Basic magnetostatic and field properties, units.

Michael Coey, *Trinity College Dublin, Ireland.*

Magnetostatics is the classical physics of the magnetic fields, forces and energies associated with distributions of magnetic material and steady electric currents. The concepts presented here underpin the magnetism of solids. The magnetic dipole moment **m** [Am^2] is the elementary magnetic quantity, and magnetization $\boldsymbol{M}(\boldsymbol{r})$ [Am^{-1}] is its mesoscopic volume average in condensed matter. The primary magnetic field is $\boldsymbol{B}$ [T], which appears in Maxwell's equation $.\boldsymbol{B} = 0$ that follows from the absence of any magnetic monopoles in Nature to act as field sources. Sources of $\boldsymbol{B}$ are electric currents, and magnetic material, which may be assimilated to a distribution of atomic currents. Unlike conduction currents $\boldsymbol{j}_c$ [Am^{-2}] the atomic currents responsible for atomic-scale magnetism $\boldsymbol{j}_m$ cannot be measured directly. Hence an indispensable second magnetic field $\boldsymbol{H}$ [Am^{-1}], defined by $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$, is introduced when dealing with magnetic or superconducting material. In static conditions where there is no time-varying electric field, $\nabla \times \boldsymbol{H} = \boldsymbol{j}_c$. It is the local distribution $\boldsymbol{H}(\boldsymbol{r})$ that determines the equilibrium distribution $\boldsymbol{M}(\boldsymbol{r})$; the atomic currents cannot act on themselves. B and H or M have different units and dimensions, and in SI, μ_0 is *defined* as $4\pi\ 10^{-7}$ T/(Am^{-1}).
The field produced by a given distribution of magnetization can be calculated by integrating the dipole field due to each volume element $\boldsymbol{M}(\boldsymbol{r})dV$, or using equivalent distributions of electric currents or magnetic charge. Magnetic charge q_m [Am] are fictional positive and negative monopoles, which offer the most convenient way of calculating $\boldsymbol{H}$. Magnetic scalar and vector potentials ϕ_m [A] and $\boldsymbol{A}$ [Tm] are defined for $\boldsymbol{H}$ and $\boldsymbol{B}$, respectively, the former only when $\boldsymbol{j}_c = 0$. Boundary conditions for the fields and potentials will be discussed.
Internal, external and demagnetizing fields are distinguished. The *H*-field produced by a magnetized body is called the stray field outside the body, and the demagnetizing field inside. The internal field may be defined on a mesoscopic or a macroscopic scale, the former in terms of the tensor relation $H_i = -\mathcal{N}_{ij} M_j$; the latter approximately in terms of a demagnetizing factor $1 \geq \mathcal{N} \geq 0$. The interaction of the magnetization with the demagnetizing field gives rise to shape anisotropy.
Magnetic forces, torques and energies are related to magnetization and external field; the thermodynamics of magnetic materials will be outlined. The apparent paradox that a magnetic field can do no work because the Lorentz force density $\boldsymbol{F}_L = \boldsymbol{j} \times \boldsymbol{B}$ [Nm^{-3}] is always perpendicular to $\boldsymbol{j}$ and cannot therefore change its magnitude will be discussed. All these concepts will be developed with examples intended to reinforce an understanding of the numerical magnitudes of the quantities involved, and an ability to calculate them. There will be some reference to spin-polarized currents.
The units and dimensions of all the quantities normally encountered in magnetism will be summarized, based on the units of mass, length, time and electric current (kg, m, s, A). Conversion to and from cgs will be mentioned, but the defects of the cgs system, which stem from the choice of μ_0 as a dimensional constant numerically equal to 1, and units of current, potential and resistance that do not feature on instruments in the Keithley catalogue, will be evident.

Background Reading;
J. M. D. Coey, *Magnetism and Magnetic Materials,* Cambridge University Press 2010, Chapter 2; Appendix B.