Mean-field approximations

Zoltán Néda

Babeș-Bolyai University, Department of Theoretical Physics, str. Kogalniceanu 1,

RO-400084, Cluj-Napoca, Romania

Statistical physics deals with systems composed of a very large number of units. Since the dynamics of such large number of particles cannot be described by integrating the equations of motion for each of them, statistical physics elaborated special methods for deriving relevant information about the system as a whole. The classical methods of statistical physics works well and exact results can be obtained for models that consider ensembles of non-interacting or weakly interacting particles. For models composed of particles between which the interactions are dominant exact solutions are rarely known. Approximate methods are used to derive the relevant statistical information. Several such methods are known, and between all these the easiest one is the mean-field approximation. In mean-field approximations, the interaction of a particle with all the others is approximated via an effective field, acting in the same manner on all particles in the system. This effective field is derived either from some simplifying assumptions or it is calculated in a self-consistent manner. Mean-field approximation works well for high-dimensional systems, but gives poor description for low-dimensional ones. Many important features of systems exhibiting phase-transitions are missed when considering a mean-field solution. Despite of this shortcoming, mean-field solutions give a first pedagogical picture about the statistical behavior of the ensemble of interacting particles, and can give precious qualitative information on the nature of the expected statistical behavior. For getting quantitatively better results other more sophisticated approximations are usually used, such as the low and high temperature expansion, renormalization, scaling or numerical methods.

In the present tutorial we first shortly review the program of statistical physics, and the problems arising in studying models with interactive particles. A brief introduction to critical phenomena is given, and basic aspects of phase-transitions (including also the Landau theory) are briefly revisited. After this, we present the elements of mean-field approximations, illustrating it in the simple case of classical gases. Ferromagnetic systems with localized vector or scalar spins are then studied with the mean-field method. We discuss also the mean-field approximation for tackling more complex magnetic structures made up by localized magnetic moments: antiferromagnets, ferrimagnets and helimagnets. We shortly discuss the mean-field solutions for the magnetism of itinerant electron system as well. Finally, mean-field results are critically discussed in the view of magnetic phase-transitions. Problems arising from the mean-field approximations are explained, and better approximation methods are suggested.

Bibiography

1. R.K. Pathria; Statistical Physics (Pergamon Press 1985, International Series in Natural Philosophy, Volume 45)
2. J.M. Yeomans; Statistical Mechanics of Phase Transitions (Oxford University Press, 1992)
3. R. M. White; Quantum Theory of Magnetism (Springer, 2006)