

# X-ray reflectivity and Grazing Incidence Small Angle X-Ray Scattering

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**European School on Magnetism**  
*New Experimental Aproaches in Magnetism*

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# Introduction

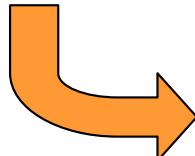
Nanoparticles, nanowires, thin films and multilayers... have  
New physical properties (e.g. magnetic, but also electronic, catalytic or photonic)



Atomic structure, size, shape & organization



Growth conditions &  
Morphology, temperature ... of the substrate surface

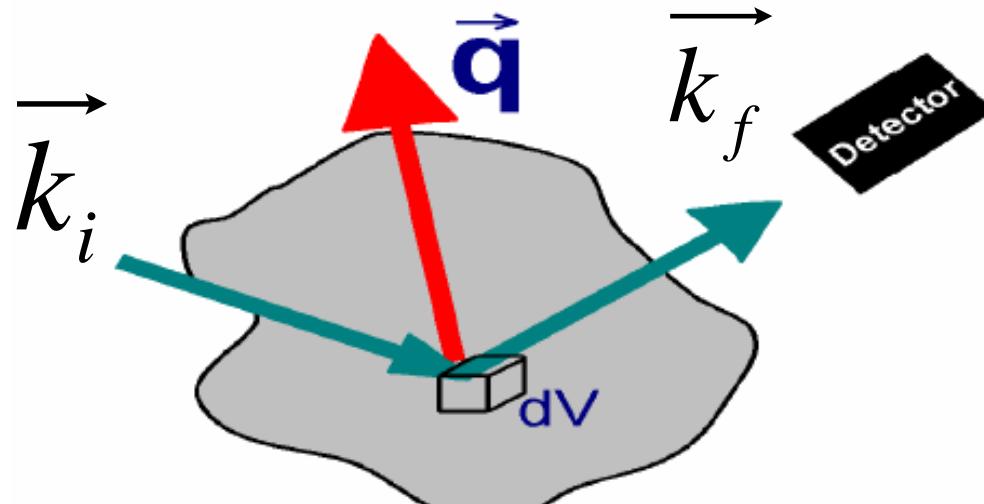


## X-ray

complementary to Near Field Microscopy

- non destructive - statistical information over mm scale
- depth sensitivity, from 20 Å up to several mm
- length scale probed : from a few Å to mm
- quantitative analysis
- following in-time: deposit - annealing - gas adsorption
- *in situ*, in UHV, during growth (and sometimes in real time)
- no charge effects : insulating samples (single crystal oxide substrates)

# X-Ray Scattering



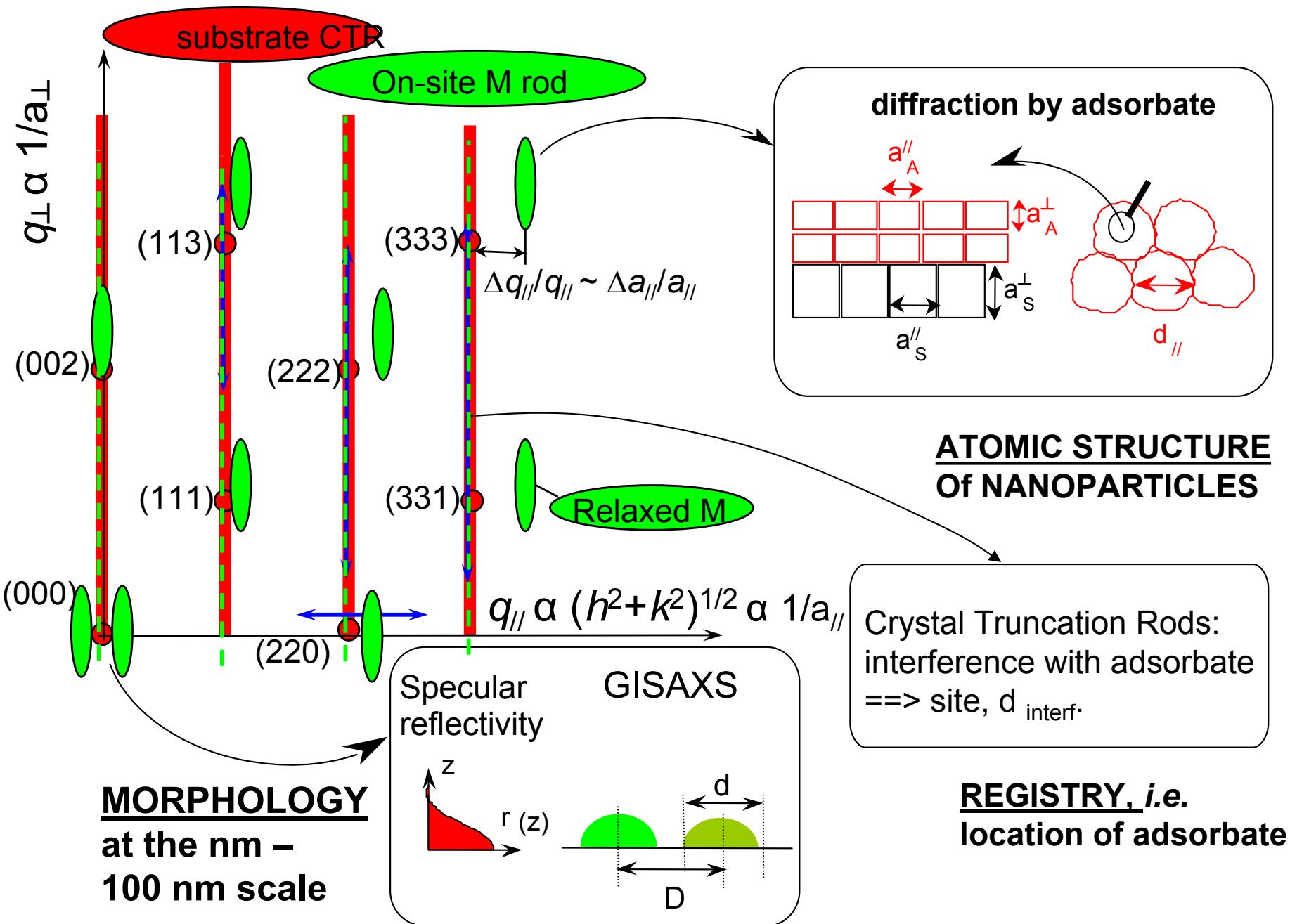
$$dI \sim \rho(r) dV$$

$$\overrightarrow{q} = \overrightarrow{k_f} - \overrightarrow{k_i} = \overrightarrow{G_{hkl}}$$

vector of the reciprocal space

Explores Reciprocal Space

# Reciprocal Space of nanostructures deposited on a substrate



## Grazing Incidence X-ray Scattering GIXS (or GID)



## Structure @ atomic scale

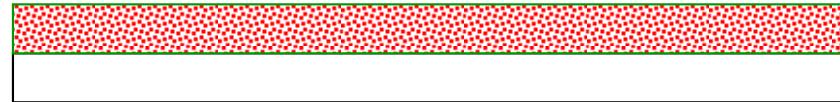
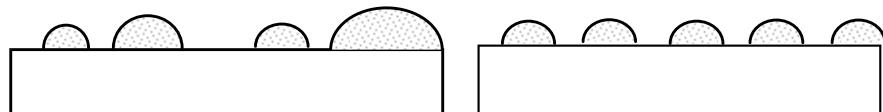


- Structure, composition
- Epitaxial relationships
- Relaxation
  - Coherent
  - Incoherent (dislocations)
- Registry / substrate lattice
- Intermixing with substrate
- Substrate distortions
- ...

## Grazing Incidence Small Angle X-ray Scattering (GISAXS) and X-R Reflectivity (XRR)



## Morphology @ nanometer scale



- Shape (facets, equilibrium shape)
- Dimensions
- Size distributions
- Organization
- Growth mode
- Density profile
- Thin film thickness
- Interface roughness
- Buried layers
- ...

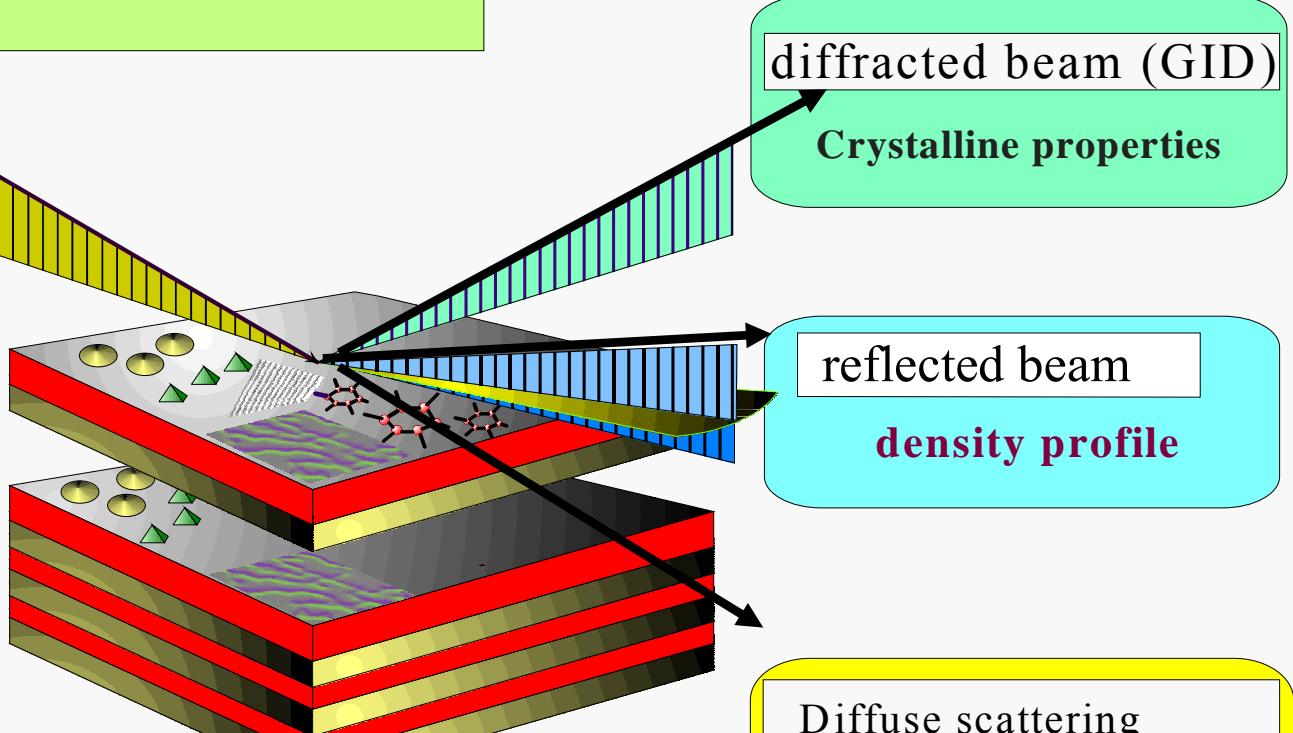
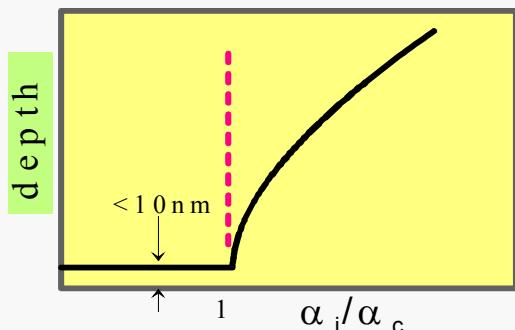
# Nanostructures (nanoparticles, nanowires, thin films, multilayers ...) & X-rays

## X-RAY METHODS AT GRAZING INCIDENCE



STRUCTURE OF THIN LAYERS ON SUBSTRATE

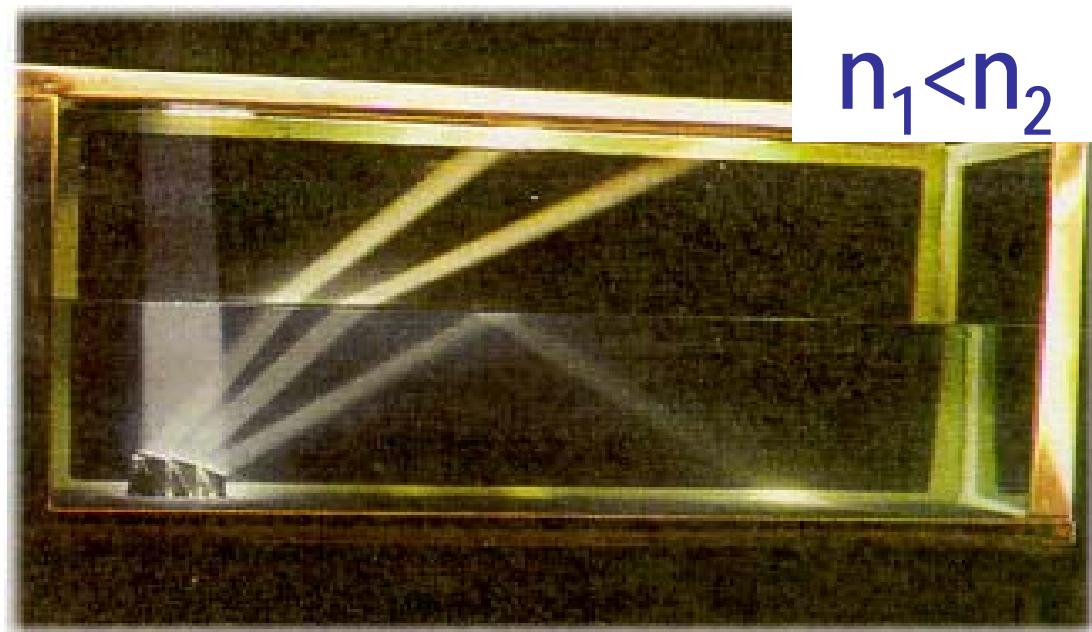
depth resolution 10nm-200nm



# X-Ray Reflectivity: Principle

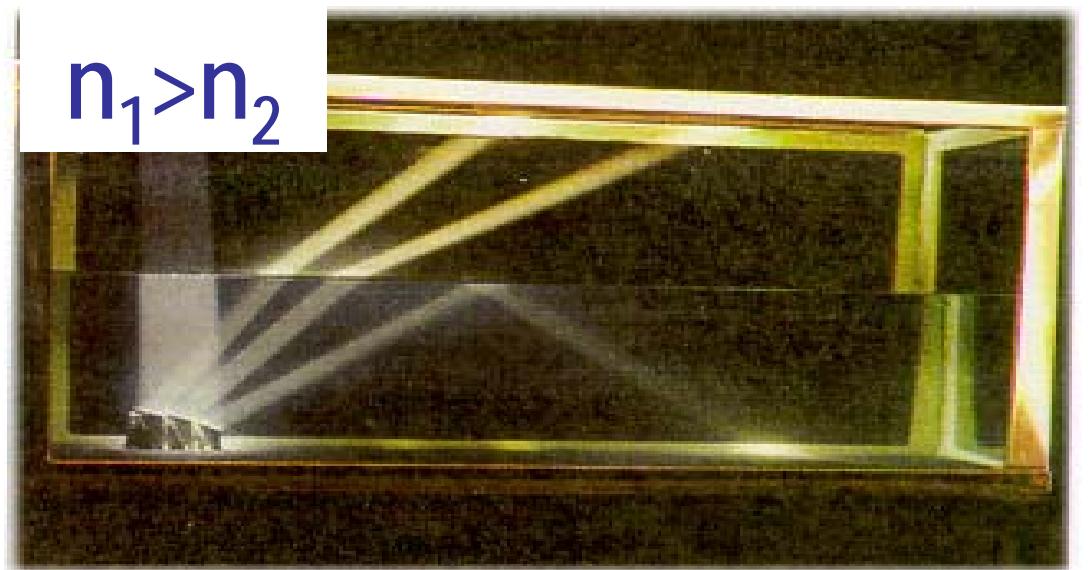
Visible Light  
Reflectivity:  
 $n_2 > 1$

$$\frac{n_1}{n_2}$$

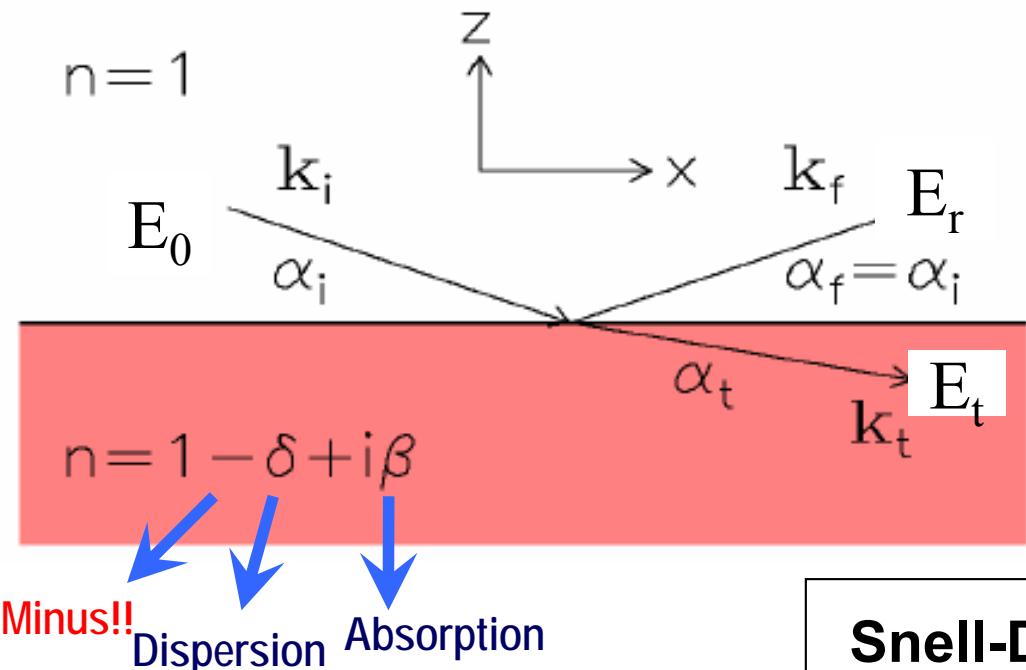


X-Ray  
Reflectivity:  
 $n_2 < 1$

$$\frac{n_1}{n_2}$$



# Reflection and refraction – Perfect surface



$$\delta = \frac{\lambda^2}{2\pi} r_0 \rho \approx 10^{-4}..10^{-6}$$

$$\beta = \frac{\lambda}{4\pi} \mu \approx 10^{-6}..10^{-9}$$

**Snell-Descartes law:**  $\cos \alpha_i = n \cos \alpha_t$

$\exists$  transmitted wave only if  $\cos(\alpha_t) \leq 1$ , i.e.  $\alpha_i \geq \alpha_c$

If  $\alpha_i \leq \alpha_c$ ,

- Incident wave totally externally reflected.
- Transmitted wave exponentially damped with  $z$ .

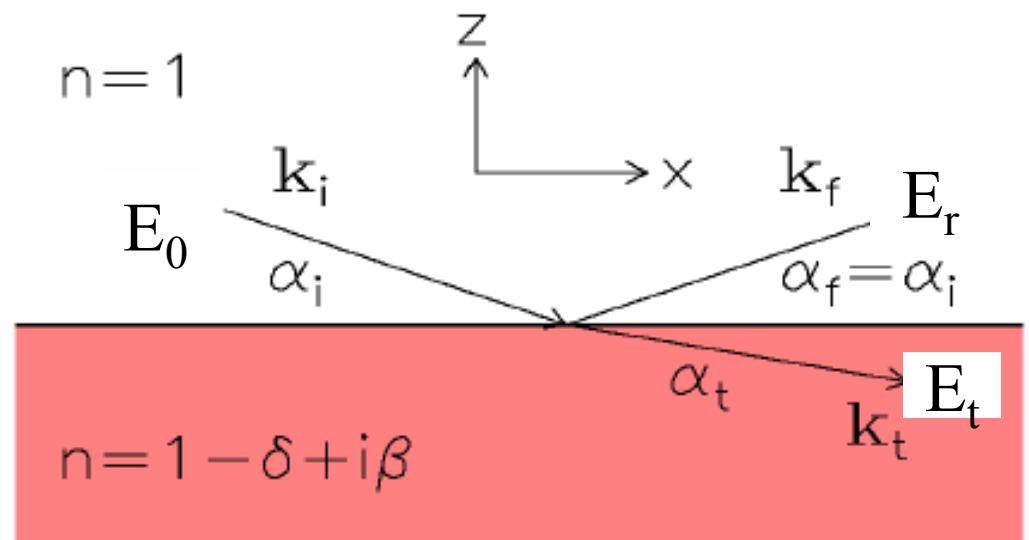
$\alpha_c$  critical angle for total external reflection of X-rays

$$\alpha_c = \sqrt{2\delta} = \sqrt{\frac{r_0}{\pi}} \times \lambda \times \sqrt{\rho} \approx 0.1 \text{ to } 0.5^\circ$$

# Reflection and refraction: perfect surface

- Fresnel equations:

Relationships between the amplitudes of incident, transmitted and reflected beam.



Reflection

Amplitude

$$r = \frac{E_r}{E_0} = \frac{\sin(\alpha_i - \alpha_t)}{\sin(\alpha_i + \alpha_t)} \approx \frac{\alpha_i - \alpha_t}{\alpha_i + \alpha_t}$$

Transmission

$$t = \frac{E_t}{E_0} = \frac{2\sin(\alpha_i)\cos(\alpha_t)}{\sin(\alpha_i + \alpha_t)} \approx \frac{2\alpha_i}{\alpha_i + \alpha_t}$$

Intensity

$$R = \left| \frac{E_r}{E_0} \right|^2$$

$$T = \left| \frac{E_t}{E_0} \right|^2$$

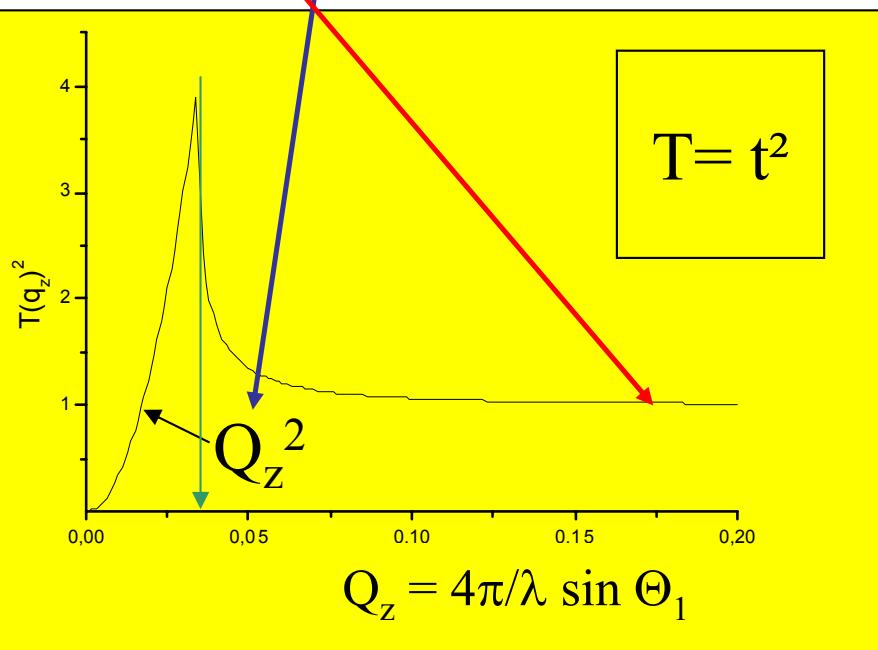
# Limiting and asymptotic values for Fresnel equations

## Transmission

$$t = \frac{2\alpha_i}{\alpha_i + \sqrt{\alpha_i^2 - 2\delta}} \approx \frac{2\alpha_i}{\alpha_i + \alpha_i(1 + \frac{1}{2} \frac{2\delta}{\alpha_i^2})}$$

$$t \approx \frac{2\alpha_i}{2\alpha_i + \frac{\delta}{\alpha_i}} \approx \frac{\alpha_i}{\delta} \dots \text{for } \alpha_i < \sqrt{2\delta}$$

$$t \approx \frac{2\alpha_i}{2\alpha_i} = 1 \dots \text{for } \alpha_i >> \sqrt{2\delta}$$

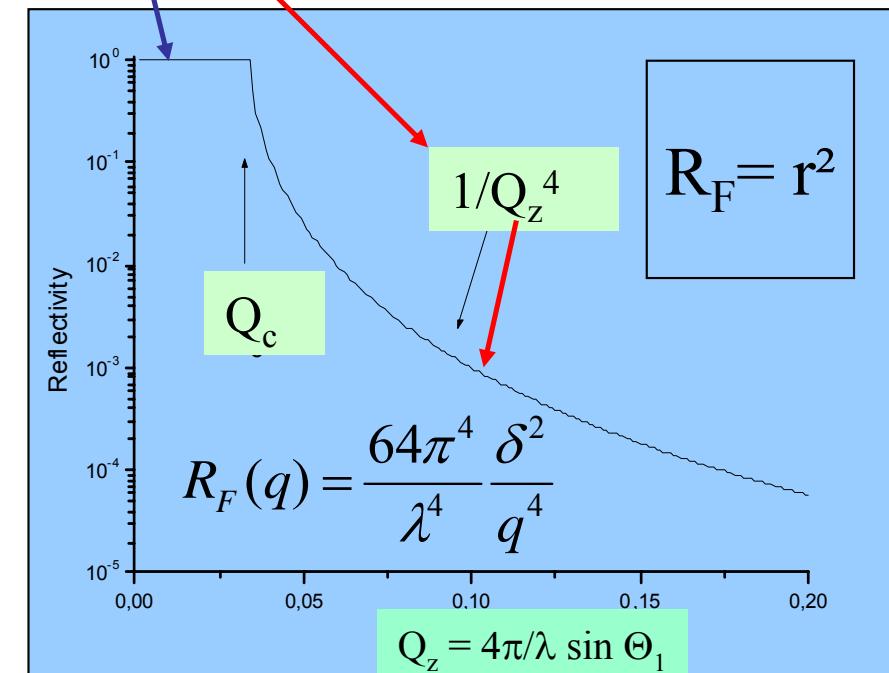
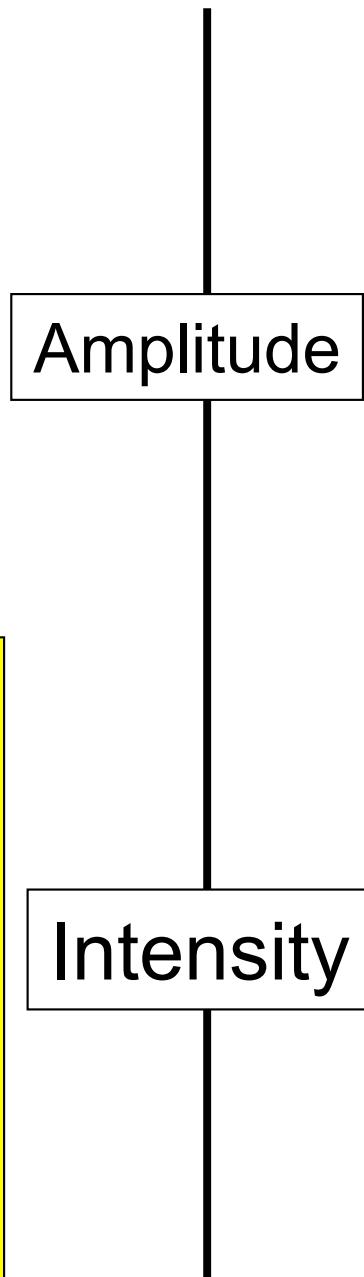


## Reflection

$$r = \frac{\alpha_i - \sqrt{\alpha_i^2 - 2\delta}}{\alpha_i + \sqrt{\alpha_i^2 - 2\delta}} \approx \frac{\alpha_i - \alpha_i(1 + \frac{1}{2} \frac{2\delta}{\alpha_i^2})}{\alpha_i + \alpha_i(1 + \frac{1}{2} \frac{2\delta}{\alpha_i^2})}$$

$$r = 1 \dots \text{for } \alpha_i \ll \sqrt{2\delta}$$

$$r = -\frac{\delta}{2\alpha_i^2} \dots \text{for } \alpha_i >> \sqrt{2\delta}$$



# Exact evaluation of Fresnel reflectivity

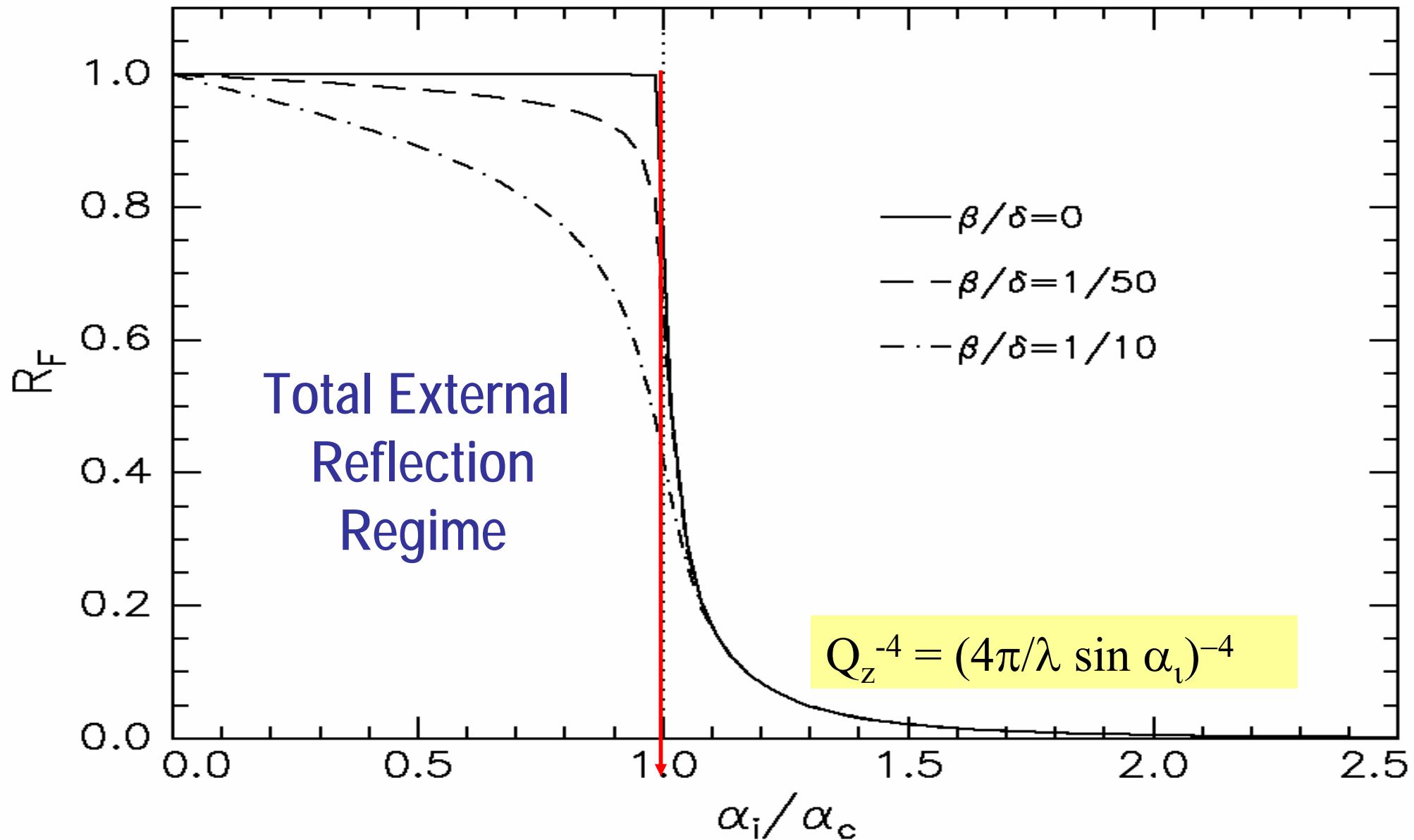
$$R_F(\alpha_i) = |r|^2 = \frac{(\alpha_i - p_+)^2 + p_-^2}{(\alpha_i + p_+)^2 + p_-^2}$$

$$\alpha_t = p_+ + i p_-$$

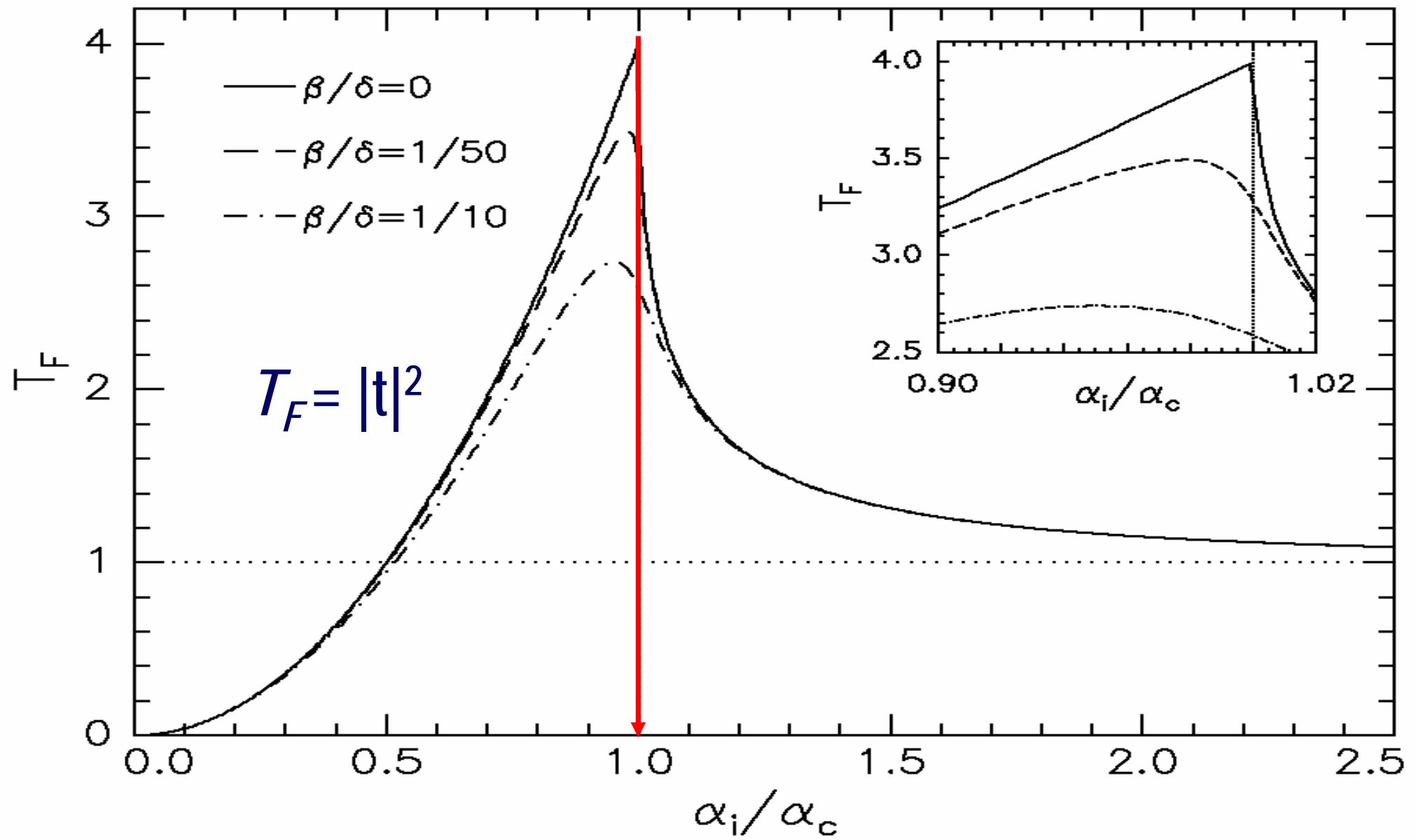
$$p_{+/-}^2 = \frac{1}{2} \left\{ \sqrt{(\alpha_i^2 - \alpha_c^2)^2 + 4\beta^2} \pm (\alpha_i^2 - \alpha_c^2) \right\}$$

→ Absorption  $\beta$  also play a significant role

# Fresnel Reflectivity: $R_F(\alpha_i)$ with absorption



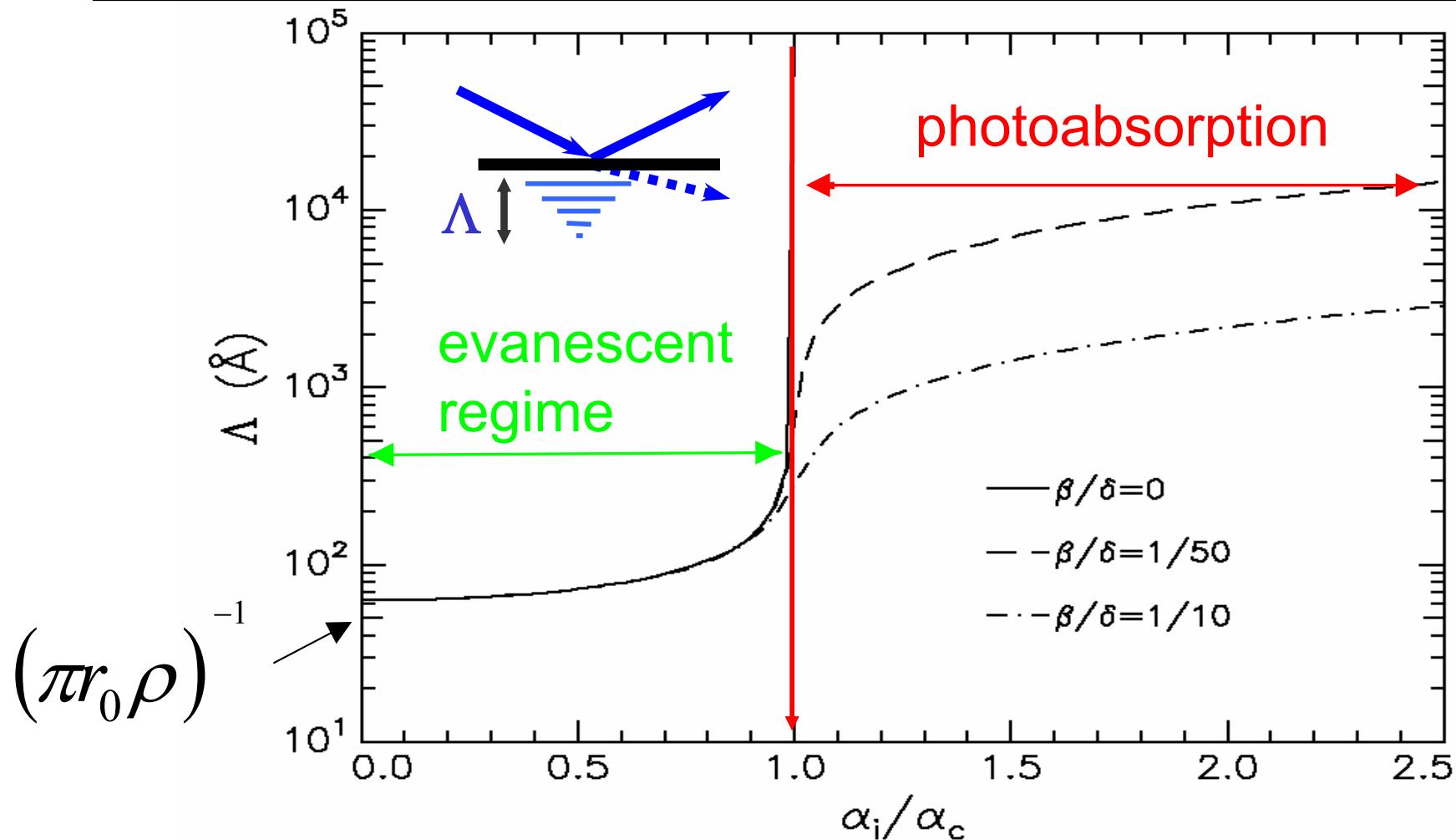
# Transmission Function with absorption



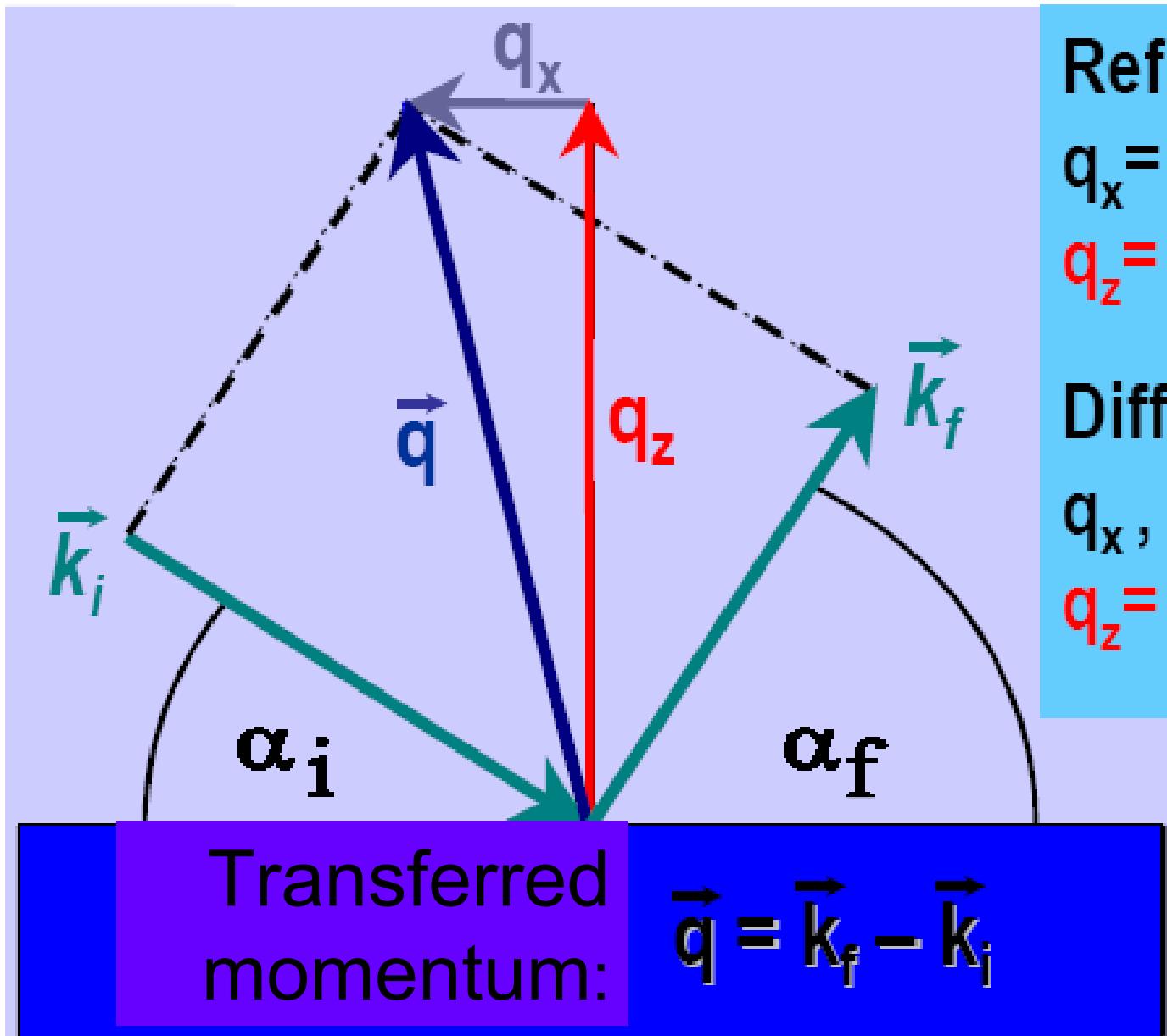
# Penetration Depth with absorption

$$L_{i,f} = |\text{Im}(k_{t,z})|^{-1} = \lambda / 2\pi l_{i,f}$$

$$l_{i,f} = \frac{1}{\sqrt{2}} \left\{ (2\delta - \sin^2 \alpha_{i,f}) + [(\sin^2 \alpha_{i,f} - 2\delta)^2 + 4\beta]^{1/2} \right\}^{1/2}$$



# The geometry of X-ray reflectivity



Reflectivity:

$$q_x = q_y = 0$$

$$q_z = (4\pi/\lambda)\sin\alpha_i$$

Diffuse Scattering:

$$q_x, q_y \neq 0$$

$$q_z = (4\pi/\lambda)\sin(\alpha_i + \alpha_f)/2$$

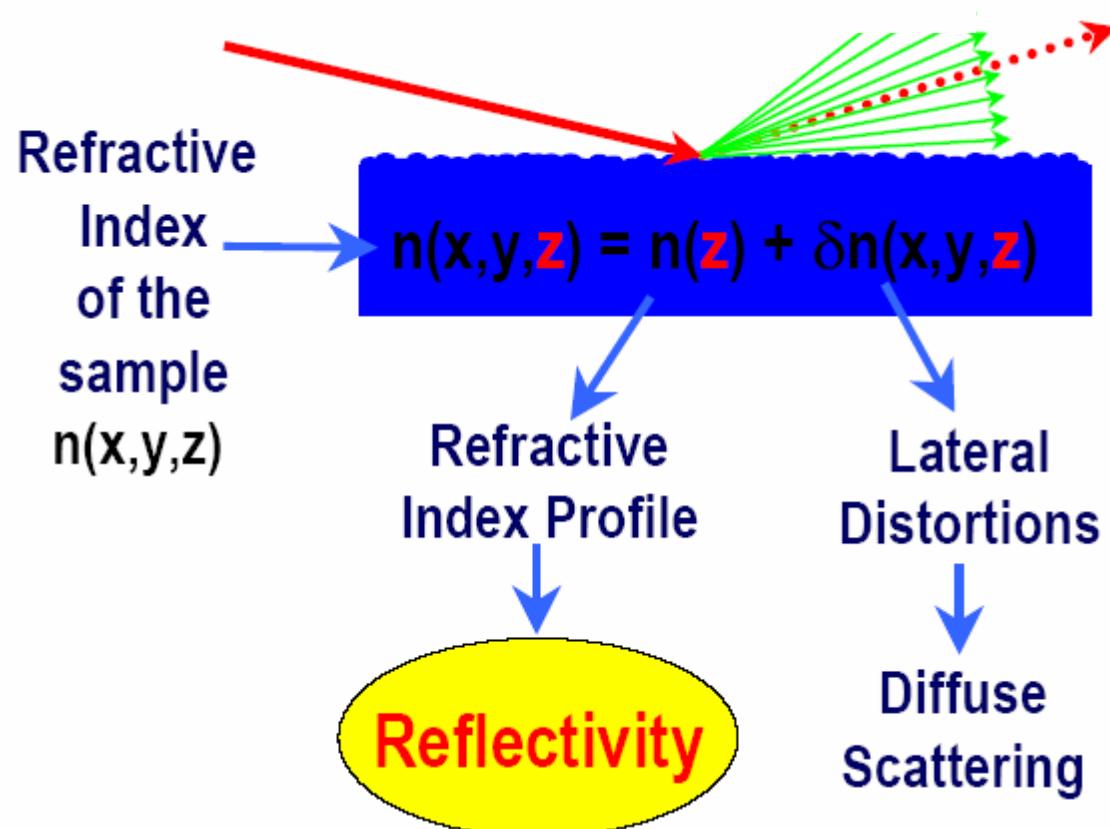
$$\alpha_i, \alpha_f < 5^\circ$$

# X-ray reflectivity: main equation

Helmholtz equation

$$\Delta E(\vec{r}) + k^2 n_x^2(\vec{r}) E(\vec{r}) = 0$$

Formal solution:

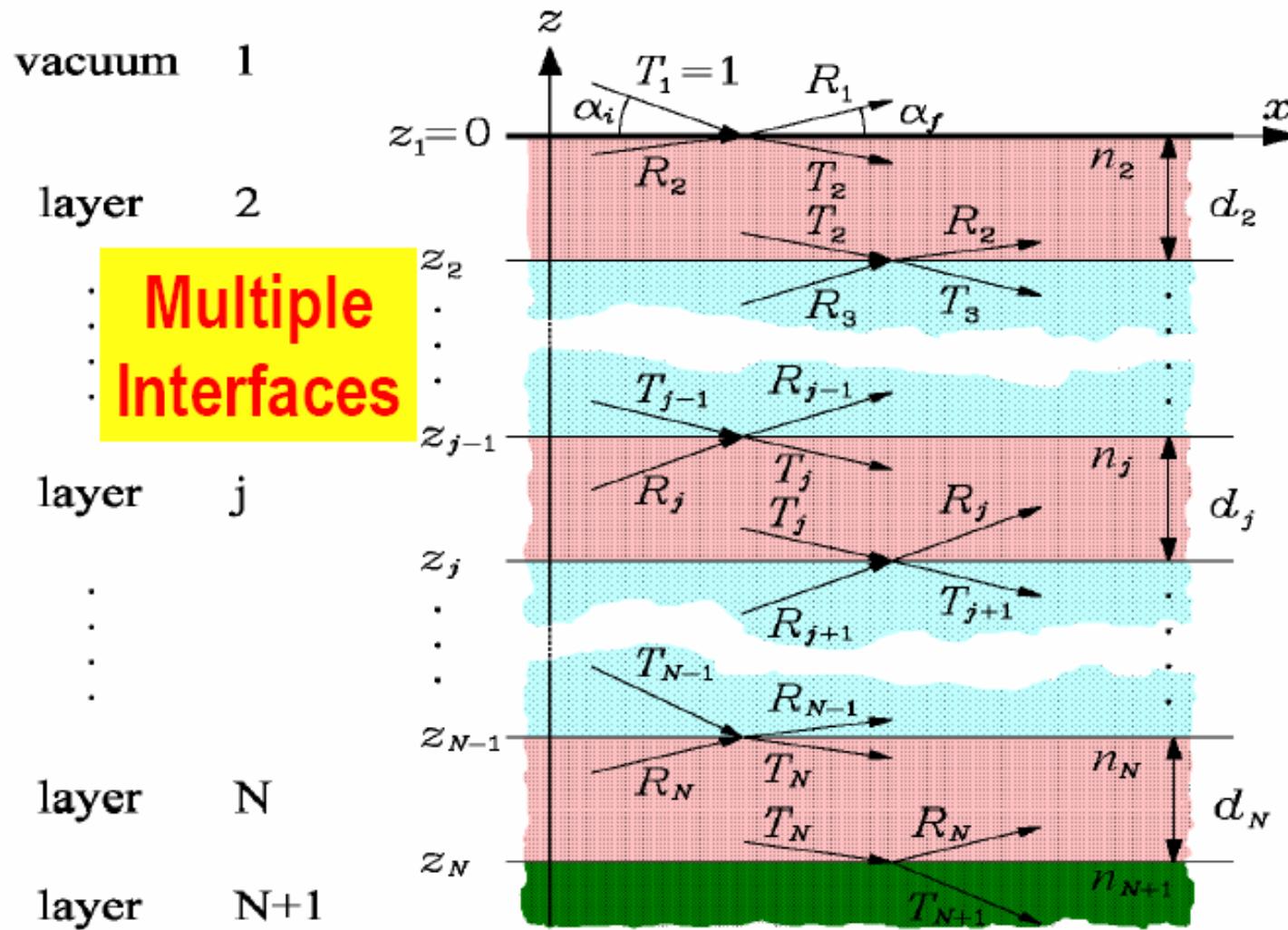


$$n(z) = 1 - \frac{\lambda^2}{2\pi} r_e \varrho(z) + i \frac{\lambda}{4\pi} \mu(z)$$

→ Electron density profile  $\varrho(z) = \langle \varrho(x, y, z) \rangle_{x,y}$

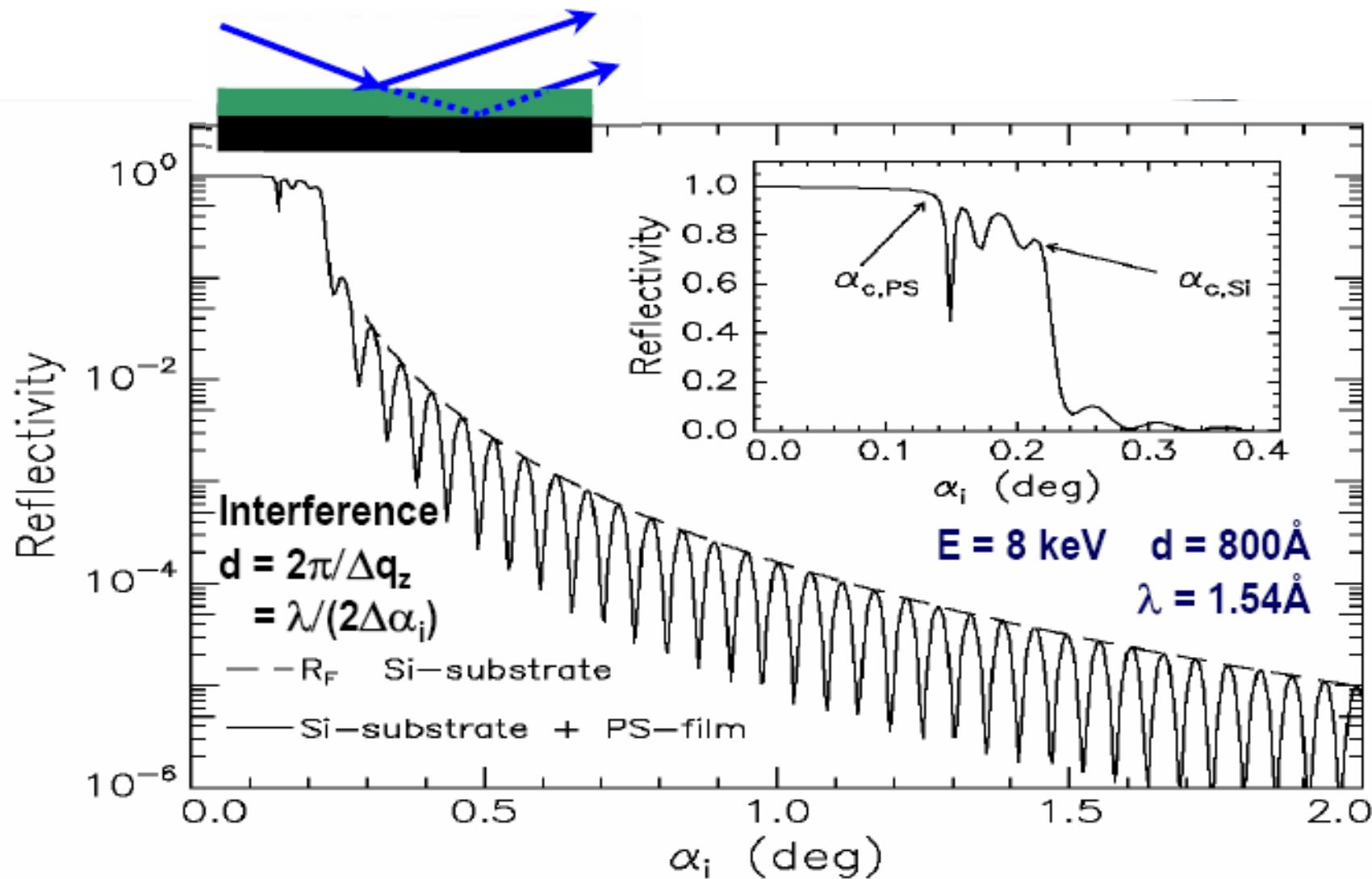
# Reflectivity from multilayers

## Multiple scattering (dynamical calculation)



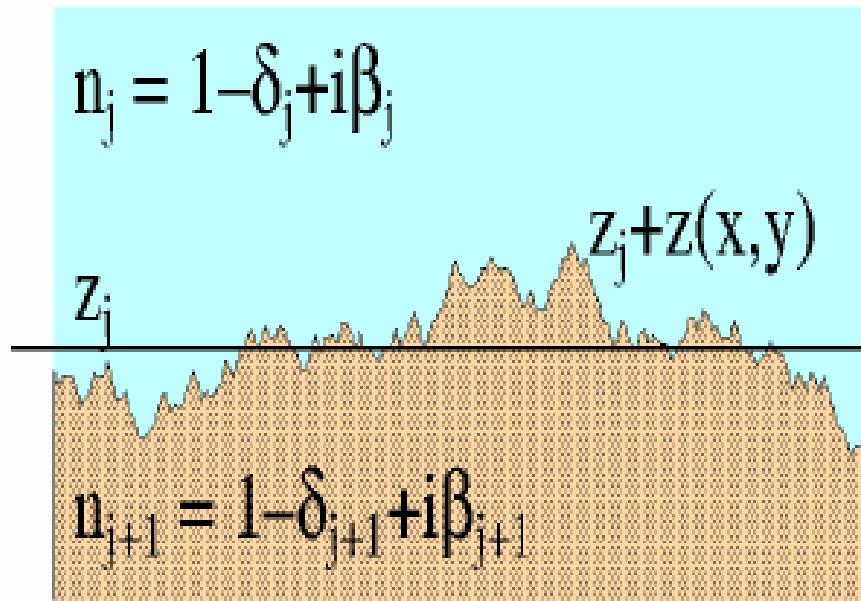
Matrix formalism:  
Parrat iterations  
Parrat, 1954

# Reflectivity from layer on substrate. Ex: PS on Si



→ Reflectivity used as an everyday laboratory tool to measure the thickness of layers deposited on a substrate

# Rough interfaces: statistics



Probability Density

$$P_j(z) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{z^2}{2\sigma_j^2}\right)$$

Integration

A graph showing the probability density  $P_j(z)$  as a function of height  $z$ . The distribution is a Gaussian curve centered at  $z_j$  with standard deviation  $\sigma_j$ . A green arrow points downwards from the text "Integration" towards the area under the curve, indicating the process of integration to find the refractive index profile.

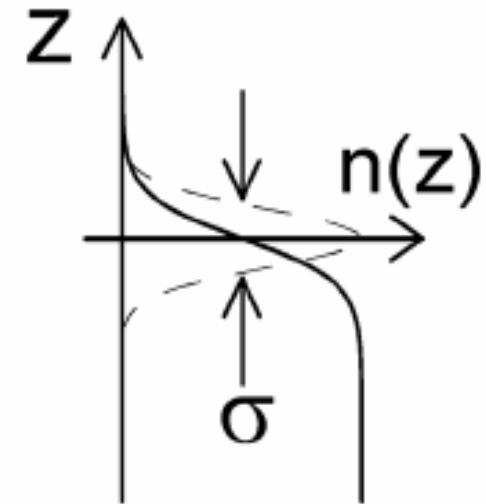
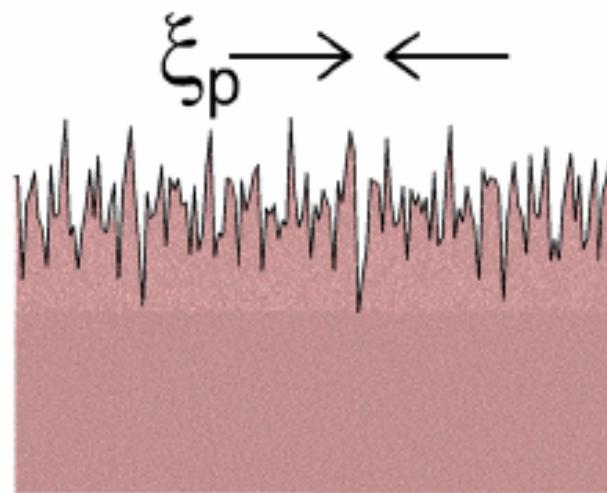
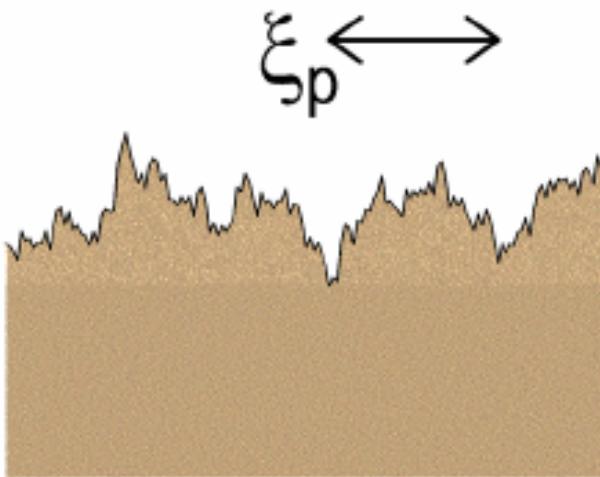
$$n_j(z) = \frac{n_j + n_{j+1}}{2} - \frac{n_j - n_{j+1}}{2} \operatorname{erf}\left(\frac{z - z_j}{\sqrt{2}\sigma_j}\right)$$

## Refractive Index Profile $n(z)$



## *Electron Density Profile $\rho(z)$*

# Reflectivity by a rough surface : which roughness ?



Same Roughness  $\sigma$  & Refractive Index Profile  $n(z)$  !

Lateral Structure Different



Different Averaging Procedures:  $\sigma/\xi_p < 1$  or  $\sigma/\xi_p > 1$

# Roughness in multilayers?

$\sigma/\xi_p < 1$

Reflection  
Coefficient

Transmission  
Coefficient

$\sigma/\xi_p > 1$

Reflection  
Coefficient

Transmission  
Coefficient

Beckmann-Spizzichino Result (1963):

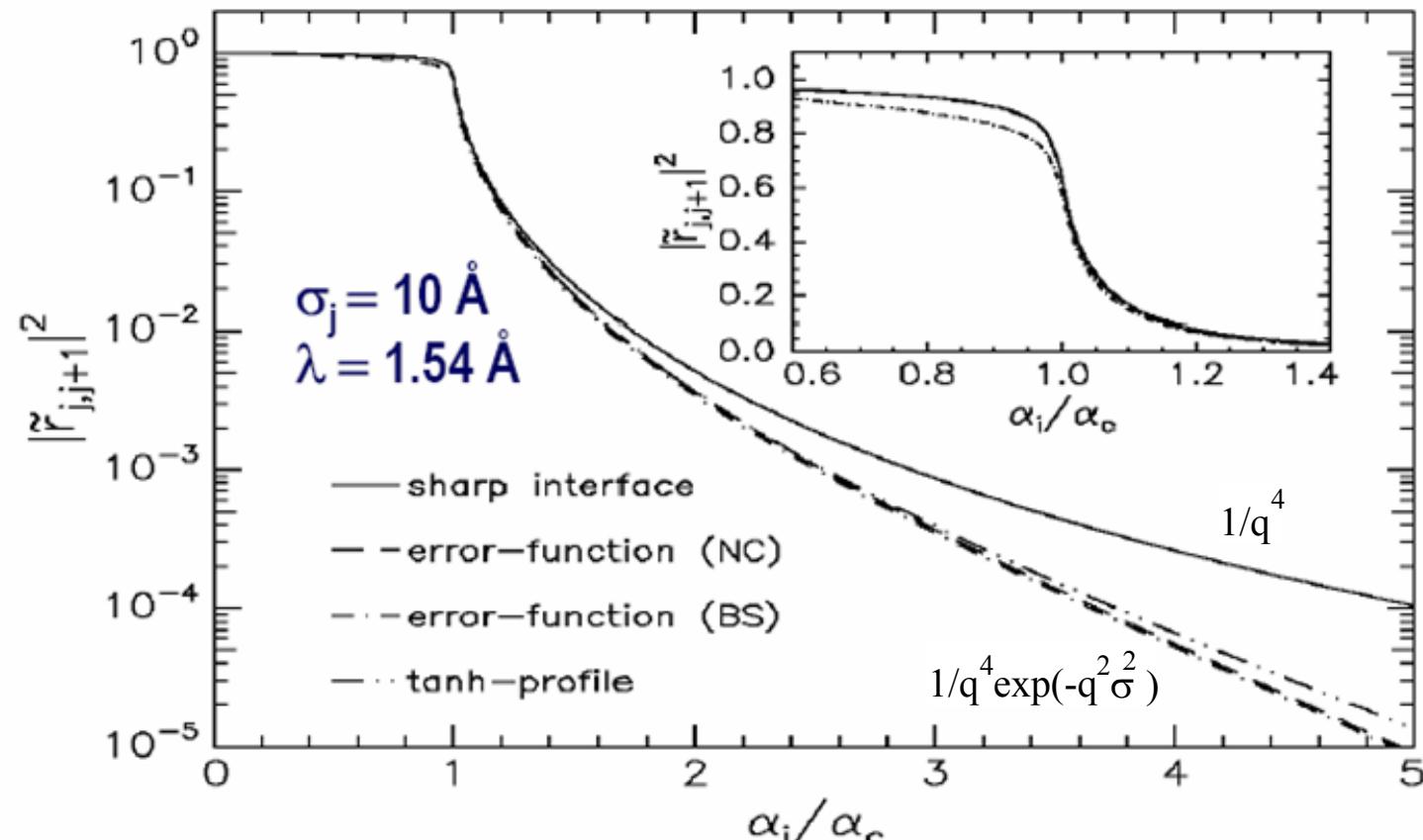
$$\tilde{r}_{j,j+1} = r_{j,j+1} e^{-2k_{z,j}^2 \sigma_j^2}$$
$$\tilde{t}_{j,j+1} = t_{j,j+1} e^{-(k_{z,j}-k_{z,j+1})^2 \sigma_j^2/2}$$

Nevot-Croce Result (1980):

$$\tilde{r}_{j,j+1} = r_{j,j+1} e^{-2k_{z,j} k_{z,j+1} \sigma_j^2}$$
$$\tilde{t}_{j,j+1} = t_{j,j+1} e^{+(k_{z,j}-k_{z,j+1})^2 \sigma_j^2/2}$$

$\sigma_j \rightarrow$  Exponential Damping  
of Reflectivity !

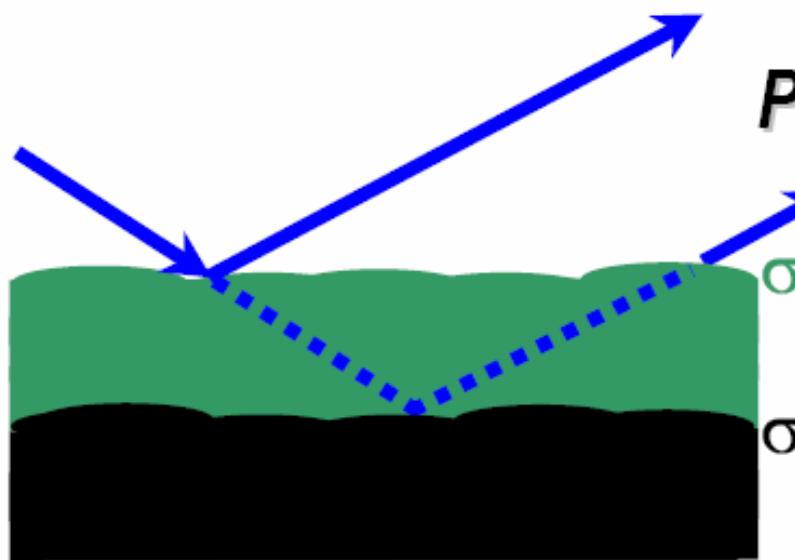
# Effect of interfacial roughness on reflectivity: single interface



$$R(q_z) = R_F(q_z) \exp(-q_z^2 \sigma^2)$$

→ Reflectivity very efficient to measure (small) (statistically averaged) roughness of surfaces or interfaces.

## Roughness at several interfaces



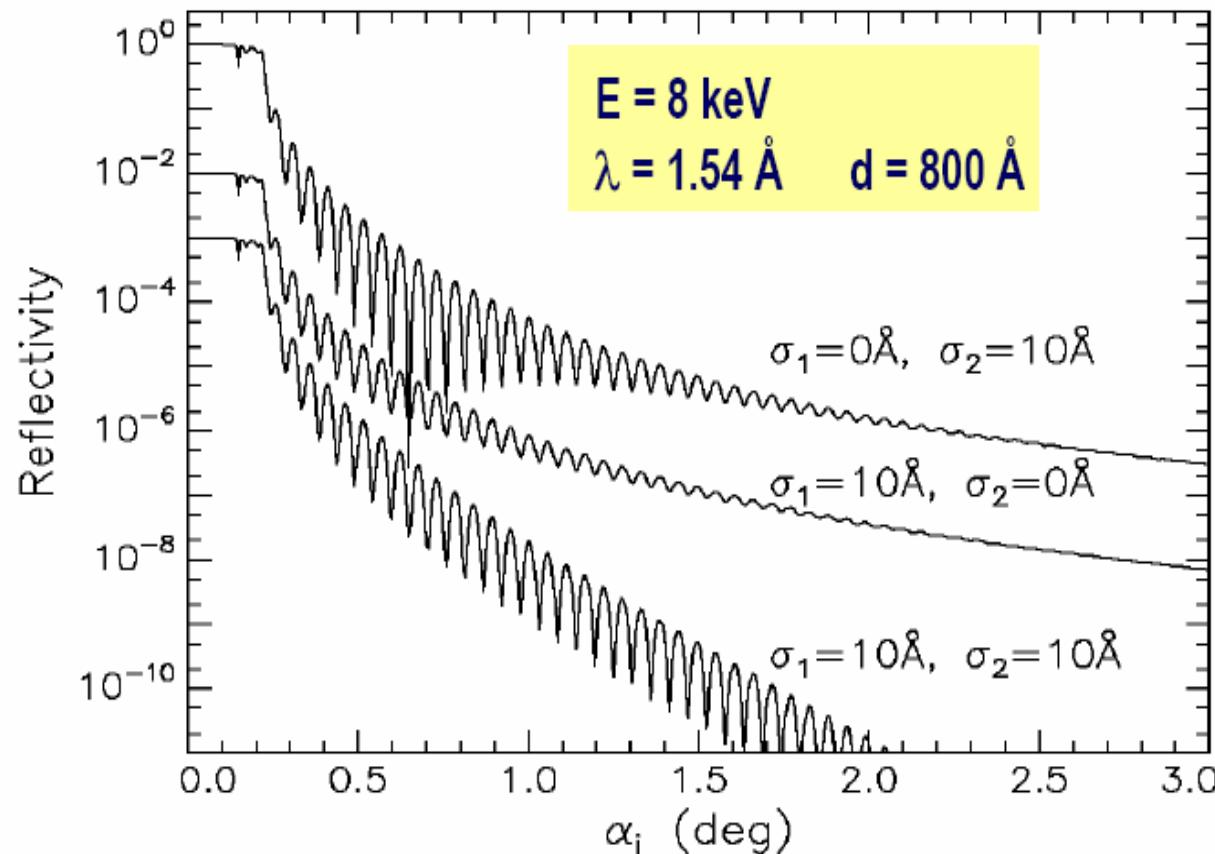
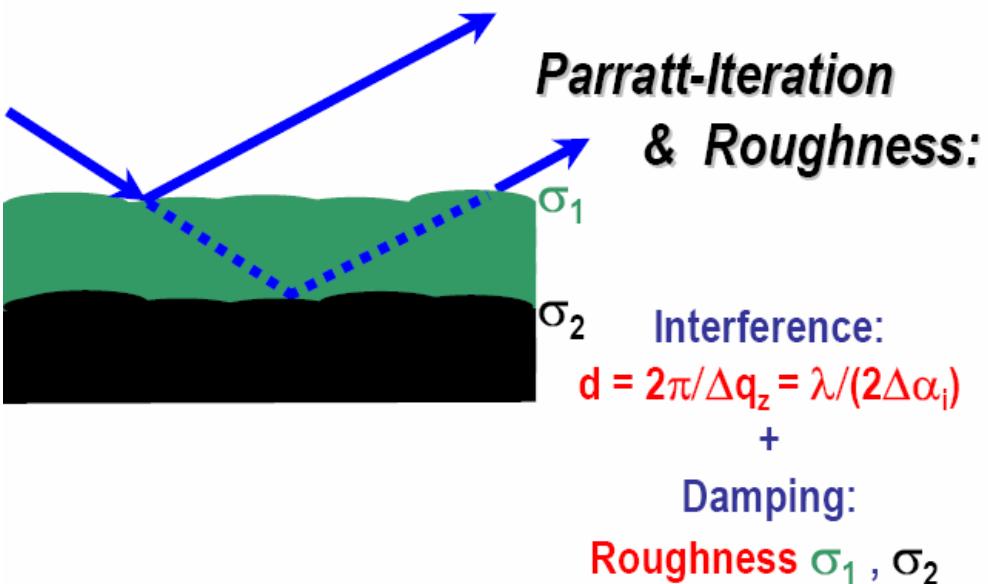
**Parratt-Iteration  
& Roughness:**

Interference:  
 $d = 2\pi/\Delta q_z = \lambda/(2\Delta\alpha_i)$   
+

Damping:

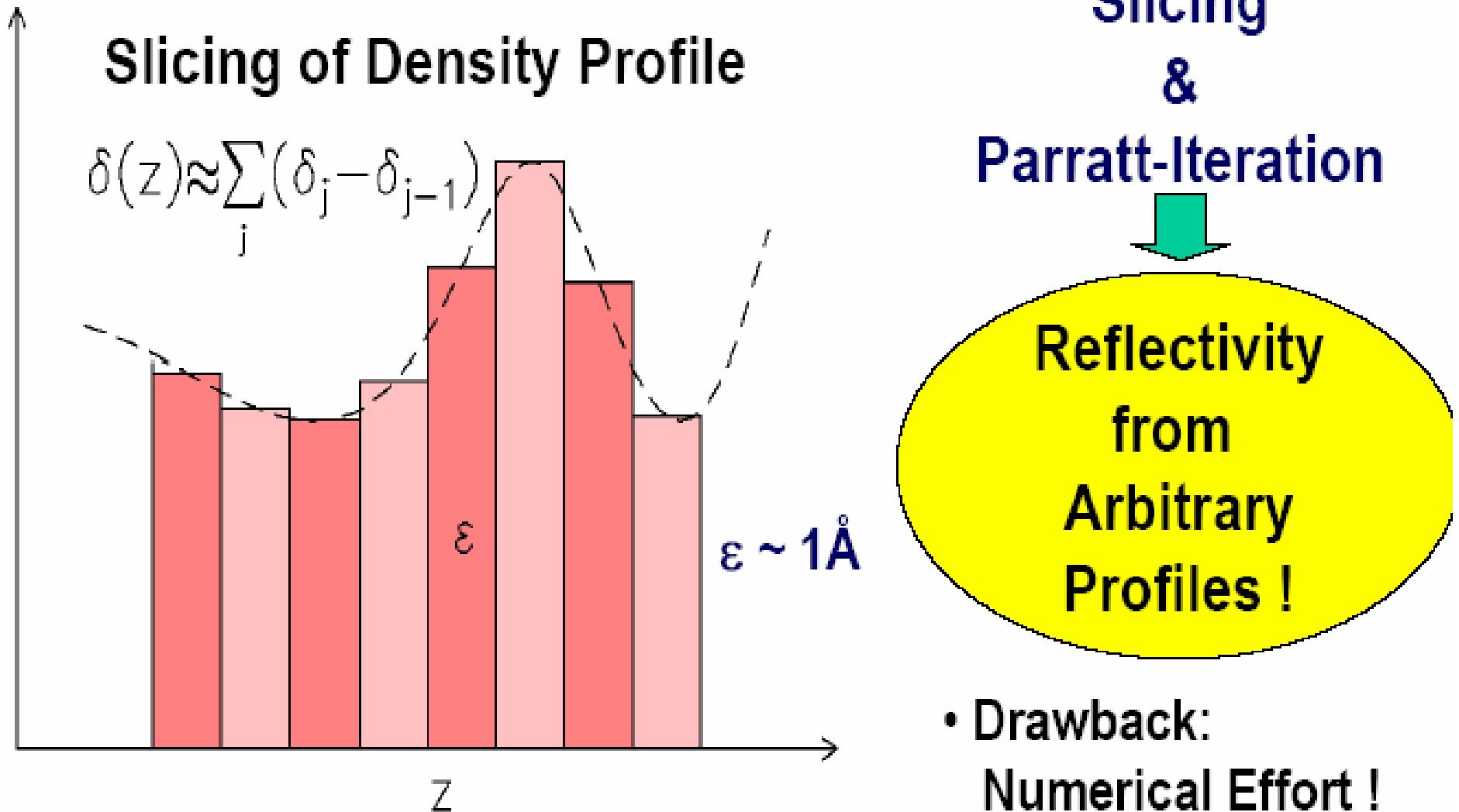
Roughness  $\sigma_1, \sigma_2$

# Thin film with surface and interface roughness. Example: PS layer on Si, with roughness

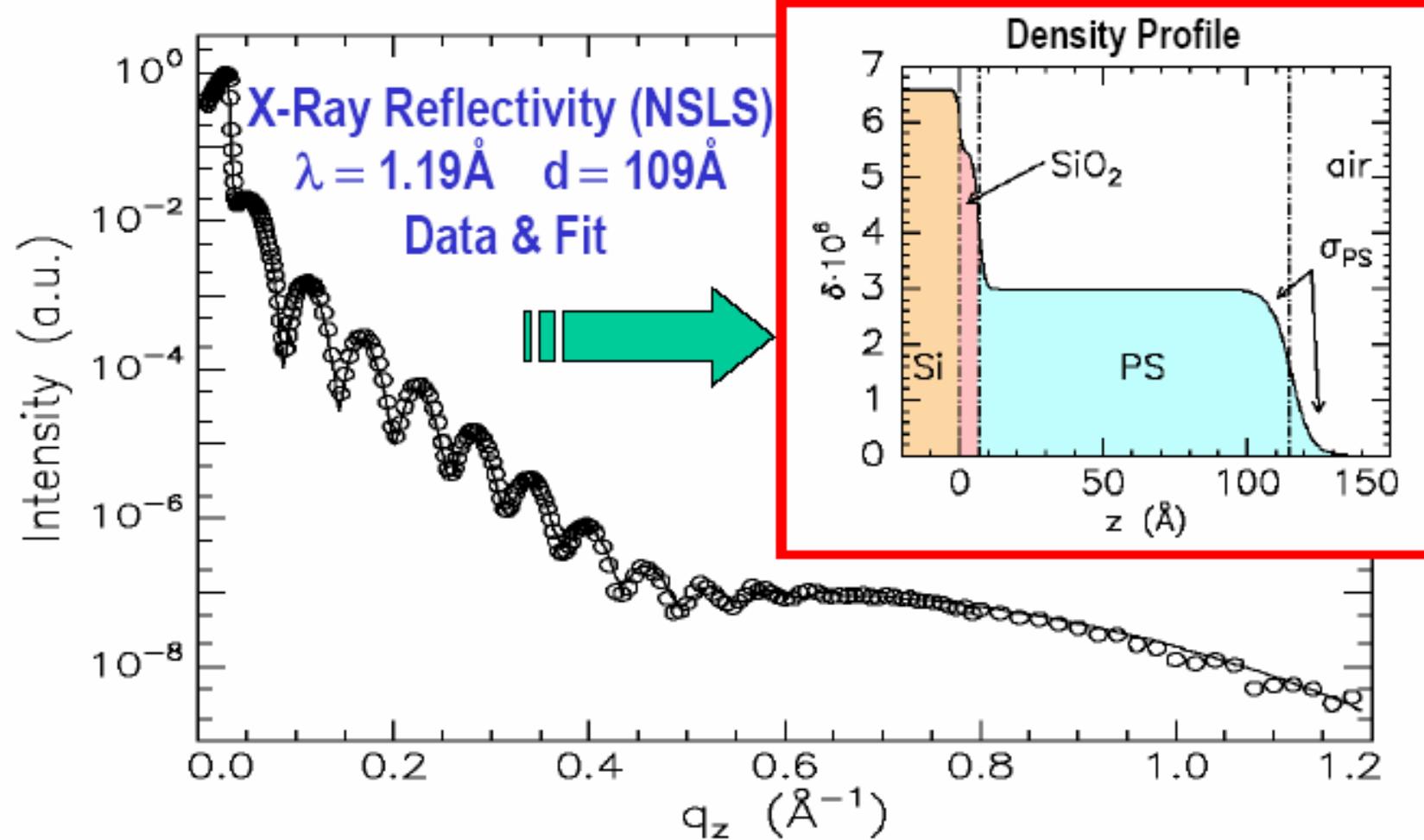


- ➔ Effects of surface and interface roughness very different
- ➔  $\sigma_1, \sigma_2$  and  $d$  can be determined independantly

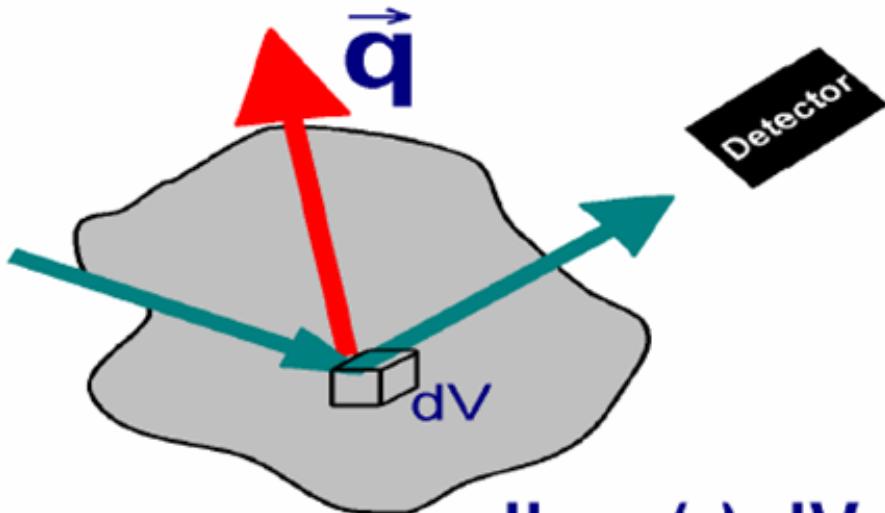
# Reflectivity calculation for arbitrary density profiles



# Example of fit of reflectivity curve:



# Simpler approach: Kinematical approximation



Multiple  
Scattering  
Negligible!

$$dI \sim \rho(r) dV$$

$$I(\vec{q}) \propto \left| \int \varrho(r) \exp(i \vec{q} \cdot \vec{r}) dr \right|^2$$

## The « master » formula

$$I(\vec{q}) \propto \left| \int \varrho(r) \exp(i \vec{q} \cdot \vec{r}) d\vec{r} \right|^2$$

### Reformulation for Interfaces

$$R(q_z) = R_F(q_z) \left| \frac{1}{\varrho_\infty} \int \frac{d\varrho(z)}{dz} \exp(i q_z z) dz \right|^2$$

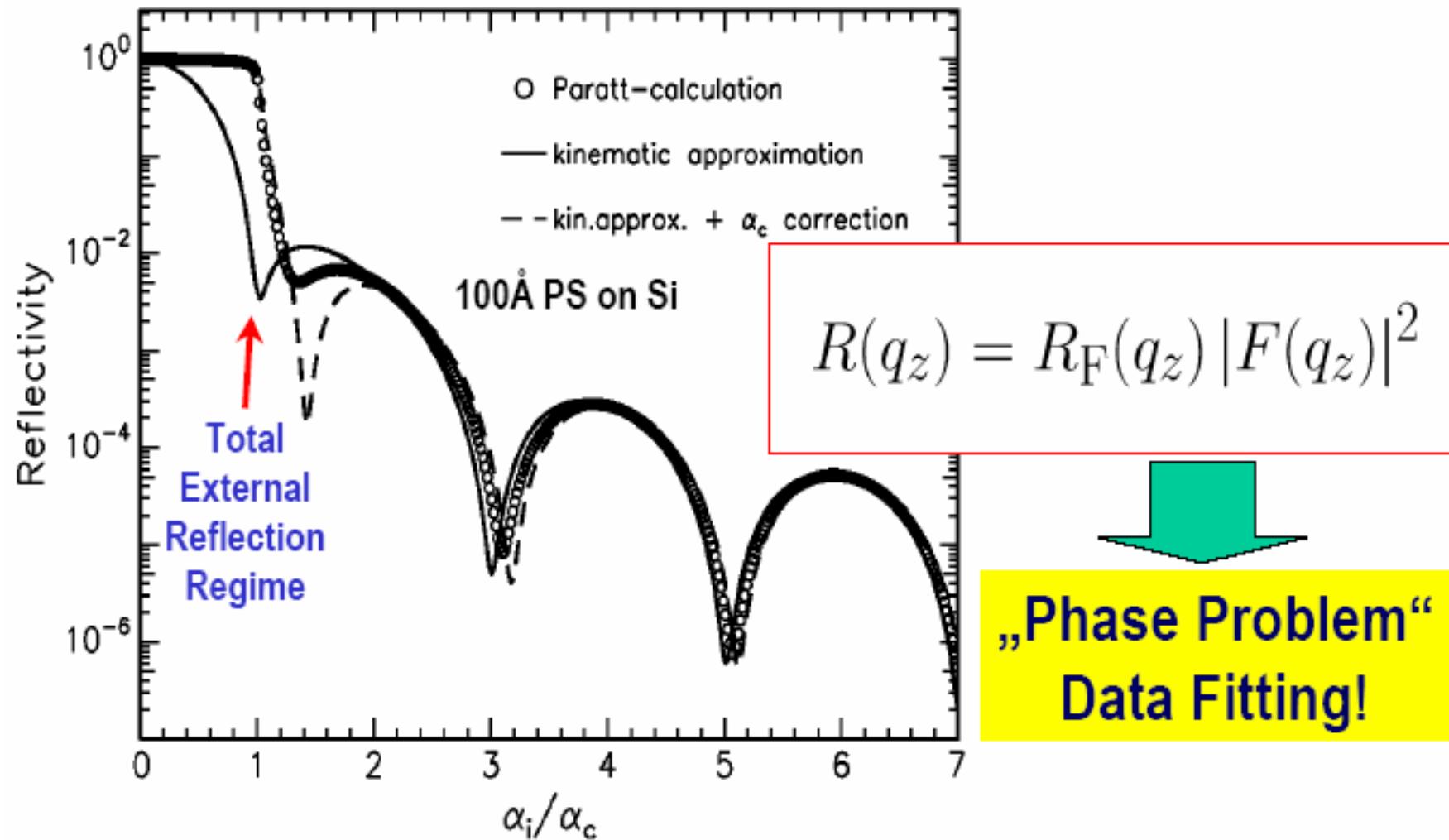
Fresnel-Reflectivity  
of the Substrate

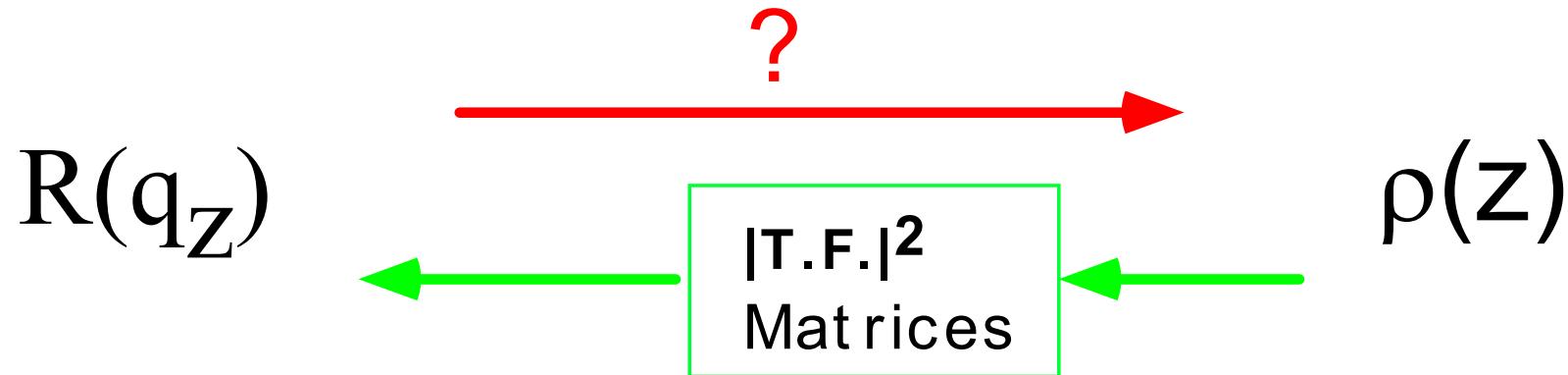
Electron Density Profile

Example: roughness

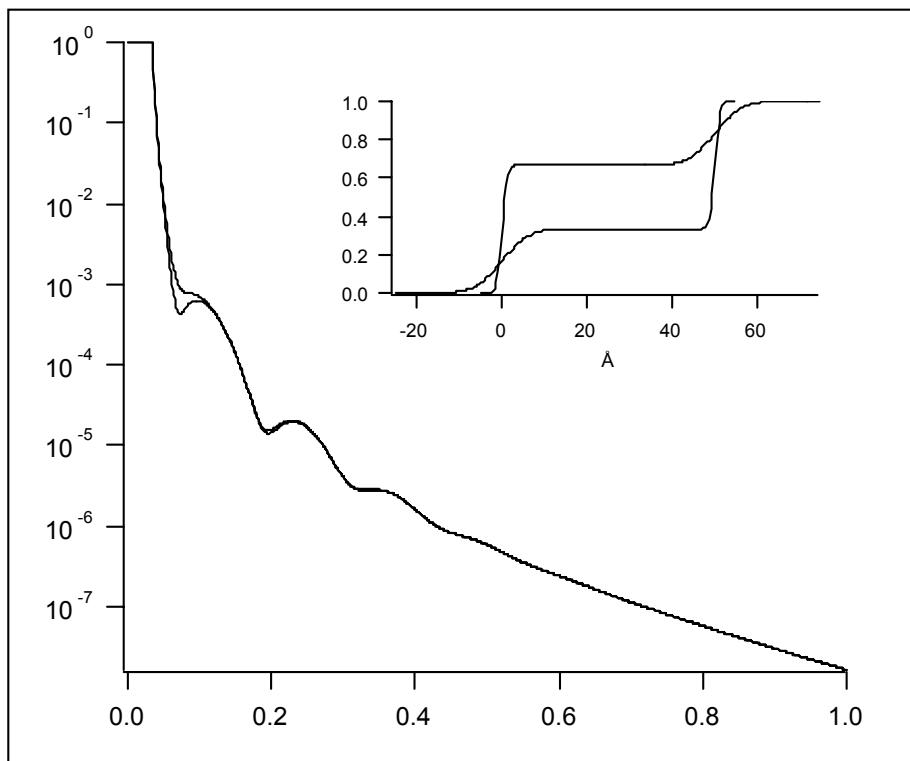
$$\rho'(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2}\right) \Rightarrow R(q) = R_F(q) \exp\left(-q^2 \sigma^2\right)$$

# Kinematical versus dynamical calculation





## Pb: Loss of the phase:

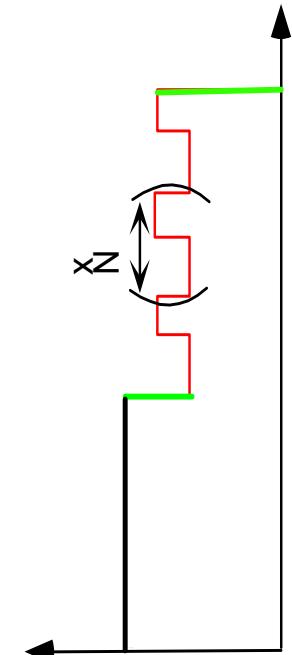
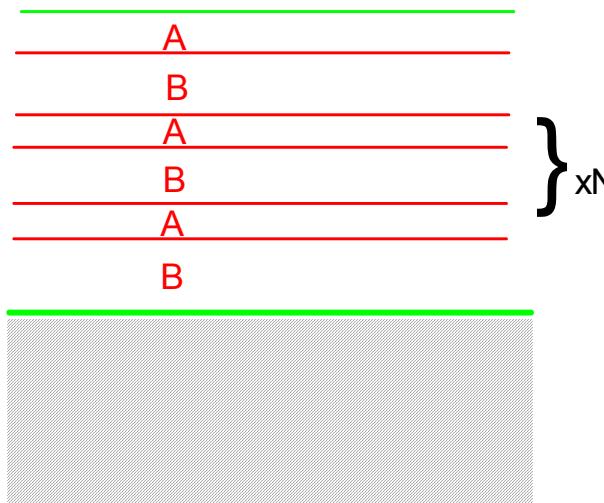
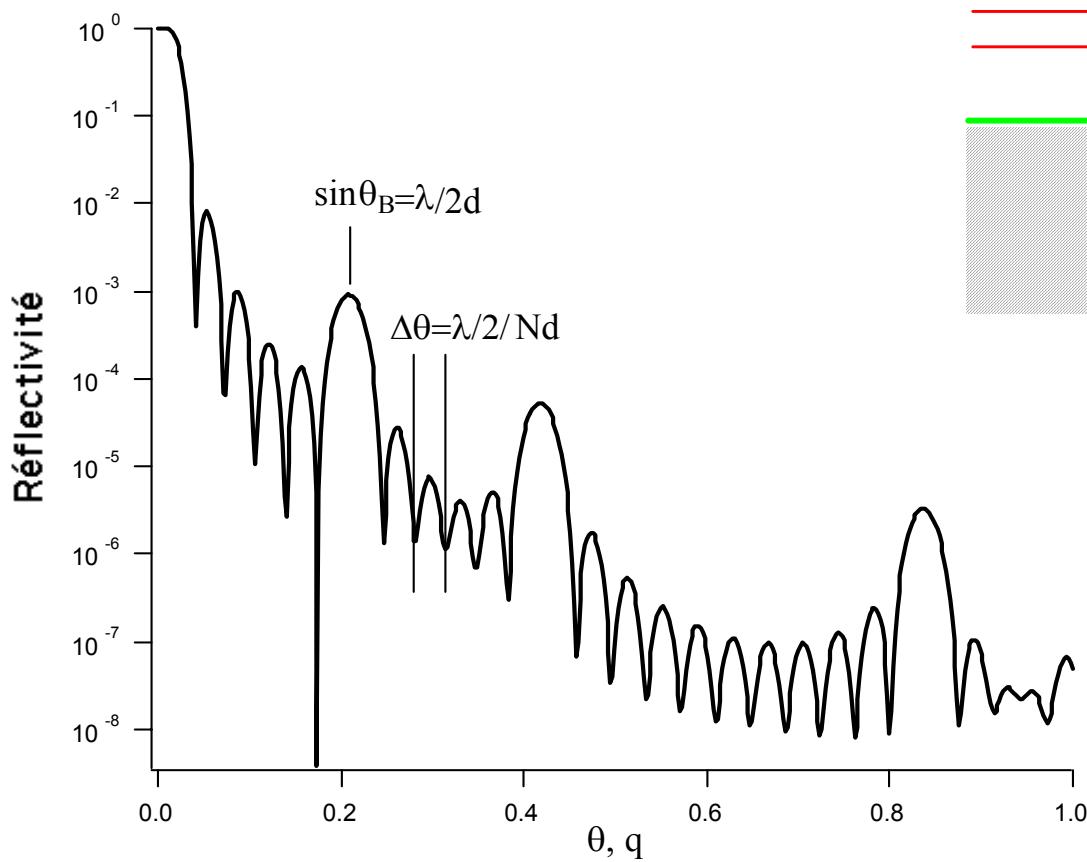


Different ways to solve the phase problem:

- Inclusion of pre-knowledge
- Anomalous reflectivity

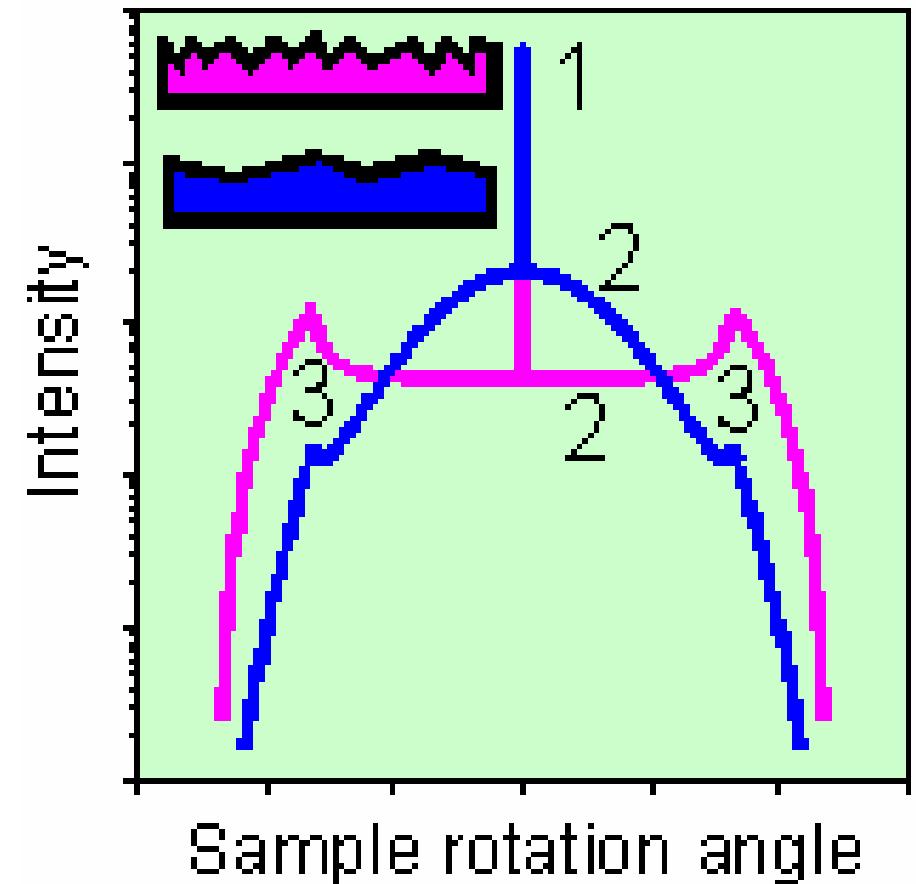
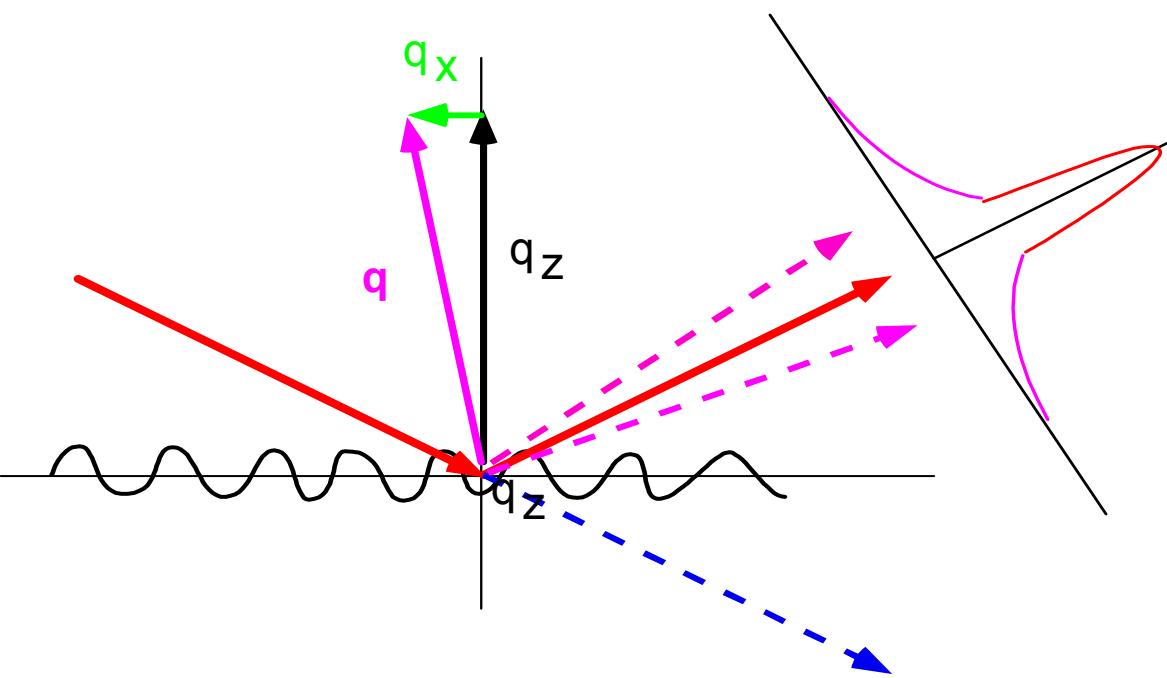
# Ex: Multilayers:

- Complex index profile



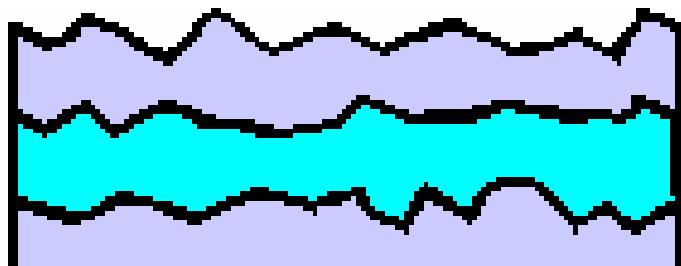
→ X-ray reflectivity used to characterize the thickness, period and roughnesses of multilayers.

# Rough surfaces → diffuse scattering

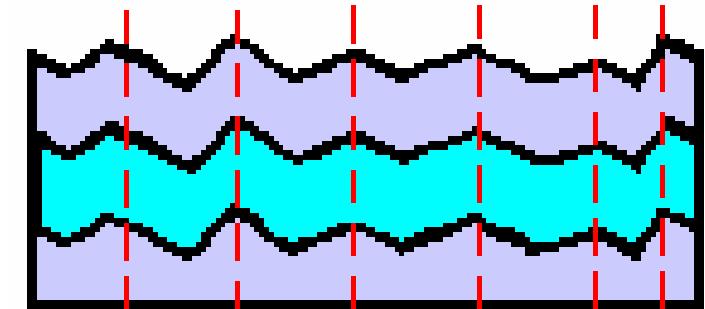


→ Lateral features of the roughness – Height-height correlations

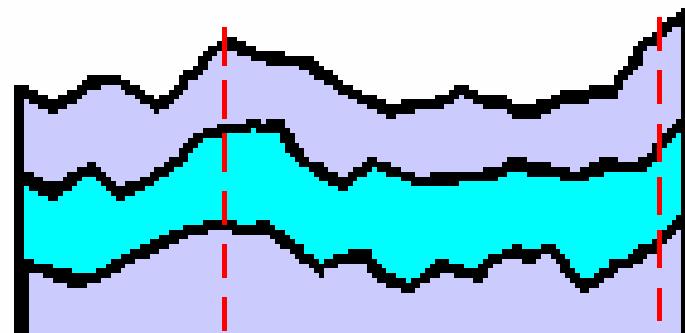
## Ex.: Roughness correlations in multilayers?



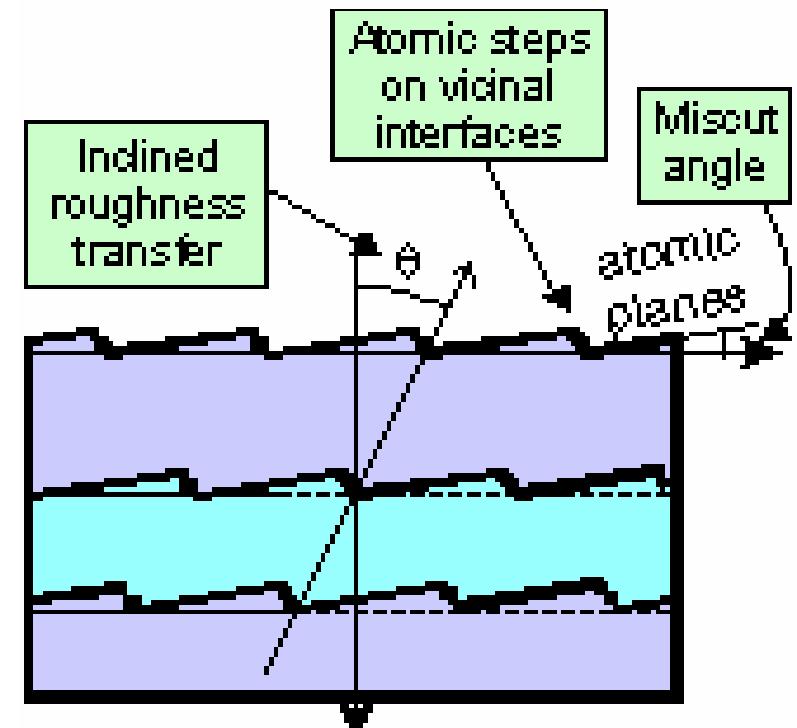
Uncorrelated roughness



Correlated (repeated) roughness



Wavelength-dependent inheritance of roughness.



# Conclusions on reflectivity

## Specular reflectivity measures

- average density (mass and electron density)
- layer thicknesses
- interface roughness

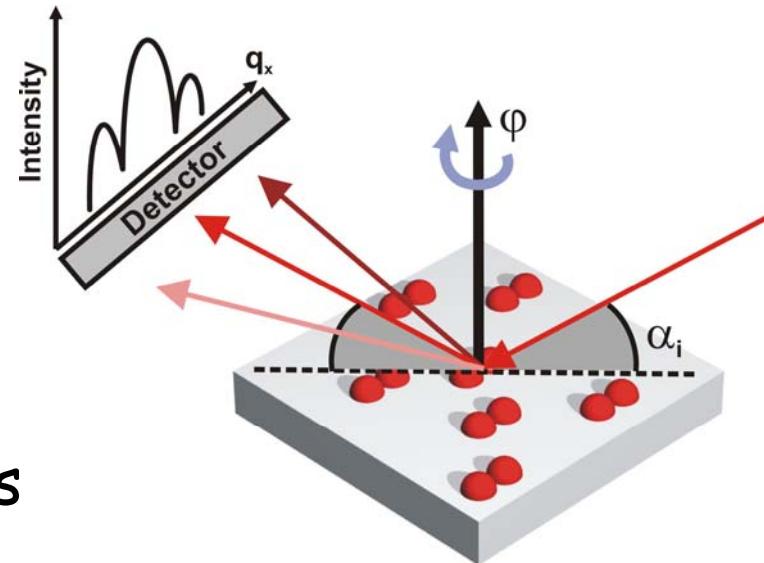
## Off-specular reflectivity probes

- Height-height correlations
  - lateral order at nanometer-micrometer scale
  - Refraction under grazing incidence
- tuneable scattering depth

# Why GISAXS ?

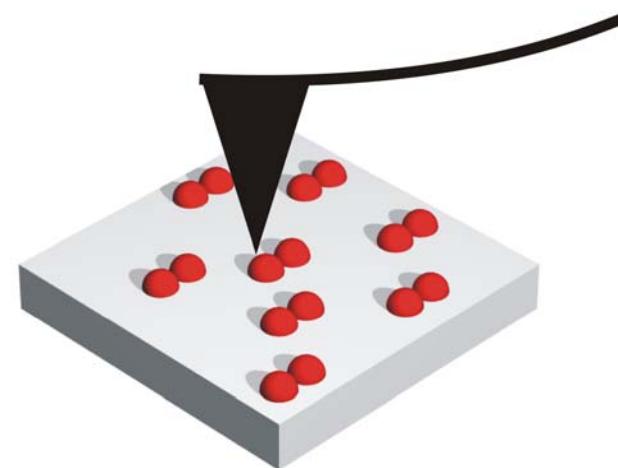
## GISAXS

- Statistical information
- Lateral and vertical correlation
- shape as seen by x-rays:  
input for diffraction experiments
- Information about buried objects



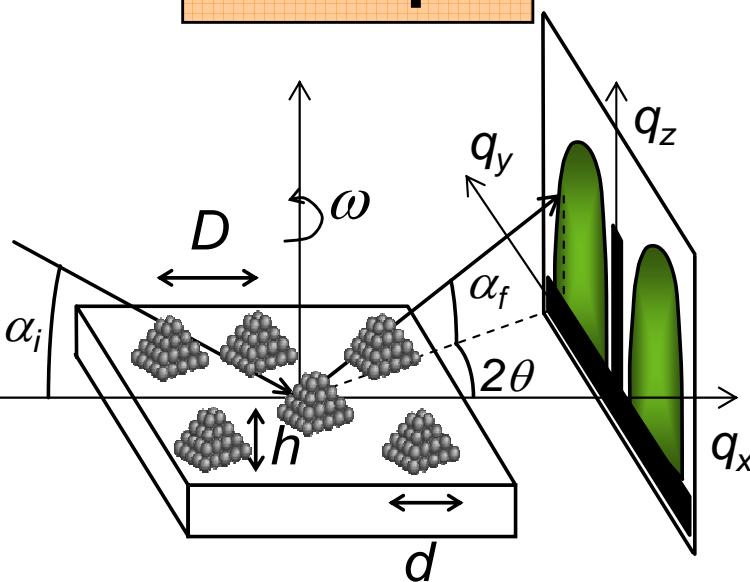
## AFM / STM

- Local information
- Detailed shape



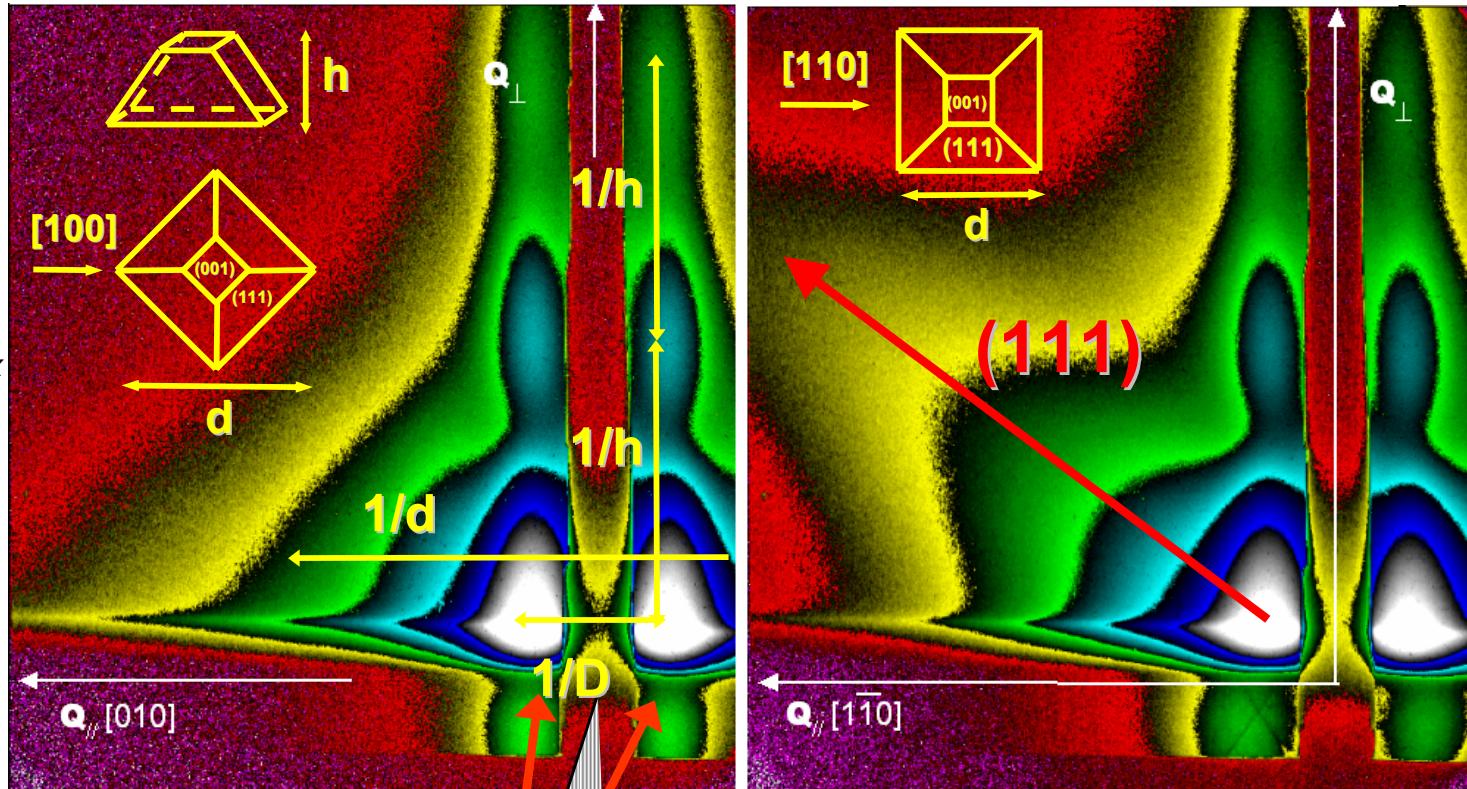
# Grazing Incidence Small Angle X-ray Scattering (GISAXS)

## Principle



## Standard 3D growth (Volmer-Weber)

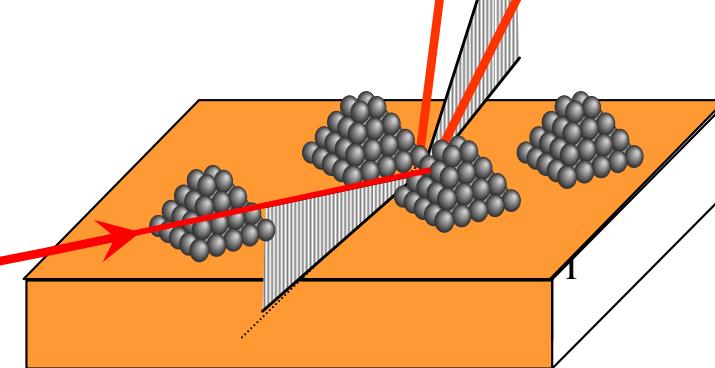
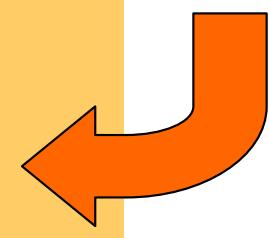
Example : 20 Å Ag/MgO(001) 500K



2D image around direct beam:  
Fourier transform of objects

## Morphology

- Shape
- Sizes
- Size distributions
- Particle-particle pair correlation function

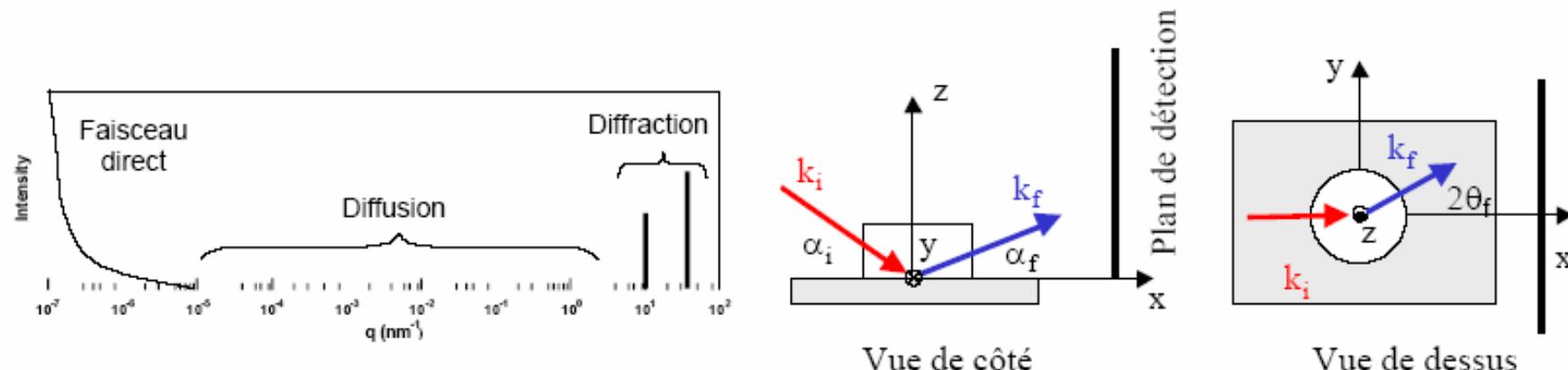


Anisotropic islands:  
truncated square pyramids  
with  $(111)$  facets

Off-specular reflectivity:

Probed length scales?

## La géométrie de diffusion : GISAXS et réflectivité



Le GISAXS ou comment mesurer des distances de l'ordre du nanomètre avec des rayons X ?

Le transfert de moment :  
grandeur pertinente ?

$$\vec{q} = \vec{k}_f - \vec{k}_i$$

Réflectivité X

$$\alpha_i = \alpha_f, 2\theta_f = 0$$

GISAXS

$$\lambda = 1 \text{ \AA}$$

$$\alpha_i = \alpha_f / 2 = 0.2^\circ$$

$$2\theta_f = 1^\circ$$

$$q_x = -0.011 \text{ nm}^{-1} \Rightarrow d_x = 580 \text{ nm}$$

$$q_y = 1.1 \text{ nm}^{-1} \Rightarrow d_y = 5.7 \text{ nm}$$

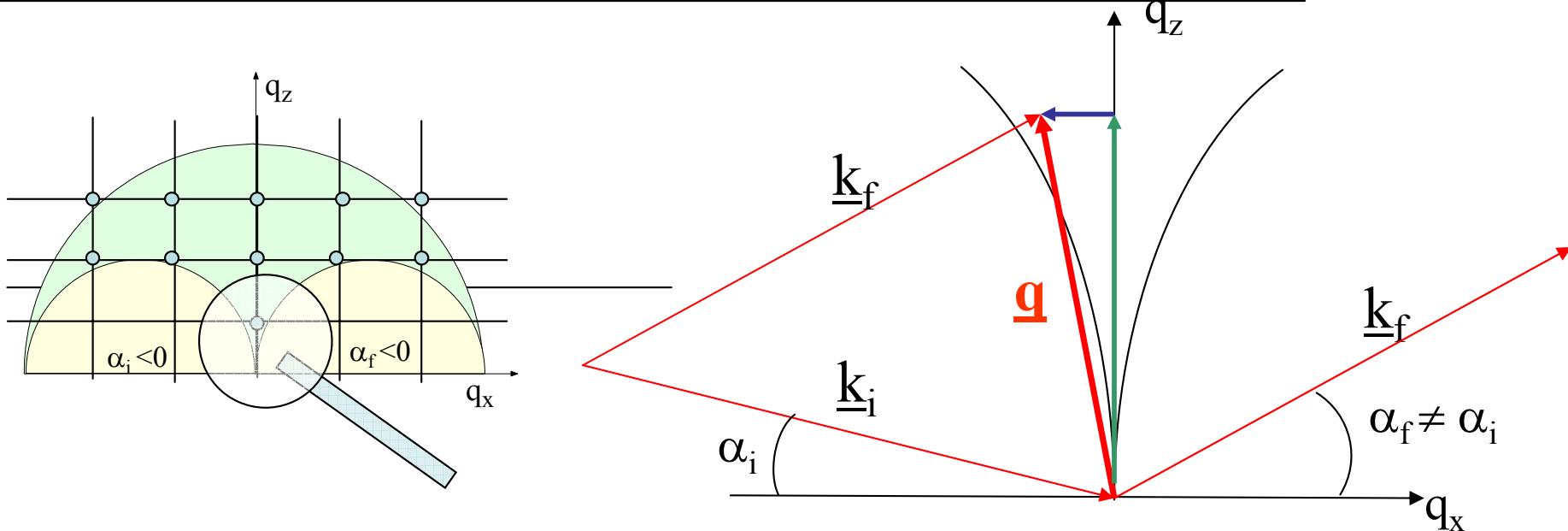
$$q_z = 0.66 \text{ nm}^{-1} \Rightarrow d_z = 9.5 \text{ nm}$$

$$q_x = \frac{2\pi}{\lambda} [\cos \alpha_f \cos 2\theta_f - \cos \alpha_i]$$

$$q_y = \frac{2\pi}{\lambda} [\cos \alpha_f \sin 2\theta_f]$$

$$q_z = \frac{2\pi}{\lambda} [\sin \alpha_f + \sin \alpha_i]$$

## Small angles of incidence and exit: plane of incidence



$$\underline{q} = \underline{k}_f - \underline{k}_i$$

Specular direction:  $\alpha_f = \alpha_i$  :  $q_z = 4\pi/\lambda(\sin \alpha_i) = 2\pi/d$

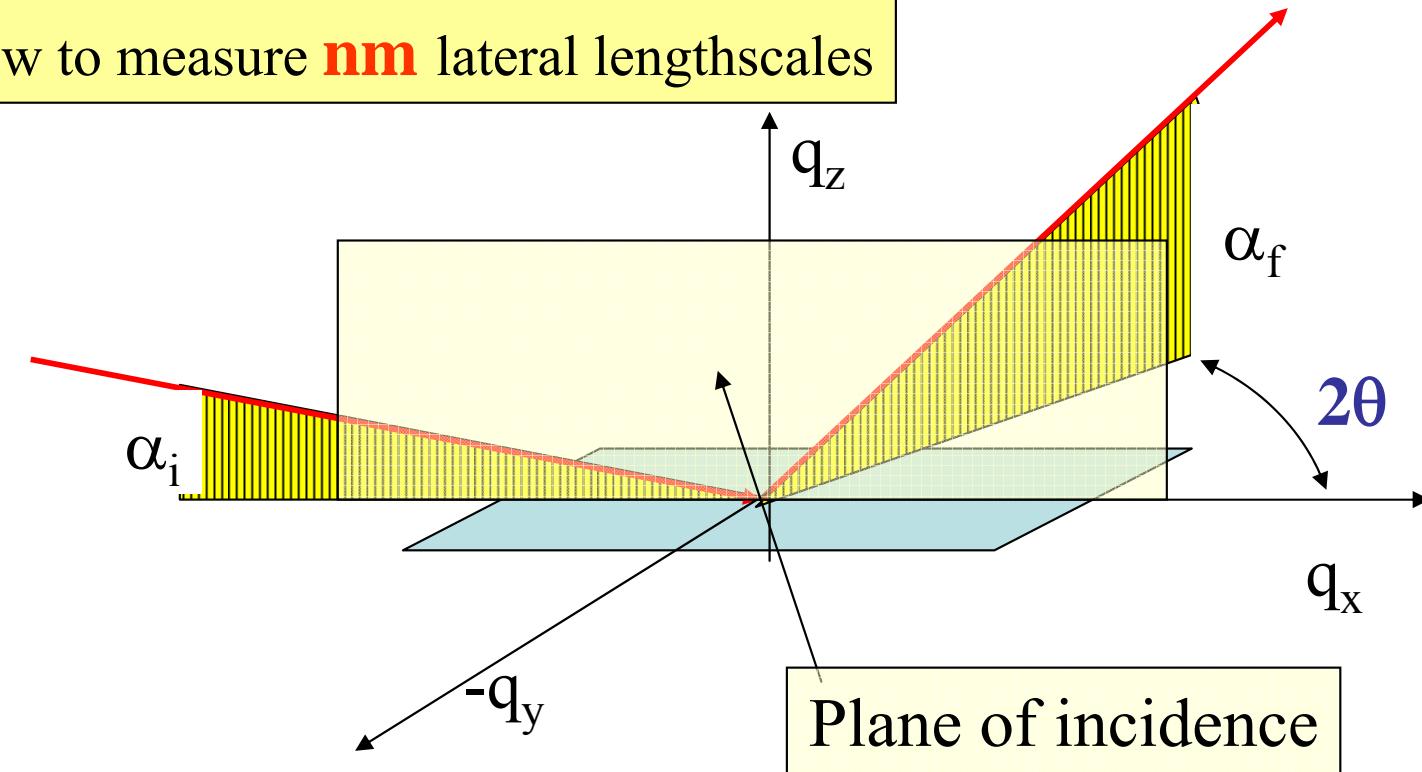
$$q_x = (2\pi/\lambda)(\cos \alpha_f - \cos \alpha_i) \quad q_z = (2\pi/\lambda)(\sin \alpha_f + \sin \alpha_i)$$

Lengthscales d: lateral and vertical

$$\lambda = 1.54 \text{\AA} \quad \alpha_i = 0.5^\circ \quad d_x = 2\pi/q_{x,\max} \leq \lambda / (\cos \alpha_f - 1) \approx 10\,000 \text{\AA}$$

$$d_z = 2\pi/q_z \approx 20 \text{\AA} \quad \text{for} \quad \alpha_f = \alpha_i$$

## GISXAS or how to measure **nm** lateral lengthscales



$$\mathbf{Q} = \mathbf{k}_f - \mathbf{k}_i$$

$$q_x = (2\pi/\lambda) (\cos \alpha_f \cos 2\theta - \cos \alpha_i)$$

$$q_y = (2\pi/\lambda) (\cos \alpha_f \sin 2\theta)$$

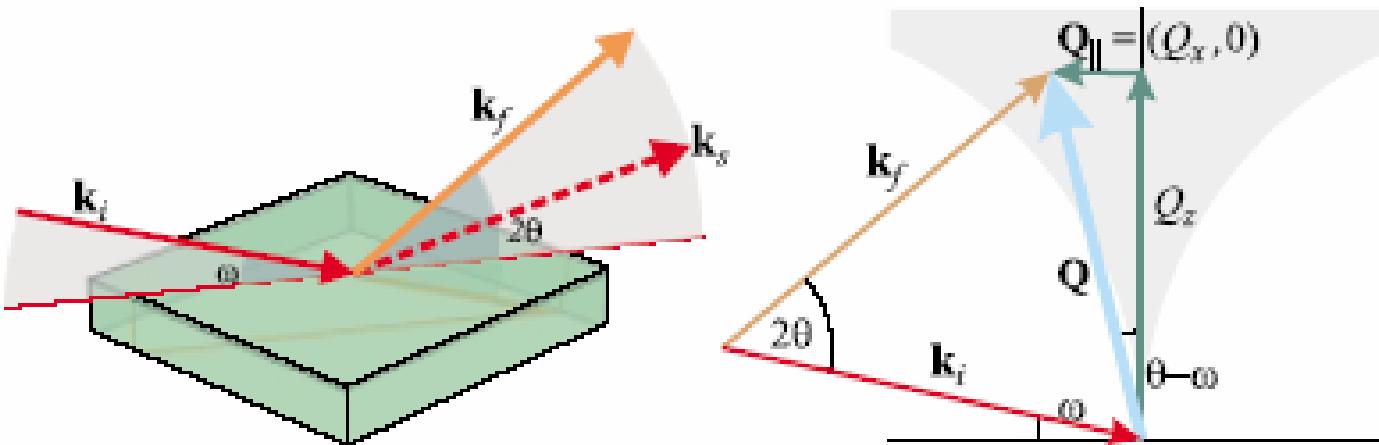
$$q_z = (2\pi/\lambda) (\sin \alpha_f + \sin \alpha_i)$$

Lengthscales d: lateral

$$\lambda = 1.54 \text{ \AA} \quad 2\theta = 2^\circ \quad \alpha_f = 0.5^\circ \quad d_y = 2\pi / q_y \leq \lambda / (\cos \alpha_f \sin 2\theta) \approx 44 \text{ \AA} !!!$$

Out-of plane scattering suited for nanostructure investigations

“No” limitation in  $2\theta$ : d from 100nm to 0.1nm



$$Q_x = 2k \sin \theta \sin(\omega - \theta)$$

$$Q_z = 2k \sin \theta \cos(\omega - \theta)$$

$$|Q_x| \simeq \frac{1}{2k} Q_z^2,$$

As the scattered intensity usually drops quite fast as a function of  $Q_z$ , the range of lateral momentum transfer is limited. Typically achievable scattering angles are in the order of  $2\theta = 3^\circ$ .<sup>3</sup> This puts an upper limit to the the accessible range of lateral structure dimension:

$$d_{\parallel} > \frac{2\pi}{|Q_{x,\max}|} \simeq \frac{\pi}{k \sin^2 \theta}.$$

Using  $2\theta = 3^\circ$  and  $k \simeq 4 \text{ \AA}^{-1}$  (for copper radiation), we obtain  $|Q_{x,\max}| \simeq 0.005 \text{ \AA}^{-1}$ , i.e., XRR is suitable only for the investigation of *lateral* structures with dimensions  $d > 1000 \text{ \AA}$  (this value depends, of course, on the wavelength and on how rapidly the intensity drops with  $Q_z$ , which depends, e.g., very sensitively on surface and interface roughnesses).

usually named  $\alpha_i$ , the exit angle is  $\alpha_f$  correspondingly. The reciprocal space coordinates are given by the relations

$$Q_x = k (\cos \alpha_i - \cos \alpha_f \cos 2\theta)$$

$$Q_y = k \cos \alpha_f \sin 2\theta$$

$$Q_z = k (\sin \alpha_i + \sin \alpha_f).$$

$Q_z$  is equivalent to the corresponding expression in XRR, Eq. (2.2), but now  $Q_z$  is virtually zero, and  $Q_y$  is finite instead. Hence for the determination of parameters of the very small sample structure, XRR and GISAXS are equivalent. However, as is obvious from Fig. 2, no restriction of  $Q_y$  due to the Laue zones exists, and consequently GISAXS is the method of choice for the investigation of small lateral structures ( $d_{\parallel} < 1000 \text{ \AA}$ ). As a "prize" for the enlarged range of lateral momentum transfer, the lateral resolution is much smaller than for XRR:

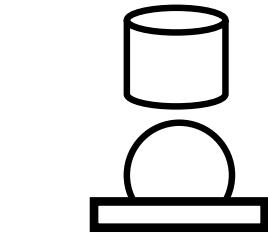
$$\begin{aligned} |\Delta Q_y| &= \left| \frac{\partial Q_y}{\partial \theta} \right| \Delta \theta + \left| \frac{\partial Q_y}{\partial \alpha_f} \right| \Delta \alpha_f = \\ &= 2k \cos \alpha_f \cos 2\theta \Delta \theta + k \sin \alpha_f \sin 2\theta \Delta \alpha_f \\ &\simeq 2k (\Delta \theta + \alpha_f \theta \Delta \alpha_f) \\ |\Delta Q_y| &\simeq 1.5 \cdot 10^{-3} \text{ \AA}^{-1}. \end{aligned}$$

# Quantitative analysis of GISAXS

$$I(q_{\parallel}, q_{\perp}) \approx \langle |F|^2 \rangle \times S(q_{\parallel})$$

Form factor :  
a kind of shape FT  
with refraction effects

$$F_{DWBA}(q_{\parallel}, k_i, k_f)$$



- Cylinder
- Troncated sphere



- Pyramid



- Tetraedron

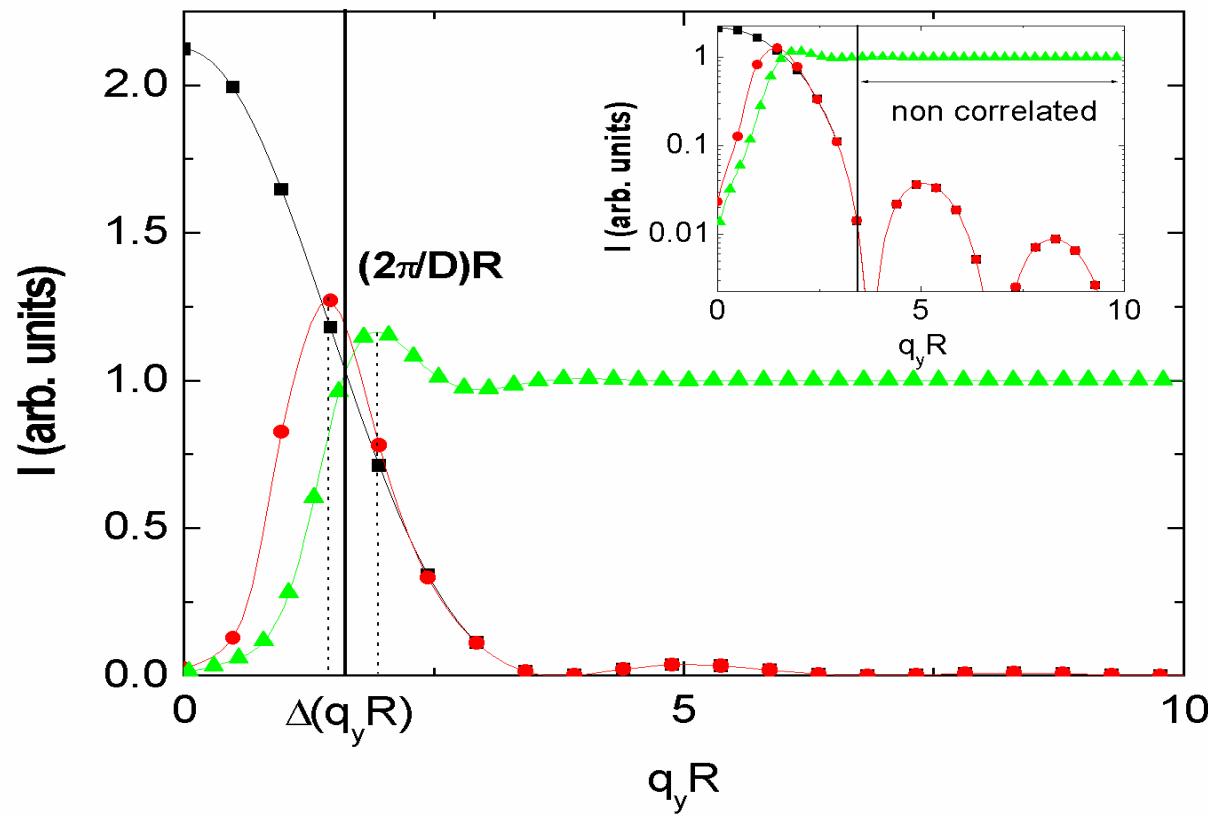


- Cubooctaedron

Interference function:  
FT of pair correlation function

*IsGISAXS program :*  
[http://www.esrf.fr/computing/scientific/joint\\_projects/IsGISAXS/](http://www.esrf.fr/computing/scientific/joint_projects/IsGISAXS/)  
 R. Lazzari, J. Appl. Cryst. 35, 406 (2002)  
 F. Leroy, R. Lazzari and G. Renaud, Acta. Cryst. A 60, 565, (2004)

$$S(q_{\parallel}) = 1 + \rho_s \int (g(r) - 1) e^{-iq_{\parallel}r} d^2r$$



# Analyse quantitative du signal GISAXS

IsGISAXS program :

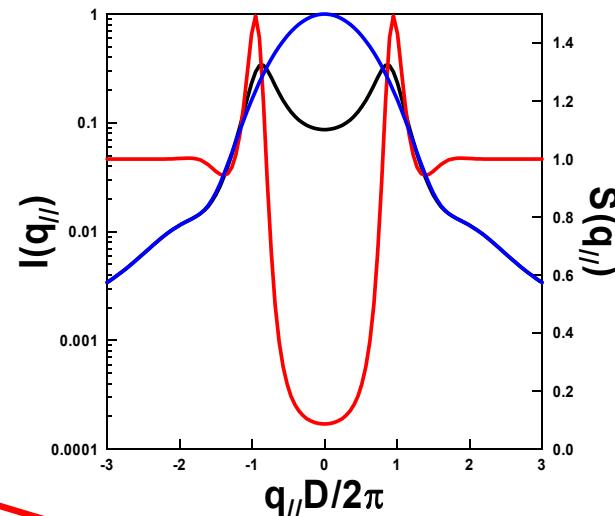
[http://www.esrf.fr/computing/scientific/joint\\_projects/IsGISAXS/](http://www.esrf.fr/computing/scientific/joint_projects/IsGISAXS/)

R. Lazzari, J. Appl. Cryst. 35, 406 (2002)

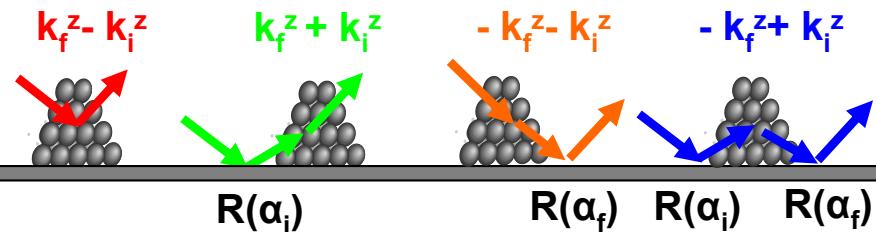
F. Leroy, R. Lazzari and G. Renaud,

Acta. Cryst. A 60, 565, (2004)

$$I(\vec{q}) = \left\langle |F|^2 \right\rangle S(q_{\parallel})$$



## Facteur de forme



## Forme, taille et orientation

- cylindre



- sphère tronquée



- pyramide



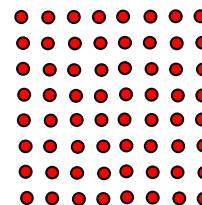
+ distributions

## Fonction d'interférence

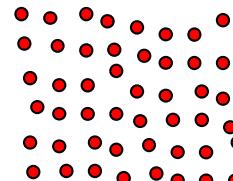
$$S(q_{\parallel}) = \text{TF} \left\{ \begin{array}{l} \text{Fonction de corrélation de} \\ \text{paires} \end{array} \right\}$$

## Arrangement spatial

- réseau



- modèle de désordre (paracristal...)



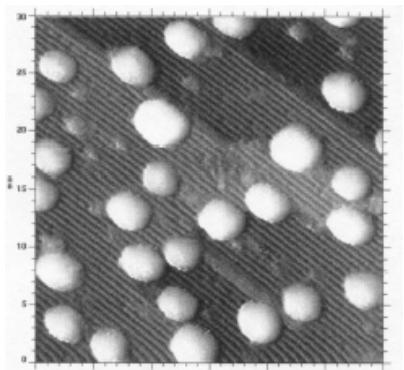
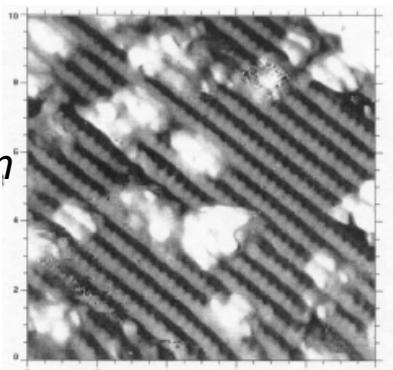
distance moyenne + distribution de distances

# Growth of Au on $\text{TiO}_2(110)$

12 Å flashed annealed @ 800K

STM pictures  
*Goodman et al.*

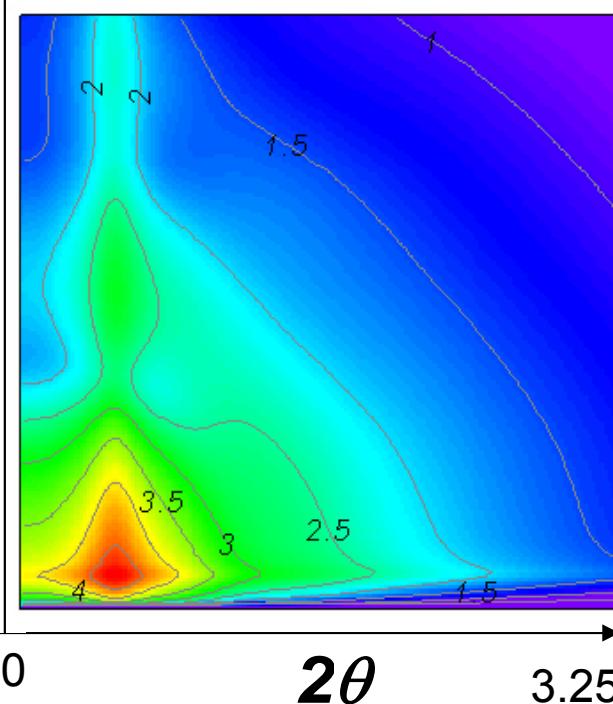
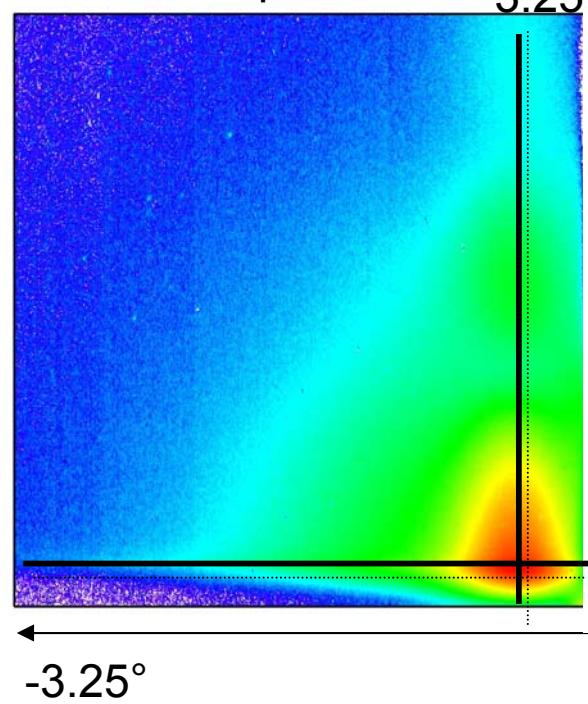
[001] → [110]



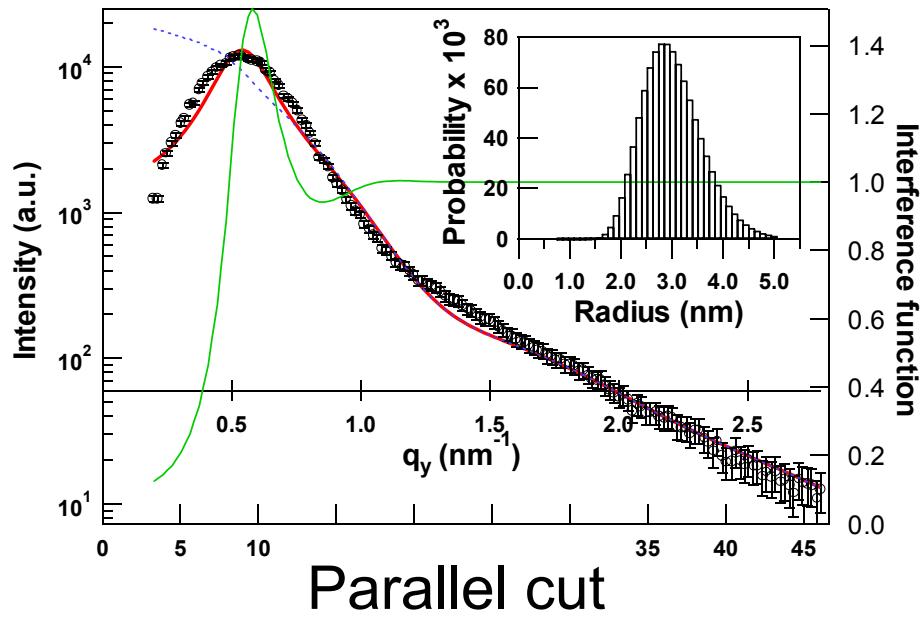
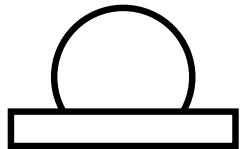
Experiment

$\alpha_f$

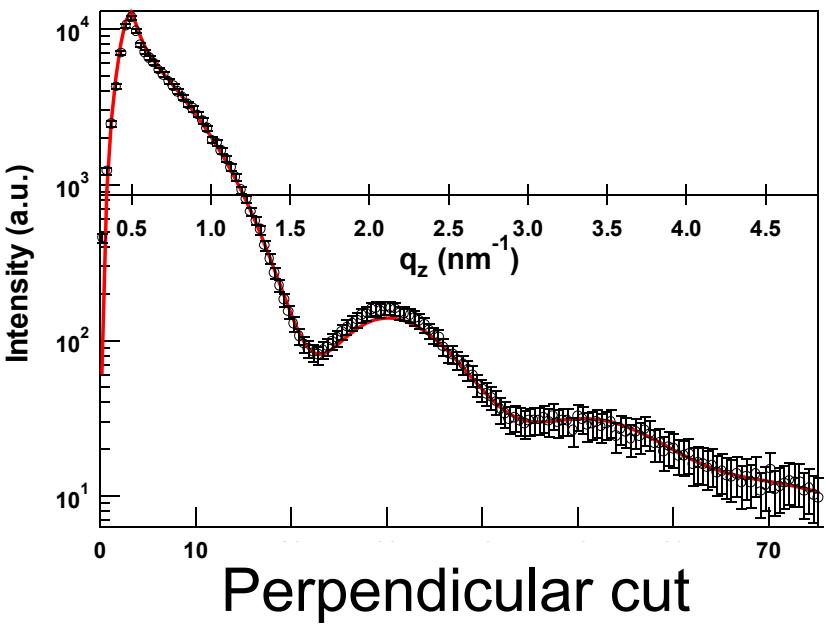
Simulation



FITS:  
Truncated  
spheres

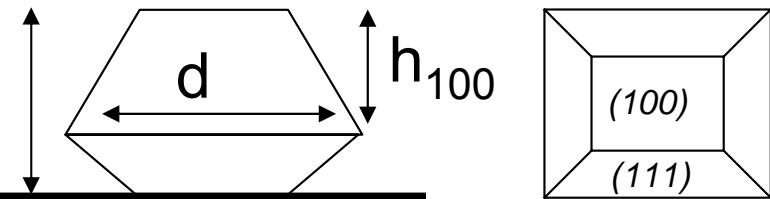


$R = 2.9 \text{ nm } s_R(\text{log normal}) = 1.2$   
 $H = 4.9 \text{ nm } s_H(\text{gaussian}) = 0.1$   
 $r = 9.1 \cdot 10^{11} \text{ part/cm}^2; q_c = 132^\circ$



# Equilibrium shape of particles. Ex: 1.5nm Pd/MgO @ 650 K

Fit with truncated octaedron



$$D=22 \pm 0.2 \text{ nm}$$

$$d=11.4 \pm 0.4 \text{ nm} - \sigma_{\text{FWHM}} = 6 \text{ nm}$$

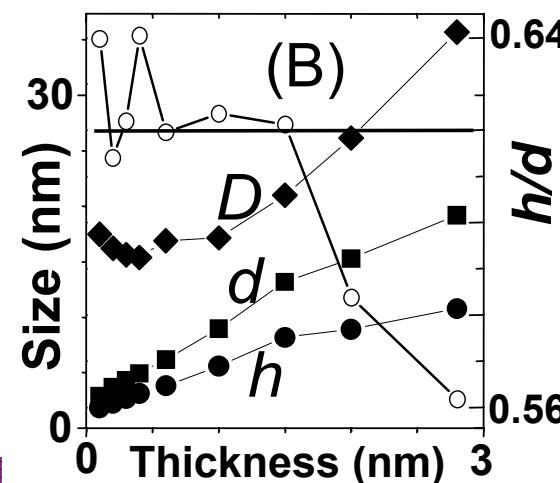
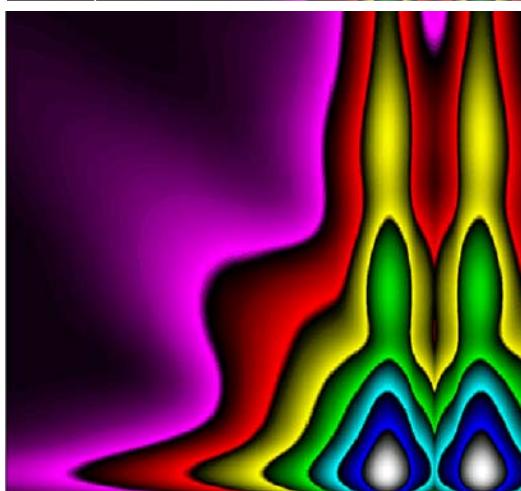
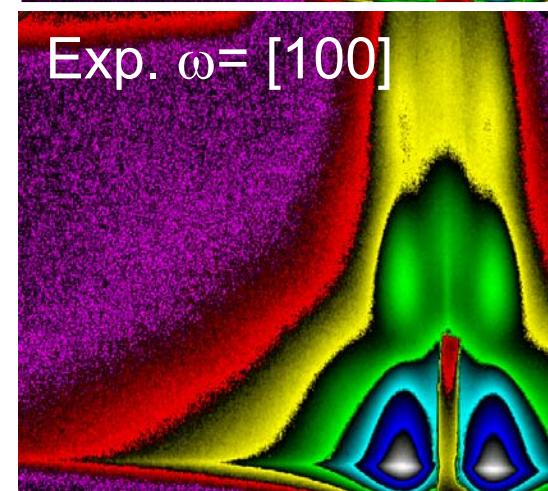
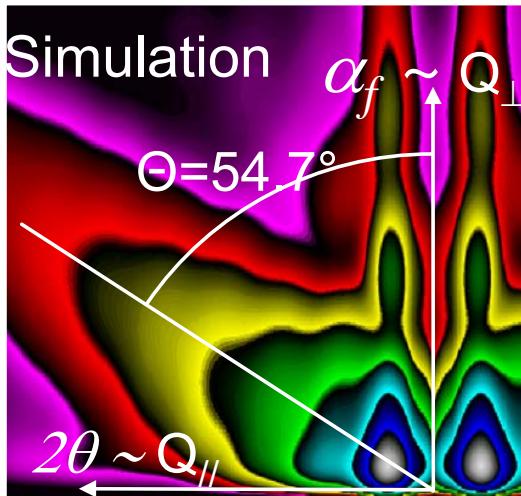
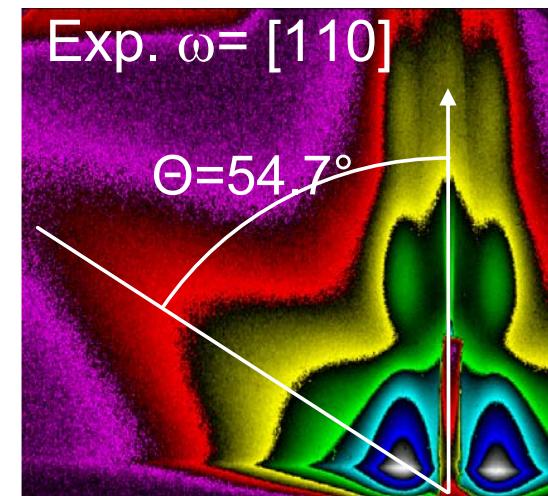
$$h=6.2 \pm 0.4 \text{ nm} - \sigma_{\text{FWHM}} = 0.8 \text{ nm}$$

$$h_{(001)}/d=0.46$$

$$h/d=0.62 \pm 0.2$$

$$H/d = \text{cte} = 0.62$$

→ Equilibrium island shape



Wulff's construction

Adhesion energy from GISAXS:  
 $\beta = 1.12 \text{ J/m}^2$

To be compared with TEM :  $\beta = 0.95 \text{ J/m}^2$

GISAXS →  
 Equilibrium shape, *in situ*,  
 non destructively during growth

G. Renaud, et al., Science 300, 1416 (2003).

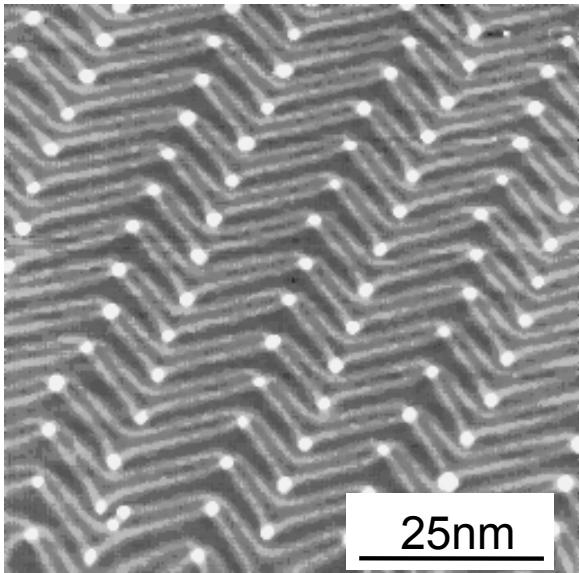
C. Revenant, F. Lazzari, F. Leroy, G. Renaud, C.R. Henry, PRB 69 (2004)

# Self-organized growth : systems

3 main types of surface structuration

Surface reconstruction

Co dots on Au(111)

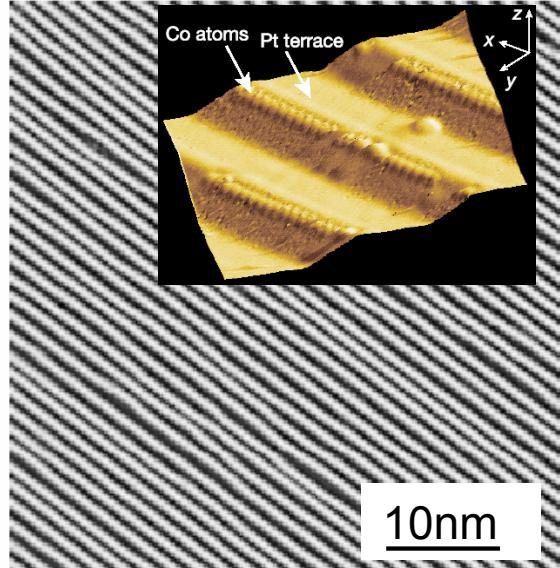


B. Voigtländer et al.

Phys. Rev. B **44** (1991)  
10354

Vicinal surface

Co wires on Pt(997)

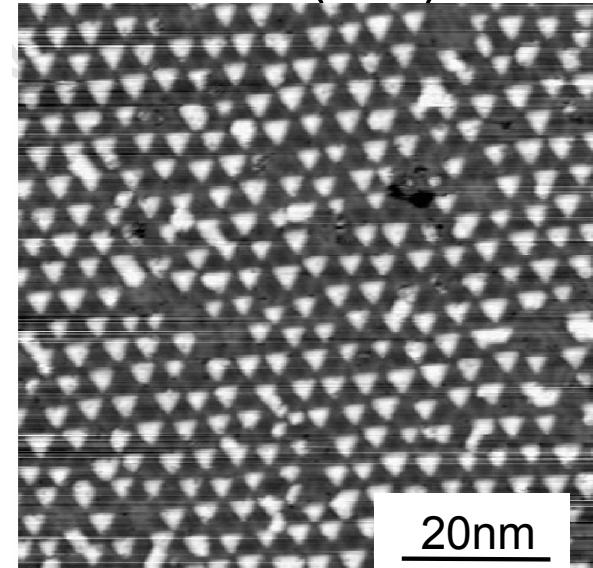


P. Gambardella et al.

Nature **416** (2002)  
301

Dislocation Network

Fe islands on a bilayer of Cu/Pt(111)



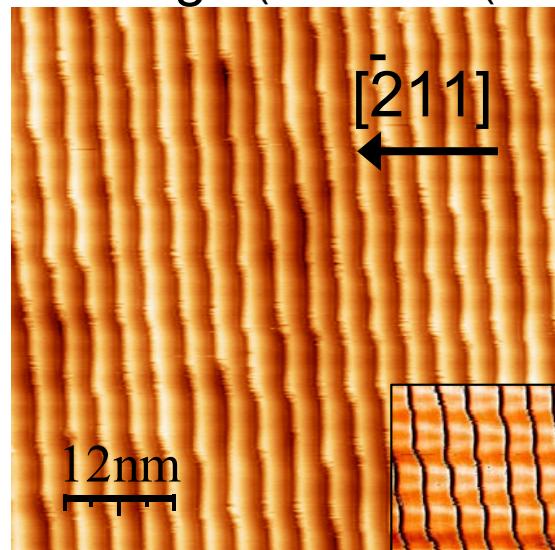
H. Brune et al.

Nature **394** (1998)  
451

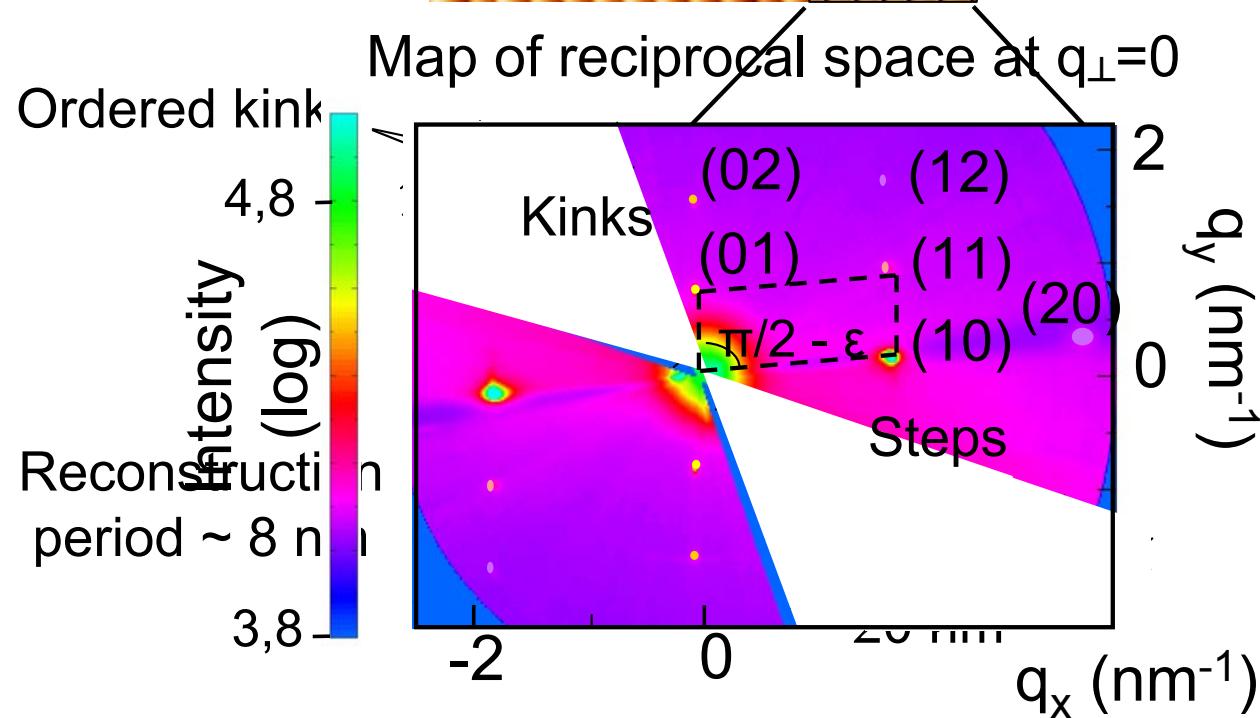
- Self-organized growth of cobalt islands on a –Au(677) kinked surface

# The kinked Au(677) surface

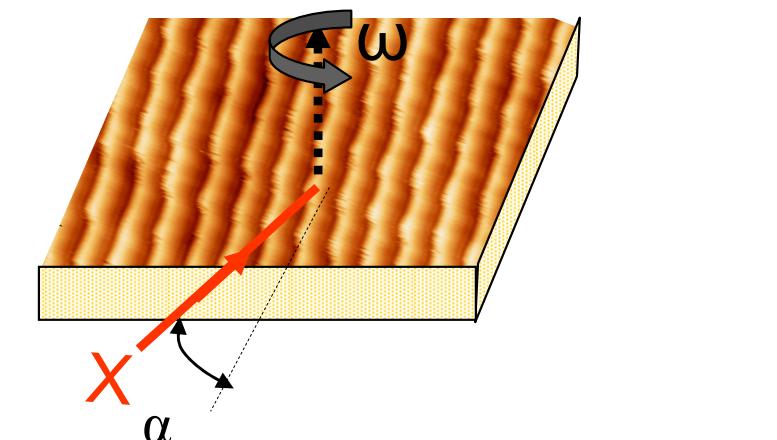
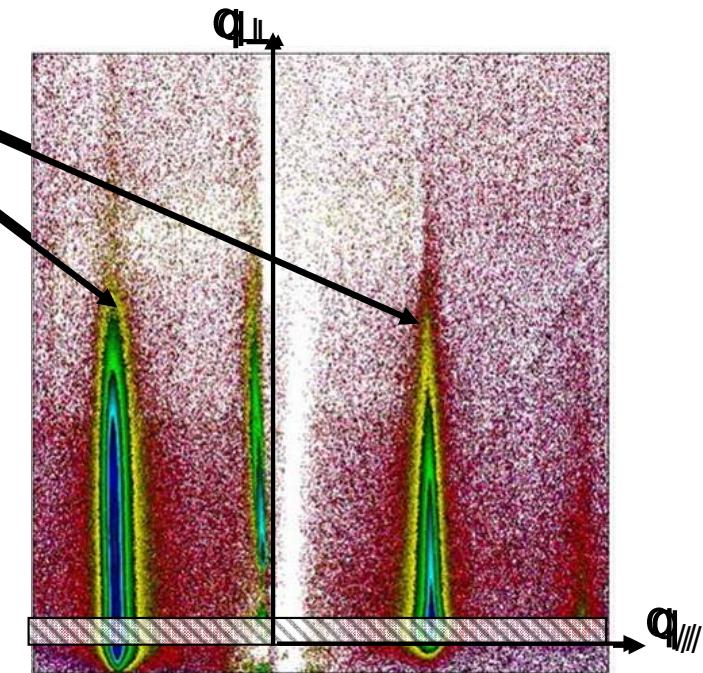
STM image (S. Rohart (GPS, Paris))



Scattering rods from steps

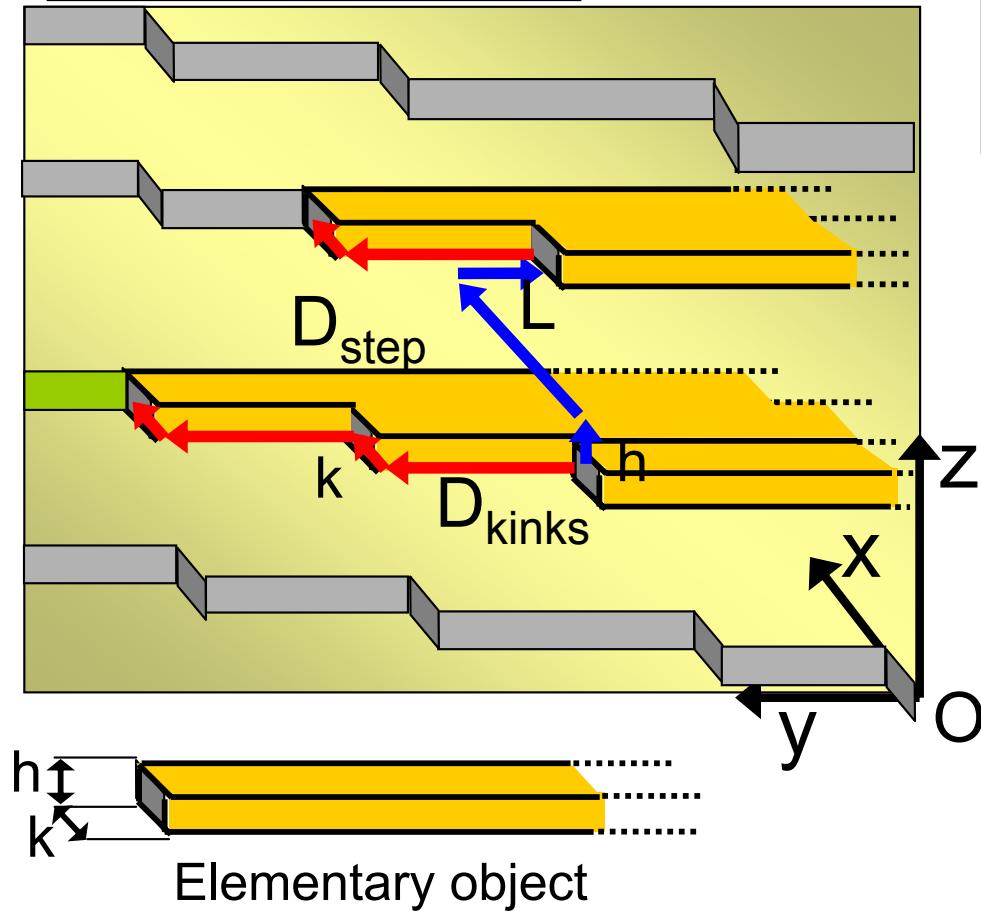


3D Measurements of reciprocal space by GISAXS



# Modelisation of a kinked Au(677) surface

Principle : Paracrystal



Steps

$$D_{\text{step}} = 3.42 \pm 0.23 \text{ nm}$$

Kinks position

$$D_{\text{kink}} = 8.04 \pm 0.62 \text{ nm}$$

$$L = 0 \pm 0.3 \text{ nm}$$

Kinks size

$$k = 0.7 \pm 0.35 \text{ nm}$$

Determined by the reconstruction

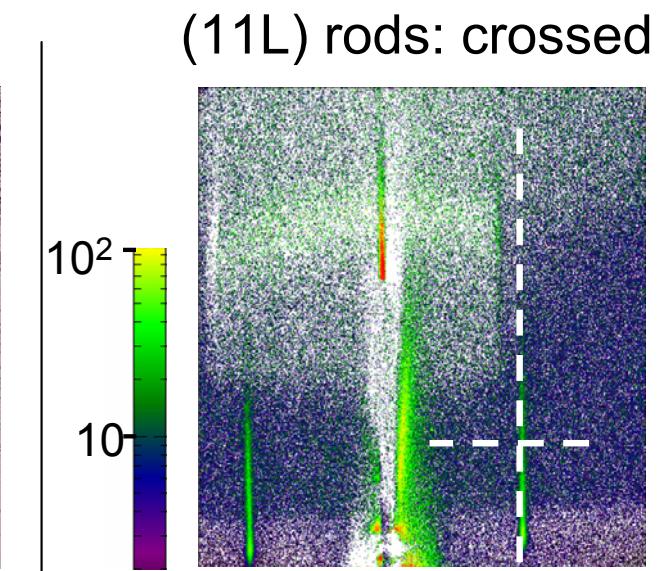
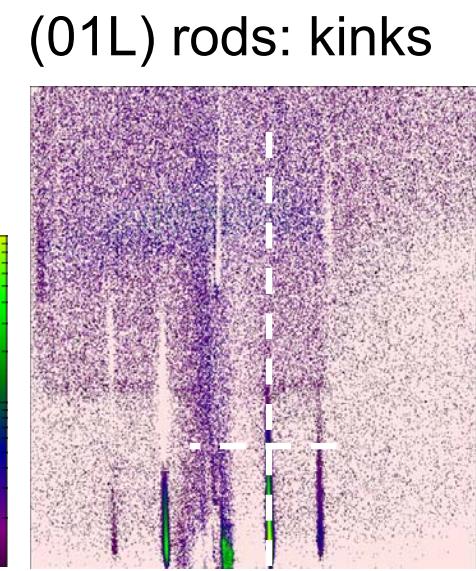
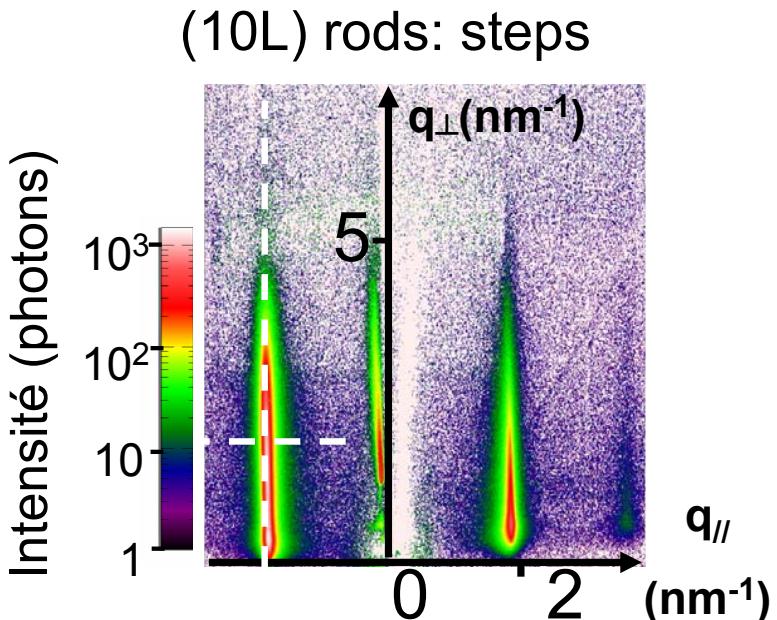
Monoatomic kinks  
Packed  
by  $3 \pm 1.5$

$$I(\vec{q}) = \left[ \frac{4 \sin\left(\frac{q_z h}{2}\right) \sin\left(\frac{q_x k}{2}\right)}{q_x q_y q_z} \right]^2$$

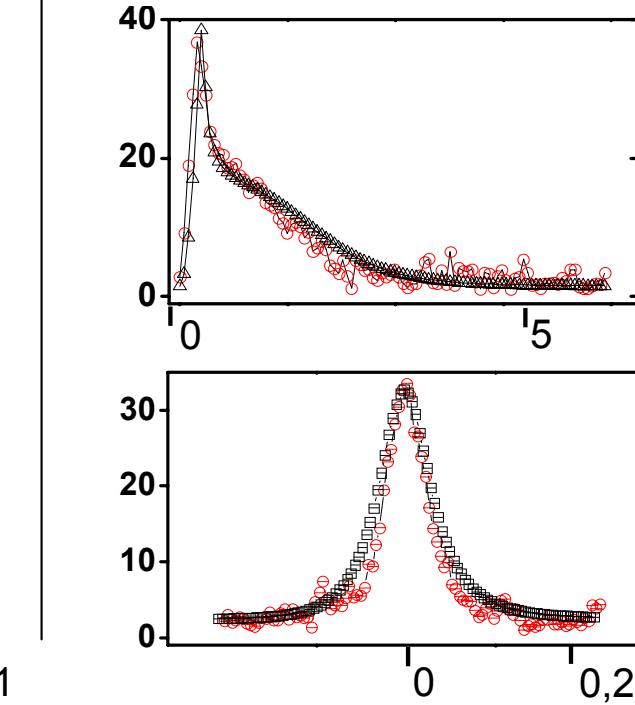
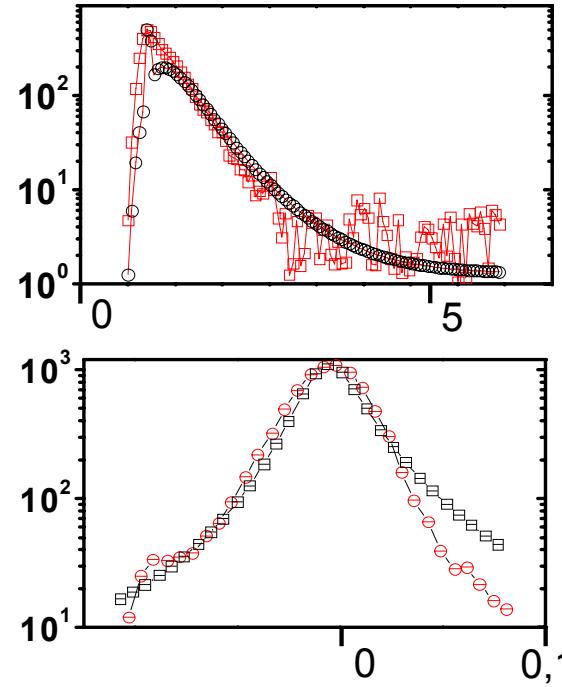
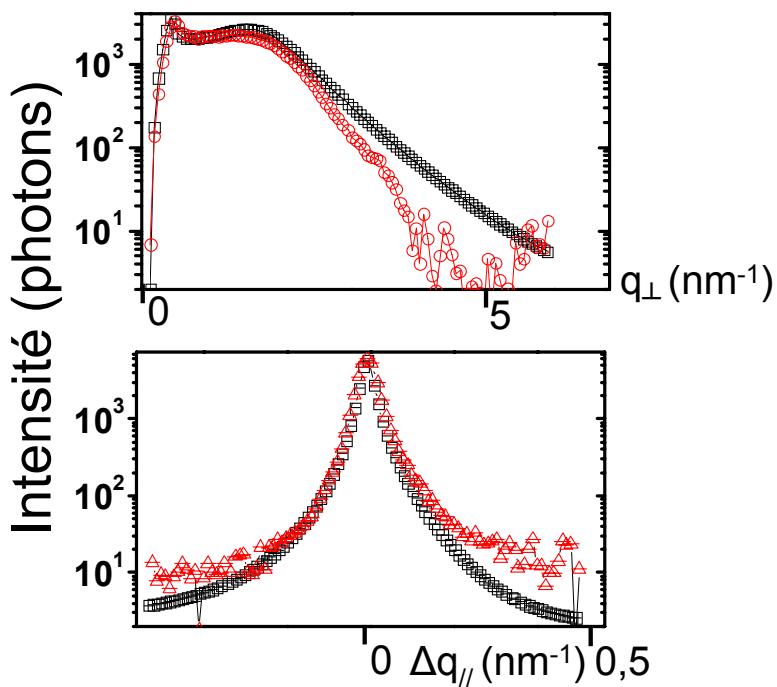
Shape and size of kinks

# GISAXS data and fits

Data



Fits



$$0 \Delta q_{\parallel}(\text{nm}^{-1}) 0.5$$

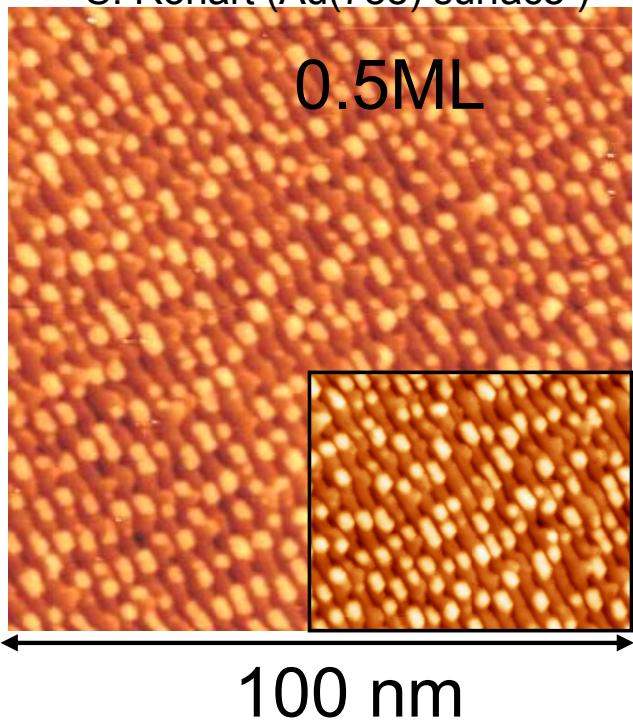
$$0 \Delta q_{\parallel}(\text{nm}^{-1}) 0.1$$

$$0 \Delta q_{\parallel}(\text{nm}^{-1}) 0.2$$

# Co growth at room temperature

## STM image

S. Rohart (Au(788) surface )



Shape  
and  
size

Position

**Island position**

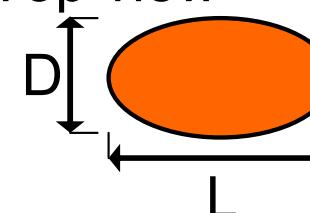
- in between kinks
- at the step edge

**Shape and size**

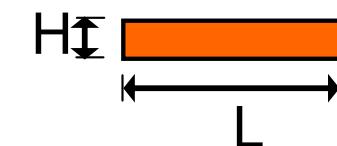
- anisotropic growth
- bilayer islands

## Model

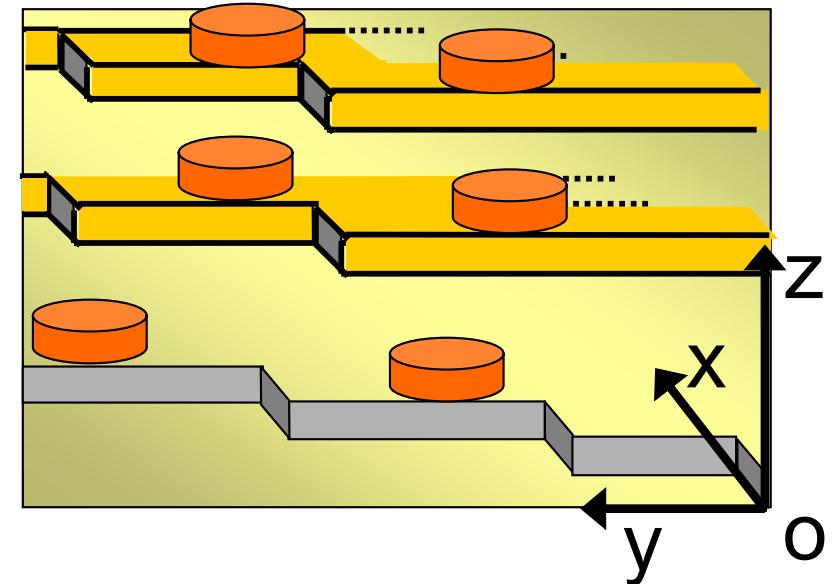
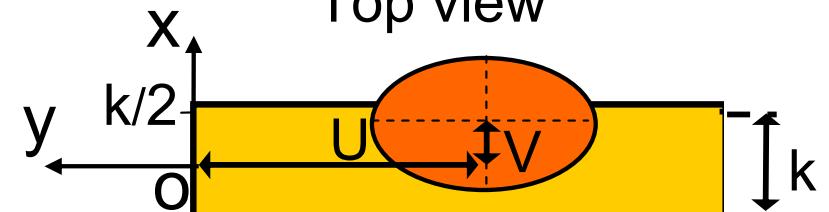
Top view



Side view



Top view

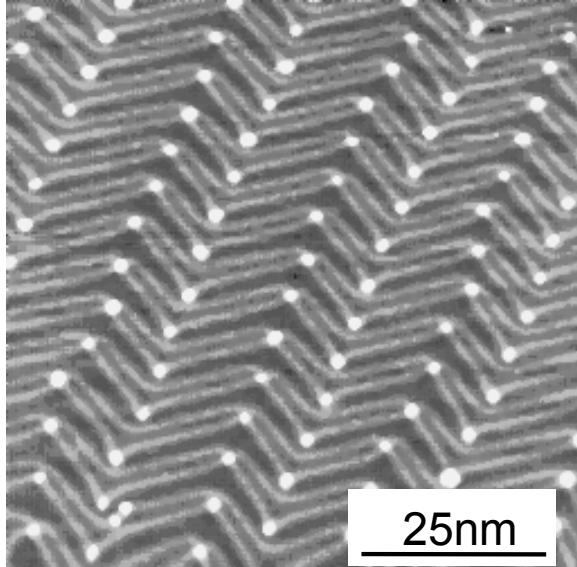


# Self-organized growth : Systems

3 main surface structuration

Surface reconstruction

Co dots on Au(111)

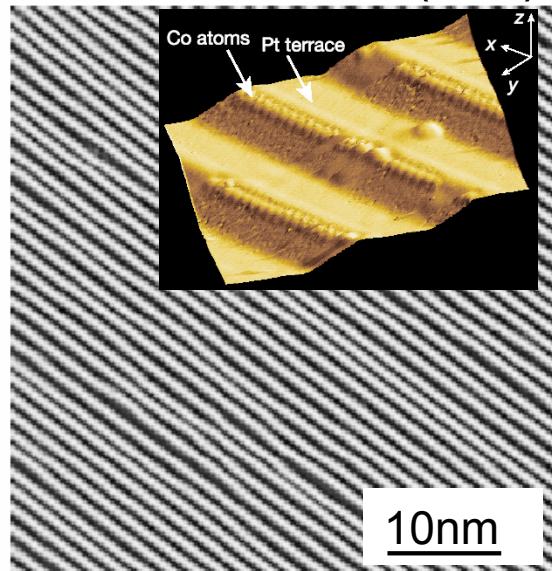


B. Voigtländer et al.

Phys. Rev. B **44** (1991)  
10354

Vicinal surface

Co wires on Pt(997)



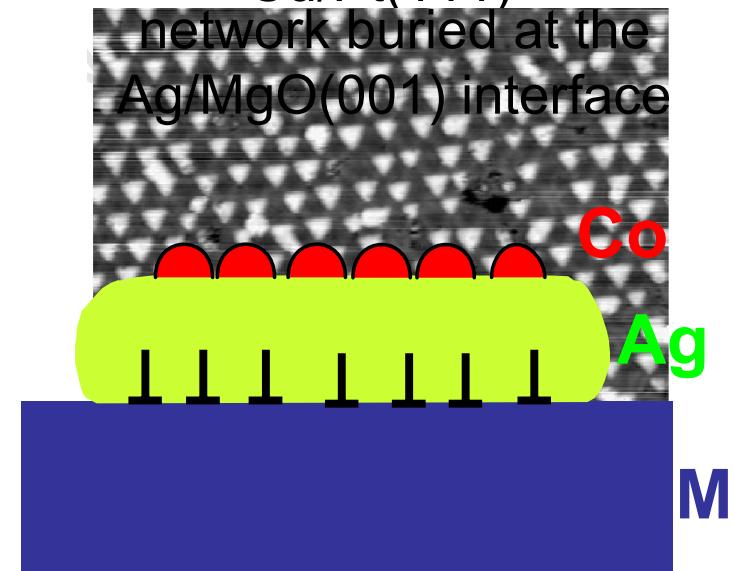
P. Gambardella et al.

Nature **416** (2002)  
301

Dislocation Network

Fe islands on a bilayer of  
Co dots on a dislocation

network buried at the  
Ag/MgO(001) interface



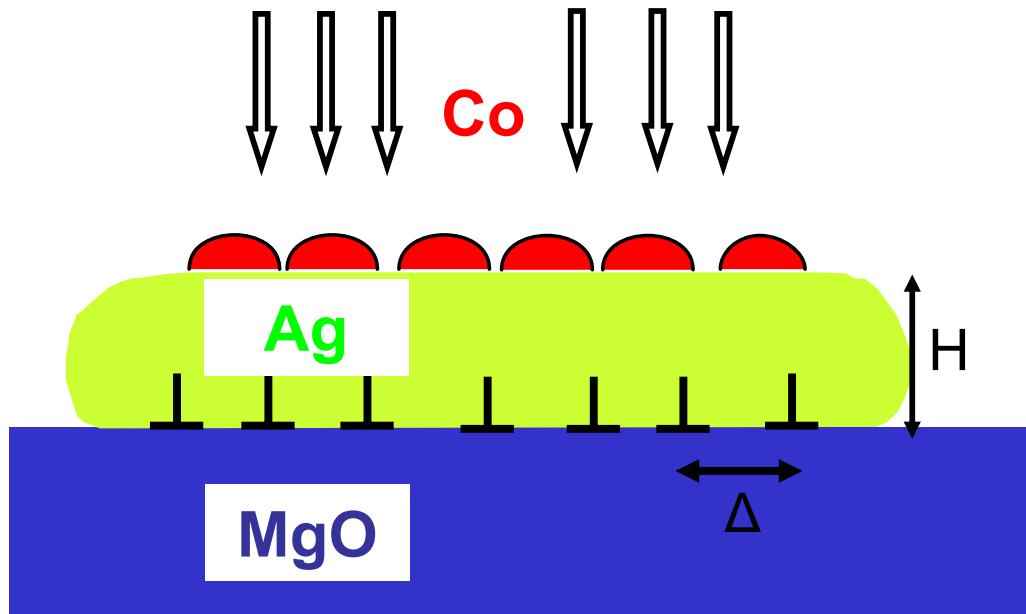
H. Brune et al.

Nature **394** (1998)  
451

- Self-organized growth of cobalt islands on a
  - dislocations network at the Ag/MgO(001) interface

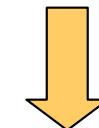
# Ordering of nanostructures induced by a dislocation network : principle

- Misfit dislocation network



Ag and MgO(001)

$$\frac{a_{\text{MgO}} - a_{\text{Ag}}}{a_{\text{Ag}}} = 3\%$$

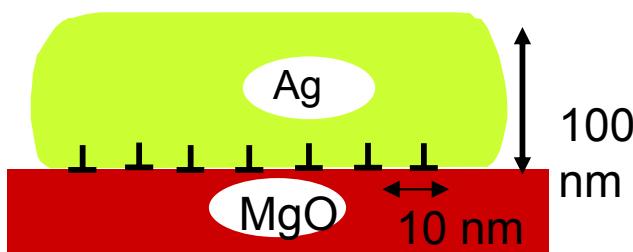


$$\Delta = 10 \text{ nm}$$

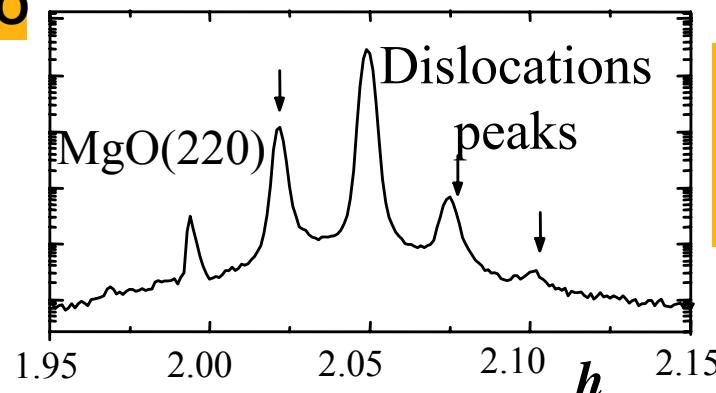
- Significant surface strain if :  $H < \Delta$

# Ag/MgO(001) ultra-thin film: *in situ* GIXS, XRR and GISAXS

## 1. Growth of 100 nm Ag(001)/MgO

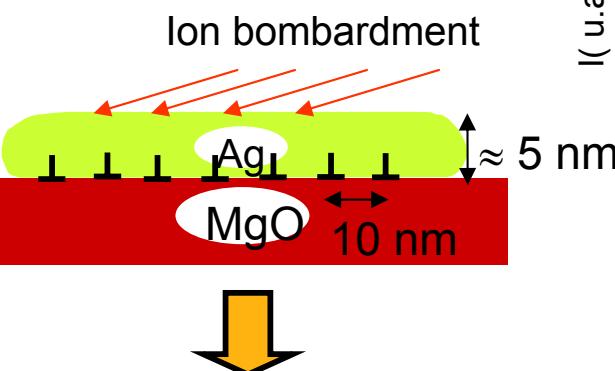


## 2. Annealing 900 K

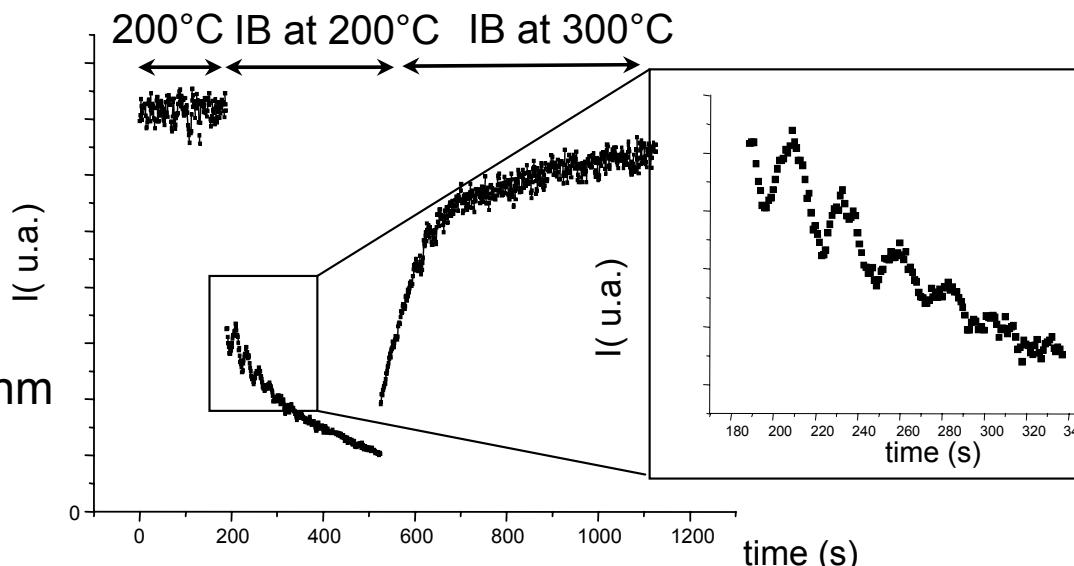


- Ordered dislocation network
- Very low roughness
- Large terraces (100nm)

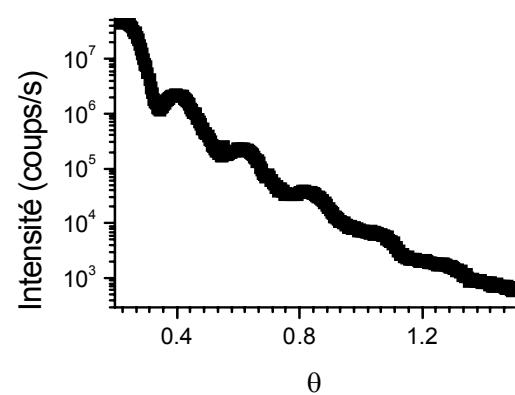
## 3. Ion Beam thinning down to 5 nm while keeping large terraces and low roughness.



## Monitored *in situ* by Anti-Bragg GIXS



and reflectivity

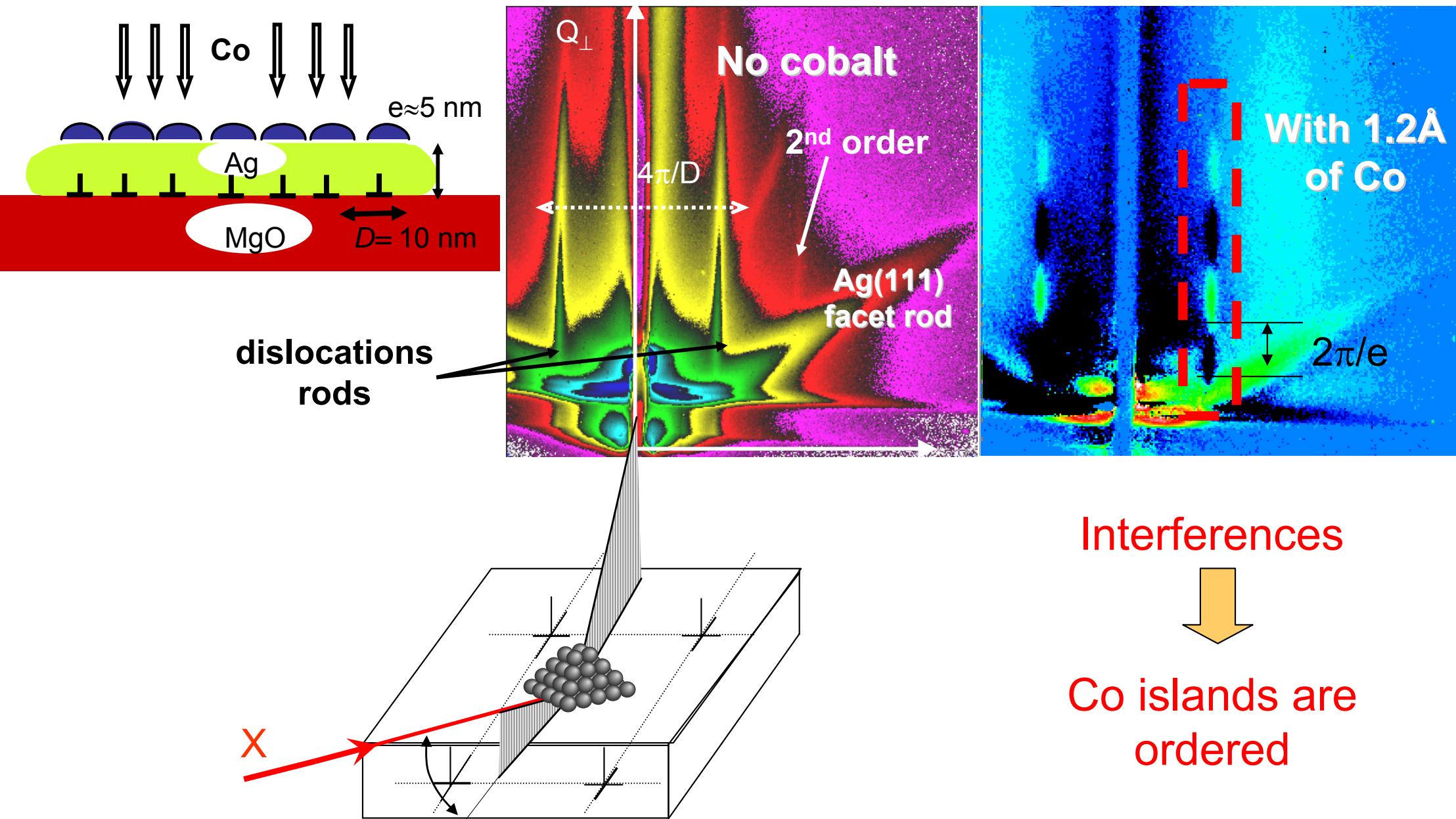


Ultra-thin (5nm) Ag film of homogeneous thickness, with an ordered array of dislocation

## 4. Co Deposition

- Room temperature : trap energy >> thermal energy
- Deposition rate ( $0.05 \text{ \AA/min}$ ): diffusion length of Co adatoms >>  $10 \text{ nm}$

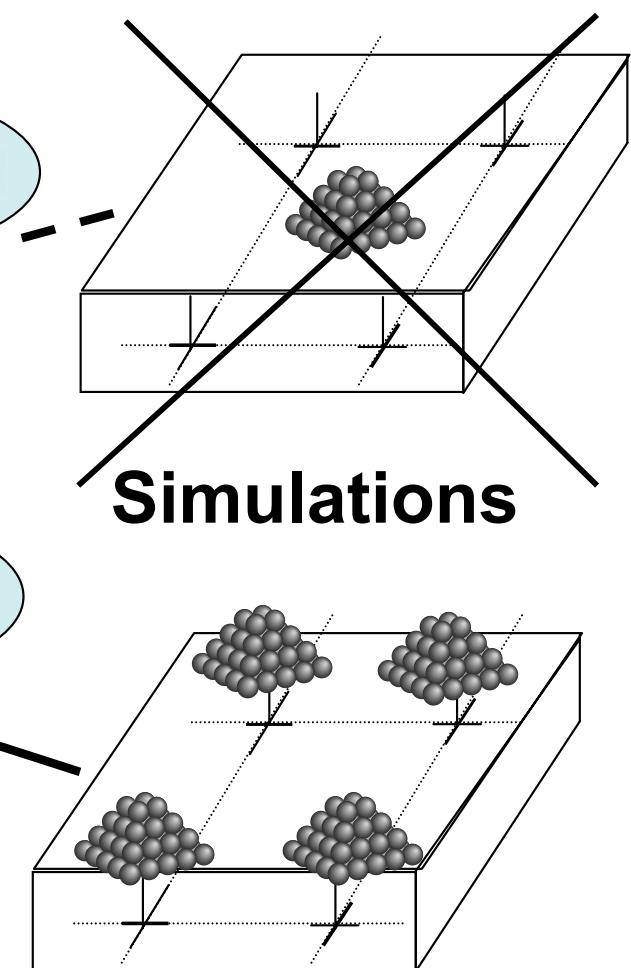
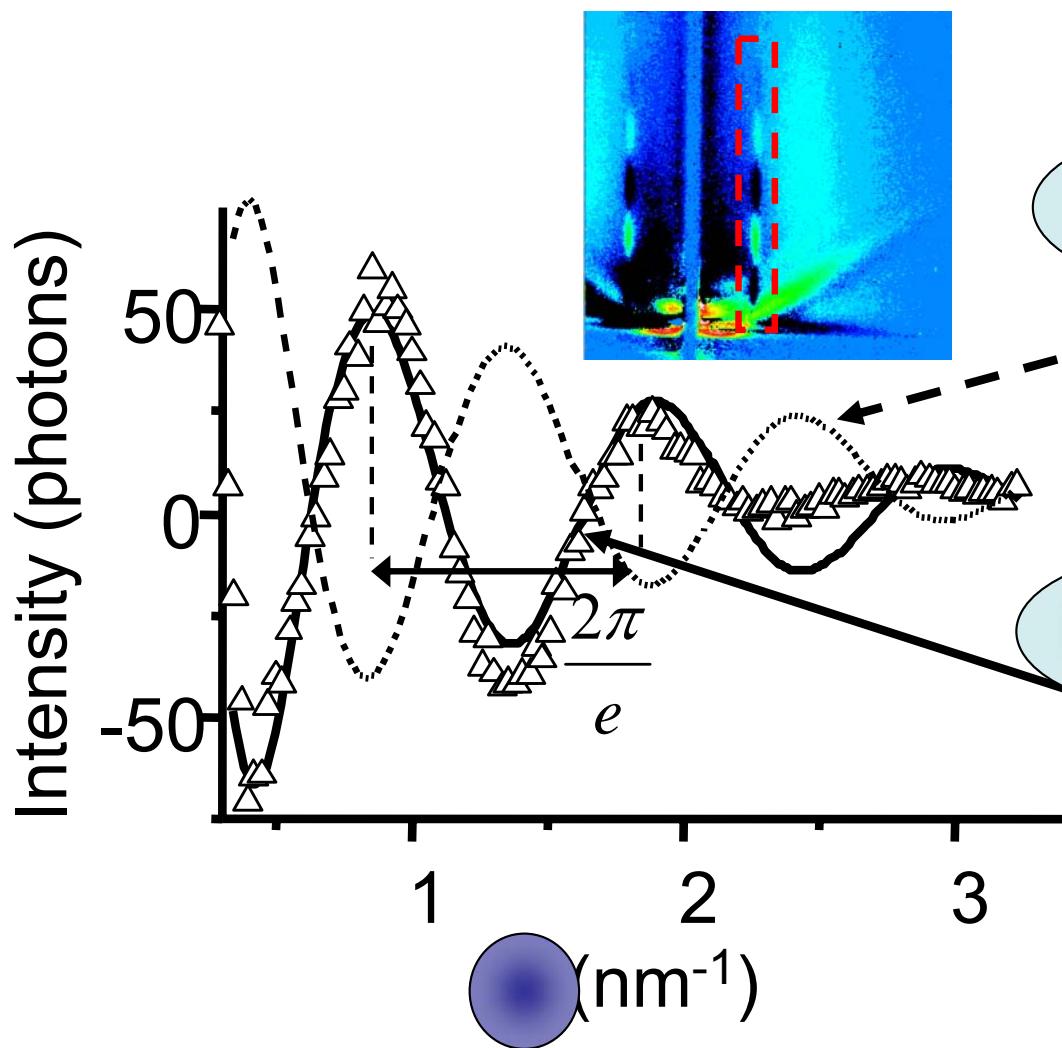
# Self-organized growth of magnetic cobalt dots on an interfacial dislocation network : Co/Ag/MgO(100)



# Position of Co islands / dislocation cores

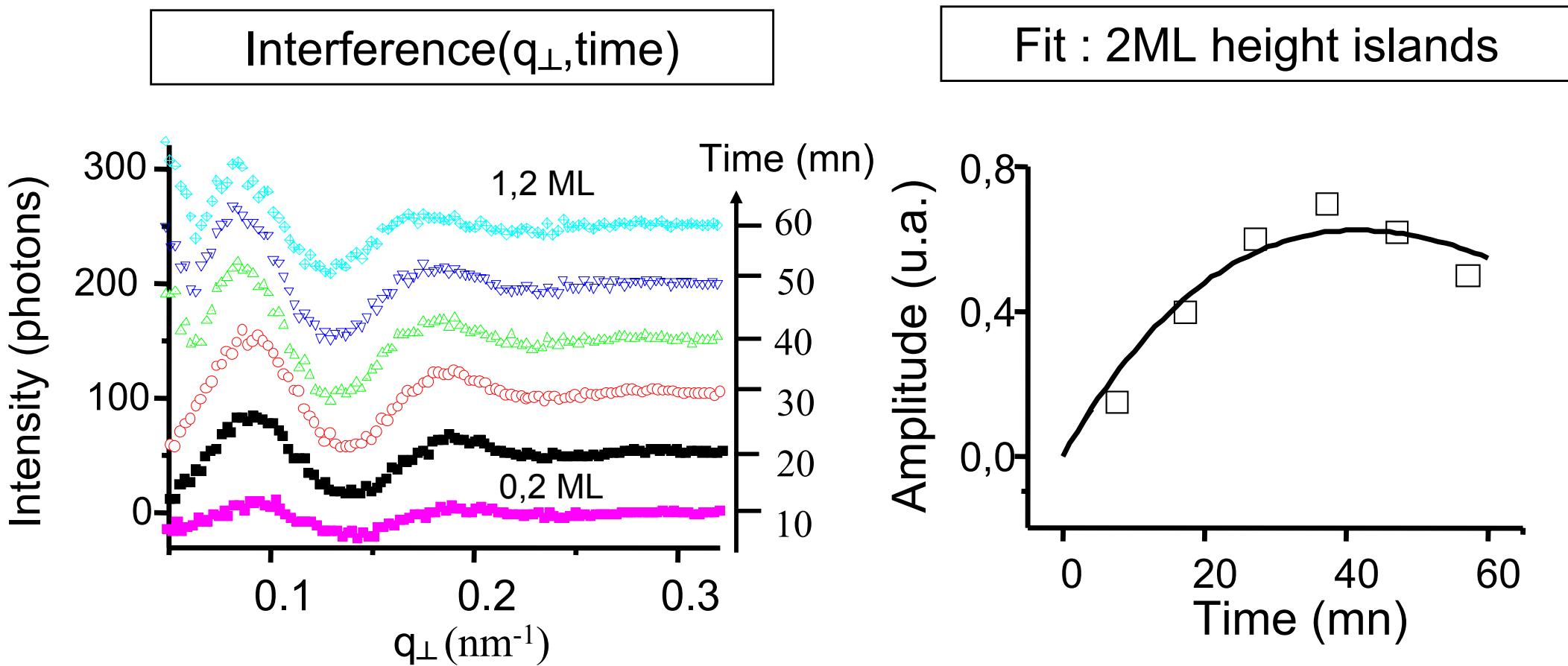
$$I(\vec{q}) - I_{disloc}(\vec{q}) = \cancel{I_{Co}(\vec{q})} + 2\sqrt{I_{disloc}(\vec{q}) \times I_{Co}(\vec{q})} \times \cos(\text{interferences})$$

After - before      ~~negligible~~



# Size and shape evolution of Co dots upon deposition time

$$I_{\text{interference}}(\vec{q}, \text{time}) = 2\sqrt{I_{\text{disloc}}(\vec{q})} \times \text{red oval} \times \cos(\vec{q}_{//} \cdot \vec{d}_{//} + q_{\perp} H)$$



# Conclusions

- GISAXS for the first time *in situ* during growth
- Combined with GIXS → Atomic structure + Morphology
- Quantitative information on nano-particles shape/size/ordering
- Very sensitive to the ordering of nanostructures

- *In situ* surface X-ray diffraction and GISAXS combined for determining conditions for ordering of Co islands on a Ag/MgO dislocation network

Determination of the nucleation site, size and shape of islands during organized growth of :

- Co on Au(111)
- Co on kinked Au(677) : in between kinks and at the step edges
- Co on Ag/MgO(001) : upon the dislocation core

# Potential and future directions

- **GISAXS extremely sensitive to the very premisses of organization**

- used to monitor organized growth in real-time and quickly reach the right thermodynamical and kinetic conditions for the organization.

- ***In situ* studies during**

- surface reactivity (e.g. catalytic reactions, annealings ...)
- growth (during MBE, (MO)CVD, LPE );
- use of gaseous, liquid or solid surfactants, at High p, T ...

- Eventually probing the shape & 2D organization of biological molecules deposited on surfaces?

- Conformation and function of selected bio-molecules?

## La section efficace de diffusion en GISAXS et l'approximation de l'onde distordue DWBA

- « petits angles »
  - pas d'effets de polarisation
  - **diffusion par des écarts  $\delta p$  à la densité électronique moyenne** = rugosité ou des variations de contraste électronique
- formulation cinématique de la diffusion (expression volumique) 
$$\frac{d\sigma}{d\Omega} \propto \left| \int \delta\rho(\vec{r}) \exp(iq \cdot \vec{r}) \right|^2$$
- $\alpha_i$  et  $\alpha_f$  proches de l'angle critique de réflexion totale externe = **effet de réfraction du faisceau ou pic de Yonéda**
- **DWBA = combinaison du traitement dynamique et cinématique de la diffusion**
  - réflexion-réfraction aux interfaces
  - traitement cinématique de la diffusion par  $\delta\rho$ , sans inclure les effets de diffusions multiples
  - approche similaire en réflexion neutronique
- section efficace calculée **au premier ordre en théorie des perturbations** par rapport au système idéal

### Historique

- 1982 : **DWBA et la diffraction en incidence rasante**, G. Vineyard Phys. Rev. B26, 4146 (1982)  
1988 : **Théorie DWBA pour les surfaces rugueuses**, S.K. Sinha et al., PRB 38, 2297 (1988)  
1993 : **DBWA et multicouches rugueuses corrélées**, V. Holy et T. Baumbach, Phys. Rev. B 47, 15896 (1993), 49, 10668 (1994)  
1994 : **DWBA d'ordre 2 en réflectivité**, D.K.G. De Boer, Phys. Rev. B 49, 5817 (1994), 51, 5297 (1995)  
1995 : **DWBA : rugosité et variation de contraste électronique**, M. Rauscher, Phys. Rev. B 52, 16855 (1995)  
1999 : **GISAXS pour des îlots sur une surface**, J. Appl. Phys., 86, 673 (1999)

## Formulation de la DWBA

Point de départ : équation de Helmholtz pour l'onde électromagnétique  $(\nabla^2 + k^2) \psi \rangle = V(r) |\psi \rangle$

Potentiel diffusant  $V(\vec{r}) = k^2 [1 - n(\vec{r})^2] = \bar{V}(\vec{r}) + \delta V(\vec{r})$

Élément de matrice de transition  $\langle f | T | i \rangle = \langle \tilde{\Psi}_f | \bar{V} | \phi_i \rangle + \langle \psi_f | \delta V | \chi \rangle \approx \underbrace{\langle \tilde{\Psi}_f | \bar{V} | \phi_i \rangle}_{\bar{V}_{if}} + \underbrace{\langle \psi_f | \delta V | \psi_i \rangle}_{\delta V_{if}}$

Les ondes

$\phi_i(\vec{r}) = \exp(i\vec{k}_i \cdot \vec{r})$  Onde incidente

$\psi_i(\vec{r}) = T_i \exp(i\vec{k}_i \cdot \vec{r}) + R_i \exp(i\vec{k}'_i \cdot \vec{r})$  Vecteurs propres du système idéal avec renversement temporel

$\tilde{\Psi}_f(\vec{r}) = T_f^* \exp(i\vec{k}_f \cdot \vec{r}) + R_f^* \exp(i\vec{k}'_f \cdot \vec{r})$

$$\frac{d\sigma}{d\Omega} \propto \left\langle |\langle i | T | f \rangle|^2 \right\rangle = \left\langle |\bar{V}_{if} + \delta V_{if}|^2 \right\rangle$$

Règle d'or de Fermi

$$\left( \frac{d\sigma}{d\Omega} \right)_{\text{spec}} \propto |\bar{V}_{if} + \langle \delta V_{if} \rangle|^2, \quad \left( \frac{d\sigma}{d\Omega} \right)_{\text{diff}} \propto \left\langle |\delta V_{if}|^2 \right\rangle - \left\langle \delta V_{if} \right\rangle^2$$

## Quelques exemples de sections efficaces en GISAXS

Développement suivant la géométrie du potentiel de diffusion

### Rugosité de surface



$$\frac{d\sigma}{d\Omega} \propto |T_i(\alpha_i)|^2 S(\vec{q}) |T_f(\alpha_f)|^2 \implies \text{Pic de Yonéda}$$

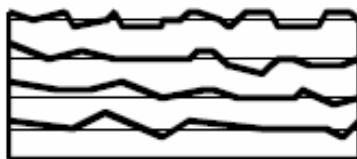
$$S(\vec{q}) = \frac{(\Delta\rho)^2}{|q_z|^2} \exp\left(-[q_x^2 + q_z^2]\sigma^2/2\right) \times \iint_S \left[ \exp\left\{q_x^2 C(\vec{r}_{||})\right\} - 1 \right] \exp(i\vec{q}_{||} \cdot \vec{r}_{||}) d^2 r_{||}$$

$$C(\vec{r}_{||}) = \langle z(\vec{r}_{||}) z(\vec{r}_{||} + \vec{r}_{||}) \rangle$$

Remarques : principe de réciprocité

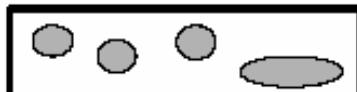
Cas limites :  $\alpha_i, \alpha_f \ll \alpha_c$ ,  $q_z \gg q_x$  et  $|T| \gg 1$  - approximation de Born valide  
 $q_z \ll \sigma$  - TF de la fonction d'autocorrélation de la rugosité

### Multicouches rugueuses corrélées



Traitement analogue mais plus complexe !  
Point de départ = optique des multicouches  
Complexité = corrélation hauteur-hauteur inter-couches

### Inclusions sous la surface



$$\frac{d\sigma}{d\Omega} \propto |T_i(\alpha_i)|^2 S(q_{||}, q_z) |T_f(\alpha_f)|^2$$

$$S(\vec{q}) = \left| \int_V \exp(i\vec{q} \cdot \vec{r}) d^3 r \right|^2$$

Transformée de Fourier de la forme de l'objet diffusant

### Îlots sur une surface



Cas approfondi par la suite !

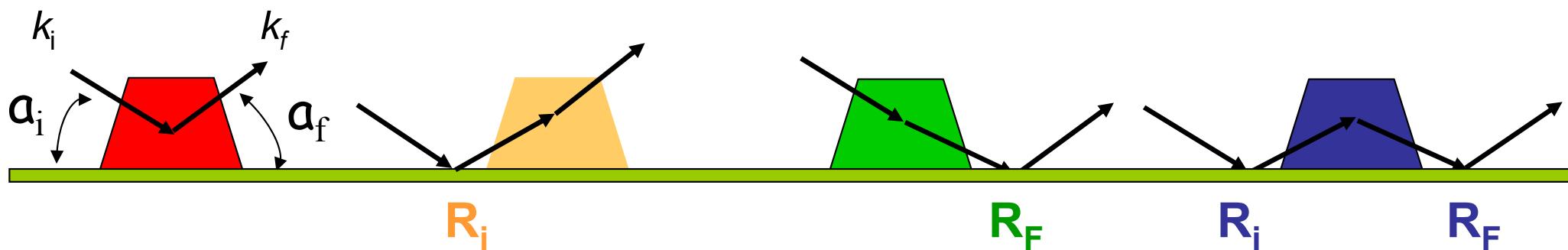
Programme  
d'analyse  
**IsGISAXS**

# Distorted Wave Born Approximation (DWBA) for supported islands

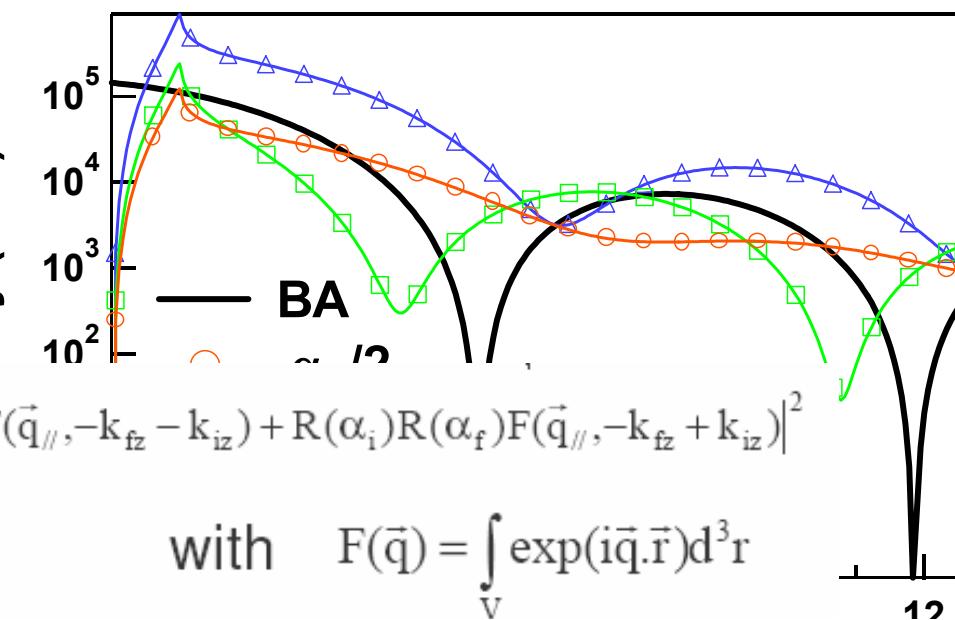
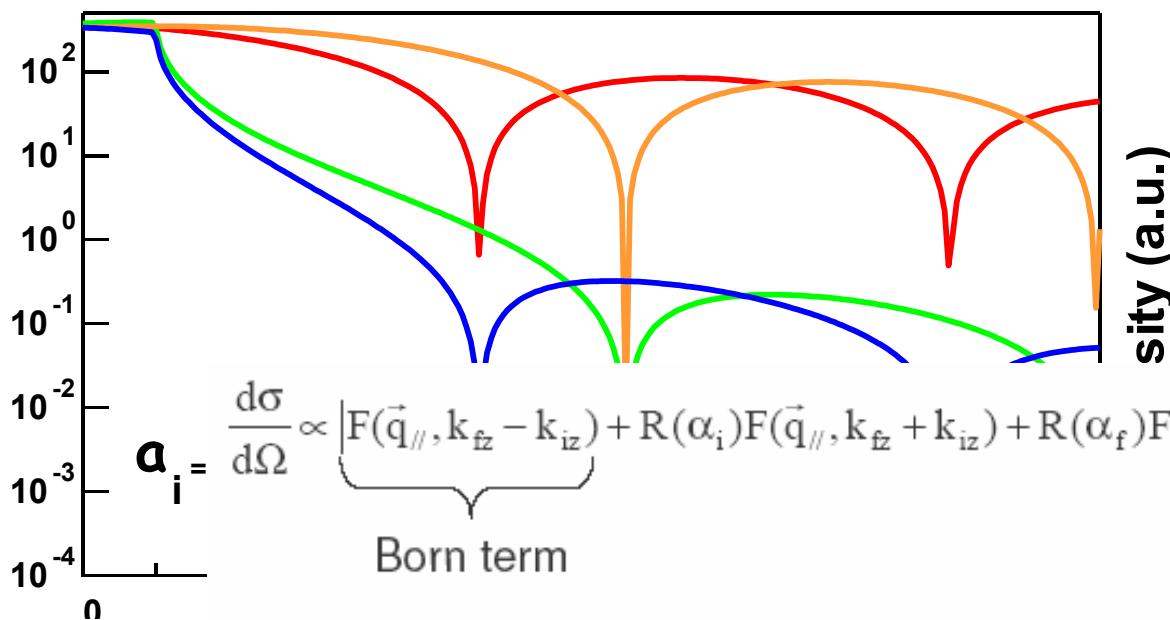
M. Rauscher, T. Salditt et H. Spohn, Phys. Rev. B 52, 16855 (1995)

M. Rauscher et al. J. Appl. Phys. 86 (12), 6763 (1999)

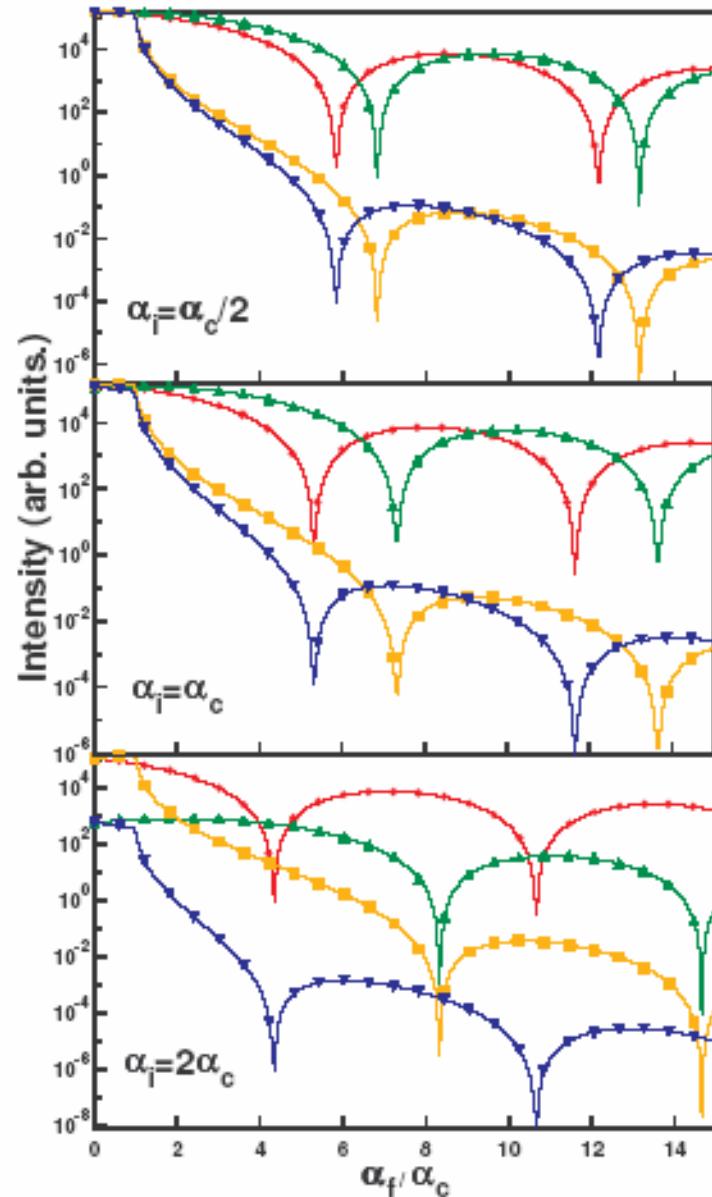
**1<sup>st</sup> term :  $q_z = k_{fz} - k_{iz}$**     **2<sup>nd</sup> term :  $q_z = k_{fz} + k_{iz}$**     **3<sup>rd</sup> term :  $q_z = -k_{fz} - k_{iz}$**     **4<sup>th</sup> term :  $q_z = -k_{fz} + k_{iz}$**



Coherent interferences between 4 waves with different  $q_z$  !



Modulus square of the each term

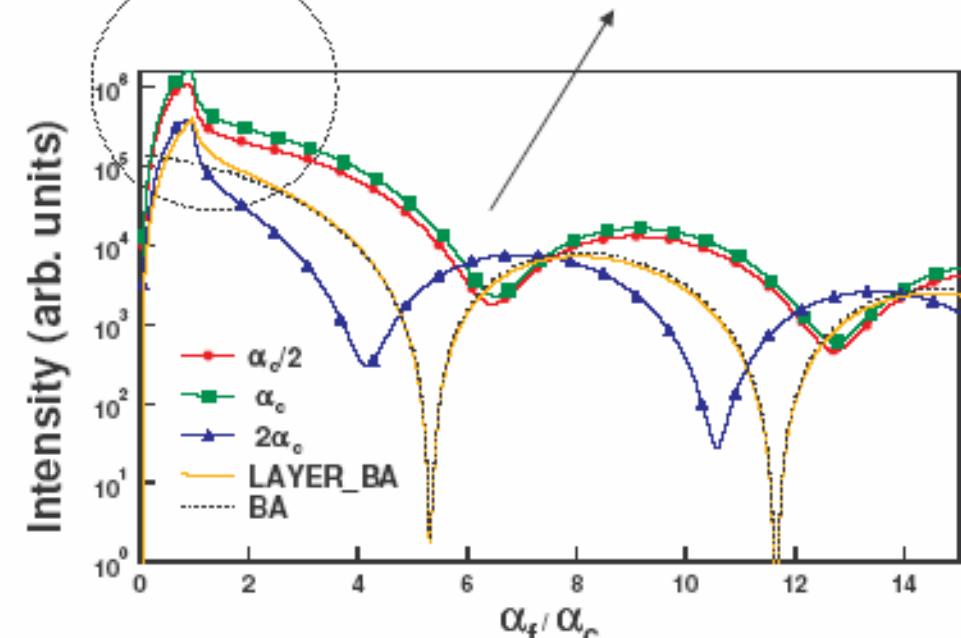


## DWBA form factor of a cylinder

+ Phases

« Yoneda peak »

Minima depend on  $\alpha_i$



Scattered intensity along  $\alpha_i$

# Diffuse Scattering due to size distributions, and sizes-distances and sizes-sizes correlations

Two usual extreme approximations neglecting correlations:

**1 – DA (Decoupling Approximation):**

Size and positions completely un-correlated

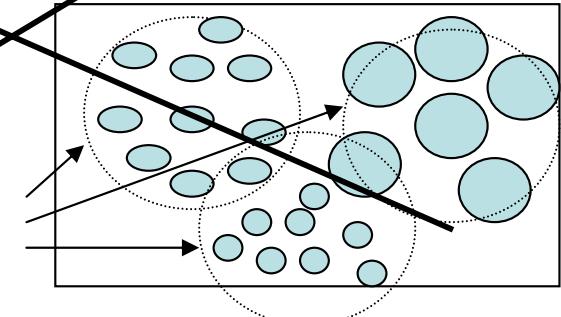
$$I(q_{\parallel}, q_{\perp}) \propto \left\langle |F|^2 \right\rangle - \left\langle |F| \right\rangle^2 + \left\langle |F| \right\rangle^2 \times S(q_{\parallel})$$

*Diffuse scattering*      *Coherent scattering*

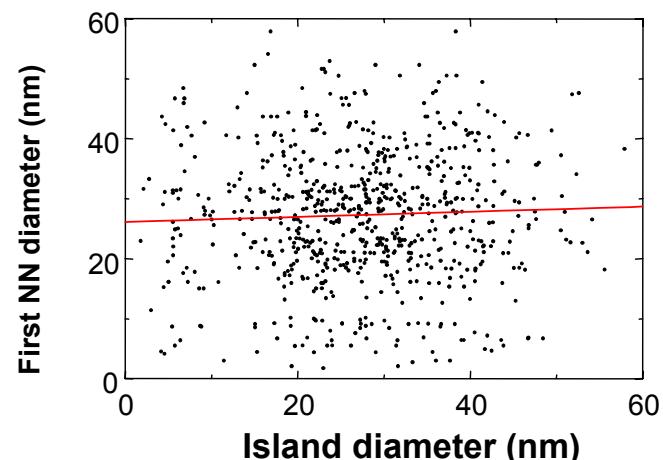
**2 - LMA : Local monodisperse approximation**

$$I(q_{\parallel}, q_{\perp}) \propto \left\langle |F|^2 \right\rangle \times S(q_{\parallel})$$

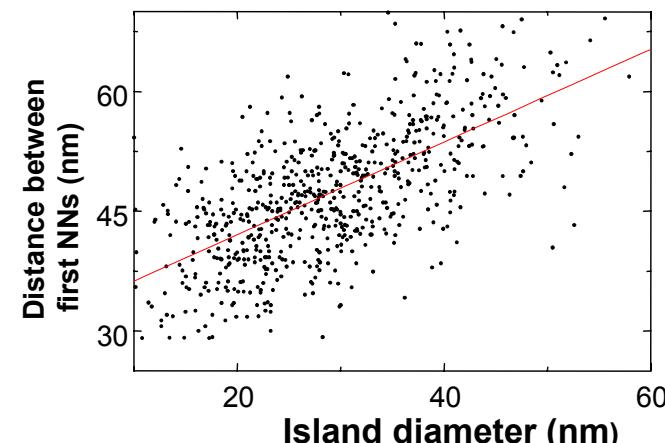
Coherency zones of the X-ray beam



•Correlations deduced from analysis of TEM images Pd/MgO(001)



No sizes-sizes correlation  LMA wrong



Sizes-separations correlations

## In plane scattering : coherent versus incoherent ?

$$\frac{d\sigma}{d\omega} \propto \frac{1}{N} \left\langle \left| \sum_i F_i(\vec{q}_{\parallel}, k_{fx}, k_{iz}) \exp(i\vec{q}_{\parallel} \cdot \vec{r}_{i\parallel}) \right|^2 \right\rangle$$

Configuration average over the coherently illuminated areas

Knowledge of all the partial pair correlation functions ?

2 simple limit cases

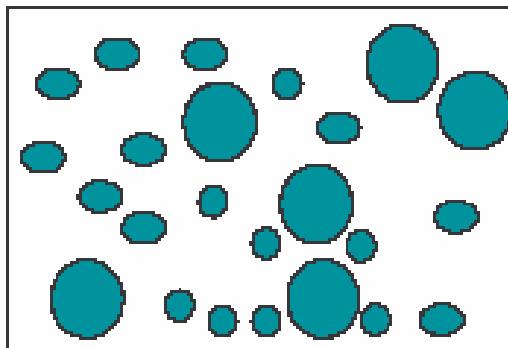
**Decoupling approximation DA**

$$\frac{d\sigma}{d\Omega} \propto \underbrace{\left\langle |F|^2 \right\rangle - |\langle F \rangle|^2}_{\text{Incoherent scattering}} + \underbrace{|\langle F \rangle|^2 \times S(q_{\parallel})}_{\text{Coherent scattering}}$$

*Incoherent scattering*

*Coherent scattering*

*Total disorder – no correlations*

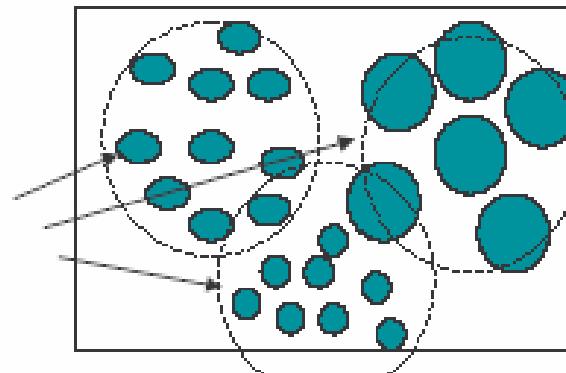


**Local Monodisperse Approximation LMA**

$$\frac{d\sigma}{d\Omega} \propto \left\langle |F|^2 \right\rangle \times S(q_{\parallel})$$

*Incoherent sum of scattering from Monodisperse domains*

X-ray  
coherent  
area



# Result from scattering theory

$$\left( \frac{d\sigma}{d\omega} \right)_{\text{diff}} = \text{const.} \sum_p \left| A_p F(\mathbf{Q}_p) \right|^2 P(\mathbf{Q}_p)$$

form factor  
of a single object

correlation function  
of the positions

over the processes

amplitude  
of the  $p$ -th process

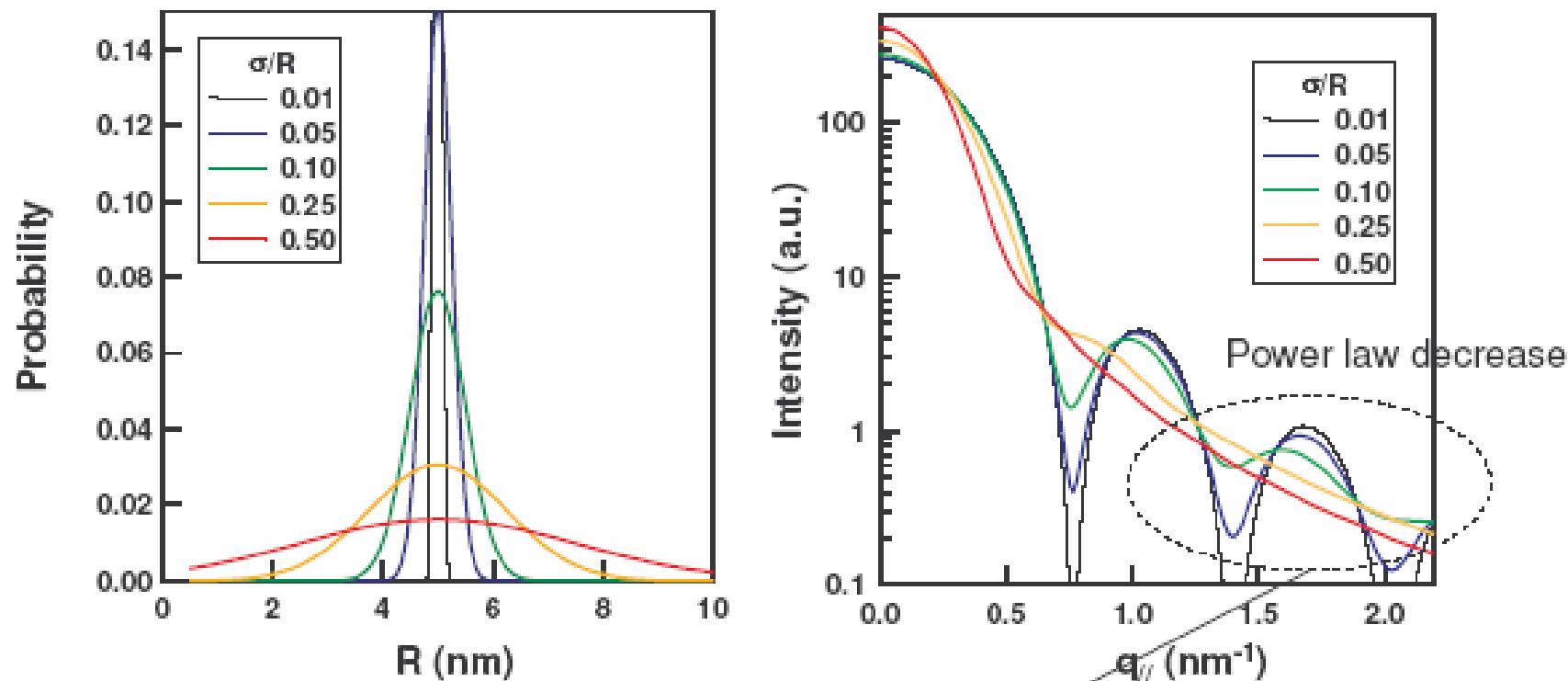
$P(\mathbf{Q}_p) = \text{FT of pair correlation in real space}$

Short range order

Long range order

## Lateral size distribution and zeros of the form factor

Cylindrical particles and gaussian distributions



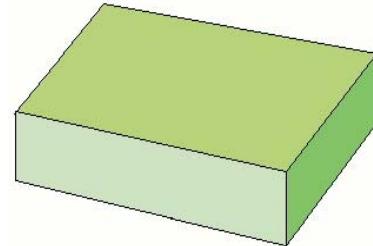
*In IsGISAXS : distributions of lateral sizes, of heights, of orientations,  
of incidence angles, of wavelengths*

Porod regime : a signature of the shape !

S from  
s with facets

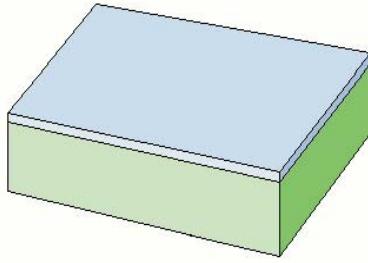
## Super template for 2D organization : nano-faceted surface

W(111), Mo(111)

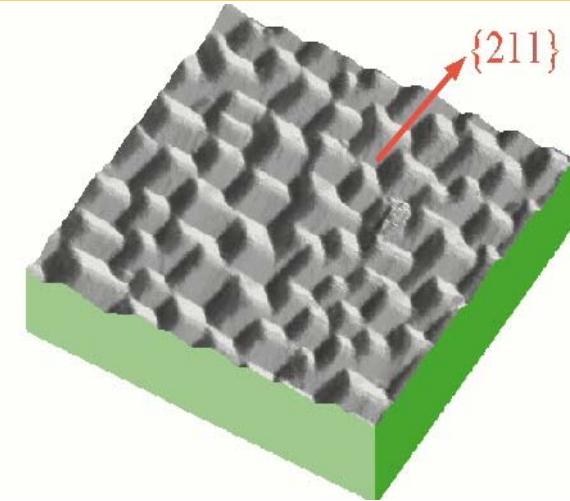


deposit  
→

Pt, Pd, Au, Rh, Ir



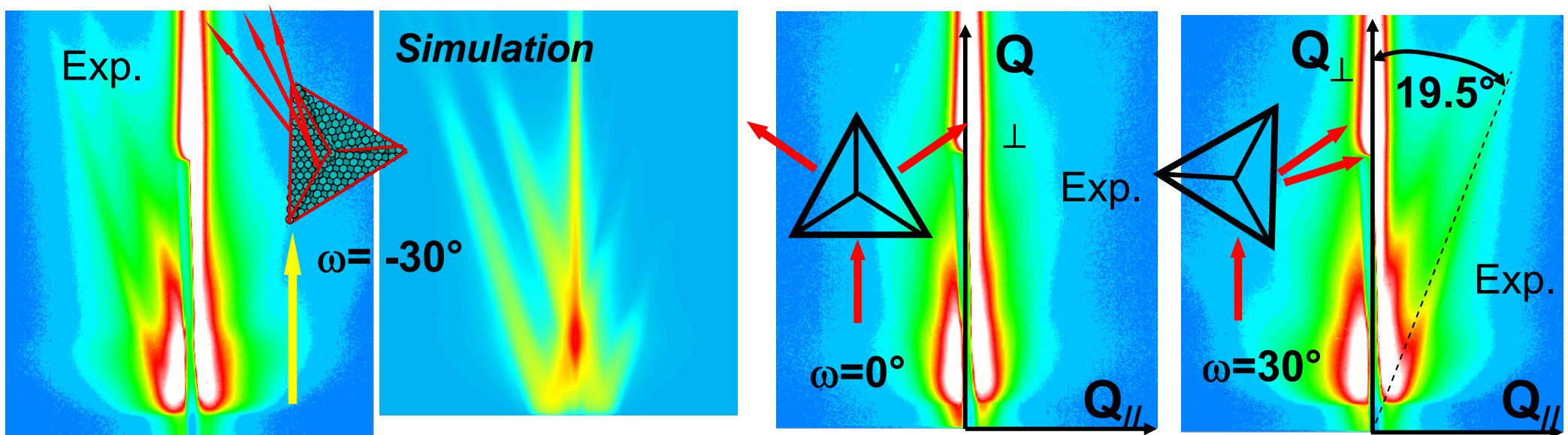
Anneal to  
 $T > 750\text{K}$



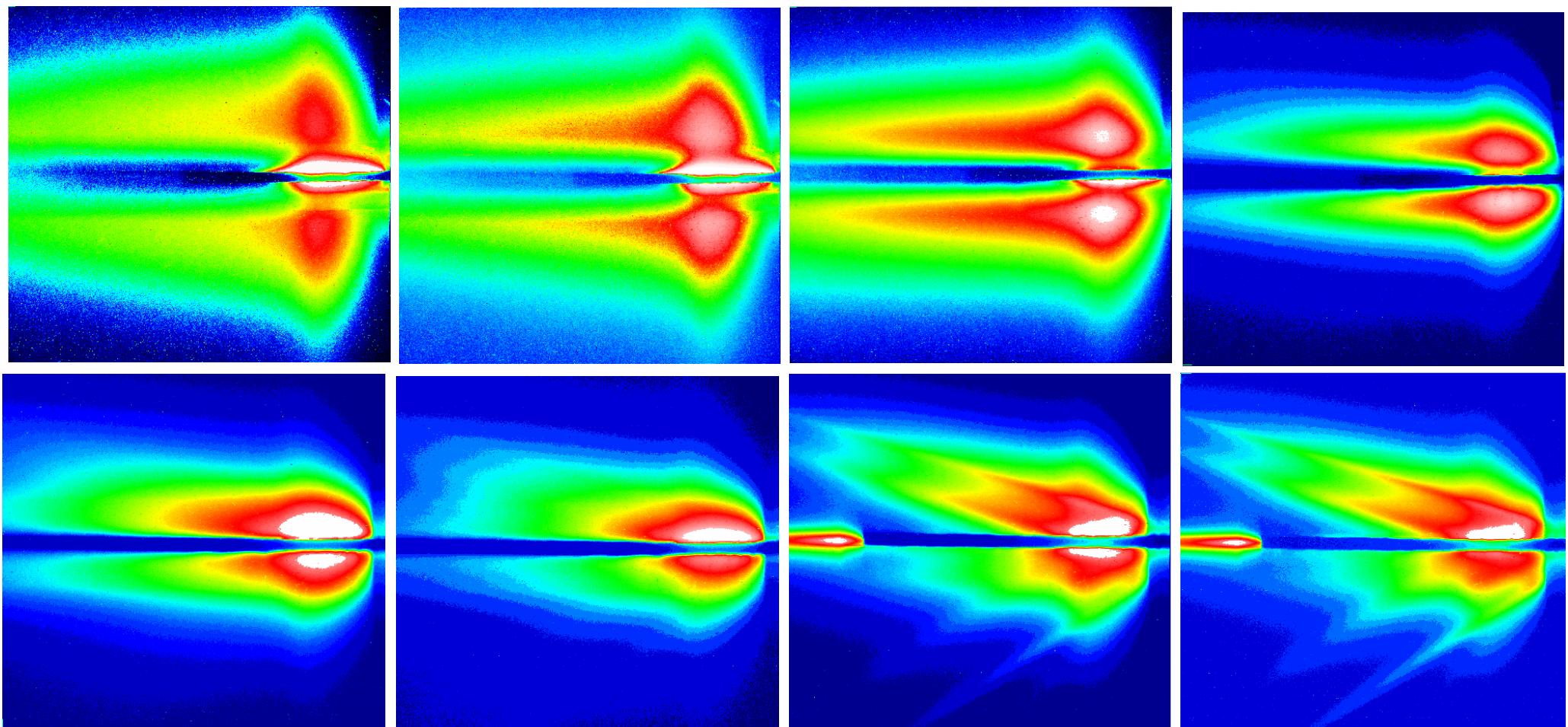
Late stages of facetting:  
Large nano-pyramids

$$I(\mathbf{Q}) = |F_{DWBA}(\mathbf{Q}, \mathbf{k}_i^z, \mathbf{k}_f^z)|^2 S(\mathbf{Q})$$

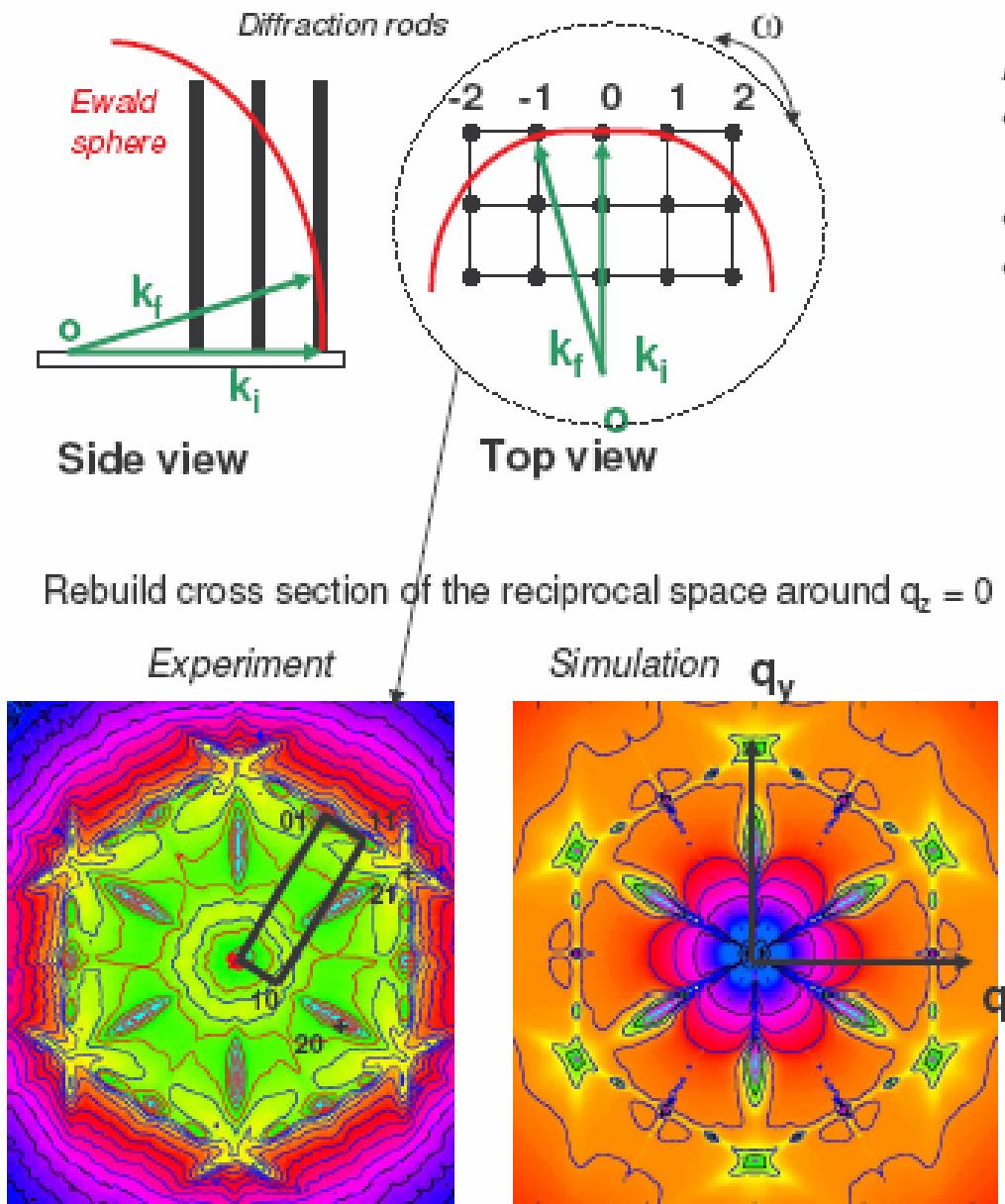
Shape:  $F_{DWBA}$



## Evolution of facetting as a function of annealing time

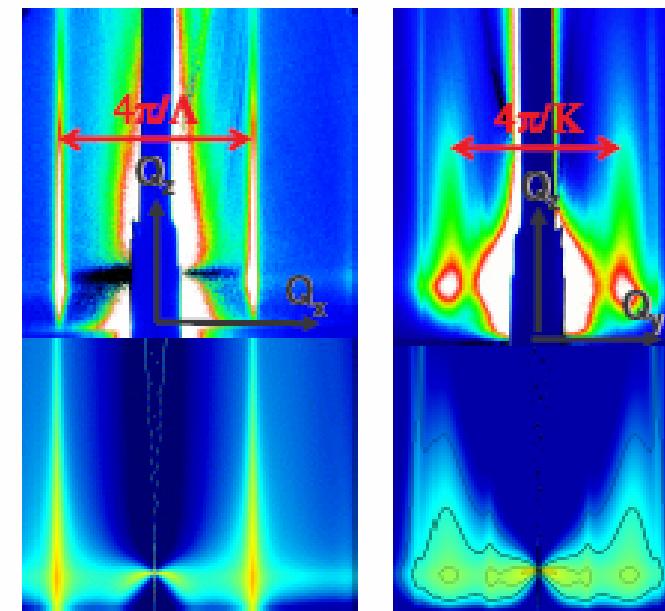
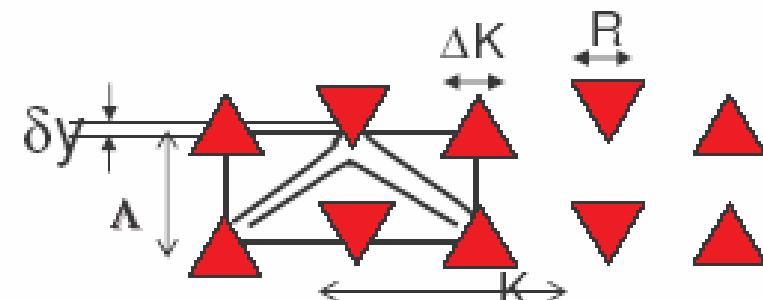


## « Super-cristallography » of the Co cluster lattice on Au(111) before coalescence

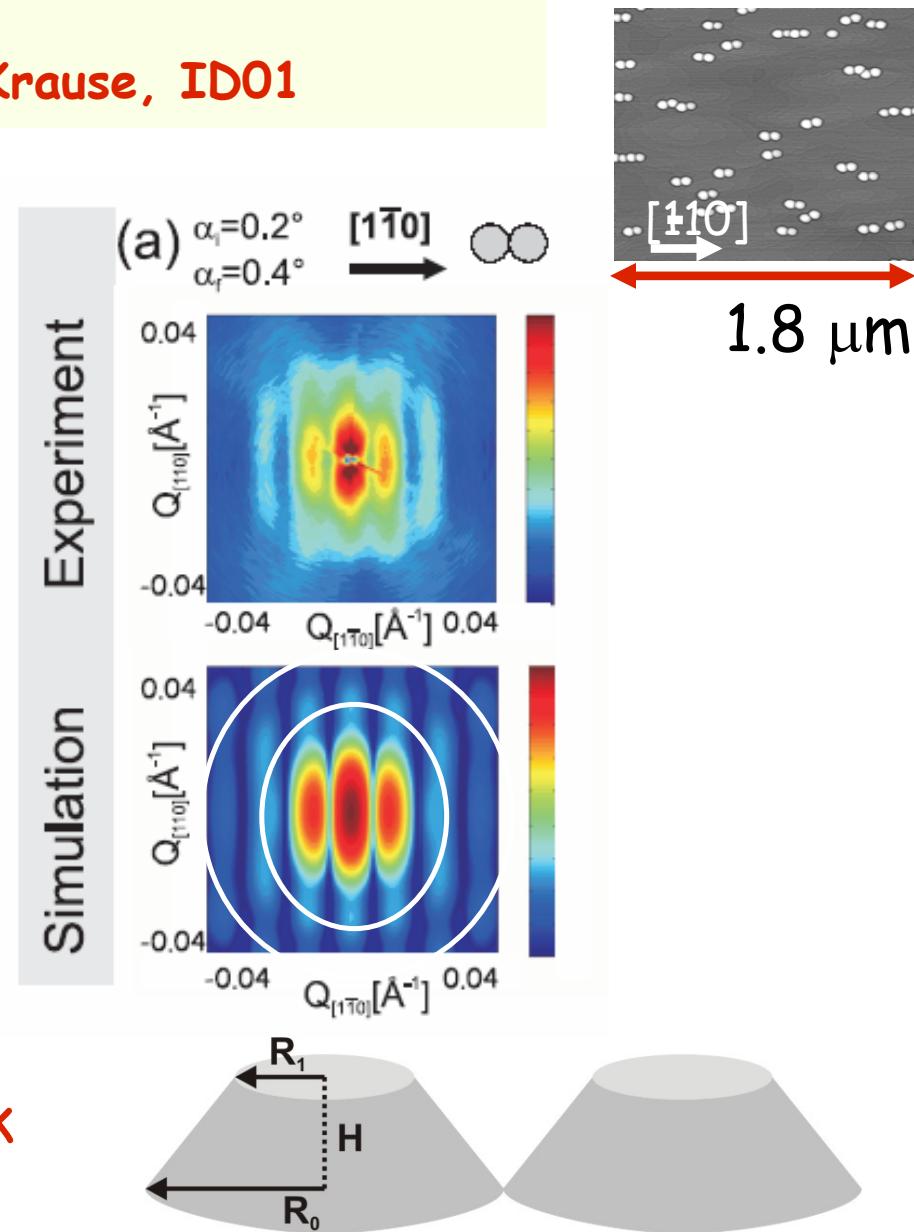
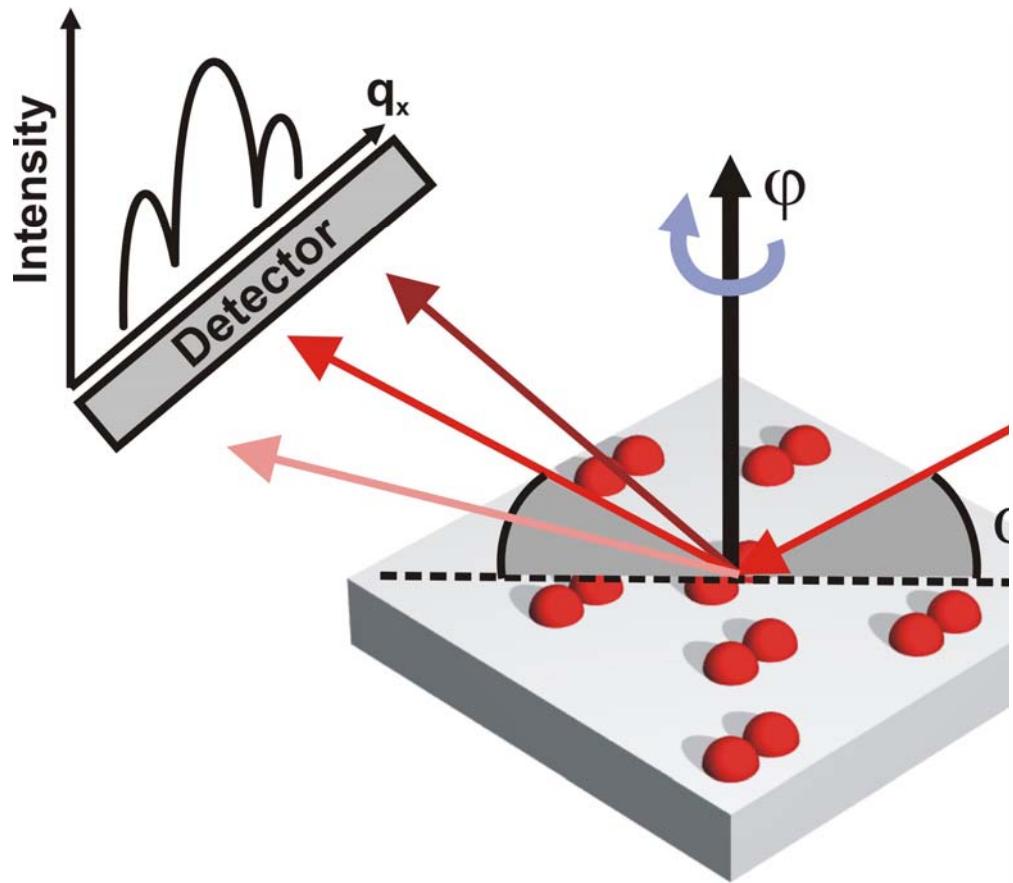


### Analysis parameters

- rectangular 2D paracrystal ( $\Delta K$ ) with three variants oriented at  $120^\circ$
- Triangular islands ( $R, H$  and size distribution)
- Centering of the mesh  $\delta y$



# GISAXS on Bimolecules B. Krause, ID01



Model: Formfactor of double-disk

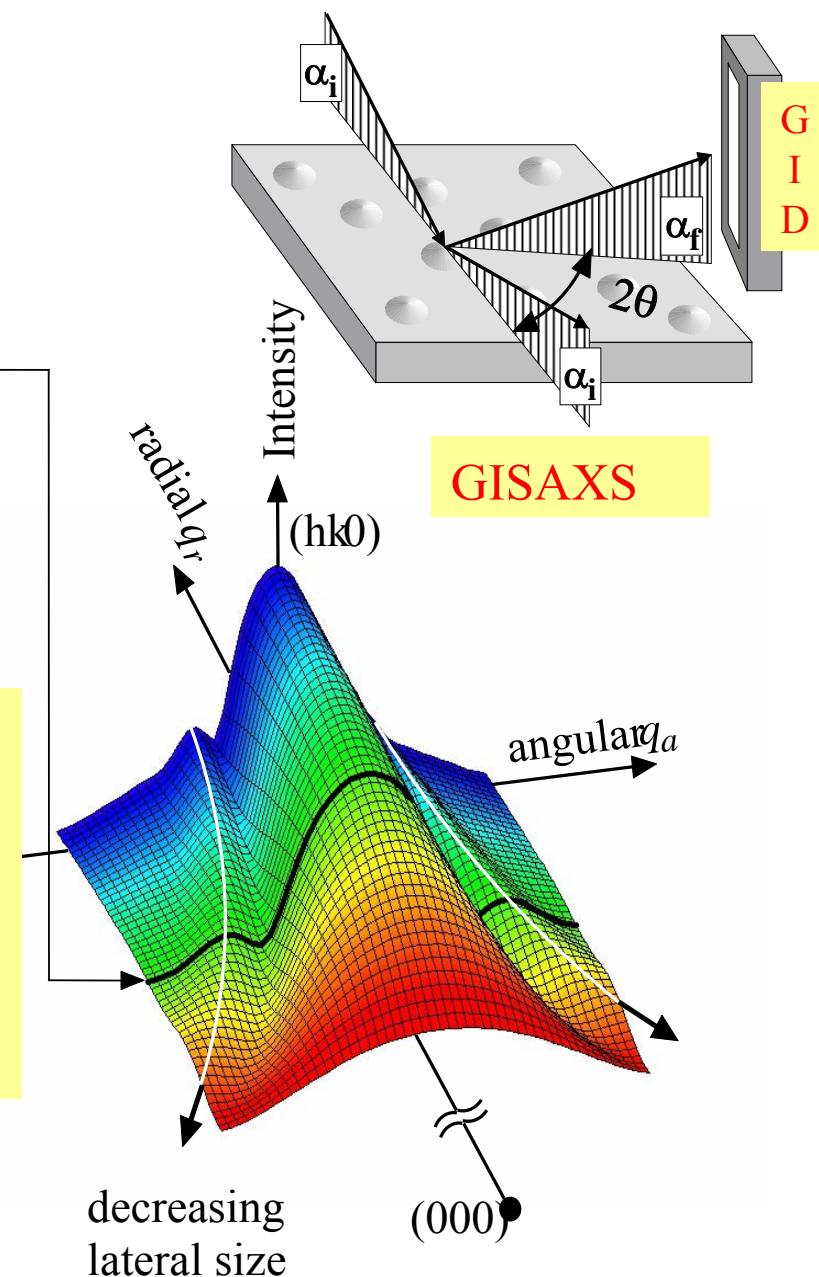
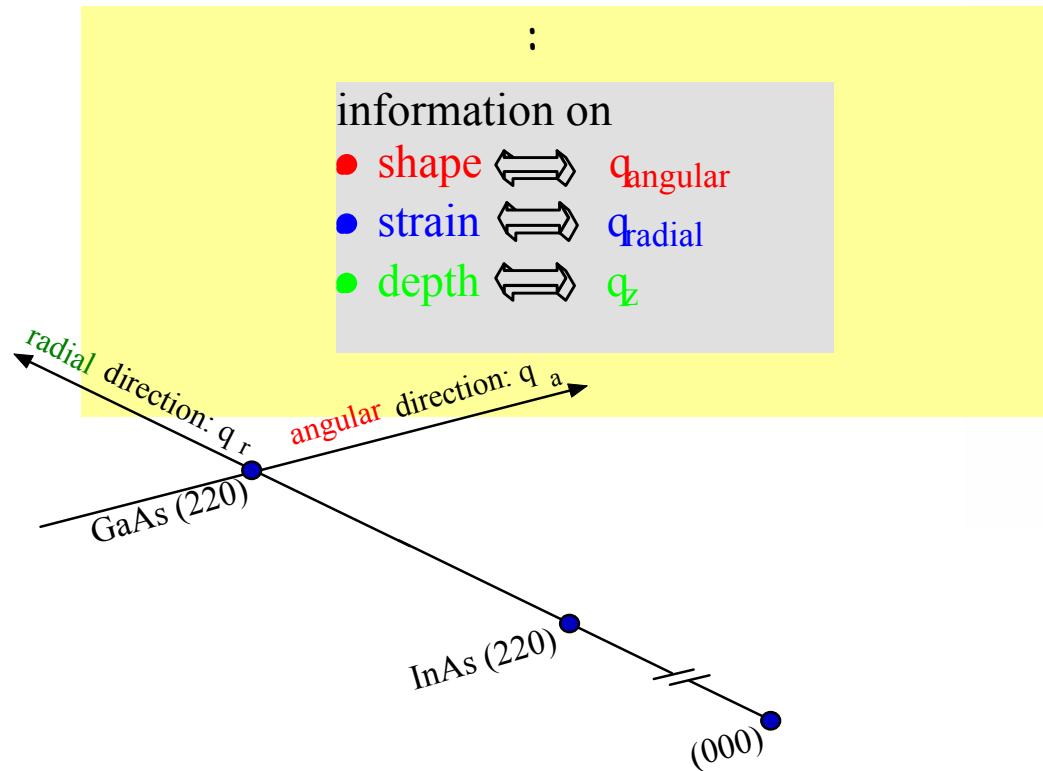
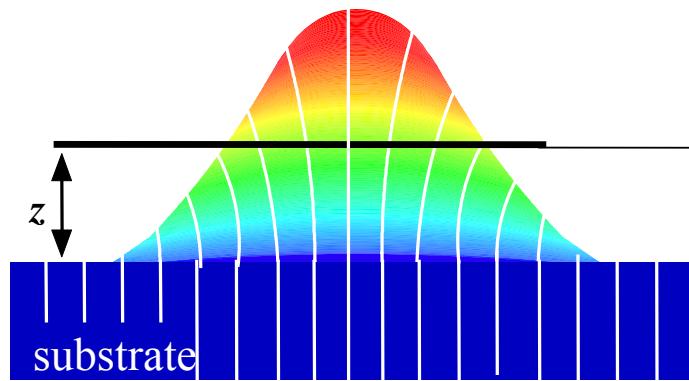
$$R_0 = 270 \text{ \AA}$$

$$H = 170 \text{ \AA}$$

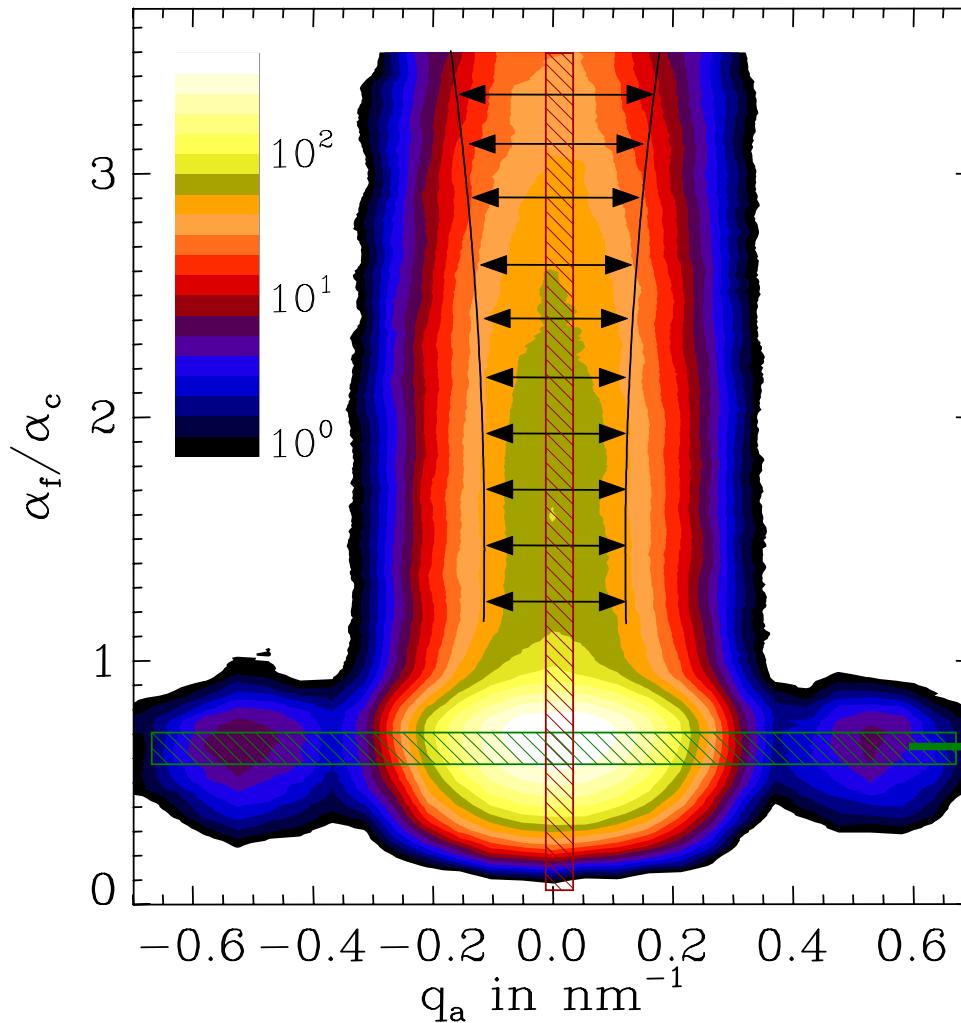
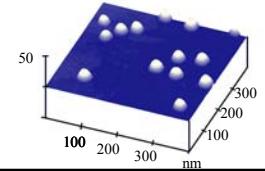
$$R_1 = 130 \text{ \AA}$$

10 % size distribution

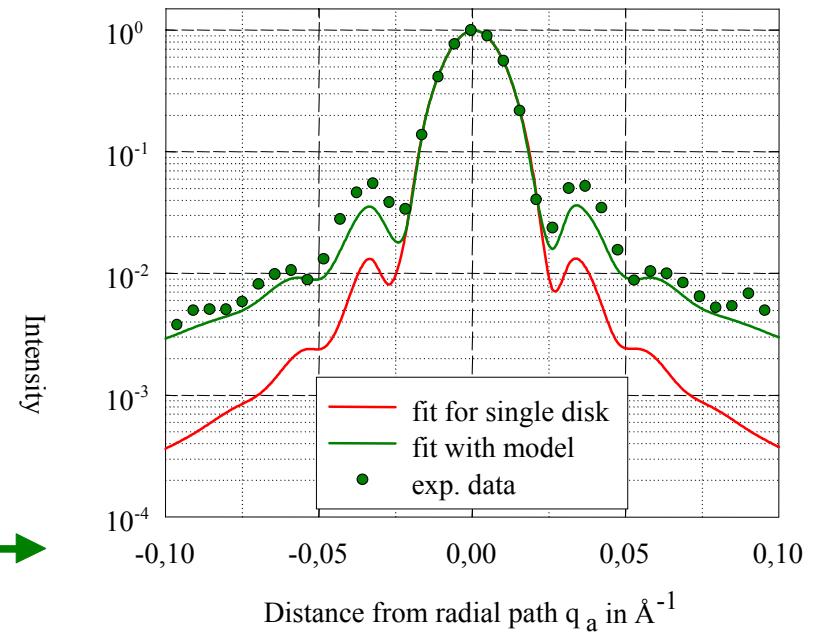
## Scans in 3D reciprocal space



# Results: data analysis ISS



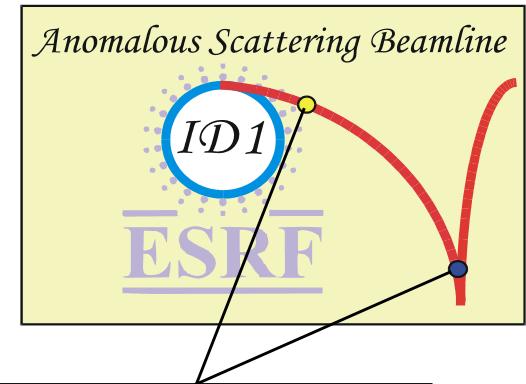
■ lateral size ■ height



Linear relation between strain and size  
is found from fits at all radial positions

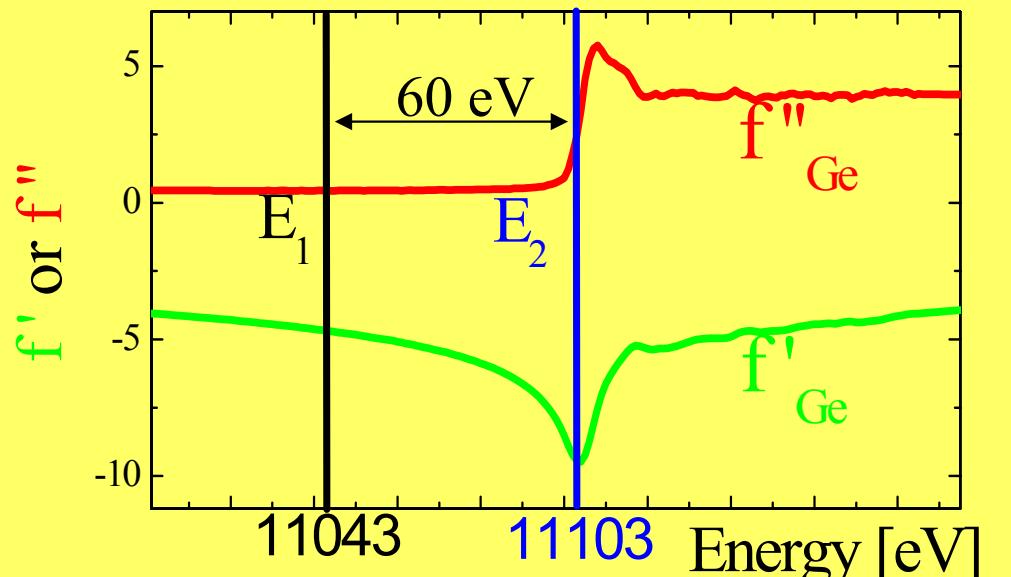
# Example III: composition of Ge domes on Si

GID with contrast variation  
using anomalous ISS



$$f(Q, E) = f_o(Q) + f'(E) + i f''(E)$$

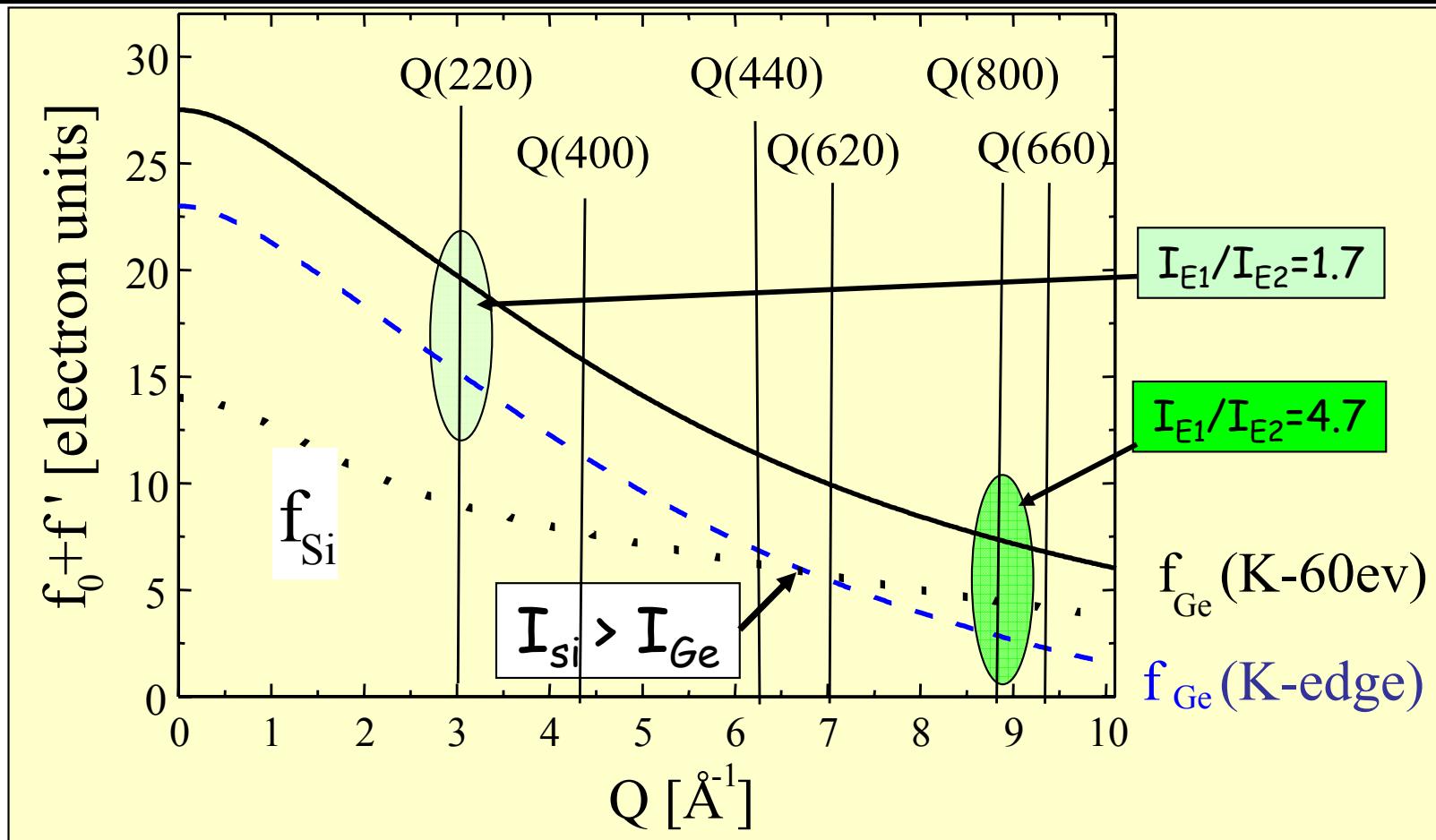
Energy dependence



For Si negligible

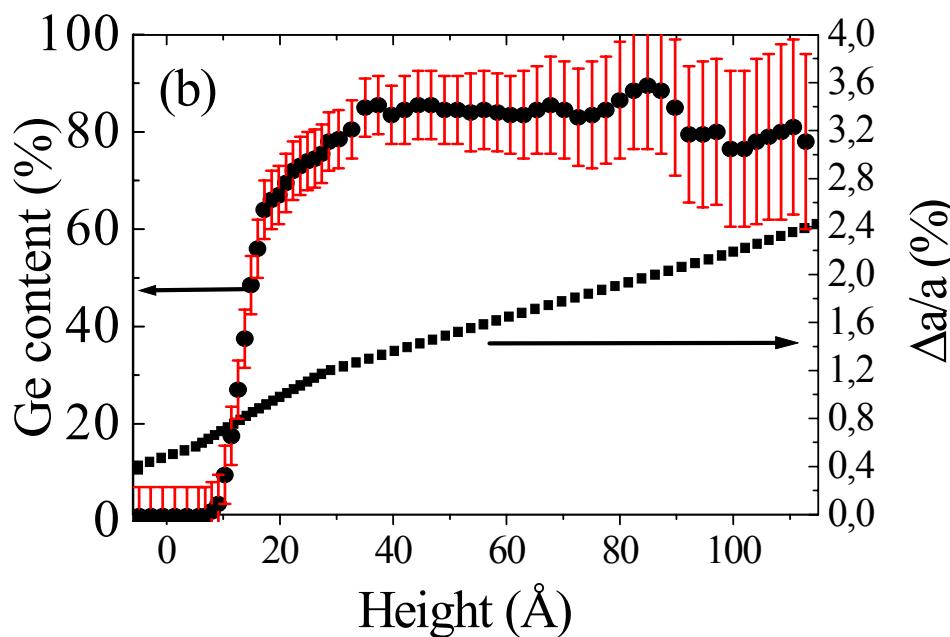
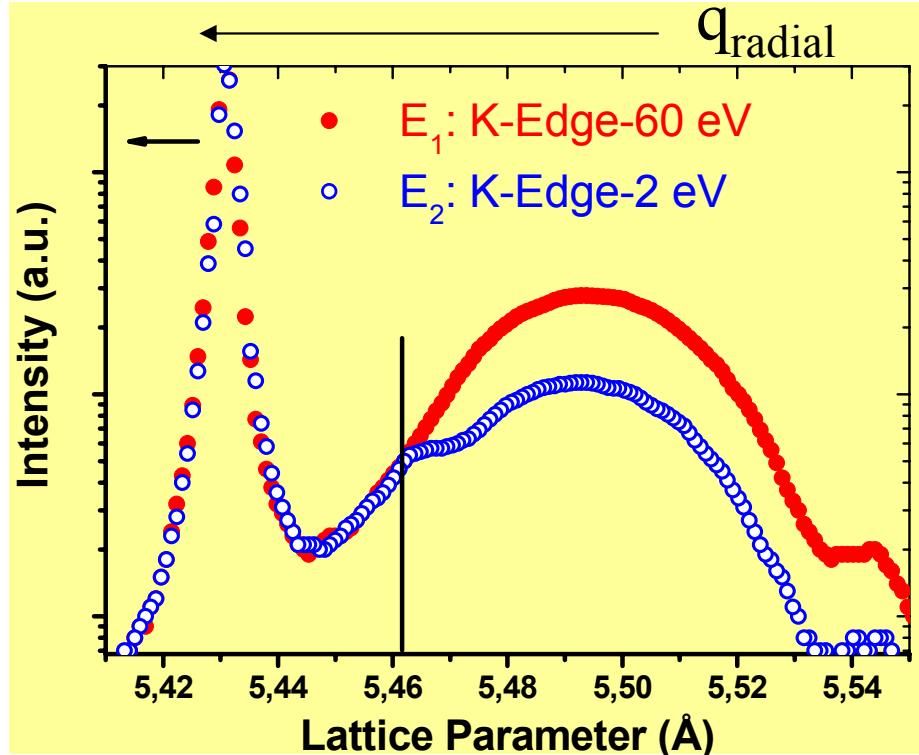
Use energy and Q dependence  
to amplify contrast:  
measure at high Q !

# Contrast variation: Q dependence



- anomalous correction enhanced at high Q
- higher resolution for  $\Delta a/a = -q_r/Q$
- Si content from deviation of  $I_{E1}/I_{E2}$  for pure Ge
- All possible GID reflections should be measured

# Si interdiffusion : vertical composition profile from (A)-GID at (800)



7 ML MBE growth at 600°C

Ge domes

Small size dispersion

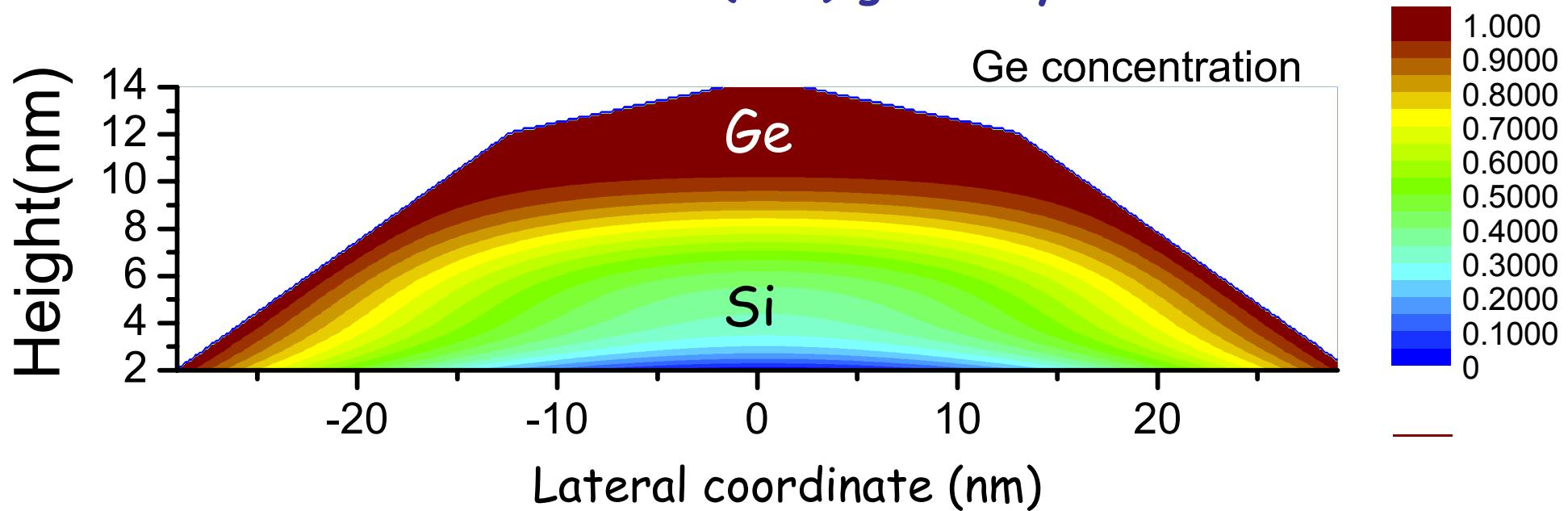
**Direct method!**

## RESULTS

- Sharp Si/Ge interface
- Ge plateau at 85%
- $\Delta a/a$  monotonic
- dot is highly strained

# "Nano -Tomography": 3D real space image shape, size, strain and composition of Ge/Si alloy islands

11.2 ML Ge domes on Si (001) grown by CVD at 600°C



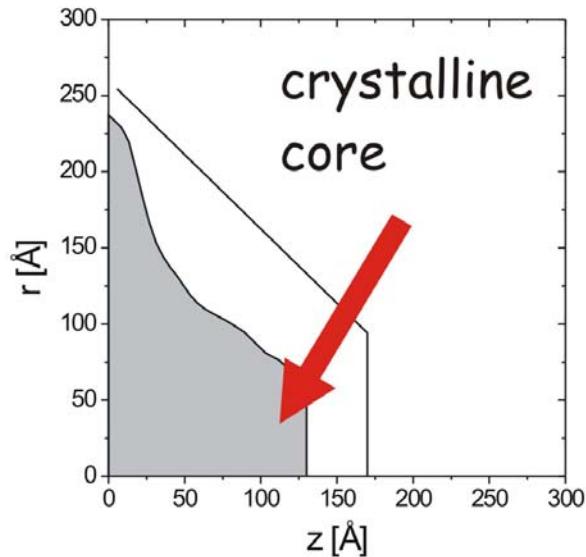
## Results:

the lateral variation of the Ge concentration changes with height  
Si rich core covered by Ge rich alloy  
concentration from pure Si at bottom to pure Ge at top

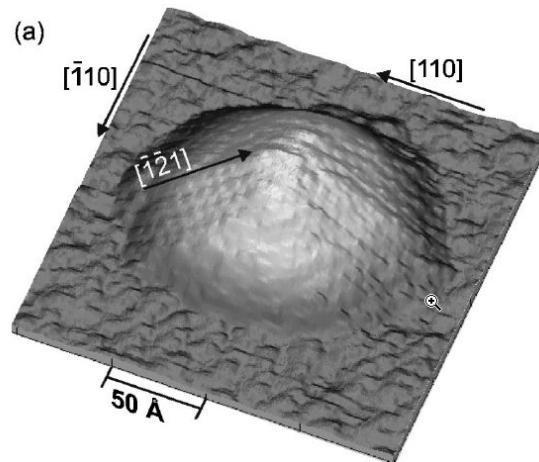
T.Schülli *et al.* PRL90, 66105 (2003)

A. Malachias *et al.* PRL91, 176101 (2003)

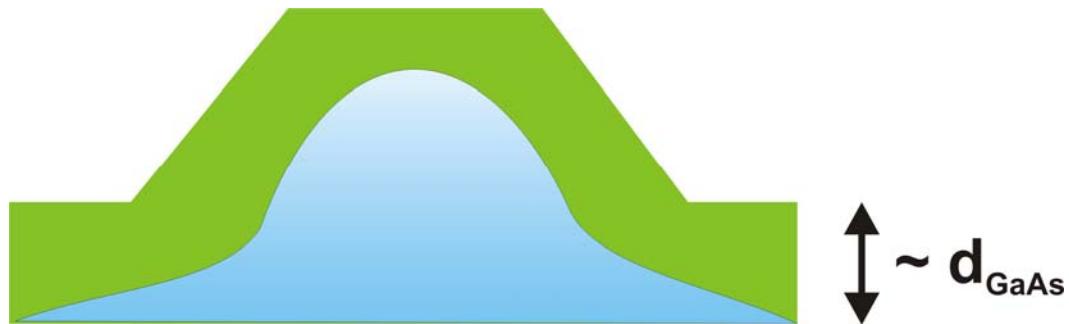
# Oxide layer and crystalline core



amorphous oxide layer  
(2-3 nm)

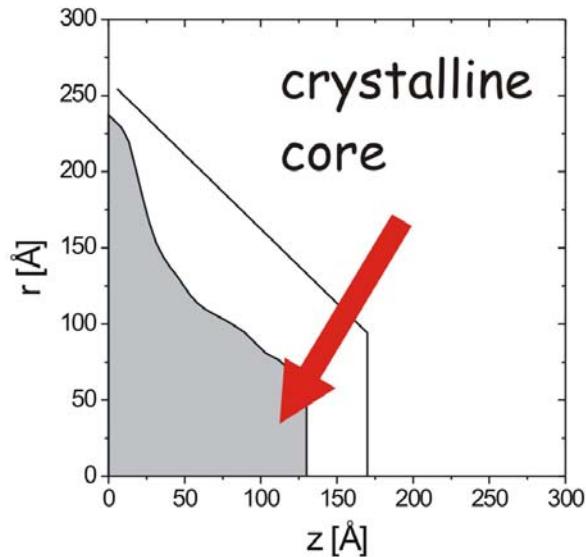


Marquez et al., APL 78 (16), 2310 (2001)

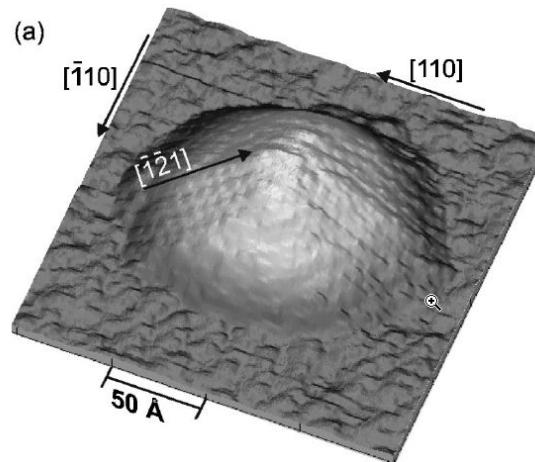


oxidation  $\Rightarrow$  foot of the dot was part of the substrate

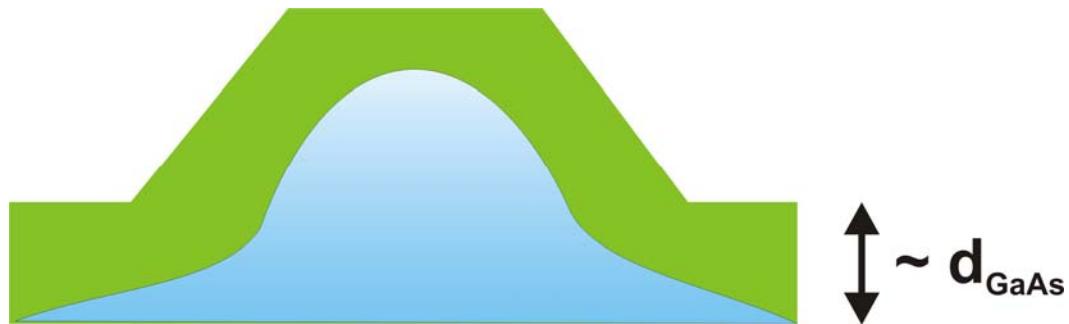
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amorphous oxide layer  
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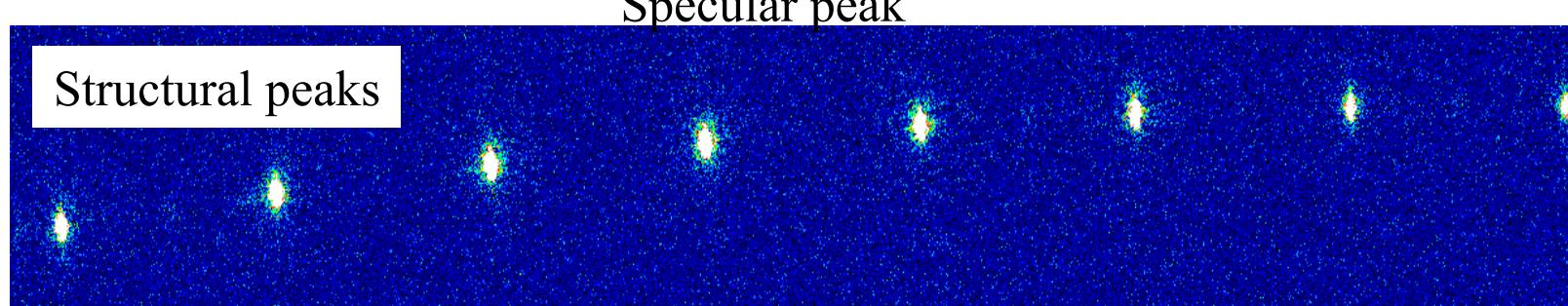
Marquez et al., APL 78 (16), 2310 (2001)



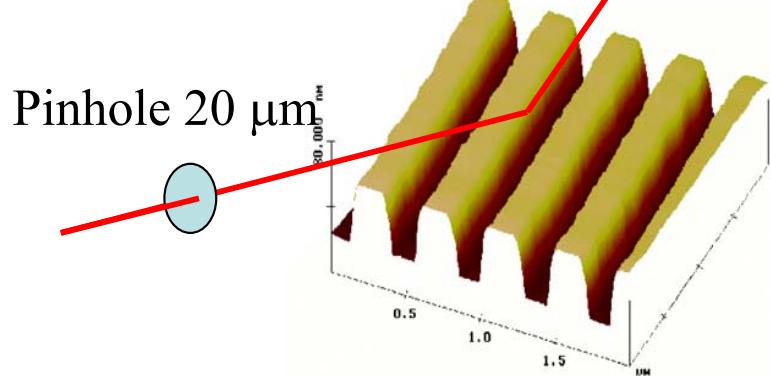
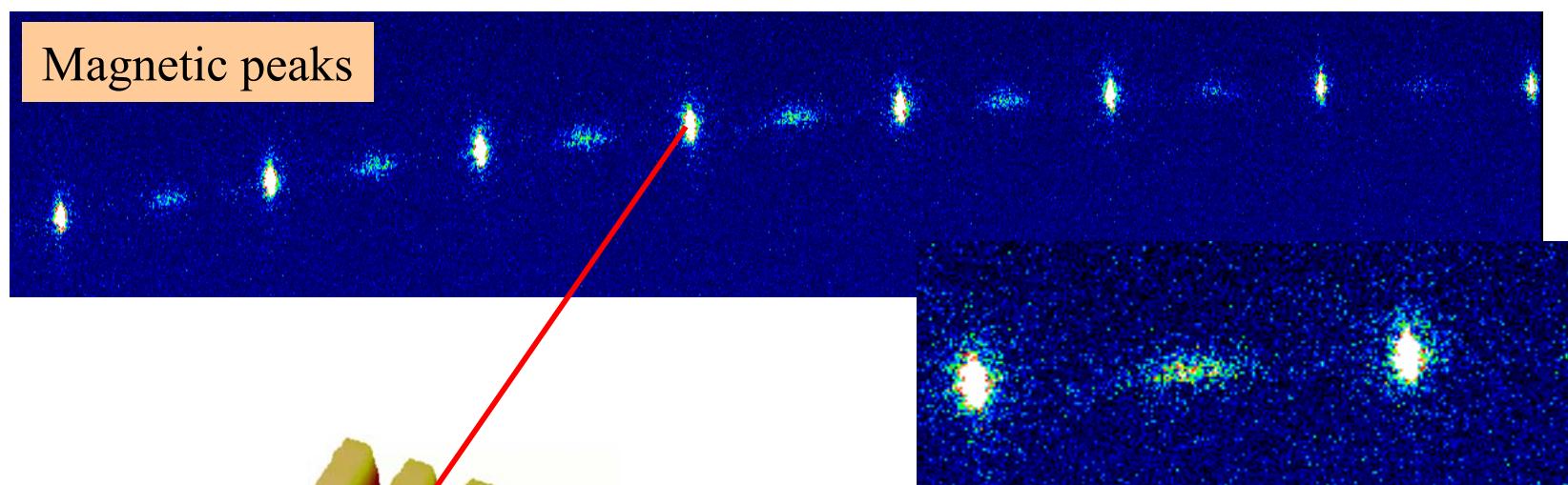
oxidation  $\Rightarrow$  foot of the dot was part of the substrate

# Magnetic resonant X-ray scattering from an array of CoPt multilayers

Away from  
 $L_3$  Co edge  
770 eV

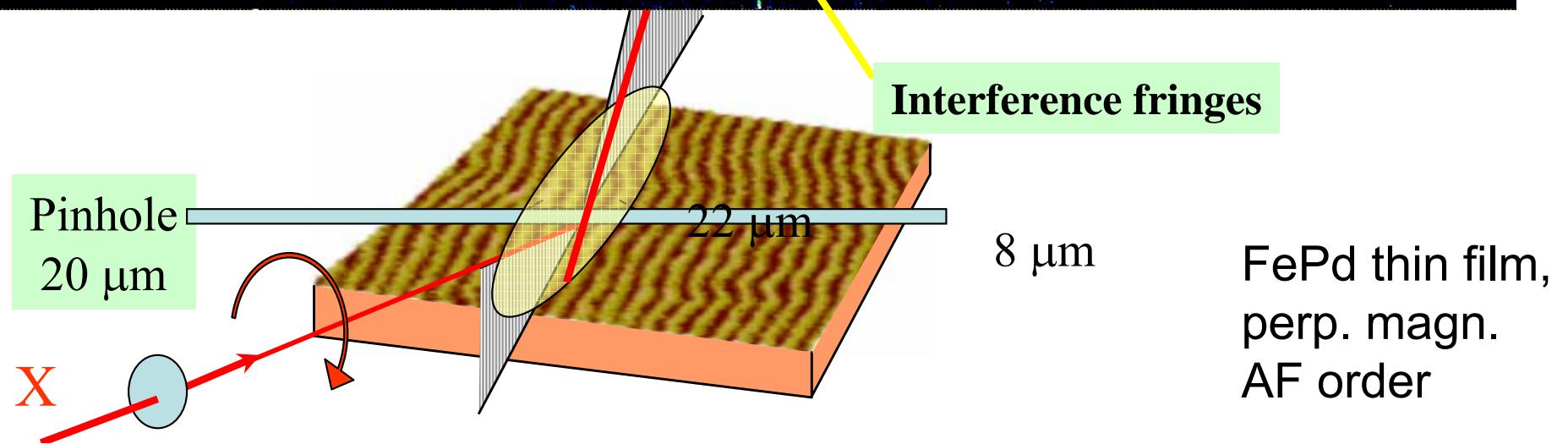
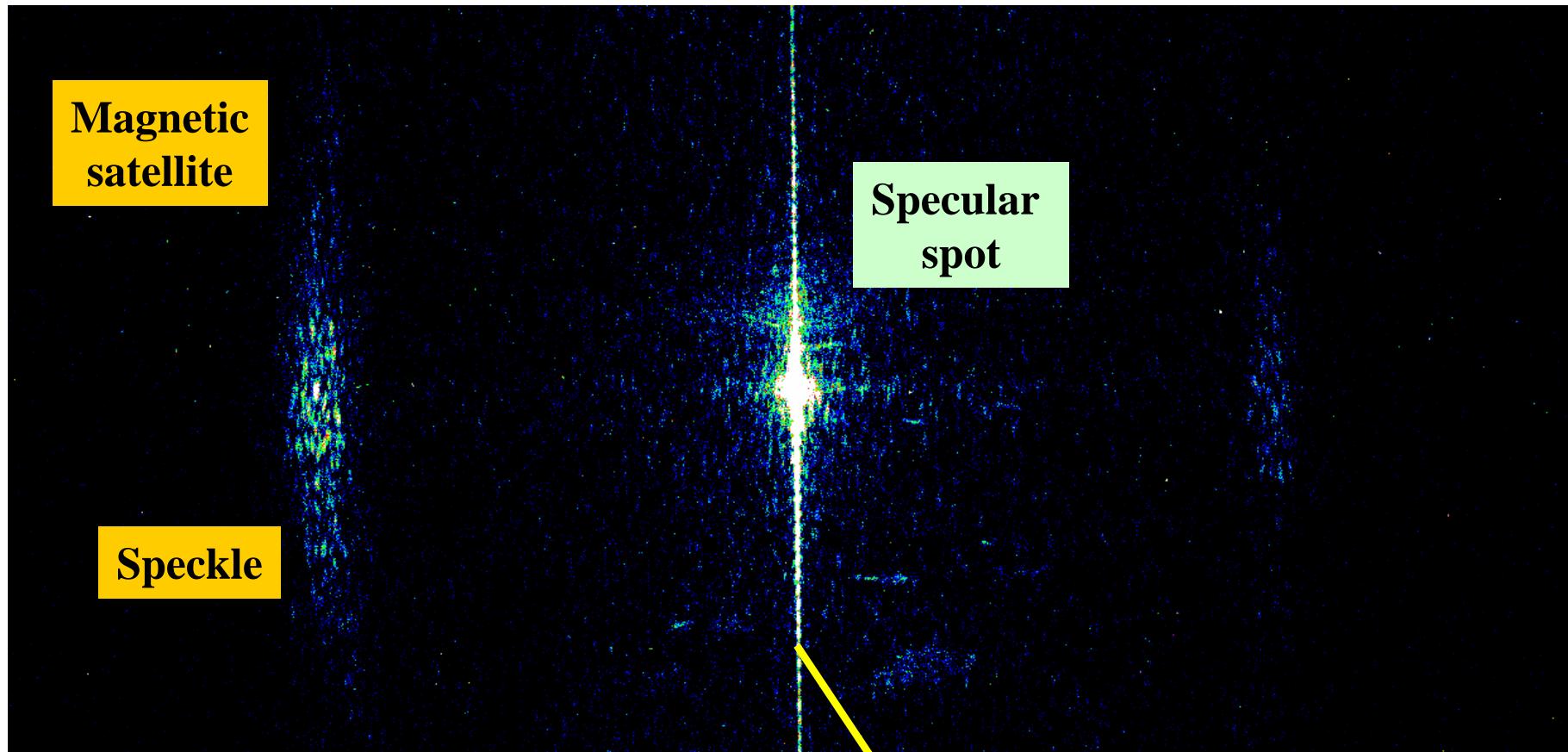


At  $L_3$  Co  
edge  
780 eV



Peak from AF magnetic order

# Coherent scattering on magnetic microstructures



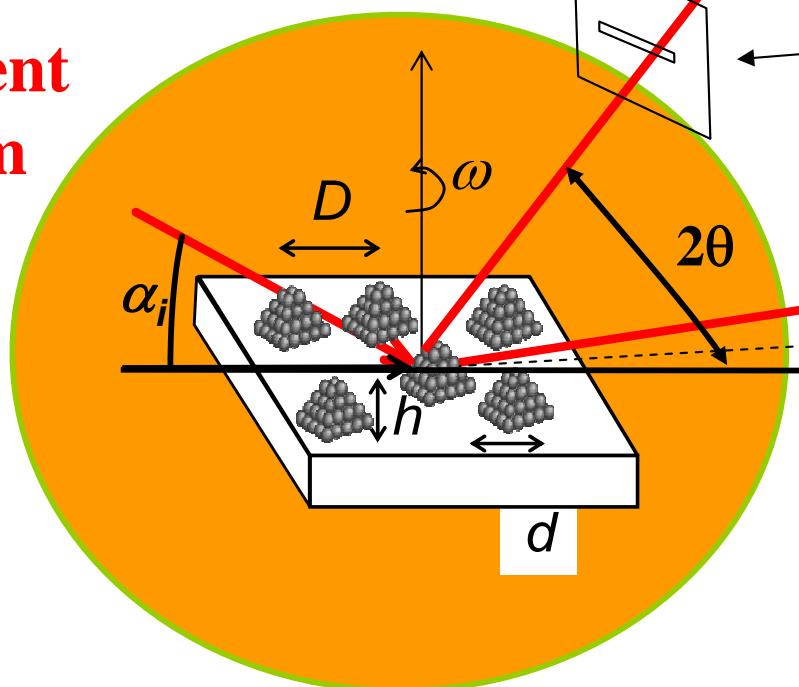
Beutier – Chesnel

CF les PRBs.

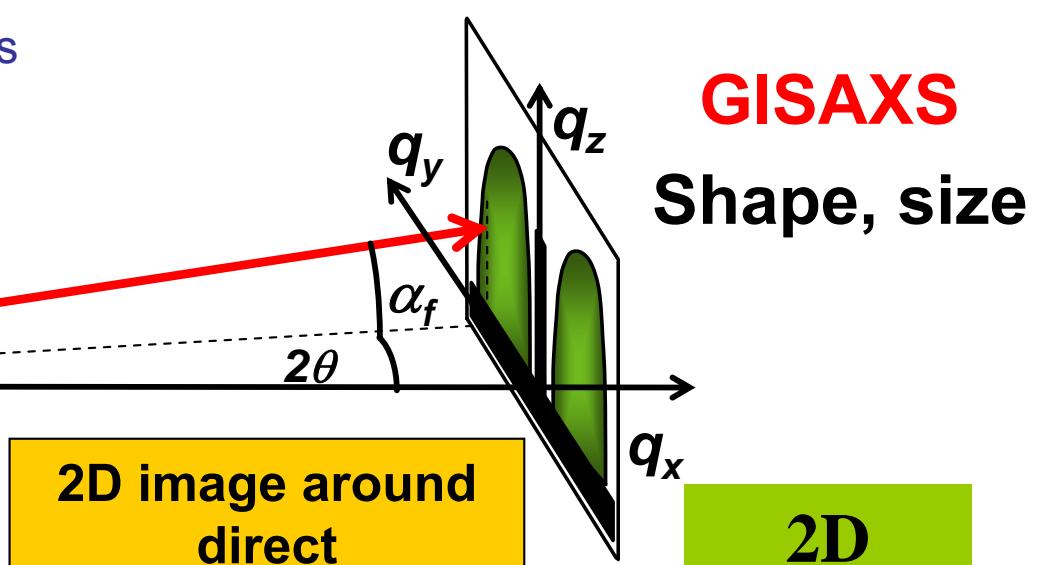
# Grazing Incidence X-ray Scattering

In UHV *in situ* during growth

Incident beam



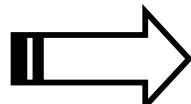
**GID**  
Strain state  
In-plane lattice parameter



2D image around  
direct  
beam:  
Fourier transform  
of the objects

**GISAXS**  
Shape, size

2D  
detector

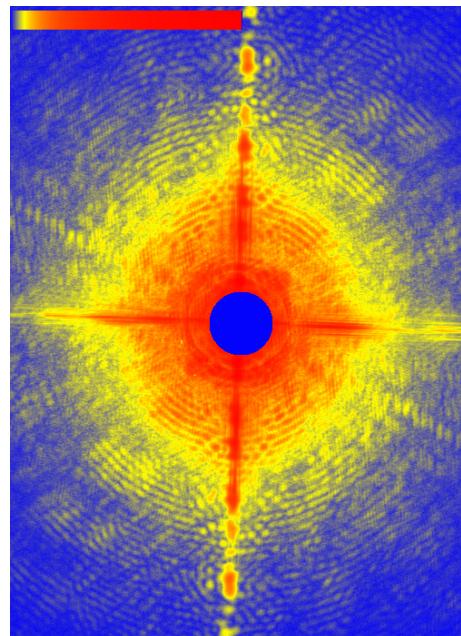
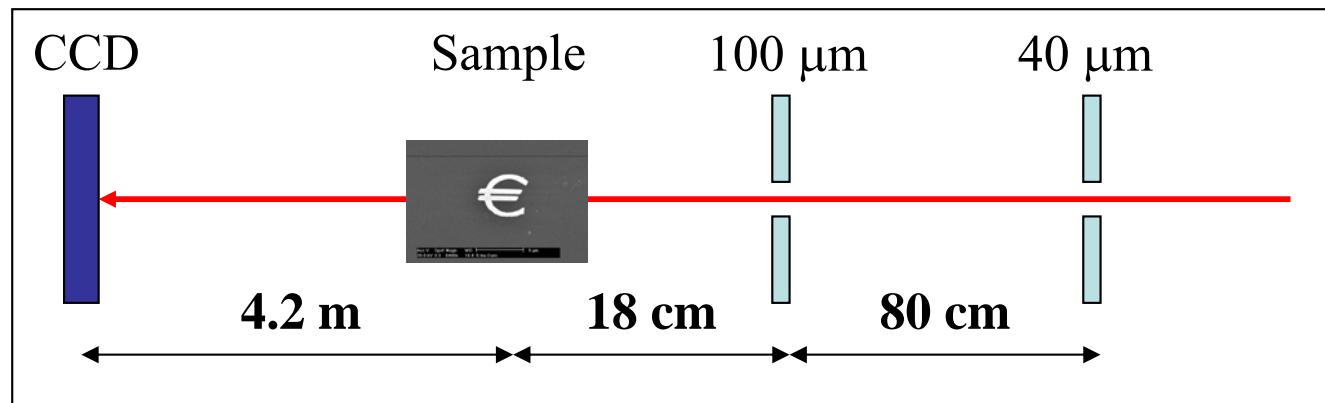


to decrease the bulk scattering as much as possible  
(to be sensitive to surface)

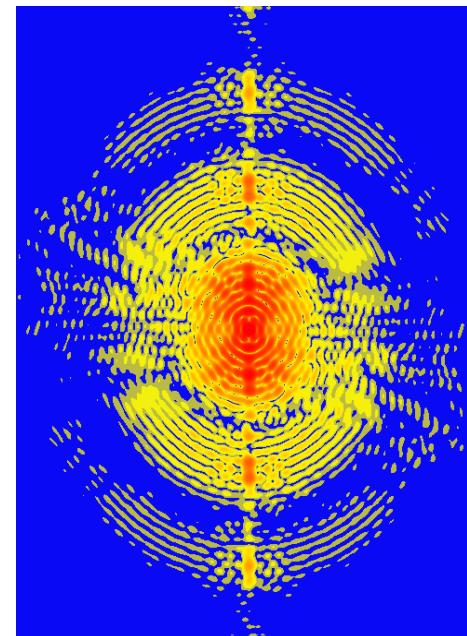
# Coherent Diffraction from a $5 \mu$ € made of Au



Christian Schroer, Edgar Weckert, Andreas Schropp, I. Vartanyants, Hasylab and ID01



Measurement



Simulation

Next steps  
Phase retrieval  
and reconstruction

## General references

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Thank you for your attention













