European School on Magnetism

New Magnetic Materials and their Functions

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Coherent Transport and Spin Effects in Quantum Dots

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Outline

1. Kondo resonance
2. Quantum interference in nanostructures
    - Fano resonance
    - Aharonov-Bohm effect
3. Many body effects in double dot systems
4. Summary

Kondo resonance

in bulk

The theory that describes the scattering of electrons from a localized magnetic impurity was initiated by the work of Jun Kondo in 1964

Minimum resistance

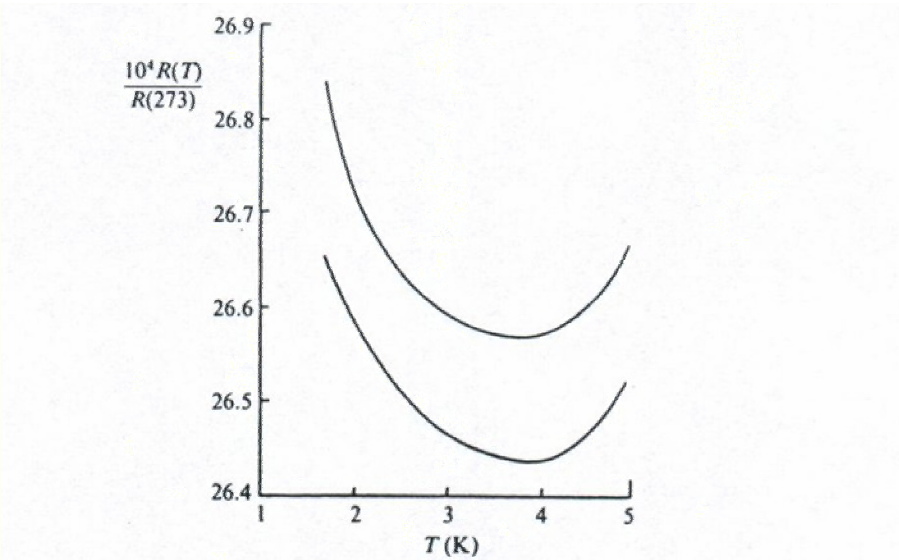


Figure 1 The minimum in the electrical resistivity of *Au* (de Haas, de Boer and van den Berg, 1934).

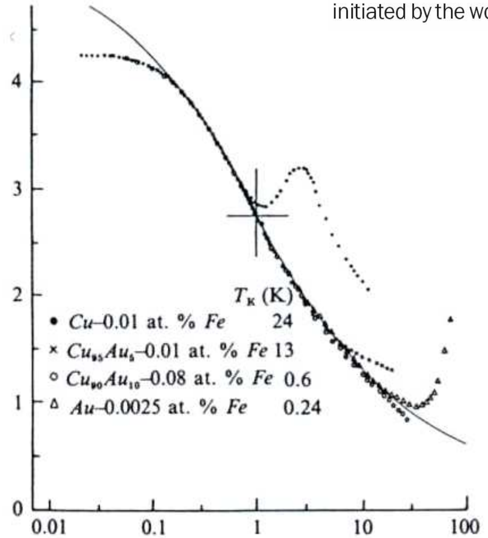


Figure 9.24 Temperature dependence of the impurity resistivity of Fe in Cu_xAu_{1-x} alloys normalized to a Kondo temperature T_K. The full line is a fit to the Hamann result (3.12) with $S = 0.77$ (Loram, Whall & Ford, 1970).

From the Boltzmann theory of the electrical resistivity

$$\rho = \frac{m}{ne^2}\frac{1}{\tau}$$

Kondo model (s-d model)

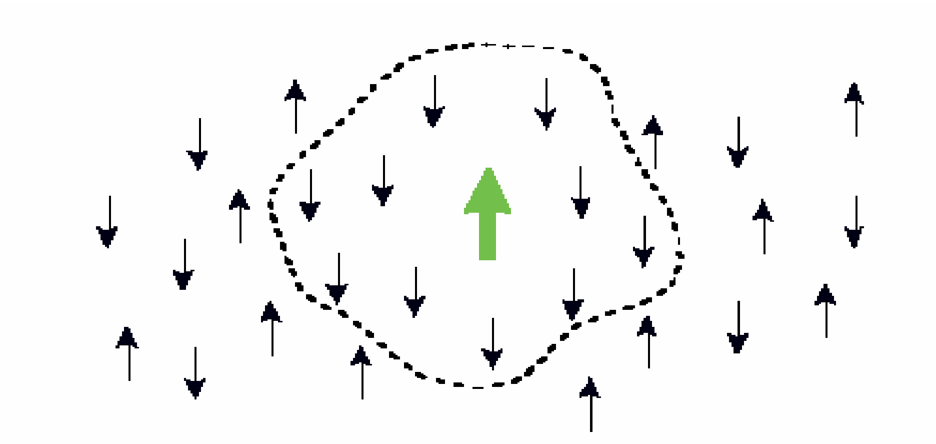
$$H_{Kondo} = \sum_{k\sigma} \varepsilon_k c^+_{k\sigma} c_{k\sigma} + J\vec{S} \cdot \sum_{kk'\sigma\sigma'} c^+_{k\sigma} \vec{\sigma}_{\sigma\sigma'} c_{k'\sigma'}$$

calulation of the relaxation time

For low temperatures

$$\rho_{spin} = \frac{3\pi m J^2 S(S+1)}{2e^2 \hbar E_F}[1 - 4J\rho(E_F)\ln(\frac{k_B T}{W})]$$

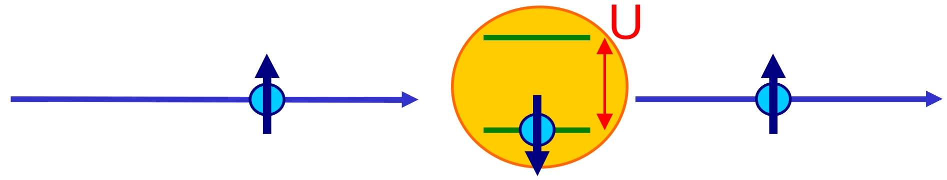
J<0



Kondo state

cloud of spins of conducting electrons screens the localized spin

Single impurity Anderson model



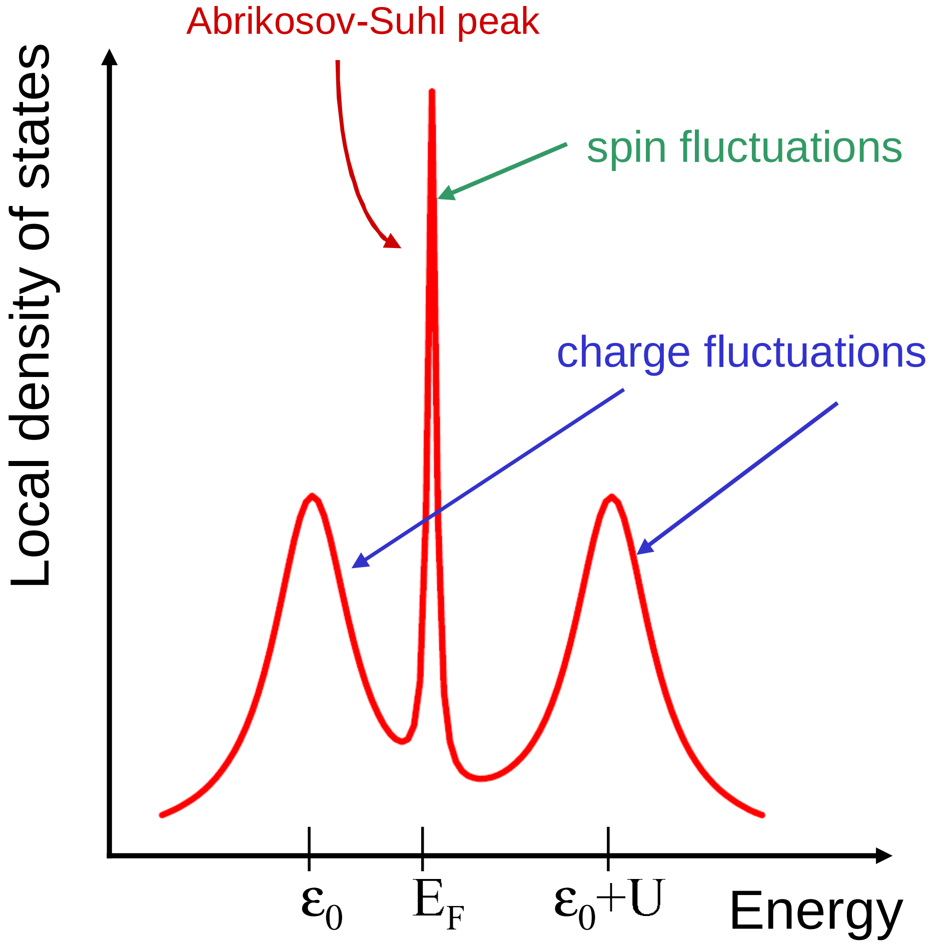
$$H_{Anderson} = \sum_{k\sigma} \varepsilon_k c^+_{k\sigma} c_{k\sigma} + \varepsilon_0 \sum_{\sigma} c^+_{0\sigma} c_{0\sigma} + U\, c^+_{0\uparrow} c_{0\uparrow} c^+_{0\downarrow} c_{0\downarrow}$$
$$+ \sum_{k\sigma} V_k (c^+_{0\sigma} c_{k\sigma} + c^+_{k\sigma} c_{0\sigma})$$

neglecting charge fluctuations

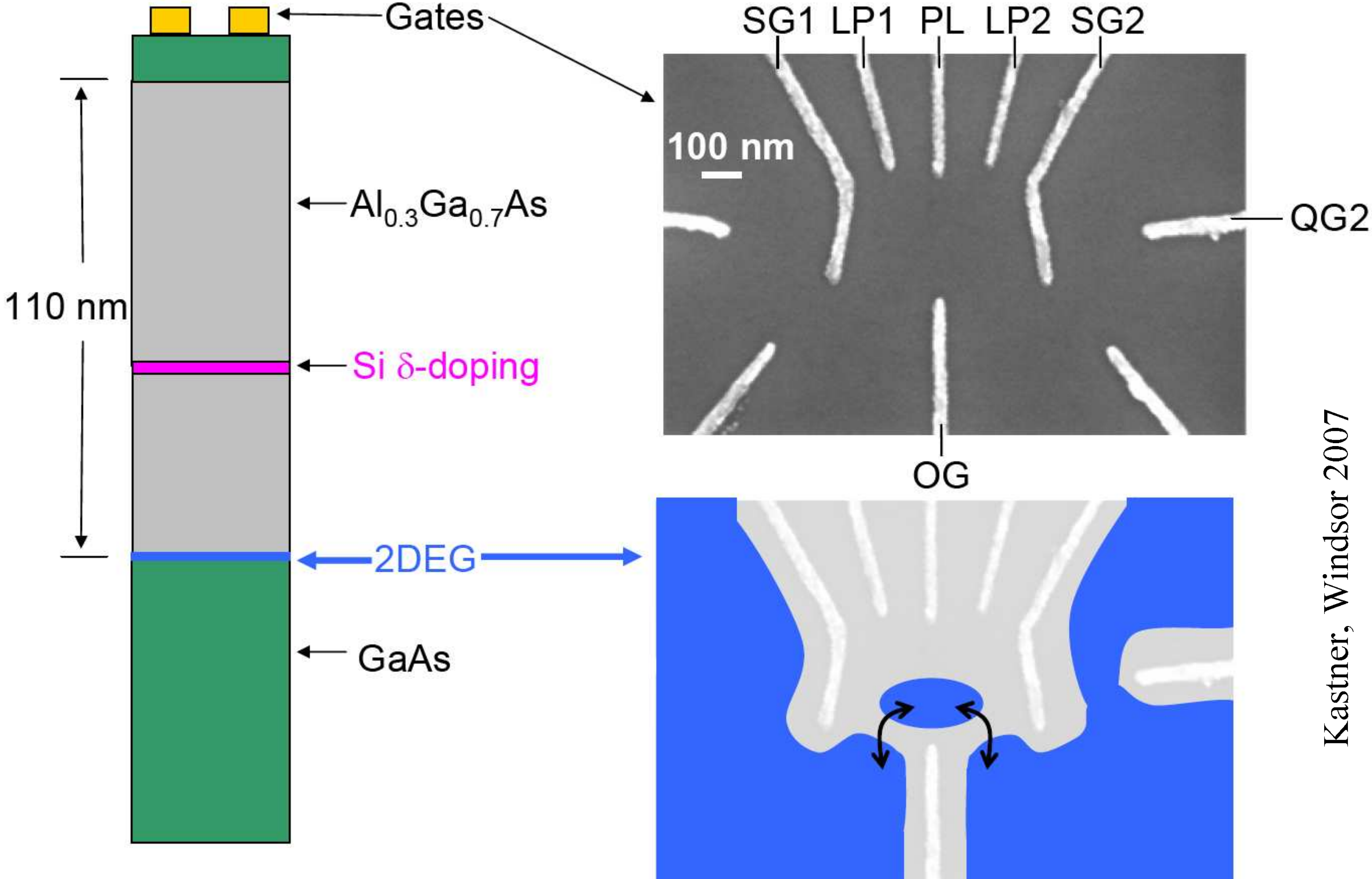
s-d model

with the effective exchange interaction

$$J_{kk'} \approx -|V_k|^2 \frac{U}{|\varepsilon_0|(U - |\varepsilon_0|)}$$



Laterally Gated Quantum Dots



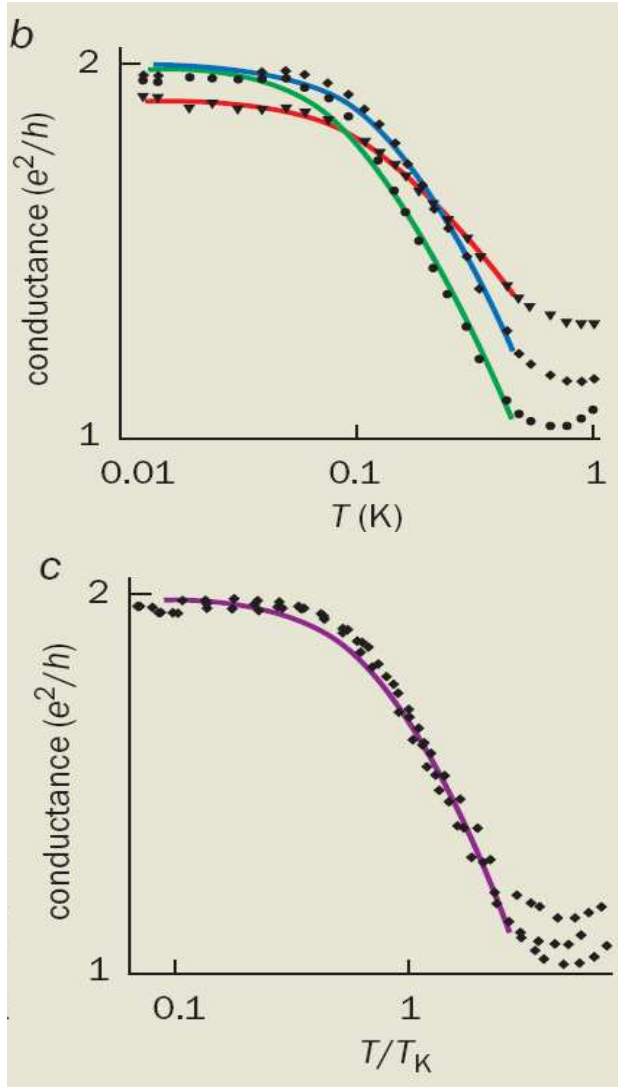
Kastner, Windsor 2007

n= 2.2×10^{11} cm^{-2}
μ= $6.4\text{x}10^5$ $cm^2/V\bullet s$

Can trap just one electron
Ciorga et al. PRB **61**, 16315 (2000)

Kondo resonance in nanostructures

Increase of conductance for $T \to 0$



Landauer approach

current

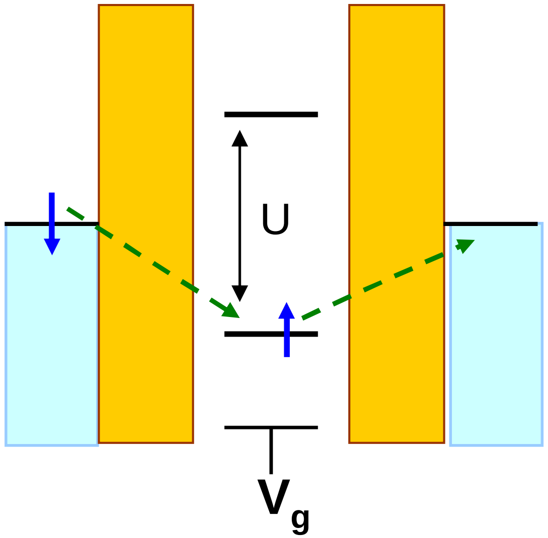
$$J = \frac{2e}{h} \int dE \; T(E) \, [f_L(E) - f_R(E)]$$

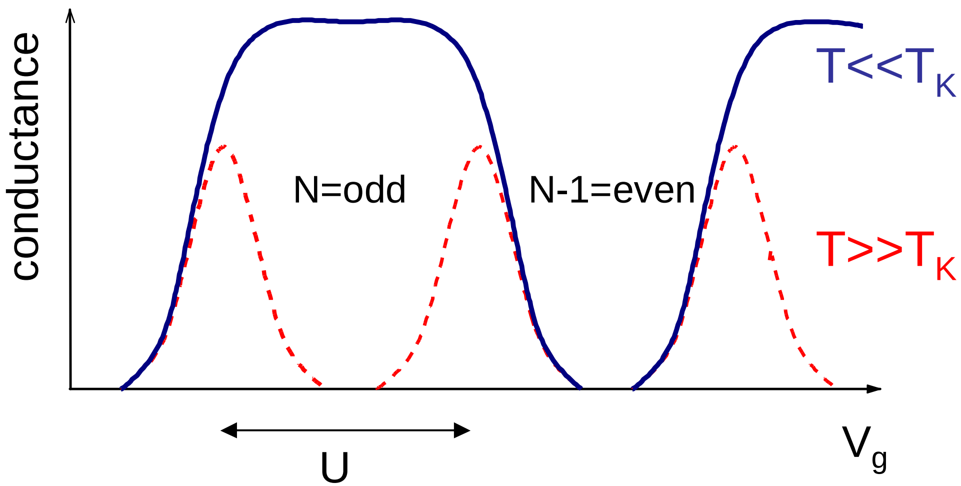
conductance (for $V_{SD} \to 0$)

$$\mathcal{G} = \frac{2e}{h} \int dE \left(-\frac{\partial f(E)}{\partial E}\right) T(E)$$

where $T(E)$ is a transmission

Coulomb blockade and Kondo effect

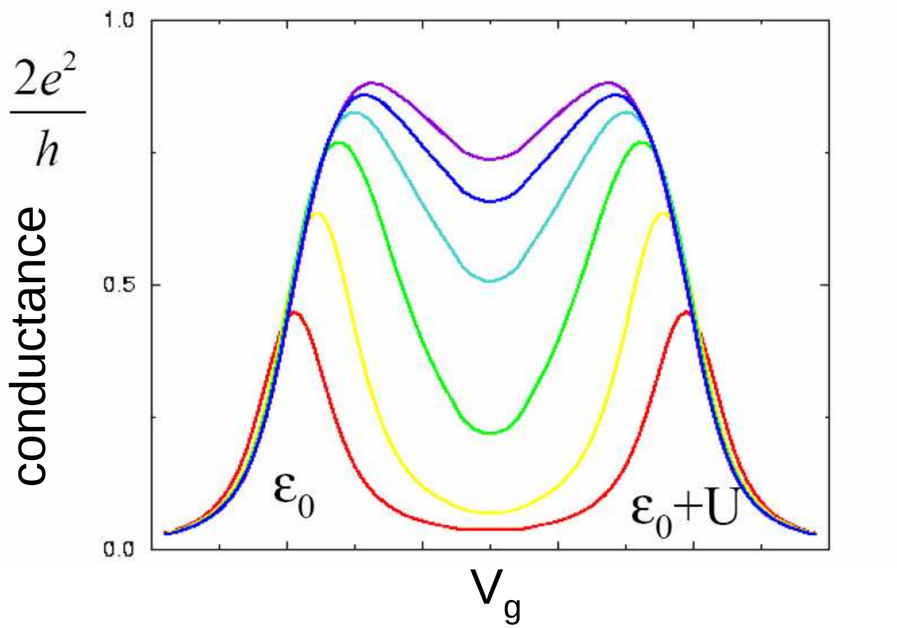




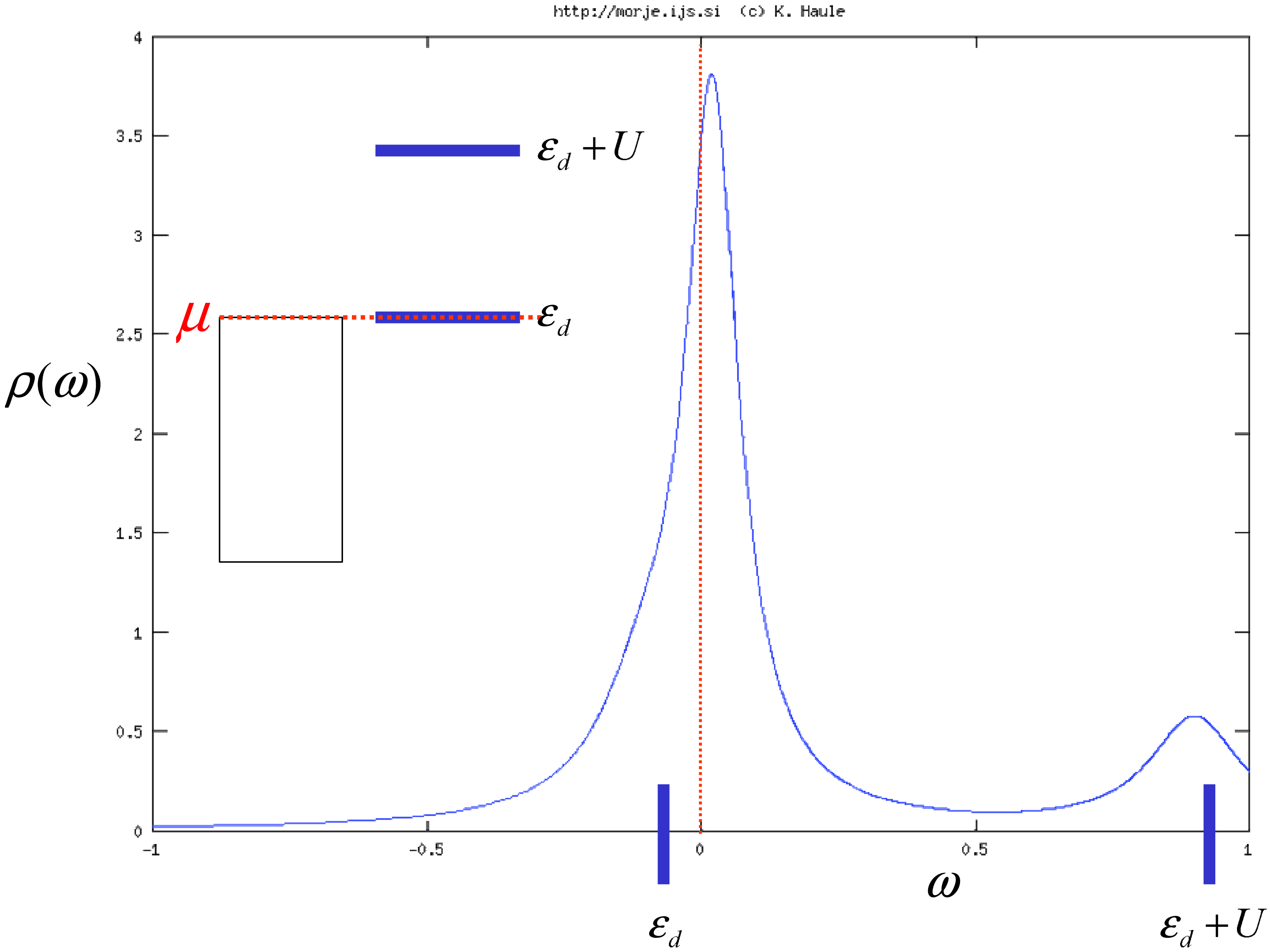
Anderson model

$$H = \sum_{k\sigma} \varepsilon_k c^+_{k\sigma} c_{k\sigma} + \sum_{k,\alpha=L,R,\sigma} t_\alpha (c^+_{0\sigma} c_{k\alpha\sigma} + c^+_{k\alpha\sigma} c_{0\sigma})$$

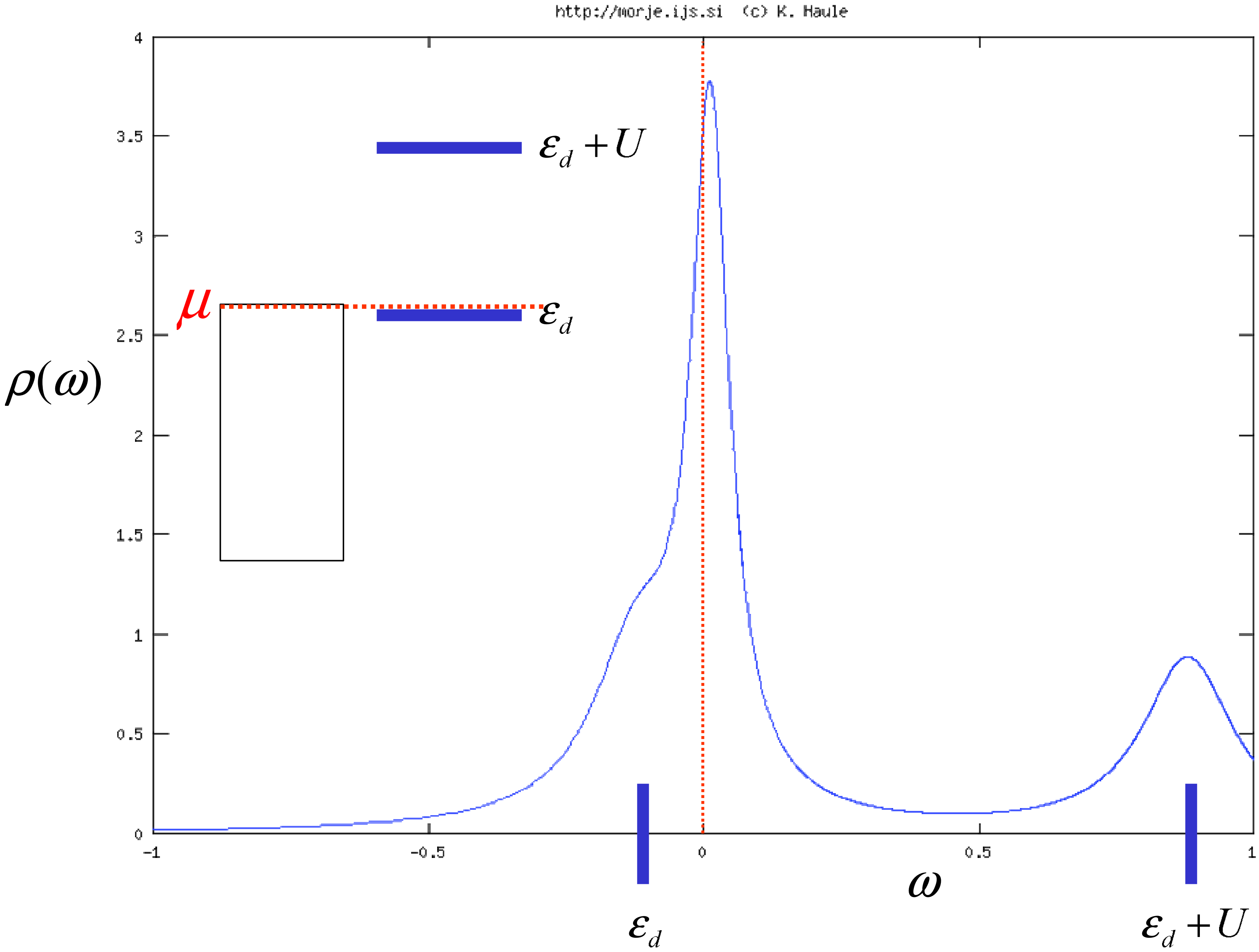
$$+ \varepsilon_0 \sum_{\sigma} c^+_{0\sigma} c_{0\sigma} + U_0 \; c^+_{0\uparrow} c_{0\uparrow} c^+_{0\downarrow} c_{0\downarrow}$$



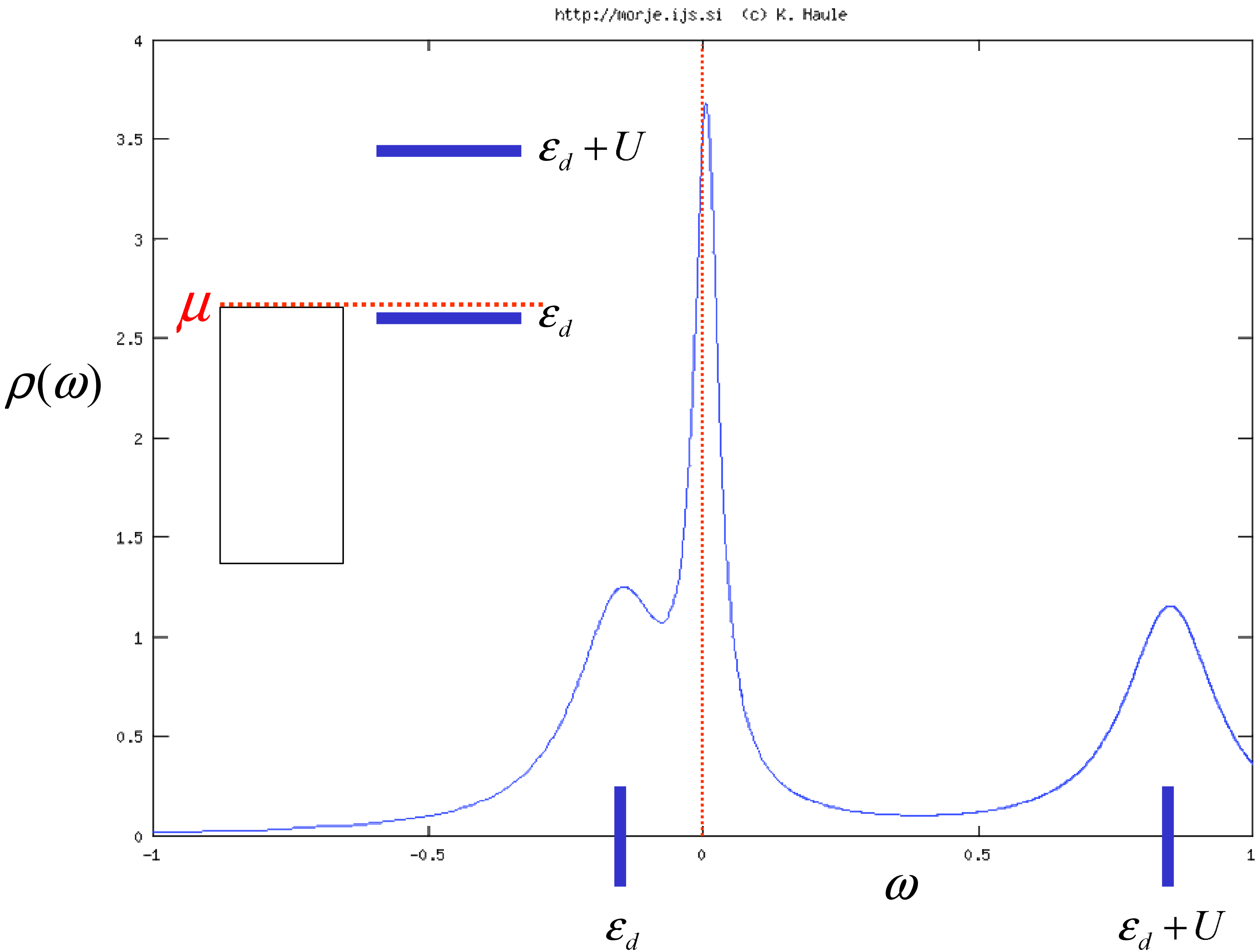
Transformation of the density of states with the gate voltage



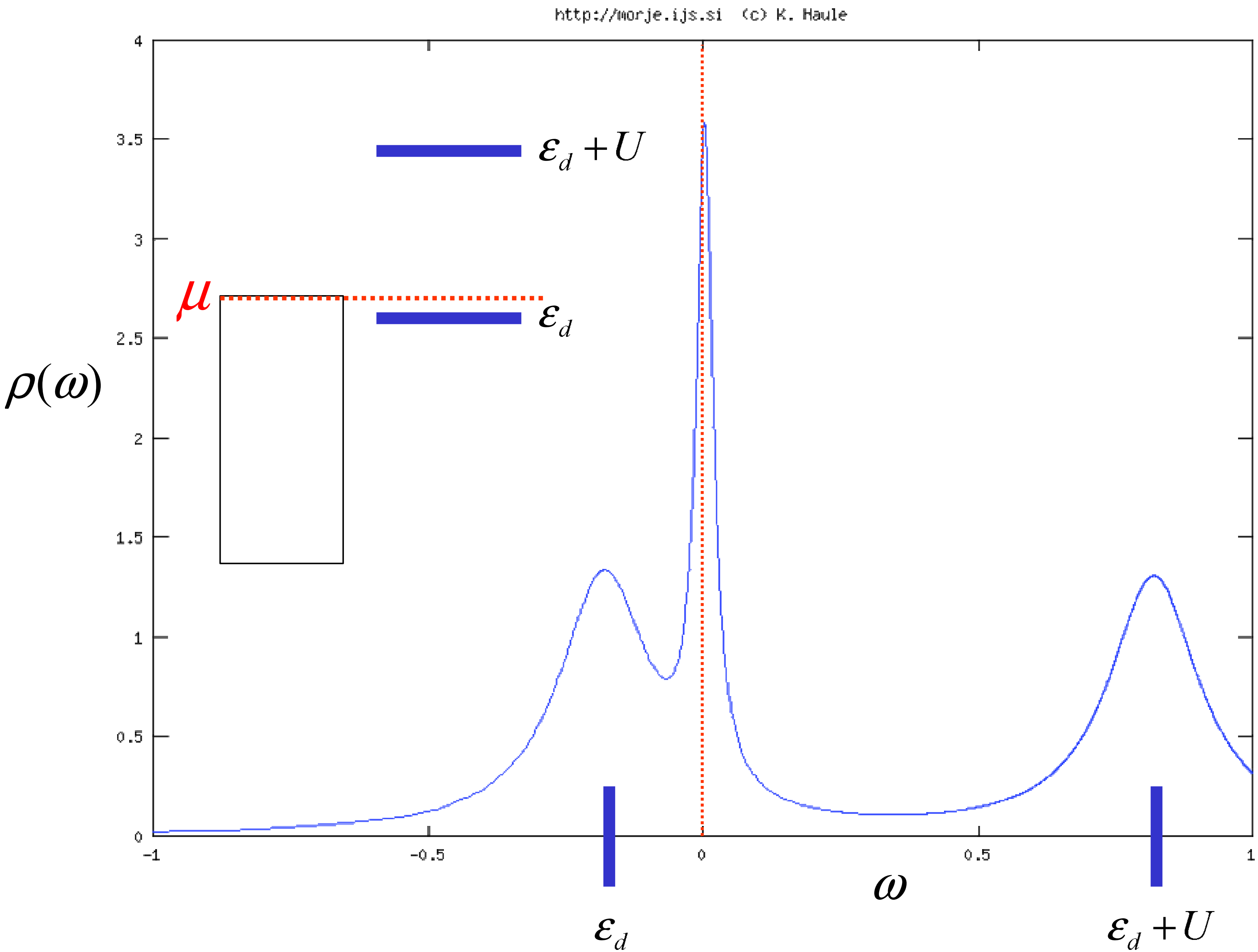
Transformation of the density of states with the gate voltage



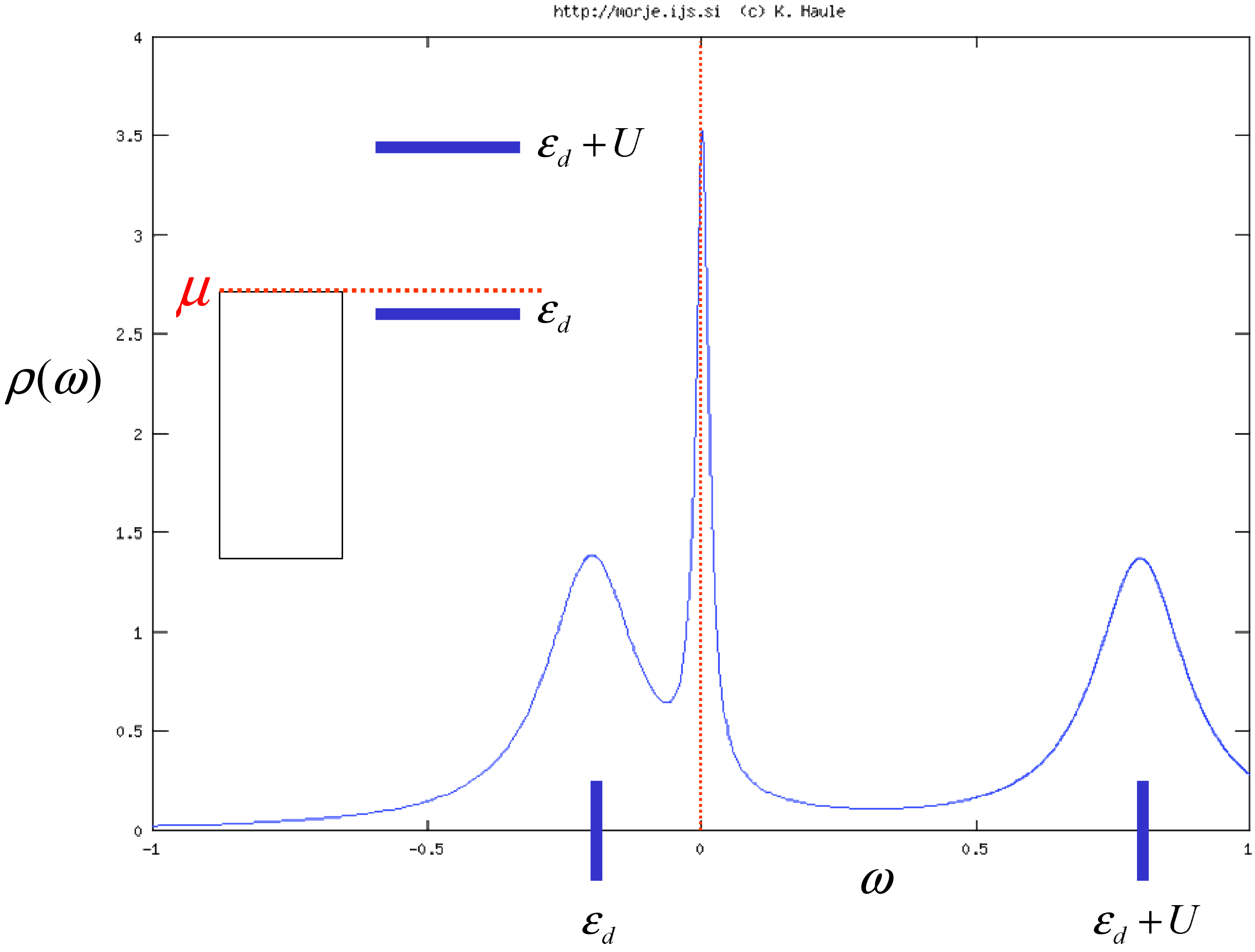
Transformation of the density of states with the gate voltage



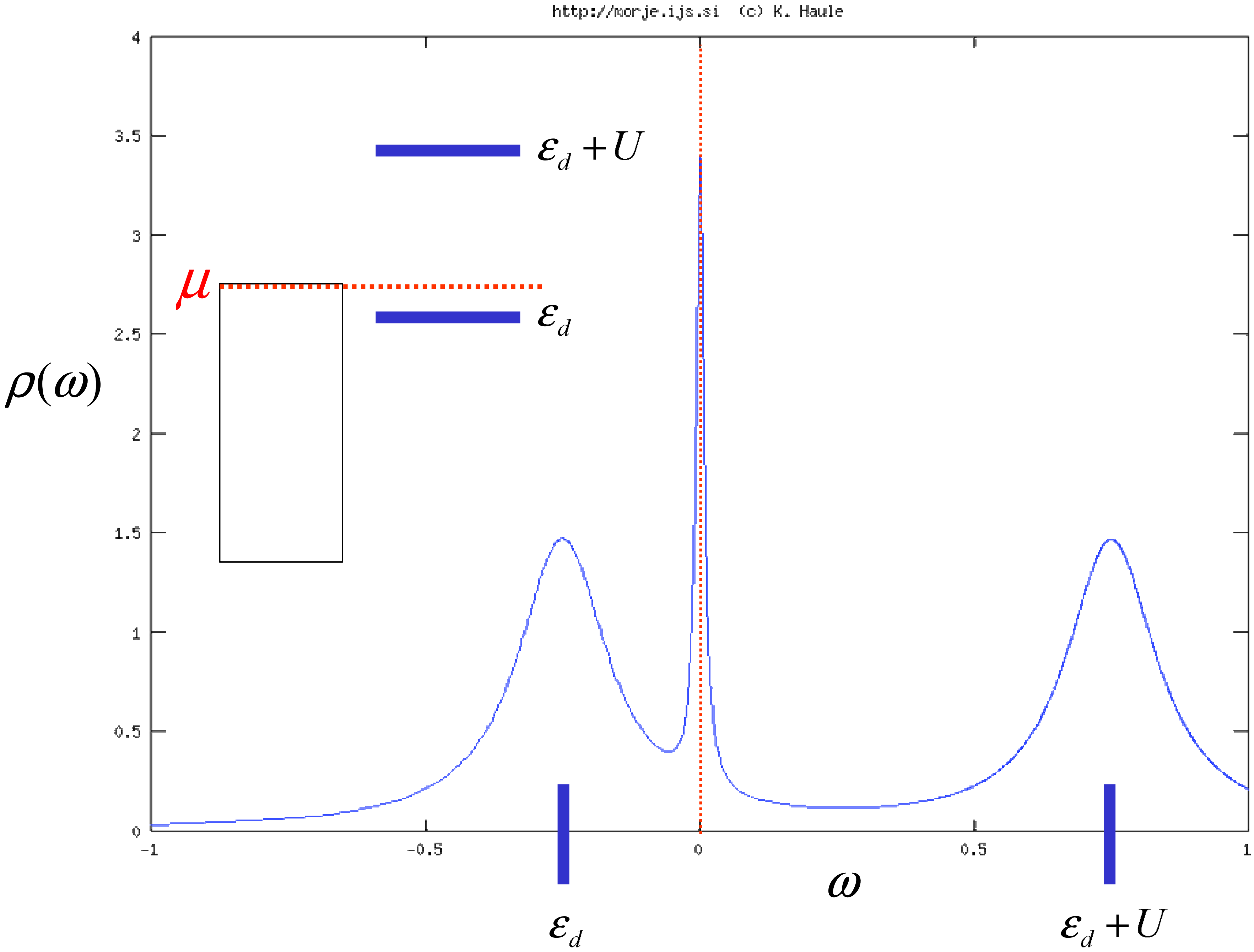
Transformation of the density of states with the gate voltage



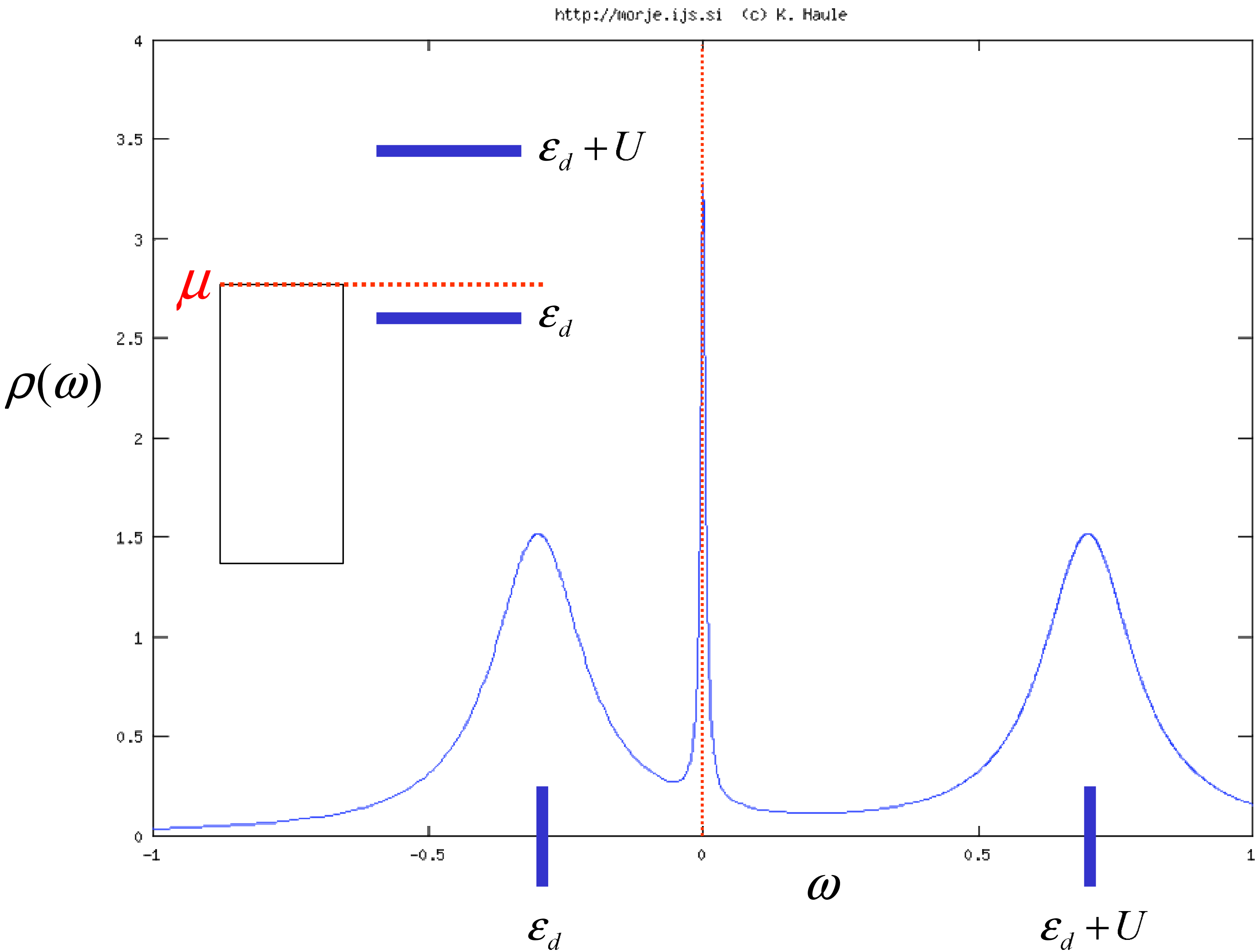
Transformation of the density of states with the gate voltage



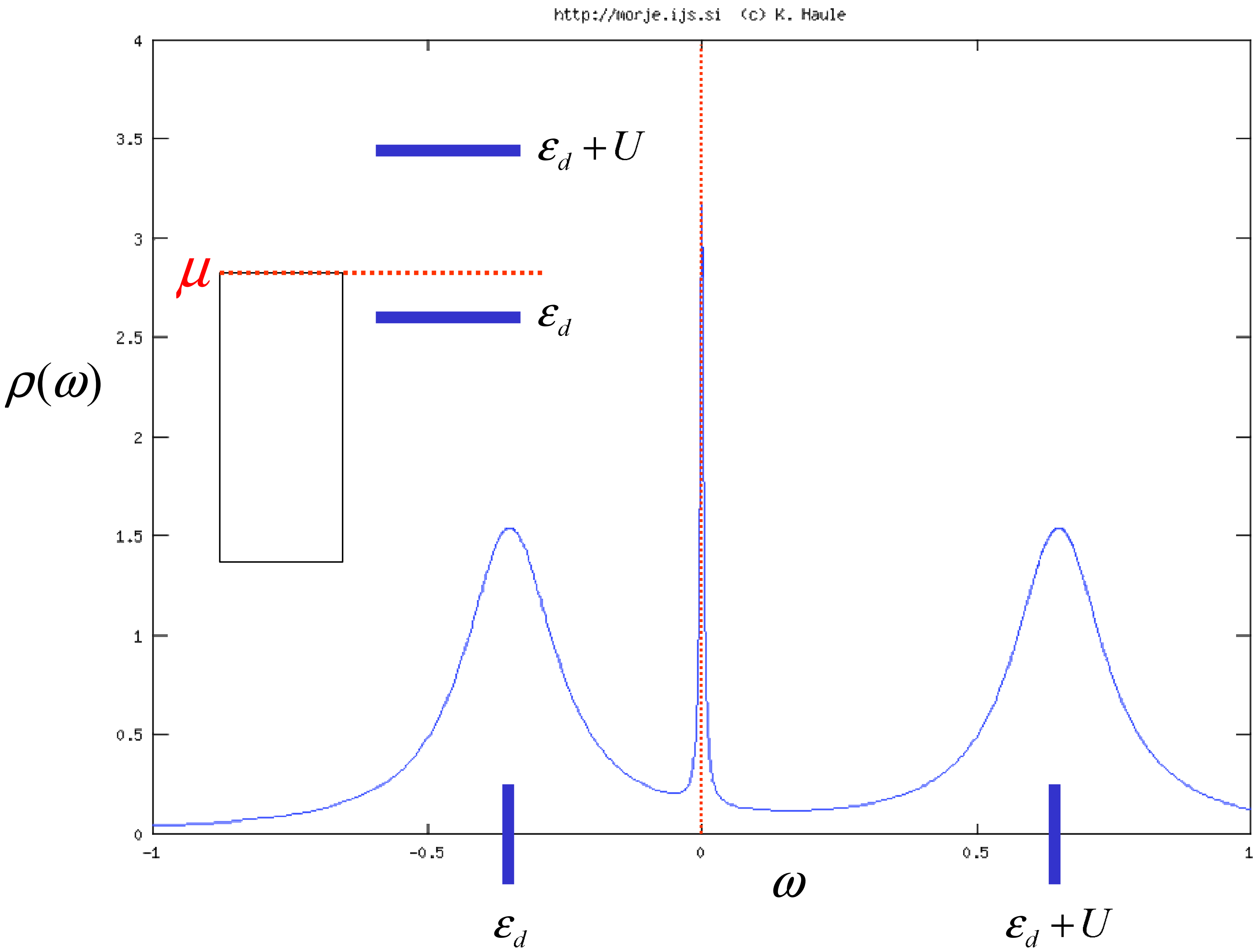
Transformation of the density of states with the gate voltage



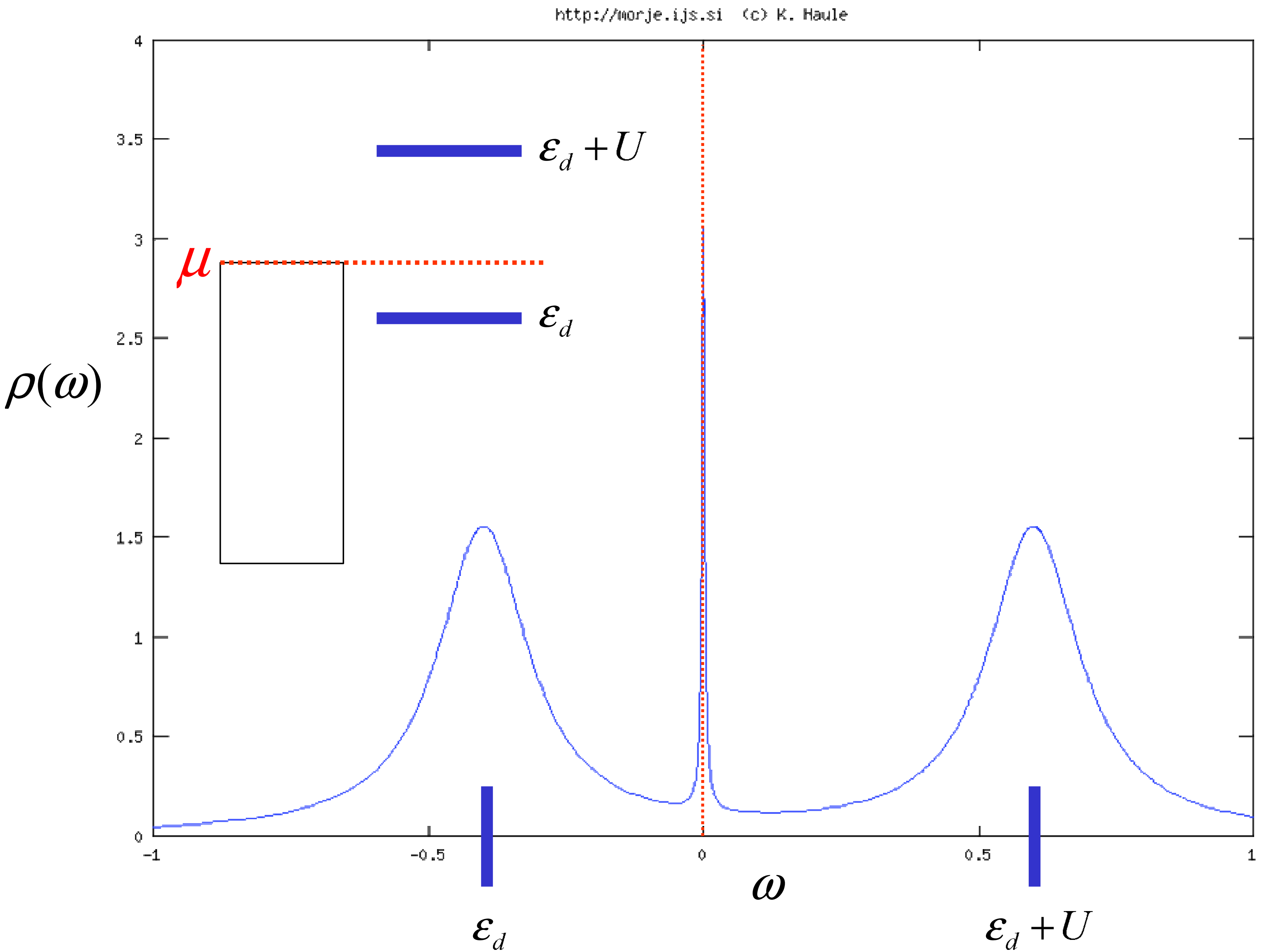
Transformation of the density of states with the gate voltage



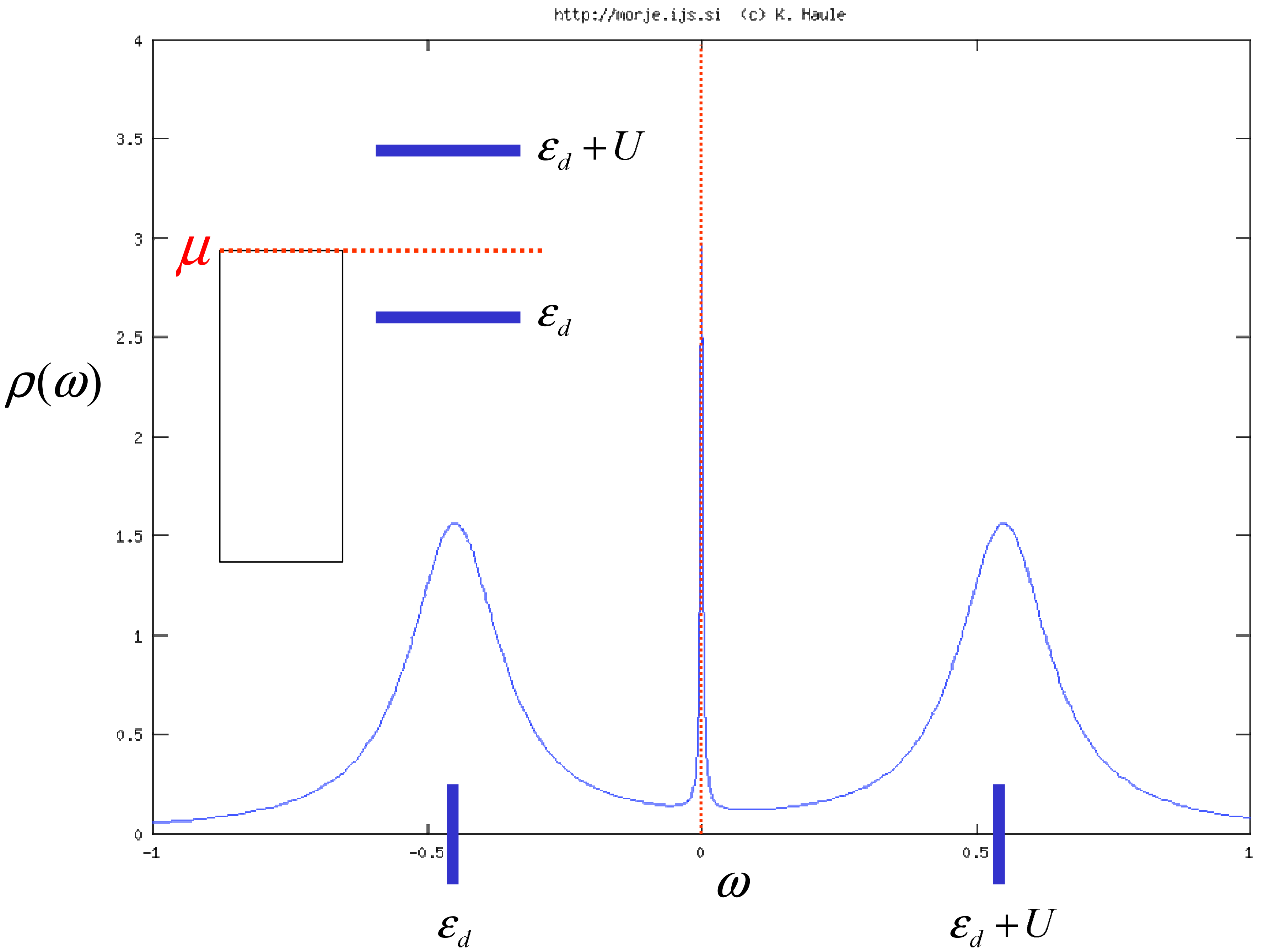
Transformation of the density of states with the gate voltage

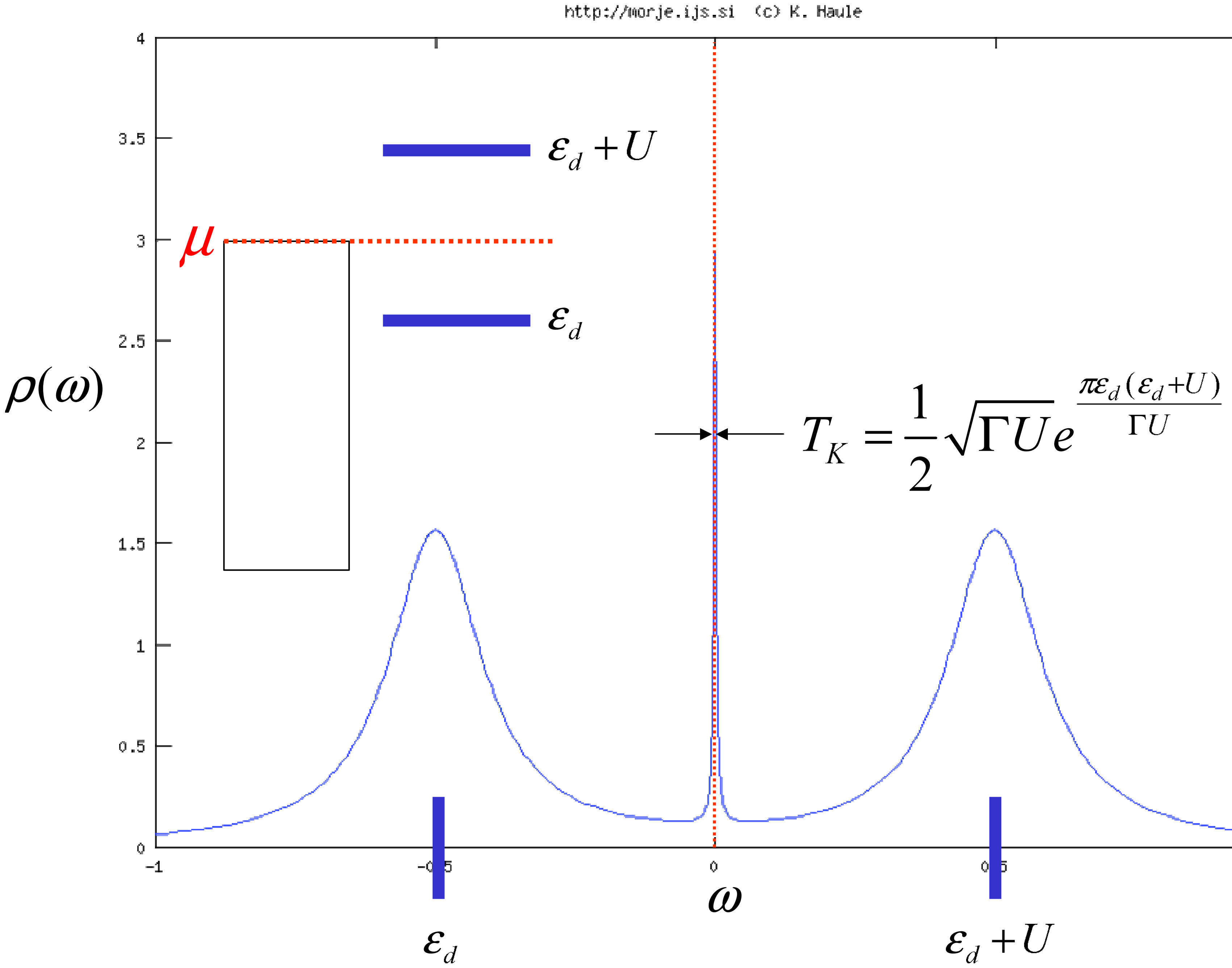


Transformation of the density of states with the gate voltage



Transformation of the density of states with the gate voltage





http://morje.ijs.si (c) K. Haule
$\rho(\omega)$
ω
μ
$\varepsilon_d + U$
ε_d
$T_K = \frac{1}{2}\sqrt{\Gamma U} e^{\frac{\pi\varepsilon_d(\varepsilon_d+U)}{\Gamma U}}$
ε_d
$\varepsilon_d + U$

Conclusion:

The Abrikosov-Suhl peak in the local density of states is pinned to the Fermi energy electrons in the electrodes, even when the local state is shifted by the gate potential

The conductance is large, when the local state is shifted by the gate potential

Gray scale map of the differential conductance vs. the source-drain and the gate voltage

A zero bias peak is a signature of the Kondo effect

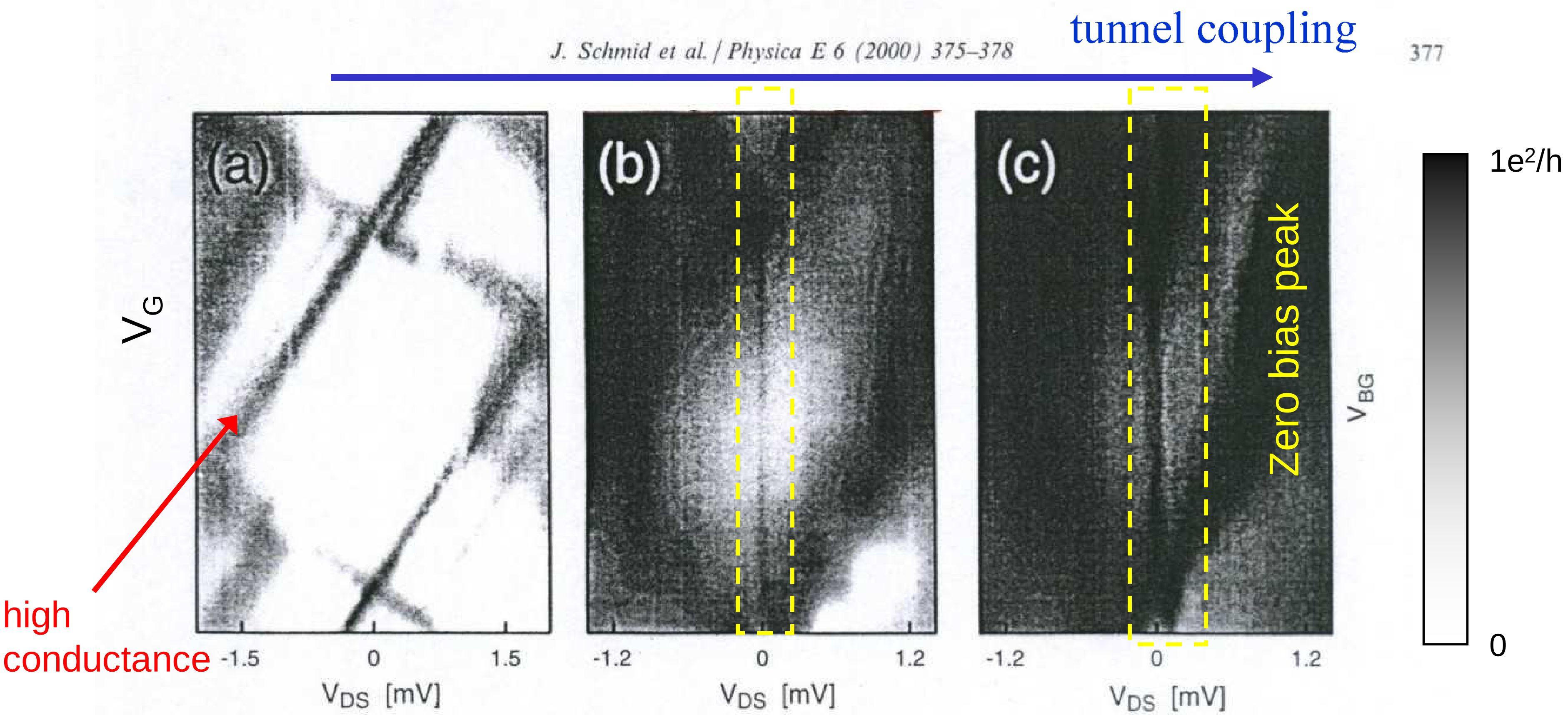
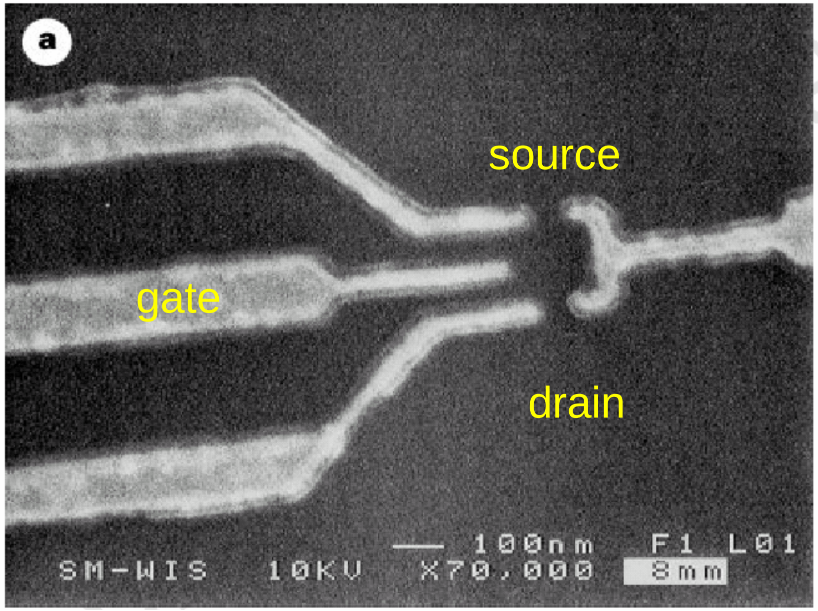


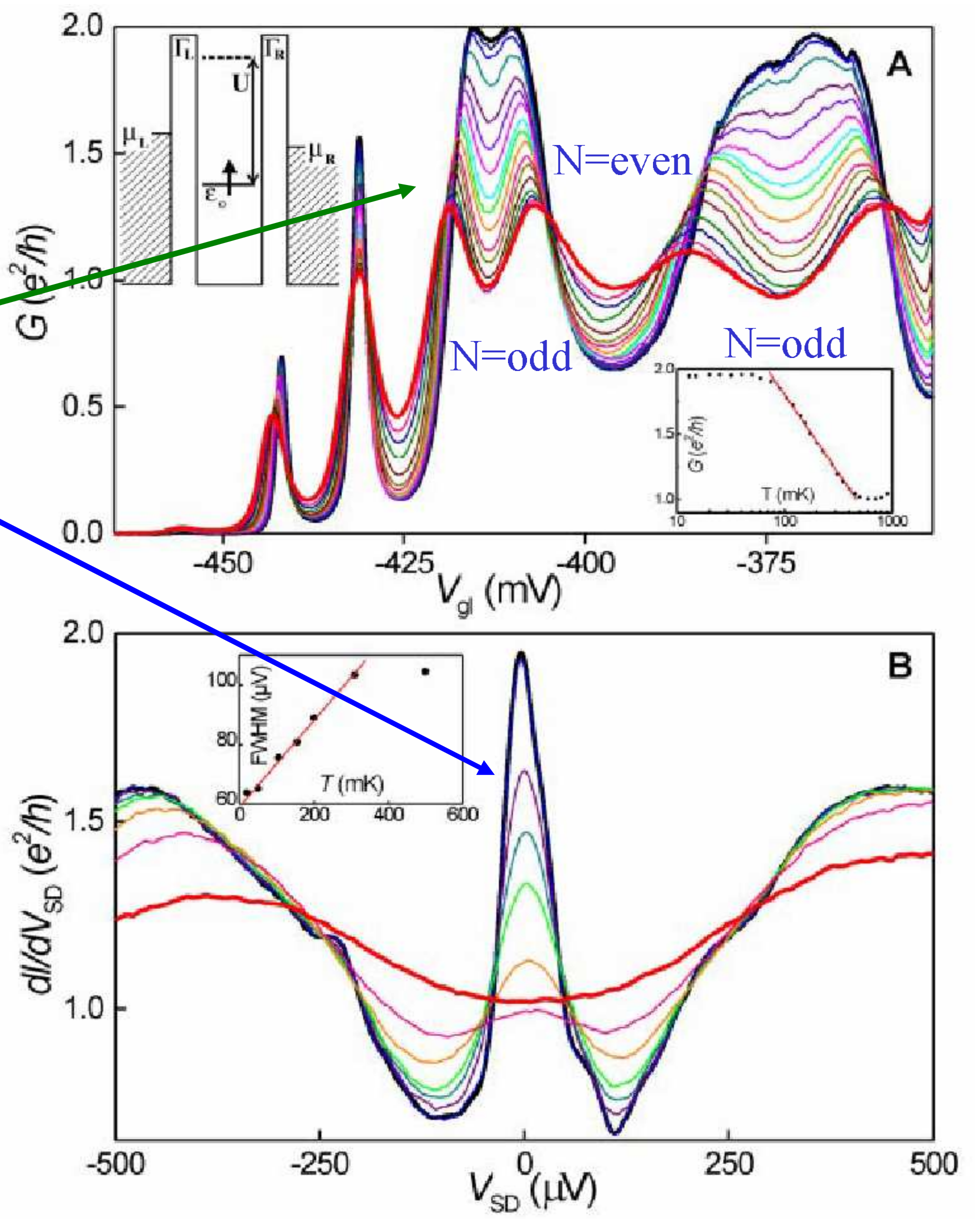
Fig. 2. Differential conductance dI/dV_{DS} (0 corresponds to white, e^2/h corresponds to black) as a function of the back-gate voltage V_{BG} and the drain–source voltage V_{DS}. Tunnel coupling increases from (a) to (c). The V_{BG} range in all plots is 30 V, the absolute values are shifted so that the number of electrons in the central Coulomb blockade region in (a)–(c) is identical.

Summary on Kondo resonance in quantum dot

Increase of the conductance for odd number of electrons

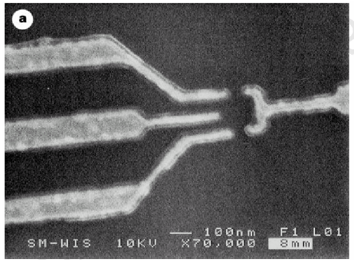
Zero bias peak pinned to the Fermi energy





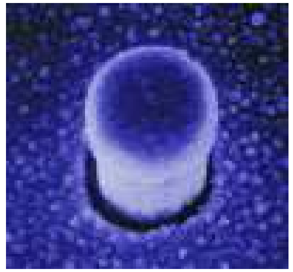
D. Goldhaber-Gordon, H.Shtrikman, D. Mahalu, D. Abusch-Magder, U. Meirav and M. A. Kastner, Nature 391 (1998) 156

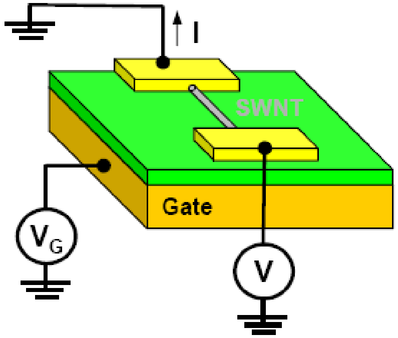
Kondo resonance in various nano-sytems



Lateral structures

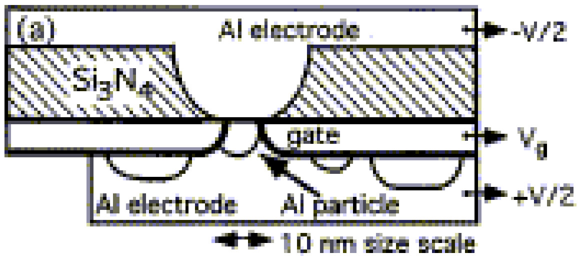
Vertical quantum dots



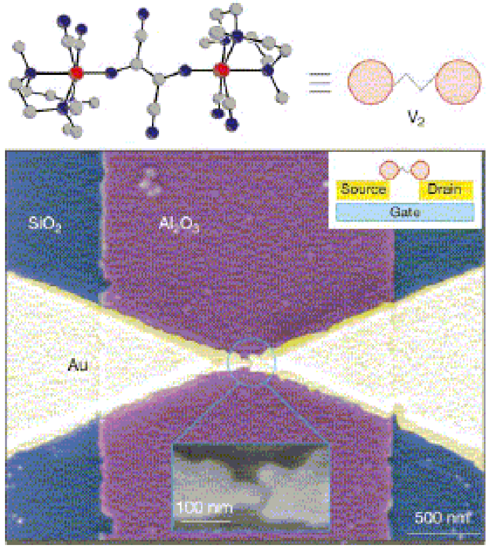


Carbon nanotubes

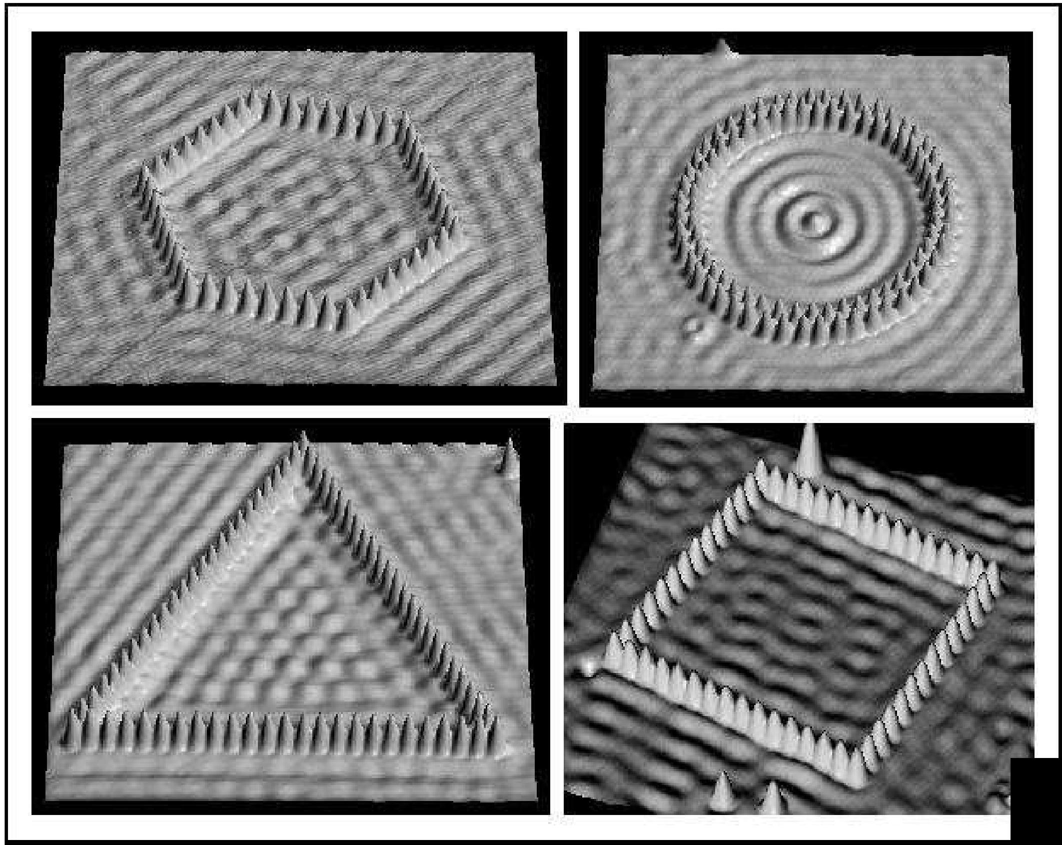
Grains



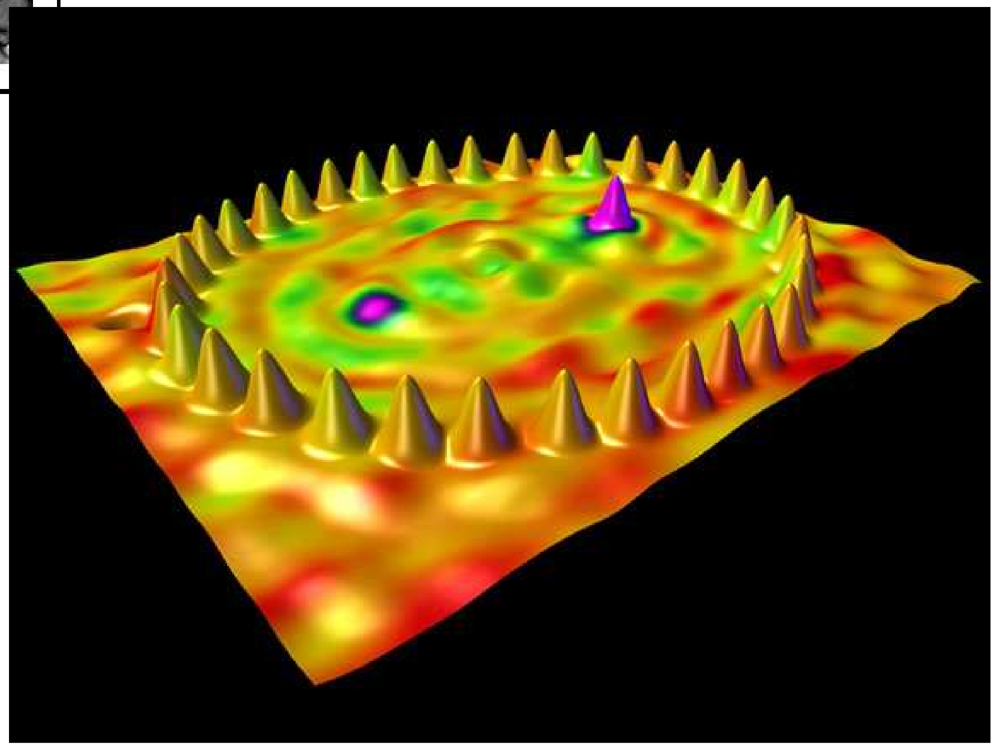
Molecules



Quantum Interference in Nanostructures



Iron atoms on copper surface
(Don Eigler, IBM).



Quantum mirage
in the ellipse of 36 cobalt atoms
(Monoharan et al., Nature 2000)

Fano resonance

Ugo Fano (1912–2001)

Outstanding interpreter of how radiation interacts with atoms and cells

U. Fano, Nuovo Cimento 12 (1935) 177
(in Italian)

PHYSICAL REVIEW VOLUME 124, NUMBER 6 DECEMBER 15, 1961

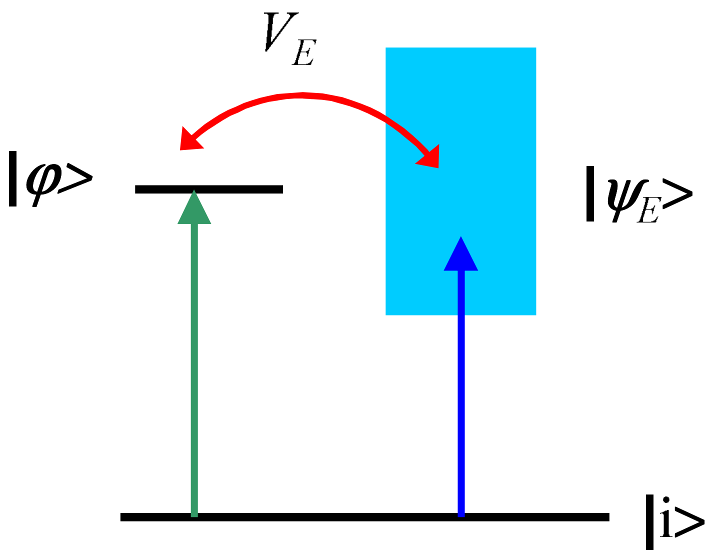
Effects of Configuration Interaction on Intensities and Phase Shifts*

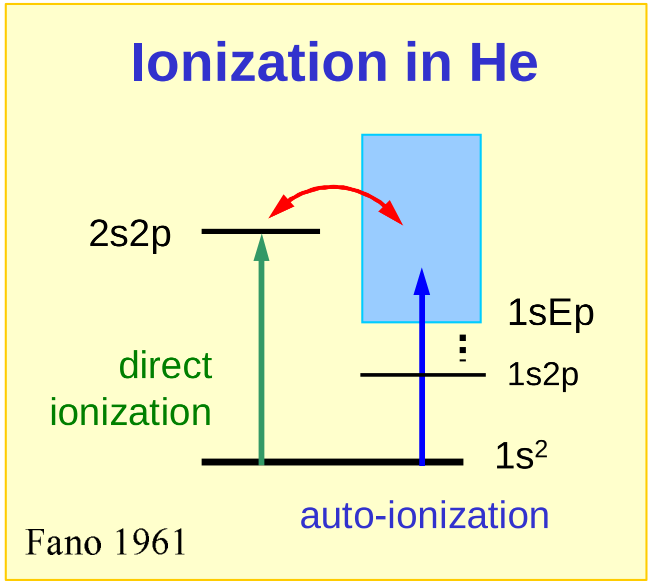
U. Fano
National Bureau of Standards, Washington, D. C.

(Received July 14, 1961)

cited > 5 000

Energy scheme for Fano resonance





Matrix elements

$$<\varphi\,|\,H\,|\,\varphi> = E_\varphi$$ for discrete state

$$<\psi_{E'}\,|\,H\,|\,\varphi> = V_{E'}$$ coupling

$$<\psi_{E''}\,|\,H\,|\psi_{E'}> = E'\,\delta(E''-E')$$ for continuum

States for the coupled system

$$\Psi_E = a\varphi + \int dE'\, b_{E'}\,\psi_{E'}$$

continuum

$$\Phi = \varphi + \mathrm{P}\int dE' \frac{V_{E'}\,\psi_{E'}}{E-E'}$$

discrete state

Modification of the autoionization absorption line (transition to the continuum)

$$\frac{|<\Psi_E\,|T|\,i>|^2}{|<\psi_E\,|T|\,i>|^2} = \frac{(q+\varepsilon)^2}{1+\varepsilon^2}$$

$$\varepsilon = (E - E_r)/\tfrac{1}{2}\Gamma$$

$\Gamma = 2\pi |V_E|^2$ – broadening of the resonant level

q is a parameter, which measures the strength of interference and is given by the ratio of direct ionization to autoionization

$$\tfrac{1}{2}\pi q^2\Gamma = \frac{|<\Phi|T|i>|^2}{|<\psi_E|T|i>|^2}$$

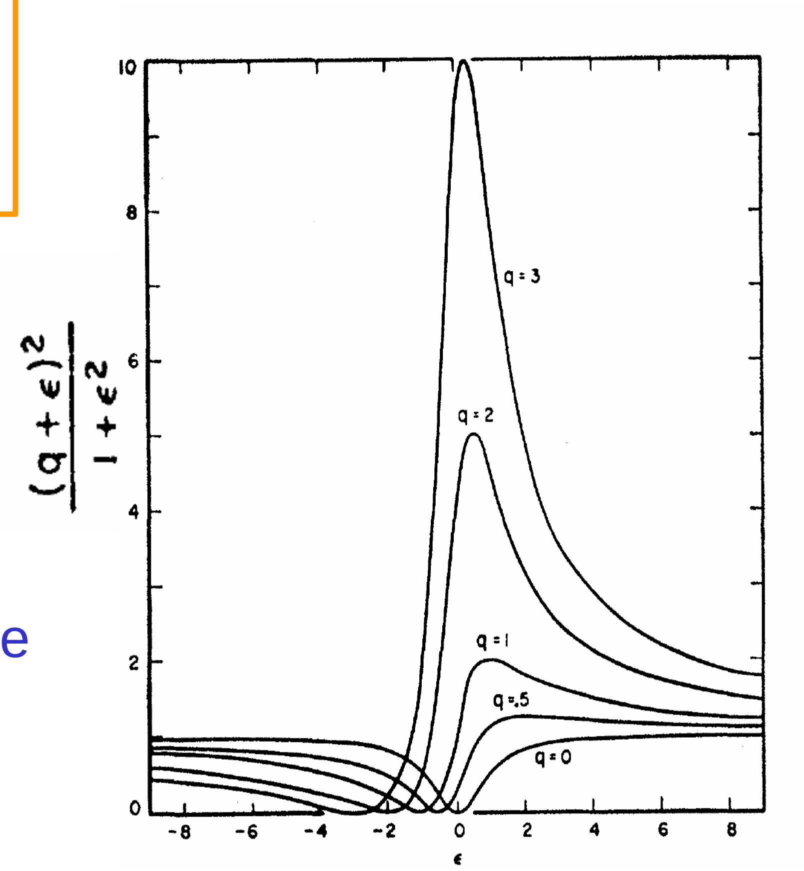
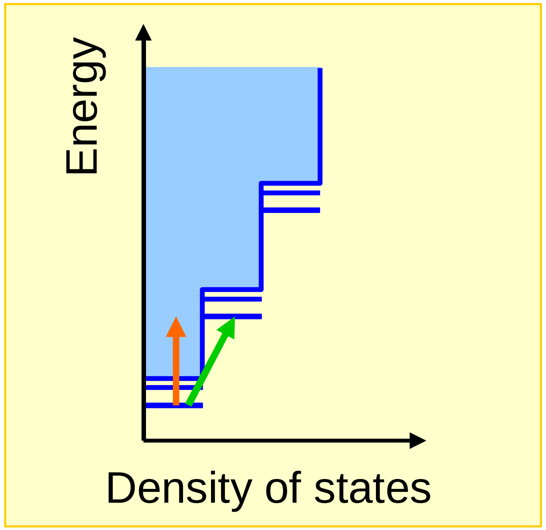


FIG. 1. Natural line shapes for different values of q. (Reverse the scale of abscissas for negative q.)

The Fano resonance is a quantum phenomenon, which was observed in systems of various states and the nature of coupling between them

Physical systems

- photoionization of rare gases
- bulk GaAs in magnetic field
- superlattice in electric field
- impurity ions in semiconductors
- electron-phonon coupling
- and many more …



Observation techniques

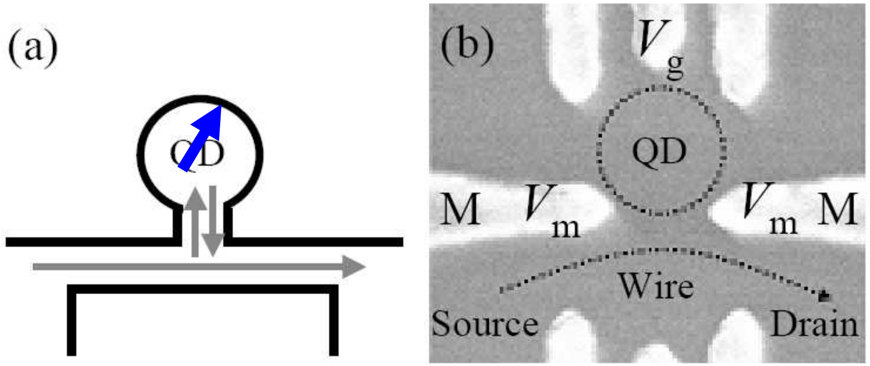
- optical absorption
- Raman spectrosopy
- luminescence
- STM
- conductance characteristics

In transport through nanostructures

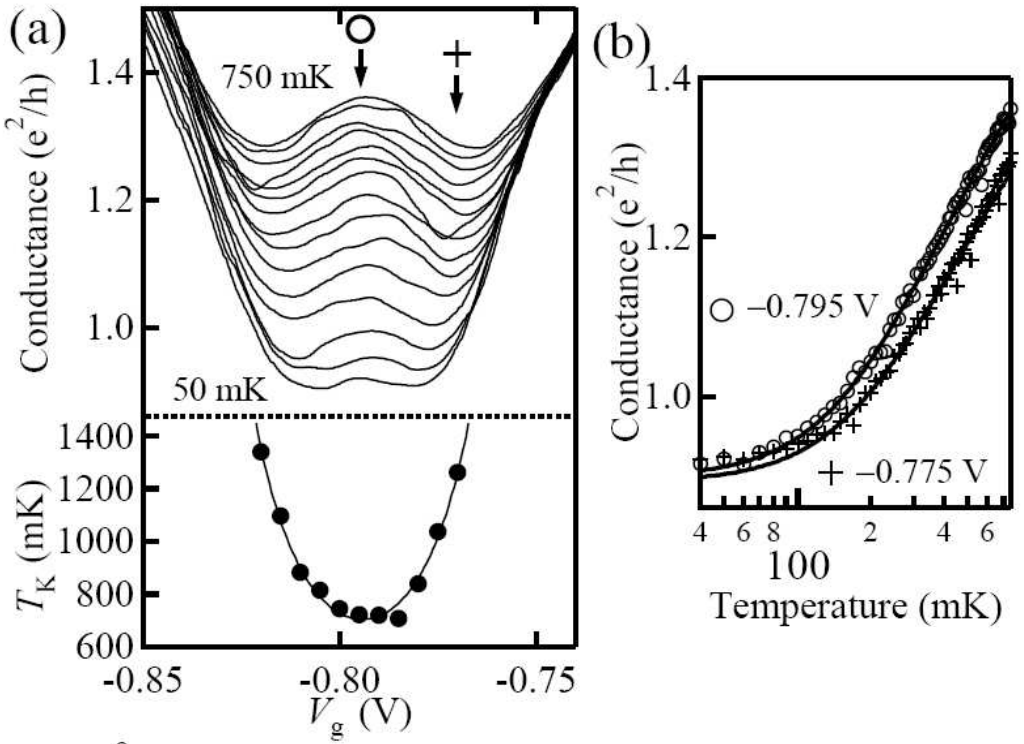
- Strongly coupled Quantum Dot
- **Side attached Quantum Dot**
- For edge states in nanorings in magnetic field

Fano resonance
in a side attached quantum dot

M. Sato, et al., PRL 2006

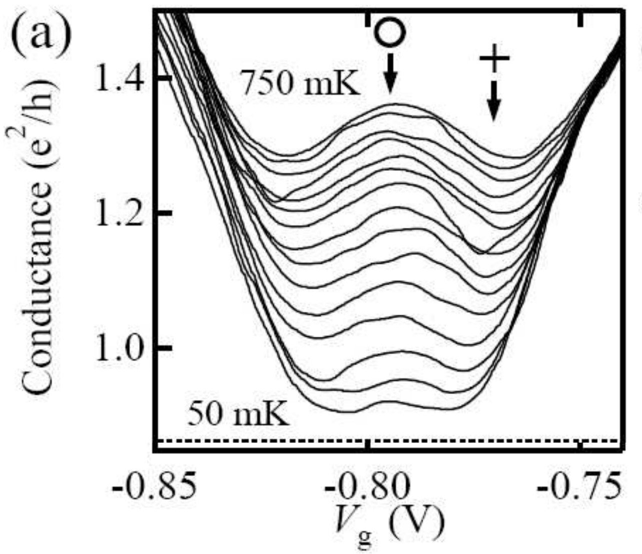


(a) Schematic diagram of a stub-resonator.
(b) Scanning electron micrograph of the device. The white areas are metallic gates made of Au/Ti. The dot and the wire are indicated by dotted lines..

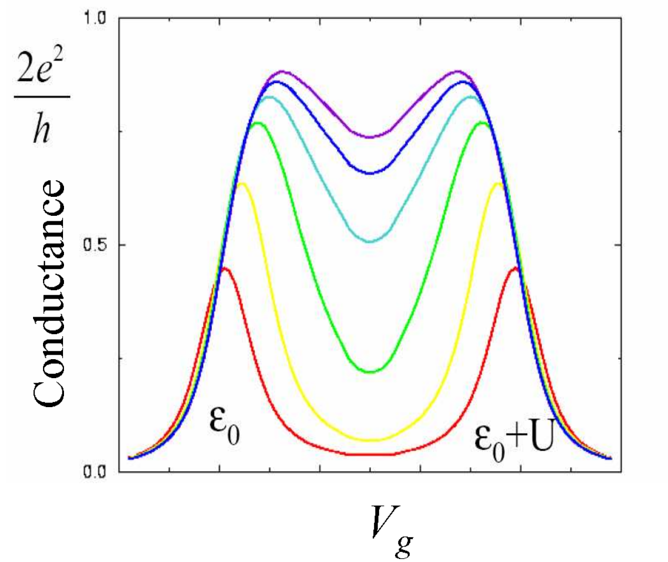


(a) Upper: Conductance as a function of gate voltage at temperatures from 750 mK to 50 mK with the temperature step of 50 mK. Lower: Kondo temperatures T_K obtained from the temperature dependence. (b) Examples of the fitting to obtain T_K. The gate voltages adopted here are indicated by arrows in (a).

M. Sato, et al., PRL2006



How to explain the experiment ?

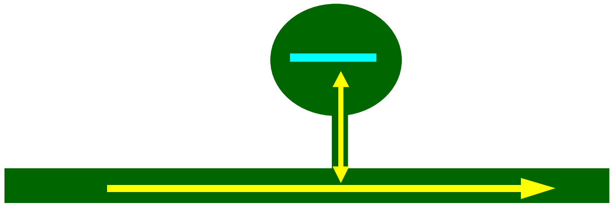


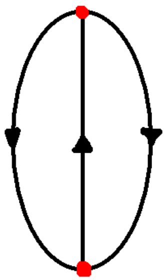
Conductance for the Kondo resonance

Conductance for the Fano resonance

Modeling of transport: quantum dot + wire

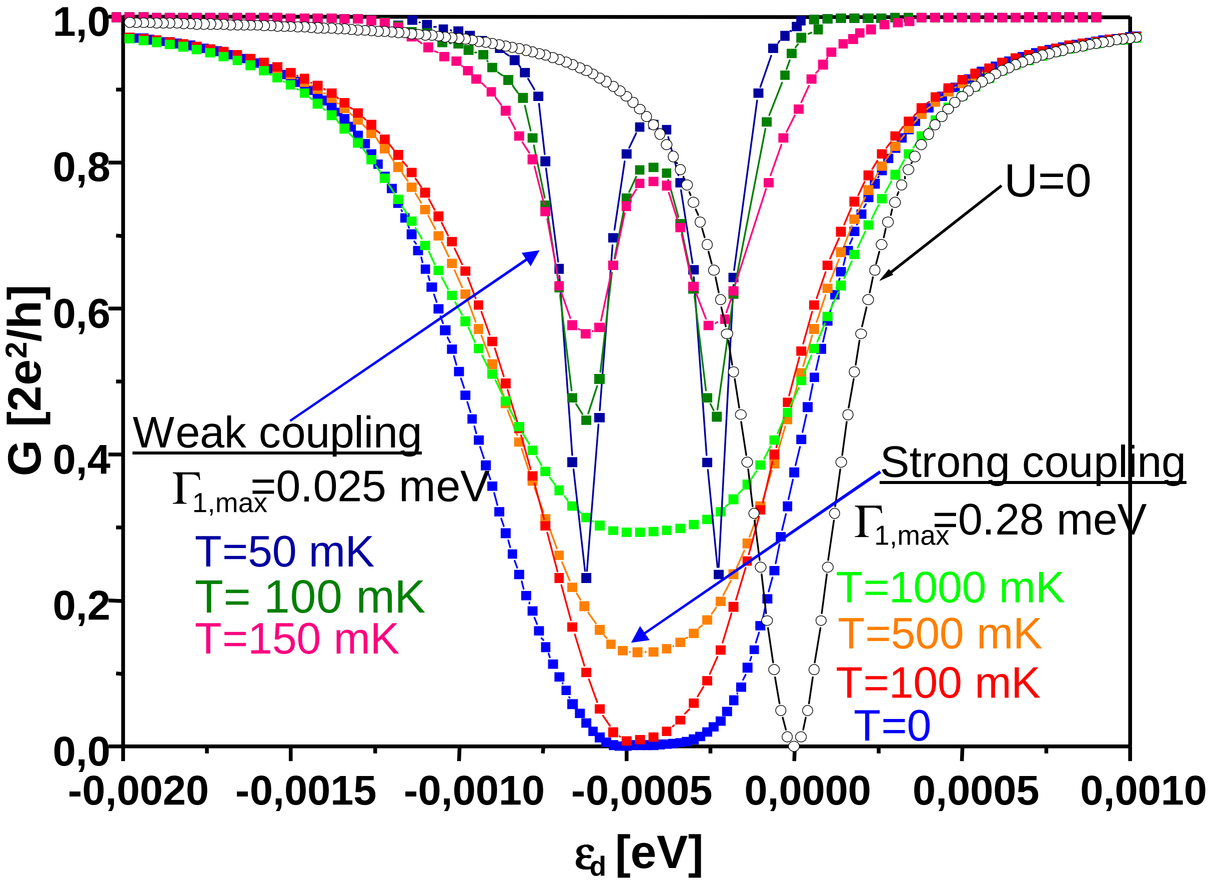
Many-body effects treated within the Interpolative Perturbative Scheme





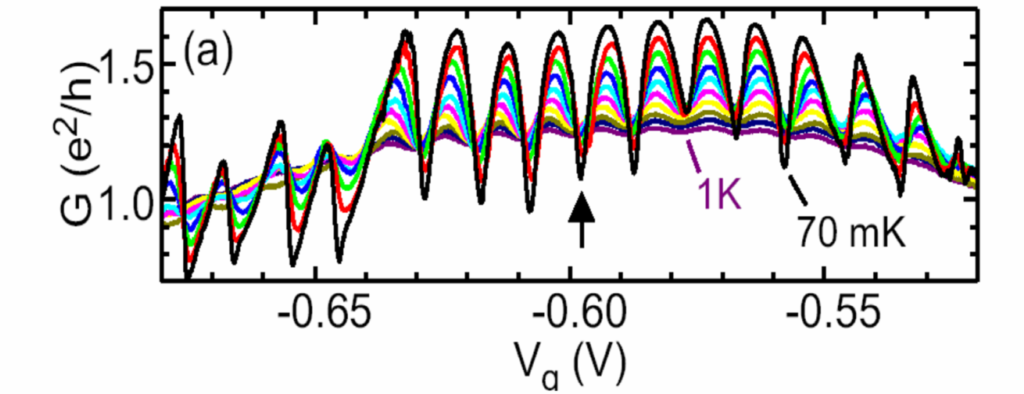
$\Sigma^{(2)}(\omega)=$

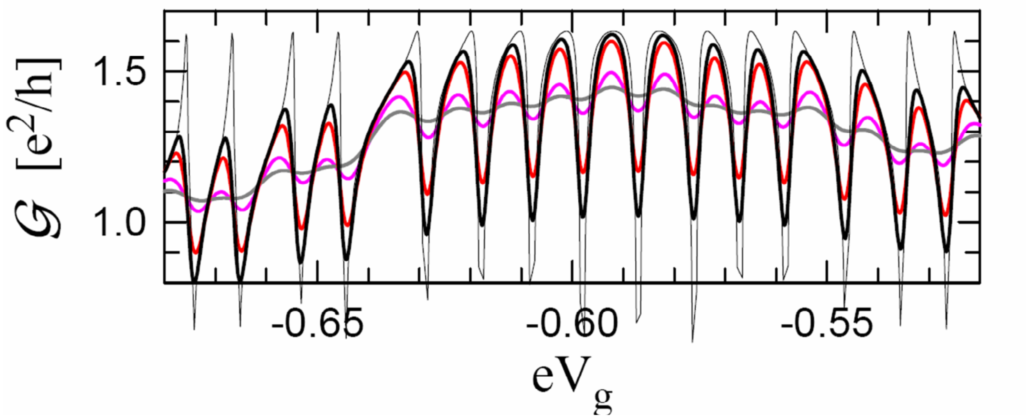
Second order term for self-energy



P. Stefański, Solid St. Commun. 128, 29 (2003)

Fano resonance in semi-open large quantum dot

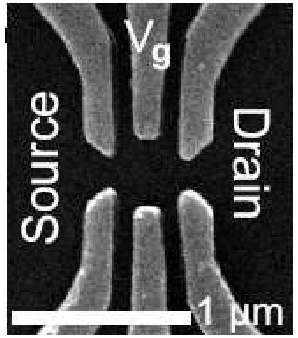


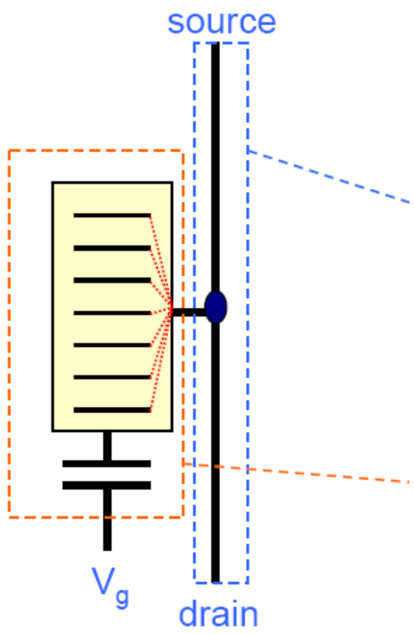


Experiment

C. Fuhner, et al., PRB **66**, 161305 (2002)

cond-mat/0307590

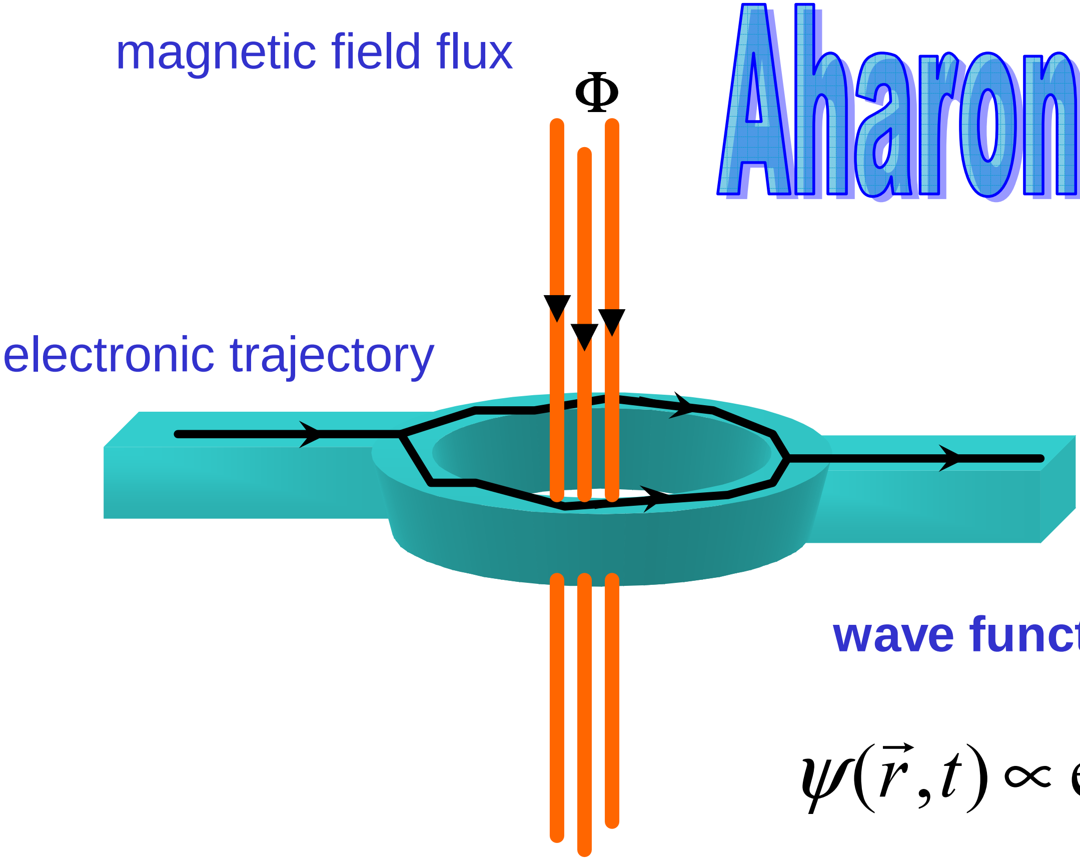




Fano condition | discret subset | continuum subset

More in: *P. Stefanski, A. Tagliacozzo, B.R.B, Phys. Rev. Lett. 93, 186805 (2004)*

Aharonov - Bohm effect



Magnteic field potential

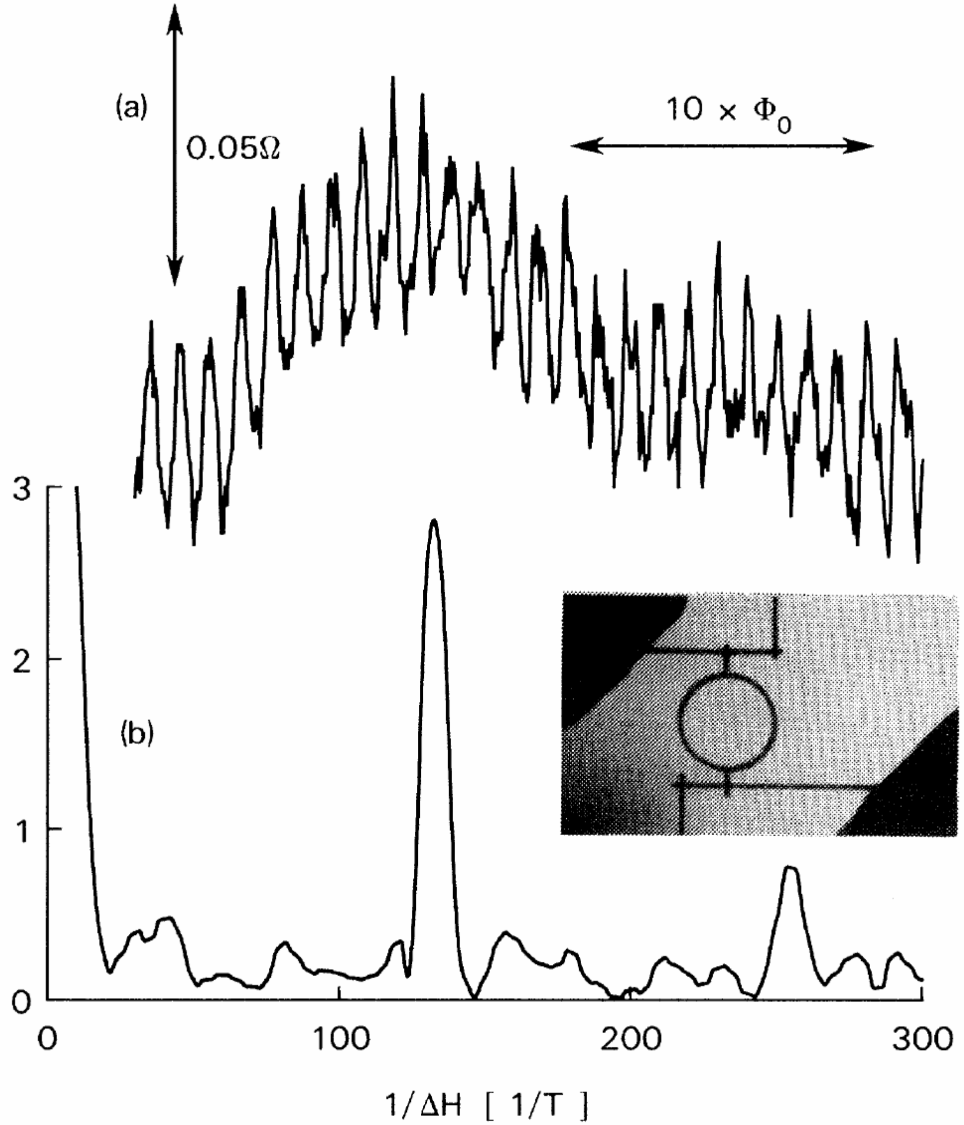
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

wave function of an electron in a magnetic field

$$\psi(\vec{r},t) \propto \exp\left[i(\vec{k} + \frac{e}{\hbar}\vec{A}) \cdot \vec{r} \right] \exp[i\omega t]$$

Schematic presentation of the Aharonov-Bohm effect in a nanoscopic metallic ring in magnetic field ***B***. The phase shift of the electronic wave traveling through the ring $\varphi = \frac{e}{\hbar}\int_L \mathbf{A} \cdot d\mathbf{s}$ depends on the trajectory of in the upper and in the lower arm of the ring and on the magnetic field potential ***A*** (***B***= **rot *A***). The traveling waves interfere, which is observed in the oscillations of the conductance with the period $\Phi_0 = e/h$.

Conductance oscillations with the period $\Phi_0 = h/e$

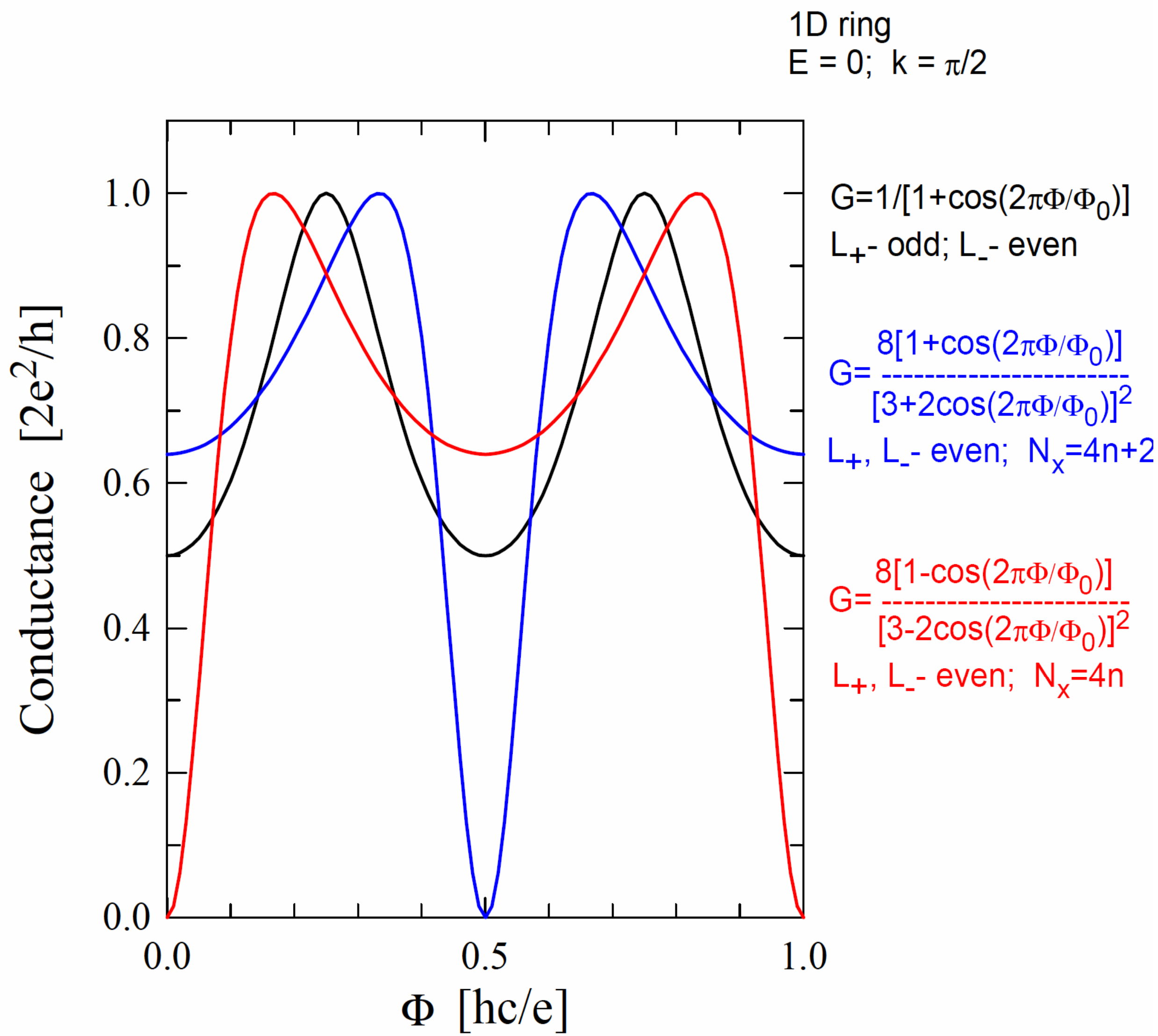


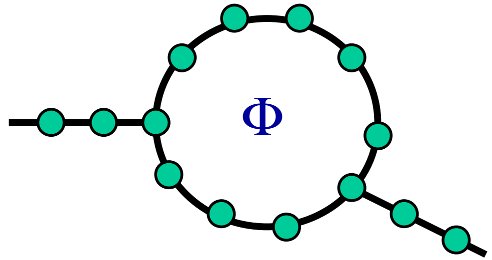
R.A. Webb, et al., PRL 54, 2696 (1985)

FIG. 1. (a) Magnetoresistance of the ring measured at $T = 0.01$ K. (b) Fourier power spectrum in arbitrary units containing peaks at h/e and $h/2e$. The inset is a photograph of the larger ring. The inside diameter of the loop is 784 nm, and the width of the wires is 41 nm.

Conductance of 1D ring vs. magnetic flux for various geometry of attached wires

B. Bulka, Erurophys. Lett. 3, 95 (1987)





Multiple reflections were taken into account

Aharonov-Bohm effect in a metallic ring with a multi-level quantum dot

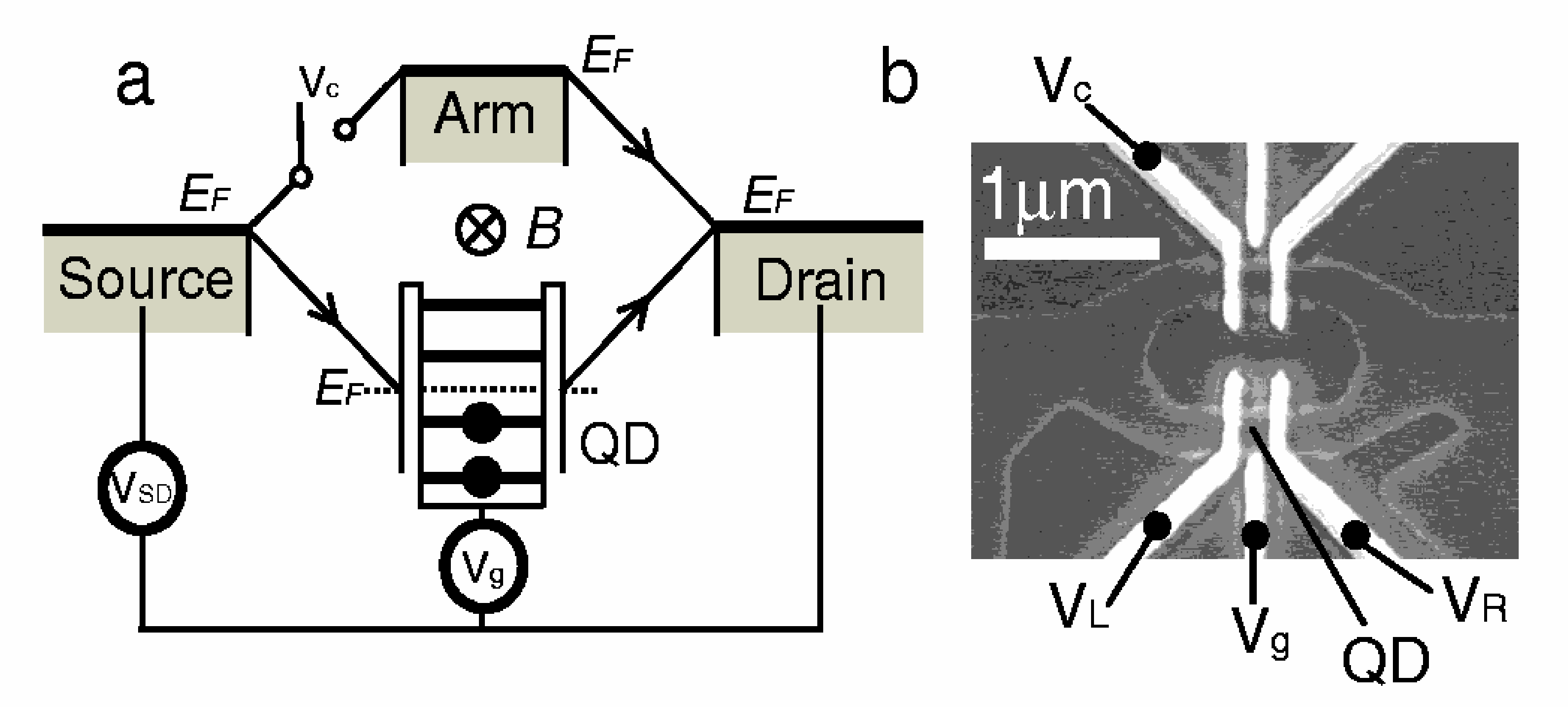


FIG. 1. (a) Schematic representation of the experimental setup. (b) Scanning electron micrograph of the correspondent device fabricated by wet etching the 2DEG at an AlGaAs_GaAs heterostructure. The white regions indicate the Au_Ti metallic gates. The three gates (V_L , V_R and V_g) at the lower arm are used for controlling the QD, and the gate at the upper arm is for V_C .

K. Kobayashi, et al, Phys. Rev. Lett. **88**, 256806 (2002)

experiment

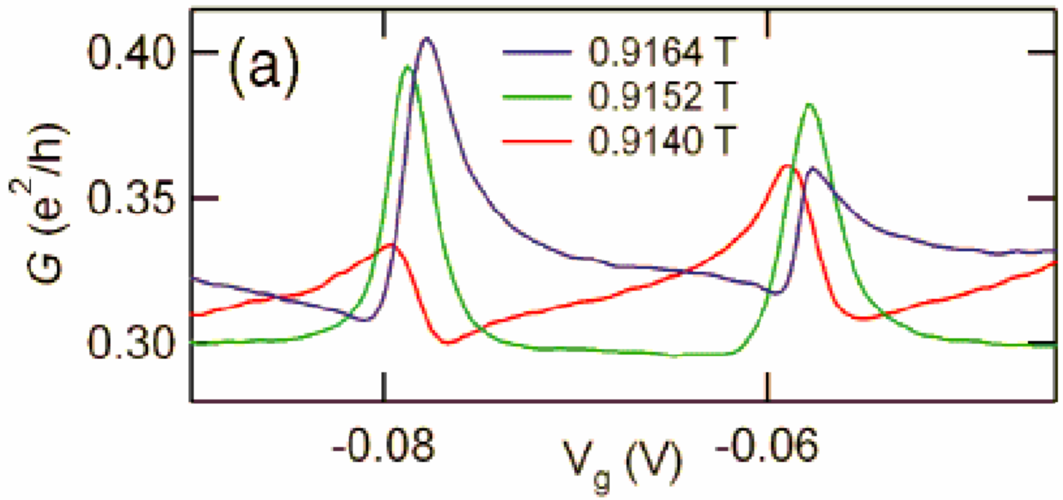
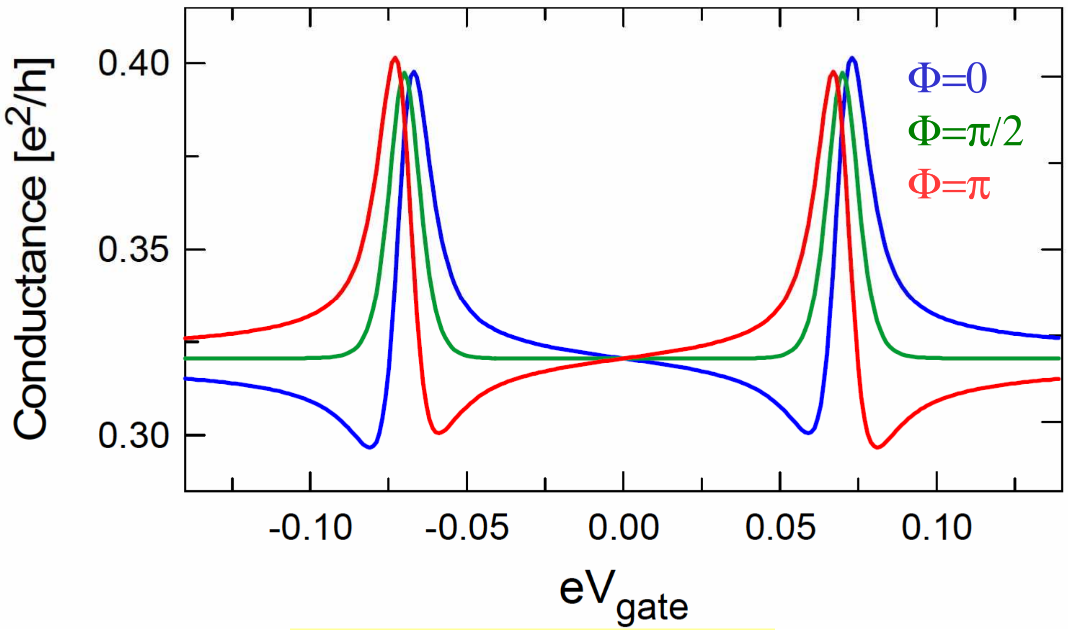


FIG. 4 (color). (a) Conductance of two Fano peaks at 30 mK at the selected magnetic fields. The direction of the asymmetric tail changes between B = 0.9140 and 0.9164 T and the symmetric shape appears in between.

K. Kobayashi, et al. Phys. Rev. Lett. 88, 256806 (2002)

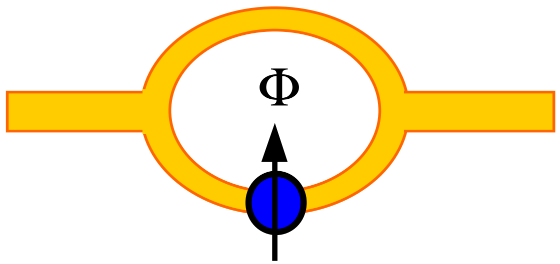


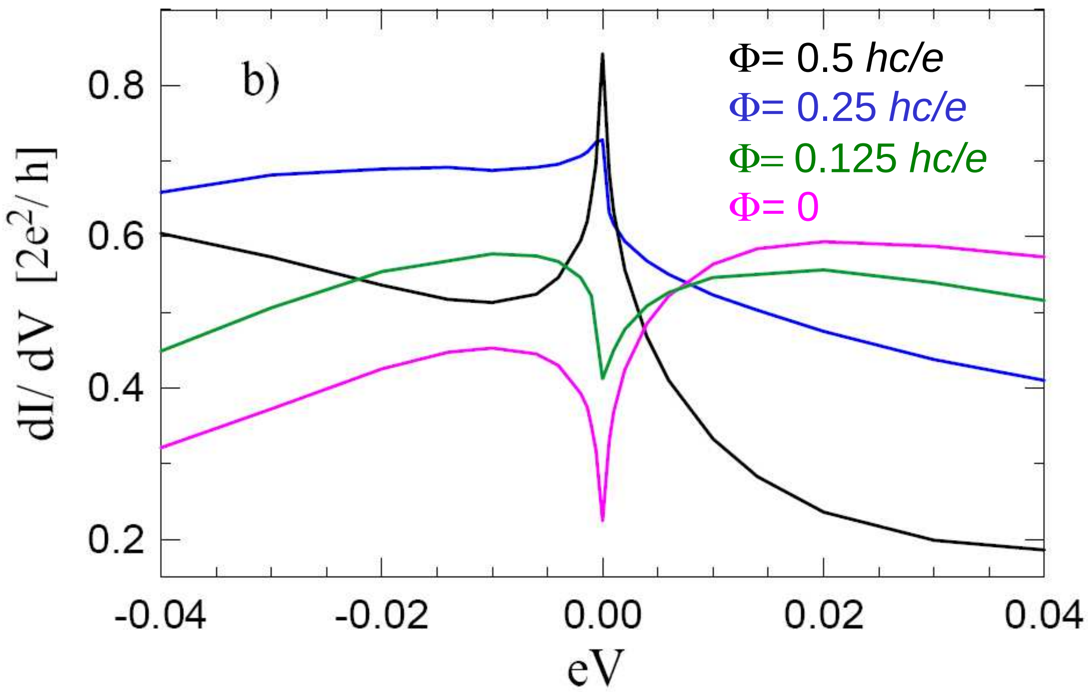
bridge model

Figure: Conductance through the metallic ring with the two-level quantum dot calculated within the bridge model. The coupling of the QD to the electrodes is symmetric $t_{Li} = t_{Rj}$ and the bridge channel was described by $t_{LR} = |t_{LR}| \exp[i\Phi]$. The parameters were taken as $t_{Li} = 0.008$, $|t_{LR}| = 0.133$, the separation of the energy levels $\Delta\varepsilon = 0.14$, temperature $T = 0.0032$ (in units the half-band width D=1). The blue, the green and the red curve corresponds to the phase shift in presence of the magnetic flux $\Phi = 0$, $\pi/2$ and π, respectively.

Bulka, et al., 2003

Change of the profile of the zero-bias anomaly due to the Aharonov-Bohm effect

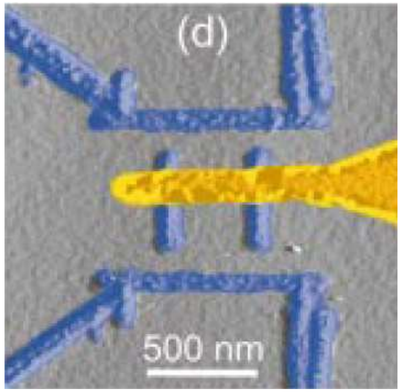




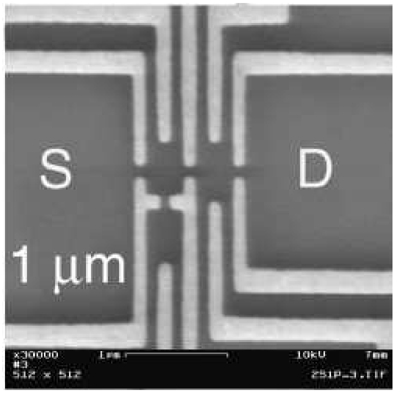
Differential conductance vs the source-drain voltage for Φ = 0.5 *hc/e* (black curve), 0.25 *hc/e* (blue curve) , 0.125 *hc/e* (green curve), and 0 (magneta curve) at T = 2 x10^{-6}, the level position $\Delta\varepsilon$ = 0.05.

Correlated transport in double dot system

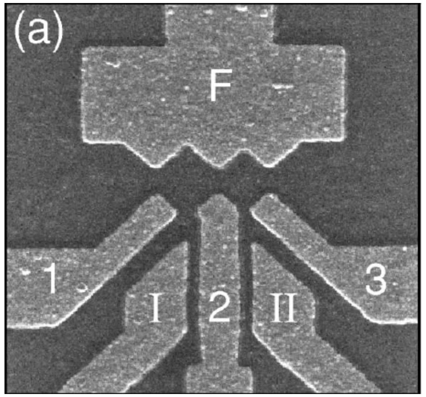
Experiments on Double Quantum Dots



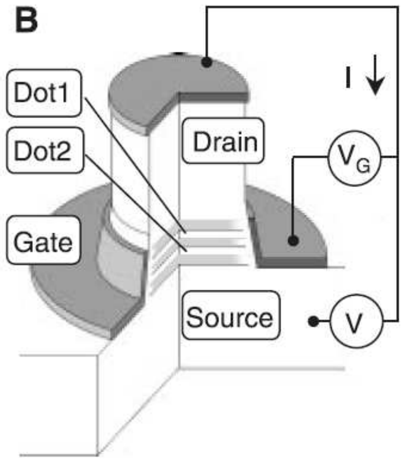
Rogge at al., APL (2003)



Brandes, et al, PRL(2001)



van der Wiel, et al., RMP(2003)



Ono, at al. Science (2002)

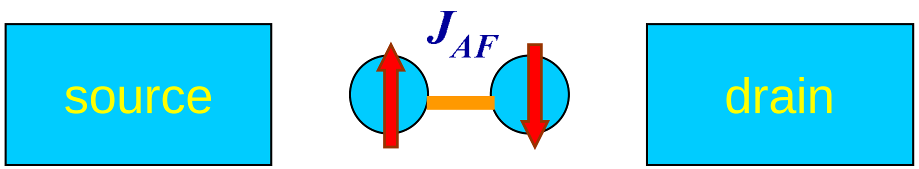
Motivation *for studies of DQD*

- **Construction of multi-dot electronic devices**
- **Construction of qubits**

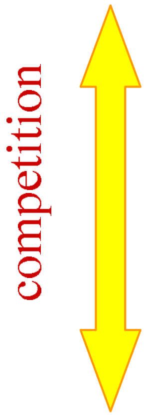
Double-Kondo system

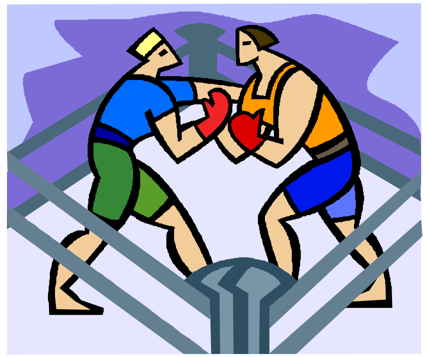
Competition:
Kondo coupling vs. Antiferromagnetic coupling

Strong inter-dot coupling

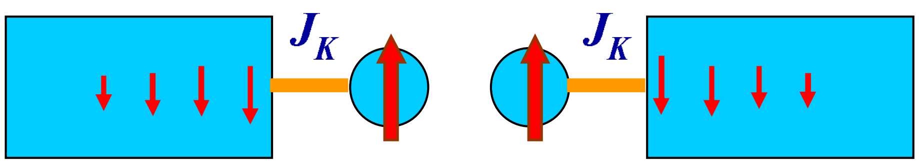


Antiferromagnetic



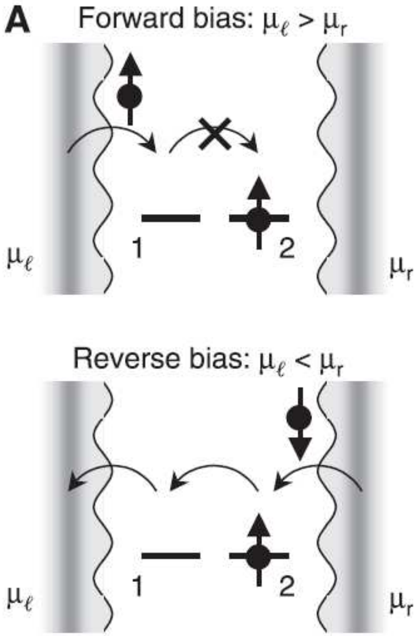


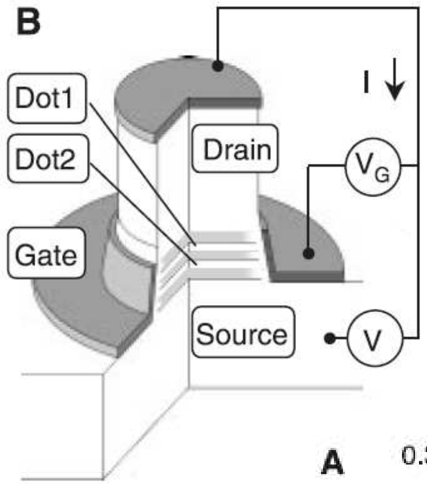
Strong dot-electrode coupling

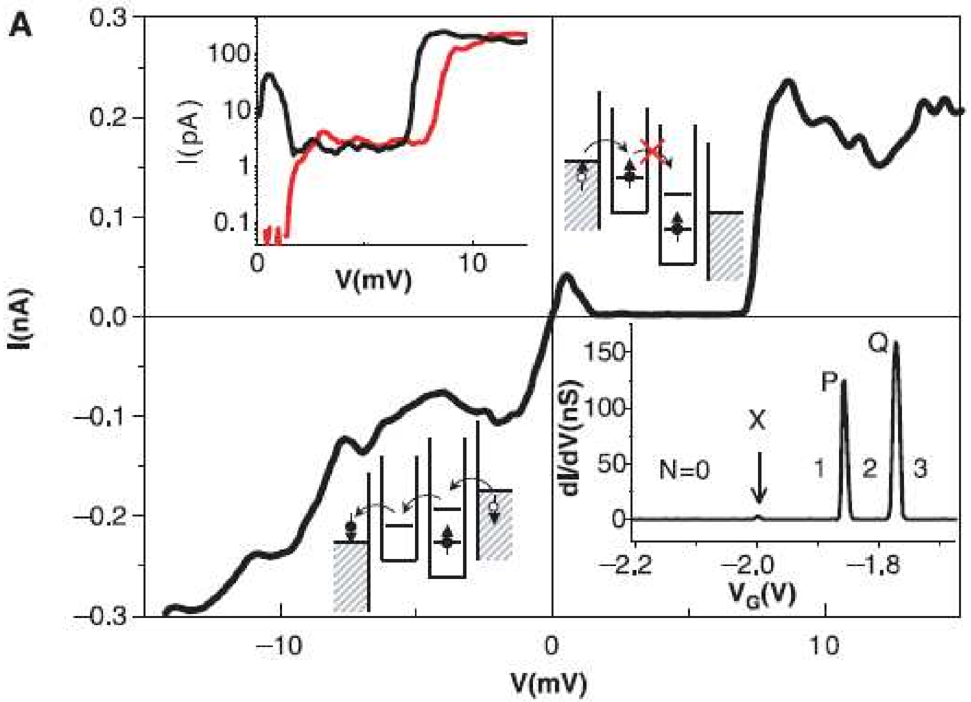


Double-Kondo

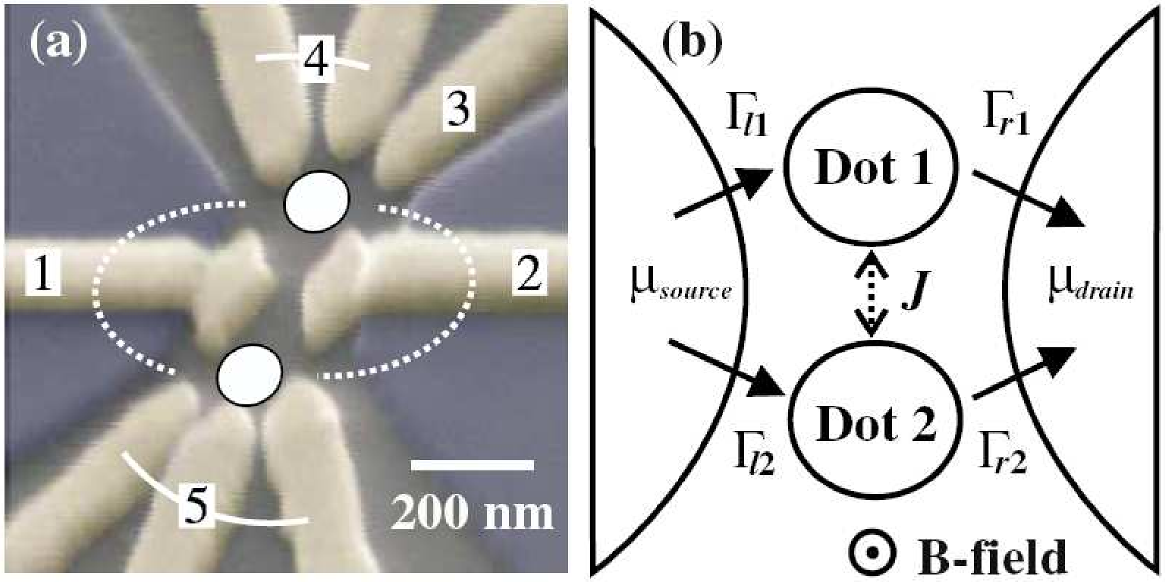
Spin-blockade in double dot system







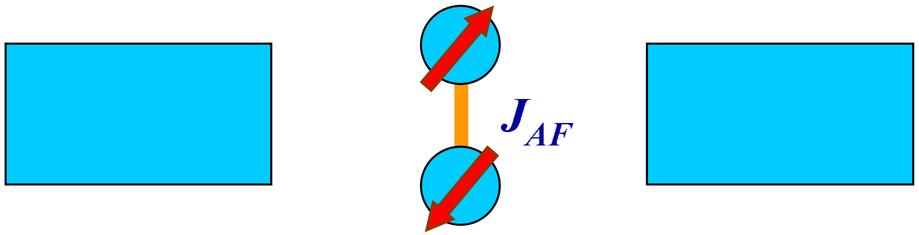
K. Ono, D. G. Austing, Y. Tokura, S. Tarucha, Science 297, 1313 (2002)



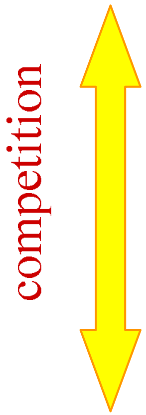
A.W. Holleitner, C.R. Decker, H. Qin, K. Eberl, and R. H. Blick
Phys. Rev. Lett. 87, 256802 (2001)

Double-Quantum Dot connected in parallel: Kondo coupling vs. Antiferromagnetic coupling

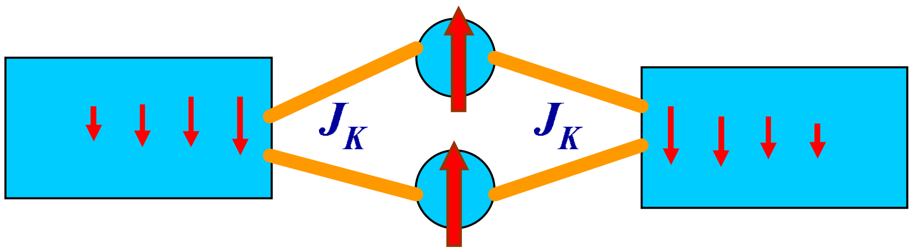
Strong inter-dot coupling



Antiferromagnetic



Strong dot-electrode coupling

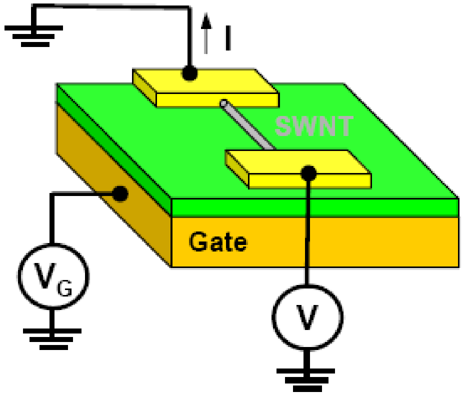


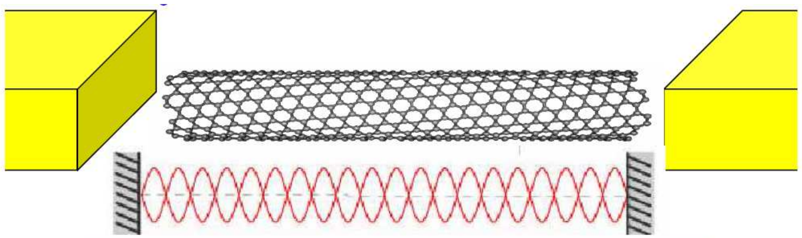
Double-Kondo

Recent experiment: Chen, Chang and Melloch, PRL (2004)

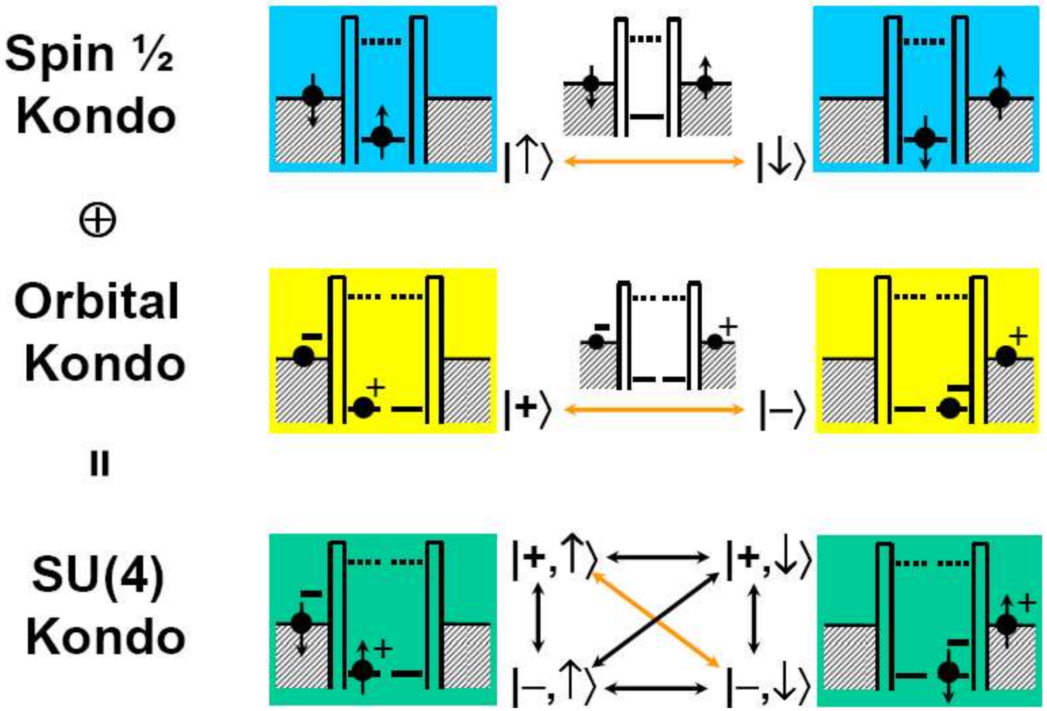
Orbital Kondo effect in carbon nanotubes

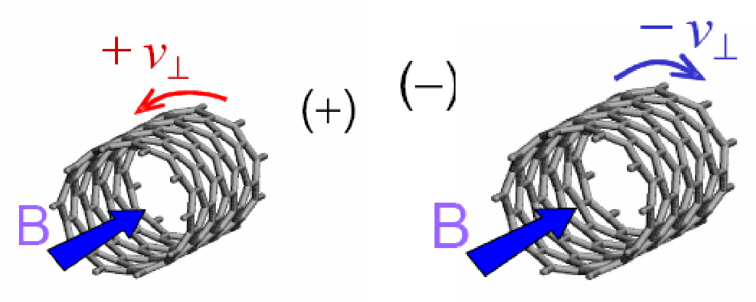
P. Jarillo-Herrero, et al., Nature 434, 484 (2005); E. Minot *et al.* Nature 428, 536 (2004); Zaric et al., Science (2004); Coskun et al., ibid





discrete spectrum:
4-fold is lifted at B≠0
(orbital splitting >> spin splitting)
[Phys. Rev. Lett. **94**, 156802 (2005)]



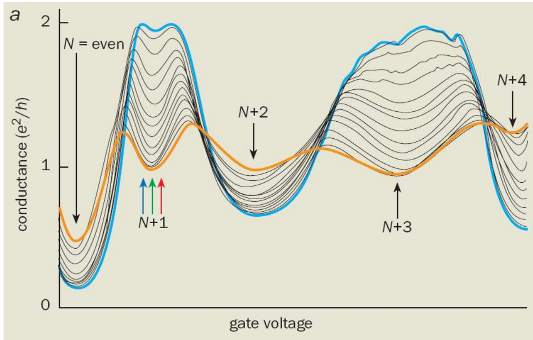


Orbital magnetic moment

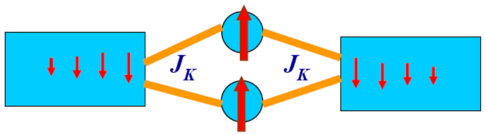
$$\mu_{orb} \approx 0.8 \text{ meV/T} \quad (>> \mu_B = 0.06 \text{ meV/T})$$

Summary

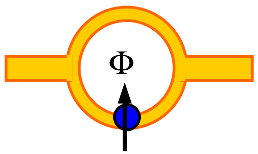
1. Spin of a single electron can be seen in quantum dots



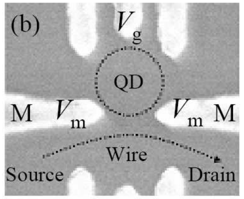
2. In multi-dot systems local spins can be coupled and form multi-electron states
Can the current switch between various configurations?



3. Quantum interference should be taken into account in construction of nanodevices

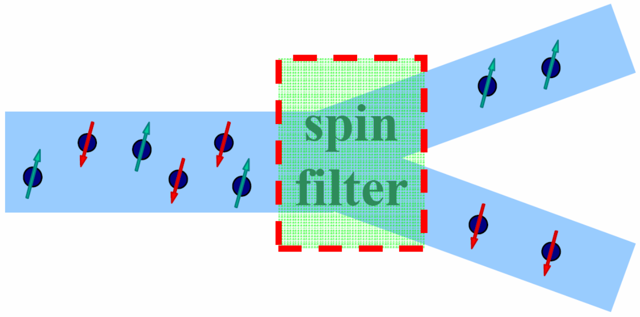


Side-attached quantum dot

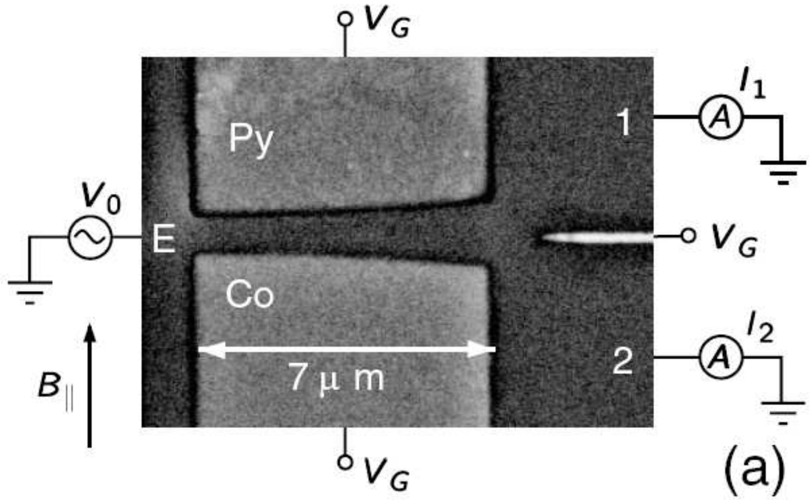


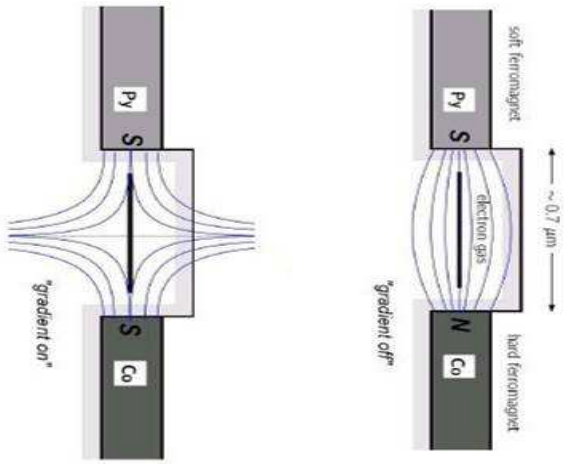
Spin Correlations in Y structures

Stern-Gerlach experiment on electrons



GaAs/GaAlAs hybrid structure

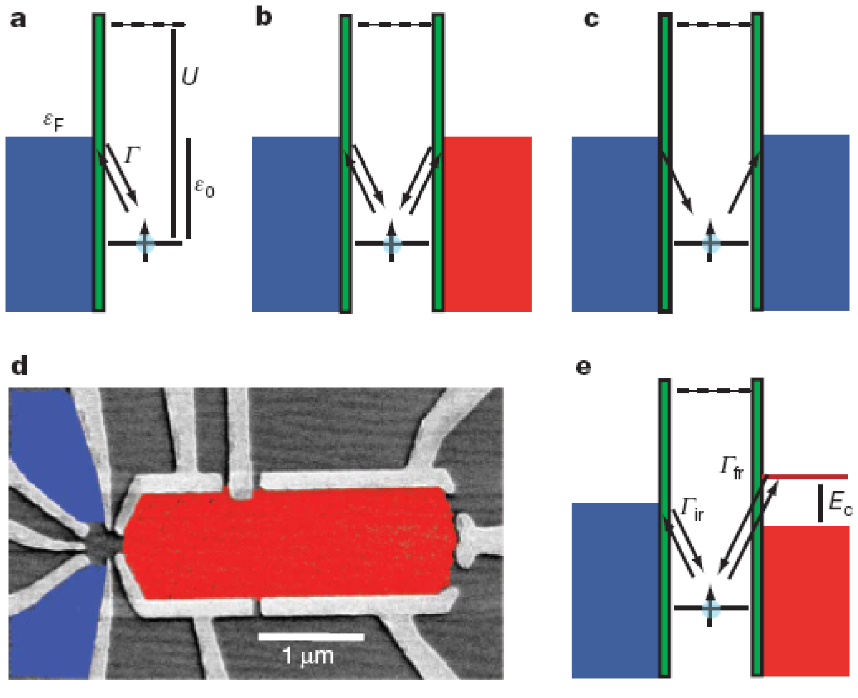




J. Wrobel, T. Dietl, A. Łusakowski, G. Grabecki, K. Fronc, R. Hey, K. H. Ploog, and H. Shtrikman, PRL **93,** 246601 (2004)

TABLE I. Ground state properties of the Kondo model with impurity spin S and number of channels n_l. C_v is the specific heat and χ the magnetic susceptibility. In the over-screened Kondo effect, electron-scattering remains inelastic in the ground state.

Kondo effect	Examples	GS	Phase shift $\delta(\epsilon)$	C_v	χ
Fully screened	$S=1/2$, $n_l=1$ $S=1$; $n_l=2$	Fermi Liquid	$\frac{\pi}{2}+\alpha\epsilon$	$C_v \sim T$	$\sim\frac{1}{T_K}$
Underscreened	$S=1$; $n_l=1$	Singular Fermi Liquid	$\frac{\pi}{2}+\frac{\alpha}{\ln(T_K/\epsilon)}$	$\sim\frac{1}{T\ln^4(T/T_K)}$	$\sim\frac{1}{B\ln^2(B/T_K)}$
Overscreened	$S=1/2$; $n_l=2$ $S=1$; $n_l=3$	Non-Fermi Liquid	inelastic scattering	$\sim T\ln T$ $\sim T^{4/5}$	$\sim\ln T$ $\sim T^{-1/5}$



R. M. Potok, I. G. Rau, Hadas Shtrikman4, Yuval Oreg4& D. Goldhaber-Gordon, Nature 446, 167 (2007)

- ❑ Kondo effect in carbon nanotubes
- ❑ Exotic SU(4) Kondo effects
- ❑ Singlet-triplet Kondo effect
- ❑ Many-level quantum dots
- ❑ Time-dependent problems effect
- ❑ Non-equilibrium Kondo effect