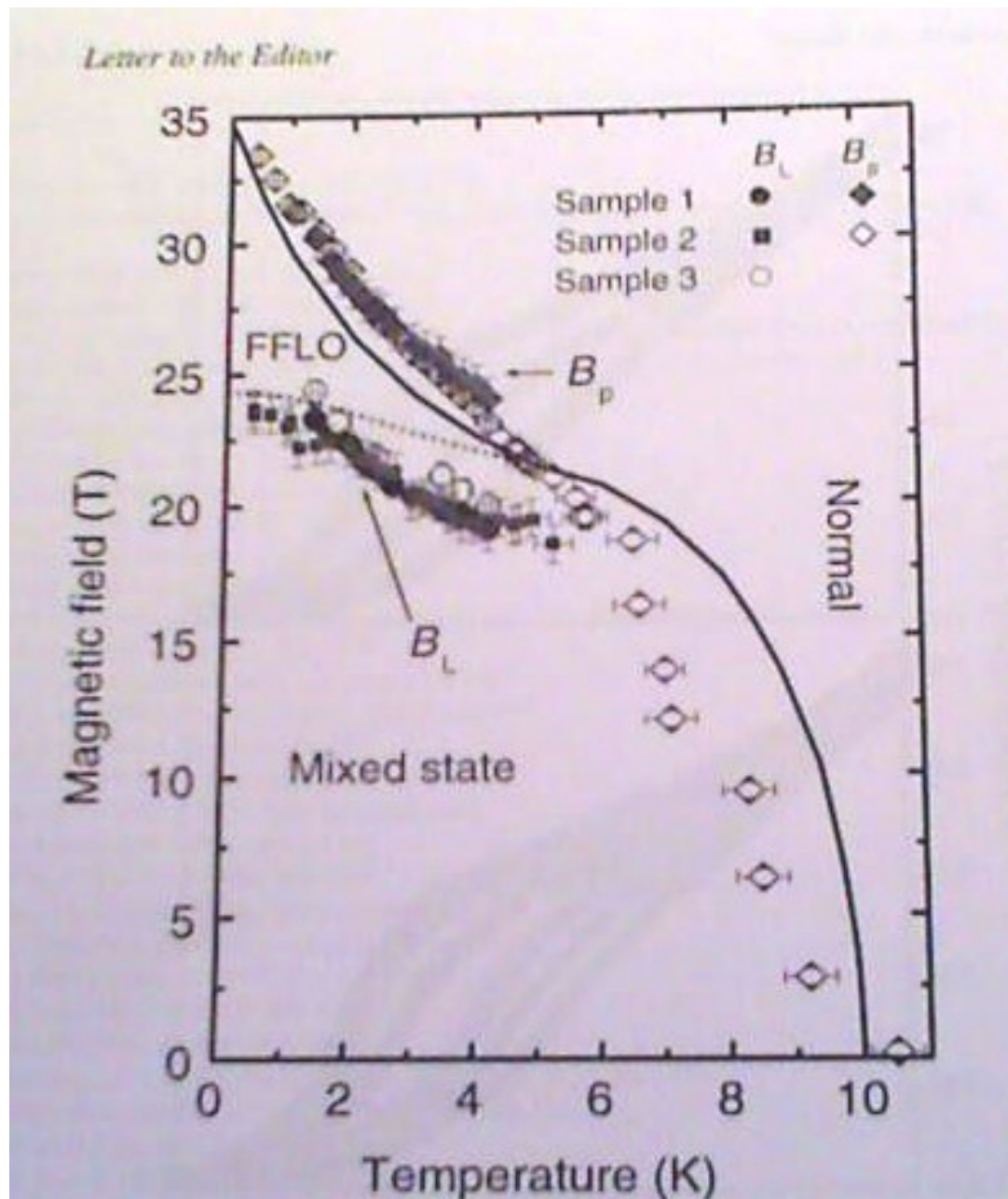


Lectures 3-4

1. **Almost localized Fermi liquid (ALFL): a paradigm for non-Landau Fermi liquid composed of strongly correlated electrons:**
 - Spin-dependent masses
 - Effective field driven by the correlations
2. **Cooper pair instability: from indistinguishable to distinguishable particles**
3. **$H_a - T$ phase diagram: BCS vs. FFLO phases stability for 3D and 2D gas**
4. **BCS vs FFLO phase for two-dimensional lattice**



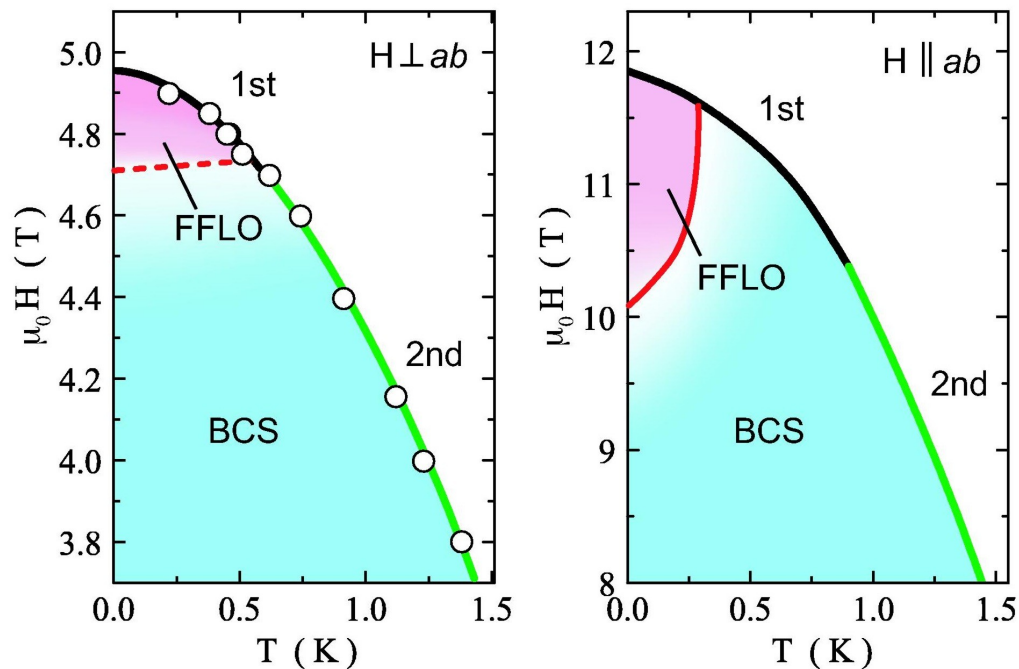
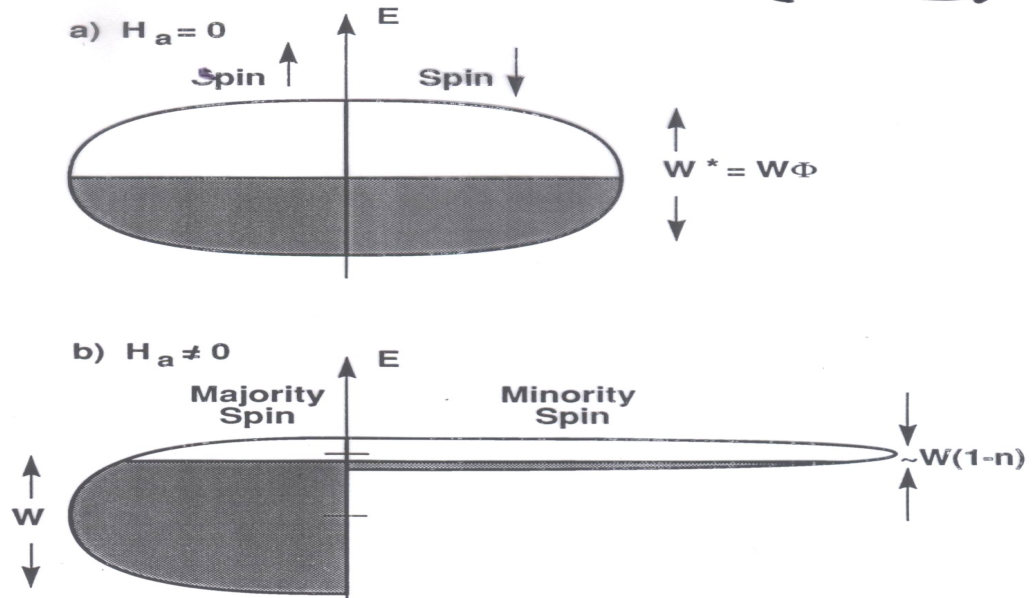


FIG. 3: H - T phase diagrams at low temperatures and high fields for $\mathbf{H} \perp ab$ (left) and for $H \parallel ab$ (right). The colored portions display the FFLO (pink) and BCS (blue) regions. The open circles are $H_{c2}^\perp$ determined by the NMR experiments. The black and green lines represent the upper critical fields which are in the first order and in the second order, respectively. The red dashed and solid lines represent the phase boundary separating the FFLO and BCS states. For $\mathbf{H} \perp ab$, precise determination of the phase boundary between the FFLO and the BCS states is difficult by the present experiments.

Spin dependent masses of quasiparticles

D. SPIN DEPENDENT EFFECTIVE MASS ($n \neq 1$)



J. Spalek & P. Gopalan,
Phys. Rev. Lett. (1990) **64**, 2823

$$\frac{m_\sigma}{m_B} = \frac{1}{q_\sigma}$$

$$q_\sigma = \frac{1}{n_\sigma(1-n_\sigma)} \left\{ \left[(n_\sigma - d^2)(1-n_\sigma + d^2) \right]^{\frac{1}{2}} + d(n_\sigma - d^2)^{\frac{1}{2}} \right\}^2$$

$$\frac{1}{N} E_G = \frac{1}{N} \sum_{\mathbf{k}\sigma} (q_\sigma \epsilon_{\mathbf{k}} - \mu_B H_a \sigma - \mu) \bar{n}_{\mathbf{k}\sigma} + U d^2$$

Spin-dependent masses quantitatively: infinite U , narrow-band limit

- $m_{\sigma}^* / m_{\text{BAND}} \underset{U \rightarrow \infty}{=} \frac{1 - n_f / \gamma}{1 - n_f} - \sigma \frac{\langle S_f^z \rangle}{1 - n_f} \Rightarrow$

- $m_{\downarrow}^* - m_{\uparrow}^* \sim \frac{1}{1 - n_f} * \text{magnetization} \Rightarrow$

- The masses depend on the field (weakly above the metamagnetic point)

- $\gamma(H_a) = \gamma_{\text{BAND}} \frac{m_{\uparrow}^* + m_{\downarrow}^*}{m_{\text{BAND}}} \rightarrow \gamma_{\text{BAND}} \quad \text{for } H_a \gg H_C$

- Masses depend on temperature:

$$\frac{m^*}{m_{\text{BAND}}} = m^*(T = 0) \left[1 + \zeta(\gamma) \left(\frac{T}{T_K} \right)^{\gamma} \right] + \dots$$

Physical origin of heavy masses:

In the magnetically polarized state the majority electrons ($\sigma=\uparrow$) scatter less effectively on a fewer spin-minority electrons. The opposite is true for the spin-minority particles.

In effect, the particles in the spin – minority subband (much) heavier than those in the majority subband

Reviews: (J. S.):

- Encyclopedia of Condensed Matter Physics, Elsevier, vol.3, 126 - 136 (2005);**
- phys. stat. sol. (b), Editor's choice, 243, 78-88 (2006);**
- Physica B (scs'05), 378-380, 1185-1190 (2006).**

Spin-dependent masses from the de Haas-van Alphen effect

	↑ m	↓ m	x ϕ
$\theta = 0^\circ$			
α_1	21.2 ± 0.2	94 ± 7	3.14
α_2	24.2 ± 0.4	94 ± 8	3.14
α_3	14.5 ± 0.6	30 ± 8	2.42
$\theta = 10^\circ$			
α_1	21.3 ± 2.4	39 ± 5	3.14
α_2	24.7 ± 3.5	30 ± 13	3.14
α_3	17.6 ± 0.8	40 ± 4	3.14

Anomalous de Haas-van Alphen Oscillations in Ce₆CoIn₅
 A. J. McCollam, S. R. Julian, P. M. C. Rourke, D. Aoki, and J. Flouquet
 Phys. Rev. Lett., 94, 186401 (2005); Physica B (2005).
 See also: I. Sheikin, et al., Phys. Rev. B 67, 094420 (2003)

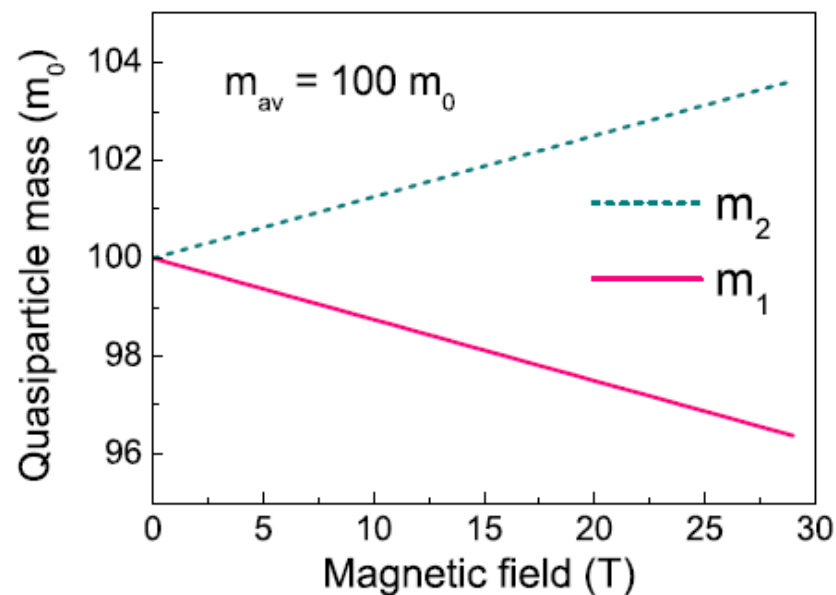
Quasiparticle states II

$$\xi_{\mathbf{k}\sigma} = \frac{\hbar^2 k^2}{2m_\sigma} - \sigma h - \mu - \sigma h_{\text{cor}}$$

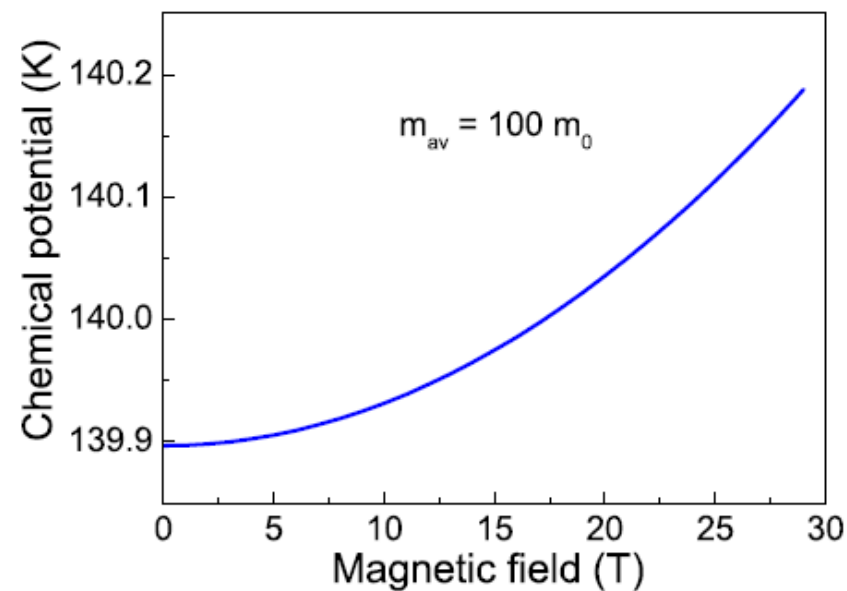
$$\frac{m_\sigma}{m_B} = \frac{1-n_\sigma}{1-n} = \frac{1-n/2}{1-n} - \sigma \frac{\bar{m}}{2(1-n)} \equiv \frac{1}{m_B} (m_{\text{av}} - \sigma \Delta m/2)$$

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} \xi_{\mathbf{k}\sigma} a_{\mathbf{k}\sigma}^\dagger a_{\mathbf{k}\sigma} - \frac{V_0}{N} \sum_{\mathbf{k}\mathbf{k}'\mathbf{Q}} a_{\mathbf{k}+\mathbf{Q}/2\uparrow}^\dagger a_{-\mathbf{k}+\mathbf{Q}/2\downarrow}^\dagger a_{-\mathbf{k}'+\mathbf{Q}/2\downarrow} a_{\mathbf{k}'+\mathbf{Q}/2\uparrow} + \frac{N}{n} \bar{m} h_{\text{cor}}$$

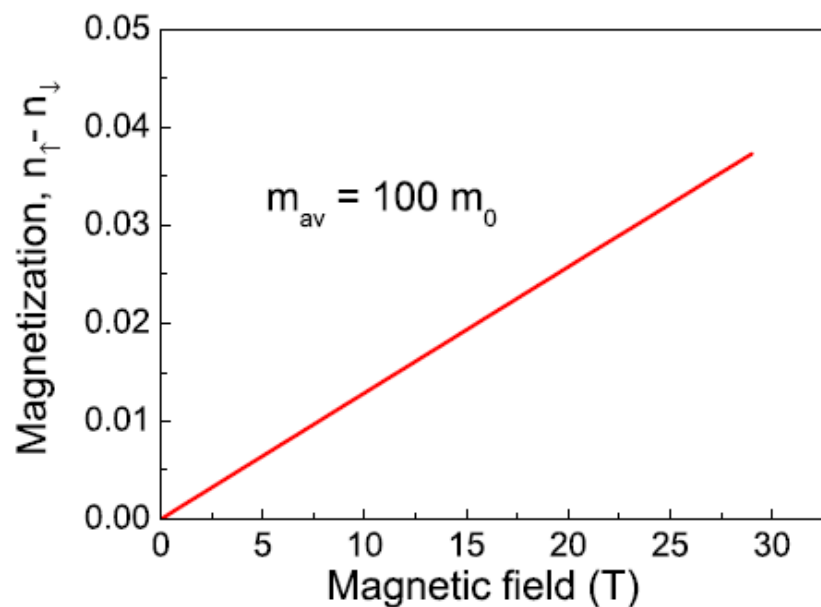
a)



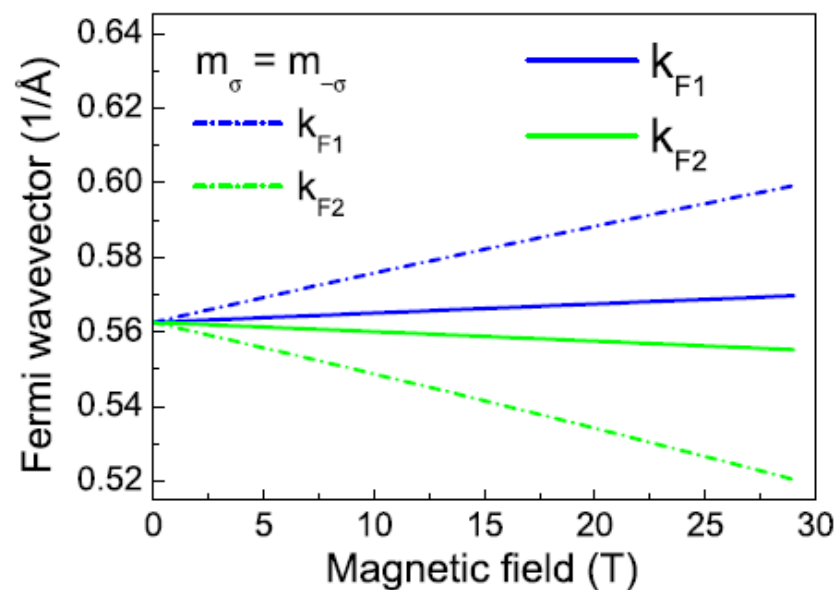
b)



c)



d)



Anderson-lattice Hamiltonian in the large U limit

$$\begin{aligned} H = & \sum_{mn} (t_{mn} - \mu \delta_{mn}) c_{m\sigma}^\dagger c_{n\sigma} + \epsilon_f \sum_{i\sigma} N_{i\sigma} (1 - N_{i\bar{\sigma}}) \\ & + \sum_{i\sigma} V_{im} * (1 - N_{i\bar{\sigma}}) (f_{i\sigma}^\dagger c_{m\sigma} + c_{m\sigma}^\dagger f_{i\sigma}) \\ & - 2 \sum_{imn} \frac{2V_{im}V_{in}}{U+\epsilon_f} b_{im}^\dagger b_{in}. \end{aligned}$$

Real-space pairing operators

$$b_{im}^\dagger = \frac{1}{\sqrt{2}} \left[a_{i\uparrow}^\dagger (1 - N_{i\downarrow}) c_{m\downarrow} - a_{i\downarrow}^\dagger (1 - N_{i\uparrow}) c_{m\uparrow} \right]$$

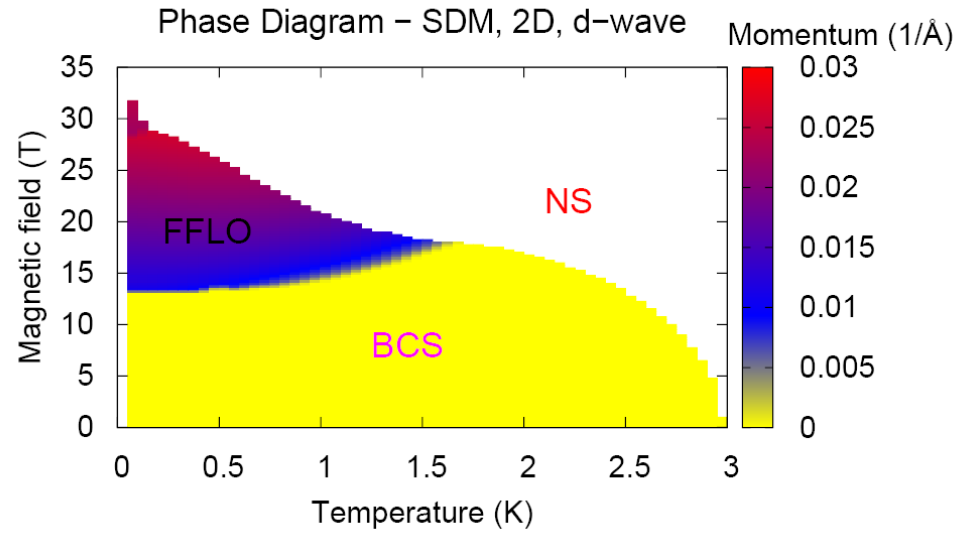
Effective quasiparticle Hamiltonian with effective pairing

(Gutzwiller approximation for quasiparticle states)

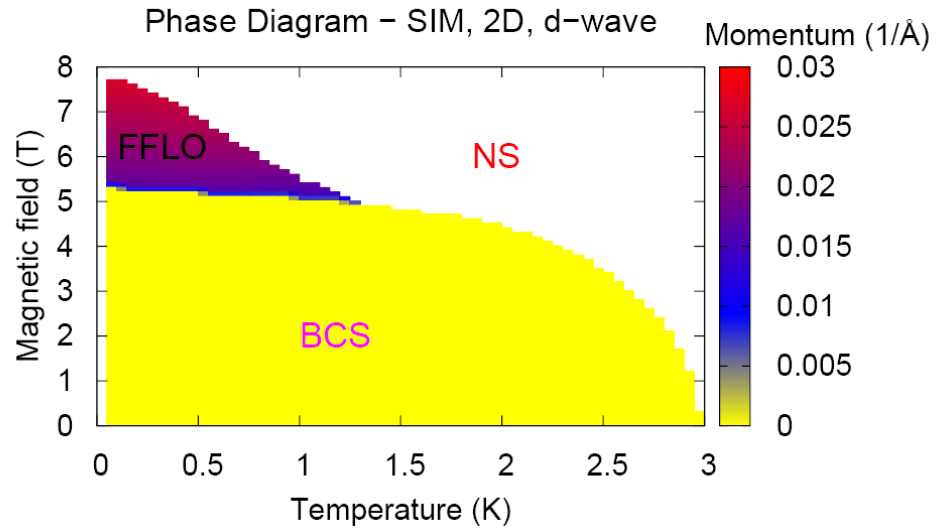
$$\mathcal{H} = \sum_{\mathbf{k}\sigma} (q_{\sigma} \epsilon_{\mathbf{k}} - \sigma \mu_B H) \psi_{\mathbf{k}\sigma}^{\dagger} \psi_{\mathbf{k}\sigma} \\ - \frac{1}{N} \sum_{\mathbf{k}\mathbf{k}'} \frac{4V_{\mathbf{k}}^2 V_{\mathbf{k}'}^2}{\epsilon_f^2 (\epsilon_f + U)} R_{\sigma\bar{\sigma}} \gamma_{\mathbf{k}} \gamma_{\mathbf{k}'} \psi_{\mathbf{k}\uparrow}^{\dagger} \psi_{-\mathbf{k}\downarrow}^{\dagger} \psi_{-\mathbf{k}'\downarrow} \psi_{\mathbf{k}'\uparrow}$$

$$\Psi_{\mathbf{k}\sigma} \approx f_{\mathbf{k}\sigma}$$

a)

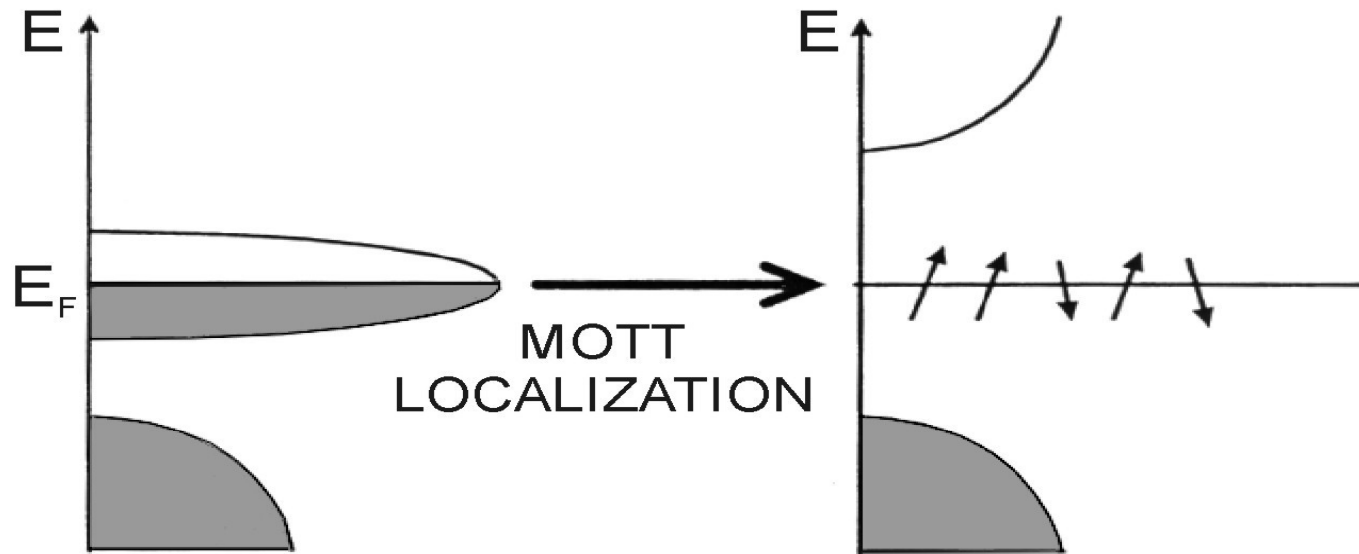


b)

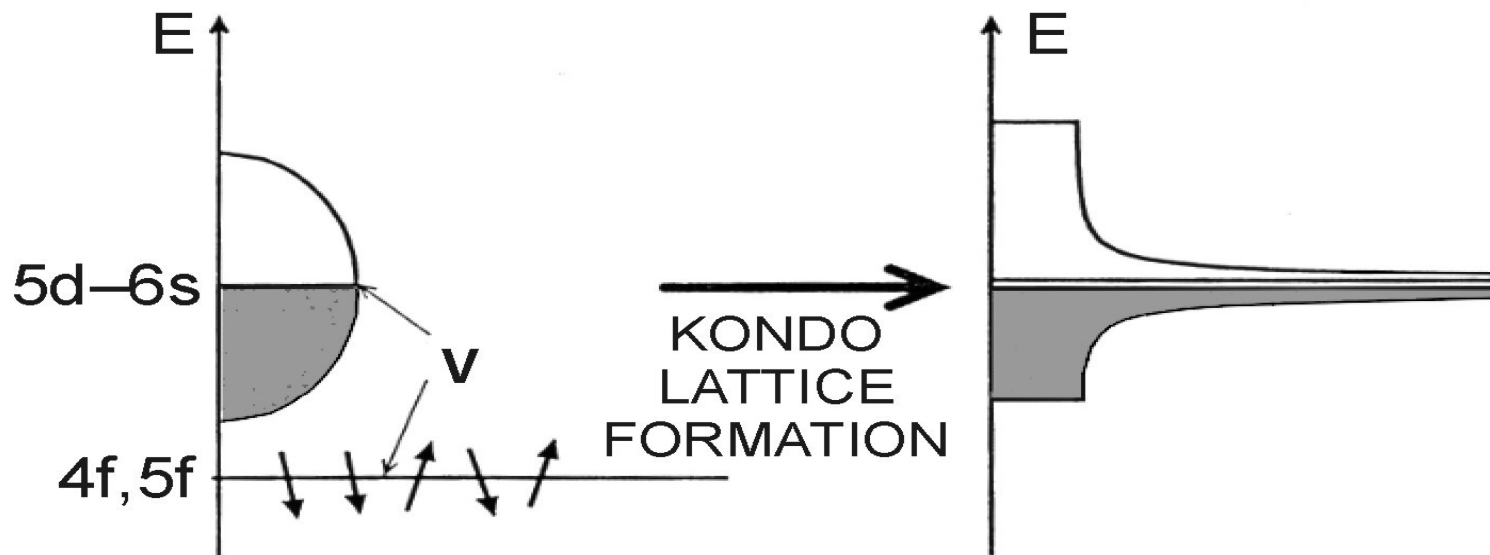


J. Kaczmarczyk and J. Spalek (2009), unpublished

Narrow-band systems (3d)



Hybridized systems (f)



Contents

- 1. Concept of spin-dependent quasiparticle mass (1990 → 2005):**

$$m_{\sigma} / m_0 = 1/Z_{\sigma} \rightarrow \infty$$

- 2. Cooper pair with the spin-dependent masses of quasiparticles:**

Układy skorelowanych fermionów:

1. Co oznacza "silnie skorelowane?"

Punkt startowy: en. oddziaływania $\gg$ en. pasmowa

$$U \gg W$$

Przybliżenie H-F nawet jakościowo niepoprawne (LDA)

Przybliżenie Gutzwillera = MFA

(Gutzwiller projection)

2. Model t-J (sieci Andersona) punktem startowym, albo model Hubbarda z $U \gg W$

Dlaczego ważne? **AF** $\rightarrow$ **SC** (ewolucja ze stanu izol. Motta)

3. Paired states:

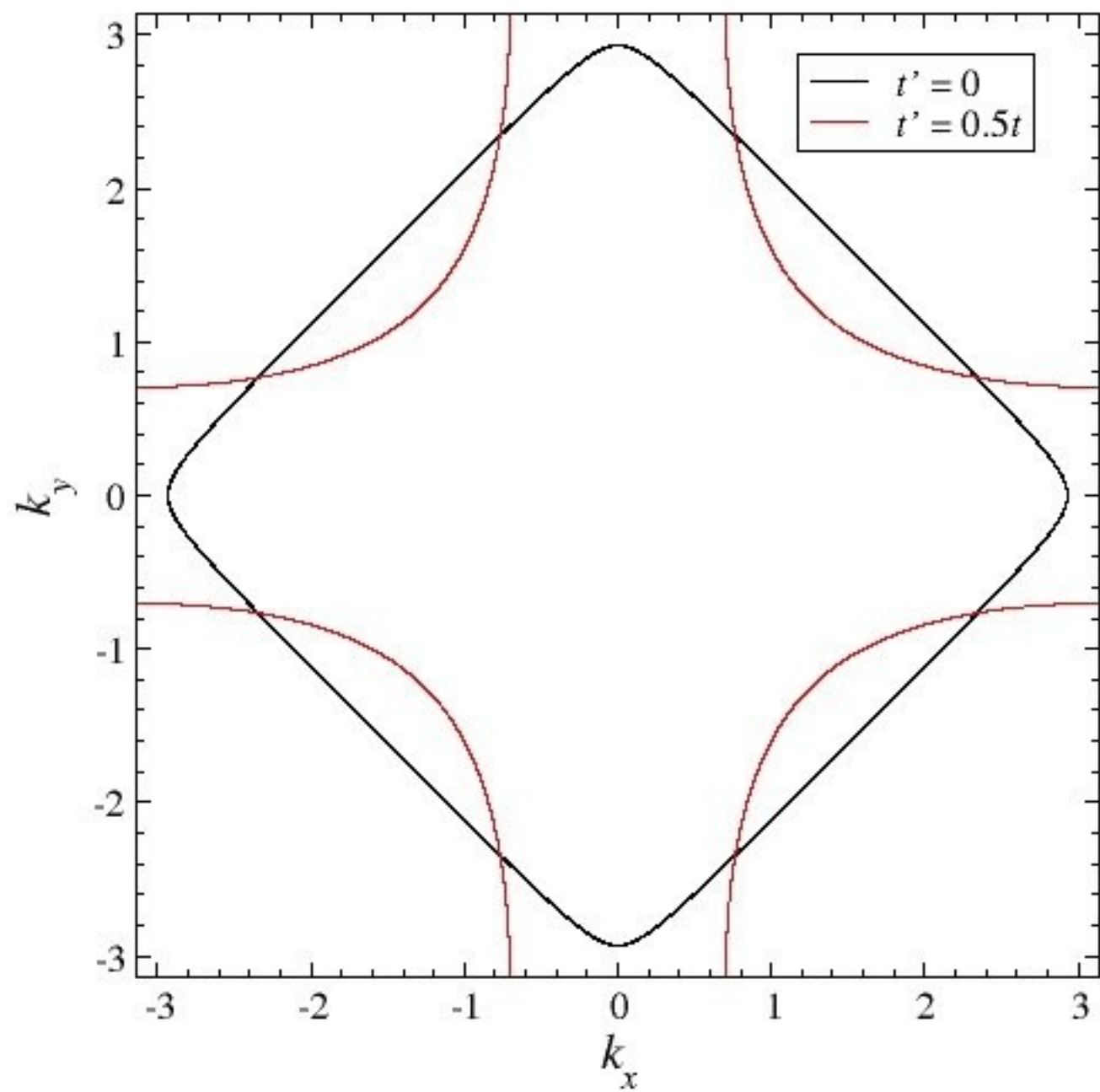
- **BCS state (condensate at rest)**
- **FFLO state (moving condensate of pairs)**

$$\begin{aligned}
H &= \sum_{ij\sigma} \left(-t_{ij\sigma} (\mathbf{A}) - \delta_{ij}\mu \right) c_{i\sigma}^\dagger c_{j\sigma} + g\mu_B H \sum_i \left(c_{i\uparrow}^\dagger c_{i\uparrow} - c_{i\downarrow}^\dagger c_{i\downarrow} \right) \\
&+ \sum_i \left(\Delta_i^* c_{i\uparrow} c_{i\downarrow} + \text{H.c.} \right)
\end{aligned}$$

$$\Delta_i = V \langle c_{i\downarrow} c_{i\uparrow} \rangle$$

$$t_{ij\sigma} (\mathbf{A}) = t_{ij\sigma} \exp \left(\frac{ie}{\hbar c} \int_{\mathbf{R}_j}^{\mathbf{R}_i} \mathbf{A} \cdot d\mathbf{l} \right)$$

$$t_{ij\sigma} = t_{ij} \frac{1-n}{1-n_\sigma}, \quad n = n_\uparrow + n_\downarrow$$



Fulde–Ferrell–Larkin–Ovchinnikov phase:

- Two-dimensional case ($H \parallel c$)
- Tight-binding approximation: $t'/t = -0.5$
- Pauli term dominant (Zeeman term only)

f

- Results: $n = \text{const}$,
 $\mu = \text{const}$ (quark-gluon plasma),
F. Wilczek, PRL (2003).

$$\begin{aligned}
H &= \sum_{\mathbf{k},\sigma} (\varepsilon_{\mathbf{k},\sigma} - \mu) c_{\mathbf{k},\sigma}^\dagger c_{\mathbf{k},\sigma} + g\mu_B H \sum_{\mathbf{k}} \left(c_{\mathbf{k},\uparrow}^\dagger c_{\mathbf{k},\uparrow} - c_{\mathbf{k},\downarrow}^\dagger c_{\mathbf{k},\downarrow} \right) \\
&+ \sum_{\mathbf{k}} \left(\Delta_{\mathbf{Q}}^* c_{\mathbf{k},\uparrow} c_{-\mathbf{k}+\mathbf{Q},\downarrow} + \text{H.c.} \right)
\end{aligned}$$

$$\Delta_{\mathbf{Q}} = \frac{V}{N} \sum_{\mathbf{k}} \langle c_{-\mathbf{k}+\mathbf{Q},\downarrow} c_{\mathbf{k},\uparrow} \rangle$$

$$\varepsilon_{\mathbf{k},\sigma} = -2t_{\sigma} (\cos k_x + \cos k_y) + 4t'_{\sigma} \cos k_x \cos k_y$$

$$\Delta_{\mathbf{Q}} = -\frac{1}{N} \sum_{\mathbf{k}} \Delta_{\mathbf{Q}} \frac{f(E_{\mathbf{k},\mathbf{Q},1}) - f(E_{\mathbf{k},\mathbf{Q},2})}{E_{\mathbf{k},\mathbf{Q},1} - E_{\mathbf{k},\mathbf{Q},2}}$$

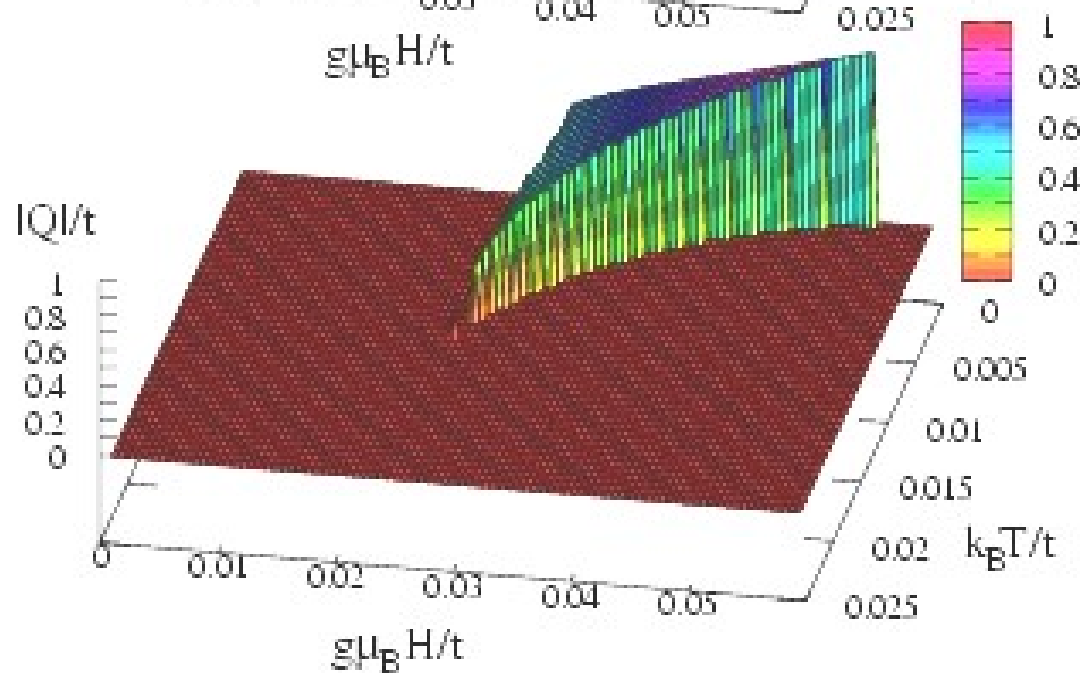
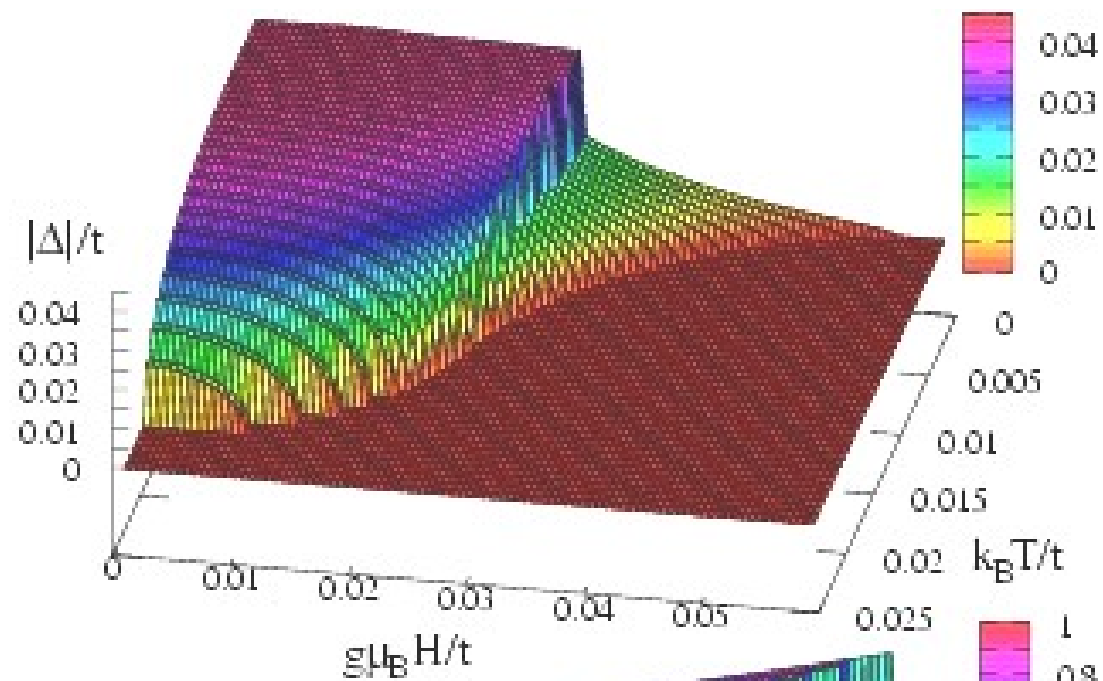
$$\begin{aligned}
E_{\mathbf{k},\mathbf{Q},1/2} &= g\mu_B H + \frac{1}{2} (\varepsilon_{\mathbf{k},\uparrow} - \varepsilon_{-\mathbf{k}+\mathbf{Q},\downarrow}) \\
&\pm \frac{1}{2} \sqrt{(\varepsilon_{\mathbf{k},\uparrow} + \varepsilon_{-\mathbf{k}+\mathbf{Q},\downarrow} - 2\mu)^2 + 4|\Delta_{\mathbf{Q}}|^2}
\end{aligned}$$

$$\begin{aligned}
n_{\uparrow} = & \frac{1}{N} \sum_{\mathbf{k}} \frac{1}{E_{\mathbf{k},\mathbf{Q},1} - E_{\mathbf{k},\mathbf{Q},2}} \\
& \times [(E_{\mathbf{k},\mathbf{Q},1} + \varepsilon_{-\mathbf{k}+\mathbf{Q},\downarrow} - \mu - g\mu_{\text{B}}H) f(E_{\mathbf{k},\mathbf{Q},1}) \\
& - (E_{\mathbf{k},\mathbf{Q},2} + \varepsilon_{-\mathbf{k}+\mathbf{Q},\downarrow} - \mu - g\mu_{\text{B}}H) f(E_{\mathbf{k},\mathbf{Q},2})]
\end{aligned}$$

$$\begin{aligned}
n_{\downarrow} = & 1 - \frac{1}{N} \sum_{\mathbf{k}} \frac{1}{E_{\mathbf{k},\mathbf{Q},1} - E_{\mathbf{k},\mathbf{Q},2}} \\
& \times [(E_{\mathbf{k},\mathbf{Q},1} - \varepsilon_{\mathbf{k},\uparrow} + \mu - g\mu_{\text{B}}H) f(E_{\mathbf{k},\mathbf{Q},1}) \\
& - (E_{\mathbf{k},\mathbf{Q},2} - \varepsilon_{\mathbf{k},\uparrow} + \mu - g\mu_{\text{B}}H) f(E_{\mathbf{k},\mathbf{Q},2})]
\end{aligned}$$

$$\begin{aligned}
\Omega(\Delta_{\mathbf{Q}}, \mathbf{Q}) = & -k_{\text{B}}T \sum_{\mathbf{k}} \sum_{\alpha=1}^2 \ln [1 + \exp(-E_{\mathbf{k},\mathbf{Q},\alpha}/k_{\text{B}}T)] \\
& + \sum_{\mathbf{k}} (\varepsilon_{-\mathbf{k}+\mathbf{Q},\downarrow} - \mu - g\mu_{\text{B}}H) + \frac{N|\Delta|^2}{V}
\end{aligned}$$

$$F(\Delta_{\mathbf{Q}}, \mathbf{Q}) = \Omega(\Delta_{\mathbf{Q}}, \mathbf{Q}) + \mu n N$$



Outlook:

1. Pair momentum Q nonzero, FFLO

a

1. $H \neq 0$: indistinguishable quasiparticles transforming into distinguishable !!!

↑

↓

2. $m \neq m$: wide range of the FFLO appearance.

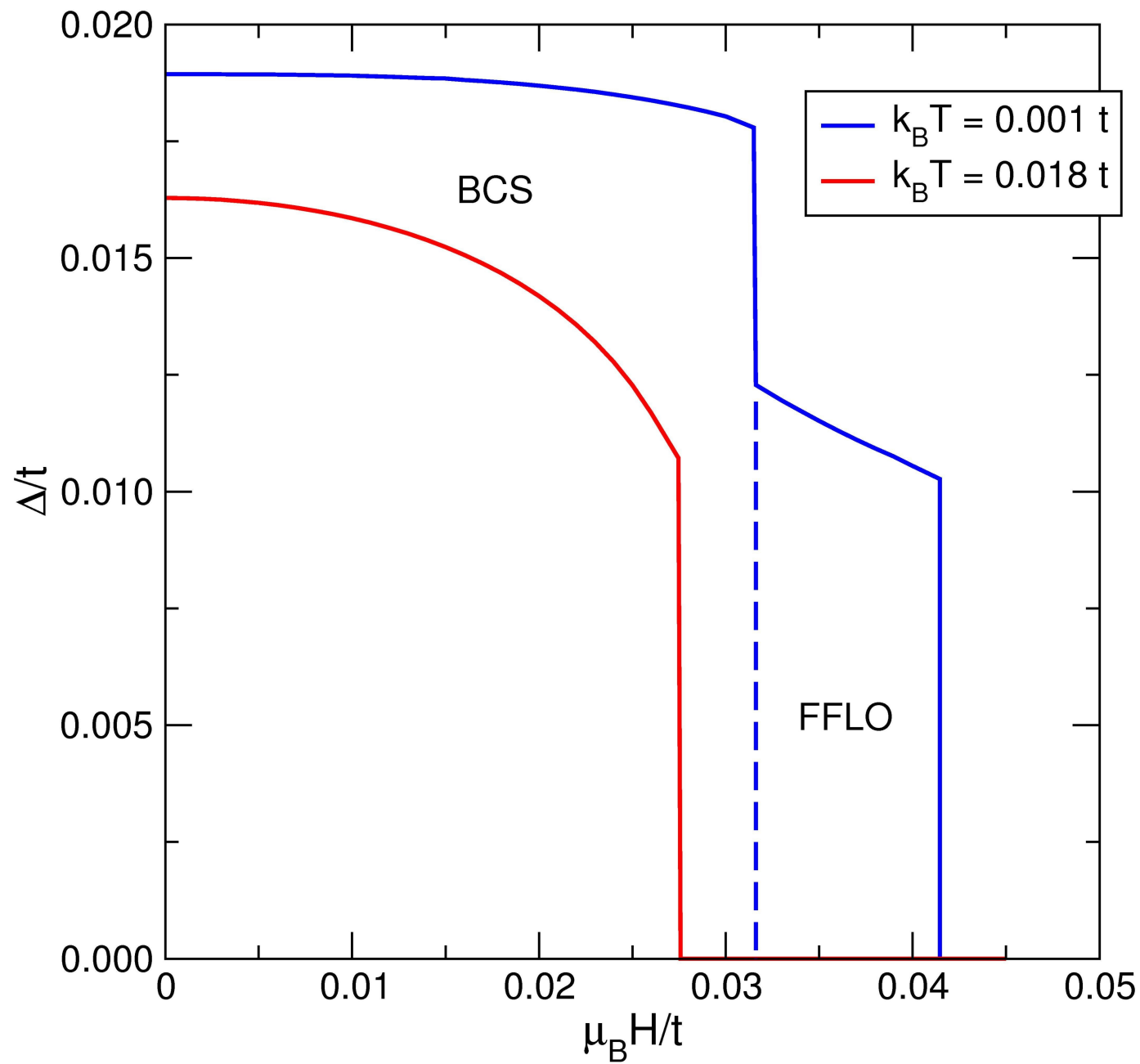
f

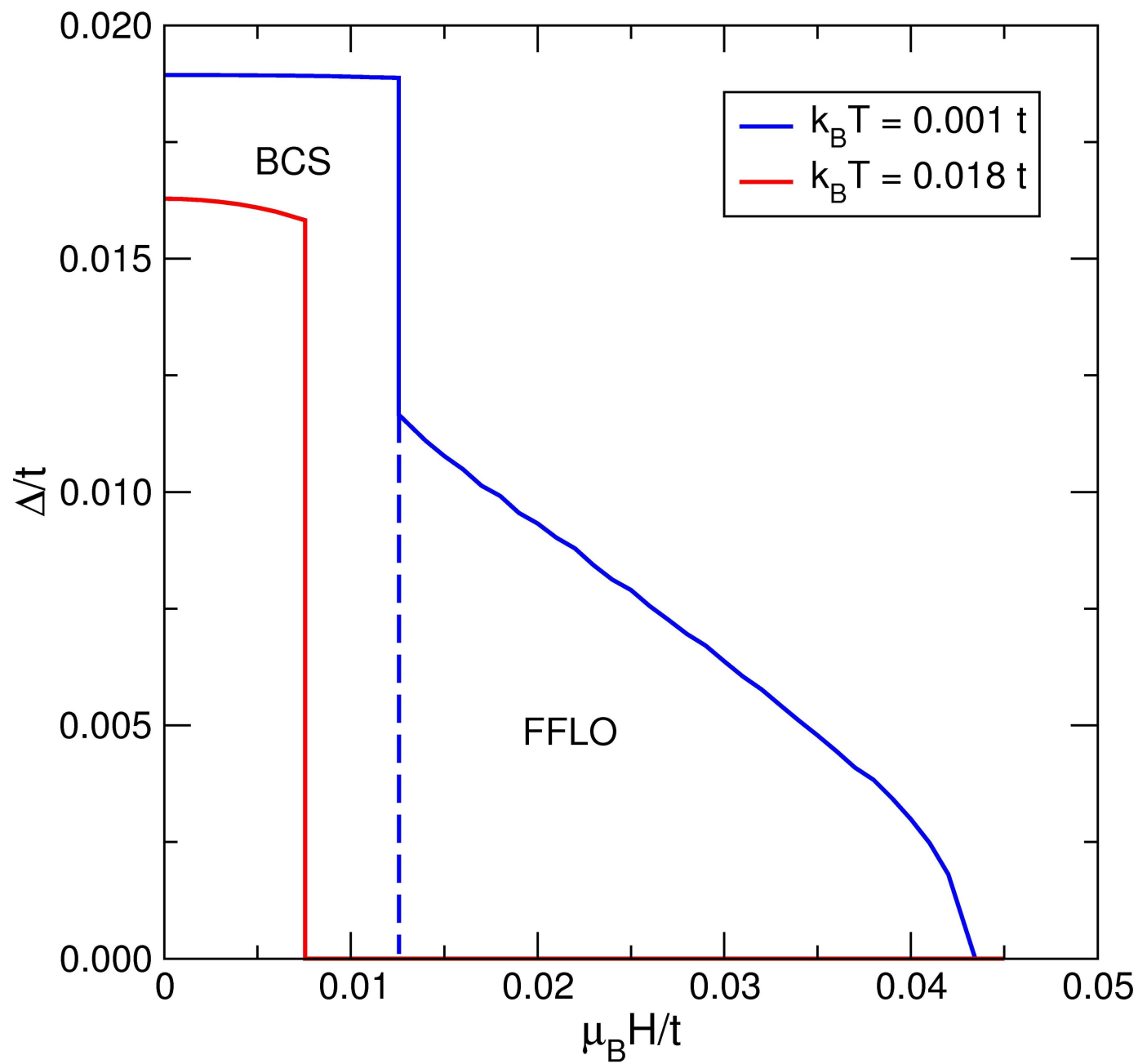
1. $\mu = \text{const}$ or $n = \text{const}$

1. s-wave or d-wave?

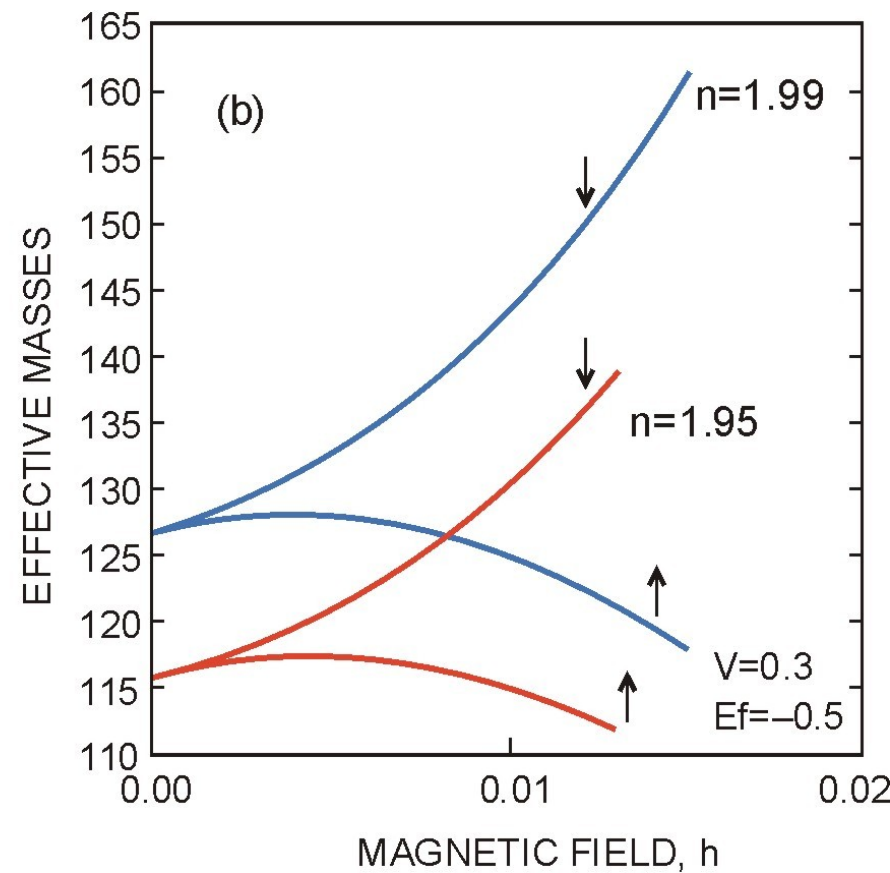
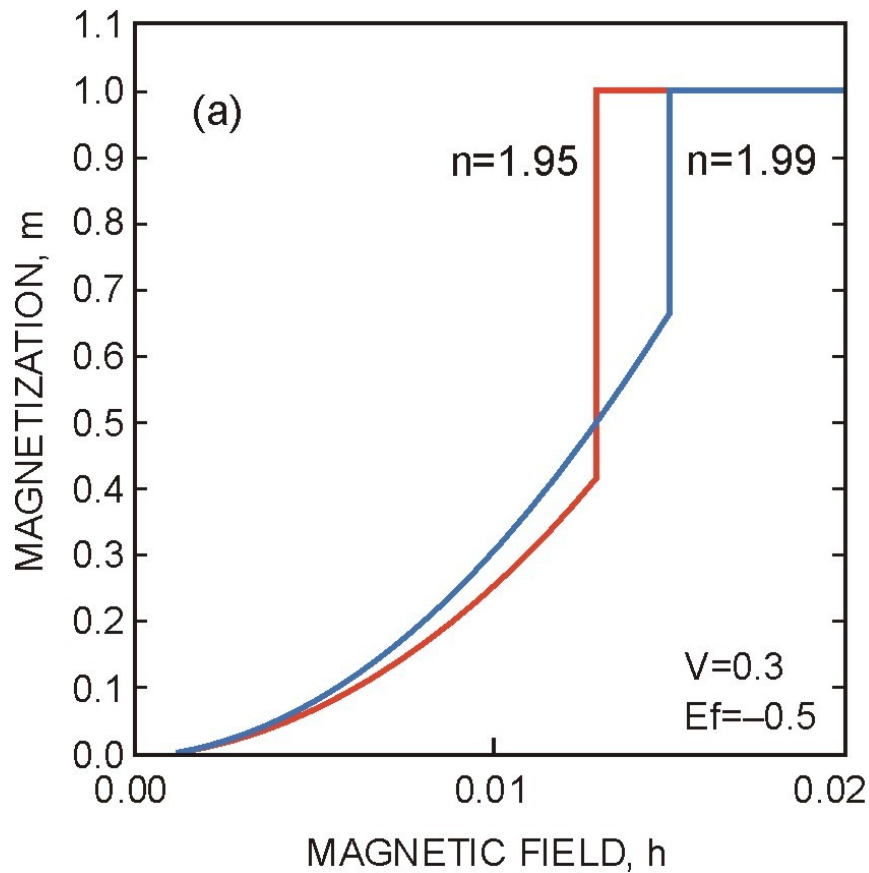
1. Cooper pair for the case with spin dependent masses:

- $Q = 0$ versus $Q \neq 0$
- Indistinguishable $\rightarrow$ distinguishable

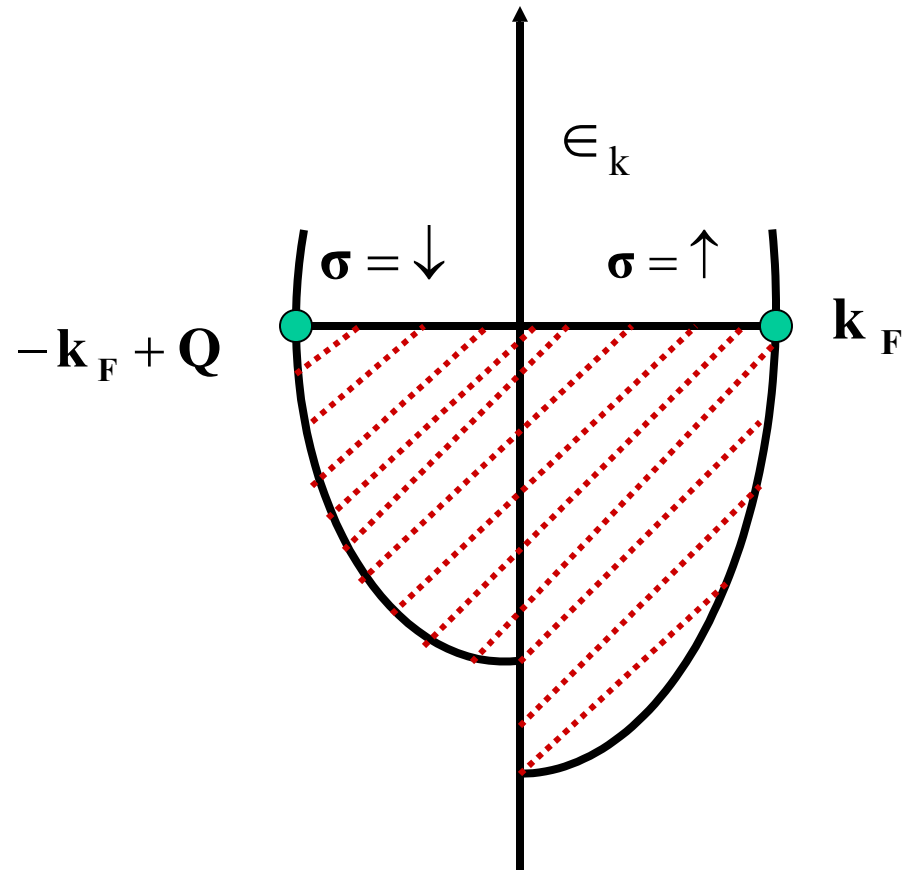




Sieć Kondo: $U \rightarrow \infty$



- (a) Magnetization curve ($m=n_{\uparrow}-n_{\downarrow}$) for $n \rightarrow 2$ and in the Kondo-lattice limit;
 (b) spin-split masses for majority ($\sigma=\uparrow$) and minority ($\sigma=\downarrow$) spin subbands



a

Fermi wavevector mismatch for $H \neq 0$:

$$\alpha_{\mathbf{k}\uparrow} = u_{\mathbf{k}} a_{\mathbf{k}+\mathbf{Q}/2\uparrow} - v_{\mathbf{k}} a_{-\mathbf{k}+\mathbf{Q}/2\downarrow}^{\dagger}$$

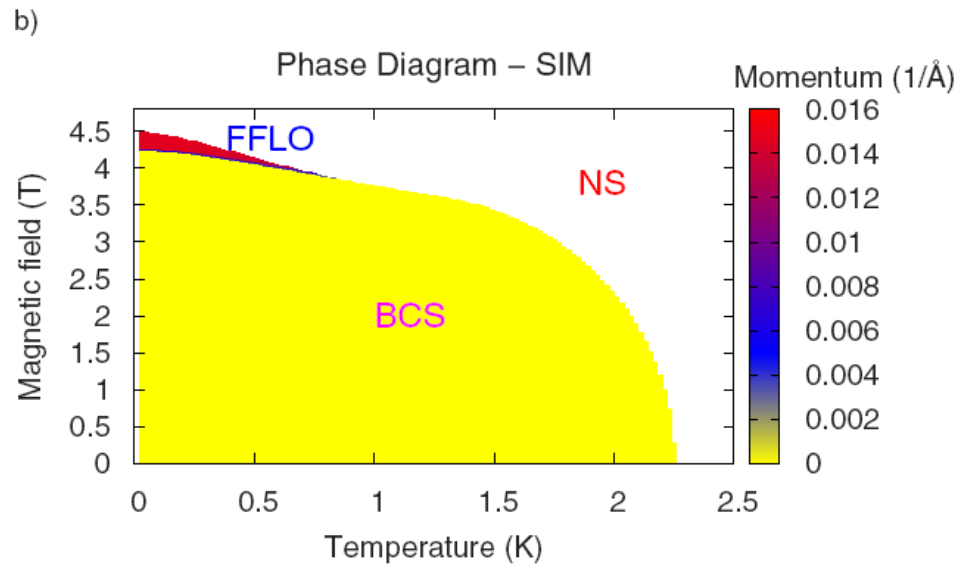
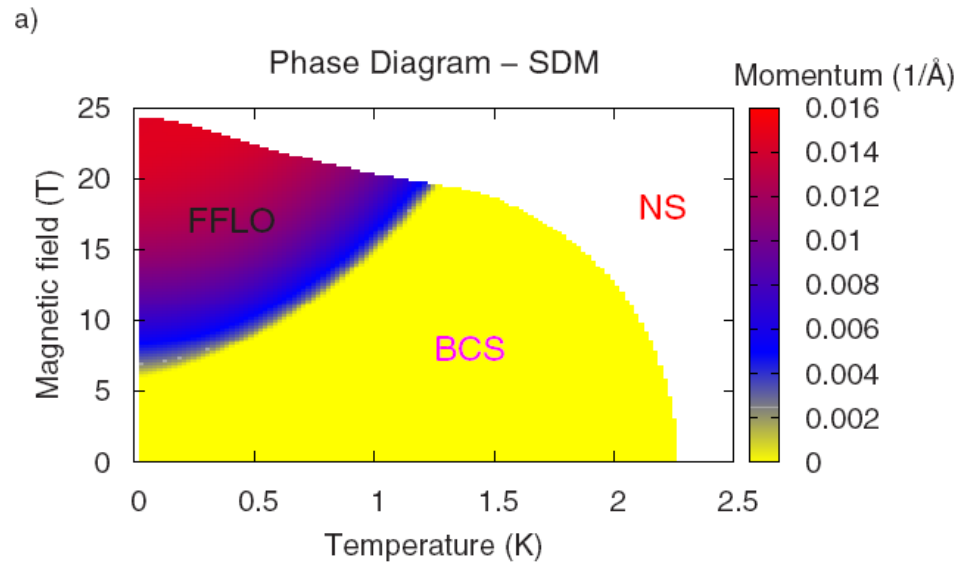
$$\alpha_{\mathbf{k}\downarrow}^{\dagger} = v_{\mathbf{k}} a_{\mathbf{k}+\mathbf{Q}/2\uparrow} + u_{\mathbf{k}} a_{-\mathbf{k}+\mathbf{Q}/2\downarrow}^{\dagger}$$

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} E_{\mathbf{k}\sigma} \alpha_{\mathbf{k}\sigma}^{\dagger} \alpha_{\mathbf{k}\sigma} + \sum_{\mathbf{k}} (\xi_{\mathbf{k}}^{(s)} - E_{\mathbf{k}}) + N \frac{\Delta_{\mathbf{Q}}^2}{V_0} + \frac{N}{n} \bar{m} h_{\text{cor}}$$

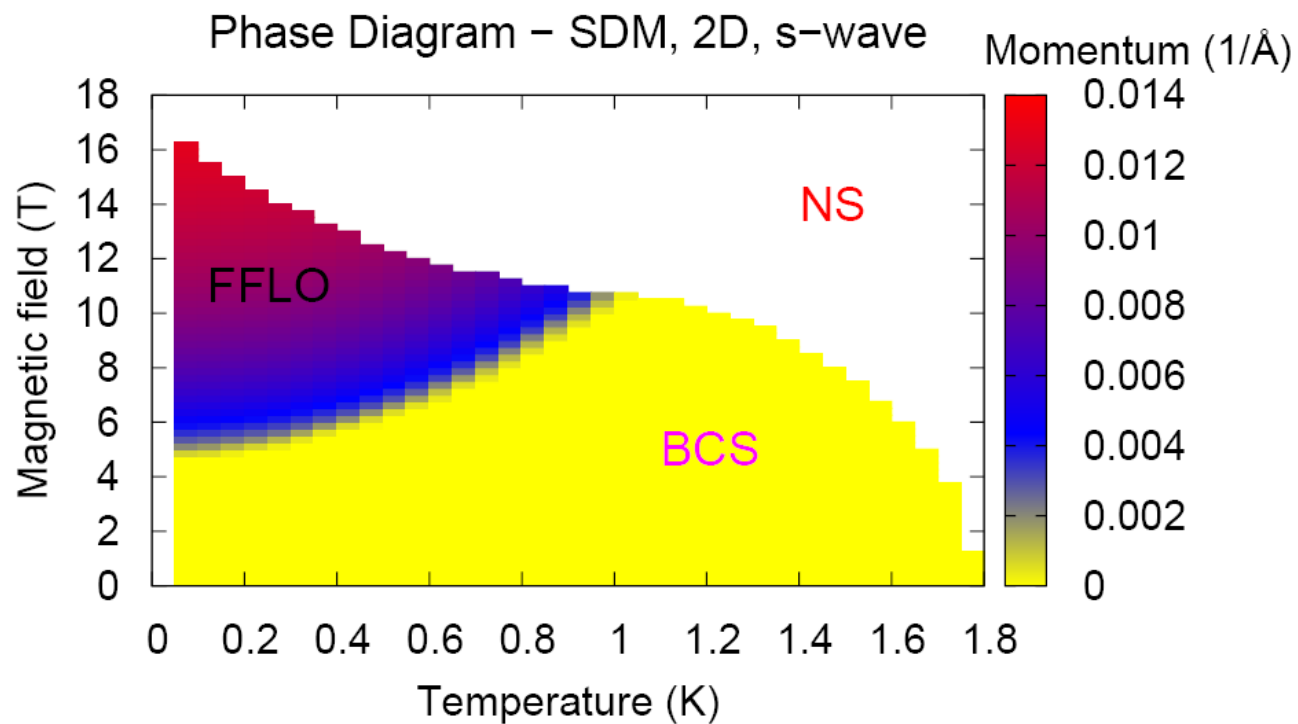
$$E_{\mathbf{k}\sigma} = E_{\mathbf{k}} + \sigma \xi_{\mathbf{k}}^{(a)}, \quad E_{\mathbf{k}} = \sqrt{\xi_{\mathbf{k}}^{(s)2} + \Delta_{\mathbf{Q}}^2}$$

$$\xi_{\mathbf{k}}^{(s)} \equiv \frac{1}{2}(\xi_{\mathbf{k}+\mathbf{Q}/2\uparrow} + \xi_{-\mathbf{k}+\mathbf{Q}/2\downarrow}), \quad \xi_{\mathbf{k}}^{(a)} \equiv \frac{1}{2}(\xi_{\mathbf{k}+\mathbf{Q}/2\uparrow} - \xi_{-\mathbf{k}+\mathbf{Q}/2\downarrow})$$

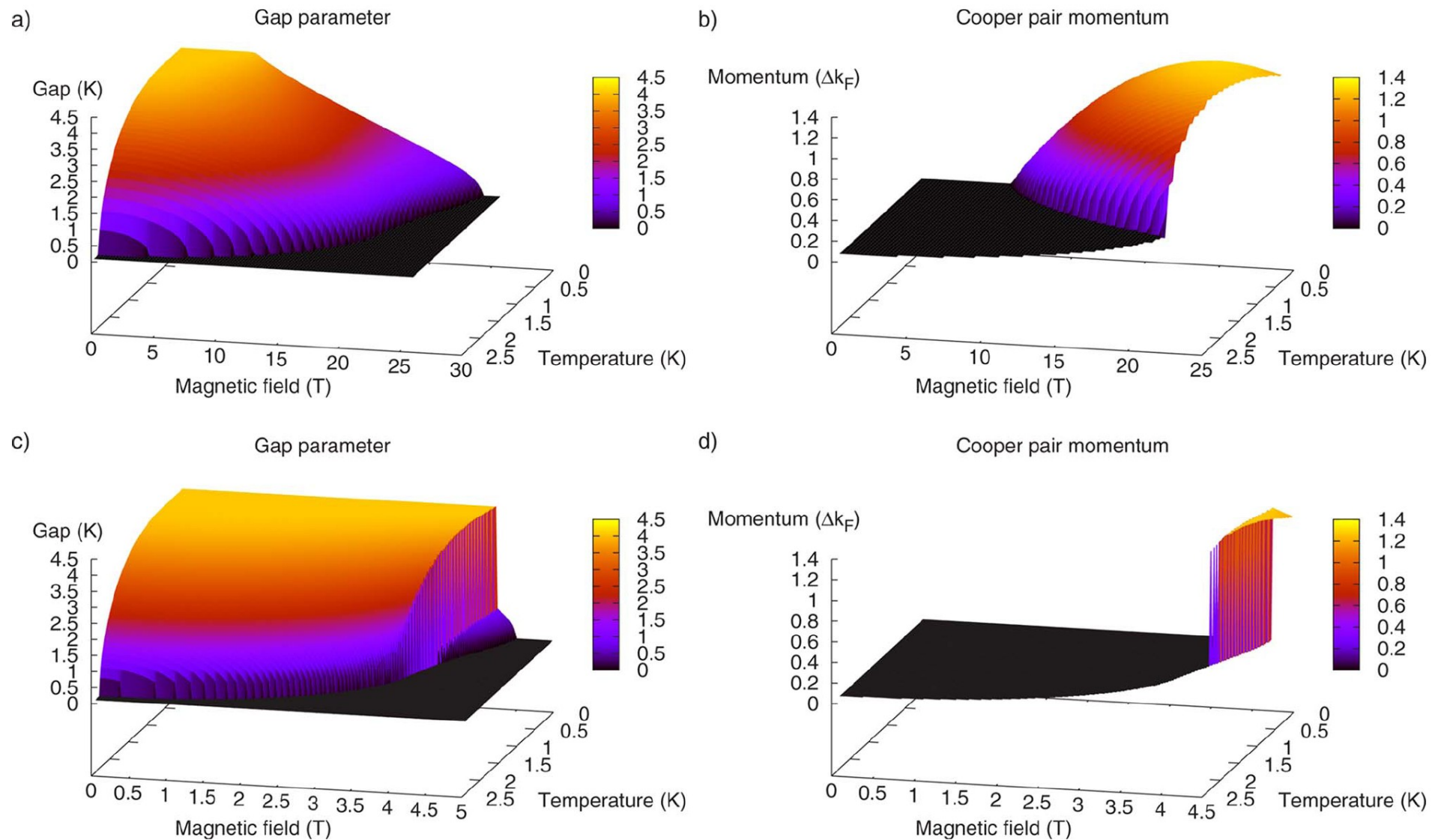
$$\Delta_{\mathbf{Q}} \equiv \frac{1}{N} \sum_{\mathbf{k}} \langle a_{-\mathbf{k}+\mathbf{Q}/2\downarrow} a_{\mathbf{k}+\mathbf{Q}/2\uparrow} \rangle$$



J. Kaczmarczyk and J. Spalek, PRB 79, 214519, pp. 1-15 (2009)



J. Kaczmarczyk and J. Spalek (2009), unpublished



J. Kaczmarczyk and J. Spalek, PRB 79, 214519, pp. 1-15 (2009)

Magnetization

0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000

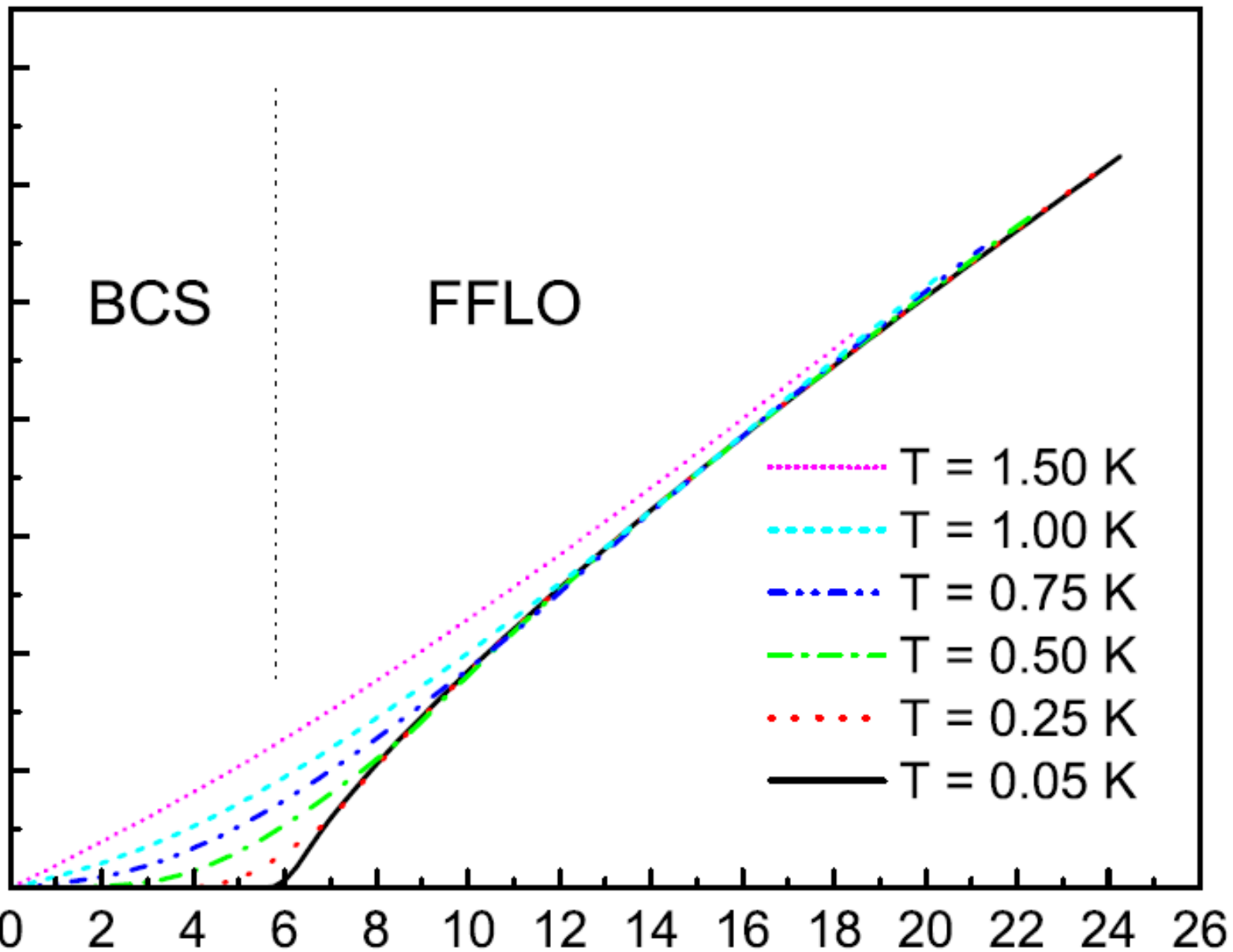
BCS

FFLO

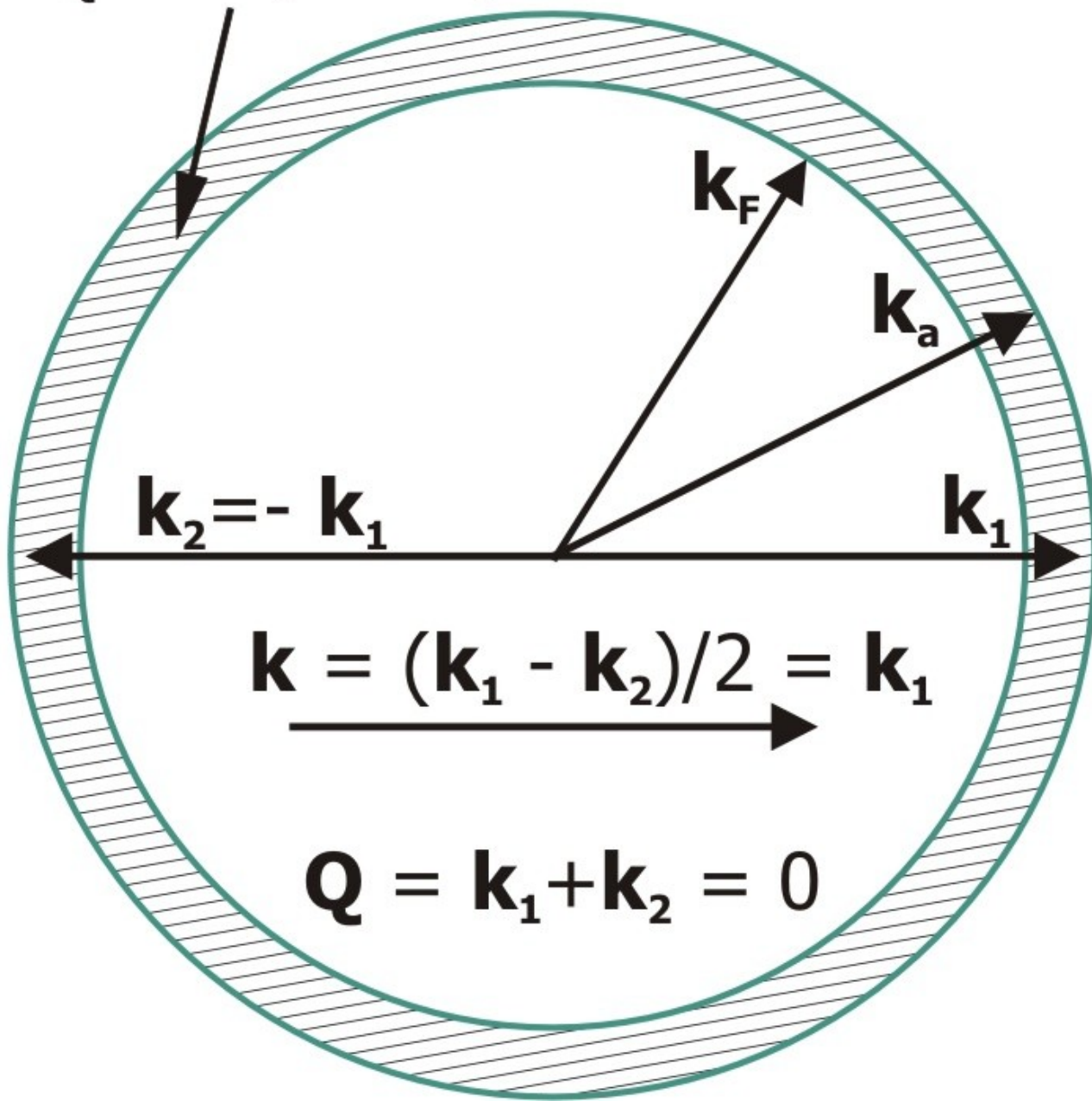
- $T = 1.50$ K
- $T = 1.00$ K
- $T = 0.75$ K
- $T = 0.50$ K
- $T = 0.25$ K
- $T = 0.05$ K

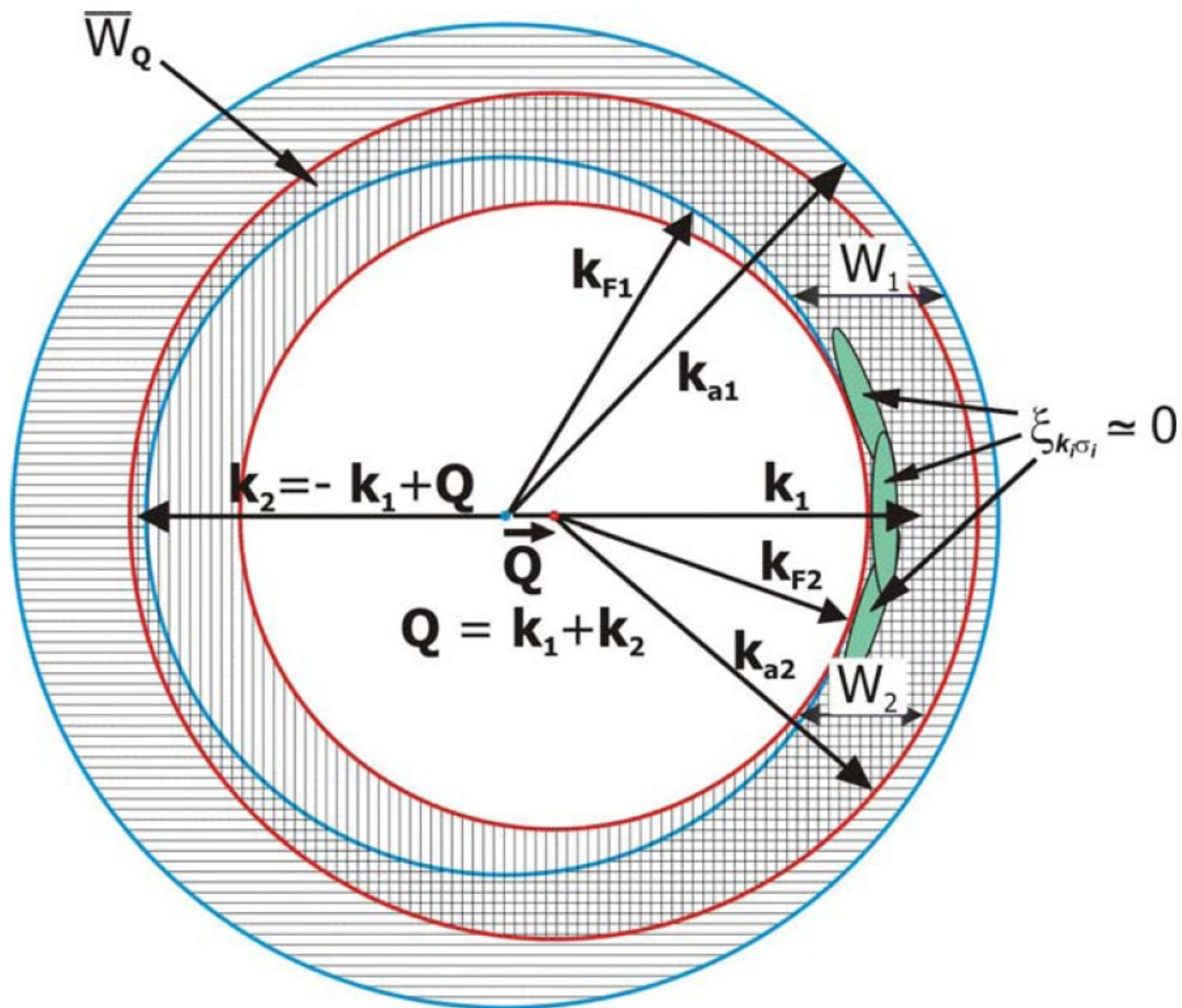
Magnetic field (T)

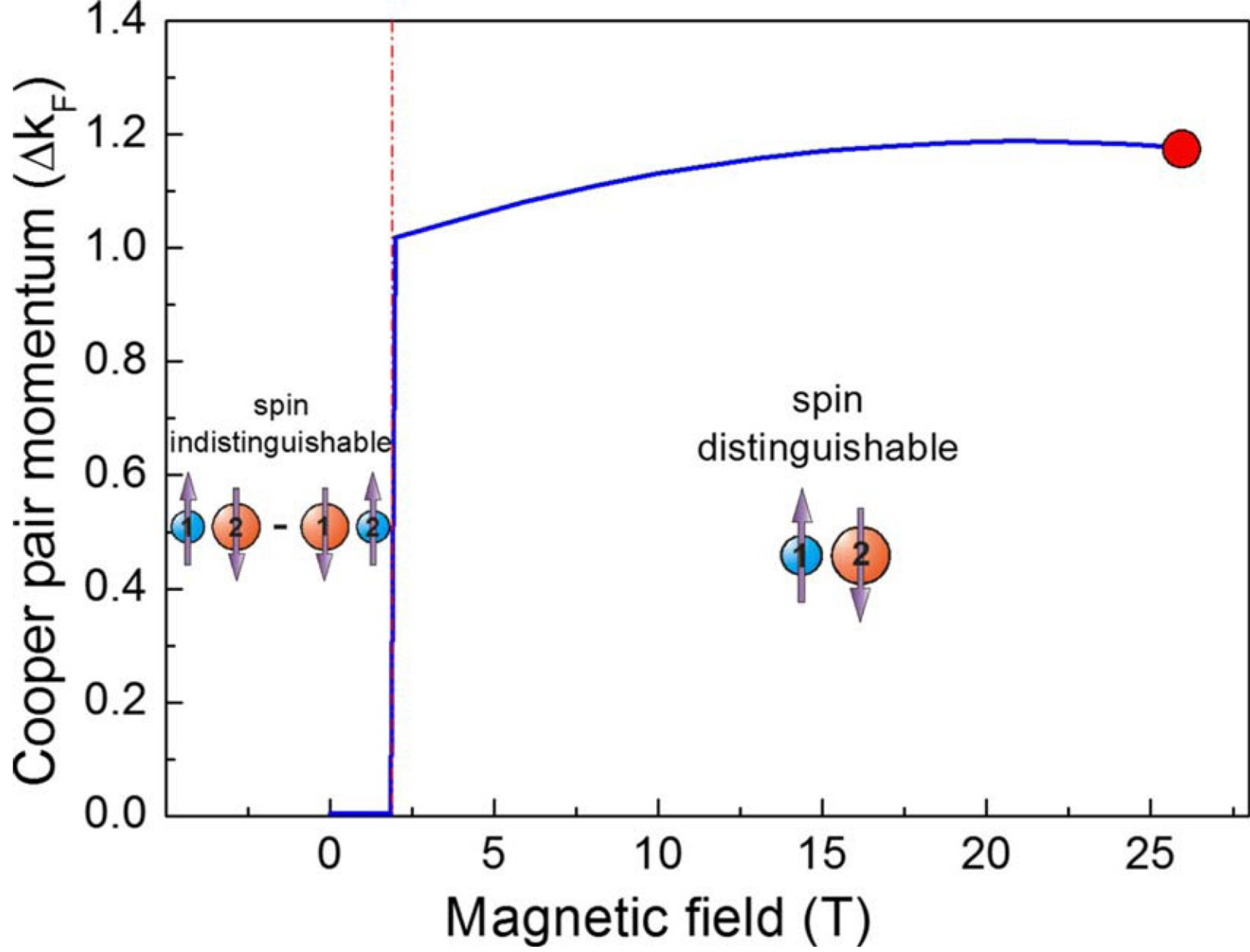
0 2 4 6 8 10 12 14 16 18 20 22 24 26

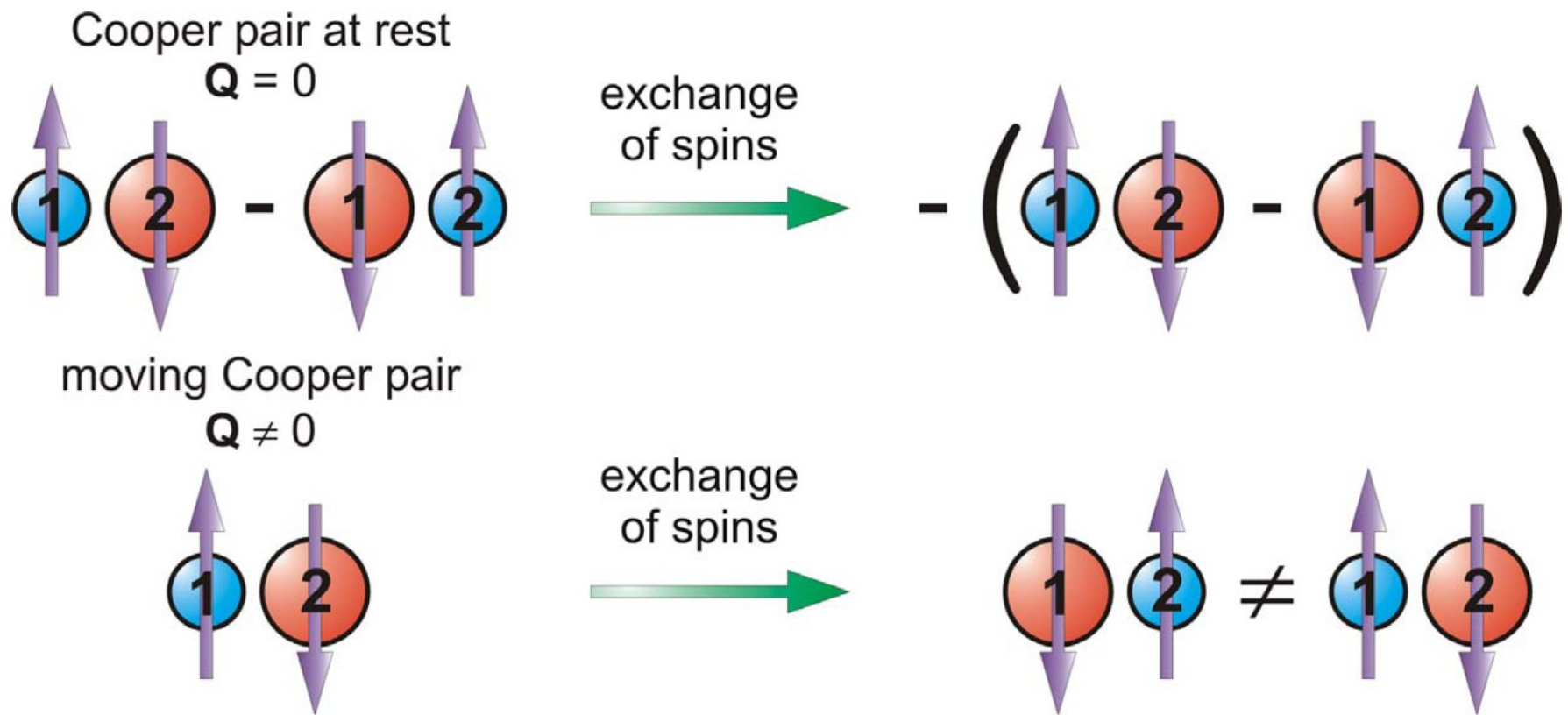


$$\overline{W}_Q = W_1 = W_2$$

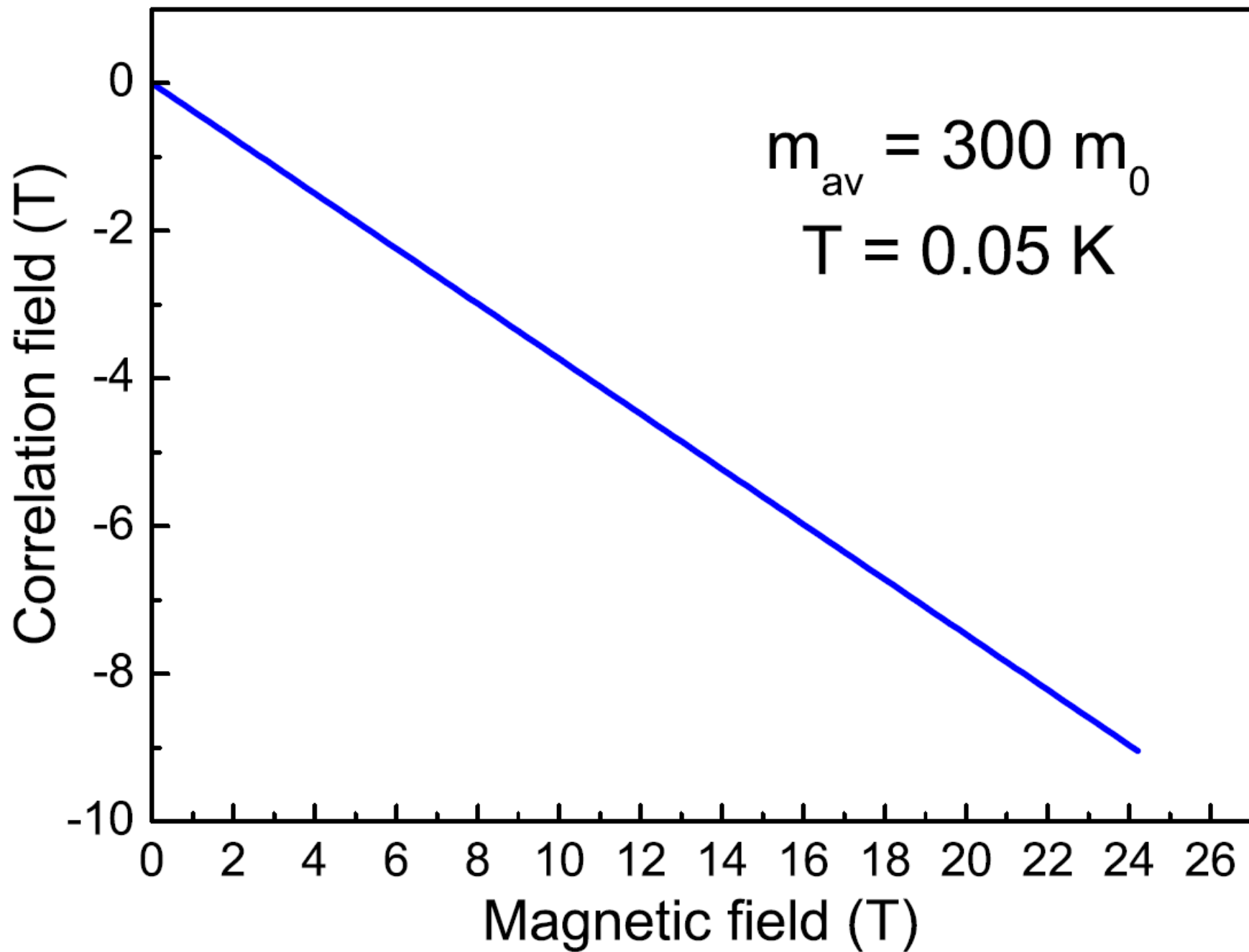


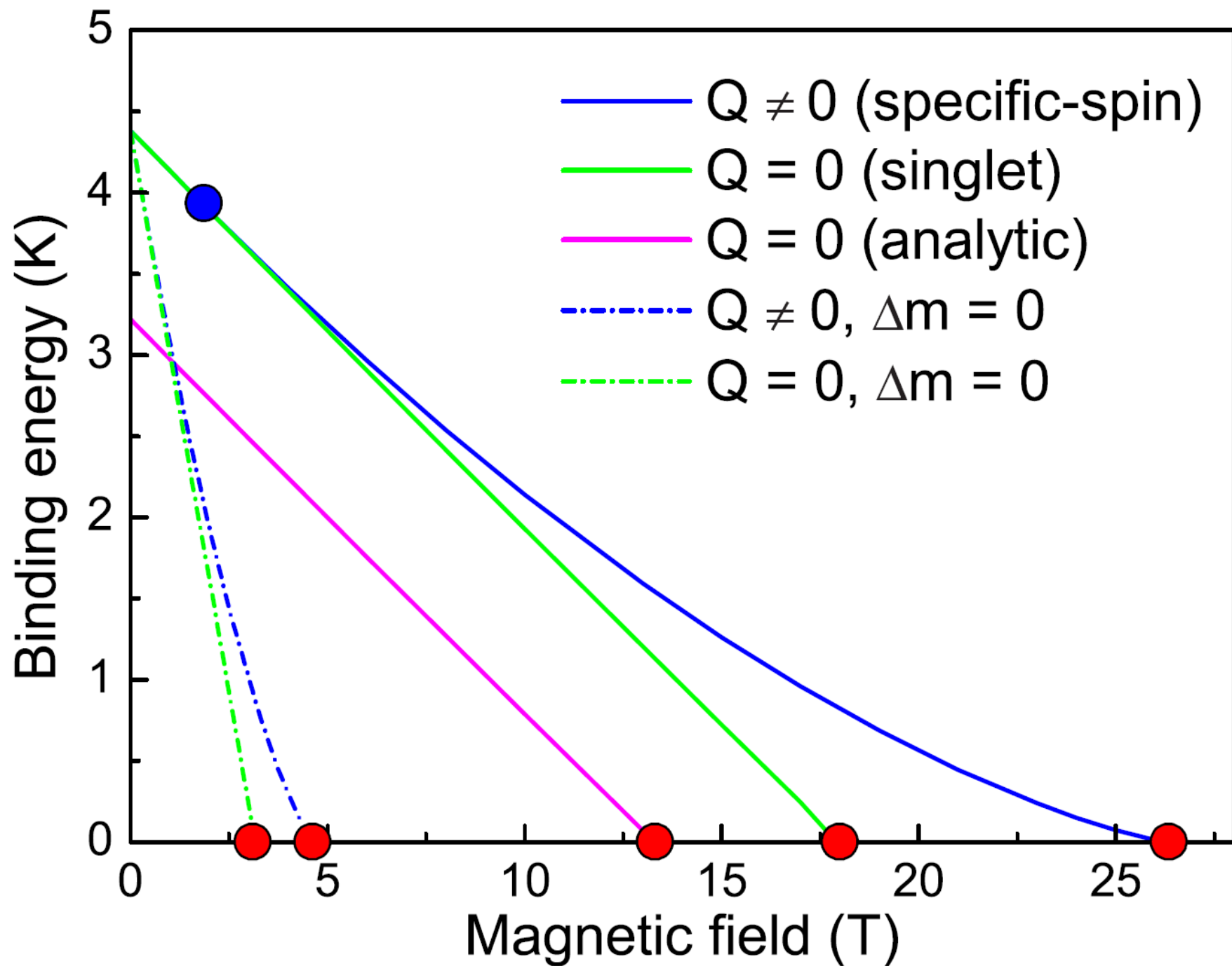


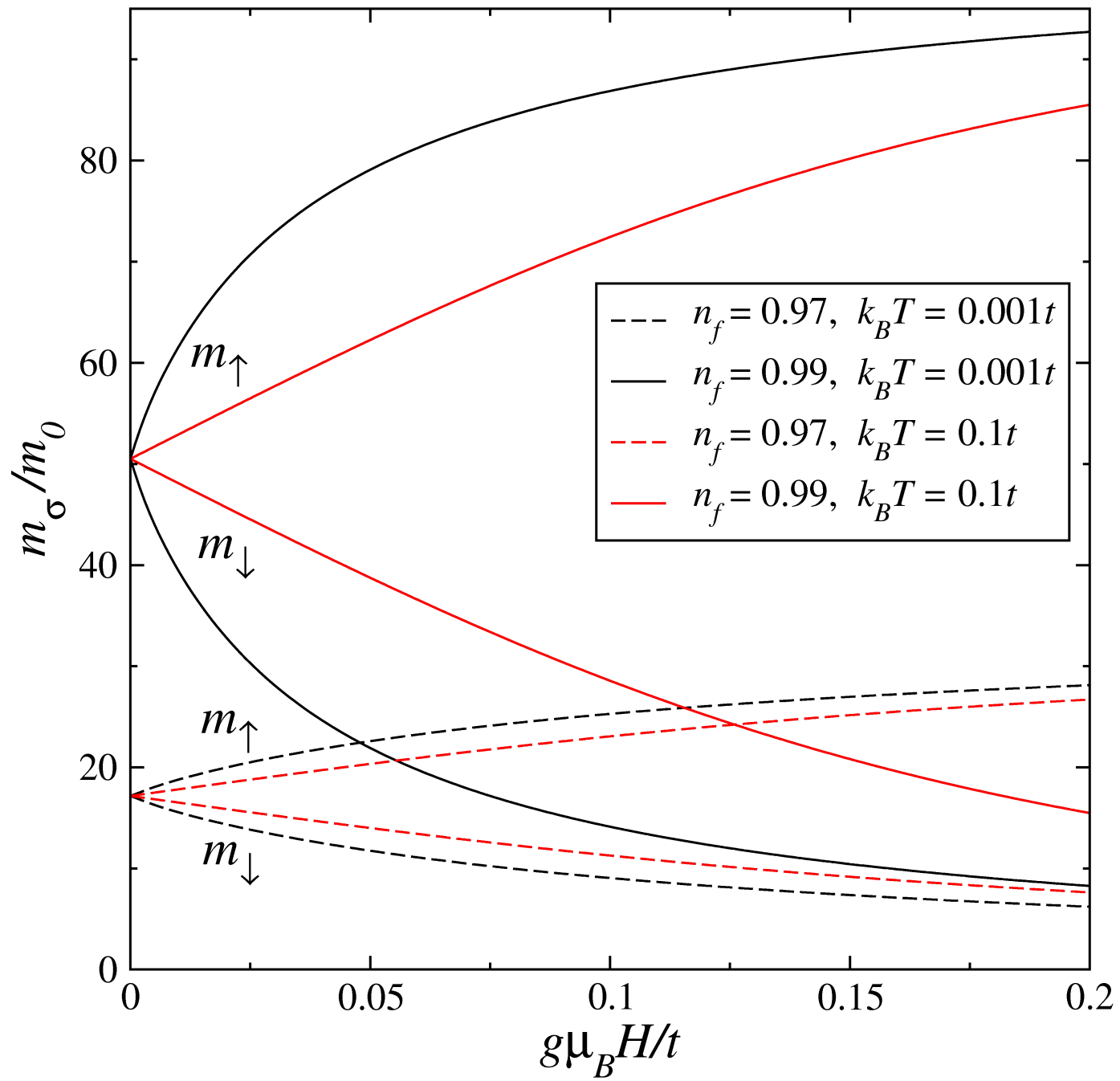


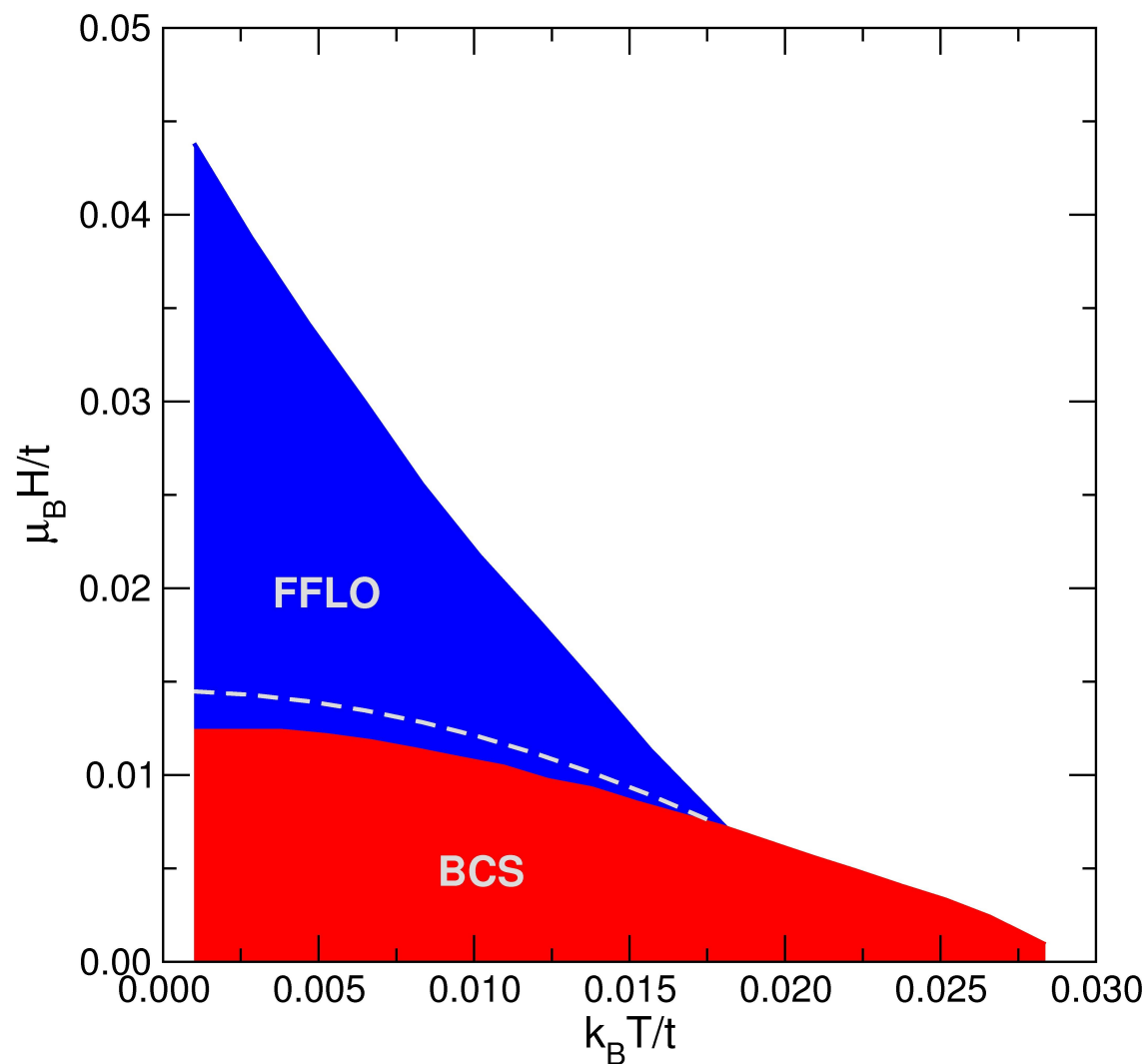


$$\chi_1(\uparrow)\chi_2(\downarrow) = \frac{1}{2}[\chi_1(\uparrow)\chi_2(\downarrow) - \chi_1(\downarrow)\chi_2(\uparrow)] + \frac{1}{2}[\chi_1(\uparrow)\chi_2(\downarrow) + \chi_1(\downarrow)\chi_2(\uparrow)]$$

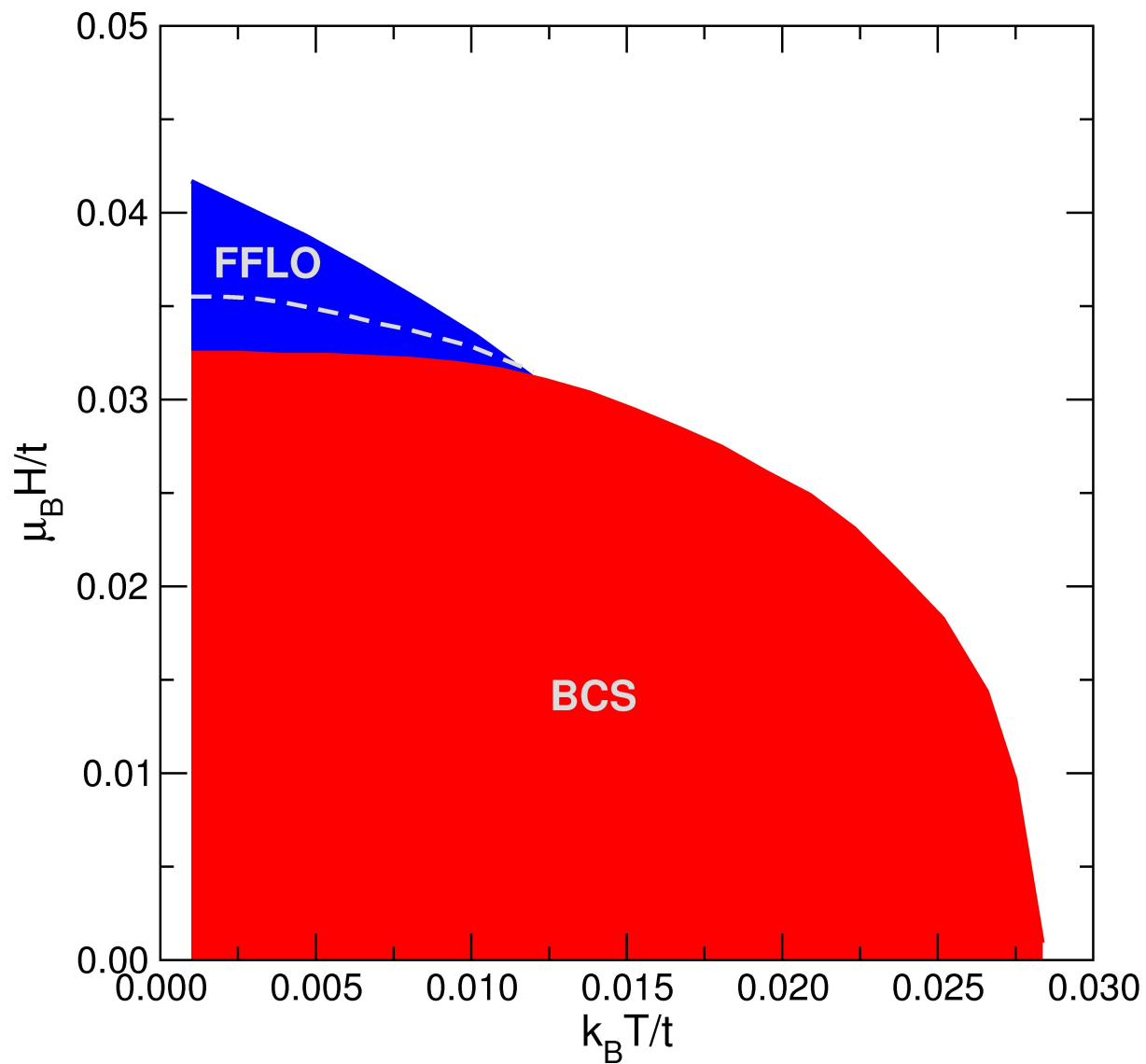




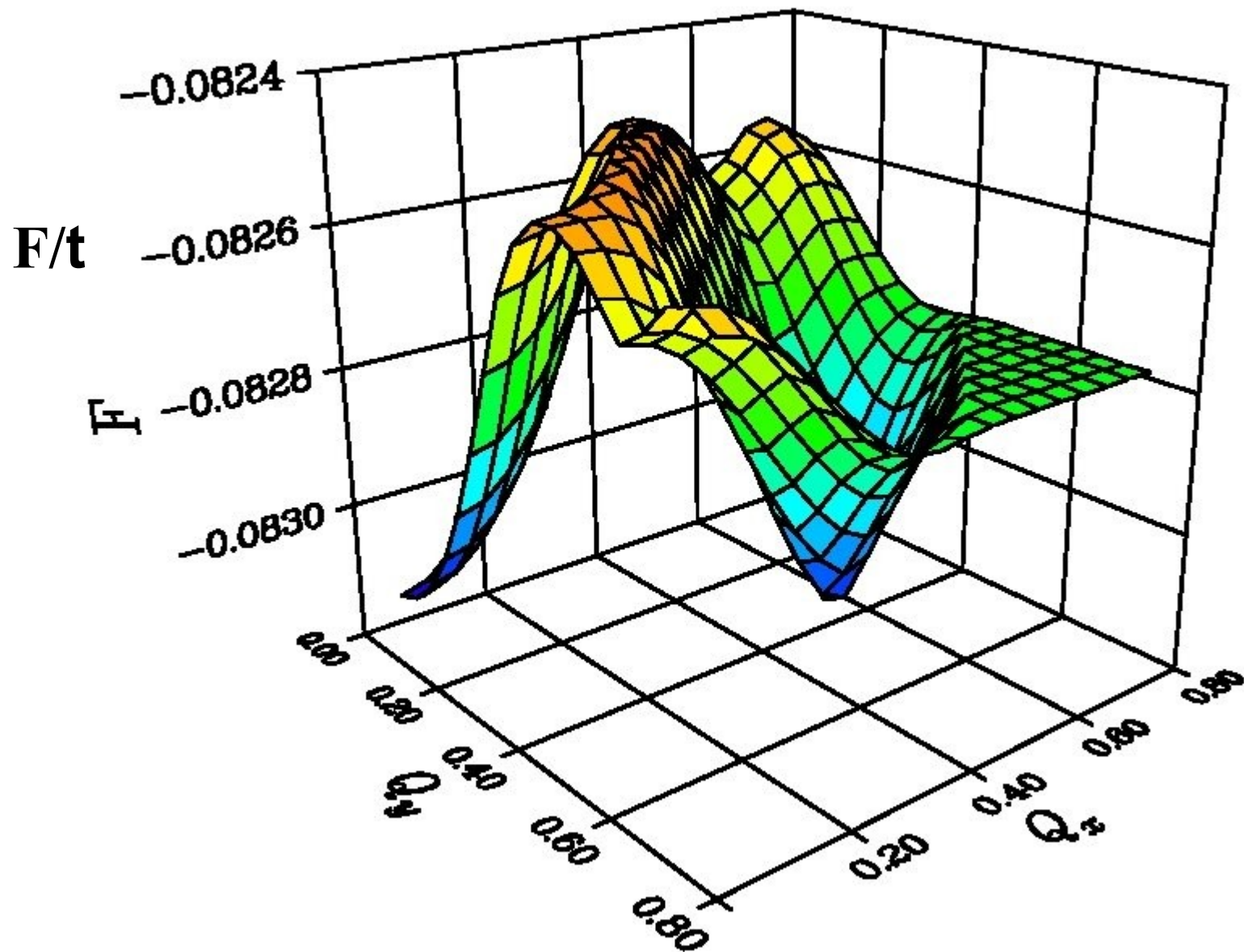




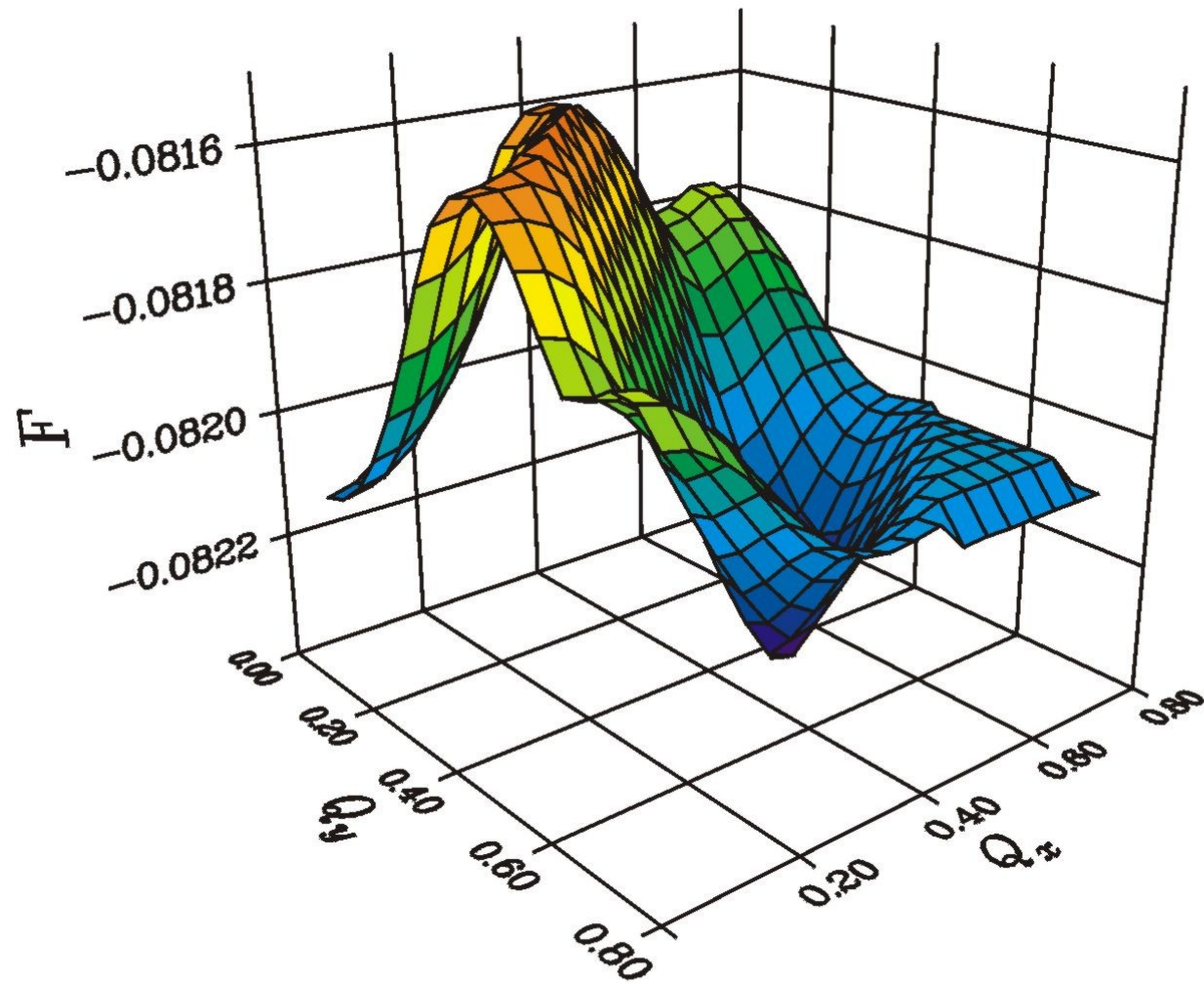
M. Mańska, M. Mierzejewski, J. Kaczmarczyk, J. S. (2009), unpubl.



M. Maśka, M. Mierzejewski, J. Kaczmarczyk, J. S. (2009), unpubl.



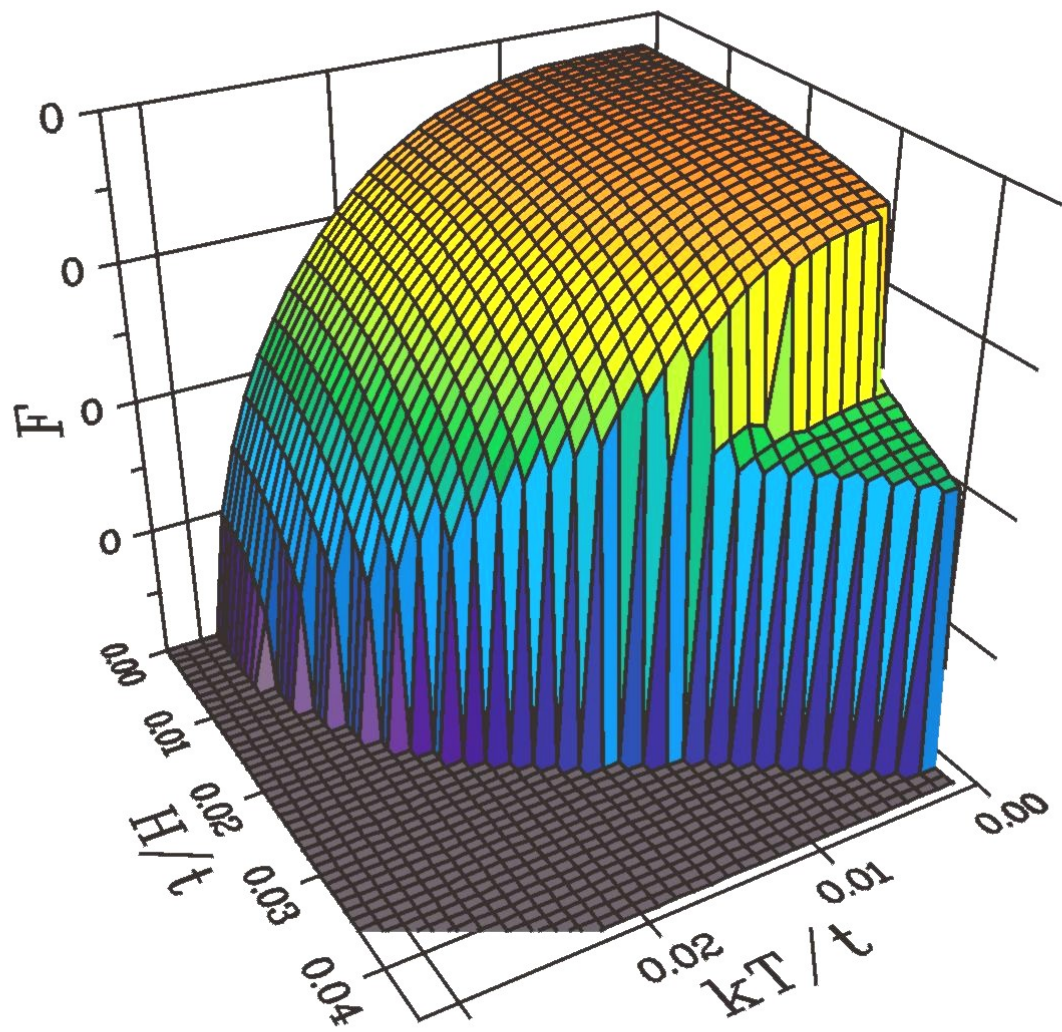
M. Maśka, M. Mierzejewski, J. Kaczmarczyk, J. S. (2009), unpubl.

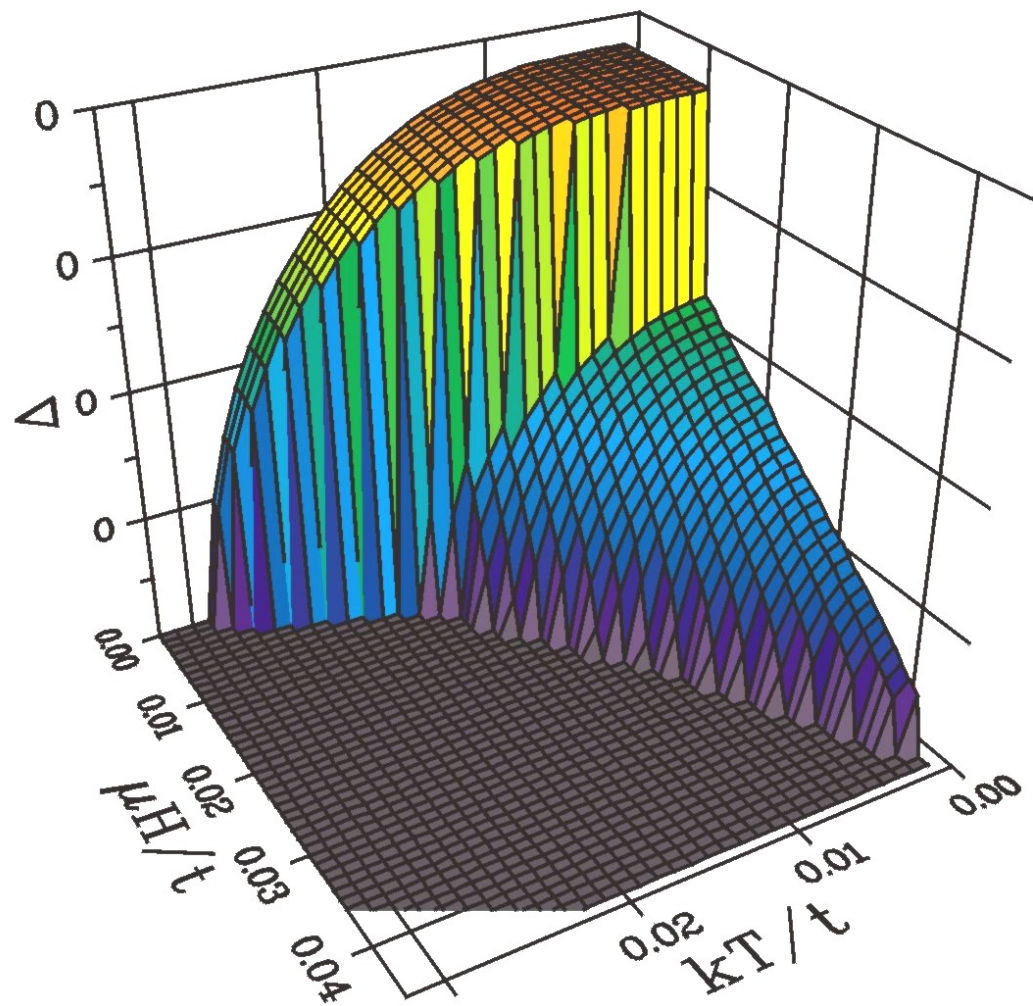


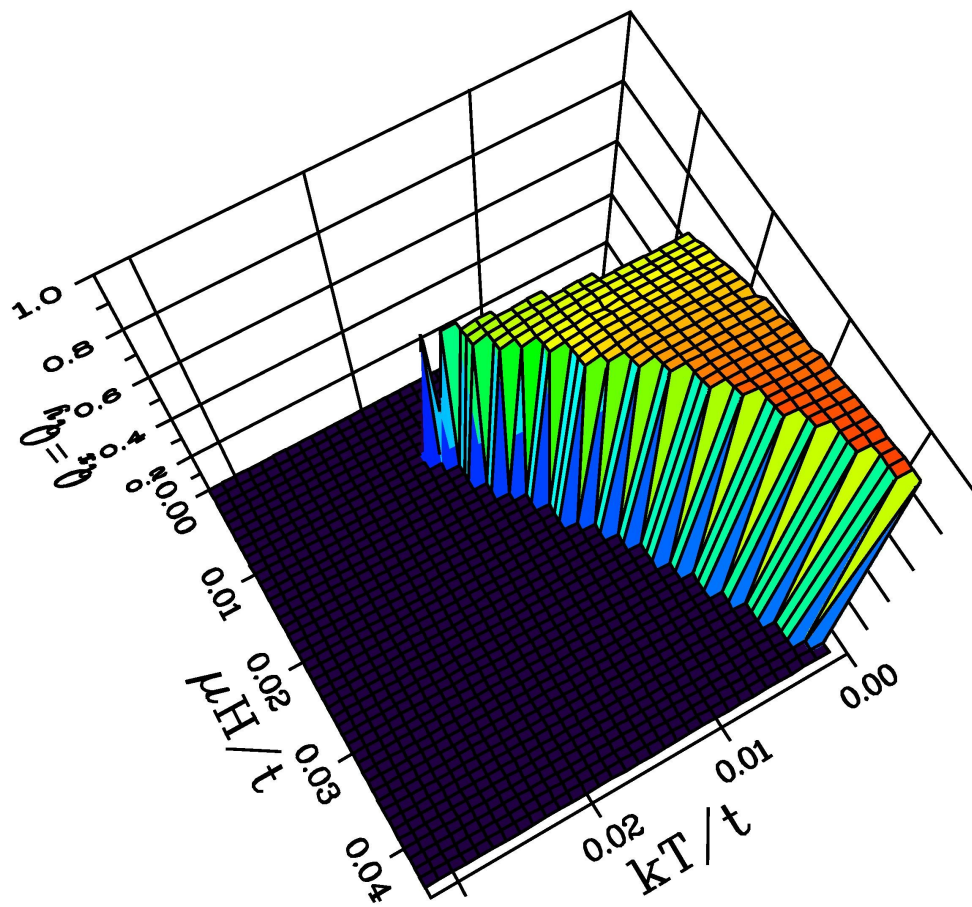
M. Maśka, M. Mierzejewski, J. Kaczmarczyk, J. S. (2009), unpubl.

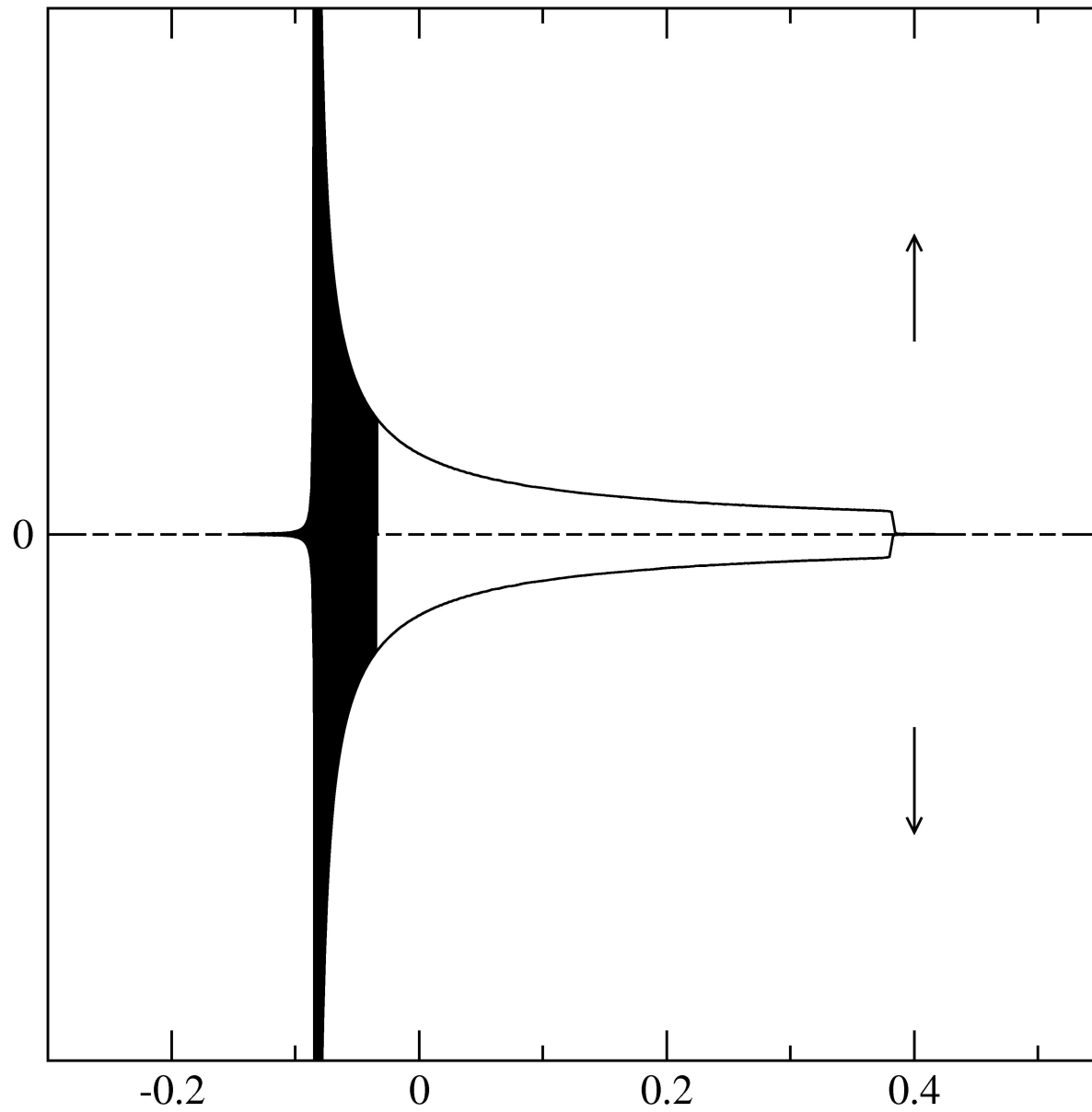
Conclusions

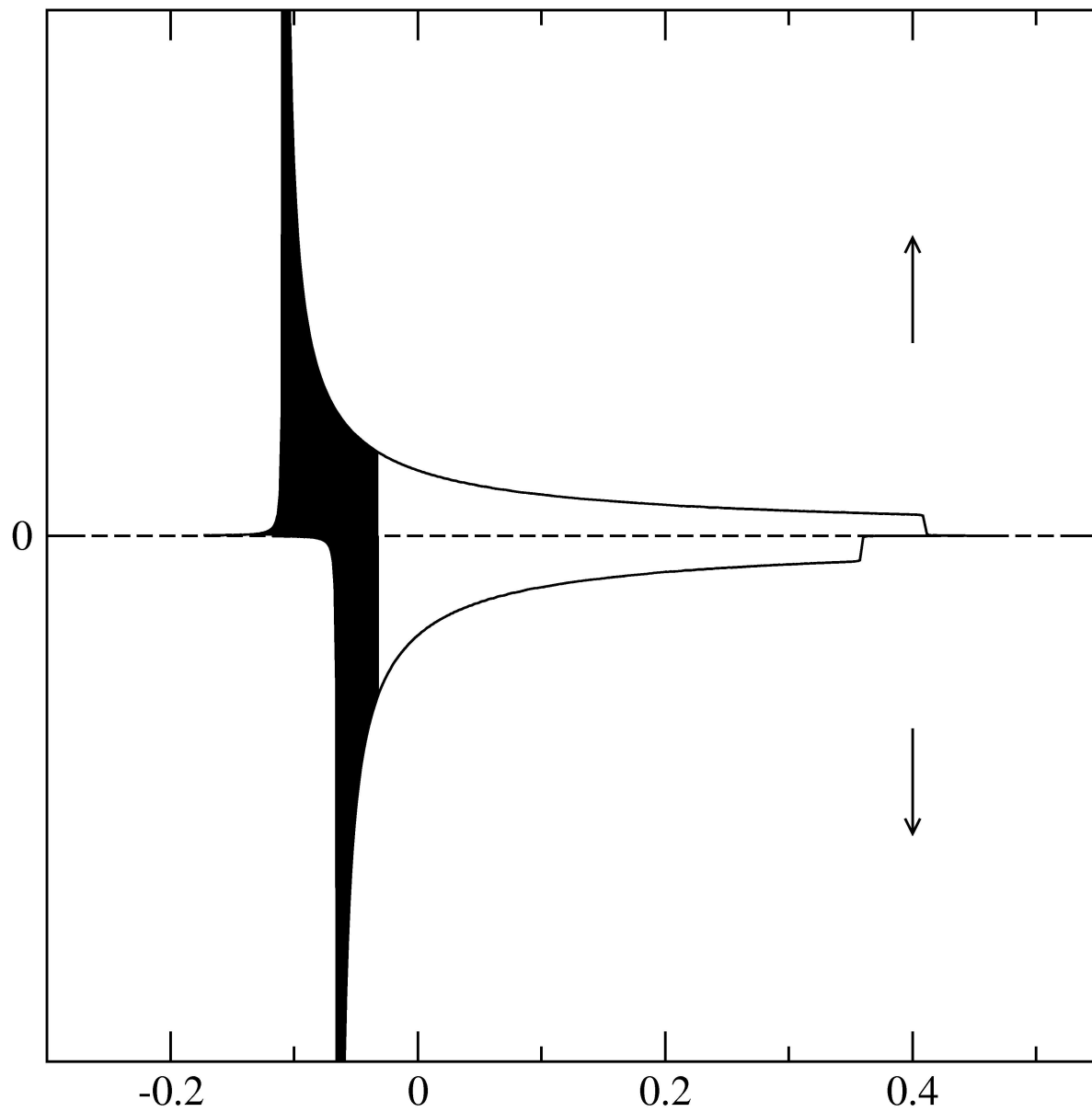
- 1. Transition to distinguishable quasiparticles in Cooper pair in an applied field**
- 2. FFLO phase is robust when the masses are spin-dependent**
- 3. Effective field driven by correlations compensates partially the applied field**
- 4. Generalization of the concept of the Landau quasiparticle**

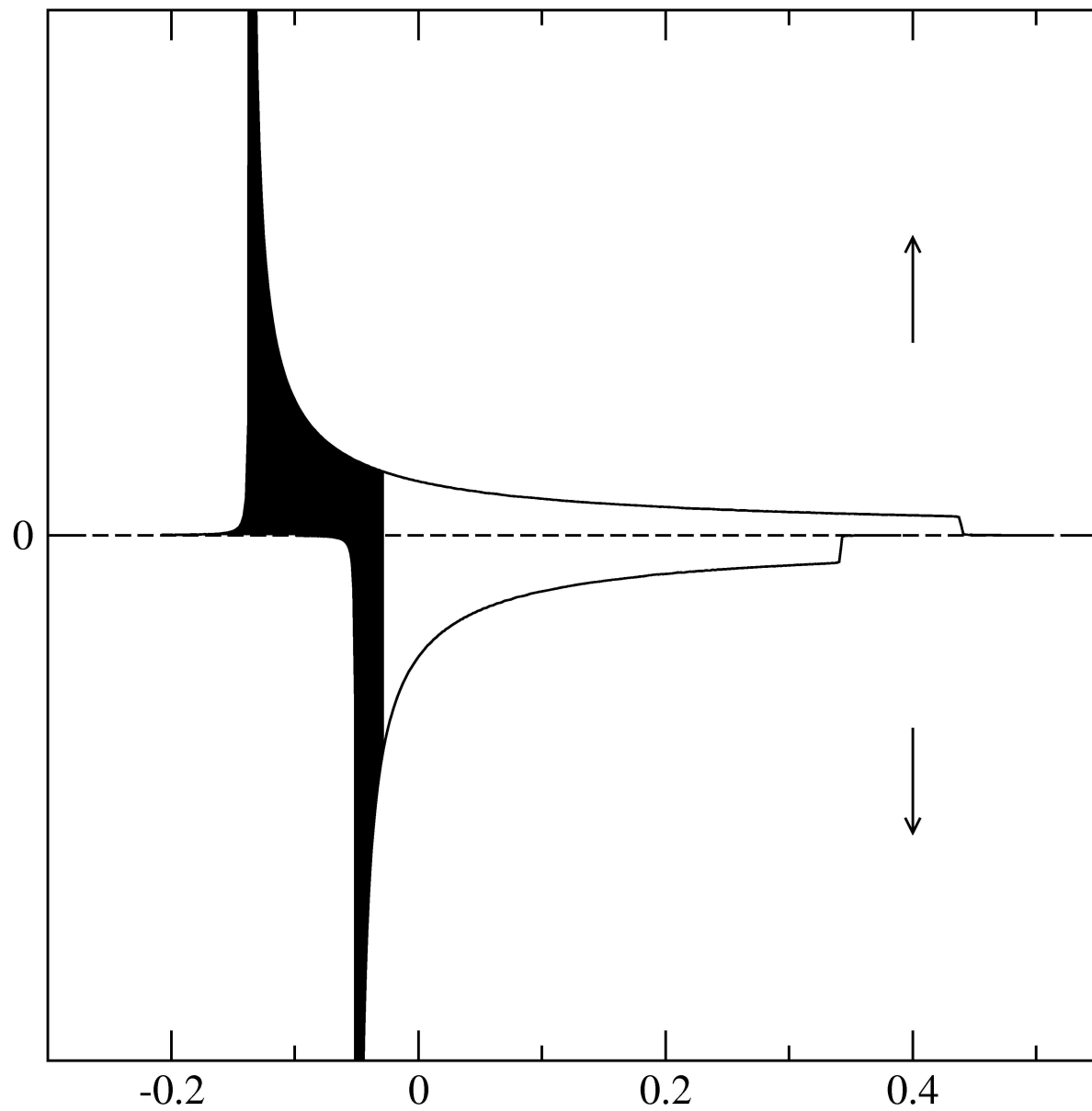


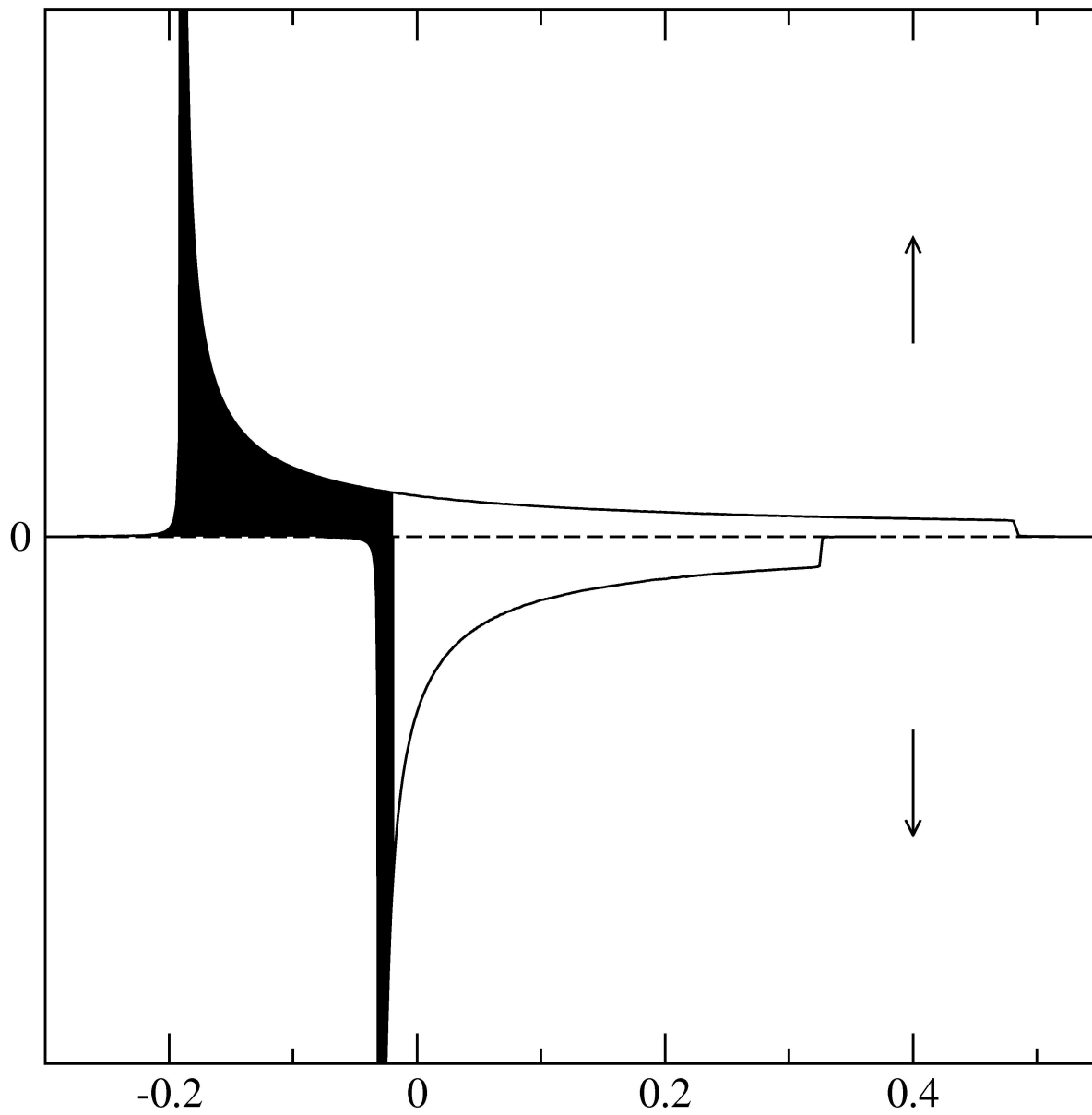


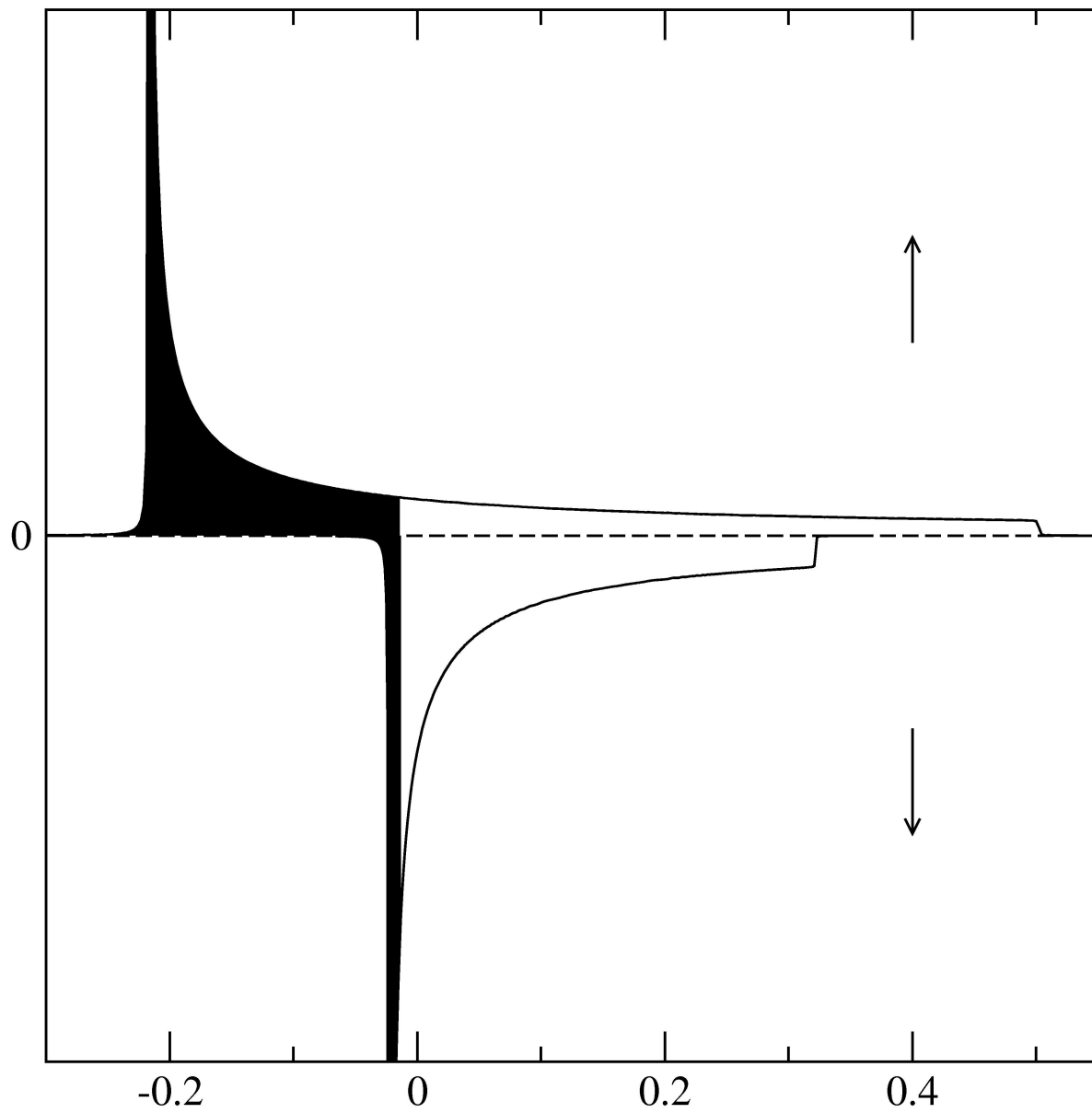


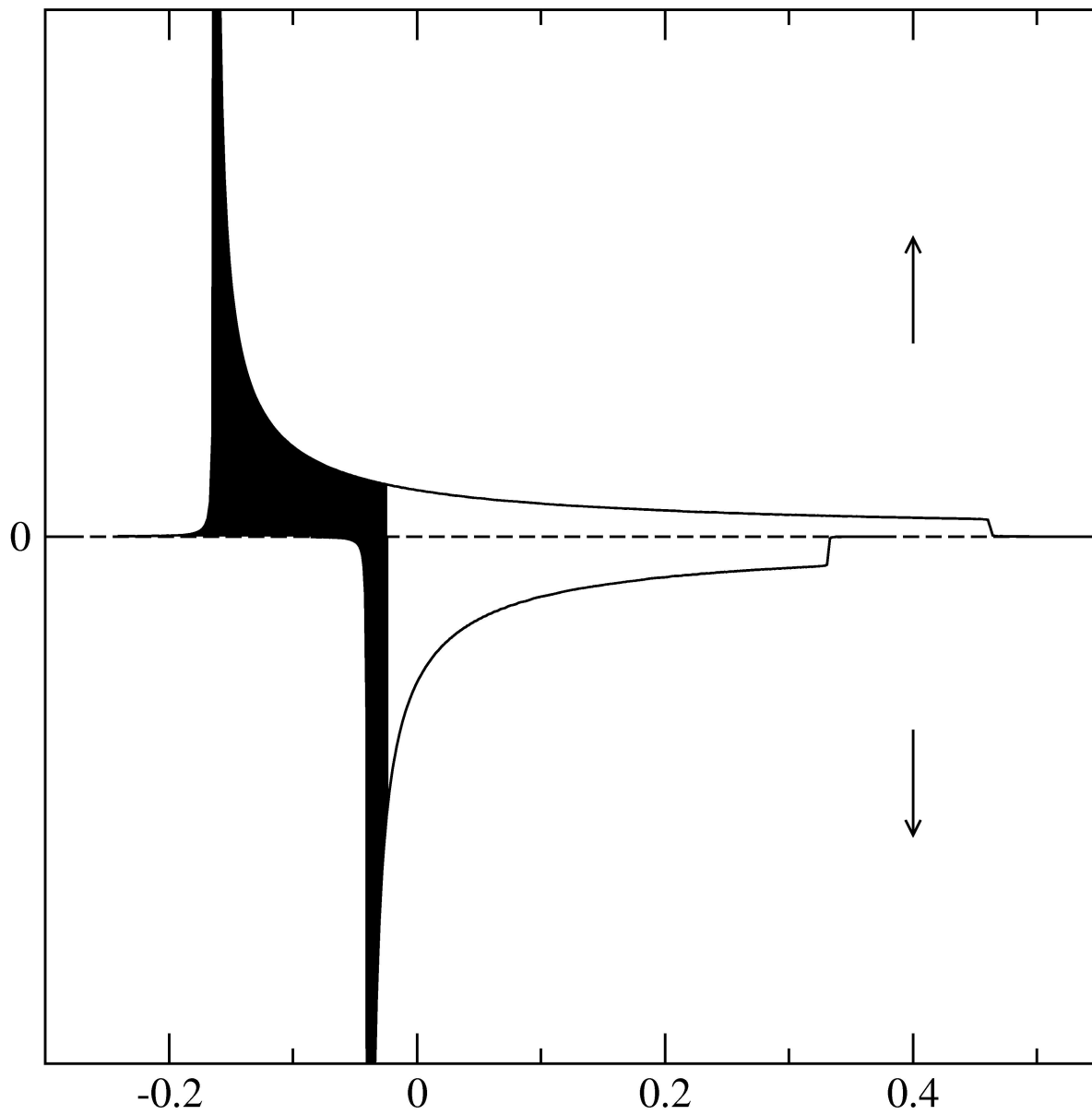












3. Struktura elektronowa? **LDA + U, LDA + DMFT, Gutzwiller LDA**

Gutzwiller + wave function readj.: beyond the parametrized models

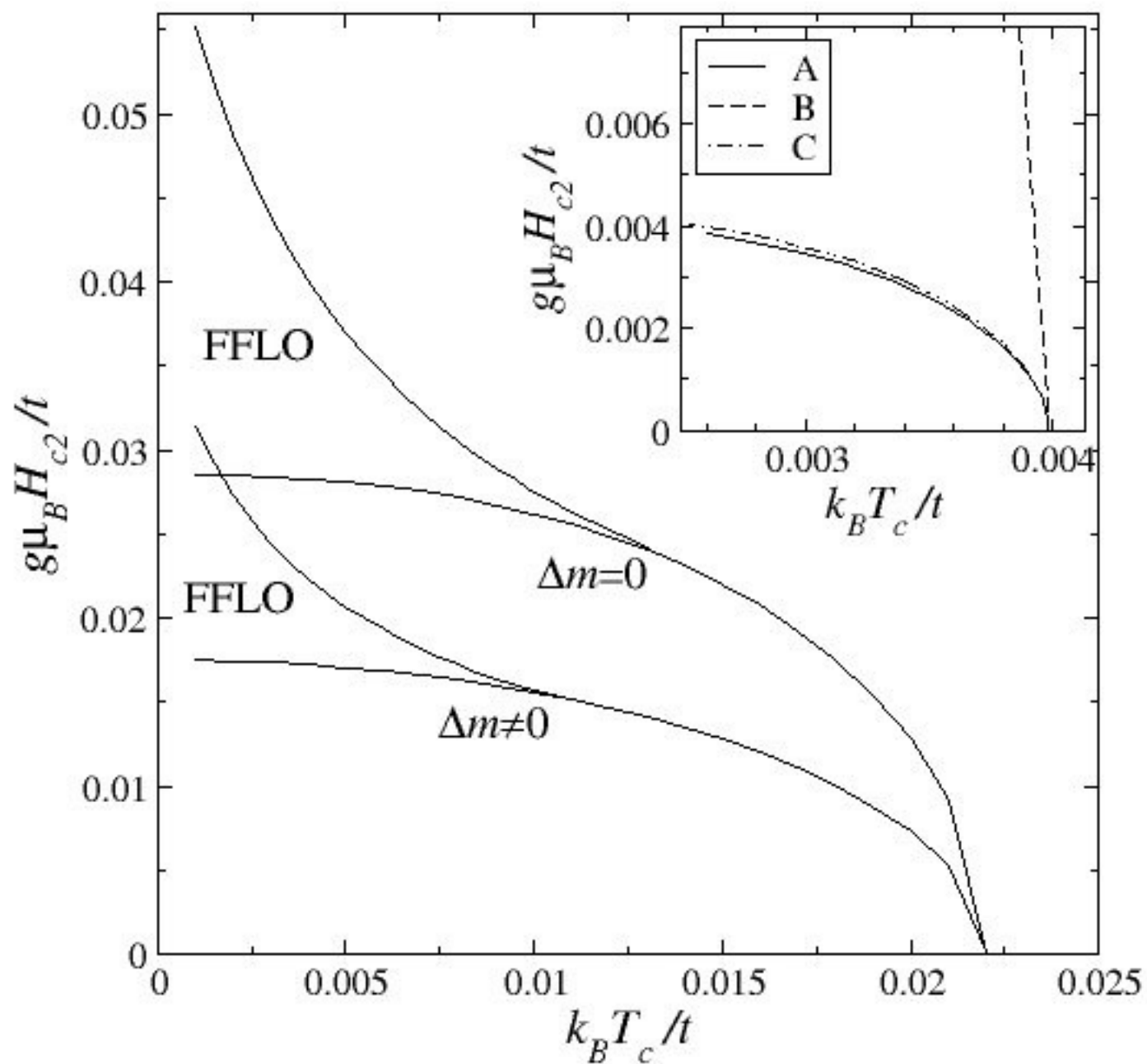
4. Efektywna temperatura Kondo: **temperatura koherencji**

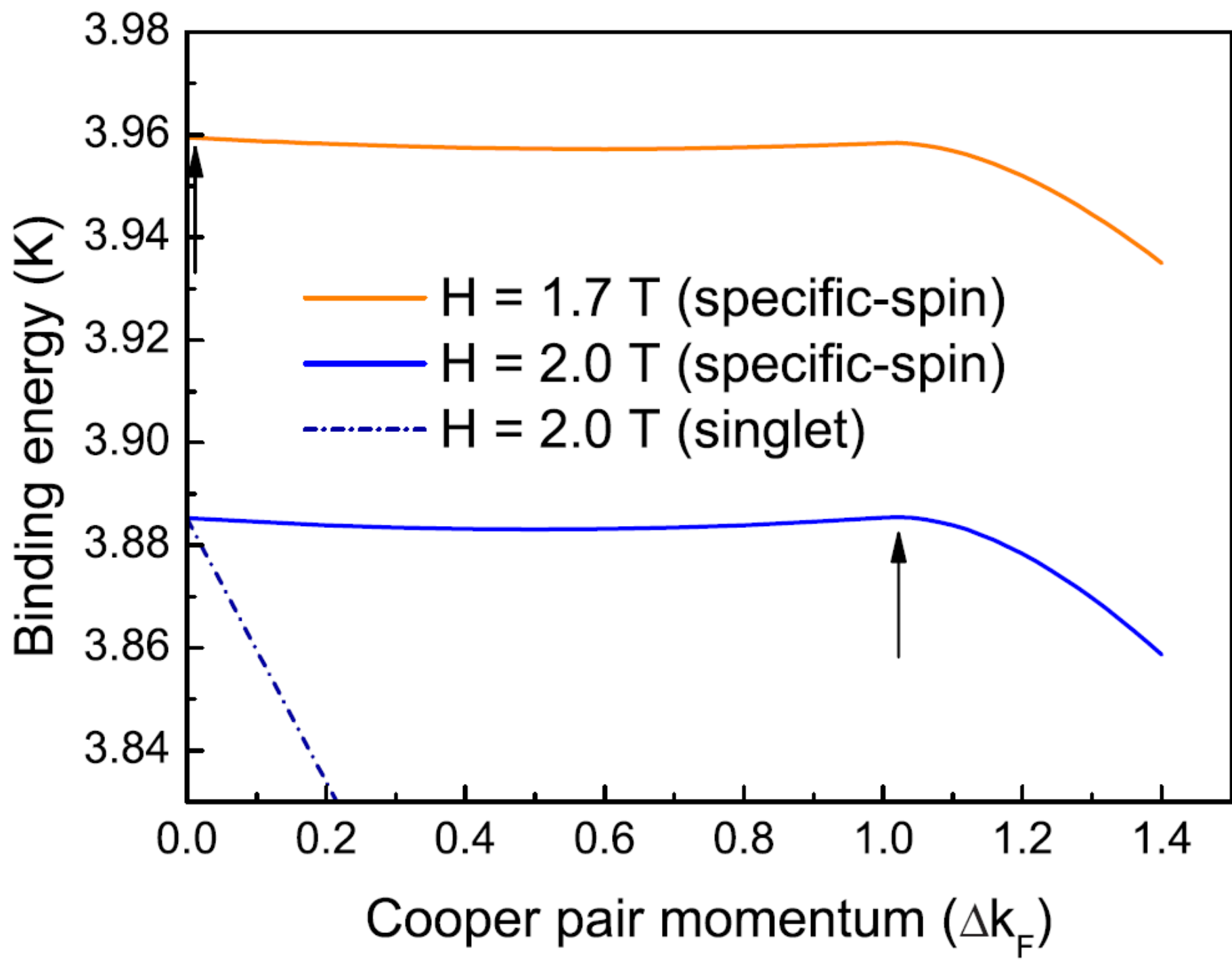
Lokalizacja zaindukowana ruchem termicznym dla rosnącej temperatury, korelacje typu Kondo (lub nie) dla $T \rightarrow 0$

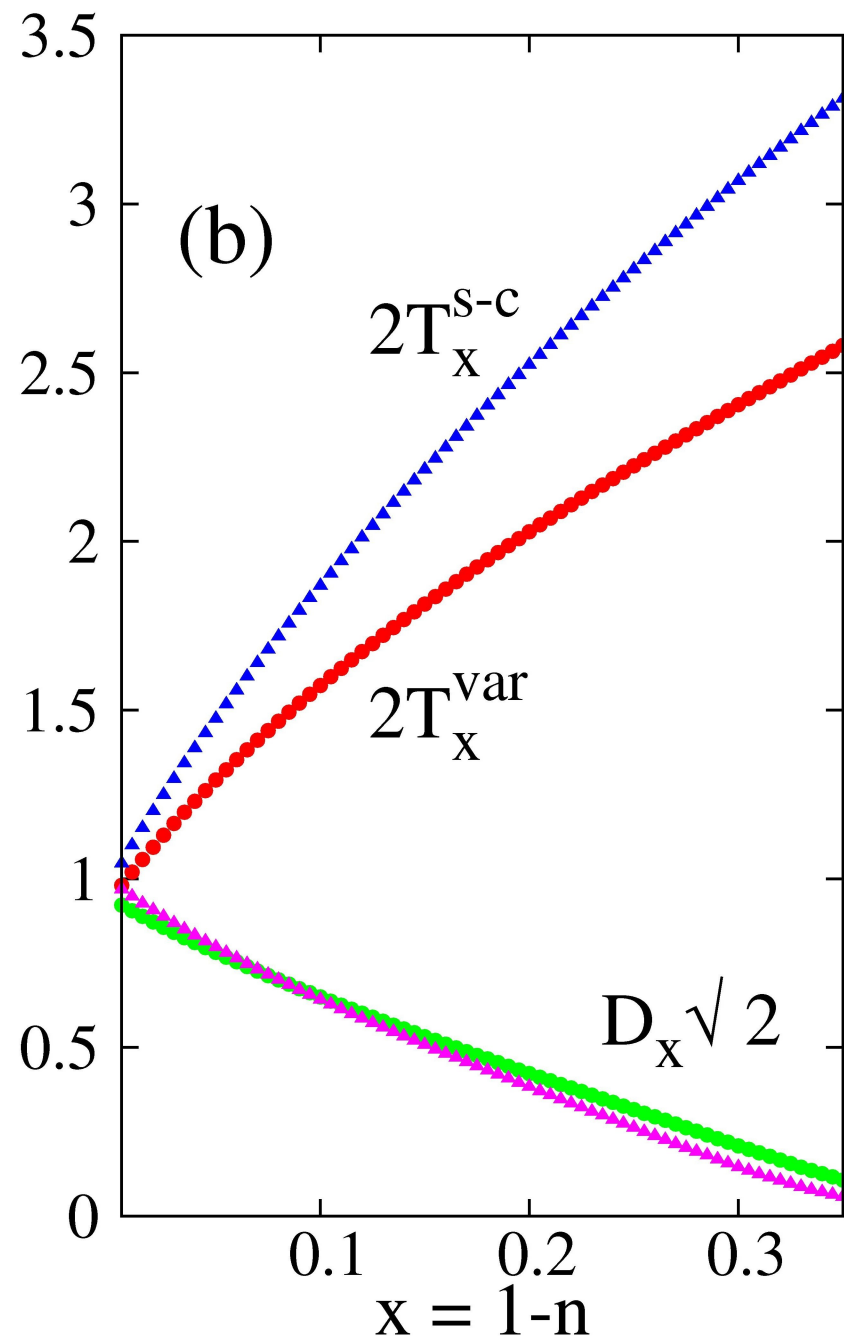
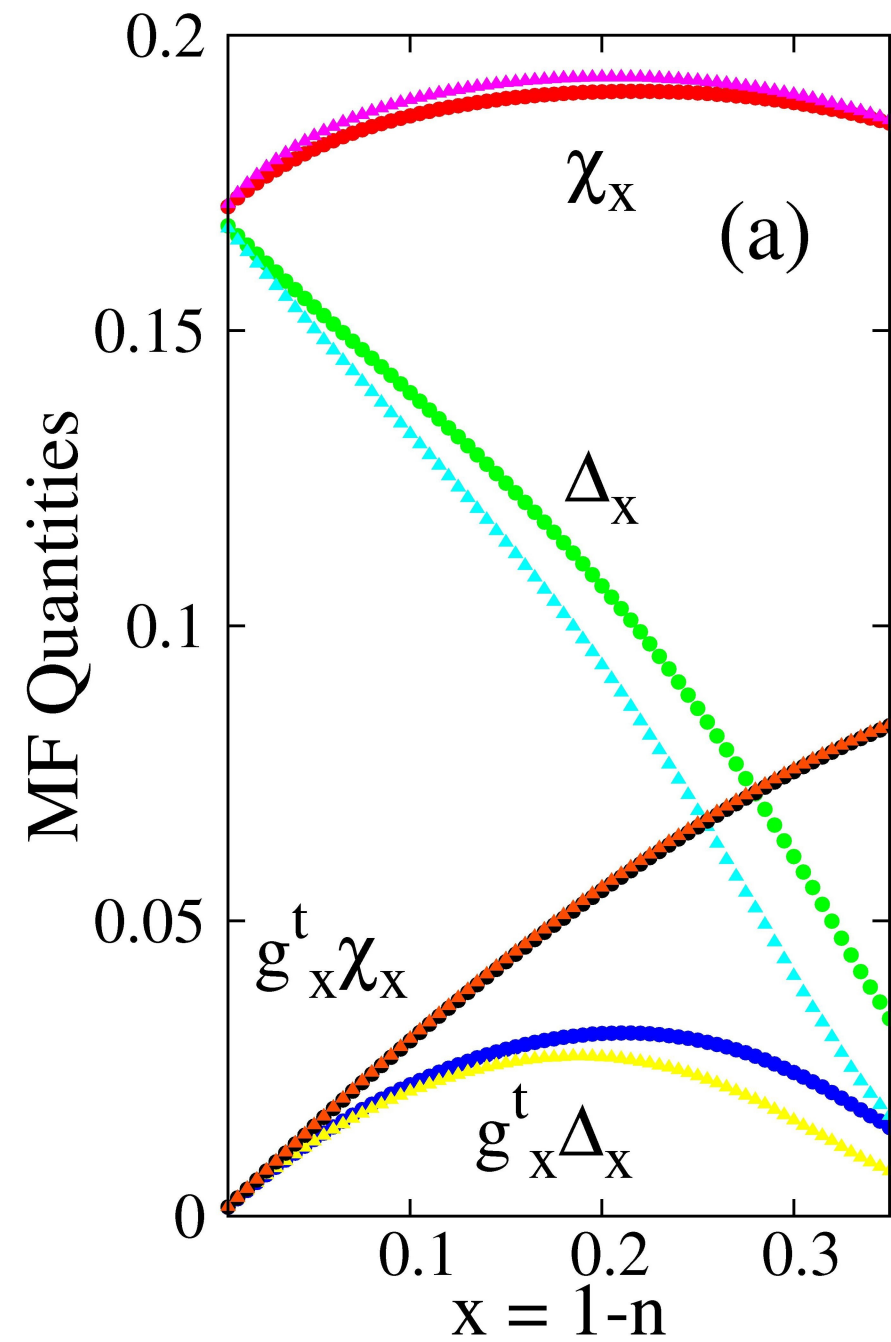
5. **Kwantowe zjawiska krytyczne** czyli wyjście poza paradygmat prawie zlokalizowanej cieczy Fermiego;

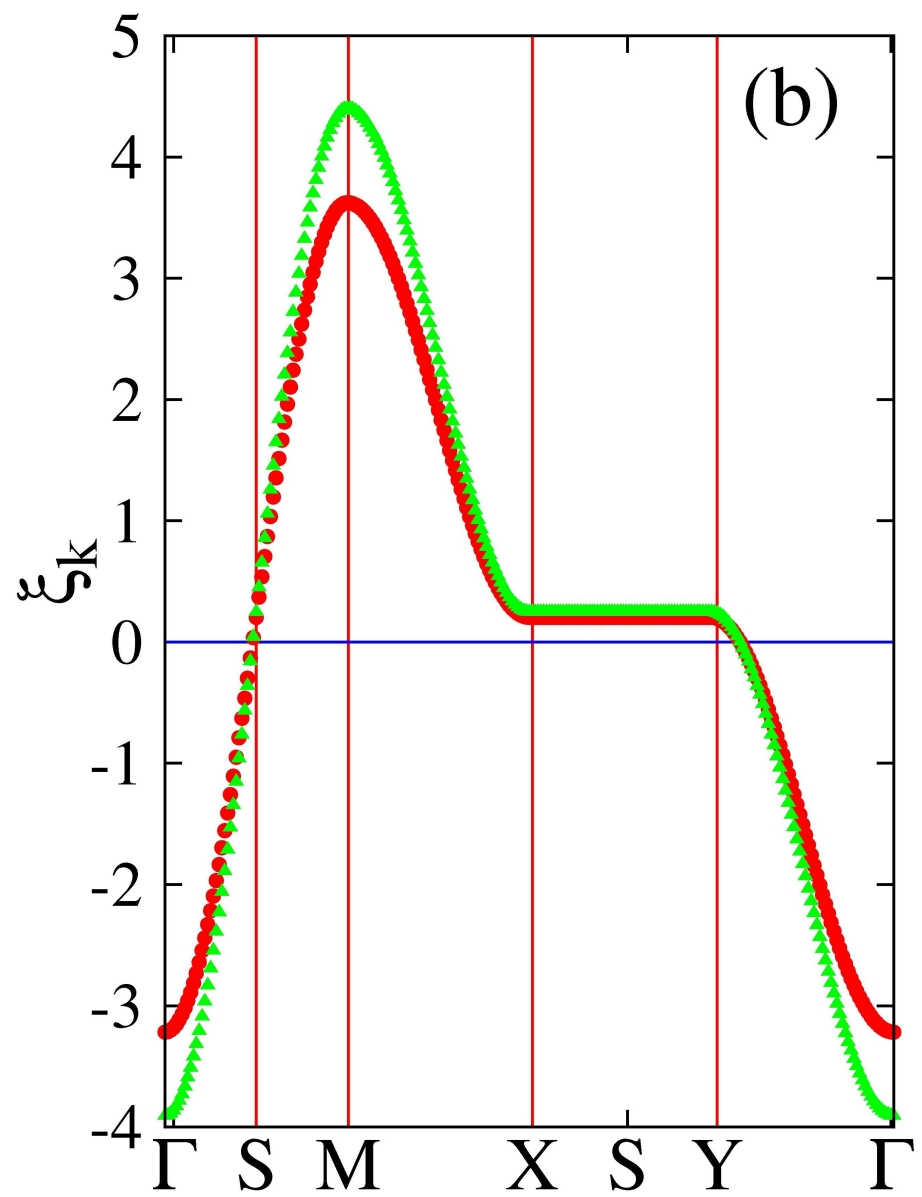
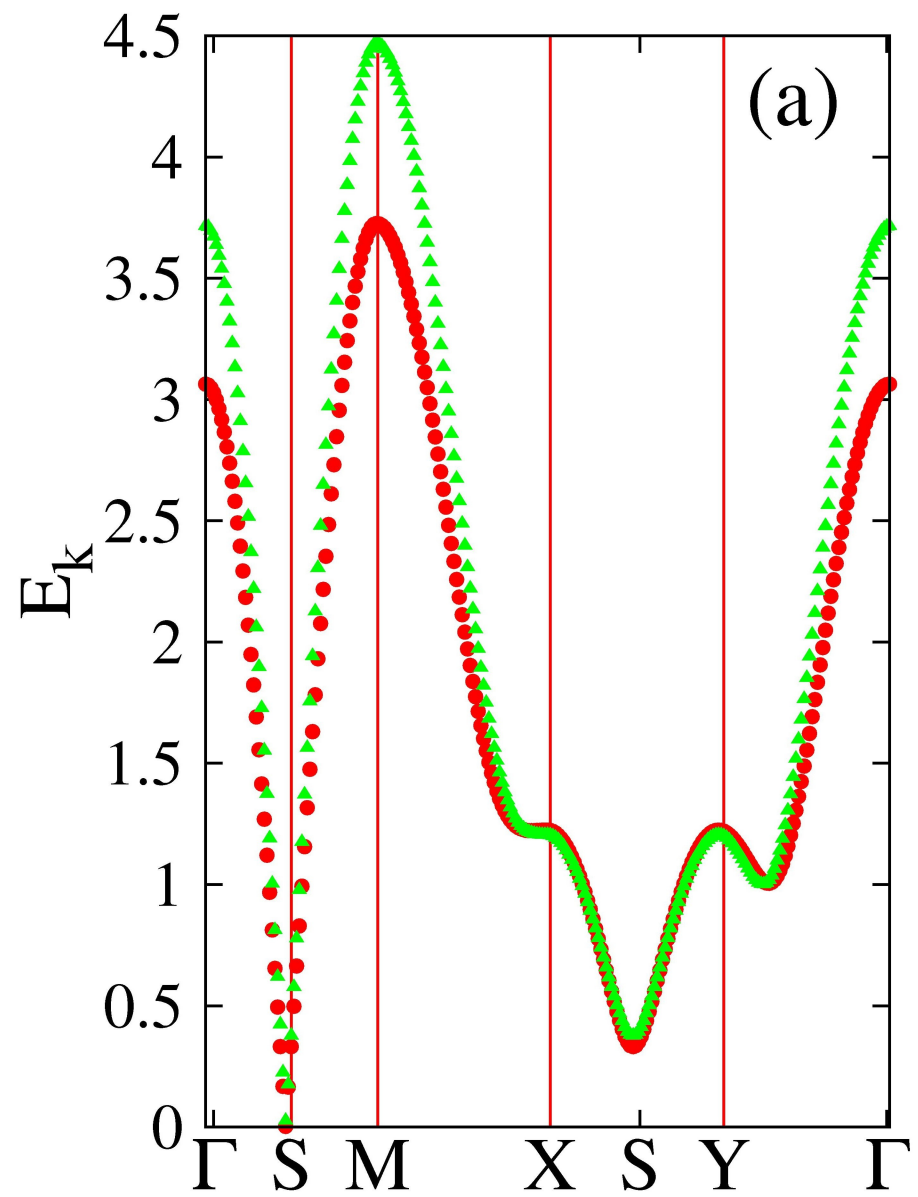
Kwantowe punkty krytyczne na progu lokalizacji Motta, metalizacja magnetytu (A. Kozłowski, jutro)

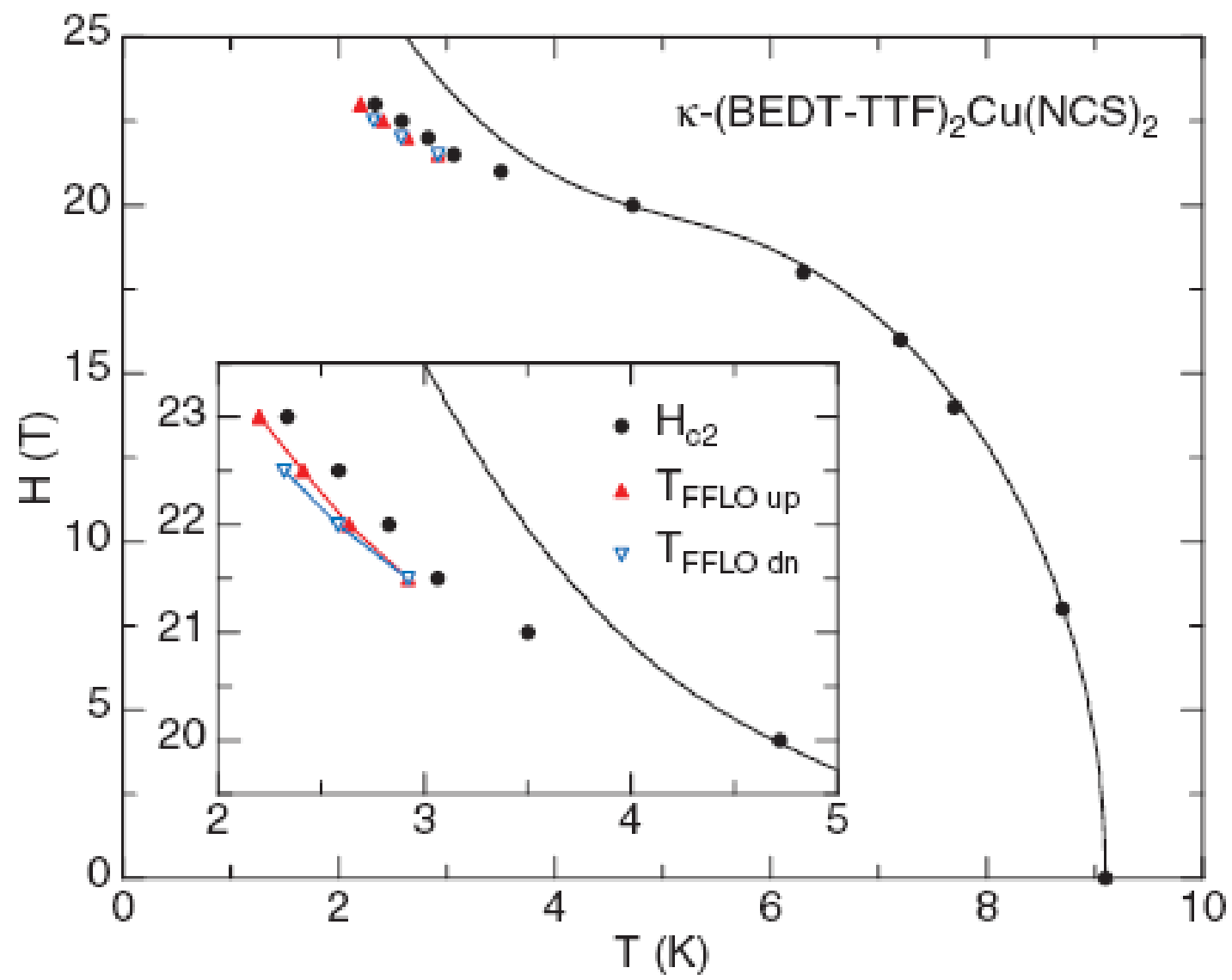
6. Uwaga metodologiczna: **wzajemne cytowanie**

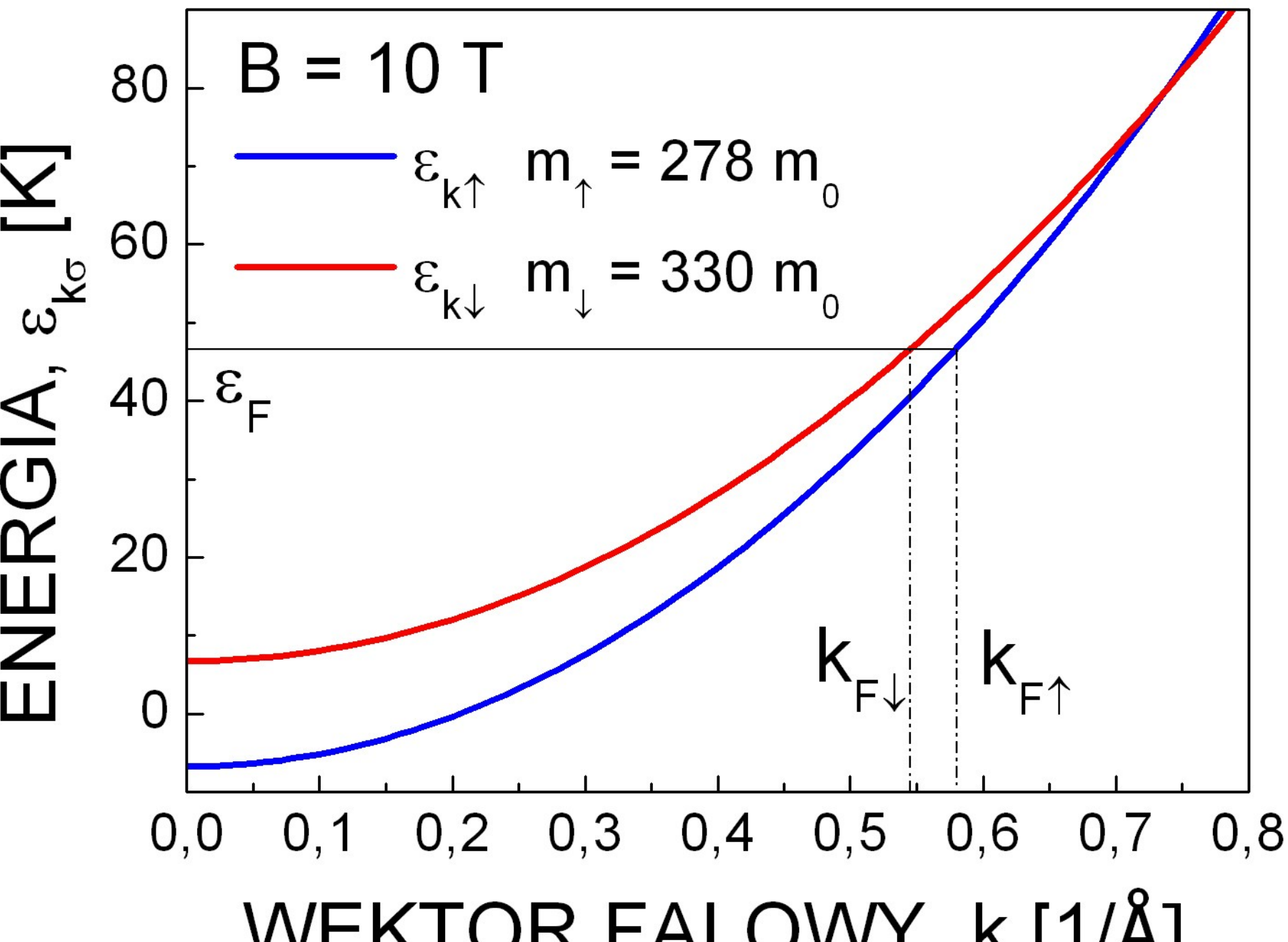






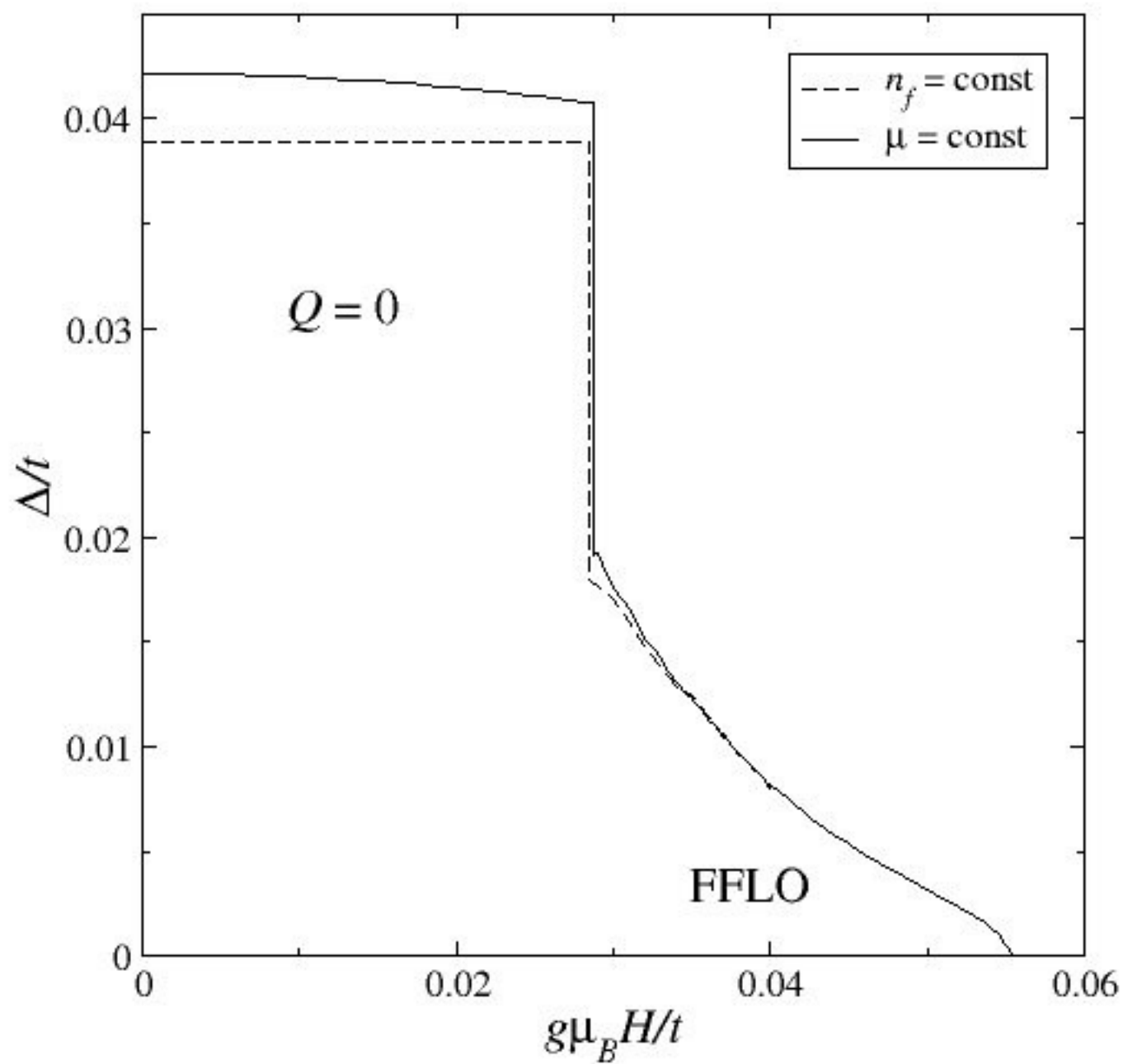


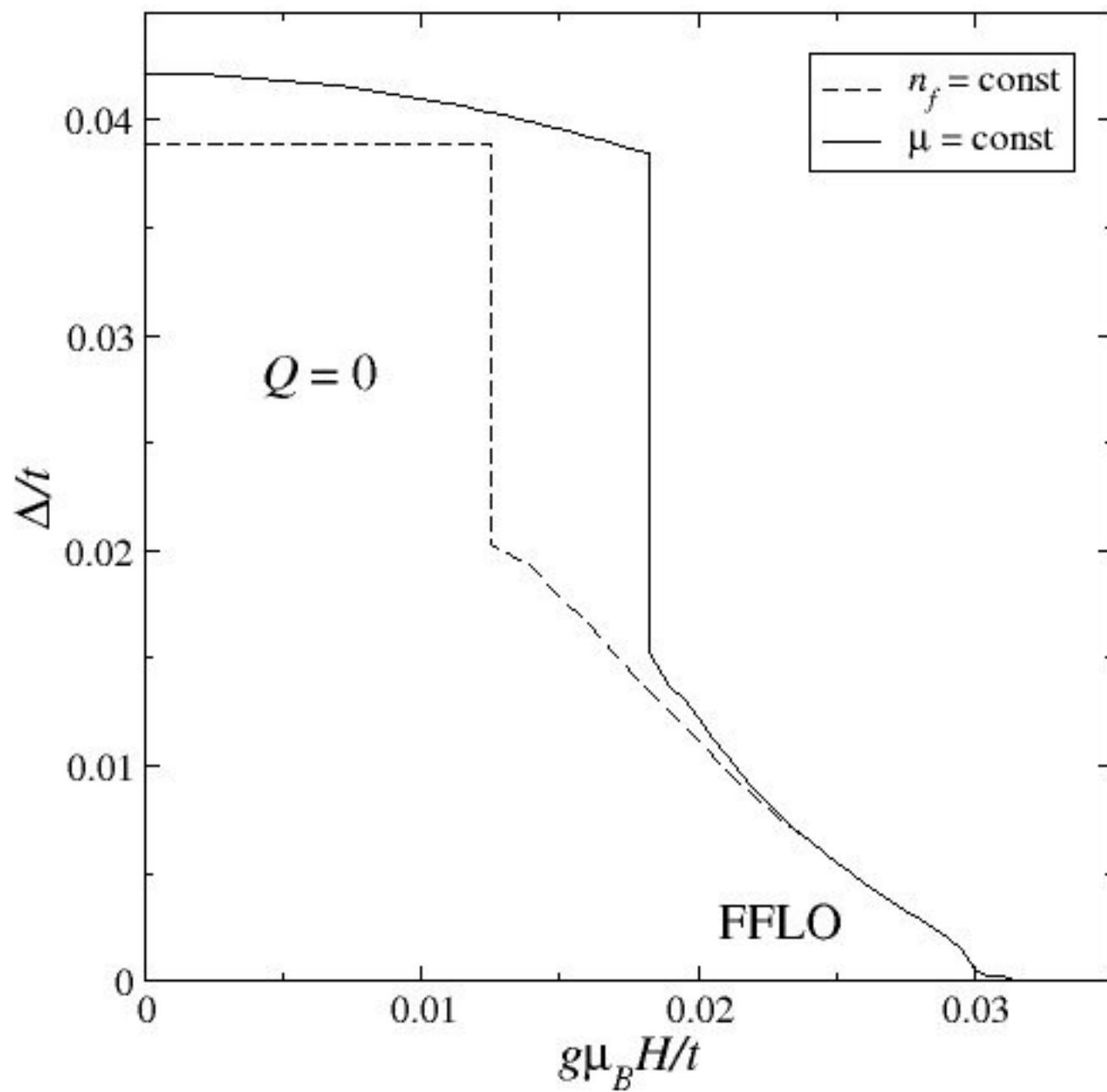


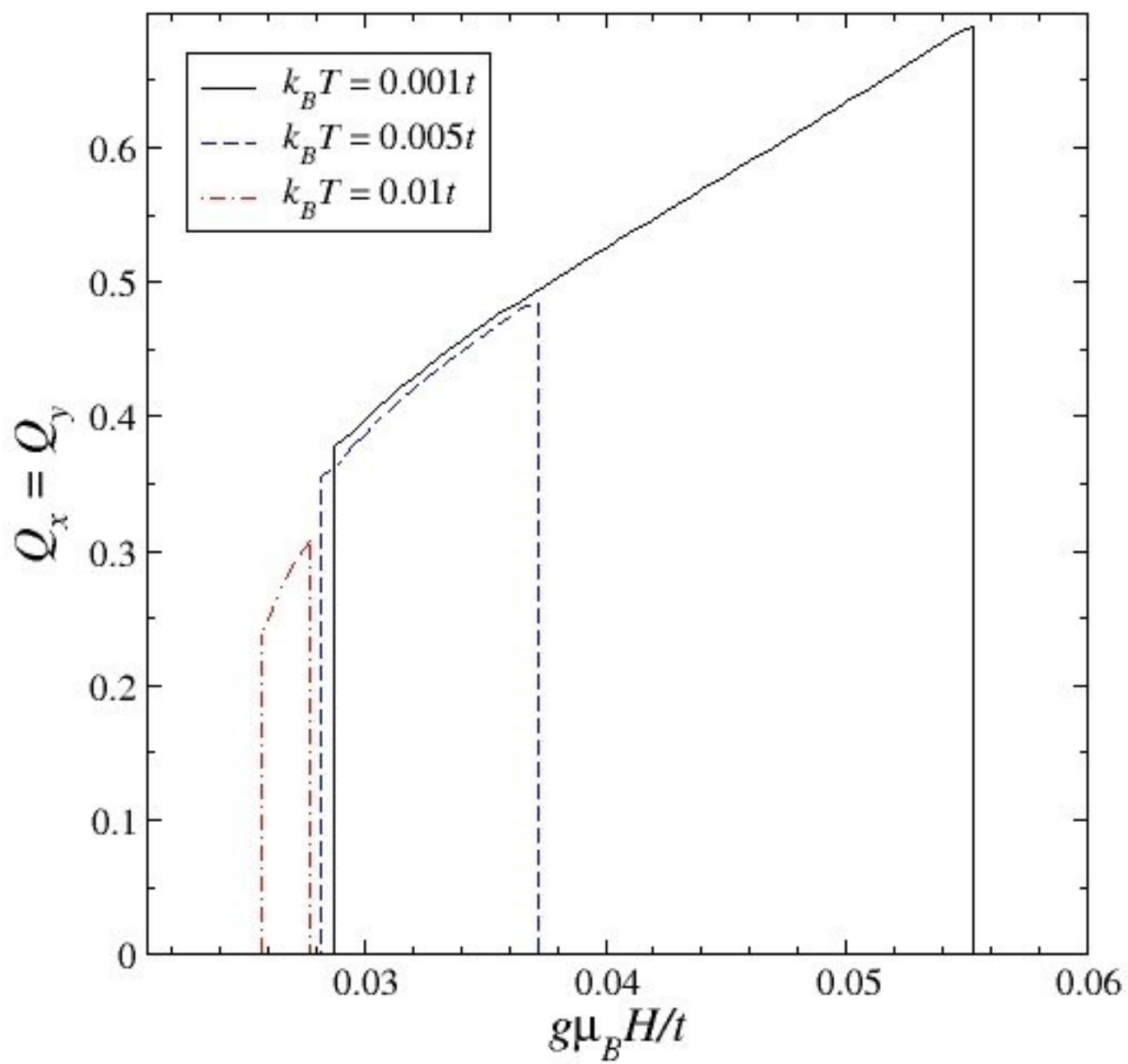


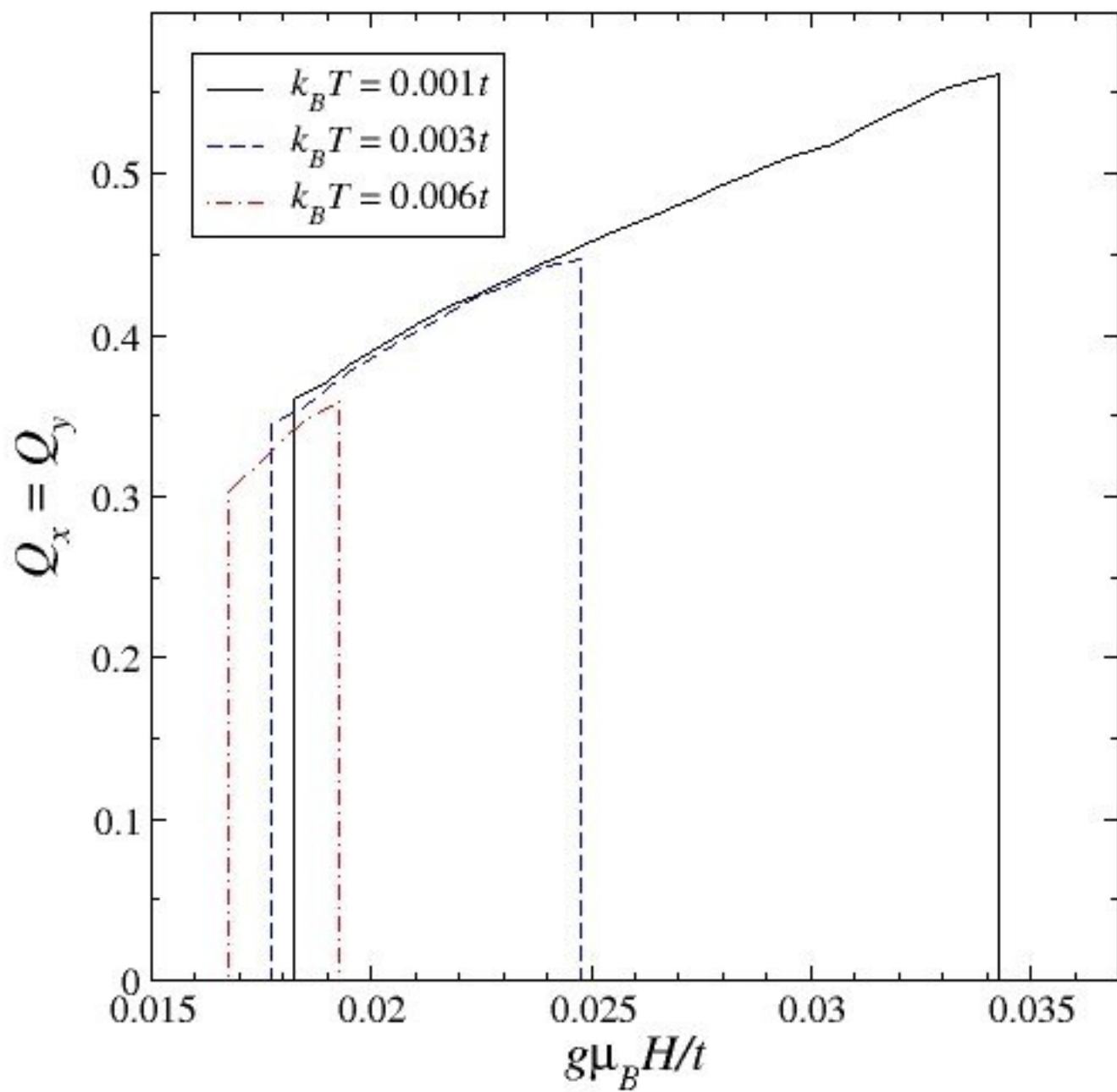
Fulde-Ferrell-Larkin-Ovchinnikov superconducting phase for paired quasiparticles with spin-dependent masses and their distinguishability











Unconventional superconductivity of quasiparticles with spin-dependent masses

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