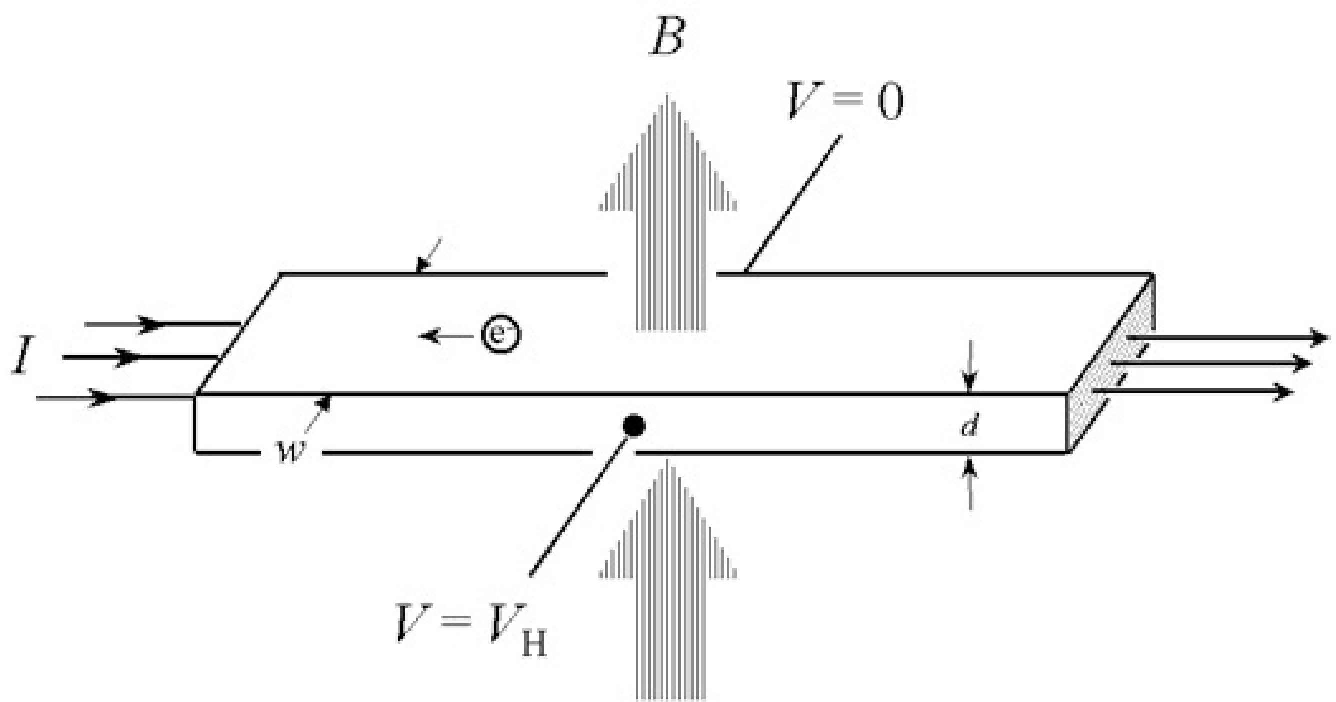
Hall effect and Giant Hall effects

Michel Viret
Service de Physique de l'Etat Condensé
CEA Saclay
France

Normal Hall effect

Geometry of measurements:
E. Hall, 1879



Simple theory

Equation of motion: $\vec{F} = m^* \left(\frac{d}{dt} + \frac{1}{\tau} \right) \vec{v} = q \left(\vec{E} + \vec{v} \times \vec{B} \right)$

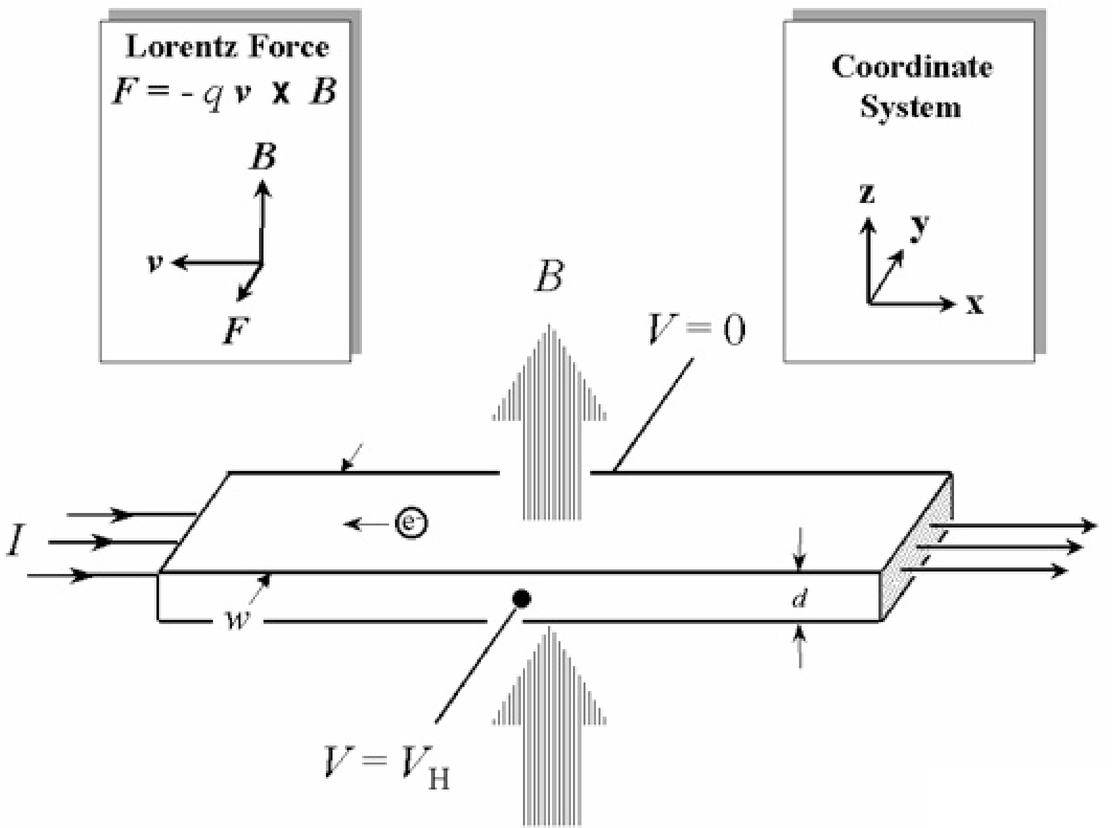
Static case: $d/dt = 0$

$$\vec{B} = B\hat{z} = (0, 0, B)$$

$$\frac{m^*}{\tau} v_x = q\left(E_x + v_y B\right)$$

$$\frac{m^*}{\tau} v_y = q\left(E_y - v_x B\right)$$

$$\frac{m^*}{\tau} v_z = qE_z$$



Special case: $v_y = 0$

$$v_x = \frac{q\tau}{m^*}E_x ; j_x = \frac{nq^2\tau}{m^*}E_x$$

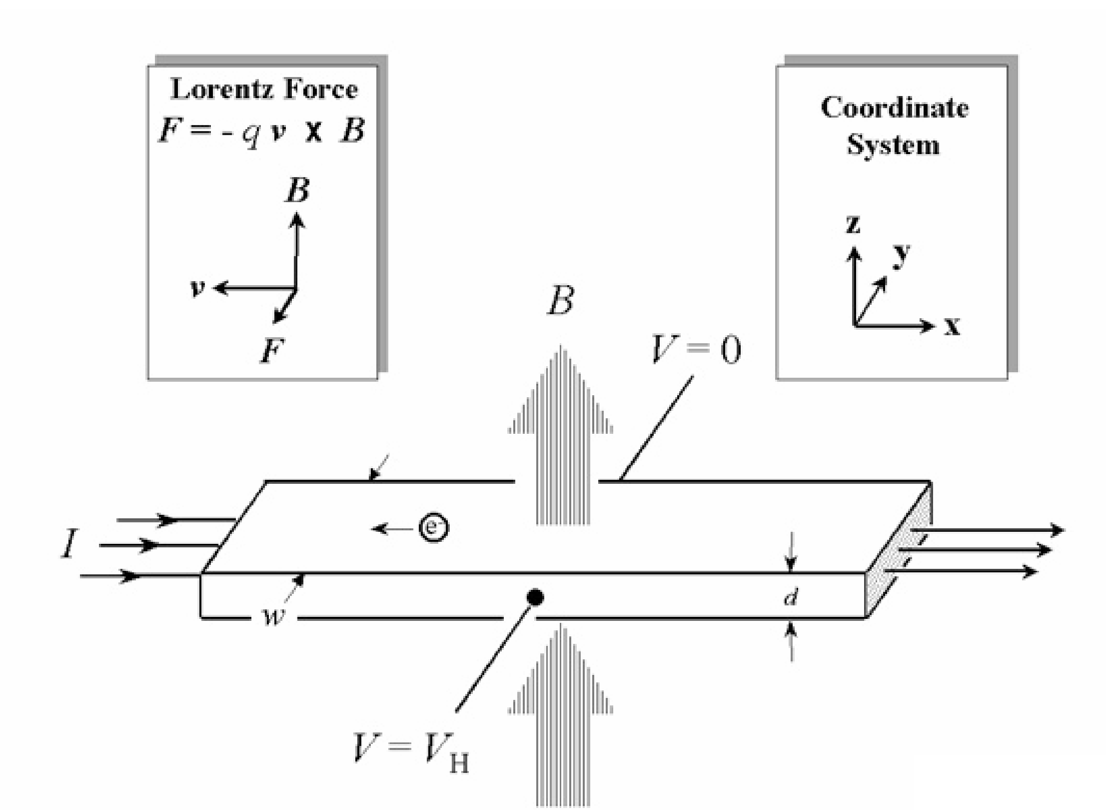
$$E_y = v_x B = \frac{q\tau B}{m^*}E_x$$

Hall Coefficient R_H:

$$R_H \equiv \frac{E_y}{\left|\vec{j} \times \vec{B}\right|} = \frac{E_y}{j_x B} = \frac{1}{nq}$$

$R_H < 0$ for electrons

$R_H > 0$ for holes



→ Routinely used to measure carrier type and concentration in conductors

→ This derivation is for simple one-band model; more complex if several bands involved

→ R_H large if n small

In semiconductors:

Related concept is mobility μ of carriers:

$$\sigma = \frac{ne^2\tau}{m^*} \equiv ne\mu \qquad \mu = \frac{e\tau}{m^*} = \sigma R_H$$

μ usually measured in cm^2/Vs, more easily understood as [cm/s]/[V/cm] or velocity per field.

$\mu_{GaAs} \approx 8000\ cm^2/Vs$, $\mu_{Si} \approx 100\ cm^2/Vs$, $\mu_{p\text{-}TCO} \approx 1\ cm^2/Vs$

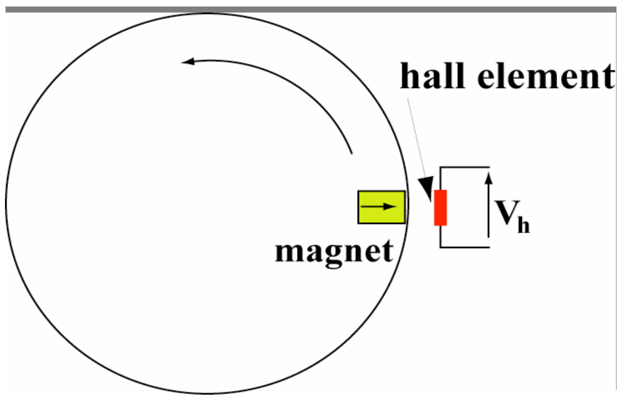
Applications: Hall sensors

Hall coefficient is rather small - of the order of 50 mV/T
Measurement of the earth's magnetic field (about 50 μT): output 2.5 μV
→ Must in almost all cases be amplified.

Advantages:

Hall voltages are easily measurable quantities.
Hall sensors are simple, linear, very inexpensive, available in arrays, can be integrated within devices.
Errors involved in measurement are mostly due to temperature and variations and the averaging effect of the Hall plate size.
A typical sensor will be a rectangular wafer of small thickness made of p or n doped semiconductor (InAs and InSb most commonly used).

Operating: current usually kept constant → output voltage proportional to the field. Very common in sensing rotation which may be used to measure position, frequency of rotation (rpm), differential position, etc…



A closer look at the Lorentz force

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right)$$

Cyclotron motion:

Free particle moves on a circular orbit of radius: $r = mv/qB$

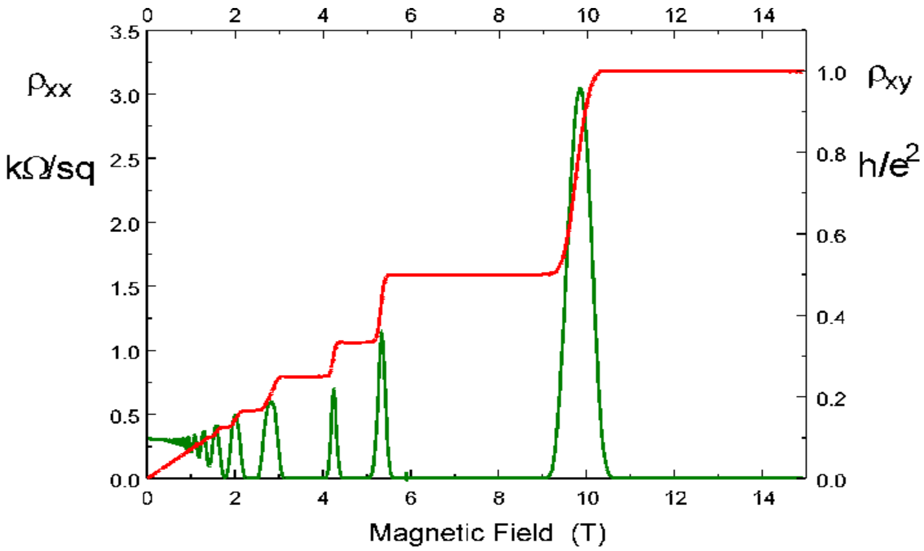
Frequency: $\omega = qB/m$

Orbit energy: $K=q^2B^2r^2/2m$

→ In solids with very large mean free paths, one could expect a significant field effect

→ 2D electron gases!

Quantum Hall effect



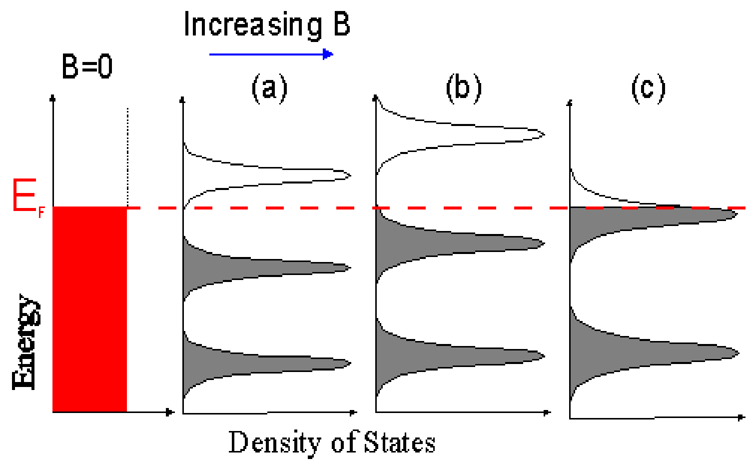
Discovered by von Klitzing in 1980 (Nobel prize 1985). Totally unexpected and initially unexplained.

Electrons confined in a thin layer at low-temperature in a high magnetic field.

Hall resistance vs. B rises in a series of quantised steps at levels given by $R=h/ie^2$ where i is an integer.

Partial explanation:

The magnetic field splits the states in a 2D electron gas into "Landau levels". The number of current carrying states in each level is eB/h. The position of the Fermi level relative to the Landau levels changes with B. So the number of charge carriers is equal to the number of filled Landau levels, i, times $eB/h \Rightarrow R=h/ie^2$.



Interesting features:

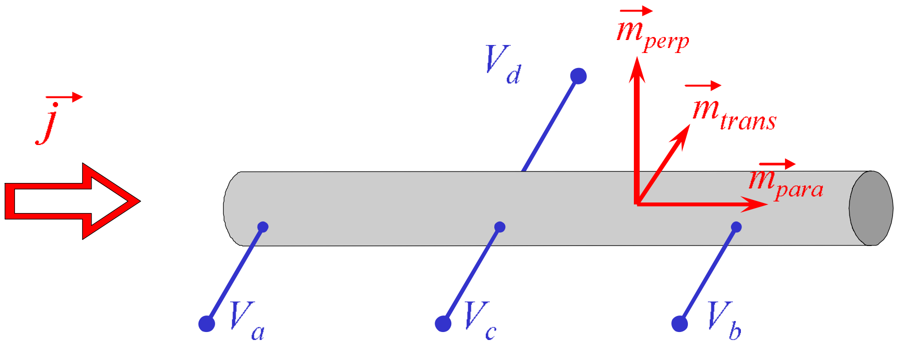
•The resistance can be precisely measured (1 in 10^8).

•It is simply related to fundamental constants. $R=h/ie^2$, $\alpha=e^2/2\varepsilon_0 hc$ so $R=1/2i\varepsilon_0 c\alpha$. Both ε_0 and c are constants without errors: $\varepsilon_0= 1/\mu_0 c^2$, $\mu_0=4\pi .\ 10^{-7}$ (NA^{-2}) and $c=299{,}792{,}458$ (ms^{-1}).

•The measurement is done at very low energy so higher order corrections are negligible.

Anomalous Hall effect in ferromagnets

Geometry :



Hall effect in Fe whiskers:
P.N. Dheer, Phys Rev (1967)

$$\rho_{xy} = R_0 . B_{perp} + R_s . \mu_0 M_{perp}$$

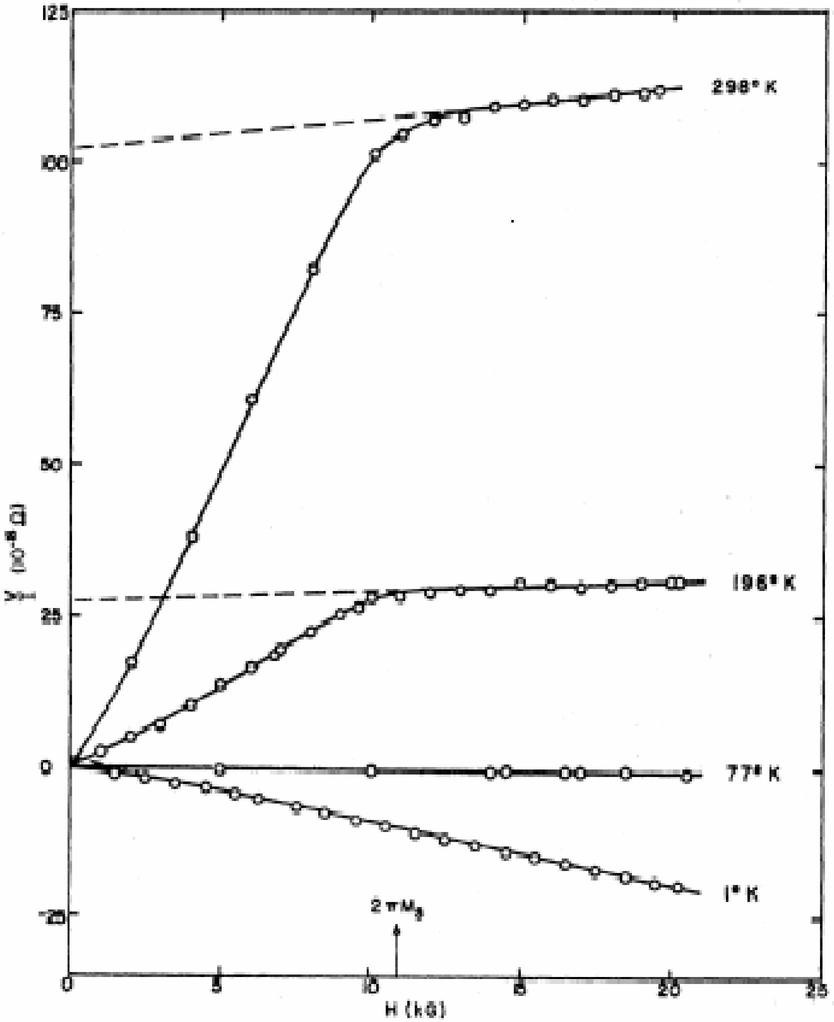
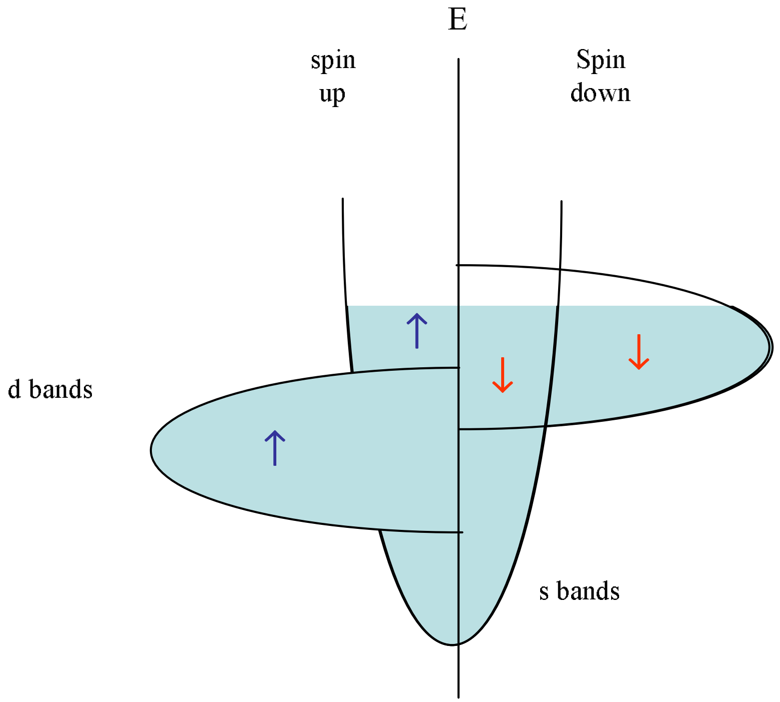


FIG. 1. Variation of the Hall resistance with applied magnetic field for whisker Fe

Spin dependent transport in ferromagnetic metals

Different DOS for up and down spins :



s electrons : low density of states
+ high mobility

d electrons : large density of states
+ low mobility

Transport is dominated by s electrons scattered into d bands
d bands split by the exchange energy
→ diffusion is spin dependent

→ **Two current model :**

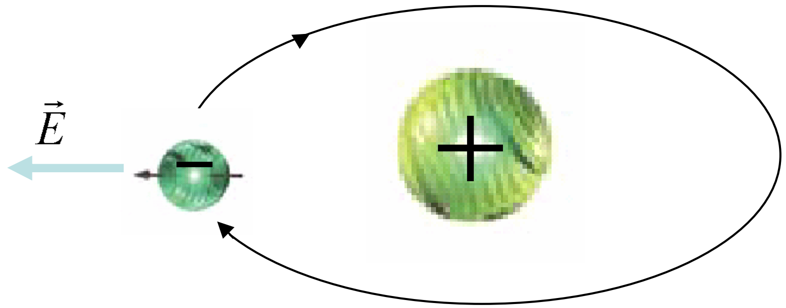
Two conduction channels in parallel with $\rho_\uparrow \neq \rho_\downarrow$

Resistivity : $$\rho = \frac{\rho_\uparrow \rho_\downarrow}{\rho_\uparrow + \rho_\downarrow}$$

or (with spin-flip) :

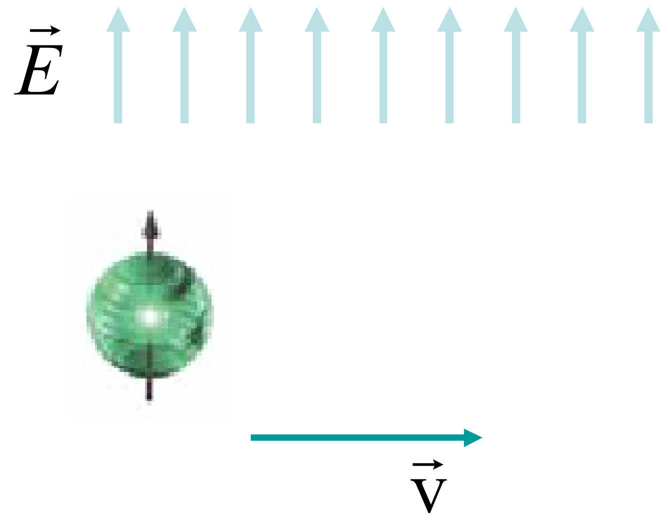
$$\rho = \frac{\rho_\uparrow \rho_\downarrow + \rho_{\uparrow\downarrow}(\rho_\uparrow + \rho_\downarrow)}{\rho_\uparrow + \rho_\downarrow + 4\rho_{\uparrow\downarrow}}$$

Relativistic Spin-Orbit Coupling



- Relativistic effect: a particle in an electric field experiences an internal effective magnetic field in its moving frame

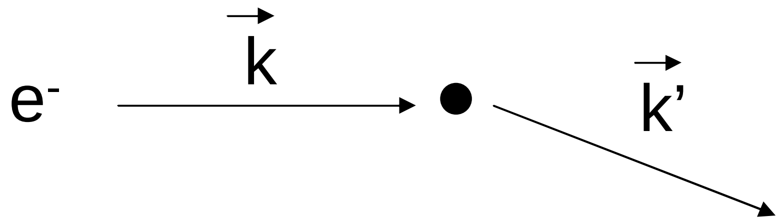
$$\vec{B}_{eff} \sim \vec{v} \times \vec{E}$$



- Spin-Orbit coupling is the coupling of spin with the internal effective magnetic field (Zeeman energy)

$$H \sim -\vec{S} \cdot \vec{B}_{eff}$$

Theories for AHE: Skew scattering



Exchange interaction unable to explain an asymmetry in scattering.
Exists only if $H_{kk'}$ is asymmetrical → terms containing the orbital angular momentum l
Origin of the asymmetry of the interaction between conduction electrons and a localized magnetic moment:
Spin-orbit coupling term associated with any scattering potential ***V(r) :***

$$H_{kk'} = (1/2m^2c^2r) \times (dV/dr) \times \mathbf{l.s}$$

but the calculation shows it is too small
→ asymmetry comes from an interaction with localized electrons possessing an orbital momentum (Kondo 1962):

$$H = -J\mathbf{s.j} + \lambda_l \mathbf{l.j}$$

s, l are the spin and orbital angular momenta of the conduction electrons, j is the total angular momentum of the localised electrons. This gives the asymmetry in scattering.

Ordinary magnetoresistance and Hall effect in Boltzman theory:

$$\sigma_{xx} = (ne^2/m)[\tau^{-1}/(\tau^{-2} + \omega_c^2)]$$
$$\sigma_{xy} = -(ne^2/m)[\omega_c/(\tau^{-2} + \omega_c^2)]$$
$$\omega_c = eH/mc$$

Magnetoresistance and Hall effect in Boltzman theory including asymmetric Hamiltonian (B. Giovannini J. Low-Temp. Phys. 1972)

$$\sigma_{xx} = \Sigma\sigma_{xx}^{\sigma}$$
$$\sigma_{xy} = \Sigma\sigma_{xy}^{\sigma}$$
$$\sigma_{xx}^{\sigma} = (ne^2/2m)[\tau_\sigma^{-1}/(\tau_\sigma^{-2} + \omega_\sigma^2)]$$
$$\sigma_{xy}^{\sigma} = -(ne^2/2m)[\omega_\sigma/(\tau_\sigma^{-2} + \omega_\sigma^2)]$$

With: $$\tau_\pm^{-1} = \tau_0^{-1} + (3\pi c_i/\varepsilon_f)\left[V^2 + \frac{2J_{ex}^2 J(J+1)}{1+e^{\pm\beta\omega_i}} \mp \langle J_z\rangle_0\left(2J_{ex}V + \frac{2J_{ex}^2}{1+e^{\pm\beta\omega_i}}\right) + \langle J_z^2\rangle_0\left(J_{ex}^2 + \frac{2}{3}\lambda^2 - \frac{2J_{ex}^2}{1+e^{\pm\beta\omega_i}}\right)\right]$$

And: $$\omega_\sigma = \omega_c + [(2\pi)^2/3]c_i(\lambda/\varepsilon_f^2(V^2\langle J_z\rangle_0 + J_{ex}^2\langle J_z^3\rangle_0 \mp 2VJ_{ex}\langle J_z^2\rangle_0)$$

→ the skew scattering term is equivalent to an effective magnetic field acting on the orbit of the conduction electrons.

Side jump mechanism

Berger (1972): Same Hamiltonian as before for the scattering of a free electron plane wave by a square potential:

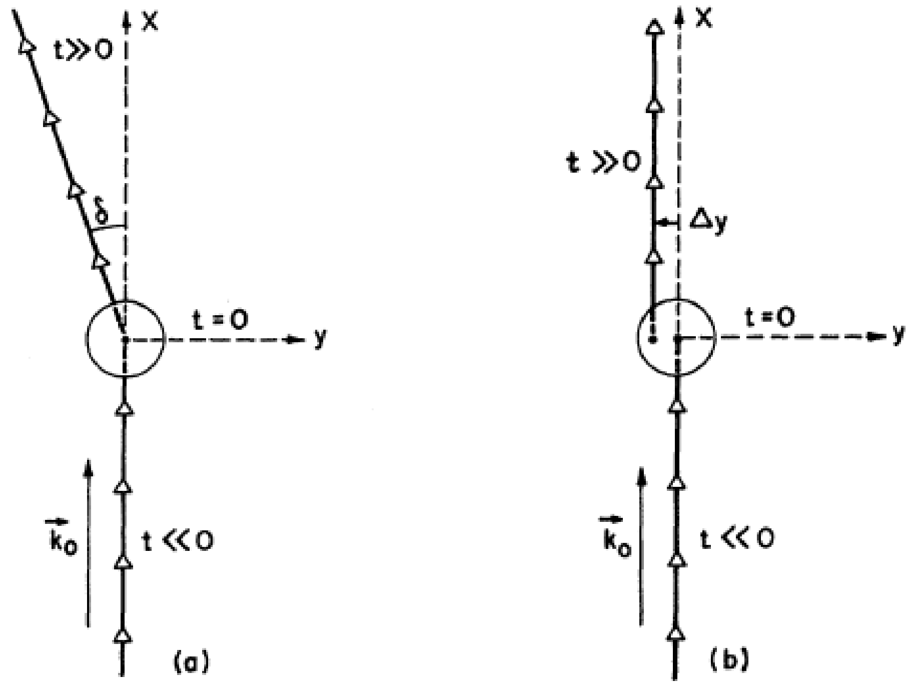
$$H_v = -(\hbar^2/2m)\nabla^2 + V(r),$$

$$H_{so} = (1/2m^2c^2)(r^{-1}\partial V/\partial r)S_z L_z,$$

$$V(r) = 0, \qquad r > R$$

$$V(r) = V_0, \qquad r < R.$$

Solving the equation of motion using this Hamiltonian results in a non-zero average angle (k,k') = skew scattering, but also to a different origin for the wave velocity = side jump



Berger (72): $\Delta y = 1/6 k_0 \lambda_c^2$

$\lambda_c = \hbar/mc$ = Compton wavelength

k_0 = incident electron wavevector

For free electrons with $k_0 = 10^{10}\ m^{-1}$, $\Delta y \approx 3.10^{-16}$ m (= small)

But, for band electrons, spin-orbit potential is added:

$$H_{so} = -(1/2m^2c^2)\mathbf{S}\cdot(\nabla U(\mathbf{x})\times\mathbf{p})$$

→ Enhancement of the side jump by a factor proportional to the spin-orbit coupling constant

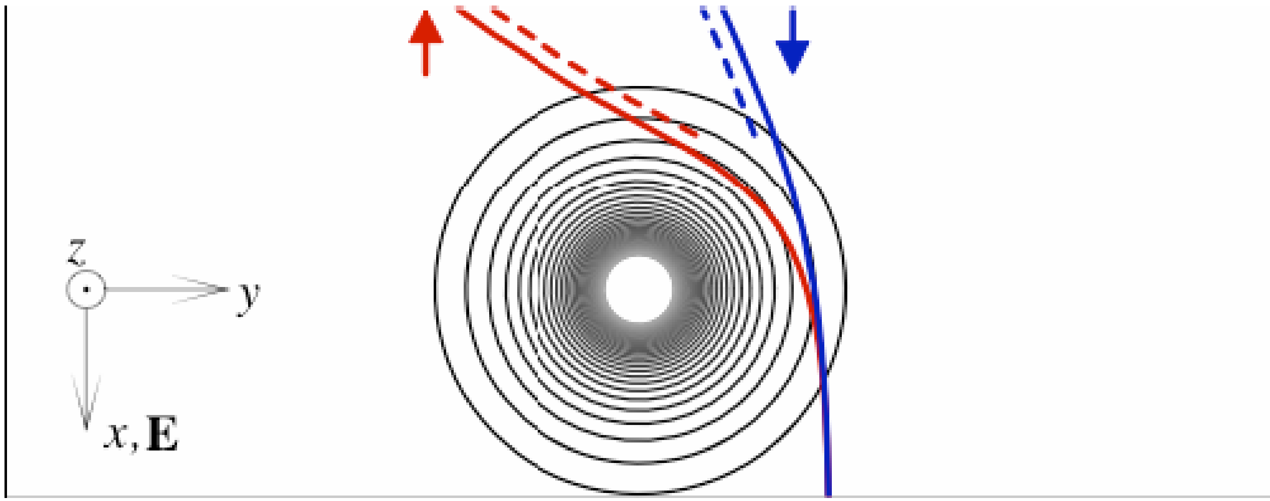
→ $\Delta y \approx 10^{-11}$ m

Nozieres-Lewiner (J. Phys. **34**, 901 (1973)) in semiconductors : Anomalous Hall current $\mathbf{J}_H$ dissipationless, indept of τ

$$\mathbf{J}_H = 2ne^2\ \lambda_{SO}\ \mathbf{E}\times\langle\mathbf{S}\rangle$$

Skew-scattering and side-jump Contributions

Modelled for 3D n-type GaAs, with ionized donors represented by attractive screened Coulomb potentials.
First order in spin-orbit coupling λ; assume Boltzmann equation



Side jump contribution to σ_H is of order $e\lambda k_F^2$, independent of τ
Skew scattering contribution is of order $egE_F\tau$, where $g=\lambda k_F^2(V\max/E_F)$
Dependence with resistivity :
Skew scattering: Hall angle constant $\Rightarrow \rho_{ss} \propto \rho_{xx}$
Side jump: Hall angle varies like $1/\tau \propto \rho \Rightarrow \rho_{sd} \propto \rho_{xx}^2$

Karplus Luttinger theory of AHE

Contribution due to the change in wave packet group velocity upon application of an electric field in a ferromagnet (Karplus, Luttinger, 1958). Not related to scattering! Topological in nature (Berry phase).

Boltzmann equation: $\mathbf{J} = 2e\sum_{\mathbf{k}} \mathbf{v}_{\mathbf{k}} \left[f_{\mathbf{k}}^0 + g_{\mathbf{k}} \right]$, $g_{\mathbf{k}} = \left(-\frac{\partial f_{\mathbf{k}}^0}{\partial \varepsilon} \right) e\,\mathbf{E}\cdot\mathbf{v}\,\tau_{\mathbf{k}}$

Anomalous velocity : $\mathbf{v}_{\mathbf{k}} = \nabla \varepsilon_{\mathbf{k}} - e\,\mathbf{E}\times\mathbf{\Omega}_{\mathbf{k}}$ (**B** = 0)

Correction: k-space Berry curvature

$\rightarrow$ Equilibrium Fermi-Dirac $f_{\mathbf{k}}^0$ distribution contributes!

Anomalous Hall current : $\mathbf{J}_{\mathrm{H}} = 2e^2\,\mathbf{E}\times\sum_{\mathbf{k}} f_{\mathbf{k}}^0\,\mathbf{\Omega}_{\mathbf{k}}$ $\rightarrow$ $\sigma_{xy}' = n\frac{e^2}{\hbar}\langle \Omega \rangle$

Independent of lifetime $\tau \Rightarrow \rho_{xy} \sim \rho^2$
+ requires sum over all **k** in Fermi sea.
Berry curvature $\mathbf{\Omega}_{\mathbf{k}}$ vanishes if time-reversal symmetry valid
$\Rightarrow$ Importance of spin-orbit coupling

Evaluation of the Berry phase contribution

Electrons hopping between atoms in a magnetic field **B** → complex factor in the quantum mechanical amplitude of the wave function with phase given by the vector potential **A** corresponding to **B** (=∇x**A**).
In magnets: analogous complex factor when electrons hop along non-coplanar spin configurations. The effective magnetic field is represented by the spin chirality, i.e. the solid angle subtended by the spins.

Calculation:

$$\mathbf{\Omega}_n(\mathbf{k}) = -\mathrm{Im}\langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} | \times | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle, \qquad \mathbf{M//z:} \quad \Omega^z(\mathbf{k}) = \sum_n f_n \Omega_n^z(\mathbf{k}).$$

u_{nk} = periodic part of the Bloch wave in the n^{th} band

Yugui Yao et al., PRL92, 037204 (2004): *ab initio* electronic structure calculation to evaluate **Ω** :
Large contribution only when the Fermi surface lies in a spin-orbit induced gap.

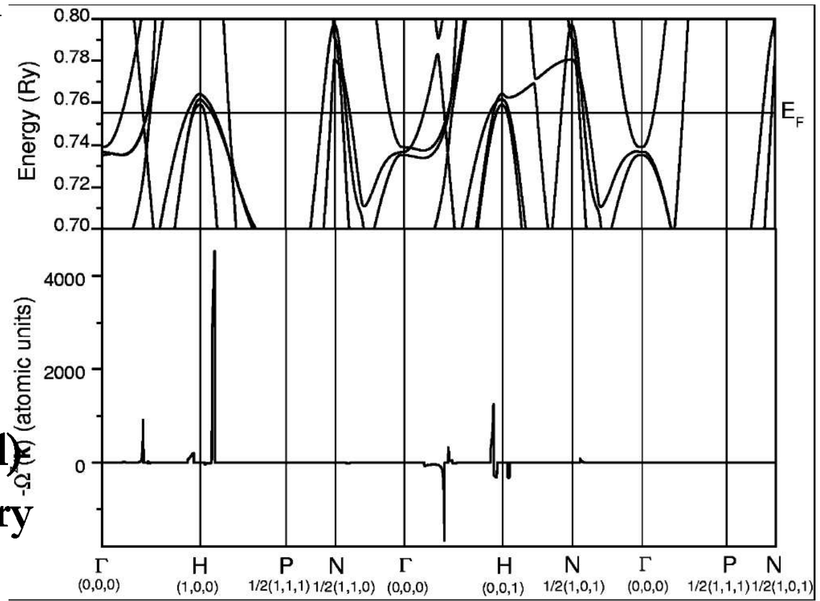
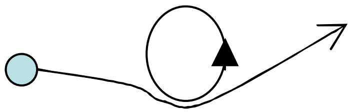
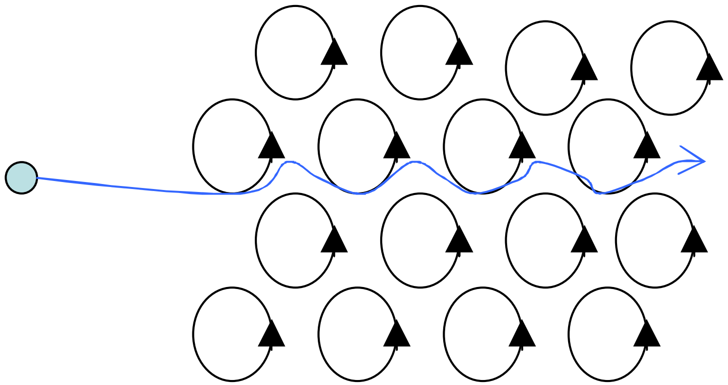


Figure: Band structure near Fermi energy (upper panel) and Berry curvature Ω_z(k) (lower panel) along symmetry lines. Total result consistent with measurements in Fe.

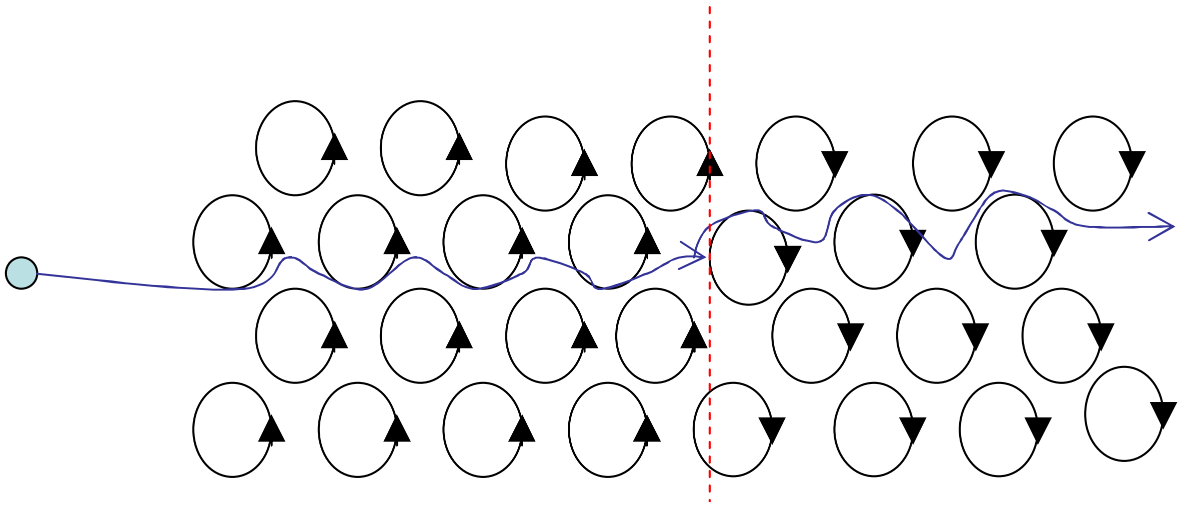
My vision of topology: hand waving considerations



Single scatterer



lattice



Two lattices with different chirality

A unified theory?

Goal: have all the sources of AHE in the same Hamiltonian to check the relevance of the different contributions (S. Onoda, N. Sugimoto and N. Nagaosa, PRL **97,** 126602 (2006)).

Notice: Topological effect important near p_0 vectors located at an anticrossing of the band structure = points with a small gap $2\Delta_0$, which is identified with the spin-orbit interaction energy ε_{SO}.

At p_0, σ_{xy} is resonantly enhanced and approaches $e^2/ha \approx 10^3\ \Omega^{-1}\ \mathrm{cm}^{-1}$ in three dimensions (a = lattice constant ≈ 4 Å).

Hamiltonian written for p_z vectors near the anticrossing p_0 :

σ= Pauli matrices

e^z =unit vector along z

$$\hat{H}_0 + \hat{H}_{imp} = -\Delta_0 \hat{\sigma}^z + \lambda \boldsymbol{p} \cdot \hat{\boldsymbol{\sigma}} \times \boldsymbol{e}^z + \frac{\boldsymbol{p}^2}{2m} + v_{imp} \sum_{\boldsymbol{r}_{imp}} \delta(\boldsymbol{r} - \boldsymbol{r}_{imp}),$$

Level splitting by S.O.

Impurity potential scattering

linear dispersion with velocity λ

Quadratic dispersion with no anisotropy

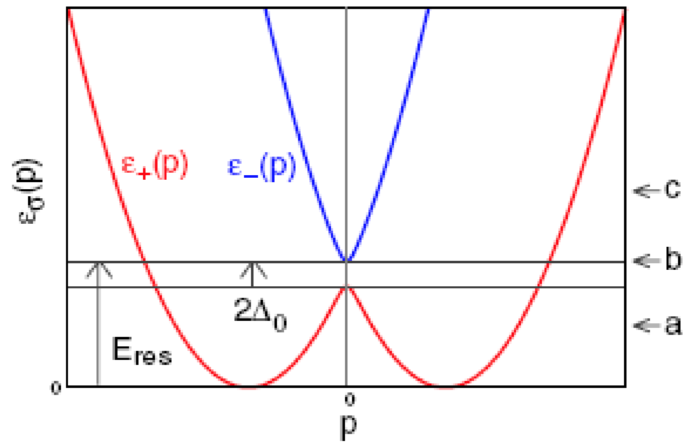


Figure: 2-band dispersions *corresponding to the Hamiltonian*

Result for the AHE:

Clean limit: skew scattering diverges:

$$\sigma_{xy}^{\text{skew}} = \sigma_{xx} S = \frac{2e^2}{ha} \frac{E_F \tau}{\hbar} S$$

$S = \sigma_{SO} \nu_{imp} / W^2$, W=bandwidth

Going away from the clean limit: Intrinsic contribution dominates. Crossover occurs at $\hbar/\tau = \varepsilon_{SO}$. For a small ratio of $\varepsilon_{SO}/E_F \approx 10^3$–$10^2$ the intrinsic AHE dominates in the usual clean metal.

Remarks: 1) effects based on scattering + SO are based on intraband matrix elements of conductivity tensor. Interband terms contain the intrinsic contribution as a part of the Berry-curvature term. 2) Side jump contribution has the same dependence as the intrinsic (σ_{xy}=cte) but its magnitude is small: $e^2/\hbar . \varepsilon_{SO}/E_F$ vs around $e^2/\hbar$. 3) In the hopping regime $\sigma_{xy} \propto \sigma_{xx}^{1.6}$.

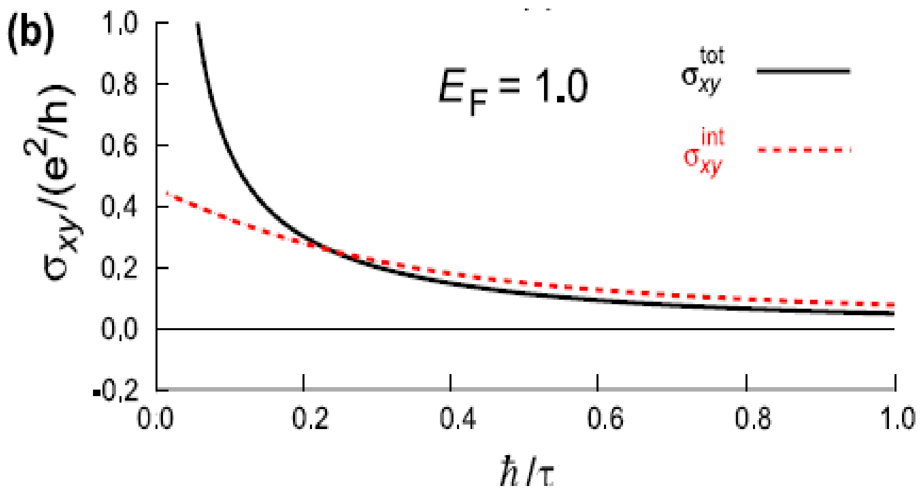


Fig.: σ^{tot}_{xy} and σ^{int}_{xy} as a function of $\hbar/\tau$ for E_F close to resonance.

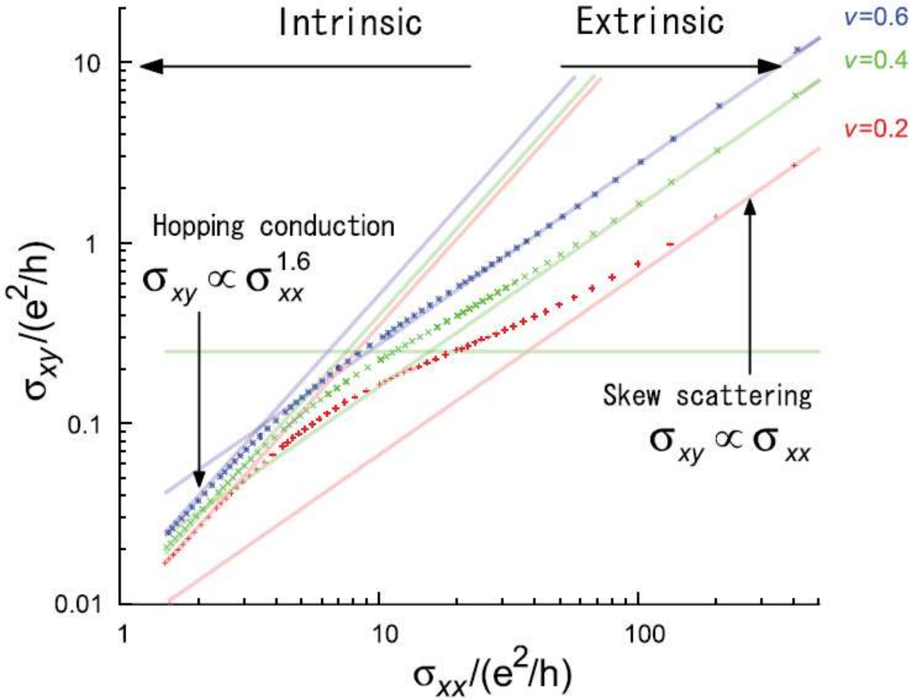
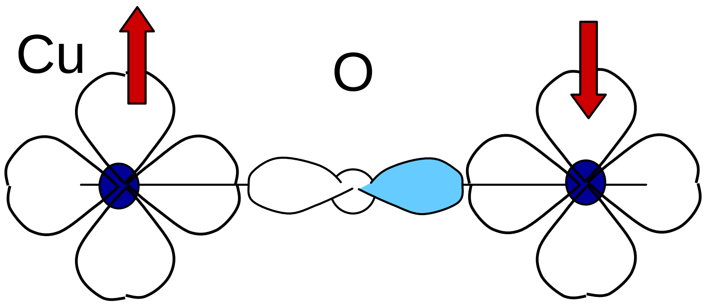
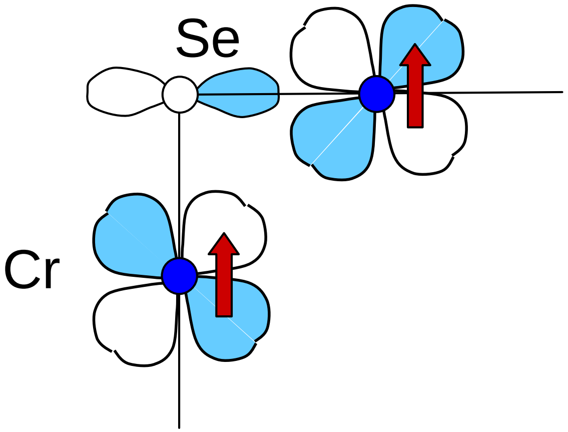


Figure: Scaling plot of σ_{xy} versus σ_{xx}

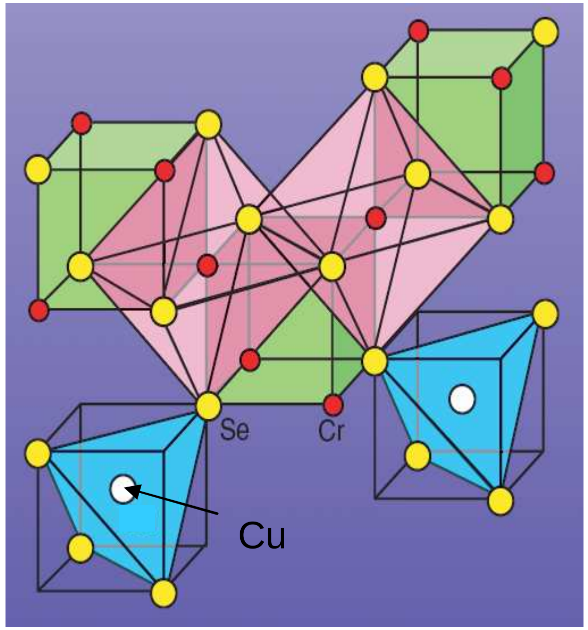
Supports KL theory: Ferromagnetic Spinel $CuCr_2Se_4$



180º bonds: AF
(superexch dominant)



90º bonds: ferromag.
(direct exch domin.)



Goodenough-Kanamori rules

Little effect of Br doping on magnetization: *380K>Tc>250K for x=0 to x=1*

At 5 K, M_{sat} ~ 2.95 μ_B /Cr for x = 1.0

Large effect on resistivity: At 5 K, ρ increases over 3 orders as x goes from 0 to 1.0.

n_H decreases linearly with x from 6.10^{20} cm^{-3} to 2.10^{20} cm^{-3} for x =1.0.

Anomalous Hall Effect scales with ρ^2 over 5 orders of magnitude:

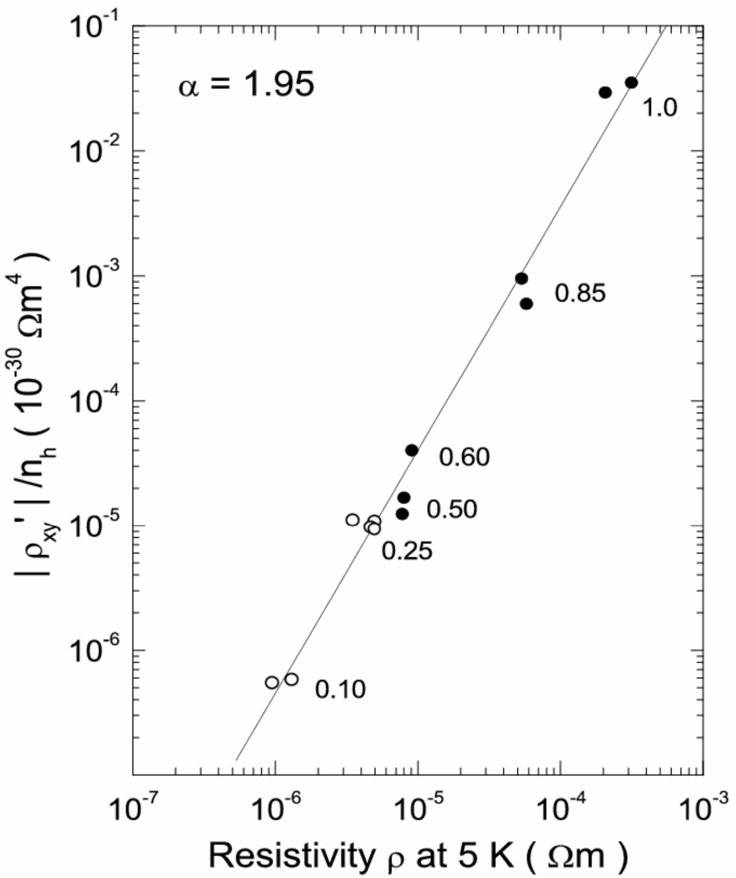
KL: $$\sigma_{xy}' = n\frac{e^2}{\hbar}\langle\Omega\rangle$$

If $\sigma'_{xy} \sim n$, then $\rho'_{xy}/n \sim 1/(n\tau)^2 \sim \rho^2$
Fit to $\rho'_{xy}/n = A\rho^2$

Observed A implies $\langle\Omega\rangle^{1/2} \sim$ 0.3 Angstrom

70-fold decrease in τ, from x=0.1 to x=0.85.
σ_{xy}/n is independent of τ
Doping has *no effect* on anomalous Hall current $\mathbf{J}_H$ per hole

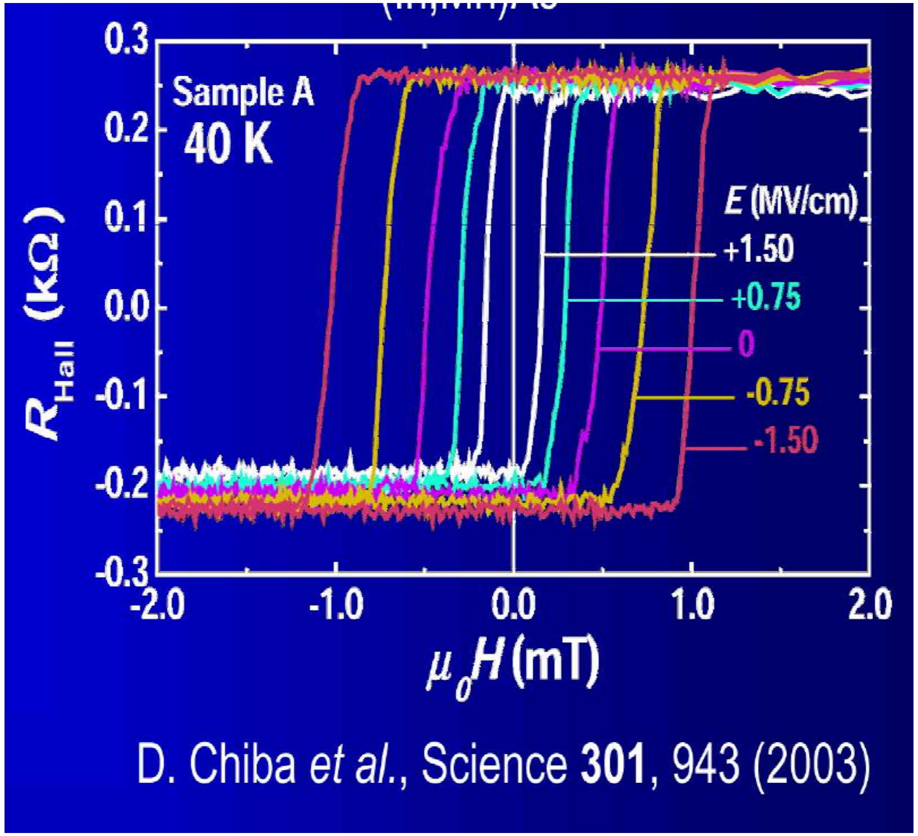
Strongest evidence to date for the anomalous-velocity theory (KL)



Wei Li Lee et al. Science (2004)

AHE in diluted magnetic semiconductors

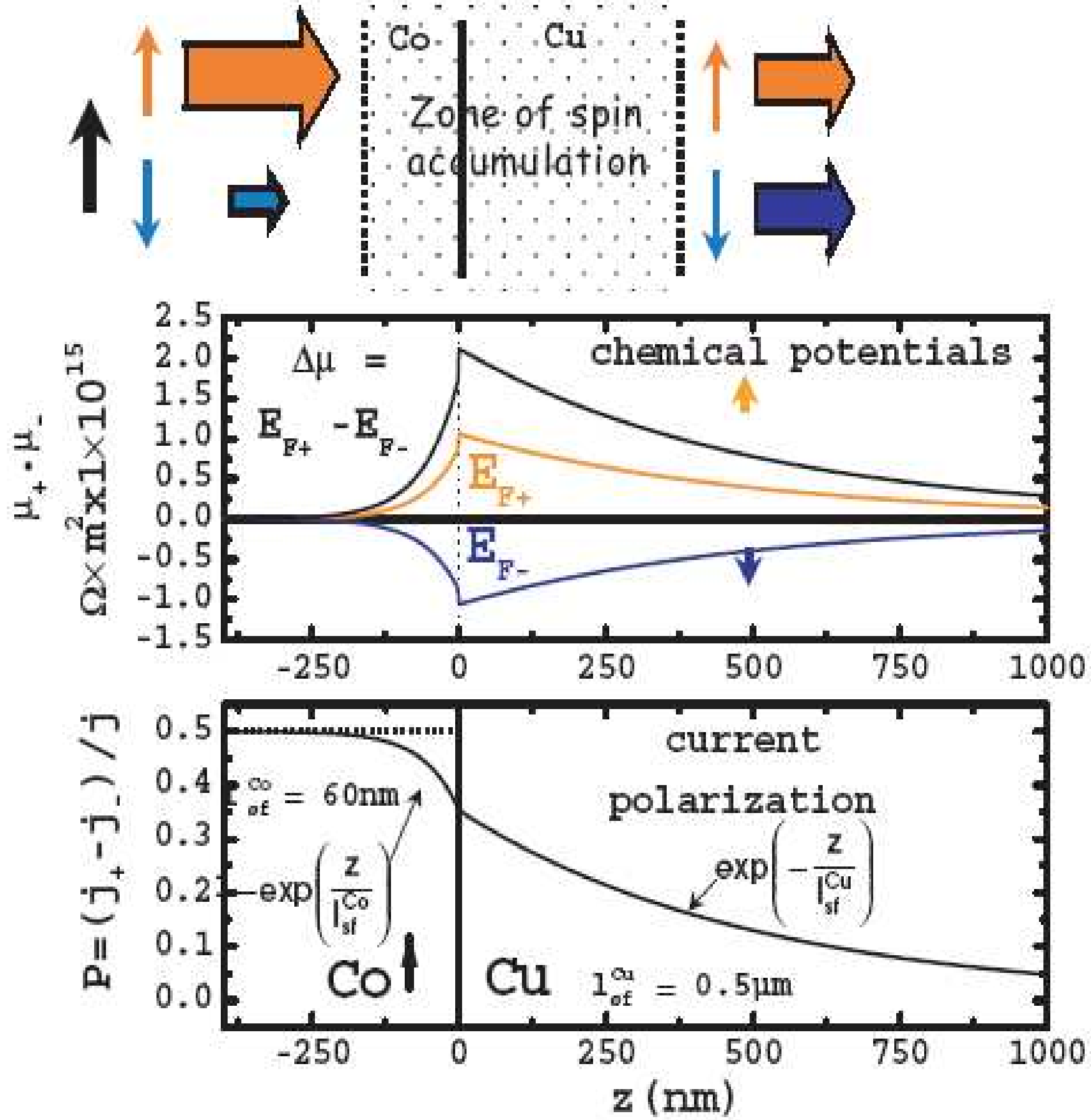
-DMS good because of large L in bands
- Scaling with ρ^2 works!



D. Chiba *et al.*, Science **301**, 943 (2003)

Hall effect in GMR systems

Current generated spin accumulation at a Ferro (Co) / Normal metal (Cu) interface:



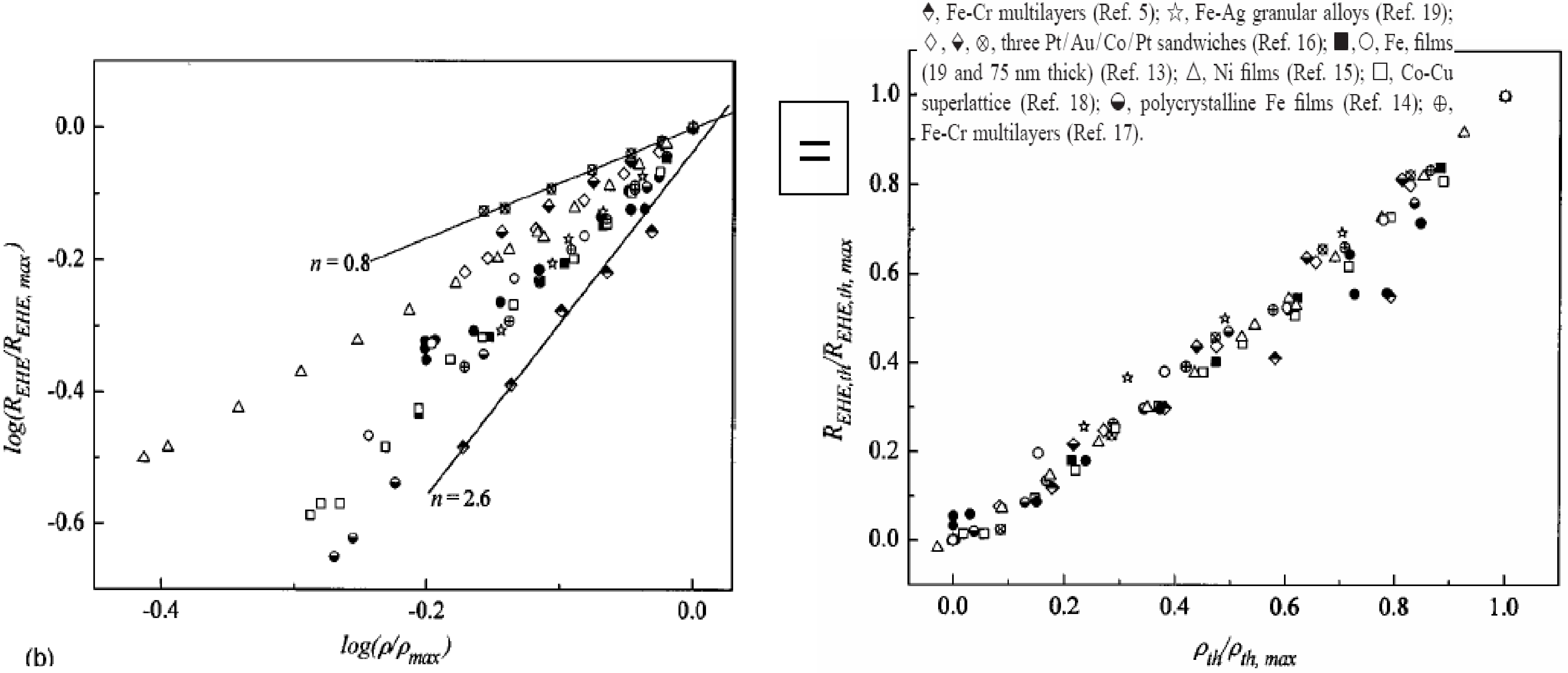
Typically, at 4.2 K :

l_{sf} (Co) ≈ 60 nm

l_{sf} (Cu) ≈ 500 nm

Systems composed of ferro/normal mixtures

Initial reports of a 'giant AHE' with temperature dependence $\rho_{AHE} \propto \rho^n$ with n>2.
But:



Normalised values of the AHE coefficients as a function of normalised resistivity.

a) Total AHE with total resistivity
b) Temperature-dependent components of the AHE coefficients $R_{AHE,th}$ with the temperature-dependent term of ρth.

From A. Gerber et al., PRB**69**, 224403 (2004).

Magnetic clusters in paramagnetic host

Co clusters in Pt

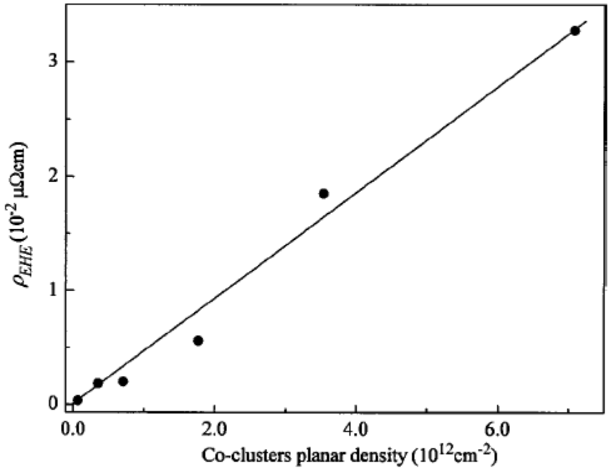


FIG. 2. The saturated EHE resistivity of planar arrays of Co nanoclusters embedded in Pt matrix as a function of Co-clusters planar density. Solid line is the guide for eyes.

→ AHE is proportional to the density of clusters

Data analysis undertaken using the skew scattering theory with the following arguments:

Total resistivity comes from scattering from ‘skew scatterers’ + events that do not break the scattering symmetry (no transverse effect).

$$\rho = \rho_0 + \rho_s$$

The transverse current density $J_\perp$ generated by electrons deflected by skew scattering is proportional to the volume density of skew centers n_s: $J_\perp = \alpha n_s J$

→ $\rho_{AHE} = E_\perp / J = \alpha n_s(\rho_0 + \rho_s)$

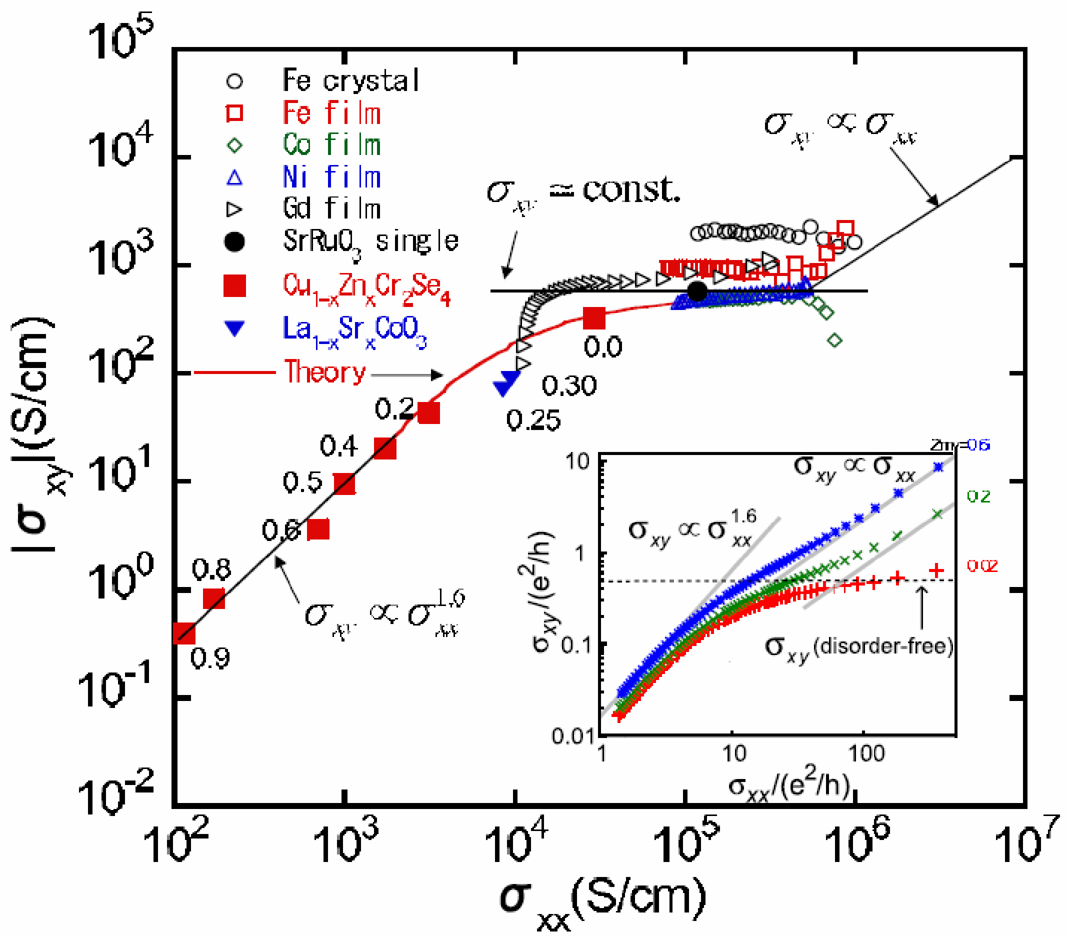
If $n_s \propto \rho_s$ then $\rho_{AHE} = \alpha \rho_s \rho_0 + \beta \rho_s^2$

When T varies, only ρ_0 varies and ρ_{AHE} is proportional to ρ_0. Consistent with data.

Crossover behaviour: support to the unified theory

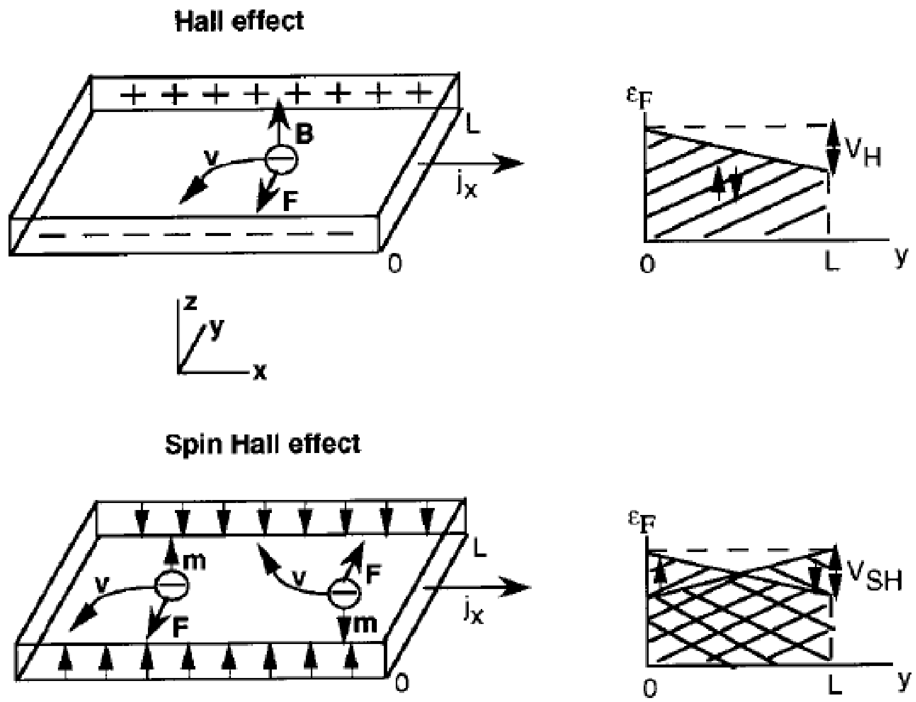
T. Miyasato et al., PRL **99,** 086602 (2007)

<u>Figure:</u> Absolute value of anomalous Hall conductivity σ_{xy} as a function of longitudinal conductivity σ_{xx} in pure metals (Fe, Ni, Co, and Gd), oxides ($SrRuO_3$ and $La_{1-x}Sr_xCoO_3$), and chalcogenide spinels ($Cu_{1-x}Zn_xCr_2Se_4$) at low temperatures. The three lines are $\sigma_{xy} \propto \sigma_{xx}^{1.6}$, σ_{xy} = const, and $\sigma_{xy} \propto \sigma_{xx}$ for the dirty, intermediate, and clean regimes, respectively. The inset shows theoretical results obtained from the same analysis (S. Onoda, N. Sugimoto and N. Nagaosa, PRL **97,** 126602 (2006)).



Spin Hall effect

Nothing very new compared to theories for the AHE, but P=0.



Observation of the Spin Hall Effect in Semiconductors
Y. K. Kato, *et al., Science* **306**, 1910 (2004);

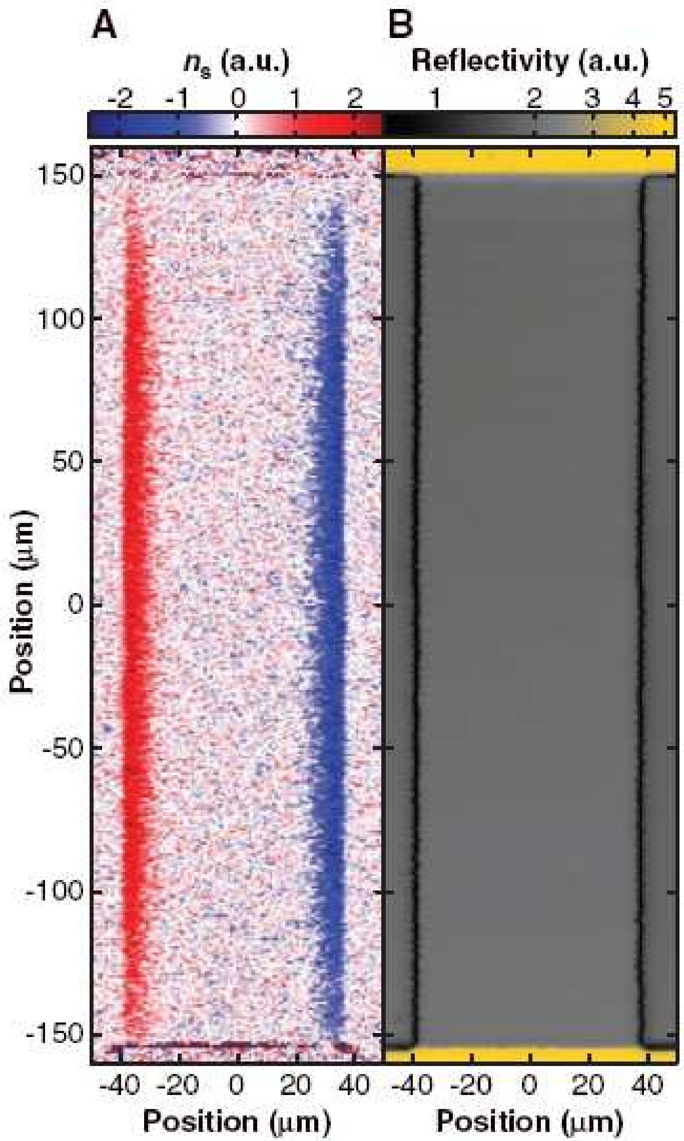


Figure A and B: Two-dimensional images of spin density ns and reflectivity R, respectively, for the unstrained GaAs sample measured at T=30 K and E=10 mV μm^{-1}.

The red curve is taken at position x=–35 μm; the blue curve is taken at x=+35 μm, corresponding to the two edges of the channel. These curves can be understood as the projection of the spin polarization along the z axis, which diminishes with an applied transverse magnetic field because of spin precession;

Conclusions

Anomalous Hall Effect : 50 years of controversy

In the process of being solved…?

Mechanisms :
Role of impurity scattering with S.O. → mainly skew scattering
Topological property of Fermi surface (with S.O.) → 'Intrinsic' effect
Side jump negligible.

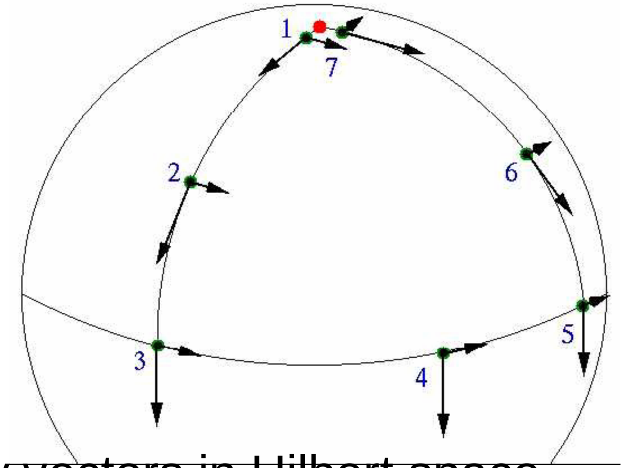
<u>Traditionally:</u> regime of skew scattering + regime of side jump.
<u>Now:</u> skew scattering + topological Berry phase

Spin Hall effect : Generation of non-dissipative spin currents…?
Can it be used in spintronics?

Berry phase

Any vector object which is parallel transported along a closed path may acquire an angle with respect to its initial orientation prior to transport. An intuitive example of such a geometric phase is the parallel transport of a vector along a loop on a sphere.

Figure: the parallel transport of two vectors on a sphere. After a closed loop of transportation from point 1 to point 7, the orientation of the vectors changed due to the geometrical phase they acquired through the transportation.



Since in quantum mechanics, states can be represented by vectors in Hilbert space, there is no reason that they should make exceptions to the general rule of acquiring phase angles after parallel transported along loops. In 1984, M. Berry [7] published a precise formulation of geometrical phase for quantum problems. Berry considered a quantum system whose Hamiltonian is slowly and adiabatically altered by varying a control parameter, as the control parameter loops back to its initial value, the system will return to its initial state except for an additional phase factor. He argued that the additional phase factor contains two parts; one is the trivial dynamical phase factor and the other is the geometrical phase factor. The crucial point about this geometrical phase factor is that it is non-integrable, i.e. it can not be expressed as a function of the control parameter and it is not single-valued under continuation around a circuit.