Information from magnetisation curves

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Fundamental properties of matter and Applications

Physics is DIFFICULT and HARD to understand

keep away of physic

Magnetic moments of the electrons

↓

Magnetic moment of the atoms

↓

Magnetism in mater

↓

Magnetic study of the mater

↓

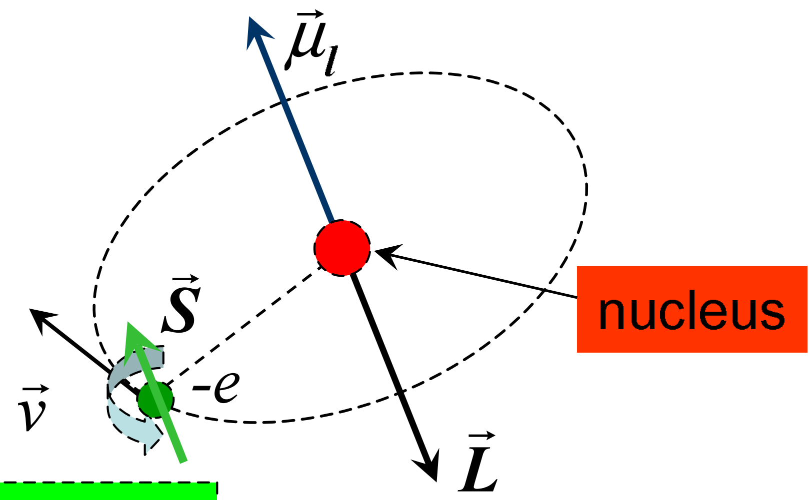
Fundamental properties and Applications

Magnetic moment of the atoms

- **Orbital kinetic moment** $\vec{L}$
- **Spin kinetic moment** $\vec{S}$

Kinetic moment of a charge

Magnetic moment: $\vec{m} = g\mu_B \vec{J}$



$\mu_B \equiv$ spin magnetic momentul of free electron

$$\mu_B = \frac{e\hbar}{2m_e} \quad (SI)$$

$\mu_B = 9{,}2742 \cdot 10^{-24}$ A·m^2

$$\mu_B = \frac{e\hbar}{2m_e c} \quad (CGS)$$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Landé factor

$$\begin{cases} \vec{m}_l = -g_l \dfrac{e}{2m_e} \vec{L}; \quad g_l = 1 \\ \vec{m}_s = -g_s \dfrac{e}{2m_e} \vec{S}; \quad g_s = 2 \end{cases}$$

magnetisation M

magnetic susceptibility χ

magnetic permeability μ

$$\vec{\mathbf{M}} = \frac{\sum \vec{\mathbf{m}}}{V}$$

$$\chi = \frac{M}{H}$$

$$\vec{\mathbf{B}} = \mu_0\left(\vec{\mathbf{H}} + \vec{\mathbf{M}}\right)$$

$$\mu = \frac{B}{H}$$

$B=\mu_0(H+\chi H)= \mu_0(1+\chi)H= \mu H$

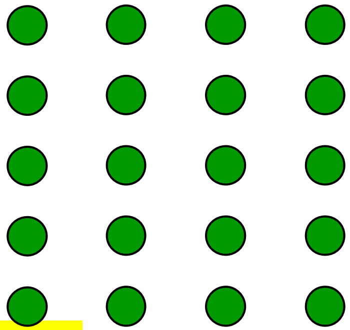
$$\mu = \mu_0\left(1 + \chi\right)$$

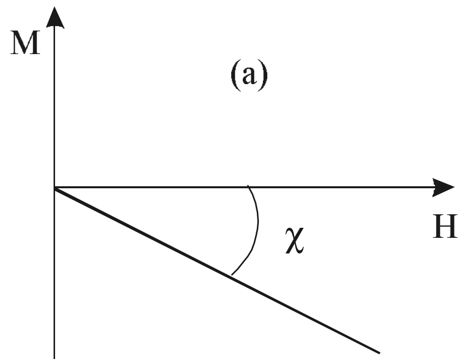
$\mu_0 = 4\pi\cdot 10^{-7}$ H/m

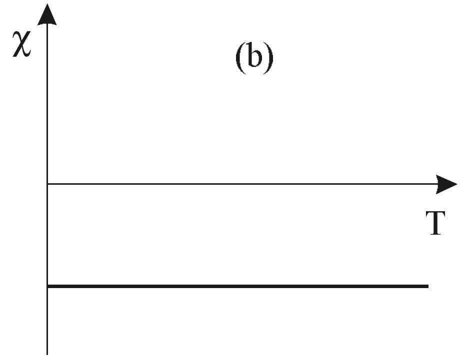
Diamagnetic

$\vec{m} = 0$

$\chi < 0$







C, Cu, Pb, H_2O, NaCl, SiO_2

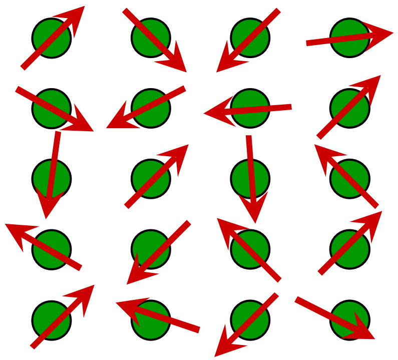
Paramagnetic

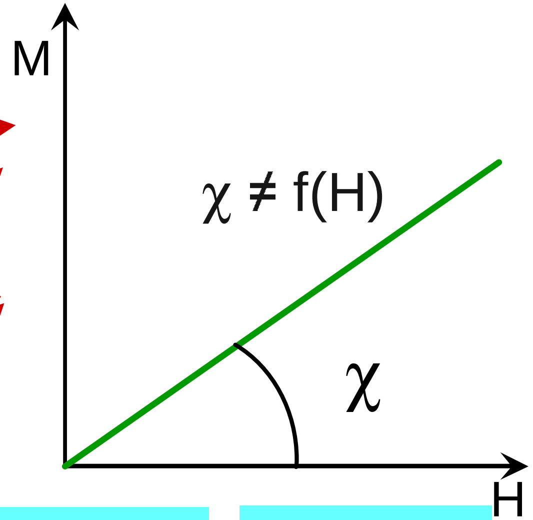
$\vec{m} \neq 0$

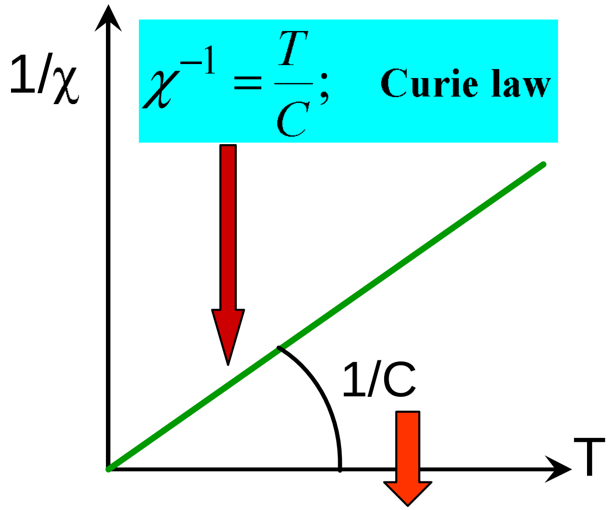
$J_{ij} = 0$

$\chi > 0$

Na, Al, $CuCl_2$







$\chi^{-1} = \frac{T}{C}$; **Curie law**

$$\mu_{eff} = g\mu_B\sqrt{J(J+1)} \Rightarrow J \Leftarrow \mu_{eff}(\mu_B) = \sqrt{8 \cdot C} \quad \mu_{eff}(\mu_B) = \sqrt{4{,}466 \cdot C} \Leftarrow \mu_{eff} = \sqrt{\frac{3k_B}{N \cdot \mu_0}C}$$

if χ(emu/mole) → $\mu_{eff}(\mu_B) = \sqrt{8 \cdot C}$

if χ(μ_B/T · f.u) → $\mu_{eff}(\mu_B) = \sqrt{4{,}466 \cdot C}$

Magnetic ordered

$\vec{m} \neq 0$

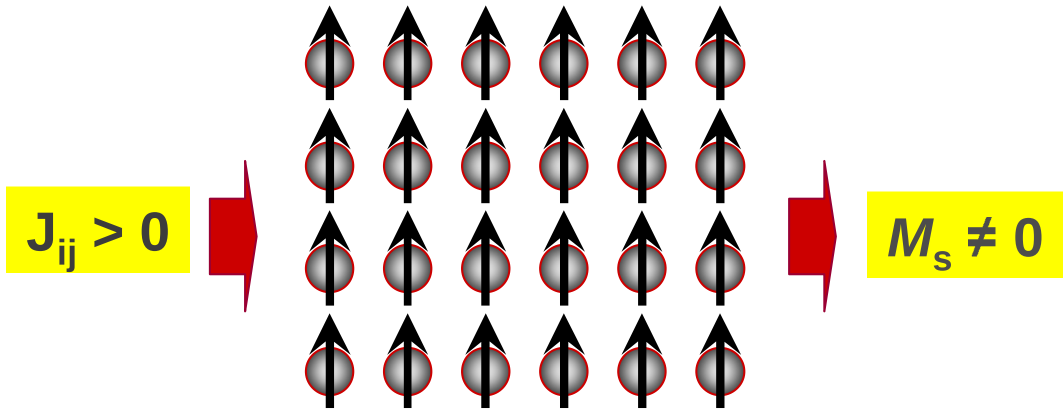
$J_{ij} \neq 0$

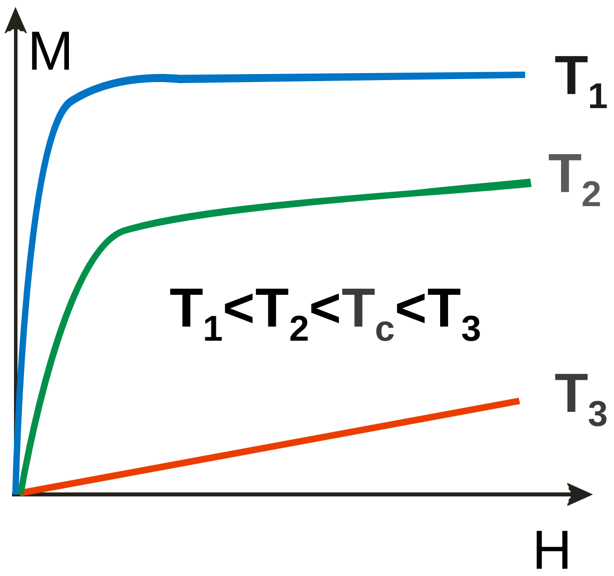
$\chi >> 0$

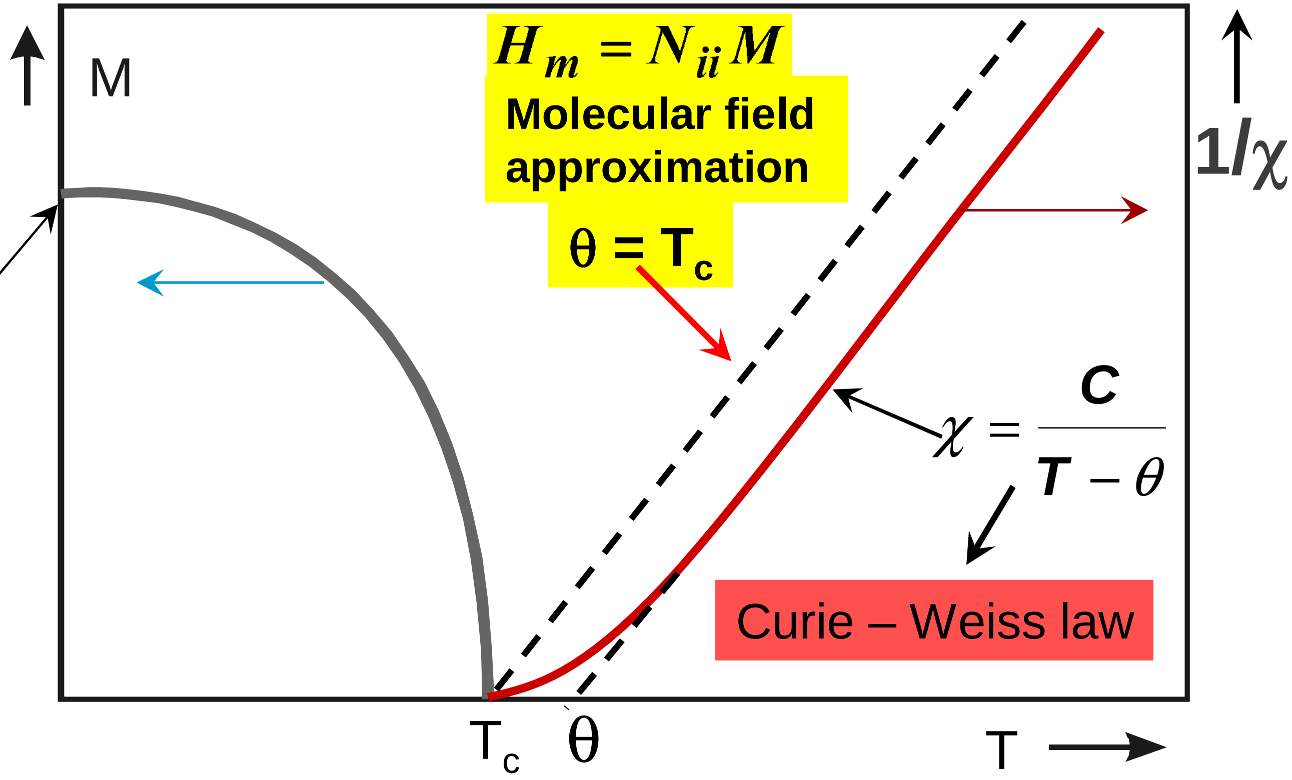
a) ferromagnetic

$$H = -2J_{ij}\ \vec{\mathbf{S}}_{\mathbf{i}} \cdot \vec{\mathbf{S}}_{j}$$

Fe, Co, Ni, Gd…





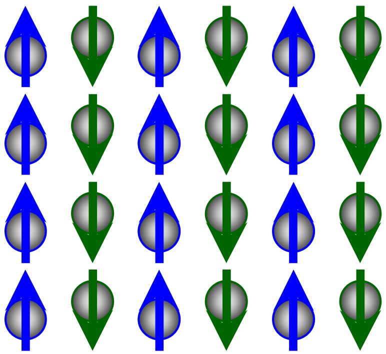


$M_s(0) = g_J \mu_B J$

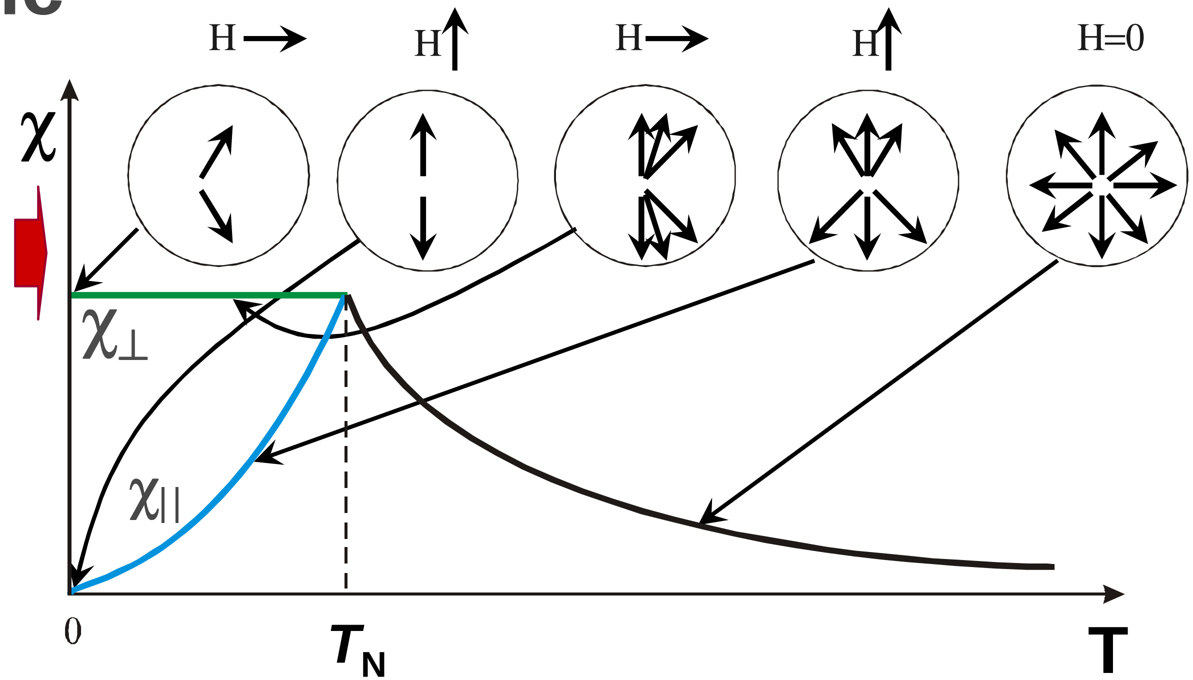
b) antiferromagnetic

$J_{ij} < 0$

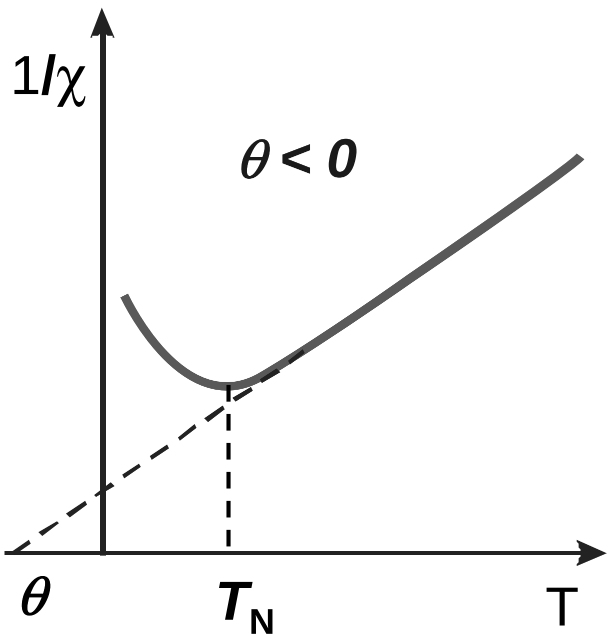
$|\vec{M}_A| = |\vec{M}_B|$



$M_s = 0$



MnO, Mn, Cr…

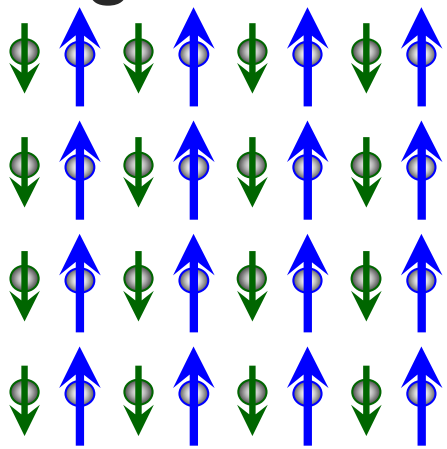


$$\chi = \frac{M_A + M_B}{H} = \frac{C}{T + \theta}$$

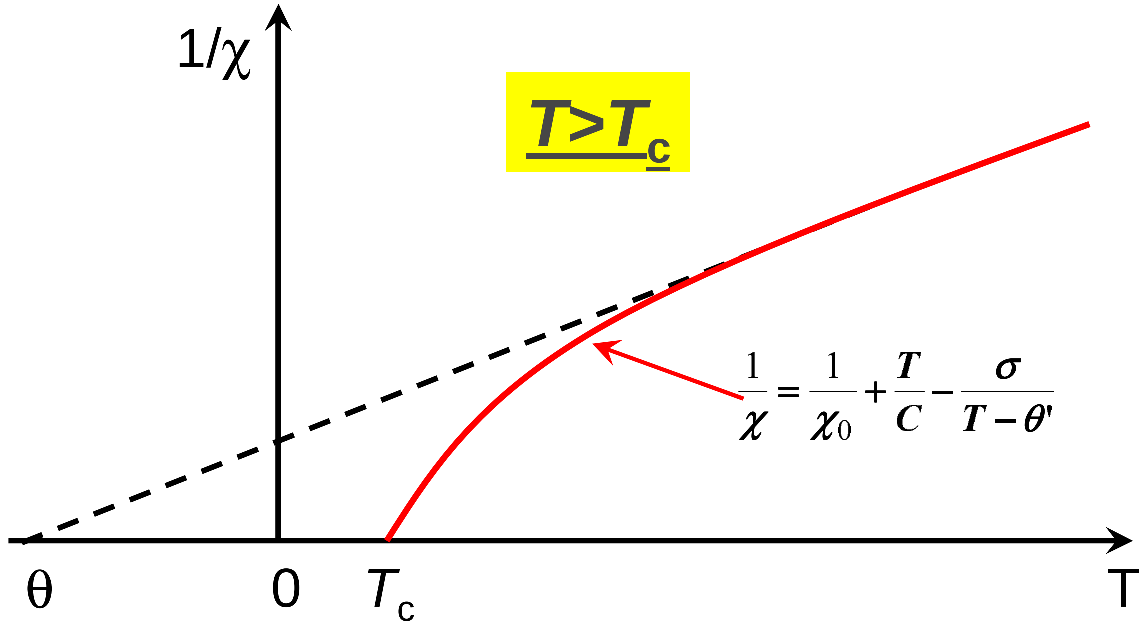
c) ferrimagnetism

$J_{ij} < 0$

$M_A \neq M_B$ ⇒ $M_s \neq 0$

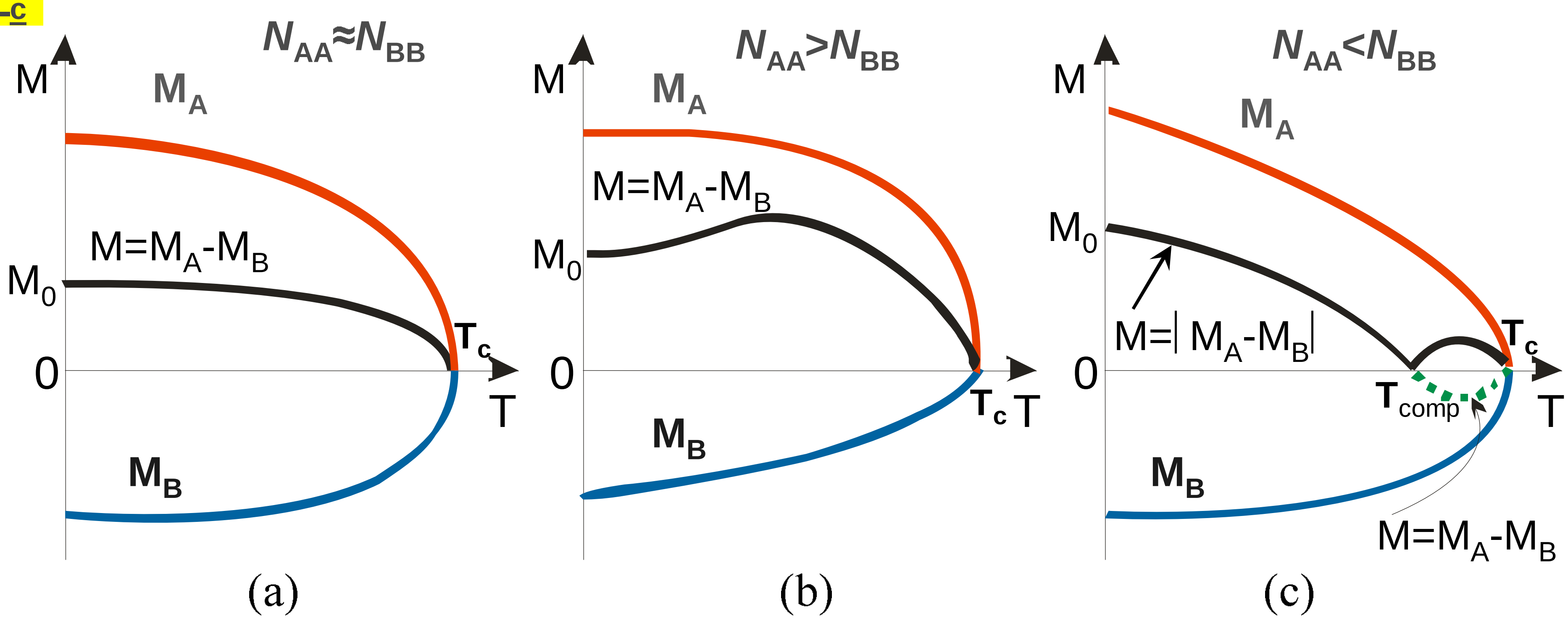


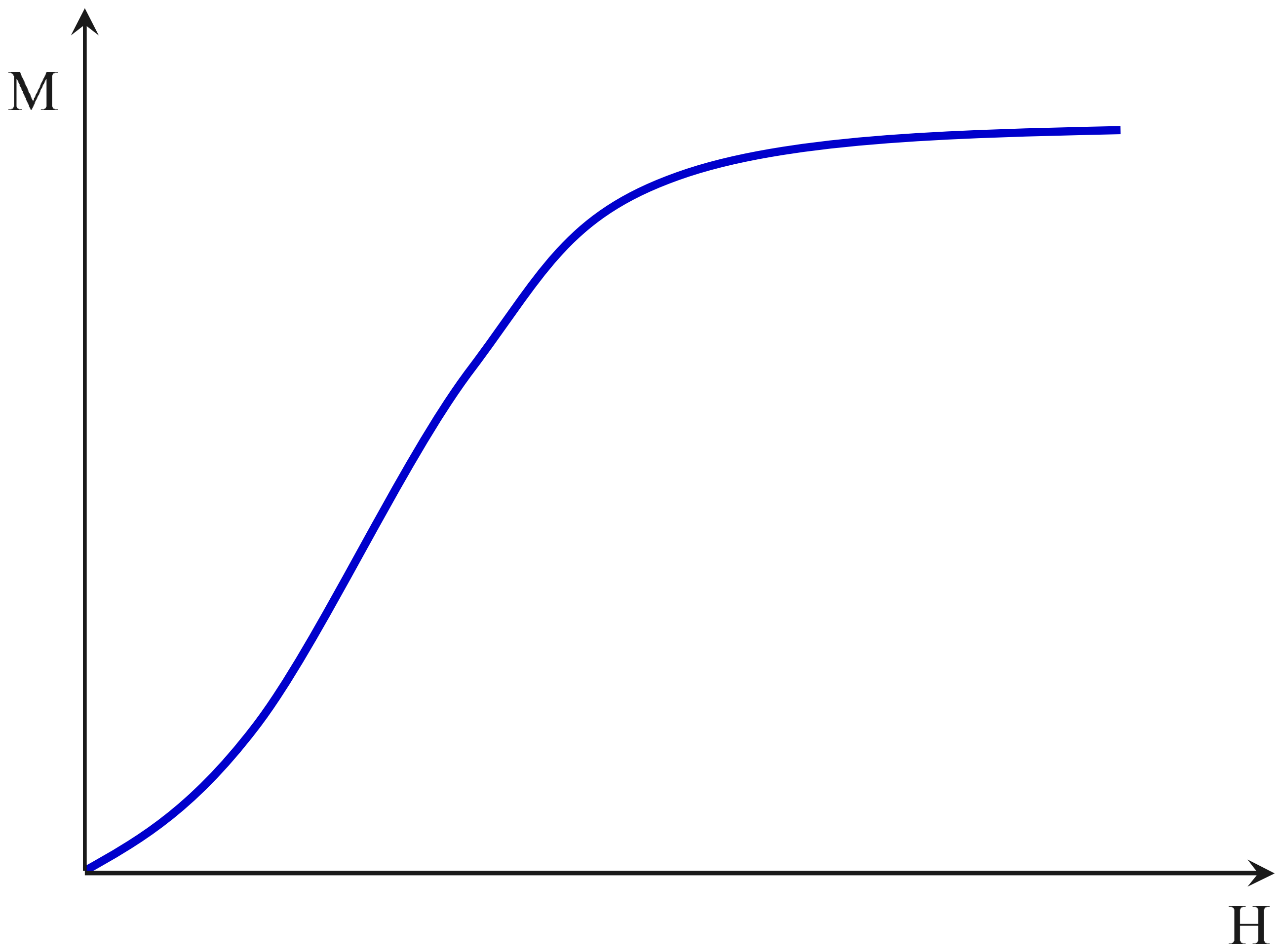
Fe_3O_4, ferrites, $GdCo_5$,…

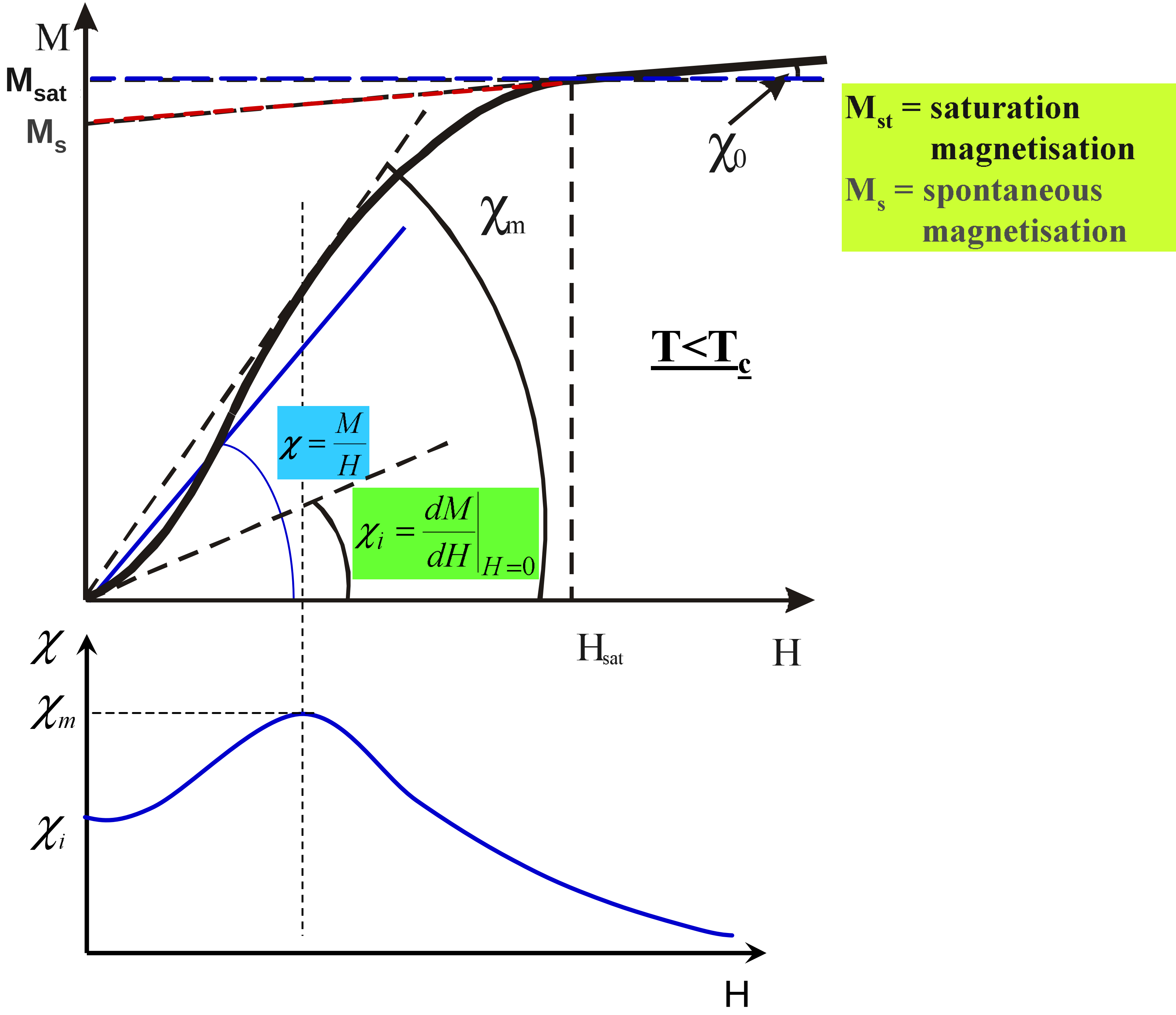


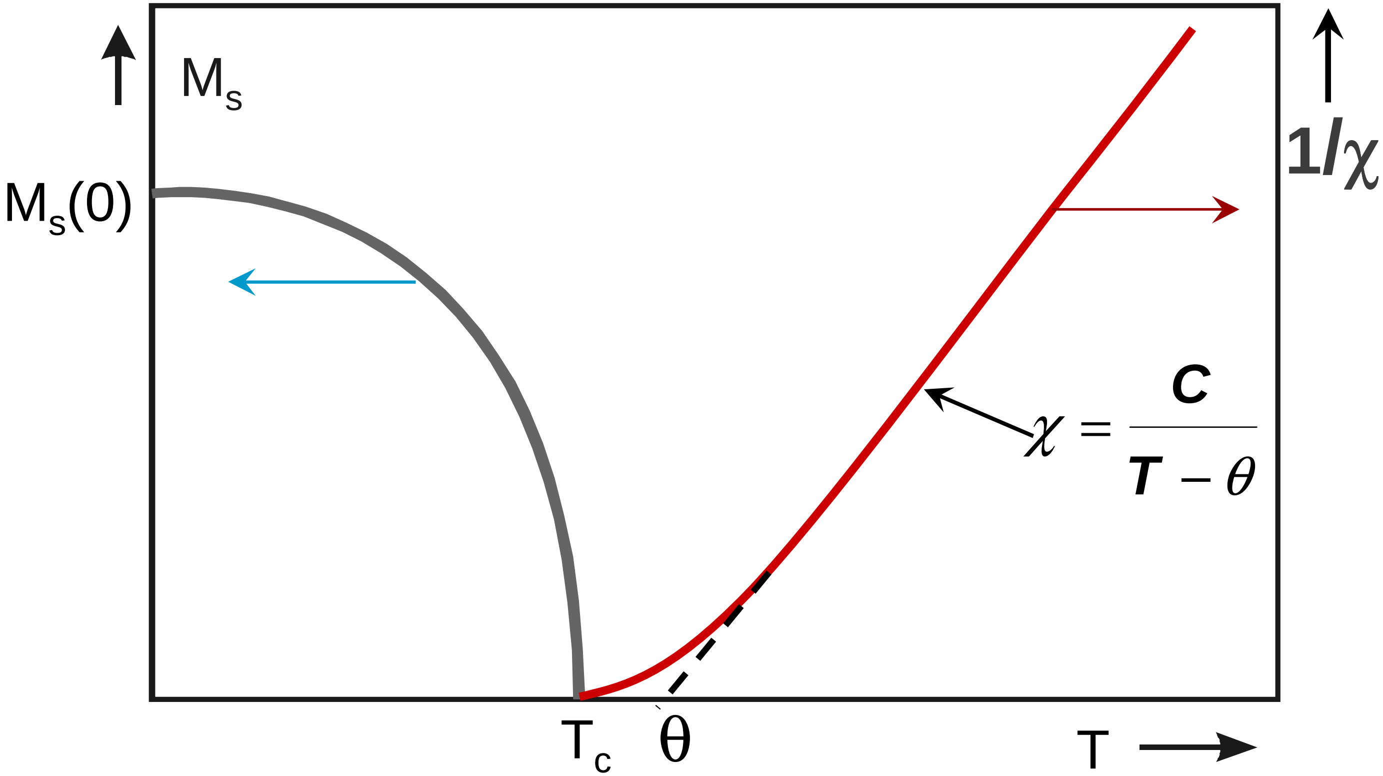
$$\frac{1}{\chi} = \frac{1}{\chi_0} + \frac{T}{C} - \frac{\sigma}{T-\theta'}$$

T<T_c

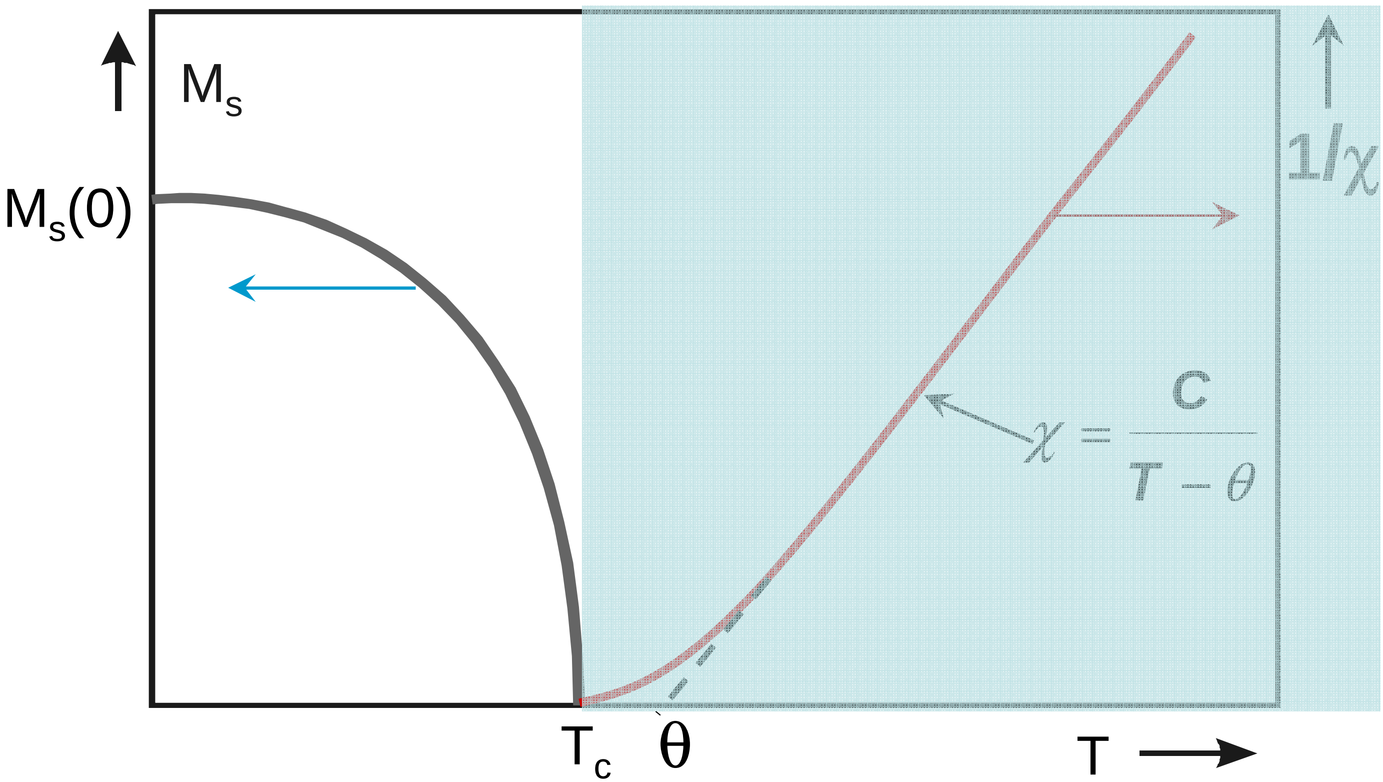



M
H

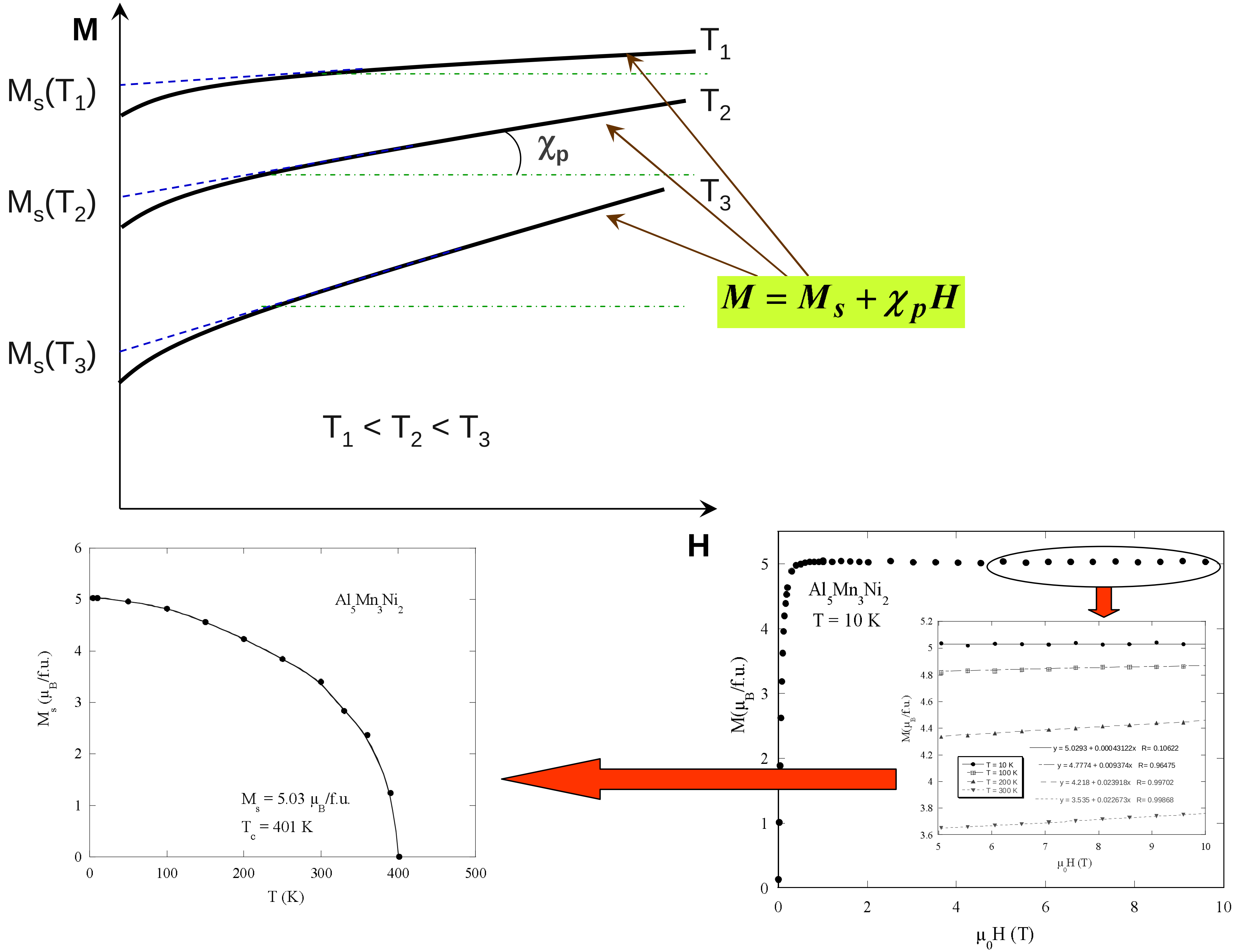

M
M_{sat}
M_s
χ_0
χ_m
M_{st} = saturation magnetisation
M_s = spontaneous magnetisation
T<T_c
$\chi = \frac{M}{H}$
$\chi_i = \left.\frac{dM}{dH}\right|_{H=0}$
H_{sat}
H
χ
χ_m
χ_i
H



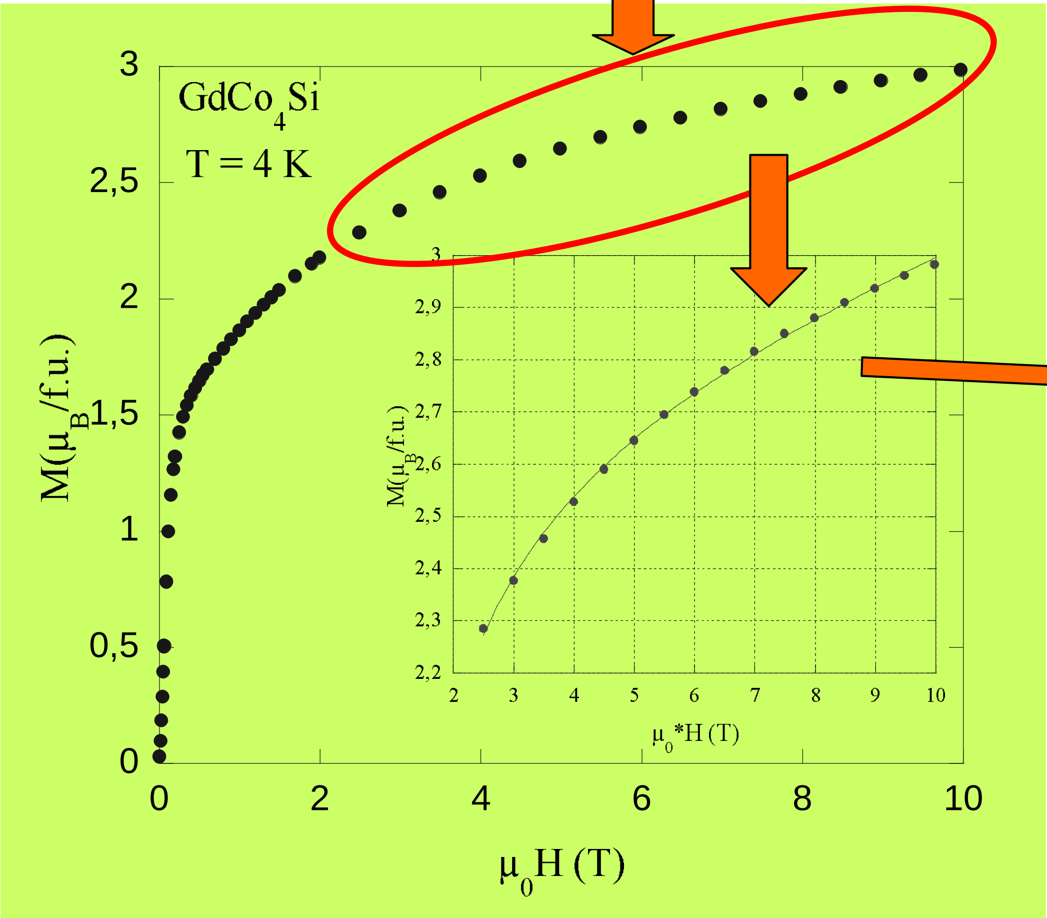
Ms
Ms(0)
1/χ
χ = C / (T − θ)
Tc
θ
T



Ms
Ms(0)
1/χ
χ = C / (T − θ)
Tc
θ
T


M
H
$M_s(T_1)$
$M_s(T_2)$
$M_s(T_3)$
T_1
T_2
T_3
χ_p
$M = M_s + \chi_p H$
$T_1 < T_2 < T_3$
$Al_5Mn_3Ni_2$
$M_s = 5.03\ \mu_B$/f.u.
$T_c = 401$ K
$M_s\ (\mu_B$/f.u.)
T (K)
$Al_5Mn_3Ni_2$
T = 10 K
$M(\mu_B$/f.u.)
$\mu_0 H$ (T)
T = 10 K
T = 100 K
T = 200 K
T = 300 K
y = 5.0293 + 0.00043122x R= 0.10622
y = 4.7774 + 0.009374x R= 0.96475
y = 4.218 + 0.023918x R= 0.99702
y = 3.535 + 0.022673x R= 0.99868

$$M = M_s\left(1-\frac{a}{H}\right)+\chi_p H$$



$M_s(T)$

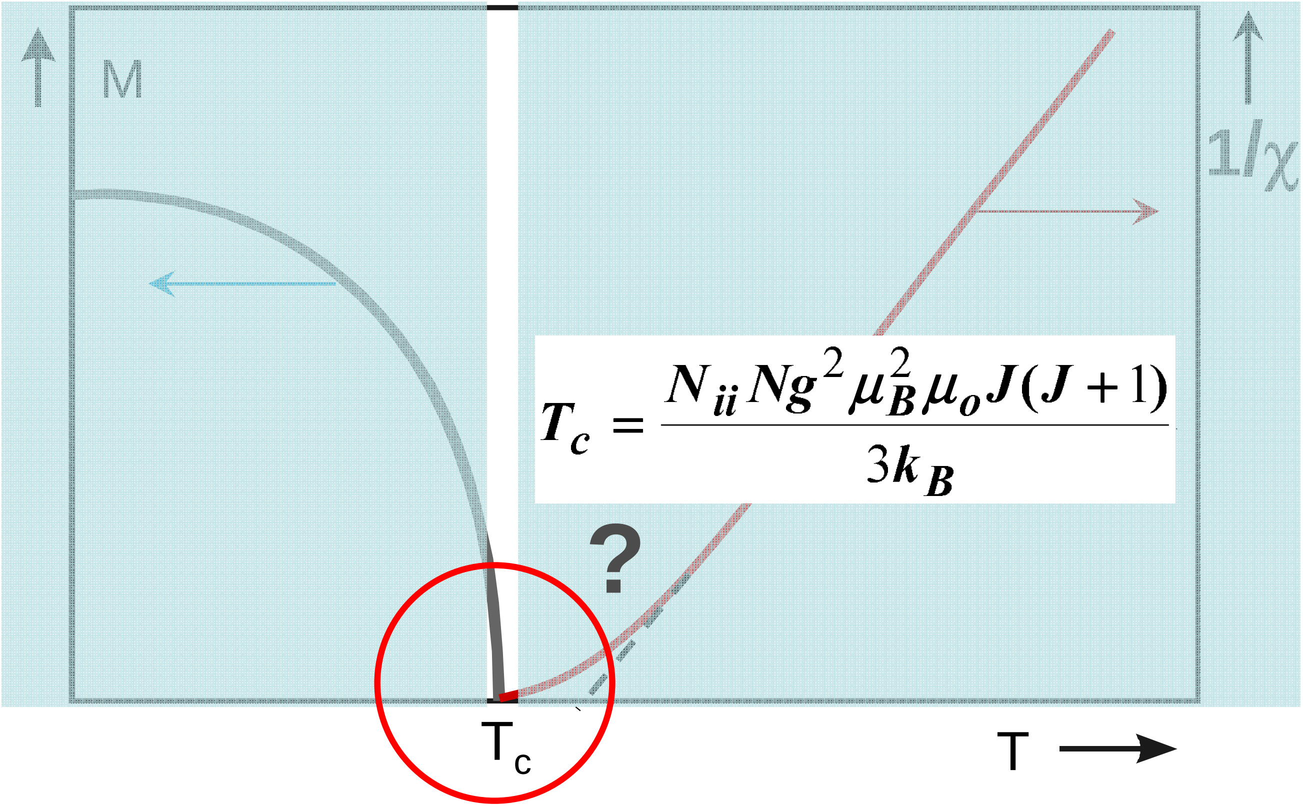
$T \rightarrow 0K$ ⇒ $M_s(0) \equiv M_0$

For 3d transition metals (Fe, Co, Ni…), the orbital moment is blocked by crystalline field:

$$M_0 = Ng\,\mu_B\,S;\; g \approx 2$$

For the rare earth (Gd for example)

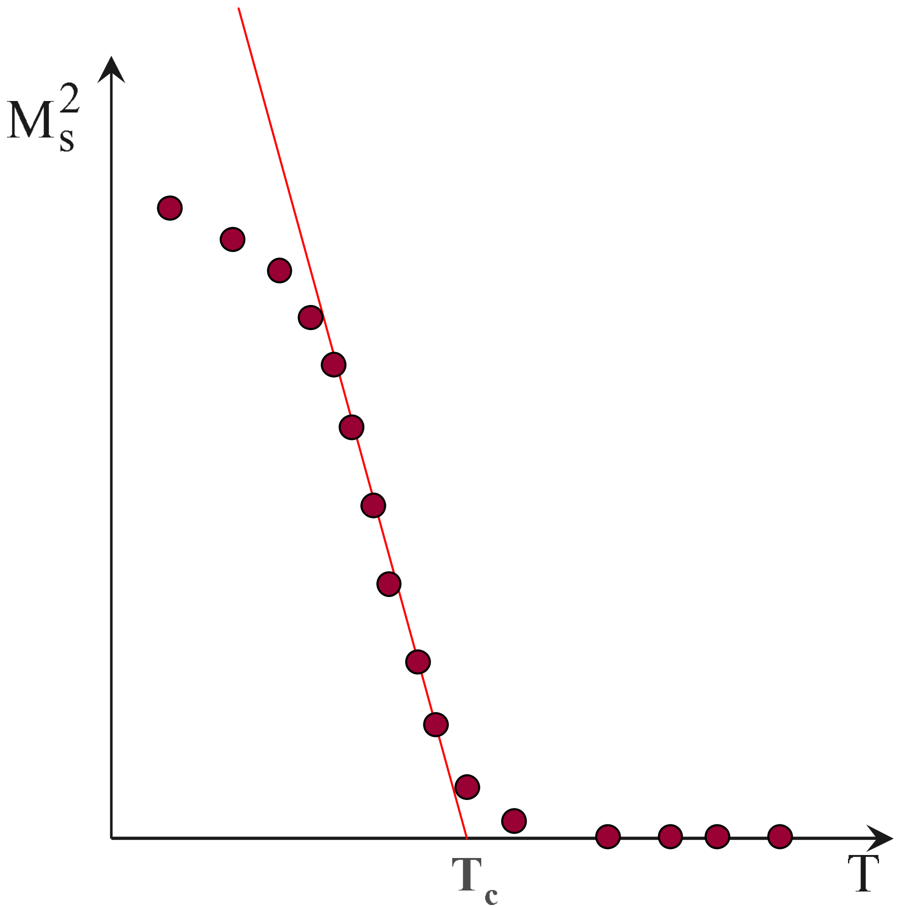
$$M_0 = Ng\,\mu_B\,J$$


M
$1/\chi$
$T_c = \frac{N_{ii} N g^2 \mu_B^2 \mu_o J(J+1)}{3k_B}$
?
T_c
T

Curie temperature evaluation

$T \rightarrow T_c$; $T < Tc$

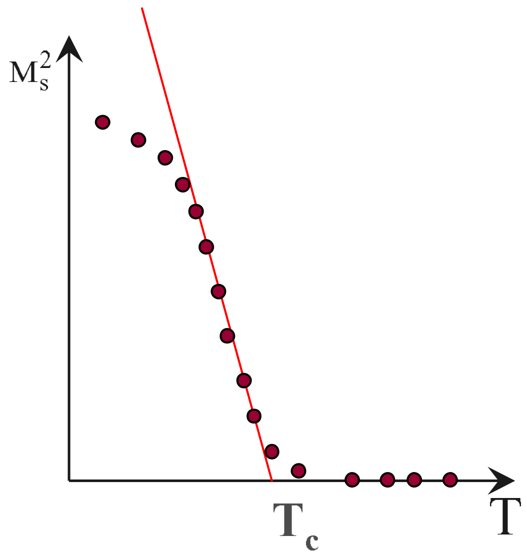
$$\left[\frac{M(T)}{M(0)}\right]^2 = \frac{10}{3} \cdot \frac{(J+1)^2}{J^2 + (J+1)^2}\left(1 - \frac{T}{T_c}\right)$$

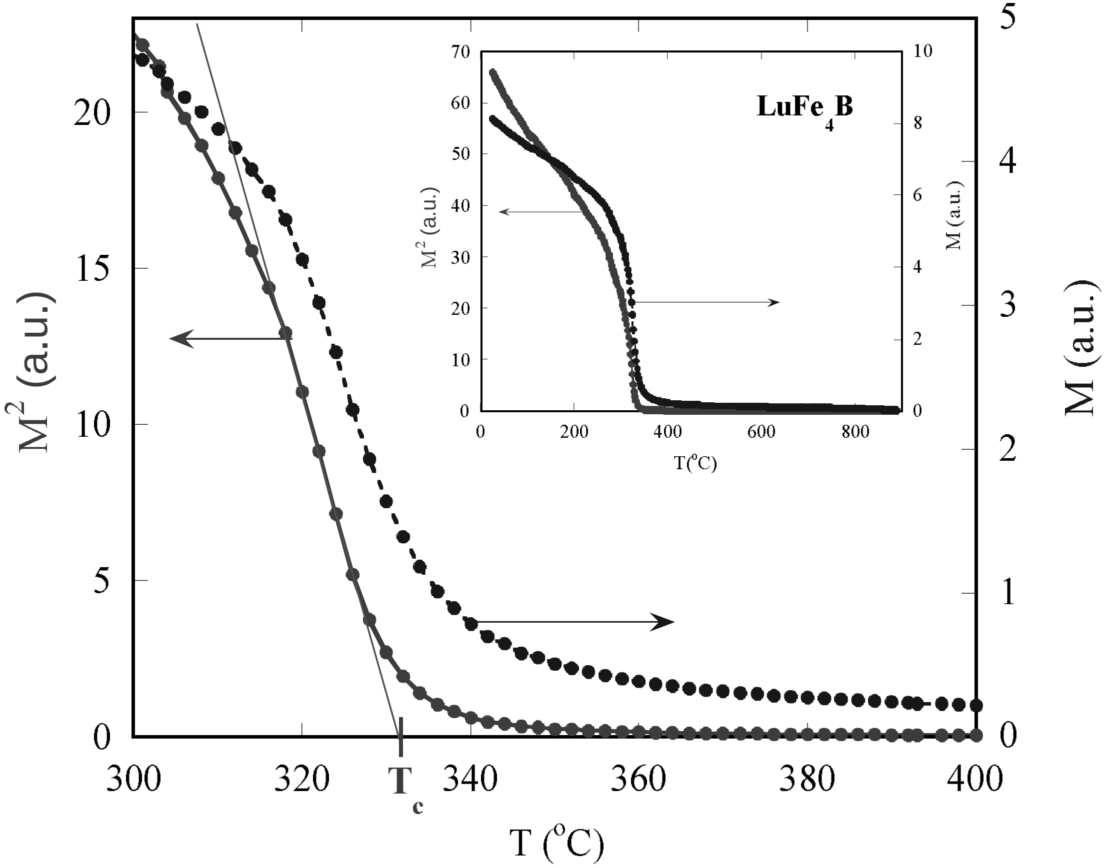


Curie temperature evaluation

$T \rightarrow T_c;\ T < T_c$

$$\left[\frac{M(T)}{M(0)}\right]^2 = \frac{10}{3} \cdot \frac{(J+1)^2}{J^2 + (J+1)^2}\left(1 - \frac{T}{T_c}\right)$$

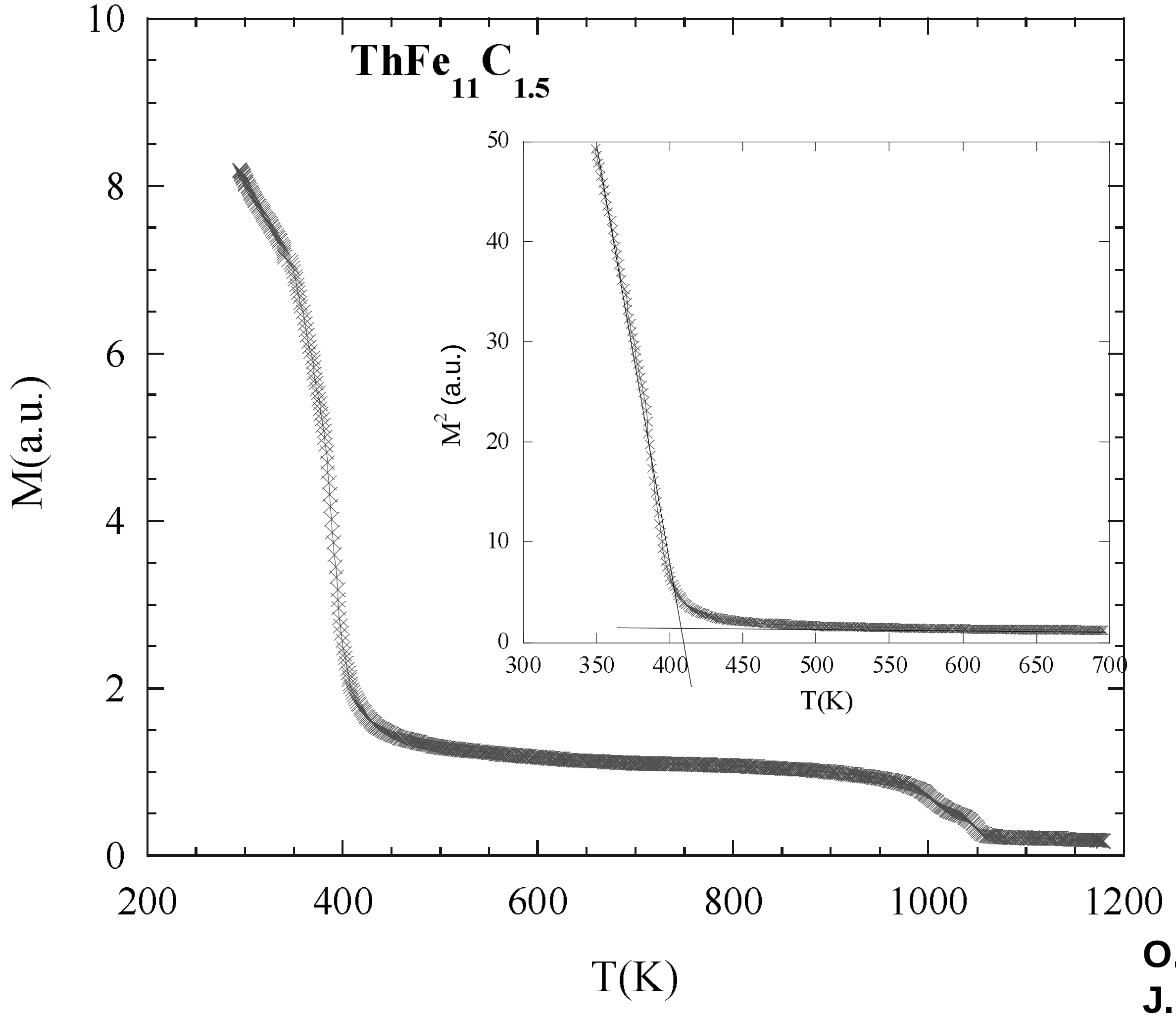




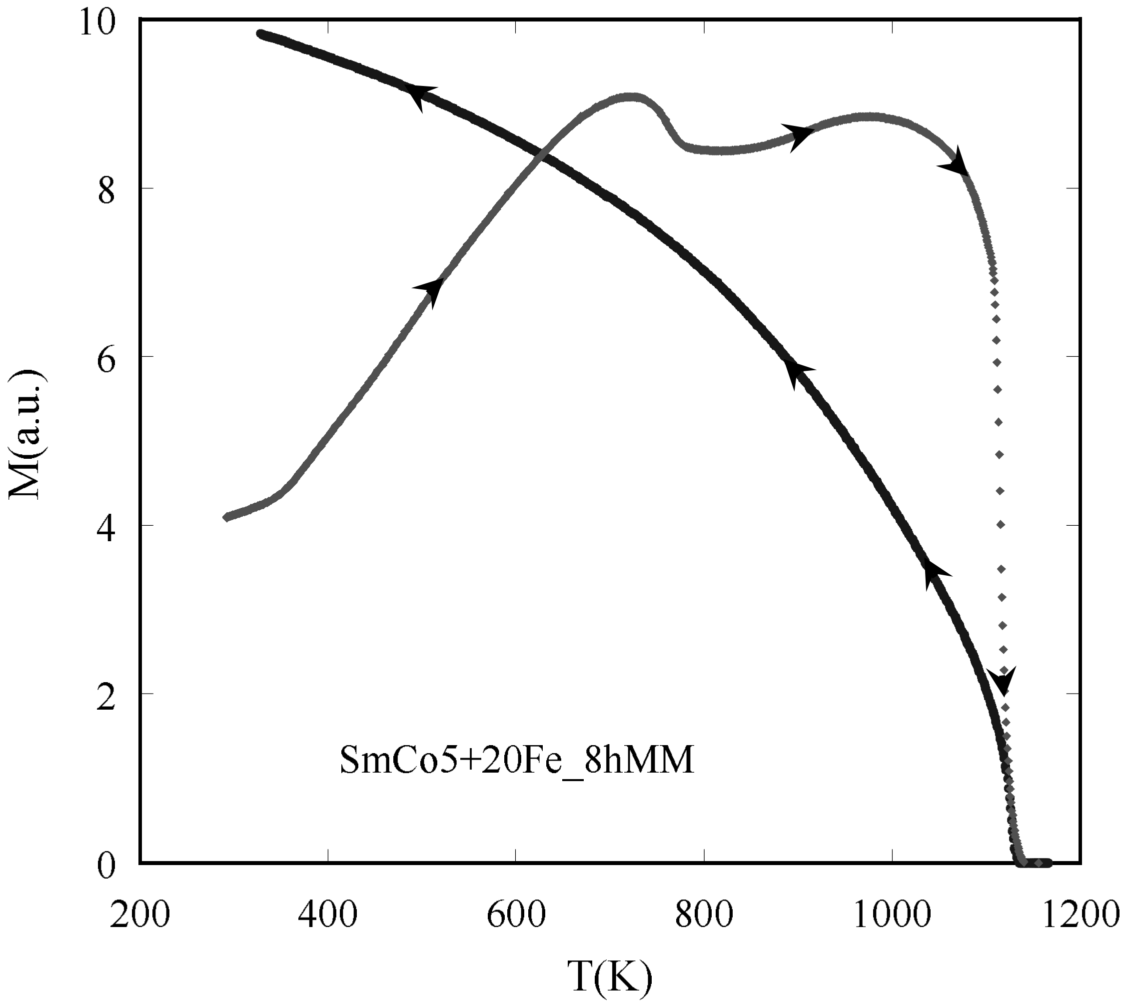
Curie temperature evaluation

$T \rightarrow T_c$; $T < Tc$

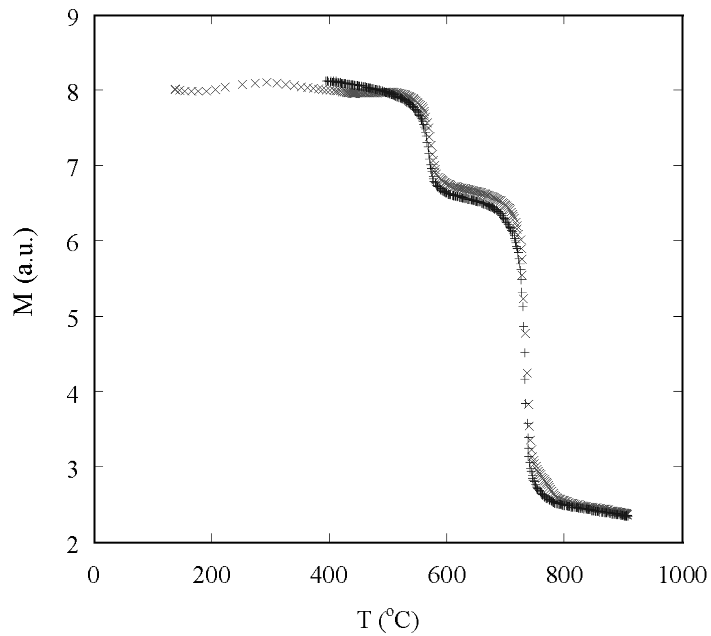
$$\left[\frac{M(T)}{M(0)}\right]^2 = \frac{10}{3} \cdot \frac{(J+1)^2}{J^2 + (J+1)^2}\left(1 - \frac{T}{T_c}\right)$$



O. Isnard, V. Pop, K.H.J. Buschow,
J. Magn. Magn. Mat. 256 (2003) 133



Phase analysis and thermal evolution



In the low magnetisation **region - for example** $T \rightarrow Tc$; $T < Tc$

$$F_m(M) = a\frac{M^2}{2} + b\frac{M^4}{4} + \cdots - \mu_0 MH$$

$$\frac{dF_m}{dM} = 0 \Rightarrow aM + bM^3 = \mu_0 H \quad \text{or} \quad M^2 = \frac{M}{H}\frac{\mu_0}{b} - \frac{a}{b}$$

molecular field approximations:

$$\frac{\mu_0 N_{ii}(T - T_c)}{T_c} M + \frac{\mu_0 3(2J^2 + 2J + 1)T}{10 M_0^2 (J+1)^2 C} M^3 = \mu_0 H$$

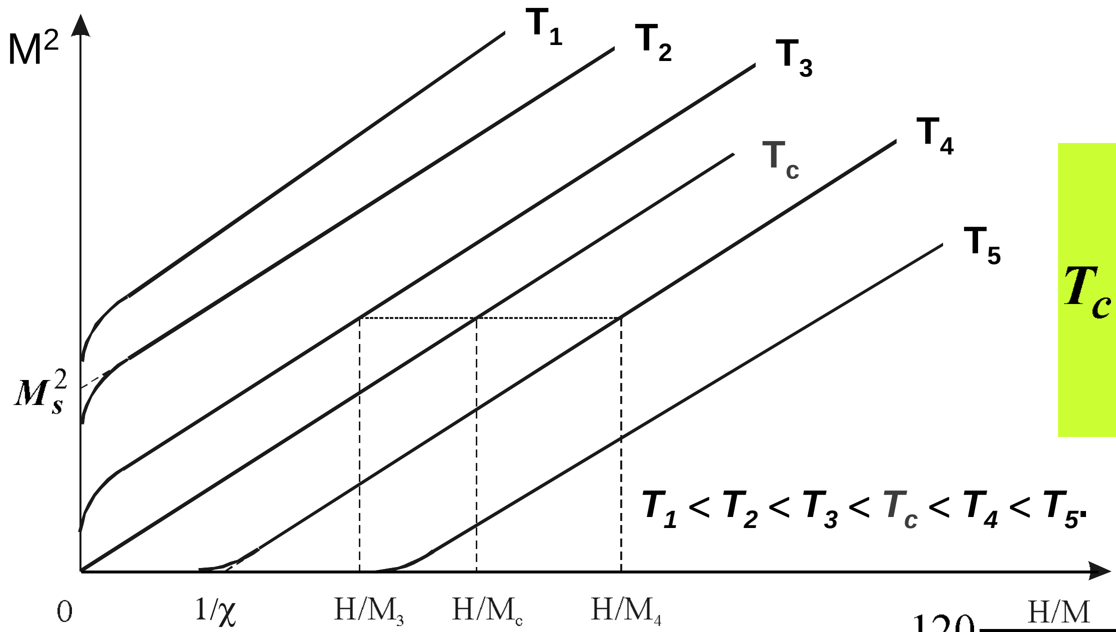
$H_m = N_{ii} M$

$N_{ii} = T_c/C$

$$a = \frac{\mu_0 N_{ii}(T - T_c)}{T_c}$$

$T < Tc$ →	$a < 0$
$T = Tc$ →	$a = 0$
$T > Tc.$ →	$a > 0$

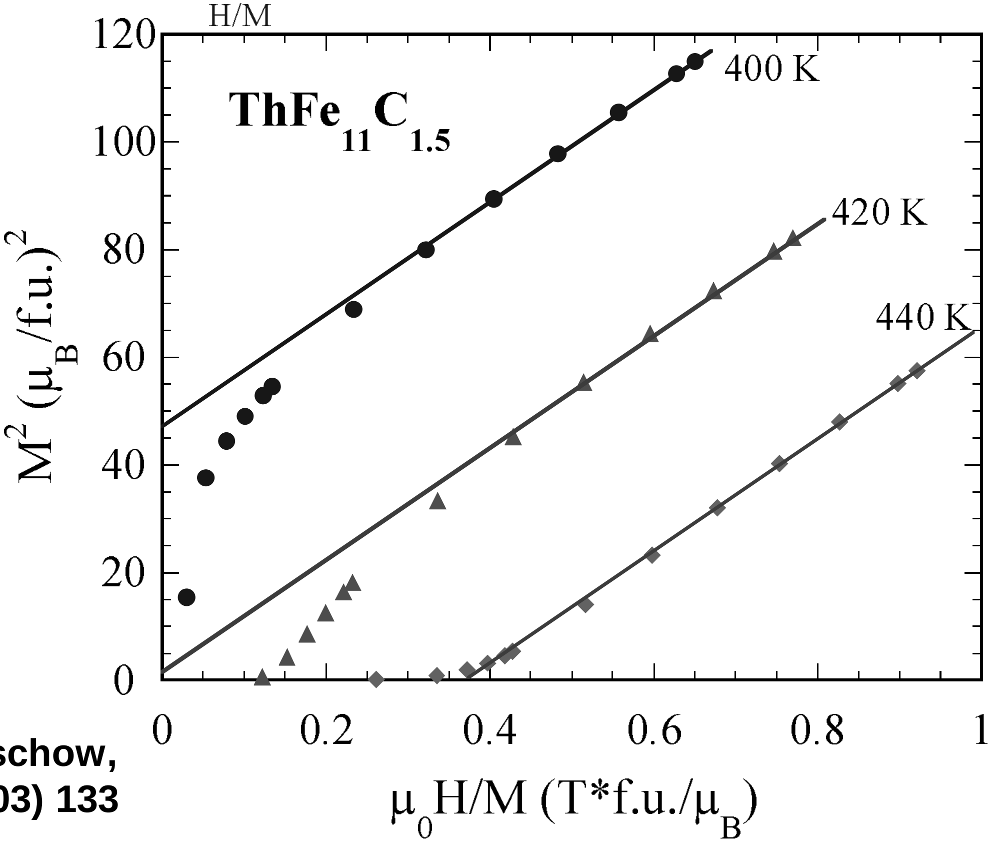
$$b = \frac{\mu_0 3(2J^2 + 2J + 1)T}{10 M_0^2 (J+1)^2 C}$$



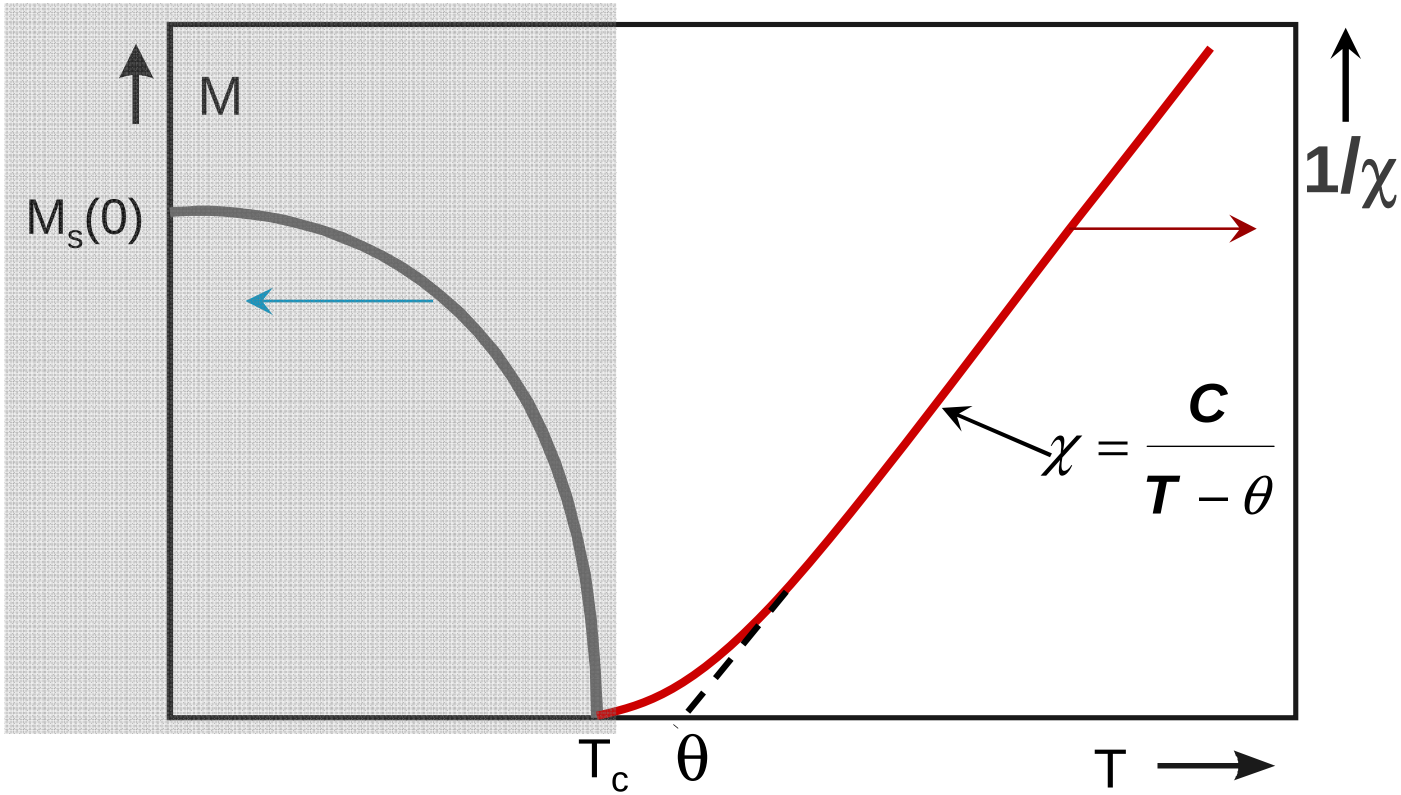
$$T_c = \frac{T_4\left(\frac{H}{M_c} - \frac{H}{M_3}\right) + T_3\left(\frac{H}{M_4} - \frac{H}{M_c}\right)}{\left(\frac{H}{M_4} - \frac{H}{M_3}\right)}$$

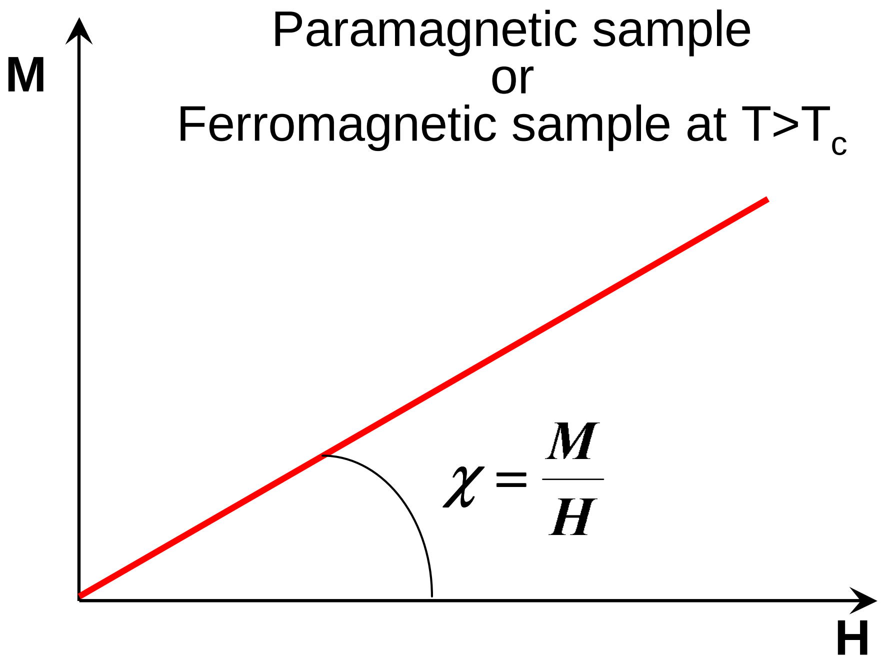
Arrott plot

A. Arrott, Phys. Rev. 108, 1394 (1957)



O. Isnard, V. Pop, K.H.J. Buschow, J. Magn. Magn. Mat. 256 (2003) 133

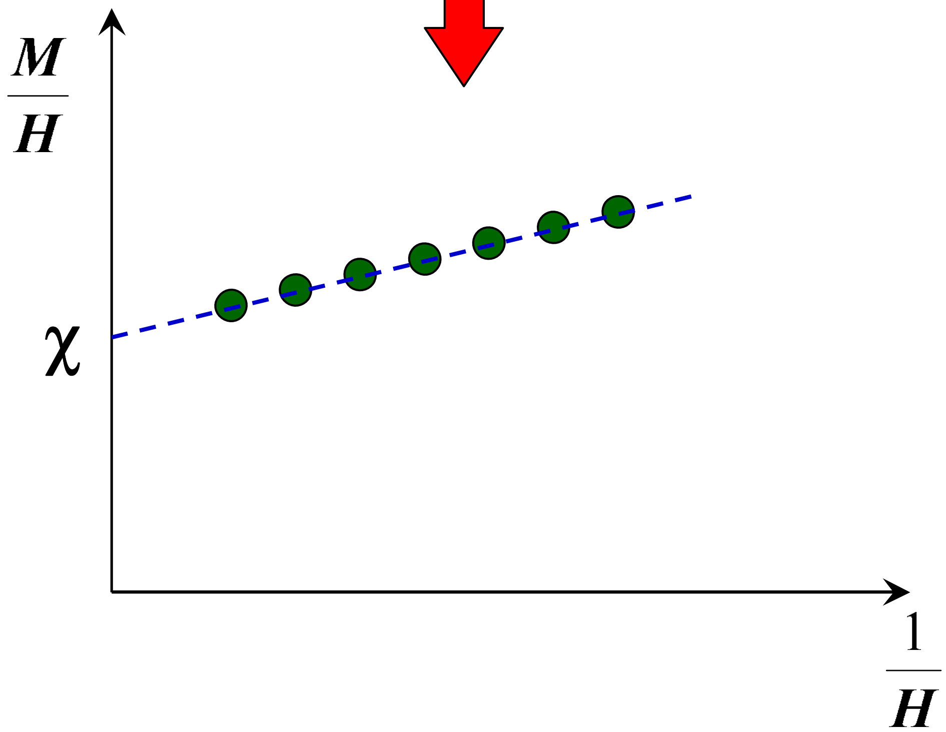

M
$M_s(0)$
$1/\chi$
$\chi = \frac{C}{T - \theta}$
T_c
θ
T

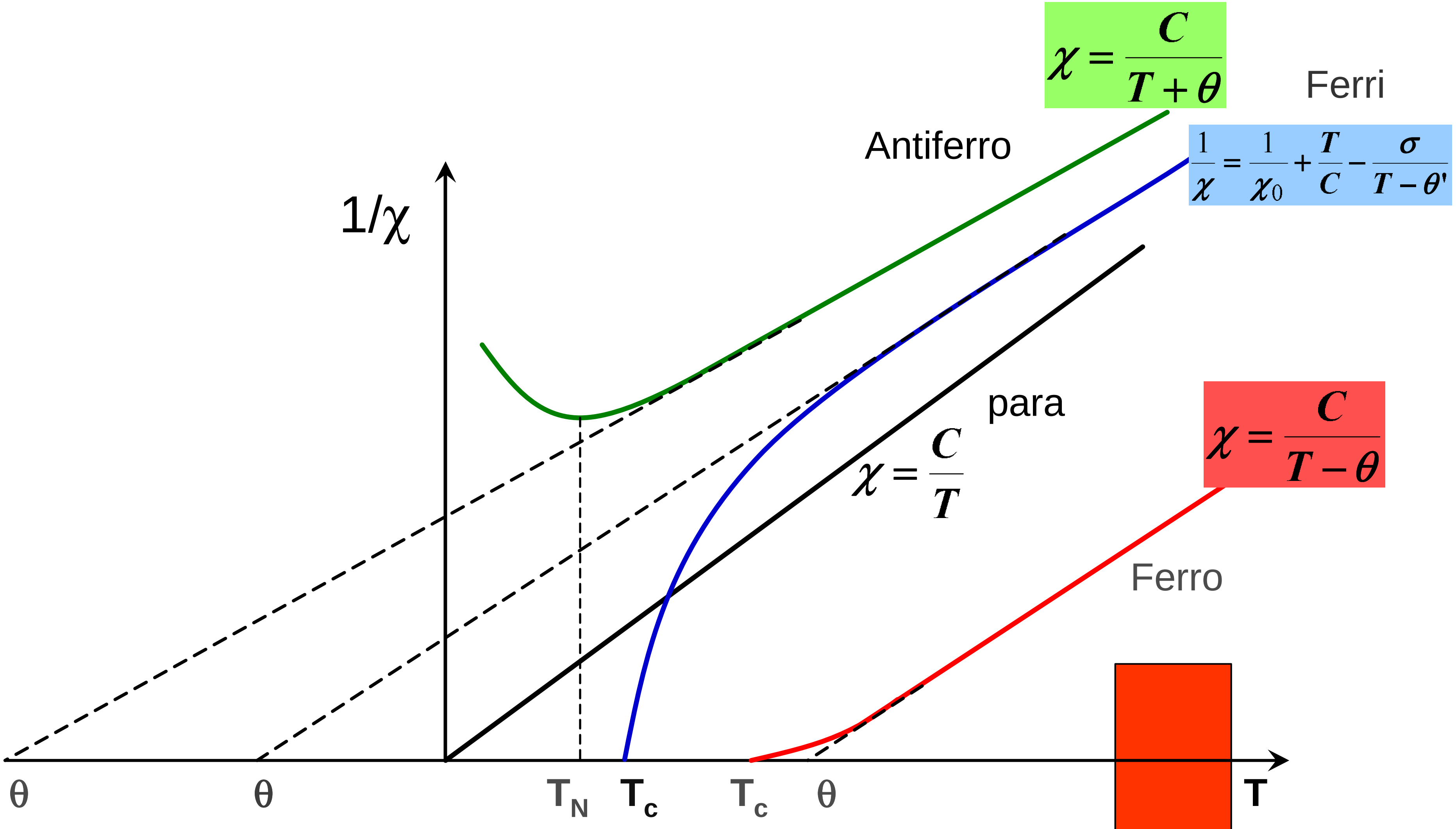


If there are some ferromagnetic impurity

$$M = \chi H + cM_s$$

$$\frac{M}{H} = \chi + c\frac{M_s}{H}$$

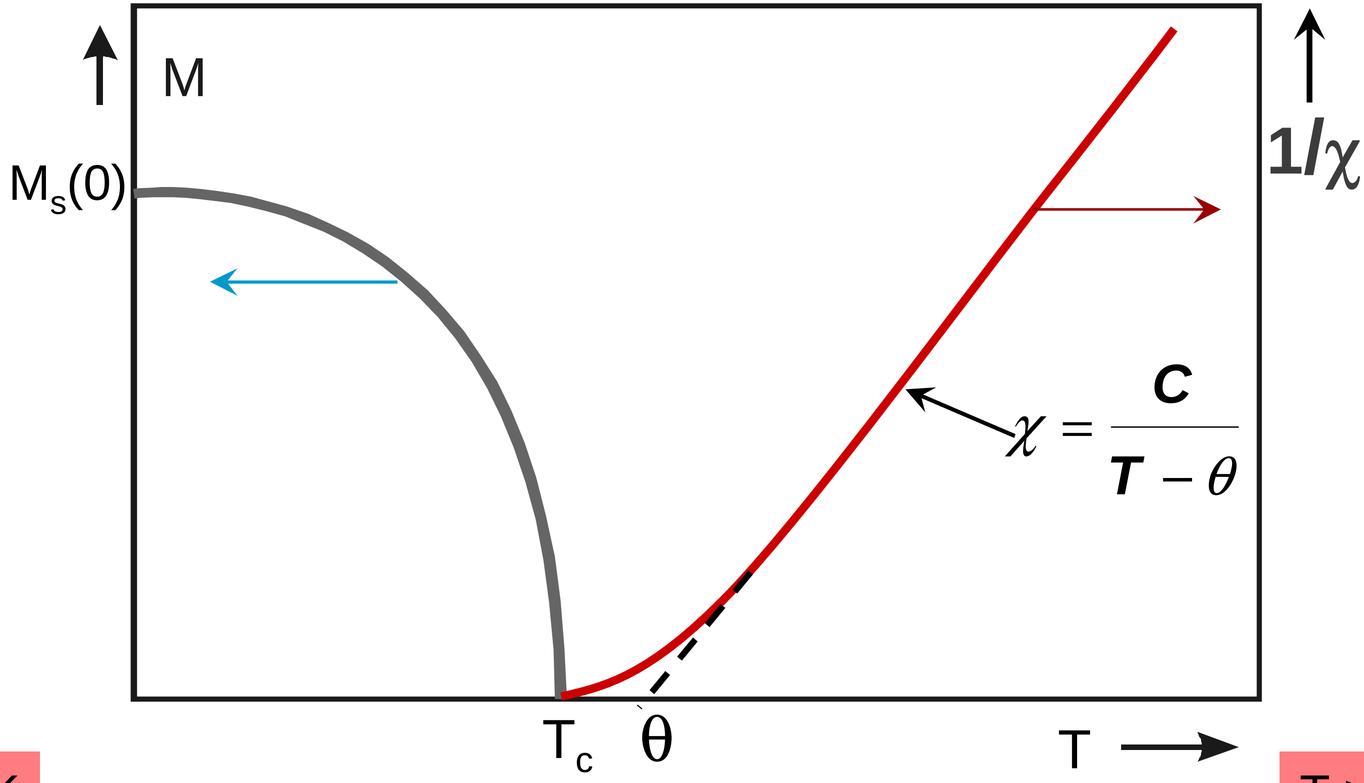




if χ(emu/mole)

if $\chi(\mu_B/T \cdot f.u)$

$$\mu_{eff} = g\mu_B\sqrt{J(J+1)} \Rightarrow J \Leftarrow \mu_{eff}(\mu_B) = \sqrt{8\cdot C} \Leftarrow \mu_{eff}(\mu_B) = \sqrt{4{,}466\cdot C} \Leftarrow \mu_{eff} = \sqrt{\frac{3k_B}{N\cdot\mu_0}C}$$



T→0K

$M_s(0) = g_J \mu_B J_0$

T > T_c

$$\mu_{eff} = g\mu_B\sqrt{J_p(J_p+1)}$$

For the rare earth (Gd for example): $\mathbf{J_0=J_p}$

For 3d transition metals (Fe, Co, Ni…), the orbital moment is blocked by crystalline field:

T→0K

$M_s(0) \approx 2\,\mu_B S_0$

$$r = \frac{S_p}{S_0} > 1$$

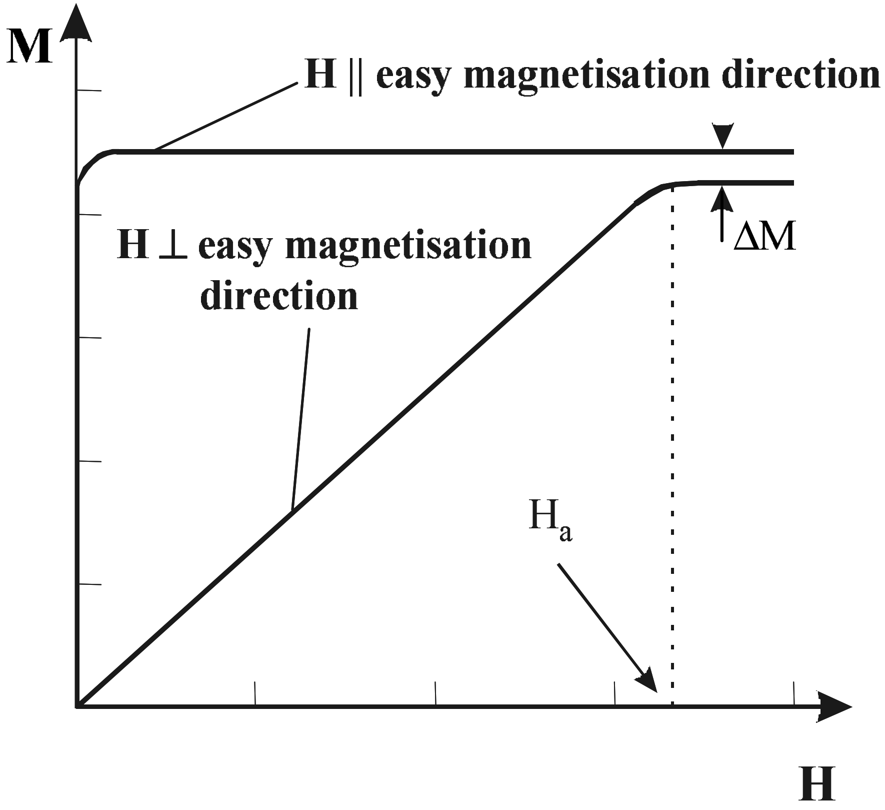
T > T_c

$$\mu_{eff} \approx g\mu_B\sqrt{S_p(S_p+1)}$$

r = 1 local moment limit
r $\to\infty$ total delocalisation limit

	Gd[1]	Fe[1]	Co[1]	$ThFe_{11}C_{1.5}$[2]	Fe_3C[3]	$HoCo_4Si$[4]	YCo_3B_2[5]
r	**1.00**	**1.01**	**1.32**	**1.5**	**1.69**	**2.03**	$\to\infty$

[1] P.R. Rhodes, E.P. Wolfarth, Proc. R. Soc. 273 (1963) 347.
[2] O. Isnard, V. Pop, K.H.J. Buschow, J. Magn. Magn. Mat. 256 (2003) 133
[3] D. Bonnenberg, K.A. Hempel, H.P.J. Wijn, Landolt-B.orsntein new series, Vol. III, 19a, Springer, Berlin, 1986, p. 142.
[4] N. Coroian, PhD thesis
[5] R. Ballou, E . Burzo, and V. Pop, J. Magn. Magn. Mat. 140-144 (1995) 945.


M
H || easy magnetisation direction
H ⊥ easy magnetisation
direction
ΔM
H
a
H

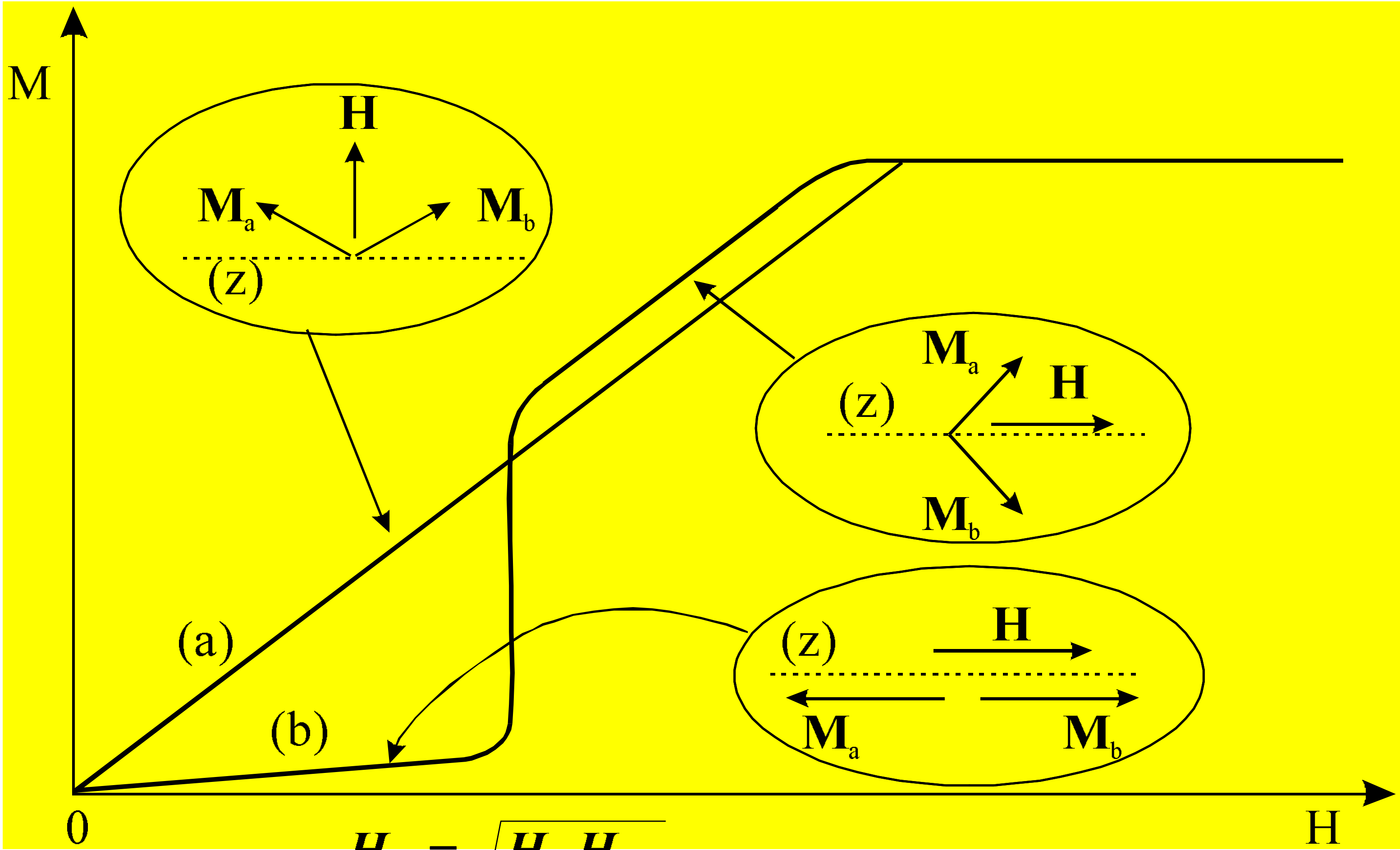
T < T_N, antiferromagnetic materials, $\chi_\perp > \chi_{||}$

Density of energy in magnetic field H,

$$E = -\chi\mu_0 H^2/2$$

low anisotropy energy

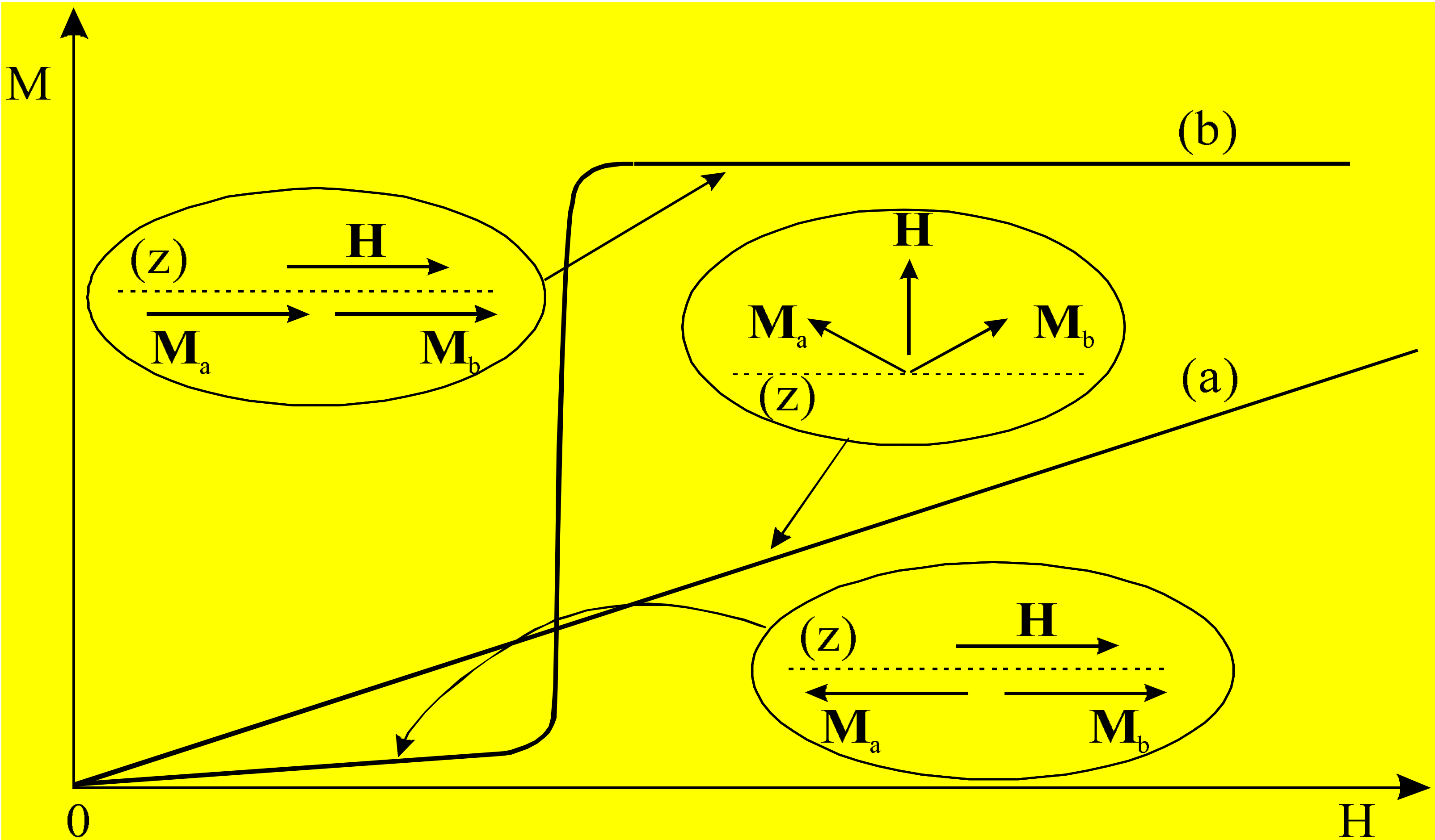
spin – flop **transition**



$$H_c = \sqrt{H_a H_{ex}}$$

E. Du Trémolet de Lacheisserie (editor),
Magnetisme, Presses Universitaires de Grenoble, 1999

High anisotropy energy
spin – flip transition

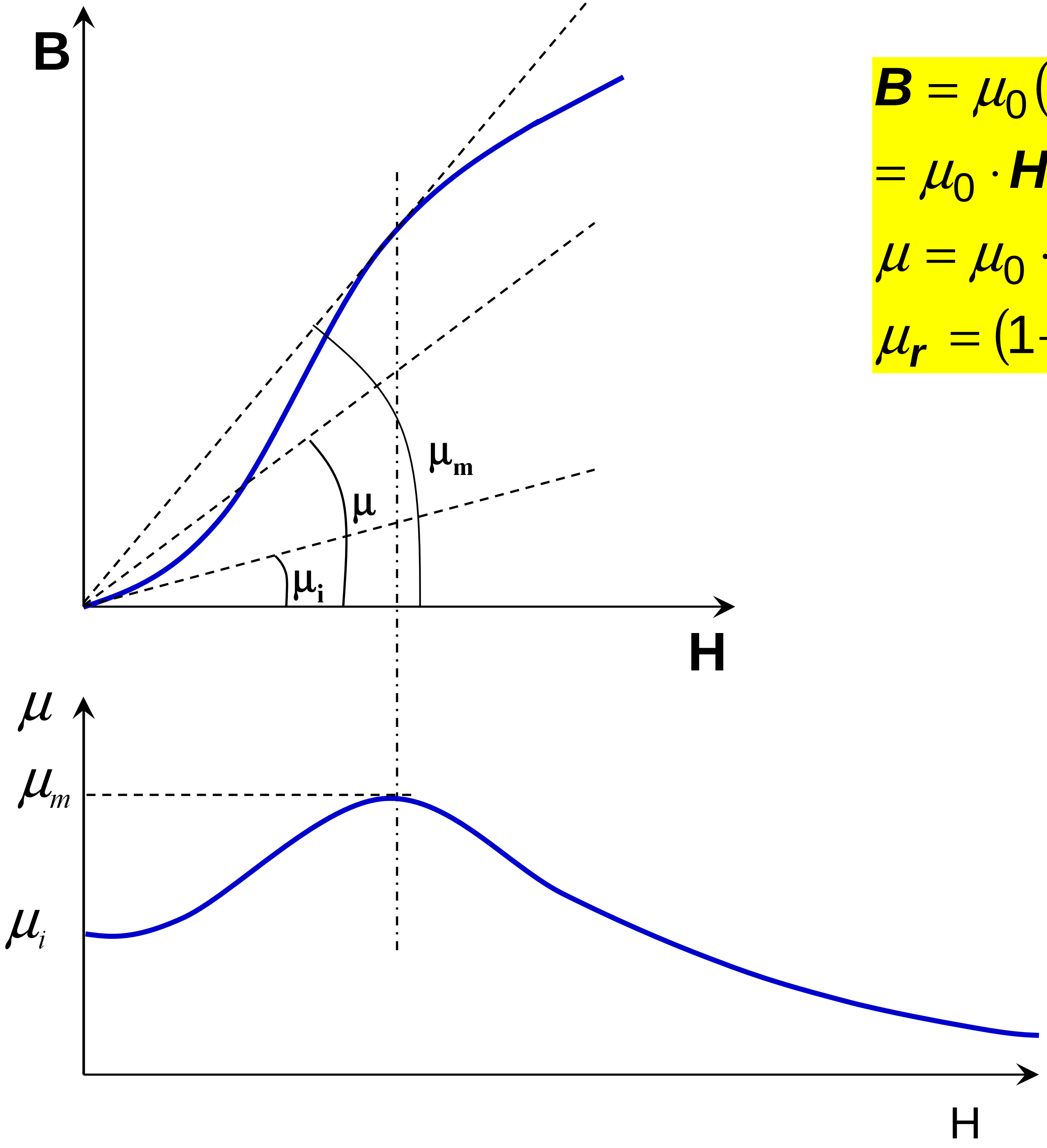


$$H_c \approx H_{ex}$$

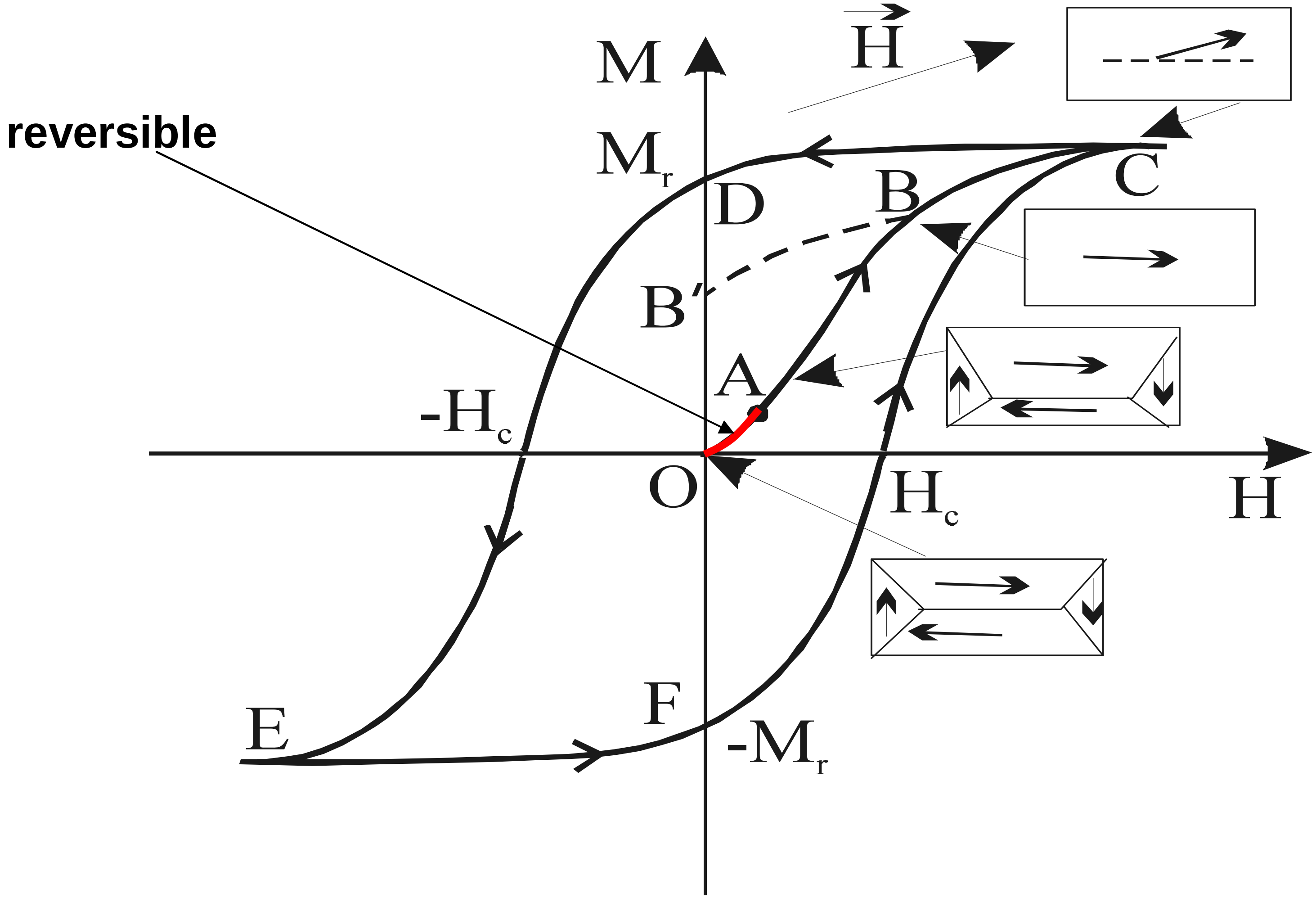
spin–flip
and
spin–flop

metamagnetic transition

also in ferrimagnetic materials



$$B = \mu_0 (H + M) = \mu_0 (H + \chi H)$$
$$= \mu_0 \cdot H (1 + \chi) = \mu \cdot H$$
$$\mu = \mu_0 \cdot (1 + \chi) = \mu_0 \cdot \mu_r$$
$$\mu_r = (1 + \chi)$$

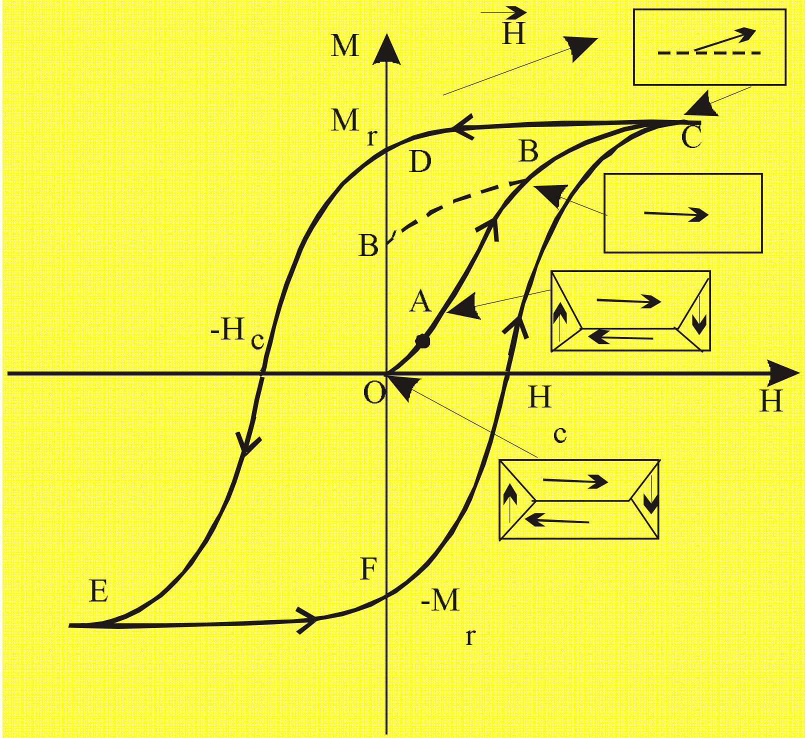


reversible
M
$\vec{H}$
M_r
D
B
C
B'
A
$-H_c$
O
H_c
H
E
F
$-M_r$

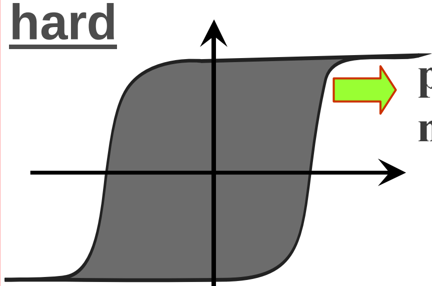
$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right)$$

Fundamental research → **M**

Application research → **B**



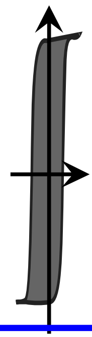
hard



permanent magnets

H_c – BIG ~$10^2 \div 10^6$ A/m

soft



μ >> 0

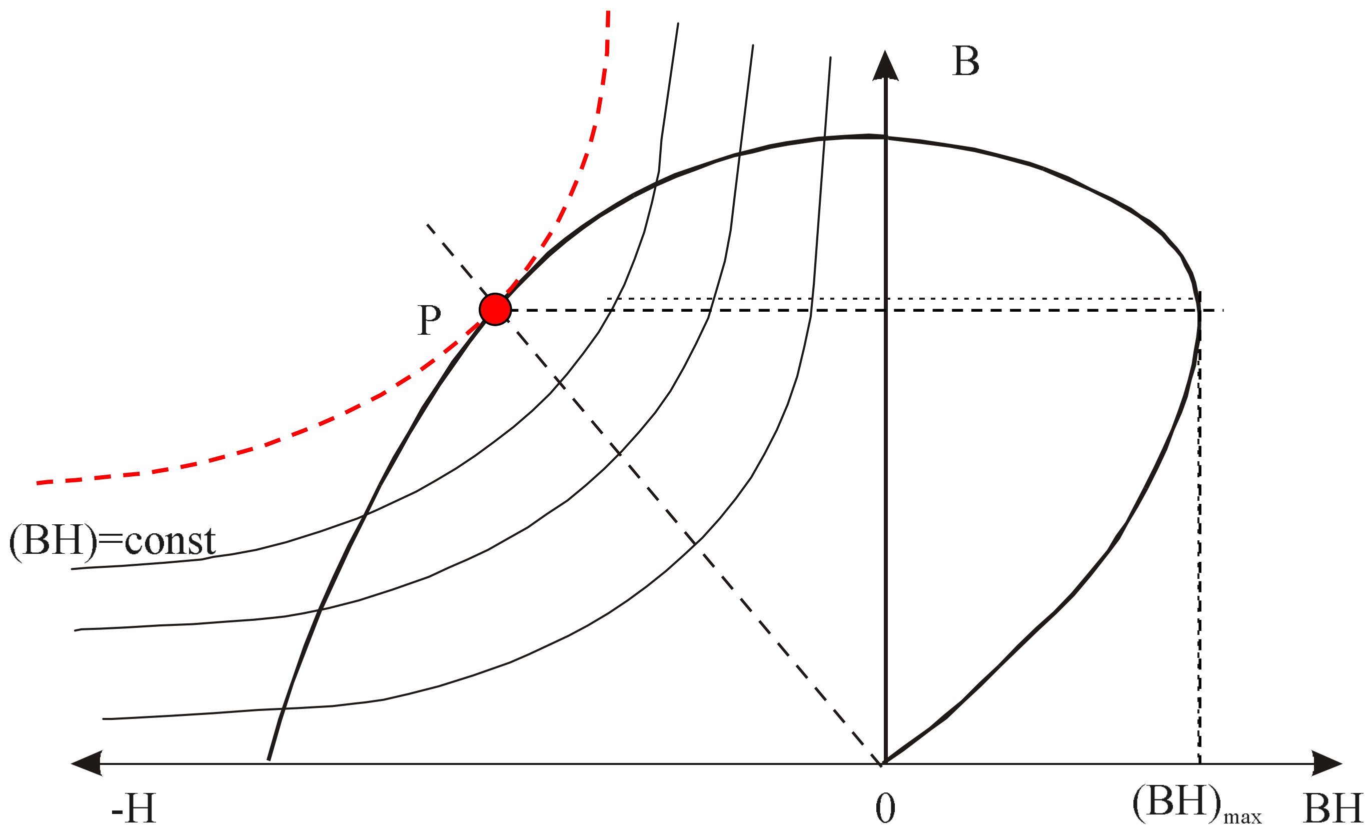
magnetic cores

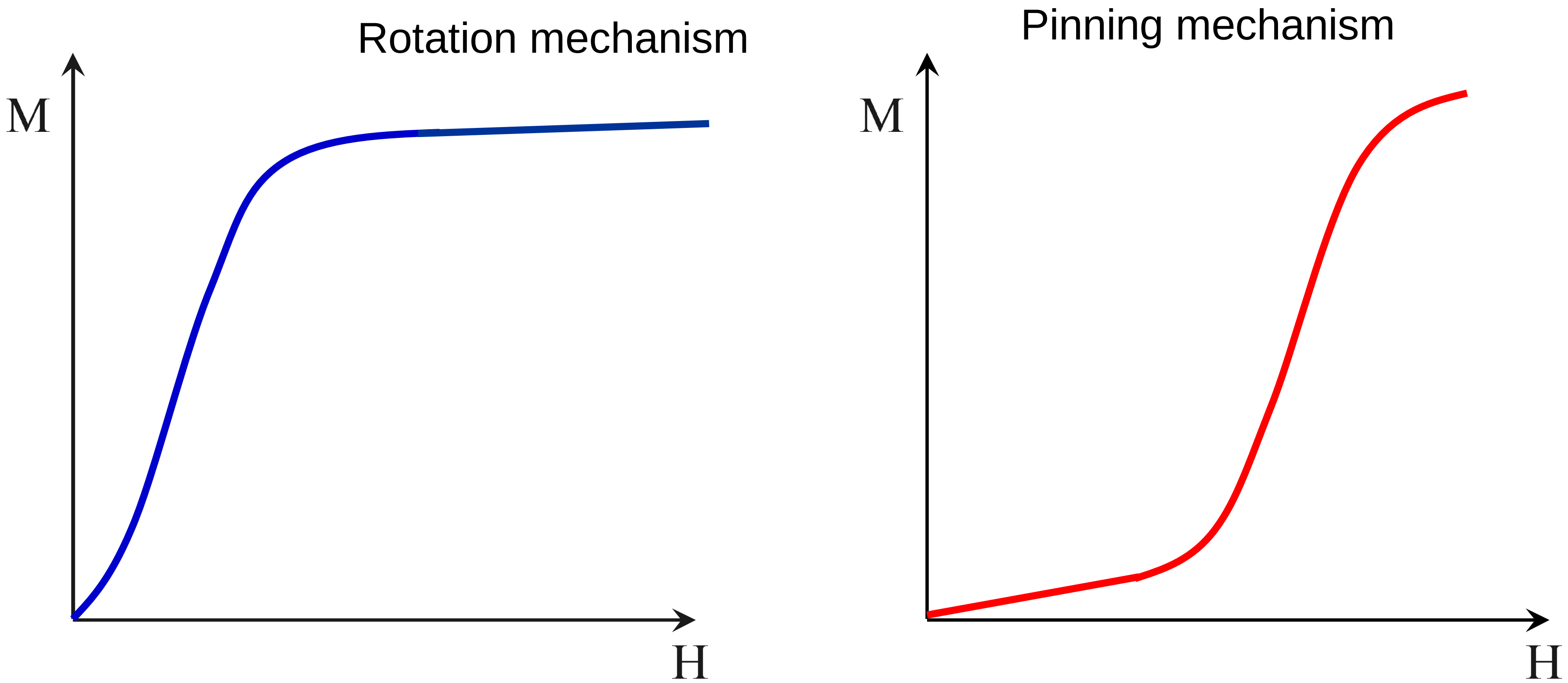
magnetic circuits

H_c – small~0,001÷10 A/m

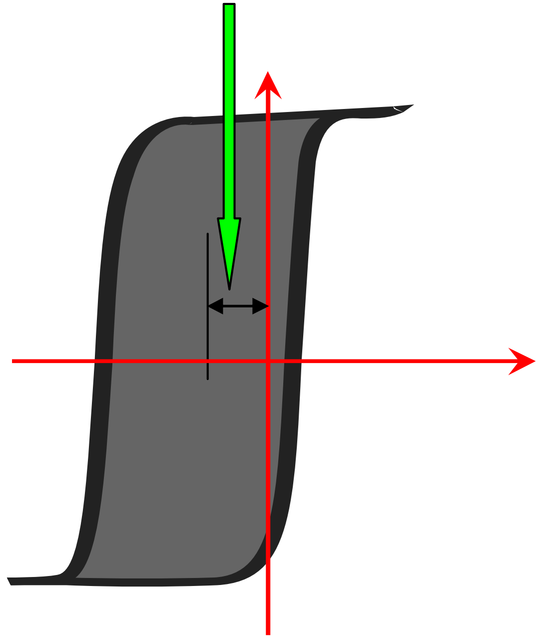
Curie temperature, T_c

Hard magnetic materials




Rotation mechanism
M
H
Pinning mechanism
M
H

Exchange bias field



E. Girgis et al, J. Appl. Phys. 97 (2005) 103911

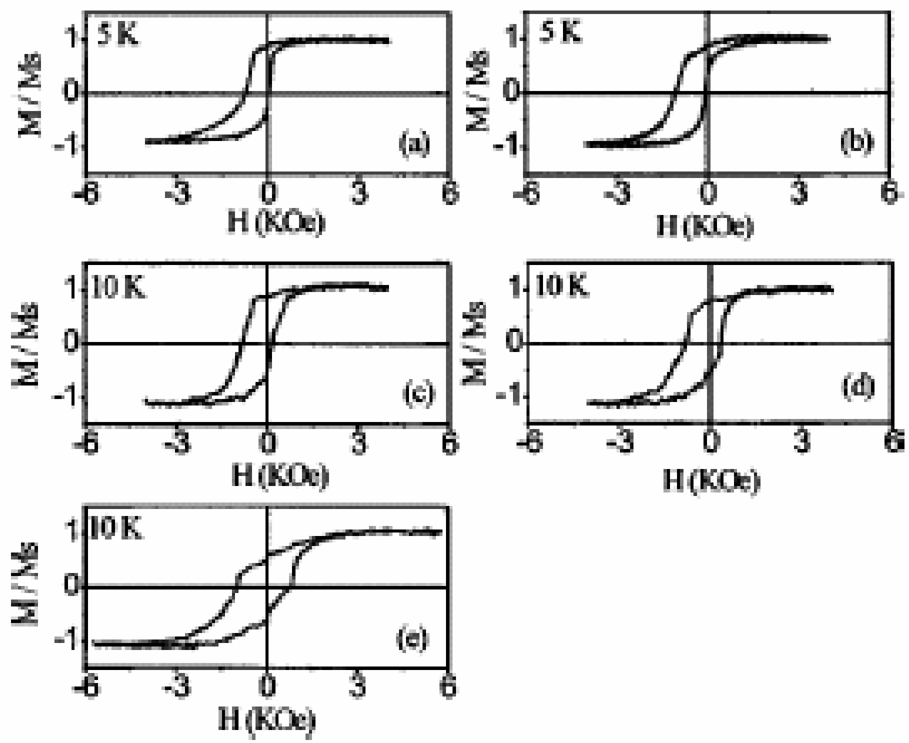
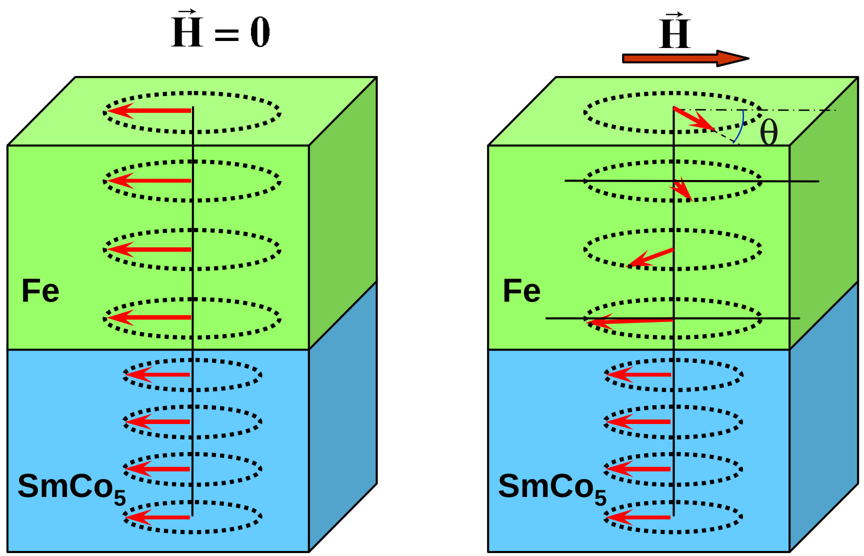
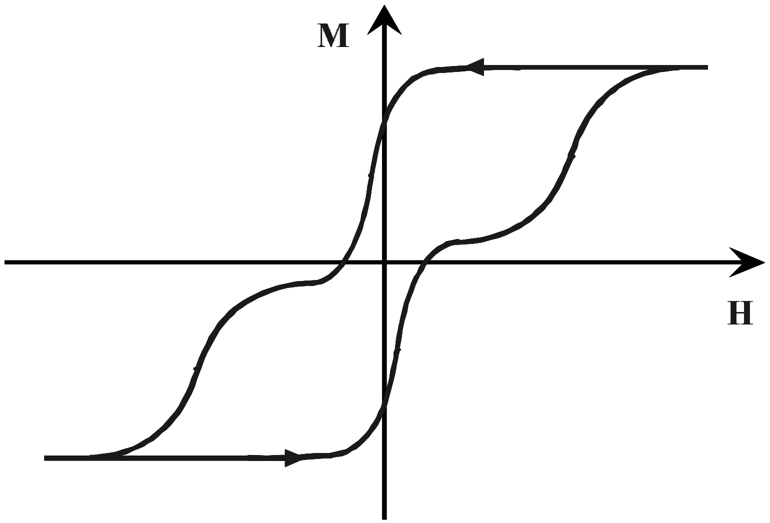
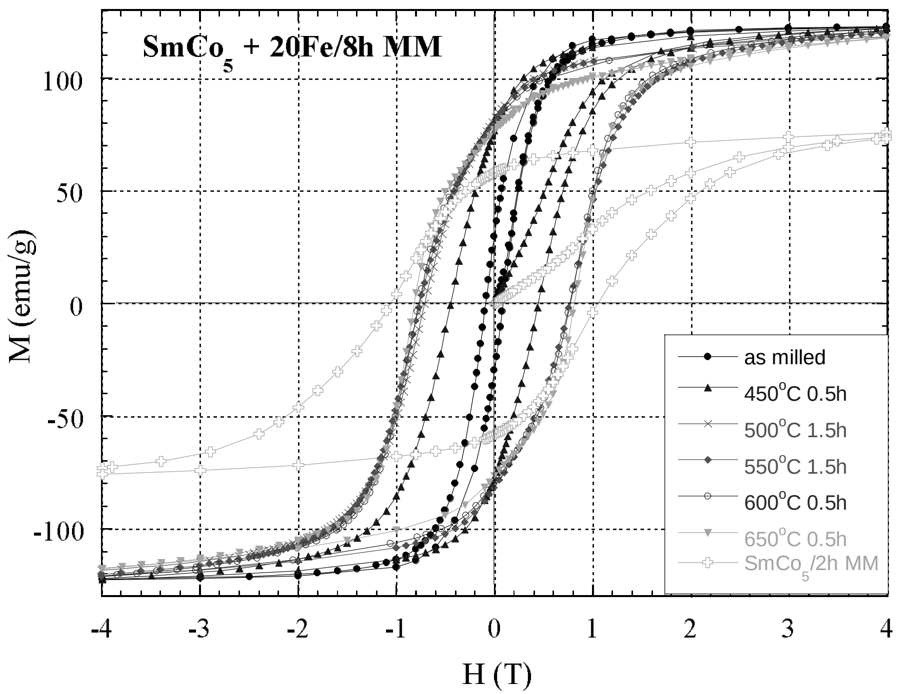


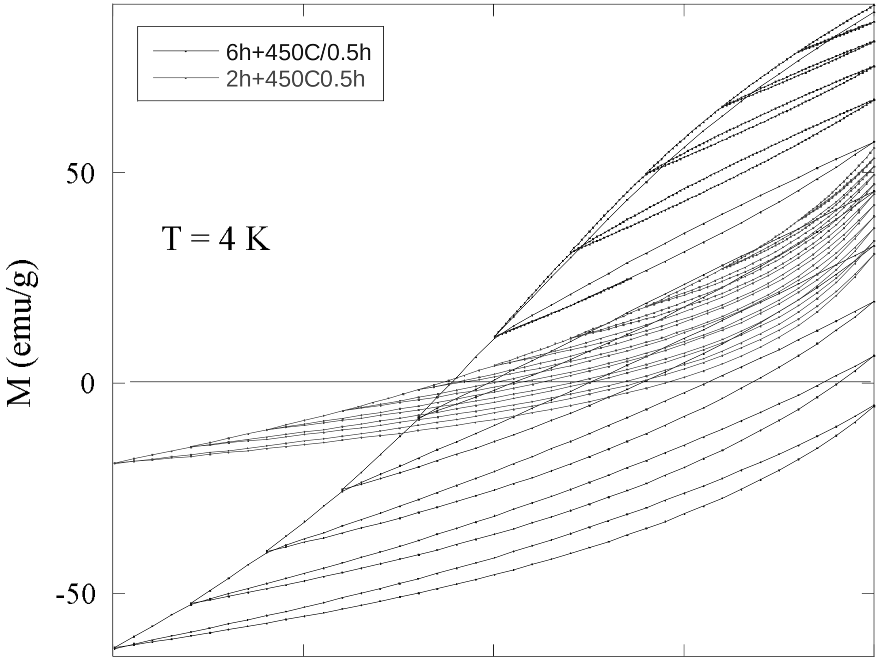
FIG. 2. Hysteresis loops of NiFe/CoO bilayer for (a) continuous film, a
patterned nanodots with dimensions of (b) 900×900, (c) 700×700, (
500×500, and (e) 300×300 nm^2.

Spring magnets

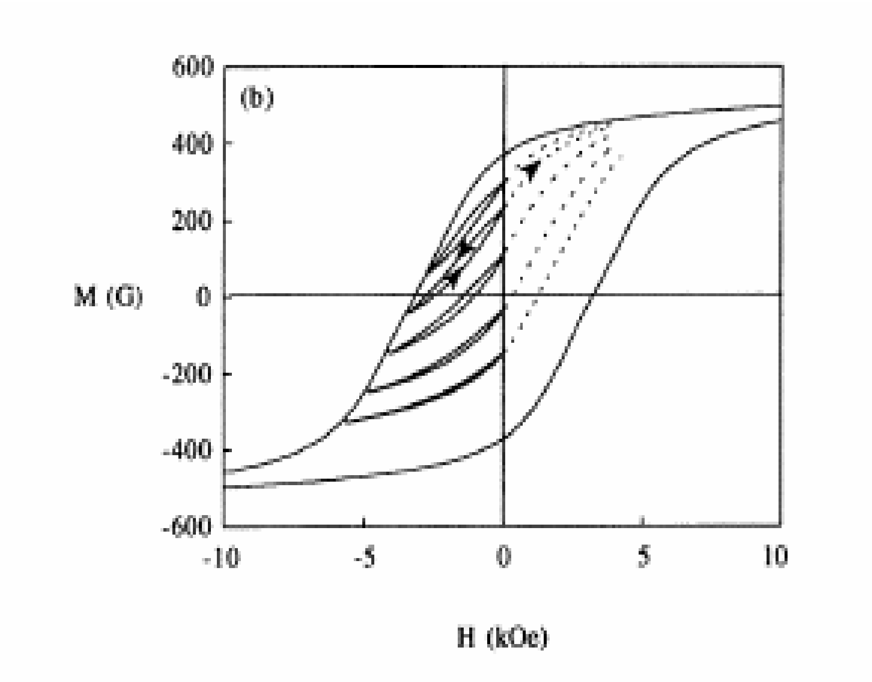


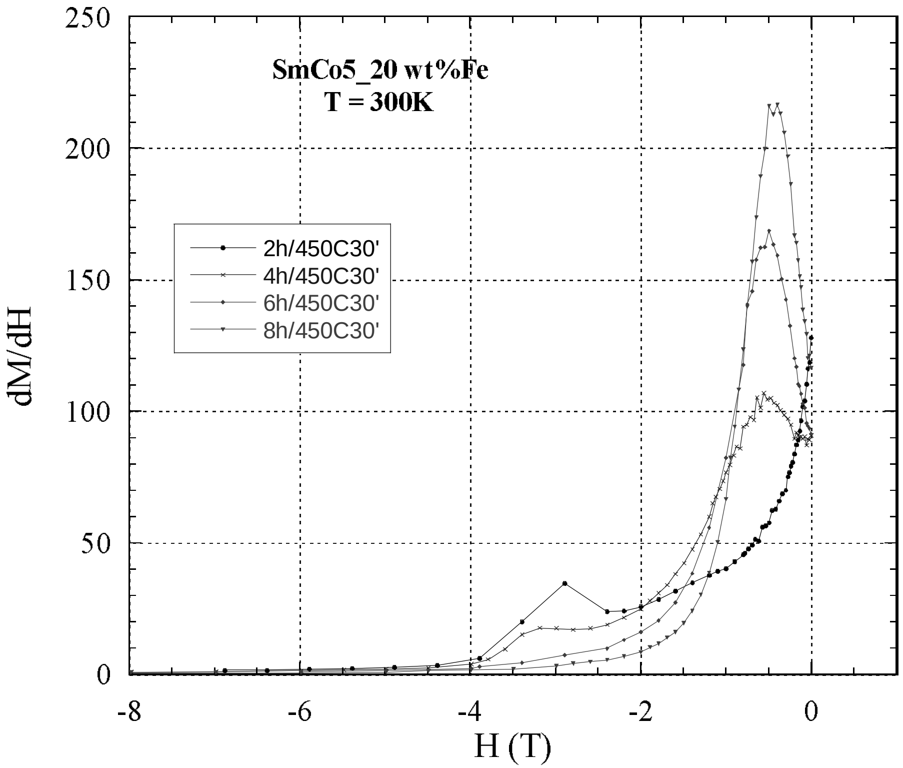


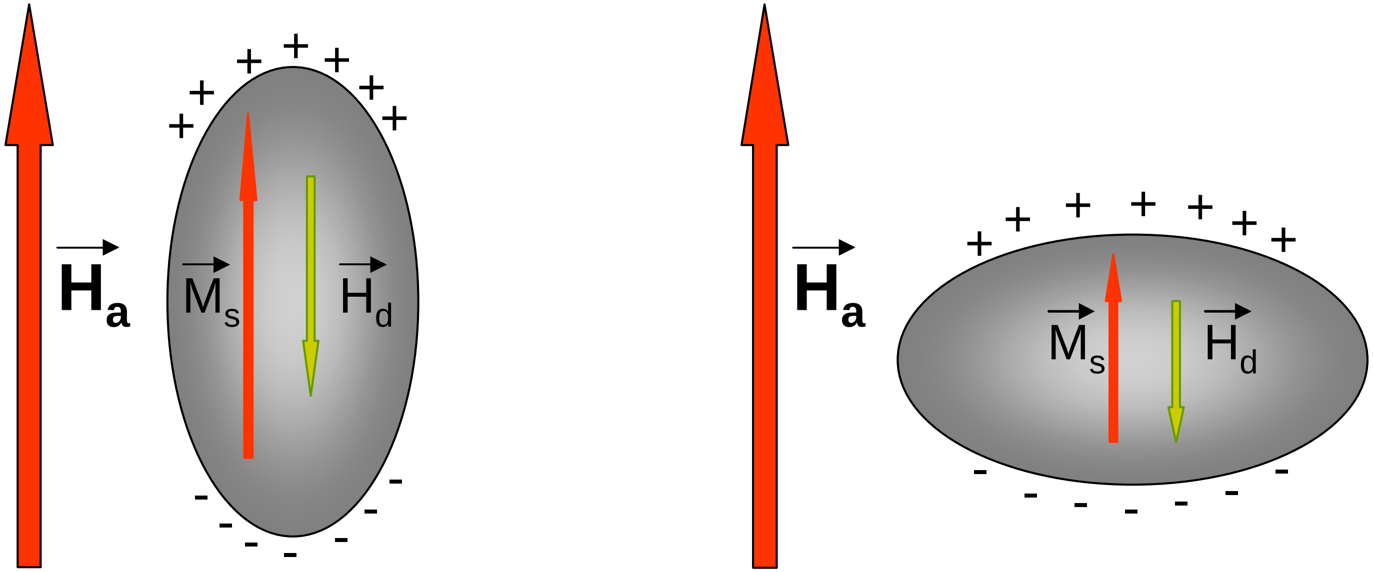




The <u>reversibility curves</u> and the <u>dM/dH variation</u> vs H are very fine instruments in qualitative evaluation of the inter-phase hard/soft exchange coupling.





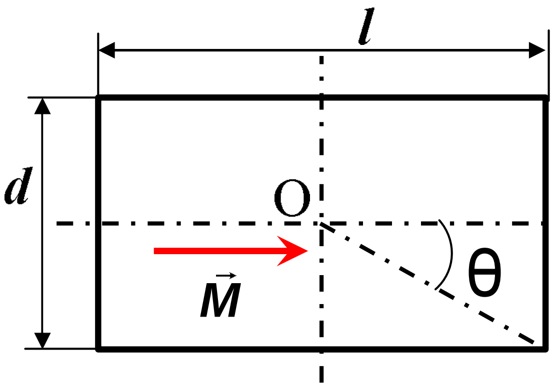


$$H_d = N_{||} M_s \Leftarrow \vec{H}_d = -N_d \vec{M} \Rightarrow H_d = N_{\perp} M_s$$

The influence of the demagnetising field on the magnetisation curves

$\vec{H}_d = N_d \vec{M}$ ⇒ $\vec{H} = \vec{H}_i = \vec{H}_a + \vec{H}_d$ H_a = applied field

sphere ⇒ $N_{dx} = N_{dy} = N_{dz} = 1/3.$



⇒ $\vec{H}_d = -(1 - \cos\theta) \cdot \vec{M}$

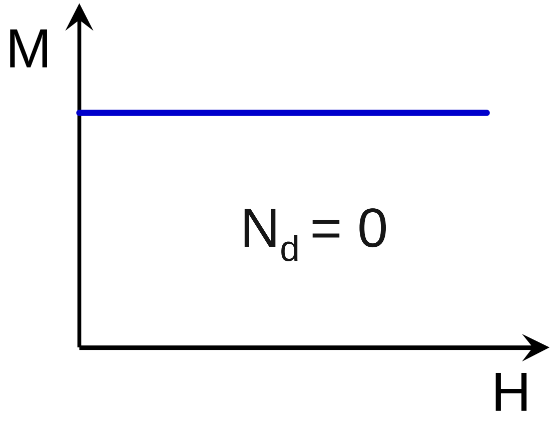
$d << l$ ⇒ $N_d = 0$

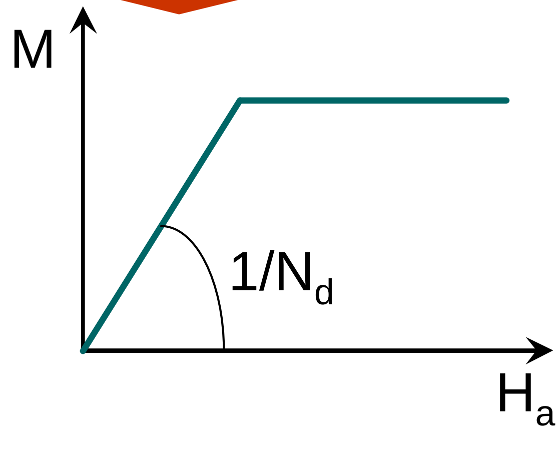
$d >> l$ ⇒ $N_d = -1$

$$M = \chi H = \chi (H_a + H_d) = \chi (H_a - N_d M)$$

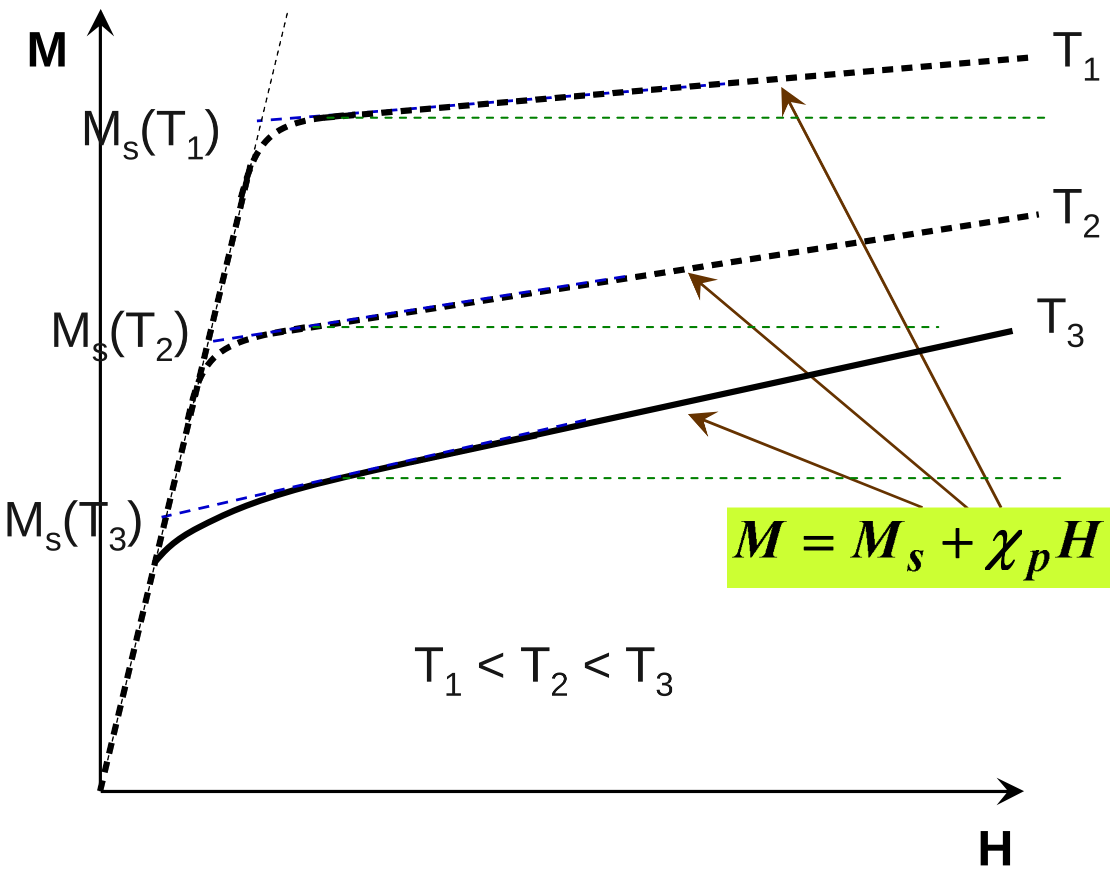
$$M = \frac{\chi}{1 + N_d \chi} H_a$$

H II easy magnetisation direction



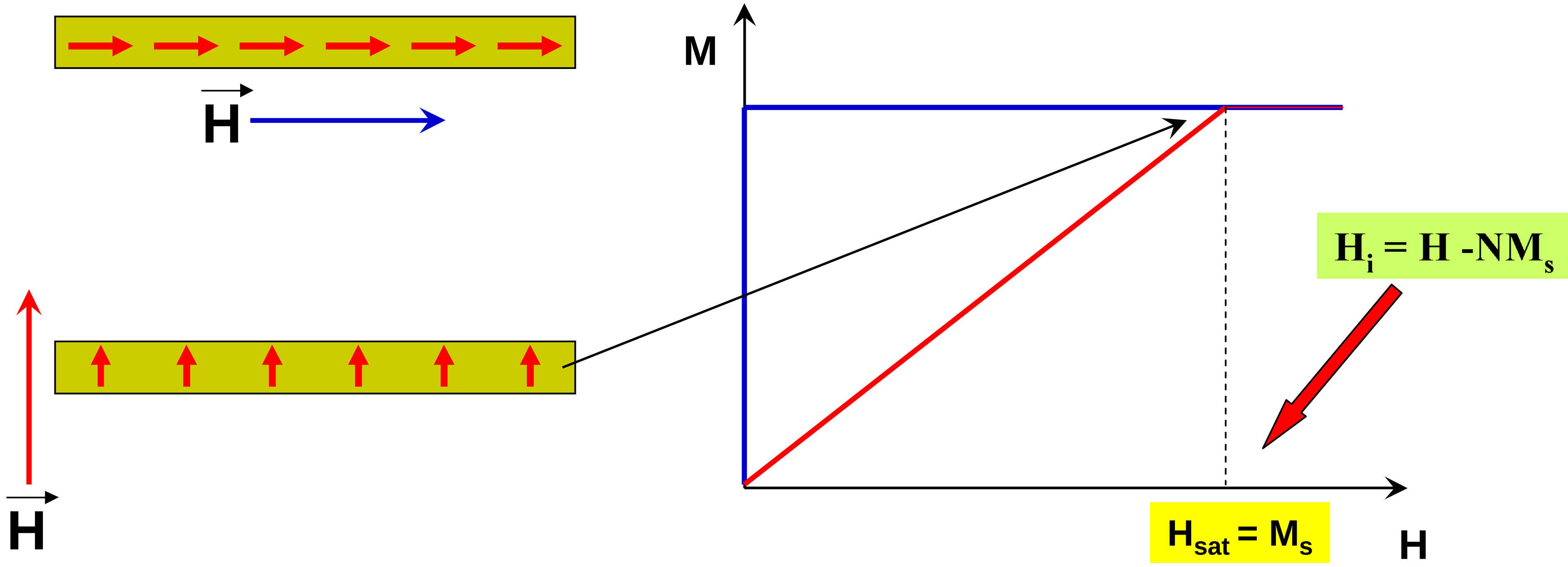


$H_d \neq 0$



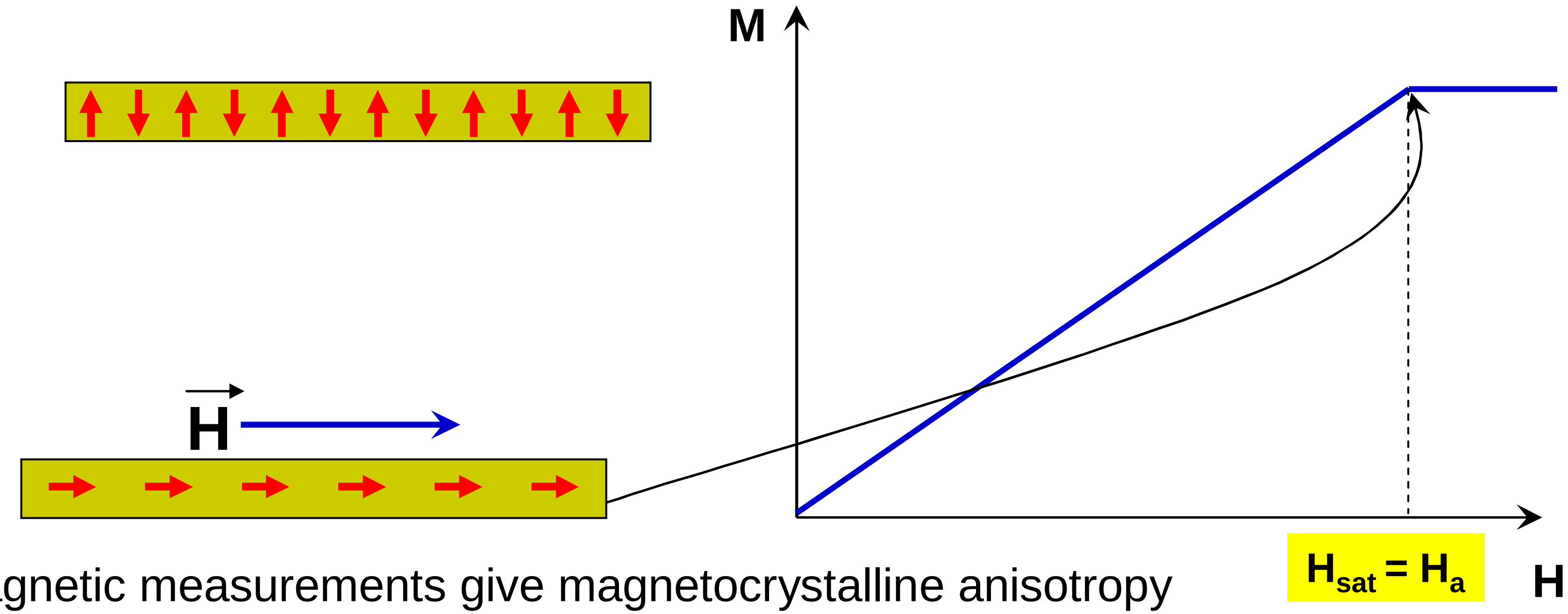
magnetic measurements on *plate shape samples*

NO MAGNETOCRYSTALLINE ANISOTROPY

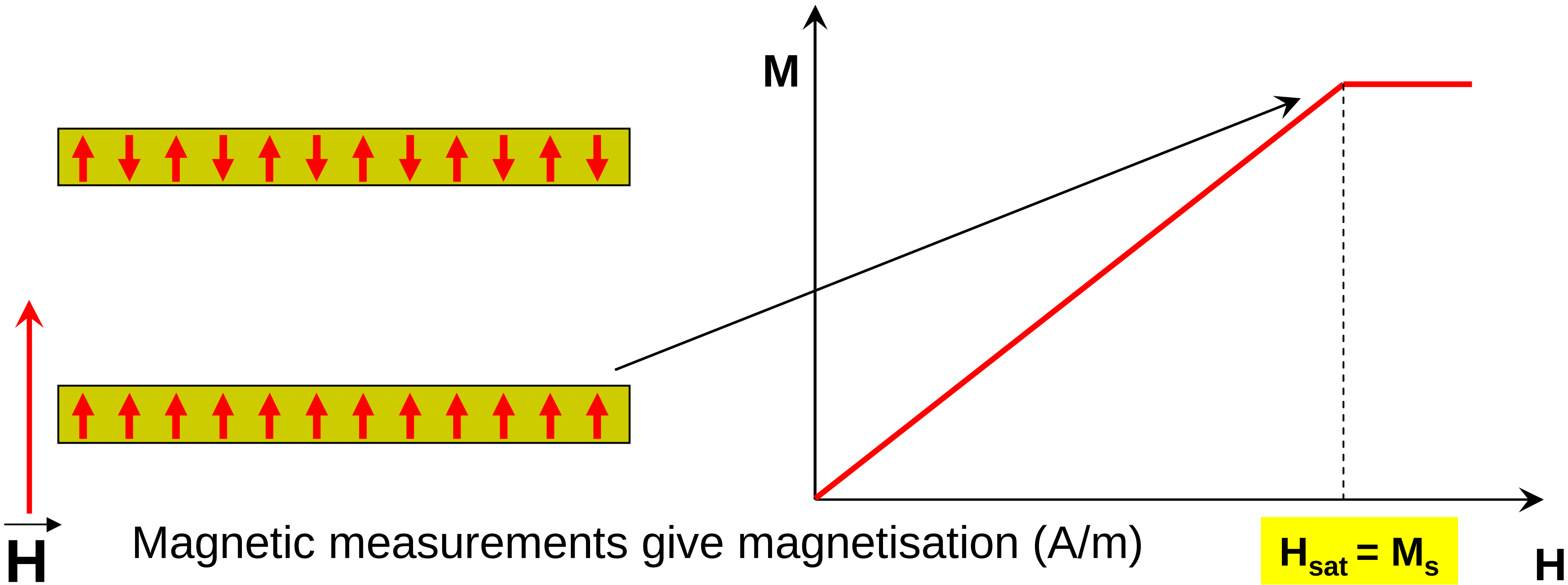


Magnetic measurements give magnetisation (A/m)

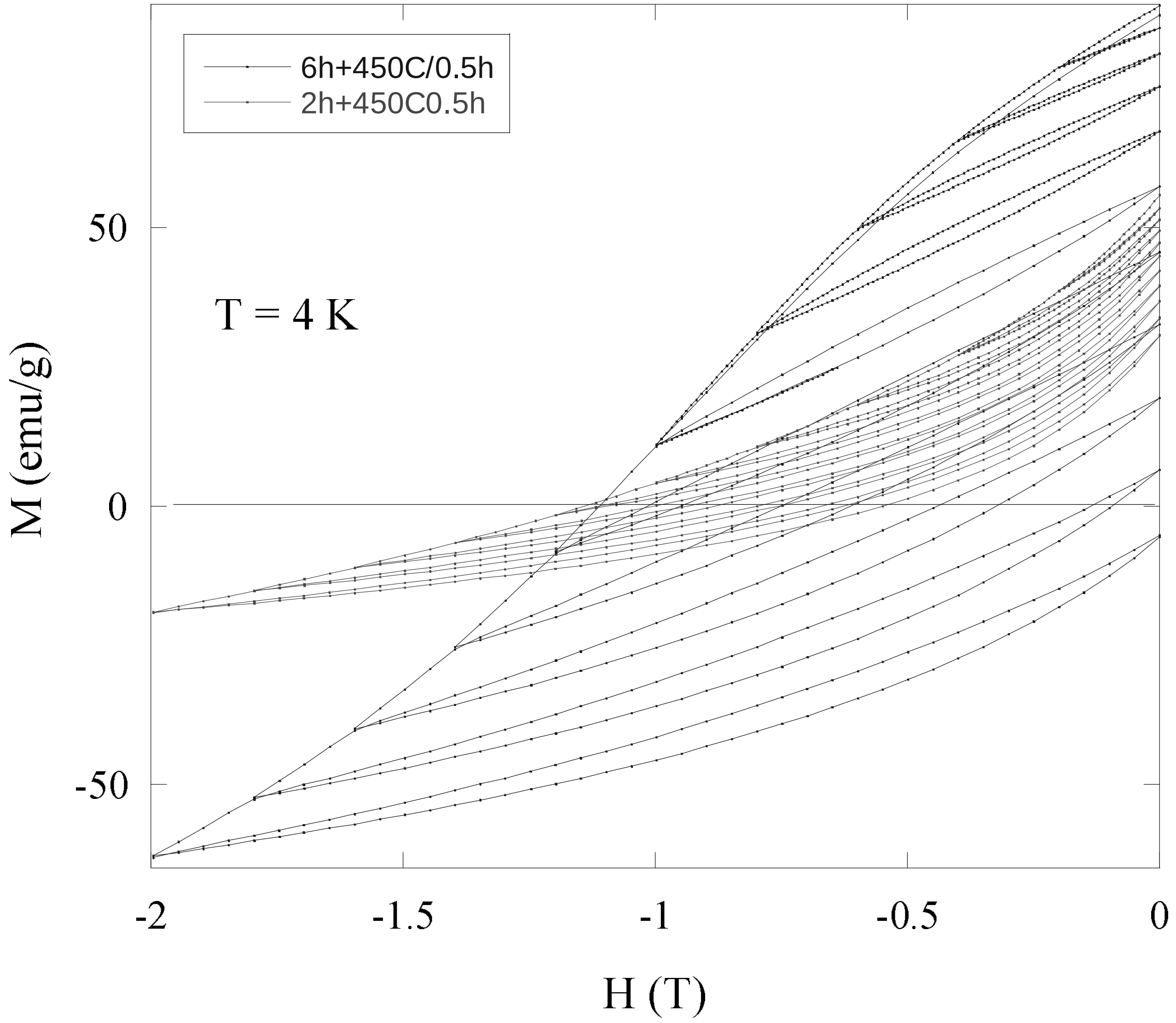
PERPENDICULAR ANISOTROPY

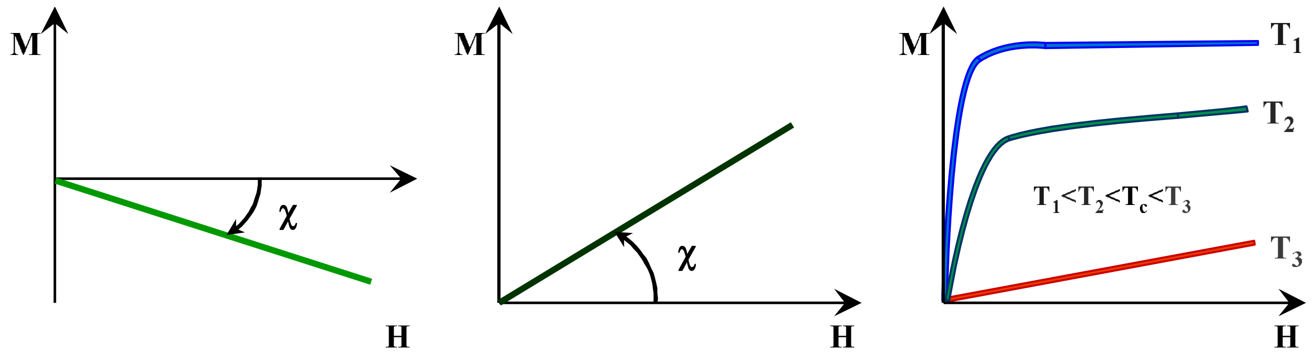


Magnetic measurements give magnetocrystalline anisotropy

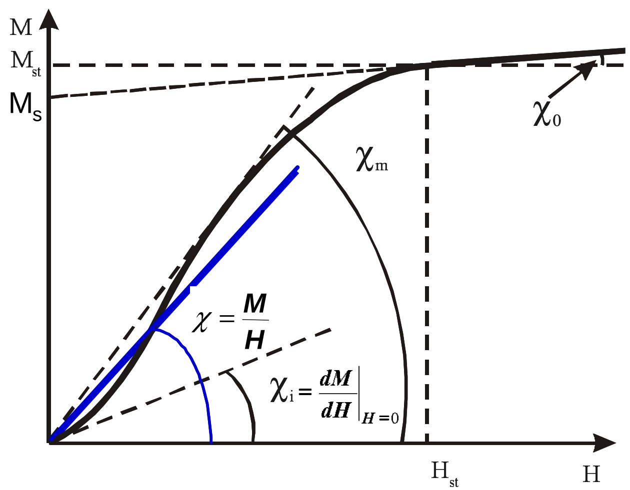


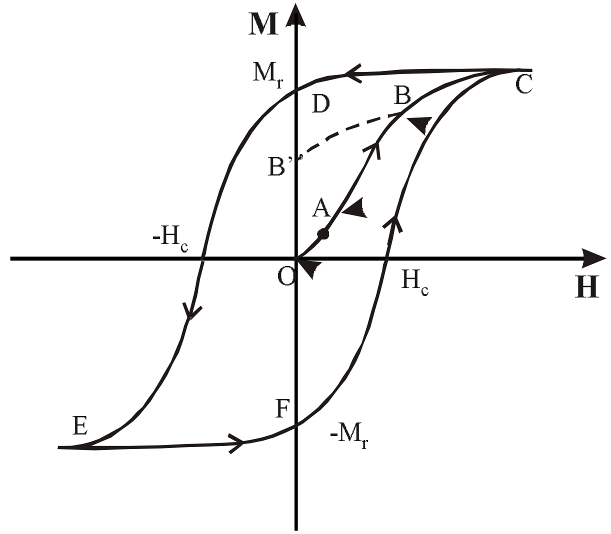
Magnetic measurements give magnetisation (A/m)

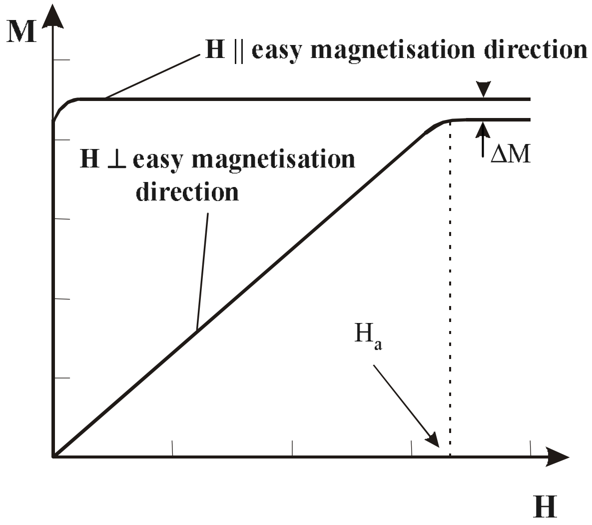

6h+450C/0.5h
2h+450C0.5h
T = 4 K
M (emu/g)
50
0
-50
-2
-1.5
-1
-0.5
0
H (T)

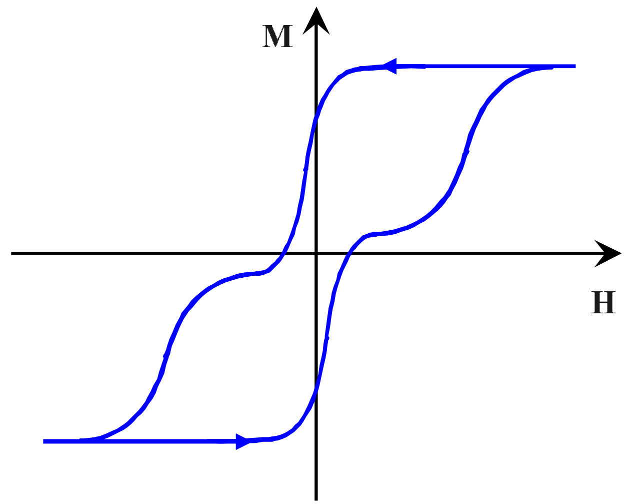

M
H
χ
M
H
χ
M
H
T1
T2
T3
T1<T2<Tc<T3

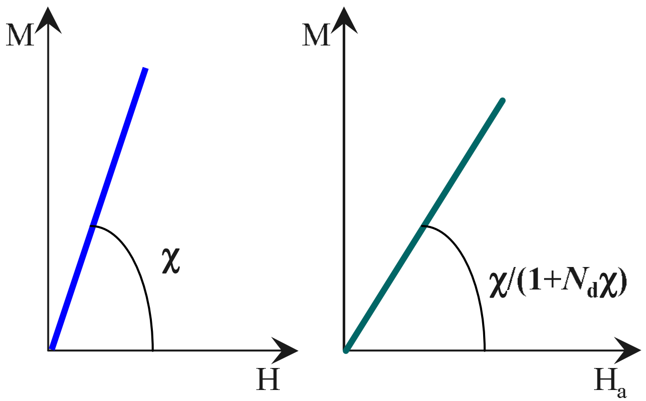
(a) (b) (c)

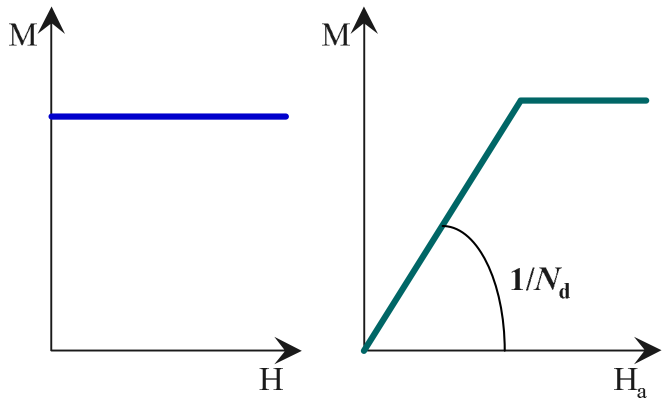

M
Mst
Ms
χ0
χm
χ = M/H
χi = dM/dH|H=0
Hst
H

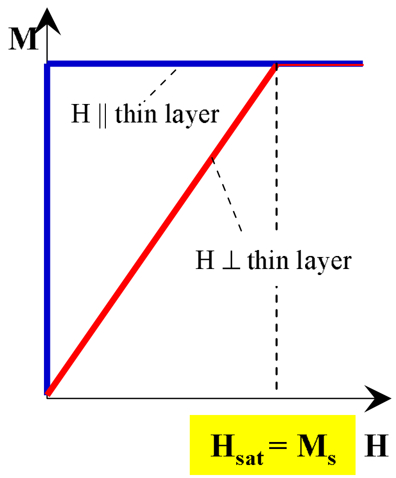

M
Mr
D
B
C
B'
A
-Hc
O
Hc
H
F
E
-Mr

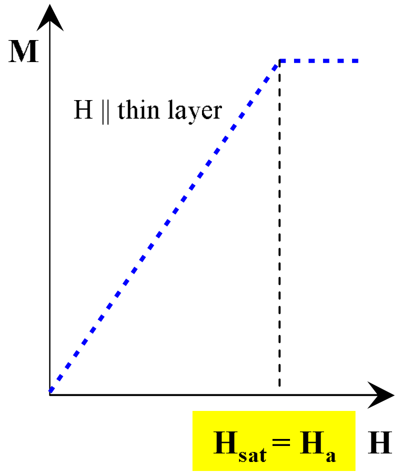

M
H || easy magnetisation direction
H ⊥ easy magnetisation direction
ΔM
H_a
H

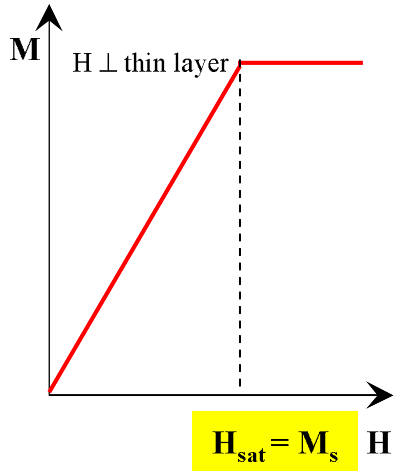

M
H

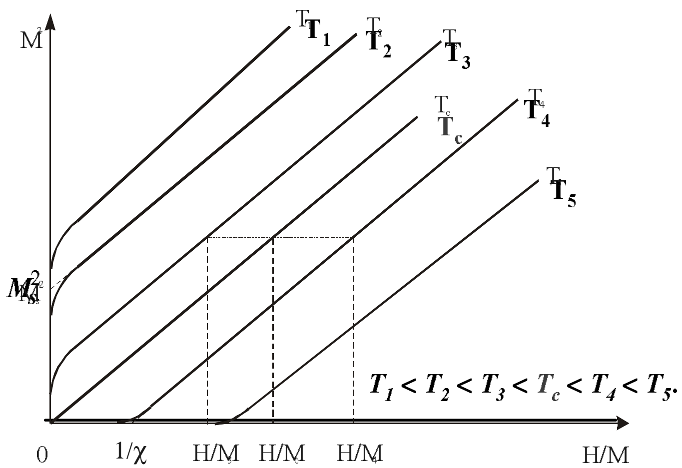

M
χ
H
M
χ/(1+N_dχ)
H_a

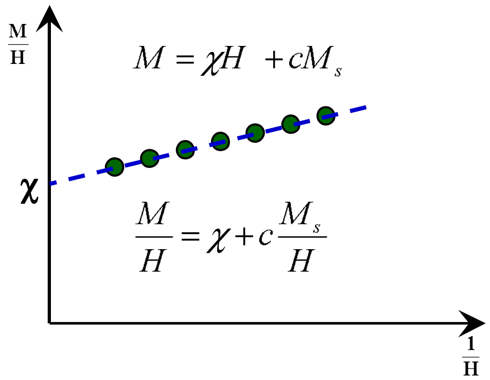

M
H
M
1/N_d
H_a

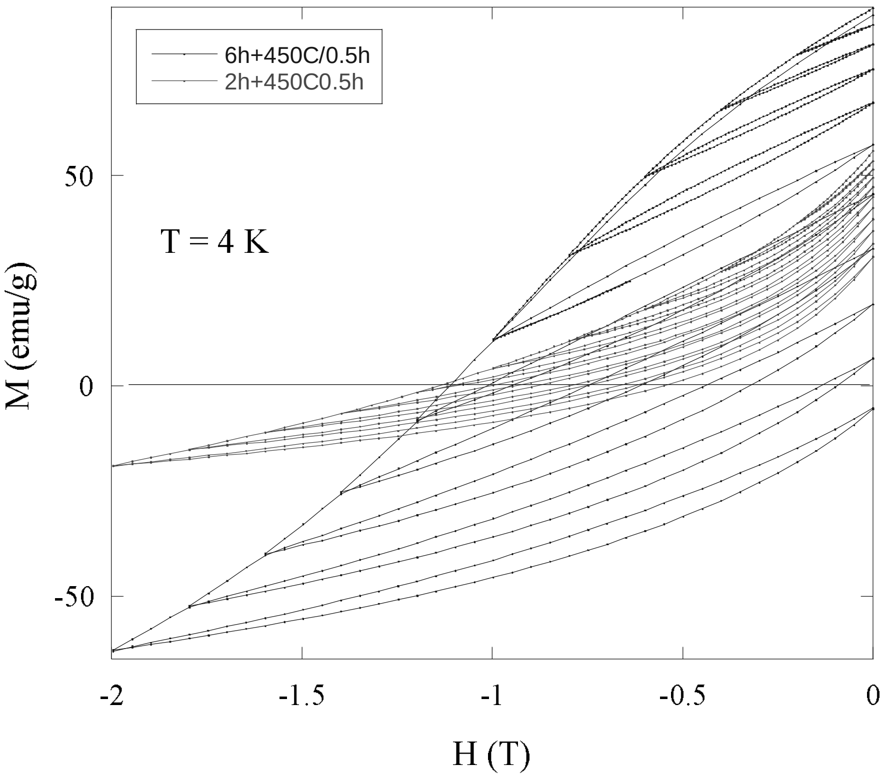

M
H || thin layer
H ⊥ thin layer
$H_{sat} = M_s$
H


M
H || thin layer
$H_{sat} = H_a$
H


M
H ⊥ thin layer
$H_{sat} = M_s$
H


M
T_1
T_2
T_3
T_c
T_4
T_5
$T_1 < T_2 < T_3 < T_c < T_4 < T_5$.
0
1/χ
H/M
H/M
H/M
H/M


$\frac{M}{H}$
$M = \chi H + cM_s$
χ
$\frac{M}{H} = \chi + c\frac{M_s}{H}$
$\frac{1}{H}$



The <u>reversibility curves</u> and the <u>dM/dH variation</u> vs H are very fine instruments in qualitative evaluation of the inter-phase hard/soft exchange coupling.