Analytical micromagnetics

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[15 septembre 2009]

The purpose of the tutorial is to get familiar with some basic knowledge and manipulation of micromagnetism. We will work around a Bloch domain wall, and a simple model of defect to explain pinning-related coercivity.

Preamble. We will use the following shortcuts : $\partial_x\theta$ for $\partial\theta/\partial x$ and $\partial_x^n\theta$ for $\partial^n\theta/\partial x^n$.

1 Framework of micromagnetism

Most of micromagnetic modeling relies on two hypotheses :

- The spatial variation of any physical quantity (magnetic moments etc.) is slow at the length scale of inter-atomic distances. This allows one to describe physical systems in a continuous medium approach and make use of the power of integral theory and differential equations.
- The resulting magnetization vector field (*i.e.* the density of magnetic moments per unit volume) has a uniform and constant magnitude : $|\mathbf{M}(\mathbf{r})| \equiv M_\mathrm{s}$, the spontaneous magnetization.

A major purpose of micromagnetism is to exhibit stable (or metastable) magnetization arrangements under static conditions. These minimize globally (resp. locally) the total energy of the system.

In most situations the density of energy comprises at most four terms : magnetic anisotropy $E_\mathrm{a} = Kf_\mathrm{a}(\theta,\varphi)$, Zeeman energy $E_\mathrm{Z} = -\mu_0\mathbf{M}_\mathrm{s}\mathbf{H}$, self-dipolar energy $E_\mathrm{d} = -(1/2)\mu_0\mathbf{M}_\mathrm{s}\mathbf{H}_\mathrm{d}$ and exchange energy, which continuous form we propose to link with microscopic quantities in this paragraph.

Let us consider exchange energy in a Heisenberg model : $\mathcal{E} = -\sum_{i>j} J\mathbf{S}_i.\mathbf{S}_j$, where the summation concerns near(est) neighbors. $J > 0$ for ferromagnets.

In the simple framework of a one-dimensional crystal with atomic spacing a, show that the density of exchange energy can be expressed as $E_\mathrm{ex} = A(\mathrm{d}_x\theta)^2$, assuming that the angle θ of the magnetization vector has a slow variation between neighboring atomic sites. A is called the exchange constant, which you will exhibit in terms of J and a.

In a three-dimensional body this energy is generalized to the expression :

$$E_\mathrm{ex} = A\left(\boldsymbol{\nabla}\mathbf{m}\right)^2. \tag{1}$$

$(\boldsymbol{\nabla}\mathbf{m})^2$ is a shortcut for $\sum_i\sum_j(\partial_{x_j}m_i)^2$ where m_i are the components of the reduced magnetization $\mathbf{m} = \mathbf{M}_\mathrm{s}/M_\mathrm{s}$.

2 Euler-Lagrange equation

We will seek to exhibit a magnetization configuration that minimizes the energy density integrated over the entire system : $\mathcal{E} = \int E(\mathbf{r})\mathrm{d}\mathbf{r}$. The problem of finding the minimum of a continuous quantity integrated over space is a common problem solved through Euler-Lagrange equation, which we will deal with in a textbook one-dimensional framework here.

Let us consider a microscopic quantity defined as $F(\theta,\mathrm{d}_x\theta)$, where x is the spatial coordinate and θ a quantity defined at each point. In the case of micromagnetism we will have :

$$F\left(\theta,\mathrm{d}_x\theta\right)=A\left(\mathrm{d}_x\theta\right)^2+E(\theta) \tag{2}$$

$E(\theta)$ may contain anisotropy, dipolar and Zeeman terms. We define the integrated quantity :

$$\mathcal{F}=\int_A^B F\left(\theta,\mathrm{d}_x\theta\right)\mathrm{d}x+E_A(\theta)+E_B(\theta). \tag{3}$$

A and B are the boundaries of the system, while $E_A(\theta)$ and $E_B(\theta)$ are surface energy terms.

Let us consider an infinitesimal function variation $\delta\theta(x)$ of θ. Show that extrema of $\mathcal{F}$ are determined by the following relationships :

$$\partial_\theta F-\mathrm{d}_x\left(\partial_{\mathrm{d}_x\theta}F\right) = 0 \tag{4}$$

$$\mathrm{d}_\theta E_A-\left.\partial_{\mathrm{d}_x\theta}F\right|_A = 0 \tag{5}$$

$$\mathrm{d}_\theta E_B+\left.\partial_{\mathrm{d}_x\theta}F\right|_B = 0 \tag{6}$$

Notice that equations Eq. (5) and Eq. (6) differ in sign because a surface quantity should be defined with respect to the unit vector normal to the surface, with a unique convention for the sense, such as the outwards normal. Here the abscissa x is outwards for point$B$ however inwards at A. An alternative microscopic explanation would be that for a given sign of $\mathrm{d}_x\theta$ the exchange torque exerted on a moment to the right (at point B) is opposite to that exerted to the left (at point A), whereas the torque exerted by a surface anisotropy energy solely depends on θ.

3 Micromagnetic Euler equation

Apply the above equations to the case of micromagnetism [Eq. (2)]. Starting from Eq. (4) exhibit a differential equation linking $E[\theta(x)]$ with $\mathrm{d}_x\theta$. Equations 5-6 are called Brown equations. $E_A(\theta)$ and $E_B(\theta)$ may be surface magnetic anisotropy, for instance. Discuss the microscopic meaning of these equations.

Comment the special case of free boundary conditions (all bulk and surface energy terms vanish at A and B), in terms of energy partition. Now on we switch back to the physics notation $\mathcal{E}$ for the total energy, instead of $\mathcal{F}$. Show that it can be expressed as :

$$\mathcal{E}=2\int_{\theta(A)}^{\theta(B)}\sqrt{AE(\theta)}\,\mathrm{d}\theta \tag{7}$$

4 The Bloch domain wall

Let us assume the following free boundary conditions, mimicking two extended domains with opposite magnetization separated by a domain wall whose profile we propose to derive here : $\theta(-\infty)=0$ and $\theta(+\infty)=\pi$. We will assume the simplest form of magnetic anisotropy, uniaxial of second order : $E(\theta)=K\sin^2\theta$.

Based on a dimensional analysis give approximate expressions for both the domain wall width δ and the domain wall energy $\mathcal{E}$. What are the SI units for $\mathcal{E}$? Discuss the form of these quantities in relation with the meaning and effects of exchange and anisotropy.

By integrating the equations exhibited in the previous section, derive now the exact profile of the domain wall

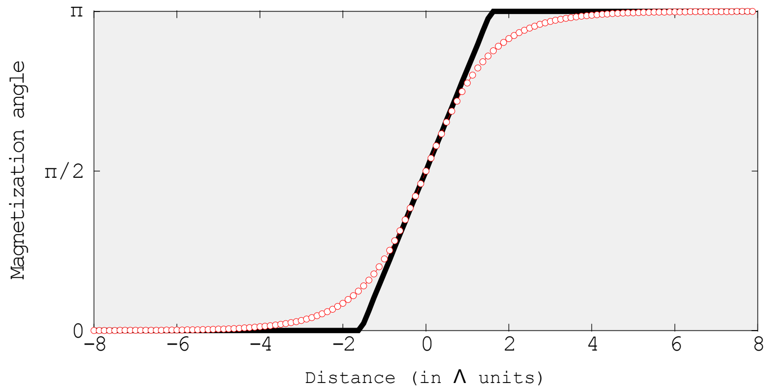

FIGURE 1 – Bloch domain wall profile : the exact solution (red dots) versus the asymptotic profile (line).

$$\theta(x) = 2\arctan[\exp(x/\Delta)] \tag{8}$$

and its energy $\mathcal{E}$.

The most common way to define the Bloch domain wall width δ_{Bl} is by replacing the exact $\theta(x)$ by its linear asymptotes (FIGURE 1). To shorten the expressions we often use the notation $\Delta = \sqrt{A/K}$, called the *Bloch parameter*. Derive δ_{Bl} as a function of Δ.

Let us stress two issues :
- As often in physics we have seen in this simple example that a dimensional analysis yields a good insight into a micromagnetic situation. It is always worthwhile starting with such an analysis before undertaking complex analytical or numerical approaches, which especially for the latter may hide the physics at play.
- We have exhibited here a characteristic length scale in magnetism. Other length scales may occur, depending on the energy terms in balance. The physics at play will often depend on the dimensions of your system with respect to the length scales relevant in your case. Starting with such an analysis is also wise.

5 An example of pinning

We remain in a one-dimensional framework. Starting from a homogeneous material let us model a local defect in the form of a magnetically-soft (*i.e.* zero anisotropy) insertion of width $\delta\ell$, located at position x. In the case where $\delta\ell \ll \Delta$ discuss what modeling of the domain wall is reasonable to make. Discuss the boundary conditions at the defect edges.

Show that the energy of the domain wall with the defect at location x reads :

$$E(x) = 4\sqrt{AK}\left[1 - \frac{1}{4}\frac{\delta\ell}{\Delta}\frac{1}{\cosh^2(x/\Delta)}\right] \tag{9}$$

Draw a schematic graph of $E(x)$ and display the characteristic length or energy scales. An external field is then applied at an angle $\cos\theta_{\mathrm{H}}$ with the easy axis direction in the domains. Assuming that the profile of the domain wall (Eq. (8)) remains unaffected by the applied field (regime of weak pinning), show that the propagation field of the domain wall over the defect reads :

$$H_{\mathrm{p}} = \frac{H_{\mathrm{a}}}{\cos\theta_{\mathrm{H}}} \frac{\Delta K}{K} \frac{\delta\ell}{\Delta} \frac{1}{3\sqrt{3}}. \tag{10}$$

Notice :
- The model of the Bloch wall was named after D. Bloch who published this model in 1932[1].
- The $1/\cos\theta_{\mathrm{H}}$ dependence of coercivity is often considered as a signature a weak-pinning mechanism, a law known as the Kondorski model[4].
- This model had been initially published in 1939 by Becker and Döring[2], and is summarized I in the nice book of Skomsky *Simple models of Magnetism*[3].
- While coercivity requires a high anisotropy, the latter is not a sufficient condition to have a high coercivity. To achieve this one must prevent magnetization reversal that can be initiated on defects (structural or geometric) and switch the entire magnetization by propagation of a domain wall. In a short-hand classification one distinguishes coercivity made possible by hindering nucleation, or hindering the propagation of domain walls. In reality both phenomena are often intermixed. Here we modeled an example of pinning.
- Simple micromagnetic models of nucleation on defects[5] were the first to be exhibited to tentatively explain the so-called *Brown paradox*, *i.e.* the fact that values of experimental values of coercivity in most samples are smaller or much smaller than the values predicted by the ideal model of coherent rotation[6].

Good references for micromagnetism are Hubert and Schäfer's book[7] (very large scope, many references), Skomski's 2003 review [3] and later book[8] and Aharoni's book[9] (a bit more mathematical and centered on the author's own contributions). For a super-light introduction to nanomagnetism you may have a look at short personal reviews[10, 11, 11] or other authors' reviews[12, 13].

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