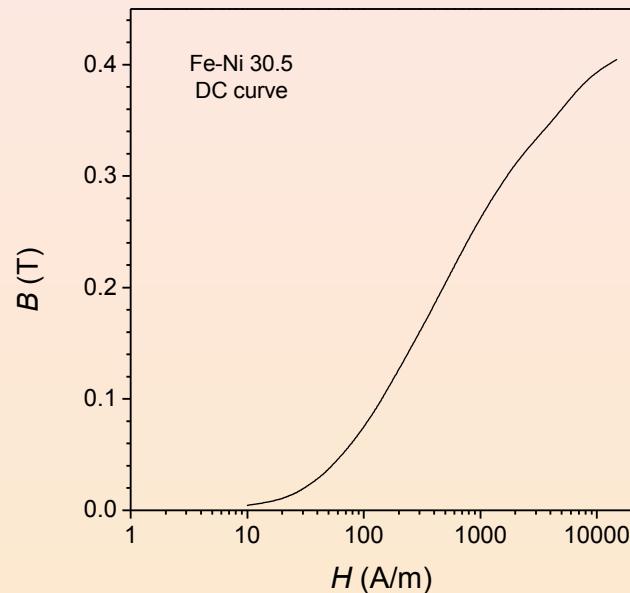
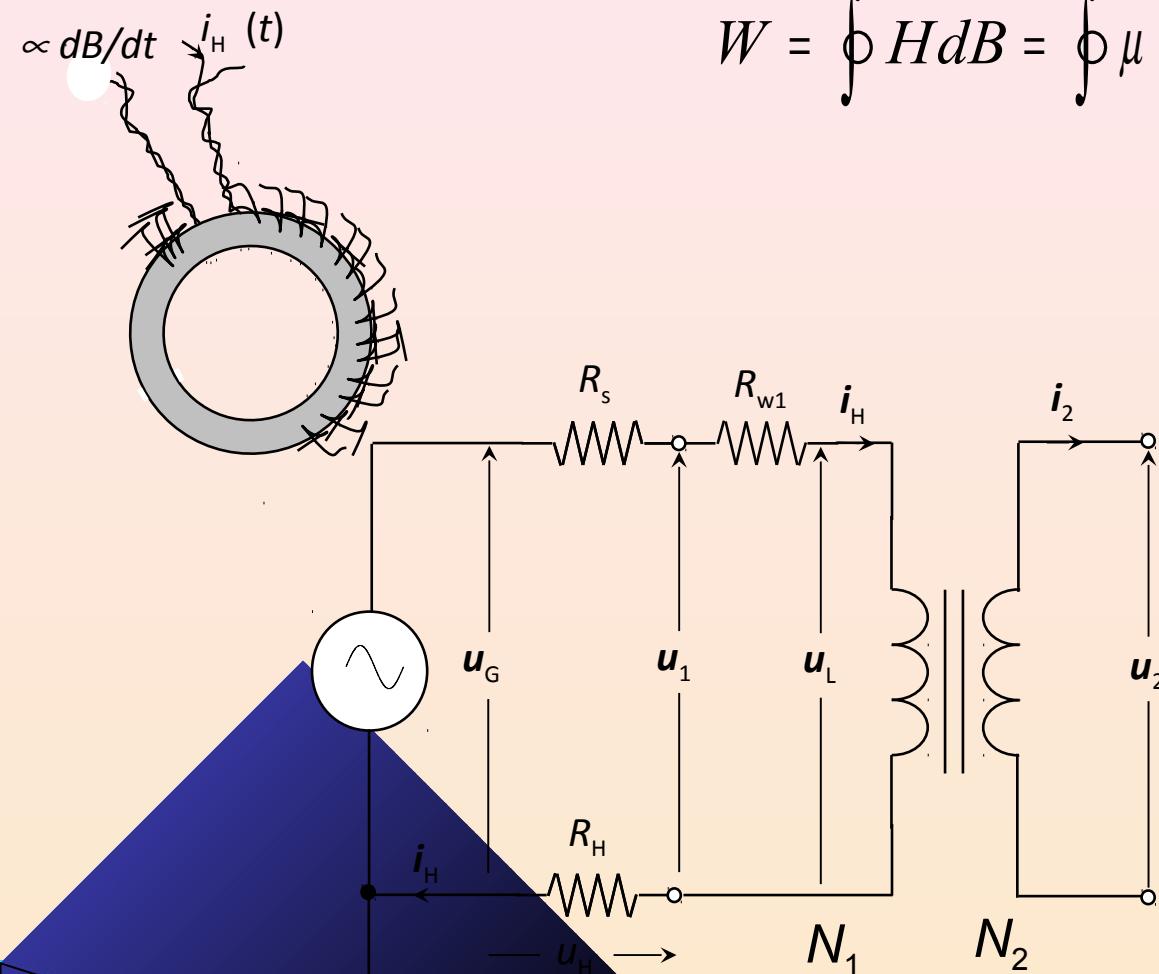


MAGNETIC LOSSES: QUESTIONS & PROBLEMS

1) Electrotechnical derivation: demonstrate that the magnetic energy loss per cycle is

$$W = \oint H dB = \oint \mu_0 H dH + \oint H dJ = \oint H dJ$$



$$u_G(t) = Ri_H(t) + d\Phi / dt$$

$$B = \mu_0 H + J$$

$$t = 0 \rightarrow B = 0; \quad t = t_0 \rightarrow B = B_p$$

$$E(t_0) = \int_0^{t_0} u_G(t) i_H(t) dt$$

$$E(t_0) = \int_0^{t_0} Ri_H^2(t) dt + \int_0^{t_0} N_1 A i_H(t) \frac{dB}{dt} dt = E_R + U$$

$$i_H(t) = (l_m / N_1) H(t)$$

$$Al_m = V$$

$$\oint H dB = \oint \mu_0 H dH + \oint H dJ$$
$$\oint \mu_0 H dH = \mu_0 \int_0^T H \frac{dH}{dt} dt = 0$$

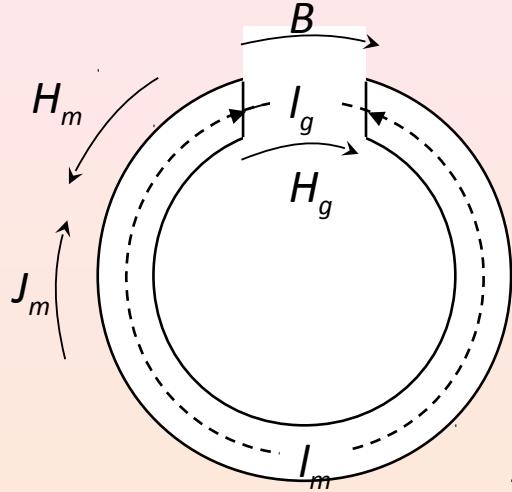
Energy delivered to the magnetic system

$$U(t_0) = V \int_0^{t_0} H(t) \frac{dB}{dt} dt = V \int_0^{B_p} H dB$$

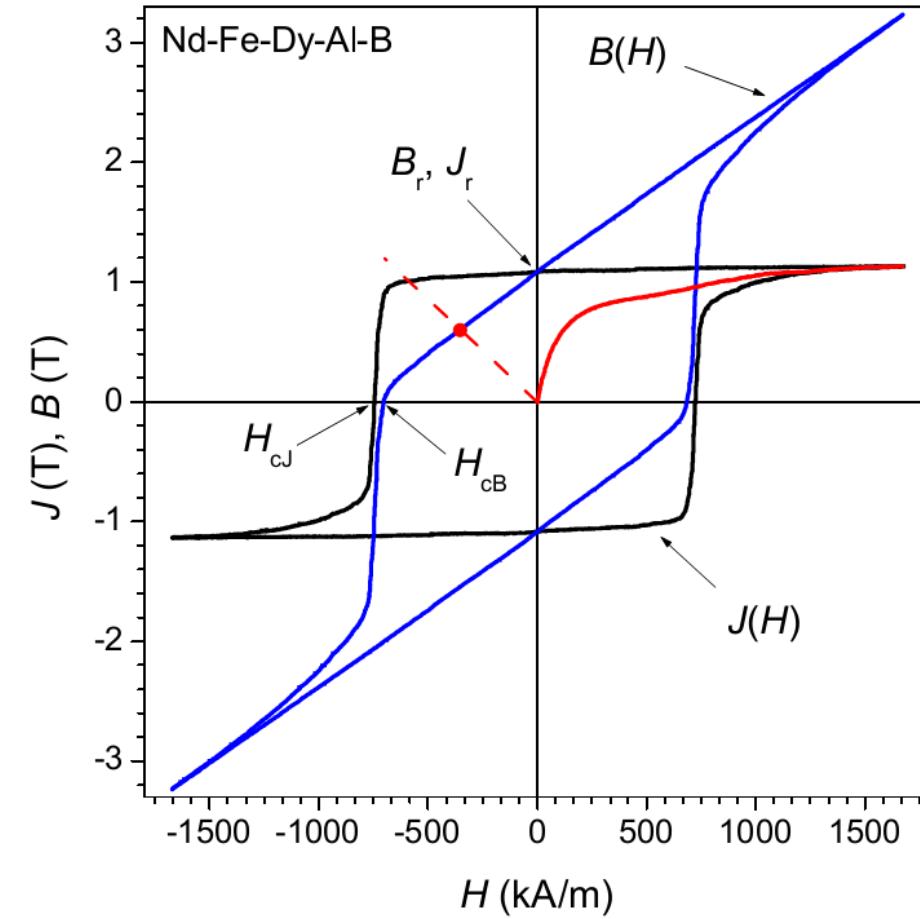
Energy loss per cycle per unit volume

$$W = \oint H dJ$$

2) Find the energy stored in the gap of a permanent magnet (the exploitable energy during magnet operation)



$$E_g = -\frac{1}{2} (BH) V$$



The whole magnetostatic energy associated with the magnet (energy of formation)

$$\mathbf{H} = \mathbf{H}_d = -\frac{N_d}{\mu_0} \mathbf{J}$$

$$E_{ms} = -\frac{1}{2} \int_V \mathbf{H} \cdot \mathbf{J} dV = \frac{1}{2\mu_0} N_d J^2 V$$

The sum of the energy stored inside the magnet and the energy stored outside it, possibly confined to a good extent inside the gap.

Energy stored in the material

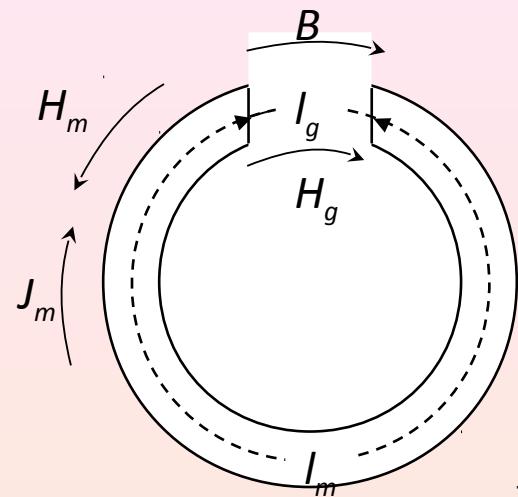
$$E_m = \frac{1}{2} \mu_0 \int_V H^2 dV = \frac{1}{2\mu_0} N_d^2 J^2 V$$

Energy stored in the gap $E_g = E_{ms} - E_m = \frac{1}{2\mu_0} J^2 (N_d - N_d^2) V$ $B = J + \mu_0 H$

$$E_g = \frac{1}{2\mu_0} N_d J (J - N_d J) V = -\frac{1}{2} H (J + \mu_0 H) V$$

$$E_g = -\frac{1}{2} (BH) V$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = 0 \quad H_g l_g = - H_m l_m$$



$$\Phi_g = \mu_0 H_g S = \Phi_m = B_m S$$

Energy inside the gap $E_g = \frac{1}{2} \mu_0 H_g^2 V_g$

$$E_g = \frac{1}{2} \mu_0 H_g^2 V_g = \frac{1}{2} \mu_0 H_g l_g \cdot H_g S = - \frac{1}{2} \mu_0 H_m l_m \cdot \frac{B}{\mu_0} S$$

$$E_g = - \frac{1}{2} (BH) V$$

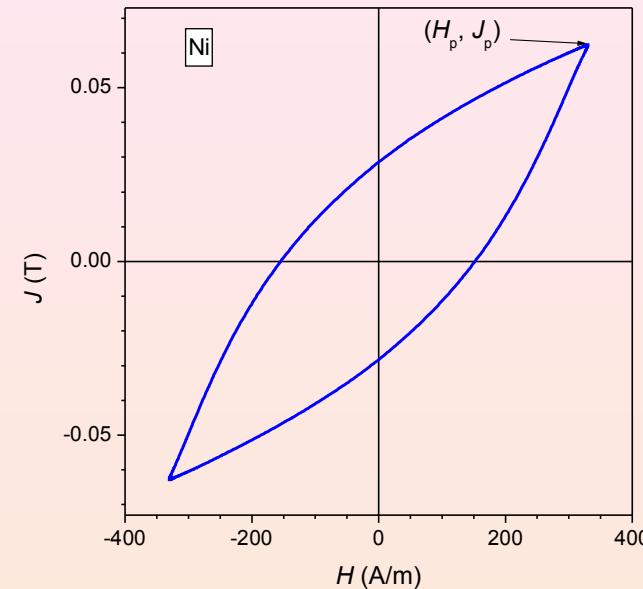


2) Find the hysteresis loss component versus H_p and J_p in the Rayleigh region.
 Find the rotational contribution to the initial permeability a for isotropic distribution of easy axis (small angle rotations).

$$J_p = aH_p + bH_p^2$$

$$J(H) = (a + bH_p)H \mp \frac{b}{2}(H_p^2 - H^2)$$

$$W(H) = \oint J(H) dH$$



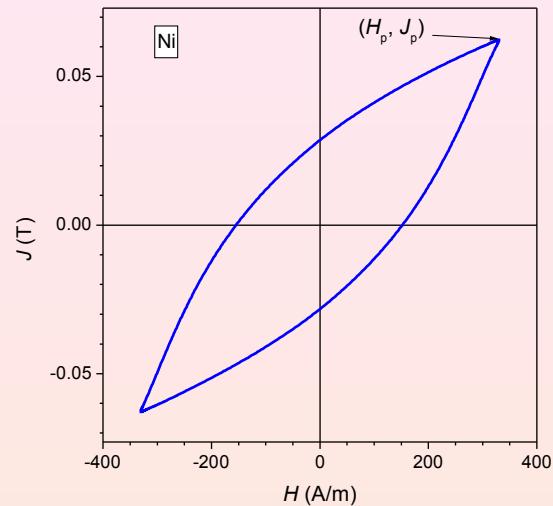
2a) Find the hysteresis loss component in the Rayleigh region.

$$J(H) = (a + bH_p)H \mp \frac{b}{2}(H_p^2 - H^2)$$

$$J_p = aH_p + bH_p^2$$

$$W(H) = \oint J(H) dH$$

The linear terms can be omitted.



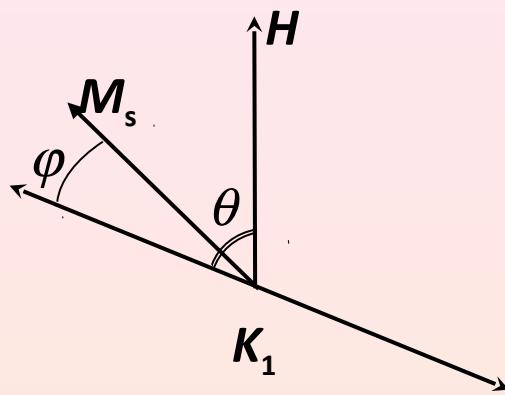
$$W(H) = \frac{b}{2} H_p^2 \int_{-H_p}^{H_p} dH - \frac{b}{2} \int_{-H_p}^{H_p} H^2 dH + \frac{b}{2} H_p^2 \int_{-H_p}^{H_p} dH - \frac{b}{2} \int_{-H_p}^{H_p} H^2 dH$$

$$W(H) = \frac{4}{3} b H_p^3 \quad \xrightarrow{\text{Red arrow}} \quad W(J_p) = \frac{1}{6b^2} \left[-a + \sqrt{a^2 + 4b J_p} \right]^3$$

$$aH_p \gg bH_p^2 \quad \xrightarrow{\text{Red arrow}} \quad W = \frac{1}{3} \frac{b}{a^3} J_p^3$$

$$aH_p \ll bH_p^2 \quad \xrightarrow{\text{Red arrow}} \quad W = \frac{4}{3\sqrt{b}} J_p^{3/2}$$

2b) Find the initial permeability associated with rotations.



$$E = -\mu_0 M_s H (\cos(\theta - \phi) + K_1 \sin^2 \phi)$$

Minimize the energy and find the equilibrium angle ϕ

Small oscillations
 $\cos \phi \approx 1$

$$\sin \phi \approx \frac{\mu_0 M_s H \sin \theta}{\mu_0 M_s H \cos \theta + 2K_1}$$

Small oscillations

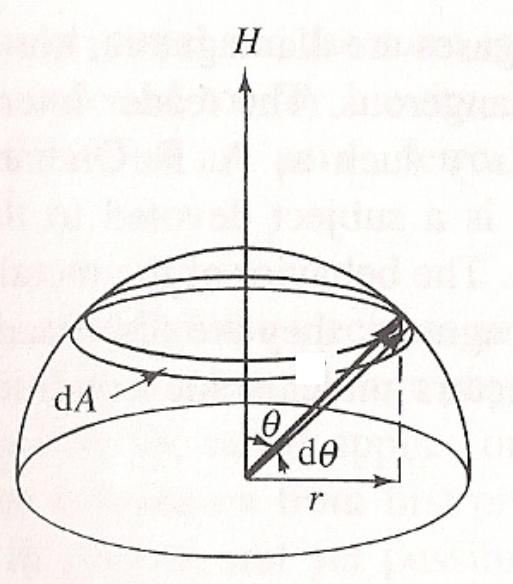
$$\mu_0 M_s H \cos \theta \ll 2K_1$$

$$\sin \phi \approx \frac{\mu_0 M_s H \sin \theta}{2K_1}$$

$$\Delta M \approx M_s \sin \theta \sin \phi$$

$$\chi = \frac{\Delta M}{H} \approx \frac{M_s \sin \theta}{H} \frac{\mu_0 M_s H \sin \theta}{2K_1}$$

$$\chi(\theta) \approx \frac{\mu_0 M_s^2}{2K_1} \sin^2 \theta$$



Uniform space distribution of easy directions.

$$\frac{dN}{N} = \frac{2\pi \sin \theta \, d\theta}{2\pi}$$

$$p(\theta) d\theta = \sin \theta \, d\theta$$

$$\int_0^{\pi/2} p(\theta) d\theta = 1$$

$$\chi(\theta) \approx \frac{\mu_0 M_s^2}{2K_1} \sin^2 \theta$$

$$\langle H_K \rangle = \frac{2K_1}{\mu_0 M_s}$$

$$\langle \chi \rangle \approx \frac{M_s}{H_K} \int_0^{\pi/2} \sin^2 \theta \, p(\theta) d\theta = \frac{M_s}{H_K} \int_0^{\pi/2} \sin^3 \theta \, d\theta$$

$$\langle \chi_{\text{rot}} \rangle \approx \frac{2}{3} \frac{M_s}{H_K}$$

Fe-(3 wt%)Si, $M_s = 6 \cdot 10^6 \text{ A/m}$, $K_1 = 38 \cdot 10^3 \text{ J/m}^3$, $H_K = 38.4 \cdot 10^3 \text{ A/m}$

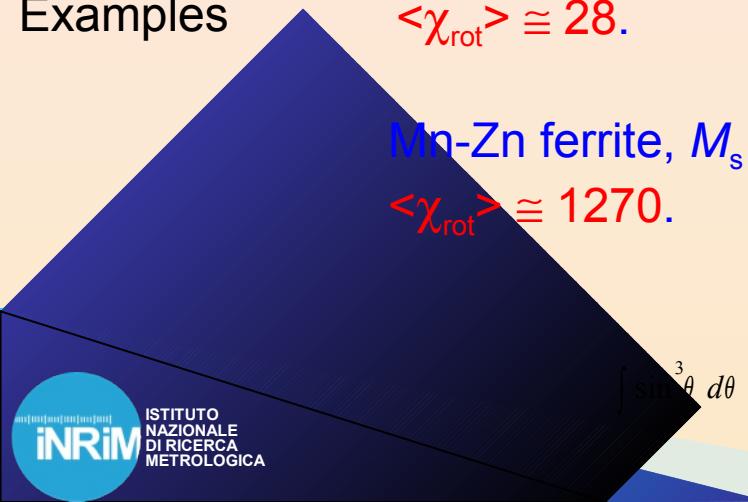
$$\langle \chi_{\text{rot}} \rangle \approx 28.$$

Mn-Zn ferrite, $M_s = 3.9 \cdot 10^5 \text{ A/m}$, $K_1 = 50 \text{ J/m}^3$, $H_K = 205 \text{ A/m}$,

$$\langle \chi_{\text{rot}} \rangle \approx 1270.$$

$$\int \sin^3 \theta \, d\theta = (1/3)\cos^3 \theta - \cos \theta$$

Examples



3) Find: 1) The complex permeability in a linear material. 2) The relationship with the energy loss. 3) The related Q factor. 4) The equivalent R-L circuit of a ring inductor.

$$H(t) = H_p \cos \omega t$$

$$B(t) = B_p \cos(\omega t - \delta) =$$

$$= B_p \cos \delta \cos \omega t + B_p \sin \delta \sin \omega t$$

in-phase

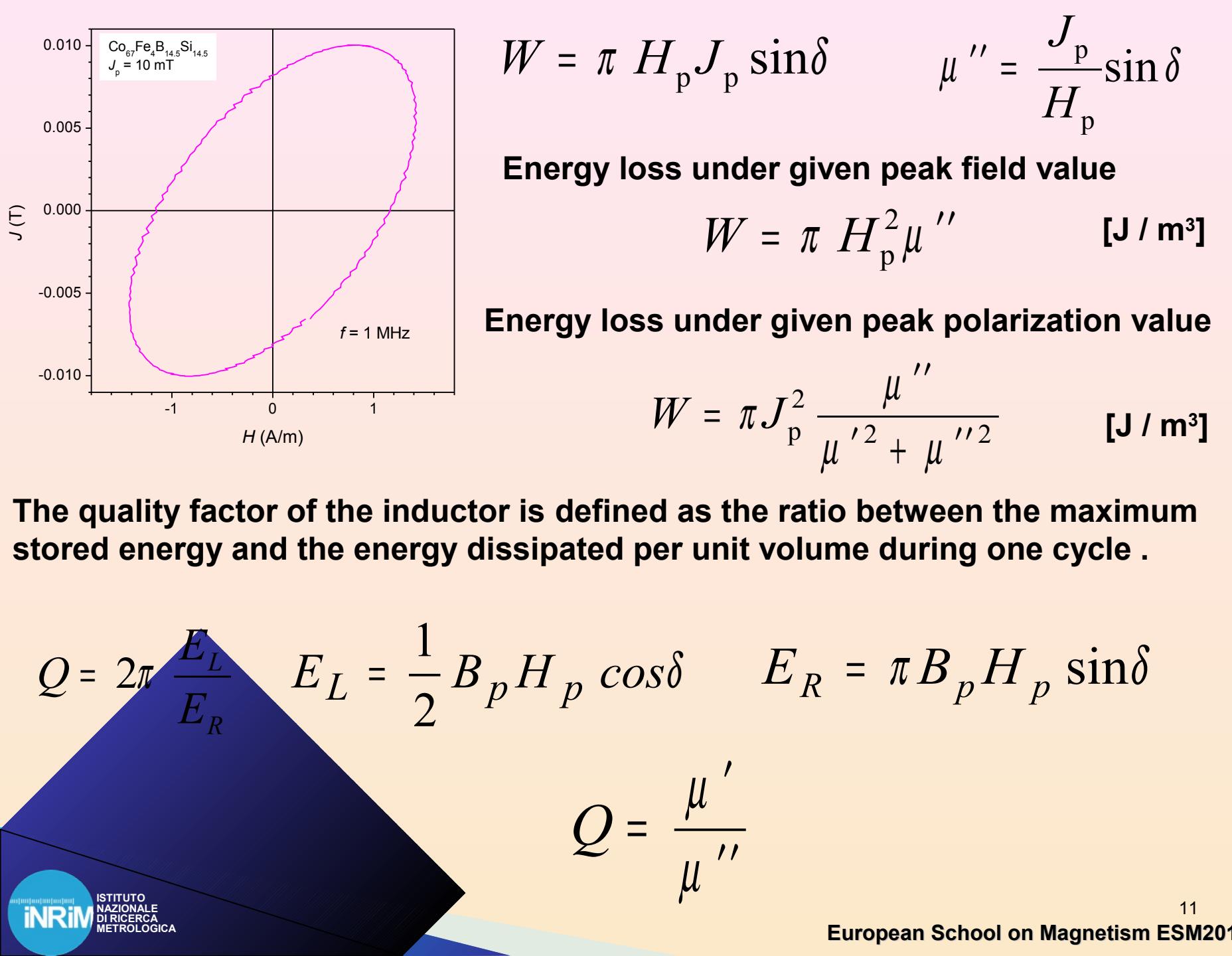
out-of-phase

$$\left\{ \begin{array}{l} \mu' = \frac{B_p}{H_p} \cos \delta \\ \mu'' = \frac{B_p}{H_p} \sin \delta \end{array} \right.$$

The 90° -delayed component of the induction is connected with the dissipation of energy.

From the definition of the energy loss per cycle W , we obtain that the power loss per unit volume is

$$P = Wf = f \int_0^T H(t) \cdot \frac{dB(t)}{dt} dt = f\pi H_p B_p \sin \delta \quad [\text{W / m}^3]$$



$$W = \pi H_p J_p \sin \delta \quad \mu'' = \frac{J_p}{H_p} \sin \delta$$

Energy loss under given peak field value

$$W = \pi H_p^2 \mu'' \quad [\text{J / m}^3]$$

Energy loss under given peak polarization value

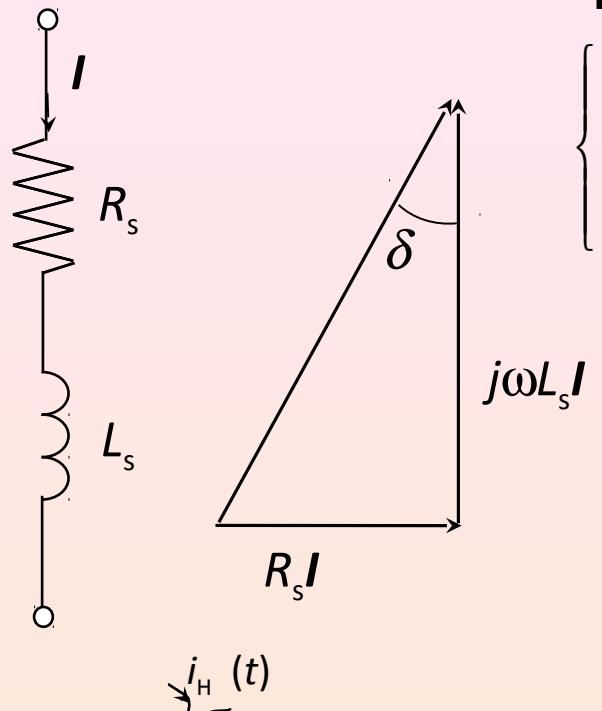
$$W = \pi J_p^2 \frac{\mu''}{\mu'^2 + \mu''^2} \quad [\text{J / m}^3]$$

The quality factor of the inductor is defined as the ratio between the maximum stored energy and the energy dissipated per unit volume during one cycle .

$$Q = 2\pi \frac{E_L}{E_R} \quad E_L = \frac{1}{2} B_p H_p \cos \delta \quad E_R = \pi B_p H_p \sin \delta$$

$$Q = \frac{\mu'}{\mu''}$$

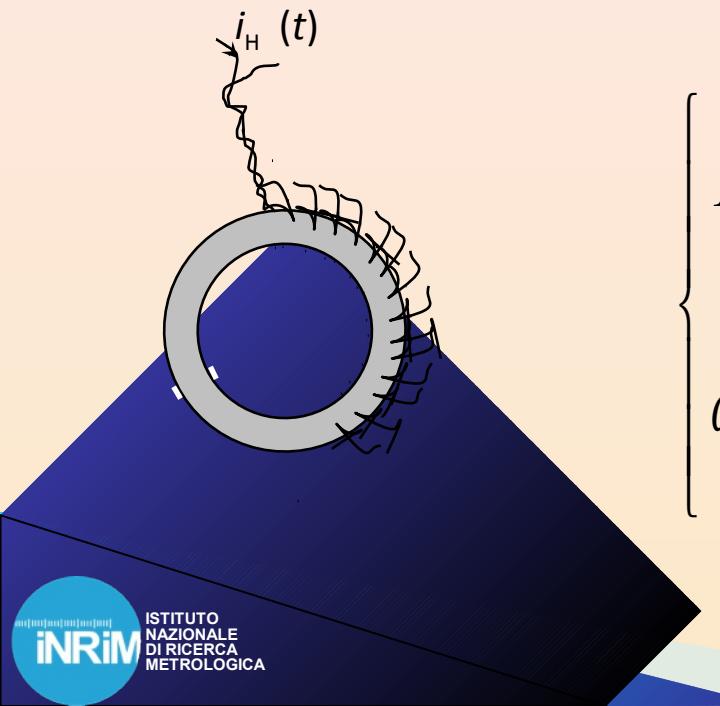
The equivalent **L - R** circuit

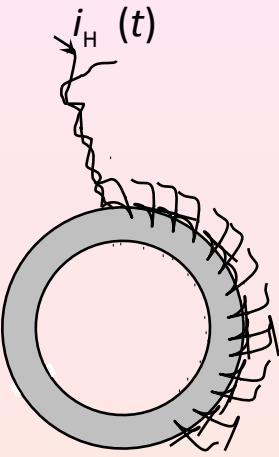


$$\begin{cases} H(t) = H_p e^{j\omega t} \\ B(t) = B_p e^{j(\omega t - \delta)} \end{cases} \xrightarrow{\hspace{1cm}} \begin{cases} I(t) = H_p \frac{l_m}{N} e^{j\omega t} \\ V(t) = j\omega B_p e^{j(\omega t - \delta)} \cdot NS \end{cases}$$

$$\begin{cases} V(t) = R_s I(t) + j\omega L_s I(t) \\ V(t) = \frac{NS\omega B_p}{I_p} (\sin \delta + j \cos \delta) I(t) \end{cases}$$

$$\begin{cases} R_s = \frac{NS\omega B_p}{I_p} \sin \delta \\ \omega L_s = \frac{NS\omega B_p}{I_p} \cos \delta \end{cases}$$





A ring core inductor of cross-sectional area **S**, mean diameter **D**, and number of turns **N**.

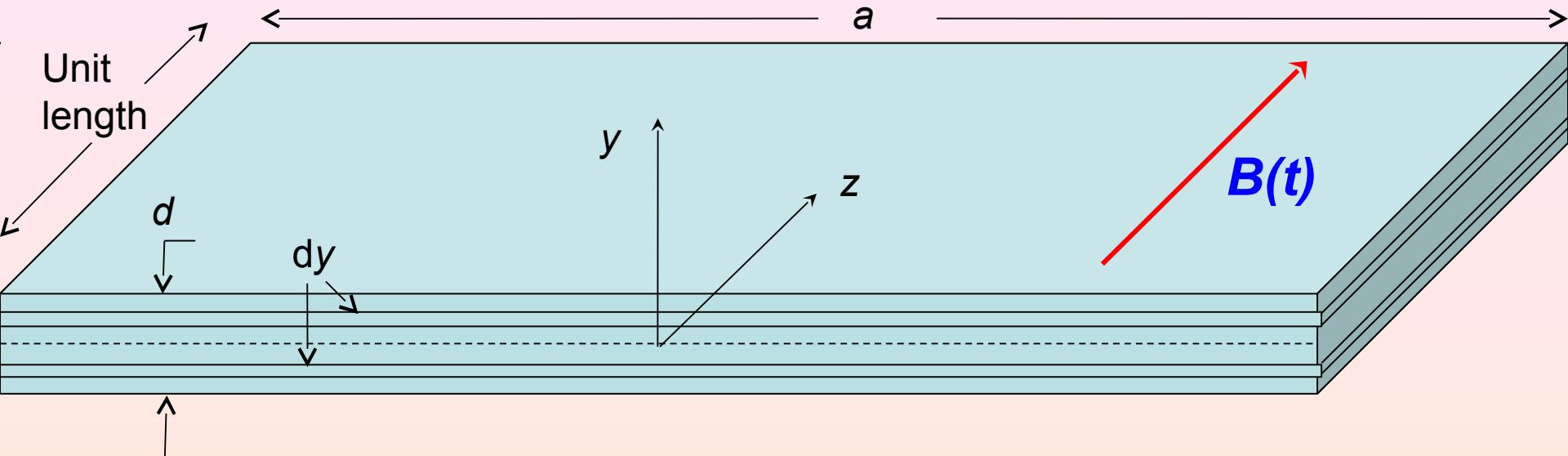
$$H_p = \frac{NI_p}{\pi D} \quad I_p = \frac{\pi D}{N} H_p \quad \left\{ \begin{array}{l} \mu' = \frac{B_p}{H_p} \cos \delta \\ \mu'' = \frac{B_p}{H_p} \sin \delta \end{array} \right.$$

$$\left\{ \begin{array}{l} R_s = \frac{NS\omega B_p}{I_p} \sin \delta \\ \omega L_s = \frac{NS\omega B_p}{I_p} \cos \delta \end{array} \right.$$

$$\left\{ \begin{array}{l} R_s = \frac{N^2 S \omega B_p}{\pi D H_p} \sin \delta \\ \omega L_s = \frac{N^2 S \omega B_p}{\pi D H_p} \cos \delta \end{array} \right.$$

$$\left\{ \begin{array}{l} R_s = \omega \frac{N^2 S}{\pi D} \mu'' \\ \omega L_s = \omega \frac{N^2 S}{\pi D} \mu' \end{array} \right.$$

Thin lamination, $d \ll \text{width}$



Instantaneous eddy current power loss per unit length in the infinitesimal strips

Sinusoidal induction $B(t) = B_p \sin \omega t$

$$dP = e^2 dG = (d\Phi / dt)^2 \sigma \frac{dy}{2a}$$

$$dP = \omega^2 B_p^2 \sin^2 \omega t \cdot 4a^2 y^2 \sigma \frac{dy}{2a}$$

$$\text{Time average } \langle dP \rangle = \frac{1}{T} \int_0^T dP = \sigma a y^2 dy \cdot \omega^2 B_p^2 a$$

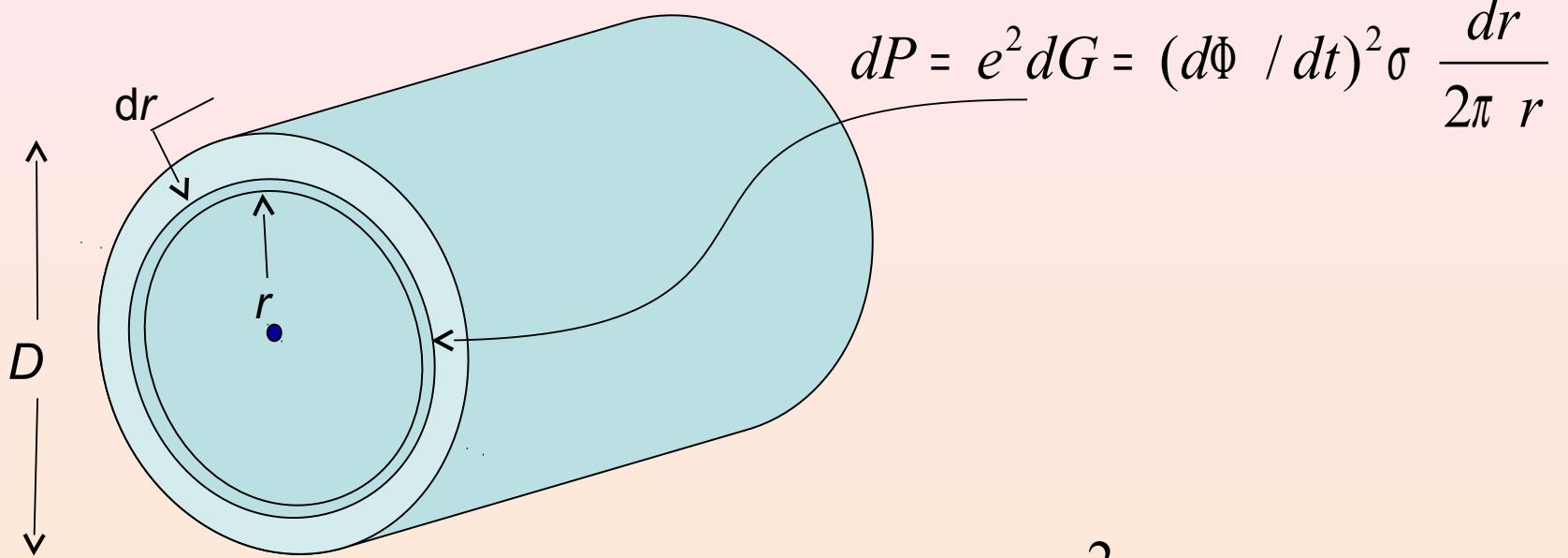
The energy loss in the sheet of thickness d

$$P = \int_0^{d/2} \langle dP \rangle = \frac{1}{3} \sigma a \omega^2 B_p^2 \frac{d^3}{8}$$

The energy loss per unit volume

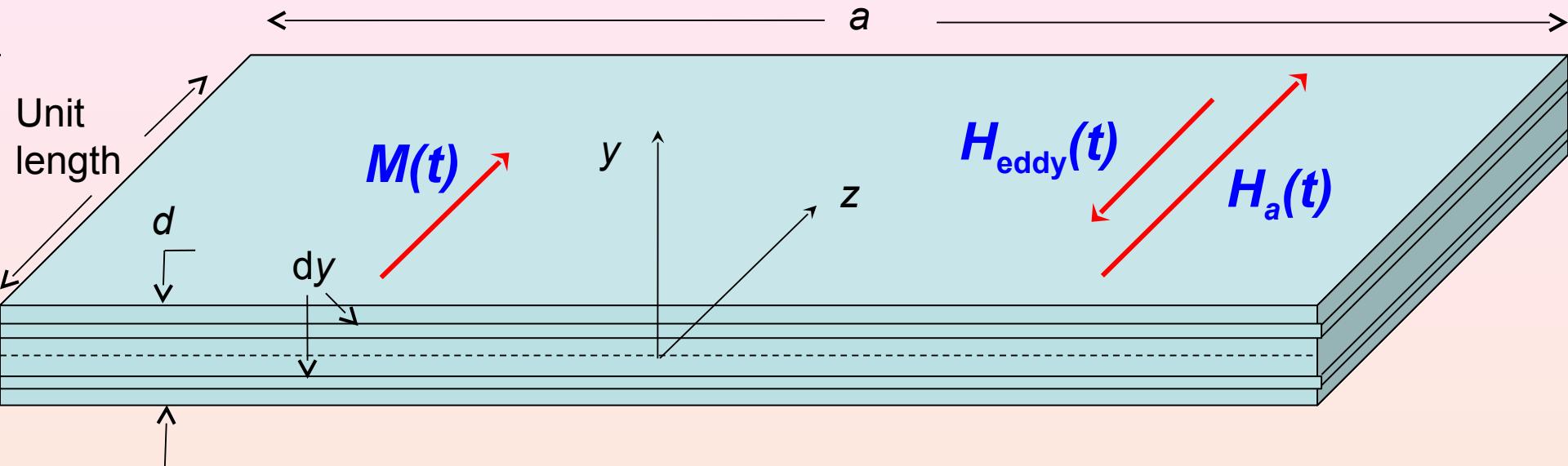
$$W = \frac{P}{daf} = \frac{\pi^2}{6} \sigma d^2 B_p^2 f$$

By the very same approach (and under the same conditions) we find the classical energy loss in cylindrical samples (sinusoidal induction $B(t) = B_p \sin \omega t$) .



$$W = \frac{\pi^2}{16} \sigma D^2 B_p^2 f$$

Thin lamination, $d \ll$ width



Response of a lamination of unit length, thickness d , conductivity σ , permeability μ to a field step.

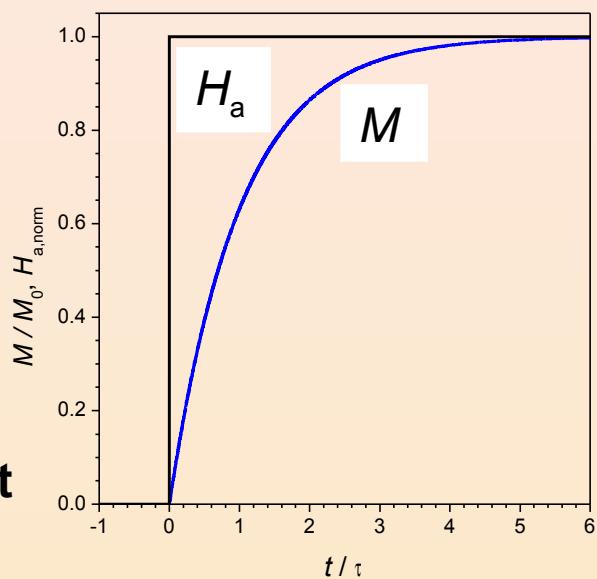
$$M(t) = \gamma H_{\text{eff}}(t)$$

$$B(t) = \mu H_{\text{eff}}(t)$$

$$\frac{dB}{dt} = \mu \frac{dH_{\text{eff}}(t)}{dt}$$

$$H_{\text{eff}} = H_a - H_{\text{eddy}}$$

We calculate the eddy current counterfield at the center of the lamination. **No skin effect.**



The electromotive force on the circuit made of the two strips of thickness dy at the coordinate y and the related eddy current

$$e(t) = - \frac{dB}{dt} \cdot 2ya \quad di(t, y) = e \, dG = - \frac{dB}{dt} \cdot 2ya \cdot \frac{\sigma}{2a} dy = dH_{\text{eddy}}(t, y)$$

$$H_{\text{eddy}}(t) = \int_0^{d/2} dH_{\text{eddy}} = - \int_0^{d/2} \frac{dB}{dt} \sigma y dy = - \dot{B} \sigma \cdot \frac{d^2}{8} = - \mu \dot{H}_{\text{eff}} \sigma \cdot \frac{d^2}{8}$$

$$H_{\text{eff}} = H_a - H_{\text{eddy}}$$

$$\dot{H}_{\text{eff}} + \frac{8}{\mu \sigma d^2} H_{\text{eff}} - \frac{8}{\mu \sigma d^2} H_a = 0$$

$$\begin{aligned} \dot{x} + ax - b &= 0 & z &= b - ax \\ \dot{x} &= -\dot{z}/a & \frac{dz}{dt} &= -az \end{aligned}$$

$$t = 0; \quad H_{\text{eff}} = 0 \quad z(t) = b \exp(-at)$$

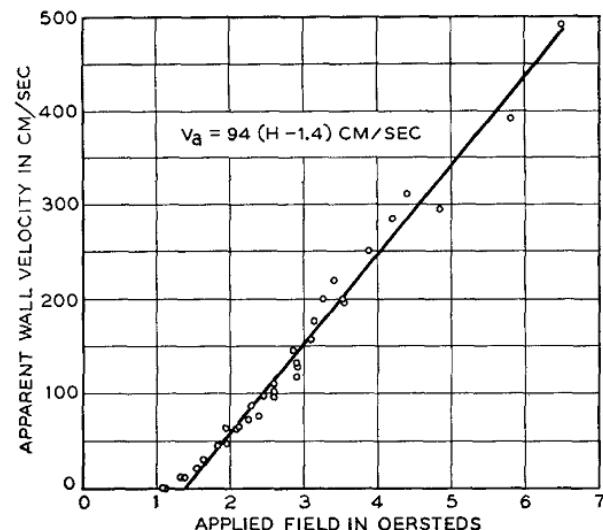
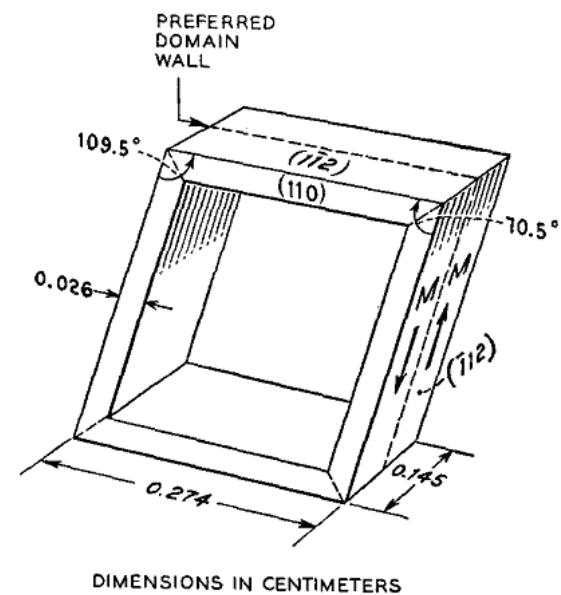
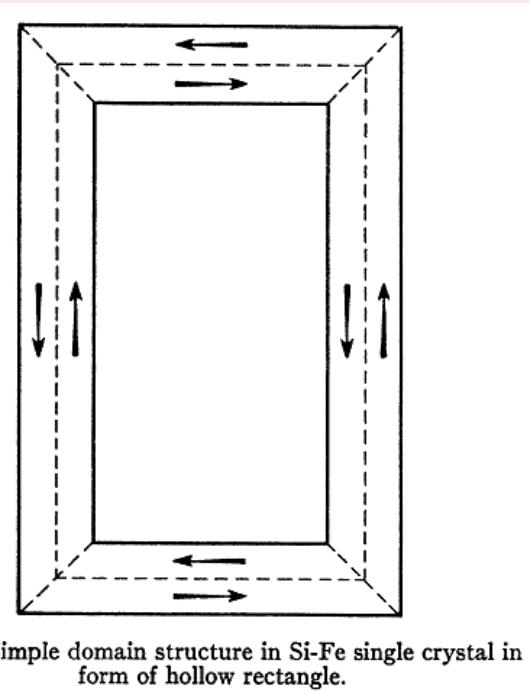
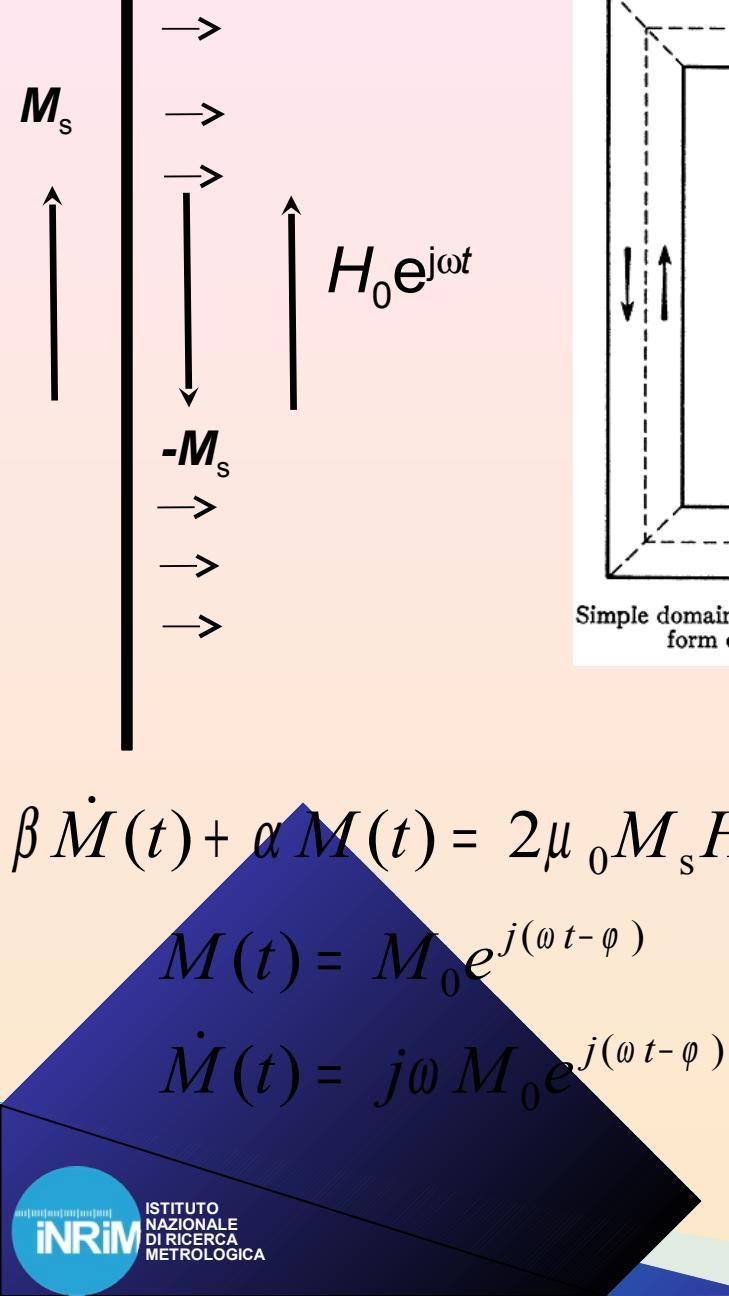
$$H_{\text{eff}}(t) = H_a \left(1 - \exp\left(-\frac{8}{\mu \sigma d^2} t\right)\right)$$

$$\tau = \frac{\mu \sigma d^2}{8}$$

$$M(t) = \chi H_{\text{eff}}(t) = \chi H_a \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$

$$\mu = 10^{-3}, \quad \sigma = 10^7 \text{ S}, \quad d = 10^{-6} \text{ m}, \quad \tau \sim 10^{-3} \text{ s}$$

Relaxation of domain wall motion. Complex susceptibility versus frequency.



$$\beta \dot{M}(t) + \alpha M(t) = 2\mu_0 M_s H_0 e^{j\omega t}$$

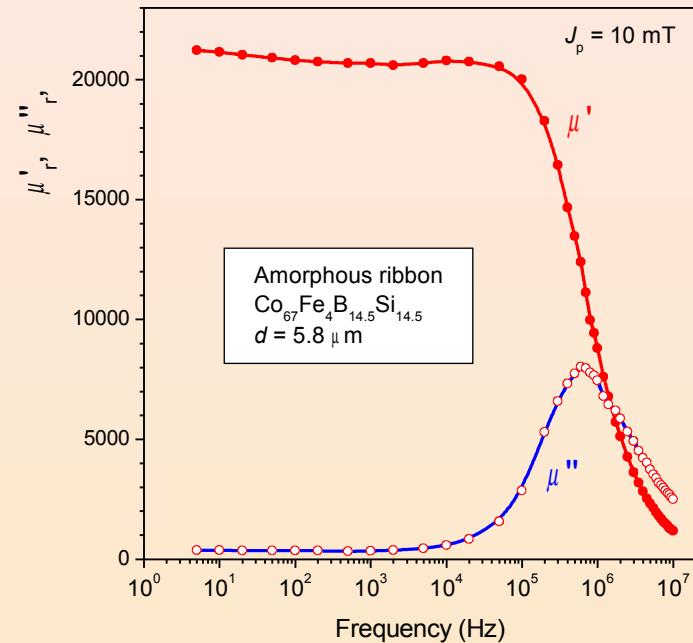
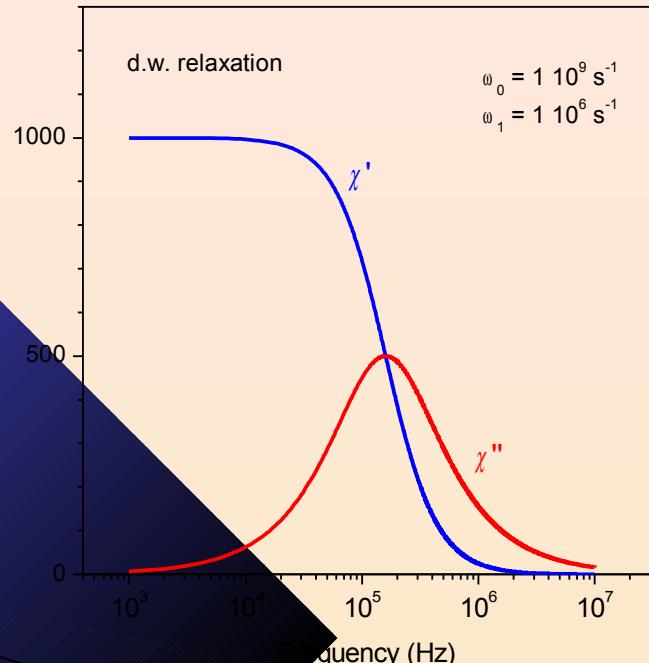
$$M(t) = M_0 e^{j(\omega t - \phi)}$$

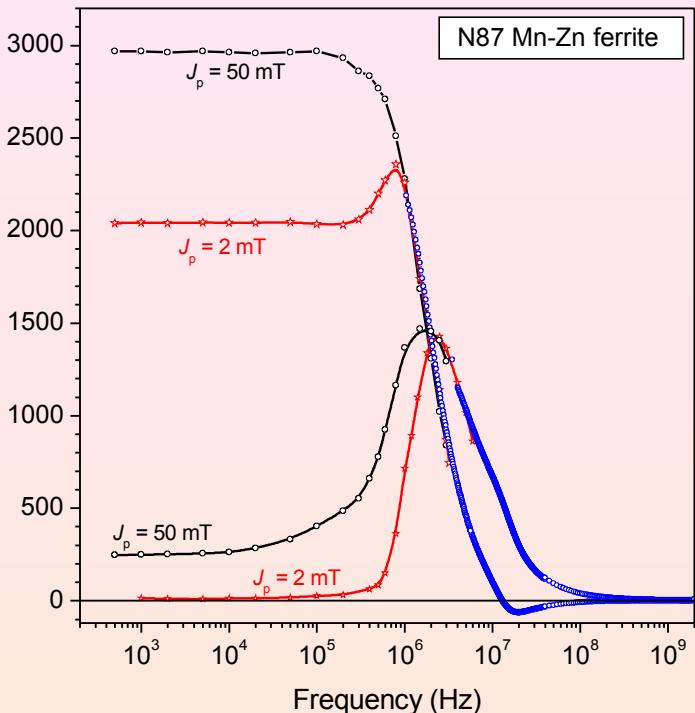
$$\dot{M}(t) = j\omega M_0 e^{j(\omega t - \phi)}$$

$$j\omega \beta M(t) + \alpha M(t) = 2\mu_0 M_s H_0 e^{j\omega t} \quad M(t) = \frac{2\mu_0 M_s H_0 e^{j\omega t}}{\alpha + j\omega \beta}$$

$$M(t) = \frac{2\mu_0 M_s H_0 e^{j\omega t}}{\alpha^2 + \omega^2} - j \frac{2\mu_0 M_s H_0 e^{j\omega t}}{\alpha^2 + \omega^2} \omega \beta$$

$$\chi' = \frac{2\mu_0 M_s}{\alpha} \frac{1}{1 + \omega^2 / \omega_1^2} \quad \chi'' = \frac{2\mu_0 M_s}{\alpha} \frac{\omega / \omega_1}{1 + \omega^2 / \omega_1^2} \quad \tau = 1 / \omega_1$$





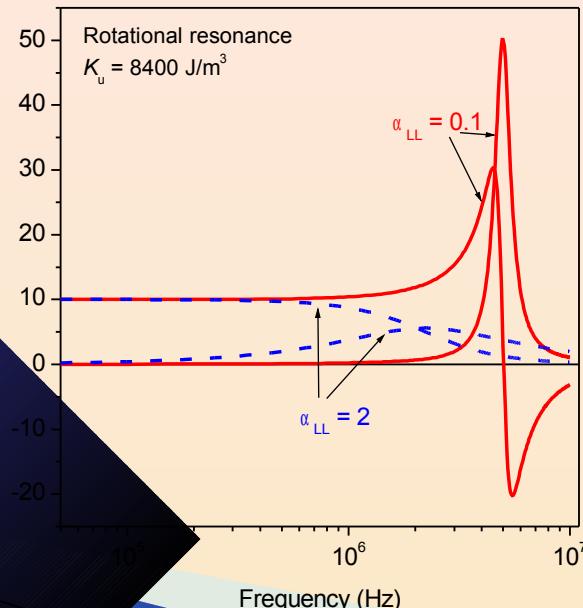
Evidence for resonant precession affecting the permeability behavior versus frequency in soft ferrites.

Landau-Lifshitz-Gilbert equation

$$\partial \mathbf{M} / \partial t = -\gamma \mu_0 \mathbf{M} \times \mathbf{H}_{\text{eff}} + (\alpha / M_s) \cdot \mathbf{M} \times \partial \mathbf{M} / \partial t$$

$$\chi'_{\text{rot}}(\omega) = \frac{\omega_J \omega_H [\omega_H^2 - \omega^2 (1 - \alpha_{LL}^2)^2]}{[\omega_H^2 - \omega^2 (1 + \alpha_{LL}^2)]^2 + 4\alpha_{LL}^2 \omega_H^2 \omega^2}$$

$$\chi''_{\text{rot}}(\omega) = \frac{\omega \omega_J \alpha_{LL} [\omega_H^2 + \omega^2 (1 + \alpha_{LL}^2)]}{[\omega_H^2 - \omega^2 (1 + \alpha_{LL}^2)]^2 + 4\alpha_{LL}^2 \omega_H^2 \omega^2}$$



Bibliography

- G. Bertotti, *Hysteresis in Magnetism*, Academic Press, 1988.
- G. Bertotti, *IEEE Trans. Magn.* 24 (1988) 621.
- G. Bertotti, *J. Appl. Phys.* 57 (1985) 2110.
- J.B. Goodenough, *IEEE Trans. Magn.* 38 (2002) 3398.
- F. Fiorillo, C. Appino, M. Pasquale, in *The Science of Hysteresis* (G. Bertotti and I. Mayergoyz, eds., Academic Press, 2006), vol III, p.1.
- R.H. Pry and C.P. Bean, *J. Appl. Phys.* 29 (1958) 532.
- H.J. Williams, W. Shockley, C. Kittel, *Phys. Rev.* 80 (1950) 1090.
- B.D. Cullity and C.D. Graham, *Introduction to magnetic materials*, Wiley, 2011.
- F. Fiorillo and A. Novikov, *IEEE Trans. Magn.* 26 (1990) 2904.
- G. Bertotti and I. Mayergoyz, eds., *The Science of Hysteresis*, Academic Press, 2006.

