Plan

1. Introduction
2. The neutron interaction with magnetism
3. The Born Approximation
4. Elastic scattering
5. Bragg scattering
6. Neutron polarization analysis
7. Diffuse scattering

Neutrons and Magnetism

I. Elastic scattering of neutrons

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No time to present:

- Sources
- Techniques and instrumentation
- Resolution
- Analysis methods

What do neutrons measure?

Magnetization and the size of the magnetic moment per atom

the spatial distribution of magnetization

Magnetic correlations in space and time

magnetic structures, from Å to µm

the influence of impurities, frustration on magnetism

magnetic phase transitions and critical exponents

Energy states associated with magnetic electrons and dynamics

eigenstates of Hamiltonians

magnetic exchange integrals

crystal field transitions

Coupling between magnetic and chemical/structural properties

superconductivity

colossal- and giant magnetoresistance

magnetostriction and INVAR effect

Limitations of neutrons

FLUX!

Measurements are limited by statistics.

Normally have to compromise on resolution.

It is difficult to measure < 0.1 μ_B per atom.

Neutrons interact with all the magnetic fields in the sample,

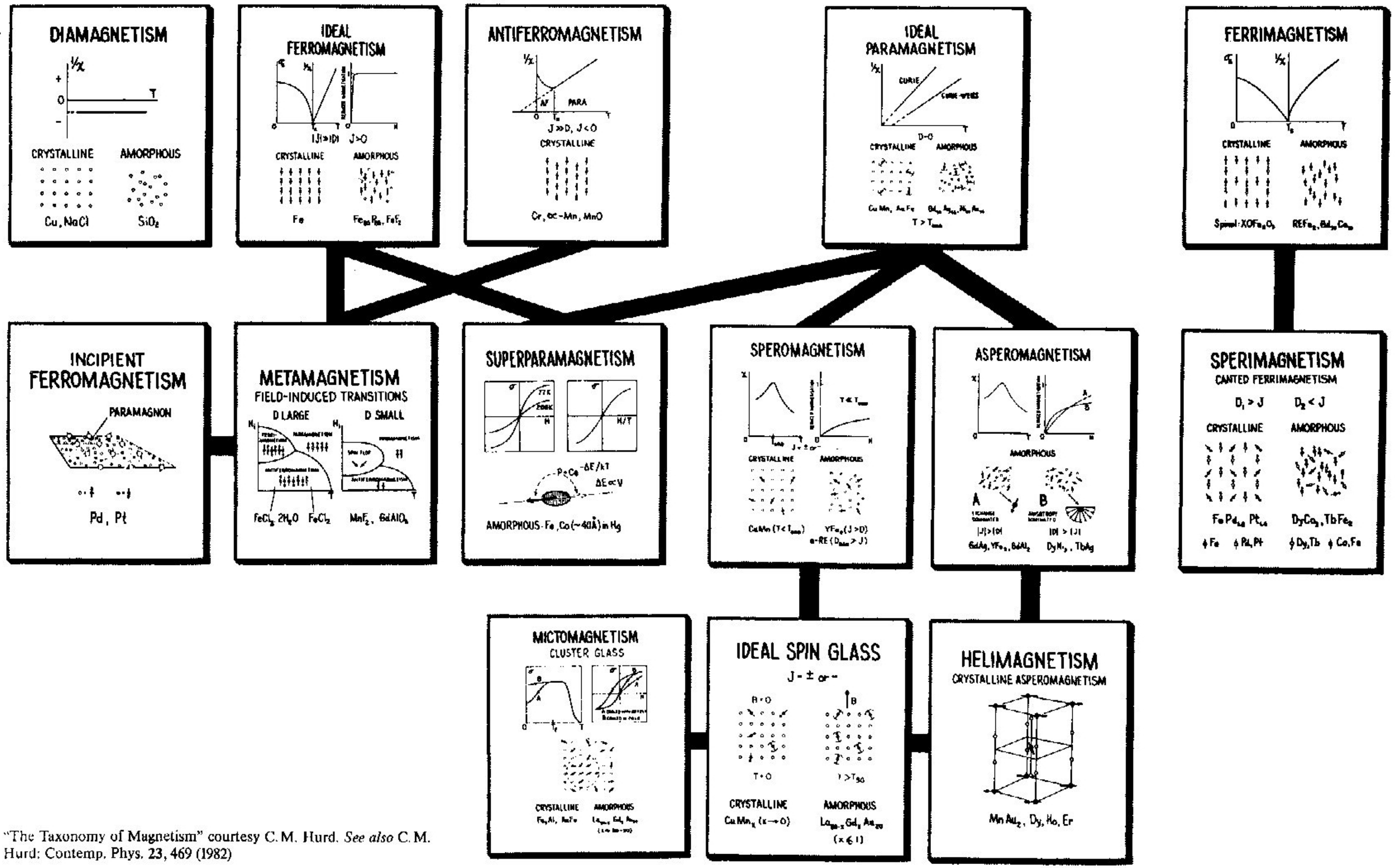
it can be difficult to separate different components.

Neutrons don't measure spatial dimensions directly,

they measure in Fourier space. Conclusions are often model-dependent

This is both a plus and a minus!

The 'Family Tree' of Magnetism



"The Taxonomy of Magnetism" courtesy C.M. Hurd. *See also* C.M. Hurd: Contemp. Phys. **23**, 469 (1982)

The properties of the neutron

Neutrons are *matter waves*.

- They have a de Broglie wavelength of λ
  ($\lambda \sim 1.8$ Å is a 'standard' for 'thermal' neutrons,
  but experiments can use ~ 0.1 to 100 Å neutrons)
- They have a momentum of $\mathbf{p} = h/\lambda = \hbar\boldsymbol{k}$
  and a kinetic energy of $E = h^2/2m\lambda^2 = \hbar^2 k^2/2m$
- They have a magnetic dipole moment given by $-\gamma\mu_N\hat{\boldsymbol{\sigma}}$
  (where $\hat{\boldsymbol{\sigma}}$ is the Pauli spin operator)
- The kinetic energy of a thermal neutron is about the same as
  the energy of a lattice or magnetic vibration

The neutron interaction with magnetism

We must solve the Wave equation for the neutron/sample ensemble.

$$\left[-\left(\hbar^2/2M\right)\nabla^2+\hat{V}(\mathbf{r})\right]\psi=E\psi$$

$\hat{V}(\mathbf{r})$ is the potential energy operator. For neutrons it is called the *Fermi pseudo-potential.*

$$\hat{V}(\mathbf{r})=V_n(\mathbf{r})+V_m(\mathbf{r})$$

where

$$V_n(\mathbf{r})=\frac{2\pi\hbar^2}{m_n}\left(b+B\hat{\mathbf{I}}\cdot\hat{\boldsymbol{\sigma}}\right)\delta(\mathbf{r})$$

$$V_m(\mathbf{r})=-\gamma\mu_N\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}(\mathbf{r})$$

b is the nuclear scattering length

$B\hat{\mathbf{I}}$ is the nuclear spin

$\mathbf{B}(\mathbf{r})$ is the magnetic induction

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$$V_m(\mathbf{r}) = -\gamma\mu_N\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}(\mathbf{r})$$

b is the nuclear scattering length

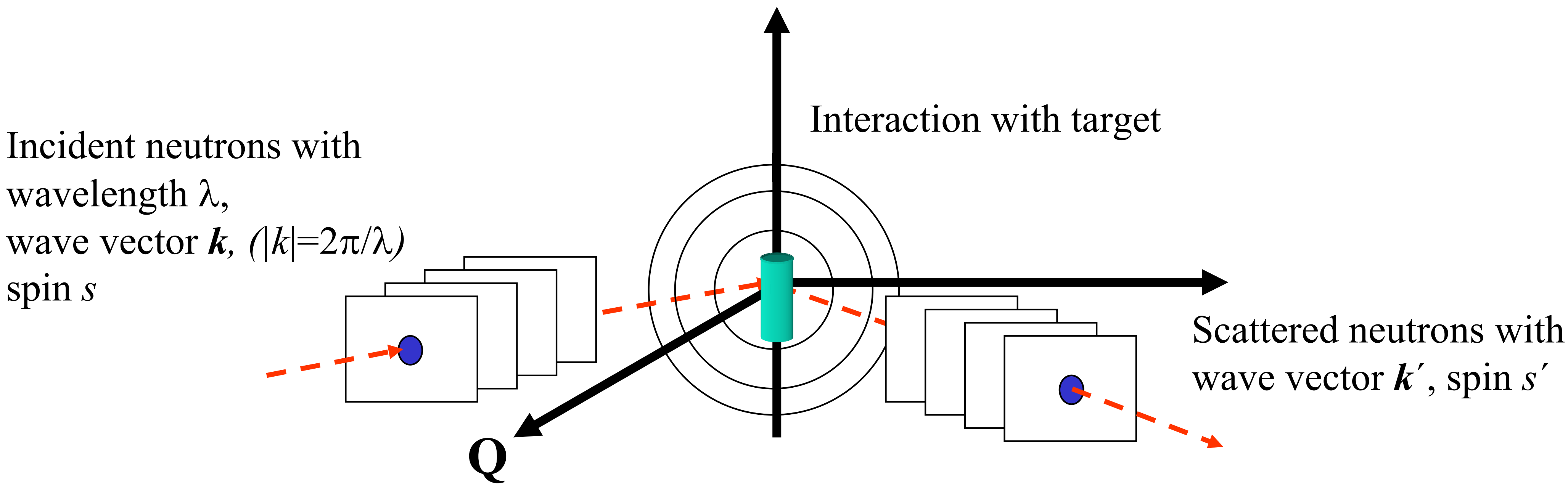
$B\hat{\mathbf{I}}$ is the nuclear spin

$\mathbf{B}(\mathbf{r})$ is the magnetic field

magnetic!

Neutron scattering

Most neutron magnetic experiments are *scattering* experiments.



Constructive and destructive interference leads to peaks in the intensity as a function of:

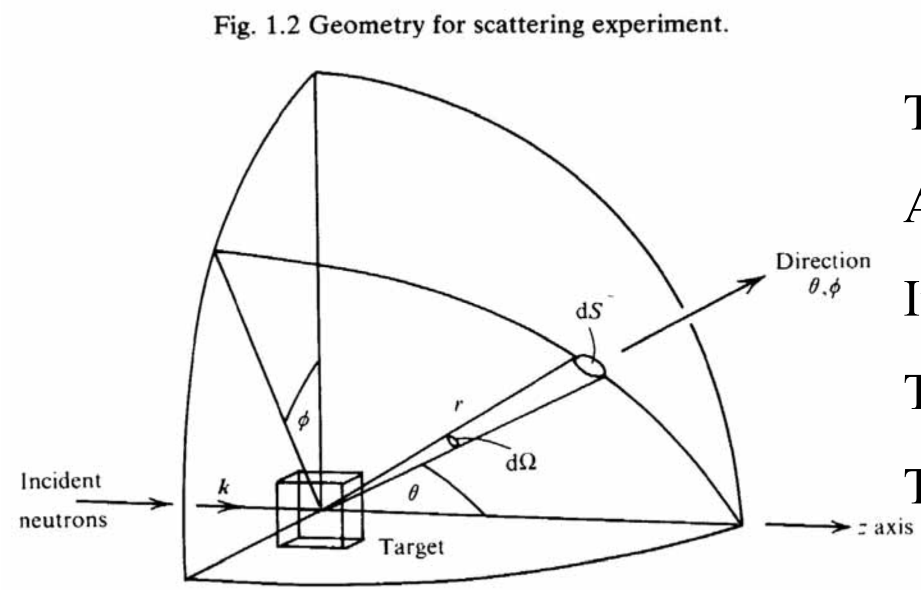
1) the momentum transfer: $\mathbf{Q} = \boldsymbol{k} - \boldsymbol{k}'$

2) the energy transfer: $\Delta E = (\hbar^2/2m)(|k|^2 - |k'|^2)$

3) the change in the neutron spin orientation

The Born approximation

Most neutron experiments are *scattering* experiments.



The target volume is initially in state ζ.

A neutron enters with wave vector ***k*** and spin *s*

It interacts with the target.

The final neutron wave vector is ***k***′ and spin *s*′.

The final target state is ζ ′.

If the neutron has a *plane wave function*, if the interaction is *weak*, then the wave equation can be solved using first order perturbation theory, i.e. Fermi's Golden Rule

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\cdot\mathrm{d}E}=\frac{k'}{k}\left(\frac{m_n}{2\pi\hbar^2}\right)^2\sum_{\zeta,s}p_\zeta p_s\sum_{\zeta',s'}\left|\left\langle \boldsymbol{k}',s',\zeta'\right|\hat{V}(\mathbf{r})\left|\boldsymbol{k},s,\zeta\right\rangle\right|^2\delta\left(\hbar\omega+E_\zeta-E_{\zeta'}\right)$$

This is known as the *cross-section*, and gives the probability that a neutron will scatter in to a certain solid angle with a certain change in energy

The two assumptions form the first *Born approximation*

The cross-section

$$\frac{d^2\sigma}{d\Omega \cdot dE} = \underbrace{\frac{k'}{k}}\left(\frac{m_n}{2\pi\hbar^2}\right)^2 \overbrace{\sum_{\zeta,s} p_\zeta p_s}^{\text{Probabilities of initial target state and neutron spin}} \sum_{\zeta',s'} \underbrace{\left|\langle \boldsymbol{k}', s', \zeta' | \hat{V}(\mathbf{r}) | \boldsymbol{k}, s, \zeta \rangle\right|^2}_{\text{The } matrix\ element\text{, which contains all the physics.}} \underbrace{\delta(\hbar\omega + E_\zeta - E_{\zeta'})}_{\text{Conservation of energy}}$$

Probabilities of initial target state and neutron spin

Conservation of energy

The *matrix element*, which contains all the physics.

Appropriate averaging over the target energy states, the positions **r**, and the neutron spin directions is necessary to find the measured cross-section

G. L. Squires, *Introduction to the theory of thermal neutron scattering*, Dover Publications, New York, 1978
W. Marshall and S. W. Lovesey, *Theory of thermal neutron scattering*, Oxford University Press, Oxford, 1971
S. W. Lovesey, *Theory of neutron scattering from condensed matter*, Oxford University Press, Oxford, 1986

Elastic scattering

If the incident neutron energy = the final neutron energy, the scattering is *elastic*.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left|\left\langle \boldsymbol{k}', s' \middle| \hat{V}(\mathbf{r}) \middle| \boldsymbol{k}, s \right\rangle\right|^2$$

Forget about the spins for the moment and integrate over all **r**:

$$\left\langle \boldsymbol{k}' \middle| \hat{V}(\mathbf{r}) \middle| \boldsymbol{k} \right\rangle = \int \hat{V}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

Momentum transfer $\mathbf{Q} = \boldsymbol{k} - \boldsymbol{k}'$

The elastic cross-section is then directly proportional to the *Fourier transform squared* of the potential. Neutron scattering thus works in Fourier space, otherwise called *reciprocal space.*

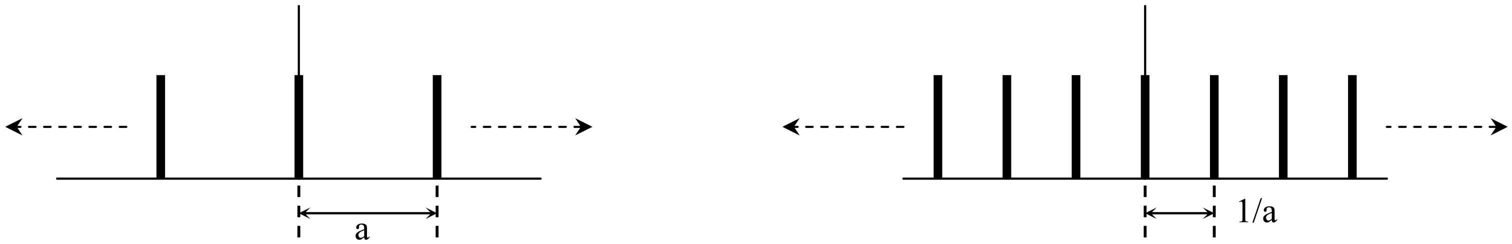
Elastic neutron scattering is also referred to as *neutron diffraction*

Fourier Transforms $F(\mathbf{Q}) = \int f(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$

$f(\mathbf{r})$ $F(\mathbf{Q})$

A Delta function

A series of Delta functions



Two Delta functions

A Gaussian

$V_{\mathrm{m}}(\mathbf{r})$, $\mathbf{B}(\mathbf{r})$ and the magnetization

$$\int V_m(\mathbf{r})e^{i\mathbf{Q}\cdot\mathbf{r}}\cdot d\mathbf{r} = -\gamma\mu_N\hat{\boldsymbol{\sigma}}\int \mathbf{B}(\mathbf{r})e^{i\mathbf{Q}\cdot\mathbf{r}}\cdot d\mathbf{r} = -\gamma\mu_N\hat{\boldsymbol{\sigma}}\cdot\mathbf{B}(\mathbf{Q})$$

$\mathbf{B}(\mathbf{Q})$ is related to the *magnetization* of the sample, $\mathbf{M}(\mathbf{r})$, through the equation:

$$\mathbf{B}(\mathbf{Q}) = \int \hat{\mathbf{Q}}\times\left(\mathbf{M}(\mathbf{r})\times\hat{\mathbf{Q}}\right)e^{i\mathbf{Q}\cdot\mathbf{r}}\cdot d\mathbf{r} = \mathbf{M}_\perp(\mathbf{Q})$$

Neutron scattering therefore probes the components of the sample magnetization that are *perpendicular* to the neutron's momentum transfer,$\mathbf{Q}$.

$$\int V_m(\mathbf{r})e^{i\mathbf{Q}\cdot\mathbf{r}}\cdot d\mathbf{r} = \mathbf{M}_\perp(\mathbf{Q})$$

and $$\frac{d\sigma_{magnetic}}{d\Omega} = \langle \mathbf{M}^*_\perp(\mathbf{Q})\rangle\langle \mathbf{M}_\perp(\mathbf{Q})\rangle$$

Neutron scattering measures the *correlations* in magnetization, i.e. the influence a magnetic moment has on its neighbours.

It is capable of doing this over all length scales, limited only by wavelength.

Elastic scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left|\left\langle \boldsymbol{k}', s' \middle| \hat{V}(\mathbf{r}) \middle| \boldsymbol{k}, s \right\rangle\right|^2$$

$$\propto \underbrace{\int \left|\left\langle \hat{V} \right\rangle\right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{\text{Long-range order}} + \underbrace{\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{\text{Short-range order}}$$

The contribution from deviations from the average structure:
Short-range order

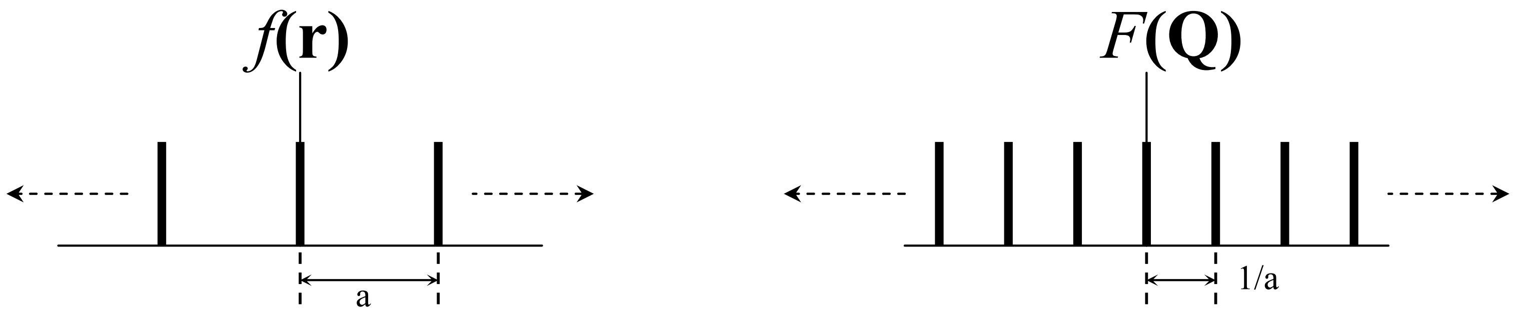
The contribution from the average structure of the sample:
Long-range order

Magnetic structure determination 1. Long-range structure

Crystalline structures

$$\frac{d\sigma}{d\Omega} \propto \int \left|\left\langle \hat{V} \right\rangle\right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

Recall the Fourier transform from a series of delta-functions

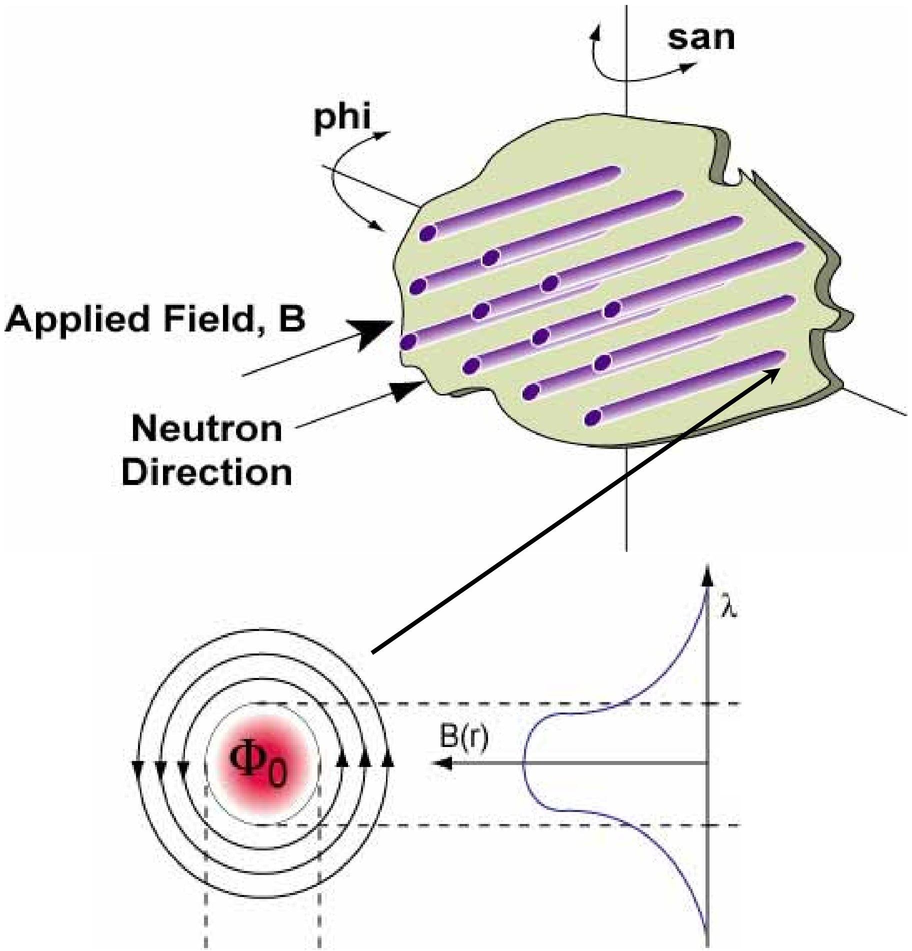


Bragg's Law: $2d\sin\theta=\lambda$

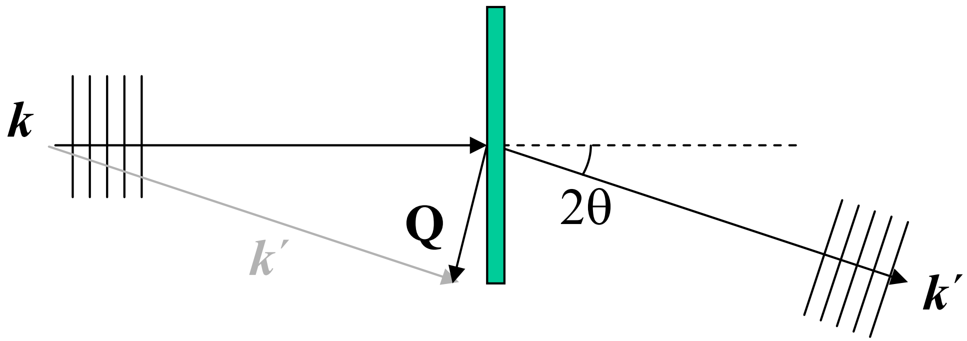
Leads to Magnetic Crystallography

A simple example of magnetic elastic scattering

MgB_2 is a superconductor below 39K, and expels all magnetic field lines (Meisner effect). Above a critical field, flux lines penetrate the sample.



Scattering geometry



The momentum transfer, **Q**, is roughly perpendicular to the flux lines, therefore all the magnetization is seen.

(recall $\frac{d\sigma_{magnetic}}{d\Omega} = \langle \mathbf{M}_\perp^*(\mathbf{Q}) \rangle \langle \mathbf{M}_\perp(\mathbf{Q}) \rangle$)

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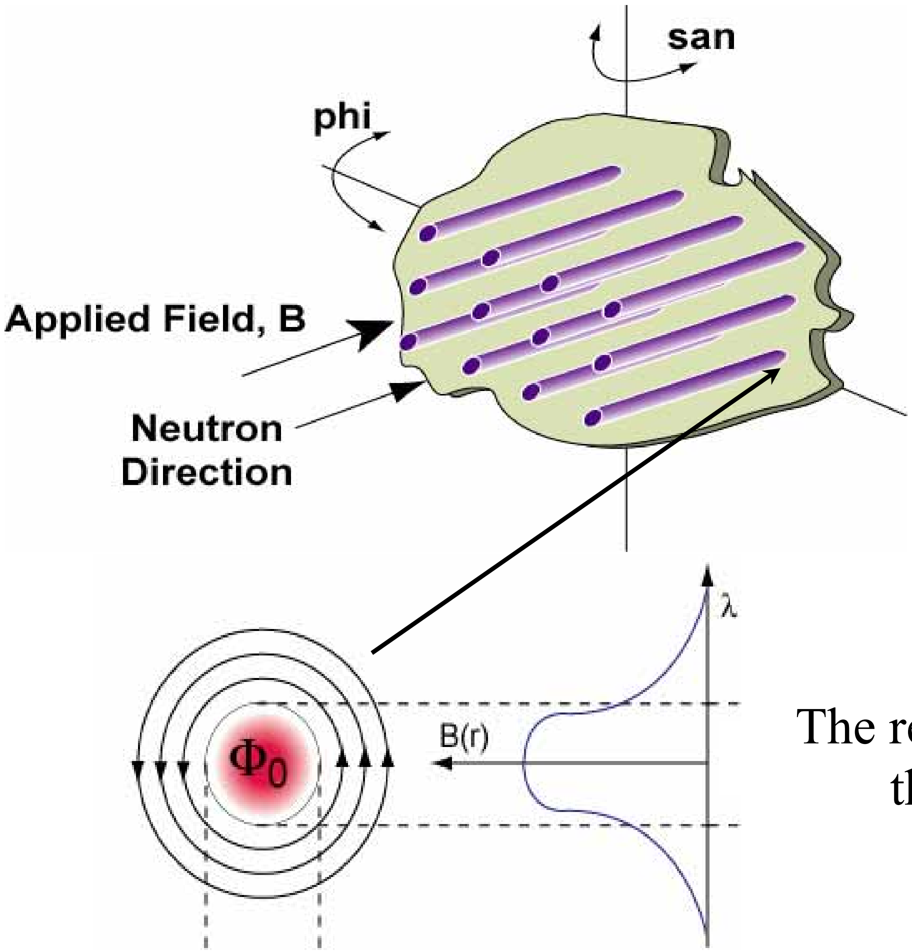

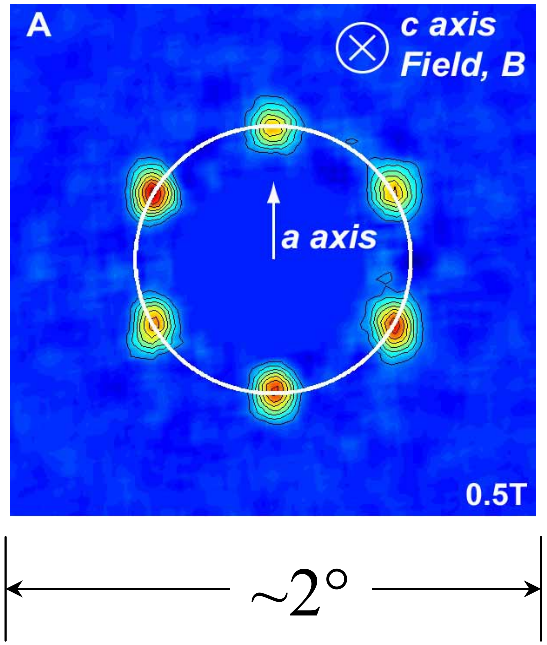

Via Bragg's Law

$2d\sin\theta=\lambda$

$\lambda = 10$ Å

$d = 425$Å

The reciprocal lattice has 60° rotational symmetry,
therefore the flux line lattice is hexagonal

Cubitt et. al. Phys. Rev. Lett. **91** 047002 (2003)

Cubitt et. al. Phys. Rev. Lett. **90** 157002 (2003)

Antiferromagnetism in MnO

Bragg peaks from crystal structure

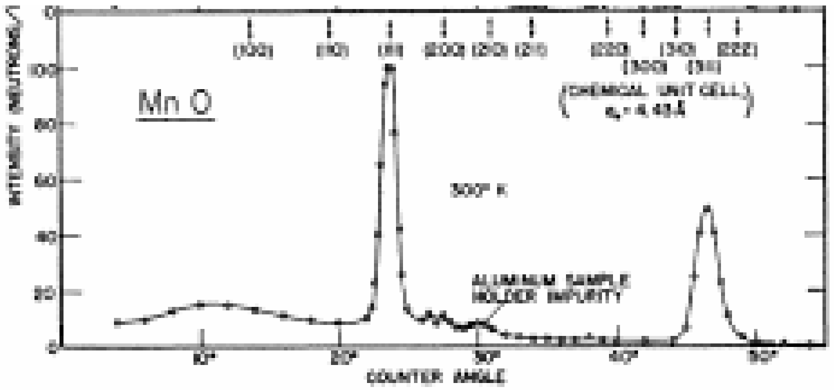

FIG. 1. Neutron diffraction patterns for MnO at room temperature and at 80°K.

300K (paramagnetic)

C. G. Shull & J. S. Smart, Phys. Rev. **76** (1949) 1256

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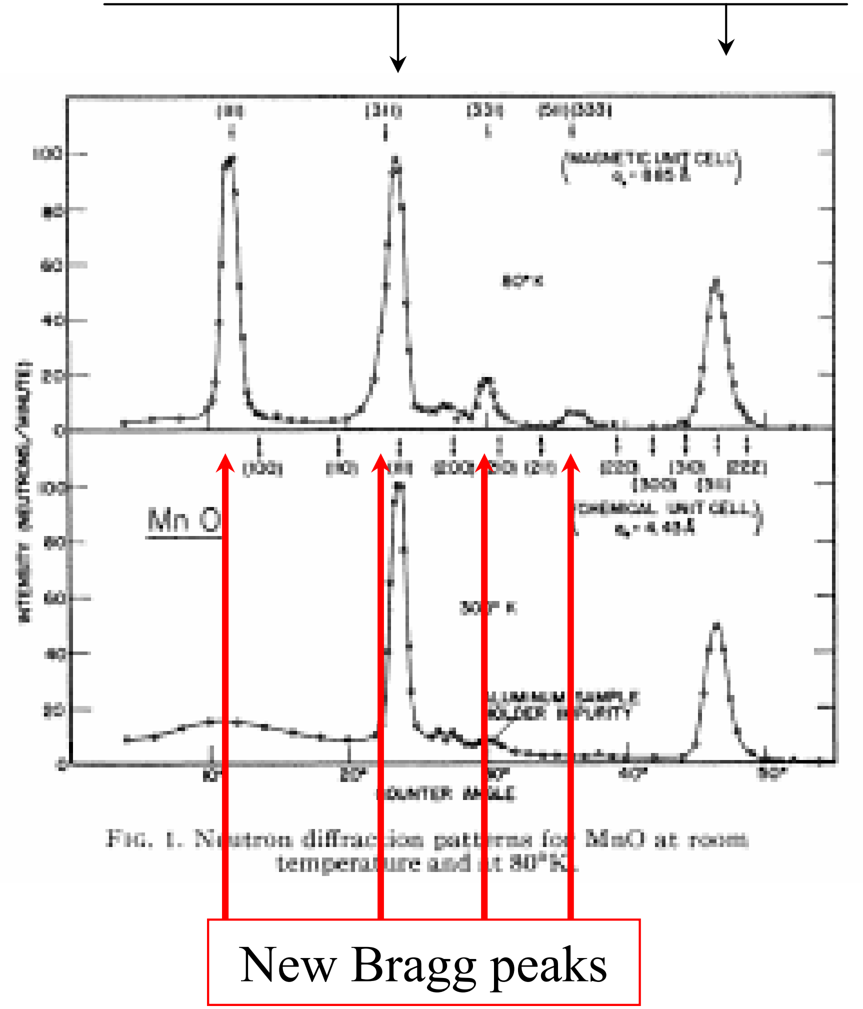

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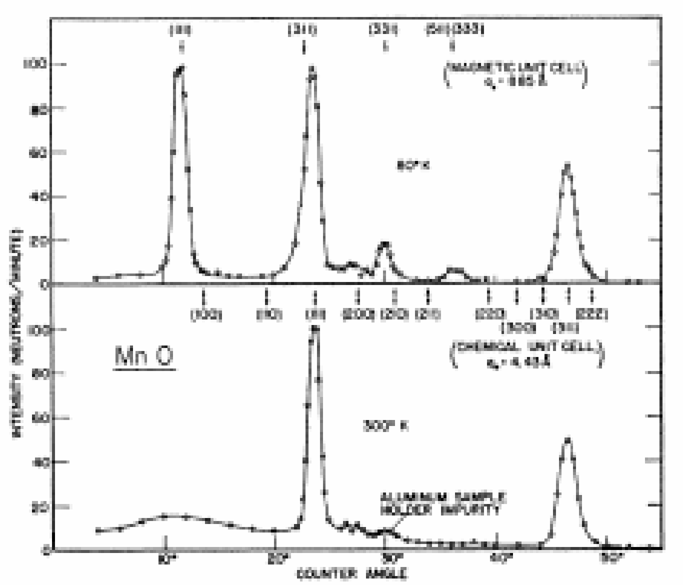
C. G. Shull & J. S. Smart, Phys. Rev. **76** (1949) 1256

Antiferromagnetism in MnO

New magnetic Bragg peaks



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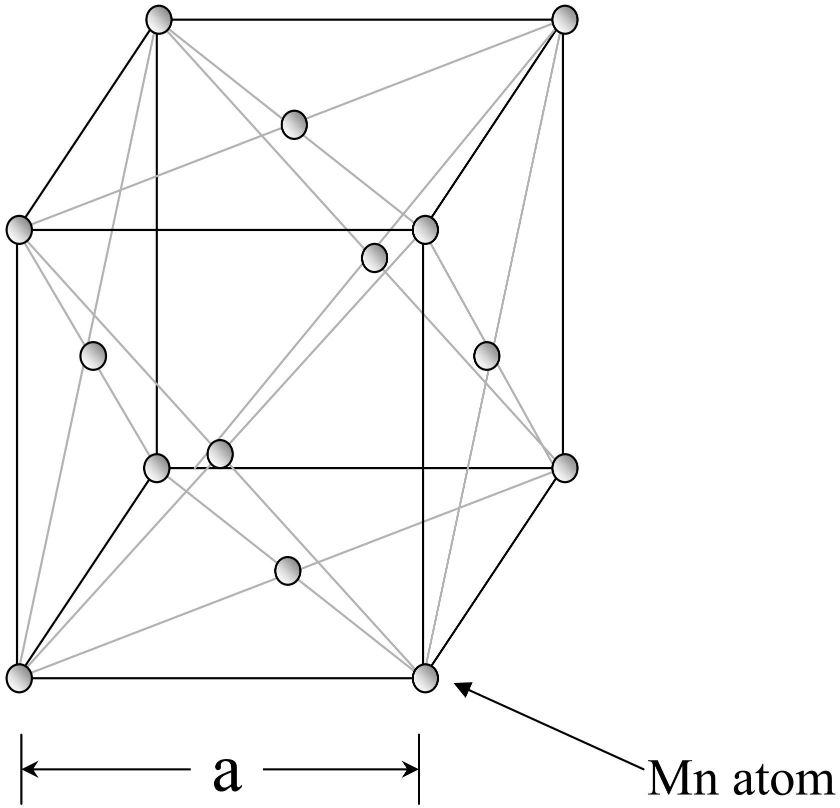
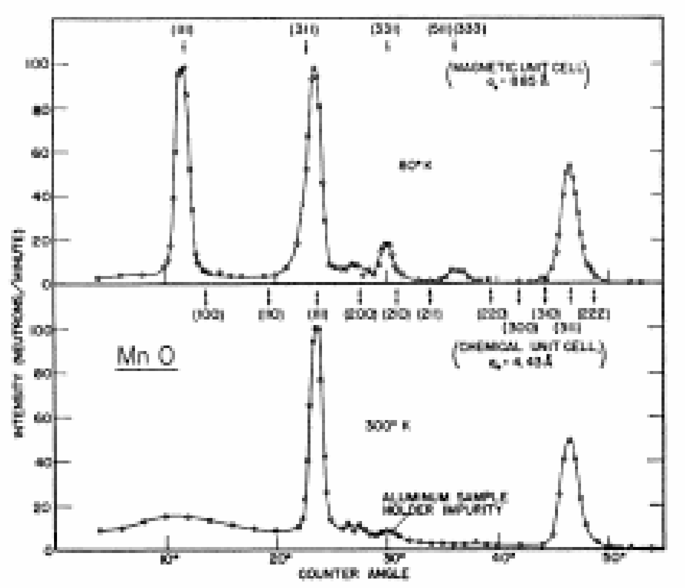
Crystal structure



Antiferromagnetism in MnO

New magnetic Bragg peaks



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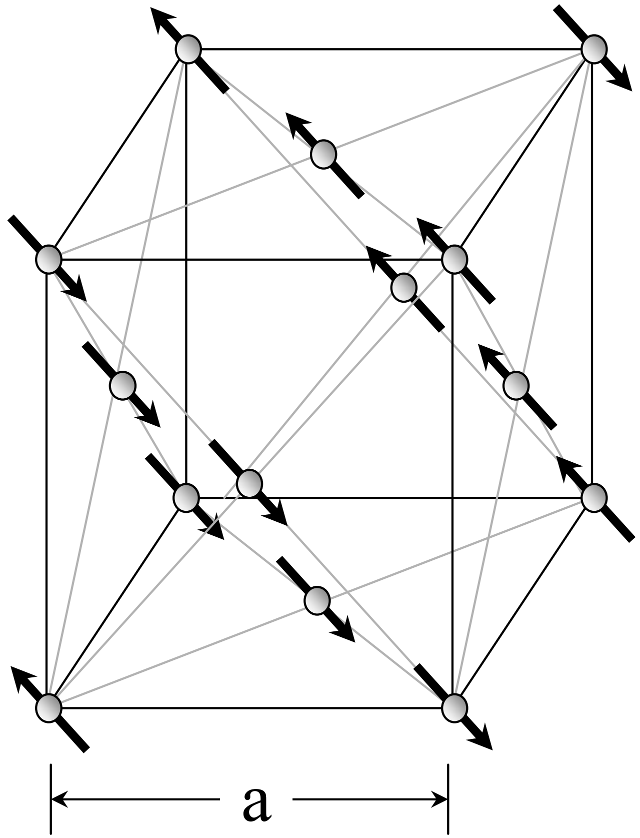
C. G. Shull & J. S. Smart, Phys. Rev. **76** (1949) 1256

Magnetic structure



C. G. Shull *et al.*, Phys. Rev. **83** (1951) 333

H. Shaked *et al.*, Phys. Rev. B **38** (1988) 11901

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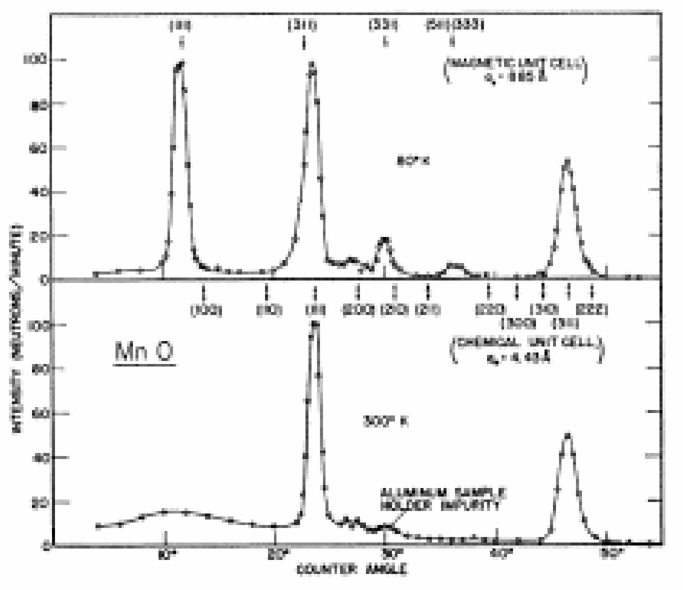

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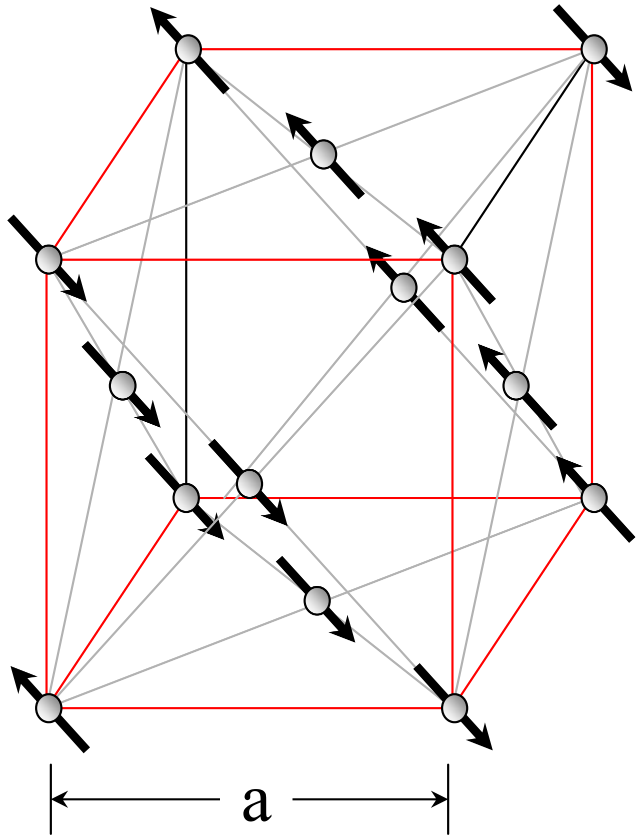
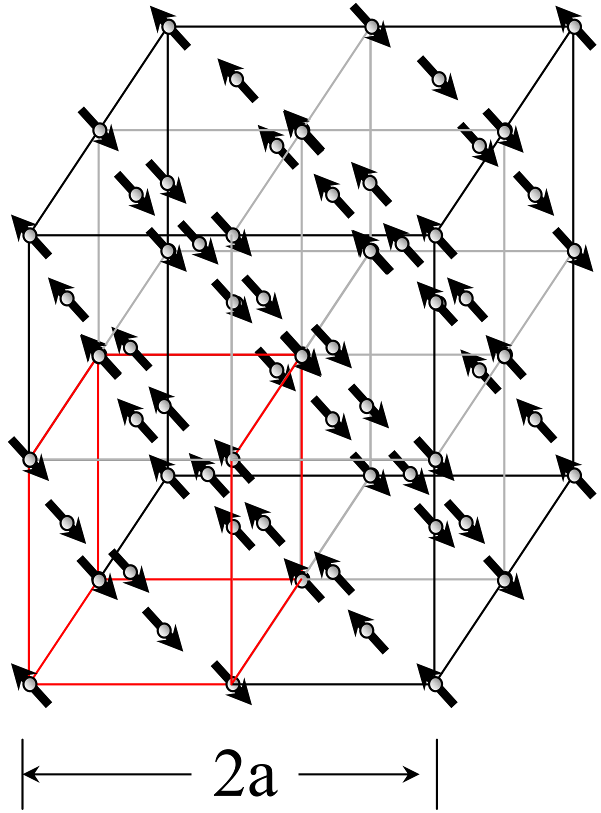


Magnetic unit cell



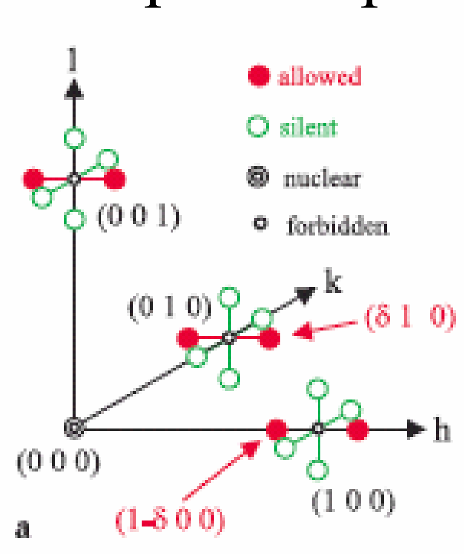
C. G. Shull *et al.*, Phys. Rev. **83** (1951) 333

H. Shaked *et al.*, Phys. Rev. B **38** (1988) 11901

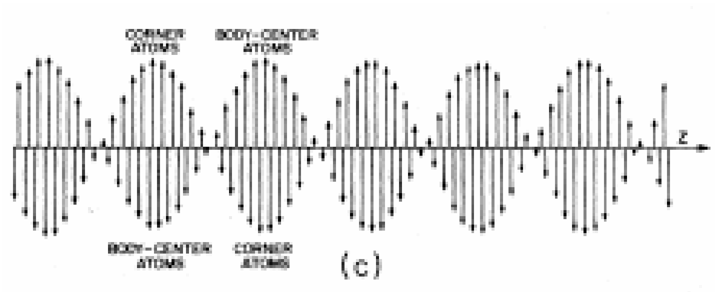
Antiferromagnetism in Chromium

Chromium is an example of an *itinerant* antiferromagnet

Reciprocal space



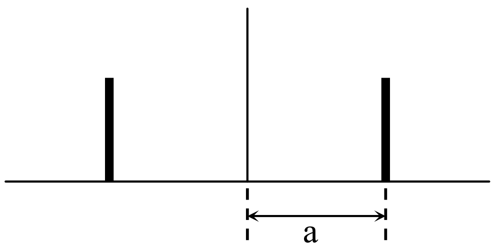
Real space



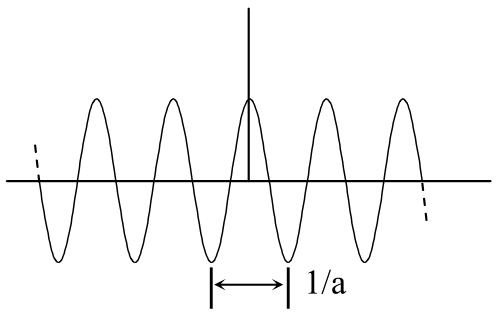
a Spin Density wave

(Recall the Fourier Transform for two Delta functions)

$f(\mathbf{r})$



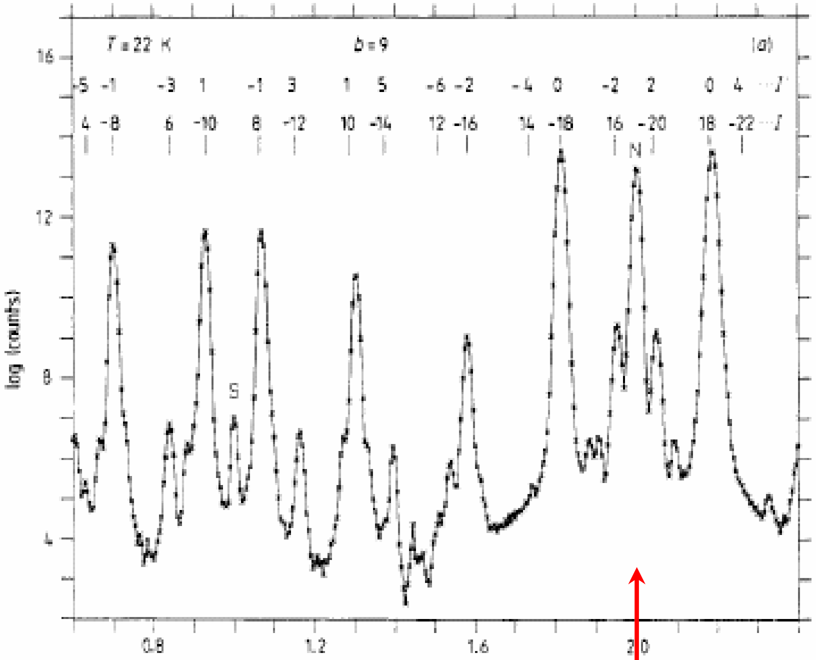
$F(\mathbf{Q})$



E.Fawcett, Rev. Mod. Phys. **60** (1988) 209

Magnetic structure of Holmium

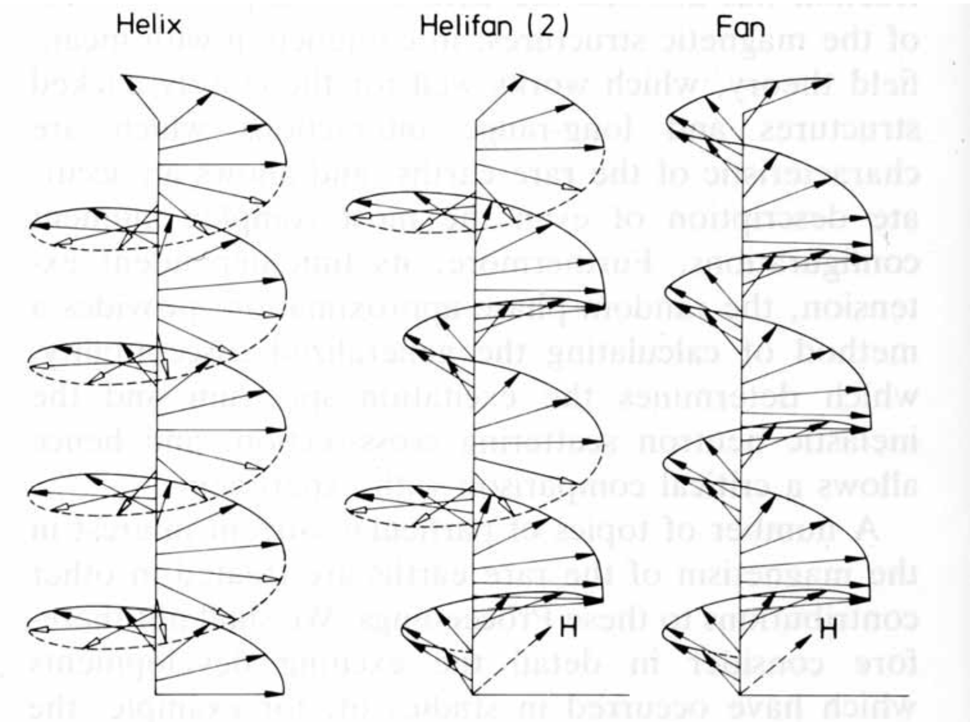
Scan along [00*l*], incommensurate peaks



l (rlu)

Nuclear peak

Real space structures



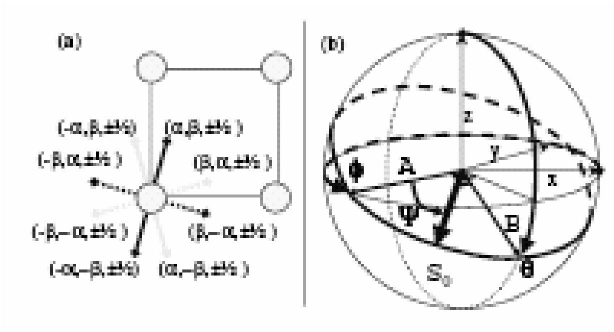
W. C. Koehler, in *Magnetic Properties of Rare Earth Metals*, ed. R. J. Elliot (Plenum Press, London, 1972) p. 81

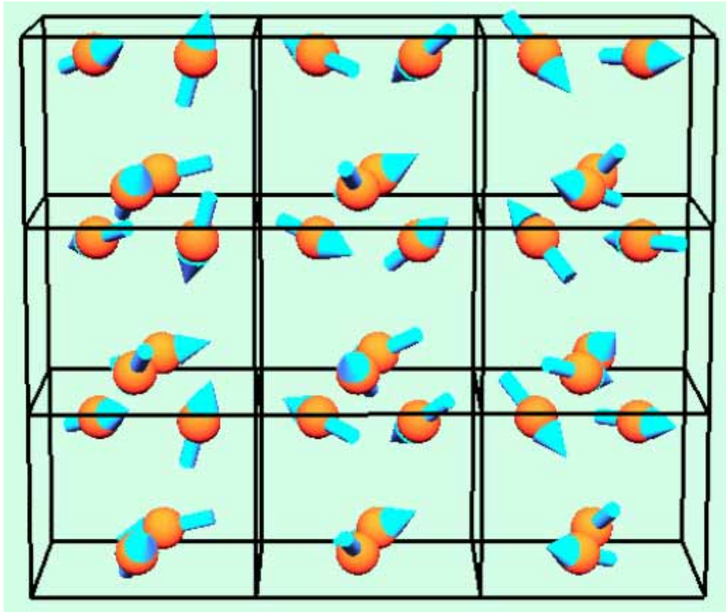
R. A. Cowley and S. Bates, J. Phys. C **21** (1988) 4113

A. R. Mackintosh and J. Jensen, Physica B **180 & 181** (1992) 1

The magnetic structure of $Cu_2Te_2O_5X_2$ (X=Cl, Br)

A map in reciprocal space…leads to the structure in real space



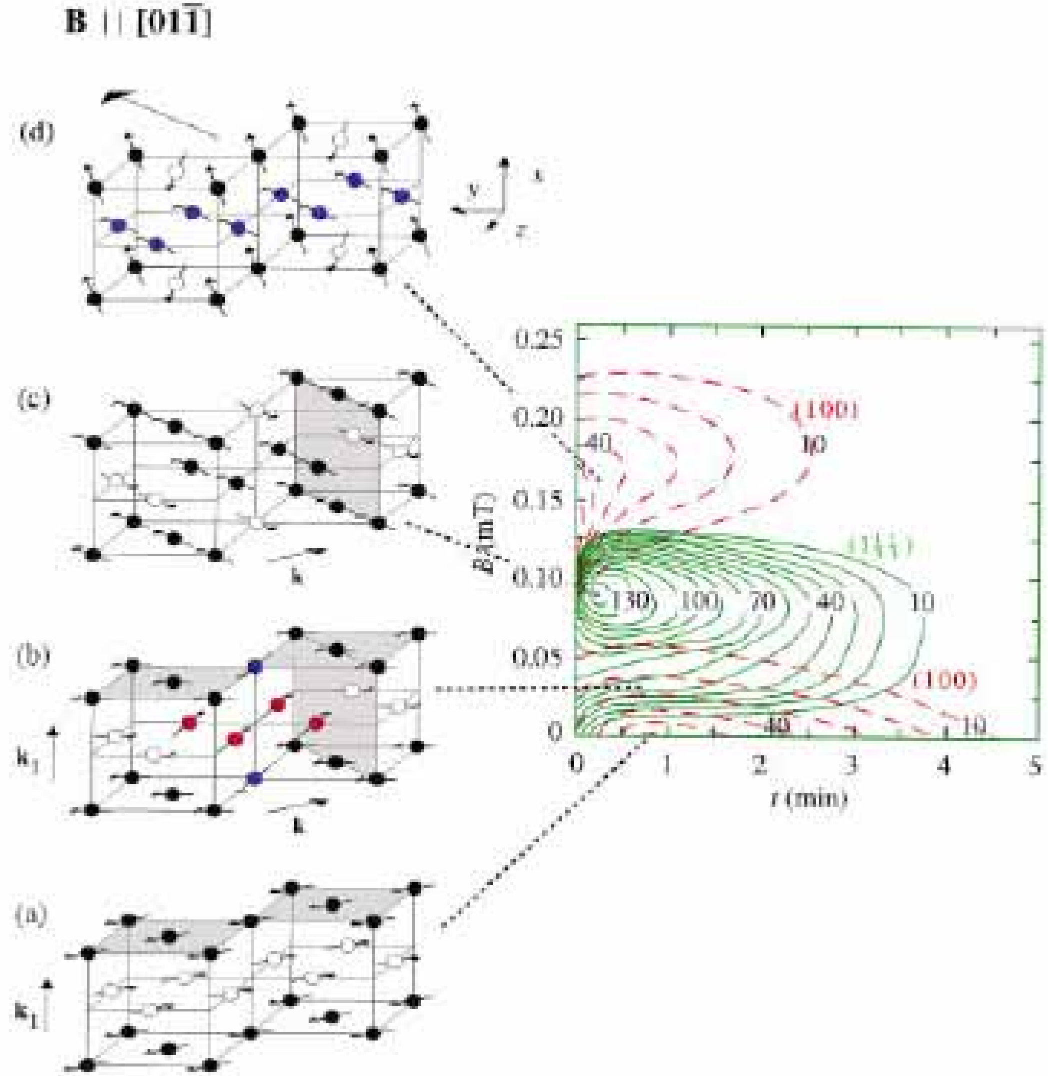


(complicated!)

O. Zaharko *et al.*, Phys. Rev. Lett. **93** (2004) 217206

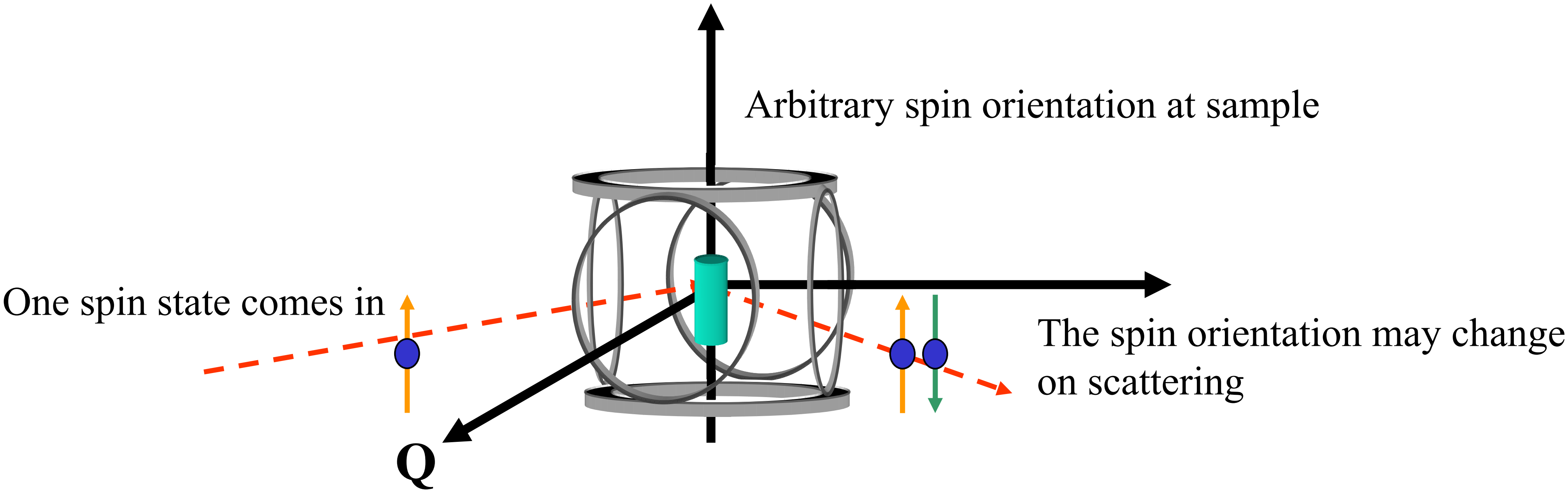
Nuclear magnetic order

The *nuclear* magnetic phase diagram as a function of field at 60 nK

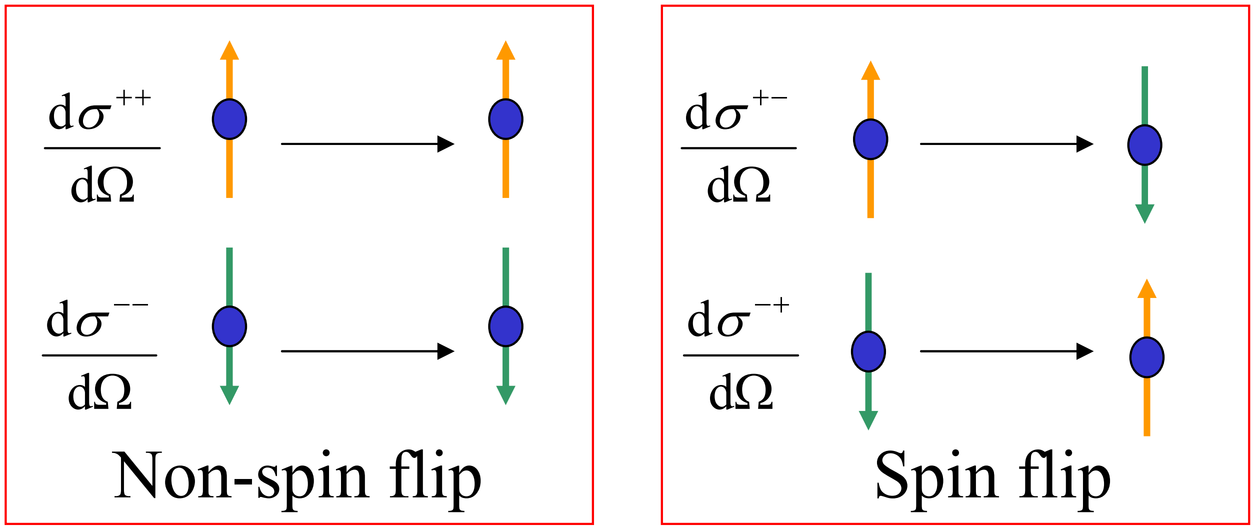


A. S. Oja and O. V. Lounasmaa, Rev. Mod. Phys. **69** (1997) 1

Polarized neutrons



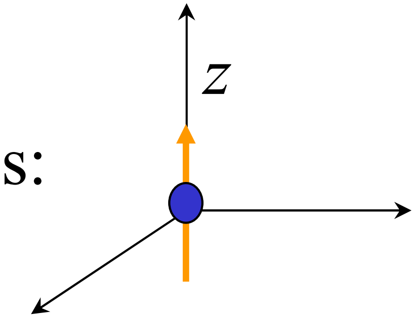
Four possibilities:



Polarized neutrons

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left| \left\langle \boldsymbol{k}', s' \left| \hat{V}(\mathbf{r}) \right| \boldsymbol{k}, s \right\rangle \right|^2$$

$$U^{ss'} = \left\langle s' \left| \hat{V} \right| s \right\rangle = \left\langle s' \left| b + B\hat{\mathbf{I}} \cdot \hat{\boldsymbol{\sigma}} - \mathbf{M}_\perp \cdot \hat{\boldsymbol{\sigma}} \right| s \right\rangle$$



Neutron polarization coordinates:
z is parallel to **P**

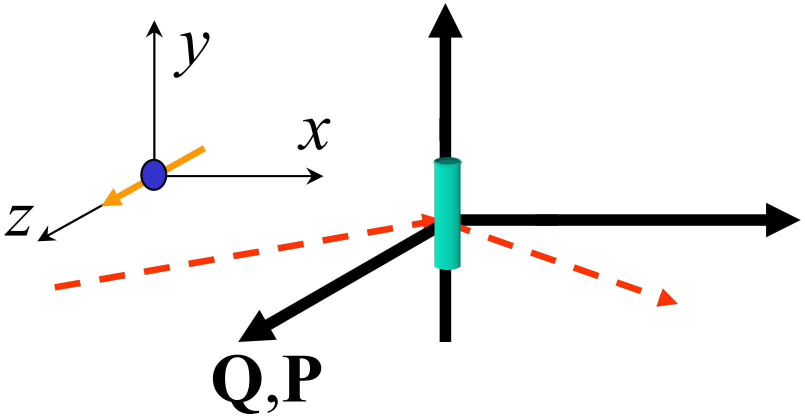
Neglect nuclear spin for the moment

$$U^{++} = b - M_{\perp z}$$
$$U^{--} = b + M_{\perp z}$$
$$U^{+-} = -\left(M_{\perp x} + iM_{\perp y}\right)$$
$$U^{-+} = -\left(M_{\perp x} - iM_{\perp y}\right)$$

REMEMBER: the only visible components of the magnetization are PERPENDICULAR to **Q**

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Elastic scattering, **P** || **Q**



$$\frac{\mathrm{d}\sigma^{\pm\pm}}{\mathrm{d}\Omega} = \left|\int b(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}\right|^2$$

$$\frac{\mathrm{d}\sigma^{\pm\mp}}{\mathrm{d}\Omega} = \left|\int \left(-M_{\perp x}(\mathbf{r}) \pm iM_{\perp y}(\mathbf{r})\right) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}\right|^2$$

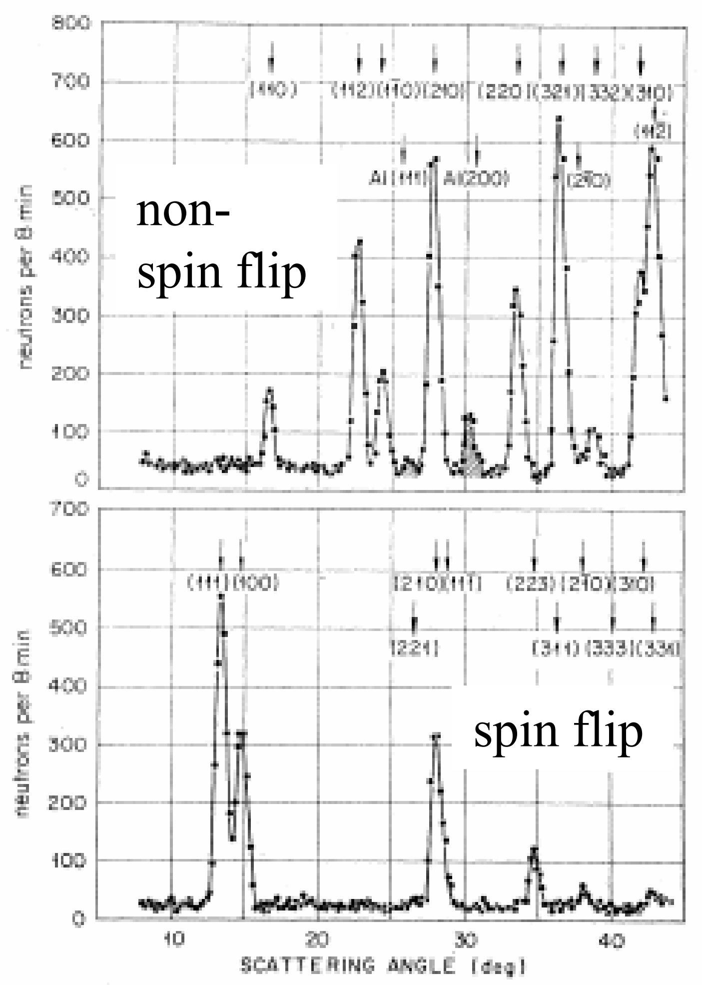
$$= \int \left[M_\perp(\mathbf{r}_i) M_\perp^*(\mathbf{r}_j) \mp i\hat{z}\left(M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j)\right)\right] e^{i\mathbf{Q}\cdot(\mathbf{r}_i - \mathbf{r}_j)} \cdot d\mathbf{r}$$

- The non-spin flip cross-sections have only nuclear components
- The spin-flip cross-sections have only the magnetic components
- There is a *complete separation* of nuclear from magnetic scattering.
- The cross term in the spin flip cross-sections usually cancels,
  i.e. $M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j) = -\left(M_\perp(\mathbf{r}_i) \times M_\perp^*(\mathbf{r}_j)\right)$

A polarization analysis measurement of α-Fe_2O_3

α-Fe_2O_3 is antiferromagnetic.

Powder diffraction gives Bragg peaks with mixed nuclear and magnetic intensities

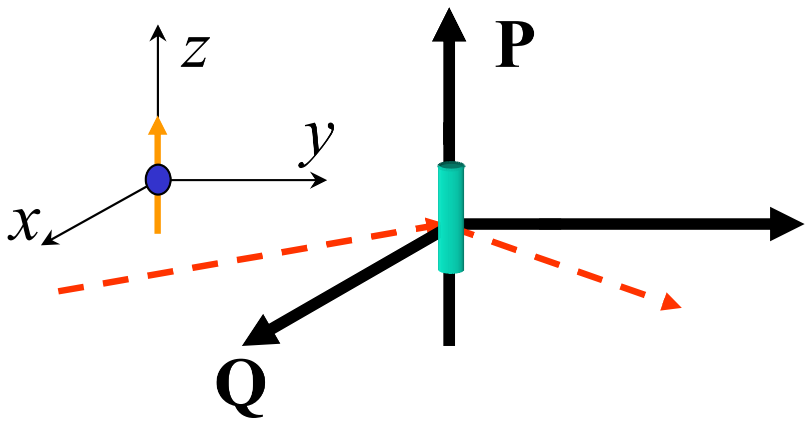


Nuclear Bragg peaks

Magnetic Bragg peaks

R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Elastic scattering, $\mathbf{P} \perp \mathbf{Q}$



$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int \left(b(\mathbf{r}) \mp M_{\perp z}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left| \int M_{\perp y}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

- The non-spin flip cross-sections probe the components of the magnetization parallel to the neutron polarization
- The spin-flip cross-sections probe the components of the magnetization that are perpendicular to *both* the polarization and to **Q**
- There is a difference between the two non-spin flip cross-sections, the difference is called nuclear-magnetic interference.
- The two spin-flip cross-sections are equivalent

Polarized neutrons and ferromagnets

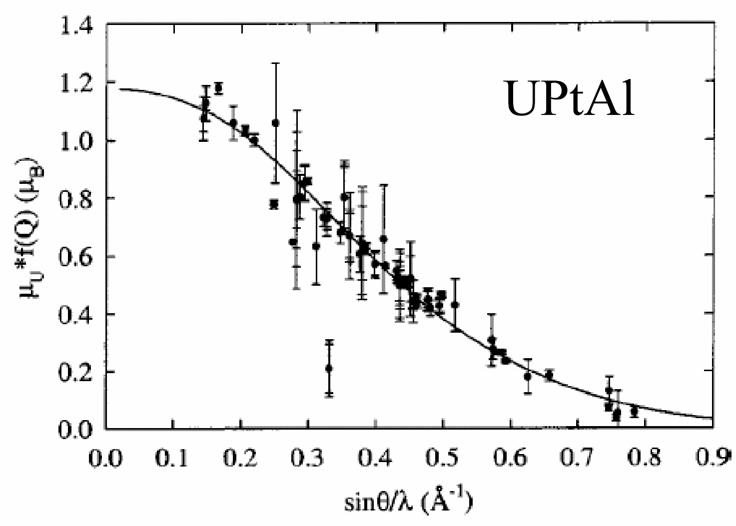
Single domain collinear ferromagnets have a magnetization that is always parallel to **P**.

If $\mathbf{P}\perp\mathbf{Q}$, $M_{\perp y}(\mathbf{r})=0$ and

$$\begin{cases} \dfrac{\mathrm{d}\sigma^{\pm\pm}}{\mathrm{d}\Omega} = \left|\int \left(b(\mathbf{r}) \mp M(\mathbf{r})\right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r}\right|^2 \\ \dfrac{\mathrm{d}\sigma^{\pm\mp}}{\mathrm{d}\Omega} = 0 \end{cases}$$

The difference between the two non-spin flip cross-sections will give information on the ferromagnetic moment.

Plot M(**Q**) as a function of Q and a characteristic line shape will emerge:



P. Javorsky *et al.*, Phys. Rev. B **67** (2003) 224429

This characteristic line shape is known as the *form factor*, $f(Q)$

Magnetic form factors

The form factor arises from the *spatial distribution* of *unpaired electrons* around a magnetic atom.

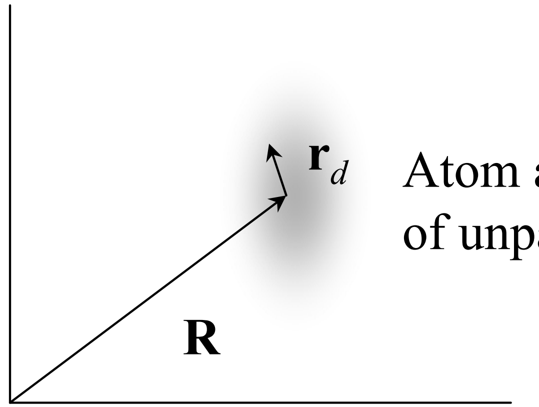
For magnetic scattering due only to electron spin:

$$\mathbf{M}(\mathbf{Q}) = \int \mathbf{M}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}$$

$$\propto \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d \int S(\mathbf{R}) e^{i\mathbf{Q}\cdot\mathbf{R}} \cdot d\mathbf{R}$$

$$= f(Q) \int S(\mathbf{R}) e^{i\mathbf{Q}\cdot\mathbf{R}} \cdot d\mathbf{R}$$

$$f(Q) = \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d$$



Atom at position **R** with a density of unpaired (magnetic) electrons

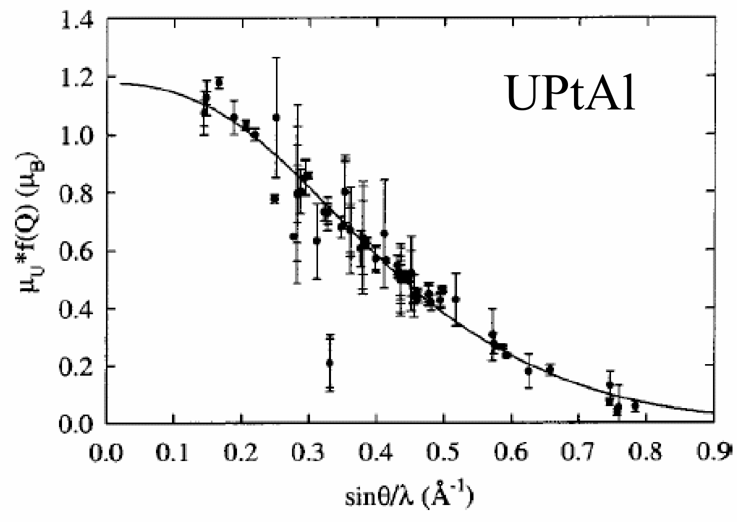
For magnetic scattering due only to electron spin:

$$\frac{d\sigma_{magnetic}}{d\Omega} = \langle \mathbf{M}_\perp^*(\mathbf{Q}) \rangle \langle \mathbf{M}_\perp(\mathbf{Q}) \rangle$$

$$\propto f^2(Q) \int S_\perp(\mathbf{R}_i) S_\perp^*(\mathbf{R}_j) e^{i\mathbf{Q}\cdot(\mathbf{R}_i - \mathbf{R}_j)} \cdot d\mathbf{R}$$

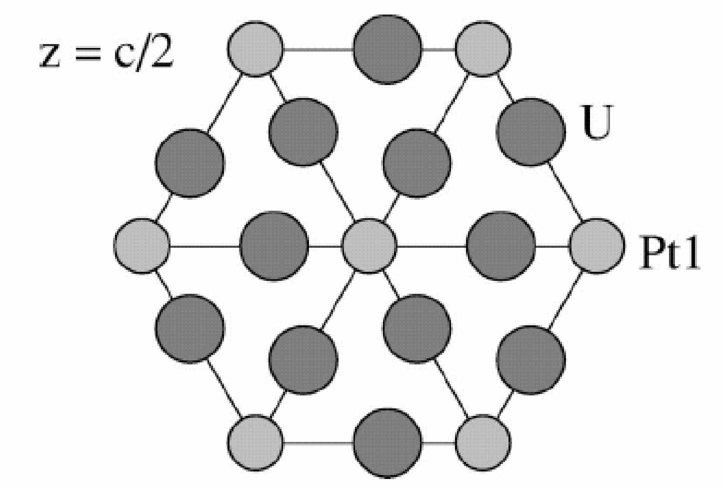
Magnetic electron density

$$f(Q) = \int s_d e^{i\mathbf{Q}\cdot\mathbf{r}_d} \cdot d\mathbf{r}_d$$

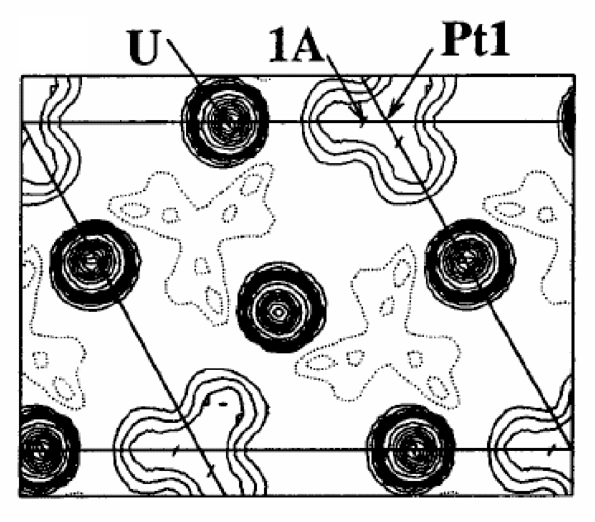


P. Javorsky *et al*., Phys. Rev. B **67** (2003) 224429

From the form factors, the magnetic moment density in the unit cell can be derived



crystal structure



magnetic moment density

Elastic scattering

$$\frac{d\sigma}{d\Omega} = \left(\frac{m_n}{2\pi\hbar^2}\right)^2 \sum_{\zeta',s'} p_s \left|\left\langle \boldsymbol{k}', s' \middle| \hat{V}(\mathbf{r}) \middle| \boldsymbol{k}, s \right\rangle\right|^2$$

$$\propto \underbrace{\int \left|\left\langle \hat{V} \right\rangle\right|^2 e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{} + \underbrace{\left(\left|\left\langle \hat{V}^2 \right\rangle\right| - \left|\left\langle \hat{V} \right\rangle\right|^2\right) \int z(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} \cdot d\mathbf{r}}_{}$$

The contribution from deviations from the average structure: *Short-range* order

The contribution from the average structure of the sample: *Long-range* order

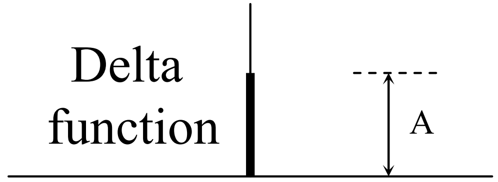
The physical meaning of $z(\mathbf{r})$

Given an atom at position $\mathbf{r}_1$,
what's the probability of finding a similar atom at position $\mathbf{r}_2$?

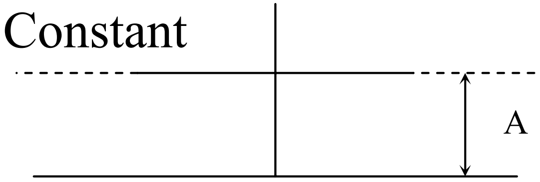
$z(\mathbf{r})$, real space | $Z(\mathbf{Q})$, reciprocal space

Zero probability?

Delta function

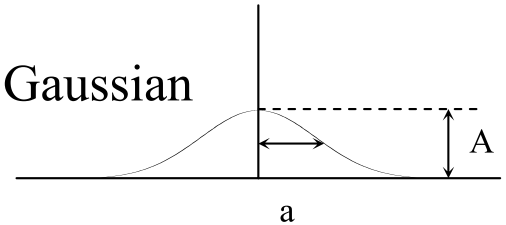


Constant

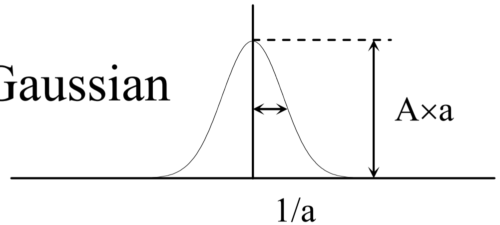


Gaussian probability?

Gaussian

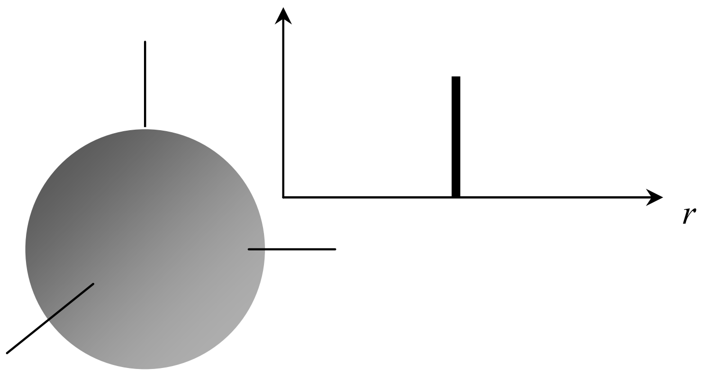


Gaussian

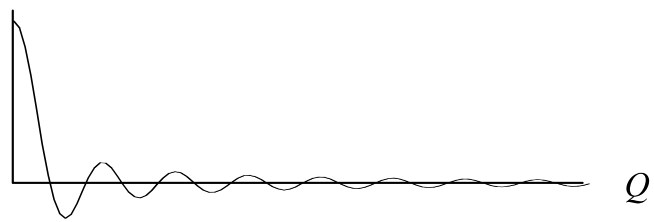


Any spherically symmetric function?
e.g. a hollow sphere

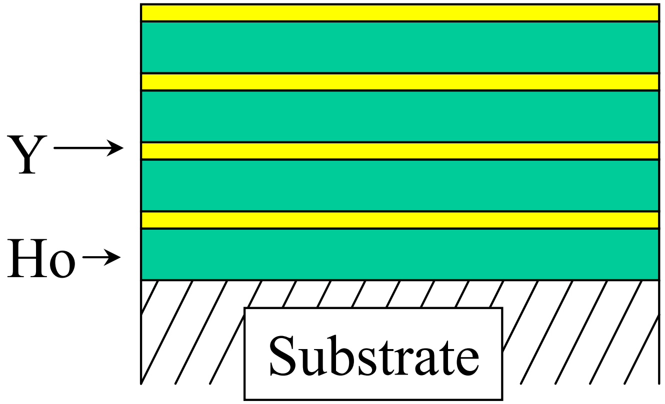
Delta function at r ≠ 0

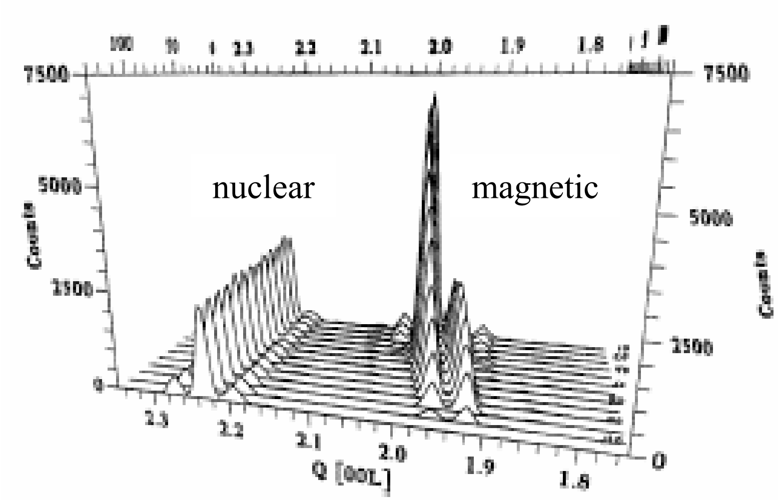


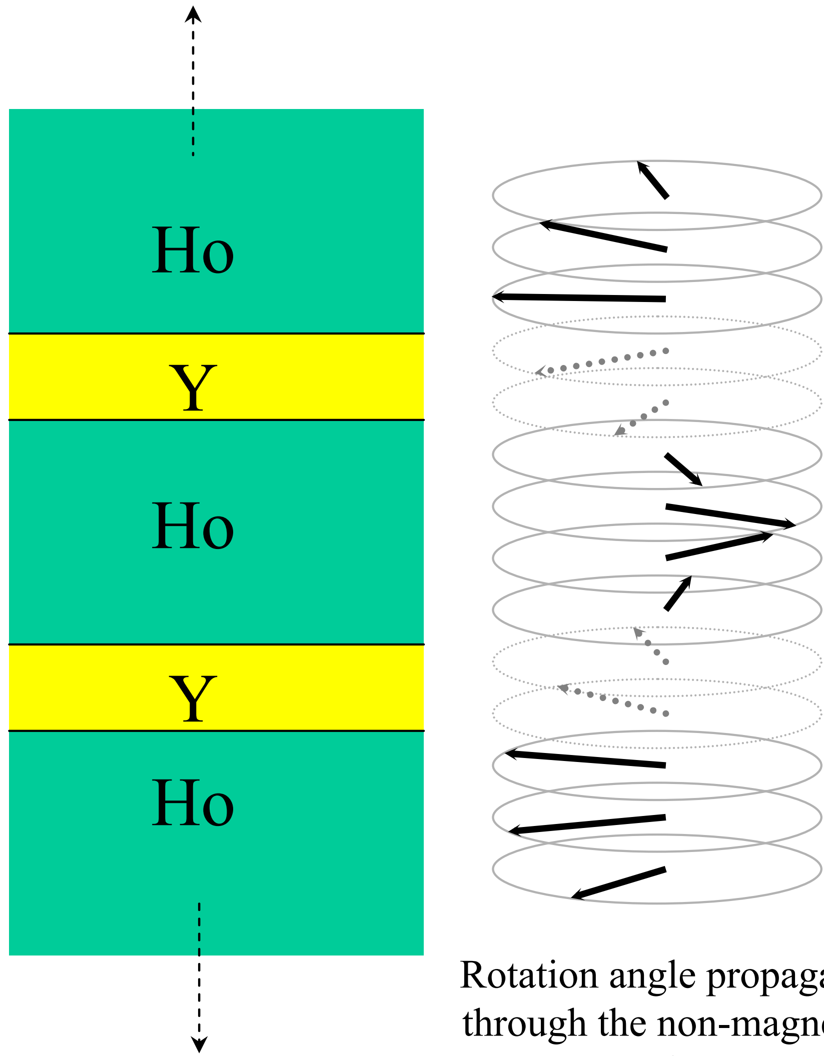
$\sin(Qr)/Qr$



Measurement of Ho_{41}/Y_{16}





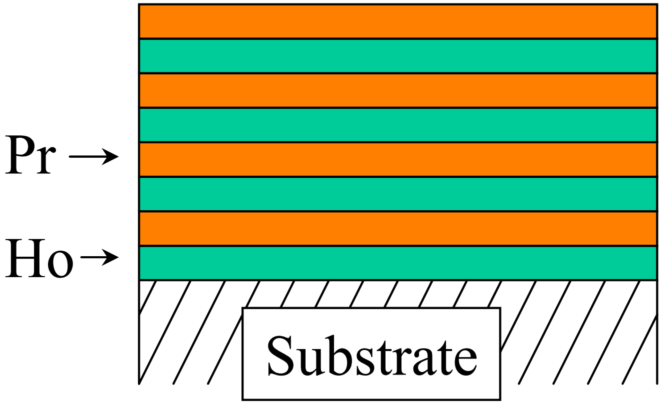


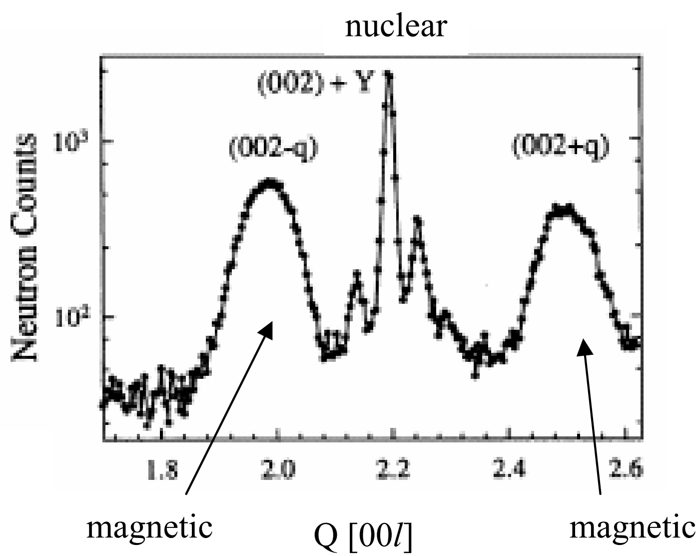
Rotation angle propagates through the non-magnetic yttrium

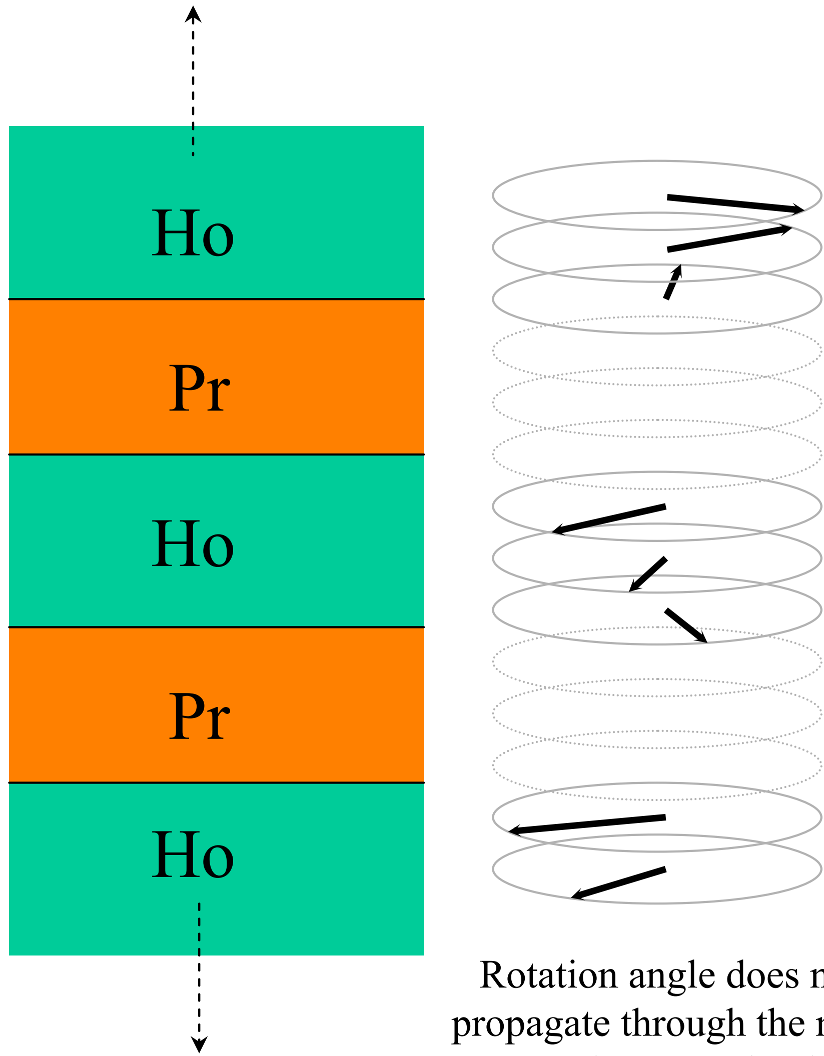
R. A. Cowley, Physica B **350** (2004) 1

Magnetic structure of Holmium superlattices

Measurement of Ho_{20}/Pr_{20}







Rotation angle does not propagate through the non-magnetic praseodymium

R. A. Cowley, Physica B **350** (2004) 1

Diffuse scattering and polarization analysis

Magnetic and nuclear short-range order are often coupled

At the very least, the diffuse scattering is often a mixture of magnetic and nuclear contributions

Neutron polarization analysis is usually essential for the measurement of magnetic short-range order

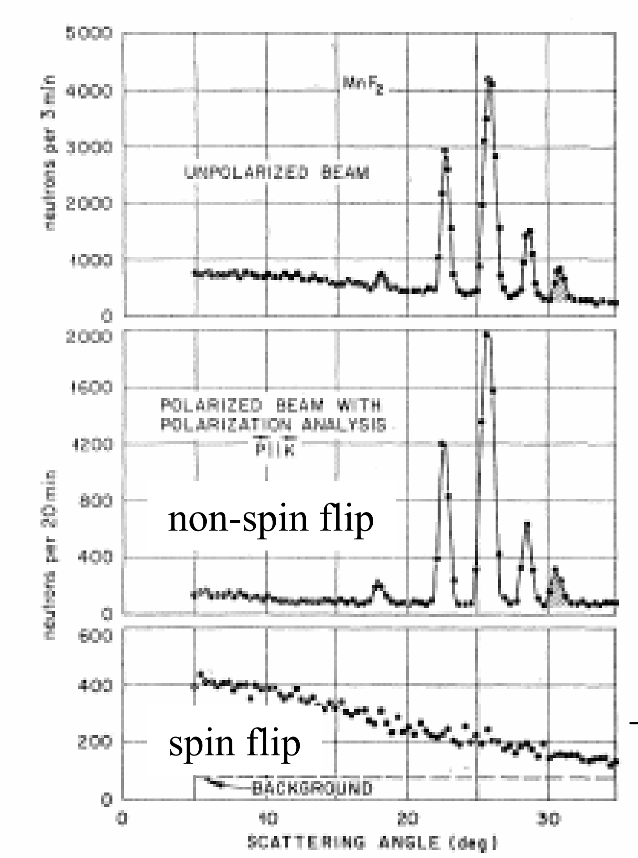
Recall: if $\mathbf{P} \parallel \mathbf{Q}$, the *non-spin flip scattering* is all *nuclear*
the *spin flip scattering* is all *magnetic*

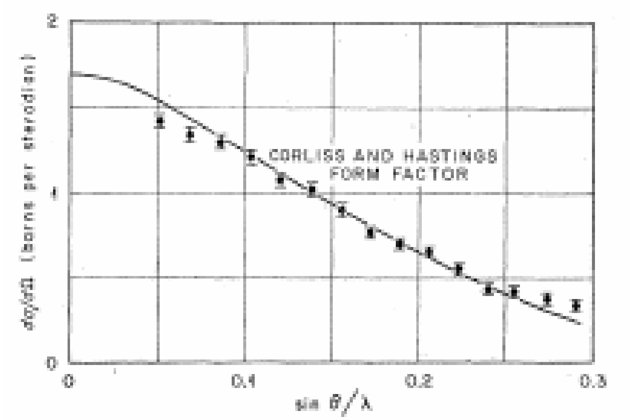
if $\mathbf{P} \perp \mathbf{Q}$, the *non-spin flip scattering* is due to *nuclear and magnetic,*
the magnetic components are *parallel to* **P**.
the *spin flip scattering* is all *magnetic*,
the magnetic components are perpendicular to **P** *and* **Q**.

T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford

Diffuse scattering from a paramagnet, MnF_2

For a paramagnet, $z(\mathbf{r}) = \delta(r)$, $$\frac{d\sigma^{\pm\mp}}{d\Omega} \propto \frac{2}{3} f^2(Q) S(S+1)$$

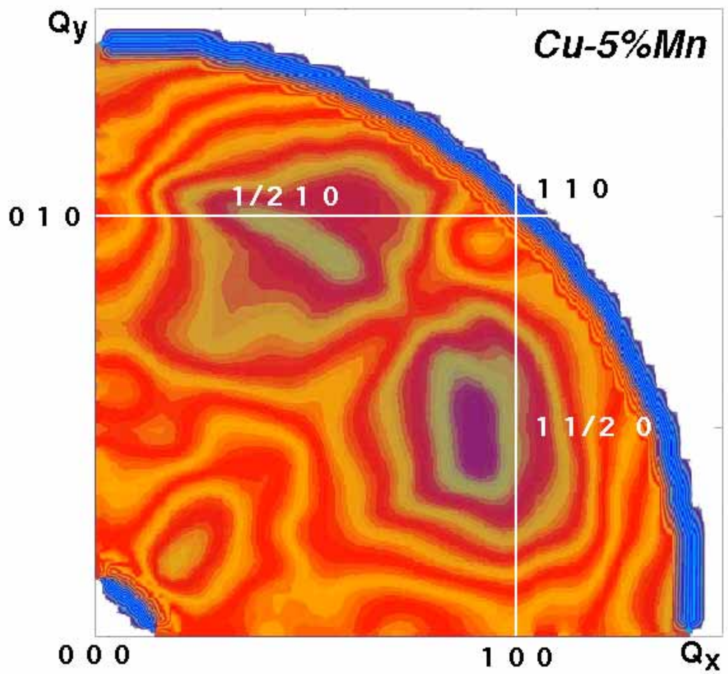




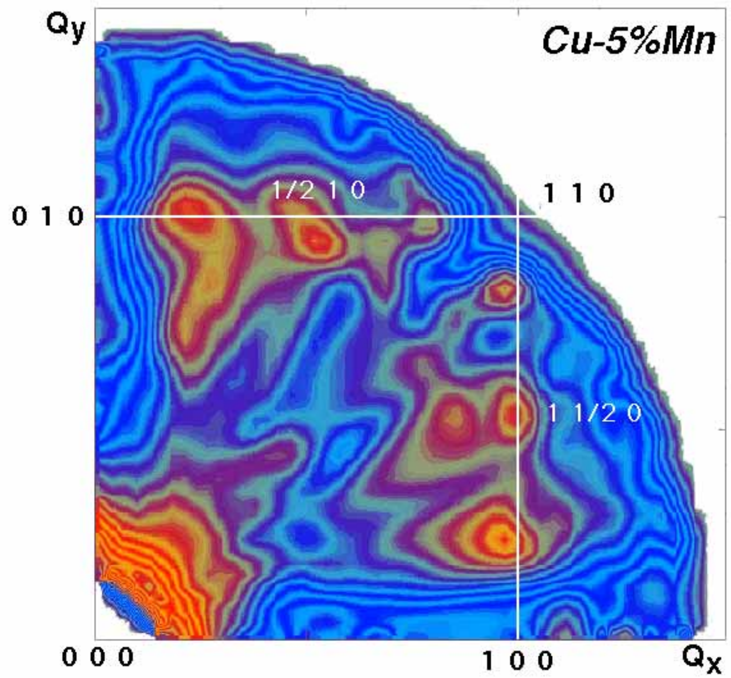
R. M. Moon, T. Riste and W. K. Koehler, Phys. Rev. **181** (1969) 920

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



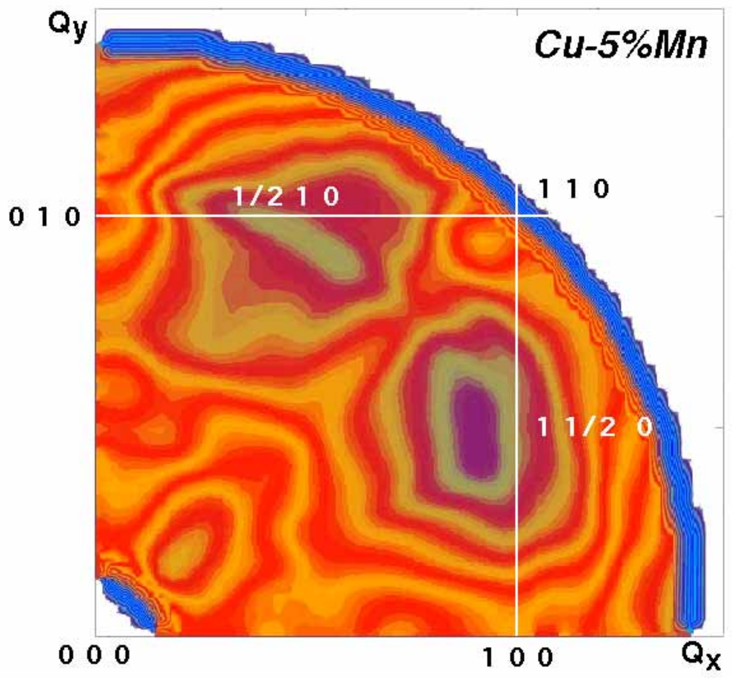
magnetic scattering



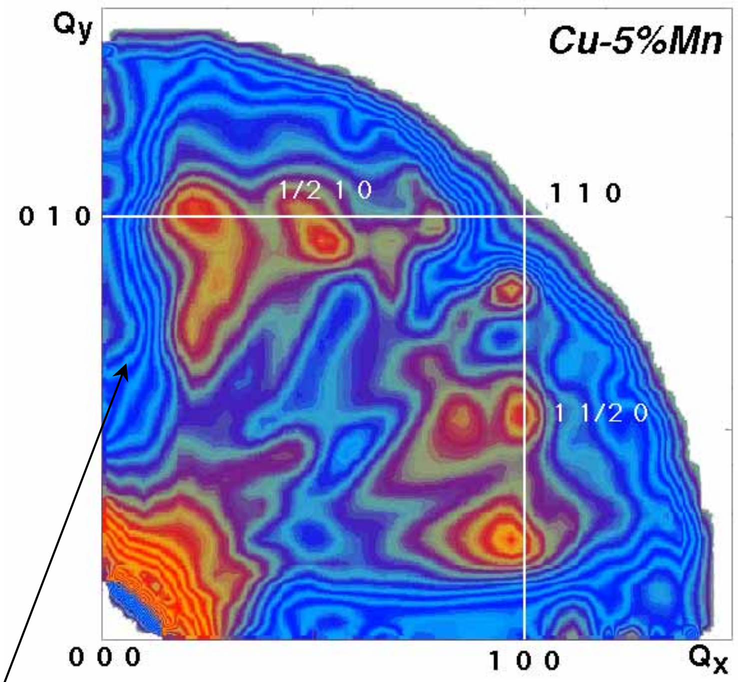
A. P. Murani *et al*., Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al*., J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



magnetic scattering

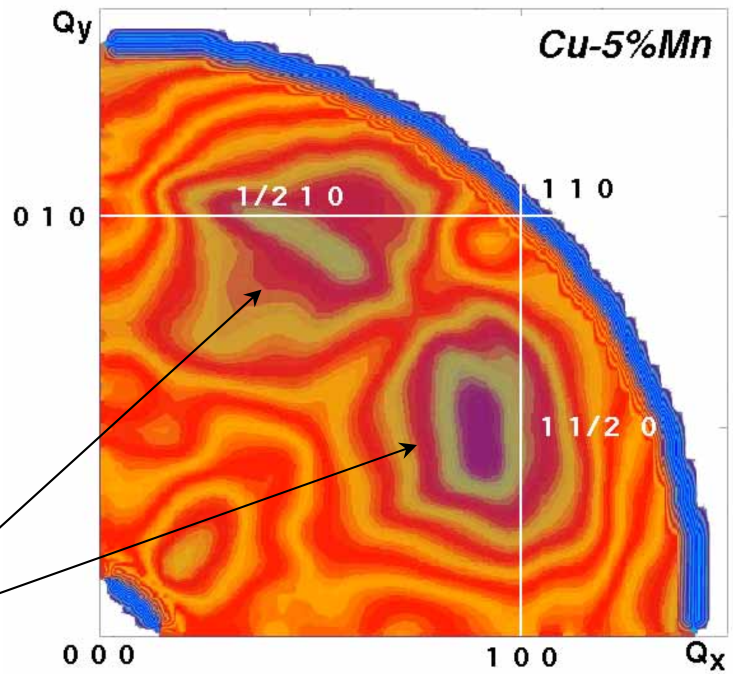


Form-factor like background, frozen moments

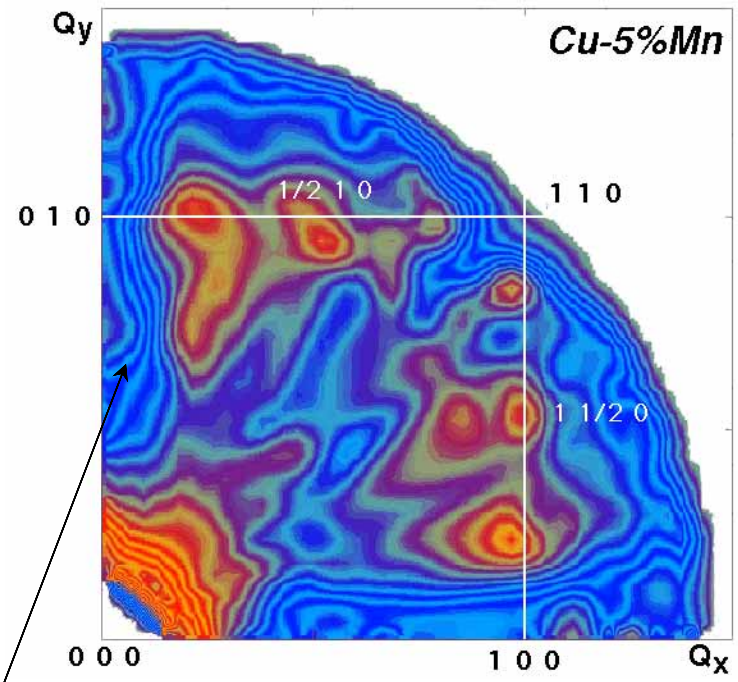
A. P. Murani *et al*., Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al*., J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



magnetic scattering



Anticlustering
by Mn atoms

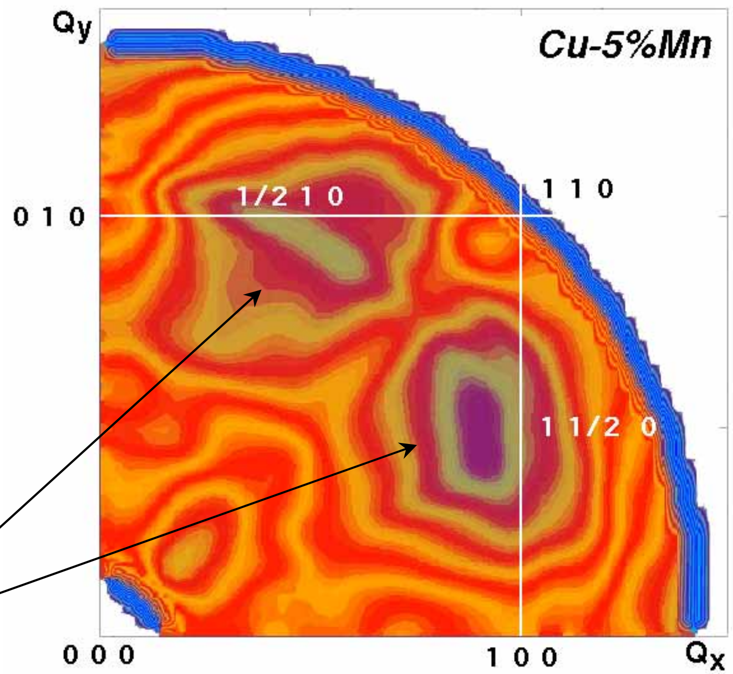
(i.e., Mn atoms want to surround themselves with Cu)

Form-factor like background, frozen moments

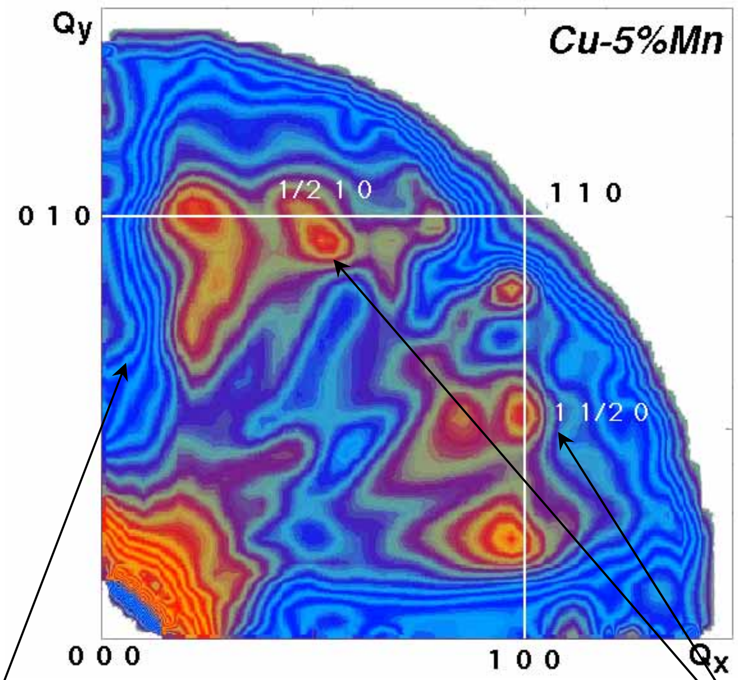
A. P. Murani *et al*., Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al*., J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



magnetic scattering



Anticlustering
by Mn atoms

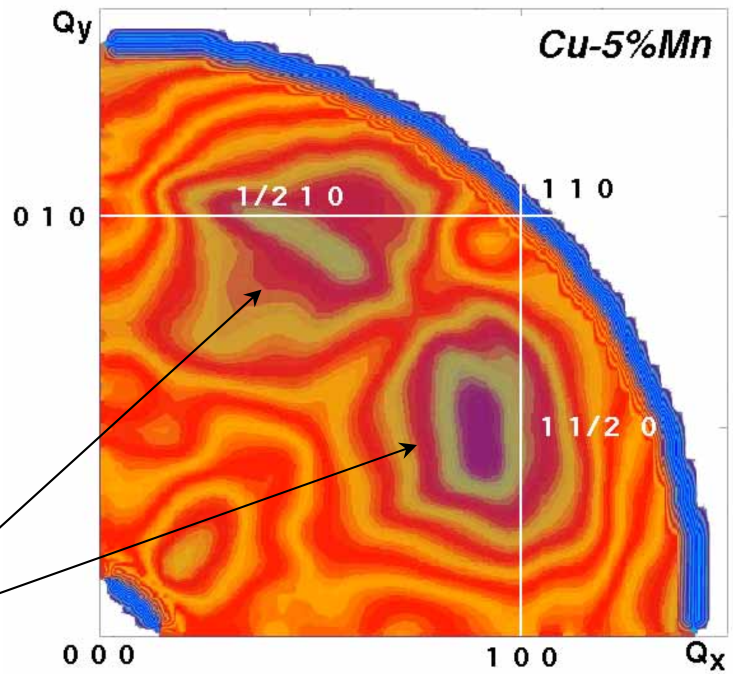
Anticlustering,
ferromagnetic

Form-factor like background, frozen moments

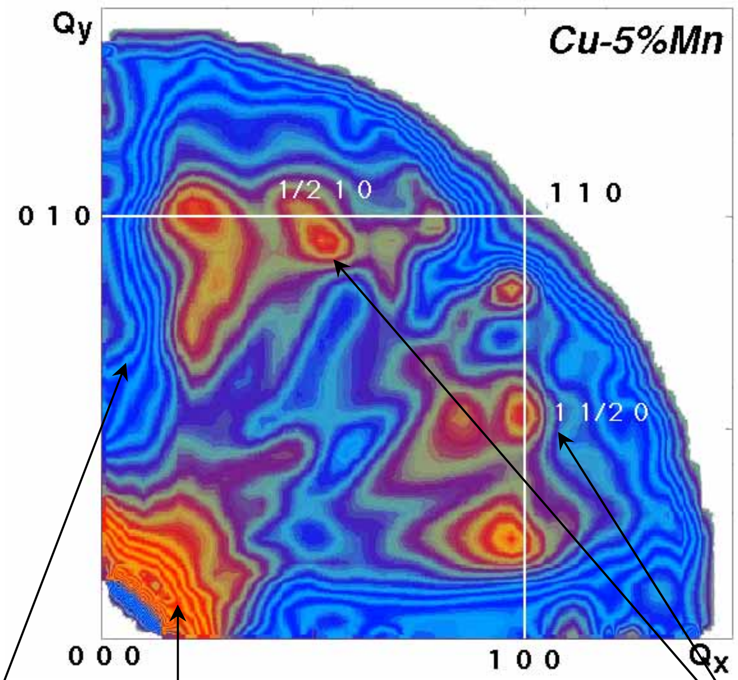
A. P. Murani *et al*., Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al*., J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



magnetic scattering



Anticlustering by Mn atoms

Anticlustering, ferromagnetic

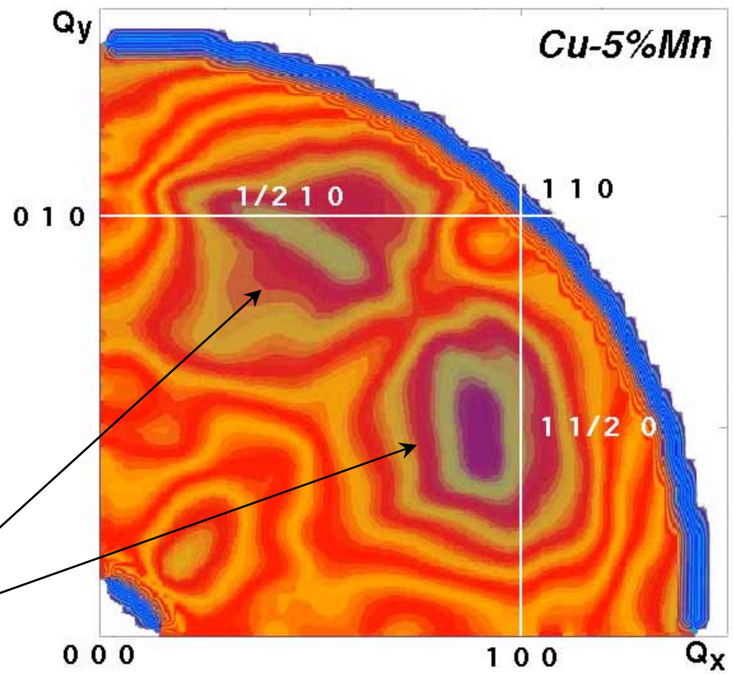
Forward scattering, ferromagnetic

Form-factor like background, frozen moments

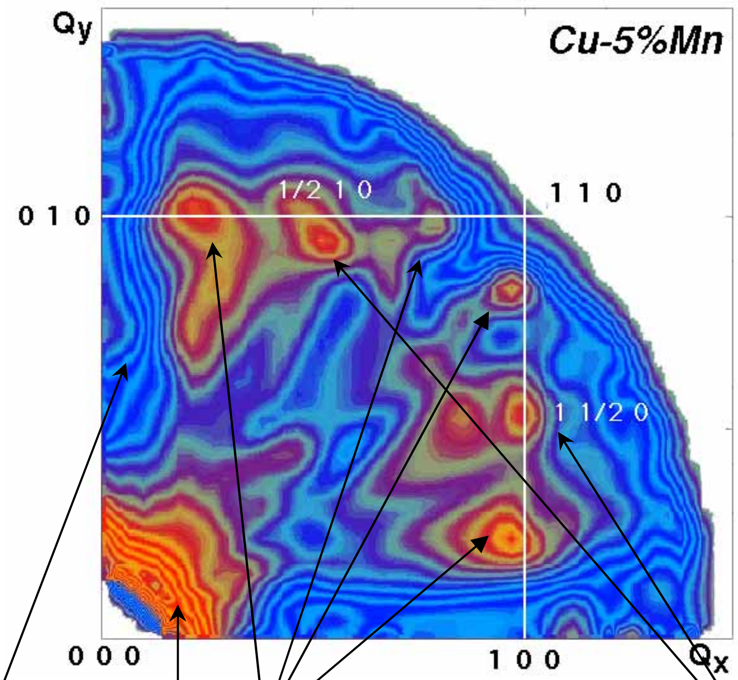
A. P. Murani *et al*., Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al*., J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from a spin-glass, $Cu_{0.95}Mn_{0.05}$

nuclear scattering



magnetic scattering



Anticlustering by Mn atoms

Spin Density Wave?

Anticlustering, ferromagnetic

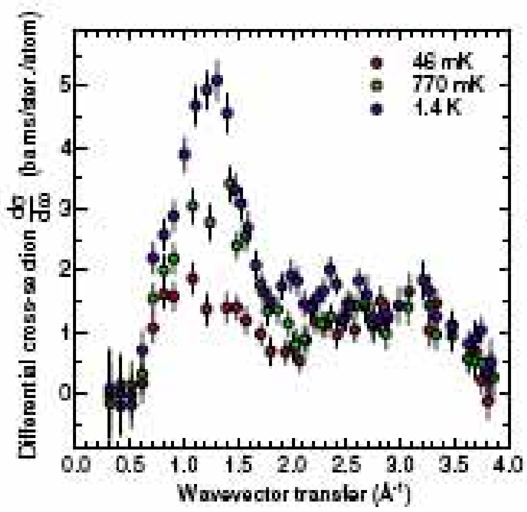
Forward scattering, ferromagnetic

Form-factor like background, frozen moments

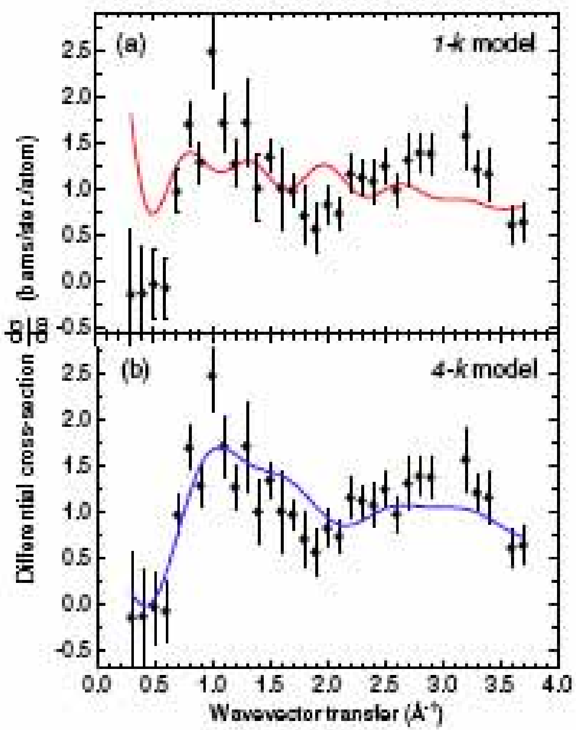
A. P. Murani *et al.*, Physica B **267-268** (1999) 131
T. J. Hicks, *Magnetism in Disorder*, 1995, Clarendon Press, Oxford
J. R. Stewart *et al.*, J. Appl. Phys. **87** (2000) 5425

Diffuse scattering from magnetic frustration, $Gd_2Ti_2O_7$

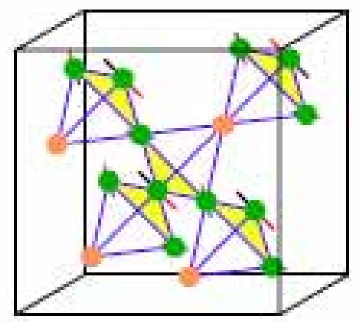
$$\frac{\mathrm{d}\sigma^{\pm\mp}}{\mathrm{d}\Omega} \propto \frac{2}{3} f^2(Q) \sum \langle \mathbf{S}_o \mathbf{S}_r \rangle \frac{\sin(Qr)}{Qr}$$



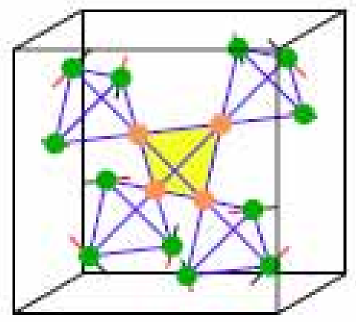
magnetic scattering



magnetic scattering at 46mK with fits



Structure model 1



Structure model 2

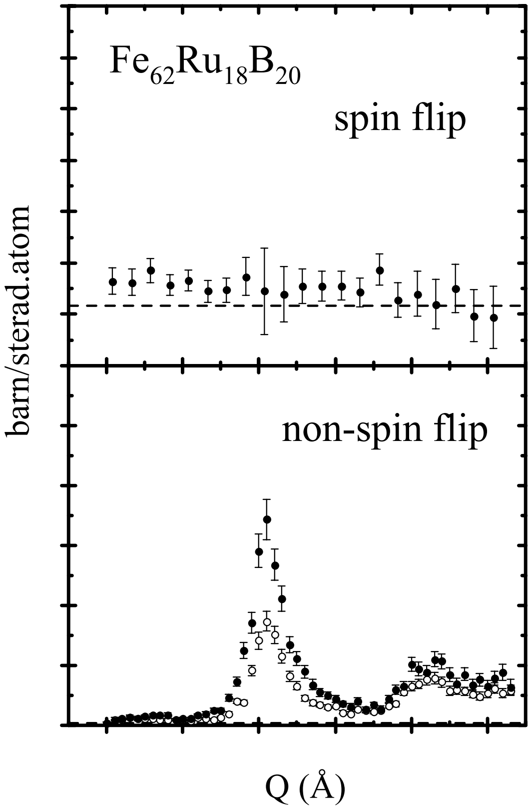
J. R. Stewart *et al.*, J. Phys.: Condens. Matter **16** (2004) L321

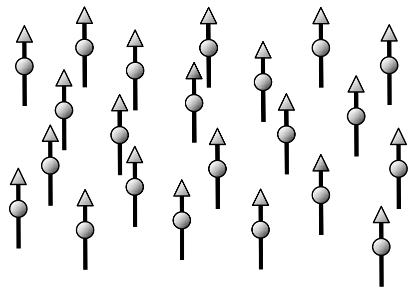
Diffuse scattering in metallic glasses, $Fe_{80-x}Ru_xB_{20}$

Metallic glasses have no long-ranged order, but each magnetic moment has a common axis. They are therefore *ferromagnetic*.

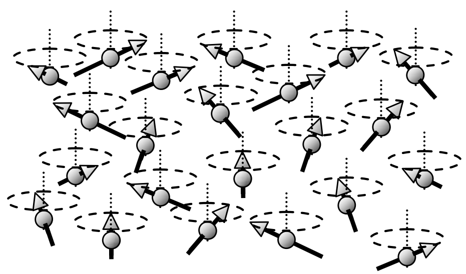
$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int \left(b(\mathbf{r}) \mp M_{\perp z}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

$$\frac{d\sigma^{\pm\mp}}{d\Omega} = \left| \int M_{\perp y}(\mathbf{r}) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

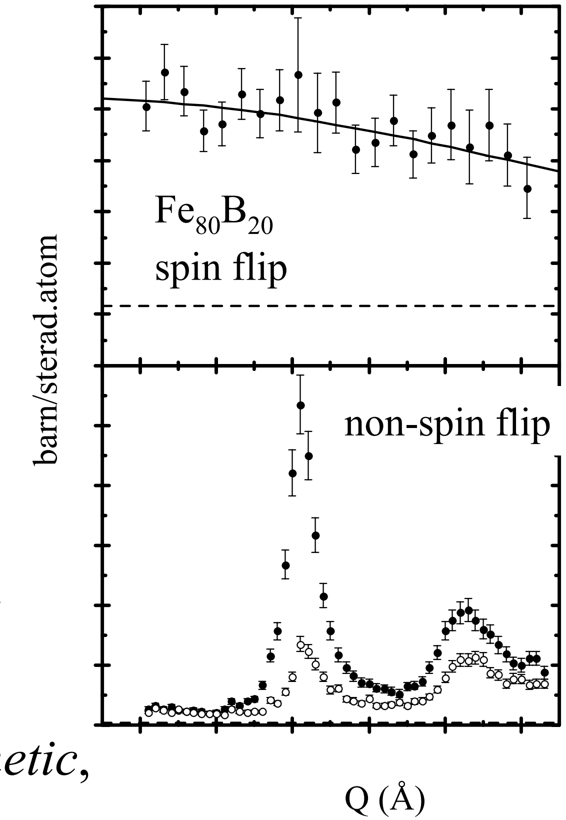




Collinear ferromagnetic.



Non-collinear ferromagnetic, asperromagnetic

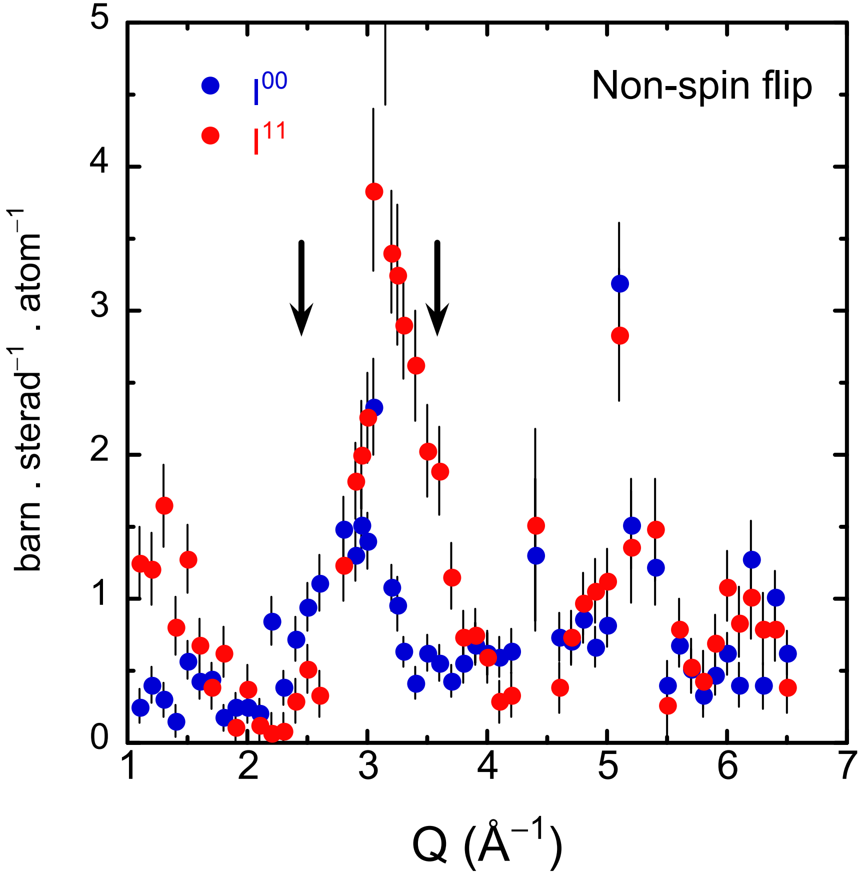


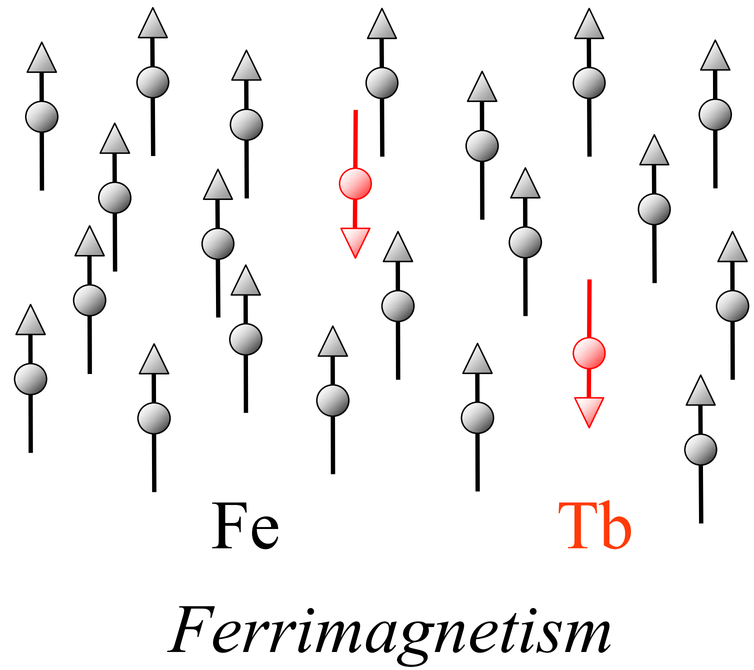
A. R. Wildes and N. Cowlam, *Recent research developments in magnetism and magnetic materials,* 2003, Transworld Research Network, India

Diffuse scattering in metallic glasses, $Fe_{73}Tb_{10}B_{17}$

$$\frac{d\sigma^{\pm\pm}}{d\Omega} = \left| \int \left(b(\mathbf{r}) \mp M_{\perp z}(\mathbf{r}) \right) e^{i\mathbf{Q}\cdot\mathbf{r}} d\mathbf{r} \right|^2$$

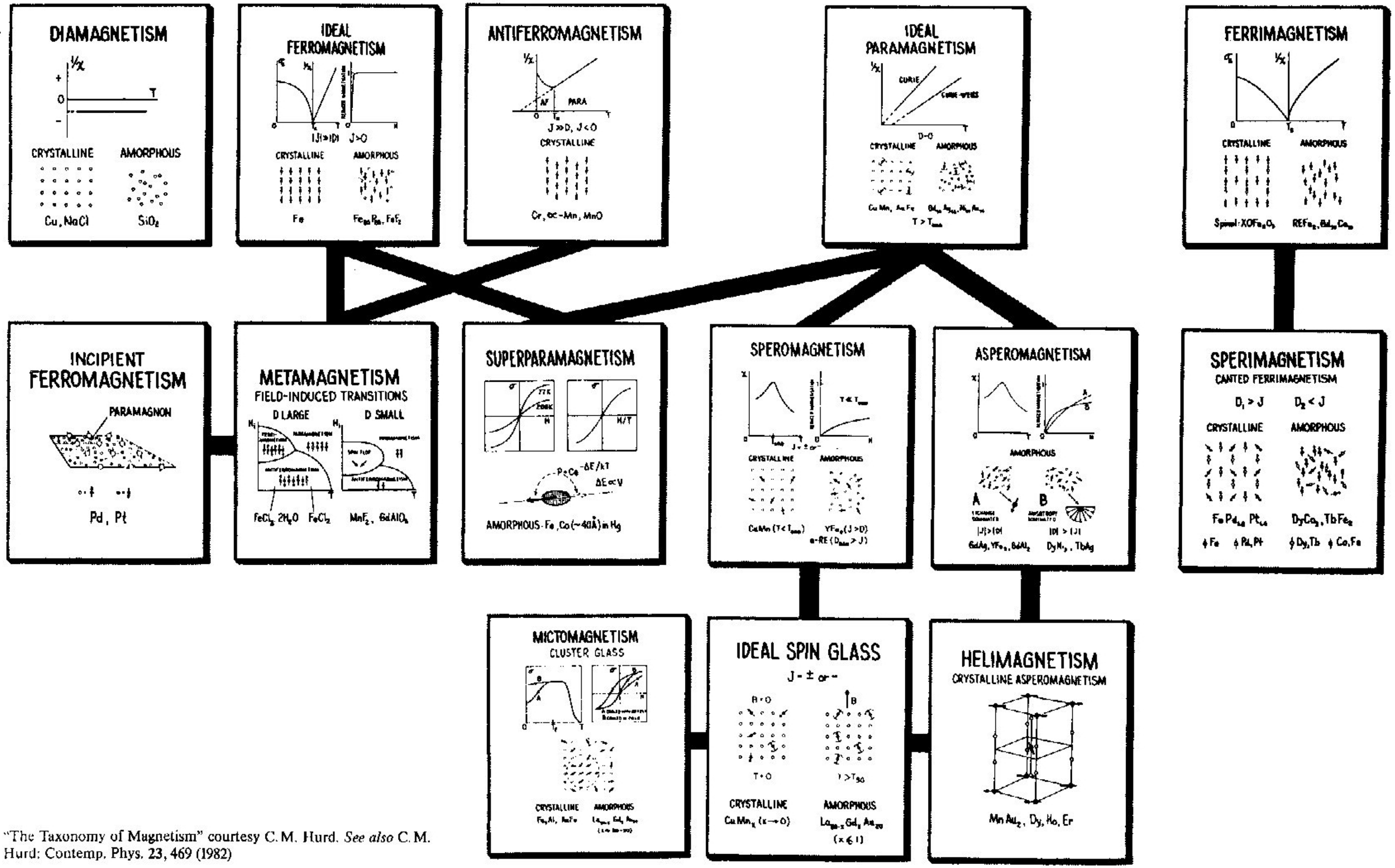
$M(\mathbf{r})$ is directional - its sign can change as a function of $\mathbf{r}$.





N. Cowlam *et al.*, J. Phys.: Condens. Matter **17** (2005) 3585

The 'Family Tree' of Magnetism



"The Taxonomy of Magnetism" courtesy C. M. Hurd. *See also* C. M. Hurd: Contemp. Phys. **23**, 469 (1982)

Inelastic scattering

Reflectivity and dynamical scattering

Neutron polarimetry