Large Angle Precessional Magnetization Dynamics
under Field and Current Excitations

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Outline

0 Introduction

I Conservative dynamics (solutions of Precession term)

II Non conservative dynamics

III Precessional reversal under transverse field pulses

IV Domain wall motion under field

V Introduction to spin transfer torque

VI Spin torque induced precession

VII (Precessional) Reversal under spin torque
**Reversal by Field**

- Stoner-Wohlfahrt Reversal under slow field

**Field Induced Precession**

- Ferromagnetic resonance

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**Precessional Reversal under Field pulses**

- Few 100 ps

**Field induced wall motion**

- Few ns for 100 nm

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- Impose a frequency
- Limited to small angles
Field provides a torque that acts on $M$

$$\frac{dM}{dt} = -\gamma M \times H$$

Bias field changes energy surface, thus acts on conservative part of the dynamics

Other means to change the magnetization state??

Spin Momentum transfer
Additional dissipative term in LLG
Spinelectronics
Charge & Spin (±1/2)

Magnetization: M → acts as Spin Filter
→ Spin polarization

Flow of electrons

5nm

Magneto-Resistance

Resistance vs. Field

Cu → acts as Spin Filter → Spin polarization
**V Spin Momentum Transfer - Concept**

**Unpolarized Electrons**

**Polarized Electrons**

**Transmitted Electrons**

Local exchange interaction between conduction electron spins and local magnetization $M$

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**Polarizer P**

**Free Layer M**

**Conduction Electrons**

Transfer of transverse moment $m$

$= \text{Spin Torque}$

---

V Spin Momentum Transfer - Concept

**Electron Flow**

**Stabilize M**

**Destabilize M**

**Torque on M**

\[
T_{//} = \gamma_0 \frac{a_j}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{P})
\]

\[\sim \sin(M\mathbf{P})\]

STT tries to align \(\mathbf{M}\) collinear to \(\mathbf{P}\)

\(a_j\)~current \(J\)

\(\mathbf{P} = \) spin polarization vector
\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{Ms} M \times \frac{dM}{dt} + \frac{\gamma a_f(\theta)}{Ms} M \times (M \times P) \]

Precession  Damping  Spin torque (ST)
0 Introduction

I Conservative dynamics (solutions of Precession term)

II Non conservative dynamics

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VII (Precessional) Reversal under spin torque
V Spin Momentum Transfer - LLG + STT

Need to add an additional term to the Landau Lifshitz Gilbert equation

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{Ms} M \times \frac{dM}{dt} + \frac{\gamma a_J(\theta)}{Ms} M \times (M \times P)
\]

Precession \hspace{2cm} \text{Damping} \hspace{2cm} \text{Spin torque (ST)}

\[
a_J = \frac{\hbar J}{2e \mu_0 M_{st}} \ g(\eta, \bar{m}, \bar{p})
\]

\[
g(\eta, \bar{m}, \bar{p}) = \frac{1}{-4 + \frac{1}{4} \frac{(1+\eta)^3}{\eta^{3/2}} (3 + \bar{p} \cdot \bar{m})}
\]

Note neglect field like term
II Non Conservative Dynamics - Summary

\[
\frac{dM}{dt} = -\gamma (M \times H_{eff}) + \frac{\alpha}{M_s} M \times \frac{dM}{dt} + \frac{\gamma a_\theta}{M_s} M \times (M \times P)
\]

Precession  Damping  Spin torque (ST)

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| **Conservative Precession Dynamics** | 1 | 0 | 2 stable foci  
1 saddle  
Closed orbits around foci  
Given by intial condition  
Non-linear frequency shift |
| **Non-conservative LLG** | 1 | <0 | 1 stable focus  
1 unstable focus  
1 saddle  
Damped oscillations around stable focus  
FMR frequencies |

**Need to re-examine solutions....**

Equilibria, stability, trajectories  
as a function of control parameters \( J, P, H \)
II Non Conservative Dynamics - Summary

\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} M \times \frac{dM}{dt} \]

Precession

Damping

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Energy minima
Saddle point
Out of plane demagnetisation energy
Energy maximum
Lines of constant energy
In-plane angle \(\phi\)
Out-of-plane angle \(\theta\)
In-plane minimum
Max demag energy
Max anisotropy energy
In-plane Maximum Saddle point
\[
\frac{d\mathbf{M}}{dt} = -\gamma (\mathbf{M} \times \mathbf{H}_{\text{eff}}) + \frac{\alpha}{M_s} \mathbf{M} \times \frac{d\mathbf{M}}{dt} + \frac{\gamma a_j(\theta)}{M_s} \mathbf{M} \times (\mathbf{M} \times \mathbf{P})
\]

Precession  
Damping  
Spin torque (ST)

Norm of \( \mathbf{M} \) conserved since

\[
\mathbf{M} \frac{d\mathbf{M}}{dt} = 0
\]
VI ST Precession - Energy Loss/Gain

\[
\begin{align*}
\frac{dE}{dt} &= -H_{\text{eff}} \frac{dM}{dt} = -\frac{\alpha}{M_s} \left( \frac{dM}{dt} \right)^2 - \frac{a_j}{M_s} \frac{dM}{dt} (M \times P) \\
\text{Damping} &\quad \text{STT} \\
\end{align*}
\]

\( > 0 \) Depending on current sign and amplitude
\( < 0 \)

\[ \rightarrow \text{STT has « dissipative » action on trajectory} \]

**Spin torque cannot be derived from a generalized energy**

\[ \rightarrow \text{ST does not change the energy surface} \]
\[ \rightarrow \text{ST does not change conservative part of LLG} \]
\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} M \times \frac{dM}{dt} + \frac{\gamma a_j(\theta)}{M_s} M \times (M \times P) \]

- **Precession**
- **Damping**
- **Spin torque (ST)**

\( a_j \sim \) current J
\( P = \) spin polarization vector

**Steady state periodic oscillations**

- **Stabilizing**
- **Destabilizing**

**Initial state**

**Constant energy trajectory**
Apply current so that Destabilisation of initial stable state

Low current amplitude

High current amplitude

Damped motion

Periodic oscillations of $M_x, M_y, M_z$

Hu=500 Oe, Hb=1000 Oe
Apply current so that Destabilisation of initial stable state

Depending on the amplitude of the current

Damped oscillation around stable focus

Oscillation away from unstable focus towards a Limit Cycle
VI ST Precession - Limit Cycles

\[ \frac{d\mathbf{M}}{dt} = -\gamma (\mathbf{M} \times \mathbf{H}_{\text{eff}}) + \frac{\alpha}{M_s} (\mathbf{M} \times \frac{d\mathbf{M}}{dt}) + \frac{a_j}{M_s} (\mathbf{M} \times (\mathbf{M} \times \mathbf{P})) \]

- Precession
- Damping
- Spin torque

- Depending on the control parameter \( J \) (current amplitude) the energy minimum can change from a stable to an unstable focus.

- When \( J > J_c \) STT moves \( \mathbf{M} \) along the energy surface until \( \mathbf{M} \) stabilizes on a limit cycle.

\[ \delta \mathbf{M} \sim e^{\Gamma t} e^{i\omega_o t} \]

\[ \Gamma_{J < J_c} < 0 \Rightarrow \Gamma_{J > J_c} > 0 \]

Note: energy surface is not changed by STT
What are the limit cycles?
Are they the same as constant energy trajectories?

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{Ms} M \times \frac{dM}{dt} + \gamma a_J(\theta) \frac{M \times (M \times P)}{Ms}
\]

Precession \quad Damping \quad Spin \ torque \ (ST)

Precession Term dominating

→ Depends on orientation of P and amplitude of current
Consider two geometries

Planar polarizer

- In-plane M
- Polarizer P

Perpendicular Polarizer

- Out-of-plane

M and P collinear

Parallel

Antiparallel

Control parameters:
- $H$ changes energy surface,
- $J, P$ change « dissipation »
Are limit cycles and constant energy trajectories the same?

**In-Plane Precession (IPP)**
- Oscillation around energy minimum
- ST tends to align $\mathbf{M}$ parallel to $\mathbf{P}$

**Out of Plane Precession (OPP)**
- Oscillation around energy maximum

If $\mathbf{P} \parallel$ symmetry axis (equilibrium point) →
Limit cycles are close to the constant energy trajectory around equilibrium point
VI ST Precession - Limit Cycles

Example perpendicular polarizer

OPP stabilized by perpendicular polarizer

ST Gilbert

Torques (theta component)

Damping torque
Precession torque
Spintorque
VI ST Precession - Limit Cycles

Example ST destabilizes parallel state (planar polarizer)

- Gilbert torque
- Spin torque

ST wants $M$ to be antiparallel

Initial state
Energy minimum

phi component
theta component

VI ST Precession – Limit Cycles
Constant energy trajectory vs limit cycle

Planar polarizer

IPP very close to constant energy trajectory but not identical

Spin transfer and damping torque cancel only on average but not at each point along the trajectory
Limit cycles can be very close to constant energy trajectories

But

**Constant energy trajectory:**
Given by initial condition \((E_0)\)

**Limit Cycle** (spin torque induced):
Independent of initial condition,
defined by \(J\)
Excitation

Spin momentum transfer

Auto-oscillations of $M$

100 nm

Readout

Magneto-Resistance

Output Voltage $U_\sim$

Spectrum Analyzer

Nanoscale Tuneable Microwave Oscillator

Spin Transfer Nano-Oscillator STNO
VI ST Precession - Experiment

Example: Planar Tunnel Junction

Typical Spectra

Increasing current

Linewidth $\sim \Gamma$

$\delta M \sim e^{\Gamma t} e^{i\omega_0 t}$

PSD (nV²/Hz)

$H_b$

$f$ (GHz)

$I$ (mA)
VI ST Precession - Experiment

Example: Planar Tunnel Junction

Typical Spectra

Increasing current

Devices Hitachi GST

Free Layer
SAF – Polarizer
AF
CoFeB / CoFe
MgO
CoFeB
Ru
CoFe
IrMn

$H_b$

Frequency

$I_c$

$I$ (mA)

$f$ (GHz)

PSD (nV²/Hz)

$f$ (GHz)

$I$ (mA)

6,7
6,6
6,5
6,4
6,3
6,2
6,1
6,0

0,0 0,2 0,4 0,6 0,8 1,0

0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000 15000

0 5 10 15
What about frequencies of trajectories?

**Conservative trajectories**

\[
\frac{d\mathbf{M}}{dt} = -\gamma (\mathbf{M} \times \mathbf{H}_{\text{eff}})
\]

Precession

\[\omega = F(H_{\text{eff}})\]

Non-linear dynamical system \(H_{\text{eff}}(\mathbf{M})\)

Frequencies depend on the precession amplitude of \(\mathbf{M}\)
Linear – small angles

\[ mL \ddot{\theta} = -mg \sin \theta \approx -mg \theta \]

\[ \theta = \theta_o e^{i \omega_o t} \quad \omega_o = \sqrt{\frac{g}{L}} \]

Non-linear – large angles

\[ mL \ddot{\theta} = -mg \sin \theta \approx -mg \theta + \frac{1}{6} mg \theta^3 \]

\[ \theta = A \sin \omega t + B \sin 3 \omega t \]

\[ \omega = \omega(\theta_o) \approx \omega_o + k \theta_o^2 \]

A. Blaquièrè “Analysis of non-linear systems” (Academic Press 1966)
How to calculate frequencies?

- Frequencies close to stable (or unstable) focus (i.e. energy minima and maxima) can be obtained by linearization of LLG (S) → Ferromagnetic Resonance section II (Imaginary part of eigenvalues $\lambda$)

$$\omega_o^2 = \gamma \left( \frac{E_{\theta\theta} E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2 \sin^2 \theta} \right)$$

$$\Gamma = \frac{\gamma \alpha}{2} \left( \frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2 \sin^2 \theta} \right)$$

- Frequencies for large amplitude trajectories and limit cycles cannot be calculated analytically easily. The torque along the trajectory varies strongly.

VI ST Precession – Frequencies

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Two possible approaches to calculate frequencies

1) Semi – analytical-numerical approach

Define \( \frac{1}{f} = T = \int dt = \int \frac{dm_i}{m_i} \)

and integrate the trajectory over one period for any of the three components \( m_i, i=x,y,z \), by using the time derivatives from the equation of motion and the parametrization for constant energy trajectories. The integral can be evaluated numerically (or in some cases analytically see Serpico et al.)

2) Numerical integration of LLG(S) and

Fourier transform of the magnetization components \( m_x(t), m_y(t), m_z(t) \).

Attention: for IPP trajectories (around X-axis as equilibrium), the frequency is double)

Here:

Numerical solution for constant energy and spin transfer induced trajectories
VI ST Precession – Frequencies

Conservative trajectories

\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) \]

Precession

Uniaxial in-plane film
Bias field \( H_b // X \)

For IPP modes:
Frequency decreases with amplitudes

For OPP modes:
Frequency increases

The frequencies depend strongly on the precession amplitude
VI ST Precession – Frequencies

Example Frequencies of Planar Polarizer

Frequencies of conservative and STT trajectories are the same
Macrospin simulations

Experiments MTJs

Tuning of frequency via current

\( \frac{df}{dl} \approx 1 \text{GHz/mA} \)

\( \phi \) C75nm
TMR 80%
Precession term in spherical coordinates:

\[ \frac{d\phi}{dt} = -\gamma \frac{dE(\theta, \phi)}{M \sin \theta} \quad ; \quad \frac{d\theta}{dt} = \frac{\gamma}{M \sin \theta} \frac{dE(\theta, \phi)}{d\phi} \]

Angular velocity \( \frac{dE}{d\theta} \) decreases at turning point.

It goes to zero when approaching the saddle point → the frequency goes to zero.
Frequencies for OPP modes vs current J

On OPP trajectory the demagnetization energy dominates

\[ \omega = \gamma H_d = \gamma 4\pi n_z \]

Conservative trajectory

\[ f \approx \frac{\gamma}{2\pi} 4\pi M_s \cos \theta \]

STT OPP trajectory of perpendicular polarizer

approximation

\[ \cos \theta \approx \frac{a_j}{\alpha 4\pi M_s} \]

\[ f \approx \frac{\gamma}{2\pi} \frac{a_j}{\alpha} \sim J \]
VI ST Precession - Summary Limit Cycles

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} M \times \frac{dM}{dt} + \frac{\gamma a_j(\theta)}{M_s} M \times (M \times P)
\]

| \(|m| \) | \(dE/dt\) | Static | Dynamic |
|--------|----------|--------|---------|
| Conservative Precession term only | 1 | 0 | 2 stable foci 1 saddle | Closed orbits around foci Given by initial condition Non-linear frequency shift |
| Non-conservative LLG | 1 | <0 | 1 stable focus 1 unstable focus 1 saddle | Damped oscillations around stable focus FMR frequencies |
| STT Dynamics LLGS | 1 | <0 | X | Damped oscillations or Limit cycles |
| | | >0 | Depend on Control Parameter J, P, H | |

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VI ST Precession – Equilibrium States

\[
\frac{d\mathbf{M}}{dt} = -\gamma (\mathbf{M} \times \mathbf{H}_{\text{eff}}) + \alpha \frac{M_s}{M_s} (\mathbf{M} \times \frac{d\mathbf{M}}{dt}) + \gamma \frac{a_j}{M_s} (\mathbf{M} \times (\mathbf{M} \times \mathbf{P}))
\]

**Precession**  
**Damping**  
**Spin torque**

What about equilibrium states?

\[
\frac{d\mathbf{M}}{dt} = 0
\]

Same energy surface

→ Are the equilibrium states the same as for the conservative part?
→ What about their stability?
→ Are there any new equilibrium states?
→ What is their dependence on current amplitude?

Remember:

STT tends to align \( \mathbf{M} \) parallel/antiparallel to \( \mathbf{P} \)
VI ST Precession - Equilibrium States

Equilibrium \( \frac{dM}{dt} = 0 \)

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) + \gamma \frac{a_j}{M_s} (M \times (M \times P))
\]

Precession \hspace{2cm} Damping \hspace{2cm} Spin torque

When \( P \) does not point into the direction of an equilibrium of the conservative part \( (M \times H_{\text{eff}} = 0) \):
\[ \Rightarrow \text{equilibrium is given by the balance between the precession and spin transfer torque} \]

Examples:
1) Perpendicular polarizer at energy minimum
2) Planar polarizer at energy maximum
VI Auto-oscillations – Equilibrium States

Example planar polarizer at in-plane energy minima

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) + \frac{a_i}{M_s} (M \times (M \times P))
\]

Precession  Damping  Spin torque

Upon increasing current \( J \)

- STT counteracts damping

- Above critical current \( J_c \) Stable focus can become an unstable focus

\[
M \sim e^{\Gamma t} e^{i\omega_0 t}
\]

\[
\Gamma_{J<J_c} < 0 \Rightarrow \Gamma_{J>J_c} > 0
\]

Stable for \( J<J_c \)

Stable/unstable depending on sign of \( J>J_c \)

Stable  unstable  stable

unstable  stable

stable  unstable
VI ST Precession – Equilibrium States

Example perpendicular polarizer

Equilibrium between precession and spin torque

\[ \frac{dM}{dt} = -\gamma (M \times H_{eff}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) + \gamma \frac{a_j}{M_s} (M \times (M \times P)) \]

Precession Torque

\[ M \times (H_u + H_b) \]

\[ \rightarrow \text{In plane rotation} \quad \phi_o = 0^\circ - 90^\circ \]

Energy Minima: no equilibrium states
Saddle Point: no equilibrium point
Energy maximum: stable \((M/P)\)
« New » state: in-plane rotation

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VI ST Precession - State Diagram

Example perpendicular polarizer

State Diagram
With in plane bias field

H_b

Energy

phi

IPS

J_c1

Spin Torque

Precession torque

OPP

J_c2

OPS
Stability analysis of equilibrium states yields critical boundaries
→ Critical Current $J_c$ as a function of bias field $H_b$

\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} \left( M \times \frac{dM}{dt} \right) + \frac{a_j}{M_s} (M \times (M \times P)) \]

- **Precession**
- **Damping**
- **Spin torque**

Equilibrium
\[ \frac{dM}{dt} = 0 \Rightarrow M_o \]

**Stability: Linearization of LLG** around equilibrium points $M_o$

\[ M = M_o + \delta M \]
\[ H_{\text{eff}}(M) = H_{\text{eff}}(M_o + \delta M) \]
\[ H_{\text{eff}}(M) = H_{\text{eff, stat}}(M_o) + h_{\text{eff, dyn}}(\delta M) \]
\[ \Rightarrow \frac{d\delta M}{dt} = A \delta M \]
Reminder Non-Conservative Dynamics

\[ \frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) \]

- **Precession**
- **Damping**

\[ \frac{d \delta M}{dt} = \frac{A \delta M}{\det(A - \lambda I)} = 0 \Rightarrow \lambda \]

\[ \delta M = \delta M_o e^{\lambda t} = \delta M_o e^{\Gamma t} e^{i \omega_o t} \]

\[ \lambda = \Gamma + i \omega_o \]

Well known solution from Ferromagnetic Resonance FMR

\[ \omega_o^2 = \gamma \left( \frac{E_{\theta\theta}E_{\varphi\varphi} - E_{\theta\varphi}^2}{M_s^2 \sin^2 \theta} \right) \]

\[ \Gamma = \frac{\gamma \alpha}{2} \left( \frac{E_{\theta\theta}}{M_s} + \frac{E_{\varphi\varphi}}{M_s^2 \sin^2 \theta} \right) \]

\[ \Gamma = \frac{\Delta \omega}{2} \]

To be evaluated at equilibrium \( M_o \Leftrightarrow \theta_o, \varphi_o \)

Can be applied to any equilibrium point
VI ST Precession – Stability & Critical Boundaries

Linearization for LLGS

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) + \frac{\alpha_j}{M_s} (M \times (M \times P))
\]

- **Precession**
- **Damping**
- **Spin torque**

For a given polarizer geometry (P constant and bias field orientation H)

The eigenvalues \( \lambda \) depend on the control parameters \( J \) and \( H_b \)

\[
\frac{d\delta M}{dt} = A \delta M
\]

\[
\det(A - \lambda I) = 0 \Rightarrow \lambda
\]

\[
\delta M = \delta M_o e^{\lambda t} = \delta M_o e^{\Gamma t} e^{i\omega_o t}
\]

\[
\lambda = \Gamma + i\omega_o
\]

Stable if \( \Gamma(H_b, J) > 0 \), exponentially decaying

Unstable if \( \Gamma(H_b, J) < 0 \), exponentially diverging

\( \rightarrow \) Boundary for \( \Gamma(J, H_b) = 0 \) \( \Rightarrow (J_c, H_b) \)
VI ST Precession – Stability & Critical Boundaries

Complex frequency including spin torque

\[
\frac{\lambda}{\gamma'} = -\frac{i}{2} \left( \frac{\Delta \omega}{\gamma} - 2a_j P'_r \right)_{\theta_o, \phi_o} \pm \sqrt{-\frac{1}{4} \left( \frac{\Delta \omega}{\gamma} + 2a_j P'_r \right)^2 + \left(1 + \alpha^2 \right) \left( \frac{\omega_o}{\gamma} \right)^2 + \left(1 + \alpha^2 \right) a_j P'_r} \right)_{\theta_o, \phi_o}
\]

- Define state diagram for arbitrary \( P \) and \( H_b \)
- Calculate FMR frequencies in damped regime around stable states

with

\[
\left( \frac{\omega_o}{\gamma} \right)^2 = \frac{E_{\theta \theta} E_{\phi \phi} - E_{\theta \phi}^2}{M_s^2 \sin^2 \theta}, \quad \frac{\Delta \omega}{\gamma} = \alpha \left( \frac{E_{\theta \theta}}{M_s} + \frac{E_{\phi \phi}}{M_s^2 \sin^2 \theta} \right)
\]

\( P'_r = P_x \sin \theta_o \cos \phi_o + P_y \sin \theta_o \sin \phi_o + P_z \cos \theta_o \)

\( a_j = \frac{\hbar J}{2e \mu_0 M_s t} g(\eta, \bar{m}, \bar{p}) \)
VI ST Precession – State Diagram

Example perpendicular polarizer

State Diagram
With in plane bias field

Energy

phi

Precession torque

Spin Torque

H_w, H_b

θ_o=90°

φ_o

J_{c1}

IPS

OPP

OPS

H_b

J_{c2}

m_x

m_y

m_z

m_x

m_y

m_z

H_w, H_b

θ_o

φ_o≈180°
VI ST Precession - State Diagram

Example planar polarizer

$H_b > H_u$

$H_b < H_u$

Reversal
No Spin torque

Current J (normalized)

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VI ST Precession - State Diagram

Example planar polarizer: New states

- H_p/H_u
- J_c1, J_c2
- P, OPP, IPP, AP
- Current J (normalized)
- H (kOe)
- « New » state
- Saddle point

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### VI ST Precession - Summary Limit Cycles

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} M \times \frac{dM}{dt} + \frac{\gamma a_J(\theta)}{M_s} M \times (M \times P)
\]

- **Precession**
- **Damping**
- **Spin torque (ST)**

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<td><strong>LLG</strong></td>
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<td>1 unstable focus</td>
<td>around stable focus</td>
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<td></td>
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<tr>
<td><strong>LLGS</strong></td>
<td></td>
<td>=0</td>
<td>New states</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;0</td>
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</tbody>
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Outline

0 Introduction

I Conservative dynamics (solutions of Precession term)

II Non conservative dynamics

III Precessional reversal under transverse field pulses

IV Domain wall motion under field

V Dynamics under spin trasnfer

VI Spin torque induced precession

VII (Precessional) Reversal under spin torque
VII Reversal under spin torque

Example planar polarizer

H_b > H_u

H_b < H_u

Reversal under ST
VII Reversal under spin torque

Example planar polarizer

Reversal under DC current

Critical Current density

\[ J_{DC}^c = k \alpha \left( H_u + H_b + 2\pi M_s \right) \]

\[ k = \frac{2e M_s t}{\hbar \eta} \]

First experiments by Albrecht et al 2000
VII Reversal under spin torque

Example planar polarizer

Reversal Time

Delay time to « wind up » and overcome the energy barrier
VII Reversal under spin torque

Example planar polarizer

Reversal Time

\[ T_{//} = \gamma_0 \frac{a_j}{M_s} M \times (M \times P) \sim \sin(MP) \]

Dynamic critical current density increases with decreasing pulse width \( \tau \)

\[ \tau \sim \left( J_{\text{pulse}}^c - J_{\text{DC}}^c \right)^{-1} \ln \frac{\theta}{\theta_o} \]

Sun PRB 62 (2000)
Example perpendicular polarizer

Stabilizes OPP trajectories under DC spin polarized current

\[ f = 2\text{-}4\text{GHz} \]
VII Reversal under spin torque – Precessional Reversal

Example perpendicular polarizer

Apply current pulse of short duration $\Delta t < 1\text{ns}$

$\Delta t=130\text{ps}$  $150\text{ps}$  $200\text{ps}$

Reversal

Non-Reversal

Current Pulse

Time

$\Delta t$
VII Reversal under spin torque – Precessional Reversal

Example perpendicular polarizer: Experiment

- Bands of Reversal/Non-Reversal when \( \Delta t \) matches precession period \( T = 1/f \)
- Since \( f \sim J : \Delta t \sim 1/J \)

Vayset APL 98, 242511 (2011)
Papusoi APL 95, 072506 (2009)
Reversal under spin transfer torque depends on the polarizer configuration

- In the **planar configuration**, there is a delay time corresponding to the time necessary for \( \mathbf{M} \) to spiral up.
- The reversal time (~ns) decreases with increasing current pulse amplitude and initial angle.

- In the **perpendicular configuration**, precessional reversal under short (<1ns) current pulses is possible, where the magnetization reverses on \( 2n+1 \) precession cycles.
Summary

\[
\frac{dM}{dt} = -\gamma (M \times H_{\text{eff}}) + \frac{\alpha}{M_s} (M \times \frac{dM}{dt}) + \gamma \frac{a_j}{M_s} (M \times (M \times P))
\]

- LLG(S) equation: Macrospin motion of \( M \) as an example of a non-linear dynamical system
- One can apply standard analysis techniques to find equilibria and orbits and their respective stability
- Three cases have been discussed (non) conservative and STT dynamics
- The external field \( H_b \) as control parameter changes the energy surface, thus the equilibrium points and the types of trajectories
- STT changes the energy loss-energy gain. For a constant field and polarizer \( P \), the stability and trajectory depend on \( J \)
  \( \rightarrow \) transition from stable to unstable points and to limit cycles

- Make use of the orbits to define strategies of fast reversal under applied field and current or to design microwave applications
- Wall dynamics follows similar principles, but a bit more complex
Thank you....
…for your …
… attention
Outline

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IV Domain wall motion under field

V Introduction to spin transfer torque

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VII (Precessional) Reversal under spin torque

VIII Domain wall motion under current
Wall motion under an applied field

- Wall displaces perpendicular to field
- Domain parallel to bias field increases in size, to minimize Zeemman energy

\[ -\gamma(M \times H_b) = \frac{dM}{dt} \]
\[ \frac{\chi}{M_s} \left( M \times \frac{dM}{dt} \right) \]
V Spin Momentum Transfer - Concept

Unpolarized Electrons

Polarizer P

Electron Flow

Polarized Electrons

Free Layer M

Transmitted Electrons

Conduction Electrons

Transfer of transverse moment $m = \text{Spin Torque}$

Local Magnetization

The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)

Conservation of angular momentum $\rightarrow$ Spin transferred to the local magnetization $\Rightarrow$ Torque on magnetization

DW motion in the direction of the e$^-$ flow
VIII Domain wall displacement under spin torque

- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
- Conservation of angular momentum \( \rightarrow \) Spin transfered to the local magnetization \( \Rightarrow \) Torque on magnetization
- **DW motion** in the direction of the e\(^-\) flow
VIII Domain wall displacement under spin torque

- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
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- The spin of the conduction electron adiabatically follow the direction of the local magnetization (large DW)
- Conservation of angular momentum $\rightarrow$ Spin transferred to the local magnetization $\Rightarrow$ Torque on magnetization
- DW motion in the direction of the e\textsuperscript{-} flow
VIII Domain wall displacement under spin torque

Modified LLG equation

\[
\frac{\partial \vec{m}}{\partial t} = -\gamma \vec{m} \times \vec{H}_{\text{eff}} + \alpha \vec{m} \times \frac{\partial \vec{m}}{\partial t} - u \frac{\partial \vec{m}}{\partial x} + \beta \mu m \times \frac{\partial \vec{m}}{\partial x}
\]

Precession \hspace{1cm} Damping \hspace{1cm} Adiabatic spin transfer torque

\[
u = \frac{J \mu_B g_m}{2eM_s}
\]

\(\beta\) term: generally much smaller, arising from spin relaxation in DW and non-adiabatic effects in narrow DW due to spin mistracking.

• $\beta = 0$, **only adiabatic term**
  - No motion for $J<J_c$
  - $J_c \ll \text{intrinsic}$ (depends on the magnetic properties of the DW)
  - **Turbulent motion** above $J_c$ with complex DW transformation

• $\beta \neq 0$
  - $\mathbf{v} \neq 0$ for perfect wire with $\mathbf{v} \sim j\beta/\alpha$
  - $J_c \ll \text{extrinsic}$ due to pinning (roughness, defect in the material, ...)
  - « Field like » torque

$\beta$ is a key parameter in the DW dynamics, but its value is still unknown and very controversial.

---

Magnetization Dynamics

- L. Perko “Differential equations and dynamical systems” Texts in Applied Mathematics
- J. Guckenheimer, P. Holmes “Non-linear oscillations, dynamical systems, and bifurcations of vector fields” (Springer, New York, 1983)

Precessional Reversal under hard axis field pulse

- R. Kikuchi “On the minimum reversal time” J. Appl. Phys. 27, 1352 (1956)
X Some papers for further reading (non exhaustive)


Domain wall motion under field
Spin transfer torque


Precession under Spin torque

Some papers for further reading (non exhaustive)

Reversal under Spin torque

Domain wall motion under Spin torque

The END
VI ST Precession - Review of Conservative Dynamics

Uniaxial thin film

\( H_u, H_b = 0 \)

Out of plane demagnetisation energy

Energy maxima

Saddle points

Energy minima
2 types of constant energy trajectories
IPP around energy minimum
OPP around energy maximum
VI ST Precession - Review of Conservative Dynamics

\( H_b = 0 \)

Maximum in plane excursion \( \varphi_{\text{max}} \leq 90^\circ \)

\( H_b > 0 \)

- \( 0 < H_b < H_u \): \( \cos \varphi_{\text{max}} = -H_b < H_u \)
- \( H_b > H_u \): \( \varphi_{\text{max}} = 180^\circ \)
VI ST Precession - Transient to Limit Cycle

System gains energy

Energy gain at turning point $\phi_{max}$ and $\theta \approx 0$
Constant energy trajectory vs limit cycle

Planar polarizer

IPP very close to constant energy trajectory but not identical

Spin transfer and damping torque cancel only on average but not at each point along the trajectory
VI ST Precession – Limit Cycles

**Constant energy trajectory vs limit cycle**

**Planar polarizer**

IPP very close to constant energy trajectory but not identical

Spin transfer and damping torque cancel only on average but not at each point along the trajectory.
VI ST Precession – Limit Cycles

Example perpendicular polarizer

OPP stabilized by perpendicular polarizer

$$m_z \approx \frac{a_j}{\alpha 4\pi M_s} \propto \frac{J}{\alpha 4\pi M_s}$$

IPP not stabilized by perpendicular polarizer

theta component  phi component
VI ST Precession - Limit Cycles

Constant energy trajectory vs limit cycle

Planar polarizer

OPP can be stabilized but they are different from constant energy trajectory

OPP Planar polarizer

Energy

ST orbit

Conservative orbit

27%
VI ST Precession – Limit Cycles

Constant energy trajectory vs limit cycle

Perpendicular polarizer

Conservative trajectory

Spintorque trajectory

OPP very close to constant energy trajectory

Energy oscillation: 0.03%
Qualitative Explanation of the nonlinear frequency shift

\[ M_z = \sqrt{M_0^2 - m_\perp^2} \approx M_0 (1 - |c|^2) \]

FMR, normal magnetization:

\[ \omega = \gamma (H_0 - 4\pi M_z) = \gamma (H_0 - 4\pi M_0) + 4\pi \gamma M_0 |c|^2 = \omega_0 + |N| |c|^2 \]

\[ N > 0 \text{ - blue frequency shift} \]

FMR, in-plane magnetization:

\[ \omega = \gamma \sqrt{H_0 (H_0 + 4\pi M_z)} \approx \]

\[ \approx \gamma \sqrt{H_0 (H_0 + 4\pi M_0)} - 2\pi \gamma M_0 \sqrt{\frac{H_0}{H_0 + 4\pi M_0}} |c|^2 = \omega_0 - |N| |c|^2 \]

\[ N < 0 \text{ – red frequency shift} \]
VI ST Precession - Frequencies

Frequencies for IPP modes vs Field

\[ J(10^{12}\text{A/m}^2) \]

\[ \text{f (GHz)} \]

\[ \text{Happ (kA/m)} \]

FMR
VI ST Precession – Frequencies

Non-sinusoidal IPP modes

- Precession term in spherical coordinates
  \[ \frac{d\varphi}{dt} = -\gamma \frac{dE(\theta, \varphi)}{d\theta} \]
  \[ \frac{d\theta}{dt} = M \sin \theta \frac{dE(\theta, \varphi)}{d\varphi} \]

- Angular velocity \( \frac{dE}{d\theta} \) decreases at turning point
  \( \rightarrow \) Non-sinusoidal trajectories

Strong variation of \( H_{\text{eff}} \) along trajectory

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VI ST Precession – Frequencies

Frequencies for OPP modes vs current J

On OPP trajectory the demagnetization energy dominates

Conservative trajectory

\[ \omega = \gamma H_d = \gamma 4\pi n_z \]

\[ f \approx \frac{\gamma}{2\pi} 4\pi M_s \cos \theta \]

STT OPP trajectory of perpendicular polarizer

Numerical approximation

\[ \cos \theta \approx \frac{a_j / \alpha}{4\pi M_s} \]

\[ f \approx \frac{\gamma}{2\pi} \frac{a_j}{\alpha} \sim J \]
VI ST Precession - State Diagram

Example perpendicular polarizer

State Diagram
With in plane bias field

Energy

Precession torque
Spin Torque

θ₀ = 90°
θ₀ = 180°

Jc1
Jc2
Jc2
Jc1
VI ST Precession - State Diagram

Example planar polarizer

$H_b < H_u$
- Out of plane demagnetisation energy
- 2 Energy minima
- Energy maximum
- Saddle point
- Energy minima

$H_b > H_u$
- 1 Energy minimum

Saddle point
$H_b < H_u$ unstable
$H_b > H_u$ stable/unstable depending on sign and strength of $J$

Energy Maximum unstable

"New" states
Canted states
All components $M_x, M_y, M_z \neq 0$ or 1

$\rightarrow$ Equilibrium between Spin and Precession torque
VI ST Precession - Non-linear Spin wave theory

Transformation of LLGS

\[
\frac{d\mathbf{M}}{dt} = -\gamma (\mathbf{M} \times \mathbf{H}_{\text{eff}}) + \frac{\alpha}{M_s} (\mathbf{M} \times \frac{d\mathbf{M}}{dt}) + \gamma \frac{a_j}{M_s} (\mathbf{M} \times (\mathbf{M} \times \mathbf{P}))
\]

- **Precession**
- **Damping**
- **Spin Torque**

\[
\frac{\partial c}{\partial t} = i\left(\omega_o + N|c|^2\right)c + \Gamma_o \left(1 + Q_o|c|^2\right)c - \Gamma_s \left(1 - Q_s|c|^2\right)c
\]

- c complex spin wave amplitude \(\sim m_{\perp}\)

\[
c(t) = |c(t)|e^{i\phi(t)}
\]

Non-linear contributions \(\sim N, Q_o, Q_s\)

Rezende et al, PRB 73, 094402 (2006);
Kim et al, PRL 100, 017207 (2008)
Oscillator Equation

\[ \frac{\partial c}{\partial t} = i(\omega_o + N|c|^2)c + \Gamma_o (1 + Q_o|c|^2)c - \Gamma_s (1 - Q_s|c|^2)c \]

- **Precession**
- **Damping**
- **Spin Torque**

= 0 for steady state

\[ |c|^2 = \frac{\Gamma_s - \Gamma_o}{\Gamma_s Q_s + \Gamma_o Q_o} \]

\[ J_c = \frac{\Gamma_o}{\sigma} \]

Non-linear contributions
\[ \sim N, Q_o, Q_s \]

Linear contributions
\[ \Gamma_s = \sigma J \]
\[ \Gamma_o \sim \alpha \]

Finite amplitude \( |c| \) due to non-linear contributions \( Q_o, Q_s \)

Rezende et al, PRB 73, 094402 (2006);
Kim et al, PRL 100, 017207 (2008)
Stability of Trajectories

\[
\frac{\partial c}{\partial t} = i(\omega + N|c|^2)c + \Gamma_o (1 + Q_o |c|^2)c - \Gamma_s (1 - Q_s |c|^2)c
\]

- **Precession**
- **Damping**
- **Spin Torque**

\[
c(t) = |c(t)|e^{i\phi(t)}
\]

\[
p = |c|^2 \quad p = p_o + \delta p
\]

\[
\frac{d\delta p}{dt} + 2\Gamma_p \delta p = h_\delta^p(t)
\]

\[
\frac{d\phi}{dt} = -\omega_g t + h_\phi^o(t) - N\delta p
\]

- **Amplitude relaxation rate** \(\Gamma_p\)
- **Phase not bounded**
- **Fluctuates due to amplitude noise**

Non-linear contributions
\(\sim N, Q_o, Q_s\)

\(\delta p(t) \sim e^{-2\Gamma_p |t|}\)

Timescale of ampl. fluctuations

\(h_n(t)\) noise or external force
VI ST Precession – Microwave Oscillators

Resonator

Attenuated Resonator

Auto-Oscillator

Attenuated Resonator & Energy feedback

Active element to supply external energy and compensate energy losses

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

STO

\[ \frac{dM}{dt} = \gamma (M \times H_{eff}) \]

\[ \frac{\alpha}{M_s} \left( M \times \frac{dM}{dt} \right) \]

Spin Transfer

Damping Precession

Spin Torque
Example: Perpendicular Polarizer

**Characteristics of OPP mode**
- Frequency Blueshift
- Saturation of Frequency due to non homogeneous magnetization

**Spectra**
- $H_{\text{eff}} = 9 \text{ Oe}$
- DC current ($I_{\text{dc}}$) vs. Frequency ($f$) graph

**Diagram**
- Stack configuration:
  - Analyzer: Co
  - Free Layer Py: Cu
  - Co/Cu/Co: Cu
  - Spacer: Cu
  - Polarizer: Pt/(Co/Pt)$_5$

- Co/Cu/Co spacing: 100 nm

**Graphs**
- PSD plot with frequency tuning for different DC currents: 0.3 mA, 0.6 mA, 0.9 mA, 1.2 mA, 1.5 mA
Nanoscale Tuneable Microwave Oscillator
Spin Transfer Nano-Oscillator STNO
Nanocontacts NC

Nanopillars NP

Cornell

Co/Py
Cu
Co

I\textsubscript{dc}

NIST

I\textsubscript{ac}

H\parallel

H\perp

NC Coupling

NIST

« Wavy » NP

CNRS/Thales

Thick Py
Cu
Thin Co

Fl Oscillation \rightarrow IEF

SAF Oscillation

SPINTEC/LETI

IEF

Perpendicular Pol

Spintec/LETI

Vortex

NP

Cornell

CNRS/Thales

NC

IEF/IMEC

CNRS/Thales
VI ST Precession – Microwave Oscillators

Electro magnet

Bottom electrode

Top electrode

100µm

Bottom Electrodes

RF probes

Spectrum Analyzer

26GHz

bias T

40 dB

ΔU~

I_{DC}

H

J

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Example: Planar Tunnel Junction

Typical Spectra

Increasing current

PSD (nV²/Hz)

f (GHz)

Increasing current
VI ST Precession – Microwave Oscillators

Example: Planar Tunnel Junction

Frequency

Power

- Frequency Redshift
- Decrease of Linewidth
- Increase of Power

Devices Hitachi GST

CoFeB / CoFe

MgO

CoFeB

Ru

CoFe

IrMn

Free Layer

SAF – Polarizer

AF

\( H_b \)

Characteristics of IPP mode
- The LLGS equation corresponds to a non-linear dynamical system that can be solved like the LLG and the precession term
- The solutions are equilibrium states and periodic orbits
- The stability of equilibrium states is analyzed via linearization
- The stability depends on the control parameters $J$, $P$, $H_b$
- The stable can be the same as in the (non) conservative case, but there can also exist new states
- Above a critical current $J_c$, the stable states become unstable and the magnetization precesses on limit cycles. These are independent of initial condition
- The limit cycles are close to constant energy trajectories when $P // \text{symmetry axis}$ and the analysis of the limit cycle properties follows the one of the conservative dynamics
Linearization and frequencies and non-linearity
Meaning of non-linearity
FMR and constant energy trajectories (difference)
Initial conditions and amplitude
VII Reversal under spin torque – Precessional Reversal

Example perpendicular polarizer

Apply current pulse of long duration $\Delta t > 1\text{ns}$

Risetime 100 ps

Stabilized

OPP

$M_x$ - $M_z$ plane

$P$ - $M$ plane

OPP