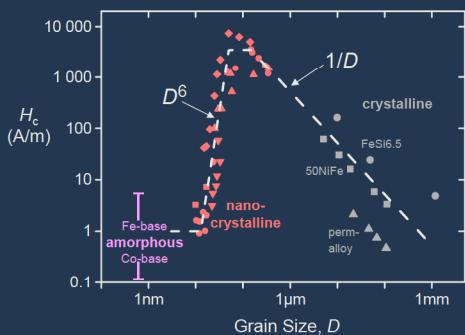




# Random Anisotropy Model

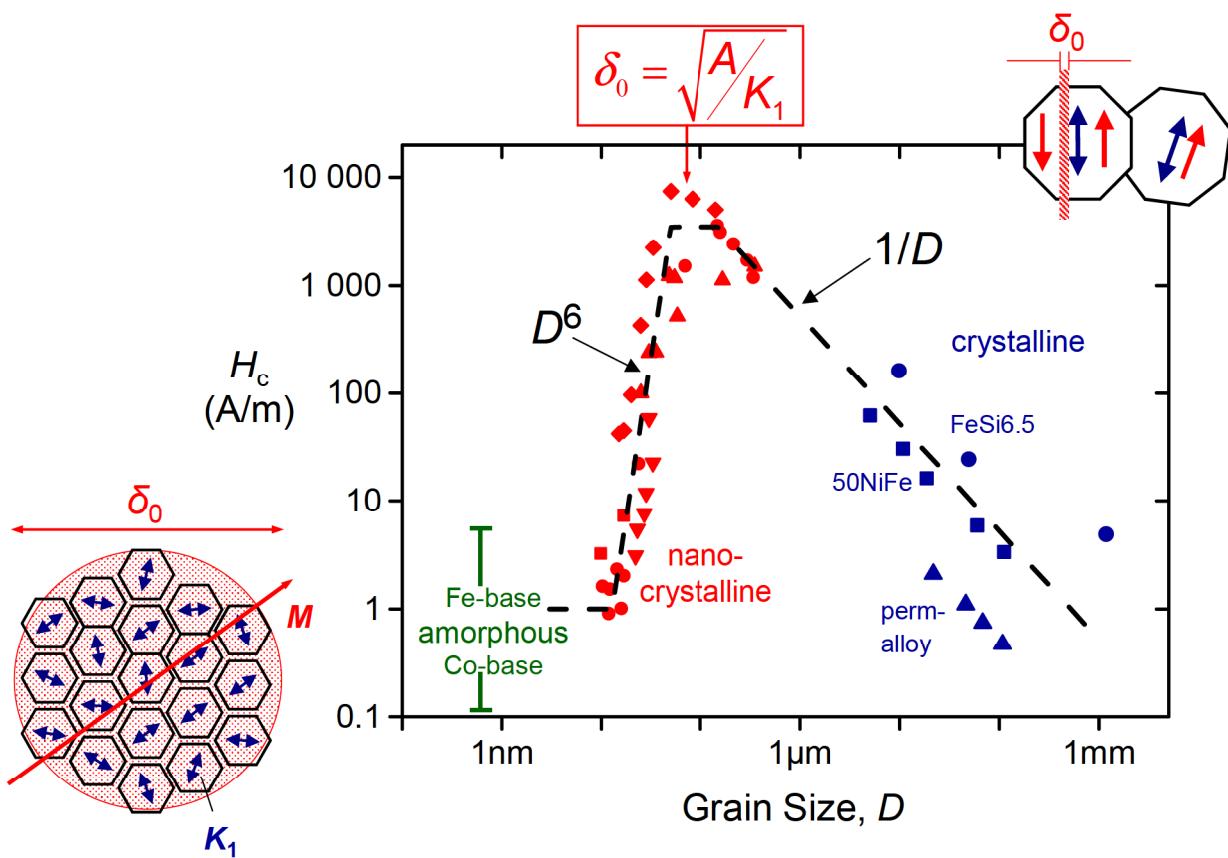
*Focus: averaging anisotropies*

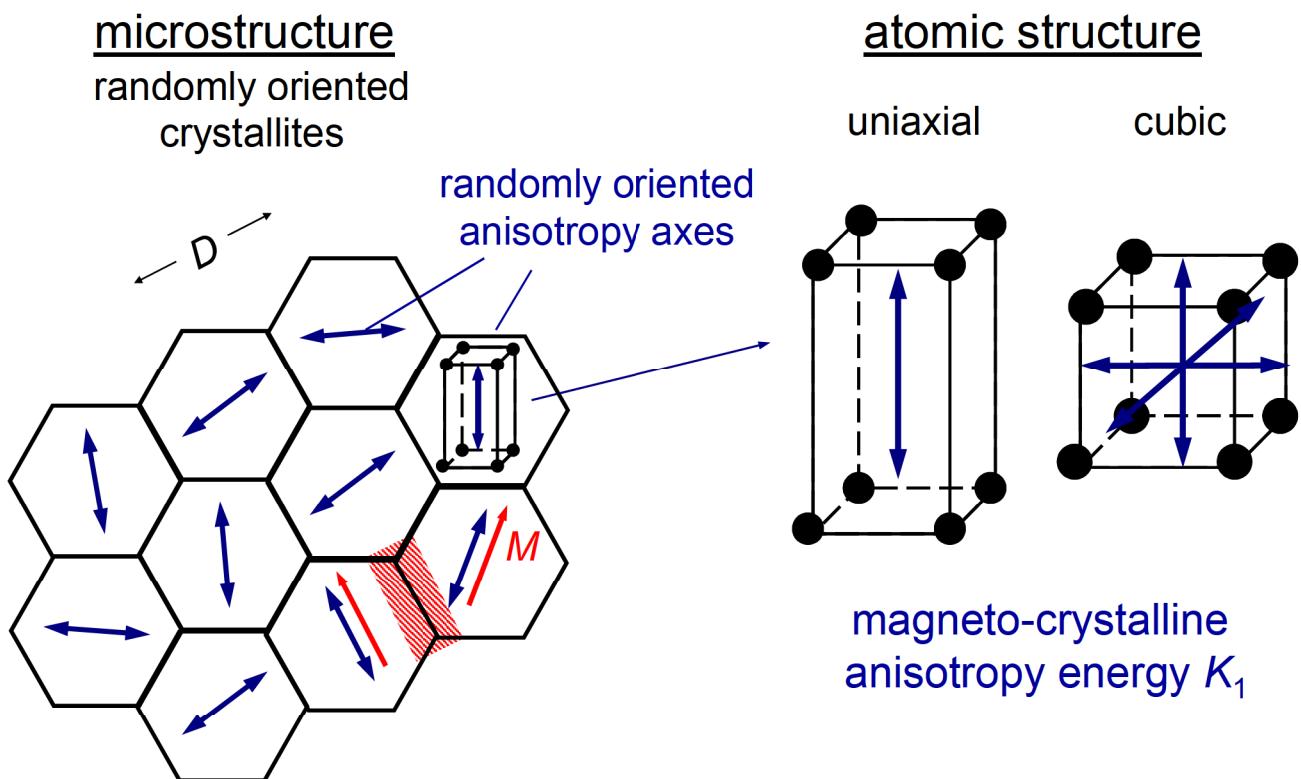
Giselher Herzer



Rapid Solidification Technology  
VACUUMSCHMELZE GmbH & Co. KG, D-63450 Hanau, Germany

## Magnetism and Microstructure

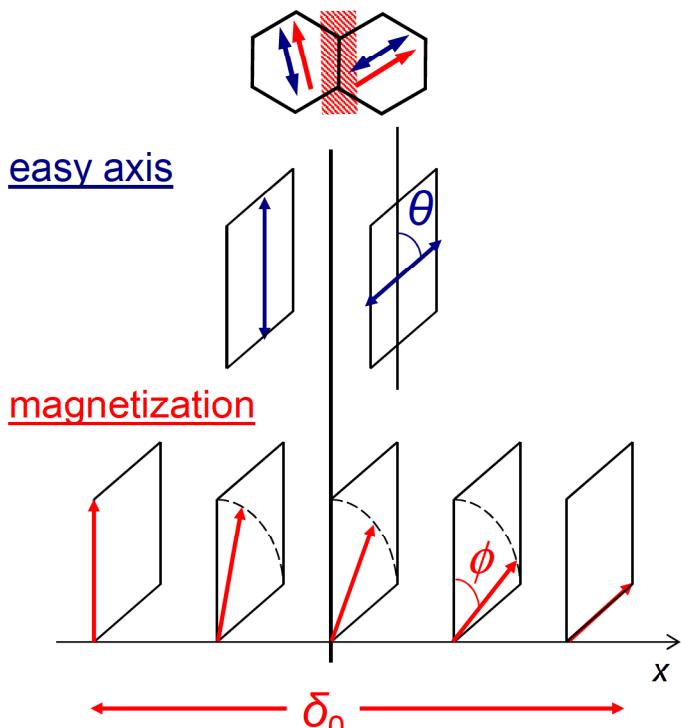




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## Anisotropy Energy versus Exchange Interaction



ferromagnetic correlation length  
minimum scale for appreciable  
variations of magnetization

• free energy density

$$e = \underbrace{A(\partial\phi/\partial x)^2}_{\text{exchange}} + \underbrace{K \sin^2(\phi - \theta)}_{\text{anisotropy}}$$

$$= K \left( \delta_0^2 (\partial\phi/\partial x)^2 + \sin^2(\phi - \theta) \right)$$

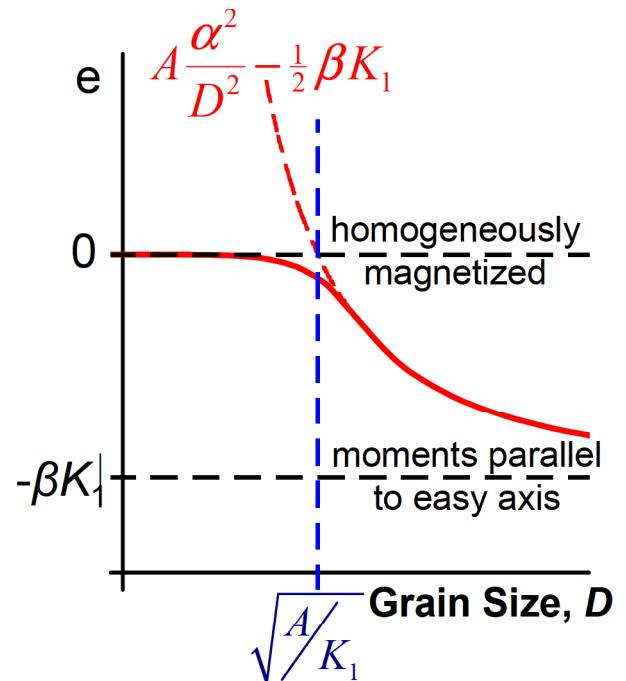
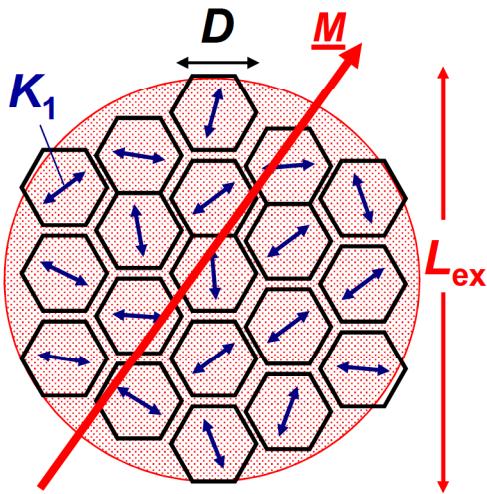
$$\delta_0 := \sqrt{A/K}$$

Example:  $\alpha\text{-Fe}_{80}\text{Si}_{20}$

$$K_1 = 8 \text{ kJ/m}^3 \rightarrow \delta_0 = 35 \text{ nm}$$

$$A = 10^{-11} \text{ J/m}$$

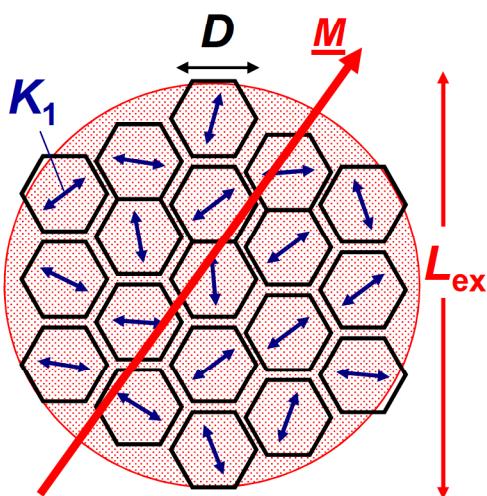
# The Random Anisotropy Model



free energy density:

$$e = \underbrace{A(\nabla \vec{m})^2}_{\text{exchange}} - \underbrace{\frac{1}{2}K_1 f(\vec{m} \cdot \vec{u}_r)}_{\text{anisotropy}}$$

$$\approx A \frac{\alpha^2}{L_{ex}^2} - \frac{1}{2}\beta \frac{|K_1|}{\sqrt{N}} \quad N = \left(\frac{L_{ex}}{D}\right)^3$$



$$\frac{\partial e}{\partial L_{ex}} = 0 \Rightarrow 0 = -2 \frac{\alpha^2 A}{L_{ex}^3} + \frac{3}{4} \frac{\beta K_1 D^{3/2}}{L_{ex}^{5/2}}$$

$$0 = -2 \frac{\alpha^2 A}{L_{ex}^3} + \frac{3}{4} \frac{\beta K_1}{\sqrt{N} L_{ex}}$$

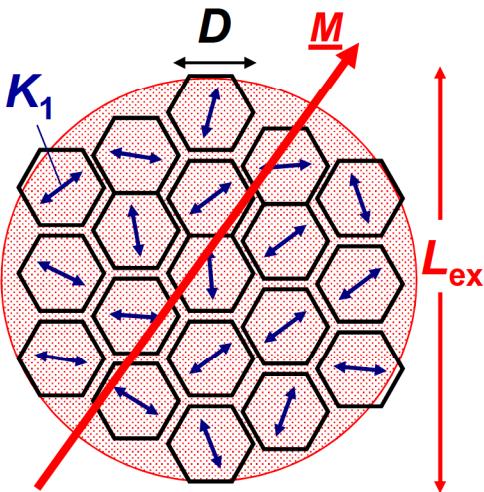
$$L_{ex} = \frac{\varphi_0}{\sqrt{\frac{8\alpha^2}{3\beta}}} \sqrt{\frac{A}{K_1/\sqrt{N}}} \quad \langle K_1 \rangle$$

free energy density:

$$e = \underbrace{A(\nabla \vec{m})^2}_{\text{exchange}} - \underbrace{\frac{1}{2}K_1 f(\vec{m} \cdot \vec{u}_r)}_{\text{anisotropy}}$$

$$\approx A \frac{\alpha^2}{L_{ex}^2} - \frac{1}{2}\beta \frac{|K_1|}{\sqrt{N}} \quad N = \left(\frac{L_{ex}}{D}\right)^3$$

# The Random Anisotropy Model



- average anisotropy constant

$$\langle K_1 \rangle = K_1 / \sqrt{N} \quad (1)$$

- number  $N$  of coupled grains

$$N = (L_{ex}/D)^3 \quad (2)$$

- exchange length (renormalized)

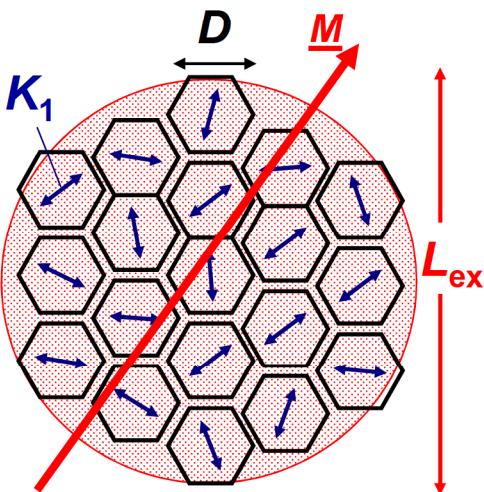
$$L_{ex} = \varphi_0 \sqrt{A/\langle K_1 \rangle} \quad (3)$$

Eqs. (2) in (1)  $\Rightarrow \langle K_1 \rangle = K_1 \left( \frac{D}{L_{ex}} \right)^{3/2} = K_1 \left( \frac{D}{\varphi_0 \sqrt{A/\langle K_1 \rangle}} \right)^{3/2}$

Resolve for  $\langle K_1 \rangle \Rightarrow \langle K_1 \rangle = \frac{K_1^4 D^6}{\varphi_0^6 A^3} = K_1 \frac{D^6}{\left( \varphi_0 \sqrt{A/K_1} \right)^6}$  **basic exchange length**

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# The Random Anisotropy Model



- average anisotropy constant

$$\langle K_1 \rangle = K_1 / \sqrt{N} \quad (1)$$

- number  $N$  of coupled grains

$$N = (L_{ex}/D)^3 \quad (2)$$

- exchange length (renormalized)

$$L_{ex} = \varphi_0 \sqrt{A/\langle K_1 \rangle} \quad (3)$$

Combination  
of Eqs. (1) - (3)  $\Rightarrow$

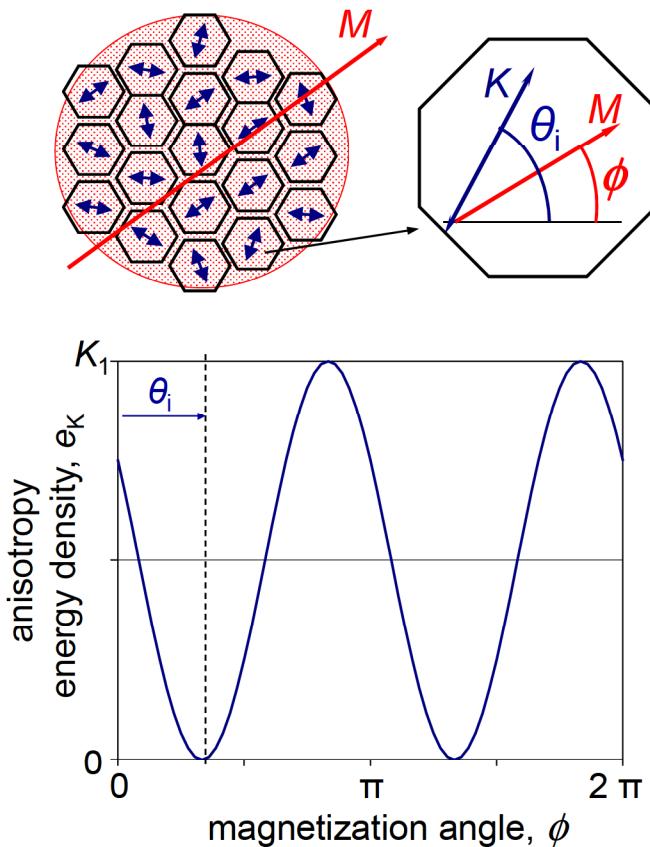
$$\langle K_1 \rangle = K_1 (D/L_0)^6$$

$$L_{ex} = L_0 (L_0/D)^3$$

$$L_0 = \varphi_0 \sqrt{A/K_1}$$

**basic** exchange length

# Averaging Random Anisotropies



anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_i \cdot \sin^2(\phi - \theta_i)$$

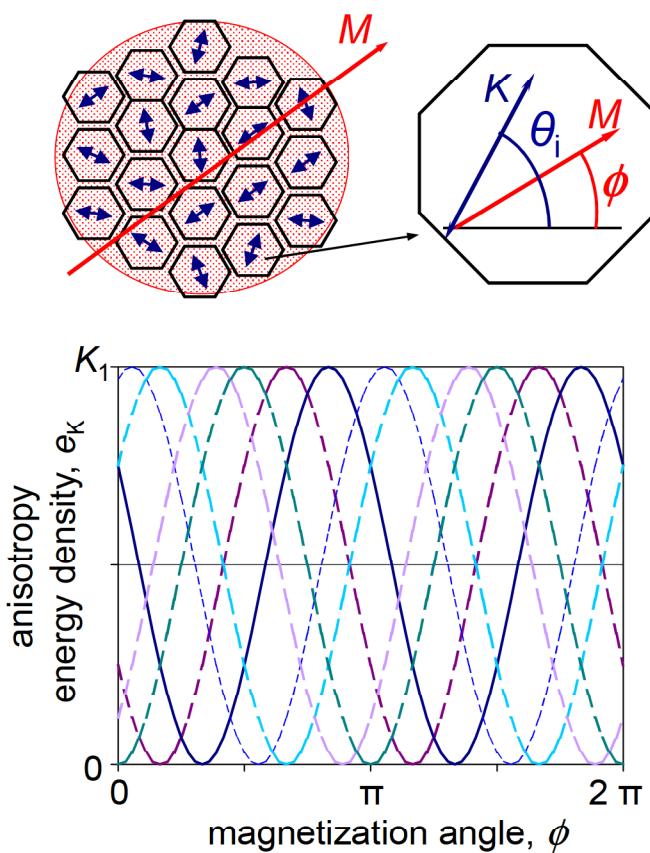
magnetization  
constant orientation  $\phi$

anisotropy  
random orientation  $\theta$

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# Averaging Random Anisotropies



anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_i \cdot \sin^2(\phi - \theta_i)$$

magnetization  
constant orientation  $\phi$

anisotropy  
random orientation  $\theta$

Open  
**Simu-Anisotropy.xls**

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# Conclusions Simulation 1 (Blackboard)

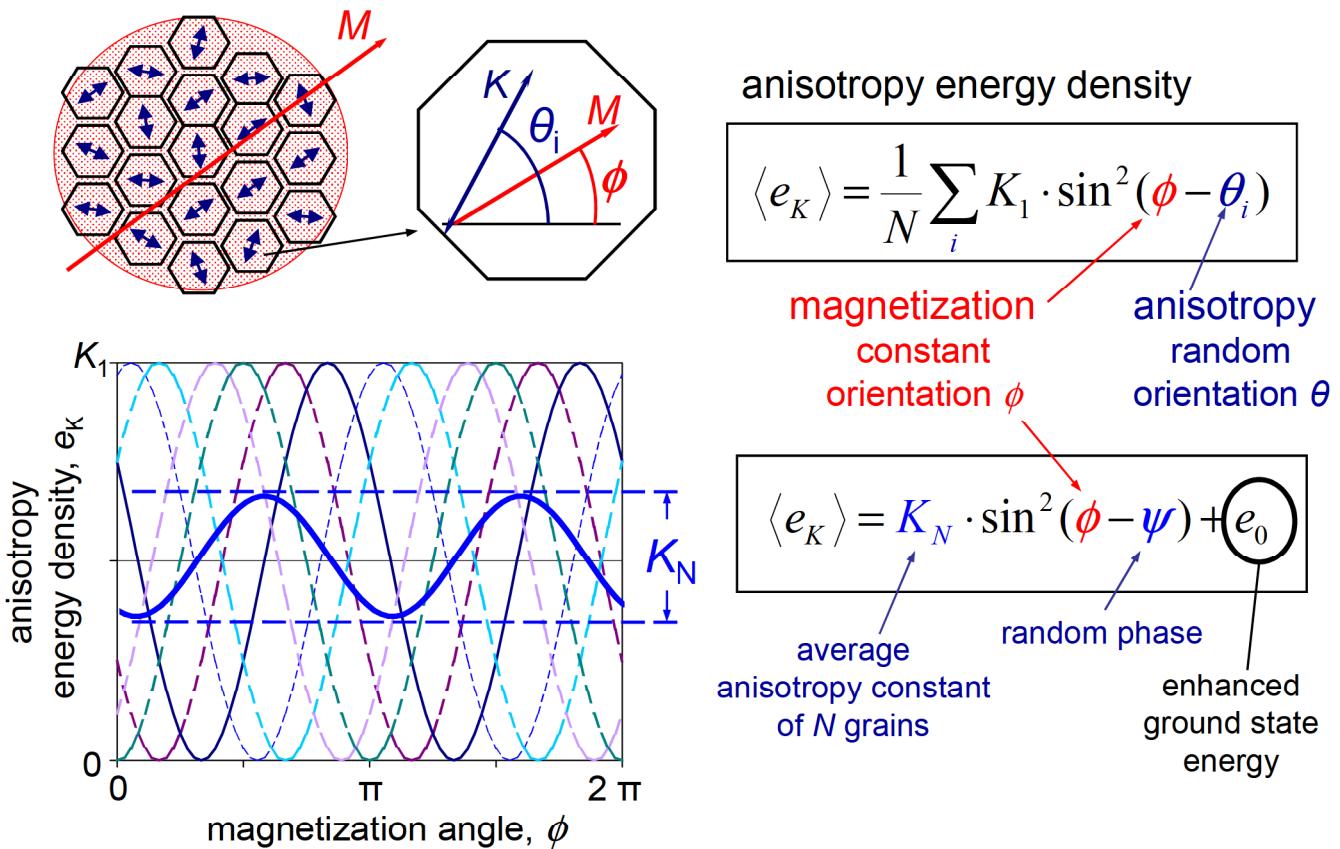
$$\begin{aligned}\langle e_K \rangle &= \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i) \\ &= K_N \cdot \sin^2(\phi - \psi) + e_0\end{aligned}$$

- anisotropy constant  $K_N$  is reduced
- $K_N$  and phase are fluctuating
- ground state energy enhanced

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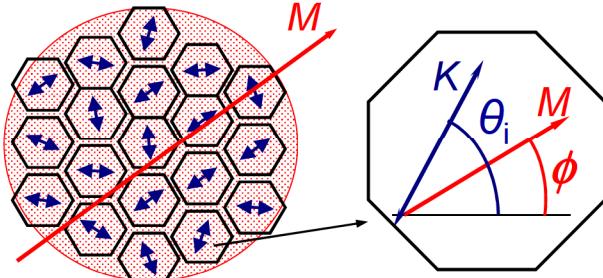
## Averaging Random Anisotropies



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## Averaging Random Anisotropies

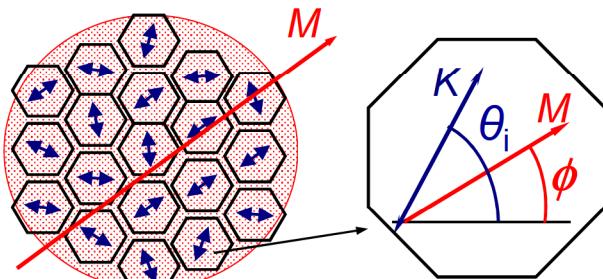


anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

$$\frac{1}{N} \sum_{i=1}^N \sin^2(\phi - \theta_i) = \frac{1}{2} (1 - \cos 2(\phi - \theta_i)) \\ = \frac{1}{2} (1 - (\cos \phi \cos \theta_i + \sin \phi \sin \theta_i))$$

## Averaging Random Anisotropies

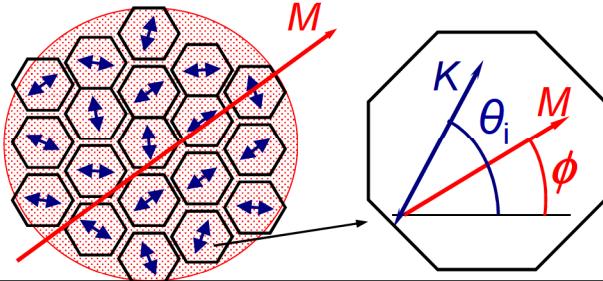


anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

$$\frac{1}{N} \sum_{i=1}^N \sin^2(\phi - \theta_i) = \frac{1}{2} \left( 1 - \frac{1}{N} \sum_{i=1}^N (\cos 2\phi \cos 2\theta_i + \sin 2\phi \sin 2\theta_i) \right) \\ = \frac{1}{2} \left( 1 - \left( \underbrace{\cos 2\phi \frac{1}{N} \sum_{i=1}^N \cos 2\theta_i}_{k_N \cos 2\psi_N} + \underbrace{\sin 2\phi \frac{1}{N} \sum_{i=1}^N \sin 2\theta_i}_{k_N \sin 2\psi_N} \right) \right) \\ k_N \sin^2(\phi - \psi_N) = \frac{1}{2} \left( k_N - \left( \underbrace{\cos 2\phi k_N \cos 2\psi_N}_{k_N \cos 2\psi_N} + \underbrace{\sin 2\phi k_N \sin 2\psi_N}_{k_N \sin 2\psi_N} \right) \right)$$

## Averaging Random Anisotropies



anisotropy energy density

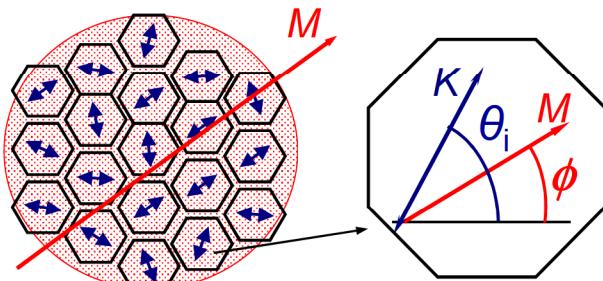
$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

$$\frac{1}{N} \sum_{i=1}^N \sin^2(\phi - \theta_i) = k_N \sin^2(\phi - \psi_N) + \frac{1}{2}(1 - k_N)$$

$$k_N \cos 2\psi_N = \frac{1}{N} \sum_{i=1}^N \cos 2\theta_i$$

$$k_N \sin 2\psi_N = \frac{1}{N} \sum_{i=1}^N \sin 2\theta_i$$

## Averaging Random Anisotropies



anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

$$\frac{1}{N} \sum_{i=1}^N \sin^2(\phi - \theta_i) = k_N \sin^2(\phi - \psi_N) + \frac{1}{2}(1 - k_N)$$

$$k_N = \frac{1}{N} \sqrt{\left( \sum_{i=1}^N \cos \theta_i \right)^2 + \left( \sum_{i=1}^N \sin \theta_i \right)^2}$$

$$\tan 2\psi_N = \frac{\sum_{i=1}^N \sin 2\theta_i}{\sum_{i=1}^N \cos 2\theta_i}$$

# Averaging Random Anisotropies

Function kN(ByVal N) As Double

'-----  
'returns the average anisotropy constant Kn  
'of N units with randomly oriented anisotropy axis  
'-----

Dim cPi As Double, Theta As Double  
Dim sum1 As Double, sum2 As Double

'initiate variables

cPi = 4# \* Atn(1#) 'the number PI  
sum1 = 0#: sum2 = 0#

'sum over n randomly oriented units

For i = 1 To N  
Theta = 2 \* Rnd \* cPi 'random phase between 0 and 2PI  
sum1 = sum1 + Cos(Theta)  
sum2 = sum2 + Sin(Theta)

Next

'calculate and return final result

kN = Sqr(sum1 \* sum1 + sum2 \* sum2) / N

End Function

$$k_N = \frac{1}{N} \sqrt{\left( \sum_{i=1}^N \cos \theta_i \right)^2 + \left( \sum_{i=1}^N \sin \theta_i \right)^2}$$

Open  
**RAM2DSimu.xls**

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## Conclusions Simulation 2

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

$$= K_N \cdot \sin^2(\phi - \psi) + e_0$$

$$K_N = \frac{K_1}{N} \sqrt{\left( \sum_{i=1}^N \cos \theta_i \right)^2 + \left( \sum_{i=1}^N \sin \theta_i \right)^2}$$

- $K_N$  is reduced as  $1/N^{1/2}$
- $K_N$  is fluctuating
- statistical deviation is in the order of  $K_N$  itself

# Separate average and statistical fluctuations

$$k_N = \frac{1}{N} \sqrt{\left( \sum_{i=1}^N \cos \theta_i \right)^2 + \left( \sum_{i=1}^N \sin \theta_i \right)^2}$$

convenient  
for numerics

$$= \frac{1}{N} \sqrt{\sum_{i=1}^N \sum_{j=1}^N (\cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j)}$$

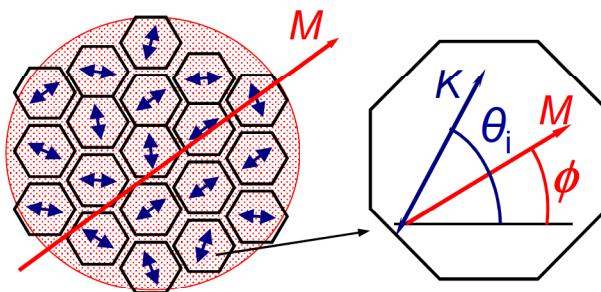
$$= \frac{1}{N} \sqrt{\sum_{i=1}^N \sum_{j=1}^N \cos 2(\theta_i - \theta_j)}$$

$$= \frac{1}{N} \sqrt{N + \sum_{i=1}^N \sum_{j \neq i}^N \cos 2(\theta_i - \theta_j)}$$

$$= \frac{1}{\sqrt{N}} \sqrt{1 + \frac{1}{N} \sum_{i=1}^N \sum_{j \neq i}^N \cos 2(\underbrace{\theta_i - \theta_j}_{\text{random phases}})}$$

more convenient  
for analytical discussion

## Averaging Random Anisotropies

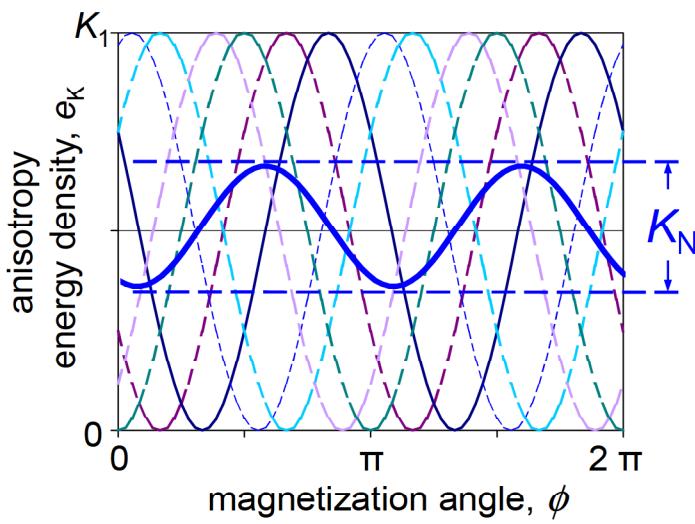


anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

magnetization  
constant  
orientation  $\phi$

anisotropy  
random  
orientation  $\theta$

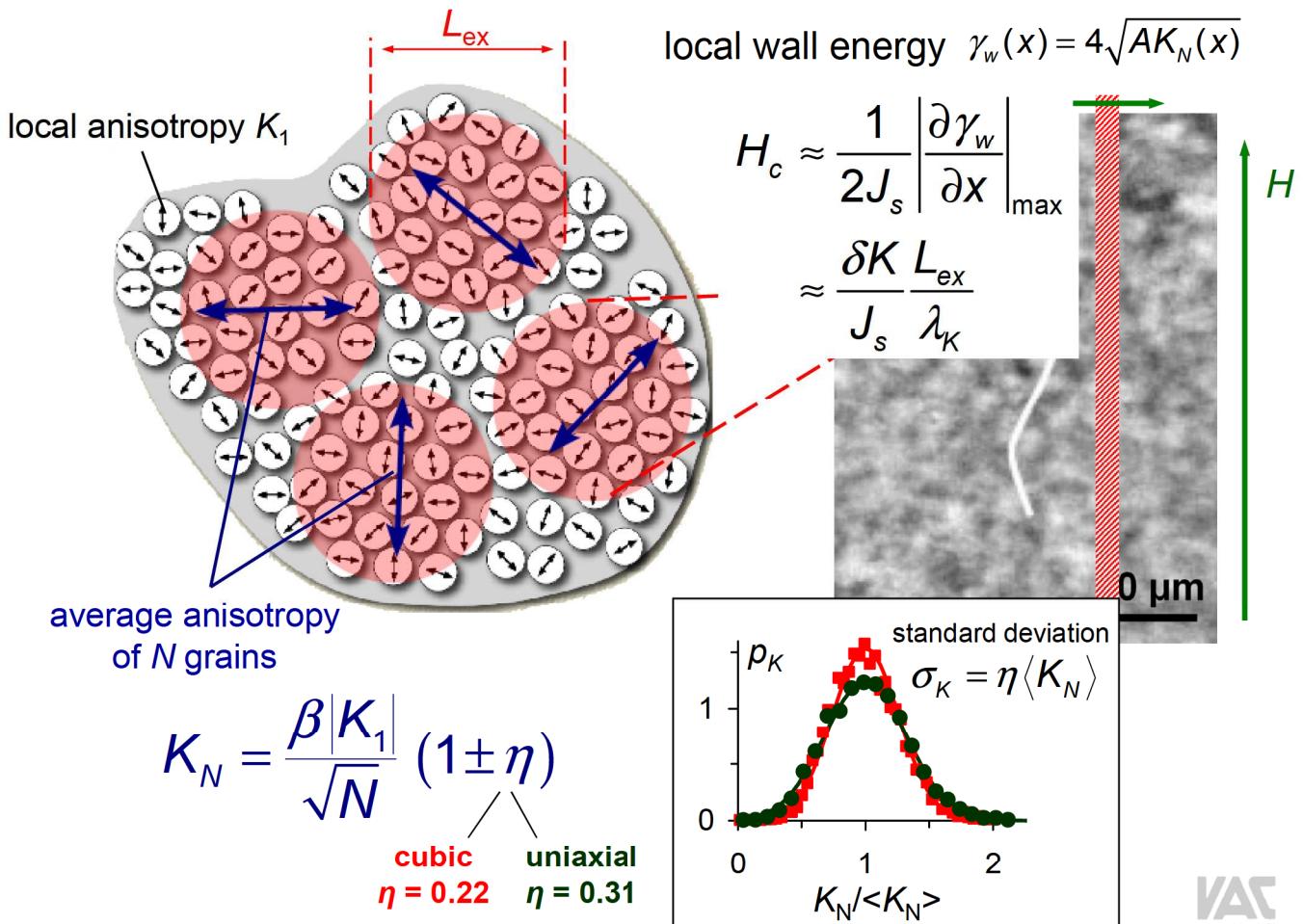


$$\langle e_K \rangle = K_N \cdot \sin^2(\phi - \psi) + e_0$$

random phases  $\frac{1}{2}(K_1 - K_N)$

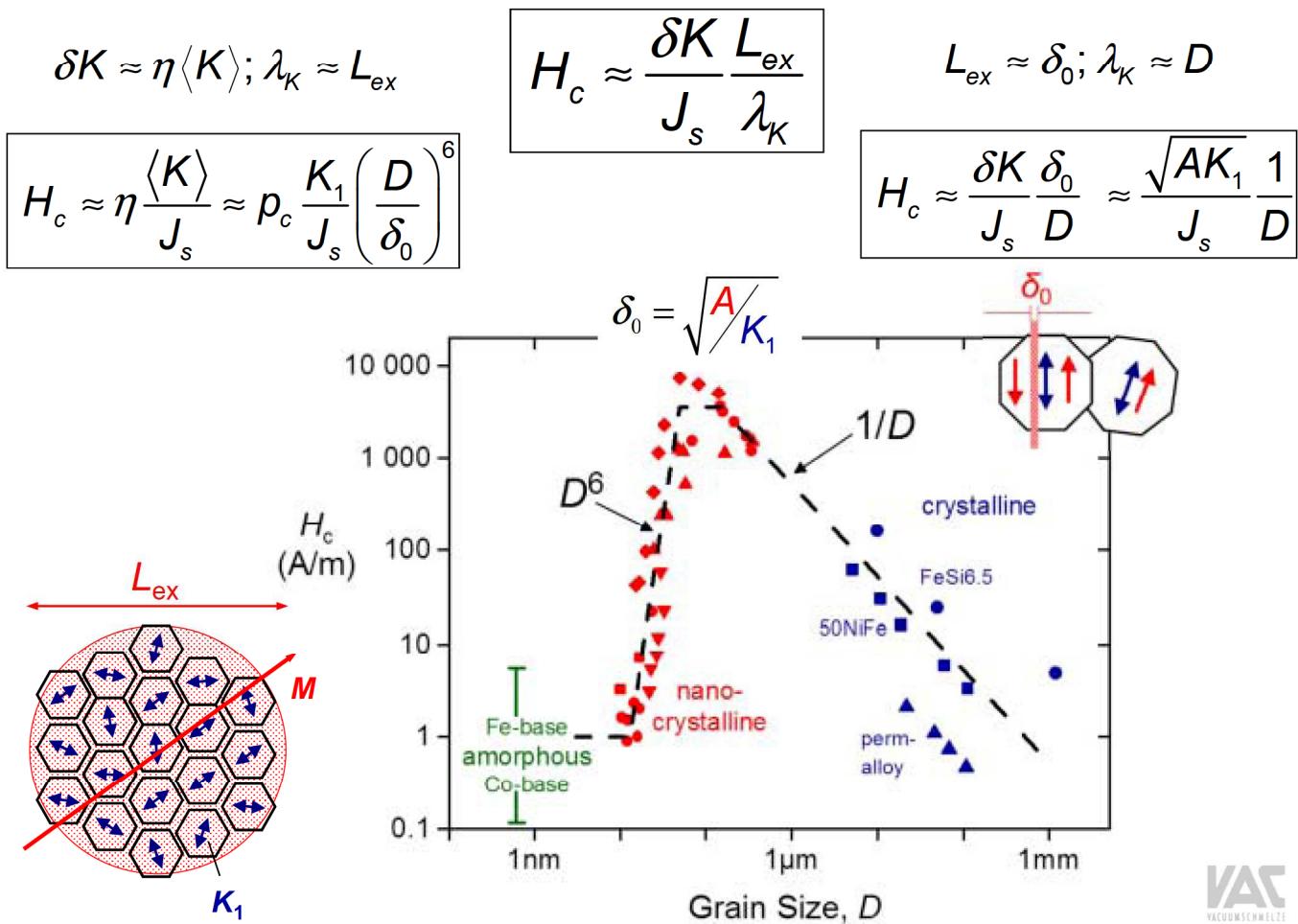
$$\frac{K_1}{\sqrt{N}} \cdot \sqrt{1 + \frac{1}{N} \sum_{i,j=1}^N \cos(2(\theta_i - \theta_j))}$$

# Anisotropy Fluctuations



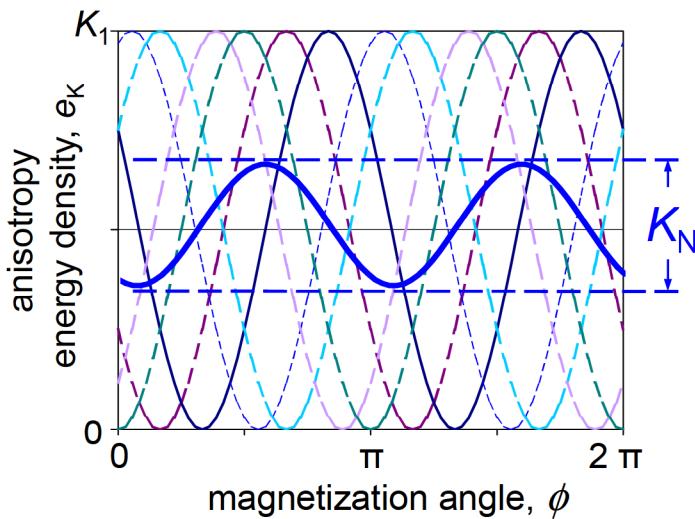
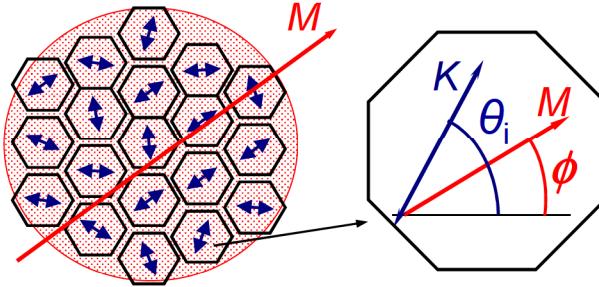
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# Magnetism and Microstructure



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# Averaging Random Anisotropies



anisotropy energy density

$$\langle e_K \rangle = \frac{1}{N} \sum_i K_1 \cdot \sin^2(\phi - \theta_i)$$

magnetization

constant orientation  $\phi$

anisotropy

random orientation  $\theta$

$$\langle e_K \rangle = K_N \cdot \sin^2(\phi - \psi) + e_0$$

random phases  $\frac{1}{2}(K_1 - K_N)$

$$\frac{K_1}{\sqrt{N}} \cdot \sqrt{1 + \frac{1}{N} \sum_{i,j=1}^N \cos(2(\theta_i - \theta_j))}$$

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# Averaging Random Anisotropies

- average

$$\begin{aligned} \langle k_N \rangle &= \left\langle \frac{1}{\sqrt{N}} \sqrt{1 + \frac{1}{N} \sum_{i,j=1}^N \cos 2(\theta_i - \theta_j)} \right\rangle \\ &= \frac{1}{\sqrt{N}} \left( 1 + \frac{1}{2} \frac{1}{N} \sum_{i,j=1}^N \langle \cos 2(\theta_i - \theta_j) \rangle - \frac{1}{8} \underbrace{\frac{1}{N^2} \sum_{i,j=1}^N \sum_{k,l=1}^N \langle \cos 2(\theta_i - \theta_j) \cos 2(\theta_k - \theta_l) \rangle}_{\frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk})} \pm \dots \right) \\ &= \frac{1}{\sqrt{N}} \left( 1 - \frac{1}{8} \pm \dots \right) \approx \frac{0.875}{\sqrt{N}} \end{aligned}$$

- standard deviation

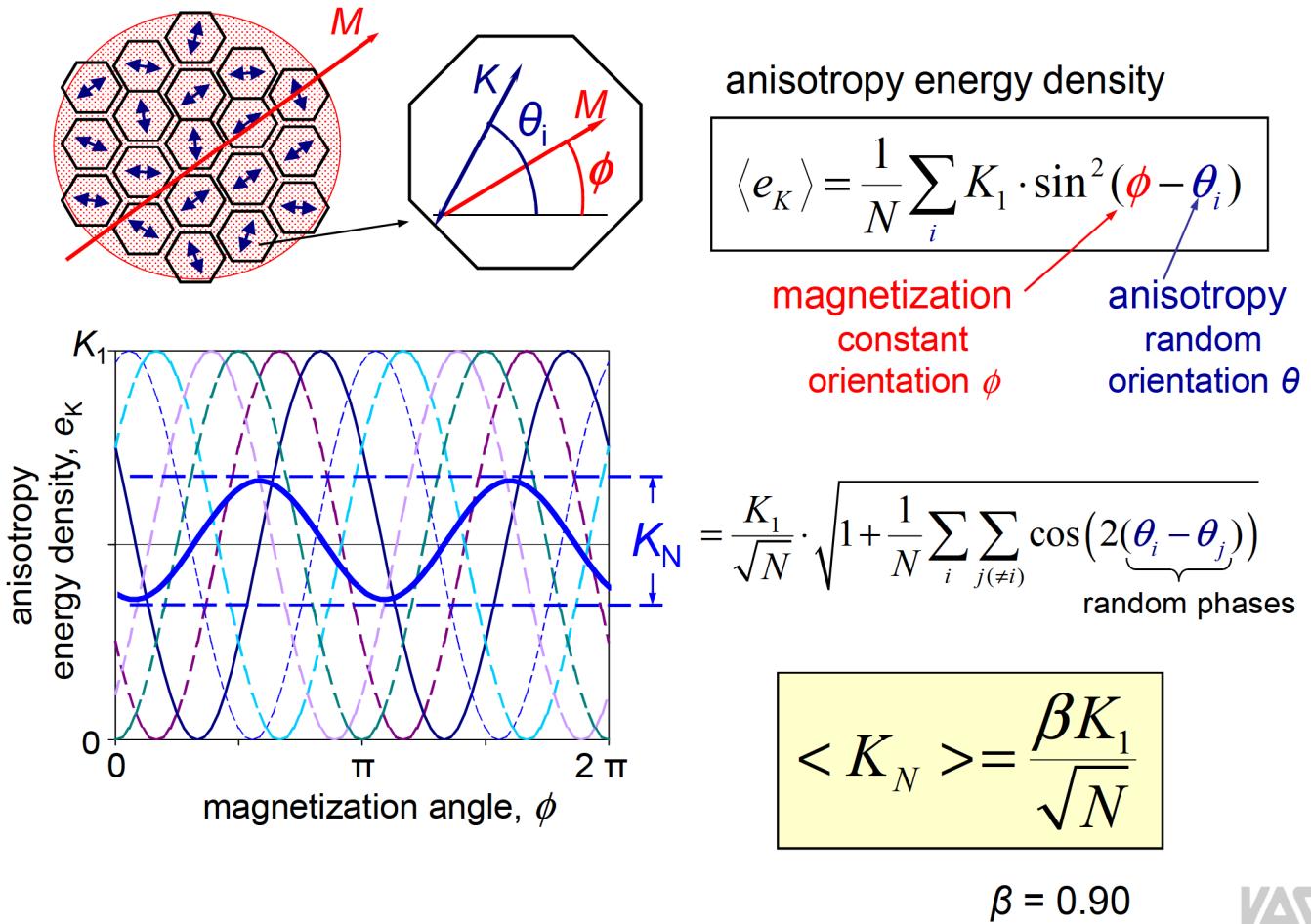
$$\begin{aligned} \sigma_K &= \sqrt{\langle k_N^2 \rangle - \langle k_N \rangle^2} \\ \langle k_N^2 \rangle &= \frac{1}{N} \left( 1 + \frac{1}{N} \sum_{i,j=1}^N \langle \cos 2(\theta_i - \theta_j) \rangle \right) \end{aligned}$$

$$\langle k_N \rangle = \frac{\beta}{\sqrt{N}} \quad \beta \approx 0.875 \quad (0.90_{\pm 0.04})^{(*)}$$

$$\begin{aligned} \sigma_K &= \eta \langle k_N \rangle \quad \eta = \sqrt{1 - \beta^2} \\ &\approx 0.484 \quad (0.50_{\pm 0.05})^{(*)} \end{aligned}$$

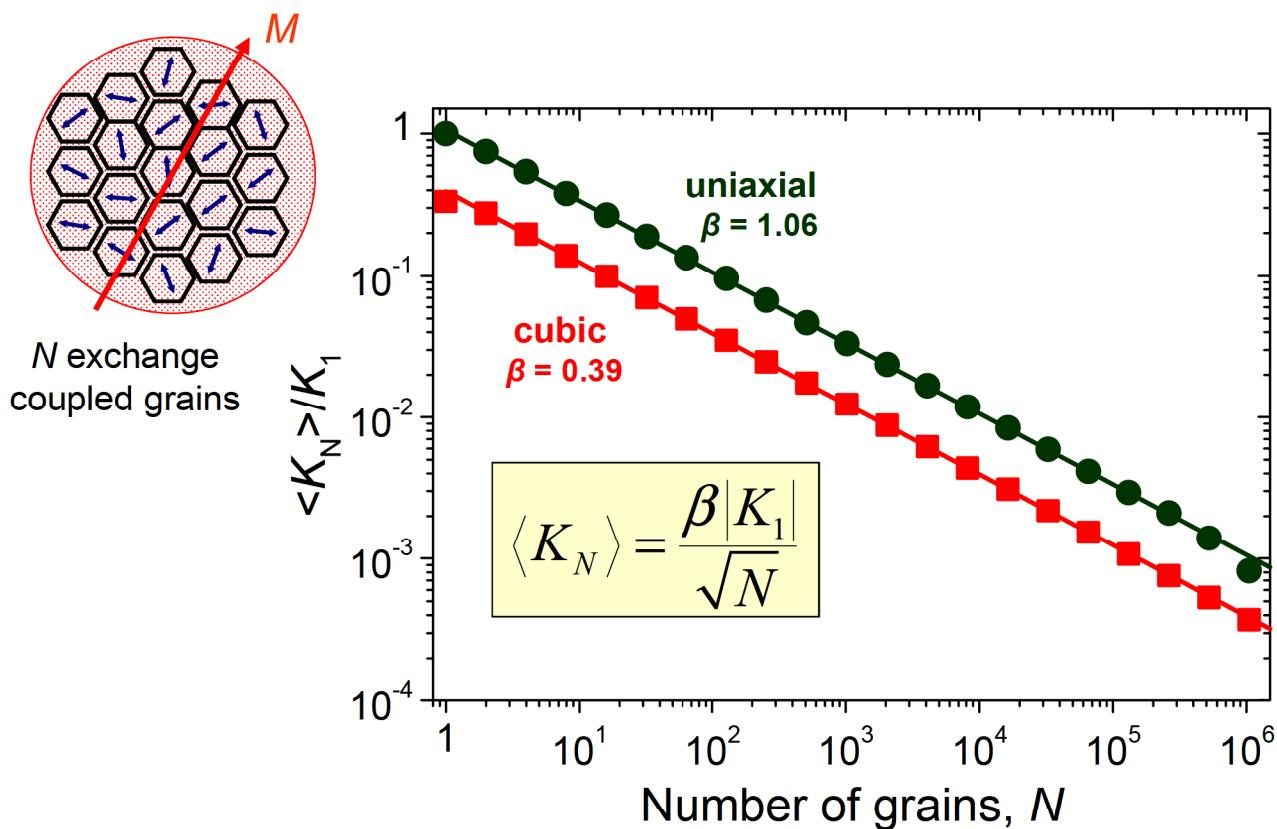
(\*) from numerical simulation

# Averaging Random Anisotropies



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## Averaged Anisotropy Constant (3D)

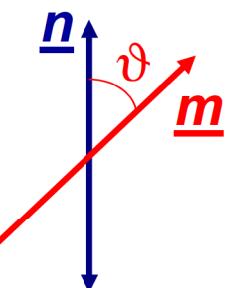
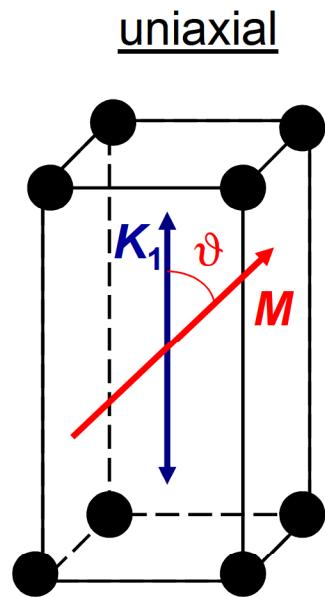


Note: for cubic anisotropy  $e_{\max} - e_{\min} = |K_1|/3$

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## Uniaxial Anisotropy (3D)



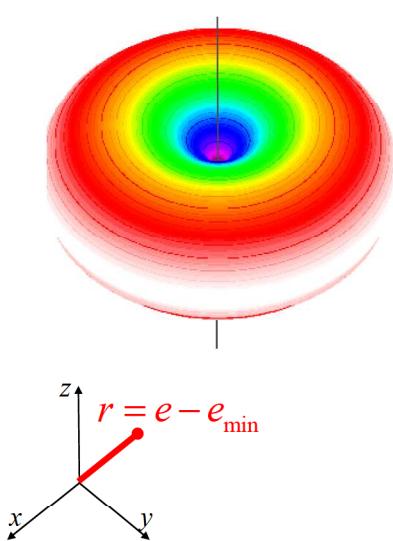
$$\begin{aligned} e_K &= K_1 \sin^2 \vartheta \\ &= K_1(1 - \cos^2 \vartheta) \\ &= K_1 - K_1(\underline{n} \cdot \underline{m})^2 \end{aligned}$$

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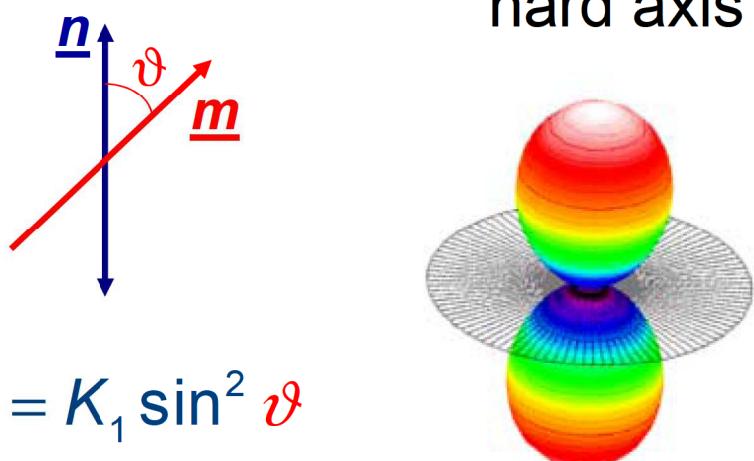
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## Uniaxial Anisotropy (3D)

$K_1 > 0$   
easy axis



$K_1 < 0$   
hard axis



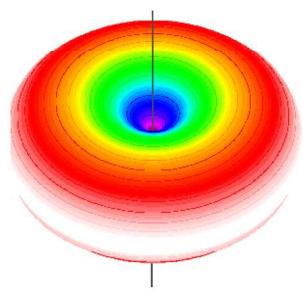
$$\begin{aligned} e_K &= K_1 \sin^2 \vartheta \\ &= K_1(1 - \cos^2 \vartheta) \\ &= K_1 - K_1(\underline{n} \cdot \underline{m})^2 \end{aligned}$$

28

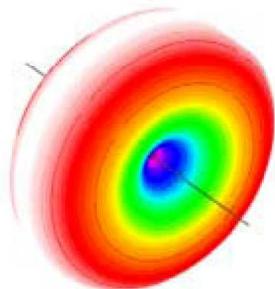
VAC  
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## Uniaxial Anisotropy (3D)

$$K_1 > 0$$

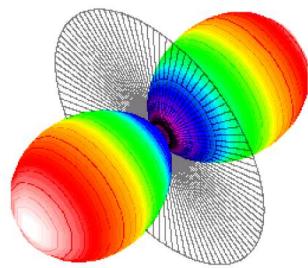


+



$$= \begin{matrix} z \\ x \\ y \end{matrix}$$

$$r = e - e_{\min}$$



$$\underbrace{-K_1(\underline{n}_z \cdot \underline{m})^2}_{m_z^2} + \underbrace{-K_1(\underline{n}_y \cdot \underline{m})^2}_{m_y^2} = -K_1 + K_1(\underline{n}_x \cdot \underline{m})^2$$

$$= 1 - (m_x^2 + m_z^2)$$

$\underbrace{\phantom{0}}$

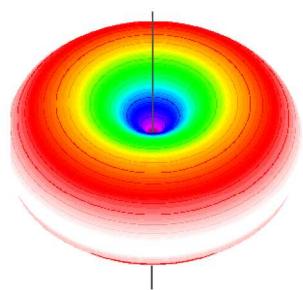
$$-K_1(1 - m_x^2)$$

29

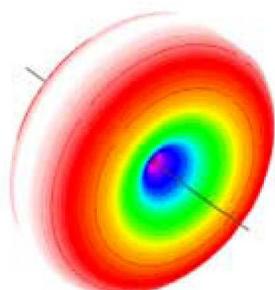
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## Uniaxial Anisotropy (3D)

$$K_1 > 0$$

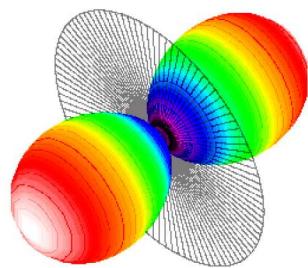


+



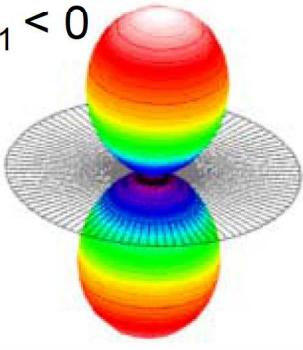
$$= \begin{matrix} z \\ x \\ y \end{matrix}$$

$$r = e - e_{\min}$$

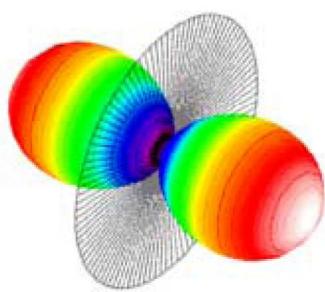


$$\underbrace{-K_1(\underline{n}_z \cdot \underline{m})^2}_{m_z^2} + \underbrace{-K_1(\underline{n}_y \cdot \underline{m})^2}_{m_y^2} = -K_1 + K_1(\underline{n}_x \cdot \underline{m})^2$$

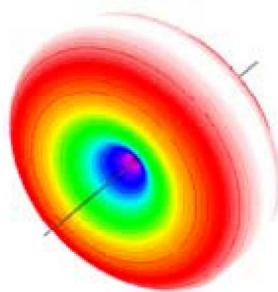
$$K_1 < 0$$



+



=



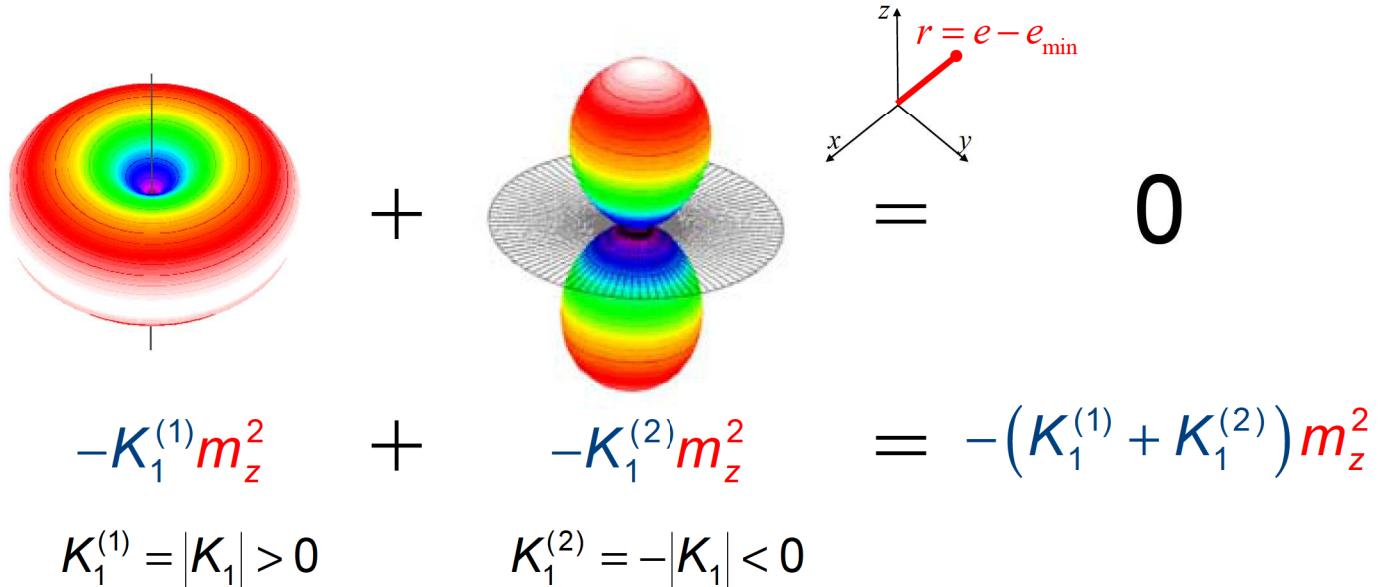
yz

xz

30

VAC  
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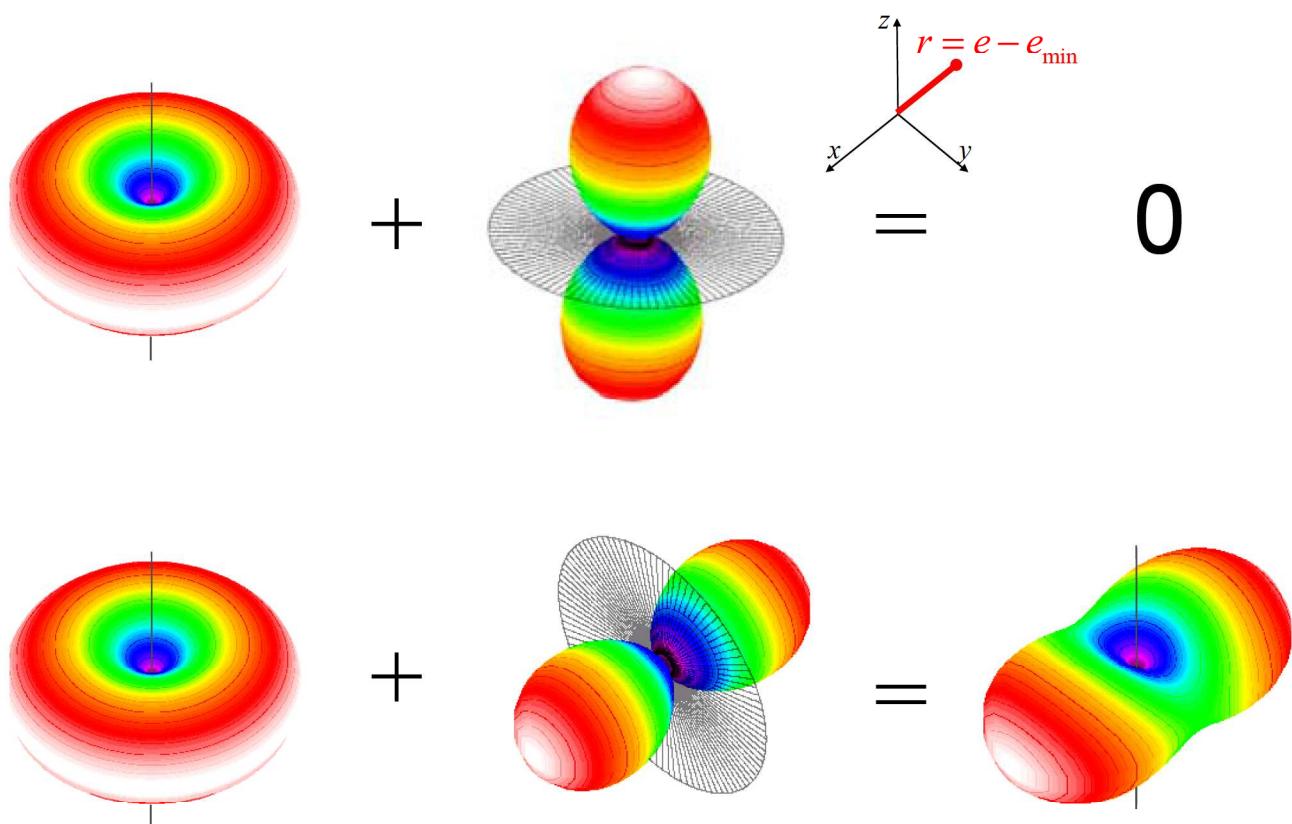
## Uniaxial Anisotropy (3D)



31



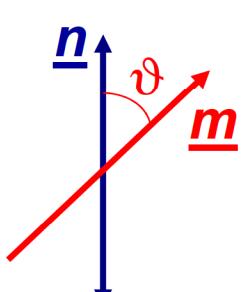
## Uniaxial Anisotropy (3D)



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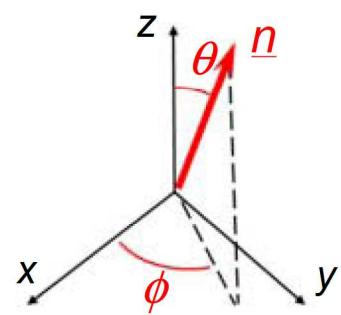
# Uniaxial Anisotropy (3D)



$$e_K = K_1 \sin^2 \vartheta$$

$$= K_1 - K_1 (\underline{n} \cdot \underline{m})^2$$

$$= \sum_{\alpha, \beta = x, y, z} K_{\alpha\beta} m_\alpha m_\beta$$



$$K_{\alpha\beta} = K_1 \begin{pmatrix} 1 - n_x n_x & -n_x n_y & -n_x n_z \\ -n_x n_y & 1 - n_y n_y & -n_y n_z \\ -n_x n_z & -n_y n_z & 1 - n_z n_z \end{pmatrix}$$

$n_x = \sin \theta \cos \phi$	$n_y = \sin \theta \sin \phi$	$n_z = \cos \theta$
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average anisotropy coefficients

$$\langle K_{\alpha\beta} \rangle_N = \frac{1}{N} \sum_{i=1}^N K_{\alpha\beta}(x_i)$$

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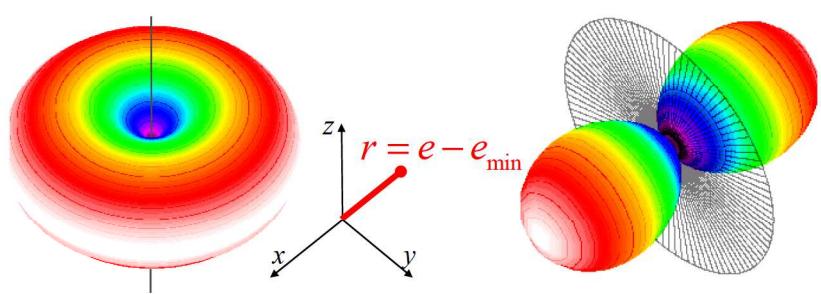
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# Random Uniaxial Anisotropy (3D)

$$e_K = \sum_{\alpha, \beta} \underbrace{\langle K_{\alpha\beta} \rangle_N}_{K_N} m_\alpha m_\beta$$

easy axis  $\frac{1}{3} \leq u_N \leq \frac{2}{3}$  easy plane

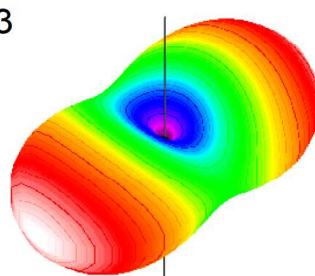
$$K_N \cdot \begin{pmatrix} u_N & 0 & 0 \\ 0 & 1 - 2u_N & 0 \\ 0 & 0 & u_N - 1 \end{pmatrix}$$



$$\langle K_N \rangle = \frac{\beta K_1}{\sqrt{N}}$$

$$\beta \approx 1.06$$

$$\langle u_N \rangle \approx 0.50$$

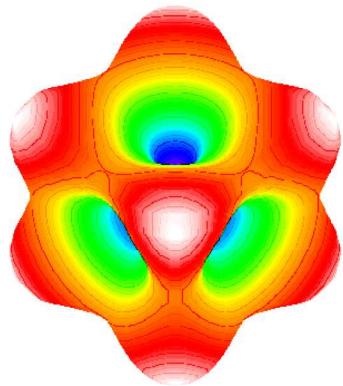


$u = 1/2$   
(random average)

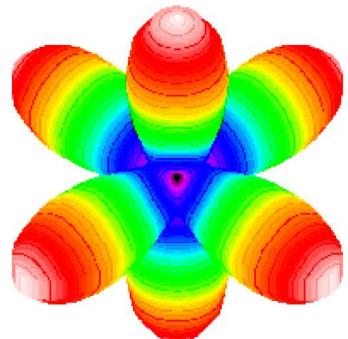
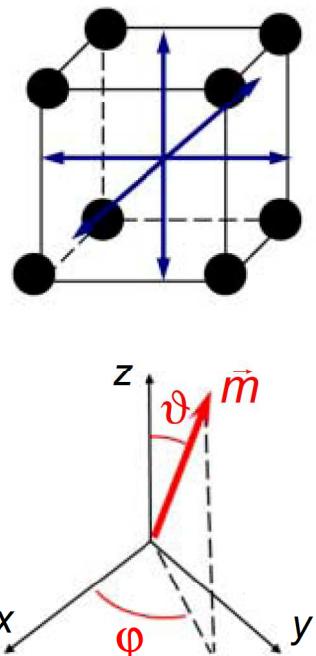
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# Cubic Anisotropy



$K_1 > 0$   
easy axes  
[100]



$K_1 < 0$   
easy axes  
[111]

$$e_K = K_1(m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2)$$

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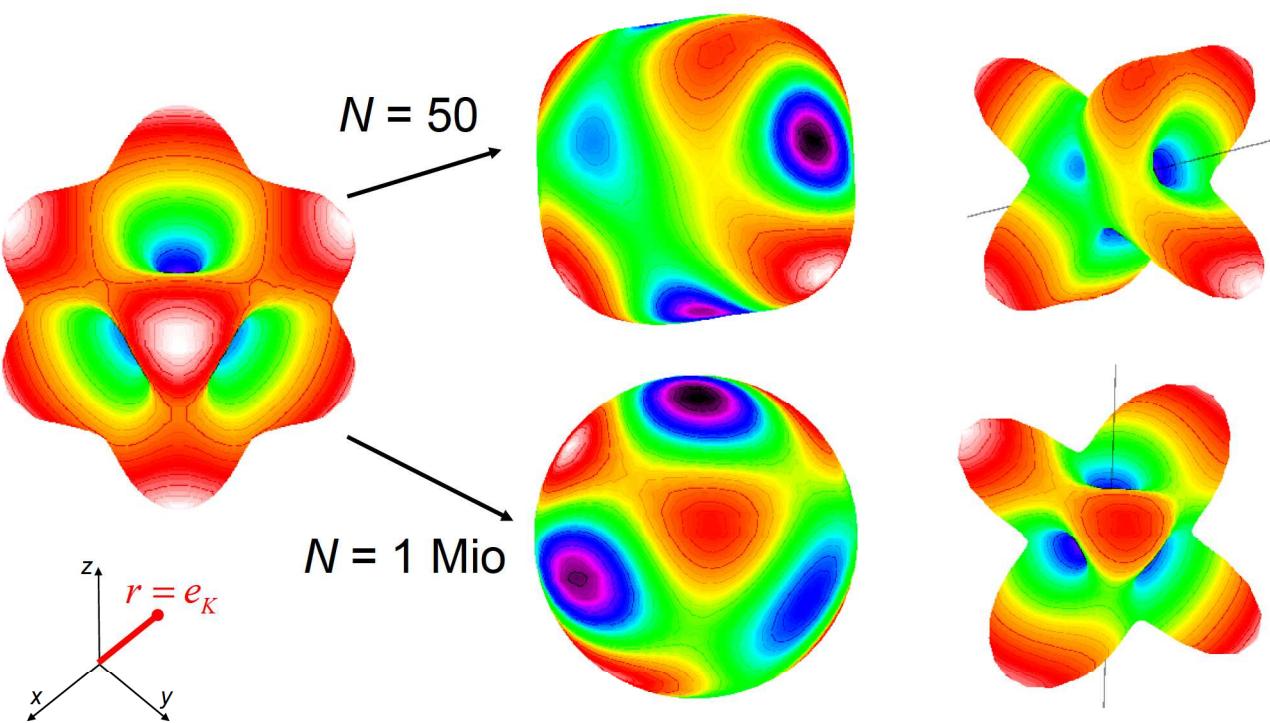
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## Random Cubic Anisotropy (3D, $K_1 > 0$ )

$$e_K = K_1(m_x^2 m_y^2 + m_y^2 m_z^2 + m_z^2 m_x^2)$$

$$\langle e_k \rangle_N$$

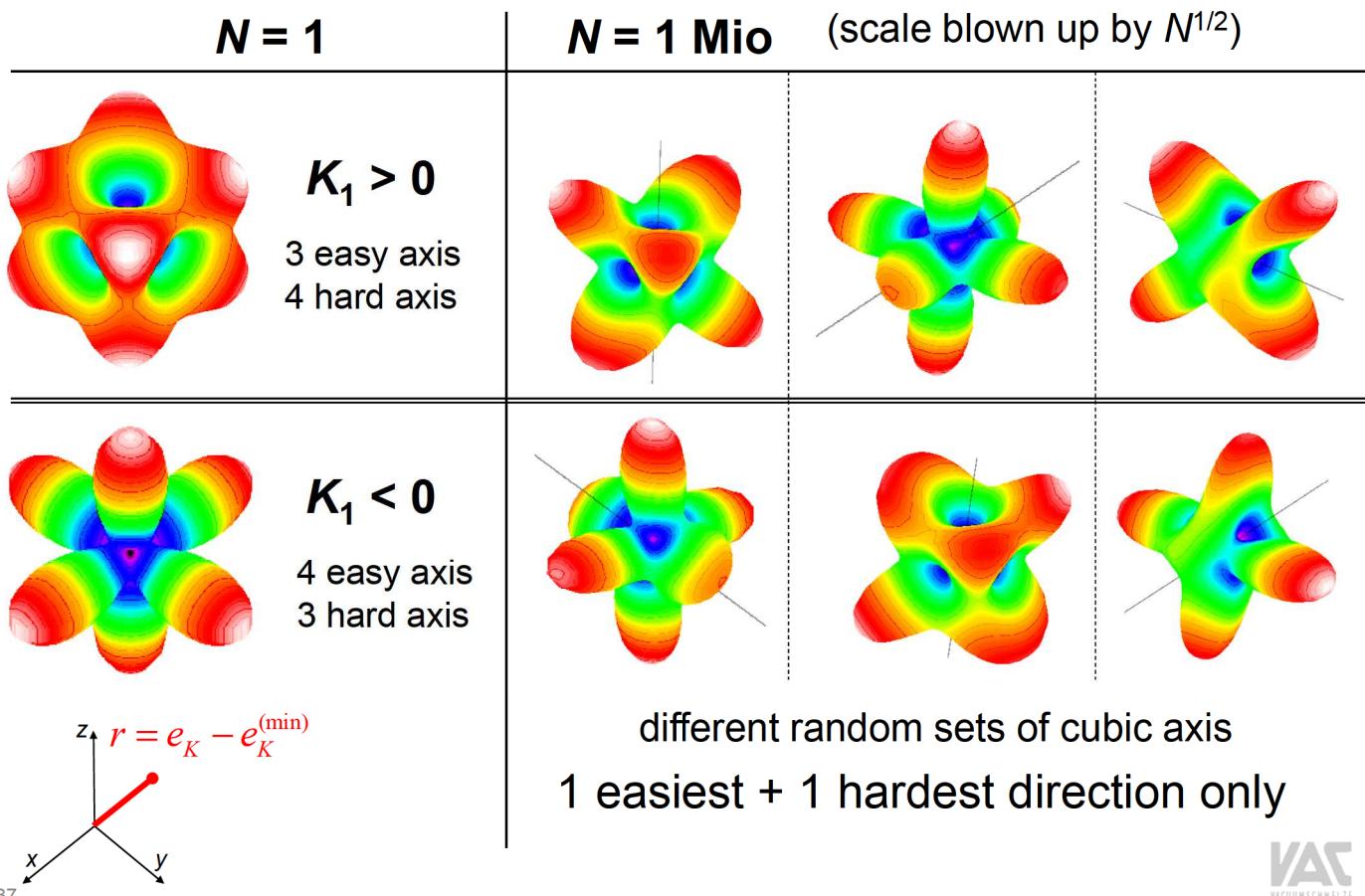
$$\left( \langle e_K \rangle - \langle e_K \rangle^{(\min)} \right) \cdot \sqrt{N}$$



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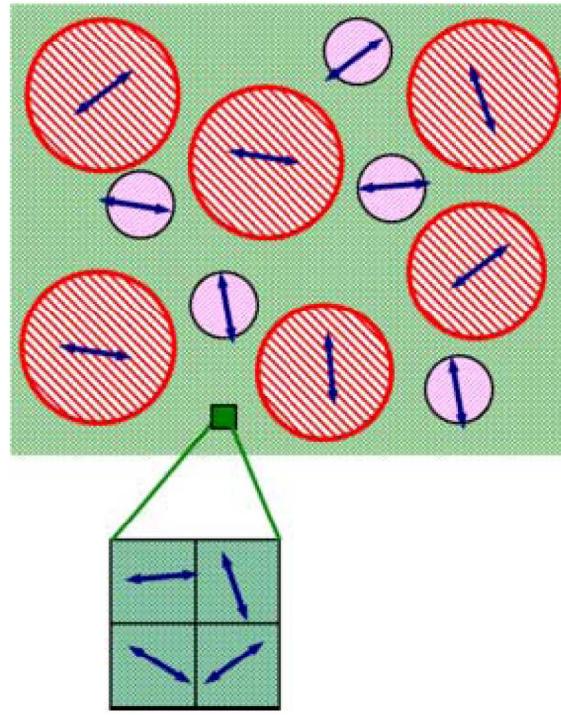
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# Random Cubic Anisotropy (3D)



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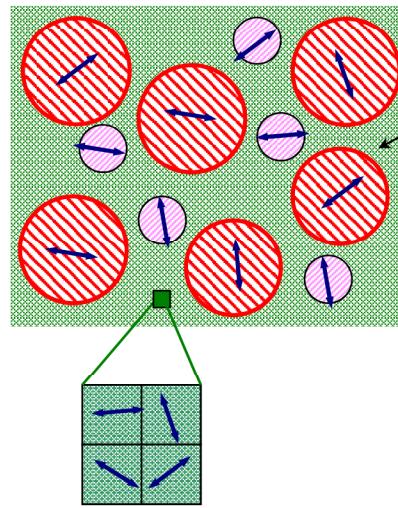




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## Extended Random Anisotropy Model



different structural phases  
with local random anisotropy  $K_{1,v}$   
grain volume  $\Omega_v = D_v^3$  and fraction  $x_v$

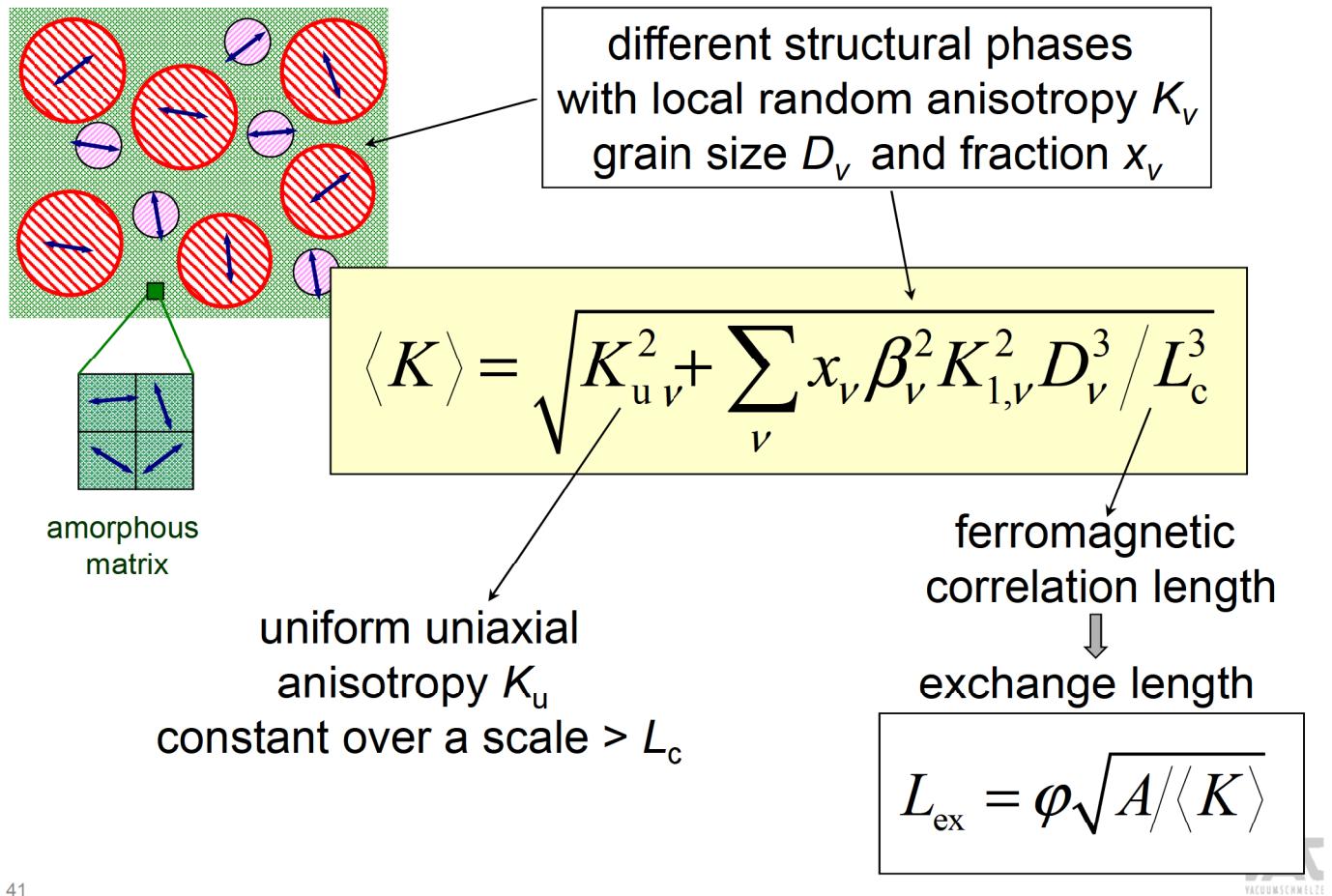
$$\begin{aligned}\langle e_K \rangle_N &= \frac{1}{V_c} \int K_1(\underline{x}) \sin^2(\phi - \theta(\underline{x})) d^3 \underline{x} \\ &= \frac{1}{V_c} \sum_{i=1}^N K_1(i) \sin^2(\phi - \theta(i)) \Omega(i) \\ &= K_N \cdot \sin^2(\phi - \psi_N) + \frac{1}{2} \left( \sum_v x_v K_{1,v} - K_N \right)\end{aligned}$$

amorphous  
matrix

$$K_N = \sqrt{\sum_v x_v K_{1,v}^2 \Omega_v / V_c + \frac{1}{V_c^2} \sum_i \sum_{j(\neq i)} K_1(i) \Omega(i) K_1(j) \Omega(j) \cos 2(\underbrace{\theta(i) - \theta(j)}_{\text{random phases}})}$$

$$\langle K_N \rangle = \beta \sqrt{\sum_v x_v K_{1,v}^2 \Omega_v / V_c} = \sqrt{\sum_v \beta_v x_v K_{1,v}^2 D_v^3 / L_c^3}$$

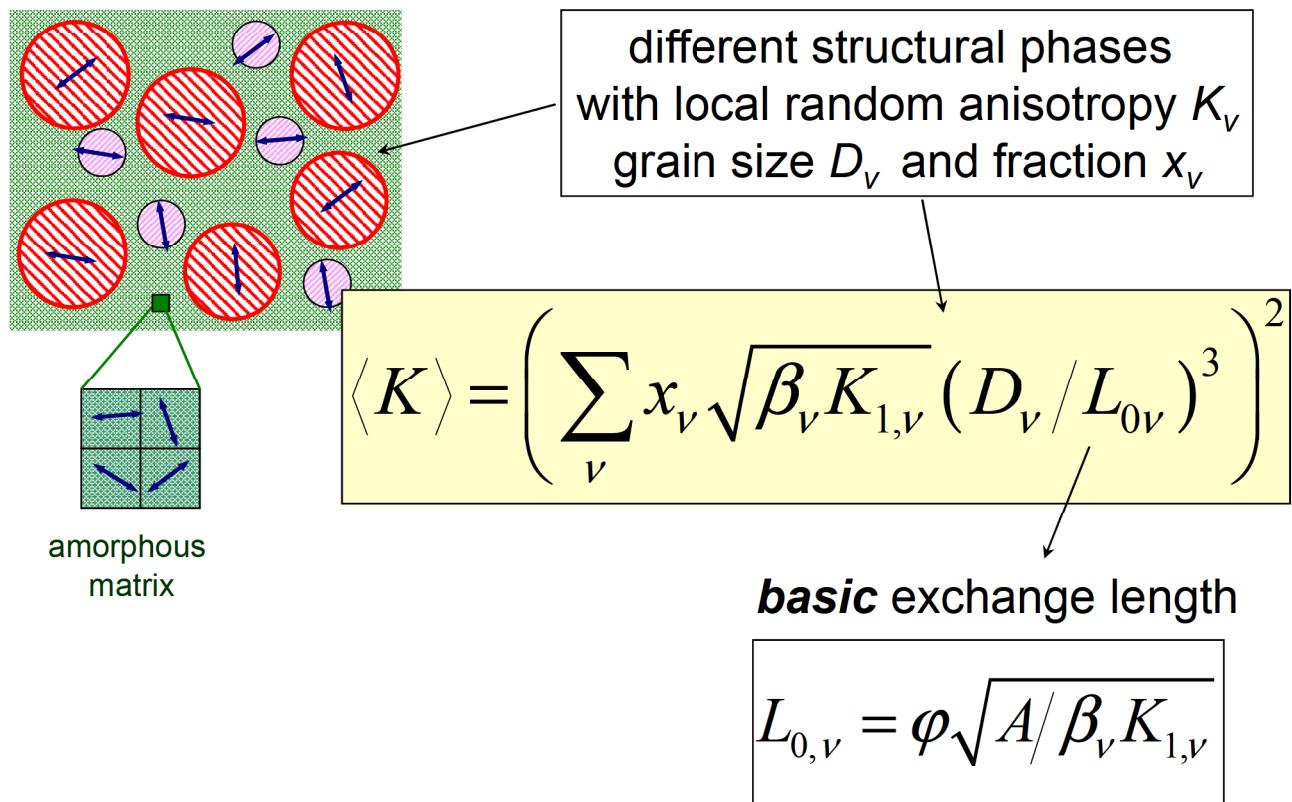
# Extended Random Anisotropy Model



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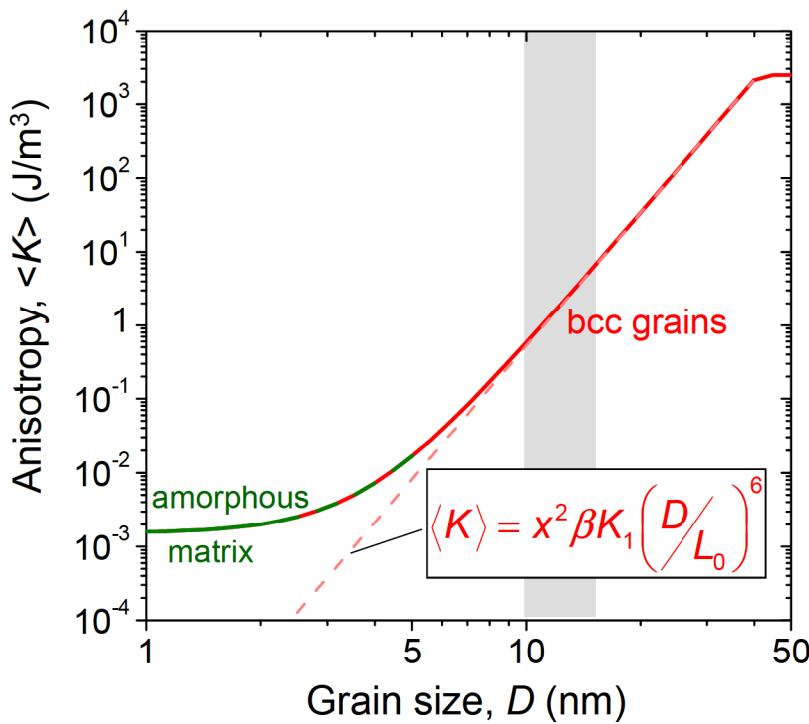
## Extended Random Anisotropy Model ( $K_u=0$ )



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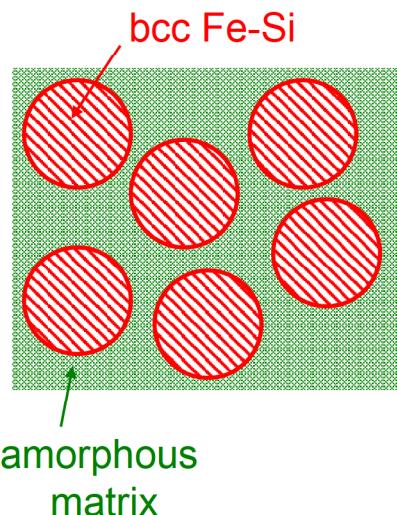
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# Contribution of Amorphous Matrix



## amorphous matrix

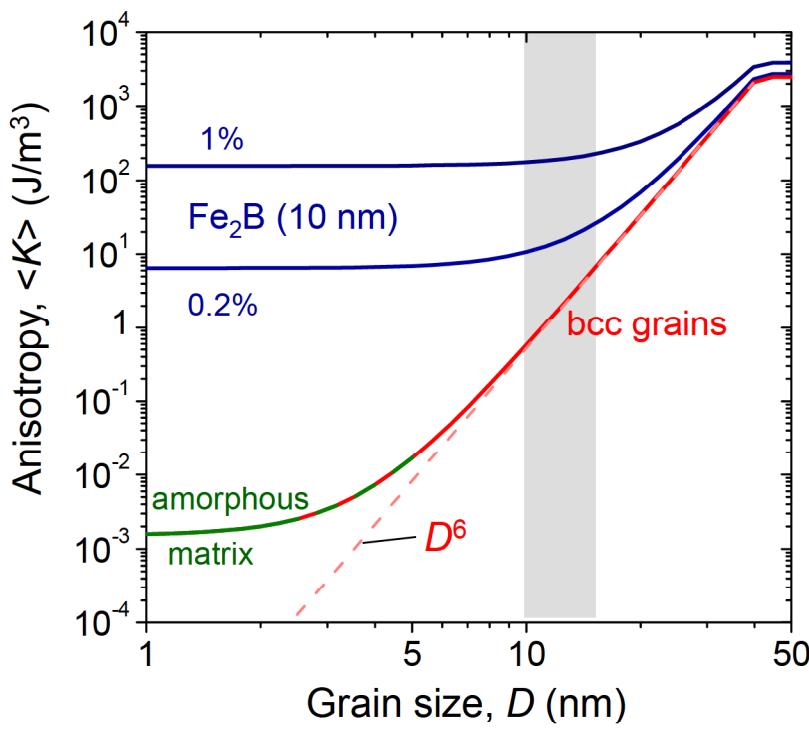
$D_{\text{am}} = 0.5 \text{ nm} !!$   
negligible, due to small structural correlation length



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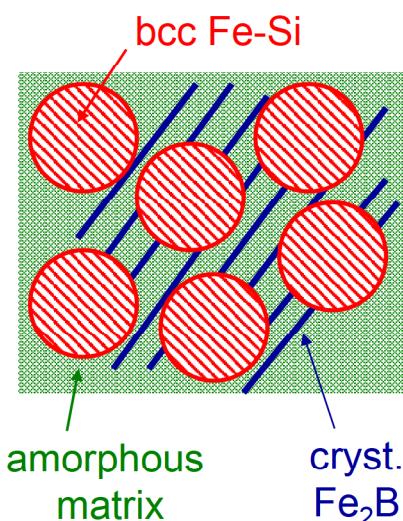
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# Effect of Boride Compounds



## FeB-compounds

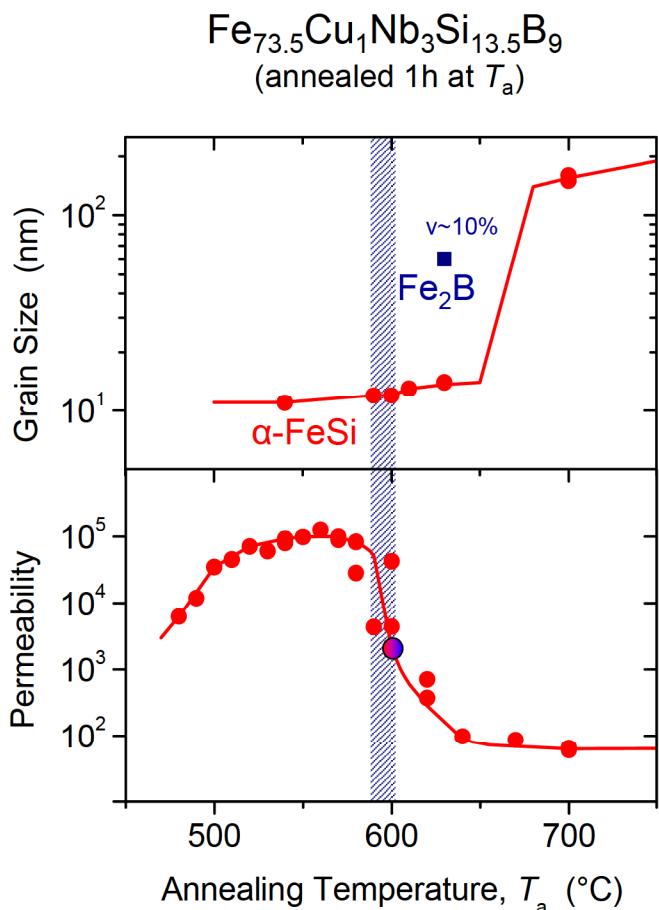
$|K_1(\text{Fe}_2\text{B})| = 430 \text{ kJ/m}^3 !!$   
effective even in smallest fractions due to huge  $K_1$



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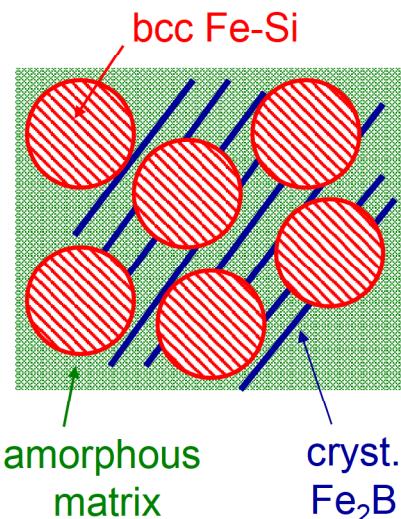
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# Effect of Boride Compounds



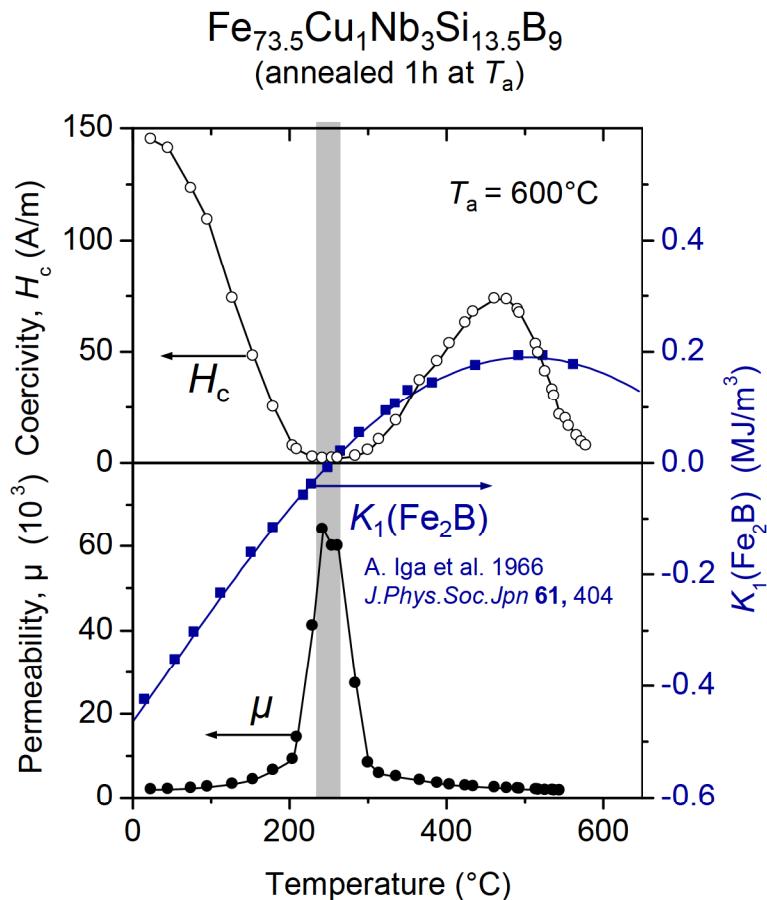
## FeB-compounds

$|K_1(\text{Fe}_2\text{B})| = 430 \text{ kJ/m}^3 !!$   
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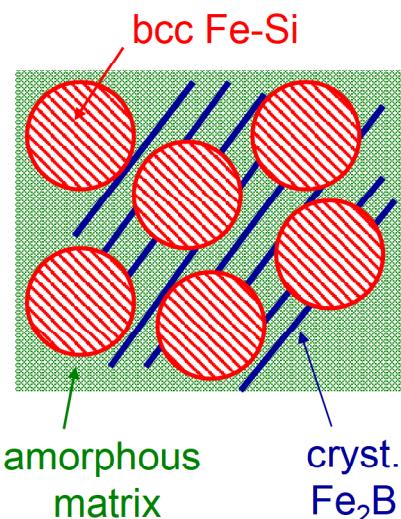
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# Effect of Boride Compounds



## FeB-compounds

$|K_1(\text{Fe}_2\text{B})| = 430 \text{ kJ/m}^3 !!$   
effective even in smallest fractions due to huge  $K_1$



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# Macroscopic and Random Anisotropies

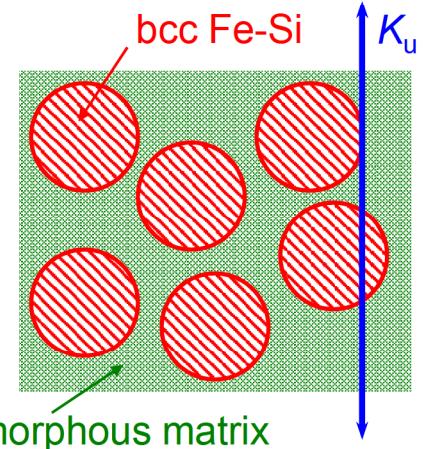
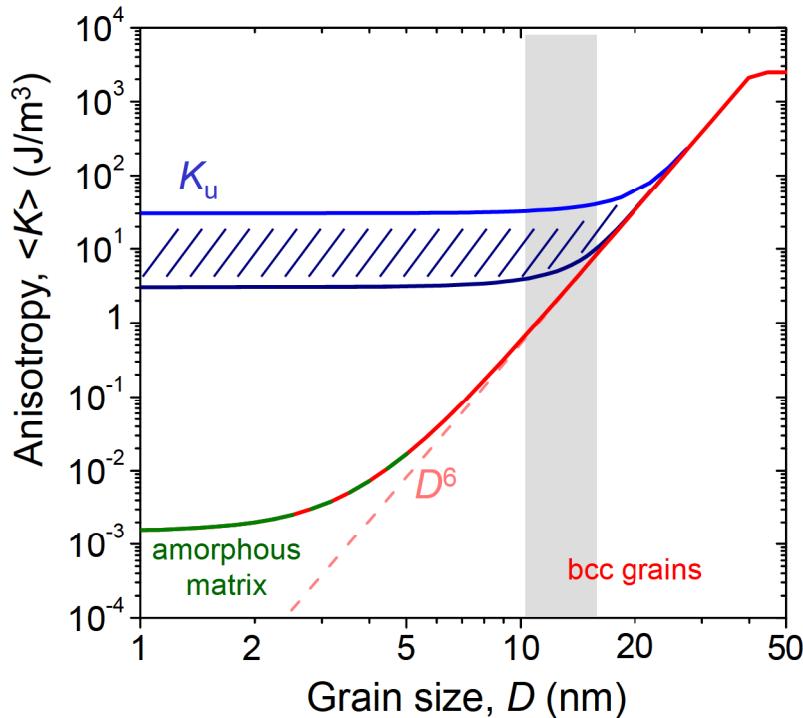
Total Anisotropy Constant

$$\langle K \rangle = \sqrt{K_u^2 + \sum_i x_i \beta_i^2 K_{1,i}^2 D_i^3 / L_{ex}^3}$$

„uniform“ anisotropy  $K_u$

constant over a scale

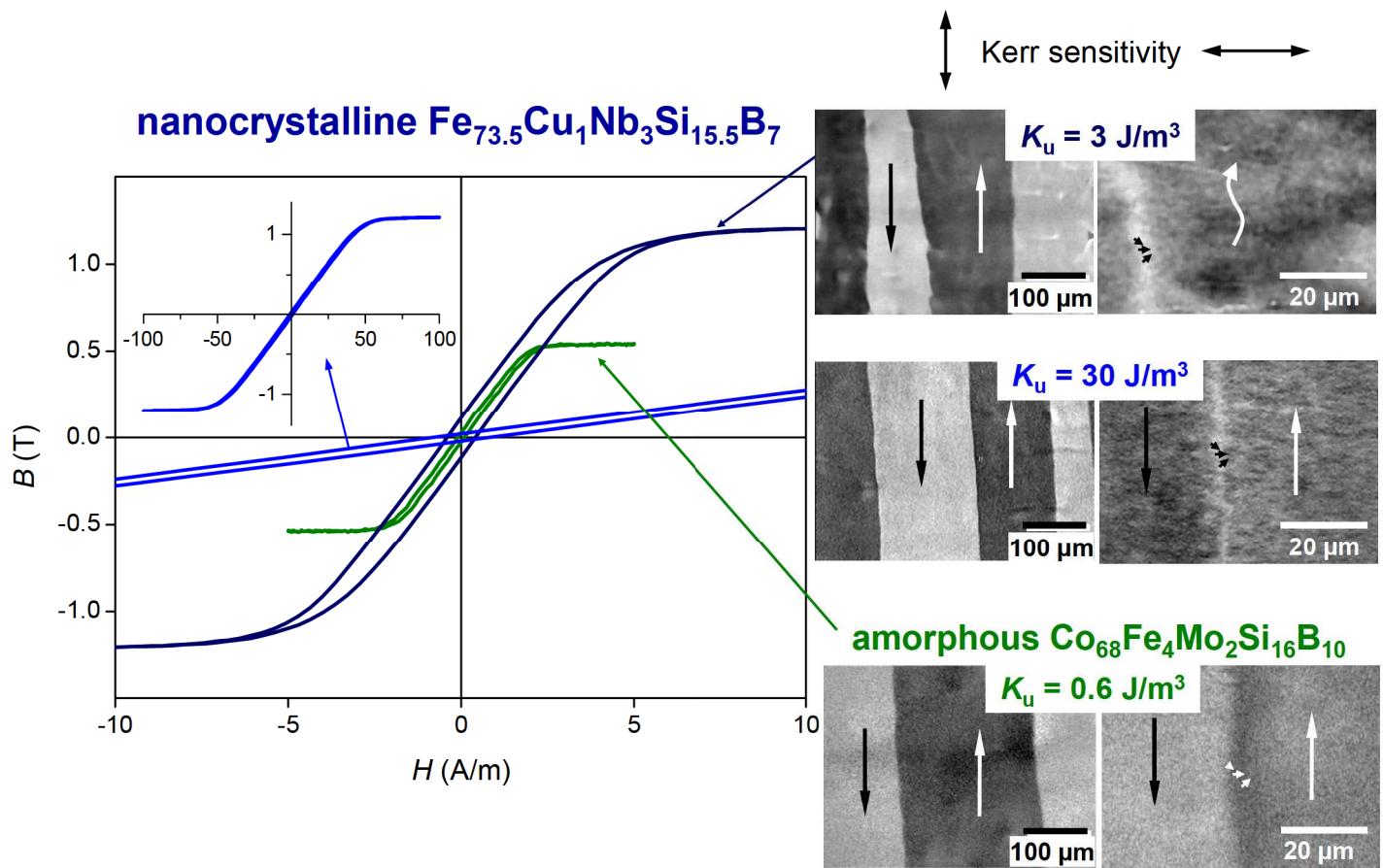
$$\gg L_{ex} = \varphi \sqrt{A/\langle K \rangle} \\ (\approx 1 \mu\text{m})$$



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## Interplay of Uniform and Random Anisotropy

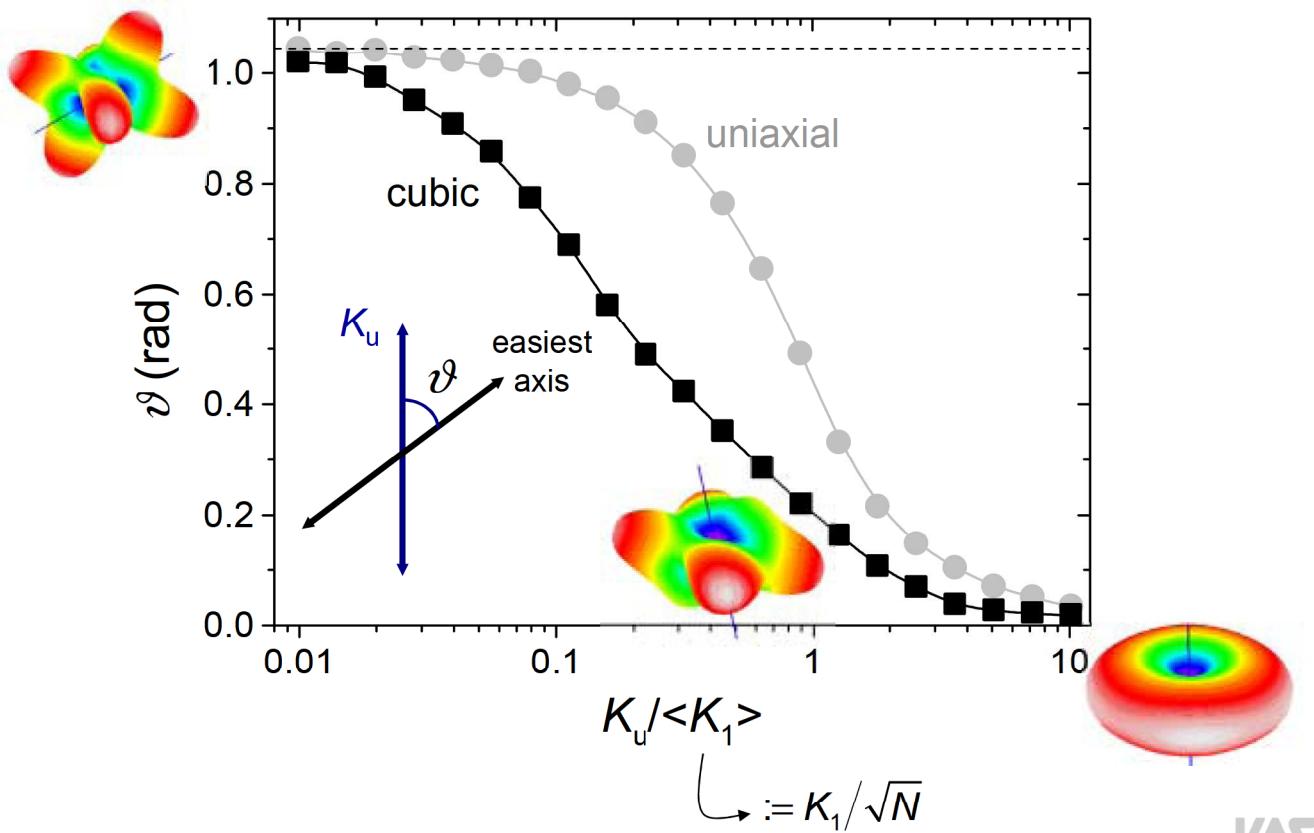


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S. Flohrer, R. Schäfer, Ch. Polak, G. Herzer, *Acta Materialia* **53** (2005) 2937

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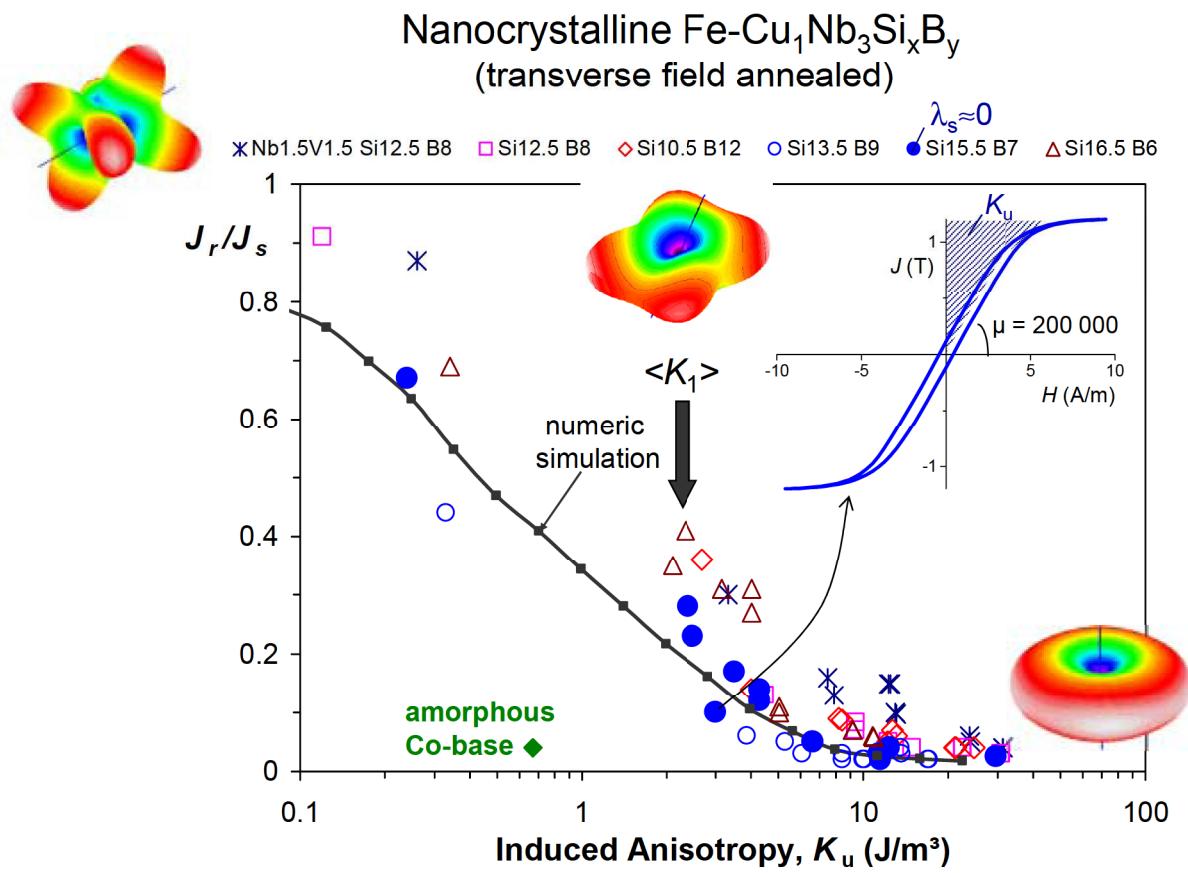
# Mix of Uniform and Random Anisotropy



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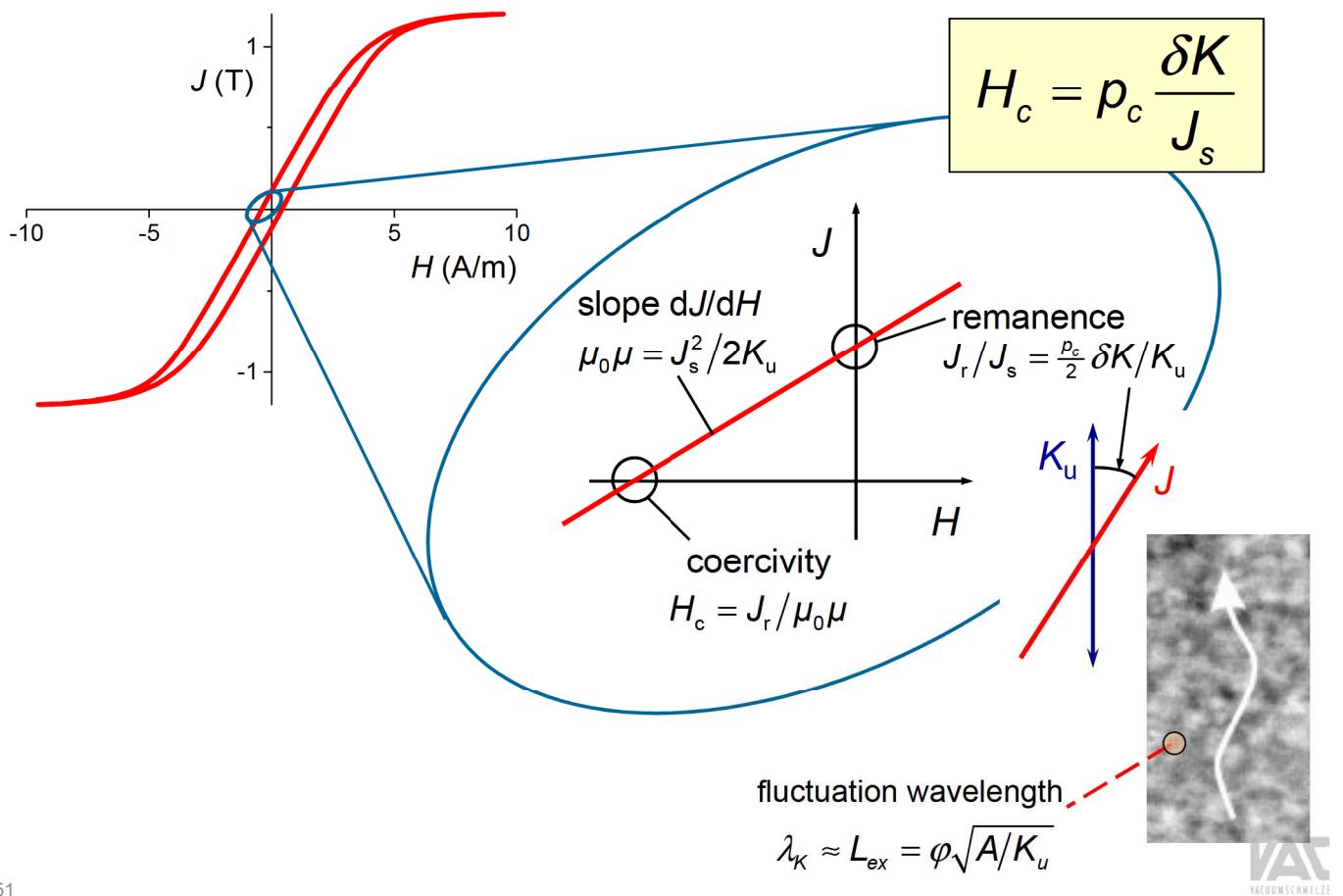
# Interplay of Uniform and Random Anisotropy



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# Coercivity Mechanism



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# Anisotropy Fluctuations $\delta K$

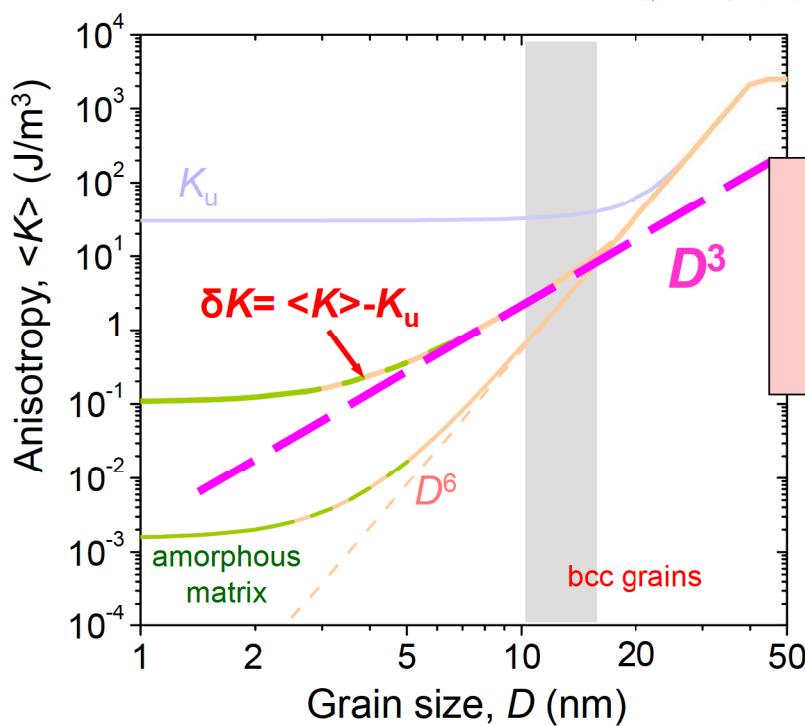
$$\delta K := \langle K \rangle - K_u \approx \frac{1}{2} \sum_i x_i \sqrt{\beta_i K_{1,i} K_u} \left( D_i / L_{0,i} \right)^3$$

$K_u \gg \delta K$

$L_{0,i} = \varphi \sqrt{A / \beta_i K_i}$

fluctuation wavelength

$$\lambda_K \approx L_{ex} = \varphi \sqrt{A / \langle K \rangle}$$



Contribution of the bcc grains

$$\delta K_1 \approx \frac{x}{2} \sqrt{\beta K_1 K_u} \left( D / L_0 \right)^3$$

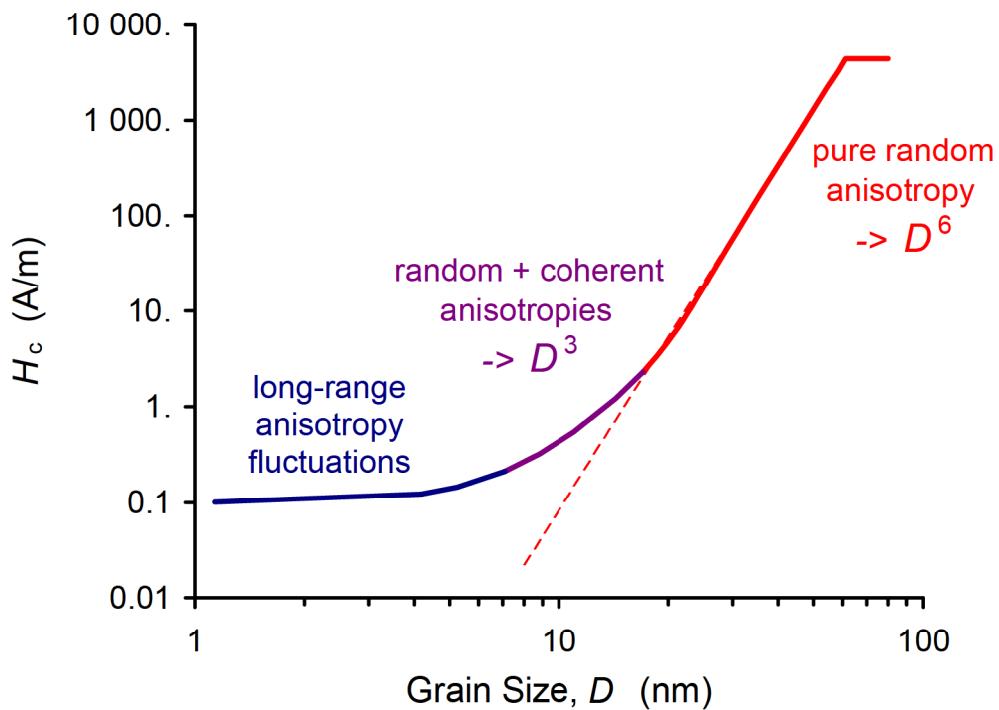
$K_u \gg \delta K$

$L_0 = \varphi_0 \sqrt{A / K_1}$

$H_c = p_c \frac{\delta K}{J_s}$

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# Conclusions



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## Open Questions

### Random Anisotropy Model

Problems:

- Adding Anisotropies

$$\langle K \rangle = \sqrt{K_u^2 + \frac{\sum_i x_i \beta_i^2 K_i^2 D_i^3}{L_c^3}}$$

✓ „easy“ part

- Exchange interaction  $A$  in multi-phase systems



- Dipolar interactions

# Literature

- G. Herzer, *The Random Anisotropy Model - A Critical Review and Update*  
In: Idzikowski B, Svec P, Miglierini M, editors. Properties and applications of nanocrystalline alloys from amorphous precursors. Dordrecht: Kluwer Academic Publishers; 2005.
- G. Herzer, Soft magnetic materials – nanocrystalline alloys, in: H. Kronmüller, S. Parkin (Eds.), *Handbook of Magnetism and Advanced Magnetic Materials*, vol. 4, John Wiley and Sons, 2007, p. 1882.
- and many references cited therein

