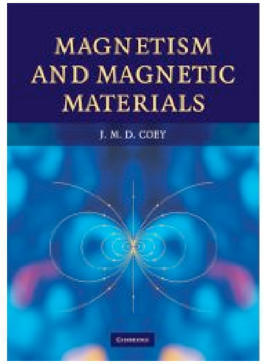
These lectures provide an account of the basic concepts of magneostatics, atomic magnetism and crystal field theory. A short description of the magnetism of the free-electron gas is provided. The special topic of dilute magnetic oxides is treated seperately.



Some useful books:

- J. M. D. Coey; *Magnetism and Magnetic Magnetic Materials*. Cambridge University Press (in press) 600 pp [You can order it from Amazon for £ 38].

- Magnétisme I and II, Tremolet de Lachesserie (editor) Presses Universitaires 2000.

- Theory of Ferromagnetism, A Aharoni, Oxford University Press 1996

- J. Stohr and H.C. Siegmann, *Magnetism*, Springer, Berlin 2006, 620 pp.

- For history, see utls.fr

Basic Concepts in Magnetism

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Ireland.

1. Magnetostatics
2. Magnetism of multi-electron atoms
3. Crystal field
4. Magnetism of the free electron gas
5. Dilute magnetic oxides

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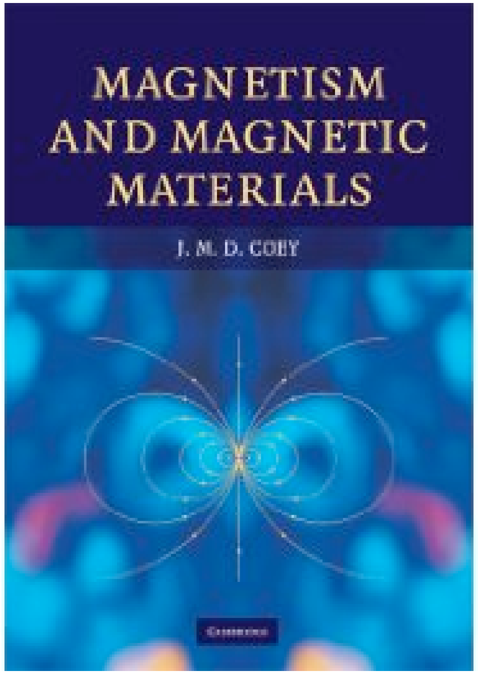
www.tcd.ie/Physics/Magnetism



Available November 2009

1 Introduction

2 Magnetostatics

3 Magnetism of the electron

4 The many-electron atom

5 Ferromagnetism

6 Antiferromagnetism and other magnetic order

7 Micromagnetism

8 Nanoscale magnetism

9 Magnetic resonance

10 Experimental methods

11 Magnetic materials

12 Soft magnets

13 Hard magnets

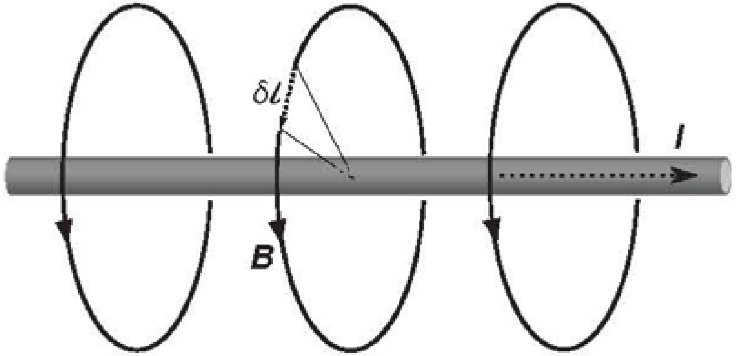
14 Spin electronics and magnetic recording

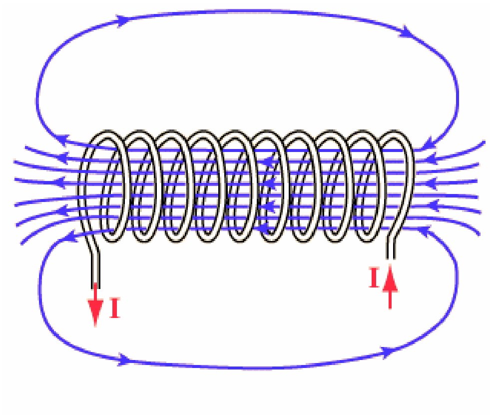
15 Other topics

1. Magnetostatics

1.1 The beginnings

The relation between electric current and magnetic field



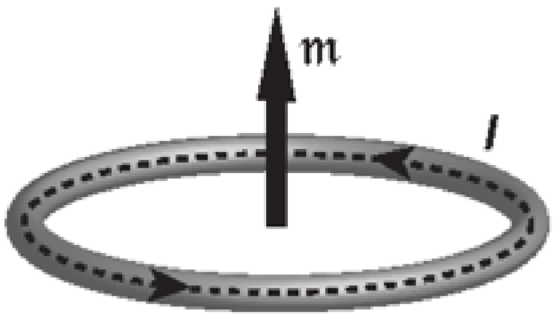


Discovered by Hans-Christian Øersted, 1820.

$$\int B dl = \mu_0 I$$ Ampère’s law

1.2 **The magnetic moment**

Ampère: A magnetic moment *m* is equivalent to a current loop.

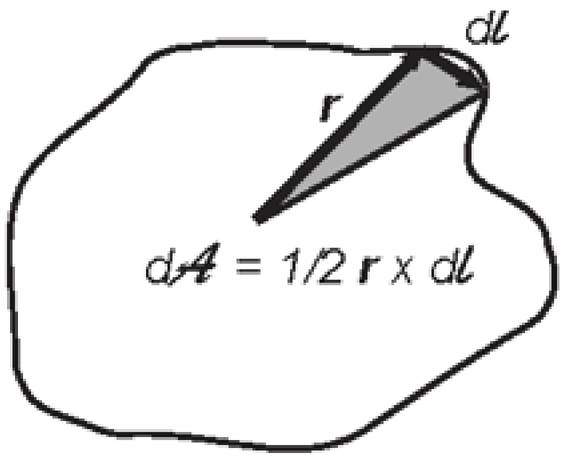


Provided the current flows in a plane

$$\boxed{\boldsymbol{m} = I\mathcal{A} \quad \textit{units } Am^2}$$

In general:

$$\boxed{\boldsymbol{m} = (1/2)\int \boldsymbol{r} \times \boldsymbol{j}(\boldsymbol{r})\mathrm{d}^3 r}$$



where $\boldsymbol{j}$ is the current density; $I = \boldsymbol{j}.\mathcal{A}$

so $\boldsymbol{m} = 1/2\int \boldsymbol{r} \times I\mathrm{d}\boldsymbol{l} = I\int \mathrm{d}\mathcal{A} = \boldsymbol{m}$

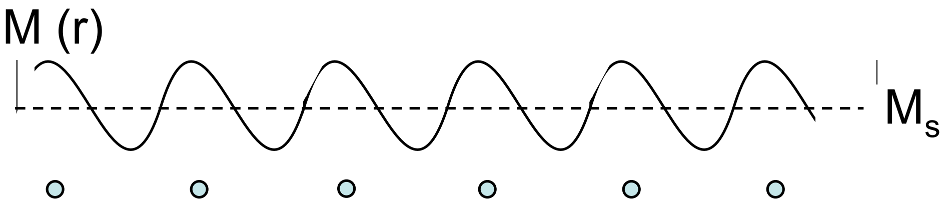
Units: Am^2

1.3 Magnetization

Magnetization $\boldsymbol{M}$ is the local moment density $\boldsymbol{M} = \delta \boldsymbol{m}/\delta V$ - it fluctuates wildly on a sub-nanometer and a sub-nanosecond scale.

Units: A m^{-1} *e.g.* for iron M = 1720 kA m^{-1}

More useful is the *mesoscopic* average, where $\delta V \sim 10$ nm^3



$$\boxed{\delta \boldsymbol{m} = \boldsymbol{M}\delta V}$$

It also fluctuates on a timescale of < 1ns. Take a time average over ~ μs.

e.g. for a fridge magnet (M = 500 kA m^{-1}, V = 2 10^6 m^3, m = 1 A m^{-1}

$\boldsymbol{M}$ can be induced by an applied field <u>*or*</u> it can arise spontaneously within a ferromagnetic domain, $\boldsymbol{M}_s$.

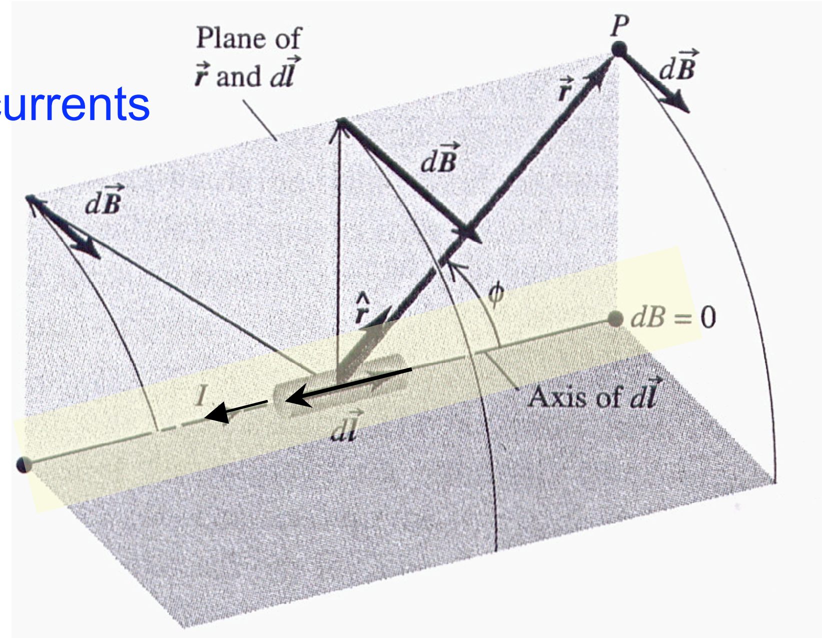
A macroscopic average magnetization is the domain average

$$\boxed{\boldsymbol{M} = \Sigma_i \boldsymbol{M}_i V_i / \Sigma_i V_i}$$

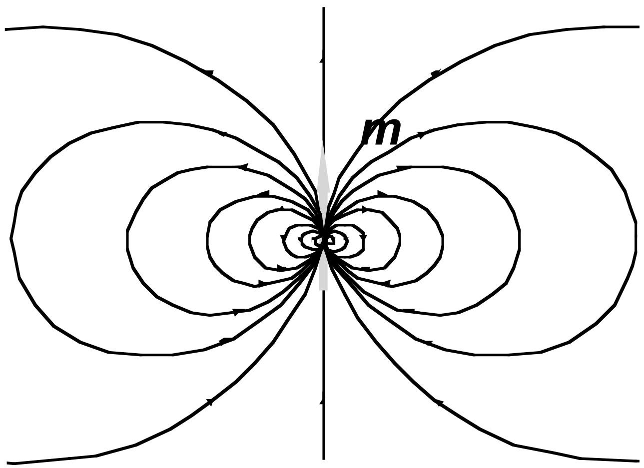
The equivalent Amperian current density is $\boldsymbol{j}_M = \nabla \times \boldsymbol{M}$

1.4 **Magnetic fields**

currents



magnets



Biot Savart law

$$\boxed{d\boldsymbol{B} = -\,\mu_0 \frac{\boldsymbol{r} \times \boldsymbol{j}\, dV}{4\pi r^3}}$$

$$= -\,\mu_0 \frac{\boldsymbol{r} \times \boldsymbol{dl}\, I}{4\pi r^3}$$

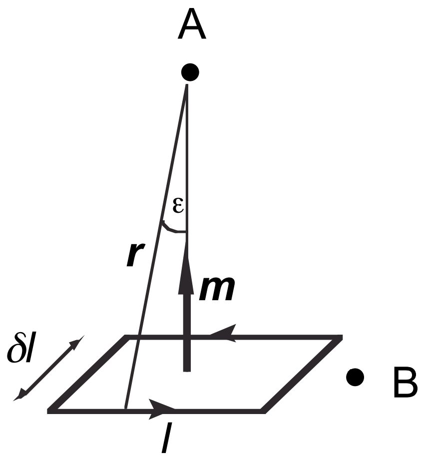
units: Tesla

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{TA^{-1}m}$

Dipole field

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\left[3\frac{(\mathbf{m}.\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\mathbf{m}}{r^3}\right]$$

Calculation of the dipole field



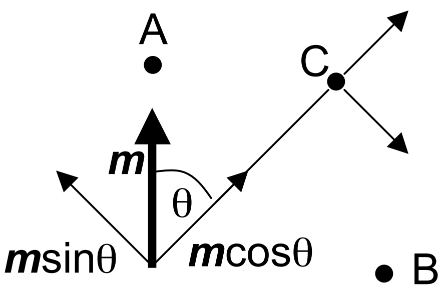
$B_A = 4\delta B \sin\varepsilon$ $\sin\varepsilon = \delta l / 2r$, $m = I(\delta l)^2$ $B_A = 2\frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3}.$

$$B_B = \frac{\mu_0}{4\pi} I\delta l \left\{ \frac{1}{(r-\delta l/2)^2} - \frac{1}{(r+\delta l/2)^2} - \frac{2\sin\epsilon}{r^2} \right\}$$

$$\approx \frac{\mu_0}{4\pi}\frac{I\delta l}{r^2}\left\{ (1+\frac{\delta l}{r}) - (1-\frac{\delta l}{r}) - \frac{\delta l}{r} \right\}$$

$$\boxed{B_A = 2\frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3}.}$$

$$\boxed{B_B = \frac{\mu_0}{4\pi}\frac{\mathbf{m}}{r^3}}$$



So at a general point C, in spherical coordinates:

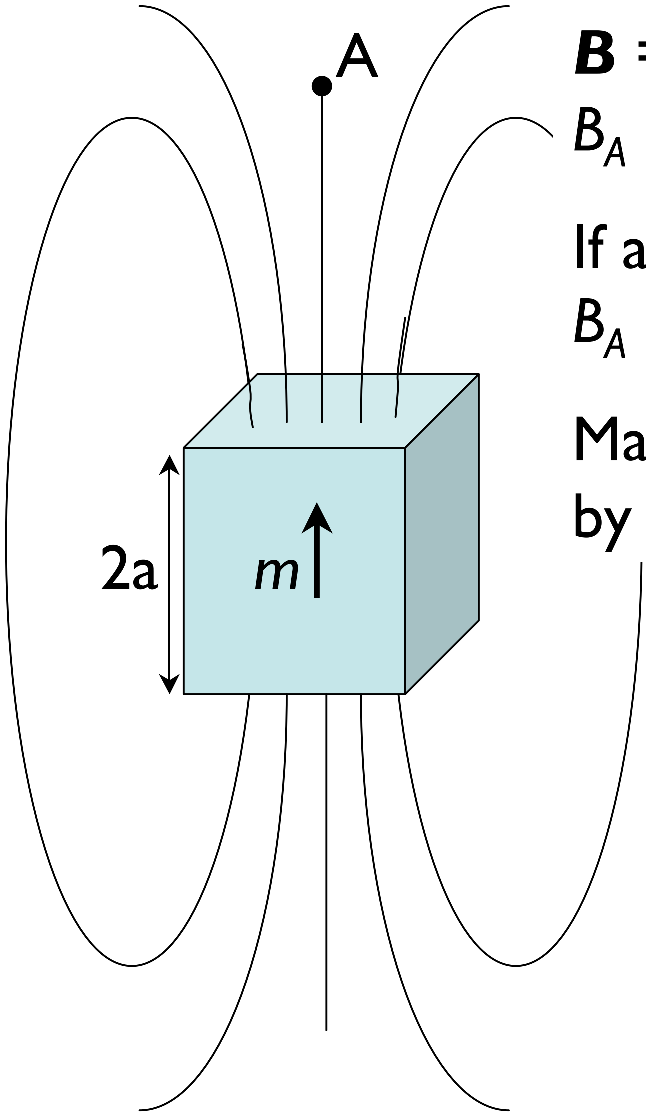
$$B_r = 2\left(\frac{\mu_0 \mathbf{m}}{4\pi r^3}\right)\cos\theta; \quad B_\theta = \left(\frac{\mu_0 \mathbf{m}}{4\pi r^3}\right)\sin\theta; \quad B_\phi = 0$$

an the equivalent form:

$$\boxed{\boldsymbol{B} = \frac{\mu_0}{4\pi}\left[3\frac{(\mathbf{m}.\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\mathbf{m}}{r^3}\right]}$$

Scaleability of magnetic devices

Why does magnetism lend itself to repeated miniaturization ?



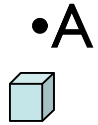
$\boldsymbol{B} = (\mu_0 m/4\pi r^3)\{2\cos\theta\,\mathbf{e}_r + \sin\theta\,\mathbf{e}_\theta\}$

$B_A = 2Ma^3/4\pi r^3;$

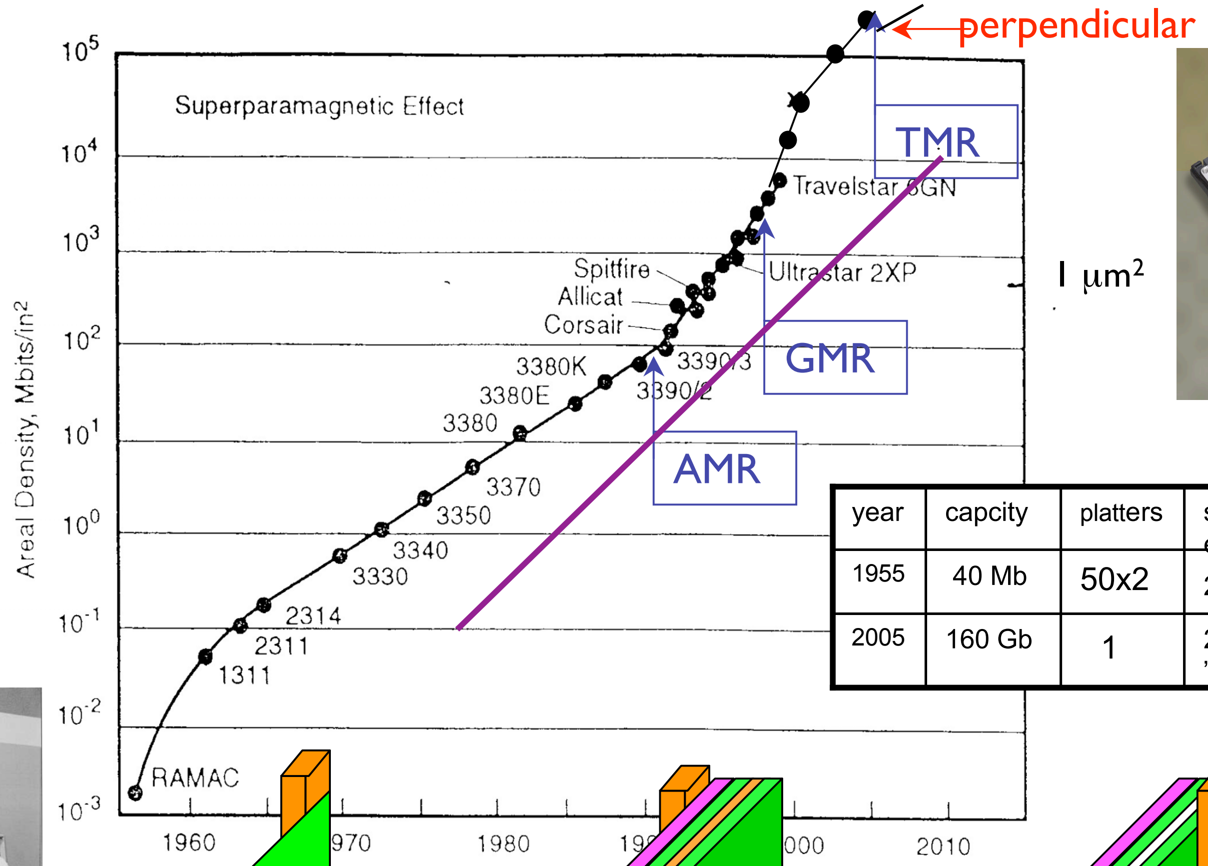
If $a = 0.1$m, $r = 4a$, $M = 1000$ kAm^{-1}

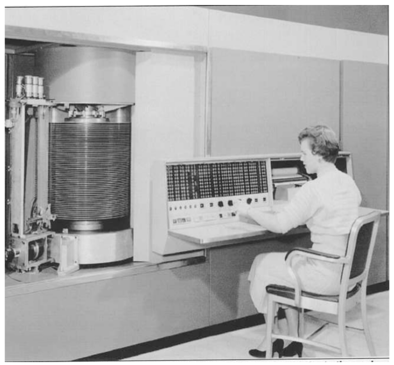
$B_A = 2\mu_0 M/16\pi = 50$ mT

Magnet-generated fields are limited by M. *They are scale-independent*

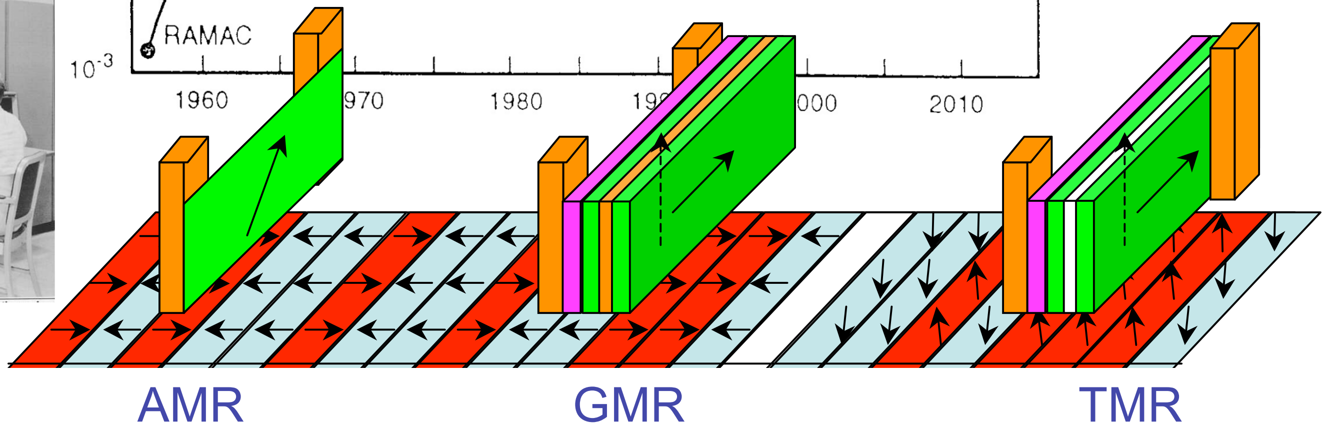


Magnetic recording is the partner of semiconductor technology in the information revolution. It provides the permanent, nonvolatile storage of information for computers and the internet. ~ 1 exobit (10^{21}bits) of data is stored





year	capcity	platters	size	rpm
1955	40 Mb	50x2	24”	1200
2005	160 Gb	1	2.5”	18000



Already, mankind produces more transistors and magnets in fabs than we grow grains of rice or wheat in fields.

1.5 **B and H**

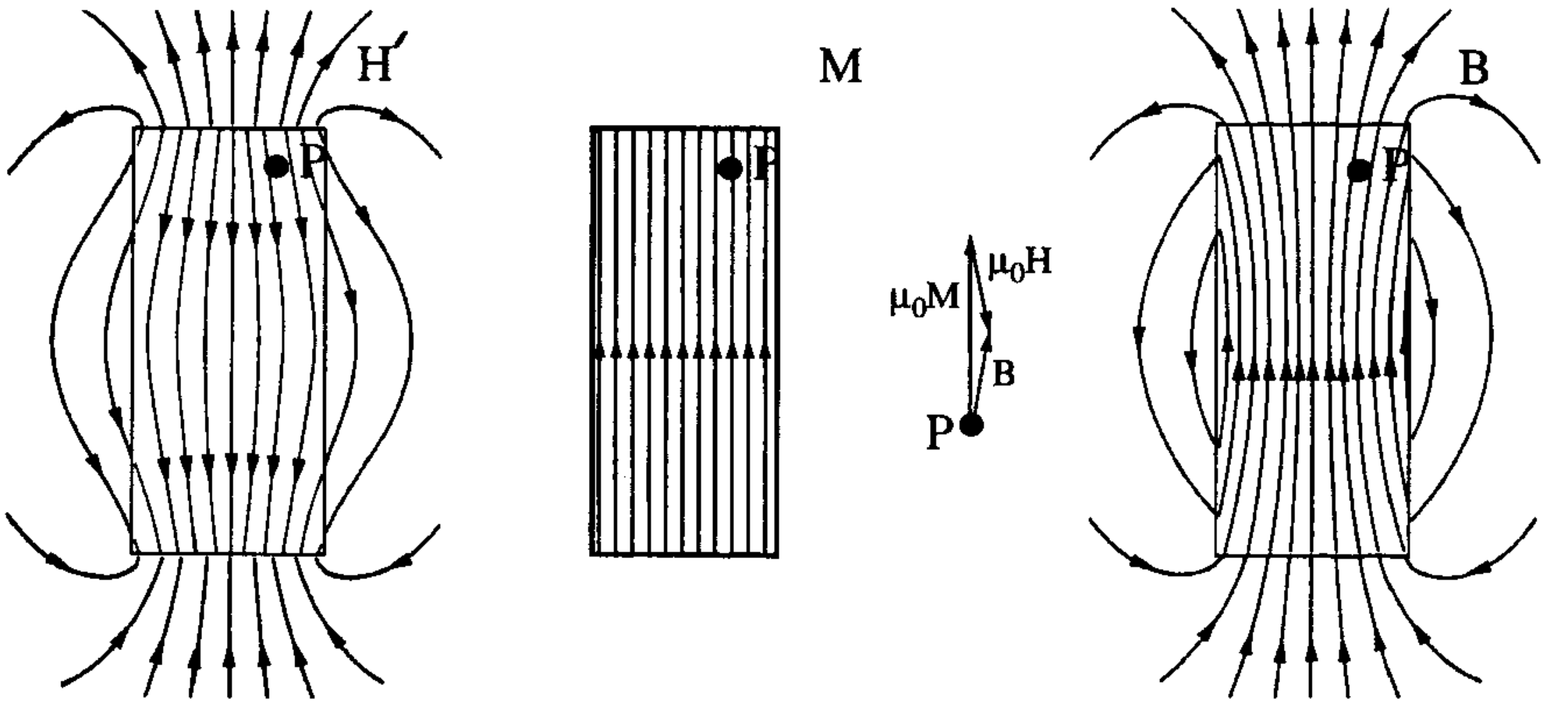
The equation used to *define* ***H*** is $\boxed{\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})}$

H_m is called the; — *stray field* outside the magnet

— *demagnetizing field,* H_d , inside the magnet

Units: Am^{-1}

The total *H*-field at any point is $\boxed{\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_m}$ where ***H´*** is the applied field



Maxwell's equations

$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \cdot \boldsymbol{D} = \rho$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \partial \boldsymbol{D}/\partial t$$
$$\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$$

From a long view of the history of mankind, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electrodynamics. Richard Feynmann

Written in terms of the four fields, they are valid in a material medium. In vacuum $\boldsymbol{D} = \varepsilon_0 \boldsymbol{E}$, $\boldsymbol{H} = \boldsymbol{B}/\mu_0$, ρ is charge density (C m^{-3}), $\boldsymbol{j}$ is current density (A m^{-2})

In vacuum they are written in terms of the two basic fields $\boldsymbol{B}$ and $\boldsymbol{E}$

Also, the force on a moving charge q, velocity $\mathbf{v}$

$$\boldsymbol{f} = q(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

Units: $\boldsymbol{H}$ A m^{-1} $\boldsymbol{B}$ kg C^{-1} s^{-1} ≡ tesla (T)

1.5.1 The B field - magnetic induction/magnetic flux density

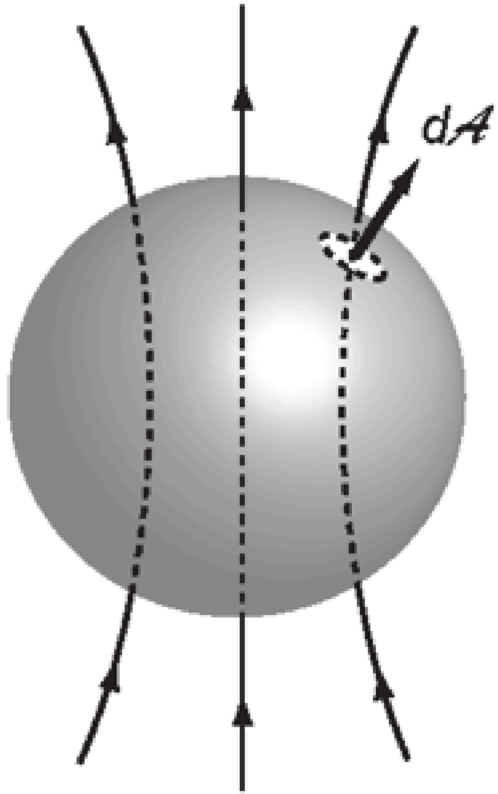
$\boxed{\nabla.\boldsymbol{B} = 0}$ There are sources or sinks of $\boldsymbol{B}$ i.e *no monopoles*

Magnetic vector potential $\boxed{\boldsymbol{B} = \nabla \times \boldsymbol{A}}$

The gradient of any scalar $\nabla\phi$ may be added to $\boldsymbol{A}$ without altering $\boldsymbol{B}$

Gauss's theorem: The net flux of B across any closed surface is zero

$$\boxed{\int_S \boldsymbol{B}.d\mathcal{A} = 0}$$



Magnetic flux $d\Phi = \boldsymbol{B}.d\mathcal{A}$ Units: Weber (Wb)

*Sources of **B***

- electric currents in conductors
- moving charges
- magnetic moments
- time-varying electric fields. (Not in *magnetostatics*)

The equation $\boxed{\nabla \times \mathbf{B} = \mu_0 \mathbf{j}}$ valid in static conditions gives:

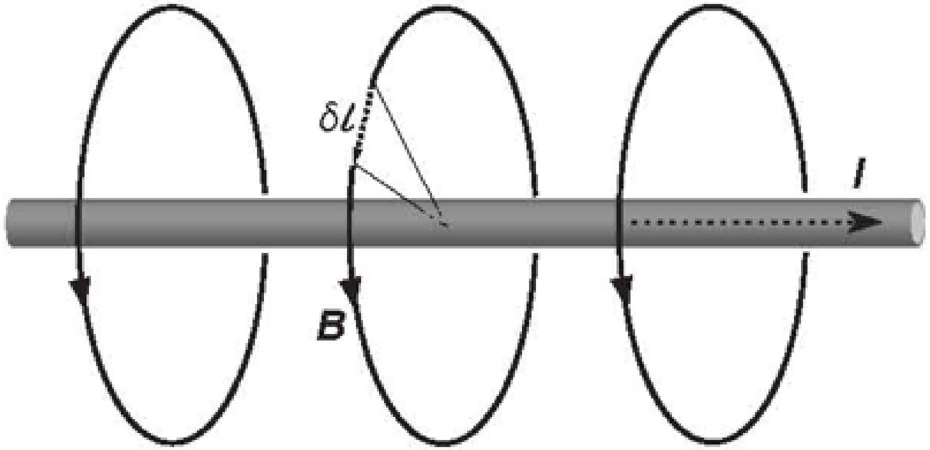
Ampere's law $\boxed{\int \mathbf{B} . d\mathbf{l} = \mu_0 I}$ for a closed path

Good for calculating the field for very symmetric current paths.

Example: the field at a distance r

from a current-carrying wire

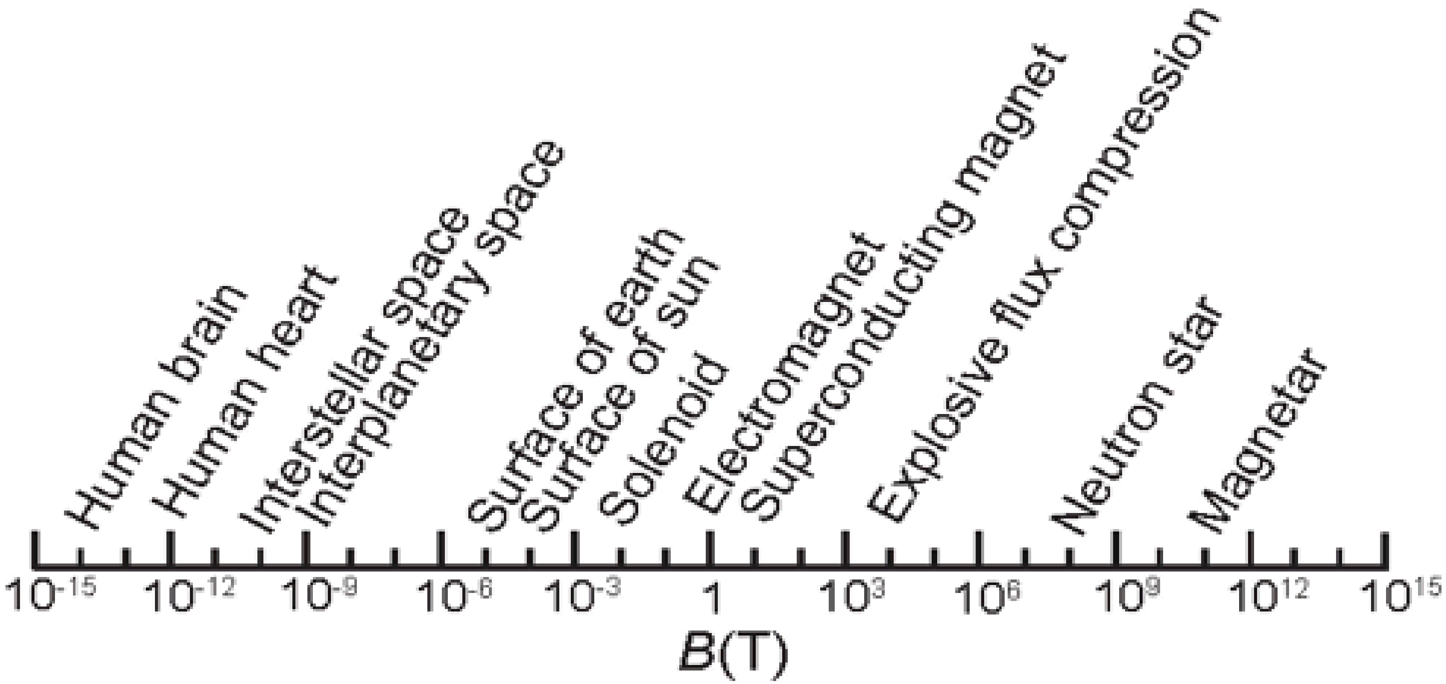
$B = \mu_0 I / 2\pi r$



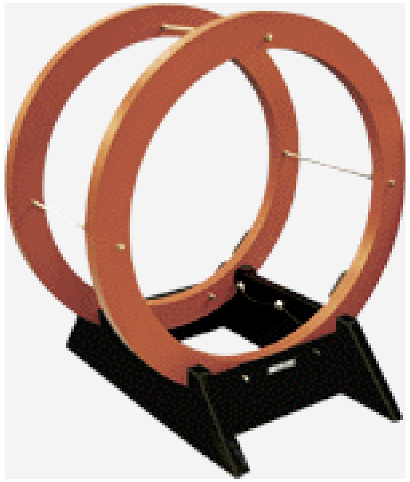
B interacts with any *moving* charge:

Lorentz force $\mathbf{f} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

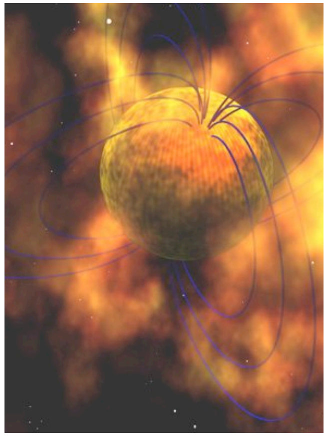
- The tesla is a very large unit
- Largest continuous field acheived in a lab is 45 T



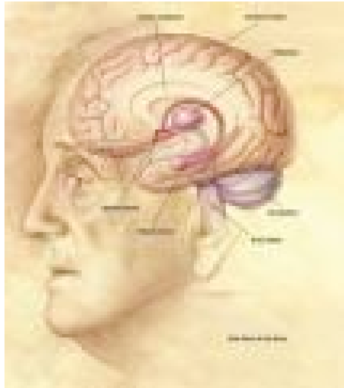
Earth 50 μT



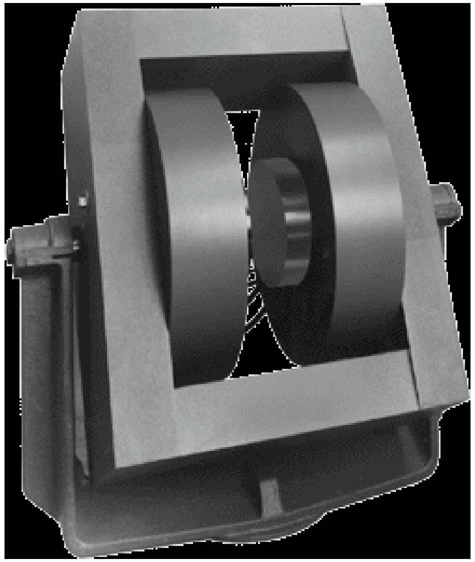
Helmholtz coils 0.01 T



Magnetar 10^{12} T



Human brain 1 fT

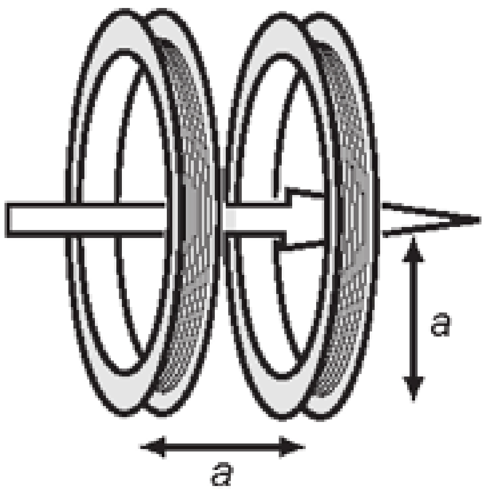


Electromagnet 1 T

Superconducting magnet 10 T

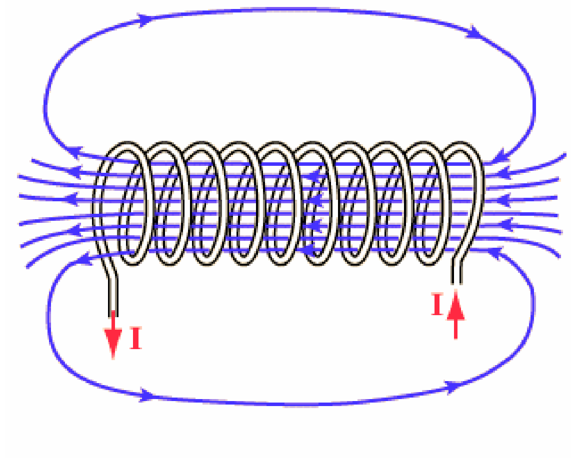
Sources of uniform magnetic fields

Helmholtz coils



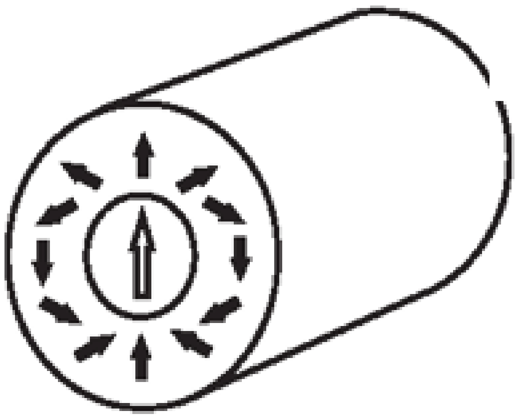
$$B = (4/5)^{3/2}\mu_0 NI/a$$

Long solenoid



$$B = \mu_0 n I$$

Halbach cylinder



$$B_r = \left(\frac{\mu_0 \lambda}{2\pi r^2}\right)\cos\theta, \quad B_\theta = \left(\frac{\mu_0 \lambda}{2\pi r^2}\right)\sin\theta, \quad B_z = 0.$$

$$B = \mu_0 M \ln(r_2/r_1)$$

1.5.2 The H field

The magnetization of a solid reflects the local value of ***H.***

$\boxed{\boldsymbol{B} = \mu_0 \boldsymbol{H}}$ In free space.

$\nabla \times \boldsymbol{B} = \mu_0(\boldsymbol{j}_c + \boldsymbol{j}_m)$ where $\boxed{\nabla \times \boldsymbol{H} = \mu_0 \boldsymbol{j}_c}$ $\boxed{\oint \boldsymbol{H}.\mathrm{d}\boldsymbol{l} = I_c}$

Coulomb approach to calculate H

H has sources and sinks associated with nonuniform magnetization

$$\boxed{\nabla.\boldsymbol{H} = - \nabla.\boldsymbol{M}}$$

Imagine ***H*** due to a distribution of magnetic charges q_m.

$$\boxed{\boldsymbol{H} = q_m \boldsymbol{r}/4\pi r^3}$$

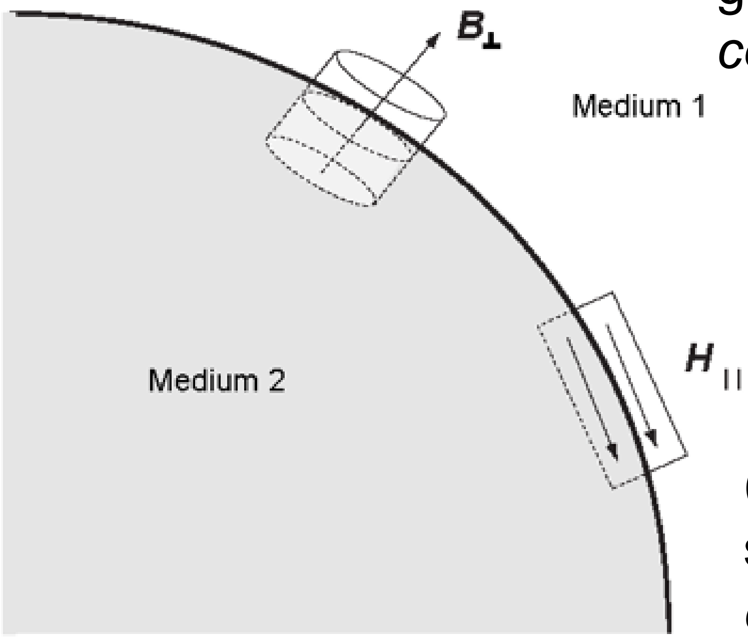
Scalar potential

When H is due only to magnets i.e $\nabla \times \boldsymbol{H} = 0$

Define a scalar potential φ_m (Units are Amps)

Such that $\boxed{\boldsymbol{H} = -\nabla \varphi_m}$ The potential of charge q_m is $\boxed{\varphi_m = q_m / 4\pi r}$

1.5.3 Boundary conditions



Gauss's law $\int_S \boldsymbol{B} \bullet d\mathcal{A} = 0$

gives that *the perpendicular component of* ***B*** *is continuous.*

$$\boxed{(\boldsymbol{B}_1 - \boldsymbol{B}_2).\boldsymbol{e}_n = 0}$$

It follows from from Ampère's law

$$\int_{loop} \boldsymbol{H}.d\boldsymbol{l} = I_c = 0$$

(there are no conduction currrents on the surface) that *the parallel component of H is continuous.*

$$\boxed{(\boldsymbol{H}_1 - \boldsymbol{H}_2) \times \boldsymbol{e}_n = 0}$$

Conditions on the potentials

Since $\int_S \boldsymbol{B} \bullet d\mathcal{A} = \int_{loop} \boldsymbol{A} \bullet d\boldsymbol{l}$ (Stoke's theorem)

$$\boxed{(\boldsymbol{A}_1 - \boldsymbol{A}_2) \times \boldsymbol{e}_n = 0}$$

The scalar potential is continuous

$$\boxed{\varphi_{m1} = \varphi_{m2}}$$

Boundary conditions in LIH media

In LIH media, $\boldsymbol{B} = \mu_0\, \mu_r\, \boldsymbol{H}$

Hence

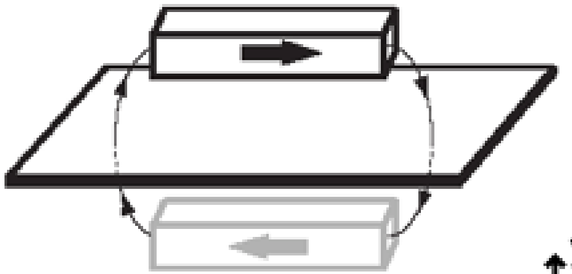
$$\boldsymbol{B}_1\boldsymbol{e}_n = \boldsymbol{B}_2\boldsymbol{e}_n$$

$$\boldsymbol{H}_1\boldsymbol{e}_n = \mu_{r2}/\mu_{r1}\, \boldsymbol{H}_2\boldsymbol{e}_n$$

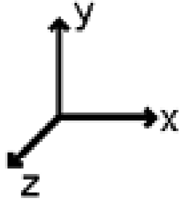
So field lies ≈ perpendicular to the surface of soft iron but parallel to the surface of a superconductor.

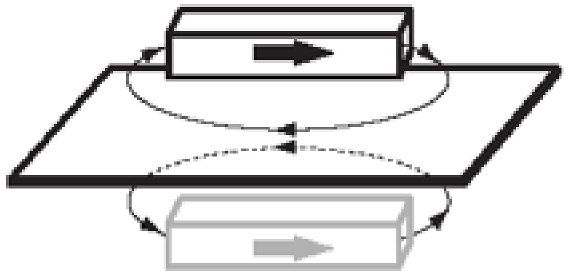
Diamagnets produce weakly repulsive images.

Paramagnets produce weakly attractive images.



Soft ferromagnetic mirror



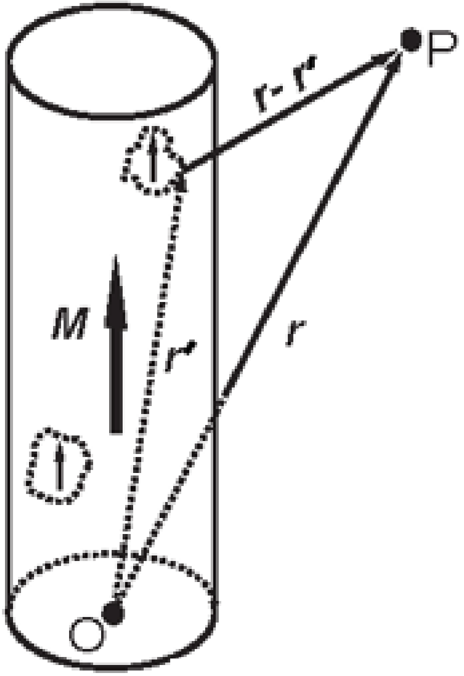


Superconducting mirror

1.6 Field calculations

Three different approaches:

*Integrate over volume distribution of **M***



Sum over fields produced by each magnetic dipole element ***M***d^3*r.*

Using

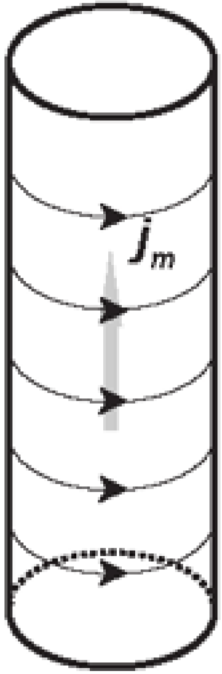
$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\left[3\frac{(\mathbf{m}.\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\mathbf{m}}{r^3}\right]$$

Gives

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left[\int\left\{\frac{3\boldsymbol{M}(\boldsymbol{r}').(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^5}(\boldsymbol{r}-\boldsymbol{r}') - \frac{\boldsymbol{M}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3} + \frac{2}{3}\mu_0\boldsymbol{M}(\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{r}')\right\}d^3r'\right]$$

(Last term takes care of divergences at the origin)

Amperian approach



Consider bulk and surface current distributions

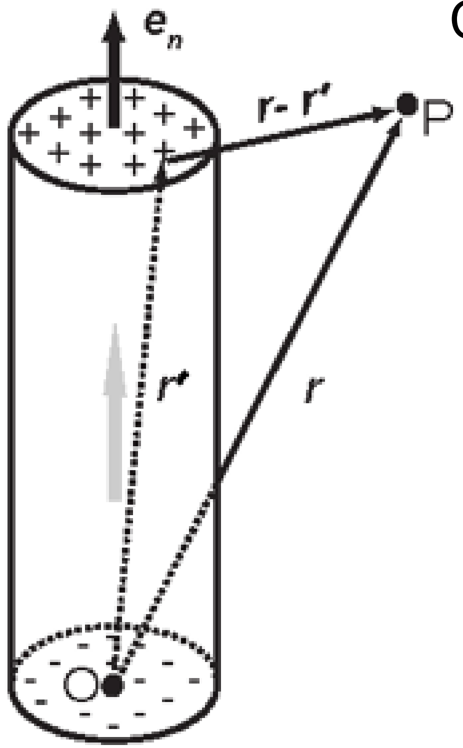
$$\boldsymbol{j}_m = \nabla \times \boldsymbol{M} \qquad \text{and} \qquad \boldsymbol{j}_{ms} = \boldsymbol{M} \times \boldsymbol{e}_n$$

Biot-Savart law gives

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{(\nabla \times \boldsymbol{M}) \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} d^3r' + \int \frac{(\boldsymbol{M} \times \mathbf{e}_n) \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} d^2r'\right\}$$

For uniform ***M***, the Bulk term is zero since $\nabla \times \boldsymbol{M} = 0$

Coulombian approach



Consider bulk and surface magnetic charge distributions

$$\rho_m = -\nabla.\boldsymbol{M} \qquad \text{and} \qquad \rho_{ms} = \boldsymbol{M}.\boldsymbol{e_n}$$

H field of a small charged volume element V is

$$\delta\boldsymbol{H} = (\rho_m \boldsymbol{r}/4\pi r^3)\ \delta V$$

So

$$\boldsymbol{H}(\boldsymbol{r}) = \frac{1}{4\pi}\left\{-\int_V \frac{(\nabla.\boldsymbol{M})(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}d^3r' + \int_S \frac{\boldsymbol{M}.\mathbf{e}_n(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}d^2r'\right\}$$

For a uniform magnetic distribution *the first term is zero*.

$\nabla.\boldsymbol{M}=0$

1.7 Demagnetising field

The H-field in a magnet depends $\boldsymbol{M}(\boldsymbol{r})$ and on the *shape* of the magnet.
$\boldsymbol{H}_d$ is uniform in the case of a *uniformly-magnetized ellipsoid*.

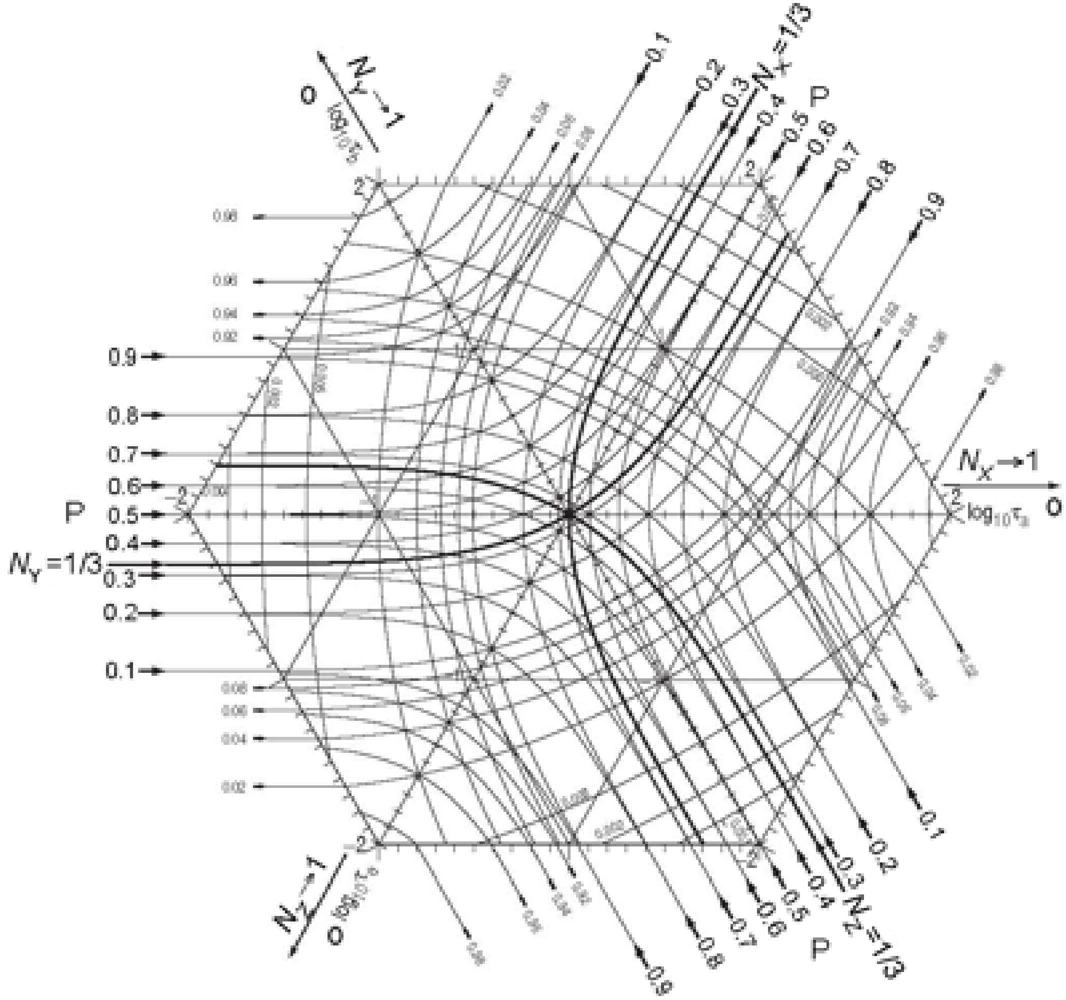
$$\boxed{(\boldsymbol{H}_d)_i = - N_{ij} \boldsymbol{M}_j} \qquad i,j = x,y,z$$

$$N_x + N_y + N_z = 1$$

Demagnetizing factors for some simple shapes

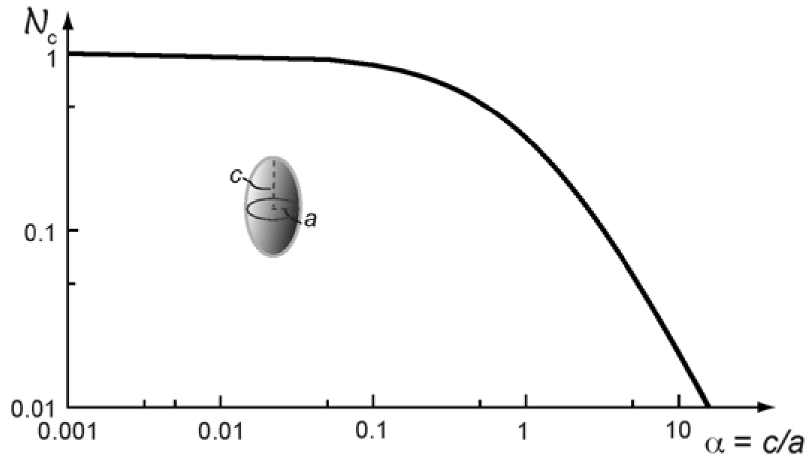
Long needle, $\boldsymbol{M}$ parallel to the long axis	0
Long needle, $\boldsymbol{M}$ perpendicular to the long axis	1/2
Sphere, $\boldsymbol{M}$ in any direction	1/3
Thin film, $\boldsymbol{M}$ parallel to plane	0
Thin film, $\boldsymbol{M}$ perpendicular to plane	1
General ellipsoid of revolution (a,a,c)	$N_c = (1 - 2N_a)$

Demagnetizing factors for general ellipsoids



Demagnetizing factors for ellipsoids of revolution

Major axes (a,a,c)

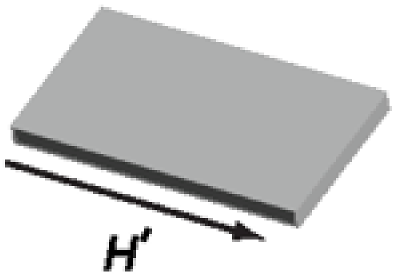


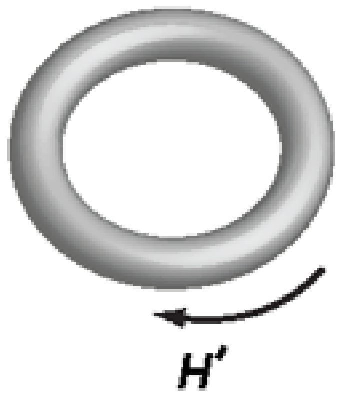
Measuring magnetization with no need for demagnetization correction

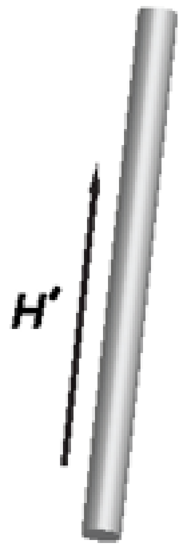
Apply a field in a direction where N=0

$$\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_m \qquad\qquad (\boldsymbol{H}_d)_i = - N_{ij} \boldsymbol{M}_j$$

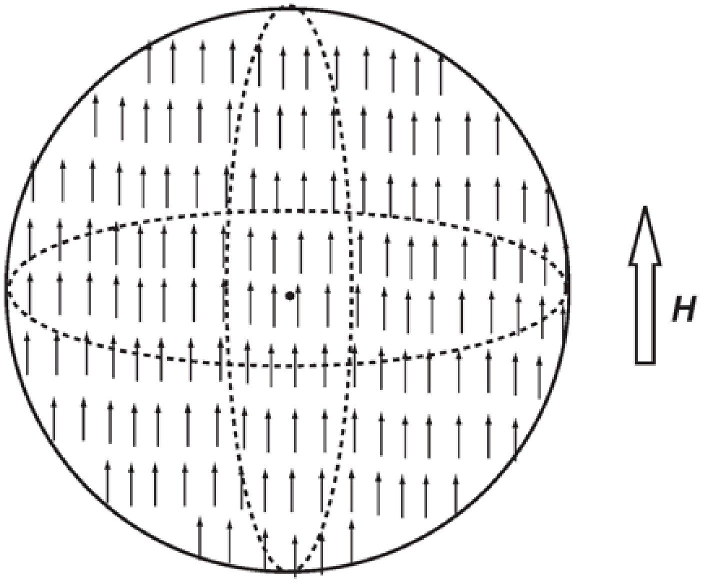
$$\boxed{\boldsymbol{H} \approx \boldsymbol{H}' - N \boldsymbol{M}}$$

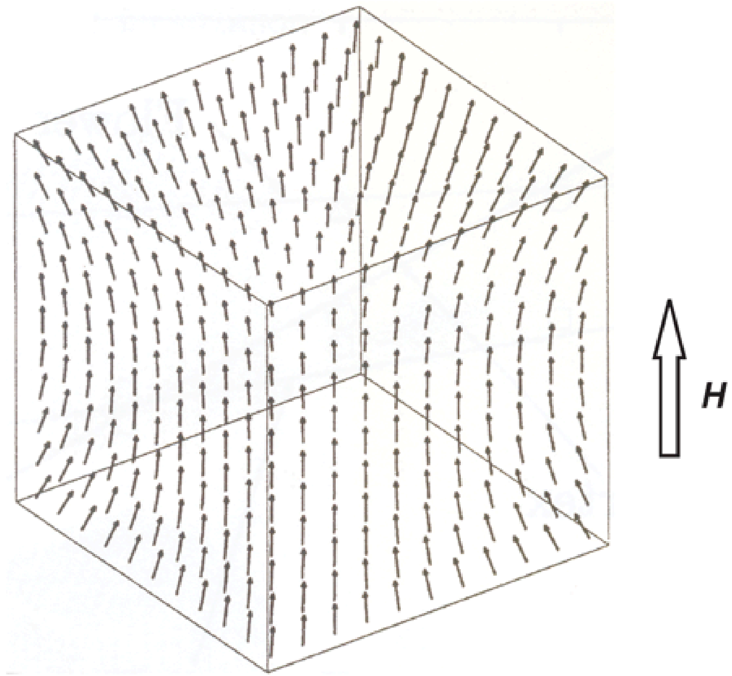






It is not possible to have a uniformly magnetized cube





When measuring the magnetization of a sample ***H*** *is taken as the independent variable,* ***M***=***M***(***H***).

1.8 Response to an applied field *H′*

Susceptibility of linear, isotropic and homogeneous (LIH) materials

$\boldsymbol{M} = \chi'\boldsymbol{H}'$ χ' is external susceptibility

$\boldsymbol{M} = \chi\boldsymbol{H}$ χ is internal susceptibility

It follows that from $\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_d$ that

$$\boxed{1/\chi = 1/\chi' - N}$$

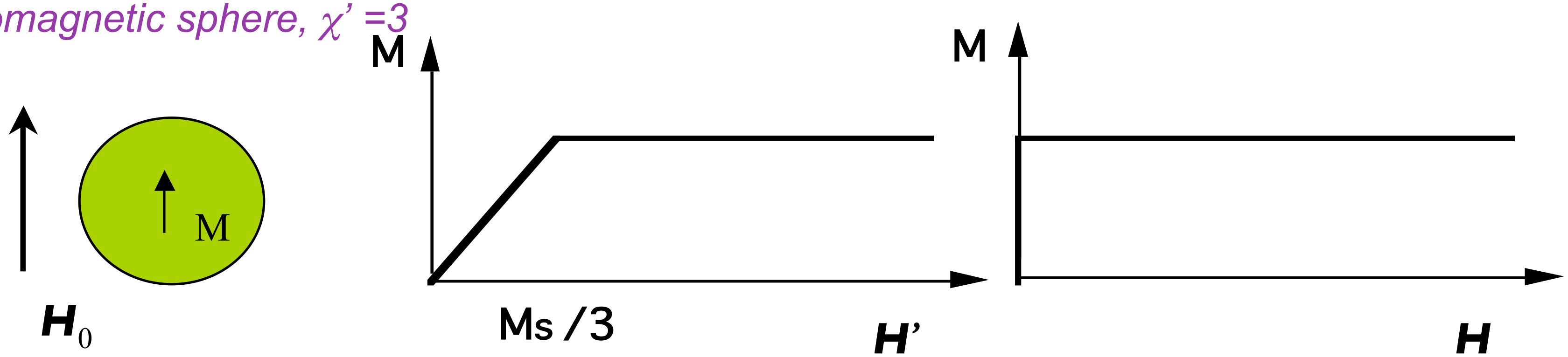
Typical paramagnets and diamagnets:

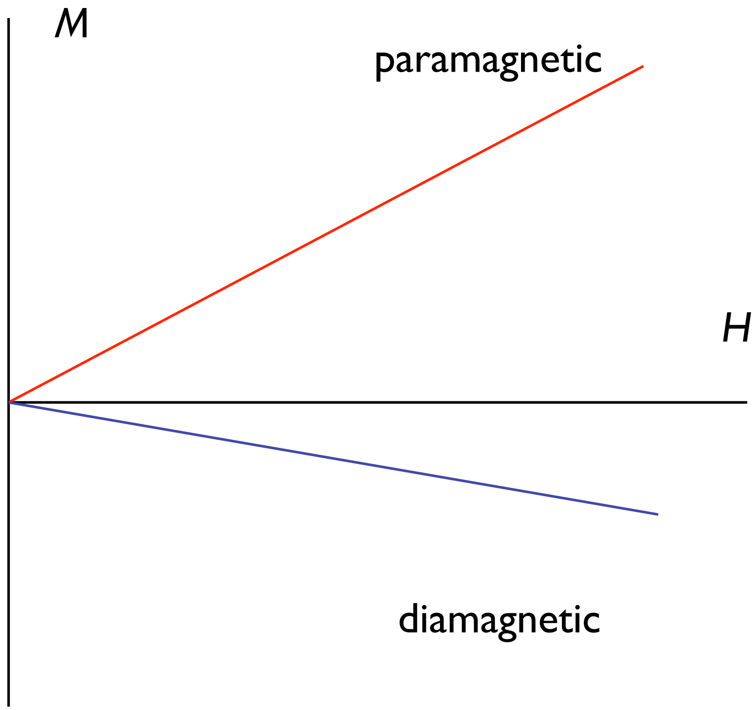
$\chi \approx \chi'$ $(10^{-5}$ to $10^{-3})$

Paramagnets close to the Curie point and ferromagnets:

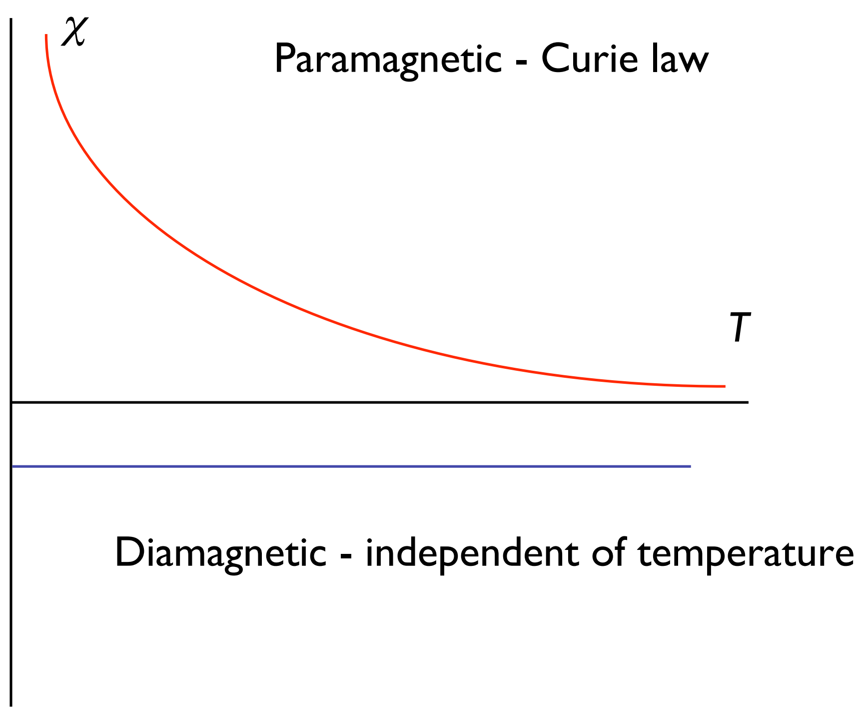
$\chi >> \chi'$ χ diverges as $T \rightarrow T_C$ but χ' never exceeds $1/N$.

Ferromagnetic sphere, χ' =3



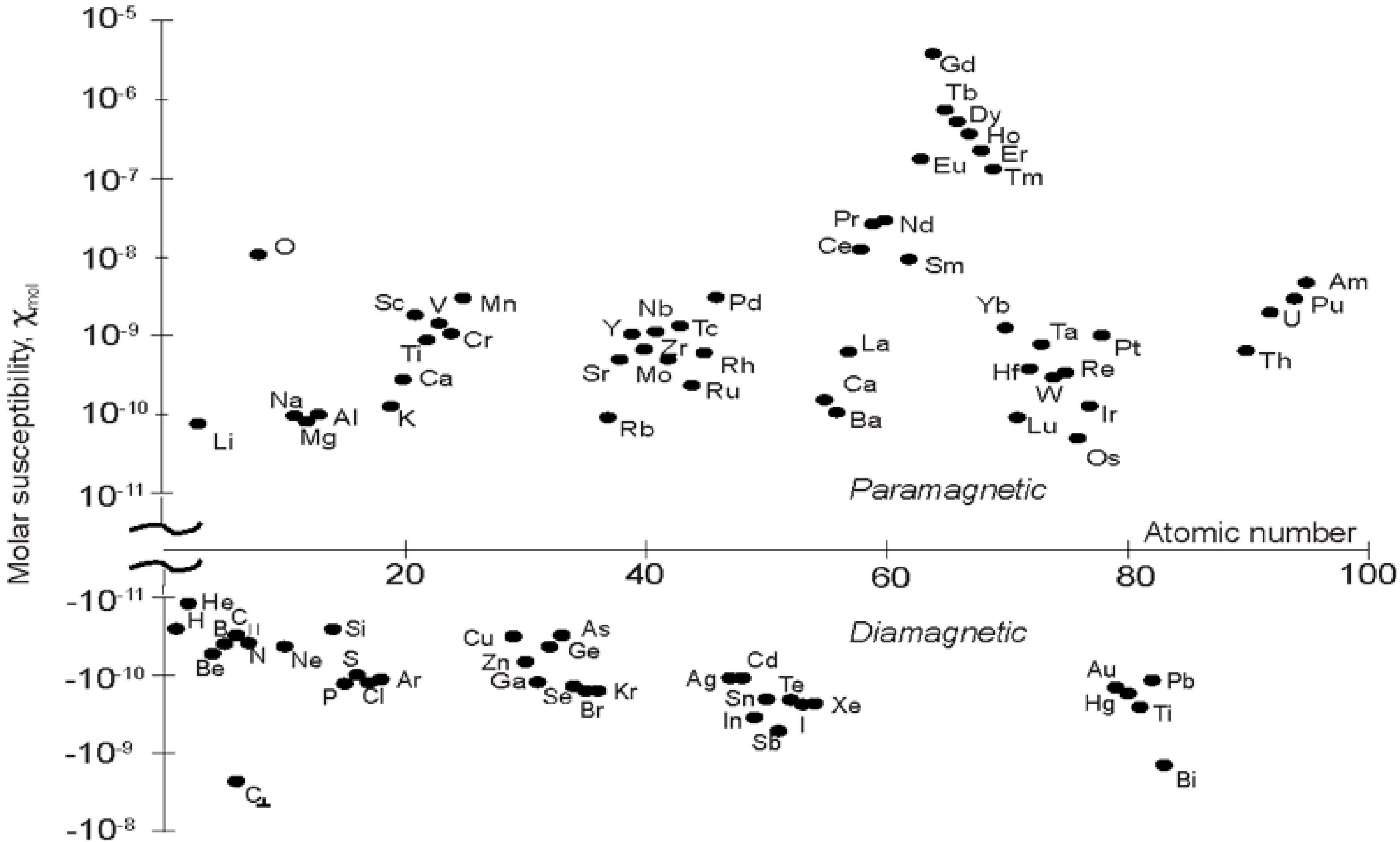


Magnetization curves



Susceptibility vs temperature

Susceptibilities of the elements



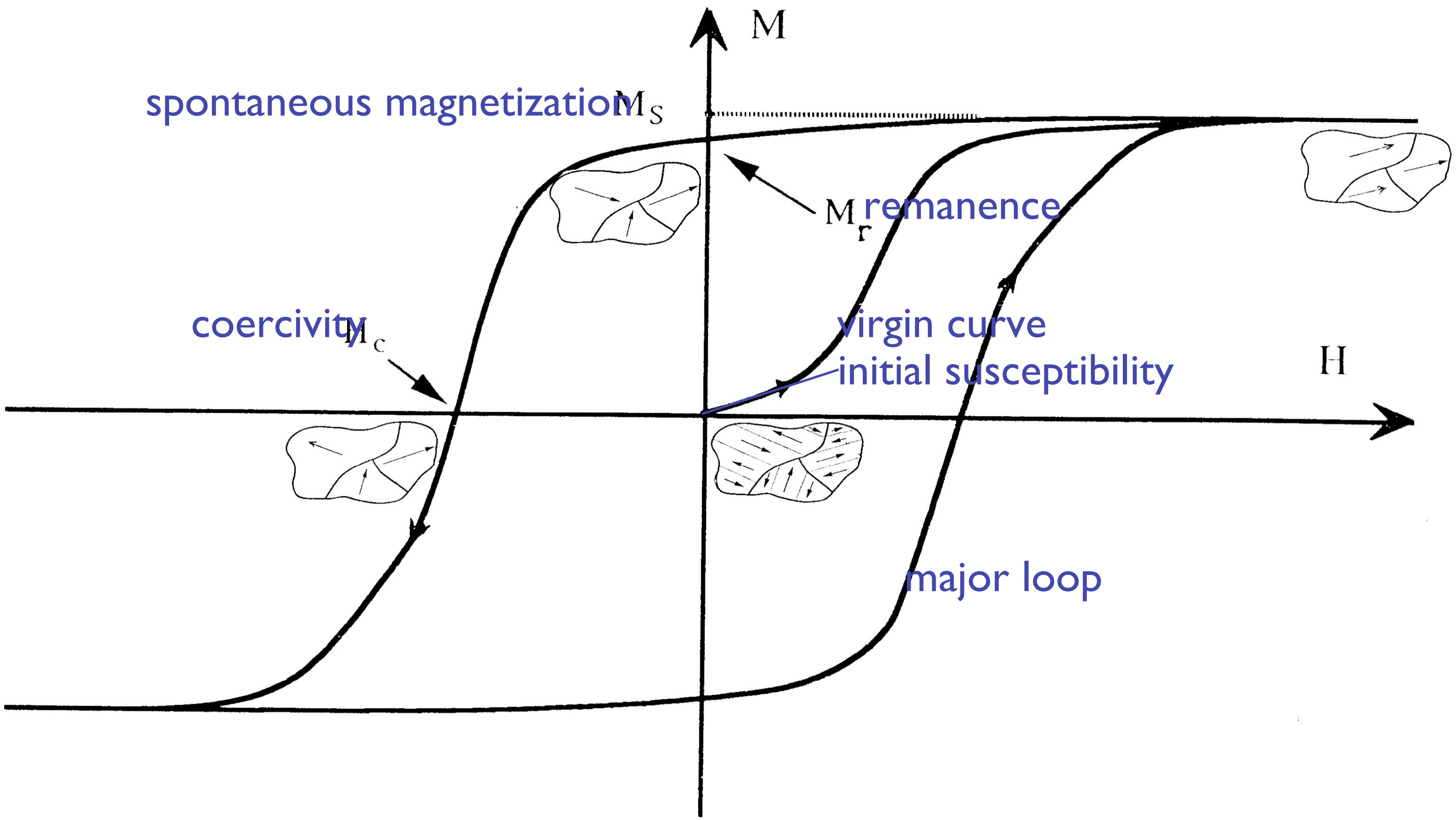
Permeability

In LIH media $\boxed{\boldsymbol{B} = \mu \boldsymbol{H}}$ Units: $TA^{-1}m$

Relative permeability $\mu_r = \mu / \mu_0$

$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ gives $\boxed{\mu_r = 1 + \chi}$

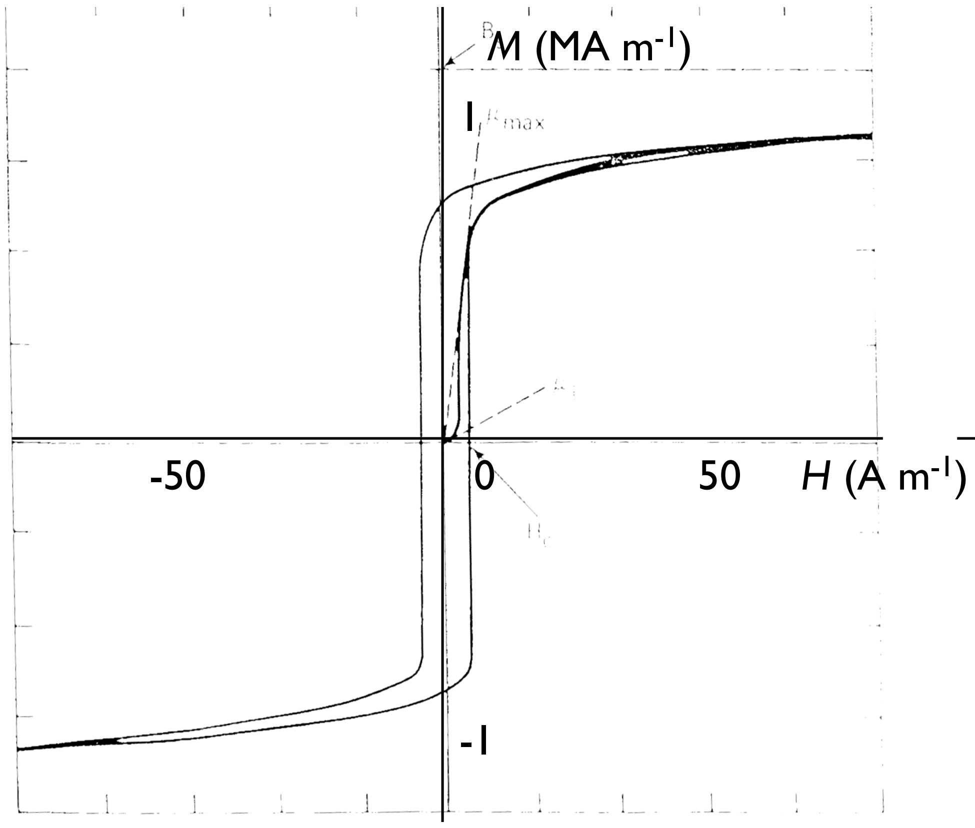
1.5 Hysteresis

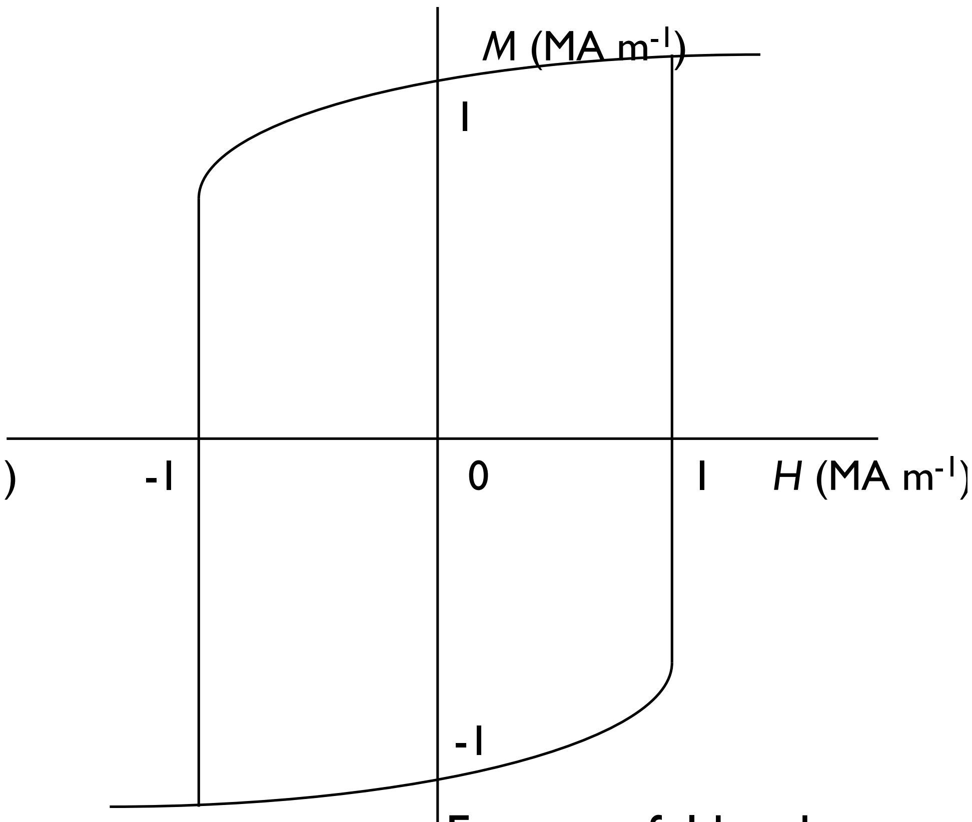


The hysteresis loop shows the irreversible, nonlinear response of a ferromagnet to a magnetic field . It reflects the arrangement of the magnetization in ferromagnetic *domains*. The magnet cannot be in thermodynamic equilibrium anywhere around the open part of the curve!

1.5.1 Soft and hard magnets.

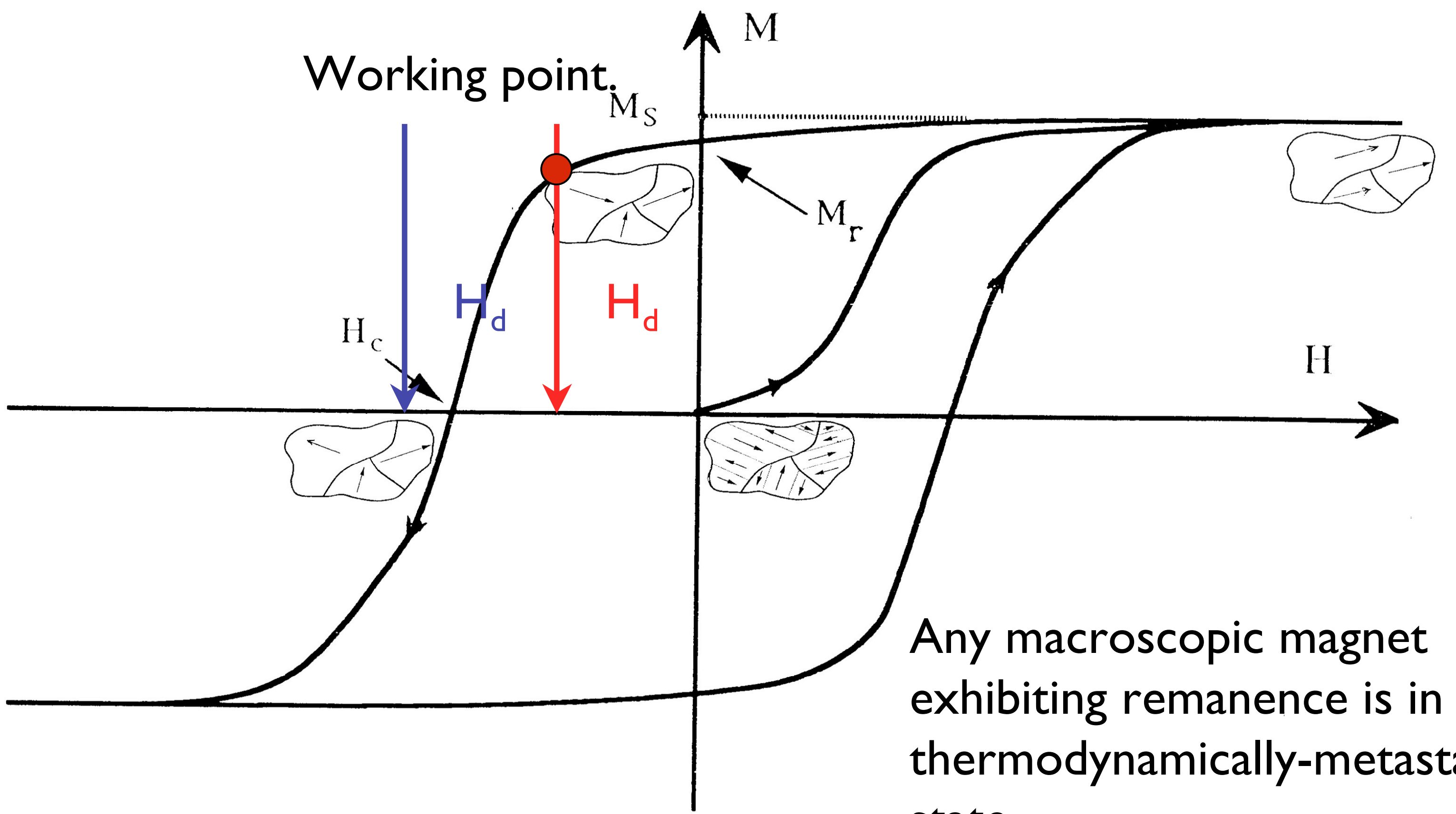
The area of the hysteresis loop represents the energy loss per cycle. For efficient *soft* magnetic materials, this needs to be as small as possible.





For a useful hard magne
$H_c > M_r/2$

1.5.2 Energy product.

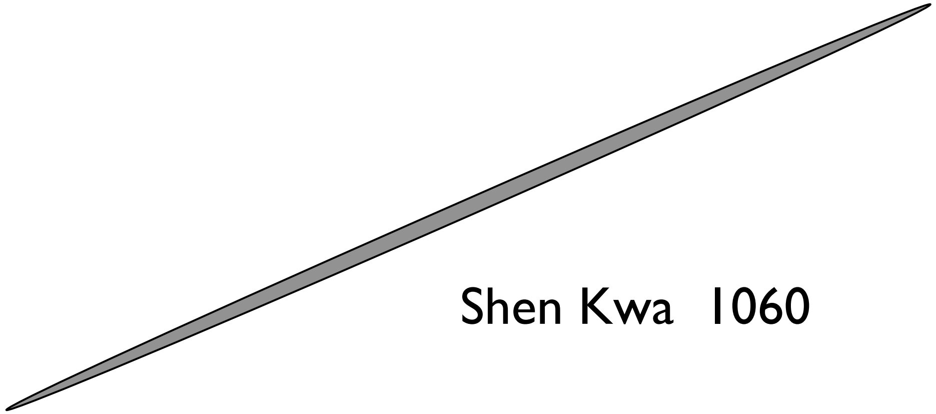


Any macroscopic magnet exhibiting remanence is in a thermodynamically-metastable state.

The shape barrier.

Daniel Bernouilli
1743

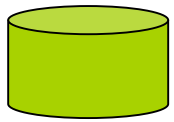
$N < 0.1$



Shen Kwa 1060

Gowind Knight
1760

New icon for permanent magnets! ⇒



1.9 Magnetostatic energy and forces

Energy of ferromagnetic bodies

• Magnetostatic (dipole-dipole) forces are long-ranged, but weak. They determine the magnetic microstructure.

$M \approx 1$ MA m^{-1}, $\mu_0 H_d \approx 1$ T, hence $\mu_0 H_d M \approx 10^6$ J m^{-3}

• Products $\boldsymbol{B.H}$, **B.M**, $\mu_0 \boldsymbol{H}^2$, $\mu_0 \boldsymbol{M}^2$ are all energies per unit volume.

• Magnetic forces *do no work* on moving charges $\boldsymbol{f} = q(\boldsymbol{v} \times \boldsymbol{B})$

• No potential energy associated with the magnetic force.

Torque and potential energy of a dipole in a field

$$\boxed{\boldsymbol{\Gamma} = \boldsymbol{m} \times \boldsymbol{B}} \qquad \boxed{\varepsilon_m = -\boldsymbol{m}.\boldsymbol{B}}$$

Force

In a non-uniform field, $\boxed{\boldsymbol{f} = -\nabla \varepsilon_m}$ $\quad \boldsymbol{f} = \boldsymbol{m}.\nabla \boldsymbol{B}$

Reciprocity theorem

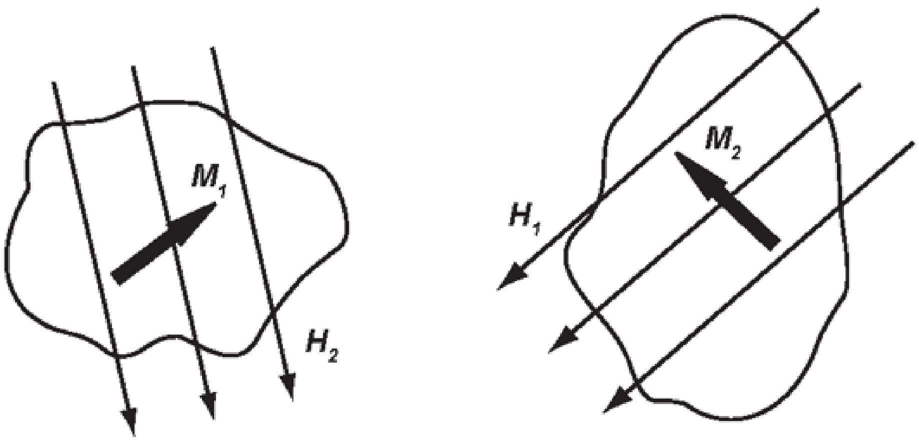
The interaction of a pair of dipoles, ε_p, can be considered as the energy of $\boldsymbol{m}_1$ in the field $\boldsymbol{B}_{21}$ created by $\boldsymbol{m}_2$ at $\boldsymbol{r}_1$ *or vice versa.*

$$\varepsilon_p = -\boldsymbol{m}_1.\boldsymbol{B}_{21} = -\boldsymbol{m}_2.\boldsymbol{B}_{12}$$

So $$\boxed{\varepsilon_p = -(1/2)(\boldsymbol{m}_1.\boldsymbol{B}_{21} + \boldsymbol{m}_2.\boldsymbol{B}_{12})}$$

Extending to magnetization distributions:

$$\varepsilon = -\mu_0 \int \boldsymbol{M}_1.\boldsymbol{H}_2 \, d^3r = -\mu_0 \int \boldsymbol{M}_2.\boldsymbol{H}_1 \, d^3r$$



Self energy

The interaction of the body with the field it creates itself, $\boldsymbol{H}_d$.

Consider the energy to bring a small moment $\delta\boldsymbol{m}$ into position *within* the magnetized body

$$\boldsymbol{H}_{loc}=\boldsymbol{H}_d+(1/3)\boldsymbol{M}$$

$$\delta\varepsilon = - \mu_0\, \delta\boldsymbol{m}.\boldsymbol{H}_{loc}$$

Integration over the whole sample gives

$$\varepsilon = -\frac{1}{2}\int_v \mu_0 \boldsymbol{H}_d.\boldsymbol{M}\, d^3r - \frac{1}{6}\int_v \mu_0 M^2 d^3r.$$

The magnetostatic self energy is defined as

$$\boxed{\varepsilon_m = \varepsilon + (1/6)\int_v \mu_0 M^2 d^3r}$$

Or equivalently, using $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$ and $\int \boldsymbol{B}.\boldsymbol{H}_d d^3r=0$

$$\boxed{\varepsilon_m = \frac{1}{2}\int \mu_0 H_d^2 d^3r,}$$

For a uniformly magnetized ellipsoid $\varepsilon_m = \frac{1}{2}\mu_0 V \mathcal{N} M^2,$

Energy associated with a field

General expression for the energy associated with a magnetic field distribution

$$\boxed{\varepsilon = \frac{1}{2}\int \mu_0 H^2 d^3r.}$$

Aim to maximize energy associated with the field created around the magnet, from previous slide:

$$\frac{1}{2}\int \mu_0 H_d^2 d^3r = -\frac{1}{2}\int_V \mu_0 \mathbf{H}_d.\mathbf{M}\, d^3r.$$

Can rewrite as:

$$\frac{1}{2}\int_o \mu_0 H_d^2 d^3r = -\frac{1}{2}\int_i \mu_0 H_d^2 d^3r - \frac{1}{2}\int_i \mu_0 \mathbf{M}.\mathbf{H}_d d^3r$$

where we want to maximize the integral on the left.

Energy product: twice the energy stored in the stray field of the magnet

$$\boxed{-\mu_0 \int_i \boldsymbol{B}.\boldsymbol{H}_d\, d^3r}$$

Work done by an external field

Elemental work δw to produce a flux change $\delta\Phi$ is $I\,\delta\Phi$

Ampere: $\int \boldsymbol{H}.\mathrm{d}\boldsymbol{l} = I$ So $\delta w = \int \delta\Phi\, \boldsymbol{H}.\mathrm{d}\boldsymbol{l}$

So in general: $\delta w = \int \delta\boldsymbol{B}.\boldsymbol{H}.\mathrm{d}^3 r$

$\boldsymbol{H} = \boldsymbol{H}' + \boldsymbol{H}_d$ $\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$

Subtract the term associated with the H-field in empty space, to give the work done on the body by the external field;

$$\delta w' = \int (\mathbf{H}.\delta\mathbf{B} - \mu_0 \mathbf{H}'.\delta\mathbf{H}) d^3 r.$$

$$\delta w' = \mu_0 \left[\int \delta(\mathbf{H}'.\mathbf{H}_d) d^3 r + \int \mathbf{H}_d \delta \mathbf{H}_d d^3 r + \int \mathbf{H}.\delta\mathbf{M}\, d^3 r \right]$$

$$\delta w' = \delta\varepsilon_m + \mu_0 \int \mathbf{H}.\delta\mathbf{M}\ d^3 r$$

$$\delta\varepsilon_m = -\frac{1}{2}\mu_0 \int (\mathbf{H}_d.\delta\mathbf{M} + \mathbf{M}.\delta\mathbf{H}_d) d^3 r,$$

$$\delta\varepsilon_m = -\mu_0 \int \mathbf{H}_d.\delta\mathbf{M}\ d^3 r,$$

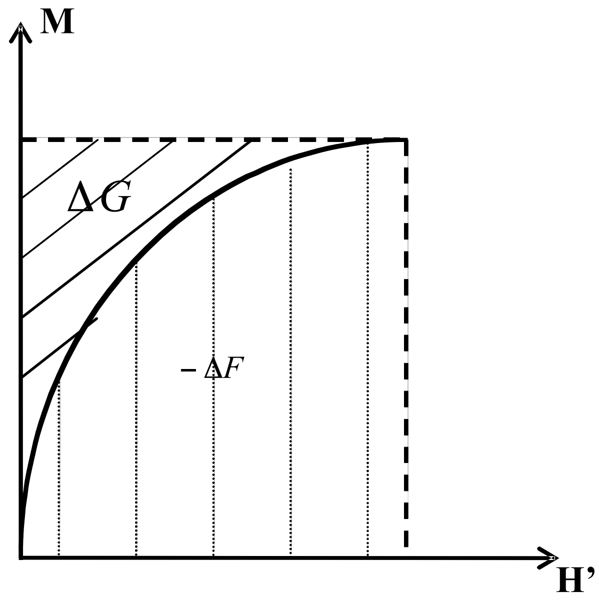
gives

$$\boxed{\delta w' = \mu_0 \int \mathbf{H}'.\delta\mathbf{M}\ d^3 r.}$$

Thermodynamics

(U, Q, F, G are in units of Jm^{-3})

First law: $dU = H_x dX + dQ$



$dQ = TdS$

Four thermodynamic potentials;

$U(X,S)$

$E(H_X,S)$

$F(X,T) = U - TS$ $\qquad dF = HdX - SdT$

$G(H_X,T) = F - H_X X$ $\qquad dG = -XdH - SdT$

Magnetic work is $H\delta B$ or $\mu_0 H'\delta M$

$$dF = \mu_0 H' dM - SdT$$
$$dG = -\mu_0 M dH' - SdT$$

$S = -(\partial G/\partial T)_{H'}$ $\quad \mu_0 M = -(\partial G/\partial H')_{T'}$

Maxwell relations

$(\partial S/\partial H')_{T'} = - \mu_0(\partial M/\partial T)_{H'}$ etc.

Magnetostatic Forces

Force density on a magnetized body at constant temperature

$$\boxed{\boldsymbol{F}_m = -\nabla G}$$

$$\boldsymbol{F}_m = \nabla(\mu_0 \boldsymbol{H}'.\boldsymbol{M}) \qquad \nabla(\boldsymbol{H}'.\boldsymbol{M}) = (\boldsymbol{H}'.\nabla)\boldsymbol{M} + (\boldsymbol{M}.\nabla)\boldsymbol{H}'$$

Kelvin force

$$\boxed{\boldsymbol{F}_m = \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}'}$$

General expression, for when ***M*** is dependent on ***H*** is

$$\boxed{\boldsymbol{F}_m = -\mu_0 \nabla \left[\int_0^H \left(\frac{\partial M \upsilon}{\partial \upsilon}\right)_{H,T} dH\right] + \mu_0(\boldsymbol{M}.\nabla)\boldsymbol{H}.}$$

$\nu = 1/d$ d is the density

1.10 **Units and dimensions**

• We use SI, with the Sommerfeld convention $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. Engineers prefer the Kenelly convention $\mathbf{B} = \mu_0\mathbf{H} + \mathbf{J}$, where the polarization $\mathbf{J}$ is $\mu_0\mathbf{M}$. Both are acceptable in SI. The polarization of iron is J = 2.16 T.

•Flux density B and polarization J are measured in telsa (also mT, μT). Magnetic moment *m* is measured in A m^2 so the magnetization M and magnetic field H are measured in A m^{-1}. From the energy relation
$E = -\boldsymbol{m}.\mathbf{B}$, it is seen that an equivalent unit for magnetic moment is J T^{-1}, so magnetization can also be expressed as J T^{-1}m^{-3}. σ, the magnetic moment per unit mass in J T^{-1}kg^{-1} or A m^2 kg^{-1} is the quantity most usually measured in practice. μ_0 is exactly $4\pi.10^{-7}$ T m A^{-1}.

•The international system is based on five fundamental units kg, m, s, K, and A.
Derived units include the newton (N) = kg.m/s^2, joule (J) = N.m, coulomb (C) = A.s, volt (V) = JC^{-1}, tesla (T) = JA^{-1}m^{-2} = Vsm^{-2}, weber (Wb) = V.s = T.m^2 and hertz (Hz) = s^{-1}.
Recognized multiples are in steps of $10^{\pm 3}$, but a few exceptions are admitted such as cm (10^{-2} m) and Å (10^{-10} m). Multiples of the meter are fm (10^{-15}), pm (10^{-12}), nm (10^{-9}), μm (10^{-6}), mm (10^{-3}) m (10^{-0}) and km (10^3).

• The SI system has two compelling advantages for magnetism:

(i) it is possible to check the dimensions of any expression by inspection and
(ii) the units are directly related to the practical units of electricity, used in the laboratory.

cgs Units

• Much of the primary literature on magnetism is still written using cgs units. Fundamental cgs units are cm, g and s. The electromagnetic unit of current is equivalent to 10 A. The electromagnetic unit of potential is equivalent to 10 nV. The electromagnetic unit of magnetic dipole moment (emu) is equivalent to 1 mA m^2. Derived units include the erg (10^{-7} J) so that an energy density such as K_1 of 1 Jm^{-3} is equivalent to 10 erg cm^{-3}. The convention relating flux density and magnetization is

$$\mathbf{B} = \mathbf{H} + 4\pi\mathbf{M}$$

where the flux density or induction B is measured in gauss (G) and field H in oersted (Oe). Magnetic moment is usually expresed as emu, and magnetization is therefore in emu/cm^3, although $4\pi M$ is frequently considered a flux-density expression and quoted in kilogauss. μ_0 is numerically equal to 1 G Oe^{-1}, but it is normally omitted from the equations.

The most useful conversion factors between SI and cgs units in magnetism are

B	1 T ≡ 10 kG	1 G ≡ 0.1 mT
H	1 kA m^{-1} ≡ 12.57 (≈12.5) Oe	1 Oe ≡ 79.58 (≈80) A m^{-1}
m	1 Am^2 ≡ 1000 emu	1 emu ≡ 1 mA m^2
M	1 kA m^{-1} ≡ 1 emu cm^{-3}	
σ	1 Am^2 kg^{-1} ≡ 1 emu g^{-1}	

The dimensionless susceptibility M/H is a factor 4π larger in SI than in cgs.

Dimensions

In the SI system, the basic quantities are mass (m), length (l), time (t), charge (q) and temperature (θ). Any other quantity has dimensions which are a combination of the dimensions of these five basic quantities, m, l, t, q and θ. In any relation between a combination of physical properties, all the dimensions must balance.

Mechanical

Quantity	symbol	unit	m	l	t	q	θ
area	A	m^2	0	2	0	0	0
volume	V	m^3	0	3	0	0	0
velocity	v	$m.s^{-1}$	0	1	-1	0	0
acceleration	a	$m.s^{-2}$	0	1	-2	0	0
density	ρ	$kg.m^{-3}$	1	-3	0	0	0
energy	E	J	1	2	-2	0	0
momentum	p	$kg.m.s^{-1}$	1	1	-1	0	0
angular momentum	L	$kg.m^2.s^{-1}$	1	2	-1	0	0
moment of inertia	I	$kg.m^2$	1	2	0	0	0
force	F	N	1	1	-2	0	0
power	p	W	1	2	-3	0	0
pressure	P	Pa	1	-1	-2	0	0
stress	S	$N.m^{-2}$	1	-1	-2	0	0
elastic modulus	K	$N.m^{-2}$	1	-1	-2	0	0
frequency	ν	s^{-1}	0	0	-1	0	0
diffusion coefficient	D	$m^2.s^{-1}$	0	2	-1	0	0
viscosity (dynamic)	η	$N.s.m^{-2}$	1	-1	-1	0	0
viscosity (kinematic)	ν	$m^2.s^{-1}$	0	2	-1	0	0
Planck's constant	h	J.s	1	2	-1	0	0

Thermal

Quantity	symbol	unit	m	l	t	q	θ
enthalpy	H	J	1	2	-2	0	0
entropy	S	$J.K^{-1}$	1	2	-2	0	-1
specific heat	C	$J.K^{-1}.kg^{-1}$	0	2	-2	0	-1
heat capacity	c	$J.K^{-1}$	1	2	-2	0	-1
thermal conductivity	κ	$W.m^{-1}.K^{-1}$	1	1	-3	0	-1
Sommerfeld coefficient	γ	$J.mol^{-1}.K^{-1}$	1	2	-2	0	-1
Boltzmann's constant	k	$J.K^{-1}$	1	2	-2	0	-1

Electrical

Quantity	symbol	unit	m	l	t	q	θ
current	I	A	0	0	-1	1	0
current density	j	$A.m^{-2}$	0	-2	-1	1	0
potential	V	V	1	2	-2	-1	0
electromotive force	ε	V	1	2	-2	-1	0
capacitance	C	F	-1	-2	2	2	0
resistance	R	Ω	1	2	-1	-2	0
resistivity	ρ	Ω.m	1	3	-1	-2	0
conductivity	σ	$S.m^{-1}$	-1	-3	1	2	0
dipole moment	p	C.m	0	1	0	1	0
electric polarization	P	$C.m^{-2}$	0	-2	0	1	0
electric field	E	$V.m^{-1}$	1	1	-2	-1	0
electric displacement	D	$C.m^{-2}$	0	-2	0	1	0
electric flux	Ψ	C	0	0	0	1	0
permitivity	ε	$F.m^{-1}$	-1	-3	2	2	0
thermopower	S	$V.K^{-1}$	1	2	-2	-1	-1
mobility	μ	$m^2V^{-1}s^{-1}$	-1	0	1	1	0

Magnetic

Quantity	symbol	unit	m	l	t	q	θ
magnetic moment	m	$A.m^2$	0	2	-1	1	0
magnetisation	M	$A.m^{-1}$	0	-1	-1	1	0
specific moment	σ	$A.m^2.kg^{-1}$	-1	2	-1	1	0
magnetic field strength	H	$A.m^{-1}$	0	-1	-1	1	0
magnetic flux	Φ	Wb	1	2	-1	-1	0
magnetic flux density	B	T	1	0	-1	-1	0
inductance	L	H	1	2	0	-2	0
susceptibility (M/H)	χ	-	0	0	0	0	0
permeability (B/H)	μ	$H.m^{-1}$	1	1	0	-2	0
magnetic polarisation	J	T	1	0	-1	-1	0
magnetomotive force	F	A	0	0	-1	1	0
magnetic ‘charge’	q_m	A.m	0	1	-1	1	0
energy product	(BH)	$J.m^{-3}$	1	-1	-2	0	0
anisotropy energy	K	$J.m^{-3}$	1	1	-2	0	0
exchange coefficient	A	$J.m^{-1}$	1	1	-2	0	0
Hall coefficient	R_H	$m^3.C^{-1}$	0	3	0	-1	0

Examples:

1) Kinetic energy of a body; $E = (1/2)mv^2$

[E] = [1, 2,-2, 0, 0]

[m] = [1, 0, 0, 0, 0]

[v^2] = 2[0, 1,-1, 0, 0]

[1, 2,-2, 0, 0]

2) Lorentz force on a moving charge; $\mathbf{F} = q\mathbf{v}\mathbf{x}\mathbf{B}$

[F] = [1, 1,-2, 0, 0]

[q] = [0, 0, 0, 1, 0]

[v] = [0, 1,-1, 0, 0]

[B] = [1, 0,-1,-1, 0]

[1, 1,-2, 0, 0]

3) Domain wall energy $\gamma_w = \sqrt{AK}$ (γ_w is an energy per unit area)

$[\gamma_w] = [EA^{-1}]$ $\qquad$ $[\sqrt{AK}] = 1/2[\ AK]$

$= [\ 1, 2,-2, 0, 0]$ $\quad$ $[\sqrt{A}]=1/2[\ 1, 1,-2, 0, 0]$

$-[\ 1, 1,-2, 0, 0]$ $\quad$ $[\sqrt{K}]=1/2[\ 1,-1,-2, 0, 0]$

$= [\ 1, 0,-2, 0, 0]$ $\qquad$ $[\ 1, 0,-2, 0, 0]$

4) Magnetohydrodynamic force on a moving conductor $f = \sigma\mathbf{v}\times\mathbf{B}\times\mathbf{B}$ (f is a force per unit volume)

$[f] = [\ FV^{-1}]$ $\qquad$ $[\sigma] = \quad [-1,-3, 1, 2, 0]$

$= [\ 1, 1,-2, 0, 0]$ $\qquad$ $[v] = \quad [\ 0, 1,-1, 0, 0]$

$-[\ 0, 3, 0, 0, 0]$ $\qquad$ $[B^2] = 2[\ 1, 0,-1,-1, 0]$

$[\ 1,-2,-2, 0, 0]$ $\qquad$ $[\ 1,-2,-2, 0, 0]$

5) Flux density in a solid $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$. (Note that quantities added or subtracted in a bracket must have the same dimensions)

$[B]= [\ 1, 0,-1,-1, 0]$ $\qquad$ $[\mu_0] = \quad [\ 1, 1, 0,-2, 0]$

$[M],[H] = \quad [\ 0,-1,-1, 1, 0]$

$[\ 1, 0,-1,-1, 0]$

6) Maxwell's equation $\nabla\times\mathbf{H} = \mathbf{j} + d\mathbf{D}/dt$.

$[\nabla\times\mathbf{H}] = [Hr^{-1}]$ $\qquad$ $[j] = [\ 0,-2,-1, 1, 0]$ $\qquad$ $[d\mathbf{D}/dt] = [Dt^{-1}]$

$= [\ 0,-1,-1, 1, 0]$ $\qquad$ $= [\ 0,-2, 0, 1, 0]$

$-[\ 0, 1, 0, 0, 0]$ $\qquad$ $-[\ 0, 0, 1, 0, 0]$

$= [\ 0,-2,-1, 1, 0]$ $\qquad$ $= [\ 0,-2,-1, 1, 0]$

Basic Concepts in Magnetism

J. M. D. Coey

School of Physics and CRANN, Trinity College Dublin

Ireland.

1. Magnetostatics
2. Magnetism of multi-electron atoms
3. Crystal field
4. Magnetism of the free electron gas
5. Dilute magnetic oxides

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SPIN ELECTRONICS

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1. Electrons in Solids — Intrinsic Properties
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