The European School on Magnetism 2017 – Practical on units

Olivier Fruchart

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In this tutorial we derive the dimensions for physical quantities of use in magnetism, and their conversions between cgs-Gauss and SI.

1 Notations

We use the following notations:

- X is a physical quantity, such as force in $F = mg$. It may be written $\mathbf{X}$ for vectors.
- $\dim X$ is the dimension of X expressed in terms of powers of fundamental dimensions, here length (L), mass (M), time (T) and electrical current (I). For example, dimensions of speed and electrical charges read: $\dim v = \mathsf{L} \cdot \mathsf{T}^{-1}$ and $\dim q = \mathsf{I} \cdot \mathsf{T}$. As a shortcut we will use here a vector matrix to summarize the dimension of quantities, with components the powers of fundamental dimensions; it will be written $[X]$ for the dimension of X. The above examples now read $[v] = [\mathsf{L}] - [\mathsf{T}] = [1\ 0\ {-1}\ 0]$ and $[q] = [\mathsf{I}] + [\mathsf{T}] = [0\ 0\ 1\ 1]$. We use shortcuts $[\mathsf{L}]$, $[\mathsf{M}]$, $[\mathsf{T}]$ and $[\mathsf{I}]$ for the four fundamental dimensions.
- In a system of units α (*e.g.* SI or cgs-Gauss) a physical quantity is evaluated numerically based on the unit physical quantities: $X = X_\alpha \langle X \rangle_\alpha$. X_α is a number, while $\langle X \rangle_\alpha$ is the standard (*i.e.*, used as unit) for the physical quantity in the system considered. For example $\langle L \rangle_{\mathrm{SI}}$ is a length of one meter, while $\langle L \rangle_{\mathrm{cgs}}$ is a length of one centimeter: $\langle L \rangle_{\mathrm{SI}} = 100 \langle L \rangle_{\mathrm{cgs}}$. For derived dimensions we use the matrix notation. For example the unit quantity for speed in system α would be written $[1\ 0\ \ -1\ 0]_\alpha$.

2 Expressing dimensions

- Based on laws for mechanics, find dimensions for force F, energy $\mathcal{E}$ and power $\mathcal{P}$, and their volume density E and P.
- Based on the above, find dimensions for electric field $\mathbf{E}$, voltage U, resistance R, resistivity ρ, permittivity ϵ_0.
- Find dimensions for magnetic moments $\boldsymbol{\mu}$, magnetic field and magnetization $\mathbf{H}$ and $\mathbf{M}$, induction $\mathbf{B}$ and flux ϕ, and permeability μ_0.

3 Conversions

Physics does not depend on the choice for a system of units, so doesn't any physical quantity X. The conversions between its numerical values X_α and X_β in two such systems is readily obtained from the relationship between $\langle X \rangle_\alpha$ and $\langle X \rangle_\beta$. In the cgs-Gauss system, the unit for length, mass and time are centimeter, gram and second. The electric current may also be considered as existing and named Biot or abampère, equivalent to 10 A. Thus we have the following conversion relationships: $\langle L \rangle_{\mathrm{SI}} = 10^2 \langle L \rangle_{\mathrm{cgs}}$. Similarly we have $\langle M \rangle_{\mathrm{SI}} = 10^3 \langle M \rangle_{\mathrm{cgs}}$, $\langle T \rangle_{\mathrm{SI}} = \langle T \rangle_{\mathrm{cgs}}$ and $\langle I \rangle_{\mathrm{SI}} = 10^{-1} \langle I \rangle_{\mathrm{cgs}}$.

In practice conversion can be formally written the following way: $X = X_\alpha \langle X \rangle_\alpha = X_\beta \langle X \rangle_\beta$. Let us consider length l as a example. $l = l_{\mathrm{SI}} \langle L \rangle_{\mathrm{SI}} = l_{\mathrm{cgs}} \langle L \rangle_{\mathrm{cgs}}$. From the above we readily have: $l_{\mathrm{SI}} = (1/100) l_{\mathrm{cgs}}$. Thus the numerical value for the length of an olympic swimming pool is 5000 in cgs, and 50 in SI. For derived units (combination of elementary units), $\langle X \rangle_\alpha$ is decomposed in elementary units in both systems, whose relationship is known. For example for speed: $\langle v \rangle_\alpha = \langle L \rangle_\alpha \langle T \rangle_\alpha^{-1}$.

Exhibit the conversion factor for these various quantities, of use for magnetism:

- Force $\mathbf{F}$, energy $\mathcal{E}$, energy per unit area E_{s}, energy per unit volume E. The units for force and energy in the cgs-Gauss system are called *dyne* and *erg*, respectively.

- Express the conversion for magnetic induction B and magnetization M, whose units in cgs-Gauss are called gauss and emu/cm^3, respectively. Express related quantities such as magnetic flux ϕ and magnetic moment μ.
- Let us recall that magnetic field is defined in SI with $B = \mu_0(H + M)$, whereas in cgs-Gauss with $B = H + 4\pi M$, with the unit called œrsted. Express the conversion for μ_0 and comment. Then express the conversion for magnetic field H.
- Discuss the cases of magnetic susceptibility and demagnetizing coefficients. In SI these are defined by $\chi = \mathrm{d}M/\mathrm{d}H$ and $H_\mathrm{d} = -NM$. What should be their definition in the cgs-Gauss system so that these dimensionless quantities have the same numerical value in both systems? Notice that definitions sometimes used in the cgs-Gauss system are: $H_\mathrm{d} = -4\pi NM$ and $H_\mathrm{d} = -DM$.

4 Further reading

- Overview of the *Système International* and conventions for writing units, to download from the Bureau International des Poids et Mesures (BIPM)[1].
- Documentation of the LaTeX siunits package[2].
- *Encyclopedia of scientific units, weight and measures*, by François Cardarelli[3].

References

[1] *Bureau International des Poids et Mesures.*
URL http://www.bipm.org/

[2] *siunits LaTeX package.*
URL https://www.ctan.org/pkg/siunits

[3] F. Cardarelli, Encyclopedia of Scientific units, weights and measures, Springer, London, 2003.