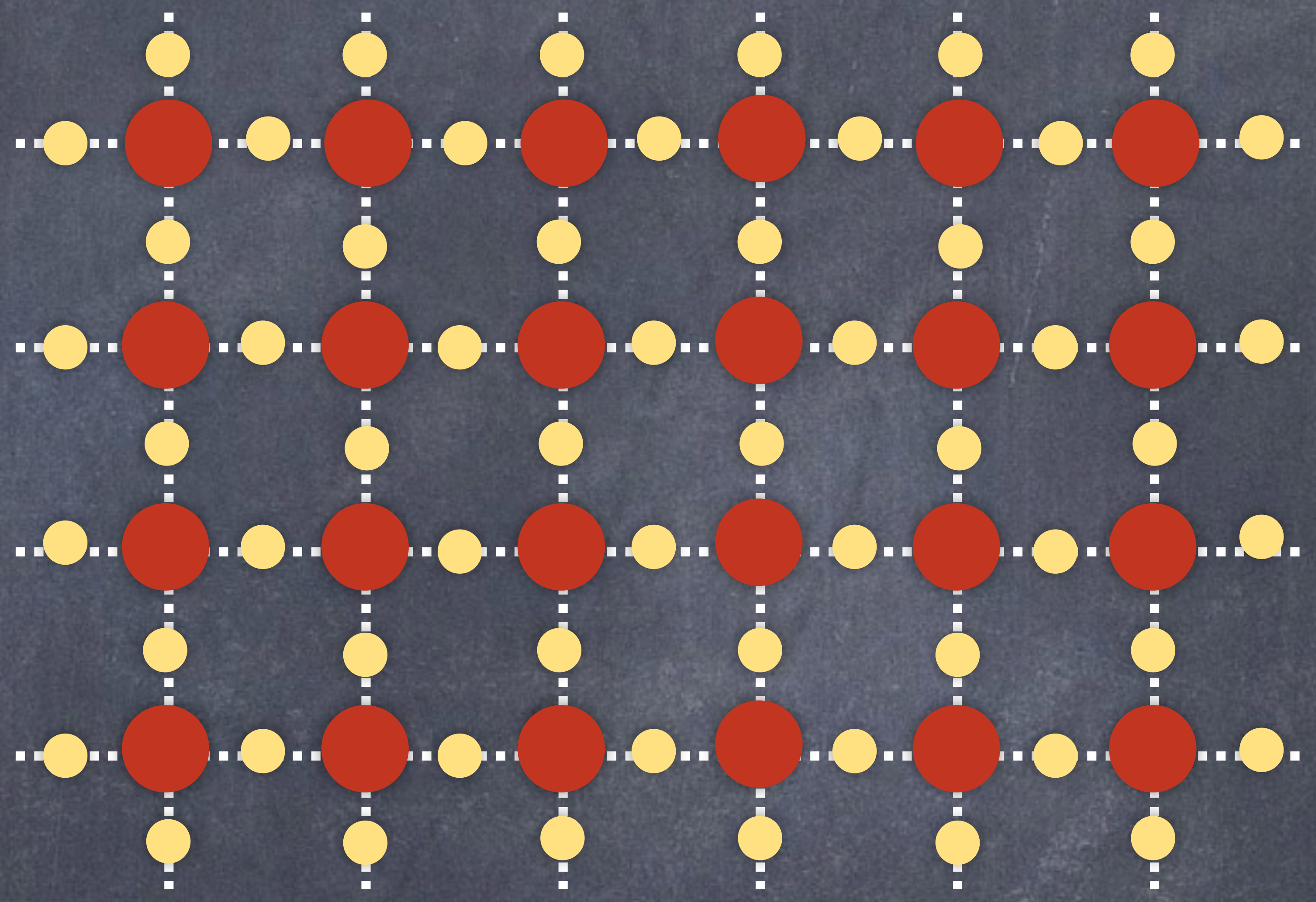


(2D translation) 6 possible configurations to tile the square lattice. Calculations are possible, but we leave thermodynamics.



Historical point of view

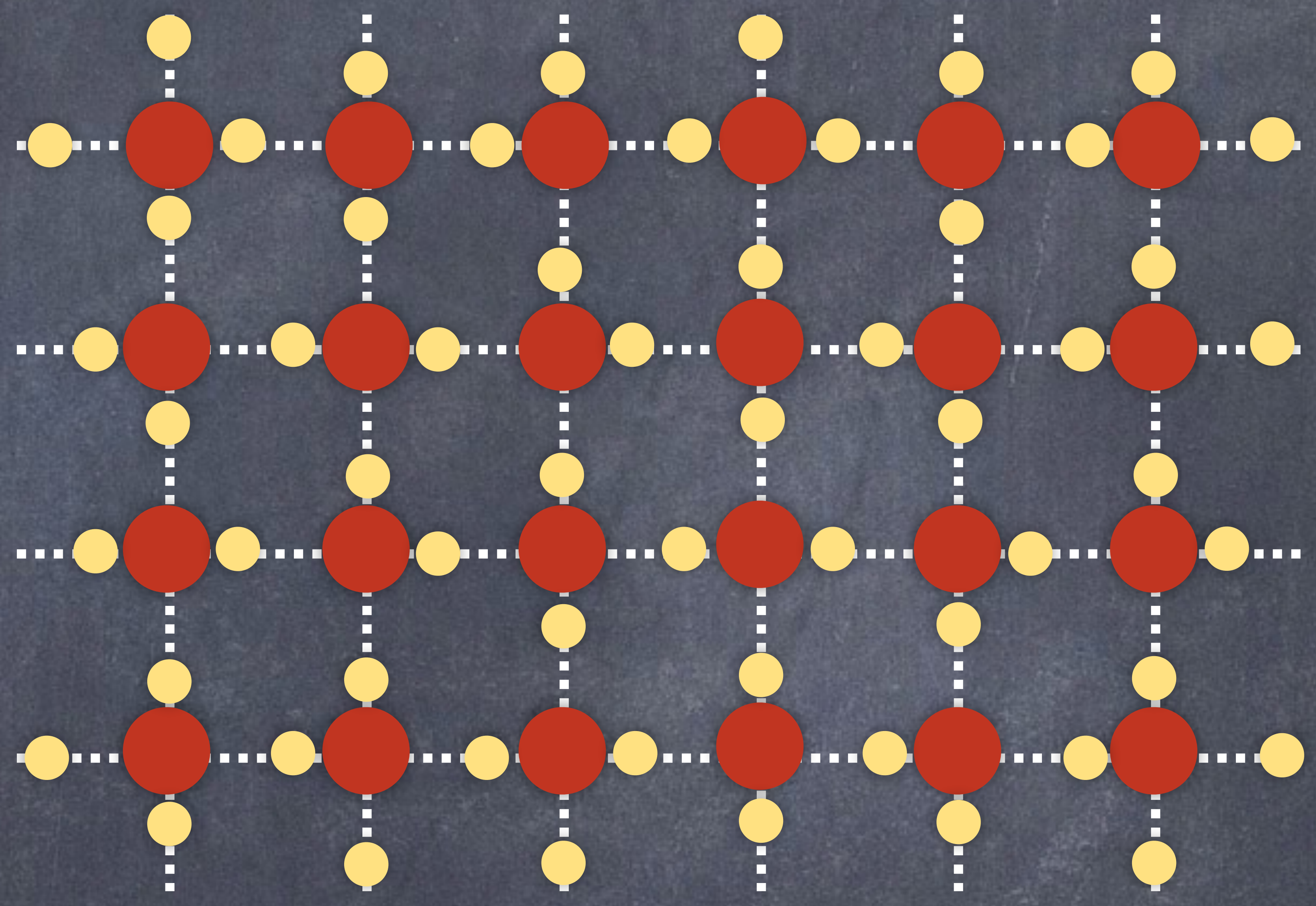
Modelling: 3D is hard, go 2D.



Historical point of view

Modelling: 3D is hard, go 2D.

Implement ice-rules, i.e.
2 near, 2 far away.



Historical point of view

Implement ice-rules, i.e.
2 near, 2 far away.

Change representation.

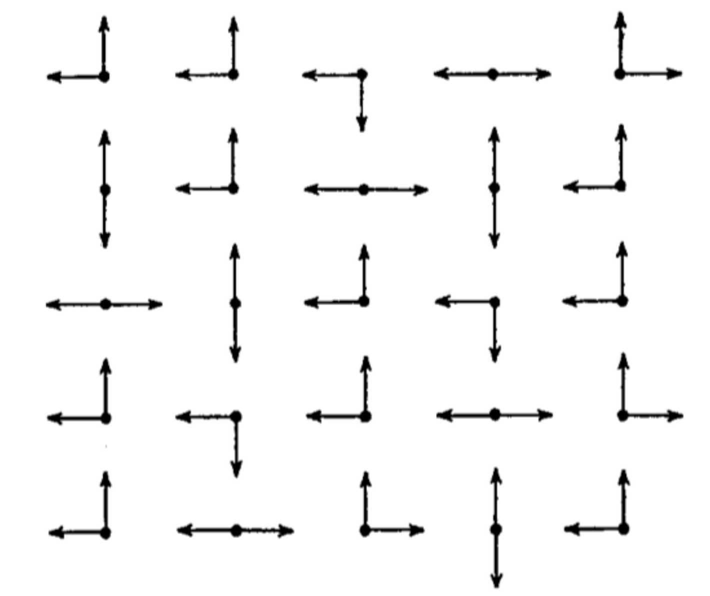
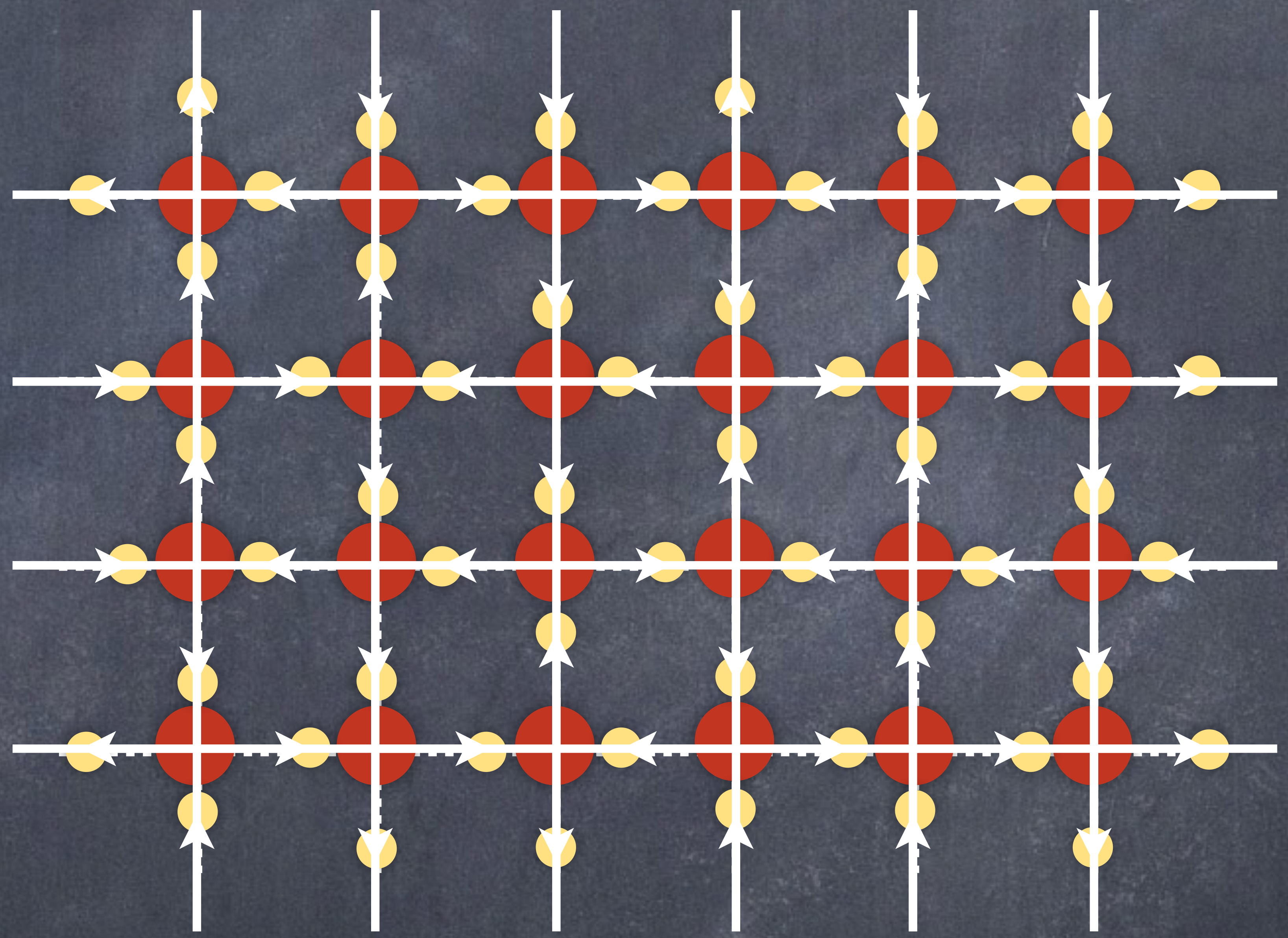
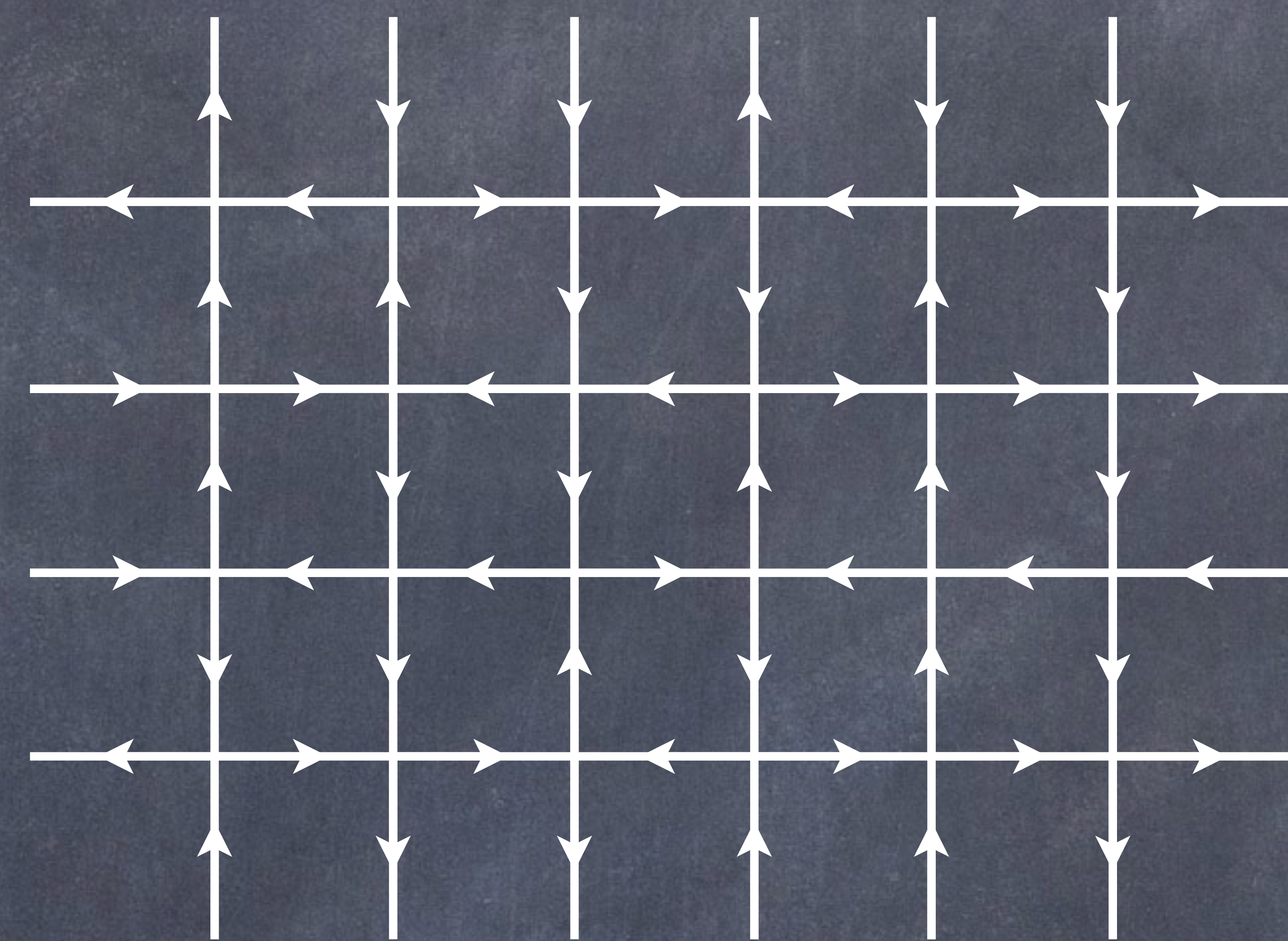


FIG. 1. Two-dimensional regular square lattice analog of real ice. The two protons bound to each oxygen atom (lattice site) are denoted by arrows. All configurations, regardless of bond angles, are regarded as equally probable.

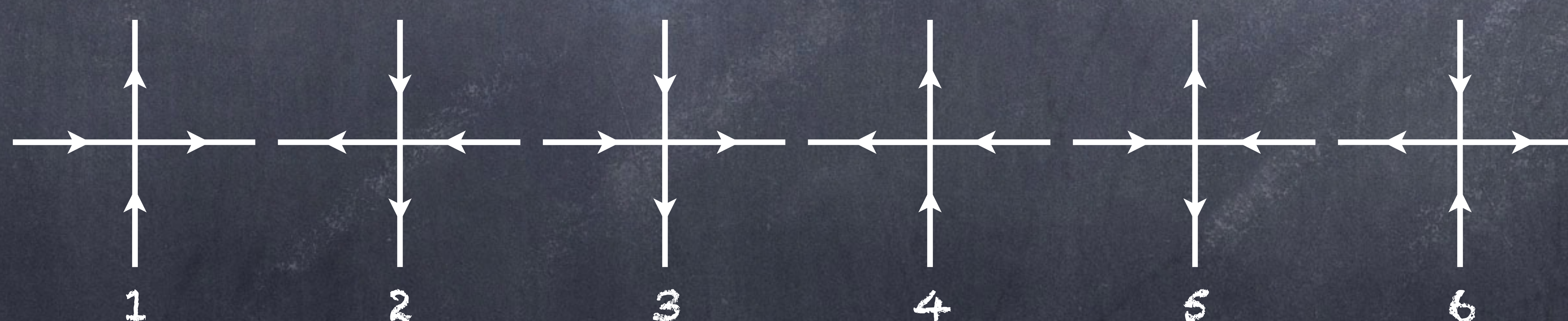
DiMarzio et al., J. Chem. Phys 40 (6), 1577 (1964)



Historical point of view

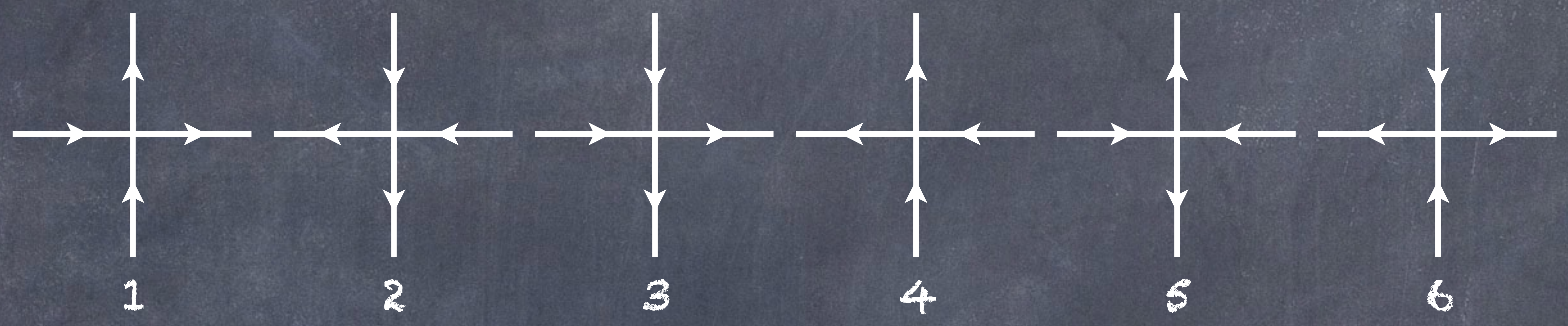


New formulation: pave the square lattice with vertices



Historical point of view

New formulation: pave the square lattice with vertices



This is the ice model: $E_1 = E_2 = E_3 = E_4 = E_5 = E_6$

It belongs to a larger class of vertex models, among which:

- Rys-F model ([1,2,3,4] - [5,6])
- KDP model ([1,2] - [3,4,5,6])

Many exact solutions are known (thermodynamics, not correlations).

E.H. Lieb and F.Y. Wu, Two Dimensional Ferroelectric Models, in « Phase Transitions and Critical Phenomena », C. Domb and M. Green eds., vol. 1, Academic Press 331-490 (1972)

Understanding Ice (H_2O) lead to a set of models of statistical physics.

Historical point of view

Time (20th century)

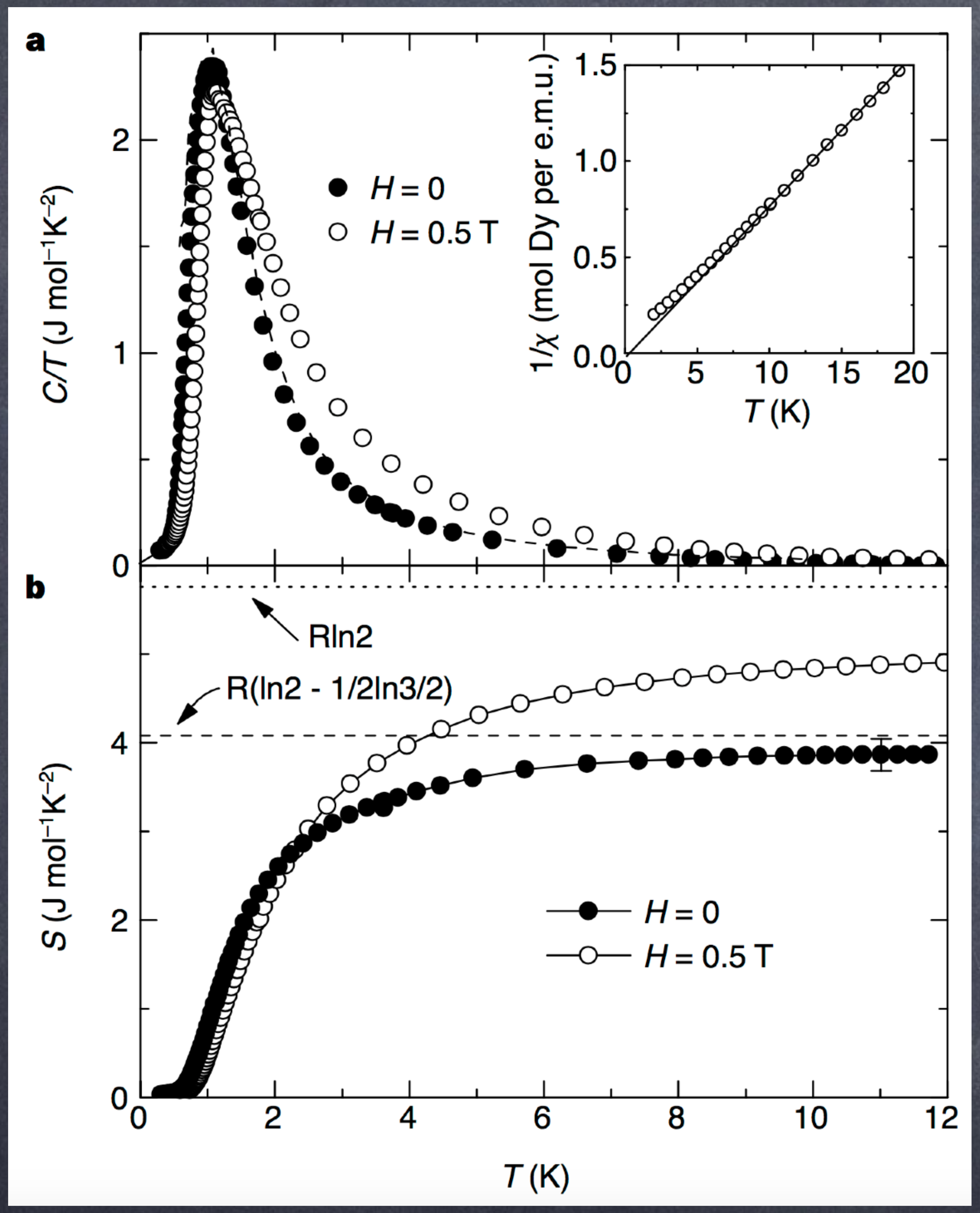
Square ice, vertex models, statistical physics, dimer models,..

Another route: condensed-matter. What is the ground state of an anti-ferromagnet?

Néel, Landau, Görter, Anderson...

$Ho_2Ti_2O_7$ (Phys. Rev. Lett., Vol. 79, p. 2554 (1997).)

Historical point of view



$Ho_2Ti_2O_7$ (Phys. Rev. Lett., Vol. 79, p. 2554 (1997).)

Zero point entropy in « spin ice »,
Nature 399, 333-335 (27 May 1999)

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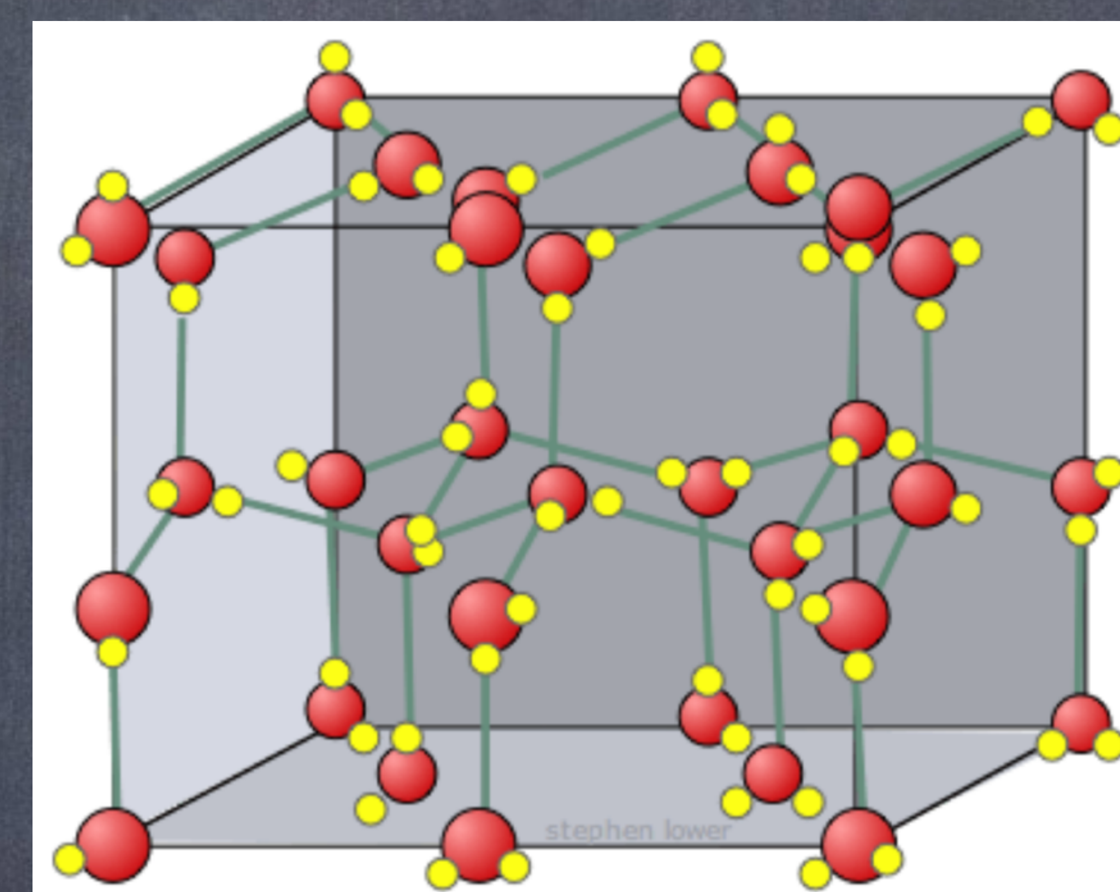
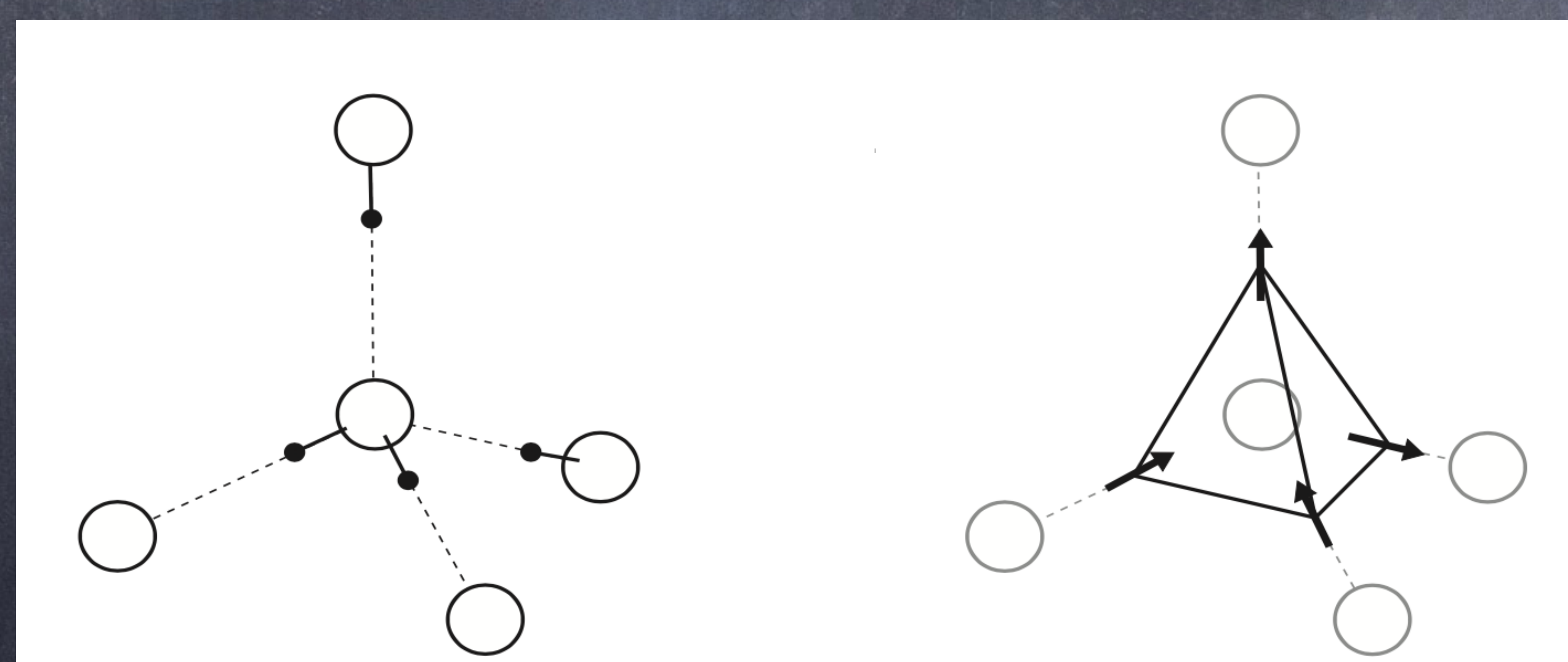
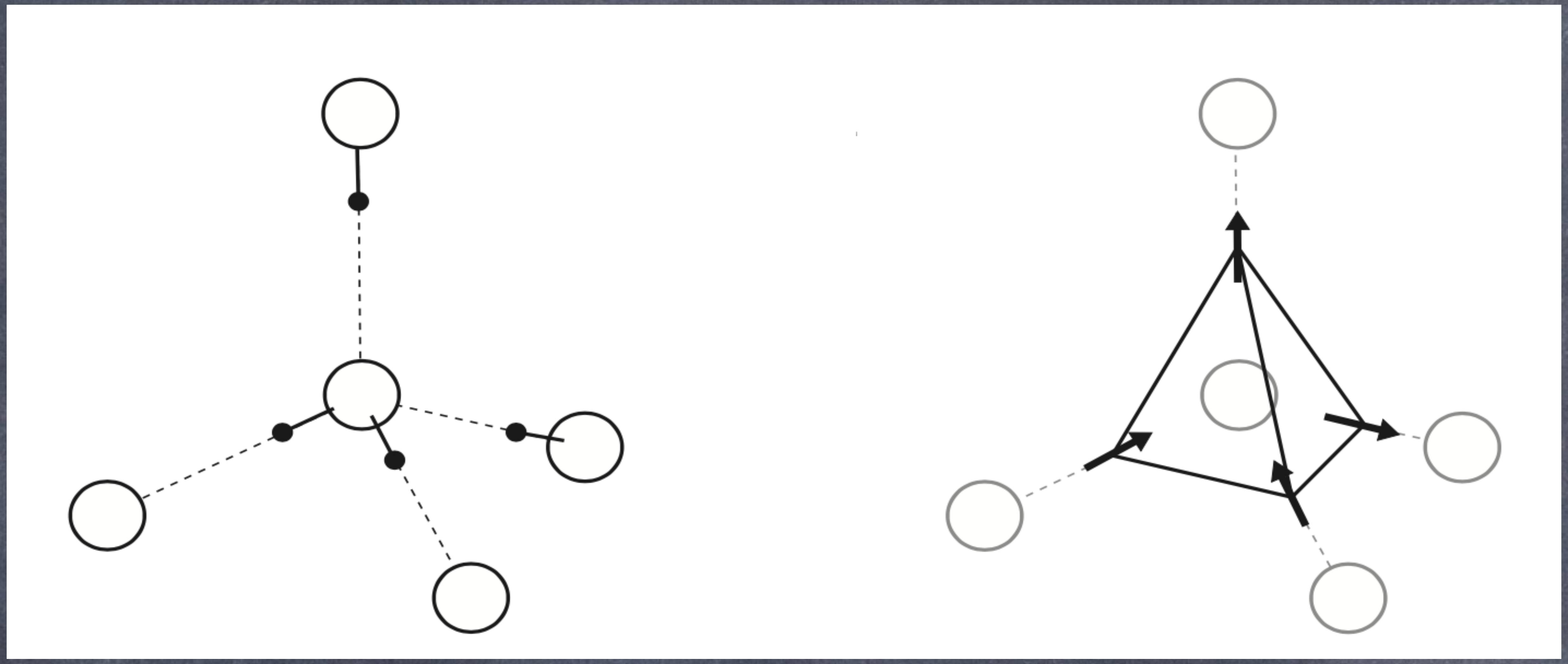
Historical point of view





Historical point of view

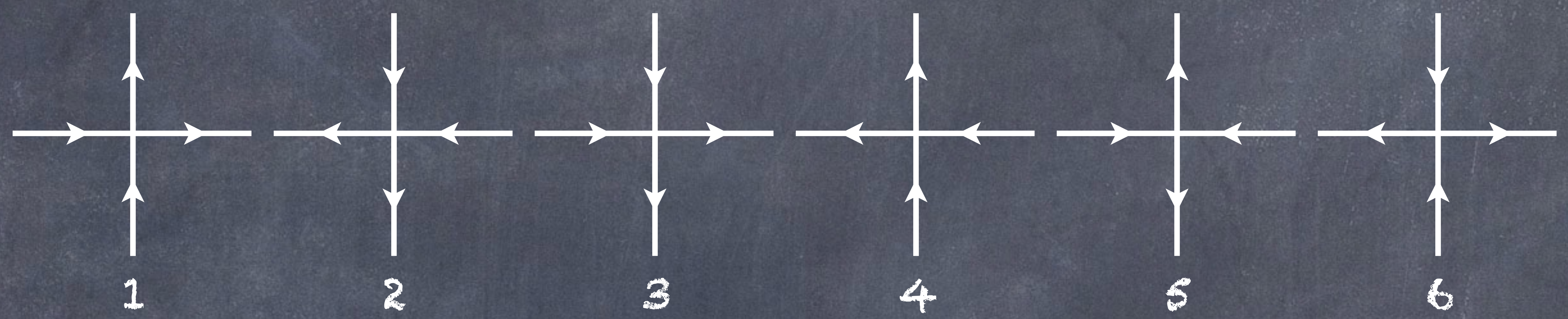


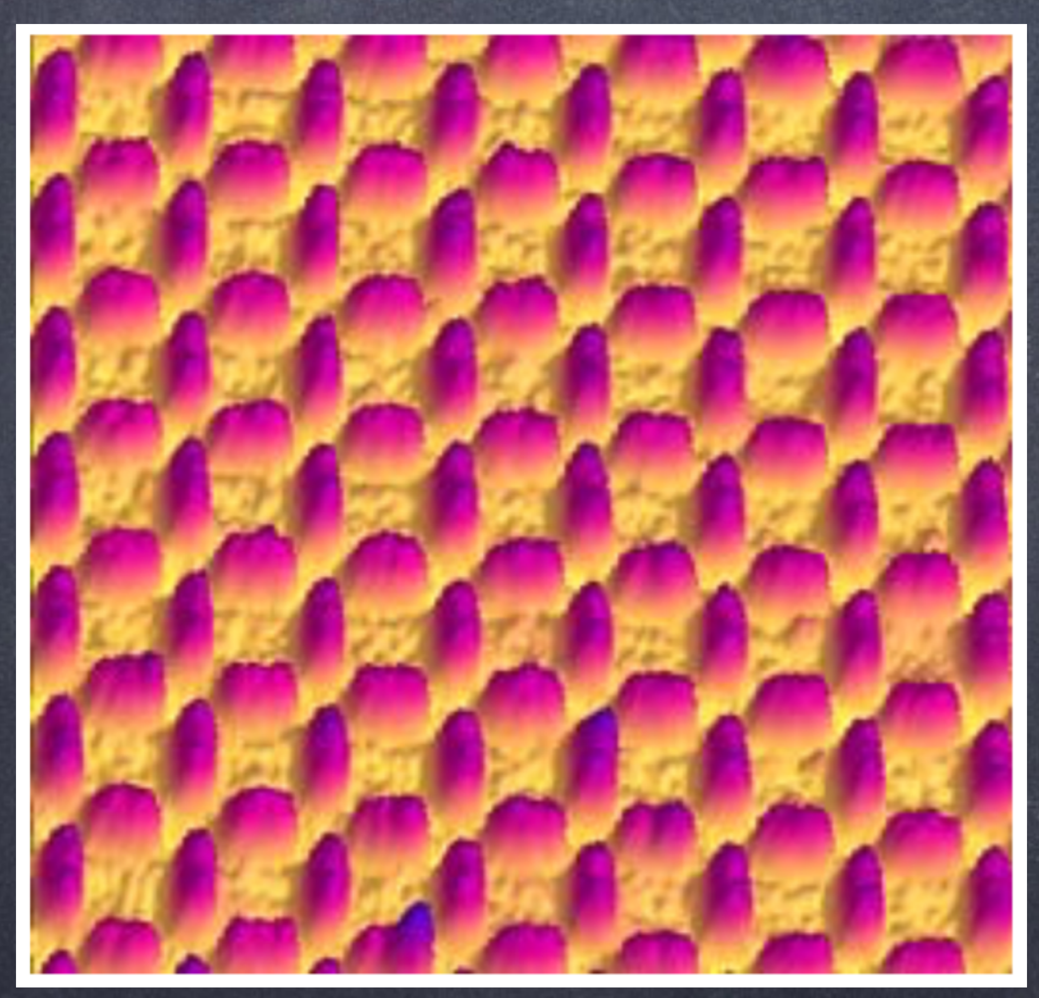
On each tetrahedron, we, again, have a 6-vertex model! But links of each vertices are local Ising degrees of freedom, magnetic degrees of freedom.

Still, 3D is tough to deal with. What about realizing a (magnetic) square ice model! I.e., what about realising the seminal Lieb square ice?

Historical point of view

Back to the square ice





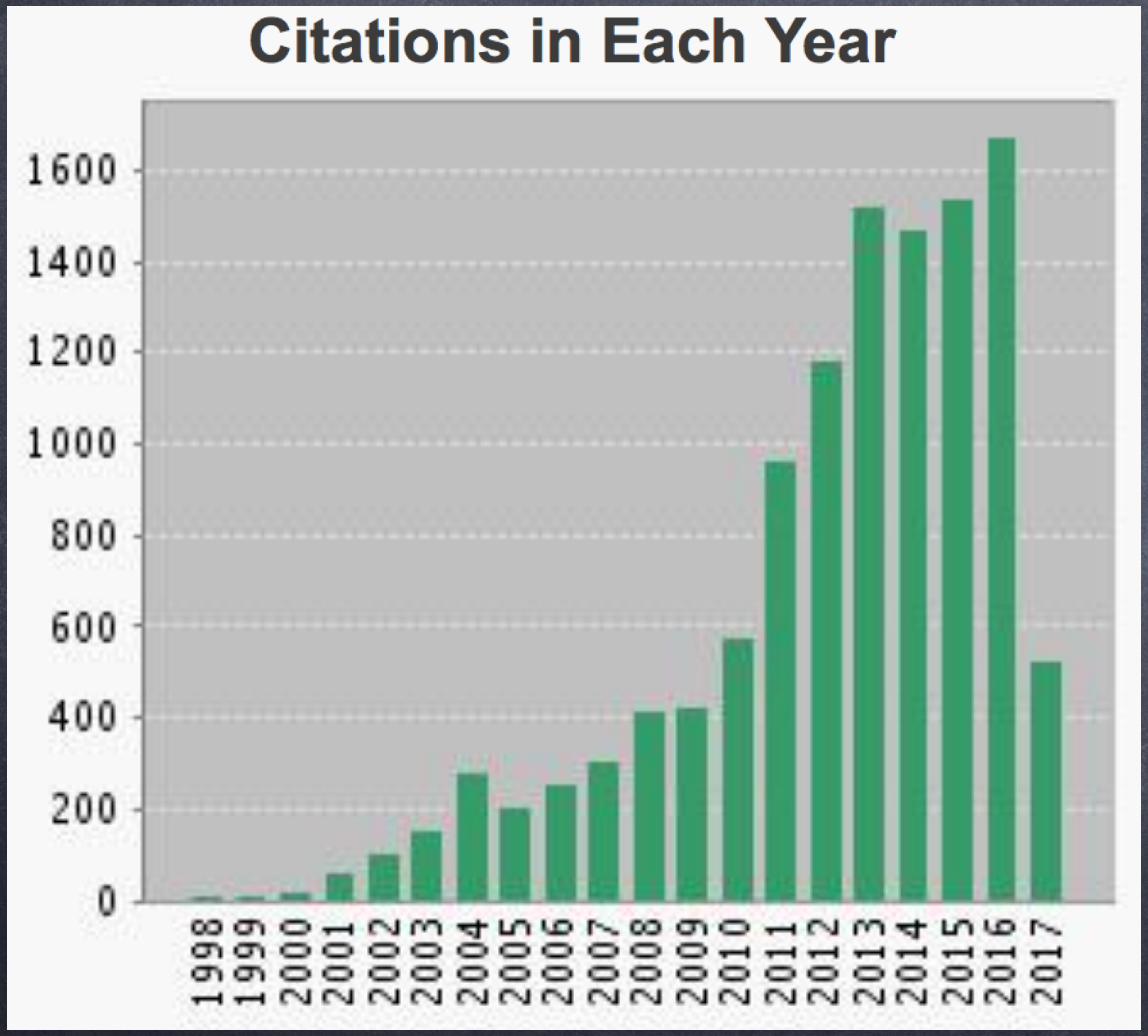
Ideally, we would like: $E_1 = E_2 = E_3 = E_4 = E_5 = E_6$

Artificial 'spin ice' in a geometrically frustrated lattice of nanoscale ferromagnetic islands
Nature 439, 303-306 (2006)

But vertices are not equivalent (we'll see later).

Historical point of view

Why such an interest in (spin)-ices?



Because the low energy manifold has a rather unexpected structure.

Historical point of view

A last word related to entropy…

« Modern » formulation of the 2nd law:

$$\langle e^{-W} \rangle = 1$$

Evans-Searles (1994), Crooks (1998), Kawasaki (1967), Seifert (2005).

This implies the older formulation (Kelvin), but now, Eddington time arrow can be reversed for small time durations!

2nd and 3rd laws are a long standing framework motivating the study of these exotic magnets.

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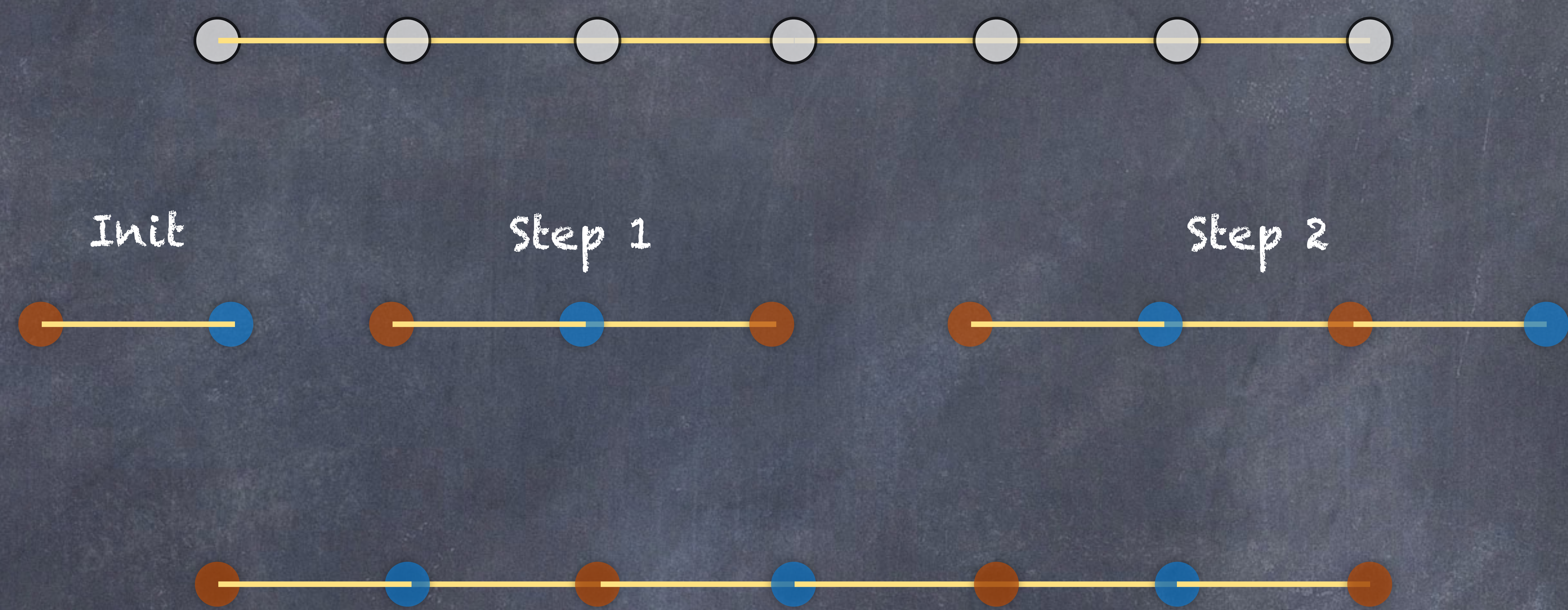
Study of an example (deeper insights during practicle)

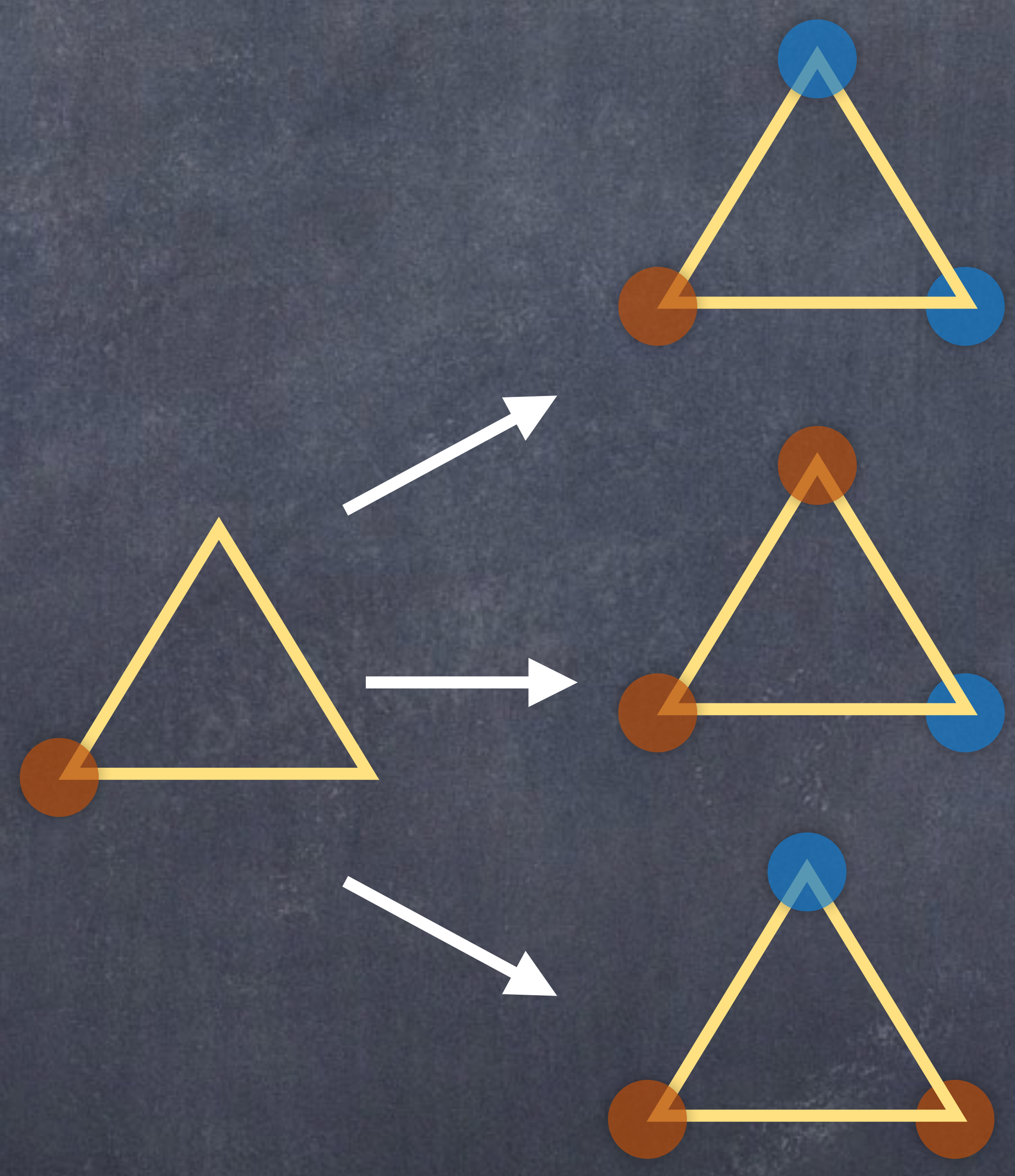
Game: maximize 2 color-bonds



Once the first color is given, only 1 coloring/configuration

Study of an example (deeper insights during practicle)

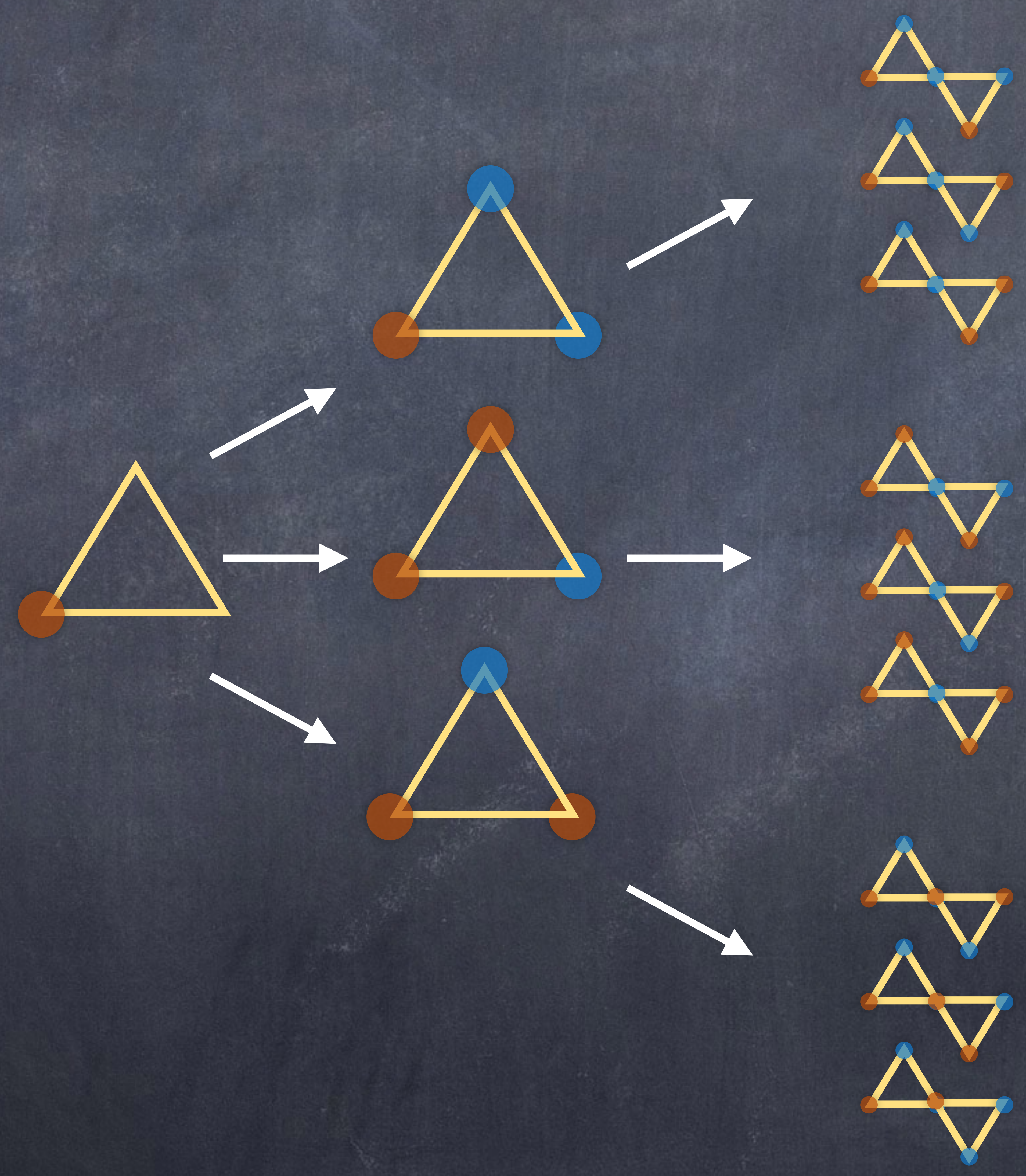
Game: maximize 2 color-bonds



These 3 configurations equally satisfy the constraint

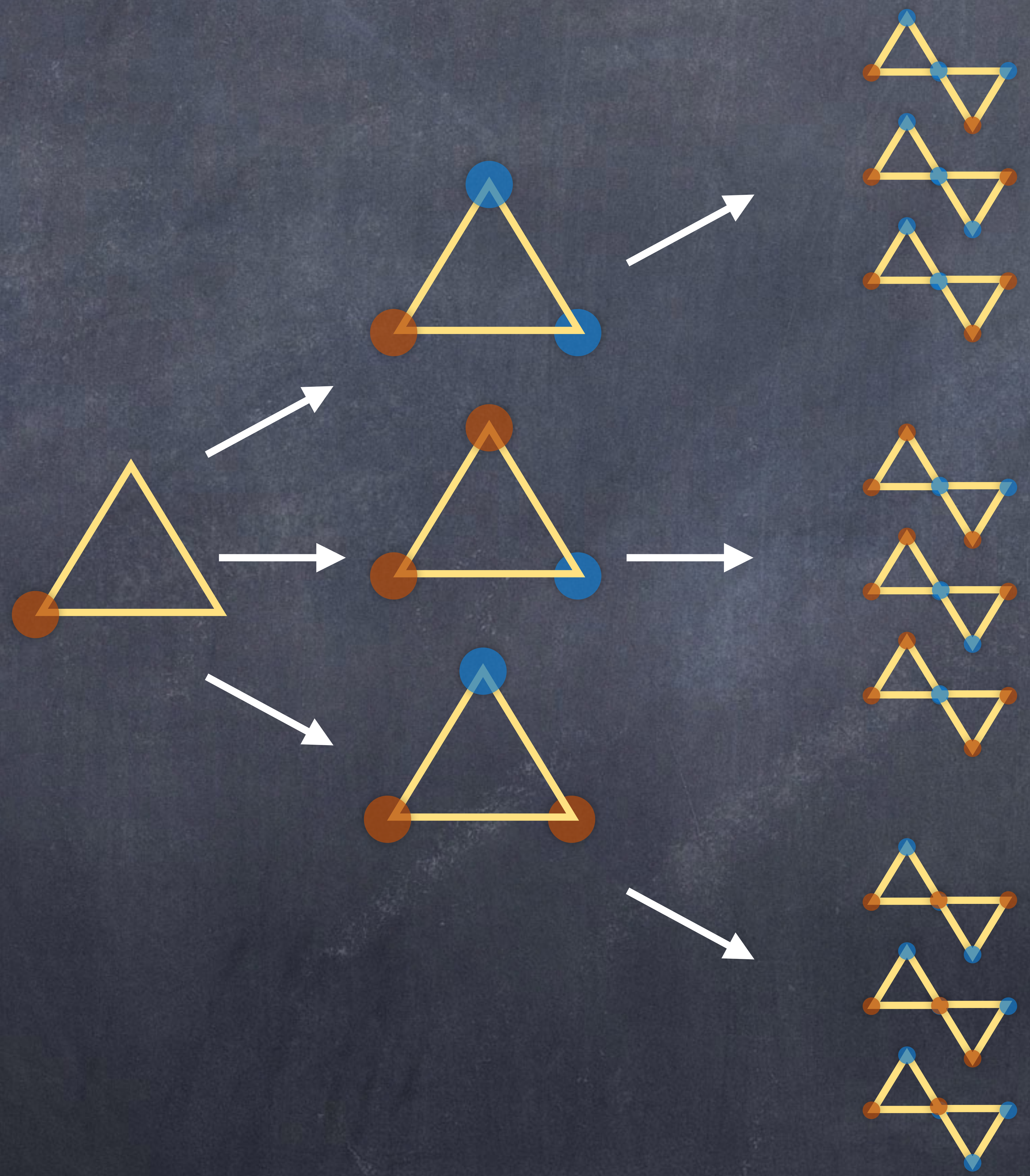
Study of an example (deeper insights during practicle)

Game: maximize 2 color-bonds



Study of an example (deeper insights during practicle)

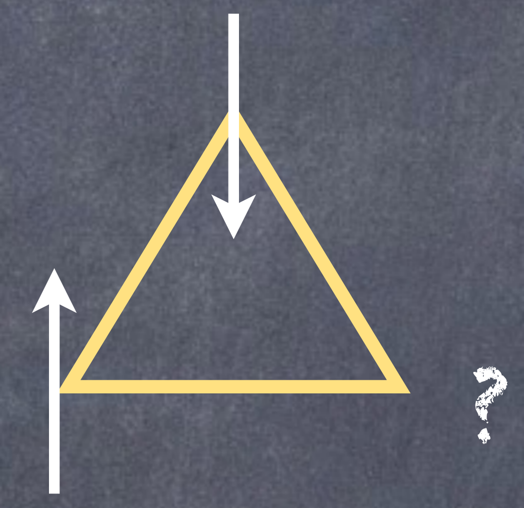
Game: maximize 2 color-bonds



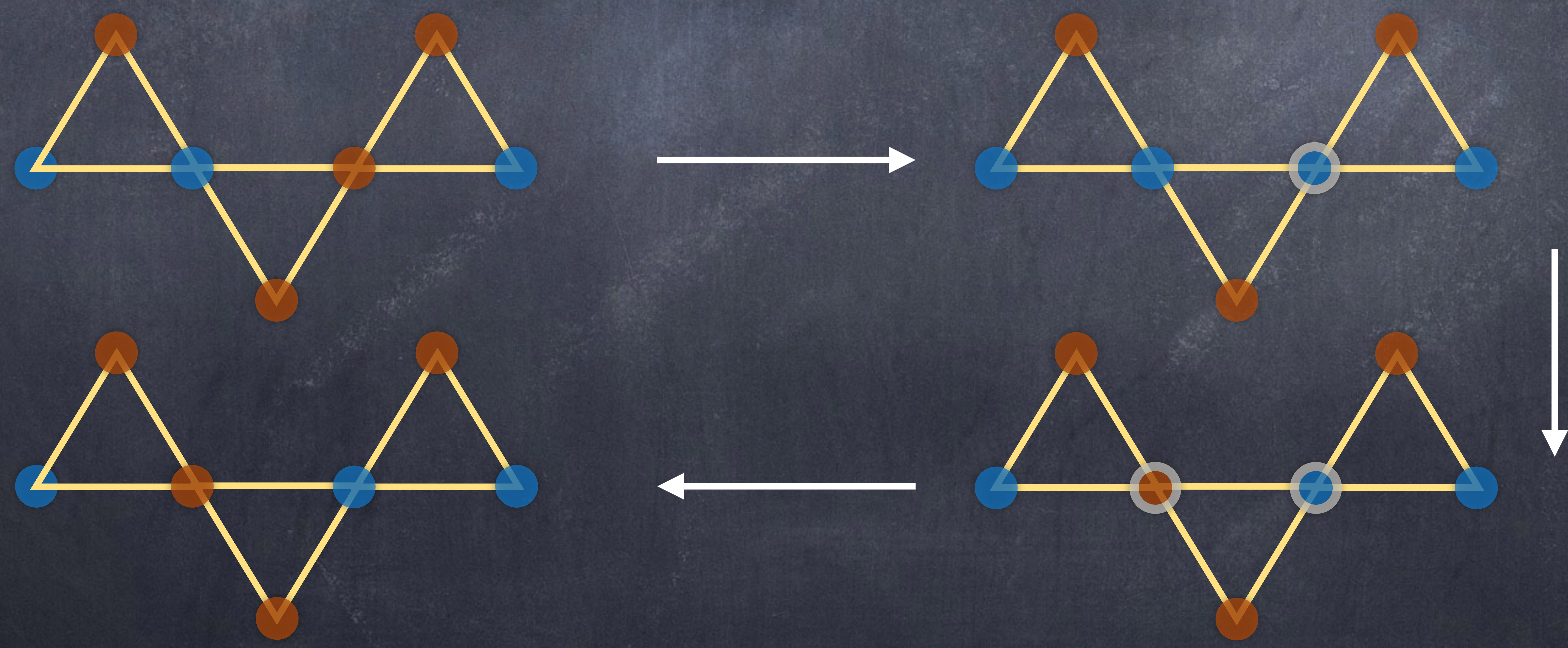
Once the first bond is given, there is an exponential number of colorings/configurations.

$$\mathcal{N}_c = 3^{N/2}$$

Note: it is sometimes written/said, that the 3rd spins does not know what to do.



It's not the case; it can dot what it « wants »! This is VERY different. Whatever its state, hence its fluctuations, the energy is the SAME.
In other words, fluctuations do not increase the energy, the ground state is no longer a point, it is a manifold, and this manifold is simply connected, through energy costless moves.



1 - the order of the moves IS important. If we dress this moves with an algebra, it is non commutative.

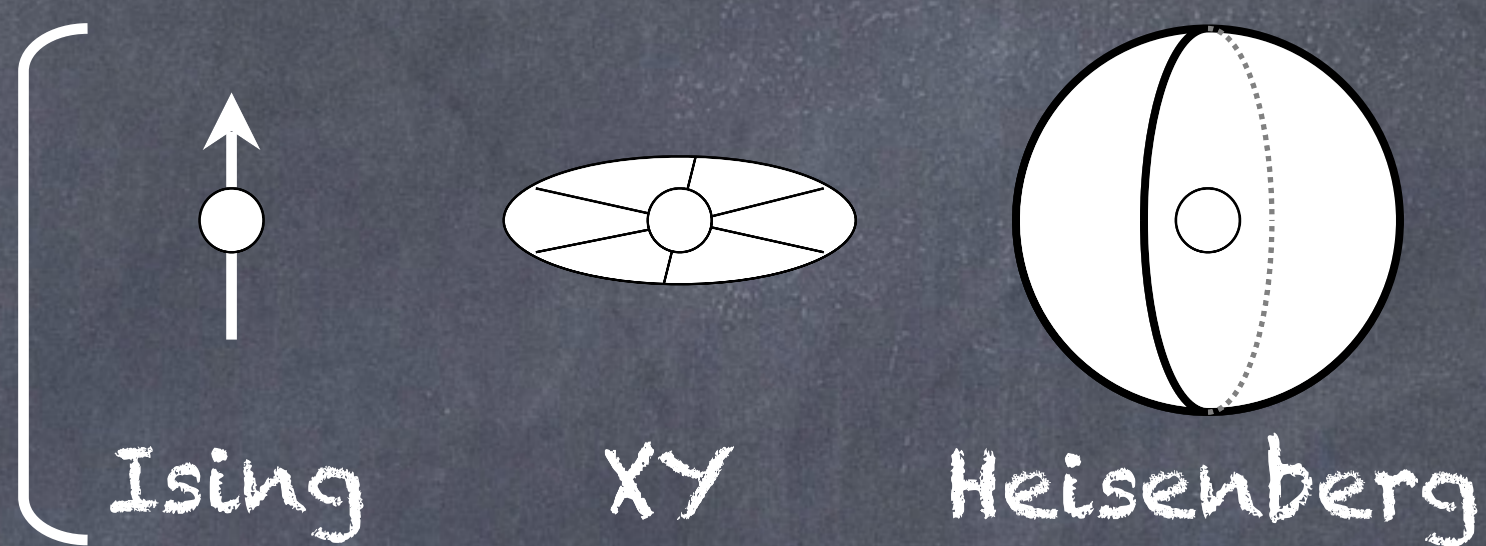
2 - in a quantum counter part, $S_i^+.S_j^- + S_i^-.S_j^+$ will do the job -> resonance. (RVB, SR-RVB physics)

From this example, we have the basic brick to try understanding what is at play and what we can play with.

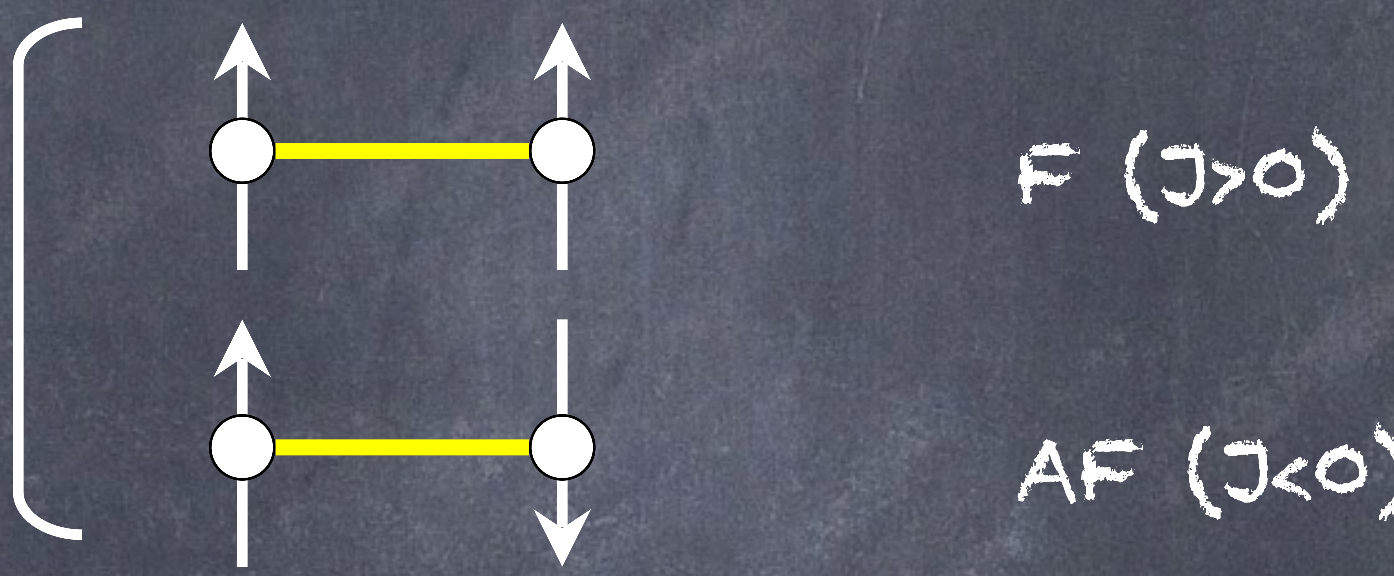
- local geometrical constraints -> edge/corner/plaquette sharing
- Cooperative geometrical constraints -> propagation of the constraints in a lattice
- Degree of freedom constraints -> Ising/XY/Heisenberg
- Interaction constraints. -> symmetric/anti-symmetric/anisotropic/Kitaev

Frustration, what can we play with?

Local magnetic degree of freedom:

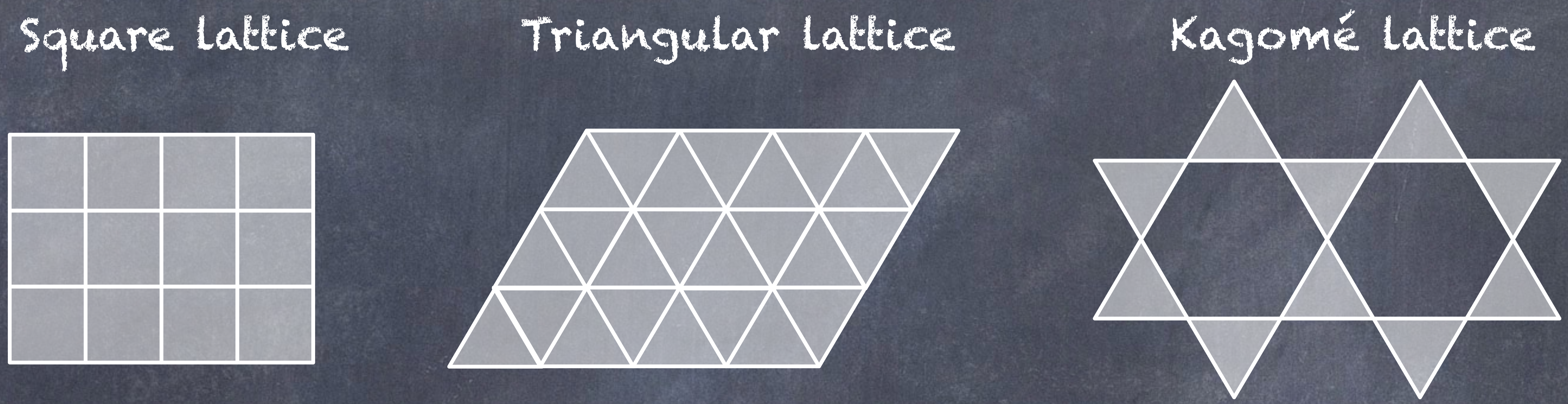


Coupling between degrees of freedom:

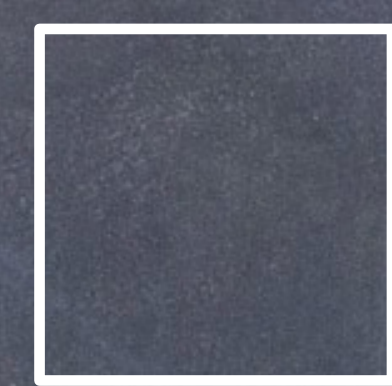
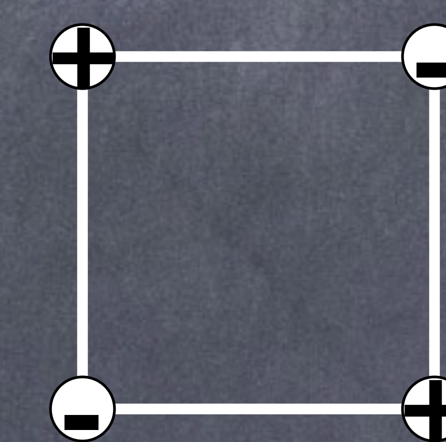


Cooperative behavior of the whole: $-J\sum_{\langle i,j\rangle}\vec{S}_i.\vec{S}_j$

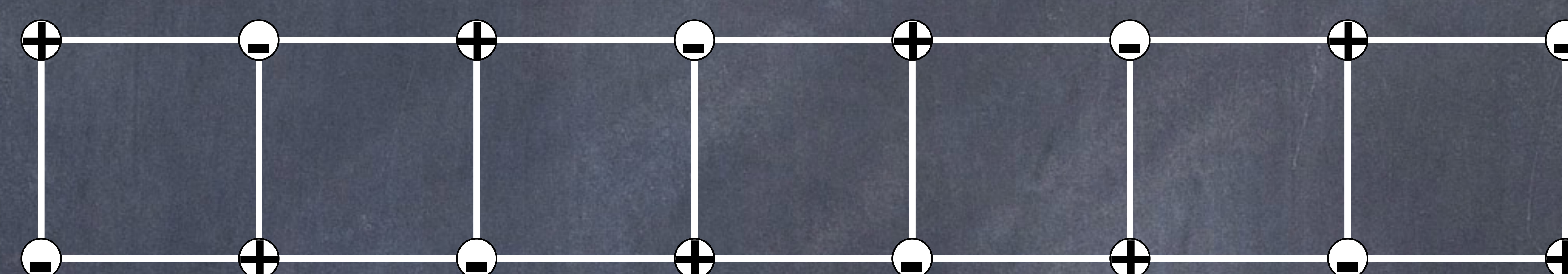
Frustration, what can we play with?



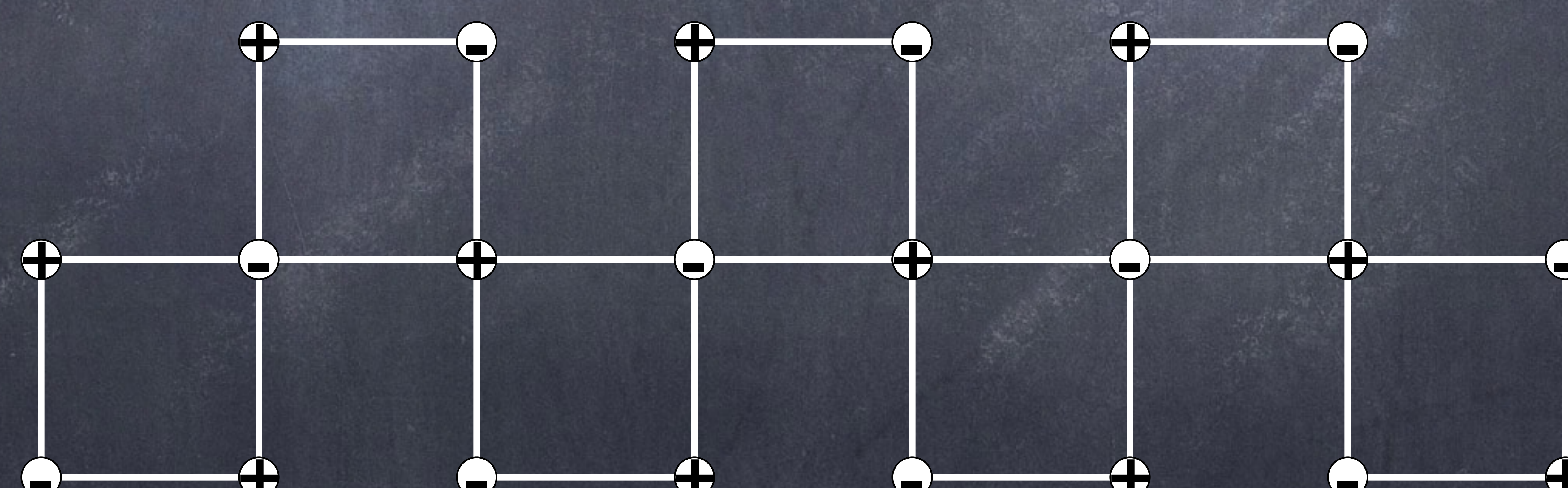
Frustration, what can we play with?

 + Ising spins AF = 

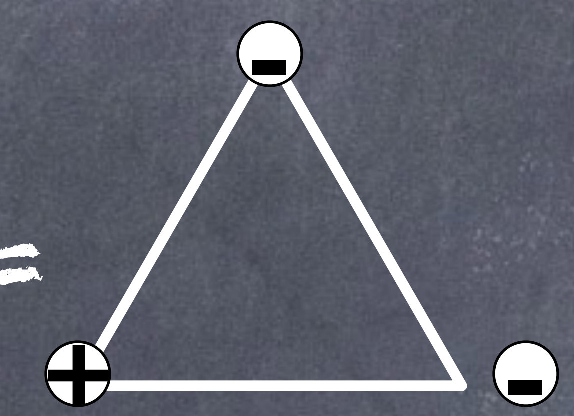
Edge sharing



Corner sharing



Frustration, what can we play with?

 + Ising spins AF = 

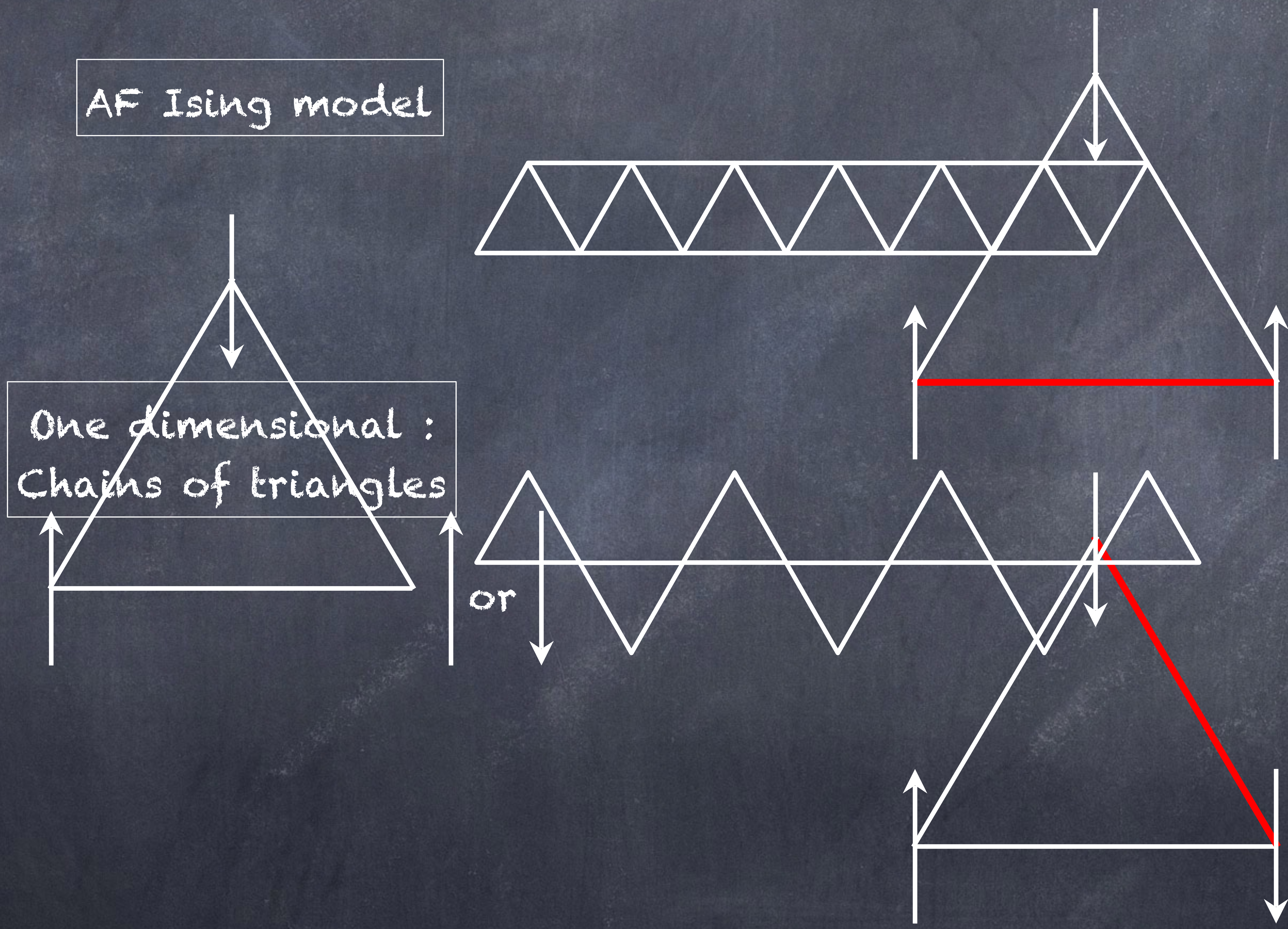
or ⊕ ? whatever...

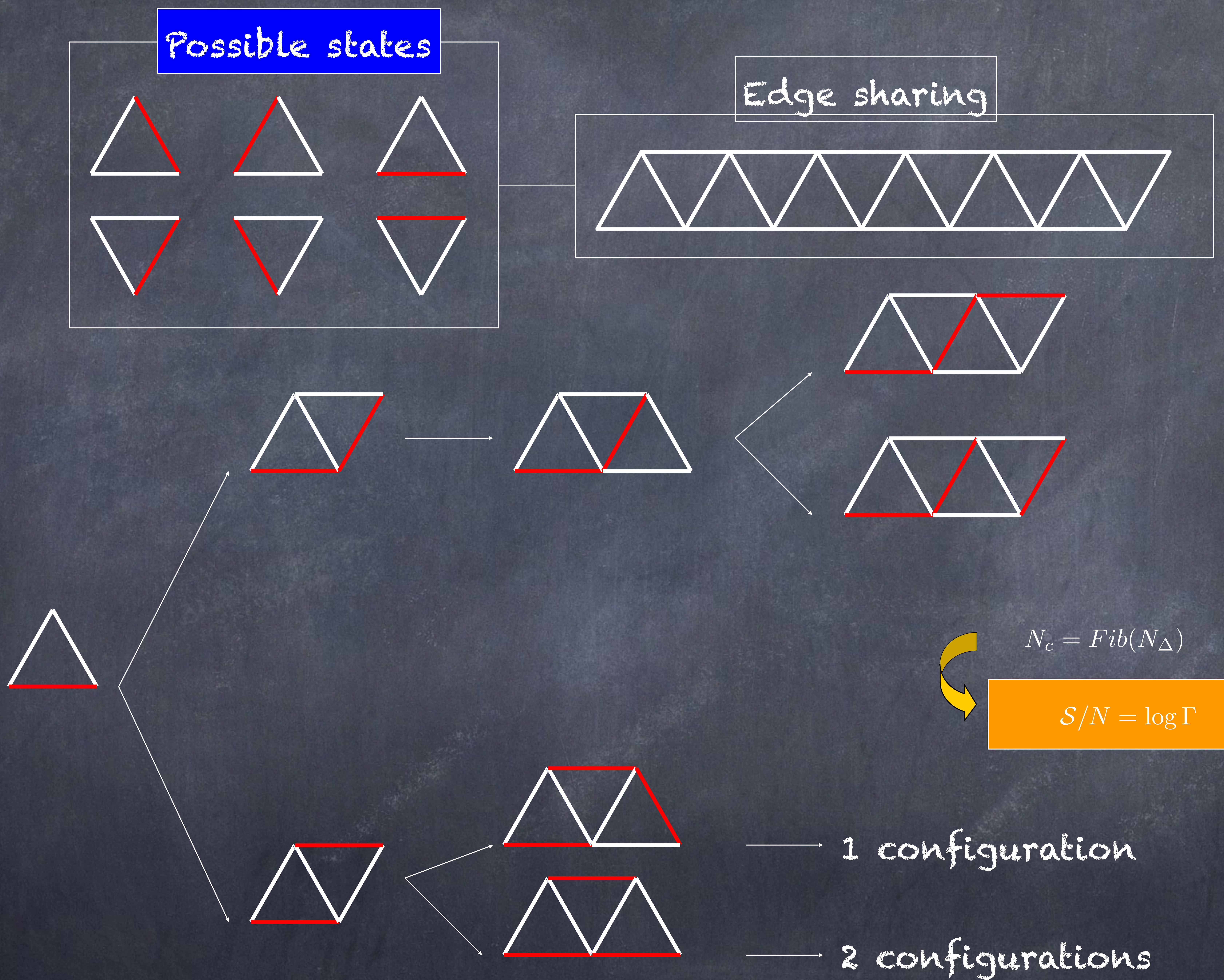
Impossible to minimize simultaneously all pairwise interactions

Local geometry (connectivity)	-	spin dimension	-	lattice effects
		Ising XY Heisenberg		Constraints related to loops

Frustration, what can we play with?
Local geometry (connectivity)

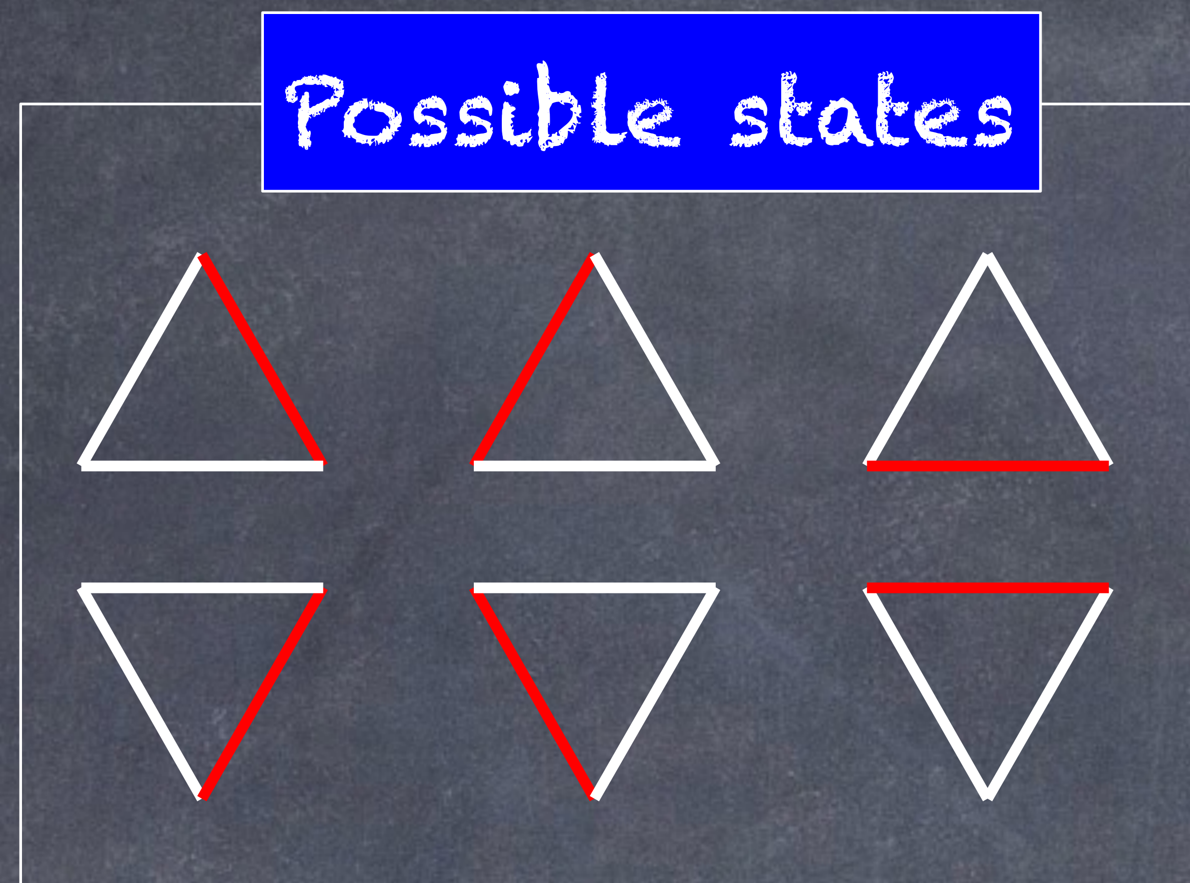
AF Ising model

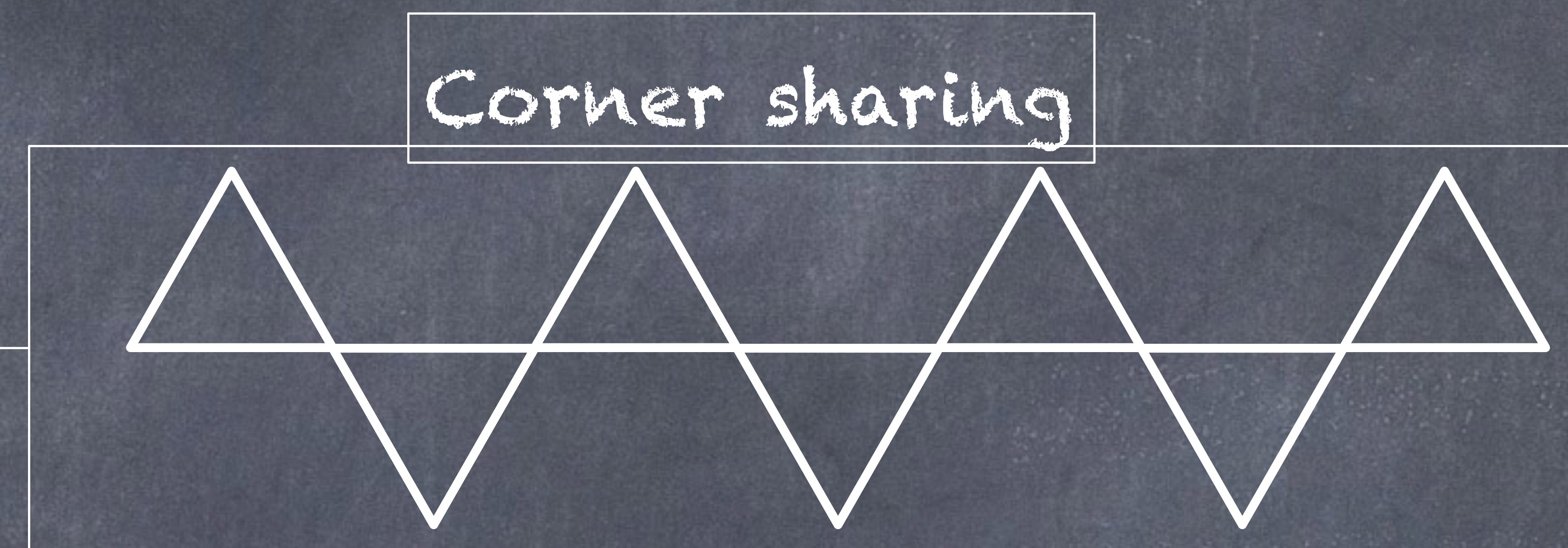




$N_c = Fib(N_\Delta)$

$\mathcal{S}/N = \log \Gamma$





3 configurations

3 configurations

3 configurations

$$N_c = 3^{N_\triangle}$$

$$\mathcal{S}/N = \frac{1}{2}\log 3$$

Frustration, what can we play with?
Local geometry (connectivity)

$\mathcal{S}/N = \log \Gamma \approx 0.48$

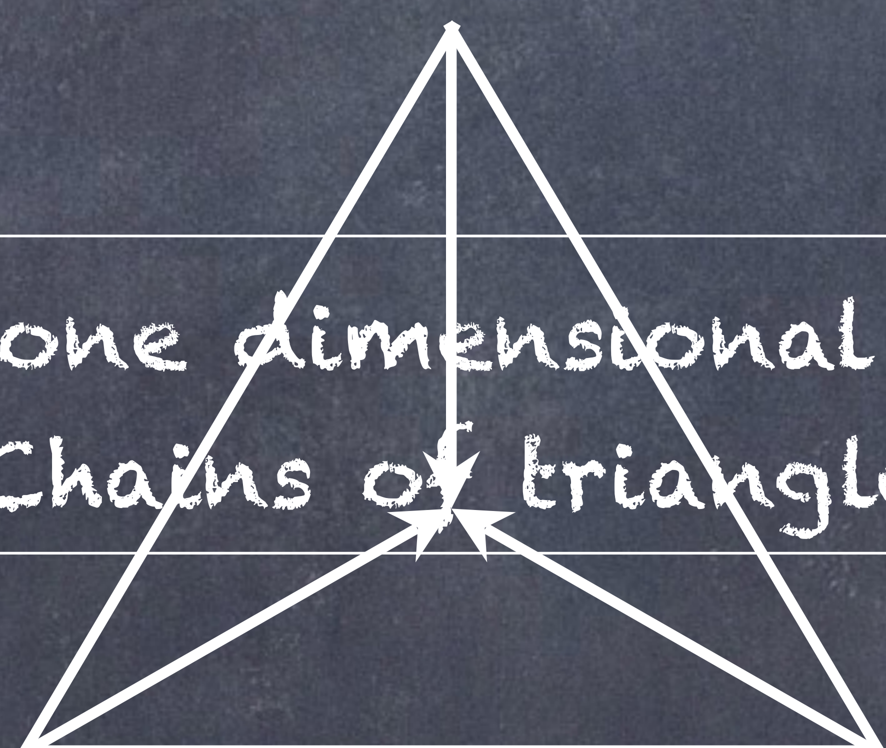
$\mathcal{S}/N = \frac{1}{2} \log 3 \approx 0.55$

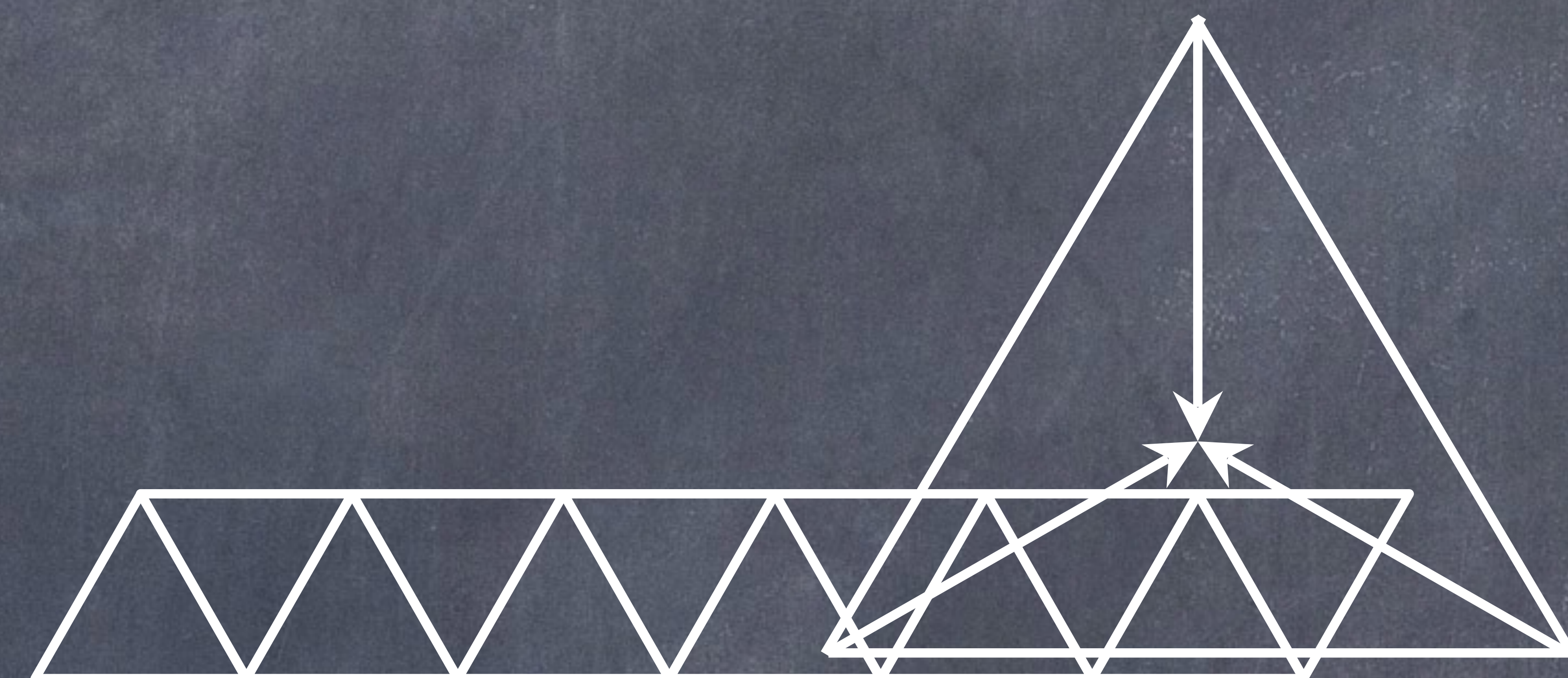
Frustration, what can we play with?

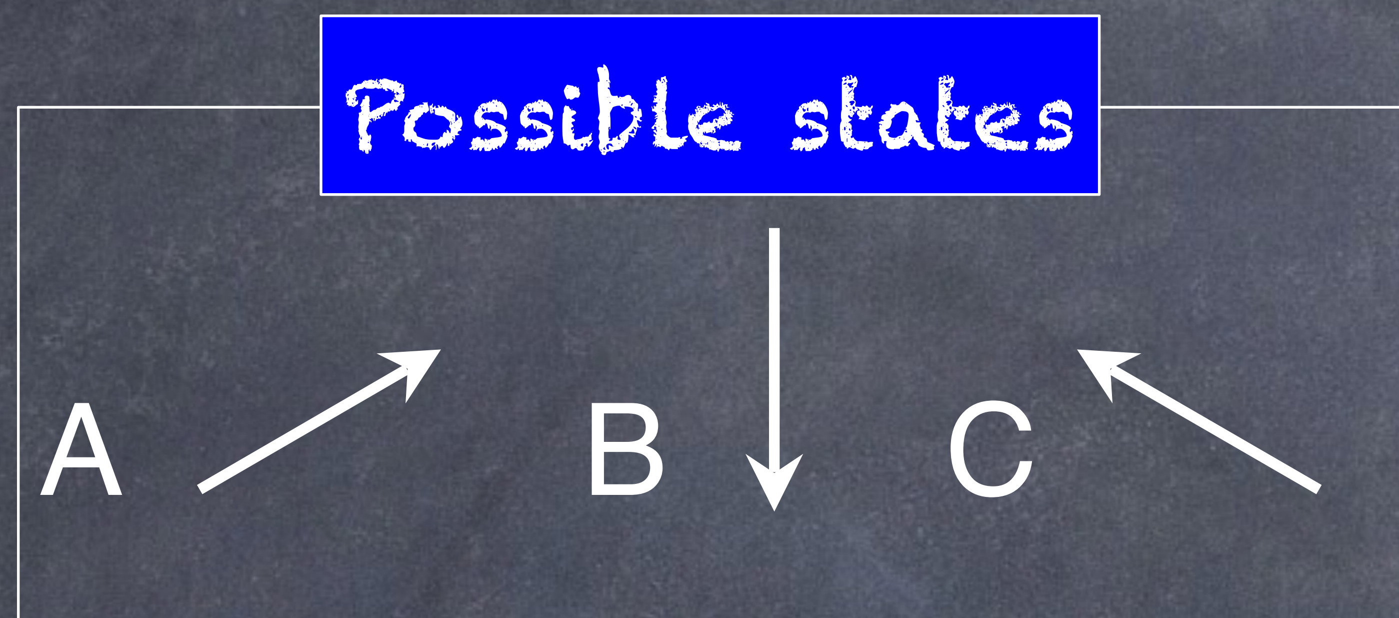
Local geometry (connectivity)

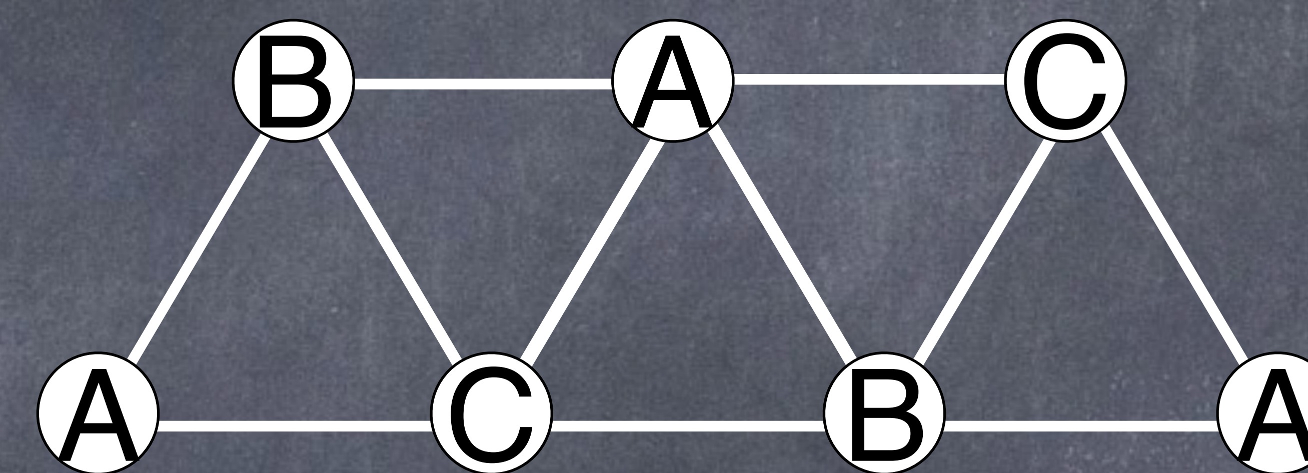
XY model

one dimensional :
Chains of triangles



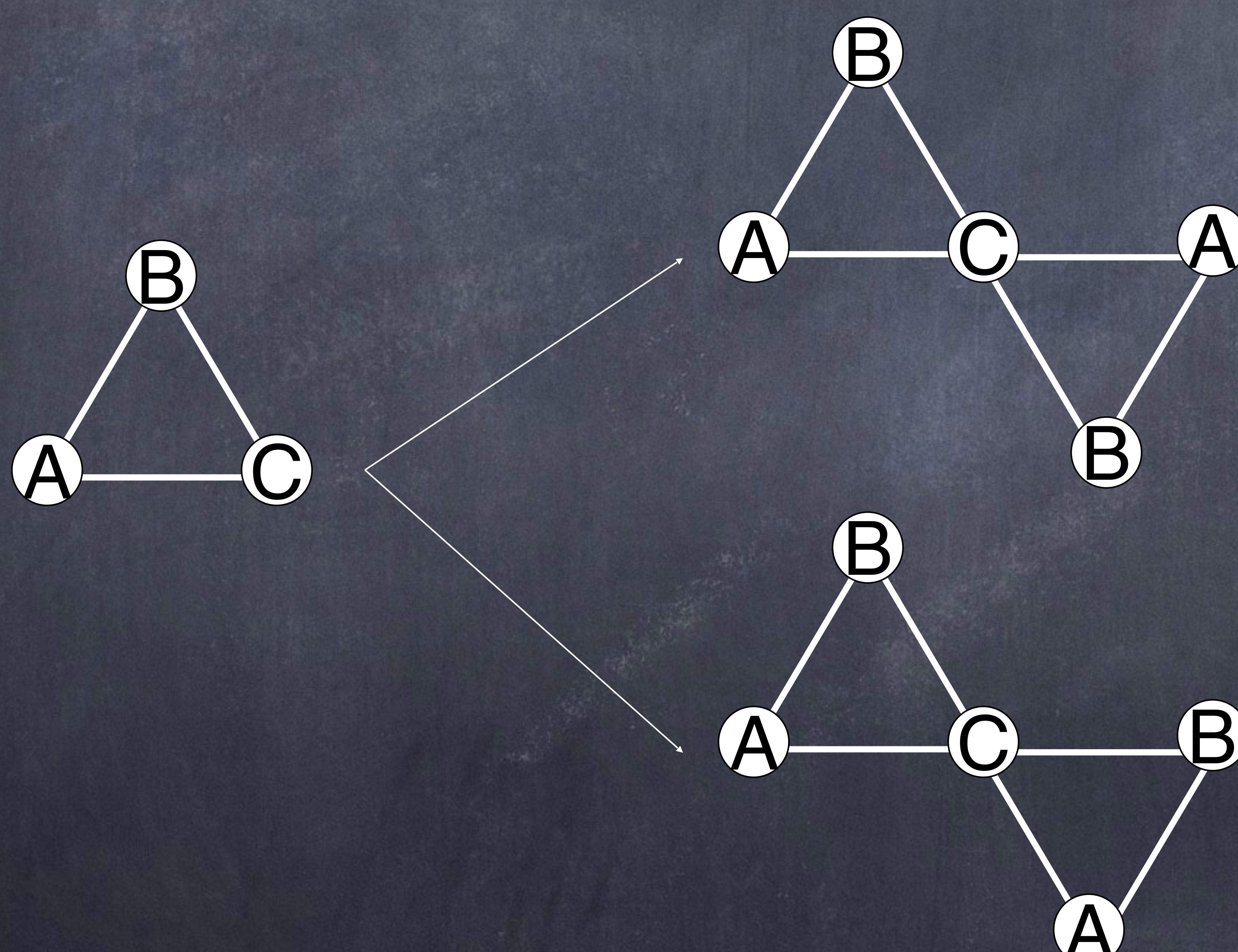






$N_c = 1$

$\mathcal{S}/N = 0$



$N_c = 2^{N_\triangle}$

$\mathcal{S}/N = \frac{1}{2}\log 2$

benjamin.canals@neel.cnrs.fr

Frustration, what can we play with?

Local geometry (connectivity) + spin dimension

$\mathcal{S}/N = 0$ — **XY model** — $\mathcal{S}/N \approx 0.34$

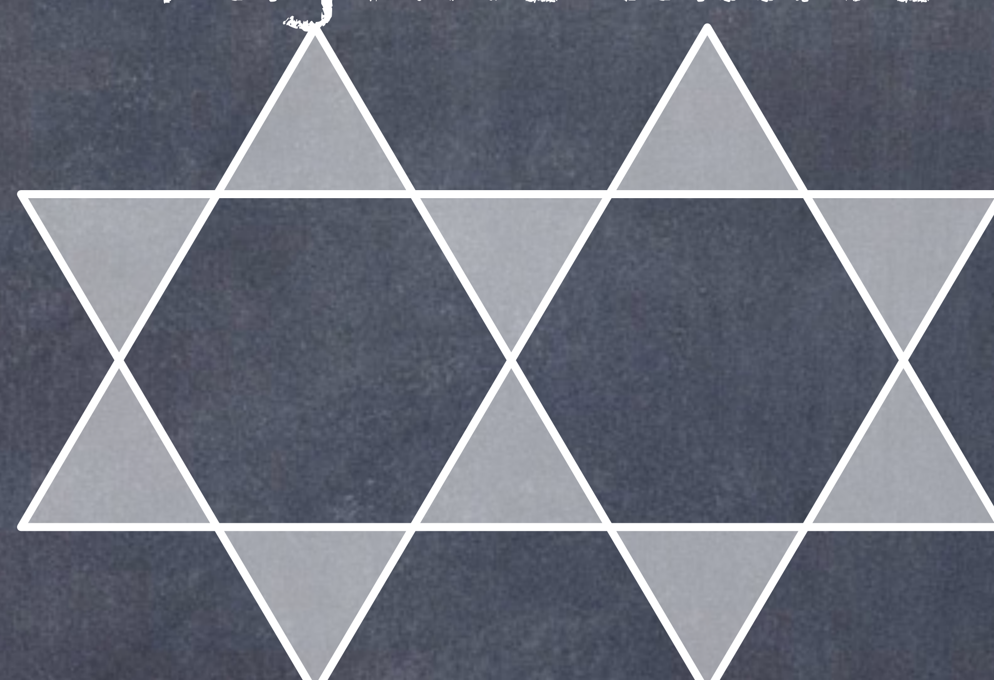
$\mathcal{S}/N \approx 0.48$ — **Ising model** — $\mathcal{S}/N \approx 0.55$

Frustration, what can we play with?

Lattice effects

Ising spins

Kagomé lattice



Triangular lattice

S/N≈0.5

[Kanô and Naya, 1953]

d↑, s→

S/N≈0.55

S/N≈0.32

[Wannier, Houttappel, 1950]

d↑, s→

S/N≈0.48

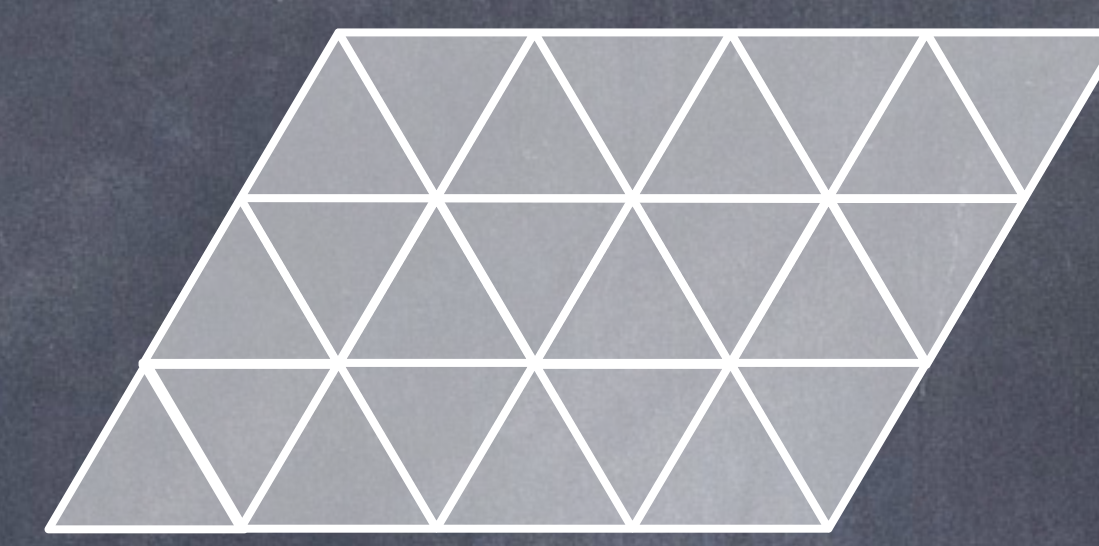
Frustration, what can we play with?

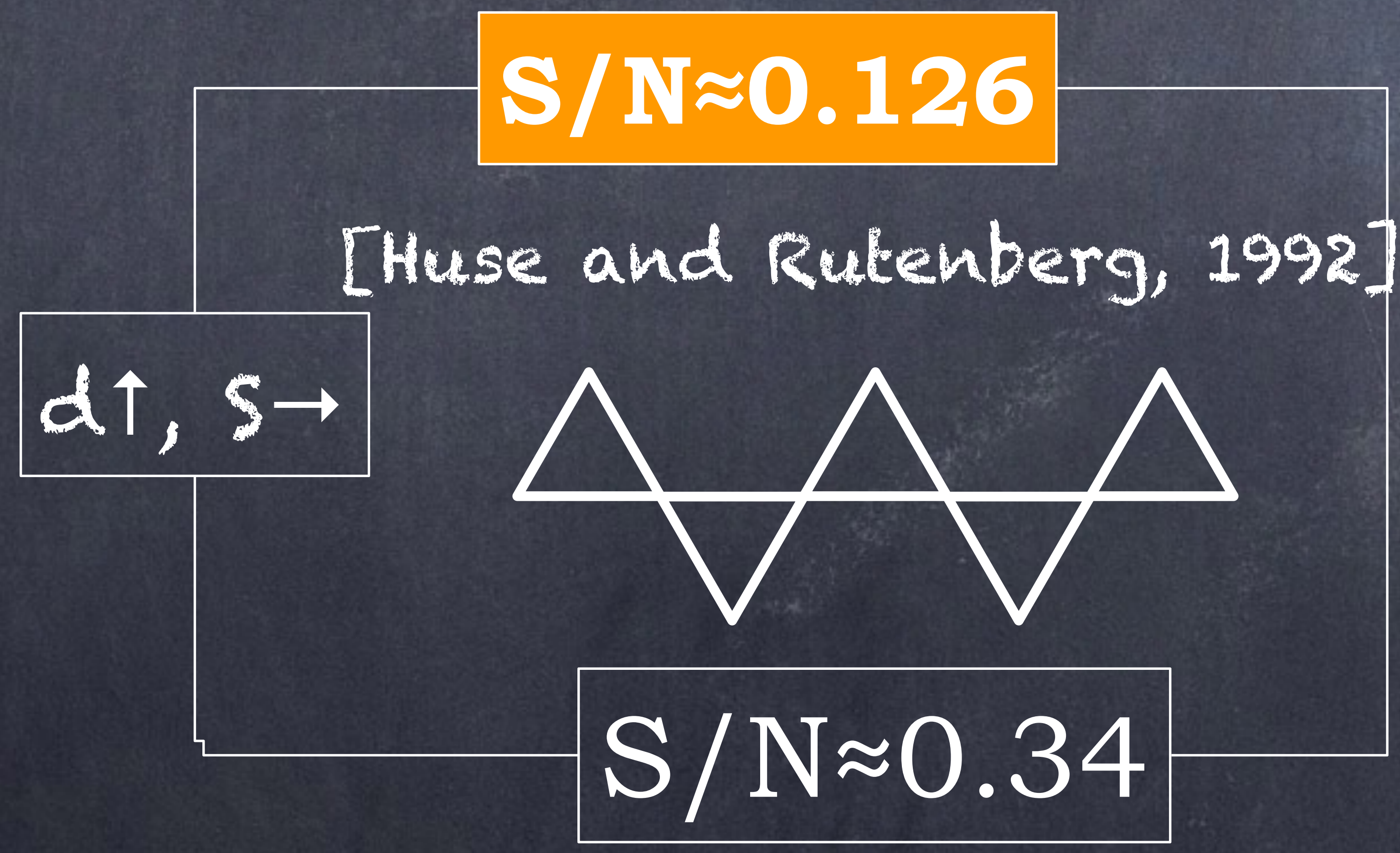
Lattice effects

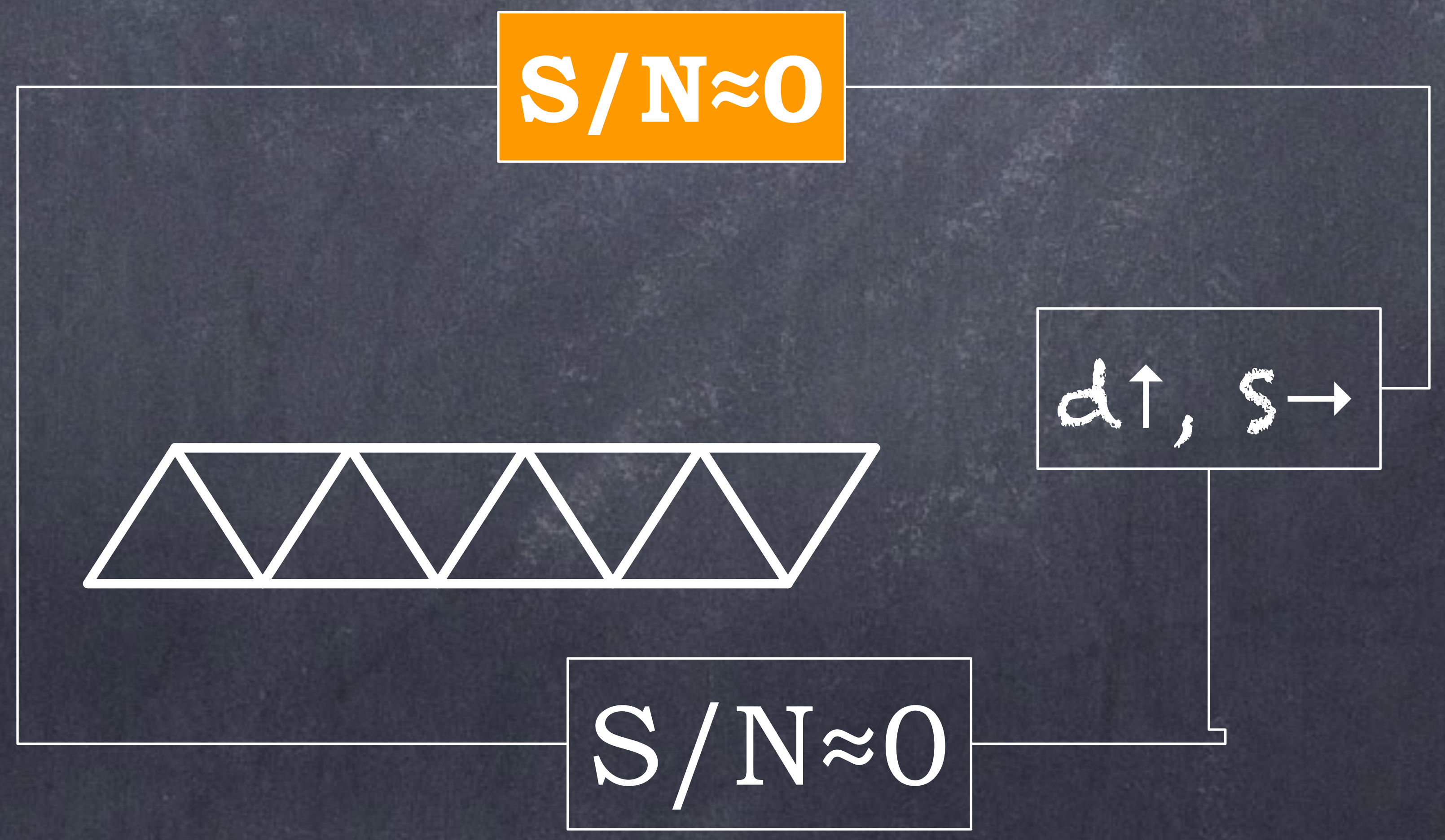
XY spins

Réseau kagomé

Réseau triangulaire

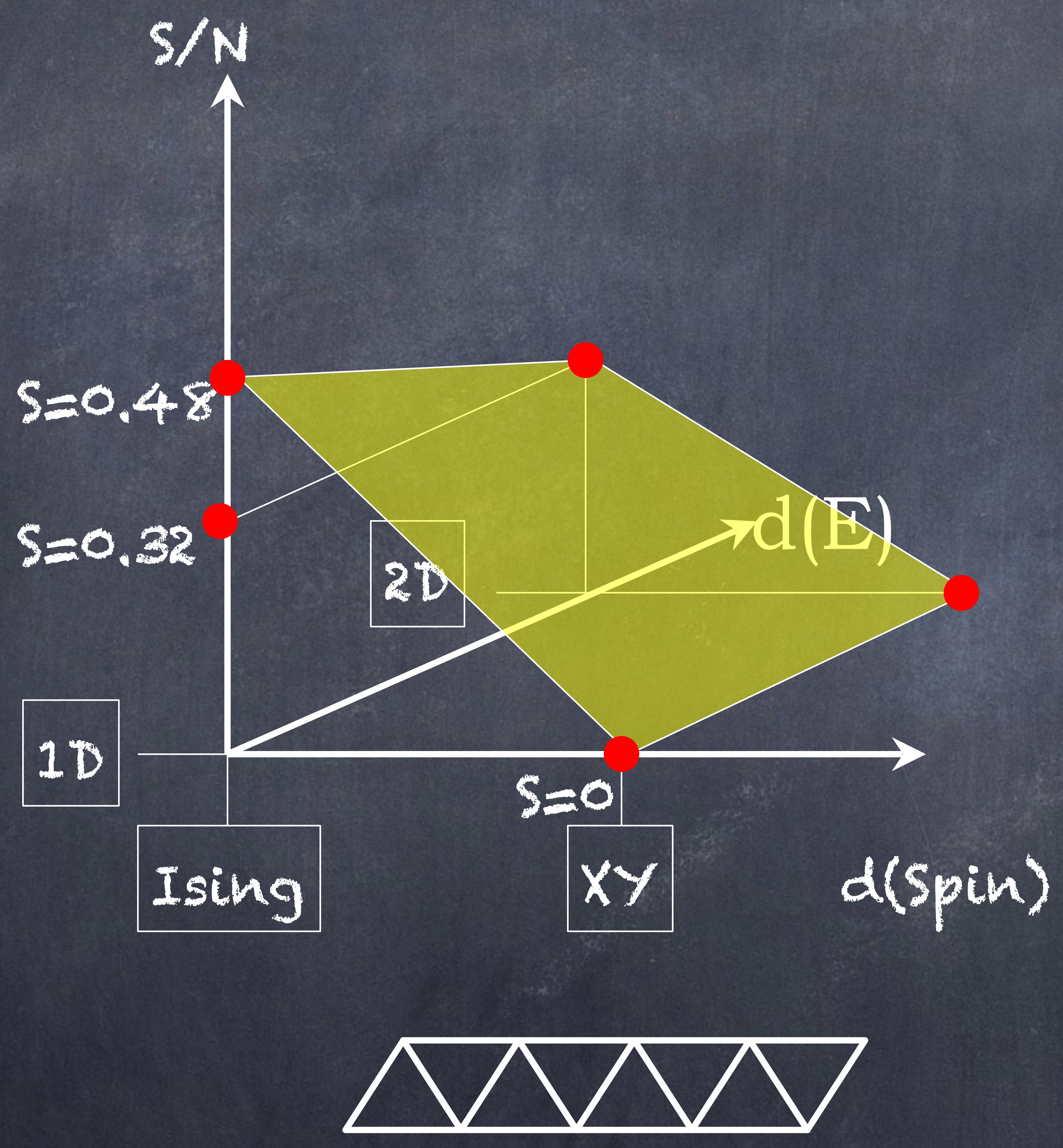


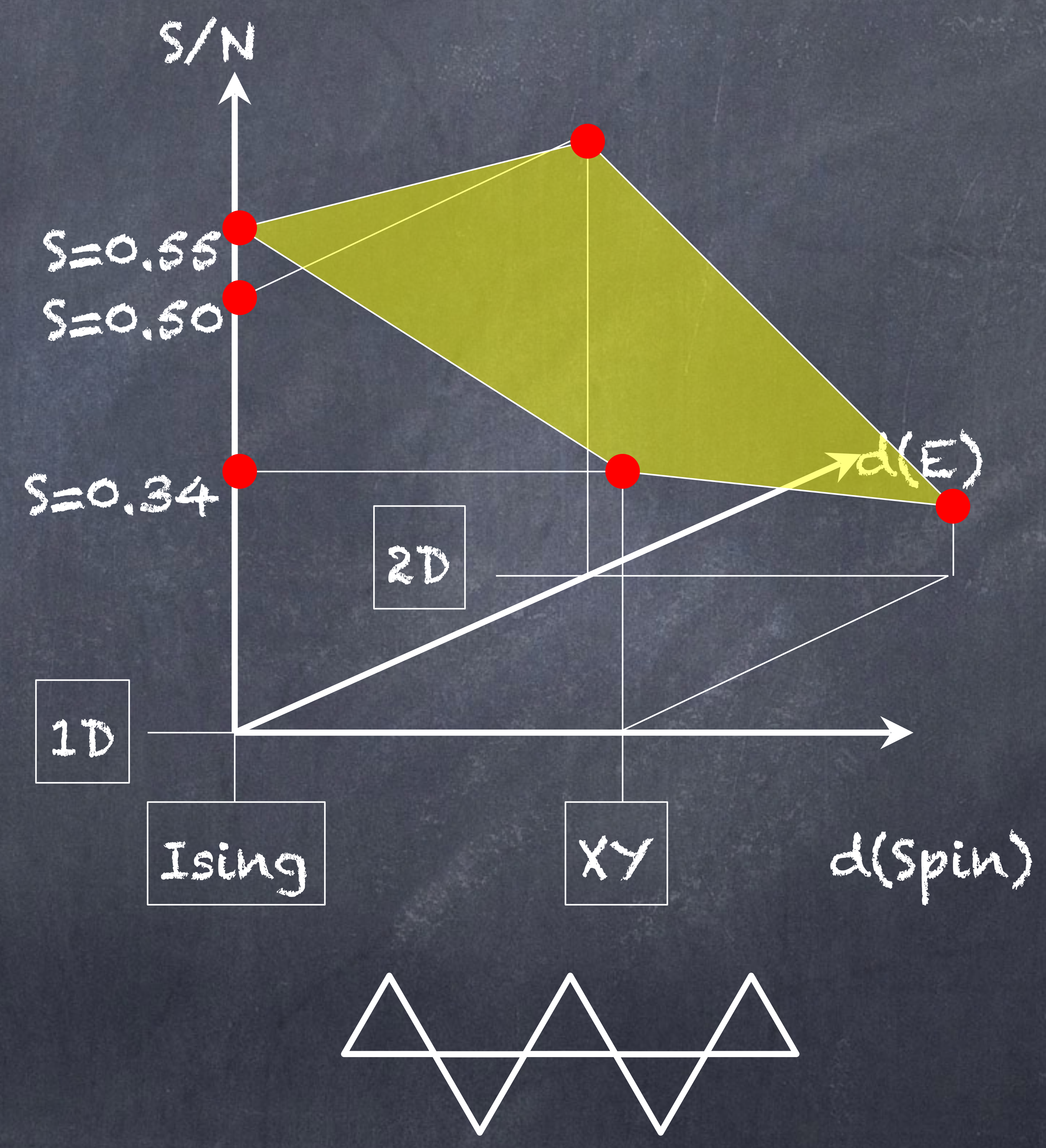




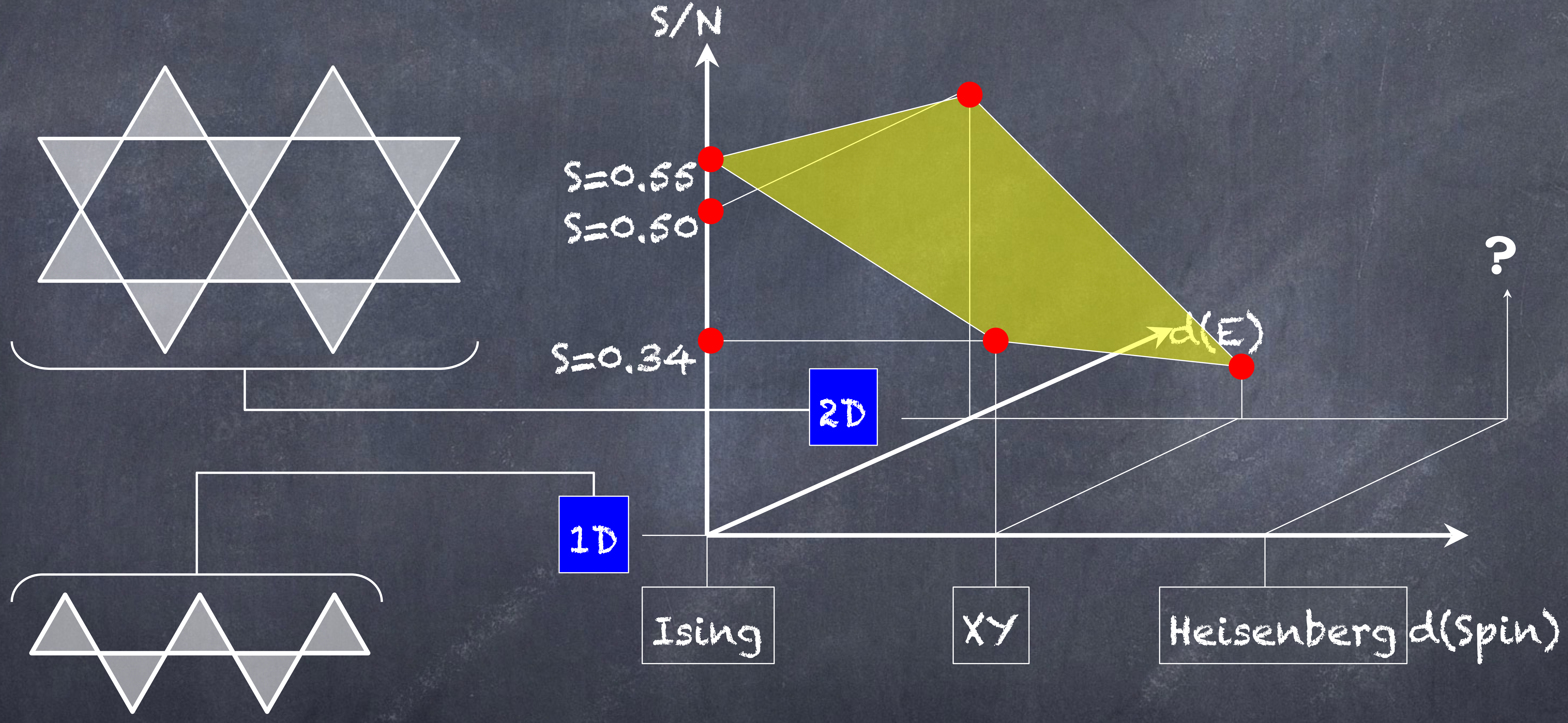
Frustration, what can we play with?

Summary





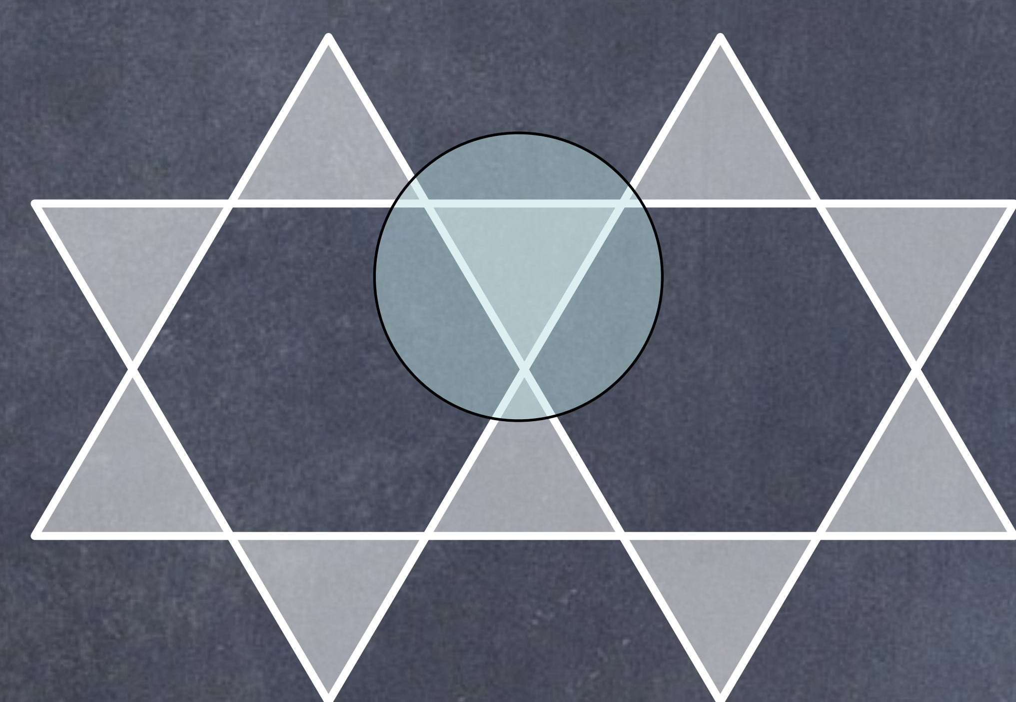
Last effort: the classical Heisenberg kagomé antiferromagnet

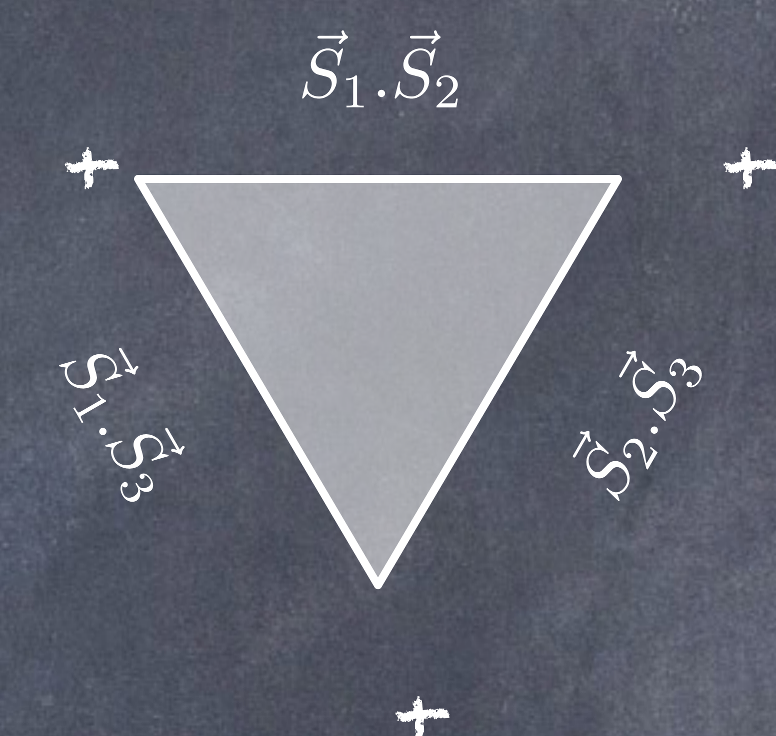


Last effort: the classical Heisenberg kagomé antiferromagnet

Factorisation and degenerate manifold

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \vec{S}_i.\vec{S}_j$$





$$\vec{S}_1.\vec{S}_2 + \vec{S}_1.\vec{S}_3 + \vec{S}_2.\vec{S}_3 = \frac{1}{2}\left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3\right)^2 - \frac{3}{2}S^2$$

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \vec{S}_i.\vec{S}_j = -\frac{J}{2}\sum_{\Delta}\left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3\right)^2 + C^{ste}$$

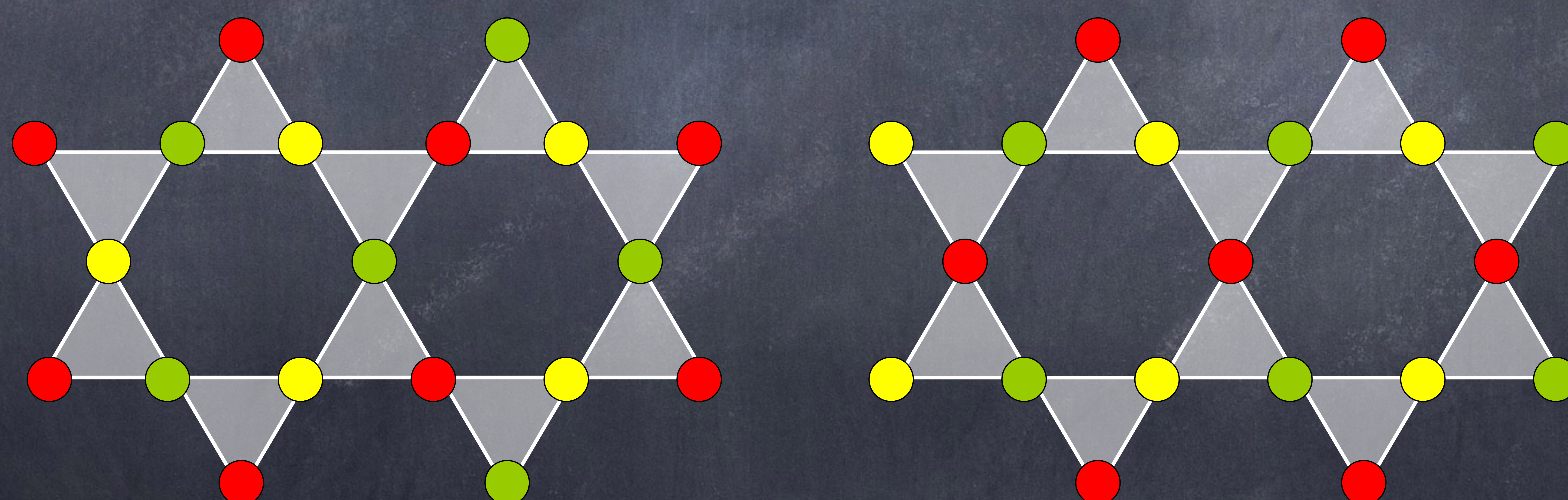
Last effort: the classical Heisenberg kagomé antiferromagnet

Factorisation and degenerate manifold

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \vec{S}_i.\vec{S}_j = -\frac{J}{2} \sum_{\triangle} \left(\vec{S}_1 + \vec{S}_2 + \vec{S}_3 \right)^2 + C^{ste}$$

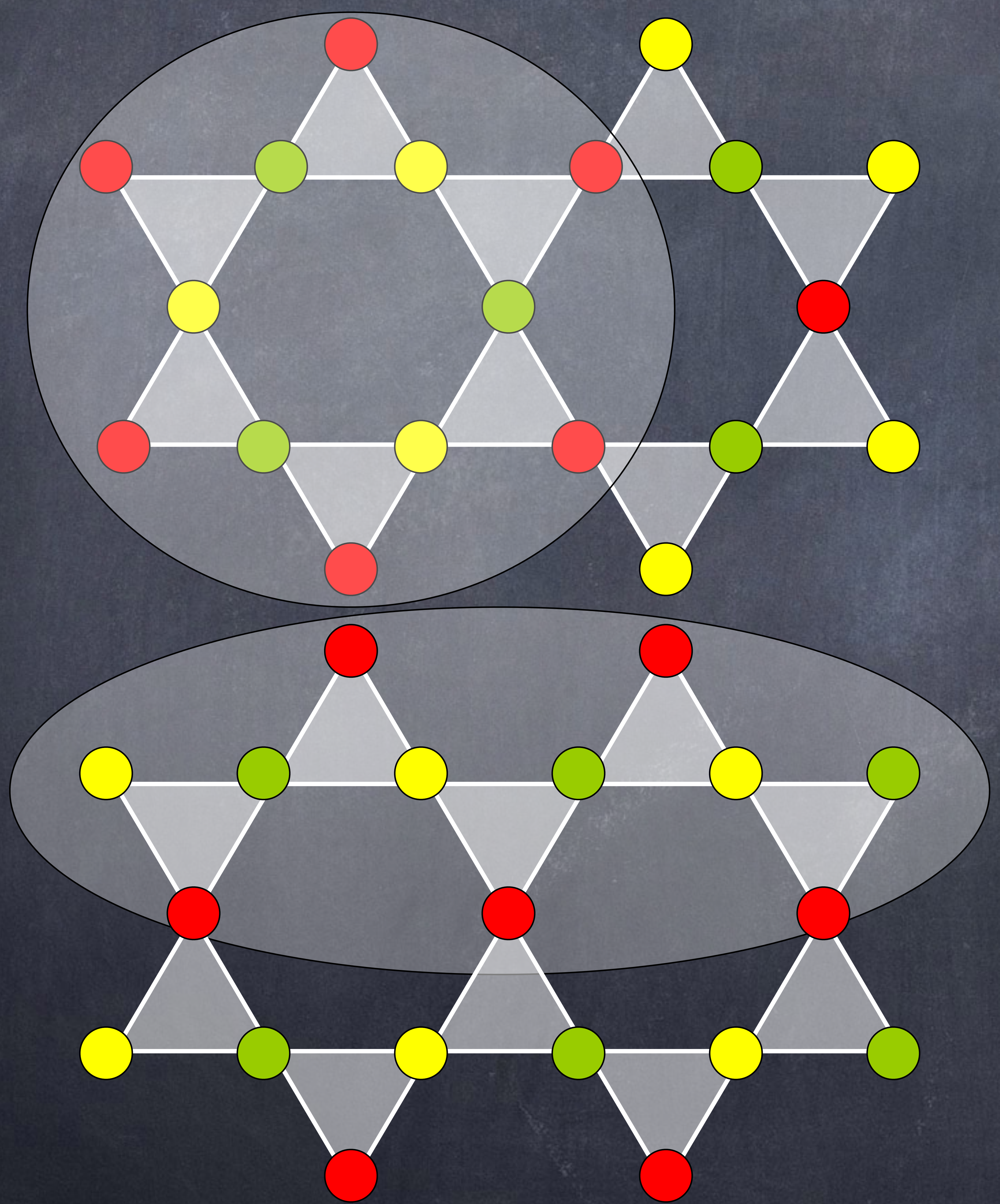
XY spin s

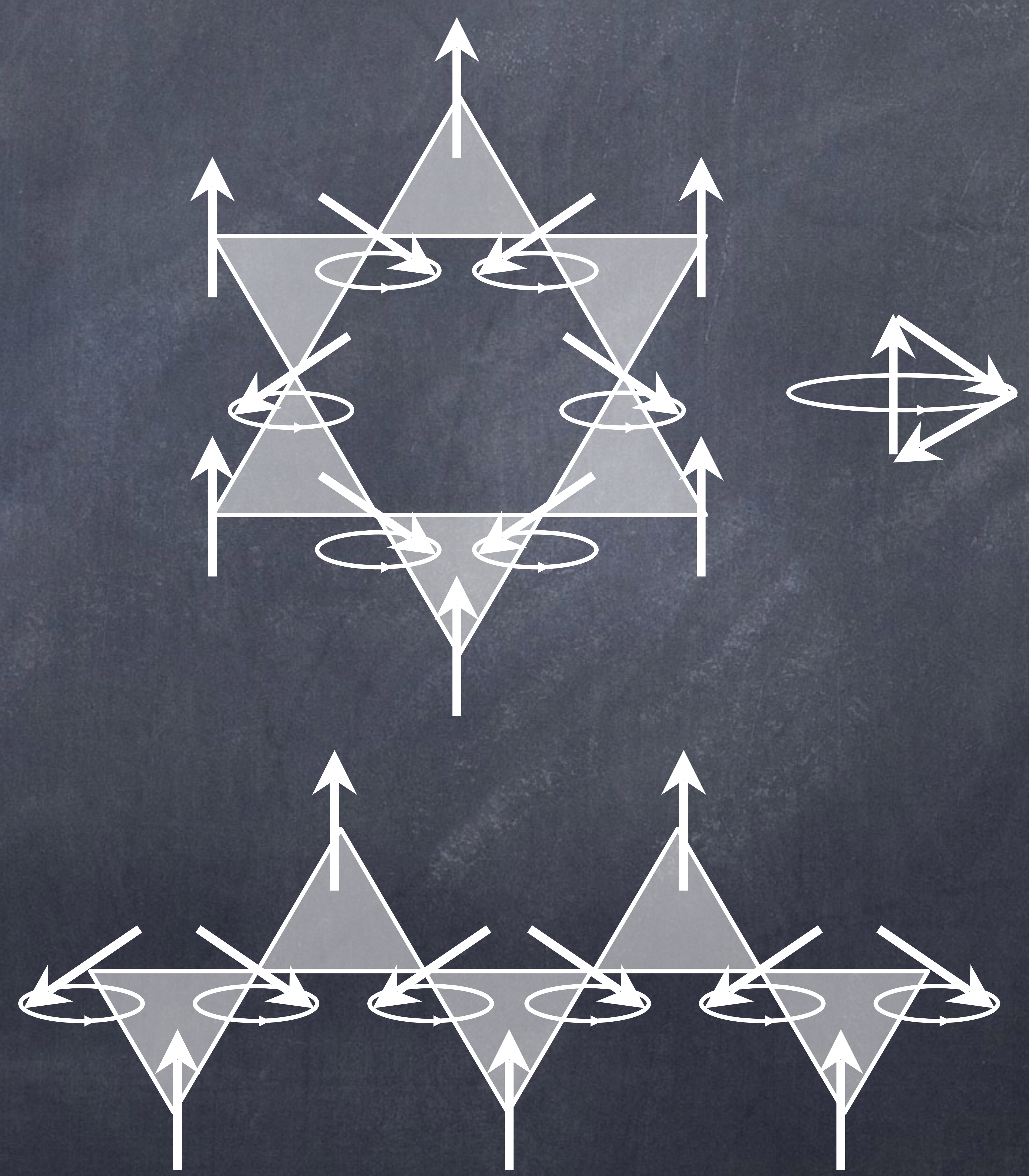
Discrete infinity of 3-colorings



Last effort: the classical Heisenberg kagomé antiferromagnet

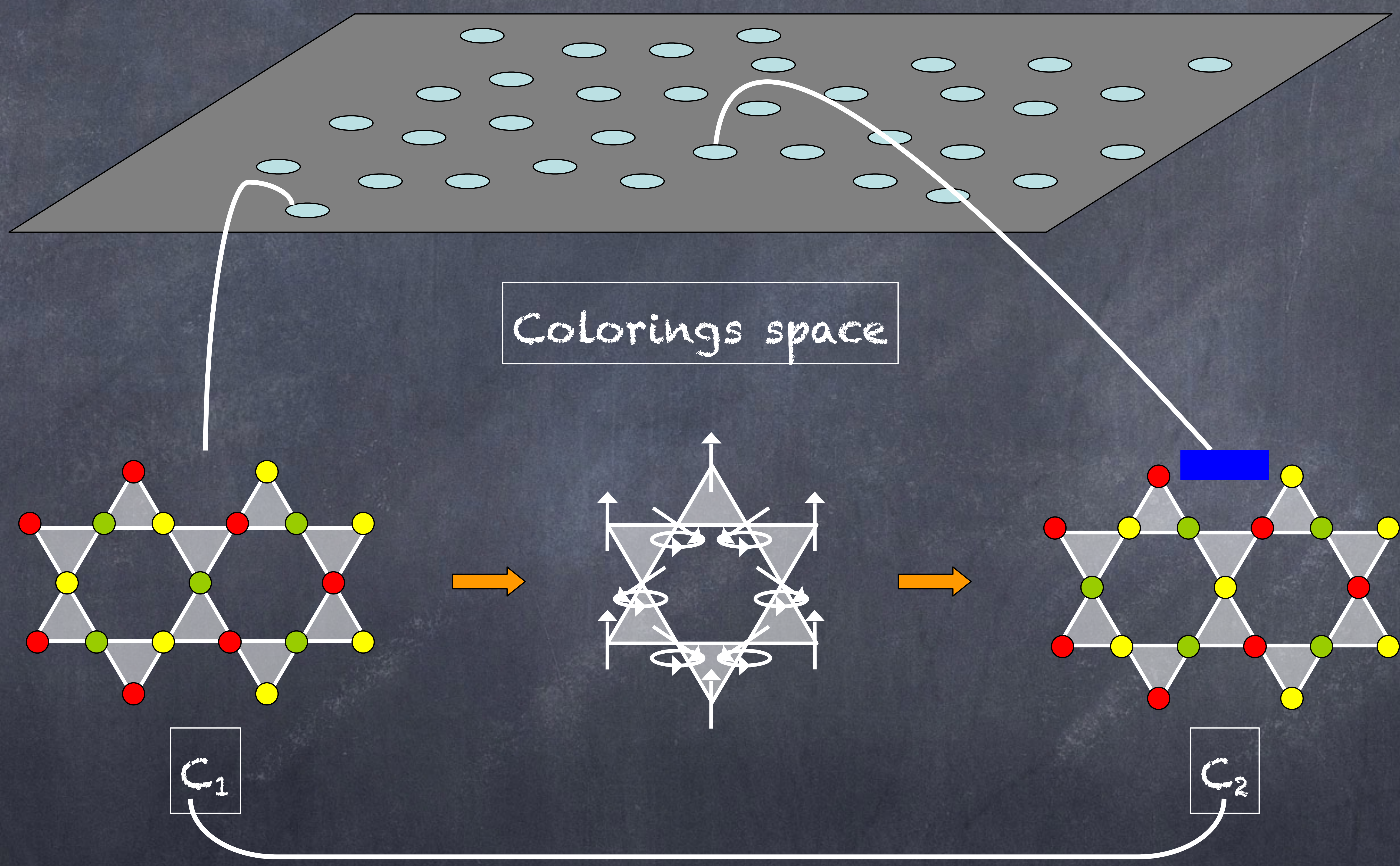
Factorisation and degenerate manifold





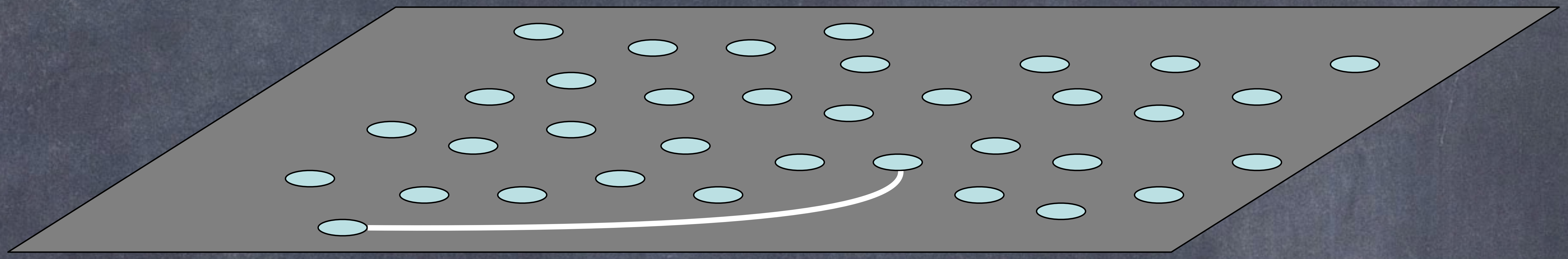
Last effort: the classical Heisenberg kagomé antiferromagnet

Local « weathervane » mode



Last effort: the classical Heisenberg kagomé antiferromagnet

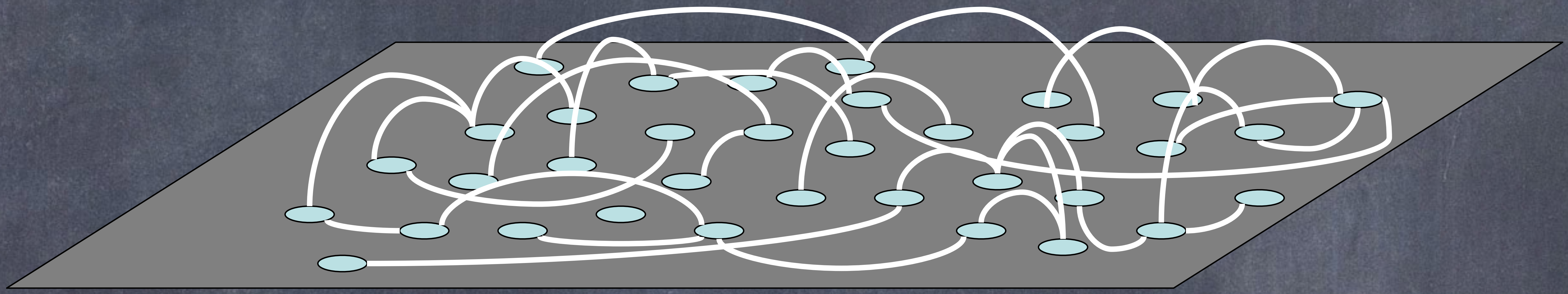
Local « weathervane » mode



Colorings space

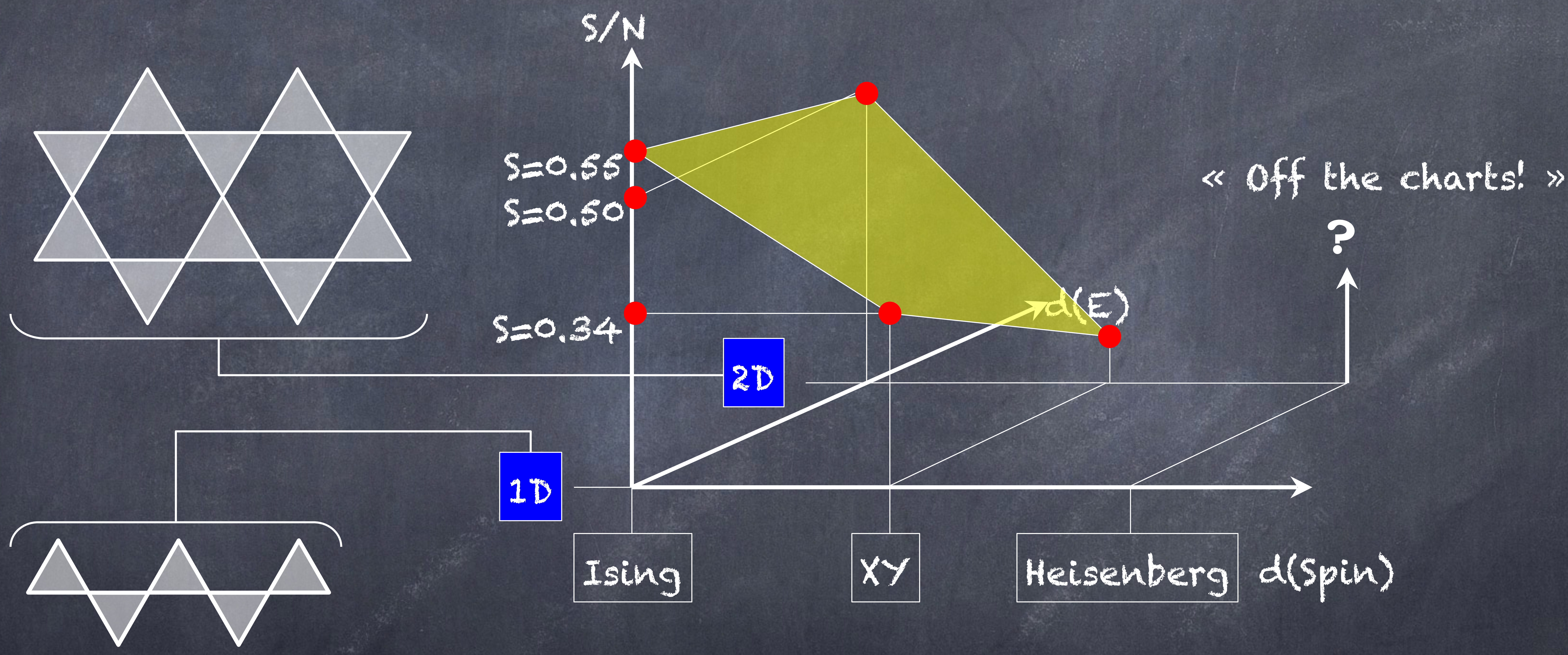
Last effort: the classical Heisenberg kagomé antiferromagnet

Local « weathervane » mode



The ground state manifold is simply connected and continuous

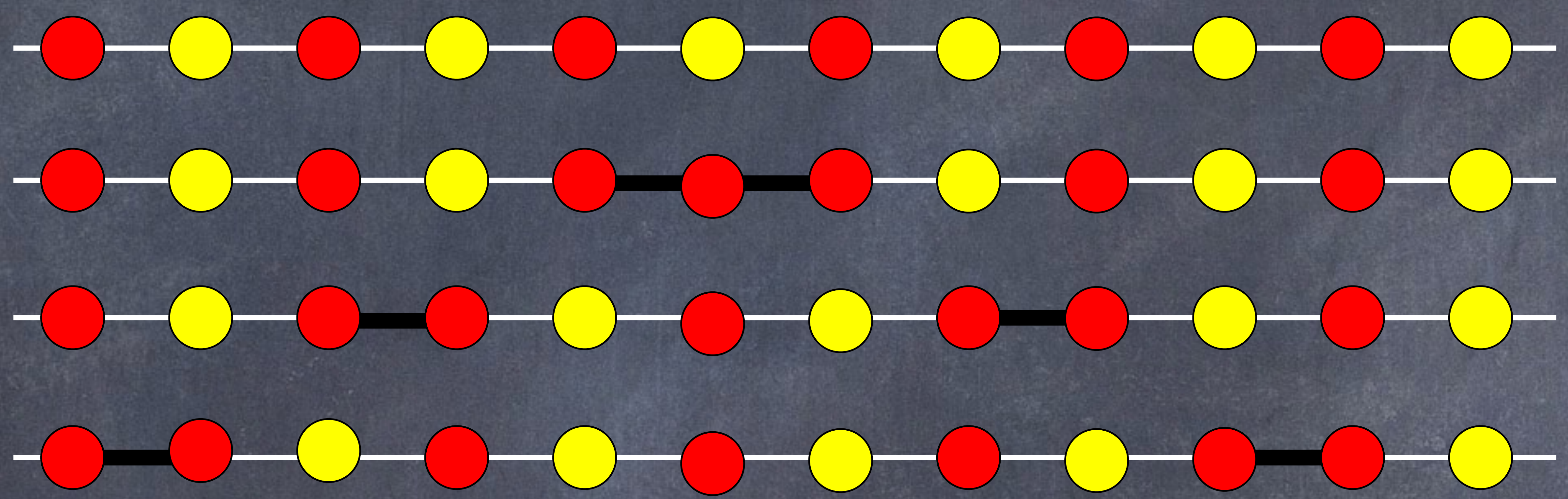
Obviously, phylogeny is not straightforward!



Intermezzo (before emergence) - a way to understand why frustration allows for high dimensional fractionalization

Kagomé based model may be prone to fractionalization -> this is a modern motivation.

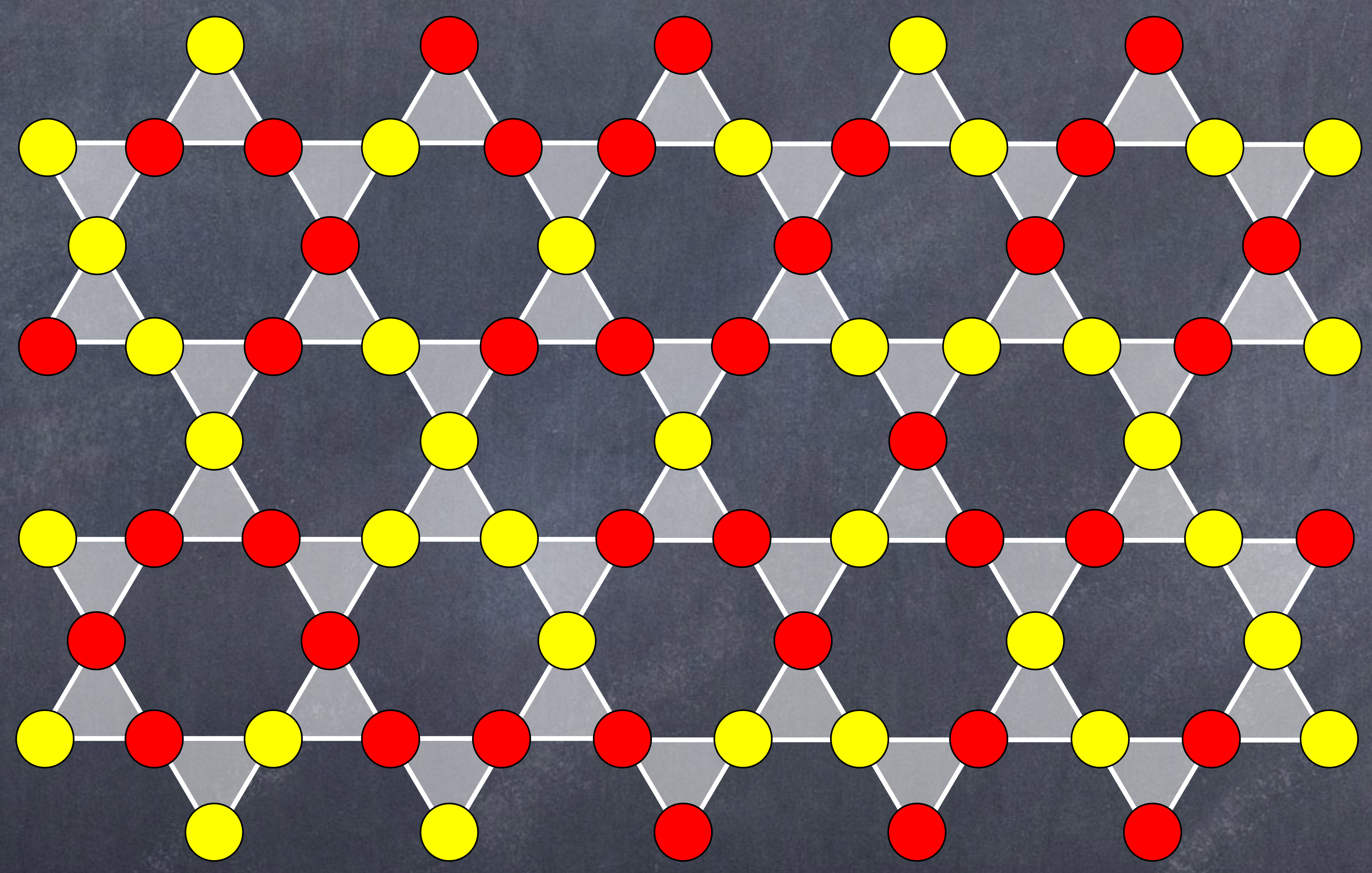
Let's come back to our 1D playground



In an AF 1D chain, the excitation may split.

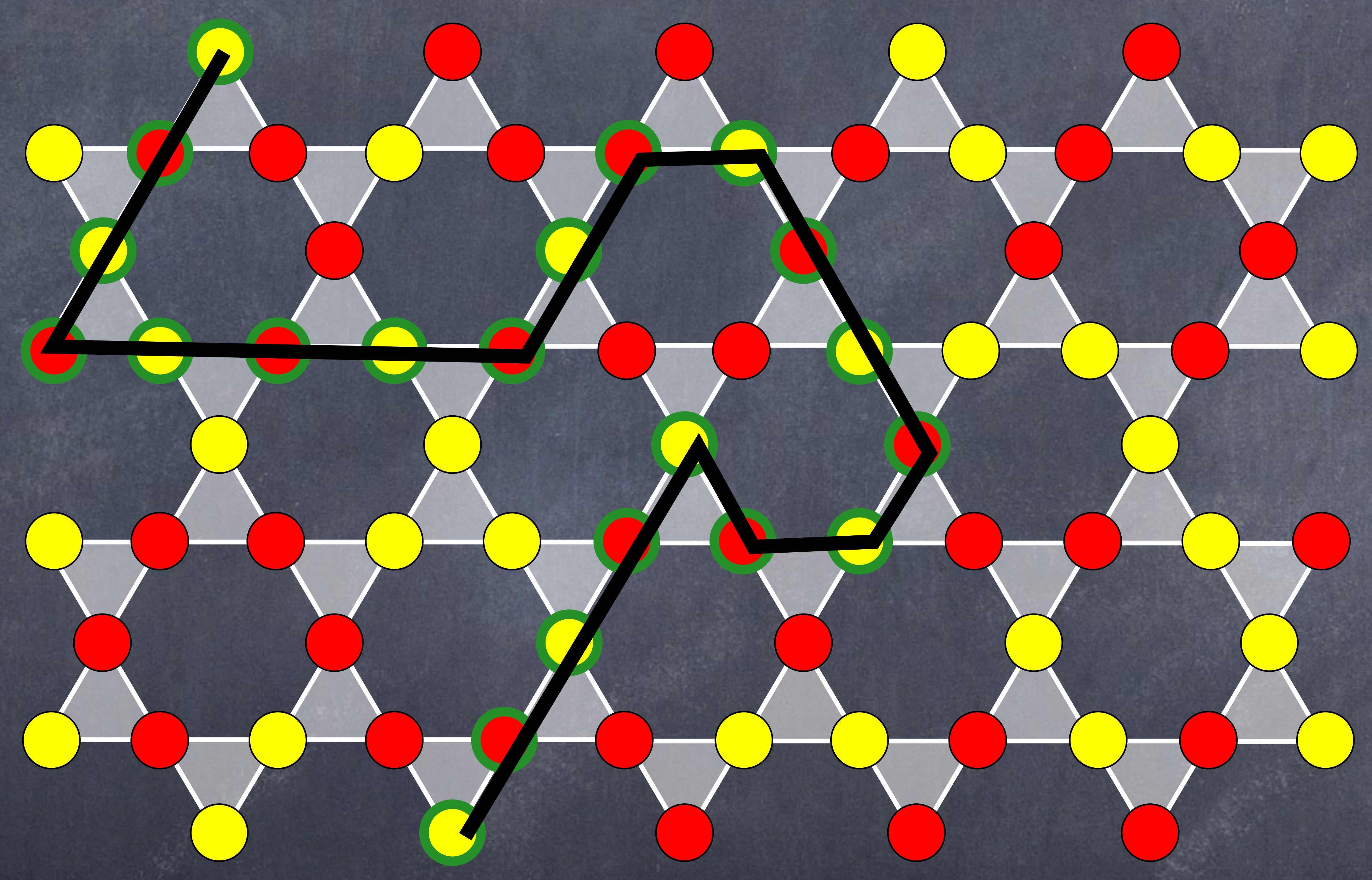
Intermezzo (before emergence) - a way to understand why frustration allows for high dimensional fractionalization

There are AF 1D chain in it!



Intermezzo (before emergence) - a way to understand why frustration allows for high dimensional fractionalization

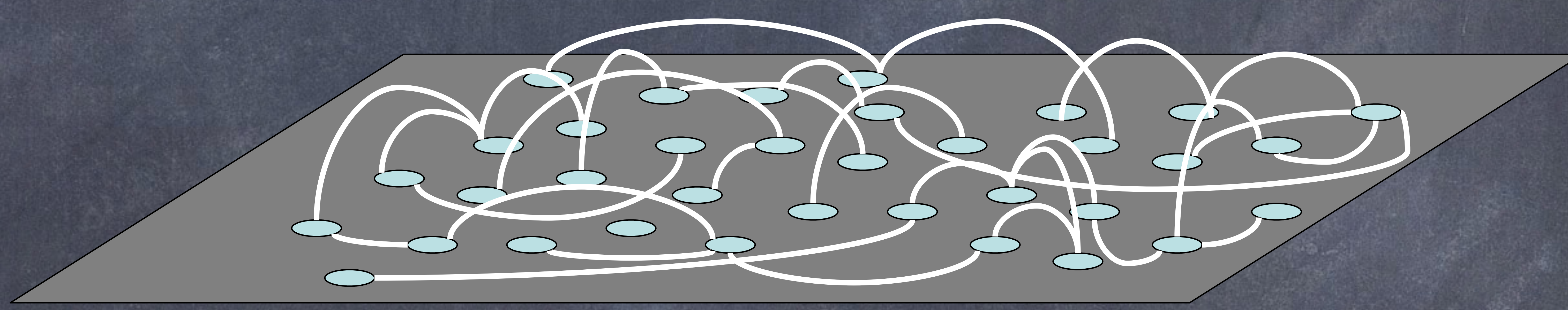
There are AF 1D chain in it!



An thanks to frustration, deconfinement takes place!

Intermezzo (before emergence) - a way to understand why frustration allows for high dimensional fractionalization

So; not only is the ground state manifold connected through e-costless moves, but excitations as well seem to be « exotic ».



Outline of the lecture

- @ On the route to frustration: ordering and time/dynamics issues of ordered magnets
  - classical case
  - quantum case
  - stability of Néel states

- @ Historical point of view
  - A first example of frustration
  - Condensed matter and statistical mechanics eventually meet
  - Entropy is interesting

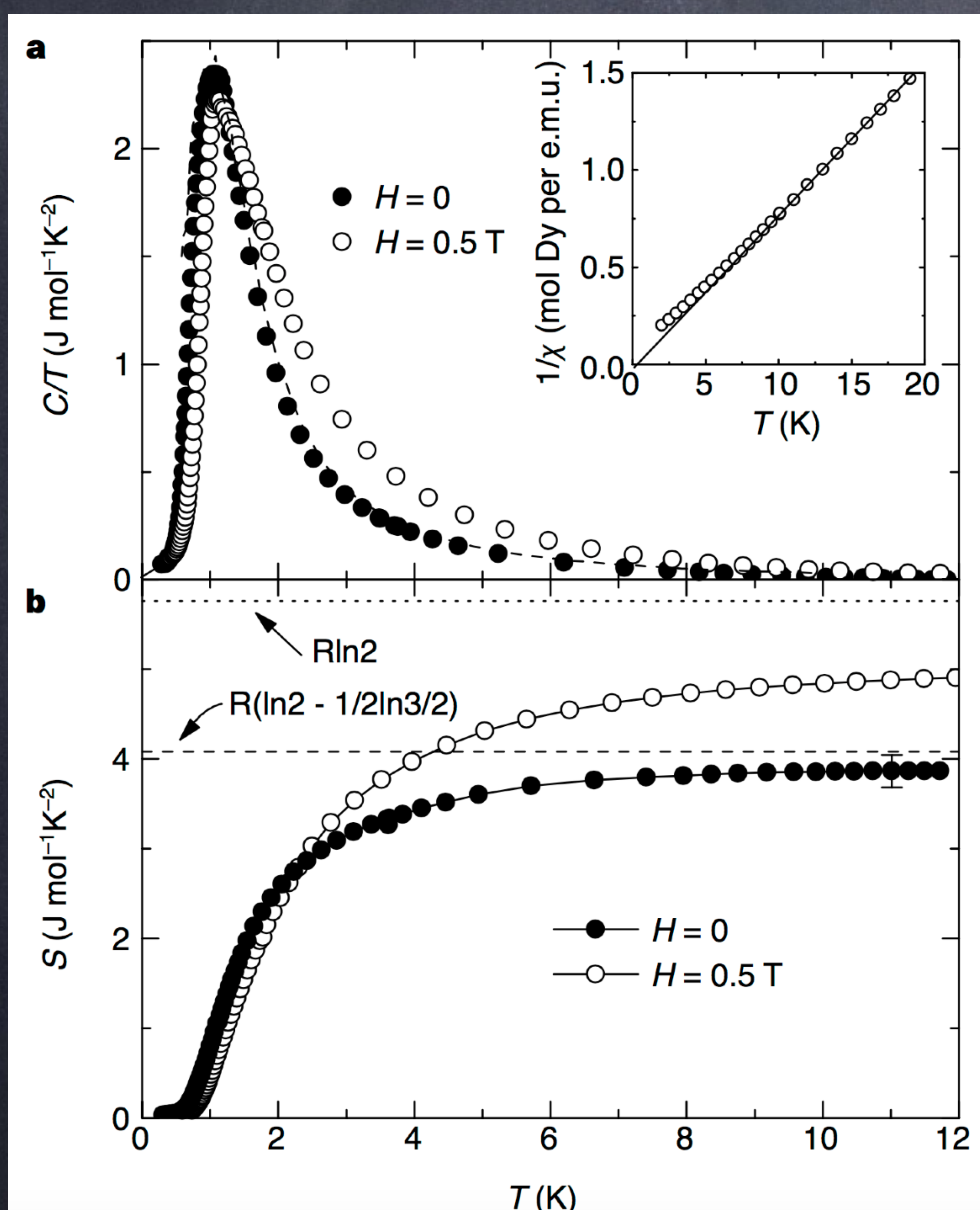
- @ Phylogeny of frustration
  - Study of a simple case
  - What can we play with
  - Well, it's not that simple...
  - But frustration helps deconfinement (fractionalization)

- @ Emergence in frustration
  - Back to spin ice
  - From spin to (magnetic) charge, and deconfinement
  - Emergent gauge structure

Emergence in frustration

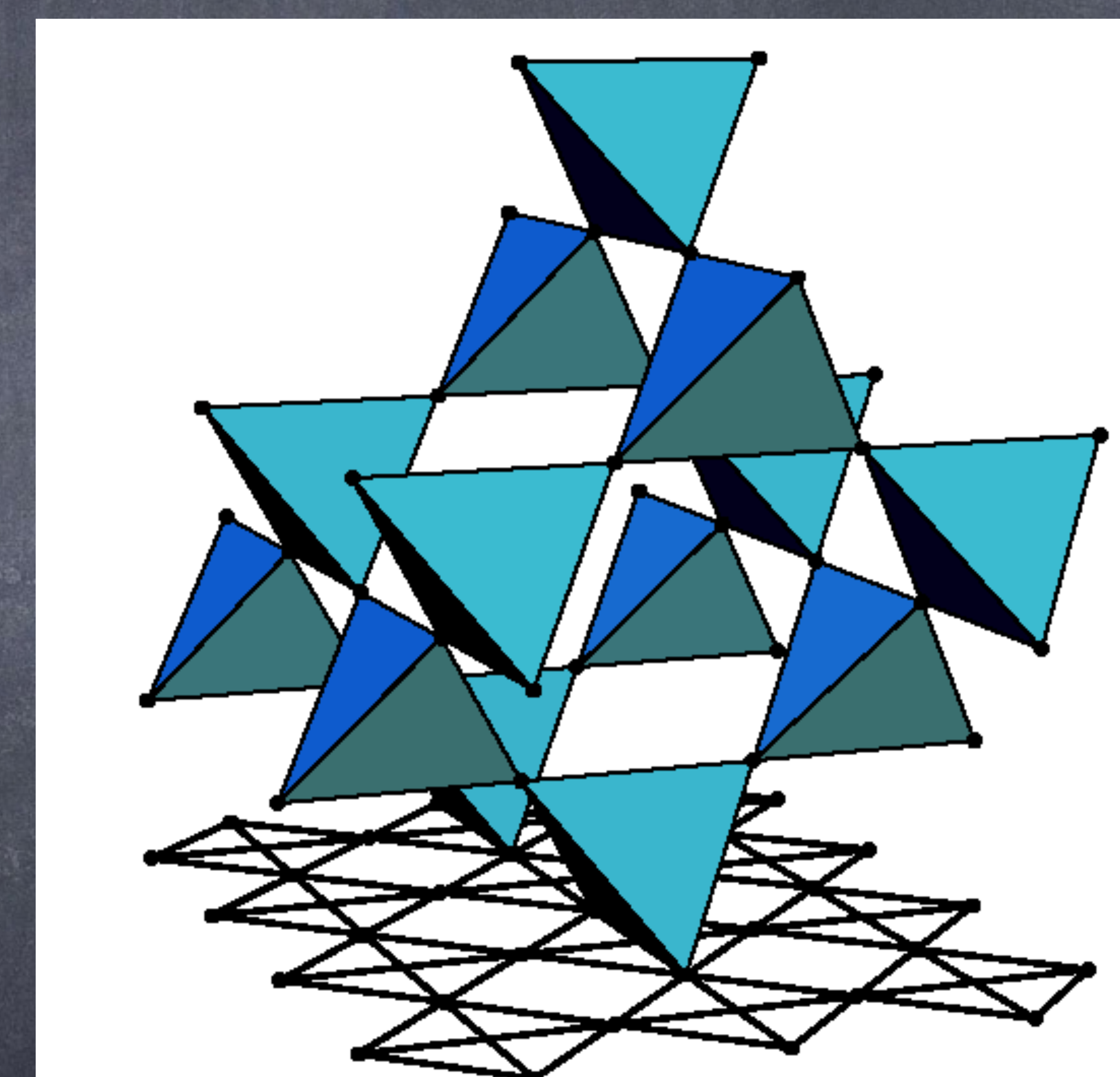
Idea: use « spin ice » physics to obtain a framework - building of a cooperative many body behavior whose low energy physics is described by degrees of freedom that are not primarily-coded in the original model/hamiltonian.

Emergence in frustration

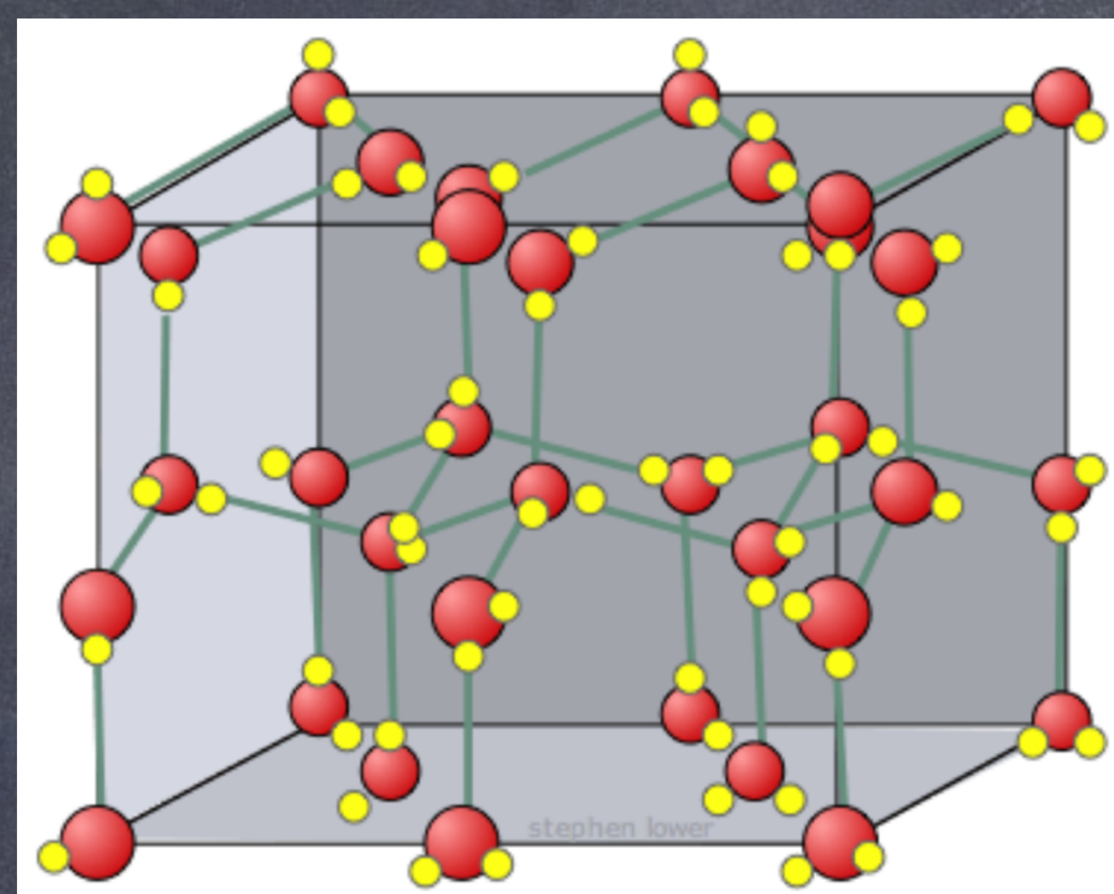


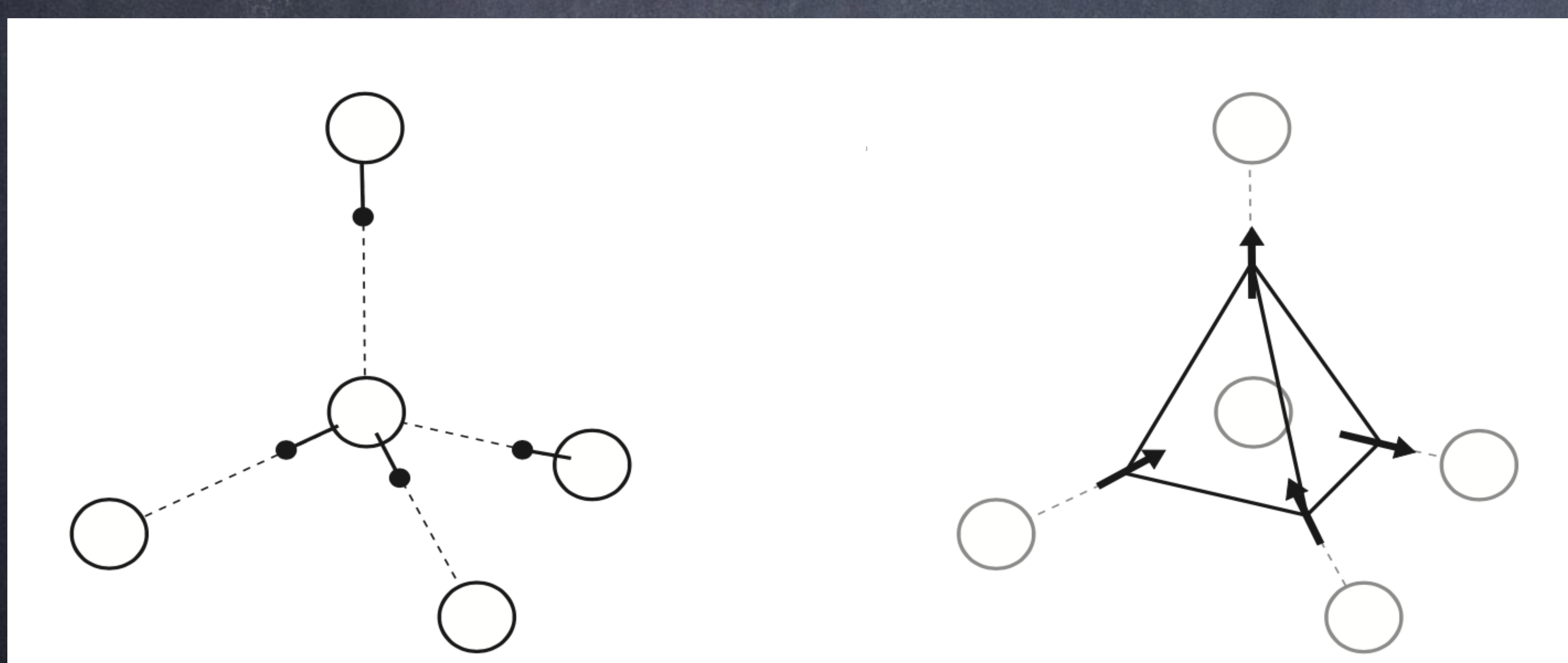
$Ho_2Ti_2O_7$ (Phys. Rev. Lett., Vol. 79, p. 2554 (1997).)

Zero point entropy in « spin ice », Nature 399, 333-335 (27 May 1999)



Emergence in frustration

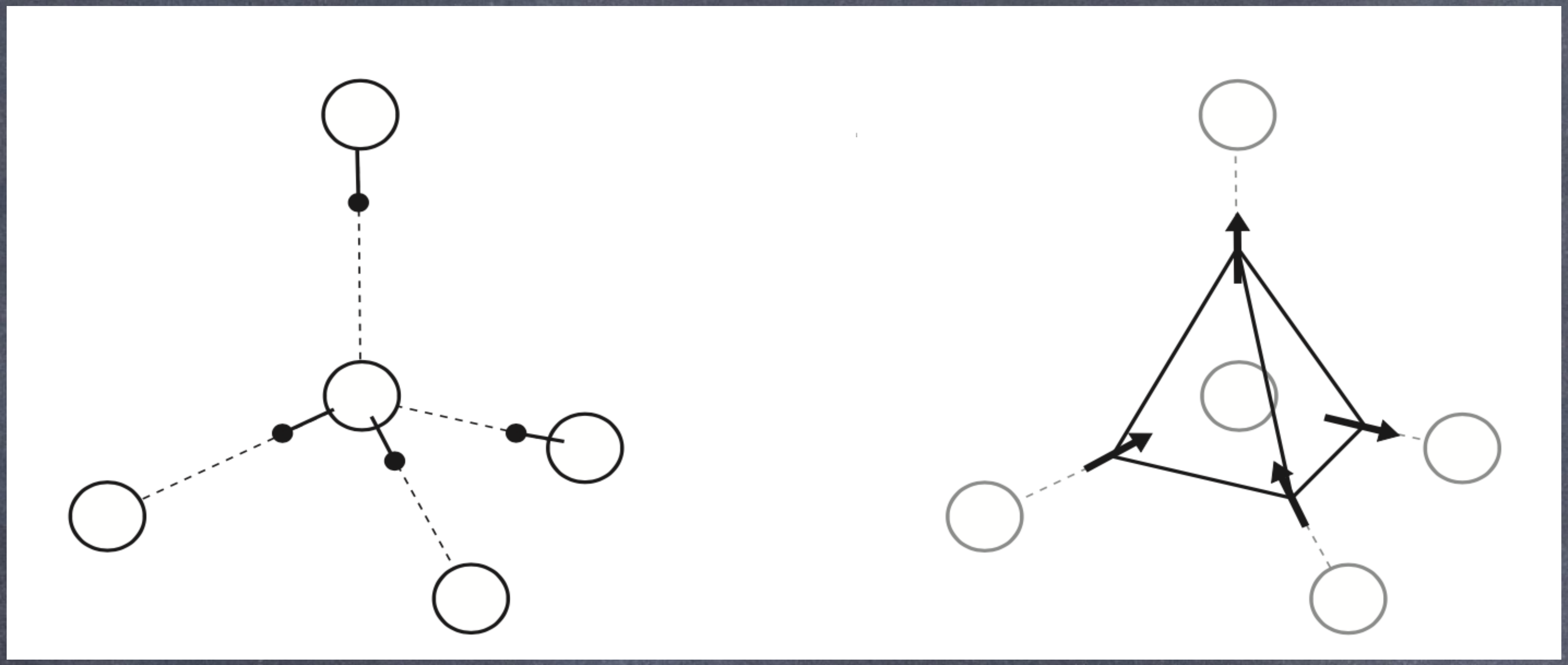




Positional directors map onto spins
Hence their name: spin ices.

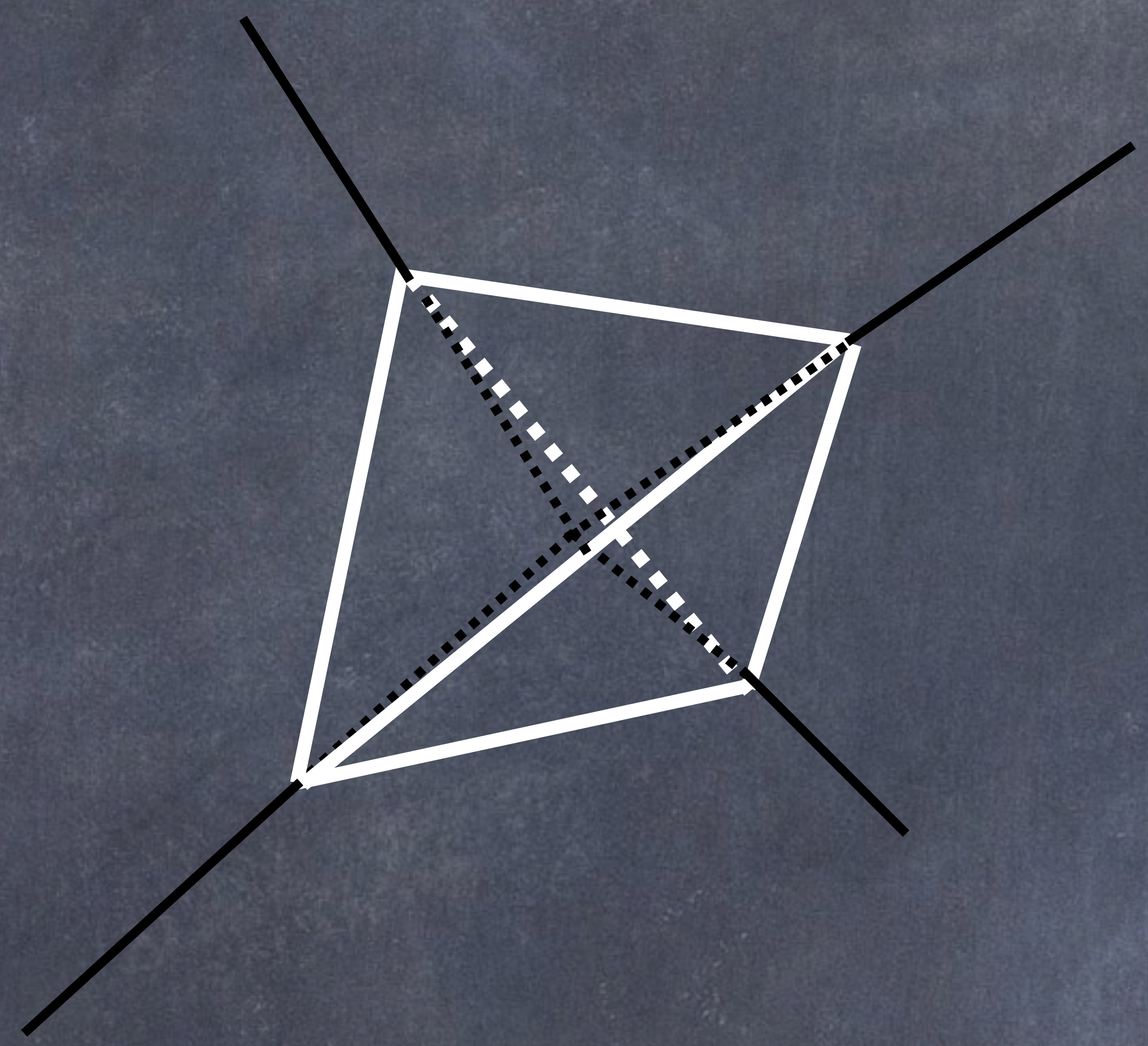
Hamiltonian factorization.
Short range F spin ice IS a short range AF spin liquid

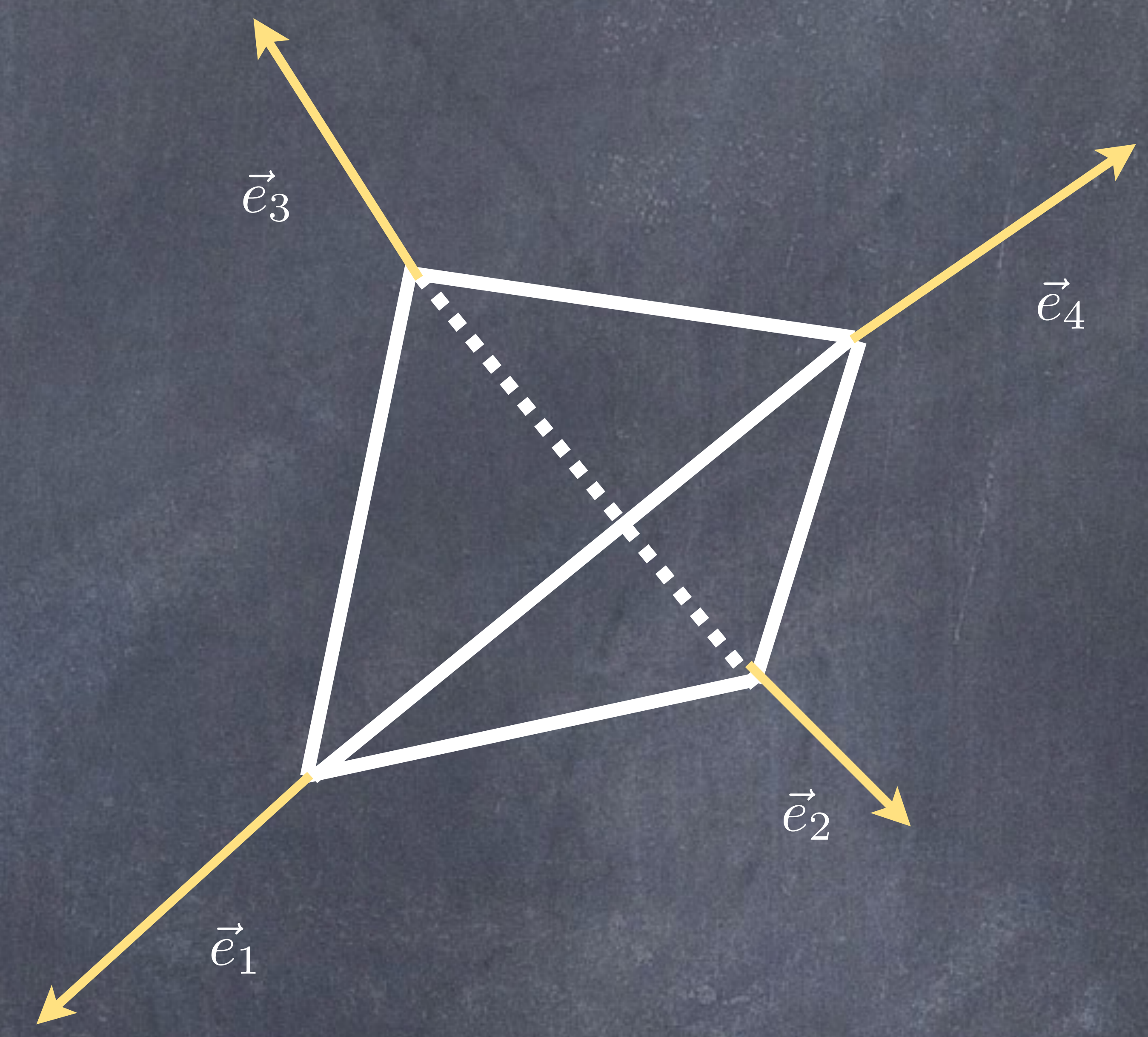
Emergence in frustration



On each tetrahedron, we, again, have a 6-vertex model! But links of each vertices are local Ising degrees of freedom, magnetic degrees of freedom.

Emergence in frustration





$$-J\vec{S}_1.\vec{S}_2 = -J\sigma_1\sigma_2\vec{e}_1.\vec{e}_2 = -J\sigma_1\sigma_2(-\frac{1}{3}) = -(-\frac{J}{3})\sigma_1\sigma_2$$

Ferromagnetic Antiferromagnetic

Multi-axial spin ice = uni axial spin liquid

Emergence in frustration

In order to describe the GS manifold: factorize the hamiltonian. [short range one… there
miracle here.]

$$
\begin{aligned}
\mathcal{H} &= -J \sum_{\langle i,j \rangle} \vec{S}_i.\vec{S}_j \\
&= -(-\frac{J}{3}) \sum_{\langle i,j \rangle} \sigma_i.\sigma_j \\
&= -(-\frac{J}{6}) \sum_{\text{tetrahedra}} (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4)^2 + C^{ste}
\end{aligned}
$$

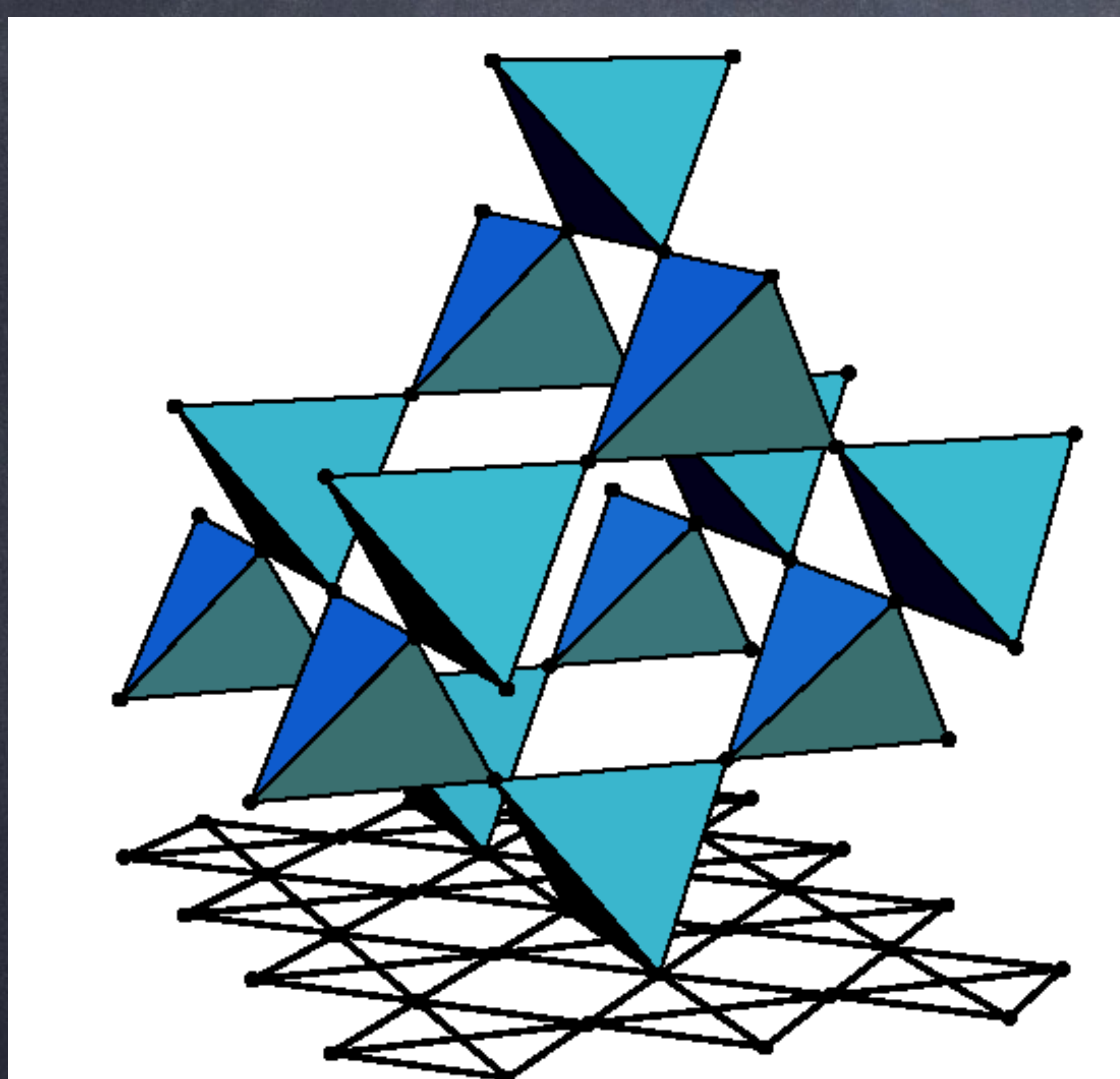
These local constraints can be fullfilled, and entropy can be estimated.
It is the Pauling estimate.

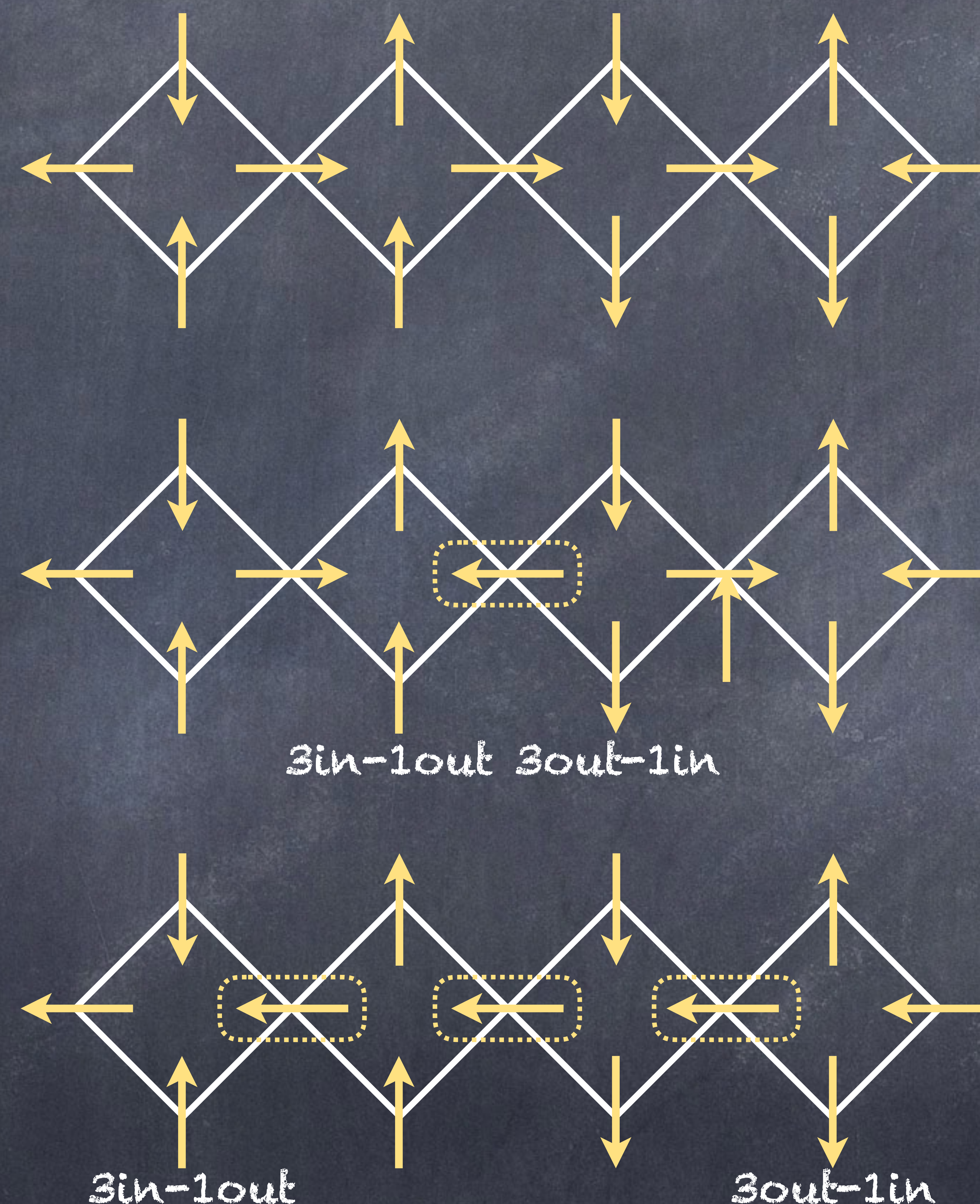
N tetrahedra; a priori 2^{2N} states, weighted by 6/16 for each tetrahedra, giving S/2N = 1/2 ln (3

Hence a highly degenerate GS manifold, though strongly correlated.

Emergence in frustration

Let's consider a chain of tetrahedra.

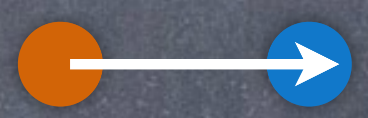


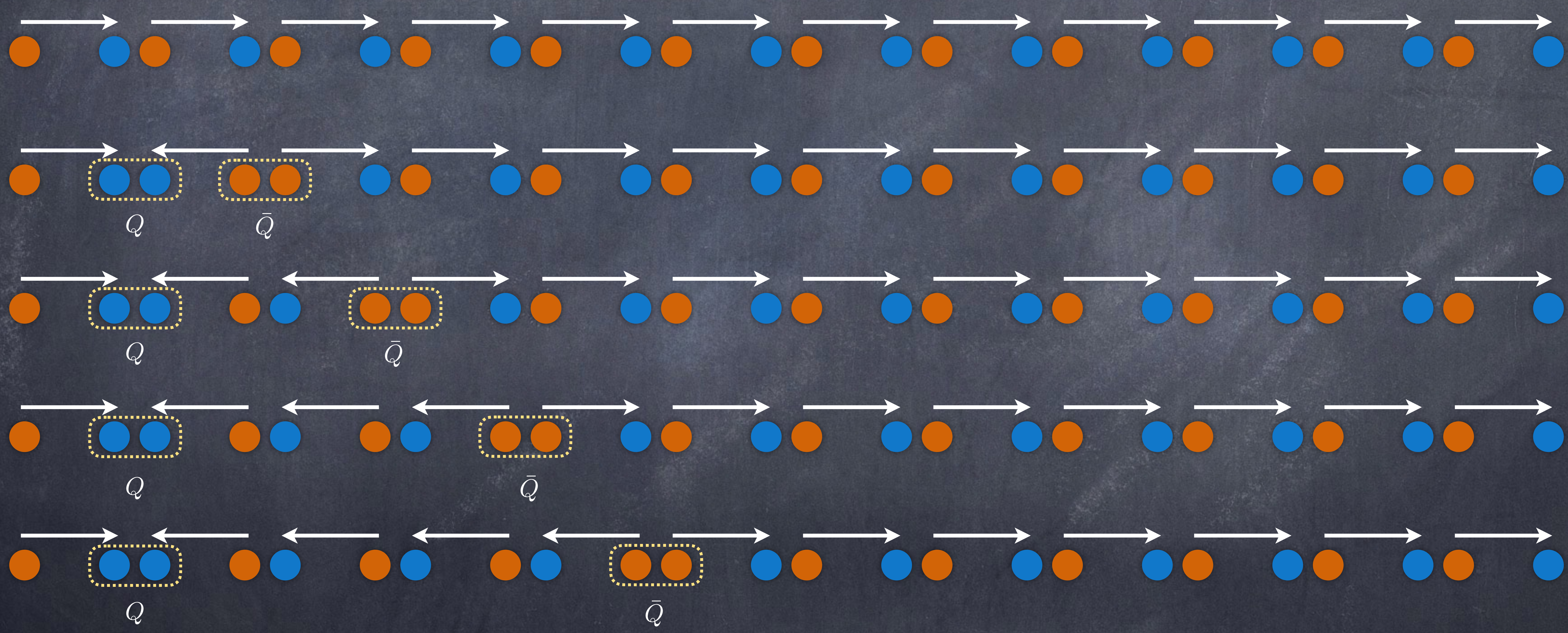


As in the kagomé case, the « defect » deconfines.

Emergence in frustration

Let's change vocabulary and notations.

Starting with the 1D case. → as a magnetic di-pole. 



Emergence in frustration

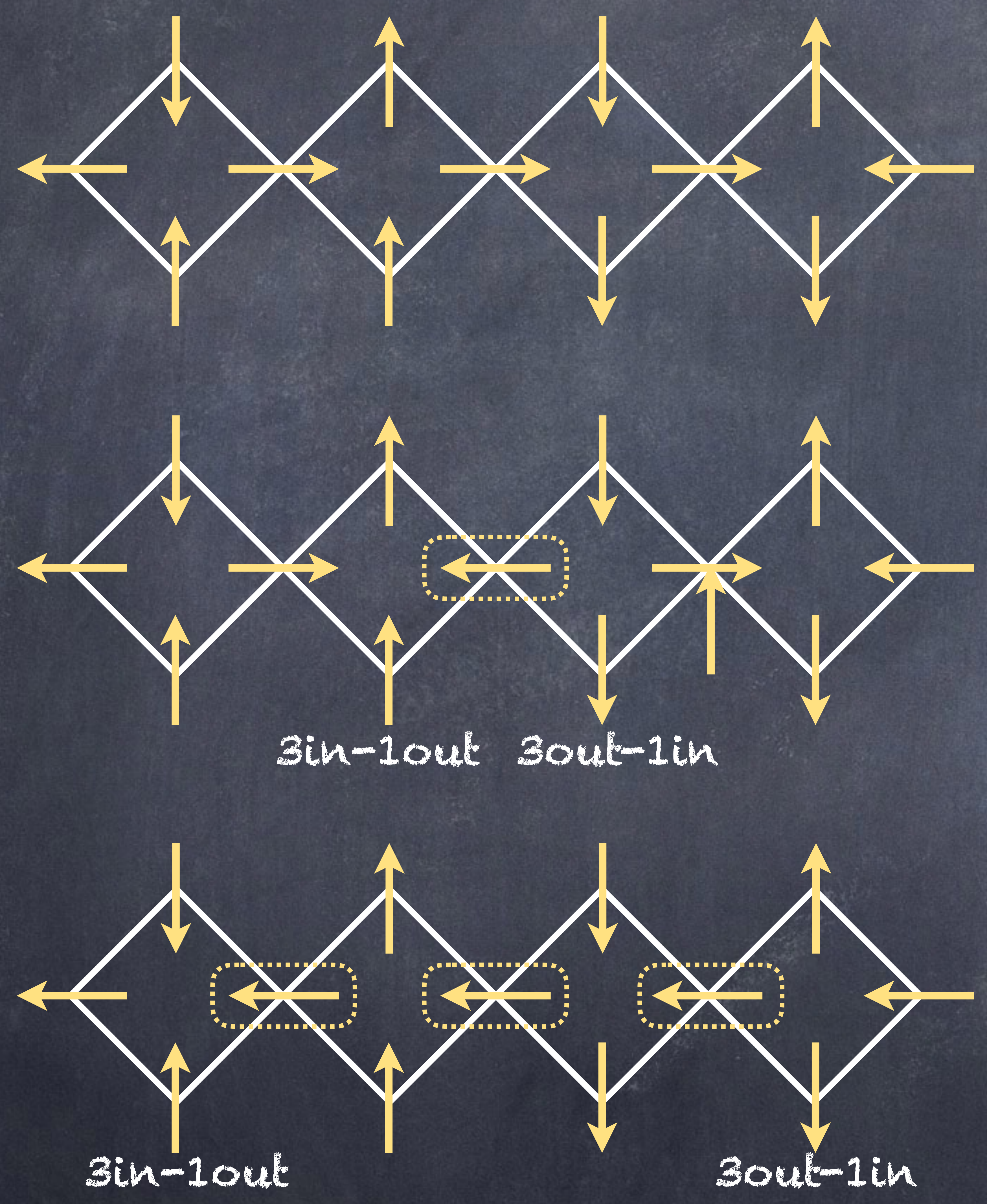
Spin GS = (magnetic) Charge vacuum.

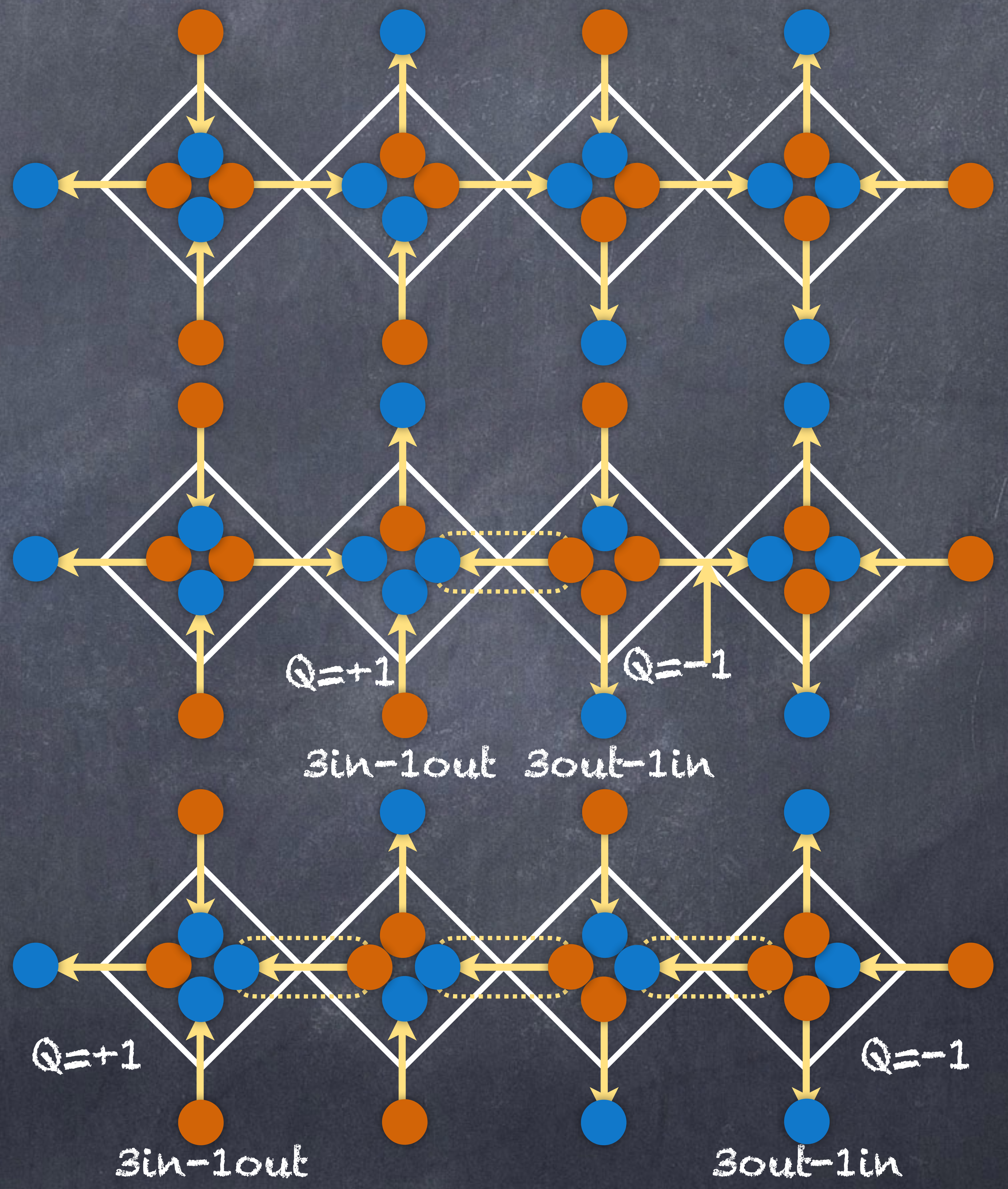
Spin flip excitation -> creation of one pair of opposite (magnetic) charges.

In 1D, they deconfine!

What about 3D?

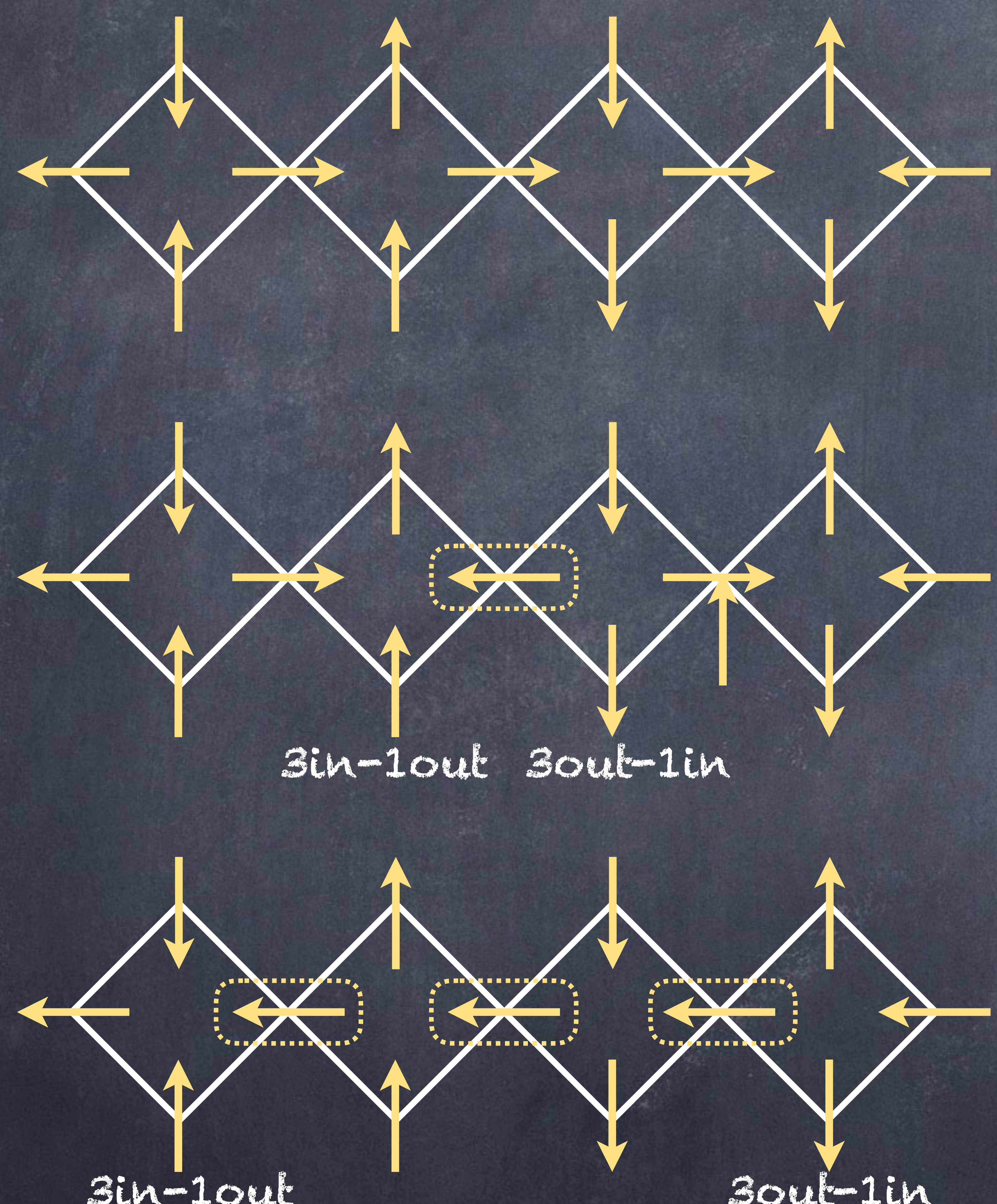
Emergence in frustration

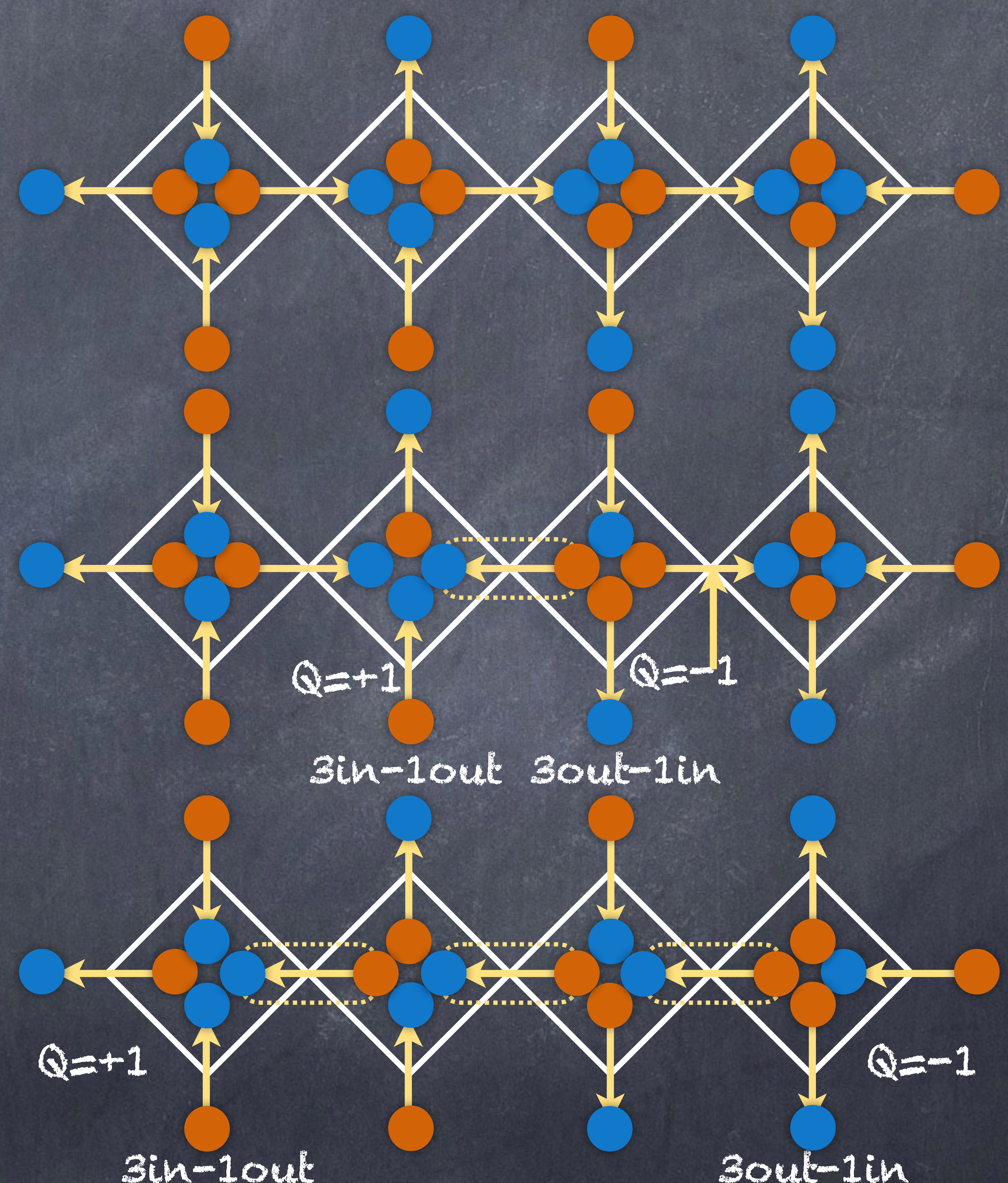




They deconfine!

Emergence in frustration





They deconfine! Really?...

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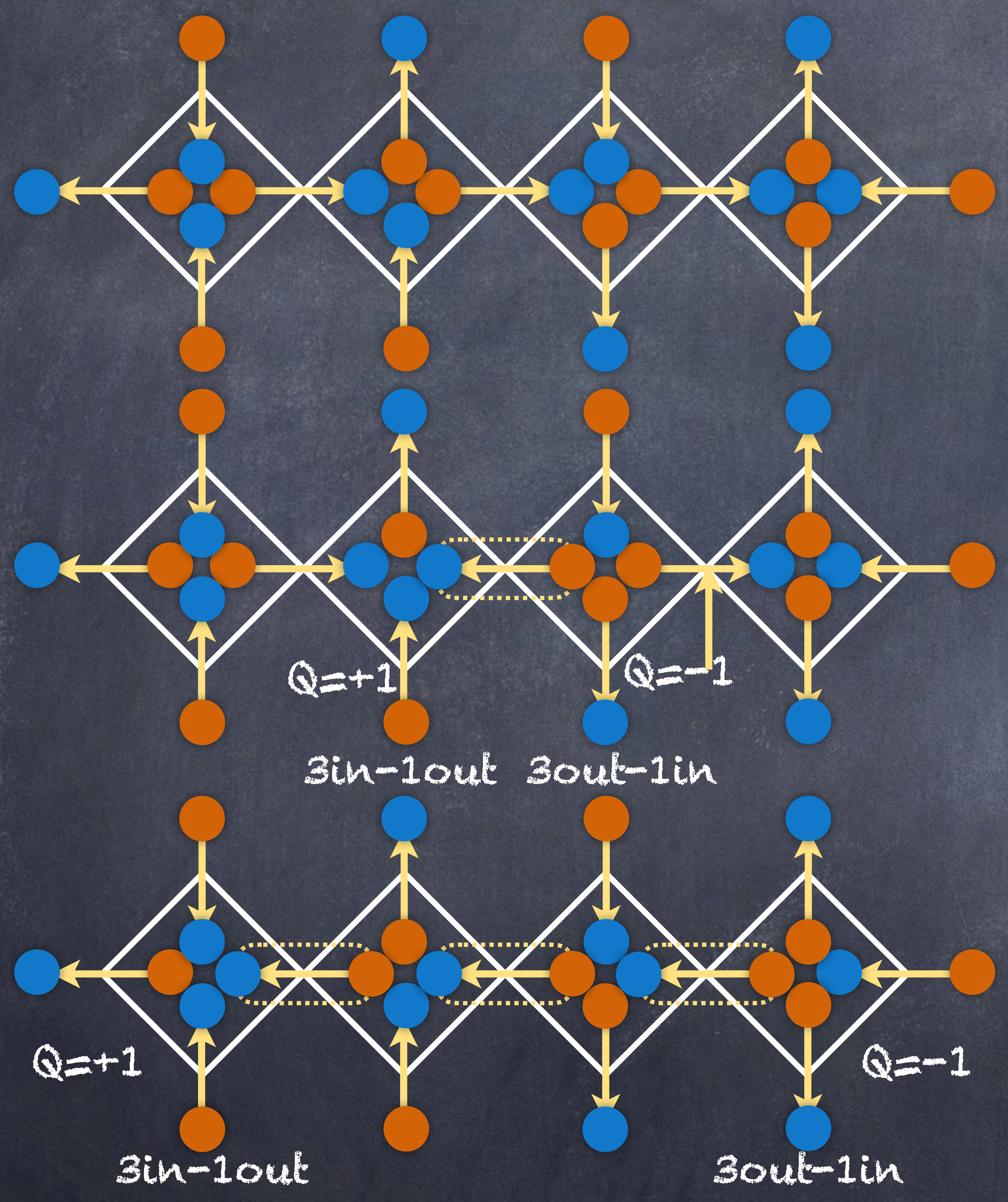
Emergence in frustration



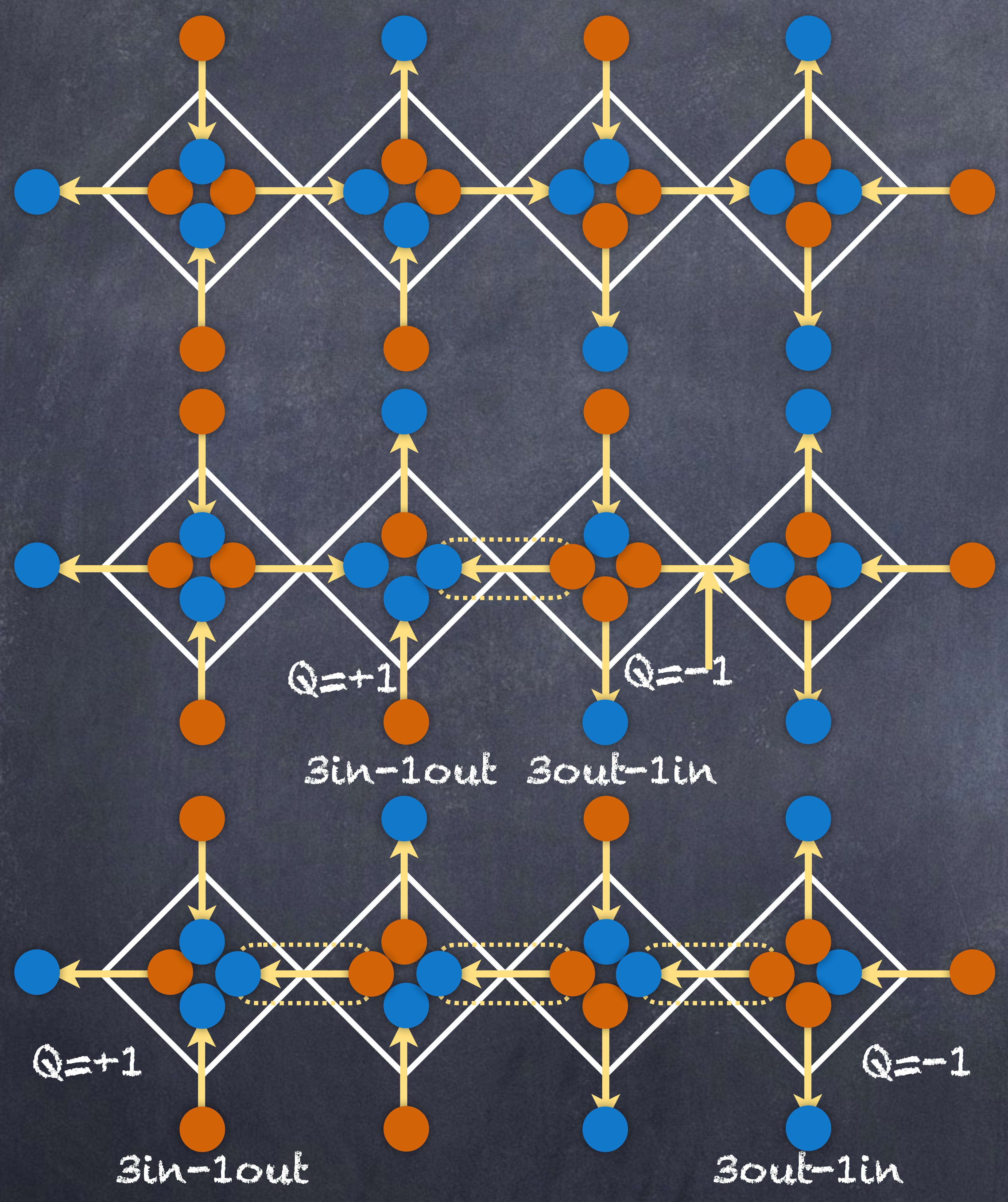
No, we must take care of two possible « corrections »:

- dipolar interactions
- Entropy

Effective rewriting:

$$E(r) = -\frac{Q^2}{r}$$

Emergence in frustration



We haver an effective way of describing the spin-model in terms of (magnetic)-charge model.

But there's more.

Think again at the constraint 2in-2out.

It looks like the lattice equivalent of a divergence free field.

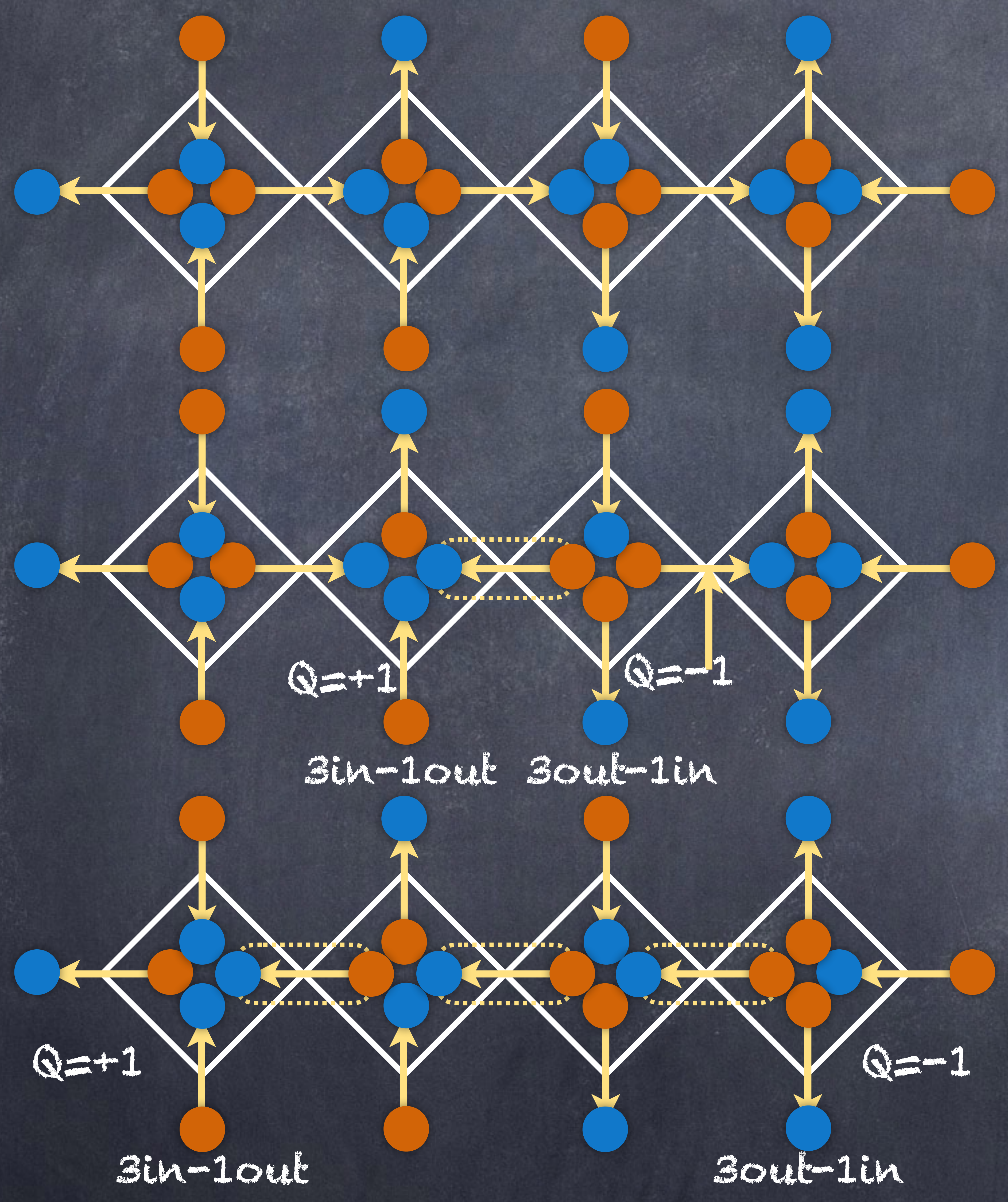
$$\vec{\nabla}.\vec{F} = 0$$

$$\vec{F} = \vec{\nabla} \times \vec{A}$$

We have an emergent gauge structure.

When we break the divergence free constraint, we have emerging charges, with Coulombic like interactions between these charges.

Emergence in frustration



In other words (see practical), we have a whole « electrostatic » like physics... with magnetic degrees of freedom.

We can go further, and build a whole artificial electrodynamics (beyond the scope of this lecture). Therefore, same algebra implies same properties, but hosted by primary, magnetic, degrees of freedom.

Conclusion

Neel like magnetism is subtle; we should be aware to that.

- Ordering is a time issue, classical or quantum
  - Statistical physics vs stochastic dynamics
  - Neel AF are fat symmetric tops
- Grounds states are few, time-disconnected

Once we know that, we understand better why frustrated magnetism is exotic. In some cases:

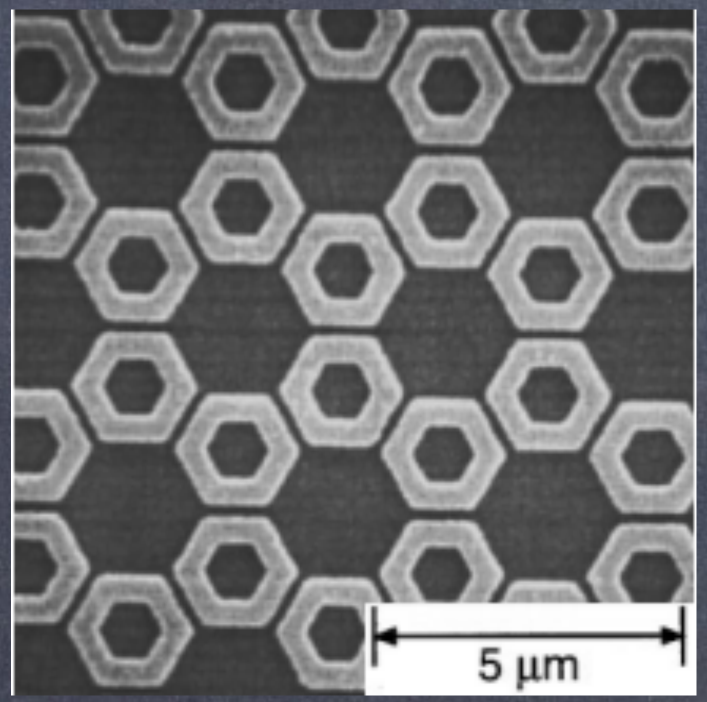
- Ground state manifold is dynamically much well connected, sometimes e-costless connected
- Ground state manifold is massively degenerate
  - 3rd law of thermodynamics must be defined with care; entropy is an important issue here!
- Grounds states support fractionalization
  - High dimensional frustated magnets allow for « spinons », and more.

Emergence:

- primary degrees of freedom define an emergent gauge structure
- This gauge structure supports secondary quasi-particles, magnetic-like

Never trust a theoretical statement, unless you fully appreciate the whole hypothesis set; remember, « there is no spoon »...

Thank you for your attention!



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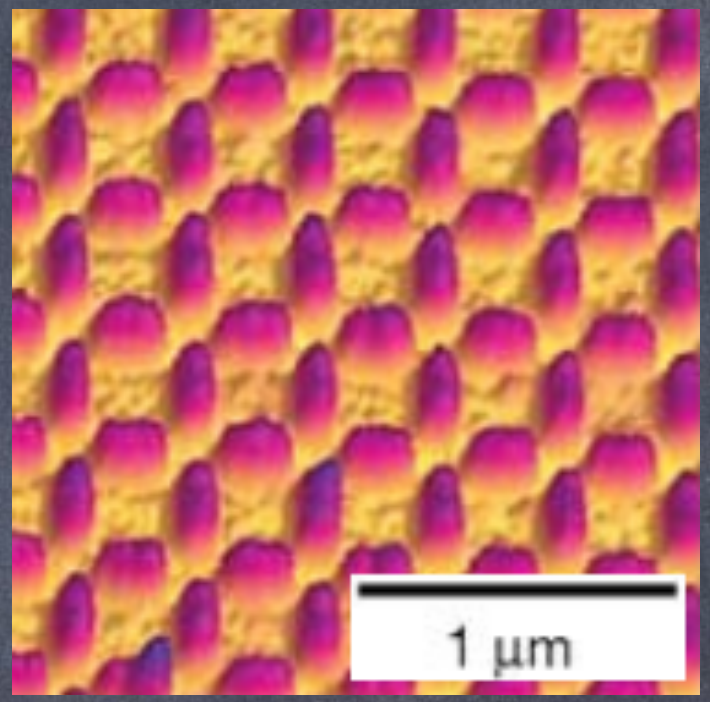

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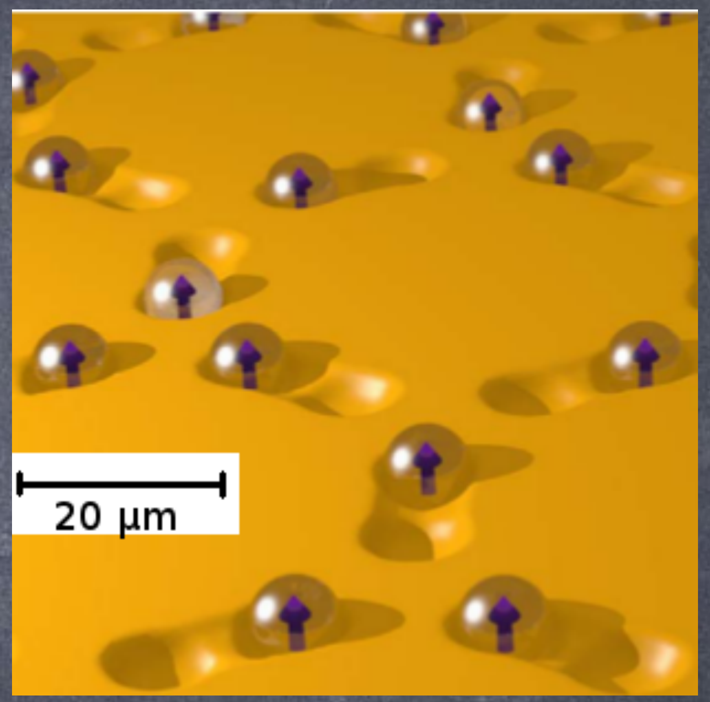

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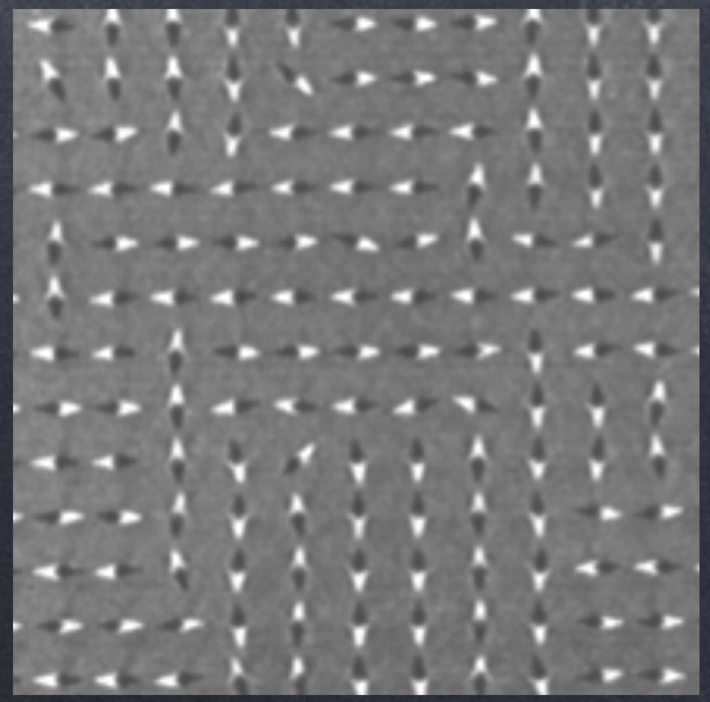

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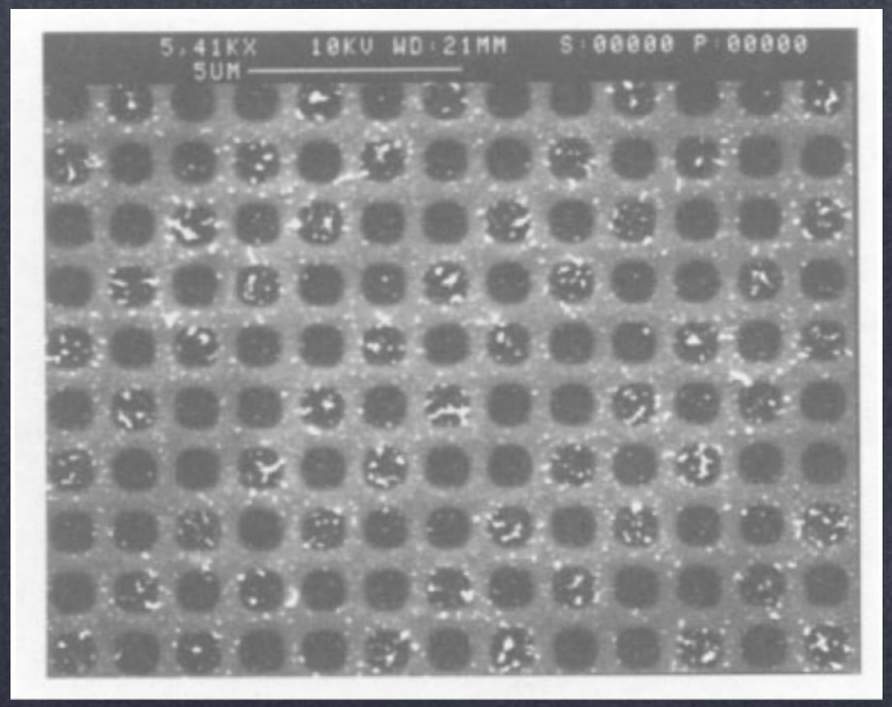

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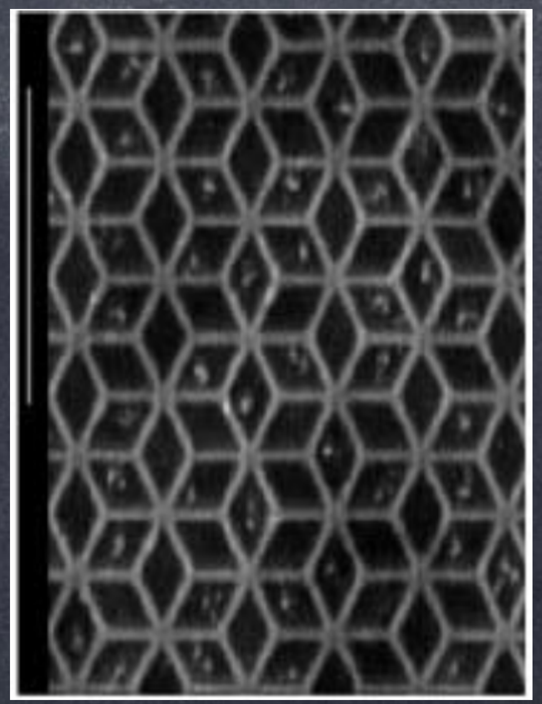

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