Damping of magnetization dynamics

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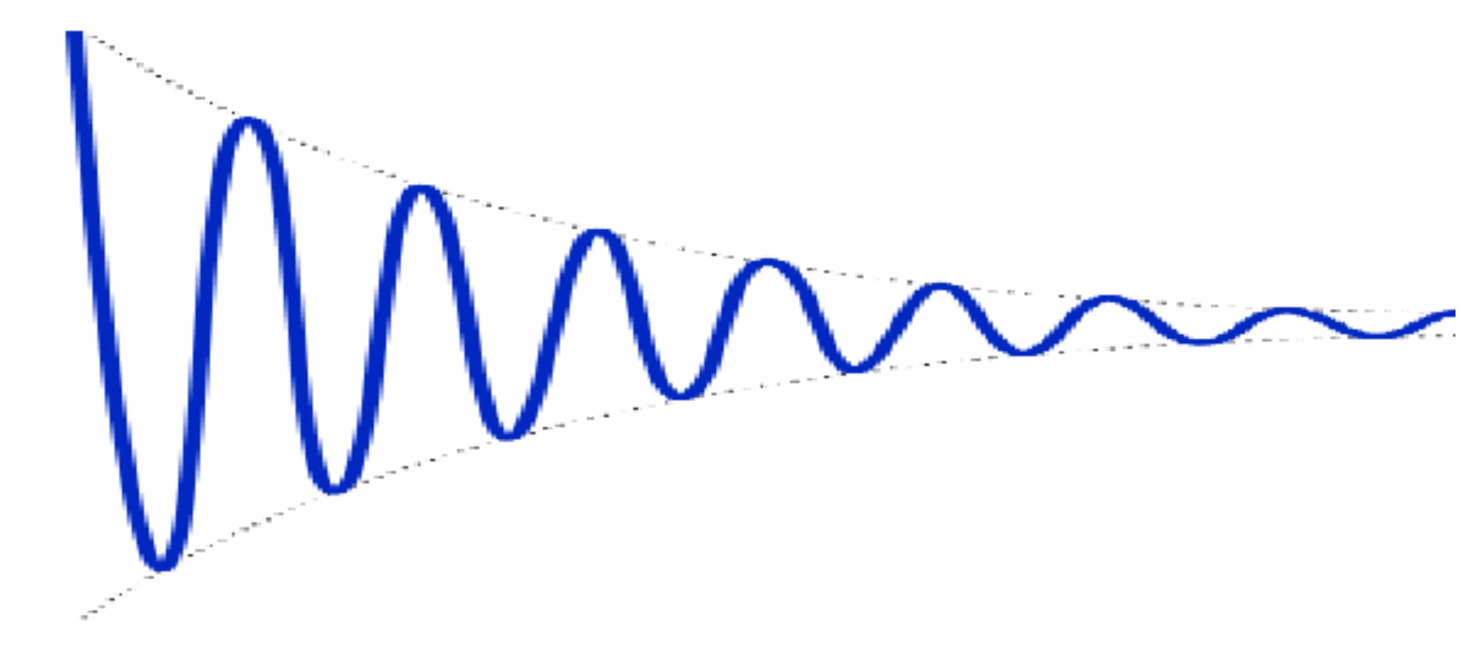

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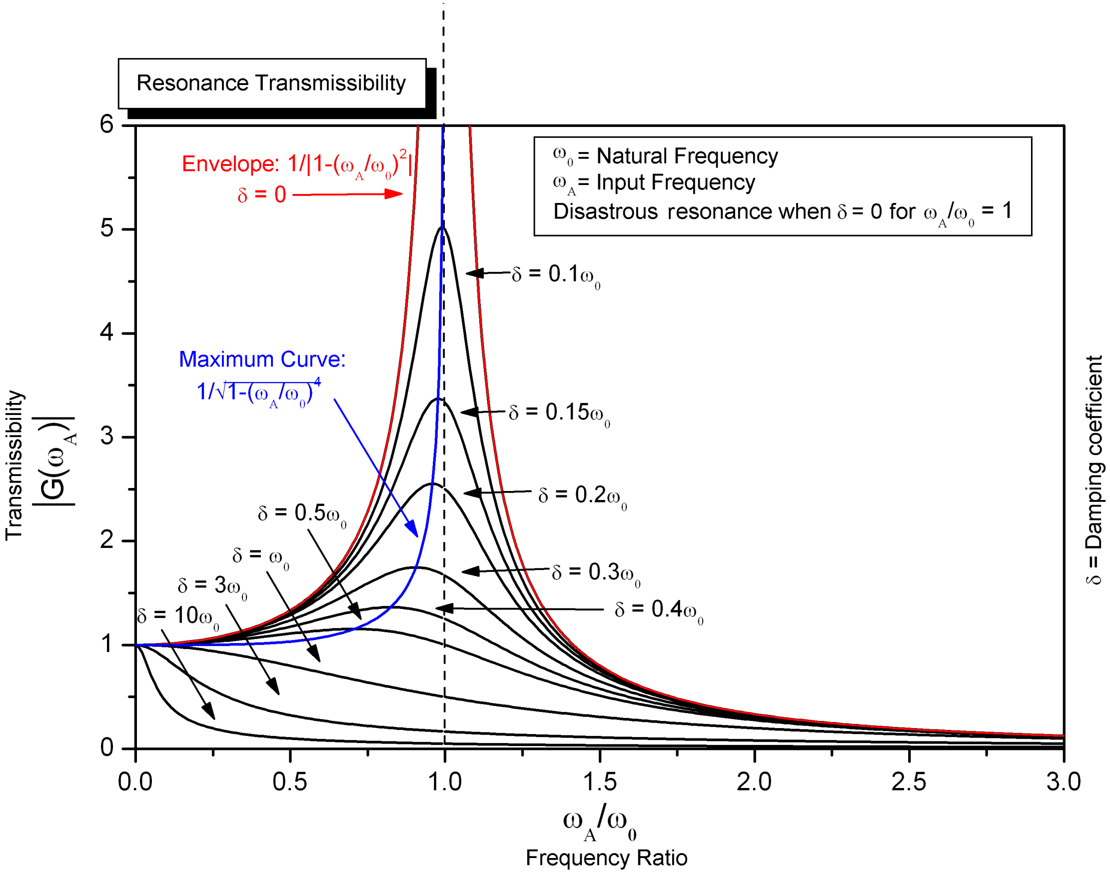
Damping

From Wikipedia, the free encyclopedia

This article is about damped harmonic oscillators. For detailed mathematical description of the harmonic oscillator including forcing and damping, see Harmonic oscillator. For damping in music, see Damping (music).

Damping is an influence within or upon an oscillatory system that has the effect of reducing, restricting or preventing its oscillations. In physical systems, damping is produced by processes that dissipate the energy stored in the oscillation. Examples include viscous drag in mechanical systems, resistance in electronic oscillators, and absorption and scattering of light in optical oscillators. Damping not based on energy loss can be important in other oscillating systems such as those that occur in biological systems.



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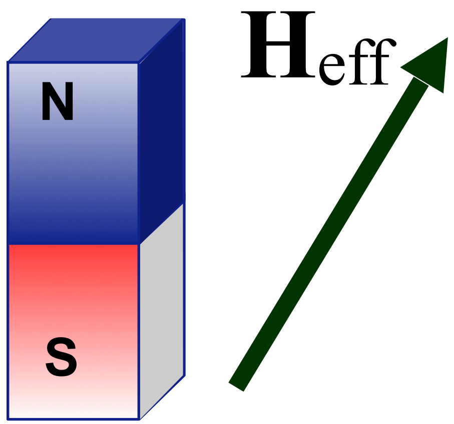
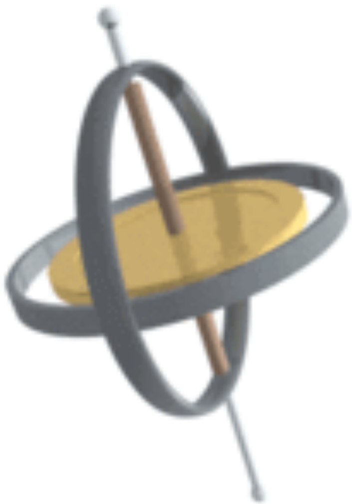
Landau-Lifshitz equation



energy gain: $E = -\mathbf{M} \cdot \mathbf{H}$

torque equation: $\dfrac{d\mathbf{L}}{dt} = \mathbf{T}$

$$\mathbf{M} = \gamma \mathbf{L} \qquad \mathbf{T} = [\mathbf{M} \times \mathbf{H}_{\text{eff}}]$$



$$\frac{d\mathbf{M}}{dt} = \gamma \, [\mathbf{M} \times \mathbf{H}_{\text{eff}}]$$

Landau & Lifshitz, 1935

$$|\gamma| = g \cdot \frac{e}{2m} = 28 \, \frac{\text{GHz}}{\text{T}}$$

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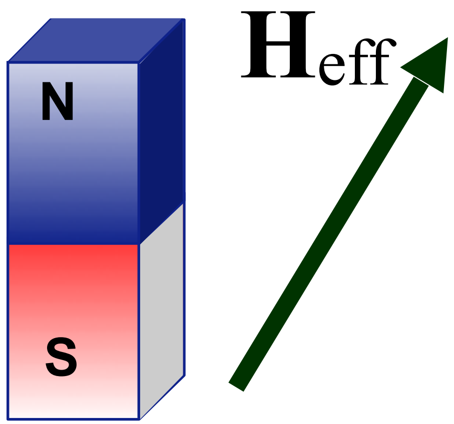

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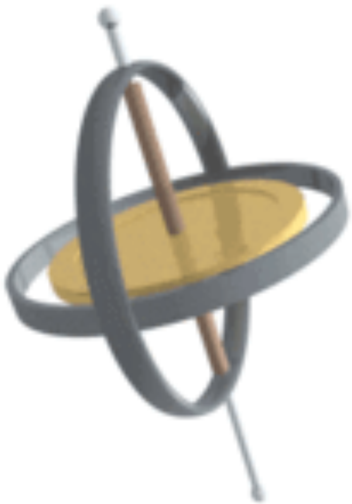

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Damping: Landau-Lifshitz vs Gilbert

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\lambda}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

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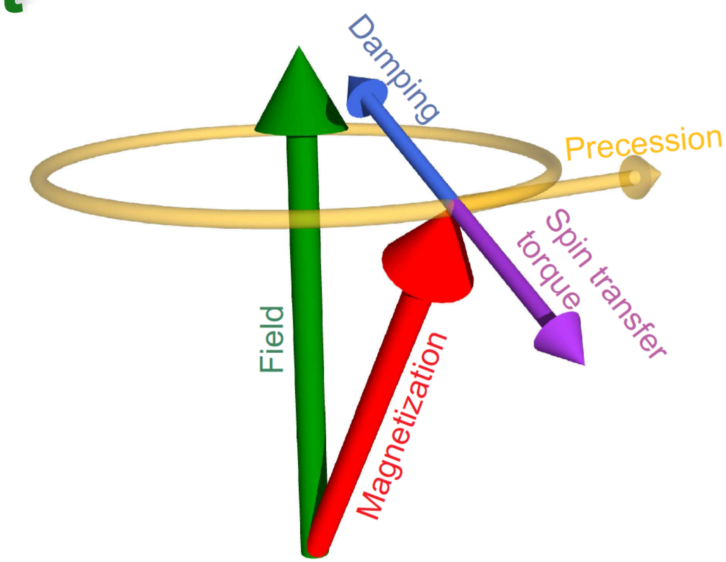

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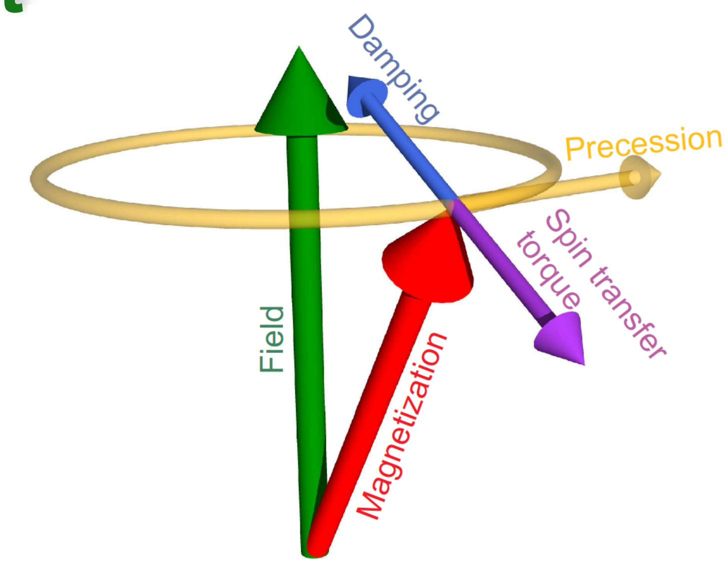
$$\frac{\partial \mathbf{M}}{\partial t} = -\frac{\gamma}{1+\alpha^2} \mathbf{M} \times \mathbf{H}_{\text{eff}} - \frac{\gamma\alpha}{(1+\alpha^2) M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})$$

$$\gamma_L = \frac{\gamma}{1+\alpha^2} \qquad \lambda = \frac{\gamma\alpha}{1+\alpha^2}$$



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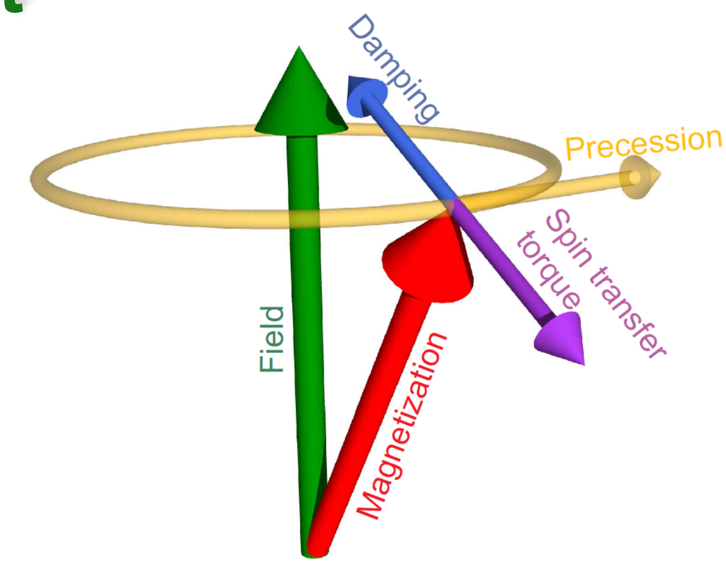

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Landau-Lifshitz vs Gilbert

$$\frac{\partial \mathbf{M}}{\partial t} \to \infty \qquad \frac{\partial \mathbf{M}}{\partial t} \to 0$$

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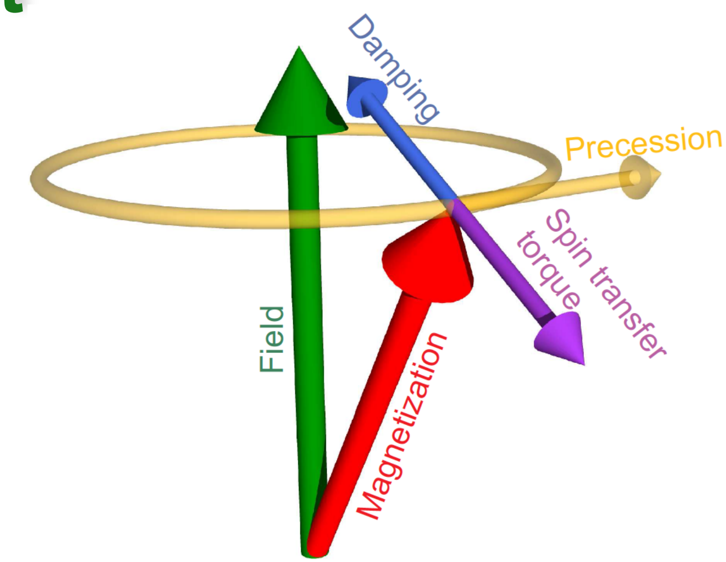

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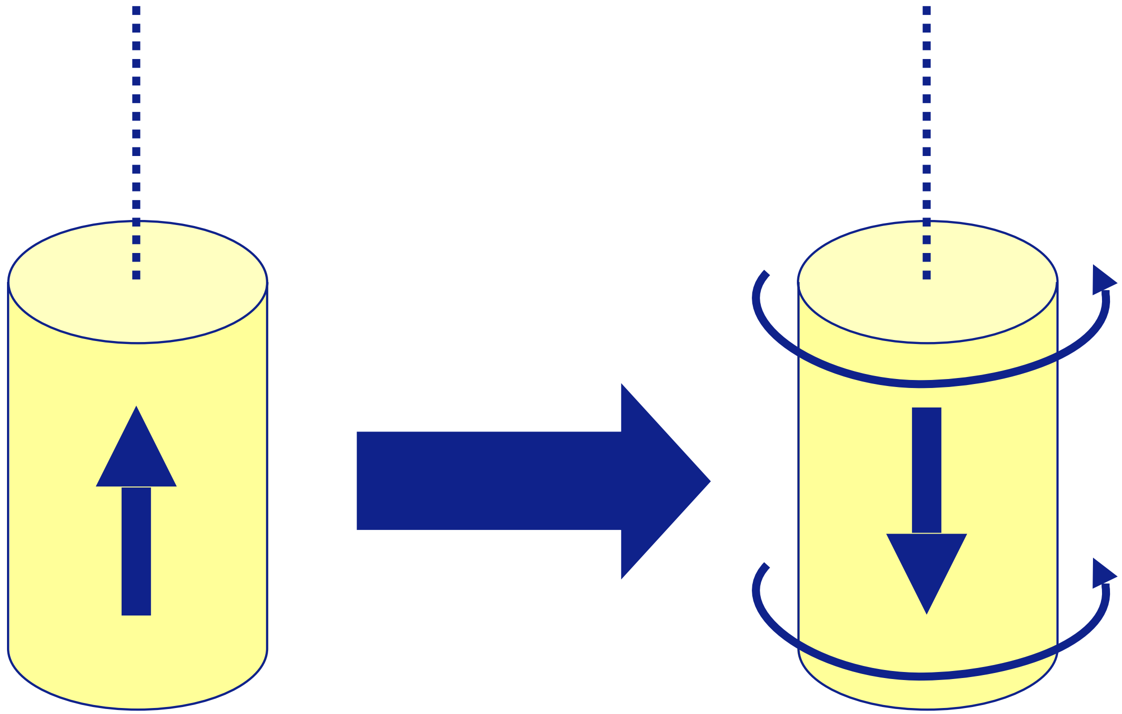
Landau-Lifshitz vs Gilbert

$$\frac{\partial \mathbf{M}}{\partial t} \to \infty \qquad \frac{\partial \mathbf{M}}{\partial t} \to 0$$

Since the second result is in agreement with the fact that a very large damping should produce a very slow motion while the first is not, one may conclude that the Landau-Lifshitz-Gilbert equation is more appropriate to describe magnetization dynamics.

To remember: magnetization = angular momentum



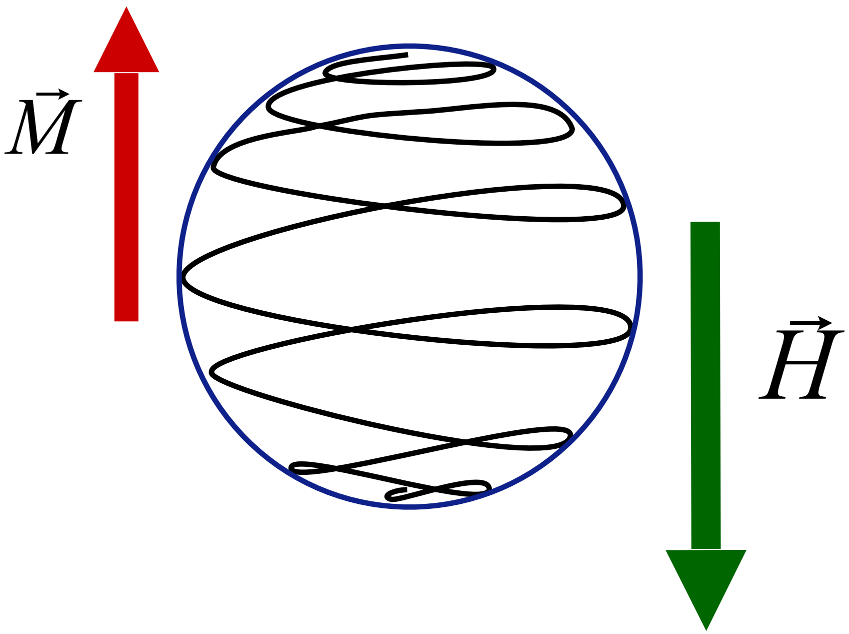
Einstein – de Haas & Barnett effects

A. Einstein & W.J. de Haas, *Experimenteller Nachweis der Amperèschen Molekülströme,* Verhandl. Deut. Phys. Ges. **17**, 152 (1915)

S.J. Barnett, *Magnetization by rotation*, Phys. Rev. **6**, 239 (1915)

Angular momentum transfer and two ways of reversal

usual (practical)



$$\frac{dM}{dt} = -|\gamma|\left(M \times H^{eff}\right) + \frac{\alpha}{M}\left(M \times \frac{dM}{dt}\right)$$

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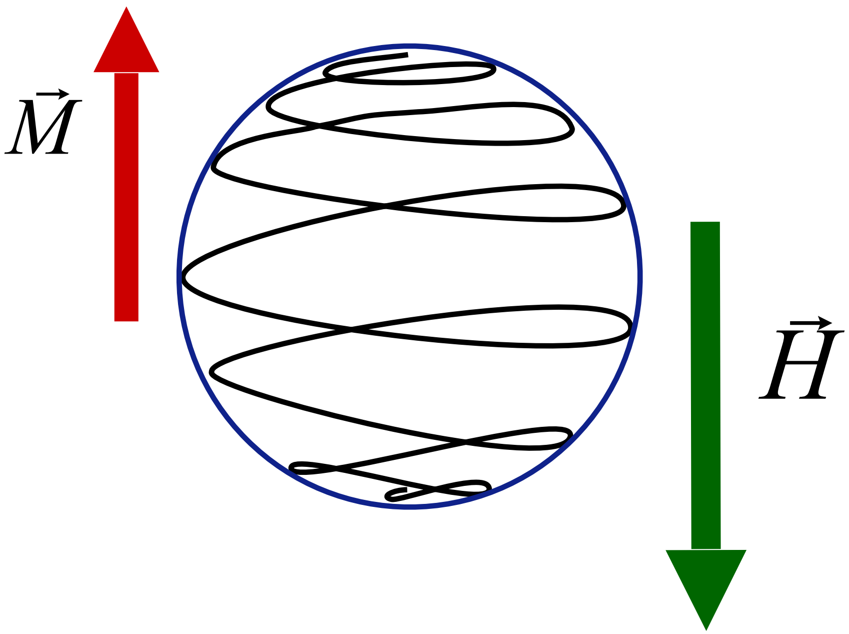

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from spins to lattice

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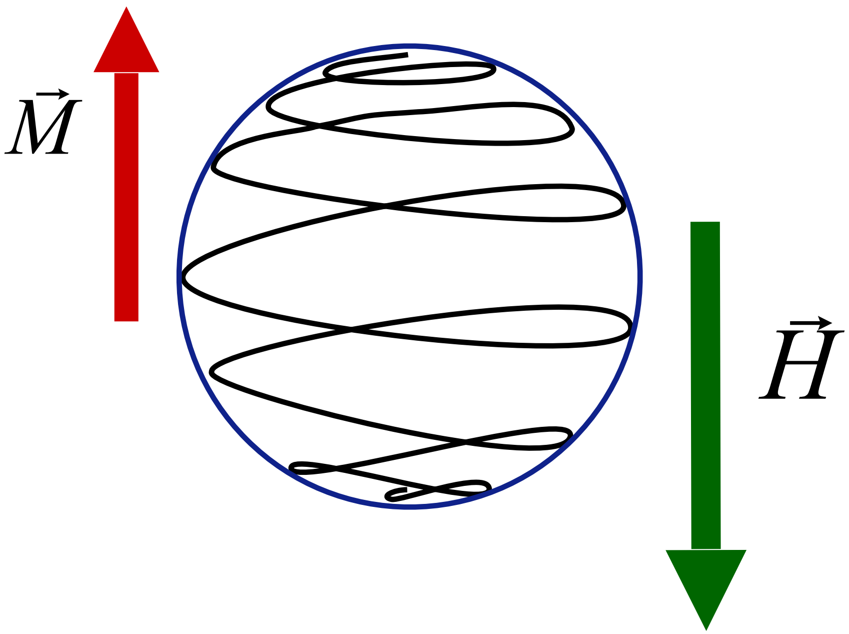

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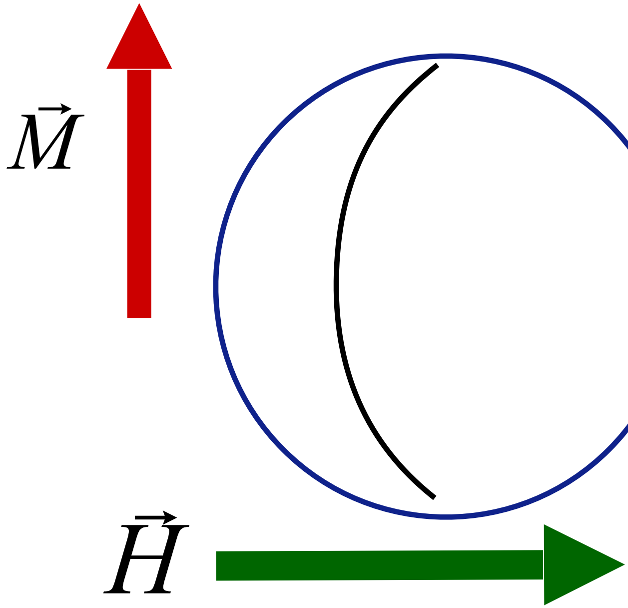
from spins to lattice

precessional (fast)



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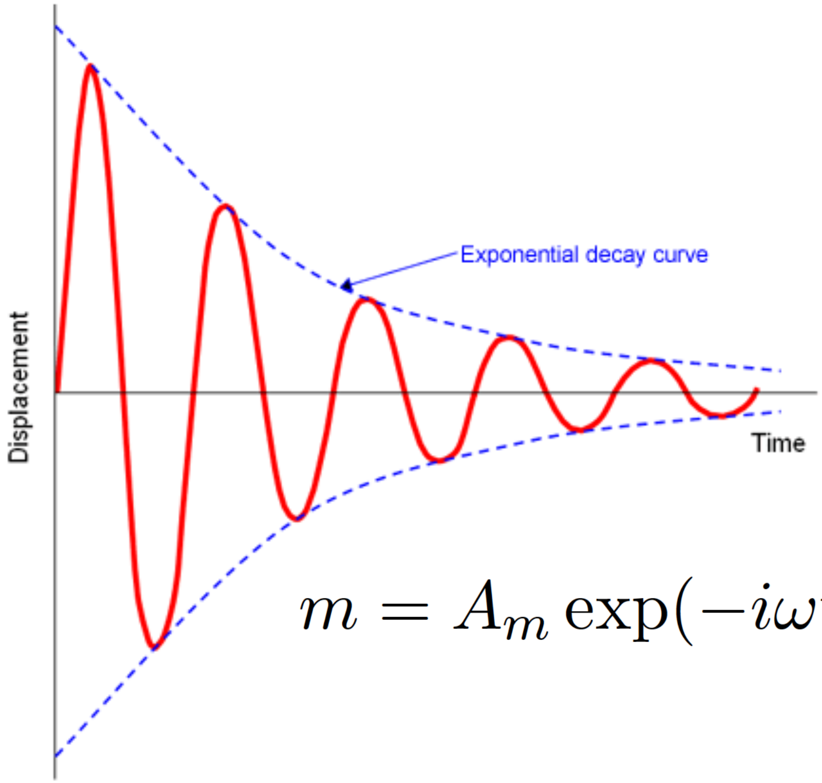
from spins to field

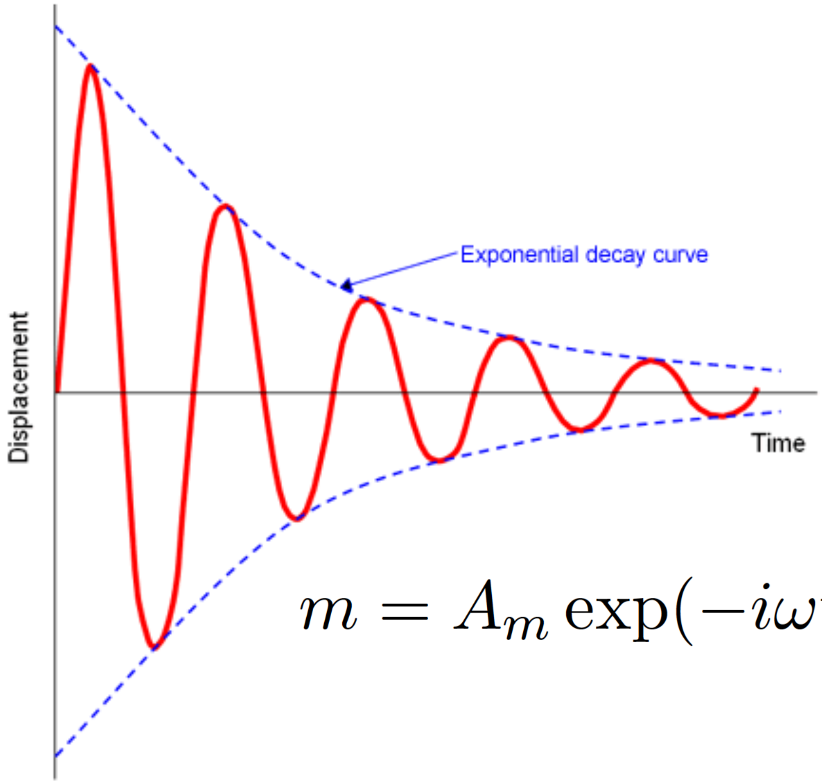
measuring the damping



$$m = A_m \exp(-i\omega t) e^{-t/\tau_\alpha}$$

$$\alpha = \frac{1}{\tau_\alpha \omega} = \frac{1}{2\pi \tau_\alpha f}$$

measuring the damping



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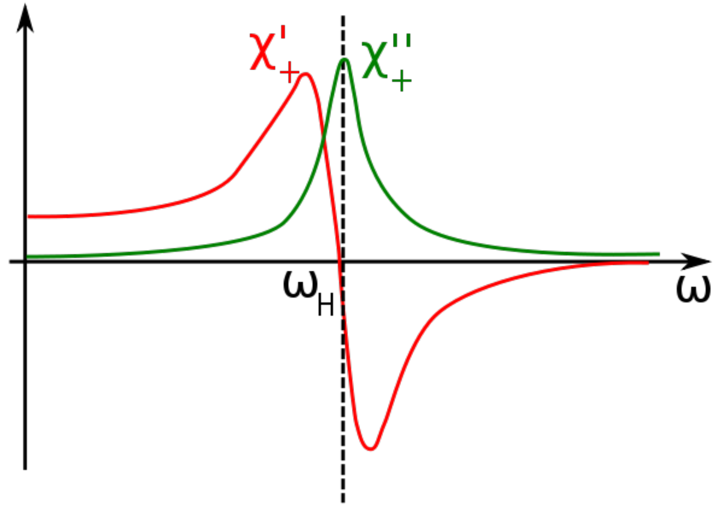
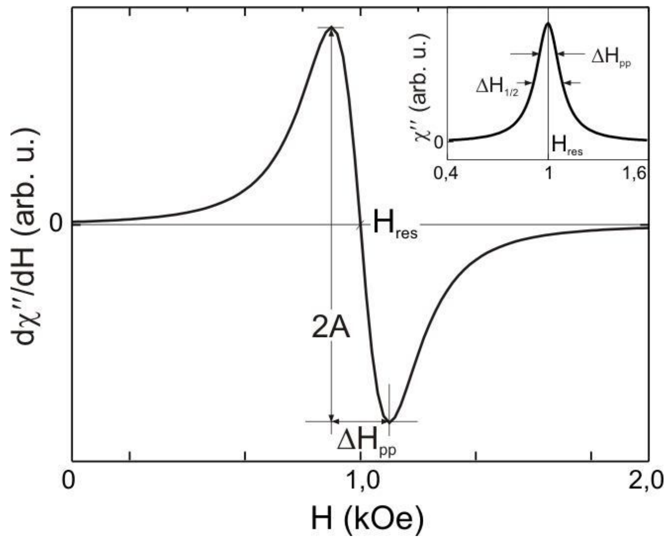
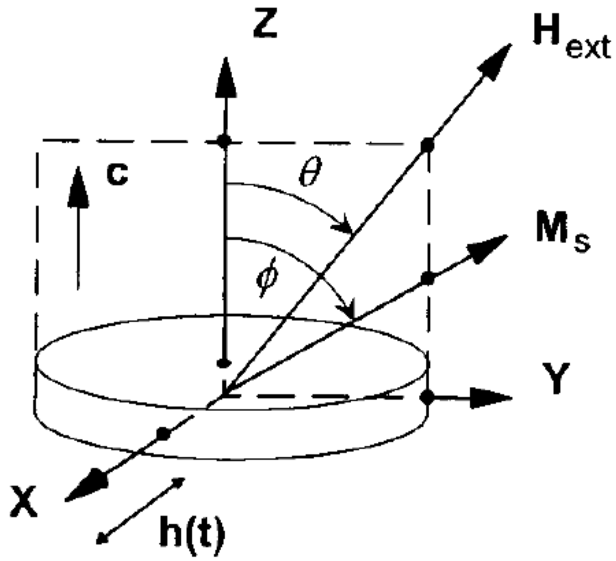




$$\Delta H = \frac{2\alpha\omega_0}{\gamma}$$

Example 1: thin film configuration



from the condition that the net torque on M is zero:

$$4H_{\text{ext}}\,\sin(\theta-\phi)=[4\pi M_s(1-3N_Z)+2H_A]\sin(2\phi)$$

FMR resonance

$$\mathbf{M}(t) \approx \mathbf{M}_s + \mathbf{m}(t)$$

$$\frac{d\mathbf{m}(t)}{dt} = -\gamma \mathbf{M}(t) \times \mathbf{H}(t) - \frac{\mathbf{m}(t)}{2T}$$

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$$\omega_{\mathrm{FMR}} = \gamma (H_x H_y)^{1/2},$$

$$H_x = H_{\mathrm{ext}} \cos(\theta - \phi) + \tfrac{1}{2}[2H_A - 4\pi M_s(3N_Z - 1)] \times \cos^2 \phi,$$

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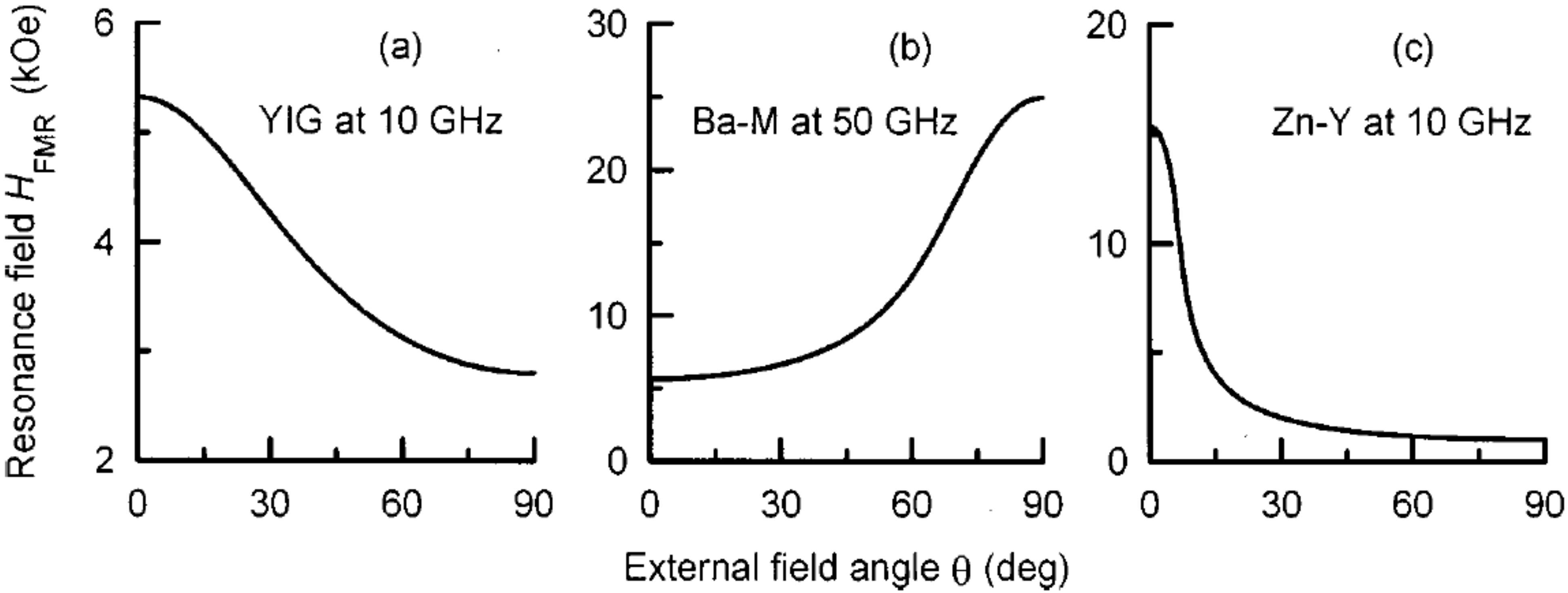
FMR versus applied field angle



isotropic | out of plane easy axis | easy plane

FMR linewidth

$$\Delta(\omega/\gamma) = \frac{1}{\gamma T}$$

$$\Delta H \approx \gamma \frac{\partial H_{\mathrm{FMR}}}{\partial \omega_{\mathrm{FMR}}} \Delta(\omega/\gamma)$$

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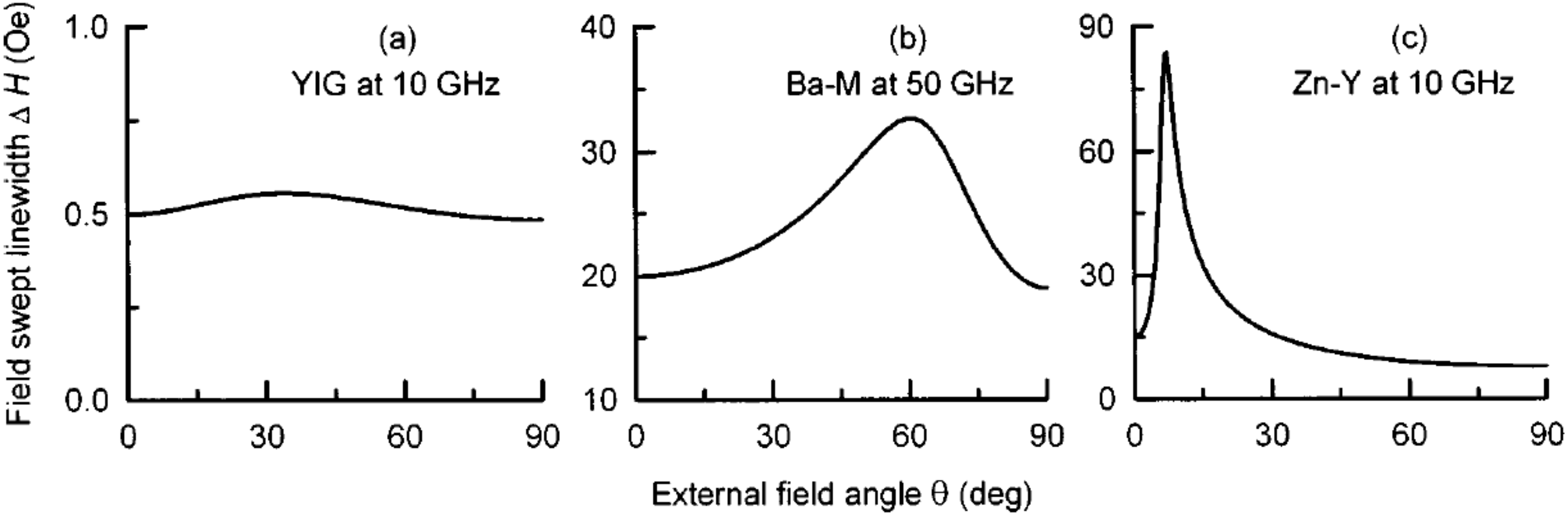
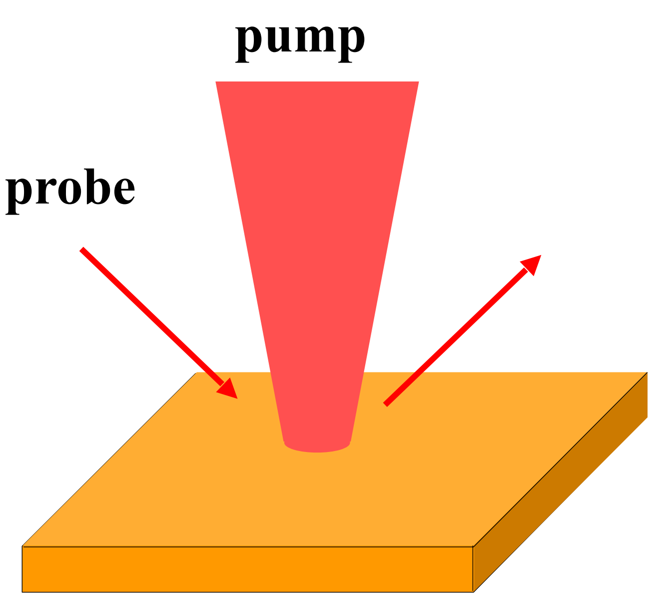


Example 2: optical pump-probe measurement

Damping in a Bi:YIG garnet film as a function of temperature



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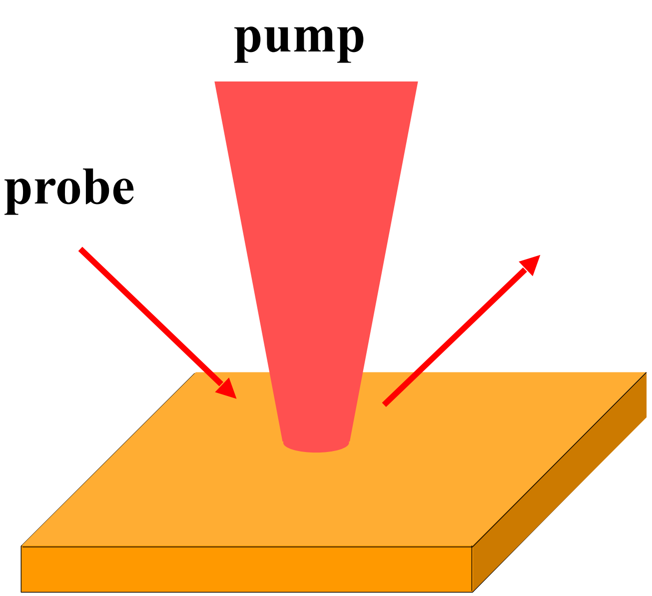
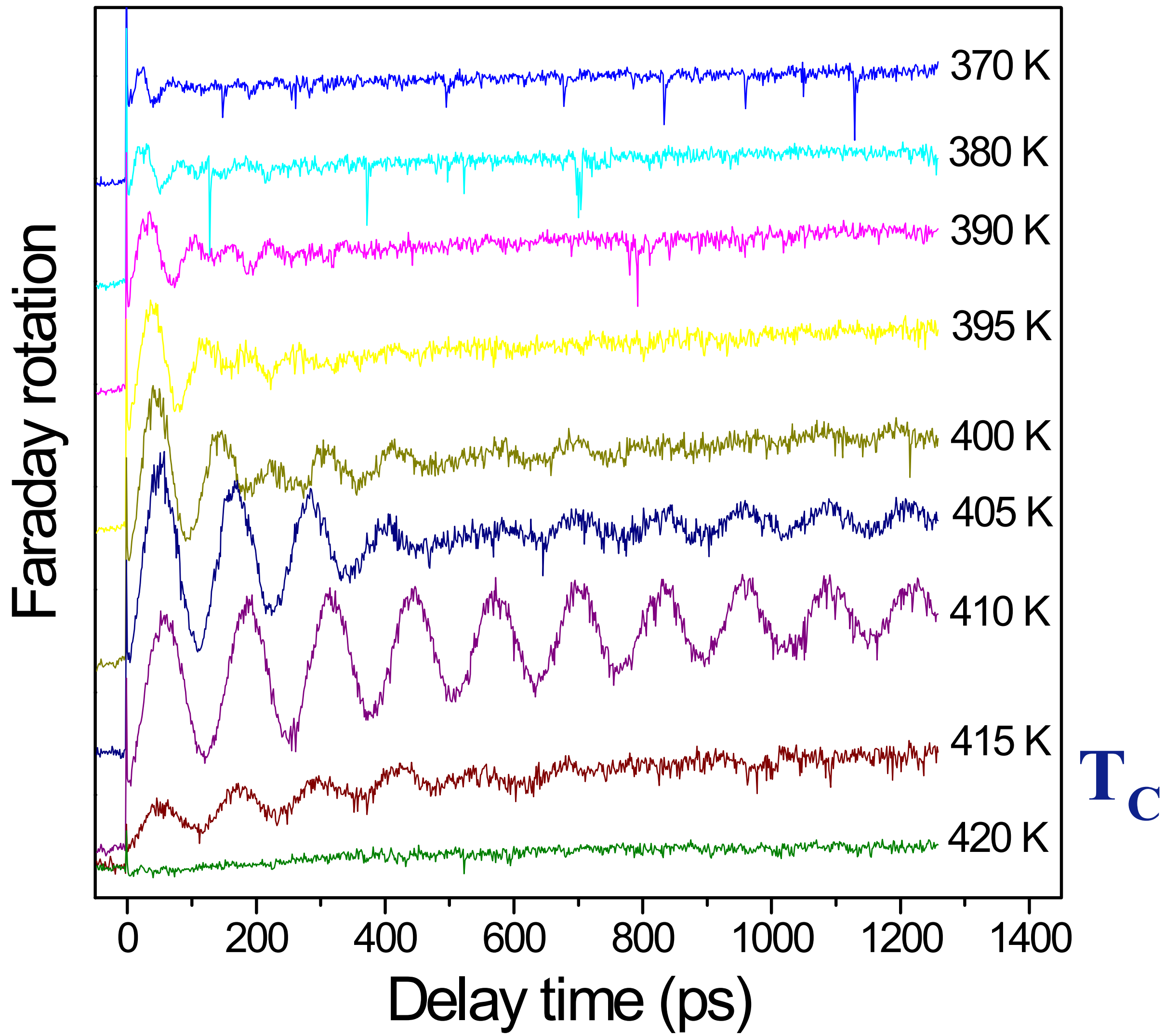
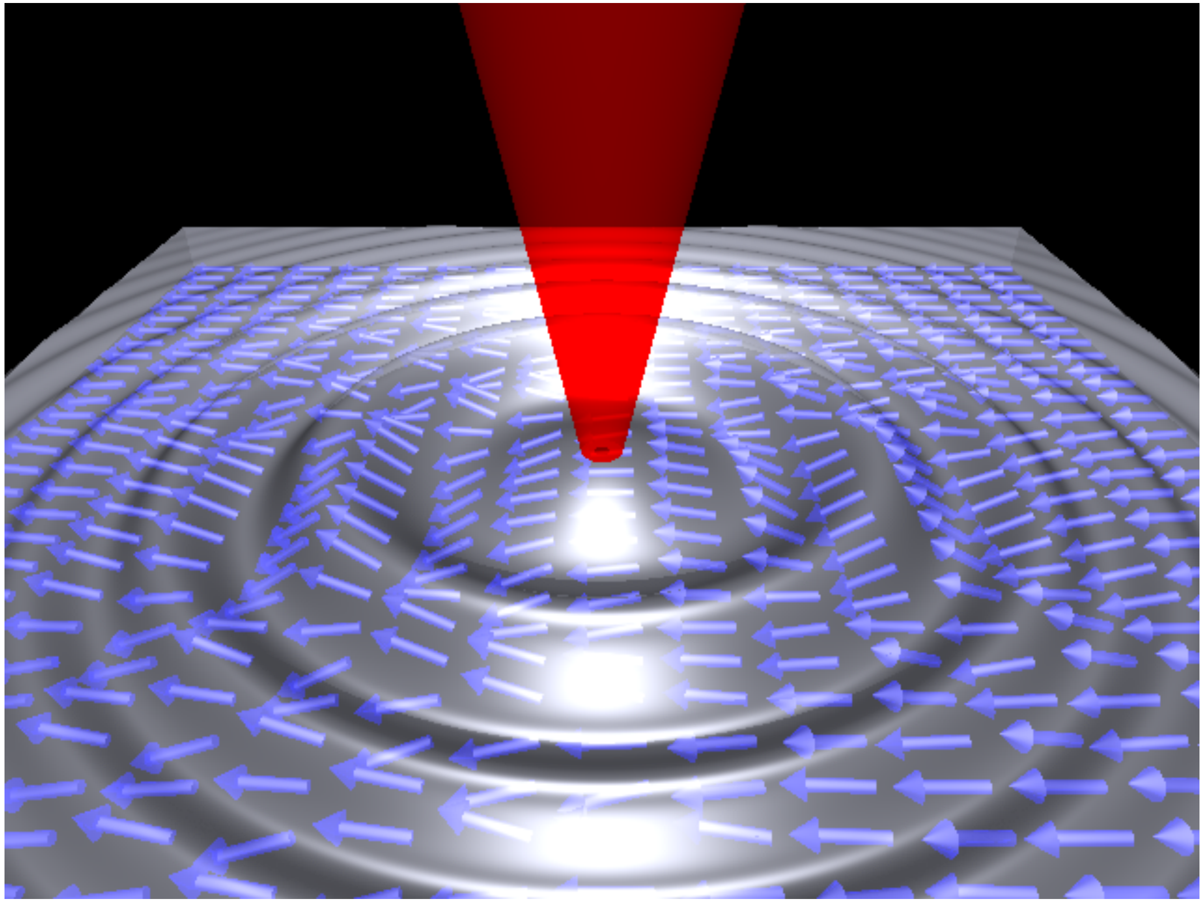




Energy flow via spin waves??



Semi-quantitative analysis

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observed τ < 200 ps;

$$\Rightarrow v > 100 \, {}^{km}\!/_{s}$$

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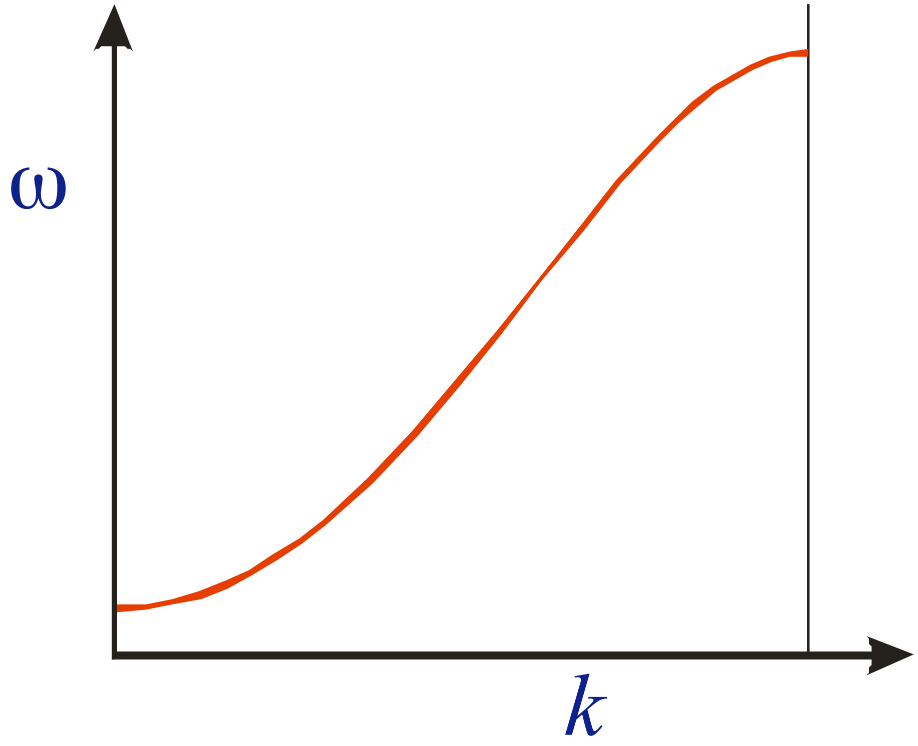

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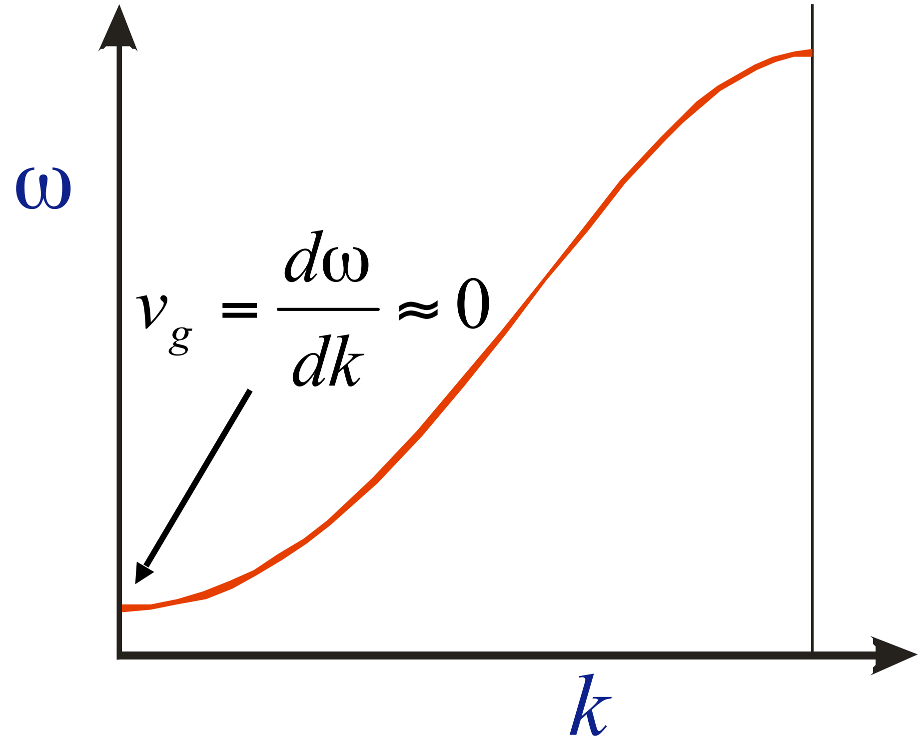

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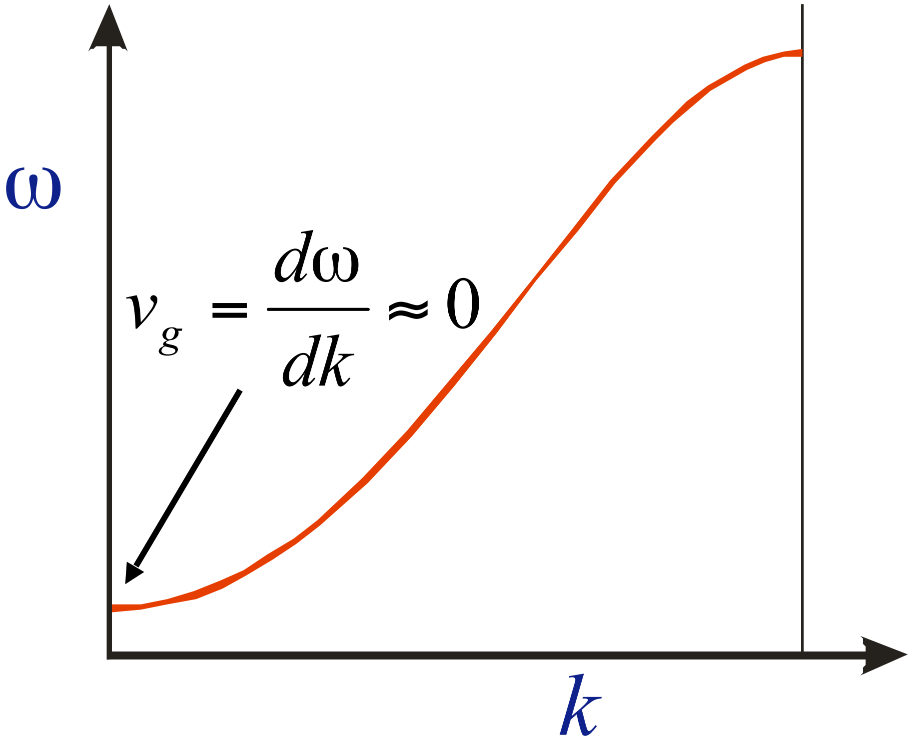

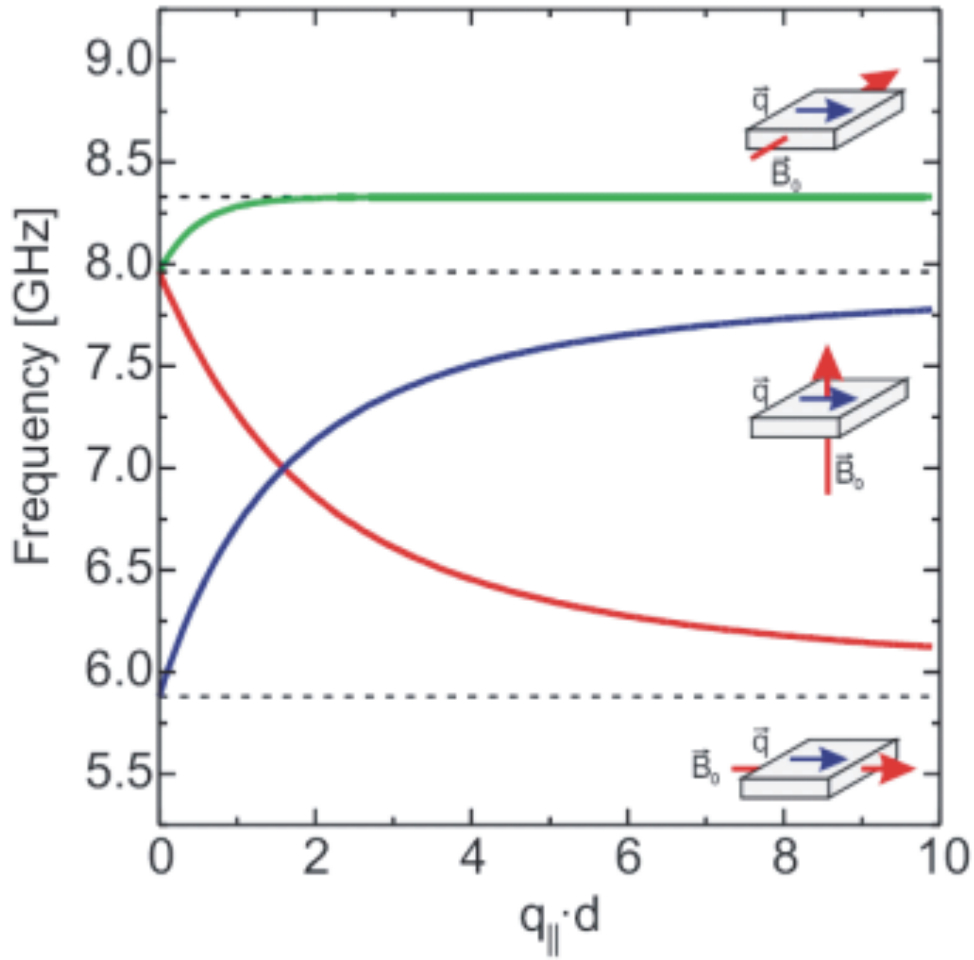

Magnetostatic modes; picture from
Demokritov & Hillebrands

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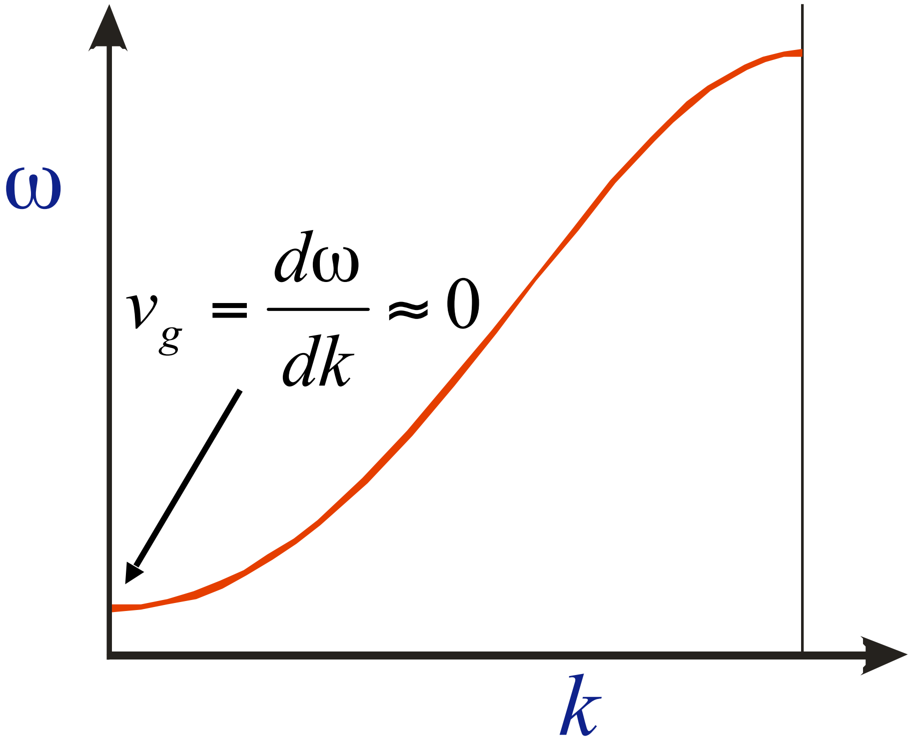
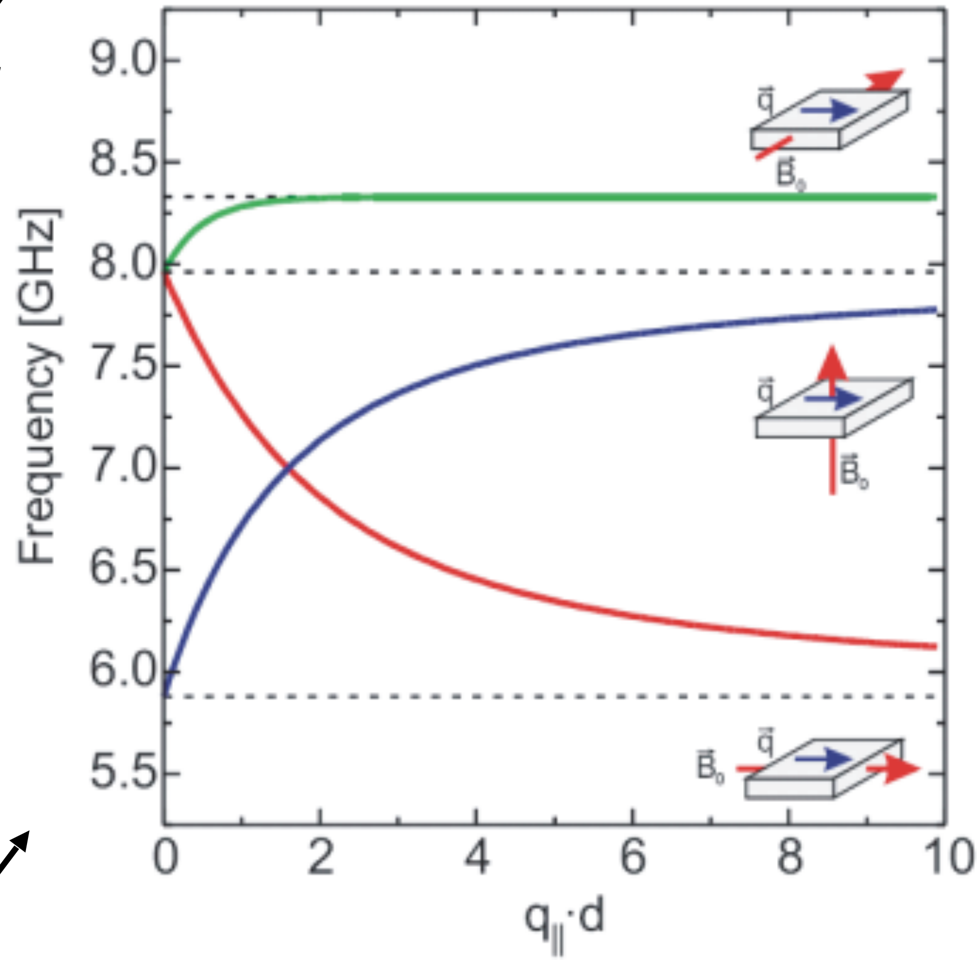


$$v_g = \frac{d\omega}{dk} \approx 0$$

$$v_g \leq 10 \, {}^{km}\!/_{s}$$



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μ-magnetic simulations [Eilers et al, PRB 74, 054411 (2006)]

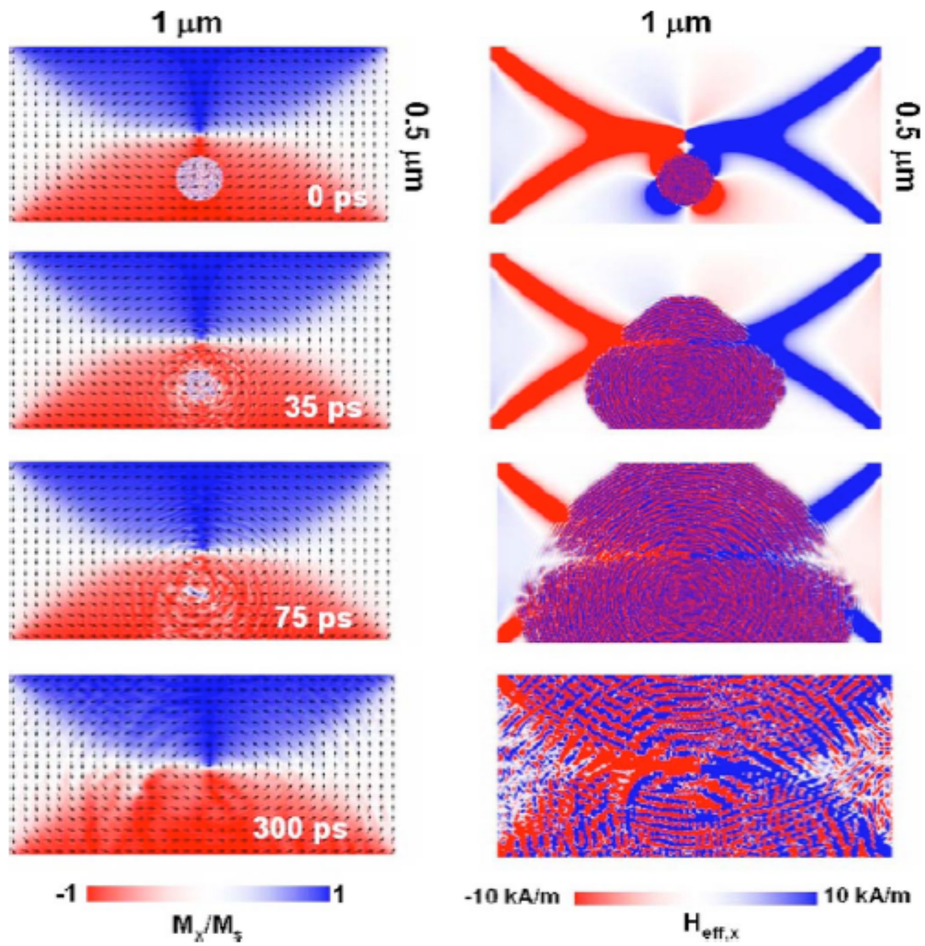

FIG. 1. (Color online) Micromagnetic simulation for a 0.5 μm × 1 μm Permalloy film structure with a 125 nm demagnetized spot diameter with 10 nm thickness. On the left the evolution of the spin-wave emission from the excited area is shown. On the right, the total effective field reflects the energy located within the domain walls and spin waves excited. The color code "red-white-blue" (white to black) indicates "positive-zero-negative" (the absolute) value of the x component.

$$v \approx \frac{0.4\mu m}{70\,ps} \approx 6\, {}^{km}\!/\!{}_{s}$$

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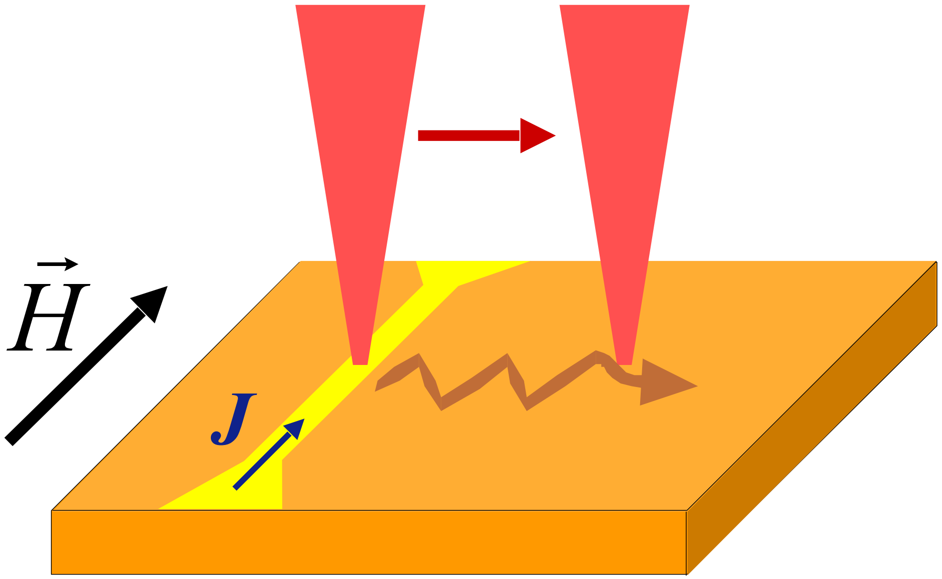
Experiment: propagation of spin waves



0.5 ns, 40 Oe pulse

part with T. Korn & U. Ebels,
SPINTEC, Grenoble

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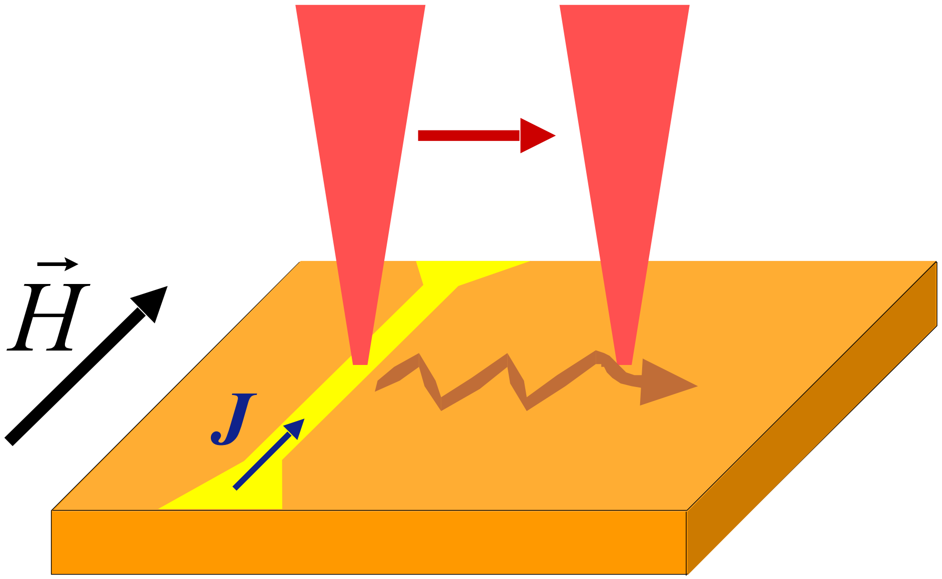

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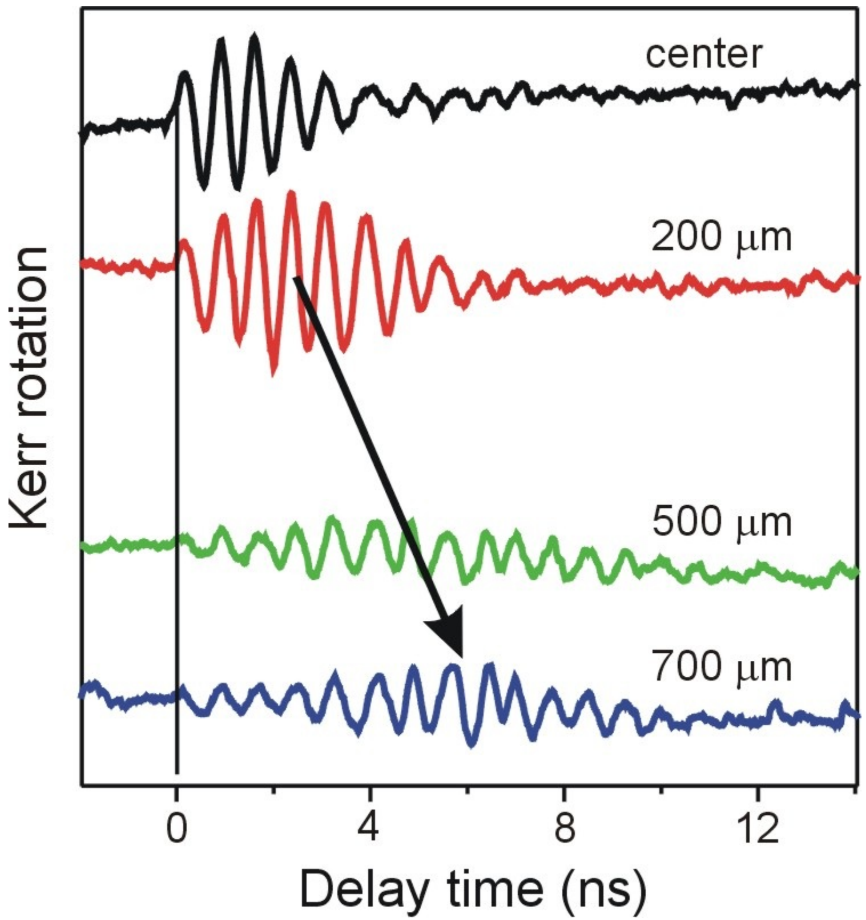

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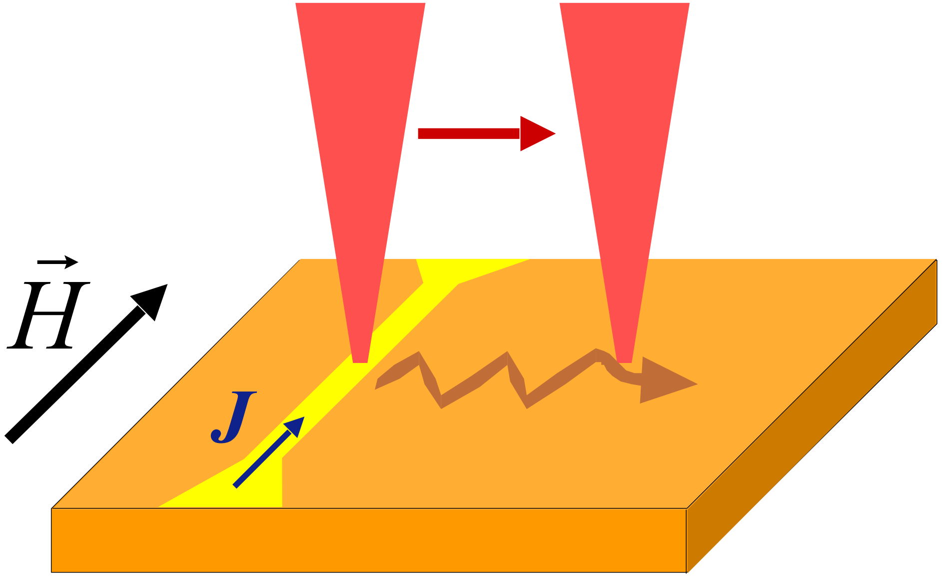

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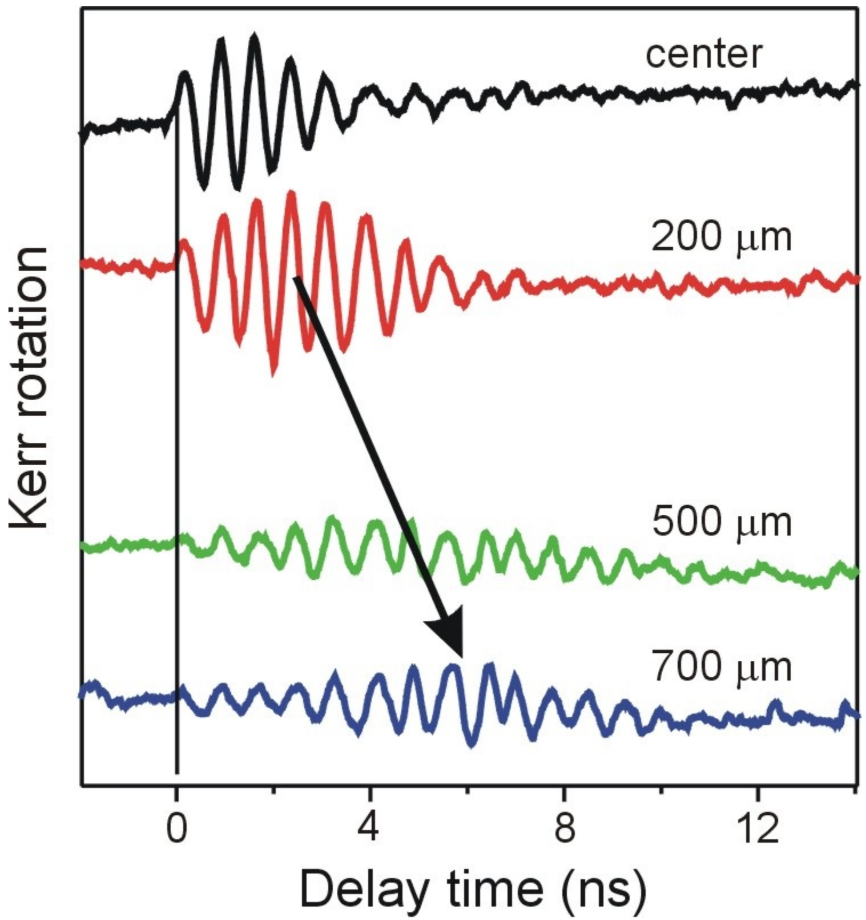

part with T. Korn & U. Ebels,
SPINTEC, Grenoble

$$v \approx \frac{500\mu m}{3.5ns} \approx 140\, {}^{km}\!/\!{}_{s}$$

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Conclusion 1

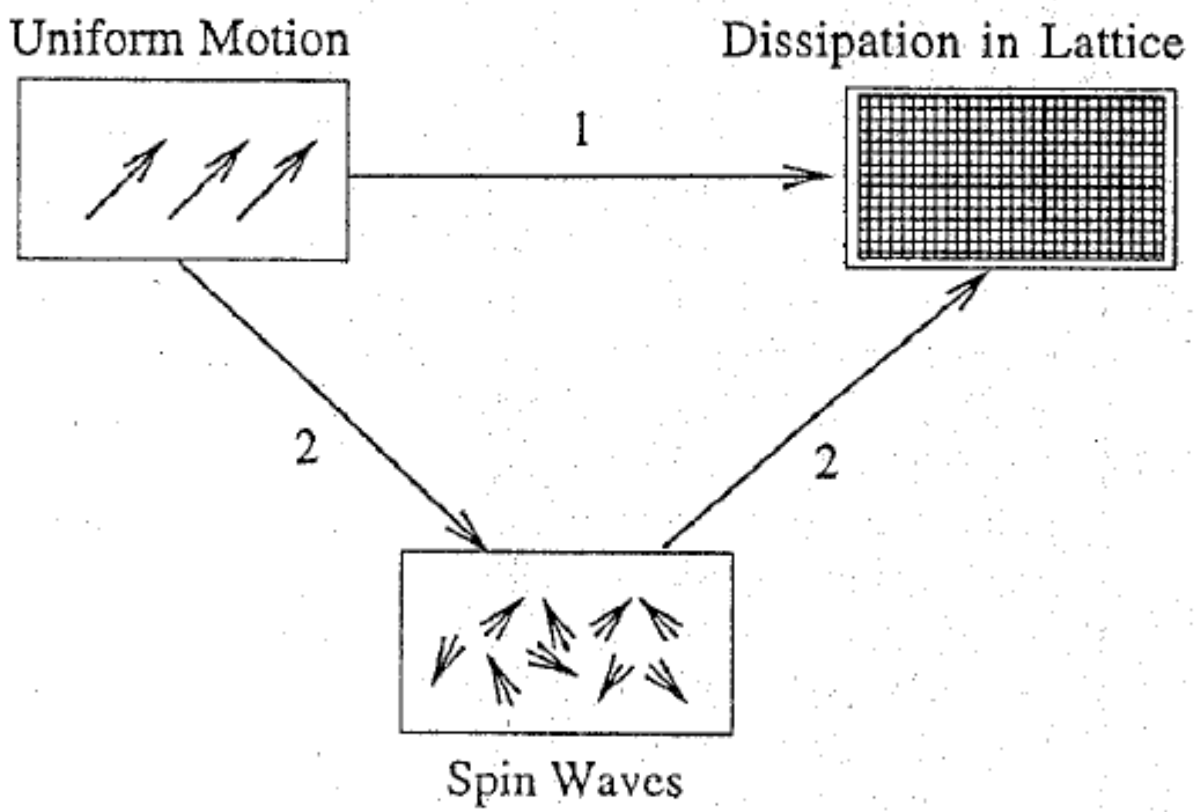
- not everything what you measure is damping!

Damping channels: intrinsic vs extrinsic

IEEE TRANSACTIONS ON MAGNETICS, VOL. 34, NO. 4, JULY 1998

THEORY OF THE MAGNETIC DAMPING CONSTANT

Harry Suhl
Department of Physics, and Center for Magnetic Recording Research, Mail Code 0319,
University of California-San Diego, La Jolla, CA 92093-0319.



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- damping via magnetoelastic interactions
- breathing Fermi-surface in metals
- extrinsic: two-magnon scattering

- **damping via magnetoelastic interactions**
- **breathing Fermi-surface in metals**
- **extrinsic: two-magnon scattering**

Phenomenology based on magneto-elasticity

$$\frac{d\mathbf{M}}{dt} = \gamma\mathbf{M} \times \mathbf{H}_{tot}$$

$$\mathbf{H}_{tot} = \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,\boldsymbol{e}}$$

the energy per unit volume u,

$$du = Tds + \mathbf{H} \cdot d\mathbf{M} + \boldsymbol{\sigma}{:}d\boldsymbol{e},$$

or the enthalpy per unit volume w,

$$dw = Tds + \mathbf{H} \cdot d\mathbf{M} - \boldsymbol{e}{:}d\boldsymbol{\sigma},$$

‘Dissipative’ part of magnetic field

$$\mathbf{H}' = \left(\frac{1}{\gamma M}\right)\tilde{\boldsymbol{\alpha}} \cdot \frac{d\mathbf{M}}{dt}$$

$$\mathbf{H}_{tot} = \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,e} + \left(\frac{1}{\gamma M}\right)\tilde{\boldsymbol{\alpha}} \cdot \frac{d\mathbf{M}}{dt}.$$

‘Dissipative’ part of magnetic field

$$\mathbf{H}' = \left(\frac{1}{\gamma M}\right)\tilde{\boldsymbol{\alpha}} \cdot \frac{d\mathbf{M}}{dt}$$

so that the total effective field is

$$\mathbf{H}_{tot} = \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,e} + \left(\frac{1}{\gamma M}\right)\tilde{\boldsymbol{\alpha}} \cdot \frac{d\mathbf{M}}{dt}.$$

Heating rate

$$\dot{Q} = \frac{d\mathbf{M}}{dt} \cdot \mathbf{H}',$$

$$\dot{Q} = \left[\frac{1}{\gamma M} \right] \frac{d\mathbf{M}}{dt} \cdot \tilde{\alpha} \cdot \frac{d\mathbf{M}}{dt},$$

$$\tilde{\alpha} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix}$$

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$$\dot{Q} = \frac{M}{\gamma} \dot{n}_i \alpha_{ij} \dot{n}_j$$

Magnetostriction

the adiabatic magnetostriction coefficients are defined as

$$2\Lambda_{ijkl} M_k = M^2 \left(\frac{\partial e_{ij}}{\partial M_l} \right)$$

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the time-varying magnetostrictive strain is then

$$\dot{e}_{ij} = 2\Lambda_{ijkl}n_k\dot{n}_l$$

Finally: the Gilbert damping tensor

thus, a changing M produce a changing strain; the crystal viscosity tensor determines the heating rate per unit volume

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$$\dot{Q} = \dot{e}_{ij}\eta_{ijkl}\dot{e}_{kl} \qquad \dot{Q} = \frac{M}{\gamma}\dot{n}_i\alpha_{ij}\dot{n}_j$$

from this, the Gilbert damping tensor is **rigorously** given by

$$\alpha_{ij} = \left[\frac{4\gamma}{M}\right]\left(\Lambda_{nmpi}n_p\right)\eta_{nmrl}\left(\Lambda_{rlqj}n_q\right)$$

Experiments vs theory

The experimental value of Gilbert damping parameter α_{exp} may be deduced from the FMR linewidth ΔH at frequency f as

$$\alpha_{\text{exp}} = \frac{\sqrt{3}}{2}\left(\frac{\gamma \Delta H}{2\pi f}\right)$$

$$\alpha_{\text{th}} = \frac{36\rho\gamma}{M\tau}\left[\frac{\lambda_{100}^2}{q_L^2} + \frac{\lambda_{111}^2}{q_T^2}\right]$$

wherein ρ is the mass density, $q_T \approx v_T \frac{M}{2\gamma A}$ is the transverse-acoustic propagation constant, q_L is the longitudinal-acoustic propagation constant, v_T is the transverse sound velocity, A is the exchange stiffness constant, λ_{100} and λ_{111} are magnetostriction constants for a cubic crystal magnetic material.

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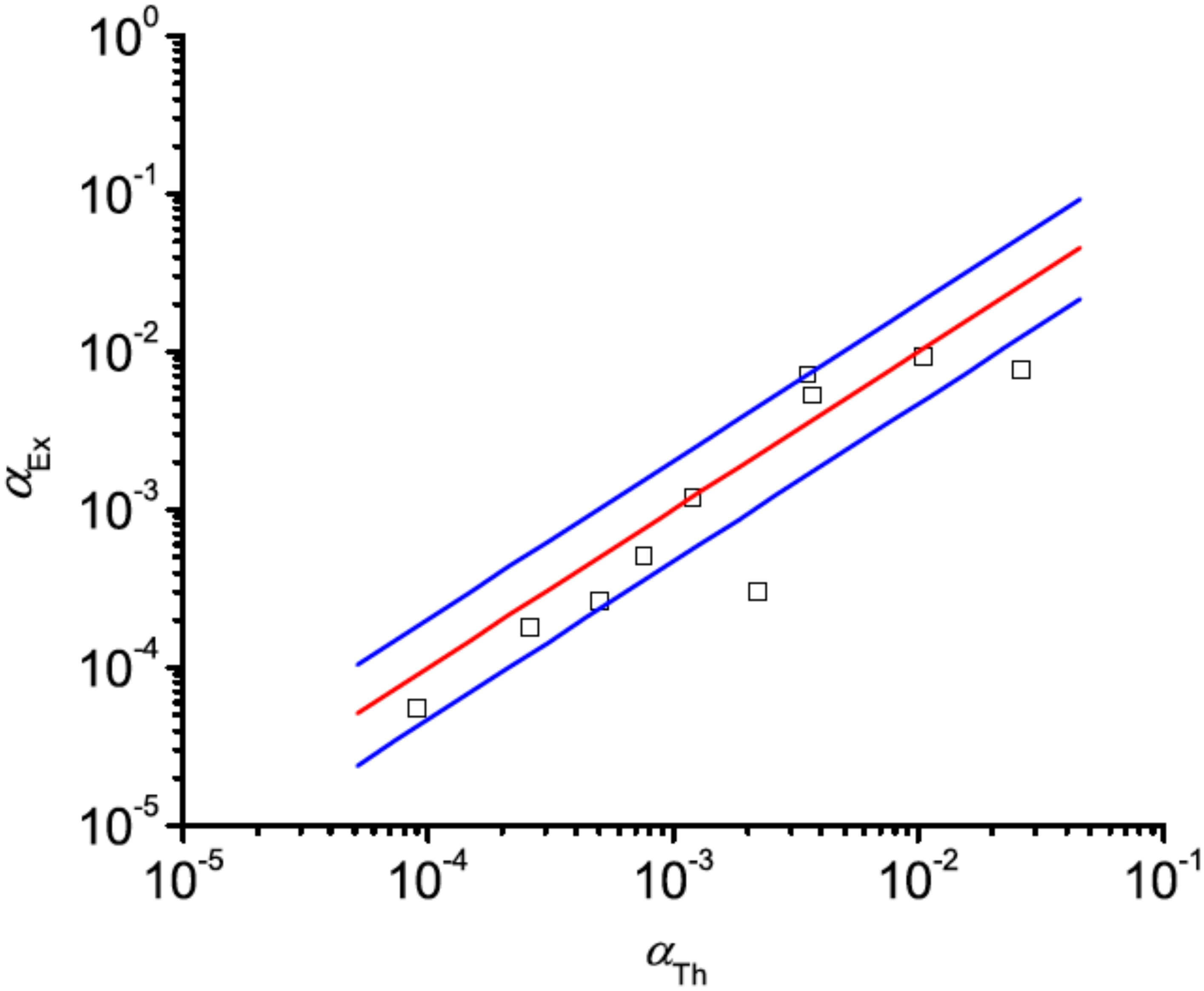
Theoretical vs measured damping parameters

Materials	M (G/4π)	A (10^{-6} erg/cm)	ΔH (Oe)	f (GHz)	τ (10^{-13} s)	α_{th} (10^{-5})	α_{exp} (10^{-5})
$Y_3Fe_5O_{12}$ [a]	139	0.40	0.33	9.53	4.4	5.56	9.0
$Y_3Fe_4GaO_{12}$ [a]	36	0.28	3.0	9.53	4.4	51	76
$Li_{0.5}Fe_{2.5}O_4$ [b]	310	0.40	2.0	9.50	1.5	26	50
$NiFe_2O_4$ [b]	270	0.40	35	24.0	1.5	710	350
$MgFe_2O_4$ [b]	90	0.1	2.3	4.9	1.5	120	120
$MnFe_2O_4$ [b]	220	0.4	238	9.2	1.5	930	1040
$BaFe_{12}O_{19}$ [c]	350	0.4	6	55	1.5	18	26
Ni[d]	484	0.75	102	9.53	1.8	770	2600
Fe[d]	1690	1.9	9	9.53	1.8	30	220
Co[d]	1400	2.78	15	9.53	1.8	530	370

[a]Garnets.
[b]Spinels.
[c]Hexagonal ferrite.
[d]Ferromagnetic materials

Theoretical vs measured damping parameters



- **damping via magnetoelastic interactions**
- **breathing Fermi-surface in metals**
- **extrinsic: two-magnon scattering**

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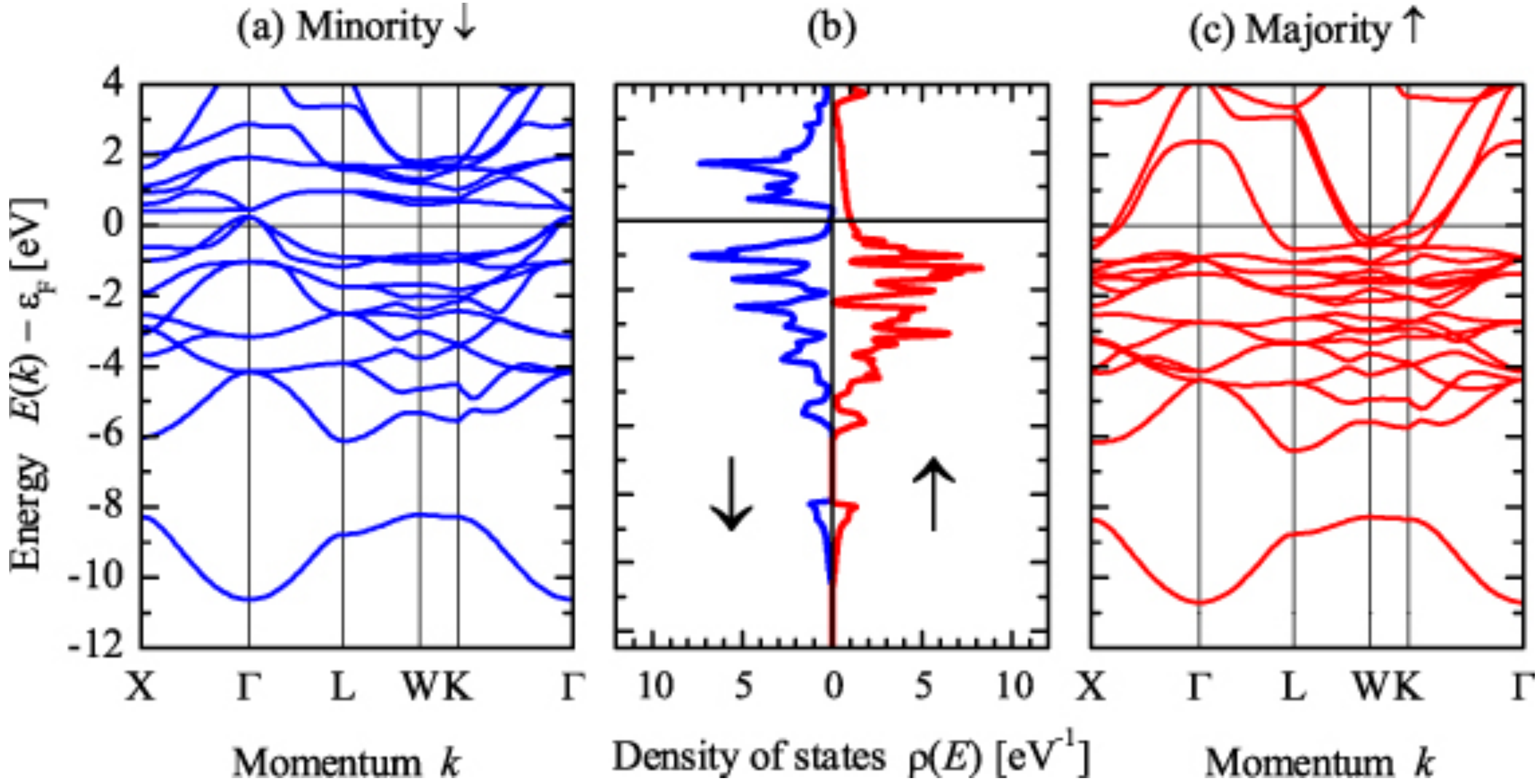
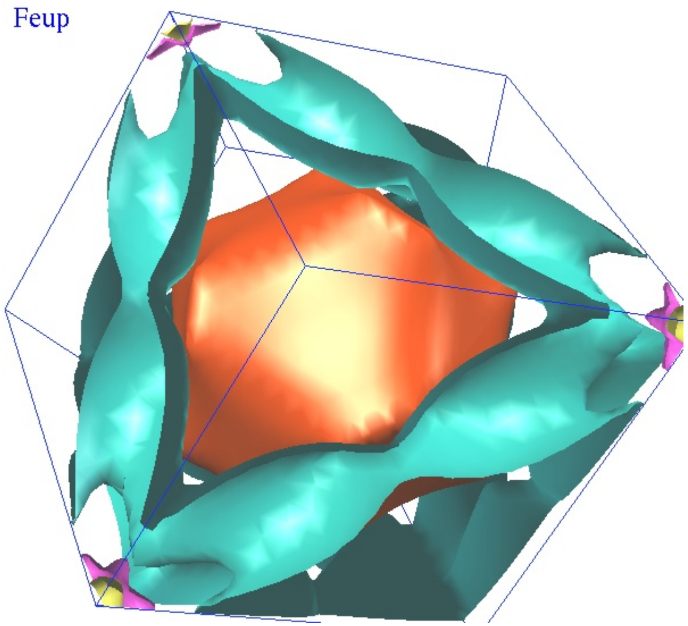
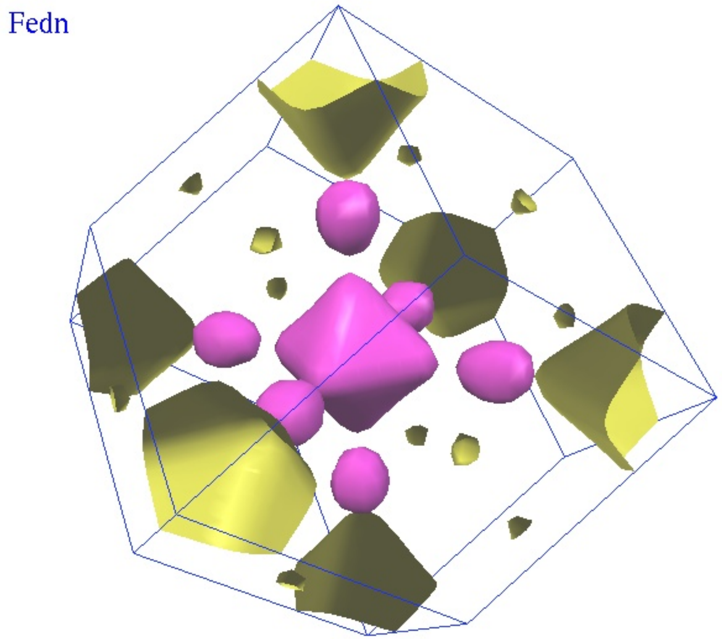
Ferromagnetism of metals



‘breathing’ Fermi-surface





following Steiauf and Fähnle, PRB **72**, 0064450 (2005);
see Kambersky, Can J. Phys. **48**, 2906 (1970);
Kunes and Kambersky, PRB **65**, 212411 (2002)

1. Adiabatic regime

we confine the treatment to the adiabatic regime:
several ps to nanoseconds
(single-electron spin fluctuations can be integrated out):

$$\mathbf{M}_{\mathrm{s},\mathbf{R}} = M_{\mathrm{s},\mathbf{R}}\mathbf{e}_{\mathrm{s},\mathbf{R}} = \int_{\Omega_{\mathbf{R}}} \mathbf{m}(\mathbf{r})d^3r.$$

2. Dissipative free-energy functional

the existence of such functional is postulated: $F_{\mathrm{diss}}[\mathbf{M}_{\mathbf{R}}]$

W. F. Brown, *Micromagnetics* (Wiley, New York, 1963).

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$$\frac{d\mathbf{e}_{\mathbf{R}}}{dt} = -\gamma(\mathbf{e}_{\mathbf{R}} \times \widetilde{\mathbf{H}}_{\mathrm{eff},\mathbf{R}}),$$

with the effective field

$$\widetilde{\mathbf{H}}_{\mathrm{eff},\mathbf{R}} = -\frac{1}{M_{\mathbf{R}}} \frac{\delta F_{\mathrm{diss}}}{\delta \mathbf{e}_{\mathbf{R}}},$$

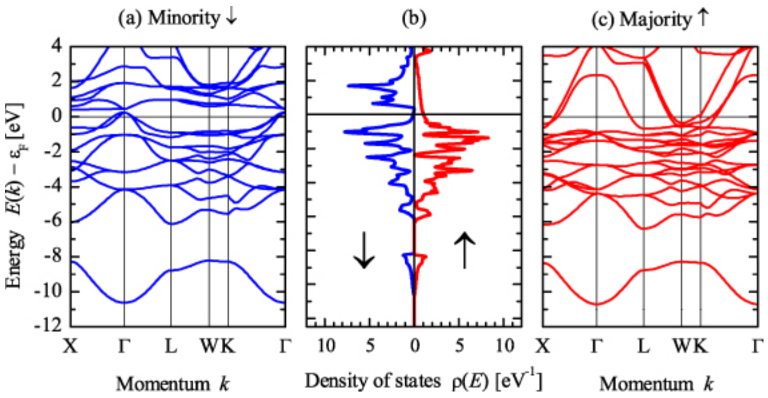
which encompasses the contributions from damping.

W. F. Brown, *Micromagnetics* (Wiley, New York, 1963).

3. Translate this to the electronic level

$$E[n,\{\mathbf{e}_{\mathbf{R}}(t)\}] = \sum_{j\mathbf{k}} n_{j\mathbf{k}}\varepsilon_{j\mathbf{k}} + E_{\mathrm{dc}}[n].$$

as outputted from the density functional theory



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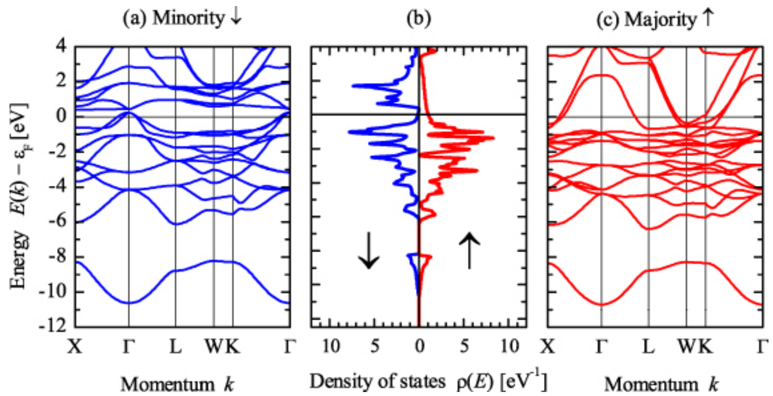

$$\delta E = \sum_{j\mathbf{k}} \delta n_{j\mathbf{k}}\varepsilon_{j\mathbf{k}} + \sum_{j\mathbf{k}} n_{j\mathbf{k}}\delta\varepsilon_{j\mathbf{k}}.$$

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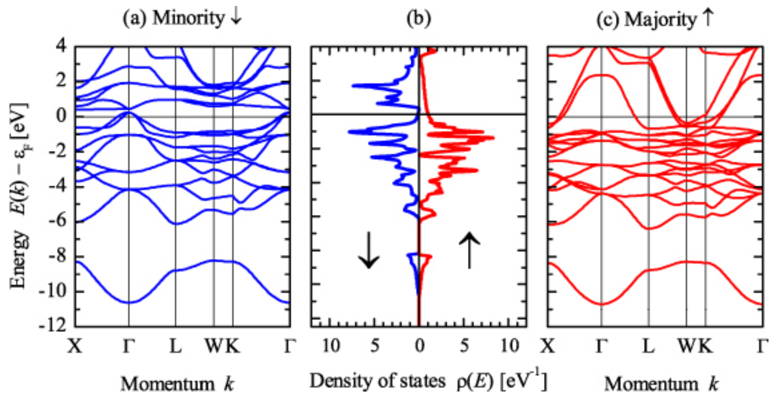

$$\delta E = \sum_{j\mathbf{k}} \delta n_{j\mathbf{k}} \varepsilon_{j\mathbf{k}} + \sum_{j\mathbf{k}} n_{j\mathbf{k}} \delta\varepsilon_{j\mathbf{k}}. \qquad \sum_{j\mathbf{k}} \delta n_{j\mathbf{k}} \varepsilon_{j\mathbf{k}} \approx \varepsilon_{\mathrm{F}} \sum_{j\mathbf{k}} \delta n_{j\mathbf{k}} = 0.$$

as the total number of states is conserved

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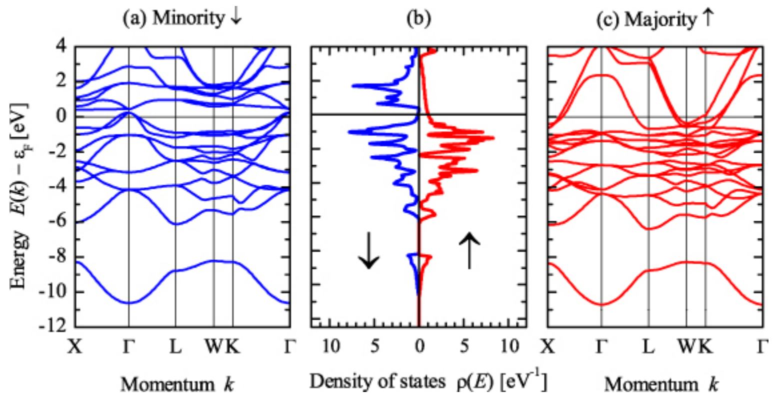

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$$\widetilde{\mathbf{H}}_{\mathrm{eff},\mathbf{R}} = -\frac{1}{M_{\mathbf{R}}}\frac{\partial E}{\partial \mathbf{e_R}} = -\frac{1}{M_{\mathbf{R}}}\sum_{j\mathbf{k}} n_{j\mathbf{k}}[\{\mathbf{e}_{\mathbf{R}'}(t)\}]\frac{\partial \varepsilon_{j\mathbf{k}}[\{\mathbf{e}_{\mathbf{R}'}(t)\}]}{\partial \mathbf{e_R}}$$

3a. Spin-orbit coupling

$$E_{s.o.} = \lambda \mathbf{s} \cdot (\nabla V(\mathbf{r}) \times \mathbf{p})$$

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for a lattice of simple cubic symmetry this gives

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N.B.: this is a difficult point, usually not much discussed!

4. Semiempirical extension of DFT

Redistribution of the occupation numbers provided by scattering processes

$$\frac{dn_{j\mathbf{k}}(t)}{dt} = -\frac{1}{\tau_{j\mathbf{k}}}[n_{j\mathbf{k}}(t) - f_{j\mathbf{k}}(t)]$$

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Approximated by

$$n_{j\mathbf{k}}(t) = f_{j\mathbf{k}}(t) - \tau_{j\mathbf{k}}\frac{df_{j\mathbf{k}}}{dt} + \cdots$$

5. Consider homogeneous situation

$$\mathbf{M_R} = \mathbf{M} = M\mathbf{e} \text{ for all sites } \mathbf{R}$$

$$\widetilde{\mathbf{H}}_{\mathrm{eff},\mathbf{R}} = \widetilde{\mathbf{H}}_{\mathrm{eff}} = \mathbf{H}_{\mathrm{aniso}} + \mathbf{H}_{\mathrm{damp}}$$

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Anisotropy and 'damping' fields:

$$\mathbf{H}_{\text{aniso}} = -\frac{1}{M}\sum_{j\mathbf{k}} f_{j\mathbf{k}} \frac{\partial \varepsilon_{j\mathbf{k}}(\mathbf{e})}{\partial \mathbf{e}}$$

$$\mathbf{H}_{\text{damp}} = -\frac{1}{\gamma M}\underline{\underline{\alpha}} \cdot \frac{d\mathbf{M}}{dt}$$

where the damping matrix:

$$\alpha_{lm} = -\frac{\gamma}{M}\sum_{j\mathbf{k}} \tau_{j\mathbf{k}} \frac{\partial f_{j\mathbf{k}}}{\partial \varepsilon_{j\mathbf{k}}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_l}\right|_{\mathbf{M}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_m}\right|_{\mathbf{M}}$$

7. Same relaxation times around the Fermi surface

In the seventh step we assume that the relaxation time $\tau_{j\mathbf{k}}$ for processes appearing at the Fermi surface are independent of the state $(j\mathbf{k})$, i.e., $\tau_{j\mathbf{k}} \equiv \tau$, yielding

$$\frac{\alpha_{lm}}{\tau} = -\frac{\gamma}{M} \sum_{j\mathbf{k}} \frac{\partial f_{j\mathbf{k}}}{\partial \varepsilon_{j\mathbf{k}}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_l}\right|_{\mathbf{M}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_m}\right|_{\mathbf{M}}$$

Finally: equation-of-motion

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{aniso}} + \frac{1}{M}\mathbf{M} \times \left(\underline{\underline{\alpha}} \cdot \frac{d\mathbf{M}}{dt} \right)$$

scalar damping parameter is

obtained only for the special case that $d\mathbf{M}/dt$ corresponds to an eigenvector of $\underline{\underline{\alpha}}(\mathbf{M})$, and then the damping scalar is given by the corresponding eigenvalue of $\underline{\underline{\alpha}}$.

sidenote: damping vs anisotropy

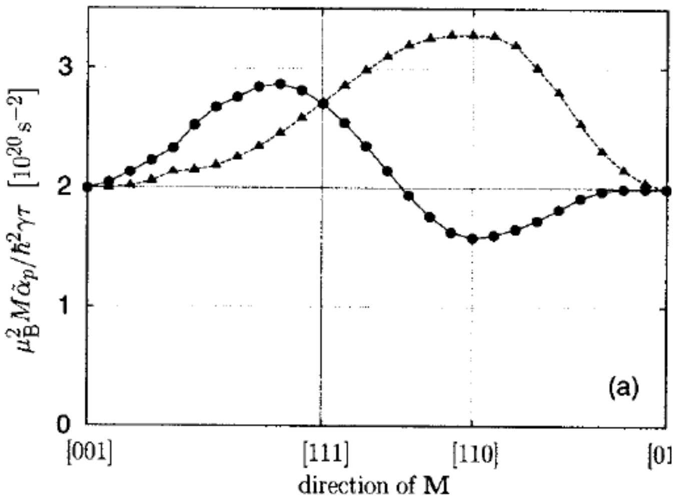
In many discussion you find the direct relation between damping and magnetocrystalline anisotropy - equations show that this is not entirely correct:

$$\mathbf{H}_{\text{aniso}} = -\frac{1}{M}\sum_{\text{jk}} f_{\text{jk}} \frac{\partial \varepsilon_{\text{jk}}(\mathbf{e})}{\partial \mathbf{e}}$$

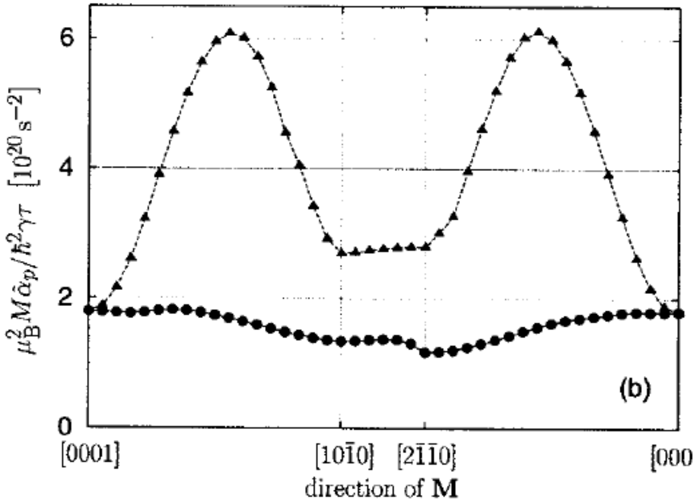
$$\alpha_{lm} = -\frac{\gamma}{M}\sum_{\text{jk}} \tau_{\text{jk}} \frac{\partial f_{\text{jk}}}{\partial \varepsilon_{\text{jk}}} \left.\frac{\partial \varepsilon_{\text{jk}}}{\partial e_l}\right|_{\mathbf{M}} \left.\frac{\partial \varepsilon_{\text{jk}}}{\partial e_m}\right|_{\mathbf{M}}$$

Radboud University

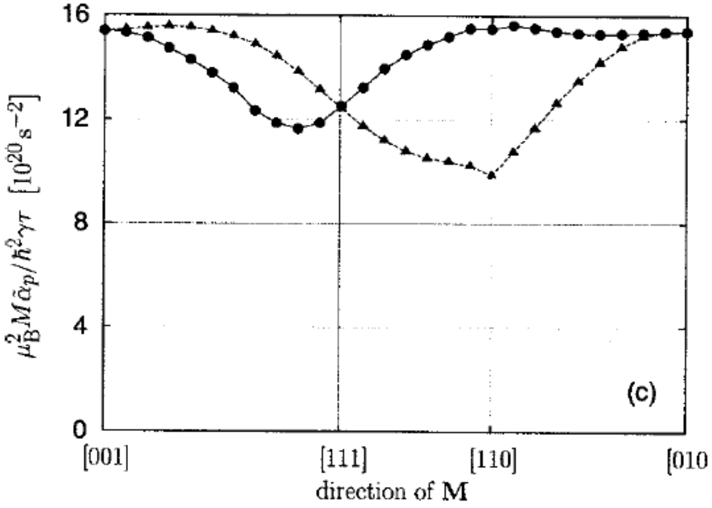
Results: Fe, Co, Ni



bcc Fe



hcp Co



fcc Ni

two eigenvalues of the damping matrix vs direction of M

Anisotropic FMR linewidth

Anisotropic ferromagnetic resonance linewidth in nickel at low temperatures

J. M. Rudd, K. Myrtle, J. F. Cochran, and B. Heinrich
Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6

We have measured the ferromagnetic resonance linewidth ΔH at 24 GHz in (110) nickel disks at 4 K and from 60 K to room temperature. Samples had a nominal purity of 99.99% and a residual resisitivy ratio of 40. The applied field was in the plane of the sample and measurements were made with the field along each of the three principal axes [100], [110], and [111]. We find $\Delta H_{[110]} > \Delta H_{[111]}$ and $\Delta H_{[100]}$ for temperatures below 200 K. At 4 K we found $\Delta H_{[100]} = 1600 \pm 50$ Oe, $\Delta H_{[111]} = 1800 \pm 50$ Oe, and $\Delta H_{[110]} = 2000 \pm 50$ Oe.

Journal of Applied Physics **57**, 3693 (1985)

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Temperature dependence of damping

$$n_{j\mathbf{k}}(t) = f_{j\mathbf{k}}(t) - \tau_{j\mathbf{k}} \frac{df_{j\mathbf{k}}}{dt} + \cdots$$

$$\frac{\alpha_{lm}}{\tau} = -\frac{\gamma}{M} \sum_{j\mathbf{k}} \frac{\partial f_{j\mathbf{k}}}{\partial \varepsilon_{j\mathbf{k}}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_l}\right|_{\mathbf{M}} \left.\frac{\partial \varepsilon_{j\mathbf{k}}}{\partial e_m}\right|_{\mathbf{M}}$$

the higher T, the shorter $\tau_{j\mathbf{k}}$ => less damping?? is this reasonable??

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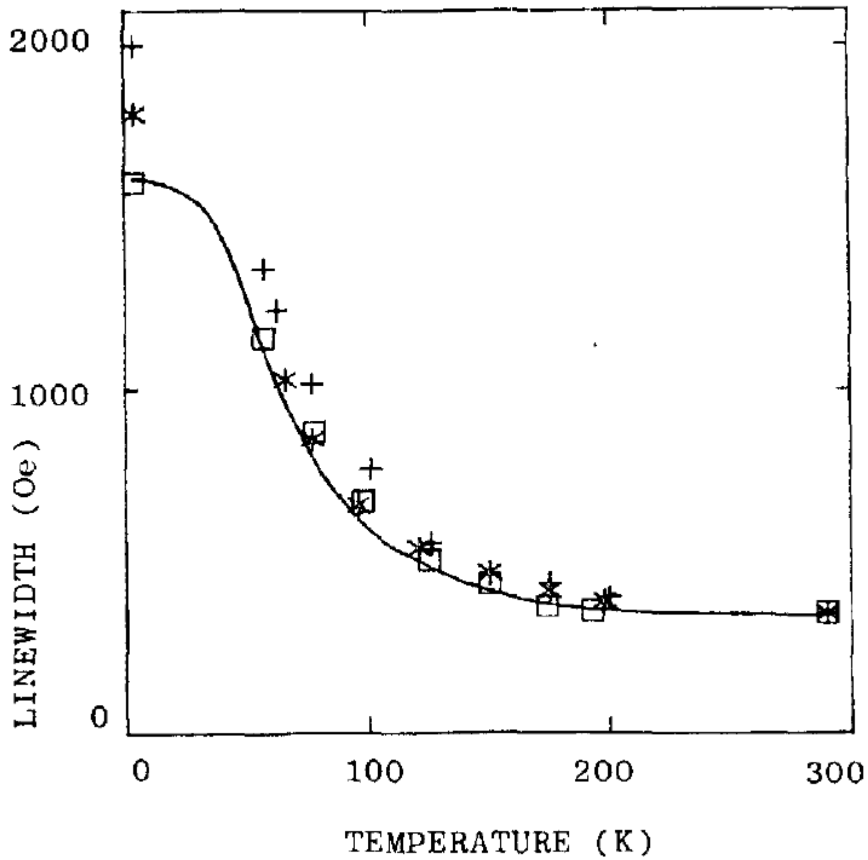
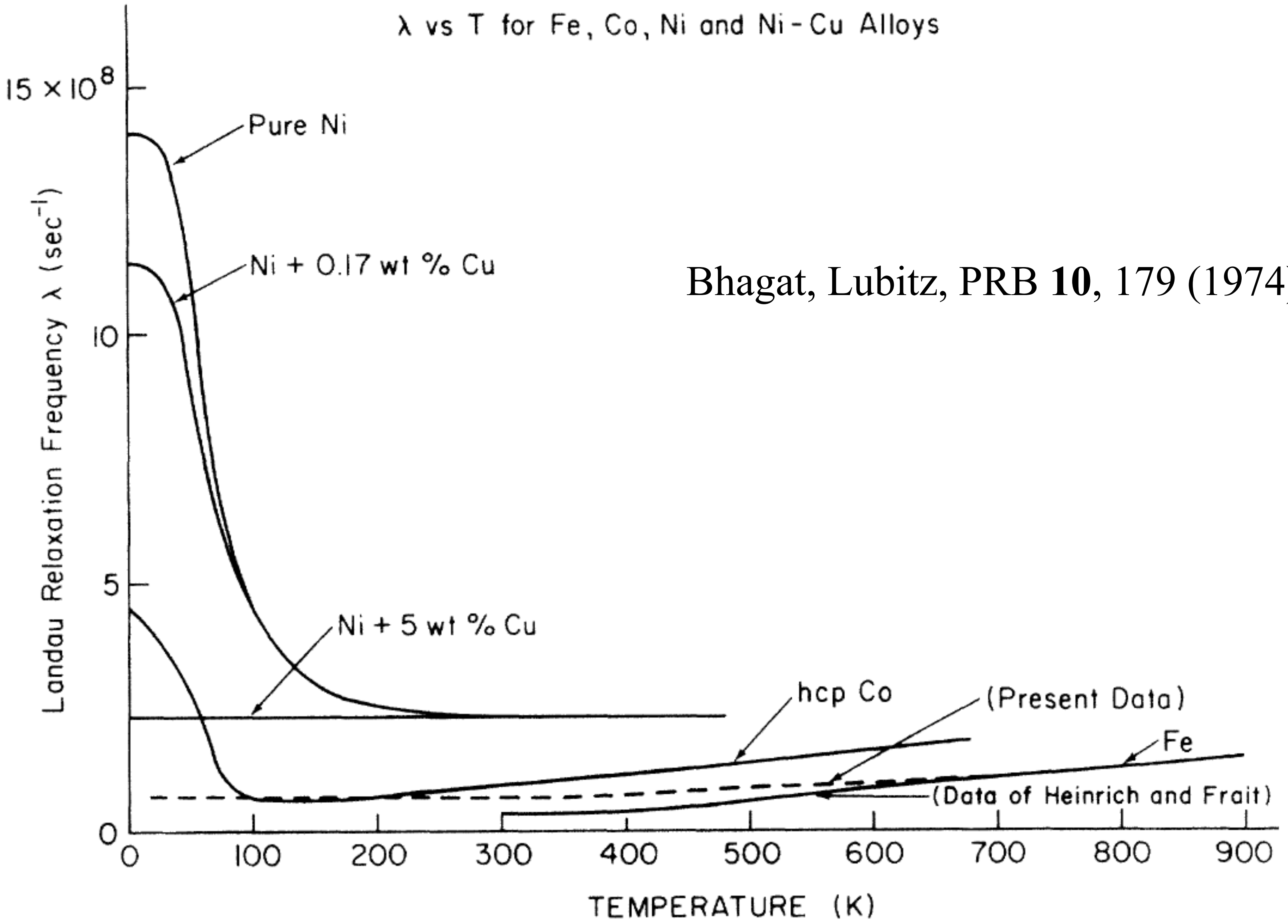


Temperature dependence of damping - 2



Bhagat, Lubitz, PRB **10**, 179 (1974)

Interband transitions at higher temperature

$$\alpha = \frac{g^2 \mu_B^2}{\hbar} \sum_{n,m} \int \frac{dk^3}{(2\pi)^3} |\Gamma_{nm}^{-}(k)|^2 W_{nm}(k)$$

note that this also includes the 'breathing Fermi surface' part for transitions inside the same band

Gilmore et al, PRL **99**, 027204 (2007)

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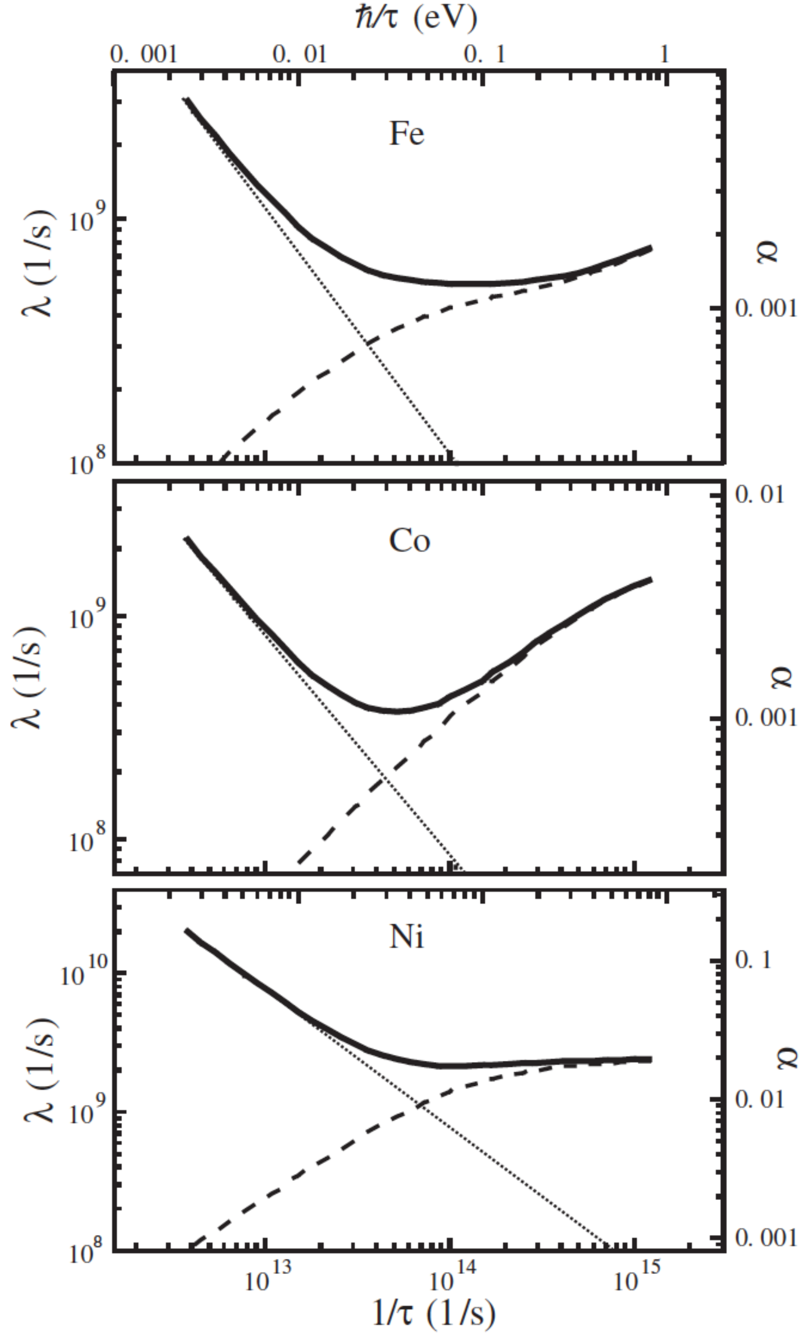

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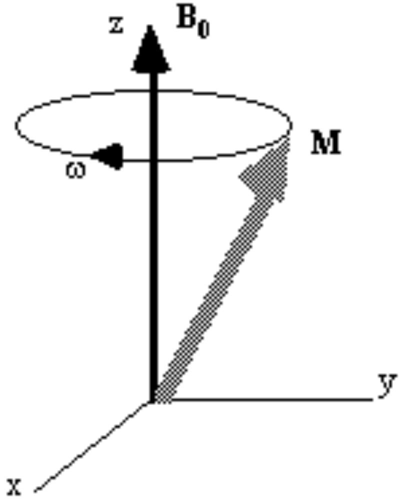
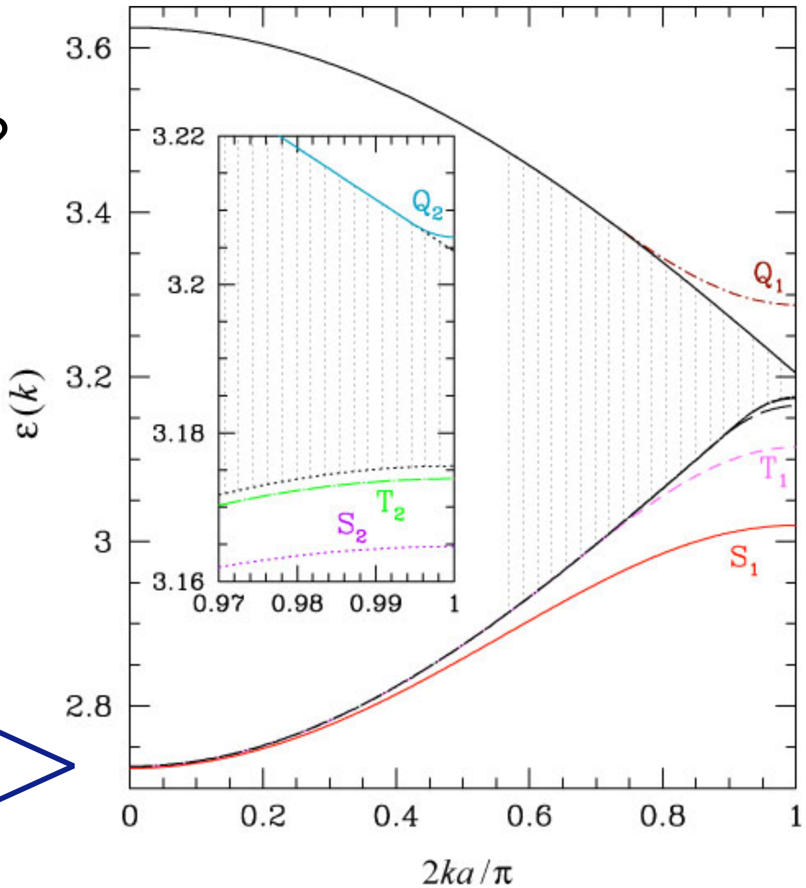
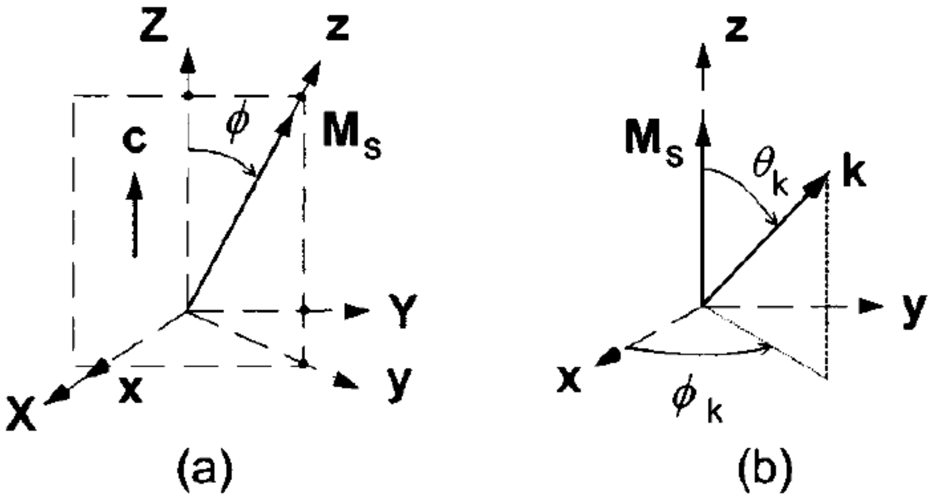
Two-magnon scattering

FMR is the lowest frequency, isn't it??



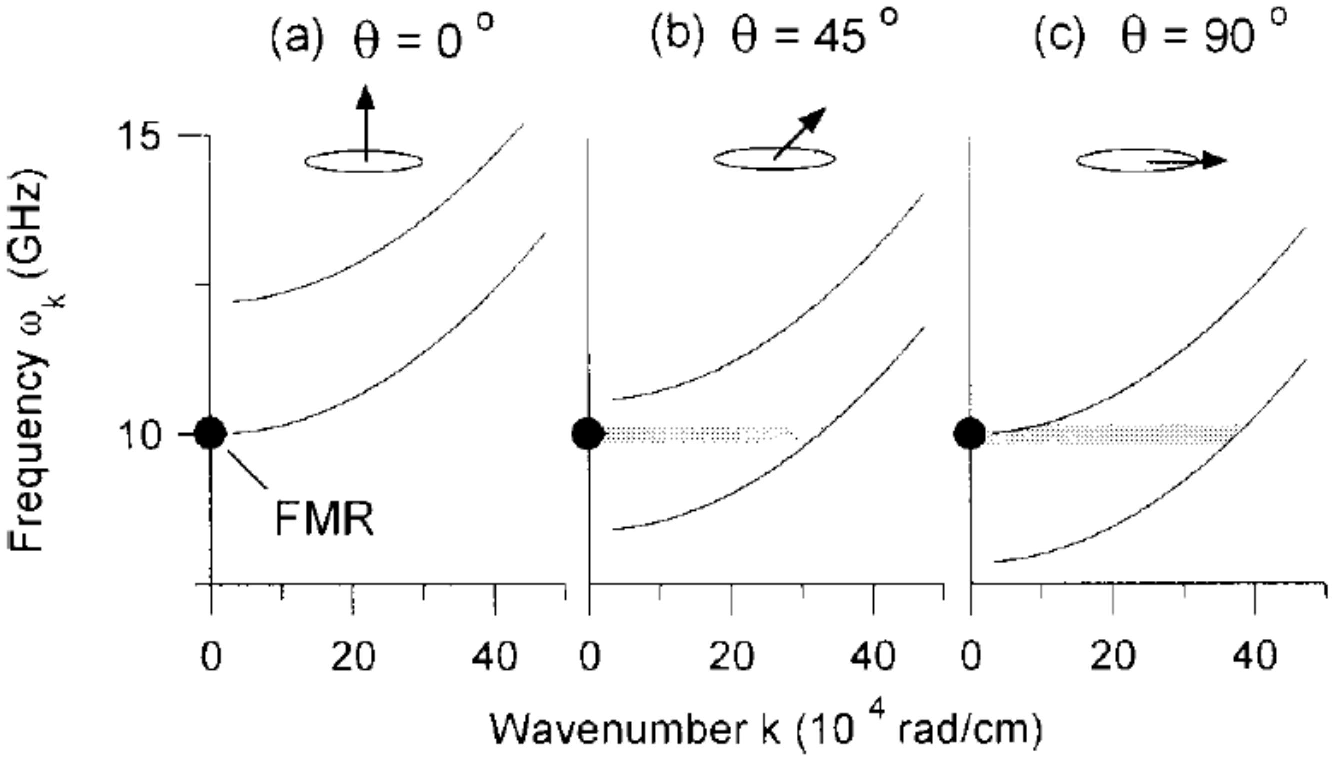


Spin waves in thin films



$$\omega_k^2 = \gamma^2 (H_i + Dk^2)(H_i + Dk^2 + 4\pi M_s \sin^2\theta_k - H_A \sin^2\phi) - \gamma^2 4\pi M_S H_A \sin^2\phi \sin^2\theta_k \cos^2\phi_k$$

Dispersion relations



$$\omega_{\max}^2 = \gamma^2(H_i + Dk^2)(H_i + Dk^2 - H_A \sin^2 \phi) + \gamma^2 4\pi M_s (H_i + Dk^2 - H_A \sin^2 \phi \cos^2 \phi_k)$$

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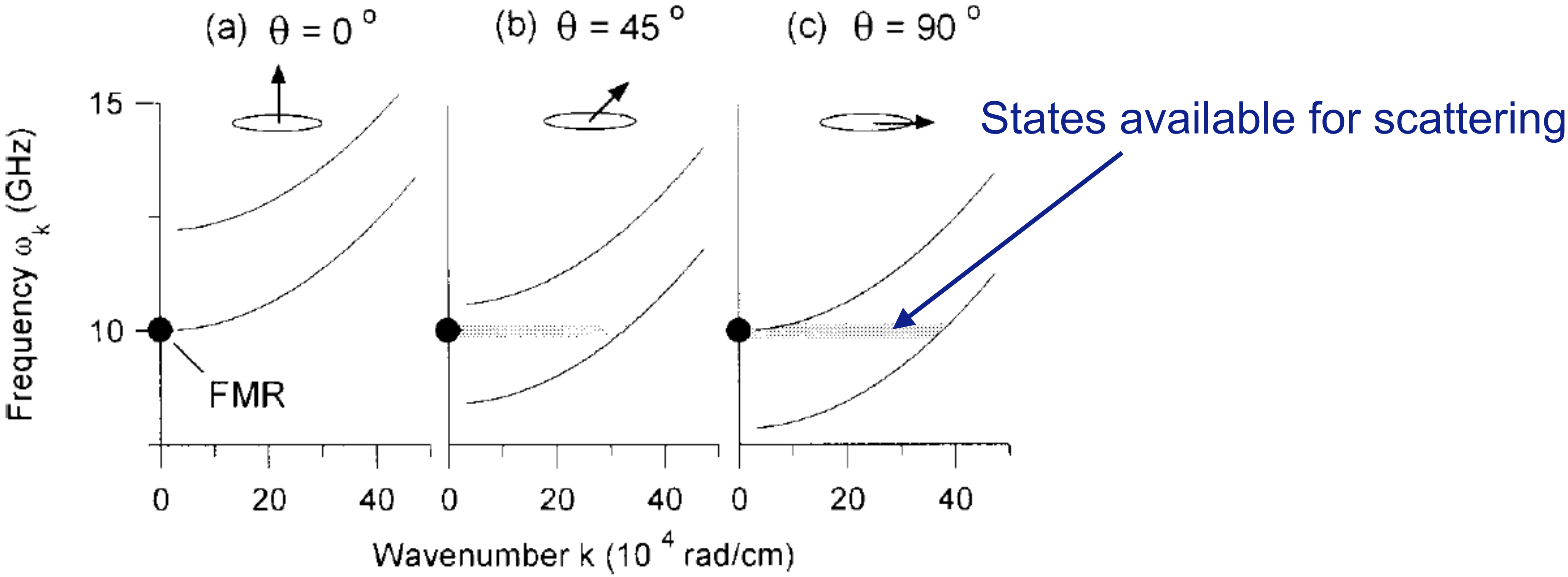
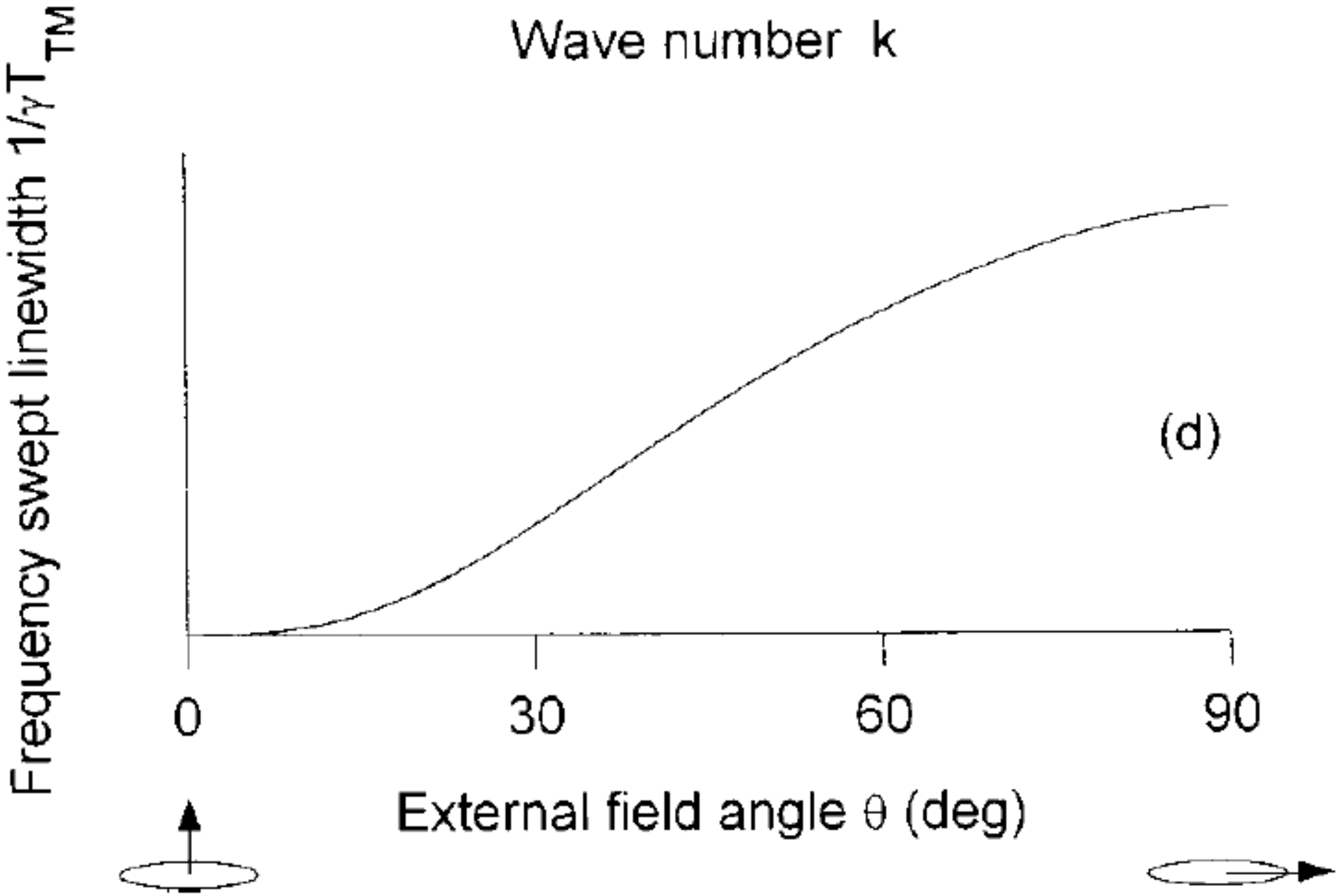
Dispersion relations



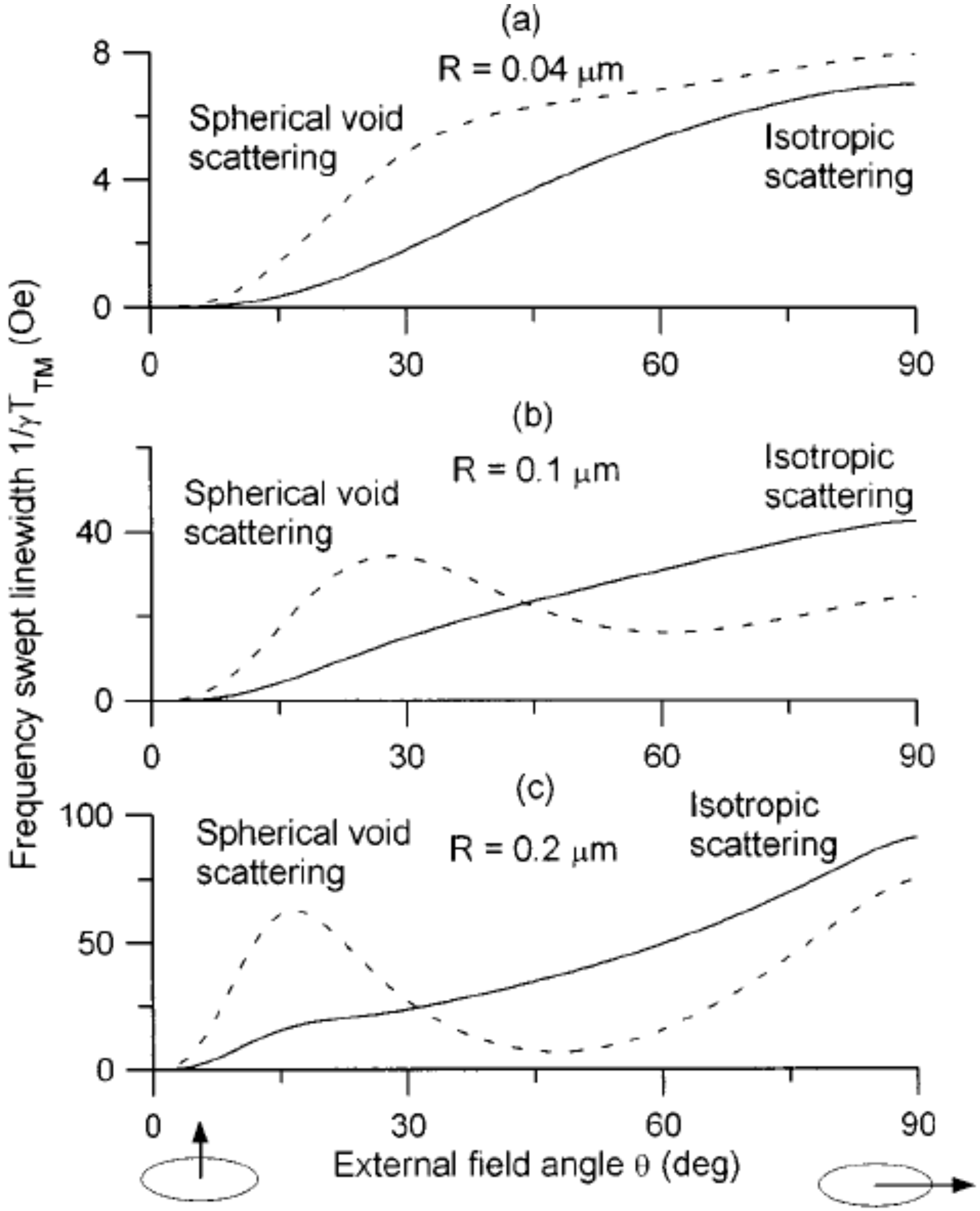
$$\omega_{\max}^2=\gamma^2(H_i+Dk^2)(H_i+Dk^2-H_A\sin^2\phi)+\gamma^2 4\pi M_s(H_i+Dk^2-H_A\sin^2\phi\cos^2\phi_k)$$

$$\omega_{\min}^2=\gamma^2(H_i+Dk^2)(H_i+Dk^2-H_A\sin^2\phi)$$

Angular dependence of 2-magnon damping



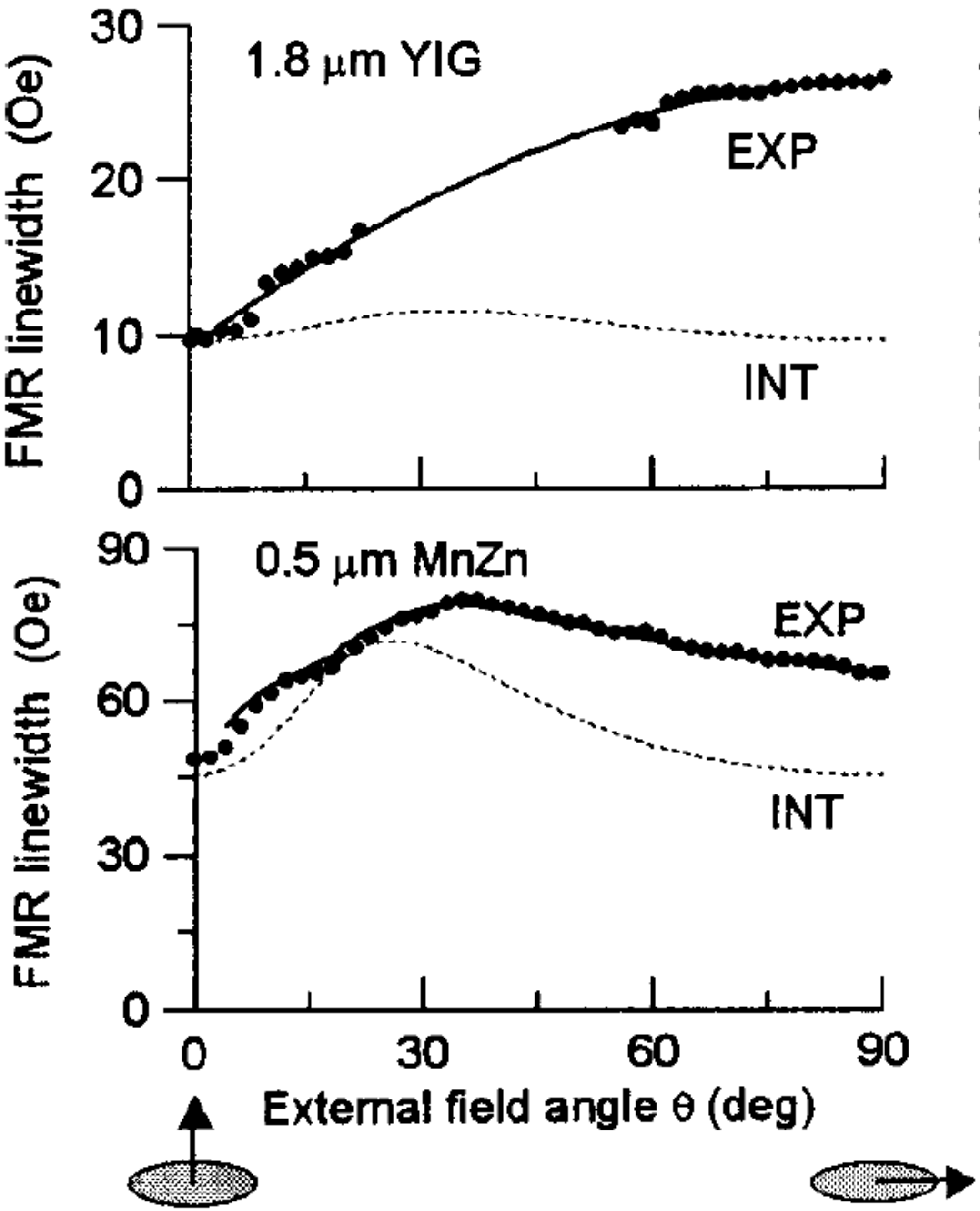
Different types of defects

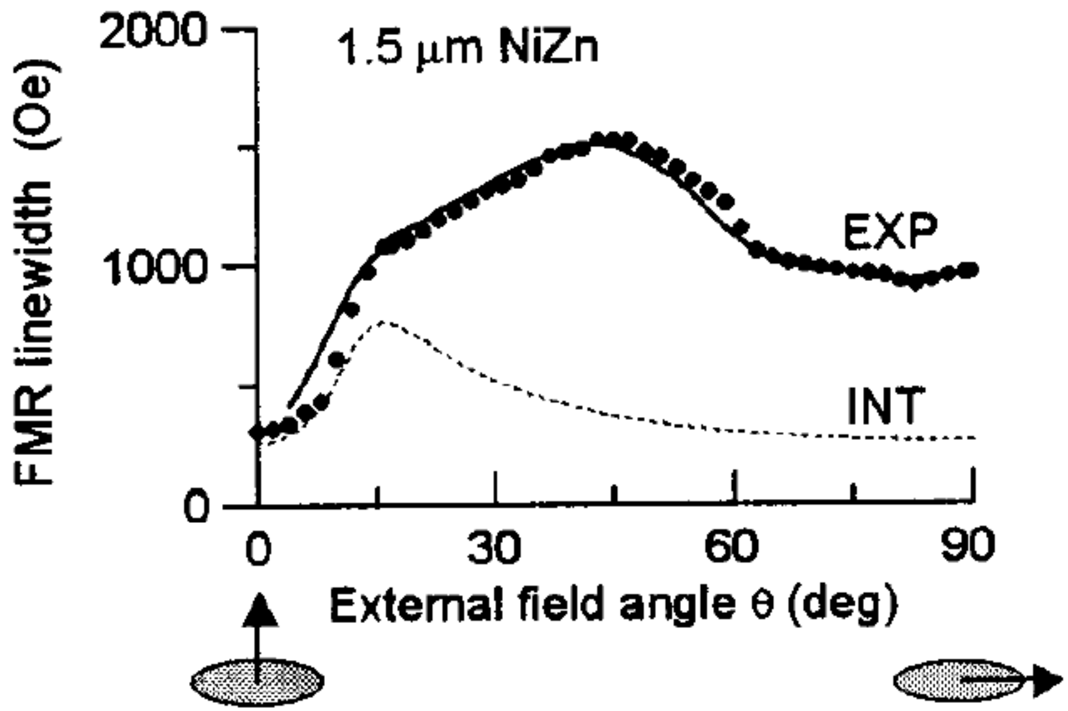


Radboud University

Experiments??

Srivastava et al, J. Appl. Phys. **85**, 7838 (1999);

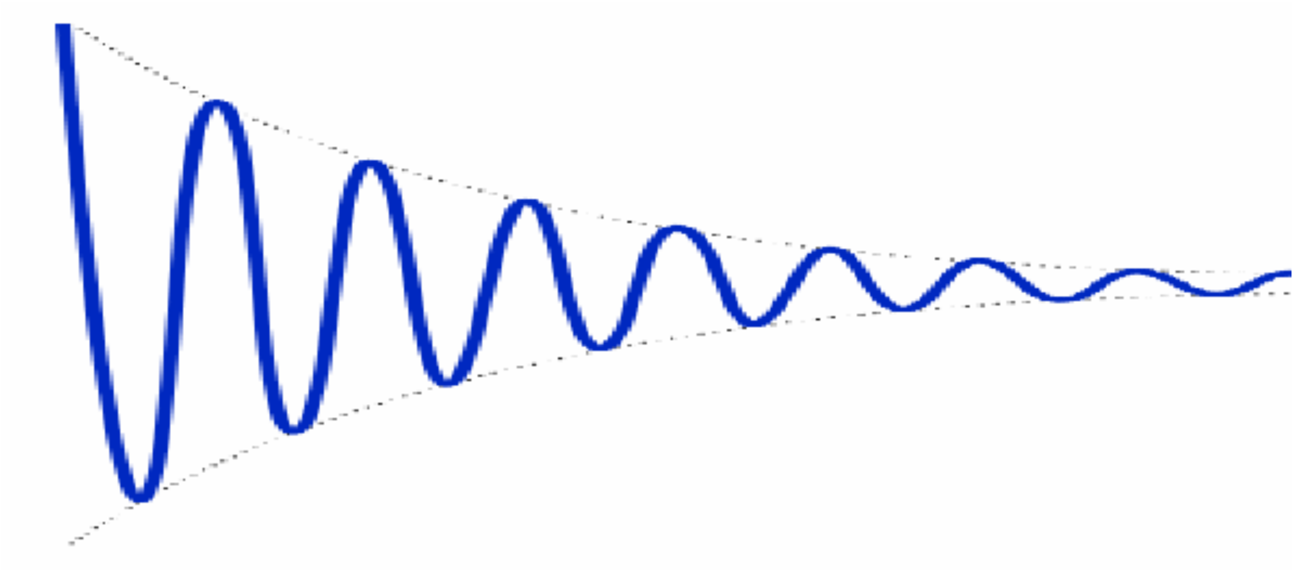




films with different anisotropy, roughly corresponding to the initially defined ones.

Summary:

- not obvious experimentally, lots of artefacts
- intrinsic versus extrinsic mechanisms
- phenomenology versus ab-initio



Functional derivative

From Wikipedia, the free encyclopedia

In the calculus of variations, a field of mathematical analysis, the **functional derivative** (or **variational derivative**)[1] relates a change in a functional to a change in a function that the functional depends on.

In the calculus of variations, functionals are usually expressed in terms of an integral of functions, their arguments, and their derivatives. In an integrand L of a functional, if a function f is varied by adding to it another function δf that is arbitrarily small, and the resulting integrand is expanded in powers of δf, the coefficient of δf in the first order term is called the functional derivative.

For example, consider the functional

$$J[f] = \int_a^b L[\,x, f(x), f\,'(x)\,]\,dx \ ,$$

where $f\,'(x) \equiv df/dx$. If f is varied by adding to it a function δf, and the resulting integrand $L(x, f + \delta f, f\,' + \delta f\,')$ is expanded in powers of δf, then the change in the value of J to first order in δf can be expressed as follows:[1][Note 1]

$$\delta J = \int_a^b \frac{\delta J}{\delta f(x)} \delta f(x)\,dx \,.$$

The coefficient of $\delta f(x)$, denoted as $\delta J/\delta f(x)$, is called the **functional derivative** of J with respect to f at the point x.[2] For this example functional, the functional derivative is the left hand side of the Euler-Lagrange equation,[3]

$$\frac{\delta J}{\delta f(x)} = \frac{\partial L}{\partial f} - \frac{d}{dx}\frac{\partial L}{\partial f'} \,.$$