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THE EUROPEAN SCHOOL ON MAGNETISM 2015

FROM BASIC MAGNETIC CONCEPTS TO SPIN CURRENTS

Quantum basis of the spin manipulation by electric fields

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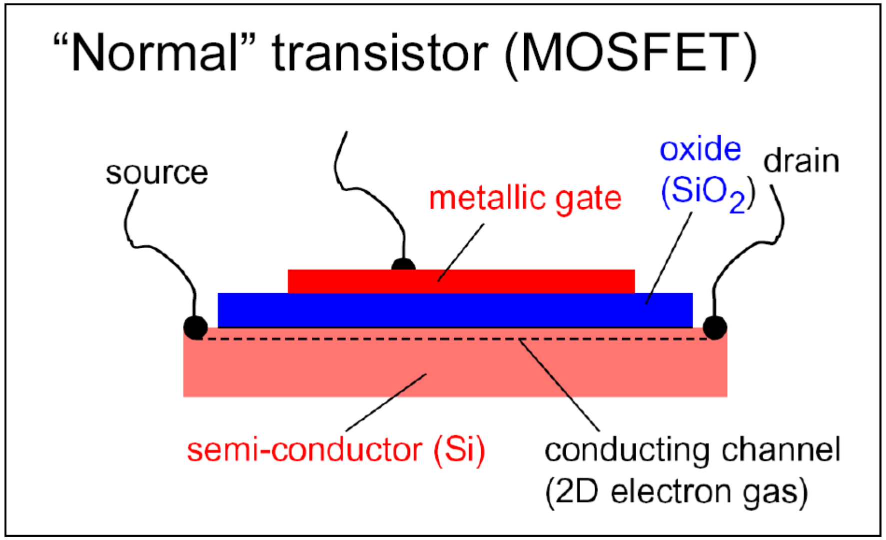
Ideea: Datta and Das Transistor

S. Datta and B. Das (1990) „*Electronic analog of the electro-optic modulator*"
Applied Physics Letters 56 (7): 665–667. (1990)

(1)



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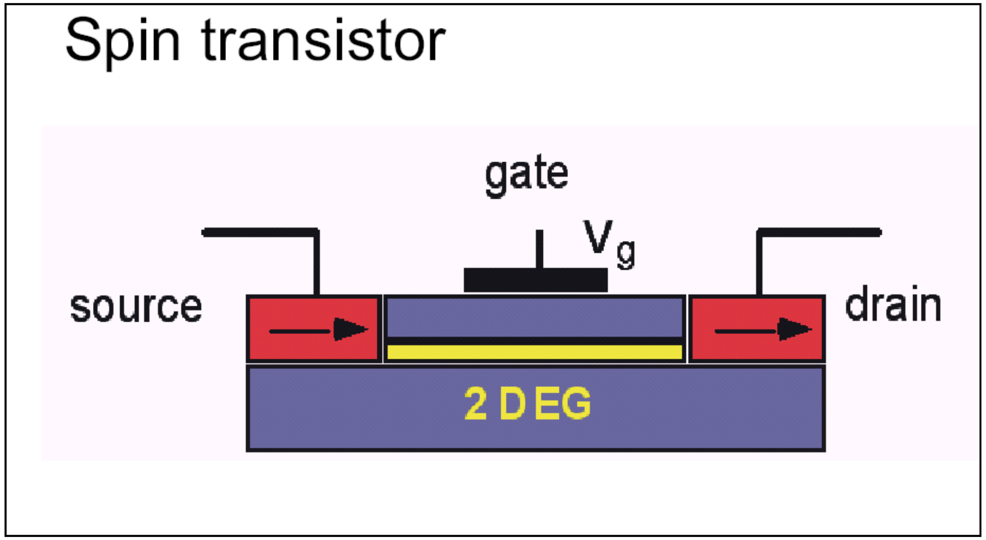
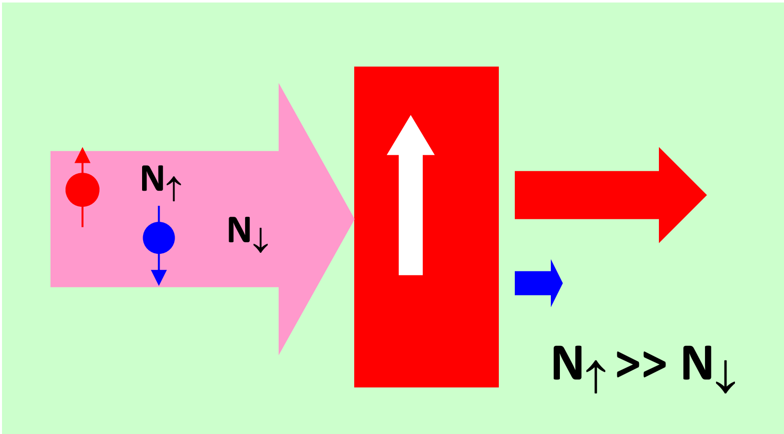


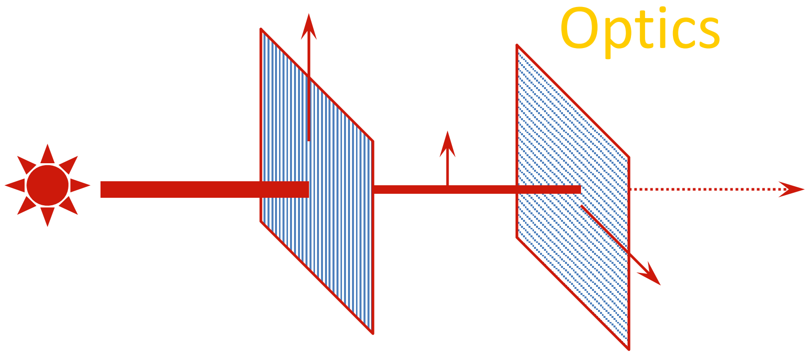
- ❑ Gate potential controls the source-drain current
- ❑ Used as modulator, amplifier, switch

- ❑ Source and drain = FM materials
- ❑ Conducton channel = 2DEG
- ❑ The gate electric field moduletes the electron spin state
- ❑ No external magnetic field

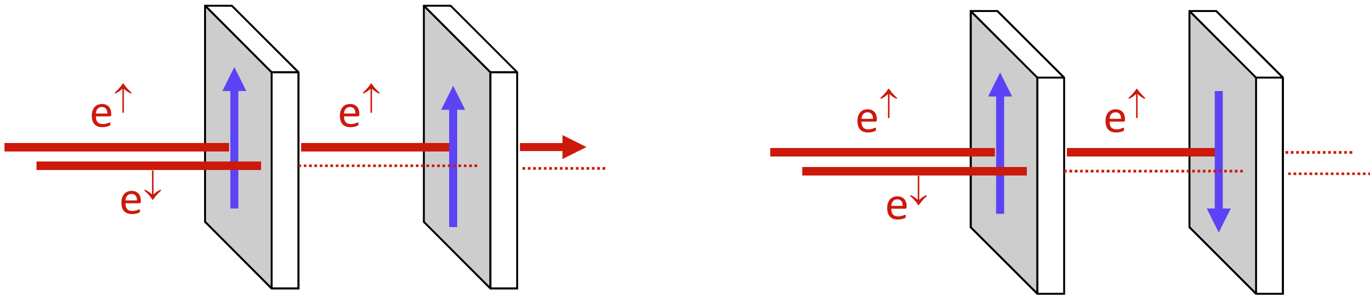
Take advantage of the electron spin as a new degree of freedom
to generate new functionalities and devices

Basic ideea: Magnetic materials can be used as **Polarizer** and **Analyzer** of electrons (spin filters)





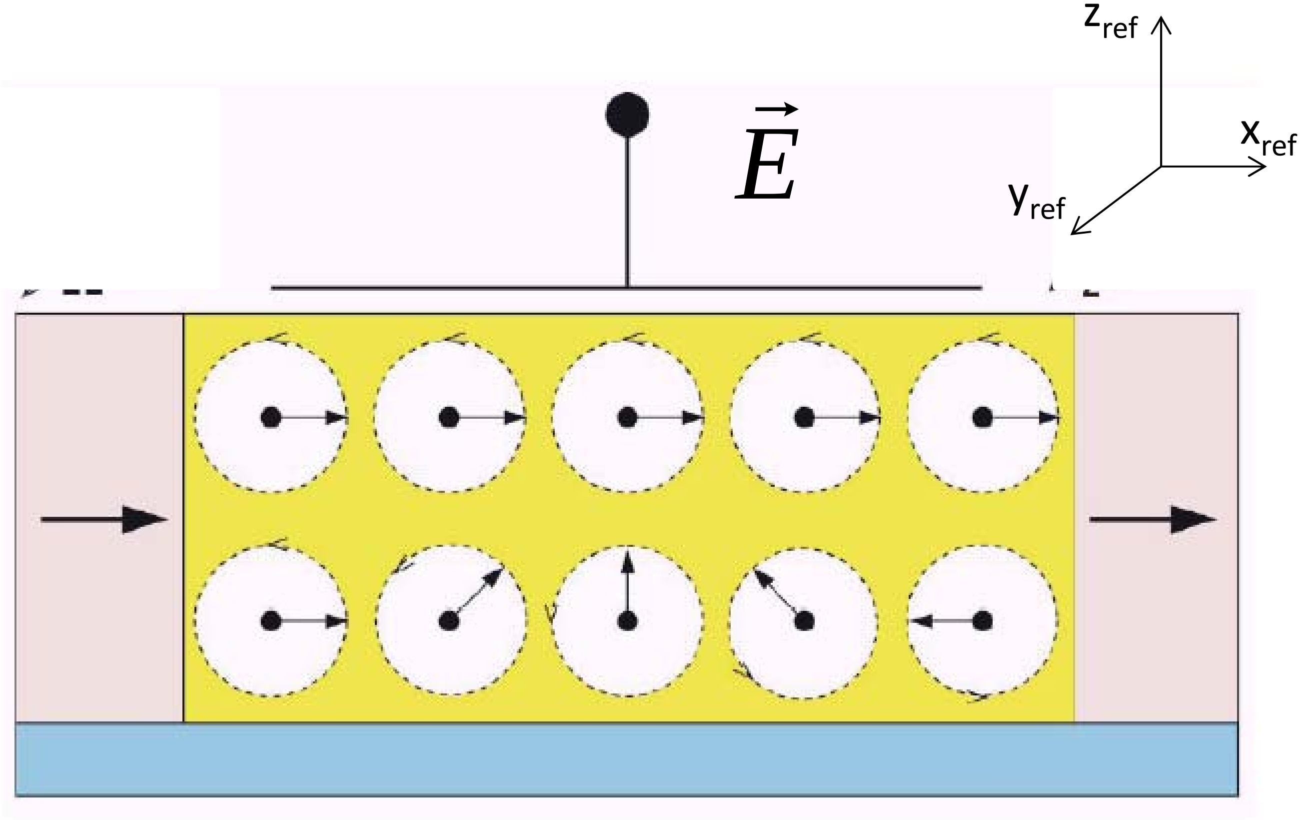
Spin filters



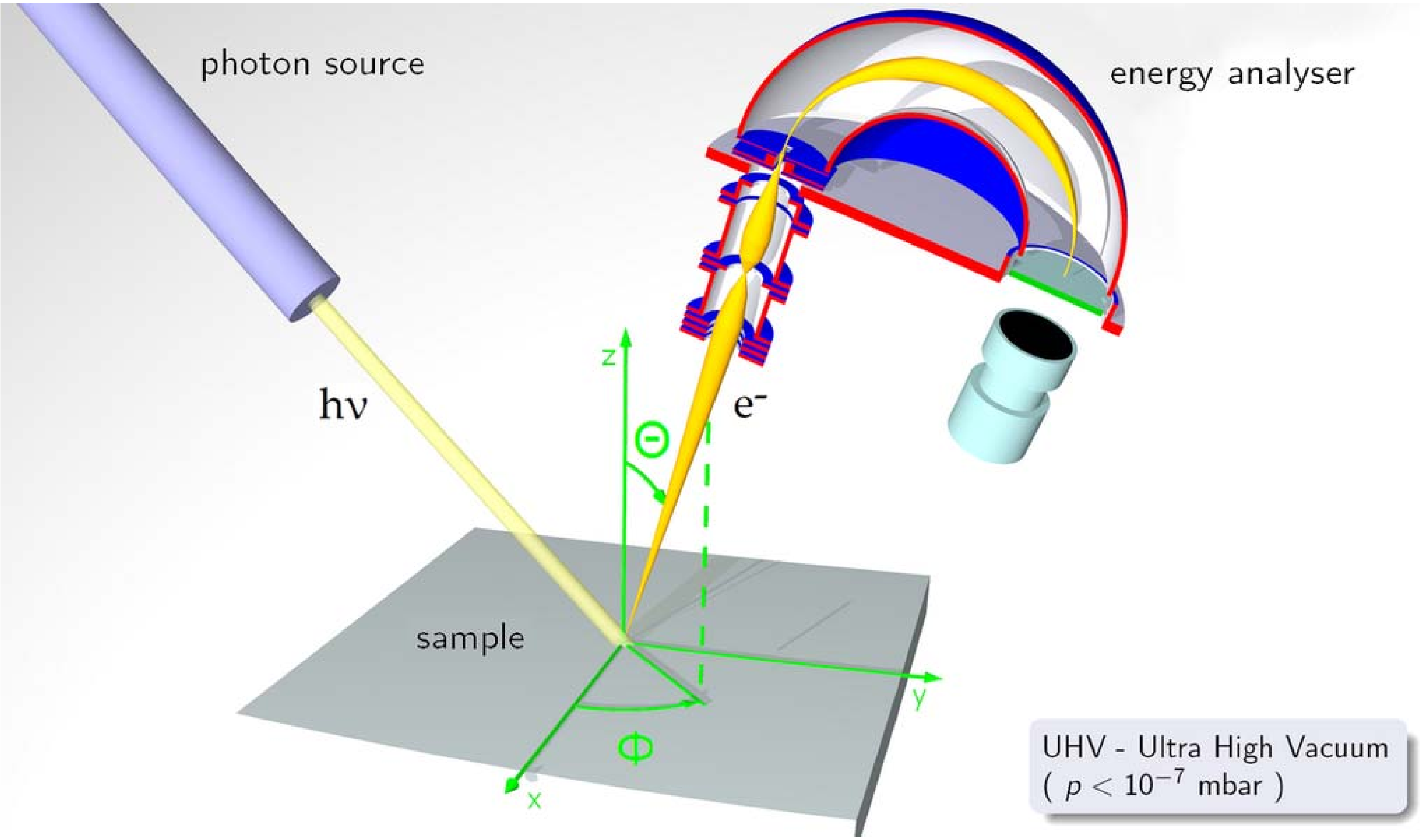
$$s_{\downarrow\downarrow} > s_{\downarrow\uparrow} \Rightarrow r_{\downarrow\downarrow} < r_{\downarrow\uparrow}$$

However, spin currents can be generated otherwise (spin-orbitronics, spin caloritronics...)...

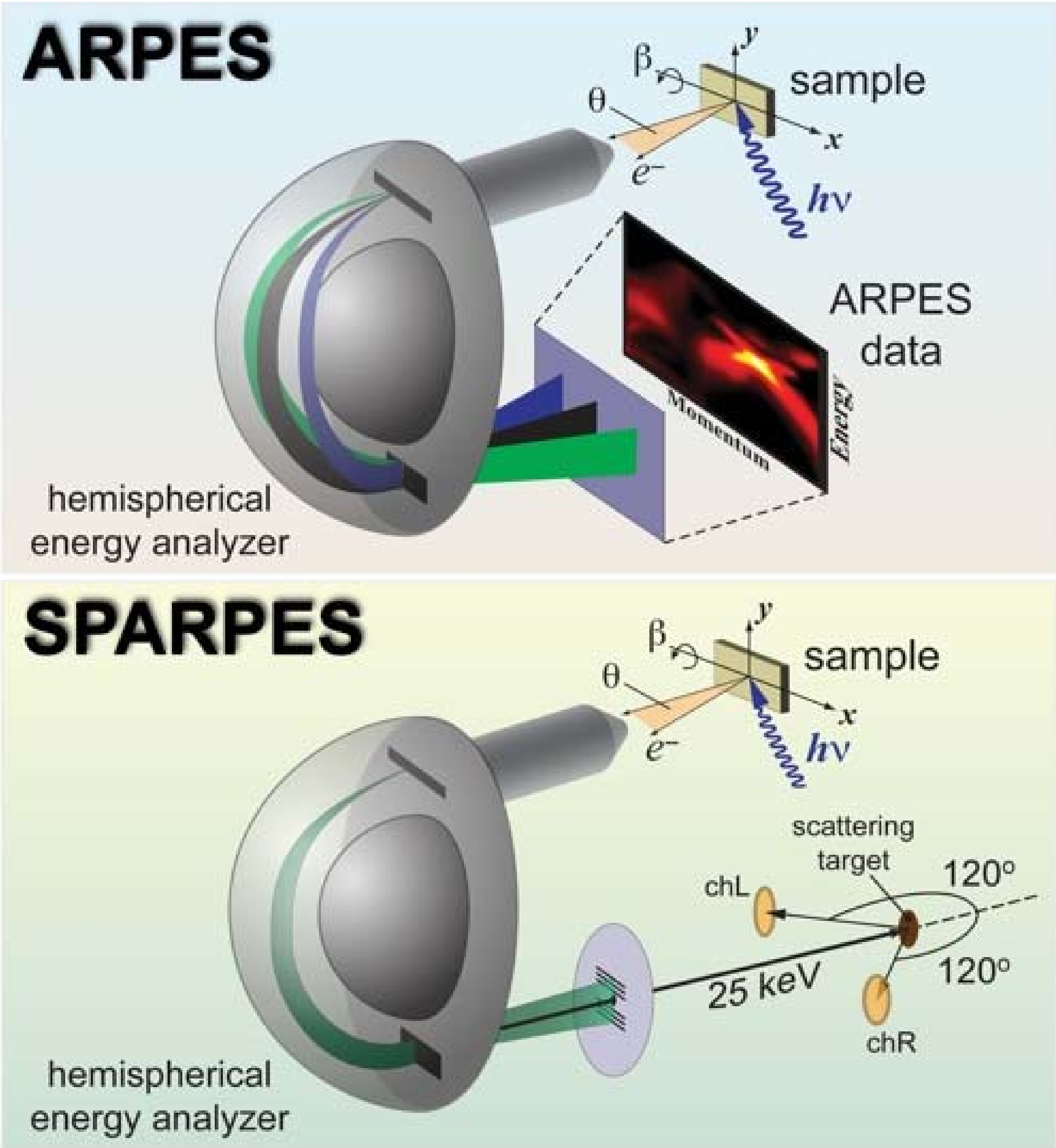
Spin FET



Photoemission/Angle-resolved photoemission spectroscopy (ARPES)

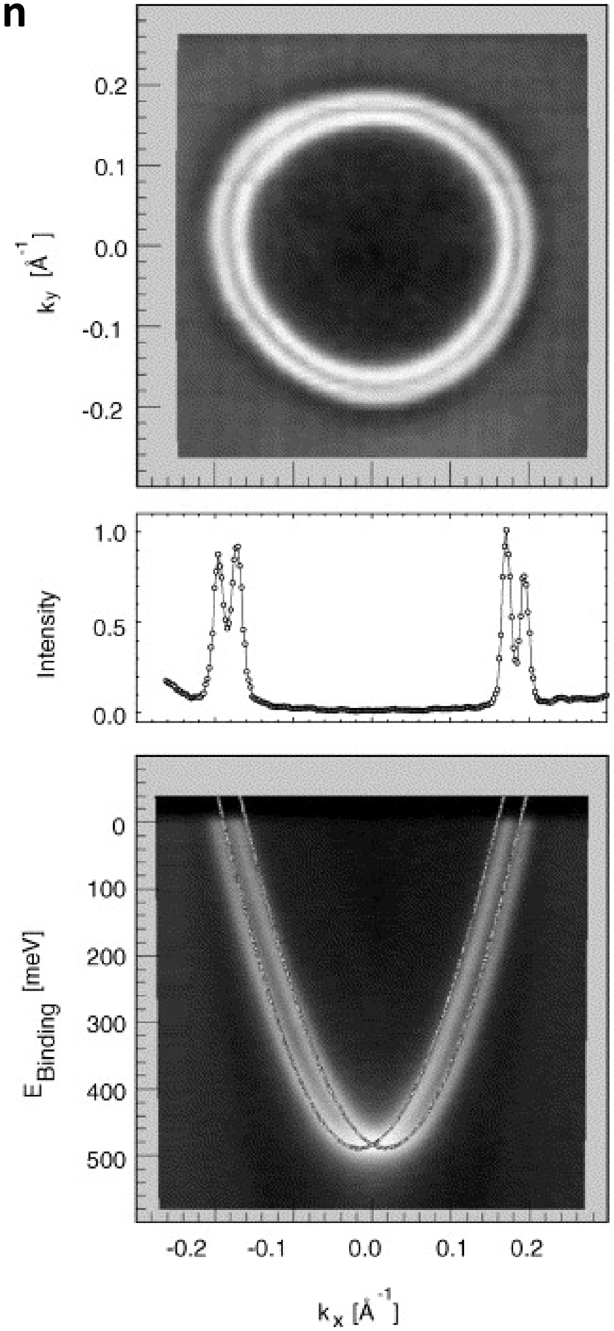


-based on photoelectric effect (X, UV) => XPS, UPS
- direct experimental technique to observe the distribution of the electrons in the reciprocal space of solids = E(k) for occupied valence states



Electron energy analyzers in ARPES use 2D CCD detector allowing to get the E(k)of the valence band states in a wide range of emission angles theta in one shot (at a fixed angle beta). The emission angle theta can be used for the calculation of wave vector component k_x of electron in solid. A rotation of the sample by angle beta produces the 3D data set of experimental photoemission intensity, $I(E_{kin},k_x,k_y)$, where E_{kin} is the kinetic energy of electron and k_y is the second in-plane component of the wave-vector calculated from the experimental geometry. In the case of spin-resolved ARPES experiments the 2D CCD detector is replaced by a spin-detector (e.g. classical Mott). This allows an effective separation of the spin-polarized electron beam into two channels: spin-up and spin-down electrons

Photoemission

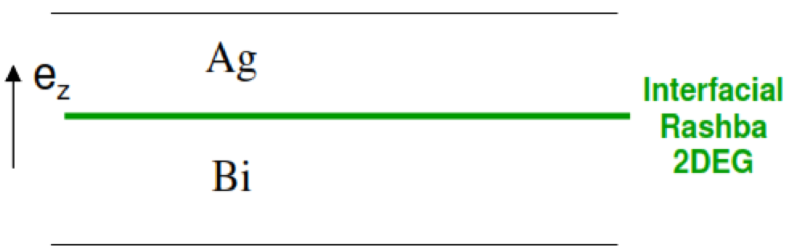


Electron photoemission experiments on a 2D electron (Shockley) state on Au(111) surface –*G. Nicolay, F. Reinert, S. Hufner, P. Blaha, Phys. Rev. B 65 (2002) 033407* respectiv *F. Reinert, G. Nicolay, S. Schmidt, D. Ehm, S. Hufner, Phys. Rev. B 63 (2001) 115415.* The experimens are taken at 30K. Bottom panel: the band structure $E(k_x)$ along the $\overline{\Gamma M}$ direction.
Middle panel: cut on (k_x, k_y) representation (top panel) at $k_y = 0$

Measuring Δk => Rashba constant α

- α_R from ARPES for materials with important potential in spin-orbitronics (when materials with significant SO are used for generation of spin currents by spin-Hall effects).

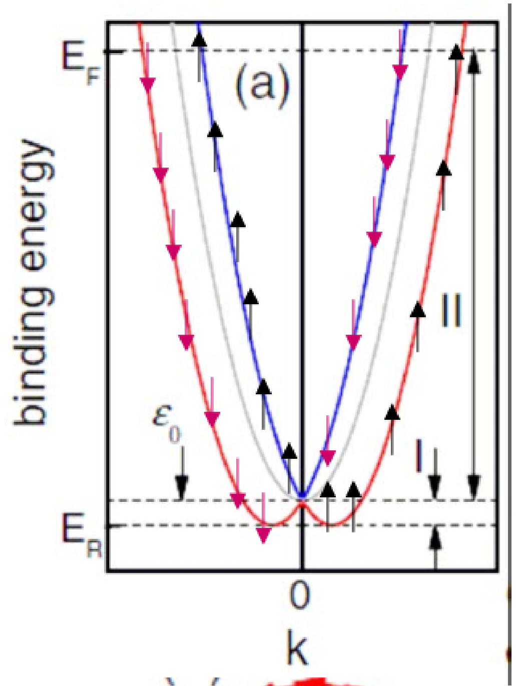
Rashba effect at interfaces or surfaces of metals

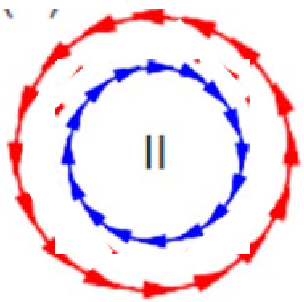


$$\hat{H}_{SO} = \alpha_R \boldsymbol{\sigma} \cdot (\boldsymbol{k}_{\|} \times \mathbf{e}_z), \quad \alpha_R \sim \frac{\partial V}{\partial z}$$

Bi/Ag(111): α_R = 3.05 eVÅ

Material	E_R (meV)	k_0 (Å^{-1})	α_R (eV Å)
InGaAs/InAlAs heterostructure	<1	0.028	0.07
Ag(111) surface state	<0.2	0.004	0.03
Au(111) surface state	2.1	0.012	0.33
Bi(111) surface state	~14	~0.05	~0.56
Bi/Ag(111) surface alloy	200	0.13	3.05





Fermi circles

From A. Fert

Plan

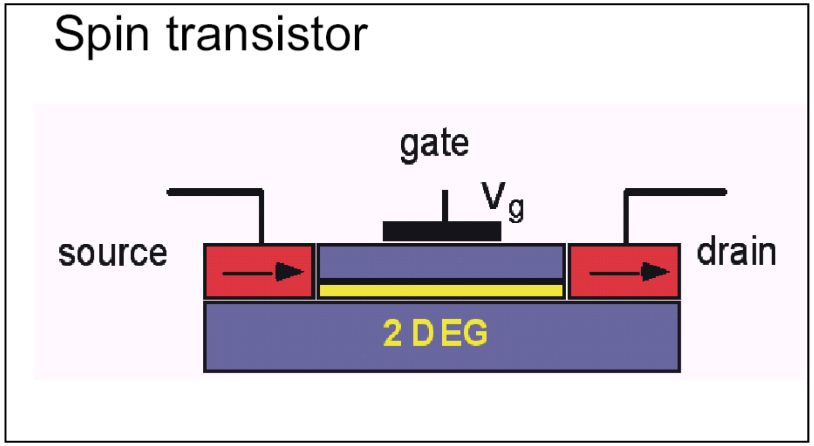
- Starting from the non-relativistic Dirac Hamiltonian, written for the case of a 2D free- electron gas with a confinement direction perpendicular to the propagation direction, we deduce the Rashba Hamiltonian. This strategy allows the direct identification of the Rashba interaction term and interaction constant alpha, as a measure of the spin-orbit interaction. One can thus understand how alpha can be controlled via the external electric field (in Datta-Das spin transistor geometry).

- Within the Heisenberg-Dirac formalism, we solve the stationary Schrodinger equation by diagonalising the spin-orbit Hamiltonian and find the eigenvalues and the stationary eigenfunctions.
  - Analyse the spin-orbit influence on the calculated parabolic *E(k)* band structure and discuss how the spin-orbit constant alpha can be extracted from ARPES experiments.
  - Illustrate with some examples of ARPES for materials with important potential in spin-orbitronics (when materials with significant SO are used for generation of spin currents by spin-Hall effects).

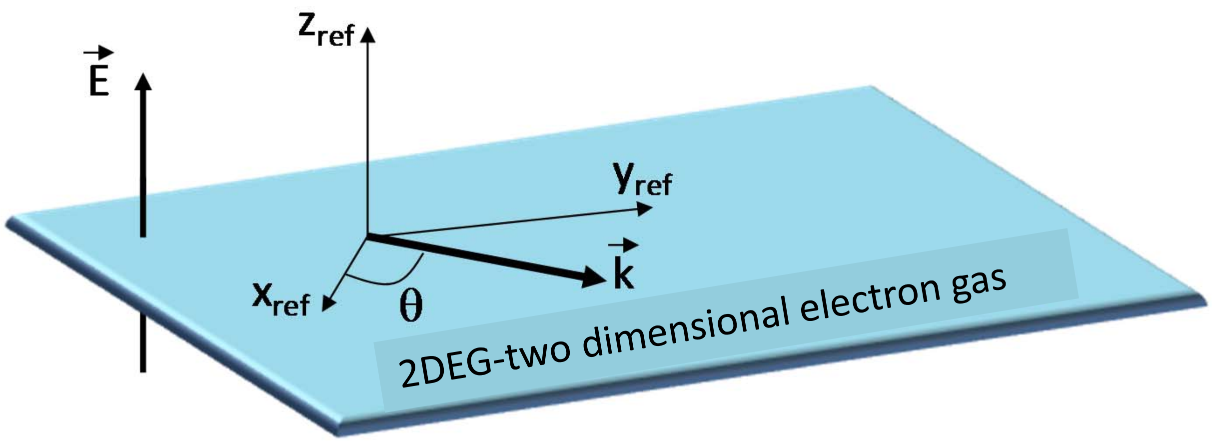
- Furthermore, we study the time evolution, solving the time dependent Schrodinger equation. Then, by calculating average values of the spin operators Sx, Sy, Sz we can demonstrate and discuss the spin precession .

Problem

- ❑ Consider the Datta@Das transitor (Fig.)
- ❑ $(x_{ref}, y_{ref}, z_{ref})$ = lab referential; z_{ref} II **E** and $z_{ref} \perp$ 2DEG plane



- ❑ Source and drain = FM materials
- ❑ Conducton channel = 2DEG
- ❑ The gate electric field moduletes the electron spin state
- ❑ No external magnetic field

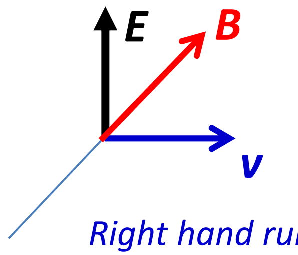


- The spin precession in external electric field is related to the **spin-orbit interaction** in the 2DEG (Rashba effect)

- The **origin** of the SPIN –ORBIT interaction is **relativistic**

- An electron moving with the velocity **v** in an external field **E** will fill in its own referential an effective magnetic field perpendicular on the direction of moving :

$$\vec{B} = -\frac{\vec{v} \times \vec{E}}{c^2}$$



This magnetic field will lead to the spin Larmor precession

Right hand rule

START

- The Hamiltonian describing the S-O interaction is obtained from the non-relativistic limit of the Dirac equation

$$H_{SO} = \frac{\hbar}{(2m_0c)^2} \vec{\nabla} V \cdot (\hat{\sigma} \times \hat{p})$$

Non-relativistic Dirac Hamiltonian

$$\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$$

Pauli matrices

For a central potential

$$H_{SO} = \lambda \vec{L} \cdot \vec{S}$$

$\vec{L}$ Orbital momentum

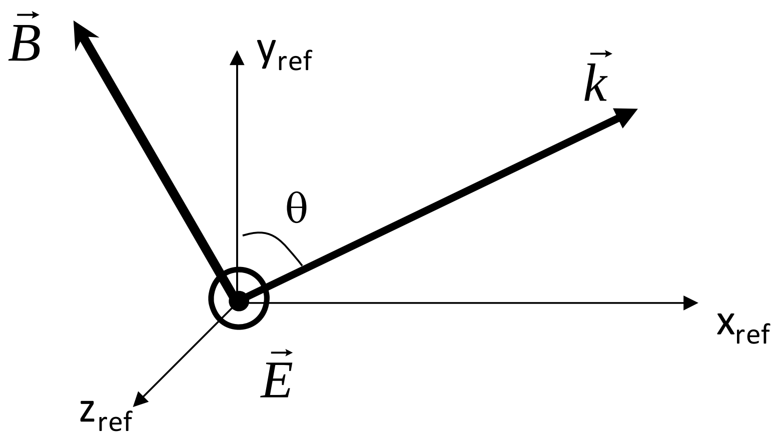
$\vec{S}$ Spin momentum

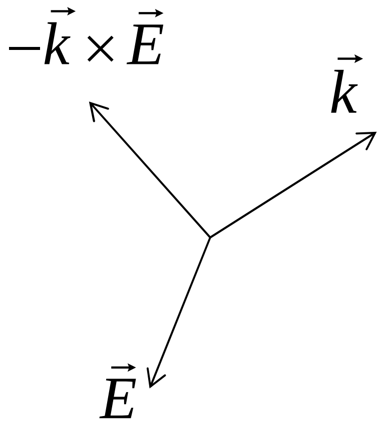
=> Gives the name of the S-O interaction

(I) Starting from the non-relativistic Dirac Hamiltonian, written for the case of a 2D free- electron gas with a confinement direction perpendicular to the propagation direction, we deduce the Rashba Hamiltonian. Then we identify the SO (Rashba) constant.

- Consider the Datta-Das transistor geometry with ***E*** along OZ_{ref}

$$\vec{B}_{ef} = -\frac{\vec{v}\times\vec{E}}{c^2}$$





$$H_{SO} = \frac{\hbar}{(2m_0c)^2}\vec{\nabla}V\cdot(\hat{\sigma}\times\hat{p})$$

Considering the 2DEG with the confinement direction perpendicular to the propagation direction, one can calculate the vector product:

$$\hat{\sigma}\times\hat{p} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \hat{\sigma}_x & \hat{\sigma}_y & \hat{\sigma}_z \\ p_x & p_y & p_z \end{vmatrix}$$

$$\hat{\sigma}\times\hat{p}=\hat{x}\left(p_z\hat{\sigma}_y-p_y\hat{\sigma}_z\right)-\hat{y}\left(p_z\hat{\sigma}_x-p_x\hat{\sigma}_z\right)+\hat{z}\left(p_y\hat{\sigma}_x-p_x\hat{\sigma}_y\right)$$

If $\vec{E}=-\vec{\nabla}V$ is applied along OZ_{ref} axis perpendicular to the 2DEG plane, we would have:

$$\vec{\nabla}V\cdot\left(\hat{\sigma}\times\hat{p}\right)=-\frac{\partial V}{\partial z}\left(\hat{p}_x\hat{\sigma}_y-\hat{p}_y\hat{\sigma}_x\right)\quad\Rightarrow\quad H_{SO}=-\frac{\hbar}{(2m_0c)^2}\frac{\partial V}{\partial z}\left(\hat{p}_x\hat{\sigma}_y-\hat{p}_y\hat{\sigma}_x\right)$$

Rashba Hamiltonian

Within the **free-electron approach**, the total Hamiltonian of an electron with the mass m= Σ **(K+SO)**

$$\hat{H}_{SO}=\frac{\hat{p}_x^{\,2}+\hat{p}_y^{\,2}}{2m}-\frac{\hbar}{(2m_0c)^2}\frac{\partial V}{\partial z}\left(\hat{p}_x\hat{\sigma}_y-\hat{p}_y\hat{\sigma}_x\right)\Leftrightarrow\hat{H}_{SO}=\frac{\hbar^2}{2m}\left(\hat{k}_x^{\,2}+\hat{k}_y^{\,2}\right)-\frac{\hbar^2}{(2m_0c)^2}\frac{\partial V}{\partial z}\left(\hat{k}_x\hat{\sigma}_y-\hat{k}_y\hat{\sigma}_x\right)$$

We denote:

$$\alpha_R=\frac{\hbar^2}{(2m_0c)^2}\frac{\partial V}{\partial z}$$

SO interaction constant (Rashba constant)

- is a measure of the spin-orbit interaction.
- alpha can be controlled via the external electric field (in Datta-Das spin transistor geometry).

$$\Rightarrow\quad\hat{H}_{SO}=\frac{\hbar^2}{2m}\left(\hat{k}_x^{\,2}+\hat{k}_y^{\,2}\right)-\alpha_R\left(\hat{k}_x\hat{\sigma}_y-\hat{k}_y\hat{\sigma}_x\right)$$

TOTAL
Rashba Hamiltonian

Discussion:

SO interaction constant (Rashba constant)

$$\alpha_R = \frac{\hbar^2}{(2m_0c)^2}\frac{\partial V}{\partial z}$$

$$-\frac{\partial V}{\partial z} = E$$

- The larger is the ***E*** felt by the electron, the larger is the SO-coupling
- In case of atom, ***E ~Ze*** => SO larger for heavy atoms: Au (Z=79), Pt (Z=78), Pd (Z=46) than for 3D atoms: Cr (Z=24), Fe (Z=26),Co (Z=27), Ni (Z=28).
- SO-coupling exacerbated at the metal surfaces: the breaking of the translational symmetry in surface is equivalent to a potential gradient felt by the electron => electric field.

(II) Within the Heisenberg-Dirac formalism, we solve the stationary Schrodinger equation by diagonalising the spin-orbit Hamiltonian and find the eigenvalues and the stationary eigenfunctions.

START:

$$\hat{H}_{SO} = \frac{\hbar^2}{2m}\left(\hat{k}_x^{\,2} + \hat{k}_y^{\,2}\right) - \alpha_R\left(\hat{k}_x\hat{\sigma}_y - \hat{k}_y\hat{\sigma}_x\right)$$

TOTAL
Rashba Hamiltonian

The $\hat{k}_x, \hat{k}_y$ operator commute with $\hat{H}$ then the eigenfunctions of the system can be:

$$|\Psi\rangle = e^{i(k_x x + k_y y)}\left(C_1|+\rangle + C_2|-\rangle\right) = e^{i\vec{k}_{\parallel}\vec{r}}\left(C_1|+\rangle + C_2|-\rangle\right)$$

$\left(|+\rangle, |-\rangle\right)$

- Represent the UP and DN states of the z component of the spin
- They are orthonormal

Obs: The **z** direction in the electron referential is given by the direction of the effective magnetic field $B_{eff.}$ This represents the magnetic field quantization axis and leads to diagonal σ_z .

- Within this representation we have:

(see Blundel 2)

$$\hat{\sigma}_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \hat{\sigma}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \hat{\sigma}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The matrix form of the Hamiltonian: $\hat{H} = \hat{H}_{FP} + \hat{H}_{SO}$

Free-particle term: $\hat{H}_{ij} = \langle i | \frac{\hbar^2 k_{\parallel}^{\ 2}}{2m} | j \rangle$ $\qquad |i\rangle, |j\rangle = |+\rangle, |-\rangle$

and take into account the orhonormized condictions

$$\Rightarrow \qquad \hat{H}_{FP} = \begin{pmatrix} \frac{\hbar^2 k_{\parallel}^{\ 2}}{2m} & 0 \\ 0 & \frac{\hbar^2 k_{\parallel}^{\ 2}}{2m} \end{pmatrix}$$

Spin-orbit term:

$$\hat{H}_{SO} = -\alpha_R \left[k_x \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} - k_y \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & \alpha_R (k_y + i k_x) \\ \alpha_R (k_y - i k_x) & 0 \end{pmatrix}$$

Total Hamiltonian:

$$\hat{H} = \begin{pmatrix} \frac{\hbar^2 k_{\parallel}^{\ 2}}{2m} & \alpha_R \left(k_y + i k_x \right) \\ \alpha_R \left(k_y - i k_x \right) & \frac{\hbar^2 k_{\parallel}^{\ 2}}{2m} \end{pmatrix}$$

Non-diagonal => $|+\rangle, |-\rangle$ are not eigenstates (stationary states) of the system

Eigenvalues:

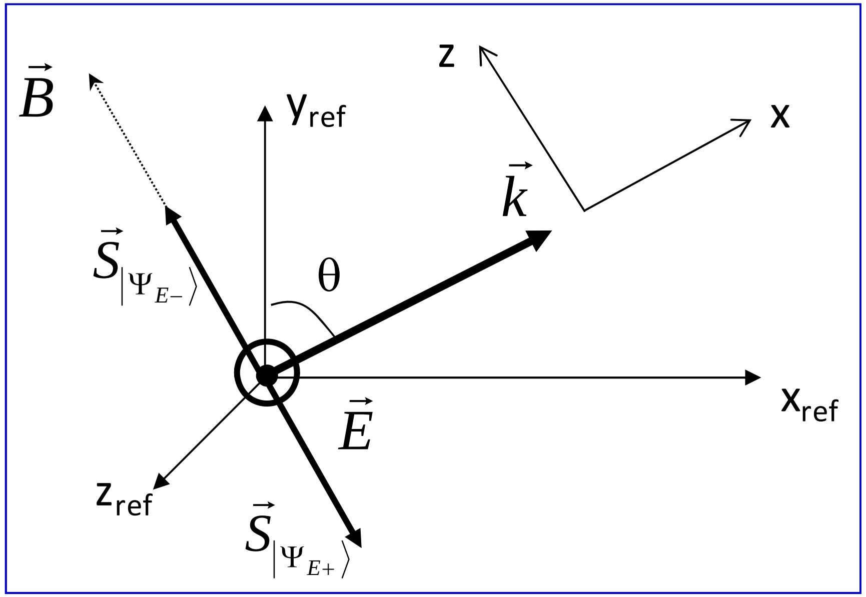
$$\det\left(\hat{H}-\lambda\hat{I}\right)=0 \quad \Leftrightarrow \quad \begin{vmatrix} \frac{\hbar^2 k_\parallel^2}{2m}-\lambda & \alpha_R\left(k_y+ik_x\right) \\ \alpha_R\left(k_y-ik_x\right) & \frac{\hbar^2 k_\parallel^2}{2m}-\lambda \end{vmatrix}=0$$

$$\left(\frac{\hbar^2 k_\parallel^2}{2m}-\lambda\right)^2=\alpha_R^2\left(k_y+ik_x\right)\left(k_y-ik_x\right)=\alpha_R^2\left(k_y^2+k_x^2\right)=\alpha_R^2 k_\parallel^2 \quad \Rightarrow \quad \lambda=\frac{\hbar^2 k_\parallel^2}{2m}\pm\alpha_R\left|k_\parallel\right|$$

$$\Rightarrow \quad E_\pm(k_\parallel)=\frac{\hbar^2 k_\parallel^2}{2m}\pm\alpha_R\left|k_\parallel\right|$$

$$\begin{cases} \measuredangle\left(\vec{k}_\parallel, S_{|\Psi-\rangle}\right)=\frac{\pi}{2} \\ \measuredangle\left(\vec{k}_\parallel, S_{|\Psi+\rangle}\right)=-\frac{\pi}{2} \end{cases}$$

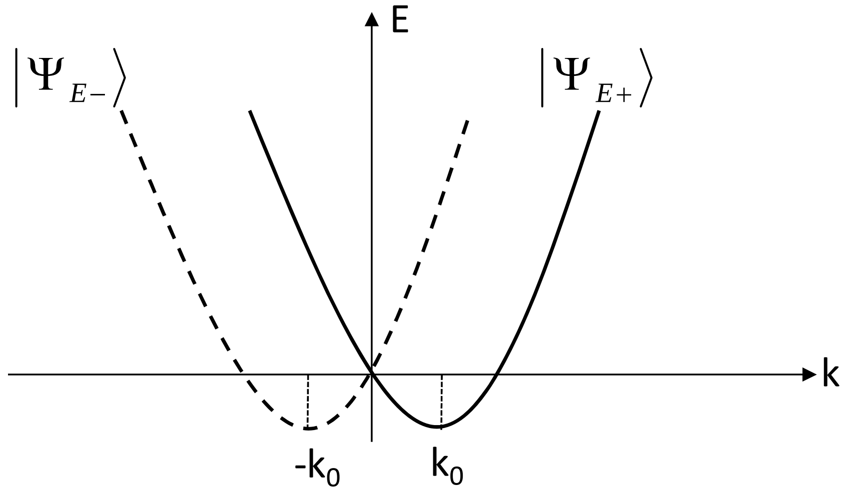
- **The eigenvalues of the Rashba Hamiltonian**
- Correspond to a 2 state system (spin parallel and antiparallel to B_{eff})



Vector diagram of eigenfunctions

- SO influence on the parabolic *E(k)* band structure
- How the spin-orbit constant alpha can be extracted from ARPES experiments.

$$E_{\pm}(k_{\parallel}) = \frac{\hbar^2 k_{\parallel}^{2}}{2m} \pm \alpha_R |k_{\parallel}|$$

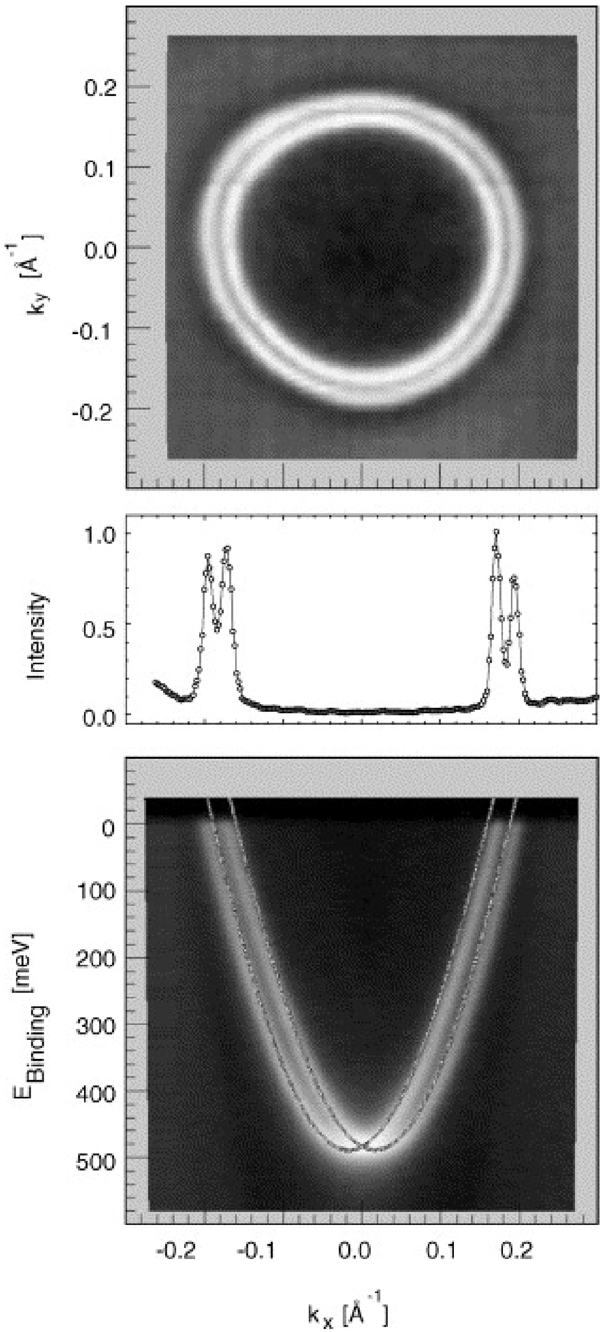


Parabola minimum get from: $\frac{\partial E}{\partial k} = 0 \Rightarrow k_0 = \frac{m\alpha_R}{\hbar^2} \Rightarrow \Delta k = 2k_0 = \frac{2m\alpha_R}{\hbar^2}$

- Measuring Δk one can get α_R

OBS: Because α_R is small (1 order of magnitude smaller than E_F) high resolution of analizer is required in photoemission experiments to observe the split

$$\alpha_R = 1meV \rightarrow \Delta k \sim 10^{-15} m^{-1} = 10^{-5} \mathring{A}^{-1}$$



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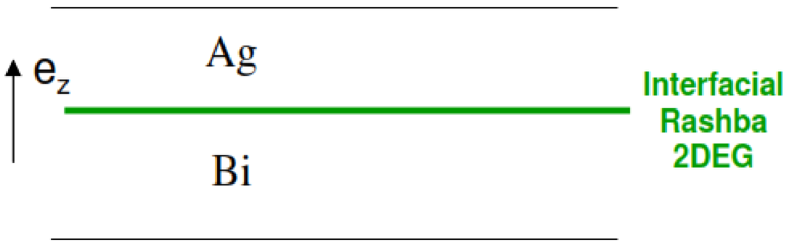
Middle panel: cut on (k_x, k_y) representation (top panel) at $k_y = 0$

- α_R from ARPES for materials with important potential in spin-orbitronics (when materials with significant SO are used for generation of spin currents by spin-Hall effects).

Rashba effect at interfaces or surfaces of metals



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Bi/Ag(111): α_R = 3.05 eVÅ

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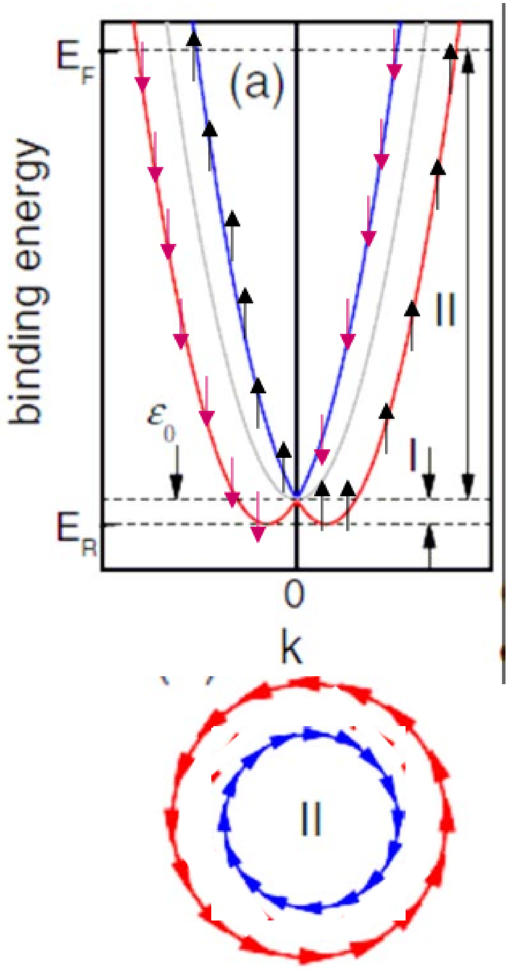


Fermi circles

From A. Fert

Eigenfunctions

The general form of the eigenfunctions within the $\{|+\rangle, |-\rangle\}$ basis is:

$$|\Psi\rangle = e^{i\vec{k}_\parallel \vec{r}}\left[u|+\rangle + v|-\rangle\right]$$

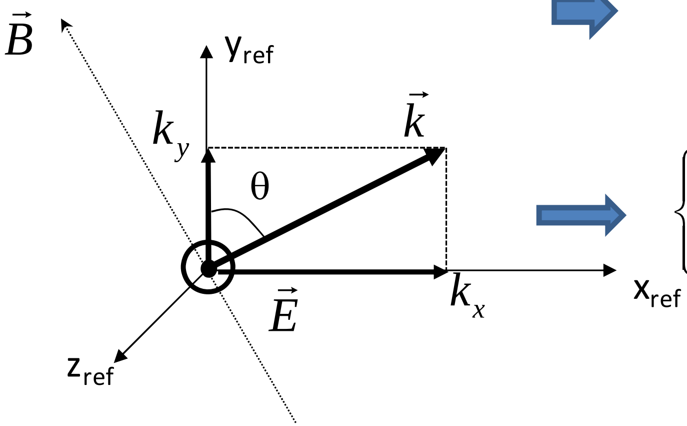
Where the amplitude probabilities ***u*** and ***v*** verify the matrix equation:

$$\left(\hat{H} - \lambda\hat{I}\right)\begin{pmatrix} u \\ v \end{pmatrix} = 0$$

For: $\lambda_1 = E_+ = \dfrac{\hbar^2 k_\parallel^2}{2m} + \alpha_R |k_\parallel|$

$$\begin{pmatrix} -\alpha_R k_\parallel & \alpha_R(k_y + ik_x) \\ \alpha_R(k_y - ik_x) & -\alpha_R k_\parallel \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0$$

Leads to 2 equivalent equations: $\begin{cases} -uk_\parallel + v(k_y + ik_x) = 0 \\ u(k_y - ik_x) - vk_\parallel = 0 \end{cases}$



$$\Rightarrow \quad u = \frac{k_y + ik_x}{k_\parallel} v$$

$$\Rightarrow \quad \begin{cases} k_x = k_\parallel \sin\theta \\ k_x = k_\parallel \cos\theta \end{cases}$$

$$\Rightarrow \quad u = ve^{i\theta}$$

Combined with the ortonormation condition: $u^2 + v^2 = 1$

$$\Rightarrow \quad u = \frac{1}{\sqrt{2}} e^{\frac{i\theta}{2}}; \quad v = \frac{1}{\sqrt{2}} e^{-\frac{i\theta}{2}}$$

$$|\Psi_{E+}\rangle = \frac{e^{i(k_x x + k_y y)}}{\sqrt{2}}\left(e^{\frac{i\theta}{2}}|+\rangle + e^{-\frac{i\theta}{2}}|-\rangle\right)$$

For: $\lambda_1 = E_- = \dfrac{\hbar^2 k_\parallel^2}{2m} - \alpha_R |k_\parallel|$ $\Rightarrow$

$$|\Psi_{E-}\rangle = \frac{e^{i(k_x x + k_y y)}}{\sqrt{2}}\left(e^{\frac{i\theta}{2}}|+\rangle - e^{-\frac{i\theta}{2}}|-\rangle\right)$$

Obs: The states $|\Psi_{E+}\rangle, |\Psi_{E-}\rangle$ are stationary states of the system

- Furthermore, we study the time evolution, solving the time dependent Schrodinger equation. Then, by calculating average values of the spin operators Sx, Sy, Sz we can demonstrate and discuss the spin precession .

To describe the time evolution of the system we use the expression of the solution of the Schrodinger equation projected on the $\left\{ |\Psi_{E+}\rangle, |\Psi_{E-}\rangle \right\}$ basis:

$$|\Psi(t)\rangle = |\Psi_{E+}\rangle e^{-\frac{i}{\hbar}E_+ t} + |\Psi_{E-}\rangle e^{-\frac{i}{\hbar}E_- t}$$

The 2 eigenstates can be written as: $E_\pm(k_\parallel) = \frac{\hbar^2 k_\parallel^2}{2m} \pm \alpha_R |k_\parallel| = \hbar\omega_0 \pm \alpha_R |k_\parallel|$

Within the $\left\{ |+\rangle, |-\rangle \right\}$ basis

$$\begin{cases} |\Psi_{E+}\rangle = \dfrac{e^{i(k_x x + k_y y)}}{\sqrt{2}} \left(e^{\frac{i\theta}{2}} |+\rangle + e^{-\frac{i\theta}{2}} |-\rangle \right) \\ |\Psi_{E-}\rangle = \dfrac{e^{i(k_x x + k_y y)}}{\sqrt{2}} \left(e^{\frac{i\theta}{2}} |+\rangle - e^{-\frac{i\theta}{2}} |-\rangle \right) \end{cases}$$

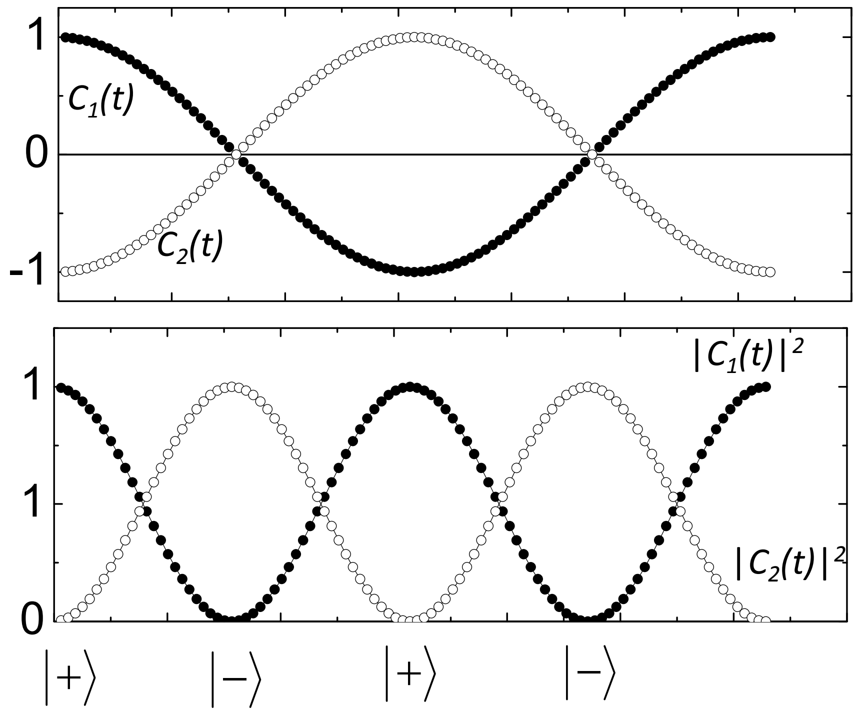
$$|\Psi(t)\rangle = \frac{e^{i\vec{k}_\parallel \vec{r}}}{\sqrt{2}} \left\{ \left(e^{\frac{i\theta}{2}} |+\rangle + e^{-\frac{i\theta}{2}} |-\rangle \right) e^{-\frac{i}{\hbar}(\hbar\omega_0 + \alpha_R k_\parallel)t} + \left(e^{\frac{i\theta}{2}} |+\rangle - e^{-\frac{i\theta}{2}} |-\rangle \right) e^{-\frac{i}{\hbar}(\hbar\omega_0 - \alpha_R k_\parallel)t} \right\}$$

$$|\Psi(t)\rangle = \frac{e^{i(\vec{k}_\parallel \vec{r} - \omega_0 t)}}{\sqrt{2}} \left\{ e^{\frac{i\theta}{2}} \left(e^{-\frac{i}{\hbar}\alpha_R k_\parallel t} + e^{+\frac{i}{\hbar}\alpha_R k_\parallel t} \right) |+\rangle + e^{-\frac{i\theta}{2}} \left(e^{-\frac{i}{\hbar}\alpha_R k_\parallel t} - e^{+\frac{i}{\hbar}\alpha_R k_\parallel t} \right) |-\rangle \right\}$$

$$\left|\Psi(t)\right\rangle = \frac{e^{i(\vec{k}_{\parallel}\vec{r}-\omega_0 t)}}{\sqrt{2}}\left\{ e^{\frac{i\theta}{2}} \cos\frac{\alpha_R k_{\parallel} t}{\hbar}\left|+\right\rangle - i e^{-\frac{i\theta}{2}} \sin\frac{\alpha_R k_{\parallel} t}{\hbar}\left|-\right\rangle \right\}$$

If one define the amplitudes of probability:

$$\left\|\Psi(t)\right\rangle\right|^2 \rightarrow \begin{cases} \left|C_1(t)\right|^2 \propto \cos^2 \dfrac{\alpha_R k_{\parallel}}{\hbar} t \\ \left|C_1(t)\right|^2 \propto \sin^2 \dfrac{\alpha_R k_{\parallel}}{\hbar} t \end{cases}$$



Flip-flop movement betwen the two un-stationary states:

$\left|+\right\rangle, \left|-\right\rangle$

Precession with Larmor frquency

In order to demonstrate the spin precession, one has to calculate the average values of the spin operatos: $\left\langle \hat{S}_x(t) \right\rangle, \left\langle \hat{S}_y(t) \right\rangle, \left\langle \hat{S}_z(t) \right\rangle$ within the basis: $\left\{ \left| \Psi_{E+} \right\rangle, \left| \Psi_{E-} \right\rangle \right\}$

$$\left| \Psi(t) \right\rangle = \frac{e^{i(\vec{k}_\| \vec{r} - \omega_0 t)}}{\sqrt{2}} \left\{ e^{\frac{i\theta}{2}} \cos \frac{\alpha_R k_\| t}{\hbar} \left| + \right\rangle - i e^{-\frac{i\theta}{2}} \sin \frac{\alpha_R k_\| t}{\hbar} \left| - \right\rangle \right\} = C_+(t) \left| + \right\rangle + C_-(t) \left| - \right\rangle$$

$$\begin{cases} C_+(t) = e^{\frac{i\theta}{2}} \dfrac{e^{i(\vec{k}_\| \vec{r} - \omega_0 t)}}{\sqrt{2}} \cos \dfrac{\alpha_R k_\| t}{\hbar} \\ C_-(t) = -i e^{-\frac{i\theta}{2}} \dfrac{e^{i(\vec{k}_\| \vec{r} - \omega_0 t)}}{\sqrt{2}} \sin \dfrac{\alpha_R k_\| t}{\hbar} \end{cases}$$

$$\left\langle \hat{S}_z \right\rangle = \left\langle \Psi \left| \hat{S}_z \right| \Psi \right\rangle = \frac{\hbar}{2} \begin{pmatrix} C_+^* & C_-^* \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \frac{\hbar}{2} \left[C_+^* C_+ - C_-^* C_- \right]$$

$$\left\langle \hat{S}_x \right\rangle = \left\langle \Psi \left| \hat{S}_x \right| \Psi \right\rangle = \frac{\hbar}{2} \begin{pmatrix} C_+^* & C_-^* \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \hbar \Re e \left[C_+^* C_- \right]$$

$$\left\langle \hat{S}_y \right\rangle = \left\langle \Psi \left| \hat{S}_x \right| \Psi \right\rangle = \frac{\hbar}{2} \begin{pmatrix} C_+^* & C_-^* \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} C_+ \\ C_- \end{pmatrix} = \left\langle \hat{S}_y \right\rangle = \hbar \Im m \left[C_+^* C_- \right]$$

$$\left\langle \hat{S}_z \right\rangle = \frac{\hbar}{2}\left[C_+^{\ *}C_+ - C_-^{\ *}C_-\right] = \frac{\hbar}{4}\left[\cos^2\left(\frac{\alpha_R k_{\parallel}}{\hbar}t\right) - \sin^2\left(\frac{\alpha_R k_{\parallel}}{\hbar}t\right)\right] = \frac{\hbar}{4}\cos\left(\frac{2\alpha_R k_{\parallel}}{\hbar}t\right)$$

$$\left\langle \hat{S}_x \right\rangle = \hbar\Re e\left[C_+^{\ *}C_-\right] = \frac{\hbar}{2}\Re e\left[e^{\frac{-i\theta}{2}}\cos\frac{\alpha_R k_{\parallel} t}{\hbar}\sin\frac{\alpha_R k_{\parallel} t}{\hbar}\left(-ie^{-\frac{i\theta}{2}}\right)\right] = -\frac{\hbar}{2}\cos\frac{\alpha_R k_{\parallel} t}{\hbar}\sin\frac{\alpha_R k_{\parallel} t}{\hbar}\Re e\left(ie^{-i\theta}\right) = -\frac{\hbar}{4}\sin\frac{2\alpha_R k_{\parallel} t}{\hbar}\sin\theta$$

$$\left\langle \hat{S}_y \right\rangle = \hbar\Im m\left[C_+^{\ *}C_-\right] = -\frac{\hbar}{4}\sin\frac{2\alpha_R k_{\parallel} t}{\hbar}\cos\theta$$

$$\Rightarrow \begin{cases} \left\langle \hat{S}_x \right\rangle = -\frac{\hbar}{4}\sin\frac{2\alpha_R k_{\parallel} t}{\hbar}\sin\theta \\ \left\langle \hat{S}_y \right\rangle = -\frac{\hbar}{4}\sin\frac{2\alpha_R k_{\parallel} t}{\hbar}\cos\theta \\ \left\langle \hat{S}_z \right\rangle = \frac{\hbar}{4}\cos\frac{2\alpha_R k_{\parallel} t}{\hbar} \end{cases}$$

If we denote by:

$$\boxed{\Omega = \frac{2\alpha_R k_{\parallel}}{\hbar}}$$

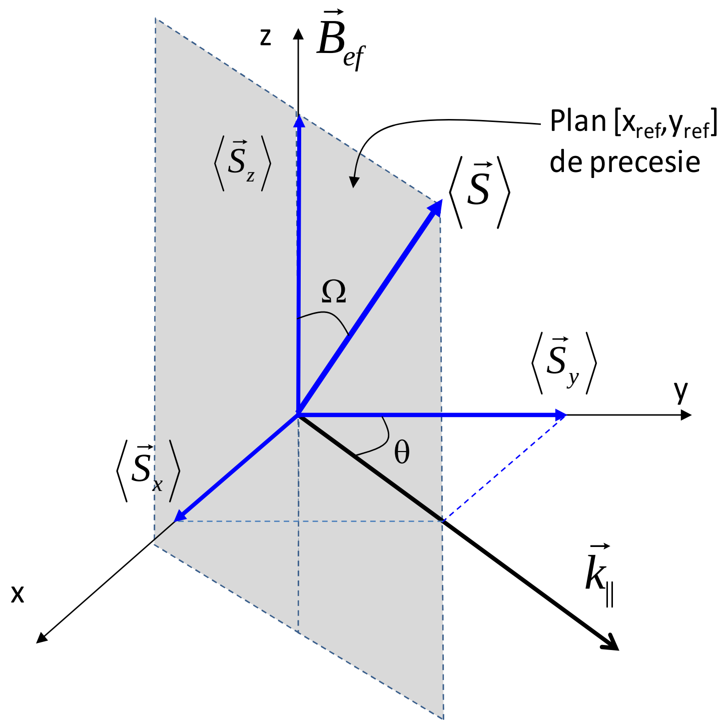
$$\Rightarrow \begin{cases} \left\langle \hat{S}_x \right\rangle \propto \sin\Omega t\sin\theta \\ \left\langle \hat{S}_y \right\rangle \propto \Omega t\cos\theta \\ \left\langle \hat{S}_z \right\rangle \propto \cos\Omega t \end{cases}$$

Precession equations of the spin angular momentum ***S*** wth the frequency Ω analoguous to the Larmor precession around B_{eff}

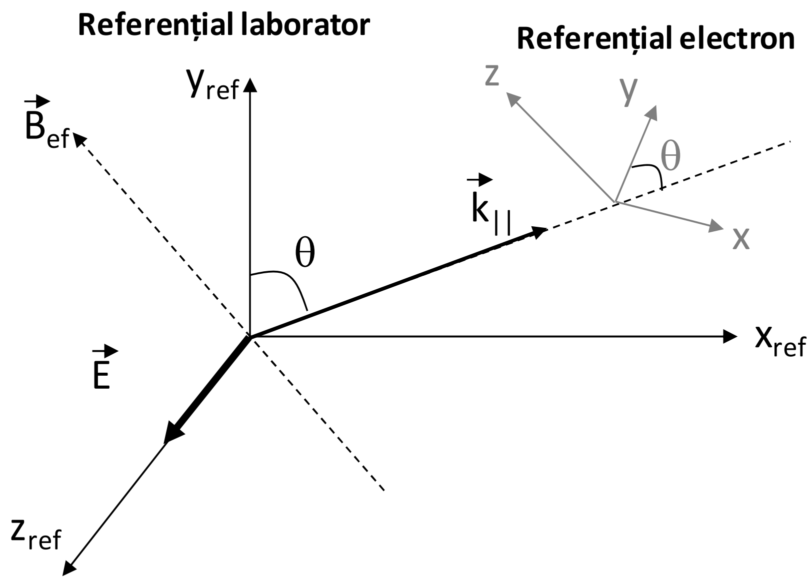
The period of this precession is: $T = \frac{2\pi}{\Omega} = \frac{\pi\hbar}{\alpha_R k_{\parallel}}$

To be dephased with π, the spin has to travel a distance corresponding to a half-period: $T_L = \frac{\pi\hbar}{2\alpha_R k_{\parallel}}$

In the Datta@Das transistor, the distance between source and drain is adjusted to insure the rotation with π whe E of the gate is turned on



Vector diagram within the electron referential



Vector diagram within the lab referential