Spin waves

Part I

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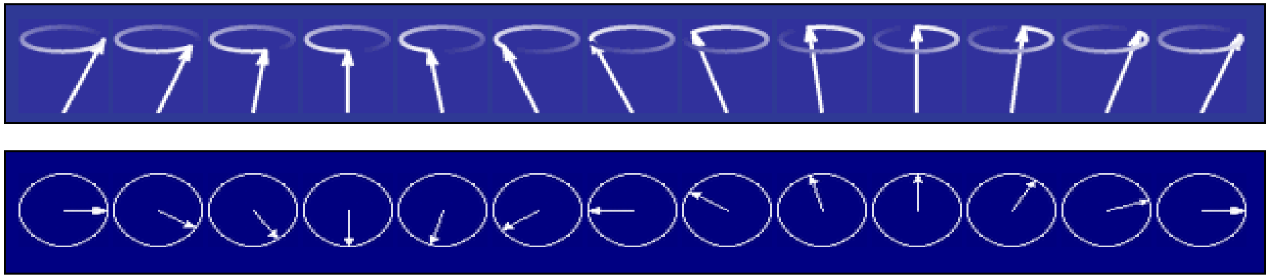
Why spin waves ?

Time-dependent phenomenon → precession of the spin



Theory developed to describe the excited states of the Heisenberg Hamiltonian

$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j}\ \vec{S}_{m,i}.\vec{S}_{n,j}$$

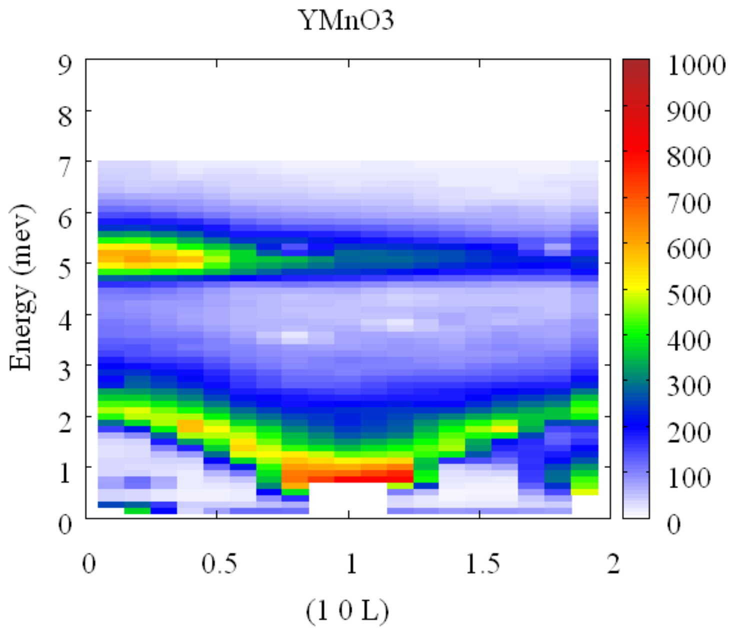
And determine exchange interaction (and anisotropies) via experiments

Why spin waves ?

Bulk systems

ω ~A few THz, meV, cm^{-1} in bulk system
k ~ 0.1 A-1

Observed by neutron scattering in (k, ω) space, but also NMR, optical techniques (Raman, ω =0)



Part I

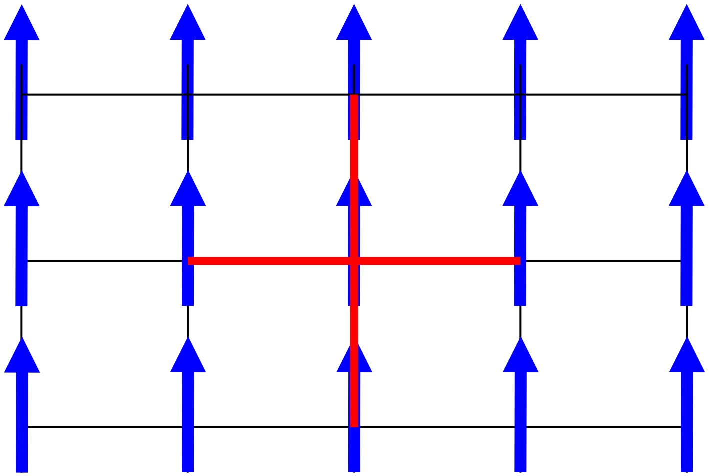
General considerations
Ferromagnet
Antiferromagnet
Failure of the theory

Part II

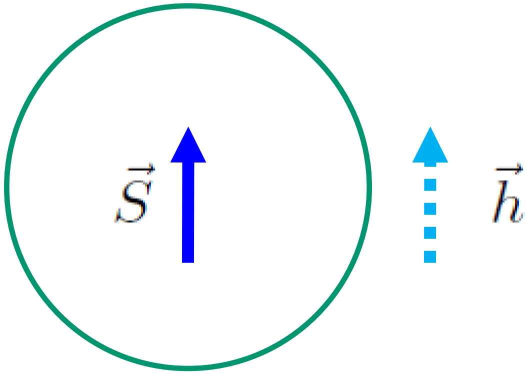
Neutron scattering
Examples

Molecular field

Heisenberg Hamiltonian $$\mathcal{H} = \sum_{m,n} J_{m,n}\ \vec{S}_m.\vec{S}_n$$



$J_{m,n}$

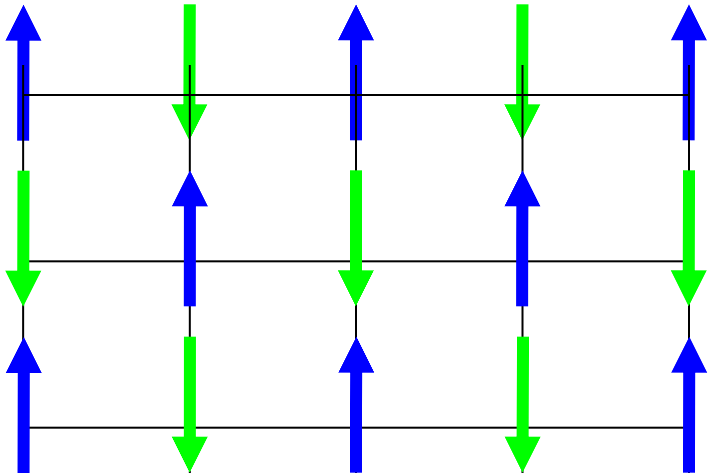


A spin experiences a molecular field due to interaction with its neighbours

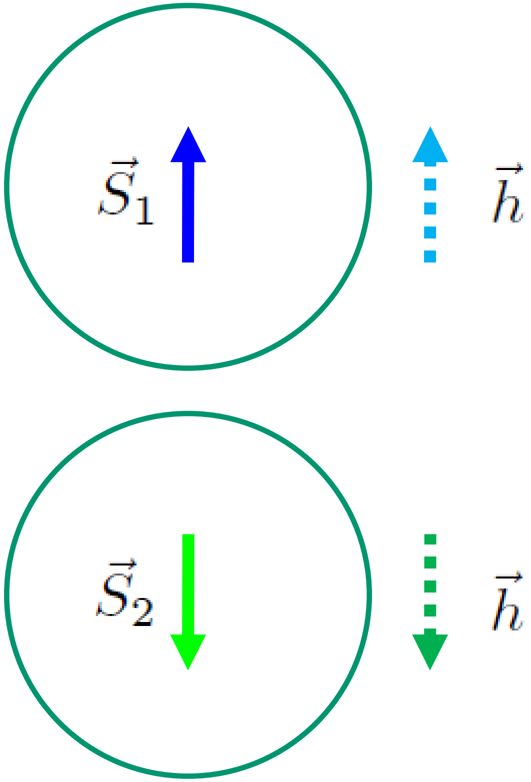
Long range ordering

Molecular field

Depending on interactions, this molecular field can induce a new periodicity

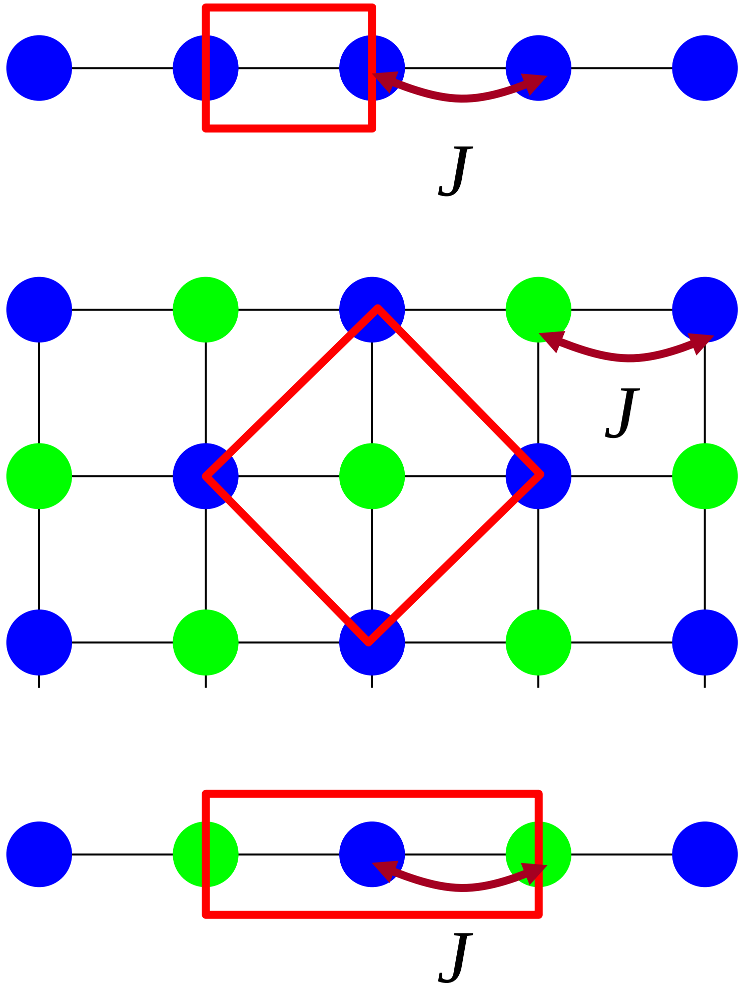


Example : AF ordering



Molecular field

New periodicity, « magnetic unit cell »



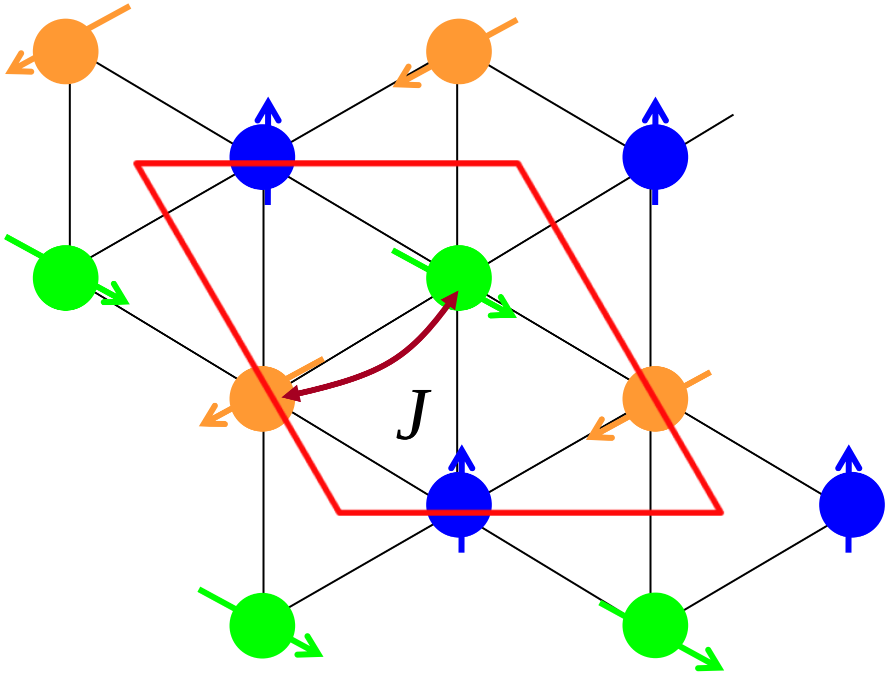
$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j}\ \vec{S}_{m,i}.\vec{S}_{n,j}$$

m labels the unit cell

i labels the ion within the unit cell

Molecular field

New periodicity, « magnetic unit cell »



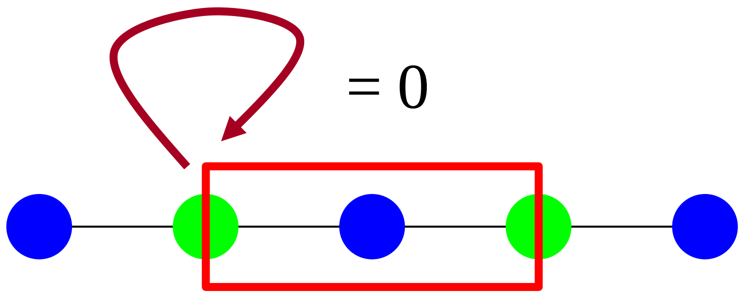
$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j} \; \vec{S}_{m,i} . \vec{S}_{n,j}$$

m labels the unit cell

i labels the ion within the unit cell

Molecular field

Define Interactions



$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j} \; \vec{S}_{m,i}.\vec{S}_{n,j}$$

$$J_{m,1,m,1}$$

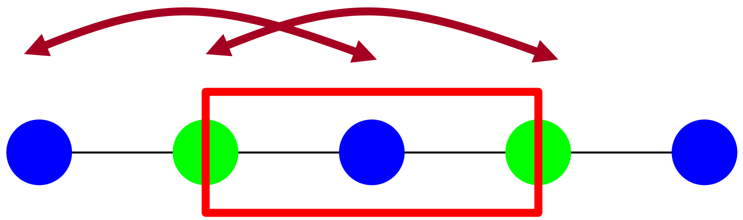
$$J_{m,1,n,1}$$

$$J_{m,1,m,2}$$

$$J_{m,1,n,2}$$

Molecular field

Define Interactions



$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j}\ \vec{S}_{m,i}.\vec{S}_{n,j}$$

$$J_{m,1,m,1}$$

$$J_{m,1,n,1}$$

$$J_{m,1,m,2}$$

$$J_{m,1,n,2}$$

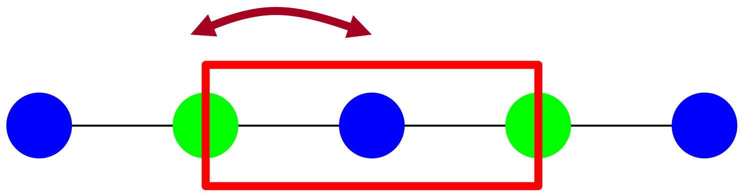
Molecular field

Define Interactions

$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j} \; \vec{S}_{m,i} . \vec{S}_{n,j}$$

$$J_{m,1,m,1}$$

$$J_{m,1,n,1}$$



$$J_{m,1,m,2}$$

$$J_{m,1,n,2}$$

Molecular field

Define Interactions

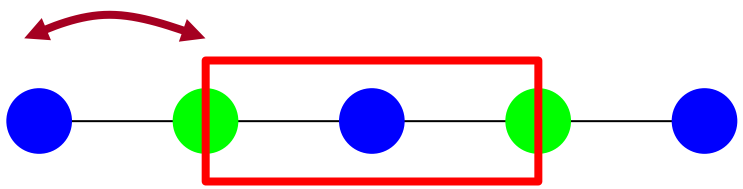
$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j} \; \vec{S}_{m,i} . \vec{S}_{n,j}$$

$$J_{m,1,m,1}$$

$$J_{m,1,n,1}$$

$$J_{m,1,m,2}$$

$$J_{m,1,n,2}$$



Molecular field

Mean field approximation

$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j}\ \vec{S}_{m,i}.\vec{S}_{n,j}$$

One site Hamiltonian, broken symmetry

$$\mathcal{H} \approx \sum_{m,i} \vec{S}_{m,i} \ \underbrace{\left(\sum_{n,j} J_{m,i,n,j}\ \langle \vec{S}_{n,j} \rangle \right)}$$

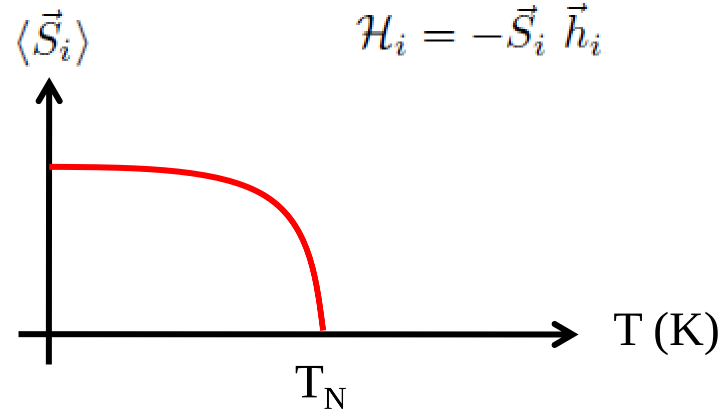
$$\mathcal{H}_i = -\vec{S}_i\ \vec{h}_i$$

Easy diagonalization

$$\langle \vec{S}_i \rangle = \frac{1}{Z_i} \text{Trace} \ \ e^{-\mathcal{H}_i/T}\ \vec{S}_i$$

$$Z = \sum_{n=-S,S} e^{\ n\ h/T}$$

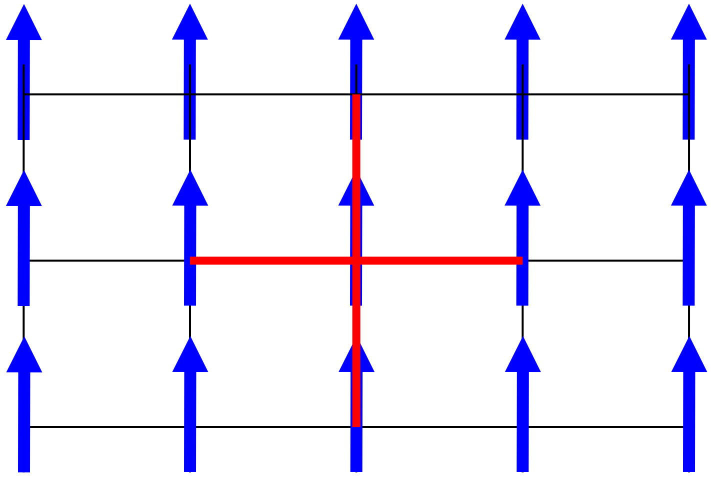
$$S^z\ |n\rangle = n|n\rangle \qquad n =, -S, .., S$$



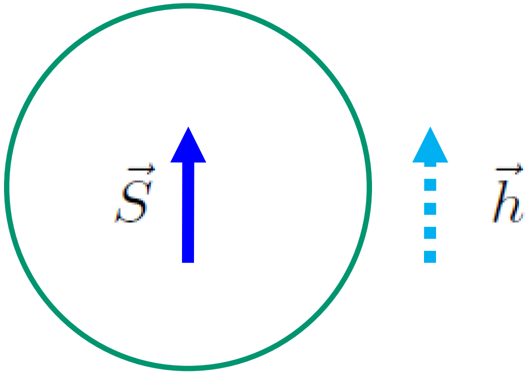
! ***Mermin and Wagner theorem*** : no spontaneous broken symmetry at finite temperature in 1and 2 dimension

Spin in a field

$$\mathcal{H} = \sum_{m,n} J_{m,n}\ \vec{S}_m.\vec{S}_n$$



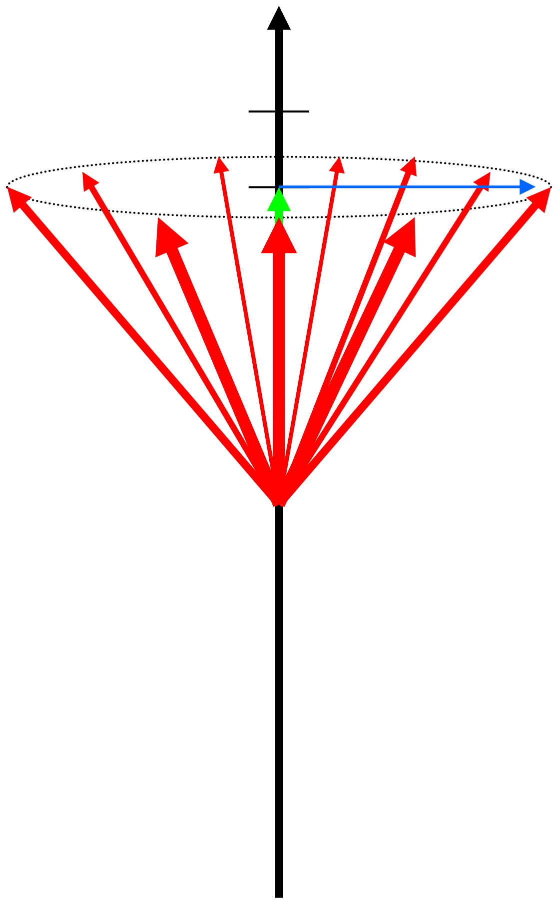
$J_{m,n}$

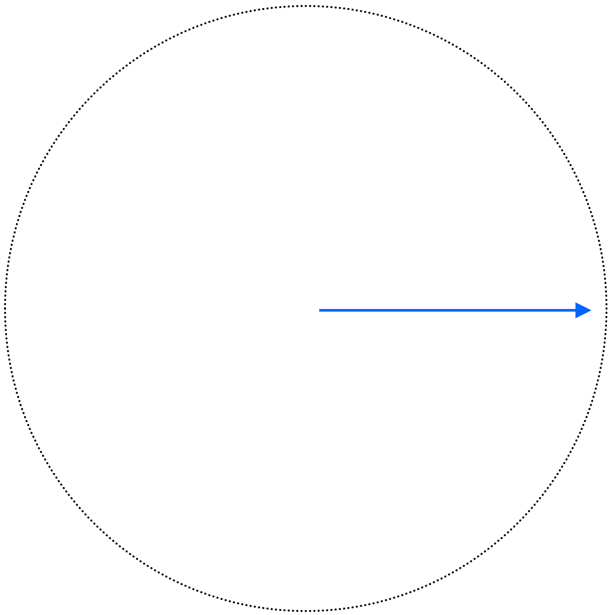


A spin experiences a molecular field
due to the interaction with its neighbors

Spin in a field

Precession of a spin in a magnetic field





Spin in a field

Classical mechanics

$$\mathcal{H} = -\vec{S}.\vec{h}$$

$$\frac{d}{dt}\,\vec{S} = \vec{S} \times \vec{h}$$ Equation of motion

$$\frac{d}{dt}\begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix} = \begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ h \end{pmatrix} = h\begin{pmatrix} S^y \\ -S^x \\ 0 \end{pmatrix}$$

$$\begin{aligned} S^+ &= S^x + i\,S^y \\ S^- &= S^x - i\,S^y \end{aligned}$$

$$\begin{aligned} S^x &= \frac{S^+ + S^-}{2} \\ S^y &= \frac{S^+ - S^-}{2i} \end{aligned}$$

Spin in a field

Classical mechanics

$$\frac{d}{dt}\begin{pmatrix} S^+ \\ S^- \\ S^z \end{pmatrix} = \begin{pmatrix} -ih & & \\ & ih & \\ & & 0 \end{pmatrix}\begin{pmatrix} S^+ \\ S^- \\ S^z \end{pmatrix}$$

$$\omega = h$$

$$\begin{aligned} S^+(t) &= S^+(t=0)\, e^{-i\omega t} \\ S^-(t) &= S^-(t=0)\, e^{+i\omega t} \\ S^z(t) &= S^z(t=0) \end{aligned}$$

The spin precesses around S^z with a frequency ω proportional to h

1 degree of freedom

$$(S^x)^2+(S^y)^2 = \frac{S^+S^- \; + \; S^-S^+}{2} = \frac{S^+(t=0)S^-(t=0) \; + \; S^-(t=0)S^+(t=0)}{2}$$

Spin in a field

Quantum mechanics

Spin operators in the local basis

$$\vec{S} = (S^1, S^2, S^3)$$

$$S^3|\ell\rangle = \ell\ |\ell\rangle \qquad \ell = -S, ..., S$$

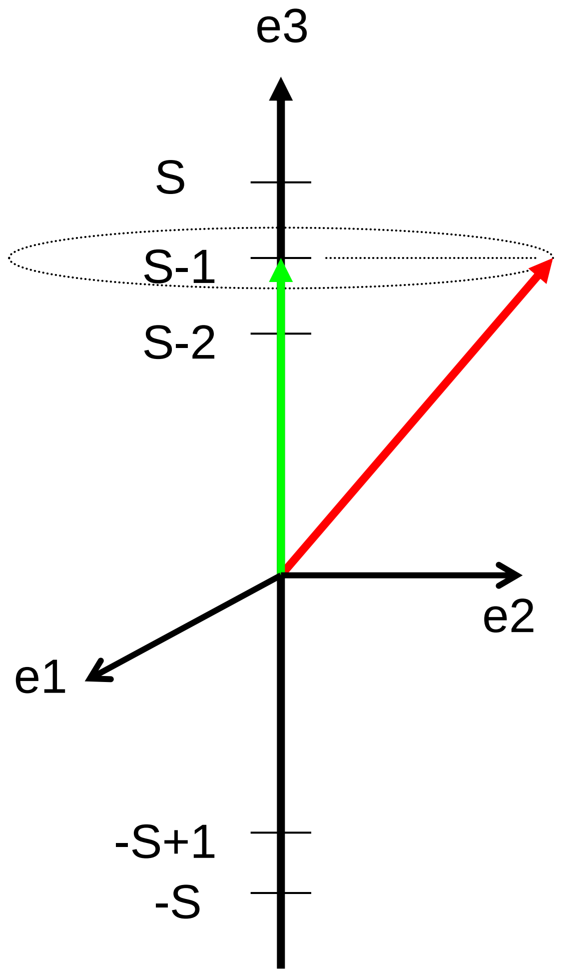
$$\begin{aligned} S^+|\ell\rangle &= \sqrt{S(S+1)-\ell(\ell+1)}\ |\ell+1\rangle \\ S^-|\ell\rangle &= \sqrt{S(S+1)-\ell(\ell-1)}\ |\ell-1\rangle \end{aligned}$$

$$[S^+, S^-] = 2\ S^3$$

Eigenvalues

$$\mathcal{H} = -\vec{S}.\vec{h} = -h\ S^3$$

$$E_\ell = h\ \ell \qquad \ell = -S, ..., S$$



Spin in a field

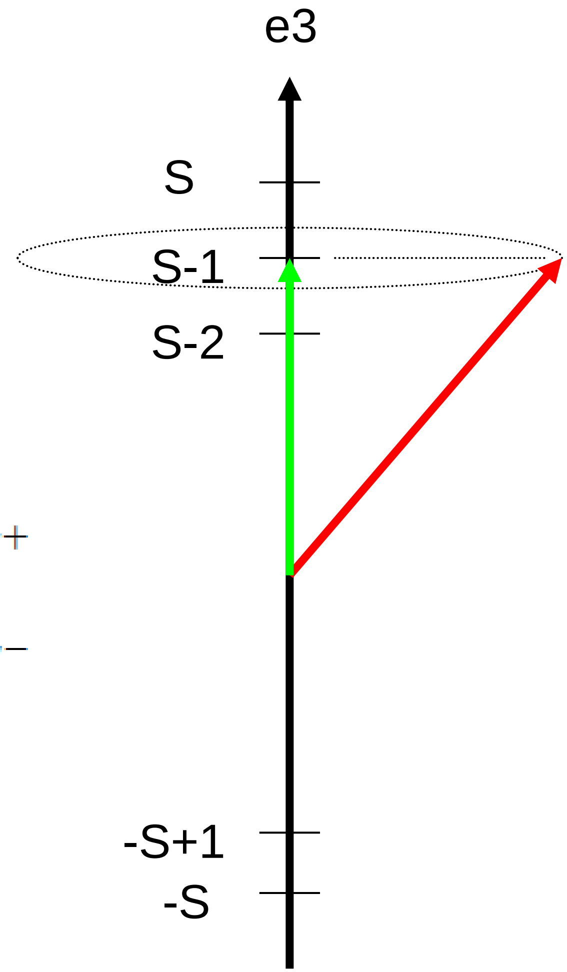
Quantum mechanics

$$\mathcal{H} = -\vec{S}.\vec{h} = -h\ S^3$$

$$\frac{d}{dt}\ \vec{S} = -i\left[\vec{S}, \mathcal{H}\right]$$ Equation of motion

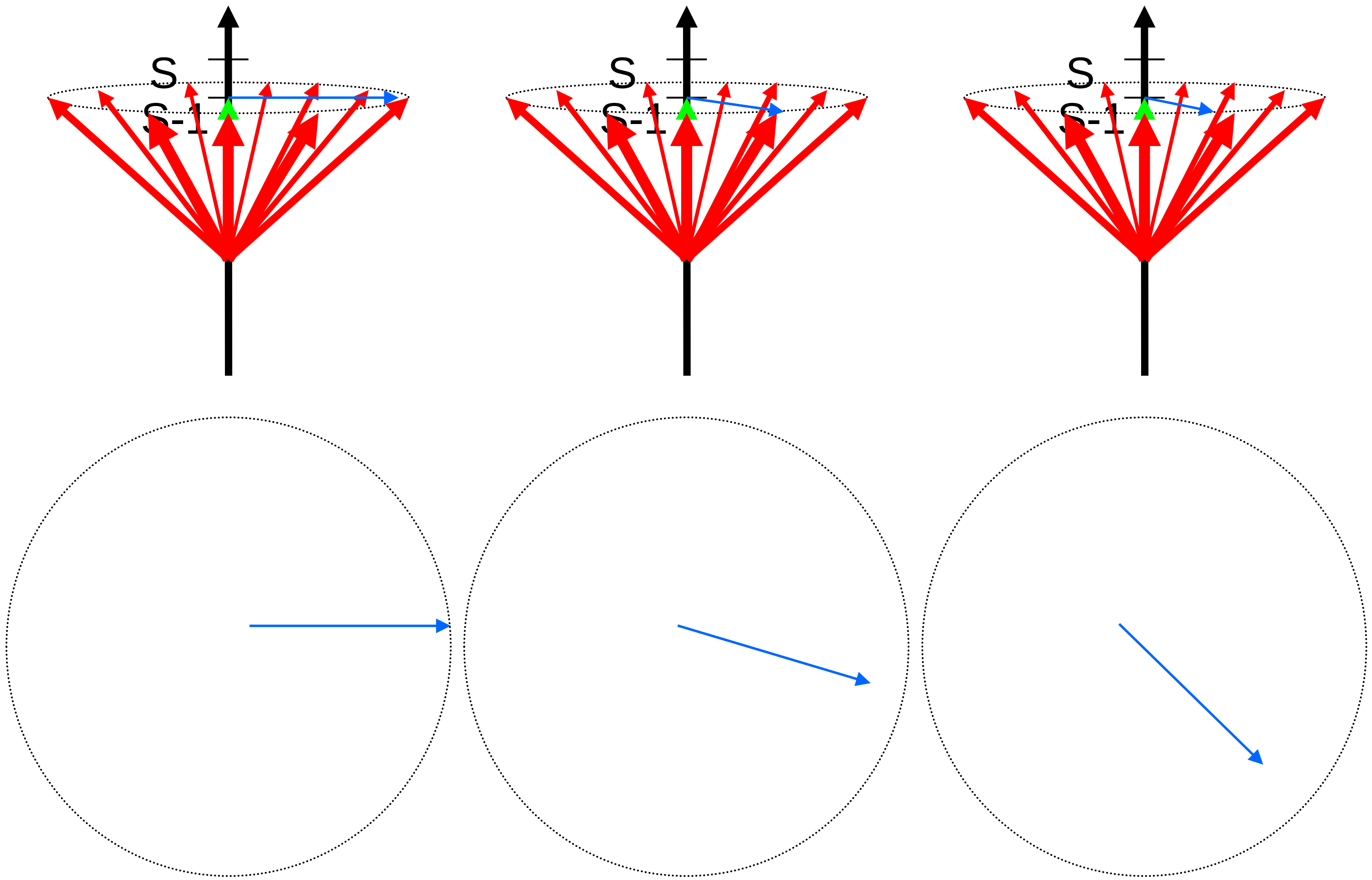
$$\begin{aligned} \frac{d}{dt}\ S^3 &= -i\ \left[S^3, \mathcal{H}\right] = 0 \\ \frac{d}{dt}\ S^+ &= -i\ \left[S^+, \mathcal{H}\right] = -i \times -h \times \left[S^+, S^3\right] = -ihS^+ \\ \frac{d}{dt}\ S^- &= -i\ \left[S^-, \mathcal{H}\right] = -i \times -h \times \left[S^+, S^3\right] = +ihS^- \end{aligned}$$

The spin rotates around S^z with a frequency ω proportional to h

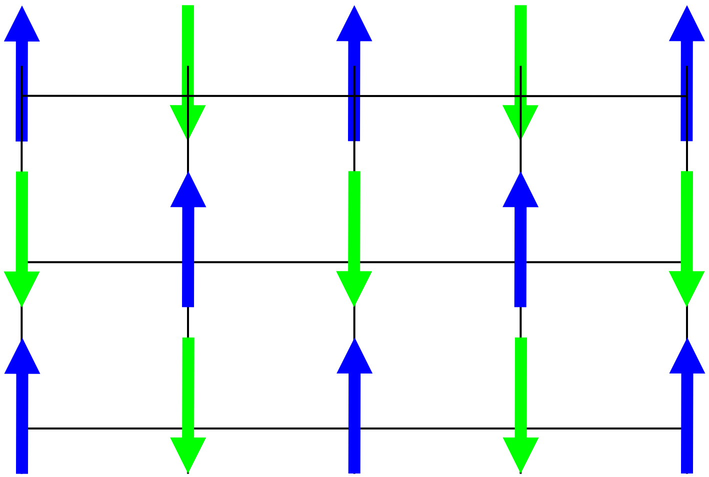


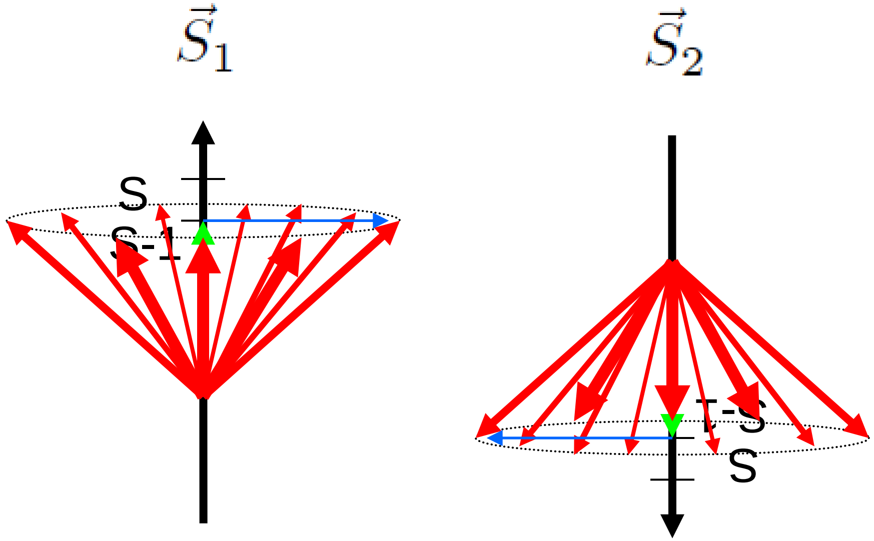
Coupled spins

Back to the problem of coupled spins …



Transformation to local basis





Cartesian coordinates

Local coordinates

$$\vec{S} = \begin{pmatrix} S^x \\ S^y \\ S^z \end{pmatrix} \longrightarrow \vec{\sigma} = \begin{pmatrix} \sigma^+ \\ \sigma^- \\ \sigma^3 \end{pmatrix}$$

$$S_i^\alpha = \sum_{\mu=1,2,3} R_i^{\alpha,\mu} \; \sigma_i^\mu$$

Equation of motion

Classical mechanics

$$\mathcal{H} = \sum_{m,n,i,j} J_{m,i,n,j}\ \vec{S}_{m,i}.\vec{S}_{n,j}$$

Equation of motion :

N coupled …

$$\frac{d}{dt}\ \vec{S}_{m,i} = \vec{S}_{m,i} \times \left(-\sum_{n,j} J_{m,i,n,j}\ \vec{S}_{n,j} \right)$$

… non linear equations

Equation of motion

Classical mechanics

1. Molecular field : small deviations around the direction of the ordered moment

$$\vec{S} = \langle \vec{S} \rangle \; + \; \delta \vec{S} \quad \longrightarrow \quad \vec{S}_{m,i} = \langle \vec{S}_{m,i} \rangle \; + \; \delta \vec{S}_{m,i}$$

2. Take advantage of the new periodicity (Fourier transform) : reduce the number of coupled equations

⚠ Magnetic unit cell

$$\vec{S}_{k,i} = \frac{1}{\sqrt{N}} \sum_m e^{ikR_m} \; \vec{S}_{m,i}$$

$$\vec{S}_{m,i} = \frac{1}{\sqrt{N}} \sum_k e^{-ikR_m} \; \vec{S}_{k,i}$$

$$\longrightarrow \quad \vec{S}_{k,i} = \langle \vec{S}_i \rangle \; \delta_{k=0} \; + \; \delta \vec{S}_{k,i}$$

3. Exchange $\quad J_{m,i,n,j} = J_{m,i,m+\Delta,j} = \sum_q e^{iq\Delta} \; J_{q,i,j}$

Equation of motion

Classical mechanics

1. Fourier transform

$$\frac{d}{dt}\,\vec{S}_{m,i} = \vec{S}_{m,i} \times \left(-\sum_{n,j} J_{m,i,n,j}\,\vec{S}_{n,j} \right)$$

$$\frac{d}{dt}\,\vec{S}_{k,i} \;=\; -\sum_{k'} \vec{S}_{k-k',i} \times \sum_{j} J_{k',i,j}\,\vec{S}_{k',j}$$

2. Ordered moment + small deviations : linearization $\quad \vec{S}_{k,i} = \langle \vec{S}_i \rangle\,\delta_{k=0} \;+\; \delta\vec{S}_{k,i}$

$$\frac{d}{dt}\,\delta\vec{S}_{k,i} \;\approx\; -\left(\sum_{k'} \delta\vec{S}_{k-k',i} \times \sum_{j} J_{k',i,j} \langle \vec{S}_j \rangle\,\delta_{k'=0} \;+\; \sum_{k'} \langle \vec{S}_i \rangle\,\delta_{k-k'=0} \times \sum_{j} J_{k',i,j}\,\delta\vec{S}_{k',j} \right)$$

$$\approx\; \sum_{j} \Bigg(\underbrace{\delta_{i,j} \sum_{\ell} J_{0,i,\ell}\,\langle \vec{S}_\ell \rangle \times \;-\; \langle \vec{S}_i \rangle \times J_{k,i,j}}_{\vec{h}_{k,i,j}} \Bigg)\;\delta\vec{S}_{k,j}$$

$\vec{h}_{k,i,j}$ effective magnetic field acting on $\delta\vec{S}_{k,j}$

Equation of motion

Classical mechanics

Effective magnetic field acting on $\delta\vec{S}_{k,j}$

$$\frac{d}{dt}\begin{pmatrix}\delta S^x_{k,i}\\ \delta S^y_{k,i}\\ \delta S^z_{k,i}\end{pmatrix} = \sum_j \vec{h}_{k,i,j} \times \begin{pmatrix}\delta S^x_{k,j}\\ \delta S^y_{k,j}\\ \delta S^z_{k,j}\end{pmatrix}$$

Local coordinates (use local transformation)

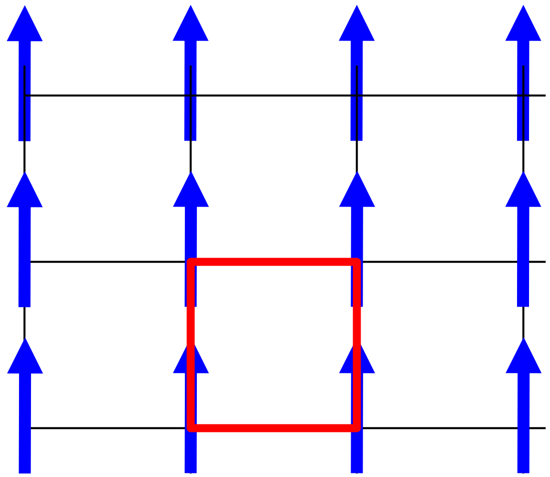
$$\frac{d}{dt}\begin{pmatrix}\delta\sigma^+_{k,i}\\ \delta\sigma^-_{k,i}\\ \delta\sigma^3_{k,i}\end{pmatrix} = \sum_j \left(R_i^T\ \vec{h}_{k,i,j} \times R_j\right)\begin{pmatrix}\delta\sigma^+_{k,j}\\ \delta\sigma^-_{k,j}\\ \delta\sigma^3_{k,j}\end{pmatrix}$$

L magnetic ions per magnetic unit cell : **L coupled linear equations**

Approximations :

1. Ordered phase
2. Small deviations around the ordered moment : « linear spin wave theory » (large S, low T)

Ferromagnet



From the general equations of motion

$$\frac{d}{dt}\,\delta\vec{S}_{k,i} \approx \sum_j \left(\delta_{i,j} \sum_\ell J_{0,i,\ell}\,\langle \vec{S}_\ell \rangle \times \; - \langle \vec{S}_i \rangle \times J_{k,i,j} \right) \delta\vec{S}_{k,j}$$

back to the simple ferromagnetic case :

$$\frac{d}{dt}\,\delta\vec{S}_k \approx (J_0 - J_k)\;\langle \vec{S} \rangle \times \;\delta\vec{S}_k$$

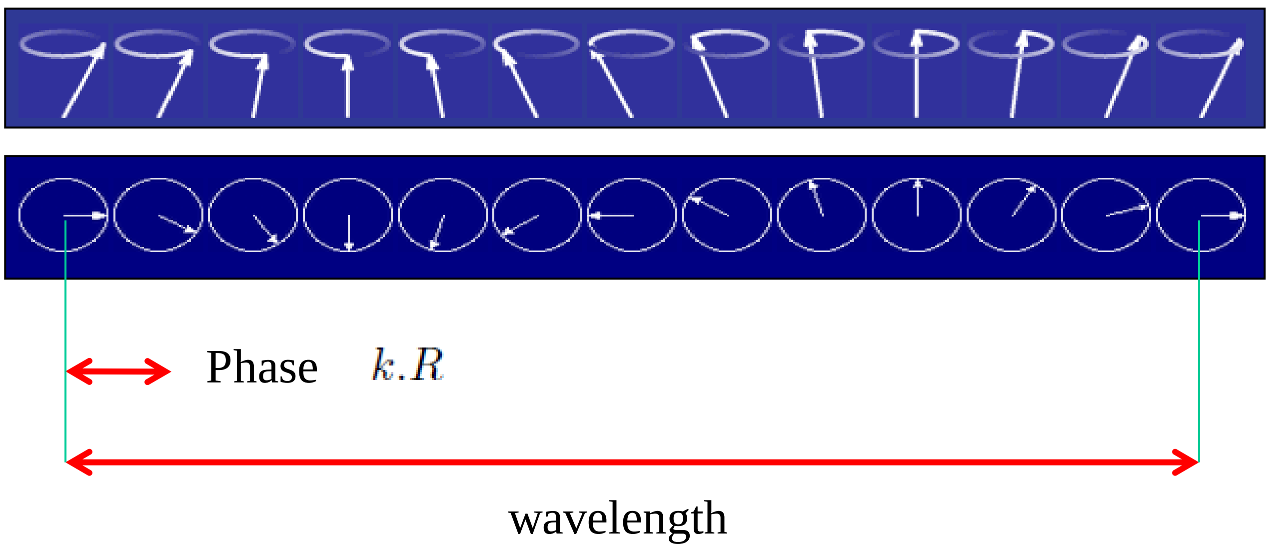
$$\frac{d}{dt}\begin{pmatrix} \delta S_k^x \\ \delta S_k^y \\ \delta S_k^z \end{pmatrix} \approx (J_0 - J_k)\begin{pmatrix} 0 \\ 0 \\ \langle S \rangle \end{pmatrix} \times \begin{pmatrix} \delta S_k^x \\ \delta S_k^y \\ \delta S_k^z \end{pmatrix} = (J_0 - J_k)\;\langle S \rangle\;\begin{pmatrix} -\delta S_k^y \\ \delta S_k^x \\ 0 \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} \delta S_k^+ \\ \delta S_k^- \\ \delta S_k^z \end{pmatrix} = \begin{pmatrix} -i\omega_k & & \\ & i\omega_k & \\ & & 0 \end{pmatrix}\begin{pmatrix} \delta S_k^+ \\ \delta S_k^- \\ \delta S_k^z \end{pmatrix}$$

Ferromagnet

$$\begin{aligned}
\delta S_k^+(t) &= \delta S_k^+(t=0)\; e^{-i\omega_k t} \\
\delta S_k^-(t) &= \delta S_k^-(t=0)\; e^{+i\omega_k t} \\
\delta S_k^z(t) &= \delta S_k^z(t=0)
\end{aligned}$$

Coupled precessions of the spins around the ordered moment; propagate through the lattice

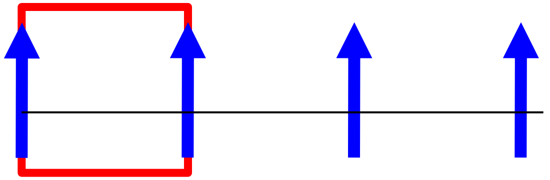


$$\lambda = \frac{2\pi}{k}$$

The dispersion relation connects the wavevector and the frequency ω (energy)

$$\omega_k = -\left(J_0 - J_k\right)\; \langle S \rangle$$

Ferromagnet

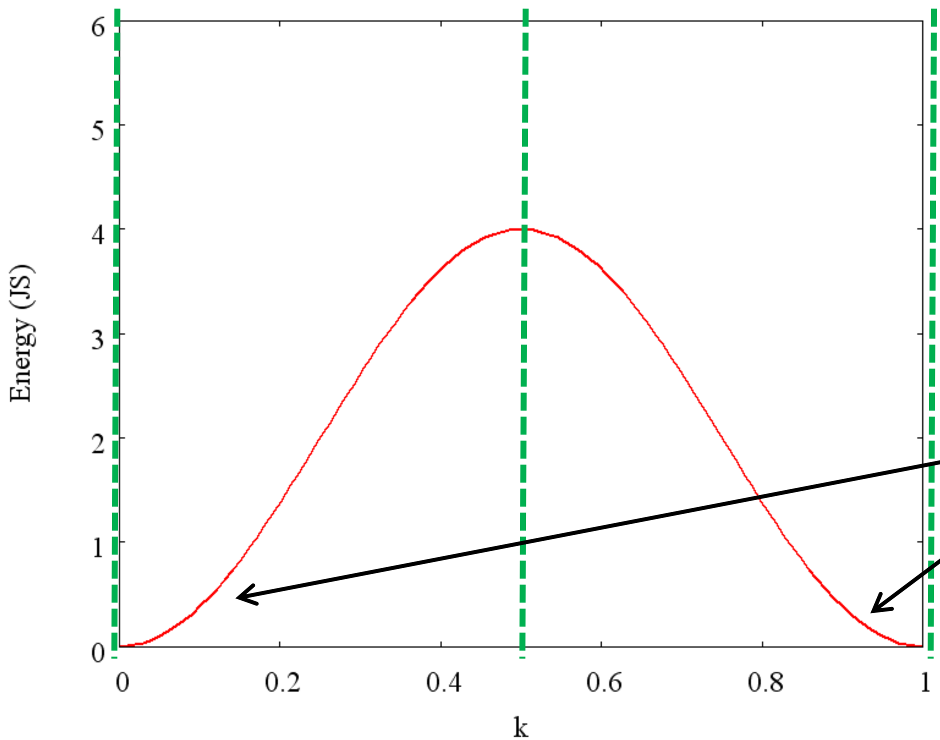


$$\omega_k = 2|J|\ \langle S \rangle\ \ (1 - \cos ka)$$

Zone center

Zone boundary

Zone center of the next Brillouin zone



$$\omega_k \sim 2\ |J|\ \langle S \rangle\ \frac{k^2 a^2}{2}$$

Parabolic dispersion

Ferromagnet

Back to quantum mechanics : spin waves are (quasi) independent Bose modes

$$\mathcal{H} = \sum_k \omega_k \; b_k^+ b_k \qquad \langle E \rangle = \sum_k \omega_k \; n_B(\omega_k) \qquad n_B(E) = \frac{1}{e^{E/k_B T} - 1}$$

Check the approximations (correction to the magnetization)

$$\left.\begin{aligned}
\langle S \rangle \;&\approx\; S - \sum_k n_B(\omega_k) \\
\sum_k &\longrightarrow \int dk^d = \int dk \, \frac{k^{d-1}}{(2\pi)^d} \\
n_B(E) &\longrightarrow \frac{k_B T}{E} \\
\omega_k &\sim 2 \; |J| \; \langle S \rangle \; \frac{k^2 a^2}{2}
\end{aligned}\right\}$$

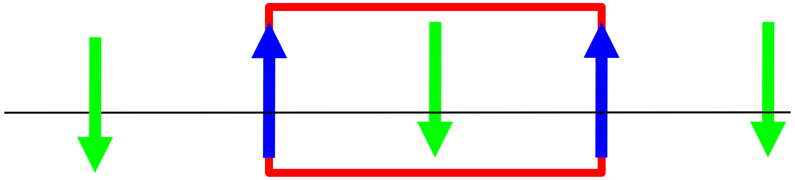
$$\sum_k n_B(\omega_k) \longrightarrow \int dk \, \frac{k^{d-1}}{(2\pi)^d} \, \frac{T}{k^2}$$

The thermal fluctuations prevents long range ordering for

$$d \leq 2$$

Breakdown of the spin wave theory is consistent with Mermin and Wagner theorem

Antiferromagnet



$$
\begin{aligned}
J_{k,1,2} &= J_k = J(1 + e^{-ika}) \\
J_{k,2,1} &= J_k^* = J(1 + e^{ika})
\end{aligned}
$$

$$J_{k,1,1} = J_{k,2,2} = 0$$

$$\langle \vec{S}_1 \rangle = -\langle \vec{S}_2 \rangle = \langle \vec{S} \rangle$$

From the general equations of motion

$$\frac{d}{dt}\, \delta \vec{S}_{k,i} \approx \sum_j \left(\delta_{i,j} \sum_\ell J_{0,i,\ell} \, \langle \vec{S}_\ell \rangle \times \; - \langle \vec{S}_i \rangle \times J_{k,i,j} \right) \delta \vec{S}_{k,j}$$

$$
\begin{aligned}
\frac{d}{dt}\, \delta \vec{S}_{k,1} &= J_0 \, \langle \vec{S}_2 \rangle \times \; \delta \vec{S}_{k,1} - J_k \, \langle \vec{S}_1 \rangle \times \delta \vec{S}_{k,2} \\
\frac{d}{dt}\, \delta \vec{S}_{k,2} &= J_0 \, \langle \vec{S}_1 \rangle \times \; \delta \vec{S}_{k,2} - J_k \, \langle \vec{S}_2 \rangle \times \delta \vec{S}_{k,1}
\end{aligned}
$$

Antiferromagnet

Local coordinates (use local transformation)

1: Sublattices 1 and 2 are still coupled

4 : There are 2 degrees of freedom

$$\frac{d}{dt}\begin{pmatrix}\delta\sigma^+_{k,1}\\ \delta\sigma^-_{k,2}\end{pmatrix} = -i\,\langle S\rangle\begin{pmatrix}J_0 & -J_k\\ J_k & -J_0\end{pmatrix}\begin{pmatrix}\delta\sigma^+_{k,1}\\ \delta\sigma^-_{k,2}\end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix}\delta\sigma^+_{k,2}\\ \delta\sigma^-_{k,1}\end{pmatrix} = -i\,\langle S\rangle\begin{pmatrix}J_0 & -J_k\\ J_k & -J_0\end{pmatrix}\begin{pmatrix}\delta\sigma^+_{k,2}\\ \delta\sigma^-_{k,1}\end{pmatrix}$$

$$\frac{d}{dt}\,\delta\sigma^3_{k,1} = \frac{d}{dt}\,\delta\sigma^3_{k,2} = 0$$

3 : Exchange the role of sublattices
1 and 2 (degenerate modes)

2 : Projection on e3 is constant

Antiferromagnet

Additional transformation to decouple sublattice 1 and 2

$$\begin{aligned} \delta\sigma^{+}_{k,1} &= u_k\,\delta A_k + v_k\,\delta B_k \\ \delta\sigma^{-}_{k,2} &= v_k\,\delta A_k + u_k\,\delta B_k \end{aligned}$$

$$\begin{aligned} \delta\sigma^{+}_{k,2} &= u_k\,\delta A_k + v_k\,\delta B_k \\ \delta\sigma^{-}_{k,1} &= v_k\,\delta A_k + u_k\,\delta B_k \end{aligned}$$

$$\begin{pmatrix} J_0 & -J_k \\ J_k & -J_0 \end{pmatrix}$$

Details of the transformation :

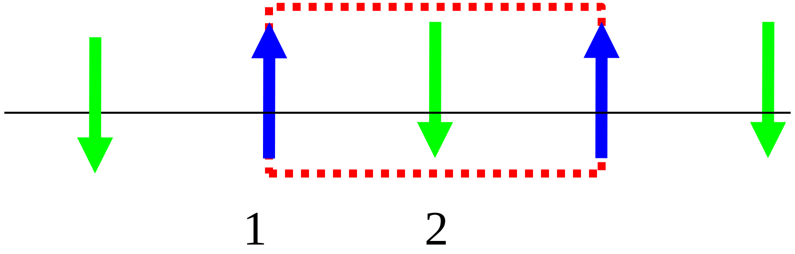
$$u_k^2 - v_k^2 = 1$$

Spin wave energies

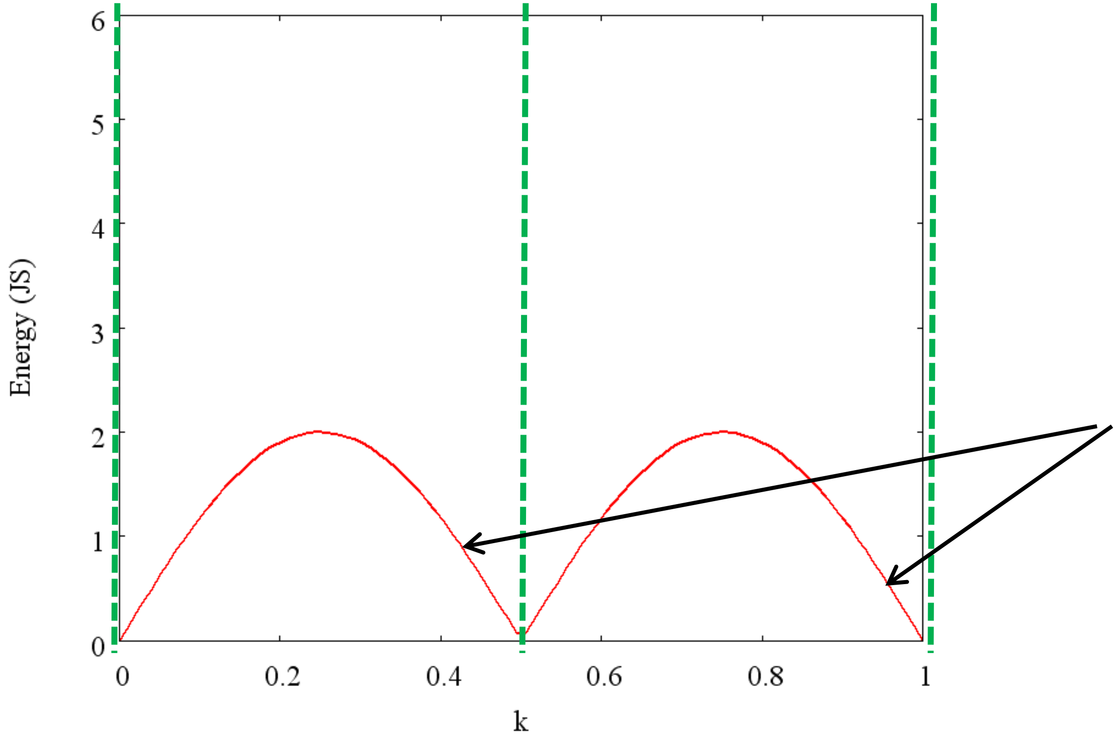
$$\omega_k = \pm \langle S\rangle \sqrt{J_0 - J_k^2}$$

$$\begin{aligned} u_k^2 &= \frac{1}{2}\left(+1 + \frac{J_0\,\langle S\rangle}{\omega_k}\right) \\ v_k^2 &= \frac{1}{2}\left(-1 + \frac{J_0\,\langle S\rangle}{\omega_k}\right) \\ u_k\,v_k &= \frac{J_k\,\langle S\rangle}{2\,\omega_k} \end{aligned}$$

Antiferromagnet



$$\omega_k = 2|J| \langle S \rangle |\sin ka|$$

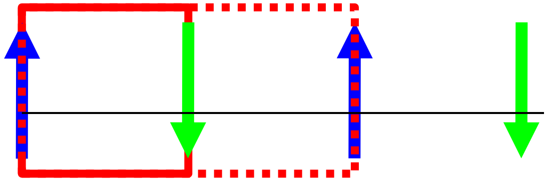


Two degenerate modes

$$\omega_k \sim 2 |J| \langle S \rangle ka$$

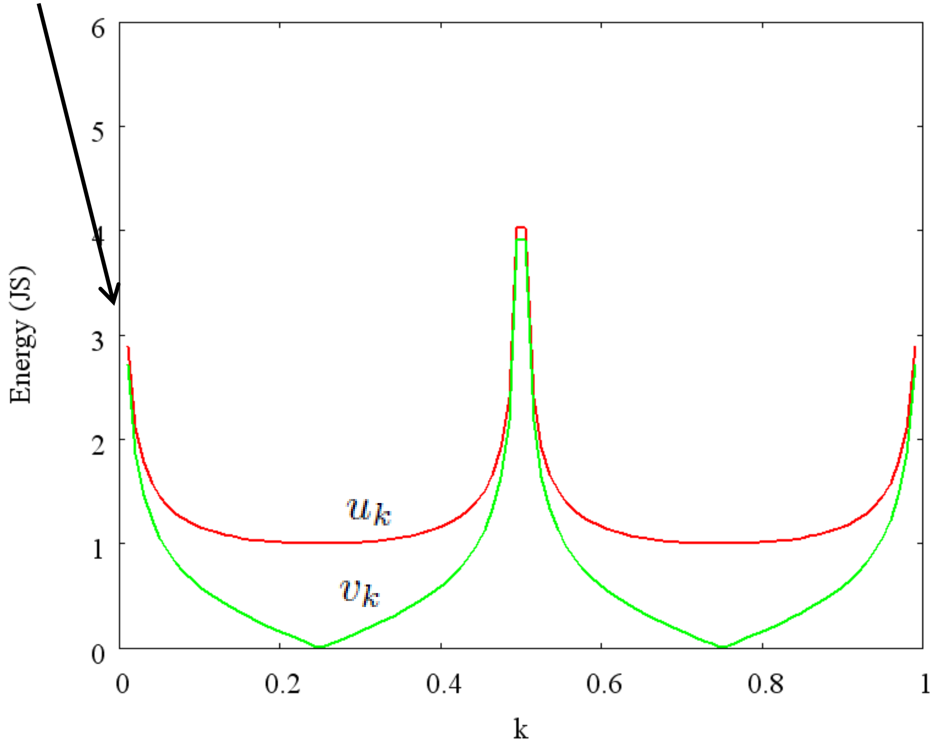
Linear dispersion

Antiferromagnet



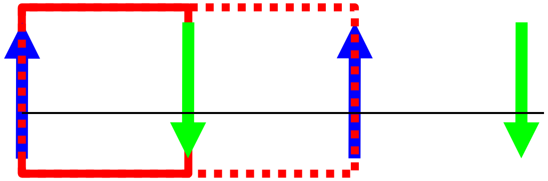
$$u_k^2 = \frac{1}{2}\left(+1 + \frac{J_0\langle S\rangle}{\omega_k}\right) \qquad v_k^2 = \frac{1}{2}\left(-1 + \frac{J_0\langle S\rangle}{\omega_k}\right)$$

Zone center



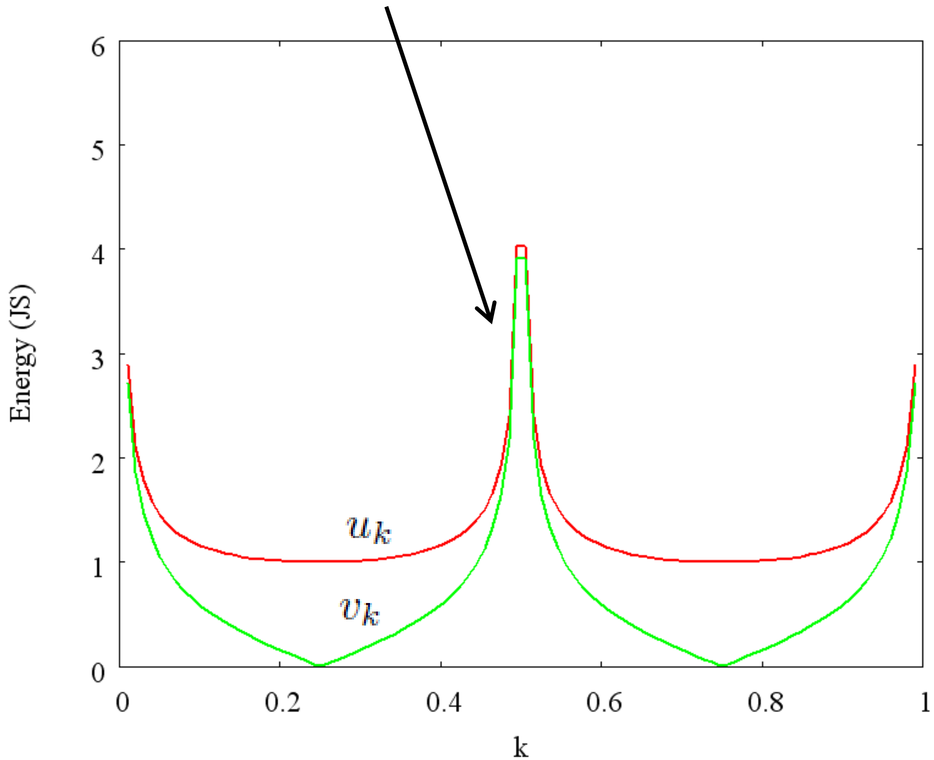
$v_k \sim u_k$

Antiferromagnet



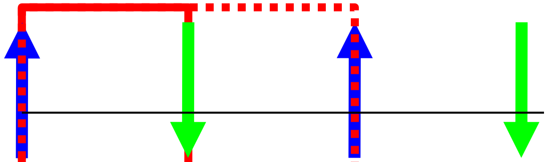
$$u_k^2 = \frac{1}{2}\left(+1 + \frac{J_0 \langle S \rangle}{\omega_k}\right) \qquad v_k^2 = \frac{1}{2}\left(-1 + \frac{J_0 \langle S \rangle}{\omega_k}\right)$$

Zone center of the magnetic unit cell,
(Zone boundray of the lattice unit cell)

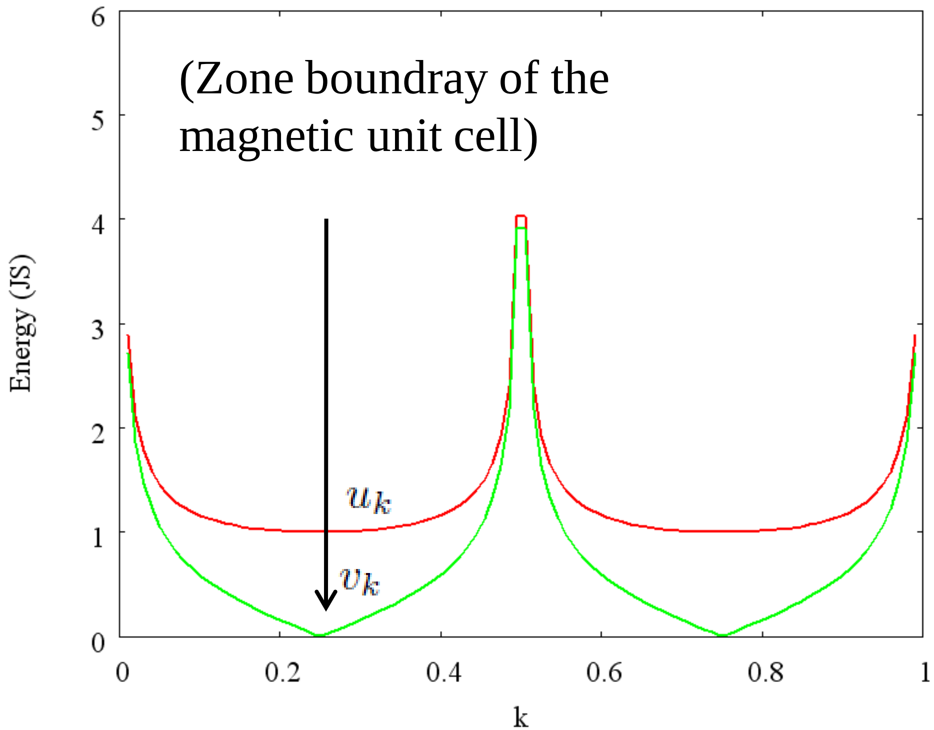


$v_k \sim -u_k$

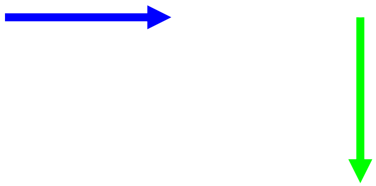
Antiferromagnet



$$u_k^2 = \frac{1}{2}\left(+1 + \frac{J_0 \langle S \rangle}{\omega_k}\right) \qquad v_k^2 = \frac{1}{2}\left(-1 + \frac{J_0 \langle S \rangle}{\omega_k}\right)$$



$v_k = 0, u_k = 1$



Antiferromagnet

Check the approximations (correction to the magnetization)

$$\langle S \rangle \approx S - \sum_k v_k^2 + (u_k^2 + v_k^2)\, n_B(\omega_k)$$

$$\langle S \rangle \approx S + \frac{1}{2} - \sum_k \frac{J_0 \langle S \rangle}{\omega_k} \left(n_B(\omega_k) + \frac{1}{2} \right)$$

Thermal fluctuations

$d \leq 2$

Quantum fluctuations

$d \leq 1$

$$\sum_k \frac{J_0 \langle S \rangle}{\omega_k} \left(n_B(\omega_k) + \frac{1}{2} \right) \longrightarrow \int dk \, \frac{k^{d-1}}{(2\pi)^d} \frac{1}{k} \left(\frac{T}{k} + \frac{1}{2} \right)$$

Breakdown of the spin wave theory is consistent with Mermin and Wagner theorem

Summary

Spin waves : excited states of the Heisenberg Hamiltonian

L ions per magnetic unit cell : L branches

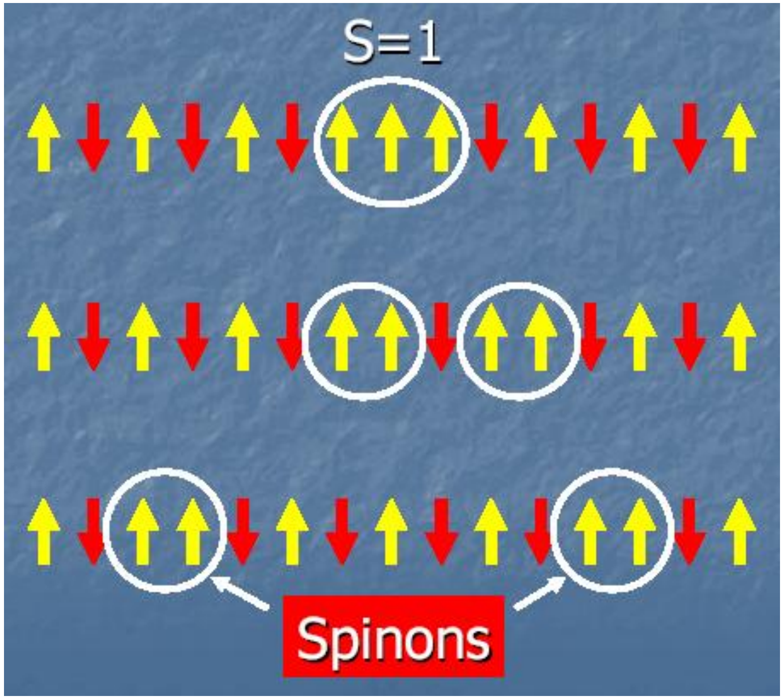
Approximations

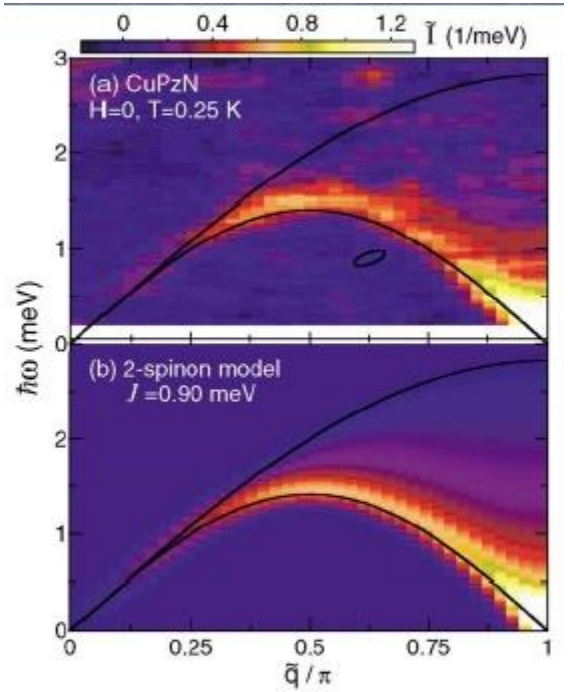
1) Ordered phase

2) Small deviations around the ordered moment : large S, low T

Quasi independent modes (bosons) and important role of quantum fluctuations (low dimension)

Beyond spin wave theory

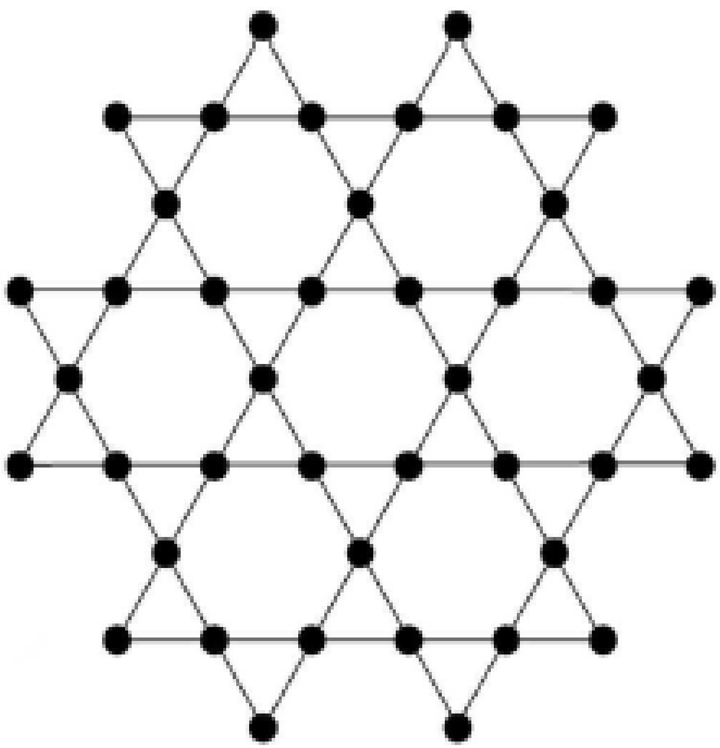




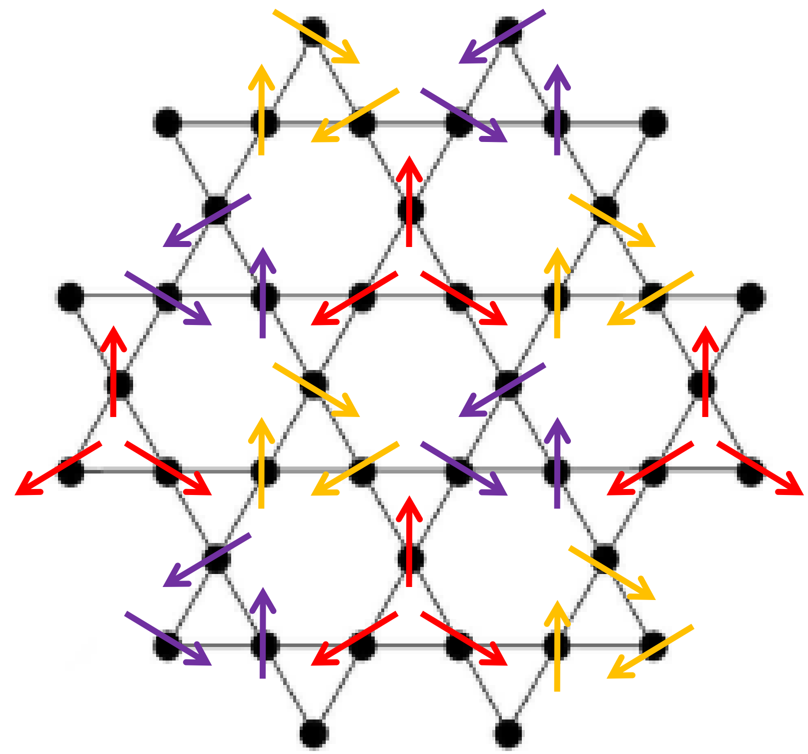
Spin ½ : no long range order, no spin waves
A spin 1 excitation = 2 spinons : continuum and no dispersion relation

Beyond spin wave theory

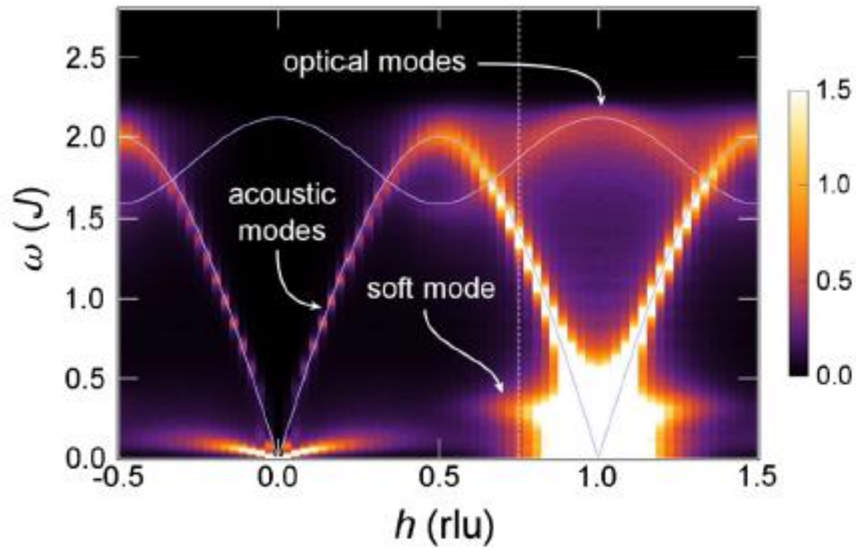
Kagome Lattice
Degenerate ground state : no long range order
The system keeps fluctuating : liquid and co-planar regimes (order by disorder)

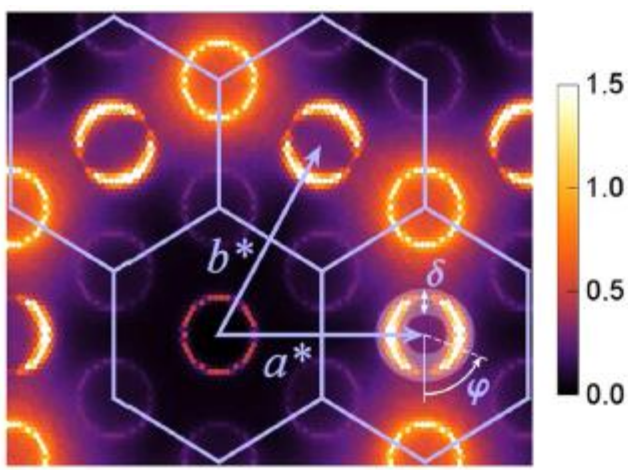


$\sqrt{3} \times \sqrt{3}$ configuration



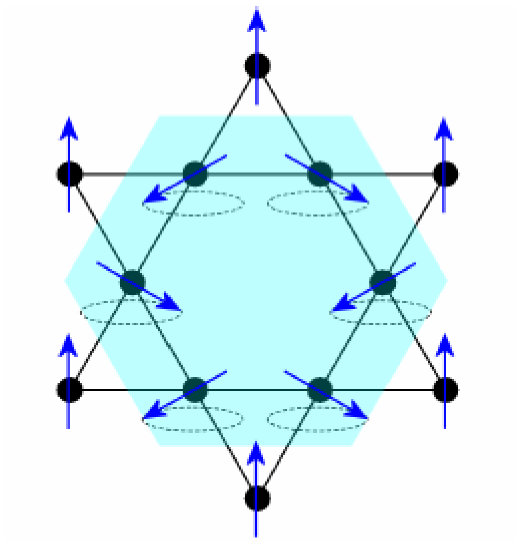
Beyond spin wave theory





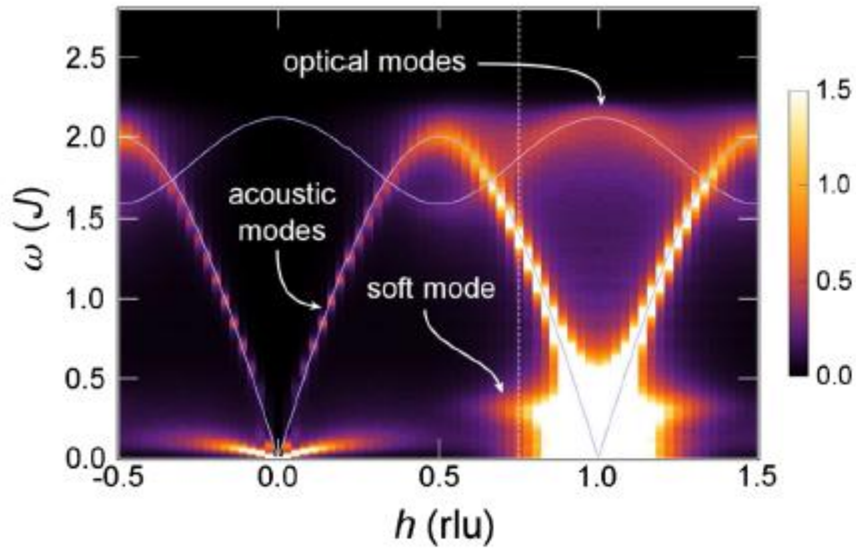
Beyond spin wave theory : calculate the equation of motion for each spin (~ molecular dynamics) in classical mechanics (no approximation):

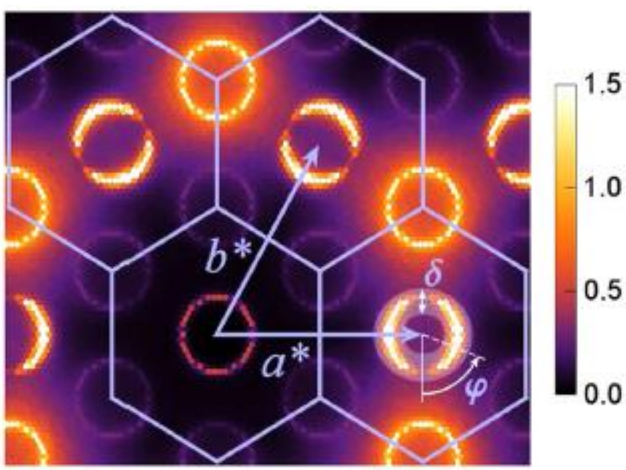
Propagative modes as well as soft modes



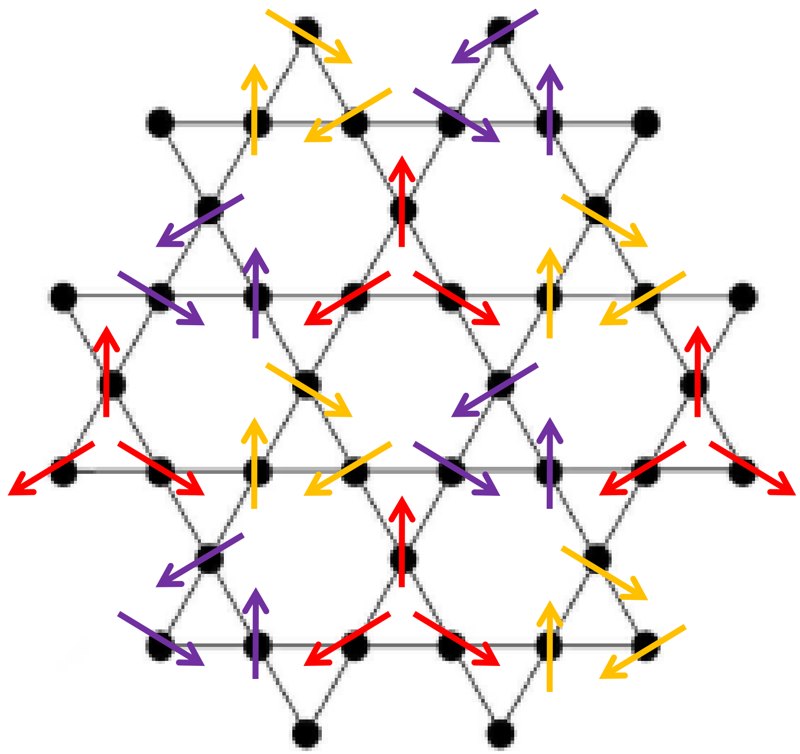
Robert et al, PRL 101, 117207 (2008)

Beyond spin wave theory

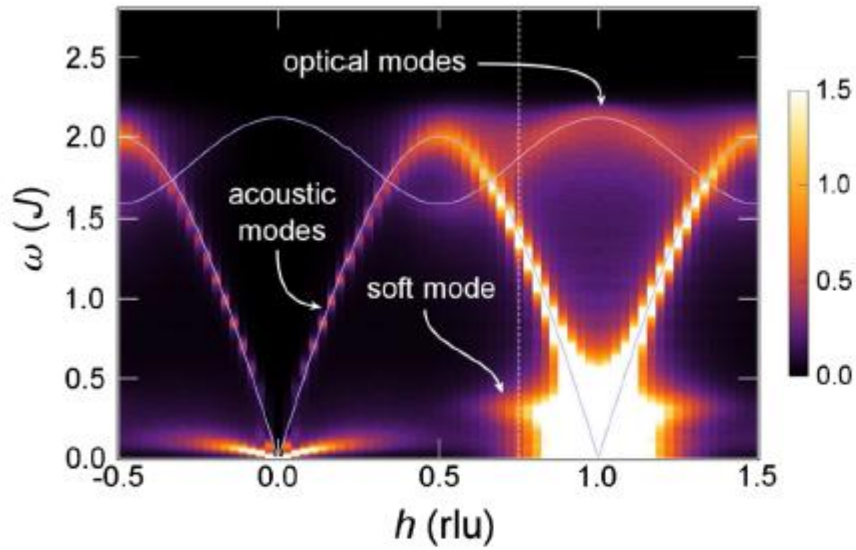


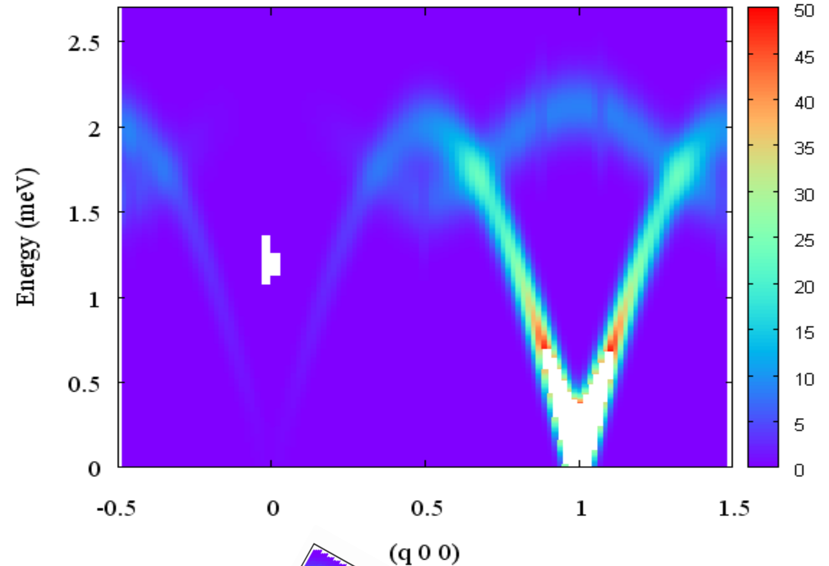


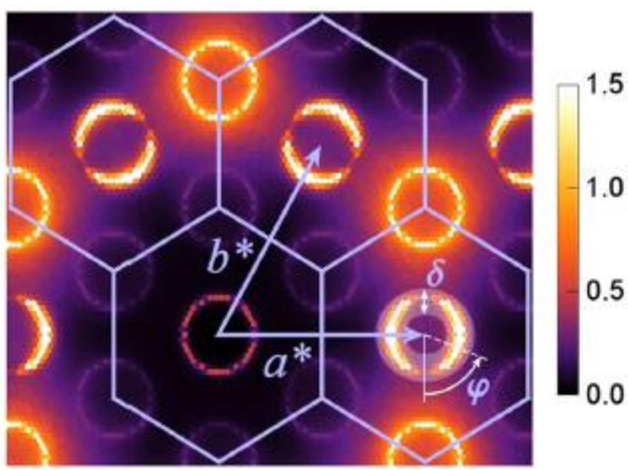
$\sqrt{3} \times \sqrt{3}$ Configuration

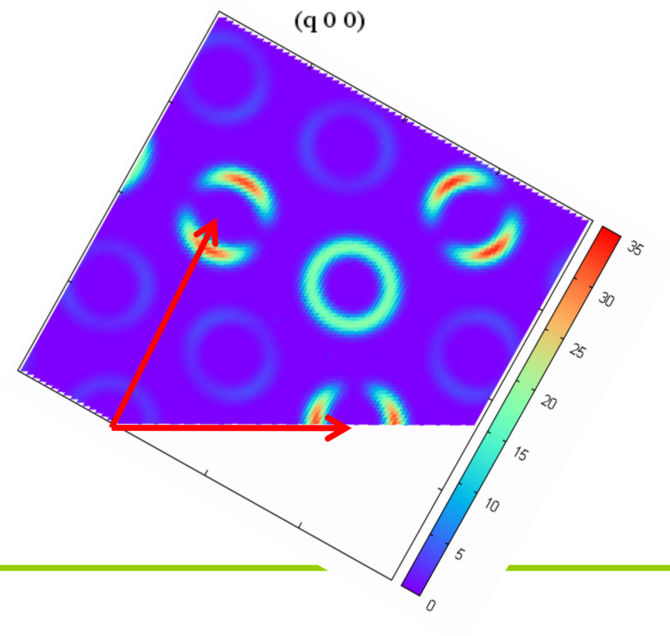


Beyond spin wave theory









Thanks for your attention

Questions

Practical

To be continued …Part II : how to observe spin waves ?

References

[1] P.W. Anderson, Phys. Rev. 83, 1260 (1951)

[2] R. Kubo, Phys. Rev. 87, 568 (1952)

[3] T. Oguchi, Phys. Rev 117, 117 (1960)

[4] D.C. Mattis, *Theory of Magnetism I*, Springer Verlag, 1988

[5] R.M. White, *Quantum Theory of Magnetism*, Springer Verlag, 1987

[6] A. Auerbach, *Interacting electrons and Quantum Magnetism*, Springer Verlag, 1994.

Quantum mechanics

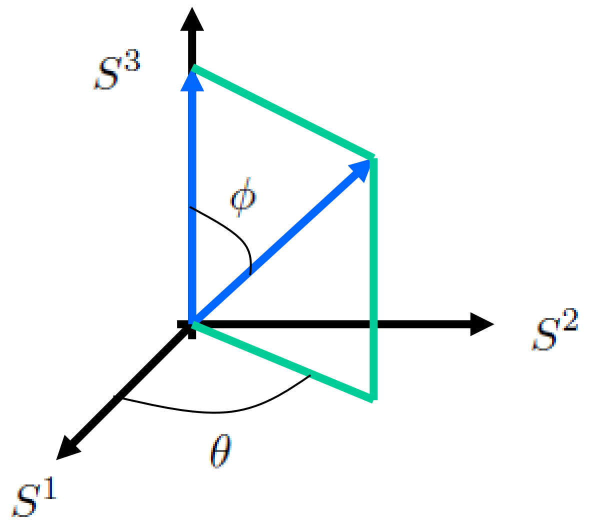
Holstein-Primakov representation of the spin « seen from the classical picture » :

$$\begin{aligned} S^1 &= S\cos\theta\sin\phi \\ S^2 &= S\sin\theta\sin\phi \\ S^3 &= S\cos\phi \end{aligned} \qquad \begin{aligned} S^+ &= S\sin\phi\; e^{+i\theta} \\ S^- &= S\sin\phi\; e^{-i\theta} \end{aligned}$$

New variable : deviation D

$$\boxed{S^3 = S - D} \qquad \cos\phi = 1 - \frac{D}{S}$$

$$\begin{aligned} S^+ &= \sqrt{2S}\sqrt{1-\frac{D}{2S}}\,\sqrt{D}\; e^{+i\theta} \\ S^- &= \sqrt{2S}\sqrt{1-\frac{D}{2S}}\,\sqrt{D}\; e^{-i\theta} \\ S^3 &= S - D \end{aligned}$$



Quantum mechanics

Deviation $D \longrightarrow n_b$

Boson field : $n_b = b^+ b = 0, 1, 2, ..., \infty \qquad [b, b^+] = 1$

$$
\begin{aligned}
S^+ &= \sqrt{2S}\,\sqrt{1-\frac{n_b}{2S}}\;b \\
S^- &= \sqrt{2S}\,b^+\,\sqrt{1-\frac{n_b}{2S}} \\
S^3 &= S - n_b
\end{aligned}
$$

$$
\begin{aligned}
S^+ &\approx \sqrt{2S}\;b \\
S^- &\approx \sqrt{2S}\;b^+ \\
S^3 &= S - n_b
\end{aligned}
$$