Large Angle Precessional Magnetization Dynamics

under Field and Current Excitations

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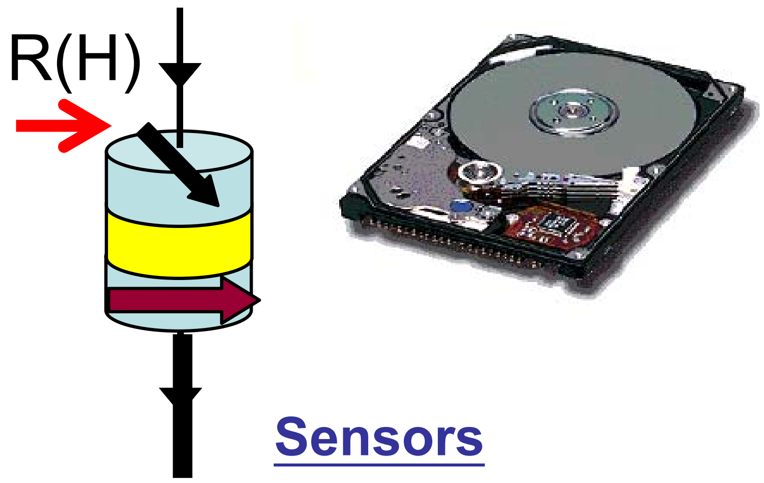
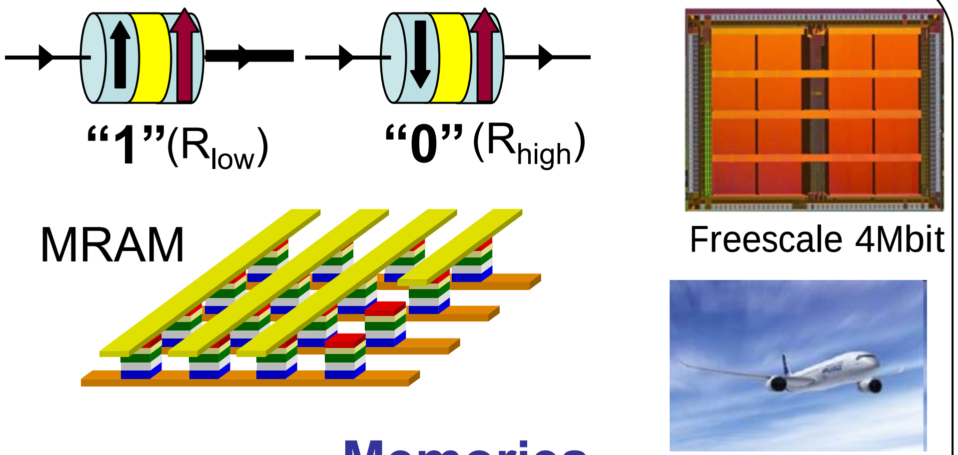
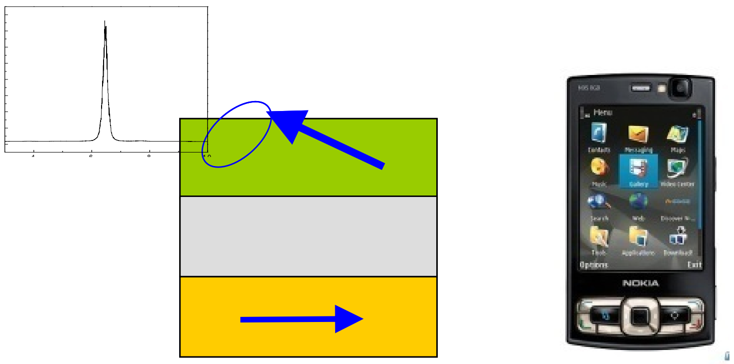
Introduction



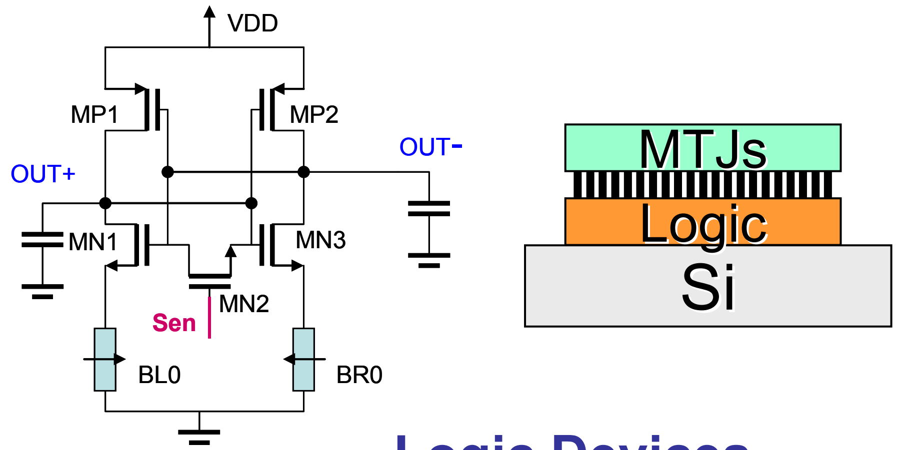
Sensors



Memories



RF Components



Logic Devices

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Magnetization Dynamics

Time Scale

1 psec	1 nsec	1 µsec	1 msec	1 sec
	1 GHz	1 MHz	1 kHz	1 Hz

- Exchange
- Antiferromagnetic Resonance
- Femtosecond dynamics

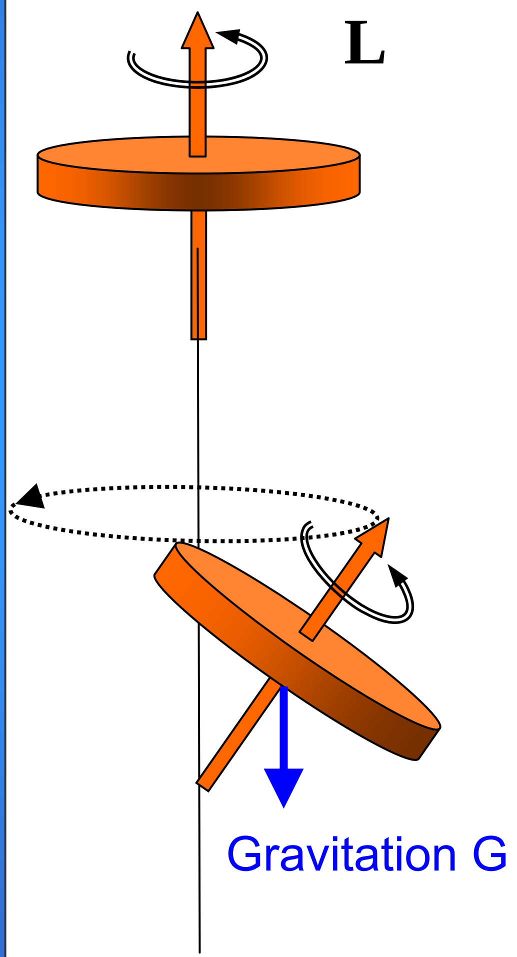
Spin waves
FMR
Domain wall motion
Vortex oscillations

10 GHz - 100 MHz

Hysteresis Loops
thermal activation (walls, small particles)
Relaxation
Wall Creep

Precessional Magnetization Dynamics

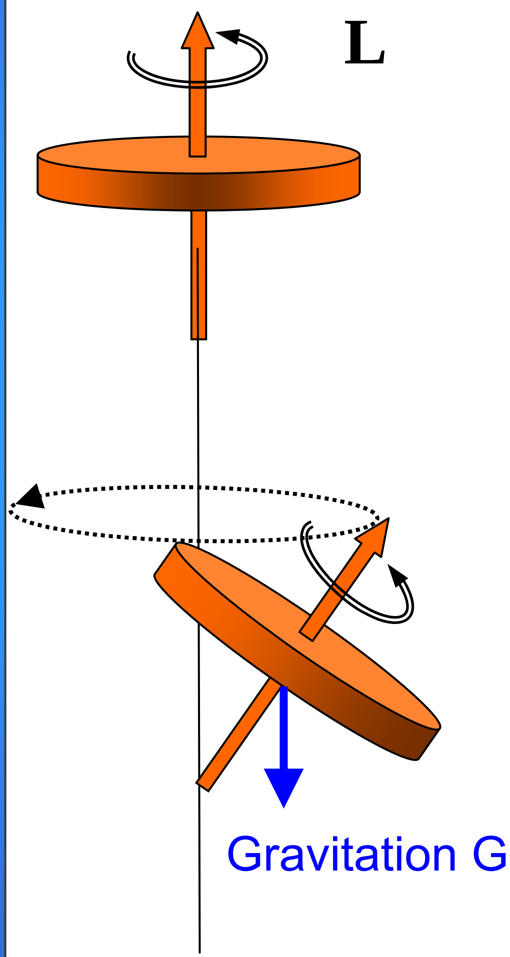
Gyroscope



$$\frac{d\mathbf{L}}{dt} = \mathbf{L} \times \mathbf{G} \quad = -\gamma \mathbf{L} \wedge \mathbf{G}$$

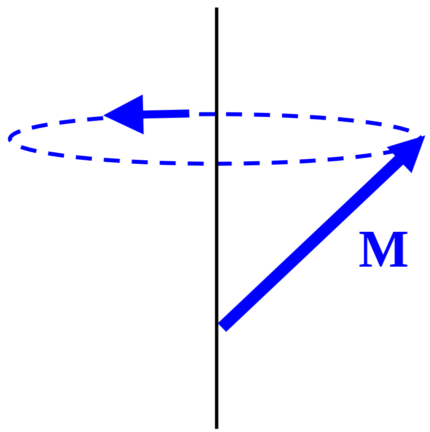
Precessional Magnetization Dynamics

Gyroscope



Magnetization is given by
Spin Angular Momentum **S**

$$\mathbf{M} = -\gamma \mathbf{S}$$



γ=gyromagnetic ratio $\gamma = g\mu_B\hbar$

Precessional Magnetization Dynamics

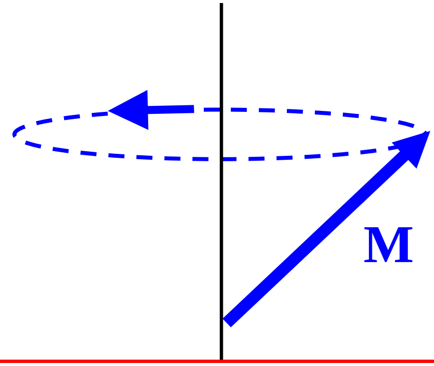
Response to perturbation is a precessional motion

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}$$

$$\gamma = 17.6 \cdot 10^6 \frac{1}{Oe \cdot \sec}$$

Magnetization is given by Spin Angular Momentum **S**

$$\mathbf{M} = -\gamma \mathbf{S}$$



Frequency given by H

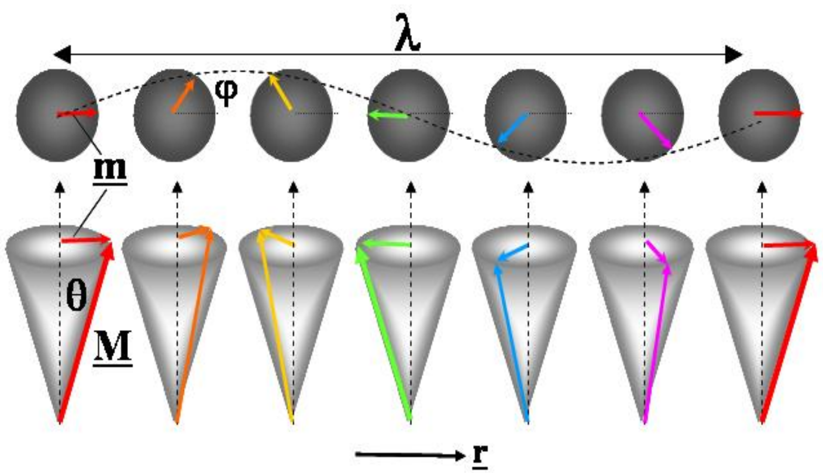
All changes of the magnetization state pass via a precessional motion

Distinguish : Intial state – transient – final state (static or dynamic)

Large Angle Precessional Magnetization Dynamics

θ small ~1°

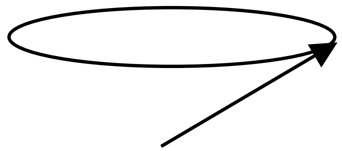
Precession around equilibrium
- Thermally excited spin waves
- Ferromagnetic resonance



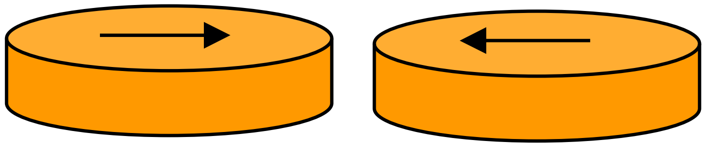
$$m = m_o \exp(ikr - \omega t)$$

θ large $\geq$ 10°

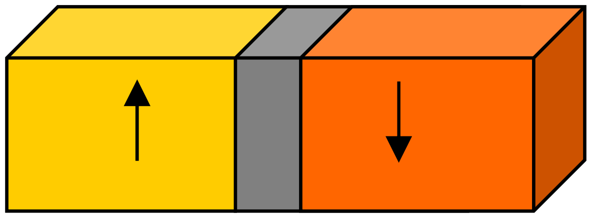
Large amplitude oscillations



Precessional Magnetization reversal



Domain wall motion



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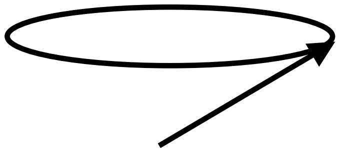
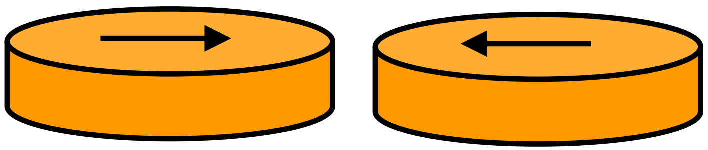
Large Angle Precessional Magnetization Dynamics

θ large ≥ 10°

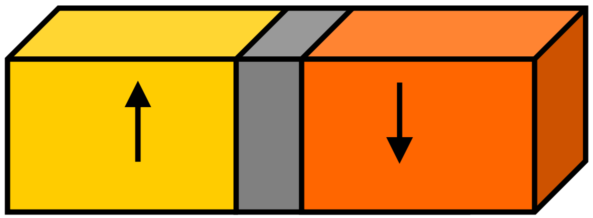
Large amplitude oscillations



Precessional Magnetization reversal



Domain wall motion



Landau-Lifschitz-Gilbert (LLG)
Equation of Motion for *M*

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M}\times\mathbf{H}_{eff}}$$

Precession

$$+\boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}} + \boxed{\frac{\gamma\, a_J(\theta)}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})}$$

Damping | Spin torque

Supplemented by
Spin Transfer Torque (STT)

Outline

0 Introduction

I Conservative dynamics

II Non conservative dynamics

III Precessional reversal under transverse field pulses

IV Domain wall motion under field

V Introduction to spin transfer torque

VI Spin torque induced precession

VII (Precessional) Reversal under spin torque

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

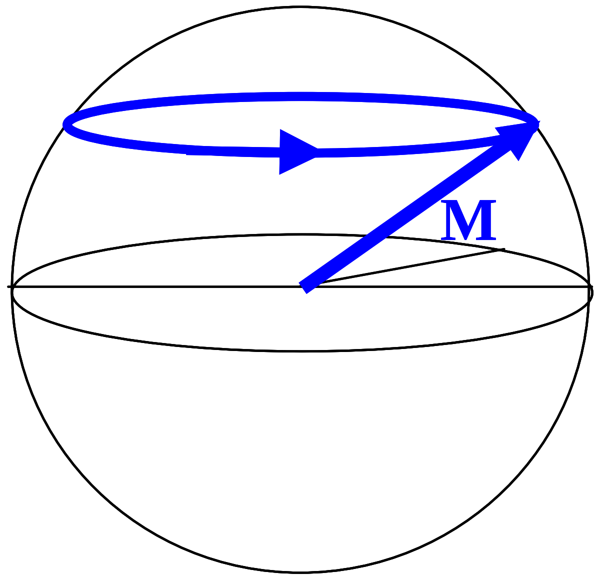
1) Conservation of M

$$\frac{d|\mathbf{M}|^2}{dt} = 2\mathbf{M} \cdot \frac{d|\mathbf{M}|}{dt} = 2\mathbf{M} \cdot \left(\mathbf{M} \times \vec{H}_{eff}\right) = 0$$

$$\Rightarrow |\mathbf{M}| = const.$$

$$\mathbf{m} = \frac{\mathbf{M}}{M_s} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \Rightarrow \boxed{m_x^2 + m_y^2 + m_z^2 = 1}$$

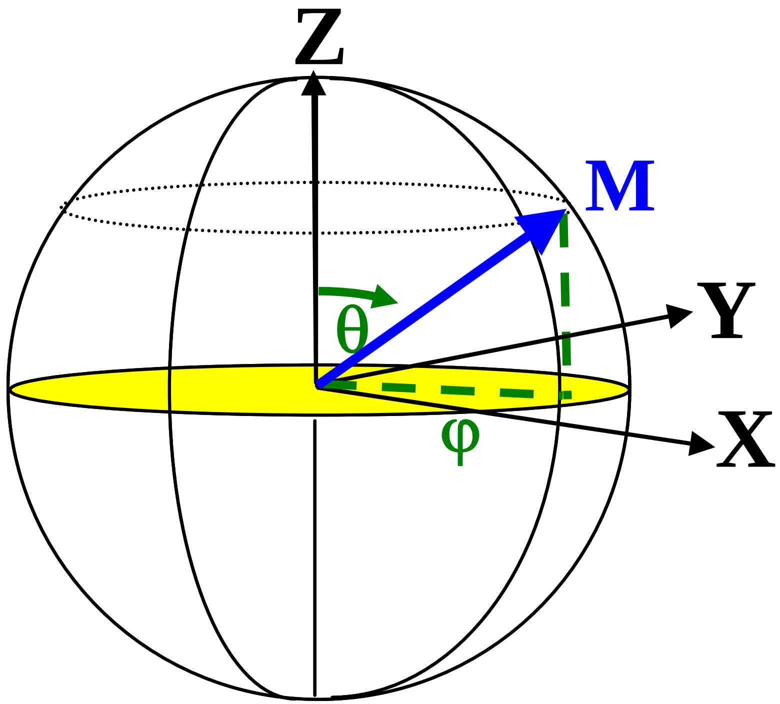
Motion of m on unit sphere



$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

Motion of m on unit sphere



$$\mathbf{m} = \frac{\mathbf{M}}{M_s} \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

Two possible descriptions

1) Cartesian coordinates (x,y,z) or

2) Spherical coordinates (θ,φ)

Only two degrees of freedom

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$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}} \qquad \mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}}$$

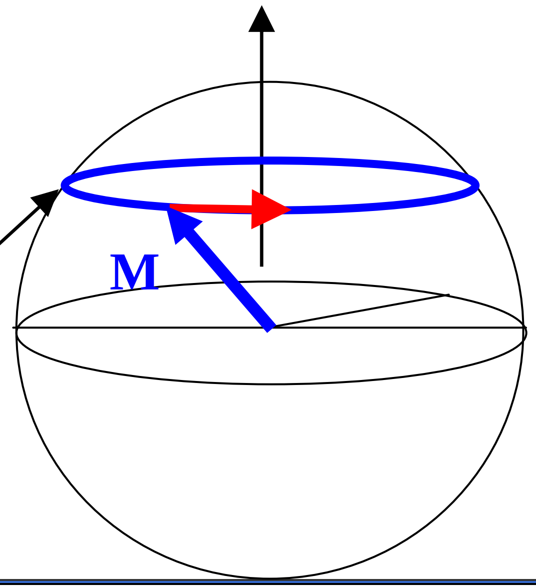
Precession

2) Conservation of energy

$$\frac{dE}{dt} = \frac{dE}{d\mathbf{M}} \frac{d\mathbf{M}}{dt} = \gamma \mathbf{H}_{eff} \left(\mathbf{M} \times \mathbf{H}_{eff} \right) = 0$$

$$\mathbf{a}(\mathbf{a} \times \mathbf{b}) = 0$$

$$\boxed{\Rightarrow E = const.}$$



$\frac{d\mathbf{M}}{dt}$ Tangential to trajectory

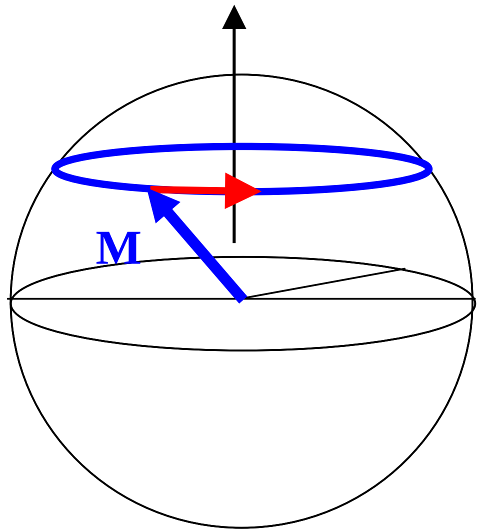
Motion of **M** along constant energy trajectory

The Precession equation corresponds to a Nonlinear Dynamical System

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}}$$



$$\frac{d\mathbf{X}}{dt} = f(\mathbf{X})$$

Its solutions are equilibrium points and dynamic orbits

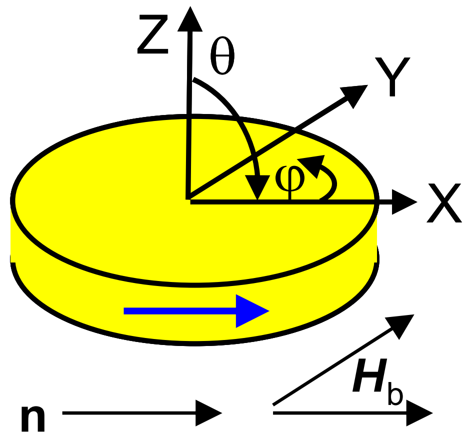
Find solutions for a given energy surface *E* and thus effective field $\mathbf{H}_{\mathrm{eff}}$

with the applied bias field $\boldsymbol{H}_b$ as the control parameter (amplitude and orientation)

Energy of an in-plane uniaxial system

$$E = K_u\left[1-(\mathbf{mn})^2\right] - M_s \mathbf{m}\mathbf{H}_b + 2\pi M_s^2\left[\mathbf{m}\left(\overleftrightarrow{\mathbf{N}}\mathbf{m}\right)\right]$$

Anisotropy　　Zeeman　　Demagnetization



$$\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}; \quad \overleftrightarrow{N} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad \mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{H}_b = \begin{pmatrix} H_b \\ 0 \\ 0 \end{pmatrix}; \quad or \quad \mathbf{H}_b = \begin{pmatrix} 0 \\ H_b \\ 0 \end{pmatrix}$$

bias field $\mathbf{H}_b$

Easy axis　　Hard axis

n unit vector of easy axis
N demagnetization tensor
M = **m** M_s
Uniaxial anisotropy field H_u

Effective Field

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}} = H_u(\mathbf{nm})\mathbf{n} + \mathbf{H}_b - H_d \overleftrightarrow{\mathbf{N}}\mathbf{m}$$

$$H_u = \frac{2K_u}{M_s}$$

$$H_d = 4\pi M_s$$

I Conservative Dynamics - Equilibrium States

Equilibrium points of a dynamical system $\frac{d\mathbf{X}}{dt} = 0$

Equilibrium points of Precession Term

$$(i)\,\frac{\partial \mathbf{M}}{\partial t} = 0 \Rightarrow \mathbf{M} \times \mathbf{H}_{eff} = 0$$

$$(ii)\,\frac{\partial E}{\partial \mathbf{M}} = 0 \Leftrightarrow \quad \frac{\partial E}{\partial \theta} = 0 \quad \wedge \quad \frac{\partial E}{\partial \varphi} = 0$$

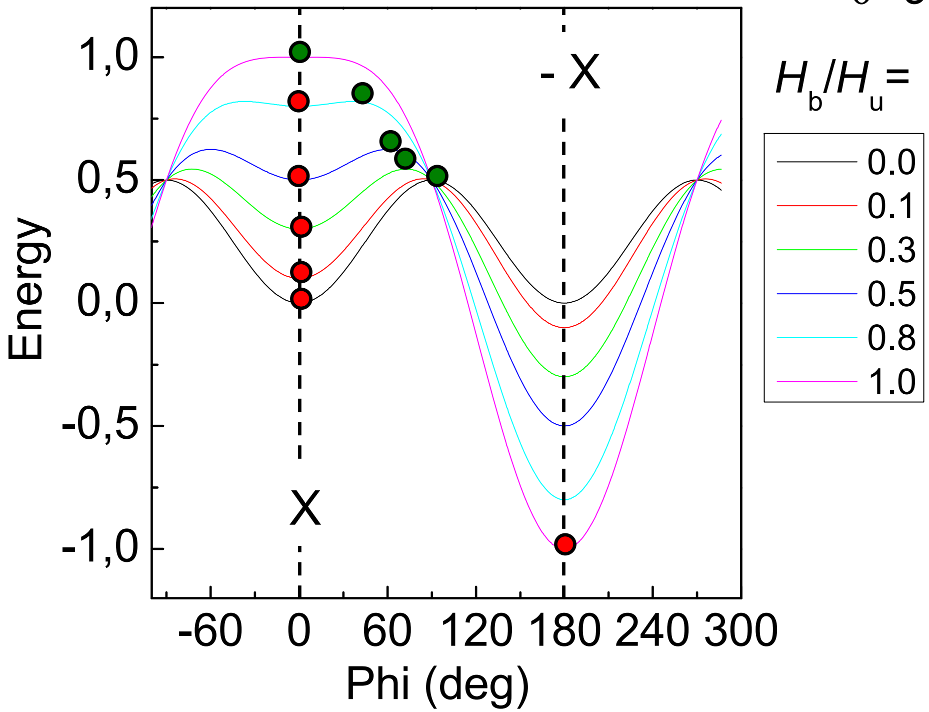
Solutions of precession term:
Minima on energy surface

I Conservative Dynamics - Energy Surface

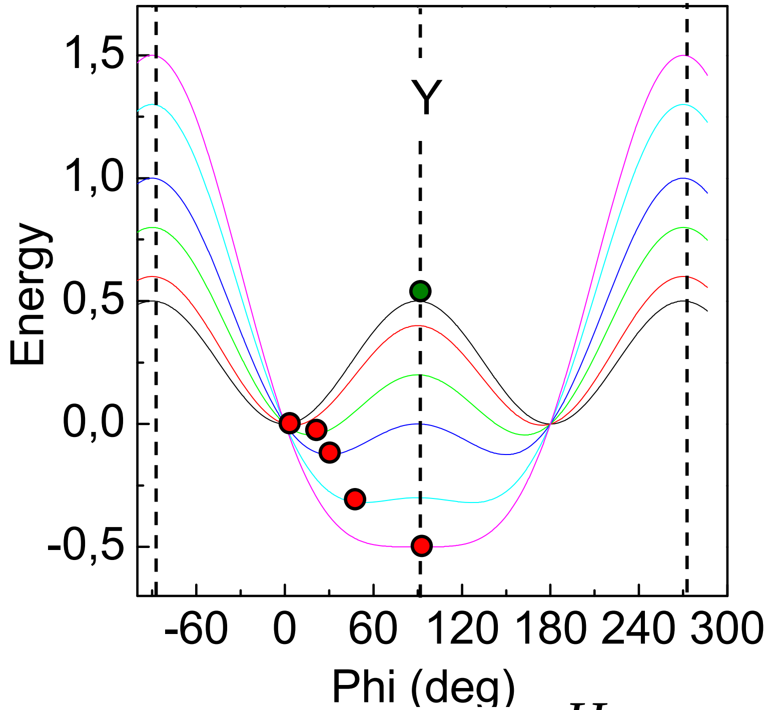
In Plane Field along

θ=90°

Easy Axis



Hard Axis



Easy Axis

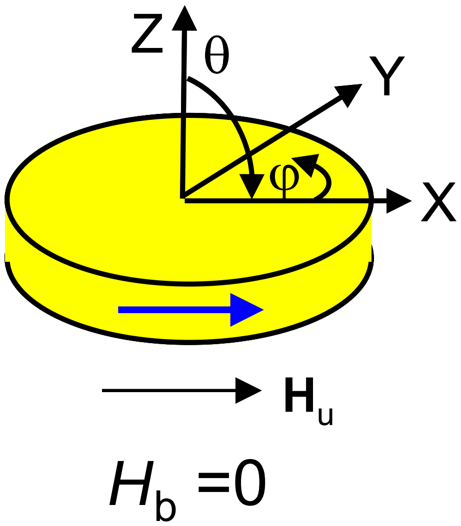
- Stable points φ=0° or φ=180°
- Unstable points $\cos\varphi = -\dfrac{H_b}{H_u}$
- Out of plane max $\cos\varphi = -\dfrac{H_b}{H_u + H_d}$

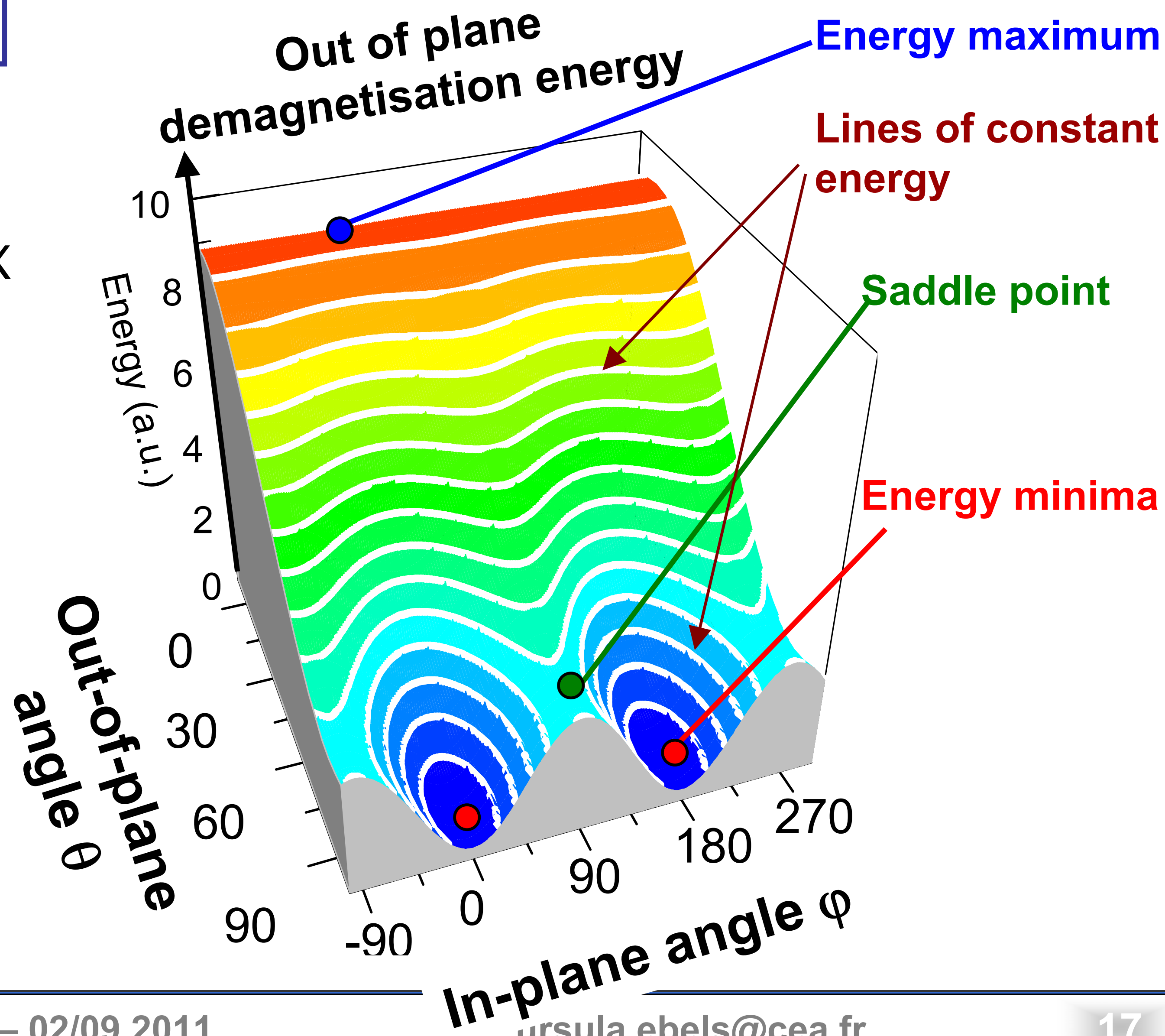
Hard Axis

- Stable points $\sin\varphi = \dfrac{H_b}{H_u}$
- Unstable points φ=90°
- Out of plane max $\sin\varphi = -\dfrac{H_b}{H_d}$

I Conservative Dynamics - Energy Surface

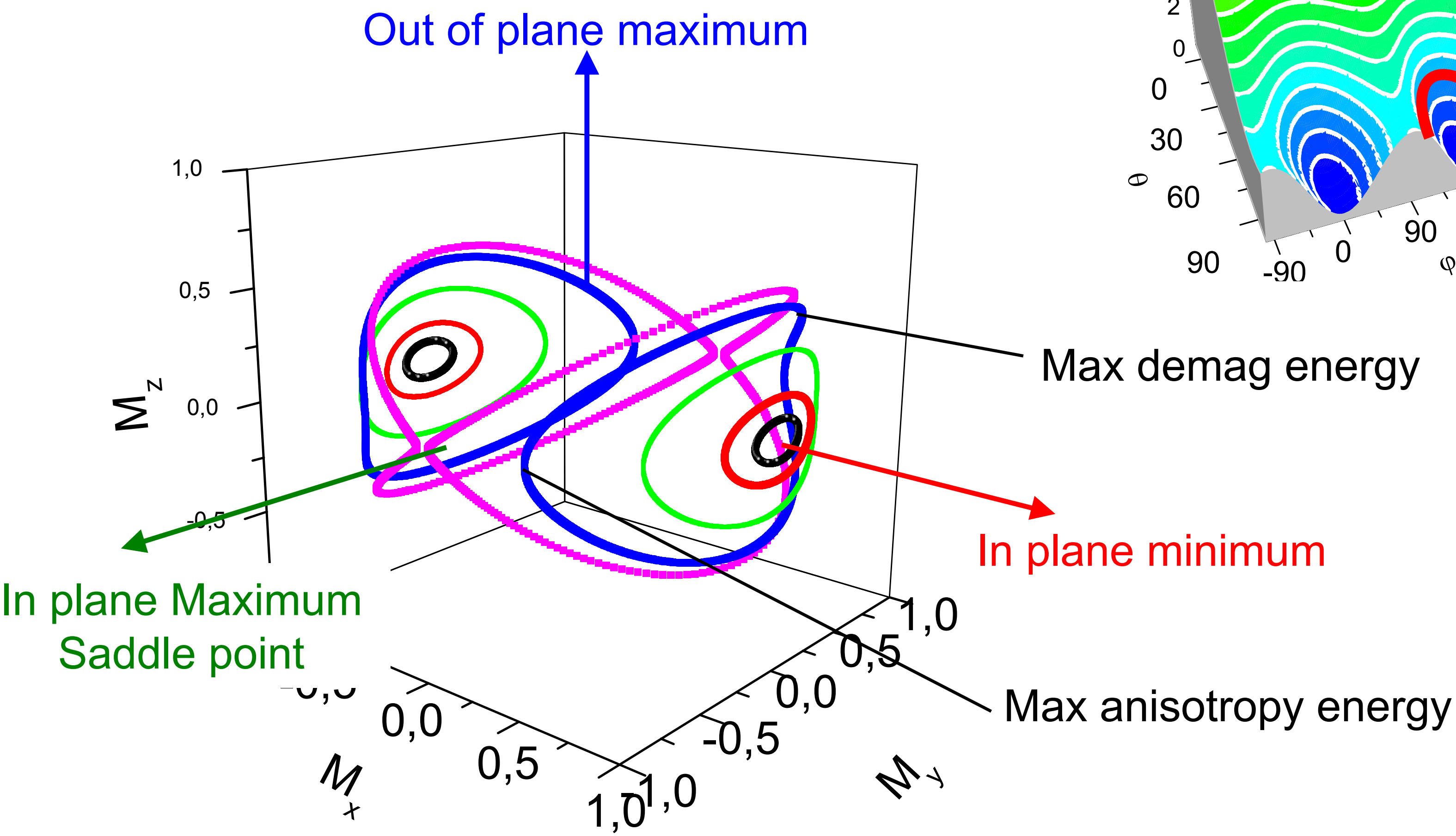
Uniaxial thin film

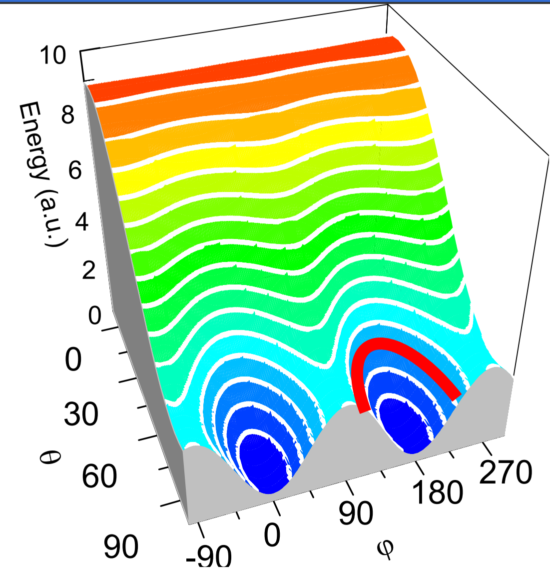




In-plane precession IPP orbit

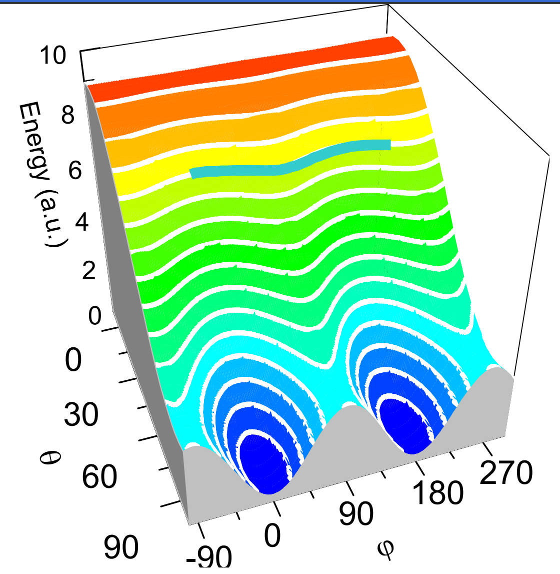
- Energy below saddle point,
- Oscillation around energy minimum

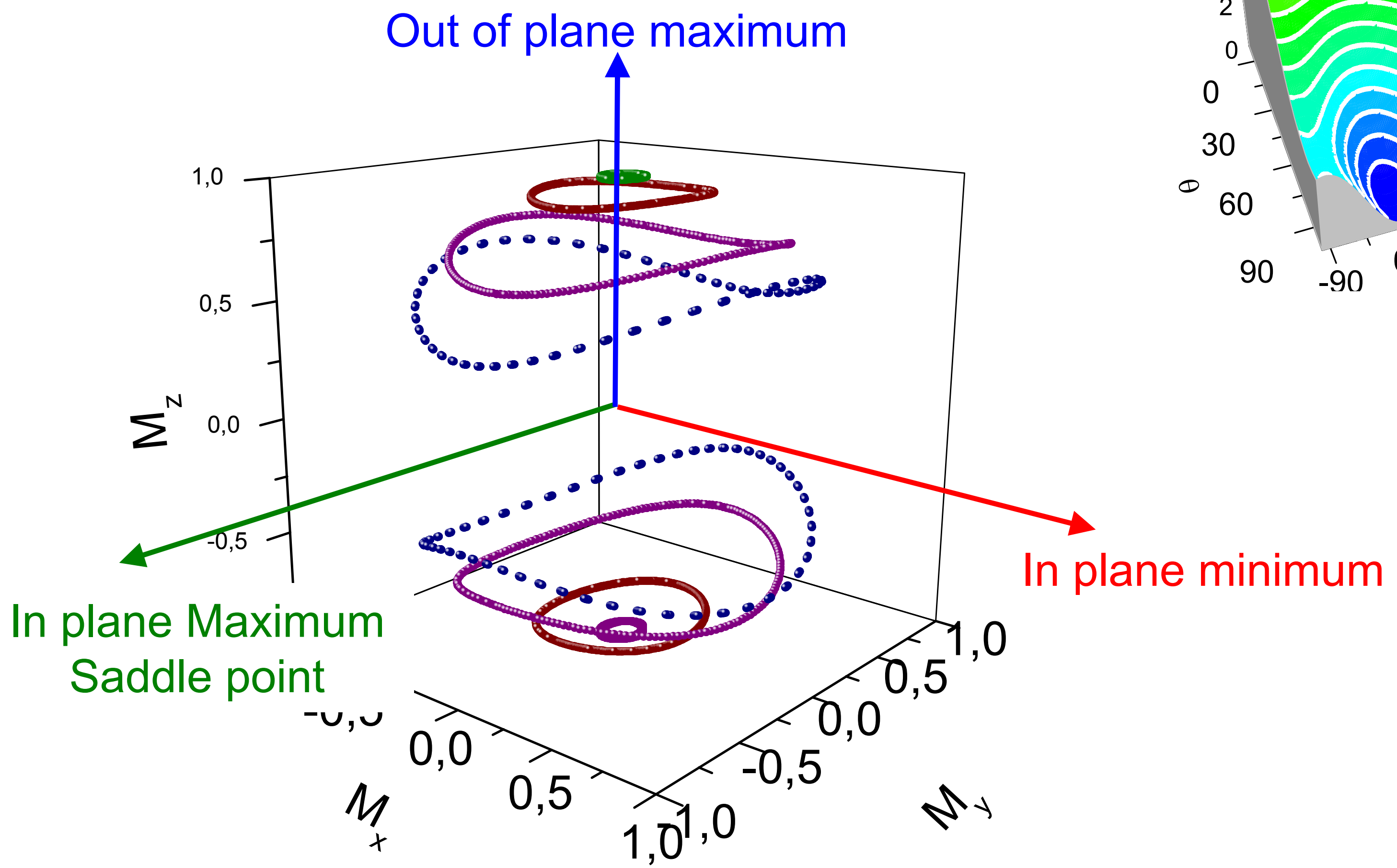




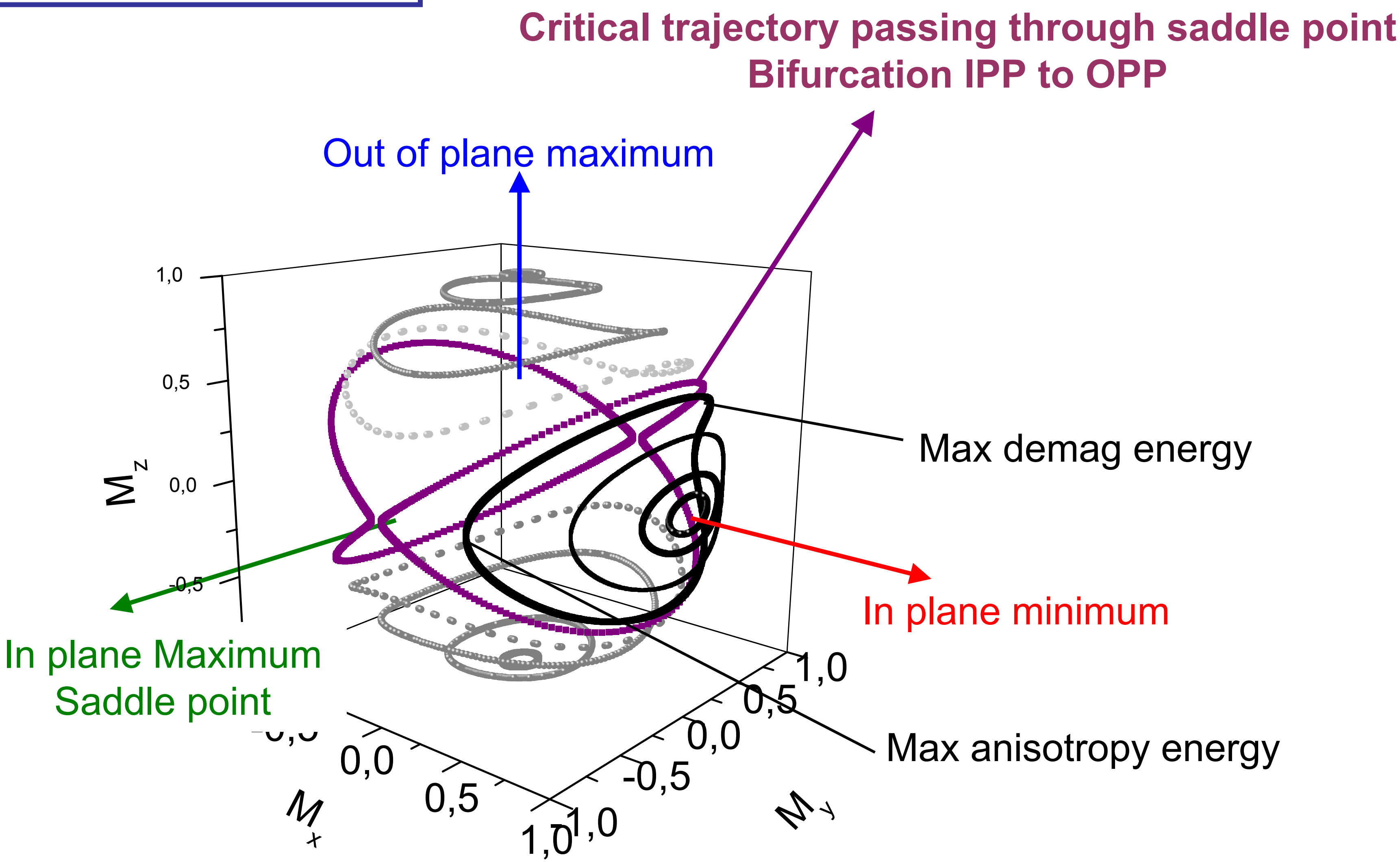
Out-of-plane precession OPP orbit

- Energy above saddle point,
- Oscillation around energy maximum





IPP & OPP orbits



I Conservative Dynamics - General Solutions

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Equilibrium points of a dynamical system $$\frac{d\mathbf{X}}{dt}=0$$

Equilibrium points of Precession Term

$$(i)\,\frac{\partial \mathbf{M}}{\partial t}=0 \Rightarrow \mathbf{M}\times\mathbf{H}_{eff}=0$$

$$(ii)\,\frac{\partial E}{\partial \mathbf{M}}=0 \Leftrightarrow \quad \frac{\partial E}{\partial \theta}=0 \quad \wedge \quad \frac{\partial E}{\partial \varphi}=0$$

Solutions of precession term:
Minima on energy surface

Dynamic orbits of precession equation

Due to energy conservation and conservation of norm

$$E=K_u\left[1-m_x^2\right]-M_sH_{bx}m_x-M_sH_{by}m_y+2\pi M_s^2m_z^2$$

$$1=m_x^2+m_y^2+m_z^2$$

2 equations and 3 unknowns: parametrize e.g. m_x and $m_y = f(m_z)$

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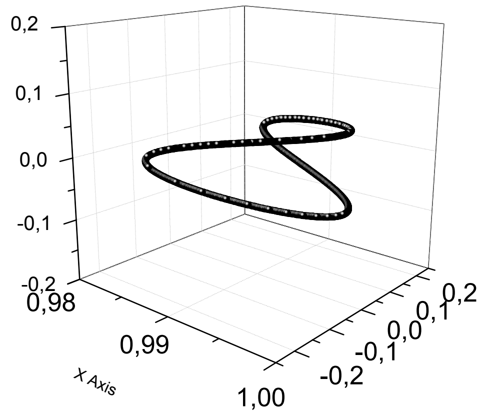
Calculation of Trajectories

$$E = K_u\left[1 - m_x^2\right] - M_s H_b m_x + 2\pi M_s^2 m_z^2$$

$$1 = m_x^2 + m_y^2 + m_z^2$$

Uniaxial thin film

H_b along easy axis



Initial energy E_o, given e.g. by φ_o=0 and θ_o

$$E_o(\varphi_o, \theta_o) = E_o(m_{xo}, m_{yo}, m_{zo}) = const$$

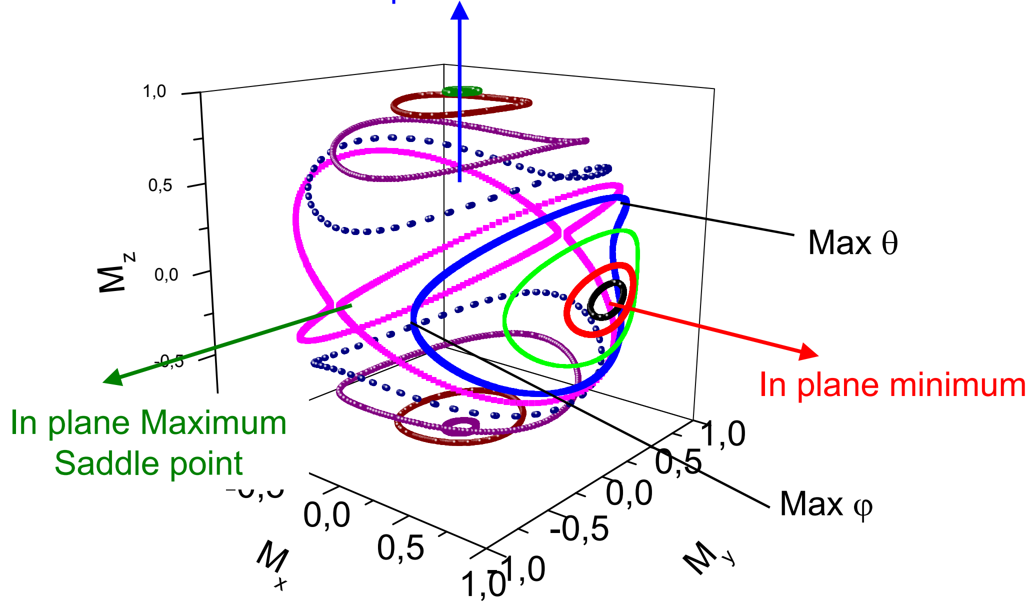
$$E = \frac{M_s H_u}{2}\left(1 - m_x^2\right) - M_s H_b m_x + \frac{M_s H_d}{2} m_z^2 = E_o = const$$

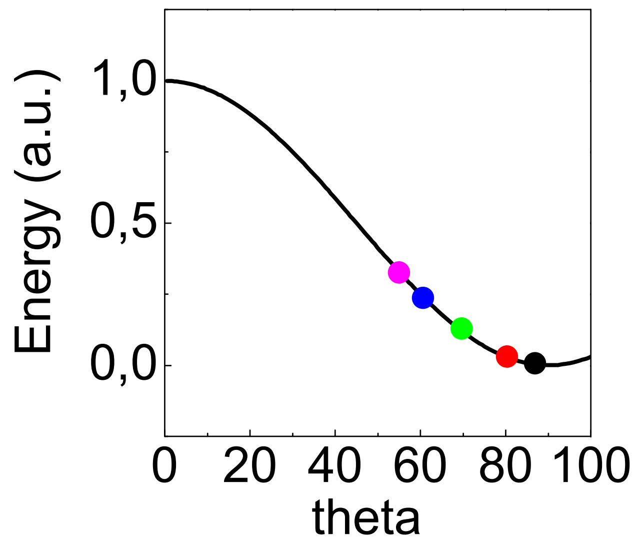
$$\Rightarrow m_x = -\frac{H_b}{H_u} \pm \sqrt{\left(\frac{H_b}{H_u}\right)^2 + \frac{H_d}{H_u} m_z^2 - \frac{2E_o}{H_u M_s} + 1} = m_x(m_z)$$

$$\Rightarrow m_y = \pm\sqrt{1 - m_x^2 - m_z^2} = m_y(m_z)$$

$$m_z \in \{-\cos\theta_o, \cos\theta_o\}$$

Amplitude and type of the trajectory given by initial condition



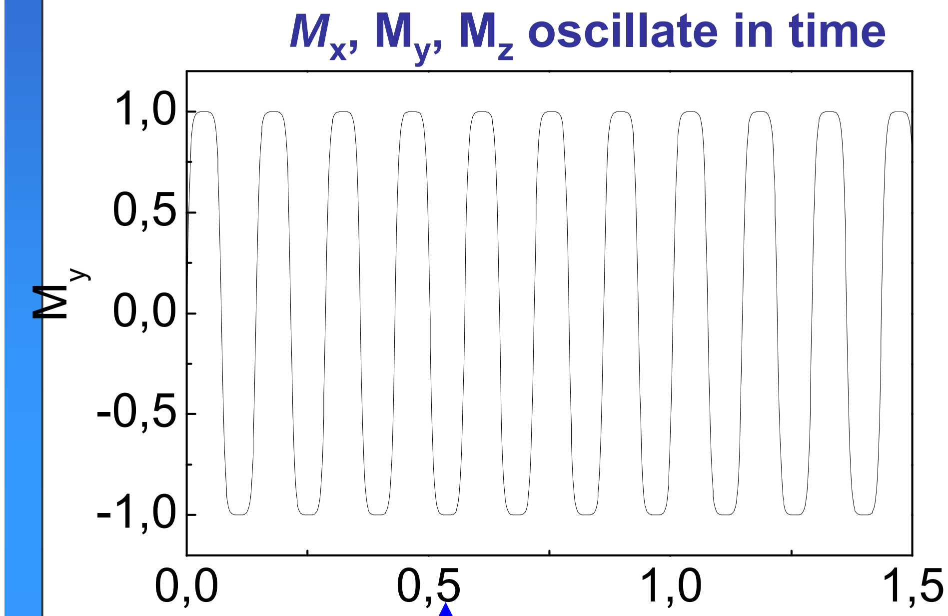


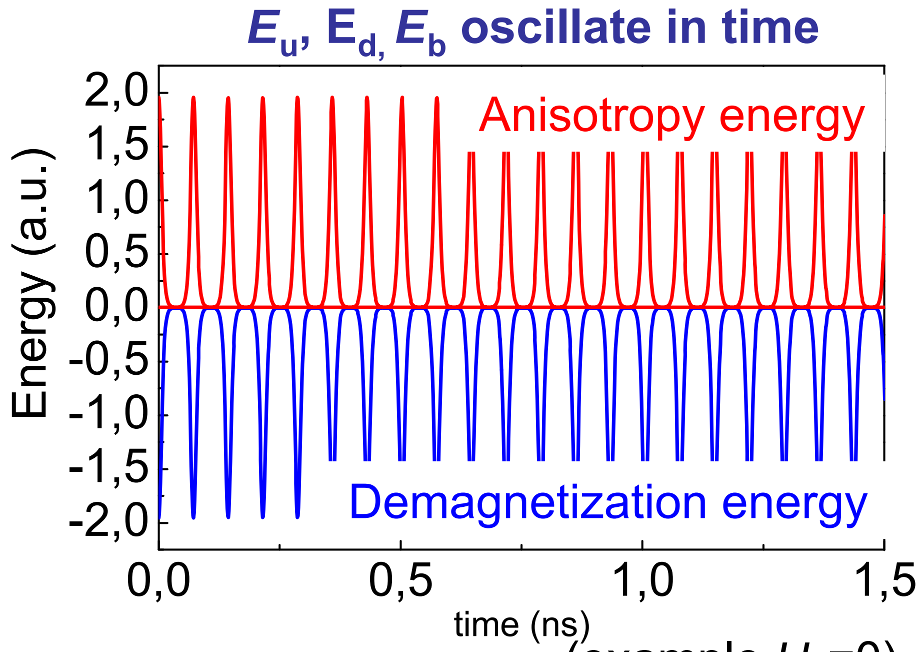
$$E_o(\varphi_o,\theta_o) = E_o(m_{xo}, m_{yo}, m_{zo}) = const$$

Initial condition:

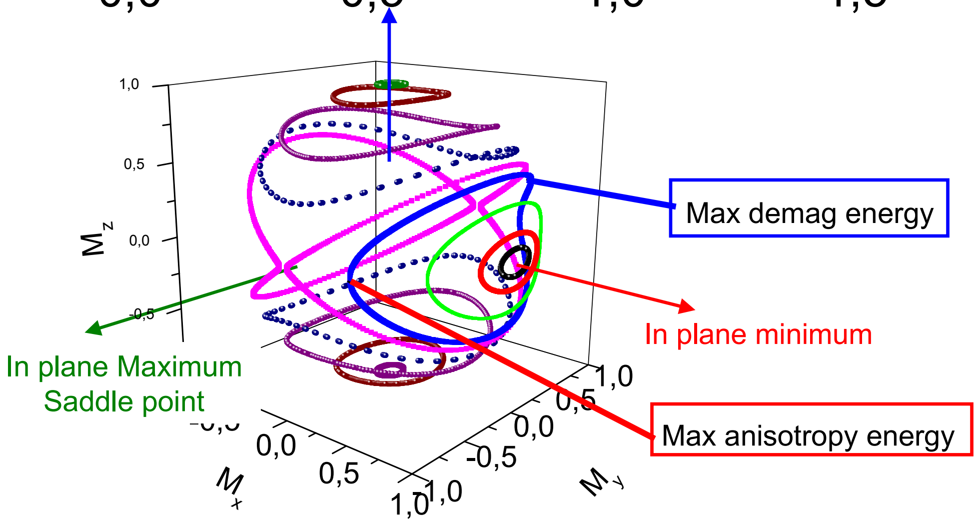
- Any point along the trajectory, since the orbits are uniquely defined (they do not cross)
- Choose for instance $\theta = \theta_{max}$ or $\varphi = \varphi_{max} \rightarrow E_o$
- Or choose $E_o \rightarrow \theta_o$ or φ_o
- With increasing energy the amplitudes θ_o, φ_o increase

I Conservative Dynamics - Energies





(example H_b=0)



Properties of the trajectories

- M_x, M_y, M_z oscillate in time
- Total energy is constant, given by E_o
- anisotropy energy E_u,
  demagnetization energy E_d
  bias field energy E_b oscillate in time

I Conservative Dynamics - Effective Field

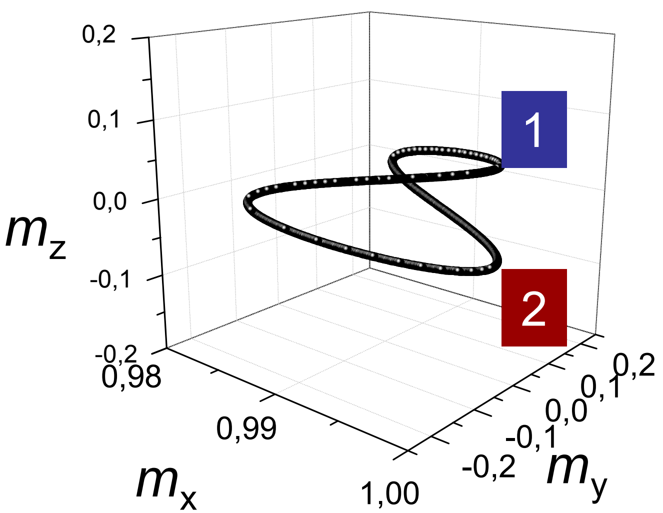
The effective field amplitude AND its ORIENTATION are NOT CONSTANT

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff}$$

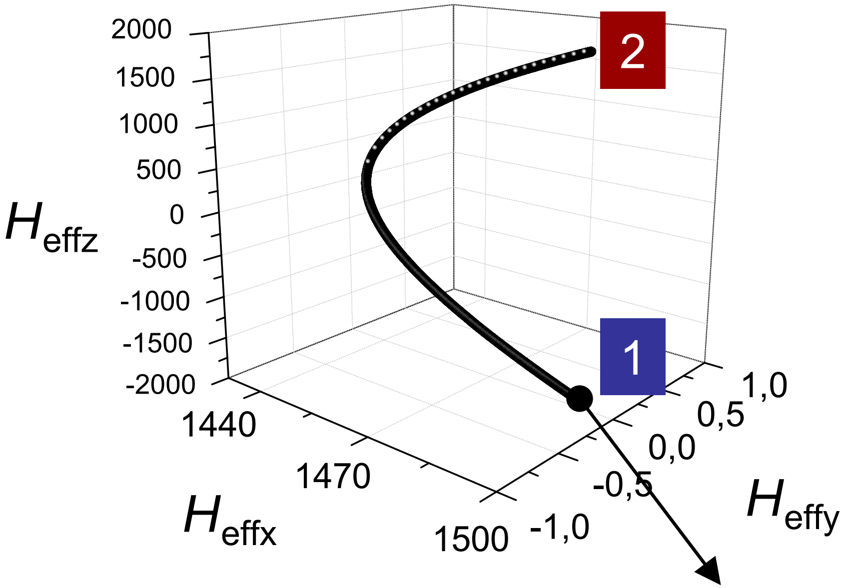
$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}} = f(\mathbf{M})$$

Thin Film

Trajectory



Effective Field



Strongest torque at maximum m_x

Demagnetization field leads to strongly elliptical trajectory

I Conservative Dynamics - Non-Linear Dynamics

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}}$$

Precession **Damping**

	|m|	dE/dt	Static	Dynamic
Conservative Precession Dynamics	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition IPP or OPP for thin films
Non-conservative LLG				

Outline

0 Introduction

I Conservative dynamics

II Non conservative dynamics

III Precessional reversal under transverse field pulses

IV Domain wall motion under field

V Introduction to spin transfer torque

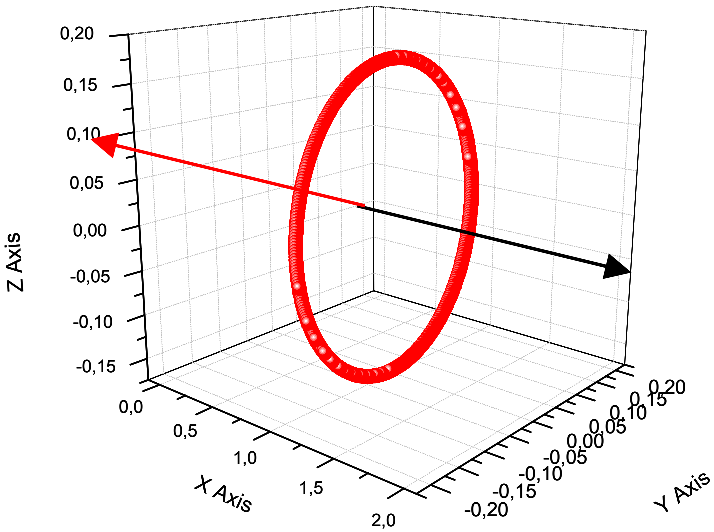
VI Spin torque induced precession

VII (Precessional) Reversal under spin torque

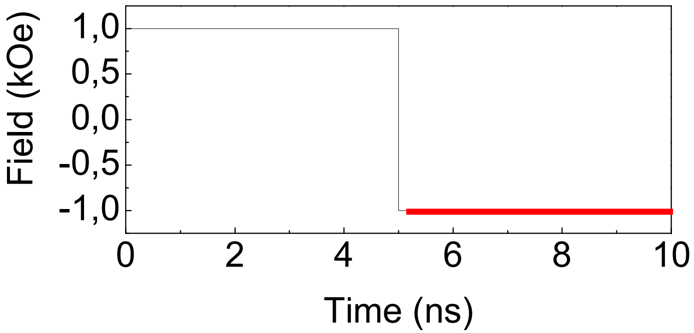
Example : Sphere with reversed field

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}$$

Precession



Switch field from + 1kOe to -1kOe

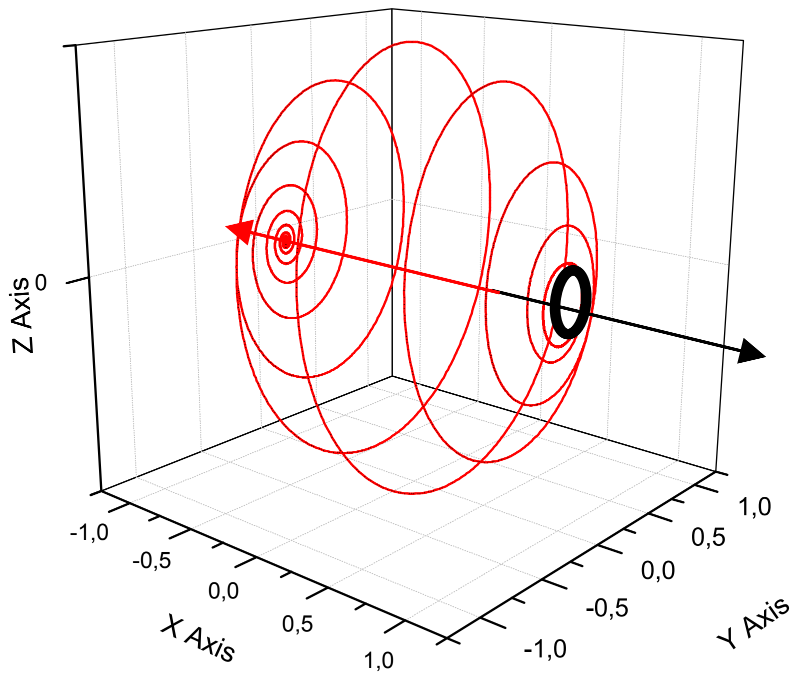


Example : Sphere with reversed field

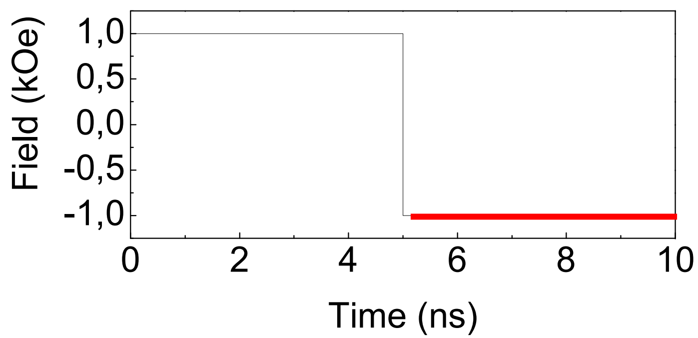
$$\frac{d\mathbf{M}}{dt} = -\boxed{\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)} + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Precession **Damping**

With damping
Magnetization Reverses

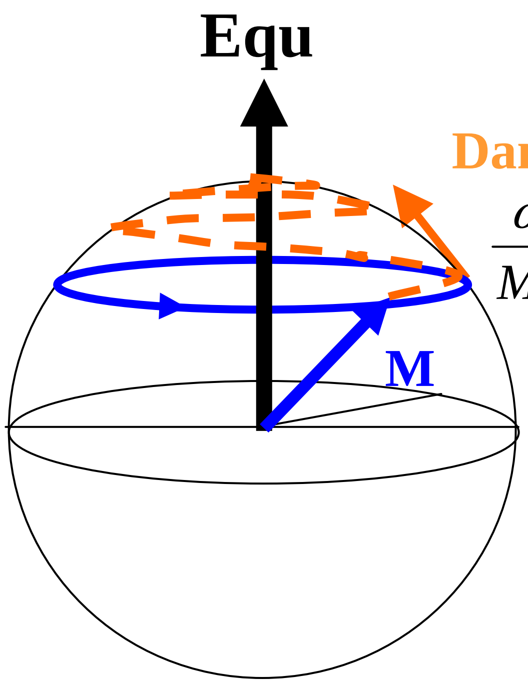


Switch field from + 1kOe to -1kOe



Landau-Lifshitz-Gilbert Equation (LLG)

$$\frac{d\mathbf{M}}{dt} = \underbrace{\boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}}_{\text{Precession}} + \underbrace{\boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}}_{\text{Damping}}$$



$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$$

Tangential,
directed towards equilibrium
(+ small component antiparallel to precession torque)

α = damping constant
typically 0.01 for metals

Time scales

Precession : order or below ns
Damping : few ns

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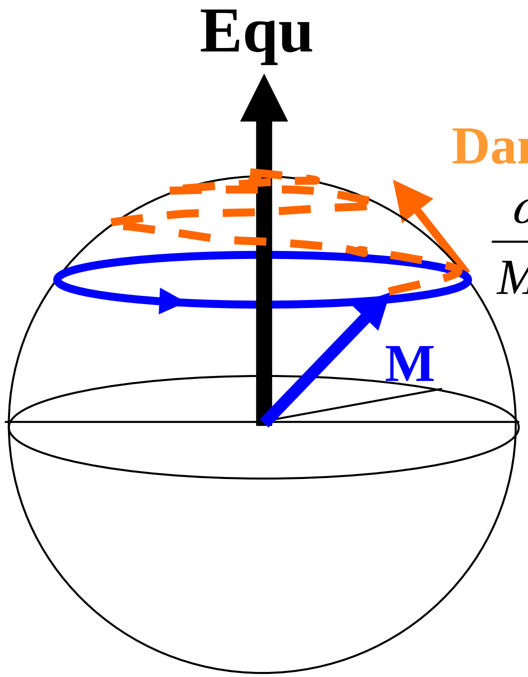
Landau-Lifshitz-Gilbert Equation (LLG)

$$\frac{d\mathbf{M}}{dt} = \underbrace{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{Precession}} + \underbrace{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}_{\text{Damping}}$$

→ Non-Linear dynamical system

Norm of **M**
Conserved since

$$\mathbf{M}\frac{d\mathbf{M}}{dt} = 0$$



Damping

$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$$

Tangential,
directed towards equilibrium
(+ small component antiparallel to precession torque)

II Non Conservative Dynamics - Energy Dissipation

Energy change

$$\frac{dE}{dt} = \frac{dE}{d\mathbf{M}}\frac{d\mathbf{M}}{dt} = -\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt}$$

$$\frac{dE}{dt} = \gamma\mathbf{H}_{eff}(\mathbf{M}\times\mathbf{H}_{eff}) - \frac{\alpha}{M_s}\mathbf{H}_{eff}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right) = -\frac{\gamma\alpha}{M_s}\left(\frac{d\mathbf{M}}{dt}\right)^2 < 0$$

$$\boxed{\frac{dE}{dt} < 0}$$ since $\alpha > 0$

- **Damping decreases the energy**

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Landau-Lifshitz-Gilbert Equation (LLG)

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)} + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Precession **Damping**

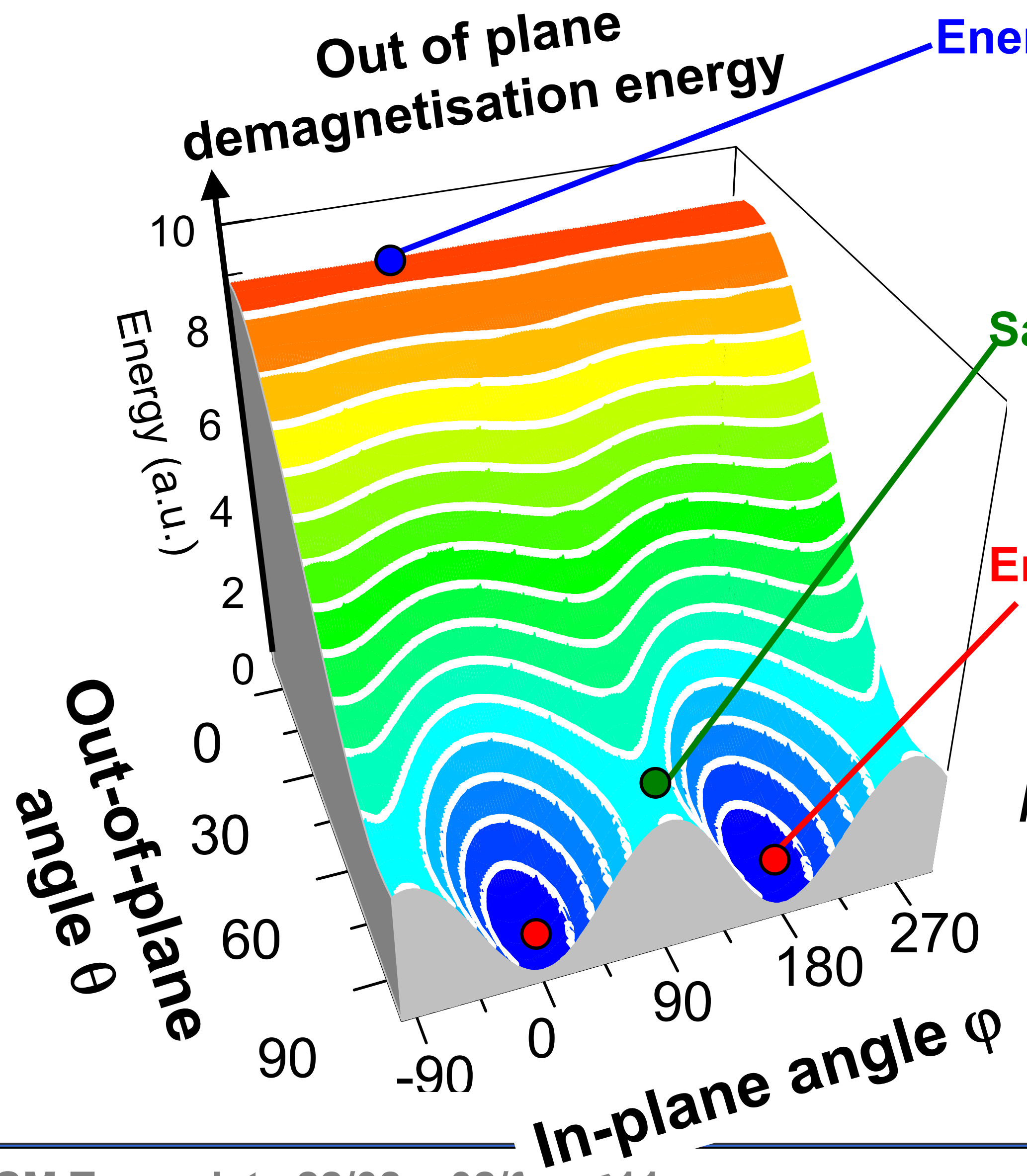
Equilibria $\mathbf{M_o}$ $$\frac{d\mathbf{M}}{dt} = 0 \Rightarrow \mathbf{M}_o \Leftrightarrow (\theta_o, \varphi_o)$$

Same as for conservative dynamics since (d**M**/dt=0)

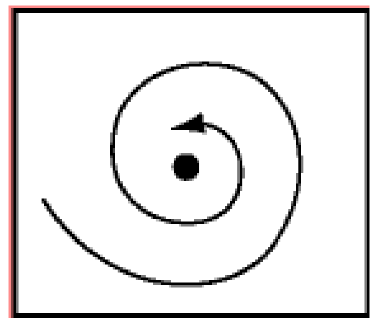
- **dE/dt < 0 : Damping decreases the energy**

→ The system relaxes towards the nearest (local) energy minimum

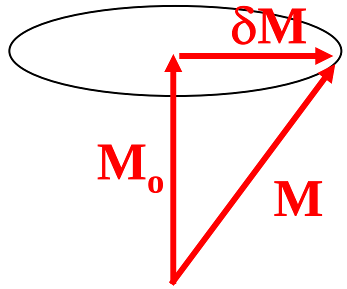
II Non Conservative Dynamics - Equilibria



$H_b = 0$



Landau-Lifshitz-Gilbert Equation (LLG)



$$\frac{d\mathbf{M}}{dt} = \underbrace{\boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}}_{\text{Precession}} + \underbrace{\boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}}_{\text{Damping}}$$

Stability: Linearization of LLG around equilibrium points $\mathbf{M}_o$

$$\mathbf{M} = \mathbf{M}_o + \delta\mathbf{M}$$

$$\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff}(\mathbf{M}_o + \delta\mathbf{M})$$

$$\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff}(\mathbf{M}_o) + \mathbf{h}_{eff}(\delta\mathbf{M})$$

$$\Rightarrow \frac{d\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\delta\mathbf{M}$$

$\underline{\underline{\mathbf{A}}}$ does not depend on $\delta\mathbf{M}$, only on $\mathbf{M}_o$ & $\mathbf{H}_{eff}(\mathbf{M}_o)$

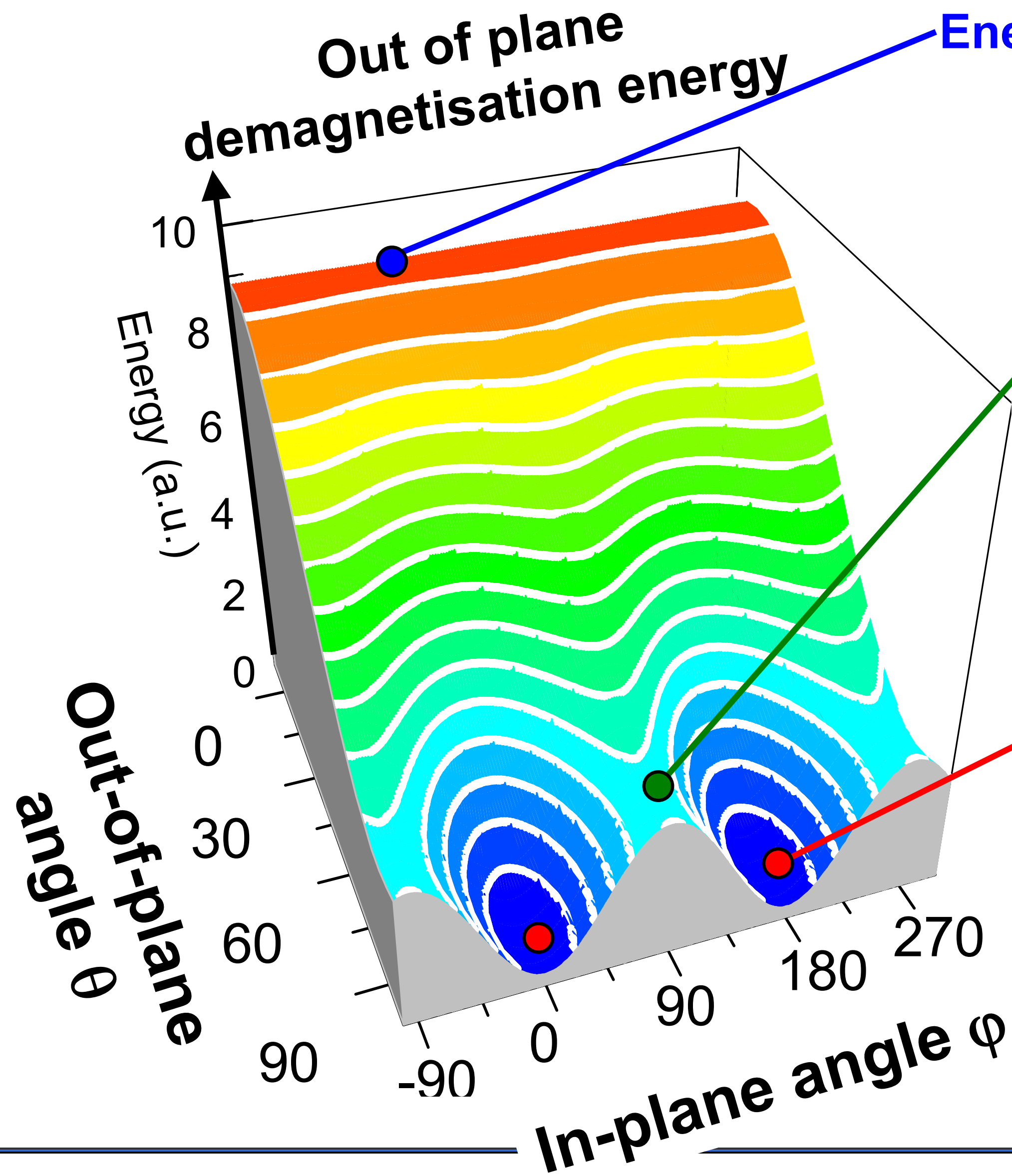
Solution

$$\boxed{\begin{aligned}\delta\mathbf{M} &= \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t} \\ \lambda &= \Gamma + i\omega_o\end{aligned}}$$

$\Gamma < 0$ stable point

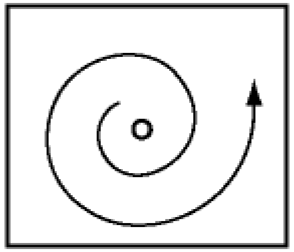
$\Gamma > 0$ unstable point

II Non Conservative Dynamics - Stability of Equilibria



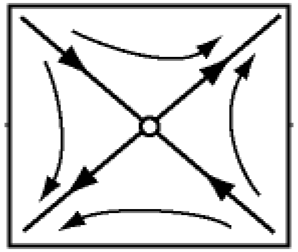
Energy maximum – unstable focus

$\Gamma > 0, \omega \neq 0$



Saddle point - unstable

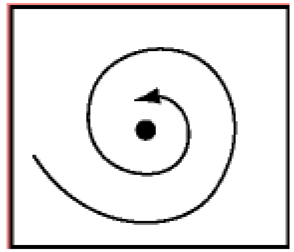
$\Gamma_1 < 0, \Gamma_2 > 0$
$\omega = 0$



Energy minima – stable focus

$H_b = 0$

$\Gamma < 0, \omega \neq 0$



$$\frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\,\delta\mathbf{M}$$

Solution

$$\boxed{\begin{aligned}\delta\mathbf{M} &= \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}\\ \lambda &= \Gamma + i\omega_o\end{aligned}}$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda \underline{\underline{\mathbf{I}}}\right) = 0 \Rightarrow \lambda$$

Γ **< 0 stable point**

Γ **> 0 unstable point**

Can be lengthy depending on the configuration.

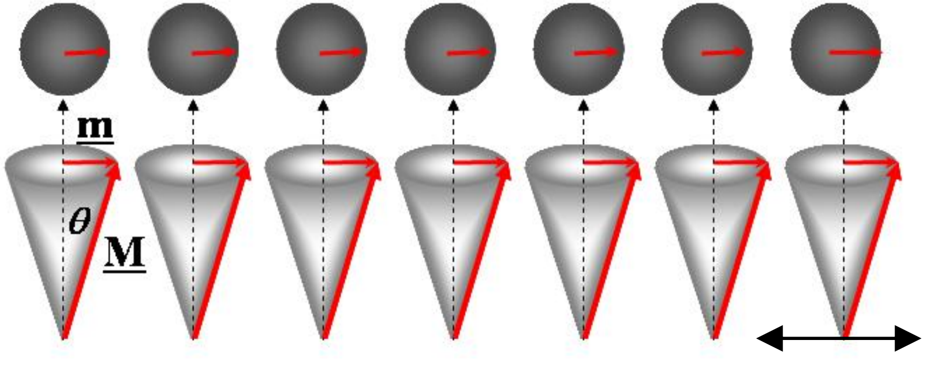
General solution in sphercial coordinates

Well known solution from Ferromagnetic Resonance FMR

$$\omega_o^{\,2} = \gamma\left(\frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2 \sin^2\theta}\right) \qquad \Gamma = \frac{\gamma\alpha}{2}\left(\frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2 \sin^2\theta}\right) \qquad \Gamma = \frac{\Delta\omega}{2}$$

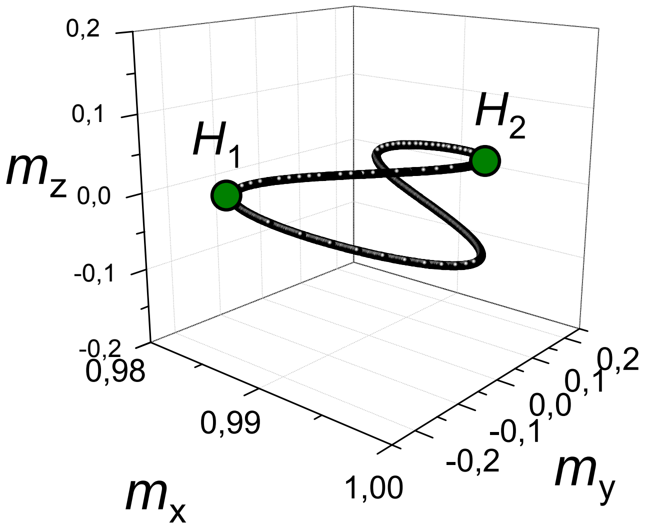
$E_{\theta\theta}, E_{\varphi\varphi}, E_{\theta\varphi}$ Second derivatives of energy to be evaluated at equilibrium $\boldsymbol{M}_o \Leftrightarrow \theta_o, \varphi_o$

Ferromagnetic Resonance – uniform mode (k=0)



M = const., **m** = const

Excitations by rf field $\mathbf{h}_{rf} \perp \mathbf{M}$



Rule of thumb for frequencies
(oscillation around a symmetry axis)

$$\frac{\omega}{\gamma} = \sqrt{H_1 \cdot H_2}$$

H_1, H_2 effective field acting on **M** at turning points

II Non Conservative Dynamics - Summary

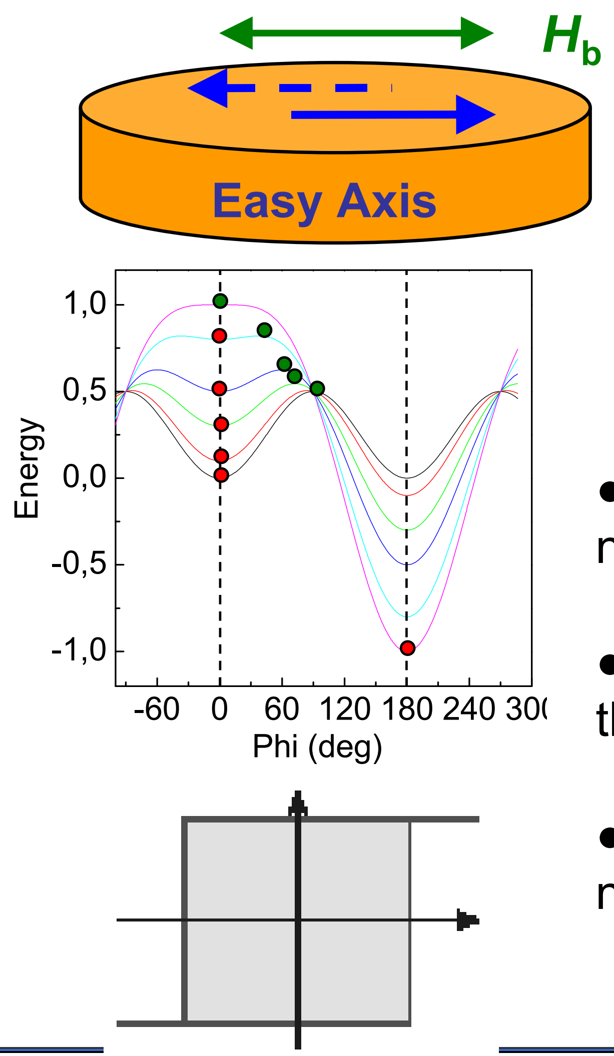
$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}}$$

Precession **Damping**

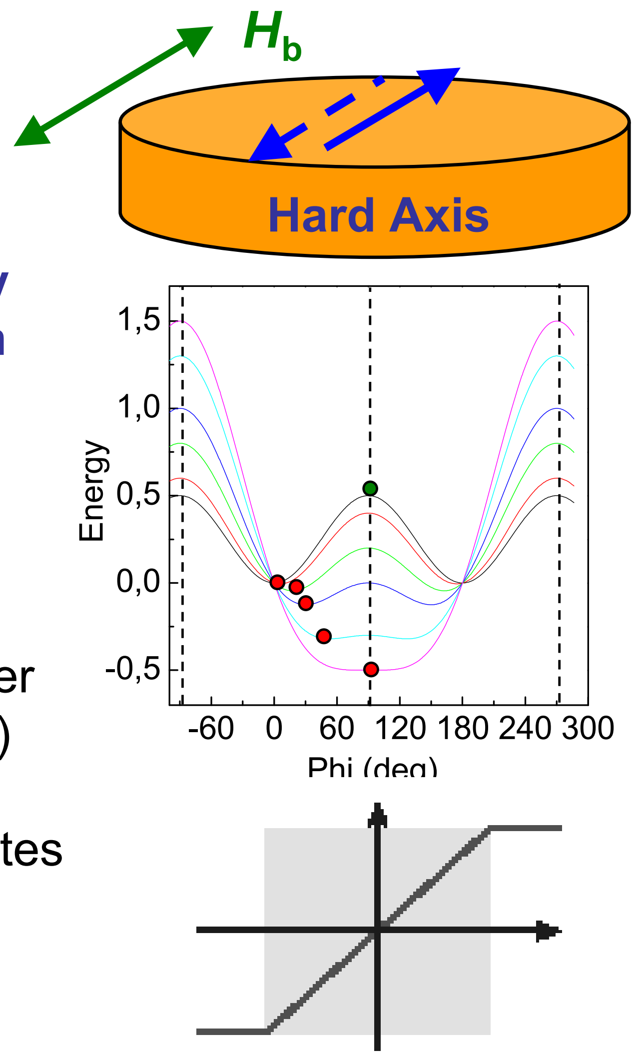
	|m|	dE/dt	Static	Dynamic
Conservative Precession Dynamics	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies

Outline

Stoner-Wohlfahrt Reversal under external bias field H_b

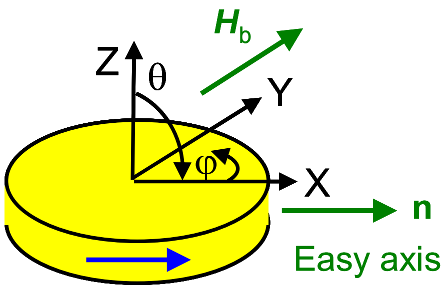


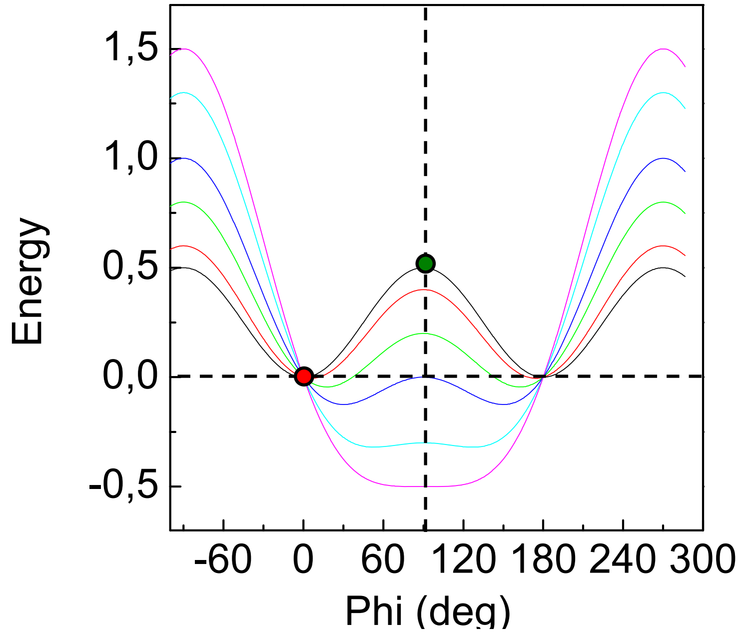
Slow Reversal between energy minima given by applied field orientation



- At each H_b, **M** relaxes to nearest local minimum.
- Relaxation (ns) much faster than rate of field sweep (ms)
- Consider only the final states not the transient

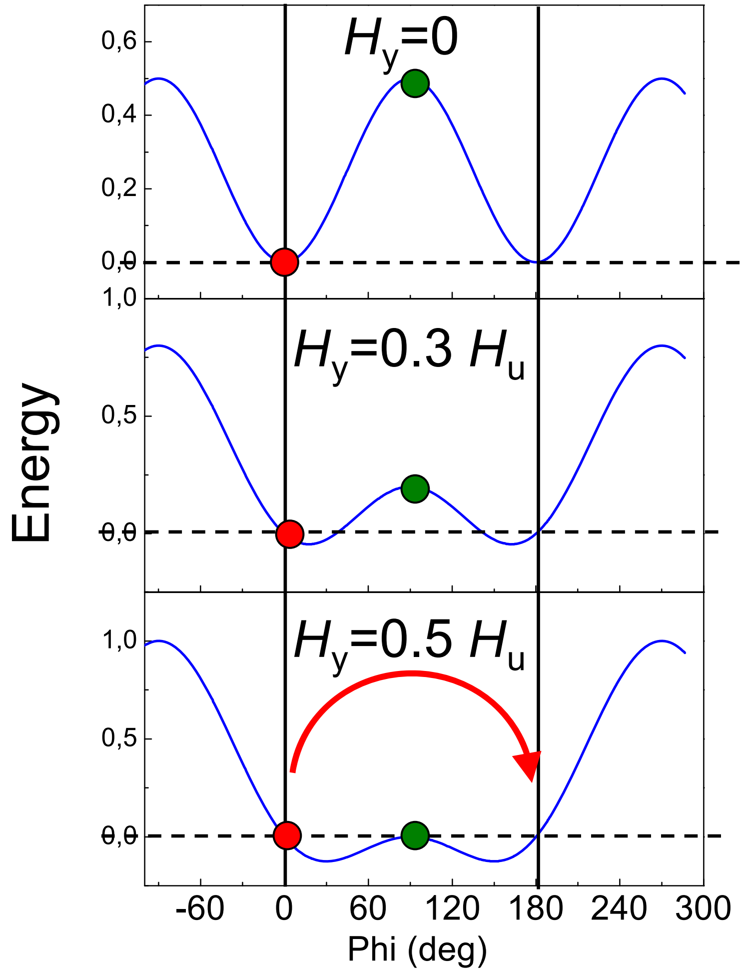
Can one switch between zero field easy axis minima using a hard axis field?





Use of hard axis field pulse of 100 ps – 1 ns
Field sweep faster than relaxation
Precessional reversal

III Precessional Reversal - Introduction



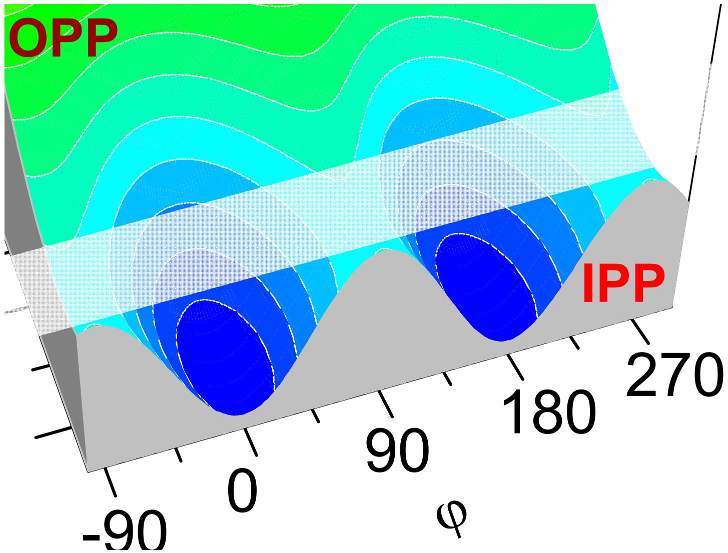
To understand the trajectory
→ need to look at solutions of
precession term with hard axis field

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

III Precessional Reversal - Hard Axis Energy Surface

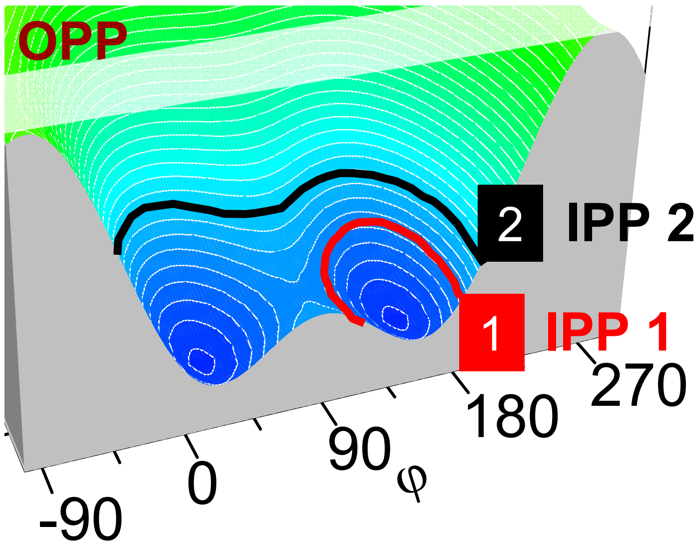
$H_y=0$



Energy < saddle point

IPP Trajectory around one energy minimum

$H_y>0$

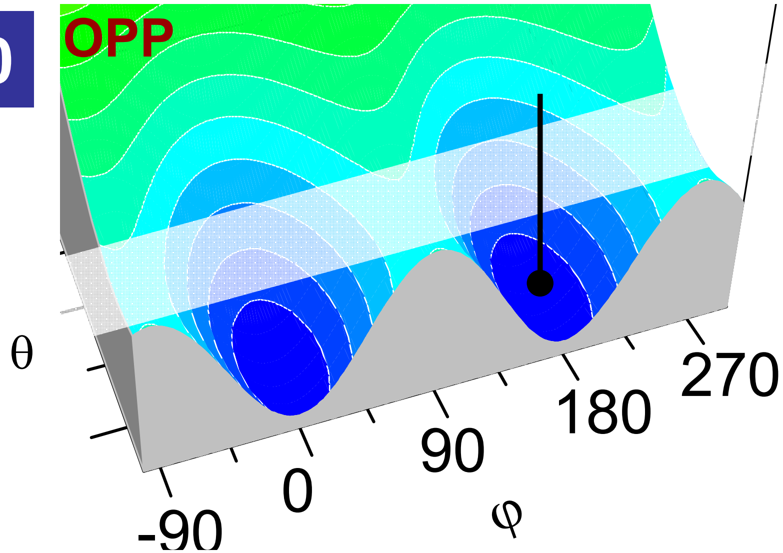


IPP 2 trajectory around both energy minima 2

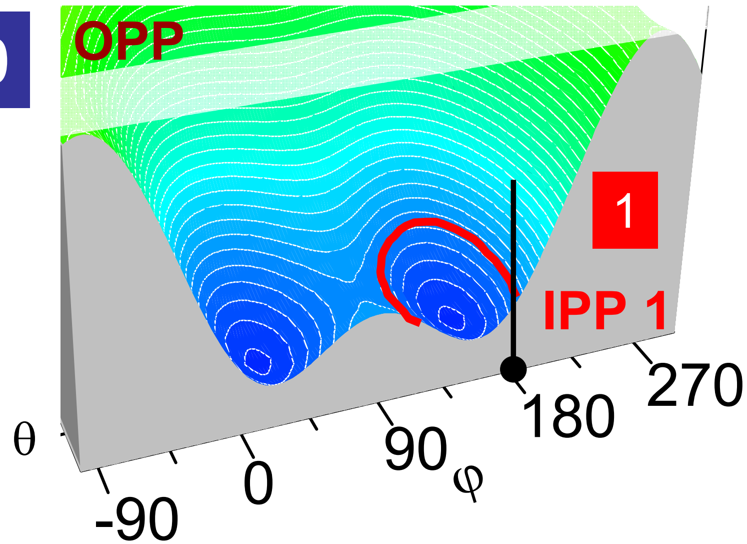
IPP 1 Trajectory around one energy minimum 1

III Precessional Reversal - Hard Axis Field Step

H_y=0

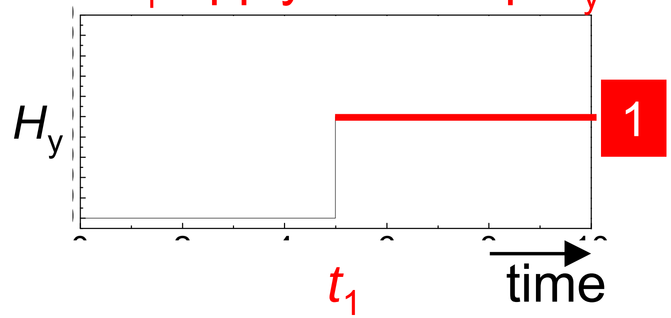


H_y>0



At t_o
M is in energy minimum φ=0°, θ=90°
- Initial energy E_o(φ=0°, θ=90°)=0

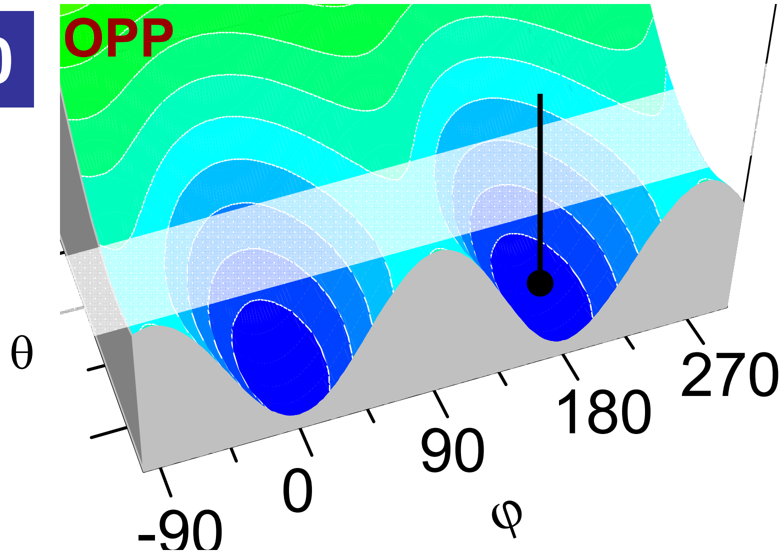
At t_1 apply field step H_y



- **M** starts to precess around new energy minimum on IPP orbit
- Trajectory is determined by initial conditions E_o(φ=0°, θ=90°)=0
- For small field step amplitude **M** precesses on IPP 1 orbit

III Precessional Reversal - Hard Axis Field Step

$H_y=0$

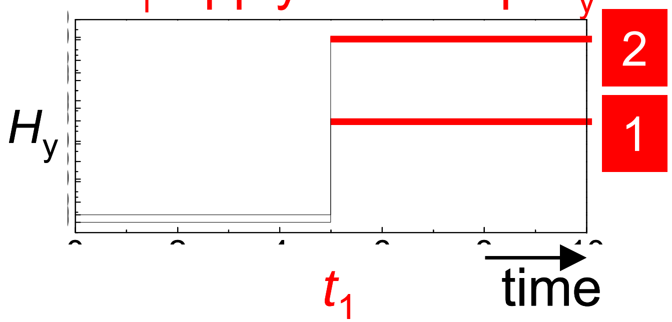


At t_o

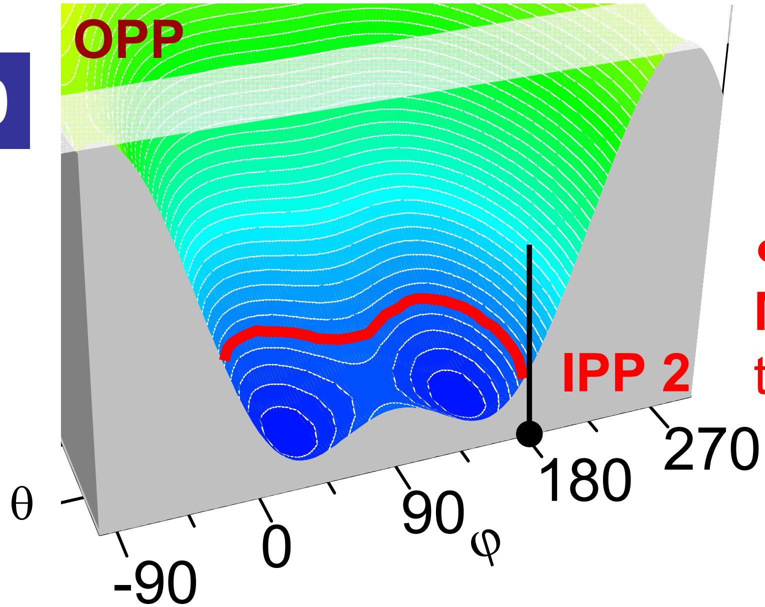
M is in energy minimum φ=0°, θ=90°

- Initial energy E_o(φ=0°, θ=90°)=0

At t_1 apply field step H_y



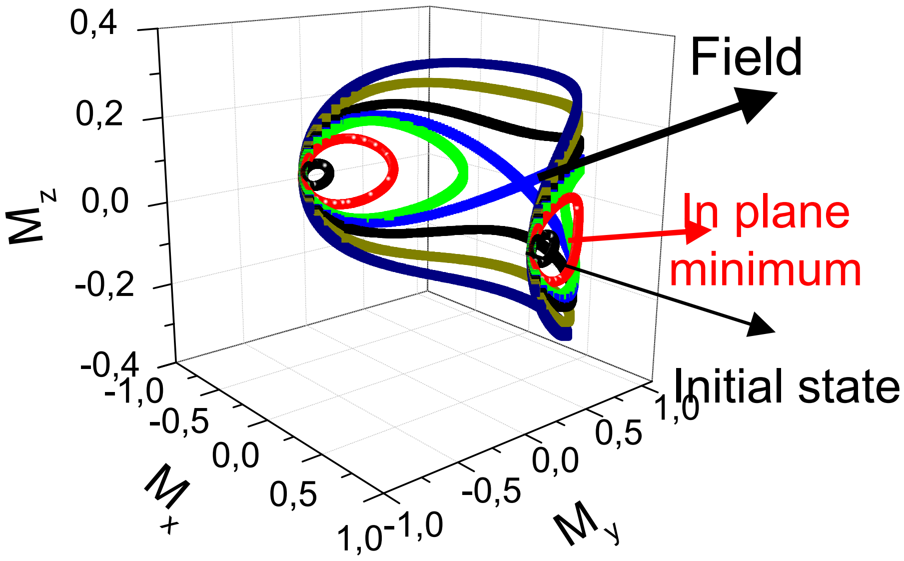
$H_y>0$

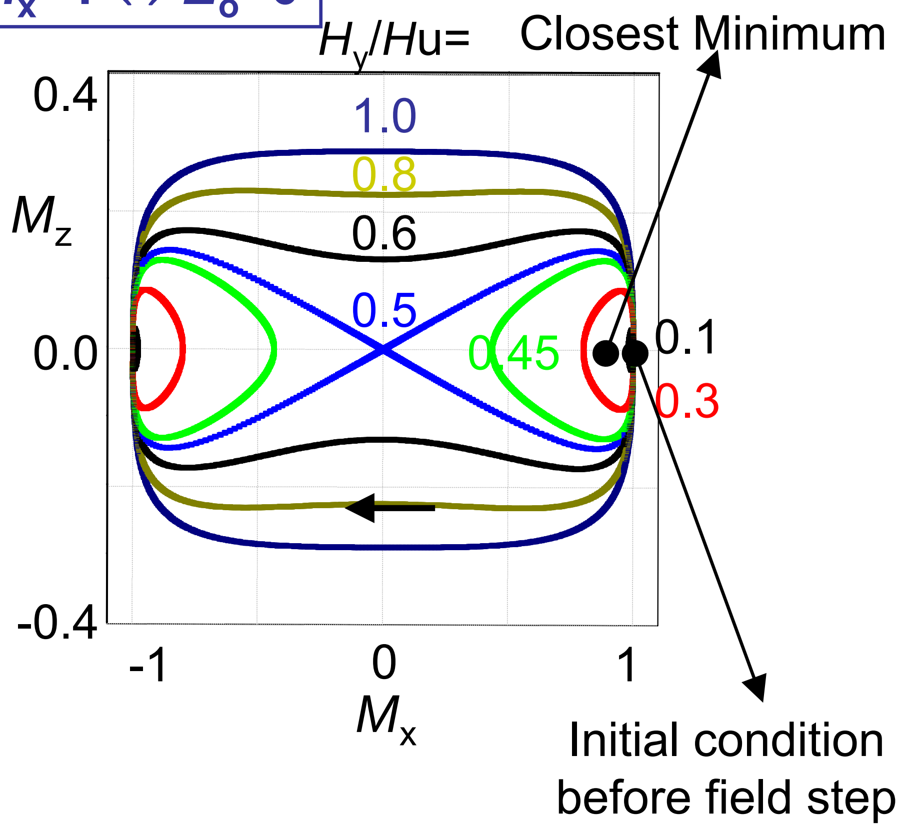


2

- For large enough field step amplitude **M** precesses on IPP 2 orbit and visits the second energy minmum

IPP trajectories for intial condition m_x=1 ⇔ E_o=0

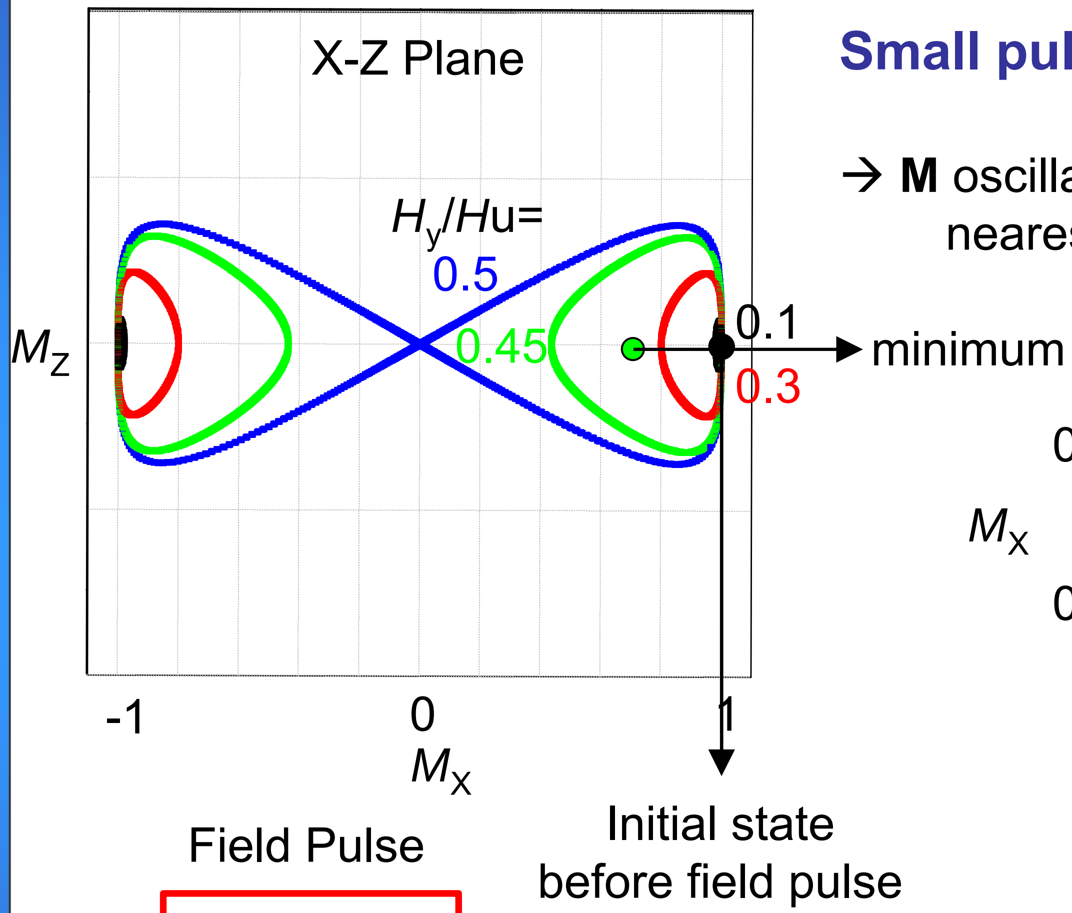




Depending on the field amplitude H_y

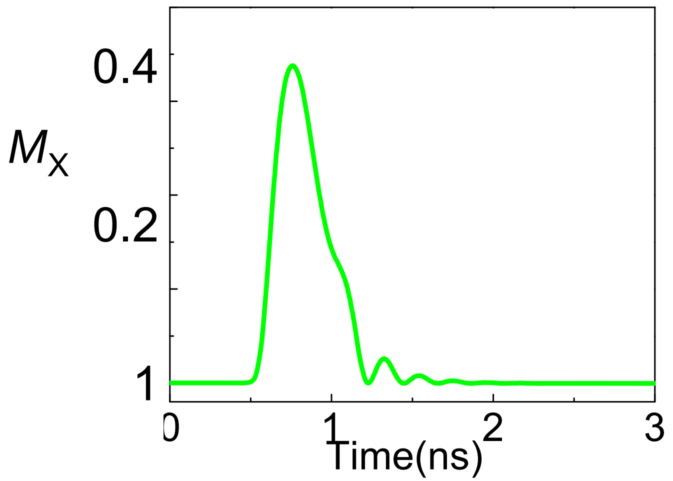
- **M** precesses on IPP 1 around closest energy minimum H_y<0.5H_u
- **M** precesses on IPP 2 around both energy minima H_y>0.5H_u
- **M** precesses on a critical trajectory H_y=0.5H_u

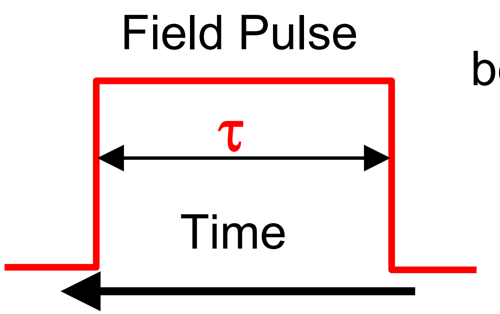
III Precessional Reversal under hard axis pulse



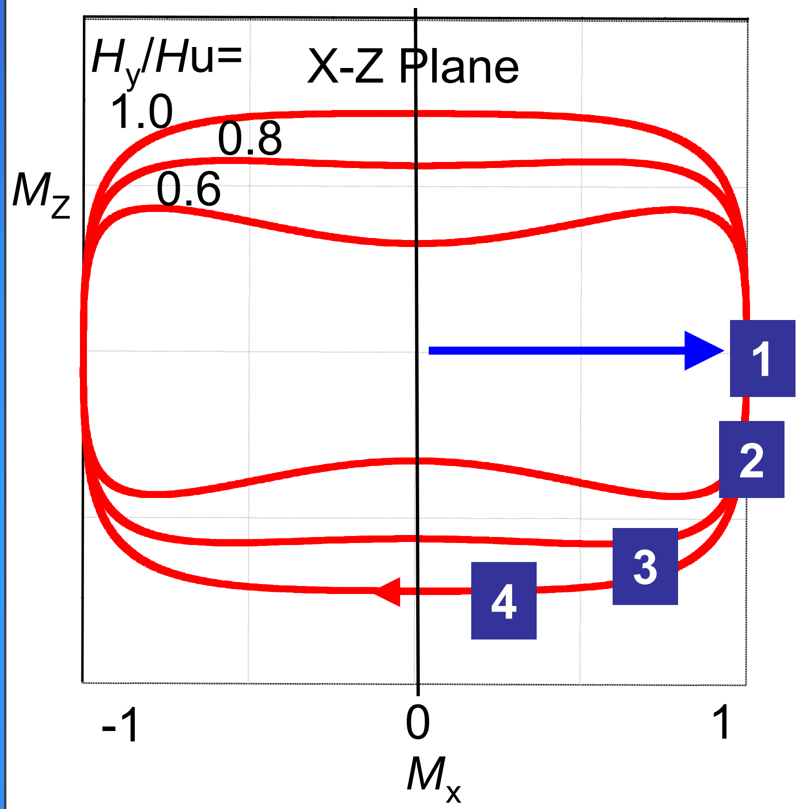
Small pulse amplitude IPP 1 orbits

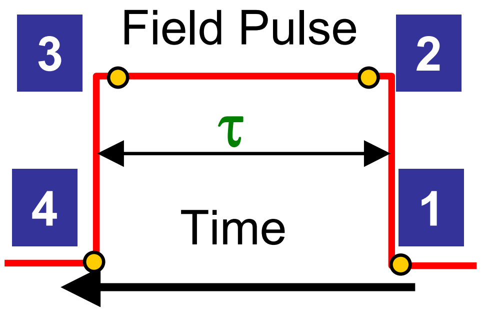
→ **M** oscillates around minimum nearest to intitial state





Keep in mind that damping will relax M into closest minimum after pulse termination

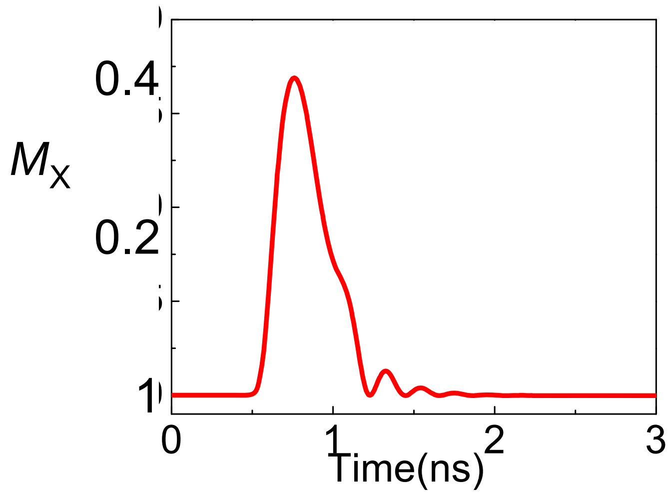




Large pulse amplitude IPP 2 orbits

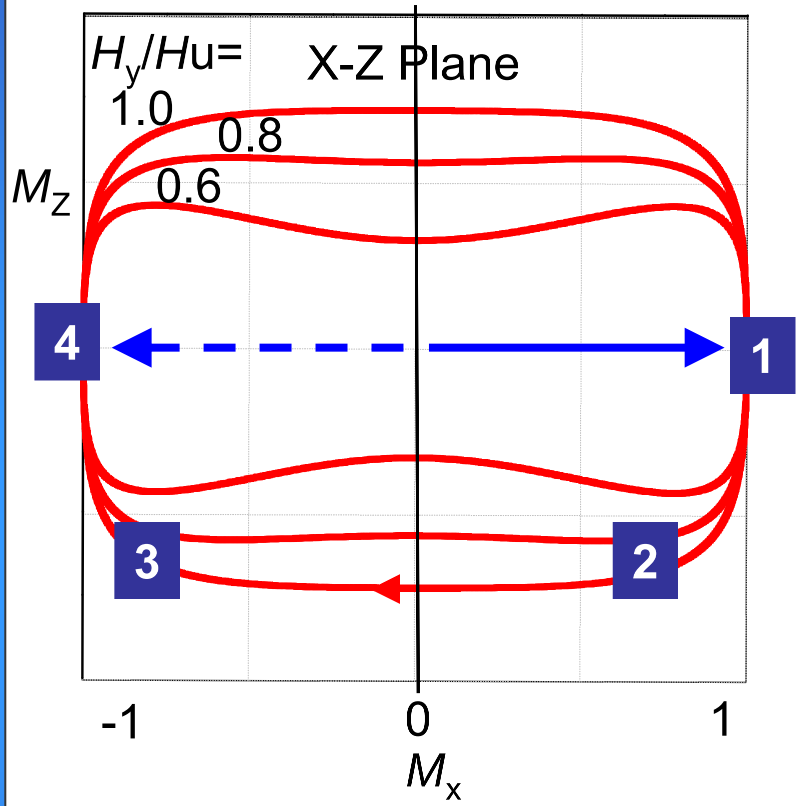
(i) Short pulse length τ

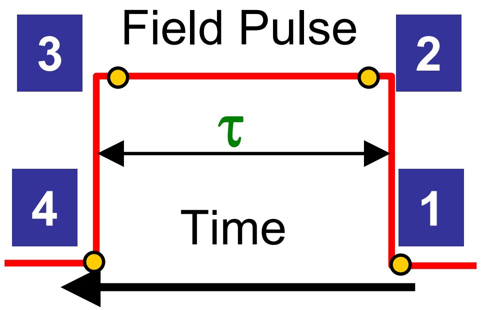
→ **M** does not cross hard axis (M_x=0) before pulse terminates and returns into intial state



Keep in mind that damping will relax **M** into closest minimum after pulse termination

III Precessional Reversal under hard axis pulse





Large pulse amplitude IPP 2 orbits

(ii) Increasing pulse length τ

→**M** visits 2nd minimum and relaxes into reversed state after pulse termination

When $\tau = (2n+1)\ T_{prec}$: **reversal**

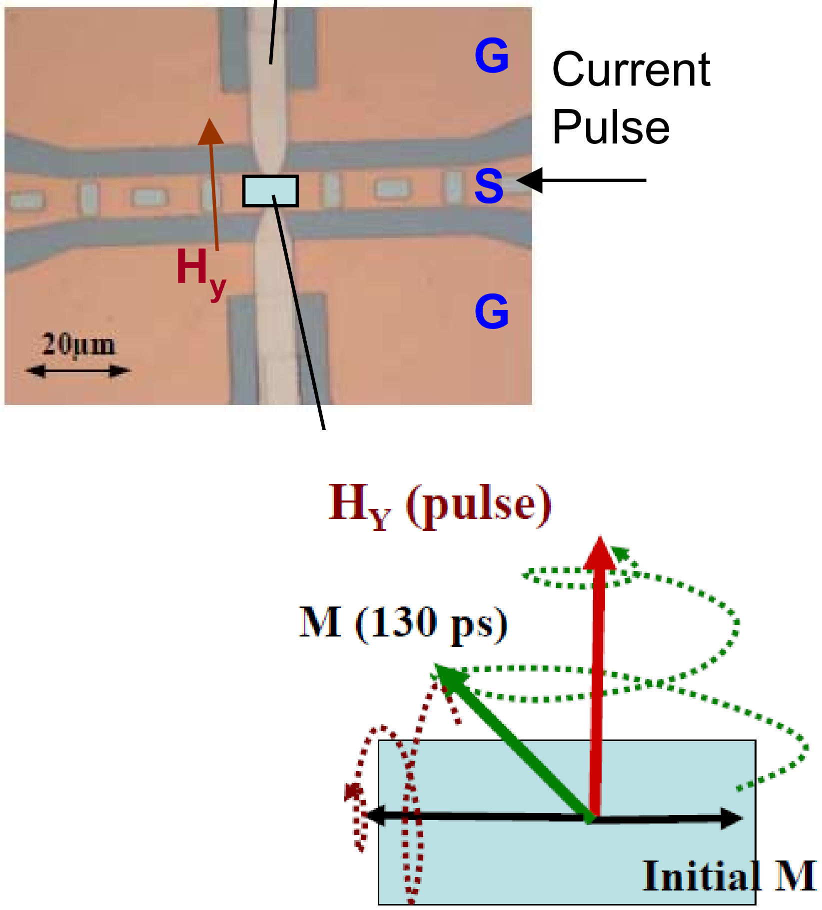
When $\tau = 2n\ T_{prec}$: **non-reversal**

with T = precession period

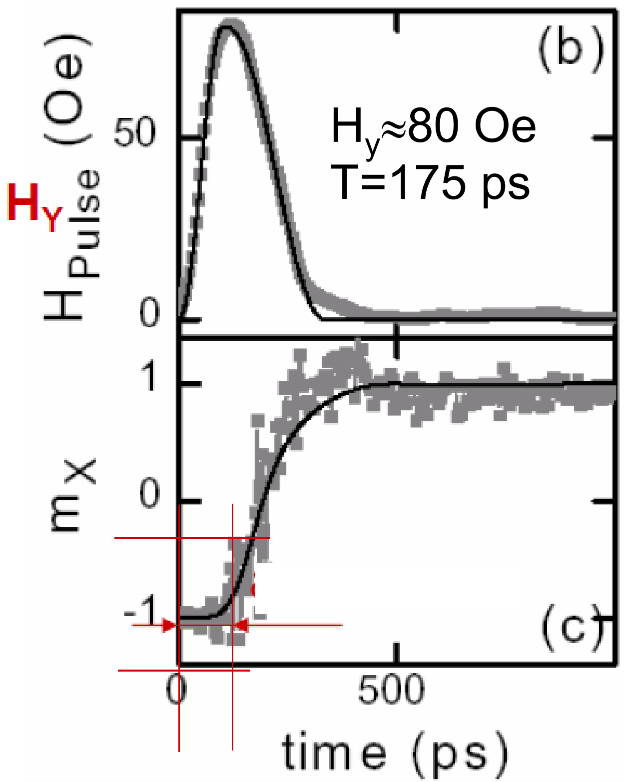
By adjusting pulse length τ and height H_y M can be reversed!

Keep in mind that damping will relax **M** into closest minimum after pulse termination

CIP contact electrodes
for CIP MR



Short pulse induces 1 reversal



IrMnCoFe/Cu/CoFe/NiFe

Spin valve element 4x2µm²

III Precessional Reversal - Experiments

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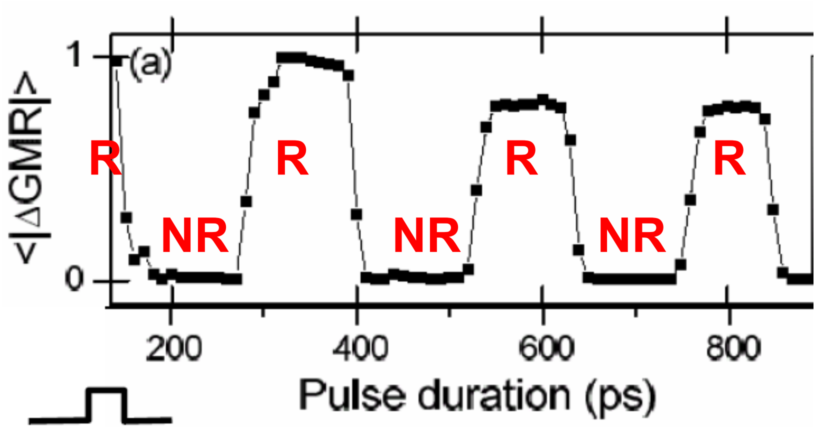
Increasing pulse length τ

Schumacher et al PRL 90, 017201 (2003)



$\tau = (2n+1)\ T_{prec}$: reversal R

$\tau = 2n\ T_{prec}$: non-reversal NR

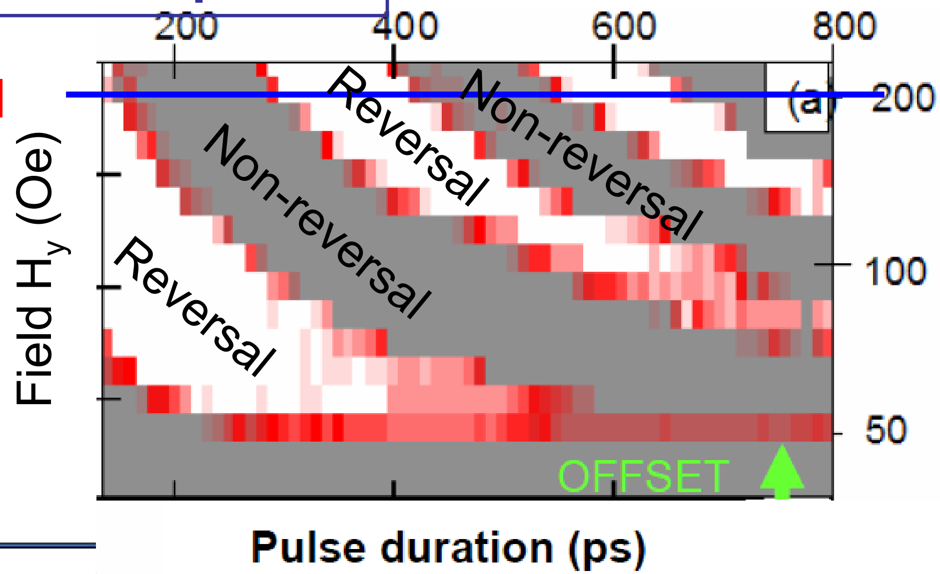
Increasing pulse length and pulse amplitude

Bands of reversal non-reversal

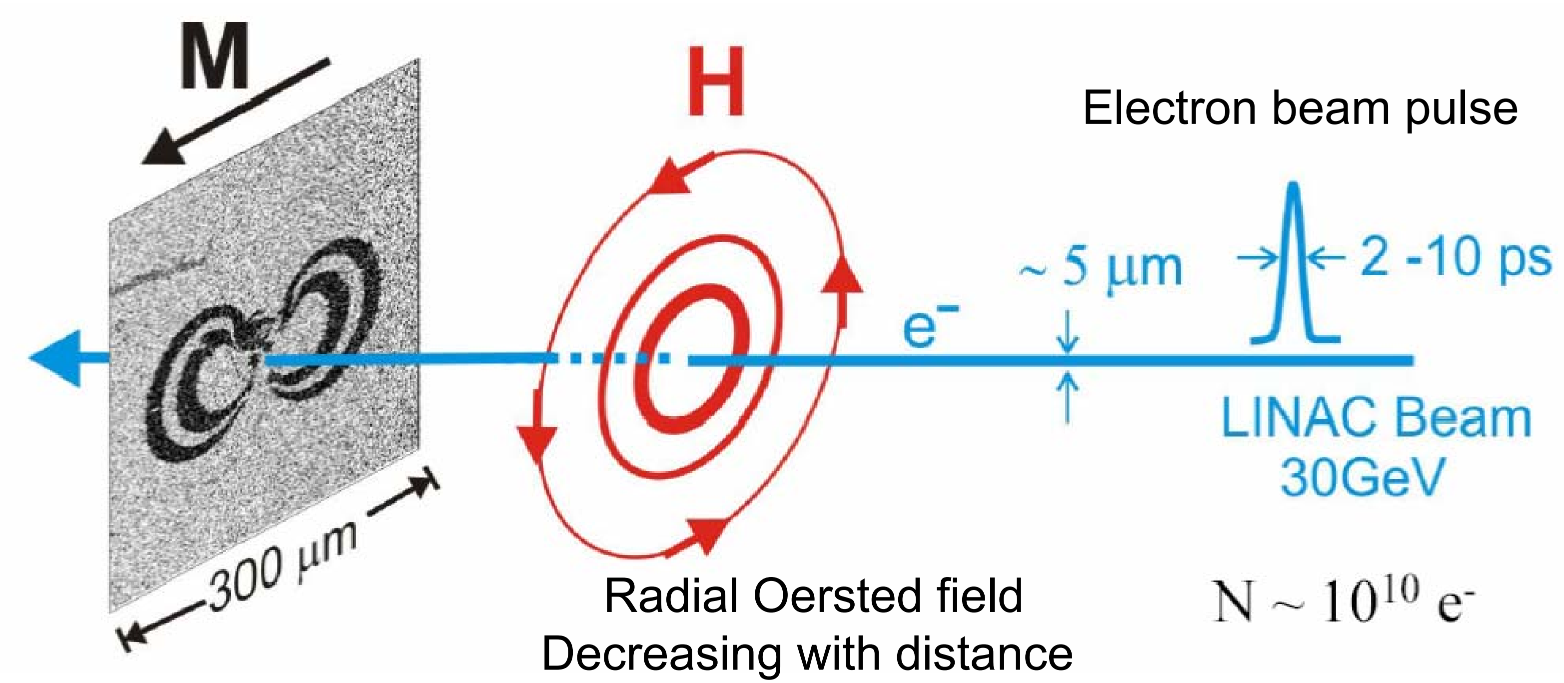
$H_y \sim 1/\tau$

$$f \approx k \frac{\gamma}{2\pi} \sqrt{H_y \cdot 4\pi M_s}$$

FMR frequency scaled by k due to large precession amplitude



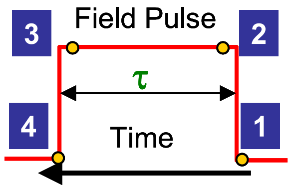
III Precessional Reversal - Experiments

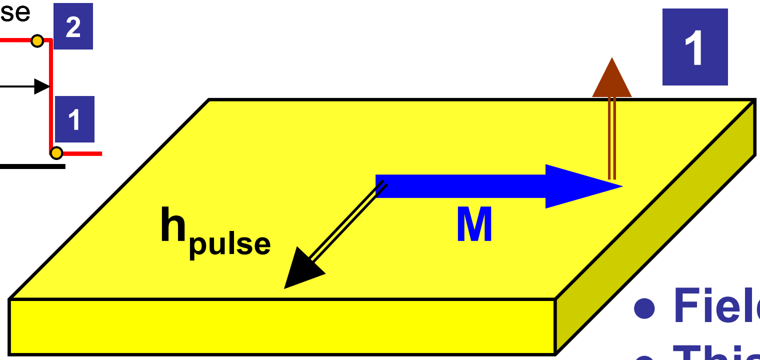


Siegmann et al., *Science* 285, 864 (1999).

III Precessional Reversal - Concept

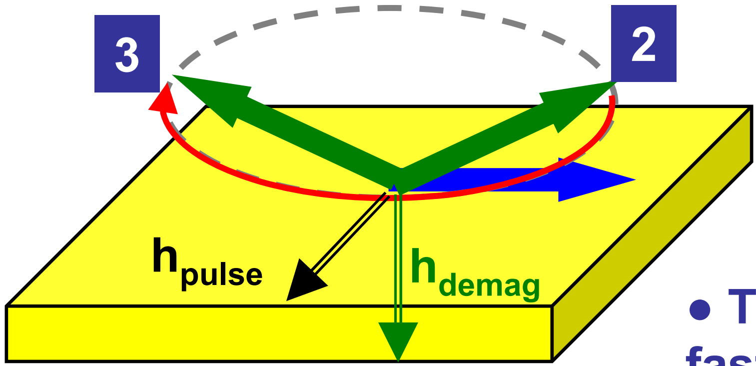
Kikuchi, JAP 27, 1352 (1956)





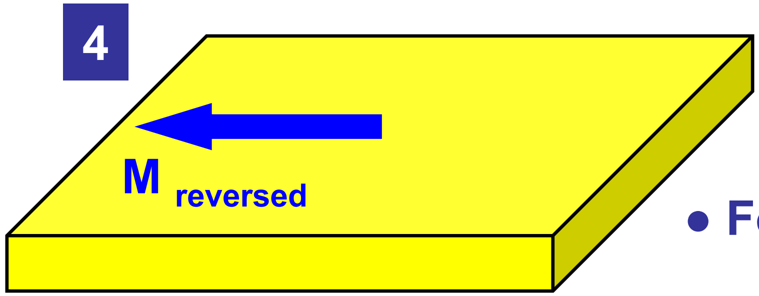
$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{h}_{pulse}$$

- Field Pulse rotates M pout of plane
- This creates a strong dipolar field

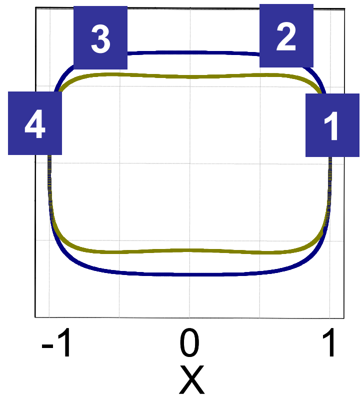


$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{h}_{demag}$$

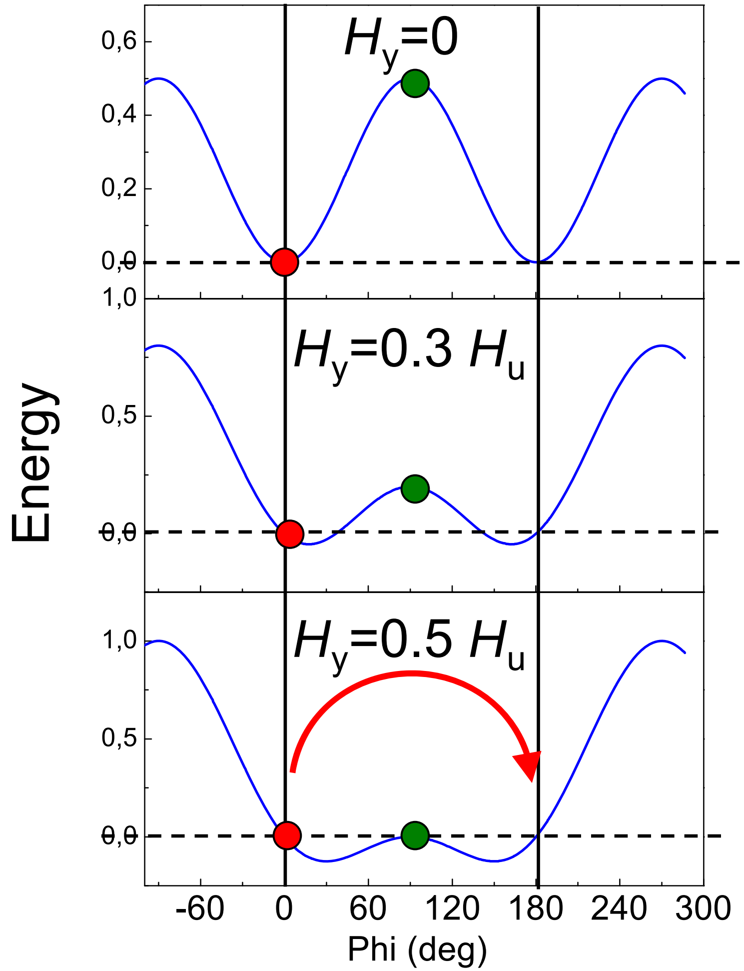
- The strong dipolar field induces fast precession



- For τ = (2n+1) T : reversal



Minimum Ampliutde H_{ymin} for IPP2 Orbits



- Initial energy, H_y=0

$$E_o = E(m_x = 1)$$

$$E_o = K_u\left[1 - m_x^2\right] + 2\pi M_s^2 m_z^2 = 0$$

- Energy at hard axis

$$E_{max} = E(\theta = 90°, \varphi = 90°)$$

$$E_{max} = K_u - M_s H_y$$

$$\boxed{E_o = E_{max} \Rightarrow H_{y\min} = \frac{H_u}{2}}$$

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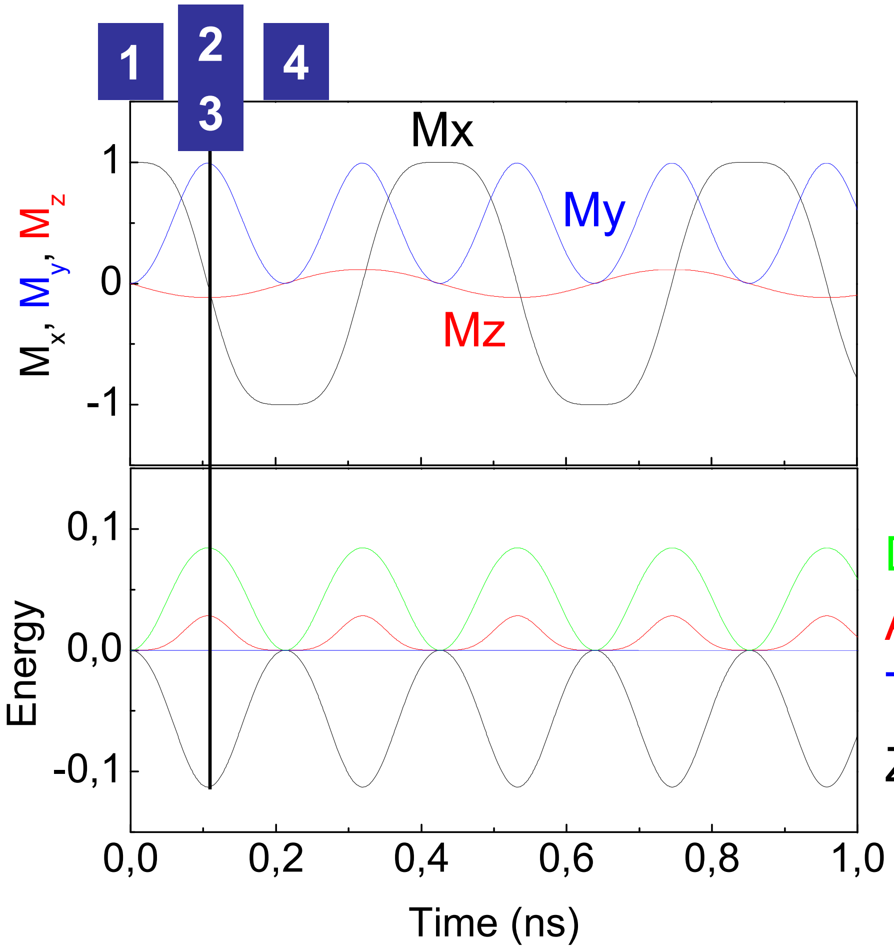
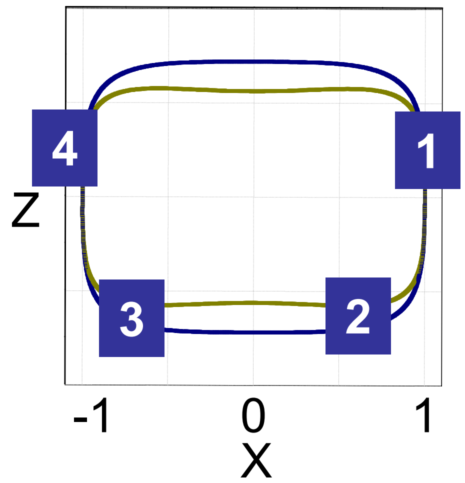
Conversion of Zeeman into demagnetization energy





Demagnetization energy

Anisotropy energy

Total Energy

Zeeman Energy

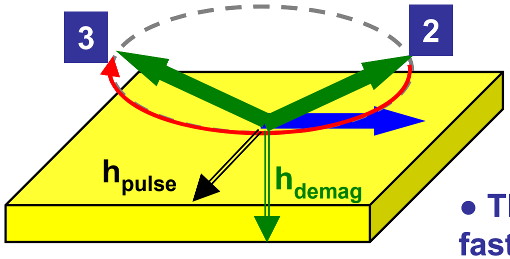
H_u = 50 Oe

H_b = 100 Oe

M_{zmax}=0.111 ⇔ 6°

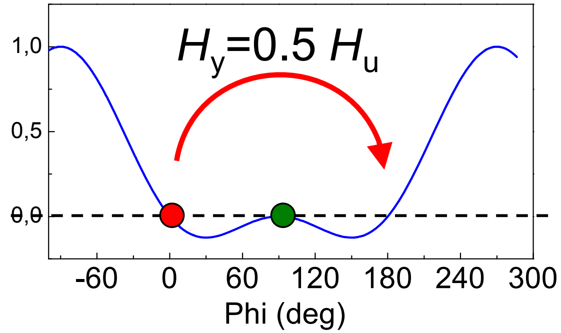
H_{dmax}=1200 Oe

III Precessional Reversal - Summary



$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{h}_{demag}$$

- **The strong dipolar field induces fast precession**



- For a thin magnetic film, a fast rising field pulse applied along the hard axis can induce a precessional reversal

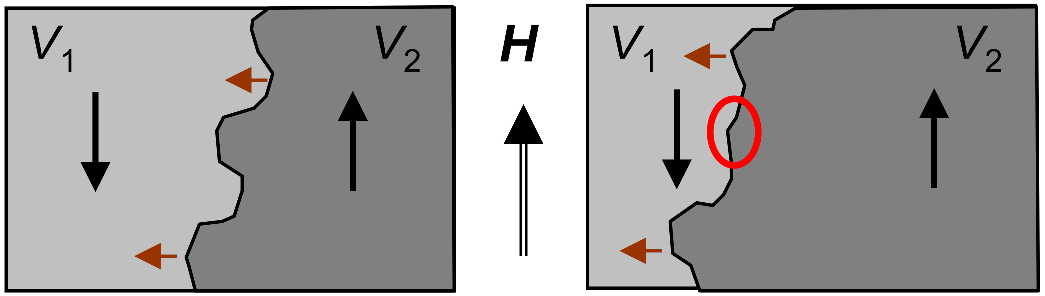
- For strong and fast enough field pulses, the magnetization follows a constant energy trajectory that goes around both energy minima

- When the pulse is turned off after 2n+1 precession cycles, M relaxes into the reversed minimum

- « Fast » means the energy gain from Zeeman has to be greater than energy loss due to damping during pulse application

Outline

Wall motion under an applied field



Time t_1 Time t_2

- Wall displaces perpendicular to field
- Domain parallel to bias field increases in size, to minimize Zeemman energy

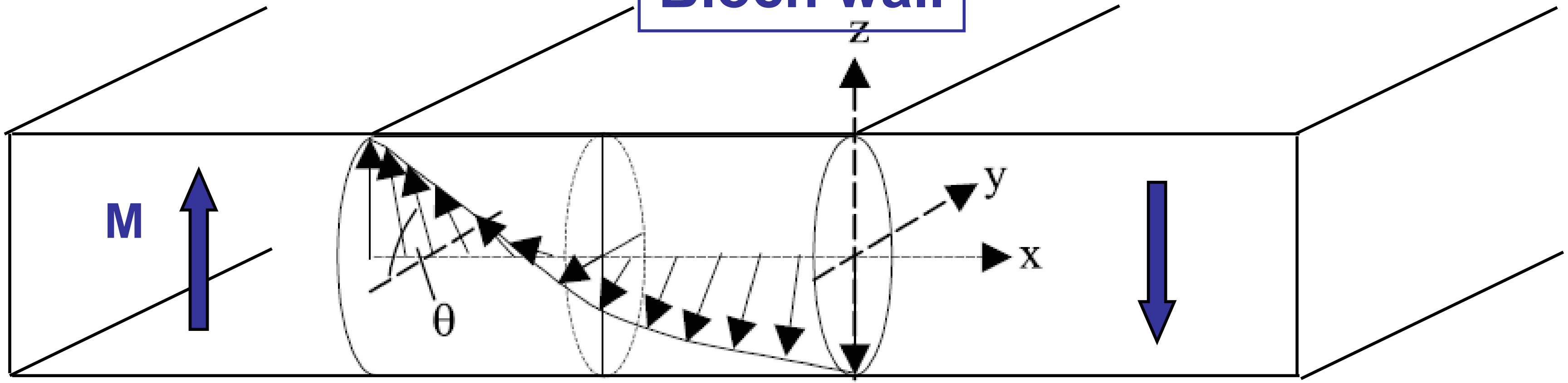
$$E = M_1 H V_1 - M_2 H V_2$$

All changes of the magnetization state pass via a precessional motion of the magnetization

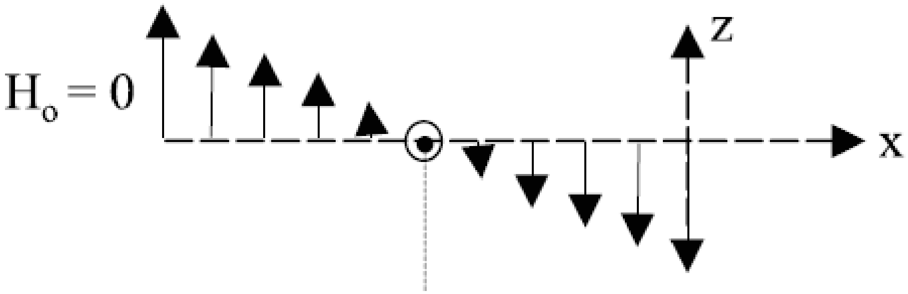
What is underlying process?

IV Domain wall Dynamics - Static Wall

Bloch wall



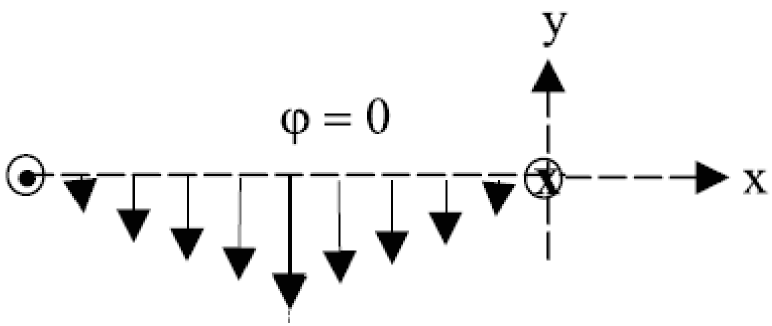
Side view



H_b

Upon application of an upward field, the wall displaces to the right

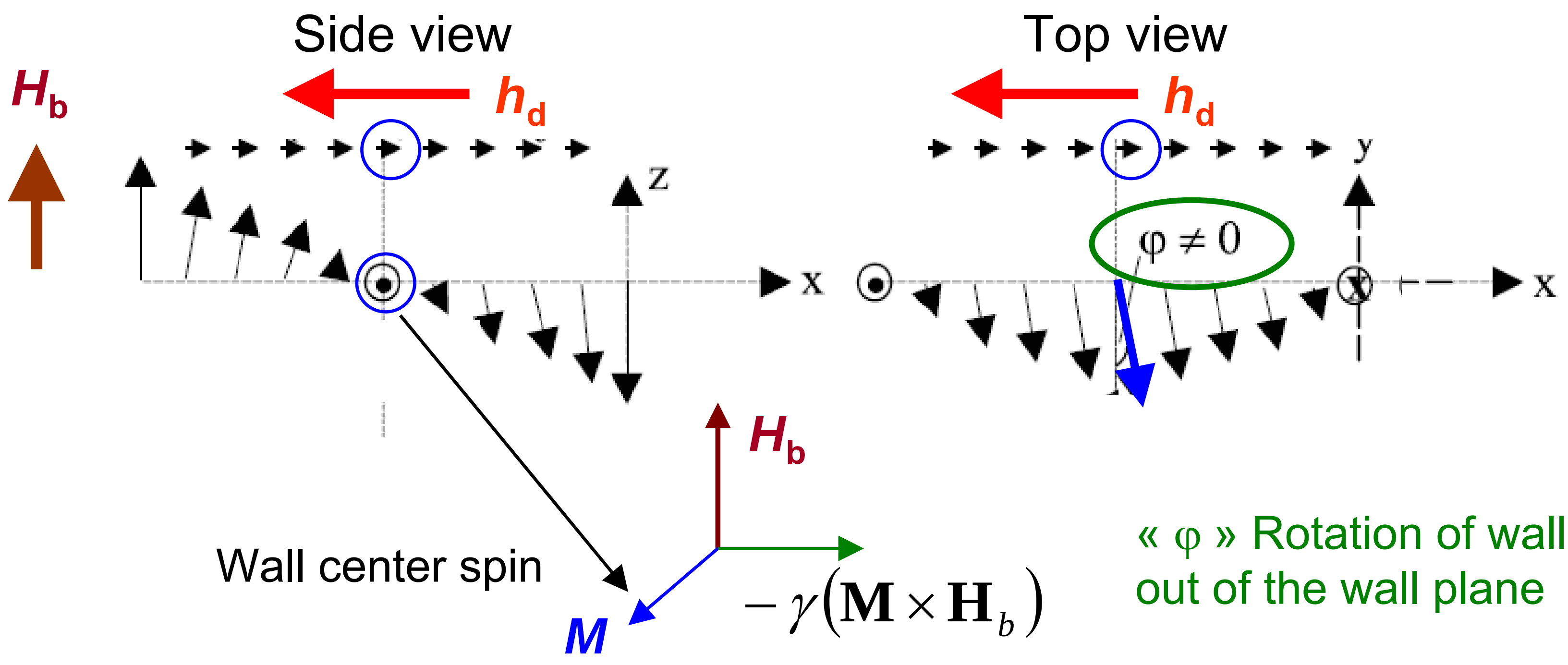
Top view



Two step reversal of magnetization inside the wall

1) Rotation of wall spins around the external field H_b

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_b\right)$$



« φ » Rotation of wall spins out of the wall plane

This rotation leads to an internal dipolar field h_d

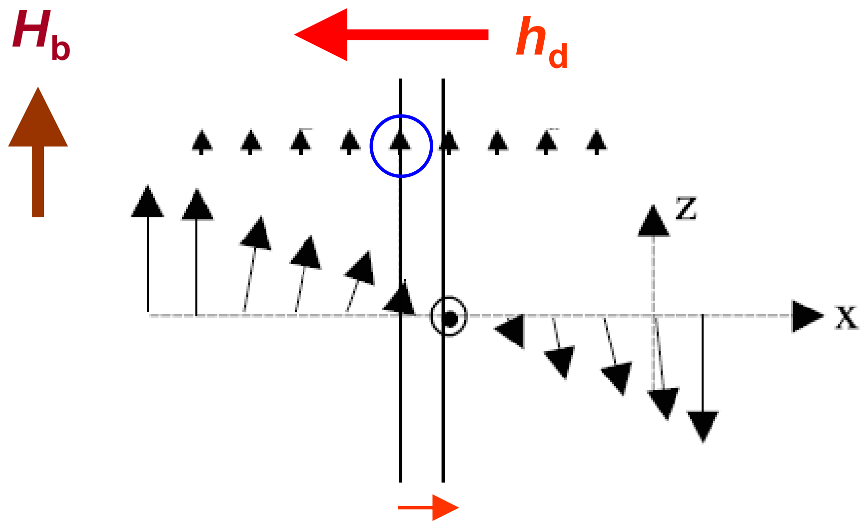
spintec SPIN IN ELECTRONICS

Two step reversal of magnetization inside the wall

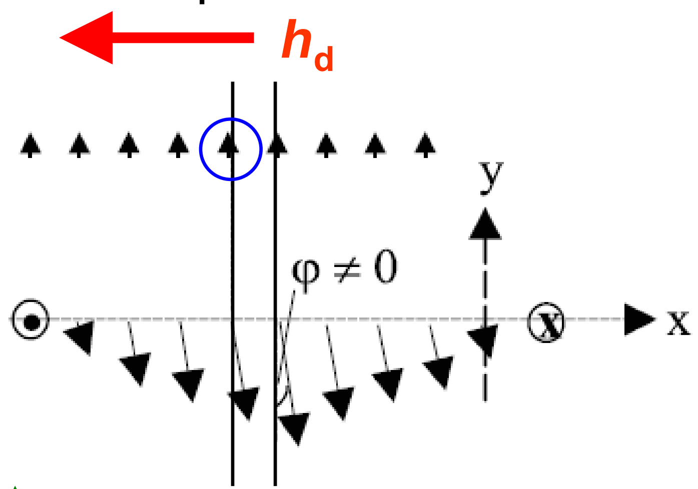
2) Rotation of wall spins around the dipolar field h_d

$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{h}_d\right)$$

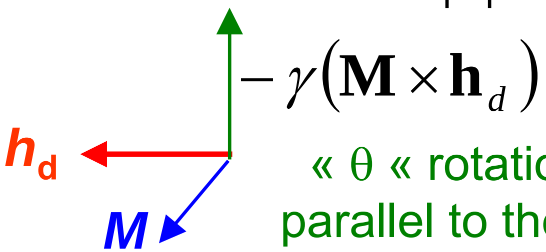
Side view



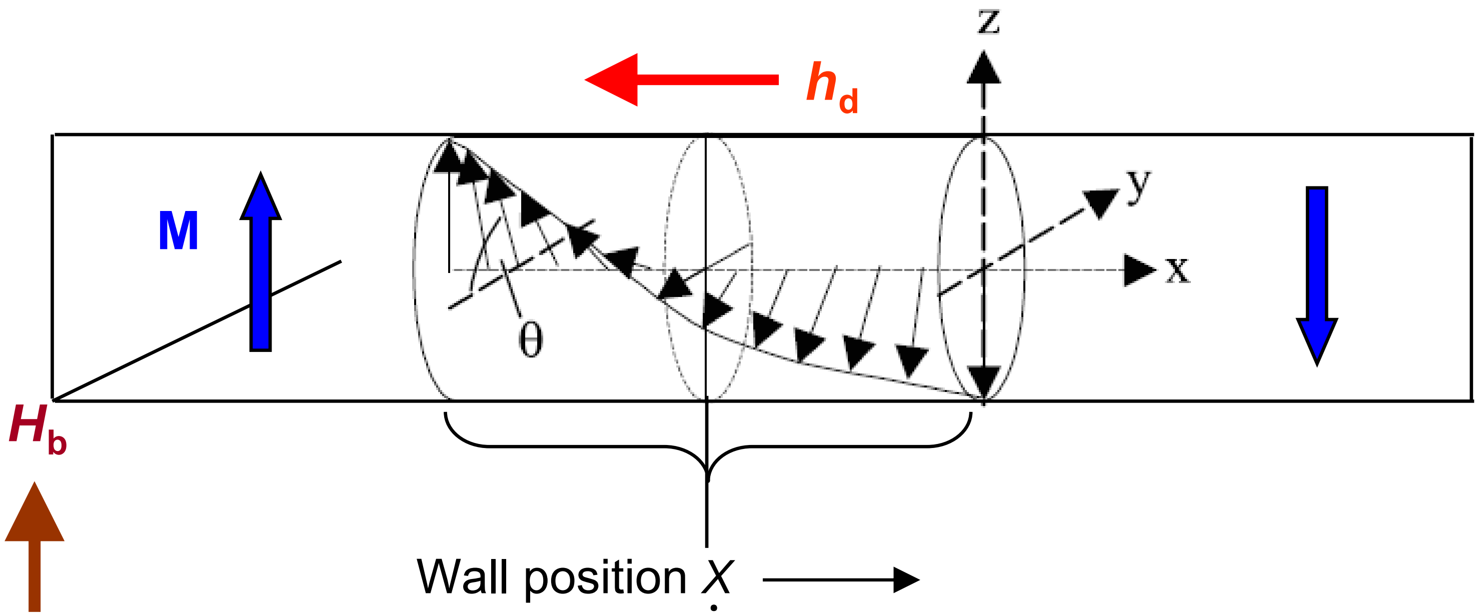
Top view



Rigid displacement of wall profile due to θ rotation



« θ « rotation of spins parallel to the wall plane



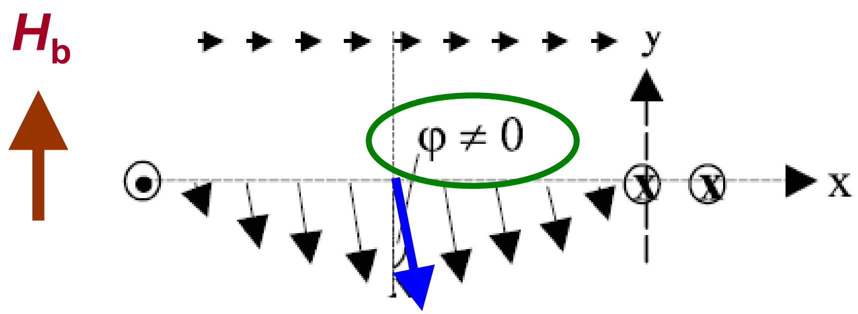
Wall position X ⟶

Wall speed $\dot{X} = v \sim \varphi \sim H_b$

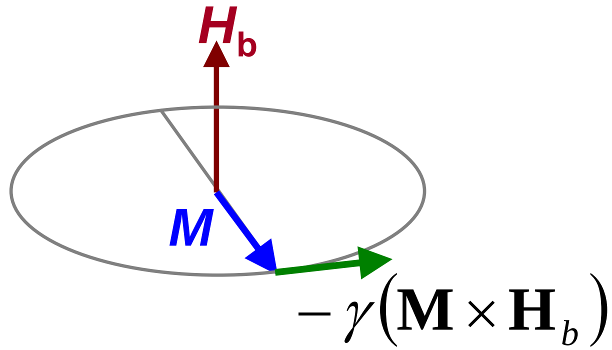
- The larger the bias field H_b, the larger the angle φ
- The larger φ, the stronger h_d and in consequence the faster the « θ » rotation
- The faster the « θ » rotation, the faster the wall displaces
- For constant φ, constant wall velocity v

- **Why is φ constant?**
- **For constant H_b, the wall spins should precess around H_b**

Top view



« φ » Rotation of spins out of the wall plane



$$\frac{d\mathbf{M}}{dt} = -\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right) + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

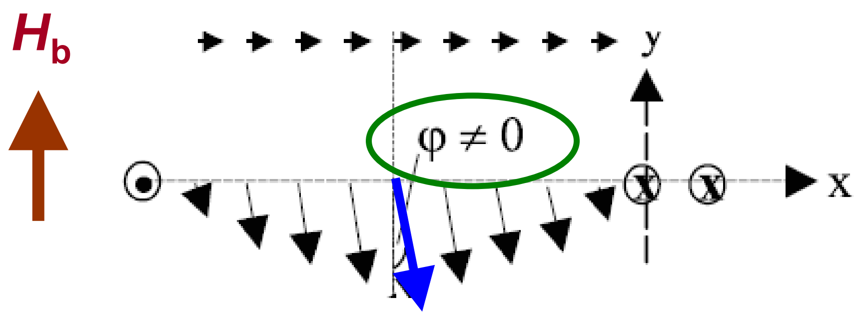
Due to damping, φ remains constant

The fast θ rotation due to h_d provides a strong damping torque that counteracts the precession torque around H_b

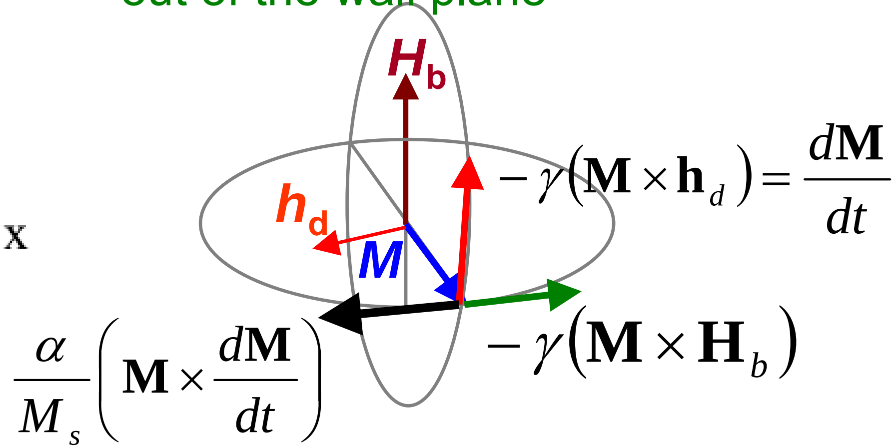
IV Domain wall Dynamics -Wall Displacement

- **Why is φ constant?**
- **For constant H_b, the wall spins should precess around H_b**

Top view



« φ » Rotation of spins out of the wall plane



$$\frac{d\mathbf{M}}{dt}=-\gamma(\mathbf{M}\times\mathbf{h}_d)+\boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Due to damping, φ remains constant

Upon field application, φ increases until damping torque due to θ rotation is strong enough to counterbalance φ rotation

spintec SPIN IN ELECTRONICS

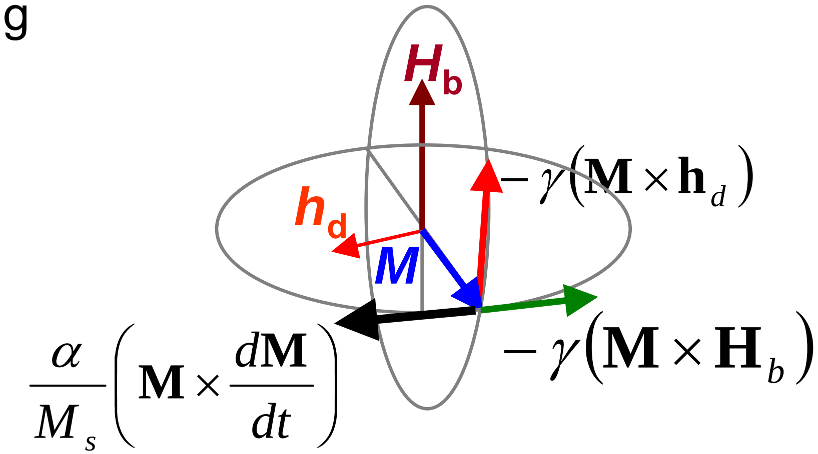
Two step reversal of magnetization inside the wall

1) Balance between precession and damping

$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)=\frac{-\gamma\alpha}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{h}_d\right)\right)$$

$$\frac{-\gamma\alpha}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{h}_d\right)\right)=-\gamma\left(\mathbf{M}\times\mathbf{H}_b\right)$$

$$\alpha h_d\cos\varphi=H_b$$



2) Precession around $\mathbf{h}_d$

$$\frac{d\mathbf{M}}{dt}=-\gamma\left(\mathbf{M}\times\mathbf{h}_d\right)$$

$$\left|\frac{d\mathbf{M}}{dt}\right|=\gamma M_s h_d\cos\varphi \qquad \left|\frac{d\mathbf{M}}{dt}\right|=M_s\dot{\theta}$$

$$\boxed{\left|\frac{d\mathbf{M}}{dt}\right|=\gamma M_s\frac{H_b}{\alpha}} \qquad \left|\frac{d\mathbf{M}}{dt}\right|=M_s\frac{\partial\theta}{\partial x}\frac{\partial x}{\partial t}=M_s\frac{\partial\theta}{\partial x}v=\boxed{M_s\frac{\pi}{\Delta}v}$$

$$\boxed{\begin{array}{l} v=\dfrac{\gamma\Delta}{\alpha\pi}H_b \\ v=\mu H_b \end{array}}$$

μ wall mobility

IV Domain wall Dynamics -Wall Displacement

spintec SPIN IN ELECTRONICS

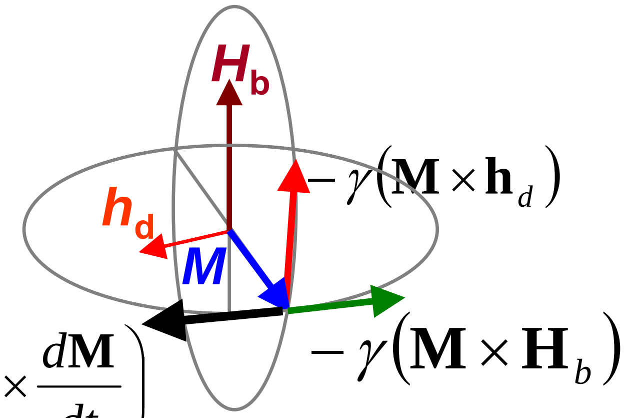
Two step reversal of magnetization inside the wall

1) Balance between precession and damping

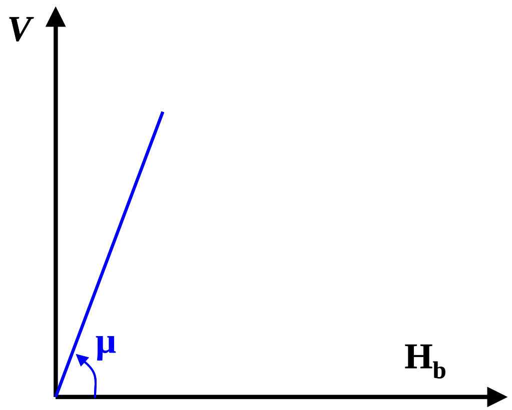
$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)=\frac{-\gamma\alpha}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{h}_d\right)\right)$$

$$\frac{-\gamma\alpha}{M_s}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{h}_d\right)\right)=-\gamma\left(\mathbf{M}\times\mathbf{H}_b\right)$$

$$\alpha h_d\cos\varphi=H_b$$



$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)$$



$$v=\frac{\gamma\Delta}{\alpha\pi}H_b$$

$$v=\mu H_b$$

μ wall mobility

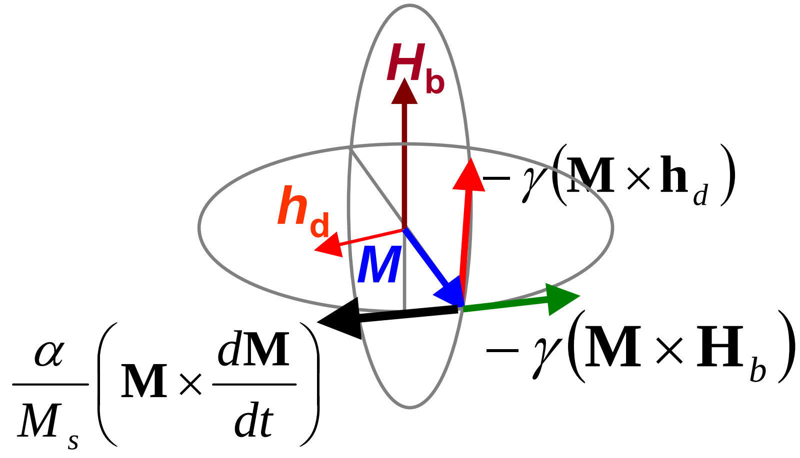
Maximum dipolar field when static rotation of $\varphi = 45°$

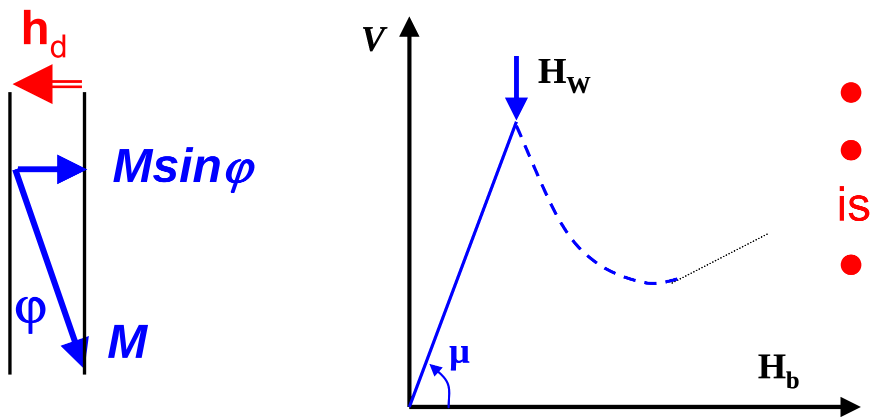
1) Balance between precession and damping

$$\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)=\frac{-\gamma\alpha}{M_s}(\mathbf{M}\times(\mathbf{M}\times\mathbf{h}_d))$$

$$\alpha h_d \cos\varphi = H_b$$

$$\boxed{\begin{aligned} h_d &= 4\pi M_s \sin\varphi \\ \alpha 4\pi M_s \sin\varphi\cos\varphi &= H_b \end{aligned}}$$





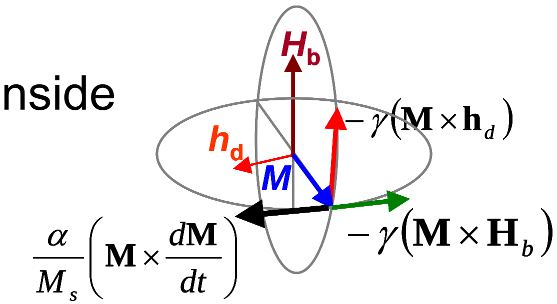
Walker breakdown

- Maximum damping torque at φ=45°.
- For larger φ, precession torque from **H**b is no more compensated
- Wall spin precess continously

$$\boxed{H_w = \alpha 2\pi M_s}$$

spintec SPIN IN ELECTRONICS

The domain wall motion under applied field passes in two steps



- The applied field creates a torque that rotates the spins inside the domain wall (distortion of wall profile):
  « φ » rotation for a Bloch wall

- The resulting dipolar fields create a torque that rotate the wall spins into the direction of the reversed domain:
  « θ » rotation for a Bloch wall

- As a result the (distorted) domain wall profile displaces at constant velocity proportional to the applied field
- The initial « φ » rotation is at a constant angle due to the balance of damping torque and precession torque
- Above a certain bias field, this damping cannot balance the precession torque: Walker breakdown

Summary

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma(\mathbf{M}\times\mathbf{H}_{eff})} + \boxed{\frac{\alpha}{Ms}\mathbf{M}\times\frac{d\mathbf{M}}{dt}}$$

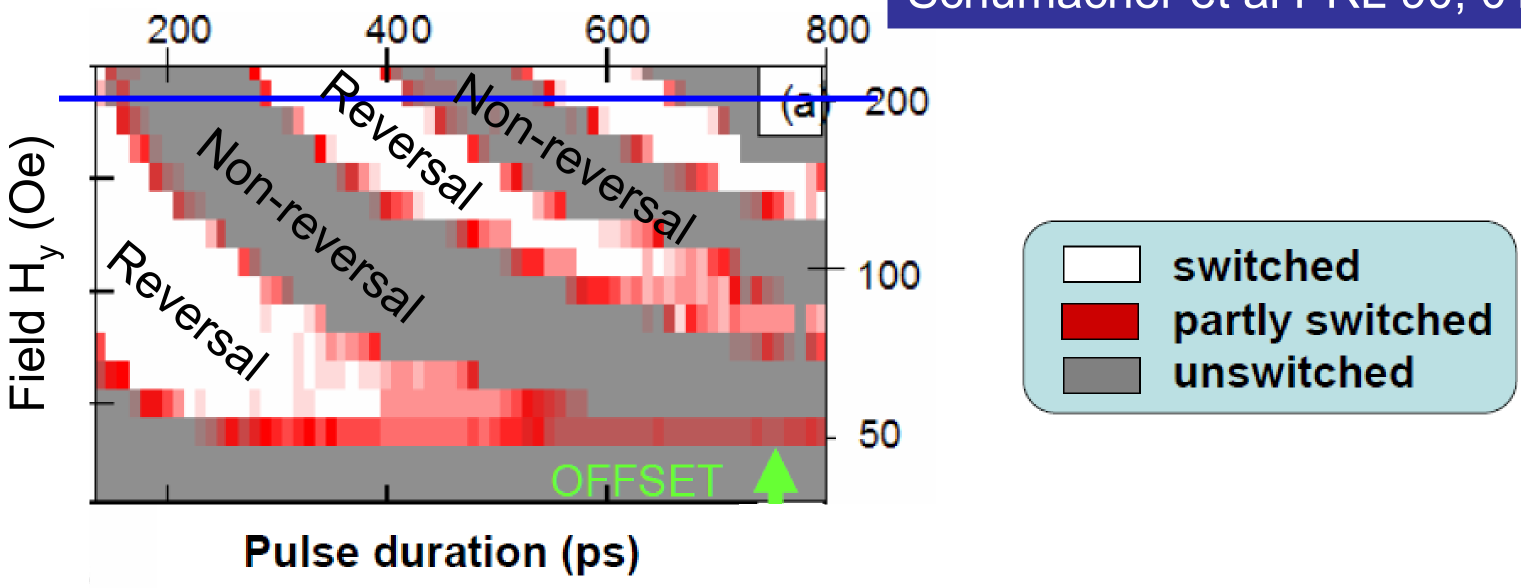
Precession **Damping**

	|m|	dE/dt	Static	Dynamic
Conservative Precession Dynamics	1	0	2 stable foci 1 saddle	Closed orbits around foci Given by intial condition Non-linear frequency shift
Non-conservative LLG	1	<0	1 stable focus 1 unstable focus 1 saddle	Damped oscillations around stable focus FMR frequencies

- Precessional reversal under fast hard axis field – along constant energy trajectory. Its a two step reversal
- Similarly, domain wall displaces under field due to large angle precession of wall spins.Two step reversal

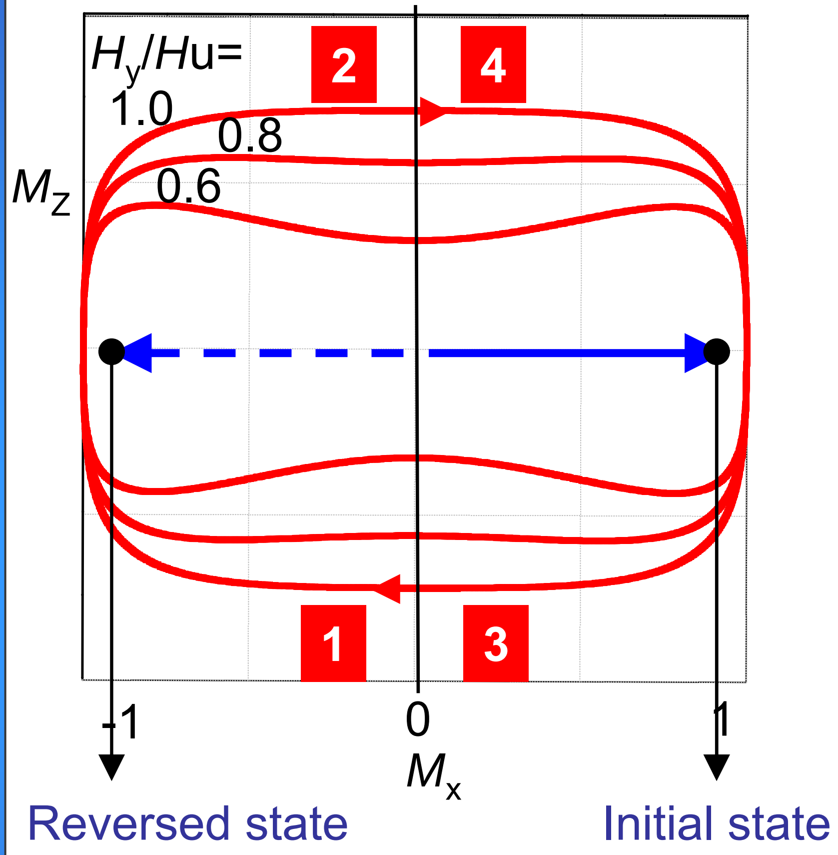
III Precessional Reversal - Characteristics

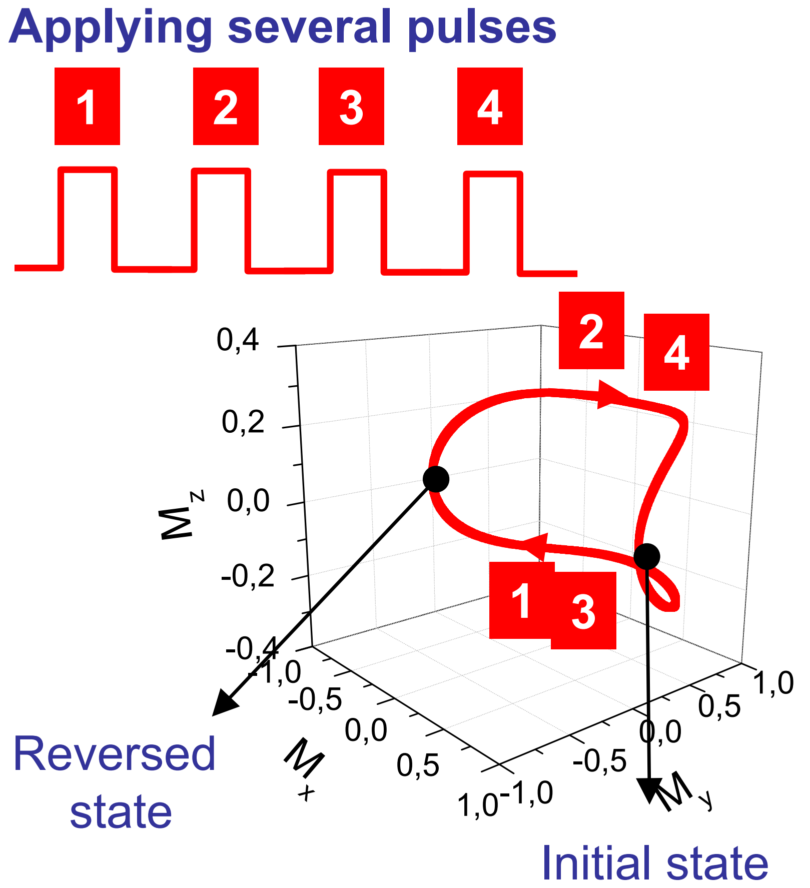
Schumacher et al PRL 90, 017201 (2003)



- **Minimum Pulse Ampliutde H_{ymin}**
- **Low probability of succesful switching at minimum H_y**
- **Low probability of succesful switching at transitions from R ⇔ NR**

III Precessional Reversal under hard axis pulse

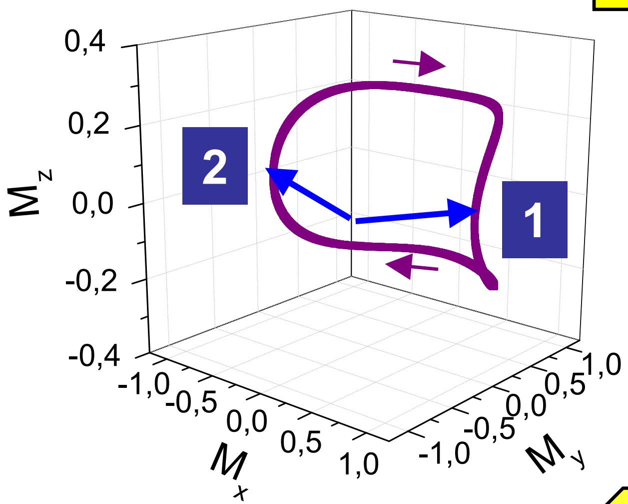


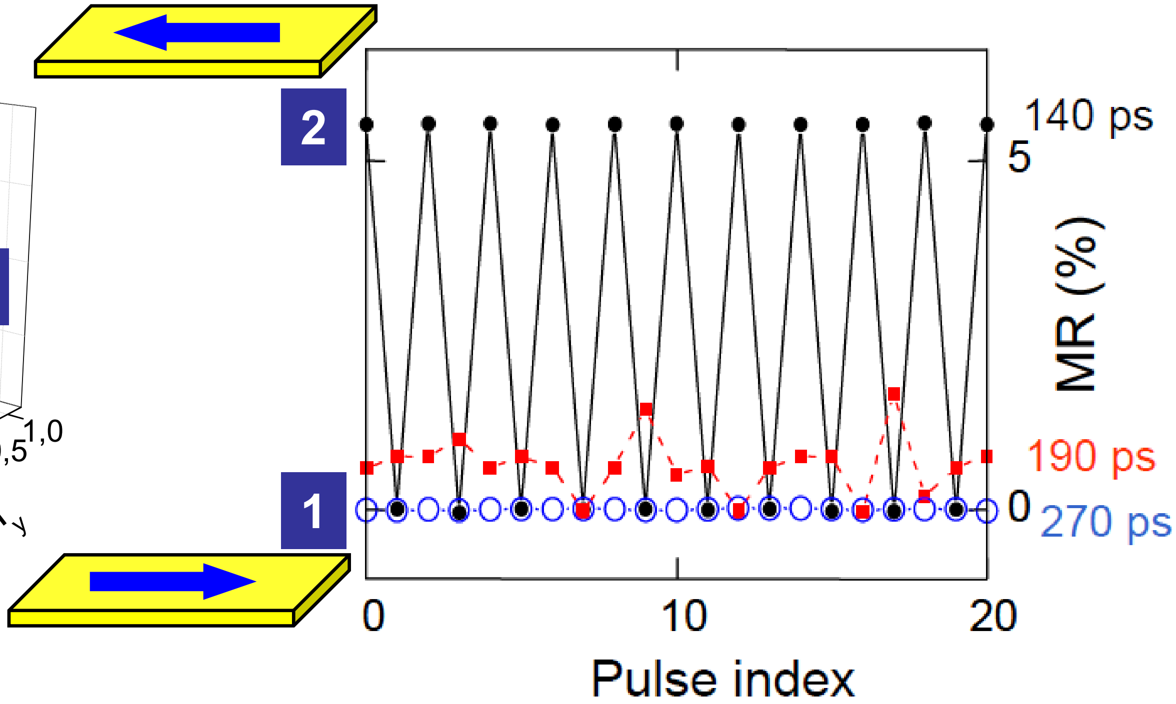


- First pulse switches **M** from initial → reversed state
- Second pulse switches back from reversed → initial state
- Switching back along same trajectory, but with M_z

III Precessional Reversal - Experiments

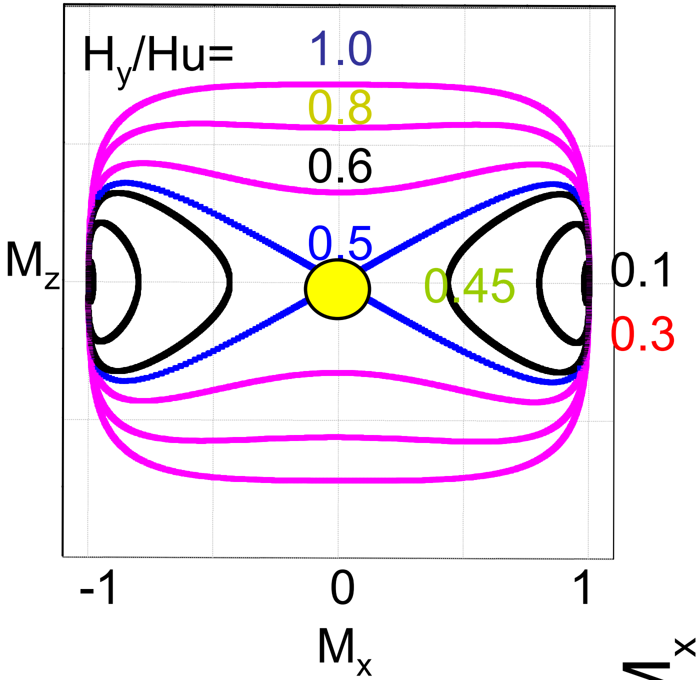
Repeated pulses induce repeated switching back and forth

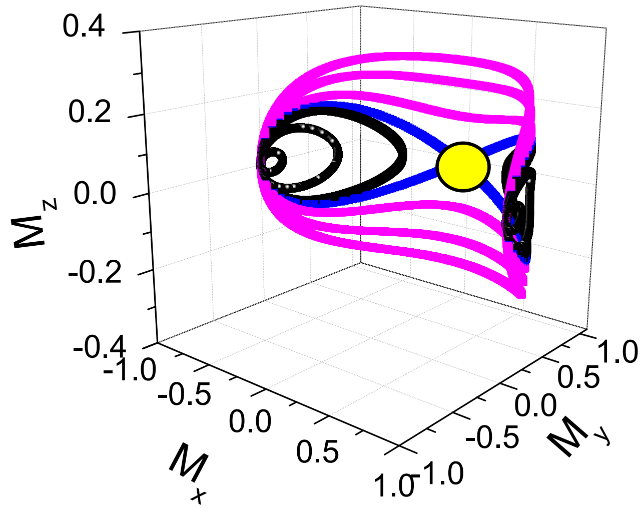




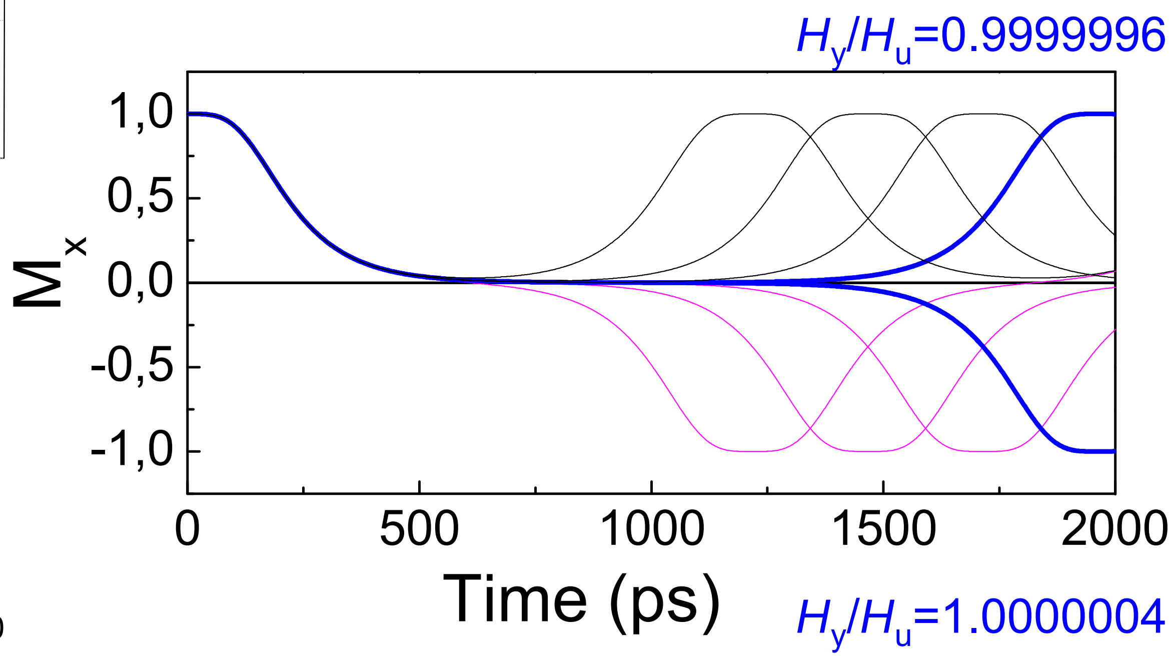
Schumacher et al PRL 90, 017201 (2003)

Low probability of succesful switching at minimum H_y

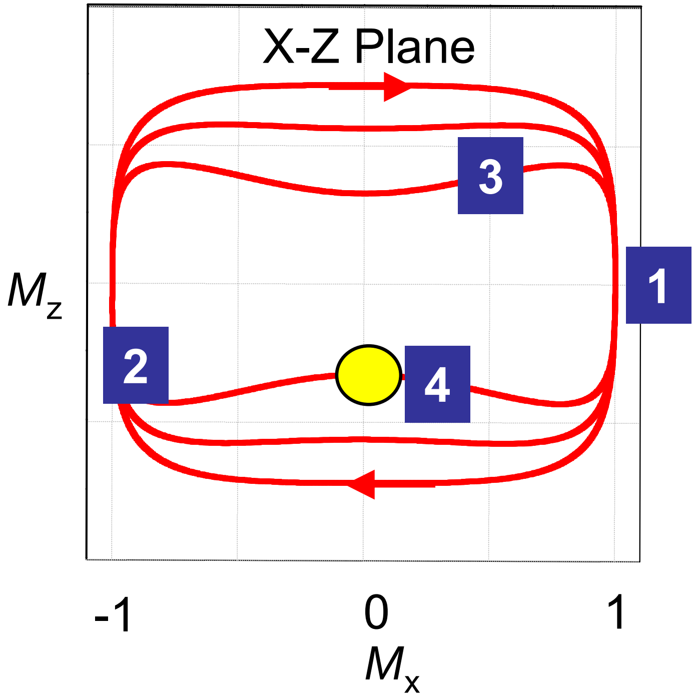


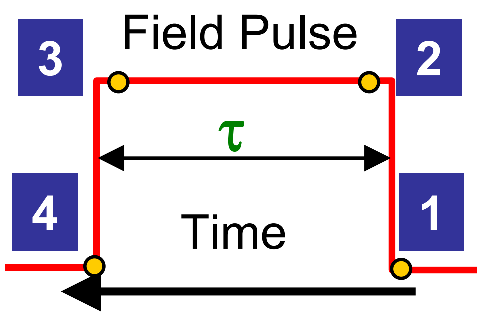


- Slow down of precession when passing saddle point (angular velocity goes to zero)
- Thermal fluctuations will have a strong effect on whether or not **M** reverses



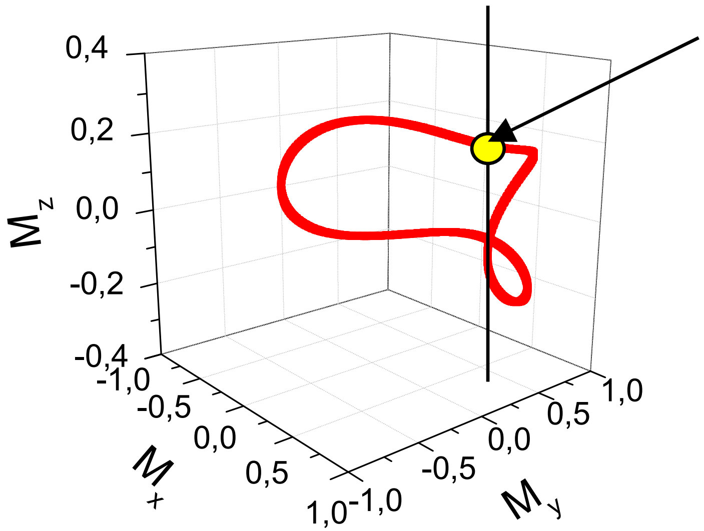
Low probability of succesful switching at transitions from R ⇔ NR





- Slow down of precession when crossing the hard axis

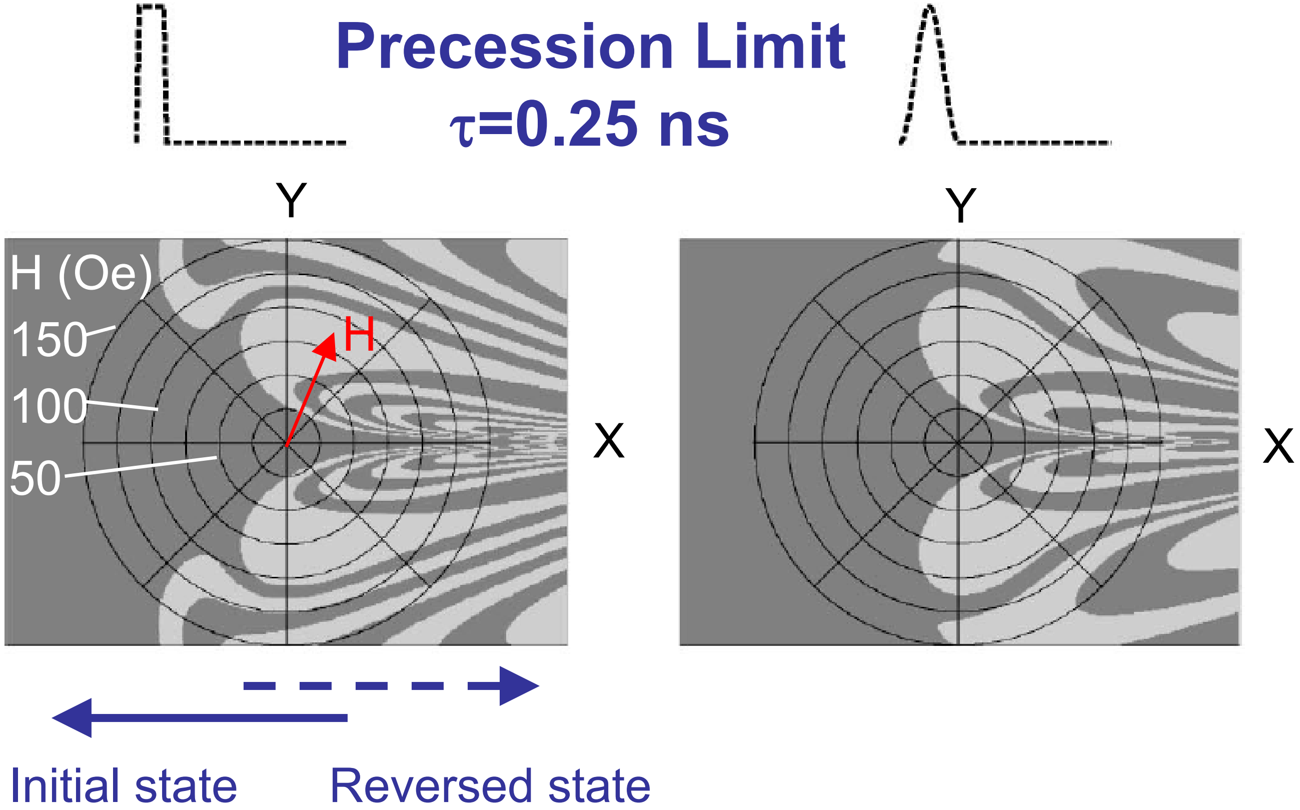
- Thermal fluctuations will have a strong effect on whether or not **M** reverses

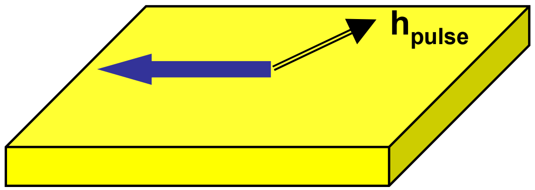


III Precessional Reversal - Pulse Shape dependence

Bauer et al PRB 61, 3410 (2000)

Precession Limit
τ=0.25 ns

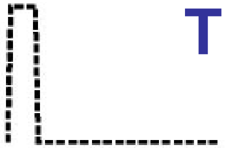




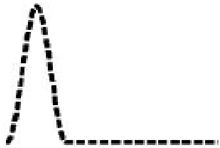
Thin Film: $N_x \ll N_y \ll N_z$
α=0.008, M_s=860emu/cm^3

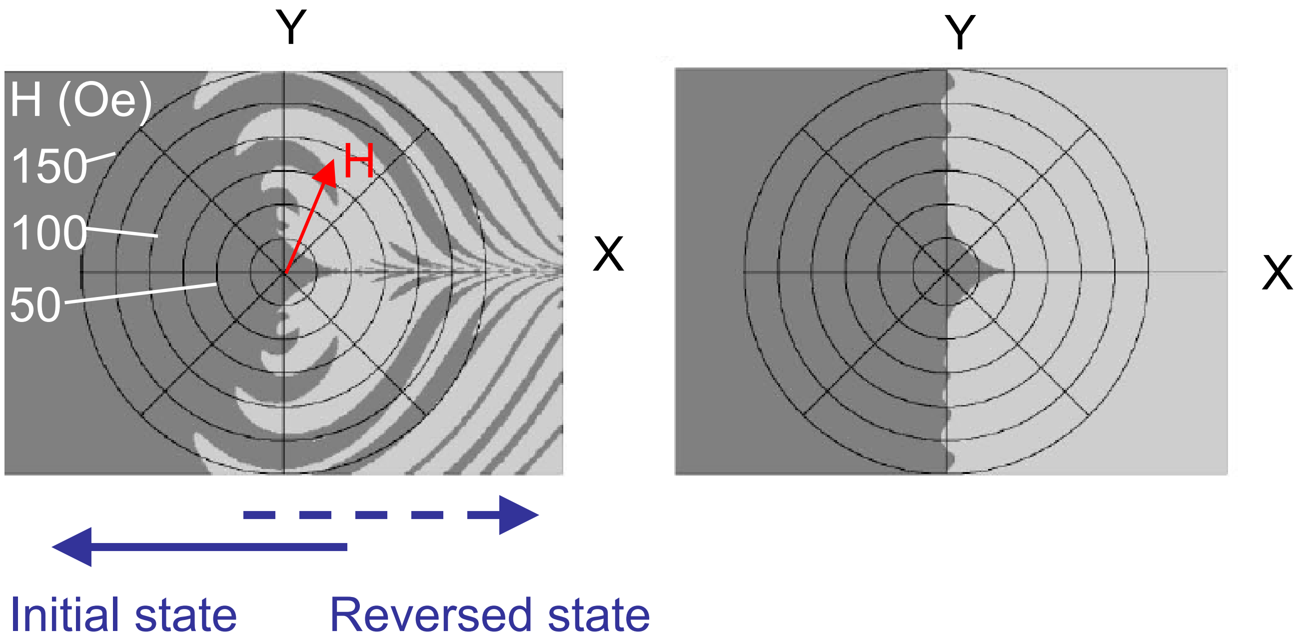
III Precessional Reversal - Pulse Shape dependence

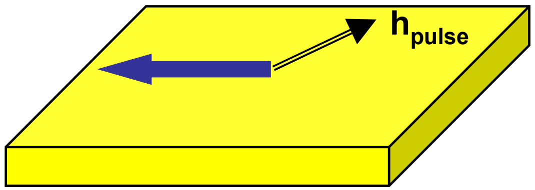
Bauer et al PRB 61, 3410 (2000)



Transition Regime
τ=1.4 ns



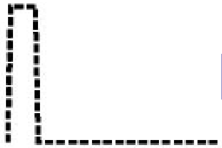




Thin Film: $N_x \ll N_y \ll N_z$
α=0.008, M_s=860emu/cm^3

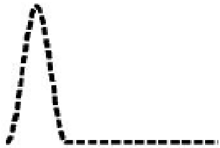
III Precessional Reversal - Pulse Shape dependence

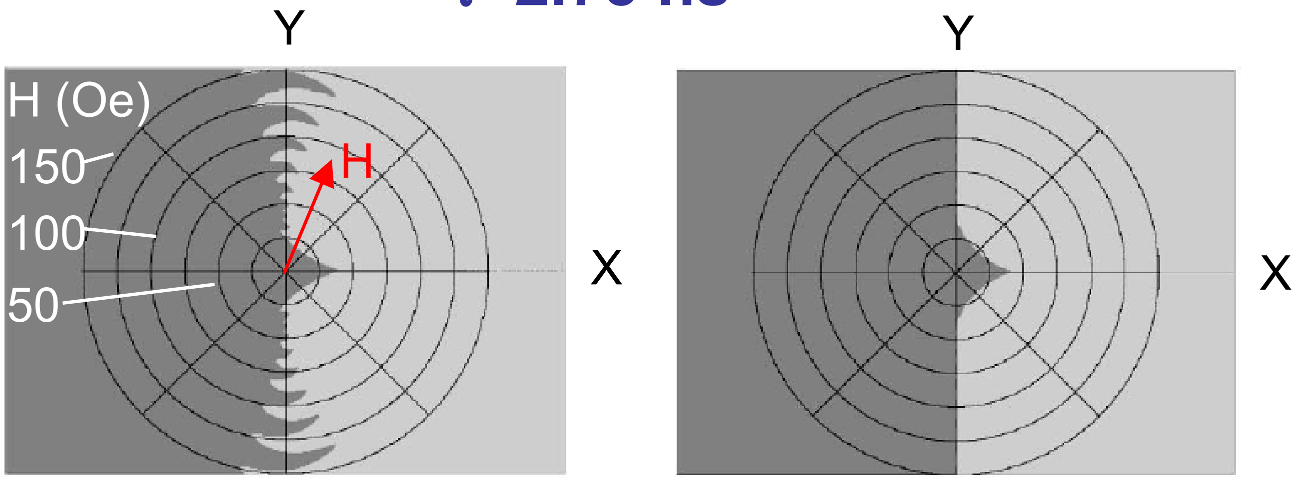
Bauer et al PRB 61, 3410 (2000)



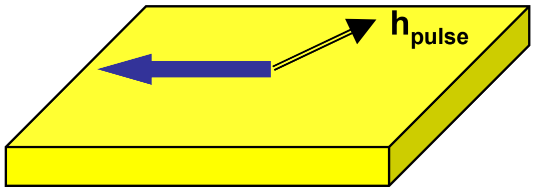
Relaxation Limit

τ=2.75 ns





Initial state Reversed state



Thin Film: $N_x \ll N_y \ll N_z$

α=0.008, M_s=860emu/cm^3

III Precessional Reversal - MOKE Experiment

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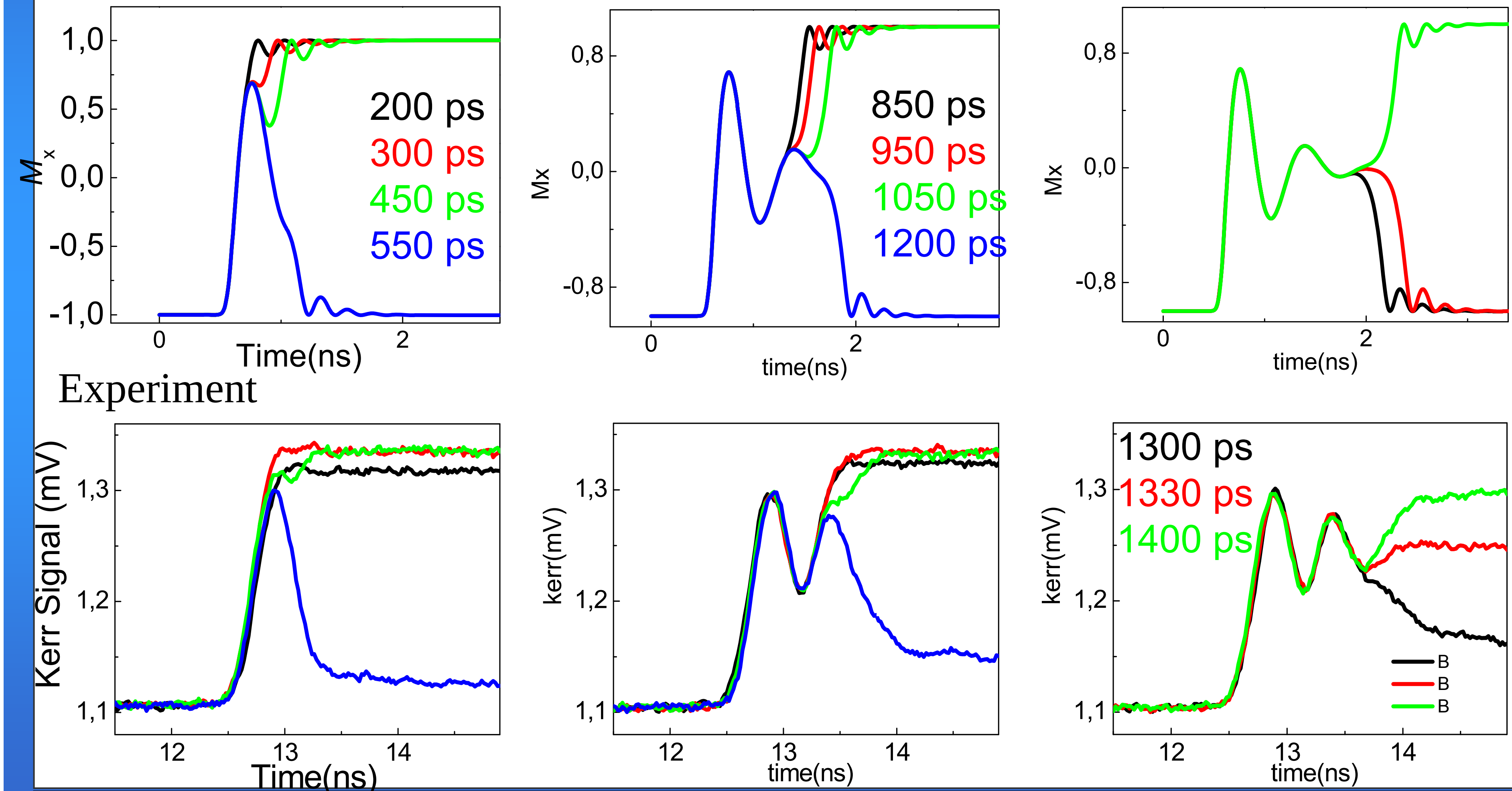
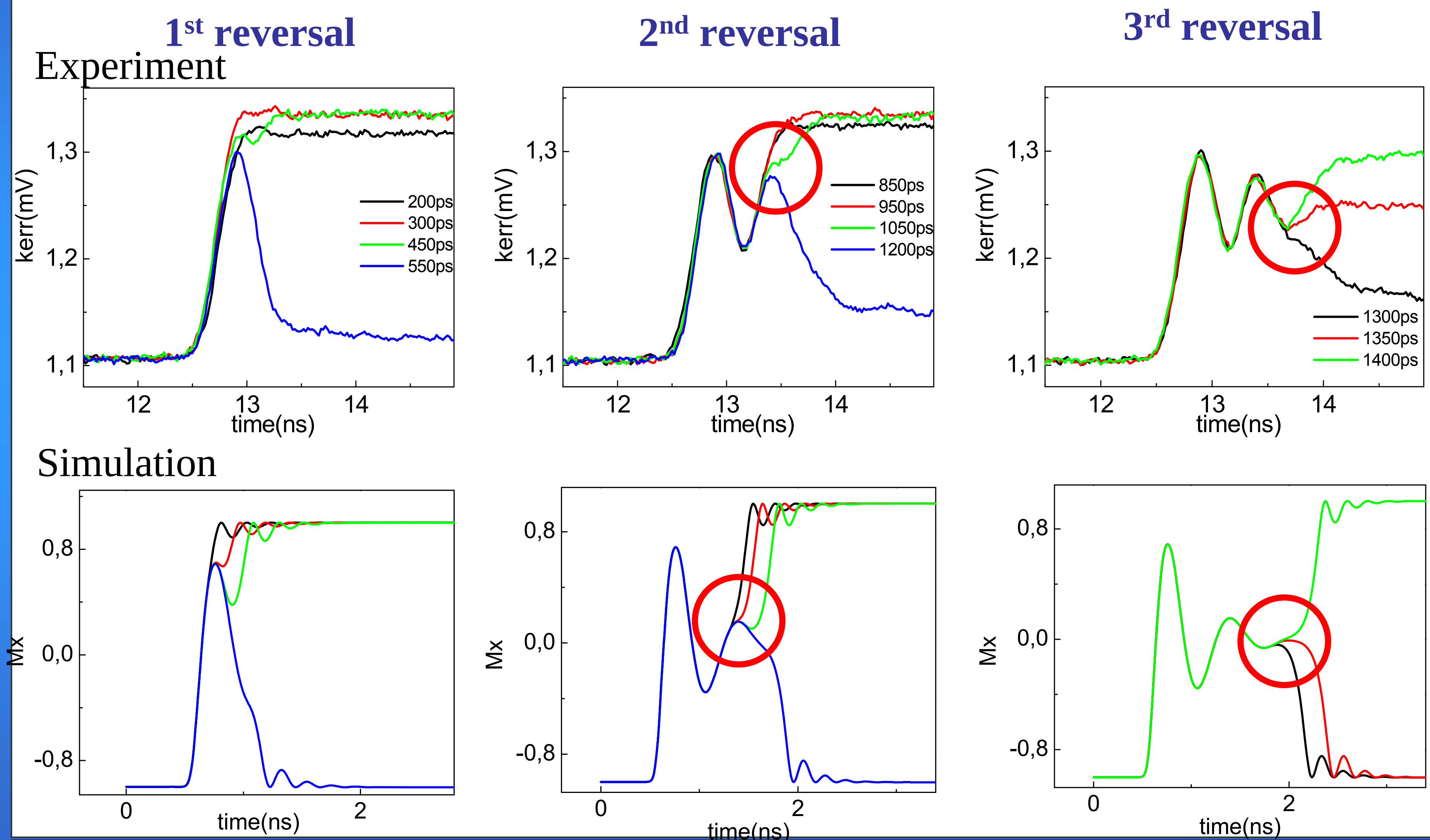
Switching for increasing pulse length

1st reversal | 2nd reversal | 3rd reversal

Simulation



Low probability of succesful switching at transitions from R ⇔ NR



Questions

- What exactly determines the shape of the trajectory ?
- Around what point does magnetization precess ?
- What is determines the amplitude of the trajecotry ?
- What happens when the applied field suddenly changes?

Questions

- What exactly determines the shape of the trajectory - Energy
- Around what point does magnetization precess – Energy Minima + Maxima
- What is determines the amplitude of the trajectocry - Intial conditions

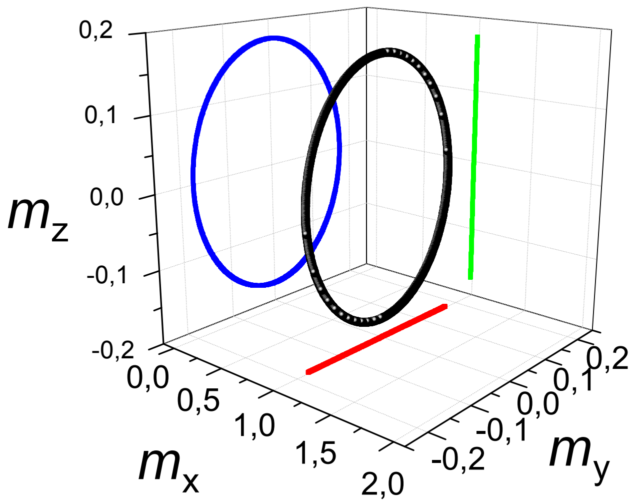
I Conservative Dynamics - Preliminaries

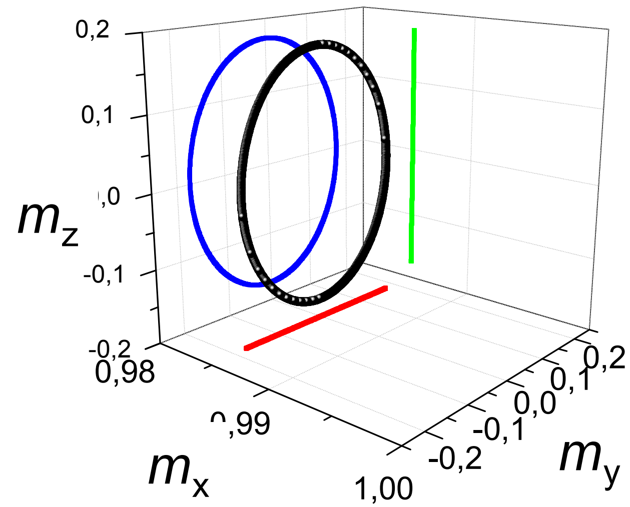
✷ What determines the shape of the trajecotry: Energy

Sphere

$N_x=N_y=N_z=1/3$

$H_b=1$ kOe //X



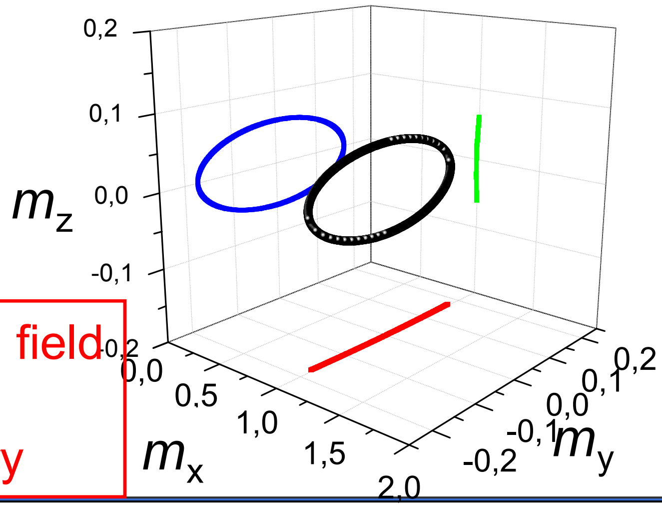


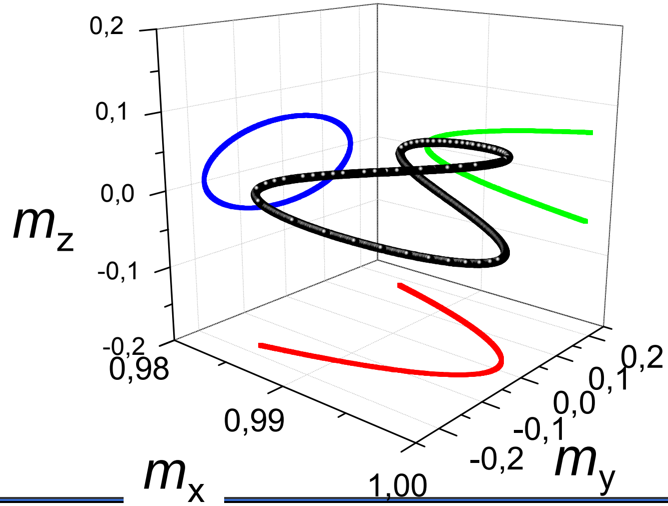
Thin Film

$N_x=N_y=0$, $N_z=1$

$H_b=1$ kOe //X

$H_u=1$ kOe //X





Demagnetization field leads to strongly elliptical trajectory

The effective field amplitude AND ORIENTATION are NOT CONSTANT

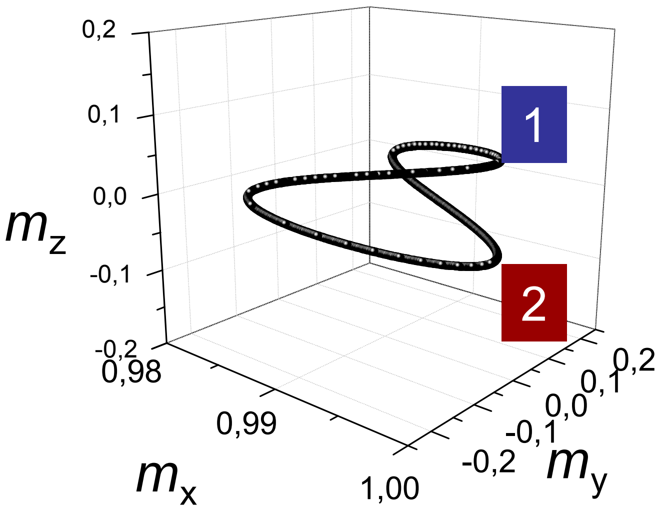
$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff}$$

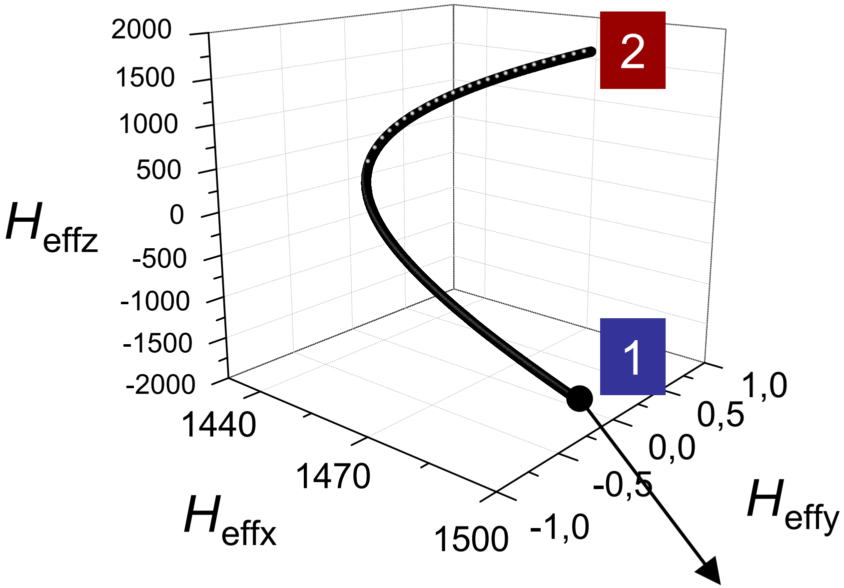
$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}} = f(\mathbf{M})$$

Thin Film

Trajectory

Effective Field





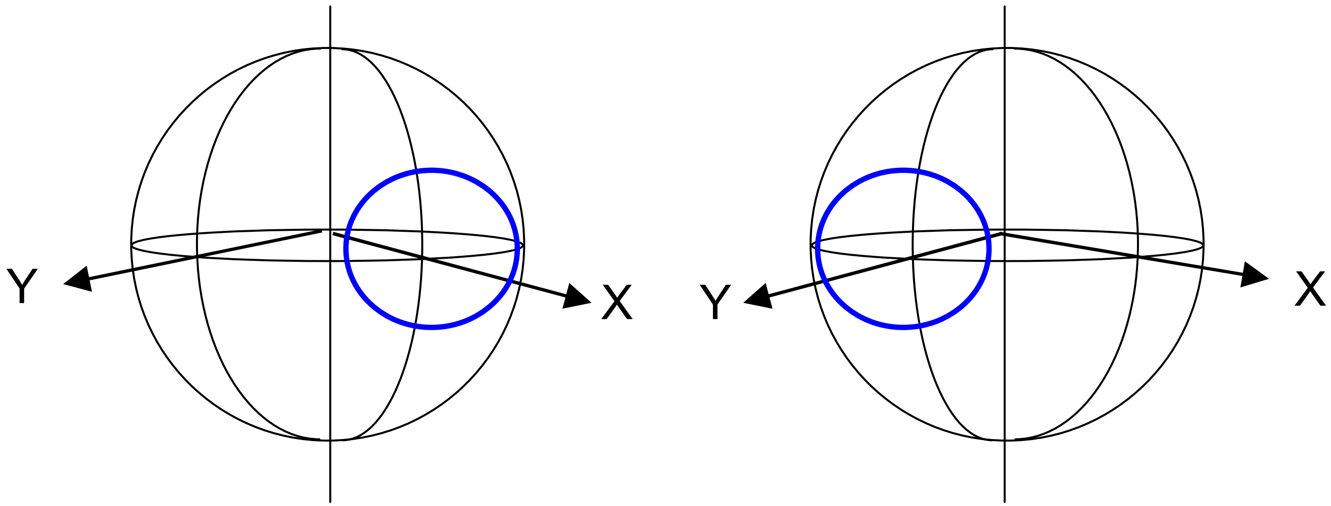
Demagnetization field leads to strongly elliptical trajectory

Strongest torque at maximum m_x

I Conservative Dynamics - Preliminaries

✷ Around what point does the magnetization precess?

Example Sphere with $N_x=N_y=N_z=1/3$, $K_u=0$ and applied field in X or Y direction



1) For a spherical system (isotropic)
   Precession around the field axis = effective field

2) For a non-spherical system (magnetic + shape anisotropy):
   Precession around the effective field

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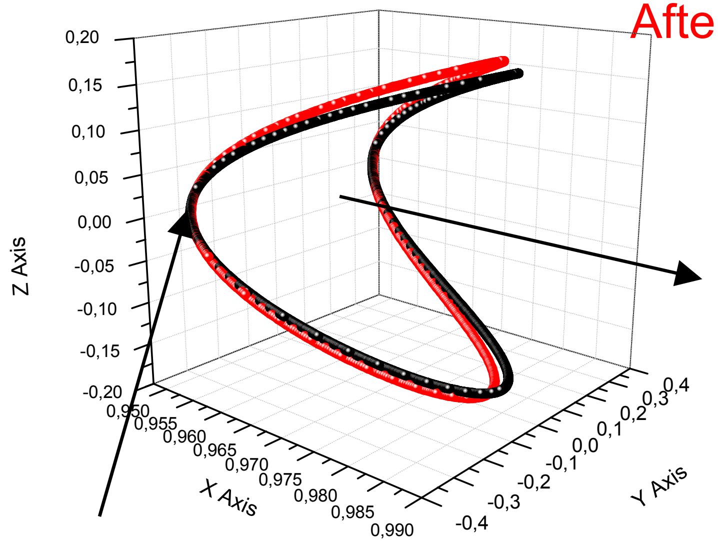
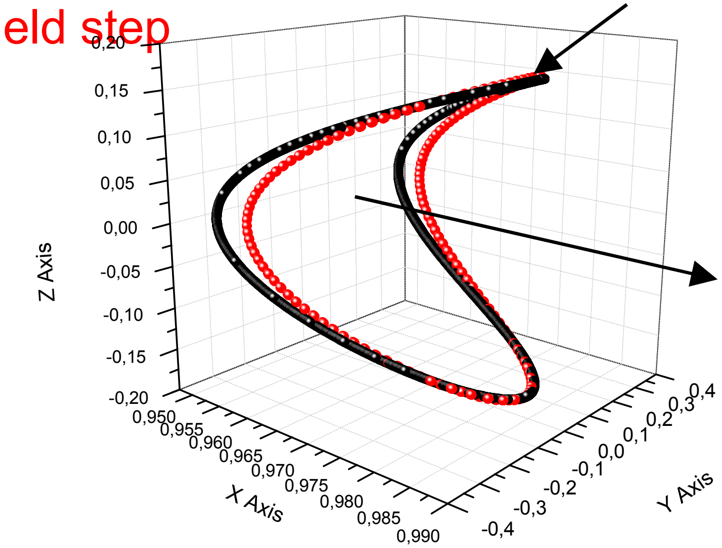
Field step along easy axis

Example Thin Film

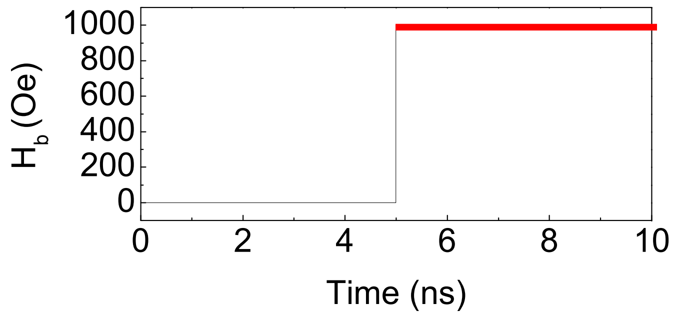
Initial trajectory
After field step

Change field here





Change field here



Field step from 0 to 1 kOe along easy axis

II Non Conservative Dynamics - Introduction

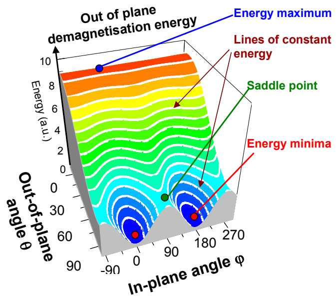
- For a sudden change of the field, **m** follows a trajectories around the new equilibrium positions depending

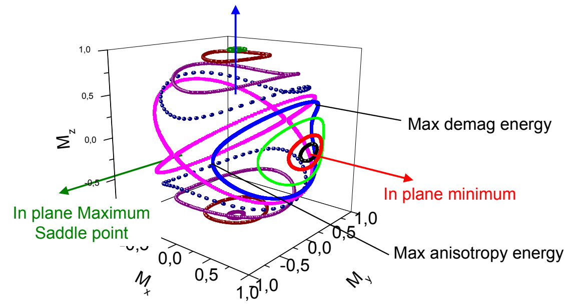
- The initial condition for this new trajectory is the position at the time when the field has changed

I Conservative Dynamics - Summary

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession



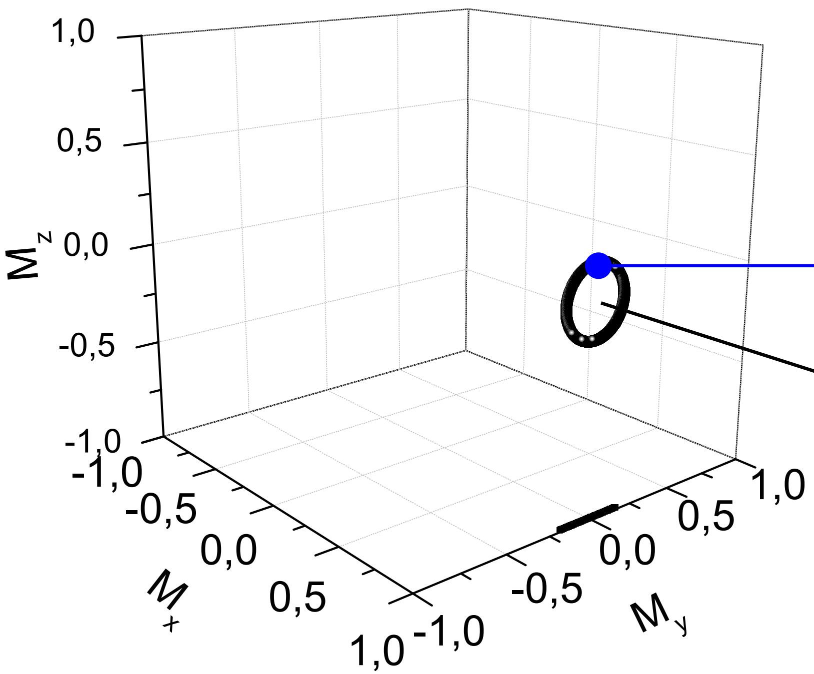


- The precession corresponds to a non-linear dynamical system
- It is characterized by the conservation of the norm of **m** and of its energy *E*(**m**)
- Its solutions are equilibria and periodic orbits
- The equilibria correspond to energy minima, maxima or saddlepoints
- The periodic orbits are constant energy trajectories
- For a uniaxial thin films there are two types of trajectories : IPP and OPP
- IPP is around the energy minimum and OPP around the energy maximum

- While the total energy is constant, the anisotropy, Zeemand and demagnetization energy oscillate
- The trajectories can be calculated analytically from the conservation of *E* and **m**
- The amplitude of the orbits is given by the initial condition (initial energy)

What happens when the applied field orientation suddenly changes?

Example Sphere



$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

Position of **M** at t_1 when field swicthes

$H_x : t_o \rightarrow t_1$

During $t_o \rightarrow t_1$ apply field along X : oscillation around H_x

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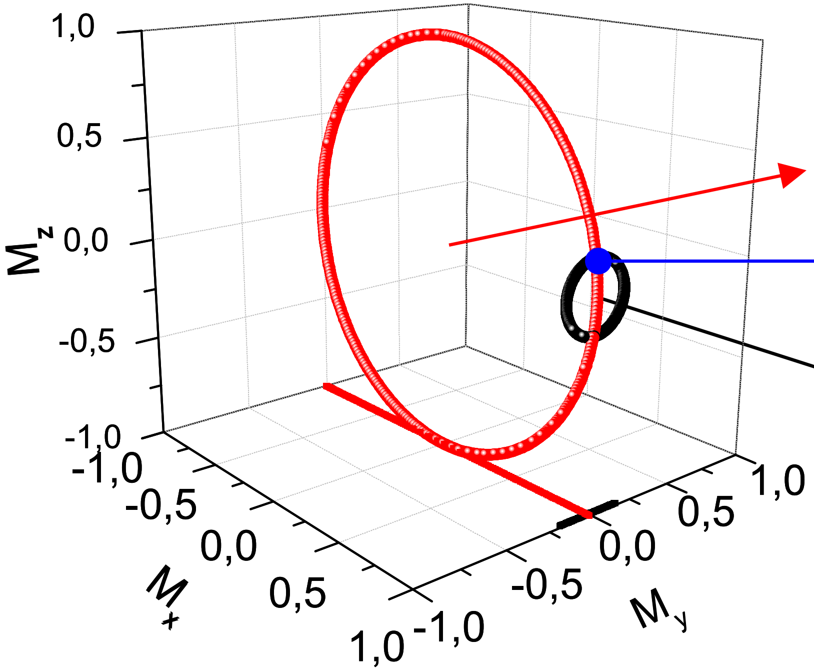
What happens when the applied field orientation suddenly changes?

Example Sphere



At t_1 switch field into Y direction: oscillation around H_y

$H_y : t_1 \rightarrow t$

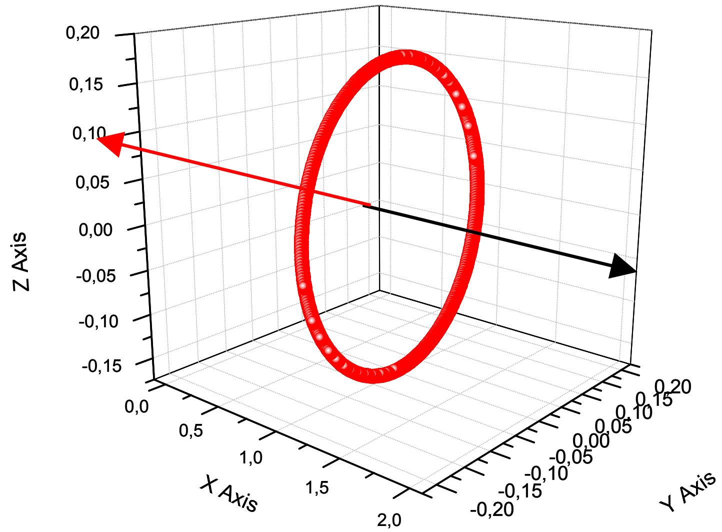
Position of **M** at t_1 when field swicthes

$H_x : t_o \rightarrow t_1$

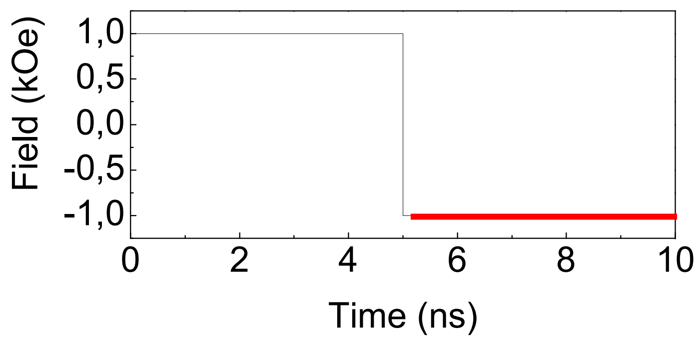
During $t_o \rightarrow t_1$ apply field along X : oscillation around H_x

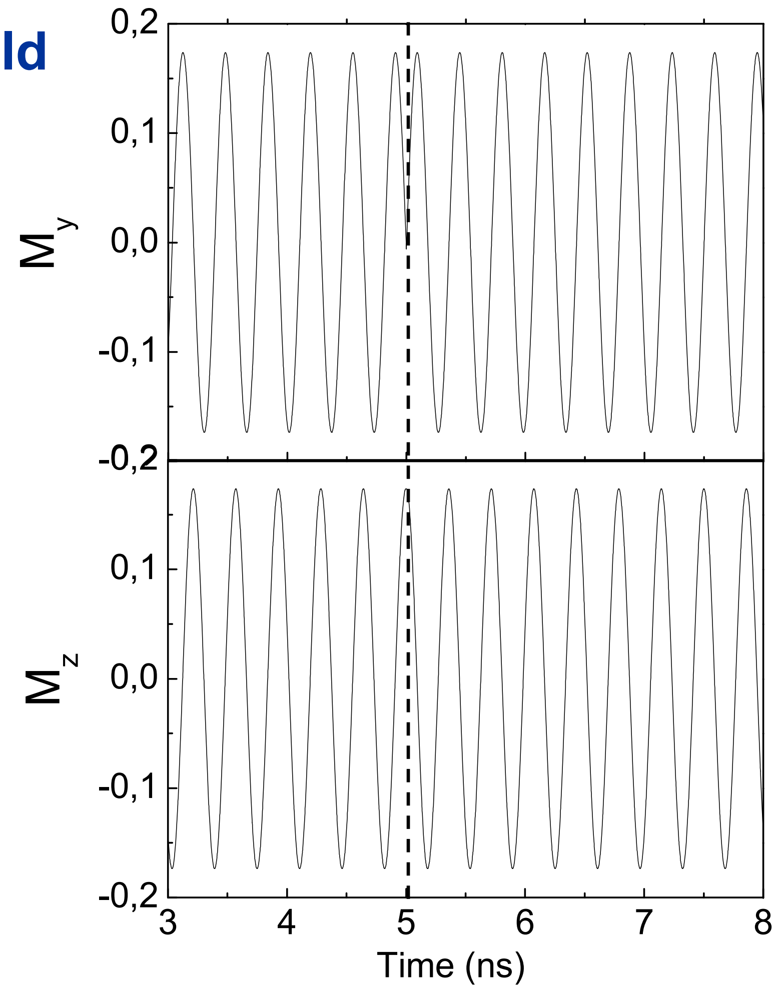
- For $t_o \rightarrow t_1$ oscillation around H_x.
- At t=t_1 switch to oscillation around H_y
- Amplitude of the new trajectory depends on where **M** has been at t_1

Example : Sphere with reversed field



Switch field from + 1kOe to -1kOe





Same trajectory but
Rotation sense changes from CCW to CW

For completeness

Relation between initial energy and maximum excursion points of θ_o and φ_o

$$E_o(\varphi_o,\theta_o)=E_o(m_{xo},m_{yo},m_{zo})=const$$

$$Cos\,\varphi_o=-\frac{H_b}{H_u}\pm\sqrt{\left(\frac{H_b}{H_u}\right)^2-\frac{2E_o}{H_uM_s}+1}$$

$$Sin\,\theta_o=-\frac{H_b}{H_u+H_d}\pm\sqrt{\left(\frac{H_b}{H_u+H_d}\right)^2+1-\frac{2E_o}{M_s(H_u+H_d)}}$$

$$\frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\delta\mathbf{M}$$

Solution

$$\boxed{\begin{aligned}&\delta\mathbf{M} = \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}\\&\lambda = \Gamma + i\omega_o\end{aligned}}$$

Γ < 0 stable point

Γ > 0 unstable point

ω_o precession frequency

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = 0 \quad \Rightarrow$$

$$\lambda^2 - tr(A)\lambda + \det(A) = 0 \quad \Rightarrow \lambda = \frac{\tau \pm \sqrt{\Delta}}{2}$$

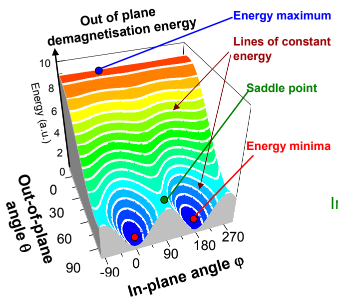
$$\tau = tr(A)$$

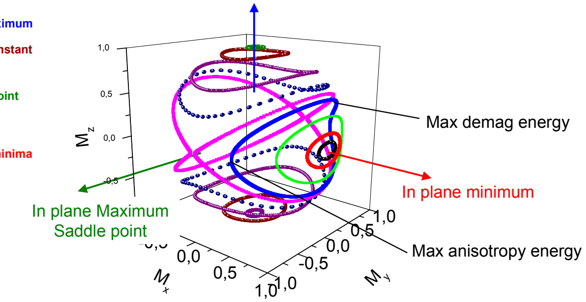
$$\Delta = \left(\frac{tr(A)}{2}\right)^2 - 4\det(A)$$

II Non Conservative Dynamics - Summary

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)} + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Precession **Damping**





- The LLG equation corresponds to a non-linear dynamical system
- It is characterized by the conservation of the norm of **m** and dE(**m**)/dt < 0
- The equilibria correspond to energy minima, maxima or saddlepoints
- The stability of equilibria is obtained via linearization of LLG
- Only the energy minima are stable
- The orbits are spirals around the energy minimum
- Independent of the initial condition the system will end up in a (local) energy minumum

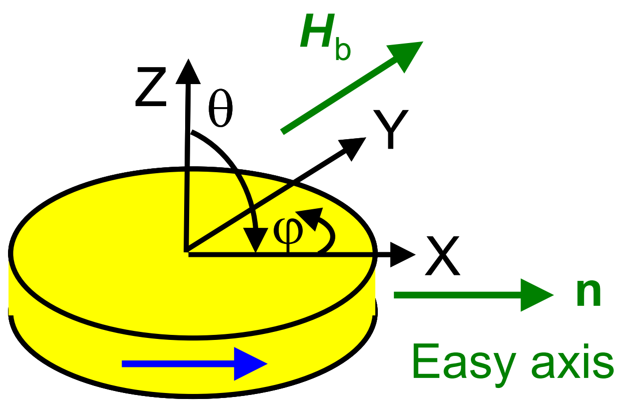
Precessional reversal at large amplitude

Let us have a look at constant energy trajectories under hard axis field

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma \mathbf{M} \times \mathbf{H}_{eff}}$$

Precession

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}}$$



$$E = K_u\left[1-(\mathbf{mn})^2\right] - M_s \mathbf{m}\mathbf{H}_b + 2\pi M_s^2\left[\mathbf{m}\left(\overleftrightarrow{\mathbf{N}}\mathbf{m}\right)\right]$$

Dynamic orbits for hard axis bias fields

Conservation of energy and of norm

$$E = K_u\left[1 - m_x^2\right] - M_s H_{by} m_y + 2\pi M_s^2 m_z^2$$

$$1 = m_x^2 + m_y^2 + m_z^2$$

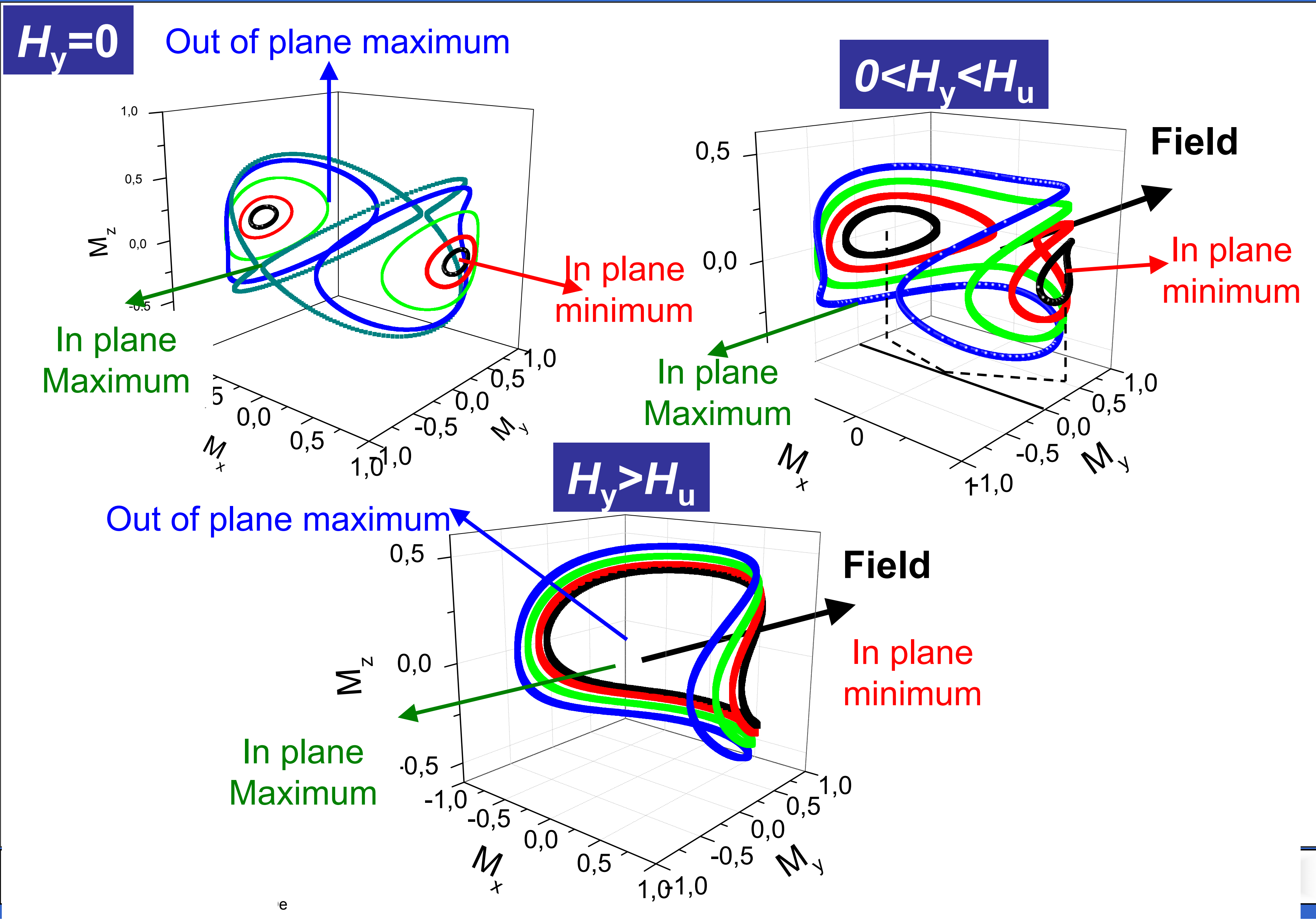
$$m_x^2 = 1 - \frac{2E_o/M_s}{H_u + H_d} - \frac{2H_b}{H_u + H_d} m_y - \frac{H_d}{H_u + H_d} m_y^2 \Rightarrow m_x(m_y)$$

$$m_z^2 = \frac{2E_o/M_s}{H_u + H_d} + \frac{2H_b}{H_u + H_d} m_y - \frac{H_u}{H_u + H_d} m_y^2 \Rightarrow m_z(m_y)$$

Special initial condition φ=0°, θ=90° (m_x=1)
→Initial energy E_o(φ=0°, θ=90°)=0

$$m_y \in \{0,1\}$$

III Precessional Reversal - Hard Axis IPP Trajectories



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Landau-Lifshitz-Gilbert Equation (LLG)

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)} + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Precession **Damping**

General analyis of non-linear dynamical systems

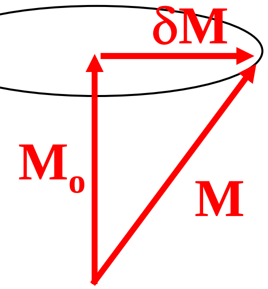
- Determine equilibria $\mathbf{M}_o$ $\quad \frac{d\mathbf{M}}{dt} = 0 \Rightarrow \mathbf{M}_o$

- Analyze stability of equilibria by linearizing the equation of motion around $\mathbf{M}_o$

Landau-Lifshitz-Gilbert Equation (LLG)

$$\frac{d\mathbf{M}}{dt} = \boxed{-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)} + \boxed{\frac{\alpha}{M_s}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)}$$

Precession Damping



Stability: Linearization of LLG around equilibrium points $\mathbf{M}_o$

$$\left.\begin{aligned}&\mathbf{M} = \mathbf{M}_o + \delta\mathbf{M}\\ &\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff}(\mathbf{M}_o + \delta\mathbf{M})\\ &\mathbf{H}_{eff}(\mathbf{M}) = \mathbf{H}_{eff}(\mathbf{M}_o) + \mathbf{h}_{eff}(\delta\mathbf{M})\end{aligned}\right\} \Rightarrow \frac{d\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\delta\mathbf{M}$$

$\underline{\underline{\mathbf{A}}}$ does not depend on $\delta\mathbf{M}$, only on $\mathbf{M}_o$ & $\mathbf{H}_{eff}(\mathbf{M}_o)$

$$\frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\delta\mathbf{M}$$

Solution

$$\delta\mathbf{M} = \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}$$
$$\lambda = \Gamma + i\omega_o$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = 0 \quad \Rightarrow$$

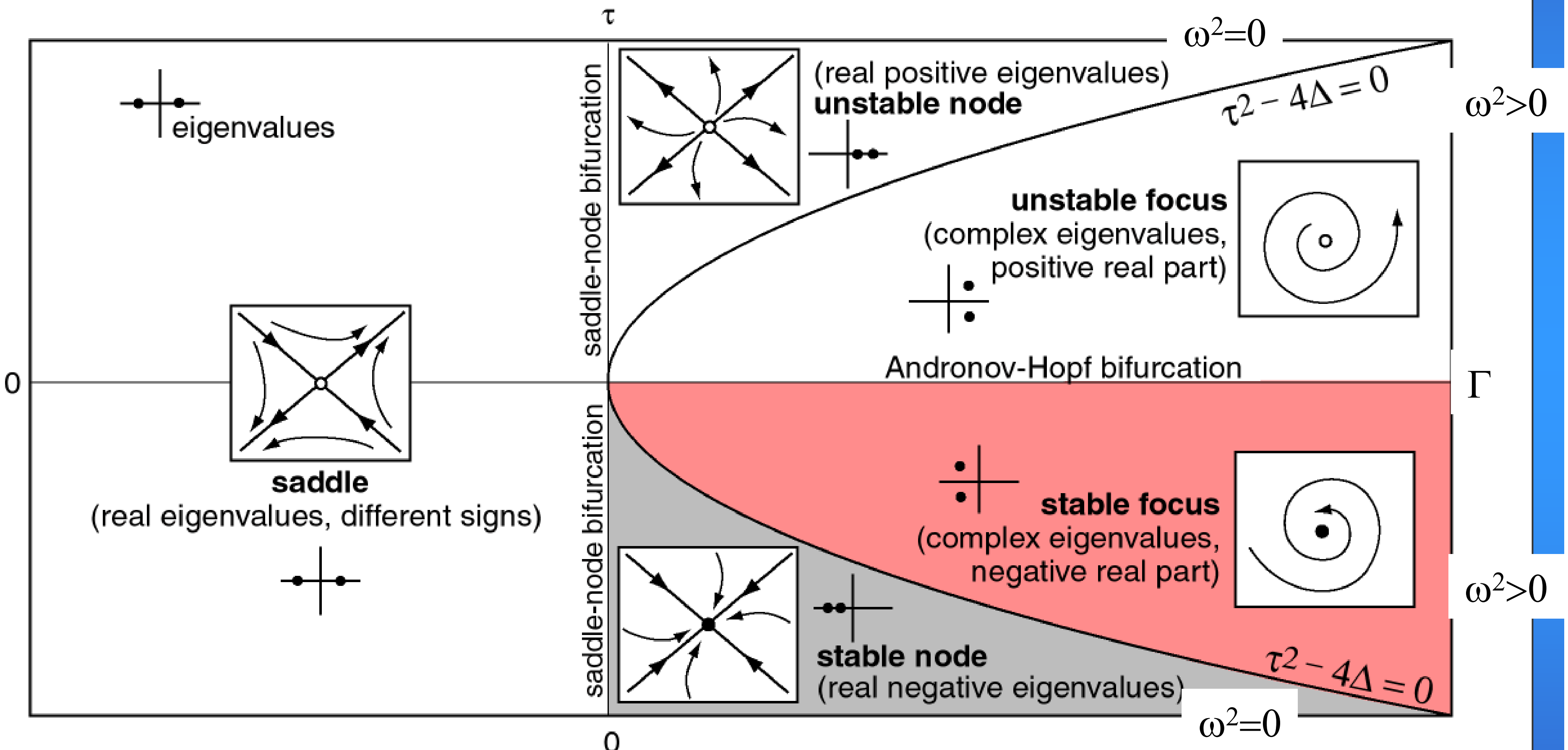
$$\lambda^2 - tr(A)\lambda + \det(A) = 0$$

$$\Rightarrow \lambda = \Gamma + i\omega_o$$

$\Gamma < 0$ stable point
$\Gamma > 0$ unstable point
$\Gamma = 0$ periodic orbit
ω_o precession frequency

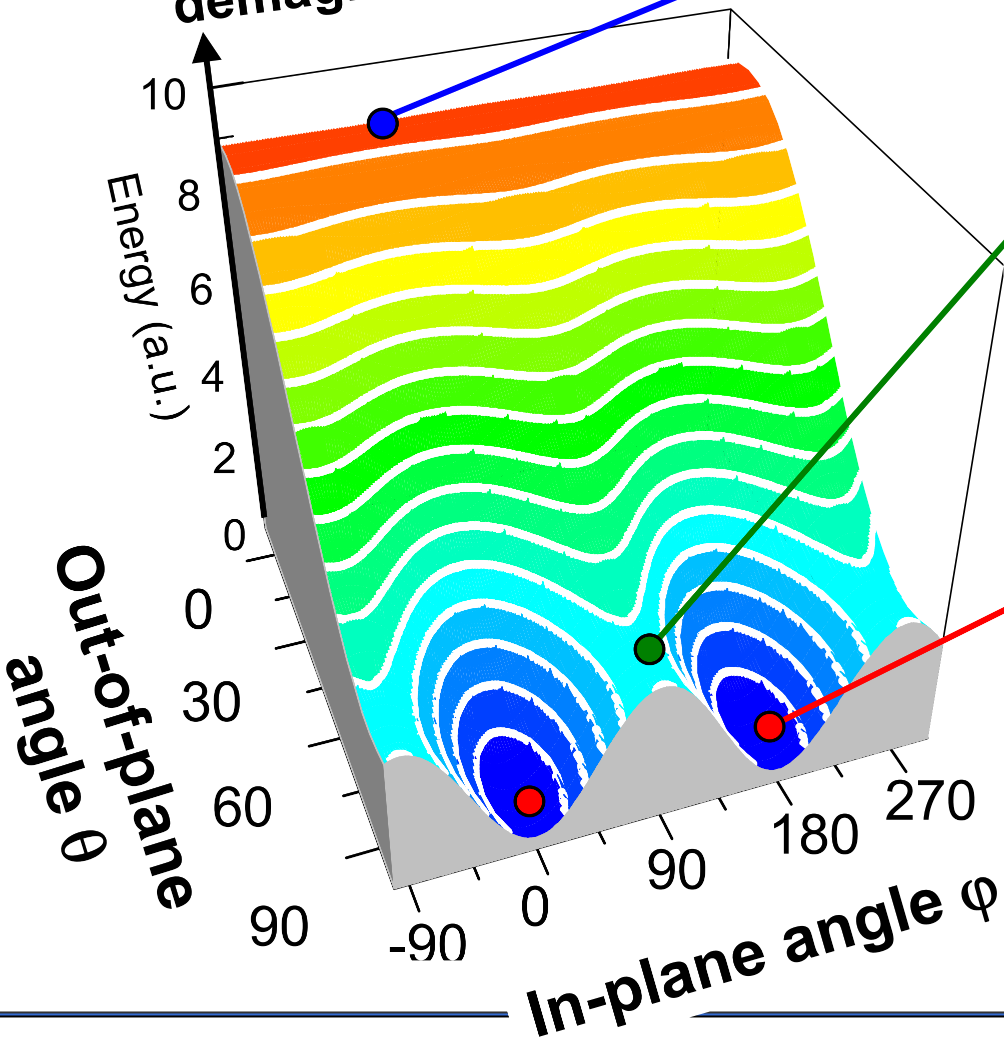
II Non Conservative Dynamics - Stability of Equilibria

Classification of equilibria for 2D dynamical systems

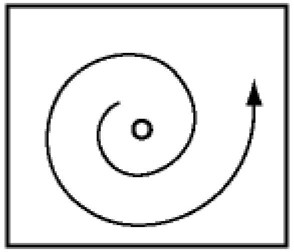


After perturbation the system either converges to or diverges from equilibrium state

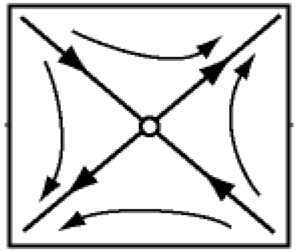
II Non Conservative Dynamics - Equilibria



Energy maximum – unstable focus

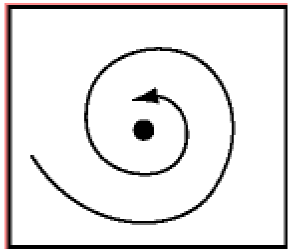


Saddle point - unstable

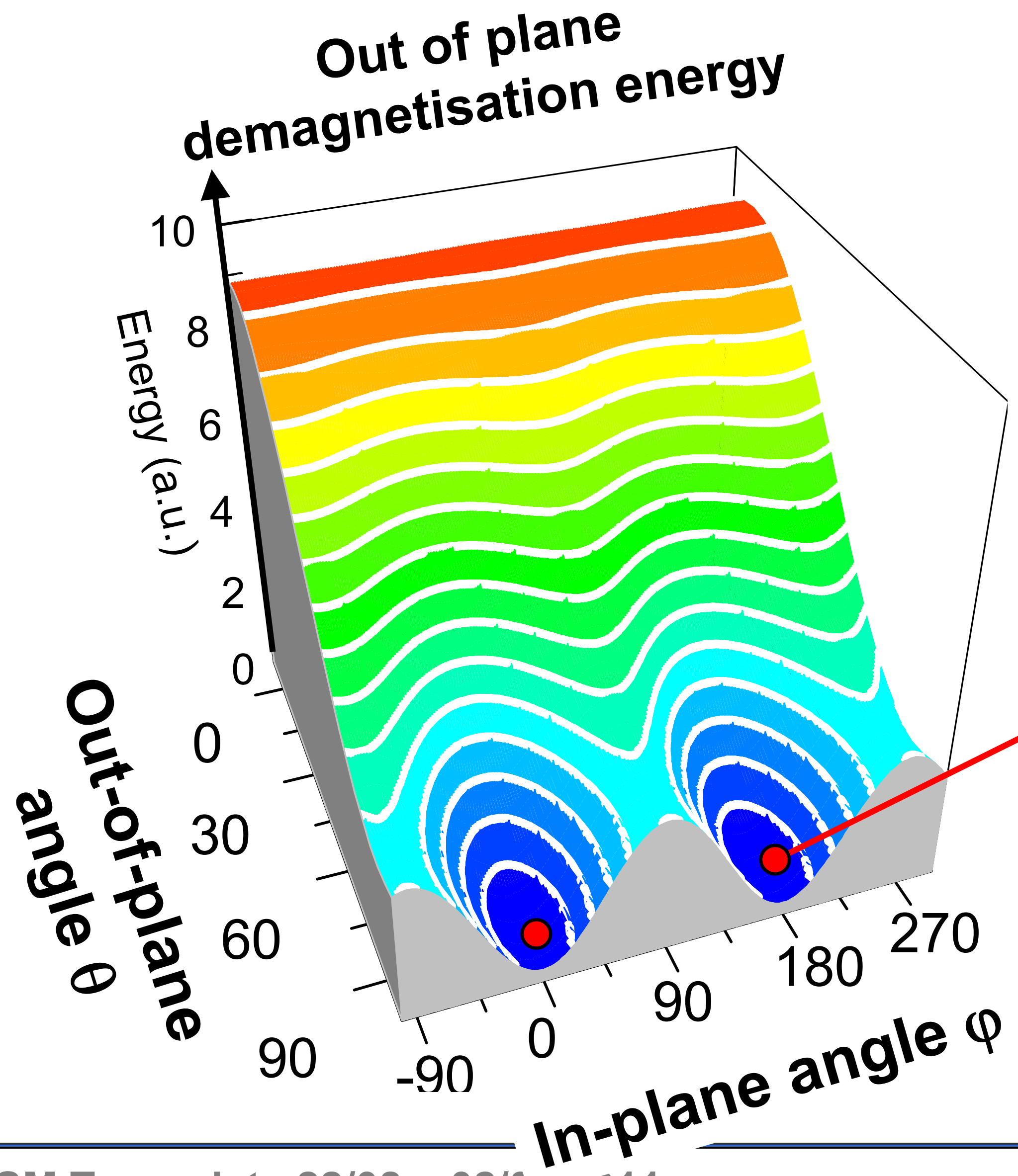


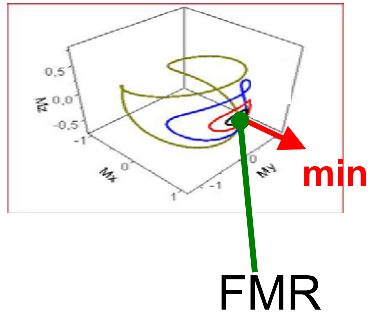
Energy minima – stable focus

$H_b = 0$



II Non Conservative Dynamics - Equilibria





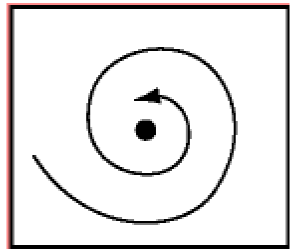
Damped oscillation

$$\mathbf{m} = \mathbf{m}_o e^{-i\omega_o t} e^{\Gamma t}$$

$\Gamma<0$

Energy minima – stable focus

H_b =0



$$\frac{d\,\delta\mathbf{M}}{dt} = \underline{\underline{\mathbf{A}}}\delta\mathbf{M}$$

Solution

$$\boxed{\begin{aligned}&\delta\mathbf{M} = \delta\mathbf{M}_o e^{\lambda t} = \delta\mathbf{M}_o e^{\Gamma t} e^{i\omega_o t}\\ &\lambda = \Gamma + i\omega_o\end{aligned}}$$

$$\det\left(\underline{\underline{\mathbf{A}}} - \lambda\underline{\underline{\mathbf{I}}}\right) = 0 \Rightarrow \lambda$$

Γ **< 0 stable point**

Γ **> 0 unstable point**

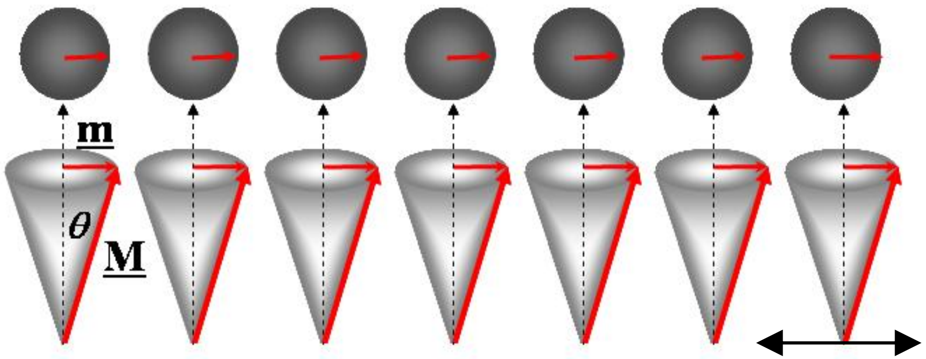
Can be lengthy depending on the configuration.
General solution in sphercial coordinates

Well known solution from Ferromagnetic Resonance FMR

$$\boxed{\omega_o^{\ 2} = \gamma\left(\frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2\sin^2\theta}\right) \qquad \Gamma = \frac{\gamma\alpha}{2}\left(\frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2\sin^2\theta}\right) \qquad \Gamma = \frac{\Delta\omega}{2}}$$

$E_{\theta\theta}, E_{\varphi\varphi}, E_{\theta\varphi}$ Second derivatives of energy
to be evaluated at equilibrium $\boldsymbol{M}_o \Leftrightarrow \theta_o, \varphi_o$

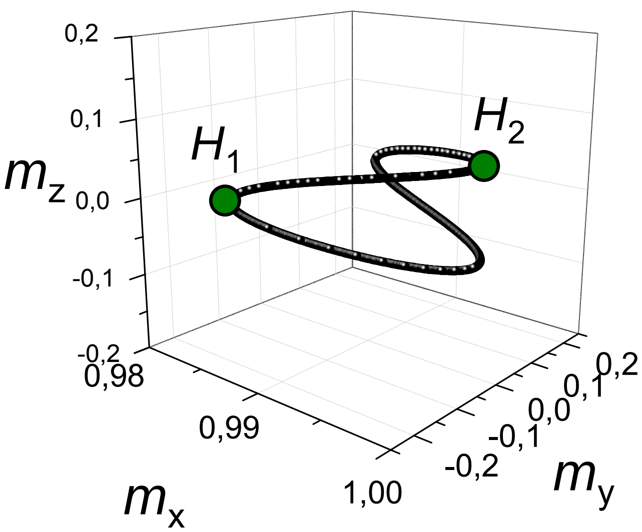
Ferromagnetic Resonance – uniform mode (k=0)



M = const., **m** = const

Excitations by rf field $\mathbf{h}_{rf} \perp \mathbf{M}$

$\mathbf{h}_{rf} \perp \mathbf{M}$



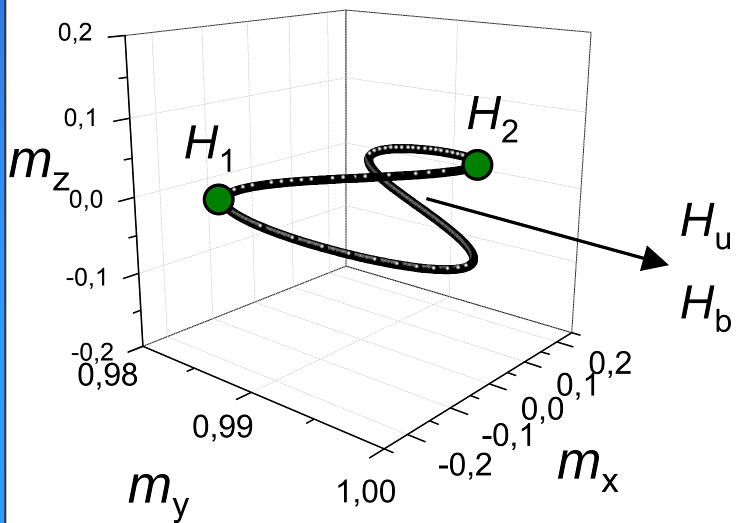
Rule of thumb for frequencies (oscillation around a symmetry axis)

$$\frac{\omega}{\gamma} = \sqrt{H_1 \cdot H_2}$$

H_1, H_2 effective field acting on **M** at turning points

II Non Conservative Dynamics - FMR

Example: uniaxial thin film, with in-plane easy axis bias field



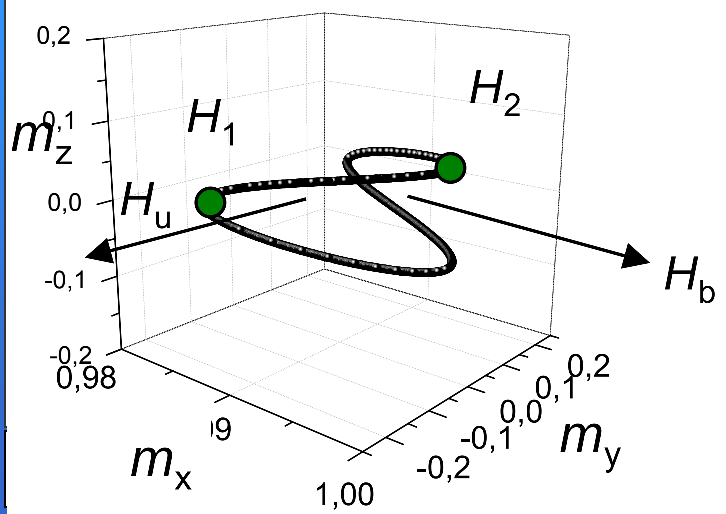
$H_1=H_u+H_b$

$H_2=H_u+H_b+H_d$

$$\frac{\omega}{\gamma}=\sqrt{(H_b+H_u)\cdot(H_u+H_b+H_d)}$$

Kittel Formula

Example: uniaxial thin film, with in-plane hard axis bias field

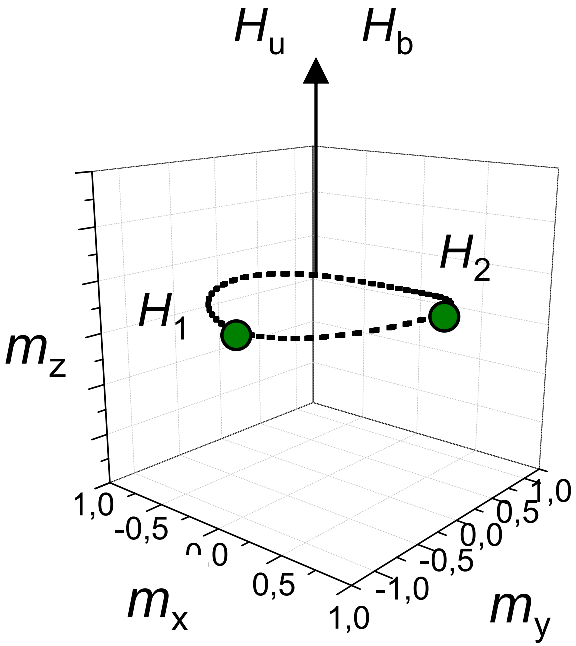


$H_1=H_b-H_u$

$H_2=H_b+H_d$

$$\frac{\omega}{\gamma}=\sqrt{(H_b-H_u)\cdot(H_b+H_d)}$$

Example: thin film with bias field and uniaxial anisotropy out-of-plane



$H_1 = H_u + H_b - H_d$

$H_2 = H_u + H_b - H_d$

$$\frac{\omega}{\gamma} = \left(H_u + H_b - H_d\right)$$