Fields, moments, units

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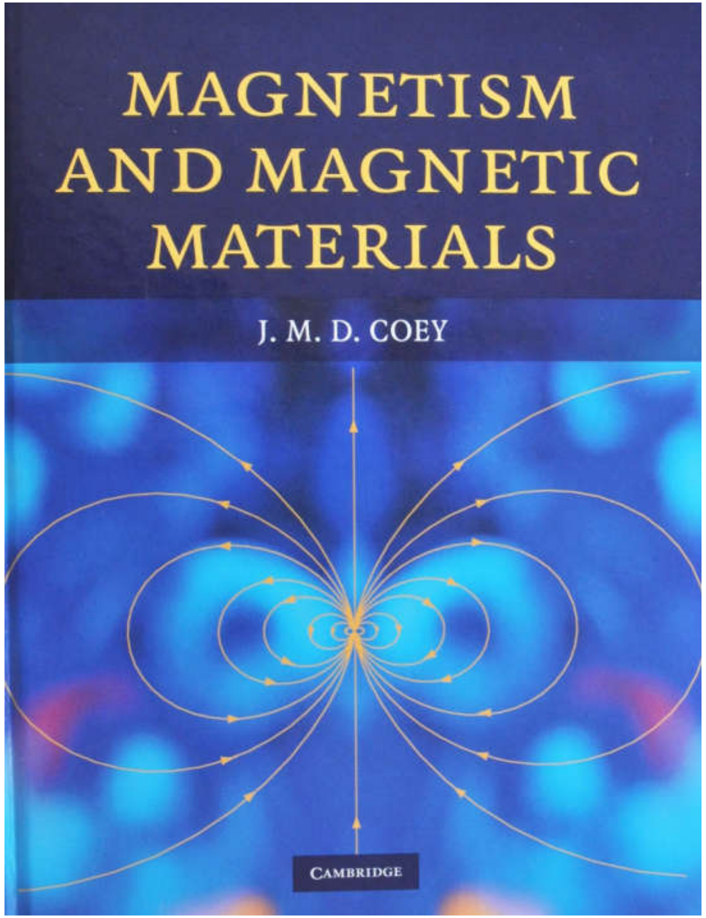
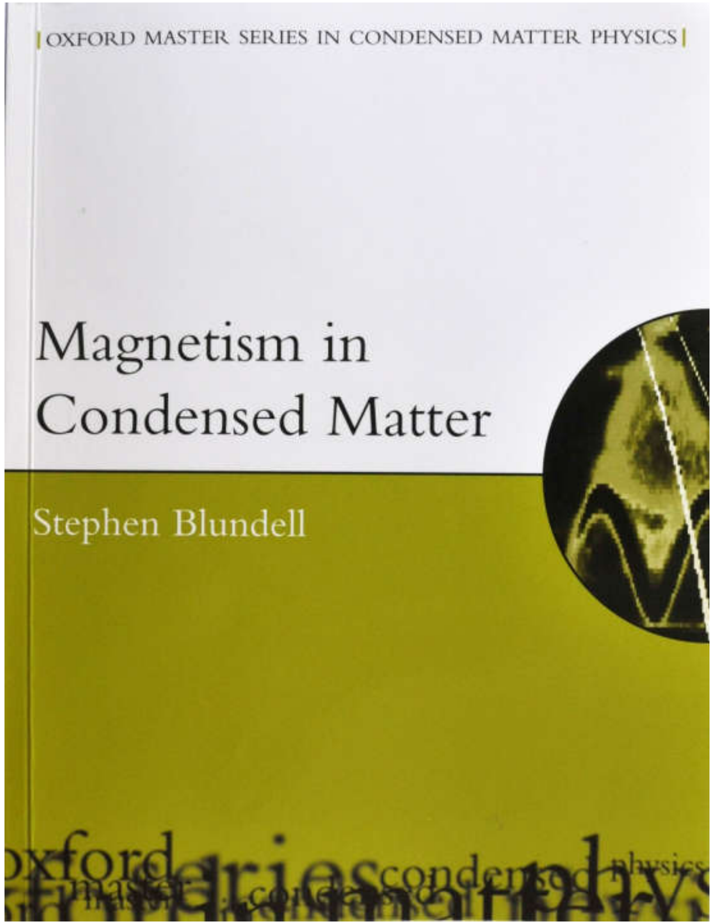
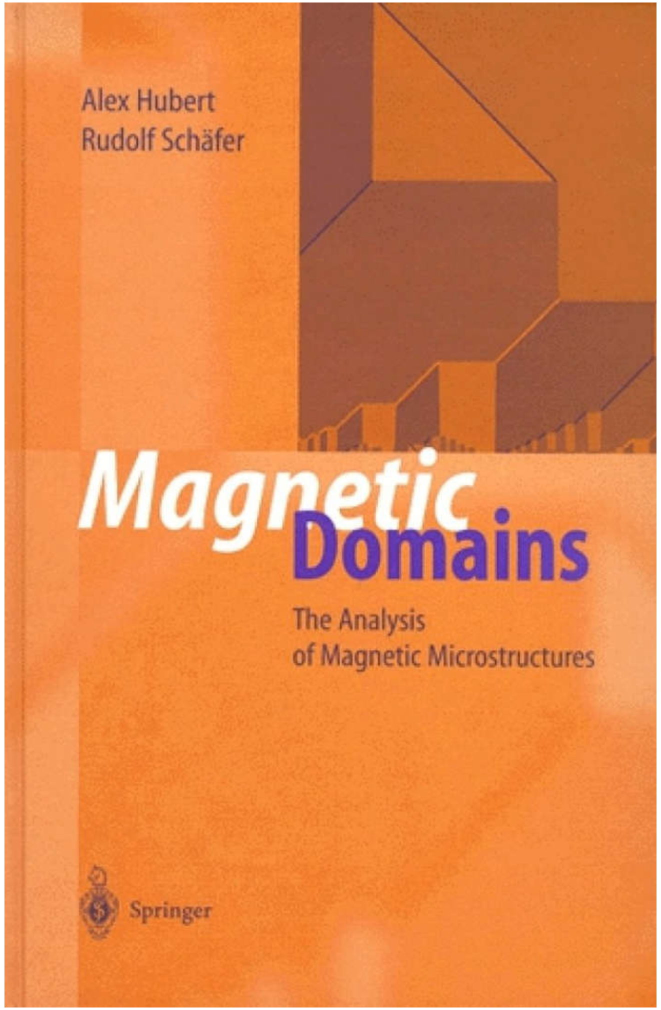
e-ESM, an online higher-education Magnetism event

Understand the deep roots of magnetism

Some references (See list of books on ESM web site)







What is a quantity?

What is a unit ?

Quantities and units in physics

Quantity

- Example: speed $\mathbf{v} = \delta\boldsymbol{\ell}/\delta t$
- Dimension: $\dim(\mathbf{v}) = \mathrm{L} \cdot \mathrm{T}^{-1}$

Units

- Why?
  - Provide a measure
  - Universality: share with others
- Possible formalism:

$$X = X_\alpha \langle X \rangle_\alpha$$

Quantity

Measure

Reference quantity

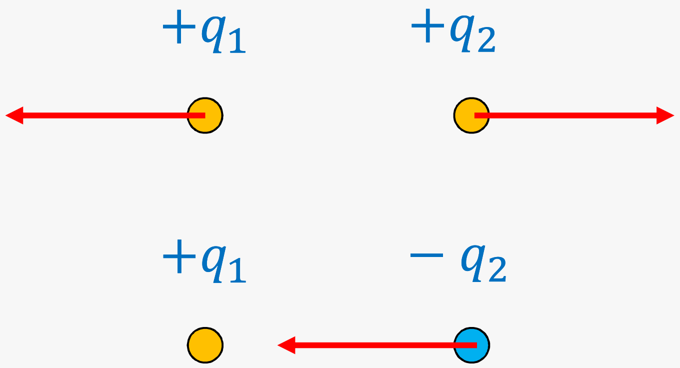
$$\langle L \rangle_{\mathrm{SI}} = \mathrm{meter} = 100 \langle L \rangle_{\mathrm{cgs}}$$

$$L = 50 \langle L \rangle_{\mathrm{SI}} = 5000 \langle L \rangle_{\mathrm{cgs}}$$

The electric charge and the electric field

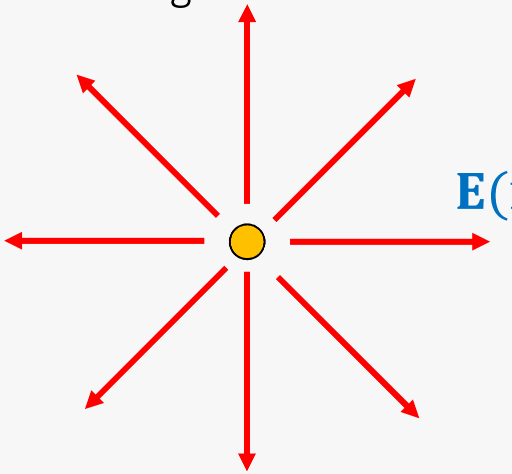
Facts: force between charges

$$\mathbf{F}_{1\to2} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\mathbf{u}_{12}$$



Modeling by the Physicist

- Electric field $\mathbf{E}_{1\to2}$ $\qquad \mathbf{F}_{1\to2} = q_2\mathbf{E}_{1\to2}$
- Charges are scalar sources of electric field

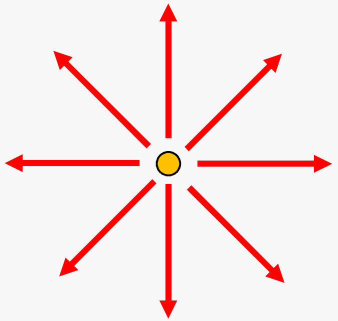


$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0 r^2}\mathbf{u}$$

The electric charge and the electric field

Macroscopic level: Gauss theroem

- Ostogradski theorem



$$\iiint_{\mathcal{V}} \boldsymbol{\nabla} \cdot \mathbf{E} \, \mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E} \cdot \mathbf{n} \, \mathrm{d}\mathcal{S}$$

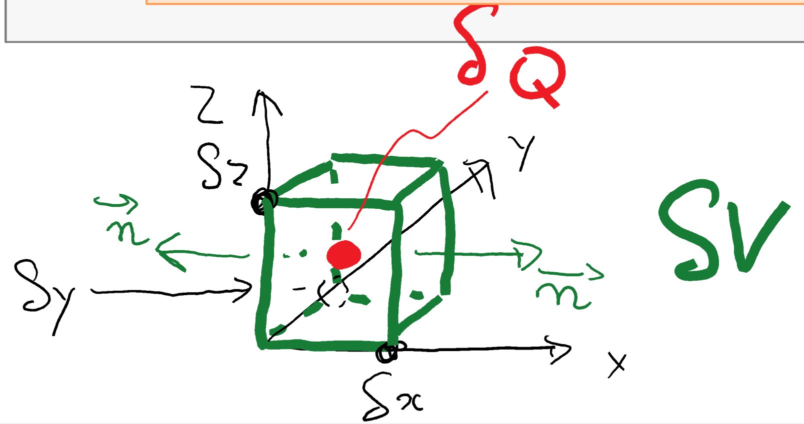
$$\Rightarrow \quad \frac{Q}{\epsilon_0} = \iiint_{\mathcal{V}} \frac{\rho}{\epsilon_0} \, \mathrm{d}\mathcal{V} = \oiint_{\partial\mathcal{V}} \mathbf{E} \cdot \mathbf{n} \, \mathrm{d}\mathcal{S}$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\rho = \frac{\delta Q}{\delta \mathcal{V}}$$ Volume density of electric charge

- Q is the scalar source of **E**

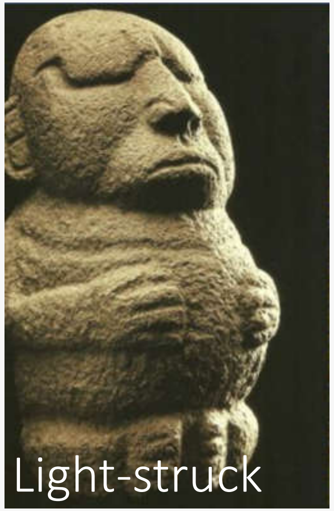


Link

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \ldots = \frac{E_x(x+\delta x) - E_x(x)}{\delta x} + \cdots$$

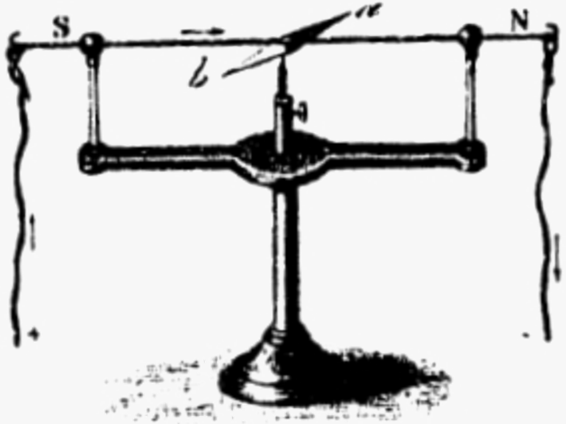
Century-old facts

- Magnetic materials (rocks)



- Magnetic field of the earth

Œrsted experiment in 1820



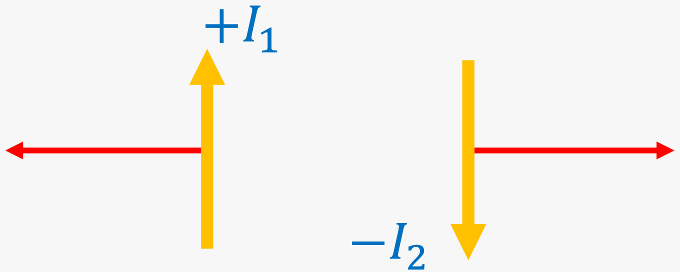
Hans-Christian Oersted, 1777–1851.

→ Birth of electromagnetism

The electric current and the magnetic induction field

Facts: force between charge currents

$$\delta\mathbf{F}_{1\to 2} = \mu_0 \frac{I_1 I_2 [\delta\boldsymbol{\ell}_2 \times (\delta\boldsymbol{\ell}_1 \times \mathbf{u}_{12})]}{4\pi r_{12}^2}$$

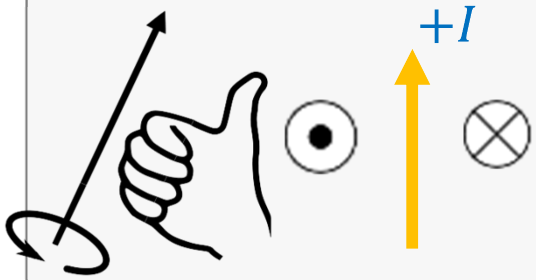


<u>Note</u>: former definition of the Ampère:

The force between two infinite wires 1m apart with current 1A is $2 \times 10^{-7}\ \mathrm{N/m}$

Modeling by the Physicist

- Magnetic induction field: Biot & Savart law



$$\delta\mathbf{B} = \frac{\mu_0 I \delta\boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Retrieve the force (Laplace)

$$\delta\mathbf{F}_2 = I_2 \delta\boldsymbol{\ell} \times \mathbf{B}(\mathbf{r}_2)$$

$$\Rightarrow \mathbf{F} = q\,\mathbf{v} \times \mathbf{B}$$

- Magnetic induction field defined through Lorentz Force

The electric current and the magnetic induction field

Macroscopic level: Ampere theorem

- Stokes theorem

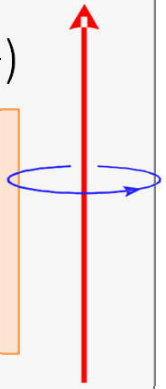
$$\iint_{\mathcal{S}} (\boldsymbol{\nabla} \times \mathbf{B}) \cdot \mathbf{n} \, \mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

$$\Rightarrow \quad I = \mu_0 \iint_{\mathcal{S}} (\mathbf{j} \cdot \mathbf{n}) \, \mathrm{d}\mathcal{S} = \oint_{\partial\mathcal{S}} \mathbf{B} \cdot \mathrm{d}\boldsymbol{\ell}$$

Microscopic level: Maxwell equation

$$\boldsymbol{\nabla} \times \mathbf{B} = \mu_0 \mathbf{j}$$

j: Volume density of current (A/m^2)

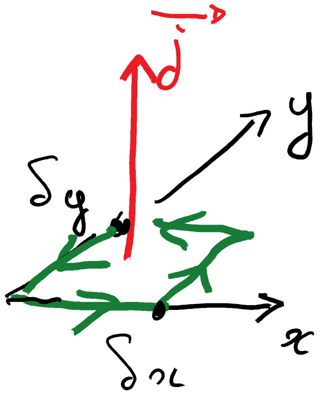


- j is the vectorial source of curl of B

Unit for B: tesla (T)

Link

$$\boldsymbol{\nabla} \times \mathbf{B} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{\partial B_y}{\partial x} - \dfrac{\partial B_x}{\partial y} \end{pmatrix} = \begin{pmatrix} \cdots \\ \cdots \\ \dfrac{B_y(x+\delta x) - B_y(x)}{\delta x} - \dfrac{B_x(y+\delta y) - B_x(y)}{\partial y} \end{pmatrix}$$



Maxwell equations (in vacuum)

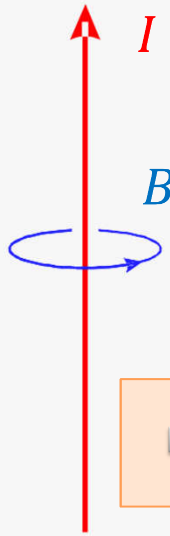
$\mathbf{\nabla} \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$	Gauss theorem
$\mathbf{\nabla} \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	Faraday law of induction
$\mathbf{\nabla} \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$	Ampère theorem
$\mathbf{\nabla} \cdot \mathbf{B} = 0$	B is divergence free (no magnetic poles)

Biot and Savart

$$\delta \mathbf{B} = \frac{\mu_0 I \delta \boldsymbol{\ell} \times \mathbf{u}}{4\pi r^2}$$

- Note: $1/r^2$ decay

Ampere theorem and Œrsted field

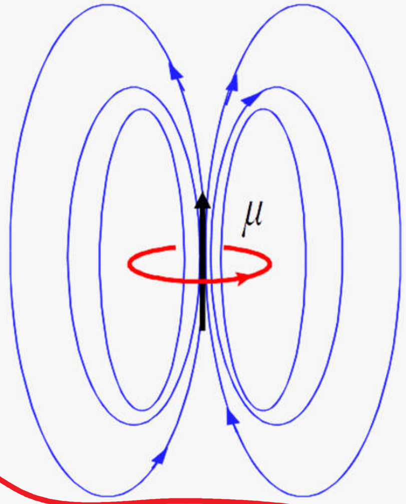


$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Integrate

- Note: $1/r$ decay

The magnetic point dipole



- Simple loop

$\boldsymbol{\mu} = I\mathcal{S}\,\mathbf{n}$ **Unit:** $\mathrm{A}\cdot\mathrm{m}^2$

- General definition

$$\boldsymbol{\mu} = \frac{1}{2}\iiint_{\mathcal{V}} \mathbf{r} \times \mathbf{j}(\mathbf{r})\,\mathrm{d}\mathcal{V}$$

Derive

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}\cdot\mathbf{r})\mathbf{r} - \boldsymbol{\mu}\right]$$

- Note: $1/r^3$ decay

$$\mathbf{B} = \frac{\mu_0}{4\pi r^3}\left(2\mu\cos\theta\,\mathbf{u}_r + \mu\sin\theta\,\mathbf{u}_\theta\right)$$

The magnetic point dipole in a magnetic induction field

Energy

$\mathcal{E} = -\boldsymbol{\mu} \cdot \mathbf{B}$ Zeeman energy (J)

Demonstration

- Work to compensate Lenz law during rise of B
- Integrate torque from Laplace force while flipping dipole in B

Torque

$$\boldsymbol{\Gamma} = \oint \mathbf{r} \times I(\mathrm{d}\boldsymbol{\ell} \times \mathbf{B}) = \boldsymbol{\mu} \times \mathbf{B}$$

- Inducing precession of dipole around the field
- It is energy-conservative, as expected from Laplace (Lorentz) force

Force

$\mathbf{F} = \boldsymbol{\mu} \cdot (\overline{\overline{\boldsymbol{\nabla}\mathbf{B}}})$

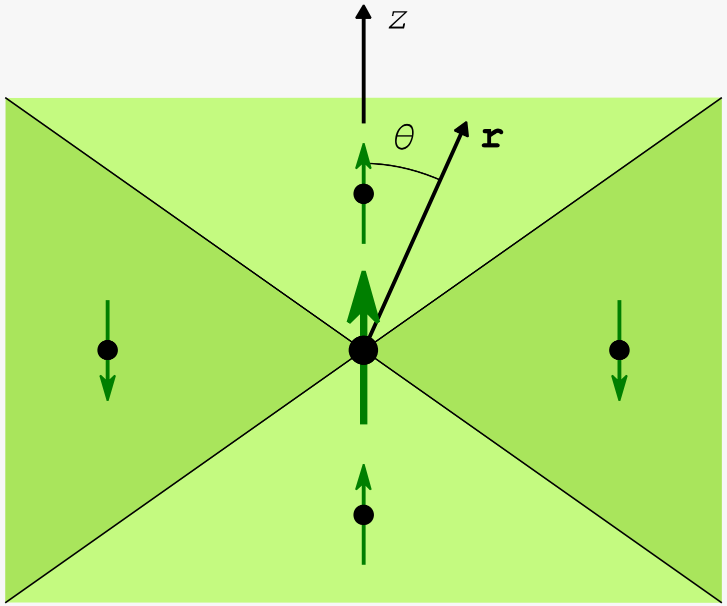
- Valid only for fixed dipole
- No force in uniform magnetic induction field

Two interacting magnetic point dipoles

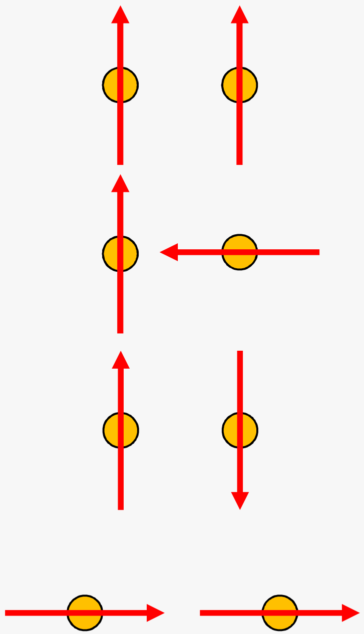
Energy

$$\mathcal{E} = -\frac{\mu_0}{4\pi r^3}\left[\frac{3}{r^2}(\boldsymbol{\mu}_1\cdot\mathbf{r})(\boldsymbol{\mu}_2\cdot\mathbf{r}) - \boldsymbol{\mu}_1\cdot\boldsymbol{\mu}_2\right]$$

- The dipole-dipole interaction is anisotropic



Examples



$$\mathcal{E} = +2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = +\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = 0$$

$$\mathcal{E} = -\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

$$\mathcal{E} = -2\frac{\mu_0\mu_1\mu_2}{4\pi r^3}$$

Magnetization

Definition

- Volume density of magnetic point dipoles

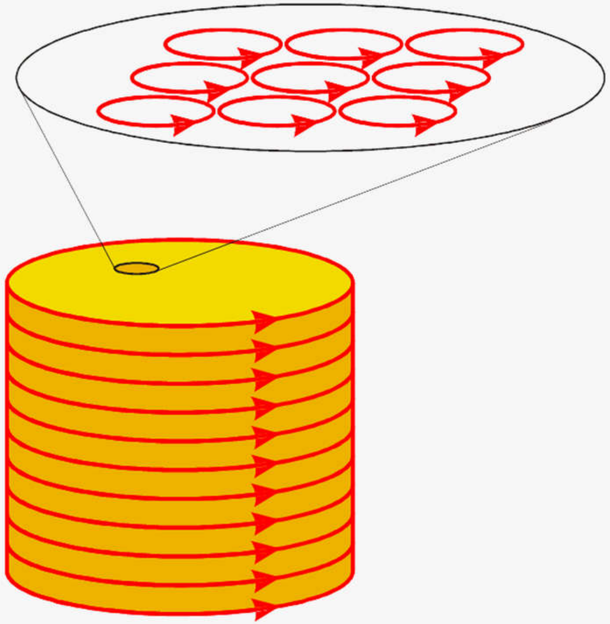
$$\mathbf{M} = \frac{\delta \boldsymbol{\mu}}{\delta \mathcal{V}} \qquad \mathrm{A/m}$$

- Total magnetic moment of a body

$$\mathcal{M} = \int_{\mathcal{V}} \mathbf{M} \, \mathrm{d}\mathcal{V} \qquad \mathrm{A \cdot m^2}$$

- Applies to: ferromagnets, paramagnets, diamagnets etc.
- Must be defined at a length scale much larger than atoms
- Is the basis for the micromagnetic theory

Equivalence with surface currents



- Name: Amperian description of magnetism
- Surface current equals magnetization $\mathrm{A/m}$

Free currents and bound currents

Back to Maxwell equations

- Disregard fast time dependence: magnetostatics

$$\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \cancel{\epsilon_0 \frac{\partial \mathbf{E}}{\partial t}} \right)$$

- Consider separately real charge current, $\mathbf{j}_\mathrm{c}$ from fictitious currents of magnetic dipoles $\mathbf{j}_\mathrm{m}$

$$\nabla \times \mathbf{B} = \mu_0 (\mathbf{j}_\mathrm{c} + \mathbf{j}_\mathrm{m})$$

- One can show: $\nabla \times \mathbf{M} = \mathbf{j}_\mathrm{m}$ $\quad \mathrm{A/m^2}$

$\mathbf{M} \times \mathbf{n} = \mathbf{j}_\mathrm{m,s}$ $\quad \mathrm{A/m}$

- Outside matter, $\mathbf{B}$ and $\mu_0 \mathbf{H}$ coincide and have exactly the same meaning.

The magnetic field H

- One has: $\nabla \times \left(\frac{\mathbf{B}}{\mu_0} - \mathbf{M} \right) = \mathbf{j}_\mathrm{c}$

- By definition: $\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M}$ $\quad \mathrm{A/m}$

$$\nabla \times \mathbf{H} = \mathbf{j}_\mathrm{c}$$

B versus H : definition of the system

- M: local (infinitesimal) part in $\delta\mathcal{V}$ of the system defined when considering a magnetic material

- H: The remaining of B coming from outside $\delta\mathcal{V}$, liable to interact with the system

Derivation of the dipolar field

The dipolar field H_d

- By definition: the contribution to H not related to free currents (possible to split as Maxwell equations are linear)

$$\nabla \times \mathbf{H}_\mathrm{d} = 0 \Rightarrow \mathbf{H}_\mathrm{d} = -\nabla \phi_\mathrm{d}$$

$$\mathbf{H} = \mathbf{H}_\mathrm{d} + \mathbf{H}_\mathrm{app}$$

External to magnetic body

Analogy with electrostatics

$$\nabla \times \mathbf{E} = 0 \Rightarrow \mathbf{E} = -\nabla \phi$$

Derive the dipolar field

Maxwell equation $\nabla \cdot \mathbf{B} = \mathbf{0} \rightarrow \nabla \cdot \mathbf{H}_\mathrm{d} = -\nabla \cdot \mathbf{M}$

$$\Rightarrow \mathbf{H}_\mathrm{d}(\mathbf{r}) = -M_\mathrm{s} \iiint_{\mathcal{V}'} \frac{[\nabla \cdot \mathbf{m}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}'$$

To lift the singularity that may arise at boundaries, a volume integration around the boundaries yields:

$$\mathbf{H}_\mathrm{d}(\mathbf{r}) = \iiint \frac{\rho(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{V}' + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3} \mathrm{d}\mathcal{S}'$$

$\rho(\mathbf{r}) = -M_\mathrm{s}\, \nabla \cdot \mathbf{m}(\mathbf{r})$ → volume density of magnetic charges

$\sigma(\mathbf{r}) = M_\mathrm{s}\, \mathbf{m}(\mathbf{r}) \cdot \mathbf{n}(\mathbf{r})$ → surface density of magnetic charges

Stray field and demagnetizing field

Vocabulary

- Generic names

  Magnetostatic field

  Dipolar field

- Inside material

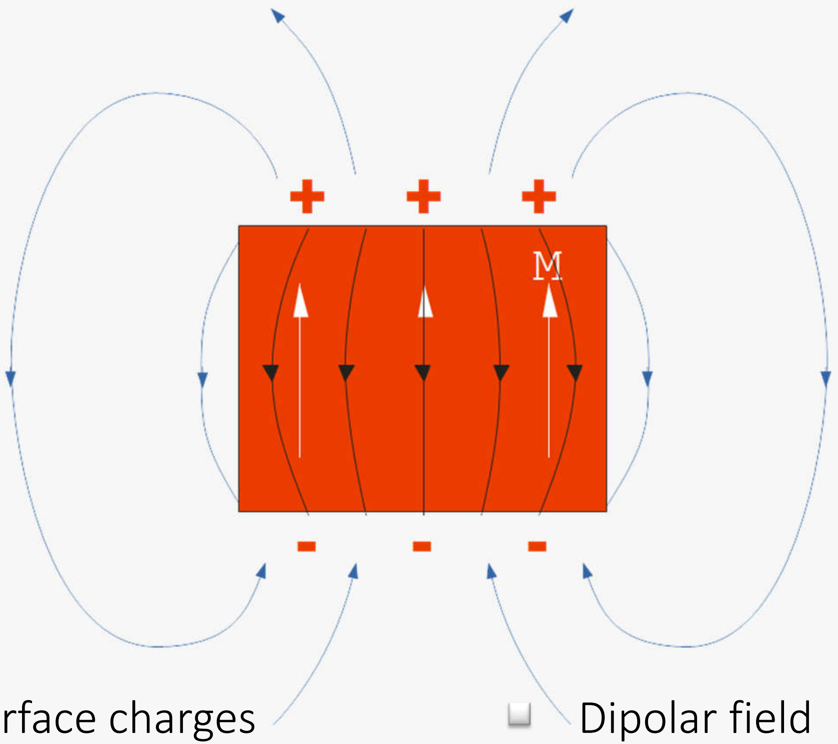
  Demagnetizing field

- Outside material

  Stray field

Example

Permanent magnet (uniformly-magnetized)



- Surface charges

$$\sigma(\mathbf{r})=M_s\mathbf{m}(\mathbf{r})\cdot\mathbf{n}(\mathbf{r})$$

- Dipolar field

$$\mathbf{H}_\mathrm{d}(\mathbf{r})=\oiint\frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}S'$$

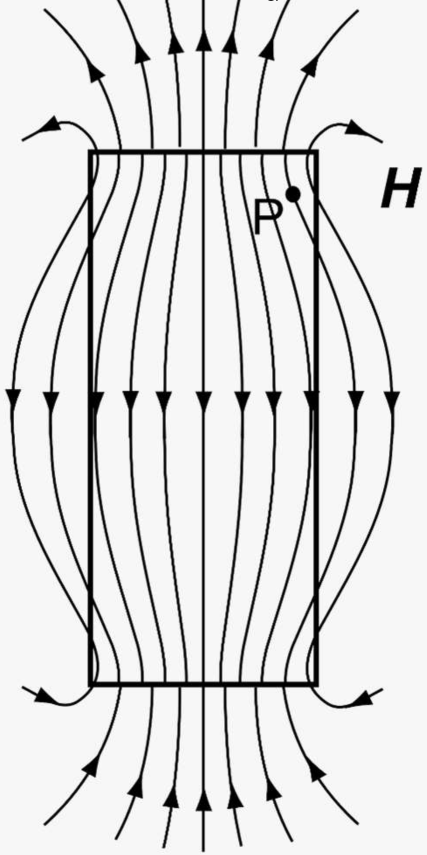
Coulombian

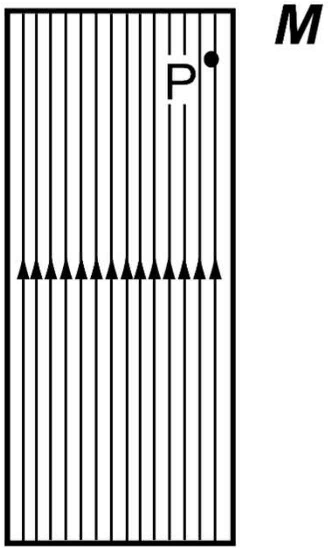
- Pseudo-charges source of H_d

$\nabla \times \mathbf{H} = \mathbf{0}$

- No closed lines
- $\Delta H_{\parallel} = 0$
- $\Delta \mathbf{H} \cdot \mathbf{n} = \sigma$

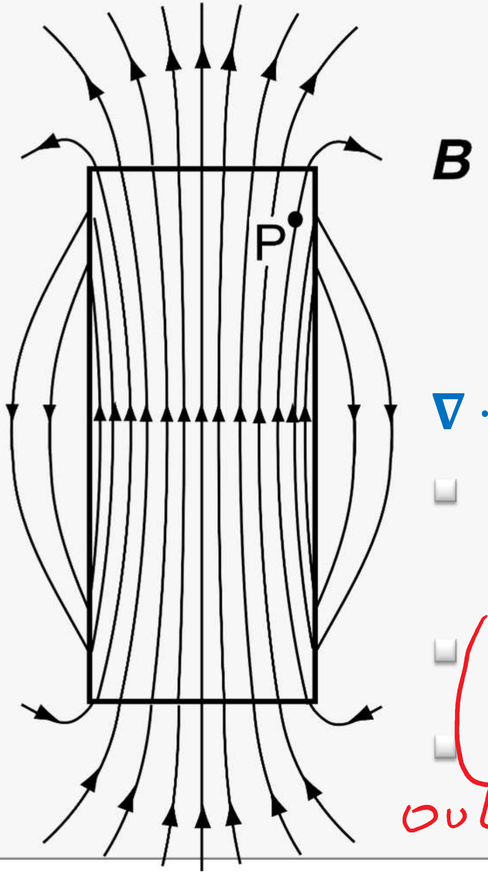
out - in





Amperian

- Fictitious currents source of B



$\nabla \cdot \mathbf{B} = \mathbf{0}$

- No magnetic monopole
- $\Delta B_{\perp} = 0$
- $\Delta \mathbf{B} = \mu_0 \mathbf{j} \times \mathbf{n}$

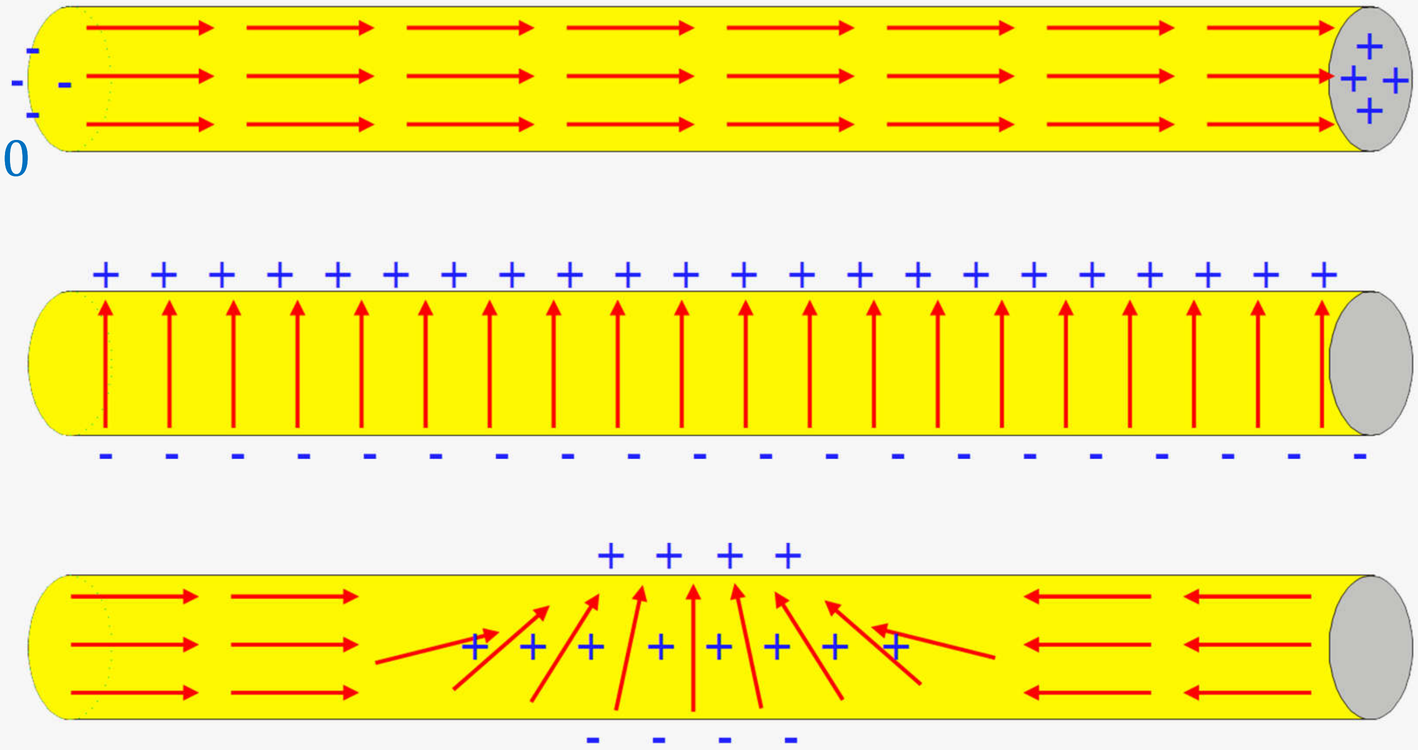
out - in

From: M. Coey's book

Examples of magnetic charges

The long cylinder

- Note for infinite cylinder: no charge $\mathcal{E}_{\mathrm{d}}=0$
- Charges on side surfaces
- Surface and volume charges



Take-away message

- Dipolar energy favors alignment of magnetization with longest direction of sample

Dipolar energy

Dipolar energy

- Zeeman energy of microscopic volume
  $\delta\mathcal{E}_{\mathrm{Z}} = -\mu_0 \mathbf{M}\delta\mathcal{V} \cdot \mathbf{H}_{\mathrm{ext}}$
- Elementary volume of a macroscopic system creating its own dipolar field
  $E_{\mathrm{d}} = \delta\mathcal{E}_{\mathrm{d}}/\delta\mathcal{V} = -\frac{1}{2}\mu_0 \mathbf{M} \cdot \mathbf{H}_{\mathrm{d}}$ mutual energy
- Total dipolar energy of macroscopic body

$$\mathcal{E}_{\mathrm{d}} = -\frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{M} \cdot \mathbf{H}_{\mathrm{d}}\, \mathrm{d}\mathcal{V}$$

$$\mathcal{E}_{\mathrm{d}} = \frac{1}{2}\mu_0 \iiint_{\mathcal{V}} \mathbf{H}_{\mathrm{d}}^2\, \mathrm{d}\mathcal{V}$$

- Always positive. Zero means minimum

Size considerations

$$\mathbf{H}_{\mathrm{d}}(\mathbf{r}) = \mathrm{Volume} + \oiint \frac{\sigma(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

- Unchanged if all lengths are scaled: homothetic.
  NB: the following is a solid angle:

$$\mathrm{d}\boldsymbol{\Omega} = \frac{(\mathbf{r}-\mathbf{r}')\mathrm{d}\mathcal{S}'}{|\mathbf{r}-\mathbf{r}'|^3}$$

- H_d does not depend on the size of the body
- Neither does the volume density of energy
- Said to be a long-range interaction

Range of dipolar interactions in low dimensionality

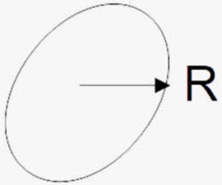
Range

- Upper bound of dipolar field in thin films

$$\|\mathbf{H}_\mathrm{d}(\mathbf{r})\| \leq M_\mathrm{s} t \int \frac{2\pi r}{r^3}\,\mathrm{dr}$$

Integration

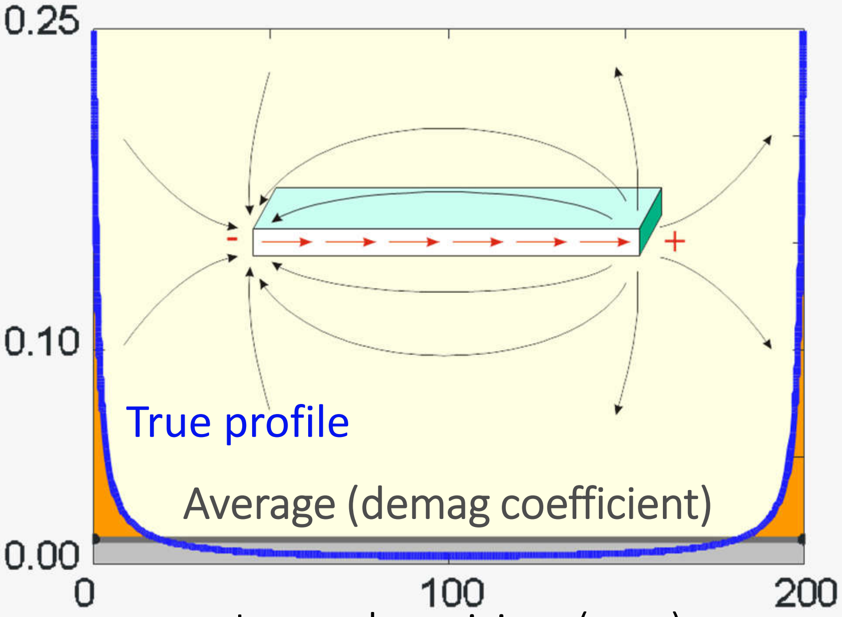
Hd for dipole



$$\|\mathbf{H}_\mathrm{d}(\mathbf{R})\| \leq \mathrm{Cste} + \mathcal{O}(1/R)$$

Non-homogeneity

- Example: flat strip with aspect ratio 0.0125



- Dipolar fields are short-ranged in low dimensions
- Dipolar fields are highly non-homogeneous in such large aspect ratio systems
- Consequences: non-uniform magnetization switching, edge modes etc.

Demagnetizing coefficients – Maths

Dipolar energy for uniform magnetization

$$\mathbf{M}(\mathbf{r}) = \mathbf{M} = M_s(m_x\hat{\mathbf{x}} + m_y\hat{\mathbf{y}} + m_z\hat{\mathbf{z}})$$

Implicit $\sum_{i=x,y,z}$

- No volume charges: $\rho(\mathbf{r}) = -M_s\,\boldsymbol{\nabla}\cdot\mathbf{m}(\mathbf{r}) = 0$
- Dipolar field: $\mathbf{H}_d(\mathbf{r}) = \oiint_{\partial\mathcal{V}} \frac{[\mathbf{M}(\mathbf{r}')\cdot\mathbf{n}(\mathbf{r}')]\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}' = M_s m_i \oiint_{\partial\mathcal{V}} \frac{\mathrm{n}_i(\mathbf{r}')\,(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$
- Dipolar energy:

$$\mathcal{E}_d = -\frac{1}{2}\mu_0\iiint_{\mathcal{V}}\mathbf{M}(\mathbf{r})\cdot\mathbf{H}_d(\mathbf{r})\mathrm{d}\mathcal{V} = -\frac{1}{2}\mu_0 M_s^2 m_i \iiint_{\mathcal{V}}\mathrm{d}\mathcal{V}\oiint_{\partial\mathcal{V}}\frac{n_i(\mathbf{r}')\,\mathbf{m}\cdot(\mathbf{r}-\mathbf{r}')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

$$\mathcal{E}_d = -K_d m_i m_j \iiint_{\mathcal{V}}\mathrm{d}\mathcal{V}\oiint_{\partial\mathcal{V}}\frac{n_i(\mathbf{r}')\,(r_j - r_j')}{4\pi\,|\mathbf{r}-\mathbf{r}'|^3}\mathrm{d}\mathcal{S}'$$

Implicit $\sum_i \sum_{j=x,y,z}$

$$\langle\mathbf{H}_d(\mathbf{r})\rangle = -M_s\,\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

$$\mathcal{E}_d = K_d V\,\mathbf{m}\cdot\bar{\bar{\mathbf{N}}}\cdot\mathbf{m}$$

See more detailed approach: M. Beleggia et al., JMMM 263, L1-9 (2003)

Demagnetizing coefficients – Take-away messages

For any shape of body

$$\langle \mathbf{H}_\mathrm{d}(\mathbf{r}) \rangle = -M_\mathrm{s}\, \overline{\overline{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\, \mathbf{m} \cdot \overline{\overline{\mathbf{N}}} \cdot \mathbf{m}$$

Dipolar anisotropy is always of second order

- $\overline{\overline{\mathbf{N}}}$ demagnetizing tensor. Always positive, and can be diagonalized. $N_x + N_y + N_z = 1$

  $$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V \left(N_x m_x^2 + N_y m_y^2 + N_z m_z^2 \right)$$

- Along main directions

  $$\langle H_{\mathrm{d},i}(\mathbf{r}) \rangle = -N_i M_\mathrm{s}$$

Hypothesis uniform **M** may be too strong
Remember: dipolar field is NOT uniform

For ellipsoids etc.

- Condition: boundary is a polynomial of the coordinates, with degree at most two

  Slabs (thin films), cylinders, ellipsoids

$$z^2 = \left(\frac{t}{2}\right)^2 \quad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \quad \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$$

$$\mathbf{H}_\mathrm{d} = -M_\mathrm{s}\, \overline{\overline{\mathbf{N}}} \cdot \mathbf{m}$$

$$\mathcal{E}_\mathrm{d} = K_\mathrm{d} V\, \mathbf{m} \cdot \overline{\overline{\mathbf{N}}} \cdot \mathbf{m}$$

- Along main directions

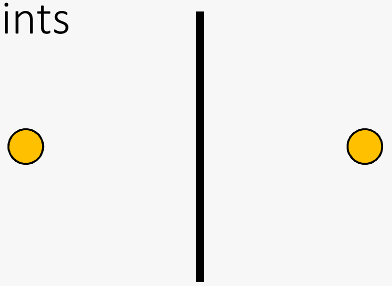
  $$H_{\mathrm{d},i} = -N_i M_\mathrm{s}$$

M and **H** may not be colinear along non-main directions

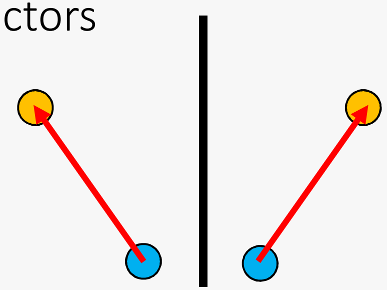
Space symmetry

Reminder about plane symmetry

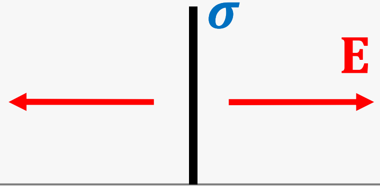
- Points



- Vectors



- Example: electric field $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$



Magnetic fields are pseudo-vectors

- Curl is a chiral operator

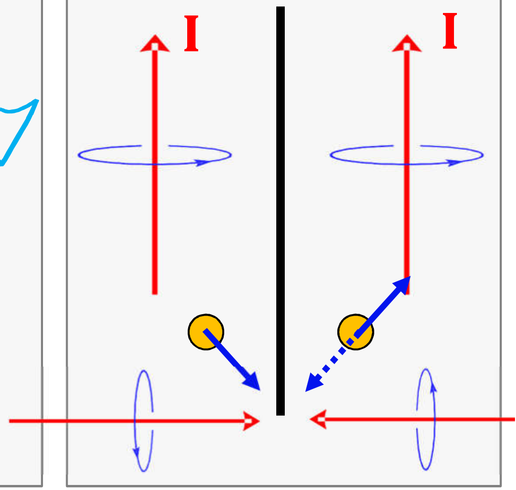
Antisymmetry

$$\delta \mathbf{B} = \frac{\mu_0 I \delta \boldsymbol{\ell} \times \mathbf{u}_{12}}{4\pi r^2}$$

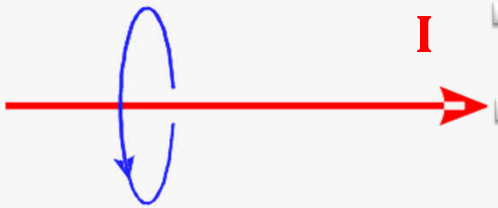
Symmetry

Antisymmetry

B is antisymmetric



What use? Example: Ampere theorem and Œrsted field



- Symmetry of I with plane containing I
- Antisymmetry of B: is azimuthal

$$B_\theta = \frac{\mu_0 I}{2\pi r}$$

Time inversion symmetry

Time inversion symmetry of Maxwell equations

- What happens with operation $t \rightarrow -t$

Unchanged $\nabla \cdot \mathbf{E} = \dfrac{\rho}{\epsilon_0}$

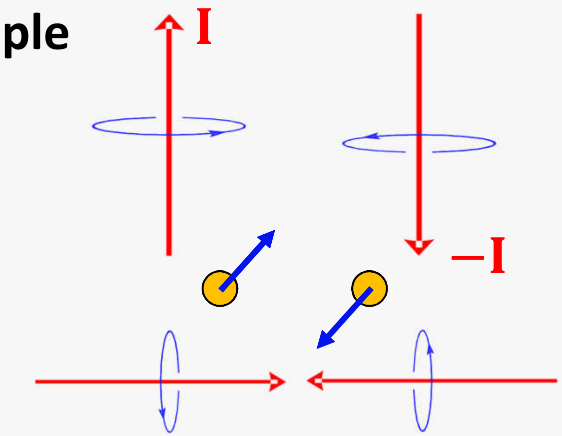
Unchanged $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$ → $-B$, → $-t$

Inversed $\nabla \times \mathbf{B} = \mu_0 \left(\mathbf{j} + \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \right)$ → $-B$, → $-j$, → $-t$

Inversed $\nabla \cdot \mathbf{B} = 0$ → $-B$

- Maxwell equations remains valid
- Solutions must comply with time-reversal symmetry

Example



What use? Magneto-crystalline anisotropy

$$E(\theta) = \cancel{K_{10} \cos\theta} + \cancel{K_{01} \sin\theta} + K_{11} \cos\theta \sin\theta + K_{20} \cos^2\theta + K_{02} \sin^2\theta + \cancel{K_{30} \cos^3\theta} + \cancel{K_{03} \sin^3\theta} + \cancel{K_{21} \cos^2\theta \sin\theta} + \cancel{K_{12} \cos\theta \sin^2\theta} + \cdots$$

- Odd terms are forbidden

	SI system	cgs-Gauss
Definitions	Meter m	Centimeter cm
	Kilogram kg	Gram g
	Second s	Second s
	Ampere A	Ab-Ampere ab-A = 10A
	$\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$	$\mathbf{B}=\mathbf{H}+4\pi\mathbf{M}$
	$\mu_0=4\pi\times10^{-7}$ SI	$\mu_0=4\pi$

Conversions

Field	$\mathbf{H}$	$1\ \mathrm{A/m}$	$4\pi\times10^{-3}\ \mathrm{Oe}$	(Oersted)
Moment	$\boldsymbol{\mu}$	$1\ \mathrm{A.m^2}$	$10^3\ \mathrm{emu}$	
Magnetization	$\mathbf{M}$	$1\ \mathrm{A/m}$	$10^{-3}\ \mathrm{emu/cm^3}$	
Induction	$\mathbf{B}$	$1\ \mathrm{T}$	$10^4\ \mathrm{G}$	(Gauss)
Susceptibility	$\chi=M/H$	1	$1/4\pi$	

Problems with cgs-Gauss

- The quantity for charge current is missing

  No check for homogeneity

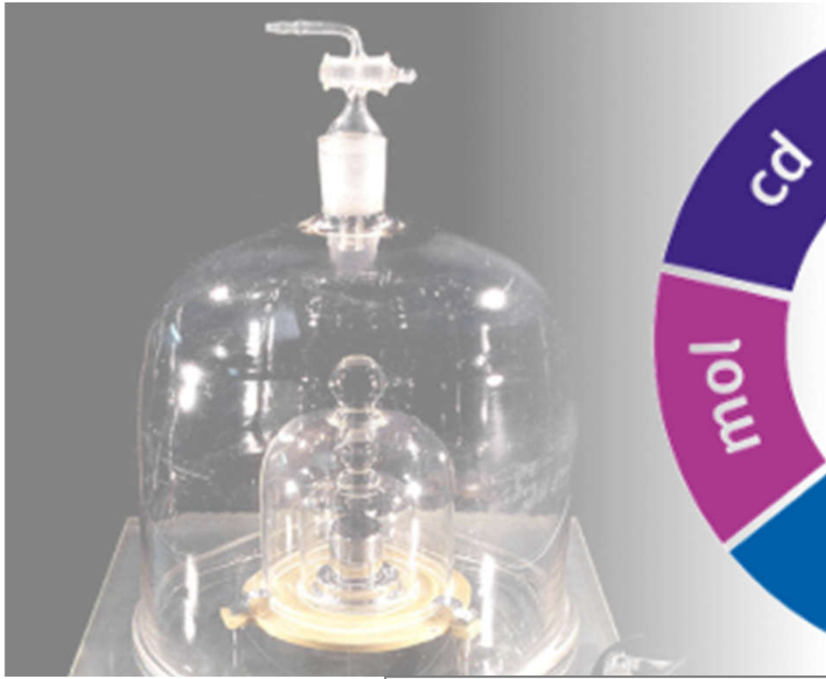
  Mix of units in spintronics

- Inconsistent definition of H

  Dimensionless quantities are effected: demag factors, susceptibility etc.

- More in the practical on units: http://magnetism.eu/esm/repository-authors.html#F

Quantum revolution in SI units in 2019



Define quantities

- Times
- Length
- Mass
- Electric charge

Fixed values

- Speed of light -> Define meter
- Planck constant -> Defines kg
- Charge of the electron

To be measured

- Magnetic permeability of vacuum

$$\mu_0 \neq 4\pi \times 10^{-7}\ \mathrm{S.I.}$$

$$\mu_0 = 4\pi[1 + 2.0(2.3) \cdot 10^{-10}] \times 10^{-7}\ \mathrm{S.I.}$$

Thank you for your attention !

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