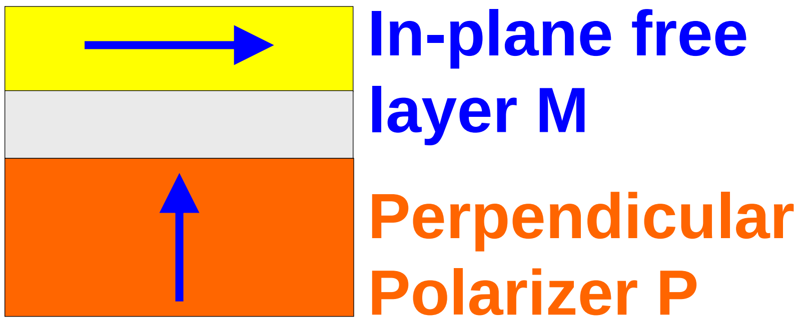
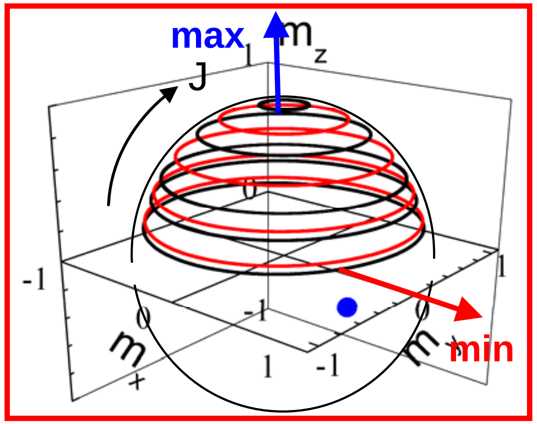
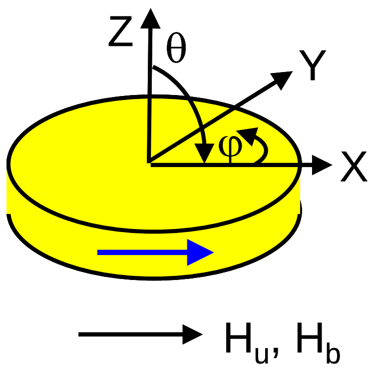
Out of plane precession modes for a perpendicular polarizer

Objective: As an illustration of some general properties and solutions of the Landau-Lifshitz-Gilbert equation including the spin transfer torque term, we derive here the out-of-plane precession (OPP) trajectories and calculate the corresponding frequencies for spin torque driven excitations under perpendicular spin polarization.





Out of plane (OPP) precession orbits



ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 1 The equation of motion under spin transfer torque is given by the Landau-Lifshitz Gilbert (LLG) equation augmented by a spin transfer torque, called here LLGS

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M}\times\mathbf{H}_{eff} + \frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{P}) \qquad a_J = \frac{\hbar}{2e}\frac{J}{M_S t}\eta$$

J=current density

Q1 By multiplying LLGS vectorially by (**M** x) convert LLGS into the Landau-Lifshitz form (with spintorque) where the damping term is written as $-\frac{\alpha\gamma}{Ms}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff})$

Use **a**x(**b** x **c**) =**b**(**ac**)-**c**(**ab**) and the conservation of the norm of **M** given by d**M**²/dt=0

In the final expression neglect terms of order $a_j\alpha$, as well as α^2

Solution LLS equation

$$\frac{d\mathbf{M}}{dt} \approx -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) - \gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff}) + \gamma\frac{a_j}{\mathrm{M_s}}(\mathbf{M}\times(\mathbf{M}\times\mathbf{P}))$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 1

Multiplying the Landau Lifschitz Gilbert STT (LLGS) equation vectorially by (**M** x)

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt}=-\gamma\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)+\frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$(i)\ \frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)=\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\left(\mathbf{M}\frac{d\mathbf{M}}{dt}\right)-\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}\mathbf{M}^2=-\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}M_s^2$$

$$(\mathrm{ii})\ \frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)=\frac{\gamma a_j}{\mathrm{M_s}}\left[\mathbf{M}\left(\mathbf{M}\left(\mathbf{M}\times\mathbf{P}\right)\right)-\left(\mathbf{M}\times\mathbf{P}\right)\mathbf{M}\mathbf{M}\right]=\frac{-\gamma a_j}{\mathrm{M_s}}M_s^2\left(\mathbf{M}\times\mathbf{P}\right)$$

$$\Rightarrow\mathbf{M}\times\frac{d\mathbf{M}}{dt}=-\gamma\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}M_s^2-\frac{\gamma a_j}{\mathrm{M_s}}M_s^2\left(\mathbf{M}\times\mathbf{P}\right)$$

insert in LLGS

$$\frac{d\mathbf{M}}{dt}=-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\frac{\alpha}{\mathrm{M_s}}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)+\frac{\gamma a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\frac{d\mathbf{M}}{dt}=-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\frac{\alpha}{\mathrm{M_s}}\left[-\gamma\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}M_s^2-\frac{\gamma a_j}{\mathrm{M_s}}M_s^2\left(\mathbf{M}\times\mathbf{P}\right)\right]+\frac{\gamma a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\Rightarrow\left(1+\alpha^2\right)\frac{d\mathbf{M}}{dt}=-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\gamma a_j\alpha\left(\mathbf{M}\times\mathbf{P}\right)+\frac{\gamma a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\Rightarrow\frac{d\mathbf{M}}{dt}=-\frac{\gamma}{\left(1+\alpha^2\right)}\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\frac{\gamma}{\left(1+\alpha^2\right)}a_j\alpha\left(\mathbf{M}\times\mathbf{P}\right)-\frac{\gamma}{\left(1+\alpha^2\right)}\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\frac{\gamma}{\left(1+\alpha^2\right)}\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\Rightarrow\boxed{\frac{d\mathbf{M}}{dt}\approx-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\gamma\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)}\quad\text{LLS}$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 2 The total energy has three contributions, a uniaxial anistropy energy (given by the uniaxial anisotropy constant K_u, the Zeeman energy due to an external bias field $\mathbf{H}_b$ and the demagnetization energy

$$E = K_u\left[1-(\mathbf{mn})^2\right] - M_s \mathbf{mH}_b + 2\pi M_s^2\left[\mathbf{m}\left(\overleftrightarrow{\mathbf{N}}\mathbf{m}\right)\right]$$

$$\mathbf{m} = \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}; \qquad \overleftrightarrow{N} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \qquad \mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Here, N is the demagnetization tensor of a thin film and **n** is the uniaxial anisotropy easy axis. We define

$$H_u = \frac{2K_u}{M_s} \qquad H_d = 4\pi M_s \qquad \mathbf{m} = \frac{\mathbf{M}}{M_s}$$

The effective field is defined by

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}} = -\frac{1}{M_s}\frac{\partial E}{\partial \mathbf{m}}$$

Q2 Deduce the general expression of the effective field from the general expression of the energy and evaluate both in the case of H_b=H_u=0 and the above given demagnetization tensor of a thin film

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 2

$$E = K_u\left[1-(\mathbf{mn})^2\right] - M_s\mathbf{m}\mathbf{H}_b + 2\pi M_s^2\left[\mathbf{m}\left(\overleftrightarrow{\mathbf{N}}\mathbf{m}\right)\right]$$

$$\mathbf{H}_{eff} = -\frac{\partial E}{\partial \mathbf{M}} = -\frac{1}{M_s}\frac{\partial E}{\partial \mathbf{m}} = \frac{2K_u}{M_s}(\mathbf{mn})\mathbf{n} + \mathbf{H}_b - 4\pi M_s\overleftrightarrow{\mathbf{N}}\mathbf{m}$$

For $H_b = H_u = 0$ and the demagnetization tensor of a thin film

$$E = 2\pi M_s^2 m_z^2$$

$$\mathbf{H}_{eff} = \begin{pmatrix} 0 \\ 0 \\ 4\pi M_s m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H_d m_z \end{pmatrix}$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 3 Spin torque driven orbits are in many situations close to constant energy trajectories that are solutions to the precession term. Quite generally, it is possible to give analytical expressions for these precession orbits upon using the conservation of the energy E (m_x, m_y, m_z) and the norm of the magnetization vector **m²**=1. (3 unknowns, 2 equations)

Q3 Give the expressions for trjacetories $m_z(m_x)$ and $m_y(m_x)$ (parametrization in m_x) for the following energy that includes a uniaxial anisotropy field and a bias field applied along the positive x-direction : Evaluate these expressions for H_b=H_u=0. What is the form of the trajectories?
Draw them schematicallly

$$E = K_u\left[1 - m_x^{\ 2}\right] - M_s m_x H_b + 2\pi M_s^2 m_z^{\ 2}$$

$$m_x^{\ 2} + m_y^{\ 2} + m_z^{\ 2} = 1$$

Solution

$$\Rightarrow m_z = \pm\sqrt{\frac{2E_o}{H_d M_s}} = const$$

$$\Rightarrow m_y = \pm\sqrt{1 - \frac{2E_o}{H_d M_s} - m_x^{\ 2}} = m_y(m_x)$$

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Answer 3

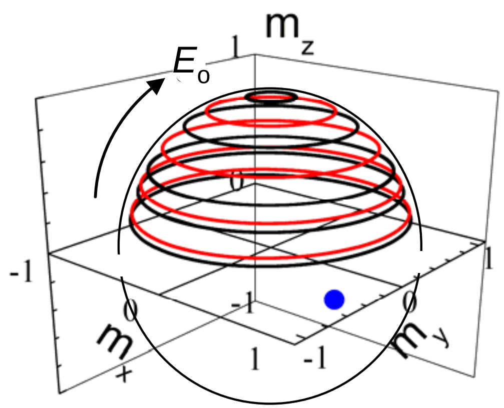
$$E = K_u\left[1-m_x^2\right] - M_s m_x H_b + 2\pi M_s^2 m_z^2 = E_o = const$$

$$m_x^2 + m_y^2 + m_z^2 = 1$$

$$\Rightarrow E_o - K_u\left[1-m_x^2\right] + M_s m_x H_b = 2\pi M_s^2 m_z^2$$

$$\Rightarrow m_z = \pm\sqrt{\frac{2E_o}{H_d M_s} + \frac{2m_x H_b}{H_d} - \frac{H_u}{H_d}\left[1-m_x^2\right]} = m_z(m_x)$$

$$\Rightarrow m_y = \pm\sqrt{1-m_x^2-m_z^2} = m_y(m_x)$$



Out of plane (OPP) precession orbits

For $H_b = H_u = 0$

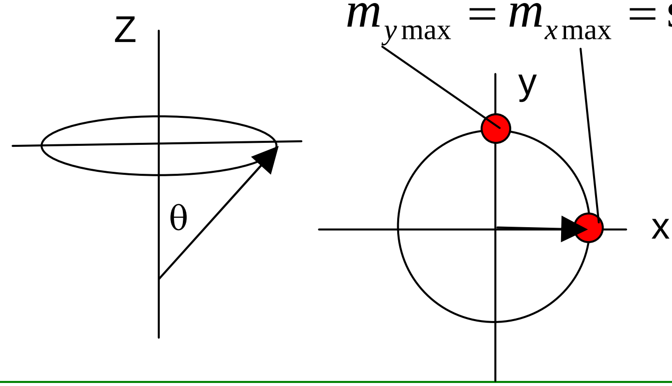
$$\Rightarrow m_z = \pm\sqrt{\frac{2E_o}{H_d M_s}} = const$$

$$\Rightarrow m_y = \pm\sqrt{1-\frac{2E_o}{H_d M_s} - m_x^2} = m_y(m_x)$$

$$\Rightarrow m_x^2 + m_y^2 = 1 - m_z^2 = 1 - \frac{2E_o}{H_d M_s}$$

$$m_z = \cos\theta$$

$$m_{y\max} = m_{x\max} = \sin\theta$$



The orbits are circles with constant m_z component.
They are out-of-plane precession (OPP) trajectories, and their center is the out of plane energy maximum
The energy E_o is for instance given by an angle θ_o

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 4 Quite general the solutions of the precession term only, are constant energy trajectories that can be determined from the procedure given under step 3. The precession term then describes the rate of change of the different components m_x, m_y, m_z. For the case H_u=H_b=0 this can be solved analytically. Do this in order to deduce the precession frequency of the constant energy trajectory.

Q4 What determines the frequency? Why is this a good approximation even when a small H_u or H_b would be considered? (only qualitative argument, no calculation)

Solution

$$\omega = \gamma H_d m_z$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 4

$$\text{Precession term}$$

$$\frac{d\mathbf{M}}{dt} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} \qquad \mathbf{H}_{eff} = \begin{pmatrix} 0 \\ 0 \\ 4\pi M_s m_z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ H_d m_z \end{pmatrix}$$

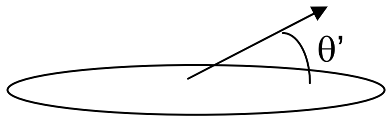
$$M_s \begin{pmatrix} \dot{m}_x \\ \dot{m}_y \\ \dot{m}_z \end{pmatrix} = -\gamma M_s \begin{pmatrix} m_x \\ m_y \\ m_z \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ H_d m_z \end{pmatrix} = -\gamma M_s \begin{pmatrix} H_d m_y m_z \\ -H_d m_x m_z \\ 0 \end{pmatrix}$$

$$\Rightarrow \dot{m}_x = -\gamma H_d m_y m_z$$

$$\Rightarrow \dot{m}_y = \gamma H_d m_x m_z$$

$$\Rightarrow m_{x,y} = m_{x,y\,\max} e^{i\omega t}$$

$$\omega = \gamma H_d m_z$$



The precession frequency is given only by the out of plane demagnetization field, scaled by the out of plane component m_z

In the presence of anisotropy or an in-plane bias field, this frequency remains a good approximation, since even for small out of plane alignements θ’=10° the corresponding demagnetization field is large. Example $4\pi M_s$ of Permalloy 10 kOe

$H_d m_z$= 10 kOe cos10° = 9850 Oe. This is large compared to anistropy fields (100-500 Oe) or in plane bias fields < 1kOe.

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 5 An alternative derivation of the precession frequency, that can be used for more complex trajectories is the following.

The frequency *f* is the inverse of the precession period T, which in turn is given by the integration over dt:

$$T = \int dt$$

This can be rewritten as

$$T = \int \frac{dm_i}{\dot{m}_i}$$

Here m_i can be any one of the three components m_x, m_y, m_z.
The time derivative of m_i is obtained from the precession equation (step 4)

Q5 For the case $H_u = H_b = 0$ this can be solved analytically. Do this, using the parametrization in m_x of step 3 and the time derivative of step 4 in order to deduce the precession frequency of the constant energy trajectory.

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} \Bigg|$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

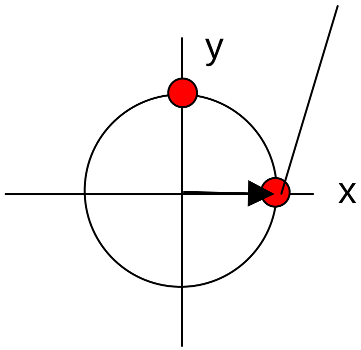
Answer 5

$$T = \int dt = \int \frac{dm_x}{\dot{m}_x} = \int \frac{dm_x}{\gamma H_d m_z \sqrt{1 - m_z^2 - m_x^2}} = 4 \frac{1}{\gamma H_d m_z} \arcsin \frac{m_x}{\sqrt{1 - m_z^2}} \Bigg|_0^{m_{x\max}}$$

$$T = \frac{4}{\gamma H_d m_z} \arcsin \frac{m_x}{\sqrt{1 - m_z^2}} \Bigg|_0^{m_{x\max}} = \frac{4\pi}{\gamma H_d m_z}$$

$$\Rightarrow \omega = \gamma H_d m_z$$

$$m_{x\max} = \sqrt{1 - m_z^2}$$



Note, for more complicated integrals this can be solved numerically.
In some cases, analytical expressions have been derived in the literature (see publications by Bertotti).

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 6A

The precession term corresponds to a conservative dynamics since it does not change the energy of the system. To see this, calculate the time derivative of the energy for the precession term only and show that this is zero

$$\frac{dE}{dt} = \frac{dE}{d\mathbf{M}}\frac{d\mathbf{M}}{dt} = -\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt} \qquad \frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{eff}$$

Step 6B

Do the same as for Step 6A, but for the full LLGS and LLS equation and show that the damping term is always negative, but that the contribution from the spin torque term can be positive or negative.

Evaluate dE/dt then for $H_b=H_u=0$ and for a perpendicular polarizer **P** = (0, 0, 1) (derivation for LLS only, use step 4).

LLGS $$\frac{d\mathbf{M}}{dt} = -\gamma\mathbf{M}\times\mathbf{H}_{eff} + \frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})$$

LLS $$\frac{d\mathbf{M}}{dt} \approx -\gamma(\mathbf{M}\times\mathbf{H}_{eff}) - \gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_{eff}) + \gamma\frac{a_j}{\mathrm{M_s}}(\mathbf{M}\times(\mathbf{M}\times\mathbf{P}))$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 6A

$$\frac{dE}{dt}=\frac{dE}{d\mathbf{M}}\frac{d\mathbf{M}}{dt}=-\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt}=\gamma\mathbf{H}_{eff}\left(\mathbf{M}\times\mathbf{H}_{eff}\right)=0$$

With $\mathbf{a}\left(\mathbf{a}\times\mathbf{b}\right)=0$

Answer 6B for LLGS

$$\frac{dE}{dt}=\frac{dE}{d\mathbf{M}}\frac{d\mathbf{M}}{dt}=-\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt}$$

$$\frac{dE}{dt}=-\mathbf{H}_{eff}\cdot\left(-\gamma\mathbf{M}\times\mathbf{H}_{eff}+\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}+\frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\frac{dE}{dt}=-\mathbf{H}_{eff}\cdot\left(\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)-\mathbf{H}_{eff}\cdot\left(\frac{\gamma a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\frac{dE}{dt}=\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\frac{\gamma a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{P}\right)\left(\mathbf{M}\times\mathbf{H}_{eff}\right)$$

Rewrite LLGS

$$\mathbf{M}\times\mathbf{H}_{eff}=-\frac{1}{\gamma}\frac{d\mathbf{M}}{dt}+\frac{\alpha}{\gamma\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}+\frac{a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)$$

$$\frac{dE}{dt}=\frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}\left(-\frac{1}{\gamma}\frac{d\mathbf{M}}{dt}+\frac{\alpha}{\gamma\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}+\frac{a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$
$$+\frac{\gamma a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{P}\right)\left(-\frac{1}{\gamma}\frac{d\mathbf{M}}{dt}+\frac{\alpha}{\gamma\mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt}+\frac{a_j}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 6B for LLGS

$$\frac{dE}{dt} = \frac{\alpha}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}\left(-\frac{1}{\gamma}\frac{d\mathbf{M}}{dt} + \frac{\alpha}{\gamma \mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{a_j}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})\right)$$

$$+\frac{\gamma a_j}{\mathrm{M_s}}(\mathbf{M}\times\mathbf{P})\left(-\frac{1}{\gamma}\frac{d\mathbf{M}}{dt} + \frac{\alpha}{\gamma \mathrm{M_s}}\mathbf{M}\times\frac{d\mathbf{M}}{dt} + \frac{a_j}{\mathrm{M_s}}\mathbf{M}\times(\mathbf{M}\times\mathbf{P})\right)$$

$$\frac{dE}{dt} = \underbrace{-\frac{\alpha}{\gamma \mathrm{M_s}}\left(\frac{d\mathbf{M}}{dt}\right)^2}_{\text{Damping part always <0}} \underbrace{-\frac{a_j}{\mathrm{M_s}}\frac{d\mathbf{M}}{dt}(\mathbf{M}\times\mathbf{P})}_{\text{STT part <0 or >0}}$$

Damping part always <0

STT part <0 or >0

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 6B for LLS

$$\frac{dE}{dt}=\frac{dE}{d\mathbf{M}}\frac{d\mathbf{M}}{dt}=-\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt}$$

$$\frac{d\mathbf{M}}{dt}\approx-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\gamma\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)$$

$$\frac{dE}{dt}=-\mathbf{H}_{eff}\frac{d\mathbf{M}}{dt}\approx-\mathbf{H}_{eff}\left(-\gamma\left(\mathbf{M}\times\mathbf{H}_{eff}\right)-\gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)+\gamma\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)\right)$$

$$\frac{dE}{dt}\approx\left(\gamma\frac{\alpha}{\mathrm{M_s}}\mathbf{H}_{eff}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{H}_{eff}\right)\right)-\gamma\frac{a_j}{\mathrm{M_s}}\mathbf{H}_{eff}\left(\mathbf{M}\times\left(\mathbf{M}\times\mathbf{P}\right)\right)\right)$$

$$\frac{dE}{dt}\approx\underbrace{-\gamma\frac{\alpha}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{H}_{eff}\right)^2}_{\text{Damping part always <0}}+\underbrace{\gamma\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{P}\right)\left(\mathbf{M}\times\mathbf{H}_{eff}\right)}_{\text{STT part <0 or >0}}$$

Damping part always <0

STT part <0 or >0

Evaluation of dE/dt for $H_b=H_u=0$ and **P**=(0,0,1).

$$\left(\mathbf{M}\times\mathbf{P}\right)=M_s\begin{pmatrix}m_x\\m_y\\m_z\end{pmatrix}\times\begin{pmatrix}0\\0\\1\end{pmatrix}=M_s\begin{pmatrix}m_y\\-m_x\\0\end{pmatrix}$$

$$\mathbf{M}\times\mathbf{H}_{eff}=M_s\begin{pmatrix}H_d m_y m_z\\-H_d m_x m_z\\0\end{pmatrix}$$

$$\frac{dE}{dt}\approx-\gamma\frac{\alpha}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{H}_{eff}\right)^2+\gamma\frac{a_j}{\mathrm{M_s}}\left(\mathbf{M}\times\mathbf{P}\right)\left(\mathbf{M}\times\mathbf{H}_{eff}\right)$$

$$\frac{dE}{dt}\approx-\gamma\frac{\alpha}{\mathrm{M_s}}M_s^2H_d^2m_z^2\left(m_x^2+m_y^2\right)+\gamma\frac{a_j}{\mathrm{M_s}}M_s^2H_d m_z\left(m_x^2+m_y^2\right)$$

$$\frac{dE}{dt}\approx-\gamma\frac{\alpha}{\mathrm{M_s}}M_s^2H_d^2m_z^2\left(1-m_z^2\right)+\gamma\frac{a_j}{\mathrm{M_s}}M_s^2H_d m_z\left(1-m_z^2\right)$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 7

In the case of spin torque driven excitations, the resulting limit cycles are in many situations close to the constant energy trajectories calculated in step 3, 4. This is in particular truE for the perpendicular polarizer that makes the magnetization to precess on limit cycles that are OPP trajectories; In spin torque driven excitations the time derivative of the energy dE/dt at a given point **M** is not zero, but the integral of dE/dt over one period vanishes. This makes sense, since in spin torque driven excitation the spin torque balances the damping torque, but only over one period.
From the vanishing of the intergral one can derive an expression for the current density, expressed here in the units of field via the prefactor a_j of the spin torque term.

$$\int \frac{dE}{dt}dt = 0 \approx -\gamma \frac{\alpha}{M_s}\int \left(\mathbf{M}\times\mathbf{H}_{eff}\right)^2 dt + \gamma \frac{a_j}{M_s}\int \left(\mathbf{M}\times\mathbf{P}\right)\left(\mathbf{M}\times\mathbf{H}_{eff}\right)dt$$

$$\Rightarrow a_j = \frac{\alpha\int \left(\mathbf{M}\times\mathbf{H}_{eff}\right)^2 dt}{\int \left(\mathbf{M}\times\mathbf{P}\right)\left(\mathbf{M}\times\mathbf{H}_{eff}\right)dt}$$

Q7 Evaluate this relation for H_b=H_u=0 and for a perpendicular polarizer **P** = (0, 0, 1) to provide a relation between the out of plane magnetization component m_z and the current density J (expressed in the units of field via the prefactor a_j). What is the critical current density a_{jc} for which the magnetization points out of plane?

Keep in mind that m_z=const on OPP trajectories, induced by a perpendicular polarizer, see step 3

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 7

$$\int \frac{dE}{dt}dt = 0 \approx -\gamma \frac{\alpha}{M_s}\int (\mathbf{M}\times\mathbf{H}_{eff})^2 dt + \gamma \frac{a_j}{M_s}\int (\mathbf{M}\times\mathbf{P})(\mathbf{M}\times\mathbf{H}_{eff})dt$$

$$\Rightarrow a_j = \frac{\alpha \int (\mathbf{M}\times\mathbf{H}_{eff})^2 dt}{\int (\mathbf{M}\times\mathbf{P})(\mathbf{M}\times\mathbf{H}_{eff})dt}$$

<u>Evaluation of dE/dt for $H_b=H_u=0$ and **P**=(0,0,1).</u>

$$a_j = \frac{\alpha H_d^2 \int m_z^2 (1-m_z^2)dt}{H_d \int m_z (1-m_z^2)dt}$$

Since m_z=const on OPP trajectories, induced by a perpendicular polarizer

$$a_j = \frac{\alpha H_d^2 m_z^2 (1-m_z^2)\int dt}{H_d m_z (1-m_z^2)\int dt}$$

$$a_j = \alpha H_d m_z \Leftrightarrow m_z = \frac{a_j}{\alpha H_d}$$

$$m_z = \frac{a_j}{\alpha H_d}$$

This expression indicates that the magnetization is stabilized on a limit cycle whose out of plane component m_z is given by the balance between the spin torque (aj) and the damping torque. The higher the current, the higher the out of plane component.

$$m_z = \frac{a_j}{\alpha H_d} \le 1 \Rightarrow a_{jc} = \alpha H_d$$

<u>Critical current for out of plane orientation</u>

ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Step 8

Now you can use the results of step 4, 5 (precession frequency of OPP trajectories) and of step 7 to calculate the frequencies as a function of the spin polarized curren a_j. Provide an interpretation.

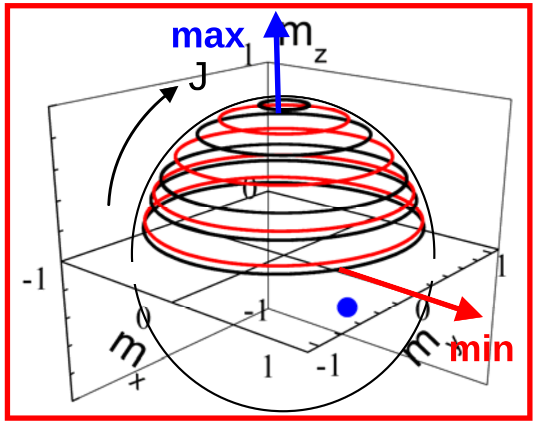
ursula.ebels@cea.fr, SPINTEC, Grenoble, France

Answer 8

Step 4, 5 $\omega = \gamma H_d m_z$

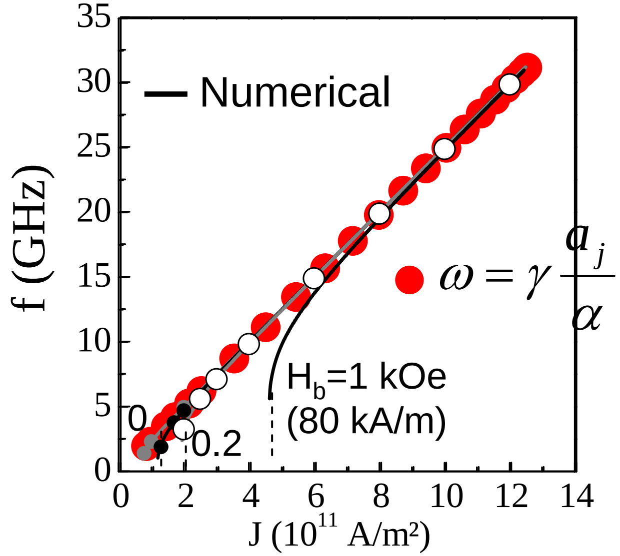
Step 7 $m_z = \dfrac{a_j}{\alpha H_d}$

$$\omega = \gamma H_d m_z = \gamma \frac{a_j}{\alpha}$$



The frequency increases linearly with current.
More spin current means, more energy gain.
More energy gain, means that the system can visit orbits of higher energy, which are characterized by higher frequencies.
Thus, the frequencies increase with current.

The above relation is a good approximation, even when an in-plane bias is applied, (except close to the critical current, where *f* drops to zero)



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Step 9

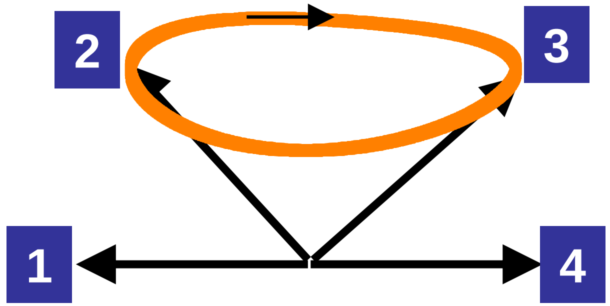
In order to reverse the magnetization using a perpendicular polarizer and a short current pulse, what is the minmum pulse duration for a current density of 10^7 A/cm^2 and a damping constant α=0.02?

Use the following relationships and values

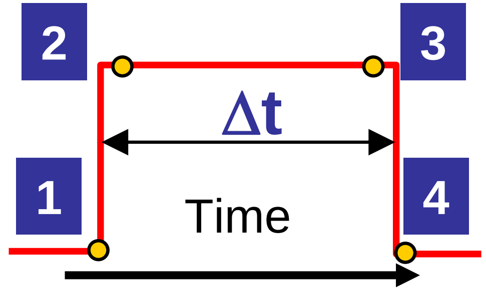
$$a_J = \frac{\hbar}{2e}\frac{J}{M_S t}\eta; \qquad J = \frac{I}{A}; \qquad \frac{\hbar}{2e} = 3.3\cdot 10^{-12}\,\frac{emuOe}{mA}$$

$$\eta \approx 0.3; \qquad M_s = 800\,\frac{emu}{cm^3}; \qquad \gamma = 17.610^{-6}\,\frac{1}{Oe\cdot s}$$

$$t = 3nm = 3\cdot 10^{-7}\,cm; \qquad A = (100nm)^2 = 10^{-10}\,cm^2$$



Current Pulse



Answer 9

$$\omega = \gamma H_d m_z = \gamma \frac{a_j}{\alpha}$$

$$\omega = \gamma \frac{a_j}{\alpha} = 35.2 \cdot 10^9 \frac{1}{s} = 2\pi f$$

$$f = \frac{35.2}{2\pi} GHz = 5.6 GHz = \frac{1}{T}$$

$$\Delta t = \frac{T}{2} = \frac{10^{-9}}{2 \cdot 5.6} s \approx 10^{-10} s = 100\, ps$$

$$a_J = \frac{\hbar}{2e} \frac{J}{M_S t} \eta$$

$$a_J = 3.3 \cdot 10^{-12} \frac{emuOe}{mA} 10^7 \frac{A}{cm^2} \frac{0.3}{800 \frac{emu}{cm^3} 3 \cdot 10^{-7} cm}$$

$$a_J = 3.3 \cdot 10^{-9} \cdot 10^7 \frac{0.3}{800 \cdot 3 \cdot 10^{-7}} Oe$$

$$a_J = 40 Oe$$

ursula.ebels@cea.fr, SPINTEC, Grenoble, France