Spin glasses, concepts and analysis of susceptibility

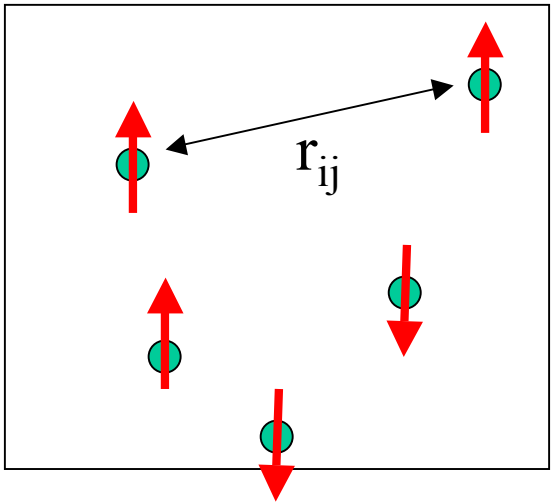
P.C.W. Holdsworth
Ecole Normale Supérieure de Lyon

1. A (brief) review of spin glasses
2. Slow dynamics, aging phenomena
3. An illustration from a simple model

My sources - Binder and Young Rev. Mod. Phys. 58,801, 1986
Spin glasses and random fields, edited by Young A. P., Vol. 12 (World Scientific, Singapore, 1998)

Spin glasses - a (very) brief overview

Noble metal doped with a few % of magnetic ions:
Cu-Mn, Au-Fe (Cannella and Mydosh (1972))



Long range RKKY interaction

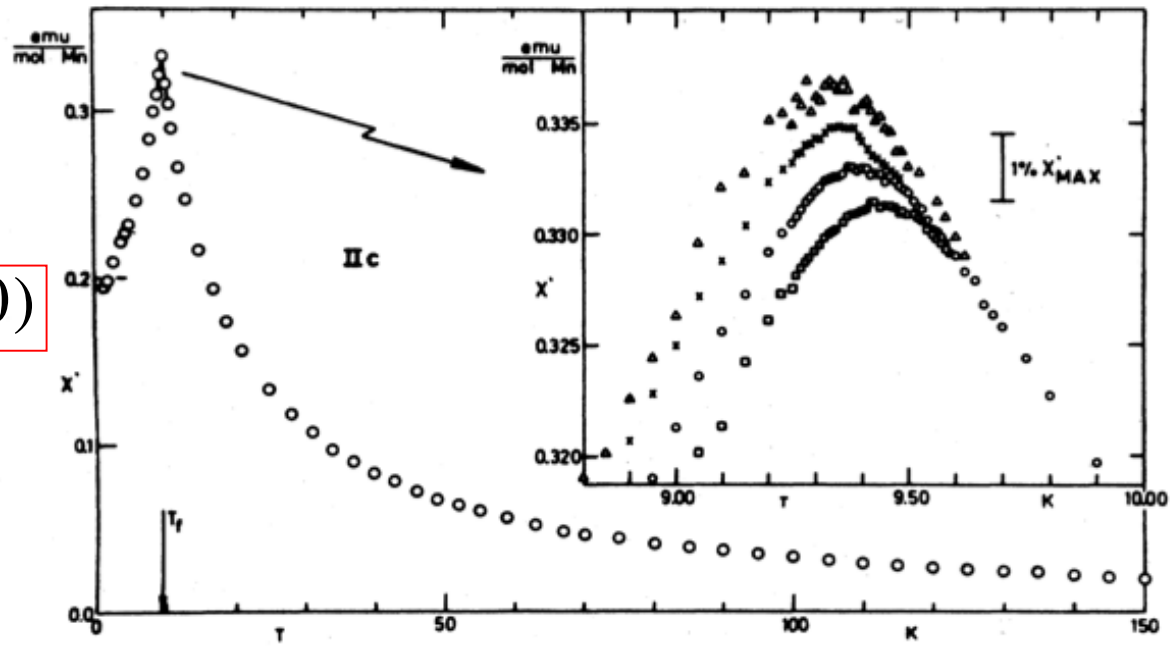
Ruderman, Kittel, Yasuya,Yosida

$$J_{ij} \sim \frac{1}{r_{ij}^3} \cos(\vec{r}_{ij} . \vec{k}_F)$$

And disorder leads to random magnetic exchange.

CuMn with 1% Mn (Mulder et al 1981)

$\chi'(\omega \to 0)$



T

Thermodynamic singularity but no long range order below T_f

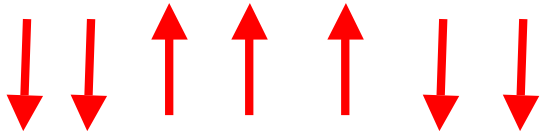
$$\chi = \frac{1}{NT}\sum_{i,j} < S_i S_j > - < S_i >< S_j >$$

$$\chi = \frac{1}{NT}\sum_{i} 1 - < S_i >^2 + \frac{1}{NT}\sum_{i \neq j} < S_i S_j > - < S_i >< S_j >$$

Curie's law

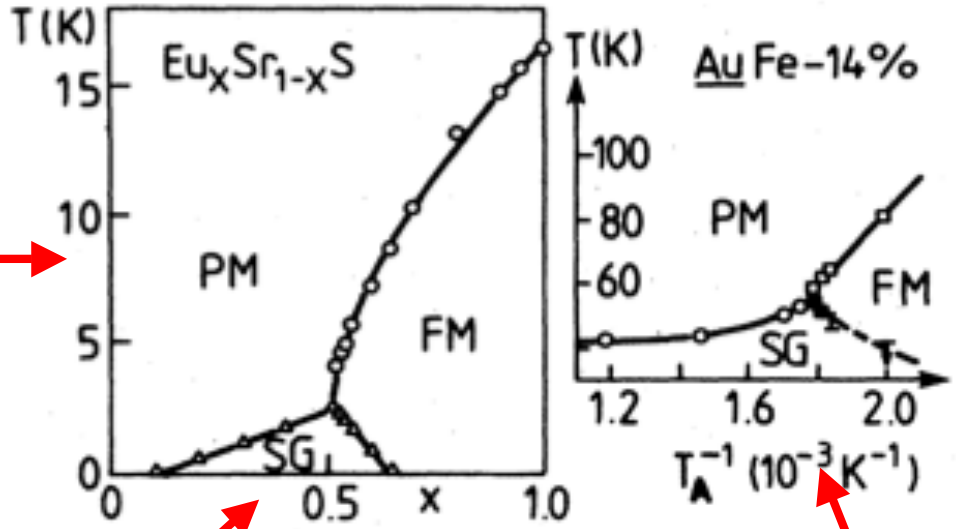
Only ferromagnet correlations here, otherwise this sums to zero

Singular spin freezing in AF or freezing spin glass



Spin glass phase diagram

EuS -ferromagnet
1st, 2nd & 3r N
interactions



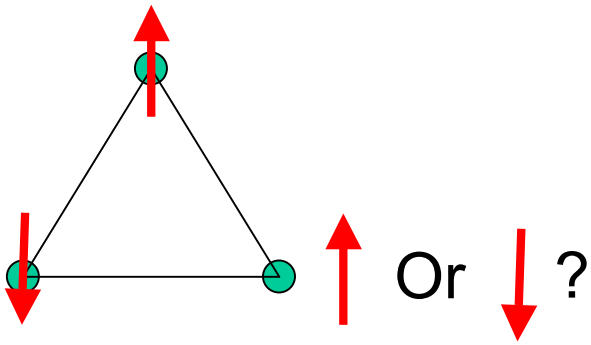
Disordered phase
characterized by
frustrated plaquettes

Low temperature annealing
removes disorder

Frustration, degeneracy and disorder:

$$H = \sum_{ij} J_{ij} S_i S_j$$

Ising antiferromagnet
on triangle, coupling $J_{ij}=J$

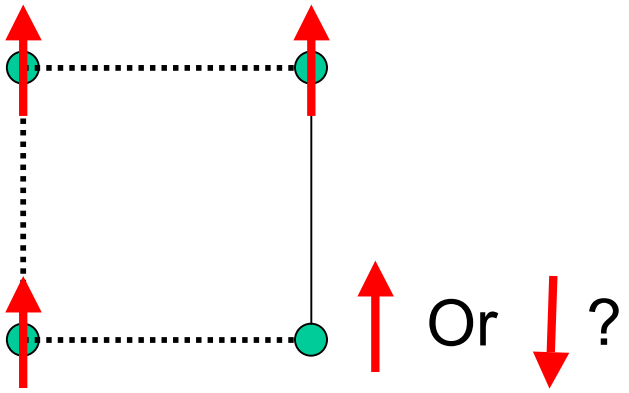


$$\Omega = 6$$

Frustrated square

Ferro $J_{ij}=-J$

Antiferro $J_{ij}=J$



$$\Omega = 8$$

Degenerate microscopic elements

In geomtrically frustrated systems the degeneracy can propagate and become macroscopic. For example the Ising triangular antiferromagnet, *G Wannier*, Phys. Rev. 79, 357, 1950.

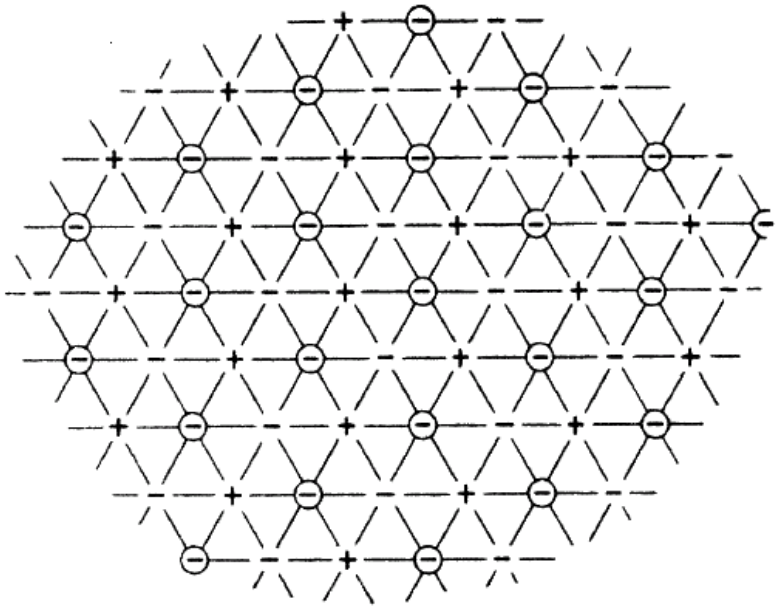


FIG. 8. An arrangement of minimum energy having finite entropy: Two sublattices have spins of fixed sign, but in the third, each sign individually may be picked at random.

$\Omega = 2^{(N/3)}$

=> $S° = R/3 \log(2) = 0.23$

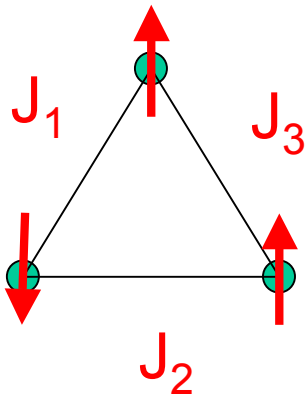
=> exact $S = 0.3383\ R$

Subset of ground states-fix two sublattices, +, - each site on the third sublattice, O, can be + or – for the same energy.

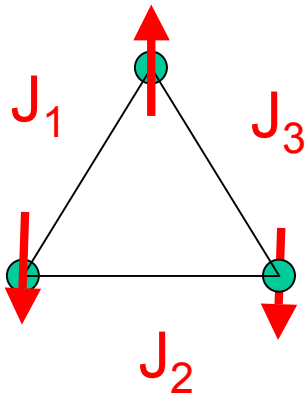
=> Exponential number of states-extensive entropy

Disorder lifts local degeneracy:

Antiferromagnetic triangle
$J_1 > J_2 > J_3$



E_a

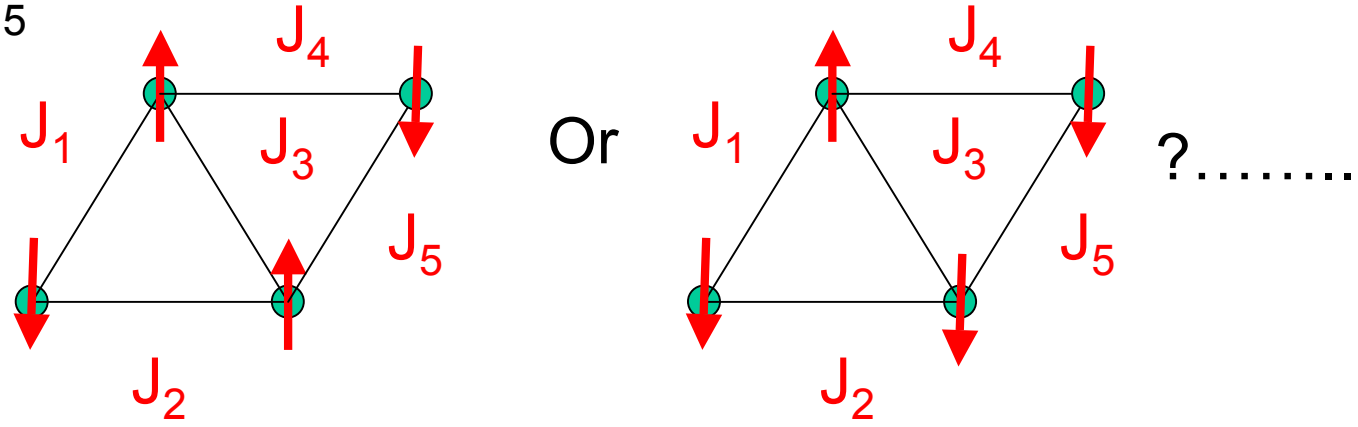


E_b

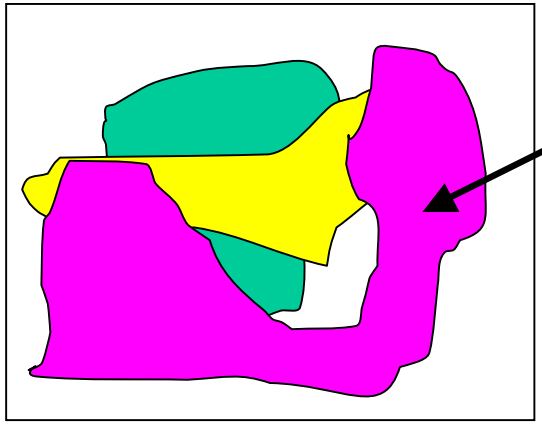
$E_a < E_b$

Fitting lowest energy elements together in disordered systems is complex - closed loops re-frustrate system at larger length scale:

$J_1 > J_2 > J_3 > J_4 > J_5$



⇒Degeneracy,
⇒metastability,
⇒energy barriers



Collective, disordered spin configuration

Spin glass phase transition

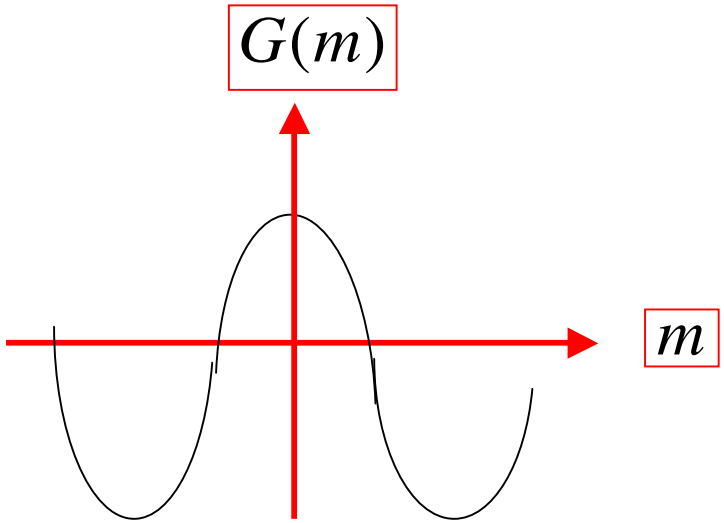
This collective “best compromise” could lead to a finite temperature phase transiton to “broken symmetry” state

Order parameter $$q_{EA} = \frac{1}{N}\sum_i < S_i >^2$$

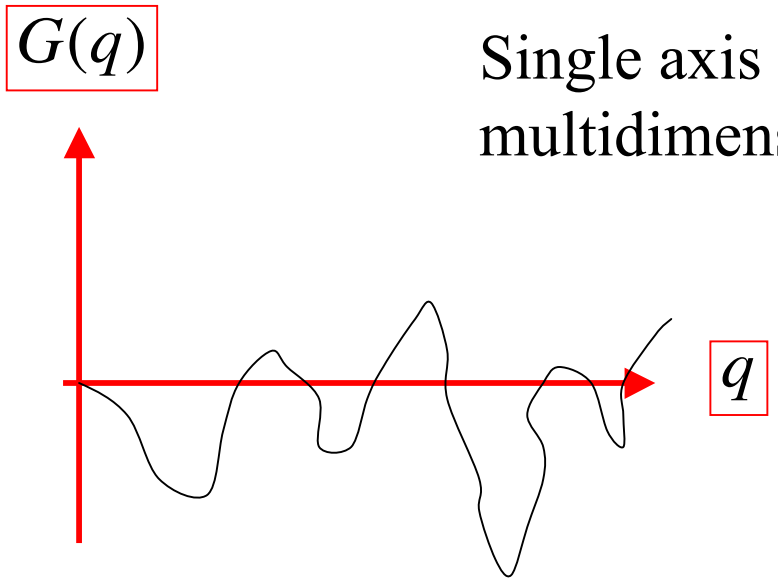
Define also $$q^{ab} = \frac{1}{N}\sum_i < S_i^a S_i^b >$$ overlap between best compromises

The famous “rough Free Energy” landscape G([q])

Ferromagnet



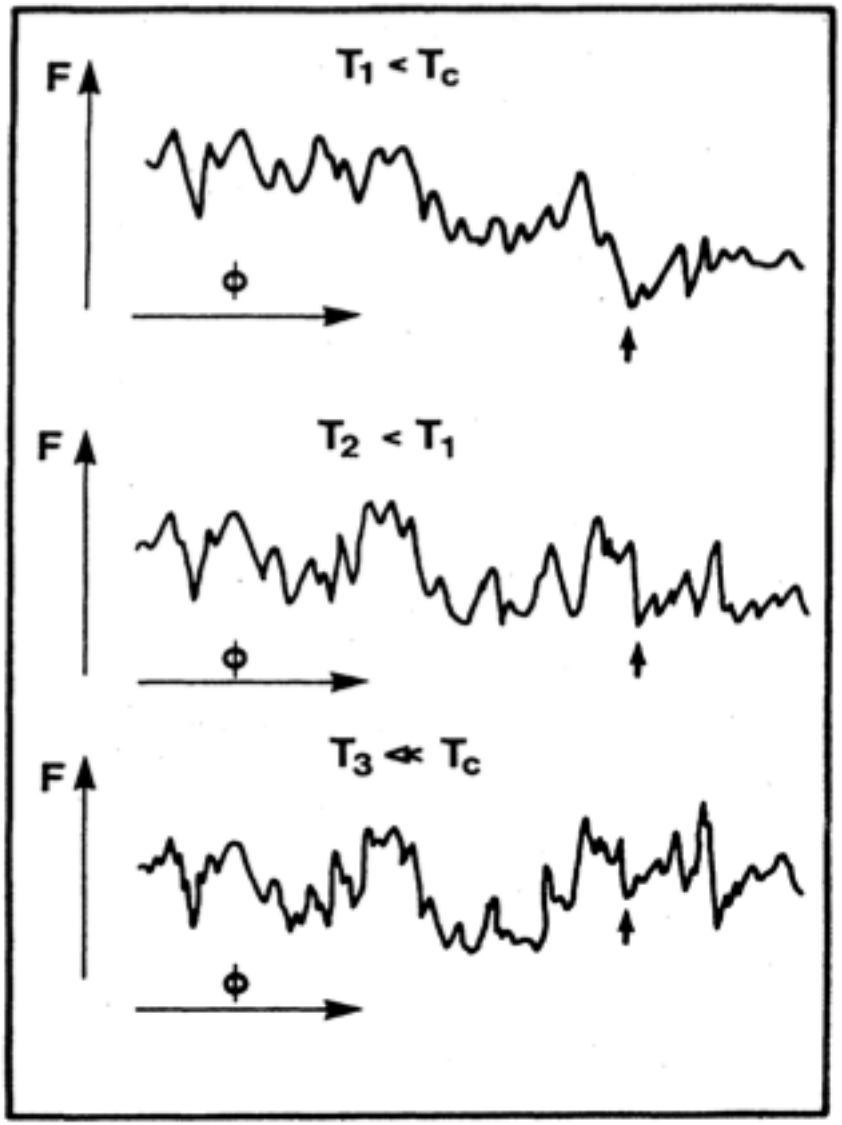
Spin Glass



Single axis of
multidimensional space

How many absolute minima - 2 ? O(N) ?

Position of minimum could evolve chaotically in temperature



Binder and Young
Rev. Mod. Phys. 58,801

Models and solutions: The Edwards Anderson models

(S. F. Edwards and P. W. Anderson, J. Phys. F 5, 965 (1975)

Random nearest neighbour interactions on (hyper)-cubic lattice

$$H = \sum_{ij} J_{ij} S_i S_j$$

$$P(J_{ij}) = A \exp\left(- \frac{(J_{ij} - J_0)^2}{2\sigma^2} \right)$$

or

$$P(J_{ij}) = p\delta(J_{ij} - J_0) + (1-p)\delta(J_{ij} + J_0)$$

Disorder leads to complex physics - even mean field theory (Sherrington-Kirkpatrick model) is a “tour de force”!

Problem is that the quenched disorder is averaged over
In free energy, not the partition function =>

$$G(N,T,J_0,\sigma) = -k_B T \int P([J_{ij}])\log(Z[J_{ij}])D(J_{ij})$$

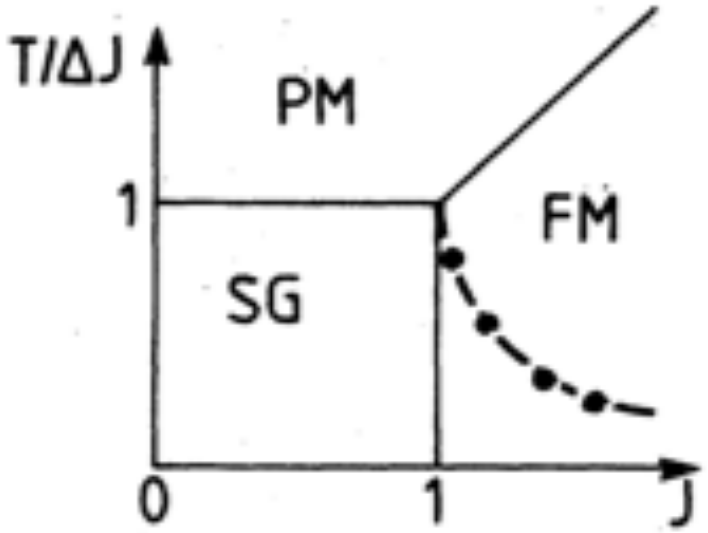
Parisi's replica trick: $$Log(Z) = Lim\ n \to 0\ \ \frac{1}{n}(Z^n - 1)$$

Take α=1,2,3…..n "replicas" of disorder.
Average over Z_n.
Take n=>0, at the same time as n becomes a continuous variable.

Create a "disorder dimension", S_i^α i=1,N α=1,n

Parisi's solution of the MFEA model (SK)

(Parisi, G., 1980a, J. Phys. A 13, 1101.) gives the spin glass transition as a "symmetry breaking" to one of Ω collectively disordered ground states

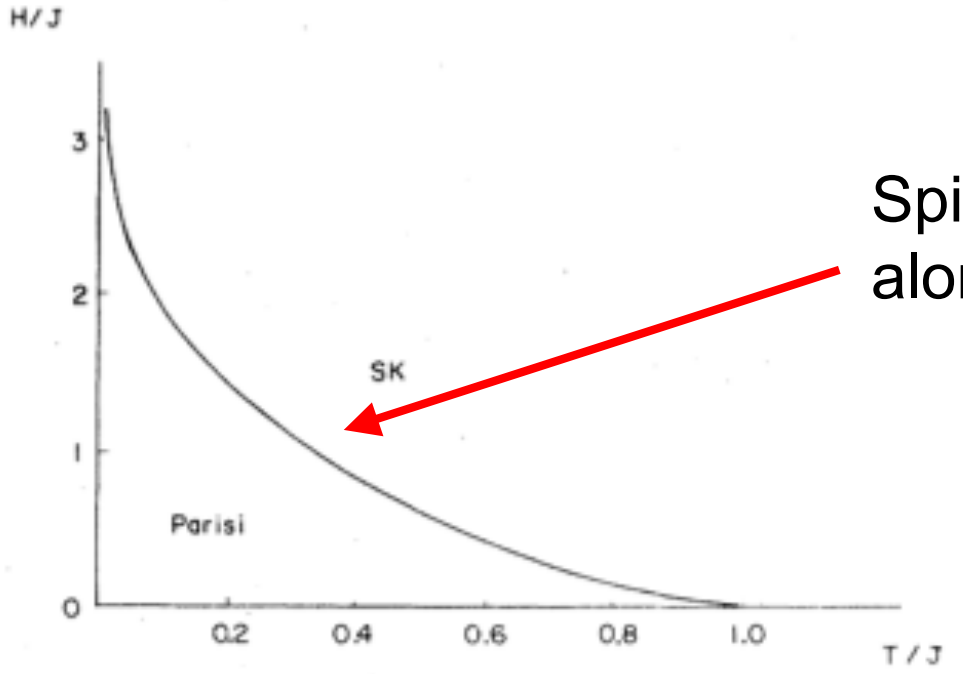


$$\log(\Omega) \sim N^{\delta} \log(2), \; 0 < \delta < 1$$

Non-extensive entropy!

Plus a hierarchy of metastable states

For many “pure” states, application of a field does not break symmetry into a single ground state. Almeida Thouless line of phase transitions in FINITE field

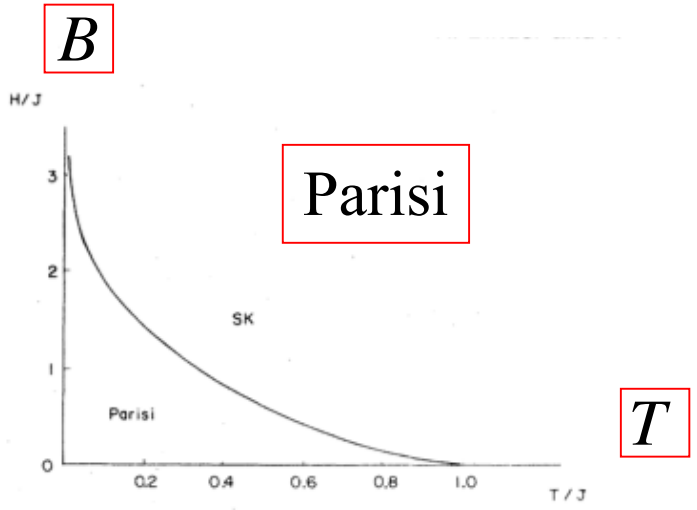


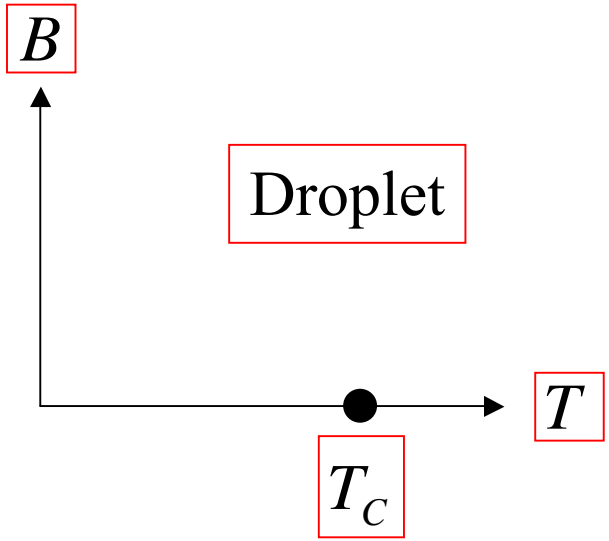
Spin glass transition along this line.

There has followed, a whole generation of intense debate concerning the reality in three dimensions

For Ising systems, in three dimensions, the Fisher/Huse school propose the“droplet picture”
(Fisher, D. S., and D. Huse, 1986, Phys. Rev. Lett. 56, 1601.)
-here the hirarchy of metastable states develops into only two symmetry related equilibrium states below T_C.

Clear distinction between these pictures comes in response to field. For droplets, field breaks symmetry in favour of one pure state - no phase transition.

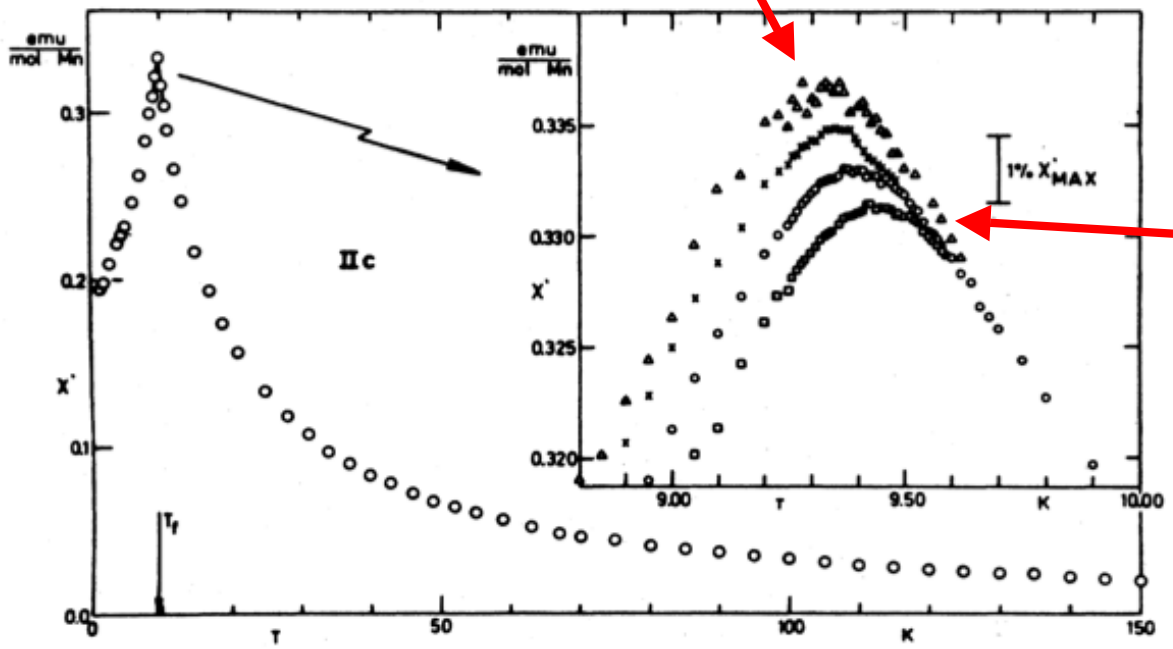




For continuous spins the debate raged over the existence of a phase transition in 3D. Now it looks as if there is one, driven by spins (Young) or effective chiral degrees of freedom (Kawamura) $\Lambda_{ij} = \vec{S}_i \wedge \vec{S}_j$

However, as spin glass - is glassy! One NEVER observes equilibrium behaviour at low temperature!

Example - this IS NOT an equilibrium kink! As ω =>0 the peak temperature moves to the left

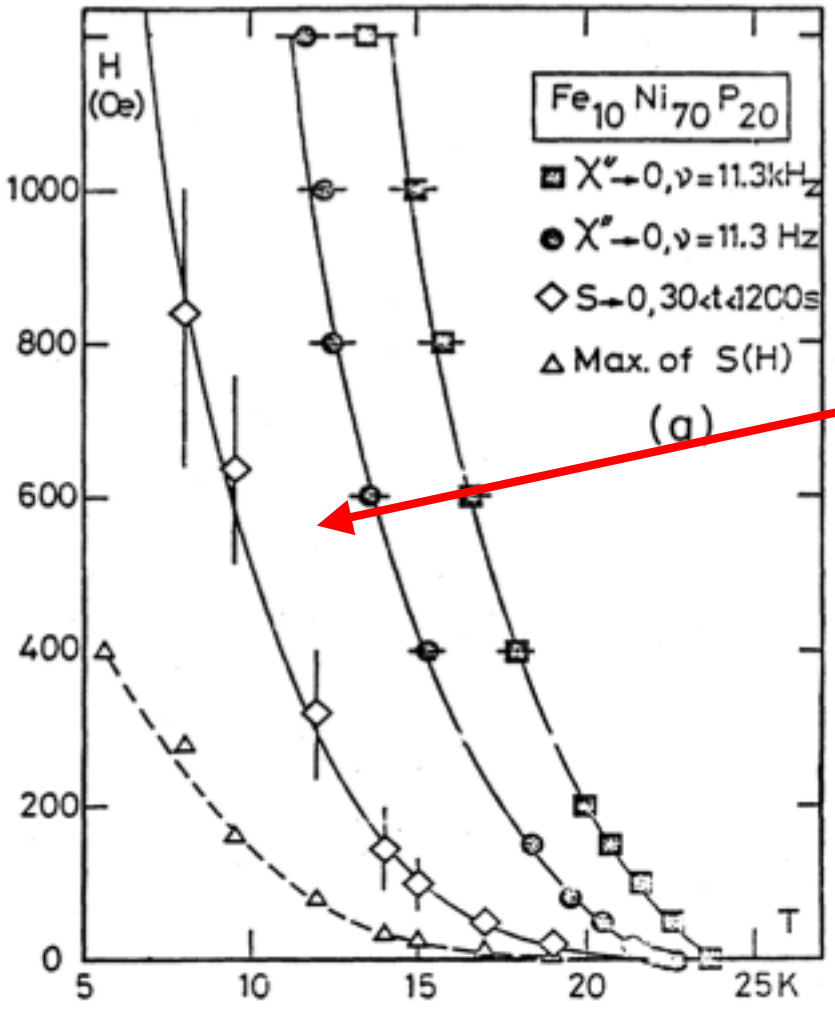


$\omega = 1000 - 1 \ Hz$

Glassy dynamics - evolution on macroscopic time scales!

Experimental Almeida -Thouless line - time dependent!

Salamon and Tholence (1983)



$\chi'(\omega)$

T

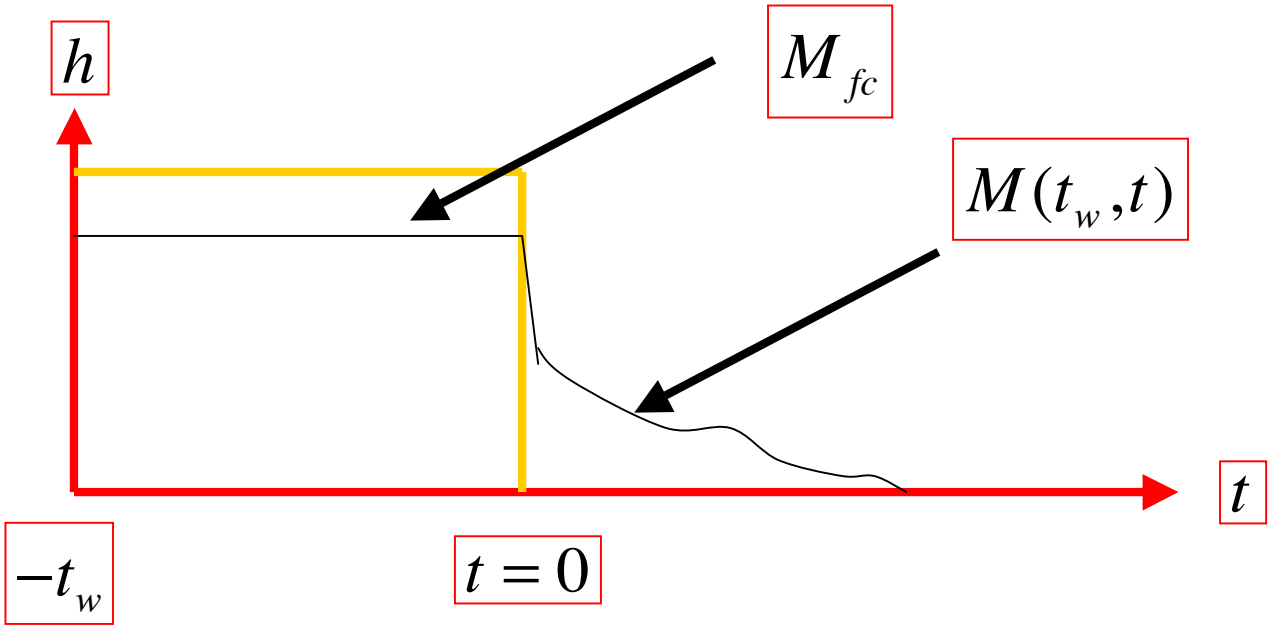
Reducing ω,
Increasing t

And for ω=0 ??

Slow dynamics: aging phenomena

In spin glasses the non-equilibrium behaviour at T_g and below results in the response time depending on the preparation time.

Aging protocol - cool in field to T<Tg. Leave to age for Waiting time t_w. Switch off field.



AgMn spin glass - relaxation depends on waiting time

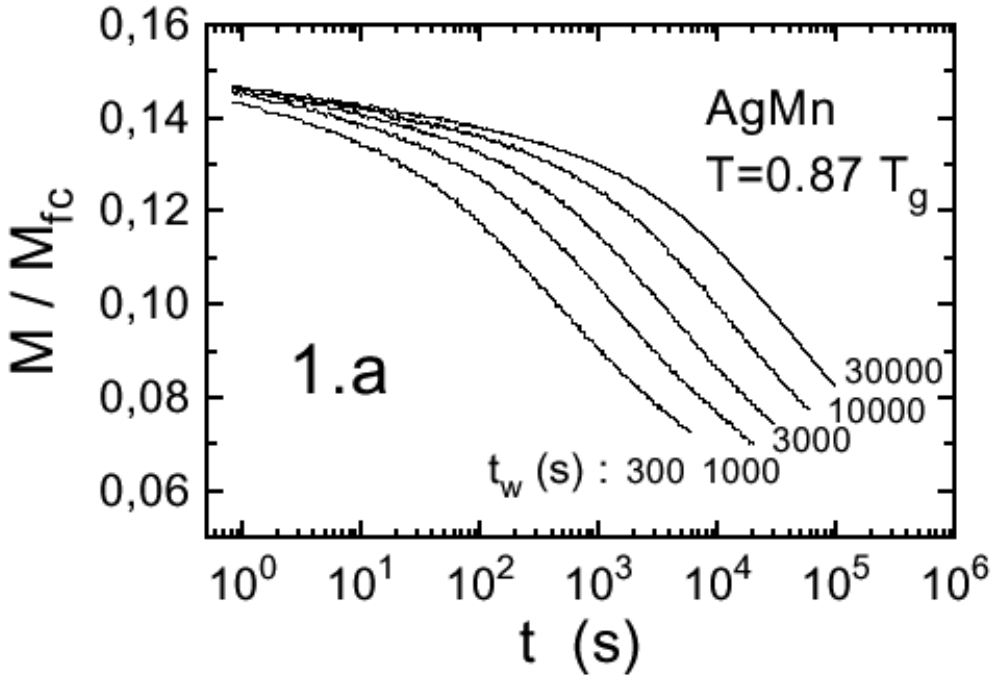


Fig. 1. a. Thermo-remanent magnetization M, normalized by the field-cooled value M_{fc}, *vs.* $t(s)$ ($\log_{10}$ scale) for the $Ag : Mn_{2.6\%}$ sample, at $T = 9K = 0.87T_g$. The sample has been cooled in a 0.1 Oe field from above $T_g = 10.4K$ to 9K; after waiting t_w, the field has been cut at $t = 0$, and the decaying magnetization recorded.

Vincent et al « Spin glasses and Random Fields, Ed. A.P. Young, 1998-Field cooled M, cut after time tw

Two-time dependence reduced to a single scaling variable- $\lambda = t/t_w$.

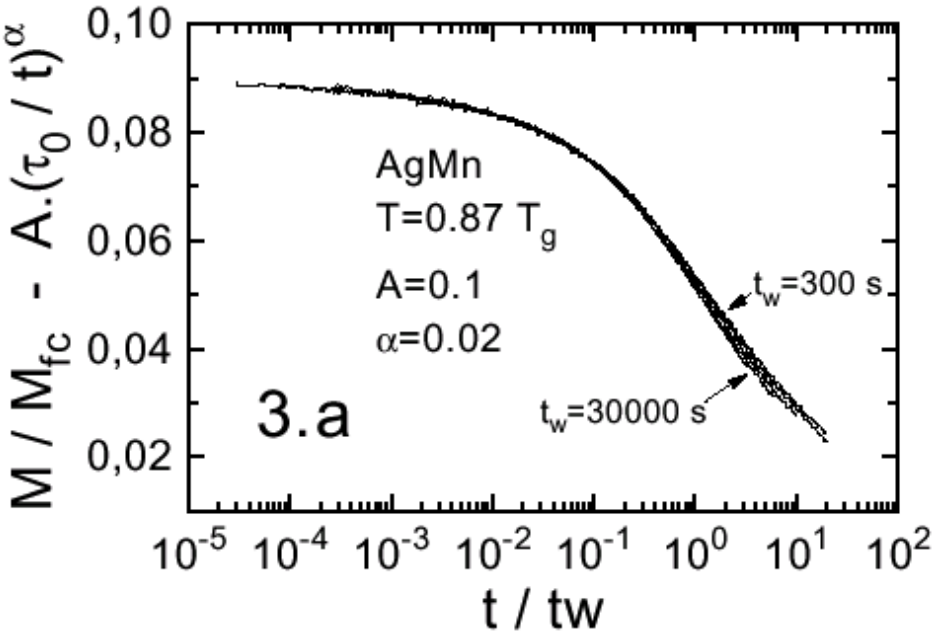
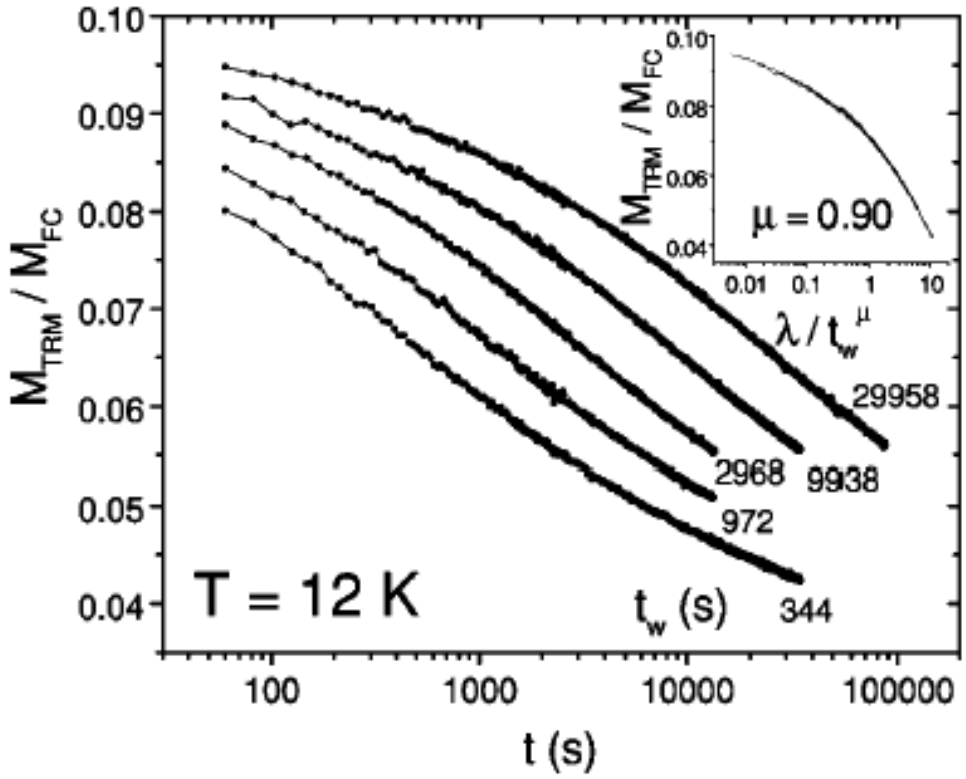


Fig. 3. a. Aging part of the TRM (Eq.28): the estimated stationary contribution has been subtracted from the full measured value. The data (same as in Fig. 1ab) is plotted *vs.* t/t_w.

Kagomé based spin glass $(H_2O)Fe_3(SO_4)(OH)_6$

Wills et al, PRB 62

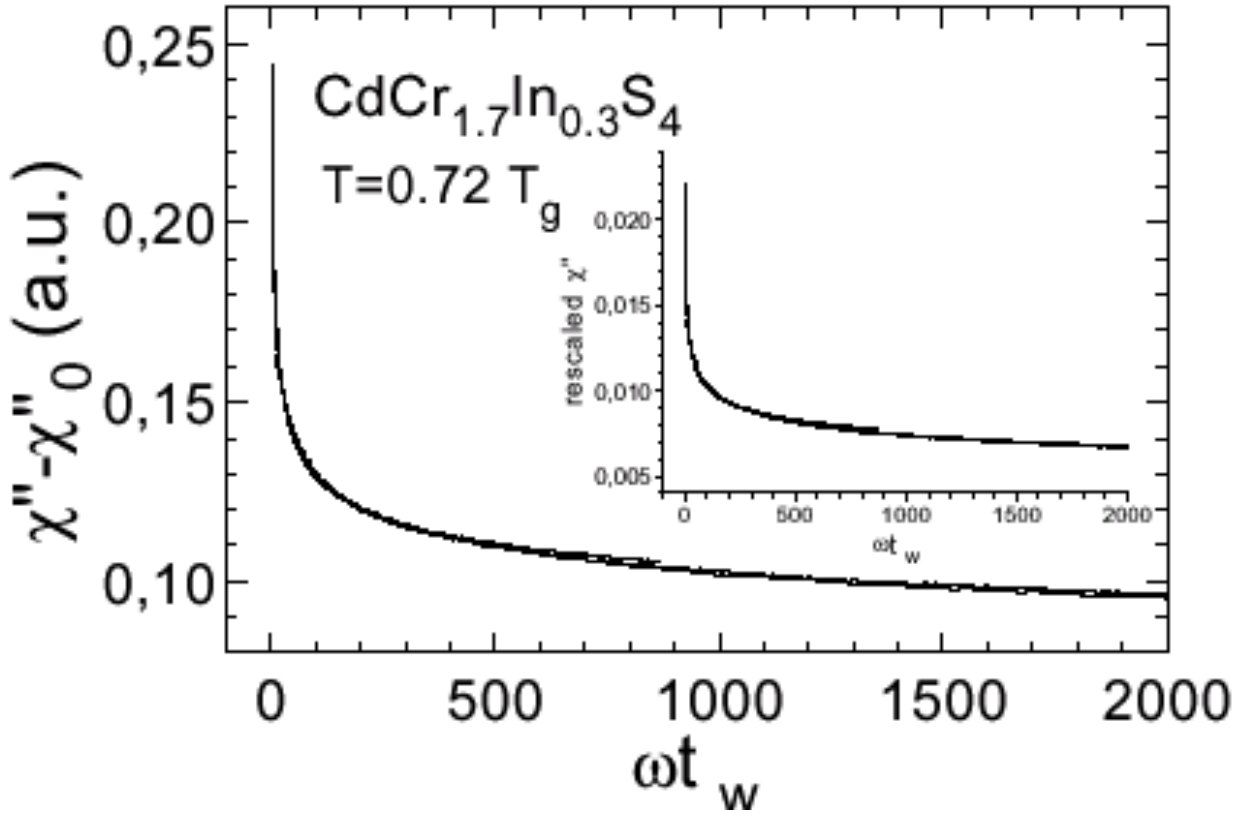
$M(t_w,t)$



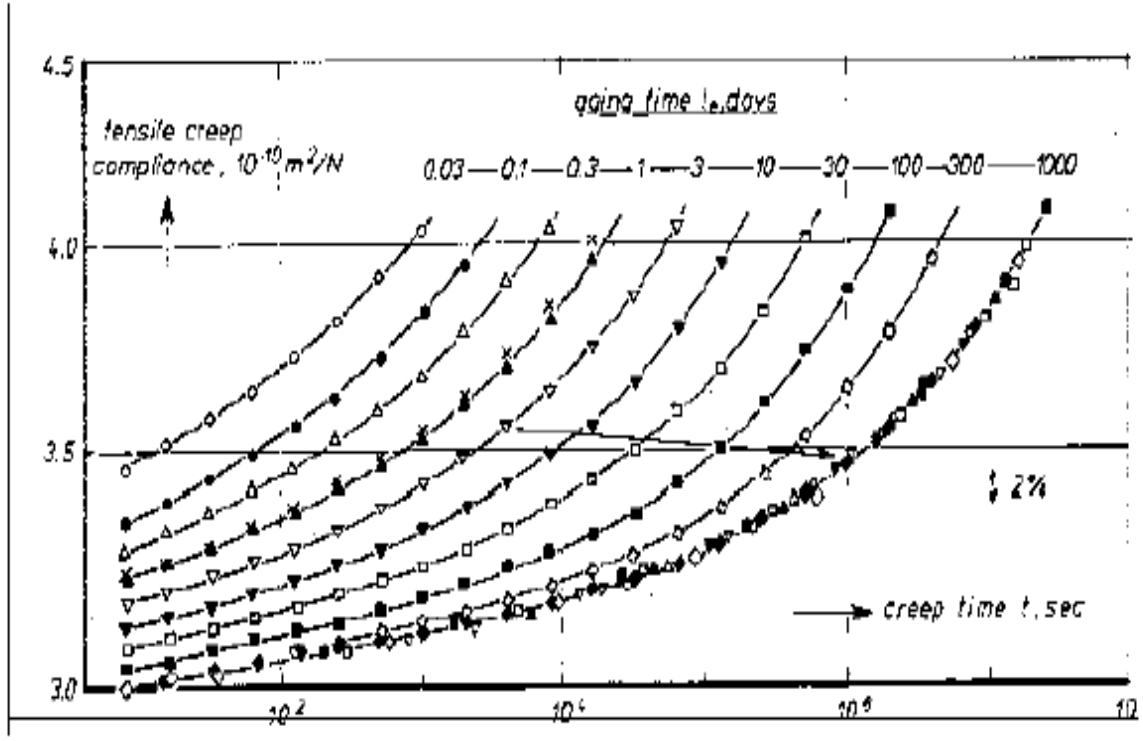
$M=f(\lambda/t_w^{\mu})$

FIG. 2. Decay of the thermoremanent magnetization at 12 K, for various values of the waiting time t_w. The inset shows the scaling of the same data as a function of the time reduced variable (Ref. 22) λ/t_w^{μ}, with $\mu=0.90$.

Imaginary part of AC susceptibility, fixed t_w
Applied field at frequencies ω= 0.01, 0.03, 0.1,1 Hz

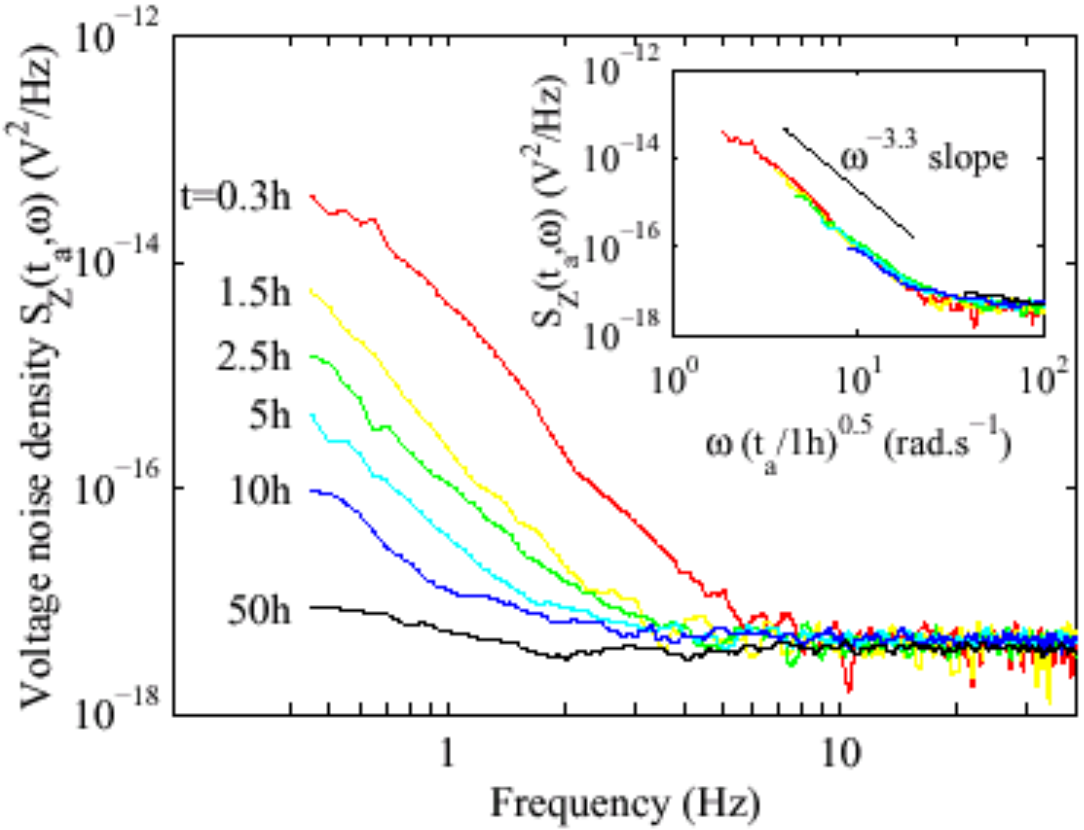


This aging phenomenon where the characteristic time depends on the sample preparation is generic to all glassy systems time- strain response to applied stress in PVC
-last plot is the scaled data.



L. Struik, « Physical ageing in amorphous polymers and other materials », Elsevier, 1978

Colloidal glass: Bellon et al, Europhys. Lett. , **51**, 551, 2002.
Voltage noise spectrum $S(t_w,\omega)$ in a lyaponite (clay) gell is a function of waiting/preparation time t_w. Scaling data by t_w gives collapse onto a master curve



Fluctuation dissipation theorem:

Define the response function: $$R(r_i, r_j, t, t') \sim \left\langle \frac{\partial S_i(t)}{\partial h_j(t')} \right\rangle$$

And correlation function: $$C(r_i - r_j, t - t') = < S(r_i, t) S(r_j, t') >$$

(for equilibrium we assume translational invariance in space and time)

FDT states $$R(r, t, t') = \frac{1}{T} \frac{\partial C(r, t - t')}{\partial t'}$$

(So that $$C(r) = < S(r) S(0) > = T \int_{-\infty}^{t} R(r, t, t') dt' = T \chi(r)$$)

Comments: $\chi(r,t) = \int_{-t}^{0} R(r,t')dt'$ $\chi(r,\omega) = \int_{0}^{\infty} \chi(r,t)\exp(-i\omega t)dt$

$\chi(\omega) = \chi'(\omega) + i\chi''(\omega)$ where $\chi''(\omega)$ is related to the energy dissipated when a perturbation is $h(t) = h_0 \cos(\omega t)$ added to the system.

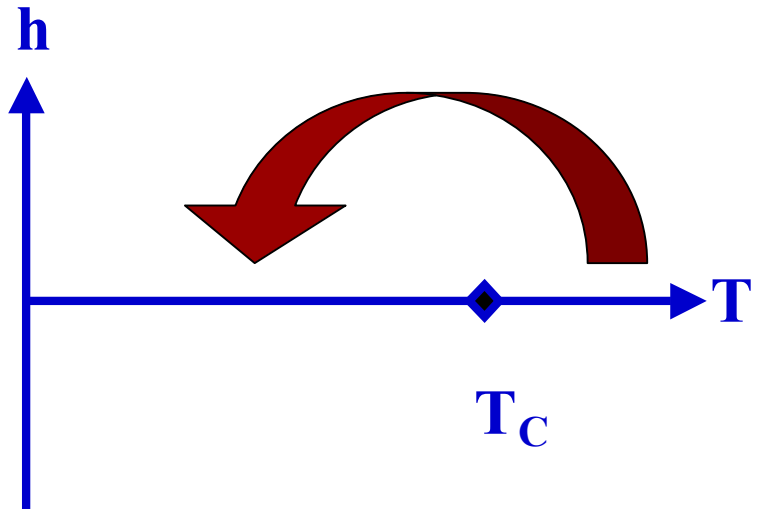
In equilibrium, measuring response tells you about fluctuations (fluctuations are very difficult to measure!)

Magnetic response of a disordered system

$$\Delta M = \int \left(\int R(r,r',t,t')h(r',t')drdr' \right) dt' \sim \int R(t,t')h(t')dt'$$

This universal behaviour invites study of model systems

Coarsening-spinodal decomposition in an Ising ferromagnet-quench from high to low temperature in zero field



Quench at $t=-t_w$.
Snapshot at t=0

Competition between two equivalent minima with spin up and spin down. Domains on characteristic **(temperature independent)** length scale $\ell(t) \sim t^{1/z}$, **z=2.**

L.Berthier, J-L. Barrat, J. Kurchan,EPJB, **11**, 635, 1999

Successive config's for increasing t_w, T=0.1J

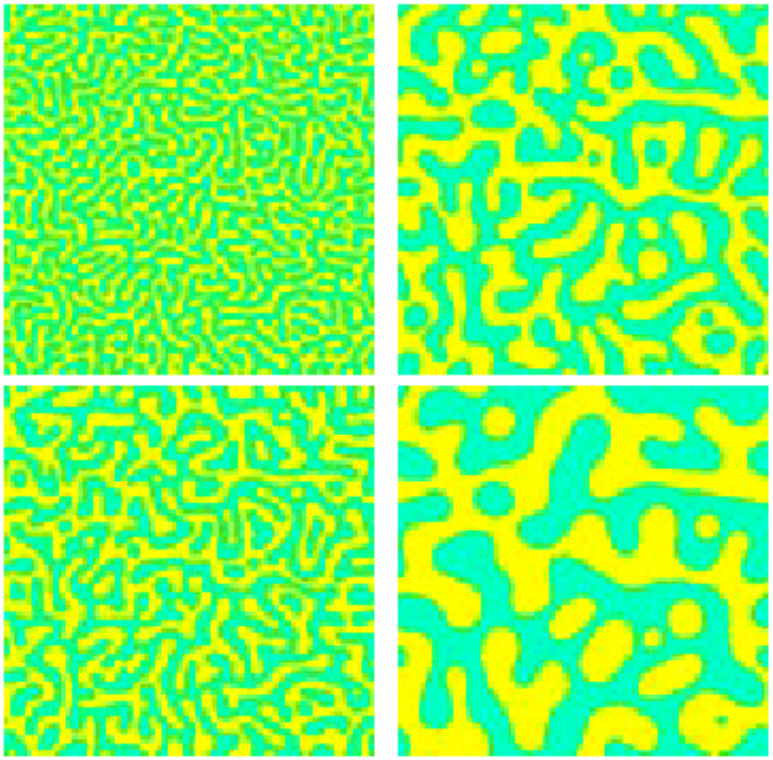
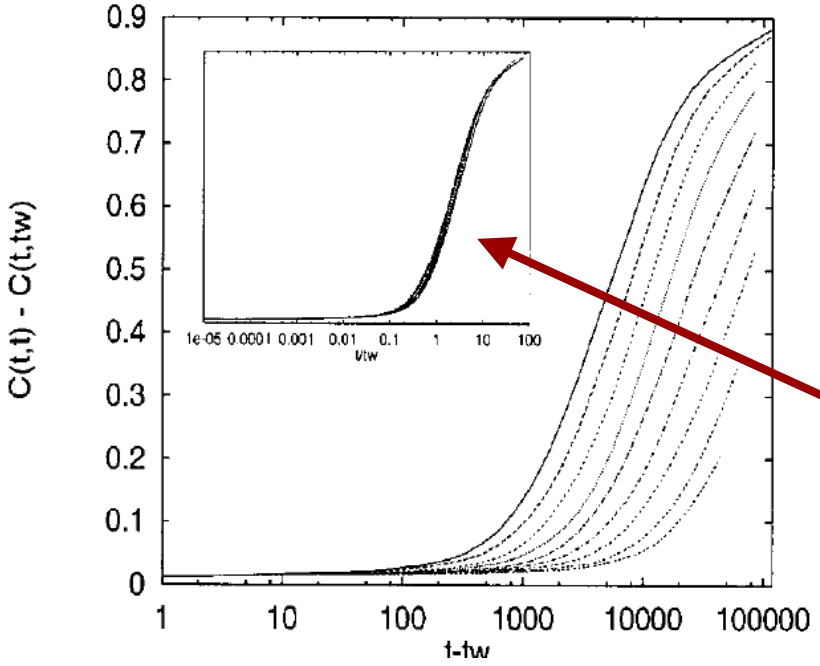


Fig. 1. Field configurations during the simulated phase separation for times 317, 1262, 5024, 20100. Each color represents one phase.

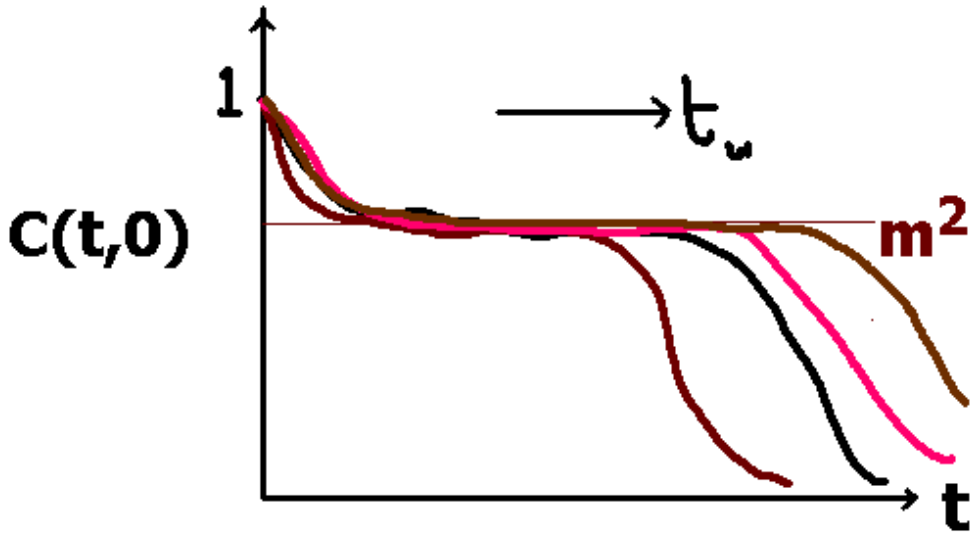
One site, two time correlation funciton

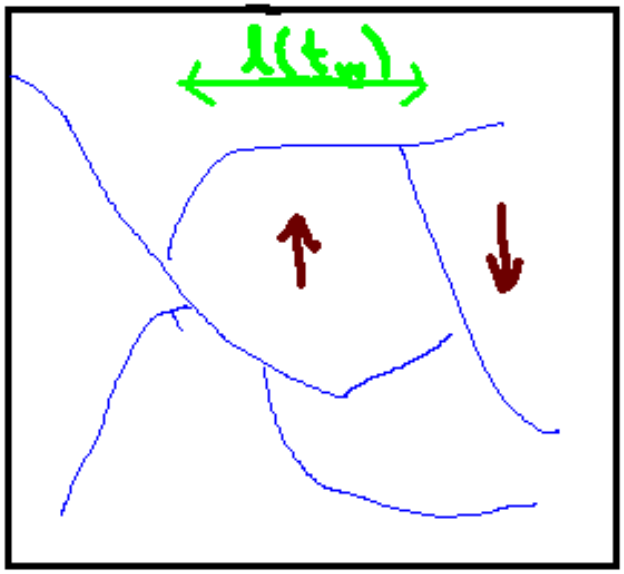
$$C(t,t') = \frac{1}{N}\sum_i S_i(t) S_i(t')$$



Scaling in terms of t/tw

$C(t,t_w)$ relaxes to an equilibrium value on a short time scale and relaxes to zero on times $t \sim t_w$





For short times correlations are within a single domain $C(t,t_w) \sim m^2$.
For longer times correlations are between different, randomly orientated domains, $C \Rightarrow 0$

Loss of equilibrium shows up in the FDT. One can write

$$R(r,t,t') = \frac{X(t,t')}{T}\frac{\partial C(r,t,t')}{\partial t'}$$

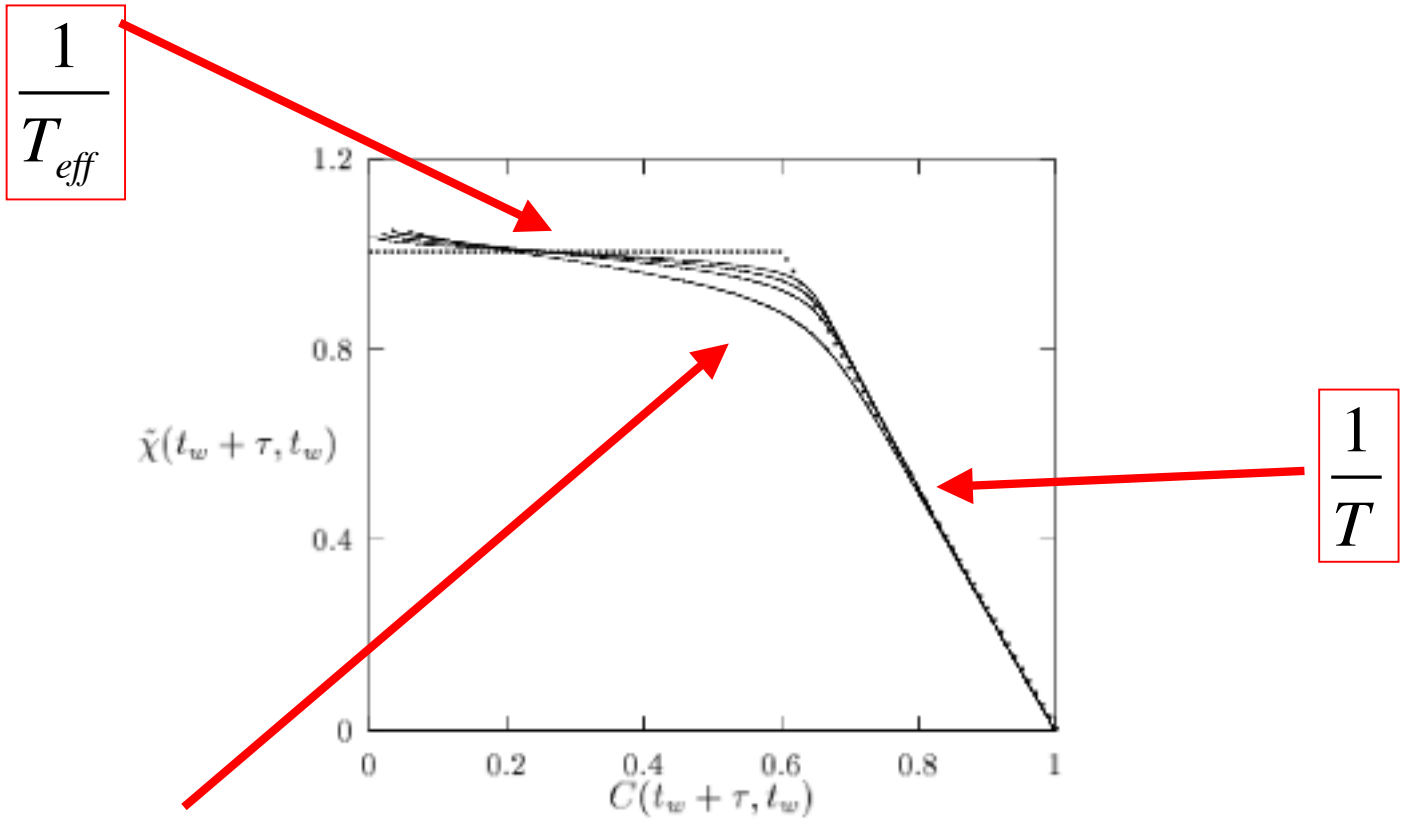
where X(t,t') is an arbitrary function, which one could interpret as an "effective temperature" T_{eff}=X/T

$$\chi(t,t_w) = \int_{t_w}^{t} dt' \frac{X(t,t')}{T}\frac{\partial C(t,t')}{\partial t'}$$

$$\chi(t,t_w) = \int_{C(t,t_w)}^{1} dC' \frac{X(C)}{T}$$

T_{eff} is the slope of a parametric plot χ vs C
(CuKu plot after Cugliandolo-Kurchan)

For aging ferro-magnet (analytic mean field) at temperature T

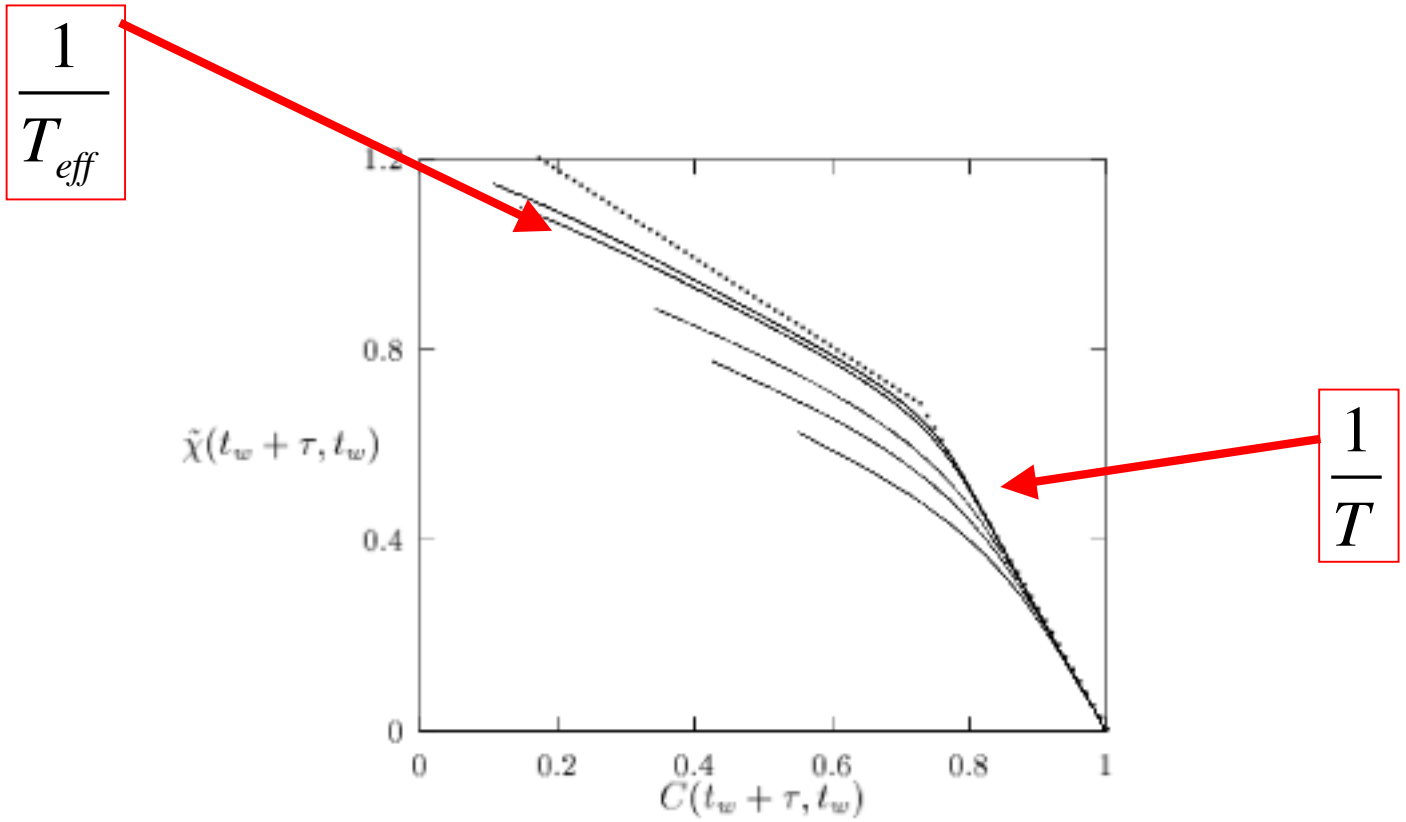


Curves for different t and t_w

$T_{eff} = 0$

t/t_w scaling

For aging mean field spin glass at temperature T



$0<T_{eff}<T$

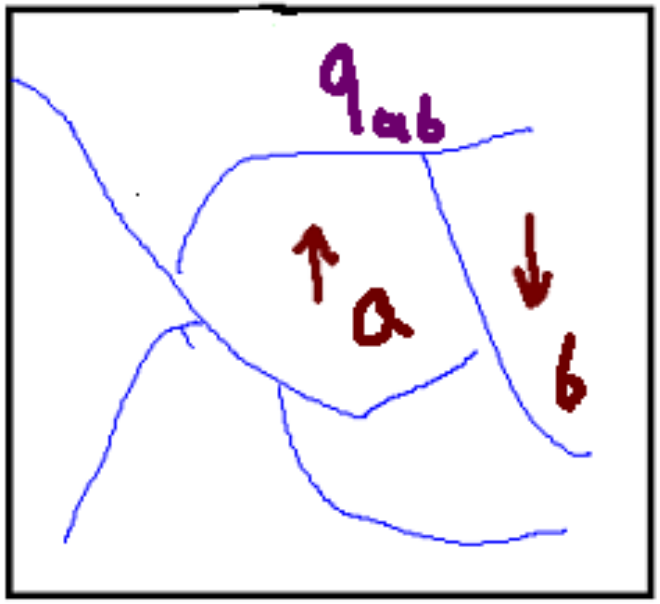
X is related to the overlap of a spin with itself in “space-time”.

In same domain X=1.

Between two domains X gives probability that overlap

Franz, Mézard, ParisiPRL, **81**, 1758, 1998.

$$q^{ab} = \left| < S^a S^b > \right| < m^2$$



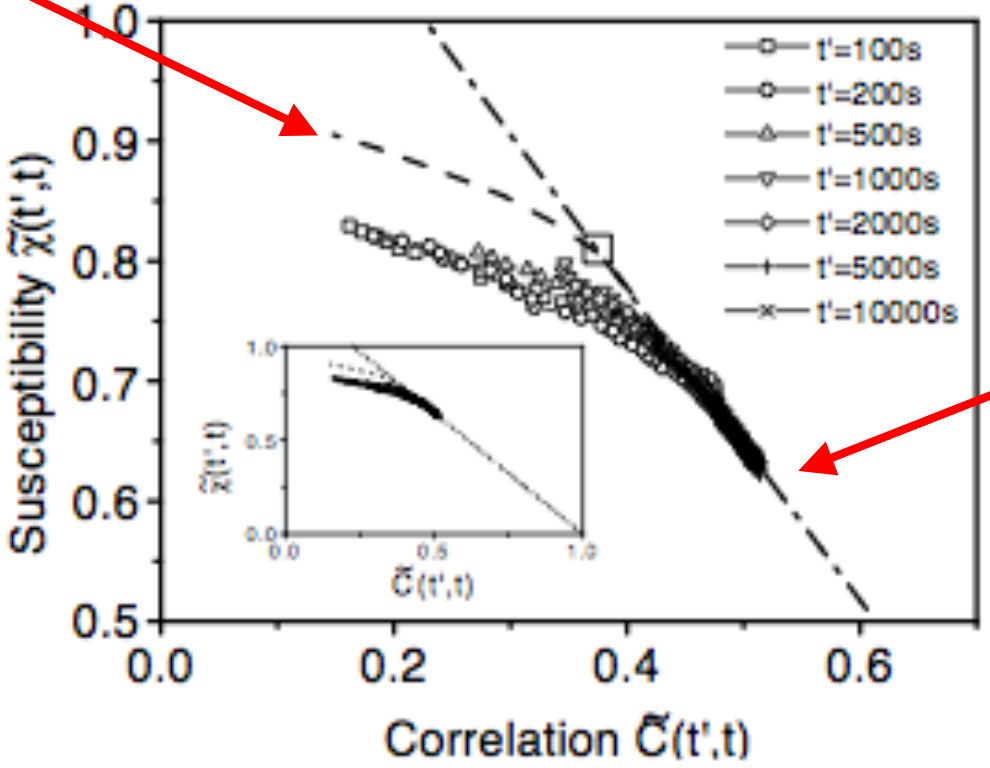
For coarsening ferromagnet, X=0 => T_{eff}=0

For coarsening spin glass with complex structure , <0X<1 => 0<T_{eff}<T

And for experiment ? Fluctuations and response in $CdCr_{1.7}In_{0.3}S_4$ spin glass

D. Hérisson and M. Ocio, Phys. Rev.Lett. 88, 257202 (2002)

$\frac{1}{T_{eff}}$

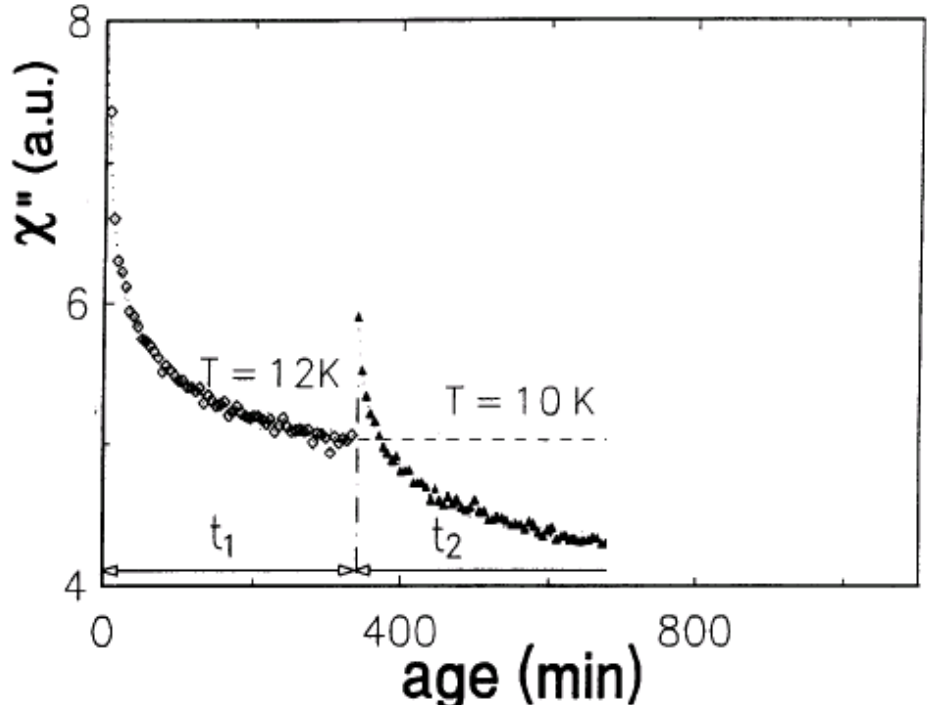


$\frac{1}{T}$

Qualitatively very like a MF spin glass with complex structure

Simple coarsening explains a lot but not everything:

Rejuvenation-after a second quench to a lower (or higher) temperature, the ageing procedure restarts from zero-
Vincent et al Spin glasses and random fields

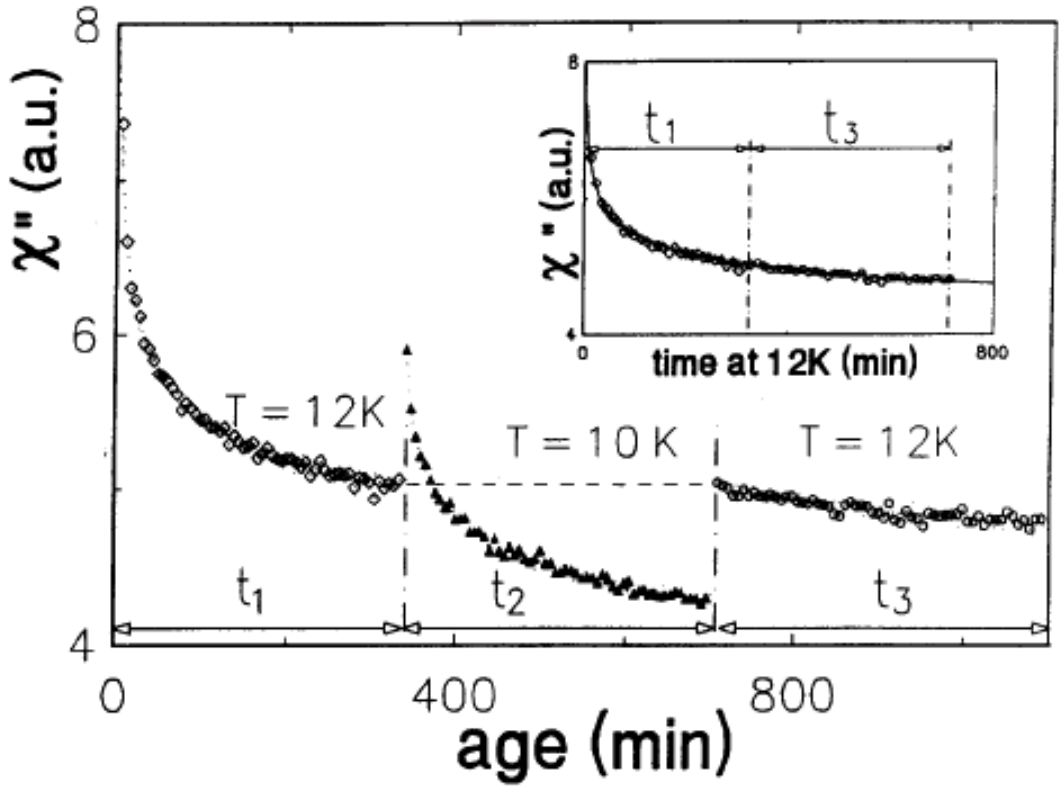


$\chi''(\omega, t_a)$ of the $CdCr_{1.7}In_{0.3}S_4$

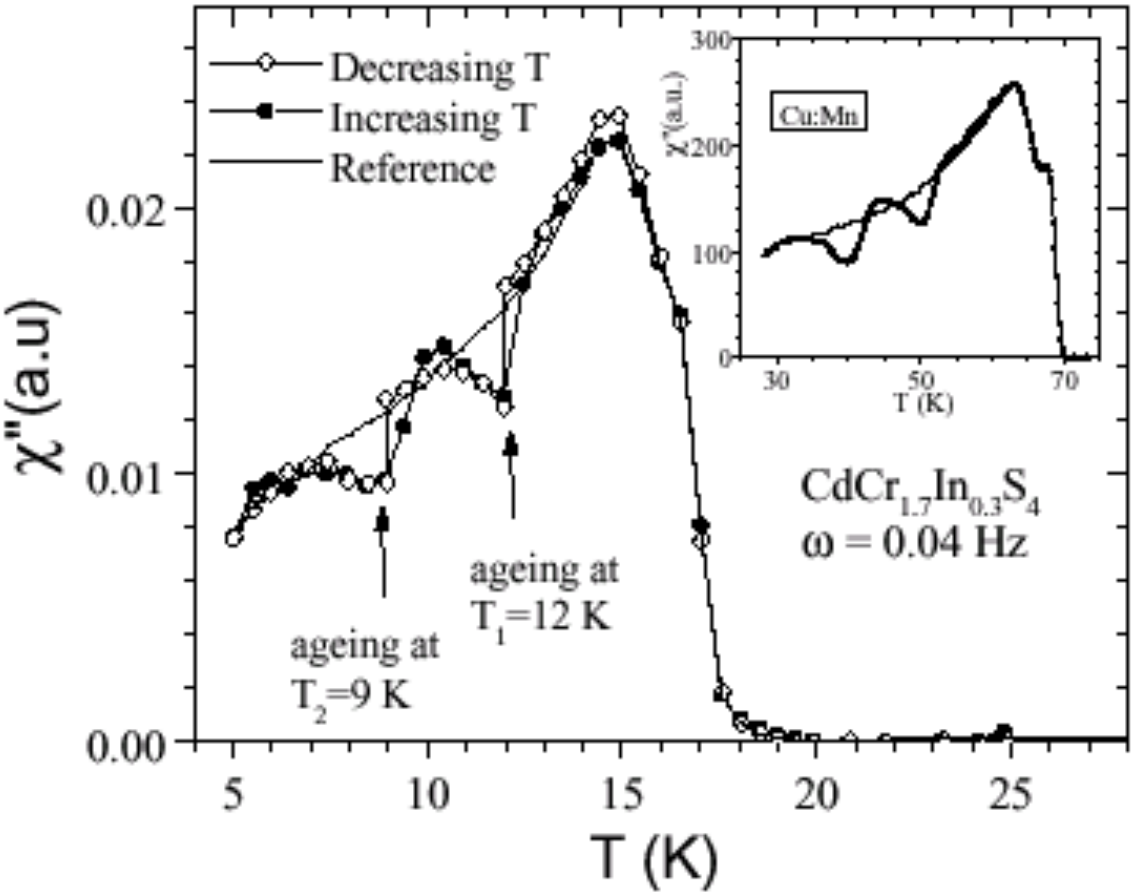
Memory effects: after a second quench to T_2, the system returns to T_1 and remembers where it left off…..

Vincent et al

$CdCr_{1.7}In_{0.3}S_4$



$CdCr_{1.7}In_{0.3}S_4$

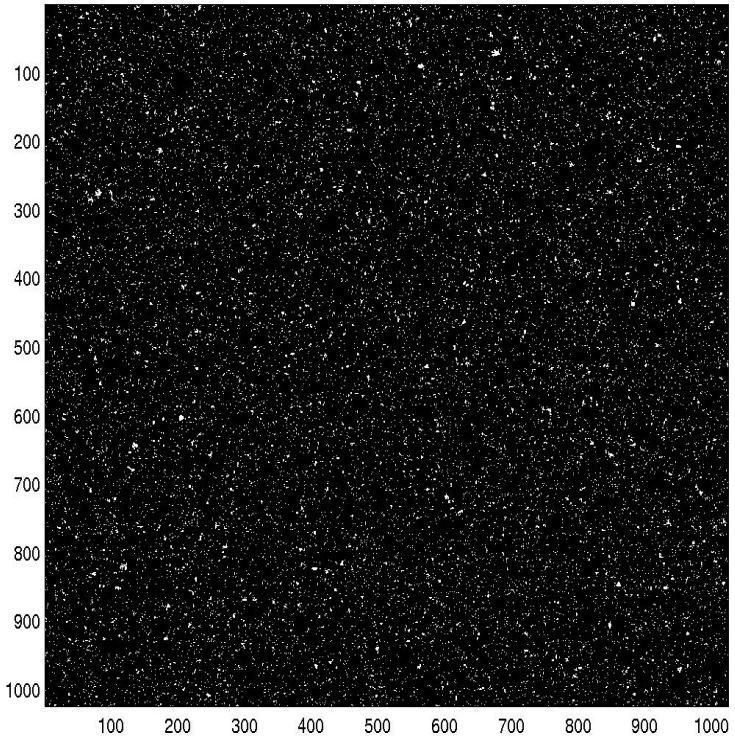


Jonason et al Phys. Rev. Lett. 81, 1998

simple coarsening can not explain rejuvenation
and memory effects.

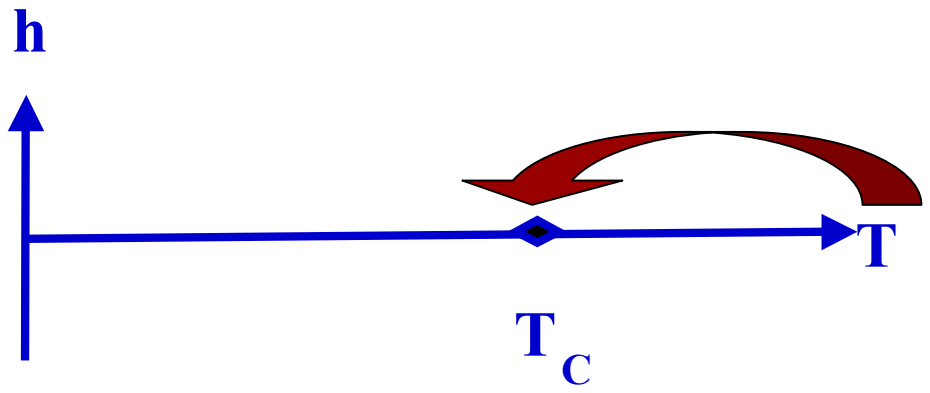
Domain length scale $\ell(t)$ is decoupled from the equilibrium correlation length ξ_{eq}. On changing T, thermal fluctuations of the bulk equilibriate on a microscopic time scale =>**No rejuvenation**

Domains continue to grow at new temperature T
=>**No memory**



Ising 2D, T<TC. ξ_{eq} is microscopic

Proposition: quench to a critical point-domain size and correlation length are locked together, $\xi(t)=l(t)$



Godreche and Luck J. Phys. A, 33, 9141, 2000

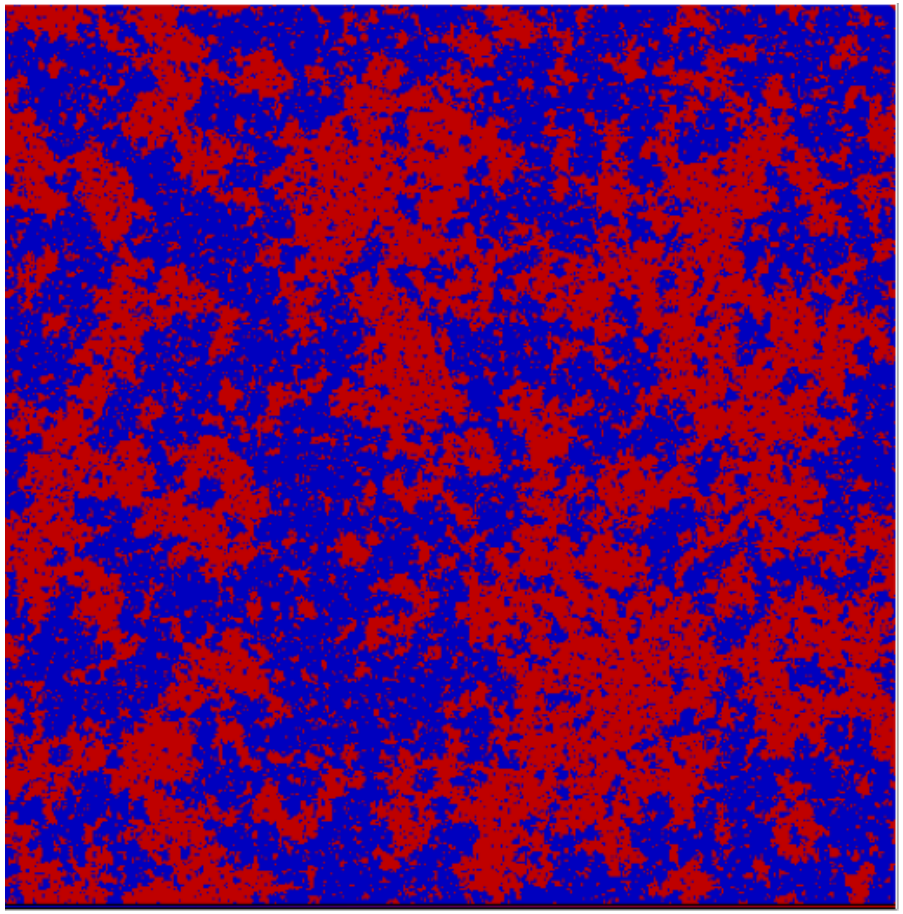
$$\xi_{eq}=\infty$$
$$\xi(t)=l(t)\sim t^{1/z}$$

Here all length scales $< \xi$ contribute to observable quantities.

Ising Model at criticality

$$m=\frac{1}{N}\sum_i S_i \quad ,S_i=\pm 1$$

The mean value $\overline{m}$ is neither $O\left(\frac{1}{\sqrt{N}}\right)$ *or* $O(1)$



Ising model $T=T_C$

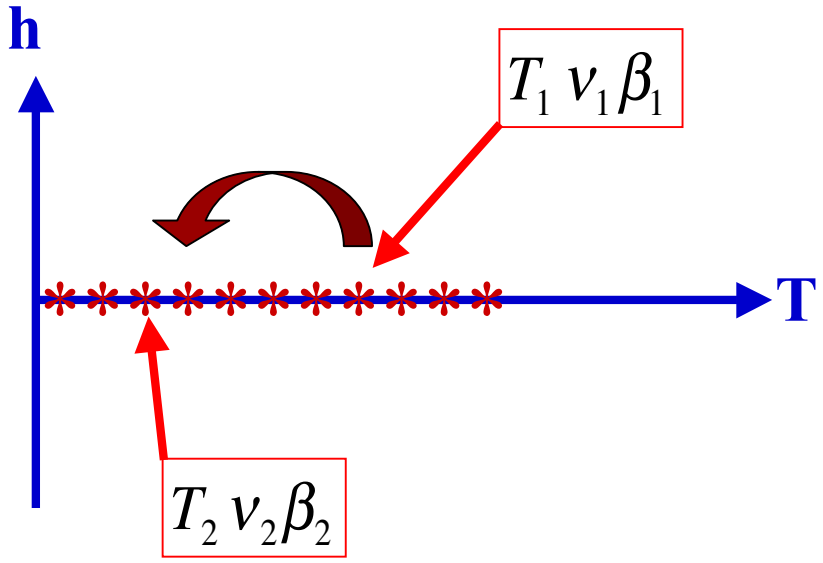
$$\overline{m}\propto L^{-\frac{\beta}{\upsilon}}, \quad \sigma\propto L^{-\frac{\beta}{\upsilon}}, \quad \Rightarrow d_f=2-\frac{\beta}{\upsilon}$$

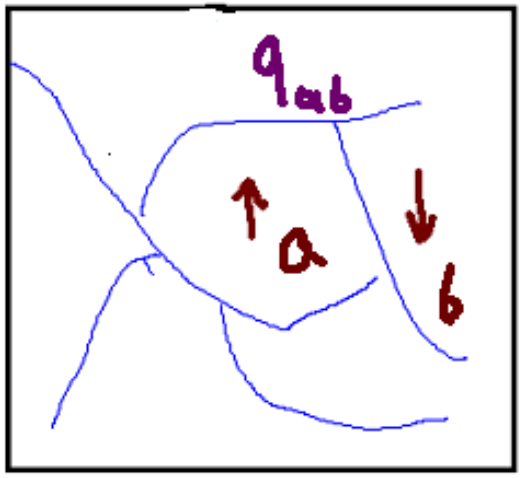
Spin up

Spin down

Surfing on a critical line

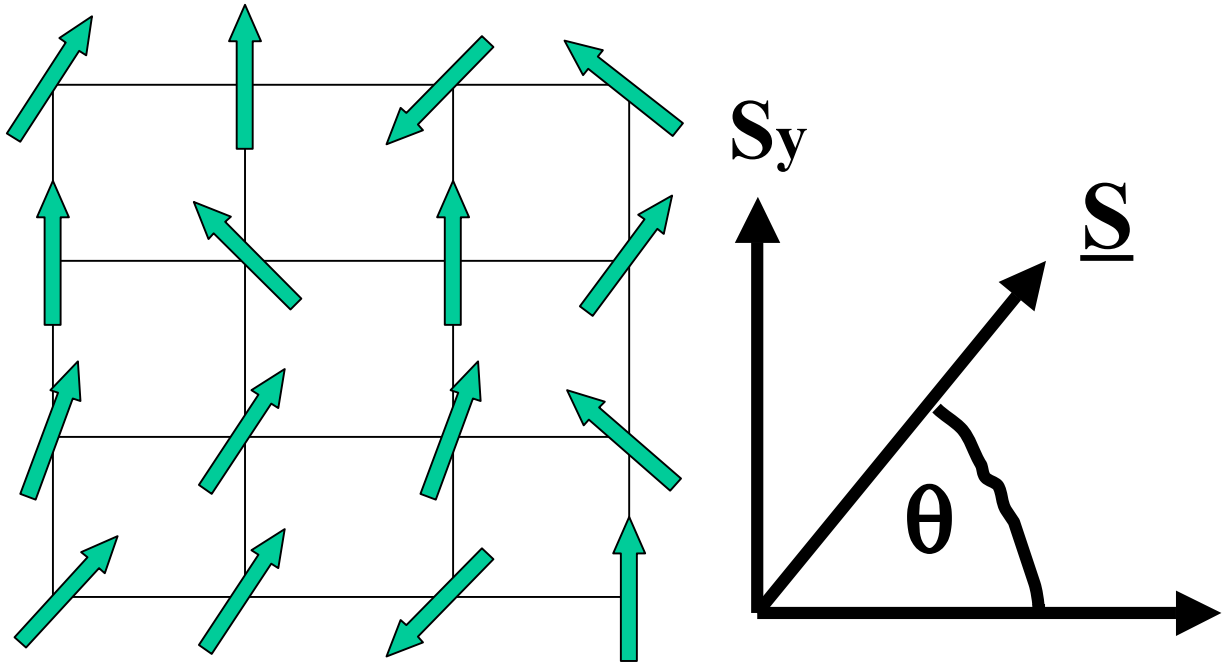
If we could do aging along a line of critical points all length scales would fall out of equilibrium when T Changes, L. Berthier, P. Holdsworth, Europhys. Lett, 58, 35, (2002).





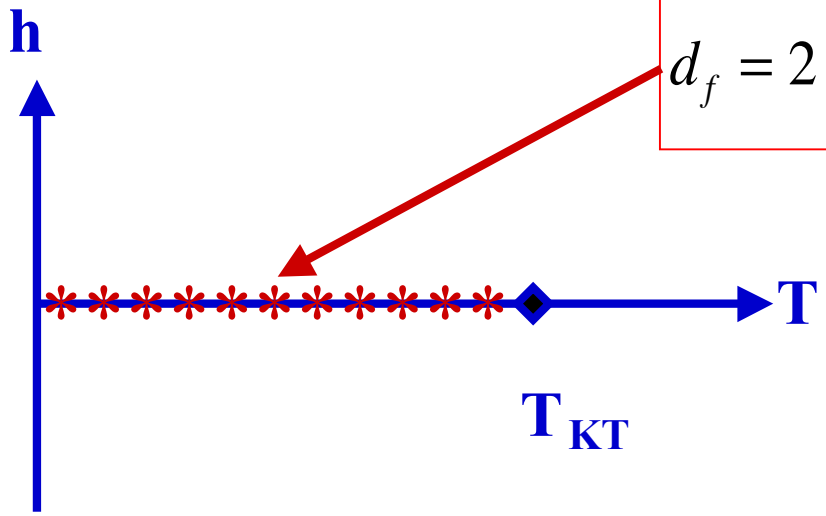
Fractal structure changes so domain a must change on all length scales

There is such a system:
The 2D-XY model



$$H = -J \sum_{<i,j>} \cos(\theta_i - \theta_j) - h \sum_i \cos \theta_i$$

$$m = |\vec{m}| = \left| \frac{1}{N} \sum_i \vec{S}_i \right|$$



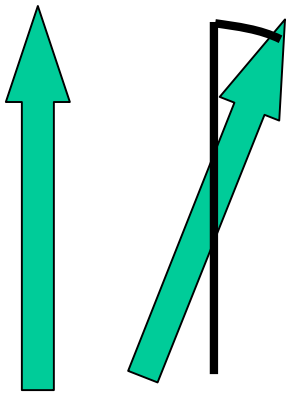
$$d_f = 2 - \frac{k_B T}{4\pi J}$$

$$H \sim \sum_q \frac{Jq^2a^2}{2}\varphi_q^2$$

Normal mode with wave vector q

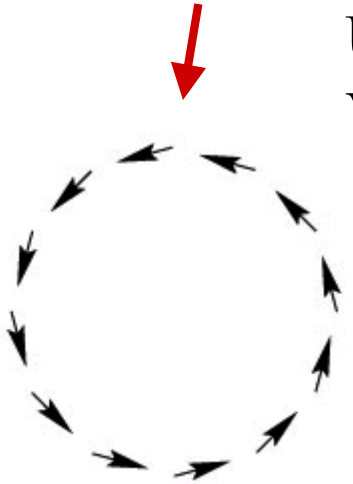
Two types of excitations, spin wave = small rotation by angle dθ, and topological defects => vortices

Spin wave



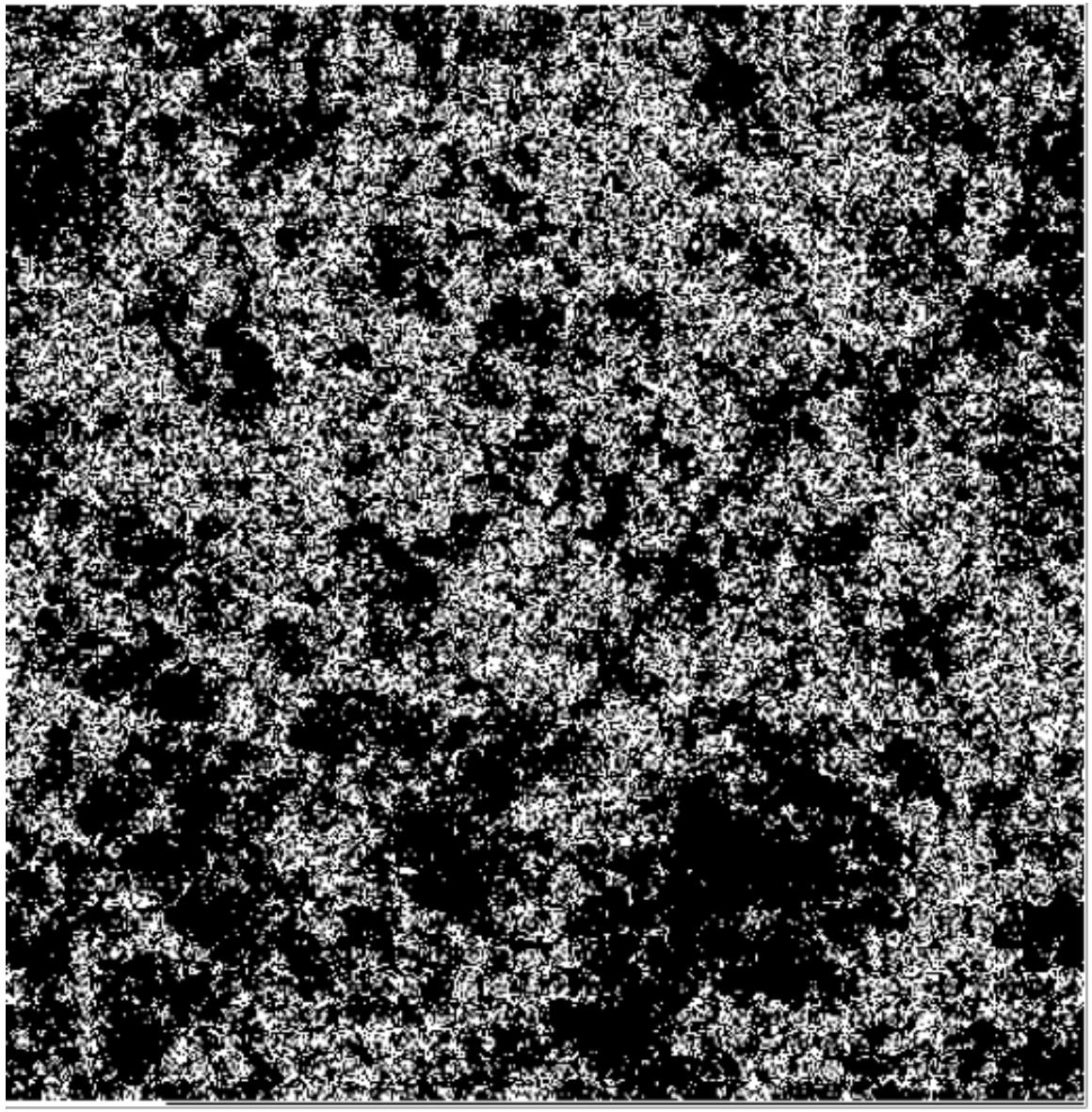
Critical phenomena below T_{KT}

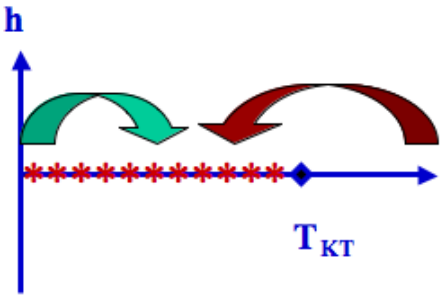
Vortex



Unbinding of Vortices at T_{KT}

Fractal structure:
2D-XY model at
T/J = 0.7, N
=512*512
Projection of θ onto
direction of <m>

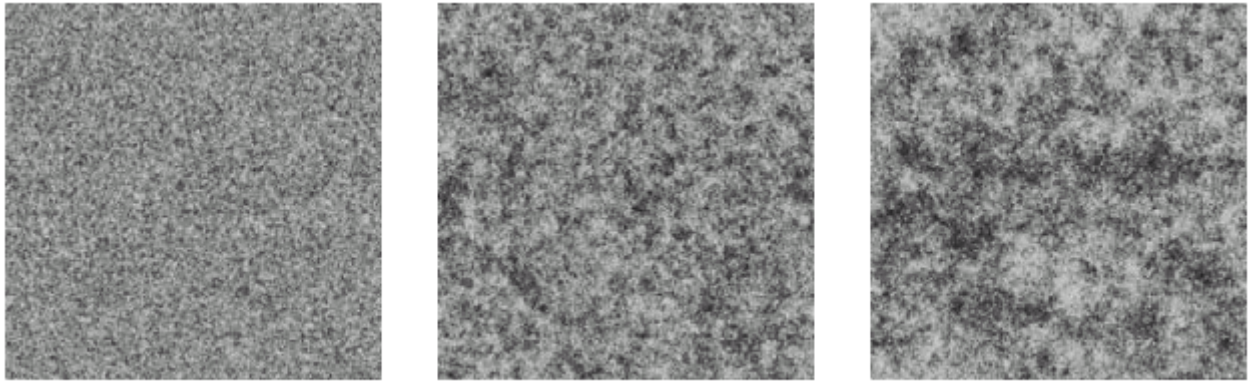




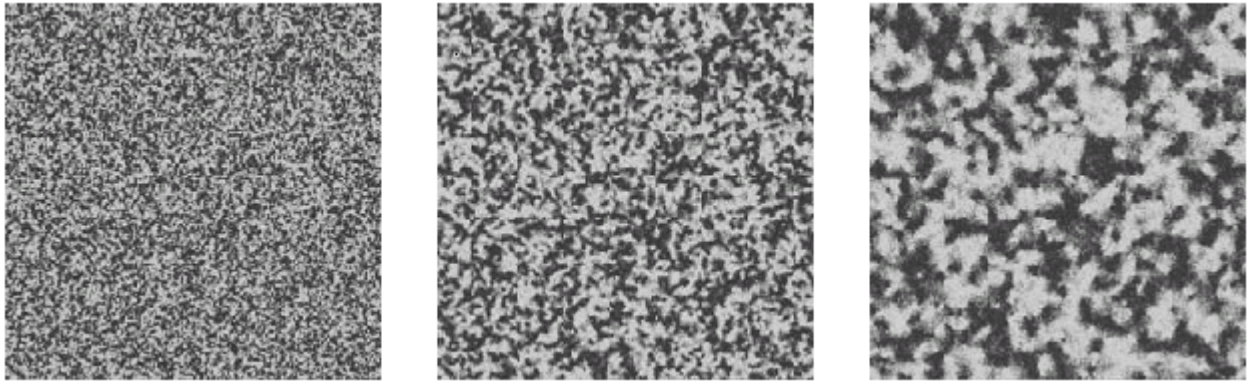
Ageing in the 2D-XY models

L. Berthier, P. Holdsworth, M. Sellitto J. Phys. A, 34, 1805, 2001

From T=0 to T=0.3J increasing tw =>



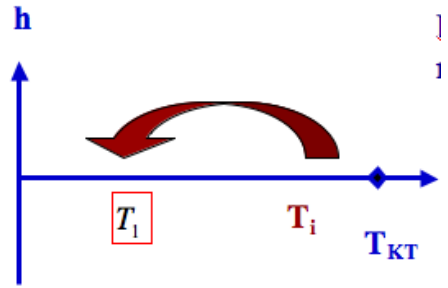
From T= infinity to T=0.3J

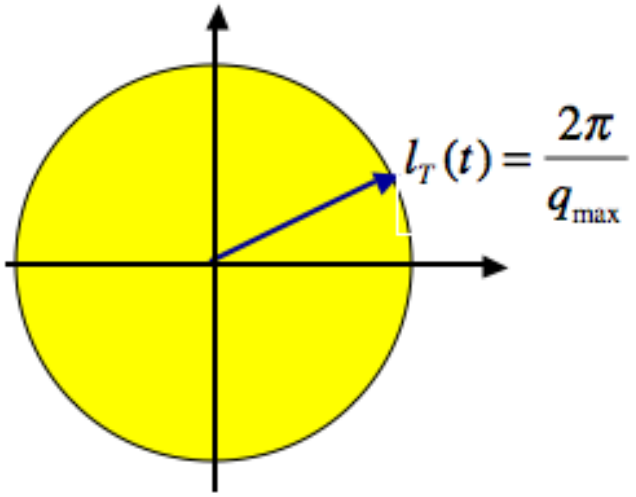


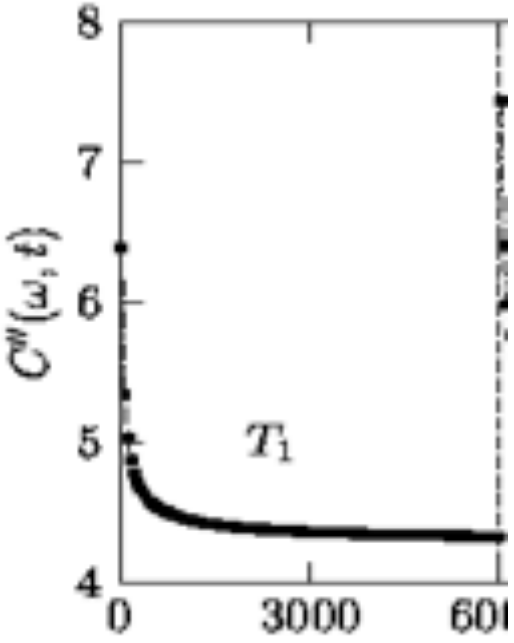
Aging. At t=0 the system in equilibrium at $T_i < T_{KT}$.

Is quenched to T_1

From Langevin dynamics

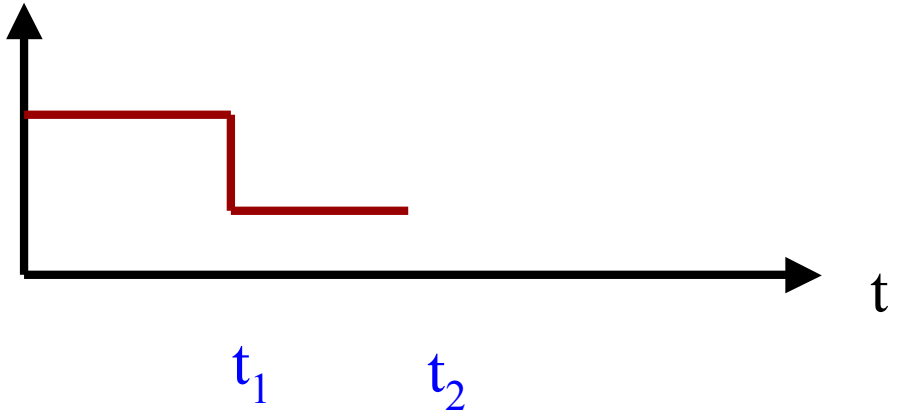


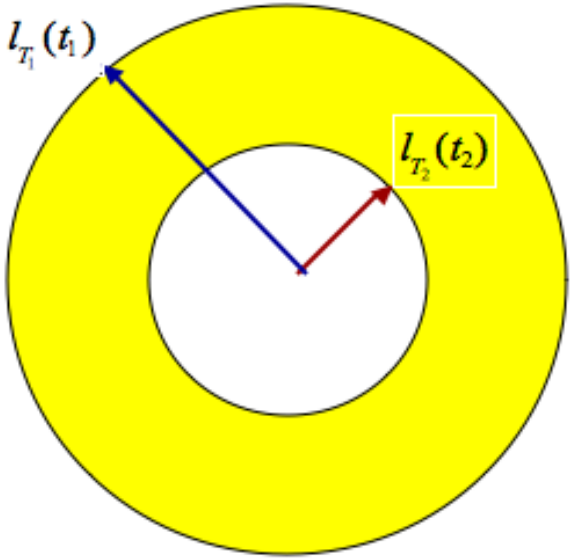


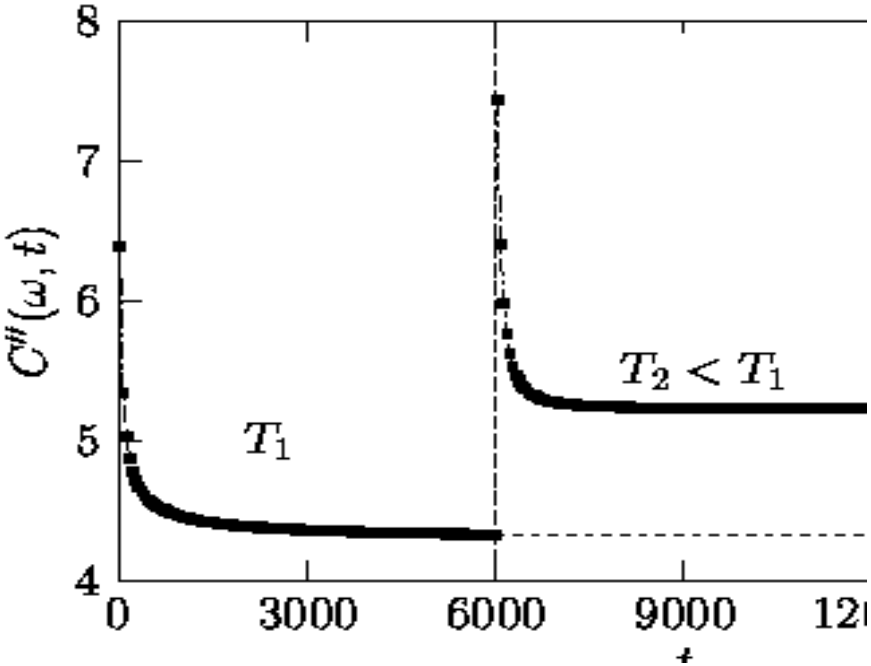


Each mode has characteristic time scale **$\tau \sim 1/(Tq^2a^2)$**. For fixed time All modes on length scales **$1/q \sim l(t) < a\, t^{1/2}$** are equilibriated, scales > **$l(t)$** are our of equilibrium. Equilibrium amplitude~T

Rejuvenation At time t_1 make a further quench to $T_2 < T_1$.



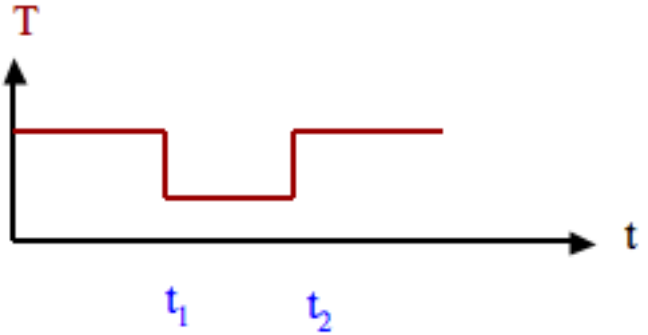


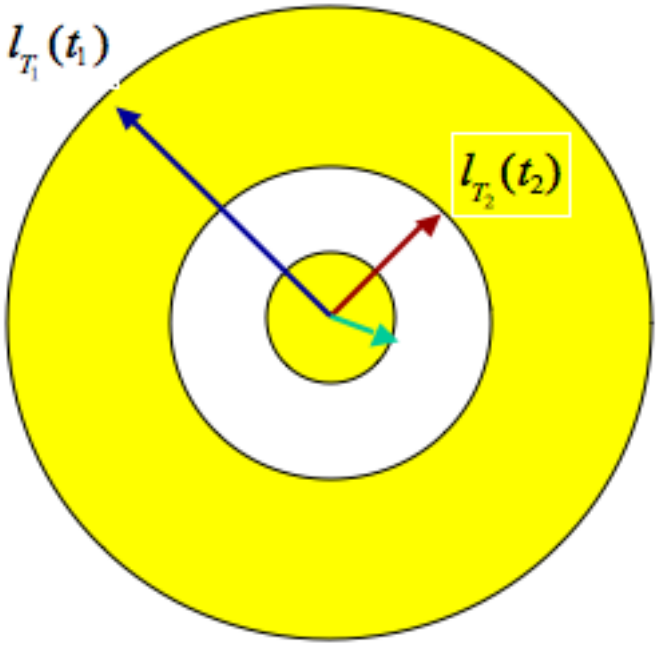


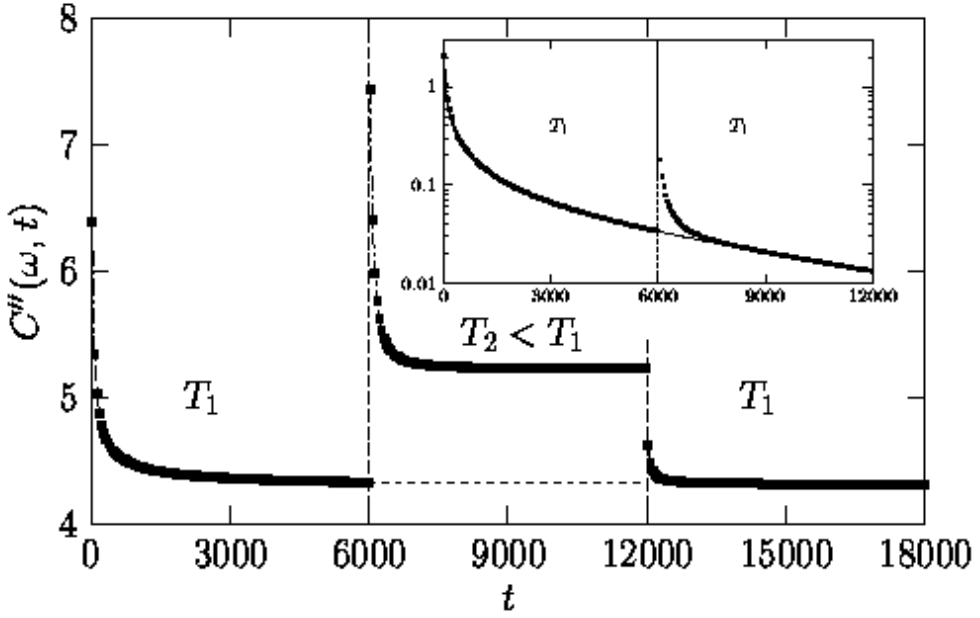
ALL length scales
are put out of equilibrium.
The clock is set to
zero and ageing
restarts-rejuvenation.

Memory

At t_2 quench back up to T_1

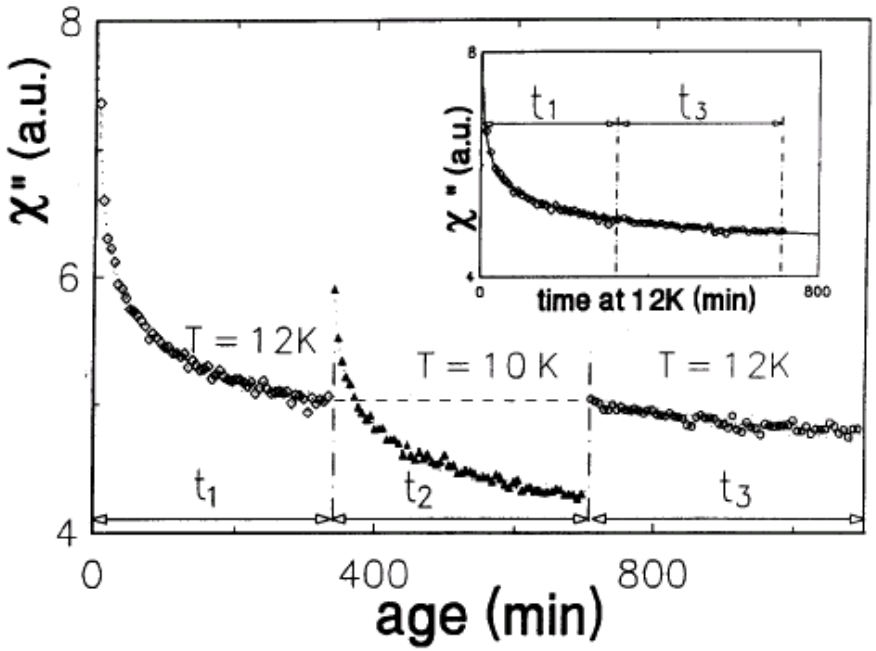




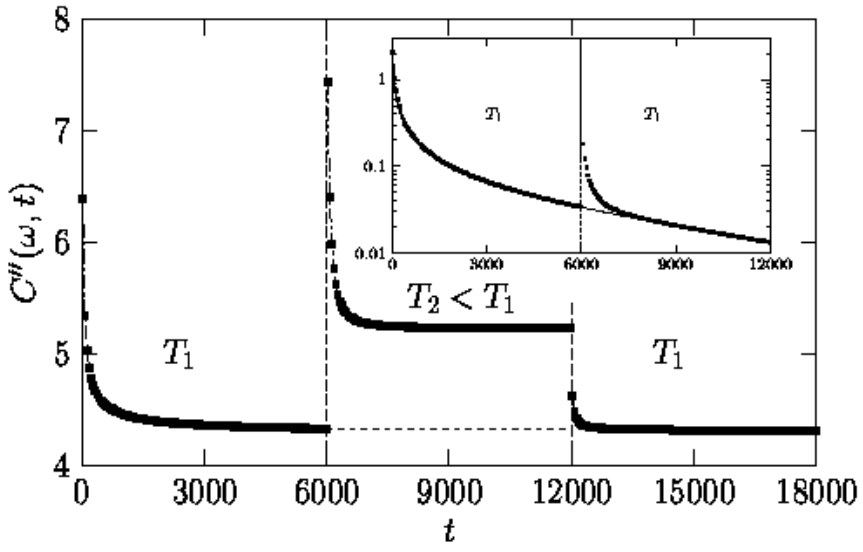


At t_2 ageing restarts from q=0-once equilibrium length reaches $\ell(t_2)$ active length scale jumps to $\ell(t_1)$-**memory**

Compare with
Experimental data
with $CdCr_{1.7}In_{0.3}S_4$



Activity on many scales
other than domain
length is the key
to rejuvenation and
memory effects!



CONCLUSION

Aging phenomena could come from
Growth of domains with internal structure.

This is the case for 2D-XY model and it provides
the correct combination of time and length scales to
describe the experiments-**Fact!**

What ever the true scenario, disorder is clearly
needed to put the non-equilibrium phenomena
within the experimental time window.

ESM Targoviste, August 2011