Charge, heat, and spin transport

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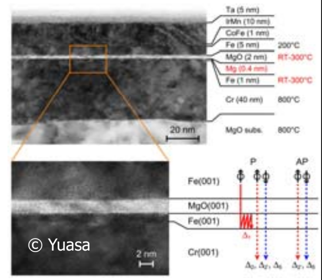
Requirements for theory of magnetoelectronics

Metals and insulators:

Fermi/screening wave length O(Å); disorder; sharp interfaces

- Size quantization effects on transport negligible (exception: MgO|Fe|Cr).
- Many-body effects negligible (exception: Kondo effect).
- Weak spin-orbit interaction.
- Quantum mechanics at the interfaces.



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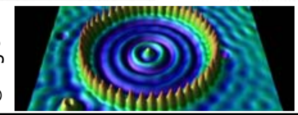


Overview first two lectures

G.D. Mahan, *Many-Particle Physics*, (Springer, 2000).
S. Doniach, E. H. Sondheimer, *Green's Functions for Solid State Physicists* (Imperial, 1998)

1 *Elementary transport theory*
(a) Linear response theory of transport
(b) Scattering theory of transport
(c) Semiclassical transport
(d) Thermoelectricity and Onsager symmetry
(e) Collinear spin transport

A. Brataas et al., Physics Reports **427**, 157 (2006)

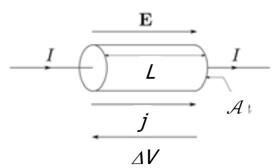
2 *DC magnetoelectronic circuit theory*
(a) Transport in non-collinear magnetization textures, non-collinear spin valves
(b) Spin transfer torque and spin mixing conductance
(c) Spintronic Kirchhoff Laws

3 *AC magnetoelectronic circuit theory*
(a) Current-induced magnetization dynamics
(b) Spin pumping and enhanced Gilbert damping
(c) DC and AC spin pumping

Y. Tserkovnyak et al., Rev. Mod. Phys., **77** 1375 (2005).

Not: domain walls, tunnel junctions, (spin-orbit interaction)

Ohm's Law

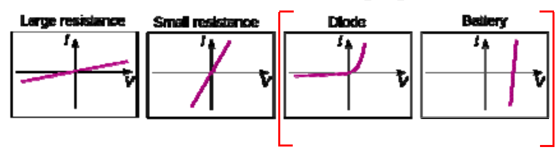


$$I = \frac{\Delta V}{R} = G\Delta V = A\frac{Gl}{A}\frac{\Delta V}{L}$$

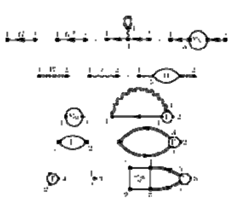
$$j = \frac{I}{A} = \frac{GL}{A}\frac{\Delta V}{L} = \sigma E = \frac{E}{\rho}$$

$$P = I\Delta V = R^2 I$$

$$[G] = \left[\frac{e^2}{h}\right] = \mathrm{S} = \frac{1}{\Omega}$$



Linear response of heat and mass transport: Fourier/Fick Laws



Perturbation theory of transport

Perturbation theory

Applied electric field **E** generates surrent <**j**> in a system H_0:

$$H = H_0 + H' \text{ where } H' = -e\mathbf{r}\cdot\mathbf{E} \qquad H_0\left|\varphi_i\right\rangle = \varepsilon_i\left|\varphi_i\right\rangle$$

$$\left|\psi_i\right\rangle = \left|\varphi_i\right\rangle + \left|\varphi_i\right\rangle' \text{ where } \left|\psi_i\right\rangle' = \sum_{j\neq i}\frac{\left\langle\varphi_j\right|H'\left|\varphi_i\right\rangle}{\varepsilon_i - \varepsilon_j}\left|\varphi_j\right\rangle$$

$$\langle\mathbf{j}\rangle = \sum_i f_i\left\langle\psi_i\right|\mathbf{j}\left|\psi_i\right\rangle = \sum_i f_i\left[\left\langle\varphi_i\right|\mathbf{j}\left|\psi_i\right\rangle' + \text{h.c.}\right]$$

f_i occupation number

$$= \sum_i f_i \sum_{i\neq j}\left[e\frac{\left\langle\varphi_i\right|\mathbf{j}\left|\varphi_j\right\rangle\left\langle\varphi_j\right|\mathbf{r}\left|\varphi_i\right\rangle}{\varepsilon_i - \varepsilon_j} + \text{h.c.}\right]\cdot\mathbf{E} = \boldsymbol{\sigma}\cdot\mathbf{E}$$

$$\left\langle\varphi_i\right|\mathbf{j}\left|\varphi_j\right\rangle = \frac{e\hbar}{2mi}\left(\varphi_i^*\nabla\varphi_j - \text{h.c.}\right)$$

$\boldsymbol{\sigma}$ conductance tensor

But $\left\langle\varphi_j\right|\mathbf{r}\left|\varphi_i\right\rangle$ is problematic for large systems.

Kubo formula

$$H=\frac{1}{2m}\left[\mathbf{p}-e\mathbf{A}\right]^2+U=\frac{p^2}{2m}+U+\mathbf{j}\cdot\mathbf{A}+\frac{eA^2}{2m}$$

$$\left(\nabla\cdot\mathbf{A}=0;\quad \mathbf{E}=-\frac{\partial\mathbf{A}}{\partial t}\right)\qquad \mathbf{A}(t)=\mathbf{A}_0e^{-i\omega t}\to\mathbf{E}=i\frac{\mathbf{A}_0}{\omega}$$

$H'=\mathbf{j}(\mathbf{r})\cdot\mathbf{A}(t)$ by *time-dependent* perturbation theory

$$(\sigma)_{ij}(\mathbf{r},\mathbf{r}')=\lim_{\omega\to0}\left\{i\frac{ne^2}{m\omega}\delta_{ij}+\frac{1}{\hbar\omega\Omega}\int_0^\infty dt\,e^{i\omega t}\left\langle\Psi_0\left|\left[j_i(\mathbf{r},t),j_j(\mathbf{r}',0)\right]\right|\Psi_0\right\rangle\right\}$$

$|\Psi_0\rangle$ ground state wavefunction (no field)

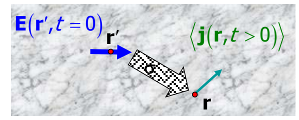
$j_i(\mathbf{r},t)$ current operator in Heisenberg picture

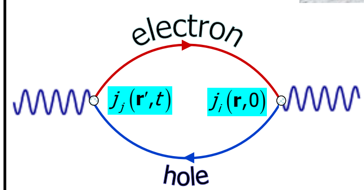
$[a,b]=ab+ba$ commutator

Fluctuation-dissipation theorem

Linear response (Kubo) formalism

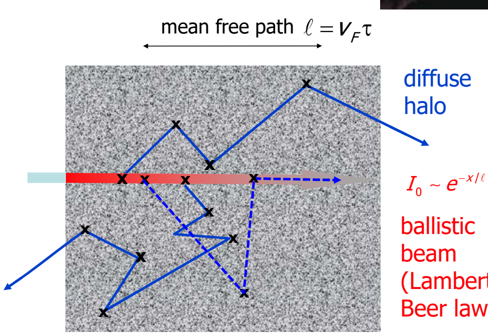
$$\langle\mathbf{j}(\mathbf{r})\rangle=\int_V d\mathbf{r}'\sigma(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')$$





Feynman diagram

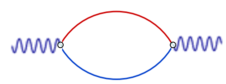
Disorder



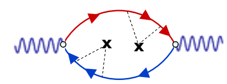
diffuse halo

ballistic beam (Lambert–Beer law)

Impurity scattering (short-range)

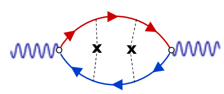


ballistic: $\sigma=\infty$

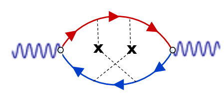


Drude: $\sigma_0=\frac{ne^2\tau}{m}$

$\tau^{-1}\sim n_iV_x^2$



diffuse scattering (ladder)



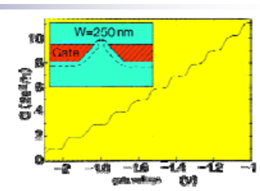
interference (cross)

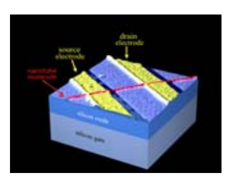
$\Delta\sigma<0$ weak localization

Scattering theory of transport

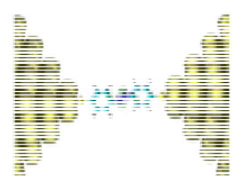
Nanostructures

quantum point contacts



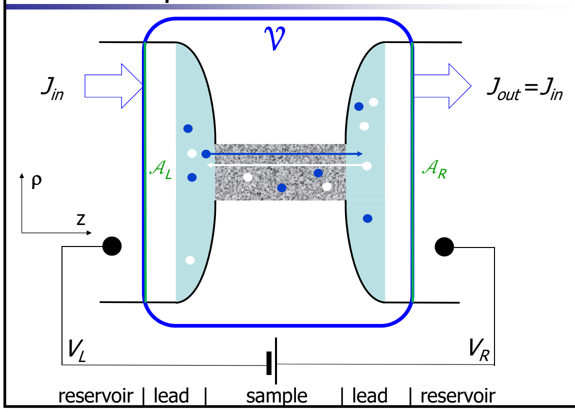


quantum wires



molecules

Nanotransport



Fisher-Lee theory

Phys. Rev. B **23**, 6851 (1981)

$$J_{out} = \int_{\mathcal{A}_R} \mathbf{j}(\boldsymbol{\rho},z)\,d\boldsymbol{\rho} = \int_{\mathcal{A}_R} d\boldsymbol{\rho} \int_{\mathcal{V}} d\mathbf{r}'\ \hat{\mathbf{z}}\cdot\sigma(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')$$

$e\mathbf{E}(\mathbf{r}) = \nabla\mu(\mathbf{r})$ In the leads: $\mathbf{E}(\mathbf{r}) = 0$

Divergence theorem and current conservation $\left(\nabla\cdot\mathbf{j}(\mathbf{r})=0\right)$:

$$\int_{\mathcal{V}} d\mathbf{r}'\ \hat{\mathbf{z}}\cdot\sigma(\mathbf{r},\mathbf{r}')\cdot\mathbf{E}(\mathbf{r}')\cdot\nabla\mu(\mathbf{r}') = \int_{\mathcal{A}_L} d\rho'\ \hat{\mathbf{z}}\cdot\sigma(\mathbf{r},\mathbf{r}')\mu(\mathbf{r}')$$

$$eJ = (\mu_R-\mu_L)\int_{\mathcal{A}_L} d\boldsymbol{\rho}' \int_{\mathcal{A}_R} d\boldsymbol{\rho}\ (\sigma)_{zz}(\mathbf{r},\mathbf{r}') = G\Delta\mu$$

independent of electric field distribution

Landauer-Büttiker formula

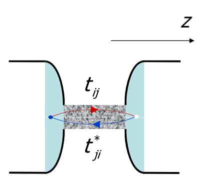
$$G = \int_{\mathcal{A}_L} d\boldsymbol{\rho} \int_{\mathcal{A}_R} d\boldsymbol{\rho}'\,(\sigma)_{zz}(\mathbf{r},\mathbf{r}')$$

Expansion into eigenstates i,j of the leads at Fermi energy

$$G = \frac{e^2}{h}\sum_i^{k_z^{(R)}>0,\mu}\ \sum_j^{k_z^{(L)}>0,\mu}\left|t_{ij}\right|^2 = \frac{e^2}{h}\sum_{ij}\left|t_{ij}\right|^2 = \frac{e^2}{h}\sum_{ij}T_{ij}$$

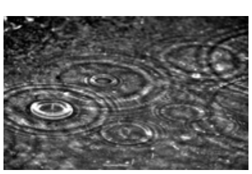
t_{ij} transmission coefficient (probability amplitude)

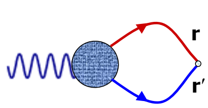
$T_{ij} = \left|t_{ij}\right|^2$ transmission probability



Semiclassics

Interference and semiclassical approximation





$$\langle\mathbf{j}(\mathbf{r})\rangle = \int_{\mathcal{V}} d\mathbf{r}'\mathbf{G}(\mathbf{r},\mathbf{r}')\cdot\mathbf{j}(\mathbf{r},\mathbf{r}')$$

Wigner representation:

$$\mathbf{G}(\mathbf{r},\mathbf{r}') \Rightarrow \mathbf{G}\left(\frac{\mathbf{r}+\mathbf{r}'}{2},\mathbf{r}-\mathbf{r}'\right)$$

$$\mathbf{G}(\mathbf{R},\mathbf{k}) = \int d(\mathbf{r}-\mathbf{r}')\mathbf{G}(\mathbf{R},\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}$$

Semiclassical approximation holds for disordered metals:

$$\mathbf{j}(\mathbf{R}) = \sum_{\mathbf{k}}\mathbf{G}(\mathbf{R},\mathbf{k})\cdot\mathbf{j}_{\mathbf{k}} \to \sum_{\mathbf{k}} f_{\mathbf{k}}(\mathbf{R})\cdot\mathbf{j}_{\mathbf{k}}$$

$$\mathbf{j}_{\mathbf{k}} = e\mathbf{v}_{\mathbf{k}} = \frac{e\partial\varepsilon_{\mathbf{k}}}{\partial\mathbf{k}} \to \frac{e\hbar}{m^*}\mathbf{k} \qquad f_{\mathbf{k}}(\mathbf{R}) > 0$$

Distribution functions

$f_{\mathbf{k}}(\mathbf{R},\delta t) = f_{\mathbf{k}+\frac{e\mathbf{E}}{\hbar}\delta t}(\mathbf{R}-\mathbf{v}_{\mathbf{k}}\delta t)$ electron distribution function

$\frac{df}{dt} = \left(\mathbf{v}_{\mathbf{k}}\cdot\frac{\partial}{\partial\mathbf{R}} + \frac{e\mathbf{E}}{\hbar}\frac{\partial}{\partial\mathbf{k}}\right)f_{\mathbf{k}}(\mathbf{R}) - \left(\frac{\partial f}{\partial t}\right)_{scatt} = 0$ steady state Boltzmann equation

$f_{\mathbf{k}}(\mathbf{R}) \approx f_0(\varepsilon,\mathbf{R})|_{\varepsilon=\varepsilon_{\mathbf{k}}} + \frac{\partial f_0(\varepsilon,\mathbf{R})}{\partial\varepsilon}g_{\mathbf{k}}(\mathbf{R})$ linearization

$f_0(\varepsilon,\mathbf{R}) = f_0^{(\alpha)}(\varepsilon,\mathbf{r})|_{\varepsilon=\varepsilon_{\mathbf{k}\alpha}} = (2\pi)^{-3}\int d\Omega_{\mathbf{k}}\frac{1}{e^{-(\varepsilon_{\mathbf{k}}-\mu(\mathbf{R}))/kT}-1}$ Fermi-Dirac distribution

$\frac{\partial f_0(\varepsilon,\mathbf{R})}{\partial\varepsilon} \xrightarrow{kT\ll\mu} -\delta(\varepsilon-\mu(\mathbf{R}))$ Fermi energy

$g_{\mathbf{k}}(\mathbf{R})$ deformation due to perturbation

Diffusion approximation

$$\left(\frac{\partial f}{\partial t}\right)_{\text{scatt}} = -\frac{g_{\mathbf{k}}(\mathbf{R})}{\tau_{\mathbf{k}}} + \sum_{\mathbf{k}'} W_{\mathbf{k}\mathbf{k}'} g_{\mathbf{k}'}(\mathbf{R}) \to -\frac{g_{\mathbf{k}}(\mathbf{R})}{\tau} + W\sum_{\mathbf{k}'} g_{\mathbf{k}'}(\mathbf{R}) = 0$$

$\tau_{\mathbf{k}} \approx \tau$ relaxation time approximation

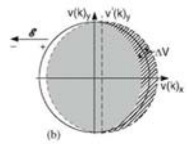
$W_{\mathbf{k}\mathbf{k}'} \approx W$ (short range impurity scattering)

$$\mathbf{v}_{\mathbf{k}} \cdot \left[\nabla_{\mathbf{R}} - e\mathbf{E}\frac{\partial}{\partial \varepsilon}\right] f_0(\varepsilon,\mathbf{R}) + \frac{\partial f_0(\varepsilon,\mathbf{R})}{\partial \varepsilon}\frac{g_{\mathbf{k}}(\mathbf{R})}{\tau} = 0$$

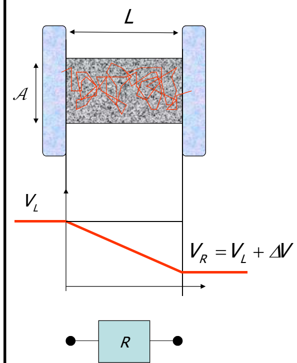
$$g_{\mathbf{k}}(\mathbf{R}) \approx -\tau \mathbf{v}_{\mathbf{k}} \cdot \left[\nabla \mu(\mathbf{R}) - e\mathbf{E}\right]$$

$$g_{\mathbf{k}} \to e\tau v_F E \cos\theta_k$$

$$\sigma \equiv \frac{e^2\tau}{3\mathcal{V}} \sum_{\mathbf{k}} v_{\mathbf{k}}^2 \left(-\frac{\partial f_0}{\partial \varepsilon_{\mathbf{k}}}\right) \to \frac{ne^2\tau}{m^*}$$



Diffusion in bulk 3D metal



Ohm's Law

$$\frac{\partial V(x)}{\partial x} = E = \rho j$$

$$\rho = \frac{m^*}{ne^2\tau} \quad \text{resistivity}$$

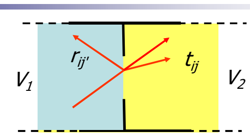
$$R = \frac{\rho L}{\mathcal{A}} \quad \text{resistance}$$

Current conservation

$$\frac{\partial}{\partial x}\frac{\partial \mu(x)}{\rho \partial x} = 0 \to \frac{eE}{\rho} = \text{const.}$$

Interfaces

Interfaces



$$I = G_{LB}^{\mathcal{I}}(V_1 - V_2)$$

$$G_{LB}^{\mathcal{I}} = \frac{e^2}{h}\sum_{ij}^{\varepsilon_i=\varepsilon_j=E_F} |t_{ij}|^2 = \frac{e^2}{h}\sum_{k=1}^{N} T_k$$

Landauer-Büttiker formula

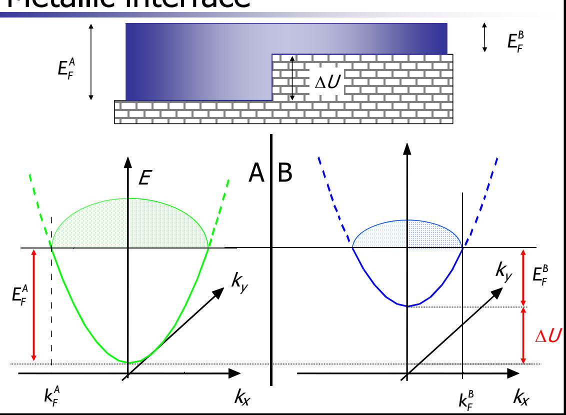
Ballistic point contact:

$$G_{LB} \xrightarrow{2=1} G_{Sharvin} = \frac{2e^2}{h}\sum_{ij}^{\varepsilon_i=\varepsilon_j=E_F} \delta_{ij} = \frac{2e^2}{h}\frac{\mathcal{A}}{(2\pi)^2}\int d\mathbf{k}_{\parallel} = \frac{2e^2}{h}\frac{\mathcal{A}k_F^2}{4\pi}$$

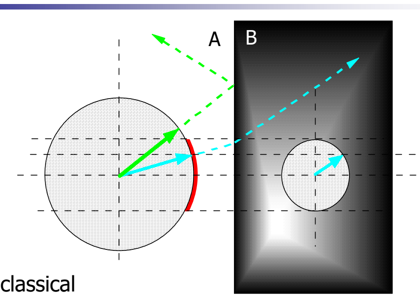
$$= \frac{2e^2}{h}N$$

Sharvin conductance

Metallic interface



Specular metallic interface



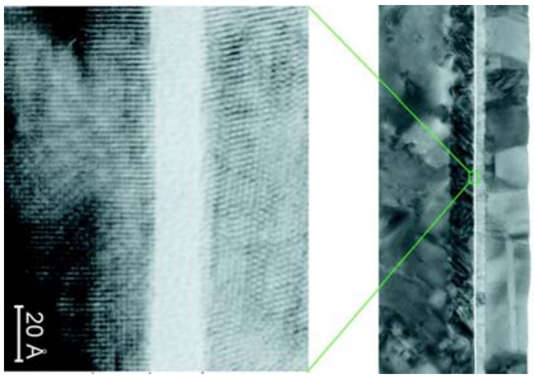
semiclassical approximation:

$$T_{\mathbf{k}_{\parallel},\mathbf{k}_{\parallel}'} \approx \delta_{\mathbf{k}_{\parallel},\mathbf{k}_{\parallel}'} \begin{cases} 1 & \text{for } E_{k_\perp} > \Delta U \\ 0 & E_{k_\perp} \le \Delta U \end{cases}$$

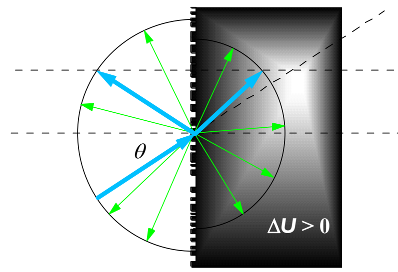
$$G = \frac{2e^2}{h}\frac{\mathcal{A}(k_F^A)^2}{4\pi}\left(1 - \frac{\Delta U}{E_F^A}\right)$$

$\mathcal{A}$ interface area

Disorder

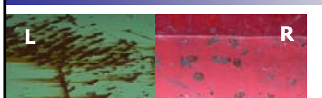


Interface roughness



"Specularity" < 1

Series resistor model

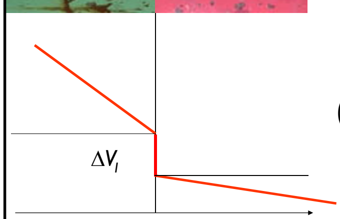


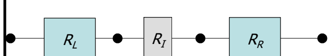
$$\frac{\partial V_{L/R}(x)}{\partial x} = E_{L/R} = -j\rho_{L/R}$$

$$j = -\frac{G_I}{A}\frac{\Delta\mu_I}{e}$$

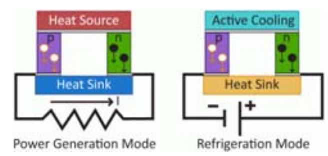
G_I interface conductance

$$\left(G^I\right)^{-1} = \left(G^I_{LB}\right)^{-1} - \frac{1}{2}\left(\left(G^A_{Sh}\right)^{-1} + \left(G^B_{Sh}\right)^{-1}\right)$$





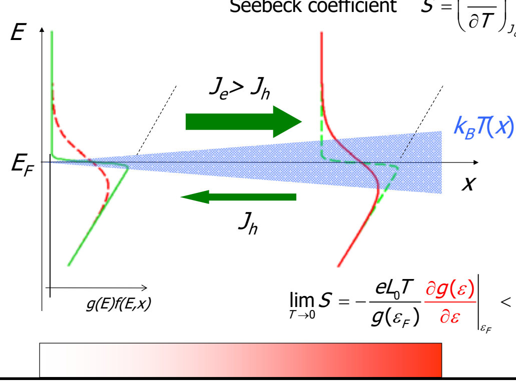
equivalent circuit



Thermoelectrics and Onsager

Heat and charge transport in metals

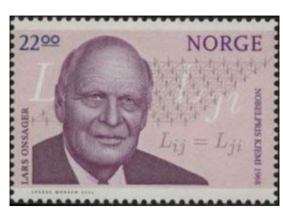
Seebeck coefficient $S = \left(\frac{\partial V}{\partial T}\right)_{J_c=0}$

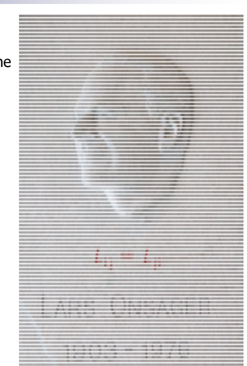


$$\lim_{T\to 0} S = -\frac{eL_0 T}{g(\varepsilon_F)}\left.\frac{\partial g(\varepsilon)}{\partial \varepsilon}\right|_{\varepsilon_F} < 0$$

Lars Onsager Memorial at NTNU Trondheim

The Nobel Prize in Chemistry 1968:
"for the discovery of the reciprocal relations bearing his name, which are fundamental for the thermodynamics of irreversible processes"





Onsager symmetry (1931)

i = {mass, charge, energy, volume, (angular) momentum, …}

X_i generalized forces

J_i generalized currents

$J_m = \sum_n L_{mn} X_n$ linear response

If: $\dot{S} = \sum_i X_i J_i$ entropy creation rate

then: $L_{ij} = L_{ji}$ **Onsager relations**

When time reversal symmetry is broken:

$$L_{ij}(\mathbf{m}, \mathbf{H}_{ext}) = \varepsilon_i \varepsilon_j L_{ji}(-\mathbf{m}, -\mathbf{H}_{ext})$$

$$\varepsilon_i = \begin{cases} 1 \\ -1 \end{cases} \text{when variable } i \begin{matrix} \text{even (charge)} \\ \text{odd (spin)} \end{matrix}$$

I[st] and 2[nd] Law of Thermodynamics

$$dU_n = dQ_n + dW_n = T_n dS_n + \mu_n dN_n$$

$$dS_n = \frac{dU_n}{T_n} - \frac{\mu_n}{T_n} dN_n$$

$$dU_1 = -dU_2 \equiv dU; \quad dN_1 = -dN_2 \equiv dN$$

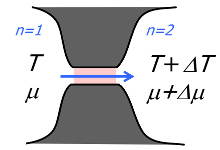
$$J_U = -\frac{dU}{dt}; \quad J_N = -\frac{dN}{dt}$$

$$dS = dS_1 + dS_2; T_1 = T; T_2 = T + \Delta T$$

$$\frac{dS}{dt} = \left(\frac{1}{T+\Delta T} - \frac{1}{T}\right) J_U + \left(\frac{\mu}{T} - \frac{\mu + \Delta\mu}{T + \Delta T}\right) J_N$$

$$T\frac{dS}{dt} \approx (J_U - \mu J_N)\left(-\frac{\Delta T}{T}\right) - J_N \Delta\mu$$

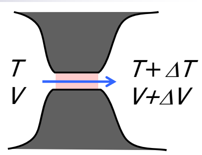
$$= J_Q\left(-\frac{\Delta T}{T}\right) + J_N(-\Delta\mu) = \sum_i J_i X_i \geq 0$$



conjugate currents and forces identified!

Thermoelectrics

$$\begin{pmatrix} J_c \\ J_Q \end{pmatrix} = \begin{pmatrix} L_{11} & L_{21} \\ L_{12} & L_{22} \end{pmatrix} \begin{pmatrix} \Delta V \\ -\frac{\Delta T}{T} \end{pmatrix}$$



$L_{12} = L_{21}$ Onsager reciprocity

$$\begin{pmatrix} \Delta V \\ J_Q \end{pmatrix} = \begin{pmatrix} R & S \\ \Pi & K \end{pmatrix} \begin{pmatrix} J_c \\ -\Delta T \end{pmatrix}$$

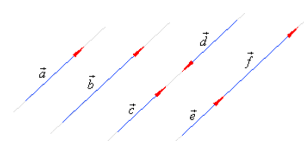
R = $1/G$ electrical resistance

K thermal conductance

S Seebeck coefficient

Π Peltier coefficient

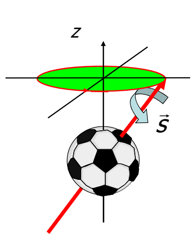
$\Pi = ST$ Onsager-Thomson (Kelvin) relation



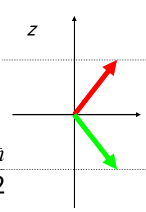
Collinear spin transport

Electron spin and quantum mechanics

$$[s_x, s_y] = i\hbar s_z$$



$$s_z = \frac{\hbar}{2} \quad |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$s_z = -\frac{\hbar}{2} \quad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$S = \sqrt{|\vec{s}|^2} = \frac{\sqrt{3}}{2}\hbar$$

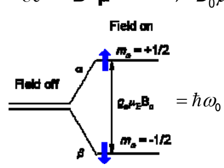
Electron spin vector operator

$$\boldsymbol{\mu} = g_e \frac{-|e|}{2m} \mathbf{s} = -\gamma \mathbf{s} = -g_e \mu_B \frac{\boldsymbol{\sigma}}{2} \qquad \mu_B = \frac{|e|\hbar}{2m} \text{ Bohr magneton}$$

$$\mathbf{s} = \frac{\hbar}{2}\boldsymbol{\sigma} \qquad \gamma = g_e \frac{\mu_B}{\hbar}$$

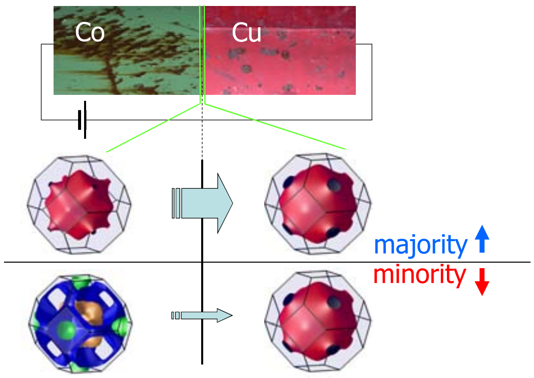
$$\boldsymbol{\sigma} = \left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \text{ Pauli matrices}$$

$$\mathcal{H} = -\mathbf{B} \cdot \boldsymbol{\mu} \xrightarrow{\mathbf{B} \| \hat{z}} -B_0 \mu_z = -\frac{g_e \mu_B B_0}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

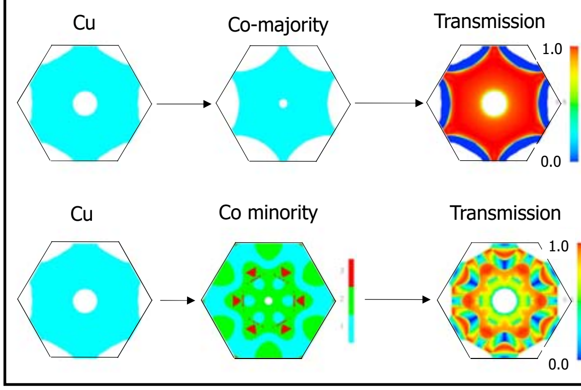


$$= \hbar\omega_0 \qquad \mathcal{H}\left|m_s = \pm\frac{1}{2}\right\rangle = \pm\hbar\omega_0 \left|m_s = \pm\frac{1}{2}\right\rangle$$

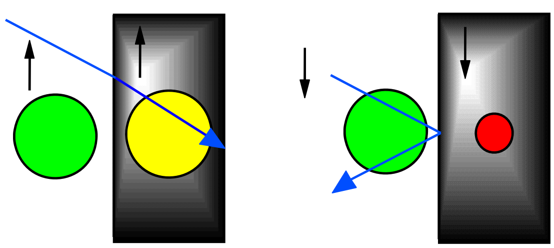
Spin-dependent interface transmissivity



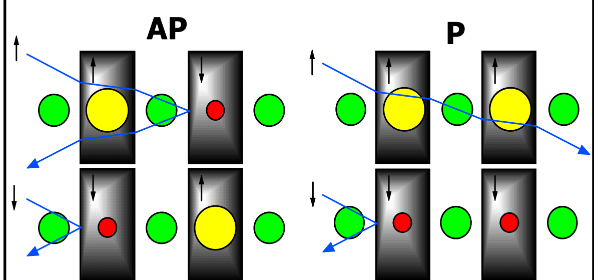
Transmission probabilities (Xia *et al.*, 2002)



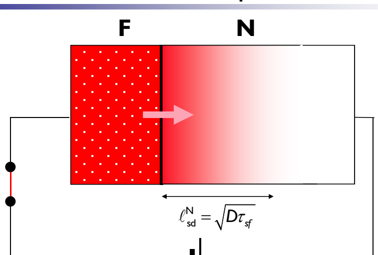
Spin dependent interface transmission



Kindergarten model of GMR



Spin-accumulation and spin-current

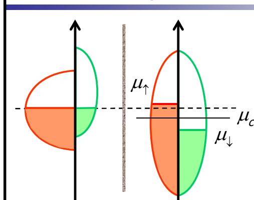


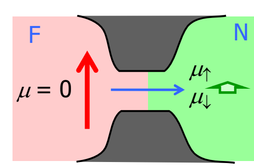
ℓ_{sd}^{N} spin diffusion length

D diffusion constant

τ_{sf} spin-flip relaxation time

Interface spin currents





$$e\begin{pmatrix}J_\uparrow\\J_\downarrow\end{pmatrix} = -\begin{pmatrix}G_\uparrow & 0\\0 & G_\downarrow\end{pmatrix}\begin{pmatrix}\mu_\uparrow\\\mu_\downarrow\end{pmatrix}$$

$$e\begin{pmatrix}J_c\\J_s\end{pmatrix} = -G\begin{pmatrix}1 & P\\P & 1\end{pmatrix}\begin{pmatrix}\mu_c\\\mu_s\end{pmatrix}$$

$$G_{\uparrow(\downarrow)} = \frac{e^2}{h}\sum_{nm}^{\varepsilon_n,\varepsilon_m=E_F}\left|t_{nm}^{\uparrow(\downarrow)}\right|^2$$

$\mu_c = \frac{\mu_\uparrow + \mu_\downarrow}{2}$ (charge) chemical potential

$\mu_s = \mu_\uparrow - \mu_\downarrow$ spin accumulation

$G_c = G_\uparrow + G_\downarrow$ (charge) conductance

$P = \frac{G_\uparrow + G_\downarrow}{G_\uparrow - G_\downarrow}$ (conductance) spin polarization

$J_c = J_\uparrow + J_\downarrow$ charge current

$J_s = J_\uparrow - J_\downarrow$ spin current

Spin diffusion in bulk metal (Valet-Fert)

Ohm's Law $j_s = -\sigma \frac{\partial \mu_s(x)}{\partial x}, \quad s = \{\uparrow, \downarrow\}$

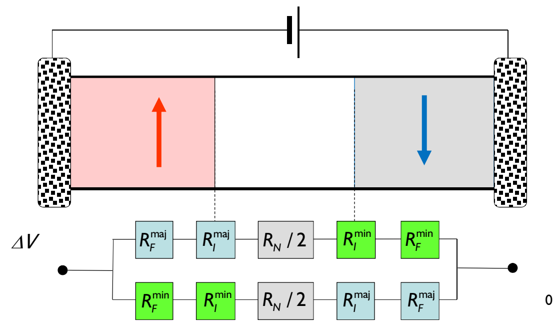
spin relaxation $\frac{\partial (j_\uparrow - j_\downarrow)}{\partial x} = \frac{e(n_\uparrow - n_\downarrow)}{\tau_{sf}}$

spin-flip diffusion $\frac{\partial^2 (\mu_\uparrow - \mu_\downarrow)(x)}{\partial x^2} = \frac{(\mu_\uparrow - \mu_\downarrow)(x)}{\ell_{sd}^2}$

$\ell_{sd} = \sqrt{D\tau_{sf}}$ spin diffusion length

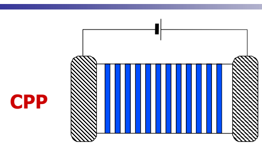
$D = \frac{1}{3} v_F^2 \tau$ averaged diffusion constant

Two-channel series resistor model



$R_F^{maj} = \frac{d_F \rho_F^\uparrow}{\mathcal{A}}$

Two-channel series resistor model



$\mathcal{A} R_s^T = N_{bl} \left(\rho_s^{(F)} d_F + \frac{\rho^{(N)} d_N}{2} + 2\mathcal{A} R_s^{(F|N)} \right)$

$\mathcal{A}$ cross section of sample

N_{bl} number of bilayers

$\rho_\uparrow^{(F)}$ spin-up resistivity of F

$R_\uparrow^{(F|N)}$ spin-up interface resistance

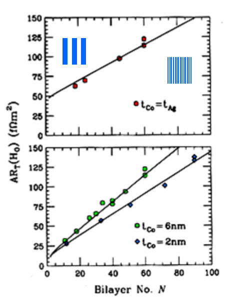
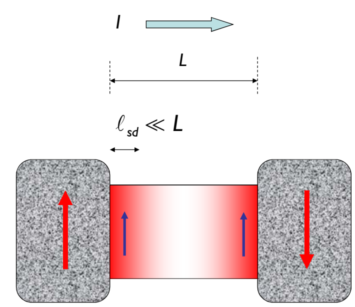


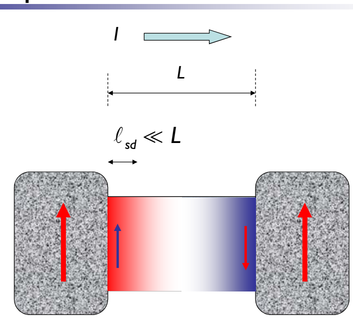
Fig. 2. $AR_T(H_0)$ versus bilayer number N for three different sets of Co/Ag multilayers, all with fixed total thickness $t \approx 720$ nm. The curves shown are fits to the two-channel model

MSU group in the 90's

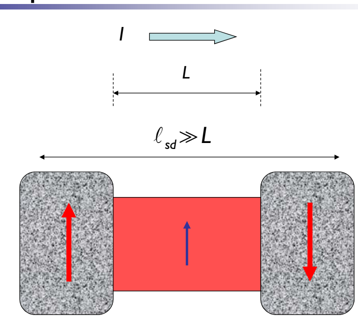
F|N|F spin valves



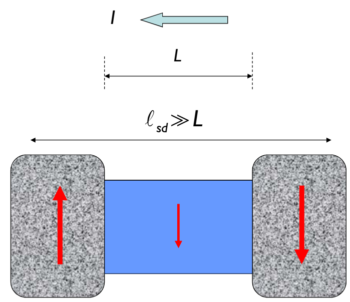
F|N|F spin valves



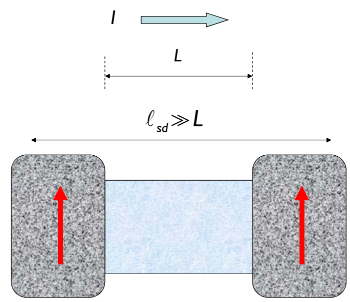
F|N|F spin valves



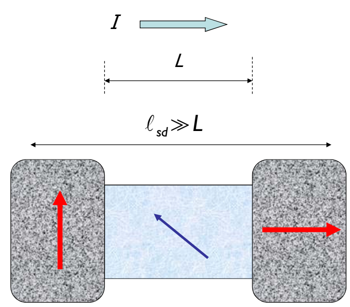
F|N|F spin valves



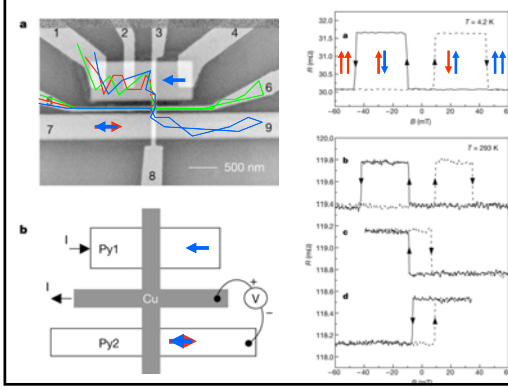
F|N|F spin valves



F|N|F spin valves: **non-collinear**



Jedema *et al.* (2001)



Tombros et *al*. (2007)