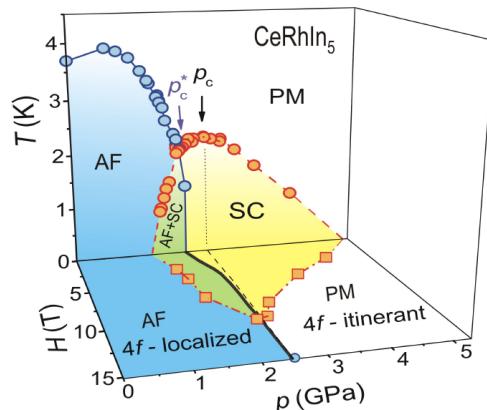


spsmsPHYSIQUE STATISTIQUE,
MAGNETISME ET
SUPRACONDUCTIVITÉ

Strongly Correlated Electron Systems

S. Raymond



SCES : broad topic with vague boundaries

- heavy fermion, transition metal compounds, organic solids
- low dimensional quantum magnetism, frustrated spin systems
- multiferroic
- nanoscale structures, topological insulators
- ultra-cold atoms

In this lecture, focus on “some” SCES that evidence failure of band theory of solids

Occurs “often” in narrow band systems (d and f electrons) : strong relation with magnetism

First part : Overview of electrons in solids : examples of SCES and some theories

Second part : Quantum critical point and unconventional Superconductivity

Tutorial : From Kondo relaxation to quantum criticality

(About dynamical spin susceptibility)

Part I

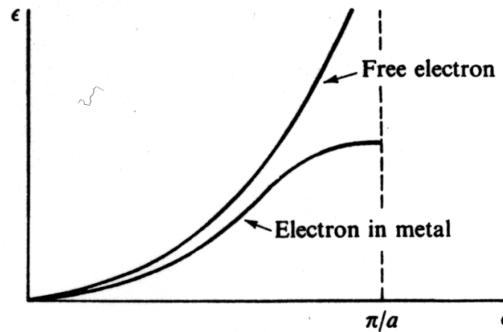
Overview of electrons in solids

- One electron approximation
- Mott insulators
- Heavy fermion systems
- Theoretical models : Hubbard model, Anderson model, DMFT

Electrons in a periodic potential : formation of energy bands

Free electrons

$$\epsilon(k) = \frac{\hbar^2 k^2}{2m_0}$$

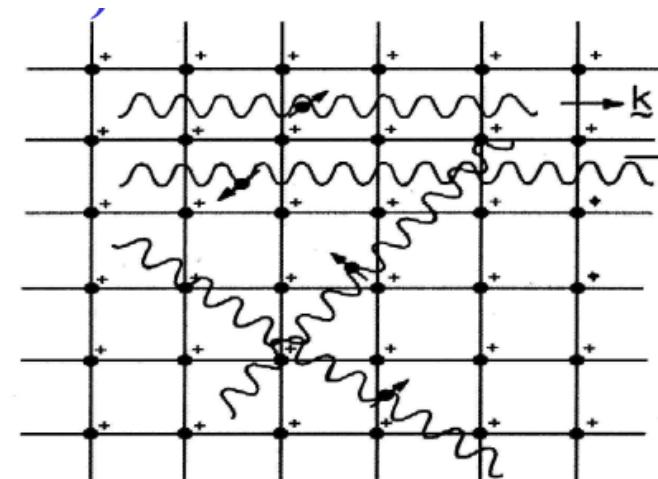


Electrons in a periodic potential

$$m_{\text{band}} = \frac{\hbar^2}{d^2\epsilon/dk^2}$$

Renormalization of the electron mass m_0 due to band formation : m_{band}

Electrons are itinerant : delocalized over the solid
wave-like description (Boch states)



Full Hamiltonian of N electrons with nuclei on a Bravais lattice R :

$$\sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \nabla_i^2 \Psi - Ze^2 \sum_R \frac{1}{|r_i - R|} \Psi \right) + \frac{1}{2} \sum_{i \neq j} \frac{e^2}{|r_i - r_j|} \Psi = E \Psi$$

leads to a complicated many-body problem !

Band theory : the kinetic energy is dominant

One electron approximation (« mean-field » approach) :

Independent electrons in the average (/effective) potential of nuclei and other electrons

Set of Shrödinger equation for one electron : $-\frac{\hbar^2}{2m} \nabla^2 \phi_i + U_{eff}(r) \phi_i = \epsilon_i \phi_i$

One must choose approximation for $U_{eff}(r)$ or for Ψ

Hartree $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \cdots \phi_N(\mathbf{r}_N)$

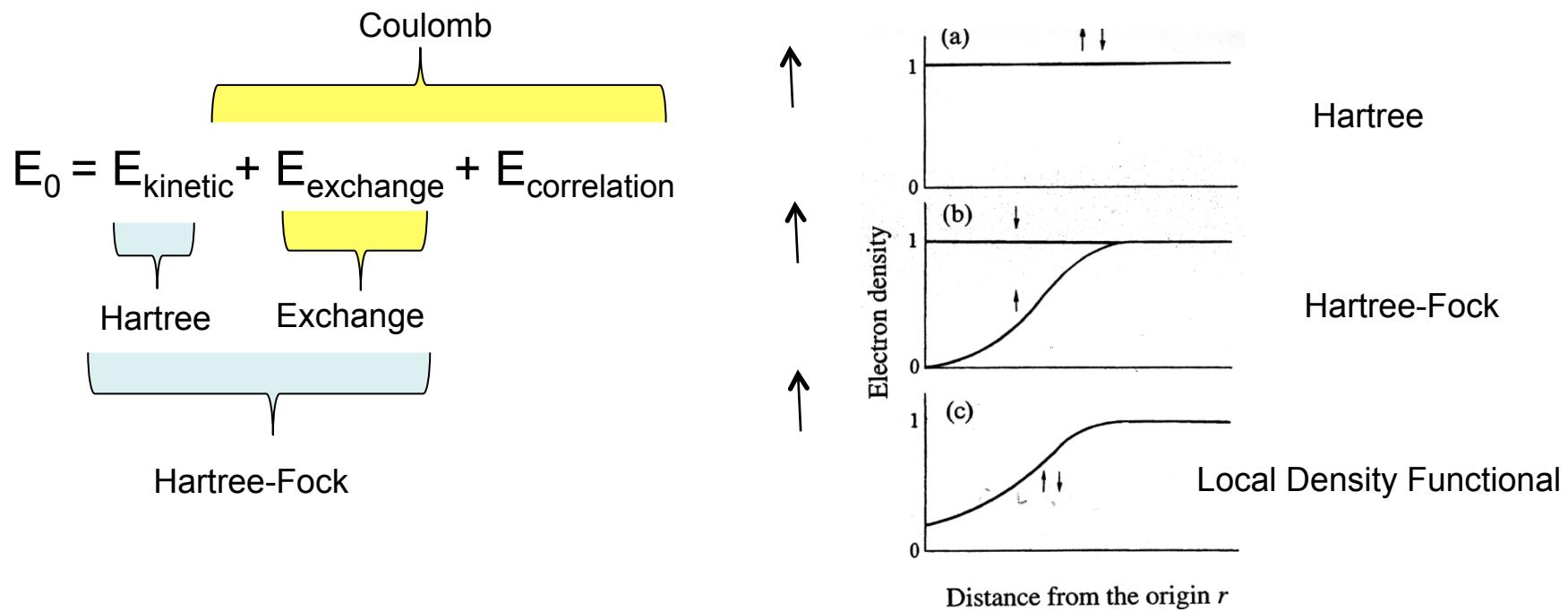
Hartree Fock : Slater determinant of one particle state

Antisymmetric function that fulfilled Pauli principle

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_1(\mathbf{r}_1) & \phi_2(\mathbf{r}_1) & \cdots & \phi_N(\mathbf{r}_1) \\ \phi_1(\mathbf{r}_2) & \phi_2(\mathbf{r}_2) & \cdots & \phi_N(\mathbf{r}_2) \\ \phi_1(\mathbf{r}_3) & \phi_2(\mathbf{r}_3) & \cdots & \phi_N(\mathbf{r}_3) \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(\mathbf{r}_N) & \phi_2(\mathbf{r}_N) & \cdots & \phi_N(\mathbf{r}_N) \end{vmatrix}$$

-> Introduce exchange term in $U_{\text{eff}}(\mathbf{r})$

One electron approximation : different levels of approximations



In SCES : $E_{\text{correlation}}$ is large

e-e interaction must be taken into account beyond one electron approximation
must take better into account the Coulomb repulsion

m^* experimentally determined can be different from calculated m_{band}

1) « small » deviation : example electron-phonon interaction

$$m^* = (1 + \lambda)m_{\text{band}} \quad \lambda \leq 1$$

2) « large » deviation : SCES

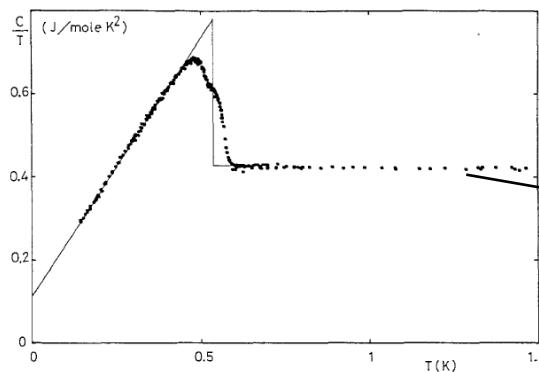
- Example 1 : UPt₃ : a « heavy fermion system » : $m^* \approx 17 m_{\text{band}}$
- Example 2 : La₂CuO₄ a « Mott Insulator » :
Must be a metal according to band theory

Failure of band theory : heavy fermion systems

Electronic specific heat $C = \gamma T$

$$\text{UPt}_3 \quad \gamma = 460 \text{ mJ / molK}^2$$

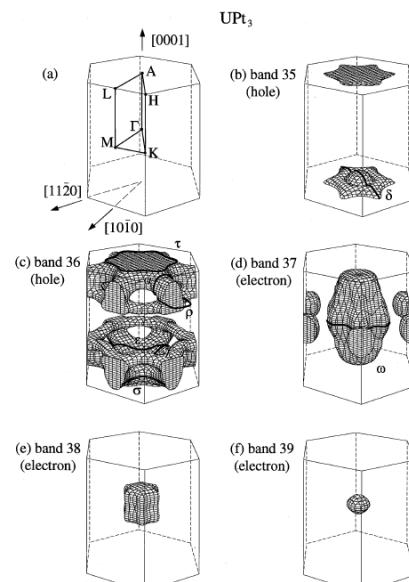
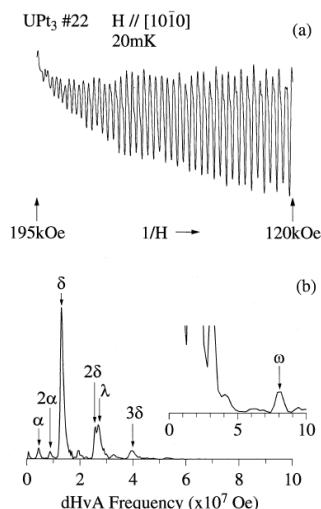
$$(\text{Cu} \quad \gamma = 0.5 \text{ mJ / molK}^2)$$



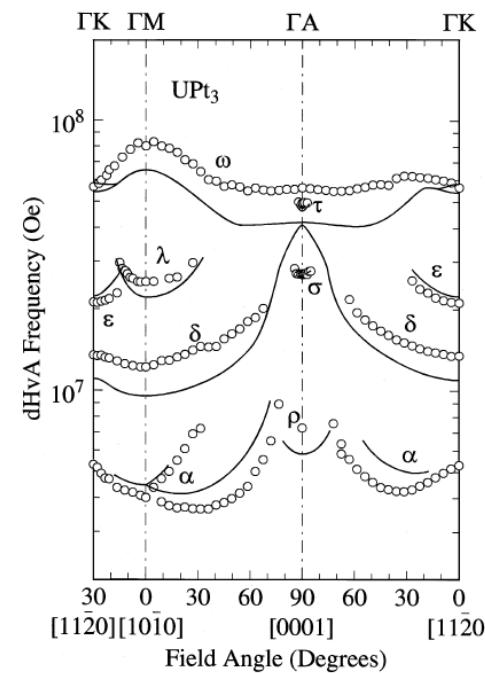
Sulpice 1986

$$\gamma = \frac{m^* k_B^2 k_F}{3\hbar^2}$$

De Haas Van Alphen effect



FLAPW calculation



$$m^* \approx 10-20 m_{\text{band}}$$

Kimura 2000

Although there are strong interactions in UPt₃

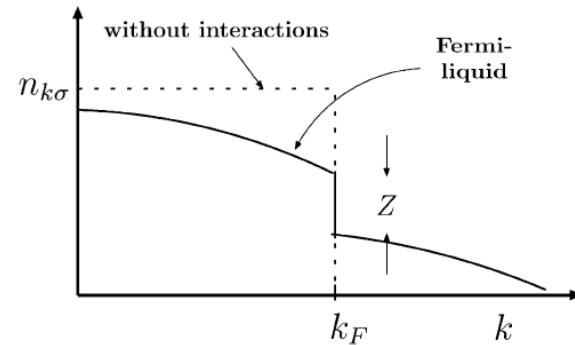
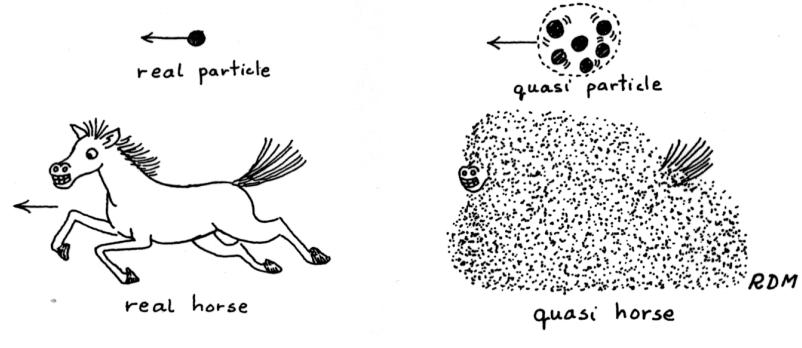
Properties look like free electrons with renormalized mass

Why ?

Properties of free electron gaz determined by elementary excitations

Fermi liquid theory (Landau) :

Excitations in non interacting electron gaz \leftrightarrow Quasiparticles

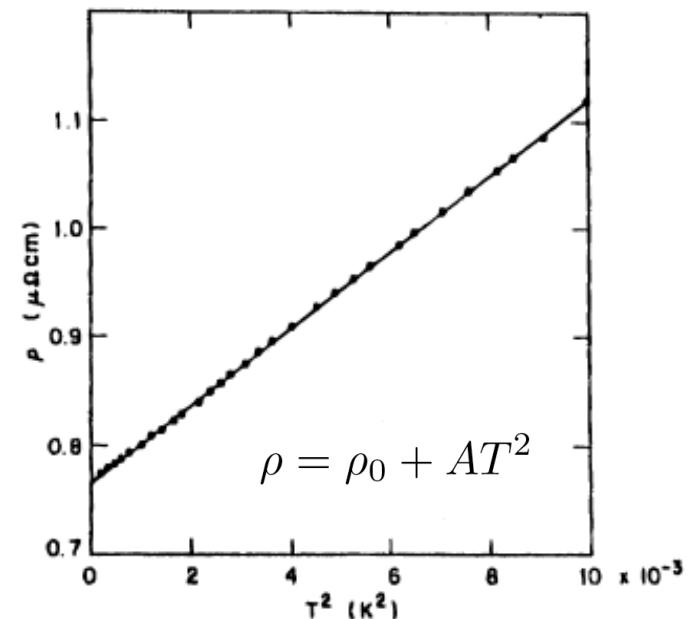
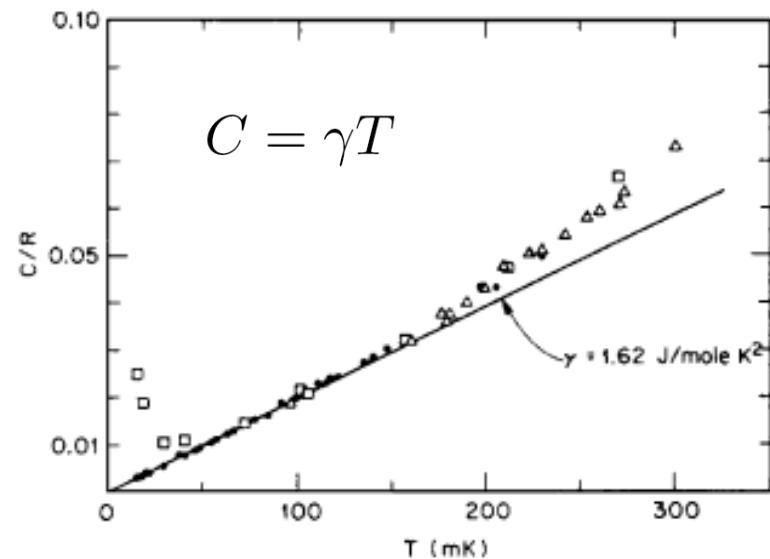


$$\delta E = \sum_k \epsilon_k \delta n_k$$

Quasiparticle (“dressed electron”) : effective mass m* and finite lifetime (lifetime \propto near k_F)

Fermi liquid behavior in some heavy fermion systems (4f and 5f electron systems)

Example : CeAl₃



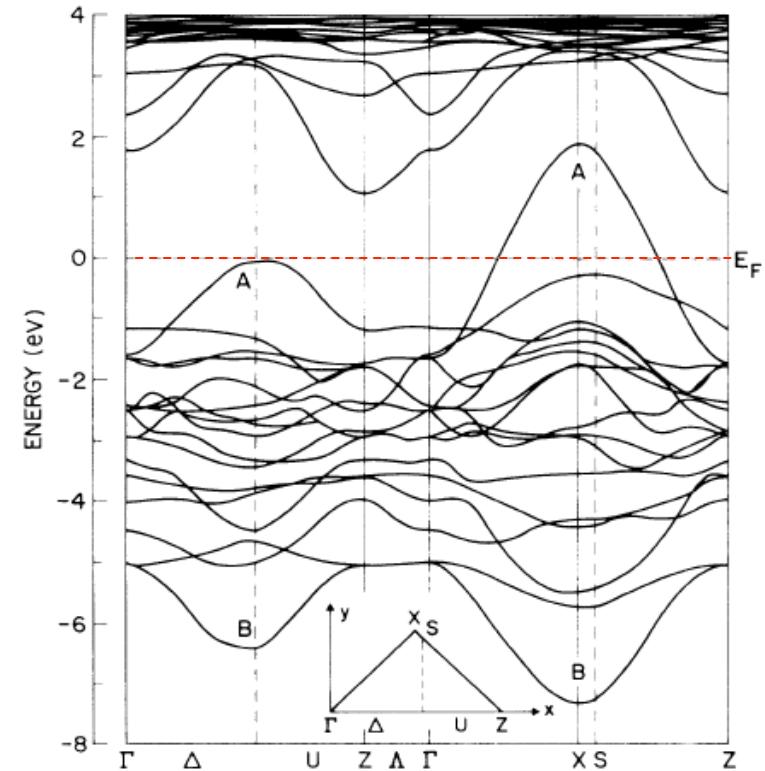
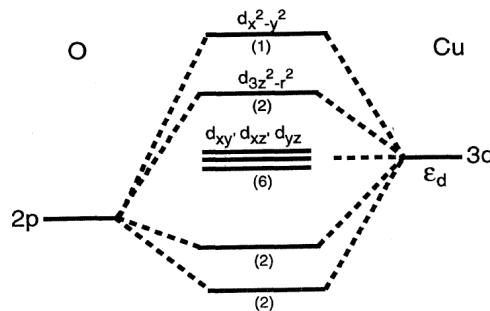
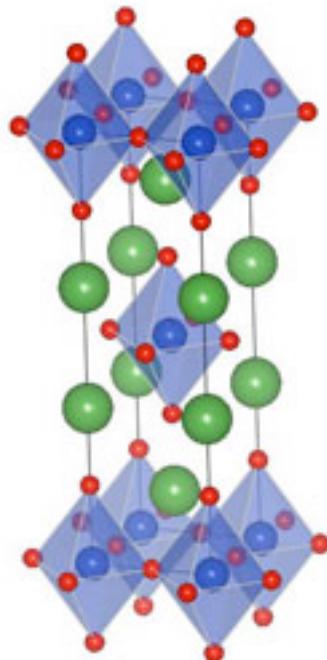
Andres 1975

$$\gamma \propto \sqrt{(A)} \propto m^*$$

Failure of band theory : Mott insulator

One electron approx. : odd number of electron per site : metal

Many such transition metal oxydes are insulating (NiO De Boer and Verwey 1937)



Mattheis 1987

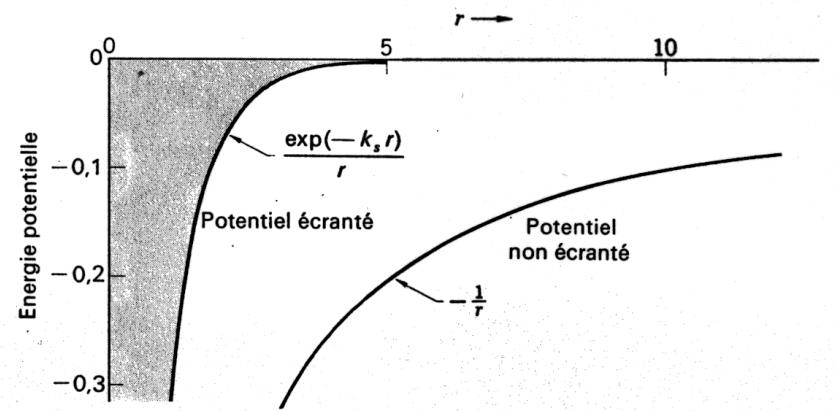
The band is half filled but La_2CuO_4 is an insulator !?!

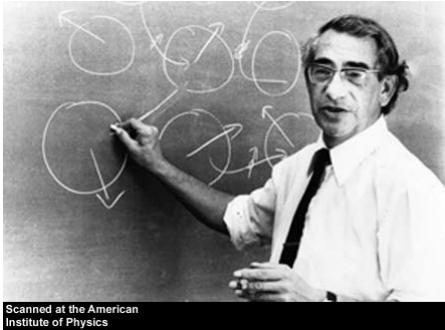
Overview of some theories in order to understand why :

- 1) some transition metal oxydes with odd number of electrons per site are insulating
- 2) m^* is so large in some 4f and 5f electron systems

What fails in one electron approximation is clearly the treatment of e-e interaction

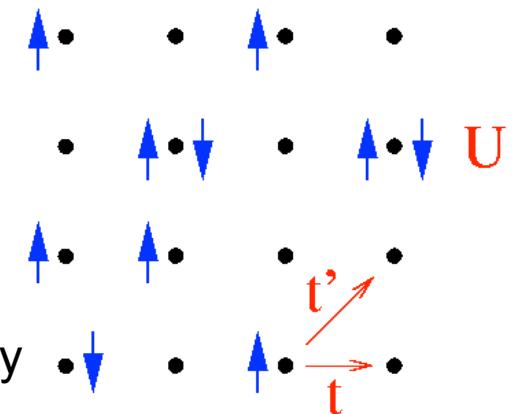
Coulomb potential is screened : short range





Hubbard Model

$$H = \sum_{i,j,\sigma} t_{i,j} c_{i,\sigma}^\dagger c_{j,\sigma} + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

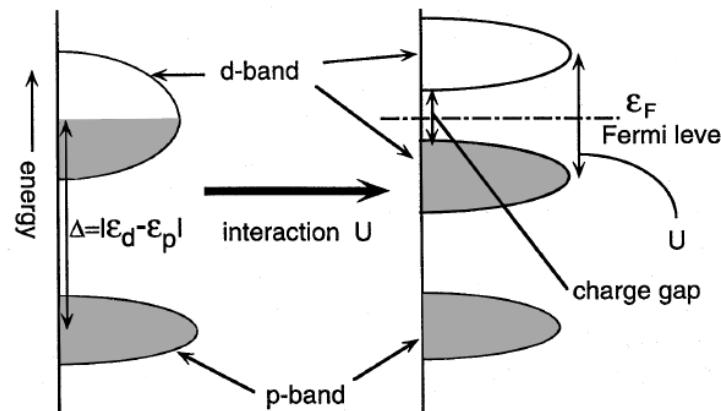


t : hopping integral ($t \sim W$ (bandwidth) $\sim 1/m^*$) : kinetic energy

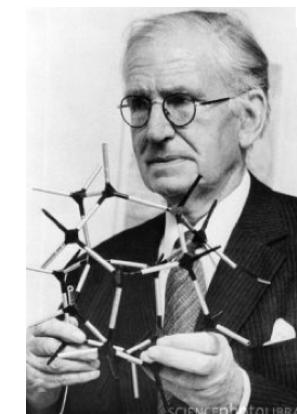
U : on-site Coulomb energy

Short range phenomena in contrast to band calculation

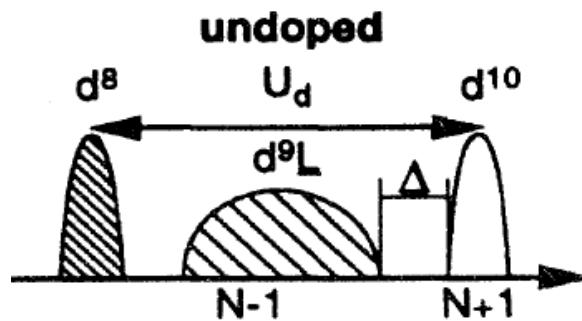
For $U \gg t$, splitting of the band in two « Hubbard bands » of opposite spins



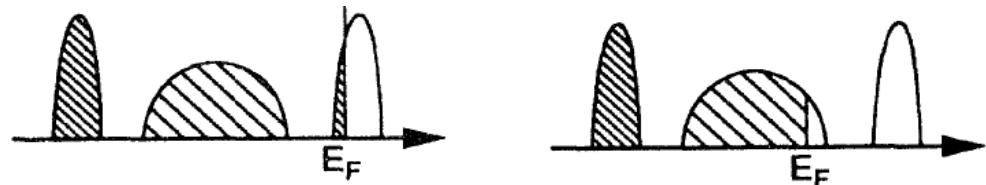
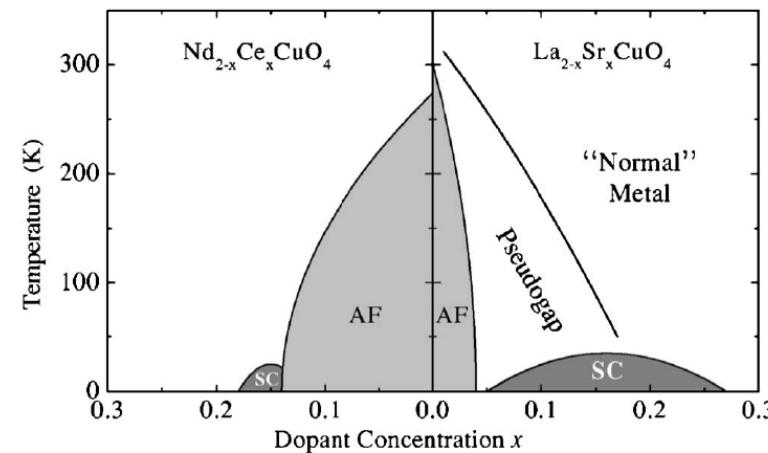
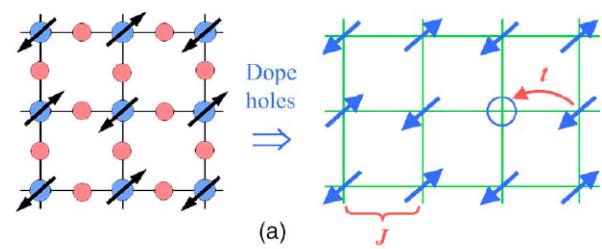
Mott-Hubbard
Insulator



La_2CuO_4 : not a « Mott-Hubbard » insulator but a « charge transfer » insulator



Δ difference between Cu and O orbitals



- Metal-Insulator transition upon doping
- Very important family of cuprate superconductors
- For $U \gg t$ near $\frac{1}{2}$ filling : exchange interaction JS_iS_j with $J = 4t^2/U$

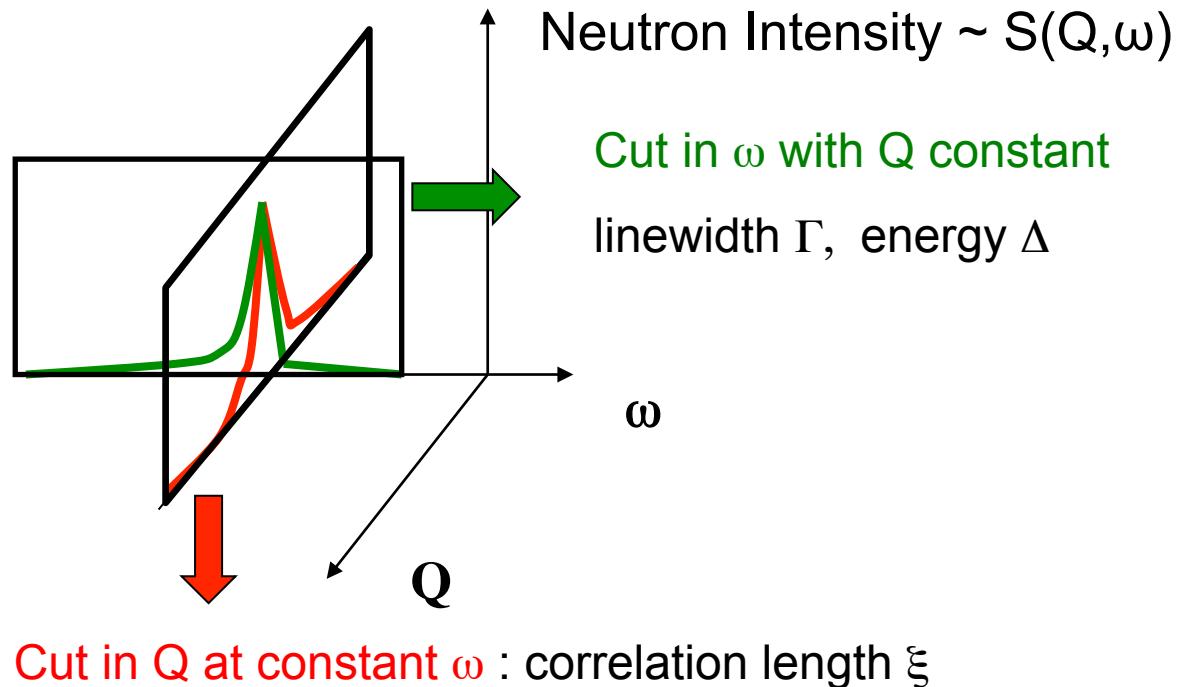
A few words about magnetic inelastic neutron scattering (See Blundell III)

Scattering function :

$$S^{\alpha\beta}(Q,\omega) = \frac{1}{h} \sum_{i,j} \int_{-\infty}^{+\infty} \exp\{iQ(R_i - R_j)\} \langle S_i^\alpha(0) S_j^\beta(t) \rangle \exp\{-i\omega t\} dt$$

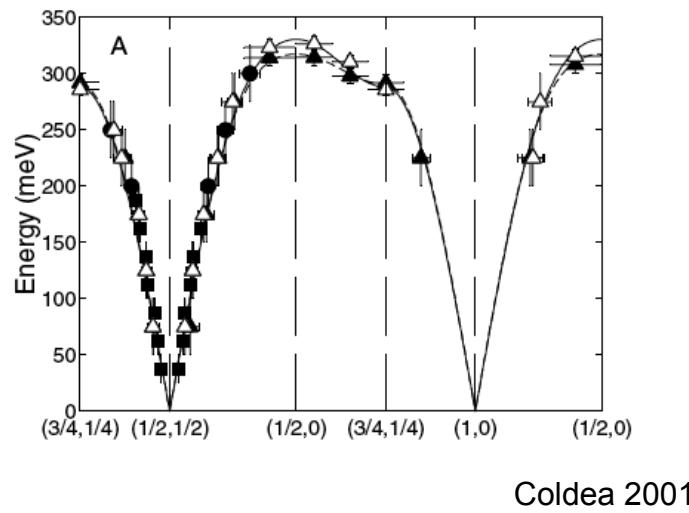
$$S^{\alpha\beta}(Q,\omega) = \frac{1}{1 - \exp(-\omega/T)} \chi''(Q,\omega)$$

Experiment on Three Axis Spectrometer using single crystal sample :



Spin dynamics across the phase diagram

Insulating antiferromagnetic La_2CuO_4 :
Well defined spin wave excitations

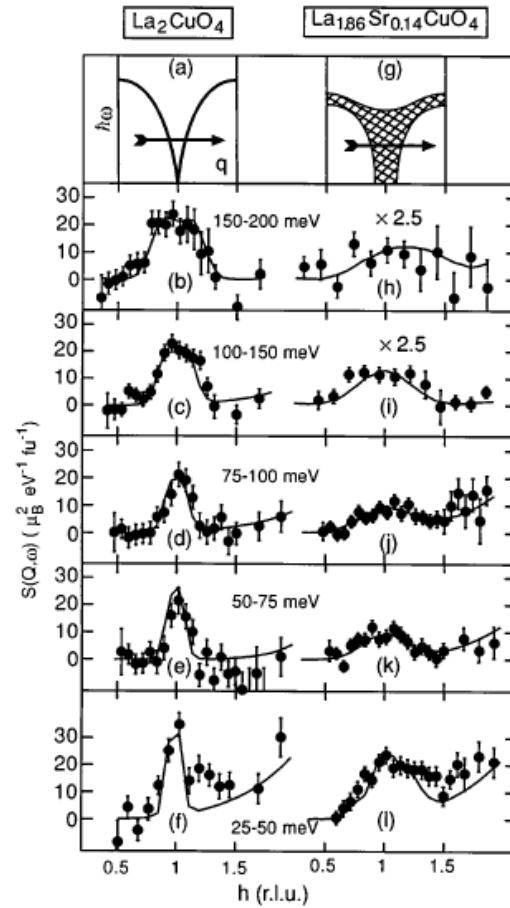


Fit of exchange parameters J , J' , J''

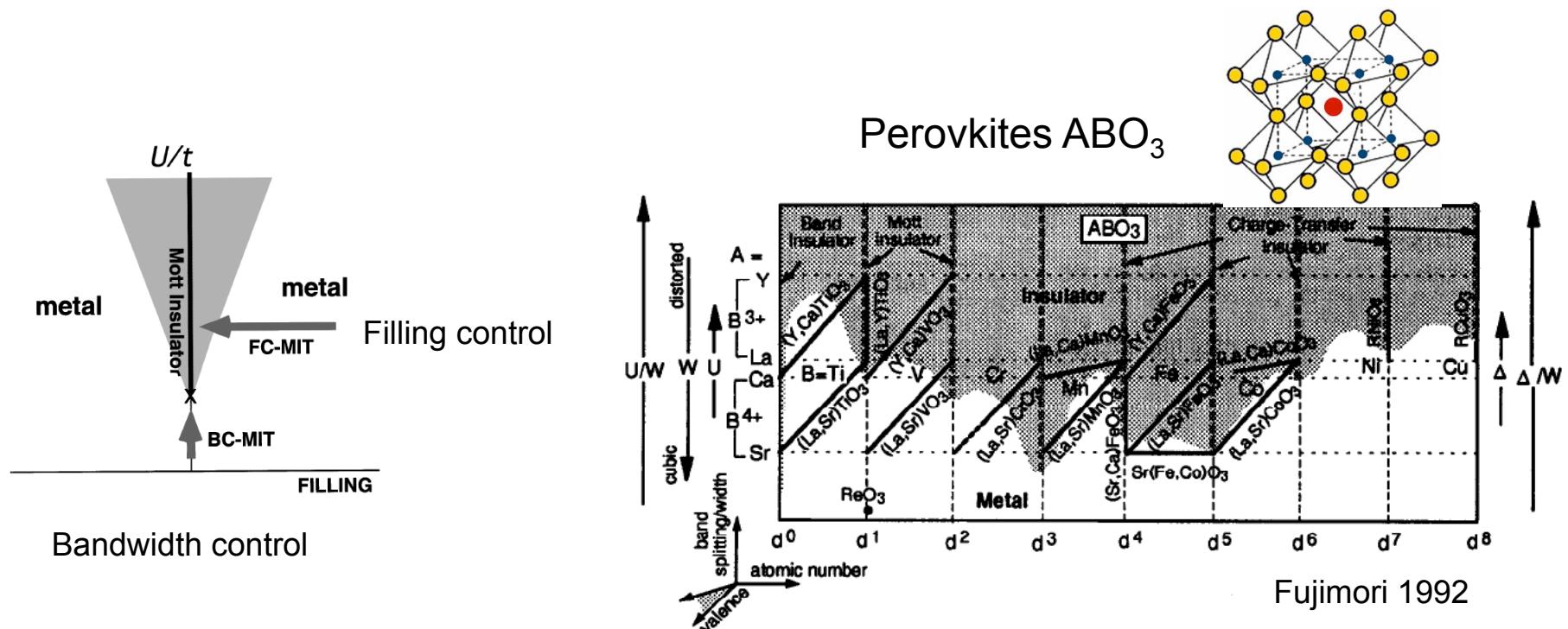
$$\rightarrow t \sim 0.3 \text{ eV}$$

$$U \sim 2 \text{ eV}$$

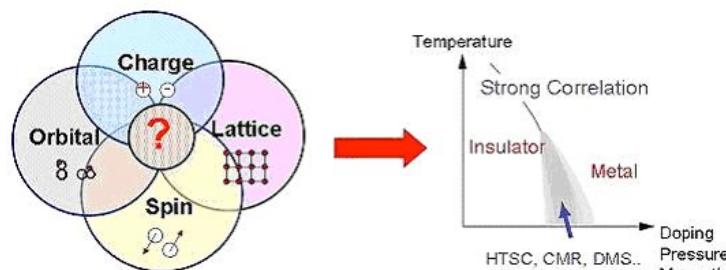
Metallic Phase : $\text{La}_{1.86}\text{Sr}_{0.14}\text{CuO}_4$
Broad magnetic fluctuations



Metal Insulator Transitions

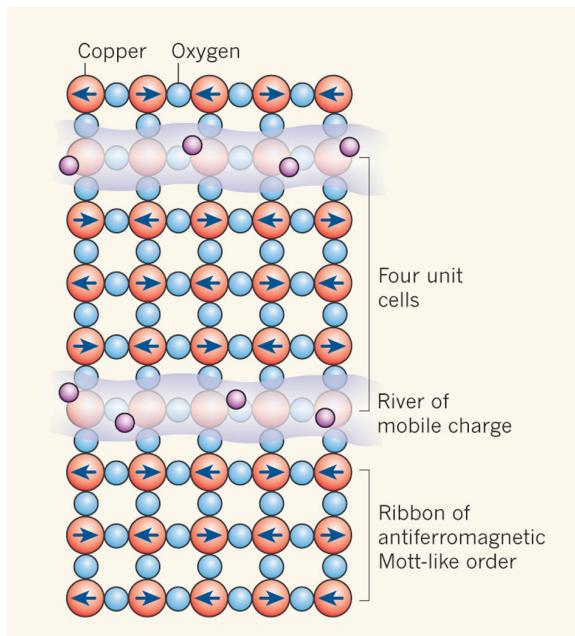


More complexity in transition metal compounds :
orbital degrees of freedom (orbital ordering), charge ordering

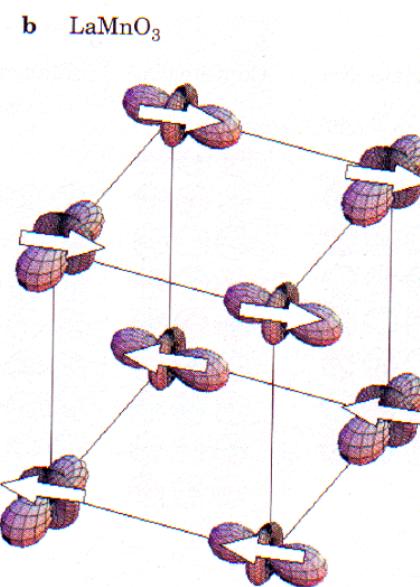


Transition Metal Oxydes : charge, orbital ordering

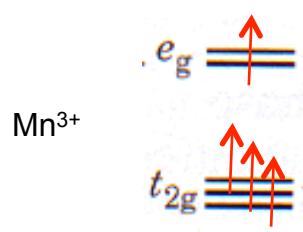
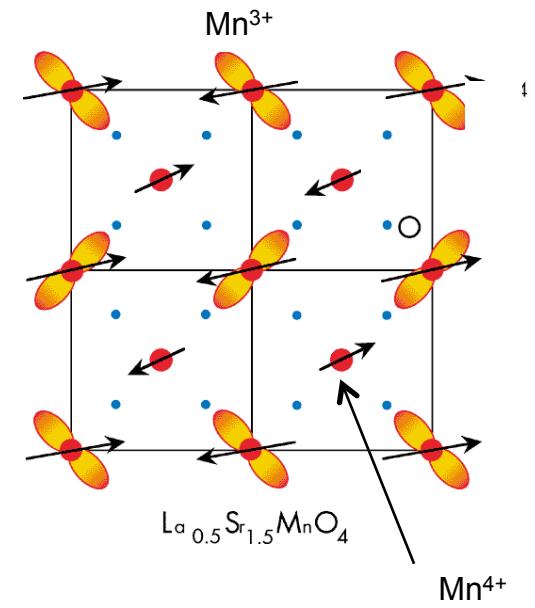
Charge ordering in cuprates



Orbital ordering

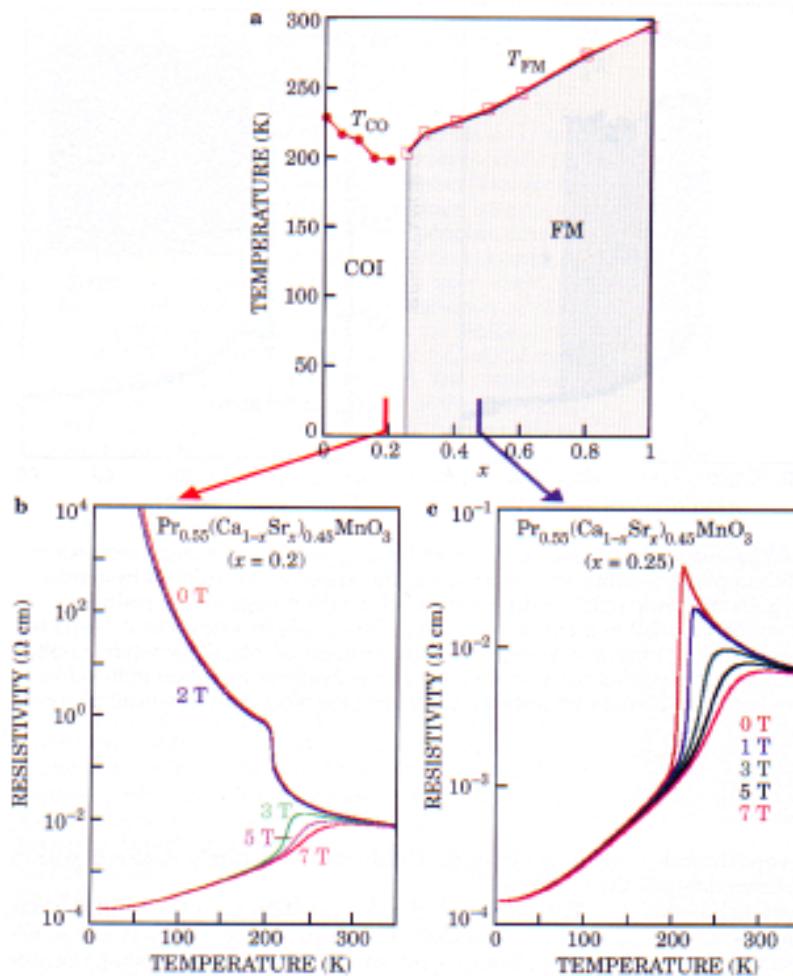


Orbital+charge ordering

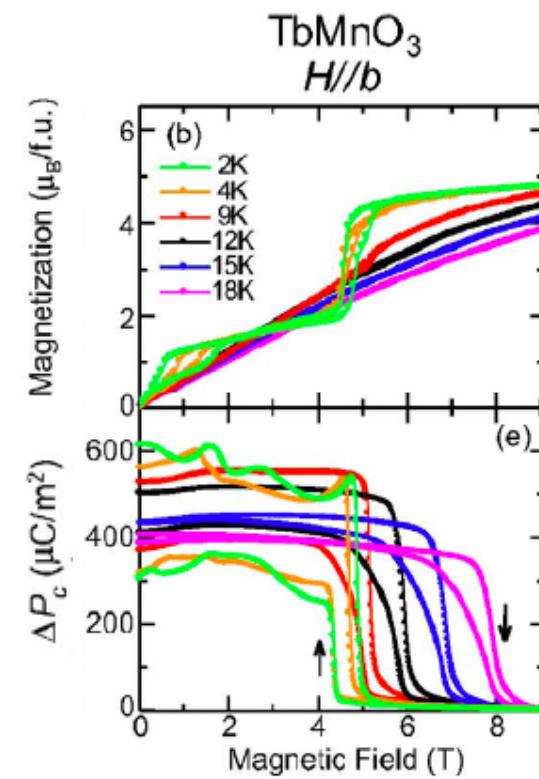


Transition Metal Oxydes : “applications”

Colossal Magnetoresistance



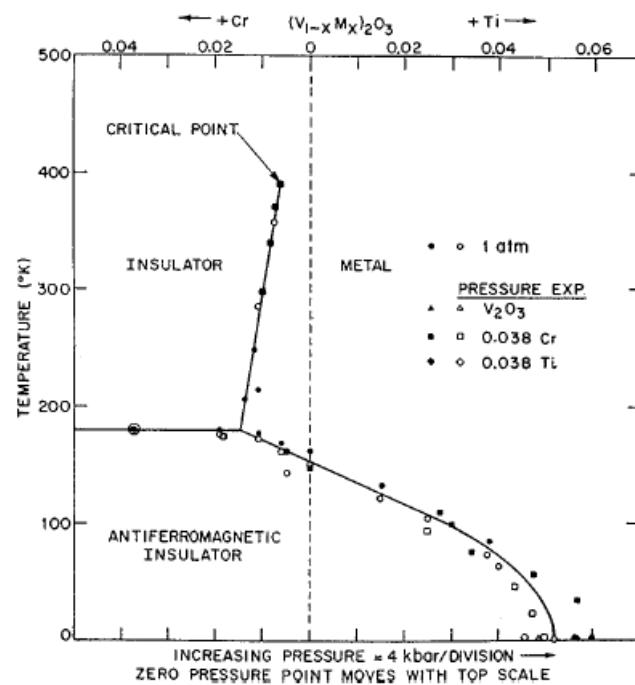
Multiferroics



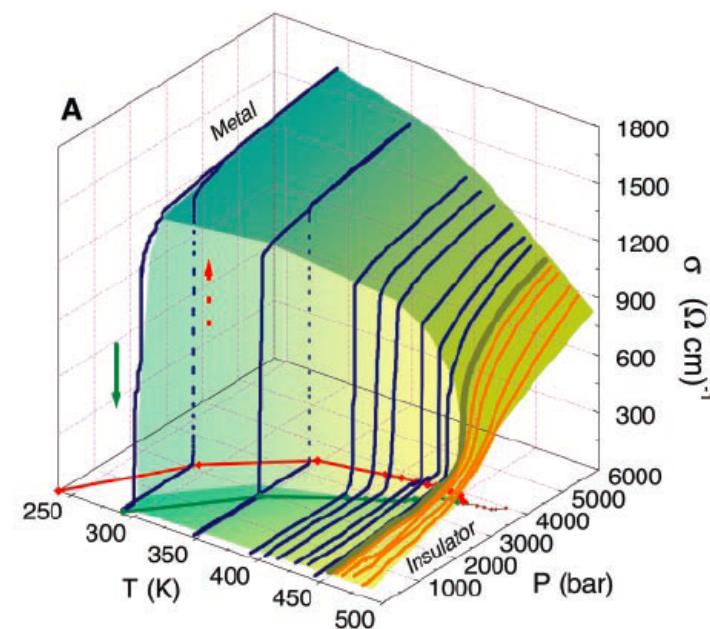
Tokura 2003

Pressure is a way to tune the bandwidth W

Example : V_2O_3



McWhan 1973



Limette 2003

Summary of Wednesday lecture

I - SCES : system for which one electron approximation “strongly” fails

(= one electron in an effective potential)

~ band description of electron

II- Examples :

1) Transition metal oxides like La_2CuO_4

Must be a metal according to band theory but is an insulator !

2) Heavy fermion systems $m^* \sim 10 m_{\text{band}}$ (However simple Fermi liquid description).

III- Introduction of the Hubbard model to explain Mott insulator

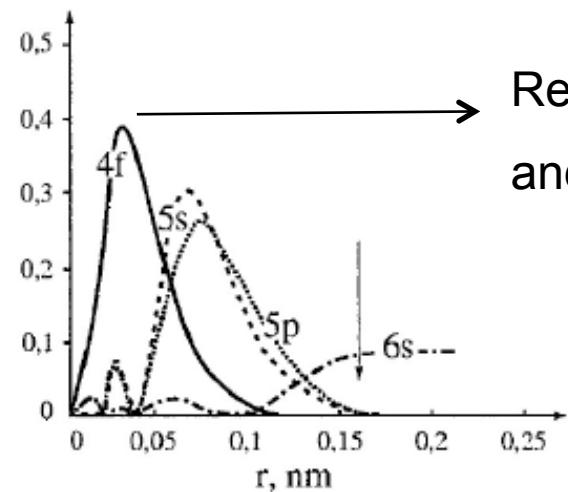
and the corresponding metal insulator transition (t / U)

Receipe to make heavy electrons

You take **f electrons** (rare-earth preferably Ce, Pr, Yb.....)
(or actinides preferably U, Np, Pu...)



You put them in a **conduction electron sea**



Remove this hidding f-electron
and let him go.....



Anderson Model

Simplest model to describe f electron systems

!!! It has nothing to do with Anderson Model of localization !!!

$$H = \sum_{i,j,\sigma} t_{i,j} c_{i,\sigma}^\dagger c_{j,\sigma} + \sum_{i,\sigma} \epsilon_f n_{i,\sigma}^f + V \sum_{i,\sigma} (c_{i,\sigma}^\dagger f_{i,\sigma} + f_{i,\sigma}^\dagger c_{i,\sigma}) + U \sum_i n_{i,\uparrow}^f n_{i,\downarrow}^f$$

↓

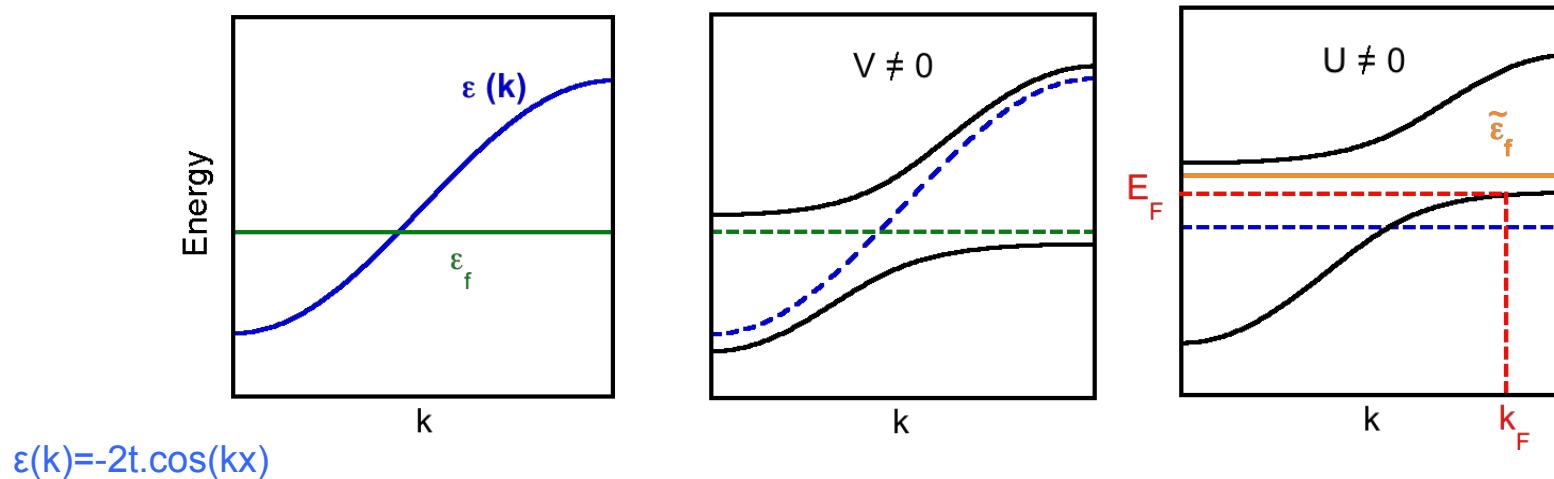
Flat f electron level

↓

Coulomb repulsion among f electrons

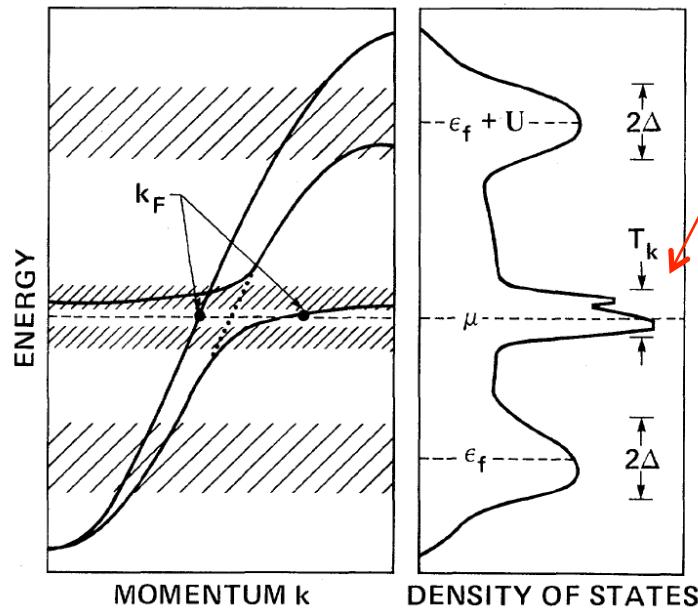
Dispersive band of conduction electron

Hybridization between conduction and f electrons



$U=0$: quasiparticle of mixed f-c character

$U \neq 0$ renormalization quasiparticles bands

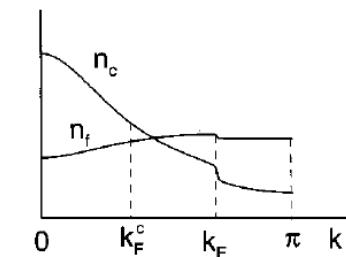
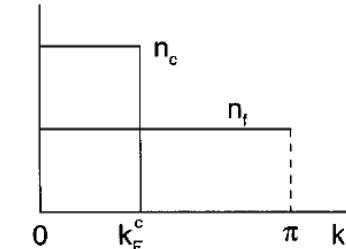


Huge density of state at the Fermi level

f electrons participate to the Fermi surface

$$V_{FS} = 4\pi^3(n_c + n_f)$$

$$m^* / m_{\text{band}} \sim 10-100$$

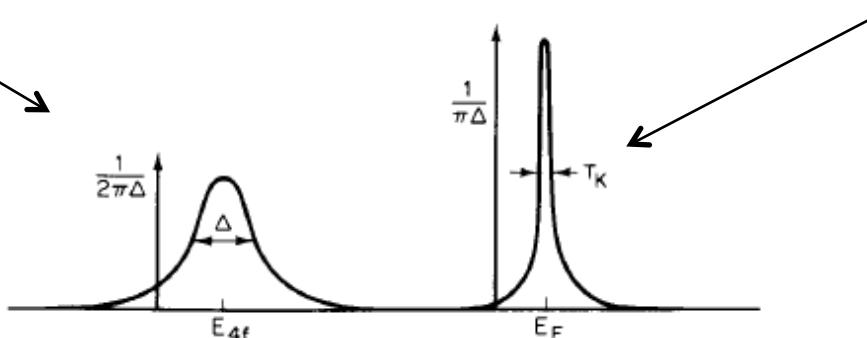


Most of physics already in the impurity Anderson model

(1 localized spin (f or d) in conduction electron sea)

- broadening of the f level : $\Delta = \pi V^2 \rho$

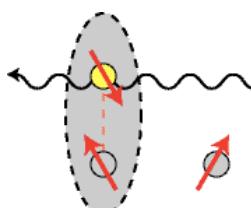
- resonance at the Fermi level at low temperature (m^* large)



Strong connection with the Kondo model : $H = -J_s \cdot S$ $J \sim \frac{8V^2}{U}$

Formation of magnetic singlet at low temperature ($T < T_K$) : screening of the local spin

$$T_K \sim \frac{1}{\rho} e^{-1/\rho J}$$

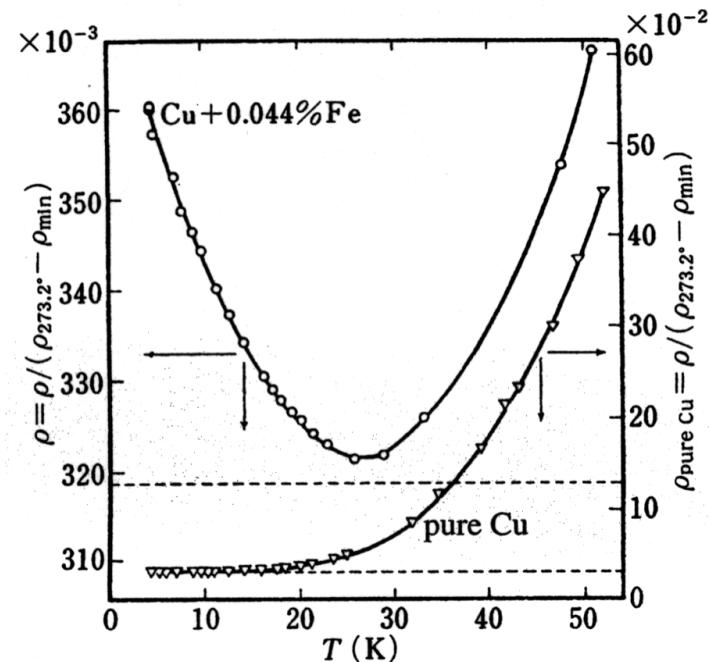


- Typical many-body problem
- Non perturbative solutions

Kondo effect : experimental manifestations

Magnetic Impurities in metal :
Minimum of resistivity

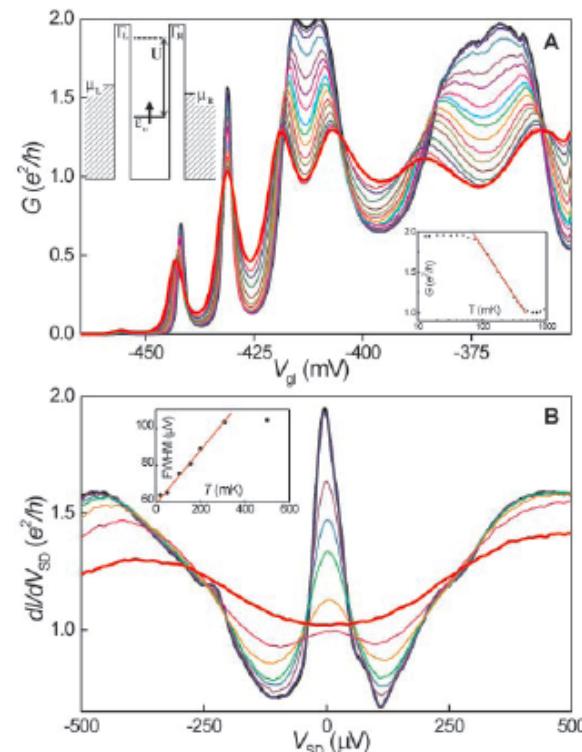
$$\rho_{impurity} \sim J^2(1 - 2J\rho \log(T/W))$$



Pearson 1955

Quantum dots

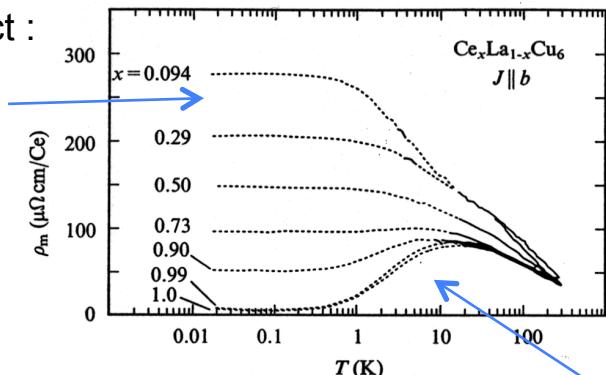
Formation of resonance for odd number of electrons facilitates transfer through the dot



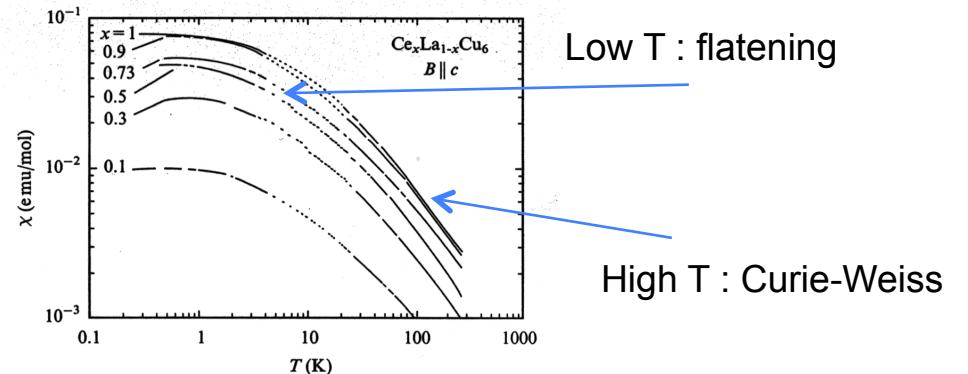
Van der Wiel 2000

From single impurity Kondo effect to the Kondo lattice and heavy fermions

Kondo effect :
impurity
scattering



Heavy fermion : coherence effect



Low T : flattening

High T : Curie-Weiss

Competition between Kondo effect and RKKY interactions : Doniach diagram

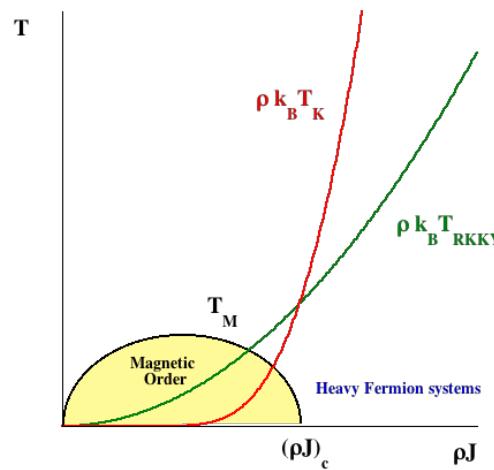
$$T_{RKKY} \sim \rho J^2$$

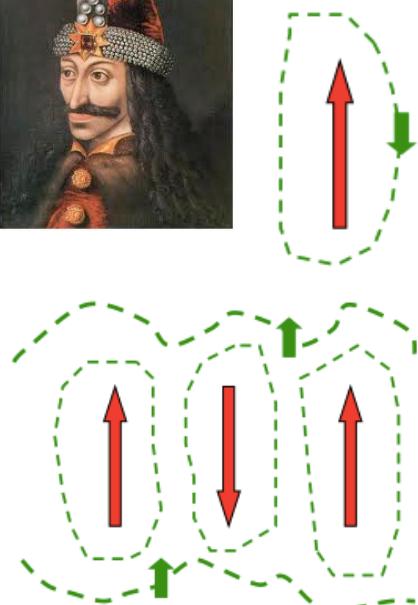
Indirect exchange :
Magnetic order

$$T_K \sim \frac{1}{\rho} e^{-1/\rho J}$$

Singlet formation

Heavy fermion near magnetic instability $(\rho J)_c$





Tutorial : Spin dynamics from Kondo to Heavy Fermion

Kondo effect : relaxation of 4f electrons by conduction electrons

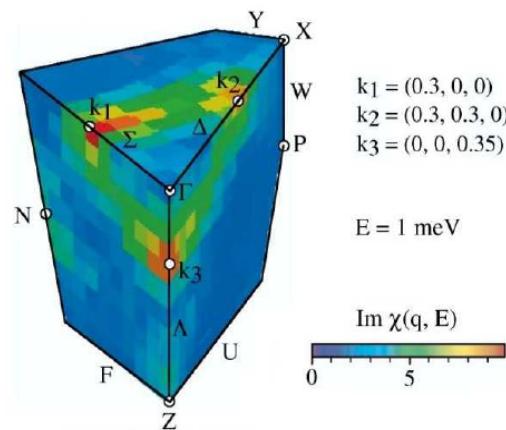
$$\chi_L(\omega) = \frac{\chi_L \Gamma_L}{\Gamma_L - i\omega}$$

Indirect RKKY exchange interaction $J_{ij}S_iS_j$ between 4f electrons

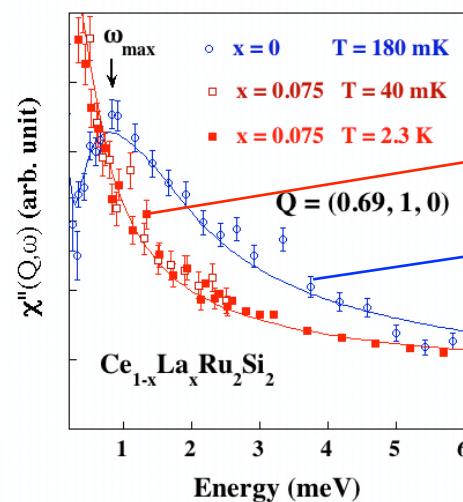
RPA approximation : $\chi(\mathbf{q}, \omega) = \frac{\chi_L(\omega)}{1 - J_{\mathbf{q}}\chi_L(\omega)}$

$Ce_{1-x}La_xRu_2Si_2$: magnetic order for $x > x_c = 0.075$

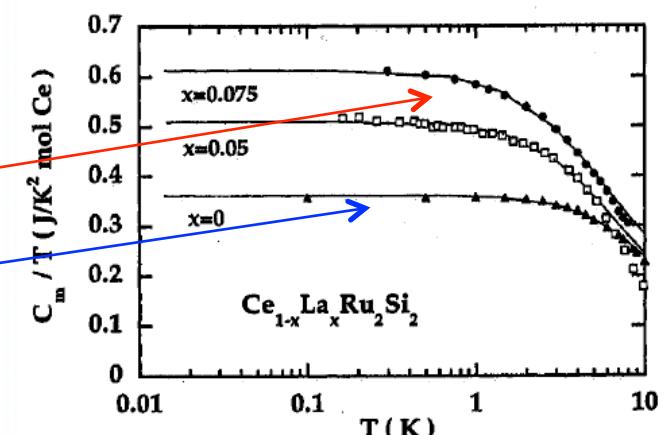
Decrease of relaxation rate \rightarrow increase of m^* when $x \rightarrow x_c$



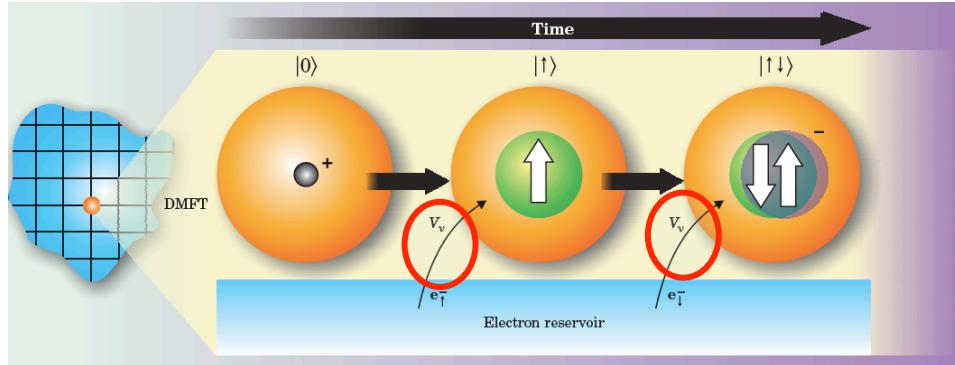
Kadowaki (2004)



ESM Targoviste 2011



Kambe (1996)



Method developed
to treat strong correlations

Metzner, D. Vollhardt
Georges, Kotliar

Lattice problem \leftrightarrow self-consistent impurity model interacting with a bath of excitations

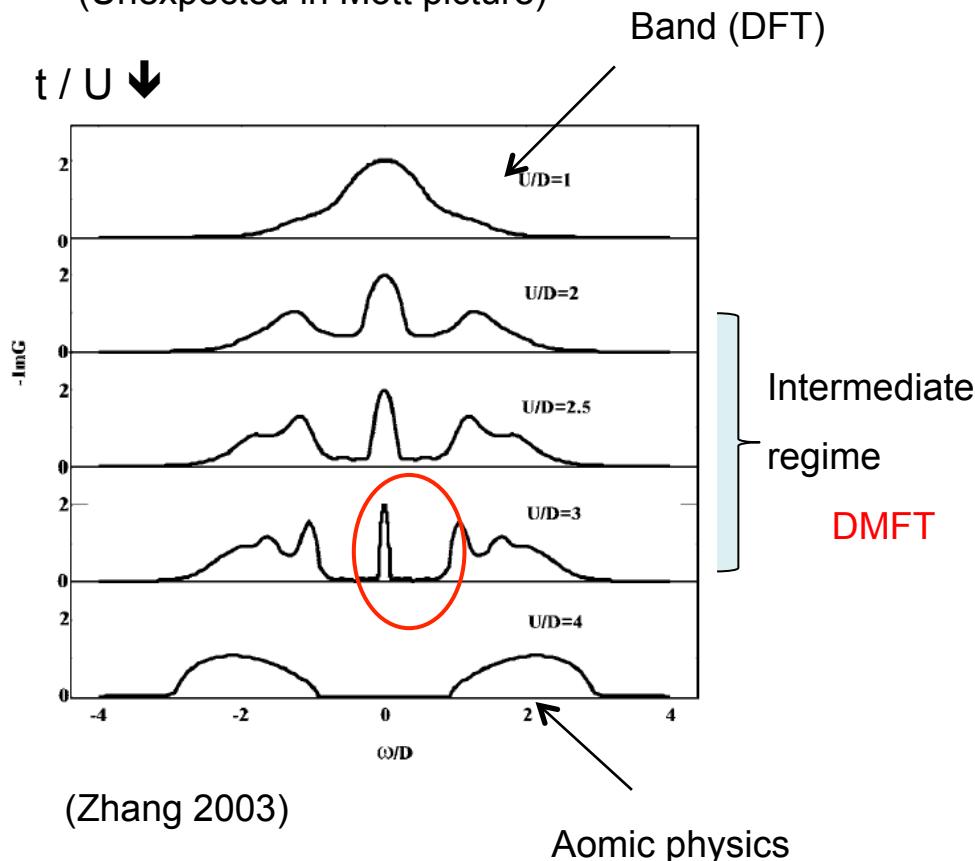
Ex : Hubbard model in $d=\infty$ mapped into an Anderson impurity model + self consistent conditions

More material specific than model Hamiltonian approach

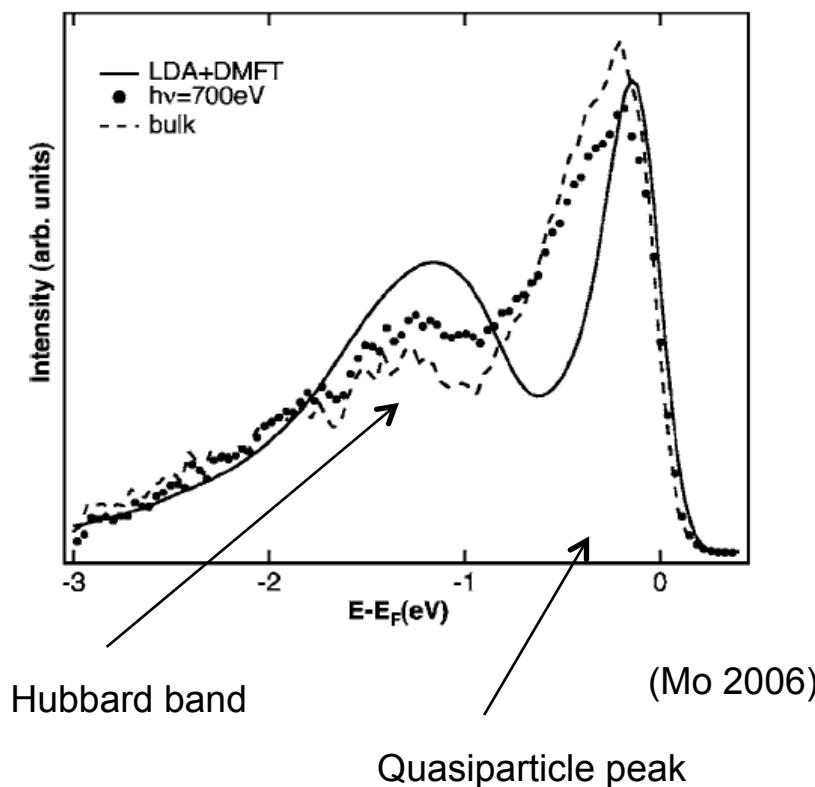
Successfully applied to describe :

- Metal insulator transition
- Volume collapse transitions in 4f (Ce) and 5f (Pu) electron systems

Three peaks structure in DOS
(Unexpected in Mott picture)

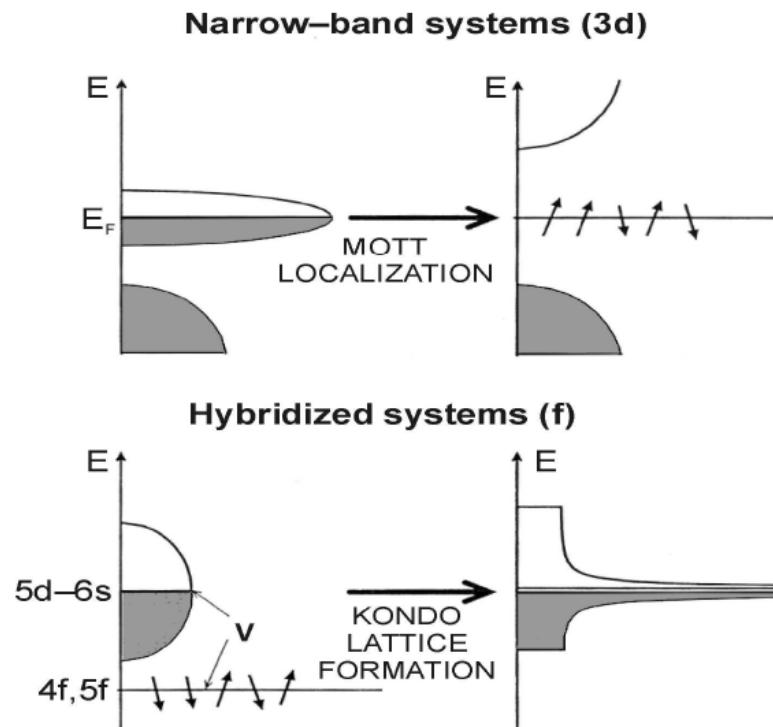


Photoemission V_2O_3
(paramagnetic metallic phase)



SCES : strong deviation from the one electron approximation / band description

- t small (kinetic energy) : narrow band (localized behavior, m_{band} large)
- other energy scales : U (Coulomb), V (hybridization)...enhance m^*



Occurs for 3d(4d) and 4f (5f) electrons as magnetism does !!

Part I : specific models for specific systems

Part II : some commonalities between SCES

Part II

Quantum Critical Point and Unconventional Superconductivity

Many phenomenon described in Part I arise at $T = 0 \text{ K}$ as a function H, P, x

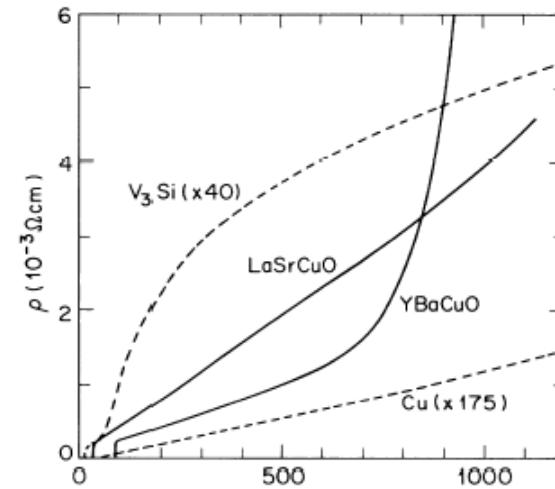
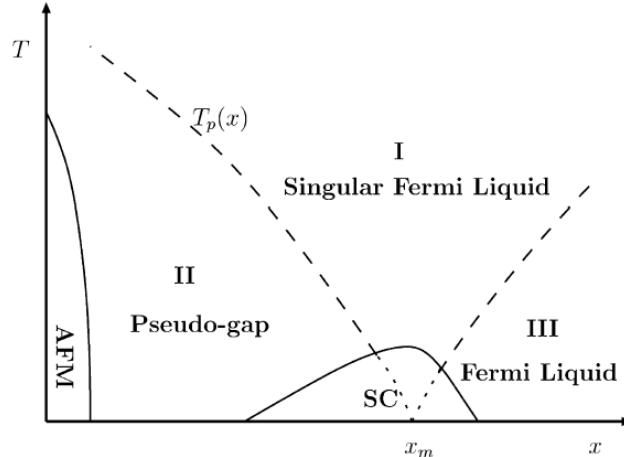
- Metal Insulator transition in oxides
- Magnetic – non magnetic transition in HF systems

For many systems intriguing physics occurs at the $T=0 \text{ K}$ phase transition :

- Non Fermi liquid behaviour
- Unconventional Superconductivity

Quantum Phase Transition : Non Fermi liquid

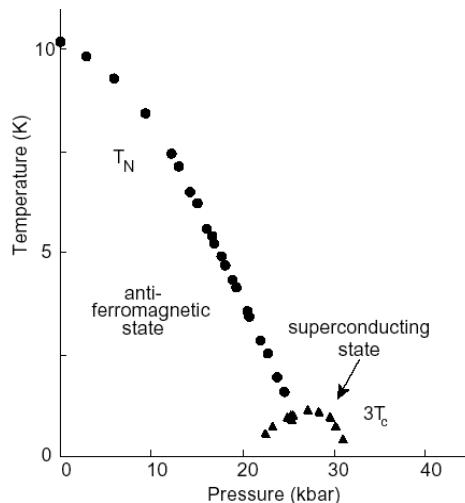
Cuprates



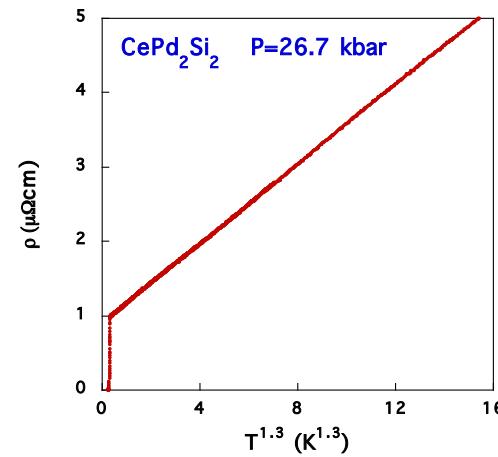
$$\rho \sim T$$

Gurvitch (1987)

Heavy Fermions



Mathur (1998)

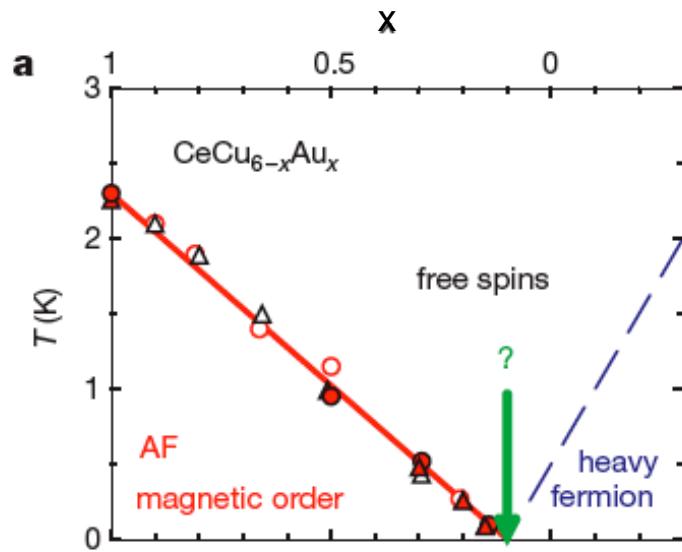


$$\rho \sim T^{1.3}$$

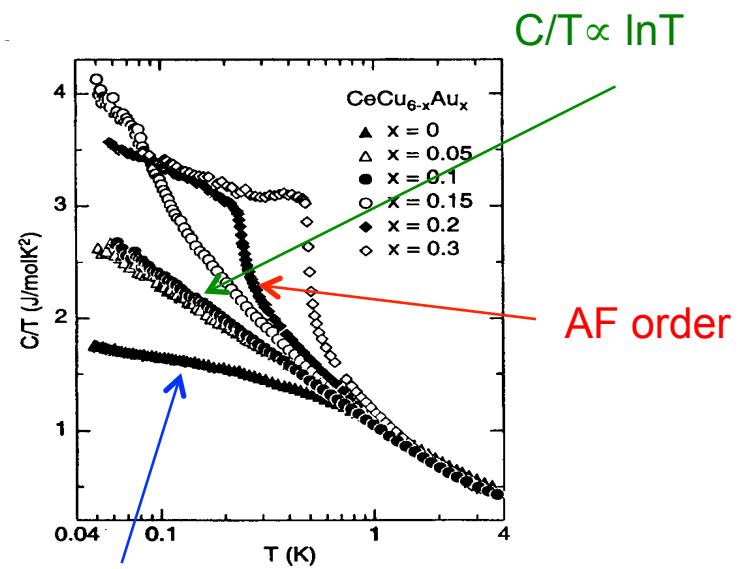
Sheikin (2001)

Quantum Phase Transition : Non Fermi liquid

Non Fermi liquid behaviour



Quantum critical



Breakdown of Fermi liquid

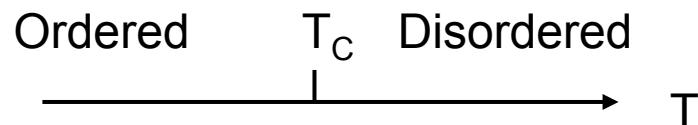
m^* diverge at x_c

H.v. Löhneysen

Classical phase transition

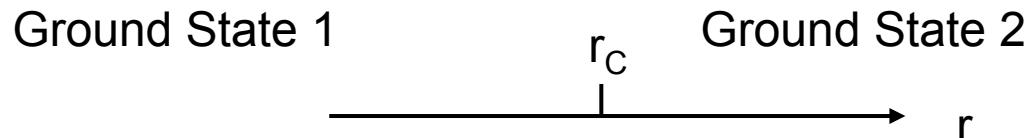
Transition from ordered to disordered phase with T

= under the action of **thermal fluctuations**



Quantum Phase Transition

Transition between two ground states at $T = 0$ K with parameter r (H , P , x) that controls **quantum fluctuations**



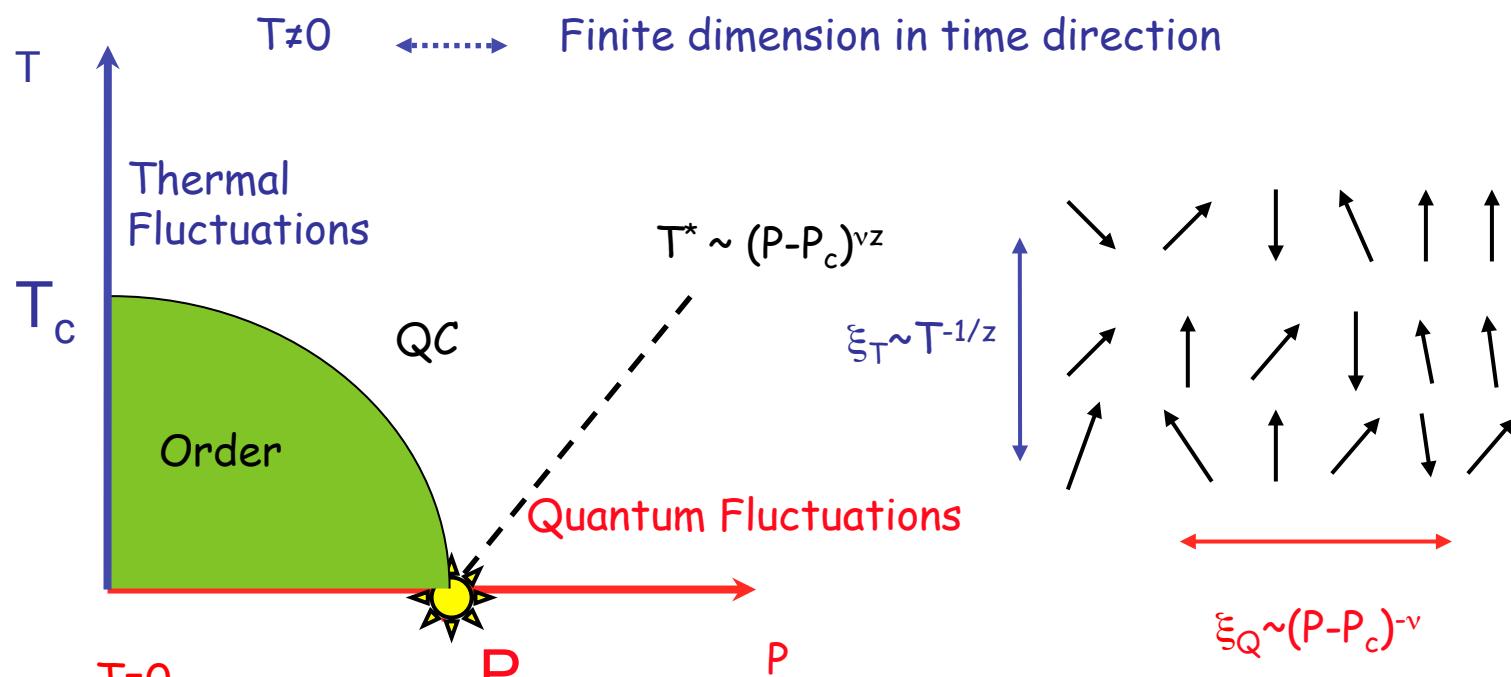
Quantum fluctuations \leftrightarrow uncertainty principle

$$\Delta A \Delta B > |<[A,B]>|/2$$

"If A is very well-defined, this lead to large fluctuations on B"

Uncertainty principle \rightarrow operators do not commute

\rightarrow You cannot separate statics and dynamics !!



Simple Problem : Quantum Physics at $d \leftrightarrow$ Classical physics at $d+z$

Sachdev (99), Continentino (01), Sondhi (97), Votja (04), Varma (02), Si (04), Senthil (04)...

Effective dimension of the T=0 K problem

Dynamical exponent

$$d_{\text{eff}} = d + z$$

↓
Dimension of the system
↑
↓

Mean field behavior is more easily achieved than for classical phase trans^{on} !!!
Landau theory applies for $d_{\text{eff}} > d_c$ (upper critical dimension)

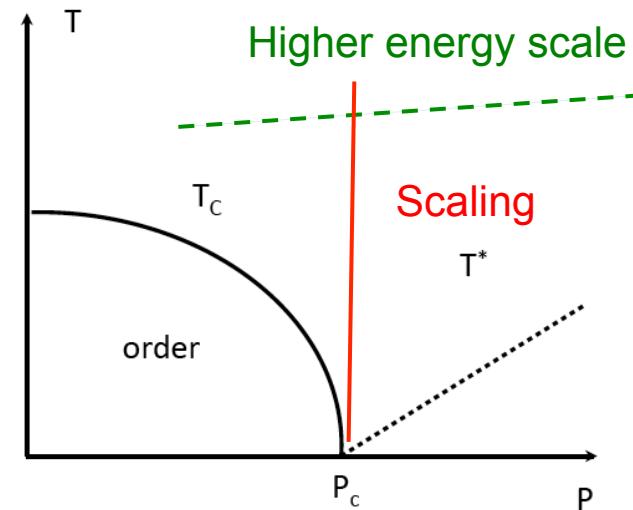
At p_c , the only energy scale is temperature itself !

→ Microscopic properties scaling in ω / T leads to Non Fermi liquid behavior

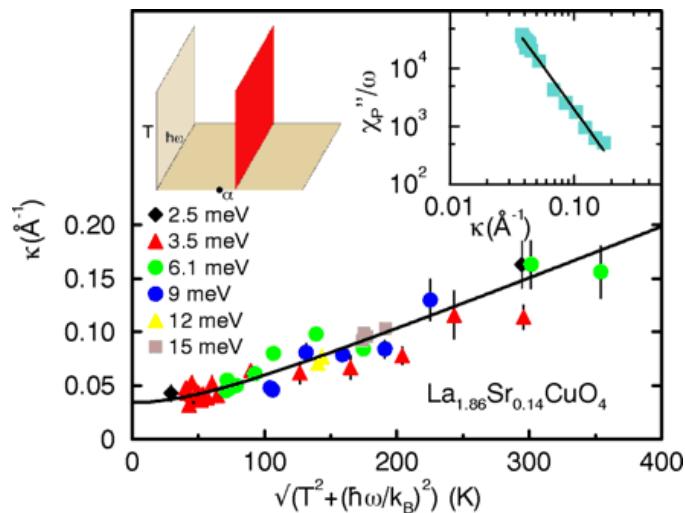
Scaling law for the dynamical spin susceptibility :

$$\chi(q, \omega) = \frac{a}{T^{(2-\eta)/z}} \Phi\left(\frac{q}{T^{1/z}}, \frac{\hbar\omega}{T}\right)$$

→ observation on large range of temperature

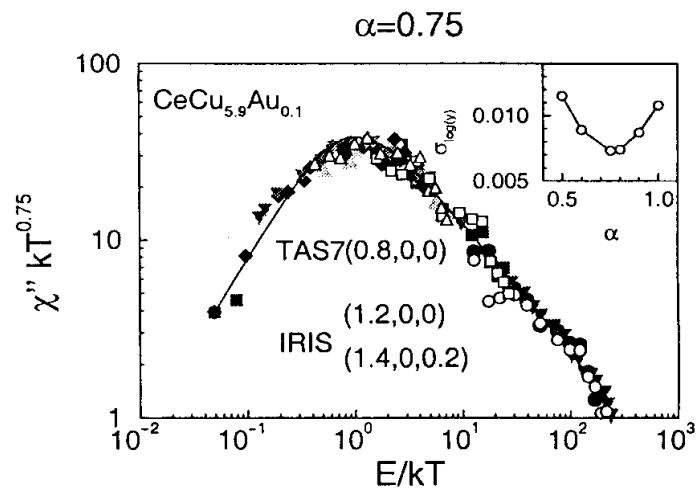


$\text{La}_{1.86}\text{Sr}_{0.4}\text{Cu O}_4$ ($d = 2$, $z=1$)



Aeppli (1997)

$\text{CeCu}_{5.9}\text{Au}_{0.1}$ ($d=3$, $z=2$)



Schröder (2000)

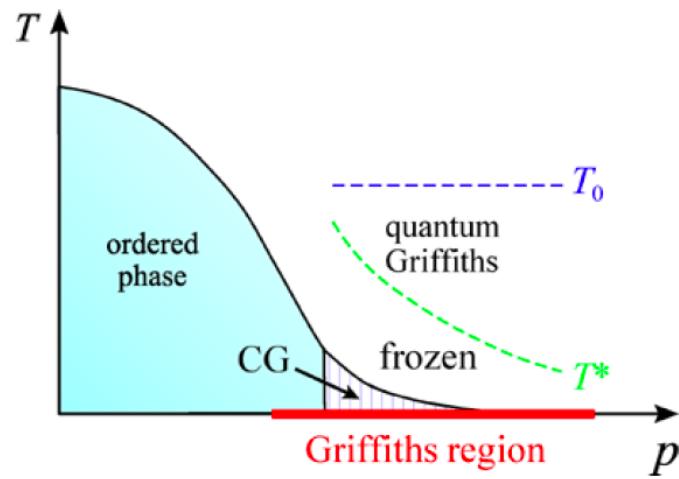
For heavy fermion systems $d_{\text{eff}}=5 > d_c=4$ (upper critical dimension)

→ **Scaling is not expected !! Subject of many works and many fights !**

Some HF systems do not show scaling at P_c

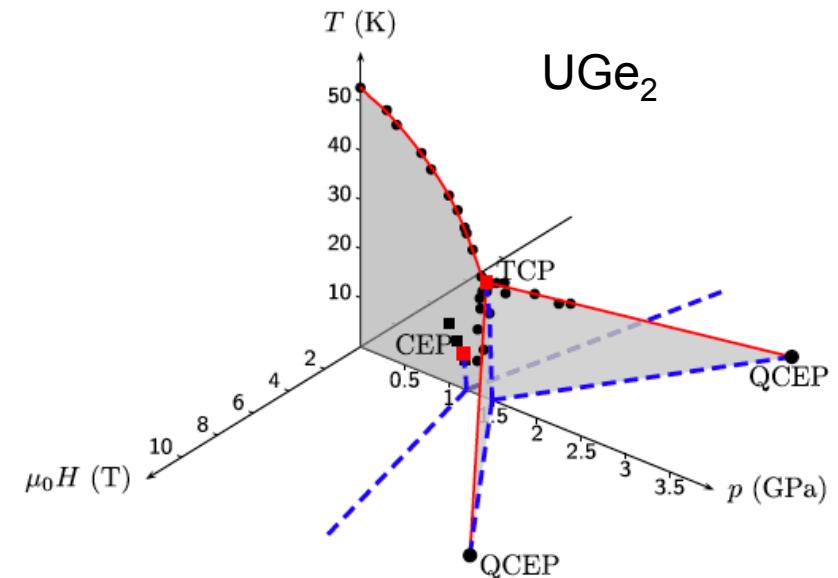
First order / second order

disorder



Vojta (2010)

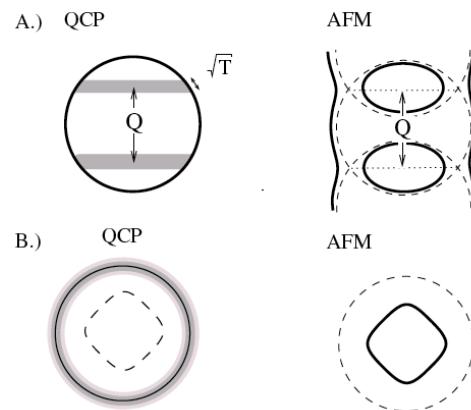
intrinsic property



Taufour (2010)

At P_c , scaling leads to non conventional behaviour (e.g. Non Fermi liquid)

Microscopic theory ? Breakdown of Fermi surface....



Above d_c : no scaling, some controversy for HF systems

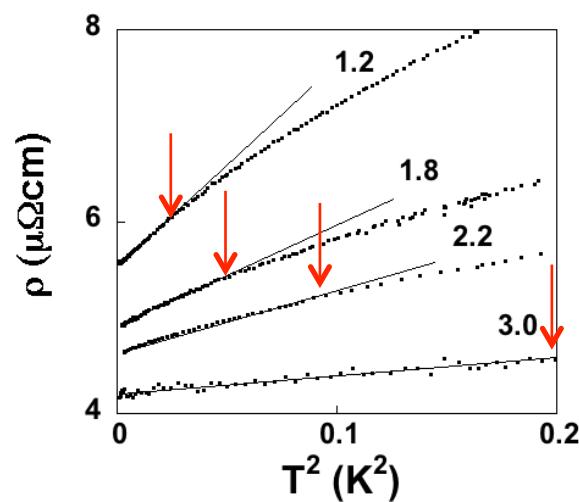
QCP = one point / one singularity

Can you achieve it ? (material purity, step in P , x , H)

For $P > P_c$

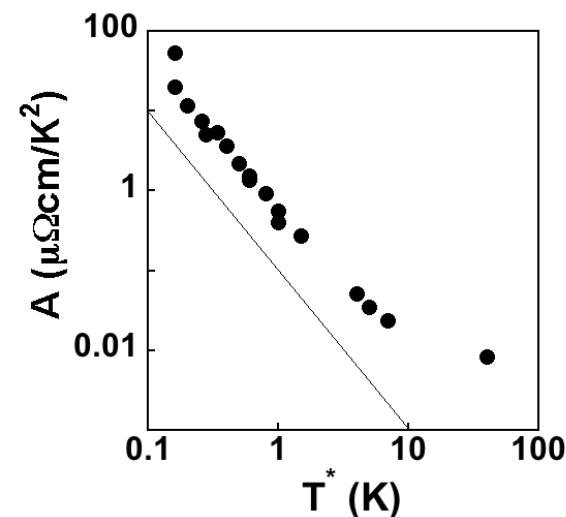
- There is a scale $T^* \sim (P - P_c)^{vz}$
- Physical properties governed $k_B T^*$ below T^*

CeCu₆



$$\rho = \rho_0 + AT^2 \text{ below } T^*$$

$$m^* \sim \sqrt{A} \sim 1/T^*$$

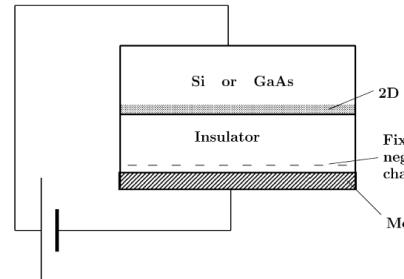


Raymond (2000)

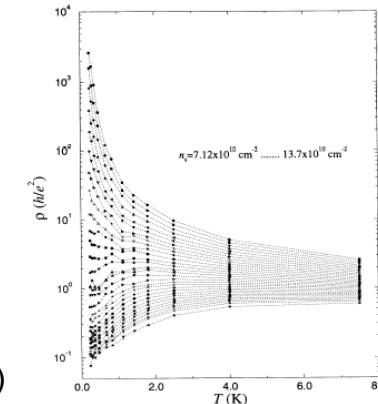
Other examples of QPT

- 2d electron gas (MOSFET)

Metal - Insulator

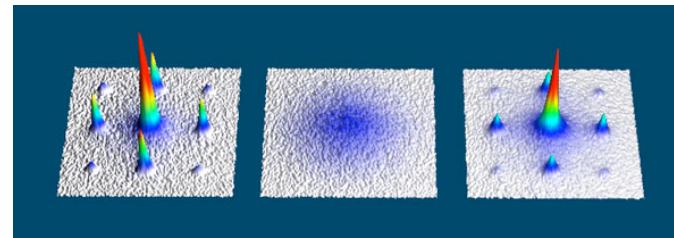


Kravchenko (1995)



- ultra cold atoms trapped in an optical lattice

Superfluid – Mott insulator



Greiner (2002)

As for classical phase transition, MAGNETISM is THE ideal PLAYGROUND

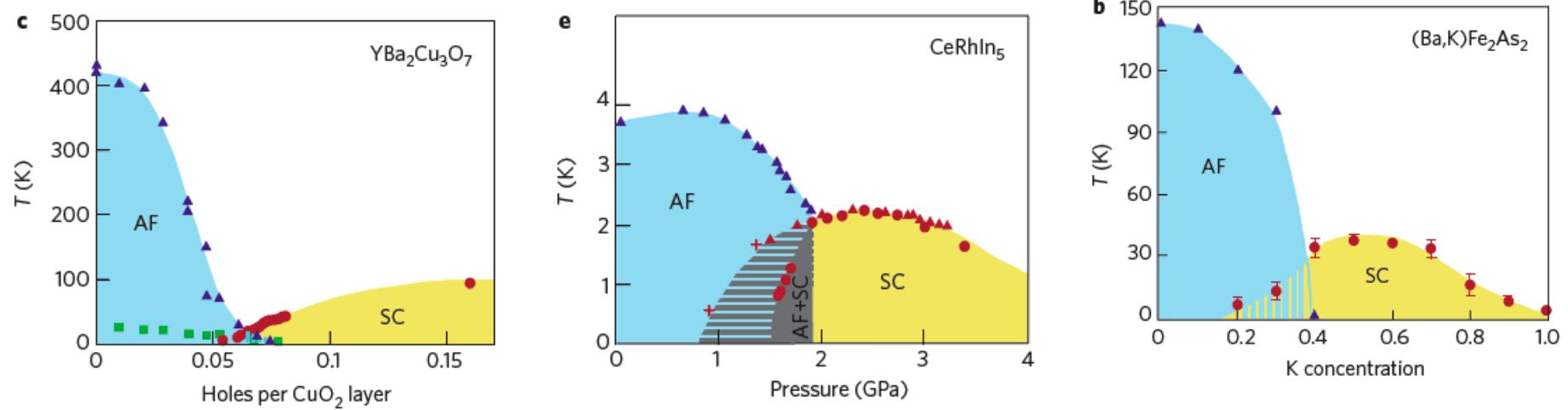
1) Textbook example of QPT in model magnets

= Ising System in a Transverse Field

2) Active research field : quantum magnetism ($S=1/2, 1\dots$) $d=0,1,2\dots$

Unconventional Superconductivity

SC often occurs near magnetic QCP : cuprates, HF, organics, Fe-based SC



Suggestion : SC originates from spin fluctuations that are enhanced near magnetic QCP

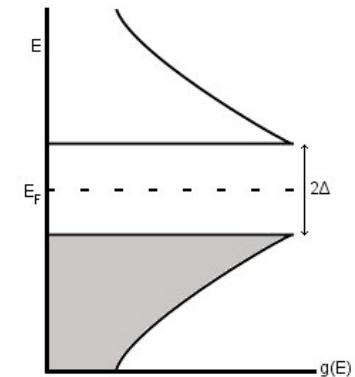
- the usual scenario of SC mediated by phonon is not likely
- anomalous properties points to unconventional superconductivity

Conventional / Unconventional

- Exotic mechanism
- Exotic symmetry of order parameter (= k dependence of gap Δ)

Physical origin : large Coulomb repulsion

→ Nodes at $r = 0$ of the SC wave-function ($L > 0$) reduces the repulsion.

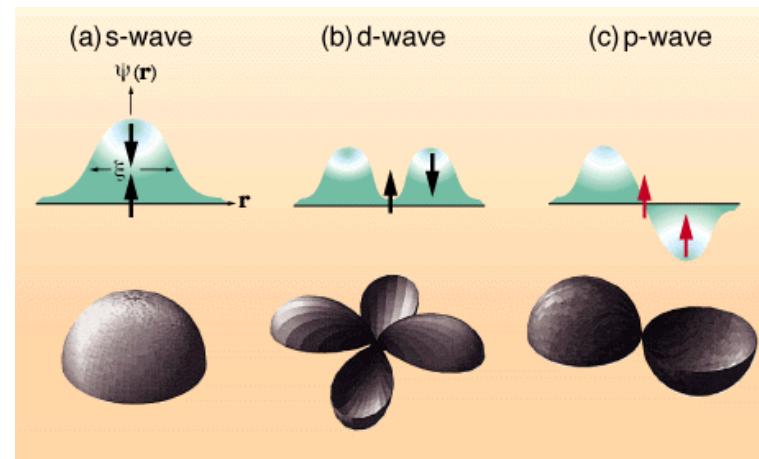


$$\Psi(1,2) = \varphi(1,2) \chi(1,2)$$

orbital spin

even $l = 0, 2, 4, \dots$; $S = 0$ (singlet)

odd $l = 1, 3, \dots$; $S = 1$ (triplet)



Ψ is also the gap Δ

In a solid : anisotropy and spin orbit must be taken into account (symmetry of Ψ by group theory)

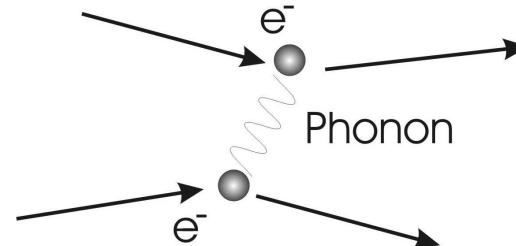
Classical mechanism of SC : Phonon mediated Superconductivity

Phonon is the glue !

$$g \equiv 2 \int_0^{\omega_0} \frac{\alpha^2(\omega) F(\omega)}{\omega} d\omega$$

Electron-phonon interaction

Phonon DOS



$$\text{BCS limit } g \ll 1 : T_c = \omega_0 e^{-1/g}$$

- Not efficient in Strongly Correlated Electron System
- Electron-phonon short range / contact interaction

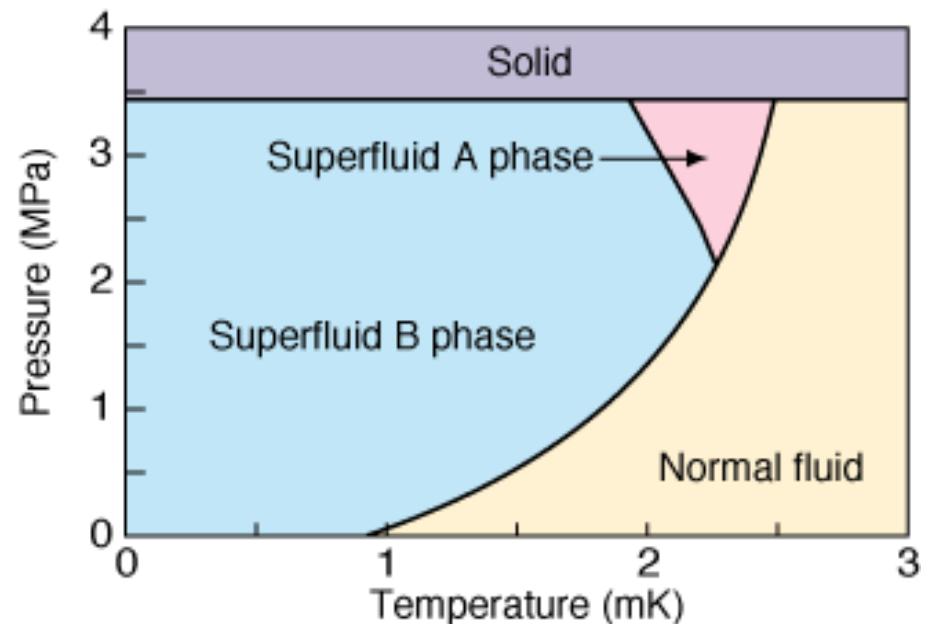
Well-known model of unconventional pairing

${}^3\text{He}$: fermion

hard-core repulsion :

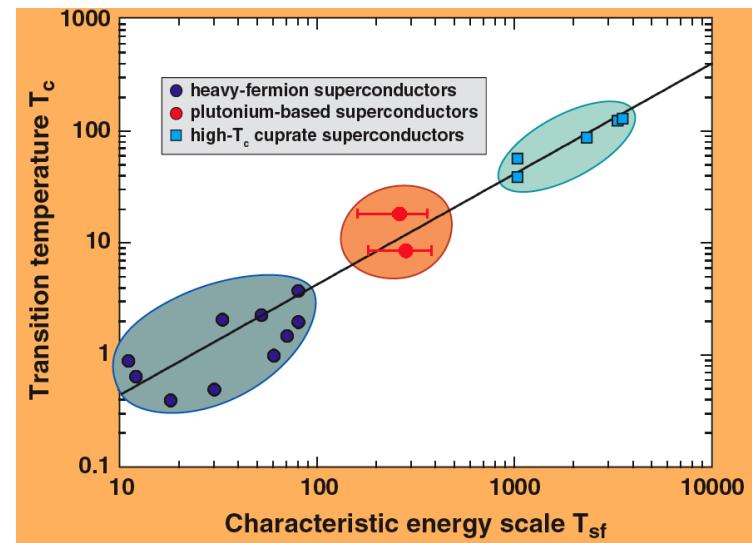
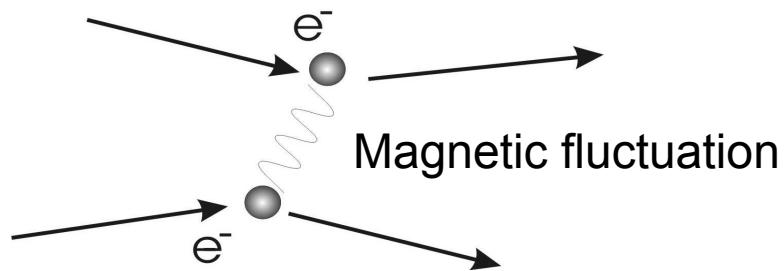
p-wave superconductivity

(+ role of magnetic fluctuations)



Exotic mechanism of SC Magnetically mediated Superconductivity

Magnetically mediated superconductivity : $V = -s_1 s_2 g^2 \chi'(q, \omega)$ and $T_c = T_{sf} e^{-(1+\lambda)/\lambda g}$



$$T_{sf} \sim 1/\gamma \quad \gamma : \text{Sommerfeld coefficient (C/T for } T \rightarrow 0\text{)}$$

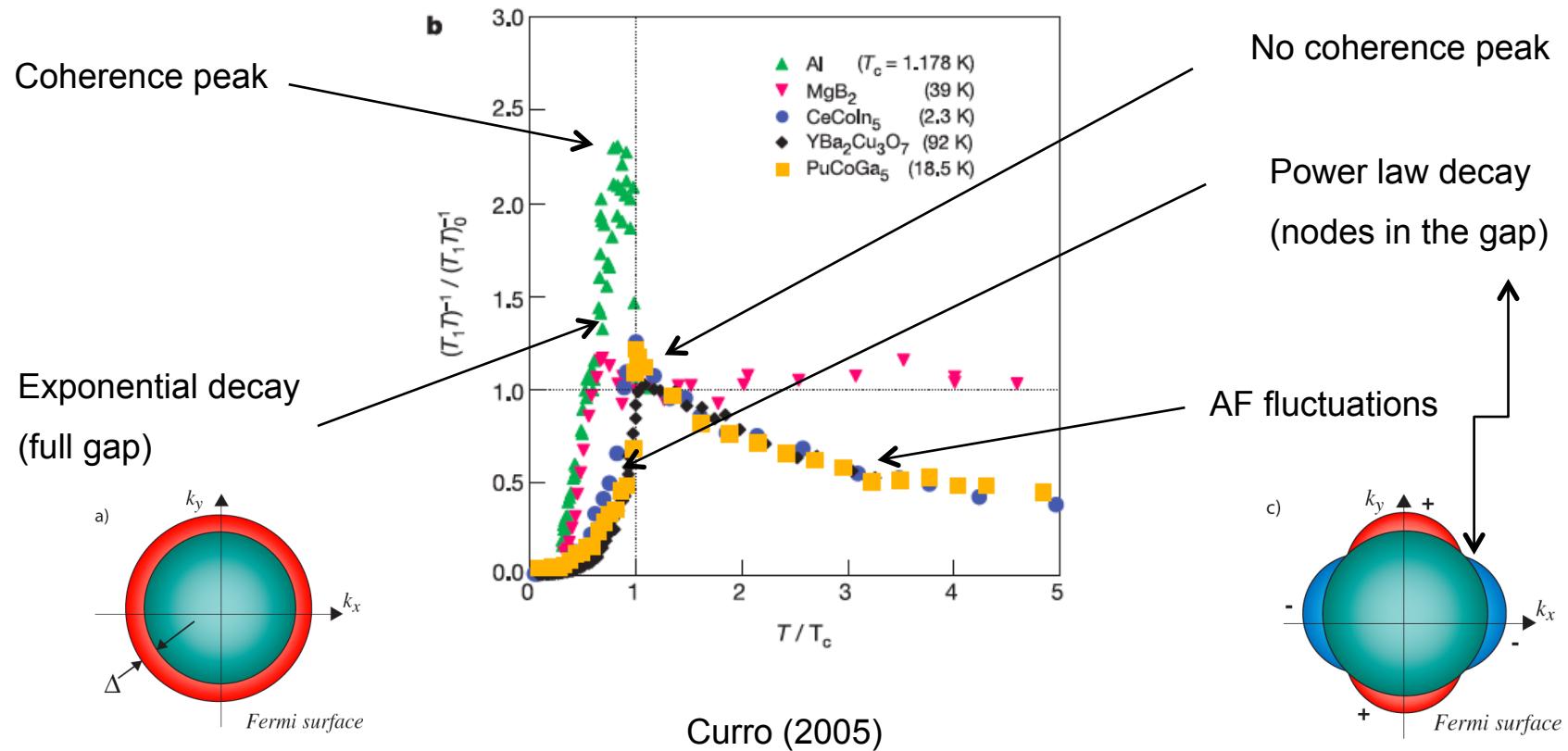
q structure of the pairing mechanism will be reflected in q structure of SC gap

AF fluctuations : even pairing

FM fluctuations : odd pairing

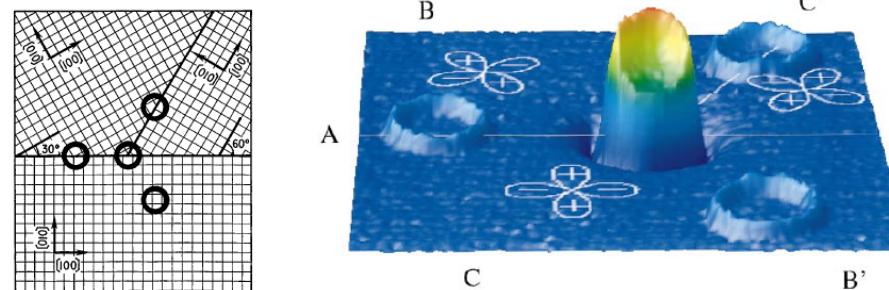
Many physical properties controlled by k dependence of gap function

NMR experiments : relaxation time measurement



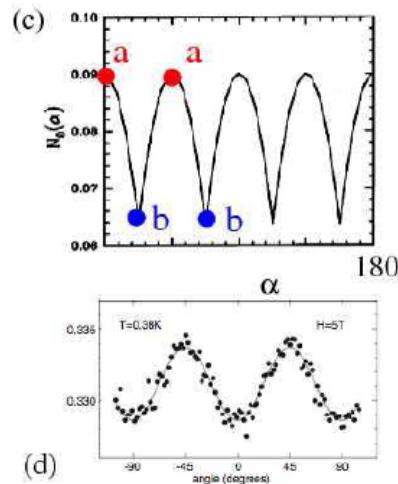
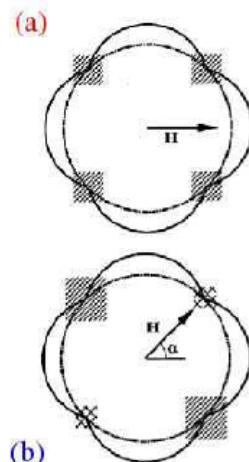
$\uparrow \downarrow$ Even pairing S=0

YBCO : probe of the phase by magnetic flux imaging in tricrystal rings



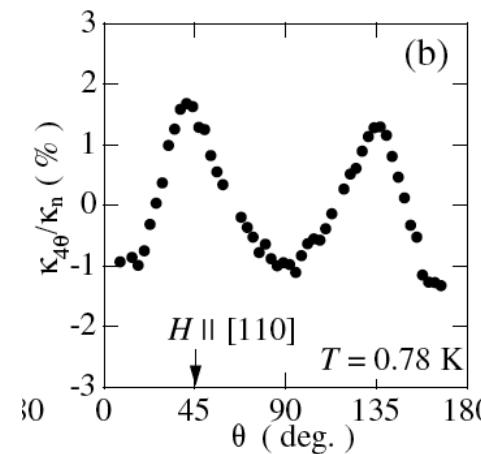
Tsuei, 1994

CeCoIn₅ : angular dependence of DOS :
C/T or thermal conductivity probe of the nodes



CeCoIn₅

Aoki (2004)

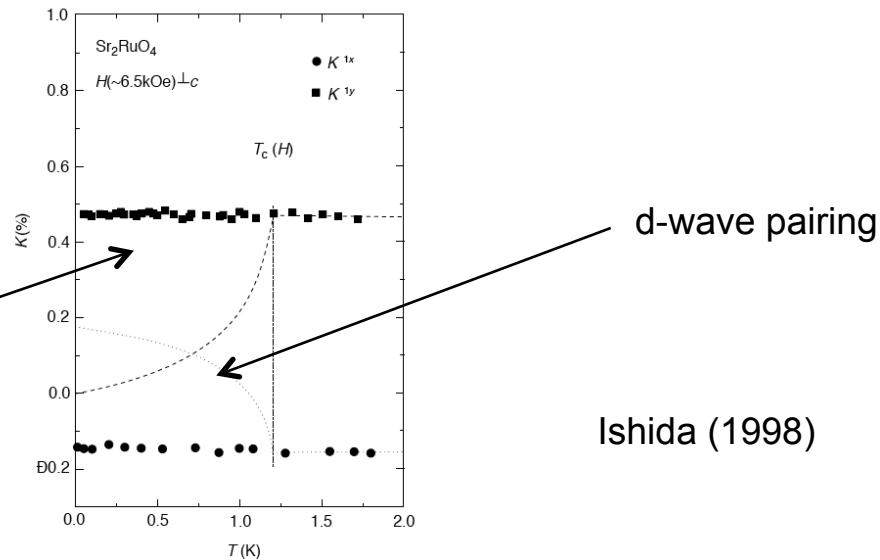


Izawa, 2001

 Odd pairing S=1

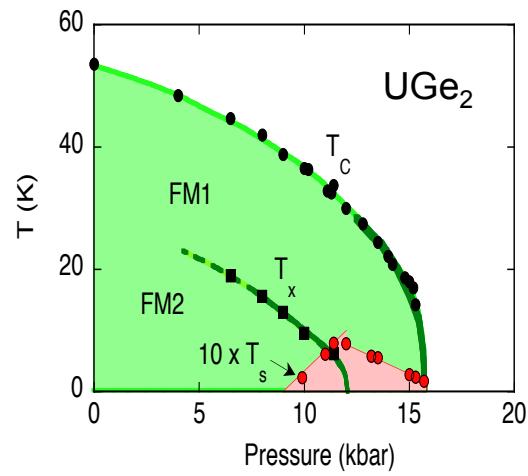
Sr_2RuO_4 : NMR Knight shift

triplet pairing

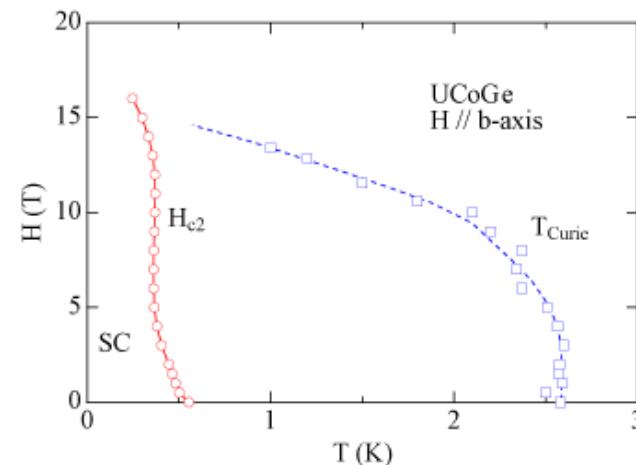


Ishida (1998)

Ferromagnetic superconductors



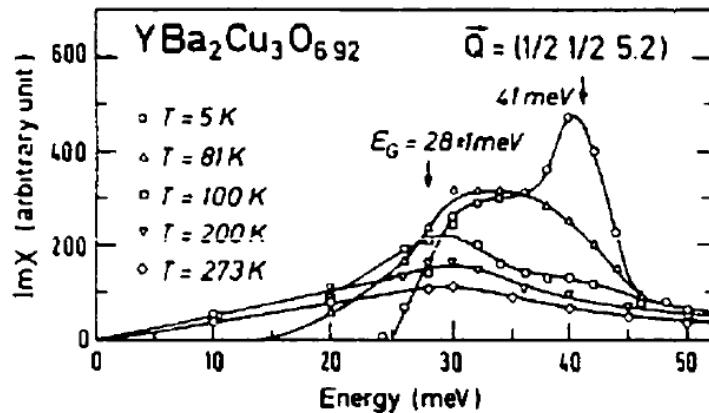
S.S. Saxena (2000)



Aoki (2009)

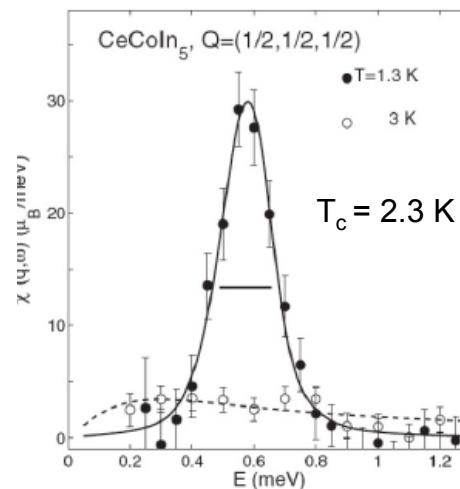
Feedback effect on magnetic excitation spectrum

Discovered in YBCO



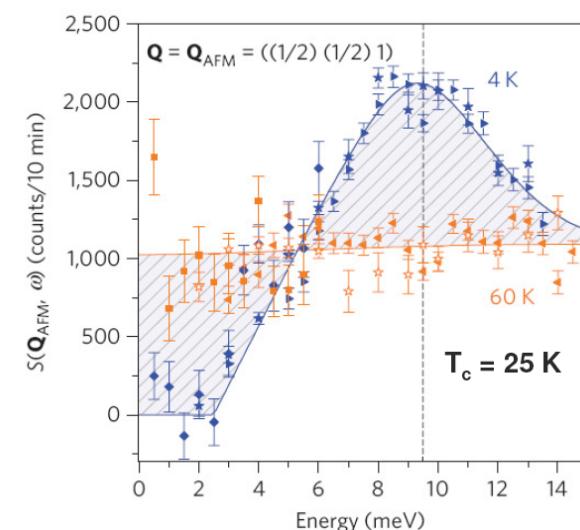
Rossat-Mignod (1991)

Heavy Fermions
CeCoIn₅



Stock (2010)

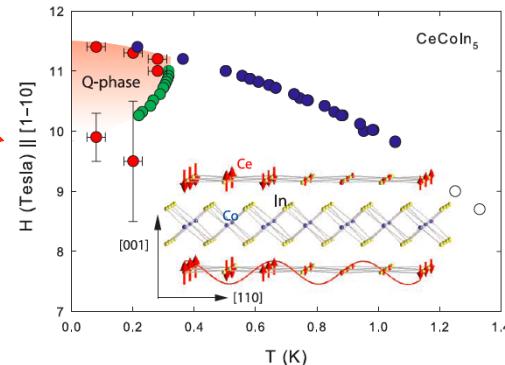
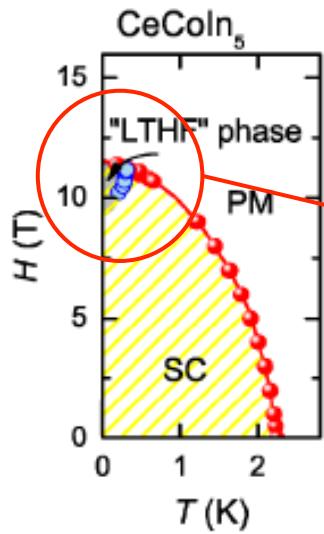
Fe superconductor
BaFe_{1.85}Co_{0.15}As₂



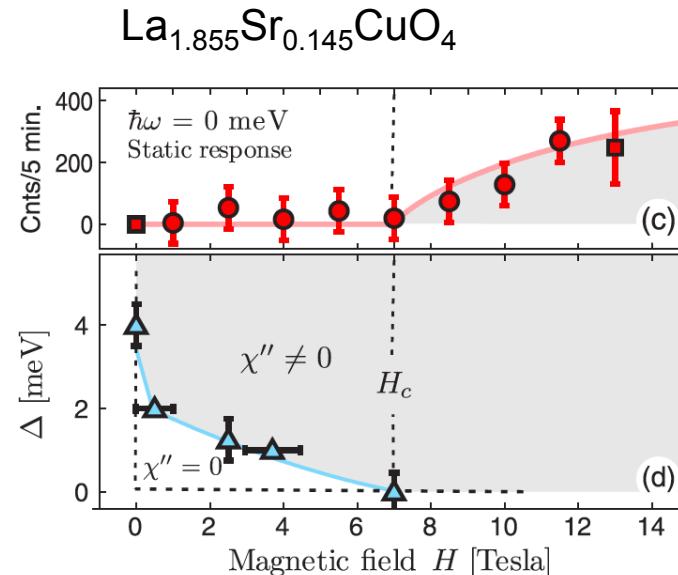
Inosov (2010)

- Dynamical spin susceptibility enhanced at Fermi Surface « hot spot » Q **and** $\Delta(Q+k) = -\Delta(k)$ (For cuprates & HF, arises from dx^2-y^2 for Fe compounds change of sign on different FS)
- « universal » value for the magnetic mode : $\Omega_{\text{res}} \approx 0.6 \Delta$ (Yu, 2009)

Field induced AF order in unconventional SC



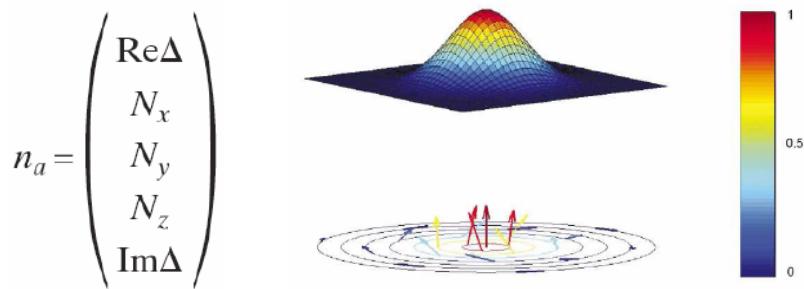
Kenzelmann (2008)



Chang (2009)

Where does AF comes from ? Vortex core ?

AF / SC = one « thing » : SO(5) theory



Part II : QCP and Unconventional Superconductivity : Summary

Commonalities between SCES phases : broader approach than model Hamiltonian
complimentary to microscopic calculations

Quantum Critical Points

- provides a framework to understand deviation from conventional behaviour
(e.g. breakdown of Fermi liquid behavior)
- Unifies many features of T=0 K phase transition (notion of $d_{\text{eff}}=d+z$)

Unconventional Superconductivity

- occurs near magnetic QCP
- mechanism related to spin fluctuations
- symmetry of the order parameter leads to peculiar physical properties
 - S=0 common to many cuprates, Fe compounds, HF
 - S=1 occurs less frequently : ^3He , Sr_2RuO_4 , ferromagnetic SC (UCoGe...)

QUESTIONS

Ising system in a transverse field

$$\Delta S_x \Delta S_y > |\langle i \hbar S_z \rangle|/2$$

T

T_c

0

$$\langle S_i \rangle_{\text{th}} = 0$$

Thermal activation

Paramagnetic

$$|\psi\rangle = |\uparrow\downarrow\uparrow\uparrow\dots\rangle + |\uparrow\uparrow\uparrow\downarrow\dots\rangle + \dots$$

H_c

H

Tunneling

$$\begin{array}{c} \uparrow \\ J_{ij} S_i^z S_j^z \\ \longrightarrow H S_i^x \end{array}$$

