Large Angle Precessional Magnetization Dynamics

under Field and Current Excitations

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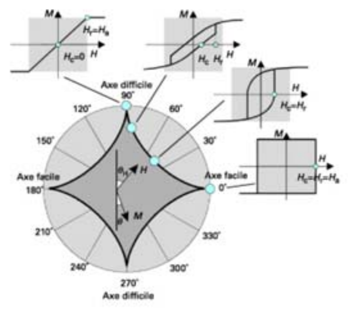
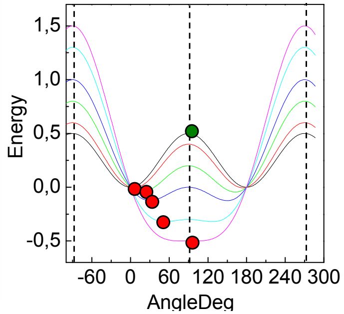
Outline

Reversal by Field

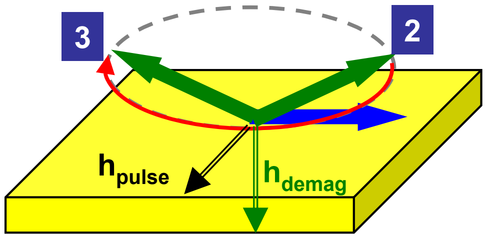
Stoner-Wohlfahrt Reversal under slow field





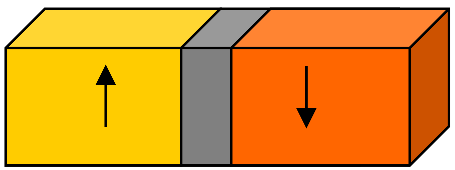
Few ms

Precessional Reversal under Field pulses



Few 100 ps

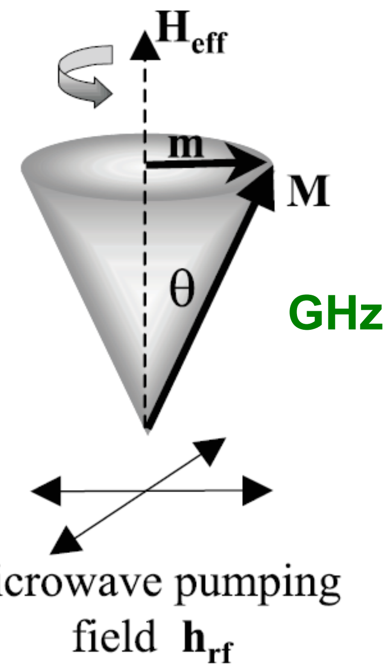
Field induced wall motion



Few ns
for 100 nm

Field Induced Precession

Ferromagnetic resonance



GHz

- Impose a frequency
- Limited to small angles

Field provides a torque that acts on M

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}$$

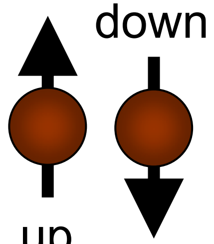
Bias field changes energy surface,
thus acts on conservative part of the dynamics

Other means to change the magnetization state??

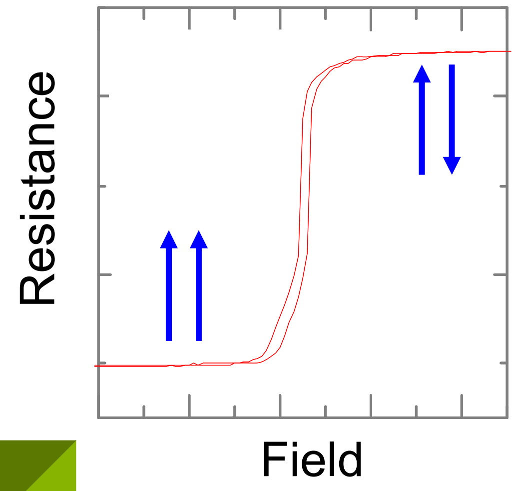
Spin Momentum transfer
Additional dissipative term in LLG

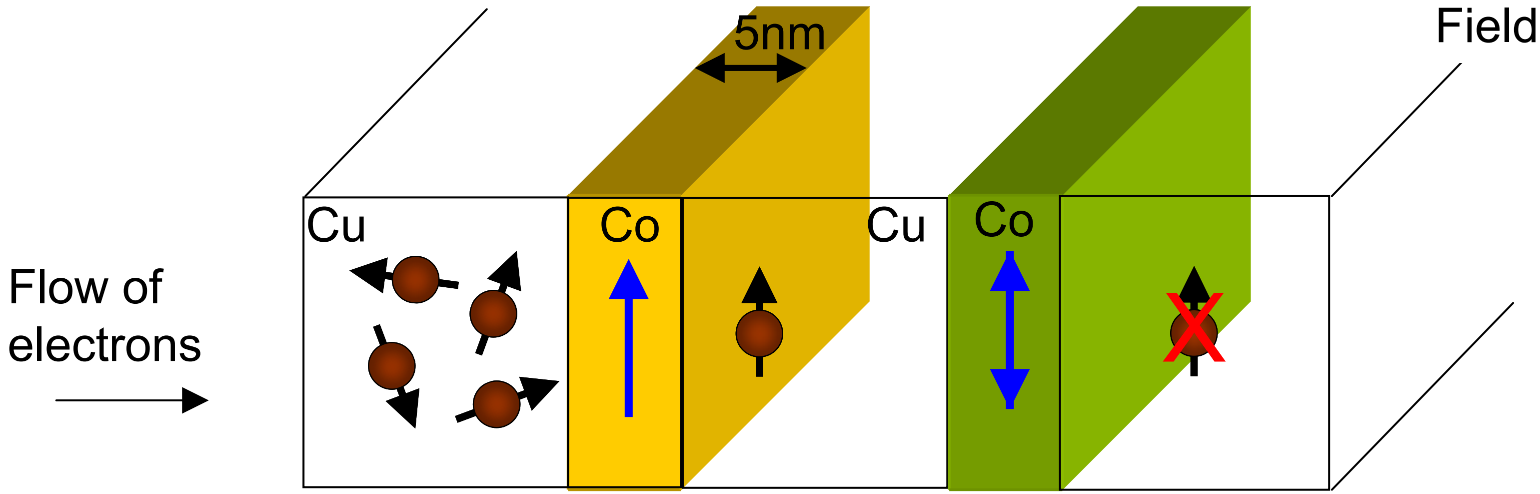
Spinelectronics

Charge & **Spin (±1/2)**



Magneto-Resistance



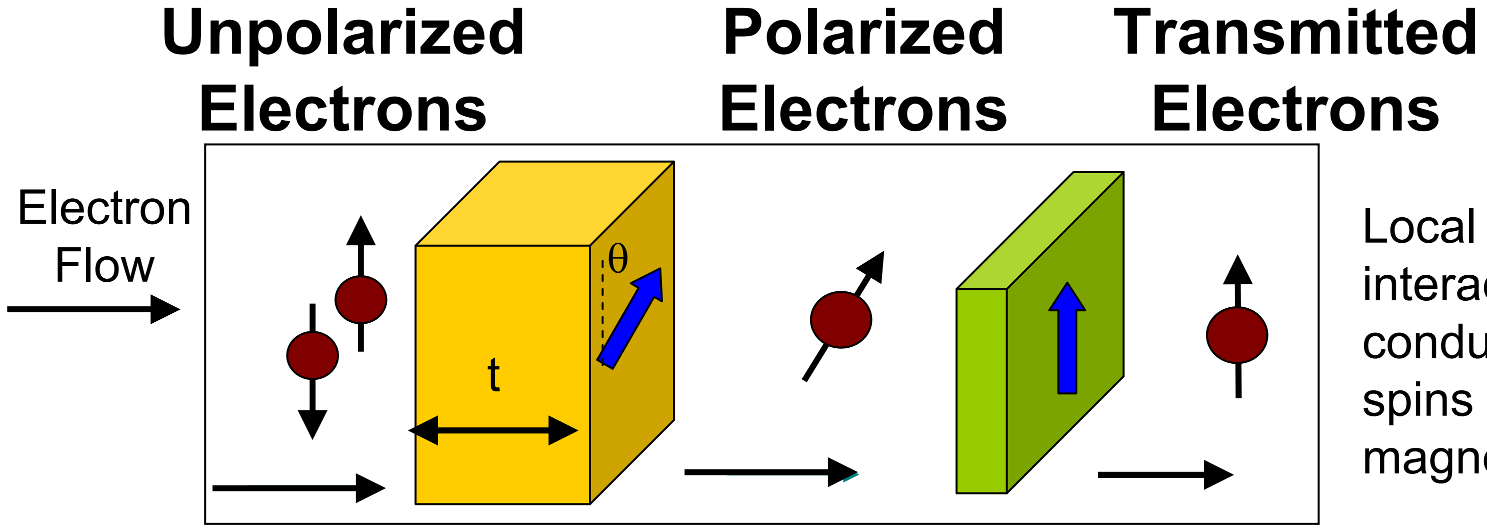


Magnetization **M** → **acts as Spin Filter**

→ Spin polarization

V Spin Momentum Transfer - Concept

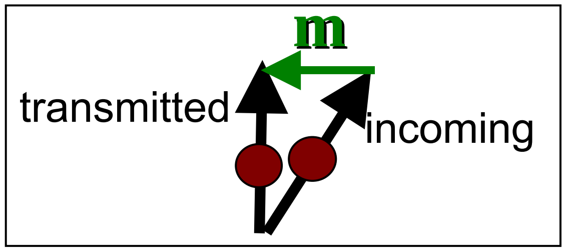
spintec SPIN IN ELECTRONICS



Local exchange interaction between conduction electron spins and local magnetization **M**

Polarizer P **Free Layer M**

Conduction Electrons

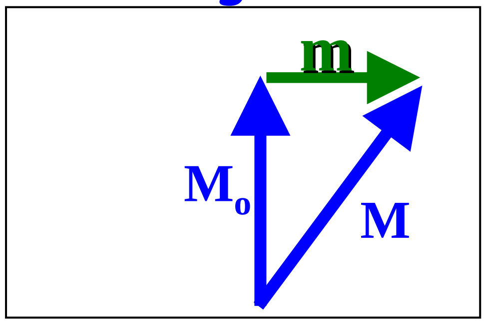


Transfer of transverse moment m

=

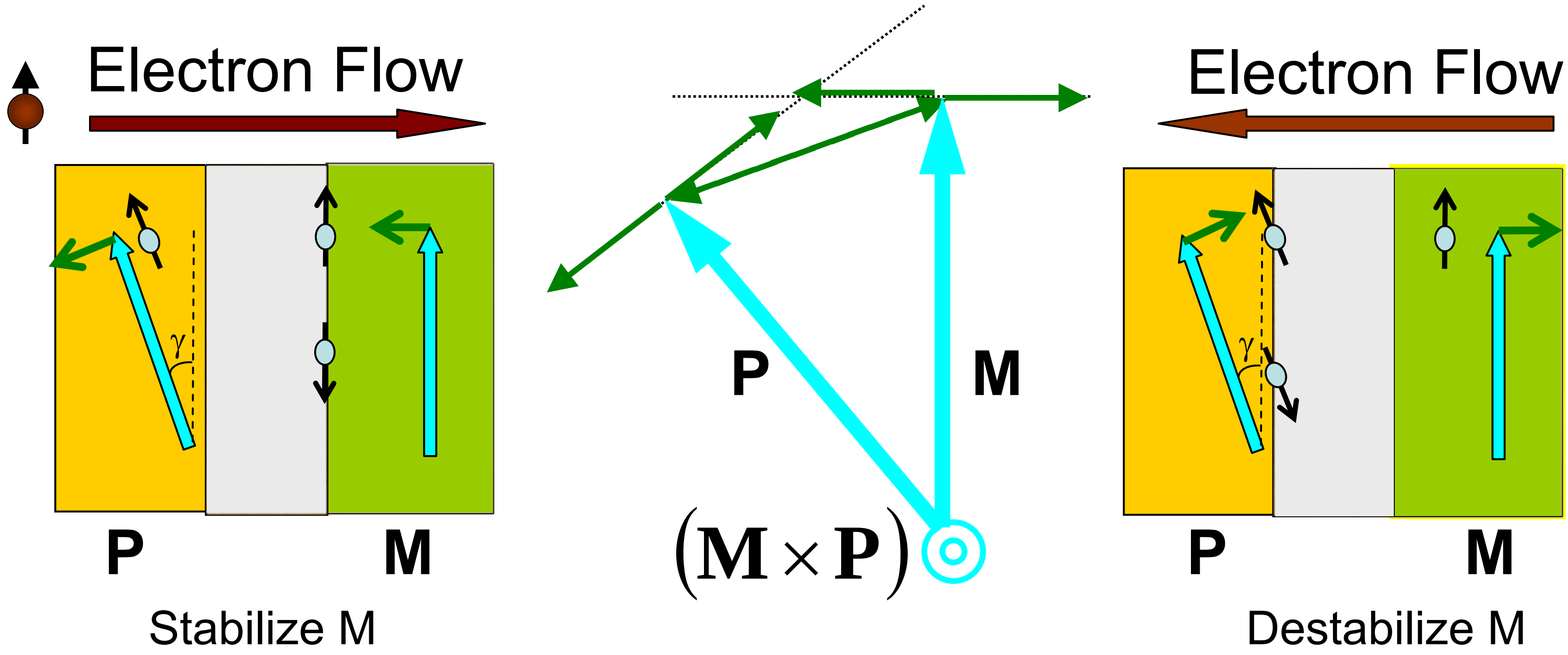
Spin Torque

Local Magnetization



J. Slonczewski, *JMMM* 159, L1 (1996)
L. Berger, *PRB* 54, 9353 (1996)

V Spin Momentum Transfer - Concept



Torque on M

$$\mathbf{T}_{//} = \gamma_0 \frac{a_j}{M_s} \mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

$\sim \sin(\mathbf{MP})$

a_J~current J

P = spin polarization vector

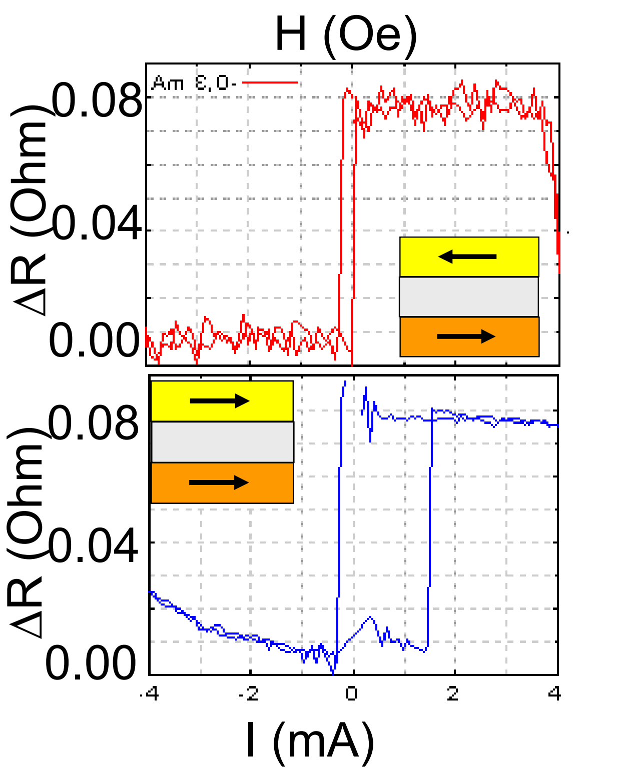
STT tries to align M collinear to P

V Spin Momentum Transfer

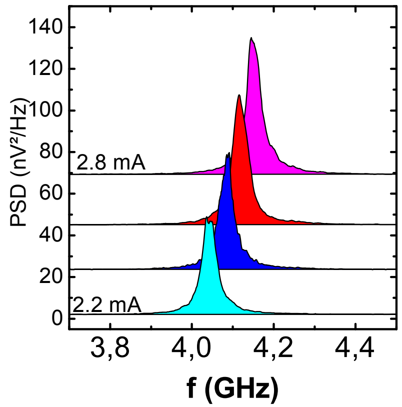
$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma\, a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

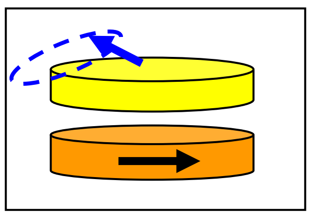
Precession **Damping** **Spin torque (ST)**

Reversal

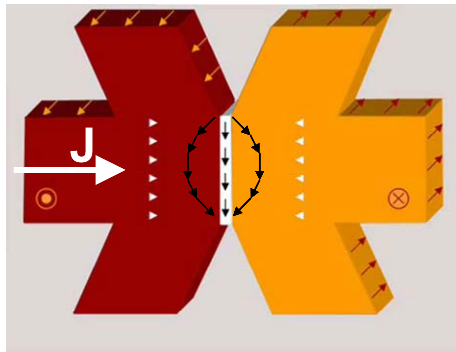


Oscillations





Wall motion



Outline

Need to add an additional term to the Landau Lifshitz Gilbert equation

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

Precession **Damping** **Spin torque (ST)**

$$a_J = \frac{\hbar}{2e}\frac{J}{\mu_0 M_S t}g(\eta,\vec{m},\vec{p})$$

$$g(\eta,\vec{m},\vec{p}) = \frac{1}{-4+\frac{1}{4}\frac{(1+\eta)^3}{\eta^{3/2}}(3+\vec{p}\cdot\vec{m})}$$

Note neglect field like term

II Non Conservative Dynamics - Summary

spintec SPIN IN ELECTRONICS

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

Precession **Damping** **Spin torque (ST)**

	|m|	dE/dt	Static	Dynamic
Conservative Precession Dynamics	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies

Need to re-examine solutions….
Equilibria, stability, trajectories
as a function of control parameters *J*, **P**, **H**

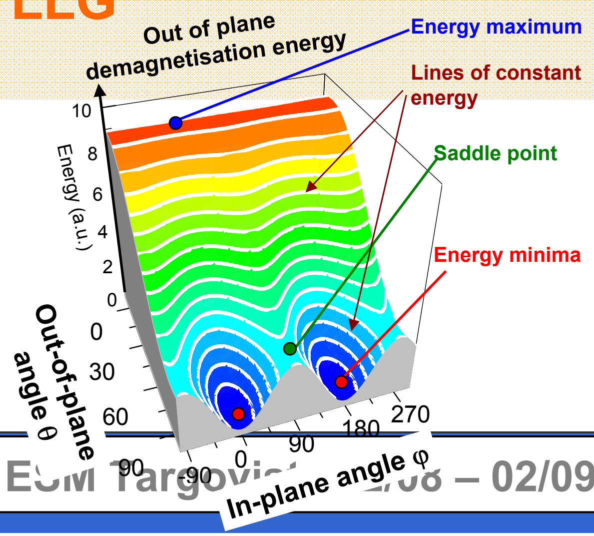
II Non Conservative Dynamics - Summary

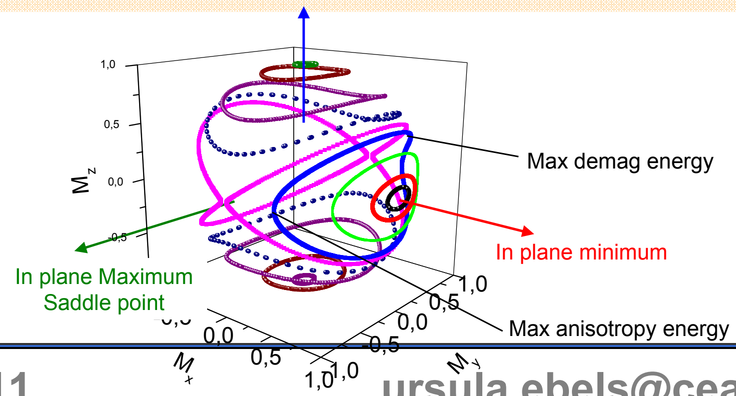
$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}}$$

Precession **Damping**

$\boldsymbol{H}_{\text{eff}}(\boldsymbol{M})$

	|m|	dE/dt	Static	Dynamic
Conservative Precession Dynamics	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies





VI ST Precession - LLG + STT

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}} + \boxed{\frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})}$$

Precession **Damping** **Spin torque (ST)**

Norm of **M**
Conserved since

$$\mathbf{M}\frac{d\mathbf{M}}{dt} = 0$$

VI ST Precession - Energy Loss/Gain

$$\frac{dE}{dt} = -\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt} = \underbrace{-\frac{\alpha}{M_s}\left(\frac{d\mathbf{M}}{dt}\right)^2}_{\text{Damping}} \underbrace{-\boxed{\frac{a_j}{M_s}\frac{d\mathbf{M}}{dt}(\mathbf{M}\times\mathbf{P})}}_{\text{STT}} \quad \left.\begin{matrix} >0 \\ <0 \end{matrix}\right.$$

Depending on current sign and amplitude

→ STT has « dissipative » action on trajectory

Spin torque cannot be derived from a generalized energy

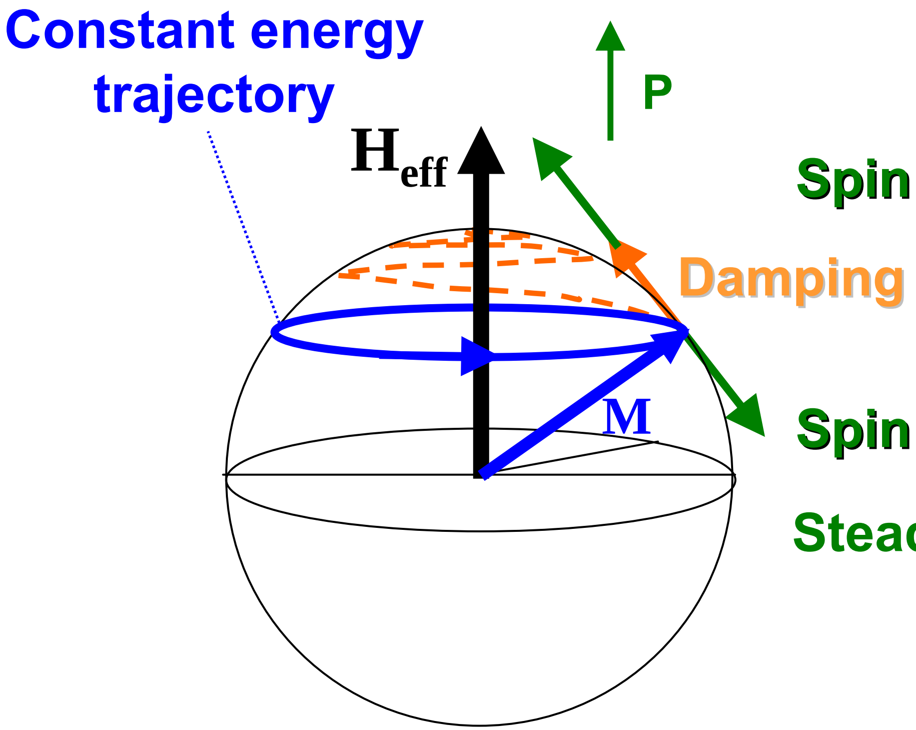
→ ST does not change the energy surface
→ ST does not change conservative part of LLG

VI ST Precession

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

Precession — Damping — Spin torque (ST)

a_J ~current J
P = spin polarization vector



Spin Torque J — Stabilizing Initial state

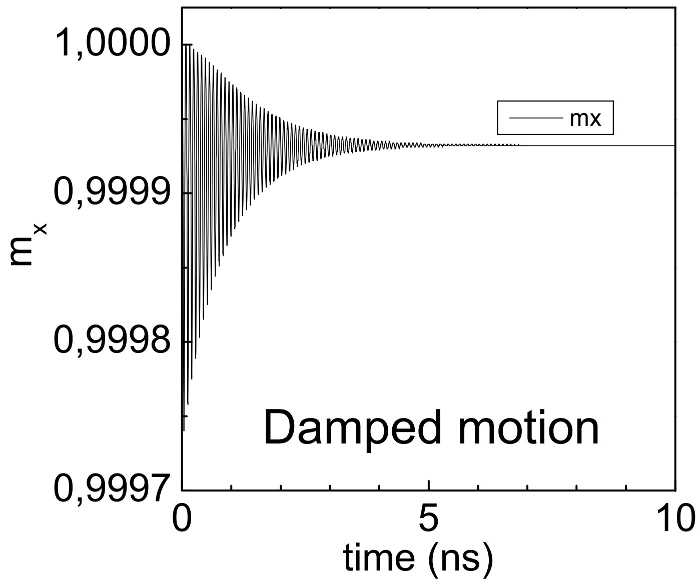
Spin Torque - J — Destabilizing Initial state

Steady state periodic oscillations (auto-oscillations)

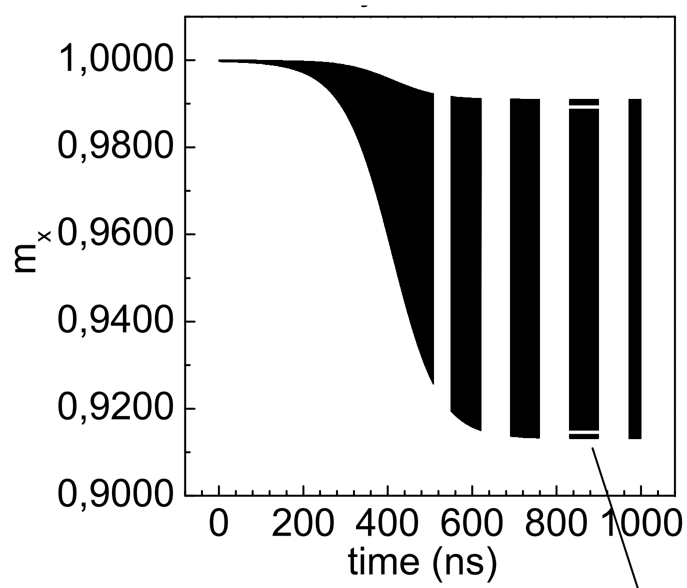
VI ST Precession - Limit Cycles

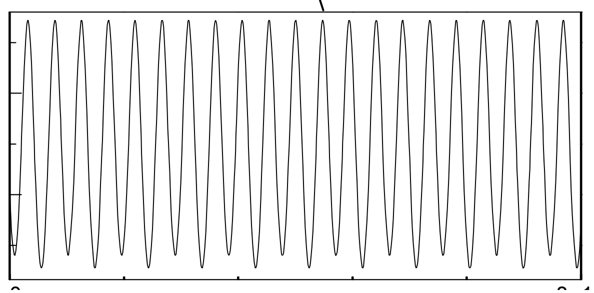
Apply current so that Destabilisation of initial stable state

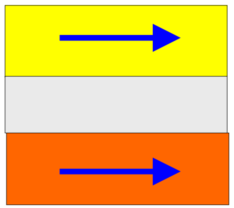
Low current amplitude



High current amplitude







M

P

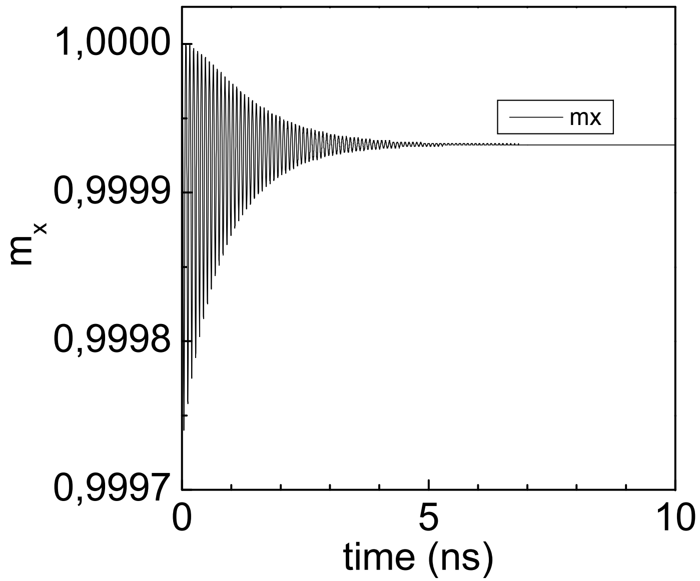
Periodic oscillations of M_x, M_y, M_z

Hu=500 Oe, Hb=1000 Oe

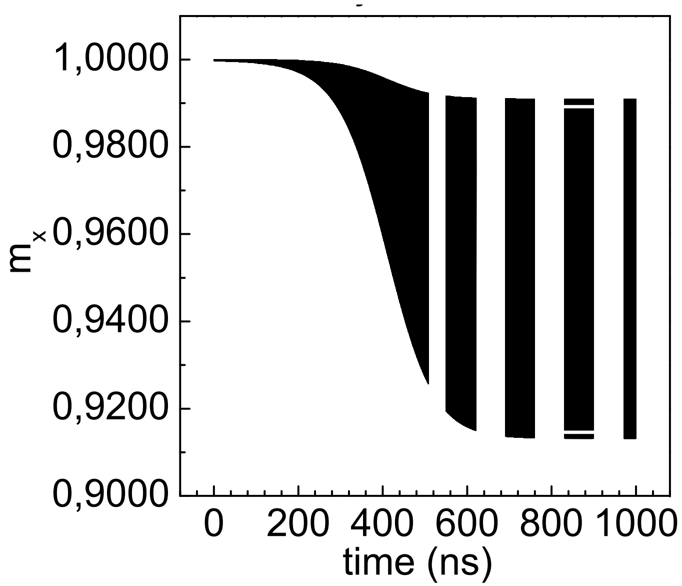
VI ST Precession - Limit Cycles

Apply current so that Destabilisation of initial stable state

Low current amplitude

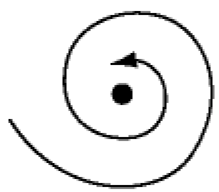


High current amplitude

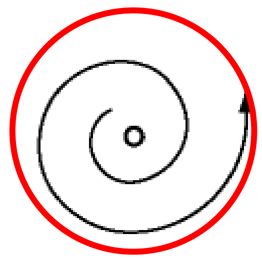


Depending on the amplitude of the current

Damped oscillation around stable focus

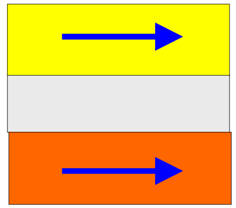


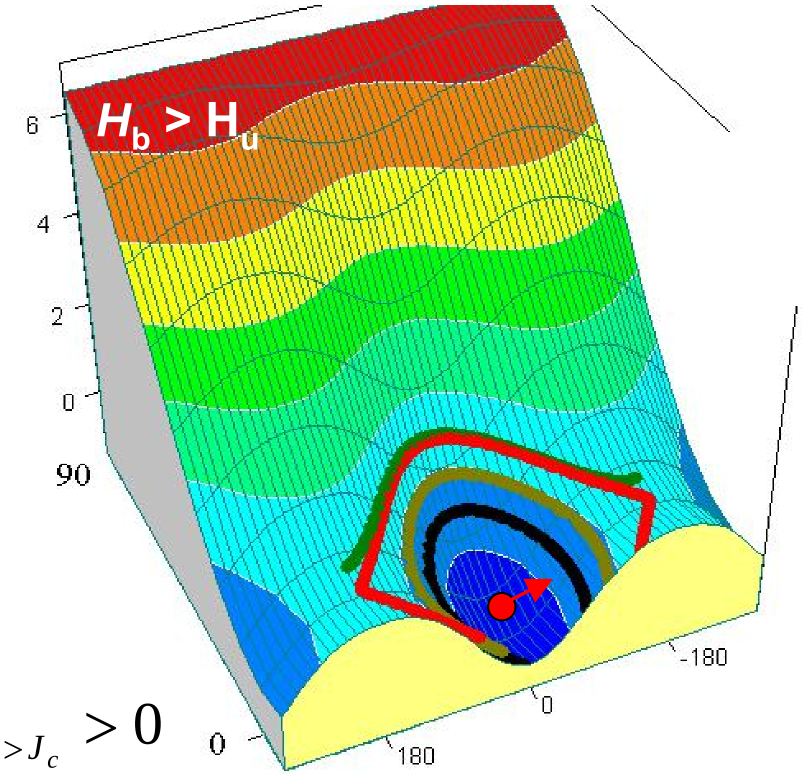
Oscillation away from unstable focus towards a Limit Cycle



VI ST Precession - Limit Cycles

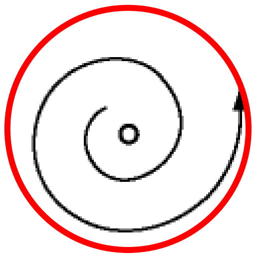
$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}_{\text{Damping}} + \underbrace{\gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)}_{\text{Spin torque}}$$





- Depending on the control parameter *J* (current amplitude) the energy minimum can change from a stable to an unstable focus

- When $J>J_c$ STT moves **M** along the energy surface until **M** stabilizes on a limit cycle



$$\delta\mathbf{M} \sim e^{\Gamma t} e^{i\omega_o t}$$

$$\Gamma_{J<J_c} < 0 \Rightarrow \Gamma_{J>J_c} > 0$$

Note: energy surface is not changed by STT

VI ST Precession - Limit Cycles

What are the limit cycles?
Are they the same as constant energy trajectories?

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}} + \boxed{\frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})}$$

Precession **Damping** **Spin torque (ST)**

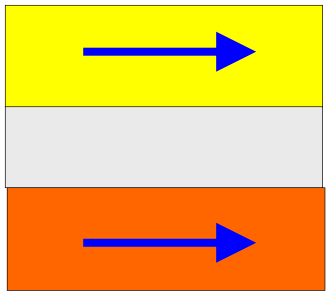
Precession Term dominating

→ **Depends on orientation of P and amplitude of current**

VI ST Precession - Polarizer Geometries

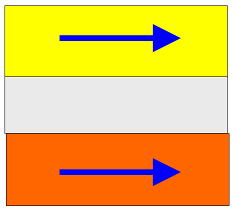
Consider two geometries

Planar polarizer

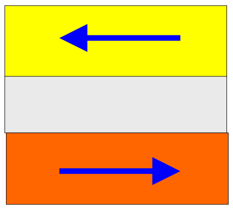


In-plane — In-plane M — Polarizer P

M and P collinear

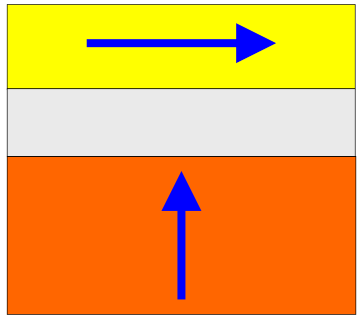


Parallel



Antiparallel

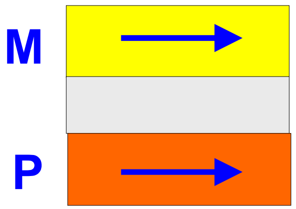
Perpendicular Polarizer



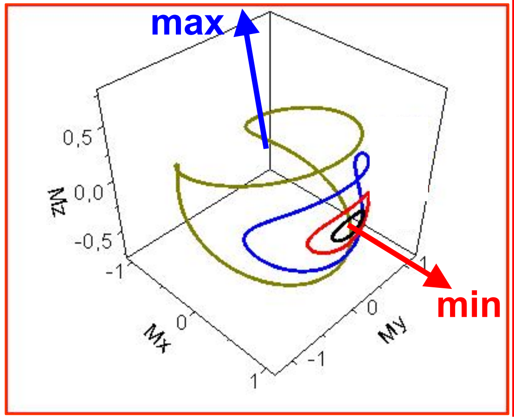
Out-of-plane

Control parameters:
H changes energy surface,
J, **P** change « dissipation »

Are limit cycles and constant energy trajectories the same ?

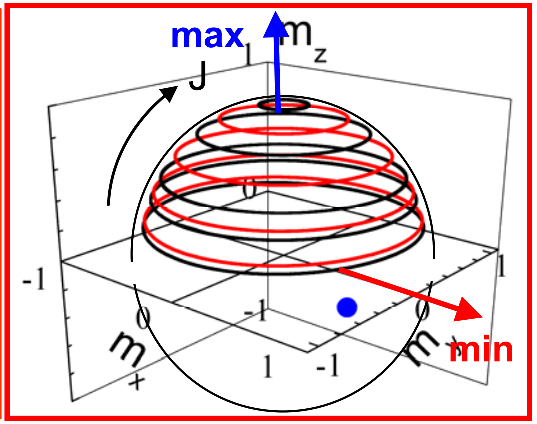


In-Plane Precession IPP

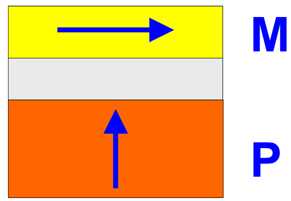


Oscillation around energy minimum

Out of Plane Precession OPP

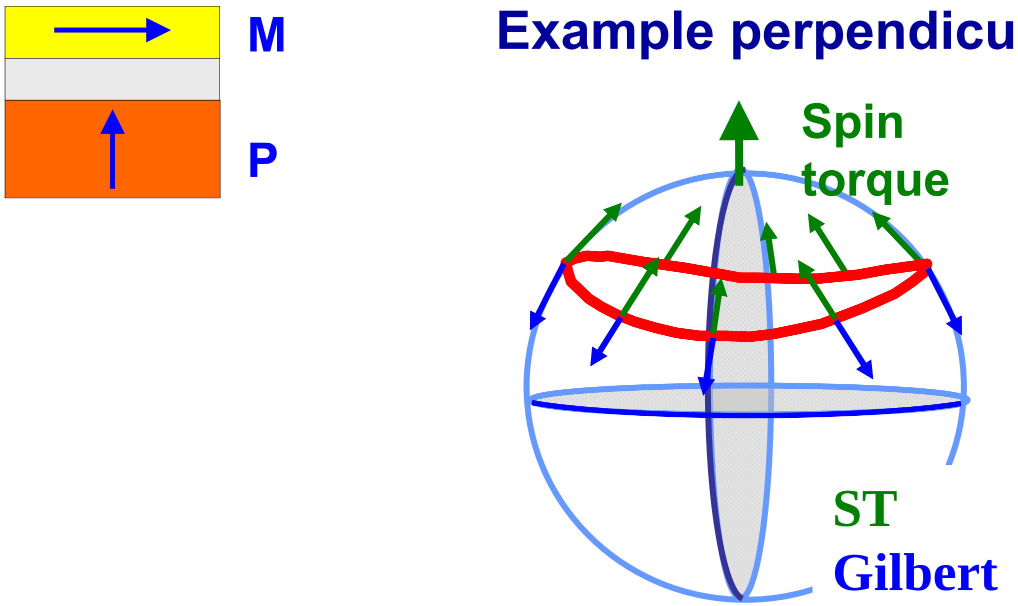


Oscillation around energy maximum



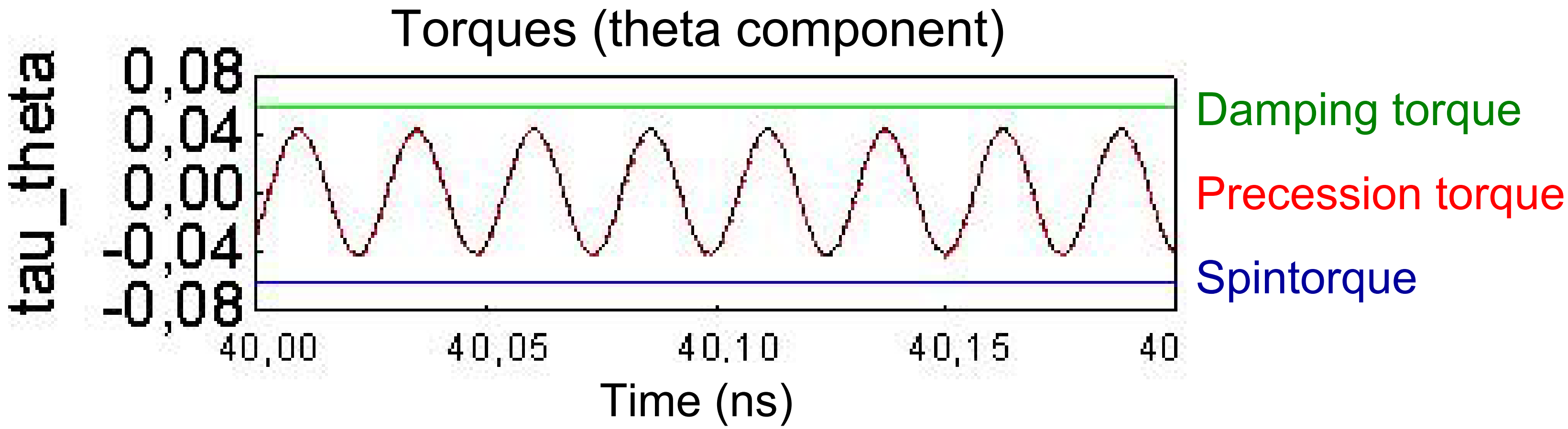
ST tends to align **M** parallel to **P**

If **P** // symmetry axis (equilibrium point) →
Limit cycles are close to the constant energy trajcectory around equilibrium point

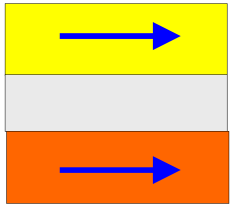


Example perpendicular polarizer

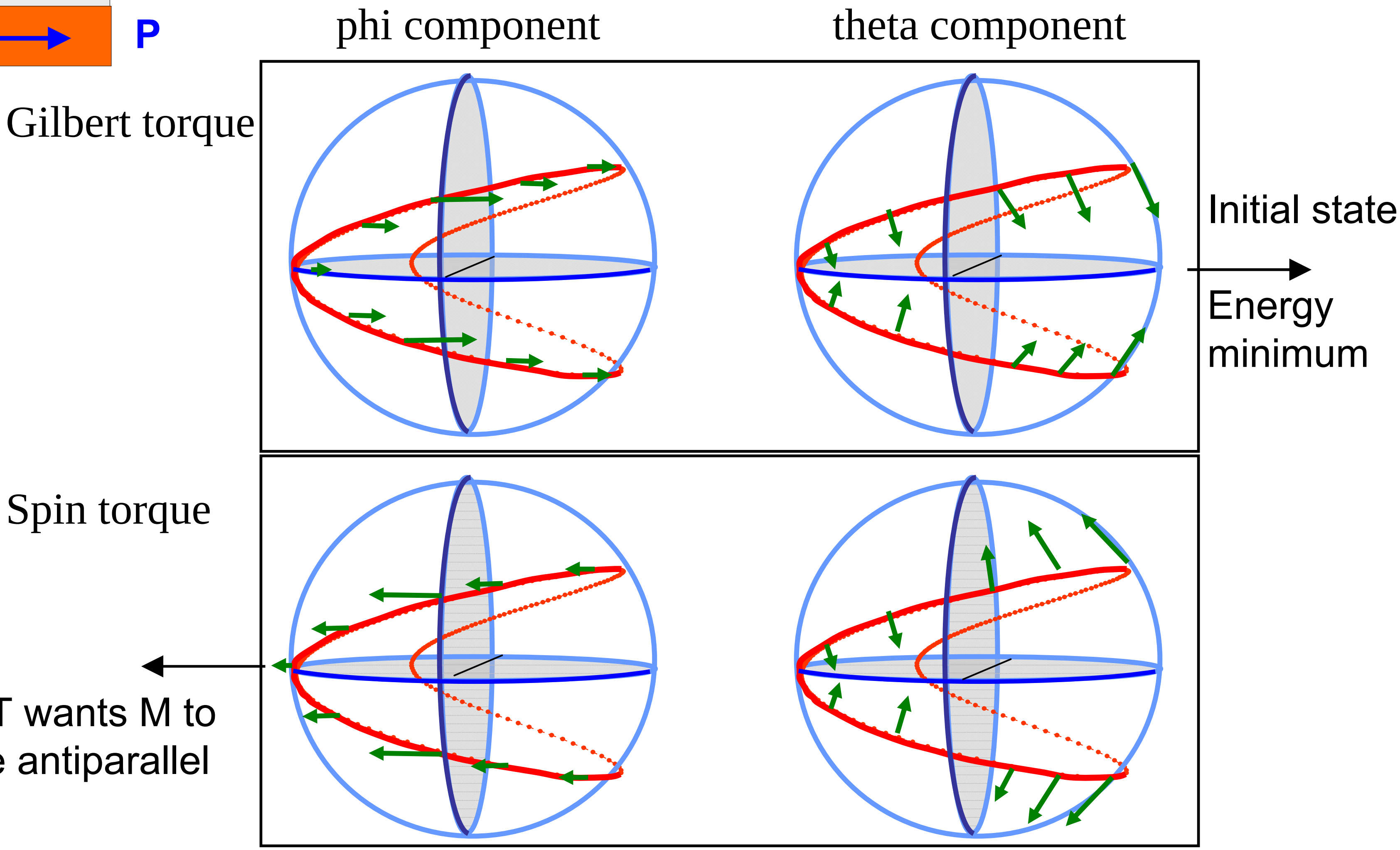
OPP stabilized by perpendicular polarizer

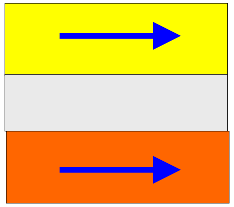


VI ST Precession - Limit Cycles



Example ST destabilizes parallel state (planar polarizer)

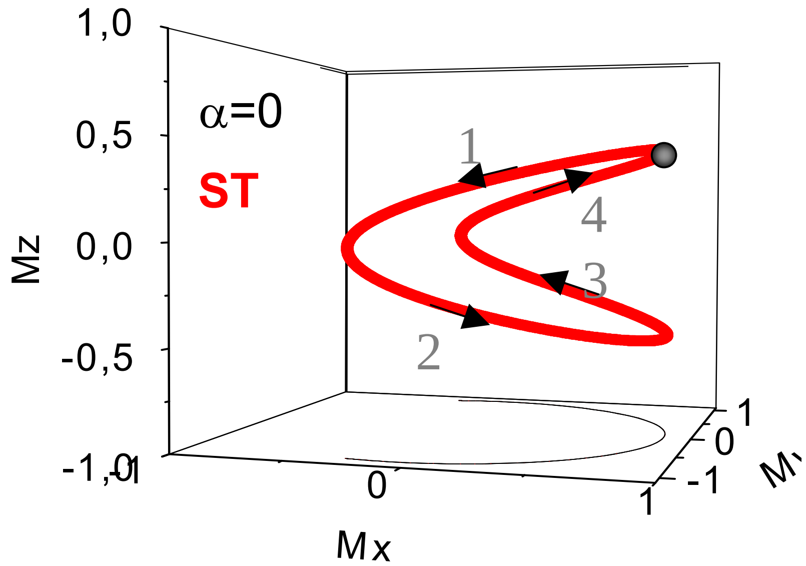


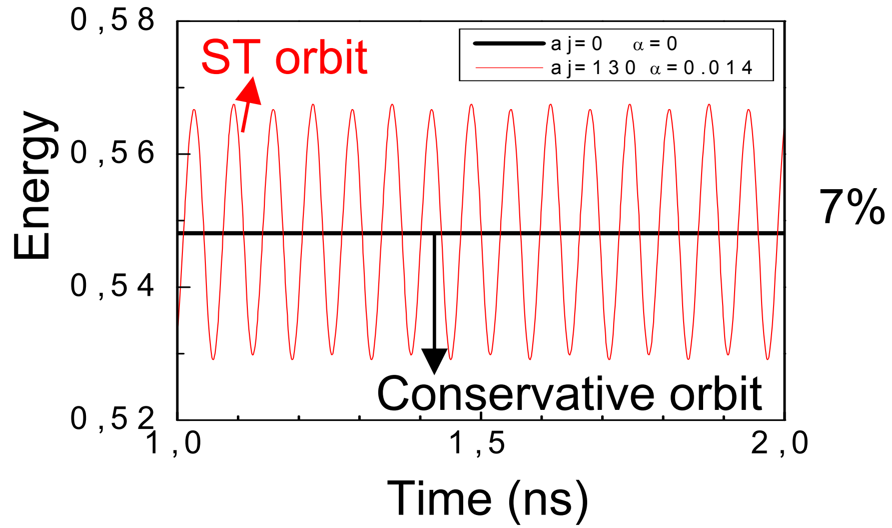


Constant energy trajectory vs limit cycle

Planar polarizer

IPP very close to constant energy trajectory but not identical

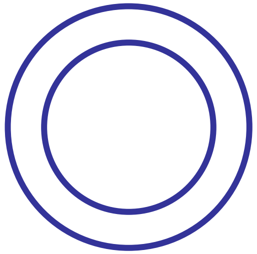




Spin transfer and damping torque cancel only on average but not at each point along the trajectory

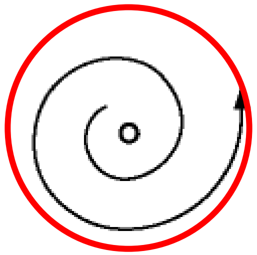
Limit cycles can be very close to constant energy trajectories

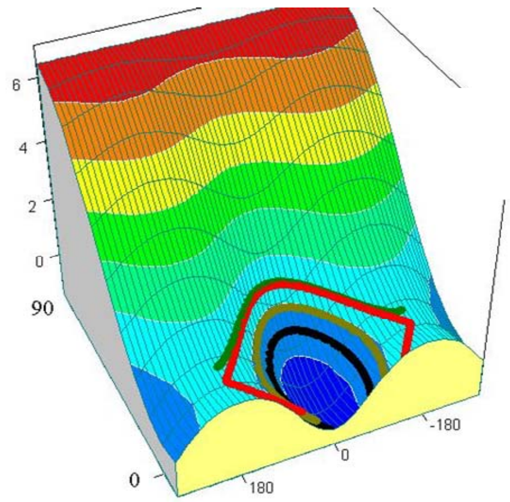
But



Constant energy trajectory:
Given by initial condition (E_o)

Limit Cycle (spin torque induced):
Independent of initial condition,
defined by J





spintec SPIN IN ELECTRONICS

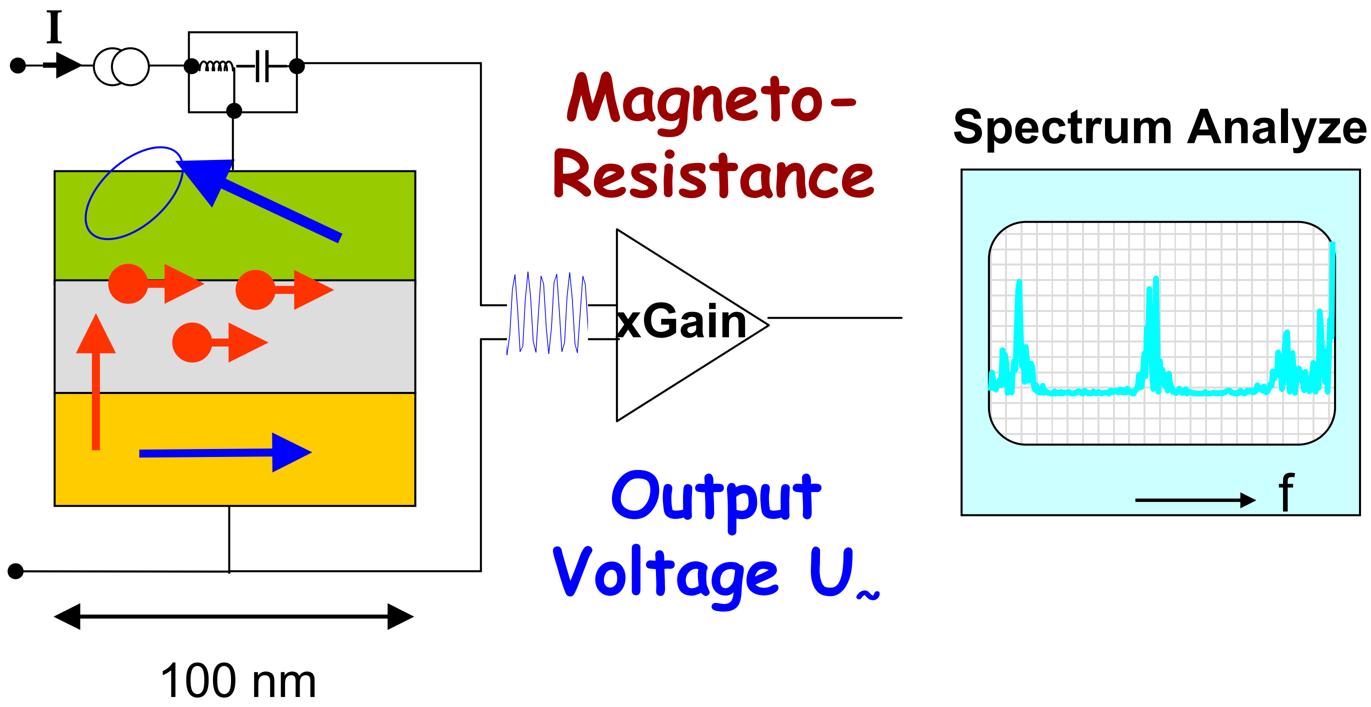
Excitation

Spin momentum transfer

↓

Auto-oscillations of M

Readout

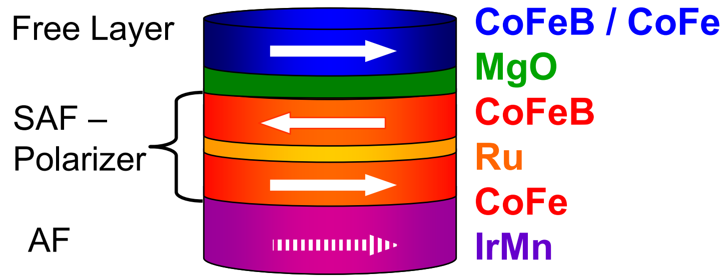


Nanoscale Tuneable Microwave Oscillator

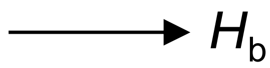
Spin Transfer Nano-Oscillator STNO

VI ST Precession - Experiment

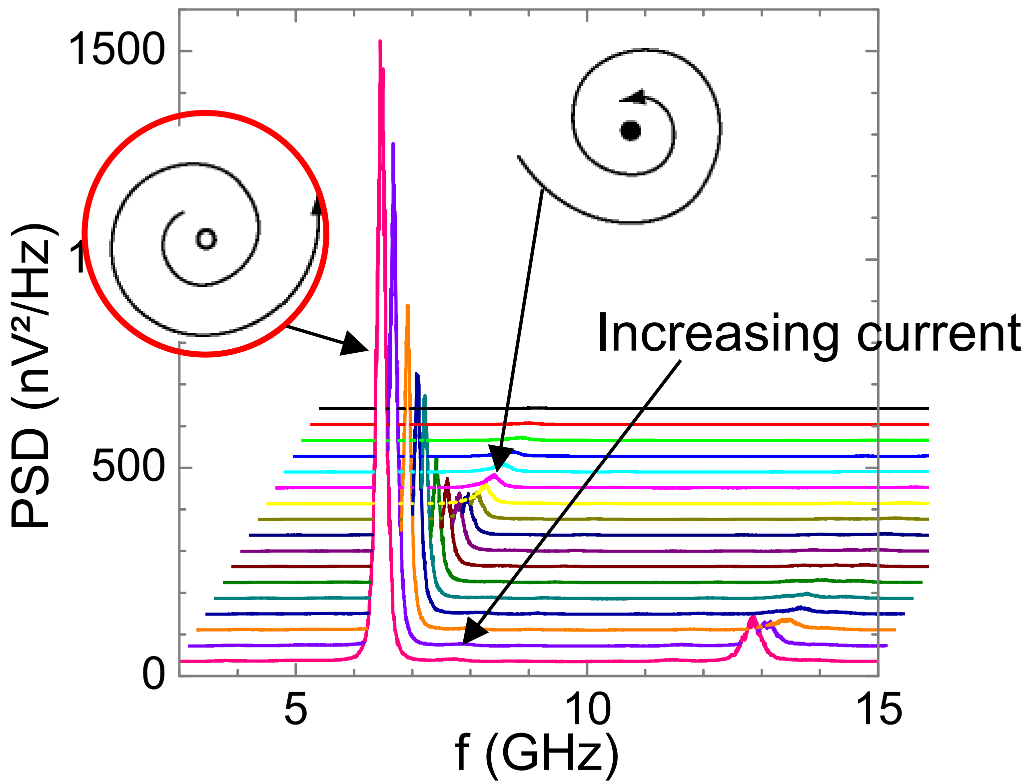
Example: Planar Tunnel Junction



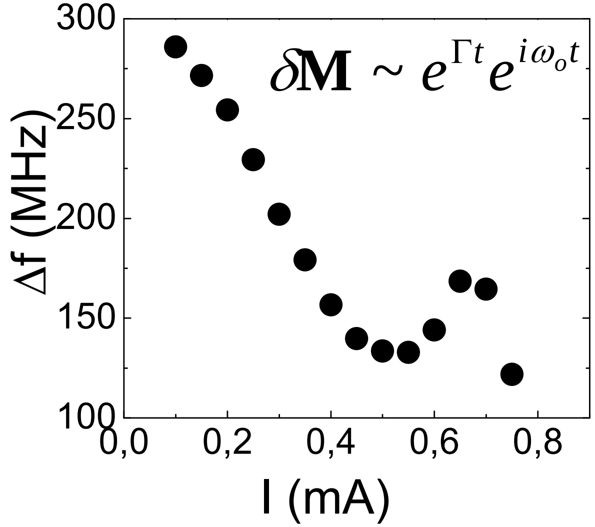
Devices Hitachi GST



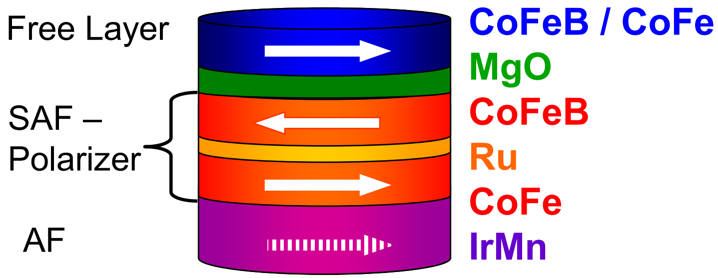
Typical Spectra



Linewidth ~ Γ

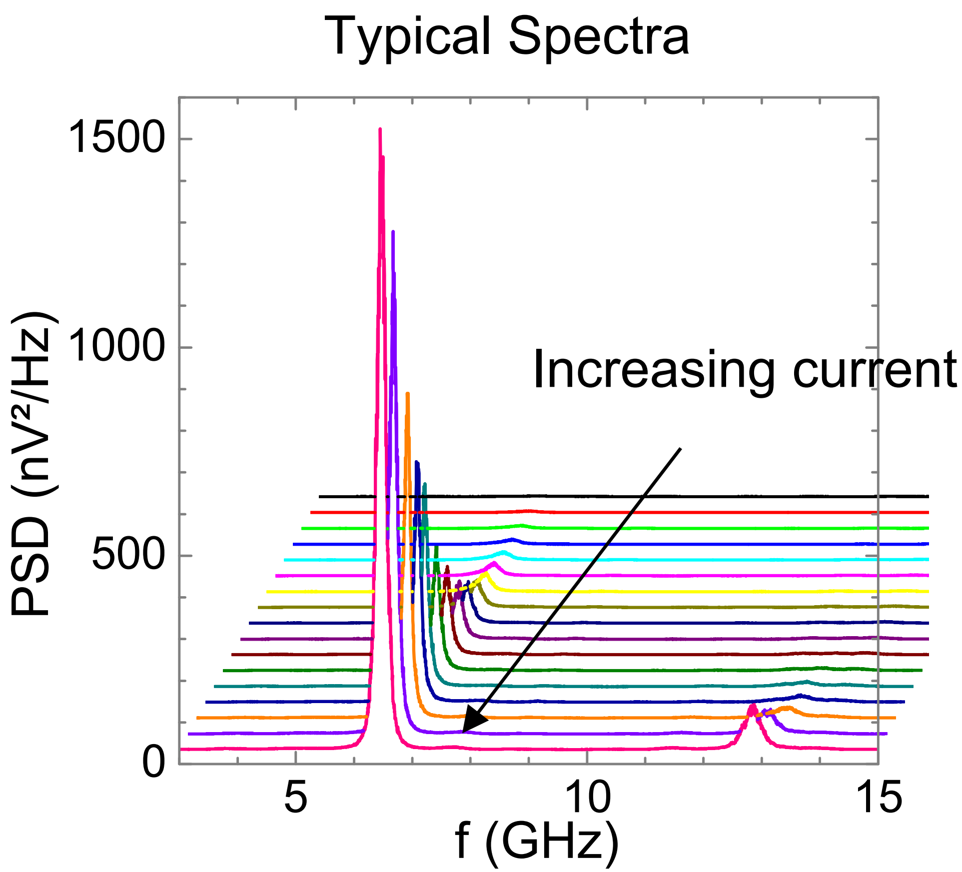


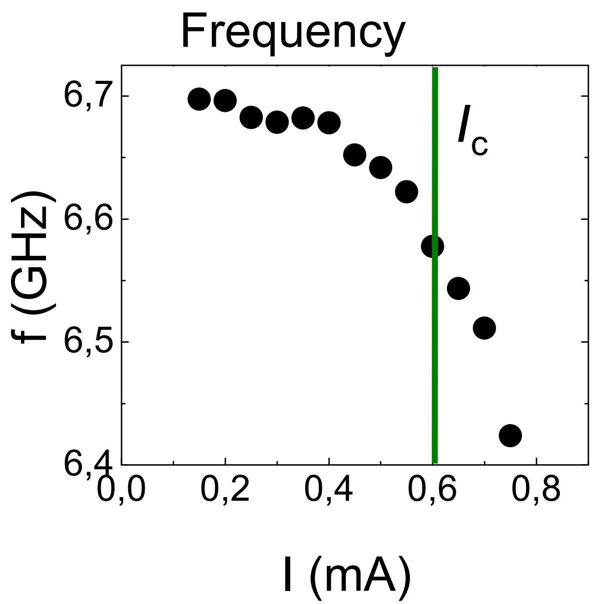
Example: Planar Tunnel Junction



Devices Hitachi GST

$\longrightarrow H_b$





What about frequencies of trajectories ?

Conservative trajectories

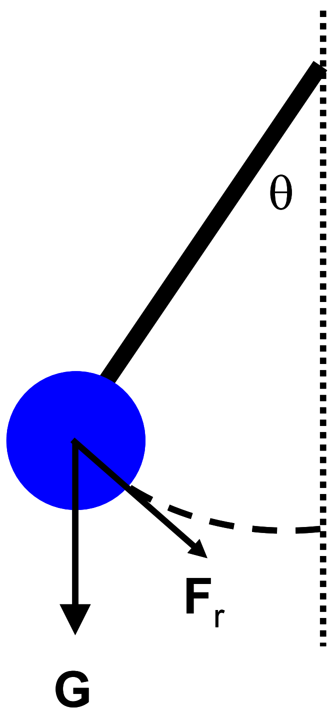
$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M} \times \mathbf{H}_{eff}) \qquad \omega = F\left(H_{eff}\right)$$

Precession

Non-linear dynamical system $\mathbf{H}_{eff}\left(\mathbf{M}\right)$

Frequencies depend on the precession amplitude of **M**

Pendulum



Linear – small angles

$$mL\ddot{\theta} = -mg\sin\theta \approx -mg\theta$$

$$\theta = \theta_o e^{i\omega_o t} \qquad \omega_o = \sqrt{\frac{g}{L}}$$

Non-linear – large angles

$$mL\ddot{\theta} = -mg\sin\theta \approx -mg\theta + \frac{1}{6}mg\theta^3$$

$$\theta = A\sin\omega t + B\sin 3\omega t$$

$$\omega = \omega(\theta_o) \approx \omega_o + k\theta_o^2$$

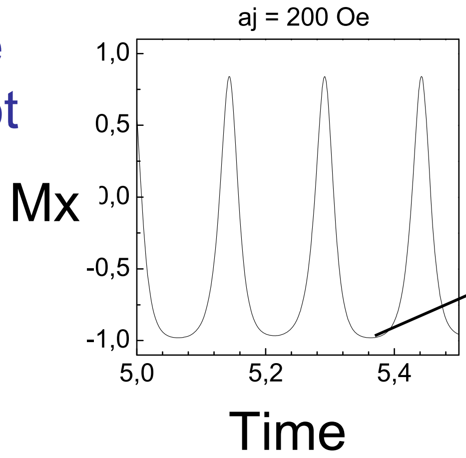
A. Blaquière "*Analysis of non-linear systems*" (Academic Press 1966)

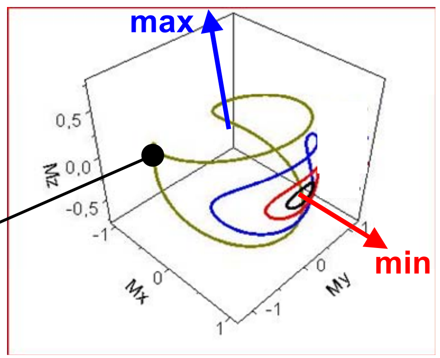
How to calculate frequencies ?

● Frequencies close to stable (or unstable) focus (i.e. energy minima and maxima) can be obtained by linearization of LLG (S) → Ferromagnetic Resonance section II (Imaginary part of eigenvalues λ)

$$\omega_o^{\,2} = \gamma\left(\frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2 \sin^2\theta}\right) \qquad \Gamma = \frac{\gamma\alpha}{2}\left(\frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2 \sin^2\theta}\right)$$

● Frequencies for large amplitude trajectories and limit cycles cannot be calculated analytically easily. The torque along the trajectory varies strongly





VI ST Precession - Frequencies

Two possible approaches to calculate frequencies

1) Semi – analytical-numerical approach

Define $$\frac{1}{f} = T = \int dt = \int \frac{dm_i}{\dot{m}_i}$$

Serpico et al., JAP 93, 6909 (2003)

and integrate the trajectory over one period for any of the three components m_i, $i=x,y,z$, by using the time derivatives from the equation of motion and the parametrization for constant energy trajectories. The integral can be evaluated numerically (or in some cases analytically see Serpico et al.)

2) **Numerical integration** of LLG(S) and
Fourier transform of the magnetization components $m_x(t)$, $m_y(t)$, $m_z(t)$.
Attention: for IPP trajectories (around X-axis as equilibrium), the frequency is double)

Here:
Numerical solution for constant energy and spin transfer induced trajectories

VI ST Precession - Frequencies

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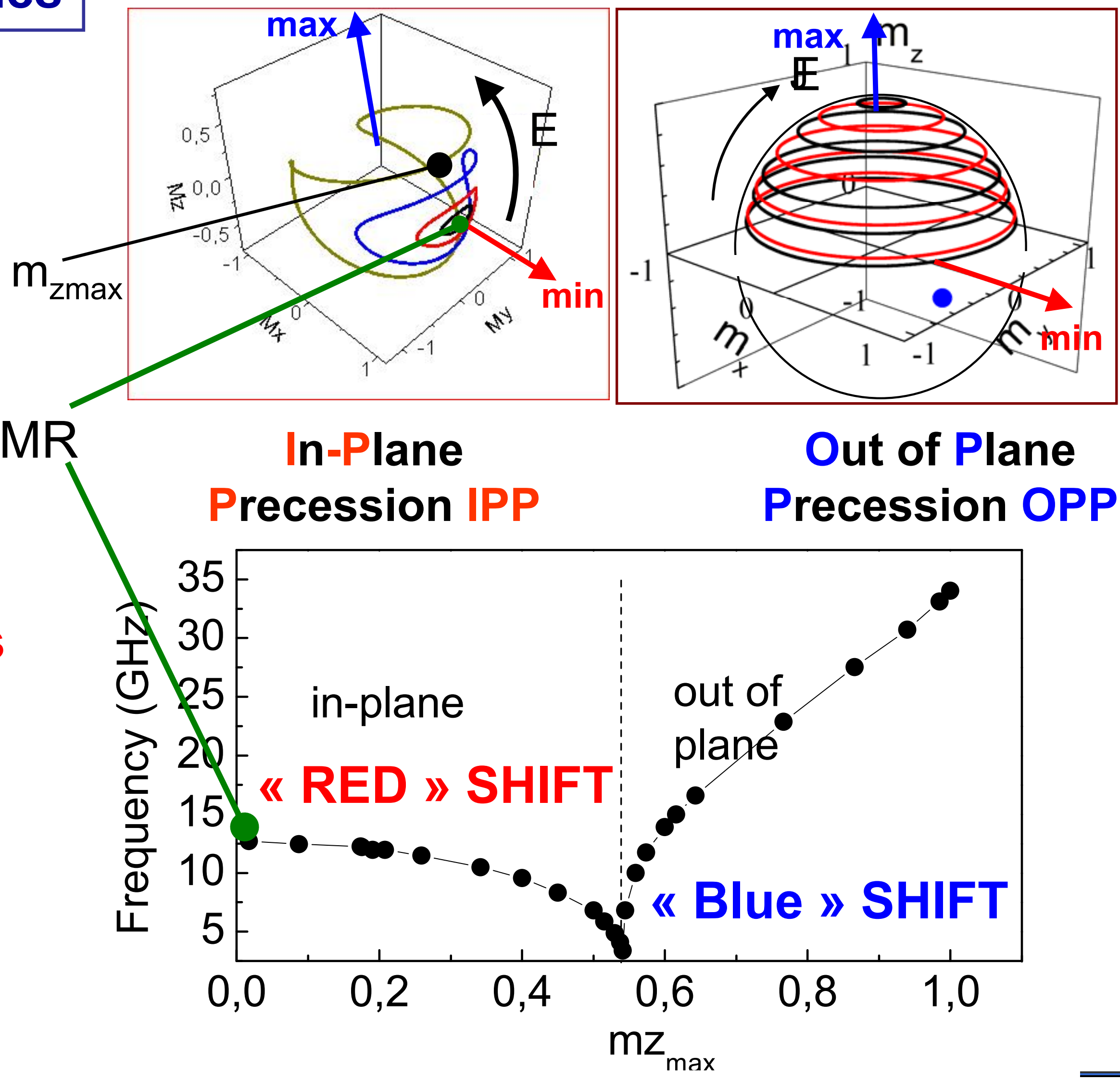
Conservative trajectories

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})}$$

Precession

Uniaxial in-plane film
Bias field $\mathbf{H}_b$ //X



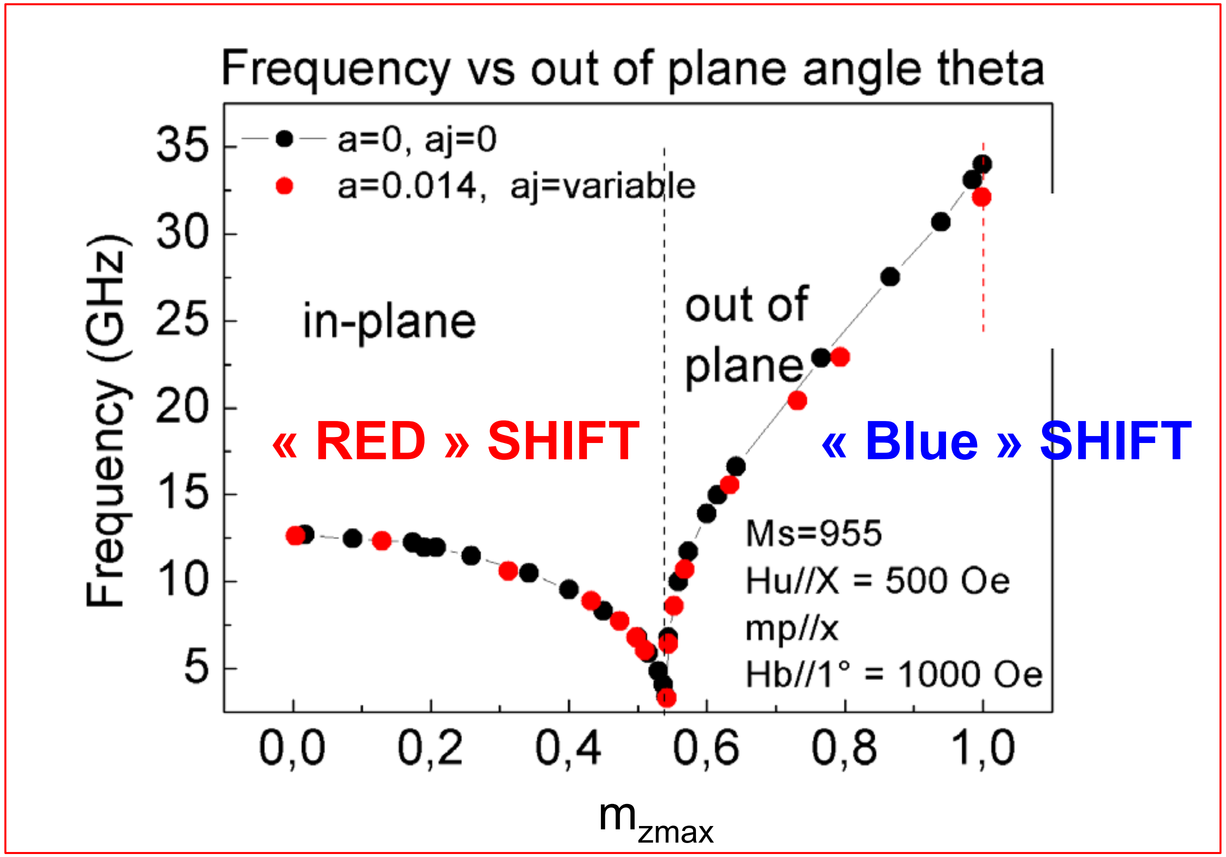
For IPP modes:
Frequency decreases with amplitudes

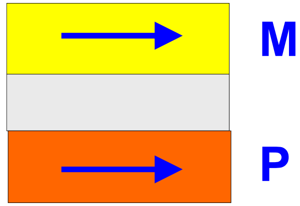
For OPP modes:
Frequency increases

The frequencies depend strongly on the precession amplitude

VI ST Precession - Frequencies

Example Frequencies of Planar Polarizer



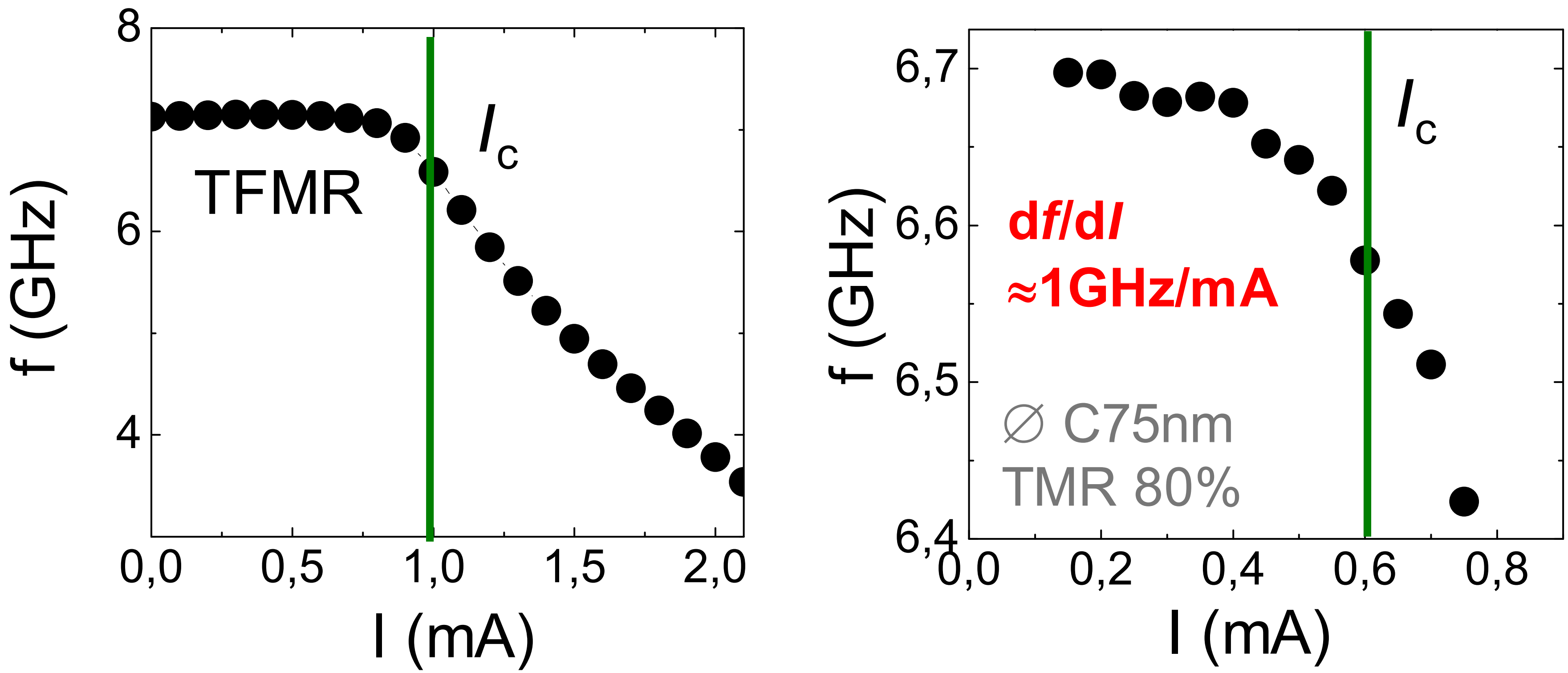


For STT orbits m_{zmax}~J

Frequencies of conservative and STT trajectories are the same

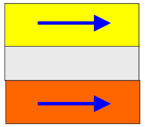
Macrospin simulations

Experiments MTJs



Tuning of frequency via current

VI ST Precession - Frequencies

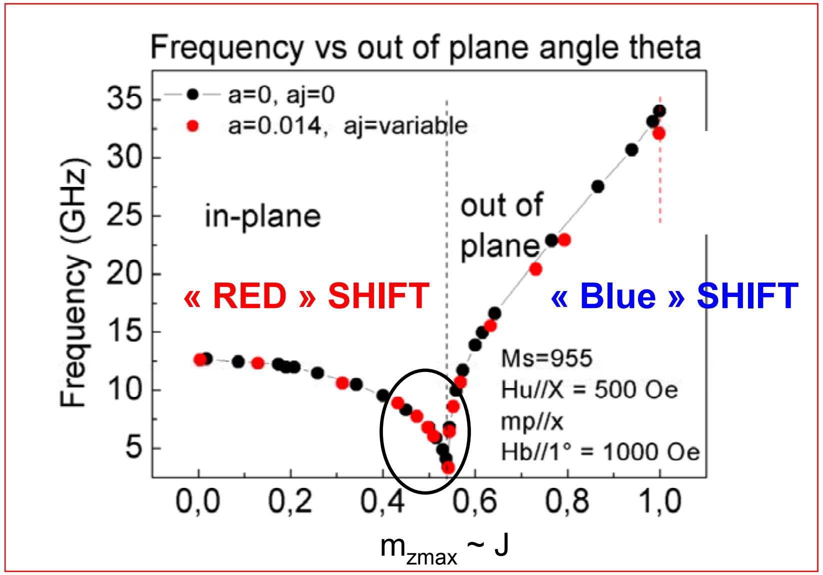


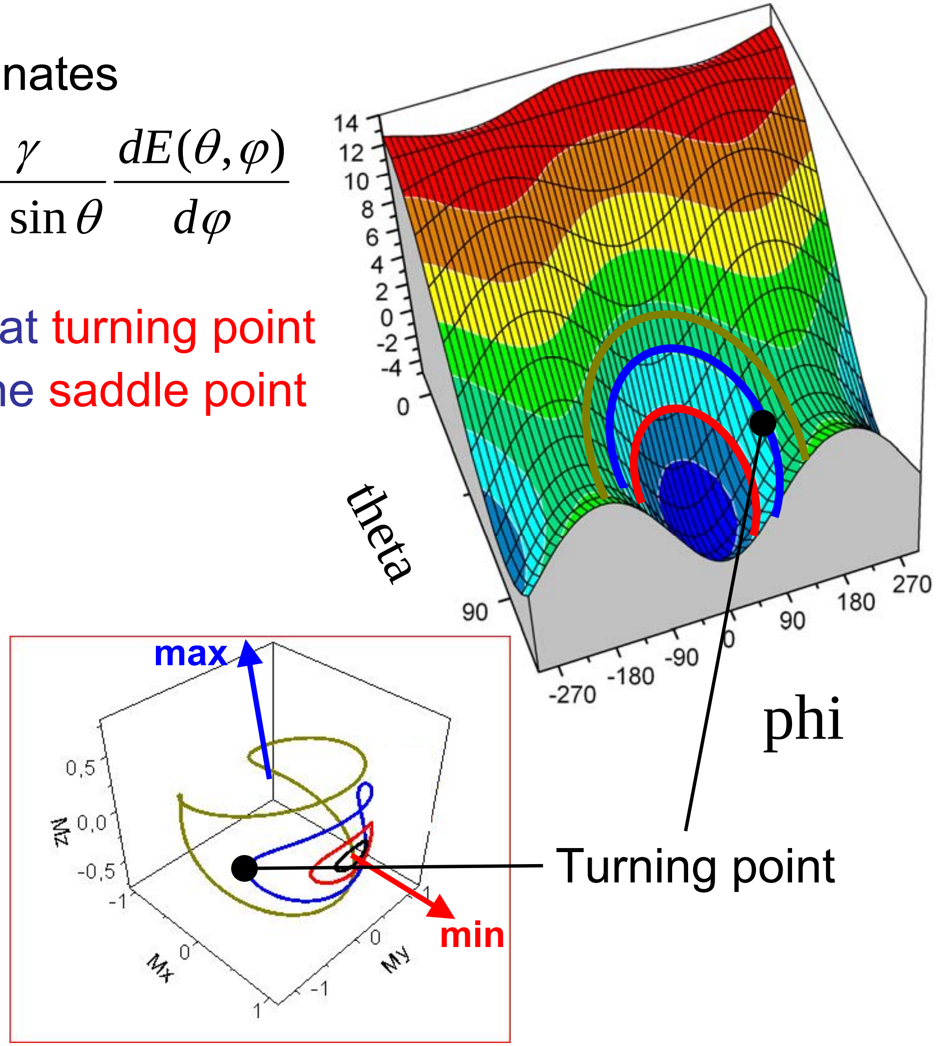
Frequencies for IPP modes vs current J

- Precession term in spherical coordinates

$$\frac{d\varphi}{dt} = \frac{-\gamma}{M\sin\theta}\frac{dE(\theta,\varphi)}{d\theta} \quad ; \quad \frac{d\theta}{dt} = \frac{\gamma}{M\sin\theta}\frac{dE(\theta,\varphi)}{d\varphi}$$

- Angular velocity $dE/d\theta$ decreases at turning point
- It goes to zero when approaching the saddle point
  → the frequency goes to zero





VI ST Precession - Frequencies

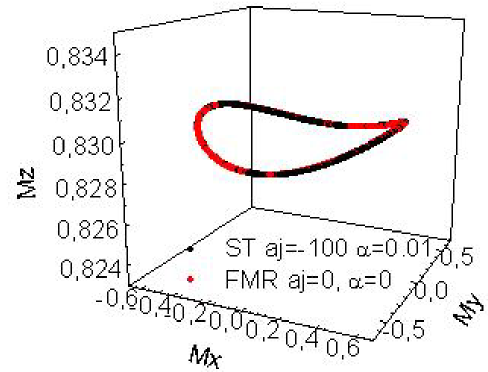
Frequencies for OPP modes vs current J

On OPP trajectory the demagnetization energy dominates

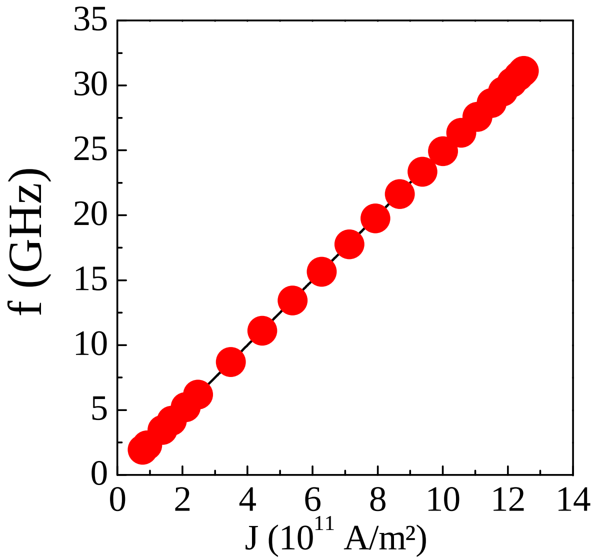
Conservative trajectory

$$\omega = \gamma H_d = \gamma 4\pi m_z$$

$$f \approx \frac{\gamma}{2\pi} 4\pi M_s \cos\theta$$



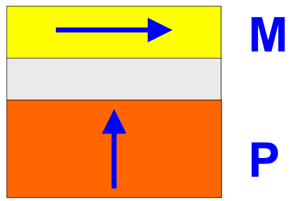
STT OPP trajectory of perpendicular polarizer



approximation

$$\cos\theta \approx \frac{a_j}{\alpha 4\pi M_s}$$

$$f \approx \frac{\gamma}{2\pi}\frac{a_j}{\alpha} \sim J$$



VI ST Precession - Summary Limit Cycles

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$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

Precession — **Damping** — **Spin torque (ST)**

	|m|	dE/dt	Static	Dynamic
Conservative Precession term only	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies
STT Dynamics LLGS	1	<0 =0 >0	X	Damped oscillations or Limit cycles

Depend on Control Parameter J, **P**, **H**

$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}_{\text{Damping}} + \underbrace{\gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)}_{\text{Spin torque}}$$

What about equilibrium states?

$$\frac{d\mathbf{M}}{dt} = 0$$

Same energy surface

→ Are the equilibrium states the same as for the conservative part?

→ What about their stability?

→ Are there any new equilibrium states?

→ What is their dependence on current amplitude?

Remember:

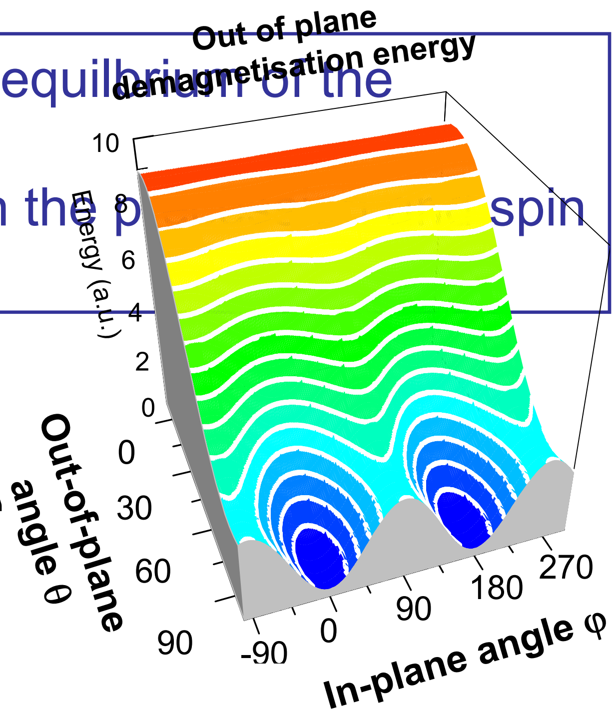
STT tends to align M parallel/antiparallel to P

Equilibrium $\frac{d\mathbf{M}}{dt} = 0$

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

Precession | **Damping** | **Spin torque**

When **P** does not point into the direction of an equilibrium of the conservative part (**M** x $\mathbf{H}_{eff}$ = 0):
⇒ equilbrium is given by the balance between the p… spin transfer torque!!

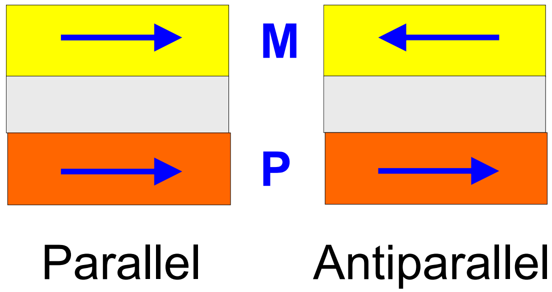


Examples:
1) Perpendicular polarizer at energy minimum
2) Planar polarizer at energy maximum

Example planar polarizer at in-plane energy minima

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

Precession — Damping — Spin torque



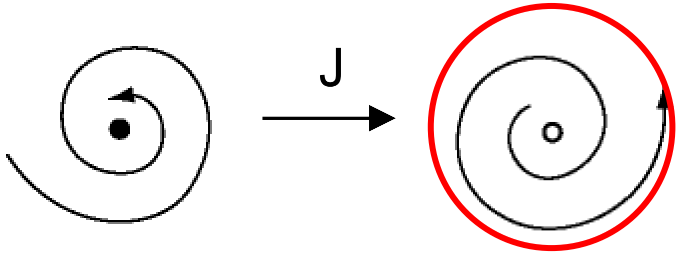
Upon increasing current J

- STT counteracts damping
- Above critical current J_c Stable focus can become an unstable focus

Stable for $J<J_c$

Stable/unstable depending on sign of $J>J_c$

	Parallel	Antiparallel
J ↑	**unstable**	**stable**
J ↓	**stable**	**unstable**



$$\delta\mathbf{M} \sim e^{\Gamma t} e^{i\omega_o t}$$

$$\Gamma_{J<J_c} < 0 \Rightarrow \Gamma_{J>J_c} > 0$$

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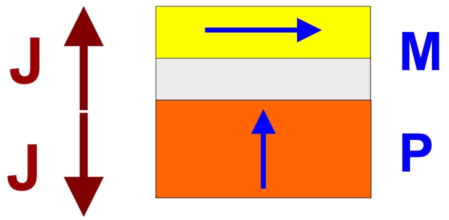
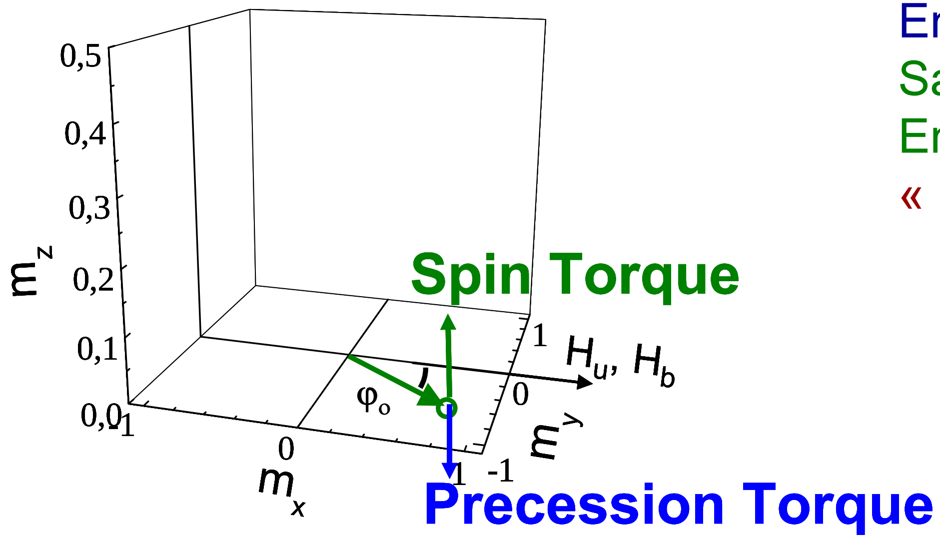
Example perpendicular polarizer



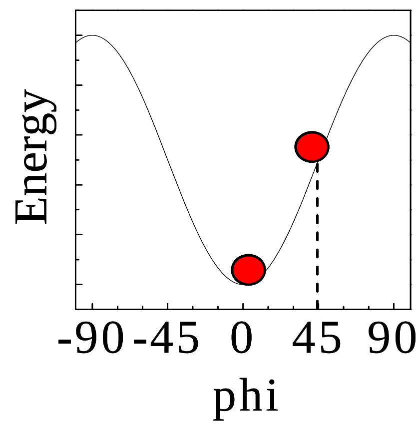
Equilibrium between precession and spin torque

$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}_{\text{Damping}} + \underbrace{\gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)}_{\text{Spin torque}}$$



Energy Minima: no equilibrium states
Saddle Point : no equilibrium point
Energy maximum: stable (**M**//**P**)
« New » state: in-plane rotation

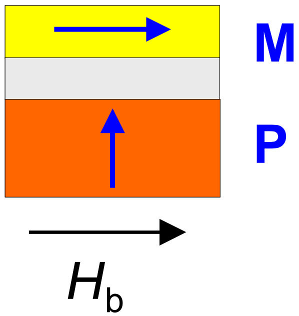
$$\mathbf{M}\times\left(\mathbf{H}_u + \mathbf{H}_b\right)$$



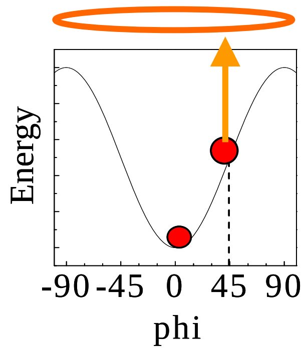
→ **In plane rotation** φ_o **=0°- 90°**

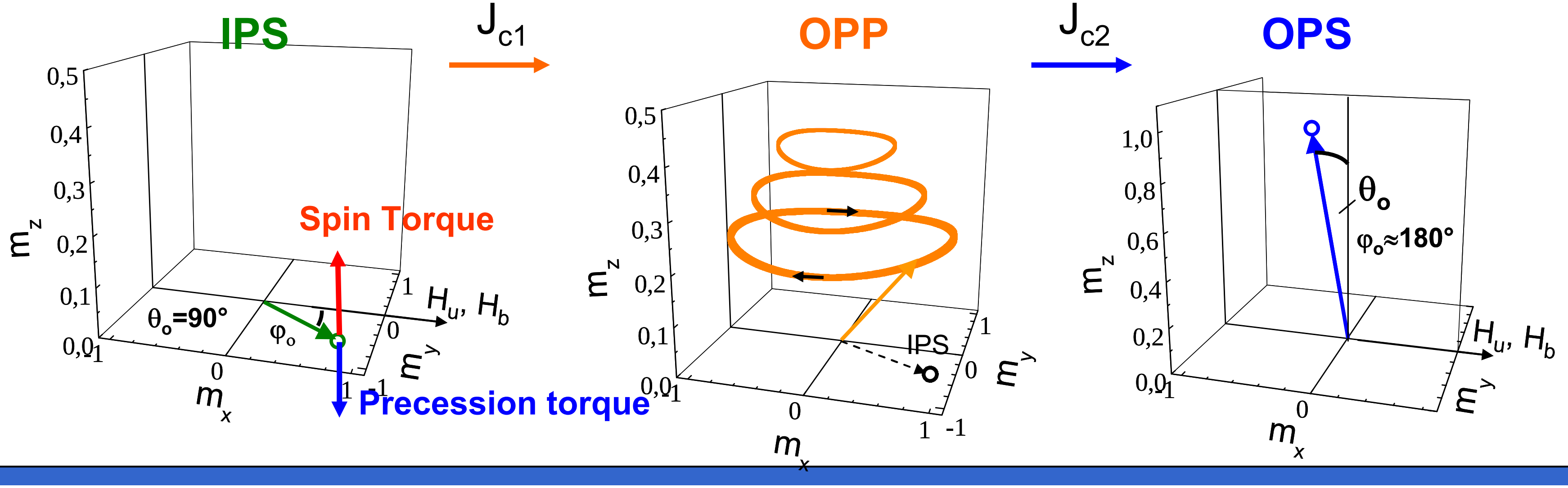
VI ST Precession – State Diagram

Example perpendicular polarizer



State Diagram
With in plane bias field





VI ST Precession - Stability & Critical Boundaries

Stability analysis of equilibrium states yields critical boundaries
→ Critical Current J_c as a function of bias field H_b

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

Precession **Damping** **Spin torque**

Equilibrium $\frac{d\mathbf{M}}{dt} = 0 \Rightarrow \mathbf{M}_o$

Stability: Linearization of LLG around equilibrium points $\mathbf{M}_o$

$$\left.\begin{aligned}
&\mathbf{M} = \mathbf{M}_o + \delta\mathbf{M} \\
&\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff}(\mathbf{M}_o + \delta\mathbf{M}) \\
&\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff_stat}(\mathbf{M}_o) + \mathbf{h}_{eff_dyn}(\delta\mathbf{M})
\end{aligned}\right\} \Rightarrow \frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\,\delta\mathbf{M}$$

Reminder Non-Conservative Dynamics

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$$

Precession — Damping

$$\frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\,\delta\mathbf{M}$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = 0 \Rightarrow \lambda$$

$$\delta\mathbf{M} = \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}$$

$$\lambda = \Gamma + i\omega_o$$

Well known solution from Ferromagnetic Resonance FMR

$$\omega_o^{\ 2} = \gamma\left(\frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2\sin^2\theta}\right) \qquad \Gamma = \frac{\gamma\alpha}{2}\left(\frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2\sin^2\theta}\right) \qquad \Gamma = \frac{\Delta\omega}{2}$$

To be evaluated at equilibrium $\boldsymbol{M}_o \Leftrightarrow \theta_o, \varphi_o$

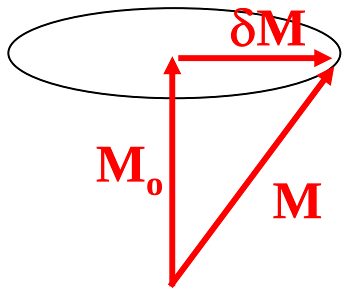
Can be applied to any equilibrium point

VI ST Precession - Stability & Critical Boundaries

Linearization for LLGS

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

Precession | **Damping** | **Spin torque**



$$\frac{d\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\,\delta\mathbf{M}$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = 0 \Rightarrow \lambda$$

$$\delta\mathbf{M} = \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}$$

$$\lambda = \Gamma + i\omega_o$$

For a given polarizer geometry (P constant and bias field orientation H)
The eigenvalues λ depend on the control parameters J and H_b

Stable if $\Gamma(H_b, J) > 0$, exponentially decaying

Untable if $\Gamma(H_b, J) < 0$, exponentially diverging

→ **Boundary for $\Gamma(J, H_b) = 0 \Rightarrow (J_c, H_b)$**

Complex frequency including spin torque

$$\frac{\lambda}{\gamma'} = -\frac{i}{2}\left(\frac{\Delta\omega}{\gamma} - 2a_j P_r'\right)\Bigg|_{\theta_o,\varphi_o} \pm \sqrt{-\frac{1}{4}\left(\frac{\Delta\omega}{\gamma} + 2a_j P_r'\right)^2 + \left(1+\alpha^2\right)\left(\frac{\omega_o}{\gamma}\right)^2 + \left(1+\alpha^2\right)\left(a_j P_r'\right)^2}\;\Bigg|_{\theta_o,\varphi_o}$$

- Define state diagram for arbitrary **P** and **H**$_b$
- Calculate FMR frequencies in damped regime around stable states

with

$$\left(\frac{\omega_o}{\gamma}\right)^2 = \frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2 \sin^2\theta}; \qquad \frac{\Delta\omega}{\gamma} = \alpha\left(\frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2 \sin^2\theta}\right)$$

$$P_r' = P_x \sin\theta_o \cos\varphi_o + P_y \sin\theta_o \sin\varphi_o + P_z \cos\theta_o$$

$$a_J = \frac{\hbar}{2e}\frac{J}{\mu_0 M_S t} g\left(\eta,\vec{m},\vec{p}\right)$$

$E_{\theta\theta}, E_{\varphi\varphi}, E_{\theta\varphi}$

Second derivatives of energy, to be evaluated at

M$_o \Leftrightarrow \theta_o, \varphi_o$

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VI ST Precession - State Diagram

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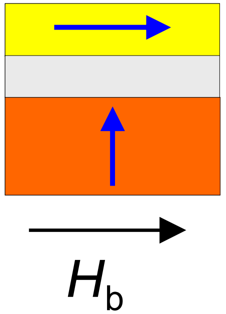
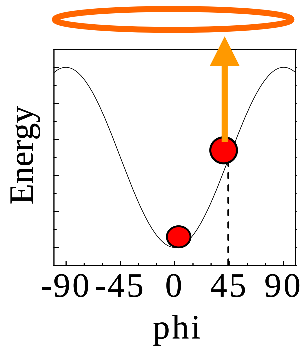
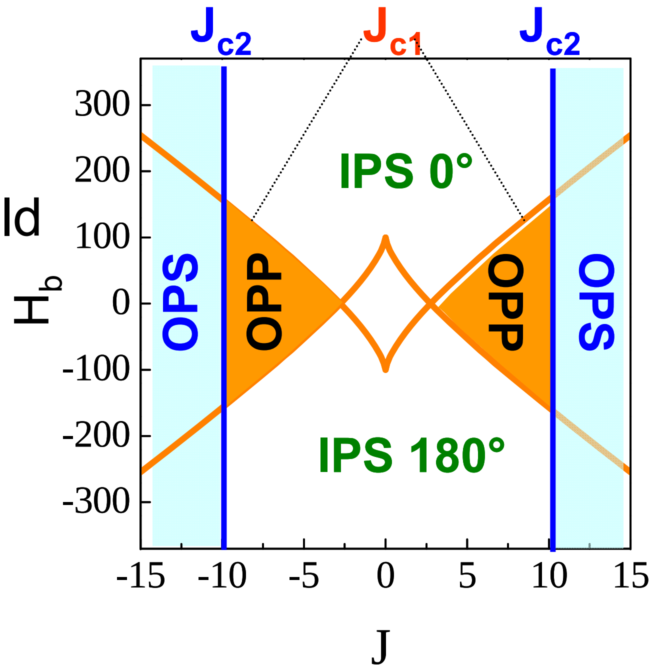
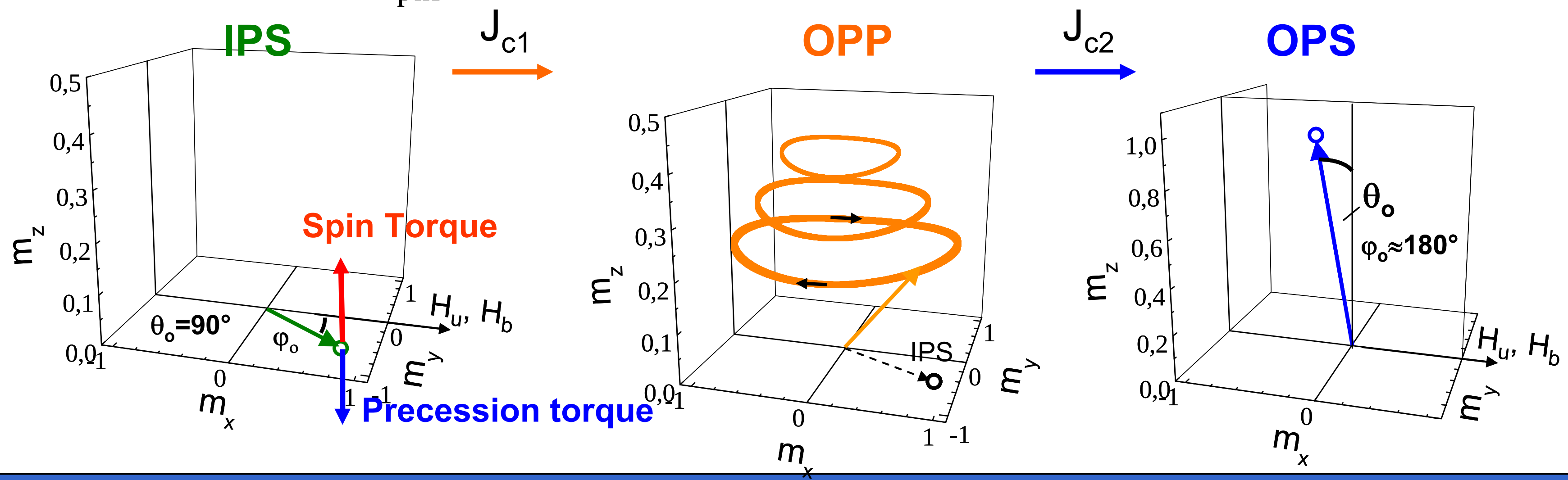
Example perpendicular polarizer



State Diagram
With in plane bias field

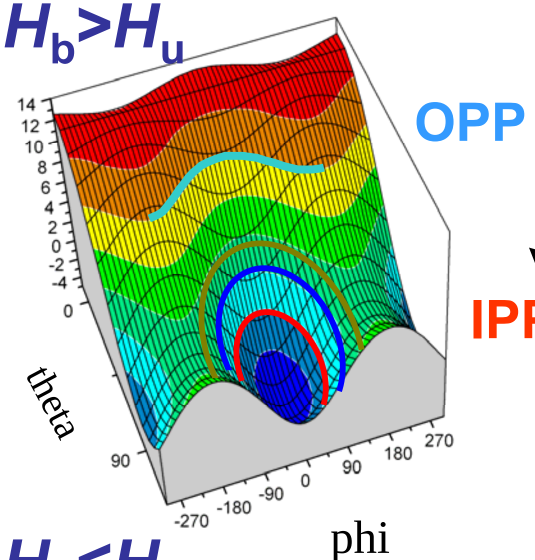




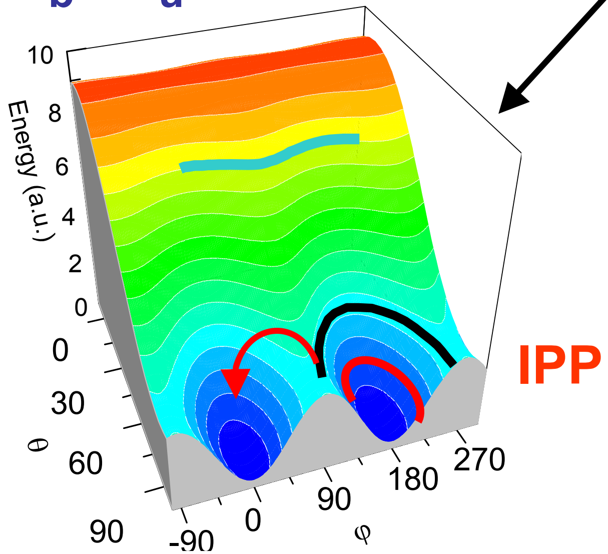


VI ST Precession – State Diagram

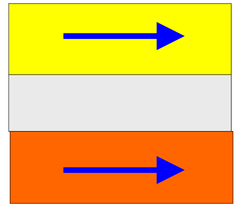
$H_b > H_u$

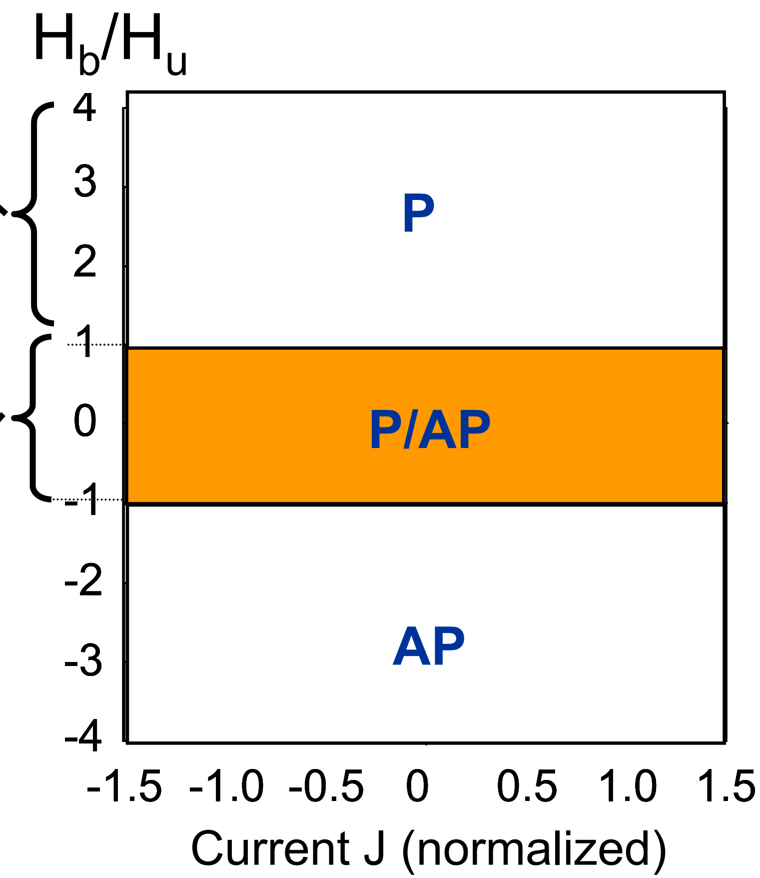


$H_b < H_u$



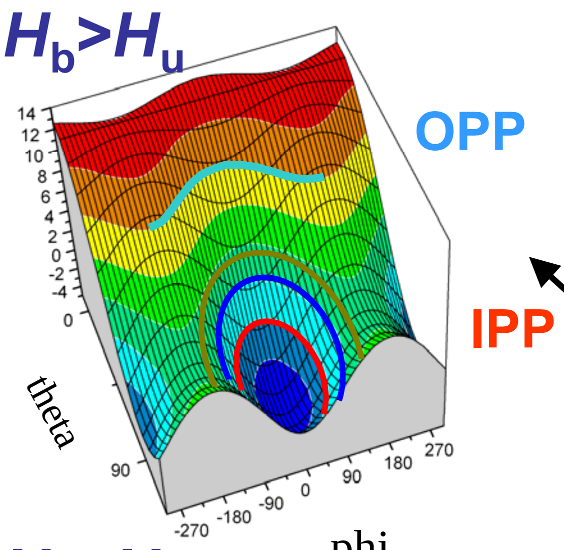
Example planar polarizer

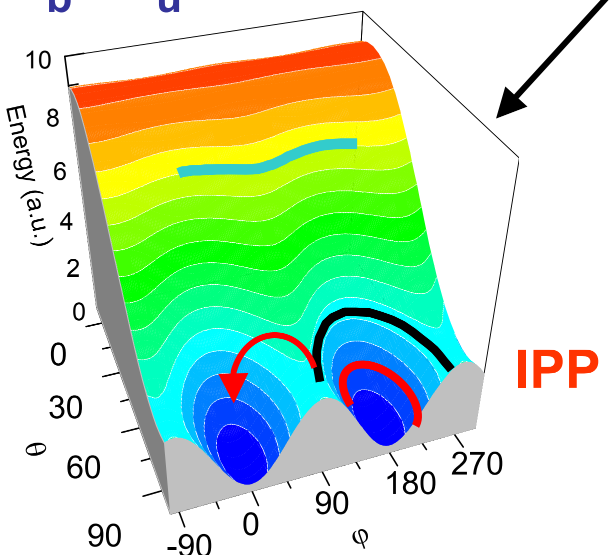




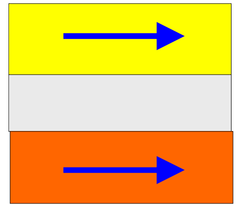
Reversal
No Spin torque

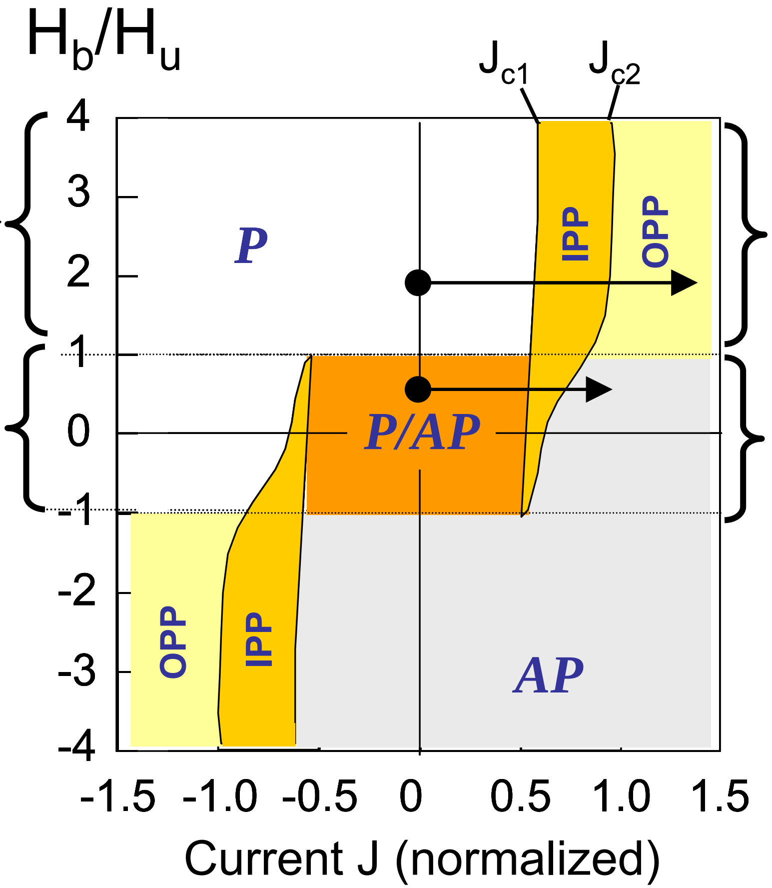
VI ST Precession – State Diagram

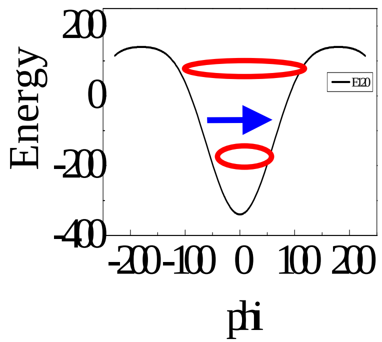


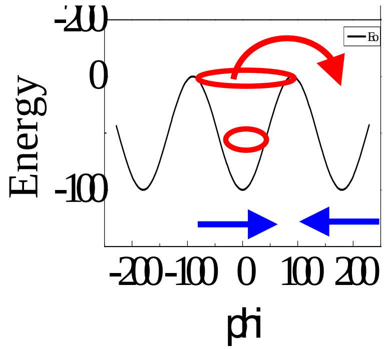


Example planar polarizer







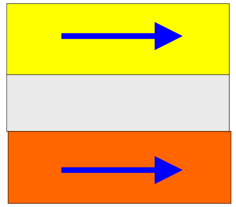


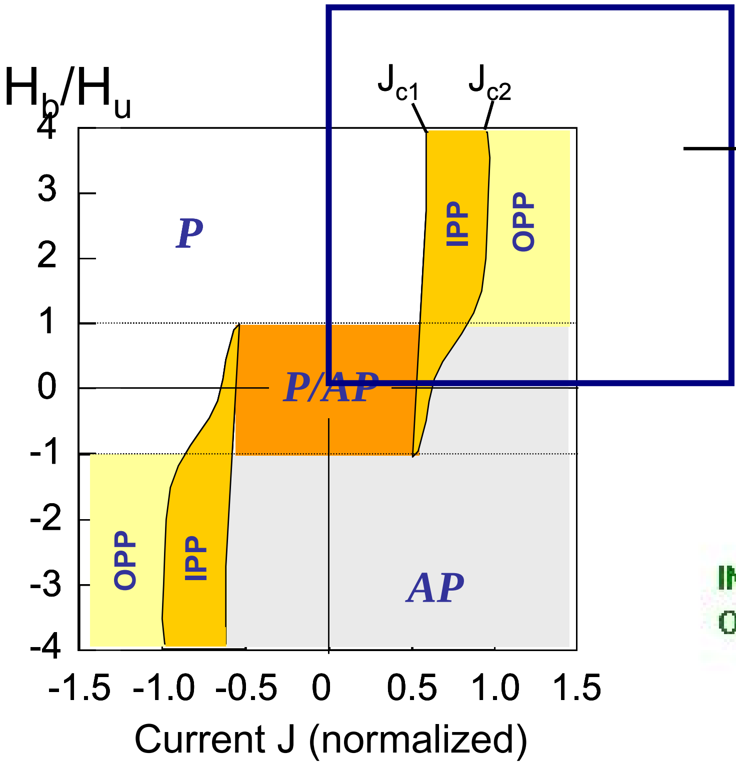
Reversal under ST

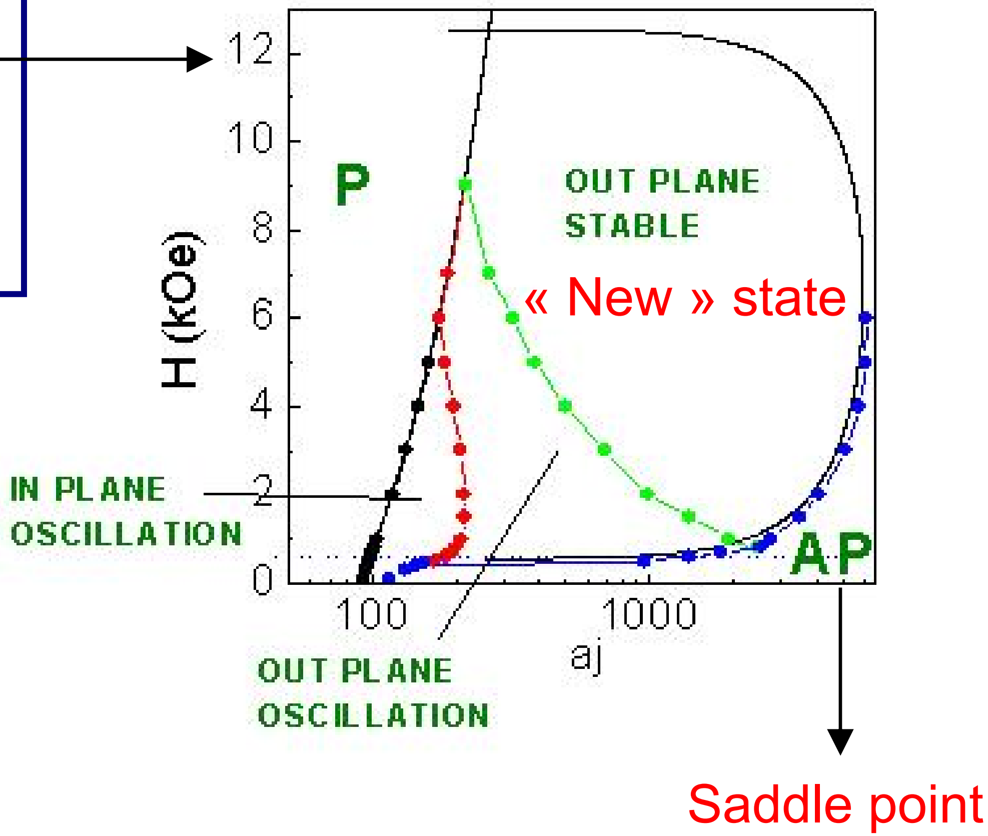
ursula.ebels@cea.fr

VI ST Precession – State Diagram

Example planar polarizer: New states







VI ST Precession – Summary Limit Cycles

$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) + \frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

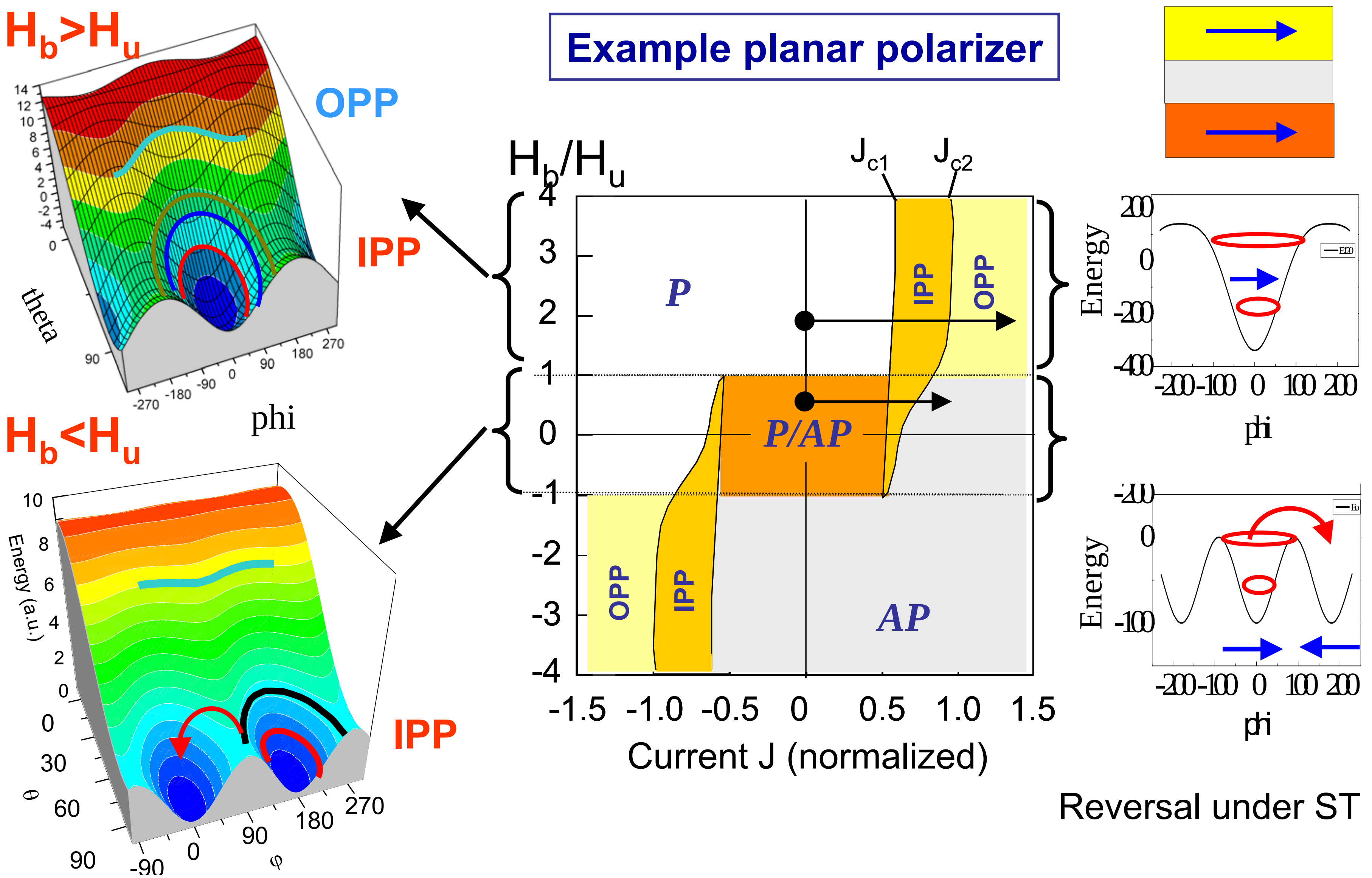
Precession **Damping** **Spin torque (ST)**

	|m|	dE/dt	Static	Dynamic
Conservative Precession term only	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies
STT Dynamics LLGS	1	<0 =0 >0	stable/unstable New states	Damped oscillations or Limit cycles

Depend on Control Parameter J, **P**, **H**

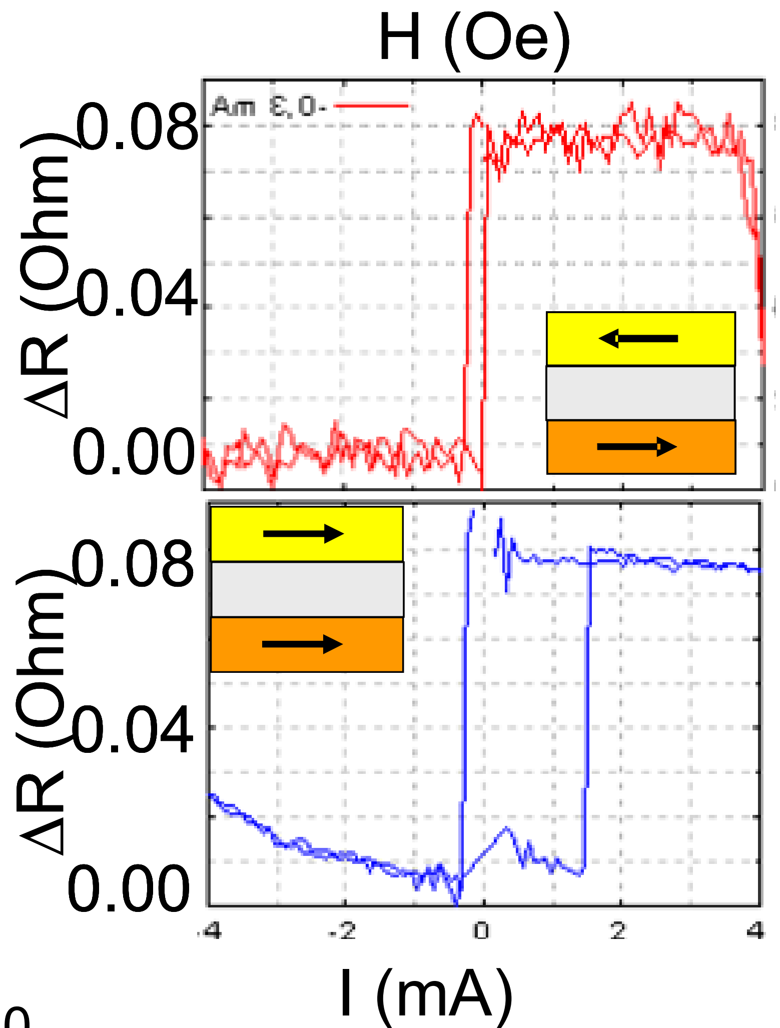
Outline

VII Reversal under spin torque



Example planar polarizer

Reversal under DC current



Reversal under DC current

Critical Current density

$$J_{DC}^{c} = k\alpha\left(H_u + H_b + 2\pi M_s\right)$$

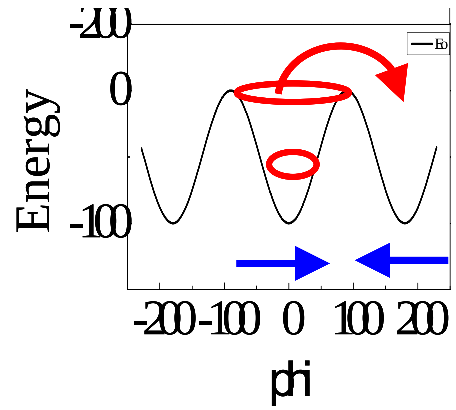
$$k = \frac{2e}{\hbar}\frac{M_S t}{\eta}$$

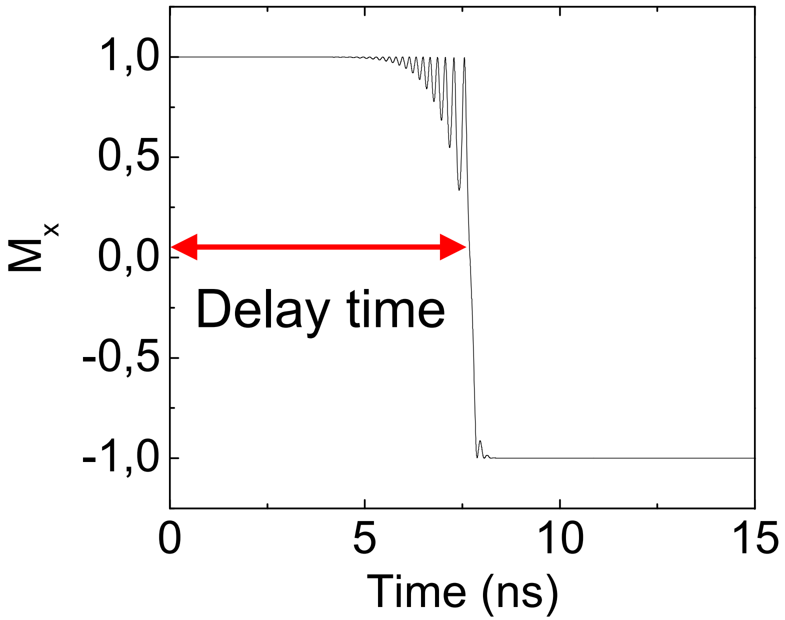
First experiments by Albrecht et al 2000

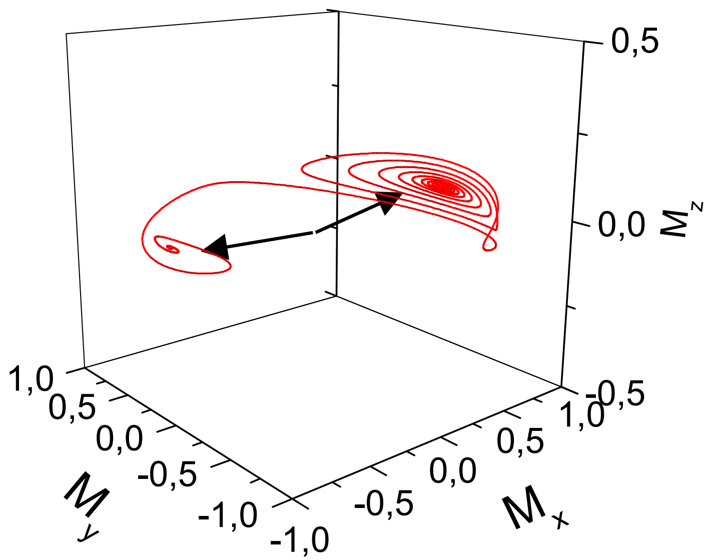
VII Reversal under spin torque

Example planar polarizer

Reversal Time



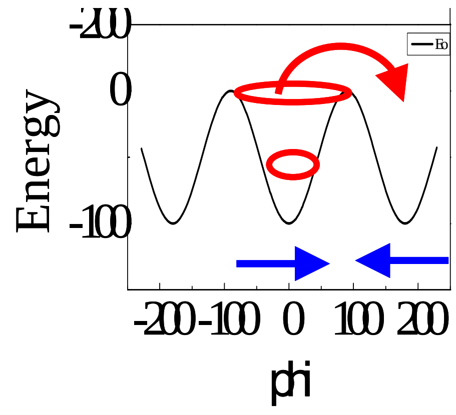


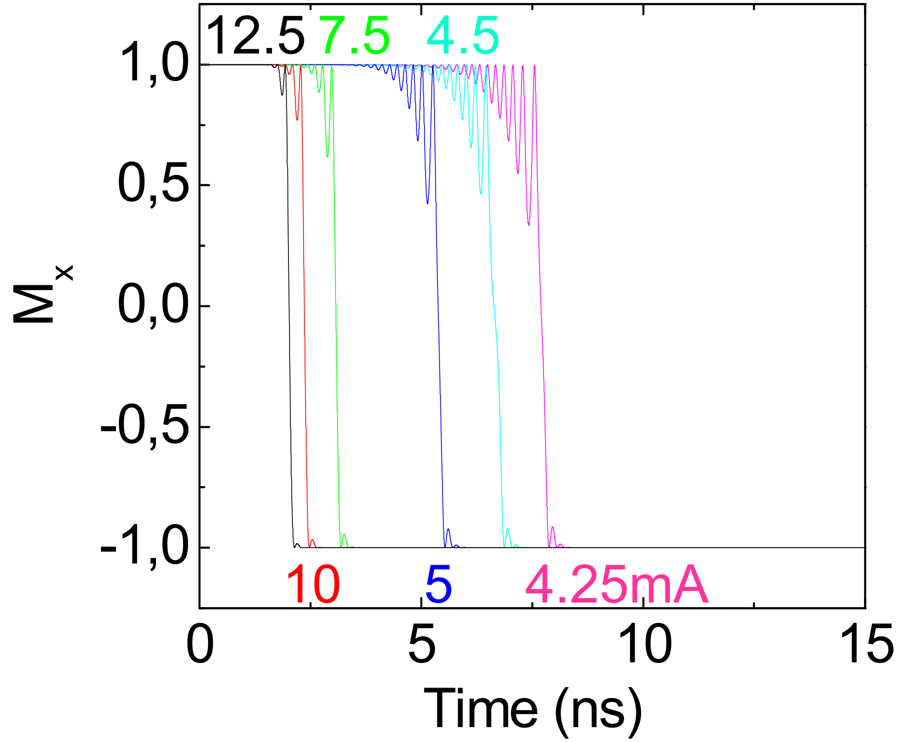


Delay time to « wind up » and overcome the energy barrier

Example planar polarizer

Reversal Time





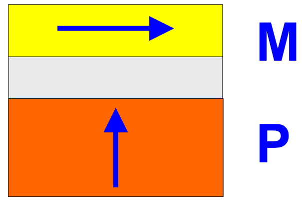
$$\mathbf{T}_{//} = \gamma_0 \frac{a_j}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{P}) \sim \sin(\mathbf{MP})$$

Dynamic critical current density increases with decreasing pulse width τ

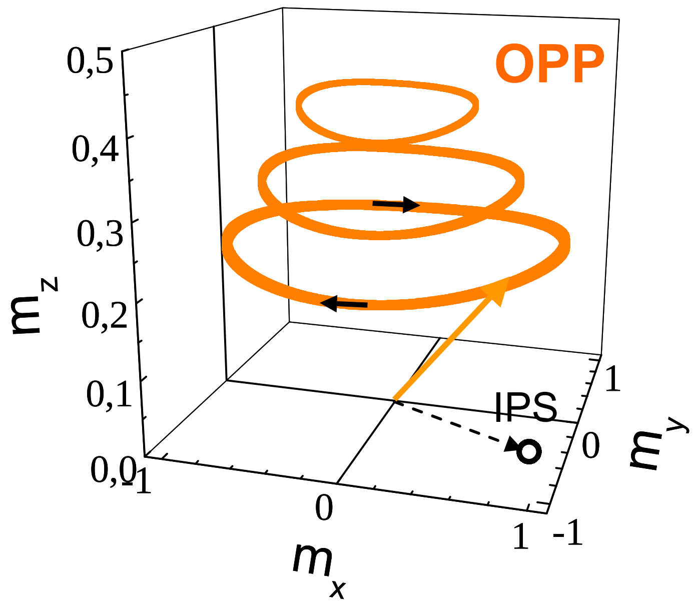
$$\tau \sim \left(J^{c}_{pulse} - J^{c}_{DC}\right)^{-1} \ln \frac{\theta}{\theta_o}$$

Sun PRB 62 (2000)

Example perpendicular polarizer



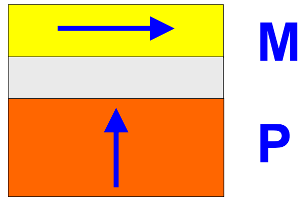
Stabilizes OPP trajectories under DC spin poloarized current



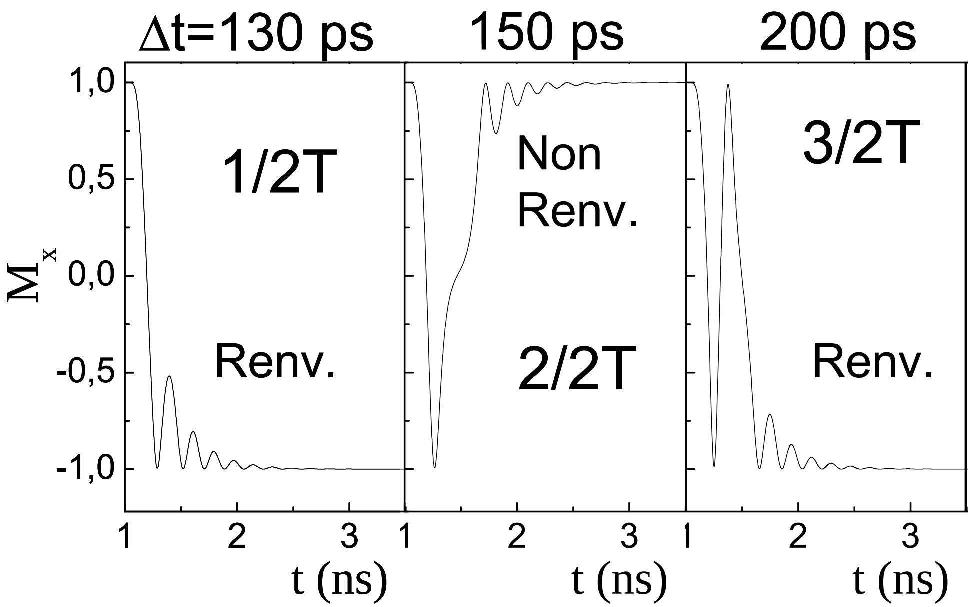
f = 2-4GHz

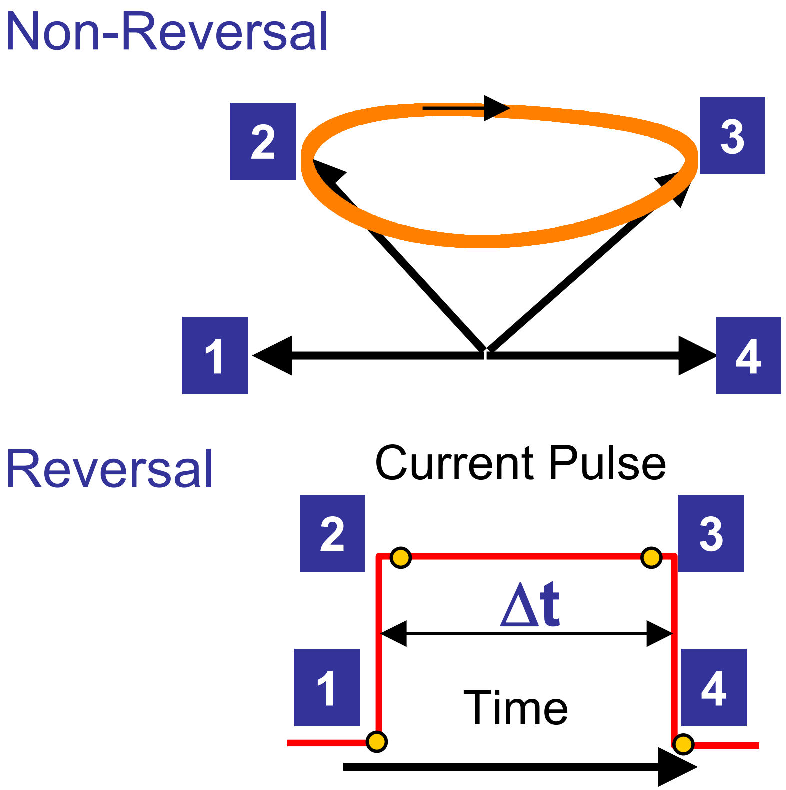
VII Reversal under spin torque - Precessional Reversal

Example perpendicular polarizer



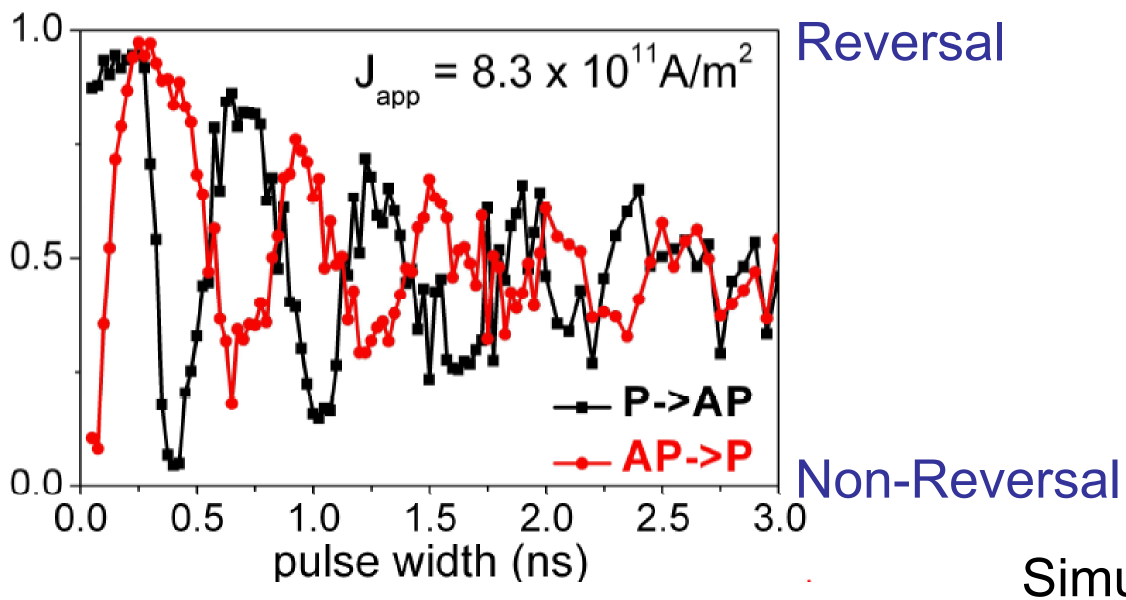
Apply current pulse of short duration $\Delta t < 1ns$

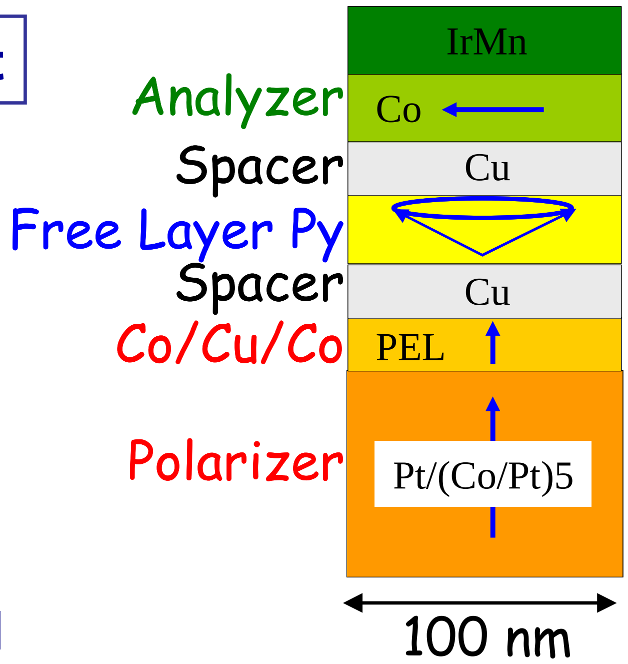




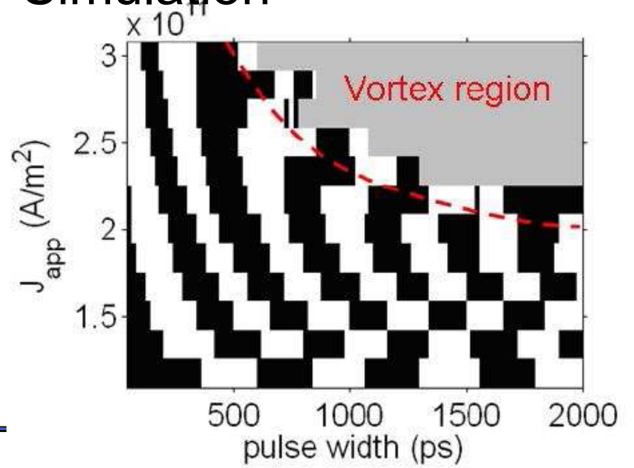
VII Reversal under spin torque - Precessional Reversal

Example perpendicular polarizer: Experiment





Simulation



- Bands of Reversal/Non-Reversal when Δt matches precession period $T=1/f$
- Since f ~ J : $\Delta t \sim 1/J$

Vayset APL **98**, 242511 (2011)
Papusoi APL **95**, 072506 (2009)

VII Reversal under spin torque - Summary

Reversal under spin transfer torque depends on the polarizer configuration

- In the **planar configuration**, there is a delay time corresponding to the time necessary for **M** to spiral up
- The reversal time (~ns) decreases with increasing current pulse amplitude and intial angle

- In the **perpendicular configuration**, precessional reversal under short (<1ns) current pulses is possible, where the magnetization reverses on 2n+1 precession cycles

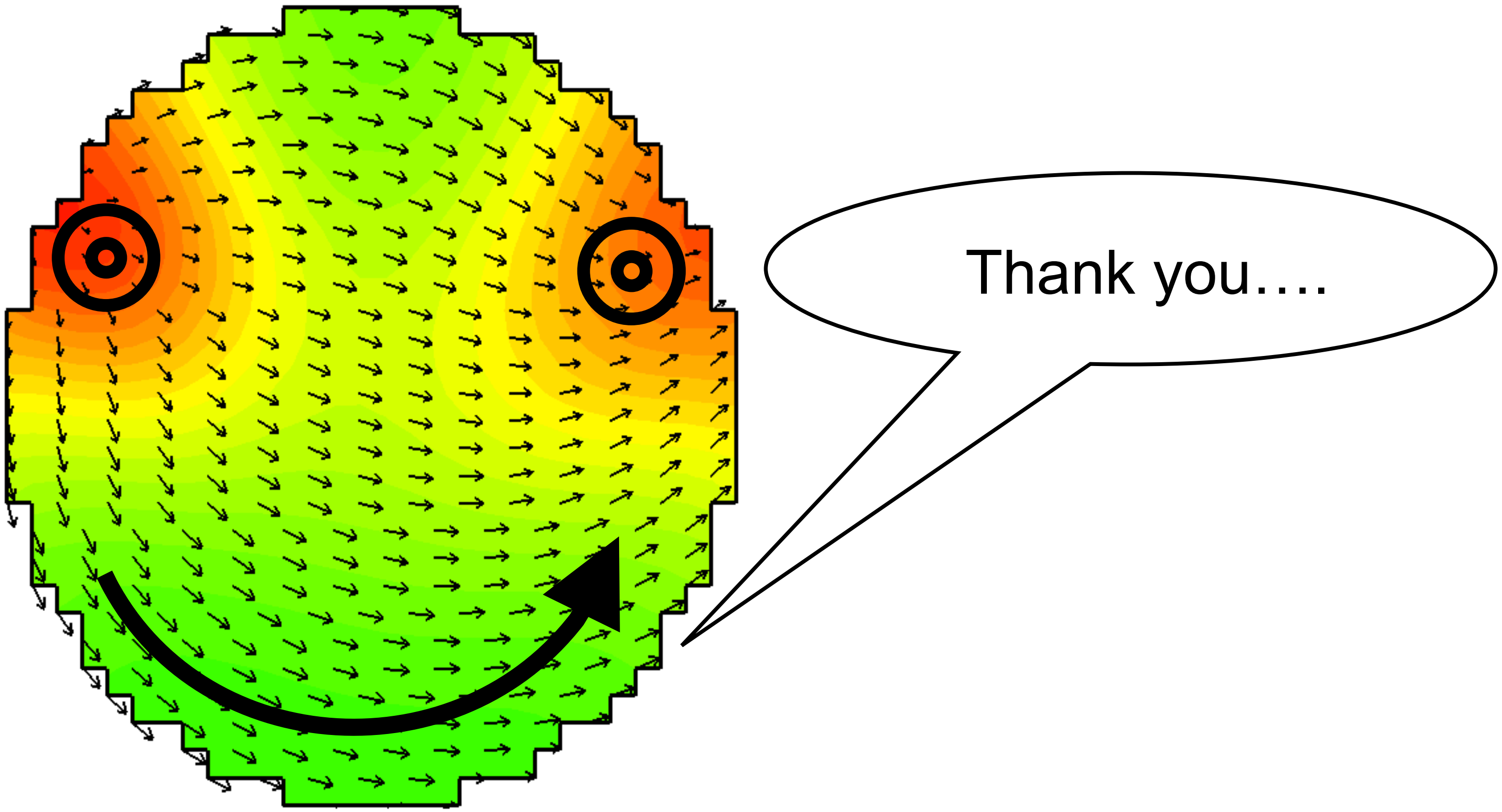
Summary

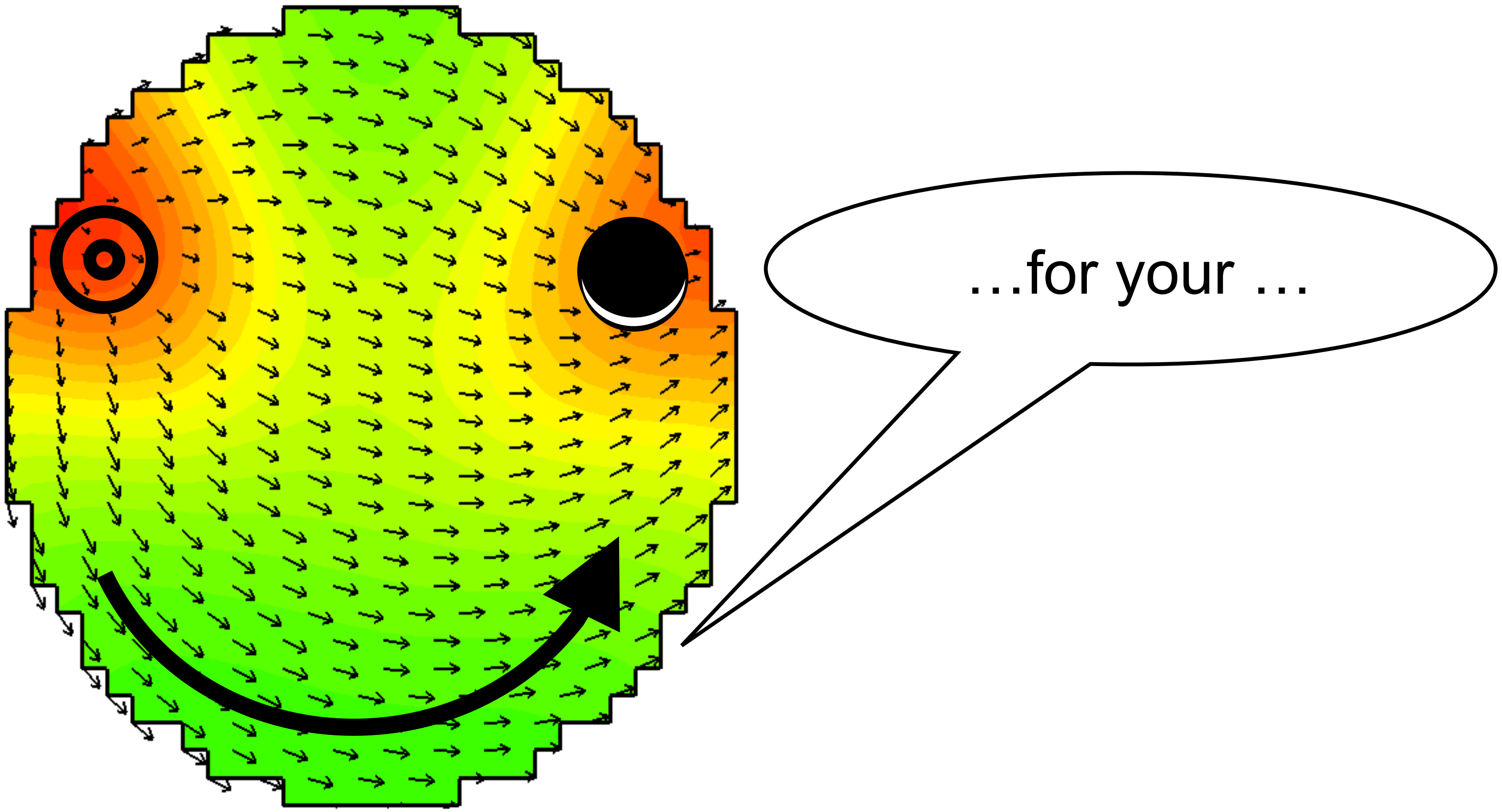
$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

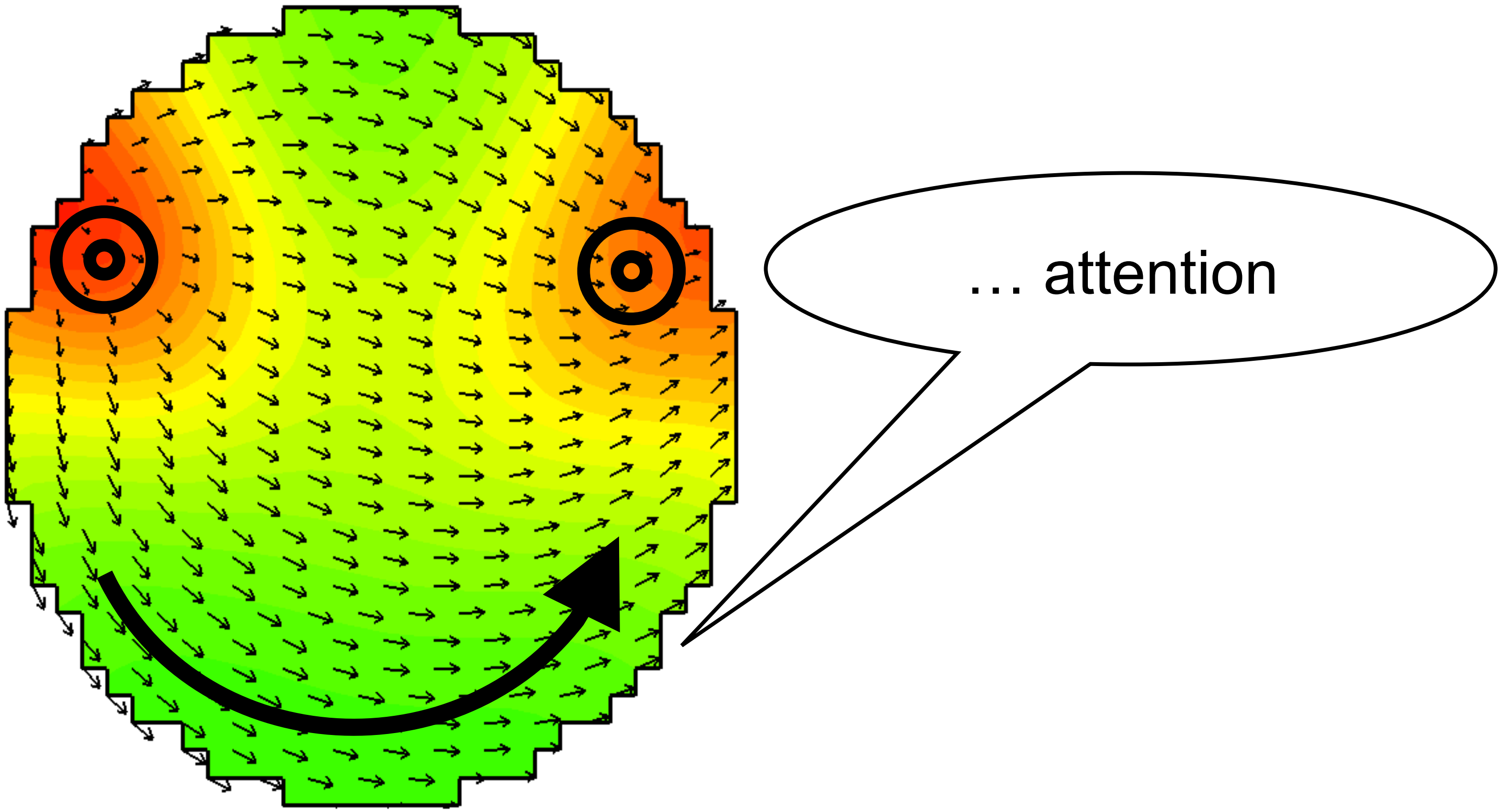
Precession **Damping** **Spin torque**

- LLG(S) equation: Macrospin motion of **M** as an example of a non linear dynamical system
- One can apply standard analysis techniques to find equilibria and orbits and their respective stability
- Three cases have been discussed (non) conservative and STT dynamics
- The external field $\mathbf{H}_b$ as control parameter changes the energy surface, thus the equilibrium points and the types of trajectories
- STT changes the energy loss-energy gain. For a constant field and polarizer **P**, the stability and trajectory depend on *J*
  → transition from stable to unstable points and to limit cycles

- Make use of the orbits to define strategies of fast reversal under applied field and current or to design microwave applications
- Wall dynamics follows similar principles, but a bit more complex


Thank you….


…for your …


… attention

Outline

0 Introduction

I Conservative dynamics (solutions of Precession term)

II Non conservative dynamics

III Precessional reversal under transverse field pulses

IV Domain wall motion under field

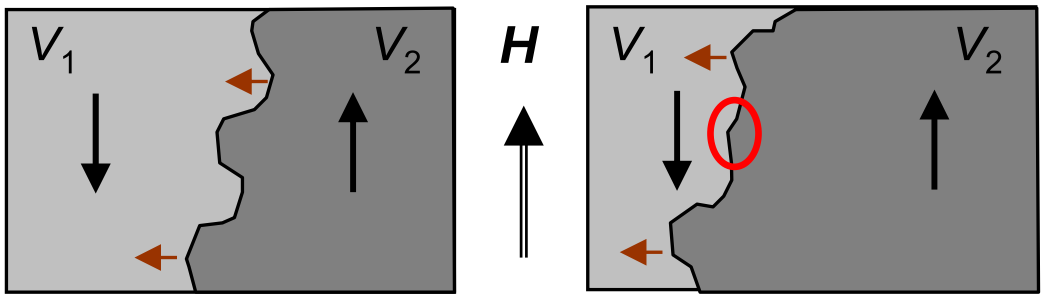
V Introduction to spin transfer torque

VI Spin torque induced precession

VII (Precessional) Reversal under spin torque

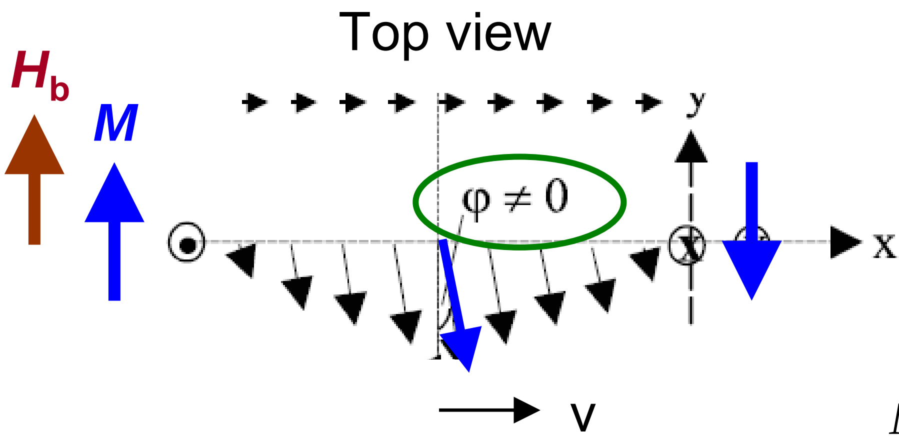
VIII Domain wall motion under current

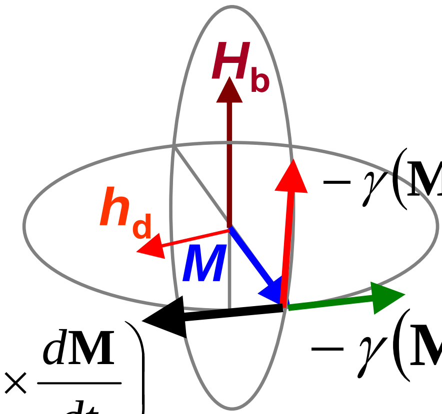
Wall motion under an applied field



Time t_1 Time t_2

- Wall displaces perpendicular to field
- Domain parallel to bias field increases in size, to minimize Zeemman energy



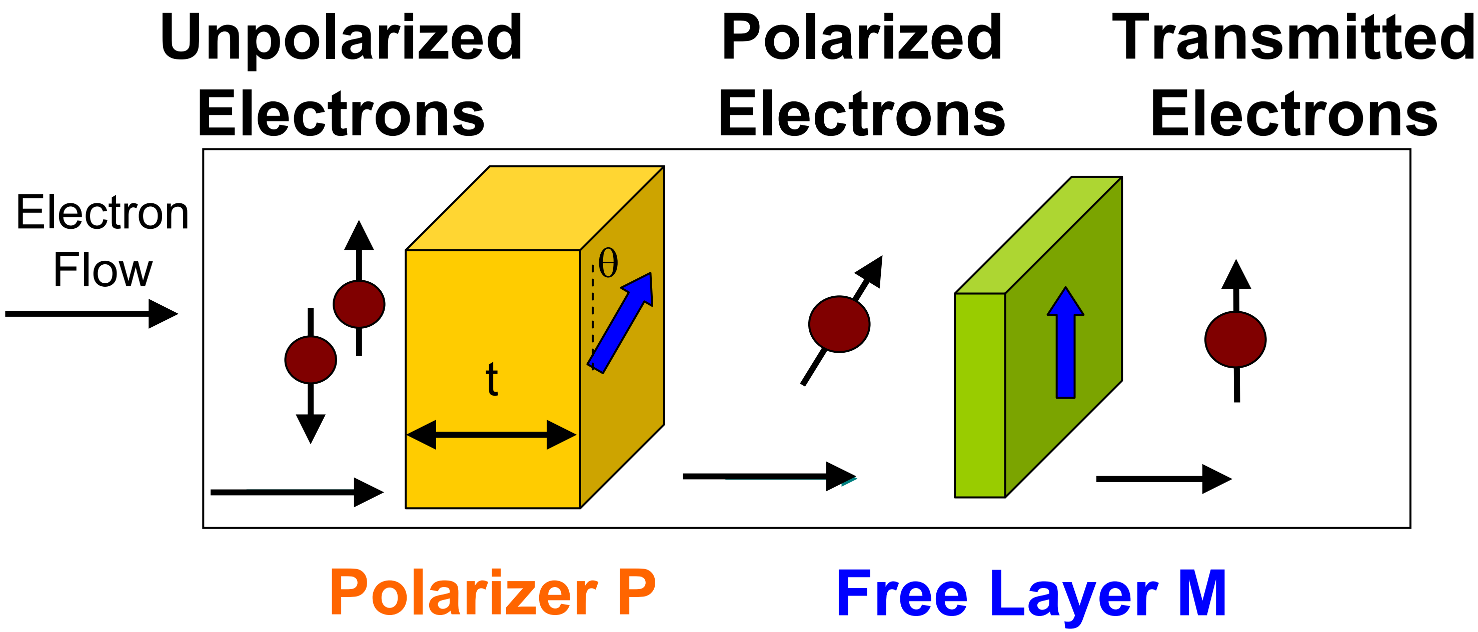


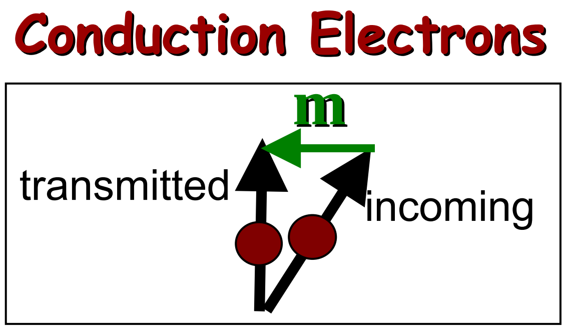
$$-\gamma(\mathbf{M}\times\mathbf{h}_d) = \frac{d\mathbf{M}}{dt}$$

$$-\gamma(\mathbf{M}\times\mathbf{H}_b)$$

$$\frac{\chi}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$$

V Spin Momentum Transfer - Concept

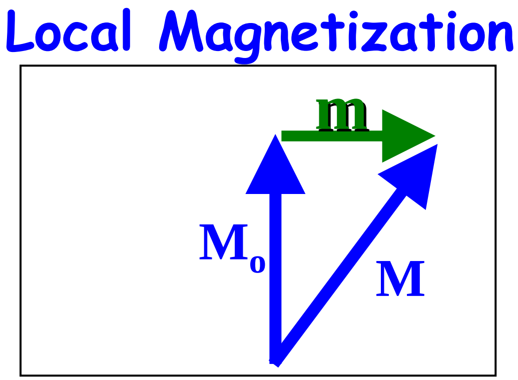




Transfer of transverse moment **m**

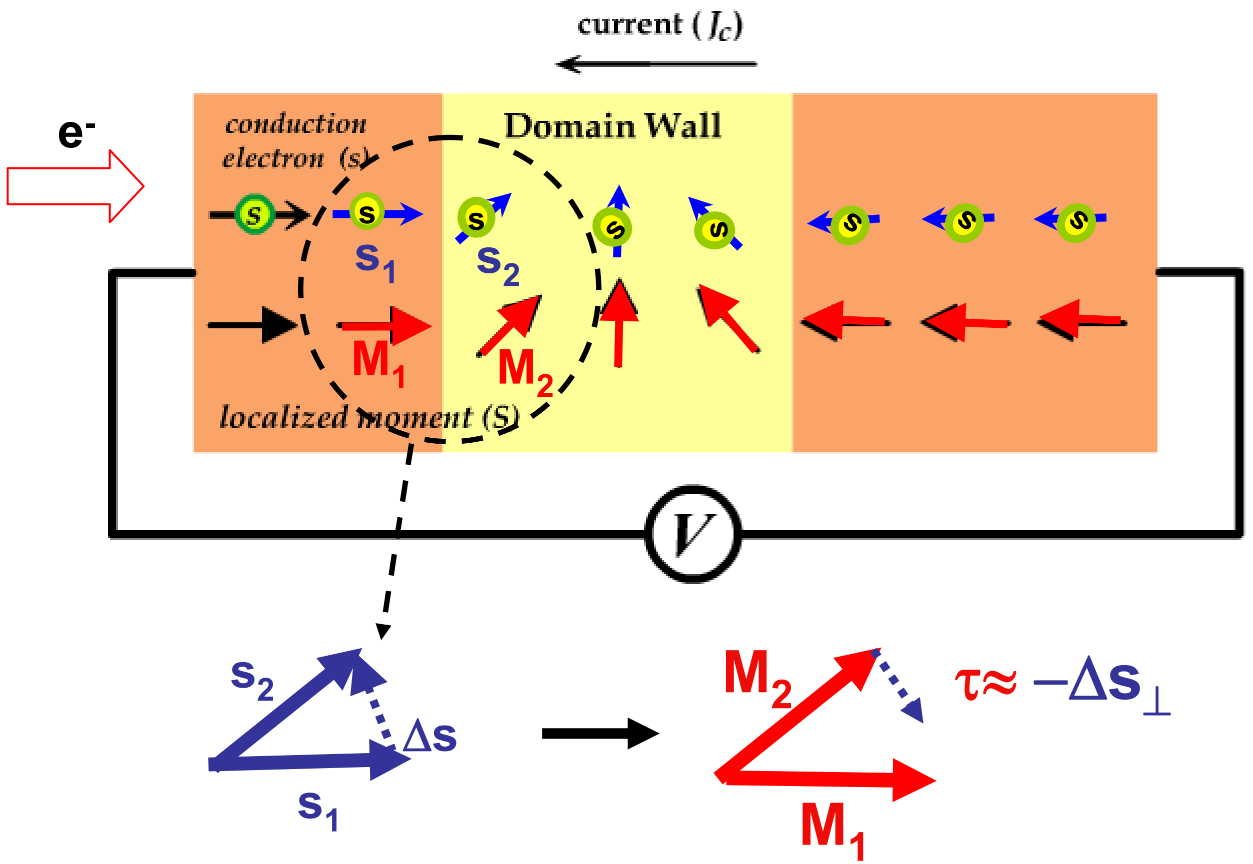
=

Spin Torque



J. Slonczewski, *JMMM* 159, L1 (1996)
L. Berger, *PRB* 54, 9353 (1996)

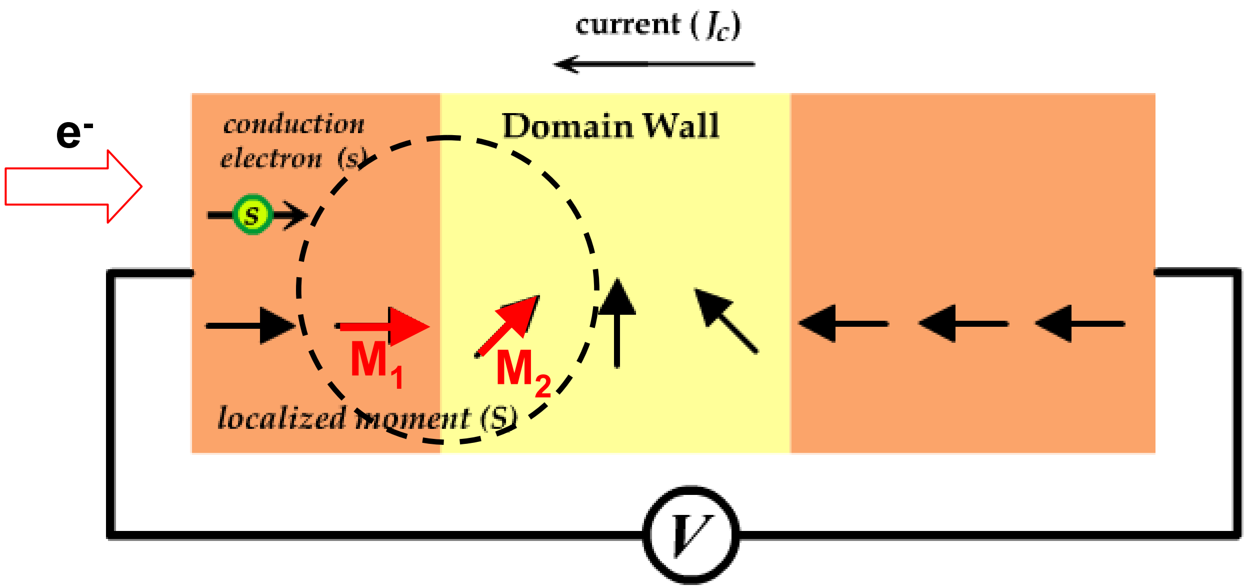
VIII Domain wall displacement under spin torque



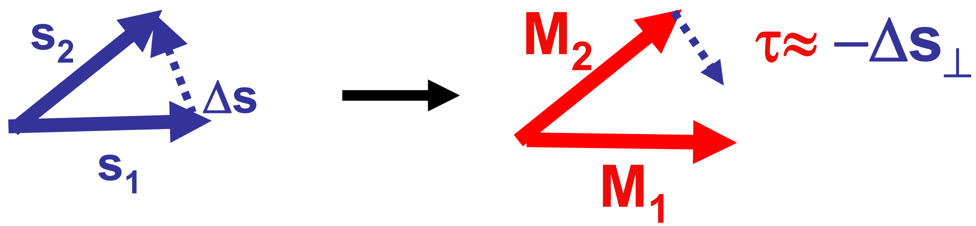
Courtesy of S. Maekawa

- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
- Conservation of angular momentum → Spin transfered to the local magnetization ⇨ **Torque** on magnetization
- **DW motion** in the direction of the e^- flow

VIII Domain wall displacement under spin torque



Courtesy of S. Maekawa



- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
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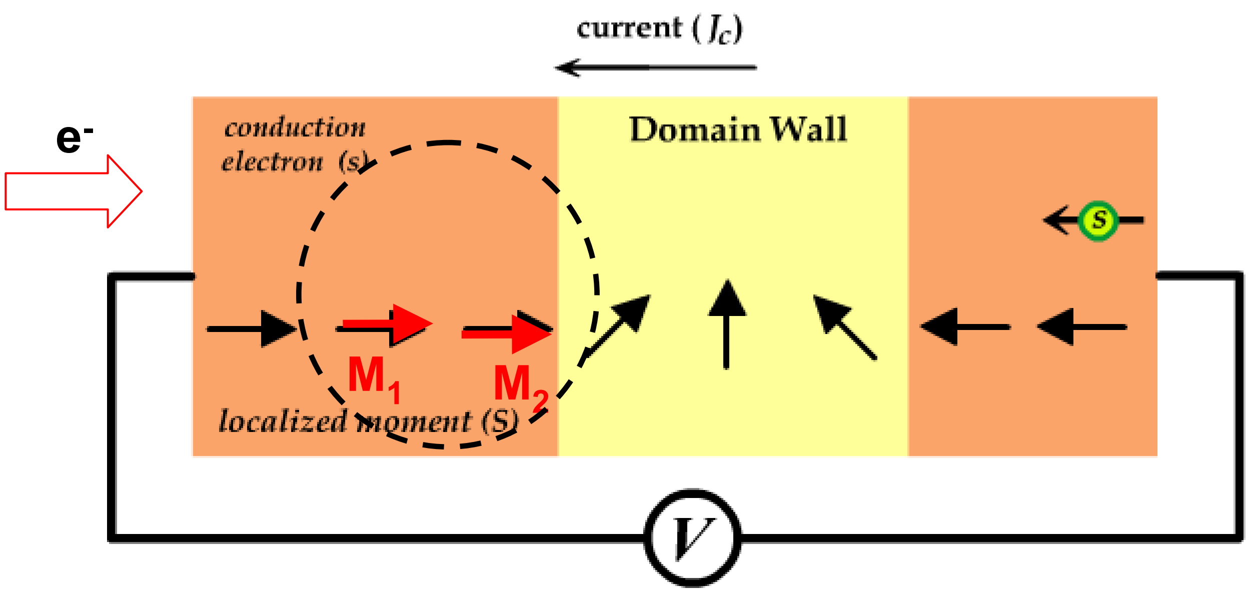
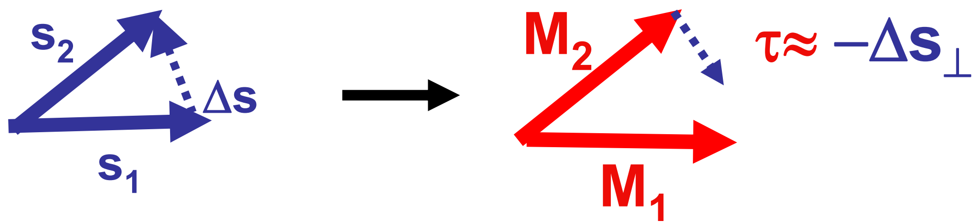
VIII Domain wall displacement under spin torque



Courtesy of
S. Maekawa



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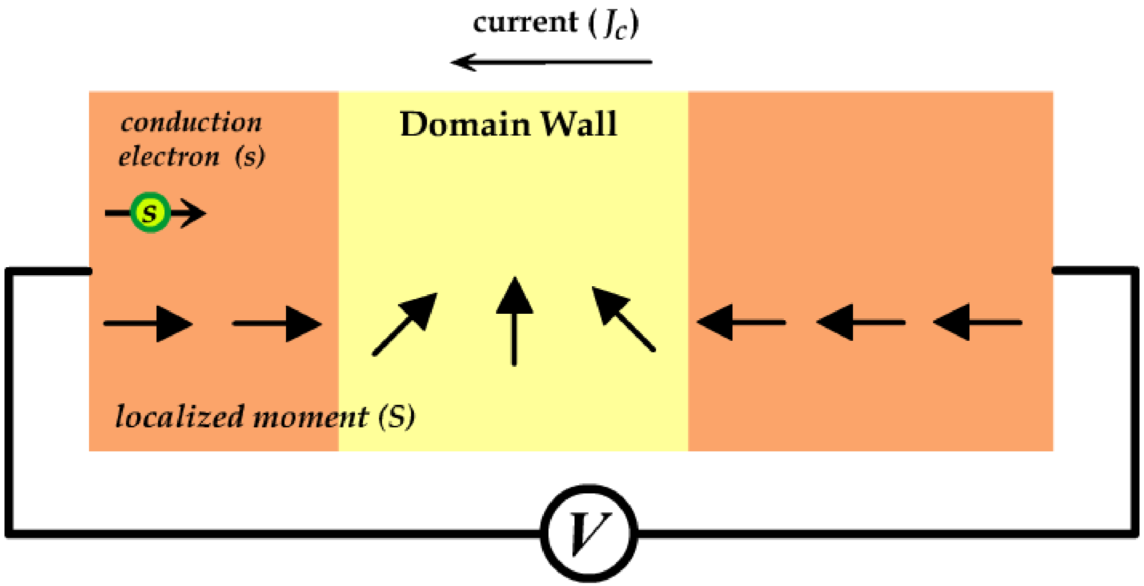
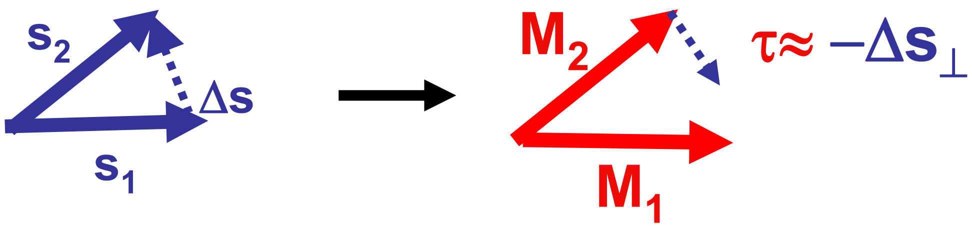
VIII Domain wall displacement under spin torque



Courtesy of S. Maekawa



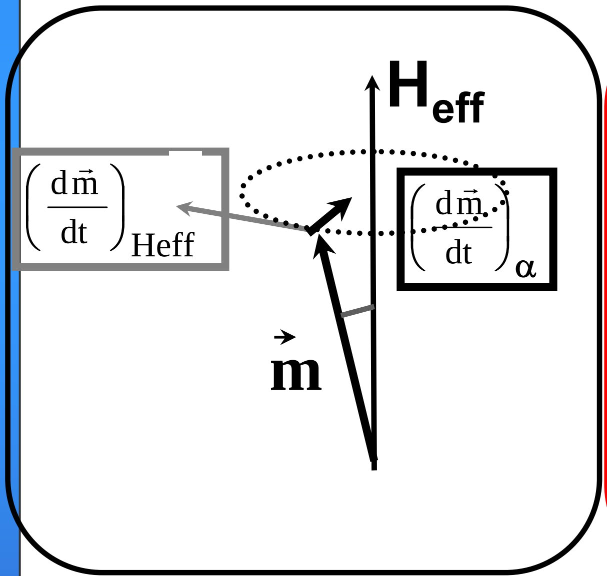
- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
- Conservation of angular momentum → Spin transfered to the local magnetization ⇨ **Torque** on magnetization
- **DW motion** in the direction of the e^- flow

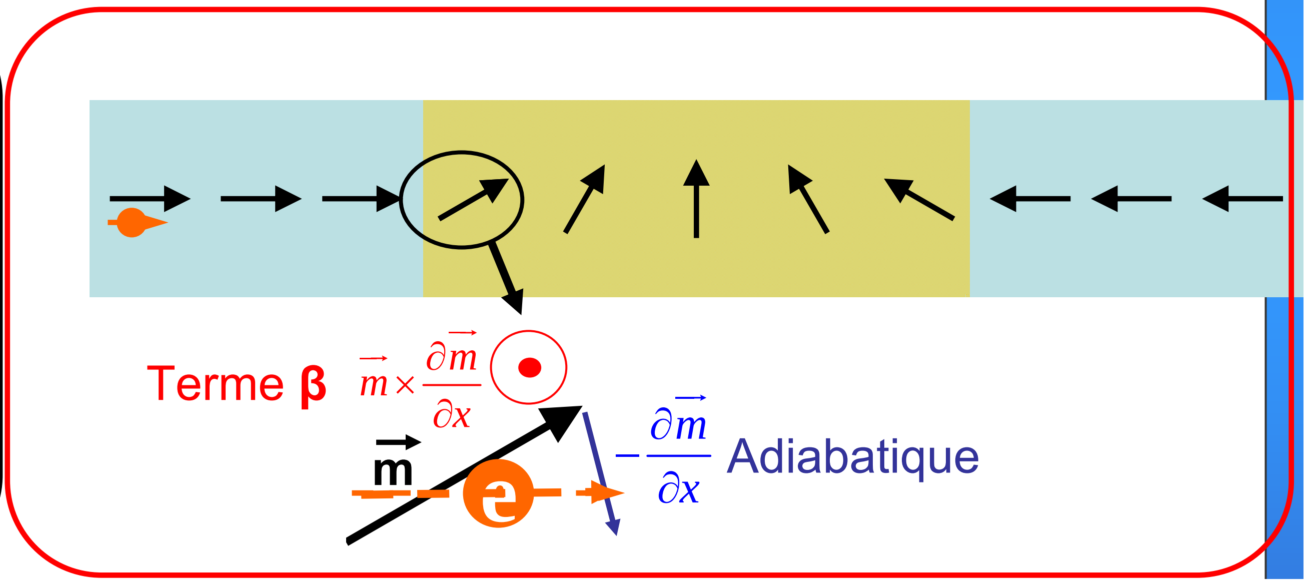
VIII Domain wall displacement under spin torque

Modified LLG equation

$$\frac{\partial \vec{m}}{\partial t} = -\gamma \vec{m} \times \vec{H}_{eff} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t} - u \frac{\partial \vec{m}}{\partial x} + \beta u \vec{m} \times \frac{\partial \vec{m}}{\partial x} \qquad u = \frac{JPg\mu_B}{2eM_s}$$

Precession — Damping — ① — ②



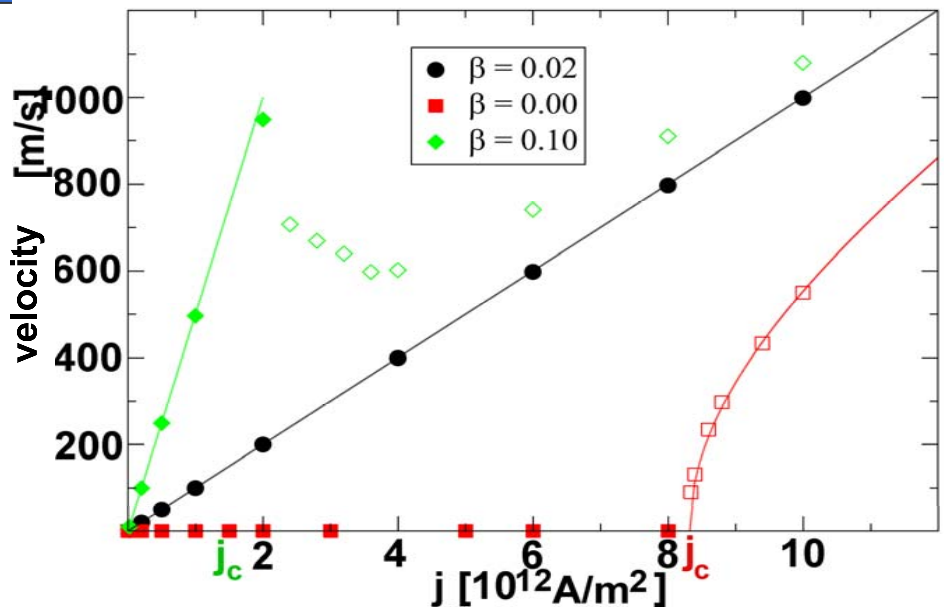


① **Adiabatic spin transfer torque**

② **β** term: generally much smaller, arising from spin relaxation in DW and non-adiabatic effects in narrow DW due to spin mistracking.

VIII Domain wall displacement under spin torque

β term



Perfect wire with no edge roughness

- **β =0**, **only adiabatic term**
  - **No motion for J<Jc**
  - **Jc « intrinsic »** (depends on the magnetic properties of the DW)
  - **Turbulent motion** above J_c with complex **DW transformation**

- **β≠0**
  - v ≠**0** for perfect wire with **v∝ jβ/α**
  - Jc **« extrinsec »** due to pinning (roughness, defect in the material,…)
  - « Field like » torque

⇨ **β is a key parameter in the DW dynamics, but its value is still unknown and very controversial.**

A. Thiaville et al., J. Appl. Phys. 95, 7049 (2004) G. Tatara et al., Phys. Rev. Lett. 92, 86601 (2004), S. Zhang and Z. Li, PRL **93**, 127204 (2004)

X Some papers for further reading (non exhaustive)

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● L. Perko “*Differential equations and dynamical systems*” Texts in Applied Matehmatics
● J. Guckenheimer, P. Holmes “*Non-linear oscillations, dynamical systems, and bifurcations of vector fields*” (Springer, New York, 1983)
● “*Non-linear Magnetization Dynamics in thin films and nanoparticles*” M d’Aquino PhD Thesis (2004) Univeristy of Napoli, http://wpage.unina.it/mdaquino/index_file/phd_thesis.html
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● R. Kikuchi “*On the minimum reversal time*” J. Appl. Phys. **27**, 1352 (1956)
● W. D. Doyle et al., “*Measurement of the switching speed limit in high coercivitiy magnetic materials*”, IEEE Trans. Mag. **29**, 3634 (1993)
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● J. Miltat “*An introduction to micromagnetics in the dynamic regime*” in Spin Dynamics in Confined magnetic Structures I, Edt B. Hillebrands & K. Ounadjela (Springer 2002)
● Y. Acreman “*Bifurcation in precessional switching*” Appl. Phys. Lett. **79**, 2228 (2001)
● C. Serpico et al., “*Analytical solutions of Landay-Lifschitz equation for precessional switching*”, J. Appl. Phys. **93**, 6909 (2003)

X Some papers for further reading (non exhaustive)

● G. Bertotti et al., "*Comparison of analytical solutions of Landay-Lifschitz equation for 'damping' and 'precessional" switching'*, J. Appl. Phys. **93**, 6811 (2003)

● C. Serpico et al., "*Nonlinear magnetization dynamics and magnetization switching in uniformly magnetized bodies*" J. Mag. Mag. Mat. **290**, 48 (2005)

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● H. W. Schumacher et al., "*Quasiballistic magnetization reversal*", Phys. Rev. Lett. **90**, 017201 (2003)

Domain wall motion under field

● A. Hubert, R. Schäfer (Edts) "*Magnetic Domains: The analysis of magnetic microstructures*" (Springer 1998)

● A. P. Malozemoff and J. C. Slonczewski "*Magnetic domain walls in bubble materials*" Applied Solid State Sciences, Adv. Mater. Dev. Res. (Academic, New York 1979)

● Textbook: S. Chikazumi "*Physics of Ferromagnetism*" (Oxford Univ. Press, Oxford 1997)

● Textbook: A. H. Morrish "*The physical principles of magnetism*" (Wiley, New York 1965)

● N. L. Schreyer, L. R. Walker, "*The motion of 180° walls in uniform dc magnetic fields*", J. Appl. Phys. 45, 5406 (1974)

● U. Ebels et al., "*Small amplitude dynamics of non-homogeneous magnetization distributions: the excitation spectrum of stripe domains*", in Spin Dynamics in Confined magnetic Structures III, Edts B. Hillebrands & K. Ounadjela (Springer 2002)

X Some papers for further reading (non exhaustive)

Spin transfer torque

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● L. Berger "*Emission of spin waves by a magnetic multilayer traversed by a Current*" Phys. Rev. B **54**, 9353 (1996)
● M. D. Stiles et al., "*Anatomy of spin-transfer torque"* Phys. Rev. B **66**, 014407 (2002)
● M. Stiles and J. Miltat, "*Spin transfer torque and dynamics*". In Spin Dynamics in Confined Magnetic Structures III, Edt B. Hillebrands & A Thiaville (Springer Berlin / Heidelberg, 2006).
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● M. Tsoi, et al., "*Excitation of a magnetic multilayer by an electric current*", Phys. Rev. Lett. **80**, 4281 (1998).
● J. A. Katine et al., "*Current-driven magnetization reversal and spin-wave excitations in Co/Cu/Co pillars*", Phys. Rev. Lett. **84**, 3149 (2000).

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● S. I. Kiselev et al., "*Microwave oscillations of a nanomagnet driven by a spin-polarized current*", Nature **425**, 380 (2003).
● W. H. Rippard et al., *"Direct current induced dynamics in Co90Fe10/Ni80Fe20 point contacts*", Phys. Rev.Lett. **92**, 027201 (2004).
● T. J. Silva, W. H. Rippard, "*Developments in nano-oscillators based upon spin-transfer point-contact devices*» **320**, 1260 (2008)
● Q. Mistral, "*Current-driven microwave oscillations in current perpendicular-to-plane spin-valve nanopillars*" Appl. Phys. Lett. **88**, 192507 (2006)

X Some papers for further reading (non exhaustive)

● Z. Li and S. Zhang, “*Magnetization dynamics with a spin-transfer torque*”, Phys. Rev. B. **68,** 024404 (2003).

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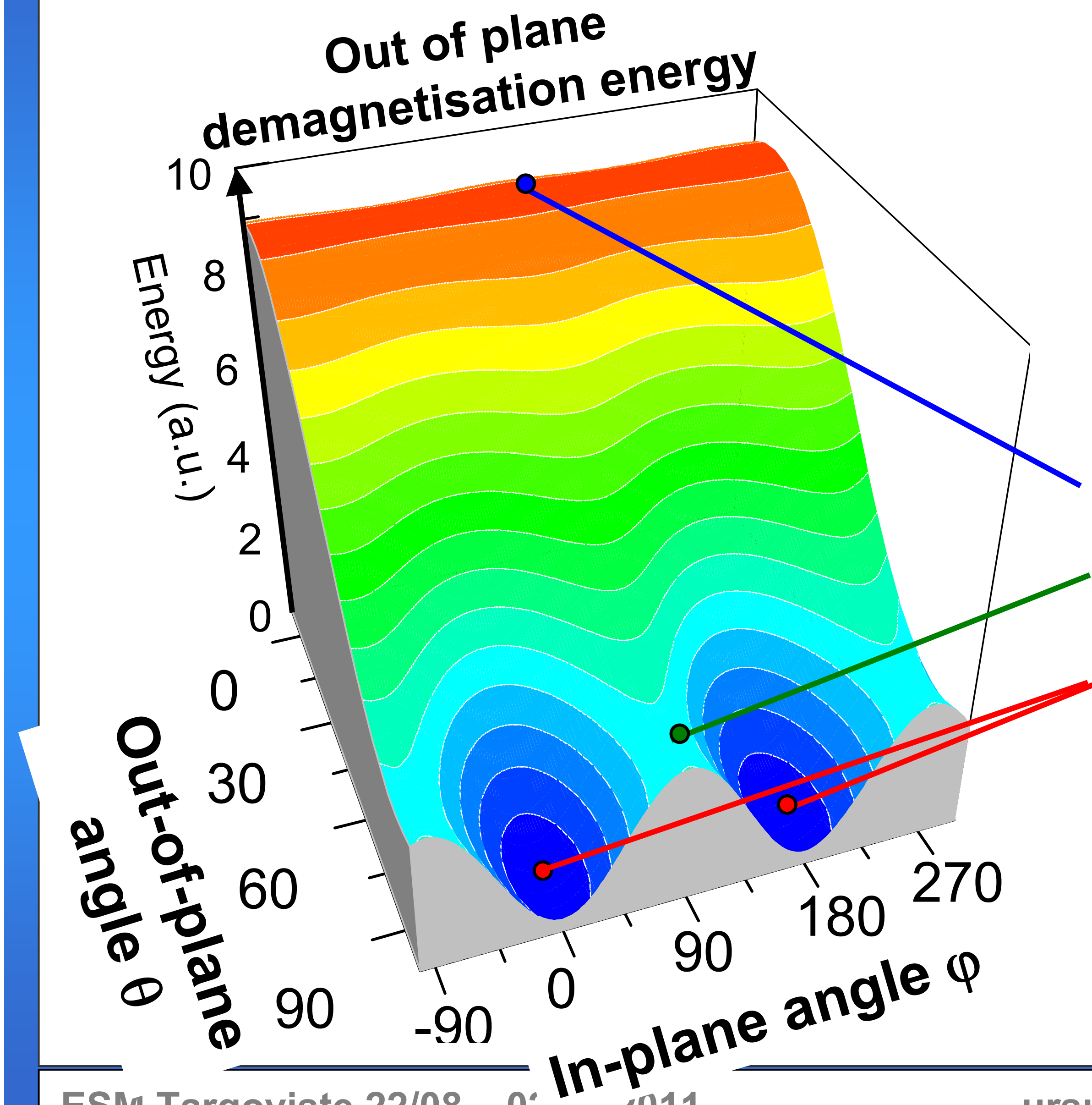
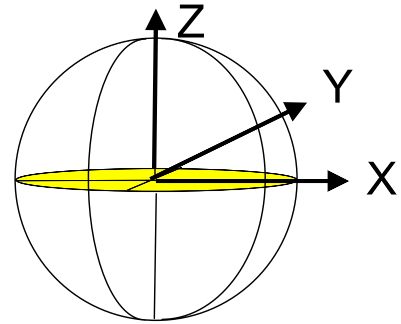
The END

VI ST Precession - Review of Conservative Dynamics



Uniaxial thin film



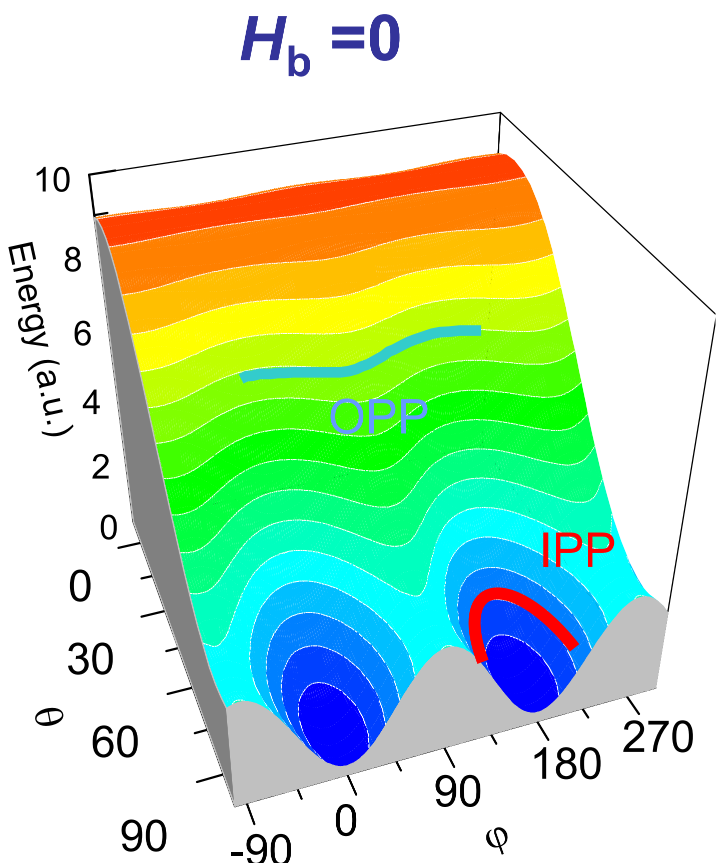
H_u, H_b =0

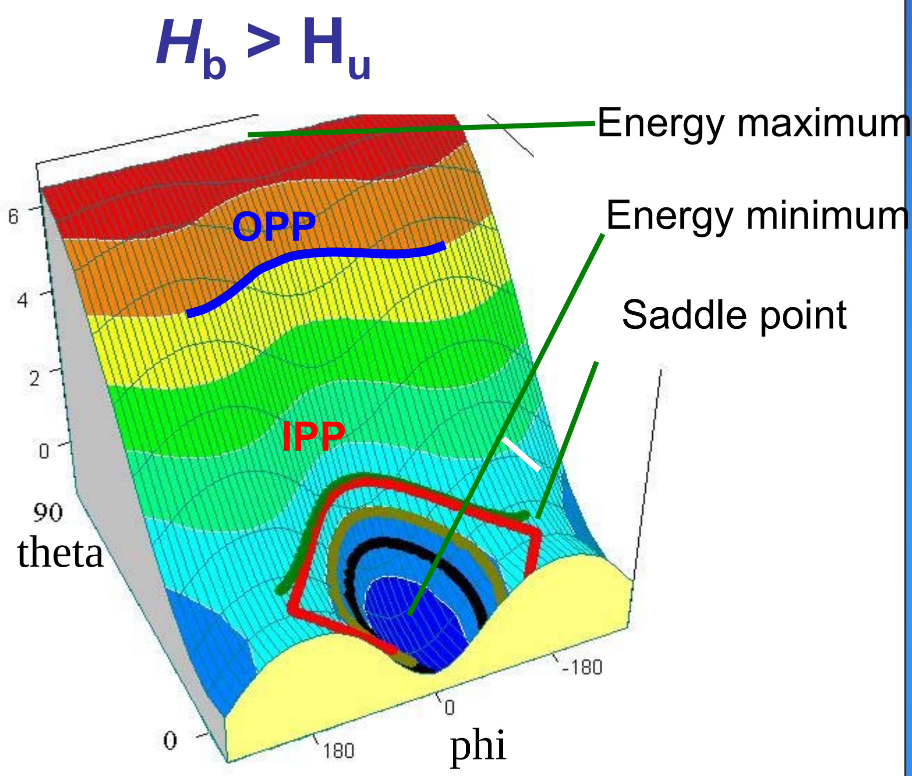
Energy maximum

Saddle point

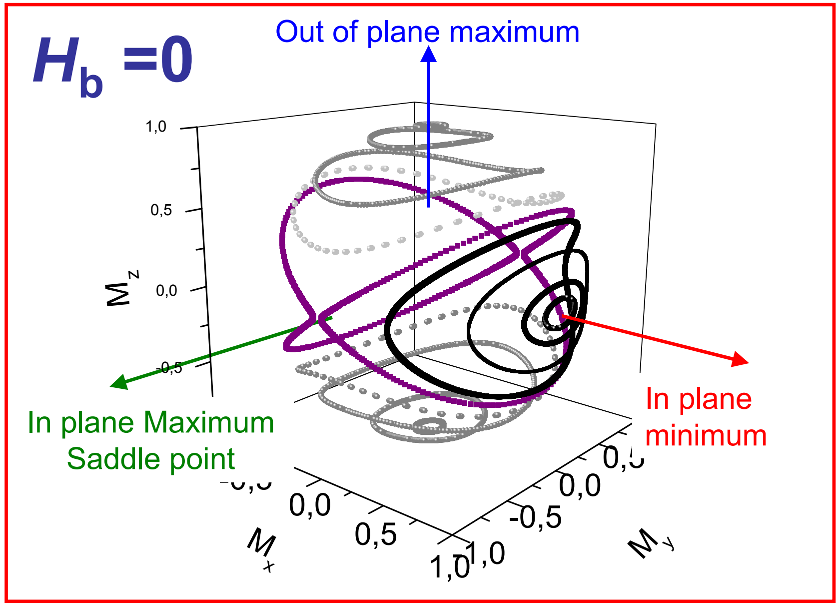
Energy minima

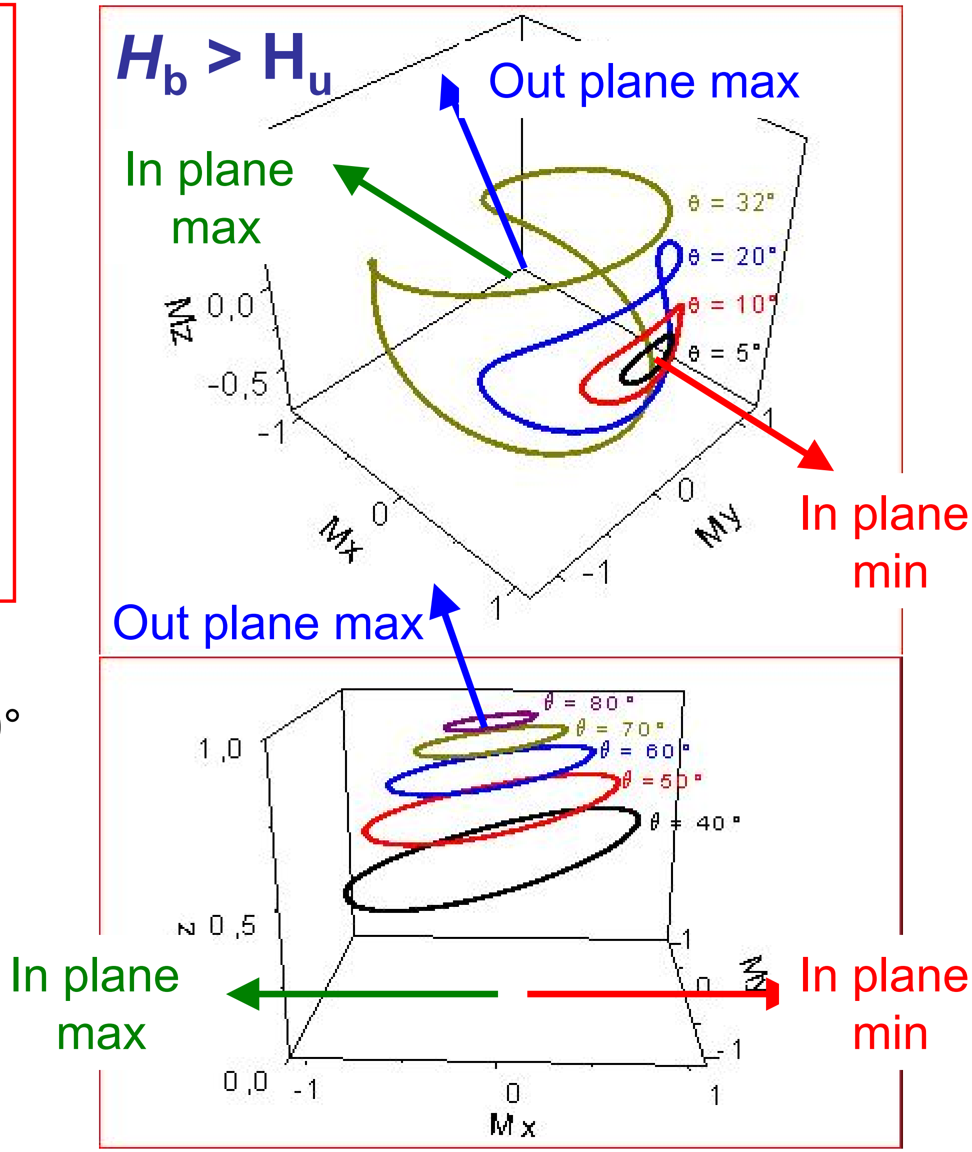
VI ST Precession - Review of Conservative Dynamics





2 types of constant energy trajectories
IPP around energy minimum
OPP around energy maximul





H_b =0

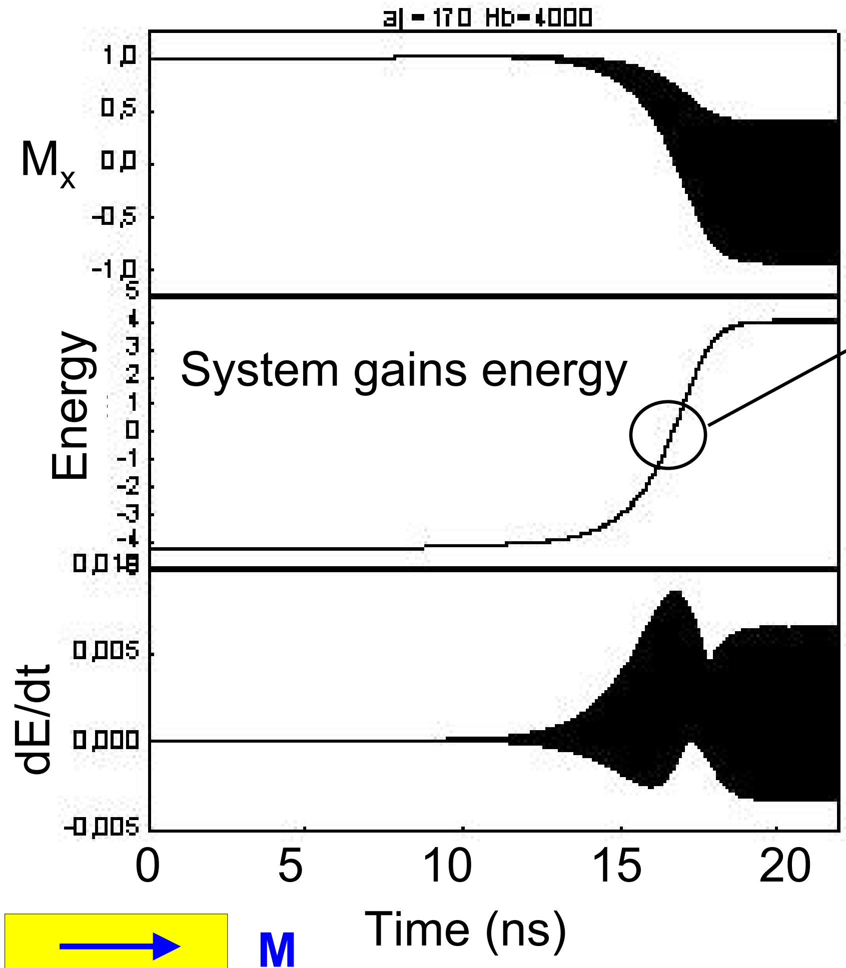
Maximum in plane excursion $\varphi_{max} \leq 90°$

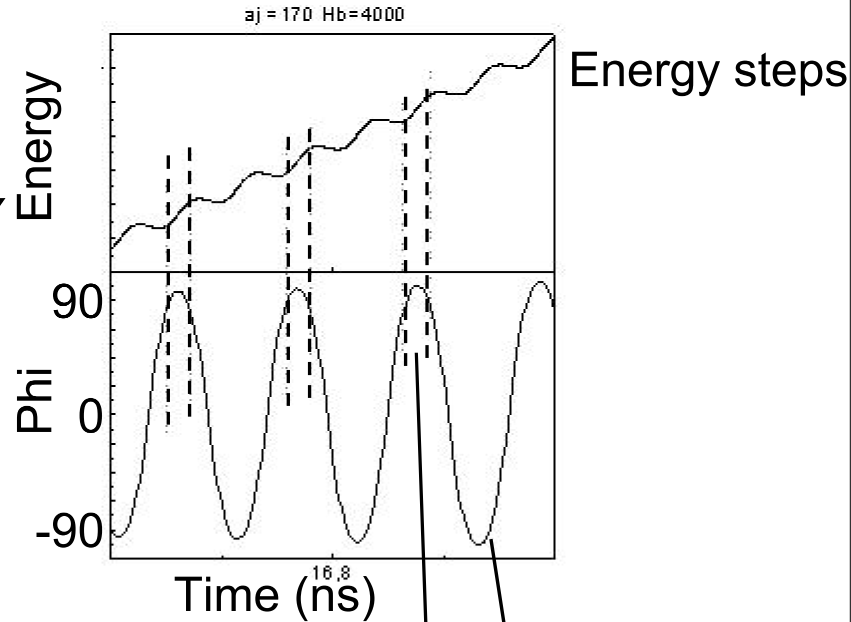
H_b > 0

$0<H_b<H_u$ $\cos\varphi_{max}=-H_b<H_u$

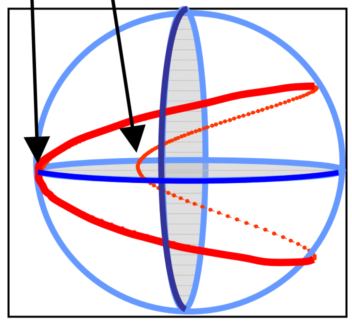
$H_b>H_u$ $\varphi_{max}=180°$

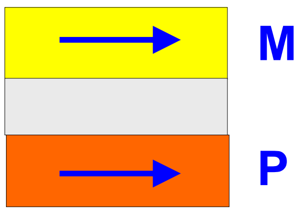
VI ST Precession - Transient to Limit Cycle



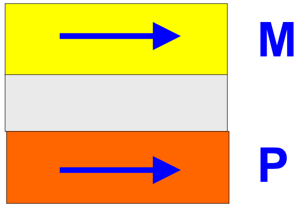


Energy gain at turning point φ_{max} and $\theta \approx 0$





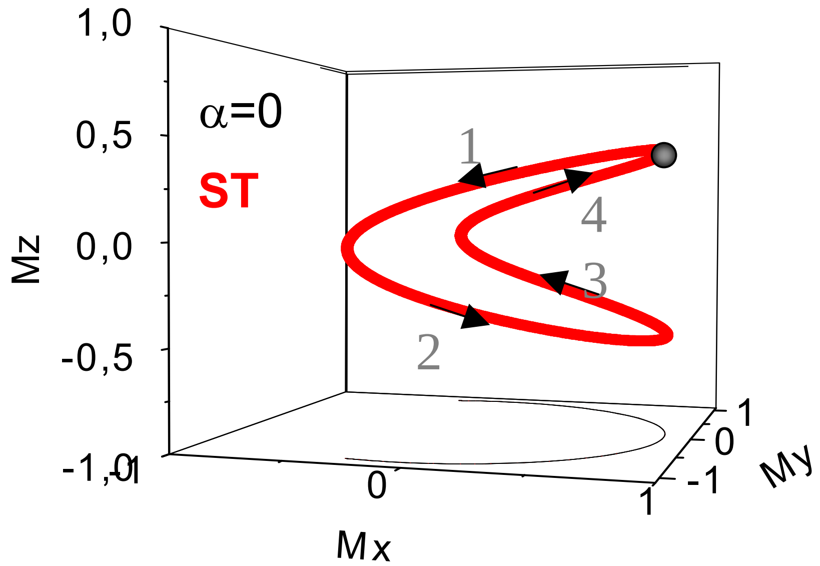
VI ST Precession - Limit Cycles

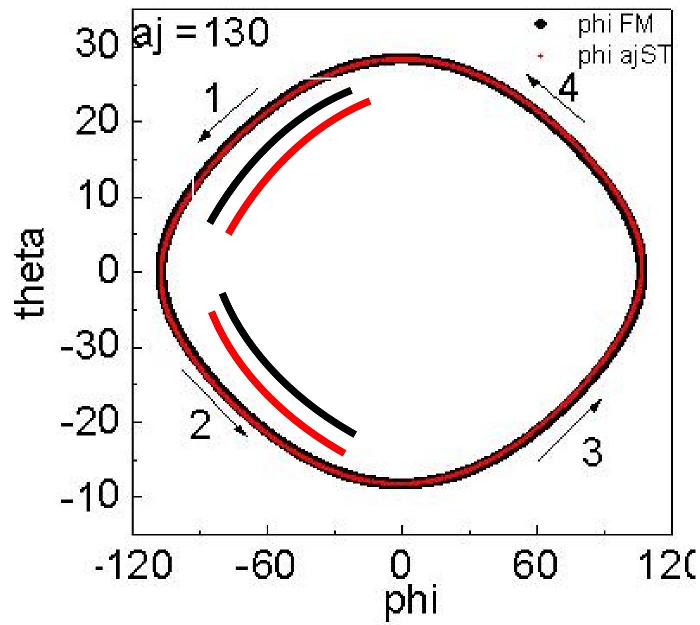


Constant energy trajectory vs limit cycle

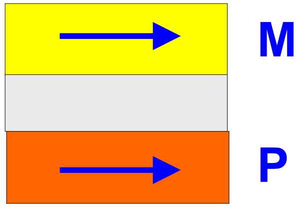
Planar polarizer

IPP very close to constant energy trajectory but not identical





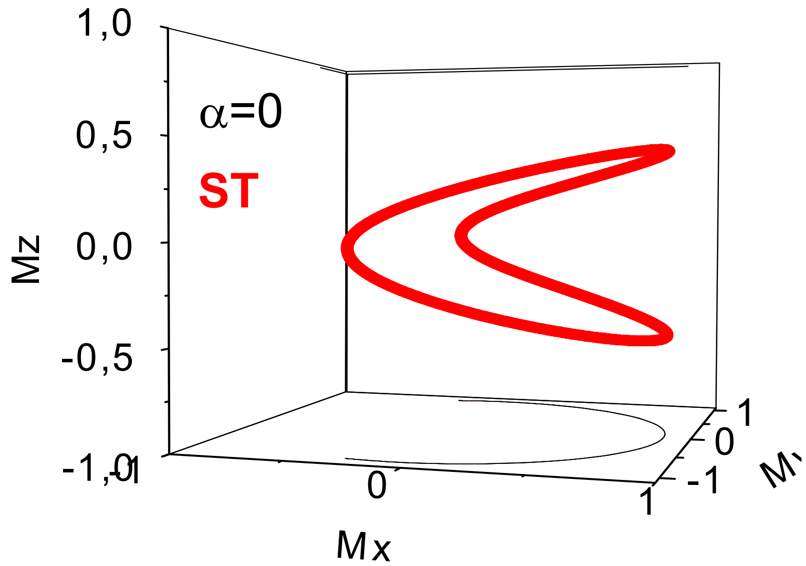
Spin transfer and damping torque cancel only on average but not at each point along the trajectory

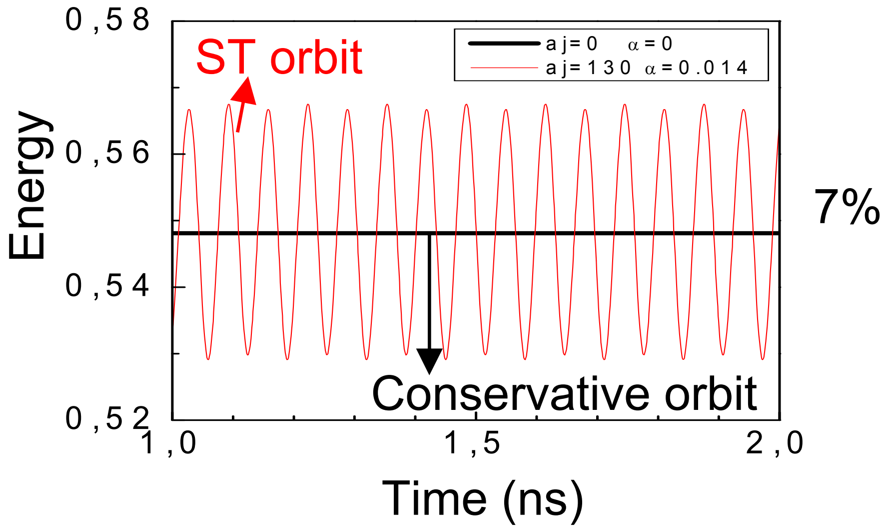


Constant energy trajectory vs limit cycle

Planar polarizer

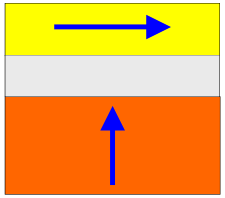
IPP very close to constant energy trajectory but not identical



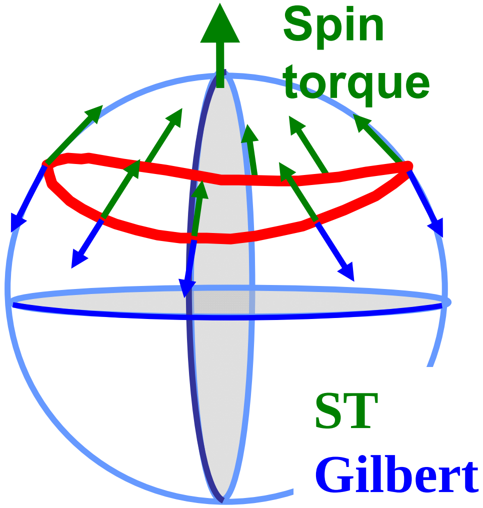


Spin transfer and damping torque cancel only on average but not at each point along the trajectory

VI ST Precession - Limit Cycles

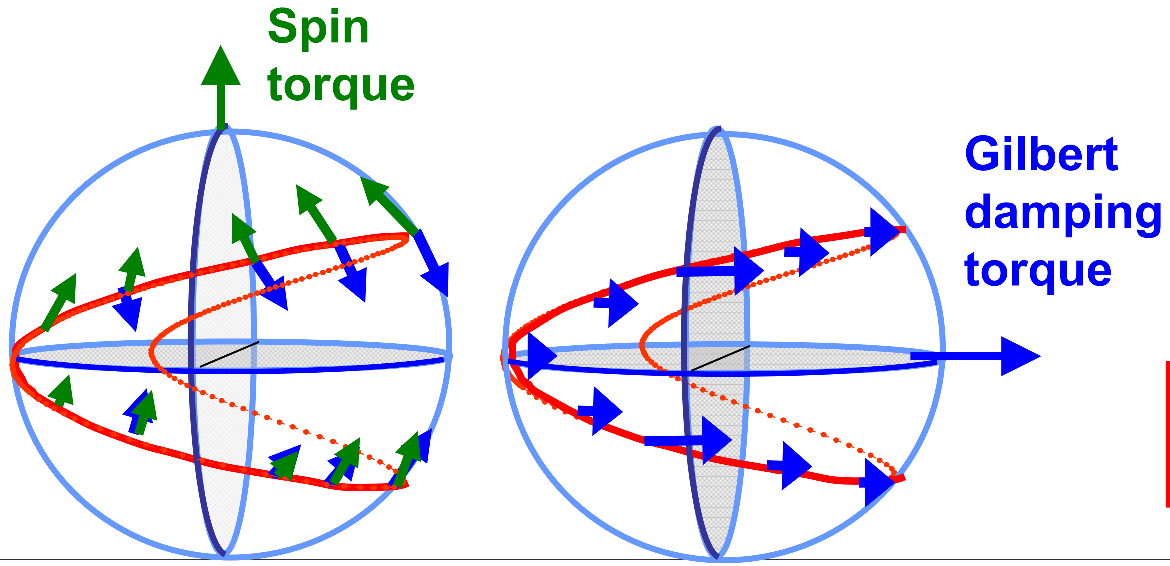


Example perpendicular polarizer



OPP stabilized by perpendicular polarizer

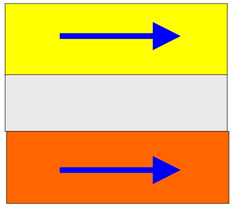
$$m_z \approx \frac{a_j}{\alpha 4\pi M_s} \propto \frac{J}{\alpha 4\pi M_s}$$



IPP not stabilized by perpendicular polarizer

theta component phi component

VI ST Precession - Limit Cycles

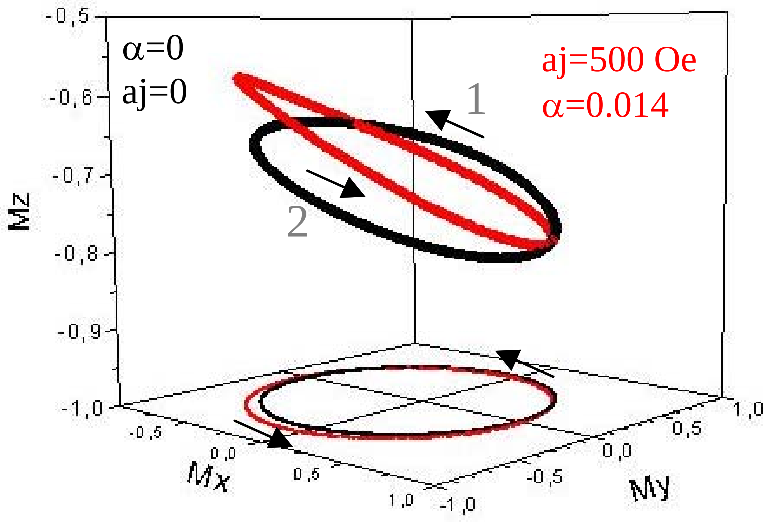


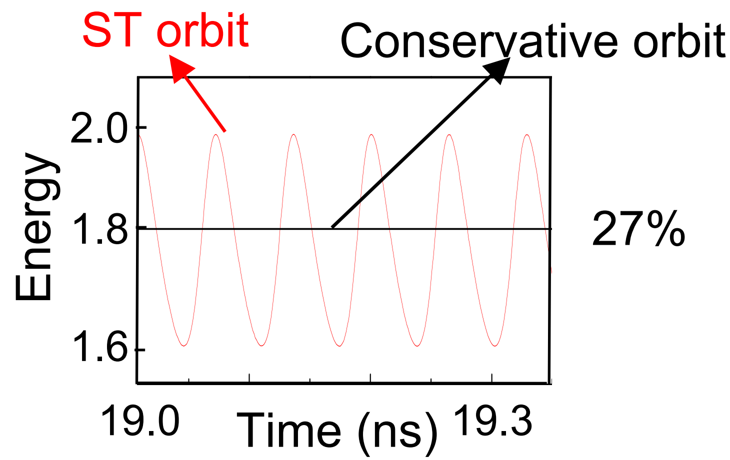
Constant energy trajectory vs limit cycle

Planar polarizer

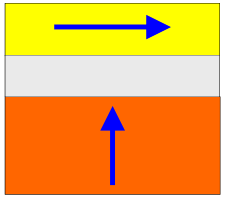
OPP can be stabilized but they are different from constant energy trajectory

OPP Planar polarizer

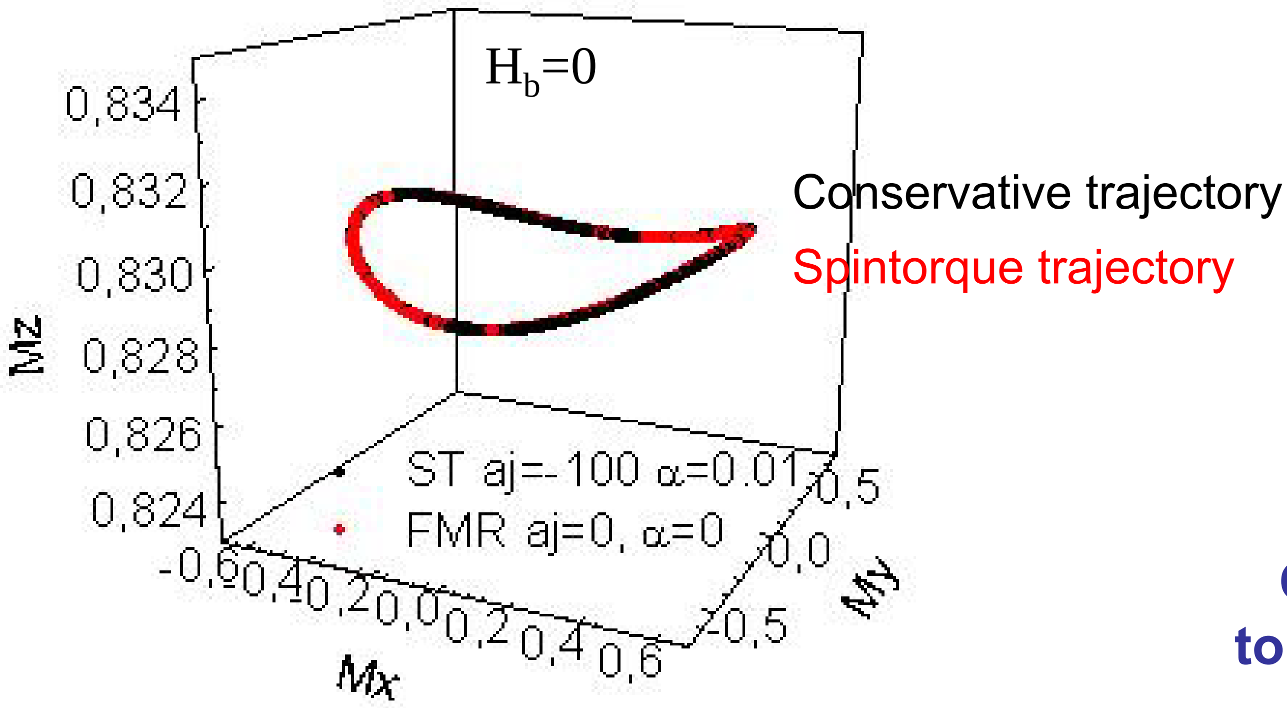




Constant energy trajectory vs limit cycle



Perpendicular polarizer



OPP very close to constant energy trajectory

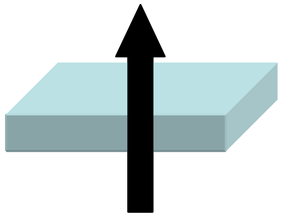
Energy oscillation: 0.03%

Qualitative Explanation of the nonlinear frequency shift

A. Slavin, V. Tiberkevich, IEEE Trans. Mag. 2009

$$M_z = \sqrt{M_0^2 - m_\perp^2} \approx M_0 (1 - |c|^2)$$

FMR, normal magnetization:



N > 0 - blue frequency shift

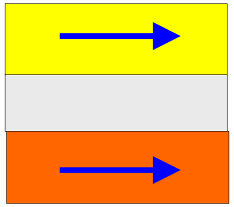
$$\omega = \gamma(H_0 - 4\pi M_z) = \gamma(H_0 - 4\pi M_0) + 4\pi\gamma M_0 |c|^2 = \boxed{\omega_0 + |N||c|^2}$$

FMR, in-plane magnetization:

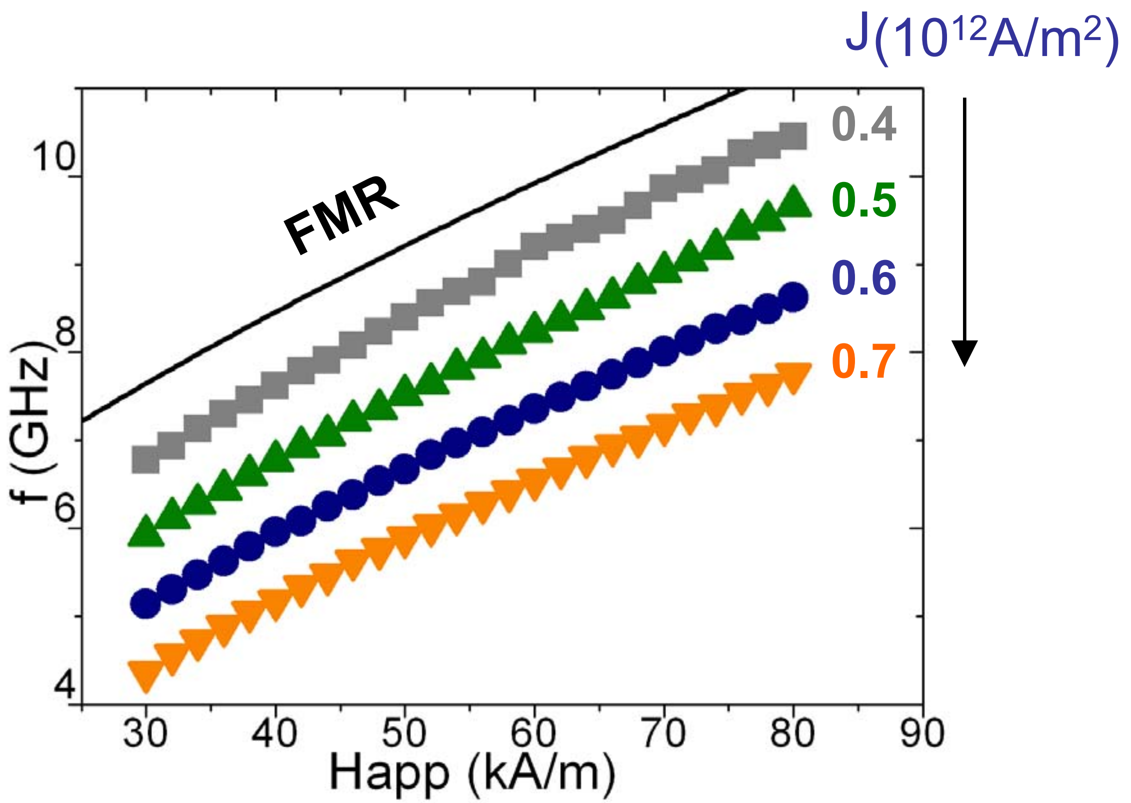
N < 0 – red frequency shift

$$\omega = \gamma\sqrt{H_0(H_0 + 4\pi M_z)} \approx$$

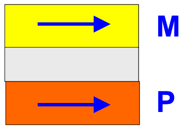
$$\approx \gamma\sqrt{H_0(H_0 + 4\pi M_0)} - 2\pi\gamma M_0 \sqrt{\frac{H_0}{H_0 + 4\pi M_0}} |c|^2 = \boxed{\omega_0 - |N||c|^2}$$



Frequencies for IPP modes vs Field



VI ST Precession - Frequencies



Non-sinusoidal IPP modes

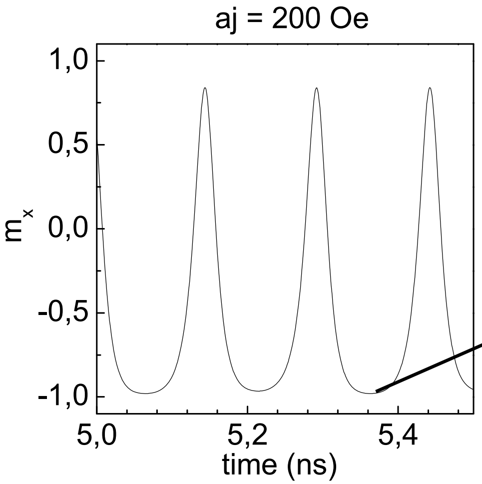
- Precession term in spherical coordinates

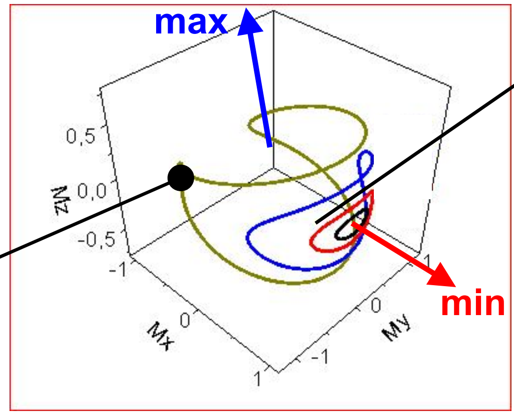
$$\frac{d\varphi}{dt} = \frac{-\gamma}{M\sin\theta}\frac{dE(\theta,\varphi)}{d\theta} \quad ; \quad \frac{d\theta}{dt} = \frac{\gamma}{M\sin\theta}\frac{dE(\theta,\varphi)}{d\varphi}$$

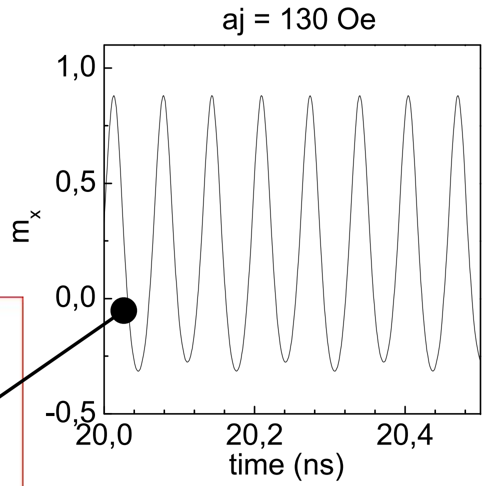
- Angular velocity $dE/d\theta$ decreases at turning point

→ Non-sinusoidal trajectories

Strong variation of H_{eff} along trajectory







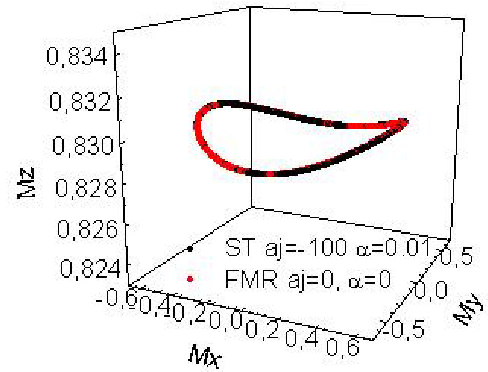
Frequencies for OPP modes vs current J

On OPP trajectory the demagnetization energy dominates

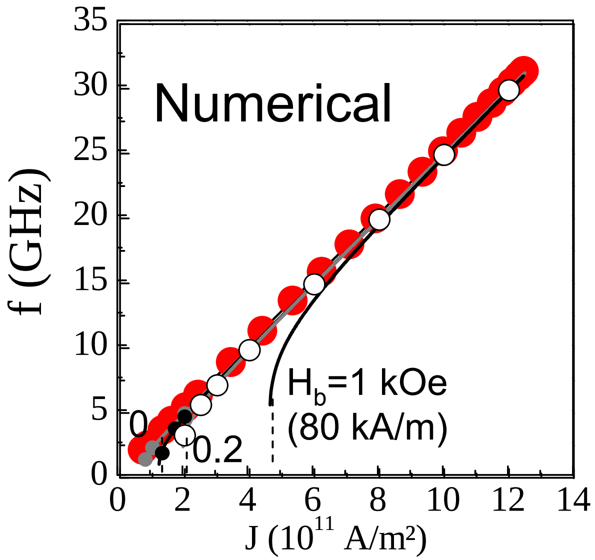
Conservative trajectory

$$\omega = \gamma H_d = \gamma 4\pi m_z$$

$$f \approx \frac{\gamma}{2\pi} 4\pi M_s \cos\theta$$



STT OPP trajectory of perpendicular polarizer



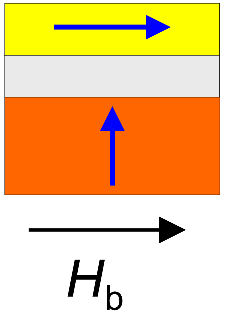
approximation

$$\cos\theta \approx \frac{a_j/\alpha}{4\pi M_s}$$

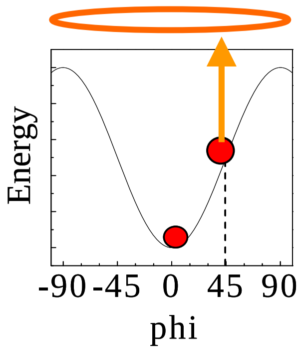
$$f \approx \frac{\gamma}{2\pi}\frac{a_j}{\alpha} \sim J$$

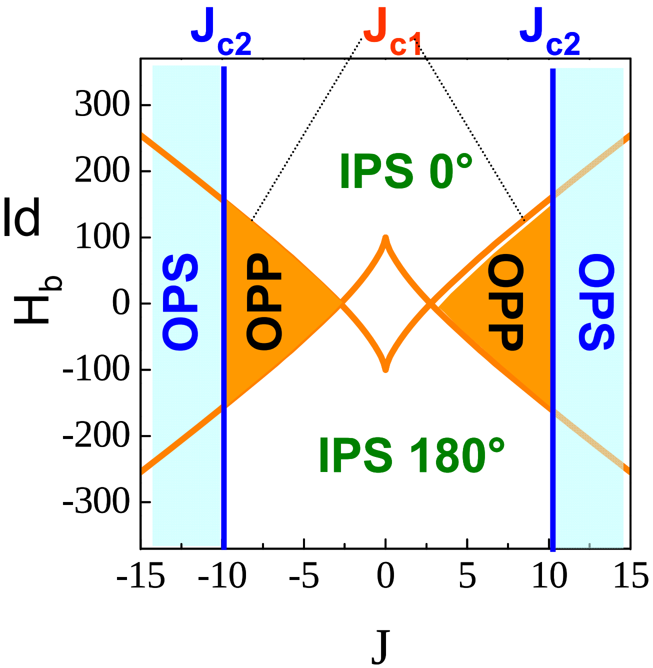
VI ST Precession – State Diagram

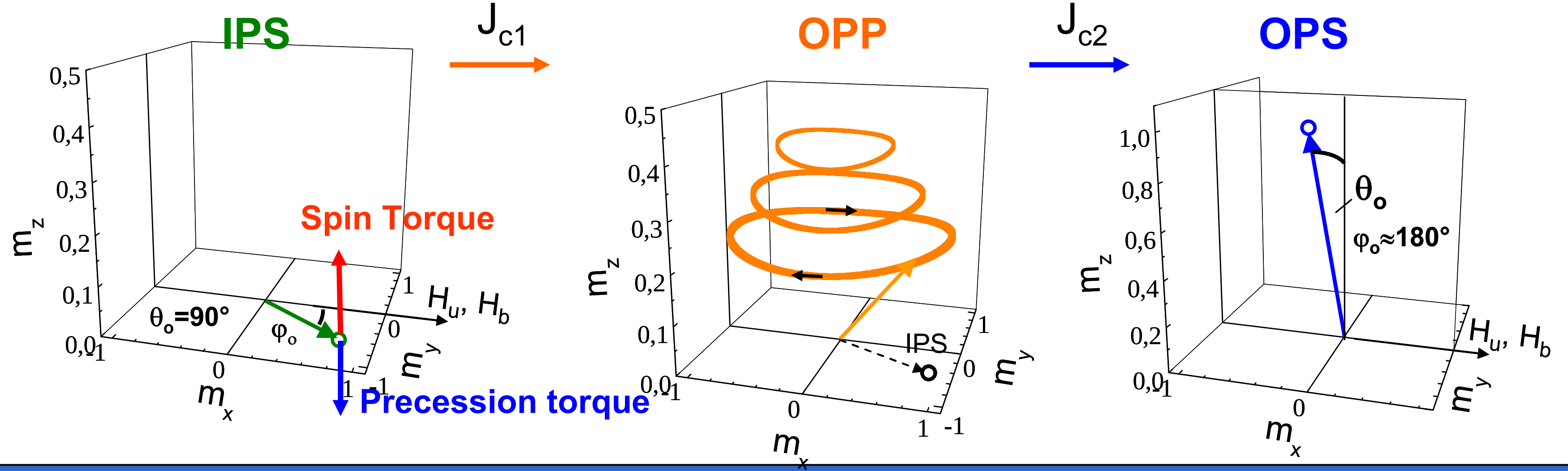
Example perpendicular polarizer



State Diagram
With in plane bias field

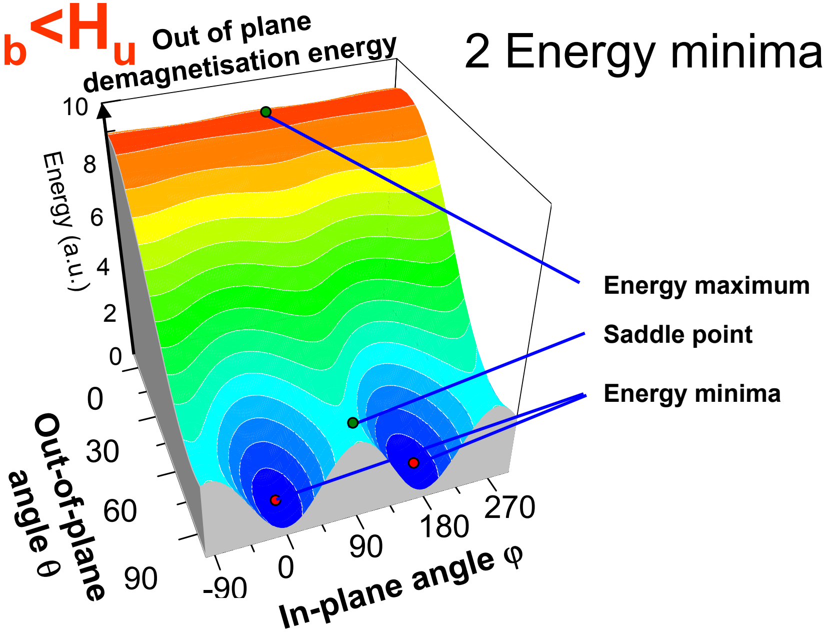






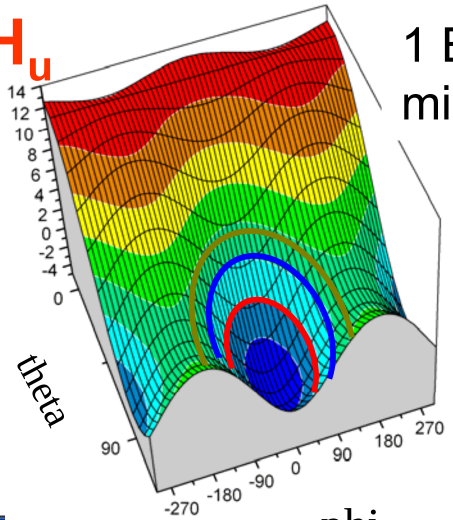
VI ST Precession – State Diagram

$H_b<H_u$



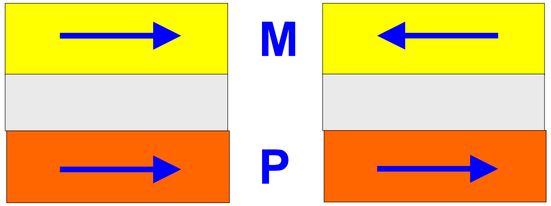
2 Energy minima

$H_b>H_u$



1 Energy minimum

Example planar polarizer



Parallel Antiparallel

Saddle point
$H_b<H_u$ unstable
$H_b>H_u$ stable/unstable
depending on sign and strength of J

Energy Maximum unstable

« New » states

Canted states
All components M_x, M_y, $M_z \neq 0$ or 1

→Equilibrium between
Spin and Precession torque

VI ST Precession - Non-linear Spin wave theory

Transformation of LLGS

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) + \gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

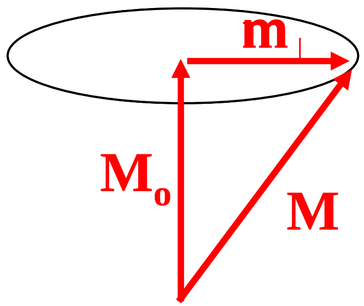
Precession **Damping** **Spin Torque**

$$\frac{\partial c}{\partial t} = i\left(\omega_o + N\left|c^2\right|\right)c + \Gamma_o\left(1+Q_o\left|c^2\right|\right)c - \Gamma_s\left(1-Q_s\left|c^2\right|\right)c$$

Non-linear contributions ~ **N**, **Q_o**, **Q_s**

c complex spin wave amplitude ~ $\mathbf{m}_\perp$

$$c(t) = \left|c(t)\right| e^{i\phi(t)}$$



Rezende et al, PRB 73, 094402 (2006);
Slavin et al, IEEE Trans Mag 41, 1264 (2005)
Kim et al, PRL 100, 017207 (2008)

VI ST Precession - Non-linear Spin wave theory

Oscillator Equation

$$\frac{\partial c}{\partial t} = i\left(\omega_o + N|c^2|\right)c + \Gamma_o\left(1 + Q_o|c^2|\right)c - \Gamma_s\left(1 - Q_s|c^2|\right)c$$

Precession **Damping** **Spin Torque**

(Damping + Spin Torque) = 0 for steady state

$$|c|^2 = \frac{\Gamma_s - \Gamma_o}{\Gamma_s Q_s + \Gamma_o Q_o} \Rightarrow J_c = \frac{\Gamma_o}{\sigma}$$

Non-linear contributions ~ **N**, **Q_o**, **Q_s**

Linear contributions

$\Gamma_s = \sigma J$

$\Gamma_o \sim \alpha$

Finite amplitude $|c|$ due to non-linear contributions Q_o, Q_s

Rezende et al, PRB 73, 094402 (2006);
Slavin et al, IEEE Trans Mag 41, 1264 (2005)
Kim et al, PRL 100, 017207 (2008)

Stability of Trajectories

$$\frac{\partial c}{\partial t} = i\left(\omega_o + N\left|c^2\right|\right)c + \Gamma_o\left(1 + Q_o\left|c^2\right|\right)c - \Gamma_s\left(1 - Q_s\left|c^2\right|\right)c$$

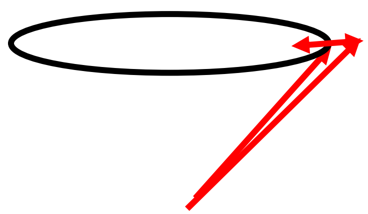
Precession **Damping** **Spin Torque**

Non-linear contributions ~ **N**, **Q_o**, **Q_s**

$$c(t) = \left|c(t)\right| e^{i\phi(t)}$$ p=|c|² $p = p_o + \delta p$

Amplitude relaxation rate Γ_p

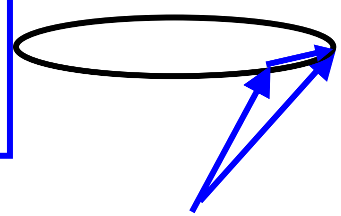
$$\frac{d\delta p}{dt} + 2\Gamma_p \delta p = h_n^{\delta p}(t)$$



$$\delta p(t) \sim e^{-2\Gamma_p |t|}$$

Timescale of ampl. fluctuations

$$\frac{d\phi}{dt} = -\omega_g t + h_n^{\varphi}(t) - N\delta p$$



Phase not bounded
Fluctuates due to amplitude noise

$h_n(t)$ noise or external force

VI ST Precession - Microwave Oscillators

VCO | **STO**

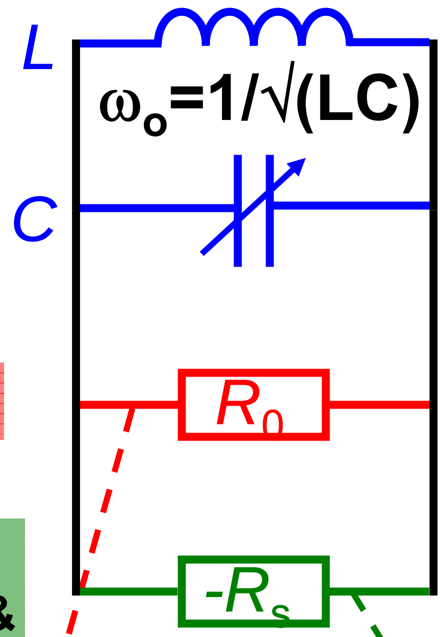
Resonator

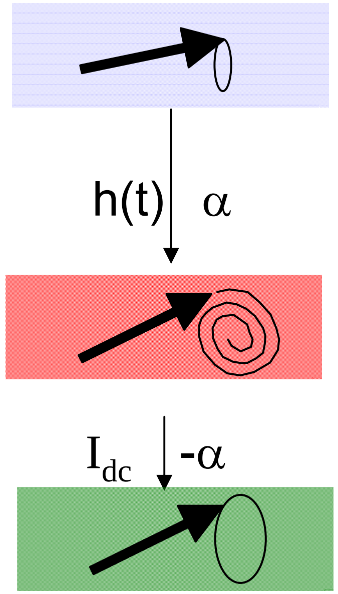
Attenuated Resonator

Auto-Oscillator
Attenuated Resonator & Energy feedback

↓

Active element to supply external energy and compensate energy losses





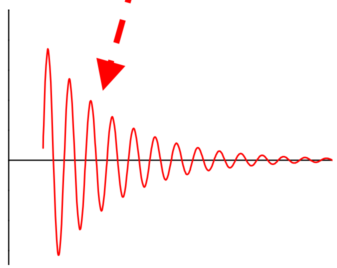
$$\frac{d\mathbf{M}}{dt} = -\gamma(\mathbf{M} \times \mathbf{H}_{eff})$$

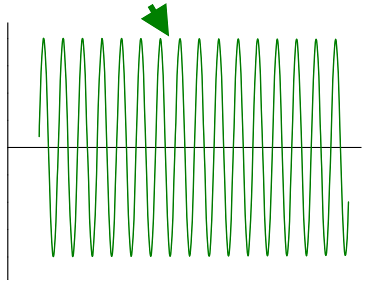
+

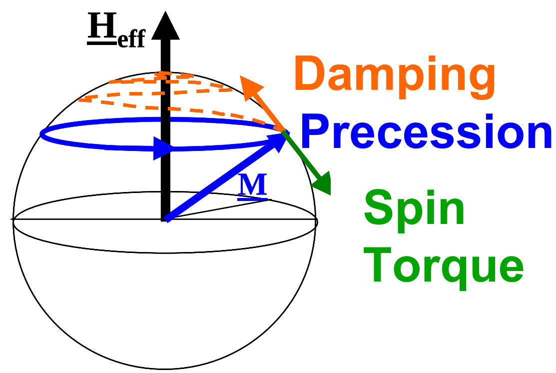
$$\frac{\alpha}{M_s}\left(\mathbf{M} \times \frac{d\mathbf{M}}{dt}\right)$$

+

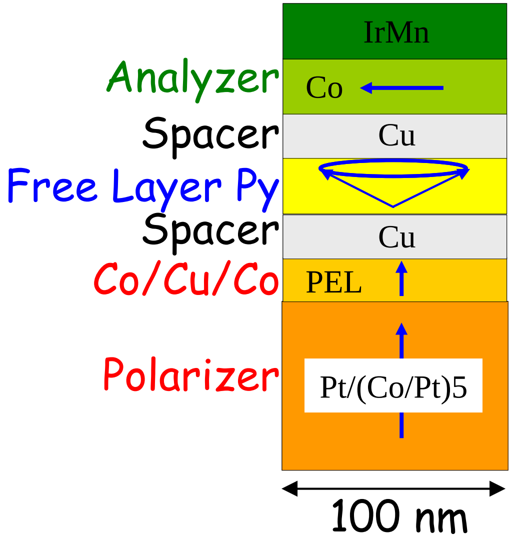
Spin Transfer

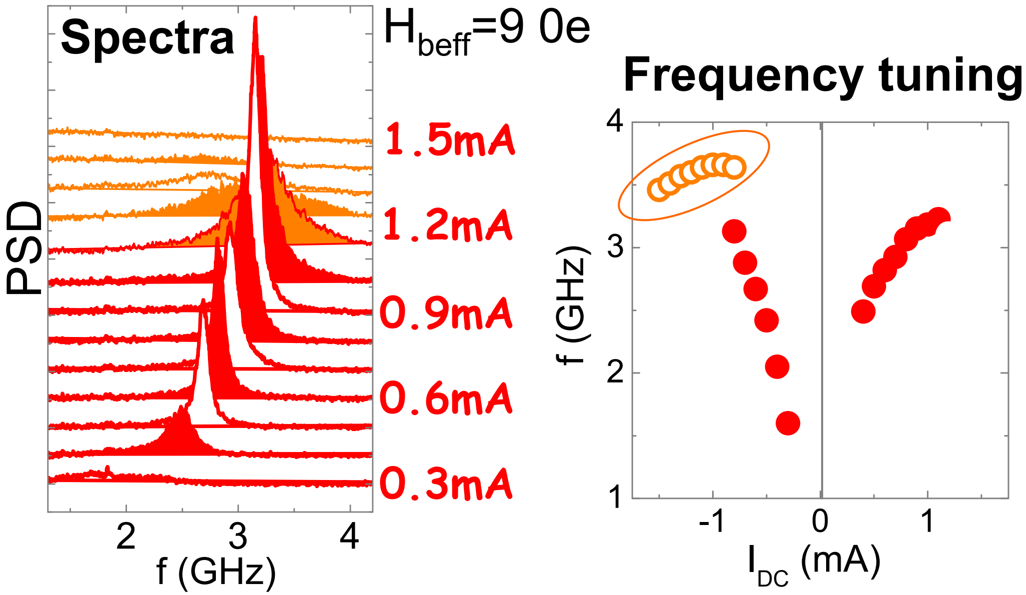






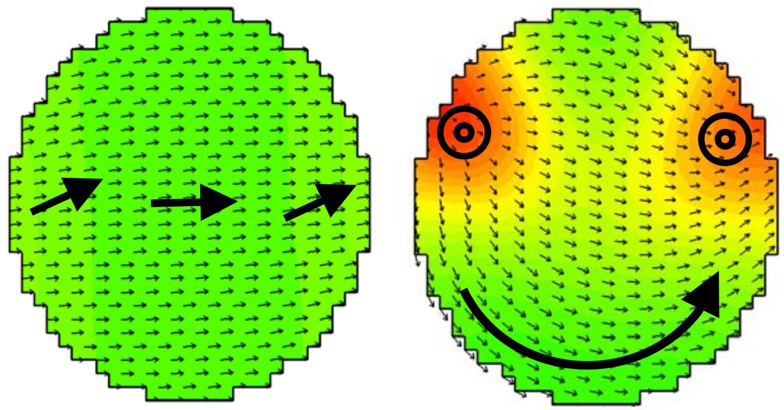
Example: Perpendicular Polarizer





Characteristics of OPP mode

- Frequency Blueshift
- Saturation of Frequency due to non homogeneous magnetization



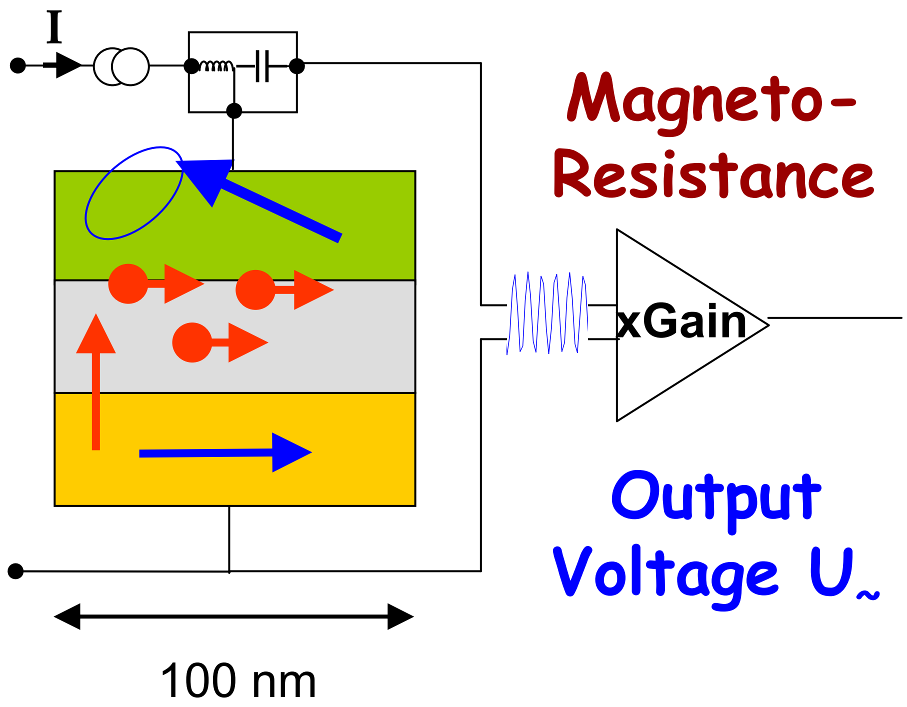
Excitation

Readout

Spin momentum transfer

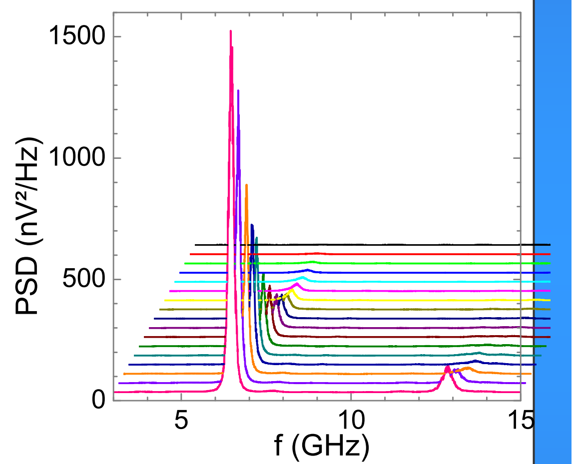
↓

Auto-oscillations of M



Magneto-Resistance

Output Voltage U~

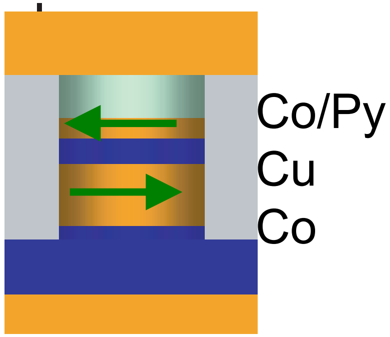


Nanoscale Tuneable Microwave Oscillator

Spin Transfer Nano-Oscillator STNO

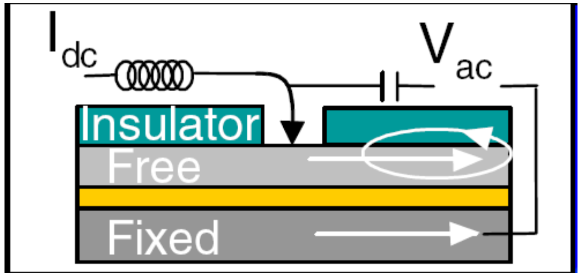
VI ST Precession - Microwave Oscillators

Nanopillars NP

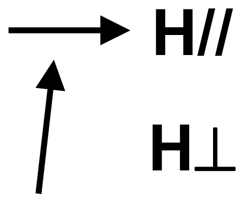


Cornell

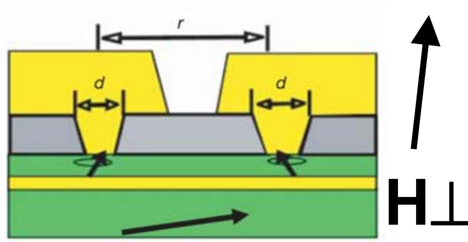
Nanocontacts NC



NIST

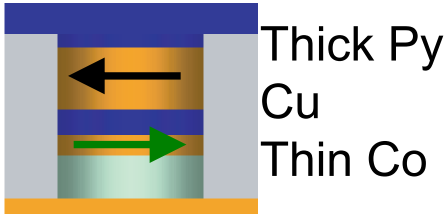


NC Coupling

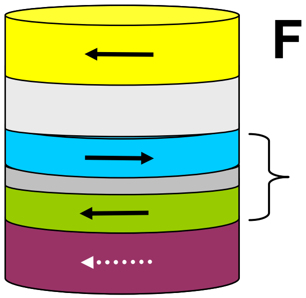


NIST

« Wavy » NP



CNRS/Thales

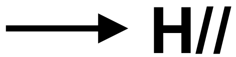


FL Oscillation → IEF

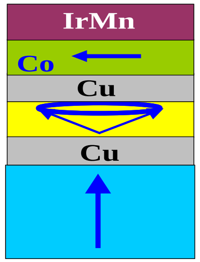
SAF Oscillation

SPINTEC/LETI

IEF

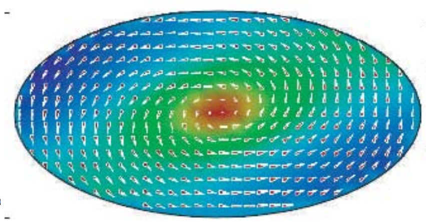


Perpendicular Pol

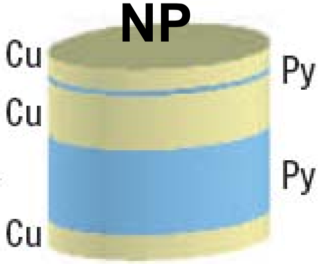


Spintec/LETI

Vortex

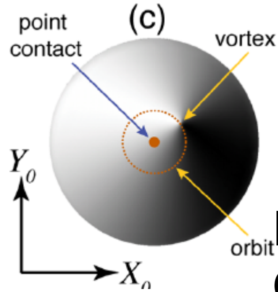


NP



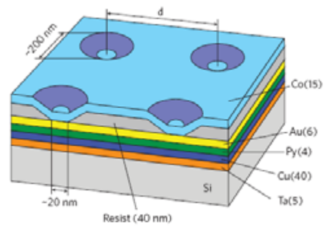
Cornell

NC



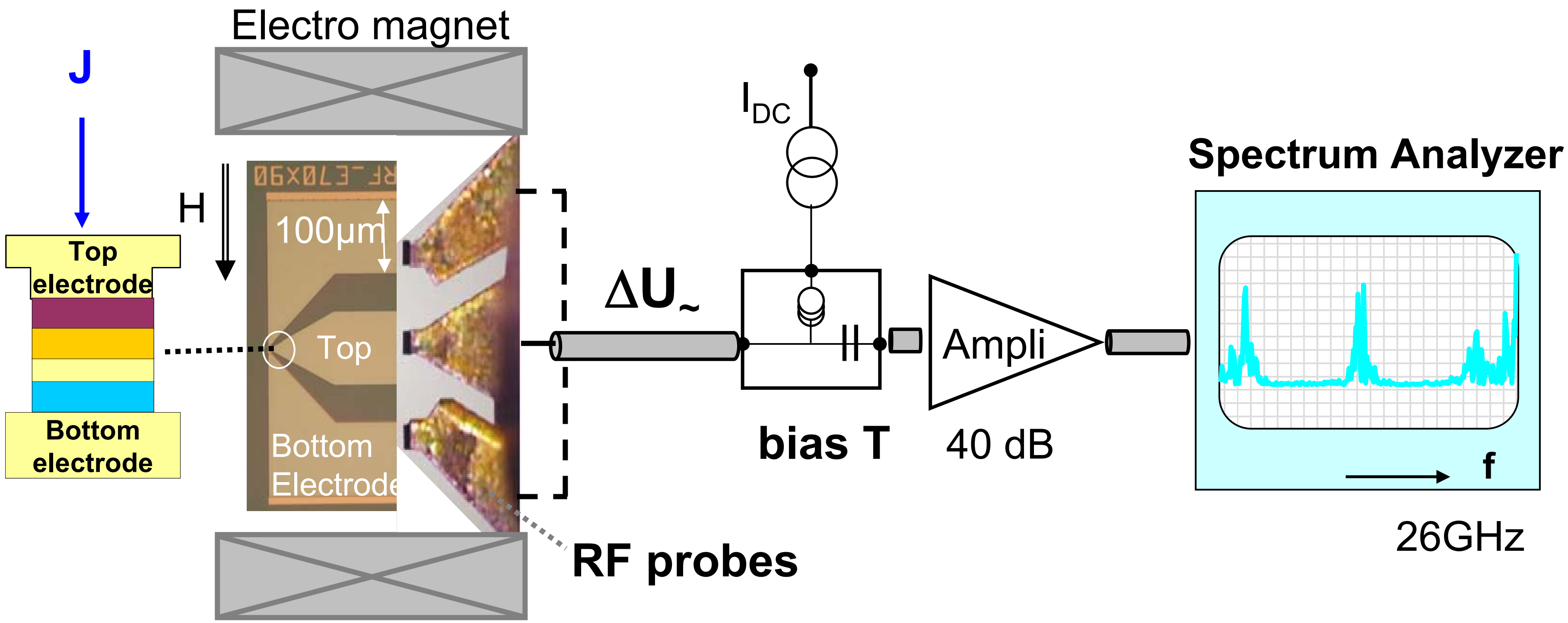
IEF/IMEC

CNRS/Thales



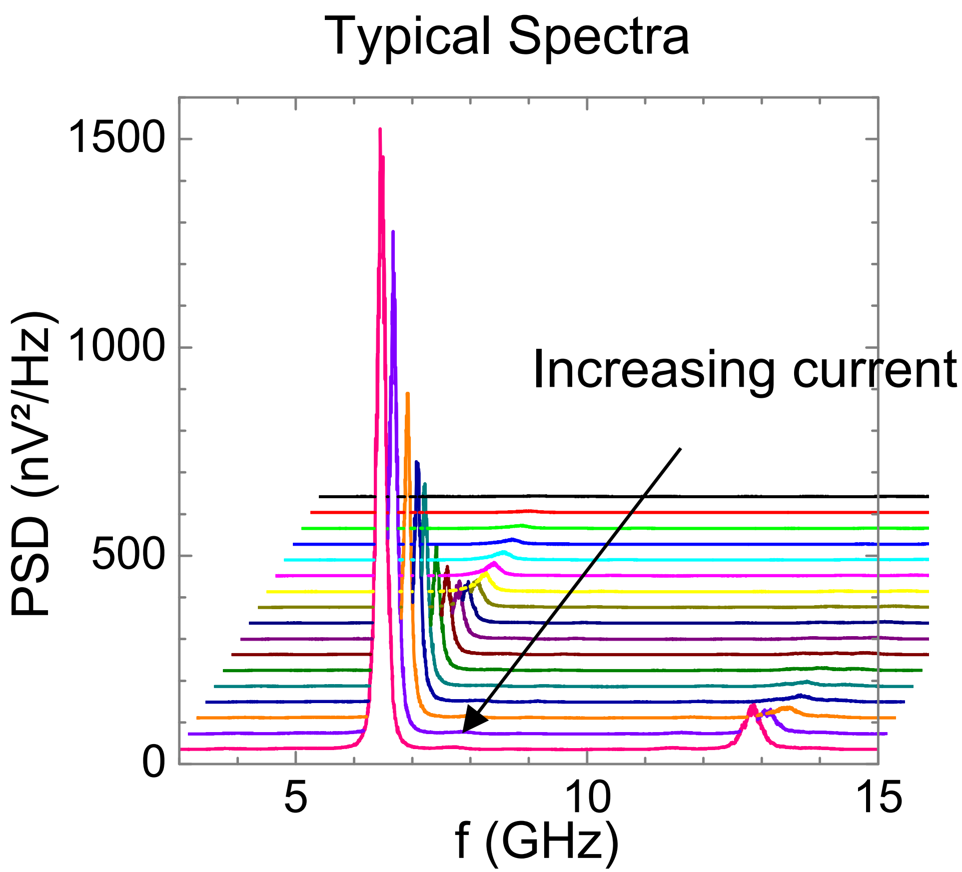
CNRS/Thales

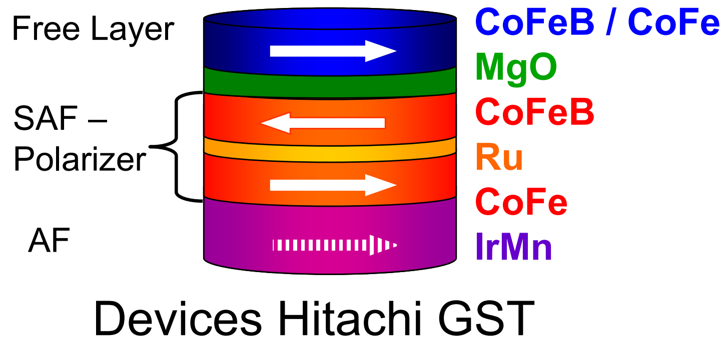
VI ST Precession - Microwave Oscillators



VI ST Precession - Microwave Oscillators

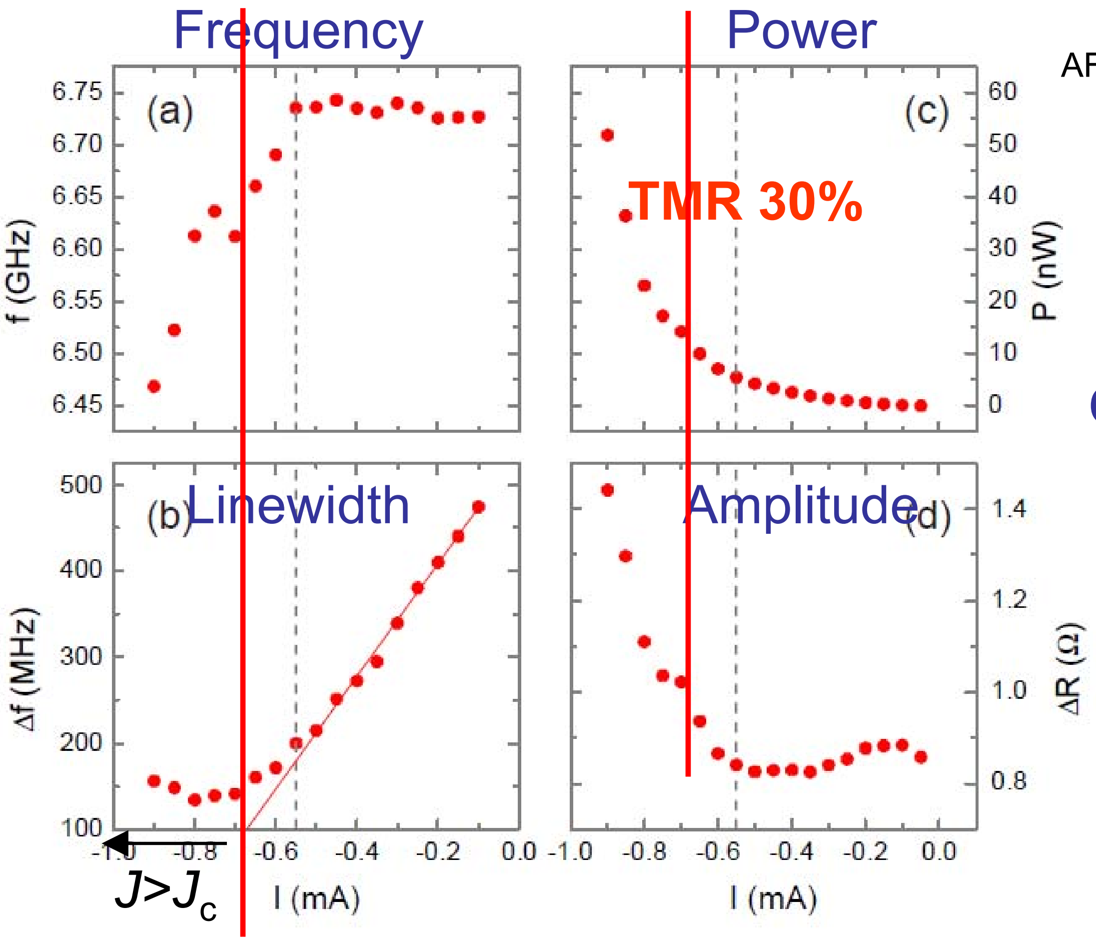
Example: Planar Tunnel Junction

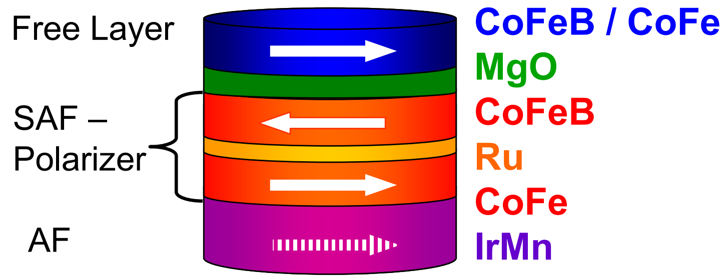




$\longrightarrow H_b$

Example: Planar Tunnel Junction





Devices Hitachi GST

H_b

Characteristics of IPP mode

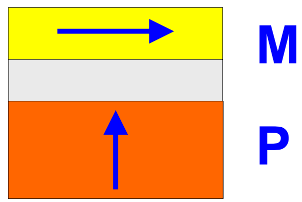
- Frequency Redshift
- Decrease of Linewidth
- Increase of Power

$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}_{\text{Damping}} + \underbrace{\gamma\frac{a_j}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)}_{\text{Spin torque}}$$

- The LLGS equation corresponds to a non-linear dynamical system that can be solved like the LLG and the precession term
- The solutions are equilibrium states and periodic orbits
- The stability of equlibrium states is analyzed via linearization
- The stability depends on the control parameters *J*, **P**, $\mathbf{H}_b$
- The stable can be the same as in the (non) conservative case, but there can also exist new states
- Above a critical current J_c, the stable states become unstable and the magnetization precesses on limit cycles. These are independent of initial condition
- The limit cycles are close to constant energy trajectories when **P** // symmetry axis and the analysis of the limit cycle properties follows the one of the conservative dynamics

LinearizatioN and frequencies and non-linearity
Meaning of non-linearity
FMR and constant energy trajectories (difference)
Initial conditions and amplitude

Example perpendicular polarizer



Apply current pulse of long duration Δt > 1ns